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PHYSICS

Waves & Thermodynamics

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B.M. Sharma

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Waves &
Thermodynamics

B.M. Sharma

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Brief Contents

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Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-facetted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.
Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma

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UNIT I
THERMAL PHYSICS

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CHAPTER

1

Thermal Properties of Matter

- Heat
- Calorimetry
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- Transmission of Heat
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- Determination of Thermal Conductivity
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- Distribution of Energy in the Spectrum of Black Body
- Wien's Displacement Law
- Temperature of the Sun and Solar Constant

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HEAT

Heat is a form of energy which appears when two bodies at different temperatures come into contact and flows from the body at higher temperature to that at lower temperature. It is energy in motion or energy in transit. Heat is not a property of a system. A system can give out or absorb heat, but does not contain heat. It is the form of energy which determines the change in thermal state of a body and is defined as the flow of energy from one body to the other due to difference in the degree of hotness of two bodies (temperature). It flows from the body which is at a high temperature to the other at low temperature.

The energy associated with configuration and random motion of the atoms and molecules within a body is called internal energy and the part of this internal energy which is transferred from one body to the other due to temperature difference is called heat.

One calorie is defined as the amount of heat energy required to raise the temperature of 1 g of water through 1°C (more specifically from 14.5°C to 15.5°C).

As heat is a form of energy it can be transformed into others and vice versa. For example, thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy.

Here it is important that whole of mechanical energy, i.e., work can be converted into heat but whole of heat can never be converted into work.

Temperature

Temperature is defined by zeroth law of thermodynamics, which states that when two bodies A and B are separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other (thermal equilibrium implies equality of temperature). Temperature is a scalar quantity which is a property of all thermodynamic systems such that the equality of temperature is necessary and sufficient for thermal equilibrium.

1. Temperature is one of the seven fundamental quantities with dimension $[\theta]$.
2. It is a scalar physical quantity with SI unit kelvin.
3. When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls, i.e., temperature can be regarded as the effect of cause 'heat'.
4. According to the kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

$$\text{Temperature} \propto \text{kinetic energy} \quad \left[\text{As } E = \frac{3}{2}RT\right]$$

5. Although the temperature of a body can be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.
6. Highest possible temperature achieved in laboratory is about 10^8 K, while the lowest possible temperature attained is 10^{-8} K.
7. Branch of physics dealing with production and measurement of temperatures close to 0 K is known as cryogenics, while that dealing with the measurement of very high temperature is called as pyrometry.
8. Temperature of the core of the sun is 10^7 K while that of its surface is 6000 K.
9. Normal temperature of human body is 310.15 K = 37°C = 98.6°F.
10. NTP or STP implies 273.15 K (0°C = 32°F).

As the temperature is measured by the value of the thermodynamic property of a substance, i.e., a property which varies *linearly* with the temperature, two fixed points are needed to define a temperature scale.

These two fixed points in modern thermometry are taken as

1. Triple point of water, i.e., the state of water where the liquid, solid and vapour phases of water coexist in equilibrium. It is characterized by unique values of temperature and pressure.
2. On this scale, the other fixed point may be taken as the absolute zero.

We then need to assign some numbers to these two fixed points. The lowest temperature may be taken as zero. The triple point of water on Celsius scale is 0.01°C. Thus the absolute temperature T for triple point of water will be given by

$$T = t_c + 273.15 = 0.01 + 273.15 = 273.16 \text{ K}$$

$$\therefore \quad T = 273.16 \text{ K (triple point of water)}$$

Thermometry

An instrument used to measure the temperature of a body is called a thermometer.

The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance.

x may be
(i) length of liquid in capillary;
(ii) pressure of gas at constant volume;
(iii) volume of gas at constant pressure and
(iv) resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 atm) are taken to define the temperature scale. In Celsius scale, freezing point of water is assumed to be 0°C while boiling point 100°C and the temperature interval between these is divided into 100 equal parts.

So, if the thermometric property at temperatures 0°C, 100°C and T_c°C is x_0, x_{100} and x, respectively, then by linear variation ($y = mx + c$) we can say that

$$0 = ax_0 + b \quad \text{(i)}$$

$$100 = ax_{100} + b \quad \text{(ii)}$$

$$T_c = ax + b \quad \text{(iii)}$$

From these equations $\dfrac{T_c - 0}{100 - 0} = \dfrac{x - x_0}{x_{100} - x_0}$

$$\therefore \quad T_c = \frac{x - x_0}{x_{100} - x_0} \times 100°C$$

In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water 273.16 K at which ice, water and water vapours coexist).

So, if the values of thermometric property at 0 K, 273.16 K and T_K K are 0, x_{T_r} and x, respectively, then by linear variation ($y = mx + c$) we can say that

$$0 = a \times 0 + b \quad \text{(i)}$$

$$273.16 = a \times x_{T_r} + b \quad \text{(ii)}$$

$$T_k = a \times x + b \quad \text{(iii)}$$

From these equations $\dfrac{T_k}{273.16} = \dfrac{x}{x_{T_r}}$

$$\therefore \quad T_K = 273.16\left[\frac{x}{x_{T_r}}\right] \text{K}$$

Measurement of Temperature

There are different systems of measurement of temperature. The lower fixed point (LFP) and the upper fixed point (UFP) in any system of units are corresponding to freezing point and boiling point of water at 1 atm.

For different system of units the LFP and UFP are given as

System of units	Units	Lower fixed point (LFP)	Upper fixed point (UFP)	Different UFP – LFP
Degree celsius (centigrade)	°C	0°C	100°C	100
Kelvin scale (SI unit)	K	273.15 K	373.5 K	100
Fahrenheit	°F	32°F	212°F	180

Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point(LFP)}} = \text{Constant for all scales}$$

The relation between Celsius (C), Kelvin (K), Fahrenheit (F) and any other new scale θ is

$$\frac{C - 0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100} = \frac{\theta - \theta_0}{n} \quad \text{(i)}$$

where n is the number of divisions between ice point and steam point on the new scale and θ_0 is the ice point on it.

Illustration 1.1 **Liquid nitrogen has a boiling point of –195.81°C at atmospheric pressure. Calculate this temperature (a) in degrees Fahrenheit and (b) in kelvin.**

Sol. We can use Eq. (i) to convert degree celsius into Fahrenheit and kelvin.

a. Temperature in Fahrenheit is given by

$$T_F = \frac{9}{5}T_C + 32°F = \frac{9}{5}(-195.81) + 32 = -320.46°F$$

b. Temperature in Kelvin T_k = 273.15 K – 195.81 K = 77.3 K

A convenient way to change one scale to another is to remember the freezing and boiling points of water in each form:

$$T_{\text{freeze}} = 32.0°F = 0°C = 273.15\text{ K}$$

$$T_{\text{boil}} = 212°F = 100°C$$

To convert from Fahrenheit to Celsius, subtract 32 (the freezing point) and then adjust the scale by the liquid range of the water.

$$\text{Scale factor} = \frac{(100-0)°C}{(212-32)°F} = \frac{5°C}{9°F}$$

A Kelvin is the same size change as a degree celsius, but the Kelvin scale takes its zero point at absolute zero, instead of the freezing point of water. Therefore, to convert from Kelvin to Celsius, subtract 273.15 K from given Kelvin temperature.

Illustration 1.2 **Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:**

Temperature	Pressure thermometer A	Pressure thermometer B
Triple point of water	1.250 × 10^5 Pa	0.200 × 10^5 Pa
Normal melting point of sulphur	1.797 × 10^5 Pa	0.287 × 10^5 Pa

a. **What is the absolute temperature of normal melting point of sulphur as read by thermometer A and B?**

b. **What do you think is the reason for slightly different answers from A and B?**

Sol.

a. For thermometer A,

$$T_{tr} = 273\text{ K}, P_{tr} = 1.250 \times 10^5\text{ Pa}$$

We have $T = \dfrac{P}{P_{tr}} \times T_{tr}$

$$= \frac{1.797 \times 10^5}{1.250 \times 10^5} \times 273 = 392.46\text{K}$$

For thermometer B,

$$T_{tr} = 273\text{ K}, \quad P_{tr} = 0.200 \times 10^5\text{ Pa}$$

We have $T = \dfrac{P}{P_{tr}} \times t_{tr}$

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$$= \frac{0.287\times10^5\times273}{0.200\times10^5} = 391.75\text{K}$$

b. The slight difference in the temperatures as read by two thermometers is due to the fact that oxygen and hydrogen do not behave like an ideal gas.

Illustration 1.3 **What will be the following temperatures on the Kelvin scale: a. 37°C, b. 80°F, c. –196°C?**

Sol.

a. Temperature on Kelvin scale T_k is related to temperature T_c on Celsius scale as

$$T_k = T_c + 273$$

or $$T_k = 37 + 273 = 310 \text{ K}$$

b. Temperatures T_k on Kelvin scale and T_F on Fahrenheit scale are related as

$$\frac{T_k - 273}{373-273} = \frac{T_F - 32}{212-32}$$

or $$\frac{T_k - 273}{100} = \frac{T_F - 32}{180}$$

or $$T_k = \frac{5}{9}(T_F - 32) + 273$$

here $T_F = 80°\text{F}$; thus

$$T_k = \frac{5}{9}(80-32) + 273 = 299.66 \text{ K}$$

c. Again from relation used in part (a)

$$T_k = T_c + 273 = -196 + 273 = 77 \text{ K}$$

CALORIMETRY

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilocalories.

One Calorie

One calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C

Mechanical Equivalent of Heat (J)

According to Joule, work may be converted into heat and vice versa. The ratio of work done (W) to heat produced (Q) by that work without any wastage is always constant.

$$W/Q = \text{constant}$$

This constant is called *mechanical equivalent of heat (J)*. The value of this constant is taken as 4.18 J/cal.

Illustration 1.4 **In the Joule experiment, a mass of 20 kg falls through 1.5 m at a constant velocity to stir the water in a calorimeter. If the calorimeter has a water equivalent of 2 g and contains 12 g of water, what is f, the mechanical equivalent of heat, for a temperature rise of 5.0°C?**

Sol. Expressing ΔPE in Joules and Q in Calories, we have

$$f = \frac{\Delta PE}{Q} = \frac{mgy}{m_w c\,\Delta t} = \frac{20(9.8)(1.5)}{(12+2)(1)(5.0)} = 4.2 \text{ J/cal}$$

Thermal Capacity and Water Equivalent

1. **Thermal capacity:** It is defined as the amount of heat required to raise the temperature of the whole body (mass m) through 1°C or 1 K.

$$\text{Thermal capacity} = H = \frac{Q}{\Delta T}$$

The value of thermal capacity of a body depends upon the nature of the body and its mass.
Dimension: $[ML^2T^{-2}\theta^{-1}]$; unit: cal/°C (practical) J/K (SI)

2. **Water equivalent:** Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by W.

If m = mass of the body, c = specific heat of body, ΔT = rise in temperature.
Then heat given to body

$$\Delta Q = mc\Delta T \qquad \text{(i)}$$

If same amount of heat is given to W grams of water and its temperature also rises by ΔT.
Then heat given to water

$$\Delta Q = W \times 1 \times \Delta T \qquad [\text{As } c_{\text{water}} = 1] \qquad \text{(ii)}$$

From Eqs. (i) and (ii), $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

∴ Water equivalent $(W) = mc$ grams

Unit: kg (SI); dimension: $[ML^\circ T^\circ]$.

Note:
- *Unit of thermal capacity is J/kg while unit of water equivalent is kg.*
- *Thermal capacity of the body and its water equivalent are numerically equal.*
- *If thermal capacity of a body is expressed in terms of mass of water it is called water equivalent of the body.*

Specific Heat

1. **Gram specific heat:** When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through 1°C (or K) is called specific heat of the material of the body.

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If Q heat changes the temperature of mass m by ΔT

$$\text{Specific heat } c = \frac{Q}{m\Delta T}$$

Units: cal/g × °C (practical), J/kg × K (SI); dimension: $[L^2T^{-2}\theta^{-1}]$

2. **Molar specific heat:** Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of 1 g mole of the substance through a unit degree; it is represented by C.
By definition, 1 mole of any substance is a quantity of the substance, whose mass M grams is numerically equal to the molecular mass M.

∴ Molar specific heat = M × gram specific heat

or
$$C = Mc$$
$$C = M\frac{Q}{m\Delta T} = \frac{1}{\mu}\frac{Q}{\Delta T}$$
$$\left[\text{As } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M}\right]$$

∴
$$C = \frac{Q}{\mu\Delta T}$$

Units: cal/mol × °C (practical), J/mol × kelvin (SI); dimension: $[ML^2T^{-2}\theta^{-1}\mu^{-1}]$

Important Points

1. Specific heat for hydrogen is maximum (3.5 cal/g × °C) and for water, it is 1 cal/g × °C.
For all other substances, the specific heat is less than 1 cal/g × °C and it is minimum for radon and actinium (≃ 0.022 cal/g × °C).
2. Specific heat of a substance also depends on the state of the substance, i.e., solid, liquid or gas.
For example, $C_{ice} = 0.5\,\text{cal/g°C}$ (solid), C_{water} = 1 cal/g × °C (liquid) and $C_{steam} = 0.47\,\text{cal/g} \times \text{°C}$ (gas)
3. The specific heat of a substance when it melts or boils at constant temperature is infinite.
As $C = \frac{Q}{m\Delta T} = \frac{Q}{m \times 0} = \infty$ (As $\Delta T = 0$)
4. The specific heat of a substance when it undergoes adiabatic changes is zero.
As $C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0$ (As $Q = 0$)
5. Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.
For example, specific heat of saturated vapours.

Illustration 1.5 **A 60 kg boy running at 5.0 m/s while playing basketball falls down on the floor and skids along on his leg until he stops. How many calories of heat are generated between his leg and the floor?**

Assume that all this heat energy is confined to a volume of 2.0 cm³ of his flesh. What will be temperature change of the flesh? Assume c = 1.0 cal/g °C and ρ = 950 kg/m³ for flesh.

Sol. The boy's kinetic energy is changed to heat energy.

Set $Q = (mv^2)/2 = [60(25)]/2 = 750\,\text{J} = 179\,\text{cal}$

from $Q = c\rho V\,\Delta T$, $179\,\text{cal} = (1.0\,\text{cal/g°C})$
$(0.950\,\text{g/cm}^3)(2.0\,\text{cm}^3)\,\Delta T$, whence $\Delta T = 94\,°\text{C}$

Illustration 1.6 **An electric heater supplies 1.8 kW of power in the form of heat to a tank of water. How long will it take to heat the 200 kg of water in the tank from 10°C to 70°C ? Assume heat losses to the surroundings to be negligible.**

Sol. The heat added is (1.8 J/s) t
The heat absorbed is $cm\,\Delta T = (4.184\,\text{kJ/kg K})(200\,\text{kg})(60\,\text{K}) =$ 5.0 × 10⁴ kJ

Equation heats, $t = 2.78 \times 10^4\,\text{s} = 7.75\,\text{h}$

Specific Heat of Solids

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume C_v.

From the graph it is clear that at $T = 0$, C_v tends to zero

With rise in temperature, C_v increases and becomes constant = $3R$ = 6 cal/mole-kelvin = 25 J/mole-kelvin at some particular temperature (Debye temperature)

For most of the solids, Debye temperature is close to room temperature.

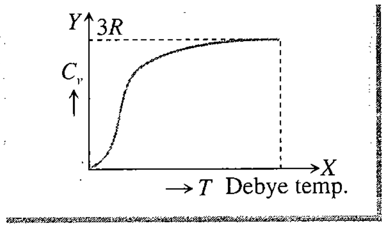

Fig. 1.1

Specific Heat of Water

The variation of specific heat with temperature for water is shown in Fig. 1.2. Usually this temperature dependence of specific heat is neglected.

From the graph:

Temperature (°C)	0	15	35	50	100
Specific heat (cal/g × °C)	1.008	1.000	0.997	0.998	1.006

As specific heat of water is very large, by absorbing or releasing large amount of heat, its temperature changes by small amount. This is why it is used in hot water bottles or as coolant in radiators.

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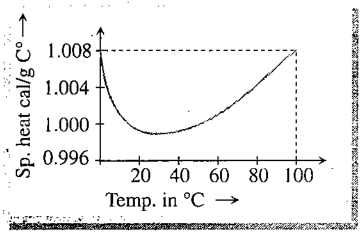

Fig. 1.2

Note: *When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general, measurements made at constant pressure are different from those at constant volume. For solids and liquids, this difference is very small and usually neglected. The specific heats of gases are quite different under constant pressure condition (c_P) and constant volume (c_V). In the chapter 'Kinetic Theory of Gases and First Law of Thermodynamics' we have discussed this topic in detail.*

Illustration 1.7 **What is wrong with the following statement: 'Given any two bodies, the one with the higher temperature contains more heat'.**

Sol. The statement shows a misunderstanding of the concept of heat. Heat is a process by which energy is transferred, not a form of energy that is held or contained. If you wish to speak of energy that is 'contained', you speak of internal energy, not heat.

Further, even, if the statement used the term 'internal energy', it would still be incorrect, since the effects of specific heat and mass are both ignored. A 1 kg mass of water at 20°C has more internal energy than a 1 kg mass of air at 30°C.

Similarly, the earth has far more internal energy than a drop of molten titanium metal.

Correct statements would be: **1.** 'Given any two bodies in thermal contact, the one with the higher temperature will transfer energy to the other by heat'. **2.** 'Given any two bodies of equal mass, the one with the higher product of absolute temperature and specific heat contains more internal energy'. All to say is that internal energy depends not only on temperature but also on mass and nature of body.

Illustration 1.8 **Two bodies have the same heat capacity. If they are combined to form a single composite body, show that the equivalent specific heat of this composite body is independent of the masses of the individual bodies.**

Sol. Let the two bodies have masses m_1, m_2 and specific heats s_1 and s_2. Then

$$m_1 s_1 = m_2 s_2 \quad \text{or} \quad m_1/m_2 = s_2/s_1$$

Let s = specific heat of the composite body.

Then $\qquad (m_1 + m_2)\, s = m_1 s_1 + m_2 s_2 = 2\, m_1 s_1$

$$s = \frac{2m_1 s_1}{m_1 + m_2} = \frac{2m_1 s_1}{m_1 + m_1(s_1/s_2)} = \frac{2s_1 s_2}{s_2 + s_1}$$

Illustration 1.9 **The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of energy by heat. The mass of bar is 525 g. Determine the specific heat of silver.**

Sol. We find its specific heat from the definition, which is contained in the equation $Q = mc_{\text{silver}}\,\Delta T$ for energy input by heat to produce a temperature change. Solving we have

$$c_{\text{silver}} = \frac{Q}{m\Delta T}$$

$$c_{\text{silver}} = \frac{1.23 \times 10^3\ \text{J}}{(0.525\,\text{kg})(10.0°\text{C})} = 234\ \text{J/kg°C}$$

Illustration 1.10 **The air temperature above coastal areas is profoundly influenced by the large specific heat of water. One reason is that the energy released when 1 m³ of water cools by 1°C will raise the temperature of a much larger volume of air by 1°C. Find this volume of air. The specific heat of air is approximately 1 kJ/kg°C. Take the density of air to be 1.3 kg/m³.**

Sol. The mass of 1 m³ of water is specified by its density,

$$m = \rho V = (1.00 \times 10^3\ \text{kg/m}^3)(1\ \text{m}^3) = 1 \times 10^3\ \text{kg}$$

When 1 m³ of water cools by 1°C, it releases energy

$$Q_C = mc\Delta T = (1\times10^3\ \text{kg})(4186\ \text{J/kg°C})(-1°\text{C}) = -4\times10^6\ \text{J}$$

where the negative sign represents heat output. When + 4 × 10⁶ J is transferred to the air, raising its temperature by 1°C, the volume of the air is given by $Q_C = mc\Delta T = \rho Vc\Delta T$

$$V = \frac{Q_c}{\rho c\Delta T} = \frac{4\times10^6\ \text{J}}{(1.3\,\text{kg/m}^3)(1\times10^3\ \text{J/kg°C})(1°\text{C})} = 3\times10^3\ \text{m}^2$$

The volume of the air is a thousand times larger than the volume of the water.

Illustration 1.11 **James Joule tested the conversion of mechanical energy into internal energy by measuring temperatures of falling water. If water at the top of a Swiss waterfall has a temperature of 10.0°C and then falls 50.0 m, what maximum temperature at the bottom would Joule expect? He did not succeed in measuring the temperature change, partly because evaporation cooled the falling water and also because his thermometer was not sufficiently sensitive.**

Sol. The temperature change can be found from the potential energy that is converted to internal energy. The final temperature is this change added to the initial temperature of the water.

The gravitational energy that can change into internal energy is $\Delta E_{\text{int}} = mgy$. It will produce the same temperature

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change as the same amount of heat entering the water from a stove, as described by $Q = mc\Delta T$. Thus, $mgy = mc\Delta T$.

$$\text{Isolating } \Delta T, \Delta T = \frac{gy}{c} = \frac{(9.80\,\text{m/s}^2)(50.0\,\text{m})}{4.186\times10^3\,\text{J/kg}^\circ\text{C}} = 0.117^\circ\text{C}$$

$$T_f = T_i + \Delta T = 10.0^\circ\text{C} + 0.117^\circ\text{C} = 10.1^\circ\text{C}$$

The final temperature might be less than we calculated since this solution does not account for cooling of the water due to evaporation as it falls.

Latent Heat

- When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.
- No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at 0°C melts into water at 0°C. Water at 100°C boils to form steam at 100°C.
- The amount of heat required to change the state of the mass m of the substance is written as: $\Delta Q = mL$, where L is the latent heat. Latent heat is also called as heat of transformation.
- Unit: cal/g or J/kg and dimension: $[L^2T^{-2}]$.
- Any material has two types of latent heats.

i. **Latent heat of fusion:** The latent heat of fusion is the heat energy required to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 kg of liquid changes to 1 kg of solid. For water at its normal freezing temperature or melting point (0°C), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{ice} \approx 80 \text{ cal/g} \approx 6 \text{ kJ/mol} \approx 336 \text{ kJ/kg}$$

ii. **Latent heat of vapourization:** The latent heat of vapourization is the heat energy required to change 1 kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state. It is also the amount of heat energy released when 1 kg of vapour changes into 1 kg of liquid. For water at its normal boiling point or condensation temperature (100°C), the latent heat of vapourization (latent heat of steam) is

$$L_v = L_{steam} \approx 540 \text{ cal/g} \approx 40.8 \text{ kJ/mol} \approx 2260 \text{ kJ/kg}$$

- In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at 100°C is more than that of water at 100°C.
- It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when 1 g of steam at 100°C gets converted to water at 100°C, then it gives out 536 cal of heat. So, it is clear that steam at 100°C has more internal energy than water at 100°C (i.e., boiling of water).
- In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.
- Latent heat of vapourization is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence, more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence, very less amount of heat is required. So, latent heat of vapourization is more than the latent heat of fusion.
- After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.
- There is more shivering effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts down, it absorbs large amount of heat from teeth.

Illustration 1.12 **Some water at 0°C is placed in a large insulated enclosure (vessel). The water vapour formed is pumped out continuously. What fraction of the water will ultimately freeze, if the latent heat of vapourization is seven times the latent heat of fusion?**

Sol. Let us learn the application of theory this illustration.

Let m = mass of water, f = fraction which freezes
L_1 = latent heat of vapourization
L_2 = latent heat of fusion

Mass of water frozen = mf
Heat lost by freezing water = $m f L_2$
Mass of vapour formed = $m(1 - f)$
Heat gained by vapours = $m(1 - f)L_1$
Now heat loss = heat gain:

$$mfL_2 = m(1 - f) \times 7L_2$$
$$f = 7 - 7f \quad \text{or} \quad f = 7/8$$

Illustration 1.13 **How many calories are required to change exactly 1 g of ice at –10°C to steam at atmospheric pressure and 120°C? [Assume the specific heat of steam at a constant pressure of 1 atm is 0.481 cal/(g°C).]**

Sol. The first stage is the warming of the ice from –10°C to the melting point (0°C).

The specific heat capacity of ice is 0.50 cal/(g°C). Therefore, the heat required in the first stage is given by $\Delta H_1 = mc_1 \Delta t_1 = (1.00\text{ g})[0.50\text{ cal}/(\text{g}^\circ\text{C})](10\,^\circ\text{C}) = 5.0\,\text{cal}$.

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The second stage is the melting of the ice at 0°C and 1 atm of pressure. The latent heat for the melting of ice is 79.8 cal/g. $\Delta H_2 = 79.8$ cal. The third stage is the heating of the water from 0°C to 100°C, the boiling point, so the heat required is given by $\Delta H_3 = mc_3 \ \Delta t_3 = (1.00\,\text{g})(1.000\,\text{cal/g}°\text{C})(100°\text{C}) = 100$ cal.

The fourth stage is the boiling of the water at a temperature of 100 °C and at a constant pressure of 1.00 atm. According to Table. the latent heat for the boiling water at 1.00 atm is 540 cal/g,

so $\qquad \Delta H_4 = mL_4 = (1.00\,\text{g})(540\,\text{cal/g}) = 540\,\text{cal}.$

The fifth and final stage is the heating of the steam from 100°C to 120°C (at a constant pressure of 1.00 atm).

Assuming that between 100°C and 120°C the specific heat capacity of steam is constant and has the value 0.481 cal/(g°C) given, we find $\Delta H_5 = mc_5\Delta t_5 = (1\,\text{g})[0.481\ \text{cal}/(\text{g}°\text{C})]\,(20°\text{C})$

$= 9.62$ cal

The total heat requirement $\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 = (5.0 + 79.8 + 100 + 540 + 9.62) = 734.4\,\text{cal}.$

Principle of Calorimetry

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat lost = Heat gained

i.e., the principle of calorimetry represents the law of conservation of heat energy.

- Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H), i.e.,

  $$T_L \le T \le T_H$$

  i.e., the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.
- When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass m is given by $Q = mc\,\Delta T$, where c is specific heat of the body and ΔT change in its temperature.
- When state of a body changes, change of state takes place at constant temperature [m.pt. or b.pt.] and heat released or absorbed is given by $Q = mL$, where L is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is released if liquid converts into solid or vapours convert into liquid.
- If two bodies A and B of masses m_1 and m_2, at temperatures T_1 and T_2 ($T_1 > T_2$) and having gram specific heat c_1 and c_2 are placed in contact,

Heat lost by A = Heat gained by B

or $\quad m_1c_1(T_1 - T) = m_2c_2(T - T_2)$

(where T = temperature of equilibrium)

$$\therefore T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

i. If bodies are of same material $c_1 = c_2$ then $T = \dfrac{m_1T_1 + m_2T_2}{m_1 + m_2}$

ii. If bodies are of same mass ($m_1 = m_2$) then $T = \dfrac{T_1c_1 + T_2c_2}{c_1 + c_2}$

iii. If bodies are of same material and of equal masses ($m_1 = m_2$, $c_1 = c_2$) then $T = \dfrac{T_1 + T_2}{2}$

Illustration 1.14 **Calculate the heat of fusion of ice from the following data for ice at 0°C added to water. Mass of calorimeter = 60 g, mass of calorimeter + water = 460 g, mass of calorimeter + water + ice = 618 g, initial temperature of water = 38°C, final temperature of the mixture = 5°C. The specific heat of calorimeter = 0.10 cal/g/°C. Assume that the calorimeter was also at 0°C initially.**

Sol. Mass of water = 460 − 60 = 400 g

mass of ice = 618 − 460 = 158 g

Heat lost by water = heat gained by ice to melt + heat gained by (water obtained from melting of ice + calorimeter) to reach 5°C

$\Rightarrow 400 \times 1 \times (38 - 5) = 158 \times L + 158 \times 1 \times 5 + 60 \times 0.1 \times 5$

(where L is the latent heat of fusion of ice)

$$\Rightarrow L = 78.35\ \text{cal/g}$$

Illustration 1.15 **A lump of ice of 0.1 kg at –10°C is put in 0.15 kg of water at 20°C. How much water and ice will be found in the mixture when it has reached thermal equilibrium? Specific heat of ice = 0.5 kcal/kg/K and its latent heat of melting = 80 kcal/kg.**

Sol. Heat released by 0.15 kg of water in being cooled to 0°C = 0.15 × 1 × 20 = 3 kcal

Heat absorbed by ice from –10°C to 0°C = 0.1 × 0.5 × 10 = 0.5 kcal

The balance heat is available for melting ice. Let m kg of ice melt.

Then $m \times 80 = 2.5$ or $m = 0.03$ kg

Thus the final temperature is 0°C with 0.07 kg of ice and 0.18 kg of water.

Illustration 1.16 **How should 1 kg of water at 5°C be divided into two parts so that if one part turned into ice at**

0°C, it would release enough heat to vapourize the other part? Latent heat of steam = 540 cal/g and latent heat of ice = 80 cal/g.

Sol. Let the mass be divided into x grams (for ice) and $(1000 - x)$ grams (for vapour).

Heat released by x grams of water $= x \times 1 \times 5 + x \times 80$

Heat absorbed by $(1000 - x)$ grams of water

$$= (1000 - x) \times 1 \times 95 + (1000 - x) \times 540$$

Assuming that the conversion of the other part takes place at 100°C.

$$85x = 95\,(1000 - x) + 540\,(1000 - x) \quad \text{or} \quad x = 882 \text{ g}$$

Thus the mass is to be divided into 882 g for conversion into ice and 118 g for conversion into vapour.

Illustration 1.17 **When a block of metal of specific heat 0.1 cal/g/°C and weighing 110 g is heated to 100°C and then quickly transferred to a calorimeter containing 200 g of a liquid at 10°C, the resulting temperature is 18°C. On repeating the experiment with 400 g of same liquid in the same calorimeter at same initial temperature, the resulting temperature is 14.5°C. Find**

a. Specific heat of the liquid.
b. The water equivalent of calorimeter.

Sol. Let s be the specific heat of the liquid and W be the water equivalent of the calorimeter.

Heat lost by the block = heat gained by (liquid + calorimeter)

For the first case:

$$\Rightarrow 110 \times 0.1 \times (100 - 18) = 200 \times s \times (18 - 10) + W \times 1 \times (18 - 10)$$

$$\Rightarrow 1600s + 8W = 902 \qquad \text{(i)}$$

For the second case:

$$\Rightarrow 110 \times 0.1 \times (100 - 14.5) = 400 \times s \times (14.5 - 10) + W \times 1 \times (14.5 - 10)$$

$$\Rightarrow 1800s + 4.5W = 940.5 \qquad \text{(ii)}$$

On solving Eqs. (i) and (ii), we get $s = 0.48$ cal/g/°C and $W = 16.6$ g.

Illustration 1.18 **The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C, respectively. The temperature when A and B are mixed is 16°C, while when B and C are mixed, it is 23°C. What would be the temperature when A and C are mixed?**

Sol. Let m = mass of each liquid, when A and B are mixed,

Heat lost by B = heat gained by A

$$\Rightarrow m\,s_B\,(19 - 16) = m\,s_A\,(16 - 12) \quad \Rightarrow 3s_B = 4s_A \qquad \text{(i)}$$

When B and C are mixed,

Heat lost by C = heat gained by B

$$\Rightarrow m\,s_C\,(28 - 23) = m\,s_B\,(23 - 19) \Rightarrow 5s_C = 4s_B \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$16s_A = 12s_B = 15s_C \qquad \text{(iii)}$$

When A and C are mixed, Let θ = final temperature

Heat lost by C = heat gained by A

$$\Rightarrow m\,s_C\,(28 - \theta) = m\,s_A\,(\theta - 12)$$

Using Eq. (iii), we get

$$\Rightarrow 15\,s_C\,(28 - \theta) = 15s_A\,(\theta - 12)$$

$$\Rightarrow 16s_A\,(28 - \theta) = 15s_A\,(\theta - 12)$$

On solving for θ, we get $\quad \theta = \dfrac{16 \times 28 + 12 \times 15}{16 + 15}$

$$\Rightarrow \theta = 20.26°\text{C}$$

Illustration 1.19 **A tube leads from a flask in which water is boiling under atmospheric pressure to a calorimeter. The mass of the calorimeter is 150 g, its specific heat capacity is 0.1 cal/g/°C, and it contains originally 340 g of water at 15°C. Steam is allowed to condense in the calorimeter until its temperature increases to 71°C, after which total mass of calorimeter and contents are found to be 525 g. Compute the heat of condensation of steam.**

Sol. Mass of calorimeter and contents before passing steam $= (150 + 340) = 490$ g

mass after passing steam = 525 g

$\Rightarrow$ mass of steam which condenses $= (525 - 490)$ g $= 35$ g

Let L = latent heat of steam.

Heat lost by steam = heat gained by water + heat gained by calorimeter

$$35L + 35 \times 1\,(100 - 71) = 340 \times 1 \times (71 - 15) + 150 \times 0.1 \times (71 - 15)$$

$$\Rightarrow L = 539 \text{ cal/g}$$

Illustration 1.20 **Determine the final result when 200 g of water and 20 g of ice at 0°C are in a calorimeter having a water equivalent of 30 g and 50 g of steam is passed into it at 100°C**

Sol. When steam is passed, the final temperature can be 0°C, between 0°C and 100°C, or 100°C.

We will consider all three possibilities.

Case I

Final temperature = 0°C

In this case, all the steam condenses and then cools down to 0°C.

Heat given out by steam

$$= 50 \times 540 + 50 \times 1 \times (100 - 0) = 32000 \text{ cal}$$

Mass of ice which will melt by this heat $= \dfrac{32000}{80} = 400$ g

But there is only 20 g of ice in the calorimeter.

Hence final temperature cannot be 0°C.

Case II

Final temperature $= \theta$ and $0 < \theta < 100$

Heat lost by steam = heat gained by (ice + water + calorimeter)

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$\Rightarrow 50 \times 540 + 50 \times 1 \times (100 - \theta) = 20 \times 80 + (20 + 200 + 30) \times 1$

$\times (\theta - 0) \Rightarrow \theta = 101.3°C$

The assumption $(0 < \theta < 100)$ is proved to be wrong. Hence, the final temperature cannot be between 0°C and 100°C

$\Rightarrow$ The final temperature will be 100°C

Case III

Let m = mass of steam condensed.

Heat lost by steam = heat gained by ice to melt + heat gained by (water + water + calorimeter) to reach 100°C

$\Rightarrow m\,(540) = 20 \times 80 + (20 + 200 + 30) \times (100 - 0)$

$\Rightarrow m = 26600/540 \approx 49$ g

$\Rightarrow$ 49 g of steam gets condensed and the final temperature is 100°C.

Illustration 1.21 **What will be the final temperature when 150 g of ice at 0°C is mixed with 300 g of water at 50°C. Specific heat of water = 1 cal/g/°C. Latent heat of fusion of ice = 80 cal/g.**

Sol. Let us assume that $T > 0°C$

Heat lost by water = heat gained by ice to melt + heat gained by water formed from ice

$\Rightarrow 300 \times 1 \times (50 - T) = 150 \times 80 + 150 \times 1 \times (T - 0)$

$\Rightarrow T = 6.7°C$

Hence our assumption that $T > 0°C$ is correct.

Illustration 1.22 **In a calorimeter (water equivalent = 40 g) are 200 g of water and 50 g of ice all at 0°C. 30 g of water at 90°C is poured into it. What will be the final condition of the system?**

Sol. Let us assume that all ice melts and temperature of water rises beyond 0°C. Thus we will assume that $T > 0$.

Heat lost by water added = heat gained by ice to melt
+ Heat to warm water formed from ice and water added
+ Heat gained by calorimeter can.

$\Rightarrow 30 \times 1 \times (90 - T) = 50 \times 80 + (50 + 200) \times 1$

$\times (T - 0) + 40 \times 1 \times (T - 0)$

$\Rightarrow 2700 - 30T = 4000 + 250T + 40T$

$\Rightarrow T = -4.1°C$

Hence our assumption that $T > 0$ is wrong, since hot water added is not able to melt all of the ice.

Therefore the final temperature will be 0°C.

Let m = mass of ice finally left in the can.

Heat lost by water = heat gained by melting ice

$\Rightarrow 30 \times 1 \times (90 - 0) = (50 - m) \times 80 \quad \Rightarrow m = 16.25$ g

Finally there is 16.25 g of ice and $(200 + 30 + 33.75) =$ 266.75 g of water at 0°C.

Heating Curve

If heat is supplied at constant rate to a given mass m of a solid, P and a graph is plotted between temperature and time, the graph is as shown in Fig. 1.3 and is called heating curve. From this curve it is clear that

- In the region OA temperature of solid is changing with time, so,

$$Q = mc_s\,\Delta T$$

or $\quad P\Delta t = mc_s\,\Delta T \qquad$ (as $Q = P\Delta t$)

But as $(\Delta T/\Delta t)$ is the slope of temperature–time curve

$$c_S \propto (1/\text{slope of line } OA)$$

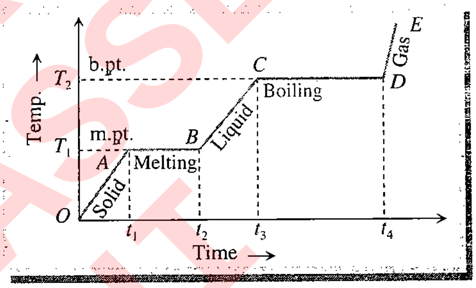

Fig. 1.3

i.e., specific heat (or thermal capacity) is inversely proportional to the slope of temperature–time curve.

- In the region AB temperature is constant, so it represents change of state, i.e., melting of solid with melting point T_1. At A melting starts and at B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion.

$$Q = mL_F \quad \text{or} \quad L_F = \frac{P(t_2 - t_1)}{m} \qquad [\text{as } Q = P(t_2 - t_1)]$$

or $\qquad L_F \propto$ length of line AB

i.e., latent heat of fusion is proportional to the length of line of zero slope. (In this region specific heat $\to \infty$)

- In the region BC temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, i.e.,

$$c_L \propto (1/\text{slope of line } BC)$$

- In the region CD temperature is constant, so it represents the change of state, i.e., boiling with boiling point T_2. At C all substance is in liquid state while at D in vapour state and between C and D partly liquid and partly gas. The length of line i is proportional to latent heat of vapourization, i.e.,

$$L_V \propto \text{Length of line } CD$$

(In this region specific heat $\to \infty$)

- The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

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Illustration 1.23 **A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph.**

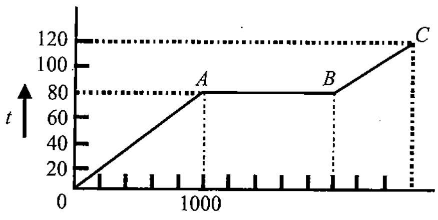

Fig. 1.4

If the relative specific heat capacity of the solid substance is 0.5, find from the graph (i) the mass of the substance; (ii) the specific latent heat of the melting process and (iii) the specific heat of the substance in the liquid state.

Specific heat capacity of water = 1000 cal/kg/K

Sol. 1000 cal of heat raises the temperature of the substance from 0°C to 80°C.

$\therefore \quad 1000 = m\,(1000 \times 0.5) \times 80$

($\because$ specific heat = relative sp. heat × of water)

or $\quad m = 0.025$ kg

Latent heat $= 200 \times 5 = 1000$ cal ($\because$ 1 div reads 200 cal) $= 0.025 \times L$

$\therefore \quad L = 40000$ cal/kg

In the liquid state temperature rises from 80°C to 120°C, that is, by 40°C after absorbing 600 cal.

$\therefore \quad 0.025\, s \times 40 = 600 \quad$ or $\quad s = 600$ cal/kg/K

Illustration 1.24 **Two bodies of equal masses are heated at a uniform rate under identical conditions. The change in temperature in the two cases is shown graphically. What are their melting points?**
Find the ratio of their specific heats and latent heats.

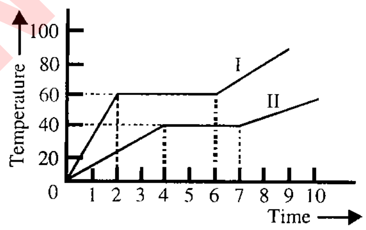

Fig. 1.5

Sol. The melting points of liquids I and II are 60°C and 40°C, respectively. Let R be the rate of supply of heat. We note from the graph that liquid I is heated through 60°C in 2 units of time and that liquid II is heated through 40°C in 4 units of time.

$\therefore \quad 2R = m \times c_1 \times 60 \quad$ and $\quad 4\,R = m \times c_2 \times 40$

Hence, $\quad \dfrac{c_1}{c_2} = \dfrac{1}{3}$

We note further that the temperature of I remains constant for 4 units of time and that of II for 2 units of time.

$\therefore \quad 4R = mL_1 \quad$ and $\quad 2R = mL_2 \Rightarrow \dfrac{L_1}{L_2} = 2$

Concept Application Exercise 1.1

1. The greater the mass of a body, the greater is its heat capacity. Is this true or false?
2. The greater the mass of a body, the greater is its latent heat capacity. Is this true or false?
3. The greater the mass of a body, the greater is its specific heat capacity. True or false?
4. Can heat be added to a substance without causing the temperature of the body to rise? If so, does this contradict the concept of heat as energy in the process of transfer because of a temperature difference?
5. Can heat be considered to be a form of stored energy?
6. Give an example of a process in which no heat is transferred to or from a system but the temperature of the system changes?
7. The latent heat of fusion of a substance is always less than the latent heat of vapourization or latent heat of sublimation of the same substance. Explain.
8. Suppose an astronaut on the surface of the moon took some water at about 20°C out of his thermos and poured it into a glass beaker. What would happen to the water?
9. Heat is added to a body. Does its temperature necessarily increase?
10. When a hot body warms a cool one, are their temperature changes equal in magnitude?
11. Steam at 100°C is passed into a calorimeter of water equivalent 10 g containing 74 cc of water and 10 g of ice at 0°C. If the temperature of the calorimeter and its contents rises to 5°C, calculate the amount of steam passed. Latent heat of steam = 540 kcal/kg, latent heat of fusion = 80 kcal/kg.
12. Ice of mass 600 g and at a temperature of –10°C is placed in a copper vessel heated to 350°C. The resultant mixture is 550 g of ice and water. Find the mass of the vessel. The specific heat capacity of copper (c) = 100 cal/kg-K.
13. When a small ice crystal is placed in overcooled water it begins to freeze instantaneously.
    i. What amount of ice is formed from 1 kg of water overcooled to –8°C? L of water = 336×10^3 J/kg and s of water = 4200 J/kg/K.
    ii. What should be the temperature of the overcooled water in order that all of it be converted into ice at 0°C?

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14. An electric heater whose power is 54 W is immersed in 650 cm^3 water in a calorimeter. In 3 min the water is heated by 3.4°C. What part of the energy of the heater passes out of the calorimeter in the form of radiant energy?
15. An ice cube whose mass is 50 g is taken from a refrigerator where its temperature was −10°C. If no heat is gained or lost from outside, how much water will freeze onto the cube if it is dropped into a beaker containing water at 0°C? Latent heat of fusion = 80 kcal/kg, specific heat capacity of ice = 500 cal/kg/K.
16. Equal volumes of three liquids of densities ρ_1, ρ_2 and ρ_3, specific heat capacities c_1, c_2 and c_3 and temperatures t_1, t_2 and t_3, respectively, are mixed together. What is the temperature of the mixture? Assume no changes in volume on mixing.
17. Victoria Falls in Africa is 122 m in height. Calculate the rise in temperature of the water if all the potential energy lost in the fall is converted into heat.
18. Equal masses of three liquids A, B and C are taken. Their initial temperatures are 10°C, 25°C and 40°C, respectively. When A and B are mixed the temperature of the mixture is 19°C. When B and C are mixed, the temperature of the mixture is 35°C. Find the temperature if all three are mixed.
19. An earthenware vessel loses 1 g of water per second due to evaporation. The water equivalent of the vessel is 0.5 kg and the vessel contains 9.5 kg of water. Find the time required for the water in the vessel to cool to 28°C from 30°C. Neglect radiation losses. Latent heat of vapourization of water in this range of temperature is 540 cal/g.
20. A certain amount of ice is supplied heat at a constant rate for 7 min. For the first 1 min, the temperature rises uniformly with time; then it remains constant for the next 4 min and again rises at a uniform rate for the last 2 min. Explain physically these observations and calculate the final temperature. L of ice = 336 × 10^3 J/kg and c_{water} = 4200 J/kg/K.
21. 1 g steam at 100°C is passed in an insulated vessel having 1 g ice at 0°C. Find the equilibrium temperature of the mixture. Neglect heat capacity of the vessel.

THERMAL EXPANSION

Figures 1.6 (a), (b) and (c) show molecules of solid, liquid and gas, respectively, in their random motions. The atoms are essentially in contact with one another. A rock is an example of a solid. It can stand alone because of the forces holding its atoms together.

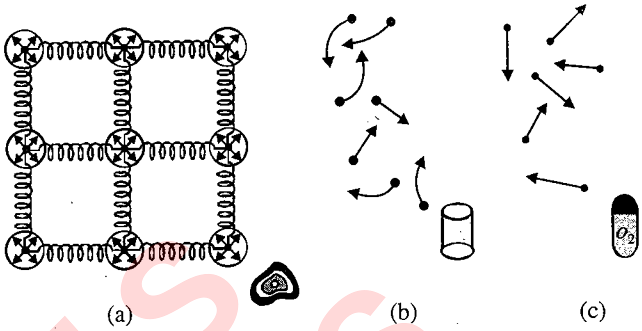

Fig. 1.6

Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. It can flow, but it is also in-close contact. It can flow, but it also remains in an open container because of the forces between its atoms.

Atoms in a gas are separated by distances that are considerably larger than their diameters. Gas must be held in a closed container to prevent it from moving out freely.

Most substances expand when their temperature is raised and contract when cooled. There is an exception to this statement: water contracts when its temperature is increased from 0°C to 4°C. Thus water has its minimum volume, and hence maximum density, at 4°C.

Atoms in solids are in close contact; the forces between them allow the atoms to vibrate but not to move freely. These forces can be thought of as springs that can be stretched or compressed. An individual molecule's motion can be modelled as a point-like particle oscillating in a parallel well caused by the inter-atomic forces, which is parabolic for a Hooke's law spring $\left(U(x)=\frac{1}{2}kx^2\right)$. The mass oscillates in simple harmonic motion between maximum and minimum positions. The potential energy curve is not symmetrical as shown in Fig.1.7. The variable r is the separation between a particle and its nearest neighbour. At temperature T_1 the total energy is E_1 and its separation from its nearest neighbour lies between $r_{1\,min}$ and r_{1max}, the average separation is r_{1av}. The $U(r)$ is not symmetrical, it is flatter to the right at larger r values. At higher temperature the total energy E is higher; the particle spends more time at r values towards less steep portion of the curve. The average separation r_{2av} increases at higher temperatures. Because $r_{2av} > r_{1av}$, the average separation of the atoms or molecules in the solid increases with increasing temperature. When matter is heated without any change in state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.

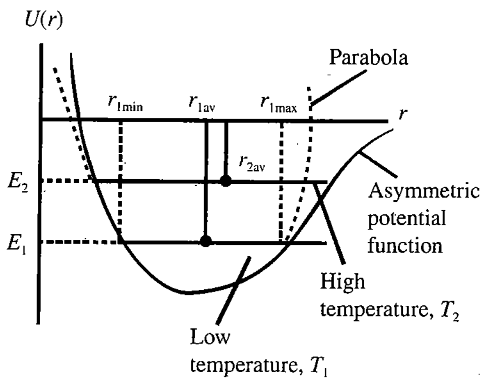

Fig. 1.7

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.
- The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly, the coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

and coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

The value of α, β and γ depends upon the nature of material. All have dimension $\left[\theta^{-1}\right]$ and unit per °C.

- As $\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$, $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$ and $\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$

$\therefore\ \Delta L = L\alpha\Delta T,\ \Delta A = A\beta\Delta T$ and $\Delta V = V\gamma\Delta T$

Final length $L' = L + \Delta L = L(1+\alpha\Delta T)$ (i)

Final area $A' = A + \Delta A = A(1+\beta\Delta T)$ (ii)

Final volume $V' = V + \Delta V = V(1+\gamma\Delta T)$ (iii)

- If L is the side of square plate and it is heated by temperature ΔT, then its side becomes L'.

The initial surface area $A = L^2$ and final surface area $A' = L'^2$

$$\therefore \frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1+\alpha\Delta T)}{L}\right)^2 = (1+\alpha\Delta T)^2 = (1+2\alpha\Delta T)$$

(using Binomial theorem)

or $\quad A' = A(1+2\alpha\Delta T)$

Comparing with Eq. (ii), we get $\beta = 2\alpha$

Similarly, for volumetric expansion

$$\frac{V'}{V} = \left(\frac{L'}{L}\right)^3 = \left(\frac{L(1+\alpha\Delta T)}{L}\right)^3 = (1+\alpha\Delta T)^3 = (1+3\alpha\Delta T)$$

(using Binomial theorem)

or $\quad V' = V(1+\gamma\Delta T)$

Comparing with Eq. (iii), we get $\gamma = 3\alpha$

So $\quad \alpha : \beta : \gamma = 1:2:3$

Some Important Points to Note

i. For the same rise in temperature
Percentage change in area = 2 × percentage change in length.
Percentage change in volume = 3 × percentage change in length.

ii. The three coefficients of expansion are not constant for a given solid. Their values depend on the temperature range in which they are measured.

iii. The values of α, β and γ are independent of the units of length, area and volume, respectively.

iv. For anisotropic solids $\gamma = \alpha_x + \alpha_y + \alpha_z$, where α_x, α_y and α_z represent the mean coefficients of linear expansion along three mutually perpendicular directions.

Illustration 1.25 **The rectangular plate shown in Fig. 1.8 has an area A_i. If the temperature increases by ΔT, each dimension increases according to $\Delta L = \alpha L\ \Delta T$, where α is the average coefficient of linear expansion. Show that the increase in area is $\Delta A = 2\alpha A_i\ \Delta T$. What approximation does this expansion assume?**

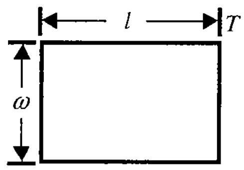

Fig. 1.8

Sol. We expect the area to increase in thermal expansion. It is neat that the coefficient of area expansion is just twice the coefficient of linear expansion.

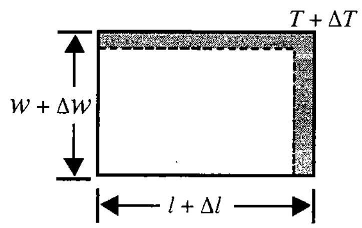

Fig. 1.9

We will use the definitions of coefficients of linear and area expansion.

From the diagram in Fig. 1.9, we see that the change in area is

$$\Delta V = l\Delta w + w\Delta l + \Delta w\Delta l$$

Since Δl and Δw are each small quantities, the product $\Delta w\Delta l$ will be very small as compared to the original or final area.

Therefore, we assume $\Delta w\, \Delta l \propto 0$

Since $\quad \Delta w = w\alpha\Delta T \quad$ and $\quad \Delta l = l\alpha\Delta T$

We then have $\quad \Delta A = lw\alpha\Delta T + wl\alpha\Delta T$

Finally, since $\quad A = lw, \quad$ we have $\quad \Delta A = 2\alpha A\Delta T$

Illustration 1.26 **A mercury thermometer is constructed as shown in Fig. 1.10. The capillary tube has a diameter of 0.004 00 cm, and the bulb has a diameter of 0.250 cm. neglecting the expansion of the glass, find the change in height of the mercury column with a temperature change of 30.0°C.**

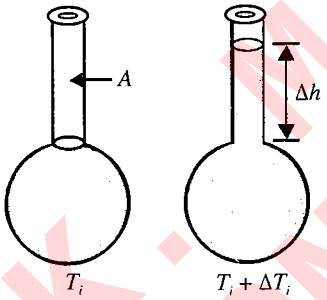

Fig. 1.10

Sol. For an easy-to-read thermometer, the column should rise by a few centimetres.

We use the definition of the coefficient of expansion.

Neglecting the expansion of the glass, the volume of liquid in the capillary will be $\Delta V = A\Delta h$, where A is the cross-sectional area of the capillary. Let V_i represent the volume of the bulb.

$$\Delta V = V_i\gamma\Delta T$$

$$\Delta h = \left(\frac{V_i}{A}\right)\gamma\Delta T = \left[\frac{\frac{4}{3}\pi R^3_{\text{bulb}}}{\pi R^2_{\text{cap}}}\right]\gamma\Delta T$$

$$\Delta h = \frac{4}{3}\frac{(0.125\text{ cm})^3}{(0.002\,00\text{ cm})^2}(1.82\times10^{-4}/°\text{C})(30.0°\text{C}) = 3.55\text{ cm}$$

This is a practical thermometer. Glass expands so little as compared to mercury that only the third digit of the answer would be affected by including the expansion of the glass in our analysis.

Illustration 1.27 **A metal rod A of 25 cm length expands by 0.05 cm when its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40 cm expands by 0.04 cm for the same rise in temperature. A third rod C of 50 cm length made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the length of each portion of composite rod C.**

Sol. From the given data for rod A, we have

$$\Delta L = \alpha_A L\Delta T$$

or $$\alpha_A = \frac{\Delta L}{L\Delta T} = \frac{0.05}{25\times100} = 2\times10^{-5}/°\text{C}$$

For rod B, we have $\quad \Delta L = \alpha_B L\Delta T$

or $$\alpha_B = \frac{\Delta L}{L\Delta T} = \frac{0.04}{40\times100} = 10^{-5}/°\text{C}$$

If rod C is made of segments of rod A and B of lengths l_1 and l_2, respectively, then we have at 0°C

$$l_1 + l_2 = 50\text{ cm} \qquad \text{(i)}$$

At $T = 50°\text{C} \quad l'_1 + l'_2 = 50.03\text{ cm}$

Thus $$\alpha_A l_1\Delta T + \alpha_B l_2\Delta T = 0.03\text{ cm}$$

or $$2\times10^{-5}\times l_1\times50 + 10^{-5}\times l_2\times50 = 0.03\text{ cm}$$

or $$2l_1 + l_2 = \frac{0.03}{50}\times10^5 = 60\text{ cm} \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get $l_1 = 10$ cm and $l_2 = 40$ cm.

Illustration 1.28 **Determine the lengths of an iron rod and a copper ruler at 0°C if the difference in their lengths at 50°C and 450°C is the same and is equal to 2 cm. The coefficient of linear expansion of iron = 12×10^{-6}/K and that of copper = 17×10^{-6} /K.**

Sol. Let x be the length of the iron rod at 0°C, y that of the copper rod at 0°C, and I the difference in lengths at t_1 and t_2°C.

Then $$I = x(1+\alpha_1 t_1) - y(1+\alpha_2 t_1) \qquad \text{(i)}$$

and $$\pm I = x(1+\alpha_1 t_2) - y(1+\alpha_2 t_2) \qquad \text{(ii)}$$

Taking +1

$$I = x(1+\alpha_1 t_2) - y(1+\alpha_2 t_2) \qquad \text{(iii)}$$

from Eqs. (i) and (iii), we get

$$x\alpha_1 = y\alpha_2 \qquad \text{(iv)}$$

from Eqs. (i) and (iv), we get

$$y = \frac{l\alpha_1}{\alpha_2-\alpha_1} = \frac{2\times12\times10^{-6}}{(17-12)\times10^{-6}} = 4.8\text{ cm}$$

and $$x = \frac{l\alpha_2}{\alpha_2-\alpha_1}$$

$\Rightarrow$ $$x = \frac{2\times17\times10^{-6}}{(17-12)\times10^{-6}}\text{ cm} = 6.8\text{ cm}$$

Taking −1

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$$y = \frac{2l + l\alpha_1 (t_1 + t_2)}{(t_2 - t_1)(\alpha_2 - \alpha_1)}; \quad x = \frac{2l + l\alpha_2 (t_1 + t_2)}{(t_2 - t_1)(\alpha_2 - \alpha_1)}$$

$$\therefore y = \frac{2\times 2 + 2\times 12\times 10^{-6}(450+50)}{(450-50)(17-12)\times 10^{-6}}\text{ cm} = 2006\text{ cm} = 20.06\text{ m}$$

$$x = \frac{2\times 2 + 2\times 17\times 10^{-6}(450+50)}{(450-50)(17-12)\times 10^{-6}}\text{ cm} = 2008.5\text{ cm} = 20.08\text{ m}$$

Illustration 1.29 **A steel ball initially at a pressure of 10^5 Pa is heated from 20°C to 120°C keeping its volume constant. Find the final pressure inside the ball. Given that coefficient of linear expansion of steel is 1.1×10^{-5}/°C and Bulk modulus of steel is 1.6×10^{11} Nt/m².**

Sol. On increasing temperature of ball by 100°C (from 20°C to 120°C), the thermal expansion in its volume can be given as

$$\Delta V = \gamma_{st} V\Delta T = 3\,\alpha_{st} V\Delta T \quad \text{(i)}$$

Here it is given that no change of volume is allowed. This implies that the volume increment by thermal expansion is compressed elastically by external pressure. Thus elastic compression in the sphere must be equal to that given in Eq. (i). Bulk modulus of a material is defined as

$$B = \frac{\text{increase in pressure}}{\text{volume strain}} = \frac{\Delta P}{\Delta V/V}$$

Here the externally applied pressure to keep the volume of ball constant is given as

$$\Delta P = B \times \frac{\Delta V}{V} = B(3\alpha_{st}\Delta T)$$

$$= 1.6 \times 10^{11} \times 3 \times 1.1 \times 10^{-5} \times 100$$

$$= 5.28 \times 10^8\text{ Nt/m}^2 = 5.28 \times 10^8\text{ Pa}$$

Thus this must be the excess pressure inside the ball at 120°C to keep its volume constant during heating.

Illustration 1.30 **A steel rail 30 m long is firmly attached to the roadbed only at its ends. The sun raises the temperature of the rail by 5°C, causing the rail to buckle. Assuming that the buckled rail consists of two straight parts meeting in the centre, calculate how much the centre of the rail rises. Coefficient of linear expansion of steel is 12×10^{-6} /K.**

Sol. As indicated in Fig. 1.11, we let the initial length be $2s$ and the final total length be $2(s + \Delta s)$.

The height of the centre of the buckled rail is denoted by y. Assuming that the standard coefficient α of linear expansion can be used (in spite of the fact that the ends are anchored), we have $\Delta s = \alpha s \Delta T$.

By the Pythagorean theorem

$$y = \sqrt{(s+\Delta s)^2 - s^2} = \sqrt{2s\Delta s + (\Delta s)^2} = s\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$$

With $s = 15.0$ m, $\alpha = 12\times 10^{-6}$/K, and $\Delta T = 50$ K, we obtain

$$y = (15.0\text{ m})\sqrt{(12\times 10^{-4}) + (6\times 10^{-4})^2} = 0.52\text{ m}$$

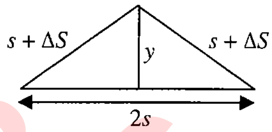

Fig. 1.11

Variation of Density with Temperature

i. Suppose ρ_0 is the density of the substance at 0°C and at any temperature t, it becomes ρ_t. As mass of the substance remains constant at any temperature, we have

$$\rho_0 V_0 = \rho_t V_t$$

Here V_0 and V_t are the volumes of the substance at 0°C and t°C, respectively.

Also $\quad V_t = V_0(1 + \gamma t)$

$$\therefore \rho_0 V_0 = \rho_t\{V_0(1+\gamma t)\}$$

or $\quad \rho_t = \dfrac{\rho_0}{(1+\gamma t)}$

For small value of γ, we can approximate it as

$$\rho_t \simeq \rho_0(1 - \gamma t)$$

ii. If ρ_1 and ρ_2 are the densities at t_1 and t_2, respectively, then we can write

$$\rho_1 V_1 = \rho_2 V_2$$

or $\quad \rho_1 V_0(1+\gamma t_1) = \rho_2 V_0(1+\gamma t_2)$

or $\quad \rho_1 = \rho_2 \dfrac{(1+\gamma t_2)}{(1+\gamma t_1)}$

$$= \rho_2(1+\gamma t_2)(1-\gamma t_1) = \rho_2[1+\gamma(t_2 - t_1)]$$

(neglecting γ_2 on being small)

$$\therefore \quad \gamma = \frac{\rho_1 - \rho_2}{\rho_2(t_2 - t_1)}$$

Illustration 1.31 **A small quantity of a liquid which does not mix with water sinks to the bottom at 20°C, the densities of the liquid and water being 1021 and 998 kg/m³, respectively. To what temperature must the mixture be uniformly heated in order that the liquid forms globules which just float on water? The cubical expansion of the liquid and water over the temperature ranges is 85×10^{-5}/K and 45×10^{-5}/K respectively.**

Sol. The liquid will float on water at the temperature at which both of them have the same densities.

$$D_{\text{common}} = \frac{1021}{1+85\times 10^{-5}\Delta\theta} = \frac{998}{1+45\times 10^{-5}\Delta\theta}$$

$$\Rightarrow 1021\,(1 + 45 \times 10^{-5}\,\Delta\theta) = 998\,(1 + 85 \times 10^{-5}\,\Delta\theta)$$

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$$\Rightarrow \Delta\theta = 59°C \Rightarrow \theta_2 - \theta_1 = 59°C$$
$$\Rightarrow \theta_2 = 59 + 20 = 79°C$$

Expansion of Liquid

Liquids also expand on heating just like solids. Since liquids have no shape of their own, they suffer only volume expansion. If the liquid of volume V is heated and its temperature is raised by $\Delta\theta$ then

$$V_L' = V(1+\gamma_L \Delta\theta)$$

(γ_L = coefficient of real expansion or coefficient of volume expansion of liquid)

As liquid is always taken in a vessel for heating, if a liquid is heated, the vessel also gets heated and it also expands.

$$V_S' = V(1+g_S\,\Delta\theta)$$

(γ_S = coefficient of volume expansion for solid vessel)

So, the change in volume of liquid relative to vessel

$$V_L' - V_S' = V(\gamma_L - \gamma_S)\Delta\theta$$

$\Delta V_{app} = V\gamma_{app}\Delta\theta$ ($\gamma_{app} = \gamma_L - \gamma_S$ = Apparent coefficient of volume expansion for liquid)

$\gamma_L > \gamma_S$	$\gamma_{app} > 0$	ΔV_{app} = positive	Level of liquid in vessel will rise on heating.
$\gamma_L < \gamma_S$	$\gamma_{app} < 0$	ΔV_{app} = negative	Level of liquid in vessel will fall on heating.
$\gamma_L = \gamma_S$	$\gamma_{app} = 0$	$\Delta V_{app} = 0$	Level of liquid in vessel will remain same.

Illustration 1.32 **A 1-L flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in the flask, given that the coefficient of linear expansion of glass = 9 × 10^{-6} /°C and the coefficient of volume expansion of Hg = 1.8 × 10^{-4} /°C?**

Sol. Since the volume above mercury remains the same at all temperatures, the expansion of the glass vessel must be the same as that of mercury in the vessel. Also, $\gamma_g = 3\,\alpha_g = 27 \times 10^{-6}$/°C. Let V be the volume of the mercury. Then, from $\Delta V = V\gamma\,\Delta T$

$$V \times (1.8 \times 10^{-4}) \times \Delta T = 10^{-3} \times 27 \times 10^{-6}\,\Delta T$$

$$(\because 1\,L = 10^{-3}\text{ m}^3 \text{ and } \gamma = 3\alpha) \quad \text{or} \quad V = 150 \times 10^{-6}\text{ m}^3$$

Illustration 1.33 **A hollow aluminium cylinder 20.0 cm deep has an internal capacity of 2.000 L at 20.0°C. It is completely filled with turpentine and then slowly warmed to 80.0°C. a. How much turpentine overflows? b. If the cylinder is then cooled back to 20.0°C, how far below the cylinder's rim does the turpentine's surface recede?**

Sol. We guess that vertical cubic centimetres of turpentine will overflow, and that the liquid level will drop about a centimetre.

We will use the definition of the volume expansion coefficient. Both the liquid and the container expand. We will need to reason carefully about original, intermediate and final volumes of each.

When the temperature is increased from 20.0°C to 80.0°C, both the cylinder and the turpentine increase in volume by $\Delta V = \gamma V \Delta T$:

a. The overflow is $V_{over} = \Delta V_{turp} - \Delta V_{Al}$

$$V_{over} = (\gamma V \Delta T)_{turp} - (\gamma V \Delta T)_{Al} = V_i \Delta T(\gamma_{turp} - 3\alpha_{Al})$$

$$V_{over} = (2.000\text{ L})(60.0°C)(9.00 \times 10^{-4}/°C - 0.720 \times 10^{-4}/°C)$$

$$= 0.099\,4\text{ L}$$

b. After warming, the whole volume of the turpentine is

$$V' = 2000\text{ cm}^3 + (9.00\times10^{-4}/°C)(2000\text{ cm}^3)(60.0°C) = 2108\text{ cm}^3$$

$$\text{The fraction lost is } \frac{99.4\text{ cm}^3}{2108\text{ cm}^3} = 4.71\times10^{-2}$$

This also is the fraction of the cylinder that will be empty after cooling. Therefore, change in level

$$\Delta h = (4.71\times10^{-2})(20.0\text{ cm}) = 0.943\text{ cm}$$

The change in volume of the container is not negligible, but is 8% of the change in volume of the turpentine.

Illustration 1.34 **A glass flask whose volume is exactly 1000 cm³ at 0°C is filled level full of mercury at this temperature. When the flask and mercury are heated to 100°C, 15.2 cm³ of mercury overflows. The coefficient of cubical expansion of Hg is 1.82 × 10^{-4}/°C. Compute the coefficient of linear expansion of glass.**

Sol. As 15.2 cm³ of Hg overflows at 100°C,
(final volume of Hg) − (final volume of glass flask) = 15.2 cm³.

$$\Rightarrow 1000(1+\gamma_l\theta) - 1000(1+\gamma_g\theta) = 15.2,$$

where θ = rise in temperature = 100 − 0 = 100°C

$$\Rightarrow \gamma_g = \gamma_l - \frac{15.2}{1000\,\theta} = 0.000182 - 0.000152$$

$$\Rightarrow \gamma_g = 3\times10^{-5}/°C \quad \Rightarrow \alpha = \frac{\gamma_g}{3} = 1\times10^{-5}/°C.$$

Illustration 1.35 **A 250 cm³ glass bottle is completely filled with water at 50°C. The bottle and water are heated to 60°C. How much water runs over if:**

a. the expansion of the bottle is neglected;
b. the expansion of the bottle is included? Given the coefficient of areal expansion of glass β = 1.2 × 10^{-5}/K and γ_{water} = 60 × 10^{-5}/°C.

Sol. Water overflow = (final volume of water) − (final volume of bottle)

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a. If the expansion of bottle is neglected:

$$\text{Water overflow} = 250(1+\gamma_l\theta) - 250$$
$$= 250 \times 60 \times 10^{-5} \times 10$$
$$\Rightarrow \text{water overflow} = 1.5 \text{ cm}^3$$

b. If the bottle (glass) expands:

Water overflow

$= \text{(final volume of water)} - \text{(final volume of glass)}$

$= 250(1+\gamma_l\theta) - 250(1+\gamma_g\theta)$

$= 250(\gamma_l - \gamma_g)\theta, \quad \text{where } \gamma_g = 3/2\beta = 1.8\times10^{-5}/°C$

$= 250(58.2\times10^{-5})\times(60-50)$

Water overflow $= 1.455$ cm^3

Effect of Temperature on Upthrust

The thrust on volume V of a body in a liquid of density σ is given by $Th = V\sigma g$

Now with rise in temperature by $\Delta\theta$ °C, due to expansion, volume of the body will increase while density of liquid will decrease according to the relations $V' = V(1+\gamma_S\Delta\theta)]$ and $\sigma' = \sigma/(1+\gamma_L\Delta\theta)$

So the thrust will become $Th' = V'\sigma' g$

$$\therefore \quad \frac{Th'}{Th} = \frac{V'\sigma' g}{V\sigma g} = \frac{(1+\gamma_S\Delta\theta)}{(1+\gamma_L\Delta\theta)}$$

and apparent weight of the body W_{app} = actual weight − thrust

As $\gamma_S < \gamma_L$, therefore, $Th' < Th$ with rise in temperature thrust also decreases and apparent weight of body increases.

Illustration 1.36 **A solid floats in a liquid at 20°C with 75% of it immersed. When the liquid is heated to 100°C, the same solid floats with 80% of it immersed in the liquid. Calculate the coefficient of expansion of the liquid. Assume the volume of the solid to be constant.**

Sol. Let m be the mass of the solid and V its volume. By the law of flotation

Weight of floating object = Buoyant force

In Case I: $\quad mg = \left(\frac{3}{4}V\right)\rho_{20}g$

where $\quad \rho_{20}$ = density of liquid at 20°C

In Case II: $\quad mg = \left(\frac{80}{100}V\right)\rho_{100}g,$

where $\quad \rho_{100}$ = density of liquid at 100°C

Considering both the cases

$$\Rightarrow \frac{3}{4}\rho_{20} = \frac{4}{5}\rho_{100} \quad \Rightarrow \frac{3}{4}\frac{\rho_0}{1+\gamma\times20} = \frac{4}{5}\frac{\rho_0}{1+\gamma\times100}$$

After solving we get

$$\gamma = \frac{1}{1180} = 8.47\times10^{-4}/°C$$

Illustration 1.37 **A sinker of weight W_0 has an apparent weight W_1 when placed in a liquid at a temperature T_1 and W_2 when weighed in the same liquid at a temperature T_2. The coefficient of cubical expansion of the material of the sinker is γ_s. What is the coefficient of volume expansion of the liquid?**

Sol. Let $\theta = T_2 - T_1$ and γ = coefficient of volume expansion of liquid.

Let density of liquid at temperatures T_1 and T_2 be ρ_1 and ρ_2, respectively.

$$\Rightarrow \rho_1 = \rho_2(1+\gamma\theta) \tag{i}$$

Let V_1 and V_2 be the volumes of the sinker at temperatures T_1 and T_2, respectively.

$$\Rightarrow V_2 = V_1(1+\gamma_s\theta) \tag{ii}$$

The loss in weight at $T_1 = V_1\rho_1 g \quad \Rightarrow W_0 - W_1 = V_1\rho_1 g \quad$ (iii)

The loss in weight at $T_2 = V_2\rho_2 g \quad \Rightarrow W_0 - W_2 = V_2\rho_2 g \quad$ (iv)

Dividing Eq. (iii) by Eq. (iv), $\dfrac{W_0 - W_1}{W_0 - W_2} = \dfrac{V_1\rho_1}{V_2\rho_2}$

Using Eqs. (i) and (ii), $\dfrac{W_0 - W_1}{W_0 - W_2} = \dfrac{1+\gamma\theta}{1+\gamma_s\theta}$

$$\Rightarrow 1+\gamma\theta = \frac{W_0 - W_1}{W_0 - W_2} + \gamma_s\left(\frac{W_0 - W_1}{W_0 - W_2}\right)\theta$$

$$\Rightarrow \gamma = \left(\frac{W_2 - W_1}{W_0 - W_2}\right)\frac{1}{T_2 - T_1} + \left(\frac{W_0 - W_1}{W_0 - W_2}\right)\gamma_s$$

Anomalous Expansion of Water

1. Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e., γ is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.
2. The anomalous behaviour of water arises due to the fact that water has three types of molecules, viz., $H_2O, (H_2O)_2$ and $(H_2O)_3$, having different volume per unit mass values and at different temperatures their properties in water are different.
3. At 4°C, density of water is maximum while its specific volume is minimum.

During winter when the water on the surface of a lake cools below 4°C by cold air, it expands and becomes lighter than water below. Therefore, the water cooled below 4°C stays on the surface and freezes when the temperature of surroundings falls below 0°C. Thus the lake freezes first on the surface and water in contact with ice has temperature 0°C while at the bottom of the lake 4°C (as density of water at 4°C is maximum) and fish and other aquatic animals remain alive in this water.

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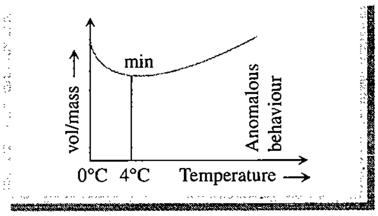

Fig. 1.12

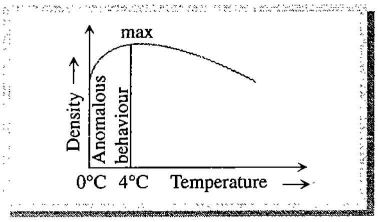

Fig. 1.13

Expansion of Gases

Gases have no definite shape; therefore, gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, gases have only real expansion.

Coefficient of Volume Expansion

At constant pressure, increase in volume per unit volume per unit degree rise of temperature is called coefficient of volume expansion.

$$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \quad \therefore \text{ Final volume } V' = V(1+\alpha\Delta T)$$

Coefficient of Pressure Expansion

$$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T}$$

$\therefore$ Final pressure $P' = P(1+\beta\Delta T)$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

i.e. $$\alpha = \beta = \frac{1}{273} /°\text{C}$$

Application of Thermal Expansion

1. **Bi-metallic strip:** Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, is called 'bi-metallic strip' and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side, i.e., convex side.

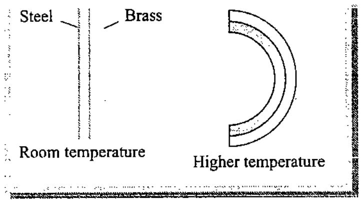

Fig. 1.14

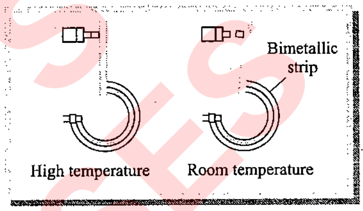

Fig. 1.15

Illustration 1.38 **A copper and a tungsten plate having a thickness δ = 2 mm each are riveted together so that at 0°C they form a flat bimetallic plate. Find the average radius of curvature of this plate at t = 200°C. The coefficients of linear expansion for copper and tungsten are α_{cu} = 1.7 × 10^{-5}/K and α_W = 0.4 × 10^{-5}/K, respectively.**

Sol. The average length of copper plate at a temperature $T = 200°\text{C}$ is $l_C = l_0(1+\alpha_C T)$, where l_0 is the length of copper plate at 0°C. The length of the tungsten plate is $l_t = l_0(1+\alpha_t T)$

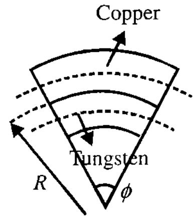

Fig. 1.16

We shall assume that the edges of plates are not displaced during deformation and that an increase in the plate thickness due to heating can be neglected.

From Fig. 1.16 we have

$$l_C = \phi(R+\delta/2) \quad \Rightarrow \quad l_t = \phi(R-\delta/2)$$

Consequently,

$$\phi(R+\delta/2) = l_0(1+\alpha_C T) \quad \text{(i)}$$

$$\phi(R-\delta/2) = l_0(1+\alpha_t T) \quad \text{(ii)}$$

To eliminate the unknown quantities, ϕ and l_0, we divide Eq. (i) by Eq. (ii) term-wise:

$$\Rightarrow \frac{(R+\delta/2)}{(R-\delta/2)} = \frac{(1+\alpha_C T)}{(1+\alpha_t T)} \quad \Rightarrow R = \delta\frac{[2+(\alpha_c+\alpha_t)T]}{[2(\alpha_c-\alpha_t)T]}$$

$$\Rightarrow R = \frac{\delta}{(\alpha_C-\alpha_t)T}$$

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neglecting $(\alpha_C + \alpha_r)$ in numerator. Substituting the values in above relation. we get: $R = 0.769$ m.

2. **Effect of temperature on the time period of a simple pendulum:** A pendulum clock keeps proper time at temperature θ. If temperature is increased to θ' $(> \theta)$ then due to linear expansion, length of pendulum and hence its time period will increase.
If l_0 be the length of the pendulum, at θ°C, then its time period

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}} \quad \text{(i)}$$

At any temperature increment $\Delta\theta$, the time period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Here, $l = l_0(1 + \alpha\Delta\theta)$

$$\therefore \quad T = 2\pi\sqrt{\frac{l_0(1+\alpha\Delta\theta)}{g}} = 2\pi\sqrt{\frac{l_0}{g}}(1+\alpha\Delta\theta)^{1/2}$$

$$= T_0(1 + \alpha\Delta\theta)^{1/2} \quad \text{(ii)}$$

$$= T_0\left(1+\frac{\alpha\Delta\theta}{2}\right) \quad \text{or} \quad \frac{T}{T_0} - 1 = \frac{\alpha\Delta\theta}{2}$$

$$\frac{T - T_0}{T_0} = \frac{\alpha\Delta\theta}{2} \quad \text{or} \quad \frac{\Delta T}{T_0} = \frac{\alpha\Delta\theta}{2}$$

$$\therefore \quad \Delta T = \left(\frac{\alpha\Delta\theta}{2}\right)T_0$$

Note:

- ***Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.***
- ***Loss of time in a time period*** $\Delta T = \frac{1}{2}\alpha\Delta\pi T_0$
- $\therefore$ ***Loss of time in any given time interval t can be given by***

$$\Delta t = \frac{1}{2}\alpha\Delta\pi t$$

- ***The clock will lose time, i.e., it will become slow if $\theta' > \theta$ (in summer).***
- ***It will gain time, i.e., it will become fast if $\theta' < \theta$ (in winter).***
- ***The gain or loss in time is independent of time period T and depends on the time interval t.***
  Time lost by the clock in a day ($t = 86400$ s)

$$\Delta t = \frac{1}{2}\alpha\Delta\pi t = \frac{1}{2}\alpha\Delta\pi\,(86400) = 43200\,\alpha\Delta\pi \text{ s}$$

- ***Since coefficient of linear expansion (α) is very small for invar, pendulums are made of invar to show the correct time in all seasons.***

Illustration 1.39 **A clock with a brass pendulum shaft keeps correct time at a certain temperature.**

a. How closely must the temperature be controlled if the clock is not to gain or lose more than 1 s a day? Does the answer depend on the period of the pendulum?

b. Will an increase of temperature cause the clock to gain or lose? ($\alpha_{brass} = 2 \times 10^{-5}$/°C)

Sol.

a. Number of seconds lost or gained per day $= \frac{1}{2}\alpha\theta \times 86400$, where θ = rise or drop in temperature; α = coeff. of linear expansion of shaft.

$$\text{We want that } \left|\frac{1}{2}\alpha\theta \times 86400\right| < 1$$

$$\Rightarrow |\theta| < \frac{2}{2\times10^{-5}\times86400} \qquad \Rightarrow |\theta| < 1.1574°\text{C}$$

Hence temperature should not increase or decrease by more than 1.1574°C. It does not depend upon time period.

b. An increase in temperature makes the pendulum slow and hence clock loses time.

Illustration 1.40 **A pendulum clock loses 12 s a day if the temperature is 40°C and goes fast by 4 s a day if the temperature is 20°C. Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.**

Sol. Let T be the temperature at which the clock is correct.
Time lost per day = 1/2 α (rise in temperature) × 86400

$$\Rightarrow 12 = 1/2\alpha(40 - T)\times 86400 \quad \text{(i)}$$

Time gained per day = 1/2 α (drop in temperature) × 86400

$$\Rightarrow 4 = 1/2\alpha(T - 20)\times 86400 \quad \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$32 = 86400\alpha(40-20) \qquad \Rightarrow \alpha = 1.85\times10^{-5}/°\text{C}$$

Dividing Eq. (i) by Eq. (ii), we get

$$12(T - 20) = 4\,(40 - T) \qquad \Rightarrow T = 25°\text{C}$$

$\Rightarrow$ Clock shows correct time at 25°C.

3. **Thermal stress in a rigidly fixed rod:** When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

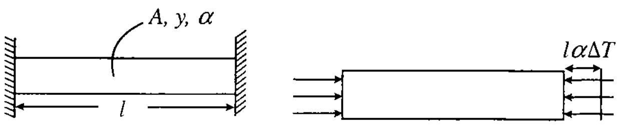

Fig. 1.17

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If temperature of rod is increased by ΔT, then change in length

$$\Delta l = l\alpha\, \Delta T$$

$$\text{strain} = \frac{\Delta l}{l} = \alpha \Delta T$$

But due to rigid support, there is no strain. Supports provide force or stresses to keep the length of rod same

$$Y = \frac{\text{stress}}{\text{strain}}$$

thermal stress = Y strain = $Y\alpha\Delta T$

$$\frac{F}{A} = Y\alpha\Delta T \quad F = AY\alpha\ \Delta T$$

Illustration 1.41 **A rod of length 2 m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C, then find stress produced when the rod is (i) fully prevented to expand, (ii) permitted to expand by 0.4 mm. $Y = 2 \times 10^{11}$ N/m²; $\alpha = 15 \times 10^{-6}$/°C.**

Sol. Free expansion of the rod = $\alpha L \Delta\theta$

$= 15 \times 10^{-6}$/°C × 2 m × (50 − 20)°C

$= 9 \times 10^{-4}$ m = 0.9 mm

i. If the expansion is fully prevented,

$$\text{then strain} = \frac{9\times10^{-4}}{2} \Rightarrow 4.5\times10^{-4}$$

∴ Temperature stress = strain × Y

$= 4.5 \times 10^{-4} \times 2 \times 10^{11} = 9 \times 10^{7}$ N/m²

ii. If 0.4 mm expansion is allowed, then length restricted to expand = 0.9 − 0.4 = 0.5 mm

$$\therefore \text{Strain} = \frac{5\times10^{-4}}{2} = 2.5\times10^{-4}$$

∴ Temperature stress = strain × $Y = 2.5 \times 10^{-4} \times 2 \times 10^{11}$
$= 5 \times 10^{7}$ N/m²

Illustration 1.42 **Two rods of different metals having the same area of cross section A are placed between the two massive walls as shown in Fig. 1.18. The first rod has a length l_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for second rod are l_2, α_2 and Y_2. The temperature of both the rods is now raised by t°C.**

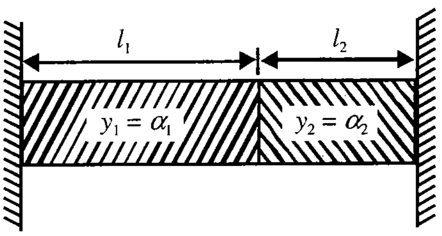

Fig. 1.18

i. Find the force with which the rods act on each other (at higher temperature) in terms of given quantities.

ii. Also find the length of the rods at higher temperature.

Sol.

i. Let t°C = increase in the temperature.
Increase in length of first rod = $l_1\alpha_1 t$
Increase in length of second rod = $l_2\alpha_2 t$
∴ Total extension in length due to rise in temperature

$$= l_1\alpha_1 t + l_2\alpha_2 t = (l_1\alpha_1 + l_2\alpha_2)t \qquad \text{(i)}$$

Since the walls are rigid, this increase in length will not happen. This will be compensated by equal and opposite forces F, F producing decrease in the lengths of the rods due to elasticity.

$$\therefore \text{Decrease in length of first rod} = \frac{F\times l_1}{Y_1\times A}$$

$$\text{And decrease in length of second rod} = \frac{F\times l_2}{Y_2\times A}$$

∴ Total decrease in length due to elastic force

$$= \frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right) = (l_1\alpha_1 + l_2\alpha_2)t$$

or
$$F = \frac{A(l_1\alpha_1 + l_2\alpha_2)t}{\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right)} \qquad \text{(iii)}$$

ii. Length of the first rod = original length + increase in length due to temperature − decrease in length due to force

$$= \left(l_1 + l_1\alpha_1 t - \frac{F}{A}\frac{l_1}{Y_1}\right)$$

$$\text{and length of second rod} = l_2 + l_2\alpha_2 t - \frac{F}{A}\frac{l_2}{Y_2}$$

∴ The total length is same = $l_1 + l_2$ at all temperatures.

Illustration 1.43 **Two rods of equal cross sections, one of copper and the other of steel, are joined to form a composite rod of length 2.0 m at 20°C; the length of the copper rod is 0.5 m. When the temperature is raised to 120°C, the length of composite rod increases to 2.002 m. If the composite rod is fixed between two rigid walls and thus not allowed to expand, it is found that the lengths of the component rods also do not change with increase in temperature. Calculate Young's modulus of steel. (The coefficient of linear expansion of copper, $\alpha_c = 1.6 \times 10^{-5}$/°C and Young's modulus of copper is 1.3×10^{13} N/m².)**

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Sol. Change in length:

For Cu rod

$$l_c\alpha_c[t_2 - t_1] = 0.5 \times \alpha_c \times (120 - 20) = 50\alpha_c$$

For steel rod

$l_s\alpha_s(t_2 - t_1) = 1.5\alpha_s\, 100 = 150\alpha_s$

$\therefore$ Total change in length $= 50\alpha_c + 150\alpha_s = 0.002$ m

$$\Rightarrow \alpha_s = \frac{4\times10^{-5} - \alpha_c}{3} = \frac{4\times10^{-5} - 1.6\times10^{-5}}{3} = 0.8 \times 10^{-5}/°C$$

Stress in steel rod, $f_s = Y_s \times \text{strain} = Y_s \times \Delta l/l$

$$= Y_s\alpha_s(t_2 - t_1) = Y_s \times \alpha_s \times 100 = 100\, Y_s\alpha_s$$

There is no change in the length of individual rod, because the length change due to stress is balanced by length change due to thermal expansion.

Similarly, stress in copper rod, $f_c = Y_c\alpha_c \times 100 = 100 Y_c\alpha_c$

Now stress is same in both:

$$Y_s = \frac{Y_c\alpha_c}{\alpha_s} = \frac{1.3\times10^{13}\times1.6\times10^{-5}}{0.8\times10^{-5}} = 2.6 \times 10^{13}\ \text{N/m}^2$$

4. **Error in scale reading due to expansion or contraction:** If a scale gives correct reading at temperature θ, at temperature $\theta'(>\theta)$ due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that

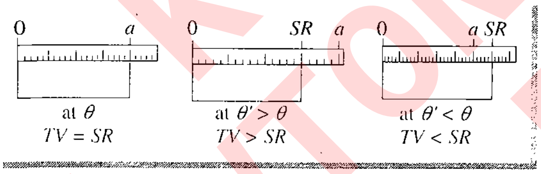

Fig. 1.19

True value = scale reading $[1 + \alpha(\theta' - \theta)]$

i.e., $TV = SR[1 + \alpha\Delta\theta]$ with $\Delta\theta = (\theta' - \theta)$

However, if $\theta' < \theta$, due to contractions of scale, scale reading will be more than true value, so true value will be lesser than scale reading and will still be given by same equation with $\Delta\theta = (\theta' - \theta)$ being negative.

Illustration 1.44 **A surveyor's 30 m steel tape is correct at a temperature of 20°C. The distance between two points, as measured by this tape on a day when the temperature is 35°C, is 26 m. What is the true distance between the point? ($\alpha_{steel} = 1.2 \times 10^{-5}/°C$)**

Sol. Let temperature rise above the correct temperature by θ.

$$\Rightarrow \theta = 35 - 20 = 15°C$$

Using the relation:

Correct length = measured length $(1 + \alpha\theta) \Rightarrow$ true distance between the points

$= 26(1 + 1.2\times10^{-5}\times15)$ $\Rightarrow$ true distance = 26.00468 m

Illustration 1.45 **A barometer with a brass scale reads 755 mm on a day when the temperature is 25°C. If the scale is correctly graduated at 0°C, find the true pressure at 0°C (in terms of height of Hg) given that the coefficient of linear expansion of brass is 18 × 10^{-6}/K. Coefficient of cubical expansion of mercury = 182 × 10^{-6}/K.**

Sol. Given that 1 mm at 0°C = 1 mm

$\therefore$ 755 mm at 25°C = 755(1 + 18 × 10^{-6} × 25) mm = 755.34 mm

Let P be the value of the atmospheric pressure.

Then $P = 755.34\,\rho_{25}g = h\rho_0 g$, where ρ_0, ρ_{25} are densities of mercury at 0°C and 25°C, respectively.

or $$h = 755.34 \times \frac{\rho_{25}}{\rho_0} = 755.34 \times \frac{\rho_0}{\rho_0(1 + 182\times10^{-6}\times25)}$$

or $$h = 751.19 \text{ mm}$$

Illustration 1.46 **At room temperature (25°C) the length of a steel rod is measured using a brass centimetre scale. The measured length is 20 cm. If the scale is calibrated to read accurately at temperature 0°C, find the actual length of steel rod at room temperature**

Sol. The brass scale is calibrated to read accurately at 0°C. This means at 0°C, each division of scale has exact 1 cm length. Thus at higher temperature the division length of scale will be more than 1 cm due to thermal expansion. Thus at higher temperature the scale reading for length measurement is not appropriate and as at higher temperature the division length is more, the length this scale reads will be lesser than the actual length to be measured. For illustration in this case the length of each division on brass scale at 25°C is

$$l_{1\text{div}} = (1\,\text{cm})[1 + \alpha_{br}(25 - 0)]$$
$$= 1 + \alpha_{br}(25)$$

It is given that at 25°C the length of steel rod measured is 20 cm. Actually it is not 20 cm, it is 20 divisions on the brass scale. Now we can find the actual length of the steel rod at 25°C as

$$l_{25°C} = (20\,\text{cm}) \times l_{1\text{div}}$$

or $$l_{act} = 20[1 + \alpha_{br}(25)] \quad \text{(i)}$$

The above expression is a general relation using which you can find the actual lengths of the objects of which lengths are measured by a metallic scale at some temperature other than the graduation temperature of the scale.

5. **Expansion of cavity:** Thermal expansion of an isotropic object may be imagined as a photographic enlargement. So if there is a hole A in a plate C (or cavity A inside a body

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C), the area of hole (or volume of cavity) will increase when body expands on heating, just as if the hole (or cavity) were solid B of the same material. Also the expansion of area (or volume) of the body C will be independent of shape and size of hole (or cavity), i.e., will be equal to that of D.

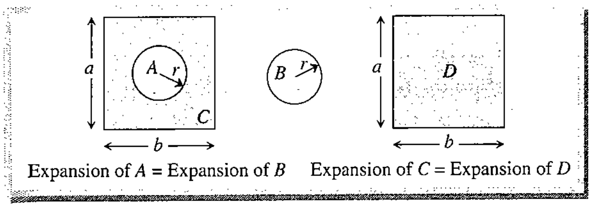

Fig. 1.20

Note: ***For a solid and hollow sphere of same radius and material, heated to the same temperature, expansion of both will be equal because thermal expansion of isotropic solids is similar to true photographic enlargement. It means the expansion of cavity is same as if it has been a solid body of the same material. But if same heat is given to the two spheres, due to lesser mass, rise in temperature of hollow sphere will be more*** $\left\{As\left(\Delta\theta = \frac{a}{mc}\right)\right\}$. ***Hence its expansion will be more.***

6. **Practical application:**

i. When rails are laid down on the ground, space is left between the ends of two rails to allow for expansion.

ii. The transmission cables are not tightly fixed to the poles.

iii. Pendulum of wall clock and balance wheel of wrist watch are made of invar (an alloy which has very low value of coefficient of expansion).

iv. Test tubes, beakers and crucibles are made of pyrex-glass or silica because they have very low value of coefficient of linear expansion.

v. The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel to ensure tight fit.

vi. A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

Concept Application Exercise 1.2

1. Does the change in volume of a body when its temperature is raised depend on whether the body has cavities inside, other things being equal?
2. Explain why some rubber-like substances contract with rising temperature.
3. Two large holes are cut in a metal sheet. If this is heated, will their diameters increase or decrease?
4. In the above question, will the distance between the holes increase or decrease on heating?
5. A long metal rod is bent to form a ring with a small gap. If this is heated, will this gap increase or decrease?
6. Two iron spheres of the same diameter are heated to the same temperature. One is solid, and the other is hollow. Which will expand more?
7. A steel rod is 3.000 cm at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide on to the rod?
8. A clock with a metallic pendulum gains 5 s each day at a temperature of 15°C and loses 10 s each day at a temperature o 30°C. find the coefficient of thermal expansion of the pendulum metal.
9. The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. Find their lengths.

   $(\alpha_{Fe} = 11 \times 10^{-6}\,°C^{-1},$
   $\alpha_{Cu} = 17 \times 10^{-6}\,°C^{-1})$
10. A metal rod of 30 cm length expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal of length 45 cm expands by 0.045 cm for the same rise in temperature. A composite rod C made by joining A and B end to end expands by 0.040 cm when its length is 45 cm and it is heated from 0°C to 50°C. Find the length of each portion of the composite rod.
11. A brass scale is graduated at 10°C. What is the true length of a zinc rod which measures 60.00 cm on this scale at 30°C?

    Coefficient of linear expansion of brass = 18×10^{-6} K^{-1}.
12. A long horizontal glass capillary tube open at both ends contains a mercury thread 1 m long at 0°C. Find the length of the mercury thread, as read on this scale, at 100°C.
13. A mercury-in-glass thermometer has a stem of internal diameter 0.06 cm and contains 43 g of mercury. The mercury thread expands by 10 cm when the temperature changes from 0°C to 50°C. Find the coefficient of cubical expansion of mercury. Relative density of mercury = 13.6 and $\alpha_{glass} = 9 \times 10^{-6}$/K.
14. A sphere of diameter 7 cm and mass 266.5 g floats in a bath of liquid. The temperature is raised, and the sphere begins to sink at 35°C. If the density of the liquid is 1.527 at 0°C, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.
15. A mercury thermometer is to be made with glass tubing of internal bore 0.5 mm diameter and the distance between the fixed point is to be 20 cm. Estimate the volume of the bulb below the lower fixed point, given that the coefficient of cubical expansion of mercury is 0.00018/K and the coefficient of linear expansion of glass is 0.000009/K.

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16. On a Celsius thermometer the distance between the readings 0°C and 100°C is 30 cm and the area of cross section of the narrow tube containing mercury is 15×10^{-4} cm^2. Find the total volume of mercury in the thermometer at 0°C. α of glass = 9×10^{-6}/K and the coefficient of real expansion of mercury = 18×10^{-5}/K.

17. The height of a mercury column measured with a brass scale, which is correct and equal to H_0 at 0°C, is H_1 at t°C? The coefficient of linear expansion of brass is α and the coefficient of volume expansion of mercury is γ. Relate H_0 and H_1.

18. A glass bulb contains air and mercury. What fraction of the bulb must be occupied by mercury if the volume of air in the bulb is to remain constant at all temperatures? The coefficient of linear expansion of glass is 9×10^{-6}/K and the coefficient of expansion of mercury is 1.8×10^{-4}/K.

19. When composite rod is free, composite length increases to 2.002 m when temperature increases from 20°C to 120°C. When composite rod is fixed between the support, there is no change in component length. Find γ and α of steel if $\gamma_{cu} = 1.5 \times 10^{13}$ N/m^2 $\alpha_{cu} = 1.6 \times 10^{-5}$/°C.

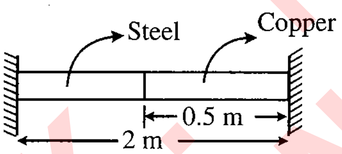

Fig. 1.21

TRANSMISSION OF HEAT

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any of the following modes.

Conduction	Convection	Radiation
Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place	Each particle absorbing heat is mobile	Heat flows without any intervening medium in the form of electromagnetic waves
Medium is necessary for conduction	Medium is necessary for convection	Medium is not necessary for radiation
It is a slow process	It is also a slow process	It is a very fast process
Path of heat flow may be zig-zag	Path may be zig-zag or curved	Path is a straight line
Conduction takes place in solids	Convection takes place in fluids	Radiation takes place in gaseous and transparent media
The temperature of the medium increases through which heat flows	In this process also the temperature of medium increases	There is no change in the temperature of the medium

Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other without dislocation of the particles from their equilibrium position is called conduction.

i. Conduction is a process which is possible in all states of matter.

ii. In solids only conduction takes place.

iii. In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules; therefore, they are poor conductors.

iv. In metallic solids free electrons carry the heat energy; therefore, they are good conductors of heat.

1. Variable and steady state: When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end. In the process of conduction each cross section of the rod receives heat from the adjacent cross section towards the hot end. A part of this heat is absorbed by the cross section itself whose temperature increases, another part is lost into atmosphere by convection and radiation and the rest is conducted away to the next cross section.

Because in this state temperature of every cross section of the rod goes on increasing; hence, rod is said to exist in variable state.

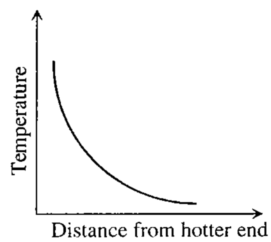

Fig. 1.22

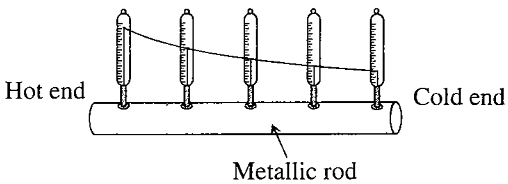

Fig. 1.23

After some time, a state is reached when the temperature of every cross section of the rod becomes constant. In this state, no heat

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is absorbed by the rod. The heat that reaches any cross section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection and radiation. This state of the rod in which no part of rod absorbs heat is called steady state.

2. Isothermal surface: Any surface (within a conductor) having its all points at the same temperature is called isothermal surface. The direction of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

i. If the material is rectangular or cylindrical rod, the isothermal surface is a plane surface.

ii. If a point source of heat is situated at the centre of a sphere the isothermal surface will be spherical.

iii. If steam passes along the axis of the hollow cylinder, heat will flow through the walls of the cylinder so that in this condition the isothermal surface will be cylindrical.

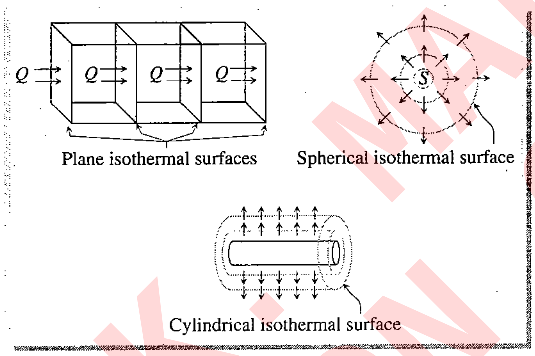

Fig. 1.24

3. Temperature gradient: The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

If the temperature of two isothermal surfaces be θ and $(\theta - \Delta\theta)$ and the perpendicular distance between them be Δx, then temperature gradient $= \dfrac{(\theta - \Delta\theta) - \theta}{\Delta x} = \dfrac{-\Delta\theta}{\Delta x}$

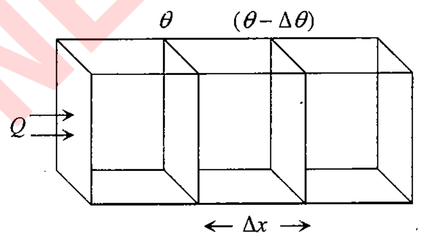

Fig. 1.25

The negative sign shows that temperature θ decreases as the distance x increases in the direction of heat flow.

Unit: K/m (SI); dimension: $[L^{-1}\,\theta]$

4. Coefficient of thermal conductivity: If L be the length of the rod, A the area of cross section and θ_1 and θ_2 be the temperatures of its two faces, then the amount of heat flowing from one face to the other face in time t is

$$Q \propto A$$
$$\propto (\theta_1 - \theta_2)$$
$$\propto t$$
$$\propto \frac{1}{l}$$

In combined form

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{l}$$

or

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

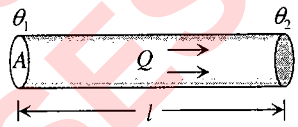

Fig. 1.26

where K is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.

This relation can also be expressed as

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

If $A = 1$ m^2, $(\theta_1 - \theta_2) = 1$°C, $t = 1$ s and $l = 1$ m, then
$Q = K$.

Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length 1 m and cross section 1 m^2 with a unit temperature difference between the opposite faces.

i. Units: cal/cm-s°C (in CGS), kcal/m-s-K (in MKS) and W/m-K (in SI)

ii. Dimension: $[MLT^{-3}\theta^{-1}]$

iii. The magnitude of K depends only on nature of the material.

iv. For perfect conductors, $K = \infty$ and for perfect insulators,

$$K = 0$$

v. Substances in which heat flows quickly and easily are known as good conductors of heat. They possess large thermal conductivity due to large number of free electrons. Example: silver, brass, etc.

vi. Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. Example: glass, wood, etc.

vii. The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

viii. Human body is a bad conductor of heat (but it is a good conductor of electricity).

Illustration 1.47 **A refrigerator door is 150 cm high, 80 cm wide, and 6 cm thick. If the coefficient of conductivity is 0.0005 cal/cm s°C, and the inner and outer surfaces are at 0°C and 30°C, respectively, what is the heat loss per minute through the door, in calories?**

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Sol. Apply the equation of thermal conductivity

$$Q = \frac{kA(t_h - t_c)(\text{time})}{d} = \frac{0.005(150\times 80)(30^\circ - 0^\circ)(60)}{4}$$
$$= 1800 \text{ cal}$$

Illustration 1.48 **An ordinary refrigerator is thermally equivalent to a box of corkboard 90 mm thick and 5.6 m² in inner surface area. When the door is closed, the inside wall is kept, on the average, 22.2°C below the temperature of the outside wall. If the motor of the refrigerator runs 15% of the time while the door is closed, at what rate must heat be taken from the interior while the motor is running? The thermal conductivity of corkboard is $k = 0.05$ W/mK.**

Sol. Consider a time interval Δt during which the door is closed. As approximation, take the heat conduction to be steady over Δt.

Then the rate of heat into the box is

$$\frac{\Delta Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right) = (0.05)(5.6)\left(\frac{22.2}{0.090}\right) = 69.1 \text{ W}$$

To remove this heat, the motor must, since it runs only for a time $(0.15)\ \Delta t$, cause heat to leave at the rate $69.1/0.15 = 460$ W.

Illustration 1.49 **Water is being boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000 cm² and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and outer surface of the bottom. K for the material of kettle is 0.5 cal/°C/s/cm, and the latent heat of steam is 540 cal/g.**

Sol. Mass of steam produced $= \frac{dm}{dt} = \frac{1}{60}$ g/s

Heat transferred per second

$$= \frac{dH}{dt} = L\frac{dm}{dt} \Rightarrow \frac{dH}{dt} = 540 \times \frac{1}{60} \text{ cal/s} = 9 \text{ cal/s}$$

Area = 3000 cm²; $K = 0.5$ cal/°C/s/cm

θ = temperature difference

d = thickness = 2 mm = 0.2 cm

$$\frac{dH}{dt} = \frac{KA\theta}{d} \Rightarrow L\frac{dm}{dt} = \frac{KA\theta}{d}$$

$$\Rightarrow 9 = \frac{0.5\times 3000\times \theta}{0.2} \Rightarrow \theta = 1.2\times 10^{-3}\,^\circ\text{C}$$

Illustration 1.50 **A closed cubical box made of perfectly insulating material has walls of thickness 8 cm and the only way for the heat to enter or leave the box is through the solid, cylindrical, metallic plugs each of cross-sectional area 12 cm² and length 8 cm fixed in the opposite walls of the box as shown in Fig. 1.27. The outer surface A is kept at 100°C while the outer surface B of other plug is kept at 4°C. K of the material of the plugs is 0.5 cal/s/C/cm. A source of energy generating 36 cal/s is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is same at all points on the inner surface.**

Sol. Let θ be the temperature of inner surface of box.

Heat transfer per second through A + heat produced by source per second = Heat transfer per second through B

$$\Rightarrow \left(\frac{dH}{dt}\right)_A + 36 \text{ cal/s} = \left(\frac{dH}{dt}\right)_B$$

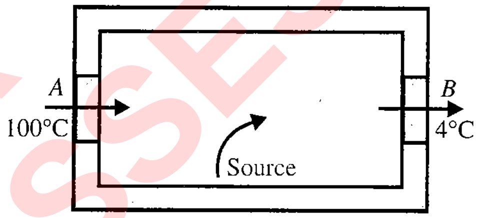

Fig. 1.27

$$\Rightarrow \frac{KA(100-\theta)}{d} + 36 = \frac{KA(\theta-4)}{d}$$

$$\Rightarrow KA(\theta - 4 - 100 + \theta) = 36\times d$$

Now, $d = 8$ cm, $A = 12$ cm^2, $K = 0.5$ cal/s/°C/cm.

$$\Rightarrow 2\theta - 104 = \frac{36\times 8}{12\times 0.5} \quad \Rightarrow = 76^\circ\text{C}$$

Illustration 1.51 **Two metal cubes A and B of same size are arranged as shown in Fig. 1.28. The extreme ends of the combination are maintained at the identical temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C, respectively. After steady state is reached, what will be the temperature T of the interface?**
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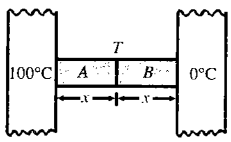

Fig. 1.28

Sol. In steady state, rate of flow of heat through A = Rate of flow of heat through B.

or $\quad K_1A\left(\frac{100-T}{x}\right) = K_2A\frac{(T-0)}{x} \quad$ or $\quad 300 - 3T = 2T$

$\therefore \quad T = 60^\circ$C

5. Relation between temperature gradient and thermal conductivity: In steady state, rate of flow of heat

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx} = -KA \text{ (temperature gradient)}$$

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If $\frac{dQ}{dt}$ is constant then temperature gradient $\propto \frac{1}{K}$.

Temperature difference between the hot end and the cold end in steady state is inversely proportional to K, i.e., in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where $K = \infty$, temperature difference in steady state will be zero.

6. Thermometric conductivity or diffusivity: It is a measure of rate of change of temperature (with time) when the body is not in steady state (i.e., in variable state).

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

Thermal capacity per unit volume $= \frac{mc}{V} = \rho c$ (as ρ is density of substance)

$$\therefore \qquad \text{Diffusivity } (D) = \frac{K}{\rho c}$$

Unit: m²/s; dimension: $[L^2T^{-1}]$

7. Thermal resistance: The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= rate of flow of heat).

Now, temperature difference $= (\theta_1 - \theta_2)$ and heat current, $H = \frac{Q}{t}$

$\therefore$ Thermal resistance

$$R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{Q/t} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$$

Unit: °C × s/cal or K × s/kcal; dimension:

$$[M^{-1}L^{-2}T^3\theta]$$

COMBINATION OF CONDUCTORS

1. Series combination: Let n slabs each of cross-sectional area A, lengths $l_1, l_2, l_3, \ldots, l_n$ and conductivities $K_1, K_2, K_3, \ldots, K_n$, respectively, be connected in the series.

Heat current is the same in all the conductors.

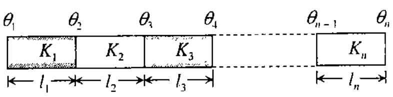

Fig. 1.29

i.e.,
$$\frac{Q}{t} = H_1 = H_2 = H_3 \cdots = H_n$$

$$\frac{K_1A(\theta_1 - \theta_2)}{l_1} = \frac{K_2A(\theta_2 - \theta_3)}{l_2} = \frac{K_3A(\theta_{-3} - \theta_{-4})}{l_3} = \cdots$$

$$= \frac{K_nA(\theta_{n-1} - \theta_n)}{l_n}$$

i. Equivalent resistance $R = R_1 + R_2 + R_3 + \cdots + R_n$

ii. If K_s is equivalent conductivity, then from relation $R = \frac{l}{KA}$

$$\frac{l_1 + l_2 + l_3 + \cdots + l_n}{K_s} = \frac{l_1}{K_1A} + \frac{l_2}{K_2A} + \frac{l_3}{K_3A} + \cdots + \frac{l_n}{K_nA}$$

$$\therefore \quad K_s = \frac{l_1 + l_2 + l_3 + \cdots + l_n}{\frac{l_1}{k_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} + \cdots + \frac{l_n}{K_n}}$$

iii. Equivalent thermal conductivity for n slabs of equal length

$$K = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \cdots + \frac{1}{K_n}}$$

For two slabs of equal length, $K = \frac{2K_1K_2}{K_1 + K_2}$

iv. Temperature of interface of composite bar: Let the two bars be arranged in series as shown in Fig. 1.30.

Then heat current is same in the two conductors.

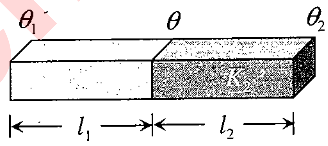

Fig. 1.30

i.e.,
$$\frac{Q}{t} = \frac{K_1A(\theta_1 - \theta)}{l_1} = \frac{K_2A(\theta - \theta_2)}{l_2}$$

By solving we get
$$\theta = \frac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

If $(l_1 = l_2 = l)$ then $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

2. Parallel combination: Let n slabs each of length l, areas $A_1, A_2, A_3, \ldots, A_n$ and thermal conductivities $K_1, K_2, K_3, \ldots, K_n$ be connected in parallel. Then

i. Equivalent resistance $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$

ii. Temperature gradient across each slab will be same.

iii. Heat current in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. i.e., $H = H_1 + H_2 + H_3 + \cdots + H_n$

$$\frac{K(A_1 + A_2 + A_3 + \cdots + A_n)(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1A_1(\theta_1 - \theta_2)}{l} + \frac{K_2A_2(\theta_1 - \theta_2)}{l} + \frac{K_3A_3(\theta_1 - \theta_2)}{l}$$

$$+ \cdots + \frac{K_nA_n(\theta_1 - \theta_2)}{l}$$

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$$\therefore \qquad K = \frac{K_1A_1 + K_2A_2 + K_3A_3 + \cdots + K_nA_n}{A_1 + A_2 + A_3 + \cdots + A_n}$$

For n slabs of equal area $K = \dfrac{K_1 + K_2 + K_3 + \cdots + K_n}{n}$

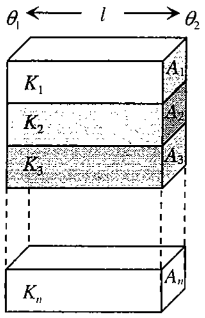

Fig. 1.31

Equivalent thermal conductivity for two slabs of equal area

$$K = \frac{K_1 + K_2}{2}$$

Electrical Analogy for Thermal Conduction

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

Electrical conduction	Thermal conduction
Electric charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge is called the electric current, i.e., $I = \dfrac{dq}{dt}$	The rate of flow of heat may be called heat current i.e., $H = \dfrac{dQ}{dt}$
The relation between the electric current and the potential difference is given by Ohm's law, i.e., $I = \dfrac{V_1 - V_2}{R}$ where R is the electrical resistance of the conductor	Similarly, the heat current may be related with the temperature difference as $H = \dfrac{\theta_1 - \theta_2}{R}$ where R is the thermal resistance of the conductor
The electrical resistance is defined as $R = \dfrac{\rho l}{A} = \dfrac{l}{\sigma A}$, where ρ = resistivity and σ = electrical conductivity	The thermal resistance may be defined as $R = \dfrac{l}{KA}$, where K = Thermal conductivity of conductor
$\dfrac{dq}{dt} = I = \dfrac{V_1 - V_2}{R} = \dfrac{\sigma A}{l}(V_1 - V_2)$	$\dfrac{dQ}{dt} = H = \dfrac{\theta_1 - \theta_2}{R} = \dfrac{KA}{l}(\theta_1 - \theta_2)$

Illustration 1.52 **Three cylindrical rods A, B and C of equal lengths and equal diameters are joined in series as shown in Fig. 1.32. Their thermal conductivities are 2 K, K and 0.5 K, respectively. In steady state, if the free ends of rods A and C are at 100°C and 0°C, respectively, calculate the temperature at the two junction points. Assume negligible loss by radiation through the curved surface. What will be the equivalent thermal conductivity?**

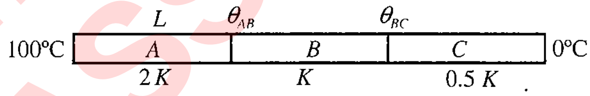

Fig. 1.32

Sol. As the rods are in series, $R_{eq} = R_A + R_B + R_C$ with $R = (L/KA)$

i.e.,
$$R_{eq} = \frac{L}{2KA} + \frac{L}{KA} + \frac{L}{0.5KA} = \frac{7L}{2KA} \qquad \text{(i)}$$

And hence,
$$H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(100-0)}{(7L/2KA)} = \frac{200KA}{7L}$$

Now in series, rate of flow of heat remains same, i.e., $H = H_A = H_B = H_C$.

So for rod A,
$$\left[\frac{dQ}{dt}\right]_A = \left[\frac{dQ}{dt}\right]$$

i.e.,
$$\frac{(100-\theta_{AB})2KA}{L} = \frac{200KA}{7L}$$

or,
$$\theta_{AB} = 100 - (100/7) = (600/7) = 87.7°\text{C}$$

And for rod C,
$$\left[\frac{dQ}{dt}\right]_C = \left[\frac{dQ}{dt}\right]$$

i.e.,
$$\frac{(\theta_{BC} - 0) \times 0.5KA}{L} = \frac{200KA}{7L}$$

or,
$$\theta_{BC} = (400/7) = 57.1°\text{C}$$

Furthermore, if K_{eq} is equivalent thermal conducitivity,

$$R_{eq} = \frac{L+L+L}{K_{eq}A} = \frac{7L}{2KA} \qquad \text{[from Eq. (i)]}$$

i.e.,
$$K_{eq} = (6/7)K$$

Illustration 1.53 **Two walls of thickness in the ratio 1 : 3 and thermal conductivities in the ratio 3 : 2 form a composite wall of a building. If the free surfaces of the wall**

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be at temperatures 30°C and 20°C, respectively, what is the temperature of the interface?

Sol. 1st method: If the temperature difference across the first wall be ΔT°C, then that across the second wall will be $(10 - \Delta T)$°C (note).

In the steady state, the rate of heat flow across the two walls will be the same.

So, considering an area A normal to the flow of heat,

$$\frac{dQ}{dt} = \frac{K_1 A(\Delta T)}{l_1} = \frac{K_2 A(10-\Delta T)}{l_2}$$

where $K_1 = K$; $K_2 = 3K$ and $l_1 = 3l$; $l_2 = 2l$

$$\frac{KA\Delta T}{3l} = \frac{3KA(10-\Delta T)}{2l} \Rightarrow 2\Delta T = 9(10-\Delta T)$$

$$\Delta T = 8.18°\text{C}$$

So, the temperature of the interface will be

$$30 - \Delta T = 30 - 8.18 = 21.82°\text{C}$$

2nd method: Using the equation, temperature of the interface

$$T = \frac{(K_1 T_1 / l_1 + K_2 T_2 / l_2)}{(K_1 / l_1 + K_2 / l_2)}$$

We have $$T = \left(\frac{K(30)}{3l} + \frac{3K(20)}{2l}\right) \Big/ \left(\frac{K}{3l} + \frac{3K}{2l}\right)$$

$$= \frac{(10+30)}{\left(\frac{1}{2}+\frac{3}{2}\right)} = 21.82°\text{C}$$

3rd method: $R_1 = \frac{l_1}{K_1 A} = \frac{3l}{KA}$ and $R_2 = \frac{l_2}{K_2 A} = \frac{2l}{3KA}$

Effective thermal resistance $R = R_1 + R_2 = \frac{11}{3}\frac{l}{KA}$

Now, thermal current $i = \frac{\Delta T}{R} = \frac{30-20}{\frac{11l}{3KA}} = \frac{30KA}{11l}$

Also, thermal current through the first slab will be

$i = \frac{(30-T)}{R_1}$ (where T = temperature of the interface)

i.e., $$\frac{30KA}{11l} = \frac{(30-T)KA}{3l} \Rightarrow 90 = 330 - 11T$$

$$\Rightarrow T = 21.82°\text{C}$$

Illustration 1.54 **One end of a uniform brass rod 15 cm long and 20 cm² cross-sectional area is kept at 100°C. The other end is at prefect contact with an iron rod of identical cross section, but length 8 cm. The lateral surface of the composite rod is surrounded by a heat insulator and the free end of the iron rod is kept in ice at 0°C. If 684 g of ice melts in 1 h, determine the thermal conductivity of iron. Thermal conductivity of brass = 0.25 cal/s-cm°C and latent heat of fusion of ice = 80 cal/g.**

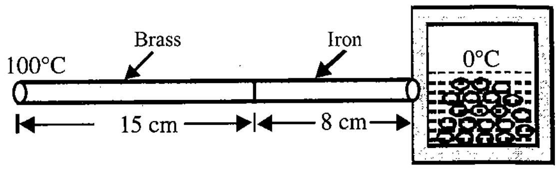

Fig. 1.33

Sol. Let the thermal conductivity of iron be K = cal/s-cm-°C. The thermal resistance of brass rod

$$R_1 = \frac{l_1}{K_1 A} = \frac{15\text{ cm}}{\frac{0.25\text{ cal}}{\text{s-cm°C}} \times 20\text{ cm}^2} = 3\text{ s-°C/cal}$$

And that of iron rod

$$R_2 = \frac{l_2}{K_2 A} = \frac{8\text{ cm}}{\frac{K\text{ cal}}{\text{s-cm°C}} \times 20\text{ cm}^2} = \frac{2}{5K}\text{ s-°C/cal}$$

Since, the two rods are arranged in series, their effective thermal resistance is given by

$$R = R_1 + R_2 = \left(3 + \frac{2}{5K}\right)\text{s-°C/cal}$$

Now, rate of heat flow through the rods $= \frac{\Delta Q}{\Delta t}$

But $$= \frac{(684\text{ g})}{3600\text{ s}}\left(80\frac{\text{cal}}{\text{g}}\right) = 15.2\text{ cal/s}$$

Since $$\frac{\Delta Q}{\Delta t} = \frac{(T_1 - T_2)}{R} \quad \therefore R = \frac{(T_1 - T_2)}{(\Delta Q / \Delta t)}$$

$$\left(3 + \frac{2}{5K}\right)\text{s°C/cal} = \frac{(100-0)°\text{C}}{15.2\text{ cal/s}}$$

$$K = 111.8 \times 10^{-3}\text{ cal/s-cm-°C}$$

Illustration 1.55 **An electric heater is used in a room of total wall area 150 m² to maintain a temperature of 30°C inside it, when the outside temperature is –5°C. The innermost layer is of wood of thickness 2.0 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 20 cm. Find the power of the electric heater. Assume heat to flow only through the walls. The thermal conductivities of wood, cement and brick are 0.150, 0.175 and 1.0 W/m°C, respectively.**

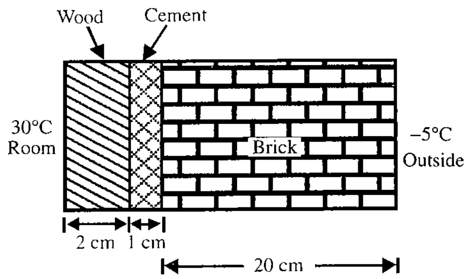

Fig. 1.34

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Sol. Evidently, the three layers (wood, cement and brick) are arranged in series.

So, the equivalent thermal resistance (R) of the combination will be

$$R = R_1 + R_2 + R_3$$

$$= \frac{1}{150\ \text{m}^2}\left[\frac{2\times10^{-2}\ \text{m}^2\,°\text{C}}{0.15\ \text{W}} + \frac{1\times10^{-2}\ \text{m}^2\,°\text{C}}{1.75\ \text{W}} + \frac{20\times10^{-2}\ \text{m}^2\,°\text{C}}{1.0\ \text{W}}\right]$$

$= 2.26\times10^{-3}$ °C/W

From, rate of heat loss $\frac{dQ}{dt} = \frac{\Delta T}{R}$, we have

$$\frac{dQ}{dt} = \frac{30-(-5)°\text{C}}{2.26\times10^{-3}°\text{C/W}} = 15.49\ \text{kW}$$

Since the heat lost from the inside of the room to the outside through the walls is compensated by the room heater, so the power of the heater is 15.49 kW.

DETERMINATION OF THERMAL CONDUCTIVITY

1. Ingen-Hausz experiment: Ingen-Hausz devised an experiment to compare the thermal conductivities of the metals. If $l_1, l_2, \ldots$, are the lengths of wax melted on the metal rods, then the ratio of thermal conductivities is

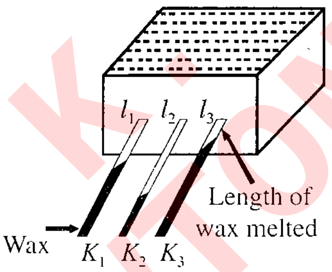

Fig. 1.35

$$K_1 : K_2 : K_3 : \ldots = l_1^2 : l_2^2 : l_3^2 : \ldots$$

2. Searle's experiment: It is a method of determination of K of a metallic rod. Here we are not much interested in the detailed description of the experimental setup. We will only understand its essence, which is the essence of solving many numerical problems.

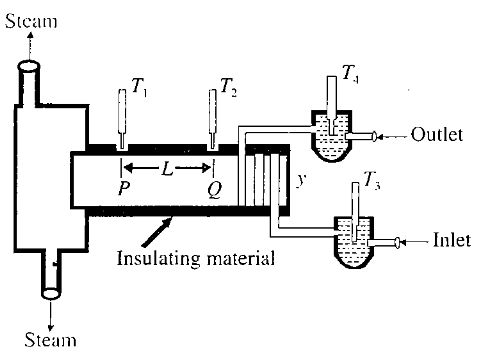

Fig. 1.36

In this experiment a temperature difference ($T_1 - T_2$) is maintained across a rod of length L and area of cross section A. If the thermal conductivity of the material of the rod is K, then the amount of heat transmitted by the rod from the hot end to the cold end in time t is given by

$$Q = \frac{KA(T_1 - T_2)t}{L} \quad \text{(i)}$$

In Searle's experiment, this heat reaching the other end is utilized to raise the temperature of certain amount of water flowing through pipes circulating around the other end of the rod. If temperature of the water at the inlet is T_3 and at the outlet is T_4, then the amount of heat absorbed by water is given by

$$Q = mc(T_4 - T_3) \quad \text{(ii)}$$

where m is the mass of the water which has absorbed this heat and temperature is raised and c is the specific heat of the water

Equating (i) and (ii), K can be determined, i.e.,

$$K = \frac{mc(T_4 - T_3)l}{A(T_1 - T_2)t}$$

Note: ***In numerical problems we may have the situation where the amount of heat travelling to the other end may be required to do some other work, e.g., it may be required to melt the given amount of ice. In that case Eq. (i) will have to be equated to mL.***

i.e.,

$$mL = \frac{KA(T_1 - T_2)t}{l}$$

GROWTH OF ICE ON LAKE

Water in a lake starts freezing if the atmospheric temperature drops below 0°C. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature be $-\theta$ °C. The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt is

$$dQ_1 = \frac{KA[0-(-\theta)]dt}{y}$$

Now, suppose the thickness of ice layer increases by dy in time dt, due to escaping of above heat. Then

$$dQ_2 = mL = \rho(dy\,A)L$$

As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho L y)$

So, the time taken by ice to grow to a thickness y is

$$t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\rho L}{2K\theta}y^2$$

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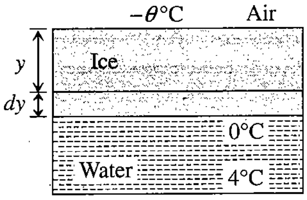

Fig. 1.37

If the thickness is increased from y_1 to y_2 then time taken

$$t=\frac{\rho L}{K\theta}\int_{y_1}^{y_2} y\,dy=\frac{\rho L}{2K\theta}(y_2^2-y_1^2)$$

i. Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

ii. Ice is a poor conductor of heat; therefore, the rate of increase of thickness of ice on ponds decreases with time.

iii. It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2, \text{ ie., } t_1 : t_2 : t_3 :: 1 : 4 : 9$$

iv. The time intervals to change the thickness from 0 to y, from y to $2y$ and so on will be in the ratio

$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2):(2^2 - 1^2):(3^2 : 2^2)$; $\Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$

CONVECTION

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.

1. **Natural convection:** This arise due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try setting down. It mostly occurs on heating a liquid/fluid.

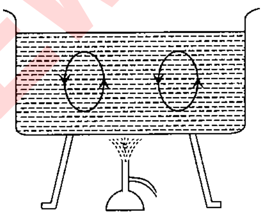

Fig. 1.38

2. **Forced convection:** If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection.

RADIATION

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum.

For example, the heat from the sun reaches the earth through radiation.

Properties of Thermal Radiation

1. The wavelength of thermal radiations ranges from 7.8×10^{-7} m to 4×10^{-4} m. They belong to infrared region of the electromagnetic spectrum. That is why thermal radiations are also called *infrared radiations*.

Radiation	Frequency	Wavelength
Cosmic rays	$>10^{21}$ Hz	$<10^{-13}$ m
Gamma rays	$10^{18} - 10^{21}$ Hz	$10^{-13} - 10^{-10}$ m
X-rays	$10^{16} - 10^{19}$ Hz	$10^{-11} - 10^{-8}$ m (0.1 Å – 100 Å)
Ultraviolet rays	$7.5 \times 10^{14} - 2 \times 10^{6}$ Hz	$1.4 \times 10^{-8} - 4 \times 10^{-7}$ m (140 Å – 4000 Å)
Visible rays	$4 \times 10^{14} - 7.5 \times 10^{14}$ Hz	$4 \times 10^{-7} - 7.8 \times 10^{-7}$ m (4000 Å – 7800 Å)
Infrared rays (Heat)	$3 \times 10^{11} - 4 \times 10^{14}$ Hz	$7.8 \times 10^{-7} - 10^{-3}$ (7800 Å – 3×10^{5} Å)
Micro-waves	$3 \times 10^{8} - 3 \times 10^{11}$ Hz	10^{-3} m – 0.1 m
Radio waves	$10^{4} - 3 \times 10^{9}$ Hz	0.1 m – 10^{4} m

2. Medium is not required for the propagation of these radiations.
3. They produce sensation of warmth in us but we can't see them.
4. Everybody whose temperature is above 0 K emits thermal radiation.
5. Their speed is equal to that of light, i.e, ($= 3 \times 10^8$ m/s).
6. Their intensity is inversely proportional to the square of distance of point of observation from the source (i.e., $I \propto 1/d^2$).
7. Just as light waves, they follow laws of reflection, refraction, interference, diffraction and polarization.
8. When these radiations fall on a surface, they exert pressure on that surface which is known as radiation pressure.
9. While travelling these radiations travel just like photons of other electromagnetic waves. They manifest themselves as heat only when they are absorbed by a substance.

Reflectance, Absorptance and Transmittance

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

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1. Reflectance or reflecting power (r): It is defined as the ratio of the amount of thermal radiations reflected (Q_r) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

2. Absorptance or absorbing power (a): It is defined as the ratio of the amount of thermal radiations absorbed (Q_a) by the body in a given time to the total amount of thermal radiatons incident on the body in that time.

3. Transmittance or transmitting power (t): It is defined as the ratio of the amount of thermal radiations transmitted (Q_t) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

From the above definitions $r = \frac{Q_r}{Q}$, $a = \frac{Q_a}{Q}$ and $t = \frac{Q_t}{Q}$. By adding we get

$$r + a + t = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = \frac{(Q_r + Q_a + Q_t)}{Q} = 1$$

$$\therefore \quad r + a + t = 1$$

a. r, a and t all are the pure ratios; so, they have no unit and dimension.

b. For perfect reflector, $r = 1$, $a = 0$ and $t = 0$.

c. For perfect absorber, $a = 1$, $r = 0$ and $t = 0$ (perfectly black body).

d. For perfect transmitter, $t = 1$, $a = 0$ and $r = 0$.

e. If body does not transmit any heat radiation, $t = 0$ $\therefore r + a = 1$ or $a = 1 - r$.

So, if r is more, a is less and vice versa. It means good reflectors are bad absorbers.

4. Monochromatic emittance or spectral emissive power: For a given surface it is defined as the radiant energy emitted per second per unit area of the surface with in a unit wavelength around λ.

$$\text{Spectral emissive power } (E_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$$

Unit: $\frac{\text{J}}{\text{m}^2 \times \text{s} \times \text{Å}}$; dimension: $[ML^{-1}T^{-3}]$

5. Total emittance or total emissive power: It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$E = \int_0^\infty E_\lambda d\lambda$$

Unit: $\frac{\text{J}}{\text{m}^2 \times \text{s}}$ or $\frac{\text{Watt}}{\text{m}^2}$; dimension: $[MT^{-3}]$

6. Monochromatic absorptance or spectral absorptive power: It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unitless quantity. It is represented by a_λ.

7. Total absorptance or total absorpting power: It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

It is also unitless and dimensionless quantity.

8. Emissivity (e): Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body ($E_{\text{practical}}$) to the total emissive power of a perfect black body (E_{black}) at that temperature.

$$e = \frac{E_{\text{practical}}}{E_{\text{black}}}$$

$e = 1$ for perfectly black body but for practical bodies emissivity (e) lies between zero and one ($0 < e < 1$).

9. Perfectly black body: A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it. As a perfectly black body neither reflects nor transmits any radiation, the absorptance of a perfectly black body is unity, i.e., $t = 0$ and $r = 0$, therefore, $a = 1$.

We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.) it emits all possible radiation so it is an example of black body.

10. Laboratory equivalent of black body: A perfectly black body can't be realised in practice. The nearest example of an ideal black body is the Ferry's black body. It is a doubled walled evacuated spherical cavity whose inner wall is blackened. There is a fine hole in it. All the radiations incident upon this hole are absorbed by this black body after successive reflections. If this black body is heated to high temperature then it emits radiations of all wavelengths.

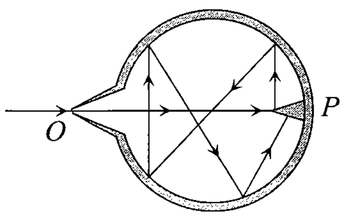

Fig. 1.39

PREVOST'S THEORY OF HEAT EXCHANGE

1. Everybody emits heat radiations at all finite temperatures (Except 0 K) as well as it absorbs radiations from the surroundings.
2. Exchange of energy along various bodies takes place via radiation.
3. The process of heat exchange among various bodies is a continuous phenomenon.
4. If the amount of radiation absorbed by a body is greater than that emitted by it then the temperature of body increases and it appears hotter.
5. If the amount of radiation absorbed by a body is less than that emitted by it, then the temperature of the body decreases and consequently the body appears colder.
6. If the amount of radiation absorbed by a body is equal to that emitted by the body, then the body will be in thermal equilibrium and the temperature of the body remains constant.
7. At absolute zero temperature (0 K or –273°C) this law is not applicable because at this temperature the heat exchange among various bodies ceases.

KIRCHOFF'S LAW

The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

When thermal radiation falls on a material surface three things may happen to it.

i. A certain amount of heat E_R will be reflected.
ii. A certain amount of heat E_A will be absorbed.
iii. A certain amount of heat E_T will be transmitted.

Thus the total amount of incident energy E_I is divided between these three parameters; therefore, we can write

$$E_I = E_R + E_A + E_T \qquad \text{(i)}$$

For a shiny surface such as silver, E_R is large and both E_T and E_A are small. For a block surface such as charcoal E_R is small, E_A is larger and E_T is small. For a transparent medium like glass E_R is small, E_A small and E_T large.

The energy absorbed by a body can be emitted later or simultaneously along with absorption, and clearly if a surface cannot absorb radiation strongly, it will be unable to emit strongly. Kirchhoff's law states that a good emitter is also a good absorber.

When thermal radiation is incident upon a body or emitted by a body equally in all directions, the radiation is said to be isotropic. Some of the radiation incident on a body may be absorbed, reflected and transmitted. In general, the incident radiation of all wavelengths that is absorbed depends on the temperature and the nature of the surface of the absorbing body. The fraction of the total incident power that is absorbed by a body is called its absorptivity. It can be given as

$$\text{Absorptivity} = \frac{\text{Total power absorbed by a body}}{\text{Total power incident on the body}}$$

When a body is in thermal equilibrium, the processes of absorption and emission of radiant power are equal and opposite. So, the total emissivity is equal to total absorptivity. Total emissivity is defined as the fraction of power provided to a general body that is emitted through a material surface as thermal radiation; here, 'total' includes all the wavelengths of electromagnetic radiation from the body. Practically it is easier to measure emissivity than absorptivity.

The emissivity of a body depends on both the temperature and the nature of emitting surface. Bodies of same temperature and size but different material emit different amount of total thermal radiation. According to Kirchoff's law, good absorbers are good emitters. For theoretical purpose we define an ideal substance capable of either absorbing all the thermal radiation falling on it or emitting all the energy provided to it in the form of thermal radiation. Such a substance is called black body.

Thus for a black body we can say that its emissivity as well as its absorptivity is 1. We can compare different substances with black body and obtain their emissivity. For example, a polished shining piece of steel emits only 9% of the thermal radiation as compared to a black body of same size and shape of same temperature; hence, its emissivity is 0.09. Similarly, emissivity of a rough oxidized steel surface is 0.81 and that of ocean water is 0.96 and some substances like lamp black, coal are there whose emissivity is very nearly unity that is these bodies behave as ideal emitters. The emissivity of a general body is denoted by e and mathematically defined as

$$e = \frac{\text{Radient power emitted by body at a given temperature}}{\text{Radient power emitted by a black body of same geometry at same temperature}}$$

Thus, the total thermal radiation power emitted by a general body is given as

$$\begin{pmatrix}\text{Radiation power}\\ \text{from a general body}\\ \text{at a given temperature}\end{pmatrix} = e\begin{pmatrix}\text{Radiation power}\\ \text{from a}\\ \text{black body}\\ \text{at same temperature}\end{pmatrix} \qquad \text{(ii)}$$

Experimentally a very good approximation to a black body is provided by a cavity enclosed by high temperature opaque walls regardless of the composition of the material of its interior walls.

Thus, if $a_{\text{practical}}$ and $E_{\text{practical}}$ represent the absorptive and emissive power of a given surface, while a_{black} and E_{black} for a perfectly black body, then according to law

$$\frac{E_{\text{practical}}}{a_{\text{practical}}} = \frac{E_{\text{black}}}{a_{\text{black}}}$$

But for a perfectly black body $a_{\text{black}} = 1$ so

$$\frac{E_{\text{practical}}}{a_{\text{practical}}} = E_{\text{black}}$$

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If emissive and absorptive powers are considered for a particular wavelength λ,

$$\left(\frac{E_\lambda}{a\lambda}\right)_{\text{practical}} = (E_\lambda)_{\text{black}}$$

Now since $(E\lambda)_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

Applications of Kirchoff's Law

1. Sand is rough black, so it is a good absorber and hence in deserts, days (when radiation from the sun is incident on sand) will be very hot. Now in accordance with Kirchoff's law, good absorber is a good emitter so nights (when sand emits radiation) will be cold. This is why days are hot and nights are cold in desert.
2. Sodium vapours, on heating, emit two bright yellow lines. These are called D_1, D_2 lines of sodium. When continuos white light from an arc lamp is made to pass through sodium vapours at low temperature, the continuous spectrum is intercepted by two dark lines exactly in the same places as D_1 and D_2 lines. Hence sodium vapours when cold absorbs the same wavelength as they emit while hot. This is in accordance with Kirchoff's law.
3. When a shining metal ball having some black spots on its surface is heated to a high temperature and is seen in dark, the black spots shine brightly and the shining ball becomes dull or invisible. The reason is that the black spots on heating absorb radiation and so emit these in dark while the polished shining part reflects radiations and absorb nothing and so does not emit radiations and becomes invisible in the dark.
4. When a green glass is heated in furnace and taken out, it is found to glow with red light. This is because red and green are complementary colours. At ordinary temperatures, a green glass appears green, because it transmits green colour and absorb red colour strongly. According to Kirchoff's law, this green glass on heating must emit the red colour, which is absorbed strongly. Similarly, when a red glass is heated to a high temperature it will glow with green light.
5. Kirchoff's law also explains the existence of Fraunhoffer lines. These are some dark lines observed in the otherwise continuous spectrum of the sun. According to Fraunhoffer, the central portion of the sun, called photosphere, is at a very high temperature and emits continuous light of all wavelengths. Before reaching us, the light passes through outer portion of the sun, called chromosphere. The chromosphere has some terrestrial elements in vapour form at lower temperature than that of photosphere. These elements absorb those wavelength which they would emit while hot. These absorbed wavelengths which are missing appear as dark lines in the spectrum of the sun and are called Fraunhoffer's lines.

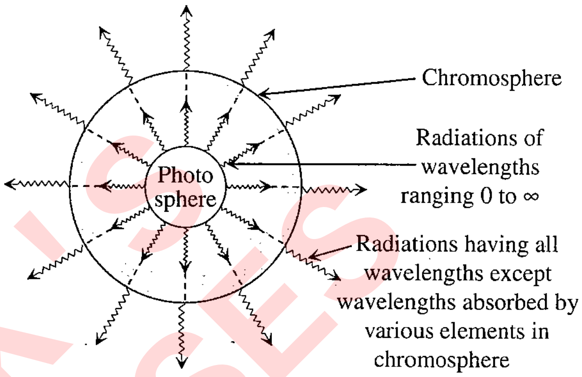

Fig. 1.40

6. A person with black skin experiences more heat and more cold as compared to a person of white skin because when the outside temperature is greater, the person with black skin absorbs more heat and when the outside temperature is less the person with black skin radiates more energy.

Illustration 1.56 **An electric heater of surface area 200 cm² emits radiant energy of 60 kJ at time interval of 1 min. Determine its emissive power. If its emissivity be 0.45, what would be the radiant energy emitted by a black body in one hour, identical to the electrical heater in all respects?**

Sol. Given $\Delta Q = 60$ kJ; $A = 200 \times 10^{-4}$ m² and $\Delta t = 60$ s

Using, emissive power $E = \left(\frac{\Delta Q}{\Delta t}\right)\frac{1}{A}$, we have

$$E = \frac{60\text{ kJ}}{200\times 10^{-4}\text{ m}^2 \times 60\text{ s}} = 50\text{ kW/m}^2$$

∴ Emissivity (of a surface)

$$e = \frac{\text{emissive power (of that surface)}}{\text{emissive power (of a black body)}}$$

∴ Emissive power of a black body

$$= \frac{50}{0.45}\text{ kW/m}^2 = 111.11\text{ kW/m}^2$$

∴ Heat radiated $\Delta Q = EA\Delta t$

$$= \left(111.11\frac{\text{kW}}{\text{m}^2}\right)(200\times 10^{-4}\text{ m}^2)(60\times 60\text{ s}) = 8000\text{ kJ}$$

STEFAN'S LAW

According to it the radiant energy emitted by a perfectly black body per unit area per second (i.e. emissive power of black body) is directly proportional to the fourth power of its absolute temperature, i.e.,

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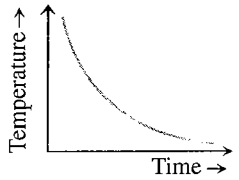
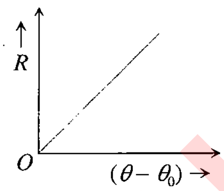
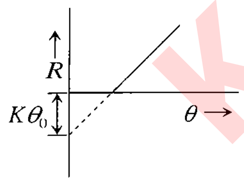
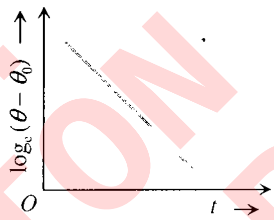
Then Newton's law of cooling becomes

$$\left[\frac{\theta_1-\theta_2}{t}\right]=K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

This form of law helps in solving numerical problems.

5. Cooling curves:

Curve between temperature of body θ and time.  **Fig. 1.43** $\theta - \theta_0 = Ae^{kt}$, which indicates temperature decreases exponentially with increasing time.	Curve between rate of cooling (R) and temperature difference between body (θ) and surrounding (θ_0)  **Fig. 1.45** $R \propto (\theta - \theta_0)$. This is a straight line passing through origin.
Curve between the rate of cooling (R) and body temperature (θ).  **Fig. 1.44** $R=K(\theta-\theta_0)=K\theta-K\theta_0$ This is a straight line intercept R-axis at $-K\theta_0$	Curve between $\log(\theta - \theta_0)$ and time  **Fig. 1.46** As $\frac{d\theta}{dt}\propto-(\theta-\theta_0)\Rightarrow\frac{d\theta}{(\theta-\theta_0)}=-Kdt$ Integrating $\log_e(\theta-\theta_0)=-Kt+C$ $\log_e(\theta-\theta_0)=-Kt+\log_e A$ This is a straight line with negative slope.

6. Determination of specific heat of a liquid: If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation), then rate of loss of heat and fall in temperature of both will be same.

i.e., $$\left(\frac{dQ}{dt}\right)_{water}=\left(\frac{dQ}{dt}\right)_{liquid}$$

$$(ms+W)\frac{(\theta_1-\theta_2)}{t_1}=(m_1s_1+W)\frac{(\theta_1-\theta_2)}{t_2}$$

or $\left[\frac{ms+W}{t_1}\right]=\left[\frac{m_1s_1+W}{t_2}\right]$ (where W = water equivalent of calorimeter)

If densities of water and liquid are ρ and ρ', respectively, then $m = V\rho$ and $m' = V\rho'$.

Illustration 1.63 **Two solid copper spheres of radii r_1 = 15 cm and r_2 = 20 cm are both at a temperature of 60°C. If the temperature of surrounding is 50°C, then find**

a. the ratio of the heat loss per second from their surfaces initially.
b. the ratio of rates of cooling initially.

Sol.
a. Ratio of heat loss $=\frac{H_1}{H_2}=\frac{(dQ/dt)_1}{(dQ/dt)_2}\Rightarrow\frac{H_1}{H_2}=\frac{KA_1(60-50)}{KA_2(60-50)}$ by using Newton's law of cooling

$$\frac{H_1}{H_2}=\frac{A_1}{A_2}=\frac{r_1^2}{r_2^2}=\left(\frac{15}{20}\right)^2=\frac{9}{16}$$

b. The ratio of initial rates of cooling $=\frac{(d\theta/dt)_1}{(d\theta/dt)_2}$

we have $\frac{H_1}{H_2}=\left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{M_1s(d\theta/dt)_1}{M_2s(d\theta/dt)_2}=\left(\frac{r_1}{r_2}\right)^2$

As the spheres have the same densities, the ratio of their masses is equal to the ratio of their volumes.

$$\Rightarrow\frac{(d\theta/dt)_1}{(d\theta/dt)_2}=\left(\frac{r_1}{r_2}\right)^2\frac{M_2}{M_1}=\left(\frac{r_1}{r_2}\right)^2\left(\frac{r_2}{r_1}\right)^3=\frac{r_2}{r_1}=\frac{20}{15}=\frac{4}{3}$$

Illustration 1.64 **Two identical spheres A and B are suspended in an air chamber which is maintained at a temperature of 50°C. Find the ratio of the heat lost per second from the surface of the spheres if**

a. A and B are at temperatures 60°C and 55°C, respectively.
b. A and B are at temperatures 250°C and 200°C, respectively.

Sol. Net heat loss per second per unit area $= e\sigma(T^4 - T_0^4)$ from Stefan's Law.

If $T - T_0$ is small as compared to the temperature of surroundings, we have

Net heat loss per second per unit area = (constant) × $(T - T_0)$. From Newton's law of cooling

a. Here the temperature difference is small and hence we can use Newton's law of cooling.

$$E_A \propto (60 - 50) \quad \text{and} \quad E_B \propto (55 - 50)$$

Hence, $$\frac{E_A}{E_B}=\frac{10}{5}=2$$

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b. As the temperature difference is not negligible as compared to the temperature of surrounding, we use Stefan's law for accurate answer.

$$E_A \propto (250+273)^4-(50+273)^4$$

$$\Rightarrow E_B \propto (200+273)^4-(50+273)^4$$

Hence, $$\frac{E_A}{E_B}=\frac{523^4-323^4}{423^2-323^4}=1.632$$

Illustration 1.65 **A body cools down from 60°C to 55°C in 30 s. Using Newton's law of cooling, calculate the time taken by same body to cool down from 55°C to 50°C. Assume that the temperature of surrounding is 45°C.**

Sol. Assume that a body cools down from temperature θ_i to θ_f in t seconds, and θ_c is the temperature of surroundings. Applying Newton's law of cooling,

According to Newton's law of cooling

$$\frac{d\theta}{dt}=-K(\theta-\theta_0);\quad \theta_0=45°\text{C};\quad \int\frac{d\theta}{\theta-45}=-\int dt$$

(K is constant)

from $t=0$ s to $t=30$ s, θ changes from 60°C to 55°C.

$$\int_{55}^{50}\frac{d\theta}{\theta-45}=-K\int_{30}^{t}dt\quad\Rightarrow \ln\left(\frac{50-45}{60-45}\right)=-K(30-0)\qquad\text{(i)}$$

$$\int_{55}^{50}\frac{d\theta}{\theta-45}=-K\int_{30}^{t}dt\quad\Rightarrow \ln\left(\frac{50-45}{60-45}\right)=-K(t-30)\qquad\text{(ii)}$$

Divide Eq. (ii) by Eq. (i) to set:

$$\frac{t-30}{30}=\frac{\ln\frac{50-45}{55-45}}{\ln\frac{55-45}{60-45}}\quad\Rightarrow\frac{t-30}{30}=\frac{\ln 2}{\ln 3/2}$$

$$\Rightarrow t=30\left(1+\frac{\ln 2}{\ln 3/2}\right)=81.28\text{ s}$$

time from θ = 55°C to θ = 50°C is $(t-30)$ = (81.28–30) = 51.28 s.

Illustration 1.66 **A body cools from 60°C to 50°C in 10 min. Find its temperature at the end of next 10 min if the room temperature is 25°C. Assume Newton's law of cooling holds.**

Sol. According to Newton's law of cooling

$$\frac{d\theta}{dt}=-k(\theta-\theta_0)$$

or $\qquad \log(\theta-\theta_0)=-kt+c$

Given at $\qquad t=0,\ \theta=60°\text{C}$

$\therefore\qquad \log(60-25)=c \quad$ or $\quad kt=\log 35-\log(\theta-25)$

$\therefore\qquad \log(\theta-25)=-kt+\log 35$

or $\qquad kt=\log 35-\log(\theta-25)$

At $\qquad t=10$ min $\theta=50°$C

$$\therefore k10=\log 35-\log 25$$

$$\Rightarrow k=\frac{1}{10}\log\frac{7}{5}$$

$$\therefore\quad \left(\frac{1}{10}\log\frac{7}{5}\right)t=\log 35-\log(\theta-25)$$

When $\qquad t=20$ min θ = ?

$$\therefore\quad \left(\frac{1}{10}\log\frac{7}{5}\right)20=\log\frac{35}{\theta-25}$$

$$\Rightarrow \log\left(\frac{7}{5}\right)^2=\log\frac{35}{\theta-25}\quad\Rightarrow\left(\frac{7}{5}\right)^2=\frac{35}{\theta-25}$$

$$\Rightarrow \theta=42.8°\text{C}$$

Illustration 1.67 **A thin brass rectangular sheet of sides 15.0 and 12.0 cm is heated in a furnace to 600°C and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.250? Neglect heat loss due to convection (Stefan–Boltzmann constant, $\sigma=5.67\times10^{-8}$ W/m² – k^4).**

Sol. Area of the both sides of the plate

$$A=2\times(15.0)\times(12.0)\times10^{-4}\text{ m}^2=3.60\times10^{-2}\text{ m}^2$$

The energy radiated by the plate

$$=e\sigma AT^4=0.250\times5.67\times10^{-8}\times3.60\times10^{-2}\times(600+273)^4$$

$$=5.10\times10^{-12}\times873^4=296.4\text{ W}$$

Illustration 1.68 **A hot body placed in air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being k times the temperature difference from the surrounding. Starting from $t=0$, find the time in which the body will lose half the maximum heat it can lose.**

Sol. We have,

$$-\frac{dT}{dt}=k(T-T_0)$$

where T_0 is the temperature of the surrounding. If T_1 is the initial temperature and T is the temperature at any time t, then

$$\int_{T_1}^{T}\frac{dT}{(T-T_0)}=-k\int_0^t dt$$

or $$\left|\ln(T-T_0)\right|_{T_1}^{T}=-kt\quad\text{or}\quad \ln\left[\frac{T-T_0}{T_1-T_0}\right]=-kt$$

or $$T=T_0+(T_1-T_0)e^{-kt}\qquad\text{(i)}$$

The body continues to lose heat till its temperature becomes equal to that of the surrounding. The loss of heat

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$$Q = mc(T_1 - T_0)$$

If the body lose half of the maximum loss that it can, then decrease in temperature

$$\frac{Q}{2} = mc\left(\frac{T_1 - T_0}{2}\right)$$

If body loses this heat in time t, then its temperature at time t' will be

$$T_1 - \left(\frac{T_1 - T_0}{2}\right) = \frac{T_1 + T_0}{2}$$

Putting these values in Eq. (i), we have

$$\frac{T_1 - T_0}{2} = T_0 + (T_1 - T_0)e^{-kt'}$$

or $\quad \dfrac{T_1 - T_0}{2} = (T_1 - T_0)e^{-kt'} \quad$ or $\quad e^{-kt'} \dfrac{1}{2} \quad$ or $\quad t' = \dfrac{\ln 2}{k}$

DISTRIBUTION OF ENERGY IN THE SPECTRUM OF BLACK BODY

Langley and later on Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths in the thermal spectrum of a black body radiation. The results obtained are shown in Fig. 1.47. From these curves it is clear that

1. At a given temperature energy is not uniformly distributed among different wavelengths.
2. At a given temperature intensity of heat radiation increases with wavelength, reaches a maximum at a particular wavelength and with further increase in wavelength it decreases.
3. With increase in temperature wavelength λ_m corresponding to most intense radiation decreases in such a way that $\lambda_m \times T =$ constant [Wien's law].
4. For all wavelengths an increase in temperature causes an increase in intensity.
5. The area under the curve $= \int E_\lambda d\lambda$ will represent the total intensity of radiation at a particular temperature. This area increases with rise in temperature of the body. It is found to be directly proportional to the fourth power of absolute temperature of the body, i.e.,

$$E = \int E_\lambda d\lambda \propto T^4 \qquad \text{[Stefan's law]}$$

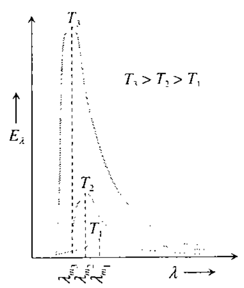

Fig. 1.47

WIEN'S DISPLACEMENT LAW

When a body is heated it emits radiations of all wavelengths. However, the intensity of radiations of different wavelengths is different.

According to Wien's law the product of wavelengths corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, i.e., $\lambda_m T = b =$ constant where b is Wien's constant and has value 2.89×10^{-3} m − K.

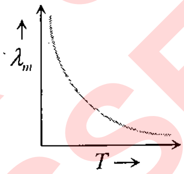

Fig. 1.48

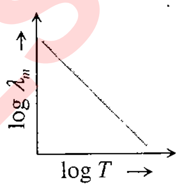

Fig. 1.49

This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m, the temperature of the star $T (= b/\lambda_m)$ is determined.

Illustration 1.69 **The spectral energy distribution of the sun has a maximum at 4753 Å. If the temperature of the sun is 6050 K, what is the temperature of a star for which this maximum is at 9506 Å?**

Sol. Given $\lambda_m = 4753$ Å, $T = 6050$ K

$\lambda'_m = 9506$ Å

If T' is the temperature of star, then

$$\lambda_m T = \lambda'_m T' \quad \text{or} \quad T' = \frac{\lambda_m T}{\lambda_m'} = \frac{4753 \times 6050}{9506} = 3025 \text{ K}$$

Illustration 1.70 **The solar radiation spectrum reveals that the intensity corresponding to a wavelength of 4750 Å is maximum. Estimate the surface temperature of the sun. (Given Wien's constant = 2.89×10^{-3} m − K)**

Sol. From Wien's displacement law, the temperature T of a body corresponding to maximum intensity wavelength λ_m is given by

$$T = \frac{b}{\lambda_m}$$

$$T = \frac{2.89 \times 10^{-3} \text{m} - \text{K}}{4750 \times 10^{-10} \text{m}} = 6084 \text{ K}$$

This temperature corresponds to the chromosphere (surface) of the sun.

Illustration 1.71 **If the filament of a 100 W bulb has an area 0.25 cm² and behaves as a perfect black body. Find the wavelength corresponding to the maximum in its energy distribution. Given that Stefan's constant is σ = 5.67 × 10^{-8} J/m² s K⁴.**

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Sol. In the bulb filament given, the energy radiated per second per m^2 of its surface area is given as

$$E = \frac{P}{A} = \frac{100}{0.25 \times 10^{-4}} = 4 \times 10^6 \text{ J/s-m}^2$$

If T is the temperature of the filament then according to Stefan's law, we have

$$E = \sigma T^4$$

or $$4 \times 10^6 = 5.67 \times 10^{-8} \times T^4$$

or $$T^4 = \frac{4 \times 10^6}{5.67 \times 10^{-8}} = 7.055 \times 10^{13}$$

or $$T = [7.055 \times 10^{13}]^{1/4} = 2898.14 \text{ K}$$

If the filament radiates the maximum energy at a wavelength λ_m, from Wein's displacement law, we have

$$\lambda_m T = b$$

or $$\lambda_m = \frac{b}{T}$$

$$= \frac{2.89 \times 10^{-3}}{2898.14} = 9971.9 \text{ Å}$$

TEMPERATURE OF THE SUN AND SOLAR CONSTANT

If R is the radius of the sun and T its temperature, then the energy emitted by the sun per second through radiation in accordance with Stefan's law will be given by

$$P = eA\sigma T^4 = 4\pi R^2 \sigma T^4$$

$$[\text{for sun } e = 1]$$

In reaching earth this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

i.e., $$T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5 \times 10^8}{7 \times 10^5}\right)^2 \times \frac{1.4 \times 10^3}{5.67 \times 10^{-8}}\right]^{1/4} \simeq 5800 \text{ K}$$

As $$r = 1.5 \times 10^8 \text{ km}, R = 7 \times 10^5 \text{ km}, S = 2\frac{\text{cal}}{\text{cm}^2 \text{ min}} = 1.4\frac{\text{kW}}{\text{m}^2}$$

and $$\sigma = 5.67 \times 10^{-8} \frac{W}{\text{m}^2\text{K}^4}$$

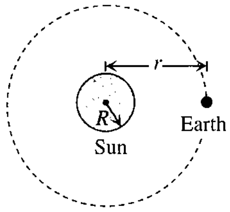

Fig. 1.50

This result is in good agreement with the experimental value of temperature of sun, i.e., 6000 K.

The difference in the two values is attributed to the fact that sun is not a perfectly black body.

Illustration 1.72 **The intensity of solar radiation just outside the earth's atmosphere is measured to be 1.4 kW/m². If the radius of the sun 7×10^8 m, while the earth–sun distance is 150×10^6 km, then find**

i. the intensity of solar radiation at the surface of the sun,
ii. the temperature at the surface of the sun assuming it to be a black body,
iii. the most probable wavelength in solar radiation.

Sol. Assuming the sun to be a 'blackbody' at a temperature T_0, we can write,

W = intensity of solar radiation on the sun's surface = σT_0^4,

where σ is the Stefan–Boltzmann constant.

i. The radiation emitted from the solar surface per unit time is spread over the surface of a sphere having a radius equal to earth–sun distance where it is received on the earth (just outside the atmosphere)

$$W \times 4\pi R_S^2 = I_0 \times 4\pi D_{SE}^2$$

where D_{SE} is the distance between the sun and the earth, and I_0 is the intensity outside the earth's atmosphere.

$$I_0 = W \times \left(\frac{R_S}{D_{SE}}\right)^2$$

Now, $$R_S = 7 \times 10^8 \text{ m}, D_{SE} = 150 \times 10^9 \text{ m}$$

and $$I_0 = 1.4 \times 10^3 \text{ W/m}^2$$

$$\therefore \quad 1.4 \times 10^3 = W \times \left(\frac{7 \times 10^8}{150 \times 10^9}\right)^2 = W \times \frac{49}{225} \times 10^{-4}$$

or $$W = 6.4 \times 10^7 \text{ W/m}^2$$

ii. Assuming the sun to be a black body,

$$6.4 \times 10^7 = \sigma T_0^4 = (5.67 \times 10^{-8}) T_0^4$$

$$\therefore \quad T_0^4 = \frac{6.4}{5.67} \times 10^{15}$$

or $$T_0 \approx 0.58 \times 10^4 \text{ K} = 5800 \text{ K}$$

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iii. Using Wien's displacement law,

$$\lambda_{mp}T_0 = 0.29 \text{ cm-K} = 2.9\times10^{-3}\text{ m-K}$$

or $$\lambda_{mp} = \frac{2.0\times10^{-3}}{5800} = 5\times10^{-7}\text{ m} = 5000\text{ Å}$$

Note: *λ_{mp} is also referred to as λ_{max}.*

Illustration 1.73 **The earth receives solar energy at the rate of 2 cal cm² per min. Assuming the radiation to be black body in character, estimate the surface temperature of the sun. Given that σ = 5.67 × 10^{-8} W/m²/K⁴ and angular diameter of the sun = 32 min of arc.**

Sol. Let surface temperature of sun be T_s. Then total energy radiated by Sun per second is given as

$$E = \sigma T_s^4 \cdot (4\pi R_s^2)$$

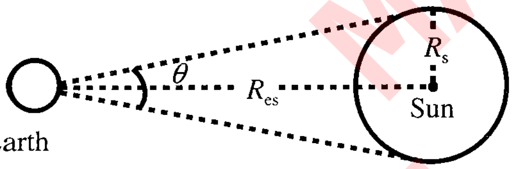

Fig. 1.51

Energy received by earth per second per square metre is give as

$$\frac{dQ}{dt} = \frac{E}{4\pi R_{es}^2} = \sigma T_s^4\left(\frac{R_s}{R_{es}}\right)^2 \quad \text{(i)}$$

It is given that angular diameter of sun as observed from earth is 32 min of arc. Thus, we have

$$\frac{R_s}{R_{es}} = \frac{\theta}{2} = \frac{1}{2}\times\frac{32}{60} = 4.655\times10^{-3}$$

and it is given that $$\frac{dQ}{dt} = \frac{2\times4.2\times10^4}{60}\text{ J/s-m}^2 \quad \text{(ii)}$$

Now from Eqs. (i) and (ii), we have

$$5.67\times10^{-8}\times T_s^4\times[4.655\times10^{-3}]^2 = \frac{2\times4.2\times10^4}{60}$$

or $$T_s^4 = \frac{2\times4.2\times10^4}{60\times5.67\times10^{-8}\times(4.655\times10^{-3})^2}$$

or $$T_s^4 = 1.14\times10^{15}$$

or $$T_s = 5810.67\text{ K}$$

Concept Application Exercise 1.3

1. Explain why the surface of a lake freezes first.
2. In Newton's law of cooling in the form
$$\frac{d\Delta\theta}{dt} = -k\Delta\theta$$
what factors does the constant k depend upon? What are the dimensions of k?
3. A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45°C and coefficient of thermal conductivity of thermocole = 0.01 J/s m-°C. Given heat of fusion of water = 335 × 10³ J/kg.
4. A brass boiler has a base area of 0.15 m² and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min, when placed on a gas. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 J/s-°C and heat of vapourization of water = 2256 J/g.
5. An electric heater is used in a room of total wall area 137 m² to maintain a temperature of +20°C inside it, when the outside temperature is −10°C. The walls have three different layers of materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m-°C, respectively.
6. An indirectly heated filament is radiating maximum energy of wavelength 2.16 × 10^{-5} cm. Find the net amount of heat energy lost per second per unit area; the temperature of the surrounding air is 13°C. Given b = 0.288 cm-K. σ = 5.77 × 10^{-5} erg/s-cm²-K⁴.
7. A uniform copper bar 100 cm long is insulated on side, and has its ends exposed to ice and steam respectively. If there is a layer of water 0.1 mm thick at each end, calculate the temperature gradient in the bar. K_{Cu} = 1.04 and K_{water} = 0.0014 in CGS units.
8. Two rods A and B of same length and cross-sectional area are connected in series and a temperature difference of 100°C is maintained across the combination as shown in Fig. 1.52. If the thermal conductivity of the rod A is 3 k and that of rod B is k, then

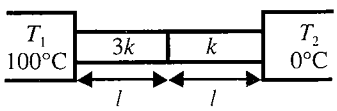

Fig. 1.52

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i. determine the thermal resistance of each rod.
ii. determine the heat current flowing through each rod.
iii. determine the heat current flowing through each rod.
iv. plot the variation of temperature along the length of the rod.

9. Two conductors A and B given in previous problem are connected in parallel as shown in Fig. 1.53.

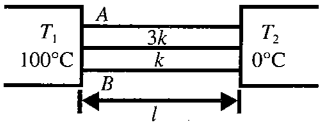

Fig. 1.53

i. Determine the equivalent thermal resistance.
ii. Determine the heat current in each rod.

10. A sphere, a cube and a thin circular pate are heated to the same temperature. If they are made of same material and have equal masses, determine which of these three objects cools the fastest and which one cools the slowest?

11. One end of a brass rod of length 2.0 m and cross section 1 cm² is kept in stream at 100°C and the other end in ice at 0°C. The lateral surface of the rod is covered by heat insulator. Determine the amount of ice melting per minute. Thermal conductivity of brass is 110 W/m-K and specific latent heat of fusion of ice is 80 cal/g.

12. Three rods AB, BC and BD having thermal conductivities in the ratio 1 : 2 : 3 and lengths in the ratio 2 : 1 : 1 are joined as shown in Fig. 1.54. The ends A, C and D are at temperatures T_1, T_2 and T_3, respectively. Find the temperature of the junction B. Assume steady state.

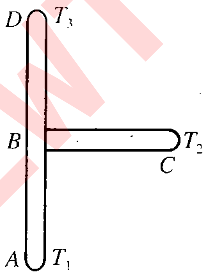

Fig. 1.54

13. A cylinder is made up of two coaxial layers, one of radius R and the other of radius $2R$. The inner and outer portions are respectively made up of substances of thermal conductivities K and $2K$. Determine the effective thermal conductivity between the flat ends of the cylinder.

14. A cylinder of radius R and length l is made up of a substance whose thermal conductivity K varies with the distance x from the axis as $K = K_1x + K_2$. Determine the effective thermal conductivity between the flat faces of the cylinder.

15. A cube and a sphere of equal edge and radius, made of the same substance are allowed to cool under identical conditions. Determine which of the two will cool at a faster rate.

16. A spherical ball of radius 1 cm coated with a metal having emissivity 0.3 is maintained at 1000 K temperature and suspended in a vacuum chamber whose walls are maintained at 300 K temperature. Find rate at which electrical energy is to be supplied to the ball to keep its temperature constant.

17. A body emits maximum energy at 4253 Å and the same body at some other temperature emits maximum energy at 2342 Å. Find the ratio of the maximum energy radiated by the body in a short wavelength range.

18. A black body at 1500 K emits maximum energy of wavelength 20000 Å. If sun emits maximum energy of wavelength 5500 Å, what would be the temperature of sun.

Solved Examples

Example 1.1 **A calorimeter contains 400 g of water at a temperature of 5°C. Then, 200 g of water at a temperature of +10°C and 400 g of ice at a temperature of –60°C are added. What is the final temperature of the contents of calorimeter?**

Specific heat capacity of water = 1000 cal/kg/K
Specific latent heat of fusion of ice = 80 × 1000 cal/kg
Relative specific heat of ice = 0.5

Sol. Let t be the common temperature above zero celsius.
Heat lost by calorimeter and water added

$$= 400 \times 10^{-3} \times 1000\,(5 - t) + 200 \times 10^{-3} \times 1000\,(10 - t)$$

Heat gained by ice $= 400 \times 10^{-3} \times 5000\,(60 + t) + 400 \times 10^{-3} \times 80000 + 400 \times 10^{-3} \times 1000t$

Since Heat lost = heat gained

$\therefore\ 400\,(5 - t) + 200\,(10 - t) = 200\,(60 + t) + 400 \times 80 + 400t$

or $t = -24.33°\text{C}$

This is contradictory to the assumption that t is above zero.
Hence, the final temperature is either 0°C or less than 0°C.
Let the temperature be t°C (less than 0°C).

Then $400 \times 10^{-3} \times 1000 \times 5 + 200 \times 10^{-3} \times 1000 \times 10 + 600 \times 10^{-3} \times 80000 + 600 \times 10^{-3} \times 500 \times t = 400 \times 500\,(60 - t)$

or $400 \times 5 + 200 \times 10 + 600 \times 80 + 300t = 200\,(60 - t)$

or $20 + 20 + 480 + 3t = 120 - 2t$

or $t = -80°\text{C}$

This is absurd because the lowest temperature is –60°C and we discard this second assumption also.
So the final temperature is 0°C.
Let x grams of water be converted into ice.

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Heat lost = $400 \times 10^{-3} \times 100 \times 5 + 200 \times 10^{-3} \times 1000 \times 10 + x \times 10^{-3} \times 8000 = 4000 + 80x$

$$\text{Heat gained} = 400 \times 10^{-3} \times 500 \times 60$$

or $$4000 + 80x = 12000$$

or $$x = 100 \text{ g}$$

Hence, the final result is 500 g of ice and 500 g of water at 0°C.

Example 1.2 **In an insulated vessel, 25 g of ice at 0°C is added to 600 g of water at 18.0°C. a. What is the final temperature of the system? b. How much ice remains when the system reaches equilibrium?**

Sol:

a. If all 250 g of ice is melted it must absorb energy

$$Q_f = mL_f = (0.250 \text{ kg})(3.33 \times 10^5 \text{ J/kg}) = 83.3 \text{ kJ}$$

The energy released when 600 g of water cools from 18.0°C to 0°C is

$$|Q| = |mc\Delta T| = (0.600 \text{ kg})(4186 \text{ J/kg°C})(18.0\text{°C}) = 45.2 \text{ kJ}$$

Since the energy required to melt 250 g of ice at 0°C exceeds the energy released by cooling 600 g of water from 18.0°C to 0°C, not all the ice melts and the final temperature of the system (water + ice) must be 0°C.

b. The originally warmer water will cool all the way to 0°C, so it loses 45.2 kJ to ice. This energy lost by the water will melt a mass of ice m, where $Q = mL_f$.

Solving for the mass $m = \dfrac{Q}{L_f} = \dfrac{45.2 \times 10^3 \text{ J}}{3.33 \times 10^5 \text{ J/kg}} = 0.136 \text{ kg}$

Therefore, the ice remaining is $m' = 0.250 \text{ kg} - 0.136 \text{ kg} = 0.114 \text{ kg}$

Step by step reasoning is essential for solving a problem like this.

Example 1.3 **A solar cooker consists of a curved reflecting surface that concentrates sunlight onto the object to be warmed. The solar power per unit area reaching the Earth's surface at the location is 6 W/m². The cooker faces the sun and has a face diameter of 0.600 m. Assume 40.0% of the incident energy is transferred to 0.500 L of water in an open container, initially at 20.0°C. Over what time interval does the water completely boil away? (Ignore the heat capacity of the container.)**

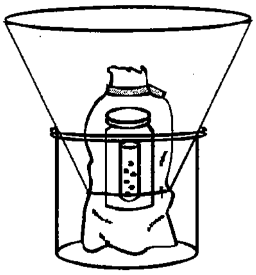

Fig. 1.55

Sol. If we point the axis of the reflecting surface toward the sun, the power incident on the solar collector is

$$P_i = IA = (600 \text{ W/m}^2)[\pi(0.300 \text{ m})^2] = 170 \text{ W}$$

For a 40.0% efficient reflector, the collected power is $P = (0.400)(170 \text{ W}) = 67.9 \text{ W} = 67.9 \text{ J/s}$

The total energy required to increase the temperature of the water to the boiling point and to evaporate it is

$$Q = mc\Delta T + mL_v$$

$$Q = (0.500 \text{ kg})(4186 \text{ J/kg°C})(80.0\text{°C}) + (0.500 \text{ kg})(2.26 \times 10^6 \text{ J/kg})$$

$$Q = 1.30 \times 10^6 \text{ J}$$

The time required is

$$\Delta t = \frac{Q}{P} = \frac{1.30 \times 10^6 \text{ J}}{67.9 \text{ J/s}} = 1.91 \times 10^4 \text{ s} = 5.31 \text{ h}$$

Example 1.4 **An iron wire AB of length 3 m at 0°C is stretched between the opposite walls of a brass casing at 0°C. The diameter of the wire is 0.6 mm. What extra tension will be set up in the wire when the temperature of the system is raised to 40°C?**

Given $\alpha_{\text{brass}} = 18 \times 10^{-6}$/K

$\alpha_{\text{iron}} = 12 \times 10^{-6}$/K

$Y_{\text{iron}} = 21 \times 10^{10}$ N/m²

Sol.

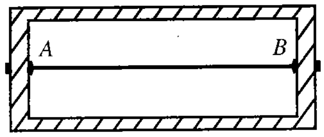

Fig. 1.56

Increase in length of the iron wire

$$= l\alpha\ \Delta t = l \times 12 \times 10^{-6} \times 40$$

$$= 48l \times 10^{-5} \text{ m}$$

Increase in length of the brass tube

$$= l\alpha\ \Delta t = l \times 18 \times 10^{-6} \times 40$$

$$= 72l \times 10^{-5} \text{ m}$$

Since the increase in length of the brass tube is greater than that of the iron wire, the latter will be tightened. Let T be the tension in the wire. Then

$$\text{Stress} = \frac{T}{\pi r^2} = \frac{T}{\pi(3 \times 10^{-4})^2} = \frac{T}{9\pi \times 10^{-8}}$$

$$\text{Strain} = \frac{(72l - 48l) \times 10^{-5}}{l} = 24 \times 10^{-5}$$

Since stress = Y × strain, $\dfrac{T}{9\pi \times 10^{-8}} = 21 \times 10^{10} \times 24 \times 10^{-5}$

$$T = 21 \times 10^{10} \times 24 \times 10^{-5} \times 9\pi \times 10^{-8} = 14.2 \text{ N}$$

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Example 1.5 **A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same area of cross section. At 25°C, the composite rod is 1 m long and the copper rod is 30 cm long. At 125°C the length of the composite rod increases by 1.91 mm. When the composite rod is prevented from expanding by holding it between two rigid walls, it is found that the constituent rods have remained unchanged in length in spite of rise of temperature. Find Young's modulus and the coefficient of linear expansion of the second rod (Y of copper = 1.3×10^{10} N/m² and α of copper = 17×10^{-6}/K).**

Sol. Since total expansion = expansion of the constituent rods

$$1.91 \times 10^{-3} = 0.3 \times 17 \times 10^{-6} \times (125 - 25) + 0.7 \times \alpha \times (125 - 25)$$

or $$1.91 \times 10^{-3} = 5.1 \times 10^{-4} + 70\alpha$$

or $$70\alpha = (1.91 - 0.51) \times 10^{-3} = 1.4 \times 10^{-3}$$

or $$\alpha = 2 \times 10^{-5}/\text{K}$$

When prevented from expanding thermal expansion is accompanied by elastic contraction. Since the lengths remain unchanged, thermal expansion is equal to elastic contraction.

$$\therefore \quad \text{Strain} = \frac{l\alpha\Delta t}{l} = \alpha\Delta t = 17 \times 10^{-6} \times 100 = 17 \times 10^{-4}$$

and $$\text{stress} = Y \times \text{strain} = 1.3 \times 10^{10} \times 17 \times 10^{-4} = 22.1 \times 10^{6}$$

For the second rod

$$\text{Strain} = \alpha\Delta t = 2 \times 10^{-5} \times 100 = 2 \times 10^{-3}$$

$$\text{Stress} = Y \times \text{strain} = Y \times 2 \times 10^{-3}$$

But the same stress is effective throughout the composite rod.

$$\therefore \quad Y \times 2 \times 10^{-3} = 22.1 \times 10^{6} \quad \text{or} \quad Y = 11.1 \times 10^{9}\ \text{N/m}^2$$

Example 1.6 **How should 1 kg of water at 50°C be divided in two parts such that if one part is turned into ice at 0°C, it would release sufficient amount of heat to vapourize the other part. Given that latent heat of fusion of ice is 3.36×10^5 J/kg, latent heat of vapourization of water is 22.5×10^5 J/kg and specific heat of water is 4200 J/kg K.**

Sol. Let x kg of water be frozen. Then the amount of heat it releases is

$$Q_1 = x \times 4200 \times 50 + x \times 3.36 \times 10^5\ \text{J} = x \times 5.46 \times 10^5\ \text{J}$$

The heat required to vapourize the $(1 - x)$ kg of water from 50°C is

$$Q_2 = (1 - x) \times 22.5 \times 10^5$$

Here, we have taken heat required to vapourize the water as only mass × latent heat of vapourization and not the heat required to first raise the temperature of $(1 - x)$ kg of water from 50° to 100° plus the mass × latent heat similar as when heat is supplied to water from an external source. It first reaches 100°C and then its vapourization starts, but when heat is taken by water itself, it vapourizes (evaporation) at 50°C as in this case. The similar case we see in our general life in cooling of water in a pitcher, drying of clothes hanging in open air, etc.

Thus heat Q_2 must be provided by first part of water. We have

$$Q_2 = Q_1$$

$$(1 - x) \times 22.5 \times 10^5 = x \times 5.46 \times 10^5$$

or $$22.5 - 22.5x = 5.46x \quad \text{or} \quad x = \frac{22.5}{25.06} = 0.812\ \text{kg}$$

Example 1.7 **A layer of ice of 0°C of thickness x_1 is floating on a pond. If the atmospheric temperature is $-T$°C, show that the time taken for thickness of the layer of ice to increase from x_1 to x_2 is given by**

$$t = \frac{\rho L}{2kT}\left(x_2^2 - x_1^1\right)$$

where ρ is the density of ice, k its thermal conductivity and L is the latent heat of fusion of ice.

Sol. When the temperature of the air is less than 0°C, the cold air near the surface of the pond takes heat (latent) from the water which freezes in the form of layers. Consequently, the thickness of the ice layer keeps increasing with time. Let x be the thickness of the ice layer at a certain time. If the thickness is increased by dx in time dt, then the amount of heat flowing through the slab in time dt is given by (see Fig. 1.57)

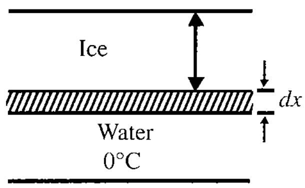

Fig. 1.57

$$Q = \frac{kA[0-(T)]dt}{x} = \frac{kATdt}{x} \qquad \text{(i)}$$

where A is the area of the layer of ice and $-T$°C is the temperature of the surrounding air. If dm is the mass of water frozen into ice, then $Q = dm \times L$. But $dm = A\rho\,dx$, where ρ is the density of ice. Hence

$$Q = A\rho L dx \qquad \text{(ii)}$$

Equating (i) and (ii), we have

$$\frac{kATdt}{x} = A\rho L dx \quad \text{or} \quad dt = \frac{\rho L}{kT} x dx$$

Integrating, we have

$$\int_0^t dt = \frac{\rho L}{kT}\int_{x_1}^{x_2} x dx \quad \text{or} \quad t = \frac{\rho L}{kT}\left|\frac{x^2}{2}\right|_{x_1}^{x_2} = \frac{\rho L}{2kT}\left(x_2^2 - x_1^1\right)$$

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Example 1.8 **A cylindrical rod of heat capacity 120 J/K in a room temperature 27°C is heated internally by heater of power 250 W. The steady state temperature attained by the rod is 37°C. Find the following:**

a. The initial rate of increase in temperature
b. The steady state rate of emission of radiant heat.
If the heater is switched off, find
c. The initial rate of decrease in temperature
d. The rate of decrease in temperature of the cylinder when its temperature falls to 31°C and
e. The maximum amount of heat lost by the cylinder

Sol. Initially, when the rod is in thermal equilibrium with the room at 27°C, from Prevost's theory of exchanges, the rate of total radiant heat emitted by the rod to the surrounding is same as the rate of total radiant energy absorbed by it from the surrounding. So, as soon as the heater is switched on, the entire heat provided by it is completely absorbed by the rod,

$$c\frac{dT}{dt} = P = \text{Power of heater}$$

$$120\ \frac{\text{J}}{\text{K}}\left(\frac{dT}{dt}\right) = 250\ \frac{\text{J}}{\text{s}}$$

$$\frac{dT}{dt} = 2.08\ \text{K/s}$$

a. Thus, the initial rate of increase in temperature of the rod is 2.08 K/s.

b. In the steady state, the absorption of the radiant energy by the rod ceases, which evidently implies that the heat energy liberated by the heater per second is emitted by the rod per second.
So, the steady state rate of emission of radiant heat from the rod = power of heater = 250 W.

c. Immediately after the heater is switched off, the rod continues emitting heat energy at the same rate, as during the steady state, by virtue of its temperature. However, the heater stops supplying the heat to the rod completely. As a result, the temperature of the rod starts falling.

From $-c\dfrac{dT}{dt} = \dfrac{dQ}{dt}$, we have

$$\frac{-dT}{dt} = \frac{250\ \text{W}}{120\ \text{J/K}} = 2.08\ \text{K/s}$$

d. Since temperature difference between the rod and the room is not too large during the process, so Newton's law of cooling becomes valid.

$$\frac{-dT}{dt} = K(T - T_0)$$

Initially, $$\left(\frac{-dT}{dt}\right)_i = K(37-27)°\text{C} = 10\,K°\text{C} \qquad \text{(i)}$$

Finally, at $T = 31$°C

$$\left(\frac{-dT}{dt}\right)_f = K(31-27)°\text{C} = 4\,K°\text{C} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\left(\frac{dT}{dt}\right)_f = \frac{4}{10}(2.08\ \text{K/s}) = 0.83\ \text{K/s} \quad \left[\because \left(\frac{-dT}{dt}\right)_i = 2.08\ \text{K/s}\right]$$

e. Since a body cannot be cooled below the temperature of the surroundings, so maximum fall in temperature of the rod can be 10°C. So, the maximum heat that the rod can lose, after the heater is switched off, by the time its temperature falls to that of the room, will be $c(\Delta T)_{max}$

$$= 120\frac{\text{J}}{\text{K}}(10\ \text{K}) = 1200\ \text{J}$$

Example 1.9 **Three rods of material *X* and three rods of material *Y* are connected as shown in Fig. 1.58. All the rods are of identical lengths and cross-sectional areas. If the rod end *A* is maintained at 60°C and the junction *E* at 10°C, calculate the temperature of junctions *B*, *C* and *D*. the thermal conductivity of *X* is 9.2 × 10⁻² kcal/m/s/°C and that of *Y* is 4.6 × 10⁻² kcal/m/s/°C. (IIT-JEE, 1978)**

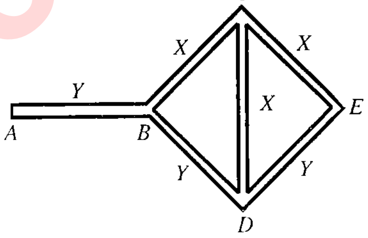

Fig. 1.58

Sol. Let k_x and k_y be the thermal conductivities of *X* and *Y* respectively, and let T_B, T_C and T_D be the temperatures of junctions *B*, *C* and *D*, respectively. Given T_A = 60°C and T_E = 10°C.

In order to solve this problem, we will apply the principle that, in the steady state, the rate at which heat enters a junction is equal to the rate at which heat leaves that junction. In Fig.1.59, the direction of flow of heat is given by arrows.

For junction *B*, we have

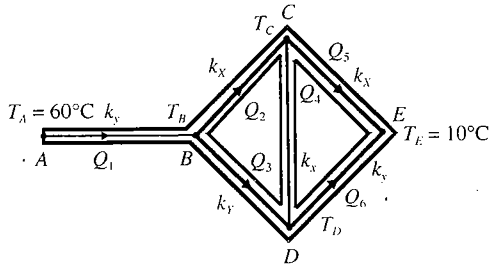

Fig. 1.59

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$$\frac{k_y A(T_A - T_B)}{d} = \frac{k_x A(T_B - T_C)}{d} + \frac{k_y A(T_B - T_D)}{d}$$

or $\quad k_y(T_A - T_B) = kx\,(T_B - T_C) + k_y(T_B - T_D)$

or $\quad (T_A - T_B) = \frac{k_x}{k_y}(T_B - T_C) + k_y(T_B - T_D)$

Given $\frac{k_x}{k_y} = \frac{9.2\times10^{-2}}{4.6\times10^{-2}} = 2$ and $T_A = 60°C$. Therefore

$$(60 - T_B) = 2(T_B - T_C) + (T_B - T_C) + (T_B - T_D)$$

or $\quad 4T_B - 2T_C - T_D = 60 \qquad \text{(i)}$

For junction C, we have

$$\frac{k_x A(T_B - T_C)}{d} = \frac{k_x A(T_C - T_D)}{d} + \frac{K_x A(T_C - T_E)}{d}$$

Solving we get

$$-T_B + 3T_C - T_D = 10 \quad [\text{At } T_E = 10°C] \qquad \text{(ii)}$$

For junction D, we have

or $\quad \frac{k_y A(T_B - T_D)}{d} + \frac{k_x A(T_C - T_D)}{d} = \frac{k_y A(T_D - T_E)}{d}$

or $\quad (T_B - T_D) + \frac{k_x}{k_y} = 2$ and $T_E = 10°C$. In this equation, we have

$$(T_B - T_D) + 2(T_C - T_D) = (T_D - 10)$$

$$T_B + 2T_C - 4T_D = -10 \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$T_B = 30°C \quad \text{and} \quad T_C = T_D = 20°C$$

Example 1.10 **A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth up to which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l showing all the steps. (IIT-JEE, 2004)**

Sol. Let l be side of cube at initial temperature and d the depth of cube submerged. Then according law of floatation

Weight of solid = weight of liquid displaced

$$Mg = l^2 d\rho_l g \qquad \text{(i)}$$

When temperature is increased, the weight remains same. Length of side of cube increases, density of liquid decrease and depth remains unchanged (as given)

$\therefore \quad Mg = (l')2\, d\rho_l g$

But $\quad l' = l(1 + \alpha_s \Delta T),\ \rho_l = \frac{\rho_l}{1 + \gamma_l \Delta T}$

Substituting these values in Eq. (iii), we get

$$l^2 d\rho_l g = l^2(1 + \alpha_s \Delta T)^2 \frac{\rho_l}{1 + \gamma_l \Delta T} g \quad \Rightarrow 1 + \gamma_l \Delta T = (1 + \alpha_s \Delta T)^2$$

As $\alpha_s \Delta T \ll 1$, using binomial theorem

$$1 + \gamma_l \Delta T = 1 + 2\alpha_s \Delta T \quad \Rightarrow \gamma_l = 2\alpha_s$$

Example 1.11 **One end of a rod of length L and cross-sectional area A is kept in a furnace at temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$, where $\Delta T \ll T_S$, T_S is the temperature of the surroundings. If $\Delta T \propto (T_1 - T_2)$, find the proportional constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. (IIT-JEE, 2004)**

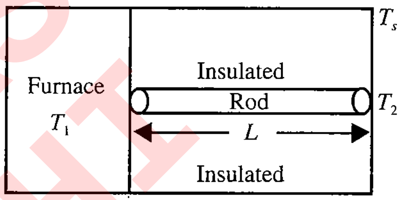

Fig. 1.60

Sol. Rate of heat conduction through rod = rate of the heat lost from right end of the rod.

$$\frac{KA(T_1 - T_2)}{L} = eA\sigma\,(T_2^4 - T_S^4) \qquad \text{(i)}$$

Given that $\quad T_2 = T_S + \Delta T$

$$T_2^4 = (T_S + \Delta T)^4 = T_S^4\left(1 + \frac{\Delta T}{T_S}\right)^4$$

Using binomial expansion, we have

$$T_2^4 = T_S^4\left(1 + 4\frac{\Delta T}{T_S}\right) \qquad (\text{as } \Delta T \ll T_S)$$

$$T_2^4 - T_S^4 = 4(\Delta T)(T_S^3)$$

Substituting in Eq. (i), we have

$$\frac{K(T_1 - T_S - \Delta T)}{L} = 4e\sigma T_S^3 \Delta T$$

$$\frac{K(T_1 - T_S)}{L} = \left(4e\sigma T_S^3 + \frac{K}{L}\right)\Delta T$$

$$\Delta T = \frac{K(T_1 - T_S)}{(4e\sigma L T_S^3 + K)}$$

Comparing with the given relation, proportionality constant

$$= \frac{K}{4e\sigma L T_S^3 + K}$$

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Example 1.12 **A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20°C is given a heat of 20000 J. Find the following:**

a. change in temperature
b. work done
c. change in internal energy

(Given: Specific heat = 400 J/kg/°C, coefficient of cubical expansion, γ = 9 × 10^{-5}/°C, density ρ = 9000 kg/m³, atmospheric pressure = 105 N/m²) (IIT-JEE, 2005)

Sol.

a. From $\Delta Q = ms\Delta T$

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20000}{1\times 400} = 50°\text{C}$$

b. $\Delta V = V_\gamma \Delta T = \left(\frac{1}{9000}\right)(9\times10^{-5})(50) = 5\times10^{-7}\ \text{m}^3$

$$W = p\cdot\Delta V = (10)^5(5\times10^{-7}) = 0.05\ \text{J}$$

c. $\Delta U = \Delta Q - W = (20000 - 0.05)$ J

$= 19999.95$ J

Example 1.13 **In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin). (IIT-JEE, 2006)**

Given, L_{fusion} **= 80 cal/g = 336 J/g**

$L_{\text{vapourization}}$ **= 540 cal/g = 2268 J/g**

S_{ice} **= 2100 J/kg-K= 0.5 cal/g-K**

And S_{water} **= 4200 J/kg-K = 1 cal/g-K**

Sol. 0.05 kg steam at 373 K $\xrightarrow{Q_1}$ 0.05 kg water at 373 K

0.05 kg water at 373 K $\xrightarrow{Q_2}$ 0.05 kg water at 273 K

0.45 kg ice at 253 K $\xrightarrow{Q_3}$ 0.45 kg ice at 273 K

0.45 kg ice at 273 K $\xrightarrow{Q_4}$ 0.45 kg water at 273 K

Q_1 = (50) (540) = 27000 cal = 27 kcal

Q_2 = (50) (1)(100) =5000 cal = 5 kcal

Q_3 = (450) (0.5)(20) = 4500 cal = 4.5 kcal

Q_4 = (450) (80) = 36000 cal = 36 kcal

Now since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273 K.

EXERCISES

Subjective Type

Solutions on page 1.69

1. A lead ball at 25°C is dropped from a height of 2 km. It is heated due to air resistance and it is assumed that all of its kinetic energy is used in increasing the temperature of ball. Find the final temperature of the ball.
2. The temperatures of equal masses of three different liquids *A*, *B* and *C* are 15°C, 20°C and 30°C, respectively. When *A* and *B* are mixed their equilibrium temperature is 180°C. When *B* and *C* are mixed, it is 22°C. What will be the equilibrium temperature when liquids *A* and *C* are mixed?
3. A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the block as thermal energy. Find the increase in temperature of block as it slides down through 60 cm. Given that specific heat of copper is 420 J/kg/K.
4. Find the result of mixing 0.5 kg ice at 0°C with 2 kg water at 30°C. Given that latent heat of ice is $L = 3.36 \times 10^5$ J/kg and specific heat of water is 4200 J/kg/K.
5. 1 g ice at 0°C is placed in a calorimeter having 1 g water at 40°C. Find equilibrium temperature and final contents. Assuming heat capacity of calorimeter is negligibly small.
6. 1 g ice at −40°C is placed in a container having 1 g water at 1°C. Find equilibrium temperature. Assume heat capacity of container is negligibly small.
7. 1 g steam at 100°C is passed in an insulated vessel having 1 g ice at 0°C. Find the equilibrium temperature of the mixture. Neglect heat capacity of the vessel.
8. A clock with a metallic pendulum is 5 s fast each day at a temperature of 15°C and 10 s slow each day at a temperature of 30°C. Find coefficient of linear expansion for the metal.
9. A rod *AB* of length *l* is pivoted at an end *A* and freely rotated in a horizontal plane at an angular speed ω about a vertical axis passing through *A*. If coefficient of linear expansion of material of rod is α, find the percentage change in its angular velocity if temperature of system is increased by ΔT.
10. A compensated pendulum shown in Fig. 1.61 is in the form of an isosceles triangle of base length l_1 = 5 cm and coefficient of linar expansion α_1 = 18 × 10^{-6} and side length l_2 and coefficient of linear expansion α_2 = 12 × 10^{-6}. Find l_2 so that the distance of centre of mass of the bob from suspension centre *O* may remain the same at all the temperature.

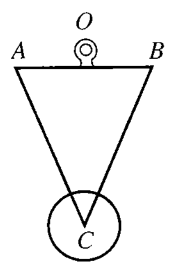

Fig. 1.61

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11. Two same-length rods of brass and steel of equal cross-sectional area are joined end to end as shown in Fig. 1.62 and supported between two rigid vertical walls. Initially the rods are unstrained. If the temperature of system is raised by Δt, find the displacement of the junction of two rods. Given that the coefficients of linear expansion and Young's modulus of brass and steel are α_b, α_s ($\alpha_b > \alpha_s$), y_b and y_s, respectively.

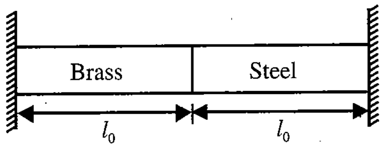

Fig. 1.62

12. A cylindrical beer can made up of aluminium contains 500 cm³ of beer. The area of cross section of can is 125 cm³ at 10°C. Find the rise in level of beer if temperature of can increases to 80°C. Given that coefficient of linear expansion of aluminium is $\alpha_{Al} = 2.3 \times 10^{-5}$/°C and that for cubical expansion of beer is $\gamma_{Beer} = 3.2 \times 10^{-4}$/°C.

13. What is the temperature of the steel–copper junction in the steady state of the system shown in Fig. 1.63. Length of the steel rod = 25 cm, length of the copper rod = 50 cm, temperature of the furnace = 300°C, temperature of the other end = 0°C. The area of cross section of the steel rod is twice that of the copper rod. (Thermal conductivity of steel = 50 J/s/m/K and of copper = 400 J/s/m/K.)

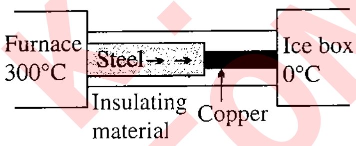

Fig. 1.63

14. We would like to increase the length of a 15-cm long copper rod of cross section 4 mm² by 1 mm. The energy absorbed by the rod if it is heated is E_1. The energy absorbed by the rod if it is stretched slowly is E_2. Then find E_1/E_2.
[Various parameters of copper are density = 9×10^3 kg/m³, thermal coefficient of linear expansion = 16×10^{-6}/K, Young's modulus = 135×109 Pa, specific heat = 400 J/kg-K]

15. The thickness of ice in a lake is 5 cm and its atmospheric temperature is –10°C. Calculate the time required for the thickness of ice to grow to 7 cm. Thermal conductivity of ice = 4×10^{-3} cal/cm-s-°C; density of ice 0.92 g/cm³ and specific latent heat of fusion for ice = 80 cal/g.

16. A uniform rod of thermal conductivity of 65 J/m-s-°C is surrounded by an insulator on its sides. One of its ends is put in a furnace, while the other end is kept exposed. If the temperature gradient of the rod is –75°C/m, find the emissive power of the exposed end.

17. A cylindrical brass boiler of radius 15 cm and thickness 1.0 cm is filled with water and placed on an electric heater. If the water boils at the rate of 200 g/s, estimate the temperature of the heater filament. Thermal conductivity of brass = 109 J/s/m/°C and heat of vapourization of water = 2.256×10^3 J/g.

18. A slab of stone of area 3600 cm² and thickness 10 cm is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on the upper surface of the slab. If in 1h 4.8 kg of ice is melted, calculate the thermal conductivity of the stone.

19. A few rods of materials X and Y are connected as shown in Fig. 1.64. The cross-sectional areas of all the rods are same. If the end A is maintained at 80°C and the end F is maintained at 10°C. Calculate the temperature of junctions B and E in steady state. Given that thermal conductivity of material X is double that of Y.

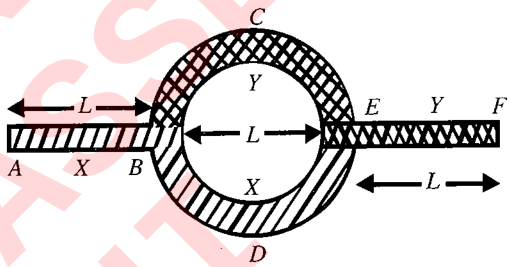

Fig. 1.64

20. A cube of mass 1 kg and volume 125 cm³ is placed in an evacuated chamber at 27°C. Initially temperature of block is 227°C. Assume block behaves like a black body, find the rate of cooling of block if specific heat of the material of block is 400 J/kg-K.

21. A solid metallic sphere of diameter 20 cm and mass 10 kg is heated to a temperature of 327°C and suspended in a box in which a constant temperature of 27°C is maintained. Find the rate at which the temperature of the sphere will fall with time. Stefan's constant = 5.67×10^{-8} W/m²/K⁴ and specific heat of metal = 420 J/kg/°C.

22. Figure 1.65 shows water in a container having 2.0 mm thick walls made of a material of thermal conductivity 0.50 W/m-°C. The container is kept in a melting ice bath at 0°C. The total surface area in contact with water is 0.05 m². A wheel is clamped inside the water and is coupled to a block of mass M as shown in the figure. As the block goes down, the wheel rotates. It is found that after some time a steady state is reached in which the block goes down with a constant speed of 10 cm/s and the temperature of the water remains constant at 1.0°C. Find the mass M of the block. Assume that the heat flows out of the water only through the walls in contact. Take g = 10 m/s².

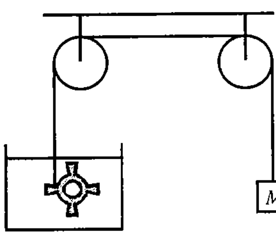

Fig. 1.65

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23. Let us assume that sun radiates like a black body with surface temperature at T_0 = 6000 K and earth absorbs radiations coming from sun only. If both earth and sun are considered perfect spheres with distance between centre of earth and centre of sun to be 200 times the radius of sun, find the temperature (in Kelvin) of surface of earth in steady state (assume radiation incident on earth to be almost parallel).

24. Water flows at the rate of 0.1500 kg/min through a tube and is heated by a heater dissipating 25.2 W. The inflow and outflow water temperatures are 15.2°C and 17.4°C, respectively. When the rate of flow is increased to 0.2318 kg/min and the rate of heating to 37.8 W, the inflow and outflow temperatures are unaltered. Find

i. the specific heat capacity of water

ii. the rate of loss of heat from the tube

25. A rod of length l with thermally insulated lateral surface is made of a material whose thermal conductivity varies as $K = C/T$, where C is a constant. The ends are kept at temperatures T_1 and T_2. Find the temperature at a distance x from the first end where the temperature is T_1 and the heat flow density.

26. A cylindrical rod of length l = 64 cm and cross-sectional radius $r = \left(2/\sqrt{\pi}\right)$ cm is placed at a distance $50r$ from a infrared point source S of power 1.25 kW as shown in Fig. 1.66. The lateral surface of the rod is perfectly insulated from the surroundings. The cross section A absorbs 80% of the incident energy, and has temperature T_A in steady state. The surface B is radiating energy into space and the wavelength emitted by it with maximum energy density is 100,000 Å. Determine the temperature of end B and find the value of T_A if conductivity varies with temperature as $K = \dfrac{T}{T_A}$. Assume that the rate of flow of heat through the rod is steady. (Wein's constant = 0.003 mK)

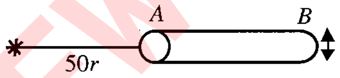

Fig. 1.66

Objective Type

Solutions on page 1.75

1. The graph AB shown in Fig. 1.67 is a plot of temperature of a body in degree celsius and degree Fahrenheit. Then

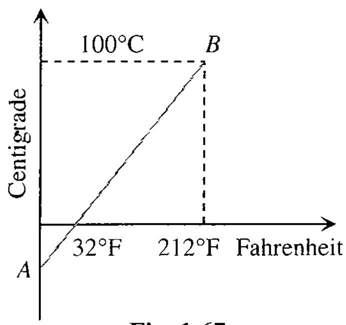

Fig. 1.67

a. Slope of line AB is 9/5
b. Slope of line AB is 5/9
c. Slope of line AB is 1/9
d. Slope of line AB is 3/9

2. The design of a physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. If $\alpha_{Fe} = 11\times10^{-6}$/°C and $\alpha_{cu} = 17\times10^{-6}$/°C, their lengths may be

a. 28.3 cm, 18.3 cm
b. 23.8 cm, 13.8 cm
c. 23.9 cm, 10.9 cm
d. 27.5 cm, 14.5 cm

3. Two rods of length L_2 and coefficient of linear expansion α_2 are connected freely to a third rod of length L_1 of coefficient of linear expansion α_1 to form an isosceles triangle. The arrangement is supported on the knife edge at the midpoint of L_1 which is horizontal. The apex of the isosceles triangle is to remain at a constant distance from the knife edge if

a. $\dfrac{L_1}{L_2} = \dfrac{\alpha_2}{\alpha_1}$ **b.** $\dfrac{L_1}{L_2} = \sqrt{\dfrac{\alpha_2}{\alpha_1}}$

c. $\dfrac{L_1}{L_2} = 2\dfrac{\alpha_2}{\alpha_1}$ **d.** $\dfrac{L_1}{L_2} = 2\sqrt{\dfrac{\alpha_2}{\alpha_1}}$

4. An iron rod of length 50 cm is joined at an end to an aluminium rod of length 100 cm. All measurements refer to 20°C. The coefficients of linear expansion of iron and aluminium are 12×10^{-6}/°C and 24×10^{-6}/°C, respectively. The average coefficient of composite system is

a. 36×10^{-6}/°C **b.** 12×10^{-6}/°C
c. 20×10^{-6}/°C **d.** 48×10^{-6}/°C

5. A brass rod and a lead rod each 80 cm long at 0°C are clamped together at one end with their free ends coinciding. The separation of free ends of the rods if the system is placed in a steam bath is ($\alpha_{brass} = 18 \times 10^{-6}$/°C and $\alpha_{lead} = 28 \times 10^{-6}$/°C)

a. 0.2 mm **b.** 0.8 mm
c. 1.4 mm **d.** 1.6 mm

6. The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel S. The coefficient of volume expansion of copper is γ_C. What is the coefficient of linear expansion of silver

a. $(C + \gamma_C + S)/3$ **b.** $(C - \gamma_C + S)/3$
c. $(C + \gamma_C - S)/3$ **d.** $(C - \gamma_C - S)/3$

7. A uniform solid brass sphere is rotating with angular speed ω_0 about a diameter. If its temperature is now increased by 100°C, what will be its new angular speed. (Given $\alpha_B = 2.0 \times 10^{-5}$ per °C)

a. $1.1\omega_0$ **b.** $1.01\omega_0$
c. $0.996\omega_0$ **d.** $0.824\omega_0$

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8. The absolute coefficient of expansion of a liquid is 7 times that the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

a. $\frac{1}{7}$ **b.** $\frac{7}{6}$

c. $\frac{6}{7}$ **d.** None of these

9. A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

a. $\frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$ **b.** $\frac{f_1 - f_2}{f_1 t_1 - f_2 t_2}$

c. $\frac{f_1 + f_2}{f_2 t_1 + f_1 t_2}$ **d.** $\frac{f_1 + f_2}{f_1 t_1 + f_2 t_2}$

10. A cylindrical metal rod of length L_0 is shaped into a ring with a small gap as shown. On heating the system

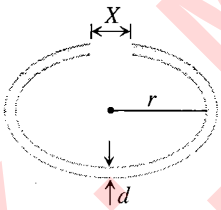

Fig. 1.68

a. x decreases, r and d increase
b. x and r increase, d decreases
c. x, r and d all increase
d. Data insufficient to arrive at a conclusion

11. Two holes of unequal diameters d_1 and d_2 ($d_1 > d_2$) are cut in a metal sheet. If the sheet is heated

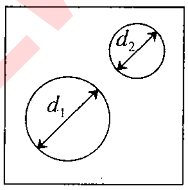

Fig. 1.69

a. Both d_1 and d_2 will decrease
b. Both d_1 and d_2 will increase
c. d_1 will increase, d_2 will decrease
d. d_1 will decrease, d_2 will increase

12. An iron tyre is to be fitted onto a wooden wheel 1.0 m in diameter. The diameter of the tyre is 6 mm smaller than that of wheel. The tyre should be heated so that its temperature increases by a minimum of (coefficient of volume expansion of iron is 3.6×10^{-5}/°C)

a. 167°C **b.** 334°C
c. 500°C **d.** 1000°C

13. A clock with a metal pendulum beating seconds keeps correct time at 0°C. If it loses 12.5 s a day at 25°C, the coefficient of linear expansion of metal of pendulum is

a. $\frac{1}{86400}$/°C **b.** $\frac{1}{43200}$/°C

c. $\frac{1}{14400}$/°C **d.** $\frac{1}{28800}$/°C

14. A wire of length L_0 is supplied heat to raise its temperature by T. If γ is the coefficient of volume expansion of the wire and Y is Young's modulus of the wire then the energy density stored in the wire is

a. $\frac{1}{2}\gamma^2 T^2 Y$ **b.** $\frac{1}{3}\gamma^2 T^2 Y^3$

c. $\frac{1}{18}\frac{\gamma^2 T^2}{Y}$ **d.** $\frac{1}{18}\gamma^2 T^2 Y$

15. Span of a bridge is 2.4 km. At 30°C a cable along the span sags by 0.5 km. Taking $\alpha = 12 \times 10^{-6}$/°C, change in length of cable for a change in temperature from 10°C to 42°C is

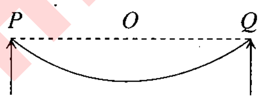

Fig. 1.70

a. 9.9 m **b.** 0.099 m
c. 0.99 m **d.** 0.4 km

16. The specific heat of a substance varies with temperature t(°C) as

$$c = 0.20 + 0.14t + 0.023t^2 \text{ (cal/g/°C)}$$

The heat required to raise the temperature of 2 g of substance from 5°C to 15°C will be
a. 24 cal **b.** 56 cal
c. 82 cal **d.** 100 c

17. Work done in converting 1 g of ice at –10°C into steam at 100°C is
a. 3045 J **b.** 6056 J
c. 721 J **d.** 6 J

18. Compared to a burn due to water at 100°C, a burn due to steam at 100°C is
a. More dangerous **b.** Less dangerous
c. Equally dangerous **d.** None of these

19. Latent heat of ice is 80 cal/g. A man melts 60 g of ice by chewing in 1 min. His power is
a. 4800 W **b.** 336 W **c.** 1.33 W **d.** 0.75 W

20. 50 g of copper is heated to increase its temperature by 10°C. If the same quantity of heat is given to 10 g of water, the rise in its temperature is (specific heat of copper = 420 J/kg/°C)
a. 5°C **b.** 6°C
c. 7°C **d.** 8°C

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21. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of
a. 3:2 b. 2:3
c. 1:1 d. 4:3

22. A beaker contains 200 g of water. The heat capacity of the beaker is equal to that of 20 g of water. The initial temperature of water in the beaker is 20°C. If 440 g of hot water at 92°C is poured in it, the final temperature (neglecting radiation loss) will be nearest to
a. 58°C b. 68°C
c. 73°C d. 78°C

23. A liquid of mass m and specific heat c is heated to a temperature $2T$. Another liquid of mass $m/2$ and specific heat $2c$ is heated to a temperature T. If these two liquids are mixed, the resulting temperature of the mixture is
a. $(2/3)T$ b. $(8/5)T$
c. $(3/5)T$ d. $(3/2)T$

24. Three liquids with masses m_1, m_2, m_3 are thoroughly mixed. If their specific heats are c_1, c_2, c_3 and their temperatures T_1, T_2, T_3, respectively, then the temperature of the mixture is

a. $\dfrac{c_1T_1+c_2T_2+c_3T_3}{m_1c_1+m_2c_2+m_3c_3}$

b. $\dfrac{m_1c_1T_1+m_2c_2T_2+m_3c_3T_3}{m_1c_1+m_2c_2+m_3c_3}$

c. $\dfrac{m_1c_1T_1+m_2c_2T_2+m_3c_3T_3}{m_1T_1+m_2T_2+m_3T_3}$

d. $\dfrac{m_1T_1+m_2T_2+m_3T_3}{c_1T_1+c_2T_2+c_3T_3}$

25. In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kilograms of steam is required per hour (specific heat of steam = 1 cal /g°C, Latent heat of vapourization = 540 cal/g)?
a. 1 g b. 1 kg
c. 10 g d. 10 kg

26. A calorimeter contains 0.2 kg of water at 30°C. 0.1 kg of water at 60°C is added to it, the mixture is well stirred and the resulting temperature is found to be 35°C. The thermal capacity of the calorimeter is
a. 6300 J/K b. 1260 J/K
c. 4200 J/K d. None of these

27. Consider two rods of same length and different specific heats (s_1 and s_2), conductivities K_1 and K_2 and areas of cross section (A_1 and A_2) and both having temperature T_1 and T_2 at their ends. If the rate of heat loss due to conduction is equal, then
a. $K_1A_1 = K_2A_2$ b. $K_2A_1 = K_1A_2$
c. $\dfrac{K_1A_1}{s_1} = \dfrac{K_2A_2}{s_2}$ d. $\dfrac{K_2A_1}{s_2} = \dfrac{K_1A_2}{s_1}$

28. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in Fig. 1.71. The points A and B are maintained at different temperatures. The ratio of the heat transferred through a cross section of a semi-circular rod to the heat transferred through a cross section of the straight rod in a given time is

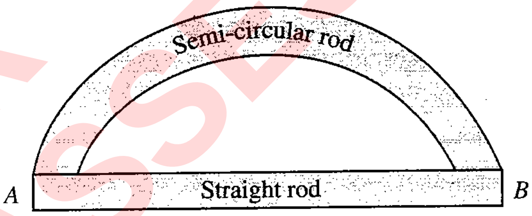

Fig. 1.71

a. $2:\pi$ b. 1:2
c. $\pi:2$ d. 3:2

29. A heat flux of 4000 J/s is to be passed through a copper rod of length 10 cm and area of cross section 100 cm². The thermal conductivity of copper is 400 W/m/°C. The two ends of this rod must be kept at a temperature difference of
a. 1°C b. 10°C
c. 100°C d. 1000°C

30. The coefficients of thermal conductivity of copper, mercury and glass are K_c, K_m and K_g, respectively, such that $K_c > K_m > K_g$. If the same quantity of heat is to flow per second per unit area of each and corresponding temperature gradients are, X_c, X_m and X_g, respectively, then
a. $X_c = X_m = X_g$ b. $X_c > X_m > X_g$
c. $X_c < X_m < X_g$ d. $X_m < X_c < X_g$

31. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and the inner surface of the shell is not to exceed T, then the thickness of the shell should not be less than

a. $\dfrac{2\pi R^2KT}{P}$ b. $\dfrac{4\pi R^2KT}{P}$

c. $\dfrac{\pi R^2KT}{P}$ d. $\dfrac{\pi R^2KT}{4P}$

32. There are three thermometers–one in contact with the skin of the man, other in between the vest and the shirt and third in between the shirt and coat. The readings of the thermometers are 30°C, 25°C and 22°C, respectively. If the vest and the shirt are of the same thickness, the ratio of their thermal conductivities is

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a. 9:25 **b.** 25:9
c. 5:3 **d.** 3:5

33. Two rods of same length and material transfer a given amount of heat in 12 s, when they are joined end to end. But when they are joined lengthwise, they will transfer same heat in same conditions in

a. 24 s **b.** 3 s
c. 1.5 s **d.** 48 s

34. Three rods of same dimensions are arranged as shown in Fig. 1.72. They have thermal conductivities K_1, K_2 and K_3. The points P and Q are maintained at different temperatures for the heat to flow at the same rate along PRQ and PQ. Which of the following options is correct?

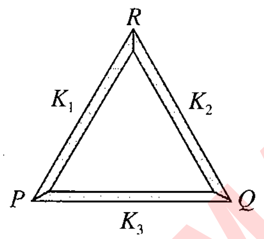

Fig. 1.72

a. $K_3 = \frac{1}{2}(K_1 + K_2)$

b. $K_3 = K_1 + K_2$

c. $K_3 = \frac{K_1 K_2}{K_1 + K_2}$

d. $K_3 = 2(K_1 + K_2)$

35. The coefficient of thermal conductivity of copper is nine times that of steel. In the composite cylindrical bar shown in Fig. 1.73, what will be the temperature at the junction of copper and steel?

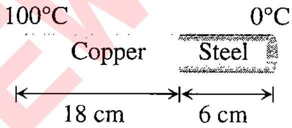

Fig. 1.73

a. 75°C **b.** 67°C
c. 33°C **d.** 25°C

36. Six identical conducting rods are joined as shown in Fig. 1.74. Points A and D are maintained at temperatures 200°C and 20°C, respectively. The temperature of junction B will be

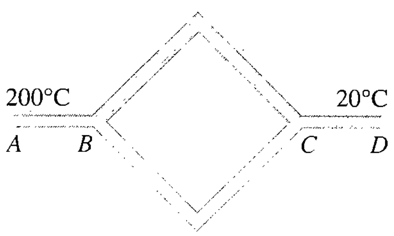

Fig. 1.74

a. 120°C **b.** 100°C
c. 140°C **d.** 80°C

37. An ice box used for keeping eatables cool has a total wall area of 1 m^2 and a wall thickness of 5.0 cm. The thermal conductivity of the ice box is $K = 0.01$ J/m°C. It is filled with large amount of ice at 0°C along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is 334×10^3 J/kg. The amount of ice melted in one day is (1 day = 86,400 s)

a. 776 g **b.** 7760 g
c. 11520 g **d.** 1552 g

38. The only possibility of heat flow in a thermos flask is through its cork which is 75 cm^2 in area and 5 cm thick. Its thermal conductivity is 0.0075 cal/cm-s-°C. The outside temperature is 40°C and latent heat of ice is 80 cal/g. Time taken by 500 g of ice at 0°C in the flask to melt into water at 0°C is

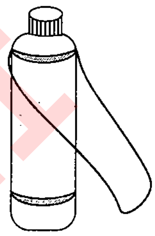

Fig. 1.75

a. 2.47 h **b.** 4.27 h
c. 7.42 h **d.** 4.72 h

39. Certain substance emits only the wavelengths $\lambda_1, \lambda_2, \lambda_3$ and λ_4 when it is at a high temperature. When this substance is at a colder temperature, it will absorb only the following wavelengths

a. λ_1 **b.** λ_2
c. λ_1 and λ_2 **d.** $\lambda_1, \lambda_2, \lambda_3$ and λ_4

40. Figure 1.76 shows two air-filled bulbs connected by a U-tube partly filled with alcohol. What happens to the levels of alcohol in the limbs X and Y when an electric bulb placed midway between the bulbs is lighted?

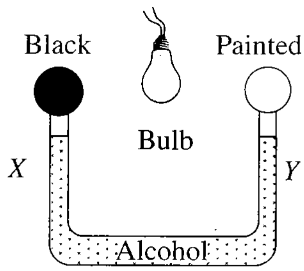

Fig. 1.76

a. The level of alcohol in limb X falls while that in limb Y rises

b. The level of alcohol in limb X rises while that in limb Y falls

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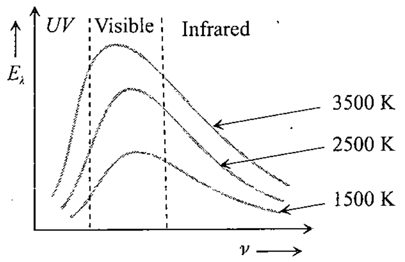
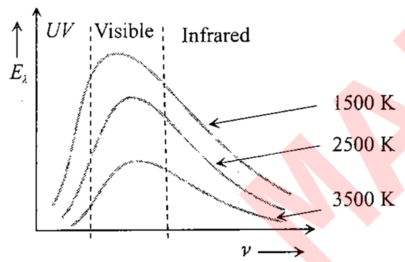
c. The level of alcohol falls in both limbs
d. There is no change in the levels of alcohol in either of the two limbs

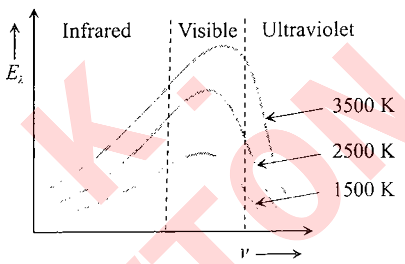
41. Following graph shows the correct variation in intensity of heat radiations by black body and frequency at a fixed temperature.



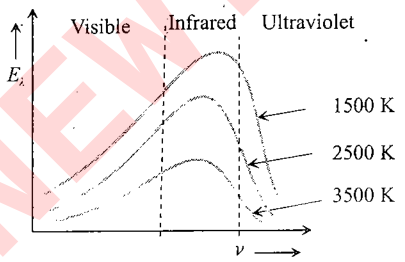
a.



b.



c.



d.

Fig. 1.77

42. A black body at 200 K is found to emit maximum energy at a wavelength of 14 μm. When its temperature is raised to 1000 K, the wavelength at which maximum energy is emitted is

a. 14 μm b. 70 μm
c. 2.8 μm d. 2.8 μm

43. The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $3\lambda_0/4$. The power radiated by the black body will now increase by a factor of

a. 256/81 b. 64/27
c. 16/9 d. 4/3

44. The wavelength of maximum energy released during an atomic explosion was 2.93×10^{-10} m. Given that Wien's constant is 2.93×10^{-3} m-K, the maximum temperature attained must be of the order of

a. 10^{-7} K b. 10^{7} K
c. 10^{-13} K d. 5.86×10^{7} K

45. Which of the following is the ν_m-T graph for a perfectly black body?

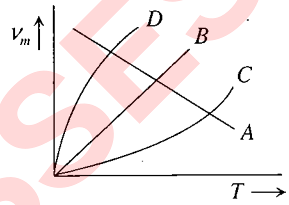


Fig. 1.78

a. A b. B
c. C d. D

46. The rectangular surface of area 8 cm × 4 cm of a black body at a temperature of 127°C emits energy at the rate of E per second. If the length and breadth of the surface are each reduced to half of the initial value and the temperature is raised to 327°C, the rate of emission of energy will become

a. $\frac{3}{8}E$ b. $\frac{81}{16}E$
c. $\frac{9}{16}E$ d. $\frac{81}{64}E$

47. A solid copper cube of edges 1 cm is suspended in an evacuated enclosure. Its temperature is found to fall from 100°C to 99°C in 100 s. Another solid copper cube of edges 2 cm, with similar surface nature, is suspended in a similar manner. The time required for this cube to cool from 100°C to 99°C will be approximately

a. 25 s b. 50 s
c. 200 s d. 400 s

48. A sphere, a cube and a thin circular plate are made of same substance and all have same mass. These are heated to 200°C and then placed in a room. Then the

a. Temperature of sphere drops to room temperature at last
b. Temperature of cube drops to room temperature at last
c. Temperature of thin circular plate drop to room temperature at last
d. Temperatures of all the three drop to room temperature at the same time

49. A sphere and a cube of same material and same volume are heated up to same temperature and allowed to cool in the same surroundings. The ratio of the amounts of radiations emitted in equal time intervals will be

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a. 1:1 **b.** $\frac{4\pi}{3}:1$

c. $\left(\frac{\pi}{6}\right)^{1/3}:1$ **d.** $\frac{1}{2}\left(\frac{4\pi}{3}\right)^{2/3}:1$

50. A bucket full of hot water cools from 75°C to 70°C in time T_1, from 70°C to 65°C in time T_2 and from 65°C to 60°C in time T_3, then

a. $T_1 = T_2 = T_3$ **b.** $T_1 > T_2 > T_3$

c. $T_1 < T_2 < T_3$ **d.** $T_1 > T_2 < T_3$

51. A cup of tea cools from 80°C to 60°C in 1 min. The ambient temperature is 30°C. In cooling from 60°C to 50°C it will take

a. 30 s

b. 60 s

c. 90 s

d. 48 s

52. A body takes T minutes to cool from 62°C to 61°C when the surrounding temperature is 30°C. The time taken by the body to cool from 46°C to 45.5°C is

a. Greater than T minutes

b. Equal to T minutes

c. Less than T minutes

d. None of these

53. The rates of cooling of two different liquids put in exactly similar calorimeters and kept in identical surroundings are the same if

a. The masses of the liquids are equal

b. Equal masses of the liquids at the same temperature are taken

c. Different volumes of the liquids at the same temperature are taken

d. Equal volumes of the liquids at the same temperature are taken

54. Hot water cools from 60°C to 50°C in the first 10 min and to 42°C in the next 10 min. The temperature of the surrounding is

a. 5°C **b.** 10°C

c. 15°C **d.** 20°C

55. There are two thin spheres A and B of the same material and same thickness. They behave like black bodies. Radius of A is double that of B and both have same temperature T. When A and B are kept in a room of temperature T_0 (< T), the ratio of their rates of cooling is (assume negligible heat exchange between A and B)

a. 2:1 b. 1:1 c. 4:1 d. 8:1

56. As shown in Fig. 1.79, AB is a rod of length 30 cm and area of cross section 1.0 cm² and thermal conductivity 336 SI units. The ends A and B are maintained at temperatures 20°C and 40°C, respectively. A point C of this rod is connected to a box D, containing ice at 0°C, through a highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is (assume latent heat of fusion for ice L_f = 80 cal/g)

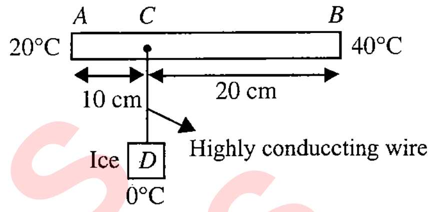

Fig. 1.79

a. 84 mg/s **b.** 84 g/s

c. 20 mg/s **d.** 40 mg/s

57. Two elastic rods are joined between fixed supports as shown in Fig.1.80. Condition for no change in the lengths of individual rods with the increase of temperature (α_1, α_2 = linear expansion coefficient, A_1, A_2 = area of rods, y_1, y_2 = Young's modulus) is

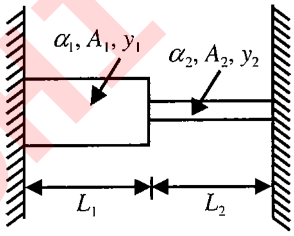

Fig. 1.80

a. $\frac{A_1}{A_2} = \frac{\alpha_1 y_1}{\alpha_2 y_2}$ **b.** $\frac{A_1}{A_2} = \frac{L_1 \alpha_1 y_1}{L_2 \alpha_2 y_2}$

c. $\frac{A_1}{A_2} = \frac{L_2 \alpha_2 y_2}{L_1 \alpha_1 y_1}$ **d.** $\frac{A_1}{A_2} = \frac{\alpha_2 y_2}{\alpha_1 y_1}$

58. A rod of length l and cross-sectional area A has a variable conductivity given by $K = \alpha T$, where α is a positive constant and T is temperatures in kelvin. Two ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$). Heat current flowing through the rod will be

a. $\frac{A\alpha(T_1^2 - T_2^2)}{l}$ **b.** $\frac{A\alpha(T_1^2 + T_2^2)}{l}$

c. $\frac{A\alpha(T_1^2 + T_2^2)}{3l}$ **d.** $\frac{A\alpha(T_1^2 - T_2^2)}{2l}$

59. A black body emits radiation at the rate P when its temperature is T. At this temperature the wavelength at which the radiation has maximum intensity is λ_0. If at another temperature T' the power radiated is P' and wavelength at maximum intensity is $\lambda_0/2$ then

a. $P'T' = 32\,PT$ **b.** $P'T' = 16\,PT$

c. $P'T' = 8\,PT$ **d.** $P'T' = 4\,PT$

60. A metallic sphere having radius 0.08 m and mass m = 10 kg is heated to a temperate of 227°C and suspended

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inside a box whose walls are at a temperature of 27°C. The maximum rate at which its temperature will fall is (take $e = 1$, Stefan's constant $\sigma = 5.8 \times 10^{-8}$ Wm^2 K^{-4} and specific heat of the metal = 90 cal/kg/deg J = 4.2 J/cal)

a. 0.055°C/s b. 0.066°C/s
c. 0.044°C/s d. 0.03°C/s

61. The coefficient of linear expansion of an inhomogeneous rod changes linearly from α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of rod is

a. $\alpha_1 + \alpha_2$ b. $\frac{\alpha_1 + \alpha_2}{2}$
c. $\sqrt{\alpha_1 \alpha_2}$ d. $\alpha_1 - \alpha_2$

62. A wire is made by attaching two segments together end to end. One segment is made of aluminium and the other is steel. The effective coefficient of linear expansion of the two segment wire is 19×10^{-6}/(°C). The fraction length of aluminium is (linear coefficients of thermal expansion of aluminium and steel are 23×10^{-6}/(°C) and 12×10^{-6}/(°C), respectively)

a. $\frac{5}{11}$ b. $\frac{6}{11}$
c. $\frac{7}{11}$ d. $\frac{8}{11}$

63. Heat is required to change 1 kg of ice at –20°C into steam. Q_1 is the heat needed to warm the ice from –20°C to 0°C, Q_2 is the heat needed to melt the ice, Q_3 is the heat needed to warm the water from 0°C to 100°C and Q_4 is the heat needed to vapourize the water. Then

a. $Q_4 > Q_3 > Q_2 > Q_1$
b. $Q_4 > Q_3 > Q_1 > Q_2$
c. $Q_4 > Q_2 > Q_3 > Q_1$
d. $Q_4 > Q_2 > Q_1 > Q_3$

64. A block of wood is floating in water at 0°C. The temperature of water is slowly raised from 0°C to 10°C. With the rise in temperature the volume of block V above water level will

a. increase
b. decrease
c. first increase and then decrease
d. first decrease and then increase

65. An incandescent lamp consuming $P = 54$ W is immersed into a transparent calorimeter containing $V = 10^3$ cm^3 of water. In 420 s the water is heated by 4°C. The percentage of the energy consumed by the lamp that passes out of the calorimeter in the form of radiant energy is

a. 81.5% b. 26%
c. 40.5% d. 51.5%

66. A thread of liquid is in a uniform capillary tube of length L, as measured by a ruler. The temperature of the tube and thread of liquid is raised by ΔT. If γ be the coefficient of volume expansion of the liquid and α be the coefficient of linear expansion of the material of the tube, then the increase ΔL in the length of the thread, again measured by the ruler will be

a. $\Delta L = L(\gamma - \alpha)\Delta T$ b. $\Delta L = L(\gamma - 2\alpha)\Delta T$
c. $\Delta L = L(\gamma - 3\alpha)\Delta T$ d. $\Delta L = L\gamma\,\Delta T$

67. A brass wire 2 m long at 27°C is held taut with negligible tension between two rigid supports. If the wire is cooled to a temperature of –33°C, then the tension developed in the wire, its diameter being 2 mm, will be (coefficient of linear expansion of brass = 2.0×10^{-5}/°C and Young's modulus of brass = 0.91×10^{11} Pa)

a. 3400 N b. 34 kN
c. 0.34 kN d. 6800 N

68. A mass m of lead shot is placed at the bottom of a vertical cardboard cylinder that is 1.5 m long and closed at both ends. The cylinder is suddenly inverted so that the shot falls 1.5 m. If this process is repeated quickly 100 times, assuming no heat is dissipated or lost, the temperature of the shot will increase by (specific heat of lead = 0.03 cal/g°C)

a. 0 b. 5°C
c. 7.3°C d. 11.3°C

69. An iron rocket fragment initially at –100°C enters the earth's atmosphere almost horizontally and quickly fuses completely in atmospheric friction. Specific heat of iron is 0.11 kcal/kg °C its melting point is 1535°C and the latent heat of fusion is 3 kcals/kg. The minimum velocity with which the fragment must have entered the atmosphere is

a. 0.45 km/s b. 1.32 km/s
c. 2.32 km/s d. zero

70. A liquid of density 0.85 g/cm^3 flows through a calorimeter at the rate of 8.0 cm^3/s. Heat is added by means of a 250 W electric heating coil and a temperature difference of 15°C is established in steady-state conditions between the inflow and the outflow points of the liquid. The specific heat for the liquid will be

a. 0.6 kcal/kgK b. 0.3 kcal/kgK
c. 0.5 kcal/kgK d. 0.4 kcal/kgK

71. An iron ball (coefficient of linear expansion = 1.2×10^{-5}/°C) has a diameter of 6 cm and is 0.010 mm too large to pass through a hole in a brass plate (coefficient of linear expansion = 1.9×10^{-5}/°C) when the ball and the plate are both at a temperature of 30°C. At what common temperature of the ball and the plate will the ball just pass through the hole in the plate?

a. 23.8°C b. 53.8°C
c. 42.5°C d. 63.5°C

72. A flask of mercury is sealed off at 20°C and is completely filled with mercury. If the bulk modulus for mercury is 250 MPa and the coefficient of volume expansion of mercury is 1.82×10^{-4}/°C and the expansion of glass is ignored, the pressure of mercury within the flask at 100°C will be

a. 100 MPa b. 72 MPa

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c. 36 MPa **d.** 24 MPa

73. The densities of wood and benzene at 0°C are 880 kg/m^3 and 900 kg/m^3, respectively. The coefficients of volume expansion of wood is 1.2×10^{-3}/°C and of benzene 1.5×10^{-3}/°C. The temperature at which a piece of this wood would just sink in benzene at the same temperature is

a. 53°C **b.** 63°C
c. 73°C **d.** 83°C

74. An iron rod and another of brass, both at 27°C differ in length by 10^{-3} m. The coefficient of linear expansion for iron is 1.1×10^{-5}/°C and for brass is 1.9×10^{-5}/°C. The temperature at which both these rods will have the same length is

a. 0°C **b.** 152°C
c. 175°C **d.** Data is insufficient

75. A refrigerator is thermally equivalent to a box of cork board 90 mm thick and 6 m^2 in inner surface area, the thermal conductivity of cork being 0.05 W/mK. The motor of the refrigerator runs 15% of the time while the door is closed. The inside wall of the door, when it is closed, is kept, on an average, 22°C below the temperature of the outside wall. The rate at which heat is taken from the interior wall while the motor is running is

a. 400 W **b.** 500 W
c. 300 W **d.** 250 W

76. A bullet of mass 5 g moving at a speed of 200 m/s strikes a rigidly fixed wooden plank of thickness 0.2 m normally and passes through it losing half of its kinetic energy. If it again strikes an identical rigidly fixed wooden plank and passes through it, assuming the same resistance in the two planks, the ratio of the thermal energies produced in the two planks is

a. 1:1 **b.** 1:2
c. 2:1 **d.** 4:1

77. A and B are two isolated spheres kept in close proximity so that they can exchange energy by radiation. The two spheres have identical physical dimensions but the surface of A behaves like a perfectly black body while the surface of B reflects 20% of all the radiations it receives. They are isolated from all other sources of radiation.

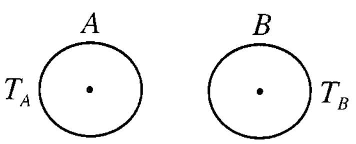

Fig. 1.81

a. If they are in thermal equilibrium and exchange equal amounts of radiation per second, then they will be at same absolute temperature, $T_A = T_B$.

b. If they are in thermal equilibrium and exchange equal amounts of radiation per second, then $T_A = (0.8)^{1/4}\, T_B$.

c. If they are not in thermal equilibrium and are each at $t = 0$ at the same temperature $T_A = T_B = T$, then the sphere A will lose thermal energy and B will gain thermal energy.

d. If they are not in thermal equilibrium and are each at $t = 0$ at the same temperature $T_A = T_B = T$, then the sphere A will gain thermal energy and B will lose thermal energy.

78. A thermally insulated piece of metal is heated under atmospheric pressure by an electric current so that it receives electric energy at a constant power P. This leads to an increase of absolute temperature T of the metal with time t as follows:

$T(t) = T_0[1 + a(t - t_0)]^{1/4}$. Here, a, t_0 and T_0 are constants. The heat capacity $C_P(T)$ of the metal is

a. $\dfrac{4P}{aT_0}$ **b.** $\dfrac{4PT^3}{aT_0^4}$ **c.** $\dfrac{2PT^3}{aT_0^4}$ **d.** $\dfrac{2P}{aT_0}$

79. A cooking vessel on a slow burner contains 5 kg of water and an unknown mass of ice in equilibrium at 0°C at time $t = 0$. The temperature of the mixture is measured at various times and the result is plotted as shown in Fig. 1.82. During the first 50 min the mixture remains at 0°C. From 50 min to 60 min, the temperature increases to 2°C. Neglecting the heat capacity of the vessel, the initial mass of the ice is

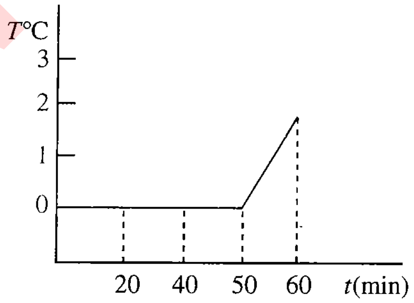

Fig. 1.82

a. $\dfrac{10}{7}$ kg **b.** $\dfrac{5}{7}$ kg **c.** $\dfrac{5}{4}$ kg **d.** $\dfrac{5}{8}$ kg

80. Liquid helium is stored at its boiling point (4.2 K) in a spherical can, separated by a vacuum space from a surrounding shield which is maintained at the temperature of liquid nitrogen (77 K). If the can is 0.1m in radius and is blacked on the outside so that it acts as a black body, how much helium boils away per hour?
(Latent heat of vapourization is 21 kJ/kg)

a. 43 g/h **b.** 43 kg/h
c. 4.3 g/h **d.** 43×10^{-3} g/h

81. A glass cylinder contains m_0 = 100 g of mercury at a temperature of t_0=0°C. When temperature becomes t_1=20°C the cylinder contains m_1 = 99.7 g of mercury. The coefficient of volume expansion of mercury $\gamma_{He} = 18 \times 10^{-5}$/°C. Assume that the temperature of the mercury is equal to that of the

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cylinder. The coefficient of linear expansion of glass α is

a. $10^{-5}/°C$ **b.** $2 \times 10^{-5}/°C$
c. $3 \times 10^{-5}/°C$ **d.** $6 \times 10^{-5}/°C$

82. A thermometer has an ordinary glass bulb and thin glass tube filled with 1 mL of mercury. A temperature change of 1°C changes the level of mercury in the thin tube by 3 mm. The inside diameter of the thin glass is ($\gamma_{Hg} = 18 \times 10^{-5}/°C$, $\alpha_{glass} = 10^{-5}/°C$)

a. 0.13 mm **b.** 0.25 mm
c. 0.40 mm **d.** 0.50 mm

83. A steel tape is placed around the earth at the equator. When the temperature is 0°C neglecting the expansion of the earth, the clearance between the tape and the ground if the temperature of the tape rises to 30°C, is nearly ($\alpha_{steel} = 11 \times 10^{-6}$ /K)

a. 1.1 km **b.** 0.5 km
c. 6400 km **d.** 2.1 km

84. Solar constant is 1370 W/m². 70% of the light incident on the earth is absorbed by the earth and the earth's average temperature is 288 K. The effective emissivity of the earth is

a. 0.2 **b.** 0.4 **c.** 0.6 **d.** 1

85. Power radiated by a black body is P_0 and the wavelength corresponding to maximum energy is around λ_0. On changing the temperature of the black body, it was observed that the power radiated becomes $\frac{256}{81}P_0$. The shift in wavelength corresponding to the maximum energy will be

a. $+\frac{\lambda_0}{4}$ **b.** $+\frac{\lambda_0}{2}$ **c.** $-\frac{\lambda_0}{4}$ **d.** $-\frac{\lambda_0}{2}$

86. Three rods AB, BC and BD of same length l and cross section A are arranged as shown. The end D is immersed in ice whose mass is 440 g and is at 0°C. The end C is maintained at 100°C. Heat is supplied at constant rate of 200 cal/s. Thermal conductivities of AB, BC and BD are K, $2K$ and $K/2$, respectively. Time after which whole ice will melt is (K = 100 cal/m-s-°C, A = 10 cm², l = 1 m)

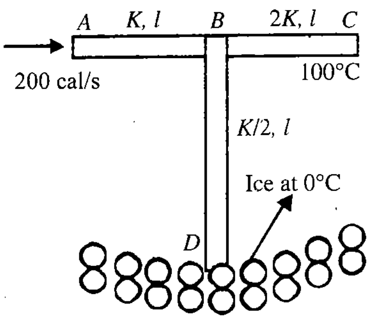

Fig. 1.83

a. 400 s **b.** 600 s
c. 700 s **d.** 800 s

87. The length of a steel rod exceeds that of a brass rod by 5 cm. If the difference in their lengths remains same at all temperatures, then the length of brass rod will be: (α for iron and brass are $12 \times 10^{-6}/°C$ and $18 \times 10^{-6}/°C$, respectively)

a. 15 cm **b.** 5 cm
c. 10 cm **d.** 2 cm

88. A container of capacity 700 mL is filled with two immiscible liquids of volume 200 mL and 500 mL with respective volume expansivities as $1.4 \times 10^{-5}/°C$ and $2.1 \times 10^{-5}/°C$. During the heating of the vessel, it is observed that neither any liquid overflows nor any empty space is created. The volume expansivity of the container is

a. $1.9 \times 10^{-5}/°C$ **b.** $1.9 \times 10^{-6}/°C$
c. $1.6 \times 10^{-5}/°C$ **d.** $1.6 \times 10^{-6}/°C$

89. The apparent coefficient of expansion of a liquid when heated, filled in vessel A and B of identical volumes, is found to be γ_1 and γ_2, respectively. If α_1 be the linear expansivity of A then that of B will be

a. $\frac{(\gamma_1-\gamma_2)}{3}-\alpha_1$ **b.** $\frac{(\gamma_2-\gamma_1)}{3}+\alpha_1$

c. $\frac{(\gamma_2-\gamma_1)}{3}-\alpha_1$ **d.** $\frac{(\gamma_1-\gamma_2)}{3}+\alpha_1$

90. A glass vessel is filled up to 3/5th of its volume by mercury. If the volume expansivities of glass and mercury be $9 \times 10^{-6}/°C$ and $18 \times 10^{-5}/°C$, respectively, then the coefficient of apparent expansion of mercury is

a. $17.1 \times 10^{-5}/°C$ **b.** $9.9 \times 10^{-5}/°C$
c. $17.46 \times 10^{-5}/°C$ **d.** $16.5 \times 10^{-5}/°C$

91. A thin circular metal disc of radius 500.0 mm is set rotating about a central axis normal to its plane. Upon raising its temperature gradually, the radius increases to 507.5 mm. The percentage change in the rotational kinetic energy will be

a. 1.5% **b.** −1.5%
c. 3% **d.** −3%

92. A platinum sphere floats in mercury. Find the percentage change in the fraction of volume of sphere immersed in mercury when the temperature is raised by 80°C (volume expansivity of mercury is $182 \times 10^{-6}/°C$ and linear expansivity of platinum is $9 \times 10^{-6}/°C$).

a. 1.24% **b.** 1.38%
c. 2.48% **d.** 2.76%

93. The variation of lengths of two metal rods A and B with change in temperature is shown in Fig. 1.84. The coefficients of linear expansion α_A for the metal A and the temperature T will be

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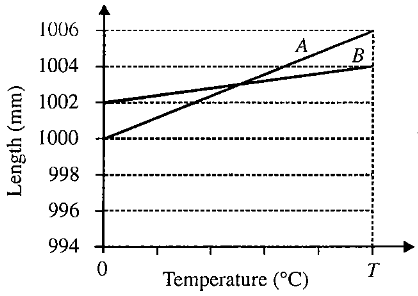

Fig. 1.84

(given $\alpha_B = 9 \times 10^{-6}$/°C)

a. $\alpha_A = 3 \times 10^{-6}$/°C, 500°C

b. $\alpha_A = 3 \times 10^{-6}$/°C, 222.22°C

c. $\alpha_A = 27 \times 10^{-6}$/°C, 500°C

d. $\alpha_A = 27 \times 10^{-6}$/°C, 222.22°C

94. The graph of elongation of a rod of a substance A with temperature rise is shown in Fig. 1.85. A liquid B contained in a cylindrical vessel made up of substance A, graduated in millilitres at 0°C is heated gradually. The readings of the liquid level in the vessel corresponding to different temperatures are shown in the figure. The real volume expansivity of liquid is

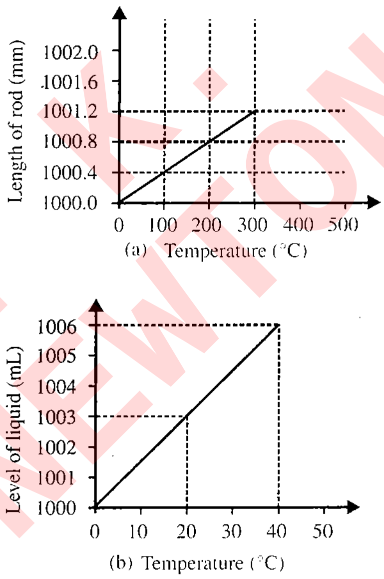

Fig. 1.85

a. 2.7×10^{-5}/°C **b.** 15.4×10^{-5}/°C

c. 16.2×10^{-5}/°C **d.** 151.2×10^{-5}/°C

95. Figure 1.86 shows the graphs of elongation versus temperature for two different metals. If these metals are employed to form a straight bimetallic strip of thickness 6 cm and heated, it bends in the form of an arc, the radius of curvature changing with temperature approximately as shown in the figure. The linear expansivities of the two metals are

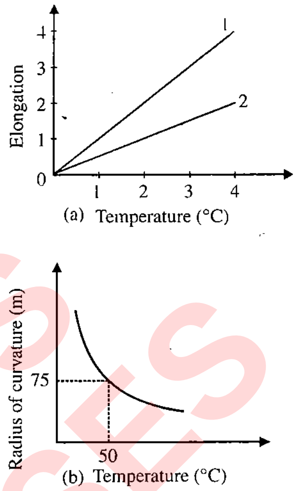

Fig. 1.86

a. 24×10^{-6}/°C and 12×10^{-6}/°C
b. 20×10^{-6}/°C and 10×10^{-6}/°C
c. 18×10^{-6}/°C and 9×10^{-6}/°C
d. 16×10^{-6}/°C and 8×10^{-6}/°C

96. Water at 0°C, contained in a closed vessel, is abruptly opened in an evacuated chamber. If the specific latent heats of fusion and vapourization at 0°C are in the ratio λ:1, the fraction of water evaporated will be

a. $\lambda/1$ **b.** $\lambda/(\lambda + 1)$
c. $(1 - \lambda)/\lambda$ **d.** $(\lambda - 1)/(\lambda + 1)$

97. A system receives heat continuously at the rate of 10 W. The temperature of the system becomes constant at 70°C when the temperature of the surroundings is 30°C. After the heater is switched off, the system cools from 50°C to 49.9°C in 1 min. The heat capacity of the system is

a. 1000 J/°C **b.** 1500 J/°C
c. 3000 J/°C **d.** None of these

98. Two identical calorimeters, each of water equivalent 100 g and volume 200 cm^3, are filled with water and a liquid. They are placed in identical constant-temperature enclosures to cool down. The temperatures are plotted at different times (the choice of units are completely arbitrary) as shown in Fig. 1.87. If the density of the liquid is 800 kgm^{-3}, then its specific heat capacity is

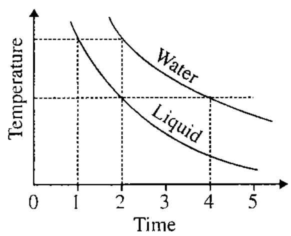

Fig. 1.87

a. 8400 J/kg°C **b.** 2100 J/kg°C
c. 1312.5 J/kg°C **d.** 1680.5 J/kg°C

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99. In a motor, the electrical power input is 500 W and the mechanical power output is 0.54 horse power. Heat developed in the motor in 1h is (assuming that all the electric energy which is not converted to mechanical energy is converted to heat) is

a. 4.18×10^4 cal **b.** 3.6×10^5 cal
c. 8.6×10^4 cal **d.** 1.28×10^5 cal

100. The molar heat capacity of a certain substance varies with temperature according to the given equation

$$C = 27.2 + (4 \times 10^{-3})\, T$$

The heat necessary to change the temperature of 2 mol of the substance from 300 K to 700 K is

a. 3.46×10^4 J **b.** 2.33×10^3 J
c. 3.46×10^3 J **d.** 2.33×10^4 J

101. The coefficient of linear expansion for a certain metal varies with temperature as $\alpha(T)$. If L_0 is the initial length of the metal and the temperature of metal is changed from T_0 to T $(T_0 > T)$, then

a. $L = L_0 \int_{T_0}^{T} \alpha(T)\, dT$

b. $L = L_0\left[1 + \int_{T_0}^{T} \alpha(T)\, dT\right]$

c. $L = L_0\left[1 - \int_{T_0}^{T} \alpha(T)\, dT\right]$

d. $L > L_0$

102. A piece of metal floats in mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2, respectively. If the temperatures of both mercury and the metal are increased by ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor of

a. $\dfrac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$ **b.** $1+\gamma_2\Delta T$

c. $1+\gamma_1\Delta T$ **d.** $\dfrac{1+\gamma_2\Delta T}{1-\gamma_1\Delta T}$

103. A uniform brass disc of radius a and mass m is set into spinning with angular speed ω_0 about an axis passing through centre of disc and perpendicular to the plane of disc. If its temperature increases from θ_1°C to θ_2°C without disturbing the disc, what will be its new angular speed? (The coefficient of linear expansion of brass is α.)

a. $\omega_0\,[1 + 2a(\theta_2 - \theta_1)]$ **b.** $\omega_0\,[1 + a(\theta_2 - \theta_1)]$

c. $\dfrac{\omega_0}{[1+2\alpha(\theta_2-\theta_1)]}$ **d.** None of these

104. Calculate the compressional force required to prevent the metallic rod of length l cm and cross-sectional area A cm^2 when heated through t°C, from expanding lengthwise. Young's modulus of elasticity of the metal is E and mean coefficient of linear expansion is α per degree celsius.

a. $EA\alpha t$ **b.** $EA\alpha t/(1+\alpha t)$
c. $EA\alpha t/(1-at)$ **d.** $El\alpha t$

105. The coefficient of linear expansion of glass is α_g per °C and the cubical expansion of mercury is γ_m per °C. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. What is the length of mercury column in capillary at T°C, if the mercury just fills the bulb at 0°C?

a. $\dfrac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+2\alpha_gT)}$ **b.** $\dfrac{V_0T(\gamma_m-3\alpha_g)}{A_0(1+2\alpha_gT)}$

c. $\dfrac{V_0T(\gamma_m+2\alpha_g)}{A_0(1+3\alpha_gT)}$ **d.** $\dfrac{V_0T(\gamma_m-2\alpha_g)}{A_0(1+3\alpha_gT)}$

106. In similar calorimeters, equal volumes of water and alcohol, when poured, take 100 and 74 s, respectively, to cool from 50°C to 40°C. If the thermal capacity of each calorimeter is numerically equal to volume of either liquid, then calculate the specific heat capacity of alcohol (given: the relative density of alcohol as 0.8 and specific heat capacity of water as 1 cal/g/°C).

a. 0.8 cal/g°C **b.** 0.6 cal/g°C
c. 0.9 cal/g°C **d.** 1 cal/g°C

107. A drilling machine of 10 kW power is used to drill a bore in a small aluminium block of mass 8 kg. If 50% of power is used up in heating the machine itself or lost to the surroundings then how much is the rise in temperature of the block in 2.5 min (given: specific heat of aluminium = 0.91 J/g°C)?

a. 103°C **b.** 130°C **c.** 105°C **d.** 30°C

108. A ball of thermal capacity 10 cal/°C is heated to the temperature of furnace. It is then transferred into a vessel containing water. The water equivalent of vessel and the contents is 200 g. The temperature of the vessel and its contents rises from 10°C to 40°C. What is the temperature of furnace?

a. 640°C **b.** 64°C **c.** 600°C **d.** 100°C

109. Two tanks A and B contain water at 30°C and 80°C, respectively; calculate the amount of water that must be taken from each tank respectively to prepare 40 kg of water at 50°C:

a. 24 kg, 16 kg **b.** 16 kg, 24 kg
c. 20 kg, 20 kg **d.** 30 kg, 10 kg

110. A steel ball of mass $m_1 = 1$ kg moving with velocity 50 m/s collides with another ball of mass $m_2 = 200$ g lying on the ground. During the collision their internal energies change equally and T_1 and T_2 are the rise in temperature of masses m_1 and m_2, respectively. If specific heat of steel is 0.105 and $J = 4.18$ J/cal, then

a. $T_1 = 7.1$°C and $T_2 = 1.47$°C
b. $T_1 = 1.47$°C and $T_2 = 7.1$°C
c. $T_1 = 3.4$ K and $T_2 = 17.0$ K
d. $T_1 = 7.1$ K and $T_2 = 1.4$ K

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111. It takes 10 min for an electric kettle to heat a certain quantity of water from 0°C to 100°C. It takes 54 min to convert this water at 100°C into steam. Then latent heat of steam is

a. 80 cal/g **b.** 540 cal/kg
c. 540 cal/g **d.** 80 cal/kg

112. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted, the temperature of flask and contents is 40°C. When a further 80 g of ice is added and has all melted, the temperature of whole becomes 10°C. Neglecting heat lost to surroundings the latent heat of fusion of ice is

a. 80 cal/g **b.** 90 cal/g
c. 70 cal/g **d.** 540 cal/g

113. The loss in weight of a solid when immersed in a liquid at 0°C is W_0 and at t°C is W. If cubical coefficients of expansion of the solid and the liquid are γ_s and γ_L, respectively, then W is equal to

a. $W_0[1 + (\gamma_s - \gamma_L)t]$ **b.** $W_0[1 - (\gamma_s - \gamma_L)t]$
c. $W_0[(\gamma_s - \gamma_L)t]$ **d.** $W_0t/(\gamma_s - \gamma_L)$

114. A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 s per day if cooled to 0°C. The coefficient of linear expansion of copper is

a. 1.7×10^{-4}/°C **b.** 1.7×10^{-5}/°C
c. 3.4×10^{-4}/°C **d.** 3.4×10^{-5}/°C

115. A glass flask is filled up to a mark with 50 cc of mercury at 18°C. If the flask and contents are heated to 38°C, how much mercury will be above the mark (α for glass is 9×10^{-6}/°C and coefficient of real expansion of mercury is 180×10^{-6}/°C)?

a. 0.85 cc **b.** 0.46 cc
c. 0.153 cc **d.** 0.05 cc

116. A flask of volume 10^3 cc is completely filled with mercury at 0°C. The coefficient of cubical expansion of mercury is 180×10^{-6}/°C and heat of glass is 40×10^{-6}/°C. If the flask is now placed in boiling water at 100°C, how much mercury will overflow?

a. 7 cc **b.** 14 cc **c.** 21 cc **d.** 28 cc

117. An aluminium measuring rod, which is correct at 5°C measures the length of a line as 80 cm at 45°C. If thermal coefficient of linear expansion of aluminium is 2.50×10^{-5}/°C, the correct length of the line is:

a. 80.08 cm **b.** 79.92 cm
c. 81.12 cm **d.** 79.62 cm

118. One end of a copper rod of uniform cross section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melting be equal to that of the steam produced in same interval of time? Assume that the whole system is insulated from surroundings. Latent heat of fusion of ice and vapourization of water are 80 cal/g and 540 cal/g, respectively.

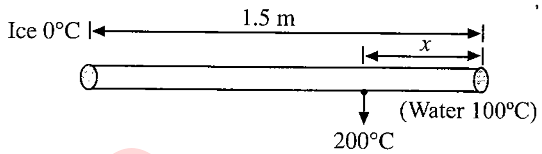

Fig. 1.88

a. 8.59 cm from ice end
b. 10.34 cm from water end
c. 10.34 cm from ice end
d. 8.76 cm from water end

119. A liquid takes 5 min to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of the surrounding is 20°C.

a. 5 min **b.** 9 min
c. 4 min **d.** 12 min

120. A 1 L glass flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in this flask if coefficient of linear expansion of glass is 9×10^{-6}/°C while of volume expansion of mercury is 1.8×10^{-4}/°C?

a. 50 cc b. 100 cc
c. 150 cc d. 200 cc

121. A piece of metal weighs 46 g in air. When immersed in a liquid of specific gravity 1.24 at 27°C it weighs 30 g. When the temperature of liquid is raised to 42°C the metal piece weighs 30.5 g. Specific gravity of liquid at 42°C is 1.20. Calculate the coefficient of linear expansion of metal:

a. 2.23×10^{-5}/°C
b. 6.7×10^{-5}/°C
c. 4.46×10^{-5}/°C
d. None of these

122. 250 g of water and equal volume of alcohol of mass 200 g are replaced successively in the same calorimeter and cool from 606°C to 55°C in 130 s and 67 s, respectively. If the water equivalent of the calorimeter is 10 g, then the specific heat of alcohol in cal/g-°C is

a. 1.30 b. 0.67
c. 0.62 d. 0.985

123. A 2 g bullet moving with a velocity of 200 m/s is brought to a sudden stoppage by an obstacle. The total heat produced goes to the bullet. If the specific heat of the bullet is 0.03 cal/g-°C, the rise in its temperature will be

a. 158.0°C b. 15.80°C
c. 1.58°C d. 0.1580°C

124. 10 g of ice at −20°C is dropped into a calorimeter containing 10 g of water at 10°C; the specific heat of

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water is twice that of ice. When equilibrium is reached, the calorimeter will contain

a. 20 g of water
b. 20 g of ice
c. 10 g ice and 10 g of water
d. 5 g ice and 15 g of water

125. A steel ball of mass 0.1 kg falls freely from a height of 10 m and bounces to a height of 5.4 m from the ground. If the dissipated energy in this process is absorbed by the ball, the rise in its temperature is (specific heat of steel = 460 K/kg/°C, g = 10 m/s²)

a. 0.01°C **b.** 0.1°C
c. 1°C d. 1.1°C

126. The earth receives on its surface radiation from the sun at the rate of 1400 W/m². The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^{8} m. Treating sun as a black body, it follows from the above data that its surface temperature is

a. 5801 K **b.** 10^6 K
c. 50.1 K **d.** 5801°C

127. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$, respectively, in the steady state. Assuming that only heat conduction takes place, temperature of point C is

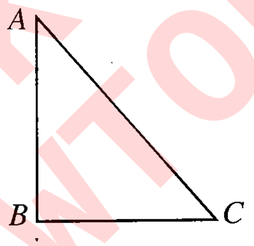

Fig. 1.89

a. $\dfrac{3T}{\sqrt{2}+1}$ **b.** $\dfrac{T}{\sqrt{2}+1}$

c. $\dfrac{T}{3(\sqrt{2}-1)}$ **d.** $\dfrac{T}{\sqrt{2}-1}$

128. The coefficient of linear expansion of crystal in one direction is α_1 and that in every direction perpendicular to it is α_2. The coefficient of cubical expansion is

a. $\alpha_1 + \alpha_2$ **b.** $2\alpha_1 + \alpha_2$
c. $\alpha_1 + 2\alpha_2$ **d.** None of these

129. A uniform metal rod is used as a bar pendulum. If the room temperature rises by 10°C, and the coefficient of linear expansion of the metal of the rod is 2×10^{-6} per °C, the period of the pendulum will have percentage increase of

a. -2×10^{-3} **b.** -1×10^{-3}
c. 2×10^{-3} **d.** 1×10^{-3}

130. An iron rod of length 50 cm is joined at an end to an aluminum rod of length 100 cm. All measurements refer to 20°C. The coefficients of linear expansion of iron and aluminum are 12×10^{-6}/°C and 24×10^{-6}/°C, respectively. The average coefficient of expansion of composite system is

a. 36×10^{-6}/°C **b.** 12×10^{-6}/°C
c. 20×10^{-6}/°C **d.** 48×10^{-6}/°C

131. The coefficient of apparent expansion of mercury in a glass vessel is 153×10^{-6}/°C and in a steel vessel is 114×10^{-6}/°C. If α for steel is 12×10^{-6}/°C, then that of glass is

a. 9×10^{-6}/C **b.** 6×10^{-6}/C
c. 36×10^{-6}/C **d.** 27×10^{-6}/C

132. A vessel is partly filled with a liquid. Coefficients of cubical expansion of material of the vessel and liquid are γ_V and γ_L, respectively. If the system is heated, then volume unoccupied by the liquid will necessarily

a. remain unchanged if $\gamma_V = \gamma_L$
b. increase if $\gamma_V = \gamma_L$
c. decrease if $\gamma_V = \gamma_L$
d. none of the above

133. An electrically heated coil is immersed in a calorimeter containing 360 g of water at 10°C. The coil consumes energy at the rate of 90 W. The water equivalent of calorimeter and coil is 40 g. The temperature of water after 10 min is

a. 4.214°C **b.** 42.14 °C
c. 30°C **d.** None of these

134. Which of the following, when mixed, would raise the temperature of 20 g of water at 30°C most?

a. 20 g of water at 40°C
b. 40 g of water at 35°C
c. 10 g of water at 50°C
d. 4 g of water at 80°C

135. A kettle with 2 L water at 27°C is heated by operating coil heater of power 1 kW. The heat is lost to the atmosphere at constant rate 160 J/s, when it is open. In how much time will water be heated to 77°C (sp. heat of water = 4.2 kJ/kg) with the lid open?

a. 8 min 20 s **b.** 6 min 2 s
c. 14 min **d.** 7 min

136. A vessel contains M grams of water at a certain temperature and water at certain other temperature is passed into it at a constant rate of m g/s. The variation of temperature of the mixture with time is shown in Fig.1.90. The values of M and m are, respectively (the heat exchanged after a long time is 800 cal)

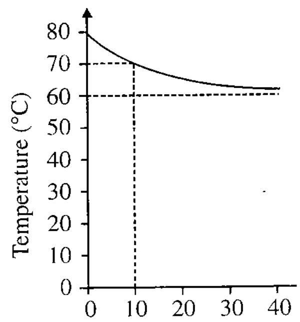

Fig. 1.90

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a. 40 & 2 **b.** 40 & 4
c. 20 & 4 **d.** 20 & 2

137. Four spheres A, B, C and D have their radii in arithmetic progression and the specific heat capacities of their substances are in geometric progression. If the ratios of heat capacities of D and B to that of C and A are as 8:27. The ratio of masses of B and A is (assume same density for all spheres)

a. 8:1 **b.** 4:1
c. 1:8 **d.** 1:4

138. Two plates identical in size, one of black and rough surface (B_1) and the other smooth and polished (A_2) are interconnected by a thin horizontal pipe with a mercury pellet at the centre. Two more plates A_1 (identical to A_2) and B_2 (identical to B_1) are heated to the same temperature and placed closed to the plates B_1, and A_2 as shown in Fig. 1.91. The mercury pellet

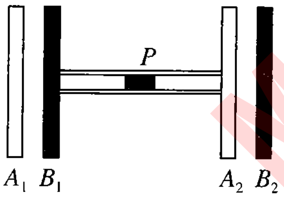

Fig. 1.91

a. moves to the right
b. moves to the left
c. remains stationary
d. starts oscillating left and right

139. Two hollow spheres of different materials, one with double the radius and one-fourth wall thickness of the other, are filled with ice. If the times taken for complete melting of ice in the larger to the smaller one are in the ratio of 25 : 16, then their corresponding thermal conductivities are in the ratio

a. 4:5 **b.** 5:4
c. 8:25 **d.** 25:8

140. The temperatures across two different slabs A and B are shown in the steady state (as shown in Fig.1.92).The ratio of thermal conductivities of A and B is

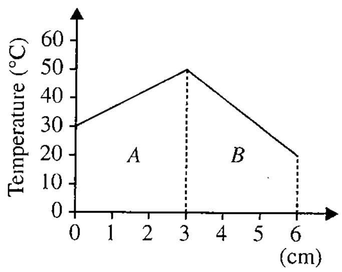

Fig. 1.92

a. 2:3 **b.** 3:2
c. 1:1 **d.** 5:3

141. Figure 1.93 shows the graph of the temperature θ of a section of a bar of length l, with distance x from the hot end, in the steady state for a metal rod polished with a poor thermal conductor on its lateral surface. Which is the correct graph?

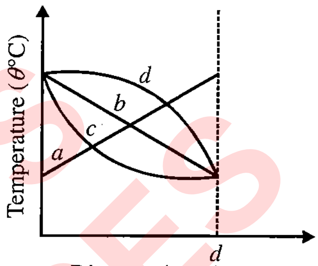

Fig. 1.93

a. a **b.** b
c. c **d.** d

142. A lead bullet just melts when stopped by an obstacle. Assuming that 25% of heat is absorbed by the obstacle, find the minimum velocity of the bullet if its initial temperature is 27°C (melting point of lead = 327°C; specific heat of lead = 0.03 cal/g °C; latent heat of fusion of lead = 6 cal/g and J = 4.2 J/cal).

a. 450 m/s **b.** 398 m/s
c. 420 m/s **d.** 410 m/s

143. An earthen pitcher loses 1 g of water per minute due to evaporation. If the water equivalent of pitcher is 0.5 kg and the pitcher contains 9.5 kg of water, calculate the time required for the water in the pitcher to cool to 28°C from its original temperature of 30°C. Neglect radiation effects. Latent heat of vapourization of water in this range of temperature is 580 cal/g and specific heat of water is 1 k cal/g C°.

a. 38.6 min **b.** 30.5 min
c. 34.5 min **d.** 41.2 min

144. 5 g of water at 30°C and 5 g of ice at –20°C are mixed together in a calorimeter. Find the final temperature of the mixture. Assume water equivalent of calorimeter to be negligible, sp. heats of ice and water are 0.5 and 1 cal/g C°, and latent heat of ice is 80 cal/g.

a. 0°C **b.** 10°C
c. –30°C **d.** >10°C

145. A body cools in 7 min from 60°C to 40°C. What will be its temperature after the next 7 min? The temperature of surroundings is 10°C.

a. 28°C **b.** 25°C
c. 30°C **d.** 22°C

146. A room at 20°C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m² and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is

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0.2 cal/m C° s and mechanical equivalent of heat is 4.2 J/cal.

a. 13.69°C **b.** 15.24°C
c. 17.85°C **d.** 19.96°C

147. A body cools from 50°C to 49°C in 5 s. How long will it take to cool from 40°C to 39.5°C? Assume the temperature of surroundings to be 30°C and Newton's law of cooling to be valid:

a. 2.5 s **b.** 10 s
c. 20 s **d.** 5 s

148. One end of a copper rod of uniform cross section and length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in the steady state, the mass of ice melting be equal to that of the steam produced in same interval of time. Assume that the whole system is insulated from surroundings:

$$[L_{ice} = 80 \text{ cal/g}, L_{steam} = 540 \text{ cal/g}]$$

a. 10.34 cm from the end at 100°C
b. 10.34 mm from the end at 100°C
c. 1.034 cm from the end at 100°C
d. 1.034 m from the end at 100°C

149. When the temperature of a black body increases, it is observed that the wavelength corresponding to maximum energy changes from 0.26 μm to 0.13 μm. The ratio of the emissive powers of the body at the respective temperatures is

a. $\frac{16}{1}$ **b.** $\frac{4}{1}$
c. $\frac{1}{4}$ **d.** $\frac{1}{16}$

150. The temperature of a room heated by a heater is 20°C when outside temperature is –20°C and it is 10°C when the outside temperature is –40°C. The temperature of the heater is

a. 80°C **b.** 100°C
c. 40°C **d.** 60°C

151. The radiation emitted by a star A is 1000 times that of the sun. If the surface temperatures of the sun and star A are 6000 K and 2000 K, respectively, the ratio of the radii of the star A and the sun is

a. 300:1 **b.** 600:1
c. 900:1 **d.** 1200:1

152. A planet radiates heat at a rate proportional to the fourth power of its surface temperature T. If such a steady temperature of the planet is due to an exactly equal amount of heat received from the sun then which of the following statements is true?

a. The planet's surface temperature varies inversely as the distance of the sun.
b. The planet's surface temperature varies directly as the square of its distance from the sun.
c. The planet's surface temperature varies inversely as the square root of its distance from the sun.
d. The planet's surface temperature is proportional to the fourth power of distance from the sun.

153. A sphere and a cube of same material and same total surface area are placed in the same evacuated space turn by turn after they are heated to the same temperature. Find the ratio of their initial rates of cooling in the enclosure.

a. $\sqrt{\frac{\pi}{6}}:1$ **b.** $\sqrt{\frac{\pi}{3}}:1$
c. $\frac{\pi}{\sqrt{6}}:1$ **d.** $\frac{\pi}{\sqrt{3}}:1$

154. A planet is at an average distance d from the sun and its average surface temperature is T. Assume that the planet receives energy only from the sun and loses energy only through radiation from the surface. Neglect atmospheric effects. If $T \propto d^{-n}$, the value of n is

a. 2 **b.** 1
c. $\frac{1}{2}$ **d.** $\frac{1}{4}$

155. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. Wien's constant $b = 2.88 \times 10^6$ nm – K, Then

a. $U_1 = 0$ **b.** $U_2 = 0$
c. $U_1 = U_2$ **d.** $U_2 > U_1$

Multiple Correct Answers Type

Solutions on page 1.94

1. Due to thermal expansion with rise in temperature:

a. metallic scale reading becomes lesser than true value
b. pendulum clock becomes fast
c. a floating body sinks a little more
d. the weight of a body in a liquid increases

2. During heat exchange, temperature of a solid mass does not change. In this process, heat

a. is not being supplied to the mass
b. is not being taken out from the mass
c. may have been supplied to the mass
d. may have been taken out from the mass

3. A metallic circular disc having a circular hole at its centre rotates about an axis passing through its centre and perpendicular to its plane. When the disc is heated:

a. its speed will decrease
b. its diameter will decrease
c. its moment of inertia will increase
d. its speed will increase

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4. A polished metallic piece and a black painted wooden piece are kept in open in bright sun for a long time.

a. the wooden piece will absorb less heat than the metallic piece
b. the wooden piece will have a lower temperature than the metallic piece
c. if touched, the metallic piece will be felt hotter than the wooden piece
d. when the two pieces are removed from the open to a cold room, the wooden piece will lose heat at a faster rate than the metallic piece

5. When m grams of water at 10°C is mixed with m grame of ice at 0°C, which of the following statements are false?

a. The temperature of the system will be given by the equation $m \times 80 + m \times 1 \times (T - 0) = m \times 1 \times (10 - T)$
b. Whole of ice will melt and temperature will be more than 0°C but lesser than 10°C
c. Whole of ice will melt and temperature will be 0°C
d. Whole of ice will not melt and temperature will be 0°C

6. A heated body emits radiation which has maximum intensity at frequency v_m. If the temperature of the body is doubled:

a. the maximum intensity radiation will be at frequency $2v_m$
b. the maximum intensity radiation will be at frequency $(1/2)\ v_m$
c. the total emitted energy will increase by a factor of 16
d. the total emitted energy will increase by a factor of 2

7. During the melting of a slab of ice at 273 K at the atmospheric pressure

a. positive work is done by the ice–water system on the atmosphere
b. positive work is done on the ice–water system by the atmosphere
c. the internal energy of the ice–water system increases
d. the internal energy of the ice–water system decreases

8. The temperature drop through a two-layer furnace wall is 900°C. Each layer is of equal area of cross section. Which of the following actions will result in lowering the temperature θ of the interface?

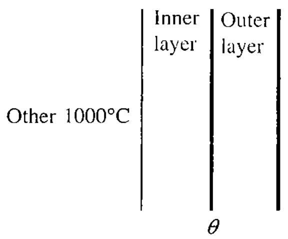

Fig. 1.94

a. By increasing the thermal conductivity of outer layer
b. By increasing the thermal conductivity of inner layer
c. By increasing thickness of outer layer
d. By increasing thickness of inner layer

9. The ends of a metal rod are kept at temperatures θ_1 and θ_2 with $\theta_2 > \theta_1$. The rate of flow of heat along the rod is directly proportional to

a. the length of the rod
b. the diameter of the rod
c. the cross-sectional area of the rod
d. the temperature difference $(\theta_2 - \theta_1)$ between the ends of the rod

10. The water equivalent of a copper calorimeter is 4.5 g. If the specific heat of copper is 0.09 cal/g/°C

a. mass of the calorimeter is 0.5 kg
b. thermal capacity of the calorimeter is 4.5 cal/°C
c. heat required to raise the temperature of the calorimeter by 8°C will be 36 cal
d. heat required to melt 15 g of ice placed in the calorimeter will be 1200 cal

11. Choose the correct statements from the following:

a. A temperature change which increases the length of a steel rod by 0.1% will increase its volume by nearly 0.3%
b. The specific heat of a solid is different when the solid is heated at (i) constant pressure and (ii) the constant volume
c. The thermal conductivity of air being less than that for wool, we prefer wool to air for thermal insulation
d. When the distance between two fixed points is measured with a steel tape, the observed reading will be less on a hot day than on a cold day

12. Choose the correct statements from the following:

a. Good reflectors are good emitters of thermal radiation
b. Burns caused by water at 100°C are more severe than those caused by steam at 100°C
c. If the earth did not have atmosphere, it would become intolerably cold
d. It is impossible to construct a heat engine of 100% efficiency

13. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.

a. percentage rise in the area of a face is 0.4%
b. percentage rise in the thickness is 0.4%
c. percentage rise in the volume is 0.6%
d. coefficient of linear expansion of copper is 0.25×10^{-4}/°C

14. A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel unoccupied by the liquid remains constant. Then the volume of the liquid (V_L) volume of the vessel (V_V), the coefficient of cubical expansion of the material of the vessel (γ_v) and of the solid (γ_L) are related as

a. $\gamma_L > \gamma_v$
b. $\gamma_L < \gamma_v$
c. $\dfrac{\gamma_v}{\gamma_L} = \dfrac{V_v}{V_L}$
d. $\dfrac{\gamma_v}{\gamma_L} = \dfrac{V_L}{V_v}$

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15. Two identical objects A and B are at temperatures T_A and T_B, respectively. Both objects are placed in a room with perfectly absorbing walls maintained at a temperature T ($T_A > T > T_B$). The objects A and B attain the temperature T eventually. Select the correct statements from the following:

a. A only emits radiation, while B only absorbs it until both attain the temperature T

b. A loses more heat by radiation than it absorbs, while B absorbs more radiation than it emits, until they attain the temperature T

c. Both A and B only absorb radiation, but do not emit it, until they attain the temperature T

d. Each object continuous to emit and absorb radiation even after attaining the temperature T

16. Seven identical rods of material of thermal conductivity k are connected as shown in Fig.1.95. All the rods are of identical length l and cross-sectional area A. If the one end B is kept at 100°C and the other end is kept at 0°C, what would be the temperatures of the junctions C, D and E (θ_C, θ_D and θ_E) in the steady state?

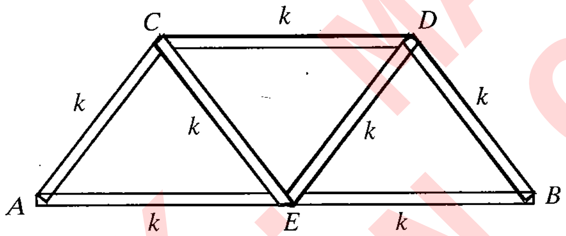

Fig. 1.95

a. $\theta_C > \theta_E > \theta_D$

b. $\theta_E = 50°C$ and $\theta_D = 37.5$ °C

c. $\theta_E = 50°C$, $\theta_C = 62.5°C$ and $\theta_D = 37.5°C$

d. $\theta_E = 50°C$, $\theta_C = 60°C$ and $\theta_D = 40°C$

17. A clock is calibrated at a temperature of 20°C. Assume that the pendulum is a thin brass rod of negligible mass with a heavy bob attached to the end ($\alpha_{brass} = 19 \times 10^{-6}$/K)

a. On a hot day at 30°C the clock gains 8.2 s

b. On a hot day at 30°C the clock loses 8.2 s

c. On a cold day at 10°C the clock gains 8.2 s

d. On a cold day at 10°C the clock loses 8.2 s

18. A rod of copper, uniform along its length l and of a rectangular cross section of sides of length a and width b ($< a$) has one end maintained at 100°C and the other end at 0°C. The rod is insulated so that no heat is lost from the sides. Let Q_l be the amount of heat per second that is transmitted along its length, Q_a the heat transmitted parallel to a and Q_b the heat transmitted parallel to b across any section after the steady state conditions are reached. Then

a. Q_l = constant, $Q_a > Q_b$ and Q_a as well as Q_b are non-zero

b. $Q_l = 0$, $Q_a = Q_b \neq 0$

c. $Q_l = 0$, $Q_a = Q_b = 0$

d. Q_l = constant, $Q_a = Q_b = 0$

19. A circular ring (centre O) of radius a, and of uniform cross section is made up of three different metallic rods AB, BC and CA (joined together at the points A, B and C in pairs) of thermal conductivities α_1, α_2 and α_3, respectively (see diagram). The junctions A, B and C are maintained at the temperatures 100°C, 50°C and 0°C, respectively. All the rods are of equal lengths and cross sections. Under steady state conditions, assume that no heat is lost from the sides of the rods. Let Q_1, Q_2 and Q_3 be the rates of transmission of heat along the three rods AB, BC and CA. Then

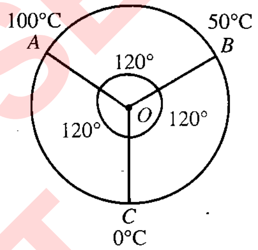

Fig. 1.96

a. $Q_1 = Q_2 = Q_3$ and all are transmitted in the clockwise sense

b. Q_1 and Q_2 flow in clockwise sense and Q_3 in the anticlockwise sense.

c. $Q_1 : Q_2 : Q_3 :: \alpha_1 : \alpha_2 : 2\alpha_3$

d. $\dfrac{Q_1}{\alpha_1} + \dfrac{Q_2}{\alpha_2} = \dfrac{Q_3}{\alpha_3}$

20. Eleven identical rods are arranged as shown in Fig.1.97. Each rod has length l, cross-sectional area A and thermal conductivity of material L. Ends A and F are maintained at temperatures T_1 and (T_2 ($< T_1$)), respectively. If lateral surface of each rod is thermally insulated, the rate of heat transfer $\left(\dfrac{dQ}{dt}\right)$ in each rod is

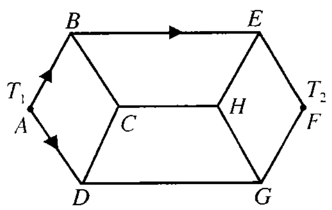

Fig. 1.97

a. $\left(\dfrac{dQ}{dt}\right)_{AB} = \left(\dfrac{dQ}{dt}\right)_{CD}$ **b.** $\left(\dfrac{dQ}{dt}\right)_{BE} = \dfrac{2}{7}\dfrac{(T_1 - T_2)KA}{l}$

c. $\left(\dfrac{dQ}{dt}\right)_{CH} \neq \left(\dfrac{dQ}{dt}\right)_{DG}$ **d.** $\left(\dfrac{dQ}{dt}\right)_{BC} = \left(\dfrac{dQ}{dt}\right)_{DC}$

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Assertion-Reasoning Type

Solutions on page 1.97

In the following questions, a statement of assertion (Statement I) is given which is followed by a corresponding statement of reason (Statement II). Examine the statements carefully and choose the correct option according to the following options.

a. Statement I is true, Statement II is true and Statement II is the correct explanation for Statement I.
b. Statement I is true, Statement II is true and Statement II is NOT the correct explanation for Statement I.
c. Statement I is true, Statement II is false.
d. Statement I is false, Statement II is true.

1. Statement I: Two solid cylindrical rods of identical size and different thermal conductivity K_1 and K_2 are connected in series. Then the equivalent thermal conductivity of two rods system is less than that value of thermal conductivity of either rod.

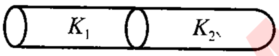

Fig. 1.98

Statement II: For two cylindrical rods of identical size and different thermal conductivity K_1 and K_2 connected in series, the equivalent thermal conductivity K is given by

$$\frac{2}{K}=\frac{1}{K_1}+\frac{1}{K_2}$$

2. Statement I: As the temperature of the black body increases, the wavelength at which the spectral intensity (E_λ) is maximum decreases.
Statement II: The wavelength at which the spectral intensity will be maximum for a black body is proportional to the fourth power of its absolute temperature.

3. Statement I: The thermal resistance of a multiple layer is equal to the sum of the thermal resistance of the individual laminas.
Statement II: Heat transferred is directly proportional to the temperature gradient in each layer.

4. Statement I: In natural convection, the fluid motion is caused due to density difference produced by temperature gradient.
Statement II: In forced convection, the fluid is forced to flow along the solid surface by means of fans or pumps.

5. Statement I: The bulb of one thermometer is spherical while that of the other is cylindrical. Both have equal amounts of mercury. The response of the cylindrical bulb thermometer will be quicker.
Statement II: Heat conduction in a body is directly proportional to cross-sectional area.

6. Statement I: The expanded length l of a rod of original length l_0 is not correctly given by (assuming α to be constant with T) $l = l_0(1+\alpha\,\Delta T)$ if $\alpha\,\Delta T$ is large.
Statement II: It is given by $l = l_0 e^{\alpha \Delta T}$, which cannot be treated as being approximately equal to $l = l_0(1+\alpha\,\Delta T)$ for large value a ΔT.

7. Statement I: A common model of a solid assumes the atoms to be points executing SHM about mean lattice positions. This model cannot explain thermal expansion of solids.
Statement II: The average distance over a time period of oscillation between the particles remains constant.

8. Statement I: A body that is a good radiator is also a good absorber of radiation at a given wavelength.
Statement II: According to Kirchoff's law the absorptivity of a body is equal to its emissivity at a given wavelength.

9. Statement I: In thermal conduction, energy is transferred due to chaotic motion of conduction electron and atomic vibrations from region of high temperature to low temperature.
Statement II: There is overall transference of particles of conducting body.

10. Statement I: Two stars S_1 and S_2 radiate maximum energy at 360 nm and 480 nm, respectively. Ratio of their absolute temperatures is 4:3.
Statement II: According to Wien's law $\lambda T = b$ (constant).

11. Statement I: Greater is the coefficient of thermal conductivity of a material, smaller is the thermal resistance of a rod of that material.
Statement II: Thermal resistance is the ratio of temperature difference between the ends of the conductor and rate of flow of heat.

12. Statement I: The coefficient of volume expansion has dimension K^{-1}.
Statement II: The coefficient of volume expansion is defined as the change in volume per unit volume per unit change in temperature.

Comprehension Type

Solutions on page 1.97

For Problems 1–3

A body cools in a surrounding of constant temperature 30°C. Its heat capacity is 2 J/°C. Initial temperature of the body is 40°C. Assume Newton's law of cooling is valid. The body cools to 38°C in 10 min.

1. In further 10 min it will cool from 38°C to _______:

a. 36°C **b.** 36.4°C **c.** 37°C **d.** 37.5°C

2. The temperature of the body in °C denoted by θ. The variation of θ versus time t is best denoted as

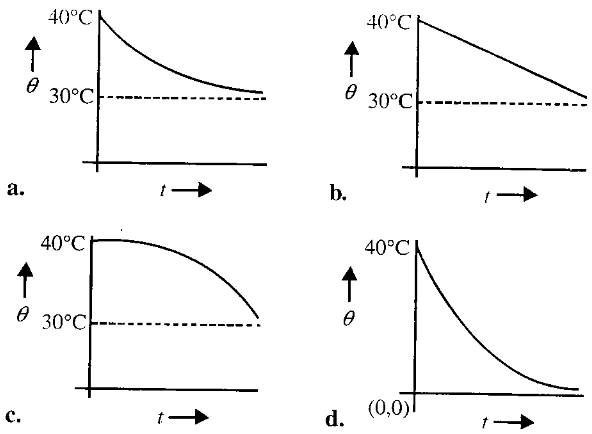

Fig. 1.99

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3. When the body temperature has reached 38°C, it is heated again so that it reaches 40°C in 10 min. The heat required from a heater by the body is
   a. 3.6 J b. 0.364 J
   c. 8 J d. 4 J

For Problems 4–6

The internal energy of a solid also increases when heat is transferred to it from its surroundings. A 5 kg solid bar is heated at atmospheric pressure. Its temperature increases from 20°C to 70°C. The linear expansion coefficient of solid bar is 1×10^{-3}/C°. The density of solid bar is 50 kg/m³. The specific heat capacity of solid bar is 200 J/kg C°. The atmospheric pressure is 1×10^5 N/m².

4. The work done by the solid bar due to thermal expansion, under atmospheric pressure, is
   a. 500 J b. 1000 J
   c. 1500 J d. 2000 J

5. The heat transferred to the solid bar is
   a. 49000 J b. 50000 J
   c. 50500 J d. 51000 J

6. The increase in the internal energy of the solid bar is
   a. 49500 J b. 48500 J
   c. 49000 J d. 50000 J

For Problems 7–9

A wire of length 1 m and radius 10^{-3} m is carrying a heavy current and is assumed to radiate as a black body. At equilibrium, its temperature is 900 K while that of surrounding is 300 K. The resistivity of the material of the wire at 300 K is $\pi^2 \times 10^{-8}$ ohm m and its temperature coefficient of resistance is 7.8×10^{-3}/°C (Stefan's constant $\sigma = 5.68 \times 10^{-8}$ W/m²K⁴).

7. The resistivity of wire at 900 K is nearly
   a. 2.4×10^7 ohm m b. 2.4×10^{-7} ohm m
   c. 1.2×10^{-7} ohm m d. 1.2×10^7 ohm m

8. Heat radiated per second by the wire is nearly
   a. 23 W b. 230 W
   c. 2300 W d. 23000 W

9. The current in the wire is nearly
   a. 0.555 A b. 5.5 A
   c. 55 A d. 550 A

For Problems 10–12

A solid aluminium sphere and a solid lead sphere of same radius are heated to the same temperature and allowed to cool under identical surrounding temperatures. The specific heat capacity of aluminium = 900 J/kg°C and that of lead = 130 J/kg°C. The density of lead = 10^4 kg/m³ and that of aluminum = 2.7×10^3 kg/m³ kg/m³. Assume that the emissivity of both the spheres is the same.

10. The ratio of rate of fall of temperature of the aluminium sphere to the rate of heat loss from the lead sphere is
    a. 1:1 b. 9:1.3
    c. 11:2.7 d. 1:4

11. The ratio of rate of fall of temperature of the aluminium sphere to the rate of fall of temperature of the lead sphere is
    a. 1000:39 b. 39:1000
    c. 1:1 d. 13:90

12. When the temperature of spheres T is not too different from the surrounding temperature, the radiating object obeys
    a. Stefan–Boltzmann law
    b. Newton's law of cooling
    c. Dulong–Petit law
    d. Planck's law

For Problems 13–15

Assume that the thermal conductivity of copper is twice that of aluminium and four times that of brass. Three metal rods made of copper, aluminium and brass are each 15 cm long and 2 cm in diameter. These rods are placed end to end, with aluminium between the other two. The free ends of the copper and brass rods are maintained at 100°C and 0°C, respectively. The system is allowed to reach the steady state condition. Assume there is no loss of heat anywhere.

13. When steady state condition is reached everywhere, which of the following statements is true?
    a. No heat is transmitted across the copper–aluminium or aluminium–brass junctions.
    b. More heat is transmitted across the copper–aluminium junction than across the aluminium–brass junction.
    c. More heat is transmitted across the aluminium–brass junction than the copper–aluminium junction.
    d. Equal amount of heat is transmitted at the copper–aluminium and aluminium–brass junctions.

14. Under steady state condition, the equilibrium temperature of the copper–aluminium junction will be
    a. 86°C b. 18.8°C
    c. 57°C d. 73°C

15. Under steady state condition, the equilibrium temperature of the aluminium–brass junction will be
    a. 57°C b. 35°C
    c. 18.8°C d. 28.5°C

For Problems 16–18

A thin copper rod of uniform cross section A square metres and of length L metres has a spherical metal sphere of radius r metre at its one end symmetrically attached to the copper rod. The thermal conductivity of copper is K and the emissivity of the spherical surface of the sphere is ε. The free end of the copper rod is maintained at the temperature T kelvin by supplying thermal energy from a P watt source. Steady state conditions are allowed to be established while the rod is properly insulated against heat loss from its lateral surface. Surroundings are at 0°C. Stefan's constant = σ W/m² K⁴.

16. After the steady state conditions are reached, the temperature of the spherical end of the rod, T_S is

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a. $T_S = T - \dfrac{PL}{KA}$ b. $T_S = 0°C$

c. $T_S = \dfrac{PL}{KA}$ d. $T_S = T - \dfrac{P(L+r)}{KA}$

17. The net power that will be radiated out, P_S, from the sphere after steady state conditions are reached is

a. $P_S = P$ b. $P_S = \dfrac{PA}{4\pi r^2}$

c. $P_S = 0$ d. $P_S = \sigma \varepsilon T_S^4$

18. If the metal sphere attached at the end of the copper rod is made of brass, whose thermal conductivity is $K_b < K$, then which of the following statements is true?

a. The temperature of the sphere will, under steady state conditions, continue to be T_B

b. The power that will be radiated out from the sphere will still be P_S

c. It will take smaller time for steady state conditions to be reached

d. The rate of thermal energy transmitted across the copper rod, under steady state, will be reduced

For Problems 19–21

An immersion heater, in an insulated vessel of negligible heat capacity brings 10 g of water to the boiling point from 16°C in 7 min. Then

19. Power of heater is nearly

a. 8.4 × 10³ b. 84 W

c. 8.4 × 10³ cal/s d. 20 W

20. The water is replaced by 200 g of alcohol, which is heated from 16°C to the boiling point of 78°C in 6 min 12 s whereas 30 g are vapourized in 5 min 6 s. The specific heat of alcohol is

a. 0.6 J/kg°C b. 0.6 cal/g°C

c. 0.6 cal/kg°C d. 6 J/kg°C

21. The heat of vapourization of alcohol is

a. 854 J/kg b. 854 × 10³ J/kg

c. 204 cal/g d. 204 cal/kg

For Problems 22–24

A body of area 0.8×10^{-2} m² and mass 5×10^{-4} kg directly faces the sun on a clear day. The body has an emissivity of 0.8 and specific heat of 0.8 cal/kg K. The surroundings are at 27°C. (solar constant = 1.4 kW/m²).

22. The rate of rise of the body's temperature is nearly

a. 0.36°C/s b. 3.6 K/s

c. 36°C/s d. 72 K/s

23. The maximum attainable temperature of the body is

a. 396 K b. 396°C

c. 85°C d. 85 K

24. The temperature that the body would reach if it lost all its heat by radiation is

a. 396 K b. 296°C

c. 85°C d. 85 K

For Problems 25–27

A copper collar is to fit tightly about a steel shaft that has a diameter of 6 cm at 20°C. The inside diameter of the copper collar at that temperature is 5.98 cm.

25. To what temperature must the copper collar be raised so that it will just slip on the steel shaft, assuming the steel shaft remains at 20°C? ($\alpha_{\text{copper}} = 17 \times 10^{-6}$/K)

a. 324°C b. 21.7°C

c. 217°C d. 32.4°C

26. The tensile stress in the copper collar when its temperature returns to 20°C is ($Y = 11 \times 10^{10}$ N/m²)

a. 1.34×10^5 N/m² b. 3.68×10^{-12} N/m²

c. 3.68×10^8 N/m² d. 1.34×10^{-12} N/m²

27. If the breaking stress of copper is 230 N/m², at what temperature will the copper collar break as it cools?

a. 20°C b. 47°C c. 94°C d. 217°C

For Problems 28–30

Two insulated metal bars each of length 5 cm and rectangular cross section with sides 2 cm and 3 cm are wedged between two walls, one held at 100°C and the other at 0°C. The bars are made of lead and silver. $K_{pb} = 350$ W/mK, $K_{Ag} = 425$ W/mK.

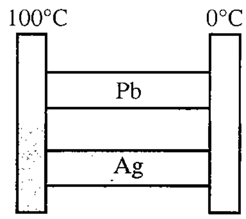

Fig. 1.100

28. Thermal current through lead bar is

a. 210 W b. 420 W

c. 510 W d. 930 W

29. Total thermal current through the two-bar system is

a. 210 W b. 420 W

c. 510 W d. 930 W

30. Equivalent thermal resistance of the two bar system is

a. 0.1 K/W b. 0.23 K/W

c 0.19 K/W d. 0.42 K/W

Matching Column Type

Solutions on page 1.100

1. A copper rod (initially at room temperature 20°C) of non-uniform cross section is placed between a steam chamber

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at 100°C and ice water chamber at 0°C. A and B are cross sections as shown in Fig.1.101. Then match the statements in Column I with results in Column II comparing only between cross section A and B. (The mathematical expressions in Column I have their usual meanings in heat transfer.)

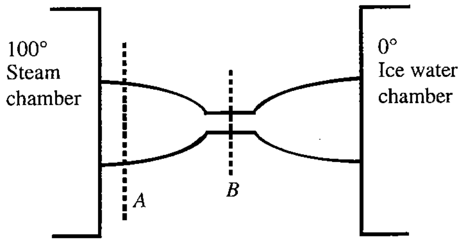

Fig. 1.101

Column I	Column II
i. Initially rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	**a.** maximum at section A
ii. At steady state rate of heat flow $\left(\frac{dQ}{dt}\right)$ will be	**b.** maximum at section B
iii. At steady state temperature gradient $\left\lvert\left(\frac{dT}{dx}\right)\right\rvert$ will be	**c.** minimum at section B
iv. At steady state rate of change of temperature $\left(\frac{dT}{dt}\right)$ will be	**d.** same for all section

2. In Column I some statements or expressions related to first law of thermodynamics are given, and corresponding process are given in Column II. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. Work done by the system on the surroundings can be non-zero in	**a.** adiabatic process
ii. $dU = nC_v\,dT$ is valid for	**b.** isothermal process
iii. dU is zero for	**c.** isothermal expansion process
iv. $dQ = nCdT$ is non zero for	**d.** polytropic process

3. The surface of a household radiator has an emissivity of 0.55 and an area of 1.5 m^2. Its equilibrium temperature is 50°C and the surroundings are at 22°C ($\sigma = 5.67\times10^{-8}$ W/m^{2}K^4).

Column I	Column II
i. At what rate is radiation emitted by the radiator?	**a.** 155
ii. At what rate is radiation absorbed by the radiator?	**b.** 509
iii. What is the net value of radiation from the radiator?	**c.** 354

4. A rod AB of uniform cross section consists of four sections AC, CD, DE and EB of different metals with thermal conductivities K, (0.8) K, (1.2) K and (1.50) K, respectively. Their lengths are respectively L, (1.2) L, (1.5) L and (0.6)L. They are joined rigidly in succession at C, D and E to form the rod AB. The end A is maintained at 100°C and the end B is maintained at 0°C. The steady state temperatures of the joints C, D and E are respectively T_C, T_D and T_E. Column I lists the temperature differences $(T_A - T_C)$, $(T_C - T_D)$, $(T_D - T_E)$ and $(T_E - T_B)$ in the four sections and Column II their values jumbled up. Match each item in Column I with its correct value in Column II.

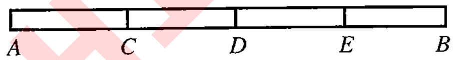

Fig. 1.102

Column I	Column II
i. $(T_A - T_C)$	**a.** 9.6
ii. $(T_C - T_D)$	**b.** 30.1
iii. $(T_D - T_E)$	**c.** 24.1
iv. $(T_E - T_B)$	**d.** 36.2

5. A piece of metal of density ρ_1 floats on mercury of density ρ_2. The coefficients of expansion of the metal and mercury are γ_1 and γ_2, respectively. The temperatures of both mercury and metal are increased by ΔT. Then match the following:

Column I	Column II
i. If $\gamma_2 > \gamma_1$	**a.** no effect on fraction of solid submerged in mercury
ii. $\gamma_2 = \gamma_1$	**b.** fraction of the volume of metal submerged in mercury increases
iii. If $\gamma_2 < \gamma_1$	**c.** the solid sinks
	d. the solid lifts up

6. In a container of negligible mass 'm' grams of steam at 100°C is added to 100 g of water that has temperature 20°C. If no heat is lost to the surroundings at equilibrium, match the items given in Column I with that in Column II.

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Column I	Column II
i. Mass of steam in the mixture, if $m = 20$ g (in g)	**a.** 114.8
ii. Mass of water in the mixture, if $m = 20$ g (in g)	**b.** 76.4
iii. If $m = 20$ g, final temperature of the mixture (in °C)	**c.** 5.2
iv. If $m = 10$ g, final temperature of the mixture (in °C)	**d.** 100

7. On the average, the temperature of the earth's crust increases 1°C for every 30 m of depth. The average thermal conductivity of the earth's crust is 0.75 J/m s K. Solar constant is 1.35 kW/m². Match the items in Column I with that in Column II:

Column I	Column II
i. Heat lost by the core of the earth per second due to conduction (in W)	**a.** 276
ii. Heat received by the earth per second from the sun (in W)	**b.** 7.5×10^{-5}
iii. If $e = 1$, average surface temperature of the earth in equilibrium (in K)	**c.** 1.3×10^{13}
iv. Ratio of heat lost by the earth to the heat received from the sun	**d.** 1.7×10^{17}

Integer Answer Type

Solutions on page 1.101

1. 2 kg of ice at −15°C is mixed with 2.5 kg of water at 25°C in an insulating container. If the specific heat capacities of ice and water are 0.5 cal/g°C and 1 cal/g°C, find the amount of water present in the container? (in kg nearest integer)

2. Four cylindrical rods of same material with length and radius $(\ell,r),(2\ell,r),(2\ell,2r)$ and $(\ell,2r)$ are connected between two reservoirs at 0°C and 100°C. Find the ratio of the maximum to minimum rate of conduction in them.

3. In two experiments with a continuous flow calorimeter to determine the specific heat capacity of a liquid, an input power of 16 W produced a rise of 10 K in the liquid. When the power was doubled, the same temperature rise was achieved by making the rate of flow of liquid three times faster. Find the power lost (in W) to the surrounding in each case.

4. 2 kg of ice at −20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water (in kg) remaining in the container.

5. A clock with a metallic pendulum at 15°C runs faster by 5 s each day and at 30°C, runs slow by 10 s. Find the coefficient of linear expansion of the metal. (nearly in 10^{-6}/°C)

6. A body is cooled in 2 min in a room at temperature of 30°C from 75°C to 65°C. If the same body is cooled from 55°C to 45°C in the same room, find the time taken (in minute).

7. Two identical conducting rods are first, connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, rods are joined end to end and are connected to the same vessels. If q_1 and q_2 (in g/s) are the rates of melting of ice in two cases, then find the ratio of q_1/q_2.

8. Two vessels connected at the bottom by a thin pipe with a sliding plug contain liquid at 20°C and 80°C respectively. The coefficient of cubic expansion of liquid is 10^{-3} K^{-1}. The ratio of heights of liquid columns in the vessel (H_{20}/H_{80}) is nearest to which integer?

ANSWERS AND SOLUTIONS

Subjective Type

1. If there is no air resistance, velocity of ball, on reaching ground is

$$V = \sqrt{2gh} = \sqrt{2\times10\times2000} = 200 \text{ m/s}$$

Kinetic energy of ball is $k = \frac{1}{2}mv^2$

$$= \frac{1}{2}\times m\times(200)^2 = 2\times10^4\times m\text{J} \qquad \text{(if mass } m \text{ is in kg)}$$

If all the kinetic energy is absorbed by the ball it gets heated. Due to this, temperature of ball is increased by ΔT, we have

$$k = ms\Delta T$$

or $$2\times10^4\times m = m\times126\times\Delta T$$

or $$\Delta T = \frac{2\times10^4}{126} = 158.73°\text{C}$$

∴ $$T_f = 25 + 158.73 = 183.73°\text{C}$$

2. Let specific heats of liquids be S_A, S_B and S_C, respectively. Then for mixing of liquids A and B, heat lost by B = heat gained by A

or $$m\times S_B\times(20-18) = m\times S_A\times(18-15)$$

$$2S_B = 3S_A \qquad \text{(i)}$$

For mixing of liquids B and C, we have
Heat lost by C = heat gained by B

$$m\times S_C\times(30-22) = m\times S_B\times(22-20)$$

$$8S_C = 2S_B \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

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$$8S_C = 3S_A \qquad \text{(iii)}$$

Now if liquids A and C are mixed, let their equilibrium temperature in T_e. Then we have

Heat lost by C = heat gained by A

$$m \times S_C \times (30 - T_e) = m \times S_A \times (T_e - 15)$$

or $$\frac{3S_A}{8} \times (30 - T_e) = S_A \times (T_e - 15)$$

or $$90 - 3T_e = 8T_e - 120$$

or $$11T_e = 210$$

or $$T_e = \frac{210}{11} = 19.09°\text{C}$$

3. As block slides at constant speed, friction on block exactly balances the gravitational pull on it, $mg \sin\theta$. Thus

$$f = mg \sin\theta = 0.2 \times 10 \times \sin 37°$$
$$= 2 \times \frac{3}{5} = 1.2 \text{ N}$$

As block slides 60 cm, work done by it against friction is

$$W = f \cdot l$$
$$= 1.2 \times 0.6 = 0.72 \text{ J}$$

This energy is only used in increasing the temperature of copper block. Thus

$$0.72 = ms\,\Delta T$$

or $$\Delta T = \frac{0.72}{ms} = \frac{0.72}{0.2 \times 420} = 0.00857°\text{C}$$

4. In such mixing problems, it is advisable to first convert all the phases of mixing substance into a single phase at a common temperature and keep the excess or required heat for this aside and finally supply or extract that amount of heat to get the final equilibrium temperature of mixture.

As in this problem 0.5 kg of ice is given at 0°C. To convert it into water at 0°C if we require Q_1 amount of heat, the we have

$$Q_1 = -mL_f = -0.5 \times 3.36 \times 10^5$$
$$= -1.68 \times 10^5 \text{ J}$$

(−ve sign for heat required)

The 2 kg of water is available at 30°C. To convert it into 0°C, it releases some heat, say Q_2. Then we have

$$Q_2 = ms\Delta T$$
$$= 2 \times 4200 \times 30 = 2.52 \times 10^5 \text{ J}$$

Thus we have the final mixture as

Final mixture = 2.5 kg water at 0° + 2.52×10^5 J − 1.68×10^5 J

= 2.5 kg water at 0° + 8.4×10^4 J

Thus finally we can supply the available heat of the 2.5 kg water at 0°C to get the final temperature of mixture as

$$2.5 \times 4200 \times (T_g - 0) = 8.4 \times 10^4$$

or $$T_f = 8°\text{C}$$

Thus final result is 8°C of 2.5 kg water after mixing.

5. The heat required to melt the ice completely

$$= mL = 1 \times 80 = 80 \text{ cal}$$

The heat available on water

$$= mc\Delta t = 1 = 1 \times 1 \times (40 - 0)$$
$$= 40 \text{ cal}$$

Threfore, entire heat of water is utilized to melt the ice and its temperature falls to 0°C. Ice is still at 0°C. So equilibrium temperature of contents remains 0°C. Let m is the amount of ice that melts by absorbing 40 cal heat. Then

$$m \times 80 = 40$$

or $$m = \frac{1}{2}\text{ g}$$

Final contents: ice = $1 - \frac{1}{2} = \frac{1}{2}$ g

Water = $1 + \frac{1}{2} = \frac{3}{2}$ g

6. The heat available when water cools from 10°C to 0°C

$$= mc\Delta T = 1 \times 1 \times (10 - 0) = 10 \text{ cal}$$

Let temperature of ice be T after taking this heat

∴ $$m_{ice} c\Delta T = 10$$

or $$1 \times 0.5 \times [T - (-40)] = 10$$

$$T + 40 = 20$$

$$T = -20°\text{C}$$

Now the system has 1 g ice at − 20°C and 1 g water at 0°C. Let m gram water get freezed to bring the ice from −20°C to 0°C,

Heat gained by ice = heat lost by water

∴ $$m_{ice} c[0 - (-20)] = m \times 80$$

or $$1 \times 0.5 \times 20 = m \times 80$$

$$m = \frac{1}{8}\text{ g}$$

Thus equilibrium temperature becomes 0°C.

Final contents: ice = $1 \text{ g} + \frac{1}{8}\text{ g} = \frac{9}{8}$ g

water = $1 - \frac{1}{8} = \frac{7}{8}$ g

7. Heat available on steam (changes into steam to water)

$$= mL = 1 \times 540 = 540 \text{ cal}$$

Heat gained by ice to change into water and then rise its temperature to 100°C

$$= m_{ice}L + m_{wat}c\Delta T$$
$$= 1 \times 80 + 1 \times 1 \times (100 - 0) = 180 \text{ cal}$$

The above calculations show that some part of steam will condense to change the ice into water of 100°C. Let m is the mass of steam condensed, then

$$m \times 540 = 180$$

or $$m = \frac{80}{540} = \frac{1}{3}\text{ g}$$

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Final contents: ice = 0 g

$$\text{water} = 1+\frac{1}{3}=\frac{4}{3}\text{ g}$$

$$\text{steam} = 1-\frac{1}{3}=\frac{2}{3}\text{ g}$$

8. Time lost or gained per second by a pendulum clock is given by

$$\delta t = \frac{1}{2}\alpha\Delta T$$

Here temperature is higher than graduation temperature thus clock will lose time and if it is lower than graduation temperature, clock will gain time.

Thus time lost or gained per day is

$$\delta t = \frac{1}{2}\alpha\Delta T\times 86400 \qquad [\text{as 1 day} = 86400\text{ s}]$$

If graduation temperature of clock is T_0. Then we have At 15°C, clock is gaining 5 s. Thus

$$5=\frac{1}{2}.\alpha.(T_0-15)\times 86400 \qquad \text{(i)}$$

At 30° clock is losing 10 s. Thus

$$10=\frac{1}{2}\alpha(30-T_0)\times 86400 \qquad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$2(T_0 - 15) = (30 - T_0)$$

or $$T_0 = 20°\text{C}$$

From Eq. (i)

$$5 = \frac{1}{2}\times\alpha\times[20 - 15]\times 86400$$

$$\alpha = 2.31\times 10^{-5}/°\text{C}$$

9. If temperature of surrounding increases by ΔT, the new length of rod becomes

$$l' = l(1+\alpha\Delta T)$$

Due to change in length, moment of inertia of rod about an end A also changes and is given as

$$I'_A = \frac{Ml'^2}{3}$$

As no external force or torque is acting on rod thus its angular momentum remains constant during heating thus we have

$$I_A\omega = I'_A\omega'$$

(If ω' is the final angular velocity of rod after heating)

or $$\frac{Ml^2}{3}\omega = \frac{Ml^2(1+\alpha\Delta T)^2}{3}\omega'$$

or $$\omega' = \omega\,(1 - 2\alpha\,\Delta T)$$

(using binomial expansion for small α)

Thus percentage change in angular velocity of rod due to heating can given as

$$= \frac{\omega-\omega'}{\omega}\times 100\% = 2\alpha\,\Delta T\times 100\%$$

10. First we must know what is a compensated pendulum. We've discussed that due to change in temperature the time period of a pendulum clock changes. Due to this, in pendulum clocks, to make a pendulum some specific metals are used which have very low coefficient of expansion so that the error introduced in their time is very small. The other alternative of minimizing the error in time measurement, is to use compensated pendulum. This is a pendulum made up of two or more metals as shown in Fig. 1.103

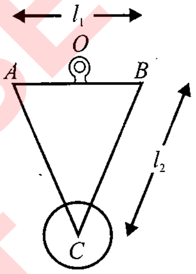

Fig. 1.103

In this case the distance of centre of mass from suspension point O is

$$h = \sqrt{l_2^2 - \frac{l_1^2}{4}}$$

When temperature is changed by Δt, new value of h will become

$$h' = \sqrt{l_2^2(1+\alpha_2\Delta t)^2 - \frac{l_1^2}{4}(1+\alpha_1\Delta t)^2}$$

As we require $h = h'$, we have

$$l_2^2 - \frac{l_1^2}{4} = l_2^2(1+2\alpha_2\Delta t) - \frac{l_1^2}{4}(1+2\alpha_1\Delta t)$$

or $$2\alpha_2 l_2^2 = \frac{l_1^2}{2}\alpha_1$$

or $$l_2 = \frac{l_1}{2}\sqrt{\frac{\alpha_1}{\alpha_2}}$$

11. Here the initial length of the two rods is l_0 and as $\alpha_b > \alpha_s$, we can directly state that the junction of the two rods is displaced toward right. Due to this brass rod is somewhat expanded but less as compared to free expansion and steel is overall compressed due to the stress developed between the two rods. Figure 1.104 shows the final situation of rods at higher temperature Δt. In the figure A is the initial position of junction and B is its final position.

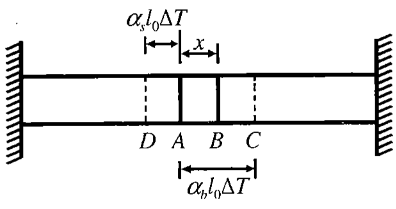

Fig. 1.104

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If only brass rod is present, expansion in brass due to increase in temperature by ΔT is

$$\Delta l_b = \alpha_b l_0 \Delta T$$

which is equal to AC. But finally the rod is expanded by x; thus the elastic compression in the rod due to stress between the two rods is BC, which is given as

$$BC = \alpha_b l_0 \Delta T - x$$

Thus elastic strain in brass rod is

$$(\text{Strain})_{\text{brass}} = \frac{\alpha_b l_0 \Delta T - x}{l_0} \qquad (\text{As } \alpha_b l_0 \Delta T \text{ is small})$$

Similarly if steel rod alone is there, it would have been expanded by

$$\Delta l_s = \alpha_s l_0 \Delta T$$

Which is equal to AD. But final steel rod is compressed by x; thus the elastic compression in steel rod due to stress between the two rods is BD, which is given as

$$BD = \alpha_s l_0 \Delta T + x$$

Thus elastic strain in steel rod is

$$(\text{Strain})_{\text{Steel}} = \frac{\alpha_s l_0 \Delta T - x}{l_0} \qquad (\text{As } \alpha_s l_0 \Delta T \text{ is small})$$

As the two rods are in contact, stress developed in the two rods must be equal; thus we have

$$(\text{Stress})_{\text{brass}} = (\text{Stress})_{\text{steel}}$$

or $$y_b \times (\text{Strain})_{\text{brass}} = y_s \times (\text{Strain})_{\text{steel}}$$

or $$y_b\left[\frac{\alpha_b l_0 \Delta T - x}{l_0}\right] = y_s\left[\frac{\alpha_s l_0 \Delta T + x}{l_0}\right]$$

or $$x = \frac{(y_b\alpha_b - y_s\alpha_s) l_0 \Delta T}{(y_b + y_s)}$$

12. It is given that at 10°C, volume of beer is 500 cm³ and the area of cross section of can is 125 cm². Thus height of bear level is

$$h = \frac{500}{125} = 4 \text{ cm}$$

Now at 80°, volume of beer becomes

$$V_{80°C} = 500\,(1 + 3.2 \times 10^{-4} \times 70) = 511.2 \text{ cm}^3$$

At 80°, area of cross section of can becomes

At $$80°C = 125\,[1 + 2\alpha_{Al} \times 70] = 125\,[1 + 2 \times 2.3 \times 10^{-5} \times 70] = 125.402 \text{ cm}^2$$

Thus new height of beer level at 80°C is

$$h' = \frac{V_{80°C}}{A_{80°C}} = \frac{511.2}{125.402} = 4.076 \text{ cm}$$

Thus rise in level of beer is

$$\Delta h = h' - h = 4.076 - 4.0 = 0.076 \text{ cm}$$

13. $$\frac{k_1 A_1 (T_1 - T)}{L_1} = \frac{k_2 A_2 (T - T_2)}{L_2}$$

$$300 - T = \left(\frac{L_1}{L_2}\right)\left(\frac{k_2}{k_1}\right)\left(\frac{A_2}{A_1}\right)(T - 0)$$

$$300 - T = 2T$$

$$T = 100°C$$

14. If temperature is increased by $\Delta\theta$ then

$$\Delta l = l\alpha\Delta\theta$$

$$\Rightarrow \Delta\theta = \frac{\Delta l}{l\alpha}$$

$$E_1 = (\rho A\, l) S \Delta\theta = \rho A l S \frac{\Delta l}{l\alpha}$$

When stretched, stress $= Y\dfrac{\Delta l}{l}$

$$E_2 = \frac{1}{2}\left(Y\frac{\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right) \times Al = \frac{Y(\Delta l)^2 A}{2l}$$

So, $$\frac{E_1}{E_2} = \frac{2\rho S l}{\alpha(\Delta l) Y} = 500$$

15. Here, $x_1 = 5$ cm, $x_2 = 7$ cm; $T = 10°C$, $K = 4 \times 10^{-3}$ cal/cm-s-°C; $L = 80$ cal/g and $\rho = 0.92$ g/cm³

Using $$\Delta t = \frac{\rho L}{2KT}(x_2^2 - x_1^2)$$

We have the required time Δt as

$$\Delta t = \frac{92 \times 10^{-2}}{2 \times 4 \times 10^{-3} \text{cal/cm-s-°C}} \frac{g}{\text{cm}^3} \times \frac{80}{10°C}\frac{\text{cal}}{g}(7^2 - 5^2) \text{ cm}$$

$$= \frac{92 \times 80 \times 24}{8} \text{ s} = 22080 \text{ s} = 6.13 \text{ h}$$

16. If $\dfrac{\Delta Q}{\Delta t}$ is the rate of heat conduction through the rod from the end at the furnace to the exposed one, then

$$\frac{\Delta Q}{\Delta t} = KA\left(\frac{-\Delta T}{l}\right)$$

where A is the cross-sectional area and $\dfrac{\Delta T}{l}$ the temperature gradient.

$$\left(\frac{\Delta Q}{\Delta t}\right)\frac{1}{A} = K\left(\frac{-\Delta T}{l}\right)$$

But $\left(\dfrac{\Delta Q}{\Delta t}\right)\dfrac{1}{A}$ is the emissive power (E) of the free end.

$$E = K\left(\frac{-\Delta T}{l}\right) = 65\frac{\text{J}}{\text{m-s-°C}} \times \frac{75°C}{\text{m}} = 4875 \text{ W/m}^2$$

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17. Rate of boiling of water = 200 g/s. Since the heat of vapourization of water is 2.256×10^3 J/g, the amount of heat energy required to boil 200 g of water is

$$2.256 \times 10^3 \text{ J/g} \times 200 \text{ g} = 4.512 \times 10^5 \text{ J}$$

Since water is boiling at the rate of 200 g/s, the rate at which heat energy is supplied by the heater to water is

$$\frac{Q}{t} = 4.512 \times 10^5 \text{ J/s}$$

Now, radius of the boiler

$$r = 15 \text{ cm} = 0.15 \text{ m}$$

$\therefore$ Base area of the boiler

$$A = \pi r^2 = 3.142 \times (0.15)^2 = 0.0707 \text{ m}^2$$

Thickness of brass

$$d = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$$

Thermal conductivity of brass $k = 109$ J/s/m/°C
Temperature of boiling water $T_w = 100$°C
If T_f is the temperature of the filament, the rate at which heat energy is transmitted through the base is given by

$$\frac{Q}{t} = \frac{kA(T_f - 100)}{1.0 \times 10^{-2}} = 4.512 \times 10^5$$

or $\quad T_f - 100 = 585.5$

or $\quad T_f = 685.5$°C

18. Assuming that heat loss from the sides of the slab is negligible, the amount of heat flowing through the slab is

$$Q = \frac{kA(T_1 - T_2)t}{d} \quad \text{(i)}$$

If m is the mass of ice and L the latent heat of fusion, then

$$Q = mL \quad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$mL = \frac{kA(T_1 - T_2)t}{d}$$

or
$$k = \frac{mLd}{A(T_1 - T_2)t} \quad \text{(iii)}$$

Given $m = 4.8$ kg, $d = 10$ cm $= 0.1$ m, $A = 3600$ cm^2 = 0.36 m^2

$T_1 = 100$°C, $T_2 = 0$°C and $t = 1$ $h = (60 \times 60)$ s

We know that $L = 80$ cal/g = 80000 cal/kg = 80000 × 4.2 J/kg = 3.36×10^5 J/kg
Substituting these values in Eq. (iii) and solving, we get
$k = 1.24$ J/s/m/°C or 1.24 W/m/K

19. We first find the thermal resistances of the different rods shown in the figure. These are given as

$$R_{AB} = \frac{1}{k_X} \cdot \frac{L}{A}$$

$$R_{BCE} = \frac{1}{k_Y} \cdot \frac{(\pi L/2)}{A}$$

$$R_{BDE} = \frac{1}{k_x} \cdot \frac{(\pi L/2)}{A}$$

$$R_{EF} = \frac{1}{k_Y} \cdot \frac{L}{A}$$

Also, $k_X = 2k_Y$. Now in steady state the amount of heat flow from end E to F remains constant as there is no absorption of heat. Then we must have that the amount of heat coming at junction B is equal to the amount of heat having B and same statement can be given for junction E. If temperatures of junctions B and E are taken as T_B and T_E, respectively, then we have for junction B.

$$\frac{T_A - T_B}{R_{AB}} = \frac{T_B - T_E}{R_{BCE}} + \frac{T_B - T_E}{R_{BDE}}$$

or
$$\frac{80 - TB}{1/k_X \cdot L/A} = \frac{T_B - T_E}{1/k_Y \cdot \pi L/2A} + \frac{T_B - T_E}{1/k_X \cdot \pi L/2A}$$

or
$$80 - T_B = \left(\frac{T_B - T_E}{\pi}\right) + \left(\frac{T_B - T_E}{2\pi}\right) \quad \text{(i)}$$

Similarly for junction E, we can write

$$\frac{T_B - T_E}{R_{BCE}} + \frac{T_B - T_E}{R_{BDE}} = \frac{T_E - 10}{R_{EF}}$$

or
$$\frac{T_B - T_E}{1/k_y \cdot \pi L/2A} + \frac{T_B - T_E}{1/k_X \cdot \pi L/2A} + \frac{T_E - 10}{1/k_Y \cdot L/A}$$

or
$$\frac{2(T_B - T_E)}{\pi} + \frac{4(T_B - T_E)}{\pi} = T_E - 10$$

or
$$\frac{3}{4\pi}(T_B - T_E) = T_E - 10 \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$T_E = 19.74\text{°C} \quad \text{and} \quad T_B = 60.52\text{°C}$$

20. In this case the rate of loss of heat by the block is given as

$$\frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$$

$= 5.67 \times 10^{-8} \times 150 \times 10^{-4} \times [(500)^4 - (300)^4]$ (surface area of cube is $6a^2 = 150$ cm^2)

If $\frac{dT}{dt}$ is rate of cooling of block then we have

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

or
$$\frac{dT}{dt} = \frac{1}{ms} \cdot \frac{dQ}{dt} = \frac{1}{1 \times 400} \times 38.556 = 0.115\text{°C/s}$$

21. The rate of loss heat by the sphere is given by

$$\frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$$

where A is the surface area of the sphere $= 4\pi r^2$, with r = 10 cm = 0.1 m, $T = 327$°C = 600 K and $T_0 = 27$°C = 300 K

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Thus $\frac{dQ}{dt} = 5.67 \times 10^{-8} \times 4\pi \times (0.1)^2 \times \{(600)^4 - (300)^4\} =$ 866 J/s

Now $dQ = msdT$, where dT is the fall in temperature in time dt.

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

or $$866 = 10 \times 420 \times \frac{dT}{dt}$$

or $$\frac{dT}{dt} = \frac{866}{4200} = 0.206°\text{C/s}$$

22. At steady state, rate of doing work by gravity on block M = Rate of energy produced

or $$\frac{d}{dt}[Mgh] = \frac{d}{dt}\left[Q + \frac{1}{2}Mv^2\right]$$

or $$Mg\left(\frac{dh}{dt}\right) = \frac{dQ}{dt} + \frac{M}{2} \times 2v\frac{dv}{dt} \quad \text{(i)}$$

As block goes down with constant velocity,

$$\frac{dh}{dt} = v \quad \text{and} \quad \frac{dv}{dt} = 0$$

$\therefore$ $$Mgv = \frac{dQ}{dt}$$

Here $$\frac{dQ}{dt} = KA\frac{\Delta T}{L}$$

$\therefore$ $$Mgv = KA\frac{\Delta T}{L}$$

or $$M \times 10 \times (0.1) = 0.50 \times 0.50\frac{(1.0 - 0)}{2 \times 10^{-3}}$$

or $$M = 12.5 \text{ kg}$$

23. Power absorbed by earth = power emitted by earth.

$$\frac{e\sigma(4\pi R_S^2)T_S^4}{4\pi r^2} \times \pi R_e^2 = e\sigma(4\pi R_e^2)T_e^4$$

$$T_e = T_S\sqrt{\frac{R_S}{2r}} = T_S\sqrt{\frac{R_S}{2 \times 200 R_S}}$$

$$T_e = \frac{T_s}{20} = 300 \text{ K}$$

24. i. $$\frac{\Delta m}{\Delta t} = \frac{0.15}{60}\text{kg/s} = 2.5 \times 10^{-3} \text{ kg/s}$$

Let P be the rate of loss of heat from the tube, and C be the specific heat capacity of water. Then

$$P + \Delta m \times C \times (17.4 - 15.2) = 25.2 \quad \text{(i)}$$

Also, $$P + \Delta m' \times C \times (17.4 - 15.2) = 37.8 \quad \text{(ii)}$$

where $$\Delta m' = \frac{0.2318}{60} = 3.8633 \times 10^{-3} \text{ kg/s}$$

Eq. (ii) − Eq. (i)

$$2.2 \times C\,[3.86 - 2.5] \times 10^{-3} = 12.6$$

$$C = 4.2 \times 10^3 \text{ J/kg°C}$$

ii. Putting the value of C in Eq. (i)

$$P + 23.1 = 25.2$$

$$P = 2.1 \text{ W}$$

25. $$\frac{dQ}{dt} = \frac{CAdT}{Tdx}$$

$$\frac{dQ}{dt}\int_0^x dx = CA\int_{T_1}^{T}\frac{dT}{T}$$

$$\frac{dQ}{dt}x = CA\ln\frac{T}{T_1}$$

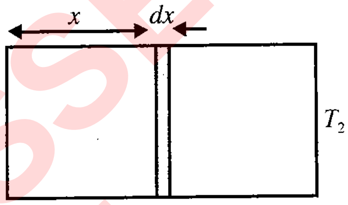

Fig. 1.105

$$\frac{dQ}{dt}l = CA\ln\frac{T_2}{T_1}$$

$$\frac{x}{l} = \frac{\ln T/T_1}{\ln T_2/T_1}$$

$$T = T_1\left(\frac{T_2}{T_1}\right)^{x/l}$$

$$\text{Heat density} = \frac{1}{A} \times \frac{dQ}{dt} = \frac{C}{l}\ln\frac{T_2}{T_1}$$

26. Intensity of the source at the cross section A

$$I = \frac{P}{4\pi(50r)^2} = \frac{1.25 \times 10^3}{4\pi(50r)^2}$$

Power absorbed by the end A

$$= 80\% \text{ of } \{I \times \pi r^2\}$$

$$= \frac{80}{100} \times \left(\frac{1.25 \times 10^3}{\pi \times 10^4 r^2}\right)(\pi r^2) = 0.1 \text{ J/s}$$

Using, Wien's displacement law, to determine the temperature of the end B

$$\lambda_m T_B = 0.003$$

$$T_B = \frac{0.003}{100000 \times 10^{-10}} = 300 \text{ K}$$

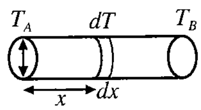

Fig. 1.106

$$\left(\frac{dQ}{dt}\right) = -\left(\frac{T}{T_A}\right)(\pi r^2)\frac{dT}{dx}$$

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$$\int_0^l \frac{0.1T_A \cdot dx}{4\times10^{-4}} = -\int_{T_A}^{T_B} TdT$$

Solving, $\quad T_A = 500$ K

Objective Type

1. **b.** Relation between Celsius and Fahrenheit scale of temperature is $\frac{C}{5} = \frac{F-32}{9}$

By rearranging we get, $C = \frac{5}{9}F - \frac{160}{9}$

By equating above equation with standard equation of line $y = mx + c$ we get $m = \frac{5}{9}$

2. **a.** Since a constant difference in length of 10 cm between an iron rod and a copper cylinder is required

$$L_{Fe} - L_{Cu} = 10 \text{ cm} \qquad \text{(i)}$$

or $\quad \Delta L_{Fe} - \Delta L_{Cu} = O \quad \therefore \Delta L_{Fe} = \Delta L_{Cu}$

i.e., linear expansion of iron rod = linear expansion of copper cylinder

$$\Rightarrow L_{Fe} \times \alpha_{Fe} \times \Delta T = L_{Cu} \times \alpha_{Cu} \times \Delta T$$

$$\Rightarrow \frac{L_{Fe}}{L_{Cu}} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11}$$

$$\therefore \quad \frac{L_{Fe}}{L_{Cu}} = \frac{17}{11} \qquad \text{(ii)}$$

From Eqs. (i) and (ii) $L_{Fe} = 28.3$ cm, $L_{Cu} = 18.3$ cm.

3. **d.** The apex of the isosceles triangle to remain at a constant distance from the knife edge. Thus, DC should remain constant before and after heating.

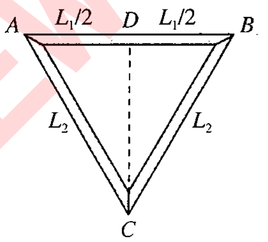

Fig. 1.107

Before expansion: In triangle ADC

$$(DC)^2 = L_2^2 - \left(\frac{L_1}{2}\right)^2 \qquad \text{(i)}$$

After expansion:

$$(DC)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]^2 \qquad \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$L_2^2 - \left(\frac{L_1}{2}\right)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]^2$$

$$L_2^2 - \frac{L_1^2}{4} = L_2^2 + L_2^2 \times 2\alpha_2 \times t - \frac{L_1^2}{4} - \frac{L_1^2}{4} \times 2\alpha_1 \times t$$

(Neglecting higher terms)

$$\Rightarrow \frac{L_1^2}{4}(2\alpha_1 t) = L_2^2(2\alpha_2 t) \quad \Rightarrow \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

4. **c.** Initially (at 20°C) length of composite system L = 50 + 100 = 150 cm

Length of iron rod at 100°C = 50[1 + 12 × 10^{-6} × (100 − 20)] = 50.048 cm

Length of aluminum rod at 100°C = 100[1 + 24 × 10^{-6} × (100 − 20)] = 100.192 cm

Finally (at 100°C) length of composite system L' = 50.048 + 100.192 = 150.24 cm

Change in length of the composite system $\Delta L = L' - L$ = 150.24 –150 = 0.24 cm

∴ Average coefficient of expansion at 100°C

$$\alpha = \frac{\Delta L}{L \times \Delta T} = \frac{0.24}{150\times(100-20)} = 20\times10^{-6}/°\text{C}$$

5. **b.** The brass rod and the lead rod will suffer expansion when placed in steam bath.

∴ Length of brass rod at 100°C

$$L'_{brass} = L_{brass}(1+\alpha_{brass}\Delta T) = 80[1+18\times10^{-6}\times100]$$

and the length of lead rod at 100°C

$$L'_{lead} = L_{lead}(1+\alpha_{lead}\Delta T) = 80[1+28\times10^{-6}\times100]$$

Separation of free ends of the rods after heating = $L'_{lead} - L'_{brass} = 80[28-18]\times10^{-4} = 8\times10^{-2}\text{ cm} = 0.8\text{ mm}$

6. **c.** Apparent coefficient of volume expansion for liquid

$$\gamma_{app} = \gamma_L - \gamma_s$$

$$\therefore \quad \gamma_L = \gamma_{app} + \gamma_s$$

where γ_s is coefficient of volume expansion for solid vessel.

When liquid is placed in copper vessel

$$\gamma_L = C + \gamma_{copper} \qquad \text{(i)}$$

(As $\gamma_{app.}$ for liquid in copper vessel = C)

When liquid is placed in silver vessel

$$\gamma_L = S + \gamma_{silver} \qquad \text{(ii)}$$

(As $\gamma_{app.}$ for liquid in silver vessel = S)

From Eqs. (i) and (ii), we get $C + \gamma_{copper} = S + \gamma_{silver}$

$$\therefore \quad \gamma_{silver} = C + \gamma_{copper} - S$$

Coefficient of volume expansion = 3 × Coefficient of linear expansion

$$\Rightarrow \alpha_{silver} = \frac{\gamma_{silver}}{3} = \frac{C+\gamma_{copper}-S}{3}$$

7. **c.** Due to increase in temperature, radius of the sphere changes.

Let R_0 and R_{100} be radius of sphere at 0°C and 100°C, respectively. $R_{100} = R_0(1+\alpha\times100)$

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Squaring both the sides and neglecting higher terms $R_{100}^2 = R_0^2(1+2\alpha\times100)$

By the law of conservation of angular momentum $I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \frac{2}{5}MR_0^2\omega_1 = \frac{2}{5}MR_{100}^2\omega_2$$

$$\Rightarrow R_0^2\omega_1 = R_0^2(1+2\times2\times10^{-5}\times100)\omega_2$$

$$\Rightarrow \omega_2 = \frac{\omega_1}{(1+4\times10^{-3})} = \frac{\omega_0}{1.004} = 0.996\omega_0$$

8. b. Apparent coefficient of volume expansion

$\gamma_{\text{app.}} = \gamma_L - \gamma_s = 7\gamma_s - \gamma_s = 6\gamma_s$ (given $\gamma_L = 7\gamma_s$)

Ratio of absolute and apparent expansion of liquid

$$\frac{\gamma_L}{\gamma_{\text{app.}}} = \frac{7\gamma_s}{6\gamma_s} = \frac{7}{6}$$

9. a. As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases.
Fraction of solid submerged at t_1°C = f_1 = Volume of displaced liquid

$$= V_0(1+\gamma t_1) \quad \text{(i)}$$

and fraction of solid submerged at t_2°C = f_2 = Volume of displaced liquid

$$= V_0(1+\gamma t_2) \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\dfrac{f_1}{f_2} = \dfrac{1+\gamma t_1}{1+\gamma t_2}$

$$\Rightarrow \gamma = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$$

10. c. On heating the system; x, r and d all increase, since the expansion of isotropic solids is similar to true photographic enlargement.

11. b. If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

12. c. Initial diameter of tyre = (1000 − 6) mm; = 994 mm, so initial radius of tyre

$$R = \frac{994}{2} = 497 \text{ mm}$$

and change in diameter ΔD = 6 mm; so

$$\Delta R = \frac{6}{2} = 3 \text{ mm}$$

Given that after increasing temperature by ΔT tyre will fit onto wheel. Increment in the length (circumference) of the iron tyre

$$\Delta L = L\times\alpha\times\Delta T = L\times\frac{\gamma}{3}\times\Delta T$$

$$\left(\text{As } \alpha = \frac{\gamma}{3}\right)$$

$$\Rightarrow 2\pi\Delta R = 2\pi R\left(\frac{\gamma}{3}\right)\Delta T$$

$$\Rightarrow \Delta T = \frac{3}{\gamma}\frac{\Delta R}{R} = \frac{3\times3}{3.6\times10^{-5}\times497}$$

[As ΔR = 3 mm and R = 497 mm]

$$\Rightarrow \Delta T = 500°\text{C}$$

13. a. Loss of time due to heating a pendulum is given as

$$\Delta T = \frac{1}{2}\alpha\Delta\theta T$$

$$\Rightarrow 12.5 = \frac{1}{2}\times\alpha\times(25-0)°\text{C}\times86400$$

$$\Rightarrow \alpha = \frac{1}{86400}/°\text{C}$$

14. d. Due to heating the length of the wire increases.

$\therefore$ Longitudinal strain is produced $\Rightarrow \dfrac{\Delta L}{L} = \alpha\times\Delta T$

Elastic potential energy per unit volume

$$E = \frac{1}{2}\times\text{Stress}\times\text{Strain} = \frac{1}{2}\times Y\times(\text{Strain})^2$$

$$\Rightarrow E = \frac{1}{2}\times Y\times\left(\frac{\Delta L}{L}\right)^2 = \frac{1}{2}\times Y\times\alpha^2\times\Delta T^2$$

or $$E = \frac{1}{2}\times Y\times\left(\frac{\gamma}{3}\right)^2\times T^2 = \frac{1}{18}\gamma^2 YT^2$$

[As $\gamma = 3\alpha$ and $\Delta T = T$ (given)]

15. c. Span of bridge = 2400 m and bridge sags by 500 m at 30° (given)

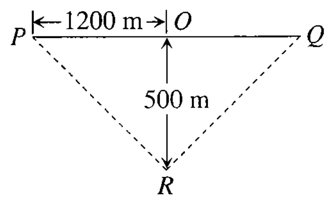

Fig. 1.108

From Figure 1.108, $L_{PRQ} = 2\sqrt{1200^2+500^2} = 2600$ m

But $L = L_0(1+\alpha\Delta t)$

(Due to linear expansion)

$$\Rightarrow 2600 = L_0(1+12\times10^{-6}\times30)$$

$\therefore$ Length of the cable L_0 = 2599 m

Now change in length of cable due to change in temperature from 10°C to 42°C

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$$\Delta L = 2599 \times 12 \times 10^{-6} \times (42-10) = 0.99 \text{ m}$$

16. c. Heat required to raise the temperature of m grams of substance by dT is given as

$$dQ = mc\,dT \quad \Rightarrow Q = \int mcdT$$

Therefore, to raise the temperature of 2 g of substance from 5°C to 15°C

$$Q = \int_5^{15} 2 \times (0.2 + 0.14t + 0.023t^2)dT$$

$$= 2 \times \left[0.2t + \frac{0.14t^2}{2} + \frac{0.023t^3}{3}\right]_5^{15} = 82 \text{ cal}$$

17. a. Work done in converting 1 g of ice at –10°C to steam at 100°C

= Heat supplied to raise temperature of 1 g of ice from –10°C to 0°C ($m \times c_{\text{ice}} \times \Delta T$)
+ Heat supplied to convert 1 g ice into water at 0°C ($m \times L_{\text{ice}}$)
+ Heat supplied to raise temperature of 1 g of water from 0°C to 100°C ($m \times c_{\text{water}} \times \Delta T$)
+ Heat supplied to convert 1 g water into steam at 100°C [$m \times L_{\text{vapour}}$]
= [$m \times c_{\text{ice}} \times \Delta T$] + [$m \times L_{\text{ice}}$] + [$m \times c_{\text{water}} \times \Delta T$] + [$m \times L_{\text{vapour}}$]
= [1 × 0.5 × 10] + [1 × 80] + [1 × 1 × 100] + [1 × 540] = 725 cal = 725 × 4.2 = 3045 J

18. a. Steam at 100°C contains extra 540 cal/g energy as compared to water at 100°C. So it's more dangerous to burn with steam than water.

19. b. Work done by man = Heat absorbed by ice = mL = 60 × 80 = 4800 cal = 20160 J

$$\therefore \quad \text{Power} = \frac{W}{t} = \frac{20160}{60} = 336 \text{ W}$$

20. a. Same amount of heat is supplied to copper and water; so

$$m_c c_c \Delta T_c = m_\omega c_\omega \Delta T_\omega$$

$$\Rightarrow (\Delta T)_\omega = \frac{m_c c_c \Delta T_c}{m_\omega c_\omega} = \frac{50 \times 10^{-3} \times 420 \times 10}{10 \times 10^{-3} \times 4200} = 5°\text{C}$$

21. c. Heat lost by A = Heat gained by B

$$\Rightarrow m_A \times c_A \times (T_A - T) = m_B \times c_B \times (T - T_B)$$

Since $m_A = m_B$ and temperature of the mixture (T) = 28°C

$$\therefore \quad c_A \times (32-28) = c_B \times (28-24)$$

$$\Rightarrow \frac{c_A}{c_B} = 1{:}1$$

22. b. Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$$\Rightarrow 440\,(92 - T) = 200 \times (T - 20) + 20 \times (T - 20)$$

$$\Rightarrow T = 68°\text{C}$$

23. d. Temperature of mixture is given by

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} = \frac{mc2T + \frac{m}{2}2cT}{mc + \frac{m}{2}2c} = \frac{3}{2}T$$

24. b. Let the final temperature be T°C.
Total heat supplied by the three liquids in coming down to $0°C$

$$= m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3 \qquad \text{(i)}$$

Total heat used by three liquids in raising temperature from $0°C$ to $T°C$

$$= m_1c_1T + m_2c_2T + m_3c_3T \qquad \text{(ii)}$$

By equating Eqs. (i) and (ii) we get

$$(m_1c_1 + m_2c_2 + m_3c_3)T = m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3$$

$$\Rightarrow T = \frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$$

25. b. Heat required by 10 kg water to change its temperature from 20°C to 80°C in one hour is

$$Q_1 = (mc\Delta T)_{\text{water}} = (10 \times 10^3) \times 1 \times (80-20) = 600 \times 10^3 \text{ cal}$$

In condensation

i. Steam releases heat when it loses its temperature from 150°C to 100°C. [$mc_{\text{steam}}\,\Delta T$]

ii. At 100°C it converts into water and gives the latent heat. [mL]

iii. Water releases heat when it loses its temperature from 100°C to 90°C. [$ms_{\text{water}}\,\Delta T$]

If m grams of steam is condensed per hour, then heat released by steam in converting to water at 90°C

$$Q_2 = (mc\Delta T)_{\text{steam}} + mL_{\text{steam}} + (ms\Delta T)_{\text{water}}$$
$$= m[1 \times (150 - 100) + 540 + 1 \times (100 - 90)] = 600 \text{ m cal}$$

According to problem, $Q_1 = Q_2 \Rightarrow 600 \times 10^3$ cal = 600 m cal
$\Rightarrow m = 10^3$ g = 1 kg

26. b. Let X be the thermal capacity of calorimeter and specific heat of water = 4200 J/kg-K.
Heat lost by 0.1 kg of water = Heat gained by water in calorimeter + Heat gained by calorimeter

$$\Rightarrow 0.1 \times 4200 \times (60 - 35)$$
$$= 0.2 \times 4200 \times (35 - 30) + X(35 - 30)$$
$$10500 = 4200 + 5X \Rightarrow X = 1260 \text{ J/K}$$

27. a. According to problem, rate of heat loss in both rods is equal, i.e., $\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$

$$\Rightarrow \frac{K_1A_1\Delta\theta_1}{l_1} = \frac{K_2A_2\Delta\theta_2}{l_2}$$

$\therefore K_1A_1 = K_2A_2$ [As $\Delta\theta_1 = \Delta\theta_2 = (T_1 - T_2)$ and $l_1 = l_2$ given]

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28. a. $\dfrac{dQ}{dt}=\dfrac{KA\Delta\theta}{l}$

For both rods K, A and $\Delta\theta$ are same.

$$\therefore \quad \frac{dQ}{dt}\propto\frac{1}{l}$$

So $$\frac{(dQ/dt)_{\text{semicircular}}}{(dQ/dt)_{\text{straight}}}=\frac{l_{\text{straight}}}{l_{\text{semicircular}}}=\frac{2r}{\pi r}=\frac{2}{\pi}$$

29. c. From $\dfrac{dQ}{dt}=\dfrac{KA\Delta\theta}{l}$

$$\Rightarrow \Delta\theta=\frac{l}{K\times A}\times\frac{dQ}{dt}=\frac{0.1}{400\times(100\times10^{-4})}\times4000=100°\text{C}$$

30. c. $\dfrac{dQ/dt}{A}=K\left(\dfrac{\Delta\theta}{\Delta x}\right)\Rightarrow$ Rate of flow of heat per unit area

= Thermal conductivity × Temperature gradient

Temperature gradient $(X)\propto\dfrac{1}{\text{Thermal conductivity (K)}}$

(As $\dfrac{dQ/dt}{A}=$ constant)

As $K_C>K_m>K_g$, therefore $X_c<X_m<X_g$

31. b. Rate of flow of heat or power $P=\dfrac{KA\Delta\theta}{\Delta x}=\dfrac{K4\pi R^2T}{\Delta x}$

$\therefore$ Thickness of shell $\Delta x=\dfrac{4\pi R^2KT}{P}$

32. d. Rate of flow of heat will be equal in both vest and shirt

$$\therefore \quad \frac{K_{\text{vest}}A.\Delta\theta_{\text{vest}}t}{l}=\frac{K_{\text{shirt}}A\Delta\theta_{\text{shirt}}t}{l}$$

$$\Rightarrow\frac{K_{\text{vest}}}{K_{\text{shirt}}}=\frac{\Delta\theta_{\text{shirt}}}{\Delta\theta_{\text{vest}}}\Rightarrow\frac{K_{\text{vest}}}{K_{\text{shirt}}}=\frac{25-22}{30-25}=\frac{3}{5}$$

33. b. $Q=KA\dfrac{\Delta\theta}{l}t \quad \therefore t\propto\dfrac{l}{A}$ (As Q, K and $\Delta\theta$ are constant)

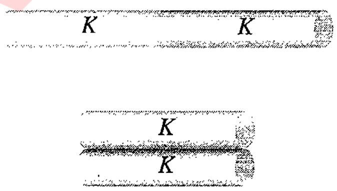

Fig. 1.109

$$\frac{t_1}{t_2}=\frac{l_1}{l_2}\times\frac{A_2}{A_1}=\left(\frac{l_1}{l_1/2}\right)\times\left(\frac{2A_1}{A_1}\right)$$

$$\frac{t_1}{t_2}=4\quad\Rightarrow t_2=\frac{t_1}{4}=\frac{12}{4}=3\text{ s}$$

34. c. Rate of flow of heat along PQ

$$\left(\frac{dQ}{dt}\right)_{PQ}=\frac{K_3A\Delta\theta}{l}\qquad\text{(i)}$$

Rate of flow of heat along PRQ

$$\left(\frac{dQ}{dt}\right)_{PRQ}=\frac{K_sA\Delta\theta}{2l}$$

Effective conductivity for series combination of two rods of same length

$$K_s=\frac{2K_1K_2}{K_1+K_2}$$

So $$\left(\frac{dQ}{dt}\right)_{PRQ}=\frac{2K_1K_2}{K_1+K_2}\cdot\frac{A\Delta\theta}{2l}=\frac{K_1K_2}{K_1+K_2}\cdot\frac{A\Delta\theta}{l}\qquad\text{(ii)}$$

Equating Eqs. (i) and (ii) $K_3=\dfrac{K_1K_2}{K_1+K_2}$

35. a. $K_1=9K_2$, $l_1=18$ cm, $l_2=6$ cm, $\theta_1=100°$C, $\theta_2=0°$C

Temperature of the junction $\theta=\dfrac{\dfrac{K_1}{l_1}\theta_1+\dfrac{K_2}{l_2}\theta_2}{\dfrac{K_1}{l_1}+\dfrac{K_2}{l_2}}$

$$\Rightarrow\theta=\frac{\dfrac{9K_2}{18}100+\dfrac{K_2}{6}\times0}{\dfrac{9K_2}{18}+\dfrac{K_2}{6}}=\frac{50+0}{8/12}=75°\text{C}$$

36. c. Let the thermal resistance of each rod be R.
The two resistances connected along two paths from B to C are equivalent to $2R$ each and their parallel combination is R.
Effective thermal resistance between B and $D=2R$

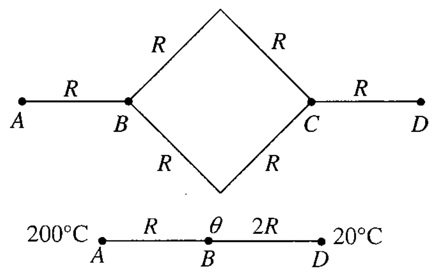

Fig. 1.110

Temperature of interface $\theta=\dfrac{R_1\theta_2+R_2\theta_1}{R_1+R_2}$

$$\theta=\frac{R\times20+2R\times200}{R+2R}=\frac{420}{3}=140°\text{C}$$

37. d. Quantity of heat transferred through wall will be utilized in melting of ice.

$$Q=\frac{KA\Delta\theta t}{\Delta x}=mL$$

$\therefore$ Amount of ice melted $m=\dfrac{KA\Delta\theta t}{\Delta xL}$

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$$\therefore m = \frac{0.01\times 1\times (30-0)\times 86400}{5\times 10^{-2}\times 334\times 10^3} = 1.552 \text{ kg or } 1552 \text{ g}$$

38. a. $mL = \dfrac{KA\Delta\theta t}{\Delta x}$

$$\Rightarrow 500\times 80 = \frac{0.0075\times 75\times (40-0)t}{5}$$

$$\Rightarrow t = 8.9\times 10^3 \text{ s} = 2.47 \text{ h}$$

39. d. If a body emits wavelength λ_1, λ_2, λ_3 and λ_4 at a high temperature then at a lower temperature it will absorb the radiation of same wavelength. This is in accordance with Kirchoff's law.

40. a. Black bulb absorbs more heat in comparison with painted bulb. So air in black bulb expands more. Hence the level of alcohol in limb X falls while that in limb Y rises.

41. c. As the temperature of body increases, frequency corresponding to maximum energy in radiation (ν_m) increases. This is shown in graph (c).

42. c. $\lambda_m T$ = constant

$$\Rightarrow \frac{(\lambda_m)_2}{(\lambda_m)_1} = \frac{T_1}{T_2} = \frac{200}{1000} = \frac{1}{5}$$

$$\Rightarrow (\lambda_m)_2 = \frac{(\lambda_m)_1}{5} = \frac{14\times \mu\text{m}}{5} = 2.8\ \mu\text{m}$$

43. a. According to Wien's law, wavelength corresponding to maximum energy decreases. When the temperature of black body increases, i.e., $\lambda_m T$ = constant

$$\Rightarrow \frac{T_2}{T_1} = \frac{\lambda_1}{\lambda_2} = \frac{\lambda_0}{3\lambda_0/4} = \frac{4}{3}$$

Now according to Stefan's law

$$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

44. b. From Wien's displacement law $\lambda_m T = b$

$$\therefore \quad T = \frac{b}{\lambda_m} = \frac{2.93\times 10^{-3}}{2.93\times 10^{-10}} = 10^7 \text{ K}$$

45. b. Wien's law $\lambda_m \propto \dfrac{1}{T}$ or $\nu_m \propto T$

ν_m increases with temperature. So the graph will be straight line.

46. d. Energy radiated by body per second $\dfrac{Q}{t} = A\sigma T^4$

or $\qquad \dfrac{Q}{t} \propto l\times b\times T^4 \qquad$ (Area = $l\times b$)

$$\therefore \frac{E_2}{E_1} = \frac{l_2}{l_1}\times\frac{b_2}{b_1}\times\left(\frac{T_2}{T_1}\right)^4 = \frac{(l_1/2)}{l_1}\times\frac{(b_1/2)}{b_1}\times\left(\frac{600}{400}\right)^4$$

$$= \frac{1}{2}\times\frac{1}{2}\times\left(\frac{3}{2}\right)^4 \quad \Rightarrow E_2 = \frac{81}{64}E$$

47. c. $\dfrac{dT}{dt} = \dfrac{eA\sigma}{mc}(T^4 - T_0^4) = \dfrac{e(6a^2)\sigma}{(a^3\times\rho)c}(T^4 - T_0^4)$

$\Rightarrow$ For the same fall in temperature, time $dt \propto a$

$$\frac{dt_2}{dt_1} = \frac{a_2}{a_1} = \frac{2 \text{ cm}}{1 \text{ cm}} \quad \Rightarrow dt_2 = 2\times dt_1 = 2\times 100 \text{ s} = 200 \text{ s}$$

(As $A = 6a_2$ and $m = V\times\rho = a_3\times\rho$)

48. a. $\dfrac{dT}{dt} = \dfrac{eA\sigma}{mc}(T^4 - T_0^4) = \dfrac{eA\sigma}{V\rho c}(T^4 - T_0^4)$

$\therefore$ Rate of cooling $R \propto A$

(As masses are equal, volume of each body must be equal because material is same)

i.e., rate of cooling depends on the area of cross section and we know that for a given volume the area of cross section will be minimum for sphere. It means the rate of cooling will be minimum in case of sphere.

So the temperature of sphere drops to room temperature at last.

49. c. $Q = \sigma At\,(T^4 - T_0^4)$

If T, T_0, σ and t are same for both bodies then

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{A_{\text{sphere}}}{A_{\text{cube}}} = \frac{4\pi r^2}{6a^2} \qquad \text{(i)}$$

But according to problem, volume of sphere = volume of cube

$$\Rightarrow \frac{4}{3}\pi r^3 = a^3$$

$$\Rightarrow a = \left(\frac{4}{3}\pi\right)^{1/3} r$$

Substituting the value of a in Eq. (i), we get

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{4\pi r^2}{6a^2}$$

$$= \frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3} r\right\}^2} = \frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3} r^2}$$

$$= \left(\frac{\pi}{6}\right)^{1/3} : 1$$

50. c. According to Newton's law of cooling the rate of cooling depends upon the difference of temperature between the body and the surrounding. It means that when the difference of temperature between the body and the surrounding is small, time required for same fall in temperature is more in comparison with the same fall at higher temperature difference between the body and surrounding. So according to problem $T_1 < T_2 < T_3$.

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51. d. According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$$

For the first condition

$$\frac{80-60}{60}\propto\left[\frac{80+60}{2}-30\right] \qquad \text{(i)}$$

and for the second condition

$$\frac{60-50}{t}\propto\left[\frac{60+50}{2}-30\right] \qquad \text{(ii)}$$

By solving Eqs. (i) and (ii), we get $t = 48$ s.

52. b. According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$$

For the first condition

$$\frac{62-61}{T}\propto\left[\frac{62+61}{2}-30\right] \qquad \text{(i)}$$

and for the second condition

$$\frac{46-45.5}{t}\propto\left[\frac{46+45.5}{2}-30\right] \qquad \text{(ii)}$$

By solving Eqs. (i) and (ii), we get $t = T$ minutes.

53. d. $\frac{dT}{dt}=\frac{\sigma A}{mc}(T^4-T_0^4)$. If the liquids are put in exactly similar calorimeters and identical surrounding then we can consider T_0 and A constant. Then

$$\frac{dT}{dt}\propto\frac{(T^4-T_0^4)}{mc} \qquad \text{(i)}$$

If we consider that equal masses of liquids (m) are taken at the same temperature then

$$\frac{dT}{dt}\propto\frac{1}{c}$$

So for same rate of cooling c should be equal, which is not possible because liquids are of different nature.
Again from Eq. (i)

$$\frac{dT}{dt}\propto\frac{(T^4-T_0^4)}{mc}$$

$$\Rightarrow \frac{dT}{dt}\propto\frac{(T^4-T_0^4)}{V\rho c}$$

Now if we consider that equal volumes of liquids (V) are taken at the same temperature then

$$\frac{dT}{dt}\propto\frac{1}{\rho c}$$

So for same rate of cooling multiplication of $\rho\times c$ for two liquids of different nature can be possible. So option (d) may be correct.

54. b. $\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$

For the first condition

$$\frac{60-50}{10}\propto\left[\frac{60+50}{2}-\theta\right]\Rightarrow 1=K[55-\theta] \qquad \text{(i)}$$

For the second condition

$$\frac{50-42}{10}\propto\left[\frac{50+42}{2}-\theta\right]\Rightarrow 0.8=K(46-\theta) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get θ = 10°C

55. b. The rate of heat loss by a thin hollow sphere of thickness Δx, mean radius r and made of density ρ is given by

$$mS\frac{dT}{dt}=-\varepsilon\sigma A(T^4-T_0^4)$$

$$(\rho 4\pi r^2\Delta x)S\frac{dT}{dt}=-\varepsilon\sigma 4\pi r^2(T^4-T_0^4)$$

$$\frac{dT}{dt}=-\frac{\varepsilon\sigma(T^4-T_0^4)}{S\Delta x} \text{ is independent of radius}$$

Hence the rate of cooling is same for both spheres.

56. d. Thermal resistance of $AC=\frac{L}{KA}=\frac{0.1}{336\times10^{-4}}=\frac{10^3}{336}=R$ (let)

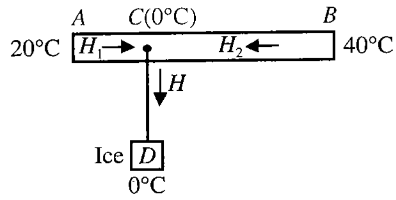

Fig. 1.111

Thermal resistance of $BC=\frac{0.2}{336\times10^{-4}}=2R$

Heat flow rates are

$$H_1=\frac{20}{R};\ H_2=\frac{40}{2R}=\frac{20}{R}$$

$$H=H_1+H_2=\frac{40}{R}=\frac{40\times336}{10^3}$$

$$=\frac{13440}{10^3}=13.44 \text{ W}$$

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Rate of melting of ice

$$= \frac{H}{L_f} = \frac{13.44/4.2}{80} \text{ g/s} = 40 \text{ mg/s}$$

57. d. Since tension in the two rods will be same,

$$A_1Y_1\alpha_1\Delta\theta = A_2Y_2\alpha_2\Delta\theta$$

$$A_1Y_1\alpha_1 = A_2Y_2\alpha_2$$

58. d. Heat current:

$$i = -kA\frac{dT}{dx}$$

$$idx = -kA\,dT$$

$$i\int_0^l dx = -A\alpha\int_{T_1}^{T_2} TdT$$

$$il = -A\alpha\frac{(T_2^2 - T_1^2)}{2} \quad i = \frac{A\alpha(T_1^2 - T_2^2)}{2l}$$

59. a. For a black body, wavelength for maximum intensity:

$$\lambda \propto \frac{1}{T} \ \& \ P \propto T^4 \quad \Rightarrow P \propto \frac{1}{\lambda^4}$$

$$P' = 16P \quad \therefore P'T' = 32\,PT$$

60. b. Rate of heat loss $= \sigma eA(T^4 - T_s^4)$

$$-ms\frac{dT}{dt} = \sigma eA(T^4 - T_S^4)$$

$$-\frac{dT}{dt} = \frac{5.8\times10^{-4}\times1\times\pi(0.08)^2\left[(500)^4 - (300)^4\right]}{10\times4.2\times90}$$

$$\frac{-dT}{dt} = 0.066\ °\text{C/s}$$

61. b. The effective value of α at a distance x from the left end is

$$\alpha_x = \alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x$$

$$\Delta L = \int_0^L \alpha_x dx\Delta t$$

$$L = \left(\frac{\alpha_1 + \alpha_2}{2}\right)L\Delta T$$

$$\alpha_{\text{eff}} = \frac{\alpha_1 + \alpha_2}{2}$$

62. c. The change in length of wire $= l_{Al}\,\alpha_{Al}\,\Delta\theta + l_{st}\,\alpha_{st}\,\Delta\theta$ associated with temperature change of $\Delta\theta$, where α_{Al} and α_{St} are the coefficient of linear expansion of aluminium and steel, respectively

$$\alpha_{\text{Al}} = 23\times10^{-6}/°\text{C}$$

$$\alpha_{\text{st}} = 12\times10^{-6}/°\text{C}$$

The effective coefficient of linear expansion of the two segments of wire $= 19 \times 10^{-6}/°\text{C}$

$$l_1\alpha_{\text{Al}}\Delta\theta + l_2\alpha_{\text{st}}\Delta\theta = (l_1 + l_2)\alpha\Delta\theta$$

$$\frac{l_1}{(l_1+l_2)} = \frac{\alpha - \dfrac{l_2}{(l_1+l_2)}\alpha_{\text{st}}}{\alpha_{Al}}$$

$$\left[\frac{l_1}{l_1+l_2} = x \quad l_1 + l_2 = l\left(\frac{l_2}{l_1+l_2}\right) = 1 - x\right]$$

$$x = \frac{\alpha - (1-x)\alpha_{\text{st}}}{\alpha_{\text{Al}}}$$

$$x = \frac{\alpha - \alpha_{\text{st}}}{\alpha_{\text{Al}} - \alpha_{\text{st}}} = \frac{19\times10^{-6} - 12\times10^{-6}}{23\times10^{-6} - 12\times10^{-6}} = \frac{7}{11}$$

63. a. $Q_4 = mL_w = 540$ cal

$Q_3 = ms_w\,(100 - 0) = 100$ cal

$Q_2 = mL_{\text{ice}} = 80$ cal

$Q_1 = ms_{\text{ice}}\,(20 - 0) = 20$ cal

$Q_4 > Q_3 > Q_2 > Q_1$

64. c. Fraction of wooden block immersed at 0°C,

$$\frac{V_1}{V_0} = \frac{(\rho_{\text{wood}})_{0°\text{C}}}{(\rho_{\text{H}_2\text{O}})_{0°\text{C}}}$$

$$f_1 = \frac{V_0 - V_1}{V_0} = \frac{(\rho_{\text{H}_2\text{O}})_{0°\text{C}} - (\rho_{\text{wood}})_{0°\text{C}}}{(\rho_{\text{H}_2\text{O}})_{0°\text{C}}}$$

V_1 – Volume of wood immersed in water at 0°C

V_0 – Volume of wood

$(\rho_{\text{wood}})_{0°\text{C}}$ – Density of wood at 0°C

When the temperature is raised to 10°C, the volume of wood immersed in water changes to V_2.

$$\frac{V_2}{V_0} = \frac{(\rho_{\text{wood}})_{10°\text{C}}}{(\rho_{\text{H}_2\text{O}})_{10°\text{C}}}$$

$$f_2 = \frac{V_0 - V_2}{V_0} = \frac{(\rho_{\text{H}_2\text{O}})_{10°\text{C}} - (\rho_{\text{wood}})_{10°\text{C}}}{(\rho_{\text{H}_2\text{O}})_{10°\text{C}}}$$

From 0°C to 4°C, the density of water increases, and from 4°C to 10°C the density of water decreases.
But for wood density decreases as temperature increases.
The volume of block above water level will first increase and then decrease.

65. b. Power sent to heat the water in the calorimeter

$$P' = \frac{ms\Delta\theta}{t}$$

$$= \frac{V\rho s\Delta\theta}{t} = \frac{10^3\times10^{-6}\times10^3\times4200\times4}{420} = 40 \text{ W}$$

Required ratio

$$= \frac{P-P'}{P} = \frac{54-40}{54} = \frac{14}{54} = 26\%$$

66. **b.** Since a ruler is used, the scale used does not expand with the tube. If the radius of the capillary be r, the increase due to thermal expansion is given by $dr = r\alpha dT$ for a temperature rise of dT. Since area of cross section is $A = \pi r^2$, we see that $dA/A = 2dr/r$ or $dA = A(2\alpha)\, dT$. Thus if the temperature is increased from T to $T + dT$, the cross-sectional area changes from A to $A\,(1 + 2\alpha dT)$. The volume expansion of the liquid gives $V' = V + dV = V(1 + \gamma dT)$, where γ is the coefficient of the volume expansion of the liquid. This causes change in length of thread and final length becomes $L' = L + dL$. The mass of liquid is constant; hence, $L'A' = V' = V\,(1 + \gamma dT) = LA\,(1 + \gamma\, dT)$.

But $\quad A' = A\,(1 + 2\alpha dT)$

Hence, $\quad L' = L\left(\dfrac{1+\gamma dT}{1+2\alpha dT}\right)$

$$= L[1+(\gamma-2\alpha)dT - 2\alpha\gamma(dT)^2]$$

The last term is negligible.

Hence, $\quad L' = L[1+(\gamma-2\alpha)dT]$

$$\Delta L = L(\gamma-2\alpha)dT$$

67. **c.** Thermal stress that is produced in an elastic wire is $Y\ \alpha\ \theta$ per unit area, where Y is Young's modulus, α coefficient of linear expansion and θ change of temperature.
Thus, tension developed in the wire is

T = Thermal stress × area of cross section
$= Y\alpha \times (\Delta T)[\pi r^2]$ N
$= 0.91 \times 10^{11} \times 2 \times 10^{-5} \times [27 - (-33)]\ [\pi \times 1 \times 10^{-6}]$ N
$= 0.91 \times 2 \times 6 \times 3.14 \times 10$ N
$= 34.4 \times 10\ N = 0.34$ kN

68. **d.** Specific heat of lead = 0.03 kcal/kg°C. Gravitational potential energy is converted into thermal energy which is absorbed by the lead shot. If T_0 is the rise in temperature of the shot, then

$$(100)\ [mgh] = (m \times s \times T)\ \text{J}$$

$$T = \frac{100 \times g \times h}{Js} = \frac{100 \times 9.8 \times 1.5}{0.03 \times 4.18 \times 10^3}\,°\text{C} = 11.3°\text{C}$$

69. **b.** The entire kinetic energy of the fragment is changed to heat. Expressing mass m in kilograms everywhere, we have

$$\frac{1}{2}mv^2 = [m(30) + m(0.11)(1535° + 100°)]4184$$

$$v^2 = (8368)\ [30 + 180] \simeq 1.76 \times 10^6$$

Hence

$$v = (\sqrt{1.76}) \times 10^3 = 1.32 \text{ km/s}$$

70. **a.** This is a problem on 'flow calorimeter' used to measure specific heat of a liquid.
Amount of heat supplied to the water per second by the heating coil = Q_S = 250 J

$$= \frac{250}{4186}\text{ kcal}$$

The volume of liquid flowing out per second = 8.0 cm³ = 8×10^{-6} m³

Mass of this liquid = (0.85) × 1000 × 8 × 10^{-6} kg

Temperature rise of this mass of liquid = 15°C

Hence, $\quad \dfrac{250}{4186} = mst = 0.85 \times 8 \times 10^{-3} \times s \times 15$

Hence, $\quad s = \dfrac{250 \times 10^3}{4186 \times 0.85 \times 8 \times 15} = 0.6$ kcal/kg K

71. **b.** Using the suffixes I and B for the iron ball and the brass plate we have L_I = 6 cm, $L_I - L_B$ = 0.001 cm at t = 30°C

Heating both the ball and the plate increases the diameters of the ball as well as the hole in the plane, with the hole diameter increasing at a faster rate, since $\alpha_B > \alpha_L$.

Now we require that $\Delta L_B - \Delta L_I$ = 0.001 cm at the desired temperature t, with $\Delta L_B = L_B\alpha_B\ \Delta t$ and $\Delta L_I = L_I\alpha_I\Delta t$.

Then $\Delta L_B - \Delta L_I = (L_B\alpha_B - L_I\alpha_I)\Delta T = 0.001$ cm
$\approx L_I(\alpha_B - \alpha_I)\Delta t$, approximating by putting $L_B \simeq L_I$ or
$(6\text{ cm})\ [1.9\times10^{-5} - 1.2\times10^{-5}]\Delta t = 0.001$ cm

Hence $\quad \Delta t = \dfrac{0.001}{6[1.9\times10^{-5} - 1.2\times10^{-5}]}\,°\text{C}$

$$\Delta t = 23.8°\text{C}$$

Hence final temperature = 30 + 23.8°C = 53.8°C

72. **c.** We have $\dfrac{\Delta V}{V} = \gamma\Delta T = (1.82\times10^{-4})(100-20)$

$$= 1.46 \times 10^{-2}$$

Now the bulk modulus, $P = B\left(\dfrac{\Delta V}{V}\right)$, where P is the pressure.

$$P = (2500) \times 1.46 \times 10^{-2}\text{ MPa} = 36\text{ MPa}$$

73. **d.** It is clear that at desired temperature, T°C, the densities of the wood and benzene must be equal for the wood to just sink.

i.e., $\quad \rho_w(T) = \rho_B(T)$

If m is the mass of wood (which is assumed to be constant) then, if $(V_0)_w$ and $(V_0)_B$ are the respective volumes at 0°C of mass m of wood and Benzene,

$$(\rho_0)_w(V_0)_w = (\rho_0)_B(V_0)_B = m$$

$$(\rho_0)_w = 880 \text{ kg/m}^3 \text{ and } (\rho_0)_B = 900 \text{ kg/m}^3$$

Hence $(V_0)_w = \frac{m}{880}(m^3)$

End $(V_0)_B = \frac{m}{900}(m^3)$

We then have, $(V_T)_W = (V_0)_W(1+1.2\times10^{-3}T)$

$$(V_T)_B = (V_0)_B(1+1.5\times10^{-3}T)$$

Thus $\frac{(V_T)_W}{(V_T)_B} = \frac{(\rho_B)_T}{(\rho_W)_T} = 1 = \frac{(V_0)_W}{(V_0)_B}\frac{1+1.2\times10^{-3}T}{1+1.5\times10^{-3}T}$

Thus $\frac{(V_0)_W}{(V_0)_B} = \frac{900}{880} = \frac{90}{88} = \frac{1+1.5\times10^{-3}T}{1+1.2\times10^{-3}T}$

Solving for T, we have $T = 83.2°C$.

74. a. The given data is normally insufficient to determine the result. Either the length of one of the rods at 0°C must be known or one has to assume that both the rods had the same length at 0°C which is not possible if they have the same length again at another temperature. Suppose the two rods had the same length L at 0°C. Then by the given problem,

$$L[1 + 1.9 \times 10^{-5} \times 27] - L[1 + 1.1 \times 10^{-5} \times 27] = 10^{-3}$$
$$L \times 0.8 \times 27 \times 10^{-5} = 10^{-3}$$

or $L = \frac{10^{-3}}{0.8\times27\times10^{-5}} \text{ m} = 4.63 \text{ m}$

Hence 0°C is a possible choice and the rods had equal length of 4.63 m at 0°C.

75. b. Consider a time interval Δt while the door is closed. Then the rate of heat into the box is

$$\frac{Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right) = (0.05)(6)\left(\frac{22}{0.09}\right) = 73.3 \text{ W}$$

To remove heat at this rate, while the motor runs only for a time $(0.15)\,\Delta t$, it must cause heat to leave at the rate,

$$H = \frac{73.3}{0.15} = 500 \text{ W}$$

76. a. Here we assume that all the work done on the bullet inside the wood is converted into thermal energy.
Thermal energy produced in the first case is

$$H_1 = \frac{1}{2}(5\times10^{-3})(200)^2 \text{ J}$$

Since the resistance in the second plank is the same as the first and also the thickness is equal, the same amount of work will be done on the bullet as in the first case and hence the thermal energy $H_2 = H_1$. The ratio is 1:1.

77. a. Since they are in thermal equilibrium, the temperatures must necessarily be equal. Temperature equality is a necessary and sufficient condition for thermal equilibrium (zeroth law of thermodynamics).

78. b. Heat given to the metal

$$dQ = P\,dt = C_P(t)\,dT \quad \text{(i)}$$

At constant pressure in time interval at

Given $T = T_0[1+a(t-t_0)]^{1/4}$

$$\frac{dT}{dt} = \frac{T_0}{4}[1+a(t-t_0)]^{-3/4}\times a \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$C_P(T) = \frac{P}{\left(\frac{dT}{dt}\right)} = \frac{4P[1+a(t-t_0)]^{3/4}}{T_0 a} = \frac{4PT^3}{aT_0^4}$$

79. b. Let m be the mass of ice.
Rate of heat given by the burner is constant.
In the first 50 min

$$\frac{dQ}{dt} = \frac{mL}{t_1} = \frac{m \text{ kg}\times(80\times4.2\times10^3)\text{ J/kg}}{(50 \text{ min})} \quad \text{(i)}$$

From 50 min to 60 min

$$\frac{dQ}{dt} = \frac{(m+5)S_{H_2O}\Delta\theta}{t_2}$$

$$= \frac{(m+5)\text{ kg }(4.2\times10^3)\text{ J/kg}\times2°\text{C}}{10 \text{ min}} \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{80m}{50} = \frac{2(m+5)}{10}$$

$$7m = 5 \Rightarrow m = \frac{5}{7}\text{ kg} \simeq 0.7 \text{ kg}$$

80. a. Heat absorbed per second by liquid helium $= \alpha A(T_0^4 - T^4)$
Heat required to boil liquid helium away

$$= \left(\frac{dm}{dt}\right)L$$

$$\left(\frac{dm}{dt}\right) = \frac{\sigma A(T_0^4 - T^4)}{L}$$

$$= \frac{5.67\times10^{-8}\times4\times3.14\times(0.1)^2(77^4-4.2^4)}{21\times10^3}$$

$$= 1.19\times10^{-5} \text{ kg/s}$$

$$= 4.3\times10^{-2} \text{ kg/h} = 43 \text{ g/h}$$

81. a. When the cylinder is heated its volume increases as

$$V_t = V_0(1+\gamma_g t) \quad \text{(i)}$$

If the densities of mercury at the temperatures t_0 and t_1 are denoted by ρ_0 and ρ_1

$$m_0 = V_0\rho_0 \quad \text{and} \quad m_t = V_t\rho_t \qquad \text{(ii)}$$

$$m_t = m_0$$

$$\rho_t = \frac{\rho_0}{(1+\gamma_{Hg}t)} \qquad \text{(iii)}$$

From Eqs. (i), (ii) and (iii)

$$\gamma_{glass} = \frac{m_t(1+\gamma_{Hg}t_t)-m_0}{m_0 t_t}$$

$$= \frac{99.7(1+18\times10^{-5}\times20)-100}{100\times20}$$

$$= 2.946\times10^{-5}/°C$$

The coefficient of linear expansion of glass

$$\alpha_{glass} = \frac{\gamma_{glass}}{3} \simeq 10^{-5}/°C$$

82. **b.** Let V_0 and V_t be volumes of mercury at 0°C and t°C, respectively; A_0 and A_t are areas of cross section of tube at 0°C and t°C, respectively.
Apparent increase in volume

$$\Delta V = \Delta V_a - \Delta V_g$$

$$= (\gamma_{Hg} - 3\alpha_g)V\Delta T$$

$$\Delta l = l\alpha_g \Delta T$$

$$A_t(\Delta l) = \Delta V = (\gamma_{Hg} - 3\alpha_g)V\Delta T$$

$$A_t = \frac{(\gamma_{Hg}-3\alpha_g)V}{\Delta l} = \frac{(18\times10^{-5}-3\times10^{-5})(10^{-6})}{3\times10^{-3}}$$

$$\pi r^2 = \frac{15}{3}\times10^{-8}$$

$$r = \left(\frac{15}{3\times3.14}\times10^{-8}\right)^{\frac{1}{2}} = 1.26\times10^{-4}\text{ m}$$

Diameter of tube = $2r$ = 0.25 mm

83. **d.** Increase in length of tape

$$\Delta l = l\alpha\Delta T$$

$$= (6400\times10^3\times11\times10^{-6}\times30) = 2113\text{ m}$$

$$\simeq 2.1\text{ km}$$

84. **c.** Energy absorbed by the earth $= \left(\frac{70}{100}\times S\right)\pi R_e^2$

Energy radiated by the earth $= e\sigma 4\pi R_e^2 T^4$

$$e = \frac{7S}{10\times4T^4\sigma}$$

$$= \frac{0.7\times1370}{4(288)^4\times5.67\times10^{-8}} \simeq 0.6$$

85. **c.** According to Wien's law $\lambda_0 T_0 = \lambda T$

According to Stefan's law $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^4$

As $\quad P = \frac{256}{81}P_0 \quad \Rightarrow \lambda = \frac{3}{4}\lambda_0$

$\therefore$ Wavelength shift $D\lambda = \lambda - \lambda_0 = -\frac{\lambda_0}{4}$

86. **d.** Let θ be the temperate of B

$$\frac{2KA(\theta-100)}{l} + \frac{\left(\frac{K}{2}\right)A(\theta-0)}{l} = 200$$

Substituting values θ = 880°C

Also from $B \to D$, $\frac{\frac{K}{2}A(\theta-0)}{l} = \frac{440}{5}\times80$

$$t = 800\text{ s}$$

87. **c.** Let l_s and l_b be the initial lengths of the steel and brass rod, respectively, and l'_s and l'_b be the corresponding lengths at any other tempeature, then,

$$l_s - l_b = l'_s - l'_b = 5\text{ cm} \qquad \text{(i)}$$

or, $\quad l_s - l_b = l_s(1+\alpha_s\Delta T) - l_b(1+\alpha_b\Delta T)$

$\Rightarrow \quad l_s\alpha_s = l_b\alpha_b$

or, $\quad \frac{l_s}{l_b} = \frac{\alpha_b}{\alpha_s} = \frac{18\times10^{-6}/°C}{12\times10^{-6}/°C} = \frac{3}{2} \qquad \text{(ii)}$

Solving Eqs. (i) and (ii), we get, l_b = 10 cm.

88. **a.** The increase in volumes of the two liquids due to an increase in temperature ΔT will be

$$\Delta V_1 = V_1\gamma_1\Delta T$$

and $\quad \Delta V_2 = V_2\gamma_2\Delta T$

$\therefore$ Total volume expansion

$$\Delta V_1 + \Delta V_2 = (V_1\gamma_1 + V_2\gamma_2)\Delta T$$

The average increase in volume per unit volume per degree rise in temperature will be

$$\frac{(\Delta V_1 + \Delta V_2)}{(V_1+V_2)\Delta T}$$

$$\gamma = \frac{V_1\gamma_1 + V_2\gamma_2}{V_1+V_2} \qquad \text{(i)}$$

But, since the cubical expansion of the two liquids compensates that of the container, γ for the container will be given by Eq. (i).

$$\therefore \quad \gamma = \frac{200\times1.4\times10^{-5}+500\times2.1\times10^{-5}}{200+50}$$

$= 1.9 \times 10^{-5}/°C$

89. b. Let V be the volume of the liquid and ΔT the rise in temperature. Since apparent expansion = true expansion − expansion of vessel

$$\therefore \quad V\gamma_1\Delta T = V\gamma\Delta T + V(3\alpha_1)\Delta T$$

or, $\gamma_1 = \gamma + 3\alpha_1$ (i) (for vessel A)

and $\gamma_2 = \gamma + 3\alpha_2$ (ii) (for vessel B)

where γ is the coefficient of real expansion of the liquid. Subtracting Eq. (ii) from Eq. (i),

$$\gamma_1 - \gamma_2 = 3(\alpha_1 - \alpha_2)$$

or, $\alpha_1 - \alpha_2 = (\gamma_1 - \gamma_2)/3$

or, $\alpha_2 = [(\gamma_1 - \gamma_2)/3] + \alpha_1$

Hence, correct option is (b).

90. d. Let V be the volume of the glass vessel. Then volume of mercury will be $(3/5)V$.

Expansion of mercury = $(3/5)\ V \times \gamma_m \Delta T$

and that of glass = $V\gamma_g \Delta T$

Net (effective or apparent) expansion of mercury

$$\Delta V = V\left(\frac{3\gamma_m}{5} - \gamma_g\right)\Delta T$$

∴ Coefficient of apparent expansion will be

$$\frac{\Delta V}{(3/5)V\times\Delta T} = \left(\frac{3}{5}\gamma_m - \gamma_g\right)\frac{5}{3}$$

91. d.

Given $\dfrac{dR}{R}\times100 = \dfrac{(507.5-500.0)}{500.0}\times100 = 1.5\%$

Now, $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2I}(I\omega)^2 = \dfrac{L^2}{2I}$

or, $K = \dfrac{L^2}{2(MR^2/2)} = \dfrac{L^2}{M}(R^{-2})$

Taking log and differentiating, $\dfrac{dK}{K} = -\dfrac{2dR}{R}$

or, % change in $K = \dfrac{dK\times100}{K} = -\dfrac{2dR}{R}\times100 = -3\%$

92. a.

$\dfrac{V'}{V} = \dfrac{\rho}{\sigma}$ = fraction of volume of sphere submerged

$= \eta$ (say)

To find % change in η, i.e., $\dfrac{\eta'-\eta}{\eta}\times100$

or, $$\left(\frac{\eta'}{\eta}-1\right)\times100 = \left[\frac{(\rho/\sigma)'}{(\rho/\sigma)}-1\right]\times100$$

$$= \left[\frac{(1+\gamma_m\Delta T)}{1+\gamma_P\Delta T}-1\right]\times100 \simeq (\gamma_m - \gamma_p)\Delta T\times100$$

$= (182 - 27) \times 10^{-6} \times 100 = 1.24\%$

93. d.

Slope of line $A = \dfrac{(1006-1000)\text{ mm}}{T°\text{C}} = \dfrac{\Delta L}{\Delta T} = L\alpha_A$

i.e., $\dfrac{6}{T}\text{ mm/°C} = (1000\text{ mm})\alpha_A$ (i)

Similarly, for line B,

$\dfrac{2}{T}\text{ mm/°C} = (1002\text{ mm})\alpha_B$ (ii)

Dividing Eq. (i) by Eq. (ii),

$3 = \dfrac{1000\alpha_A}{1002\alpha_B} \simeq \alpha_A = 3\alpha_B$ (iii)

From Eq. (iii), $\alpha_A = 3 \times 9 \times 10^{-6} = 27 \times 10^{-6}/°C$

From Eq. (i), $T = \dfrac{6}{1000\alpha_A} = \dfrac{6\times10^6}{1000\times27}$

$= 222.22°C$

94. c. From Fig. 1.85(a), the slope of line is

$$\frac{\Delta L}{\Delta T} = \frac{(1001.2-1000.0)\text{ mm}}{(300-0)°\text{C}}$$

$= 4\times10^{-6}$ m/°C

But $\dfrac{\Delta L}{\Delta T} = L\alpha$

$\therefore \quad \alpha = \dfrac{4\times10^{-6}}{1\text{ m}}$ m/°C

$= 4\times10^{-6}/°C$

As shown in Fig. 1.85(b) in the question, the slope of line is

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$$\frac{\Delta V_a}{\Delta T} = \frac{(1003-1000)\text{ mL}}{(20-0)°\text{C}} = \frac{3}{20}\text{ mL/°C}$$

But, $\frac{\Delta V_a}{\Delta T} = V\gamma_a$

$\therefore$ $\gamma_a = \frac{3}{20\times 10^3}$/°C $= 150\times 10^{-6}$/°C

Since, $\gamma_a = \gamma_r - \gamma_c$

$\therefore$ $\gamma_r = \gamma_a + \gamma_c = \gamma_a + 3\alpha$

$= (150 + 12) \times 10^{-6}$/°C

$= 16.2 \times 10^{-5}$/°C

95. d. The slope of lines are

$$\left(\frac{\Delta L}{\Delta T}\right)_1 = L\alpha_1 = \frac{4-0}{4-0} = 1 \quad \text{(For the first metal)}$$

and $\left(\frac{\Delta L}{\Delta T}\right)_2 = L\alpha_2 = \frac{2-0}{4-0} = \frac{1}{2}$ (For the second metal)

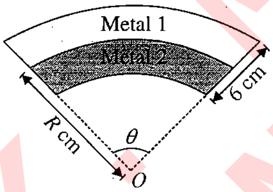

Fig. 1.112

$L_0(1+\alpha_1 T) = (R + 1.5)\,\theta$

and $L_0(1+\alpha_2 T) = (R - 1.5)\,\theta$

$\therefore$ Dividing, we get, $\frac{1+\alpha_1 T}{1+\alpha_2 T} = \frac{R+1.5}{R-1.5}$

or, $(\alpha_1 - \alpha_2)T \simeq \frac{3}{R}$ and $\alpha_1 - \alpha_2 = \frac{3}{RT}$

From the graph [shown in Fig. 1.87(a)]

$$\alpha_1 - \alpha_2 = \frac{3}{7500\times 50} = 8\times 10^{-6}\text{/°C}$$

But $\alpha_1/\alpha_2 = 2$

$\therefore$ $\alpha_1 = 16\times 10^{-6}$/°C and $\alpha_2 = 8\times 10^{-6}$/°C

96. b.

Given $\frac{\text{Latent heat of fusion}}{\text{Latent heat of vapourization}} = \frac{\lambda}{1} = \frac{L_f}{L_v}$ (say)

i.e., $L_f = \lambda L_v$

Now, if k is required fraction.

Then, $(1-k)L_f = kL_v$

or, $\frac{(1-k)}{k} = \frac{1}{\lambda}$ or, $\frac{1}{k} = \frac{1}{\lambda} + 1$

or, $k = \frac{\lambda}{\lambda+1}$

97. c. At 70°C, the system attains steady state.

i.e., Rate of heat generated = Rate of heat loss

or, 10 W = k (70 − 30)°C

(From Newton's law of cooling)

or, k = (1/4) W/°C

At 50°C, rate of heat loss should be k (50 − 30)°C = 5 W,
But rate of heat loss = (heat capacity) × (rate of cooling)

i.e., $\frac{-dQ}{dt} = c\left(\frac{-dT}{dt}\right) - 5\text{ W} = c\left[\frac{49.9-50}{60}\right]$°C/s

or 3000 J/°C

98. c. From the graph for the same temperature drop, (ΔT say), the respective time taken by the liquid and water are 1 and 2 units, respectively. Average rate of heat losses for the two containers should be the same.

$\therefore$ (100 g × 4200 J/kg°C + 160 g × s) ($\Delta T/1$)

= (100 g + 200 g) 4200 J/kg°C ($\Delta T/2$)

$\Rightarrow s$ = 1312.5 J/kg°C

99. c.

$$\text{Efficiency} = \frac{0.54\times 746}{500} = 0.80 \quad \text{or} \quad 80\%$$

(0.5 kW = 500 W and 0.54 hp = 0.54 × 746 W)

$\therefore$ 80% of the electrical energy is converted to mechanical energy and the rest 20% is converted to heat energy.

$\therefore \frac{20}{100}\times 500 = 100$ W of power is converted to heat

$\therefore$ Heat produced in 1 h (or 3600 s)

= 100 × 3600 = 36 × 10^4 J

$= \frac{36\times 10^4}{4.18}$ cal $= 8.6\times 10^4$ cal

100. d.

Initial temperature, T_i = 300 K
Final temperature, T_f = 700 K

$$Q = \int_{300}^{700} nC\,dT = \int_{300}^{700} 2\times(27.2 + 4\times 10^{-3}\times T)\,dT$$

$$= \left[54.4T + 4\times 10^{-3}T^2\right]_{300}^{700}$$

$= 2.33 \times 10^4$ J

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101. b.

As, $$\frac{dL}{L_0} = \alpha(T)\,dT$$

$$\int_{L_0}^{L} dL = L_0 \int_{T_0}^{T} \alpha(T)\,dT$$

$$L - L_0 = L_0 \int_{T_0}^{T} \alpha(T)\,dT$$

$$L = L_0\left[1 + \int_{T_0}^{T} \alpha(T)\,dT\right]$$

102. a.

As, $$\gamma_1 = \frac{\Delta V_1}{V_1 \Delta T} \quad \text{and} \quad \gamma_2 = \frac{\Delta V_2}{V_2 \Delta T}$$

$$\Rightarrow \Delta V_1 = \gamma_1 V_1 \Delta T$$

and $$\Delta V_2 = \gamma_2 V_2 \Delta T$$

Fraction of volume submerged before temperature is raised is given by $f = \rho_1/\rho_2$.

Fraction of volume submerged after the temperature is raised is given by $f' = \rho'_1/\rho'_2$.

$$\alpha' = \left(\frac{\rho_1}{1+\gamma_1\Delta T}\right)\left(\frac{1+\gamma_2\Delta T}{\rho_2}\right) = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)$$

$$\Rightarrow \alpha' = f\left(\frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}\right)$$

$$\frac{\alpha'}{f} = \frac{1+\gamma_2\Delta T}{1+\gamma_1\Delta T}$$

103. c. From the conservation principle of angular momentum

$$L_i = L_f$$

$$I_0\omega_0 = I\omega$$

Here, $$I = I_0[1 + 2\alpha(\theta_2 - \theta_1)]$$

$$\therefore \omega = \frac{I_0\omega_0}{I_0[1+2\alpha(\theta_2-\theta_1)]} = \frac{\omega_0}{[1+2\alpha(\theta_2-\theta_1)]}$$

104. b. The change in natural length $= \Delta l_t = l\alpha t$

The natural length of rod at temperature t°C $= l + l\alpha t$

The decrease in natural length due to developed stress $= \Delta l$

But the length of rod remains constant.

$$\therefore \Delta l_t - \Delta l = 0$$

$$\therefore \Delta l = \Delta l_t = l\alpha t$$

$$\therefore E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\frac{-\Delta l}{l+\Delta l_t}}$$

$$\therefore \quad E = -\frac{EA\Delta l}{l+\Delta l_t} = -\frac{EA\Delta l_t}{l+\Delta l_t}$$

$$= -\frac{EAl\alpha t}{l+l\alpha t} = -\frac{EA\alpha t}{(1+\alpha t)}$$

105. b. Expansion in mercury $= V_0\gamma_m T$

Expansion in glass bulb $= V_0(3\alpha)T$.

Apparent expansion in mercury $= V_0\gamma_m T - V_0(3\alpha)T$

$$\therefore \quad \Delta l = \frac{V_0\gamma_m T - V_0 3\alpha T}{A_0(1+2\alpha_g T)}$$

Length of mercury column in capillary is

$$h = \Delta l = \frac{V_0 T}{A_0}\frac{(\gamma_m - 3\alpha_g)}{(1+2\alpha_g T)}$$

106. b. Let V be volume of either liquid

Mass of water $= V \times 1$ g

Mass of alcohol $= V \times 0.8 = 0.8\,V$ g

Rate of cooling of the water calorimeter

$$= \frac{1}{100}[V\times(50°-40°) + V\times 1\times(50°-40°)]$$

$$= (1/5)\,V \text{ cal/s}$$

Rate of cooling of alcohol calorimeter

$$= \frac{1}{74}[V\times(50°-40°) + 0.5V\times s(50°-40°)]$$

$$= (1/74)(10\,V + 8Vs) \text{ cal/s}$$

As, rate of cooling of both is same

$$5V = (1/74)(10V + 8Vs)$$

$$s = 0.6 \text{ cal/g°C}$$

107. a.

$$P = W/t$$

Total work done by drill machine in 2.5×60 s

$$= (10 \times 10^3)(2.5 \times 60) = 15 \times 10^5 \text{ J}$$

$\therefore$ Energy lost $= 50\%$ of 15×10^5 J $= 7.5 \times 10^5$ J

Energy taken by its surroundings, i.e., aluminium block

$$\Delta Q = mc\Delta t = 8 \times 10^3 \times 0.91 \times \Delta T \text{ J}$$

Energy given = Energy taken

$$7.5 \times 10^5 = 8 \times 10^3 \times 0.91 \times \Delta T$$

$$\Rightarrow \Delta T = 103°\text{C}$$

108. a. Thermal capacity of ball $= mc = 10$ cal/°C

Let T be the furnace temperature.

Water eq. of vessel and contents $= mc = 200$ g.

Resultant temperature $= 40$°C.

According to principle of calorimetry.

Heat lost by hot body = heat gained by cold body.

$$m_1c_1(T - 40) = m_2c_2(40 - 10)$$

$$10(T - 40) = 200 \times 30$$

$$T = 640°\text{C}$$

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109. a.

$$m_1\times1\times(50-30)=m_2\times1\times(80-10)$$

$$m_1\times20=m_2\times30 \quad \text{or} \quad \frac{m_1}{n_2}=\frac{3}{2}$$

Mass of water from tank $A=\frac{3}{5}\times40=24$ kg

Mass of water from tank $B=\frac{2}{5}\times40=16$ kg

110. c. Half of KE is attained as heat by each ball.

$$\frac{1}{2}\text{KE}=m_1s_1T_1=m_2s_2T_2=\frac{1}{2}\times1\times(50)^2$$

$$=1\times0.105\times418\times10^3\times T_1$$

$$T_1=\frac{50\times50}{2\times0.105\times4.18\times10^3}$$

$$=\frac{25}{2.1\times4.18}=\frac{25}{8.778}\cong3.4\text{ K}$$

As $\quad m_2=\frac{m_1}{5}$, so $T_2=5T_1=17$ K

111. c. Let m be the mass of water.
Quantity of heat absorbed by water in 10 min.

$=ms\,\Delta T=m\times1\times100=100$ m (in calories)

Quantity of heat absorbed by water in 54 min

$$=\frac{100m\times54}{10}$$

Quantity of heat required to convert water into steam $=mL$

Hence, $\frac{100m\times54}{10}=mL$ or $L=540$ cal/g

112. b. According to principle of calorimetry,

$$ML_F+Ms\,\Delta T=(ms\,\Delta T)_{\text{water}}+(ms\,\Delta T)_{\text{flask}}$$

$$50L_F+50\times1\times(40-0)$$
$$=200\times1\times(70-40)+W(70-40)$$
$$50L_F+2000=(200+W)\,30$$
$$5L_F=400+3W \qquad \text{(i)}$$

Now the system contains (200 + 50) g of water at 40°C, so when further 80 g of ice is added

$$80L_F+80\times1\times(10-0)$$
$$=250\times1\times(40-10)+W(40-10)$$
$$80L_F=670+3W \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii),

$$L_F=90\text{ cal/g} \quad \text{and} \quad W=\frac{50}{3}\text{ g}$$

113. a.

$$U_0=V_0\sigma_L^0g=W_0 \quad \text{and} \quad U_t=V_t\sigma_L^tg=W$$

$$\frac{W}{W_0}=\frac{V_t}{V_0}\times\frac{\sigma_L^t}{\sigma_L^0}=\frac{(1+\gamma_B\,\Delta\theta)}{(1+\gamma_L\,\Delta\theta)}$$

$$=(1+\gamma_S\Delta\theta)\,(1+\gamma_L\,\Delta\theta)^{-1}=1+\gamma_s\,\Delta\theta-\gamma_L\,\Delta\theta$$
$$=W=W_0[1+(\gamma_S-\gamma_L)\,\Delta\theta)]$$
$$=W_0\,[1+(\gamma_S-\gamma_L)t]$$

114. b. $\frac{T_{20}-T_0}{T_0}=\frac{1}{2}\times\alpha\times20=10\alpha$

Here T_{20} is the correct time period. The time period at 0°C is smaller so that the clock runs fast. The time gained in 24 h.

$$=24\text{ h}\times\alpha\times10$$
$$\Rightarrow 15\text{ s}=24\text{ h}\times\alpha\times10$$

$$\alpha=\frac{15\,s}{86400\,s\times10°\text{C}}=1.7\times10^{-5}\text{ /°C}$$

115. c. Volume of mercury at 18°C

$$V_0=50\text{ cc}$$

Volume of mercury at 38°C
$$(V_{38})_r=V_0(1+\gamma_m\,\Delta\theta)$$

Volume of flask at 38°C
$$(V_{38})_f=V_0(1+\gamma_f\,\Delta\theta)$$

Volume of mercury at 38°C above the tank

$$=V_0(1+\gamma_m\,\Delta\theta)-V_0(1+\gamma_f\,\Delta\theta)$$
$$=V_0\,(\gamma_m-\gamma_t)\,\Delta\theta=50\,[180\times10^{-6}-3\times9\times10^{-6}](38-18)$$
$$=0.153\text{ cc}$$

116. d. Increase in volume of flask
$$=40\times10^{-6}\times10^3\times10^2=4\text{ cc}$$

Increase in volume of mercury
$$=180\times10^{-6}\times10^3\times10^2=18\text{ cc}$$

Volume of mercury overflow
$$=18-4=14\text{ cc}$$

117. a. The 80 cm mark on the aluminium rod is really at a greater distance from the zero position than indicated because of the increase in temperature $\Delta\theta=40°$C. The increased length is

$$\Delta L=\alpha_{Al}L_{Al}\Delta\theta$$
$$=(2.50\times10^{-5})(80)(40)$$
$$=0.08\text{ cm}$$

The correct length of the line is

$$L=80+0.08=80.08\text{ cm}$$

118. b. If the point is at a distance x from water at 100°C, heat conducted to ice in time t,

$$Q_{\text{ice}}=KA\frac{(200-0)}{(1.5-x)}\times t$$

So ice melted by this heat

$$m_{ice} = \frac{Q_{ice}}{L_F} = \frac{KA}{80}\frac{(200-0)}{(1.5-x)}\times t$$

Similarly heat conducted by the rod to the water at 100°C in time t,

$$Q_{water} = KA\frac{(200-100)}{x}t$$

Steam formed by this heat

$$m_{steam} = \frac{Q_{water}}{L_V} = KA\frac{(200-100)}{540\times x)}t$$

According to given problem $m_{ice} = m_{steam}$,

i.e., $\quad \dfrac{200}{80(1.5-x)} = \dfrac{100}{540\times x} \quad x = \dfrac{6}{58}m = 10.34$ cm

i.e., 200°C temperature must be maintained at a distance 10.34 cm from water at 100°C.

119. b. According to Newton's law of cooling

$$\left(\frac{\theta_1-\theta_2}{t}\right) = K\left[\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)\right]$$

So that $\quad \left(\dfrac{80-50}{5}\right) = K\left[\left(\dfrac{80+50}{2}-20\right)\right]$

And $\quad \left(\dfrac{60-30}{t}\right) = K\left[\left(\dfrac{60+30}{2}\right)-20\right]$

Solving these for t, we get $t = 9$ min.

120. c. It is given that the volume of air in the flask remains the same. This means that the expansion in volume of the vessel is exactly equal to the volume expansion of mercury.

i.e., $\quad \Delta V_G = \Delta V_L$

or $\quad V_G\gamma_G\,\Delta\theta = V_L\gamma_L\,\Delta\theta$

$\therefore \quad V_L = \dfrac{V_G\gamma_G}{\gamma_L} = \dfrac{1000\times(3\times9\times10^{-6})}{1.8\times10^{-4}} = 150$ cc

121. a. Let volume of metal piece be V_1 at t_1°C (= 27°C) and V_2 at t_2°C (= 42°C).

Given weight of metal piece in liquid at 27°C = 30 g.

As weight of metal piece in air = 46 g, loss of weight of metal piece in liquid = 46 − 30 = 16 g = weight of liquid displaced = −volume of liquid displaced × density.

$\therefore \quad 16 = V_1\times1.24 \quad$ or $\quad V_1 = \dfrac{16}{1.24}$ cm^3

Similarly $\quad V_2 = \dfrac{46-30.5}{1.20} = \dfrac{15.5}{1.20}$ cm^3

Now $\quad V_{42} = V_{27}(1+\gamma\,\Delta\theta)$

or $\quad V_2 = V_1(1+\gamma\Delta\theta) = V_1(1+\gamma\times15)$

$\therefore \quad 1+15\gamma = \dfrac{V_2}{V_1} = \dfrac{15.5/1.20}{16/1.24} = 1.0010$

$\therefore \quad \gamma = 6.7\times10^{-5}$/°C

$\therefore \quad \alpha = \dfrac{\gamma}{3} = 2.23\times10^{-5}$/°C

122. c. Mass of water = 250 g
Mass of alcohol = 200 g
Water equivalent of calorimeter, W = 10 g
Fall of temperature = 60 − 55 = 5°C
Time taken by water to cool = 130 s
Time taken by alcohol to cool = 67 s
Heat lost by water and calorimeter

= (250 + 10) 5 = 260 × 5 = 1300 cal

Rate of loss of heat $= \dfrac{1300}{130} = 10$ cal/s

Heat lost by alcohol and calorimeter = (200s + 10)5

Rate of loss of heat $= \dfrac{(200s+10)5}{67}$ cal/s

Rates of loss of heat in the two cases are equal

$\therefore \quad \dfrac{(200s+10)s}{67} = 10 \quad$ or $\quad s = 0.62$ cal/g°C

123. a. The kinetic energy of the bullet will be utilized to melt the bullet

$$\frac{1}{2}mv^2 = (ms\Delta\theta)\text{ J}$$

$$\frac{1}{2}\times2\times10^{-3}\times(200)^2 = 2\times0.03\times\Delta\theta\times4.2$$

$$\Delta\theta = 158°\text{C}$$

124. c. Q_1 = 10 × 1 × 10 = 100 cal
Q_2 = 10 × 0.50 [0 − (−20)] + 10 × 80
= (100 + 800) cal = 900 cal

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of the ice from − 20°C to 0°C, hence mixture in equilibrium will consist of 10 g of ice and 10 g of water, both at 0°C.

125. b. Loss in energy = $mg(h - h')$

= 0.1 × 10 × (10 − 5.4)
= 4.6 J

Now, $\quad$ 4.6 J = $ms\,\Delta\theta$
= 0.1 × 460 × $\Delta\theta$

$\therefore \quad \Delta\theta$ = 0.1°C

126. a. According to Stefan's law,

$$E = \sigma T^4$$

Total surface area of the sun $=4\pi R_s^2$.
Therefore, the total energy radiated per second by the sun per unit solid angle

$$=\frac{\sigma T^4\times 4\pi R_s^2}{4\pi}=\sigma T^4 R_s^2$$

Let R_{es} be the distance of earth from the sun. Hence, intensity of radiation of earth

$$I=\sigma T^4 R_s^2/R_{es}^2$$

$$\therefore \quad 1400=(5.6\times10^{-8})T^4\left[\frac{7.0\times10^8}{1.5\times10^{11}}\right]^2$$

$$T=5801 \text{ K}$$

127. a. As $T_B > T_A$, heat flows from B to A through both paths BA and BCA.
Rate of heat flow in BC = Rate of heat flow in CA

$$\frac{KA(\sqrt{2}T-T_c)}{l}=\frac{KA(T_c-T)}{\sqrt{2}l}$$

Solving this, we get $\quad T_c=\frac{3T}{\sqrt{2}+1}$

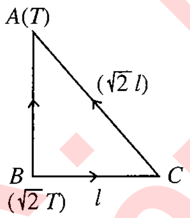

Fig. 1.113

128. c

$$V=V_0(1+\gamma\,\Delta\theta)$$

or $\quad L^3=L_0(1+\alpha_1\,\Delta\theta)\,L_0^2\,(1+\alpha_2\,\Delta\theta)^2$

$\Rightarrow \quad L^3=L_0^3\,(1+\alpha_1\,\Delta\theta)\,(1+\alpha_2\,\Delta\theta)^2$

Since $\quad L_0^3=V_0$, hence

$$1+\gamma\,\Delta\theta=(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$$
$$\cong(1+\alpha_1\,\Delta\theta)(1+2\alpha_2\,\Delta\theta)$$
$$\cong 1+\alpha_1\,\Delta\theta+2\alpha_2\,\Delta\theta$$

$\therefore \quad \gamma=\alpha_1+2\alpha_2$

129. d. We Know that

$$T=2\pi\sqrt{\frac{L}{g}}$$

$$\therefore \quad \Delta T=\frac{2\pi}{2}\left(\frac{1}{Lg}\right)^{1/2}\Delta L$$

or $\quad \frac{\Delta T}{T}=\frac{\Delta L}{2L}=\frac{1}{2}\alpha\,\Delta\theta=\frac{2\times10^{-6}\times10}{2}=10^{-5}$

$$\therefore \quad \frac{\Delta T}{T}\times100=1\times10^{-5}\times100=10^{-3}\%$$

130. c. Length of iron rod at 100°C,

$$L_1=50[1+12\times10^{-6}\times(100-20)]=50.048 \text{ cm}$$

Length of aluminum rod at 100°C,

$$L_2=100[1+24\times10^{-6}\times(100-20)]=100.192 \text{ cm}$$

The length of composite system at 20°C

$$=50+100=150 \text{ cm}$$

and length of composite system at 100°C

$$=50.048+100.192=150.24 \text{ cm}$$

$$\therefore \text{ Average } \alpha=\frac{0.24 \text{ cm}}{150 \text{ cm}\times(100-20)}$$
$$=20\times10^{-6}/°\text{C}$$

The above shortcut is applicable only if α varies linearly.

131. a. $\gamma_{\text{real}}=\gamma_{\text{app}}+\gamma_{\text{vessel}}$

$\Rightarrow \quad 153\times10^{-6}+(\gamma_{\text{vessel}})_{\text{glass}}=144\times10^{-6}+(\gamma_{\text{vessel}})_{\text{steel}}$

Further $\quad (\gamma_{\text{vessel}})_{\text{steel}}=3\alpha=3\times(12\times10^{-6})$
$$=36\times10^{-6}/°\text{C}$$

$\therefore \quad 153\times10^{-6}+(\gamma_{\text{vessel}})_{\text{glass}}=144\times10^{-6}+36\times10^{-6}$

$\therefore \quad (\gamma_{\text{vessel}})_{\text{glass}}=27\times10^{-6}/°\text{C}$

or $\quad \alpha=\frac{\gamma_{\text{glass}}}{3}=9\times10^{-6}/°\text{C}$

132. b. Since the vessel is partly filled, volume of the vessel is greater than that of the liquid. When a body having volume V is heated through $\Delta\theta$, then increase in its volume is given by

$$\Delta V=V\cdot\gamma\cdot\Delta\theta$$

Since, $\gamma_V=\gamma_L$, therefore $\Delta V\propto V$. Hence, on heating expansion of vessel will be greater than that of liquid. It means unoccupied volume will necessarily increase. So, option (b) is correct.

133. b. Energy supplied by the heater to the system in 10 min

$$Q_1=P\times t=90 \text{ J/s}\times10\times60 \text{ s}$$

$$=54000 \text{ J}=\frac{54000}{4.2}\text{cal}=12857 \text{ cal}$$

Now if θ is the final temperature of the system, energy absorbed by it to change its temperature from 10°C to θ°C is

$$Q_2=(ms\,\Delta T)_{\text{water}}+(ms\,\Delta T)_{\text{coil + calorimeter}}$$
$$=360\times1\times(\theta-10)+40(\theta-10)$$
$$=400\,(\theta-10)$$

According to problem, $Q_1=Q_2$

So $12857=400(\theta-10)$ or $\theta=42.14°\text{C}$

134. d. Let m grams of water whose temperature is θ_0 (> 30°C) and specific heat is 1 cal/g-°C be added to

20 g of water at 30°C and let θ be the final temperature of mixture.

$$m(1)(\theta_0-\theta)(20)(1)(\theta-30)$$

$$\therefore \quad \theta=\frac{600+m\theta_0}{20+m}$$

The right-hand side is maximum for option (d). Therefore, the correct answer is (d).

135. a. By the law of conservation of energy, energy given by heater must be equal to the sum of energy gained by water and energy lost from the lid.

$$Pt = ms\Delta\theta + \text{energy lost}$$

i.e., $\quad 1000t = 2 \times (4.2 \times 10^2) \times 50 + 160t$

or $\quad 840t = 8.4 \times 10^3 \times 50$

or $\quad t = 500$ s

$\quad = 8$ min 20 s

136. b. Evidently the initial temperature of the water contained in the vessel (Mg) is 80°C, and the temperature of the water passed into it is 60°C, as the final temperature of the mixture tends to attain a value of 60°C.

$$M \times 1\ (80-70) = m \times 10 \times 1\ (70-60)$$

or, $\quad M/m = 10$

Since the heat exchanged after a long time is 800 cal.

$$(Mg)\ (1\ \text{cal/m°C})\ (80-60°\text{C}) = 80\ \text{cal}$$

or, $\quad M = 40$ g

$\Rightarrow m = 4$ g

137. a. Let the radii of the spheres be R, $R+a$, $R+2a$ and $R+3a$, where a is a constant and the specific heat capacities be s, sr, sr^2 and sr^3 where r is another constant.

Given, $\left(\dfrac{\text{heat capacity of } D}{\text{heat capacity of } B}\right):\left(\dfrac{\text{heat capacity of } C}{\text{heat capacity of } A}\right) = 8:27$

or, $\left[\dfrac{(R+3a)^3 sr^3}{(R+a)^3 sr}\right]:\left[\dfrac{(R+2a)^3 sr^2}{Rs}\right] = 8:27$

or, $\left(1+\dfrac{2a}{R+a}\right):\left(1+\dfrac{2a}{R}\right) = 2:3$

or, $\dfrac{2a}{R+a}:\dfrac{2a}{R} = 1:2$

or, $R = a$

$$\frac{m_2}{m_1}=\frac{(4/3)\pi(R+R)^3\rho}{(4/3)\pi(R)^3\rho}=\frac{8}{1}$$

138. c. Smooth and polished plates are poor radiators of heat. Hence, heat coming out from A is small, even though B being a black and rough plate is a good absorber. Effectively the heat coming to the left of pellet P is small.

Black and rough plates are good radiators of heat. Hence, plate B_2 radiates heat to a satisfactory level; however, plate A_2, being smooth and polisher, is a bad absorber. Effectively, the heat coming to the right of P is also small.

139. c.

$$\frac{\Delta Q}{\Delta t}=\frac{KA\Delta T}{\Delta x} \qquad \Delta Q = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$$

Assuming the thickness of the spheres to be small, we have

For smaller sphere:

(rate of heat flow) (time) = (volume of ice melted) (ρL)

$$\text{i.e., } K_1(4\pi r^2)\frac{\Delta\theta}{d}\cdot 16=\frac{4}{3}\pi r^3\rho L \qquad \text{(i)}$$

For larger sphere:

$$K_2[4\pi(2r)^2]\frac{\Delta\theta}{d/4}\cdot 25=\frac{4\pi}{3}(2r)^3\rho L \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i),

$$K_2/K_1 = 8/25.$$

140. b.

$$\left(\frac{dQ}{dt}\right)\times\frac{1}{A}=K_A\frac{(50-30)}{3} \qquad \text{(for slab } A\text{)}$$

$$=K_B\frac{(50-20)}{3} \qquad \text{(for slab } B\text{)}$$

$$2KA = 3KB$$

or, $\quad KA/KB = 3/2$

141. b. In the steady state for a polished rod, the thermal current is constant.

$$i=\frac{dQ}{dt}=-KA\left(\frac{d\theta}{dx}\right)$$

or, $\quad d\theta \cong c\,dx$

where, c is a +ve constant $= i/KA$

i.e., $\quad \theta \cong -cx + c_0$

a straight line having a negative slope.

142. c. If mass of the bullet is m grams, heat absorbed by it to raise its temperature from 27° C to 327°C

$$mc\ \Delta T = m\times 0.03\times(327-27) = 9m \text{ cal}$$

And heat required by the bullet to melt

$$mL = m\times 6 = 6\,m \text{ cal}$$

So, the total heat required by the bullet

$$Q_1 = (9m+6m) = 15\,m \text{ cal} = (15m\times 4.2)\text{ J}$$

(as 1 cal = 4.2 J)

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Now when the bullet is stopped by the obstacle, loss in its mechanical energy.

$$ME=\frac{1}{2}(m\times10^{-3})v^2\text{ J} \qquad (\text{as } m \text{ gram} = m\times 10^{-3}\text{ kg})$$

As 25% of the energy is absorbed by the obstacle, the energy absorbed by the bullet

$$Q_2=\frac{75}{100}\times\frac{1}{2}mv^2\times10^{-3}=\frac{3}{8}mv^2\times10^{-3}\text{ J}$$

Now the bullet will melt if

$$Q_2\geq Q_1$$

$$\frac{3}{8}mv^2\times10^{-3}\geq15m\times4.2$$

$$v\geq\sqrt{(4\times4.2)}\times10^2$$

$$v_{\min}=410\text{ m/s}$$

143 a. As water equivalent of pitcher is 0.5 kg, i.e., pitcher is equivalent to 0.5 kg of water, heat to be extracted from the system of water and pitcher for decreasing its temperature from 30 to 28°C is

$$Q_1=(m+M)c\Delta T$$

$$=(9.5+0.5)\text{ kg }(1\text{ kcal/kgC°})(30-28)\text{°C}$$

$$=20\text{ kcal}$$

And heat extracted from the pitcher through evaporation in t minutes

$$Q_2=mL=\left[\frac{dm}{dt}\times t\right]L=\left[\frac{1\text{ g}}{\text{min}}\times t\right]580\frac{\text{cal}}{\text{g}}$$

$$=580\times t\text{ cal}$$

According to given problem $Q_2=Q_1$, i.e., $580\times t=20\times10^3$

$$t=34.5\text{ min}$$

144. a. Here ice will absorb heat while hot water will release it. So if T is the final temperature of the mixture, heat given by water

$$Q_1=mc\Delta T=5\times1\times(30-T)$$

And heat absorbed by ice

$$Q_2=5\times(1/2)\,[0-(-20)]+5\times80+5\times1(T-0)$$

So, by principle of calorimetry $Q_1=Q_2$, i.e.,

$$150-5T=450+5T$$

$$T=-30\text{°C}$$

Which is impossible as a body cannot be cooled to a temperature below the temperature of cooling body. The physical reason for this discrepancy is the heat remaining after changing the temperature of ice from –20 to 0°C with some ice left unmelted and we are taking it for granted that heat is transferred from water at 0°C to ice at 0°C so that temperature of system drops below 0°C.

However, as heat cannot flow from one body (water) to the other (ice) at same temperature (0°C), the temperature of system will not fall below 0°C.

145. a. According to Newton's law of cooling,

$$\left[\frac{\theta_1-\theta_2}{t}\right]=K\left[\left(\frac{\theta_1+\theta_2}{2}\right)-\theta_0\right]$$

So that $$\left[\frac{60-40}{7}\right]=K\left[\left(\frac{60+40}{2}\right)-10\right]$$

$$\Rightarrow K=\frac{1}{14} \qquad \text{(i)}$$

Now if after cooling from 40°C to 7 min the temperature of the body becomes θ, according to Newton's law of cooling,

$$\left[\frac{40-\theta}{7}\right]=K\left[\left(\frac{40+\theta}{2}\right)-10\right]$$

Which in the light of Eq. (i), i.e., $K=(1/14)$, gives

$$\left[\frac{40-\theta}{7}\right]=\frac{1}{14}\left[\left(\frac{20+\theta}{2}\right)\right]$$

$$160-4\theta=20+\theta\,;\theta=28\text{°C}$$

146. b. If θ is the temperature of outside, heat passing per second through the glass window,

$$\frac{dQ}{dt}=KA\frac{(\theta_1-\theta_2)}{L}$$

$$=\frac{0.2\times1\times(20-\theta)\text{ cal}}{0.2\times10^{-2}}=100(20-\theta)$$

And heat produced per second by the heater in the room

$$P=\frac{V^2}{R}\frac{\text{J}}{\text{s}}=\frac{V^2}{RJ}\frac{\text{cal}}{\text{s}}$$

$$=\frac{200\times200}{20\times4.2}=476.2\frac{\text{cal}}{\text{s}}$$

Now as the temperature of the room is constant, the heat produced per second by heater must be equal to the heat conducted through the glass window.

$$100(20-\theta)=476.2\,;\theta=15.24\text{°C}$$

147. b. According to Newton's law of cooling, the ratio of cooling is directly proportional to the temperature difference. When the average temperature difference is halved, the rate of cooling is also halved. So, the time taken is 10 s.

148. a. If the point is at a distance x from water at 100°C, heat transferred to ice in time t to melt it is

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$$m_1 L_1 = \frac{KA(200-0)t}{(1.5-x)}$$

or $$m_1 = \frac{KA \times 200t}{80(1.5-x)}$$

Similarly, heat conducted by the rod to water at 100°C in time t is

$$Q = \frac{KA(200-100)t}{x} = m_S L_S$$

$$\therefore m_S = \frac{KA(200-100)t}{xL_S} = \frac{KA \times 100t}{x \times 540}$$

According to problem, $m_l = m_s$

i.e., $$\frac{KA \times 200t}{80(1.5-x)} = \frac{KA \times 100t}{x \times 540}$$

or $$\frac{2}{8(1.5-x)} = \frac{1}{54x}$$

Solving it, we get, $x = 0.1034$ m or 10.34 cm

149. d. Suppose the temperatures of junctions B, C, D are θ_1, θ_2 and θ_3, respectively. Let Q_1, Q_2, Q_3, Q_4, Q_5 and Q_6 be the amounts of heat flowing from A to B, B to C, B to D, C to D to E and C to E per second, respectively. Temperature of junctions A and E are 60°C and 10°C, respectively.

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1} = \frac{0.26}{0.13} = 2$$

$$\therefore \quad T_2 = 2T_1$$

By Stefan's law, emissive power $E = \sigma T^4$

$$E_1 = \sigma T_1^4; \quad E_2 = \sigma T_2^4$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \frac{T_1^4}{(2T_1)^4} = \frac{1}{16}$$

150. d. Under steady state condition, heat released to the room = heat dissipated out of the room. Let θ be the temperature of heater. Then

$$\theta - 20 = \alpha[20-(-20)] \quad \text{(i)}$$

and $$\theta - 10 = \alpha[10-(-40)] \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\theta = 60°\text{C}$$

151. c. According to Stefan–Boltzmann law, the energy radiated per second through the surface of area A is given by

$$E = \sigma A T^4$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{A_1}{A_2}\left(\frac{T_1}{T_2}\right)^4$$

or $$10000 = \frac{r_1^2}{r_2^2}\left(\frac{2000}{6000}\right)^4$$

or $$\frac{r_1^2}{r_2^2} = (30)^4$$

or $$r_1 : r_2 = 900:1$$

152. c. Rate of loss of energy by unit area of the planet = σT^4, where σ is the Stefan's constant. Let Q be the total energy emitted by the sun every second. If d is the distance of the planet from sun, then Q falls uniformly over the inner surface of the sphere of radius d. Rate of gain of heat by unit area of planet

$$= \frac{Q}{4\pi d^2}$$

For steady temperature of plant

$$\sigma T^4 = \frac{Q}{4\pi d^2}$$

$$T^4 = \frac{Q}{4\pi\sigma d^2} \quad \text{or} \quad T = \left(\frac{Q}{4\pi\sigma d^2}\right)^{1/4}$$

or $$T \propto \frac{1}{\sqrt{d}}$$

153. a. Rate of emission of energy = $\sigma T^4 s$

Let m_1 be the mass of sphere, C is specific heat and $(d\theta/dt)$, the rate of cooling.

For sphere

$$\sigma T^4 S = m_1 C\left(\frac{d\theta}{dt}\right)_S \quad \text{(i)}$$

Let m_2 be the mass of cube, C its specific heat and $(d\theta/dt)$, the rate of cooling.

For cube

$$\sigma T^4 S = m_2 C\left(\frac{d\theta}{dt}\right)_S \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{(d\theta/dt)_s}{(d\theta/dt)_c} = \frac{m_2}{m_1} = \frac{R_s}{R_c}$$

or $$\frac{a^3\rho}{(4/3)\pi r^2 \rho} = \frac{R_s}{R_c}$$

where a is the side of cube and r is the radius of sphere, ρ is the density.

$$\therefore \quad \frac{R_s}{R_c} = \frac{3a^3}{4\pi r^3}$$

But since S is the same,

$$6a^2 = 4\pi r^2$$

or $$a^2 = (2/3)\pi r^2$$

$$\therefore \quad \frac{R_s}{R_c}=\frac{3(2\pi r^2/3)^{3/2}}{4\pi r^3}=\frac{2\pi\sqrt{2\pi}}{\sqrt{3}(4\pi)}$$

$$=\sqrt{\frac{2\pi}{12}}=\sqrt{\frac{\pi}{6}}$$

154. c. Let P be the power radiated by the sun and R be the radius of planet. Energy radiated by planet

$$=4\pi R^2\times(\sigma T^4)$$

For thermal equilibrium

$$\frac{P}{4\pi d^2}\times\pi R^2=4\pi R^2(\sigma T^4)$$

$$\therefore \quad T^4\propto\frac{1}{d^2} \quad \text{or} \quad T\propto d^{-1/2}$$

Hence $\quad n=\frac{1}{2}$

155. d. From Wien's law, $\lambda_m T$ = constant, where T is the temperature of black body and λ_m is the wavelength corresponding to maximum energy of emission. Energy distribution of black body radiation is given below:

i. U_1 and U_2 are not zero because a black body emits radiations of nearly all wavelengths.
ii. Since U_1 corresponding to lower wavelength, U_3 corresponds to higher wavelength and U_2 corresponds to medium wave length, hence $U_2 > U_1$.

Multiple Correct Answers Type

1. **a., c., d.**
True value = scale reading $[1+\alpha(\theta'-\theta)]$
a. If $\theta' > \theta$, then
Scale reading < true value
b. $\Delta t=\frac{1}{2}\alpha\,\Delta\theta t$

If $\theta' > \theta$ or $\Delta\theta$ is positive, i.e., clock will lose time, i.e., will become slow. Hence, not correct.
c. With rise in temperature, upthrust decreases. As weight remains same, hence, a floating body sinks a little more.
d. As upthrust on the body decreases with rise in temperature, hence, weight of a body in a liquid ($=W_0$ − th) increases.

2. **c., d.** This situation arises during melting (alternative (c)) or freezing (alternative (d)) of the mass.

3. **a., c.** When the disc with a central hole is heated, diameter of hole as well as outer diameter of disc both increases. As a result of this, mass of the disc will be distributed more away from its axis which means that moment of inertia will increase on heating. Now according to the law of conservation of angular momentum, as $I\omega$ = constant so ω will decrease.

4. **c., d.** From metallic piece, more heat is conducted into the body than from a wooden piece. So, alternative (c). is correct. Wooden piece will act as a black body. It will absorb more heat as compared to a polished metallic piece when placed in open in bright sun. Now, according to Kirchhoff's law, as a good absorber is also a good emitter, hence, wooden piece will lose heat at a faster rate than the metallic piece.

5. **a., b., c.** Q_1 = heat given out by water if it was to cool up to 0°C

$$=mg\times 1\ \text{cal/g-°C}\times(10-0)=10\ \text{m cal}$$

Q_2 = heat required by ice to melt completely into water at 0°C

$$=m\times L_F=mg\times 80\ \text{cal/g}=80\ \text{m cal}$$

As $Q_1 < Q_2$, equilibrium temperature will be 0°C and whole of ice will not melt.

6. **a., c.** According to Wien's law,

$$\lambda_m T=\text{constant}$$

or $$\frac{T}{v_m}=\text{constant}$$

So, if T is doubled, v_m is also doubled
Further, according to Stefan's law,

$$E\propto T^4$$

When T is doubled, E increases by a factor of 16.

7. **b., c.** Melting involves an increase in potential energy and hence an increase in internal energy.

8. **a., d.**

$$H=\text{rate of heat flow}$$

$$=\frac{900}{\frac{l_i}{K_iA}+\frac{l_0}{K_0A}}$$

Now, $$1000-\theta=\frac{Hl_i}{K_iA}$$

or, $$\theta=1000-\left[\frac{900}{\frac{l_i}{K_iA}+\frac{l_0}{K_0A}}\right]\frac{l_i}{K_iA}$$

$$=100-\frac{900}{1+\frac{l_0}{K_0}\frac{K_i}{l_i}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (l_i).

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9. c., d. The rate of flow of heat is given by

$$\frac{Q}{t}=\frac{KA(\theta_2-\theta_1)}{l}$$

where K is the thermal conductivity. A is the cross-sectional area and l the length of the rod. Hence, the correct choices are (c) and (d).

10. b., c., d.

$$W=ms \quad \text{or,} \quad m=\frac{W}{s}=\frac{4.5}{0.09}=50\text{ g}$$

The thermal capacity and the water equivalent of a body have the same numerical value.

Also, $$Q=4.5\times 8=36\text{ cal}$$

Since, the temperature remains constant, during the process of melting, no heat is exchanged with the calorimeter and hence,

$$Q=15\times 80=1200\text{ cal}$$

Hence, the correct choices are (b), (c) and (d).

11. a., b., d. Statement (a) is correct because the coefficient of volume expansion is very nearly three times the coefficient of linear expansion.

Statement (b) is also correct. When a solid is heated at constant pressure, it does expand a little and some heat is required for doing the mechanical work associated with this expansion. The difference between the two specific heats (at constant pressure and at constant volume) is small enough to be generally neglected for solids and liquids. This difference is large for gases.

Statement (c) is incorrect. The thermal conductivity of air is less than that of wool. Yet we prefer wool which traps pockets of air between its fibres. There air pockets cannot carry heat by convection either.

Statement (d) is correct because of the expansion of steel tape on a hot day, the markings on the tape would be farther apart than on a cold day.

12. c, d. Statement (a) is incorrect. According to Kirchhoff's law, the ratio of the emissive power e. and absorptive power a. is constant for all substances at any given temperature and for radiation of the same wavelength,

i.e., $$\frac{e}{a}=\text{constant}$$

Thus, if e is large, a must also be large, i.e., if a body is a good emitter of a radiation of a particular wavelength, it is also a good absorber of that radiation. Conversely, if a body is a poor emitter of radiation, it is also a poor absorber (and hence, a good reflector) of the radiation.

Statement (b) is also incorrect. The latent heat of steam is very high (2.25×10^6 J/kg). This means that 1 kg of steam at 100°C gives out 2.25×10^6 J of heat energy to convert into 1 kg of boiling water at 100°C. Hence, the burns caused by steam are more severe than those caused by boiling water. For the same reason, heating systems based on circulation of steam are more efficient in warming a house than those based on circulation of hot water.

Statement (c) is correct. The thermal radiation from the sun warms the earth during the day. Since, air is a poor conductor of heat, the atmosphere acts as a blanket for the earth and keeps the earth warm during the night. Moon is very cold because it has no atmosphere.

Statement (d) is also correct. The efficiency of an ideal heat engine is given by

$$\eta=1-\frac{T_2}{T_1}$$

where T_2 is the temperature of the sink. To have an efficiency η = 1 (or 100%), T_2 = 0 K. Since, absolute zero cannot be achieved, even an ideal heat engine cannot have a 100% efficiency.

13. a., c., d.

$$\frac{\Delta A}{A}\times 100=2\left(\frac{\Delta l}{A}\right)\times 100$$

% increase in area = 2 × 0.2 = 0.4

$$\frac{\Delta V}{V}\times 100=3\times 0.2=0.6\%$$

Since $$\Delta l=l\,\alpha\,\Delta T$$

$$\frac{\Delta l}{l}\times 100=\alpha\ \Delta T\times 100=0.2$$

$$\alpha=0.25\times 10^{-4}/°\text{C}$$

14. a., d. $\Delta V_L=\Delta V_V$

$$Y_L V_L=Y_V V_V \quad \text{or} \quad \frac{Y_L}{Y_V}=\frac{V_V}{V_L}$$

$$V_V>V_L \quad \Rightarrow Y_L>Y_V$$

15. b., d. Every object emits and absorbs the radiation simultaneously. If energy emitted is more than energy absorbed, temperature falls and vice versa.

16. a., c. This problem can be solved like electric current problem.

Let R_1, R_2, R_3, R_4, R_5, R_6 and R_7 be the rates of heat flow through AE, EB, AC, CD, CE, ED and DB, respectively.

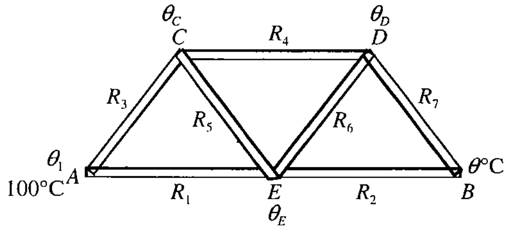

Fig. 1.114

Since $$R_1=R_2 \quad \theta_E=50°\text{C} \qquad \text{(i)}$$

$$R_5=R_6 \quad R_3=R_4+R_5=R_7 \qquad \text{(ii)}$$

$$R_4+R_6=R_7$$

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$$\frac{kA(\theta_c-50)}{l}=\frac{kA}{l}(50-\theta_D)$$

$$\frac{kA}{l}(100-\theta_c)=\frac{kA}{l}(\theta_c-50)+\frac{kA}{l}(\theta_c-\theta_D)=\frac{kA}{l}\theta_D$$

$$\theta_C+\theta_D=100$$

$$2\theta_C-2\theta_D=50\Rightarrow\theta_C=62.5°\text{C}$$

$$\theta_D=37.5°\text{C}$$

$$\therefore\quad \theta_C>\theta_E>\theta_D$$

17. b., c. $T=2\pi\sqrt{\frac{l}{g}}=2\pi\sqrt{\frac{l_0+\alpha l_0\Delta\theta_0}{g}}$

$$=T_0\left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

At 30°C, fraction loss of time $=\frac{T_{30°}-T_{20°}}{T_{20°}}$

$$=5\alpha=5\times19\times10^{-6}$$

Time lost in 24 h

$$=86400\times95\times10^{-6}=8.2\text{ s}$$

On a cold day at 10°C, fraction gain of time

$$=\frac{T_{10°}-T_{20°}}{T_{20°}}=-5\alpha$$

Time graph in 24 h = 8.2 s

18. c., d. The temperature gradient (which causes the heat to flow) is only along the length. The temperature at any point of a given cross section is the same and so $Q_a=Q_b=0$. In the steady state, Q_1 is constant $\neq 0$.

19. b., c., d.

The flow of heat will always be in the direction of the temperature gradient from higher to lower temperature. Hence Q_1 in rod AB, Q_2 in rod BC will both be in clockwise sense while Q_3 in CA will be in anti-clockwise sense. Also, we have if L is the length of each rod and A its area of cross-section,

$$Q_1=\frac{\alpha_1A(100-50)}{L}=(50\alpha_1)\frac{A}{L}$$

$$Q_2=\frac{\alpha_2A(50-0)}{L}=(50\alpha_2)\frac{A}{L}$$

$$Q_3=\frac{\alpha_3A(100-0)}{L}=(100\alpha_3)\frac{A}{L}$$

Hence $Q_1:Q_2:Q_3::\alpha_1:\alpha_2:2\alpha_3$

Also,

$$\frac{Q_1}{\alpha_1}+\frac{Q_2}{\alpha_2}=\left(50\frac{A}{L}\right)+\left(50\frac{A}{L}\right)=\frac{100A}{L}=\frac{Q_3}{\alpha_3}$$

20. b., c., d.

Resistance of each rod $R=\frac{l}{kA}$

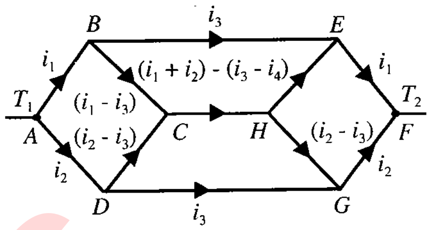

Fig. 1.115

In steady state $T_B=T_D$

$$T_E=T_G$$

Thermal current $\left(\frac{dQ}{dt}\right)=i$

$$i_1R+i_3R+i_1R=(T_1-T_2)$$

$$2i_1+i_3=\frac{(T_1-T_2)}{R}\quad\text{(i)}$$

$$2i_1+i_3=\frac{T_1-T_2}{R}\quad\text{(ii)}$$

$$I_1=i_2\quad\text{(iii)}$$

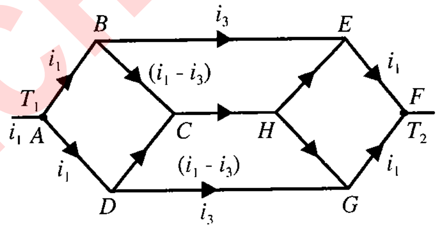

Fig. 1.116

For the path ABCHGF

$$i_1R+(i_1-i_3)R+2(i_1-i_3)R+(i_1-i_3)R+i_1R=(T_1-T_2)$$

$$6i_i-4i_3=\frac{(T_1-T_2)}{R}=2i_1+i_3\quad\text{(iv)}$$

$$4i_i=5i_3\Rightarrow i_3=\frac{4}{5}i_i\quad\text{(v)}$$

From Eqs. (ii) and (v)

$$2i_i+\frac{4}{5}i_1=\frac{(T_1-T_2)}{R}$$

$$\frac{14i_i}{5}=\frac{(T_1-T_2)}{R}$$

Equivalent thermal resistance $\frac{(T_1-T_2)}{2i_1}=\frac{7}{5}R$

$$\left(\frac{dQ}{dt}\right)_{AB}=i_1=\frac{5(T_1-T_2)KA}{14l}$$

$$\left(\frac{dQ}{dt}\right)_{BE}=\frac{2}{7}\frac{(T_1-T_2)KA}{l}$$

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$$\left(\frac{dQ}{dt}\right)_{BC}=\frac{(T_1-T_2)KA}{14l}$$

$$\left(\frac{dQ}{dt}\right)_{CH}=\frac{(T_1-T_2)KA}{7l}$$

Assertion-Reasoning Type

1. **d.** Equivalent thermal conductivity of two identical rods in series is given by

$$\frac{2}{K}=\frac{1}{K_1}+\frac{1}{K_2}$$

If $K_1 < K_2$. Then $K_1 < K < K_2$
Hence statement I is false.

2. **c.** From Wein's law $\lambda_m T$ = constant i.e., peak emission wavelength $\lambda_m \propto \frac{1}{T}$. Hence as T increases λ_m decreases.
3. **d.** The correct reason is because under steady-state conditions, when temperature becomes constant, the rate of conduction of heat across every lamina is the same.
4. **b.** Statements I and II are true but Statement II is not correct explanation for Statement I.
5. **a.** Both are true and Statement II explains Statement I because for same volume, surface area of the cylindrical bulb will be more.
6. **a.** $\alpha=\frac{1}{l}\frac{dl}{dT}$

$$\Rightarrow \alpha\int_{T_0}^{T} dT=\int_{l_0}^{l}\frac{dl}{l}$$

$$\Rightarrow l=l_0\, e^{\alpha\Delta T}$$

7. **a.** The correct model is one in which the atoms fling away from the equilibrium position through a greater distance than the one by which they come closer.
8. **a.** According to Kirchoff's law $\frac{e_\lambda}{a_\lambda}=E_\lambda$

For a particular wavelength $E_\lambda=1\Rightarrow e_\lambda=a_\lambda$
$\therefore$ Emissivity = Absorptivity

9. **c.** There is only energy transfer and not matter transfer.
10. **a.** On comparison

$$\frac{T_1}{T_2}=\frac{\lambda_2}{\lambda_1}=\frac{480}{360}=\frac{4}{3}$$

11. **b.** $R=\frac{(\theta_1-\theta_2)}{\omega/t}=\frac{l}{KA}\quad\Rightarrow R\propto\frac{1}{K}$
12. **a.** As $r=\frac{\Delta V}{V\Delta t}$, i.e., unit of coefficient of volume expansion is K^{-1}.

Comprehension Type

For Problems 1–3

1. **b.** We have $\theta-\theta_s=(\theta_0-\theta_s)e^{-kt}$,

where θ_0 = initial temperature of body = 40°C
θ = temperature of body after time t.

Since body cools from 40 to 38 in 10 min, we have

$$38-30=(40-30)\,e^{-10k}\qquad\text{(i)}$$

Let after 10 min, the body temperature be θ

$$\theta-30°=(38-30)\,e^{-10k}\qquad\text{(ii)}$$

$$\frac{\text{Eq. (i)}}{\text{Eq. (ii)}}\text{ gives }\frac{8}{\theta-30}=\frac{10}{8},\quad \theta-30=6.4$$

$$\theta=36.4\ °\text{C}$$

2. **a.** Self-explanatory.
3. **c.** During heating process from 38°C to 40°C in 10 min, the body will lose heat in the surrounding which will be exactly equal to the heat lost when it cooled from 40°C to 38°C in 10 min, which is equal to $ms\,\Delta\theta=2\times2=4$ J. During heating process heat required by the body $ms\,\Delta\theta=4$ J.
$\therefore$ Total heat required = 8 J.

For Problems 4–6

4. **c.** $\Delta V=\gamma V\Delta T$

$$=3\alpha\ V\Delta T=3\times1\times10^{-3}\times\left(\frac{5\text{ kg}}{50\text{ kg/m}^3}\right)\times50$$

$$=15\times10^{-3}\text{ m}^3$$

$$W=P\Delta V$$

$$=\left(1\times10^5\frac{\text{N}}{\text{m}^2}\right)\times(15\times10^{-3}\text{ m}^3)=15\times10^2$$

$$=1500\text{ J}$$

5. **b.** $Q=mC\Delta T=(5\text{ kg})\times(200\text{ J/kg°C})\times(50°\text{C})=50000$ J
6. **b.** $\Delta U=Q-W=50000-1500=48500$ J

For Problems 7–9

7. **b.** $\rho_{300}=\pi^2\times10^{-8}$ ohm m

$$\rho_{900}=\frac{\rho_{300}(1+900\alpha)}{(1+300\alpha)}$$

$$=\frac{\pi^2(10^{-8})(1+900\times7.8\times10^{-3})}{(1+300\times7.8\times10^{-3})}$$

$$=\pi^2\times10^{-8}\times\frac{8.02}{3.34}$$

$$=2.4\,\pi^2\times10^{-8}$$

$$\simeq 2.4\times10^{-7}\text{ ohm m}\quad(\pi^2\simeq10)$$

8. **b.** Heat radiated per second, $E=\sigma A(T^4-T_0^4)$

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$$= \sigma(2\pi rl)(T^4 - T_0^4)$$
$$= 5.68 \times 10^{-8} \times 2\pi \times 10^{-3} \times 1\,[(900^4 - 300^4)]$$
$$= 11.36\,\pi\,(6480) \times 10^{-3} \approx 230 \text{ W}$$

9. c. Heat dissipated per second = I^2R

$I^2R = E$

$$I = \sqrt{\frac{E}{R}} = \sqrt{\frac{E\pi r^2}{\rho_{900} l}} = \sqrt{\frac{231 \times .14 \times 10^{-6}}{2.4 \times 10^{-7} \times 1}} \simeq 55\text{A}$$

For Problems 10–12

10. a. Heat emitted by the surface of sphere per unit time $P_r = \sigma T^4(\pi R^2)$
Since the radius of both spheres is equal, the rate of heat loss by aluminium sphere = rate of heat loss by lead sphere.

11. b. Let $d\theta_1/dt$ be the rate of fall of temperature of aluminium sphere and $d\theta_2/dt$ be the rate of fall of temperature of lead sphere.

$$\frac{P_1}{P_2} = \frac{m_1 S_1 \dfrac{d\theta_1}{dt}}{m_2 S_2 \dfrac{d\theta_2}{dt}} = \frac{V_1 d_1 S_1 \dfrac{d\theta_1}{dt}}{V_2 d_2 S_2 \dfrac{d\theta_2}{dt}} = 1$$

$$\frac{\dfrac{d\theta_1}{dt}}{\dfrac{d\theta_2}{dt}} = \frac{S_2}{S_1}\frac{d_2}{d_1} = \frac{130 \times 2.7}{900 \times 10} = \frac{39}{1000}$$

$$[\because V_1 = V_2]$$

12. b. Power radiated by an object to the surrounding temperature T_0

$$P = e\sigma A(T^4 - T_0^4)$$
$$= \sigma eA(T^2 + T_0^2)(T_0 + T_0)(T - T_0)$$
$$= 4e\sigma A\, T_0^3\, \Delta T$$
$$P \propto \Delta T$$

The net power radiated is approximately proportional to the temperature difference, in agreement with Newton's law of cooling; if $(T - T_0)$ is small.

For Problems 13–15

13. d. Under steady state conditions, the temperatures at all sections in the system remain constant and maintain a constant temperature gradient for a given material. The temperature gradient in copper, aluminium and brass will not be same however, the rate of heat conducted across all sections whether in copper or aluminium or brass will be the same.

14. a. Since heat transmitted per second in the steady state is $Q = \dfrac{KA(T_2 - T_1)}{L}$ and the dimensions of copper, aluminium and brass rods are identical, we must have

$$K_1(100 - T_{ca}) = K_2(T_{ca} - T_{ab}) = K_3(T_{ab} - 0)$$

where K_1, K_2 and K_3 are thermal conductivities of copper, aluminium and brass, respectively, and T_{ca} and T_{ab} are the steady state temperatures of copper–aluminium and aluminium–brass junctions, respectively.

Now $\qquad K_1 = 2K_2 = 4K_3 \quad$ (given)

Hence, $\qquad 4(100 - T_{ca}) = 2(T_{ca} - T_{ab}) = T_{ab}$

Solving for T_{ab} and T_{ca} we obtain,

$$14\, T_{ca} = 1200$$

$$\Rightarrow T_{ca} = \frac{1200}{14} = 85.7 = 86°\text{C}$$

Also $\qquad T_{ab} = \dfrac{2}{3}T_{ca} = \dfrac{2 \times 86}{3} = 57°\text{C}$

15. a. See solution above.

For Problems 16–18

16. d. The distance from the end A of the copper rod (where power is supplied) and centre O of the metal sphere is $(L + r)$. Hence, in the steady state conditions,

$$P = \frac{KA(T - T_S)}{L + r}$$

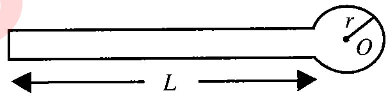

Fig. 1.117

Which gives $\qquad T_S = T - \dfrac{P(L+r)}{KA}$

(The heat is transmitted up to centre of the spherical end, and the sphere loses energy by radiation out of its spherical surface.)

17. a. Under steady state conditions, all temperatures remain constant including T_S, the temperature at centre of sphere. Hence no thermal energy accumulates inside the sphere. Hence $P_S = P$ = energy received from source.

18. b. The temperature T_S will change to a new value. The transmitted thermal energy will be the same.

For Problems 19–21

19. b. In 7 min, temperature of 100 g of water is raised by $(1000 - 16°\text{C}) = 84°\text{C}$. The amount of the heat provided by heater

$$Q_W = C_W m_W \Delta T = (1\text{ cal/g°C})(100\text{ g})(84°\text{C})$$
$$= 8.4 \times 10^3\text{ cal} = (8.4 \times 10^3 \times 4.186)\text{ J}$$
$$\simeq 3.5 \times 10^4\text{ J}$$

Power of heater $= \dfrac{Q_W}{t_1}$

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$$= \frac{8.4\times10^3\,\text{cal}}{(7\times60)\,\text{s}} = 20\,\text{cal/s}$$

$$= (20\times4.18)\ \text{J/s} = 83.6\ \text{W} \approx 84\ \text{W}$$

20. b. With 200 g of alcohol in the vessel, the temperature rises from 16°C to 78°C in 6 min 12 s.

Quantity of heat absorbed by alcohol

$$Q_a = C_a m_a(\theta_f - \theta_i)$$

$$= C_a \times (200\,\text{g})(78-16)°\text{C}$$

Heat given by heater

$$Q_a = Pt_2 = (20\ \text{cal/s})\times 372\ \text{s}$$

Specific heat of alcohol

$$C_a = \frac{20\times372}{200\times62} = 0.6\ \text{cal/g°C}$$

$$= \frac{0.6\times4.186\,\text{J}}{10^{-3}\,\text{kg°C}} = 2.5\times10^3\ \text{J/kg°C}$$

$$= 2.5\ \text{K/g°C}$$

21. b. Heat required to vapourize it at 78°C

$$Q_v = Pt_3 = (20\,\text{cal})(306\,\text{s})$$

If L is the heat of vapourization of alcohol.

$$Q_v = m_3 L$$

$$L = \frac{(20\times306)\,\text{cal}}{30\,\text{g}} = 204\ \text{cal/g} = 854\ \text{J/g}$$

$$= 854\times10^3\ \text{J/kg}$$

For Problems 22–24

22. b. We consider the leaf to be a black body. The rate of energy radiated at any instant.

$$\left(\frac{dQ}{dt}\right)_e = \sigma\, e\, A\, T_0^4 \qquad \text{(i)}$$

If m is the mass of the body and C is its specific heat then the heat gained

$$\frac{dQ}{dt} = mc\frac{dT}{dt} \qquad \text{(ii)}$$

Since intensity of sun beam is uniform, power incident on the body is SA heat absorbed $\left(\frac{dQ}{dt}\right)_{\text{ab}} = SA\ e$

Rate of increase of temperature

$$= \frac{(\text{net power absorbed})}{\text{thermal capacity of body}}$$

$$= \frac{(SA\alpha - \sigma AeT^4)}{mc}$$

Given $\sigma = 5.67\times10^{-8}\ \text{J/s m}^2\,\text{K}^4$

$A = 0.8\times10^{-2}\,\text{m}^2;\ \alpha = e = 0.8$

$S = 1.4\times10^3\ \text{W/m}^2$

$T = 300\ \text{K}$

$m = 5\times10^{-4}\ \text{kg}$

$C = 0.8\ \text{kcal/kg K} = 0.8\times4.2\ \text{kJ/kg K}$

$$\frac{dT}{dt} = \frac{[1.4\times10^3\times0.8\times10^{-2}\times0.8 - 5.67\times10^{-8}\times0.8\times10^{-2}\times0.8\times(300)^4]}{5\times10^{-4}\times(0.8\times4.2\times10^3)}$$

$$= \frac{(8.96-2.9)}{1.68} = 3.6°\text{C/s} = 3.6\ \text{K/s}$$

23. a. The temperature becomes maximum when net rate of absorption of energy becomes equal to zero, i.e., at any instant $P = (S - \sigma T^4)\, eA = 0$.

$$T_{\max} = \left(\frac{S}{\sigma}\right)^{1/4} = \left(\frac{1.4\times10^3}{5.67\times10^{-8}}\right)^{1/4} = 396\,\text{K}$$

24. c. At maximum temperature $\frac{dT}{dt} = 0$

Initially $\frac{dT}{dt} = 3.6°\text{C/s}$

$$\left(\frac{dT}{dt}\right)_{\text{av}} = \frac{3.6}{2}\,°\text{C/s}$$

Rate of loss of heat

$$= ms\left(\frac{dT}{dt}\right)_{\text{av}} = \sigma\, A\, e\left(T^4 - T_0^4\right)$$

$$T^4 = \frac{\left(5\times10^{-4}\times0.8\times4.2\times10^3\times\frac{3.6}{2}\right)}{5.67\times10^{-8}\times0.8\times10^{-2}\times0.8} + 300^4$$

$$T = 358\ \text{K} = 85°\text{C}$$

For Problems 25–27

25. c. $l_t = l_0(1+\alpha t)$

$$\frac{2\pi\times6}{2} = l_0(1+\alpha t)$$

$$\frac{2\pi\times5.98}{2} = l_0(1+20\alpha)$$

$$\frac{6}{5.98} = \frac{1+\alpha t}{1+20\alpha} = \frac{(1+17\times10^{-6}t)}{1+20\times17\times10^{-6}}$$

$$\therefore\quad t = 216.8°\text{C} \approx 217°\text{C}$$

26. c. Tensile stress $= Y\alpha\Delta\theta$

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$$=11\times10^{10}\times17\times10^{-6}\times(217-20)$$

$$=3.68\times10^{+8}\ \text{N/m}^2$$

27. **c.** $\alpha L\Delta T+L\dfrac{F/A}{Y}=0$

$$\Delta T=-\frac{(F/A)}{\alpha Y}=-\frac{230\times10^6\ \text{N/m}^2}{(17\times10^{-6}\text{K}^{-1})(110\times10^9\text{N/m}^2)}$$

$$=-123\ \text{K}=-123°\text{C}$$

Final temperature at which the copper collar breaks

$$T_f=T_i+\Delta T=217°\text{C}-123°\text{C}=94°\text{C}$$

For Problems 28–30

28. **b.** Resistance of lead bar

$$R_{\text{Pb}}=\frac{l_{\text{Pb}}}{K_{\text{Pb}}A_{\text{Pb}}}=\frac{5\times10^{-2}}{350\times6\times10^{-4}}$$

$$=\frac{10}{21}=\frac{5}{21}\text{K/W}$$

Thermal current through lead bar

$$I_{\text{Pb}}=\frac{\Delta T}{R_{\text{Pb}}}=\frac{100\times21}{5}=420\ \text{W}$$

29. **d.** Resistance of silver bar

$$R_{\text{Ag}}=\frac{l_{\text{Ag}}}{K_{\text{Ag}}A_{\text{Ag}}}=\frac{5\times10^{-2}}{425\times6\times10^{-4}}$$

$$=\frac{100}{85\times6}=\frac{20}{17\times6}=\frac{10}{51}\ \text{K/W}$$

Thermal current through silver bar

$$I_{\text{Ag}}=\frac{\Delta T}{R_{\text{Ag}}}=\frac{100\times51}{10}=510\ \text{W}$$

Total thermal current $I_{\text{Pb}}+I_{\text{Ag}}=930\ \text{W}$

30. **a.** $\dfrac{1}{R_{\text{eq}}}=\dfrac{1}{R_{\text{Pb}}}+\dfrac{1}{R_{\text{Ag}}}=\dfrac{21}{5}+\dfrac{51}{10}$

$$=\frac{42+51}{10}=\frac{93}{10}$$

$$R_{\text{eq}}=\frac{10}{93}=0.107\ \text{W}\simeq0.1\ \text{K/W}$$

Matching Column Type

1. **i. → a., c.; ii. → d.; iii. → b.; iv. → d.**

Initially temperature greatest at A will be more and also area is more at A. So the rate of flow of heat is maximum at A.

At steady state, rate of flow of heat is constant at all sections as all sections are connected in series.

As $\left(\dfrac{dQ}{dt}\right)=KA\left(-\dfrac{dT}{dx}\right)$

$$\left|\frac{dT}{dx}\right|\propto\frac{1}{A}$$

At steady state, the temperature at each at every section is constant. Hence $\dfrac{dT}{dt}=0$ at each section.

2. **i. → a., b., c., d.; ii. → a., b., c., d.; iii. → b., c.; iv. → b., c., d.**

Work done by the system can be non-zero in any of the process.

The relation $dU=nC_v\,dT=n\left(\dfrac{R}{r-1}\right)dT$ is valid for all the process.

In isothermal process $dT=0$.

In adiabatic process $dQ=0$ and non-zero for any other process.

3. **i → b.; ii → c.; iii → a.**

i. Rate at which heat is radiated from the body

$$=Q_r\ \text{J/s}$$

$$=e\sigma AT_1^4=0.55\times5.67\times10^{-8}\times1.5\times(323)^4\ \text{J/s}=509\ \text{W}$$

ii. Rate at which heat radiation is absorbed by the body =

$$=Q_a\ \text{J/s}$$

$$=e\sigma AT_2^4=0.55\times5.67\times10^{-8}\times1.5\times(295)^4\ \text{J/s}$$

$$=354\ \text{W}$$

iii. Rate at which net radiation is emitted by the body

$$=Q_n\ \text{J/s}=Q_r-Q_a=(509-354)\ \text{W}=155\ \text{W}$$

4. **i. → c.; ii. → d.; iii. → b.; iv. → a.**

We have for the four sections, AB, BC, CD and DE with (dQ/dt) as the steady state thermal, energy transmitted per second (A being the area of cross section)

$$\frac{dQ}{dt}=\frac{KA(100-T_C)}{L}=\frac{A(0.8)K(T_C-T_D)}{(1.2)L}$$

$$=\frac{(1.2)KA(T_D-T_E)}{(1.5)L}=\frac{(1.5)KAT_E}{(0.6)L}$$

These give

$$(100-T_C)=\left(\frac{0.8}{1.2}\right)(T_C-T_D)$$

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$$=\left(\frac{1.2}{1.5}\right)(T_D-T_E)=\left(\frac{1.5}{0.6}\right)T_E$$

$$6(100-T_C)=4(T_C-T_D)=(4.8)(T_D-T_E)=15T_E$$

Solving for the differences $(100 - T_C)$, $(T_C - T_D)$, $(T_D - T_E)$ and T_E remaining that the sum of these differences is 100, we obtain

$$(T_A-T_C)=24.1,\ (T_C-T_D)=36.2$$

$$(T_D-T_E)=30.1 \text{ and } (T_E-T_B)=T_E=9.6$$

5. **i. → b.; ii. → a.; iii. → d.**

Fraction of volume submerged $f=\frac{V_i}{V}=\frac{\rho_1}{\rho_2}$

After increasing the temperature

$$f'=\frac{V_i'}{V'}=\frac{\rho_1(1-\gamma_1\Delta T)}{\rho_2(1-\gamma_2\Delta T)}>f \text{ (because } \gamma_2>\gamma_1)$$

If $\gamma_2=\gamma_1$, then $f'=f$

If $\gamma_2 < \gamma_1$, then $f' < f$; it means fraction of volume submerged decreases and solid lifts up.

6. **i. → c.; ii. → a.; iii. → d.; iv. → b.**

Let Q be the heat required to convert 100 g of water at 20°C to 100°C

Then $mc\Delta\theta = (100)\,(1)\,(100 - 20)$
$Q = 8000$ cal

Now suppose m_0 mass of steam converts into water to liberate this much amount of heat. Then

$$m_0=\frac{Q}{L}=\frac{8000 \text{ cal}}{540 \text{ cal/g}}=14.8 \text{ g}$$

Since it is less than m = 20 g, the temperature of the mixture is 100°C.
Mass of steam in the mixture = (20 − 14.8) = 5.2 g
Mass of water in the mixture = (100 + 14.8) = 114.8 g
If m = 10 g, the amount of heat liberated by steam = mL = 10 × 540 = 5400
Let θ be the final temperature of the mixture.

$$m_{H_2O}\, s_{H_2O}(\theta-20)=m_{\text{steam}}L+ms_{H_2O}(100-\theta)$$

$$100\times1(\theta-20)=10\times540+10\times1(100-\theta)$$

$$110\theta = 5400 + 1000 + 2000$$

$$\theta = 76.4°\text{C}$$

7. **i. → c.; ii. → d.; iii. → a.; iv. → b.**

Solar constant = 1.35 kW/m²
Thermal conductivity of earth's crust = 0.75 J/s mK
Heat transferred per second is

$$\frac{dQ}{dt}=-K(4\pi r^2)\frac{dT}{dr}$$

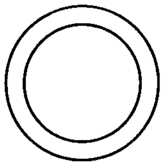

Fig. 1.118

$$r=R_e=6400 \text{ km}$$

$$-\frac{dT}{dr}=-\frac{1°\text{C}}{30\,\text{m}}$$

Heat lost by the earth per second due to conduction from the core

$$\frac{dQ}{dt}=\left(\frac{0.75\text{ J}\times4\pi}{msK}\right)\times(6400\times10^3\text{ m})^2\,\frac{1°\text{C}}{30\,\text{m}}$$

$$P_1=\left[\frac{0.75\times4\pi\times(6400\times10^3)^2}{30}\right]\text{J/s}$$

$$=1.286\times10^{13}\text{ J/s}\approx1.3\times10^{13}\text{ J/s}$$

Heat absorbed from the sun $= S\pi R_e^2$

$$P_2=\left(1.35\times10^3\,\frac{\text{W}}{\text{m}^2}\right)\pi\,(6400\times10^3\text{m})^2$$

$$= 1.7 \times 10^{17}\text{ W}$$

Heat lost by the earth by radiation if e = 1

$$P_2=e\sigma A'T^4$$

$$(A'=4\pi R_e^2)$$

$$1.7\times10^{17}=1\times5.67\times20^{-8}\times4\pi\times(6400\times10^3)^2\ T^4$$

$$T^4=\frac{1.7\times10^{17}}{5.67\times10^{-8}\times4\pi\times(6400\times10^3)^2}$$

$$= 5.8 \times 10^9 = 58 \times 10^8$$

Surface temperature T of earth

$$=(58)^{1/4}\times10^2=(7.6)^{1/2}\times10^2$$

$$=2.76\times100=276\text{ K}$$

$$\frac{P_1}{P_2}=\frac{1.3\times10^{13}}{1.7\times10^{17}}=7.5\times10^{-5}$$

Integer Answer Type

1. **(3)** Energy released by water from 25°C to 0°C
= 2500 × 1 × 25 = 62500 cal
Energy to bring ice to 0°C

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$= 2000 \times \frac{1}{2} \times 15 = 15000$ cal

Energy used to melt ice of m gram = $m80$ cal

$\therefore$ Ice melt $m = \left(\frac{62500-15000}{80}\right) = 593.75$ g

So, mass of water = (2500 + 593.75) g
= 3093.75 g $\simeq$ 3 kg

2. **(8)** Rate of conduction $R \propto \frac{r^2}{\ell}$

The ratio of conduction in them is

$\frac{r^2}{\ell} : \frac{r^2}{2\ell} : \frac{2r^2}{\ell} : \frac{4r^2}{\ell}$, i.e. 2 : 1 : 4 : 8

So, the ratio of maximum to minimum conduction rate is 8 : 1.

3. **(8)** Let power lost to surrounding is Q

$$16 - Q = \left(\frac{dm}{dt}\right) S(10)$$

and $$32 - Q = 3\left[\left(\frac{dm}{dt}\right) S(10)\right]$$

$\Rightarrow$ $\frac{32-Q}{16-Q} = 3 \Rightarrow Q = 8$ W

4. **(6)** Energy with 5 kg of H_2O at 20°C to become ice at 0°C

$E_1 = 5000 \times 1 \times 20 = 100000$ cal

Energy to raise the temperature of 2 kg ice from −20°C to 0°C

$E_1 = 5000 \times 0.5 \times 20 = 20000$ cal

$(E_1 - E_2) = 80000$ cal is available to melt ice at 0°C.

So only 1000 g or 1 kg of ice would have melt.

So, the amount of water available 1 + 5 = 6 kg

5. **(2)**

Loss or gain per day $= dT = \frac{1}{2}\alpha dt \times 86400$

Since $T = 86400$ s for each day

At 15°C, $5 = \frac{1}{2}\alpha(t-15) \times 86400$

At 30°C $10 = \frac{1}{2}\alpha(30-t) \times 86400$

$\therefore$ $\frac{30-t}{t-15} = 2 \Rightarrow 3t = 60°C \Rightarrow t = 20°C$

$\therefore$ $\alpha = \frac{10}{(t-15)\times 86400} = \frac{10}{5 \times 86400} = 0.000023$
$= 2.3 \times 10^{-5}/°C = 2$

6. **(4)** From $\frac{(\theta_1 - \theta_2)}{t} = \alpha\left(\frac{\theta_1+\theta_2}{2} - \theta_0\right)$, in the first case we have,

$$\frac{75-65}{2} = \alpha\left[\frac{75+65}{2} - 30\right] \quad \text{(i)}$$

For the second case,

$$\frac{55-45}{t} = \alpha\left[\frac{55+45}{2} - 30\right] \quad \text{(ii)}$$

Divide Eq. (i) by Eq. (ii), $\frac{t}{2} = 2$

$\Rightarrow$ $t = 4$ min

7. **(4)** $\frac{\text{Temperature difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$

$\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$

$q \propto \frac{1}{R}$

The rods are in parallel in the first case and they are in series in the second case.

$\frac{q_1}{q_2} = \frac{2R}{(R/2)} = 4$

8. **(1)** Because the sliding plug stays in the connecting pipe, the pressure in both the vessels at the level of the pipe must be the same.

$h_{20}d_{20} = h_{80}d_{80} \Rightarrow \frac{h_{20}}{h_{80}} = \frac{d_{80}}{d_{20}}$

$\Rightarrow$ $\frac{d_0(1-80\gamma)}{d_0(1-20\gamma)} = 0.94$

CHAPTER
2

Kinetic Theory of Gases and First Law of Thermodynamics

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KINETIC THEORY OF GASES

In gases, the intermolecular forces are very weak and the molecules may fly apart in all directions. So the gas is characterized by the following properties:

i. It has no shape and size and can be obtained in a vessel of any shape or size.
ii. It expands indefinitely and uniformly to fill the available space.
iii. It exerts pressure on its surroundings.

Assumption of Kinetic Theory of Gases

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature, etc.) to the microscopic properties of the gas molecules (such as speed, momentum, kinetic energy of molecule, etc.).

Actually, the kinetic theory of gases attempts to develop a model of the molecular behaviour which should result in the observed behaviour of an ideal gas. It is based on the following assumptions:

1. Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical, but are different from those of another gas.
2. The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
3. Their molecular size is negligible in comparison to intermolecular distance (10^{-9} m).
4. The volume of molecules is negligible in comparison to the volume of a gas. (The volume of molecules is only about 0.014% of the volume of the gas.)
5. In a gas, molecules are moving in all possible directions with all possible speeds in accordance with 'Maxwell's distribution law' shown graphically in Fig. 2.1(a, b). In graphs, V_{mp} = Most probable speed, V_{av} = Average speed, V_{rms} = Root-mean-square speed.

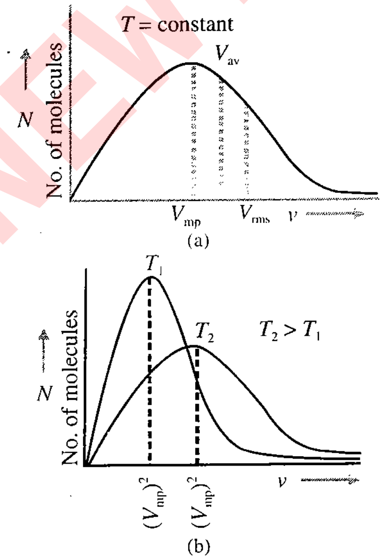

Fig. 2.1

6. The speed of gas molecules lies between zero and infinity (very high speed).
7. The number of molecules moving with most probable speed is maximum.
8. The gas molecules keep on colliding among themselves as well as with the walls of the containing vessel. These collisions are perfectly elastic (i.e., the total energy before collision = total energy after the collision).
9. The molecules move in a straight line with constant speeds between successive collisions.
10. The distance covered by the molecules between two successive collisions is known as free path and mean of all free paths is known as mean free path.
11. The time spent in a collision between two molecules is negligible in comparison to the time between two successive collisions.
12. The number of collisions per unit volume in a gas remains constant.
13. No attractive or repulsive force acts between gas molecules.
14. Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
15. The molecules constantly collide with the walls of the container due to which their momenta change. The change in momentum is transferred to the walls of the container. Consequently, pressure is exerted by gas molecules on the walls of the container.
16. The density of a gas is constant at all points of the container.

Pressure of an Ideal Gas

Consider an ideal gas (consisting of N molecules each of mass m) confined in a box of side a, b and c as shown in Fig. 2.2.

If a molecule hits the wall A moving with velocity v, the velocity resolves into components v_x, v_y and v_z along the x-, y- and z-axis respectively.

The change in momentum along the x-axis due to collision of the molecules will be

$$\Delta p = mv_x - (-mv_x) = 2mv_x$$

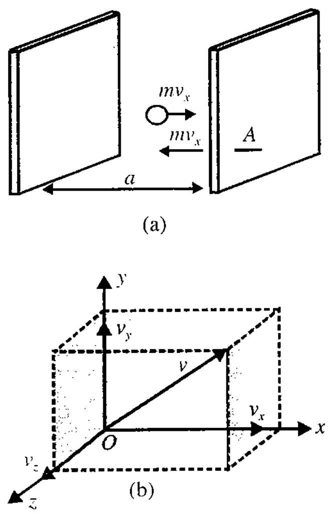

Fig. 2.2

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Note: *As time interval between successive collisions on the same wall A is $(2a/v_x)$, the number of collisions per second due to a single molecule on the same face will be $(v_x/2a)$.*

Hence, the change in momentum per second at face A due to the collision of a single molecule

$$= (v_x/2a)\times(2mv_x) = (mv_x^2/a)$$

So the change in momentum per second due to all the molecules striking the face A will be $\Sigma(mv_x^2/a)$.

But according to Newton's II law, the rate of change of momentum is the force, i.e.,

$$F_x = \sum\frac{mv_x^2}{a} = \frac{m}{a}\sum v_x^2$$

Now as pressure is defined as force per unit area,

$$P_x = \frac{F}{A} = \frac{F_x}{bc} = \frac{m}{abc}\sum v_x^2 = \frac{m}{V}\sum v_x^2 \qquad (\text{as } abc = V)$$

Similarly, $p_y = \frac{m}{V}\sum v_y^2$ and $p_z = \frac{m}{V}\sum v_z^2$

As a result,

$$P_x + P_y + P_z = \frac{m}{V}\sum\left(v_x^2 + v_y^2 + v_z^2\right)$$

But as

$P_x = P_y = P_z = P$ and $v_x^2 + v_y^2 + v_z^2 = v^2$, we get

$3P = m/V\sum v^2$

Now if $(\bar{v})^2$ is the mean square velocity, i.e.,

$$(\bar{v})^2 = (v_1^2 + v_2^2 + \cdots)/N = \sum v^2/N$$

The above equation reduces to

$$3P = (m/V)N(\bar{v})^2$$

$$PV = \frac{1}{3}mN(\bar{v})^2 \qquad \text{(i)}$$

$$P = \frac{1}{3}\frac{mN}{V}(\bar{v})^2$$

Important Points

i. Relation between pressure, volume, mass and temperature

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 \quad \text{or} \quad P \propto \frac{(mN)T}{V} \qquad (\text{as } v_{rms}^2 \propto T)$$

- If volume and temperature of a gas are constant, then $P \propto mN$, i.e., pressure $\propto$ (mass of gas). That is, if mass of a gas is increased, number of molecules and hence number of collision per second will also increase, i.e., pressure will increase.
- If mass and temperature of a gas are constant, then $P \propto (1/V)$, i.e., if volume decreases, the number of collisions per second will increase due to lesser effective distance between the walls resulting in greater pressure.
- If mass and volume of gas are constant, then $p \propto (v_{rms})^2 \propto T$. That is, if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure. Therefore,

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

(As $M = mN$ = total mass of the gas)

$$\therefore \quad p = \frac{1}{3}\rho v_{rms}^2 \qquad (\text{as } \rho\ M/V)$$

ii. Relation between pressure and kinetic energy

Kinetic energy $= 1/2\, M v_{rms}^2$

Therefore, kinetic energy per unit volume

$$(E) = (1/2)(M/V)v_{rms}^2 = (1/2)\rho v_{rms}^2 \qquad \text{(i)}$$

We know

$$P = \frac{1}{3}\rho v_{rms}^2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $P = 2/3E$.

That is, the pressure exerted by an ideal gas is numerically equal to two-thirds of the mean kinetic energy of translation per unit volume of the gas.

Illustration 2.1 **The mass of hydrogen molecule is 3.23×10^{-27} kg. If 10^{23} hydrogen molecules strike 2 cm² of a wall per second at an angle of 45° with the normal when moving with a speed of 10^5 cm s⁻¹, what pressure do they exert on the wall? Assume collision to be elastic.**

Sol. Velocity of each molecule along the normal $= v\cos\theta$ where θ is the angle made by the velocity of the molecule with the normal.

Therefore, change in velocity

$$= v\cos\theta - (-v\cos\theta) = 2v\cos\theta$$

Change in momentum $= m \times 2v\cos\theta = 2mv\cos\theta$

Therefore, rate of change of momentum $= 2mv\cos\theta \times N$ where N is the number of molecules striking the wall each second.

Force exerted by molecules $= 2mvN\cos\theta$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{2mvN\cos\theta}{A} = \frac{2\times3.32\times10^{-27}\times10^3\times10^{23}\times\cos45°}{2\times10^{-4}}$$

$$= 2.35 \times 10^3 \text{ N m}^{-2}$$

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Illustration 2.2 **A parallel beam of nitrogen molecules moving with velocity $v = 400$ m s^{-1} impinges on a wall at an angle $\theta = 30°$ to its normal. The concentration of molecules in the beam is $n = 9 \times 10^{18}$ cm^{-3}. Find the pressure exerted by the beam on the wall, assuming that collisions are perfectly elastic.**

Sol. All the molecules contained in the cylinder of length v and inclined on the wall at an angle θ to the normal will strike the wall every second. Let ΔS be the base area of the cylinder.

The volume of the cylinder = $\Delta Sv \cos\theta$.

Therefore, the number of molecules striking ΔS every second = $(\Delta Sv \cos\theta \times n)$.

Change of momentum per molecule = $2mv\cos\theta$.

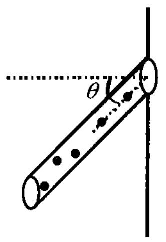

Fig. 2.3

$$\therefore \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{(2mv\cos\theta)(\Delta Sv\cos\theta \times n)}{\Delta S} = 2mv^2n\cos^2\theta$$

$$\therefore p = 2\times\left(\frac{28}{6.0\times10^{26}}\right)(9\times10^{24})\times400^2\times\cos^2 30°$$

$= 1.00 \times 10^5$ N m^{-2} = 1 atm

Illustration 2.3 **(a) Calculate (i) root-mean-square speed and (ii) the mean energy of 1 mol of hydrogen at STP (given that density of hydrogen is 0.09 kg /m³).**

(b) Given that the mass of a molecule of hydrogen is 3.34 × 10^{-27} kg, calculate Avogadro's number.

(c) Calculate Boltzmann's constant.

Sol.

a. (i) We know that the pressure of a gas is given as

$$P = \frac{1}{3}\rho v_{\text{rms}}^2$$

$$v_{\text{rms}} = \sqrt{\left(\frac{3P}{\rho}\right)}$$

$$= \sqrt{\left(\frac{3\times0.76\times13.6\times10^3\times9.8}{0.09}\right)}$$

= 1837 m/s = 1.837 km/s

(ii) Kinetic energy $= \frac{1}{2}mv_{\text{rms}}^2$

Here M = 2 g = 2 × 10^{-3} kg

or KE = $\frac{1}{2}$ × 2 × 10^{-3} × $(1837)^2$

= 3374.56 J

b. Mass of one molecule of H_2 = 3.34 × 10^{-27} kg

Molecular mass of hydrogen = 2 × 10^{-3} kg

Avogadro's number N_A, which is the number of molecules in one gram molecule of hydrogen, is given by

$$N_A = \frac{2\times10^{-3}}{3.34\times10^{-27}} = 5.988 \times 10^{23} \text{ molecules}$$

c. We know that $k = R/N_A = 8.3/(5.988\times10^{23})$

$= 1.37 \times 10^{-23}$ J/mol K

Ideal Gas Equation

A gas that strictly obeys the gas laws is called perfect or an ideal gas. The size of the molecule of an ideal gas is zero, i.e., each molecule is a point mass with no dimension. There is no force of attraction or repulsion amongst the molecules of the gas. All real gases are not perfect gases. However, at extremely low pressure and high temperature, the gases such as hydrogen, nitrogen and helium, are nearly perfect gases.

The equation that relates the pressure (P), volume (V) and temperature (T) of the given state of an ideal gas is known as gas equation.

Ideal gas equations	
For 1 mole or N_A molecule or M grams or 22.4 L of gas	$PV = RT$
For μ moles of gas	$PV = \mu RT$
For 1 molecule of gas	$PV = (R/N_A)T = kT$
For N molecules of gas	$PV = NkT$
For 1 g of gas	$PV = (R/M)T = rT$
For n grams of gas	$PV = nrT$

1. **Universal gas constant (R):** Dimension ($ML^2T^{-2}\theta^{-1}$)

$$R = \frac{PV}{\mu T} = \frac{\text{Pressure}\times\text{Volume}}{\text{No. of moles}\times\text{Temperature}}$$

$$= \frac{\text{Work done}}{\text{No. of moles}\times\text{Temperature}}$$

Thus, universal gas constant signifies the work done by (or on) a gas per mole per kelvin.

STP value: $8.31\frac{\text{joule}}{\text{mole}\times\text{kelvin}} = 1.98\frac{\text{cal}}{\text{mole}\times\text{kelvin}}$

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$$= 0.8221 \frac{\text{litre} \times \text{atm}}{\text{mole} \times \text{kelvin}}$$

2. **Boltzmann's constant (k):** Dimension ($ML^2T^{-2}\theta^{-1}$)

$$k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} \text{ J/K}$$

3. **Specific gas constant (r):** Dimension ($L^2T^{-2}\theta^{-1}$)

$$r = \frac{R}{M}; \text{ unit: } \frac{\text{joule}}{\text{g} \times \text{kelvin}}$$

Since the value of M is different for different gases. Hence, the value of r is different for different gases.

Various Speeds of Gas Molecules

The motion of molecules in a gas is characterized by any of the following three speeds, such as root-mean-square speed, most probable speed and average speed.

1. **Root-mean-square speed:** It is defined as the square root of mean of squares of the speed of different molecules, i.e.,

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \cdots}{N}}$$

(i) From the expression for pressure of ideal gas,

$$P = \frac{1}{3}\frac{mN}{V} v_{\text{rms}}^2$$

$$v_{\text{rms}} = \sqrt{\frac{3PV}{mN}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3P}{\rho}}$$

(as ρ = mass of gas / V)

(ii) $v_{\text{rms}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3\mu RT}{\mu M}} = \sqrt{\frac{3RT}{M}}$

(as M is the molecular weight of gas; $PV = \mu RT$ and mass of gas $= \mu M$)

(iii) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_AkT}{N_Am}} = \sqrt{\frac{3kT}{m}}$

(as $M = N_Am$ and $R = N_Ak$)

Therefore, root-mean-square velocity

$$V_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

Important Points

i. With rise in temperature, rms speed of gas molecules increases as $v_{\text{rms}} \propto \sqrt{T}$.

ii. With the increase in molecular weight, rms speed of gas molecule decreases as $v_{\text{rms}} \propto 1/\sqrt{M}$. For example, rms speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.

iii. rms speed of gas molecules is of the order of km/s. For example, at NTP for hydrogen gas

$$(v_{\text{rms}}) = \sqrt{3RT/M} = \sqrt{(3 \times 8.31 \times 273)/(2 \times 10^3)} = 1840 \text{ m/s}.$$

iv. rms speed of gas molecules is $\sqrt{3/\gamma}$ times that of speed of sound in gas.

As $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_s = \sqrt{\frac{\gamma RT}{M}}$

$$\therefore \quad v_{\text{rms}} = \frac{3}{\gamma} v_s$$

v. rms speed of gas molecules does not depend on the pressure of gas (if temperature remains constant) because $P \propto \rho$ (Boyle's law). If pressure is increased n times, then density will also increase by n times hence v_{rms} remains constant.

vi. Moon has no atmosphere because v_{rms} of gas molecules is more than escape velocity (v_e) on moon. A planet or satellite will have atmosphere only and only if $v_{\text{rms}} < v_e$

vii. At $T = 0$, $v_{\text{rms}} = 0$, i.e., the rms speed of molecules of a gas is zero at 0 K. This temperature is called absolute zero.

2. **Most probable speed:** The particles of a gas have a range of speeds. This is defined as the speed which is possessed by the maximum fraction of total number of molecules of the gas. For example, if speeds of 10 molecules of a gas are 1, 2, 2, 3, 3, 3, 4, 5, 6, 6 km/s, then the most probable speed is 3 km/s, as the maximum fraction of total molecules possess this speed.

Most probable speed $v_{\text{mp}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$

3. **Average speed:** It is the arithmetic mean of the speed of molecules in a gas at given temperature, i.e.,

$$v_{\text{av}} = \frac{v_1 + v_2 + v_3 + v_4 + \cdots}{N}$$

According to kinetic theory of gases

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Average speed $v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$

Note:

- $v_{rms} > v_{av} > v_{mp}$
- $v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$

Illustration 2.4 **Given molecular weight of hydrogen molecule is $M = 2.016 \times 10^{-3}$ kg/mol. Calculate the root-mean-square speed of hydrogen molecules (H_2) at 373.15 K (100°C).**

Sol. The mass of an H_2 molecule may be calculated from the molecular weight as

$$m = \frac{M}{N_0} = \frac{2.016\times10^{-3}\text{ kg/mol}}{6.02\times10^{23}\text{ mol}^{-1}} = 3.35\times10^{-27}\text{ kg}$$

Then $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})(373.15)}{3.35\times10^{-27}}} = 2.15\text{ km/s}$$

Illustration 2.5 **One mole of oxygen occupies a volume of 24 L at 20°C and 1 atm pressure. What is the rms speed of an oxygen molecule? Find the total translational kinetic energy and the total momentum of the molecules.**

Sol.

$$pV = \frac{1}{3}Mv_{rms}^2$$

or $$v_{rms} = \sqrt{\frac{3pV}{M}}$$

$$= \sqrt{\frac{3\times(1.013\times10^5)\times24\times10^{-3}}{32\times10^{-3}}} = 477\text{ m s}^{-1}$$

$$U = \frac{3}{2}RT = \frac{3}{2}\times8.3\times(273+20) = 3647.8\text{ J}$$

Momentum $= M\bar{c} = Mv_{rms} \quad (\bar{c} \approx v_{rms})$

$$= (32\times10^{-3}) \times 477 = 15\text{ kg m s}^{-1}$$

Kinetic Energy of Ideal Gas

Molecules of ideal gases possess only translational motion. So they possess only translational kinetic energy.

Quantity of gas	Kinetic energy
Kinetic energy of a gas molecule ($E_{molecule}$)	$= \frac{1}{2}mv_{rms}^2 = \frac{1}{2}m\left(\frac{3kT}{m}\right) = \frac{3}{2}kT$ $\left(\text{as } v_{rms} = \sqrt{\frac{3kT}{m}}\right)$
Kinetic energy of 1 mole (M gram) gas (E_{mole})	$= \frac{1}{2}Mv_{rms}^2 = \frac{1}{2}M\frac{3RT}{M} = \frac{3}{2}RT$ $\left(\text{as } v_{rms} = \sqrt{\frac{3RT}{M}}\right)$
Kinetic energy of 1g of gas (E_{gram})	$= \frac{3}{2}\frac{R}{M}T = \frac{3}{2}\frac{kN_A}{mN_A}T$ $= \frac{3}{2}\frac{k}{m}T = \frac{3}{2}rT$

Here m, mass of each molecule; M, molecular weight of gas; N_A, Avogadro number = 6.023 × 10^{23}.

Important Points

1. Kinetic energy per molecule of gas does not depend upon the mass of the molecule but only depends upon the temperature of the gas.

   As $E = \frac{3}{2}kT$ or $E \propto T$, i.e., molecules of different gases such as He, H_2 and O_2 at same temperature will have same translational kinetic energy, though their rms speeds are different. $\left[v_{rms} = \sqrt{\frac{3kT}{m}}\right]$
2. Kinetic energy per mole of gas depends only upon the temperature of gas.
3. Kinetic energy per gram of gas depends upon the temperature as well as molecular weight (or mass of one molecule) of the gas.

   $$E_{gram} = \frac{3}{2}\frac{k}{m}T \qquad \therefore\ E_{gram} \propto \frac{T}{m}$$

   From the above expressions, it is clear that higher the temperature of the gas, more will be the average kinetic energy possessed by the gas molecules. At $T = 0$, $E = 0$, i.e., at absolute zero the molecular motion stops.

Illustration 2.6 **One gram-mole of oxygen at 27°C and 1 atm pressure is enclosed in a vessel. Assuming the molecules to be moving with v_{rms}, find the number of collisions per second which the molecules make with 1 m² area of the vessel wall.**

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Sol. From gas law, we have

$$PV = \frac{m}{M}RT$$

or $$PV = \frac{m'N}{m'N_A}RT$$ (m', mass of one mole; N, total number of molecules)

or $$PV = NkT \qquad \left(\text{as } \frac{R}{N_{AV}} = k\right)$$

or $$P = n_0 kT \qquad \text{(i)}$$

Here $n_0 = N/V$ the number of molecules per unit volume of gas. Students should note that Eq. (i) relates the gas pressure, temperature and molecular number density. This expression can be used as a standard equation in several numerical cases as an alternative form of gas law.

Thus, here molecular number density is given as

$$n_o = \frac{P}{kT}$$

$$= \frac{(1\text{ atm})}{(1.38\times10^{-23})\times300}$$

$$= \frac{10^5}{1.38\times10^{-23}\times300} = 2.44\times10^{25}\ \text{m}^{-3}$$

The rms speed of gas molecules is given as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3\times8.314\times300}{32\times10^{-3}}} = 483.4\ \text{m/s}$$

Now the number of collision per square metre of vessel wall is given as

$$N = \frac{n_0 v_{\text{rms}}}{6} = \frac{2.44\times10^{25}\times483.4}{6} = 1.97\times10^{27}\,\text{m}^{-2}$$

Gas Laws

Boyle's Law

For a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure. That is,

$$v \propto 1/P \quad \text{or} \quad PV = \text{constant} \quad \text{or} \quad P_1V_1 = P_2V_2$$

(m and T are constant)

where P is the pressure, V volume and the value of constant depends on the gas temperature and its quantity. We can alternatively write Boyle's law as follows:

i. $PV = P(m/\rho) = \text{constant}$ (as volume = m/ρ)

$\therefore P/\rho = \text{constant}$ or $P_1/\rho_1 = P_2/\rho_2$ (as m = constant)

ii. $PV = P(N/n) = \text{constant}$

(as number of molecules per unit volume $n = N/V \therefore V = N/n$)

$\therefore P/n = \text{constant}$ or $P_1/n_1 = P_2/n_2$

(as N = constant)

iii. According to kinetic theory of gases $P = (1/3)(nM/V)v_{\text{rms}}^2$

$$\therefore\ P \propto \frac{(\text{mass of gas})}{V}\times T$$

(as $v_{\text{rms}} \propto \sqrt{T}$ and mN = mass of the gas)

If mass and temperature of gas remain constant then $P \propto 1/V$. This is in accordance with Boyle's law.

iv. Graphical representation: If m and T are constants, we get the following graphs.

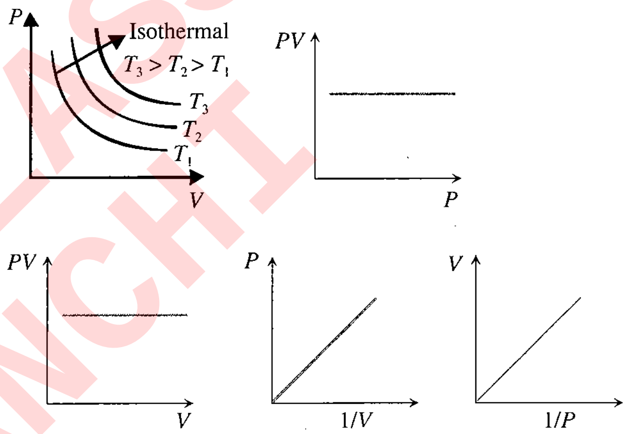

Fig. 2.4

Charles's Law

In Boyle's law, we related the temperature and volume of a gas at constant pressure. Here, we will investigate the effect on the volume of a gas at constant pressure.

If the gas is heated, and if pressure remains constant then gas expands to maintain the pressure. Now after the gas is heated at different temperatures and recording its data for temperature and corresponding volume, we plot a graph. The respective graph is shown in Fig. 2.5. Whatever gas we use, the behaviour is the same. The graph will always be a straight line although slope will be different for different conditions.

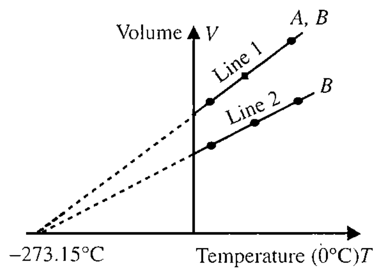

Fig. 2.5

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Here, we can see that these straight lines if extended intersect with temperature axis at –273.15°C. Theoretically, it shows that the volume of the gas becomes zero at this temperature. The reason already we have assumed that the size of molecules is negligible and at –273.15°C or 0 K temperature, the kinetic energy of molecules becomes zero or all motions are frozen at 0 K temperature, thus no movement is there in gas molecules of negligible size at this temperature. If the graph shown in Fig. 2.5 is again plotted with Kelvin scale, we get the graph as shown in Fig. 2.6.

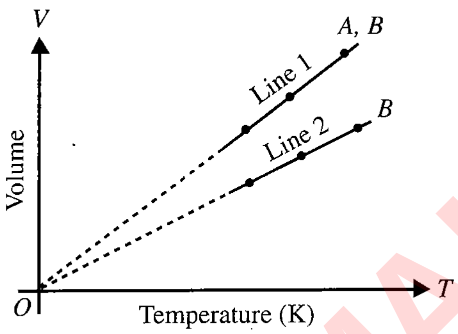


Fig. 2.6

This scale is absolute scale with the sense that its zero (–273.15°C) is the lower limit for temperature and in practical nature to attain a temperature below this is not possible due to the reason discussed above. In further analysis of gases, we will use Kelvin scale.

i. If the pressure remains constant, the volume of the given mass of a gas increases or decreases by 1/273.15 of its volume at 0°C for each 1°C rise or fall in temperature.

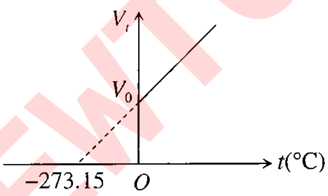


Fig. 2.7

$V_t = V_0\,[1 + (1/273.15)t]$. This is Charles's law for centigrade scale.

$$V_t = V_0\left[\frac{273.15+t}{273.15}\right] = V_0\left[\frac{T}{T_0}\right] \Rightarrow \frac{V_t}{V_0} = \frac{T}{T_0}$$

ii. If the pressure remains constant, the volume of the given mass of a gas is directly proportional to its absolute temperature. That is,

$$V \propto T \quad \text{or} \quad \frac{V}{T} = \text{constant} \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(if m and P are constant)

iii. $\dfrac{V}{T} = \dfrac{m}{\rho T} = \text{constant}$ (as volume $V = m/\rho$)

or $\rho T = \text{constant}$ or $\rho_1 T_1 = \rho_2 T_2$ (as m = constant)

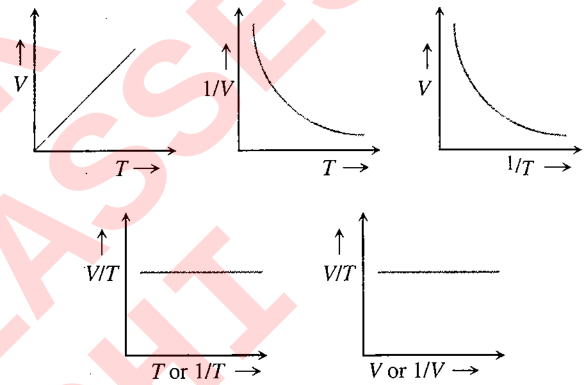
iv. According to kinetic theory of gases

$$P = \frac{1}{3}\frac{mN}{V} v_{rms}^2$$

or $$P \propto \left(\frac{\text{Mass of gas}}{V}\right) T$$

If mass and pressure of the gas remain constant, then $V \propto T$. This is in accordance with Charles's law.

v. Graphical representation: If m and P are constant, we get the following graphs.



(all temperature T are in kelvin)

Fig. 2.8

Gay-Lussac's Law or Pressure Law

i. When the temperature of a gas is changed keeping the volume constant, the pressure of a given mass of gas increases or decreases by 1/273.15 of its pressure at 0°C for each 1°C rise or fall in temperature. We get

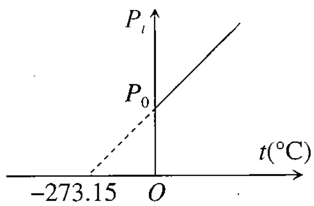


Fig. 2.9

$$P_t = P_0\left[1 + \frac{1}{273.15}t\right]$$

This is pressure law for centigrade scale.

$$P_t = P_0\left[\frac{273.15+t}{273.15}\right] = P_0\left[\frac{T}{T_0}\right] \Rightarrow \frac{P_t}{P_0} = \frac{T}{T_0}$$

ii. If the volume remains constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature. That is,

$$P \propto T \quad \text{or} \quad \frac{P}{T} = \text{constant} \quad \text{or} \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(if m and V are constant)

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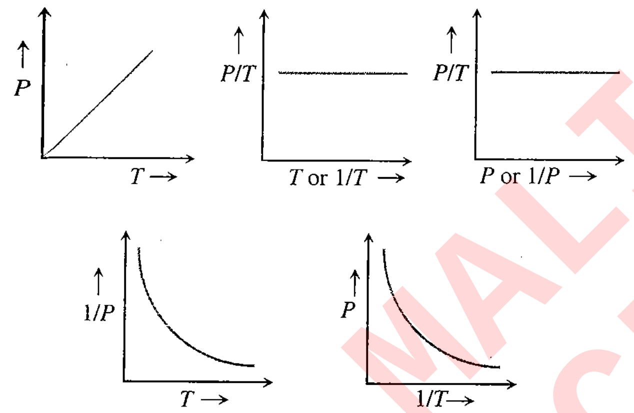
iii. According to kinetic theory of gases

$$P = \frac{1}{3}\frac{mN}{V}v_{\text{rms}}^2 \qquad (\text{as } v_{\text{rms}}^2 \propto T)$$

or
$$P \propto \frac{\text{mass of gas}}{V}T$$

If mass and volume of gas remain constant, then $P \propto T$. This is in accordance with Gay-Lussac's law.

iv. Graphical representation: If m and V are constants, we get the following graphs.



(temperature T is in kelvin)

Fig. 2.10

Avogadro's Law

Avogadro stated that equal volume of all the gases under similar conditions of temperature and pressure contain equal number of molecules. This statement is called Avogadro's hypothesis.

According to kinetic theory of gases

$$PV = \frac{1}{3}mN\,v_{\text{rms}}^2$$

For the first gas, we get

$$PV = \frac{1}{3}m_1N_1v_{\text{rms}(1)}^2 \qquad \text{(i)}$$

For the second gas, we get

$$PV = \frac{1}{3}m_2N_2v_{\text{rms}(2)}^2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$m_1N_1v_{\text{rms}1}^2 = m_2N_2v_{\text{rms}2}^2 \qquad \text{(iii)}$$

As the two gases are at the same temperature, we get

$$\frac{1}{2}m_1v_{\text{rms}1}^2 = \frac{1}{2}m_2v_{\text{rms}2}^2 = \frac{3}{2}kT$$

$$\Rightarrow m_1v_{\text{rms}1}^2 = m_2v_{\text{rms}2}^2 \qquad \text{(iv)}$$

So from Eq. (iii), we can say that $N_1 = N_2$. This is Avogadro's law.

i. Avogadro's number (N_A): The number of molecules present in 1 g mole of a gas is defined as Avogadro's number. That is,

$$N_A = 6.023\times10^{23} \text{ per gram mole}$$
$$= 6.023\times10^{26} \text{ per kg mole.}$$

ii. At STP or NTP (T = 273 K and P = 1 atm) 22.4 L of each gas has 6.023×10^{23} molecules.

iii. One mole of any gas at STP occupies 22.4 L of volume.

Note: ***32 g of oxygen, 28 g of nitrogen and 2 g of hydrogen occupy the same volume at STP.***

iv. For any gas 1 mole = M grams = 22.4 L = 6.023×10^{23} molecules.

Illustration 2.7 **A glass bulb of volume 400 cm³ is connected to another bulb of volume 200 cm³ by means of a tube of negligible volume. The bulbs contain dry air and are both at a common temperature and pressure of 20°C and 1.000 atm, respectively. The larger bulb is immersed in steam at 100°C and the smaller in melting ice at 0°C. Find the final common pressure.**

Sol. Let n_1 and n_2 denote the number of moles of gas in the large and small bulbs, in the final configuration, respectively. Denoting the final temperatures by T_1 and T_2 and the final pressure by P_f, the ideal gas law implies that

$$p_fV_1 = n_1RT_1 \qquad \text{(i)}$$

and
$$p_fV_2 = n_2RT_2 \qquad \text{(ii)}$$

where p_0, $V_0 = V_1 + V_2$ and T_0 are the initial pressure, volume, and temperature, respectively.

Using Eqs. (i) and (ii) in the equation $n_1 + n_2 = n$, we get

$$\frac{p_fV_1}{RT_1} + \frac{p_fV_2}{RT_2} = \frac{p_0V_0}{RT_0}$$

Solving for P_f, we obtain
$$p_f = \frac{p_0V_0}{T_0\left(\dfrac{V_1}{T_1} + \dfrac{V_2}{T_2}\right)}$$

Inserting the numerical values; p_0 = 1.00 atm, T_0 = 293.15 K (t_0 = 20°C),

$$T_1 = 373.15\text{ K}\,(t_1 = 100°\text{C}), \quad \text{and} \quad T_2 = 237.15\text{ K}$$

$$(t_2 = 0°\text{C}),$$

we find
$$p_f = \frac{(1.00)(600)}{(293.15)\left[\left(\dfrac{400}{373.15}\right) + \left(\dfrac{200}{273.15}\right)\right]} = 1.13\,\text{atm}$$

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Illustration 2.8 **Explain whether (a) $T_2 > T_1$, (b) $P_2 > P_1$ and (c) $V_2 > V_1$ or otherwise in Fig. 2.11 (a), (b) and (c), which represent isothermal, isobaric, and isochoric processes for the same mass of an ideal gas, respectively.**

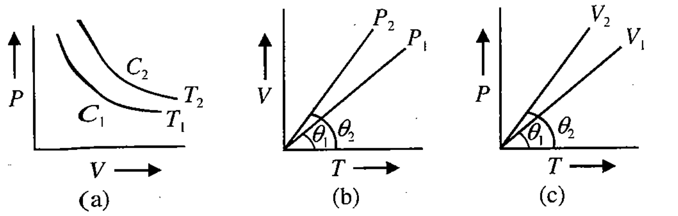

Fig. 2.11

Sol.

a. As the given curve is rectangular hyperbola, its equation will be

$$PV = C \quad \text{as} \quad xy = \text{constant}$$

So, gas equation $PV = \mu RT$ in the light of above yields

$$T = \frac{PV}{\mu R} = \frac{C}{\mu R}$$

Now from Fig. 2.11(a), $C_2 > C_1$ (as for same x, $y_2 > y_1$), $T_2 > T_1$.

b. As the given curve is a straight line passing through the origin with positive slope, its equation will be

$$V = (\tan\theta)T \qquad (as \;\; y = mx)$$

So the gas equation $PV = \mu RT$ in the light of above yields

$$P = \frac{\mu RT}{V} = \frac{\mu RT}{T\tan\theta} = \frac{\mu R}{\tan\theta}$$

Now as from Fig. 2.11(b), we get, $\theta_2 > \theta_1$, i.e., $\tan\theta_2 > \tan\theta_1$, so $P_2 < P_1$.

c. As the given curve is a straight line passing though the origin with positive slope, its equation will be

$$P = T\tan\theta \quad (\text{as } y = mx)$$

So gas equation $PV = \mu RT$ in the light of above yields

$$V = \frac{\mu RT}{P} = \frac{\mu RT}{T\tan\theta} = \frac{\mu R}{\tan\theta}$$

Now as from Fig. 2.11(c), we get $\theta_2 > \theta_1$, $\tan\theta_2 > \tan\theta_1$. So $V_2 < V_1$.

Illustration 2.9 **The three diagrams below depict three different processes for a given mass of an ideal gas. What information can be drawn regarding the change of (a) pressure, (b) volume and (c) temperature of the gas from the plots as shown in Fig. 2.12 (a), (b) and (c), respectively.**

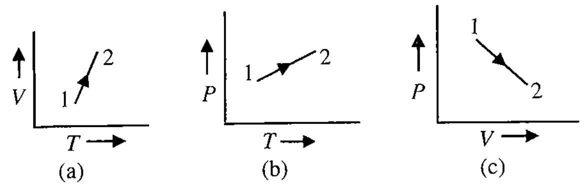

Fig. 2.12

Sol.

a. As the given curve is a straight line with positive slope and negative intercept with the V-axis, its equation will be

$$V = aT - b \quad (\text{as } y = mx - c)$$

So the gas equation $PV = \mu RT$ in the light of above yields

$$P = \frac{\mu RT}{V} = \frac{\mu RT}{aT - b} = \frac{\mu R}{a - (b/T)}$$

Now from Fig. 2.14(a) $T_2 > T_1$, so $P_2 < P_1$.

b. As the given curve is a straight line with positive slope and positive intercept with the P-axis, its equation will be

$$P = aT + b \quad (\text{as } y = mx + c)$$

So the gas equation $PV = \mu RT$ in the light of above yields

$$V = \frac{\mu RT}{P} = \frac{\mu RT}{aT + b} = \frac{\mu R}{a + (b/T)}$$

Now from Fig. 2.12(b), $T_2 > T_1$ so $V_2 > V_1$.

c. As the given curve is a straight line with negative slope and positive intercept with the P-axis, its equation will be

$$P = -aV + b \quad (\text{as } y = -mx + c)$$

So the gas equation $PV = \mu RT$ in the light of above yields $(-aV + b)V = \mu RT$.

This is the equation of a parabola as shown in Fig. 2.13. So during expansion, the temperature of gas first increases, reaches a maximum and then decreases.

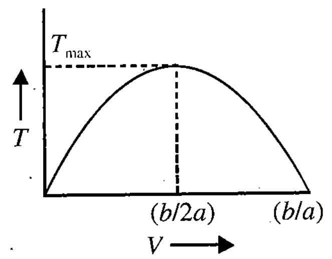

Fig. 2.13

Illustration 2.10 **Draw the P–T and V–T diagrams of an isochoric process of n moles of an ideal gas from pressure P_0, volume V_0 to pressure $4P_0$, indicating the pressures and temperatures of the gas in the initial and the final states.**

Sol. From the equation of state for an ideal gas $PV = nRT$, we get the following.

a. $P = \dfrac{nRT}{V}$, i.e, $P \propto T$ (V = constant)

At $P = P_0$, and $V = V_0$, we get $T = P_0V_0/nR$

And at $P = 4P_0$, $V = V_0$, we get $T = 4P_0V_0/nR$

The graph is a straight line (passing through the origin, when produced) shown in Fig. 2.14.

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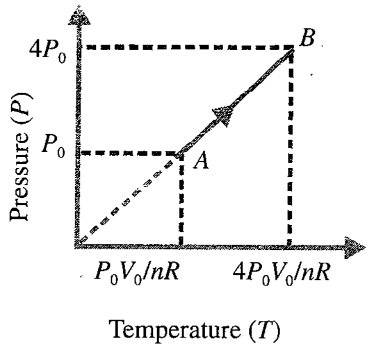

Fig. 2.14

b. Since temperature increases from $\dfrac{P_0V_0}{nR}$ to $\dfrac{4P_0V_0}{nR}$

Volume remains constant; so, the graph of V–T will be as shown in Fig. 2.15.

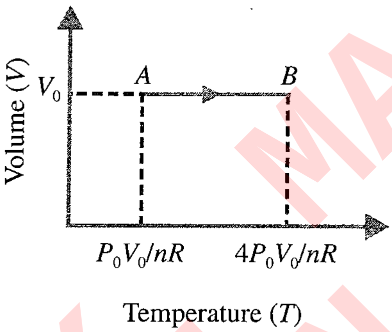

Fig. 2.15

Illustration 2.11 **Draw the P–T and V–T diagrams for an isobaric process of expansion, corresponding to n moles of an ideal gas at a pressure P_0, from V_0 to $2V_0$.**

Sol. From the equation state for an ideal gas $PV = nRT$,

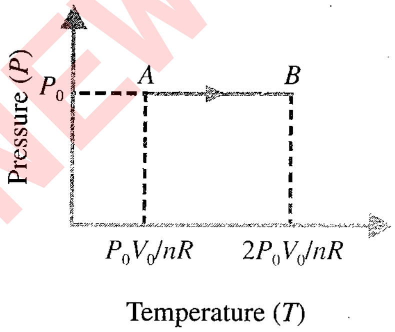

Fig. 2.16

We have $\quad V = (nR/P_0)T \quad$ or $\quad V \propto T$

At $V = V_0$, we get $T_1 = \dfrac{P_0V_0}{nR}$

And at $V = 2V_0$, we get

$$T_2 = \frac{2P_0V_0}{nR}$$

For the graph of P versus T the variation is a straight line normal to the pressure axis, the temperature varies from T_1 to T_2 as shown in the Fig. 2.16.

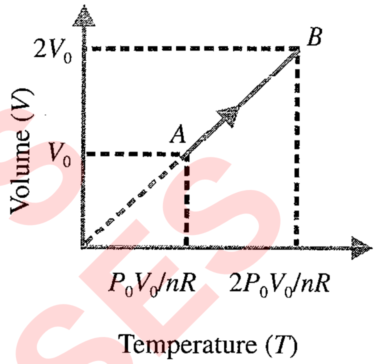

Fig. 2.17

For the graph of V versus T, the equation $V = (nR/P_0)T$ or $V = kT$ shows that the volume varies directly with the temperature (Charlie's law). So, the graph is a straight line inclined to the (V–T) axes, and passes through the origin (when produced) as shown in Fig. 2.17.

Illustration 2.12 **A cyclic process $ABCA$ shown in V–T diagram (Fig. 2.18) is performed with a constant mass of an ideal gas. Show the same process on a P–V diagram.**

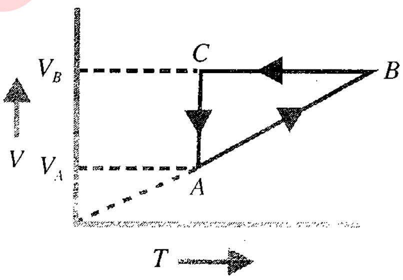

Fig. 2.18

According to given diagram, we notice the following:

Sol.

a. For line AB, $V = aT$ and both V and T increase so the gas equation $PV = \mu RT$ in the light of above yields $P(aT) = \mu RT$, i.e., $P = (\mu R/a) =$ constant, with V increasing. So in P–V diagram, line AB will be a straight line parallel to the V-axis (with V increasing)

b. For line BC, $V =$ constant and T decreasing; so, the gas equation $PV = \mu RT$ in the light above yields.

$$P = \frac{\mu RT}{\text{Constant}}, \quad \text{where } P \propto T \text{ with } T \text{ decreasing}$$

i.e., along line BC, P decreases with $V =$ constant. So in P–V diagram, line BC will be a straight line parallel to the P-axis (with P decreasing).

c. $\quad P = \dfrac{\mu RT}{V}, \quad$ where $PV =$ constant with V decreasing

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So in P–V curve line, CA will be a hyperbola (with P increasing).

The complete cycle on P–V diagram is shown in Fig. 2.19.

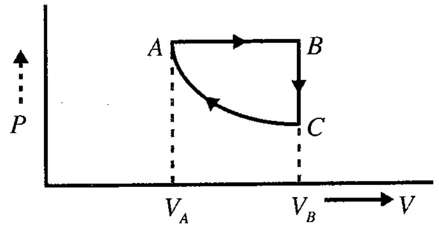

Fig. 2.19

Degree of Freedom

Degree of freedom is the minimum number of variables required to completely specify the state of system.

For thermodynamic system (moving particles), these are the total number of independent terms of energy. The independent possible motions are translational, rotational and vibrational, so there are three types of degrees of freedom.

i. **Translational degrees of freedom:** The maximum number of translational degrees of freedom can be three. These are $1/2mv_x^2, 1/2mv_y^2, 1/2mv_z^2$.

ii. **Rotational degrees of freedom:** The maximum number of rotational degrees of freedom can be three. These are $1/2I_x\omega_x^2, 1/2I_y\omega_y^2, 1/2I_z\omega_z^2$.

iii. **Vibrational degrees of freedom:** Their numbers depend on atoms in the molecule and their arrangement. These degrees of freedom are considered at a very high temperature only.

Note: ***At room temperature only translational and rotational degrees of freedom are taken into account.***

i. **Monatomic gas:** It has been assumed that the molecules of a gas are negligible in size, so moment of inertia and hence rotational kinetic energy of monatomic gas molecules about the axis passes through itself will be zero. The degrees of freedom of monatomic gas molecules are due to three independent translation along the x-, y- and z-axis. The degrees of freedom are $1/2mv_x^2, 1/2mv_y^2, 1/2mv_z^2$.

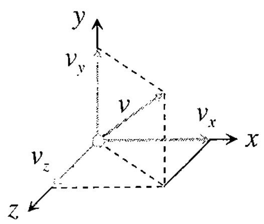

Fig. 2.20

ii. **Diatomic gas:** In diatomic gases, the molecules are assumed to be in the shape of dumbbells; two atoms of negligible size are at some separation. In addition to translation motion, the molecule can rotate about an axis, so the degrees of freedom of diatomic gas molecules are due to translation and due to rotation. If the line joining the two atoms (particles) is taken as the z-axis, then moment of inertia and hence rotational kinetic energy about the z-axis becomes zero. The molecule has three degrees of freedom of translation and two degrees of rotation. These are

$$1/2mv_x^2, 1/2mv_y^2, 1/2mv_z^2, 1/2I_x\omega_x^2, 1/2I_y\omega_y^2$$

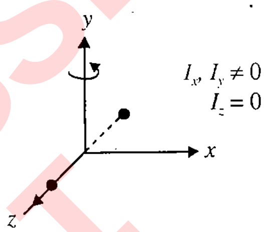

Fig. 2.21

Note: ***If vibrational degrees of freedom are taken into account, then the total number of degrees of freedom of diatomic molecule becomes seven. These are $1/2mv_x^2$, $1/2mv_y^2$, $1/2mv_z^2$, $1/2I_x\omega_x^2$, $1/2I_y\omega_y^2$, $1/2\mu v^2$, $1/2kr^2$. Here, $1/2\mu v^2$ corresponds to kinetic energy of vibration (μ is the reduced mass) and $1/2kr^2$ corresponds to potential energy of vibration (k is the constant, r is the separation between the atoms).***

Triatomic or polyatomic gas: A non-linear molecule has non-zero moment of inertia about any axis, so there are three rotational degrees of freedom. A total number of degrees of freedom are six. The corresponding energies are $1/2mv_x^2$, $1/2mv_y^2$, $1/2mv_z^2$, $1/2I_x\omega_x^2$, $1/2I_y\omega_y^2$, $1/2I_z\omega_z^2$.

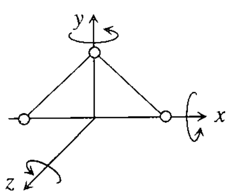

Fig. 2.22

Note:

- ***At high temperature, in case of diatomic or polyatomic molecules, the atoms within the molecule may also vibrate with respect to each other. In such***

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cases, the molecule will have an additional degrees of freedom due to vibrational motion.

- *An object that vibrates in one dimension has two additional degree's of freedom. One for the potential energy and one for the kinetic energy of vibration.*
- *A diatomic molecule that is free to vibrate (in addition to translation and rotation) will have 7 (2 + 3 + 2) degrees of freedom.*
- *Though an atom in a solid has no degree of freedom for translational and rotational motion, due to vibration along 3 axes, it has 3 × 2 = 6 degrees of freedom*
- *When a diatomic or polyatomic gas dissociates into atoms, it behaves as monatomic gas whose degrees of freedom are changed accordingly.*

Law of Equipartition of Energy

For any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom. And the energy associated with each molecule of the system per degree's of freedom of the system is $1/2kT$, where k (Boltzmann's constant) $= 1.38 \times 10^{-23}$ J/K, T = absolute temperature of the system.

If the system possesses number of degrees of freedom f, then we study equipartition of energy as given below.

Total energy associated with each molecule	$(f/2)kT$
Total energy associated with N molecules	$N(f/2)kT$
Total energy associated with each mole	$(f/2)RT$
Total energy associated with μ mole	$(\mu f/2)RT$
Total energy associated with each gram	$(f/2)rT$
Total energy associated with M_0 gram	$M_0(f/2)rT$

Illustration 2.13 **Calculate (a) the average kinetic energy of translation of an oxygen molecule at 27°C, (b) the total kinetic energy of an oxygen molecule at 27°C, (c) the total kinetic energy in joule of one mole of oxygen at 27°C. Given Avogadro's number = 6.02×10^{23} and Boltzmann's constant = 1.38×10^{-23} J/(mol-K).**

Sol.

a. An oxygen molecule has three translational degrees of freedom, thus the average translational kinetic energy of an oxygen molecule at 27°C is given as

$$E_T = \frac{3}{2}kT = \frac{3}{2}\times 1.38\times 10^{-23}\times 300$$

$$= 6.21 \times 10^{-21} \text{ J/molecule}$$

b. An oxygen molecule has total five degrees of freedom, hence its total kinetic energy is given as

$$E_T = \frac{5}{2}kT = \frac{5}{2}\times 1.38\times 10^{-23}\times 300$$

$$= 10.35 \times 10^{-21} \text{ J/molecule}$$

c. Total kinetic energy of one mole of oxygen is its internal energy, which can be given as

$$U = \frac{f}{2}\mu RT = \frac{5}{2}\times 1\times 8.314\times 300 = 6235.5 \text{ J/mole}$$

Specific Heat or Specific Heat Capacity

It characterizes the nature of the substance in response to the heat supplied to the substance. Specific heat can be defined by two following ways: gram specific heat and molar specific heat.

Gram Specific Heat

Gram specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance by unit degree.

Gram specific heat $c = \dfrac{\Delta Q}{m\Delta T}$

Units: $\dfrac{\text{cal}}{\text{g}\times{}^\circ\text{C}}, \dfrac{\text{cal}}{\text{g}\times\text{K}}; \dfrac{\text{J}}{\text{kg}\times\text{K}};$ dimension: $(L^2T^{-2}\theta^{-1})$

Molar Specific Heat

Molar specific heat of a substance may be defined as the amount of heat required to raise the temperature of 1 g mole of the substance by a unit degree, it is represented by C.

$$C = \frac{\Delta Q}{\mu\Delta T}$$

Units: $\dfrac{\text{cal}}{\text{mole}\times{}^\circ\text{C}}, \dfrac{\text{cal}}{\text{mole}\times\text{K}}$ or $\dfrac{\text{J}}{\text{mole}\times\text{K}}$

Important Points

i. $C = Mc = (M/m)(\Delta Q/\Delta T)$

$= (1/\mu)(\Delta Q/\Delta T)$ $\quad$ (as $\mu = m/M$)

It means that molar specific heat of the substance is M times the gram specific heat, where M is the molecular weight of that substance.

ii. Specific heat for hydrogen is maximum, i.e., $C = 3.5$(cal/g°C).

iii. In liquids, water has maximum specific heat $C = 1$(cal/g°C).

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iv. Specific heat of a substance also depends on the state of substance, i.e., solid, liquid or gas. For example,

$$C_{ice} = 0.5\frac{\text{cal}}{\text{g}^\circ\text{C}}, \quad C_{water} = 1\frac{\text{cal}}{\text{g}^\circ\text{C}}, \quad C_{steam} = 0.47\frac{\text{cal}}{\text{g}^\circ\text{C}}$$

v. Specific heat also depends on the conditions of the experiment, i.e., the way in which heat is supplied to the body. In general, experiments are made either at constant volume or at constant pressure.

In case of solids and liquids, due to small thermal expansion, the difference in measured values of specific heats is very small and is usually neglected. However, in case of gases, specific heat at constant volume is quite different from that at constant pressure.

Specific Heat of Gases

In case of gases, heat energy supplied to a gas is spent not only in increasing the temperature of the gas but also in expanding the gas against atmospheric pressure.

Hence, specific heat of a gas, which is the amount of heat energy required to raise the temperature of 1 g of gas through a unit degree, shall not have a single or unique value.

Consider a gas enclosed in a cylinder which is fitted with an airtight and frictionless piston.

i. If the gas is compressed suddenly and no heat is supplied from outside, i.e., $\Delta Q = 0$, the temperature of the gas increases on the account of compression

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = 0, \quad \text{i.e., } C = 0$$

Such a process is called 'adiabatic'.

ii. If the gas is heated and allowed to expand at such a rate that the rise in temperature due to heat supplied is exactly equal to the fall in temperature due to expansion of the gas, i.e., $\Delta T = 0$

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = \frac{\Delta Q}{0} = \infty, \quad \text{i.e. } C = \infty$$

Such a process is called 'isothermal'.

iii. If the rate of expansion of the gas was slow, the fall in temperature of the gas due to expansion would be smaller than the rise in temperature of the gas due to heat supplied. Therefore, there will be some net rise in temperature of the gas, i.e., ΔT will be positive.

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = \text{positive}, \quad \text{i.e., } C = \text{positive}$$

iv. If the gas is heated and allowed to expand very fast, the fall of temperature of the gas due to expansion would be greater than the rise in temperature due to heat supplied. Therefore, there will be some net fall in temperature of the gas, i.e., ΔT will be negative.

$$C = \frac{\Delta Q}{m(-\Delta T)} = \text{negative}, \quad \text{i.e., } C = \text{negative}$$

Hence, the specific heat of gas can have any positive value ranging from zero to infinity. Further, it can even be negative. The exact value depends upon the mode of heating the gas. Out of many values of specific heat of a gas, two are of special significance.

1. Specific heat of a gas at constant volume (c_v): The specific heat of a gas at constant volume is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its volume is kept constant, i.e.,

$$c_v = \frac{(\Delta Q)_v}{m\Delta T}$$

If instead of unit mass, 1 mole of gas is considered, the specific heat is called molar specific heat at constant volume and is represented by C_v.

$$C_v = Mc_v = \frac{M(\Delta Q)_v}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_v}{\Delta T} \qquad (\text{as } \mu = m/M)$$

2. Specific heat of a gas at constant pressure (c_p): The specific heat of a gas at constant pressure is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its pressure is kept constant, i.e.,

$$c_p = \frac{(\Delta Q)_p}{m\Delta T}$$

If instead of unit mass, 1 mole of gas is considered, the specific heat is called molar specific heat at constant pressure and is represented by C_p.

$$C_p = Mc_p = \frac{M(\Delta Q)_p}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_p}{\Delta T} \qquad (\text{as } \mu = m/M)$$

Mayer's Formula

Out of two principle specific heats of a gas, C_p is more than C_v because in case of C_v, volume of gas is kept constant and heat is required only for raising the temperature of 1 g mole of the gas through 1°C or 1 K.

No heat, whatsoever, is spent in the expansion of the gas. It means that the heat supplied to the gas increases its internal energy only, i.e.,

$$(\Delta Q)_v = \Delta U = \mu C_v \Delta T \qquad \text{(i)}$$

while in case of C_p the heat is used in two ways:

i. In increasing the temperature of the gas by ΔT,

ii. In doing work (ΔW), due to expansion at constant pressure. Therefore,

$$(\Delta Q)_p = \Delta U + \Delta W = \mu C_p \Delta T \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\mu C_p \Delta T - \mu C_v \Delta T = \Delta W$$

$$\Rightarrow \mu \Delta T (C_p - C_v) = P\Delta V$$

(for constant P, $\Delta W = P\Delta V$)

$$\Rightarrow C_p - C_v = \frac{P\Delta V}{\mu \Delta T}$$

(from $PV = \mu RT$. At constant pressure $P\Delta V = \mu R \Delta T$)

$$\Rightarrow C_p - C_v = R$$

This relation is called the Mayer's formula and shows that $C_p > C_v$, i.e., molar specific heat at constant pressure is greater than that at constant volume.

Specific Heat in Terms of Degree of Freedom

We know that kinetic energy of one mole of the gas, having f degrees of freedom, can be given by

$$E = \frac{f}{2} RT \quad \text{(i)}$$

where T is the temperature of the gas, but from the definition of C_v, if dE is a small amount of heat energy required to raise the temperature of 1 g mole of the gas at constant volume through a temperature dT, then

$$dE = \mu C_v dT = C_v dT \quad \text{or} \quad C_v = \frac{dE}{dT} \quad (\text{as } \mu = 1) \quad \text{(ii)}$$

Putting the value of E from Eq. (i), we get

$$C_v = \frac{d}{dT}\left(\frac{f}{2}RT\right) = \frac{f}{2}R$$

$$\therefore \quad C_v = \frac{f}{2}R$$

From Mayer's formula, we get

$$C_p - C_v = R$$

$$\Rightarrow C_p = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2}+1\right)R$$

$$\therefore \quad C_p = \left(\frac{f}{2}+1\right)R$$

Ratio of C_p to C_v

$$\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2}+1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$$

$$\therefore \quad \gamma = 1 + \frac{2}{f}$$

Important Points

i. Value of γ is always more than 1. So we can always say that $C_p > C_v$.

ii. Value of γ is different for monatomic, diatomic and triatomic gases.

iii. As $\gamma = 1 + \frac{2}{f} \Rightarrow \frac{2}{f} = \gamma - 1 \Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1}$

$$\therefore \quad C_v = \frac{f}{2}R = \frac{R}{\gamma - 1}$$

and $C_p = \left(\frac{f}{2}+1\right)R = \left(\frac{1}{\gamma-1}+1\right)R = \left(\frac{\gamma}{\gamma-1}\right)R$

Specific Heat and Kinetic Energy for Different Gases

		Monatomic	Diatomic	Triatomic non-linear	Triatomic linear
Atomicity	A	1	2	3	3
Restriction	B	0	1	3	2
Degree of freedom	$f = 3A - B$	3	5	6	7
Molar specific heat at constant volume	$C_v = (f/2)R = R/(\gamma - 1)$	$3/2R$	$5/2R$	$3R$	$7/2R$
Molar specific heat at constant pressure	$C_p = [(f/2)+1]R = (\gamma/\gamma - 1)R$	$5/2R$	$7/2R$	$4R$	$9/2R$
Ratio of C_p to C_v	$\gamma = C_p/C_v = 1 + (2/f)$	$5/3 \simeq 1.66$	$7/5 \simeq 1.4$	$4/3 \simeq 1.33$	$9/7 \simeq 1.28$
Kinetic energy of 1 mole	$E_{mole} = (f/2)RT$	$(3/2)RT$	$(5/2)RT$	$3RT$	$(7/2)RT$
Kinetic energy of 1 molecule	$E_{molecule} = (f/2)kT$	$(3/2)kT$	$(5/2)kT$	$3kT$	$(7/2)kT$
Kinetic energy of 1 g	$E_{gram} = (f/2)rT$	$(3/2)rT$	$(5/2)rT$	$3rT$	$(7/2)rT$

Illustration 2.14 **An ideal gas has a specific heat at constant pressure $C_P = (5/2)\,R$. The gas is kept in a closed vessel of volume 8.3×10^{-3} m³ at a temperature of 300 K and a pressure of 1.6×10^6 N/m². An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas ($R = 8.3$ J/mol K).**

Sol. As here volume of gas remains constant, we have

$$(\Delta Q)_V = \mu C_V\, \Delta T$$

Here, $$C_V = C_P - R = (5/2)R - R = (3/2)R$$

$$\mu = \frac{PV}{RT} = \frac{1.6\times10^6 \times 8.3\times10^{-3}}{8.3\times300} = \frac{16}{3}$$

$$2.49\times10^4 = \frac{16}{3}\times\frac{3}{2}\times 8.3\Delta T$$

i.e., $\Delta T = 375$ or $T_2 - T_1 = 375$, i.e., $T_2 = (375 + 300) = 675$ K

Now as for a given mass of an ideal gas at constant volume, $P \propto T$

$$(P_2/P_1) = (T_2/T_1)$$

$$P_2 = \frac{675}{300}\times1.6\times10^6 = 3.6\times10^6 \text{ N/m}^2$$

Illustration 2.15 **A vessel of volume 0.2 m³ contains hydrogen gas at temperature 300 K and pressure 1 bar. Find the heat required to raise the temperature to 400 K. The molar heat capacity of hydrogen at constant volume is 5 cal/mol K.**

Sol. As here volume of gas remains constant, we get

$$(\Delta Q)_V = \mu C_V\, \Delta T$$

Here, $$C_v = 5 \text{ cal/(mol K)}$$

And $$\Delta T = (400 - 300) = 100\,\text{K}$$

And so for ideal gas $PV = \mu RT$,

$$\mu = \frac{(10)^5\times(0.2)}{8.31\times300} = 8 \text{ mol}$$

$$(\Delta Q)_V = 8\times5\times100 = 4 \text{ kcal}$$

Illustration 2.16 **About 0.014 kg of nitrogen is enclosed in a vessel at temperature of 27°C. How much heat has to be transferred to the gas to double the rms speed of its molecules? ($R = 2$ cal/mol K)**

Sol. As gas is enclosed in the cylinder, V = constant

$$(\Delta Q)_V = \mu C_V\, \Delta T$$

Here, $$\mu = (0.014\times10^3)/28 = (1/2) \text{ mol}$$

And as nitrogen is diatomic, $C_V = (5/2)\,R$,

Further, as according to the given problem,

$$\frac{(v_{\rm rms})_2}{(v_{\rm rms})_1} = \sqrt{\frac{T_2}{T_1}} = 2, \quad \text{i.e., } T_2 = 4T_1$$

$$\Delta T = 4T_1 - T_1 = 3T_1 = 3\times300 = 900 \text{ K}$$

$$(\Delta Q)_V = \frac{1}{2}\times\frac{5}{2}\times2\times900 = 2250 \text{ cal}$$

Gaseous Mixture

If two non-reactive gases are enclosed in a vessel of volume V. In the mixture μ_1 moles of one gas are mixed with μ_2 moles of another gas. If N_A is Avogadro's number then number of molecules of first gas $N_1 = \mu_1 N_A$ and number of molecules of second gas $N_2 = \mu_2 N_A$.

i. Total number of moles $\mu = (\mu_1 + \mu_2)$.

ii. If M_1 is the molecular weight of first gas and M_2 that of second gas. Then molecular weight of mixture will be

$$M = \frac{\mu_1 M_1 + \mu_2 M_2}{\mu_1 + \mu_2}$$

iii. Specific heat of the mixture at constant volume will be

$$C_{V_{\rm mix}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} = \frac{\mu_1\left(\dfrac{R}{\gamma_1 - 1}\right) + \mu_2\left(\dfrac{R}{\gamma_2 - 1}\right)}{\mu_1 + \mu_2}$$

$$= \frac{R}{\mu_1 + \mu_2}\left(\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}\right)$$

$$\therefore\ C_{V_{\rm mix}} = \frac{R}{\dfrac{m_1}{M_1} + \dfrac{m_2}{M_2}}\left(\frac{m_1/M_1}{\gamma_1 - 1} + \frac{m_2/M_2}{\gamma_2 - 1}\right)$$

iv. Specific heat of the mixture at constant pressure will be

$$C_{P_{\rm mix}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 + \mu_2}$$

$$\Rightarrow C_{P_{\rm mix}} = \frac{\mu_1\left(\dfrac{\gamma_1}{\gamma_1 - 1}\right)R + \mu_2\left(\dfrac{\gamma_2}{\gamma_2 - 1}\right)R}{\mu_1 + \mu_2}$$

$$= \frac{R}{\mu_1 + \mu_2}\left[\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

$$\therefore\ C_{P_{\rm mix}} = \frac{R}{\dfrac{m_1}{M_1} + \dfrac{m_2}{M_2}}\left[\frac{m_1}{M_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \frac{m_2}{M_2}\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

v. $$\gamma_{\rm mix} = \frac{C_{P_{\rm mix}}}{C_{V_{\rm mix}}} = \frac{\dfrac{(\mu_1 C_{P_1} + \mu_2 C_{P_2})}{\mu_1 + \mu_2}}{\dfrac{(\mu_1 C_{V_1} + \mu_2 C_{V_2})}{\mu_1 + \mu_2}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 C_{V_1} + \mu_2 C_{V_2}}$$

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$$= \frac{\left[\mu_1\left(\frac{\gamma_1}{\gamma_1-1}\right)R+\mu_2\left(\frac{\gamma_2}{\gamma_2-1}\right)R\right]}{\left[\mu_1\left(\frac{R}{\gamma_1-1}\right)+\mu_2\left(\frac{R}{\gamma_2-1}\right)\right]}$$

$$\therefore \qquad \gamma_{\text{mix}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1-1}+\frac{\mu_2\gamma_2}{\gamma_2-1}}{\frac{\mu_1}{\gamma_1-1}+\frac{\mu_2}{\gamma_2-1}} = \frac{\mu_1\gamma_1(\gamma_2-1)+\mu_2\gamma_2(\gamma_1-1)}{\mu_1(\gamma_2-1)+\mu_2(\gamma_1-1)}$$

Illustration 2.17 **A gaseous mixture enclosed in a vessel consists of 1 g mole of gas A with ($\gamma_1 = 5/3$) and another gas B with ($\gamma_2 = 7/5$) at a temperature T. The gases A and B do not react with each other and assume to be ideal. Find the number of gram moles of the gas B, if γ for the gaseous mixture is (19/13).**

Sol. As for ideal gas $C_P - C_V = R$ and $\gamma = (C_p/C_v)$,

$$\gamma - 1 = \frac{R}{C_V} \quad \text{or} \quad C_V = \frac{R}{(\gamma-1)}$$

$$(C_V)_1 = \frac{R}{(5/3)-1} = \frac{3}{2}R$$

$$(C_V)_2 = \frac{R}{(7/5)-1} = \frac{5R}{2}$$

and
$$(C_V)_{\text{mix}} = \frac{R}{(19/13)-1} = \frac{13}{6}R$$

Now from the conservation of energy, i.e., $\Delta U = \Delta U_1 + \Delta U_2$, we get

$$(\mu_1+\mu_2)(C_V)_{\text{mix}}\Delta T = [\mu_1(C_V)_1 + \mu_2(C_V)_2]\Delta T$$

$$(C_V)_{\text{mix}} = \frac{\mu_1(C_V)_1+\mu_2(C_V)_2}{\mu_1+\mu_2}$$

$$\frac{13}{6}R = \frac{1\times\frac{3}{2}R+\mu_2\times\frac{5}{2}R}{1+\mu_2} = \frac{(3+5\mu_2)R}{2(1+\mu_2)}$$

or $\qquad 13 + 13\mu_2 = 9 + 15\mu_2$, i.e., $\mu_2 = 2$ g mole

Illustration 2.18 **How much heat energy should be added to the gaseous mixture consisting of 1 g of hydrogen and 1 g of helium to raise its temperature from 0°C to 100°C**
a. at constant volume,
b. at constant pressure ($R = 2$ cal/mol K)?

Sol. As hydrogen is diatomic and has molecular weight 2,

$$(C_V)_H = \frac{5}{2}R$$

and
$$\mu_H = \frac{1}{2}$$

While He is monatomic and has molecular weight 4,

$$(C_V)_{He} = \frac{3}{2}R \quad \text{and} \quad \mu_{He} = \frac{1}{4}$$

So, by conservation of energy, we get

$$(C_V)_{\text{mix}} = \frac{\mu(C_V)_1+\mu_2(C_V)_2}{\mu_1+\mu_2} = \frac{\frac{1}{2}\times\frac{5}{2}R+\frac{1}{4}\times\frac{3}{2}R}{\frac{1}{2}+\frac{1}{4}}$$

$$(C_V)_{\text{mix}} = \frac{13}{8}\times\frac{4}{3}R = \frac{13}{6}R$$

a. So

$$(\Delta Q)_V = \mu C_V \Delta T = \left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{13}{6}\times 2\times(100-0) = 325 \text{ cal}$$

b. Now as $(C_P)_{\text{mix}} = (C_V)_{\text{mix}} + R = \frac{13}{6}R + R = \frac{19}{6}R$

$$(\Delta Q)_P = \mu C_P \Delta T = \left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{19}{6}\times 2\times(100-0) = 475 \text{ cal}$$

Illustration 2.19 **Calculate the specific heat capacity C_v of a gaseous mixture consisting of ν_1 moles of a gas of adiabatic exponent γ_1 and ν_2 moles of another gas of adiabatic exponent γ_2.**

Sol. The internal energy of an ideal gas of mass m is given by

$$U = \frac{pV}{\gamma-1}$$

Internal energy is an extensive property.

$$\therefore \qquad U_{\text{mix}} = U_1 + U_2 \quad \Rightarrow \quad \frac{p_{\text{mix}}}{\gamma-1} = \frac{P_1}{\gamma-1} + \frac{P_2}{\gamma_2-1}$$

(as volume is the same for all)

From the formula $pV = nRT$

$$p_{\text{mix}}V = (\nu_1+\nu_2)RT; \quad p_1V = \nu_1RT; \quad p_2V = \nu_2RT$$

$$\therefore \qquad p_1 = \frac{\nu_1}{\nu_1+\nu_2}p_{\text{mix}} \quad \text{and} \quad p_2 = \frac{\nu_2}{\nu_1+\nu_2}p_{\text{mix}}$$

$$\therefore \qquad \frac{1}{\gamma-1} = \frac{1}{\nu_1+\nu_2}\left(\frac{\nu_1}{\gamma_1-1}+\frac{\nu_2}{\gamma_2-1}\right)$$

$$C_v = \frac{R}{\gamma-1} = \frac{R}{\nu_1+\nu_2}\left(\frac{\nu_1}{\gamma_1-1}+\frac{\nu_2}{\gamma_2-1}\right)$$

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Concept Application Exercise 2.1

1. The average kinetic energy per molecule of all monatomic gases is the same at the same temperature. Is this true or false?
2. What happens to the random motion when an ideal gas undergoes free expansion (i.e., expansion into a vacuum)?
3. The highest vacuum attained so far is of the order of 10^{-11} mm of mercury. How many molecules are there in 1 cc of a vessel under such a high vacuum at 0°C?
4. A thermally insulated vessel with gaseous nitrogen at a temperature of 27°C moves with velocity 100 m/s^{-1}. How much (in percentage) and in what way will the gas pressure change if the vessel is brought to rest suddenly?
5. Find the molar mass and the number of degrees of freedom of molecules in a gas if its heat capacities are $C_v = 650$ J kg^{-1} K^{-1} and $C_p = 910$ J kg^{-1} K^{-1}.
6. At 127°C and 1.00×10^{-2} atm pressure, the density of a gas is 1.24×10^{-2} kg m^{-3}.
   a. Find v_{rms} for the gas molecules.
   b. Find the molecular weight of the gas and identify it.
7. The mass of a gas molecule can be computed from the specific heat at constant volume. Take $C_v = 0.075$ kcal kg^{-1} k^{-1} for argon and calculate
   a. the mass of an argon atom
   b. the atomic weight of argon.
8. In a certain gas, 2/5th of the energy of the molecules is associated with the rotation of the molecules, and the rest is associated with the motion of their centres of mass. What is the average translational energy of one such molecule when the temperature is 27°C? How much energy is required to raise the temperature by 1°C?
9. If the water molecules in 1 g of water were distributed uniformly over the surface of the earth, how many molecules would there be in 1 m^2 of the earth's surface (radius of the earth = 6400 km)?
10. How many degrees of freedom do the molecules of a gas have if under standard conditions the gas density $\rho = 1.3$ kg m^{-3} and the velocity of sound propagation in it is $v = 330$ m/s^{-1}?
11. The temperature of a gas consisting of rigid diatomic molecules is $T = 300$ K. Calculate the angular root-mean-square velocity of a rotating molecule if its moment of inertia is $I = 2.0 \times 10^{-40}$ kg m^2.
12. The molar specific heat capacity of all monatomic gases is the same. Is this true or false? (Assume ideal nature.)
13. One mole of a monatomic ideal gas is mixed with one mole of a diatomic ideal gas. What is the molar specific heat of the mixture at constant volume?
14. A gas has been subjected to isochoric–isobaric processes 1–2–3–4–1 (see Fig. 2.23). Plot this process on T–V and P–T diagrams.

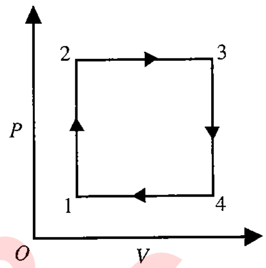

Fig. 2.23

15. A gas has been subjected to an isothermal–isochoric cycle 1–2–3–4–1 (see Fig. 2.24). Represent the same cycle on P–V and P–T diagrams.

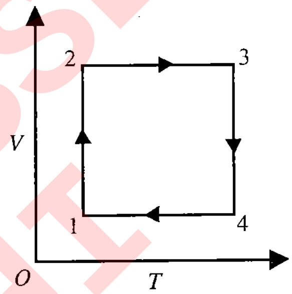

Fig. 2.24

16. Calculate γ of a gaseous mixture consisting of 3 moles of nitrogen and 2 moles of carbon dioxide.
17. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at 27°C temperature and 1.0×10^5 N/m^2 pressure. The gram-molecular weights of neon and argon are 20 and 40, respectively. Find the masses of the individual gases in the container. Assuming them to be ideal ($R = 8.314$ J/mol-K). Total mass of the mixture is 28 g.
18. Equal masses of a gas are sealed in two vessels, one of volume V_0 and other of volume $2V_0$. If the first vessel is kept at temperature 300 K and the other is at 600 K. Find the ratio of pressures in the two vessels.
19. A glass container encloses a gas at a pressure of 8×10^5 Pa and 300 K temperature. The container walls can bear a maximum pressure of 10^6 Pa. If the temperature of container is gradually increased, find the temperature at which the container will break.
20. Calculate the number of molecules in 1 cc of an ideal gas at 27°C and a pressure of 10 mm of mercury. Mean kinetic energy of molecule at 27°C is 4×10^{-14} erg; the density of mercury is 13.6 g/cc.

THERMODYNAMICS

Thermodynamics is a branch of science which deals with exchange of heat energy between bodies and conversion of the heat energy into mechanical energy and vice versa.

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Thermodynamic System

Thermodynamics basically deals with the exchange of heat between a body and the surrounding along with the other processes accompanying it, such as the work involved and the changes in the internal energy, taking place in the body. Such studies are primarily carried out in terms of a few macroscopic properties, such as pressure, volume, temperature, etc. Any alternation in 'one or more' of the macroscopic properties of a body renders specific changes in other related properties, and the body is said to have been subjected to a thermodynamic process (or is said to have undergone a thermodynamic process). The 'body' which is subjected to a thermodynamic process is known as a thermodynamic system. The macroscopic parameters in terms of which the study of a thermodynamic process is carried out are known as thermodynamic variables. For example, gas enclosed in a cylinder fitted with a piston forms the thermodynamic system but the atmospheric air around the cylinder, movable piston, burner, etc., is the surroundings.

Thermodynamic system is classified into the following three systems:

i. **Open system:** It exchanges both energy and matter with the surrounding.
ii. **Closed system:** It exchanges only energy (not matter) with the surroundings.
iii. **Isolated system:** It exchanges neither energy nor matter with the surrounding.

Thermodynamic Variables and Equation of State

A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For μ moles of an ideal gas, equation of state is $PV = \mu RT$ and for 1 mole of an it ideal gas is $PV = RT$.

Thermodynamic Equilibrium

When the thermodynamic variables attain a steady value, i.e., they are independent of time, the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.

i. **Mechanical equilibrium:** There is no unbalanced force between the system and its surroundings.
ii. **Thermal equilibrium:** There is a uniform temperature in all parts of the system and is same as that of surrounding.
iii **Chemical equilibrium:** There is a uniform chemical composition throughout the system and the surrounding.

Thermodynamic Process

The transaction of heat between the surroundings and a thermodynamic system can be achieved in several ways, effecting the work done by or on it, and/or allowing an alternation in its internal energy. Each of the ways executed is said to be a process (thermodynamic). Obviously, there is no limitation to the variety of thermodynamic processes.

The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system. This is known as thermodynamic process. Some important processes are (i) isothermal process, (ii) adiabatic process, (iii) isobaric process, (iv) isochoric (isovolumic process), (v) cyclic and non-cyclic process and (vi) reversible and irreversible process.

Zeroth Law of Thermodynamics

If systems A and B are separately in thermal equilibrium with a third system C, then A and B are also in thermal equilibrium with each other.

i. The zeroth law leads to the concept of temperature. All bodies in thermal equilibrium must have a common property which has the same value for all of them. This property is called the temperature.
ii. The zeroth law came to light long after the first and second laws of thermodynamics had been discovered and numbered. Because the concept of temperature is fundamental to those two laws, the law that establishes temperature as a valid concept should have the lowest number. Hence, it is called zeroth law.

First Law of Thermodynamics

Consider a thermodynamic system at a certain thermodynamic state with a certain external pressure bearing on its surface (due to surrounding). Let an infinitely small quantity of heat dQ be imparted to it externally. In the general case, the temperature of the system increases and it also expands.

An increase in temperature signifies an increase in the randomness and intensity of molecular motion, i.e., an increase in the kinetic energy of the molecules, which is the modern concept of increase in temperature, according to the kinetic theory's interpretation. Let the increase in internal kinetic energy be dK.

The increased volume of the system also denotes some work done against the intermolecular forces of attraction, which is the work of dispersion, and is stored in the system itself as an increased internal potential energy. Let it be denoted by dU'.

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The increase in volume also involves an appreciable amount of work done against the external forces, the work of expansion often spoken of as external work. Let it be denoted by dW.

Ignoring, any other type of changes (electrical, chemical, hysterical, etc.) by the law of conservation of energy, and the principle of the equivalence of heat and work, the heat energy balance equation can be written as

$$dQ = dK + dU' + DW$$

Now, $dK + dU'$ is evidently the sum total of the increase in the internal kinetic and potential energy of the system, which can be more briefly stated as the increase in the internal energy of the system and can be denoted by dU.

The above equation can be rewritten

$$dQ = dU + dW \qquad \text{(i)}$$

Equation (i) establishes the first law of thermodynamics, which states that the heat imparted to a body is, in general, expanded in two ways. A part of it is used to increase its internal energy and the remaining is utilized to do work against external pressure.

While substituting the values of dU, dQ and dW in Eq. (i) the following sign convention should be adopted:

1. If heat is imparted or supplied to a system, then dQ is positive; if heat is extracted from a system, then dQ is negative.
2. If there is an increase in volume, i.e., dV is positive, then the system does work against the external pressure bearing upon the system's surface and accordingly dW is positive and work is said to be done by the system; on the other hand, if there is a decrease in volume, i.e., dV is negative, then dW is negative and work is said to have been done on the system.
3. $dU = U_f - U_i$, where U_i and U_f are the initial and final internal energies of the system, respectively. If the internal energy increases (i.e., $U_f > U_i$), then dU is positive, while if the internal energy decreases during the process (i.e., $U_f < U_i$), then dU is negative.
4. For an ideal gas, internal energy depends on temperature only.

Illustration 2.20 **Figure 2.25 shows a cylinder closed by a movable piston containing an ideal gas. The piston is quickly pushed down from position 2. The piston is held at position 2 until the gas is again at 0°C and then is slowly raised back to position 1. Represent the whole operation on a *P–V* diagram. If 100 g of ice melts during the cycle, how much work is done on the gas?**

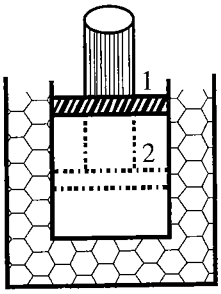

Fig. 2.25

Sol. A quick process is generally adiabatic and a slow process isothermal. Since 100 g of ice melts, heat given out by the system (gas in the cylinder) is equal to the required latent heat.

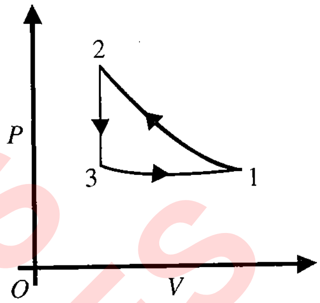

Fig. 2.26

$\therefore \qquad \Delta Q = -100 \times 10^{-3} \times 336 \times 10^3$

$\qquad\qquad = -3360$ J

$\qquad \Delta U = 0 \quad$ (since it is a cyclic process)

But $\qquad \Delta Q = \Delta U + \Delta W \quad$ (always)

$\therefore \qquad -3360 = \Delta W$

$\therefore$ Work done on the system $= -\Delta W = 3360$ J

Quantities Involved in First Law of Thermodynamics

Heat (ΔQ)

It is the energy that is transferred between a system and its environment because of the temperature difference between them. Heat always flows from a body at higher temperature to a body at lower temperature till their temperatures become equal.

Important Points

i. Heat is a form of energy, so it is a scalar quantity with dimension (ML^2T^{-2}).

ii. Unit: joule (SI), calorie (practical unit), and 1 calorie = 4.2 J.

iii. Heat is a path dependent quantity, i.e., heat required to change the temperature of a given gas at a constant pressure is different from that required to change the temperature of same gas through same amount at constant volume.

iv. For solids and liquids: $\Delta Q = mL$ (for change in state) and $\Delta Q = mc\Delta T$ (for change in temperature)

For gases when heat is absorbed and temperature changes:

$$(\Delta Q)_V = \mu C_V \Delta T \qquad \text{(for constant volume)}$$

$$(\Delta Q)_P = \mu C_P \Delta T \qquad \text{(for constant pressure)}$$

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Work Done by a Gas

Consider an ideal gas container in a cylindrical vessel fitted with a frictionless piston as shown in Fig. 2.27, the upward thrust of the gas on the piston is balanced by the downward force due to the weights kept on the piston. Let the piston be raised up by a small distance dx (by heating or by other means). If P be the gas pressure (equal to the external pressure due to the load on the piston, the latter being assumed to be in equilibrium) and A be the cross-sectional area of the piston, then the force against which the gas performs work will be PA.

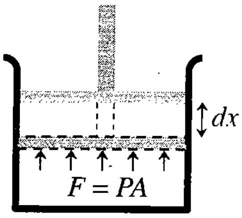


Fig. 2.27

Therefore, work done along the small distance dx is

$$dW = PAdx$$

However, Adx represents the small change in volume undergone by the gas. If this change in volume be denoted by dV, then $dW = PdV$.

The total work done by the gas, while its volume change from V_1 to V_2, can be obtained by adding up (integrating) the small works dW performed by the gas in stages during each element change in volume dV. Thus, the total work done will be

$$W = \int dW = \int_{V_1}^{V_2} PdV$$

If the pressure P remains constant during the process of change in volume, then

$$W = P\int_{V_1}^{V_2} dV$$

(Pressure being constant can be taken out of the integrating sign)

$$W = P[V]_{V_1}^{V_2} = P(V_2 - V_1) = P\Delta V$$

where ΔV represents the total change in volume. If, on the other hand, the pressure changes, then the relationship between the pressure and the volume should be known (i.e., the pressure should be explicit as a function of volume to evaluate the integral)

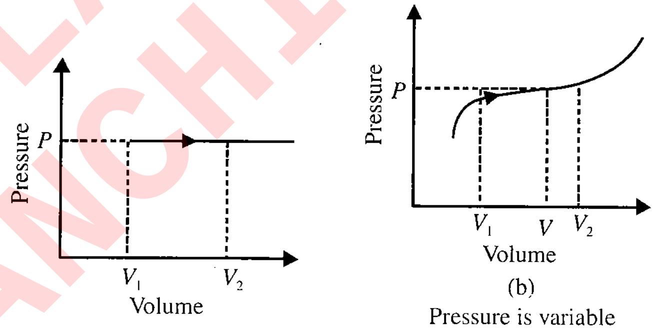
$$W = \int_{V_1}^{V_2} P\, dV$$

Note:

i. If an expansion takes place in the gas (i.e., $V_1 < V_2$), then ΔV is positive and the work done by the gas is also positive, pressure being always positive. However, if the gas contracts (i.e., $V_2 > V_2$), then ΔV is negative and hence work done is also negative.

ii. During the expansion of a gas, the movement dx of the piston (upward) being opposite to the direction of the application of the external force (downward), the work done by the external force will be equal in value to that performed by the gas but opposite in sign, i.e., negative. Similarly, during the compression of a gas, the external work done is positive.

If the pressure P (taken along the y-axis) be plotted against the volume V (taken along the x-axis), then the work done by the gas is numerically equal to the area under the pressure–volume curve, between the ordinates at V_1 and V_2.



(a)

Pressure is constant

$W = P(V_2 - V_1)$

(b)

Pressure is variable

$W = \int_{V_1}^{V_2} P\, dV$

Fig. 2.28

Important Points

- From $\Delta W = P\Delta V = P(V_f - V_i)$
  ΔW = positive if V_f – V_i, i.e., system expands against some external force.
  ΔW = negative if V_f – V_i, i.e., system contracts because of some external force exerted by the surrounding.
- In P–V diagram or indicator diagram, the area under P–V curve represents work done.

  W = area under P–V diagram

  It is positive if volume increases (for expansion).
  It is negative if volume decreases (for compression).

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- In a cyclic process, work done is equal to the area under the cycle.
  It is positive if the cycle is clockwise.
  It is negative if the cycle is anticlockwise.

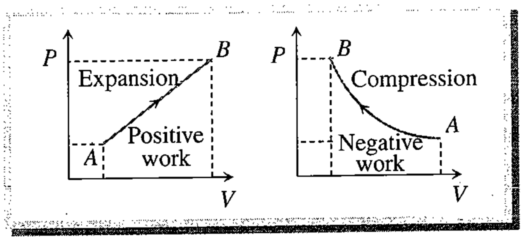

Fig. 2.29

- If a massless piston is attached to a spring of force constant K and a mass m is placed over the piston. The external pressure P_0 is applied and due to the expansion of the gas the piston moves up through a distance x, then

Total work done by the gas is $W = W_1 + W_2 + W_3$

where W_1 = work done against external pressure (P_0)

W_2 = work done against spring force (Kx)

W_3 = work done against gravitational force (mg)

$$\therefore \quad W = P_0 V + \frac{1}{2}Kx^2 + mgx$$

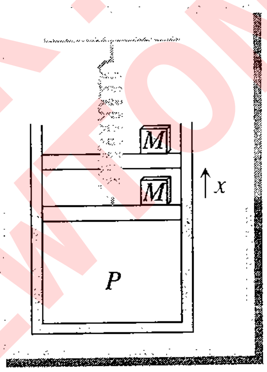

Fig. 2.30

- If the gas expands in such a way that the other side of the piston is vacuum, then work done by the gas will be zero.

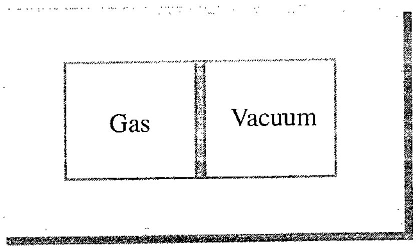

Fig. 2.31

As $\quad W = P\Delta V = 0 \quad$ (here $P = 0$)

Illustration 2.21 **A quantity of gas occupies an initial volume V_0 at pressure p_0 and temperature T_0. It expands to a volume V (a) at constant temperature and (b) at constant pressure. In which case does the gas do more work?**

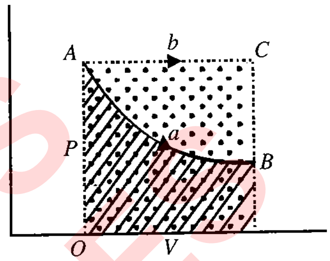

Fig. 2.32

Sol. The work done is given by the area between the P–V plot and the V-axis. Here, the area between the P–V plot and the V-axis is less in the case (a) than in the case (b). Hence, the work done at constant pressure is greater than the work done at constant temperature.

Illustration 2.22 **A sample of ideal gas is expanded to twice its original volume of 1.00 m³ in a quasi-static process for which $P = \alpha V^2$, with α = 5.00 atm/m⁶, as shown in Fig. 2.33. How much work is done on the expanding gas?**

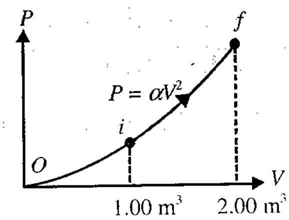

Fig. 2.33

Sol. The final pressure is 5 × 4 = 20 atm = 2 MPa.

The work done on the gas is the negative of the area under the curve $P = \alpha V^2$ from V_i to V_f. The work on the gas is negative, which means that the expanding gas does positive work. We find its value as

$$W = -\int_{V_i}^{V_f} P dV \quad \text{with} \quad V_f = 2V_i = 2(1.00 \text{ m}^3) = 2.00 \text{ m}^3$$

$$W = \int_{V_i}^{V_f} \alpha V^2 dV = -\frac{1}{3}\alpha(V_f^3 - V_i^3)$$

$$W = -\frac{1}{3}(5.00 \text{ atm/m}^6)(1.013 \times 10^5 \text{ Pa/atm}) \times [(2.00 \text{ m}^3)^3 - (1.00 \text{ m}^3)^3] = 1.18 \times 10^6 \text{ J}$$

Our estimate was good. To do the integral of PdV, we start by identifying how P depends on V, just as to do the integral of ydx in mathematics you start by identifying how y depends on x.

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Illustration 2.23 **Determine the work done by an ideal gas doing $1 \to 4 \to 3 \to 2 \to 1$.**

Given $P_1 = 10^5$ Pa, $P_0 = 3 \times 10^5$ Pa, $P_3 = 4 \times 10^5$ Pa and $V_2 - V_1 = 10$ L.

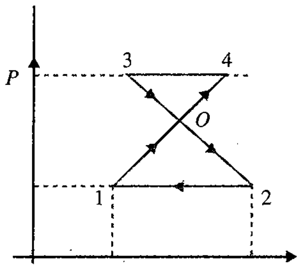

Fig. 2.34

Sol. From Fig. 2.34,

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_3 - P_0}{P_0 - P_1} \Rightarrow \frac{V_4 - V_3}{10} = \frac{4\times10^5 - 3\times10^5}{3\times10^5 - 10^5}$$

$$V_4 - V_3 = 5 \text{ L}$$

Now, work done

$$W = \left(\frac{1}{2}\times10\times2\times10^5 - \frac{1}{2}\times5\times1\times10^5\right)\times10^{-3} = 750 \text{ J}$$

Illustration 2.24 **An ideal gas is enclosed in a cylinder with a movable piston on top of it. The piston has a mass of 8000 g and an area of 5.00 cm² and is free to slide up and down, keeping the pressure of the gas constant. How much work is done on the gas as the temperature of 0.200 mol of the gas is raised from 20.0°C to 300°C?**

Sol. The gas is expanding and doing positive quantity of work in lifting the piston. The work on the gas is this number of joules but negative. The original internal energy of the gas is some multiple of nRT_i = (0.2 mol) (8.314 J/mol K) (293 K) = 10^3 J. The work done by the gas does not come just from the internal energy, but from some of the energy that must be put into the sample by heating to raise its temperature. But still we can estimate the work as on the order of -1 kJ.

The integral of PdV is easy because the pressure is constant. Then the equation of state will let us evaluate what we need about the pressure and the change in volume.

For constant pressure,

$$W = -\int_i^f PdV = -P\Delta V = -P(V_f - V_i)$$

Rather than evaluating the pressure numerically from atmospheric pressure plus the pressure due to the weight of the piston, we can just use the ideal gas law to express in the volumes obtaining

$$W = -P\left(\frac{nRT_h}{P} - \frac{nRT_c}{P}\right) - nR(T_h - T_c)$$

Therefore, $W = -nR\Delta T$

$= -$ (0.200 mol) (8.314 J/mol/K) (280 K)

$= -$ 466 J

We did not need to use the mass of the piston or its face area. The answer would be the same if the gas had to lift a heavier load over a smaller volume change or a lighter load through a larger volume change. The calculation turned out to be simple, but only because the pressure stayed constant during the expansion. Do not try to use $-P\Delta V$ for work in any case other than constant pressure, and never try to use $-V\Delta P$.

Internal Energy (U)

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and energy due to molecular configuration is called internal potential energy U_P. Therefore, total internal energy

$$U = U_K + U_P$$

i. For an ideal gas, as there is no molecular attraction $U_p = 0$, i.e., internal energy of an ideal gas is totally kinetic and is given by $U = U_K = (3/2)\mu RT$ and change in internal energy $U = (3/2)\mu R\Delta T$

ii. In case of gases, whatever may be the process

$$\Delta U = \mu\frac{f}{2}R\Delta T = \mu C_v\Delta T$$

$$= \mu\frac{R}{(\gamma - 1)}\Delta T = \frac{\mu R(T_f - T_i)}{\gamma - 1}$$

$$= \frac{\mu RT_f - \mu RT_i}{\gamma - 1} = \frac{(P_fV_f - P_iV_i)}{\gamma - 1}$$

iii. Change in internal energy does not depend on the path of the process. So it is called a point function, i.e., it depends only on the initial and final states of the system, i.e., $U = U_f = U_i$

iv. Change in internal energy in a cyclic process is always zero as for cyclic process $U_f = U_i$. Therefore,

$$\Delta U = U_f = U_i = 0$$

Illustration 2.25 **When a thermodynamic system is taken from an initial state *I* to a final state *F* along the path *IAF*, as shown in Fig. 2.35, the heat energy absorbed by the system is $Q = 55$ J and the work done by the system is $W = 25$ J. If the same system is taken along the path *IBF*, the value of $Q = 35$ J.**

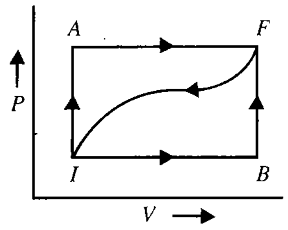

Fig. 2.35

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a. Find the work done along the path *IBF*.

b. If $W = -15$ J for the curved path *FI*, how much heat energy is lost by the system along this path?

c. If $U_I = 10$ J, what is U_F?

d. If $U_B = 20$ J, what is Q for the process *BF* and *IB*?

Sol. The first law of thermodynamics states that

$$\Delta Q = \Delta U + \Delta W \quad \text{or} \quad Q = (U_F - U_I) + W$$

Here U_I and U_F are the internal energies in the initial and the final state, respectively. Given that for path *IAF*, $Q = 55$ J and $W = 25$ J. Therefore,

$$\Delta U = U_F - U_I = Q - W = 55 - 25 = 30 \text{ J}$$

The internal energy is independent of the path; it depends only on the initial and final states of the system. Thus, the internal energy between I and F states is 30 J irrespective of the path followed by the system.

a. For path *IBF*, $Q = 35$ J and $\Delta U = 30$ J. Therefore, $W = Q - \Delta U = 35 - 30 = 5$ J

b. $W = -15$ J, but $\Delta U = -30$ J. Therefore, $Q = W + \Delta U = -15 - 30 = -45$ J.

c. Given $U_I = 10$ J. Therefore, $U_F = \Delta U + U_I = 30 + 10 = 40$ J.

d. The process *BF* is isochoric, i.e., the volume is constant. Hence, $W = 0$. Therefore, $Q = (\Delta U)_{BF} = U_F - U_B = 40 - 20 = 20$ J. The process *IB* is isobaric (constant pressure). Therefore, $Q = (Q)_{IBF} - (Q)_{BF} = 35 - 20 = 15$ J.

Joule's Law

Whenever heat is converted into mechanical work or mechanical work is converted into heat, the ratio of work done to heat produced always remains constant. That is,

$$W \propto Q \quad \text{or} \quad \frac{W}{Q} = J$$

This is Joule's law and J is called mechanical equivalent of heat.

Illustration 2.26 **How much work is done on the steam when 1.00 mol of water at 100°C boils and becomes 1.00 mol of steam at 100°C at 1.00 atm pressure? Assuming the steam to behave as an ideal gas, determine the change in the internal energy of the material as it vapourizes.**

Sol. Consider the liquid water vapourizing inside a cylinder with a piston. It expands greatly to lift the piston and do the positive work. Many hundreds of joules of negative work is done on the material. Even more positive energy is put into it by heating, to produce a net increase in its internal energy.

We use the idea of the negative integral of PdV to find the work, identifying the volumes of the liquid and the gas as we do so. Then mL will tell us the heat input and the first law of thermodynamics will tell us the change in internal energy.

For a constant pressure process,

$$W = -P\Delta V = -P(V_s - V_w)$$

where V_s is the volume of the steam and V_w is the volume of the liquid water.

We can PV_s and PV_w, find, respectively, from $PV_s = nRT$ and $V_W = m/\rho = nM/\rho$

Calculating each work term, we get

$$PV_s = (1.00 \text{ mol})\left(8.314 \frac{\text{J}}{\text{K mol}}\right)(373 \text{ K}) = 3101 \text{ J}$$

$$PV_w = (1.00 \text{ mol})(18.0 \text{ g/mol})\left(\frac{1.013 \times 10^5 \text{ N/m}^2}{1.00 \times 10^6 \text{ g/m}^3}\right) = 1.82 \text{ J}$$

Thus the work done is $W = -3.10$ kJ.

The energy input by heat is

$$Q = mL_v = (18.0 \text{ g})(2.26 \times 106 \text{ J/kg}) = 40.7 \text{ kJ}.$$

So the change in internal energy is $\Delta U = Q + W = 37.6$ kJ.

Steam at 100°C is on the point of liquefaction, so it probably behaves differently from the ideal gas we were told to assume. Still, the volume increase by more than a thousand times means that a significant amount of the 'heat of vapourization' is coming out of the sample as it boils, as the work done on the environment in its expansion. Remember that we start with 18 cubic centimetres of liquid water. Visualize it!

Illustration 2.27 **An ideal gas initially at 30 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m³ to 3.00 m³ and 12.5 kJ is transferred to the gas by heat, what are (a) the change in its internal energy and (b) its final temperature?**

Sol. The gas pressure is much less than one atmosphere. This could be managed by having the gas in a cylinder with a piston on the bottom end, supporting a constant load that hangs from the piston. The (large) cylinder is put into a warmer environment to make the gas expand.

The first law of thermodynamics will tell us the change in internal energy. The ideal gas law will tells us the final temperature.

a. $\Delta U = Q + W$

where $W = -P\Delta V$ for a constant pressure process.

So
$$\Delta U = Q - P\Delta V$$
$$\Delta U = 1.25 \times 10^4 \text{ J} - (2.50 \times 10^3 \text{ N/m}^2) \times (3.00 \text{ m}^3 \times 1.00 \text{ m}^3) = 7500 \text{ J}$$

b. Since pressure and quantity of the gas are constant, we have from the equation of state

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

and
$$T_2 = \left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{3.00 \text{ m}^3}{1.00 \text{ m}^3}\right)(300 \text{ K}) = 900 \text{ K}$$

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Important Points

i. First law of thermodynamics makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased by adding either heat to it or doing work on it or both.

ii. ΔQ and ΔW are the path functions, but ΔU is the point function.

iii. In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in Joule or in calorie.

iv. Sign conventions are as follows:

ΔQ	Positive	When heat is supplied to a system
	Negative	When heat is drawn from the system
ΔW	Positive	When work is done by the gas (expansion)
	Negative	When work is done on the gas (compression)
ΔU	Positive	When temperature increases, internal energy increases
	Negative	When temperature decreases, internal energy decreases

v. Limitation: First law of thermodynamics does not indicate the direction of heat transfer. It does not tell anything about the conditions under which heat can be transformed into work and also it does not indicate as to why the total heat energy cannot be converted into mechanical work continuously.

Illustration 2.28 **The temperature of 3 kg of nitrogen is raised from 10°C to 100°C. Compute the heat added, the work done, and the change in internal energy if (a) this is done at constant volume and (b) if the heating is at constant pressure. For nitrogen**

$$C_P = 1400 \text{ J kg}^{-1}\text{ K}^{-1} \quad \text{and} \quad C_V = 740 \text{ J kg}^{-1}\text{ K}^{-1}.$$

Sol.

a. At constant volume $\Delta W = 0$.

Now ΔU is always given by $\Delta U = mC_V\Delta T$

$$= 3 \times 740 \times (100 - 10) = 199800 \text{ J}$$

According to the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \Delta Q = \Delta U = 199800 \text{ J}$$

b. At constant pressure $\Delta Q = mC_P\Delta T = 3 \times 1040 \times (100 - 10) = 280800$ J

ΔU is always given by $\Delta U = mc_V\Delta T = 199800$ J

According to the first law of thermodynamics $\Delta Q = \Delta U + \Delta W$

or $\quad \Delta W = \Delta Q - \Delta U = 280800 - 199800 = 81000$ J

Illustration 2.29 **Gaseous hydrogen contained initially under standard conditions in a sealed vessel of volume V = 5 L was cooled by ΔT = 55 K. Find how much the internal energy of the gas will change and what amount of heat will be lost by the gas.**

Sol. By the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Here $\Delta W = 0$ as the volume remains constant.

$$\therefore \quad \Delta Q = \Delta U$$

Now $\quad \Delta U = \frac{m}{M}C_v(-\Delta T)$

$$= -\frac{m}{M}\frac{R}{\gamma - 1}\Delta T \qquad \left(\text{as } C_v = \frac{R}{\gamma - 1}\right)$$

We have $\quad p_0V = \frac{m}{M}RT_0$

$$= -\frac{1.013\times 10^5 \times (5\times 10^{-3})\times 55}{273(1.4-1)} = -255 \text{ J}$$

Illustration 2.30 **The volume of a monatomic ideal gas increases linearly with pressure, as shown in Fig. 2.36 Calculate (a) increase in internal energy, (b) work done by the gas and (c) heat supplied to the gas.**

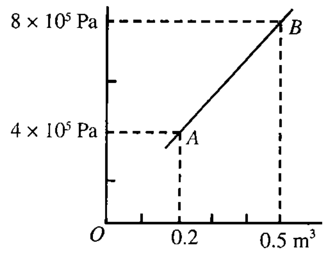

Fig. 2.36

Sol. We know that for a perfect gas, internal energy is given by

$$U = \frac{pV}{\gamma - 1}$$

$$\Rightarrow \Delta U = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$$

$$\therefore \quad \Delta U = \frac{8\times 10^5 \times 0.5 - 4\times 10^5 \times 0.2}{5/3 - 1} = 4.8\times 10^5 \text{ J}$$

Now, $\quad \Delta W = \int_{0.2}^{0.5} p\,dV$

Here, p varies with V along a straight line.

Therefore, one may say $p = kV + V_0$

At $\quad p = 4 \times 10^5$, $V = 0.2$ m³ and $p = 8 \times 10^5$, $V = 0.5$ m³

$4 \times 10^5 = k \times 0.2 + V_0 \quad 8 \times 10^5 = k \times 0.5 + V_0$

Hence, $\quad k = \frac{4}{3}\times 10^6$ and $V_o = \frac{4}{3}\times 10^5$

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$\therefore \quad p=\frac{4}{3}\times10^5(10V+1)$

$\therefore \quad \Delta W=\int_{0.2}^{0.5}\frac{4}{3}\times10^5\left(\frac{V^2}{2}\right)_{0.2}^{0.5}+\frac{4}{3}\times10^5(V)_{0.2}^{0.5}$

$$=\frac{4}{3}\times10^6\times\frac{0.21}{2}+\frac{4}{3}\times10^5\times0.3=1.8\times10^5\text{ J}$$

By the first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\therefore \quad \Delta Q = 4.8\ 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5$ J

Illustration 2.31 **A vessel contains helium, which expands at constant pressure when 15 kJ of heat is supplied to it. What will be the variation of the internal energy of the gas? What is the work performed in the expansion?**

Sol. Here ΔQ = 15000 J (given)

In an isobaric process, we have

$\Delta Q = C_p\Delta T,\ \Delta U = C_v\Delta T$ (always)

$\Delta W = (C_p - C_v)\,\Delta T = R\Delta T$

(from $\Delta Q = \Delta U + \Delta W$ relation)

But $\quad C_p=\left(\frac{\gamma R}{\gamma-1}\right)$ and $C_v=\left(\frac{R}{\gamma-1}\right)$

$\therefore \quad \Delta Q:\Delta U:\Delta W=\left(\frac{\gamma R}{\gamma-1}\right):\left(\frac{R}{\gamma-1}\right):R=\gamma:1:\gamma-1$

Helium is monatomic, hence its γ = 5/3

$\therefore \quad \Delta Q:\Delta U:\Delta W=\left(\frac{5}{3}\right):1:\left(\frac{2}{3}\right)=5:3:2$

$\therefore \quad \frac{\Delta U}{\Delta Q}=\frac{3}{5}$ or $\Delta U=\frac{3}{5}\times15\text{ kJ}=9\text{ kJ}$

$\frac{\Delta W}{\Delta Q}=\frac{2}{5}$ or $\Delta W=\frac{2}{5}\times15\text{ kJ}=6\text{ kJ}$

Illustration 2.32 **The volume of one mole of an ideal gas with adiabatic exponent γ is varied according to the law $V = a/T$, where a is constant. Find the amount of heat obtained by the gas in this process, if the temperature is increased by ΔT.**

Sol. $\Delta W=\int_T^{T+\Delta T}pdV;\ pV=RT$ (always) and here $V = a/T$

$\therefore \quad \Delta W=\int_T^{T+\Delta T}\frac{RT^2}{a}\left(-\frac{a}{T^2}dT\right)=-\int_T^{T+\Delta T}RdT=-R\Delta T$

$\Delta U=\int_T^{T+\Delta T}C_vdT=\frac{R}{\gamma-1}\Delta T \quad \gamma-1 \quad \left(\because C_v\frac{R}{\gamma-1}\right)$

$\therefore \quad \Delta Q=\Delta U+\Delta W=\frac{R}{\gamma-1}\Delta T-R\Delta T=\frac{2-\gamma}{\gamma-1}R\Delta T$

Isothermal Process

When a thermodynamic system undergoes a physical change in such a way that its temperature remains constant, then the change is known as isothermal change.

In this process, P and V change but T = constant, i.e., change in temperature $\Delta T = 0$

1. **Essential condition for isothermal process:**

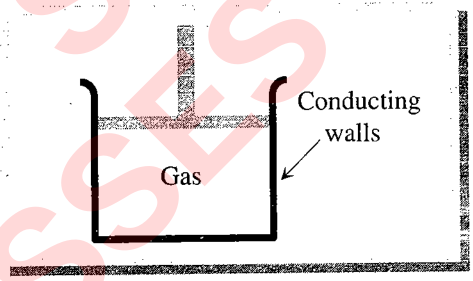

Fig. 2.37

   i. The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and its surrounding.

   ii. The process of compression or expansion should be so slow so as to provide time for the exchange of heat.

   Since these two conditions are not fully realized in practice, no process is perfectly isothermal.

2. **Equation of state:** From ideal gas equation $PV = \mu RT$, we observe that if temperature remains constant, then PV = constant, i.e., in all isothermal process Boyle's law is obeyed.

   Hence, equation of state is PV = constant.

3. **Example of isothermal process:**

   i. Melting process (ice melts at constant temperature 0°C)

   ii. Boiling process (water boils at constant temperature 100°C)

4. **Indicator diagram:**

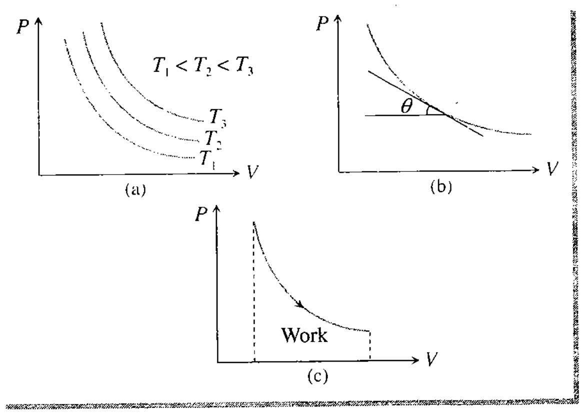

Fig. 2.38

   i. Curves obtained on PV graph are called isotherms and they are hyperbolic in nature.

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ii. Slope of isothermal curve: By differentiating PV = constant. We get

$$PdV + VdP = 0 \quad \Rightarrow P\,dV = -VdP \quad \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

$$\therefore \qquad \tan\theta = \frac{dP}{dV} = -\frac{P}{V}$$

iii. Area between the isotherm and volume axis represents the work done in isothermal process.

If volume increases ΔW = + area under curve and if volume decreases ΔW = –area under curve

5. Specific heat: Specific heat of gas during isothermal change is infinite.

As $$C = \frac{Q}{m\Delta T} = \frac{Q}{m\times 0} \to \infty \qquad (\text{as } \Delta T = 0)$$

6. Isothermal elasticity: For isothermal process PV = constant

Differentiating both sides we get

$$PdV + VdP = 0$$

$$\Rightarrow PdV = -VdP$$

$$\Rightarrow P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_\theta$$

$\therefore E_\theta = P$, i.e., isothermal elasticity is equal to pressure.

At NTP, isothermal elasticity of gas = atmospheric pressure = $1.01\times 10^5\,\text{N/m}^2$

7. Work done in isothermal process:

$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{\mu RT}{V}dV \qquad (\text{as } PV = \mu RT)$$

$$W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = 2.303\mu RT \log_{10}\left(\frac{V_f}{V_i}\right)$$

or $$W = \mu RT \log_e\left(\frac{P_i}{P_f}\right) = 2.303\mu RT \log_{10}\left(\frac{P_i}{P_f}\right)$$

8. First law of thermodynamics in isothermal process:

$$\Delta Q = \Delta U + \Delta W \quad \text{but} \quad \Delta U \propto \Delta T$$

$$\therefore \qquad \Delta U = 0 \qquad (\text{as } \Delta T = 0)$$

$\therefore \Delta U = \Delta W$, i.e., heat supplied in an isothermal change is used to do work against the external surrounding, or if the work is done on the system, then equal amount of heat energy will be liberated by the system.

Illustration 2.33 **A 2.00 mol sample of helium gas initially at 300 K and 0.400 atm is compressed isothermally to 1.20 atm. Noting that the helium behaves as an ideal gas, find (a) the final volume of the gas, (b) the work done on the gas and (c) the energy transferred by heat.**

Sol. The final volume will be one-third of the original, for the temperature to be constant, the sample must liberate as many joules by heat as it takes in by work.

The ideal gas law can tell us the original and final volumes. The negative integral of PdV will tell us the work input, and the first law of thermodynamics will tell us the energy output by heat.

a. Rearranging $PV = nRT$, we get $V_i = \dfrac{nRT}{P_i}$

The initial volume is

$$V_i = \frac{(2.00\,\text{mol})(8.314\,\text{J mol K})(300\,\text{K})}{(0.400\,\text{atm})(1.013\times 10^5\,\text{Pa/atm})}\left(\frac{1\,\text{Pa}}{\text{N/m}^2}\right) = 0.123\,\text{m}^3$$

For isothermal compression, PV is constant, so $P_iV_i = P_fV_f$ and the final volume is

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = (0.123\,\text{m}^3)\left(\frac{0.400\,\text{atm}}{1.20\,\text{atm}}\right) = 0.0410\,\text{m}^3$$

b. $$W = -\int PdV = -\int \frac{nRT}{V}dV = -nRT\ln\left(\frac{V_f}{V_i}\right)$$

$$= -(4988\,\text{J})\ln\left(\frac{1}{3}\right) = +5.48\,\text{kJ}$$

c. The ideal gas keeps constant temperature so $\Delta E_{\text{int}} = 0 = Q + W$ and the heat is $Q = -5.48$ kJ.

Isobaric Process

When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, the change is known as isobaric process.

In this process, V and T change but P remains constant. Hence, Charles's law is obeyed in this process.

1 Equation of state: From ideal gas equation, we get $PV = \mu RT$

If pressure remains constant, then $V \propto T$

or $$V_1/T_1 = V_2/T_2 = \text{Constant}$$

2 Indicator diagram: In Fig. 2.39, graph I represents isobaric expansion, and graph II represents isobaric compression.

Slope of indicator diagram $dP/dV = 0$.

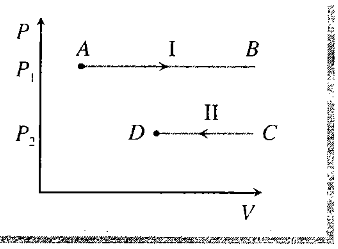

Fig. 2.39

3 Specific heat: Specific heat of gas during isobaric process

$$C_p = \left(\frac{f}{2}+1\right)R$$

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4. **Bulk modulus of elasticity:**

$$K = \frac{\Delta P}{-\Delta V/V} = 0 \quad (\text{as } \Delta P = 0)$$

5. **Work done in isobaric process:**

$$\Delta W = \int_{V_i}^{V_f} P dV = P\int_{V_i}^{V_f} dV = P[V_f - V_i] \quad (\text{as } P = \text{constant})$$

$$\therefore \ \Delta W = P(V_f - V_i) = \mu R[T_f - T_i] = \mu R \Delta T$$

6. **First law of thermodynamics in isobaric process:**

$$\Delta U = \mu C_v \Delta T = \mu \frac{R}{(\gamma - 1)} \Delta T \quad \text{and} \quad \Delta W = \mu R \Delta T$$

From the first law of thermodynamics, we get

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \ \Delta Q = \mu \frac{R}{(\gamma - 1)} \Delta T + \mu R \Delta T$$

$$= \mu R \Delta T \left[\frac{1}{\gamma - 1}\right] = \mu R \Delta T \frac{\gamma}{\gamma - 1}$$

$$= \mu \left(\frac{\gamma}{\gamma - 1}\right) R \Delta T$$

$$\Delta Q = \mu C_p \Delta T$$

7 **Examples of isobaric process:**

i. Conversion of water into vapour phase (boiling process): When water gets converted into vapour phase, its volume increases. Hence, some part of absorbed heat is used up to increase the volume against external pressure and remaining amount of heat is used up to increase the internal potential energy of the molecules (because inter-atomic forces of attraction takes place between the molecules of system and when the distance between them increases, its potential energy increases. It must be noted that during the change of state since temperature remains constant, there will be no increase in internal kinetic energy of the molecules).

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = \Delta U_K + \Delta U_P + \Delta W = 0 + \Delta U_P + P\Delta V$$

$$\therefore \quad \Delta U_P = mL - P[V_f - V_i]$$

$$\text{or} \quad \Delta U_P = mL - P[V_f - V_i] \quad (\text{as } \Delta Q = mL)$$

ii. Conversion of ice into water

$$\Delta Q = \Delta U + \Delta W = \Delta U_P + \Delta U_K + \Delta W$$

$$mL = \Delta U_p + 0 + P[V_f - V_i]$$

($\Delta U_K = 0$ as there is no change in temperature)

$\Delta U_p = m_L$ (when ice converts into water, then change in volume is negligible)

Note:

- ***In isobaric compression, temperature increases and internal energy flows out in the form of heat energy, while in isobaric expansion, temperature increases and heat flows into the system.***
- ***Isobaric expansion of the volume of a gas is given by $V_t = V_0(1 + \gamma_V t)$ where $\gamma_V = 1/273$ per °C = coefficient of volume expansion.***

Illustration 2.34 **As a result of isobaric heating by ΔT = 72 K, one mole of a certain ideal gas obtains an amount of heat Q = 1.60 kJ. Find the work done by the gas, the increment in its internal energy, and the value of γ.**

Sol. By the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

In an isobaric process

$$\Delta Q = C_p \Delta T$$

and $\quad \Delta U = C_v \Delta T \quad$ (always)

$\therefore \quad C_p \Delta T = C_v \Delta T + \Delta W$

or $\quad \Delta W = (C_p - C_v) \Delta T$

or $\quad \Delta W = R\Delta T \quad (\because C_p - C_v = R)$

$\therefore \quad \Delta W = 8.3 \times 72 = 597.6$ J

$$\Delta Q = C_p \Delta T$$

$\therefore \quad 1.6 \times 1000 = C_p \times 72$

$$\Rightarrow C_p = \frac{1.6 \times 1000}{72} = 22.2 \text{ J mol}^{-1} \text{ K}^{-1}$$

$\Delta U = \Delta Q = \Delta W = 1.6 \times 1000 - 597.6 =$ 1002.4 J

But $\quad \Delta U = C_v \Delta T$

$$\therefore \quad C_v = \frac{1002.4}{72} = 13.9 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{22.2}{13.9} = 1.60$$

Isochoric or Isometric Process

When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, the change is known as isochoric process.

In this process P and T changes, but V = constant. Hence Gay-Lussac's law is obeyed in this process.

1. **Equation of state:** From ideal gas equation, we get $PV = \mu RT$. If volume remains constant, than $P \propto T$ or $P_1/T_1 = P_2/T_2$ = constant.

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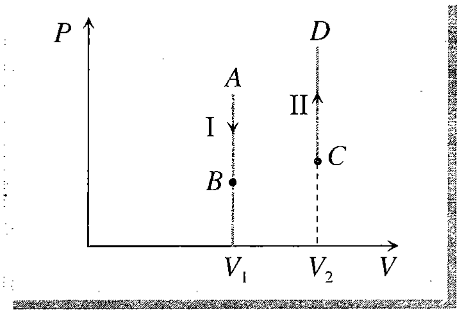

Fig. 2.40

2. **Indicator diagram:** In Fig. 2.40, graphs I and II represent isometric decrease in pressure at volume V_1 and isometric increase in pressure at volume V_2, respectively, and slope of indicator diagram $dP/dV = \infty$.
3. **Specific heat:** Specific heat of gas during isochoric process $C_v = (f/2)R$.
4. **Bulk modulus of elasticity:** $K = \Delta P/(-DV/V) = \Delta P/0 = \infty$.
5. **Work done in isobaric process:** $W = P\Delta V = P[V_f - V_i]$ (as $V = 0$).

$$\therefore \qquad W = 0$$

6. **From the first law of thermodynamics in isochoric process:** $\Delta Q = \Delta U + \Delta W = \Delta U$ (as $\Delta W = 0$)

$$\Delta Q = \mu C_v \Delta T = \mu \frac{R}{\gamma - 1}\Delta T = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

Note: ***Isometric expansion of the pressure of a gas is given by $P_t = P_0\,(1 + \gamma_P t)$ where $\gamma P = (1/273)$ per °C coefficient of volume expansion.***

Illustration 2.35 **Six grams of hydrogen gas at a temperature of 273 K was isothermally expanded to five times its initial volume and then isochorically heated so that the pressure in the final state becomes equal to that in the initial state. Find the total amount of heat absorbed by the gas during the entire process.**

Sol. The processes represent on the P–V diagram by the curve 1 to 2 (isothermal) and the line 2 to 3 (isochoric). Here $P_1 = P_3$

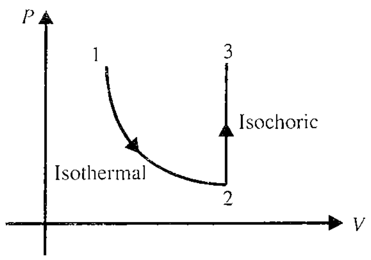

Fig. 2.41

For the process 1 to 2 (isothermal)

$$T_2 = T_1,\ V_2 = 5V_1$$

$$\Delta H = \Delta U + W = 0 + W$$

$$\Rightarrow \Delta H = 2.303\, nRT_1 \log\left(\frac{V_2}{V_1}\right)$$

$$\Delta H = 2.303 \times \left(\frac{6}{2}\right) \times 8.3 \times 273 \times \log 5 = 10942.4\text{J}$$

State 1 and state 3 are at the same pressure.

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_3}{T_3} \quad \Rightarrow T_3 = \frac{V_3}{V_1}T_1 = 5 \times 273 = 1365\text{ K}$$

Process 2 to 3: $\Delta H = \Delta U + W$

$$\Rightarrow \Delta H = nC_v \Delta T + 0$$

$$\Delta H = \frac{6}{2} \times \frac{5}{2} R \times (T_3 - T_2)$$

$$\Rightarrow \Delta H = 7.5 \times 8.3 \times (1365 - 273)\text{J} = 67977\text{J}$$

Thus total heat absorbed $= \Delta H_{1\to 2} + \Delta H_{2\to 3} = 78919.4$ J

Illustration 2.36 **Two moles of an ideal gas at temperature $T_0 = 300$ K was cooled isochorically so that the pressure was reduced to half. Then, in an isobaric process, the gas expanded till its temperature got back to the initial value. Find the total amount of heat absorbed by the gas in the process.**

Sol. In an isochoric process, $\Delta W = 0$ $(\Delta V = 0)$

and $\quad \Delta U = C_v \Delta T = \dfrac{R}{\gamma - 1}\Delta T \qquad \left(\because C_v = \dfrac{R}{\gamma - 1}\right)$

$$\Rightarrow \Delta U = \frac{\Delta(RT)}{\gamma - 1} = \frac{\Delta(pV)}{\gamma - 1}$$

$$= \frac{V}{\gamma - 1}\Delta p = \frac{V}{\gamma - 1}\left(\frac{1}{2}p - p\right) = -\frac{1}{2}\frac{pV}{\gamma - 1}$$

Now $\qquad \Delta Q = \Delta U + \Delta W \qquad$ (always)

$$\therefore \qquad \Delta Q = -\frac{1}{2}\frac{pV}{\gamma - 1} + 0 = -\frac{1}{2}\frac{pV}{\gamma - 1} = -\frac{1}{2\gamma}\frac{RT_0}{-1}$$

The negative sign shows that heat is not added but subtracted from the gas in the process.

In the isobaric process

$$\Delta W = \int_{V_i}^{V_f} \frac{p}{2}dV = \frac{p}{2}(V_f - V_i)\left[\text{In this process pressure is } \frac{P}{2}\right]$$

In an isochoric process, V remains constant, so $p \propto T$ and temperature is reduced to $T_0/2$.

In the isobaric process, the original temperature is restored.

$$\Delta T = T_0 - T_0/2 = T_0/2$$

$$\Delta U = C_v \Delta T = \frac{R}{\gamma - 1}\frac{T_0}{2} \qquad \left(\because C_v = \frac{R}{(\gamma - 1)} \text{and } \Delta T = \frac{T_0}{2}\right)$$

$$\therefore \Delta Q = \Delta U + \Delta W \frac{1}{2}\frac{RT_0}{\gamma - 1} + \frac{1}{2}RT_0$$

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$\therefore$ Net heat added is

$$\left[-\frac{1}{2}\frac{RT_0}{\gamma-1}\right]+\left[\frac{1}{2}\frac{RT_0}{\gamma-1}+\frac{1}{2}RT_0\right]=\frac{1}{2}RT_0$$

This is the heat added when the number of moles is one. When there are 2 moles, heat added is

$$2\times\frac{1}{2}RT_0=RT_0 = 8.3\times 300 = 2490\text{ J}$$

Adiabatic Process

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between it and the surroundings, the process is known as adiabatic process.

In this process P, V and T change, but $\Delta Q = 0$.

1. Essential conditions for adiabatic process:

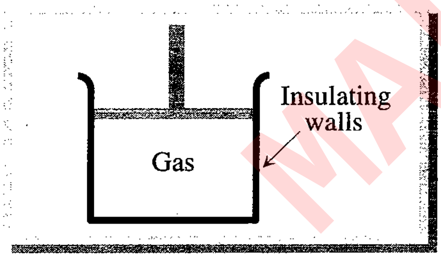

Fig. 2.42

i. There should not be any exchange of heat between the system and its surroundings. All walls of the container and the piston must be perfectly insulating.

ii. The system should be compressed or allowed to expand suddenly so that there is no time for the exchange of heat between the system and its surroundings.

Since, these two conditions are not fully realized in practice, no process is perfectly adiabatic.

2. Examples of some adiabatic processes:

i. Sudden compression or expansion of a gas in a container with perfectly non-conducting walls.

ii. Sudden bursting of the tube of bicycle tyre.

iii. Propagation of sound waves in air and other gases.

iv. Expansion of steam in the cylinder of steam engine.

3. First law of thermodynamics in adiabatic process: $\Delta Q=\Delta U+\Delta W$

but for adiabatic process $\Delta Q=0$ $\therefore \Delta U+\Delta W=0$

If ΔW = positive, then ΔU = negative so temperature decreases, i.e., adiabatic expansion produces cooling.

If ΔW = negative, then ΔU = positive so temperature increases, i.e., adiabatic compression produces heating.

4. Equation of state: As in case of adiabatic change, the first law of thermodynamics reduces to

$$\Delta U+\Delta W=0,\quad \text{i.e.,}\quad dU+dW=0 \qquad \text{(i)}$$

But as for an ideal gas

$$dU = \mu C_v dT \text{ and } dW = PdV$$

Equation (i) becomes

$$\mu C_v dT + PdV = 0 \qquad \text{(ii)}$$

But for a gas as

$$PV=\mu RT, PdV+VdP=\mu RdT \qquad \text{(iii)}$$

So eliminating dT between Eqs. (ii) and (iii), we get

$$\mu C_v\frac{(PdV+VdP)}{\mu R}+PdV=0$$

or $$\frac{(PdV+VdP)}{(\gamma-1)}+PdV=0 \qquad \left(\text{as } C_v=\frac{R}{(\gamma-1)}\right)$$

or $$\gamma PdV+VdP=0,\quad \text{i.e.,}\quad \gamma\frac{dV}{V}+\frac{dP}{P}=0$$

which on integration gives

$$\gamma\log_e V+\log_e P=C$$

$$\Rightarrow \log(PV^\gamma)=C$$

$$\Rightarrow PV^\gamma = \text{constant} \qquad \text{(iv)}$$

Equation (iv) is called equation of state for adiabatic change and can also be rewritten as

$$TV^{\gamma-1} = \text{constant} \qquad (\text{as } P = (mRT/V)) \qquad \text{(v)}$$

and $$\frac{T^\gamma}{(P^{\gamma-1})} = \text{constant} \qquad \left(\text{as } V=\frac{\mu RT}{P}\right) \qquad \text{(vi)}$$

5. Indicator diagram:

i. Curve obtained on PV graph are called adiabatic curve.

ii. Slope of adiabatic curve: From PV^γ = constant.

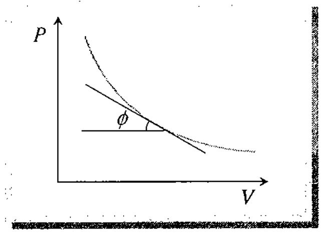

Fig. 2.43

By differentiating, we get $dPV^\gamma+P_\gamma V^{\gamma-1}dV=0$

$$\frac{dP}{dV}=-\gamma\frac{PV^{\gamma-1}}{V^\gamma}=-\gamma\left(\frac{P}{V}\right)$$

Therefore, slope of adiabatic curve $\tan\phi=-\gamma(P/V)$.

But we also know that slope of isothermal curve $\tan\phi=-P/V$.

So, $$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}}=\frac{-\gamma(P/V)}{-(P/V)}=\gamma=\frac{C_p}{C_v}>1$$

6. Specific heat: Specific heat of a gas during adiabatic change is zero.

$$C=\frac{Q}{(m\Delta T)}=\frac{0}{(m\Delta T)}=0 \qquad (\text{as } Q=0)$$

7. Adiabatic elasticity: For adiabatic process PV^γ = constant.

Differentiating both sides, we get

$$dPV^\gamma+P\gamma V^{\gamma-1}dV=0$$

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$$\gamma P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_\phi$$

$$E_\phi = \gamma P$$

i.e., adiabatic elasticity is γ times that of pressure but we know isothermal elasticity $E_\theta = P$

So $$\frac{E_\phi}{E_\theta} = \frac{\text{Adiabatic elasticity}}{\text{Isothermal elasticity}} = \frac{\gamma P}{P} = \gamma$$

i.e., the ratio of two elasticities of gases is equal to the ratio of two specific heats.

9. Work done in adiabatic process:

$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{K}{V^\gamma} dV \qquad \left(\text{as } P = \frac{K}{V^\gamma}\right)$$

or $$= \frac{1}{[1-\gamma]}\left[\frac{K}{V_f^{\gamma-1}} - \frac{K}{V_i^{\gamma-1}}\right] \qquad \left(\text{as} \int V^{-\gamma} dV = \frac{V^{-\gamma+1}}{(-\gamma+1)}\right)$$

or $$= \frac{[P_f V_f - P_i V_i]}{(1-\gamma)} \qquad (\text{as } K = PV^\gamma = P_f V_f^\gamma = P_i V_i^\gamma)$$

or $$= \frac{\mu R}{(1-\gamma)}[T_f - T_i] \quad (\text{as } P_f V_f = \mu R T_f \text{ and } P_i V_i = \mu R T_i)$$

So $$= \frac{[P_i V_i - P_f V_f]}{(\gamma - 1)} = \frac{\mu R (T_i - T_f)}{(\gamma - 1)}$$

9. Free expansion: Free expansion is adiabatic process in which no work is performed on or by the system. Consider two vessels placed in a system which is enclosed with thermal insulation (asbestos-covered). One vessel contains a gas and the other is evacuated. The two vessels are connected by a stopcock. When suddenly the stopcock is opened, the gas rushes into the evacuated vessel and expands freely. The process is adiabatic as the vessels are placed in thermal insulating system ($dQ = 0$). Moreover, the walls of the vessel are rigid and hence no external work is performed ($dW = 0$).

Now according to the first law of thermodynamics $dU = 0$, if U_i and U_f be the initial and final internal energies of the gas, then

$$U_f - U_i = 0 \qquad (\text{as } U_f = U_i)$$

Thus, the final and initial energies are equal in free expansion.

10. Special cases of adiabatic process:

$$PV^\gamma = \text{constant}$$

$$\therefore \quad P \propto \frac{1}{V^\gamma}$$

$$PT^{\lambda/(1-\lambda)} = \text{constant}$$

$$\therefore \quad P \propto T^{\gamma/(\gamma-1)} \quad \text{and} \quad TV^{\gamma-1} = \text{constant}$$

$$\therefore \quad T \propto \frac{1}{(V^{\gamma-1})}$$

Type of gas	$P \propto \frac{1}{V^\gamma}$	$P = T^{\gamma/(\gamma-1)}$	$T \propto \frac{1}{(V^{\gamma-1})}$
Monatomic $\gamma = 5/3$	$P \propto \frac{1}{(V^{5/3})}$	$P \propto T^{5/2}$	$T \propto \frac{1}{(V^{2/3})}$
Diatomic $\gamma = 7/5$	$P \propto \frac{1}{(V^{7/5})}$	$P \propto T^{7/2}$	$T \propto \frac{1}{(V^{2/5})}$
Polyatomic $\gamma = 4/3$	$P \propto \frac{1}{(V^{4/3})}$	$P \propto T^4$	$T \propto \frac{1}{(V^{1/3})}$

11. Comparison between isothermal and adiabatic process:

i. Compression: If a gas is compressed isothermally and adiabatically from volume V_i to V_f, then from the slope of the graph it is clear that graph 1 (Fig. 2.44) represents adiabatic process, whereas graph 2 represents isothermal process.

Work done	$W_{\text{adiabatic}} > W_{\text{isothermal}}$
Final pressure	$P_{\text{adiabatic}} > P_{\text{isothermal}}$
Final temperature	$T_{\text{adiabatic}} > T_{\text{isothermal}}$

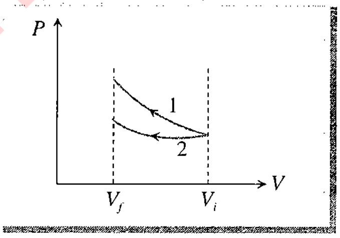

Fig. 2.44

ii. Expansion: If a gas expands isothermally and adiabatically from volume V_i to V_f, then from the slope of the graph it is clear that graph 1 (Fig. 2.45) represents isothermal process, whereas graph 2 represents adiabatic process.

Work done	$W_{\text{isothermal}} > W_{\text{adiabatic}}$
Final pressure	$P_{\text{isothermal}} > P_{\text{adiabatic}}$
Final temperature	$T_{\text{isothermal}} > T_{\text{adiabatic}}$

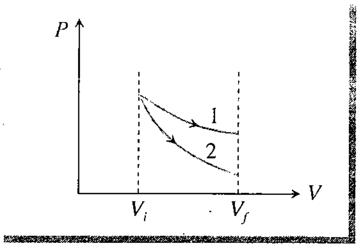

Fig. 2.45

Illustration 2.37 **In a cylinder, 2.0 moles of an ideal monatomic gas initially at 1.0×10^6 Pa and 300 K expands**

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until its volume doubles. Compute the work done if the expansion is (a) isothermal, (b) adiabatic and (c) isobaric.

Sol. Let $P_1 = 1\times10^6\,\text{Pa} = 10^6\,\text{N/m}^2$; $T_1 = 300$ K; $n = 2$ moles.
Final volume = 2 (initial volume) $\Rightarrow V_2 = 2V_1$

a. $W_{\text{isothermal}} = 2.303\, nRT \log(V_2/V_1)$

$$\Rightarrow W = 2.303 \times 2 \times 8.3 \times 300 \log 2$$

$$W = 3452 \text{ J}$$

b. $W_{(\text{adiabatic})} = \dfrac{nR(T_1 - T_2)}{\gamma - 1}$

For adiabatic process: $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_2 = T_1 = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 300 \times \left(\frac{1}{2}\right)^{(5/3)-1} \Rightarrow T_2 = 189\text{K}$$

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1} = \frac{2\times8.3\times(300-189)}{(5/3)-1} \Rightarrow W = 2764 \text{ J}$$

c. $W_{(\text{isobaric})} = P(V_2 - V_1) \quad \Rightarrow P_1V_1 = nRT_1$

$$V_1 = \frac{nRT_1}{P_1} = \frac{2\times8.3\times300}{10^6} \Rightarrow V_1 = 4.98\times10^{-3}\,\text{m}^3$$

$$W = P(V_2 - V_1) = P(2V_1 - V_1) = PV_1$$

$$\Rightarrow W = 10^6 \times 4.98\times10^{-3}\text{ J} \quad \Rightarrow W = 4980 \text{ J}$$

Illustration 2.38 **One mole of a perfect gas, initially at a pressure and temperature of 10^5 N/m² and 300 K, respectively, expands isothermally until its volume is doubled and then adiabatically until its volume is again doubled. Find the final pressure and temperature of the gas. Find the total work done during the isothermal and adiabatic processes. Given γ = 1.4. Also draw the *P–V* diagram for the process.**

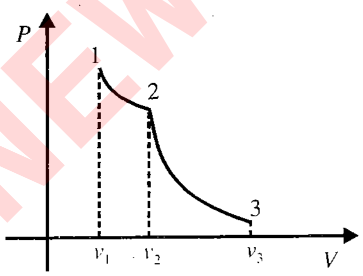

Fig. 2.46

Sol. Let P_1, V_1, T_1 be the initial pressure, volume and the temperature of the gas, respectively. For isothermal expansion (1 to 2), we have

$$V_2 = 2V_1; \quad P_1V_1 = P_2V_2$$

$$P_2 = P_1(V_1/V_2) = P_1/2 = 0.5 \times 10^5 \text{ N/m}^2$$

$$T_2 = T_1 = 300 \text{ K}$$

For adiabatic expansion (2 to 3),

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\Rightarrow V_3 = 2V_2$$

$$\Rightarrow P_3 = P_2\left(\frac{V_2}{V_3}\right)^{\gamma} = 0.5\times10^5\times(0.5)^{1.4}$$

$$\Rightarrow P_3 = 1.9\times10^4\,\text{N/m}^2$$

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

$$\Rightarrow T_3 = T_2\left(\frac{V_2}{V_3}\right)^{\gamma-1} = 300\left(\frac{1}{4}\right)^{0.4} = 227.35 \text{ K}$$

Hence, final pressure and temperatures are 1.9×10^4 N/m² and 227.35 K, respectively.

Work done is $W = W_{1\to2} + W_{2\to3}$.

$$W = 2.303\, nRT_1 \log\left(\frac{V_2}{V_1}\right) + \frac{nR(T_2 - T_3)}{(\gamma - 1)}$$

$$W = R(2.303\times300\times\log 2) + \frac{R(300-227.35)}{0.4}$$

$$\Rightarrow W = 3233.56 \text{ J}$$

Polytropic Process

A process PV^r = constant is called polytropic process, in which $r \neq 1$ or γ.

i. Work done in polytropic process: As we have calculated in adiabatic process, here also the work done is

$$W = \frac{vR}{(r-1)}[T_f - T_i]$$

$$= \frac{-nR}{(r-1)}[T_f - T_i]$$

or $$= -\frac{nR\Delta T}{(r-1)}$$

For one mole of a gas $n = 1$

$$W = -\frac{-R\Delta T}{(r-1)}$$

ii. Specific heat: If C is the molar specific heat, then heat required to increase the temperature of one mole of a gas by ΔT is

$$Q = C\Delta T$$

From the first law of thermodynamics

$$Q = \Delta U + W$$

or $$C\Delta T = C_v\Delta T - \frac{R\Delta T}{(r-1)}$$

$$\therefore \quad C = C_v - \frac{R}{(r-1)} = \frac{R}{(\gamma-1)} - \frac{R}{(r-1)}$$

Illustration 2.39 **An ideal gas ($C_P/C_V = \gamma$) is taken through a process in which the pressure and the volume**

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vary as $P = aV^b$. Find the value of b for which the specific heat capacity in the process is zero.

Sol. Given that $P = aV^b$

or $PV^{-b} = a$

Comparing with PV^r = constant, we have

$$r = -b$$

We know that $C = C_v - \dfrac{R}{r-1}$

Here, $C = 0 = C_v - \dfrac{R}{\gamma - 1}$

$\therefore$ $0 = \dfrac{R}{\gamma-1} - \dfrac{R}{-b-1}$

or $b = -\gamma$

Reversible and Irreversible Processes

Reversible Process

A reversible process is one which can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order and inverse sense and no change is left in any of the bodies taking part in the process or in the surroundings. For example, if heat is absorbed in the direct process, the same amount heat should be given out in the reverse process. If work is done on the working substance in the direct process, then the same amount of work should be done by the working substance in the reverse process. The conditions for reversibility are as follows:

i. There must be complete absence of dissipative forces such as friction, viscosity, electric resistance, etc.
ii. The direct and reverse processes must take place infinitely slowly.
iii. The temperature of the system must not differ appreciably from its surroundings.

Some examples of reversible process are as follows:

a. All isothermal and adiabatic changes are reversible if they are performed very slowly.
b. When a certain amount of heat is absorbed by ice, it melts. If the same amount of heat is removed from it, the water formed in the direct process will be converted into ice.
c. An extremely slow extension or contraction of a spring without setting up oscillations.
d. When a perfectly elastic ball falls from some height on a perfectly elastic horizontal plane, the ball rises to the initial height.
e. If the resistance of a thermocouple is negligible, there will be no heat produced due to Joule's heating effect. In such a case, heating or cooling is reversible. At a junction where a cooling effect is produced due to Peltier effect when current flows in one direction, and equal heating effect is produced when the current is reversed.
f. Very slow evaporation or condensation.

It should be remembered that the conditions mentioned for a reversible process can never be realized in practice. Hence, a reversible process is only an ideal concept. In actual process, there is always loss of heat due to friction, conduction, radiation, etc.

Irreversible Process

Any process which is not reversible exactly is an irreversible process. All natural processes such as conduction, radiation and radioactive decay are irreversible. All practical processes such as free expansion, Joule–Thomson expansion, electrical heating of a wire are also irreversible. Some examples of irreversible processes are given below:

i. When a steel ball is allowed to fall on an inelastic lead sheet, its kinetic energy changes into heat energy by friction. The heat energy raises the temperature of lead sheet. No reverse transformation of heat energy to kinetic energy occurs.
ii. The sudden and fast stretching of a spring may produce vibrations in it. Now a part of the energy is dissipated. This is the case of irreversible process.
iii. Sudden expansion or contraction and rapid evaporation or condensation are examples of irreversible processes.
iv. Produced by the passage of an electric current through a resistance is irreversible.
v. Heat transfer between bodies at different temperatures is also irreversible.

Illustration 2.40 **A reversible heat engine carries 1 mole of an ideal monatomic gas around the cycle 1–2–3–1. Process 1–2 takes place at constant volume, process 2–3 takes place at constant volume, process 2–3 is adiabatic, and process 3–1 takes place at constant pressure. Compute the values for the heat ΔQ, the change in internal energy ΔU, and the work done ΔW, for each of the three processes and for the cycle as a whole.**

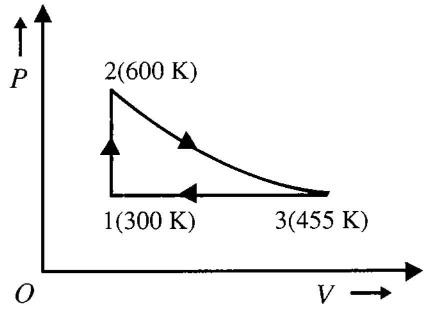

Fig. 2.47

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Sol. For the process 1–2, we have $\Delta W = 0$ (since volume remains constant)

$$\Delta U = C_v \Delta T = \frac{R}{(\gamma - 1)}\Delta T = \frac{8.3}{\left(\frac{5}{3}\right)-1} \times (600 - 300) = 3735 \text{ J}$$

For the process 2–3 we have $\Delta Q = 0$ (since the process is adiabatic)

$$\Delta W = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{(\gamma - 1)} = \frac{8.3(600 - 455)}{\left(\frac{5}{3}\right)-1} = 1805 \text{ J}$$

$$\Delta U = \Delta Q - \Delta W = 0 - 1805 = -1805 \text{ J}$$

For the process 3–1, we have $\Delta Q = C_p \Delta T = \frac{\gamma R}{(\gamma - 1)} \times \Delta T$

or $$\Delta Q = \frac{\left(\frac{5}{3}\right) \times 8.3}{\left(\frac{5}{3}\right)} \times (300 - 455) = -3216 \text{ J}$$

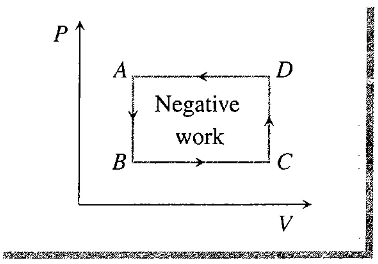
$$\Delta U = C_v \Delta T = \frac{R}{(\gamma - 1)}\Delta T = \frac{8.3}{\left(\frac{5}{3}\right)-1}(300 - 455) = -1930 \text{ J}$$

$$\Delta W = \Delta Q - \Delta U = -3216 - (-1930) = -1286 \text{ J}$$

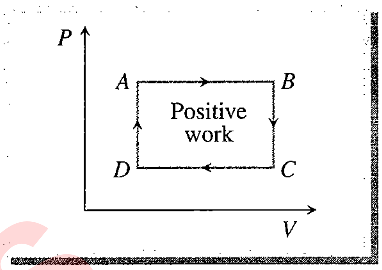
Cyclic and Non-Cyclic Processes

A cyclic process consists of a series of changes that return the system back to its initial state. In a non-cyclic process, the series of changes involved do not return the system back to its initial state.

1. In case of cyclic process as $U_f = U_i$. Therefore
$$\Delta U = U_f - U_i = 0$$
i.e., change in internal energy for cyclic process is zero and also $\Delta U \propto \Delta T$
$\therefore \Delta T = 0$, i.e., temperature of system remains constant.
2. From the first law of thermodynamics
$$\Delta Q = \Delta U + \Delta W$$
$\Delta Q = \Delta W$, i.e., heat supplied is equal to the work done by the system (as $\Delta U = 0$).
3. For cyclic process, P–V graph is a closed curve and area enclosed by the closed path represents the work done.



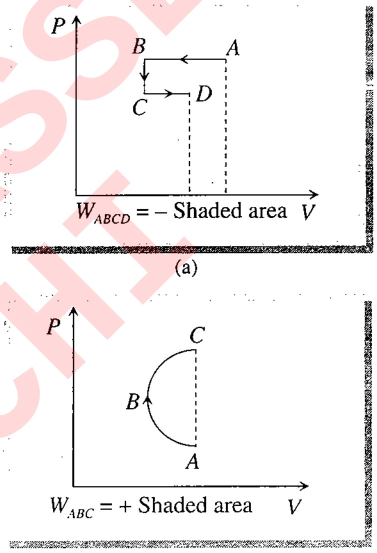
(a)



(b)

Fig. 2.48

If the cycle is clockwise work done is positive and if the cycle is anticlockwise work done is negative.



(b)

Fig. 2.49

4. Work done in non-cyclic process depends upon the path chosen or the series of changes involved and can be calculated by the area covered between the curve and volume axis on P–V diagram.
   Here, it is worth mentioning that the loop can be of any possible shape. Further, it can ever intersect.

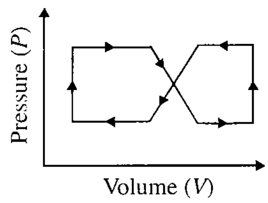


Fig. 2.50

The working substance in heat engines, meant to draw a continuous supply of work at the cost of heat, is subjected to cyclic process repeatedly.

Efficiency of a Cyclic Process

When a system is subjected to a cyclic process, heat is applied during some part of the process, while heat is abstracted during other part.

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Evidently, the net heat applied will be the work done by the system ($\because \Delta Q = \Delta W$). However, the gross heat applied will be more than that of the net heat.

Efficiency (η) of a cycle is defined as the ratio of the work performed (net heat given) to the gross heat supplied to the system per cycle. Thus,

$$\eta = \frac{\text{Work done per cycle}}{\text{Gross heat supplied per cycle}}$$

Graphical representation of various processes

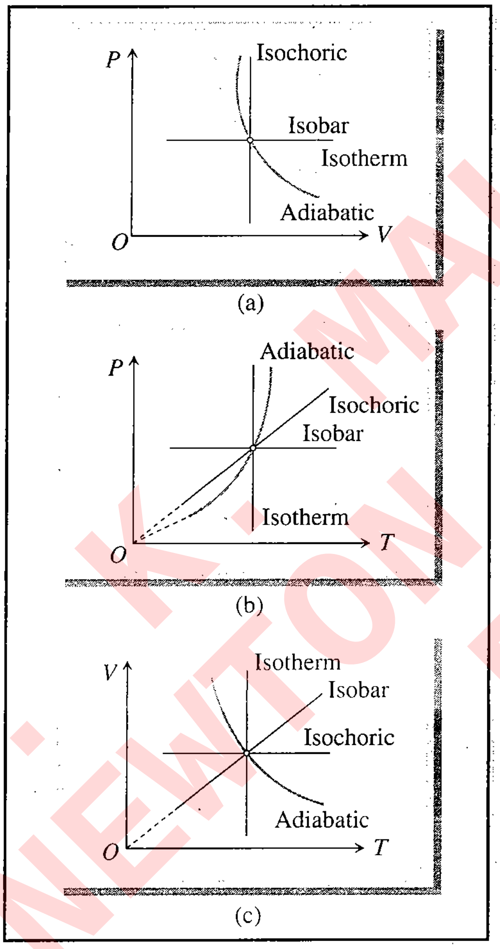

Fig. 2.51

Illustration 2.41 **Figure 2.52 shows the indicator diagram corresponding to n moles of an ideal gas taken along the cyclic process $ABCDA$. Find the efficiency of the cycle.**

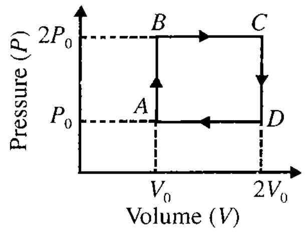

Fig. 2.52

Sol. Evidently, work done by the gas during the cyclic process is

$$W = (2P_0 - P_0)(2V_0 - V_0) \text{ unit} = P_0V_0 \text{ unit}$$

For the process A to B which is isochoric, the heat supplied will be given by

$$\Delta Q_1 = \Delta U + \Delta W = nC_V\Delta T + 0$$

$$= n\left(\frac{3R}{2}\right)\left[\frac{V_0(2P_0 - P_0)}{nR}\right] (\because nR\Delta T = V\Delta P)$$

$$= \frac{3}{2}P_0V_0 \text{ unit}$$

And for the process B to C which is isobaric, the heat absorbed will be given by

$$\Delta Q_2 = nC_P\Delta T$$

$$= n\left(\frac{5R}{2}\right)\left[\frac{2P_0(2V_0 - V_0)}{nR}\right] \quad (\because nR\Delta T = P(\Delta V))$$

$$= 5P_0V_0 \text{ unit}$$

For the processes C to D and D to A, the heat given will be obviously negative, which implies that heat is abstracted from the system. Therefore, efficiency (η) of the cycle

$$= \frac{\text{Work done per cycle}}{\text{Total heat given per cycle}}\%$$

$$= \frac{P_0V_0}{\left(\frac{3P_0V_0}{2}\right) + 5P_0V_0} = \frac{2}{13} \times 100 = \frac{200}{13}\% = 15.38\%$$

Illustration 2.42 **Three moles of an ideal gas ($C_p = 7R/2$) at pressure P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at a constant pressure to its original volume. Finally, the gas is compressed at constant volume to the original pressure P_A.**

a. Sketch P–V and P–T diagrams for the complete process.

b. Calculate the net work done by the gas and net heat supplied to the gas during the complete process.

Sol.

a.

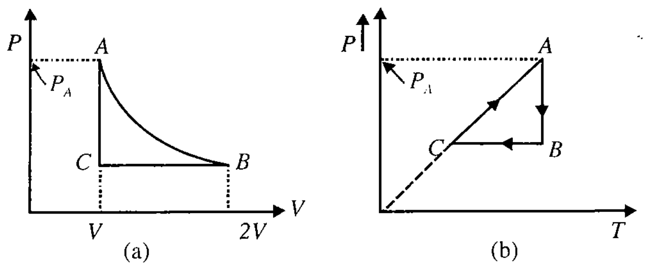

Fig. 2.53

b. Process $A \to B$: $(P_A, V_A, T_A) \to (P_B, 2V_A = V_B, T_B = T_A)$

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$$\therefore \frac{P_A V_A}{T_A} = \frac{P_B 2V_A}{T_A} \quad \Rightarrow P_B = P_A/2$$

$$\Delta U = 0 \quad (\because \Delta T = 0) \quad \text{and} \quad \Delta W = \int_{V_A}^{V_B} pdV = RT_A \ln 2$$

$$\therefore \Delta Q = \Delta U + \Delta W = \Delta W = RT_A \ln 2$$

Process $B \to C$: $(P_A/2, T_A, 2V_A) \to (P_A/2, T_C, V_A)$

$$\therefore \quad \frac{(P_A/2)2V_A}{T_A} = \frac{(P_A/2)V_A}{T_C} \quad \Rightarrow T_C = T_A/2$$

$$\therefore \quad \Delta W = \int_{2V_A}^{V_A} pdV = \frac{1}{2} p_A (V_A - 2V_A)$$

$$= -\frac{1}{2} p_A V_A = -\frac{1}{2} RT_A$$

$$\therefore \quad \Delta Q = \Delta U + \Delta W = -\frac{5}{4} RT_A - \frac{1}{2} RT_A = -\frac{7}{4} RT_A$$

Process $C \to A$: $\left(\frac{1}{2} p_A, \frac{1}{2} T_A, V_A\right) \to (p_A, T, V_A)$

$$\frac{\frac{1}{2} p_A \times V_A}{\frac{1}{2} T_A} = \frac{p_A V_A}{T} \quad \Rightarrow T = T_A$$

$$\Delta U = C_V \left(T_A - \frac{1}{2} T_A\right) = \frac{5}{2} R(T_A/2) = \frac{5}{4} RT_A$$

$$\Delta W = 0 \quad \therefore \Delta Q = \Delta U + \Delta W = \left(\frac{5}{4}\right) RT_A$$

$\therefore$ Net work done by the gas is

$$3\left[RT_A \ln 2 - \frac{1}{2} RT_A + 0\right] = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

Net heat absorbed is

$$3\left[RT_A \ln 2 - \frac{7}{4} RT_A + \frac{5}{4} RT_A\right] = 3RT_A \left(\ln 2 - \frac{1}{2}\right)$$

Illustration 2.43 **An ideal gas is taken round a cyclic thermodynamic process $ABCA$ as shown in Fig. 2.54. If the internal energy of the gas at point A is assumed zero while at B it is 50 J. The heat absorbed by the gas in the process BC is 90 J.**

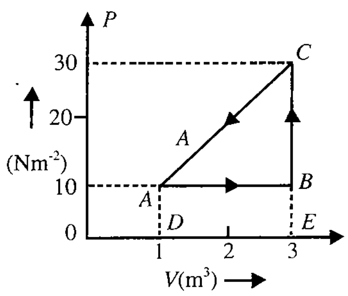

Fig. 2. 54

a. What is the internal energy of the gas at point C?

b. How much heat energy is absorbed by the gas in the process AB?

c. Find the heat energy rejected or absorbed by the gas in the process CA.

d. What is the net work done by the gas in the complete cycle $ABCA$?

Sol. Given that $U_A = 0$, $U_B = 50$ J and $Q_{BC} = 90$ J.

Also $\quad P_A = P_B = 10\ \text{Nm}^{-2}$, $P_C = 300\ \text{Nm}^{-2}$, $V_A = 1\ \text{m}^3$
and $\quad V_B = V_C = 3\ \text{m}^3$

a. In process BC as volume of gas remains constant, work done by gas in this process is zero, thus

$$W_{BC} = 0$$

Heat absorbed by the gas is $Q_{BC} = 90$ J. From the first law of thermodynamics.

$$(\Delta U)B_C = U_C - U_B = Q_{BC} - W_{BC} = 90\ \text{J} - 0 = 90\ \text{J}$$
$$U_C = (\Delta U)_{BC} + U_B = 90\ \text{J} + 50\ \text{J} = 140\ \text{J}$$

b. In process AB, we have

$$(\Delta U)_{AB} = U_B - U_A$$
$$= 50 - 0 = 50\ \text{J}$$

Work done is given as

$$W_{AB} = \text{area under } AB \text{ in } P\text{–}V \text{ diagram}$$
$$= \text{area of rectangle } ABED$$
$$= AB \times AD = (3\ \text{m}^3 - 1\ \text{m}^3) \times 10\ \text{Nm}^{-2}$$
$$= 20\ \text{J}$$

Thus, heat absorbed by the system is

$$Q_{AB} = (\Delta U)_{AB} + W_{AB} = 50 + 20 = 70\ \text{J}$$

c. For process CA

$$(\Delta U)_{CA} = U_A - U_C = 0 - 140 = -140\ \text{J}$$

Work done is given as

$$W_{CA} = \text{area } ACED$$
$$= \text{area of triangle } ACB + \text{area of rectangle } ABED$$
$$= 1/2 \times AB \times BC + AB \times AD$$
$$= 1/2 \times (3 - 1)\ \text{m}^3 \times (30 - 10)\text{Nm}^{-2} + 20$$
$$= 20 + 20 = 40\ \text{J}$$

In this process, the volume decreases, the work is done on the gas. Hence, the work done is negative,
Thus,

$$W_{CA} = -40\ \text{J}.$$

Thus heat rejected by the gas is

$$Q_{CA} = (\Delta U)_{CA} + W_{CA} = -140 - 40 = -180\ \text{J}$$

d. Net work done in the complete cyclic process $ABCA$ is

$$W = \text{area of triangle } ABC = \frac{1}{2} \times 2 \times 20 = 20\ \text{J}$$

As the cycle is anticlockwise, net work is done on the gas.

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Illustration 2.44 A fixed mass of oxygen gas performs a cyclic process $ABCA$ as shown. Find the efficiency of the process.

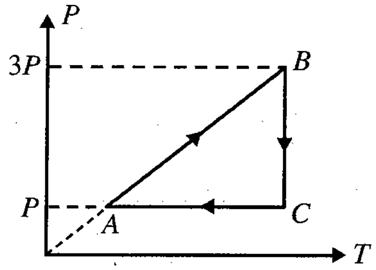

Fig. 2. 55

Sol. $A \to B$ isochoric

$$W_{AB} = 0$$

$$Q_{AB} = nC_V(3T_A - T_A)$$

$$= \left(\frac{5}{2}\right) nR2T_A = 5nRT_A$$

$$W_{BC} = nR3T_A \ln\left(\frac{P_B}{P_C}\right) = 3nRT_A \ln 3$$

$$= 3nR\,T_A \ln 3 = Q_{BC}$$

$$W_{CA} = P(V_A - V_C) = nR\,(T_A - 3T_A) = -2n\,RT_A$$

Efficiency of a cyclic process is given by η

$$= \frac{\text{Work done in cyclic process}}{\text{Heat supplied to the system}} = \frac{W}{Q_{in}}$$

$$\text{Efficiency } \eta = \frac{3nRT_A \ln 3 - 2nRT_A}{5nRT_A + 3nRT_A \ln 3}\ \%$$

$$= \frac{3\ln 3 - 2}{5 + 3\ln 3}\ \%$$

Concept Application Exercise 2.2

1. Explain why the temperature of a gas drops in an adiabatic expansion, on the basis of the kinetic theory of gases.
2. Explain why a gas cools on sudden expansion on the basis of the principles of thermodynamics.
3. A mass M = 15 g of nitrogen is enclosed in a vessel at temperature T = 300 K. What amount of heat has to be transferred to the gas to increase the root-mean-square velocity of molecules 2 times?
4. A gas consisting of rigid diatomic molecules (degrees of freedom $r = 5$) under standard conditions ($p_0 = 10^5$ Pa and T_0 = 273 K) was compressed adiabatically η = 5 times. Find the mean kinetic energy of a rotating molecule in the final state.
5. Can one distinguish between the internal energy of a body acquired by heat transfer and that acquired by the performance of work on it by an eternal agent?
6. A thermos flask contains coffee. It is vigorously shaken, considering the coffee as the system. (a) Does its temperature rise? (b) Has heat been added to it? (c) Has work been done on it?
7. Does a gas do any work when it expands adiabatically? If so, what is the source of the energy needed to do this work?
8. A block returns to its initial position after dissipating mechanical energy in the form of heat through friction. Is this process reversible or irreversible?
9. Explain why the temperature of a gas drops in an adiabatic expansion.
10. In what process is the heat added entirely converted into internal energy of the system?
11. When a gas is compressed adiabatically, it becomes more elastic. Is this true?
12. A cylinder contains 3 moles of oxygen at a temperature of 27°C. The cylinder is provided with a frictionless piston which maintains a constant pressure of 1 atm on the gas. The gas is heated until its temperature rises to 127°C.
    a. How much work is done by the gas in the process?
    b. What is the change in the internal energy of the gas?
    c. How much heat was supplied to the gas?
    For oxygen C_p = 7.03 cal mol^{-1}°C^{-1}.
13. What amount of heat is to be transferred to nitrogen in an isobaric heating process so that the gas may perform 2 J work?
14. As a result of the Isobaric heating by ΔT = 72 K, one mole of a certain ideal gas receives heat Q = 1.6 kJ. Find the work performed by the gas, the increment of its internal energy, and the value of its adiabatic exponent γ.
15. A certain mass of a gas is compressed first adiabatically, and then isothermally. In both cases, the initial state of the gas is the same. Is the work done W_1 in the first case greater than the work done W_2 in the second case? Explain.

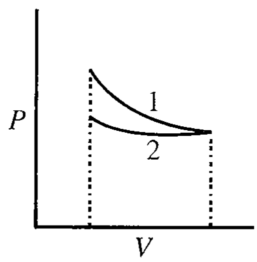

Fig. 2. 56

16. A cylindrical vessel of 28 cm diameter contains 20 g of nitrogen compressed by a piston supporting a weight of 75 kg. The temperature of the gas is 17°C. What work will the gas do if it is heated to a temperature of 250°C? What amount of heat should be supplied? To what distance will the weight be raised? The process should be assumed to be isobaric; the heating of the vessel as well as the external pressure is negligible.

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17. One mole of oxygen is heated at constant pressure starting at 0°C. How much heat energy must be added to the gas to double its volume?

18. An ideal gas expands from an initial temperature T_1 to a final temperature T_2. Prove that the work done by the gas is C_v ($T_1 - T_2$).

19. An ideal gas whose adiabatic exponent equals γ expands so that the amount of heat transferred to it is equal to the decrease of its internal energy. Find
    i. the molar heat capacity of the gas, and
    ii. the T–V equation for the process.

20. One mole of argon expands polytropically, the polytropic constant being 1.5, that is, the process proceeds according to the law $pV^{1.5}$ = constant. In the process, its temperature changes by $\Delta T = -26$ K. Find
    a. the amount of heat obtained by the gas,
    b. the work performed by the gas.

Solved Examples

Example 2.1 **A vessel containing 1 g of oxygen at a pressure of 10 atm and a temperature of 47°C. It is found that because of a leak, the pressure drops to 5/8th of its original value and the temperature falls to 27°C. Find the volume of the vessel and the mass of oxygen that is leaked out.**

Sol. The pressure, temperature and the number of moles of oxygen in the vessel change due to leak, while the volume remains fixed. Hence using $PV = nRT$, we have

$$\left(\frac{P_1}{n_1T_1}\right)=\left(\frac{P_2}{n_2T_2}\right) \Rightarrow n_2=\frac{320}{300}\times\frac{5}{8}\times\frac{1}{32}=\frac{1}{48}\text{ moles}$$

$$\text{Mass leaking out} = 1 - n_2(32) = 1-(2/3) = \frac{1}{3}\text{ g}$$

$$\text{Volume of vessel} = V = \frac{n_1RT_1}{P_1}$$

$$\Rightarrow V = \frac{1\times 0.082\times 320}{32\times 10} = 0.082\text{ L}$$

Example 2.2 **An ideal gas (2.0 moles) is carried round a cycle as shown. If the process $b \to c$ is isothermal and C_v = 3 cal/mol/K. Determine**
   a. work done,
   b. change in internal energy,
   c. heat supplied to the system during processes $a \to b$, $b \to c$ and $c \to a$.

Sol.

a. Work done: W
   Process $a \to b$ is clearly isobaric at P = 4 atm.
   $W = P(V_b - V_a) = 4(16.4 - 8.2) = 32.8$ L atm

$$\Rightarrow W_{a-b} = 32.8 - 101 = 3313\text{ J}$$

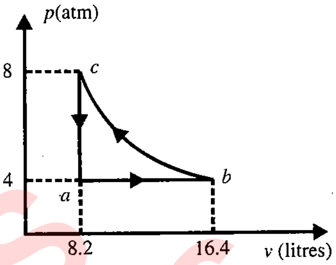

Fig. 2.57

Process $b \to c$ is isothermal (given)

$$W = 2.303\, nRT \log(V_c/V_b)$$
$$\Rightarrow W = 2.303(P_b V_b)\log(V_c/V_b)$$
$$W = 2.303(4\times 16.4\times 101)\log(8.2/16.4)$$
$$W_{b\to c} = -4592.5\text{ J}$$

Process $c \to a$ is isochoric(volume is constant)

$$W_{c\to a} = 0\text{ J}$$

b. Change in internal energy ΔU
   Process $c \to a$: $\Delta U = nC_v\Delta T$

$$\Delta U = nC_v\left(\frac{P_bV_b}{nR}-\frac{P_aV_a}{nR}\right) \qquad (\text{since } pV = nRT)$$

$$=\frac{C_v}{R}(P_0V_0 - P_aV_a) = \frac{3}{0.0821}(4\times 16.4 - 4\times 8.2)$$

$$\Rightarrow \Delta U = 1200\text{ cal}$$

Process $b \to c$: isothermal, $\Delta T = 0$

$$\Rightarrow \Delta U = 0$$

Process $c \to a$: ΔU for complete cyclic process is zero because it depends on initial and final states, i.e., temperature difference only.

$$(\Delta U)_{ab} + (\Delta U)_{bc} + (\Delta U)_{ca} = 0$$
$$\Rightarrow 1200 + 0 + (\Delta U)_{ca} = 0$$
$$\Rightarrow (\Delta U)_{ca} = -1200\text{ cal}$$

c. Heat supplied: ΔH
   Using the first law of thermodynamics:

$$(\Delta H)_{ab} = (\Delta U)_{ab} + W_{ab}$$
$$= (1200 + 3313/4.18)\text{ cal} = 1992.5\text{ cal}$$
$$\Rightarrow (\Delta Q)_{ab} = 1992.5\text{ cal}$$
$$\Rightarrow (\Delta H)_{bc} = (\Delta U)_{bc} + W_{bc}$$
$$= (0 - 4592.5/4.18)\text{ cal} = 1098.7\text{ cal}$$

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$$\Rightarrow (\Delta H)_{bc} = 1098.7 \text{ cal}$$

$$(\Delta H)_{ca} = (\Delta U)_{ca} + W_{ca} = -1200 + 0 = -1200 \text{ cal}$$

$$\Rightarrow \Delta H = 1200 \text{ cal}$$

Example 2.3 **Figure 2.58 shows three processes for an ideal gas. The temperature at '*a*' is 600 K, pressure 16 atm, and volume 1 L. The volume at '*b*' is 4 L. Out of two processes, *ab* and *ac*, one is adiabatic and other is isothermal. The ratio of specific heats of the gas is 1.5.**

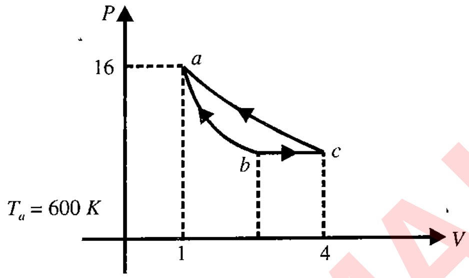

Fig. 2.58

a. **Which of the processes *ab* and *ac* is adiabatic? Why?**
b. **Compute the pressure of the gas at *b* and *c*.**
c. **Compute the temperature at *b* and *c*.**
d. **Compute the volume at *c*.**

Sol.

a. The process *ab* is adiabatic because it is steeper than *ac*.

b. $P_a = 16$ atm, $V_b = 4$ L, $V_a = 1$ L

$$\Rightarrow T_a = 600 \text{ K}$$

Process '*ab*' (adiabatic)

$$\Rightarrow P_a V_a^{\gamma} = P_b V_b^{\gamma}$$

$$P_b = P_a \left(\frac{V_a}{V_b}\right)^{\gamma} = 16 \times \left(\frac{1}{4}\right)^{1.5}$$

$$\Rightarrow P_b = 2 \text{ atm}$$

As *bc* is isobaric,

$$P_b = P_c = 2 \text{ atm}$$

c. $T_c = T_a = 600$ K because *ac* is isothermal.

Process '*ab*' is adiabatic hence

T_b can be calculated using $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$

$$\Rightarrow \left(\frac{P_a V_a}{T_a}\right) = \left(\frac{P_b V_b}{T_b}\right)$$

Hence, $$T_b = T_a \left(\frac{V_a}{V_b}\right)^{\gamma-1} = 600\left(\frac{1}{4}\right)^{0.5} = 300 \text{ K}$$

d. Process '*ac*' (isothermal)

$$\Rightarrow P_a V_a = P_c V_c$$

$$V_c = \frac{P_a V_a}{P_c} = \frac{(16 \times 1)}{2} = 8 \text{ L}$$

Example 2.4 ***N* molecules each of mass *m* of gas *A* and 2*N* molecules each of mass 2*m* of gas *B* are contained in the same vessel which is maintained at a temperature *T*. The mean square of the velocity of the molecules of *B* type is denoted by v^2 and the mean square of the *x*-component of the velocity of *A* type is denoted by ω^2. What is the ratio of ω^2/v^2 = ?**

Sol. The mean square velocity of gas molecule is given by

$$v^2 = \frac{3kT}{m}$$

For gas *A*,

$$v_A^2 = \frac{3kT}{m} \tag{i}$$

For a gas molecule, $v^2 = v_x^2 + v_y^2 + v_z^2$

$$= 3v_x^2 \; (v_x^2 = v_y^2 = v_z^2)$$

or $$v_x^2 = \frac{v^2}{3}$$

From Eq. (i), we get

$$v_x^2 = \frac{v_A^2}{3} = \left[\frac{(3kT/m)}{3}\right] = \frac{kT}{m} \tag{ii}$$

For gas *b*,

$$v_B^2 = v^2 = \frac{3kT}{m_B} = \frac{3kT}{m} \tag{iii}$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\omega^2}{v^2} = \frac{kT/m}{3kT/2m} = \frac{2}{3}$$

Example 2.5 **Two moles of an ideal monatomic gas is taken through a cycle *ABCA* as shown in the *P*–*T* diagram. During the process *AB*, pressure and temperature of the gas vary such that *PT* = constant. If $T_1 = 300$ K, calculate**

a. **the work done of the gas in the process *AB*, and**
b. **heat absorbed or released by the gas in each of the process. Give answers in terms of the gas constant *R*.**

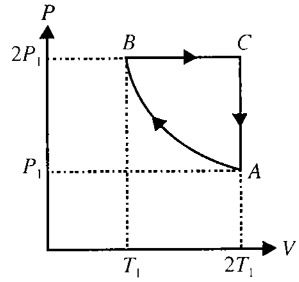

Fig. 2.59

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Sol. For the process A–B, it is given that

$$PT = \text{constant}$$

Differentiating above equation partially, we have

$$PdT + TdP = 0 \quad \text{(i)}$$

Equation of state for two moles of a gas

$$PV = 2RT \quad \text{or} \quad P = \frac{2RT}{V} \quad \text{(ii)}$$

After differentiating Eq. (ii) partially, we get

$$PdV + VdP = 2R\,dT$$

From Eq. (ii) partially, we get

$$PdV + VdP = 2R\,dT \quad \text{(iii)}$$

From Eqs. (i) and (ii), we have

$$\left(\frac{2RT}{V}\right)dT + T\,dP = 0$$

or $$2RTdT + VTdP = 0$$

$$VdP = -2\,RdT \quad \text{(iv)}$$

Now from Eqs. (iii) and (iv), we have

$$-2RdT + VdP = 2RdT$$

or $$PdV = 4\,RdT \quad \text{(v)}$$

a. The work done in the process AB

$$W_{AB}\int PdV = \int_{600}^{300} 4R\,dT$$

$$= 4R|T|_{600}^{300} = 4R(300-600)$$

$$= -1200\,R$$

b. i. As process $B \to C$ is isobaric, so

$$Q_{BC} = nC_P\Delta T = 2\times\frac{5R}{2}\times(600-300)$$

$$= 1500\,R$$

ii. Process $C \to A$ is isothermal, so $\Delta U = 0$

$$Q_{CA} = \Delta U + W_{CA} = W_{CA}$$

$$W_{CA} = nRT\ln\left(P_C/P_A\right)$$

$$= 2R\times 600\ \ln\left(2P_1/P_1\right) = 1200\,R\ln 2$$

$$Q_{CA} = 1200\,R\ln 2$$

Again for process $A \to B$

$$Q_{AB} = \Delta U + W_{AB}$$

$$= nC_V\Delta T + W_{AB}$$

$$= 2\times\left(\frac{3R}{2}\right)\times(300-600)-1200R$$

$$= -900\,R - 1200\,R = -2100R$$

Example 2.6 **A smooth vertical tube having two different cross sections is open from both the ends but closed by two sliding pistons as shown in Fig. 2.60 and tied with an inextensible string. One mole of an ideal gas is enclosed between the piston. The difference in cross-sectional areas of the two pistons is given ΔS. The masses of piston are m_1 and m_2 for larger and smaller one, respectively. Find the temperature by which tube is raised so that the pistons will be displaced by a distance l. Take atmospheric pressure equal to P_0.**

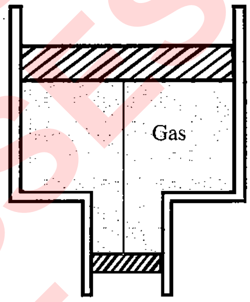

Fig. 2.60

Sol. If initial pressure of gas is P and let S_1 and S_2 be the cross-sectional area of the larger and the smaller piston, respectively, then for equilibrium of the two pistons, we have

For larger piston

$$P_0S_1 + m_1g + T = PS_1 \quad \text{(if } T \text{ is the tension in string)} \quad \text{(i)}$$

For smaller piston

$$PS_2 + m_2g = T + P_0S_2 \quad \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$P_0(S_1 - S_2) + m_1g + m_2g = P(S_1 - S_2) + m_1g + m_2g = P(S_1 - S_2)$$

or $$P_0 + \left(\frac{m_1+m_2}{\Delta S}\right)g = P \quad \text{(iii)}$$

If gas temperature is increased from T_1 to T_2 the volume of gas increase from V to $V + l\Delta S$ as l is the displacement of pistons, then from gas law we must have

$$PV = RT_1 \quad \text{(for initial state)} \quad \text{(iv)}$$

$$P(V + l\,\Delta S) = RT_2 \quad \text{(for final state)} \quad \text{(v)}$$

According to Eq. (iii), we find that pressure of the gas does not change, as it does not depend on temperature.

From Eqs. (iv) and (v), if we subtract these equation, we get

$$Pl\Delta S = R(T_2 - T_1)$$

or $$T_2 - T_1 = \frac{Pl\Delta S}{R}$$

$$= \left(P_0 + \frac{(m_1+m_2)}{\Delta S}g\right)\frac{l\Delta S}{R}$$

$$= [P_0\Delta S + (m_1+m_2)g]\frac{1}{R}$$

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Example 2.7 **Two moles of helium gas ($\gamma = 5/3$) is initially at temperature 27°C and occupies a volume of 20 L. The gas is first expanded at constant pressure until the volume is doubled. Then, it undergoes an adiabatic change until the temperature returns to its initial value.**

i. Sketch the process on a P–V diagram.

ii. What are the final volume and pressure of the gas?

iii. What is the work done by the gas?

(IIT-JEE, 1988)

Sol. For a perfect gas

$$PV = nRT$$

Given that $V = 20 \text{ L} = 20 \times 10^{-3} \text{ m}^3$

$T = 27°\text{C} = 300$ K and number of molecules $n = 2$

Thus, initial pressure is given as

$$P = \left(\frac{nRT}{V}\right) = \frac{(2 \times 8.3 \times 300)}{(20 \times 10^{-3})} = 2.5 \times 10^5 \text{ N/m}^2$$

i. Figure 2.61 shows the indicator diagram of the complete process.

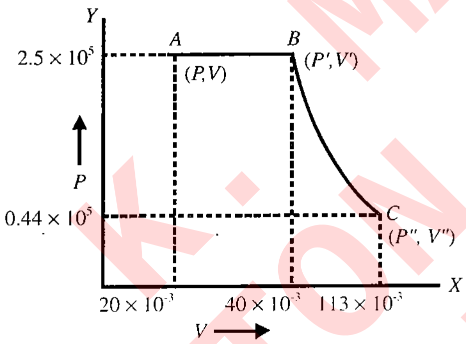

Fig. 2.61

ii. At point B,

Pressure $P' = P = 2.5 \times 10^5$ N/m^2, and

$V' = 2V = 40 \times 10^{-3}$ m^3

As pressure is constant in the process AB, making its volume doubled, its temperature will also be doubled.

Thus, temperature at point B is $T' = 600$ K.

The gas now undergoes adiabatic expansions to cool down at $T'' = T = 300$ K

We know for an adiabatic process $TV^{\gamma-1}$ = constant

$$T'(V')^{\gamma-1} = T''(V'')^{\gamma-1}$$

$$\left(\frac{V''}{V'}\right) = \left(\frac{T'}{T''}\right)^{1/(\gamma-1)} = \left(\frac{600}{300}\right)^{\frac{1}{2.5-1}} = (2)^{3/2} = 2\sqrt{2}$$

Thus, final volume is

$$V'' = (2\sqrt{2})V'$$

$2 \times 1.414 \times 40 \times 10^{-3} = 113.14 \times 10^{-3}$ m^3

Similarly, final pressure is given by process equation as

$$P'V'^{\gamma} = P''V''^{\gamma}$$

or $$P'' = P'\left(\frac{V'}{V''}\right)^{\gamma}$$

$$= 2.5 \times 10^5 \times \left(\frac{40 \times 10^{-3}}{113.14 \times 10^{-3}}\right)^{5/3}$$

$$= 4.42 \times 10^4 \text{ Pa}$$

iii. Work done under isobaric process AB is

or $W_1 = P\Delta V$

or $W_1 = 2.5 \times 10^5 \times (40 - 20) \times 10^{-3}$

or $= 4980$ J

Work done during adiabatic process BC is given as

or $$W_2 = \left(\frac{nR}{\gamma - 1}\right)[T_1 - T_2]$$

or $$= \frac{(2 \times 8.3)}{[1-(5/3)]}[300 - 600] = 7470 \text{ J}$$

total work done $= W_1 + W_2 = 4980 + 7470 = 12450$ J

Example 2.8 **An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross section 8×10^{-3} m^2. Initially, the gas is at 300 K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed (unstretched, uncompressed) state (see Fig. 2.62). The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joule) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure 1×10^5 N/m^2. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heart. The heat capacity of the heater coil is negligible. Assume the spring to be massless.** **(IIT-JEE, 1989)**

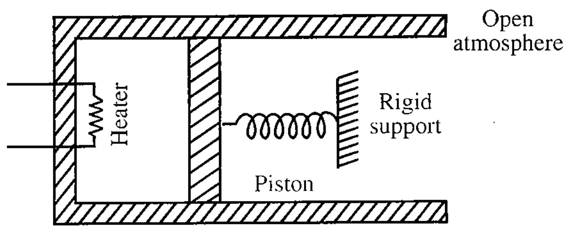

Fig. 2.62

Sol. Initially, the pressure of the gas in the cylinder is atmospheric pressure as spring is in relaxed state. Therefore,

P_1 = atmospheric pressure = 1.0×10^5 N/m^2

V_1 = initial volume = 2.4×10^3 m^3

T_1 = initial temperature = 300 K

When the heat is supplied by the heater, the piston is compressed by 0.1 m. The reaction force of compression of spring is equal to kx which acts on the piston or on the gas as

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$$F = kx = 8000 \times 0.1 \times 800\ \text{N}$$

Pressure exerted on the piston by the spring is

$$\Delta P = \frac{F}{A} = \frac{800}{8\times10^{-3}} = 1\times10^{5}\ \text{N/m}^2$$

The total pressure P_2 of the gas inside cylinder is

$$P_2 = P_{\text{atm}} + \Delta P = 1 \times 10^5 + 1 \times 10^5 = 2 \times 10^5\ \text{N/m}^2$$

Since the piston has moved outwards, there has been an increase of ΔV in the volume of the gas, i.e.,

$$\Delta V = A \times x = (8 \times 10^{-3}) \times (0.1)$$
$$= 8 \times 10^{-4}\ \text{m}^3$$

The final volume of the gas

$$V_2 = V_1 + \Delta V = 2.4 \times 10^{-3} + 8 \times 10^{-4} = 3.2 \times 10^{-3}\ \text{m}^3$$

Let T_2 be the final temperature of gas. Then

$$(P_1V_1/T_1) = (P_2V_2/T_2) \Rightarrow T_2 = (P_2V_2/P_1V_1)T_1$$
$$= 300\times(2\times10^5\times3.2\times10^{-3})/(10^5\times2.4\times10^{-3})$$
$$= 800\ \text{K}$$

Let the heat supplied by the heater be Q. This is used in two parts, i.e., a part is used in doing external work W due to expansion of the gas and the other part is used in increasing h internal energy of the gas. Hence,

$$Q = W + \Delta U$$

Now $$W = \int_{V_1}^{V_2} P dV = \int_0^x \left(P_{\text{atm}} + \frac{kx}{A}\right) A dx$$

$$\left[\text{as pressure is } (P_{\text{atm}} + kx/A \text{ and } dV = A dx)\right]$$

or $$W = P_{\text{atm}} Ax + \left(\frac{kx^2}{2}\right)$$

$$= \left[10^5\times(8\times10^{-3})(0.1) + \frac{8000\times(0.1)^2}{2}\right] = 120\ \text{J}$$

Further, $\Delta U = nC_v\Delta T$

Number of moles of gas can be obtained from initial conditions and gas law as

$$n = \left(\frac{PV}{RT}\right) = \left(\frac{1\times10^5\times2.4\times10}{8.314\times300}\right) = 0.096\ \text{mol}$$

Thus, change in internal energy of gas is given as

$$\Delta U = n\left(\frac{3}{2}R\right)\Delta T \qquad \left(\text{as for monatomic gas } Cv = \frac{3}{2}R\right)$$

or $$U = 0.096 \times \left(\frac{3}{2}\right) \times 8.314 \times 500 = 598.6\ \text{J}.$$

Heat supplied by the heater $= (120 + 598.6) = 718.6$ J.

Example 2.9 **Three moles of an ideal gas $(C_P = 7/2R)$ at pressure P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally, the gas is compressed at constant volume to its original pressure P_A.**

a. **Sketch P–V and P–T diagrams for the complete process.**

b. **Calculate the new work done by the gas and net heat supplied to the gas during the complete process.**

(IIT–JEE 1991)

Sol.

a. Diagrams of P–V and P–I are shown in Figs. 2.63(a) and (b)

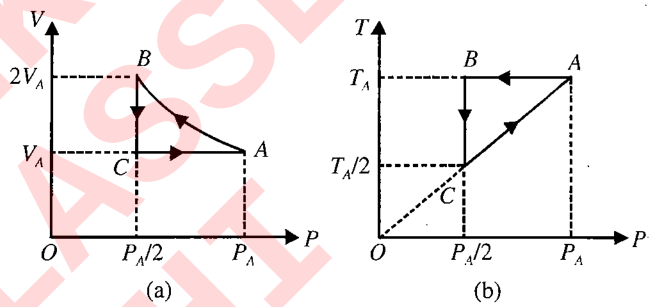

Fig. 2.63

The initial state of isothermal expansion is represented by A where pressure is P_A and volume V_A. Let the final state be represented by B where volume V_B is twice of V_A. Let the pressure be P_B. Then

$$P_AV_A = P_BV_B = P_B(2V_A)$$

or $$P_B = P_A(V_A/2V_A) = P_A/2$$

When the molecule is compressed to initial volume, the process is represented by BC. Finally, the gas is compressed at constant volume to its original pressure. The process is shown by curve CA.

Similarly, P–T diagram can be drawn.

b. Work done in the process AB is given by

$$W_1 = nRT\ln(V_B/V_A)$$
$$= 3 \times 8.314 \times T_A \times \ln 2$$
$$= 3 \times 8.314 \times T_A \times 0.693 = 17.29\ T_A$$

Work done in the process BC is given by

$$W_2 = P\Delta V = P_B \times (V_C - V_B)$$
$$= \left(\frac{P_A}{2}\right)\times(V_A - 2V_A)$$
$$= -P_AV_A/2 = -nRT_A/2$$
$$= -3RT_A/2 = -3 \times 8.314\ T_A/2 = -12.471\ T_A$$

Work done during process C_A is given by

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$$W_3 = P\Delta V = 0 \qquad (\text{as } \Delta V = 0)$$

Net work $\quad W = W_1 + W_2 + W_3$

$$= 17.26T_A - 12.45\,T_A = 4.81\,T_A$$

As initial and final states of the gas are same

$$\Delta U = U_A - U_A = 0$$

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = 0 + \Delta W$$

$$Q = W \text{ joules}$$

Example 2.10 **Two moles of helium gas undergoes a cyclic process as shown in Fig. 2.64. Assuming the gas to be ideal, calculate the following quantities in this process. (IIT–JEE, 1992)**

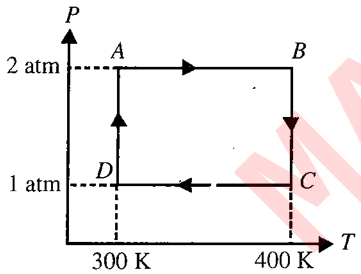

Fig. 2.64

a. The net change in the heat energy.
b. The net work done.
c. The net change in internal energy.

Sol. As we know in a cyclic process, the change in heat energy or heat supplied to the gas is equal to the net work done by the gas.

Here, AB is isobaric process. Hence, work done during this process from A to B is

$$W_{AB} = P(V_2 - V_1) = nR(T_2 - T_1)$$

or $\quad W_{AB} = 2 \times 8.314 \times (400 - 300) = 1662.8 \text{ J}$

Work done during isothermal process from B to C is

$$W_{BC} = nRT_C \ln(V_2/V_1) = nRT_C \ln(P_1/P_2)$$

$$= 2 \times 8.314 \times 400 \times \ln(2) = 2 \times 8.314 \times 400 \times 0.693$$
$$= 4610.2 \text{ J}$$

Work done during isobaric process from C to D

$$W_{CD} = nR(T_D - T_C) = 2 \times 8.314 \times (300 - 400)$$
$$= -1662.8 \text{ J}$$

Work done during isothermal process from D to A

$$W_{DA} = nRT_D \ln(P_D/P_A)$$
$$= nRT_D \ln(2)$$
$$= -2 \times 8.314 \times 300 \times 0.693$$
$$= -3457.7 \text{ J}$$

Net work done

$$= W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= 1662.8 + 4610.2 - 1662.5 - 3457.7 = 1152.5 \text{ J}$$

Now from the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Here $\Delta U = 0$, thus we have

$$\Delta Q = \Delta W = 1152.5 \text{ J}$$

So the heat given to the system is 1152.5 J

As the gas returns to its original state, there is no change in internal energy.

Example 2.11 **One mole of monatomic ideal gas is taken through the cycle as shown in Fig. 2.65. (IIT–JEE, 1993)**

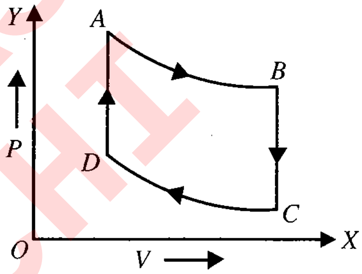

Fig. 2.65

$A \to B$ adiabatic, expansion
$B \to C$ cooling at constant volume
$C \to D$ adiabatic compression
$D \to A$ heating at constant volume

The pressure and temperature at A, B, etc., are denoted by P_A, P_B, . . . and, T_A, T_B . . . T, respectively. Given that $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$.

Calculate the following quantities:

i. The work done by the gas in process $A \to B$.
ii. The work done by the gas in process $B \to C$.
iii. The temperature T_D.

(Given that $(2/3)^{2/5} = 0.85$)

Sol.

i. The work done in adiabatic process AB is given by

$$W_1 \frac{R(T_A - T_B)}{\gamma - 1} = \frac{R(T_A - T_B)}{(5/3) - 1}$$

(as for monatomic gas $\gamma = 5/3$)

or $\quad = \frac{3}{2} R(T_A - T_B) \qquad \text{(i)}$

For adiabatic change, we have

$$P_A^{\gamma-1} T_A^{\gamma} = P_B^{\gamma-1} T_B^{-\gamma}$$

or $\quad \left(\frac{P_A}{P_B}\right) = \left(\frac{T_A}{T_B}\right)$ or $\left(\frac{T_A}{T_B}\right) = \left(\frac{P_A}{P_B}\right)^{2/5}$

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or $$\left(\frac{T_A}{T_B}\right)=\left(\frac{3}{2}\right)^{2/3} \qquad (\text{as } P_B = (2/3)P_A)$$

or $$T_B = T_A\left(\frac{2}{3}\right)^{2/3} \qquad \text{(ii)}$$

$$= 1000 \times 0.85 = 850 \text{ K}$$

From Eq. (i)

$$W_1 = \left(\frac{3}{2}\right) \times 8.31 \times (1000 - 850) = 1869.83 \text{ J}$$

ii. Heat lost by the gas in process BC is given by

$$C_V(T_B - T_C) = \left(\frac{R}{\gamma - 1}\right)(T_B - T_C) = \left(\frac{3}{2}\right)R(T_B - T_C) \qquad \text{(iii)}$$

Process BC is under constant volume; hence

$$\left(\frac{P_B}{T_B}\right)=\left(\frac{P_C}{T_C}\right) \quad \text{or} \quad T_C = \left(\frac{P_C}{P_B}\right)\times T_B$$

or $$T_C = \frac{(P_A/3)}{2P_A/3}T_B = \left(\frac{T_B}{2}\right) = 425 \text{ K} \qquad \text{(iv)}$$

From Eq. (ii), we get

$$\text{Heat lost} = \left(\frac{3}{2}\right) \times 8.31 \times (850 - 425) = 5297.63 \text{ J}$$

iii. For path AB

$$P_A^{\gamma-1}T_A^{-\gamma} = P_B^{\gamma-1}T_B^{-\gamma}$$

or $$\left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^{\gamma} \qquad \text{(v)}$$

For path BC

$$\left(\frac{P_B}{T_B}\right)=\left(\frac{P_C}{T_C}\right) \quad \text{or} \quad \left(\frac{P_B}{P_C}\right)=\left(\frac{T_B}{T_C}\right) \qquad \text{(vi)}$$

For path CD

$$\left(\frac{P_D}{T_B}\right)^{\gamma-1} = \left(\frac{T_D}{T_C}\right)^{\gamma} \qquad \text{(vii)}$$

For path AD

$$\left(\frac{P_A}{P_D}\right)=\left(\frac{T_A}{T_C}\right) \qquad \text{(viii)}$$

Dividing Eq. (v) by Eq. (vii), we get

$$\left(\frac{P_A}{P_B}\times\frac{P_C}{P_D}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\times\frac{T_C}{T_D}\right)^{\gamma} \qquad \text{(ix)}$$

Dividing Eq. (viii) by Eq. (iv), we get

$$\left(\frac{P_A}{P_B}\times\frac{P_C}{P_D}\right)=\left(\frac{T_A}{T_B}\times\frac{T_C}{T_D}\right) \quad \text{or} \quad \left(\frac{P_A}{P_B}\times\frac{P_C}{P_D}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\times\frac{T_C}{T_D}\right)^{\gamma-1} \qquad \text{(x)}$$

$$\left(\frac{T_A}{T_B}\times\frac{T_C}{T_D}\right)^{\gamma} = \left(\frac{T_A}{T_B}\times\frac{T_C}{T_D}\right)^{\gamma-1}$$

$$T_AT_C = T_DT_B$$

$$1000 \times 425 = T_D \times 850$$

$$T_D = 500 \text{ K}$$

Example 2.12 **A sample of 2 kg of monatomic helium (assumed ideal) is taken through the process *ABC* and another sample of 2 kg of the same gas is taken through the process *ADC* as shown in Fig. 2.66. Given molecular mass of helium = 4.**

a. What is the temperature of helium in each of the states *A*, *B*, *C* and *D*?

b. Is there any way of telling afterwards which sample of helium went through the process *ABC* and which went through the process *ADC*? Write yes or no.

c. How much is the heat involved in each of the processes *ABC* and *ADC*? (IIT–JEE, 1997)

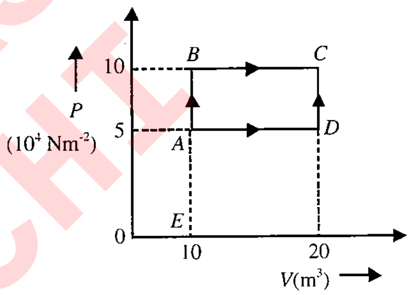

Fig. 2.66

Sol. Given that mass of helium used in the process is $m = 2$ kg, number of moles can be given as

$$n = \left(\frac{m}{M}\right) = \frac{2}{(2\times 10^{-3})} = 500$$

At different states, from Fig. 2.66, the pressure and volume of gas are also givens as

$$P_A = P_D = 5 \times 10^4 \text{ N/m}^2$$

$$P_B = P_C = 10^5 \text{ N/m}^2$$

$$V_A = V_B = 10 \text{ m}^3$$

$$V_C = V_D = 20 \text{ m}^3$$

a. From gas law, we have

$$T_A = \frac{(P_AV_A)}{(nR)} = \frac{(5\times 10^4 \times 10)}{(500\times 8.314)} = 120.3 \text{ K}$$

$$T_B = \frac{(P_BV_B)}{(nR)} = \frac{(10^5 \times 20)}{(500\times 8.314)} = 481.11 \text{ K}$$

$$T_D = \frac{(P_DV_D)}{(nR)} = \frac{(5\times 10^4 \times 20)}{(500\times 8.314)} = 120.3 \text{ K}$$

b. Since the gas is taken from same initial state to same final state C, no matters whatever be the path, the answer is no.

c. In process ABC, the change in internal energy is

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$$\Delta U_{ABC} = U_C - U_A = (f/2)nR(T_C - T_A)$$

$$= (3/2) \times 500 \times 8.314\,(481.11 - 120.3) = 2.25 \times 10^6 \text{ J}$$

Net work done in process ABC is

$$W_{ABC} = W_{AB} + W_{BC}$$
$$= 0 + \text{area below curve } BC$$
$$= 0 + 10^5 \times 10$$
$$= 10^6 \text{ J}$$

Thus, from the first law of thermodynamics, heat supplied in process ABC is

$$Q = W + \Delta U$$

or
$$Q = 10^6 + 2.25 \times 10^6$$
$$= 3.25 \times 10^6 \text{ J}$$

Similarly, in process ADC as being a state function, change in internal energy remains same as initial and final states are same. Thus,

$$\Delta U_{ADC} = 2.25 \times 10^6 \text{ J}$$

Thus, work done by the gas in process ADC is

$$W_{ADC} = W_{AD} + W_{DC}$$
$$= \text{area below curve } AD + 0$$
$$= 5 \times 10^4 \times 10$$
$$= 0.5 \times 10^6 \text{ J}$$

Thus, from the first law of thermodynamics, heat supplied in the process ADC is given as

$$Q = W + \Delta U$$

or
$$= 0.5 \times 10^6 + 2.25 \times 10^6$$
$$= 2.75 \times 10^6 \text{ J}$$

Example 2.13 **One mole of an ideal monatomic gas is taken round the cyclic process $ABCA$ as shown in Fig. 2.67. Calculate**

a. the work done by the gas,
b. the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB,
c. the net heat absorbed by the gas in the path BC,
d. the maximum temperature attained by the gas during the cycle. (IIT–JEE, 1998)

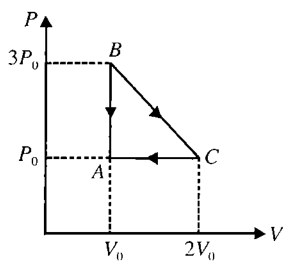

Fig. 2.67

Sol.

a. The work done by the gas is equal to the area under the closed curve. Thus, work done in cycle is

$$W = \frac{1}{2}(2V_0 - V_0)(3P_0 - P_0)$$

$$= \frac{1}{2}V_0 \times 2P_0 = P_0V_0$$

b. Heat rejected in path CA is given as

$$Q_{CA} = nC_P\,\Delta T = nC_P\,(T_C - T_A)$$

$$= 1 \times (5/2)\,R\left[\frac{P_0 2V_0}{R} - \frac{P_0V_0}{R}\right] = \frac{5}{2}P_0V_0$$

(as $n = 1$ mole)

Heat absorbed in path AC is

$$Q_{AC} = nC_P\,(T_B - T_A)$$

$$= 1 \times \frac{3}{2}R \times \left[\frac{3P_0V_0}{R} - \frac{P_0V_0}{R}\right] = 3P_0V_0$$

c. For cycle ABC, we have
heat supplied = work done by the gas

or
$$-\left(\frac{5}{2}\right)P_0V_0 + 3P_0V_0 + Q_{BC} = P_0V_0$$

Heat supplied in path BC is given by

$$Q_{BC} = P_0V_0 + \left(\frac{5}{2}\right)P_0V_0 - 3P_0V_0 = P_0V_0/2$$

d. We know that PV/T = constant. So, when PV is maximum, T is also maximum. PV is maximum for part BC. Hence, temperature will be maximum between B and C.
Let equation of BC be

$$P = kV + k'$$

satisfying both the point B and C
For point B,

$$3P_0 = kV_0 + k'$$

For point C,

$$P_0 = k(2V_0) + k'$$

Solving these equations, we get

$$k = -2(P_0/V_0) \text{ and } k' = 5P_0$$

So the equation for time BC is

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$$P = -2\left(\frac{P_0}{V_0}\right) \times V + 5P_0$$

or $$\left(\frac{RT}{V}\right) = -\left(\frac{2P_0V}{V_0}\right) + 5P_0$$

or $$T = \frac{P_0}{R}\left[5V - 2\left(\frac{V^2}{V_0}\right)\right] = 0 \qquad \text{(i)}$$

For maximum, $dT/dV = 0$

So $$\frac{dT}{dV} = \left(\frac{P_0}{R}\right)\left[5 - \left(\frac{4V}{V_0}\right)\right] = 0$$

Hence, $$5 - \left(\frac{4V}{V_0}\right) = 0 \quad \text{or} \quad 5V_0 - 4V = 0$$

or $$V = \left(\frac{5}{4}\right)V_0 \qquad \text{(ii)}$$

Substituting the value of V from Eq. (ii) in Eq. (i), we get

$$T_{\max} = \frac{P_0}{R}\left[5 \times \left(\frac{5}{4}V_0\right) - 2\left(\frac{5V_0}{4}\right)^2 \frac{1}{V_0}\right] = \frac{P_0}{R}\left[\frac{25V_0}{4} - \frac{25V_0}{8}\right]$$

$$= \frac{P_0}{R} \times \frac{25V_0}{8} = \frac{25P_0V_0}{8R}$$

Example 2.15 **Figure 2.68 shows three isothermals at temperature $T_1 = 4000$ K, $T_2 = 2000$ K and $T_3 = 1000$ K. When 1 mole of an ideal monatomic gas is taken through the paths AB, BC, CD and DA, find the change in internal energy ΔU, the work done by the gas W, and the heat Q absorbed by the gas in each path. Also find these quantities for complete cycle $ABCDA$.**

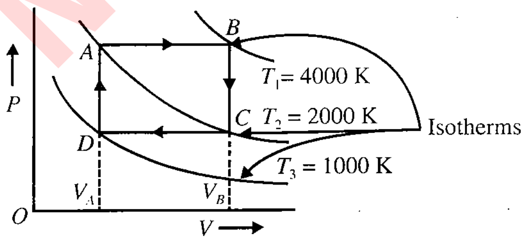

Fig. 2.68

Sol. When 1 mole of an ideal gas undergoes a change in temperature ΔT, the change in its internal energy equals $C_V\Delta T$, where C_V is the molar specific heat of the gas at constant volume. For monatomic gas $C_V = (3/2)R$. Thus, the change of internal energy is given by

$$\Delta U_{AB} = C_V(T_B - T_A)$$
$$= (3/2)\,R\,(4000 - 2000) = 3000\,R$$
$$= 3000 \times 8.31 = 24.93 \times 10^3 \text{ J}$$
$$\Delta U_{BC} = C_V(T_C - T_B)$$
$$= (3/2)\,R(2000 - 4000) = -3000\,R$$
$$= 24.93 \times 10^3 \text{ J}$$
$$\Delta U_{CD} = C_V(T_D - T_C)$$
$$= (3/2)R(1000 - 2000) = -1500\,R$$
$$= 12.465 \times 10^3 \text{ J}$$
$$\Delta U_{DA} = C_V(T_A - T_D)$$
$$= (3/2)R(2000 - 1000) = 1500R$$
$$= +\,12.465 \times 10^3 \text{ J}$$

Total $\Delta U = 0$

ii. Work done by gas during path AB under constant pressure is

$$P(V_2 - V_1) = R(T_1 - T_2) = 8.314 \times 2000$$
$$= 16.62 \times 10^3 \text{ J}$$

No work is done during BC and DA.

Work done on the gas during CD is

$$R(T_2 - T_1) = 8.314 \times 1000 = 8.314 \times 10^3 \text{ J}$$

Net work done by the gas during the cycle is

$$(16.62 \times 10^3 - 78.314 \times 10^3) = 8.314 \times 10^3 \text{ J}$$

iii. Heat absorbed during AB is $Cp\,(T_1 - T_2)$

$$= \frac{R\gamma}{\gamma - 1} \times 2000 = \frac{8.314 \times (5/3)}{(5/3) - 1} \times 200$$

(as $\gamma = 5/3$ for monatomic gas)

$$= 8.314 \times \frac{3}{2} \times 2000 = 24.93 \times 10^3 \text{ J}$$

Heat released during CD is

$$\frac{R\gamma}{\gamma - 1} \times 1000 = 8.314 \times \frac{5}{2} \times 1000 = 20.775 \times 10^3 \text{ J}$$

Heat absorbed during DA is

$$\frac{R}{\gamma - 1} \times 1000 = 8.314 \times \frac{3}{2} \times 10^3 \text{ J} = 12.465 \times 10^3 \text{ J}$$

Net heat absorbed by the gas during the cycle is

$$(12.465 + 41.55 - 20.775 - 24.93) \times 10^3 \text{ J} = 8.314 \times 10^3 \text{ J}$$

Example 2.16 **A massless piston divides a closed, thermally insulated cylinder into two equal parts. One part contains $M = 28$ g of nitrogen. At this temperature, one-third of molecules are dissociated into atoms and the other part is evacuated. The piston is released and the gas fills the whole volume of the cylinder at temperature T_0. Then, the piston is slowly displaced back to its initial position. Calculate the increase in internal energy of the gas. Neglect further dissociation of molecules during, the motion of the piston.**

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Sol. Molar mass of N_2 gas is $M = 28$ g

Number of moles of N_2, $n_0 = (m/M) = 1$

One-third molecules are dissociated into atoms.

No. of moles of diatomic $N_2 = 2/3$

No. of moles of monatomic nitrogen $= 2\times(1/3)$

For monatomic gas $C_{V_1} = (3/2)R$

For diatomic gas $C_{V_2} = (5/2)R$

Average molar specific heat at constant volume C_V is

$$\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2} = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)R+\left(\frac{2}{3}\right)\left(\frac{5}{2}\right)R}{\frac{2}{3}+\frac{2}{3}} = 2R$$

$$C_P - C_V = R \quad \Rightarrow C_P = 3R$$

$$\gamma = \frac{C_P}{C_V} = 1.5$$

Piston is displaced back to its initial position, and during the adiabatic compression, volume of the gas mixture is halved.

For adiabatic compression,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

where $T_1 = T_0$; $V_1 = V_0$;

$T_2 = ?$; $V_2 = V_0/2$

$$T_0V_0^{\gamma-1} = T_2\left(\frac{V_0}{2}\right)^{\gamma-1} \Rightarrow T_2 = T_0\sqrt{2}$$

$$\Delta U = (n_1+n_2)C_V(T_2-T_0)$$

$$= \frac{4}{3}\times 0.2R(T_0\sqrt{2}-T_0)$$

$$\Delta U = \frac{8}{3}RT_0(\sqrt{2}-1)$$

EXERCISES

Subjective Type

Solutions on page 2.78

1. Five grams of helium having rms speed of molecules 1000 m/s and 24 g of oxygen having rms speed of 1000 m/s are introduced into a thermally isolated vessel. Find the rms speeds of helium and oxygen individually when thermal equilibrium is attained. Neglect the heat capacity of the vessel.

2. Consider a vertical tube open at both ends. The tube consists of two parts, each of different cross sections and each part having a piston which can move smoothly in respective tubes. The two pistons are joined together by an inextensible wire. The combined mass of the two pistons is 5 kg and area of cross section of the upper piston is 10 cm² greater than that of the lower piston. Amount of gas enclosed by the pistons is 1 mol. When the gas is heated slowly, pistons move by 50 cm. Find the rise in the temperature of the gas in the form X/R K, where R is universal gas constant. Use $g = 10$ m/s² and outside pressure $= 10^5$ N/m²).

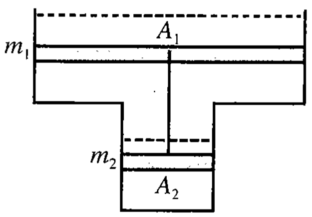

Fig. 2.69

3. Figure 2.70 shows the variation of internal energy (U) with the pressure (P) of 2.0 mole gas in cyclic process *abcda*. The temperatures of gas at c and d are 300 and 500 K, respectively. Calculate the heat absorbed by the gas during the process.

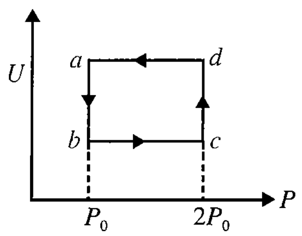

Fig. 2.70

4. Two moles a monatomic gas in state 'A' having critical pressure P_0 and temperature $3T_0$ is taken to a state B having pressure $3P_0$ and temperature $T_0/3$ by the process of equation P^2T = constant. Then state B is taken to state C keeping the volume constant and it comes back to initial state 'A' keeping temperature constant.

   i Plot a P and T graph. (P on the y-axis and T on the x-axis).

   ii Find the net work done and heat supplied to the gas during the complete cycle.

5. A gaseous mixture enclosed in a vessel consists of 1 g mole of a gas A with $\gamma = 5/3$ and another B with $\gamma = 7/5$ at a temperature T. The gases A and B do not react with each other and are assumed to be ideal. Find the number of gram moles of gas if γ of the gaseous mixture is 19/13.

6. A gas expands in a piston–cylinder device from volume V_1 to V_2, the process being described by $P = a/V + b$, a and b are constants. Find the work done in the process.

7. Find the minimum attainable pressure of an ideal gas in the process $T = T_0 + \alpha V^2$, where T_0 and α are positive constants and V is the volume of one mole of gas. Draw the approximate P–V plot of this process.

8. Air is contained in a piston–cylinder arrangement as shown in Fig. 2.71 with a cross-sectional area of 4 cm² and an initial volume of 20 cc. The air is initially at a pressure of 1 atm and temperature of 20°C. The piston is connected to a spring whose spring constant is $k = 10^4$ N/m, and the spring is initially undeformed. How much heat must be added to the air inside cylinder to increase the pressure to 3 atm (for air, CV = 718 J/kg °C, molecular mass of air 28.97)?

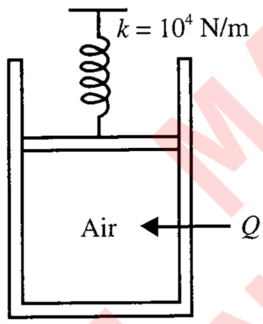

Fig. 2.71

9. Consider the cyclic process $ABCA$, shown in Fig. 2.72, performed, on a sample of 2.0 moles of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done by the gas during the part BC.

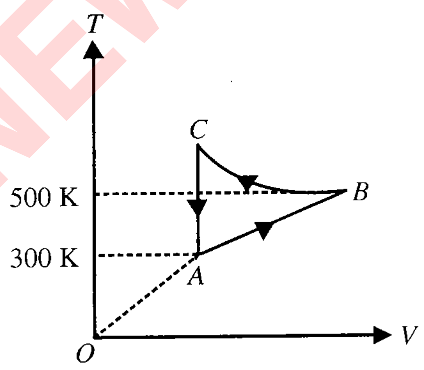

Fig. 2.72

10. Figure 2.73 shows the variation in the internal energy U with the volume V of 2.0 moles of an ideal gas in cyclic process abcda. The temperatures of the gas at b and c are 500 K and 300 K, respectively. Calculate the heat absorbed by the gas during the process.

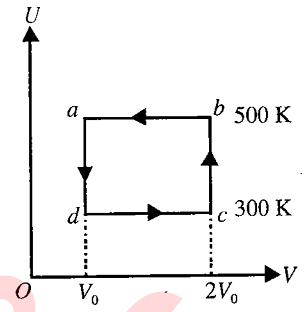

Fig. 2.73

11. One mole of a gas is carried through the cycle shown in Fig. 2.74. The gas expands at constant temperature T from volume V to 2V. It is then compressed to the initial volume at constant pressure and is finally brought back to its original state by heating at constant volume. Calculate the work done by the gas in complete cycle.

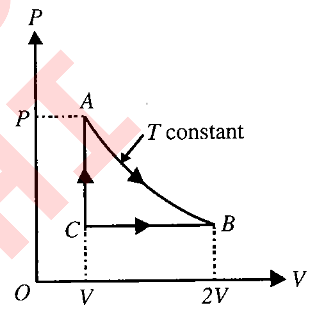

Fig. 2.74

12. One mole of a monatomic gas is taken from a point A to another point B along the path ACB. The initial temperature at A is T_0. Calculate the heat absorbed by the gas in the process $A \to C \to B$.

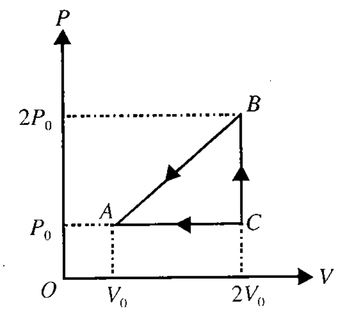

Fig. 2.75

13. Figure 2.76 shows an adiabatic cylindrical tube of volume V_0, divided in two parts by a frictionless adiabatic separator. Initially, the separator is kept in the middle, an ideal gas at pressure P_1 and temperature T_1 is injected into the left part and another ideal gas at pressure P_2 and temperature T_2 is injected into the right part. $C_P/C_V = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find

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a. the volumes of the two parts,
b. the heat given to the gas in the left part and
c. the final common pressure of the gases.

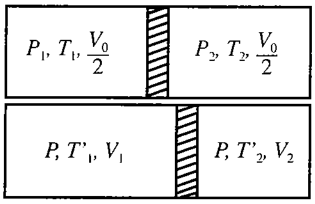

Fig. 2.76

14. Figure 2.77 shows a horizontal cylindrical container of length 30 cm, which is partitioned by a tight-fitting separator. The separator is diathermic but conducts heat very slowly. Initially the separator is in the state shown in the figure. The temperature of left part of cylinder is 100 K and that on right part is 400 K. Initially the separator is in equilibrium. As heat is conducted from right to left part, separator displaces to the right. Find the displacement of separator after a long when gases on the two parts of cylinder are in thermal equilibrium.

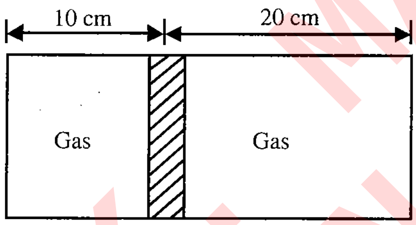

Fig. 2.77

15. An adiabatic vessel contains $n_1 = 3$ mole of diatomic gas. Moment of inertia of each molecule is $I = 2.76 \times 10^{-46}$ kg m^2 and root-mean-square angular velocity is $\omega_0 = 5 \times 10^{12}$ rad/s. Another adiabatic vessel contains $n_2 = 5$ mole of a monatomic gas at a temperature 470 K. Assume gases to be ideal, calculate root-mean-square angular velocity of diatomic molecules when the two vessels are connected by a thin tube of negligible volume. Boltzmann constant $k = 1.38 \times 10^{-23}$ J/molecule.

16. Figure 2.78 shows an ideal gas changing its state A to state C by two different paths ABC and AC.

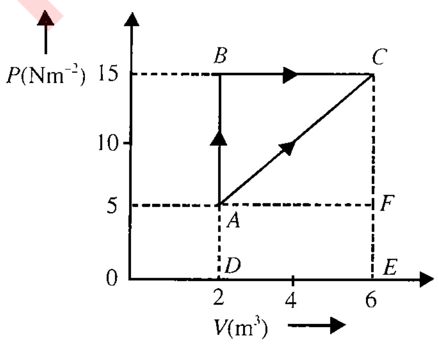

Fig. 2.78

a. Find the path along which the work done is the least.

b. The internal energy of the gas at A is 10 J and the amount of heat supplied to change its state to C through the path AC is 200 J. Find the internal energy at C.

c. The internal energy of the gas at state B is 20 J. Find the amount of heat supplied to the gas to go from state A to state B.

17. Figure 2.79 shows a process $ABCA$ performed on 1 mole of an ideal gas. Find the net heat supplied to the gaseous system during the process.

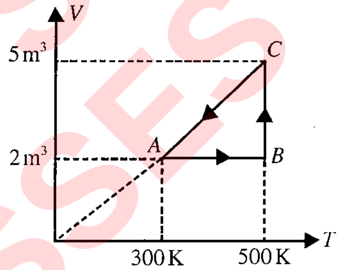

Fig. 2.79

18. Calculate the heat absorbed by a system in going through the cyclic process shown in Fig. 2.80.

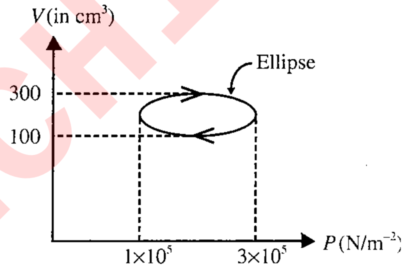

Fig. 2.80

19. An ideal gas has a volume 0.3 m^3 at 150 kPa. It is confined by a spring-loaded piston in a vertical cylinder. Initially, the spring is in relaxed state. If the gas is heated to a final state of 0.5 m^3 and pressure 600 kPa, find the work done on the spring (atmospheric pressure, $P_0 = 1 \times 10^5$ N/m^2).

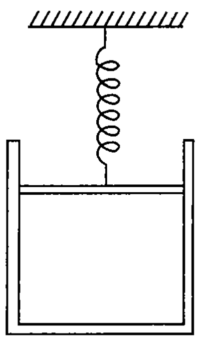

Fig. 2.81

20. One mole of an ideal monatomic gas undergoes the process $P = aT$, where a is a constant.

a. Find the work done by the gas if its temperature increases by 50 K.

b. Also, find the molar specific heat of the gas.

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21. Two moles of monatomic ideal gas is taken through a cyclic process shown on P–T diagram in Fig. 2.82. Process CA is represented as PT = constant. If efficiency of given cyclic process is

$$1-\frac{x}{12\ln 2+15}$$

then find x.

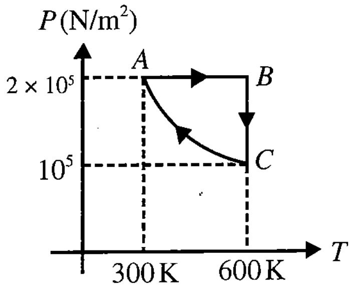

Fig. 2.82

22. One mole of a gas is enclosed in a cylinder and occupies a volume of 1.5 L at a pressure 1.5 atm. It is subjected to strong heating due to which temperature of the gas increases according to the relation $T = \alpha V^2$, where α is a positive constant and V is volume of the gas.

a. Find the work done by air in increasing the volume of gas to 9 L.

b. Draw the P–V diagram of the process.

c. Determine the heat supplied to the gas (assuming $\gamma = 1.5$).

Objective Type

Solutions on page 2.84

1. The temperature of a gas is raised while its volume remains constant, the pressure exerted by the gas on the walls of the container increases because its molecules

a. lose more kinetic energy to the wall
b. are in contact with the wall for a shorter time
c. strike the wall more often with higher velocities
d. collide with each other with less frequency

2. A cylinder of capacity 20 L is filled with H_2 gas. The total average kinetic energy of translatory motion of its molecules is 1.5×10^5 J. The pressure of hydrogen in the cylinder is

a. 2×10^6 N/m^2 **b.** 3×10^6 N/m^2
c. 4×10^6 N/m^2 **d.** 5×10^6 N/m^2

3. A flask is filled with 13 g of an ideal gas at 27°C and its temperature is raised to 52°C. The mass of the gas that has to be released to maintain the temperature of the gas in the flask at 52°C, the pressure remaining the same is

a. 2.5 g **b.** 2.0 g
c. 1.5 g **d.** 1.0 g

4. Air is filled at 60°C in a vessel of open mouth. The vessel is heated to a temperature T so that 1/4th part of air escapes. Assuming the volume of vessel remaining constant, the value of T is

a. 80°C **b.** 444°C
c. 333°C **d.** 171°C

5. If the intermolecular forces vanish away, the volume occupied by the molecules contained in 4.5 kg water at standard temperature and pressure will be given by

a. 5.6 m^3 **b.** 4.5 m^3
c. 11.2 L **d.** 11.2 m^3

6. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line D. Then the expansion of the same ideal gas of mass $2m$ at a pressure $P/2$ is given by the straight line

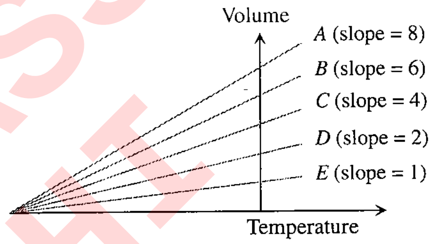

Fig. 2.83

a. E **b.** C
c. B **d.** A

7. Two identical glass bulbs are interconnected by a thin glass tube. A gas is filled in these bulbs at NTP. If one bulb is placed in ice and another bulb is placed inside hot bath, then the pressure of the gas becomes 1.5 times. The temperature of hot bath will be

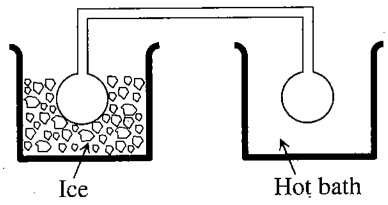

Fig. 2.84

a. 100°C **b.** 182°C
c. 256°C **d.** 546°C

8. Two containers of equal volume contain the same gas at pressures P_1 and P_2 and absolute temperatures T_1 and T_2, respectively. On joining the vessels, the gas reaches a common pressure P and common temperature T. The ratio P/T is equal to

a. $\dfrac{P_1}{T_1}+\dfrac{P_2}{T_2}$ **b.** $\dfrac{P_1T_1+P_2T_2}{(T_1+T_2)^2}$

c. $\dfrac{P_1T_2+P_2T_1}{(T_1+T_2)^2}$ **d.** $\dfrac{P_1}{2T_1}+\dfrac{P_2}{2T_2}$

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9. An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross-section $8\times10^{-3}\,\text{m}^2$. Initially the gas is at 300 K and occupies a volume of $2.4\times10^{-3}\,\text{m}^3$ and the spring is in a relaxed state. The gas is heated by a small heater coil H. The force constant of the spring is 8000 N/m and the atmospheric pressure is 1.0×10^5 Pa. The cylinder and piston are thermally insulated. The piston and the spring are massless and there is no friction between the piston and cylinder. There is no heat loss through heater coil wire leads and thermal capacity of the heater coil is negligible. With all the above assumptions, if the gas is heated by the heater until the piston moves out slowly by 0.1 m, then the final temperature is

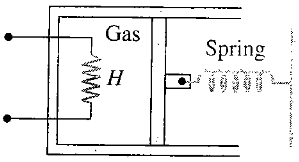


Fig. 2.85

a. 400 K **b.** 800 K
c. 1200 K **d.** 300 K

10. At the top of a mountain a thermometer reads 7°C and a barometer reads 70 cm of Hg. At the bottom of the mountain these read 27°C and 76 cm of Hg, respectively. Ratio of density of air at the top with that of bottom is

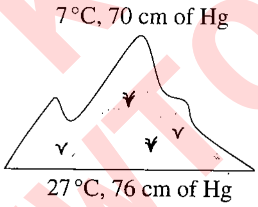


Fig. 2.86

a. 75/76 **b.** 70/76
c. 76/75 **d.** 76/70

11. A box contains N molecules of a perfect gas at temperature T_1 and pressure P_1. The number of molecules in the box is doubled keeping the total kinetic energy of the gas same as before. If the new pressure is P_2 and temperature T_2, then

a. $P_2 = P_1,\ T_2 = T_1$ **b.** $P_2 = P_1,\ T_2 = \dfrac{T_1}{2}$

c. $P_2 = 2P_1,\ T_2 = T_1$ **d.** $P_2 = 2P_1,\ T_2 = \dfrac{T_1}{2}$

12. An air bubble of volume V_0 is released by a fish at a depth h in a lake. The bubble rises to the surface. Assume constant temperature and standard atmospheric pressure P above the lake. The volume of the bubble just before touching the surface will be (density of water is ρ)

a. V_0 **b.** $V_0(\rho gh/P)$

c. $\dfrac{V_0}{\left(1+\dfrac{\rho gh}{P}\right)}$ **d.** $V_0\left(1+\dfrac{\rho gh}{P}\right)$

13. Hydrogen gas is filled in a balloon at 20°C. If temperature is made 40°C, pressure remaining same, what fraction of hydrogen will come out?

a. 0.07 **b.** 0.25
c. 0.5 **d.** 0.75

14. The expansion of unit mass of a perfect gas at constant pressure is shown in Fig. 2.87. Here

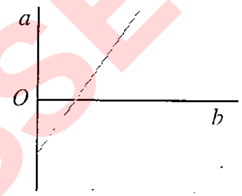


Fig. 2.87

a. a = volume, b = °C temperature
b. a = volume, b = K temperature
c. a = °C temperature, b = volume
d. a = K temperature, b = volume

15. A gas is filled in the cylinder shown in Fig. 2.88. The two pistons are joined by a string. If the gas is heated, the right piston will

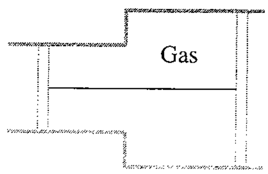
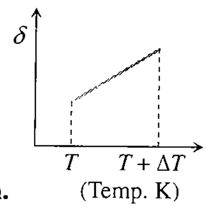


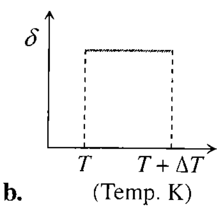
Fig. 2.88

a. move towards left
b. move towards right
c. remain stationary
d. none of these

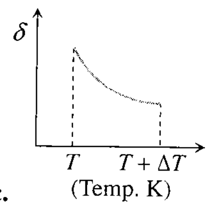
16. An ideal gas is initially at a temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V/V\Delta T$ varies with temperature as



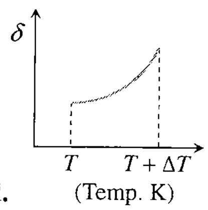
a.



b.



c.



d.

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17. If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1°C, the initial temperature must be

a. 250 K **b.** 250°C
c. 2500 K **d.** 25°C

18. Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes is plotted as shown in Fig. 2.89. Choose the correct alternative.

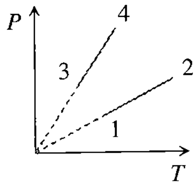

Fig. 2.89

a. $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$
b. $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$
c. $V_1 = V_2 = V_3 = V_4$
d. $V_4 > V_3 > V_2 > V_1$

19. The capacity of a vessel is 3 L. It contains 6 g oxygen, 8 g nitrogen and 5 g CO_2 mixture at 27°C. If $R = 8.31$ J/mol K, then the pressure in the vessel in N/m^2 will be (approx.)

a. 5×10^5 **b.** 5×10^4
c. 10^6 **d.** 10^5

20. Two gases occupy two containers A and B; the gas in A, of volume $0.10\,\text{m}^3$, exerts a pressure of 1.40 MPa and that in B, of volume $0.15\,\text{m}^3$, exerts a pressure 0.7 MPa. The two containers are joined by a tube of negligible volume and the gases are allowed to intermingle. Then if the temperature remains constant, the final pressure in the container will be (in MPa)

a. 0.70 **b.** 0.98
c. 1.40 **d.** 2.10

21. A closed vessel contains 8 g of oxygen and 7 g of nitrogen. The total pressure is 10 atm at a given temperature. If now oxygen is absorbed by introducing a suitable absorbent, the pressure of the remaining gas in atm will be

a. 2 **b.** 10
c. 4 **d.** 5

22. Energy of all molecules of a monatomic gas having a volume V and pressure P is $3/2\,PV$. The total translational kinetic energy of all molecules of a diatomic gas at the same volume and pressure is

a. 1/2 PV **b.** 3/2 PV
c. 5/2 PV **d.** 3 PV

23. Forty calories of heat is needed to raise the temperature of 1 mol of an ideal monatomic gas from 20°C to 30°C at a constant pressure. The amount of heat required to raise its temperature over the same interval at a constant volume ($R = 2\,\text{cal mol}^{-1}\text{K}^{-1}$) is

a. 20 cal **b.** 40 cal
c. 60 cal **d.** 80 cal

24. For a gas the difference between the two specific heats is 4150 J/kg K. What is the specific heat at constant volume of gas if the ratio of specific heats is 1.4

a. 8475 J/kg-K
b. 5186 J/kg-K
c. 1660 J/kg-K
d. 10375 J/kg-K

25. The specific heat at constant volume for the monatomic argon is 0.075 kcal/kg-K, whereas its gram molecular specific heat is $C_v = 2.98$ cal/mol/K. The mass of the argon atom is (Avogadro's number $= 6.02\times10^{23}$ molecules/mol)

a. 6.60×10^{-23} g
b. 3.30×10^{-23} g
c. 2.20×10^{-23} g
d. 13.20×10^{-23} g

26. The temperature of 5 mol of a gas which was held at constant volume was changed from 100°C to 120°C. The change in internal energy was found to be 80 J. The total heat capacity of the gas at constant volume will be equal to

a. $8\,\text{JK}^{-1}$ **b.** $0.8\,\text{JK}^{-1}$
c. $4\,\text{JK}^{-1}$ **d.** $0.4\,\text{JK}^{-1}$

27. A gas is heated at a constant pressure. The fraction of heat supplied used for external work is

a. $\frac{1}{\gamma}$ **b.** $\left(1-\frac{1}{\gamma}\right)$
c. $\gamma - 1$ **d.** $\left(1-\frac{1}{\gamma^2}\right)$

28. A monatomic gas expands at constant pressure on heating. The percentage of heat supplied that increases the internal energy of the gas and that is involved in the expansion is

a. 75%, 25% **b.** 25%, 75%
c. 60%, 40% **d.** 40%, 60%

29. The average degrees of freedom per molecule for a gas are 6. The gas performs 25 J of work when it expands at constant pressure. The heat absorbed by gas is

a. 75 J **b.** 100 J
c. 150 J **d.** 125 J

30. Certain amount of an ideal gas is contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The rise in temperature of the gas when the vessel is suddenly stopped is ($\gamma = C_P / C_V$)

a. $\frac{Mv^2(\gamma-1)}{2R(\gamma+1)}$ **b.** $\frac{Mv^2(\gamma-1)}{2R}$
c. $\frac{Mv^2}{2R(\gamma+1)}$ **d.** $\frac{Mv^2}{2R(\gamma-1)}$

31. The density of a polyatomic gas in standard conditions is $0.795\,\text{kg m}^{-3}$. The specific heat of the gas at constant volume is

a. 930 J-kg^{-1} K^{-1} **b.** 1400 J-kg^{-1} K^{-1}
c. 1120 J-kg^{-1} K^{-1} **d.** 1600 J-kg^{-1} K^{-1}

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32. The value of $C_p - C_v = 1.00R$ for a gas in state A and $C_p - C_v = 1.06R$ in another state. If P_A and P_B denote the pressure and T_A and T_B denote the temperatures in the two states, then

a. $P_A = P_B, T_A > T_B$ **b.** $P_A > P_B, T_A = T_B$
c. $P_A < P_B, T_A > T_B$ **d.** $P_A = P_B, T_A < T_B$

33. If 2 moles of diatomic gas and 1 mole of monatomic gas are mixed, then the ratio of specific heats for the mixture is

a. $\frac{7}{3}$ **b.** $\frac{5}{4}$
c. $\frac{19}{13}$ **d.** $\frac{15}{19}$

34. Twenty-two grams of CO_2 at 27°C is mixed with 16 g of O_2 at 37°C. The temperature of the mixture is about

a. 31.5°C **b.** 27°C
c. 37°C **d.** 30.5°C

35. A gas mixture consists of 2 mol of oxygen and 4 mol of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

a. $4RT$ **b.** $15RT$
c. $9RT$ **d.** $11RT$

36. A thermodynamic system is taken through the cyclic $PQRSP$ process. The net work done by the system is

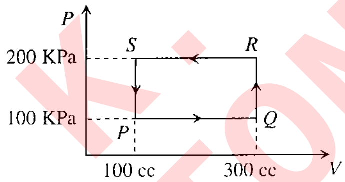

Fig. 2.90

a. 20 J **b.** −20 J
c. 400 J **d.** −374 J

37. An ideal gas is taken around $ABCA$ as shown in the above P–V diagram. The work done during a cycle is

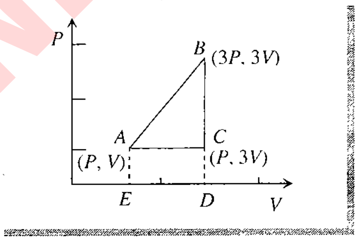

Fig. 2.91

a. $2PV$ **b.** PV
c. $1/2PV$ **d.** Zero

38. An ideal gas of mass m in a state A goes to another state B via three different processes as shown in Fig. 2.92. If Q_1, Q_2 and Q_3 denote the heat absorbed by the gas along the three paths, then

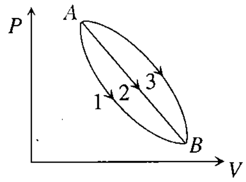

Fig. 2.92

a. $Q_1 < Q_2 < Q_3$ **b.** $Q_1 < Q_2 = Q_3$
c. $Q_1 = Q_2 > Q_3$ **d.** $Q_1 > Q_2 > Q_3$

39. The relation between the internal energy U and adiabatic constant γ is

a. $U = \frac{PV}{\gamma - 1}$ **b.** $U = \frac{PV^\gamma}{\gamma - 1}$
c. $U = \frac{PV}{\gamma}$ **d.** $U = \frac{\gamma}{PV}$

40. A thermodynamic process is shown in Fig. 2.93. The pressures and volumes corresponding to some points in the figure are: $P_A = 3 \times 10^4$ Pa, $P_B = 8 \times 10^4$ Pa and $V_A = 2 \times 10^{-3}$ m³, $V_D = 5 \times 10^{-3}$ m³.
In process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in process AC would be

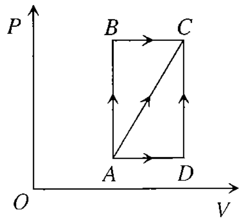

Fig. 2.93

a. 560 J **b.** 800 J **c.** 600 J **d.** 640 J

41. If R = universal gas constant, the amount of heat needed to raise the temperature of 2 mol of an ideal monatomic gas from 273 K to 373 K when no work is done is

a. $100R$ **b.** $150R$ **c.** $300R$ **d.** $500R$

42 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p / C_v for the gas is

a. $\frac{3}{2}$ **b.** $\frac{4}{3}$
c. 2 **d.** $\frac{5}{3}$

43. An ideal gas at 27 °C is compressed adiabatically to 8/27 of its original volume. If γ = 5/3, then the rise in temperature is

a. 450 K **b.** 375 K **c.** 225 K **d.** 405 K

44. Four curves A, B, C and D are drawn in Fig. 2.94 for a given amount of gas. The curves which represent adiabatic and isothermal changes are

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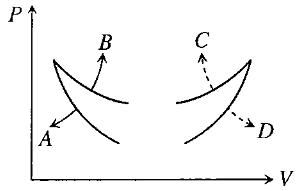

Fig. 2.94

a. C and D, respectively
b. D and C, respectively
c. A and B, respectively
d. B and A, respectively

45. A thermally insulated container is divided into two parts by a screen. In one part the pressure and temperature are P and T for an ideal gas filled. In the second part it is vacuum. If now a small hole is created in the screen, then the temperature of the gas will

a. decrease **b.** increase
c. remain same **d.** none of these

46. Two samples A and B of a gas initially at the same pressure and temperature are compressed from volume V to $V/2$ (A isothermally and B adiabatically). The final pressure of A is

a. greater than the final pressure of B
b. equal to the final pressure of B
c. less than the final pressure of B
d. twice the final pressure of B

47. $1\,cm^3$ of water at its boiling point absorbs 540 cal of heat to become steam with a volume of $1671\,cm^3$. If the atmospheric pressure is $1.013\times10^5\ N/m^2$ and the mechanical equivalent of heat = 4.19 J/cal, the energy spent in this process in overcoming intermolecular forces is

a. 540 cal **b.** 40 cal **c.** 500 cal **d.** zero

48. Five moles of hydrogen gas are heated from 30°C to 60°C at constant pressure. Heat given to the gas is (given R = 2 cal/mol degree)

a. 750 cal **b.** 630 cal **c.** 1050 cal **d.** 1470 cal

49. When an ideal gas (γ = 5/3) is heated under constant pressure, what percentage of given heat energy will be utilized in doing external work?

a. 40% **b.** 30% **c.** 60% **d.** 20%

50. At 100°C the volume of 1 kg of water is $10^{-3}\ m^3$ and volume of 1 kg of steam at normal pressure is $1.671\ m^3$. The latent heat of steam is 2.3×10^6 J/kg and the normal pressure is $10^5\ N/m^2$. If 5 kg of water at 100°C is converted into steam, the increase in the internal energy of water in this process will be

a. 8.35×10^5 J **b.** 10.66×10^6 J
c. 11.5×10^6 J **d.** zero

51. In the following pressure–volume diagram, the isochoric, isothermal and isobaric parts, respectively, are

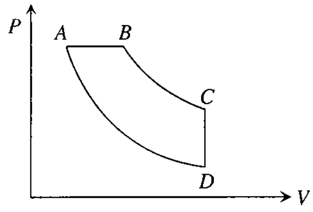

Fig. 2.95

a. BA, AD, DC **b.** DC, CB, BA
c. AB, BC, CD **d.** CD, DA, AB

52. The P–V diagram of a system undergoing thermodynamic transformation is shown in Fig. 2.96. The work done on the system in going from $A \to B \to C$ is 50 J and 20 cal heat is given to the system. The change in internal energy between A and C is

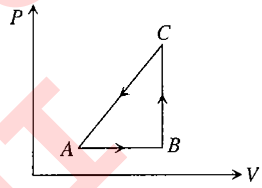

Fig. 2.96

a. 34 J **b.** 70 J **c.** 84 J **d.** 134 J

53. In the following indicator diagram, the net amount of work done will be

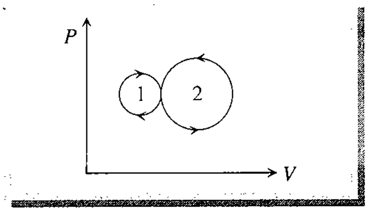

Fig. 2.97

a. positive **b.** negative **c.** zero **d.** infinity

54. A cyclic process for 1 mole of an ideal gas is shown in Fig. 2.98 in the V–T, diagram. The work done in AB, BC and CA, respectively, is

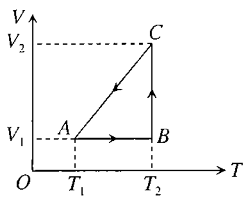

Fig. 2.98

a. $0, RT_2 \ln\left(\frac{V_1}{V_2}\right), R(T_1 - T_2)$

b. $R(T_1 - T_2), 0, RT_1 \ln\left(\frac{V_1}{V_2}\right)$

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c. $0, RT_2 \ln\left(\frac{V_2}{V_1}\right), R(T_1 - T_2)$

d. $0, RT_2 \ln\left(\frac{V_2}{V_1}\right), R(T_2 - T_1)$

55. A cyclic process *ABCD* is shown in the following *P–V* diagram. Which of the following curves represents the same process?

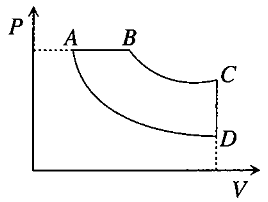
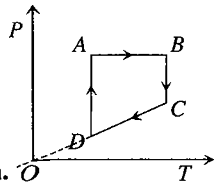
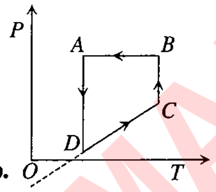
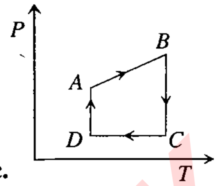
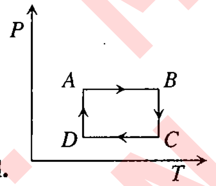


Fig. 2.99

a.


b.


c.


d.


56. An ideal gas expands in such a manner that its pressure and volume can be related by equation PV^2 = constant. During this process, the gas is

a. heated
b. cooled
c. neither heated nor cooled
d. first heated and then cooled

57. A cyclic process *ABCA* is shown in the *V–T* diagram. Process on the *P–V* diagram is

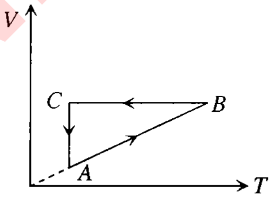
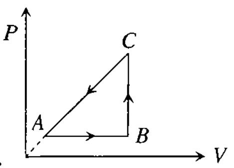
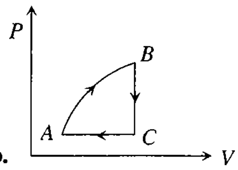
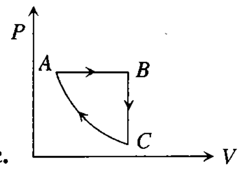
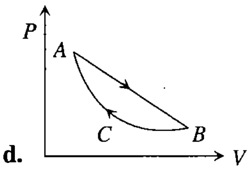


Fig. 2.100

a.


b.


c.


d.


58. One mole of a diatomic gas undergoes a process $P = P_0/[1 + (V/V_0)^3]$ where P_0 and V_0 are constants. The translational kinetic energy of the gas when $V = V_0$ is given by

a. $5P_0V_0/4$ **b.** $3P_0V_0/4$ **c.** $3P_0V_0/2$ **d.** $5P_0V_0/2$

59. If 50 cal of heat is supplied to the system containing 2 mol of an ideal monatomic gas, the rise in temperature is (R = 2 cal/mol-K)

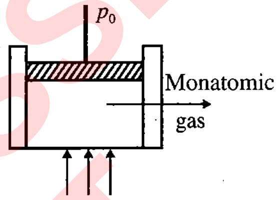


Fig. 2.101

a. 50 K **b.** 5 K **c.** 10 K **d.** 20 K

60. In the given elliptical *P–V* diagram

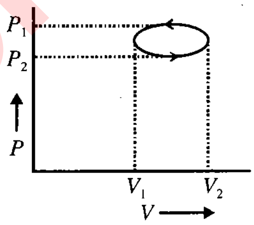


Fig. 2.102

a. the work done is positive
b. the change in internal energy is non-zero
c. the work done $= -(\pi/4)(P_2 - P_1)(V_2 - V_1)$
d. the work done $= \pi(V_1 - V_1)^2 - \pi(P_1 - P_1)^2$

61. Three processes compose a thermodynamic cycle shown in the accompanying *P–V*, diagram of an ideal gas.

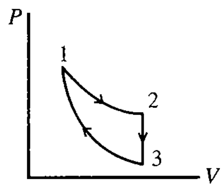


Fig. 2.103

Process 1→2 takes place at constant temperature, during this process 60 J of heat enters the system.
Process 2→3 takes place at constant volume. During this process 40 J of heat leaves the system.
Process 3→1 is adiabatic.
What is the change in internal energy of the system during process 3→1?

a. −40 J **b.** −20 J **c.** +20 J **d.** +40 J

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62. When an ideal gas is taken from state a to b, along a path acb, 84 kJ of heat flows into the gas and the gas does 32 kJ of work. The following conclusions are drawn. Mark the one which is not correct.

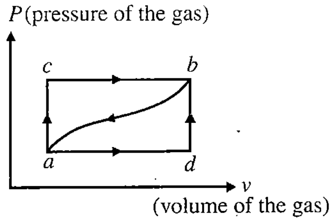


Fig. 2.104

a. If the work done along the path adb is 10.5 kJ, the heat that will flow into the gas is 62.5 kJ.
b. When the gas is returned from b to a along the curved path, the work done on the gas is 21 kJ, and the system absorbs 73 kJ of heat.
c. If $U_a = 0$, $U_d = 42$ kJ, and the work done along the path adb is 10.5 kJ then the heat absorbed in the process ad is 52.5 kJ.
d. If $U_a = 0$, $U_d = 42$ kJ, heat absorbed in the process db is 10 kJ.

63. A sample of an ideal gas is taken through the cyclic process $ABCA$ shown in Fig. 2.105. It rejects 50 J of heat during the part AB, does not absorb or reject the heat during BC, and accepts 70 J of heat during CA. Forty joules of work is done on the gas during the part BC. The internal energies at B and C, respectively, will be

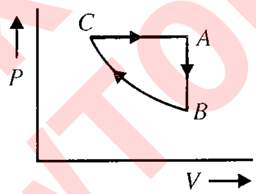


Fig. 2.105

a. 1450 J and 1410 J
b. 1550 J and 1590 J
c. 1450 J and 1490 J
d. 1550 J and 1510 J

64. A ring-shaped tube contains two ideal gases with equal masses and molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α is

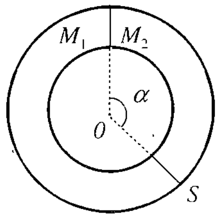


Fig. 2.106

a. 182° **b.** 170°
c. 192° **d.** 180°

65. Variation of internal energy with density of 1 mole of monatomic gas is depicted in Fig. 2.107. Corresponding variation of pressure with volume can be depicted as (assume the curve is rectangular hyperbola)

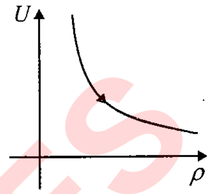
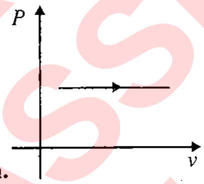
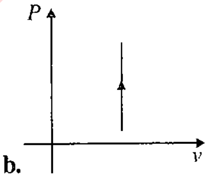
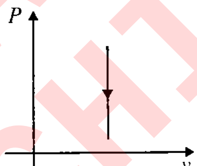
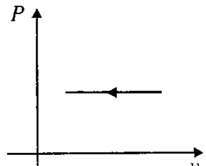


Fig. 2.107

a.


b.


c.


d.


66. One mole of an ideal gas is taken along the process in which PV^x = constant. The graph shown represents the variation of molar heat capacity of such a gas with respect to 'x'. The values of c' and x', respectively, are given by

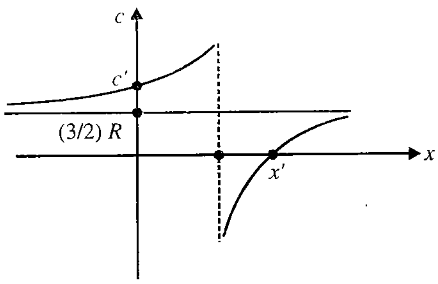


Fig. 2.108

a. $\frac{5}{2}R, \frac{5}{2}$ **b.** $\frac{5}{2}R, \frac{5}{3}$ **c.** $\frac{7}{2}R, \frac{7}{2}$ **d.** $\frac{5}{2}R, \frac{7}{5}$

67. A vessel of volume 20 L contains a mixture of hydrogen and helium at temperature of 27°C and pressure 2.0 atm. The mass of the mixture is 5 g. Assuming the gases to be ideal, the ratio of the mass of hydrogen to that of helium in the given mixture will be

a. 1:2 **b.** 2:3 **c.** 2:1 **d.** 2:5

68. A cylinder of ideal gas is closed by an 8 kg movable piston (area 60 cm²) as shown in Fig. 2.109. Atmospheric pressure is 100 kPa. When the gas is heated from 30°C to 100°C, the piston rises by 20 cm. The piston is then fixed in its placed and the gas is cooled back to 30°C. Let ΔQ_1 be the heat

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added to the gas in the heating process and $|\Delta Q_2|$ the heat lost during cooling. Then the value of $[\Delta Q_1 - |\Delta Q_2|]$ will be

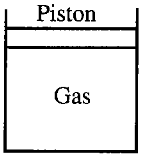

Fig. 2.109

a. zero **b.** 136 J **c.** −136 J **d.** −68 J

69. A certain ideal gas undergoes a polytropic process PV^n = constant such that the molar specific heat during the process is negative. If the ratio of the specific heats of the gas be γ, then the range of values of n will be

a. $0 < n < \gamma$ **b.** $1 < n < \gamma$
c. $n = \gamma$ **d.** $n > \gamma$

70. A cylindrical chamber A of uniform cross section is divided into two parts X and Y by a movable piston P which can slide without friction inside the chamber. Initially part X contains 1 mol of a monochromatic gas and Y contains 2 mol of a diatomic gas, and the volumes of X and Y are in the ratio 1:2 with both parts X and Y being at the same temperature T. Assuming the gases to be ideal, the work W that will be done in moving the piston slowly to the position where the ratio of the volumes of X and Y is 2:1 will be

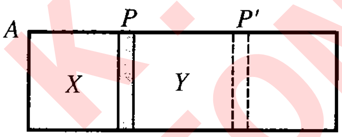

Fig. 2.110

a. $-5.8T$ **b.** $8.3T$ **c.** $12.3T$ **d.** zero

71. An ideal gas (1 mol, monatomic) is in the initial state P (see Fig. 2.111) on an isothermal A at temperature T_0. It is brought under a constant volume $(2V_0)$ process to Q which lies on an adiabatic B intersecting the isothermal A at (P_0, V_0, T_0). The change in the internal energy of the gas during the process is (in terms of T_0) $(2^{2/3} = 1.587)$

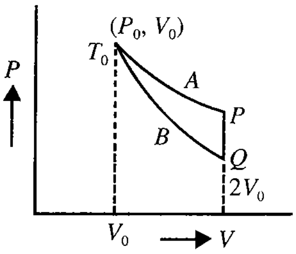

Fig. 2.111

a. $2.3T_0$ **b.** $-4.6T_0$
c. $-2.3T_0$ **d.** $4.6T_0$

72. Figure 2.12 shows an isochore, an isotherm, an adiabatic and two isobars of two gases on a work done versus heat supplied curve. The initial states of both gases are the same and the scales for the two axes are same.

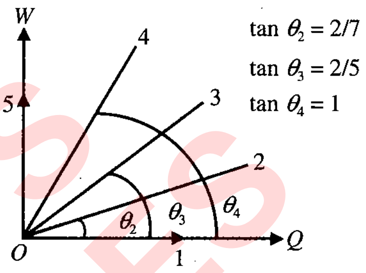

Fig. 2.112

Which of the following statements is incorrect?

a. Straight line 1 corresponds to an isochoric process.
b. Straight line 2 corresponds to an isobaric process for diatomic gas.
c. Straight line 4 corresponds to an isothermal process.
d. Straight line 5 corresponds to an isothermal process.

73. A stationary cylinder of oxygen used in a hospital has the following characteristics at room temperature 300 K, gauge pressure 1.38×10^7 Pa, volume 16 L. If the flow area, measured at atmospheric pressure, is constant at 2.4 L/min, the cylinder will last for nearly

a. 5 h **b.** 10 h **c.** 15 h **d.** 20 h

74. A sample of ideal gas is expanded to twice its original volume of 1 m³ in a quasi-static process for which $P = \alpha V^2$ with $\alpha = 3 \times 10^5$ Pa/m⁶ as shown in Fig. 2.113. Work done by the expanding gas is

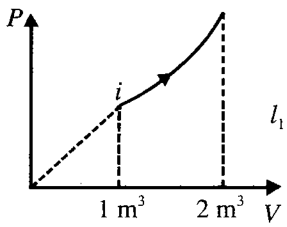

Fig. 2.113

a. 8×10^5 J **b.** 7×10^5 J
c. 6×10^5 J **d.** 3×10^5 J

75. One mole of air $(C_V = 5R/2)$ is confined at atmospheric pressure in a cylinder with a piston at 0°C. The initial volume occupied by gas is V. After the equivalent of 13200 J of heat is transferred to it, the volume of gas V' is nearly (1 atm = 10^5 N/m³)

a. 37 L **b.** 22 L
c. 60 L **d.** 30 L

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76. A thermodynamic process is shown in Fig. 2.114. The pressures and volumes corresponding to some points in the figure are

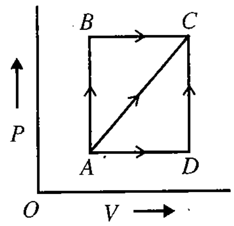


Fig. 2.114

$P_A = 3 \times 10^4$ Pa $\quad V_A = 2 \times 10^{-3}$ m^3
$P_B = 8 \times 10^4$ Pa $\quad V_D = 5 \times 10^{-3}$ m^3

In the process AB, 600 J of heat is added to the system. The change in internal energy of the system in the process AB would be

a. 560 J **b.** 800 J
c. 60 J **d.** 640 J

77. Carbon monoxide is carried around a closed cyclic process abc, in which bc is an isothermal process, as shown in Fig. 2.115. The gas absorbs 7000 J of heat as its temperature is increased from 300 K to 1000 K in going from a to b. The quantity of heat ejected by the gas during the process ca is

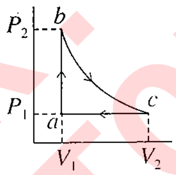


Fig. 2.115

a. 4200 J **b.** 500 J
c. 9000 J **d.** 9800 J

78. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J, $Q_4 = 3645$ J, respectively. The corresponding works involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4, respectively. The value of W_4 is

a. 1315 J **b.** 275 J
c. 765 J **d.** 675 J

79. Pressure versus temperature graph of an ideal gas is as shown in Fig. 2.116.

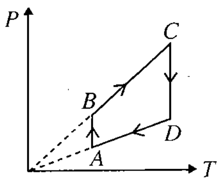
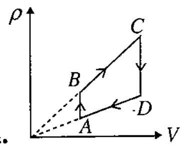
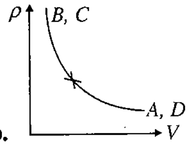
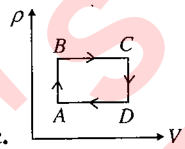


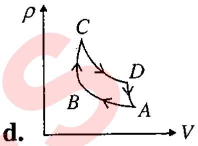
Fig. 2.116

Corresponding density (ρ) versus volume (V) graph will be

a. 

b. 

c. 

d. 

80. An ideal gas is taken through $A \to B \to C \to A$, as shown in Fig. 2.117. If the net heat supplied to the gas in the cycle is 55 J, the work done by the gas in the process $C \to A$ is

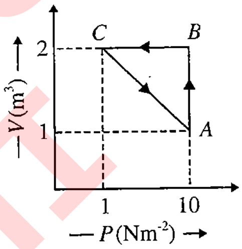


Fig. 2.117

a. –5 J **b.** –10 J **c.** –15 J **d.** –20 J

81. Two identical containers A and B have frictionless pistons. They contain the same volume of an ideal gas at the same temperature. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. The change in the pressure in A and B, respectively, is Δp and $1.5\ \Delta p$. Then

a. $4m_A = 9m_B$ **b.** $2m_A = 3m_B$
c. $3m_A = 2m_B$ **d.** $9m_A = 4m_B$

82. Argon gas is adiabatically compressed to half its volume. If P, V and T represent the pressure, volume and temperature of the gaseous system, respectively, at any stage, then the correct equation representing the process is

a. $TV^{2/5} =$ constant **b.** $VP^{5/3} =$ constant
c. $TP^{-2/5} =$ constant **d.** $PT^{2/5} =$ constant

83. A fixed mass of helium gas is made to undergo a process in which its pressure varies linearly from 1 kPa to 2 kPa, in relation to its volume as the latter varies from 0.2 m^3 to 0.4 m^3. The heat absorbed by the gas will be

a. 300 J **b.** 900 J
c. 1200 J **d.** 1500 J

84. Oxygen gas is made to undergo a process in which its molar heat capacity C depends on its absolute temperature T as $C = \alpha T$. Work done by it when heated from an initial temperature T_0 to a final temperature $2T_0$, will be

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a. $4\alpha T_0^2$
b. $(\alpha T_0 - R)\frac{3T_0}{2}$
c. $(3\alpha T_0 - 5R)\frac{T_0}{2}$
d. none of these

85. If the ratio of specific heat of a gas at constant pressure to that at constant volume is γ, the change in internal energy of the mass of gas, when the volume changes from V to $2V$ at constant pressure p, is
a. $R/(\gamma - 1)$
b. pV
c. $pV/(\gamma - 1)$
d. $\gamma pV/(\gamma - 1)$

86. A gas is at 1 atm pressure with a volume 800 cm^3. When 100 J of heat is supplied to the gas, it expands to 1 L at constant pressure. The change in its internal energy is
a. 80 J
b. −80 J
c. 20 J
d. −20 J

87. A fixed mass of a gas is first heated isobarically to double the volume and then cooled isochorically to decrease the temperature back to the initial value. By what factor would the work done by the gas decreased, had the process been isothermal?
a. 2
b. 1/2
c. ln 2
d. ln 3

88. An ideal heat engine has an efficiency η. The coefficient of performance of the engine when driven backward will be
a. $1 - (1/\eta)$
b. $\eta/(1-\eta)$
c. $(1/\eta) - 1$
b. $1/(1-\eta)$

89. Two moles of helium gas are taken along the path $ABCD$ (as shown in Fig. 2.118). The work done by the gas is

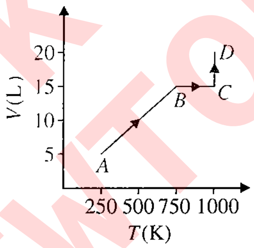

Fig. 2.118

a. $2000R\left(\frac{1}{2}+\ln\frac{4}{3}\right)$
b. $500R(3+\ln 4)$
c. $500R\left(2+\ln\frac{16}{9}\right)$
d. $1000R\left(1+\ln\frac{16}{9}\right)$

90. Figure 2.119 shows the adiabatic curve for n moles of an ideal gas; the bulk modulus for the gas corresponding to the point P will be

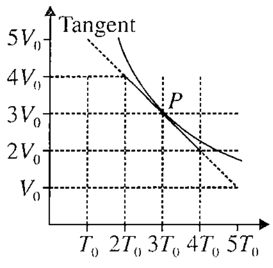

Fig. 2.119

a. $\frac{5nRT_0}{3V_0}$
b. $nR\left(2+\frac{T_0}{V_0}\right)$
c. $nR\left(1+\frac{T_0}{V_0}\right)$
d. $\frac{2nRT_0}{V_0}$

91. Two moles of an ideal gas is contained in a cylinder fitted with a frictionless movable piston, exposed to the atmosphere, at an initial temperature T_0. The gas is slowly heated so that its volume becomes four times the initial value. The work done by the gas is
a. zero
b. $2RT_0$
c. $4RT_0$
d. $6RT_0$

92. The equation of state for a gas is given by $PV = \eta RT + \alpha V$, where η is the number of moles and α a positive constant. The initial pressure and temperature of 1 mol of the gas contained in a cylinder is P_0 and T_0, respectively. The work done by the gas when its temperature doubles isobarically will be
a. $\frac{P_0T_0R}{P_0-\alpha}$
b. $\frac{P_0T_0R}{P_0+\alpha}$
c. $P_0T_0R\ln 2$
d. none of these

93. A sound wave passing through air at NTP produces a pressure of 0.001 dyne/cm^2 during a compression. The corresponding change in temperature (given $\gamma = 1.5$ and assume gas to be ideal) is
a. 8.97×10^{-4} K
b. 8.97×10^{-6} K
c. 8.97×10^{-8} K
d. none of these

94. A container of volume 1 m^3 is divided into two equal compartments by a partition. One of these compartments contains an ideal gas at 300 K. The other compartment is vacuum. The whole system is thermally isolated from its surroundings. The partition is removed and the gas expands to occupy the whole volume of the container. Its temperature now would be
a. 300 K
b. 250 K
c. 200 K
d. 10 K

95. The molar specific heat of oxygen at constant pressure $C_P = 7.03$ cal/mol °C and $R = 8.31$ J/mol °C. The amount of heat taken by 5 mol of oxygen when heated at constant volume from 10° C to 20°C will be approximately
a. 25 cal
b. 50 cal
c. 250 cal
d. 500 cal

96. Four moles of hydrogen, 2 moles of helium and 1 mole of water vapour form an ideal gas mixture. What is the molar specific heat at constant pressure of mixture?
a. $\frac{16}{7}R$
b. $\frac{7R}{16}$
c. R
d. $\frac{23}{7}R$

97. In an adiabatic process pressure is increased by 2/3% if $C_p/C_V = 3/2$. Then the volume decreases by about
a. $\frac{4}{9}\%$
b. $\frac{2}{3}\%$
c. 4%
d. $\frac{9}{4}\%$

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98. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is

a. 30 K **b.** 18 K **c.** 50 K **d.** 42 K

99. A gas under constant pressure of 4.5×10^5 Pa, when subjected to 800 kJ of heat, changes the volume from 0.5 m^3 to 2.0 m^3. The change in internal energy of the gas is

a. 6.75×10^5 J **b.** 5.25×10^5 J
c. 3.25×10^5 J **d.** 1.25×10^5 J

100. If for hydrogen $C_p - C_v = m$ and for nitrogen $C_p - C_v = n$, where C_p and C_v refer to specific heats per unit mass respectively at constant pressure and constant volume, the relation between m and n is (molecular weight of hydrogen = 2 and molecular weight of nitrogen = 14)

a. $n = 14m$ **b.** $n = 7m$
c. $m = 7n$ **d.** $m = 14n$

101. Five moles of hydrogen ($\gamma = 7/5$), initially at STP, is compressed adiabatically so that its temperature becomes 400°C. The increase in the internal energy of the gas in kilojoules is (R = 8.30 J/mol-K):

a. 21.55 **b.** 41.50
c. 65.55 **d.** 80.55

102. One mole of gas having $\gamma = 7/5$ is mixed with 1 mole of a gas having $\gamma = 4/3$. What will be the γ for the mixture?

a. $\frac{15}{11}$ **b.** $\frac{5}{13}$
c. $\frac{5}{11}$ **d.** $\frac{15}{13}$

103. The specific heats of argon at constant pressure and constant volume are 525 J/kg and 315 J/kg, respectively. Its density at NTP will be

a. 1.77 kg/m^3 **b.** 0.77 kg/m^3
c. 1.77 g/m^3 **d.** 0.77 g/m^3

104. An ideal gas expands isothermally from volume V_1 to V_2 and is then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then

a. $P_3 > P_1$, $W > 0$ **b.** $P_3 < P_1$, $W < 0$
c. $P_3 > P_1$, $W < 0$ **d.** $P_3 = P_1$, $W = 0$

105. Internal energy of n_1 mol of hydrogen of temperature T is equal to the internal energy of n_2 mol of helium at temperature $2T$. The ratio n_1/n_2 is

a. $\frac{3}{5}$ **b.** $\frac{2}{3}$ **c.** $\frac{6}{5}$ **d.** $\frac{3}{7}$

106. An ideal gas ($\gamma = 1.5$) is expanded adiabatically. How many times has the gas to be expanded to reduce the root-mean-square velocity of molecules becomes half?

a. 4 times **b.** 16 times **c.** 8 times **d.** 2 times

107. If 2 mol of an ideal monatomic gas at temperature T_0 are mixed with 4 mol of another ideal monatomic gas at temperature $2T_0$, then the temperature of the mixture is

a. $\frac{5}{3}T_0$ **b.** $\frac{3}{2}T_0$ **c.** $\frac{4}{3}T_0$ **d.** $\frac{5}{4}T_0$

108. Three samples of the same gas A, B and C ($\gamma = 3/2$) initially have equal volume. Now the volume of each sample is doubled. The process is adiabatic for A, isobaric for B and isothermal for C. If the final pressures are equal for all the three samples, the ratio of their initial pressures is

a. $2\sqrt{2}:2:1$ **b.** $2\sqrt{2}:1:2$
c. $\sqrt{2}:1:2$ **d.** $2:1:\sqrt{2}$

109. P–V diagram of an ideal gas is as shown in Fig. 2.120. Work done by the gas in the process $ABCD$ is

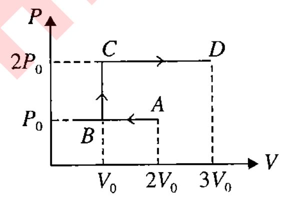

Fig. 2.120

a. $4P_0V_0$ **b.** $2P_0V_0$ **c.** $3P_0V_0$ **d.** P_0V_0

110. n moles of an ideal gas undergoes a process $A \rightarrow B$ as shown in Fig. 2.121. Maximum temperature of the gas during the process is:

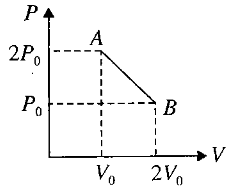

Fig. 2.121

a. $\frac{3P_0V_0}{2nR}$ **b.** $\frac{9P_0V_0}{4nR}$
c. $\frac{9P_0V_0}{2nR}$ **d.** $\frac{9P_0V_0}{nR}$

111. The relation between internal energy U, pressure P and volume V of a gas in an adiabatic process is

$$U = a + bPV$$

where a and b are constants. What is the effective value of adiabatic constant γ?

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a. $\frac{a}{b}$ b. $\frac{b+1}{b}$

c. $\frac{a+1}{a}$ d. $\frac{b}{a}$

112. One mole of an ideal gas at temperature T_1 expands according to the law (P/V) = constant. Find the work done when the final temperature becomes T_2.

a. $R(T_2 - T_1)$ b. $(R/2)(T_2 - T_1)$
c. $(R/4)(T_2 - T_1)$ d. $PV(T_2 - T_1)$

113. Two moles of an ideal gas at 300 K were cooled at constant volume so that the pressure is reduced to half the initial value. Then as a result of heating at constant pressure, the gas expands till it attains the original temperature. Find the total heat absorbed by the gas, if R is the gas constant.

a. $150R$ J b. $300R$ J
c. $75R$ J d. $100R$ J

114. An ideal gas is taken around the cycle $ABCA$ shown in P–V diagram. The net work done by the gas during the cycle is equal to

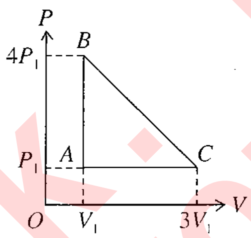


Fig. 2.122

a. $12P_1V_1$ b. $6P_1V_1$
c. $3P_1V_1$ d. P_1V_1

115. Heat energy absorbed by a system in going through a cyclic process shown in Fig. 2.123 is

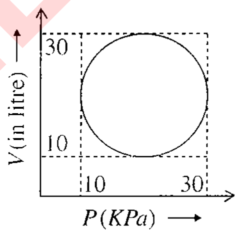


Fig. 2.123

a. $10^7\pi$ J b. $10^4\pi$ J
c. $10^2\pi$ J d. $10^{-3}\pi$ J

116. A diatomic ideal gas is heated at constant volume until the pressure is doubled and again heated at constant pressure until the volume is doubled. The average molar heat capacity for the whole process is

a. $\frac{13R}{6}$ b. $\frac{19R}{6}$
c. $\frac{23R}{6}$ d. $\frac{17R}{6}$

117. One mole of an ideal gas is taken from state A to state B by three different processes (a) ACB, (b) ADB and (c) AEB as shown in the P–V diagram. The heat absorbed by the gas is

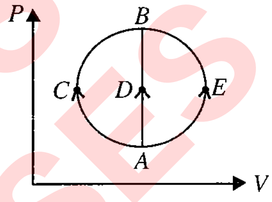
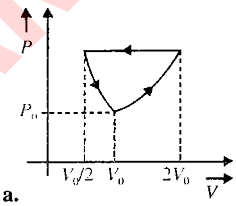


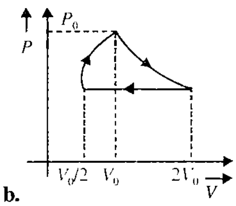
Fig. 2.124

a. greater in process (b) than in (a)
b. the least in process (b)
c. the same in (a) and (c)
d. less in (c) than in (b)

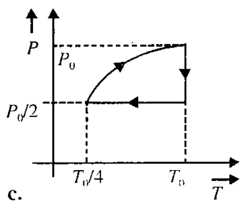
118. One mole of an ideal gas at pressure P_0 and temperature T_0 is expanded isothermally to twice its volume and then compressed at constant pressure to $(V_0/2)$ and the gas is brought back to original state by a process in which $P \propto V$ (pressure is directly proportional to volume). The correct temperature of the process is



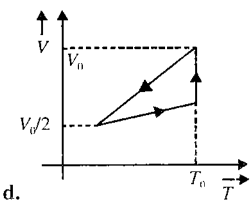
a.



b.



c.



d.

119. P–T diagram is shown in Fig. 2.125. Choose the corresponding V–T diagram.

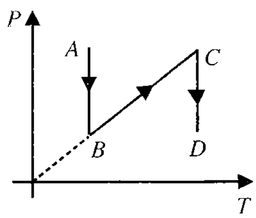


Fig. 2.125

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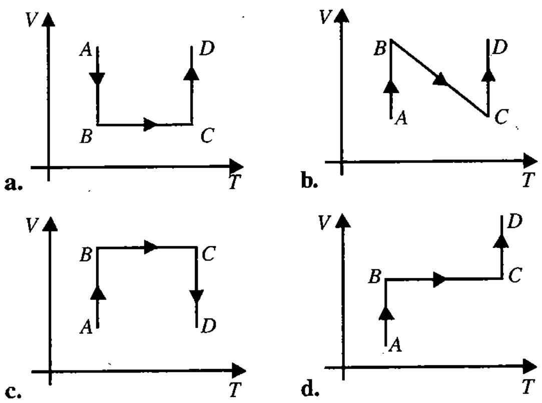

120. A diatomic ideal gas undergoes a thermodynamic change according to the P–V diagram shown in Fig. 2.126. The total heat given to the gas is nearly

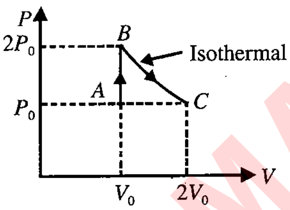

Fig. 2.126

a. $2.5P_0V_0$ **b.** $1.4P_0V_0$
c. $3.9P_0V_0$ **d.** $1.1P_0V_0$

121. Find the amount of work done to increase the temperature of 1 mol of an ideal gas by 30°C if it is expanding under the condition $V \propto T^{2/3}$.

a. 166.2 J **b.** 136.2 J
c. 126.2 J **d** none of these

122. A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric process, process 2 is isothermal process and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas in these three processes. Then

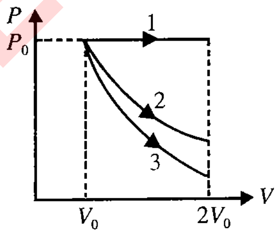

Fig. 2.127

a. $\Delta U_1 > \Delta U_2 > \Delta U_3$
b. $\Delta U_1 < \Delta U_2 < \Delta U_3$
c. $\Delta U_2 < \Delta U_1 < \Delta U_3$
d. $\Delta U_2 < \Delta U_3 < \Delta U_1$

123. Logarithms of readings of pressure and volume for an ideal gas were plotted on a graph as shown in Fig. 2.128. By measuring the gradient, it can be shown that the gas may be

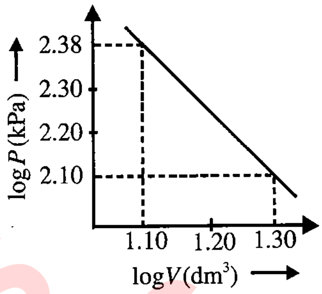

Fig. 2.128

a monatomic and undergoing an adiabatic change
b monatomic and undergoing an isothermal change
c. diatomic and undergoing an adiabatic change
d. triatomic and undergoing an isothermal change

124. Two moles of an ideal monatomic gas undergoes a cyclic process as shown in Fig. 2.129. The temperatures in different states are given as $6T_1 = 3T_2 = 2T_4 = T_3 = 1800$ K. Determine the work done by the gas during the cycle.

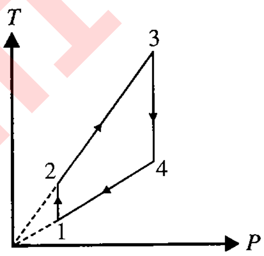

Fig. 2.129

a. –10 kJ **b.** –20 kJ
c. –15 kJ **d.** –30 kJ

125. n moles of gas in a cylinder under a piston is transferred infinitely slowly from a state with a volume of V_0 and a pressure $3P_0$ to a state with a volume of $3V_0$ and a pressure P_0 as shown in Fig. 2.130. The maximum temperature that the gas will reach in this process is

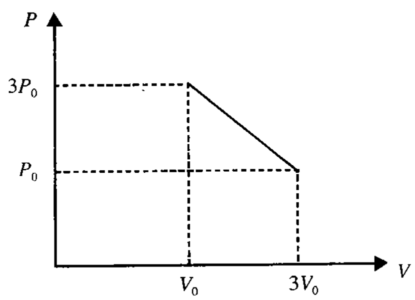

Fig. 2.130

a. $\dfrac{P_0V_0}{nR}$ **b.** $\dfrac{3P_0V_0}{nR}$
c. $\dfrac{4P_0V_0}{nR}$ **d.** $\dfrac{2P_0V_0}{nR}$

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126. A vessel contains a mixture of 7 g of nitrogen and 11 g of carbon dioxide at temperature $T = 300$ K. If the pressure of the mixture is 1 atm (1×10^5 N/m^2), its density is (gas constant $R = 25/3$ J/mol K)

a. 0.72 kg/m^3
b. 1.44 kg/m^3
c. 2.88 kg/m^3
d. 5.16 kg/m^3

127. The mass of a gas molecule can be computed from the specific heat at constant volume. C_v for argon is 0.075 kcal/kg K. The molecular weight of an argon atom is (R = 2 cal/mol K).

a. 40 kg
b. 40×10^{-3} kg
c. 20 kg
d. 20×10^{-3} kg

128. Figure 2.131 shows five paths traversed by a gas on a P–V diagram. ΔU_1, ΔU_2, ΔU_3, ΔU_4 and ΔU_5 are the change in internal energy of the gas in paths 1, 2, 3, 4 and 5, respectively. Then

a. $\Delta U_5 > \Delta U_3$ **b.** $\Delta U_3 > \Delta U_5$
c. $\Delta U_1 > \Delta U_2$ **d.** $\Delta U_2 > \Delta U_5$

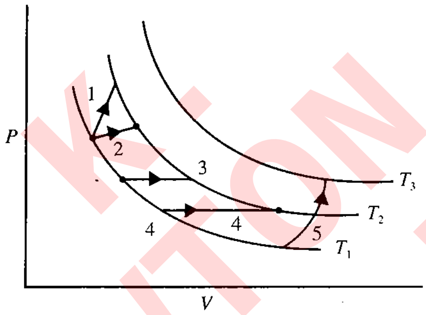


Fig. 2.131

129. The molar heat capacity varies as $C = C_v + \beta V$. Then the equation of the process for an ideal gas is given as

a. $T^{\frac{\beta}{RV}}$ = constant

b. $V^{\frac{\beta T}{R}}$ = constant

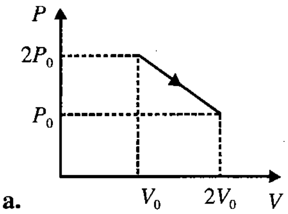
c. $T^{-\frac{R}{\beta V}}$ = constant

d. $V^{\frac{R}{\beta T}}$ = constant

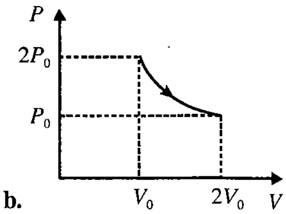
130. The P–V equation for a process of an ideal gas is given as

$$P = \frac{12P_0}{V_0}V - \frac{4P_0}{V_0^2}V^2 - 7P_0$$

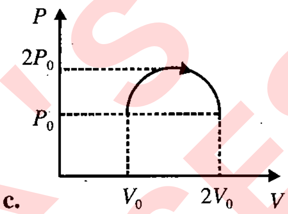
The graphical representation of the above process is shown as



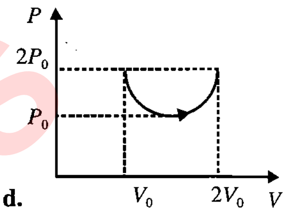
a.



b.



c.



d.

131. Three processes compose a thermodynamic cycle shown in the accompanying P–V diagram. Process 1→2 takes place at constant temperature, process 2→3 takes place at constant volume and process 3→1 is adiabatic. During the complete cycle the total amount of work done is 10 J. During process 3→1, 20 J of work is done on the system. Which of the following statements is incorrect?

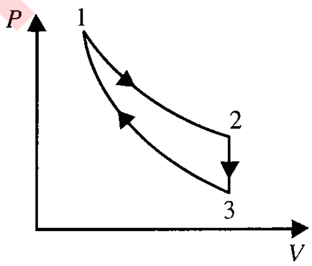


Fig. 2.132

a. $(\Delta U)_{\text{cycle}} = 0$
b. $(\Delta Q)_{\text{cycle}} = 10$ J
c. $(\Delta Q)_{1\to 2} = 30$ J
d. During process 1→ 2, work is done on the system.

132. A certain balloon maintains an internal gas pressure of $P_0 = 100$ kPa until the volume reaches $V_0 = 20$ m^3. Beyond a volume of 20 m^3, the internal pressure varies as $P = P_0 + 2k(V - V_0)^2$ where P is in kPa, V is in m^3 and k is a constant ($k = 1$ kPa/m^3). Initially the balloon contains helium gas at 20°C, 100 kPa with a 15 m^3 volume. The balloon is then heated until the volume becomes 25 m^3 and the pressure is 150 kPa. Assume ideal gas behaviour for helium. The work done by the balloon for the entire process in kJ is

a. 1256 **b.** 1414
c. 1083 **d.** 1512

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133. A spherical balloon contains air at temperature T_0 and pressure P_0. The balloon material is such that the instantaneous pressure inside is proportional to the square of the diameter. When the volume of the balloon doubles as a result of heat transfer, the expansion follows the law

a. PV = constant
b. $PV^{2/5}$ = constant
c. PV^{-1} = constant
d. $PV^{-2/3}$ = constant

Multiple Correct Answers Type

Solutions on page 2.100

1. Select the correct alternatives for an ideal gas:

a. The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.

b. The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.

c. The internal energy does not change in an isothermal process.

d. No heat is added or removed in an adiabatic process.

2. In the cyclic process shown in Fig. 2.133, ΔU_1 and ΔU_2 represent the change in internal energy in process A and B, respectively. If ΔQ be the net heat given to the system in the process and ΔW be the work done by the system in the process, then

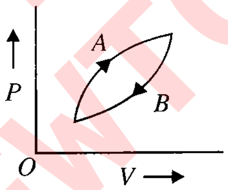

Fig. 2.133

a. $\Delta U_1 + \Delta U_2 = 0$
b. $\Delta U_1 - \Delta U_2 = 0$
c. $\Delta Q - \Delta W = 0$
d. $\Delta Q + \Delta W = 0$

3. An ideal gas is taken from state A (pressure P, volume V) to state B (pressure $P/2$, volume $2V$) along a straight line path in the P–V diagram. Select the correct statements from the following:

a. The work done by the gas in the process A to B exceeds the work done that would be done by it if the system were taken from A to B along an isotherm.

b. In the T–V diagram, the path AB becomes a part of a parabola.

c. In the P–T diagram, the path AB becomes a part of hyperbola.

d. In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

4. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S into two equal parts A and B. Part A has an ideal gas at pressure P_0 and temperature T_0 and part B is vacuum. A massless spring of force constant K is connected with the piston and the wall of the container as shown. Initially the spring is unstretched. The gas inside chamber A is allowed to expand. Let in equilibrium the spring be compressed by x_0. Then

Fig. 2.134

a. final pressure of the gas is $\dfrac{Kx_0}{S}$

b. work done by the gas is $\dfrac{1}{2}Kx_0^2$

c. change in internal energy of the gas is $\dfrac{1}{2}Kx_0^2$

d. temperature of the gas is decreased

5. In the arrangement shown in Fig. 2.135, gas is thermally insulated. An ideal gas is filled in the cylinder having pressure P_0 (> atmospheric pressure P_a). The spring of force constant K is initially unstretched. The piston of mass m and area S is frictionless. In equilibrium, the piston rises up by distance x_0, then

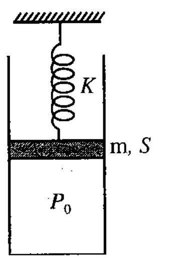

Fig. 2.135

a. final pressure of the gas is $P_0 + \dfrac{Kx_0}{S} + \dfrac{mg}{S}$

b. work done by the gas is $\dfrac{1}{2}Kx_0^2 + mgx_0$

c. decrease in internal energy of the gas is

$\dfrac{1}{2}Kx_0^2 + mgx_0 + P_0Sx_0$

d. all of the above

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6. A gas undergoes change in its state from position A to position B via three different paths as shown in Fig. 2.136. Select the correct alternatives:

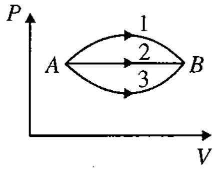


Fig. 2.136

a. Change in internal energy in all the three paths is equal.
b. In all the three paths heat is absorbed by the gas.
c. Heat absorbed/released by the gas is maximum in path (1).
d. Temperature of the gas first increases and then decreases continuously in path (1).

7. An ideal gas undergoes a thermodynamic cycle as shown in Fig. 2.137. Which of the following statements are correct?

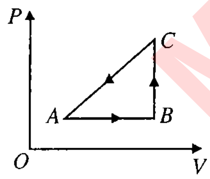


Fig. 2.137

a. Straight line AB cannot pass through O.
b. During process AB, temperature decreases while during process BC it increases.
c. During process BC, work is done by the gas against external pressure and temperature of the gas increases.
d. During process CA, work is done by the gas against external pressure and heat supplied to the gas is exactly equal to this work.

8. At ordinary temperatures, the molecules of an ideal gas have only translational and rotational kinetic energies. At high temperatures they may also have vibrational energy. As a result of this, at higher temperature

a. $C_v = \frac{3R}{2}$ for a monatomic gas
b. $C_v > \frac{3R}{2}$ for a monatomic gas
c. $C_v < \frac{5R}{2}$ for a diatomic gas
d. $C_v > \frac{5R}{2}$ for a diatomic gas

9. The molar heat capacity for an ideal gas cannot
a. be negative
b. be equal to either C_v or C_P
c. lie in the range $C_v \le C \le C_p$
d. it may have any value between $-\infty$ and $+\infty$

10. A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times that of B and mass of gas A contained in the vessel is 2 times that of B. Which of the following statements are correct?
a. Average kinetic energy per molecule of A is equal to that of B.
b. Root-mean-square value of translational velocity of B is four times that of A.
c. Pressure exerted by B is eight times of that exerted by A.
d. Number of molecules of B, in the cylinder, is eight times that of A.

11. An ideal gas undergoes a thermodynamic cycle as shown in Fig. 2.138. Which of the following graphs represent the same cycle?

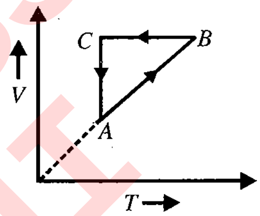
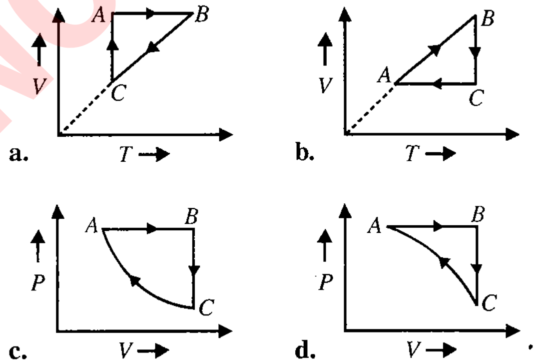


Fig. 2.138



12. Which of the following statements are correct?
a. Two bodies at different temperatures T_1 and T_2 are brought in thermal contact. When thermal equilibrium is attained, the temperature of each body is $(T_1 + T_2)/2$.
b. The coolant used in a car or a chemical or nuclear plant should have high specific heat.
c. Vapour in equilibrium with its liquid at a constant temperature does not obey Boyle's law.
d. Two vessels A and B of equal capacity are connected to each other by a stop cock. Vessel A contains a gas at 0° C and 1 atm pressure. Vessel B is completely evacuated. When the stop cock is opened, the final pressure of gas in each vessel will be 0.5 atm.

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13. Figure 2.139 shows the P–V diagram for a Carnot cycle. In this diagram:

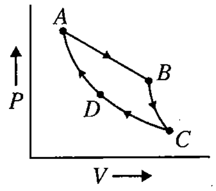

Fig. 2.139

a. curve AB represents isothermal process and BC adiabatic process
b. curve AB represents adiabatic process and BC isothermal process
c. curve CD represents isothermal process and DA adiabatic process
d. curve CD represents adiabatic process and DA isothermal process

14. Figure 2.140 shows an indicator diagram. During path 1–2–3, 100 cal is given to the system and 40 cal worth work is done. During path 1–4–3, the work done is 10 cal. Then

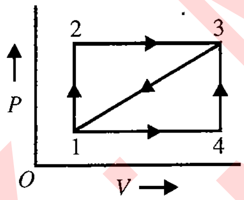

Fig. 2.140

a. heat given to the system during path 1–4–3 is 70 cal
b. if the system is brought from 3 to 1 along straight line path 3–1, work done is worth 25 cal
c. along straight line path 3–1, the heat ejected by the system is 85 J
d. the internal energy of the system in state 3 is 140 cal above that in state 1

15. One mole of an ideal monatomic gas has initial temperature T_0, is made to go through the cycle $abca$ shown in Fig. 2.141. If U denotes the internal energy, then choose the correct alternative.

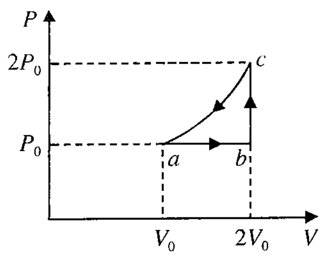

Fig. 2.141

a. $U_c > U_b > U_a$
b. $U_c - U_b = 3RT_0$
c. $U_c - U_a = \dfrac{9RT_0}{2}$
d. $U_b - U_a = \dfrac{3RT_0}{2}$

16. P–V diagram of a cyclic process $ABCA$ is as shown in Fig. 2.142. Choose the correct alternative.

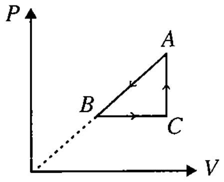

Fig. 2.142

a. $\Delta Q_{A\to B}$ is negative
b. $\Delta U_{B\to C}$ is positive
c. $\Delta U_{C\to A}$ is negative
d. ΔW_{CAB} is negative

17. During the process AB of an ideal gas

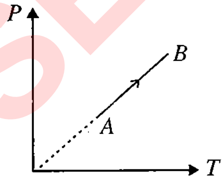

Fig. 2.143

a. work done on the gas is zero
b. density of the gas is constant
c. slope of line AB from the T-axis is inversely proportional to the number of moles of the gas
d. slope of line AB from the T-axis is directly proportional to the number of moles of the gas

18. Temperature versus pressure graph of an ideal gas is shown in Fig. 2.144. During the process AB

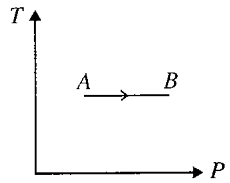

Fig. 2.144

a. internal energy of the gas remains constant
b. volume of the gas is increased
c. work done by the atmosphere on the gas is positive
d. pressure is inversely proportional to volume.

19. An ideal gas undergoes the cyclic process shown in a graph below:

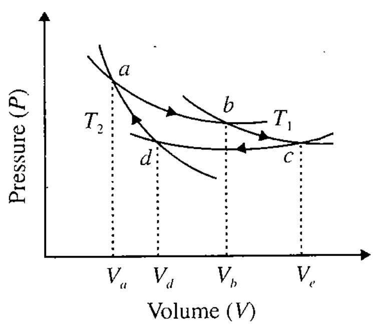

Fig. 2.145

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a. $T_1 = T_2$ **b.** $T_1 > T_2$

c. $V_aV_c = V_bV_d$ **d.** $V_aV_b = V_cV_d$

20. The indicator diagram for two processes 1 and 2 carrying on an ideal gas is shown in Fig. 2.146. If m_1 and m_2 be the slopes (dP/dV) for Process 1 and Process 2, respectively, then

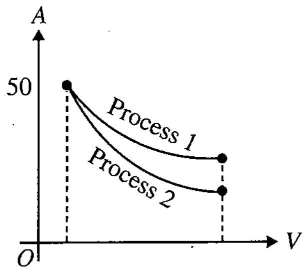


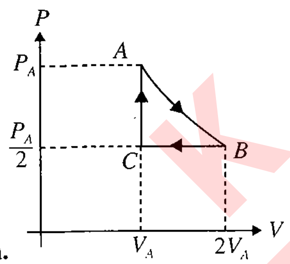
Fig. 2.146

a. $m_1 = m_2$ **b.** $m_1 > m_2$

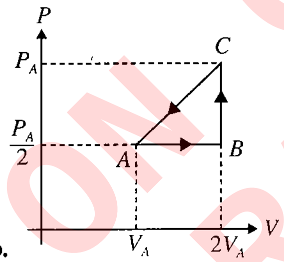
c. $m_1 < m_2$ **d.** $m_2C_V = m_1C_P$

21. Three moles of an ideal gas $C_P = 7/2R$ at pressure P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally the gas is compressed at constant volume to the original pressure P_A. The correct P–V and P–T diagrams indicating the process are

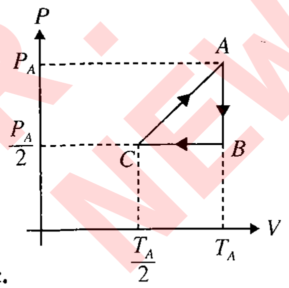
a.



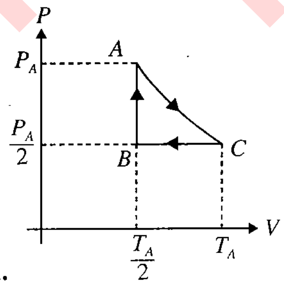
b.



c.



d.



22. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V_0$ along a straight line path in the P–V diagram. Select the correct statement(s) from the following:

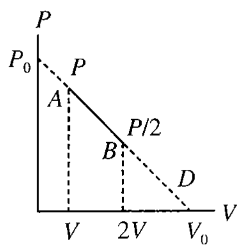


Fig. 2.147

a. The work done by the gas in process AB is greater than the work that would be done if the system were taken from A to B along the isotherm

b. In the T–V diagram, the path AB becomes a part of parabola.

c. In the P–T diagram, the path AB becomes a part of hyperbola

d. In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases

23. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statements is/are correct, with reference to the final equilibrium position?

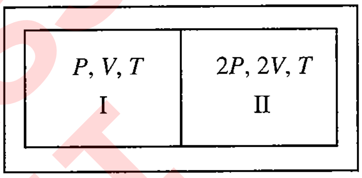


Fig. 2.148

a. The pressures in the two compartments are equal.

b. Volume of compartment I is $3V/5$

c. Volume of compartment II is $12V/5$

d. Final pressure in compartment I is $5P/3$

24. During an experiment, an ideal gas is found to obey a condition P^2/ρ = constant (ρ = density of the gas). The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\rho/2$.

a. The pressure of the gas changes to $\sqrt{2}P$.

b. The temperature of the gas changes to $\sqrt{2}T$.

c. The graph of the above process on the P–T diagram is parabola.

d. The graph of the above process on the P–T diagram is hyperbola.

25. Pick the correct statement(s):

a. The rms translational speed for all ideal gas molecules at the same temperature is not the same but it depends on the mass.

b. Each particle in a gas has average translational kinetic energy and the equation $1/2\, mv^2_{max} = 3/2\, kT$ establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas.

c. If the temperature of an ideal gas is doubled from 100°C to 200°C, the average kinetic energy of each particle is also doubled.

d. It is possible for both pressure and volume of a monatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

26. An ideal gas undergoes an expansion from a state with temperature T_1 and volume V_1 through three different polytropic processes A, B and C as shown in the P–V

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diagram. If $|\Delta E_A|$, $|\Delta E_B|$ and $|\Delta E_C|$ be the magnitude of changes in internal energy along the three paths respectively, then:

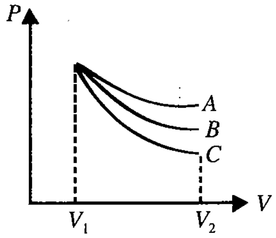

Fig. 2.149

a. $|\Delta E_A| < |\Delta E_B| < |\Delta E_C|$ if temperature in every process decreases

b. $|\Delta E_A| > |\Delta E_B| > |\Delta E_C|$ if temperature in every process decreases

c. $|\Delta E_A| > |\Delta E_B| > |\Delta E_C|$ if temperature in every process increases

d. $|\Delta E_B| < |\Delta E_A| < |\Delta E_C|$ if temperature in every process increases

27. A system undergoes three quasi-static processes sequentially as indicated in Fig. 2.150. $1-2$ is an isobaric process, $2-3$ is a polytropic process with $\gamma = 4/3$ and $3-1$ is a process in which PV = constant. $P_2 = P_1 = 4 \times 10^5$ N/m², $P_3 = 1 \times 10^5$ N/m² and $V_1 = 1$ m³.
The heat transfer for the cycle is ΔQ, the change in internal energy is ΔU and the work done is ΔW. Then

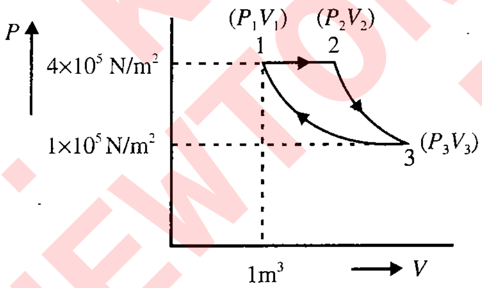

Fig. 2.150

a. $\Delta W = 0$ **b.** $\Delta Q = 1.08 \times 10^5$J
c. $\Delta U = 0$ **d.** $\Delta Q > \Delta W$

28. An insulated 0.2 m³ tank contains helium at 1200 kPa and 47°C. A valve is now opened, allowing some helium to escape. The valve is closed when one-half of the initial mass has escaped. The temperature of the gas is $(\sqrt[3]{4} = 1.6)$.

a. 100 K **b.** 200 K
c. 73°C **d.** − 73°C

29 A gas in container A is in thermal equilibrium with another gas in container B. Both contain equal masses of the two gases. Which of the following can be true?

a. $P_A V_A = P_B V_B$ **b.** $P_A = P_B, V_A \neq V_B$
c. $P_A \neq P_B, V_A = V_B$ **d.** $\dfrac{P_A}{V_A} = \dfrac{P_B}{V_B}$

Assertion-Reasoning Type

Solutions on page 2.105

In the following questions, a statement of assertion (Statement I) is given which is followed by a corresponding statement of reason (Statement II). Examine the statements carefully and choose the correct option according to the following options.

a. Statement I is true, Statement II is true and Statement II is the correct explanation for Statement I.
b. Statement I is true, Statement II is true and Statement II is NOT the correct explanation for Statement I.
c. Statement I is true, Statement II is false.
d. Statement I is false, Statement II is true.

1. Statement I: A quasi-static process is so called because it is a sudden and large change of the system.
Statement II: An adiabatic process is not quasi-static because it is a sudden and large change of the system.

2. Statement I: The work done on an ideal gas in changing its volume from V_1 to V_2 under a polytropic process is given by the integral $\int_{V_1}^{V_2} P \cdot dV$ taken along the process.
Statement II: No work is done under an isochoric process of the gas.

3. Statement I: When an ideal gas is taken from a given thermodynamics state A to another given thermodynamic state B by any polytropic process, the change in the internal energy of the system will be the same in all processes.
Statement II: Internal energy of the gas depends only upon its absolute temperature.

4. Statement I: The specific heat of a gas in an adiabatic process is zero but it is infinite in an isothermal process.
Statement II: Specific heat of a gas is directly proportional to heat exchanged with the system and inversely proportional to change in temperature.

5. Statement I: Work done by a gas in isothermal expansion is more than the work done by the gas in the same expansion adiabatically.
Statement II: Temperature remains constant in isothermal expansion but not in adiabatic expansion.

6. Statement I: A gas is expanded from a volume V to $2V$, first through adiabatic process then through isothermal process. Work done in isothermal process is more if final stage (i.e., pressure and volume) in both the cases is same.

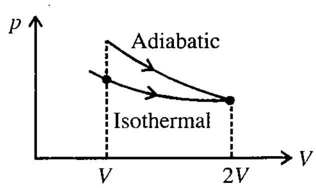

Fig. 2.151

Statement II: Work done by the gas is equal to area under p–V curve.

7. Statement I: In an isothermal process whole of the heat energy supplied to the body is converted into internal energy.

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Statement II: According to the first law of thermodynamics $\Delta Q = \Delta U + P\Delta V$.

8. **Statement I:** In an adiabatic process, change in internal energy of a gas is equal to work done on or by the gas in the process.
Statement II: Temperature of gas remains constant in an adiabatic process.

9. **Statement I:** In an adiabatic process, change in internal energy of a gas is equal to work done on or by the gas in the process.
Statement II: Temperature of gas remains constant in a adiabatic process.

10. **Statement I:** An ideal gas is enclosed within a container fitted with a piston. When volume of this enclosed gas is increased at constant temperature, the pressure exerted by the gas on the piston decreases.
Statement II: In the above situation the rate of molecules striking the piston decreases. If the rate at which molecules of a gas having same average speed striking a given area of the wall decreases, the pressure exerted by gas on the wall decreases.

Comprehension Type

Solutions on page 2.106

For Problems 1–3

A fixed mass of gas is taken through a process $A \to B \to C \to A$. Here $A \to B$ is isobaric, $B \to C$ is adiabatic and $C \to A$ is isothermal.

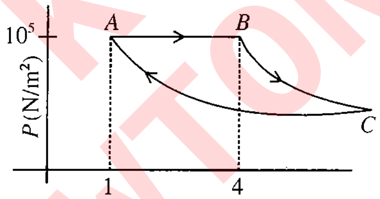


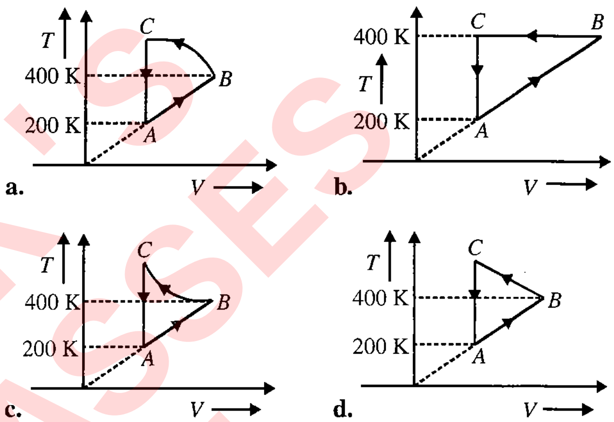
Fig. 2.152

1. Find pressure at C.
   a. $\frac{10^5}{64}\,\mathrm{N/m^2}$ **b.** $\frac{10^5}{32}\,\mathrm{N/m^2}$
   c. $\frac{10^5}{12}\,\mathrm{N/m^2}$ **d.** $\frac{10^5}{6}\,\mathrm{N/m^2}$
2. Find volume at C.
   a. 32 m^3 **b.** 100 m^3
   c. 64 m^3 **d.** 25 m^3
3. Find work done in the process (take $\gamma = 1.5$).
   a. 4.9×10^5 J **b.** 3.2×10^5 J
   c. 1.2×10^5 J **d.** 7.2×10^5 J

For Problems 4–6

Four moles of an ideal gas is initially in state A having pressure 2×10^5 N/m^2 and temperature 200 K. Keeping the pressure constant the gas is taken to state B at temperature of 400 K. The gas is then taken to a state C in such a way that its temperature increases and volume decreases. Also from B to C, the magnitude of dT/dV increases. The volume of gas at state C is equal to its volume at state A. Now gas is taken to initial state A keeping volume constant. A total of 1000 J of heat is withdrawn from the sample in the cyclic process. Take $R = 8.3$ J/K/mol.

4. Which graph between temperature T and volume V for the cyclic process is correct.



5. The work done in path B to C is
   a. 1000 J **b.** –1000 J
   c. –7640 J **d.** 5640 J
6. The volume of gas at state C is
   a. 0.0332 m^3 **b.** 0.22 m^3
   c. 0.332 m^3 **d.** 3.32 m^3.

For Problems 7–9

A process in which work performed by an ideal gas is proportional to the corresponding increment of its internal energy is described as a polytropic process. If we represent work done by a polytropic process by W and increase in internal energy as ΔU then

$$W \propto \Delta U$$

or
$$W = K_1 \Delta U \qquad \text{(i)}$$

For this process, it can be demonstrated that the relation between pressure and volume is given by the equation

$$PV^{\eta} = K_2 \text{ (constant)} \qquad \text{(ii)}$$

We know that a gas can have various values for molar specific heats. The molar specific heat 'C' for an ideal gas in polytropic process can be calculated with the help of first law of thermodynamics. In polytropic process the variation of molar specific heat 'C' with η for a monatomic gas is plotted as in the graph shown.

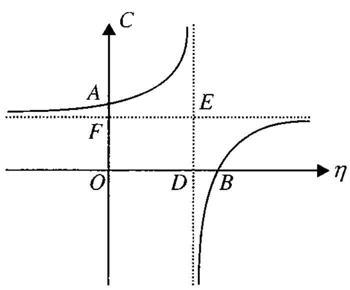


Fig. 2.153

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7. In the graph shown, the y-coordinate of point A is (for monatomic gas)

a. $3R/2$ b. $5R/2$
c. $7R/2$ d. $4R$

8. In the graph shown, the x-coordinate of point B is (for monatomic gas)

a. $7/5$ b. $5/3$
c. $2/3$ d. $8/3$

9. For a monatomic gas, the values of polytropic constant η for which molar specific heat is negative is

a. $1<\eta<\frac{2}{3}$ b. $1<\eta<\frac{8}{3}$
c. $1<\eta<\frac{5}{3}$ d. $\frac{2}{3}<\eta<\frac{8}{3}$

For Problems 10–12

Figure 2.154 shows the variation of potential energy (U) of 2 mol of Argon gas with its density in a cyclic process $ABCA$. The gas was initially in the state A whose pressure and temperature are P_A = 2 atm, T_A = 300 K, respectively. It is also stated that the path AB is a rectangular hyperbola and the internal energy of the gas at state C is 3000 R. Based on the above information answer the following questions:

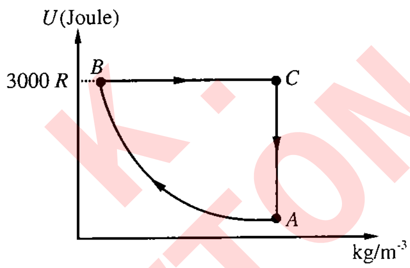

Fig. 2.154

10. a. The process AB is isobaric, BC is adiabatic and CA is isochoric.
b. The process AB is adiabatic, BC is isothermal and CA is isochoric.
c. The process AB is isochoric, BC is isothermal and CA is isobaric
d. The process AB is isobaric, BC is isothermal and CA is isochoric.

11. The heat supplied to the gas in the process AB is

a. $700R$ b. $3500R$
c. $4400R$ d. $1600R$

12. Heat supplied in the process CA is

a. $-1400R$ b. $1400R$
c. $2100R$ d. $-2100R$

For Problems 13–15

One mole of an ideal gas has an internal energy given by $U = U_0 + 2\,PV$, where P is the pressure and V the volume of the gas. U_0 is a constant. This gas undergoes the quasi-static cyclic process $ABCD$ as shown in the U–V diagram.

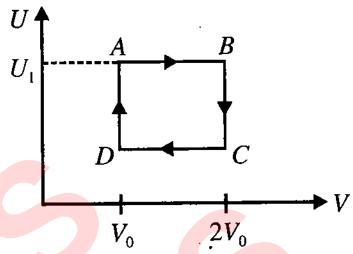

Fig. 2.155

13. The molar heat capacity of the gas at constant pressure is

a. $2R$ b. $3R$
c. $\frac{5}{2}R$ d. $4R$

14. The work done by the ideal gas in the process AB is

a. zero b. $\frac{U_1-U_0}{2}$
c. $\frac{U_0-U_1}{2}$ d. $\frac{U_1-U_0}{2}\log_e 2$

15. The gas must be

a. monatomic
b. diatomic
c. a mixture of mono and diatomic gases
d. a mixture of di- and tri-atomic gases

For Problems 16–18

An ideal diatomic gas is expanded so that the amount of heat transferred to the gas is equal to the decrease in its internal energy.

16. The molar specific heat of the gas in this process is given by C whose value is

a. $-\frac{5R}{2}$ b. $-\frac{3R}{2}$
c. $2R$ d. $\frac{5R}{2}$

17. The process can be represented by the equation TV^n = constant, where the value of n is

a. $n=\frac{7}{5}$ b. $n=\frac{1}{5}$
c. $n=\frac{3}{2}$ d. $n=\frac{3}{5}$

18. If in the above process, the initial temperature of the gas be T_0 and the final volume be 32 times the initial volume, the work done (in joules) by the gas during the process will be

a. RT_0 b. $\frac{5RT_0}{2}$
c. $2RT_0$ d. $\frac{RT_0}{2}$

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For Problems 19–21

A cylinder containing an ideal gas (see Fig. 2.156) and closed by a movable piston is submerged in an ice–water mixture. The piston is quickly pushed down from position (1) to position (2) (process AB). The piston is held at position (2) until the gas is again at 0°C (process BC). Then the piston is slowly raised back to position (1) (process CA).

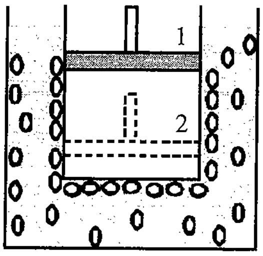
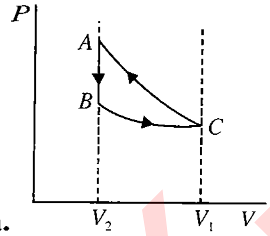
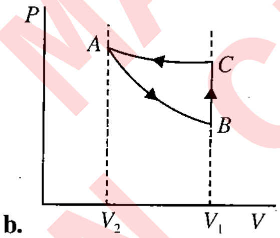
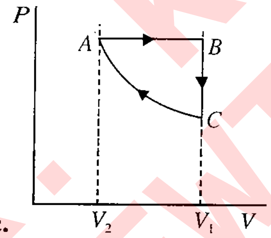


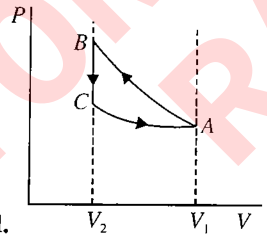
Fig. 2.156

19. Which of the following P–V diagrams will correctly represent the processes AB, BC and CA and the cycle $ABCA$?

a.


b.


c.


d.


20. If 100 g of ice is melted during the cycle $ABCA$, how much work is done on the gas?

a. 8 kcal **b.** 5 kcal
c. 2.1 kJ **d.** 4.2 kJ

21. If the change in the volume is $(V_1 - V_2) = V\text{ m}^3$, the work done (in N/m²) during the cycle is, with P being the atmospheric pressure acting on the piston.

a. $\dfrac{PV}{2}$ J **b.** $\dfrac{2PV}{3}$ J

c. PV J **d.** None of these

For Problems 22–24

A reversible heat engine carries 1 mol of an ideal monatomic gas around the cycle $ABCA$, as shown in the diagram. The process BC is adiabatic. Call the processes AB, BC and CA as 1, 2 and 3 and the heat $(\Delta Q)_r$, change in internal energy $(\Delta U)_r$ and the work done $(\Delta W)_r$, r = 1, 2, 3 respectively. The temperature at A, B, C are T_1 = 300 K, T_2 = 600 K and T_3 = 455 K. Indicate the pressure and volume at A, B and C by P_r and V_r, r = 1, 2, 3, respectively. Assume that initially pressure P_1 = 1.00 atm.

22. Which of the following represents the correct values of the quantities indicated?

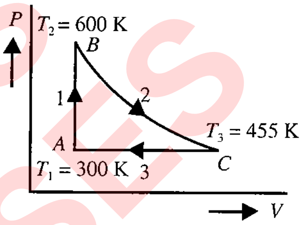


Fig. 2.157

a. $(\Delta Q)_1 = 450R$ J, $(\Delta U)_1 = 450R$ J, $(\Delta W)_1 = 300R$ J
b. $(\Delta Q)_2 = 0$, $(\Delta U)_2 = 450R$ J, $(\Delta W)_2 = -217.5\,R$ J
c. $(\Delta Q)_3 = 0$, $(\Delta U)_3 = -232.5R$ J, $(\Delta W)_3 = 0$
d. $(\Delta Q)_1 = 450R$ J, $(\Delta U)_3 = -232.5R$ J, $(\Delta W)_2 = 217.5R$ J

23. Which of the following represents the correct values for the quantities indicated?

a. $V_1 = 3 \times 10^{-3}\,R$ m³, P_2 = 0.5 atm
b. $V_2 = 3 \times 10^{-3}\,R$ m³, P_3 = 1.0 atm
c. $V_3 = 3 \times 10^{-3}\,R$ m³, P_2 = 2.0 atm
d. $V_2 = 3 \times 10^{-3}\,R$ m³, P_3 = 2.0 atm

24. Suppose the gas had been taken along an isothermal process from B instead of along the adiabatic process BC shown, so as to reach the same volume V_3, and the total heat Q_i is taken during the complete cycle involving the isothermal, while Q_a is the total heat taken in the cycle ABCA shown in the diagram. Then

a. $Q_i < Q_a$
b. $Q_i = Q_a$
c. $Q_i > Q_a$
d. Q_i will be greater or less than Q_a depending upon the value of T_3

For Problems 25–26

A monatomic ideal gas undergoes the shown cyclic process in which path of the process 2→3 is a semicircle. If 2 mol of gas is taken, find

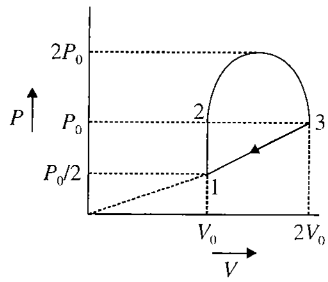


Fig. 2.158

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25. Heat given to the system in semicircular process

a. $\frac{P_0V_0}{2}\left(7-\frac{\pi}{4}\right)$ b $\frac{P_0V_0}{3}\left(7+\frac{\pi}{4}\right)$

c. $\frac{P_0V_0}{2}\left(7+\frac{\pi}{4}\right)$ d $\frac{P_0V_0}{3}\left(7-\frac{\pi}{4}\right)$

26. Total heat rejected in one cycle

a. $\frac{P_0V_0(32-\pi)}{4}$ b $\frac{P_0V_0(32+\pi)}{4}$

c. $\frac{P_0V_0(32+\pi)}{8}$ d $\frac{P_0V_0(32-\pi)}{8}$

For Problems 27–28

In a thermodynamic process a gas expands such that the heat transferred to the gas (dQ) = decrease in internal energy ($-dU$). Find

27. The molar heat capacity

a. $\frac{R}{1-\gamma}$ b. $\frac{2R}{\gamma-1}$ c. $\frac{R}{2\gamma-1}$ d. $\frac{R}{\gamma+2}$

28. The equation of the process in variables T and V

a. $T^2 V^{\frac{(\gamma-1)}{2}}$ = constant b. $TV^{\gamma-1}$ = constant

c. $T^{\gamma} V^{\frac{\gamma-1}{2}}$ = constant d. $T\ V^{\frac{(\gamma-1)}{2}}$ = constant

For Problems 29–30

A gas takes part in two thermal processes in which it is heated from the same initial state to the same final temperature. The processes are shown on the P–V diagram by straight lines 1 → 3 and 1 → 2. From above description following conclusions can be taken:

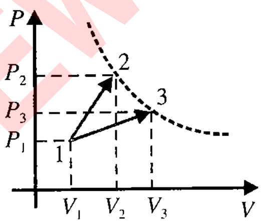

Fig. 2.159

29. a. $W_{13} > W_{12}$ b. $W_{12} > W_{13}$
c. $W_{13} = W_{12}$ d. None of above

30 a. $Q_{13} < Q_{12}$ b. $Q_{13} > Q_{12}$
c. $Q_{13} = Q_{12}$ d. None of above

For Problems 31–36

A monatomic ideal gas of 2 mol is taken through a cyclic process starting from A as shown in Fig. 2.160. The volume ratios are $V_B/V_A = 2$ and $V_D/V_A = 4$. If the temperature T_A at A is 27°C, and gas constant is R. Calculate

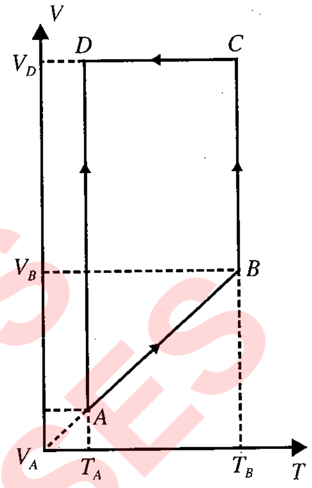

Fig. 2.160

31. The temperature of the gas at point B
a. 500 K b. 700 K
c. 600 K d. 300 K

32. Heat absorbed or released by the gas in process $A \to B$.
a. 1500R, added
b. 1200R ln (2), added
c. 900R, rejected
d. 1200R ln (2), rejected

33. Heat absorbed or released by the gas in process $B \to C$
a. 1500R, added
b. 1200R ln (2), added
c. 900R, rejected
d. 1200R ln (2), rejected

34. Heat absorbed or released by the gas in process $C \to D$.
a. 1500R, added
b. 1200R ln (2), added
c. 900R, rejected
d. 1200R ln (2), rejected

35. Heat absorbed or released by the gas in process $D \to A$.
a. 1500R, added
b. 1200R ln (2), added
c. 900R, rejected
d. 1200R ln (2), rejected

36. The total work done by the gas during the complete cycle.
a. 500R J
b. 600R J
c. 700R J
d. 300R J

For Problems 37–38

One mole of an ideal monatomic gas undergoes thermodynamic cycle 1 → 2 → 3 → 1 as shown in Fig. 2.161. Initial temperature of gas is T_0 = 300 K.

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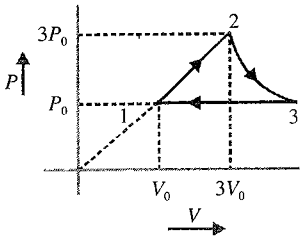

Fig. 2.161

Process 1 → 2: $P = aV$
Process 2 → 3: PV = Constant
Process 3 → 1: P = Constant
(Take ln |3| = 1.09)

37. Find the net work done by the cycle.

a. $3.27RT_0$ **b.** $6.83RT_0$
c. $4.53RT_0$ **d.** $5.81RT_0$

38. Determine the heat capacity of each process.

a. 20.75 J/mol-K **b.** 10.23 J/mol-K
c. 22.37 J/mol-K **d.** 15.96 J/mol-K

For Problems 39–42

A monatomic gas undergoes a cycle consisting of two isothermals and two isobarics. The minimum and maximum temperatures of the gas during the cycle are T_1 = 400 K and T_2 = 800 K, respectively, and the ratio of maximum to minimum volume is 4.

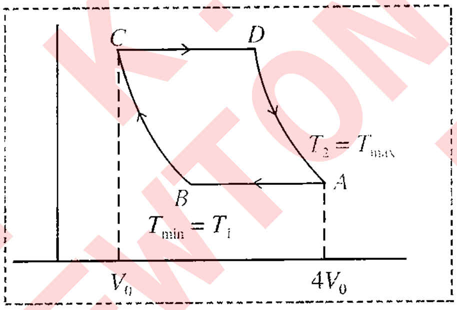

Fig. 2.162

39. The volume at B is

a. $1.5V_0$ **b.** $2V_0$
c. $3V_0$ **d.** $2.5V_0$

40. The volume at D is

a. $1.5V_0$ **b.** $2V_0$
c. $3V_0$ **d.** $2.5V_0$

41. The heat is extracted from the system in process

a. $A \to B$, $B \to C$ **b.** $C \to D$, $D \to A$
c. $A \to B$, $C \to D$ **d.** $B \to C$, $C \to D$

42. The efficiency of the cycle is

a. $\frac{3\ln 2}{5+4\ln 2}\times 100\%$ **b.** $\frac{2\ln 2}{5+4\ln 2}\times 100\%$

c. $\frac{3\ln 3}{5+4\ln 2}\times 100\%$ **d.** $\frac{2\ln 2}{3+4\ln 2}\times 100\%$

For Problems 43–45

Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.

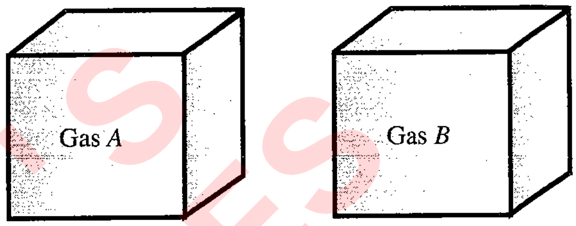

Fig. 2.163

Experiment 1: When the two containers are weighed W_A = 225g, W_B = 160 g and mass of evacuated container W_C = 100 g.
Experiment 2: When the two containers are given same amount of heat, same temperature rise is recorded. The pressure changes found are ΔP_A = 2.5 atm, ΔP_B = 1.5 atm.
Required data for unknown gas:

Mono (molar mass)	He 4 g	Ne 20 g	Ar 40 g	Kr 84 g	Xe 131 g	Rd 222 g
Di (molar mass)	H_2 2 g	F_2 19 g	N_2 28 g	O_2 32 g	Cl_2 71 g	

43. Identify the type of gas filled in containers A and B, respectively.

a. Mono, Mono **b.** Dia, Dia
c. Mono, Dia **d.** Dia, Mono

44. Identify the gas filled in the container A and B.

a. N_2, Ne **b.** He, H_2 **c.** O_2, Ar **d.** Ar, O_2

45. The gases have initial temperature 300 K and they are mixed in an adiabatic container having the same volume as the previous container. Now if the temperature of the mixture is T and pressure is P, then

a. $P > P_A$, $T > 300$ K **b.** $P > P_B$, $T > 300$ K
c. $P > P_A$, $T = 300$ K **d.** $P > P_A$, $T < 300$ K

For Problems 46–50

Figure 2.164 shows an insulated cylinder of volume V containing monatomic gas in both the compartments. The piston is diathermic. Initially the piston is kept fixed and the system is allowed to acquire a state of thermal equilibrium. The initial pressures and temperatures are as shown in the figure. Calculate

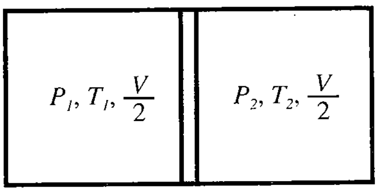

Fig. 2.164

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46. The final temperature

a. $\dfrac{(P_1-P_2)T_1T_2}{P_1T_2-P_2T_1}$ b. $\dfrac{(P_1+P_2)T_1T_2}{P_1T_1-P_2T_2}$

c. $\dfrac{(P_1+P_2)T_1T_2}{P_1T_2+P_2T_1}$ d. $\dfrac{(P_1+P_2)T_1T_2}{P_1T_1-P_2T_2}$

47. The final pressures of left and right compartment respectively are

a. $\dfrac{P_1(P_1+P_2)T_2}{P_1T_2+P_2T_1}, \dfrac{P_2(P_1+P_2)T_1}{(P_1T_2+P_2T_1)}$

b. $\dfrac{P_1(P_1+P_2)T_1}{P_1T_2+P_2T_2}, \dfrac{P_2(P_1+P_2)T_1}{(P_1T_2+P_2T_1)}$

c. $\dfrac{P_1(P_1+P_2)T_2}{P_1T_2-P_2T_1}, \dfrac{P_2(P_1-P_2)T_1}{(P_1T_2+P_2T_1)}$

d. $\dfrac{P_1(P_1+P_2)T_2}{P_1T_2+P_2T_1}, \dfrac{P_2(P_1-P_2)T_1}{(P_1T_2+P_2T_1)}$

48. The heat that flows from RHS to LHS, given $T_2 > T_1$.

a. $\dfrac{4}{3}P_1P_2V\dfrac{(T_2-T_1)}{P_1T_2+P_2T_1}$ b. $\dfrac{3}{4}P_1P_2V\dfrac{(T_2-T_1)}{P_1T_2+P_2T_1}$

c. $\dfrac{3}{4}P_1P_2V\dfrac{(T_2+T_1)}{P_1T_2-P_2T_1}$ d. $\dfrac{3}{4}P_1P_2V\dfrac{(T_2-T_1)}{P_1T_2-P_2T_1}$

Now, the pin which was keeping the piston fixed is removed and the piston is set free to move. The piston is allowed to slide slowly, such that a state of mechanical equilibrium is also achieved and $T_1P_2 > P_1T_2$. Find

49. Final volume of the left compartment.

a. $\dfrac{V}{2}\left(\dfrac{P_1T_2}{P_2T_1+P_1T_2}\right)$ b. $V\left(\dfrac{P_1T_2}{P_2T_1-P_1T_2}\right)$

c. $V\left(\dfrac{P_1T_2}{P_2T_1+P_1T_2}\right)$ d. $V\left(\dfrac{2P_1T_2}{P_2T_1+P_1T_2}\right)$

50. Final volume of right compartment.

a. $\dfrac{V}{2}\left(\dfrac{P_2T_1}{P_2T_1+P_1T_2}\right)$ b. $V\left(\dfrac{P_2T_1}{P_2T_1-P_1T_2}\right)$

c. $V\left(\dfrac{P_2T_1}{P_2T_1+P_1T_2}\right)$ d. $V\left(\dfrac{2P_2T_2}{P_2T_1-P_1T_2}\right)$

For Problems 51–52

The insulated box shown in Fig. 2.165 has an insulated partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has 1 mol of a monatomic ideal gas ($\gamma = 5/3$) at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater so that its gas, pushing the partition, expands until the final pressure in both the chambers becomes $243P_0/32$. Determine

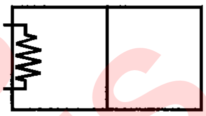

Fig. 2.165

51. Final temperature of the gas in each chamber.

a. $4/9T_0$ b. $4/7T_0$ c. $1/5T_0$ d. $9/4T_0$

52. Work done on the gas in the right chamber

a. $\dfrac{15}{8}RT_0$ b. $-\dfrac{15}{8}RT_0$

c. $\dfrac{7}{4}RT_0$ d. $-\dfrac{7}{4}RT_0$

For Problems 53–55

A container of volume $4V_0$ made of a perfectly non-conducting material is divided into two equal parts by a fixed rigid wall whose lower half is non-conducting and upper half is purely conducting. The right side of the wall is divided into equal parts (initially) by means of a massless non-conducting piston free to move as shown. Section A contains 2 mol of a gas while the section B and C contain 1 mol each of the same gas ($\gamma = 1.5$) at pressure P_0. The heater in left part is switched on till the final pressure in section C becomes $125/27\ P_0$. Calculate

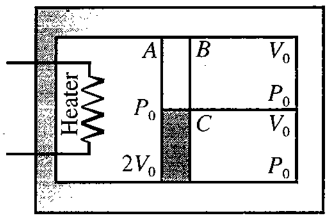

Fig. 2.166

53. Final temperature in part A.

a. $\dfrac{205P_0V_0}{27R}$ b. $\dfrac{P_0V_0}{R}$

c. $\dfrac{105P_0V_0}{13R}$ d. $\dfrac{12P_0V_0}{13R}$

54. Final temperature in part C.

a. $\dfrac{P_0V_0}{R}$ b. $\dfrac{5P_0V_0}{3R}$

c. $\dfrac{P_0V_0}{3R}$ d. $\dfrac{5P_0V_0}{R}$

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55. The heat supplied by the heater.

a. $\frac{368}{9}P_0V_0$ b. $\frac{113}{5}P_0V_0$

c. $\frac{316}{9}P_0V_0$ d. $\frac{405}{8}P_0V_0$

For Problems 56–57

Figure 2.167 shows two identical cylinders A and B which contain equal amounts of an ideal gas with adiabatic exponent γ. The piston of cylinder A is free and that of cylinder B is fixed.

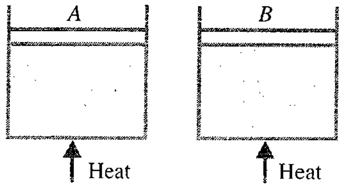

Fig. 2.167

56. If the same amount of heat is absorbed by each cylinder, then in which cylinder the temperature rise is more?
 a. Temperature of A will be more.
 b. Temperature of B will be more.
 c. The temperature of both A and B will be same.
 d. Information is not sufficient to reach the conclusion.

57. If the temperature rise in cylinder A is T_0, then determine the temperature rise in cylinder B.

a. $2T_0$ b. T_0

c. γT_0 d. $\frac{T_0}{\gamma}$

For Problems 58–60

An ideal gas at NTP is enclosed in an adiabatic vertical cylinder having an area of cross section $A = 27$ cm^2 between two light movable pistons as shown in Fig. 2.168. Spring with force constant $k = 3700$ N/m is in a relaxed state initially. Now the lower position is moved upwards a distance $h/2$, h being the initial length of gas column. It is observed that the upper piston moves up by a distance $h/16$. Final temperature of gas $= 4/3 \times 273$ K. Take γ for the gas to be 3/2.

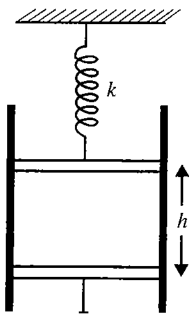

Fig. 2.168

58. When the lower piston is moved upwards a distance $h/2$, the compression
 a. leads to cooling
 b. takes place isothermally
 c. takes place adiabatically
 d. leads to heating

59. Which of the following statements is correct?
 a. Heat given in compression is used to do work against elastic force only.
 b. Heat given in compression is used to do work against elastic force and atmospheric force.
 c. No work is done during compression.
 d. There is no change in internal energy.

60. The value of h is

a. 1 m b. 1.4 m

c. 1.6 m d. 2 m

For Problems 61–63

Piston cylinder device initially contains 0.5 m^3 of nitrogen gas at 400 kPa and 27°C. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120 V source. Nitrogen expands at constant pressure and a heat loss of 2800 J occurs during the process.

$$R = 25/3 \text{ kJ/kmol-K}$$

61. Electric work done on the nitrogen gas is

a. 72 kJ b. 36 kJ

c. 18 kJ d. 9 kJ

62. Number of moles of nitrogen gas is

a. 0.8 k mol b. 0.08 k mol

c. 0.8 mol d. 0.08 mol

63. The final temperature of nitrogen is

a. 2.6°C b. 56.6°C

c. 29.6°C d. 5.67°C

Matching Column Type

Solutions on page 2.113

1. When a sample of a gas is taken from state f along path iaf, heat supplied to the gas is 50 cal and work done by the gas is 20 cal. If it is taken by path ibf, then heat supplied is 36 cal.

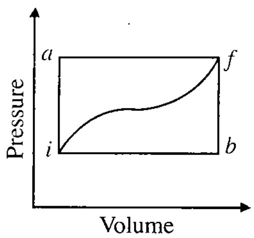

Fig. 2.169

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Column I	Column II
i. Work done by the gas along path *ibf* is	**a.** 6 cal
ii. If work done upon the gas is 13 cal for the return path *fi*, then heat rejected by the gas along path *fi* is	**b.** 18 cal
iii. If internal energy of the gas at state *i* is 10 cal, then internal energy at state *f* is	**c.** 40 cal
iv. If internal energy at state *b* is 22 cal and at *i* is 10 cal then heat supplied to the gas along path *ib* is	**d.** 43 cal

2. Find the change in the internal energy of the system in each of situation in Column I.

Column I	Column II
i. A system absorbs 500 cal of heat and at the same time does 400 J of work.	**a.** −5000 J
ii. A system absorbs 300 cal and at the same time 420 J of work is done on it.	**b.** 1700 J
iii. Twelve hundred calories is removed from a gas held at constant volume.	**c.** 1680 J

3. n kmol of a monochromatic ideal gas is taken quasi-statically from state A to state C along the straight line shown in Fig. 2.170. Alternatively, the same gas is taken quasi-statically from A to C along the path ABC. Express all answers in terms of P_A and V_A.

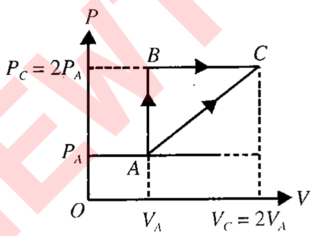


Fig. 2.170

Column I	Column II
i. The heat ΔH added to the gas along the straight line path AC	**a.** $2P_AV_A$
ii. Change in internal energy of the gas along the straight line path AC	**b.** $13/2P_AV_A$
iii. The work done by the gas along the path ABC	**c.** $6P_AV_A$
iv. The heat $\Delta H'$ added to the gas along the path ABC	**d.** $9/2P_AV_A$

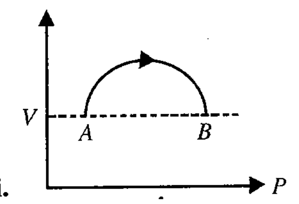
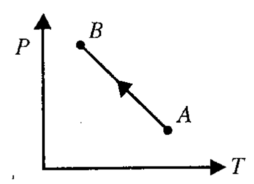
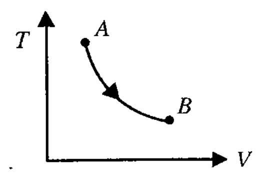
4. The heat capacity of a certain amount of a particular gas at constant pressure is greater than that at constant volume by 29.1 J/K. Match the items given in Column I with the items given in Column II.

Column I	Column II
i. If the gas is monatomic, heat capacity at constant volume	**a.** 131 J/K
ii. If the gas is monatomic, heat capacity at constant pressure	**b.** 43.7 J/K
iii. If the gas is rigid diatomic, heat capacity at constant pressure	**c.** 72.7 J/K
iv. If the gas is vibrating diatomic, heat capacity at constant pressure	**d.** 102 J/K

5. An ideal monatomic gas undergoes different types of processes which are described in Column I. Match the corresponding effects in Column II. The letters have their usual meanings.

Column I	Column II
i. $P = 2V^2$	**a.** If volume increases then temperature will also increase
ii. PV^2 = constant	**b.** If volume increases then temperature decreases
iii. $C = C_V + 2R$	**c.** For expansion, heat will have to be supplied to the gas
iv. $C = C_V - 2R$	**d.** If temperature increases then work done by gas is positive

6. A sample of gas goes from state A to state B in four different manners, as shown by the graphs. Let W be the work done by the gas and ΔU be change in internal energy along the path AB. Correctly match the graphs with the statements provided.

Column I	Column II
i. 	**a.** Both W and ΔU are positive
ii. 	**b.** Both W and ΔU are negative
iii. 	**c.** W is positive whereas ΔU is negative

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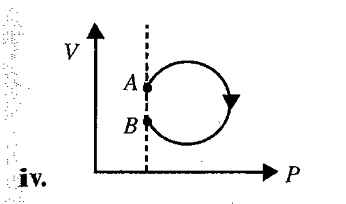
iv. 	**d.** W is negative whereas ΔU is positive

7. A horizontal cylinder with adiabatic walls is closed at both ends and is divided into two parts by a frictionless piston that is also insulating. Initially, the value of pressure and temperature of the ideal gas on each side of the cylinder are V_0, P_0 and T_0, respectively. A heating coil in the right-hand part is used to slowly heat the gas on that side until the pressure in both parts reaches $64P_0/27$. The heat capacity C_V of the gas is independent of temperature and $C_P/C_V = \gamma = 1.5$. Take $V_0 = 16\ \text{m}^3$, $T_0 = 324$ K, $P_0 = 3 \times 10^5$ Pa.

Column I represents the physical parameters of the gas and Column II gives their corresponding values Match Column I with Column II.

Column I	Column II
i. Final left-hand volume (in m^3)	**a.** 432
ii. Final left-hand temperature (in K)	**b.** 9
iii. Final right-hand temperature (in K)	**c.** 1104
iv. Work done (in kJ) on the left-hand gas	**d.** 3200

8. An ideal gas (molar specific heat $C_v = 5R/2$) is taken along paths *acb*, *adb* and *ab*. $P_2 = 2P_1$, $V_2 = 2V_1$. Along *ab*, $P = kV$ where k is a constant. The various parameters are shown in Fig. 2.171. Match Column I with the corresponding options of Column II.

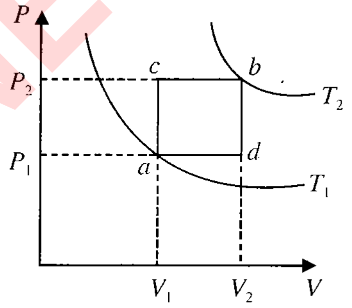


Fig. 2.171

Column I	Column II
i. W_{acb}	**a.** $15RT_1/2$
ii. W_{adb}	**b.** $-15RT_1/2$
iii. ΔU_{ab}	**c.** RT_1
iv. ΔU_{bca}	**d.** $2RT_1$

9. An ideal monatomic gas at initial temperature T_0 expands from initial volume V_0 to volume $2V_0$ by each of the five processes indicated in the T–V diagram in Fig. 2.172.

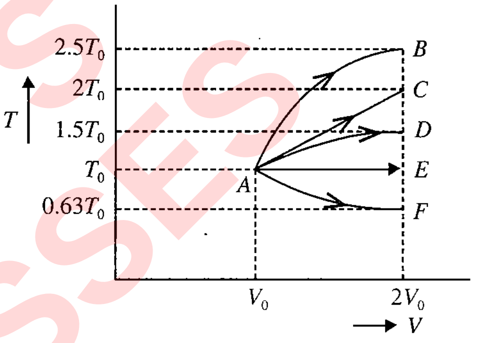


Fig. 2.172

Match Group I with Group II process

Column I	Column II
i. Isothermal	**a.** AB
ii. Isobaric	**b.** AC
iii. Adiabatic	**c.** AD
	d. AE
	e. AF

Integer Answer Type

Solutions on page 2.115

1. Calculate the pressure exerted by a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide in a container of 10 L at a temperature of 27°C [in $10^5\ \text{N/m}^2$ and to the nearest integer].

2. A vessel contains 1 mole O_2 gas at a temperature T. An identical vessel contains one mole of He gas at a temperature $2T$. Find the ratio of their pressure (He to O_2).

3. If P-V diagram of a diatomic gas is plotted, it is a straight line passing through origin. The molar heat capacity of the gas in the process is nR where n is an integer. Find value of n.

4. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. Find the ratio of average rotational kinetic energies per O_2 molecule to per N_2 molecule.

5. A vessel of volume $3V$ contains a gas at pressure $4P_0$ and another vessel of volume $2V$ contains the same gas at pressure $1.5P_0$. Both vessels have same temperature. When both vessels are connected by a tube of negligible

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volume, the equilibrium is IP_0, where I is an integer. Find the value of I.

6. A vessel of volume 0.2 m^3 contains hydrogen gas at temeperature 300 K and pressure 1 bar. Find the heat (in Kcal) required to raise the temperature to 400 K. The molar heat capacity of hydrogen at constant volume is 5 cal/mol K.

7. A vessel contains helium, which expands at constant pressure when 15 kJ of heat is supplied to it. What will be the variation of the internal energy of the gas? (in kJ)

8. A certain mass of gas is taken from an initial thermodynamics state A to another state B by process I and II. In process I for the gas does 5 J of work and absorbs 4 J of heat energy. In process II, the gas absorbs 5 joules of heat. The work done by the gas in process II is

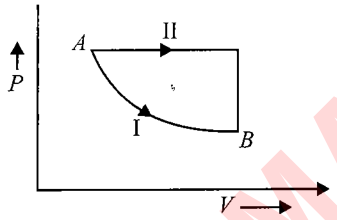

Fig. 2.173

9. The two conducting cylinder–piston systems shows below are linked. Cylinder 1 is filled with a certain molar quantity of a monatomic ideal gas, and cylinder 2 is filled with an equal molar quantity of a diatomic ideal gas. The entire apparatus is situated inside an oven whose temperature is T_a = 27°C. The cylinder volumes have the same initial value V_0 = 100 cc. When the oven temperature is slowly raised to T_b = 127°C. What is the volume change ΔV (in cc) of cylinder 1?

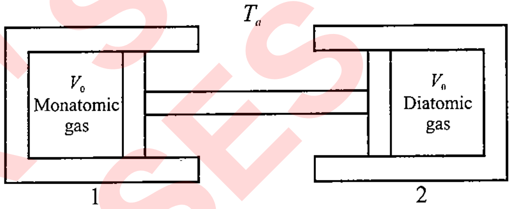

Fig. 2.174

10. A long container has air enclosed inside at room temperature and atmospheric pressure (10^5 Pa). It has a volume of 20000 cc. The area of cross section is 100 cm^2 and force constant of spring is k_{spring} = 1000 N/m. We push the right piston isothermally and slowly till it reaches the original position of the left piston which is movable. Final length of air column is found to be 25h cm. Assume that spring is initially relaxed. Find h.

ANSWERS AND SOLUTIONS

Subjective Type

1. For helium, $V = \sqrt{\dfrac{3RT}{M}}$

$$1000 = \sqrt{\frac{3RT_1}{4\times10^{-3}}} \Rightarrow 3RT_1 = 4000$$

For oxygen,

$$1000 = \sqrt{\frac{3RT_2}{32\times10^{-3}}} \Rightarrow 3RT_2 = 32000$$

By internal energy conservation $U_1 + U_2 = U_1^{|} + U_2^{|}$

$$U = \frac{f}{2}nRT;\quad \frac{3}{2}\times\frac{5}{4}RT_1 + \frac{5}{2}\times\frac{24}{32}RT_2$$

$$= \left(\frac{3}{2}\times\frac{5}{4} + \frac{5}{2}\times\frac{24}{32}\right)RT$$

$$T = \frac{T_1 + T_2}{2}$$

After mixing rms speed of helium,

$$v_1^{|} = \sqrt{\frac{3RT}{mw}} = \sqrt{\frac{3R\,(T_1+T_2)}{2\quad mw}}$$

$$= \sqrt{\frac{4000+32000}{2\times4\times10^{-3}}}$$

$$v_1^{|} = 1500\sqrt{2}\ \text{m/s}$$

For oxygen,

$$v_2^{|} = \sqrt{\frac{3RT\,(T_1+T_2)}{2mw}} = \sqrt{\frac{4000+32000}{2\times32\times10^{-3}}}$$

$$v_2^{|} = 750\,\text{m/s}$$

2. Because the heating pressure inside is not changed, let inside pressure be ρ. Then for equilibrium of the system,

$$P(A_1 - A_2) = P_0(A_1 - A_2) + (m_1 + m_2)g$$

$$P\Delta V = (\rho_0 \Delta A + mg)l$$

l is the displacement of the piston.

$$P\Delta V = nR\Delta T$$

$$\Delta T = \frac{P\Delta V}{nR} = \frac{(\rho_0\Delta A + mg)l}{nR}$$

$$= \frac{(10^5\,\text{Pa}\times10^{-3}\,\text{m}^2 + 5\times10)(50\times10^{-2})}{1\times R}$$

$$\Delta T = \frac{75}{R}\ \text{K}$$

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3. Change in internal energy for cyclic process $(\Delta U) = 0$
For process $a \to b$, (P – constant)

$$W_{a\to b} = P\Delta V = nR\Delta T = -400R$$

For process $b \to c$ (T – constant)

$$W_{b\to c} = -2R(300)\ln 2$$

For process $c \to d$ (P – constant)

$$W_{c\to d} = +400R$$

For process $d \to a$ (T – constant)

$$W_{d\to a} = +2R(500)\ln 2$$

Net work,

$$(\Delta W) = W_{a\to b} + W_{b\to c} + W_{c\to d} + W_{d\to a}$$

$$\Delta W = 400R\ln 2$$

$dQ = dU + dW$, first law of thermodynamics.

$$dQ = 400R\ln 2$$

4. i.

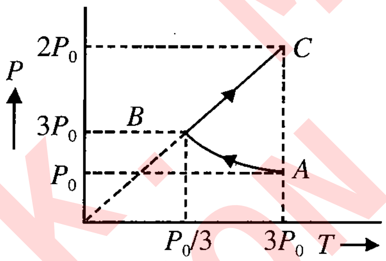

Fig. 2.175

ii. Work done in process AB

$$\Delta W_{AB} = \frac{2r\left[\frac{T_0}{3} - 3T_0\right]}{\left(\frac{1}{3} - 1\right)}$$

$$W = -8RT_0$$

$$\Delta Q_{AB} = du + \Delta w = -nC_v\Delta T - 8RT_0$$

$$\Delta Q_{AB} = -16RT_0 \qquad \text{(ii)}$$

For process $B \to C$, $W_{BC} = 0$

$$du = 8RT_0, \quad \Delta Q_{BC} = 8RT_0$$

and $C \to A$ (isothermal process)

$$\Delta W_{CA} = 2R(3T_0)\ln\frac{V_f}{V_i} \quad \left(\frac{V_f}{V_i} = 27\right)$$

$$\Delta W_{CA} = 18RT_0\ln 3$$

$$\Delta Q_{CA} = 18RT_0\ln 3$$

Net work done,

$$\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$$

$$= -8RT_0 + 0 + 18RT_0\ln 3$$

$$\Delta W = RT_0 + (18\ln 3 - 8)$$

$$\Delta Q_{net} = \Delta Q_{BC} + \Delta Q_{CA}$$

$$= -8RT_0 + 18RT_0\ln 3$$

$$\Delta Q_{net} = RT_0(18\ln 3 + 8)$$

5. For two gases, we can write

$$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_{mean} - 1}$$

Here, $n_1 = 1$ and $n_2 = ?$

$\gamma_1 = 5/3$ and $\gamma_2 = 7/5$

Substituting these values in the above equation, we get

$$\frac{1}{\frac{5}{3} - 1} + \frac{n_2}{\frac{7}{5} - 1} = \frac{1 + n_2}{\frac{19}{13} - 1}$$

or $$\frac{3}{2} + \frac{5n_2}{2} = \frac{13(1 + n_2)}{6}$$

or $$3(3 + 5n_2) = 13(1 + n_2)$$

$$9 + 15n_2 = 13 + 13n_2$$

$$n_2 = 2 \text{ g mol}$$

6. Work done,

$$W = \int_{C_1}^{V_2} PdV$$

$$= \int_{V_1}^{V_2}\left(\frac{a}{V} + b\right)dV$$

$$= |a\ln V + bV|_{V_1}^{V_2}$$

$$= (a\ln V_2 + bV_2) - (a\ln V_1 + bV_1)$$

$$= a\ln\left(\frac{V_2}{V_1}\right) + b(V_2 - V_1)$$

7. Given,

$$T = T_0 + \alpha V^2 \qquad \text{(i)}$$

For 1 mol of a gas, $PV = RT$

or $$V = \frac{RT}{P}$$

Substituting this value in Eq. (i), we get

$$T = T_0 + \alpha\left(\frac{RT}{P}\right)^2 = T_0 + \alpha\frac{R^2T^2}{P^2}$$

or $$TP^2 = T_0P^2 + \alpha R^2T^2$$

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or $$P = \sqrt{\alpha} RT(T - T_0)^{-1/2} \quad \text{(ii)}$$

After differentiating, we get

$$\frac{dP}{dT} = \sqrt{\alpha} R\left[(T - T_0)^{-1/2} - \frac{1}{2}T(T - T_0)^{-3/2}\right]$$

For minimum pressure,

$$\frac{dP}{dT} = 0$$

$$\therefore \quad 0 = \sqrt{\alpha} R\left[(T - T_0)^{-1/2} - \frac{1}{2}T(T - T_0)^{-3/2}\right]$$

After solving, $T = 2T_0$

From Eq. (ii),

$$P_{\min} = \sqrt{\alpha} R 2T_0 (2T_0 - T_0)^{-3/2}$$
$$= 2R\sqrt{\alpha T_0}$$

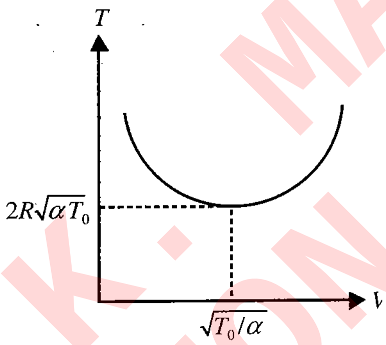

Fig. 2.176

8. When pressure changes from 1 atm to 3 atm, the change in pressure

$$P = 2 \text{ atm}$$
$$= 2 \times 1 \times 10^5 \text{ N/m}^2$$

The force exerted on the piston

$$F = PA = 2 \times 10^5 \times 4 \times 10^{-4}$$
$$= 80 \text{ N}$$

The compression of the spring

$$x = \frac{F}{k} = \frac{80}{10^4} = 0.008 \text{ m}$$

The change in volume of air due to displacement of piston by x

$$\Delta V = Ax = 4 \times 10^{-4} \times 0.008$$
$$= 3.2 \times 10^{-6} \text{ m}^3$$

$\therefore$ Final volume, $V_2 = V_1 + \Delta V$

$$= 20 \times 10^{-6} + 3.2 \times 10^{-6}$$

By equation of state

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$T_2 = \frac{P_2V_2T_1}{P_1V_1} = \left(\frac{3}{1}\right) \times \frac{(23.2 \times 10^{-6})}{(20 \times 10^{-6})} \times (273 + 20)$$
$$= 1020 \text{ K}$$

The change in internal energy of air

$$\Delta U = mC_v\Delta T$$
$$= (2.38 \times 10^{-5}) \times 718 \times (1020 - 293)$$
$$= 12.42 \text{ J}$$

Work done in compressing the spring by x

$$W = \frac{1}{2}kx^2 = \frac{10^4}{2} \times (0.008)^2 = 0.32 \text{ J}$$

From the first law of thermodynamics

$$Q = \Delta U + W = 12.42 + 0.32 = 12.74 \text{ J}$$

9. In a cyclic process

$$\Delta U = 0$$

From the first law of thermodynamics, for the cyclic process

$$Q = \Delta U + W$$
$$\therefore \quad W = Q - \Delta U = -1200 - 0$$
$$= -1200 \text{ J}$$

From C to A, $\Delta V = 0$

$$\therefore \quad W_{CA} = 0$$

For the whole cycle

$$W_{AB} + W_{BC} + W_{CA} = W$$
$$= -1200 \text{ J}$$

As $W_{CA} = 0$

$$\therefore \quad W_{AB} + W_{BC} = -1200 \text{ J} \quad \text{(i)}$$

Work done from A to B:
In the process $V \propto T$, so pressure remains constant.

We know that $PV = nRT$

or $P\Delta V = nR\Delta T$

$$\therefore \quad W_{AB} = P\Delta V = nR\Delta T$$
$$= 2 \times 8.31 \times (500 - 300) = 3324 \text{ J}$$

Substituting this value in Eq. (i), we get

$$3324 + W_{BC} = -1200$$
$$\therefore \quad W_{BC} = -4524 \text{ J}$$

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10. In the process a to b and c to d

as $\Delta U = 0$, therefore $\Delta T = 0$ or $T =$ constant

$$W = \int_{V_i}^{V_f} P dV$$

We have,

$$PV = nRT \quad \Rightarrow P = \frac{nRT}{V}$$

$$\therefore \quad W = \int_{V_i}^{V_f} (nRT)\frac{RV}{V}$$

$$= nRT \left| \ln V \right|_{V_i}^{V_f} = nRT \ln \frac{V_f}{V_i}$$

$$W_{ab} = nRT_b \ln \frac{2V_0}{V_0} = 2R \times 500 \ln 2 = 1000 R \ln 2$$

and $W_{cd} = nRT_c \ln \dfrac{V_0}{2V_0} = 2R \times 300 \ln \dfrac{1}{2} = -600 R \ln 2$

There is no volume change from b to c and from d to a, so

$$W_{bc} = V_{da} = 0$$

The work done in complete cycle

$$W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$

$$= 1000 R \ln 2 + 0 - 600 R \ln 2 + 0$$

$$= 400 R \ln 2$$

11. Work done in isothermal process A to B

$$W_{AB} = nRT \ln \frac{V_f}{V_i} = RT \ln \frac{2V}{V} [\because n = 1]$$

If pressure at B is P_B, then

$$P_A V_A = P_B V_B$$

or $\quad PV = P_B \times 2V \quad \Rightarrow P_B = P/2$

Work done in isobaric process from B to C at constant pressure $P/2$

$$W_{BC} = P_B (V_f - V_i)$$

$$= \frac{P}{2}(V - 2V) = -\frac{PV}{2}$$

$$= \frac{-RT}{2}$$

Work done in isochoric process C to A, $W_{CA} = 0$
Therefore work done in the whole cycle,

$$W = W_{AB} + W_{BC} + W_{CA}$$

$$= RT \ln 2 - \frac{RT}{2} + 0$$

$$= RT\left(\ln 2 - \frac{1}{2}\right)$$

12. If T_B be the temperature at B, then by gas law

$$\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B}$$

$$\therefore \quad T_B = \frac{P_B V_B}{P_A V_A} T_A = \frac{(2P_0)(2V_0)}{P_0 V_0} T_0$$

The change in internal energy from A to B

$$\Delta U = nC_v \Delta T = 1 \times \frac{3R}{2} \times (4T_0 - T_0)$$

$$= \frac{9RT_0}{2}$$

Work done in the process A to C

$$W_{AC} = P\Delta V = P_0 (2V_0 - V_0)$$
$$= P_0 V_0 = RT_0$$

and $\quad W_{CB} = 0$

$\therefore$ Total work done from $A \to C \to B$

$$W_{AC} + W_{CB} = RT_0 + 0 = RT_0$$

From the first law of thermodynamics,

$$Q = \Delta U + W$$

$$= \frac{9RT_0}{2} + RT_0$$

$$= \frac{11RT_0}{2}$$

Thus heat absorbed by the gas from $A \to C \to B$ is

$$\frac{11RT_0}{2}$$

13. The separator will stop in the position when pressures of both the parts of the tube become equal. Let it is P. If V_1 and V_2 are the volumes of two parts, then

$$V_1 + V_2 = V_0 \qquad \text{(i)}$$

a. The process in each part is adiabatic, so

$$P_1\left(\frac{V_0}{2}\right)^{\gamma} = PV_1^{\gamma} \qquad \text{(ii)}$$

$$P_2\left(\frac{V_0}{2}\right)^{\gamma} = PV_2^{\gamma} \qquad \text{(iii)}$$

Dividing Eq. (i) by Eq. (iii), we have

$$\frac{P_1}{P_2} = \frac{V_1^{\gamma}}{V_2^{\gamma}}$$

or $$V_1 = \left(\frac{P_1}{P_2}\right)^{1/\gamma} V_2 \qquad \text{(iv)}$$

Substituting this value in Eq. (i), we get

$$V_2 = \left[\frac{V_0 P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right] \text{ and } V_1 = \left[\frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right]$$

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b. As cylindrical tube and separator both are adiabatic, no heat is given in the process.

c. Substituting the value of V_1 in Eq. (ii), we get

$$P = \left[\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2^\gamma}\right]$$

14. It is given that initially the separator is in equilibrium; thus pressure on both sides of the gas is equal say, it is P_i. If A be the area of cross section of cylinder, number of moles of gas in left and right part, n_1, and n_2 can be given as

$$n_1 = \frac{P_i(10A)}{R(100)}$$

and $$n_2 = \frac{P_i(20A)}{R(400)}$$

Finally if separator is displaced to right by a distance x, we have

$$n_1 = \frac{P_f(10+x)A}{RT_f}$$

and $$n_2 = \frac{P_f(20-x)A}{RT_f}$$

Here P_f and T_f are the final pressure and temperature on both sides after a long time.

Now if we equate the ratio of moles n_1/n_2 in initial and final states, we get

$$\frac{n_1}{n_2} = \frac{(10A/100)}{(20A/400)} = \frac{(10+x)A}{(20-x)A}$$

$$2(20 - x) = 10 + x$$

$$x = 10 \text{ cm}$$

Thus in final state when gases in both parts are in thermal equilibrium, the piston is displaced 10 cm rightward from its initial position.

15. We know according to law of equipartition of energy, each gas molecule has 1/2 kT energy associated with each of its degrees of freedom. As a diatomic gas molecule has two rotational degrees of freedom, final temperature of the system is

$$T_f = \frac{f_1 n_1 T_1 + f_2 n_2 T_2}{f_1 n_1 + f_2 n_2} \quad \text{(i)}$$

Here for diatomic gas,

$$f_1 = 5, \quad n = 3 \quad \text{and} \quad T_1 = 250 \text{ K}$$

For monatomic gas,

$$f_2 = 3, \quad n_2 = 5 \quad \text{and} \quad T_2 = 470 \text{ K}$$

Thus from Eq. (i),

$$T_f = \frac{5\times3\times250+3\times5\times470}{5\times3+3\times5} = 360 \text{ K}$$

Thus final mixture of the two gases is at temperature 360 K. If final rms angular velocity of diatomic gas molecules is $\omega_{\text{rms}\,f}$, according to law of equipartition of energy, we have

$$\frac{1}{2} I\omega_{\text{rms}\,f}^2 = kT$$

or $$\omega_{\text{rms}\,f} = \sqrt{\frac{2kT}{I}}$$

$$= \sqrt{\frac{2\times1.38\times10^{-23}\times360}{2.76\times10^{-46}}}$$

$$\omega_{\text{rms}\,f} = 6\times10^{12} \text{ rad/s}$$

16. a. We know work done is given by the area below PV curve thus by observing the two PV curves, AC and ABC, we can say that work done in path AC is less than that in path ABC.

b. Work done in path AC by the gas is

$$W_{AC} = \text{area of } ACFEDA$$
$$= \text{area of } ACF + \text{area of } AFED$$
$$= \frac{1}{2}\times(15-5)\times(6-2)+(6-2)\times5$$
$$= 20 + 20 = 40 \text{ J}$$

It is given that heat supplied in process AC is

$$Q_{AC} = 200 \text{ } T$$

Thus change in internal energy of gas in path AC is, from the first law of thermodynamics, given as

$$Q_{AC} = W_{AC} + \Delta U_{AC}$$

or $$\Delta U_{AC} = Q_{AC} - W_{AC} = 200 - 40 = 160 \text{ J}$$

As it is given that at state A, internal energy of gas is 10 J thus at state C, its internal energy is

$$\Delta U_{AC} = U_C - U_A$$

or $$U_C = \Delta U_{AC} + U_A = 16 + 10 = 170 \text{ J}$$

c. As in process AB, no volume change takes place thus no work is done by or on the gas during path AB. Thus according to the first law of thermodynamics, we have

$$Q_{AB} = \Delta U_{AB} + W_{AB}$$

or $$Q_{AB} = U_B - U_A + 0$$

or $$Q_{AB} = 20 - 10 = 10 \text{ J}$$

17. As we know in a cyclic process gas finally returns to its initial state and total change in internal energy of gas is zero; thus the total heat supplied to the gas is equal to work done by the gas. Now we find the work done by the gas in different paths of the cycle.

In process AB

As shown in graph, during process AB, volume of gas remains constant; thus work done by gas is zero.

$$W_{AB} = 0$$

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In Process *BC*

In process BC, volume of gas changes from $V_1 = 2$ m^3 to $V_2 = 5$ m^3 thus work done can be obtained as

$$W_{BC} = \int_2^5 P dV$$

As in process *BC*, temperature of gas remains constant at 500 K, we can write pressure of gas from gas law as

$$P = \frac{RT}{V} = \frac{500R}{V} \qquad (\text{as } n = 1 \text{ mol})$$

Now work done is

$$W_{BC} = \int_2^5 \frac{500R}{V} dV = 500R\ln\left(\frac{5}{2}\right)$$

In process *CA*

As in this process, path is a straight line passing through origin thus $V \propto T$ or pressure of gas remains constant and we know if gas pressure is constant work done is given as

$$W_{CA} = nR(T_2 - T_1) = nR(300 - 500) = -200R$$

This is negative as gas is being compressed from volume 5 m^3 to 2 m^3 or work is done on the gas.

Now we can find the total work done by the gas in the complete cycle *ABCA* as

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$

$$= -0 + 500R\ln\left(\frac{5}{2}\right) - 200R - R\left(500\ln\frac{5}{2} - 200\right)$$

$$= 2146.22 \text{ J} = \text{heat supplied to the gas}$$

18. The cycle is clockwise; thus net work is done by the gas. As in a cyclic process no change in internal energy takes place, heat supplied is equal to the work done by the gas in one complete cycle. So in this case heat supplied to the gas is given as

$$Q = \text{work done by the gas, } W$$
$$= \text{area of ellipse}$$
$$= -\pi ab$$

where a and b are semi-major and semi-minor axes of the ellipse, respectively, which are given from the figure as

$$a = 1.0 \times 10^5 \text{ N/m} \quad \text{and} \quad b = 100 \times 10^{-6} \text{ m}^3$$

Thus area of ellipse is

$$Q = W = \pi \times 1.0 \times 10^5 \times 100 \times 10^{-6}$$
$$= 3.14 \times 10 = 3.14 \text{ J}$$

19. $P_{at} = 10^5$ Pa

Initial pressure of gas,

$$P_1 = P_{1\,at} + P_{1\,\text{spring}} + P_{1\,\text{piston}}$$

P_{spring}: pressure due to spring
P_{piston}: pressure due to piston
Initially, the spring is in relaxed position,

$$P_{1\,\text{spring}} = 0$$
$$P_1 = 150 \text{ kPa} = 1.5 \times 10^5 \text{ Pa}$$
$$1.5 \times 10^5 = 10^5 + 0 + P_{\text{piston}}$$

Pressure due to piston $= mg/A = 0.5 \times 10^5$ N/m^2

In the final condition,

$$P' = P_0 + P'_{\text{spring}} + P'_{\text{piston}} \; (P_{\text{piston}} = P'_{\text{piston}})$$
$$P'_{\text{spring}} = 6 \times 10^5 - 1 \times 10^5 - 0.5 \times 10^5$$
$$= 4.5 \times 10^5 \text{ N/m}^2$$

Initial load in the spring, $F_1 = 0$
Final load in the spring,

$$F_2 = P'_{\text{spring}} A = (4.5 \times 10^5 A) \text{ N}$$

Work done on the spring

$$= (\text{Average force}) \, (\text{distance moved})$$

$$= \frac{(F_1 + F_2)}{2} \frac{(V_2 - V_1)}{A}$$

$$= \left[\left(\frac{0 + 4.5\times10^5}{2}\right) A \times \left(\frac{0.5 - 0.3}{A}\right)\right]$$

$$= 0.45 \times 10^5 \text{ J} = 45 \text{ kJ}$$

20. $P = \alpha T^{1/2}$ (i)

where α is constant
$n = 1$ mol, monatomic

$$W = \int P\,dV = \int (\alpha T^{1/2}) dV$$

From Eq. (i),

$$T = \frac{\alpha T^{1/2} V}{nR} \quad \Rightarrow T^{1/2} = \frac{\alpha V}{nR}$$

Differentiating both sides,

$$\frac{1}{2\sqrt{T}} dT = \frac{\alpha\, dV}{nR} \quad \Rightarrow dV = \frac{nR}{\alpha 2\sqrt{T}} dT$$

$$W = \int \alpha T^{1/2} \frac{nR}{2T^{1/2}} dT$$

$$= \frac{n\alpha R}{2} \int_{T_1}^{T_2} dT = \frac{R}{2} \times 50 = 25R = 207.7 \text{ J}$$

b. $Q = \Delta U + W$

$$C(T_2 - T_1) = \frac{R}{(\gamma - 1)}(T_2 - T_1) + \frac{R}{2}(T_2 - T_1)$$

$$C = \frac{R}{(\gamma - 1)} + \frac{R}{2} = \left(\frac{\gamma + 1}{\gamma - 1}\right)\frac{R}{2}$$

21. For process *AB*,

$$T_A = 300 \text{ K}, T_B = 600 \text{ K}$$
$$W = nR\Delta T = nR(T_B - T_A) = 300nR = 600R$$

$$Q = nC_p\Delta T = 2 \times \frac{5}{2} R(300) = 1500R$$

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For process BC,

$$W = nRT\ln\frac{V_f}{V_i} = nRT\ln\frac{p_i}{p_f}$$

$$= nRT\ln 2 = 1200R\ln 2$$

For process CA,

$$W = \int P\,dV = \int_{600}^{300}\frac{K}{T}\frac{2nRT}{K}dT$$

$$= -2nR(300) = -1200R$$

$$Q = nC_V\Delta T + W$$

$$= 2\times\frac{3}{2}R(-300) - 1200R$$

$$= -900R - 1200R = -2100R$$

$$\eta = \frac{600R + 1200R\ln 2 - 1200R}{1500R + 1200R\ln 2}$$

$$\eta = 1 - \frac{21}{12\ln 2 + 15}$$

Comparing with given equation, $x = 21$

22. a.

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2}\frac{RT}{V}dV = R\int_{V_1}^{V_2}\frac{\alpha V^2}{V}dV$$

$$= \alpha R\int_{V_1}^{V_2} V dV = \frac{\alpha R}{2}(V_2^2 - V_1^2)$$

$$W = \frac{R}{2}(T_2 - T_1) \qquad \text{(i)}$$

Now, $T_1 = \alpha V_1^2$

$T_2 = \alpha V_2^2$

$$T_2 = T_1\left[\frac{V_2}{V_1}\right]^2 = 36\,T_1$$

Hence from Eq. (i), we have

$$W = \frac{R}{2}[36T_1 - T_1] = \frac{35RT_1}{2} = \frac{35}{2}P_1V_1$$

$$= \frac{35}{2}\times(1.5\times10^{-3})\times(1.2\times10^5)\,\text{J} = 3150\text{ J}$$

b. We know,

$$PV = RT = R\alpha V^2$$

$$P \propto V$$

Hence P–V graph is a straight line.

c.

$$\Delta U = nC_V\Delta T = n\left(\frac{R}{\gamma - 1}\right)(T_2 - T_1)$$

$$= \frac{R}{0.5}\times 35T_1 = 70\,RT_1 = 70\,P_1V_1$$

$$= 70\times(1.2\times10^5)\times(1.5\times10^{-3})\,\text{J} = 12600\text{ J}$$

$$\Delta Q = \Delta U + W = 12600 + 3150 = 15750\text{ J}$$

Objective Type

1. c. Due to increase in temperature root-mean-square velocity of gas molecules increases. So they strike the wall more often with higher velocity. Hence the pressure exerted by a gas on the walls of the container increases.

2. d. Kinetic energy $E = 1.5\times10^5$ J, volume, $V = 20\,L = 20\times10^{-3}\text{ m}^3$

Pressure

$$= \frac{2}{3}\frac{E}{V} = \frac{2}{3}\left(\frac{1.5\times10^5}{20\times10^{-3}}\right) = 5\times10^6\text{ N/m}^2$$

3. d. $PV \propto$ Mass of gas $\times$ Temperature

In this problem pressure and volume remain constant, so $M_1T_1 = M_2T_2 =$ constant

$$\therefore \frac{M_2}{M_1} = \frac{T_1}{T_2} = \frac{(27+273)}{(52+273)} = \frac{300}{325} = \frac{12}{13}$$

$$\Rightarrow M_2 = M_1\times\frac{12}{13} = 13\times\frac{12}{13}\text{ g} = 12\text{ g}$$

i.e., the mass of gas released from the flask = 13 g – 12 g = 1 g.

4. d.

$$M_1 = M,\quad T_1 = 60 + 273 = 333\text{ K}$$

$$M_2 = M - \frac{M}{4} = \frac{3M}{4}$$

(as 1/4th part of air escapes)

If pressure and volume of the gas remain constant, then $MT =$ constant

$$\therefore \frac{T_2}{T_1} = \frac{M_1}{M_2} = \left(\frac{M}{3M/4}\right) = \frac{4}{3}$$

$$\Rightarrow T_2 = \frac{4}{3}\times T_1 = \frac{4}{3}\times333 = 444\text{ K} = 171°\text{C}$$

5. a.

$$\mu = \frac{\text{Mass of water}}{\text{Molecular wt. of water}} = \frac{4.5\text{ kg}}{18\times10^{-3}\text{ kg}} = 250$$

$T = 273$ K and $P = 10^5$ N/m^2 (STP)

From $PV = \mu RT$,

$$\Rightarrow V = \frac{\mu RT}{P} = \frac{250\times8.3\times273}{10^5} = 5.66\text{ m}^3$$

6. d. $PV \propto MT$ or $\dfrac{V}{T} \propto \dfrac{M}{P}$

Here (M/P) represents the slope of curve drawn on volume and temperature axis.

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For the first condition slope (M/P) graph is D (given in the problem)
For the second condition slope

$$\frac{2M}{P/2} = 4\left(\frac{M}{P}\right)$$

i.e., slope becomes four times so graph A is correct in this condition.

7. **d.** Quantity of gas in these bulbs is constant, i.e., initial no. of moles in both the bulbs = final number of moles

$$\mu_1 + \mu_2 = \mu_1' + \mu_2'$$

$$\frac{PV}{R(273)} + \frac{PV}{R(273)} = \frac{1.5PV}{R(273)} + \frac{1.5PV}{R(T)}$$

$$\Rightarrow \frac{2}{273} = \frac{1.5}{273} + \frac{1.5}{T}$$

$$\Rightarrow T = 819\text{ K} = 546°\text{C}$$

8. **d.** Number of moles in the first vessel

$$\mu_1 = \frac{P_1V}{RT_1}$$

Number of moles in the second vessel

$$\mu_2 = \frac{P_2V}{RT_2}$$

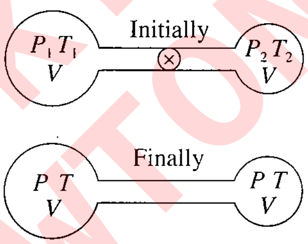

Fig. 2.177

If both vessels are joined together, then quantity of gas remains same, i.e., $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT} = \frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}$$

$$\frac{P}{T} = \frac{P_1}{2T_1} + \frac{P_2}{2T_2}$$

9. **b.**

$$V_1 = 2.4\times10^{-3}\text{ m}^3, \quad P_1 = P_0 = 10^5\text{ N/m}^2$$

and $\quad T_1 = 300$ K (given)

If area of cross section of piston is A and it moves through distance x, then increment in volume of the gas = Ax.

If force constant of a spring is k, then force $F = kx$ and pressure = F/A = kx/A

$$V_2 = V_1 + Ax = 2.4\times10^{-3} + 8\times10^{-3}\times0.1 = 3.2\times10^{-3}$$

and $$P_2 = P_0 + \frac{kx}{A} = 10^5 + \frac{8000\times0.1}{8\times10^{-3}} = 2\times10^5$$

From ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{10^5\times2.4\times10^{-3}}{300} = \frac{2\times10^5\times3.2\times10^{-3}}{T_2}$$

$$\Rightarrow T_2 = 800\text{ K}$$

10. **a.** Ideal gas equation, in terms of density,

$$\frac{P_1}{\rho_1T_1} = \frac{P_2}{\rho_2T_2} = \text{constant}$$

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{P_1}{P_2}\times\frac{T_2}{T_1}$$

$$\frac{\rho_{Top}}{\rho_{Bottom}} = \frac{P_{Top}}{P_{Bottom}}\times\frac{T_{Bottom}}{T_{Top}} = \frac{70}{76}\times\frac{300}{280} = \frac{75}{76}$$

11. **b.** Kinetic energy of N molecules of gas,

$$E = \frac{3}{2}NkT$$

Initially

$$E_1 = \frac{3}{2}N_1kT_1$$

and finally

$$E_2 = \frac{3}{2}N_2kT_2$$

But according to the problem $E_1 = E_2$ and $N_2 = 2N_1$

$$\frac{3}{2}N_1kT_1 = \frac{3}{2}(2N_1)kT_2$$

$$\Rightarrow T_2 = \frac{T_1}{2}$$

Since the kinetic energy is constant

$$\frac{3}{2}N_1kT_1 = \frac{3}{2}N_2kT_2$$

$$\Rightarrow N_1T_1 = N_2T_2$$

$$\therefore \quad NT = \text{constant}$$

From ideal gas equation of N molecules $PV = NkT$

$$\Rightarrow P_1V_1 = P_2V_2$$

$$\therefore \quad P_1 = P_2$$

(as $V_1 = V_2$ and NT = constant)

12. **d.** According to Boyle's law, multiplication of pressure and volume will remain constant at the bottom and top.

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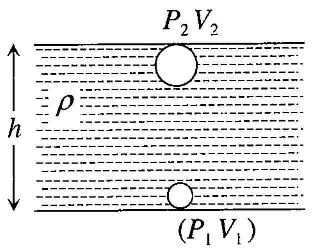

Fig. 2.178

If P is the atmospheric pressure at the top of the lake and the volume of bubble just before touching surface is V, then from $P_1V_1 = P_2V_2$

$$(P + h\rho g)V_0 = PV \quad \Rightarrow V = \left(\frac{P + h\rho g}{P}\right)V_0$$

$$\therefore \quad V = V_0\left[1 + \frac{\rho g h}{P}\right]$$

13. a. As $V \propto T$

$$\therefore \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow V_2 = \left(\frac{313}{293}\right)V_1$$

Fraction of gas that comes out

$$= \frac{V_2 - V_1}{V_1} = \frac{\left(\frac{313}{293}\right)V_1 - V_1}{V_1} = \frac{20}{293} = 0.07$$

14. c. In the given graph, line has a positive slope with the x-axis and negative intercept on the y-axis.
So we can write the equation of line as

$$y = mx - c \qquad \text{(i)}$$

According to Charles's law

$$V_t = \frac{V_0}{273}t + V_0$$

by rewriting this equation, we get

$$t = \left(\frac{273}{V_0}\right)V_t - 273 \qquad \text{(ii)}$$

By comparing Eqs. (i) and (ii), we can say that time is represented on the y-axis and volume on the x-axis.

15. b. When temperature of gas increases, it expands. As the cross-sectional area of right piston is more, greater force will work on it (because $F = PA$). So piston will move towards right.

16. c. From ideal gas equation

$$PV = RT \qquad \text{(i)}$$

$$P\Delta V = R\Delta T \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \quad \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \delta \quad \text{(given)}$$

$$\therefore \quad \delta = \frac{1}{T}$$

So the graph between δ and T will be a rectangular hyperbola.

17. a.

$$P_1 = P, \quad T_1 = T, \quad P_2 = P + (0.4\% \text{ of } P)$$

$$\Rightarrow P_2 = P + \frac{0.4}{100}P = P + \frac{P}{250} \quad \text{and} \quad T_2 = T + 1$$

From Gay-Lussac's law

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{P}{P + \frac{P}{250}} = \frac{T}{T+1}$$

(as V = constant for closed vessel)

By solving, we get $\quad T = 250$ K

18. a. From ideal gas equation,

$$PV = \mu RT$$

$$\therefore \quad P = \frac{\mu R}{V}T$$

Comparing, this equation with $y = mx$
Slope of line, $\tan\theta = m = \mu R/V$

$$\text{i.e.,} \quad V \propto \frac{1}{\tan\theta}$$

It means line of smaller slope represents greater volume of gas.

For the given problem figure

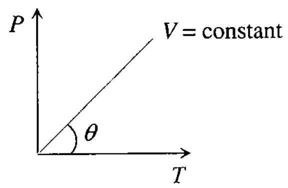

Fig. 2.179

Points 1 and 2 are on the same line, so they will represent same volume, i.e., $V_1 = V_2$.

Similarly points 3 and 4 are on the same line, so they will represent same volume, i.e., $V_3 = V_4$.

But $V_1 > V_3 (= V_4)$ or $V_2 > V_3 (= V_4)$ as slope of line 1–2 is less than that of 3–4.

19. a. By Dalton's law

$$P = P_1 + P_2 + P_3 = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \frac{\mu_3 RT}{V}$$

$$= \frac{RT}{V}[\mu_1 + \mu_2 + \mu_3] = \frac{RT}{V}\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3}\right]$$

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$$= \frac{8.31\times 300}{3\times 10^{-3}}\left[\frac{6}{32}+\frac{8}{28}+\frac{5}{44}\right]$$

$$= 498\times 10^3 = 500\times 10^3 = 5\times 10^5 \text{ N/m}^2$$

20. **b.** As the quantity of gas remains constant, $\mu_A + \mu_B = \mu$

$$\frac{P_A V_A}{RT}+\frac{P_B V_B}{RT}=\frac{P(V_A+V_B)}{RT}$$

$$P=\frac{P_A V_A + P_B V_B}{V_A+V_B}=\frac{1.4\times 0.1+0.7\times 0.15}{0.1+0.15}$$

$$\Rightarrow P = 0.98 \text{ MPa}$$

21. **d.** From Dalton's law, final pressure of the mixture of nitrogen and oxygen is

$$P = P_1 + P_2$$

$$=\frac{\mu_1 RT}{V}+\frac{\mu_2 RT}{V}=\frac{m_1}{M_1}\frac{RT}{V}+\frac{m_2}{M_2}\frac{RT}{V}$$

$$=\frac{8}{32}\frac{RT}{V}+\frac{7}{28}\frac{RT}{V}=\frac{RT}{2V} \quad \Rightarrow 10=\frac{RT}{2V} \qquad \text{(i)}$$

When oxygen is absorbed then for nitrogen let pressure is

$$P=\frac{7}{28}\frac{RT}{V}$$

$$\Rightarrow P=\frac{RT}{4V} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get pressure of the nitrogen $P = 5$ atm

22. **b.** Energy of 1 mol of gas

$$=\frac{f}{2}RT=\frac{f}{2}PV$$

where f = degree of freedom
Monatomic or diatomic gases possess equal degree of freedom for translational motion and that is equal to 3, i.e., $f = 3$

$$\therefore \quad E=\frac{3}{2}PV$$

23. **a.** At constant pressure $(\Delta Q)_p = \mu C_p \Delta T = 1 \times C_p \times (30 - 20) = 40$

$$\Rightarrow C_p = 4 \text{ cal/mol-K}$$

$$\therefore \quad C_v = C_p - R = 4 - 2 = 2 \text{ cal/mol-K}$$

Now $\quad (\Delta Q)_v = \mu C_v \Delta T = 1\times 2\times(30-20) = 20$ cal

24. **d.** Given $\quad c_p - c_v = 4150 \qquad$ (i)
and $\quad c_p/c_v = 1.4 \quad \Rightarrow c_p = 1.4\, c_v \qquad$ (ii)

By substituting the value of c_p in Eq. (i), we get

$$1.4\, c_v - c_v = 4150$$

$$\Rightarrow 0.4 c_v = 4150$$

$$\therefore \quad c_v=\frac{4150}{0.4}=10375 \text{ J/kg-K}$$

25. **a.** Molar specific heat = molecular weight × gram specific heat

$$C_v = M \times c_v$$

$$2.98 \text{ cal/mol-K} = M \times 0.075 \text{ kcal/kg-K}$$

$$= M\times\frac{0.075\times 10^3}{10^3} \text{cal/g-K}$$

∴ Molecular weight of argon

$$M=\frac{2.98}{0.075}=39.7 \text{ g}$$

i.e., mass of 6.023×10^{23} atom = 39.7 g
Therefore, mass of single atom

$$=\frac{39.7}{6.023\times 10^{23}}=6.60\times 10^{-23} \text{ g}$$

26. **c.** At constant volume the total energy will be utilized in increasing the temperature of gas

i.e., $\quad (\Delta Q)_v = \mu C_v \Delta T = \mu C_v (120 - 100) = 80$

$$\Rightarrow \mu C_v = \frac{80}{20} = 4 \text{ J/K}$$

This is the heat capacity of 5 mol gas.

27. **b.** We know fraction of given energy that goes to increase the internal energy $= 1/\gamma$.
So we can say the fraction of given energy that is supplied for external work = $1 - (1/\gamma)$.

28. **c.** Fraction of energy supplied for increment in internal energy = $1/\gamma$ = 3/5 (as γ = 5/3 for monatomic gas)
Therefore, percentage energy = 30/5 = 60%.
Fraction of energy supplied for external work done

$$=1-\frac{1}{\gamma}=\frac{\gamma-1}{\gamma}=\frac{\frac{5}{3}-1}{\frac{5}{3}}=\frac{2}{5}$$

∴ Percentage energy

$$=\frac{2}{5}\times 100\% = 40\%$$

29. **b.** As f = 6 (given), therefore

$$\gamma = 1+\frac{2}{f}=1+\frac{2}{6}=\frac{4}{3}$$

Fraction of energy given for external work

$$\frac{\Delta W}{\Delta Q}=\left(1-\frac{1}{\gamma}\right)$$

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$$\Rightarrow \frac{25}{\Delta Q} = \left(1 - \frac{1}{4/3}\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow Q = 25 \times 4 = 100 \text{ J}$$

30. b. If m is the total mass of the gas, then its kinetic energy $= 1/2\, mv^2$.

When the vessel is suddenly stopped, total kinetic energy will increase the temperature of the gas (because process will be adiabatic), i.e.,

$$\frac{1}{2}mv^2 = \mu C_v \Delta T$$

$$= \frac{m}{M} C_v \Delta T$$

$$\Rightarrow \frac{m}{M}\frac{R}{\gamma - 1}\Delta T = \frac{1}{2}mv^2 \qquad \left(\text{As } C_v = \frac{R}{\gamma - 1}\right)$$

$$\Rightarrow \Delta T = \frac{Mv^2(\gamma - 1)}{2R}$$

31. b. Ideal gas equation for m grams of gas is

$$PV = mrT \quad (\text{where } r = \text{specific gas constant})$$

or
$$P = \frac{m}{V} rT = \rho rT$$

$$\Rightarrow r = \frac{P}{\rho T} = \frac{1.013 \times 10^5}{0.795 \times 273} = 466.7$$

Specific heat at constant volume

$$c_v = \frac{r}{\gamma - 1} = \frac{466.7}{\frac{4}{3} - 1} = 1400 \text{ J/kg-K}$$

($\gamma = 4/3$ for polyatomic gas)

32. c. For state A, $C_p - C_v = R$, i.e., the gas behaves as an ideal gas.

For state B, $C_p - C_v = 1.06\, R (\neq R)$, i.e., the gas does not behave like an ideal gas.

We know that at high temperature and at low pressure, nature of gas may be ideal.

So we can say that $P_A < P_B$ and $T_A > T_B$.

33. c. $\mu_1 = 1, \gamma_1 = 5/3$ (for monatomic gas) and $\mu_2 = 2, \gamma_2 = 7/5$ (for diatomic gas).

From formula

$$\gamma_{\text{mixture}} = \frac{\dfrac{\mu_1\gamma_1}{\gamma_1 - 1} + \dfrac{\mu_2\gamma_2}{\gamma_2 - 1}}{\dfrac{\mu_1}{\gamma_1 - 1} + \dfrac{\mu_2}{\gamma_2 - 1}}$$

$$= \frac{\dfrac{1 \times \frac{5}{3}}{\frac{5}{3} - 1} + \dfrac{2 \times \frac{7}{5}}{\frac{7}{5} - 1}}{\dfrac{1}{\frac{5}{3} - 1} + \dfrac{2}{\frac{7}{5} - 1}} = \frac{\frac{5}{2} + 7}{\frac{3}{2} + 5} = \frac{19}{13}$$

34. a. Let t be the temperature of mixture.

Heat gained by CO_2 = Heat lost by O_2

$$\Rightarrow \mu_1 C_{v_1} \Delta T_1 = \mu_2 C_{v_2} \Delta T_2$$

$$\Rightarrow \frac{22}{44}(3R)(t - 27) = \frac{16}{32}\left(\frac{5}{2}R\right)(37 - t)$$

$$\Rightarrow 3(t - 27) = \frac{5}{2}(37 - t)$$

By solving we get $t = 31.5°C$

35. d. Total internal energy of system

$$= U_{\text{oxygen}} + U_{\text{argon}} = \mu_1 \frac{f_1}{2} RT + \mu_2 \frac{f_2}{2} RT$$

$$= 2\frac{5}{2}RT + 4\frac{3}{2}RT = 5RT + 6RT = 11RT$$

(As $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon))

36. b. Work done by the system = area of shaded portion on P–V diagram.

$$= (300 - 100)10^{-6} \times (200 - 100) \times 10^3 = 20 \text{ J}$$

And direction of process is anticlockwise so work done will be negative, i.e., $\Delta W = -20$ J.

37. a. Work done = area enclosed by triangle ABC

$$= \frac{1}{2} AC \times BC = \frac{1}{2} \times (3V - V) \times (3P - P) = 2PV$$

38. a. Area enclosed by curve 1 < Area enclosed by curve 2 < Area enclosed by curve 3

$\therefore\ Q_1 < Q_2 < Q_3$ (As ΔU is same for all the curves)

39. a. Change in internal energy

$$\Delta U = \mu C_v \Delta T \quad \Rightarrow U_2 - U_1 = \mu C_v (T_2 - T_1)$$

Let initially $T_1 = 0$ so $U_1 = 0$ and finally $T_2 = T$ and $U_2 = U$

$$U = \mu C_v T = \mu T \times C_v = \frac{PV}{R} \times \frac{R}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

(As $PV = \mu RT$, $\therefore \mu T = PV/R$ and $C_v = R/(\gamma - 1)$)

40. a. By the graph, $W_{AB} = 0$ and $W_{BC} = 8 \times 10^4 [5 - 2] \times 10^{-3} = 240$ J

$$\therefore \quad W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240 \text{ J}$$

Now,

$$\Delta Q_{AC} = \Delta Q_{AB} + \Delta Q_{BC} = 600 + 200 = 800 \text{ J}$$

From the first law of thermodynamics,

$$\Delta Q_{AC} = \Delta U_{AC} + \Delta W_{AC}$$

$$\Rightarrow 800 = \Delta U_{AC} + 240 \quad \Rightarrow \Delta U_{AC} = 560 \text{ J}$$

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41. c. $\Delta Q = \Delta U$

$$= \mu C_v \Delta T = \mu\left(\frac{R}{\gamma-1}\right)\Delta T = 2\times\frac{R}{\frac{5}{3}-1}[373-273] = 300R$$

(as for monatomic gas $\gamma = 5/3$)

42. a. Given $P \propto T^3$. But for adiabatic process $P \propto T^{\gamma/\gamma-1}$. So,

$$\frac{\gamma}{\gamma-1} = 3 \quad \Rightarrow \gamma = \frac{3}{2} \quad \Rightarrow \frac{C_p}{C_v} = \frac{3}{2}$$

43. b. For an adiabatic process $TV^{\gamma-1}$ = constant. Therefore,

$$\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma-1}$$

$$\Rightarrow T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

$$= 300\left[\frac{27}{8}\right]^{5/3-1} = 300\left[\frac{27}{8}\right]^{2/3} = 675\text{ K}$$

$$\Rightarrow \Delta T = 675 - 300 = 375\text{ K}$$

44. c. As we know that slopes of isothermal and adiabatic curves are always negative and slope of adiabatic curve is always greater than that of isothermal curve, in the given graph curves A and curve B represent adiabatic and isothermal changes, respectively.

45. c. In second part there is a vacuum, i.e., $P = 0$. So work done in expansion $= P\Delta V = 0$. Also, $\Delta Q = 0$. From the first law of thermodynamics, $\Delta U = 0$ i.e., temperature of an ideal gas remains same due to free expansion.

46. c. For isothermal process

$$P_1 V = P_2'\frac{V}{2} \quad \Rightarrow P_2' = 2P_1 \qquad \text{(i)}$$

For adiabatic process

$$P_1 V^\gamma = P_2\left(\frac{V}{2}\right)^\gamma$$

$$\Rightarrow P_2 = 2^\gamma P_1 \qquad \text{(ii)}$$

Since $\gamma > 1$, $P_2 > P_2'$

47. c. Energy spent in overcoming inter-molecular forces, $\Delta U = \Delta Q - \Delta W$

$$= \Delta Q - P(V_2 - V_1)$$

$$= 540 - \frac{1.013\times10^5(1671-1)\times10^{-6}}{4.2}$$

$$\approx 500\text{ cal}$$

48. c.

$$(\Delta Q)_p = \mu C_p \Delta T = \mu\left(\frac{\gamma}{\gamma-1}\right)R\Delta T$$

$$\therefore (\Delta Q)_p = 5\times\left(\frac{\frac{7}{5}}{\frac{7}{5}-1}\right)\times2\times30$$

$$= 5\times2\times\frac{7}{5}\times\frac{5}{2}\times30 = 1050\text{ cal}$$

(as μ = 5 mol and γ = 7/5 for H_2)

49. a. $$\frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q}$$

$$= \frac{C_p - C_v}{C_p} = 1 - \frac{1}{\gamma} = 1 - \frac{1}{1-\frac{5}{3}} = \frac{2}{5}$$

i.e., percentage energy utilized in doing external work $= \frac{2}{5}\times100 = 40\%$

50. b. Heat required to convert 5 kg of water into steam

$$\Delta Q = mL = 5\times2.3\times10^6 = 11.5\times10^6\text{ J}$$

Work done in expanding volume,

$$\Delta W = P\Delta V$$

$$= 5\times10^5(1.671-10^{-3}) = 0.835\times10^6\text{ J}$$

Now by the first law of thermodynamics $\Delta U = \Delta Q - \Delta W$

$$\Rightarrow \Delta U = 11.5\times10^6 - 0.835\times10^6 = 10.66\times10^6\text{ J}$$

51. d. Process CD is isochoric as volume is constant, process DA is isothermal as temperature is constant and process AB is isobaric as pressure is constant.

52. d. Heat given, ΔQ

$$= 20\text{ cal} = 20\times4.2 = 84\text{ J}$$

Work done, $\Delta W = -50$ J

(as the process is anticlockwise)

By the first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W = 84 - (-50) = 134\text{ J}$$

53. b. Work done during process 1 is positive while during process 2 it is negative, because process 1 is clockwise, while process 2 is anticlockwise. But area enclosed by P–V graph (i.e., work done) in process 1 is smaller, so net work done will be negative.

54. c. Process AB is isochoric; therefore

$$W_{AB} = P\Delta V = 0$$

Process BC is isothermal; therefore

$$W_{BC} = RT_2\ln\left(\frac{V_2}{V_1}\right)$$

Process CA is isobaric; therefore

$$W_{CA} = P\Delta V = R\Delta T = R(T_2 - T_1)$$

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55. **a.** AB is isobaric process, BC is isothermal process, CD is isochoric process and DA is isothermal process.

These processes are correctly represented by graph (a).

56. **b.** For an isothermal process, PV = constant and for the given process PV^2 = constant.

Therefore the gas is cooled because volume expands by a greater exponent than in an isothermal process.

57. **c.** From the given V–T diagram, we can see that in process AB, $V \propto T$. Therefore pressure is constant (as quantity of the gas remains same).

In process BC, V = constant and in process CA, T = constant.

Therefore these processes are correctly represented on P–V diagram by graph (c).

58. **b.** $P = \dfrac{P_0}{1+(V/V_0)^3} = \dfrac{P_0}{2}$

$$T = \frac{P_0V_0}{2R}$$

Therefore translational kinetic energy is equal to

$$\frac{3}{2}RT = \frac{3R}{2}\frac{P_0V_0}{2R} = \frac{3P_0V_0}{4}$$

59. **b.**

$$\frac{C_v}{C_p} \times Q = nC_v dT$$

$$dT = \frac{Q}{nC_p} = \frac{50}{2\times\frac{5}{2}\times R} = 5\text{ K}$$

60. **c.** The work done = area of P–V diagram

$$a = \frac{V_2 - V_1}{2}, \quad b = \frac{P_2 - P_1}{2}$$

$$W = -\pi\left(\frac{V_2 - V_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$$

But the cyclic process is anticlockwise. Hence, the work done is negative.

61. **d.**

$1 \to 2$: isothermal, $\Delta U_{12} = 0$
$2 \to 3$: isochoric, $\Delta W = 0$
$\Rightarrow \Delta Q_{23} = \Delta U_{23} \quad \Rightarrow -40 = \Delta U_{23}$

For a cyclic process, $\Delta U = 0$

$$\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0$$
$$0 + (-40) + \Delta U_{31} = 0$$
$$0 + (-40) + \Delta U_{31} = 0$$
$$\Delta U_{31} = +40\text{ J}$$

62. **b.** For path acb:

$$\Delta Q = \Delta U + \Delta W$$
$$\Rightarrow \quad 84 = \Delta U + 32 \Rightarrow \Delta U = 52\text{ kJ}$$

Hence $\Delta U_{acb} = \Delta U_{ab} = \Delta U_{adb} = 52$ kJ

For path adb:

$$\Delta Q = \Delta U + \Delta W$$
$$= 52 + 10.5 = 62.5\text{ kJ}$$

so option (a) is correct

for process ba, system will release the heat. So option (b) is wrong.

For path ad:

$$\Delta W_{adb} = \Delta W_{ad} + \Delta W_{db}$$
$$\Rightarrow \quad 10.5 = \Delta W_{ad} + 0$$
$$\Rightarrow \quad \Delta W_{ad} = 10.5\text{ kJ}$$
$$\Delta Q_{ad} = \Delta U_{ad} + \Delta W_{ad}$$
$$= (42 - 0) + 10.5$$
$$= 52.5\text{ kJ}$$

So option (c) is correct

$$\Delta Q_{adb} = 52 + 10.5 = 62.5\text{ kJ}$$
$$\Delta Q_{db} = \Delta Q_{adb} - \Delta Q_{ad}$$
$$= 62.5 - 52.5$$
$$= 10\text{ kJ}$$

So option (d) is correct

Hence answer of this question is (b)

63. **c.** Along path AB

$$\Delta Q = \Delta U + \Delta W = U_B - U_A + \Delta W$$

or $\quad -50 = U_B - U_A = U_B - 1500$

or $\quad U_B = 1450\text{ J}$

Along path BC

or $\quad \Delta Q = U_C - U_B + \Delta W$

or $\quad 0 = U_C - U_B + \Delta W = U_C - U_B + (-40)$

or $\quad 0 = U_C - 1450 - 40 \quad$ or $\quad U_C = 1490\text{ J}$

64. **c.** $PV = \dfrac{m}{M}RT$ (for ideal gas)

$$\therefore MV = \frac{mRT}{P}$$

In the position of equilibrium of stopper S,

$$P_1 = P_2, \quad T_1 = T_2, \quad m_1 = m_2$$
$$\therefore \quad MV = \text{constant}$$
$$M_1V_1 = M_1V_2$$
$$\Rightarrow A \times 32\,(360 - \alpha) = 28\alpha \times A$$
$$\alpha = 192^\circ$$

65. **d.** Since $U\rho$ = constant,

$$\frac{P}{\rho} = \frac{RT}{M}$$

P = constant since ρ is increasing, therefore V is decreasing.

66. **b.** At $x = \infty$, $C = \dfrac{3}{2}R$

from PV^x = constant

$\Rightarrow P^{1/x}V =$ another constant

so at $x = \infty$, $V =$ constant

hence $C = C_v = \frac{3}{2}R$

and then $C_p = C_v + R = \frac{5}{2}R$

at $x = 0$, $P =$ constant and $C = C'$

hence $C' = C_p = \frac{5}{2}R$

at $x = x'$, $C = 0$, so the process is adiabatic, hence

$$x' = \frac{C_p}{C_v} = \frac{5}{3}$$

67. **d.** Suppose there are n_1 moles of hydrogen and n_2 moles of helium in the given mixture. Then the pressure of the mixture will be

$$P = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\frac{RT}{V}$$

$$2\times 101.3\times 10^3 = (n_1 + n_2)\frac{(8.3)(300)}{20\times 10^{-3}}$$

$$(n_1 + n_2) = \frac{2\times 101.3\times 10^3 \times 20\times 10^{-3}}{(8.3)(300)} = 1.62$$

The mass of the mixture is (in grams):

$$n_1 \times 2 + n_2 \times 4 = 5$$

$$(n_1 + 2n_2) = 2.5$$

Solving Eqs. (i) and (ii), $n_1 = 0.74$, $n_2 = 0.88$

Hence $\quad \frac{m_{H}}{m_{He}} = \frac{0.74\times 2}{0.88\times 4} = \frac{1.48}{3.52} \rightarrow 2:5$

68. **b.** The gas pressure

$$= \frac{\text{Weight of piston}}{\text{Area of cross-section}} + \text{atm. pressure}$$

$$= \frac{8\times 9.8}{60\times 10^{-4}} + 1.00\times 10^5 \text{ N/m}^2$$

$$= 1.13\times 10^5 \text{ N/m}^2$$

During the heating pressure, the internal energy is changed by ΔU_1 and work ΔW_1 is done.

Therefore, $\quad \Delta Q_1 = \Delta U_1 + \Delta W_1 = \Delta U_1 + PdV$

$$= \Delta U_1 + (1.13\times 10^5)(0.20\times 60\times 10^{-4})$$

$$= \Delta U_1 + 136 \text{ J}$$

During the cooling process, no work is done as volume is constant, $\Delta W = 0$.

Hence, $\Delta Q_2 = \Delta U_2$. But ΔU_2 is negative as the temperature decreases, and since the gas returns to its original temperature, $\Delta U_2 = -\Delta U_1$

Hence

$$\left[\Delta Q_1 - \left|\Delta Q_2\right|\right] = (\Delta U_1 + 136 - \Delta U_1) = +136\text{ J}$$

69. **b.** Since $PV^n =$ constant and also $PV = RT$, taking 1 mol of the gas for simplicity,

$$dU = C_v\, dT$$

where $C_V \rightarrow$ molar specific heat at constant volume.

Now the molar specific heat in a polytropic process $PV^n =$ constant is given by

$$C_V = \left(\frac{R}{\gamma - 1}\right) - \left(\frac{R}{n-1}\right) = \frac{(n-\gamma)R}{(n-1)(\gamma-1)} \qquad \text{(i)}$$

From this equation we see that C_v will be negative when $n < \gamma$ and $n > 1$, simultaneously, i.e., $1 < n < \gamma$. Since γ for all ideal gases is greater than 1, if $n > \gamma$ or $n < 1$, then C_v will be positive.

70. **a.** Since the piston is moved slowly we assume isothermal condition for both the gases as thermal equilibrium is maintained throughout.

Let the total volume of chamber be V. Then volume of gas in X increases (expands) from $V/3$ to $2V/3$. The work done is positive and given (for 1 mol of monatomic gas) by

$$W_X = +RT \log_e \frac{2V/3}{V/3} = RT \log_e 2$$

The volume of gas in Y decreases (compressed) from $2V/3$ to $V/3$.

The work done (isothermally) is negative and given (for 2 mol of diatomic gas) by

$$W_Y = 2RT \log_e \frac{V/3}{2V/3} = -2RT \log_e 2$$

Hence the total work done on the system is

$$W = W_X + W_Y = -RT \log_e 2$$

Substituting $R = 8.3$, $\log_e 2 = 0.6996$ and simplifying $W = -5.8\,T$.

71. **b.** Temperature at state $P = T_0$, since P lies on the isothermal of temperature T_0. If T be the temperature at Q, then for the adiabatic process B, we have, $T_0 V_0^{\gamma-1} = T(2V_0)^{\gamma-1}$

$$T = \frac{T}{2^{\gamma-1}} = \frac{T_0}{2^{2/3}}$$

Change in the internal energy of the gas is

$$\Delta U = C_V(T - T_0) = \left(\frac{R}{\gamma-1}\right)\left(\frac{T_0}{2^{2/3}} - T_0\right)$$

$$= \frac{3RT_0(1-2^{2/3})}{2\times 2^{2/3}} = -4.6\,T_0$$

72. **d.** From the first law of thermodynamics

$$Q = W + \Delta U$$

For an isobaric process

$$W = R\Delta T, \quad Q = C_P\Delta T = \left(\frac{f+2}{2}\right)R\Delta T$$

$$\frac{W}{Q} = \frac{2}{f+2}$$

where f is the number of degrees of freedom.

For an isochoric process,

$$W = 0, \quad \frac{W}{Q} = 0$$

The straight line 1 corresponds to an isochoric process. For monoatomic gas $f = 3$ and for diatomic gas $f = 5$.

$$\frac{W}{Q} = \frac{2}{5} \quad \text{(for monatomic gas)}$$

$$= \frac{2}{7} \quad \text{(for diatomic gas)}$$

Straight line 2 corresponds to isobaric process for diatomic gas and straight line 3 corresponds to isobaric process for monatomic gas.

For isotherm $W = Q$, $\tan\theta_4 = 1$ straight line 4 corresponds to isothermal process because $W = Q$ only if $\Delta T = 0$ i.e. $\Delta U = 0$. For an adiabatic process $Q = 0$, the straight line 5 corresponds to it.

73. **c.**

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{(1.38\times10^7\,\text{Pa})(16\,\text{L})}{300\,\text{K}} = \frac{(10^5\,\text{Pa})\left(2.4\frac{\text{L}}{\text{m}}t\right)}{300\,\text{K}}$$

$$t = \frac{1.38\times10^7\times16}{10^5\times2.4} = 920 \text{ min}$$

$$= \frac{920}{60}\text{ h} = 15 \text{ h (approx.)}$$

74. **b.** Work done, $W = \int_{V_1}^{V_2} \alpha V^2 dV$

where $\alpha = 3\times10^5$ Pa/m^6

$$\Rightarrow \quad W = \left[\frac{\alpha V^3}{3}\right]_{V_1=1\text{m}^3}^{V_2=2\text{m}^3} = \left[\frac{3\times10^5 V^3}{3}\right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = 10^5\left(V_2^3 - V_1^3\right)$$

$V_1 = 1$ m^3 $\quad V_2 = 2$ m^3

$W = 10^5 (8 - 1) = 7 \times 10^5$ J

75. **c.** $P = 1$ atm $= 10^5$ N/m^2

$T = 0°\text{C} = 273$ K

$$V = \frac{nRT}{P} = \frac{1\times8.3\times273}{10^5} = 0.0227 \text{ m}^3 = 22.7 \text{ L}$$

$$C_v = \frac{5}{2}R; \quad C_p = \frac{7}{2}R$$

Heat transferred

$$\Delta Q = nC_p\Delta T = n\frac{7R}{2}\Delta T = 13200 \text{ J}$$

Work done

$$nR\Delta T = \frac{13200\times2}{7}$$

$$= P(V_f - V_i) = 3771$$

$$V_f - V_i = 3771\times10^{-5} = 0.0377 \text{ m}^3$$

$$V_f = V_i + 37.7 \text{ L}$$

$= 22.7$ L $+ 37.7$ L

$= 60.4$ L ≈ 60 L

76. **a.** No work is done along path AB because the process is isochoric.

$\therefore$ Work done $= P_B(V_D - V_A)$

$= 8 \times 10^4(5 \times 10^{-3} - 2 \times 10^{-3}) = 240$ J

$(\Delta Q)_{AC} = (\Delta Q)_{AB} + (\Delta Q)_{BC} = 600 + 200 = 800$ J

$\therefore$ $(\Delta U)_{AC} = (\Delta Q) - (\Delta W) = 800 - 240 = 560$ J

77. **d.**

$$(\Delta Q)_{ab} = +7000 = \mu C_v(1000 - 300) \quad \text{(i)}$$

For the process ca:

$T_a = 300$ K, $\quad T_c = T_b = 1000$ K

$(\Delta Q)_{ca} = \mu C_p(300 - 1000) = -\mu C_p \times 700$

$= -\mu(C_v + R)700 \quad$ (ii)

For carbon monoxide:

$T_a = 300$ K, $\quad T_c = T_b = 1000$ K

$(\Delta Q)_{ca} = \mu C_p (300 - 1000) = -\mu C_p \times 700$

$= -\mu(C_v + R)\,700$

For carbon monoxide:

$$\gamma = \frac{7}{5}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2} \quad \text{(iii)}$$

Hence, from Eq. (i)

$$7000 = \mu\frac{5R}{2}\times700 \quad \text{or} \quad \mu R = \frac{20}{5} = 4$$

$$(\Delta Q)_{ca} = -(7000 + 4 \times 700) = -9800 \text{ J}$$

Negative sign shows that heat is ejected.

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78. c.

$$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4$$
$$= 5960 - 5585 - 2980 + 3645 = 1040 \text{ J}$$
$$\Delta W = W_1 + W_2 + W_3 + W_4$$
$$= 220 - 825 - 1100 + W_4 = 275 + W_4$$

For a cyclic process, $U_f = U_f$

$$\Delta U = Uf - U_i = 0$$

From the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
$$1040 = 0 = 275 + W_4 \quad \text{or} \quad W_4 = 765 \text{ J}$$

79. b. Process AB is an isothermal process, i.e., $P \propto 1/V$ and since $\rho \propto 1/V$, $\rho - V$ graph will be a rectangular hyperbola. Pressure is increasing; therefore, volume will increase. Process BC is an isochoric process. Therefore, V = constant and since $\rho = m/V$, density is also constant. i.e., $\rho - V$ graph is a dot. Process CD is inverse of process AB and process DA is inverse is BC.

80. a. $dU = 0$

Therefore by the first law of thermodynamics

$$dQ_{\text{Cyclic}} = dW_{\text{Cyclic}}$$

Since $B \to C$ is an isochoric process

$$\Rightarrow dW_{a\to C} = 0$$
$$\Rightarrow 5 = dW_{A\to B} + dW_{a\to C} + dW_{C\to A}$$
$$\Rightarrow 5 = 10\,(2-1) + 0 + dW_{c\to A}$$
$$\Rightarrow dW_{C\to A} = -5 \text{ J}$$

81. c. For the gas in container A

$$\Delta P = (P_A)_{\text{final}} - (P_A)_{\text{initial}} = \frac{n_A RT}{2V} - \frac{n_A RT}{V}$$

$$\Delta P = -\frac{n_A RT}{2V} \qquad \text{(i)}$$

For gas in container B

$$1.5\,\Delta P = (P_B)_{\text{final}} - (P_B)_{\text{initial}} = \frac{n_B RT}{2V} - \frac{n_B RT}{V}$$

$$1.5\,\Delta P = -\frac{n_B RT}{2V} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$n_B = 1.5\,n_A$$
$$\Rightarrow 2n_B = 3n_A$$
$$\Rightarrow 2m_B = 3m_A$$

82. c. For an adiabatic process,

$$PV^{\gamma} = \text{constant}$$
$$TV^{\gamma-1} = \text{constant}$$

and $\quad T^{\gamma}P^{1-\gamma} = \text{constant}$

Putting, $\gamma = 5/3$ (argon being a monatomic gas), the equation becomes:

$$PV^{5/3} = \text{constant}$$
$$TV^{-2/3} = \text{constant}$$
$$T^{5/3}P^{-2/3} = \text{constant} \Rightarrow TP^{-2/5} = \text{constant}$$

83. c.

$$\Delta W = \frac{1}{2}(1+2)\times 10^3 \text{ N/m}^2 \times (0.4-0.2) \text{ m}^3 = 300 \text{ J}$$

$$\Delta U = nC_V\Delta T = \frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

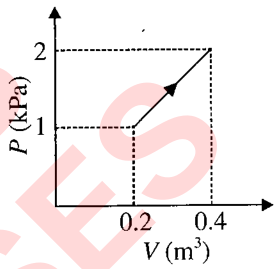

Fig. 2.180

$$= (3/2)\,(0.4\times 2 - 0.2\times 1)\times 103 \text{ J} = 900 \text{ J}$$
$$\Delta Q = \Delta W + \Delta U = 1200 \text{ J}$$

84. c. $C = C_V + W'$

where W' is the work done by the gas per unit mole per unit rise in temperature. So

$$W' = \alpha T - C_V = \alpha T - (5R/2)$$

$$\Delta W = \int W' dT = \int_{T_0}^{2T_0}\left(\alpha T - \frac{5R}{2}\right)dT$$

$$= (3\alpha T_0 - 5R)\frac{T_0}{2}$$

85. c. $\Delta U = nC_v\,\Delta T$

Also, $\quad \dfrac{C_p}{C_v} = \gamma$

Hence $\quad \dfrac{C_p - C_v}{C_v} = \gamma - 1$

$$\Rightarrow C_v = R/(\gamma - 1)$$

$$\Delta U = \frac{nR}{\gamma - 1}\Delta T = \frac{p\Delta V}{\gamma - 1} = \frac{p(2V - V)}{\gamma - 1} = \frac{pV}{\gamma - 1}$$

86. a. Work done by the gas at constant pressure

$$\Delta W = P\Delta V$$
$$= (1\times 10^5 \text{ Nm}^{-2})\,(1000 - 800)\times 10^{-6} \text{ m}^3 = 20 \text{ J}$$
$$\Delta U = \Delta Q - \Delta W$$
$$\Delta U = 100 \text{ J} - 20 \text{ J} = 80 \text{ J}$$

87. c.

$$\Delta W_{ABC} = 2P(2V - V) = 2PV$$
$$\Delta W_{ADC} = nRT\ln(2V/V)$$
$$= 2PV\ln 2$$

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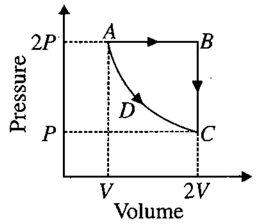

Fig. 2.181

∴ The required factor will be

$$= \frac{2PV \ln 2}{2PV} = \ln 2$$

88. c.

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\omega = \frac{T_2}{T_1 - T_2} = \frac{T_2/T_1}{1-(T_2/T_1)} = \frac{(1-\eta)}{\eta} = \frac{1}{\eta} - 1$$

89. a. $A \to B$ is an isobaric process,

$$V \propto T$$

so, $\Delta W_{AB} = nR\Delta T = 2 \times R \times (750 - 250) = 1000R$

$B \to C$ is an isochoric process

∴ $\Delta W_{BC} = 0$ and

$C \to D$ is an isothermal process

$$\Delta W_{CD} = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$= 2 \times R \times 1000 \ln\left(\frac{20}{15}\right) = 2000R \ln\left(\frac{4}{3}\right)$$

Total work done, $\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD}$

90. d. For adiabatic process:
Bulk modulus: $B = \gamma P$

for point p: $P = \frac{nRT}{V} = \frac{nR3T_0}{3V_0} = \frac{nRT_0}{V_0}$

$$\Rightarrow \quad B = \frac{\gamma nRT_0}{V_0} \quad \text{(i)}$$

Now $TV^{\gamma-1}$ = constant
$\Rightarrow (\gamma - 1)TdV + VdT = 0$

$$\Rightarrow \frac{dV}{dT} = \frac{-V}{(\gamma-1)T}$$

for point $P \to$

$$\frac{-3V_0}{3T_0} = \frac{-(3V_0)}{(\gamma-1)(3T_0)}$$

$\Rightarrow \gamma = 2$

so from Eq. (i), $B = \frac{2nRT_0}{V_0}$

91. d. Since the gas is slowly heated, it remains in equilibrium (more or less) with the atmosphere, i.e., the process takes place at a constant pressure.

Now, from $\quad PV = nRT, \quad PdV = nRdT$
or, $\quad P\Delta V = nR\Delta T \quad$ (i)

But $P\Delta V$ is the work done by the gas.

So, $\Delta W = nR\Delta T = (2 \text{ mol})(R)(4T_0 - T_0) = 6RT_0$

[From Eq. (i) $\Delta V \propto \Delta T$, i.e., if $\Delta V = 3V_0$, $\Delta T = 3T_0$]

92. a.

$$\Delta W = P\Delta V; \text{ given: } PV = \mu RT + \alpha V$$

∴ $\quad P\Delta V = \mu R\Delta T + \alpha \Delta V$

or, $\quad \Delta V = [(\mu R\Delta T)/(P_0 - \alpha)]$

i.e., $\quad \Delta W = \frac{P_0RT_0}{P_0 - \alpha} \quad (\mu = 1)$

93. c. Differentiating

$$\gamma T^{\gamma-1} dT\, P^{1-\gamma} + T^{\gamma}(1-\gamma)P^{-\gamma} dP = 0$$

or, $\quad dT = \frac{(\gamma-1)T}{\gamma P} dP$

or, $\quad dT = \left(\frac{1.5-1}{1.5}\right)\left(\frac{273}{76 \times 13.6 \times 981} \times 0.001\right)$

$$= 8.97 \times 10^{-8} \text{ K}$$

94. a. It is free expansion, temperature will remain constant.

95. c. $Q = nC_v \Delta T = n(C_P - R)\Delta T$

$$= 5\left(7.03 - \frac{8.31}{4.2}\right) \times (20 - 10) = 250 \text{ cal}$$

96. d. C_v for hydrogen $= 5R/2$, C_V for helium $= 3R/2$, C_v for water vapour $= 6R/2$

$$\therefore \quad (C_v)_{\text{mix}} = \frac{4 \times \frac{5R}{2} + 2\frac{3R}{2} + 1 \times 3R}{4+2+1} = \frac{16R}{7}$$

$$\therefore \quad C_p + C_v + R = \frac{16R}{7} + R = \frac{23R}{7}$$

97. a. For an adiabatic process, $PV^{\gamma} = K$

Here, $\quad \gamma = 3/2$ and K = constant

∴ $\quad PV^{3/2} = K$

$$\log P + \frac{3}{2}\log V = \log K$$

$$\frac{\Delta P}{P} + \frac{3}{2}\frac{\Delta V}{V} = 0$$

$$\therefore \quad \frac{\Delta V}{V} = -\frac{2}{3}\frac{\Delta P}{P}$$

$$\frac{\Delta V}{V} \times 100 = -\left(\frac{2}{3}\right)\left(\frac{\Delta P}{P} \times 100\right) = -\frac{2}{3} \times \frac{2}{3} = -\frac{4}{9}$$

Therefore volume decreases by about (4/9)%.

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98. d. For A: As piston is free to move, the process is isobaric.

$$\Delta Q = \mu C_p (\Delta T)_1$$

For B: As piston is held fixed, the process is isochoric.

$\therefore$ $$\Delta Q = \mu C_v (\Delta T)_2$$

Now $$C_p(\Delta T)_1 = C_v(\Delta T)_2$$

$$\frac{7R}{2} \times 30 = \frac{5R}{2}(\Delta T)_2$$

$\therefore$ $$\Delta T_2 = 42 \text{ K}$$

99. d. $P = 4.5 \times 10^5$ Pa; $dQ = 800$ kJ

$$V_1 = 0.5 \text{ m}^3; \quad V_2 = 2 \text{ m}^3$$

$$dW = P(V_2 - V_1) = 4.5 \times 10^5 (2 \times 0.5)$$

$$= 6.75 \times 10^5 \text{ J}$$

Change in internal energy

$$dU = dQ - dW$$

$$= 800 \times 10^3 - 6.75 \times 10^5 = 1.25 \times 10^5 \text{ J}$$

100. c. $C_p - C_v = $ m, for hydrogen ($M_1 = 2$)

$C_p - C_v = n$, for nitrogen ($M_2 = 14$)

For hydrogen, $$C_p - C_v = \frac{1}{M_1}\frac{dQ}{dT} = m$$

For nitrogen, $$C_p - C_v = \frac{1}{M_2}\frac{dQ}{dT} = n$$

$\therefore$ $$\frac{m}{n} = \frac{\frac{1}{M_1}\frac{dQ}{dT}}{\frac{1}{M_2}\frac{dQ}{dT}} = \frac{M_2}{M_1} = \frac{14}{2} = 7$$

$\therefore$ $$m = 7n$$

101. b.

$$\frac{C_P}{C_v} = \gamma = \frac{7}{5}$$

$$\text{Work done} = \frac{\mu R}{\gamma - 1} \times \Delta T$$

$$= \frac{8.3 \times 400 \times 5}{\frac{7}{5} - 1} = 41.5 \text{ J}$$

Work done $=$ change in internal energy
($\because \Delta Q = 0$ for adiabatic process)

Therefore, change in internal energy

$$= 41.5 \text{ kJ}$$

102. a.

For $$\gamma = \frac{7}{5}, \quad C_v = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2}$$

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{(7/5)R}{\frac{7}{5} - 1} = \frac{7R}{2}$$

For $$\gamma = \frac{4}{3}, \quad C_v = 3R, \quad C_p = 4R$$

$\therefore$ $$\gamma_{\text{mix}} = \frac{\frac{7}{2} + 4}{\frac{5}{2} + 3} = \frac{15}{11}$$

103. a.

$$c_p - c_v = \frac{R}{M} = \frac{PV}{TM} = \frac{P}{Td}$$

$\therefore$ $$d = \frac{P}{T(c_p - c_v)}$$

$$= \frac{1.013 \times 10^5}{273(525 - 315)} = 1.77 \text{ kg/m}^3$$

104. c. The two processes are shown in the following P–V diagram:

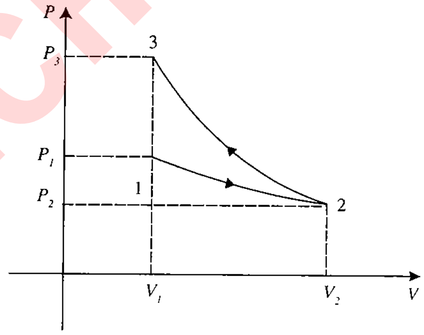

Fig. 2.182

For isothermal process:

$$P_1V_1 = P_2V_2$$

i.e., $$P_1 = \left(\frac{V_2}{V_1}\right)P_2$$

For adiabatic process:

$$P_3V_1^\gamma = P_2V_2^\gamma$$

i.e., $$P_3 = \left(\frac{V_2}{V_1}\right)^\gamma P_2$$

As $\gamma > 1$, hence $P_3 > P_1$.

Further, as slope of adiabatic curve is greater than that of isothermal process curve, adiabatic curve will lie above the isothermal curve. That is, area under adiabatic curve > area under isothermal curve

i.e., Negative work > Positive work

i.e., $W < 0$

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105. c. Internal energy of n moles of an ideal gas at temperature T is given by

$$U = \frac{f}{2}nRT \qquad (f = \text{degrees of freedom})$$

$$U_1 = U_2$$

$$f_1 n_1 T_1 = f_2 n_2 T_2$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{f_2 T_2}{f_1 T_1} = \frac{3\times 2}{5\times 1} = \frac{6}{5}$$

Here, f_2 = degrees of freedom of He = 3
and, f_1 = degrees of freedom of H_2 = 5

106. b.

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

or $$v_{rms} \propto \sqrt{T}$$

Since v_{rms} becomes half, so temperature is reduced to one-fourth of its previous value.

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process,

$$TV^{\gamma-1} = T'V'^{\gamma-1}$$

or $$\frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}}$$

$$= (4)^{\frac{1}{1.5-1}} = 4^2 = 16$$

$$\therefore \quad V' = 16V$$

107. a. Let T be the temperature of the mixture.

Then $U = U_1 + U_2$

or $$\frac{f}{2}(n_1+n_2)RT = \frac{f}{2}(n_1)(R)(T_0) + \frac{f}{2}(n_2)(R)(2T_0)$$

or $$(2+4)T = 2T_0 + 8T_0$$

$$(\because n_1 = 2, \quad n_2 = 4)$$

$$\therefore \quad T = \frac{5}{3}T_0$$

108. b. Let the initial pressure of the three samples be P_A, P_B and P_C, then

$$P_A(V)^{3/2} = (2V)^{3/2}P \quad (\because P_B = P)$$

or $$P_A = P(2)^{3/2}$$

$$P_C(V) = P(2V)$$

or $$P_C = 2P$$

$$\therefore \quad P_A : P : P_C$$

$$= (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$$

109. c.

$$W_{AB} = -P_0V_0$$

$$W_{BC} = 0$$

and $$W_{CD} = 4P_0V_0$$

$$\therefore \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$$

110. b. Since P–V graph of the process is a straight line and two points $(V_0, 2P_0)$ and $(2V_0, P_0)$ are known, its equation will be

$$(P - P_0) = \frac{(2P_0 - P_0)}{(V_0 - 2V_0)}(V - 2V_0) = \frac{P_0}{V_0}(2V_0 - V)$$

$$\therefore \quad P = 3P_0 - \frac{P_0 V}{V_0}$$

According to equation for ideal gas,

$$T = \frac{pV}{nR}$$

$$= \left(3P_0 - \frac{P_0V}{V_0}\right)\frac{V}{nR}$$

$$= \frac{3P_0V_0V - P_0V^2}{nRV_0} \qquad \text{(i)}$$

For T to be maximum, $\frac{dT}{dV} = 0$

$$3P_0V_0 - 2P_0V = 0$$

or $$V = \frac{3V_0}{2} \qquad \text{(ii)}$$

Putting this value in Eq. (i), we get

$$T_{max} = \frac{3P_0V_0\left(\frac{3V_0}{2}\right) - P_0\left(\frac{9}{4}V_0^2\right)}{nRV_0} = \frac{9P_0V_0}{4nR}$$

111. b. For an adiabatic process,

$$0 = dU + PdV$$

or $$d(a + bPV) + PdV = 0$$

or $$bP\,dV + bV\,dP + p\,dV = 0$$

or $$(b+1)PdV + bV\,dP = 0$$

or $$(b+1)\frac{dV}{V} + b\frac{dP}{P} = 0$$

or $$(b+1)\log V + b\log P = \text{constant}$$

$$V^{b+1}p^b = \text{constant}$$

or $$PV^{\frac{b+1}{b}} = \text{constant}$$

$$\therefore \quad \gamma = \frac{b+1}{b}$$

112. b.

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} KV\,dV$$

$$\left(\because \frac{p}{V} = K = \text{constant}\right)$$

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$\therefore \quad W = \frac{1}{2}k(V_2^2 - V_1^2)$

$PV = RT$

But $\quad p = KV$

$\therefore \quad KV^2 = RT$

or $\quad K(V_2^2 - V_1^2) = R(T_2 - T_1)$

$\therefore \quad W = \frac{R}{2}(T_2 - T_1)$

113. b. For 1 mol of gas,

$$\Delta Q = C_v\Delta T + P\Delta T$$

At constant volume, $\Delta T = 0$
For 2 moles of gas,

$$\Delta = 2C_v\Delta T$$

From $\quad PV = nRT = 2R \times 300$

and $\quad \frac{P}{2}V = 2RT_f$

$\therefore \quad T_f = 150$ K

$\therefore \quad \Delta Q = 2C_v(T_f - T_i) = 2C_v(150 - 300)$
$= -300C_v$ J

In the next process,

$$\Delta Q = 2C_p\Delta T = 2C_p\,(300 - 150)$$
$$= 300C_p \text{ J}$$

$\therefore \quad$ Net heat absorbed $= -300C_v + 300C_p$
$= 300(C_p - C_v) = 300R$ J

114. c. Work done = area of the ΔABC

$$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times (3V_1 - V_1) \times (4P_1 - P_1)$$
$$= \frac{1}{2} \times 2V_1 \times 3P_1 = 3P_1V_1$$

115. c. As $\Delta U = 0$ in a cyclic process,

$$\Delta Q = \Delta W = \text{area of circle} = \pi r^2$$

or $\quad \Delta W = 10^2\pi$ J

116. b. Let initial pressure, volume and temperature be P_0, V_0 and T_0, respectively, indicated by state A in P–V diagram. The gas is then isochorically taken to state B ($2P_0$, V_0, $2T_0$) and then taken from state B to state C ($2P_0$, $2V_0$, $4T_0$) isobarically.

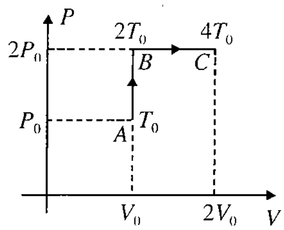

Fig. 2.183

Total heat absorbed by 1 mol of gas

$$\Delta Q = C_v(2T_0 - T_0) + C_P(4T_0 - 2T_0)$$
$$= \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0 = \frac{19}{2}RT_0$$

Total change in temperature from series A to C is $\Delta T = 3T_0$
Therefore,

$$\text{Molar heat capacity} = \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$$

117. d. Heat absorbed by gas in three processes is given by

$$Q_{ACB} = \Delta U + W_{ACB}$$
$$Q_{ADB} = \Delta U$$
$$Q_{AEB} = \Delta U + W_{AEB}$$

The change in internal energy in all the three cases is same and W_{ACB} is positive, W_{AEB} is negative.

Hence $\quad Q_{ACB} > Q_{ADB} > Q_{AEB}$

118. c. Process AB is isothermal expansion, BC is isobaric compression and in process CA

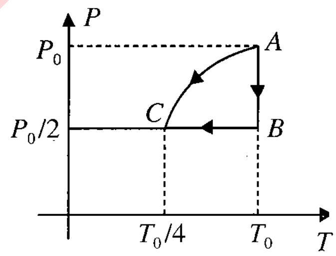

Fig. 2.184

$$P \propto \frac{nRT}{P} \quad \Rightarrow P^2 \propto T$$

119. d. BC is isochoric. $V_B > V_A$, $\quad V_B = V_C$, $\quad V_D > V_C$

120. c.

$$Q_{AB} = \Delta U_{AB} + W_{AB}$$
$$W_{AB} = 0$$
$$\Delta U_{AB} = \frac{f}{2}nR\Delta T$$

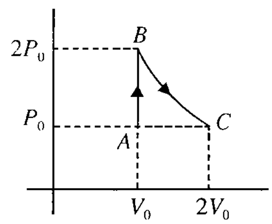

Fig. 2.185

$$\frac{f}{2}(\Delta\, PV)\Delta U_{AB} = \frac{5}{2}(\Delta\ PV)$$

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$$Q_{AB} = 2.5\, P_0 V_0$$

Process BC, $Q_{BC} = \Delta U_{BC} + W_{BC}$

$$Q_{BC} = 0 + 2\, P_0 V_0 \log 2 = 1.4\, P_0\, V_0$$

$$Q_{net} = Q_{AB} + Q_{BC} = 3.9\, P_0 V_0$$

121. a. $PV = RT$ for 1 mol

$$W = \int P dV = \int \frac{RT}{V}\, dV$$

$$V = CT^{2/3}$$

$$dV = \frac{2}{3} CT^{-1/3} dT \quad \text{or} \quad \frac{dV}{V} = \frac{2}{3}\frac{dT}{T}$$

$$W = \int_{T_1}^{T_2} RT\left(\frac{2}{3}\right)\frac{dT}{T} = \frac{2}{3} R(T_2 - T_1) = 166.2 \text{ J}$$

122. a. Change in temperature in process 1 will be greater and in process 3 will be least.

123. c. $\log P = m \log V + C_1$, where C_1 is positive, m is slope

$$m = \frac{2.38 - 2.10}{1.1 - 1.3} = -1.4$$

$$\log P = -1.4 \log V + C_1$$

$$\log PV^{1.4} = C_1$$

$$PV^{1.4} = k$$

Thus, it represents an ideal diatomic gas undergoing adiabatic change.

124. a. $6T_1 = 3T_2 = 2T_4 = T_3 = 1800$ K

$$T_1 = 300 \text{ K}; \quad T_2 = 600 \text{ K}$$

$$T_4 = 900 \text{ K}; \quad T_3 = 1800 \text{ K}$$

$4 \to 1$ and $2 \to 3$ are isochoric processes in which work done = 0

$$W_{12} = P(V_2 - V_1) = nR(T_2 - T_1)$$

$$= 2 \times R\,(600 - 300) = 600R$$

$$W_{34} = P(V_4 - V_3) = nR(T_4 - T_3)$$

$$= 2 \times R\,(900 - 1800) = -1800R$$

$$W_{Total} = 600R - 1800R = -1200R = -10000 \text{ J}$$

125. c.

$$\frac{P - P_1}{P_1 - P_2} = \frac{V - V_1}{V_1 - V_2}$$

$$(P - P_1)(V_1 - V_2) = (V - V_1)(P_1 - P_2)$$

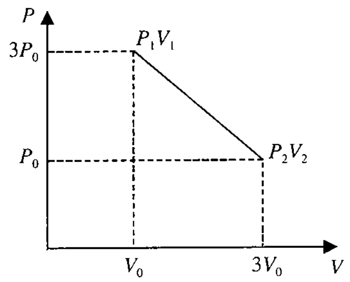

Fig. 2.186

$$(P - 3P_0)(V_0 - 3V_0) = (V - V_0)(3P_0 - P_0)$$

$$(P - 3P_0)(-2V_0) = (V - V_0)(2P_0)$$

$$-2V_0P + 6P_0V_0 = 2VP_0 - 2P_0V_0$$

$$2VP_0 + 2V_0P - 8P_0V_0 = 0$$

$$VP_0 + \frac{V_0 nRT}{V} - 4P_0V_0 = 0$$

$$V^2P_0 - 4P_0V_0 + V_0 nRT = 0$$

$$T = \frac{P_0(-V^2 + 4VV_0)}{V_0 nR}$$

For maximum or minimum value of T,

$$\frac{dT}{dV} = -2V + 4V_0 = 0 \quad \Rightarrow V = 2V_0$$

$$\frac{d^2T}{dV^2} = -2$$

It is negative so

T is maximum at $V = 2V_0$

$$T_{max} = \frac{P_0\left(-4V_0^2 + 8V_0^2\right)}{V_0 nR} = \frac{4P_0V_0}{nR}$$

126. b. Density of mixture, $\rho_{mix} = \dfrac{PM_{mix}}{RT}$

Mass of nitrogen gas, $m_N = 7$ g $= 7 \times 10^{-3}$ kg

Mass of carbon dioxide, $m_{CO_2} = 11$ g $= 11 \times 10^{-3}$ kg

Molecular weight of nitrogen gas, $M_N = 28 \times 10^{-3}$ kg

Molecular weight of carbon dioxide, $M_{CO_2} = 44 \times 10^{-3}$ kg

$$M_{mix} = \frac{n_N M_N}{n_N + n_{CO_2}} = \frac{\frac{m_N}{M_N} M_N + \frac{m_{CO_2}}{M_{CO_2}} M_{CO_2}}{\frac{m_N}{M_N} + \frac{m_{CO_2}}{M_{CO_2}}}$$

$$= \frac{m_N + m_{CO_2}}{\frac{m_N}{M_N} + \frac{m_{CO_2}}{M_{CO_2}}} = \frac{(7 + 11) \times 10^{-3}}{\left(\frac{7}{28} + \frac{11}{44}\right)}$$

$$= \frac{18 \times 10^{-3}}{\frac{1}{4} + \frac{1}{4}} = 36 \times 10^{-3} \text{ kg}$$

$$\rho = \frac{(1 \times 10^5)(36 \times 10^{-3})}{\frac{25}{3} \times 300} = 1.44 \text{ kg/m}^3$$

127. b.

$$C_v = \frac{3R}{2} \text{ J/mol K} = \frac{3R}{2M} \text{ J/kg K}$$

$$= \frac{3R}{2M \times 4.2} \text{ cal/kg K} = 0.075 \text{ kcal/kg K}$$

Molecular weight of argon atom

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$$M = \frac{3R}{2M \times 4.2 \times 0.075 \times 10^3}$$

$$= \frac{3 \times \frac{25}{3}}{2 \times 4.2 \times 75} = 40 \times 10^{-3} \text{ kg}$$

128. a. For paths 1, 2, 3 and 4, initial temperature of the gas is T_1 and final temperature of the gas is T_2

i.e., $\Delta U_1 = \Delta U_2 = \Delta U_3 = \Delta U_4$

$$= nC_V \Delta T = nC_v (T_2 - T_1)$$

$$T_3 > T_2 > T_1$$

For path 5, $(T_3 - T_1) > (T_2 - T_1)$

$$\Delta U_5 > \Delta U_3$$

129. a.

$$C = C_v + \beta V$$

$$\frac{dQ}{dT} = \frac{dU}{dT} + P\frac{dV}{dT} \Rightarrow C = C_V + \frac{PdV}{dT}$$

comparing, $P\dfrac{dV}{dT} = \beta V$

$$\frac{RT}{V}\frac{dV}{dT} = \beta V \quad \Rightarrow \frac{dV}{V^2} = \frac{\beta}{R}\frac{dT}{T}$$

On integration

$$\frac{-1}{V} = \frac{\beta}{R}\ln T \quad \Rightarrow -\ln T = \frac{R}{\beta V}$$

$\Rightarrow$ $T^{\beta V/R} = \text{constant}$

130. c. The P–V equation is given as

$$P = \frac{12P_0}{V_0}V - \frac{4P_0}{V_0^2}V^2 - 7P_0$$

putting $P = P_0$, we get

$$V^2 - 3V_0V + 2V_0^2 = 0$$

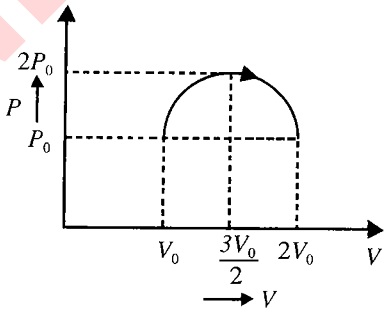

Fig. 2.187

$$V = \frac{+3V_0 \pm \sqrt{9V_0^2 - 8V_0^2}}{2} = 2V_0 \quad \text{or} \quad V_0$$

At $P = 2P_0, \quad V = \dfrac{3V_0}{2}$

$$\frac{dP}{dV} = 0 \quad \text{at} \quad V = \frac{3V_0}{2}$$

$$\frac{d^2P}{dV^2} = \text{negative}$$

P attains the maximum value at $V = 3V_0/2$

131. d. Work done on the system is negative.

For the process $1 \to 2$

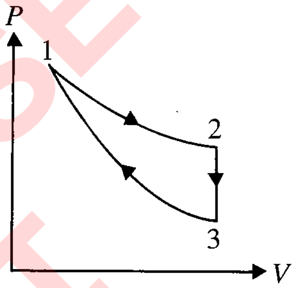

Fig. 2.188

Process	ΔQ	ΔW	ΔU
1–2			
2–3			−20 J
3–1		−20 J	
Cycle		10 J	

$$\Delta U_{1\to2} = 0$$

$\therefore$ $\Delta Q_{1\to2} = \Delta W_{12}$

For the process $2 \to 3$

$$\Delta W_{2\to3} = 0$$

$$\Delta Q_{2\to3} = -20 \text{ J}$$

For the process $3 \to 1$

$$\Delta Q_{3\to1} = 20 \text{ J}$$

$$\Delta U = \Delta U_{1\to2} + \Delta U_{2\to3} + \Delta U_{3\to1} = 0$$

$$\Delta W = \Delta W_{1\to2} + \Delta W_{2\to3} + \Delta W_{3\to1}$$

$$= \Delta W_{1\to2} + 0 - 20 \text{ J} = 10 \text{ J}$$

Work done in the process $1 \to 2 = 30$ J

$$\Delta Q = \Delta Q_{1\to2} + \Delta Q_{2\to3} + \Delta Q_{3\to1} = 30 \text{ J} - 20 \text{ J} = 10 \text{ J}$$

For the process $1 \to 2$, work is positive, i.e., work is done by the system.

132. c. Work done by the balloon

$$W = \int_{15\text{ m}^3}^{20\text{ m}^3} P_0 dV + \int_{20\text{ m}^3}^{25\text{ m}^3} PdV$$

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$$= \int_{15\,m^3}^{20\,m^3} P_0 dV + \int_{20\,m^3}^{25\,m^3} [P_0 + 2(V-V_0)^2] dV$$

$$= \left[100\times5+100\times5+2\times\frac{(25-20)^3}{3}\right] \text{kJ} = 1083\text{ kJ}$$

133. d. Pressure is directly proportional to square of the diameter of the balloon,

$$\frac{P_2}{P_1} = \frac{D_2^2}{D_1^2} \quad \text{(i)}$$

When $\quad V_2 = 2V_1 \quad \Rightarrow D_2^3 = 2D_1^3 =$

$$\Rightarrow \quad \frac{D_2}{D_1} = \left(\frac{V_2}{V_1}\right)^{1/3} \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{2/3}$$

$$\Rightarrow \quad PV^{-2/3} = \text{constant}$$

Multiple Correct Answers Type

1. a, b, c, d.

a. $\Delta U = Q - W = nC_P\Delta T - P\Delta V$

$$= nC_P\Delta T - nR\Delta T = n(C_P - R)\Delta T$$

$$= nC_v\Delta T = nC_v(T_2 - T_1)$$

b. $\Delta Q = \Delta U + \Delta W$

But $\Delta Q = 0$ for adiabatic process; hence

$$\Delta U = -\Delta W$$

or, $\quad |\Delta U| = |\Delta W|$

c. $\Delta U = nC_v\Delta T = 0 \quad (\because \Delta T = 0)$

d. $\Delta Q = 0$ (in adiabatic change)

2. a., c.

For a cyclic process, $\quad \Delta U = 0$

i.e., $\quad \Delta U = \Delta U_1 + \Delta U_2 = 0$

From relation $\quad \Delta Q = \Delta U + \Delta W$

As $\quad \Delta U = 0$

Hence, $\quad \Delta Q = \Delta W$

or $\quad \Delta Q - \Delta W = 0$

3. a, b, d. Figure 2.189 shows the straight line path along with the corresponding isothermal path. Since the work done by the gas is equal to area under the curve (such as shown in the figure by the shaded portion for the isothermal path), it is obvious that the gas does more work along the straight line path as compared with that for the isothermal path.

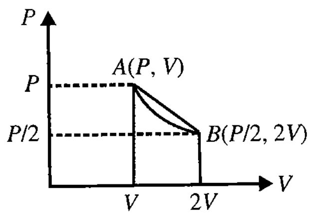

Fig. 2.189

As the volume is increased from V to $2V$, the difference of pressure between the straight line path and isothermal path initially increases and then decreases after attaining a maximum value. The same trend is observed in the case of temperature ($P \propto T$, $\therefore$ V is constant)

Now, the slope of straight line path is

$$m = \frac{P - P/2}{V - 2V} = -\frac{P}{2V}$$

or $\quad P = -2Vm$

Putting this in the ideal gas equation,

$$PV = nRT$$

or $\quad [-2Vm]\,V = nRT$

$$V^2 = -\frac{nR}{2m}T$$

$$V^2 = kT$$

Which is the equation of a parabola.

Similarly, eliminating V from ideal gas equation, we get

$$P\left[-\frac{P}{2m}\right] = nRT$$

or $\quad P^2 = (\text{constant})T$

Which is again an equation of a parabola.

4. a, b, c, d. Equilibrium of piston gives $PS = Kx_0$

$$P = \frac{Kx_0}{S}$$

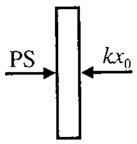

Fig. 2.190

Since the chamber is thermally insulated, $\Delta Q = 0$

$\therefore \quad$ Elastic PE of spring = Work done by gas

or, $\quad$ work done by gas $= \frac{1}{2}Kx_0^2$

This work is done at the expense of internal energy of the gas. Therefore, internal energy of the gas is decreased by $(1/2)Kx_0^2$.

5. a, c. Equilibrium of piston gives

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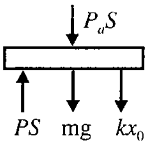

Fig. 2.191

$$PS = P_aS + mg + Kx_0$$

$$P = P_a + \frac{mg}{S} + \frac{Kx_0}{S}$$

(P = final pressure of gas)

Work done by the gas
= work done against atmospheric pressure + elastic potential energy stored in the spring + increase in gravitational potential energy of the piston

$$= P_a\Delta V + \frac{1}{2}Kx_0^2 + mgx_0 = P_aSx_0 + \frac{1}{2}Kx_0^2 + mgx_0$$

There occurs decrease in internal energy of the gas, because the gas is thermally insulated and hence, work is done at the expense of internal energy of the gas.

6. **a, b, c.** Internal energy (U) depends only on the initial and final states. Hence, ΔU will be same in all the three paths. In all the three paths, work done by the gas is positive and the product PV or temperature T is increasing. Therefore, internal energy is also increasing. So, from the first law of thermodynamics, heat will be absorbed by the gas. Further, area under $P-V$ graph is maximum in path 1 while ΔU is same for all the three paths. Therefore, heat absorbed by the gas is maximum in path 1. For temperature of the gas, we can see that product PV first increases in path 1 but whether it is decreasing or increasing later on we cannot say anything about it unless the exact values are known to us.

7. **b, d.** For an isothermal process, PV = constant

$$P = \frac{\text{constant}}{V} \quad \text{or} \quad P \propto \frac{1}{V}$$

It means, for an isothermal process the graph between P and $1/V$ will be a straight line passing through origin. Hence, the straight line AB will pass through origin. Hence, option (a) is wrong.

During process AB, the pressure P remains constant but $1/V$ increases. It means, volume V decreases. Hence, AB is isobaric compression. Since volume of the gas decreases at constant pressure, therefore its temperature decreases. But temperature at C is equal to that of A. Hence during the process BC, the temperature of the gas increases. In fact, process BC is an isochoric heating. Therefore, option (b) is correct. Since during the process BC, volume remains constant, no work is done by the gas against external pressure. Therefore option (c) is wrong.

During process CA, $1/V$ decreases, it means volume V increases. Since the volume increases, work is done by the gas against external pressure.

Since process CA is an isothermal process, no change in internal energy of the gas takes place. Hence, according to the first law of thermodynamics, $Q = W + \Delta U$; heat supplied during this process is equal to work done by the gas against external pressure. Hence option (d) is correct.

8. **a, d.** Vibrational kinetic energy of a monatomic gas = 0 at all temperatures. So, $C_v = 3R/2$ for a monoatomic gas at high temperatures also. In case of a diatomic gas $C_v = 5R/2$ at low temperatures while, $C_v > 5R/2$ at high temperatures due to vibrational KE.

9. **a, b, c.** The molar heat capacity has the general definition

$$C = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

where n = number of moles
ΔQ = heat absorbed by the gas
ΔT = rise in temperature of gas

It is possible to obtain almost any set of values for ΔQ and ΔT by proper selection of a process.

10. **a, b, c, d.** Since both the gases are contained in the same vessel, temperature of both the gases is same.

Average KE per molecule of a diatomic gas is 5/2 KT. Hence, average KE per molecule of both the gases is same. Therefore, option (a) is correct.

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Hence,

$$\frac{(v_{rms})_2}{(v_{rms})_1} = \sqrt{\frac{M_1}{M_2}} = \sqrt{16} = 4$$

Hence, option (b) is correct.

Let molar mass of B be M, then that of A will be equal to $16M$.

Let mass of gas B in the vessel be m; then that of A will be $2m$. The number of moles of a gas, in the vessel will be $n = m/M$. Hence, number of moles of gases A and B will be

$$n_1 = \frac{2m}{16M} \quad \text{and} \quad n_2 = \frac{m}{M}$$

Hence,

$$\frac{n_1}{n_2} = \frac{1}{8}$$

Hence, option (d) is correct.

Partial pressure exerted by a gas is

$$P = \frac{nRT}{V}$$

Hence,

$$\frac{P_2}{P_1} = \frac{n_2}{n_1} = 8$$

Therefore, option (c) is also correct.

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11. **a, c.** Process AB in the given figure is an isobaric process. During this process

$$V \propto T$$

But $$PV = nRT$$

or $$PV \propto T$$

Therefore during this process, pressure P remains constant. Process BC is an isochoric cooling. During this process, volume of the gas remains constant but the temperature decreases.

Process CA is an isothermal process. During this process, volume decreases and temperature remains constant. Hence, pressure increases during this process. Hence, on P–V diagram, process AB will be a straight line parallel to the V-axis, process BC will be a straight line parallel to the P-axis and CA will be a rectangular hyperbola. Hence, option (c) is correct, while (d) is wrong.

On $T-V$ diagram, process AB will be a straight line parallel to the T-axis, during which temperature increases. Process BC will be a straight line passing through origin, during which temperature and pressure both decrease and process CA will be a straight line parallel to the P-axis during which pressure increases. Hence, option (a) is correct while (b) is wrong.

12. **b, c, d.** Statement (a) is incorrect. It is true only if the two bodies have the same thermal capacity.

Statement (b) is correct. The coolant is used to prevent the engine or the nuclear plant from becoming too hot. The heat absorbed by a substance per unit mass is directly proportional to its specific heat. Therefore if a coolant has a high specific heat, it will remove a large amount of heat from the engine or the nuclear plant.

Statement (c) is correct. If the volume of vapour is decreased, at a constant temperature a part of the vapour will condense into liquid such that the vapour pressure remains unchanged. In other words, the pressure does not increase but the volume of vapour decreases.

Statement (d) is also correct. Since the vessels are of the same capacity, the volume occupied by the gas is doubled, hence the pressure reduces to half.

13. **a, c.** For an adiabatic process, PV^{γ} = constant. Differentiating w.r.t. V, we get

$$\frac{dP}{dV}V^{\gamma} + P\gamma V^{\gamma-1} = 0$$

or $$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

For isothermal process, PV = constant.

Hence,

$$\frac{dP}{dV} = -\frac{P}{V}$$

Now, dP/dV is the slope of the $(P-V)$ graph. Thus, the slope of the $(P-V)$ graph for an adiabatic process is γ times that for an isothermal process. Hence, curves BC and DA both represent adiabatic process and curves AB and CD both represent isothermal process. Thus, the correct choices are (a) and (c).

14. **a, b, c.** The critical step is that work in path $1-3$ is mean of that for $1-2-3$ and $1-4-3$ (considering areas).

For (a) $$100 - 40 = x - 10$$

or, $$x = 70 \text{ cal}$$

(b) It is also correct as half of (40 + 10) is 25 cal

(c) dU is -60, work is -25

So $$-60 = dQ - (-25)$$

or $$dQ = -85 \text{ cal}$$

(d) $$dU = 100 - 40 = 60 \text{ cal}$$

Hence, option (d) is not correct.

15. **a, b, c, d.** One mole of an ideal monatomic gas (initial temperature T_0) is made to go through the cycle $abca$ shown in Fig. 2.192. U denotes the internal energy.

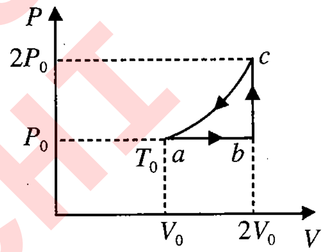

Fig. 2.192

For the process ab, $\dfrac{P_0V_0}{T_0} = \dfrac{2P_0V_0}{T_0}$

$$T_b = 2T_0$$

$$T_b > T_a \Rightarrow U_b > U_a$$

$$U_b - U_a = C_v\Delta T = \frac{3R}{2}(2T_0 - T_0) = \frac{3RT_0}{2}$$

For the process bc,

$$\frac{P_0(2V_0)}{2T_0} = \frac{2P_0(2V_0)}{T_C} \Rightarrow T_C = 4T_0$$

$$T_c > T_0$$

$$U_c - U_b = \frac{3R}{2}(4T_0 - 2T_0) = 3RT_0$$

For the process ca,

$$U_c - U_a = \frac{3R}{2}(4T_0 - T_0) = \frac{9RT_0}{2}$$

16. **a, b, d.** During process A and B, pressure and volume both are decreasing. Therefore, temperature and hence internal energy of the gas will decrease ($T \propto PV$) or $\Delta V_{A \to B}$ = negative. Further, $\Delta W_{A \to B}$ is negative.

In process B to C, pressure of the gas is constant while volume is increasing. Hence, temperature should increase

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or $\Delta U_{B \to C}$ = positive. During C to A volume is constant while pressure is increasing.

Therefore, temperature and hence internal energy of the gas should increase or $\Delta U_{C \to A}$ = positive. During process CAB, volume of the gas is decreasing. Hence, work done by the gas is negative.

17. a, b, d. P–T graph is a straight line passing through origin. Therefore, V = constant.

$\therefore$ Work done on the gas = 0.

Further, $$\rho = \frac{m}{V} \propto \frac{1}{V}$$

Volume of the gas is constant. Therefore, density of gas is also constant.

$$PV = nRT$$

or $$P = \left(\frac{nR}{V}\right)T$$

i.e., slope of P–T line $\propto n$

18. a, c, d.

$$T = \text{constant}$$
$$PV = \text{constant (Boyle's law)}$$

or $$P \propto \frac{1}{V}$$

Pressure of the gas is increasing. Therefore, volume should decrease. Work done by the gas is negative or work done on the gas is constant. Therefore, internal energy will remain constant.

19. b, c. For adiabatic process 'bc'

$$T_1V_b^{g-1} = T_2V_c^{g-1} \quad \text{(i)}$$

For adiabatic process 'da'

$$T_2V_d^{\gamma-1} = T_1V_a^{\gamma-1} \quad \text{(ii)}$$

Multiplying Eqs. (i) and (ii)

$$\Rightarrow T_1T_2(V_bV_d)^{\gamma-1} = T_1T_2(V_aV_c)^{\gamma-1}$$
$$\Rightarrow V_bV_d = V_aV_c$$

Since adiabatic expansion leads to cooling,

so $$T_1 > T_2$$

20. c, d. During expansion, an isotherm lies above an adiabat

Also $$\begin{pmatrix}\text{Slope of} \\ \text{an adiabat}\end{pmatrix} = \gamma \begin{pmatrix}\text{Slope} \\ \text{of an} \\ \text{isotherm}\end{pmatrix}$$

$$\Rightarrow m_2 = \frac{C_p}{C_V}(m_1)$$

$$\Rightarrow m_2C_v = m_1C_p$$

Since $\gamma - 1$,

$$\Rightarrow m_2 > m_1$$

21. a, c.

Sol. Let the process start from initial pressure P_A, volume V_A and temperature T_A.

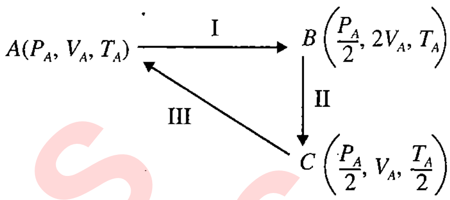

Fig. 2.193

I: Isothermal expansion (PV = constant) at temperature T_A to twice the initial volume V_A.

II: Compression at constant pressure $P_A/2$ to original volume V_A (i.e., $V \propto T$)

III: Isochoric process (at volume V_A) to initial conditions (i.e., $P \propto T$)

22. a, b, d.

(i) Work done = area under the curve,

$$W_1 = \left(P + \frac{P}{2}\right)(2V - V) = \frac{3}{2}PV$$

Work done under isothermal process,

$$W_2 = RT \times 2.3026 \log\left(\frac{2V}{V}\right)$$
$$= 0.693\ RT = 0.693PV$$

$\therefore W_1 > W_2$, i.e., option (a) is correct.

(ii) Let P_0 and V_0 be the intercepts on the P and V axes. Now the equation of straight line would be

$$P = -\frac{P_0}{V_0} \times V + P_0$$

$$(\because y = mx + c)$$

Further $\quad PV = RT \quad$ or $\quad P = \frac{RT}{V}$

(This represents parabola, so option (b) is also correct).

23. a, b, c, d. In the equilibrium position, the net force on the partition will be zero.

Hence pressure on both sides is same.

Hence, (a) is correct.

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{PV}{RT}$$

$$n_2 = \frac{(2P)(2V)}{RT} = 4\frac{PV}{RT} \quad \Rightarrow n_2 = 4n_1$$

Moles remain conserved.

Finally pressure becomes equal in both the parts.

Using, $$P_1V_1 = n_1RT_1$$
$$P_2V_2 = n_2RT_2$$
$$P_1 = P_2 \quad \text{and} \quad T_1 = T_2$$

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$$\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{1}{4}$$

$$V_2 = 4V_1$$

Also, $V_1 + V_2 = 3V \Rightarrow V_1 + 4V_1 = 3V$

$$V_1 = \frac{3}{5}V \quad \text{and} \quad V_2 = \frac{12}{5}V$$

Hence options (b) and (c) are correct.
In compartment I:

$$P_1'V_1 = n_1RT_1$$

$$P_1'\left(\frac{3V}{5}\right) = \left(\frac{PV}{RT}\right)R(T)$$

$$P_1' = \frac{5PV}{3V} = \frac{5}{3}P$$

Hence option (d) is also correct.

24. b, d.

$$\frac{P^2}{\rho} = k \Rightarrow \frac{P^2RT}{PM} = k$$

$$PT = \left(\frac{kM}{R}\right) \Rightarrow P \propto \frac{1}{T} \qquad \text{(i)}$$

$$\frac{P^2}{\rho} = \frac{P'^2}{\rho/2} \Rightarrow P' = \frac{P}{\sqrt{2}}$$

Hence from Eq. (i) $T' = T\sqrt{2}$
PT = constant, hence $P-T$ curve is a parabola.

25. a, d.

$$V_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Since $PV = nRT$, therefore P and V both can change simultaneously keeping the temperature constant.

26. a, c. Initial state is same for all the three processes (say initial internal energy $= E_0$).

In the final state, $V_A = V_B = V_C$

and $$P_A > P_B > P_C$$
$$P_AV_A > P_BV_B > P_CV_C$$
$$E_A > E_B > E_C$$

If $T_1 < T_2$, then $E_0 > E_f$ for all the three processes and hence $(E_0 - E_A) < (E_0 - E_B) < (E_0 - E_C)$

$$|\Delta E_A| < |\Delta E_B| < |\Delta E_C|$$

If $T_1 < T_2$, then $E_0 < E_f$ for all the three processes and hence $(E_A - E_0) > (E_B - E_0) > (E_C - E_0)$

$$|\Delta E_A| > |\Delta E_B| > |\Delta E_C|$$

27. a, b, c. For the process 3 → 1

$$P_1V_1 = P_3V_3 \Rightarrow V_3 = \frac{P_1V_1}{P_3} = \frac{4\times10^5\times1}{1\times10^5} = 4\ \text{m}^3$$

For the process 2 → 3

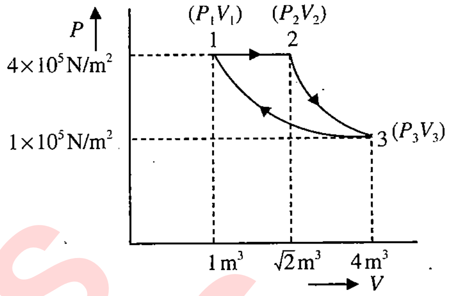

Fig. 2.194

$$P_2V_2^\gamma = P_3V_3^\gamma \Rightarrow V_2 = \left(\frac{P_3}{P_2}\right)^{\frac{1}{\gamma}} V_3 = 4\left(\frac{1}{4}\right)^{\frac{3}{4}}$$

$$= 4^{\frac{1}{4}}\ \text{m}^3 = \sqrt{2}\ \text{m}^3$$

For the process 1 → 2,

$$W_{12} = P_1(V_2 - V_1)$$
$$= (\sqrt{2}-1)4\times10^5\ \text{J}$$

For the process 2 → 3,

$$W_{23} = \frac{(P_2V_2 - P_3V_3)}{(\gamma-1)}$$

$$= \frac{(4\sqrt{2} - 1\times4)\times10^5}{\left(\frac{4}{3}-1\right)} = 12(\sqrt{2}-1)\times10^5\ \text{J}$$

For the process 3 → 1,

$$W_{31} = -P_1V_1 \ln\frac{V_3}{V_1}$$

$$= -4\times10^5\times1\times\ln\left(\frac{4}{1}\right)$$

$$\Delta U = 0$$

$$\Delta Q_{1\to2\to3\to1} = \Delta U + \Delta W = 4\times10^5(\sqrt{2}-1)$$
$$+ 12\times10^5(\sqrt{2}-1) - 4\times10^5\times1.386$$
$$\simeq 1.08\times10^5\ \text{J}$$
$$\Delta Q = \Delta W \simeq 1.08\times10^5\ \text{J}$$

28. b, d. Initial mass of air, $m = \frac{P_1V}{RT_1}$

Final mass of air

$$= \frac{m}{2} = \frac{P_2V}{RT_2}$$

$$\frac{P_1V}{2RT_1} = \frac{P_2V}{RT_2}$$

$$\left(\frac{T_2}{T_1}\right) = 2\left(\frac{P_2}{P_1}\right)$$

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As the tank is insulated, the process is adiabatic with $\gamma = \frac{5}{3}$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \left(\frac{P_2}{P_1}\right)$$

$$\frac{T_2}{T_1} = 2\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} \Rightarrow T_2 = \frac{T_1}{2^{\frac{2}{3}}}$$

$$= \frac{320}{2^{\frac{2}{3}}} = 200\text{ K} = -73°\text{C} \quad [\text{Given } \sqrt[3]{4} = 1.6]$$

29. b, c. According to the problem, mass of gases is equal so number of moles will not be equal, i.e., $\mu_A \neq \mu_B$

From ideal gas equation, $PV = \mu RT$

$\therefore \frac{P_A V_A}{\mu_A} = \frac{P_B V_B}{\mu_B}$ (as temperatures of the containers are equal)

From this relation it is clear that if $P_A = P_B$; then

$$\frac{V_A}{V_B} = \frac{\mu_A}{\mu_B} \neq 1$$

i.e., $V_A \neq V_B$

Similarly, if $V_A = V_B$ then

$$\frac{P_A}{P_B} = \frac{\mu_A}{\mu_B} \neq 1$$

i.e., $P_A \neq P_B$

Assertion-Reasoning Type

1. b. It is true that for real gases, and for the real procedures in the laboratory, an adiabatic process is a sudden and large change of the system obtained in a small time. However, for an ideal gas, and for an ideal procedure, we assume for such a gas, an ideal adiabatic process is considered quasi-static, in which the system passes through a continuous succession of equilibrium states, which we can plot on a $P-V$ diagram. Such a process is also assumed to be reversible though, of course, a quasi-static process may or may not be reversible. Such ideal conditions as we assume may not be obtained in particular but almost all real systems deviate little from the results of our ideal procedures, which may therefore be considered approximations to real processes.

2. c. Work is done only in expansions or compressions of the gas where volume changes occur. In a quasi-static condition, a small change of volume dV under pressure P involves the work $dW = PdV$. In a succession of quasi-static equilibrium state, total work done can be obtained by integration.

3. c. Any given thermodynamics state has a definite temperature T. The internal energy of a system in an ideal gas is a function of only the absolute temperature and is independent of its pressure and volume. The internal energy per mole of a gas is directly proportional to its absolute temperature.

4. c.

$$C = \frac{\Delta Q}{M\Delta T}$$

In adiabatic process, $\Delta Q = 0$.
In isothermal process, $\Delta T = 0$.

5. a.

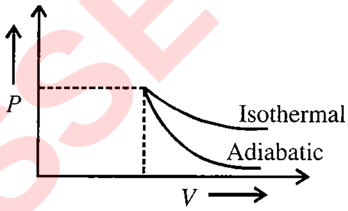

Fig. 2.195

The slope of adiabatic curve is seven times the slope of an isothermal curve and slope of both is negative. Thus, area under adiabatic curve is smaller than that under isothermal curve.

6. a. When final stage is same with same initial volume, the work done in adiabatic expansion is more.

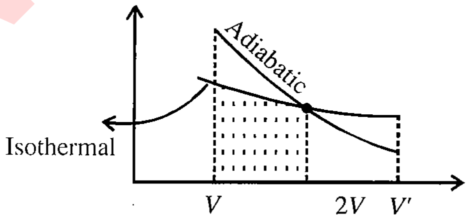

Fig. 2.196

7. a. As there is no change in internal energy of the system during an isothermal change, hence the energy taken by the gas is utilized by doing work against external pressure.

8. a. In an adiabatic process, there is no exchange of heat.

i.e., $\Delta Q = 0$

$$\Delta Q = \Delta U + \Delta W = 0$$
$$\Rightarrow \Delta U = -\Delta W$$

9. c. In an adiabatic process, no exchange of heat is permissible, i.e., $dQ = 0$

As $dQ = dU + dW = 0$

$\therefore$ $dU = -dW$

10. a. If the rate at which molecules (of same mass, having same rms velocity) strike a wall decrease, then the rate at which momentum is imparted to the wall also decreases.

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This results in lowering of pressure. Hence, Statement II is correct.

In Statement I the rms velocity of gas remains same on increasing the volume of container by piston, since the given process is isothermal. Now the piston is at a greater distance from opposite wall and hence time taken by gas molecules from near the opposite wall to reach the piston will be more. Thus rate of molecules striking the piston decreases. Hence, Statement I is correct and Statement II is the correct explanation for Statement I.

Comprehension Type

For Problems 1–3

1., a., 2., c., 3. a.

Sol.

1. **a.** For adiabatic process BC,

$$P_B = P_C \qquad \text{(i)}$$

For isothermal process CA,

$$P_A V_A = P_C V_C \qquad \text{(ii)}$$

From Eqs. (i) and (ii)

$$V_C = \left[\frac{V_B^{\gamma}}{V_A}\right]^{\frac{1}{\gamma-1}} = 64\ \text{m}^3$$

$$P_C = \frac{P_A V_A}{V_C} = \frac{10^5}{64}\ \text{N/m}^2$$

b. Work done, $W = W_{AB} + W_{BC} + W_{CA}$

$$= P(V_B - V_A) + \frac{1}{\gamma - 1}[PV_B - P_C V_C + PV_A \ln(V_A/V_C)]$$

Putting the values,

$$W = 4.9 \times 10^5\ \text{J}$$

For Problems 4–6

4.c., 5.c., 6.a.

Sol.

4. **c.** An AB is isobaric, in $B \to C$ magnitude of $\frac{dT}{dv}$ increases and OA is isochoric process.
Hence option (c) is correct.

5. **c.** In cyclic process

$$\Delta U = 0$$

$$W_{AB} + W_{BC} + W_{CA} = -1000\ \text{J}$$

$$4.R\,200 + W_{BC} + 0 = -1000\ \text{J}$$

$$W_{BC} = -7640\ \text{J}$$

6. **a.** The volume of gas at state C is

$$V_C = V_A = \frac{nRT_A}{P_A}$$

$$= 0.0332\ \text{m}^3$$

For Problems 7–9

7. b., 8. b., 9., c.

Sol.

7. **b.** For point A, $\eta = 0$

$$\therefore \quad C = \frac{R(0-\gamma)}{(0-1)(\gamma-1)} = \frac{\gamma R}{\gamma - 1} = \frac{5}{2}R$$

8. **b.** At point B, $C = 0$

$$T_2 = m_1 g = 50\ \text{N}$$

$$\text{As } C = \frac{R(\eta - \gamma)}{(\eta - 1)(\gamma - 1)}$$

$$\eta = \gamma \text{ at point } B$$

For monatomic gas, $\gamma = 5/3$

9. **c.** $C \to \pm\alpha$
is negative for $1 < \eta < \gamma$

For Problems 10–12

10. d., 11. b., 12. d.

Sol.

10. **d.** Since AB is rectangular hyperbola, therefore

$$\therefore \quad U\rho = \text{Constant}$$

$$\Rightarrow nC_v T \frac{PM}{RT} = C$$

$$\Rightarrow P = \text{Constant}$$

$\therefore$ AB $\to$ isobaric
BC $\to$ isothermal as U is constant
CA $\to$ isochoric as ρ is constant

11. **b.** Given $U_C = 3000\,R = nC_v T_C$

$$\Rightarrow 3000\,R = 2 \times \frac{3}{2}R \times T_C$$

$$\Rightarrow T_C = 1000\ \text{K}$$

Also $\quad U_B = U_C$

and $\quad U_A = 2 \times \frac{3}{2}R \times 300 = 900R$

$$W_{AB} = P\Delta V = nR\Delta T$$
$$= 2 \times R\,(1000 - 300)$$
$$= 1400\,R$$

$$U_{AB} = U_B - U_A = 3000R - 900R$$
$$= 2100R$$

$$\therefore \quad \Delta Q = \Delta W + \Delta U$$
$$= 1400R + 2100R = 3500R$$

12. **d.** CA is isochoric. Therefore W_{CA} is zero.

$$\therefore \quad Q_{CA} = U_{CA}$$
$$= U_A - U_C$$
$$= 900\,R - 3000\,R$$
$$= -2100\,R$$

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For Problems 13–15
13. b., 14. d., 15. c.

Sol.
13. b. $\Delta U = 2\Delta (PV) = 2R\Delta T$

$$\Rightarrow C_V = 2R$$
$$\Rightarrow C_P = 3R$$

14. d. $W = RT \log_e \dfrac{2V_0}{V_0} = PV \log_e 2 = \dfrac{U_1 - U_0}{2} \log_e 2$

15. c. $C_V = 2R$, which is between the C_V values of mono and diatomic gases.

For Problems 16–18
16. a., 17. b., 18. b.

Sol.
16. a. It is given that there is decrease in the internal energy while the gas expands absorbing some heat, which is numerically equal to the decrease in internal energy.

Hence, $dQ = -\,dU$

Thus, $dQ = dW + dU = -\,dU$

$dw = -2dU$ (remembering that both dW and dU are negative)

$$PdV = 2CdT \quad (dU = -\,CdT) \qquad \text{(i)}$$

where C is the molar specific heat of the gas.
Since the gas is ideal, we have

$$P = \frac{RT}{V} \qquad \text{(ii)}$$

Hence Eq. (i) gives

$$\frac{RT}{V} dV = 2CdT$$

$$\frac{dV}{V} = \frac{2C}{R}\frac{dT}{T}$$

Solving we get

$$(VT^{-2C/R}) = \text{constant} \qquad \text{(iii)}$$

Also since $dQ = -dU$,

$$C = \left(\frac{dQ}{dT}\right)_{\text{constant volume}} = \left(\frac{dU}{dT}\right) = -C_v$$

Hence,

$$C = -C_V = -\frac{R}{\gamma - 1} = -\frac{R}{\frac{7}{5} - 1} = -\frac{5R}{2}$$

Since the gas is diatomic.

17. b. From Eq. (iii)

$$TV^{\frac{-R}{2C}} = \text{constant}$$

Putting $C = -5R/2$, $TV^{\frac{1}{5}} = \text{constant} \qquad \text{(iv)}$

18. b. Under the given process, the final volume V is 32 times the initial volume V_0.

Thus, $W = \displaystyle\int_{V_0}^{32V_0} \frac{PdV}{2} = \int_{V_0}^{32V_0} \frac{RT}{2V} dV$ (see note below)

$$= \int_{V_0}^{32V_0} \frac{R}{2V} TdV$$

Now,

$$TV^{\frac{1}{5}} = T_0 V_0^{\frac{1}{5}} \quad \text{gives} \quad T = \frac{T_0 V_0^{\frac{1}{5}}}{V^{\frac{1}{5}}}$$

Hence,

$$W = \int_{V_0}^{32V_0} \frac{R}{2V} \frac{T_0 V_0^{\frac{1}{5}}}{V^{1/5}} dV$$

$$= RT_0 V_0^{1/5} \int_{V_0}^{32V_0} \frac{V^{-6/5}}{2} dV$$

$$= RT_0 V_0^{1/5} \left[\frac{-5V^{-1/5}}{2}\right]_{V_0}^{32V_0}$$

$$W = -\frac{5RT_0}{2}$$

Work done by the gas $= \dfrac{5RT_0}{2}$

For Problems 19–21
19. d., 20. a., 21. d.

Sol.
19. d. Since the piston is pushed very quickly from position 1 to position 2, the process AB is an adiabatic compression. Process BC is isochoric and process CA is an isothermal expansion. These are shown on the P–V diagram correctly in option (d).

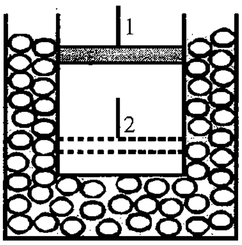

Fig. 2.197

20. a. If ΔW is the work done in the whole cycle, we have

$$\Delta Q = \Delta W + \Delta U$$

But $\Delta U = 0$ for the entire cycle.

Hence $\Delta W = \Delta Q = mL$

$$= 100 \times 80 = 8000 \text{ cal} = 8 \text{ kcal}$$

21. d. Work done during the entire cycle = 8 kcal = W

But $W = W_{ad} = W_v + W_{iso}$

$$= \frac{R(T_2 - T_1)}{\gamma - 1} + 0 + RT\ln\frac{V_2}{V_1}$$

This is not equal to any of options given.

For Problems 22–24

22. d., 23. b., 24. a.

Sol.

22. d. Process 1 is isochoric. Now work is done $dW = 0$

Hence $(\Delta Q)_1 = dU = C_V(T_2 - T_1)$

$$= \frac{3R}{2}[600 - 300] = 450R \text{ J}$$

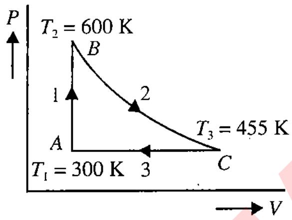

Fig. 2.198

Process 2 is adiabatic expansion

$$(\Delta Q)_2 = 0, (\Delta W)_2 = -(\Delta U)_2$$
$$(\Delta W)_2 = -[C_v(T_3 - T_2)]$$

$$= -\left[\frac{3R}{2}(455 - 600)\right]$$

$$= \frac{3 \times 145}{2} R = 217.5R \text{ J}$$

$$(\Delta U)_2 = -217.5R \text{ J}$$

Process 3 is isobaric compression.
Value of pressure = 1 atm $= 1.013 \times 10^5$ N/m^2

$$(\Delta W)_3 = 1.013 \times 10^5 \ (V_C - V_A) \ \text{J} \neq 0$$

$$(\Delta W)_3 = C_v(300 - 455) = \frac{-3R}{2} \times 155 = -232.5R$$

23. b. It is given that $P_1 = 1$ atm $= 1.013 \times 10^5$ N/m^2

and $T_1 = 300$ K

Hence

$$V_1 = \frac{1 \times RT_1}{P_1} = \frac{R \times 300}{1.013 \times 10^5} \text{ m}^3$$

$$= 3 \times 10^{-3} R \text{ m}^3$$

Also,

$$P_2 = \frac{RT_2}{V_2} = \frac{RT_2}{V_1} = \left(\frac{RT_1}{V_1}\right) \times \frac{T_2}{T_1} = 2P_1 = 2 \text{ atm}$$

$$V_2 = V_1 = 3 \times 10^{-3} R \text{ m}^3$$

$$V_3 \neq 3 \times 10^{-3} R \text{ m}^3$$

Also $P_3 = P_1 = 1$ atm

24. a. The isothermal process at $T = 600$ K is shown by 4 (see Fig. 2.199). Now the thermal energy Q_1 is the same through both cycles. When Q_2 is zero for the adiabatic expansion, Q_4 is positive and finite. Also, the isobaric compression will involve more work done on the gas for the process PA than for CA. Also temperature at P is 600 K. More heat is emitted by the gas in the change PA than in the change CA. Hence the cycle $ABPA$ involving the isothermal will take less total heat than the cycle $ABCA$, i.e., $Q_i < Q_a$.

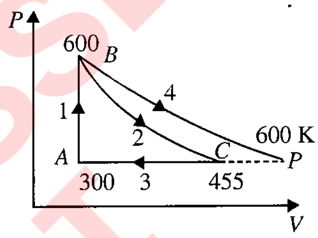

Fig. 2.199

For Problems 25–26

25. c., 26. d.

Sol. Process 1→2 (Isochoric process)

$$W_{12} = 0$$
$$Q_{12} = \Delta U_{12} = nC_V(T_2 - T_1)$$
$$= n\frac{3}{2}R(T_2 - T_1)$$
$$= \frac{3}{2}[P_0V_0 - (P_0/2)V_0]$$
$$= \frac{3P_0V_0}{4} \text{ (heat absorbed)}$$

Process 3→1,

$$W_{3\to1} = \text{Area under process } 3 \to 1$$
$$= -\left(\frac{P_0}{2} + P_0\right) \cdot \frac{V_0}{2} = -\frac{3P_0V_0}{4}$$

Work done during 3→1 will be negative as volume is decreasing.

$$\Delta U_{3\to1} = n\frac{3R}{2}(T_1 - T_3) = \frac{3}{2}\left(\frac{P_0V_0}{2} - P_0 2V_0\right) = -\frac{9P_0V_0}{4}$$

$$Q_{3\to1} = \Delta U_{3\to1} + W_{3\to1}; = -3P_0V_0 \text{ (heat rejected)}$$

Process 2→3,
The temperature will be maximum at $x = 2P_0$, $T_x = \dfrac{3P_0V_0}{2R}$

$$W_{2\to x} = W_{x\to3} = \frac{1}{4}\pi\frac{P_0V_0}{2} + \frac{P_0V_0}{2} = \frac{P_0V_0}{2}\left(\frac{\pi}{4} + 1\right)$$

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$$W_{2\to3} = \frac{P_0V_0}{4}(\pi+4)$$

$$\Delta U_{2\to x} = n\frac{3}{2}R(T_x - T_2) = \frac{3}{2}(3P_0V_0 - P_0V_0) = 3P_0V_0$$

$$\Delta U_{x\to3} = n\frac{3}{2}R(T_3 - T_x) = \frac{3}{2}(2P_0V_0 - 3P_0V_0)$$

$$= -\frac{3}{2}P_0V_0$$

$$\therefore \Delta Q_{2\to x} = \Delta U_{2\to x} + W_{2\to x}$$

$$= 3P_0V_0 + \frac{P_0V_0}{2}\left(\frac{\pi}{4}+1\right)$$

$$= P_0V_0\left(3+\frac{1}{2}+\frac{\pi}{8}\right) = \frac{P_0V_0}{2}\left(7+\frac{\pi}{4}\right)$$

$$\Delta Q_{x\to3} = \Delta U_{x\to3} + W_{x\to3}$$

$$= -\frac{3}{2}P_0V_0 + \frac{P_0V_0}{2}\left(\frac{\pi}{4}+1\right) = \frac{-P_0V_0}{2}\left(2-\frac{\pi}{4}\right)$$

Total heat rejected in one cycle

$$= 3P_0V_0 + \frac{P_0V_0}{2}\left(2-\frac{\pi}{4}\right) = 4P_0V_0 - \frac{\pi P_0V_0}{8}$$

$$= P_0V_0\left(4-\frac{\pi}{8}\right) = \frac{P_0V_0(32-\pi)}{8}$$

For Problems 27–28
27. a., 28., d.

Sol.
27. **a.** (i) Change in internal energy when temperature changes from T to $T + \Delta T$, for 1 mol of gas, is

$$\Delta U = \int_T^{T+\Delta T} C_v dT = \int_T^{T+\Delta T} \frac{R}{\gamma-1}dT = \frac{R\Delta T}{\gamma-1}$$

Heat transferred to system for 1 mol of gas,

$$Q = C\Delta T = -\Delta U = \left[-\frac{R}{(\gamma-1)}\right]\Delta T$$

$$C = \frac{-R}{\gamma-1}$$

28. **d.** (ii) From the first law of thermodynamics,

$$dQ = dU + dW$$

But $\quad dQ = -dU$

Hence $\quad 2dQ = dW$

$$2C\,dT = PdV$$

$$\left(\frac{-2R}{\gamma-1}\right)dT = P\,dV$$

From ideal gas equation,

$$PV = RT \text{ (1 mol of gas)}$$

Hence

$$\frac{RT}{V}dV + \frac{2R\,dT}{\gamma-1} = 0$$

$$\left(\frac{2}{\gamma-1}\right)\frac{dT}{T} + \frac{dV}{V} = 0$$

$$\frac{dT}{T} + \left(\frac{\gamma-1}{2}\right)\frac{dV}{V} = 0$$

$$\log_e T + \left(\frac{\gamma-1}{2}\right)\log_e V = \text{constant}$$

Therefore

$$TV^{\frac{(\gamma-1)}{2}} = \text{constant}$$

For Problems 29–30
29. a., 30. b.

Sol. Since the temperature remains constant,

$$\Delta T = 0$$

Now

$$W_{12} = \frac{1}{2}(P_1+P_2)(V_2-V_1)$$

$$W_{13} = \frac{1}{2}(P_1+P_3)(V_3-V_1)$$

$$W_{12} - W_{13} = \frac{1}{2}\,[P_1(V_2 - V_3) + (P_3 - P_1)\,V_1] < 0$$

$$W_{13} > W_{12};\ Q_{13} > Q_{12}$$

For Problems 31–36
31. c., 32., a 33. b., 34. c., 35. d., 36. b.

Sol. Gas is monatomic; hence

$$C_v = \frac{3}{2}R \quad \text{and} \quad C_p = \frac{5}{2}R$$

Number of moles, $n = 2$.

$$T_A = 27°\text{C} = 300\,\text{K}$$

Process $A \to B$ is a straight line passing through origin; hence it is an isobaric process.

$$\frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$T_B = \left(\frac{V_B}{V_A}\right)T_A$$

$$= 2\times 300\,\text{K} = 600\,\text{K}$$

Process $A \to B$ is isobaric.

$$Q_P = nC_P(T_B - T_A)$$

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$$Q_p = 2\left(\frac{5}{2}R\right)(600-300) = 1500\ R$$

Heat is added to the system.
Process $B \to C$ is isothermal.
From the first law of thermodynamics,

$$Q_{BC} = W_{BC}$$

(as $\Delta U = 0$)

$$W = nRT_B\ \ln\left(\frac{V_C}{V_B}\right)$$

$$= (2)(R)(600)\ln\left(\frac{4V_A}{2V_A}\right)$$

$$= (1200R)\ \ln\ (2)$$

Heat is added to the system.
Process $C \to D$ is isochoric.
From the first law of thermodynamics,

$$Q_{CD} = nC_v(T_D - T_C) = n\left(\frac{3}{2}R\right)(T_A - T_B)$$

$$= 2\left(\frac{3}{2}R\right)(300-600) = -900\,R$$

Negative sign implies that heat is rejected by the system.
Process $D \to A$ is isothermal.

$$Q_{DA} = W_{DA} = nRT_D\ \ln\left(\frac{V_A}{V_D}\right)$$

$$= (2)(R)300\ \ln\left(\frac{V_A}{4V_A}\right) = 600\,R\ \ln\left(\frac{1}{4}\right)$$

Negative sign implies that heat is rejected by the system.
In a cyclic process, $\Delta U = 0$

$$Q_{\text{net}} = W_{\text{net}}$$

$$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$= 1500\,R + 831.6\,R - 900\,R - 831.6\,R = 600\,R$$

For Problems 37–38
37. d., 38. a.

Sol. For process $1 \to 2$

$$W_{12} = \int_1^2 \alpha V\,dV = \alpha\int_{V_0}^{3V_0} V\,dV = \frac{\alpha}{2}(9V_0^2 - V_0^2)$$

$$= 4\alpha\,V_0^2$$

Using gas law, $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$$T_2 = \frac{P_2V_2}{P_1V_1}T_1 = \frac{V_2^2}{V_1^2}T_1 = \left(\frac{3V_0}{V_0}\right)^2 T_0 = 9T_0$$

For process $2 \to 3$

$$W_{23} = RT_2\log\left|\frac{P_2}{P_3}\right| = R(9T_0)\ \log\left|\frac{3P_0}{P_0}\right|$$

$$= 9RT_0\log|3| = 9.81\ RT_0$$

For isothermal process: $P_2V_2 = P_3V_3$

Therefore, $V_3 = \dfrac{P_2V_2}{P_3} = \dfrac{3P_0}{P_0}(3V_0) = 9\ V_0$

Also, $W_{31} = P_0(V_1 - V_3) = P_0\ (V_0 - 9V_0) = -\ 8P_0V_0 = -\ 8RT_0$
Applying gas law in process $1 \to 2$

$$P_0V_0 = R\,T_0 \quad \text{or} \quad \alpha V_0^2 = RT_0$$

The net work is

$$W_{\text{net}} = W_{12} + W_{23} + W_{31}$$

$$= 4RT_0 + 9.81RT_0 - 8RT_0$$

$$= 5.81RT_0$$

For process $1 \to 2$

$$\Delta U_{12} = C_V(T_2 - T_1)$$

$$= 4RT_0 + 12RT_0 = 16RT_0$$

Since,

$$\Delta U_{12} = C_{12}(T_2 - T_1) = 8C_{12}T_0$$

$$C_{12} = 2R = 16.6\ \text{J/mol-K}$$

For the process $2 \to 3$: $C_{23} = \infty$
For the process $3 \to 1$: $C_{31} = C_P + R$

$$= 5R/2 = 20.75\ \text{J/mol-K}$$

For Problems 39–42
39. b., 40. b., 41. a., 42. b.

Sol. Process CB is done at minimum temperature, while process DA is done at maximum temperature.
Process
$A \to B$ (isobaric)

$$\frac{V_A}{V_B} = \frac{T_A}{T_B} = \frac{800}{400} = 2$$

hence $V_B = 2V_0$
$B \to C$ (isothermal)
$C \to D$ (isobaric)

$$\frac{V_D}{V_C} = \frac{T_D}{T_C} \Rightarrow V_D = 2V_0$$

$D \to A$ (isothermal)

$$W_{AB} = P(V_B - V_A) = nR\ \text{D}T$$

$$= nR\ (-\ 400) = -\ 400nR$$

$$W_{BC} = nRT\ln\frac{V_C}{V_B} = nR400\ \ln\frac{V_0}{2V_0}$$

$$= -\ 400\ nR\ \ln 2$$

$$W_{CD} = P(V_D - V_C) = nR\ (400) = 400nR$$

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$$W_{DA} = nRT \ln \frac{V_A}{V_D}$$

$$= nR \times 800 \times \ln \frac{2V_0}{V_0} = 800\, nR \ln 2$$

$$\Delta W = 400\, nR \ln 2$$

ΔQ heat is extracted

$$Q_{CD} = nC_P \Delta T = n\frac{5}{2}R\, 400 = 1000 nR$$

$$= 800 nR \ln 2 = 1000 nR + 800 nR \ln 2$$

$$\text{Efficiency} = \frac{\Delta W}{\Delta Q}$$

$$\eta = \frac{2 \ln 2}{5 + 4 \ln 2} \times 100\%$$

For Problems 43–45

43. c., 44. d., 45. b.

Sol. 43. c. Heat given for gas A, $\Delta Q = n_1 C_{v_1} \Delta T$

For gas $B - \Delta Q = n_2 C_{v_2} \Delta T$

($\because$ For same heat given, temperature rises by same value for both the gases)

$$n_1 C_{v_1} = n_2 C_{v_2} \quad \text{(i)}$$

Also, $(\Delta P_B)V = n_2 R \Delta T$ and $(\Delta P_A)V = n_1 R \Delta T$

$$\frac{n_1}{n_2} = \frac{\Delta P_A}{\Delta P_B} = \frac{2.5}{1.5} = \frac{5}{3}$$

$$n_1 = \frac{5}{3} n_2$$

Substituting in Eq. (i),

$$\frac{5}{3} n_2 C_{v_1} = n_2 C_{v_2} \Rightarrow \frac{C_{v_2}}{C_{v_1}} = \frac{5}{3} = \frac{\left(\frac{5}{2}R\right)}{\left(\frac{3}{2}R\right)}$$

Hence, gas B is diatomic and gas A is monatomic.

44. d. Since $n_1 = 5/3\ n_2$,

$$\frac{125}{M_A} = \frac{5}{3}\left(\frac{60}{M_B}\right)$$

(From experiment 1: W_A = 25 g and W_B = 60 g)

$$5M_B = 4M_A$$

The above relation holds if gas A is Ar and gas B is O_2

45. c. It could also be seen directly that temperature finally will be 300 K, since no heat exchange takes place between those gases as their initial temperatures are same.

Since volume remains same but number of moles increases, pressure increases.

For Problems 46–50

Sol. 46. c., 47. a., 48. b., 49. c., 50. c.

47. a. Applying the first law of thermodynamics to the cylinder,

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta Q = \Delta W = 0$$
$$\Rightarrow \Delta U = 0$$
$$n_1 C_V (T - T_1) + n_2 C_V (T - T_2) = 0$$

$$T = \frac{(P_1 + P_2) T_1 T_2}{P_1 T_2 + P_2 T_1}$$

For left compartment,

$$\frac{P_1}{T_1} = \frac{P'_1}{T} \Rightarrow P'_1 = \frac{P_1 (P_1 + P_2) T_2}{P_1 T_2 + P_2 T_1}$$

Similarly, $$P'_2 = \frac{P_2 (P_1 + P_2) T_1}{(P_1 T_2 + P_2 T_1)}$$

48. b. Heat flowing from right to left

$$Q = n_1 C_V (T - T_1) = \frac{P_1 V}{2RT_1} \frac{3}{2} R \left\{\frac{(P_1 + P_2) T_1 T_2}{P_1 T_2 + P_2 T_1} - T_1\right\}$$

$$= \frac{3}{4} P_1 P_2 V \frac{(T_2 - T_1)}{P_1 T_2 + P_2 T_1}$$

49. c., 50. c.

$$P'_1 = \frac{P_1 T_2 (P_1 + P_2)}{(P_1 T_2 + P_2 T_1)} \quad \text{and} \quad P'_2 = \frac{P_2 T_1 (P_1 + P_2)}{(P_1 T_2 + P_2 T_1)}$$

As $P_2 T_1 > P_1 T_2$ (given)

Hence, $P'_2 > P'_1$

Let ΔV be the change in volume of any compartment.

$$\frac{P((V/2) - \Delta V)}{T} = \frac{P_1 V}{2T_1}$$

$$\frac{P(V/2) + \Delta V)}{T} = \frac{P_2 V}{2T_2}$$

Dividing, we get,

$$\frac{((V/2) + \Delta V)}{((V/2) - \Delta V)} = \frac{P_2 T_1}{P_1 T_2}$$

$$\Delta V = \frac{V}{2}\left(\frac{P_2 T_1 - P_1 T_2}{P_2 T_1 + P_1 T_2}\right)$$

Volume of left compartment

$$V'_1 = \frac{V}{2} - \Delta V = V\left(\frac{P_1 T_2}{P_2 T_1 + P_1 T_2}\right)$$

Volume of right compartment

$$V'_2 = \frac{V}{2} + \Delta V = V\left(\frac{P_2 T_1}{P_2 T_1 + P_1 T_2}\right)$$

For Problems 51–52

51. d., 52. b.

Sol. For the right part,

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$$P_0V_0^\gamma = PV_2^\gamma$$

$$V_2 = \left(\frac{P_0}{P}\right)^{1/\gamma} V_0 = \left(\frac{32}{243}\right)^{3/5} V_0$$

$$V_2 = \frac{8}{27}V_0$$

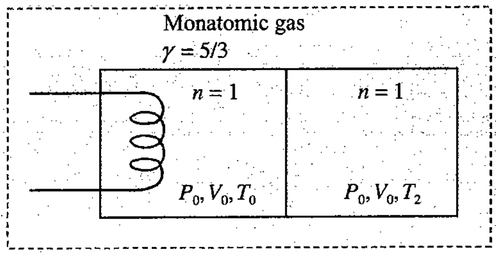

Fig. 2.200

$$V_1 = 2V_0 - \frac{8}{27}V_0 = \frac{46}{27}V_0$$

For left part,

$$\frac{P_0V_0}{T_0} = \frac{PV_1}{T_1}$$

$$T_1 = \frac{PV_1}{P_0V_0}T_0 = \frac{\frac{243P_0}{32}\times\frac{46}{27}}{P_0V_0}T_0$$

$$T_1 = \frac{207}{16}T_0$$

Now applying ideal gas equation in right part,

$$T_2 = \frac{PV_2}{P_0V_0}T_0 = \frac{9}{4}T_0$$

For right part,

$$\Delta Q = 0 = dU + dW$$

$$dW = -dU = -1\, C_V RT$$

$$= -\frac{3}{2}R\left(\frac{9}{4}T_0 - T_0\right) = -\frac{15}{8}RT_0$$

For Problems 53–55

53. a., 54. b., 55. a.

Sol. For 'C', $P_0V_0^\gamma = \left(\frac{125}{27}\right)P_0V_C^\gamma$

$$V_C = V_0\left(\frac{9}{25}\right)$$

$$V_B = 2V_0 - \frac{9V_0}{25} = \frac{(50-9)}{25}V_0$$

$$V_B = \frac{41}{25}V_0$$

$$T_{0A} = T_{0B} = \frac{P_0V_0}{R}$$

$$T_C = \frac{\frac{125}{27}P_0\times\frac{9}{25}V_0}{R} = \frac{5P_0V_0}{3R}$$

$$T_B = \frac{\frac{125}{27}P_0\times\frac{41}{25}V_0}{R} = \frac{205}{27}\frac{P_0V_0}{R} = T_A$$

$$Q = \Delta U$$

$$Q = n_AC_V(T_A - T_0) + n_BC_V\Delta T_B + n_CC_V\Delta T_C$$

$$Q = 2.2R\left(\frac{205}{27}-1\right)\frac{P_0V_0}{R} + 1.2R\left(\frac{205}{27}-1\right)\frac{P_0V_0}{R} + 1.2R\left[\frac{5}{3}\frac{P_0V_0}{R} - \frac{P_0V_0}{R}\right]$$

$$= 2P_0V_0\left[\frac{178\times2}{27} + \frac{178}{27} + \frac{2}{3}\right] = \frac{368}{9}P_0V_0$$

For Problems 56–57

56. b., 57. c.

Sol. The heat absorbed by cylinder A is utilized in two ways: increasing the internal energy (or temperature) and performing work during expansion.

While the heat absorbed by cylinder B is used up only in increasing the internal energy (or temperature).

Therefore, the temperature rise in cylinder B is more than that in cylinder A.

At constant pressure heat absorbed by the system is

$$Q = nC_P\Delta T$$

and internal energy change is

$$\Delta U = nC_V\Delta T$$

Therefore, the fraction,

$$\frac{\Delta U}{Q} = \frac{nC_V\Delta T}{nC_P\Delta T} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

Since $\quad \Delta T \propto \Delta U$

$$\frac{\Delta T_B}{\Delta T_A} = \frac{\Delta U_B}{\Delta U_A} = \frac{Q}{Q/\gamma} = \gamma$$

$$\Delta T_B = \gamma\Delta T_A$$

Since $\quad \Delta T_A = T_0$, therefore $\Delta T_B = \gamma T_0$

For Problems 58–60

58. c., d., 59. b., 60. c.

Sol.

58. **c.** First adiabatic compression takes place

$$\gamma = 1.5$$

$$V_0 = Ah, \quad V_1 = \frac{Ah}{2}$$

$$T_0V_0^{\gamma-1} = T_1V_1^{\gamma-1} \quad \Rightarrow T_1 = \sqrt{2}T_0 > T_0$$

59. **b.** Heat given in adiabatic compression is used to work against elastic force and against atmospheric force.

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60. c. Final pressure,

$$P_f = \left(P_0 + \frac{Kh/16}{A}\right)$$

$$\frac{P_f V_f}{T_f} = \frac{P_0 V_0}{T_0}$$

$$\frac{\left(P_0 + \frac{kh}{16A}\right) A \times \left(\frac{h}{2} + \frac{h}{16}\right)}{T_f} = \frac{P_0 Ah}{T_0}$$

$$\frac{\left(10^5 + \frac{3700h}{16 \times 27 \times 10^{-4}}\right)\frac{9}{16}}{\frac{4}{3} \times 273} = \frac{10^5}{273}$$

$$(1 + 0.856\, h) = \frac{16}{9} \times \frac{4}{3}$$

Solving, $h = 1.6$ m

For Problems 61–63

61. a., 62. b., 63. b.

Sol.

61. a. Electric work done on the nitrogen

$$W_e = -VI\Delta T$$
$$= -120 \times 2 \times 5 \times 60 = 72 \times 10^3 \text{ J}$$
$$= -72 \text{ kJ}$$

(Negative sign is added because the work is done on the system.)

62. b. Number of moles of nitrogen gas

$$n = \frac{PV}{RT} = \frac{(400\,\text{kPa})(0.5\ \text{m}^3)}{\left(\frac{25}{3}\ \text{kJ/k mol-K}\right)(300\ \text{K})} = 0.08 \text{ kmol}$$

63. b. Heat loss, $Q_1 = 2800$ J

Work done at constant pressure $= nC_p\Delta T$

$$\left(\because C_p = \frac{7R}{2} = \frac{7}{2} \times \frac{25}{3} = \frac{175}{6}\ \text{kJ/kmol-K}\right)$$

$$Q - W_e = nC_p\Delta T$$

$$-2.8 \text{ kJ} - (-72 \text{ kJ}) = \frac{0.08 \times 175}{6}\Delta T$$

$$\Rightarrow \Delta T = \frac{(69.2 \text{ kJ})}{(0.08 \text{ kmol})\left(\frac{175}{6} - \text{kJ/kmol-K}\right)}$$

$$= 29.6 \text{ K}$$

Final temperature = 27 + 29.6

= 56.6°C

Matching Column Type

1. i→a.; ii→d.; iii→c.; iv→b.

$$W_{iaf} + u_{if} = H_{iaf}$$
$$20 + u_{ij} = 50$$
$$u_{if} = 30 \text{ and } w_{ibf} + u_{if} = H_{ibf}$$
$$w_{ibf} + 30 = 36$$
$$w_{ipf} = 6$$

i. $w_{ibf} = 6$

ii. $H_{fi} = W_{fi} + u_{fi} = 13 + 30 = 43$

iii. $u_{if} = 30\ u_i = 10\ u_f = 40$

iv. $H_{ib} = w_{ib} + u_{ib} = 6 + (22 - 10) = 18$

2. i→b.; ii→c.; iii→a.

i. $dQ = dU + dw$

Here

$$dQ = +\frac{500 \times 4184}{1000}\text{ J} = 2092 \text{ J}$$

$dW = +400$ J. Since the work is done by the system,

$dU = dQ - dW = 2092 - 400 = 1700$ J

ii. $\Delta U = \Delta Q - \Delta W = (300)(4.184) - (-420) = 1680$J (approx.)

iii. $\Delta U = (\Delta Q - \Delta W) = (-1200)(4.184) - 0 = -5000$ J

(Note that ΔQ is positive when heat is added to the system and ΔW is positive when the system does the work.)

3. i→c.; ii→d.; iii→a.; iv→b.

i. Heat, ΔH added to the gas along the straight line path AC.

(i) Work ΔW_{AC} done by the gas along the straight line path AC = area $\left(\int_A^C p dv\right)$ under the path.

$$\Delta W_{AC} = P_A V_A + \frac{1}{2}P_A V_A = \frac{3}{2}P_A V_A$$

(ii) Increase of internal energy ΔU_{AC} is

$$\Delta U_{AC} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(P_C V_C - P_A V_A) = \frac{9}{2}P_A V_A$$

Hence heat added,

$$\Delta H_{AC} = \Delta W_{AC} + \Delta U_{AC} = \frac{3}{2}P_A V_A = \frac{9}{2}P_A V_A = 6P_A V_A$$

ii. From above,

$$U_{AC} = \frac{9}{2}P_A V_A$$

iii. Work done

$$\Delta W_{ABC} = \Delta W_{AB} + \Delta W_{BC} = 0 + P_C(V_C - V_A) = 2P_A V_A$$

iv. Heat ΔH added to the gas along ABC,

$$\Delta H_{ABC} = \Delta H_{AB} + \Delta H_{BC} = \frac{9}{2}P_A V_A + 2P_A V_A$$

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$$=\frac{13}{2}P_AV_A$$

4. i→b.; ii→c.; iii→d.; iv→a.

$$C_p-C_V=29.1\text{ J/K}=nR$$

For molar heat capacities

$$C_P-C_V=R$$

Number of moles in the gas,

$$n=\frac{29.1}{8.31}=3.5\text{ mol}$$

For monatomic gas,

$$\frac{C_P}{C_V}=\frac{5}{3}$$

$$\frac{5}{3}C_P-C_V=29.1\text{J/K}$$

$$C_P=\left(\frac{3}{2}\times 29.1\right)\text{J/K}=43.7\text{ J/K}$$

$$C_P=72.7\text{ J/K}$$

For rigid diatomic gas the number of degree of freedom is 5.

$$\frac{C_P}{C_V}=\frac{7}{5}$$

$$C_P=\frac{5}{7}C_P=29.1$$

$$C_p=102\text{ J/K}$$

For vibrating diatomic ideal gas, the total number of degrees of freedom is 7.
Internal energy,

$$U=\frac{7}{2}nRT$$

$$C_v=\frac{7}{2}nR\quad\text{and}\quad C_P=\frac{9}{2}nR$$

$$C_p-C_v=nR\quad\Rightarrow\ C_P=\frac{9}{2}\times 29.1=131\text{ J/K}$$

5. i→a., c., d.; ii→b.; iii→a., c., d.; iv→b., c.

i. If $P=2V^2$, from ideal gas equation, we get

$$2V^3=nRT$$

(1) Hence as volume increases temperature will also increase.
(2) $dW=$ positive
Hence $dQ=dU+dW=$ positive
ii. If $PV^2=$ constant, from ideal gas equation, we get $VT=K$ (constant)
Hence with increase in volume, temperature decreases.

Now $dQ=dU+PdV$

$$=nC_VdT-\frac{PK}{T^2}dT$$

$$\left[PdV=-\frac{K}{T^2}dT\right]$$

$$=nC_VdT-\frac{PVT}{T^2}dT=n(C_v-R)dT\quad[\because K=VT]$$

As temperature increases, $dT=$ positive.
And since $C_V>R$ for monatomic gas, $dQ=$ positive as temperature is increased.

iii. $dQ=nCdT=nC_v+PdV$

$$n(C_v+2R)dT=nC_vdT+pdV$$

$$2nRdT=PdV$$

$$\therefore\quad\frac{dV}{dT}=+\text{ve}$$

Hence with increase in temperature, volume increases and vice versa.

$$\therefore\quad dQ=\Delta U+dW+PdV$$

$$n(C_v-2R)dT=nC_vdT+pdV$$

$$-2nRdT=PdV$$

$$\therefore\quad\frac{dV}{dT}=-\text{ve}$$

Therefore, with increase in volume, temperature decreases.

Also $\quad dQ=n\,(C_v-2R)\,dT$

With increase in temperature $dT=+$ve but $C_v<2R$ for monatomic gas. Therefore $dQ=-$ve with increase in temperature.

6. i→d.; ii→b.; iii→c.; iv→b.

In (i), V is on the vertical axis.

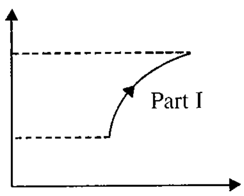

Fig. 2.201

As V is increasing, W is positive

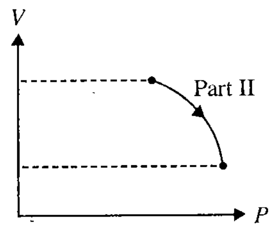

Fig. 2.202

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V is decreasing, W is negative

As negative work in Part II is greater than positive work in Part I, net work during the process is negative. Using $PV = nRT$ and as V remains same for initial and final points of the process, it is obvious that final temperature is greater than initial temperature as pressure has increased. Therefore dU is positive. Hence option (d) is connected with (i).

Similar arguments can be applied to other graphs.

7. i→b.; ii→a.; iii→c.; iv→d.

The compression in the left-hand side is adiabatic,

$$P_0V_0^{\gamma} = P_1V_1^{\gamma}$$

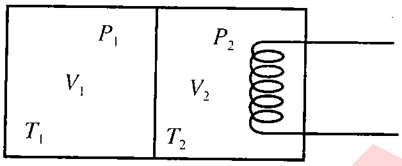

Fig. 2.203

$$V_1 = V_0\left(\frac{P_0}{P_1}\right)^{2/3} = V_0\left(\frac{27}{64}\right)^{2/3} = \frac{9V_0}{16} = 9\,\text{m}^3$$

Also

$$\frac{P_0V_0}{T_0} = \frac{P_1V_1}{T_1}$$

$$T_1 = \frac{P_1V_1}{P_0V_0} = \frac{4T_0}{3} = 432\text{ K}$$

$$P_2 = \frac{64\,P_0}{27}$$

And

$$\frac{P_0V_0}{T_0} = \frac{P_2V_2}{T_2}$$

$$V_2 = 2V_0 - V_1 = 23\,\text{m}^3$$

$$T_2 = \frac{92T_0}{27} = 1104\text{ K}$$

Work done on the left-hand side gas is

$$W = \frac{P_1V_1 - P_0V_0}{\gamma - 1} = \frac{\left(\frac{64}{27}\times\frac{9}{16} - 1\right)P_0V_0}{\frac{3}{2} - 1}$$

$$= \frac{2}{3}P_0V_0 = 3200\text{ kJ}$$

8. i→d.; ii→c.; iii→a.; iv→b.

$$W_{acb} = W_{ac} + W_{cb} = 0 + P_2(V_2 - V_1)$$

$$= P_2V_1 = 2P_1V_1 = 2RT_1$$

$$W_{adb} = W_{ad} + W_{db}$$

$$= P_1(V_2 - V_1) + 0 = P_1V_1 = RT_1$$

$$U_{ab} = U_{ac} + U_{cb} = (Q_{ac} - W_{ac}) + (Q_{cb} - W_{cb})$$

$$= C_v(T_c - T_1) + C_v(T_2 - T_c) = \frac{5R}{2}(T_2 - T_1)$$

$$C_v = \frac{5R}{2} \quad \text{(given)}$$

For an ideal gas,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow T_2 = 4T_1$$

$$U_{ab} = \frac{5R}{2}(4T_1 - T_1) = \frac{15RT_1}{2}$$

For the process, $U_{bca} = -U_{ab}$

$$= \frac{-15RT_1}{2}$$

9. i→d.; ii→b.; iii→c.

d. AE is isothermal process

$$\frac{V_0}{T_0} = \frac{2V_0}{2T_0} \to AC \;-\; \text{isobaric process}$$

For adiabatic process,

$$T_0V_0^{\gamma-1} = T(2V_0)^{\gamma-1}, \quad \gamma = \frac{5}{3}$$

$$T = T_0\left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\frac{2}{3}}} = 0.63T_0 \to AF \;-\; \text{adiabatic process}$$

Integer Answer Type

1. (3) Since $PV = nRT = \frac{m}{M}RT$ we have

$$P = \frac{RT}{V}\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3}\right]$$

where m's are mass of given gas and M is its molecular mass

$$P = \frac{8.3\times 300}{10\times 10^{-3}}\left[\frac{8}{32} + \frac{14}{28} + \frac{22}{44}\right]$$

$$= 3.11 \times 10^5\ \text{N/m}^2$$

In 10^5 N/m², $P = 3$

2. (2) Using $P = \frac{nRT}{V}$, we get

$$\frac{P_{He}}{P_{O_2}} = \frac{2T}{V}\frac{V}{T} = 2$$

3. (3) $P \propto V$ since graph is a straight line

PV^{-1} = constant

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For the process PV^x = Constant, molar heat capacity is given by $C = \frac{R}{(\gamma-1)} + \frac{R}{(1-x)}$

But here $\gamma = 1.4$ and $x = -1$

So, $C = \frac{R}{(1.4-1)} + \frac{R}{(1+1)} = 3R = nR$ (Given)

So, $n = 3$

4. **(1)** Both gases will have same number of rotational degree of freedom (equal to 2). Since both are diatomic and are at the same temperature. Average rotational kinetic energy per molecule for both the gases $= 2\left(\frac{1}{2}KT\right)$.
Required ratio = 1

5. **(3)** No change of temperature
So, $(n_1 + n_2)_i = (n_1 + n_2)_f$

$$\frac{P_1V_1}{RT} + \frac{P_2V_2}{RT} = \frac{PV_1}{RT} + \frac{PV_2}{RT}$$

So, $P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$

$$= \frac{(4P_0)(3V) + (1.5P_0)(2V)}{5V} = 3P_0 = IP_0 \quad \text{(given)}$$

So, $I = 3$

6. **(4)** As here volume of gas remains constant,
$(\Delta Q)_V = \mu C_V \Delta T$, Here $C_V = 5$ cal/mol K
And $\Delta T = (400 - 300) = 100$ K
And so for ideal gas $PV = \mu RT$,

$$\mu = \frac{(10)^5 \times (0.2)}{8.31 \times 300} = 8 \text{ mol}$$

$(\Delta Q)_V = 8 \times 5 \times 100 = 4$ kcal

7. **(9)** Here $\Delta Q = 15000$ J (given)
In an isobaric process
$\Delta Q = nC_p\Delta T$, $\Delta U = nC_V\Delta T$ (always)

$$\frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_p\Delta T} = \frac{1}{\gamma} \Rightarrow \Delta U = \frac{\Delta Q}{\gamma}$$

or $\Delta U = \frac{3}{5} \times 15 = 9$ kJ

8. **(2)** For process I : $dQ = dU + dW$
$\Rightarrow 4 = dU + 5 \Rightarrow dU = -1$
For process II : $5 = dU + dW$
$\Rightarrow 5 = -1 + dW \Rightarrow dW = +6$ J

9. **(0)** $P_0V_0 = nRT_a$
$(P_0 + \Delta P)(V_0 + \Delta V) = nRT_b$
$(P_0 + \Delta P)(V_0 - \Delta V) = nRT_b$
$\Delta V = 0$
Pressure depends on number of moles and is independent of nature of gas.

10. **(4)** $P_2V_2 = P_1V_1$
$V_1 = 20000$ cc $\quad P_1 = 10^5$ Pa
$V_2 = Ax$, $\quad P_2 = P_0 + \frac{kx}{A}$

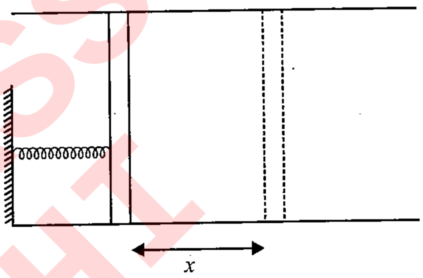

Fig. 2.204

$\Rightarrow 10^5 \times 20000 \times 10^{-6}$

$$= \left(10^5 + \frac{1000x}{100 \times 10^{-4}}\right) 100x \times 10^{-4}$$

$\Rightarrow 2000 = 1000\,[x\,(1 + x)]$
$x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1+8}}{2} = 1 \text{ m} = 100 \text{ cm}$$

$\Rightarrow 25\,h = 100 \Rightarrow h = 4$

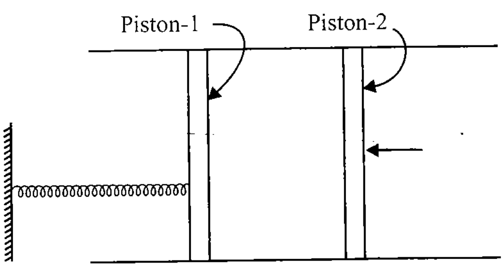

Fig. 2.205

CHAPTER

3

Archives on Chapters 1 and 2

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EXERCISES

Archives

Solutions on page 3.11

Fill in the Blank Type

1. One mole of a monatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is ____. **(IIT-JEE, 1984)**
2. The variation of temperature of a material as heat is given to it at a constant rate is shown in Fig. 3.1. The material is in solid state at point O. The state of the material at point P is ____. **(IIT-JEE, 1985)**

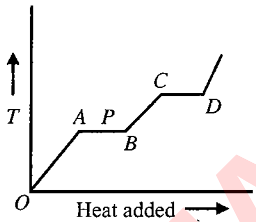

Fig. 3.1

3. During an experiment, an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at a temperature T and volume V. When it expands to a volume $2V$, the temperature becomes——°C. **(IIT-JEE, 1987)**
4. 300 grams of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is ____. **(IIT-JEE, 1989)**
5. The earth receives on its surface radiation from the sun at the rate of 1400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11}, and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is ____ K. **(IIT-JEE, 1989)**
6. A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required for the temperature of the sphere to drop to 100 K is ____. **(IIT-JEE, 1991)**
7. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than ____. **(IIT-JEE, 1991)**
8. A substance of mass M kilograms requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The latent heat of fusion of the substance is ____. **(IIT-JEE, 1992)**
9. A container of volume 1 m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300 K and the other part is vacuum. The whole system is thermally isolated from the surrounding. When the partition is removed, the gas expands to occupy the whole volume. Its temperature now will be ____. **(IIT-JEE, 1993)**
10. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is P_a. The ratio of the specific heats of the gas is 1.67. The ratio p_a/p_i is ____. **(IIT-JEE, 1994)**
11. Two metal cubes A and B of same size are arranged as shown in Fig. 3.2. The extreme ends of the combination are maintained at the indicated temperature. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C, respectively. After steady state is reached, the temperature t of the interface will be ____. **(IIT-JEE, 1996)**

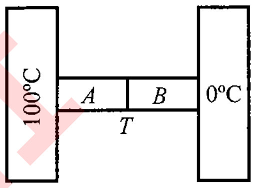

Fig. 3.2

12. A ring-shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in Fig. 3.3 is ____. **(IIT-JEE, 1997)**

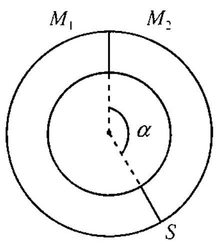

Fig. 3.3

13. A gas thermometer is used as a standard thermometer for measurement of temperature. When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads 3.0×10^4 N/m^2. When the gas container of the same thermometer is immersed in another system, the gas pressure reads 3.5×10^4 N/m^2. The temperature of this system is therefore ____°C. **(IIT-JEE, 1997)**
14. Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 g, the time taken to melt the ice will be ____ minutes (latent heat of fusion of ice = 3.3×10^5 J/kg). **(IIT-JEE, 1997)**

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True/False Type

1. The root mean square speeds of the molecules of different ideal gases maintained at the same temperature are the same. **(IIT-JEE, 1981)**
2. The volume V versus temperature T graphs for a certain amount of a perfect gas at two pressures p_1 and p_2 are as shown in Fig. 3.4. It follows from the graphs that p_1 is greater than p_2. **(IIT-JEE, 1982)**

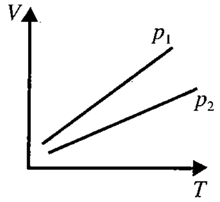

Fig. 3.4

3. Two different gases at the same temperature have equal root mean square velocities. **(IIT-JEE, 1982)**
4. The curves A and B in Fig. 3.5 show P–V graphs for an isothermal and an adiabatic process of an ideal gas. The isothermal process is represented by the curve A. **(IIT-JEE, 1985)**

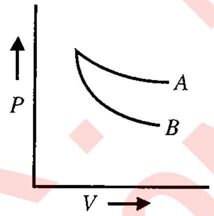

Fig. 3.5

5. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. **(IIT-JEE, 1985)**
6. At a given temperature, the specific heat of a gas at constant pressure is always greater than its specific heat at constant volume. **(IIT-JEE, 1987)**
7. Two spheres of the same material have radii 1 m and temperatures 4000 K and 2000 K, respectively. The energy radiated per second by the first sphere is greater than that by the second. **(IIT-JEE, 1988)**

Single Correct Answer Type

1. A wall has two layers A and B, each made of different materials. Both the layers have the same thikness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer A is **(IIT-JEE, 1980)**

   a. 6°C **b.** 12°C
   c. 18°C **d.** 24°C

2. An ideal monatomic gas is taken round the cycle $ABCDA$ as shown in the P–V diagram (see Fig. 3.6). The work done during the cycle is **(IIT-JEE, 1983)**

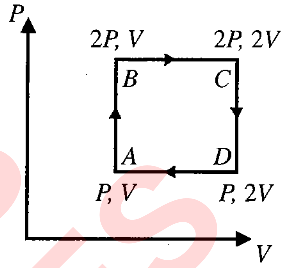

Fig. 3.6

   a. PV **b.** $2PV$
   c. 1/2 **d.** zero

3. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is **(IIT-JEE, 1984)**

   a. H_2 **b.** F_2
   c. O_2 **d.** Cl_2

4. 70 calories of heat are required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is **(IIT-JEE, 1985)**

   a. 30 **b.** 50
   c. 70 **d.** 90

5. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilograms is **(IIT-JEE, 1986)**

   a. 0.130 **b.** 0.065
   c. 0.260 **d.** 0.135

6. If 1 mole of a monatomic gas ($\gamma = 5/3$) is mixed with 1 mole of a diatomic gas ($\gamma = 7/5$), the value of γ for the mixture is **(IIT-JEE, 1988)**

   a. 1.40 **b.** 1.50
   c. 1.53 **d.** 3.07

7. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in the steady state. The effective thermal conductivity of the system is **(IIT-JEE, 1988)**

   a. $K_1 + K_2$ **b.** $\dfrac{K_1K_2}{(K_1+K_2)}$
   c. $\dfrac{(K_1+3K_2)}{4}$ **d.** $\dfrac{(3K_1+K_2)}{4}$

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8. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is **(IIT-JEE, 1990)**

a. $\frac{2}{5}$ **b.** $\frac{3}{5}$

c. $\frac{3}{7}$ **d.** $\frac{5}{7}$

9. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1, that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is **(IIT-JEE, 1992)**

a. $\frac{(v_1+v_2)}{2}$ **b.** v_1

c. $(v_1 v_2)^{1/2}$ **d.** $\sqrt{3kT/M}$

10. Three rods of identical cross-sectional area are made from the same metal and form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$, respectively. In the steady state, the temperature of the point is T_c. Assuming that only heat conduction takes place, T_c/T is **(IIT-JEE, 1995)**

a. $\frac{1}{2(\sqrt{2}-1)}$ **b.** $\frac{3}{\sqrt{2}+1}$

c. $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ **d.** $\frac{1}{\sqrt{2}+1}$

11. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is **(IIT-JEE, 1995)**

a. $\frac{1}{3}$ **b.** $\frac{1}{\sqrt{3}}$

c. $\frac{\sqrt{3}}{1}$ **d.** $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

12. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes **(IIT-JEE, 1996)**

a. $4v$ **b.** $2v$

c. $v/2$ **d.** $v/4$

13. The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m/s, respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour) **(IIT-JEE, 1997)**

a. 12.42×10^{-21} J, 968 m/s
b. 8.78×10^{-21} J, 684 m/s
c. 6.21×10^{-21} J, 968 m/s
d. 12.42×10^{-21} J, 684 m/s

14. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, the ratio of the surface temperature of the sun and the North Star is **(IIT-JEE, 1997)**

a. 1.46 **b.** 0.69

c. 1.21 **d.** 0.83

15. The average translational kinetic energy of O_2 (relative molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (relative molar mass 28) molecules in eV at that temperature is **(IIT-JEE, 1997)**

a. 0.0015 **b.** 0.003

c. 0.048 **d.** 0.768

16. A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing 1 mole of He gas (relative molar mass 4) at a temperature $2T$ has a pressure of **(IIT-JEE, 1997)**

a. $P/8$ **b.** P

c. $2P$ **d.** $8P$

17. A spherical black body with a radius of 12 cm radiates 450 W power at 50 K. If the radius were halved and the temperature doubled, the power radiated in watts would be **(IIT-JEE, 1997)**

a. 225 **b.** 450

c. 900 **d.** 1800

18. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is **(IIT-JEE, 1998)**

a. 1:1
b. 1:2
c. 2:1
d. depends on the moment of inertia of the two molecules

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be Δp and $1.5\,\Delta p$, respectively. Then **(IIT-JEE, 1998)**

a. $4\,m_A = 9\,m_B$ **b.** $2\,m_A = 3\,m_B$

c. $3\,m_A = 2\,m_B$ **d.** $9\,m_A = 4\,m_B$

20. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the

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rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is **(IIT-JEE, 1998)**

a. 30 K **b.** 18 K
c. 50 K **d.** 42 K

21. A black body is at a temperature of 2880 K. The energy of radiation emitted by this body with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm, is U_2 and between 1499 nm and 1500 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nm-K. Then **(IIT-JEE, 1998)**

a. $U_1 = 0$ **b.** $U_3 = 0$
c. $U_1 > U_2$ **d.** $U_2 > U_1$

22. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is **(IIT-JEE, 1999)**

a. $4RT$ **b.** $15RT$
c. $9RT$ **d.** $11RT$

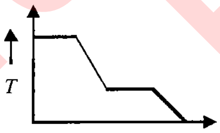
23. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion, respectively, then T_1/T_2 is given by **(IIT-JEE, 2000)**

a. $\left(\dfrac{L_1}{L_2}\right)^{2/3}$ **b.** $\dfrac{L_1}{L_2}$

c. $\dfrac{L_2}{L_1}$ **d.** $\left(\dfrac{L_2}{L_1}\right)^{2/3}$

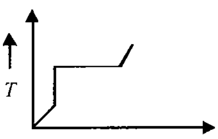
24. A block of ice at $-10°C$ is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively? **(IIT-JEE, 2000)**

a.



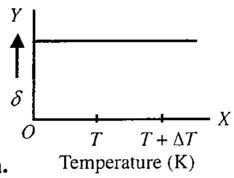
b.

c.

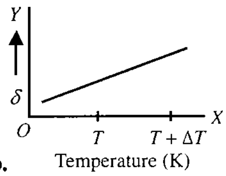


d.

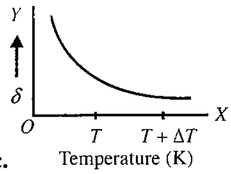
25. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta T / V\Delta T$ varies with temperature as **(IIT-JEE, 2000)**



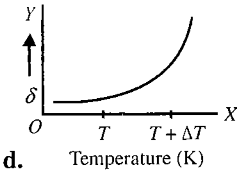
a.



b.



c.



d.

26. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then **(IIT-JEE, 2000)**

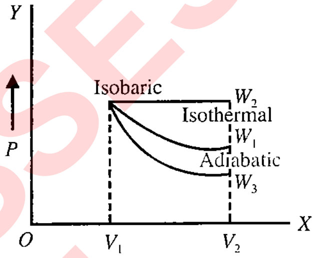


Fig. 3.7

a. $W_2 > W_1 > W_3$ **b.** $W_2 > W_3 > W_1$
c. $W_1 > W_2 > W_3$ **d.** $W_1 > W_3 > W_2$

27. The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3, respectively are as shown. Their temperatures are such that **(IIT-JEE, 2000)**

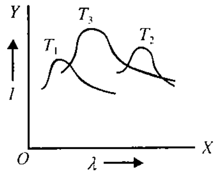


Fig. 3.8

a. $T_1 > T_2 > T_3$ **b.** $T_1 > T_3 > T_2$
c. $T_2 > T_3 > T_1$ **d.** $T_3 > T_2 > T_1$

28. Three rods made of same material and having the same cross section have been joined as shown in Fig. 3.9. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C, respectively. The temperature of the junction of the three rods will be **(IIT-JEE, 2001)**

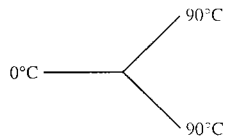


Fig. 3.9

a. 45°C **b.** 60°C **c.** 30°C **d.** 20°C

29. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas **(IIT-JEE, 2001)**

a. the temperature will decrease
b. the volume will increase
c. the pressure will remain constant
d. the temperature will increase

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30. $P-V$ plots for two gases during adiabatic processes are shown in Fig. 3.10. Plots 1 and 2 should correspond, respectively, to **(IIT-JEE, 2001)**

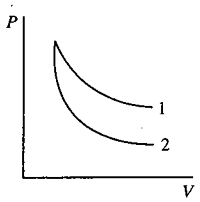


Fig. 3.10

a. He and O_2 **b.** O_2 and He
c. He and Ar **d.** O_2 and N_2

31. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in Fig. 3.11. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is **(IIT-JEE, 2002)**

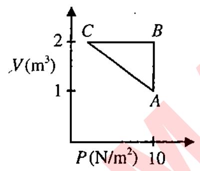
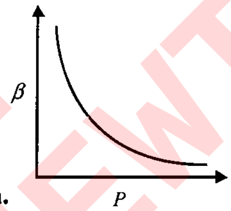


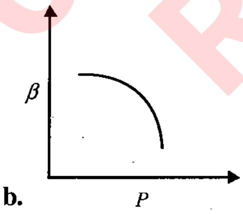
Fig. 3.11

a. –5 J **b.** –10 J
c. –15 J **d.** –20 J

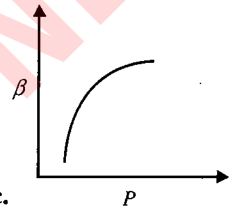
32. Which of the following graphs correctly represents the variation of $\beta = (dV/dP)1/V$ with P for an ideal gas at constant temperature? **(IIT-JEE, 2002)**



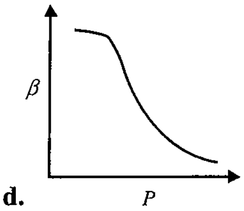
a.



b.



c.



d.

33. An ideal black body at room temperature is thrown into a furnace. It is observed that **(IIT-JEE, 2002)**

a. initially it is the darkest body and later the brightest
b. it is the darkest body at all times
c. it cannot be distinguished at all times
d. initially it is the darkest body and later it cannot be distinguished

34. The graph, shown in the diagram represents the variation of temperature (T) of two bodies, x and y, having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of two bodies. **(IIT-JEE, 2003)**

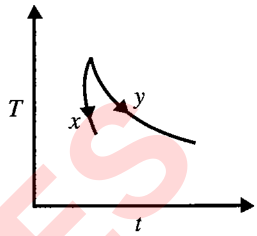


Fig. 3.12

a. $E_x > E_y$ and $a_x < a_y$
b. $E_x < E_y$ and $a_x > a_y$
c. $E_x > E_y$ and $a_x > a_y$
d. $E_x < E_y$ and $a_x < a_y$

35. Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s, respectively. If the length of each rod increases by the same amount when their temperatures are raised by t°C, then find the ratio $\frac{l_1}{l_1 + l_2}$. **(IIT-JEE, 2003)**

a. α_s/α_a **b.** α_a/α_s
c. $\alpha_s/(\alpha_a + \alpha_s)$ **d.** $\alpha_a/(\alpha_s + \alpha_s)$

36. The $P-T$ diagram for an ideal gas is shown in Fig. 3.13, where AC is an adiabatic process, find the corresponding $P-V$ diagram. **(IIT-JEE, 2003)**

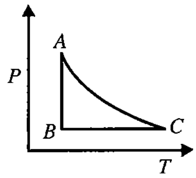
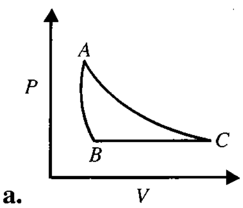
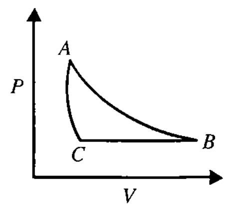


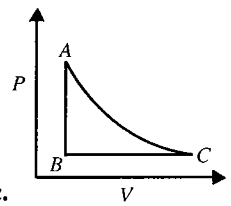
Fig. 3.13



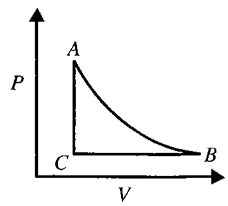
a.



b.



c.



d.

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37. 2 kg of ice at $-20°C$ is mixed with 5 kg of water at $20°C$ in an insulating vessel having negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg/°C and 0.5 kcal/kg/°C respectively, while the latent heat of fusion of ice is 80 kcal/kg.

(IIT-JEE, 2003)

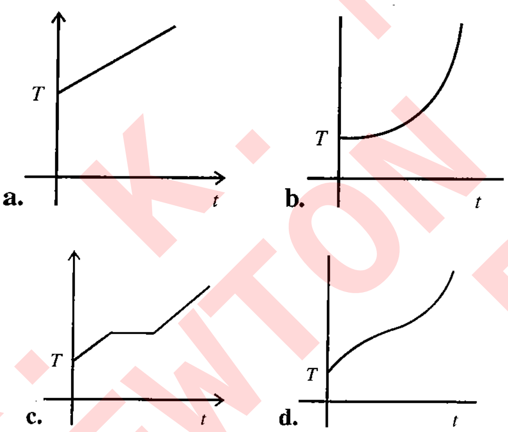
a. 7 kg **b.** 6 kg
c. 4 kg **d.** 2 kg

38. Three discs A, B and C having radii 2, 4 and 6 cm, respectively, are coated with carbon black. Wavelengths for maximum intensity for the three discs are 300, 400 and 500 nm, respectively. If Q_A, Q_B and Q_C are powers emitted by A, B and C, respectively, then

(IIT-JEE, 2004)

a. Q_A will be maximum
b. Q_B will be maximum
c. Q_C will be maximum
d. $Q_A = Q_B = Q_C$

39. If liquefied oxygen at 1 atmospheric pressure is heated from 50 K to 300 K by supplying heat at constant rate, the graph of temperature vs. time will be **(IIT-JEE, 2004)**



40. Two identical rods are connected between two containers one of them is at 100°C and another is at 0°C. If rods are connected in parallel then the rate of melting of ice is q_1 g/s. If they are connected in series then the rate is q_2. The ratio q_2/q_1 is **(IIT-JEE, 2004)**

a. 2 **b.** 4
c. 1/2 **d.** 1/4

41. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in the complete process, which of the following is true?

(IIT-JEE, 2004)

a. $W > 0$; $P_3 > P_1$ **b.** $W < 0$; $P_3 > P_1$
c. $W > 0$; $P_3 < P_1$ **d.** $W < 0$; $P_3 < P_1$

42. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc is a function of its wavelength as shown in Fig. 3.14. Which of the following option is the correct match? **(IIT-JEE, 2005)**

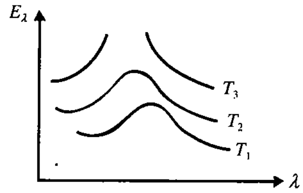


Fig. 3.14

a. Sun – T_3, tungsten filament – T_1, welding arc – T_2
b. Sun – T_2, tungsten filament – T_1, welding arc – T_3
c. Sun – T_3, tungsten filament – T_2, welding arc – T_1
d. Sun – T_1, tungsten filament – T_2, welding arc – T_3

43. In which of the following processes, convection does not take place primarily?

(IIT-JEE, 2005)

a. sea and land breeze
b. boiling of water
c. heating air around a furnace
d. warming of glass of bulb due to filament

44. A spherical body of area A and emissivity $e = 0.6$ is kept inside a perfectly black body. Total heat radiated by the body at temperature T is

(IIT-JEE, 2005)

a. $0.4\,\sigma AT^4$ **b.** $0.8\,\sigma AT^4$
c. $0.6\,\sigma AT^4$ **d.** $1.0\,\sigma AT^4$

45. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under which of the following conditions? **(IIT-JEE, 2005)**

a. From 14.5°C to 15.5°C at 760 mm of Hg
b. From 98.5°C to 99.5°C at 760 mm of Hg
c. From 13.5°C to 14.5°C at 76 mm of Hg
d. From 3.5°C to 4.5°C at 76 mm of Hg

46. Two litres of water in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at the rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C (given specific heat of water is 4.2 kJ/kg)? **(IIT-JEE, 2005)**

a. 7 min **b.** 6 min 2 s
c. 8 min 20 s **d.** 14 min

47. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(IIT-JEE, 2008)

a. $\frac{1}{T}$ **b.** $\frac{2}{T}$
c. $\frac{3}{T}$ **d.** $\frac{4}{T}$

48. A real gas behaves like an ideal gas if its

a. pressure and temperature are both high
b. pressure and temperature are both low
c. pressure is high and temperature is low
d. pressure is low and temperature is high

(IIT-JEE, 2010)

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49. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is **(IIT-JEE, 2011)**

a. $\frac{9}{8}RT_1$ b. $\frac{3}{2}RT_1$
c. $\frac{15}{8}RT_1$ d. $\frac{9}{2}RT_1$

Multiple Correct Answers Type

1. For an ideal gas: **(IIT-JEE, 1989)**
 a. The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nCv(T_2-T_1)$, where Cv is the molar specific heat at constant volume and n the number of moles of the gas.
 b. The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.
 c. The internal energy does not change in an isothermal process.
 d. No heat is added or removed in an adiabatic process.

2. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the $P-V$ diagram. Select the correct statement(s) from the following: **(IIT-JEE, 1993)**
 a. The work done by the gas in process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.
 b. In the $T-V$ diagram, the path AB becomes a part of a parabola.
 c. In the $P-T$ diagram, the path AB becomes a part of a hyperbola.
 d. In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81, respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. Wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A, by 1.00 μm. If the temperature of A is 5802 K **(IIT-JEE, 1994)**
 a. the temperature of B is 1934 K
 b. $\lambda_B = 1.5\ \mu$m
 c. the temperature of B is 11604 K
 d. the temperature of B is 2901 K

4. From the following statements concerning ideal gas at any given temperature T, select the correct one(s). **(IIT-JEE, 1995)**
 a. The coefficient of volume expansion at constant pressure is the same for all ideal gases.
 b. The average translational kinetic energy per molecule of oxygen gas is $3kT$, k being the Boltzmann constant.
 c. The mean free path of molecules increases with decrease in pressure.
 d. In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different.

5. During the melting of a slab of ice at 273 K at atmosphere pressure, **(IIT-JEE, 1998)**
 a. positive work is done by the ice–water system on the atmosphere
 b. positive work is done on the ice–water system by the atmosphere
 c. the internal energy of the ice–water system increases
 d. the internal energy of the ice–water system decreases

6. Let $\bar{v}$, v_{rms} and v_p, respectively, denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then **(IIT-JEE, 1998)**
 a. no molecule can have a speed greater than $\sqrt{2}v_{rms}$
 b. no molecule can have speed less than $v_p/\sqrt{2}$
 c. $v_p < \bar{v} < v_{rms}$
 d. the average kinetic energy of a molecule is $3/4mv_p^2$

7. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is **(IIT-JEE, 1999)**
 a. proportional to ΔT
 b. inversely proportional to ΔT
 c. proportional to $|\alpha_B - \alpha_C|$
 d. inversely proportional to $|\alpha_B - \alpha_C|$

8. A black body of temperature T is inside the chamber of T_0 temperature initially. Sun rays are allowed to fall from a hole in the top of the chamber. If the temperatures of black body (T) and chamber (T_0) remain constant, then **(IIT-JEE, 2006)**

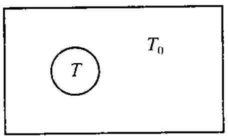

Fig. 3.15

 a. Black body will absorb radiation.
 b. Black body will absorb less radiation.
 c. Black body will emit more energy.
 d. Black body will emit energy equal to energy absorbed by it.

9. Cv and Cp denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **(IIT-JEE, 2009)**
 a. $Cp - Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas.
 b. $Cp + Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas.

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c. Cp/Cv is larger for a diatomic ideal gas than for a monatomic ideal gas.

d. $Cp \cdot Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas.

10. Figure 3.16 shows the $P-V$ plot of an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then **(IIT-JEE, 2009)**

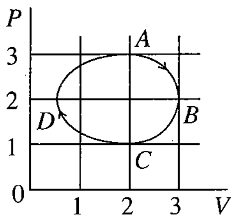

Fig. 3.16

a. the process during the path $A \to B$ is isothermal

b. heat flows out of the gas during the path $B \to C \to D$

c. work done during the path $A \to B \to C$ is zero

d. positive work is done by the gas in the cycle $ABCDA$

11. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in Fig. 3.17. Its pressure at A is P_0. Choose the correct option(s) from the following

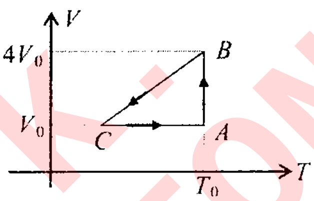

Fig. 3.17

a. Internal energies at A and B are the same

b. Work done by the gas in process AB is $P_0V_0 \ln 4$

c. Pressure at C is $P_0/4$

d. Temperature at C is $T_0/4$

12. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in Fig. 3.18. All slabs are of same width. **(IIT-JEE, 2011)**

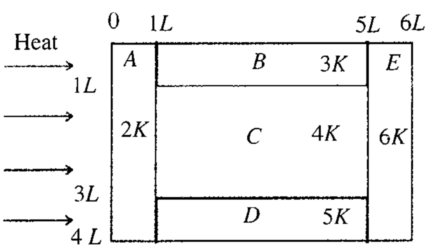

Fig. 3.18

a. Heat flow through slabs A and E is same

b. Heat flow through slab E is maximum

c. Temperature difference across slab E is smallest.

d. Heat flow through C = heat flow through B + heat flow through D.

Assertion-Reasoning Type

In the following question, assertion (A) is given by corresponding statement of reason (R) of the statements, mark the correct answer.

a. If both assertion and reason are true and the reason is correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of assertion.

c. If assertion is true, but the reason is false.

d. If assertion is false, but the reason is true.

1. **Assertion:** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.
Reason: The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. **(IIT-JEE, 2007)**

Comprehension Type

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in Fig. 3.19. The atmospheric pressure is p_0. **(IIT-JEE, 2007)**

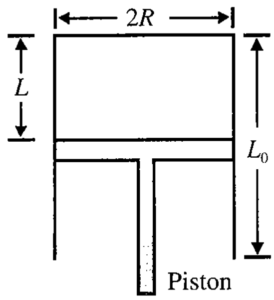

Fig. 3.19

1. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

a. p_0 **b.** $\dfrac{p_0}{2}$

c. $\dfrac{p_0}{2} + \dfrac{Mg}{\pi R^2}$ **d.** $\dfrac{p_0}{2} - \dfrac{Mg}{\pi R^2}$

2. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

a. $\left(\dfrac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)(2L)$ **b.** $\left(\dfrac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)(2L)$

c. $\left(\dfrac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)(2L)$ **d.** $\left(\dfrac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$

3. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below

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the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in Fig. 3.20. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies

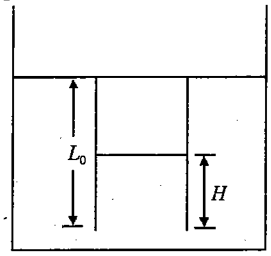


Fig. 3.20

a. $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$
b. $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$
c. $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$
d. $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

Matching Column Type

1. Heat given to process is positive. Match the following options of Column I with the corresponding options in Column II: **(IIT-JEE, 2006)**

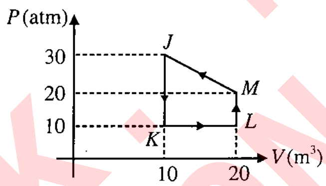


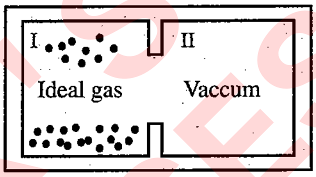
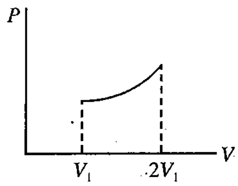
Fig. 3.21

Column I	Column II
i. JK	**a.** $\Delta W > 0$
ii. KL	**b.** $\Delta Q < 0$
iii. LM	**c.** $\Delta W < 0$
iv. MJ	**d.** $\Delta Q > 0$

2. Column I gives some devices and Column II gives some processes on which the functioning of these devices depends. Match the devices in Column I with the processes in Column II. **(IIT-JEE, 2007)**

Column I	Column II
i. Bimetallic strip	**a.** Radiation from a hot body
ii. Steam engine	**b.** Energy conversion
iii. Incandescent lamp	**c.** Melting
iv. Electric fuse	**d.** Thermal expansion of solids

3. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. **(IIT-JEE, 2008)**

Column I	Column II
i. An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and Chamber II has vacuum. The valve is opened.  Fig. 3.22	**a.** The temperature of the gas decreases
ii. An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^2$, where V is the volume of the gas.	**b.** The temperature of the gas increases or remains constant
iii. An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^{4/3}$, where V is its volume.	**c.** The gas loses heat
iv. An ideal monatomic gas expands such that its pressure P and volume V follow the behaviour shown in the graph.  Fig. 3.23	**d.** The gas gains heat

4. One mole of a monatomic ideal gas is taken through a cycle $ABCDA$ as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I. **(IIT-JEE, 2011)**

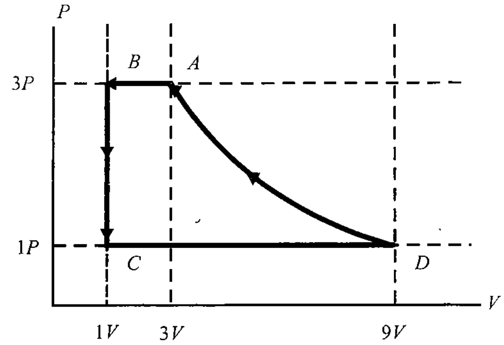


Fig. 3.24

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Column I	Column II
i. Process $A \to B$	**a.** Internal energy decreases
ii. Process $B \to C$	**b.** Internal energy increases
iii. Process $C \to D$	**c.** Heat is lost
iv. Process $D \to A$	**d.** Heat is gained
	e. Work is done on the gas

Integer Answer Type

1. A metal rod AB of length $10x$ has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surrounding.) **(IIT-JEE, 2009)**

2. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **(IIT-JEE, 2010)**

3. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is **(IIT-JEE, 2010)**

4. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C. Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then the value of 'm' in kg is nearly.

ANSWERS AND SOLUTIONS

Fill in the Blank Type

1. $$C_V = \frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2} = \frac{(1)\left(\frac{3}{2}R\right)+(1)\left(\frac{5}{2}R\right)}{1+1} = 2R$$

2. AB represents a process when physical state changes from solid to liquid and the temperature remains unchanged. Since P is a point between A and B, the material is partly solid and partly liquid.

3. $PV = RT$ (ideal gas equation)

$$\Rightarrow P = \frac{RT}{V} \quad \text{(i)}$$

Given that

$$VP^2 = \text{constant} \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$V \times \frac{R^2T^2}{V^2} = \text{constant}$$

$$\therefore \quad \frac{T^2}{V} = \text{constant}$$

$$\therefore \quad \frac{T_1^2}{V_1} = \frac{T_2^2}{V_2}$$

$$\Rightarrow T_2 = T_1\sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2}T$$

4. The heat required for 100 g of ice at 0°C to change its temperature to 0°C

$$= mL = 100 \times 80 \times 4.2 = 33600 \text{ J} \quad \text{(i)}$$

The heat released by 300 g of water at 25°C to change its temperature to 0°C

$$= mc\Delta = 300 \times 4.2 \times 25 = 31{,}500 \text{ J} \quad \text{(ii)}$$

Since the energy in Eq. (ii) is less than that of Eq. (i), the final temperature will be 0°C.

5. The energy received per second per unit area from sun at a distance of 1.5×10^{11} m is 1400 J/sm^2. The total energy released by sun per second

$$= 1400 \times 4\pi \times (1.5 \times 10^{11})^2$$

Therefore, the total energy released per second per unit surface area of the sun

$$= \frac{1400 \times 4\pi \times (1.5\times 10^{11})^2}{4\pi \times (7\times 10^8)^2}$$

This energy E is also equal to $E = \sigma T^4$

$$\Rightarrow \quad T = \left[\frac{1400 \times 4\pi \times (1.5\times 10^{11})^2}{4\pi \times (7\times 10^8)^2 \times 5.67\times 10^{-8}}\right]^{1/4} = 5803 \text{ K}$$

6. The energy emitted per second when the temperature of the copper sphere is T and the surrounding temperature T_0

$$= \sigma(T^4 - T_0^4) \times A$$

where A = surface area

$$= \sigma T^4 A \quad \text{(i)}$$

Here $\quad T_0 = 0$ K

We know that

$$dQ = mcdt$$

$$\therefore \quad \frac{dQ}{dt} = -mc\frac{dT}{dt} \quad \text{(ii)}$$

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Here the –ve sign shows that the themperature is decreasing with time.
Energy emitted per second from Eqs. (i) and (ii)

$$\sigma T^4 A = -mc\frac{dT}{dt}$$

$$\Rightarrow dt = -\frac{mcdT}{\sigma T^4 A} = -\frac{\rho \times \frac{4}{3}\pi r^3 cdT}{\sigma T^4 \times 4\pi r^2}$$

$$\left[\therefore m = \rho \times \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow dt = -\frac{\rho rc}{3\sigma}\frac{dT}{T^4}$$

Integrating both sides,

$$\int_0^t dt = -\frac{\rho rc}{3\sigma}\int_{200}^{100}\frac{dT}{T^4} = -\frac{\rho rc}{3\sigma}\left[-\frac{1}{3T^3}\right]_{200}^{100}$$

$$t = \frac{\rho rc}{9\sigma}\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

$$t = \frac{7\rho rc}{(72\times 10^6)\sigma}$$

7. When the spherical shell is thin, $t \ll R$
In this case, the rate of flow of heat from the sphere to the surroundings

$$P = \frac{K(4\pi R^2)T}{t}$$

where T is the temperature difference and t is the thickness of steel.

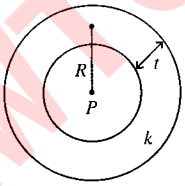

Fig. 3.25

$$t = \frac{4\pi R^2 KT}{P}$$

8. Since P joules per second of heat is supplied to keep the substance in molten state, it means that the substance in the molten state at its meeting point releases P joules of heat in one second.

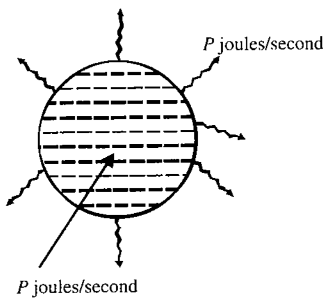

Fig. 3.26

When the power is turned off, the heat input becomes zero. But heat output continues. It takes t seconds for the substance to solidify (given). Therefore total heat released in t seconds = $P \times t$

This is equal to $ML_{\text{fusion}} \Rightarrow ML_{\text{fusion}} = P \times t$

$$L_{\text{fusion}} = \frac{P\times t}{M}$$

9. This is a case of free expansion. Here $\Delta Q = 0$ and $\Delta W = 0$, because gas is doing work on vacuum. So $\Delta U = 0$. Hence no change in temperature, it will remain 300 K.

10. For isothermal expansion,

$$P\times V = P_i 2V \Rightarrow P_1 = \frac{P}{2}$$

For adiabatic expansion,

$$PV^\gamma = P_a \times (2V)^\gamma \Rightarrow P_a = \frac{P}{2^\gamma} = \frac{P}{2^{1.67}}$$

$$\therefore \quad \frac{P_a}{P_i} = \frac{P}{2^{1.67}} \times \frac{2}{P} = \frac{1}{2^{0.67}}$$

11. The heat temperature through A per second

$$Q_1 = K_1 A\,(100 - t)/l$$

The heat transferred through B per second

$$Q_2 = K_2 A(t - 0)/l$$

At steady state, $K_1 A(100 - t)/l = K_2 A(t - 0)/l$

$$\Rightarrow \quad 300\,(100 - t) = 200(t - 0)$$

$$\Rightarrow \quad 300 - 3t = 2t \Rightarrow t = 60°\text{C}$$

12. Pressure on both sides will be equal

$$p_1 = p_2$$

i.e., $$\frac{n_1 RT}{V_1} = \frac{n_2 RT}{V_2} \qquad \left(n = \frac{m}{M}\right)$$

$$\frac{m}{M_1 V_1} = \frac{m}{M_2 V_2} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$

but $$\frac{V_2}{V_1} = \frac{\alpha}{360 - \alpha}$$

Solve to get $\alpha = \left(\dfrac{360°}{8+7}\right)\times 8 = 192°$

13. In case of gas thermometer, the volume of the gas container remains constant. Therefore

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{3\times 10^4}{273.16} = \frac{3.5\times 10^4}{T_2}$$

$$\Rightarrow T_2 = 318.6 \text{ K}$$

14. Solar power received by earth = 1400 W/m^2
Solar power received by 0.2 m^2 area = (1400 W/m^2) (0.2 m^2) = 280 W

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Mass of ice = 280 g = 0.280 kg

Heat required to melt ice

$= (0.280)\,(3.3 \times 10^5)$

$= 9.24 \times 10^4$ J

If t is the time taken for the ice to melt, we will have

$$(280)\,t = 9.24\times10^4 \text{ J} \left[\therefore P = \frac{E}{t}\right]$$

$$t = \frac{9.24\times10^4}{280}\text{ s} = 330\text{ s} = 5.5\text{ min}$$

True/ False Type

1. **False.** At the same temperature $\bar{c} \propto \frac{1}{\sqrt{M}}$

   i.e., dependent on molar mass and hence $\bar{c}$ will be different for different ideal gases.

2. **False.** $\frac{PV}{T} = \text{constant}$

   For a particular temperature T_1

   We have $p \propto \frac{1}{V}$

   Since, $V_1 > V_2$, $p_2 < p_1$

   $\Rightarrow p_1 > p_2$

3. **False.** We know that $c_{rms} = \sqrt{\frac{3RT}{M}}$

   For a particular temperature, $c_{rms} \propto \sqrt{\frac{1}{M}}$

   i.e., c_{rms} will have different values for different gases.

4. **True.** The slope of $p-V$ curve is more for adiabatic process than for isothermal process. From the graph it is clear that slope for B is greater than the slope for A.

5. **False.** $v_{rms} = \sqrt{\frac{3RT}{M}}$

   $$v_{rms} \propto \sqrt{\frac{T}{M}} \qquad \text{(i)}$$

   When oxygen gas dissociates into atomic oxygen, its atomic mass M will become half. Temperature is doubled. So, from Eq. (i) v_{rms} will become two times.

6. **True.** $C_p > C_v$

   This is because at constant pressure when heat is supplied to the gas for increasing temperature, some heat is used up in doing work for increasing volume.

7. **True.** $E \propto T^4$

Single Correct Answer Type

1. **b.** $H = \frac{2kA(T_1 - T)}{L} = \frac{kA(T - T_2)}{L}$

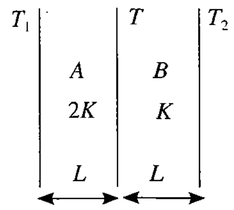

Fig. 3.27

$\Rightarrow 2T_1 - 3T = -T_2$

adding T_1 on both sides:

$3T_1 - 3T = T_1 - T_2$

$\Rightarrow T_1 - T = \frac{T_1 - T_2}{3}$

$= \frac{36}{3} = 12°\text{C}$

2. **a.** Work done during the cycle = area enclosed in the curve = $(2P - P)\,(2V - V) = PV$

3. **a.** $v_{rms} = \sqrt{\frac{3RT}{M}}$

   Room temperature, $T \approx 300$ K

   $$1930 = \sqrt{\frac{3\times8.31\times10^3\times300}{M}}$$

   $M = 2$ g

   or the gas is H_2.

4. **b.**

   $Q_1 = nC_P\Delta T$

   $Q_2 = nC_V\Delta T$

   $\frac{Q_2}{Q_1} = \frac{C_V}{C_P} = \frac{1}{\gamma}$

   $$Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50\text{ cal}$$

5. **a.** Heat required,

   $Q = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15) = 72800$ cal

   Let m gram of steam is condensed, then heat loss

   $= m \times 540 + m \times 1 \times 20$

   $= 560$ m

   Heat loss = Heat gain

   $\Rightarrow 560$ m = 72800

   $\Rightarrow m = 130$ g = 0.130 kg

6. **b.** $\gamma_1 = 5/3$ means gas is monatomic or $C_{V_1} = \frac{3}{2}R$

   $\gamma_2 = 7/5$ means gas is diatomic or $C_{V_2} = 5/2\,R$

   C_V (of the mixture)

   $$= \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

   C_P (of the mixture) $= C_V + R = 3R$

   $$\gamma_{\text{mixture}} = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

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7. **c.** Let R_1 and R_2 be the thermal resistances of inner and outer portions. Since temperature difference at both ends is same, the resistances are in parallel. Hence,

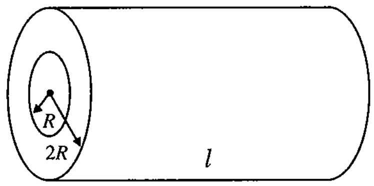

Fig. 3.28

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

$$\frac{K(4\pi R^2)}{l}=\frac{K_1(\pi R^2)}{l}+\frac{K_2(3\pi R^2)}{l} \Rightarrow K=\frac{3K_2+K_1}{4}$$

8. **d.** The desired fraction is

$$f=\frac{\Delta U}{\Delta Q}=\frac{nC_V\Delta T}{nC_P\Delta T}=\frac{C_V}{C_P}=\frac{1}{\gamma}$$

$$f=\frac{5}{7} \qquad \left(\text{as } \gamma=\frac{7}{5}\right)$$

9. **b.** The average speed of molecules of an ideal gas is given by

$$<v>=\sqrt{\frac{8RT}{\pi M}}$$

i.e., $<v> \propto \sqrt{T}$ for same gas.
Since temperature of A and C are same, average speed of O_2 molecules will be equal in A and C, i.e., v_1.

10. **b.** $\left(\frac{\Delta Q}{\Delta t}\right)_{BC}=\left(\frac{\Delta Q}{\Delta t}\right)_{CA}$

$$\Rightarrow \frac{kA(\sqrt{2}T-T_C)}{a}=\frac{kA(T_C-T)}{\sqrt{2}a}$$

Solve to get

$$\frac{T_C}{T}=\frac{3}{\sqrt{2}+1}$$

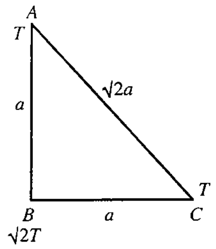

Fig. 3.29

11. **d.** According to Stefan's law

$$\Delta Q = e\sigma AT^4\Delta t$$

also $\Delta Q = mc\Delta T \Rightarrow \Delta Q = mc\Delta T = e\sigma AT^4\Delta t$

$$\Rightarrow \frac{\Delta T}{\Delta t}=\frac{e\sigma AT^4}{mc}=\frac{e\sigma T^4}{mc}\left[\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right]$$

$$=k\left(\frac{1}{m}\right)^{1/3}$$

$$\therefore \quad \frac{\Delta T_1/\Delta t_1}{\Delta T_2/\Delta t_2}=\left(\frac{m_2}{m_1}\right)^{1/3}=\left(\frac{1}{3}\right)^{1/3}$$

12. **b.** $v_{rms}=\sqrt{\frac{3RT}{M}}$

$v_{rms} \propto \sqrt{T}$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times, i.e., $2v$.

13. **d.** The formula for average kinetic energy is

$$\overline{(\text{KE})}=\frac{3}{2}KT$$

$$\therefore \quad \frac{\overline{(\text{KE})}_{600K}}{\overline{(\text{KE})}_{300K}}=\frac{600}{300}$$

$$\Rightarrow \overline{(\text{KE})}_{600K}=2\times 6.21\times 10^{-21}\text{ J}=12.42\times 10^{-21}\text{ J}$$

Also the formula for rms velocity is

$$C_{rms}=\sqrt{\frac{3KT}{m}}$$

$$\therefore \quad \frac{(C_{rms})_{600K}}{(C_{rms})_{300K}}=\sqrt{\frac{600}{300}}$$

$$\Rightarrow (C_{rms})_{600K}=\sqrt{2}\times 484=684 \text{ m/s}$$

14. **b.** According to Wein's displacement law, $\lambda_m T$ = constant where λ_m is the wavelength for which intensity of radiation emitted is maximum

$$(\lambda_m)_s T_s=(\lambda_m)_{NS}\times T_{NS}$$

S = sun
NS = north star

$$\Rightarrow \frac{T_s}{T_{NS}}=\frac{(\lambda_m)_{NS}}{(\lambda_m)_S}=\frac{350}{510}=0.69$$

15. **c.** Average translational kinetic energy of an ideal gas molecule is $3/2KT$ which depends on temperature only. Therefore if the temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

16. **c.** $PV=nRT$ or $P=\frac{nRT}{V}$ or $P\propto T$

If V and n are same. Therefore, if T is doubled, pressure also becomes two times, i.e., $2P$.

17. **d.** The energy radiated per second by a black body is given by Stefan's law,

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$$\frac{E}{t} = \sigma T^4 \times A$$

where A is the surface area of the black body

$$\frac{E}{t} = \sigma T^4 \times 4\pi r^2$$

Since black body is a sphere, $A = 4\pi r^2$

Case (i)

$$\frac{E}{t} = 450, \quad T = 500 \text{ K}, \quad r = 0.12 \text{ m},$$

$$\therefore \quad 450 = 4\pi\sigma(500)^4(0.12)^2 \qquad \text{(i)}$$

Case (ii)

$$\frac{E}{t} = ?, \quad T = 1000 \text{ K}, \quad r = 0.06 \text{ m} \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{E/t}{450} = \frac{(1000)^4(0.06)^2}{(500)^4(0.12)^2} = \frac{2^4}{2^2} = 4$$

$$\Rightarrow \frac{E}{t} = 450 \times 4 = 1800 \text{ W}$$

18. a. Average kinetic energy per molecule per degree of freedom = $1/2kT$. Since both the gases are diatomic and are at same temperature (300 K), both will have the same number of rotational degree of freedom, i.e., two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$$= 2 \times \frac{1}{2}kT = kT$$

Thus, the ratio will be 1:1.

19. c. Process is isothermal. Therefore, T = constant. Volume is increasing; therefore, pressure will decrease $\left(p \propto \frac{1}{V}\right)$.

In chamber $A \rightarrow$

$$\Delta p = (p_A)_i - (p_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V}$$

$$= \frac{n_A RT}{2V} \qquad \text{(i)}$$

In chamber $B \rightarrow$

$$1.5\Delta p = (p_B)_i - (p_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V}$$

$$= \frac{n_B RT}{2V} \qquad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3}$$

$$\frac{m_A/M}{m_B/M} = \frac{2}{3}$$

$$\frac{m_A}{m_B} = \frac{2}{3}$$

$$3m_A = 2m_B$$

20. d. A is free to move; therefore, heat will be supplied at constant pressure.

$$dQ_A = nC_P dT_A \qquad \text{(i)}$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$dQ_B = nC_V dT_B \qquad \text{(ii)}$$

But $\quad dQ_A = dQ_B$

$$nC_P dT_A = nC_V dT_B$$

$$dT_B = \left(\frac{C_P}{C_V}\right) dT_A$$

$$= \gamma(dT_A) \qquad (\gamma = 1.4 \text{ (diatomic)}) \quad (dT_A = 30 \text{ K})$$

$$= (1.4)(30 \text{ K})$$

$$dT_B = 42 \text{ K}$$

21. d. Wien's displacement law is

$$\lambda_m T = b \qquad (b = \text{Wien's constant})$$

$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows:

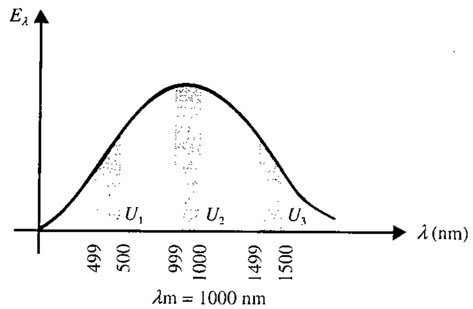

Fig. 3.30

From the graph it is clear that

$$U_2 > U_1 \qquad \text{(in fact } U_2 \text{ is maximum)}$$

22. d. The internal energy of n moles of a gas is $u = \frac{1}{2}nFRT$

where F = number of degrees of freedom.

The internal energy of 2 moles of oxygen at temperature T is

$$u_1 = \frac{1}{2} \times 2 \times 5RT = 5RT \qquad (F = 5 \text{ for oxygen molecule})$$

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Total internal energy of 4 moles of argon at temperature T is

$$= u_2 = \frac{1}{2} \times 4 \times 3RT = 6RT$$

Total internal energy $= u_1 + u_2 = 11RT$

23. d. Here $TV^{\gamma-1}$ = constant

As $\gamma = 5/3$, hence $TV^{2/3}$ = constant

Now $\quad T_1 L_1^{2/3} = T_2 L_2^{2/3} \quad (\because V \propto L)$

Hence $\quad \dfrac{T_1}{T_2} = \left(\dfrac{L_2}{L_1}\right)^{2/3}$

24. a.

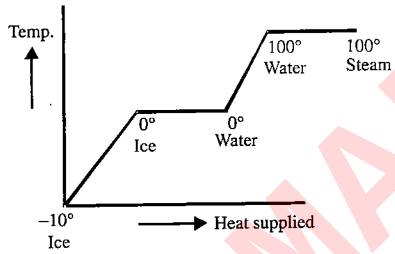

Fig. 3.31

25. c. We know that V/T = constant

$$\frac{V+\Delta V}{T+\Delta T} = \frac{V}{T} \quad \text{or} \quad VT + T\Delta V = VT + V\Delta T$$

or $\quad T\Delta V = V\Delta T \quad$ or $\quad \dfrac{\Delta V}{V\Delta T} = \dfrac{1}{T}$

This is represented by graph (c).

26. a. Work done is equal to area under the curve on PV diagram. (a) is the correct option.

27. b. According to Wien's law, λT = constant. From the graph

$$\lambda_1 < \lambda_3 < \lambda_2$$

$$\therefore \quad T_1 > T_3 > T_2$$

28. b. Let θ °C be the temperature at B. Let Q be the heat flowing per second from A to B on account of temperature difference by conductivity.

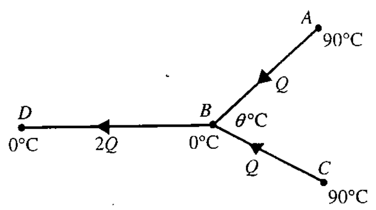

Fig. 3.32

$$\therefore \quad Q = \frac{KA(90-\theta)}{l} \qquad \text{(i)}$$

where k = thermal conductivity of the rod, A = Area of cross section of the rod, l = length of the rod. By symmetry, the same will be the case for heat flow from C to B.

$\therefore$ The heat flowing per second from B to D will be

$$2Q = \frac{KA(\theta - 0)}{l} \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$2 = \frac{\theta}{90-\theta} \quad \Rightarrow \theta = 60°\text{C}$$

29. a. From the first law of thermodynamics

$$dQ = dU + dW$$

Here $dW = 0$ (given)

$$\therefore \quad dQ = dU$$

Now since $dQ < 0$ (given)

$\therefore \quad dQ$ is negative $\Rightarrow dU = -$ve

$\Rightarrow \quad dU$ decreases $\Rightarrow$ Temperature decreases.

$\therefore$ The correct option is (a).

30. b. For adiabatic process, PV^{γ} = constant

For monatomic gas, $\gamma = \dfrac{C_p}{C_v} = 1.67$

For diatomic gas, $\quad \gamma = 1.4$

Since $\gamma_{\text{diatomic}} < \gamma_{\text{monatomic}}$, so with increase in volume, decrease in pressure will be more for monatomic gas.

$\Rightarrow$ Graph 1 is for diatomic and Graph 2 is for monatomic. Correct option is (b).

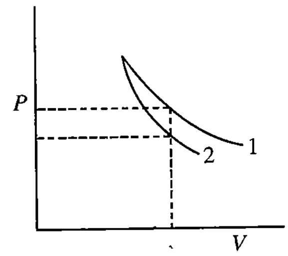

Fig. 3.33

31. a. For cyclic process,

$$Q_{\text{cyclic}} = W_{AB} + W_{BC} + W_{CA}$$

$$= 10\text{ J} + 0 + W_{CA} = 5\text{ J} \quad \Rightarrow W_{CA} = -5\text{ J}$$

32. a. PV = constant

Differentiating, $\quad \dfrac{PdV}{dP} = -V$

$$\beta = -\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right) = \left(\frac{1}{P}\right) \quad \Rightarrow \beta \times P = 1$$

Therefore, the graph between β and P will be a rectangular hyperbola. (a) is the correct option.

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33. a. When the temperature of black body becomes equal to the temperature of furnace, it will radiate maximum energy, so it will be brightest. Initially it will absorb all radiations, so it will be darkest.

34. c. The graph shows that for the same temperature difference $(T_2 - T_1)$, less time is taken for x. This means the emissivity is more for x. According to Kirchhoff's law, a good emitter is a good absorber as well.

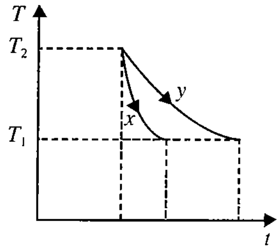

Fig. 3.34

35. c. The length of each rod increases by the same amount

$$\therefore \quad \Delta l_a = \Delta l_s \quad \Rightarrow l_1\alpha_a t = l_2\alpha_s t$$

$$\Rightarrow \frac{l_2}{l_1} = \frac{\alpha_a}{\alpha_s} \quad \Rightarrow \frac{l_2}{l_1} + 1 = \frac{\alpha_a}{\alpha_s} + 1$$

$$\Rightarrow \frac{l_2 + l_1}{l_1} = \frac{\alpha_a + \alpha_s}{\alpha_s} \quad \Rightarrow \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

36. None. Process AC cannot be adiabatic, because in adiabatic expansion, temperature decreases.

37. b. Heat required to convert 5 kg of water at 20°C to 5 kg of water at 0°C = $mC_w\Delta T = 5 \times 1 \times 20 = 100$ kcal.
Heat released by 2 kg ice at −20°C to convert 2 kg of ice at 0°C = $mC_{\text{ice}}\Delta T = 2 \times 0.5 \times 20 = 20$ kcal.
How much ice at 0°C will convert into water at 0°C for giving another 80 kcal of heat $Q = mL$
$\Rightarrow 80 = m \times 80 \Rightarrow m = 1$ kg
Therefore, the amount of water at 0°C = 5 kg + 1 kg = 6 kg. Thus, at equilibrium we have
(6 kg water at 0°C + 1 kg ice at 0°C)

38. b. We know that $\lambda_m T = c \quad \lambda_A < \lambda_B < \lambda_C$

So, $\quad T_A > T_B > T_C$

$$\left\{\because T_A = \frac{C}{3\times10^{-7}},\quad T_B = \frac{C}{4\times10^{-7}},\quad T_C = \frac{C}{5\times10^{-7}}\right\}$$

$$Q = e\sigma AT^4 \qquad (e = 1 \text{ black body})$$

$$\therefore \quad Q = \sigma AT^4$$

$$\therefore \quad Q_A = \sigma\pi(2\times10^{-2})^2 \times \frac{C^4}{27\times10^{-28}}$$

and $\quad Q_B = \sigma\pi(4\times10^{-2})^2 \times \dfrac{C^2}{64\times10^{-28}}$

$$Q_C = \sigma\pi(6\times10^{-2})^2 \times \frac{C^2}{625\times10^{-28}}$$

From comparison, Q_B is maximum.

39. c. Temperature of liquid oxygen will first increase till boiling point.
Then phase change will take place from liquid to gas during which temperature remains same. After this temperature of gaseous oxygen will further increase. All this is correctly shown by graph (c).

40. d.

$$q_1 = \frac{K2A(100)}{l}$$

$$q_2 = \frac{A(100)}{\frac{l}{k} + \frac{l}{k}} = \frac{KA(100)}{2l}$$

$$\therefore \quad \frac{q_2}{q_1} = \frac{KA(100)}{2l} \times \frac{l}{K2A(100)} = \frac{1}{4}$$

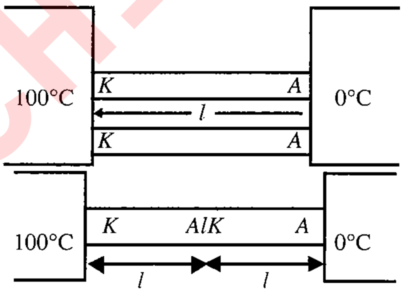

Fig. 3.35

41. b. In the first process W is +ve as ΔV is +ve, in the second process W is –ve as ΔV is –ve and area under the curve of second process is more. Therefore, the net work <0 and also $P_3 > P_1$.

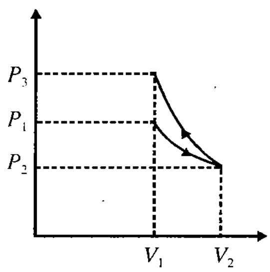

Fig. 3.36

42. a. According to Wien's displacement law

$$\lambda_m \times T = \text{constant}$$

here $\quad \lambda_{m3} < \lambda_{m2} < \lambda_{m1}$

$$\Rightarrow T_3 > T_2 > T_1$$

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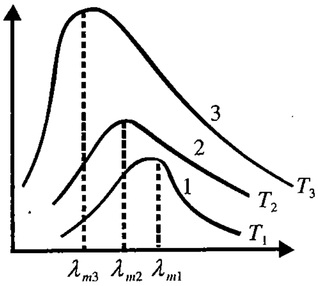

Fig. 3.37

The temperature of the sun is higher than that of the welding arc which in turn is greater than the tungsten filament. (a) is the correct option.

43. **d.** Heating of glass bulb through filament is through radiation.

44. **c.** Heat radiated at temperature T:
$Q = e\sigma AT^4 = 0.6\sigma AT^4$

45. **a.** One calorie is the heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C at 760 mm of Hg.

46. **c.** As shown in Fig. 3.38, the net heat absorbed by the water to raise its temperature = (1000 – 160) = 840 J/s. Now, the heat required to raise the temperature of water from 27°C to 77°C is $Q = mc\Delta T = 2 \times 4200 \times 50$ J. Therefore the time required

$$t = \frac{Q}{840} = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s}$$

$$= 8 \text{ min } 20 \text{ s}$$

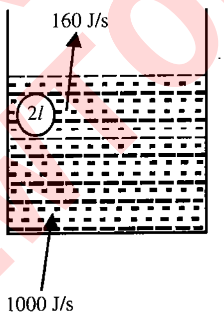

Fig. 3.38

47. **c.**

$$pT^2 = \text{constant}$$

$$\left(\frac{nRT}{V}\right)T^2 = \text{constant}$$

$$T^3V^{-1} = \text{constant}$$

Differentiating the equation, we get

$$\frac{3T^2}{V}dT - \frac{T^3}{V^2}dV = 0$$

$$3dT = \frac{T}{V}dV \qquad \text{(i)}$$

From the equation, $dV = V\gamma\, dT$

γ = coefficient of volume expansion of gas = dV/VdT.

From Eq. (i), $\gamma = \dfrac{dV}{VdT} = \dfrac{3}{T}$

48. **d.** Theory based

49. **a.** Number of moles of He = 5.6/22.4 = 1/4

Now $\quad T(5.6)^{\gamma-1} = T_2(0.7)^{\gamma-1}$

$$T_1 = T_2\left(\frac{1}{8}\right)^{2/3} \Rightarrow 4T_1 = T_2$$

$$\text{Work done} = -\frac{nR[T_2 - T_1]}{\gamma - 1} = \frac{\frac{1}{4}R[3T_1]}{\frac{2}{3}} = -\frac{9}{8}RT_1$$

Multiple Correct Answers Type

1. **a, b, c, d.** $\Delta U = nC_v\Delta T$ → for any process
In adiabatic process: $\Delta Q = 0$, so $\Delta U = -\Delta w \Rightarrow |\Delta U| = |\Delta w|$
In isothermal process, $\Delta T = 0$, so $\Delta U = 0$

2. **a, b, d.** Work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm. This is because the work done is the area under the P–V indicator diagram. As shown, the area under the graph in the first diagram will be more than that in the second diagram. When we extrapolate the graph shown in Fig. 3.39(a), let P_0 be the intercept on the P-axis and V_0 be the intercept on the V-axis. The equation of the line AB can be written as

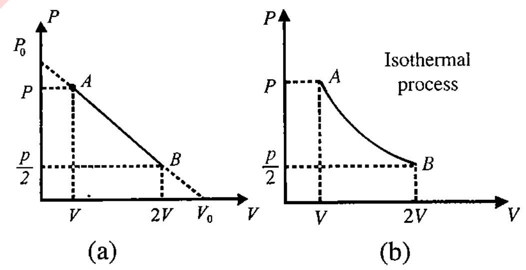

Fig. 3.39

$$P = -\frac{P_0}{V_0}V + P_0 \quad [\because y = mx + c] \qquad \text{(i)}$$

To find a relationship between P and T, we use

$$PV = RT \quad \Rightarrow V = \frac{RT}{P} \qquad \text{(ii)}$$

From Eqs. (i) and (ii),

$$P = -\frac{P_0}{V_0} \times \frac{RT}{P} + P_0$$

$$\Rightarrow P^2V_0 - PP_0V_0 = -P_0RT \qquad \text{(iii)}$$

Relation between P and T is the equation of a parabola.

Also $\qquad PV = RT$

$$\therefore \qquad P = \frac{RT}{V} \qquad \text{(iii)}$$

From Eqs. (i) and (ii),

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$$\frac{RT}{V} = -\frac{P_0}{V_0}V + P_0$$

$$\Rightarrow RT = -\frac{P_0}{V_0}V^2 + P_0V \qquad \text{(iv)}$$

The above equation is of a parabola (between T and V)

$$T = -\frac{P_0}{V_0R}V^2 + \frac{P_0}{R}V$$

Differentiating the above equation w.r.t. V we get

$$\frac{dT}{dV} = -\frac{P_0}{V_0R}\times 2V + \frac{P_0}{R}$$

when $\frac{dT}{dV} = 0,$

then $\frac{P_0}{V_0R}\times 2V = \frac{P_0}{R} \Rightarrow V = \frac{V_0}{2}$

Also $\frac{d^2T}{d^2V} = \frac{-2P_0}{V_0R} = -\text{ve}$

$\Rightarrow V = V_0/2$ is the value of maxima of temperature

Also $P_AV_A = P_BV_B \Rightarrow T_A = T_B$ (From Boyle's law)

$\Rightarrow$ In going from A to B, the temperature of the gas first increases to a maximum (at $V = V_0/2$) and the decreases and reaches back to the same value.

3. a, b. Energy emitted per second by body $A = \varepsilon_A\sigma T_A{}^4A$ where A is the surface area.

Energy emitted per second by body $B = \varepsilon_B\sigma T_B{}^4A$

Given that power radiated is equal

$$\varepsilon_A\sigma T_A{}^4A = \varepsilon_B\sigma T_B{}^4A, \quad \varepsilon_AT_A{}^4 = \varepsilon_BT_B{}^4$$

$$\Rightarrow T_B = \left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{1/4} T_A = 1934 \text{ K}$$

According to Wien's displacement law $(\lambda_m) \propto \frac{1}{T}$

Since temperature of A is more, therefore $(\lambda_m)_A$ is less

$$\therefore \quad (\lambda_m)_B - (\lambda_m)_A = 1\times 10^{-6}\text{m} \quad \text{(given)} \qquad \text{(i)}$$

Also according to Wien's displacement law

$$(\lambda_m)_AT_A = (\lambda_m)_BT_B$$

$$\Rightarrow \frac{(\lambda_m)_A}{(\lambda_m)_B} = \frac{T_B}{T_A} = \frac{1934}{5802} = \frac{1}{3} \qquad \text{(ii)}$$

On solving Eqs. (i) and (ii),

we get $\lambda_B = 1.5\times 10^{-6}$ m.

4. a, c. For 1 mole of an ideal gas

$$pV = RT \qquad \text{(i)}$$

at constant pressure:

$$PdV = RdT \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dV}{V} = \frac{dT}{T}$$

The coefficient of volume expansion at constant pressure is given by

$$\frac{dV}{VdT} = \frac{1}{T}$$

same for all gases at same temperature.

The average translational kinetic energy per molecule is $(3/2)kT$ and not $3kT$. With decrease in pressure, volume of the gas increases so its mean free path increases. [Option (c)]

The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.

5. b, c. There is a decrease in volume during melting of an ice slab at 273 K. Therefore, negative work is done by ice–water system on the atmosphere or positive work is done on the ice–water system by the atmosphere. Hence option (b) is correct. Second, heat is absorbed during melting (i.e., dQ is positive) and as we have seen, work done by ice–water system is negative (dW is negative.) Therefore, from the first law of thermodynamics, $dU = dQ - dW$, with change in internal energy of ice–water system, dU will be positive or internal energy will increase.

6. c, d. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}},\ \bar{v} = \sqrt{\frac{8}{\pi}\cdot\frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}}$

and $v_p = \sqrt{\frac{2RT}{M}}$

From these expressions we can see that

$$v_p < \bar{v} < v_{\text{rms}}$$

Second, $v_{\text{rms}} = \sqrt{\frac{3}{2}}\,v_p$

and average kinetic energy of a gas molecule

$$= \frac{1}{2}mv_{\text{rms}}^2$$

$$= \frac{1}{2}m\left(\sqrt{\frac{3}{2}}v_p\right)^2 = \frac{3}{4}mv_p^2$$

7. b, d. The expression of radius of curvature R is

$$R = \frac{d}{(\alpha_1 - \alpha_2)\Delta t}$$

Thus, $R \propto \frac{1}{\Delta t}$ and $R \propto \frac{1}{|\alpha_B - \alpha_C|}$

8. a, d. Since the sun rays fall on the black body, it will absorb radiations and since its temperature is constant, it will emit radiations. The temperature will remain same only when energy emitted is equal to energy absorbed.

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9. b, d.
For monatomic gas,

$$C_v = \frac{3}{2}R, \quad C_p = \frac{5}{2}R$$

For diatomic gas,

$$C_v = \frac{5}{2}R, \quad C_p = \frac{7}{5}R$$

10. b, d. $\Delta Q = \Delta U + W$
For process $B \to C \to D$
ΔU is negative and W is also negative, so ΔQ is also negative, hence heat flows out during this process.
A to B, work is +ve. B to C work is –ve. But +ve work is more, so net work is not zero in process $A \to B \to C$.

11. a, b. Process AB is isothermal. Temperature at A and B is same, so internal energy at A and B is same.
$\Delta W_{AB} = nRT_0 \ln(V_B/V_A)$
For point A: $P_0V_0 = nRT_0$
So $\Delta W_{AB} = P_0V_0 \ln(4V_0/V_0) = P_0V_0 \ln(4)$
Nothing can be said about pressure and temperature at C.

12. a., c., d.
a. At steady state, heat flow through A and E are same. So option (a) is correct and option (b) is incorrect.
c. $\Delta T = H \times R$, where H is heat current.
'H' is same for A and E but R is smallest for E. So temperature difference across slab E is smallest. So option (c) is correct

d. $H_B = \frac{\Delta T}{R_B}$, $H_C = \frac{\Delta T}{R_C}$ and $H_D = \frac{\Delta T}{R_D}$,
From this option if, $H_C = H_B + H_D$

$$\Rightarrow \frac{1}{R_C} = \frac{1}{R_B} + \frac{1}{R_D}$$

$$\Rightarrow \frac{4K(2Lb)}{4L} = \frac{3K(Lb)}{4L} + \frac{5K(Lb)}{4L}$$

which is true. Here b is the width of slab.
Hence option (d) is also correct.

Assertion-Reasoning Type

1. b. Total translational kinetic energy

$$= \frac{3}{2}nRT = \frac{3}{2}pV = 1.5\ pV$$

Comprehension Type

1. a. Since it is open from top, pressure will be p_0.
Therefore, option (a) is correct.

2. d. Let p be the pressure in equilibrium, then
$pA = p_0A - Mg$

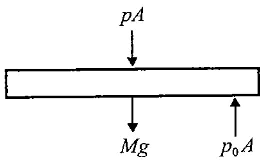

Fig. 3.40

$$p = p_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

Applying $\quad p_1V_1 = p_2V_2$

$$L' = \frac{2p_0L}{p} = \left(\frac{p_0}{p_0 - \frac{Mg}{\pi R^2}}\right)(2L)$$

$$\left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$$

Therefore, option (d) is correct.

3. c. $p_1 = p_2$

$$p_0 + \rho g(L_0 - H) = p \qquad \text{(i)}$$

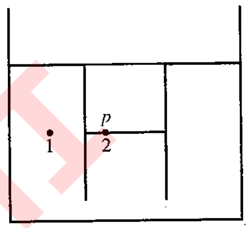

Fig. 3.41

Now, applying $p_1V_1 = p_2V_2$ for the air inside the cylinder, we have

$$p_0(L_0) = p(L_0 - H)$$

$$p = \frac{p_0L_0}{L_0 - H}$$

Substituting in Eq. (i), we have

$$p_0 + \rho g(L_0 - H) = \frac{p_0L_0}{L_0 - H}$$

$$\Rightarrow \qquad \rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$$

Therefore, option (c) is correct.

Matching Column Type

1. i → b; ii → a, d; iii → d; iv → b, c.
In process $J \to K$: V is constant whereas p is decreasing. Therefore, T should also decrease.

$$W = 0, \quad \Delta U = \text{negative} \quad \text{and} \quad Q < 0$$

In process $K \to L$: p is constant while V is increasing. Therefore, temperature should also increase.

$$W > 0, \quad \Delta U > 0 \quad \text{and} \quad Q > 0$$

In process $L \to M$: This is inverse of process $J \to K$.

$$W = 0, \quad \Delta U > 0 \quad \text{and} \quad Q > 0$$

In process $M \to J$: V is decreasing.

Therefore, $W < 0$.

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$$(pV)_J < (pV)_M$$

$$T_J < T_M$$

$$\Delta U < 0$$

Therefore, $Q < 0$.

2. **i $\longrightarrow$ d; ii $\rightarrow$ b; iii $\rightarrow$ a, b; $\rightarrow$ iv $\rightarrow$ b, c.**

Bimetallic strip is based on thermal expansion of materials.

In steam engine, internal energy of fuel (say coal) is converted into mechanical work.

In incandescent lamp and fuse also energy is converted from electrical to heat.

Fuse is based on melting of fuse wire if suddenly current increases.

3. **i $\longrightarrow$ b; ii $\rightarrow$ a, c; iii $\rightarrow$ a, d; iv $\rightarrow$ b, d.**

i. In case of free expansion under adiabatic condition, change in internal energy $\Delta U = 0$.

Therefore, internal energy and temperature will remain constant.

ii. $p \propto \dfrac{1}{V^2}$

$$\therefore \quad pV^2 = \text{constant} \qquad \text{(i)}$$

$$\left(\frac{nRT}{V}\right)\cdot V^2 = \text{constant}$$

$$T \propto \frac{1}{V} \qquad \text{(ii)}$$

If volume is doubled, temperature will decrease as per Eq. (ii).

Further, molar heat capacity in process pV^x = constant.

$$C = C_V + \frac{R}{1-x}$$

From Eq. (i), $x = 2$

$$C = \frac{3}{2}R + \frac{R}{1-2} = +\frac{R}{2}$$

Since molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative, or gas loses heat if temperature is decreasing.

iii. $p \propto \dfrac{1}{V^{4/3}}$

$$pV^{4/3} = \text{constant}$$

$$\left(\frac{nRT}{V}\right)V^{4/3} = \text{constant}$$

$$T \propto \frac{1}{V^{1/3}}$$

Further, with increase in volume, temperature will decrease.

Here, $T \propto \dfrac{1}{V^{1/3}}$

$$C = \frac{3}{2}R + \frac{R}{1-\frac{4}{3}} = -15\,R$$

As molar heat capacity is negative, Q will be positive if ΔT is negative. The gas gains heat with decrease in temperature.

iv. $T \propto pV$

In expansion from V_1 to $2V_2$, product of pV is increasing. Therefore, temperature will increase or ΔU = positive. Further, in expansion, work done is also positive.

Hence, $Q = W + \Delta U$ = positive, gas gains heat.

4. **i $\rightarrow$ a., c., e.; ii $\rightarrow$ a., c.; iii $\rightarrow$ b., d.; iv $\rightarrow$ c., e.**

i. $A \rightarrow B$: $V\downarrow$, P constant $\rightarrow T\downarrow$, $U\downarrow$ and ΔW is –ve.

ii. $B \rightarrow C$: V is same, $P\downarrow T\downarrow$

No work is done.

iii. $C \rightarrow D$: $V\uparrow \Rightarrow T\uparrow$, $\Delta U \Rightarrow$ +ve

ΔW = +ve

iv. $D \rightarrow A$: V decreases so $\Delta W \Rightarrow$ –ve

Final temperature is same so $\Delta U = 0$

and then $\Delta Q \Rightarrow$ –ve.

Integer Answer Type

1. **(9)**

$$\frac{dm_{\text{ice}}}{dt} = \frac{dm_{\text{vapour}}}{dt}$$

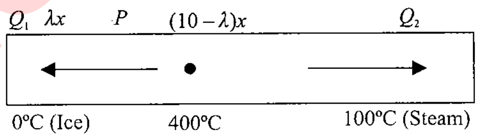

Fig. 3.42

$$Q_1 = \frac{KA400}{t} = m \times 80$$

$$Q_1 = \frac{KA(400-100)t}{(10-\lambda)x}t = m \times 540$$

Dividing both, $\lambda = 9$

2. **(9)**

$\lambda_m T$ = constant $\lambda_A T_A = \lambda_B T_B$

Now rate of total energy radiated $\propto AT^4$

3. **(4)**

$TV^{\lambda-1}$ = constant

$$TV^{7/5-1} = aT\left(\frac{V}{32}\right)^{7/5-1} \Rightarrow a = 4$$

4. **(3)**

$$\frac{F}{A} = Y\frac{\Delta L}{L} \Rightarrow \frac{mg}{A} = Y(\alpha\Delta\theta)$$

$$m = \frac{Ay\alpha(\Delta\theta)}{g} = \frac{\pi r^2 y\alpha(\Delta\theta)}{g}$$

$$= \frac{\pi(10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi \approx 3 \text{ kg}$$

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UNIT II
OSCILLATION AND WAVES

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CHAPTER
4

Linear and Angular Simple Harmonic Motion

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PERIODIC MOTION

Periodic motion is motion of an object that regularly returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motions in everyday life. Your car returns to the driveway each afternoon. You return to the dinner table each night to eat. A bumped chandelier swings back and forth, returning to the same position at a regular rate. The earth returns to the same position in its orbit around the sun each year, resulting in the variation among the four seasons.

In addition to these everyday examples, numerous other systems exhibit periodic motion. The molecules in a solid oscillate about their equilibrium positions; electromagnetic waves such as light waves, radar and radio waves are characterized by oscillating electric and magnetic field vectors; in alternating-current electrical circuit, voltage, current and electric charge vary periodically with time.

It T is the period of motion after which it repeats itself, then the frequency f of the periodic motion is the number of cycles performed in 1 s and it is given as $f = 1/T$.

Units of f are s^{-1} or per second. A special name is given to the unit of frequency, hertz (Hz) after the discoverer of radio waves.

$$1\ \text{Hz} = 1\ \text{cycle per second}$$

OSCILLATIONS

An oscillation is a spherical type of periodic motion in which a particle moves to and fro about a fixed point called mean position of particle. Oscillations are commonly seen in general life in our surroundings. As discussed, in all types of oscillations, there is always a mean position about which the particle can oscillate. This is the position where the particle is in equilibrium, that is, net force on the particle at this position is zero. If particle is displaced from the mean position, due to this displacement some forces appear on it which act on the particle in a direction directed toward its equilibrium position, these forces are called restoring forces as these forces tend the particle to move towards its equilibrium position. Due to restoring forces, particle starts moving toward the mean position and when it reaches the mean position, it gains some KE due to work done by the restoring forces and it will o vershoot from this point with some velocity in other direction; again restoring forces appear on the particle toward mean position and now the particle is retarded and will stop after travelling some distance. It will return toward the mean position and start accelerating and in such a way motion is continued which we call oscillation. The maximum displacement of particle from its mean position, where it will come to rest or from where it started with zero initial speed, is called as amplitude of oscillations.

SIMPLE HARMONIC MOTION

It is a special case of oscillatory motion in which the acceleration of the vibrating particle (or body), at any position directly as its displacement from a fixed point (which may or may not lie along the line of motion) and is always directed towards that fixed point. Thus, simple harmonic motion (abbreviated as SHM) is a case of variable acceleration; however, the variation takes place in a regular and periodic fashion.

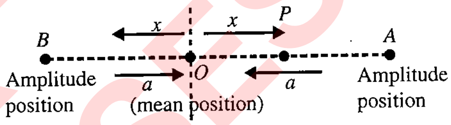

Fig. 4.1

Suppose a particle P performs oscillatory motion between two fixed points A and B. Let O be the mid-point of A and B. If the particle P oscillates about point O in such a way that its acceleration 'a' at any position when its displacement from O is x can be mathematically expressed as $a \propto x$ and is directed towards O, then its motion will be SHM. Here, the point 'O' is known as mean or stable or equilibrium or neutral position, and in case of 'simple' harmonic motion, the maximum displacement of the particle on either side of the mean position is the same, i.e., $OA = OB$

Now, taking the motion of the particle to be along the x-axis, SHM can be mathematically expressed as

$$a \propto x \quad \text{or} \quad \vec{a} = \left(-\omega^2\right)\vec{x} \tag{i}$$

Here, the negative sign stands for the fact that the direction of acceleration is opposite to that of displacement, and ω^2 is a positive constant.

Again, if m be the mass of the oscillating particle (or body), then multiplying both side of Eq. (i) by m, we have

$$m\vec{a} = \left(-m\omega^2\right)\vec{x} \quad \text{or} \quad F = (-k)\vec{x} \tag{ii}$$

where $m\vec{a} = \vec{F}$ is the force acting on the particle and $k = m\omega^2$ is a positive constant.

Eqs. (i) and (ii) can equally be used to show that a given motion is SHM.

From Eq. (ii) it is evident that at $x = 0$ and $F = 0$, which shows that the particle experiences no force at mean position. Thus, the particle in itself will not oscillate. However, if it is disturbed even slightly by external force, then new forces (restoring forces) will be set up in the system which will tend to bring the particle to its mean position. Depending upon the nature of restoring forces, we have several types of SHM. The restoring forces can be electrical, gravitational, magnetic, elastic, etc.

In this section, we discussed a special type of oscillation called simple harmonic motion. A general oscillation can be regarded as SHM if it satisfies the following basic conditions:

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i. The oscillation amplitude of the particle must be very small as compared to its surrounding dimensions (dimensions of bodies with which it can interact).

ii. During oscillation the acceleration of the particle toward mean position, due to net restoring forces, must be directly proportional to its displacement for mean position.

A FEW TERMS RELATED TO SHM

a. **Amplitude:** It is maximum displacement of the particle executing SHM from its mean position.

Thus, amplitude $= |x_{max}| = a$

b. **Time period or (period of oscillation):** We have discussed earlier that all SHMs are periodic motions which repeat themselves in equal time intervals. This minimum time interval is known as time period for the oscillations.

c. **Angular frequency:** The number of revolutions (expressed in radian) performed per unit time is known as angular frequency.

(Each oscillation corresponds to one revolution; see more in the next section.)

Therefore, in a time T (time period), no. of revolutions covered = 1

Therefore in a time of 1 s, no. of revolutions covered = $1/T$

Therefore, angle described in each revolution = 2π

Therefore, angle described in $1/T$ revolution = $\dfrac{2\pi}{T}$

Thus, angular frequency = $\dfrac{2\pi}{T} = \omega$

The number of oscillations described per unit time is known as the frequency of oscillations.

Evidently, frequency $n = \dfrac{1}{T}$.

EQUATION OF SIMPLE HARMONIC MOTION

The differential equation of simple harmonic motion is given by $d^2x/dt = \omega^2 x$.

The necessary and sufficient condition for a motion to be simple harmonic is that the net restoring force (or torque) must be linear, i.e., $F = ma = -kx$ (where k is a constant)

$$\Rightarrow \quad a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

where x is the instantaneous displacement. Multiplying both sides by dx/dt and integrating with respect to t,

$$\left[\frac{d}{dt}\left(\frac{dx}{dt}\right)\right]\frac{dx}{dt} = \left[\frac{-kx}{m}\right]\frac{dx}{dt}$$

$$\Rightarrow \quad \int v\frac{dv}{dt} = \int\frac{-k}{m}\frac{xdx}{dt} \qquad \left[\because v = \frac{dx}{dt}\right]$$

$$\frac{v^2}{2} = \frac{-k}{m}\frac{x^2}{2} + c$$

We get

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{kx^2}{2m} + C \qquad \text{(i)}$$

where C is a constant of integration. Now when x is maximum dx/dt will be zero. The maximum displacement x_{max} of the particle from the mean position is called amplitude and is represented by A, then the value of C comes out to be

$$C = \frac{k}{m}\frac{A^2}{2}$$

Here $$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \frac{k}{2m}\left(A^2 - x^2\right)$$

Putting $k/m = \omega^2$, we get

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2} \qquad \text{(ii)}$$

This equation gives the velocity of the particle in SHM

$$\frac{dx}{\sqrt{A^2 - x^2}} = \omega\, dt \qquad \text{(iii)}$$

Integrating this equation with respect to t, we get

$$\sin^{-1}\frac{x}{A} = \omega t + \phi$$

where ϕ is another constant of integration which depends on initial conditions. Thus

$$x = A\sin(\omega t + \phi) \qquad \text{(iv)}$$

Here ω is called angular frequency and ϕ is called initial phase or epoch constant, whose value depends upon initial conditions.

Graphical Representation of SHM

Consider a pen attached to a vibrating mass and a sheet of paper moved at a steady rate beneath it; the pen would trace the curve that looks sinusoidal (such as cosine or sine) as a function of time and its height is the amplitude A (Fig. 4.2).

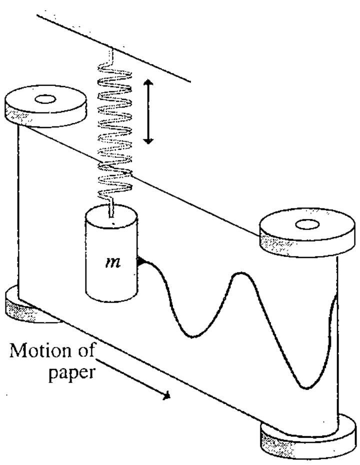

Fig. 4.2 (a)

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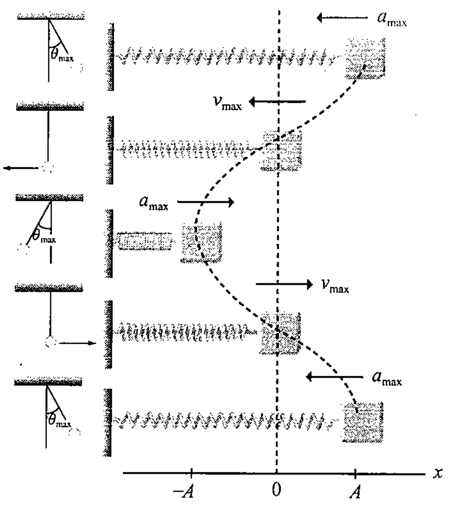

Fig. 4.2(b)

MOTION OF AN OBJECT ATTACHED TO A SPRING

As a model for simple harmonic motion, consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface. When the spring is neither stretched nor compressed, the block is at rest at the position called the equilibrium position of the system, which we identify as $x = 0$. We know from experience that such a system oscillates back and forth if disturbed from its equilibrium position.

We can understand the oscillating motion of the block in Fig. 4.3 qualitatively by first recalling that when the block is displaced to a position x, the spring exerts on the block a force that is proportional to the position and given by Hooke's law

$$F_s = -kx \quad \text{(i)}$$

We call F_s a restoring force because it is always directed toward the equilibrium position and therefore opposite the displacement of the block from equilibrium. That is, when the block is displaced to the right of $x = 0$ in Fig. 4.4(a) the position is positive and the restoring force is directed to the left. Figure 4.4(b) shows the block at $x = 0$, where the force on the block is zero. When the block is displaced to the left of $x = 0$ as in Fig. 4.4(c), the position is negative and the restoring force is directed to the right.

Applying Newton's second law to the motion of the block, with Eq. (i) providing the net force in the x direction, we obtain

$$-kx = ma_x \quad a_x = -\frac{k}{m}x = -\omega^2 x \quad \text{(ii)}$$

Here ω is a positive constant to $\omega = \sqrt{\dfrac{k}{m}}$

That is, the acceleration of the block is proportional to its position, and the direction of the acceleration is opposite the direction of the displacement of the block from equilibrium.

Systems that behave in this way are said to exhibit simple harmonic motion. An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

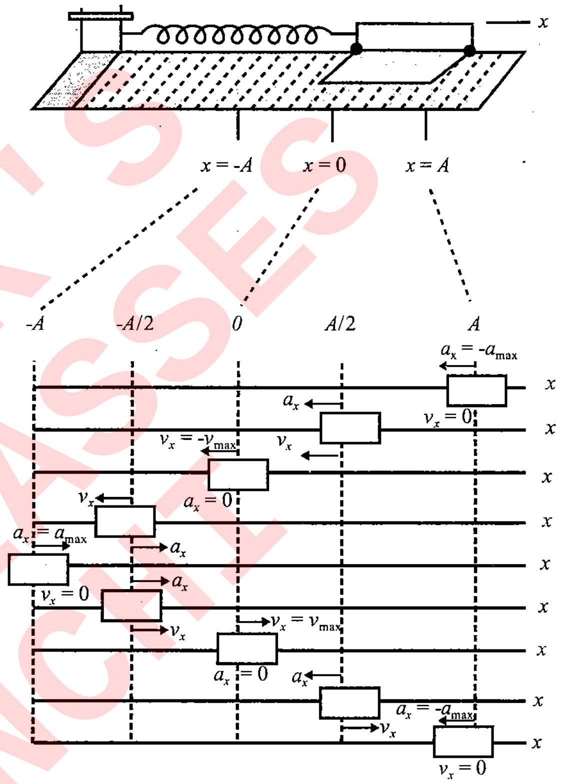

Fig. 4.3

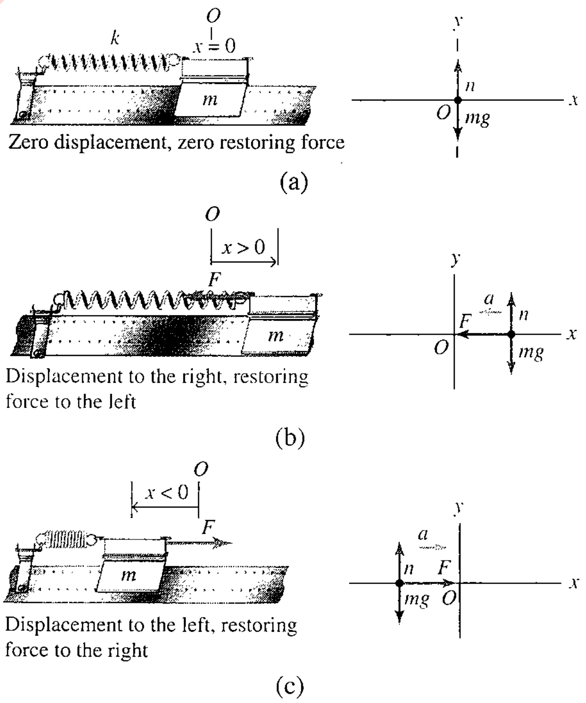

Fig. 4.4

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COMPARING SIMPLE HARMONIC MOTION WITH UNIFORM CIRCULAR MOTION

Some common devices in our everyday life exhibit a relationship between oscillatory motion and circular motion. For example, the pistons in an automobile engine go up and down—oscillatory motion—yet the net result of this motion is circular motion of the wheels. In an old-fashioned locomotive, the drive shaft goes back and forth in oscillatory motion, causing a circular motion of the wheels.

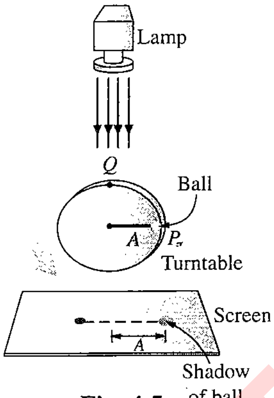

Fig. 4.5

In this section, we explore this interesting relationship between these two types of motion. Figure 4.5 is a view of an experimental arrangement that shows this relationship. A ball is attached to the rim of a turntable of radius A, which is illuminated from the side by a lamp. The ball casts a shadow on a screen. As the turntable rotates with constant angular speed, the shadow of the ball moves back and forth in simple harmonic motion.

Consider a particle located at point P on the circumference of a circle of radius A as in Fig. 4.6(a) with the line OP making an angle ϕ with the x-axis at $t = 0$; we call this circle a reference circle for comparing simple harmonic motion with uniform circular motion, and we choose the position of P at $t = 0$ as our reference position. If the particle moves along the circle with constant angular speed ω until OP makes an angle ϕ with the x-axis as in Fig. 4.6(b), at some time $t > 0$ the angle between OP and the x-axis is $\theta = \omega t + \phi$. As the particle moves along the circle, the projection of P on the x-axis, labelled as point Q, moves back and forth along the x-axis between the limits $x = \pm a$. Notice that points P and Q always have the same x-coordinate. From the right triangle OPQ, we see that this x-coordinate is

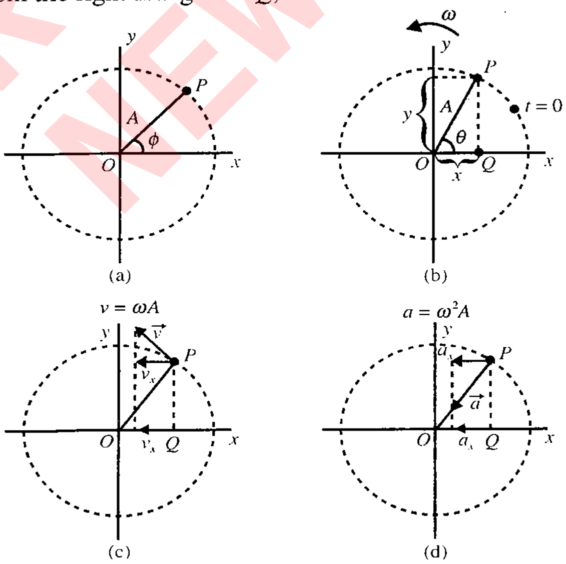

Fig. 4.6

$$x(t) = A\cos(\omega t + \phi) \qquad \text{(i)}$$

This expression is the same as Eq. (i) and shows that the point Q moves with simple harmonic motion along the x-axis. Therefore, simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.

If we take projection of point P on y-axis, the y-coordinate is given by

$$y(t) = A\sin(\omega t + \phi) \qquad \text{(ii)}$$

This geometric interpretation shows that the time interval for one complete revolution of point P on the reference circle is equal to the period of motion T for simple harmonic motion between $x = \pm a$. That is, the angular speed ω of P is the same as the angular frequency ω of simple harmonic motion along the x-axis (which is why we use the same symbol).

We may choose any position of the particle as initial position by pressing the stop watch on. It means, from any position of the particle you may start calculating time; for this, we can use $y = A\sin(\omega t + \phi)$ or $y = A\cos(\omega t + \phi)$.

If you start calculating time from extreme position use $y = A\cos\omega t$

The position V_S time graph will be

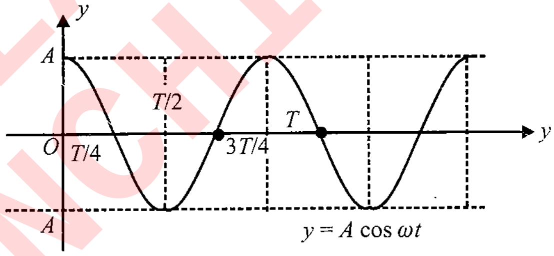

Fig. 4.7

If you start calculating time from mean (stable equilibrium) position then use $y = A\sin\omega t$.

Position V_S time graph

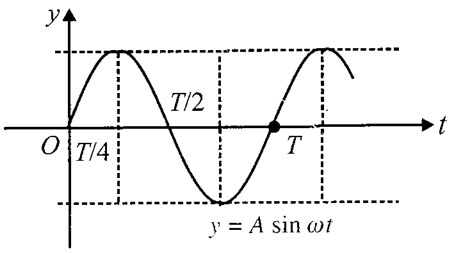

Fig. 4.8

If you choose to start calculation of time when the particle stays in any intermediate position, the variation of displacement 'y' with time can be given as 'sin' or 'cosine' function.

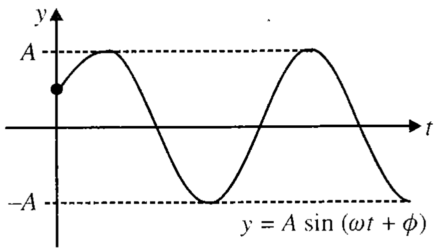

Fig. 4.9

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The displaceemnt is always measured from mean position whereas time 't' can be calculated from any position of the oscillating particle.

If we differentiate Eq. (i) or (ii) twice, we get differential equation of SHM.

Differentiating Eq. (i), we get

$$\frac{dy}{dt} = A\omega\cos(\omega t+\phi) \qquad \text{(iii)}$$

Again differentiating Eq. (iii),

$$\frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t+\phi) = -\omega^2 y \text{ i.e., } \frac{d^2y}{dt^2}+\omega^2 y = 0$$

This is the basic equation of SHM.

Illustration 4.1 **Identify which of the following functions represent simple harmonic motion.**

i. $y = Ae^{i\omega t}$ ii. $y = ae^{-\omega t}$
iii. $y = a\sin^2\omega t$ iv. $y = a\sin\omega t + b\cos\omega t$
v. $y = \sin\omega t + \cos 2\omega t$

Sol. i. According to given equation in problem, differentiating with respect to time, we get $\frac{dy}{dt} = iA\omega e^{i\omega t}$

Differentiating again with respect to time, we get

$$\frac{d^2y}{dt^2} = -\omega^2 Ae^{i\omega t} = -\omega^2 y \qquad [\text{as } y = Ae^{i\omega t}]$$

Thus we have $\frac{d^2y}{dt^2}+\omega^2 y = 0$

This is the basic differential equation of SHM.

ii. The function $y = ae^{-\omega t}$ is not harmonic as it is not expressed in terms of sine and cosine functions. So, it cannot be simple harmonic.

Moreover, this function is not periodic.

iii. $y = a\sin^2\omega t$

The function $y = a\sin^2\omega t$ is harmonic.

To become simple harmonic $\frac{d^2y}{dt^2} \propto y$

Here, $\frac{dy}{dt} = 2a\omega\sin\omega t\cos\omega t$

$$\frac{d^2y}{dt^2} = 2a\omega^2[\cos^2\omega t - \sin^2\omega t] = 2a\omega^2[1-2\sin^2\omega t]$$

$$\frac{d^2y}{dt^2} = 2a\omega^2\left[1-\frac{2y}{a}\right]$$

The function is not simple harmonic.

iv. $y = a\sin\omega t + b\cos\omega t$

The function $y = a\sin\omega t + b\cos\omega t$ is simple harmonic.

Because $\frac{dy}{dt} = \omega a\cos\omega t - \omega b\sin\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2 a\sin\omega t - \omega^2 b\cos\omega t \Rightarrow \frac{d^2y}{dt^2} = -\omega^2 y$$

This is the basic differential equation of SHM,

v. $y = \sin\omega t + \cos 2\omega t$

The function $y = \sin\omega t + \cos 2\omega t$ is not simple harmonic.

Because $\frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin 2\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2\sin\omega t - 4\omega^2\cos 2\omega t = -\omega^2[\sin\omega t + 4\cos 2\omega t]$$

$$\frac{d^2y}{dt^2} \neq -\omega^2 y$$

The function is not simple harmonic.

VELOCITY AND ACCELERATION IN SHM

We can write the equation of motion of a particle performing SHM is

$$y = A\sin(\omega t+\phi) \qquad \text{(i)}$$

Differentiatintg y with respect to 't' given velocity of the particle

$$v = \frac{dy}{dt} = A\omega\cos(\omega t+\phi)$$

From Eq. (i), we can write $\sin(\omega t+\phi) = \frac{y}{A}$

and we know that

$$\cos(\omega t+\phi) = \sqrt{1-\sin^2(\omega t+\phi)} = \sqrt{1-\frac{y^2}{A^2}} = \frac{1}{A}\sqrt{A^2-y^2}$$

Substituting the value of $\cos(\omega t+\phi)$ in Eq. (i), we get

$$v = A\omega\left[\frac{1}{A}\sqrt{A^2-y^2}\right] = \omega\sqrt{A^2-y^2} \qquad \text{(ii)}$$

Differentiating Eq. (ii) again w.r.t. time.

$$a = \frac{dv}{dt} = -A\omega^2\sin\omega t = -\omega^2 y$$

Note:

- ***For a particle performing SHM.***
- ***Velocity of a particle maximum at mean position and zero at amplitude positions.***
- ***Acceleration of a particle is zero at mean position and maximum at amplitude positions.***
- ***We have differential equation of SHM as***

$$a = -\omega^2 y \Rightarrow \frac{d^2y}{dt^2} = -\omega^2 y$$

$$v\frac{dv}{dy} = -\omega^2 y \Rightarrow v\,dv = -\omega^2 y\,dy$$

Integrating both sides $\int v\,dv = -\omega^2\int y\,dy$

$$\frac{v^2}{2} = -\omega^2\frac{y^2}{2} + \text{constant}$$

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$$v^2 + \omega^2 y^2 = \text{constant} \quad (i)$$

The above equation is also an important equation to express SHM.

Illustration 4.2 **A particle executes SHM with an angular frequency $\omega = 4\pi$ rad/s. If it is at its extreme position initially, then find the instants when it is at a distance $\sqrt{3}/2$ times its amplitude from the mean position.**

Sol. Let a be the amplitude and ϕ the phase constant for the oscillation.

The displacement equation can be written as

$$x = a \sin(4\pi t + \phi) \qquad [\because \omega = 4\pi \text{ rad/s}]$$

Since at $t = 0$; $x = +a$, therefore

$$4\pi(0) + \phi = \frac{\pi}{2} \quad \Rightarrow \phi = \frac{\pi}{2}$$

$$\therefore \quad x = a \sin(4\pi t + \pi/2) = a\cos(4\pi t)$$

Now let t_n be the instant when the particle's displacement is $\sqrt{3}a/2$, nth

Evidently $\pm\dfrac{\sqrt{3}a}{2} = a\cos(4\pi t_n)$

$$\Rightarrow 4\pi t_n = n\pi \pm \frac{\pi}{6} \quad \Rightarrow t_n = \frac{n}{4} \pm \frac{1}{24}$$

Taking only the positive values for 't',

$$t_1 = \frac{1}{24}\text{ s}, t_2 = \frac{5}{24}\text{ s}, t_3 = \frac{7}{24}\text{ s}; t_4 = \frac{11}{24}\text{ s}, t_5 = \frac{13}{24}\text{ s, etc.}$$

Illustration 4.3 **A particle executes SHM with an amplitude 8 cm and a frequency 10 s⁻¹. Assuming the particle to be at a displacement 4 cm initially, in the positive direction, determine its displacement equation and the maximum velocity and acceleration.**

Sol. Given $a = 8$ cm and $\omega = 2\pi n = 20\pi$ rad/s

Let the phase constant be ϕ.

The displacement equation can be written as

$$x = 8\sin(20\pi t + \phi)$$

Given, at $t = 0$; $x = 4$ cm, therefore

$$4 = 8\sin(20\pi(0) + \phi) \quad \Rightarrow \sin(\phi) = \frac{1}{2} \quad \Rightarrow \phi = \frac{\pi}{6}$$

$\therefore$ The displacement equation $x = 8\sin\left(20\pi t + \dfrac{\pi}{6}\right)$

Differentiating the above equation w.r.t. time 't'

$$\frac{dx}{dt} = v = 160\pi\cos\left(20\pi t + \frac{\pi}{6}\right)$$

$$\therefore \quad v_{max} = \pm 160\pi \text{ cm/s} \qquad [\text{when } \cos(20\pi t + \pi/6) = \pm 1]$$

Differentiating once again w.r.t. time t.

$$\frac{d^2x}{dt^2} = f = -3200\pi^2 \sin\left(20\pi t + \frac{\pi}{6}\right)$$

$$f_{max} = \pm 3200\pi^2$$

Illustration 4.4 **A particle executing SHM oscillates between two fixed points separated by 20 cm. If its maximum velocity be 30 cm/s. Find its velocity when its displacement is 5 cm from its mean position.**

Sol. Evidently, the maximum displacement = 20/2 = 10 cm. For a particle executing SHM, the speed at a displacement 'x' is given by

$$v = \omega\sqrt{a^2 - x^2} \qquad (i)$$

and $$v_{max} = \omega a \qquad (ii)$$

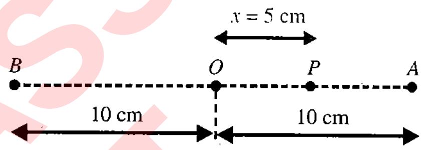

Fig. 4.10

Dividing Eq. (i) by Eq. (ii), we get $\dfrac{v}{v_{max}} = \dfrac{a}{\sqrt{a^2 - x^2}}$

$$\Rightarrow \quad \frac{v}{v_{max}} = \frac{10}{\sqrt{10^2 - 5^2}} \quad \Rightarrow \frac{10}{5\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\therefore \quad v = 30 \times \frac{2\sqrt{3}}{3} = 20\sqrt{3} \text{ cm/s}$$

Illustration 4.5 **A particle executes SHM with time period of 2 s; find the time taken by it to move from one amplitude position to half the amplitude position.**

Sol. Let 'a' be the amplitude of oscillation then evidently

$$\omega = \frac{2\pi}{T} = \pi \text{ rad/s}$$

The displacement equation can bewritten as $x = a\sin(\pi t + \phi)$ Let, for simplicity, the time be reckoned from the instant when the particle was at its extreme position.

Thus, at $t = 0$; $x = a$. Therefore,

$$a = a\sin(\phi) \quad \Rightarrow \phi = \frac{\pi}{2}$$

Let at $t = t_1$, $x = a/2$,

$$\therefore \quad \frac{a}{2} = a\sin\left(\pi t_1 + \frac{\pi}{2}\right)$$

$$\Rightarrow \left(\pi t_1 + \frac{\pi}{2}\right) = \frac{5\pi}{6} \quad \Rightarrow \pi t_1 = \frac{\pi}{3} \quad \Rightarrow t_1 = \frac{1}{3}\text{ s}$$

PHASE AND PHASE DIFFERENCE IN SIMPLE HARMONIC MOTION

Phase: We have seen earlier that the displacement, velocity and acceleration of a particle executing SHM vary periodically with the angle $(\omega t + \phi)$ associated with the

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sine or cosine term. Knowing this angle, we can be sure of its position as well as state of motion as to how and where it is oscillating. This angle $(\omega t+\phi)$ is known as the 'phase' of the oscillating particle. Since the phase is a time dependent factor, it will be more worthwhile to speak in terms of instantaneous phase (i.e., phase at any instant).

Taking the displacement equation as $x = A\sin(\omega t+\phi)$, we have

at $\omega t+\phi=0$; $x=0$ (mean position)
at $\omega t+\phi=\pi/2$; $x=A$ (amplitude position)
at $\omega t+\phi=\pi$; $x=0$ mean position
at $\omega t+\phi=3\pi/2$; $x=-A$ and so on

Notice that as time varies indefinitely, phase keeps on changing, 0 to 2π; the reason being $(\omega t+\phi)=(\omega t+\phi)+2n\pi$, where $n\in I$.

Phase constant (Epoch): The phase of a particle executing SHM, initially (i.e., at the instant when time was reckoned), is known as the initial phase or phase constant or epoch.

Since instantaneous phase $=\omega t+\phi$, so initial phase can be obtained by putting $t=0$

$\therefore$ Phase constant $=\phi$

PHASE DIFFERENCE IN TWO SIMPLE HARMONIC MOTIONS

We can easily analyse phase or phase difference using reference circle.

Figure 4.11 shows two particles P' and Q' in SHM with same angular frequency ω. P and Q are the corresponding particles in circular motion for SHM of P' and Q'. **P' and Q' are the projection of images of P and Q on y-axis.**

To visualize same phase: Let P and Q both start their circular motion at the same time $t=0$, then at the same instant P' and Q' start their motion in upward directions as shown. As the frequency of both are equal, both will reach their amplitude positions (topmost point) at the same time and will again reach their mean position simultaneously at time $t=T/2$. (T = Time period of SHM $=2\pi/\omega$) and will move in downward direction together or we can state that the oscillations of P' and Q' are exactly parallel and at every instant the phases of both P' and Q' are equal, thus phase difference in these two SHMs is zero. These SHMs are called same phase SHMs.

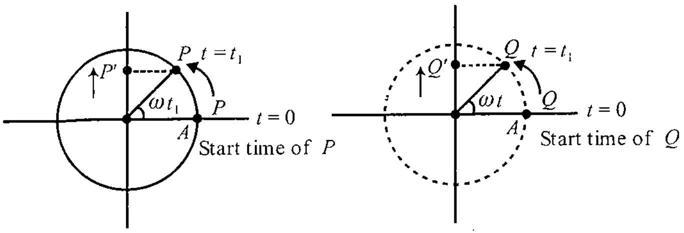

Fig. 4.11

Difference in phase: Now consider the reference circle figure where we assume if P' starts its motion at $t=0$ but Q' starts at time t_1. In this duration from $t=0$ to $t=t_1$, P' will move ahead in phase by ωt_1 radians while Q' was at rest. Now Q' starts at time t_1 and move with same angular frequency ω. It can never catch P' as both are oscillating at same angular frequency. Thus here Q' will always lag in phase by ωt_1 than P' or we say P' is leading in phase by ωt_1 than Q' and as ω of both are constant, their phase difference will also remain constant. So in two SHMs of same angular frequency, if they have same phase difference, it will always remain constant.

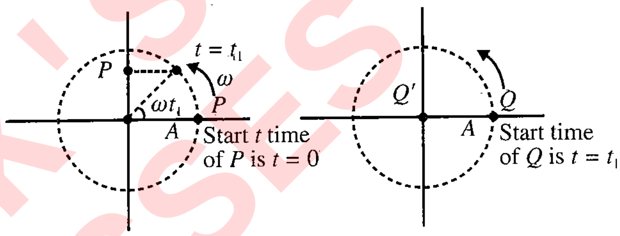

Fig. 4.12

Opposite phase: Now we consider a case when time lag between the starting of two SHMs is $T/2$, i.e., half of time period. Consider Fig. 4.10, here we assume that particle P' starts at $t=0$ and Q' at $t=T/2$ when P' completes its half oscillation. Here we can see that the phase difference in the two SHM is π by which Q' is lagging. Here when Q' starts its oscillation in upward direction, P' moves in downward direction. As angular velocity of P and Q are same, both complete their oscillation in same time. Thus when P' reaches its bottom extreme position, Q' will reach its upper extreme position and then after Q' starts moving downward, P' starts moving upward and both of these will reach their mean position simultaneously but in opposite directions; P' has completed its one oscillation where as Q' is at half of its oscillation due to a phase lag of π.

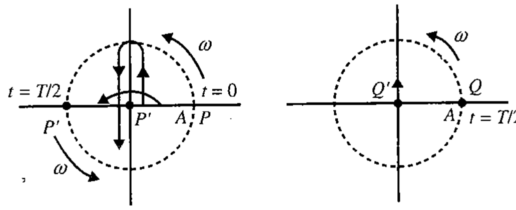

Fig. 4.13

Thus if we observe both oscillations simultaneously, we can see that oscillations of the two particles P' and Q' are exactly anti-parallel, i.e., when P' goes up, Q' comes down and at all instants of time their displacements from mean position are equal but in opposite directions, if there amplitudes are equal. Such SHMs are called opposite phase SHMs.

Relation between Phase Difference and Time Difference

Let the phase of a particle at time t_1 be ϕ_1, then

$$\phi_1=\omega t_1+\phi \qquad \text{(i)}$$

Let the phase of the particle at time t_2 changes to ϕ_2, then

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$$\phi_2 = \omega t_2 + \phi \qquad \text{(ii)}$$

Subtracting Eq. (i) from Eq. (ii), phase difference,

$$\phi_2 - \phi_1 = \omega(t_2 - t_1) \text{ or } \Delta\phi = \omega\ \Delta t = \frac{2\pi}{T}\cdot\Delta t$$

Putting $\Delta t = T$ and $\omega = 2\pi/T$ we have $\Delta\phi = 2\pi$

Thus, a phase difference of 2π is equivalent to a time difference of T.

Similarly, a phase difference of π is equivalent to a time difference of $T/2$, and so on.

Illustration 4.6 **A particle of mass m = 1 kg oscillates simple harmonically with angular frequency 1 rad/s. Find the phase of the particle at t = 1 s and 2 s. Start calculating time when the particle moves up passing through the mean position.**

Sol. We need to find $\phi = \omega t + \phi_0$, where ω (angular frequency of SHM)

Since, the particle moves up at the equilibrium position at $t = 0$, we have $\phi_0 = 0$. Then, $\phi = t$ rad.

Substituting, t = 1 s, 2 s; we have ϕ = 1 rad, 2 rad

Illustration 4.7 **Figure 4.14 shows the displacement–time graph of a particle executing SHM with a time period T. Four points 1, 2, 3 and 4 are marked on the graph where the displacement is half that of the amplitude.**

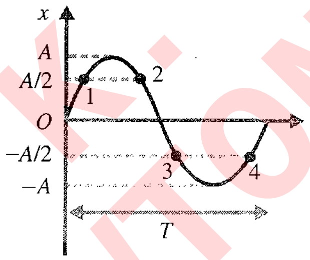

Fig. 4.14

a. **Identify the points of same displacement but with opposite direction of motion. Find the time difference between them.**
b. **Identify the points where the particles move in the same direction. Find the time difference between them.**

Sol. a. Points 1 and 2 have same displacements $x = +A/2$ with equal but opposite velocities.

Similarly points 3 and 4 have same displacements, $x = -A/2$ with equal but opposite velocities.

The phase difference between 1 and 2 and 3 and 4 is

$$\Delta\phi = \frac{2}{3}\pi$$

Therefore the time difference is

$$\Delta t = \frac{\Delta f}{\omega} = -\frac{T}{2\pi}\left(\frac{2\pi}{3}\right) = \frac{T}{3}$$

b. Points 1 and 4 have same direction of motion. Similarly, points 2 and 3 have same direction of motion.

The phase difference between 1 and 4 is

$$\Delta\phi = 2\pi - 2\left(\frac{\pi}{6}\right) = \frac{5\pi}{3}$$

Therefore, the time difference is $\Delta t = \dfrac{\Delta\phi}{\omega} = \dfrac{T}{2\pi}\left(\dfrac{5\pi}{3}\right) = \dfrac{5T}{3}$

The phase difference between 2 and 3 is $\Delta\phi = 2\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{3}$

Thus, the time difference is $\Delta t = \dfrac{\Delta\phi}{\omega} = \dfrac{T}{2\pi}\left(\dfrac{\pi}{3}\right) = \dfrac{T}{6}$

Illustration 4.8 **Two particles execute SHM with same frequency and amplitude along the same straight line. They cross each other, at a point midway between the mean and the extreme position. Find the phase difference between them.**

Sol. Method 1

Let 'a' be the amplitude and ϕ_1 and ϕ_2 their respective phases at any instant, when they cross each other.

Then their displacement equations can be written as

$$x = a \sin(\text{phase})$$

For the two particles we have $\dfrac{a}{2} = a\sin\phi_1$ and $\dfrac{a}{2} = a\sin\phi_2$

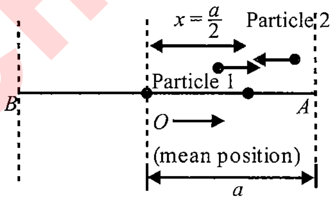

Fig. 4.15

$\Rightarrow$ Taking the least value, $\phi_1 = \dfrac{\pi}{6}$

and taking the next possible value $\phi_2 = \dfrac{5\pi}{6}$

Moreover, from $\vec{v} = a\omega\cos(\text{phase})$

If $\vec{v}_1$ and $\vec{v}_2$ be the respective velocities of the two particles. The particles cross each other their velocities should be in opposite directions. If we take velocity of the particle 1 positive then velocity of particle should be negative. The velocity of th particle 2 will be negative when it has phase $\phi_2 = 5\pi/6$.

Therefore, the required phase difference $= \phi_2 - \phi_1 \Rightarrow \dfrac{2\pi}{3}$ ad

Note: ***The phase difference can in general be***

$$\boldsymbol{\frac{2\pi}{3} + 2n\pi;\ n \in I^+}$$

Method 2

Figures 4.16(a) and (b) show that two particle P' and Q' in SHM along with their corresponding partices in circular motion. Let P' moves in upward direction whn crossing Q' at $A/2$ as shown in Fig. 4.16(a).

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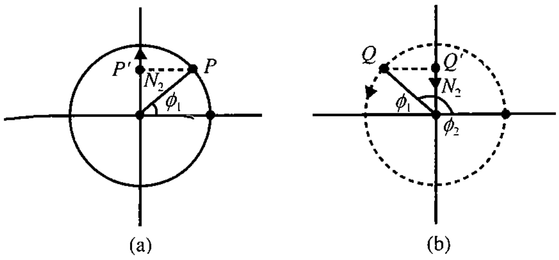

Fig. 4.16

At this instant, phase of P' is

$$\phi_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \qquad \text{(i)}$$

Similarly as shown in Fig. 4.13(b), particle Q' moves in downward direction (opposite P') at $A/2$, this implies its circular motion particle is in second quadrant thus its phase angle is

$$\phi_2 = \pi - \phi_1 = \pi - \sin^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \qquad \text{(ii)}$$

As both are oscillating at same angular frequency, their phase difference remains constant which can be given from Eqs. (i) and (ii), as $\Delta\phi = \phi_2 - \phi_1 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$

Note: ***In a particular position we have one displacement, one acceleration but two velocities, one is away from mean position other is towards mean position. It means, we have two different phases at a position. Hence the phase is not only decided by the position but it is also decided by the velocity of the particle.***

ENERGY OF A BODY IN SIMPLE HARMONIC MOTION

The total energy of a harmonic oscillator consists of two parts, potential energy (PE) and kinetic energy (KE). The former being due to its displacement from the mean position and later due to its velocity. Since the position and velocity of the harmonic oscillator are continuously changing, PE and KE also change but their sum, i.e., the total energy (TE) must have the same value at all times.

Simple harmonic motion is defined by the equation

$$F = -kx$$

The work done by the force F during the displacement from x to $x + dx$ is $dW = Fdx = -kx\,dx$.

The work done in the displacement from $x = 0$ to x is

$$W = \int_0^x (-kx)\,dx = -\frac{1}{2}kx^2$$

Let $U(x)$ be the potential energy of the system when the displacement is x. The change in potential energy corresponding to a force is negative of the work done by this force,

$$U(x) - U(0) = -W = \frac{1}{2}kx^2$$

Let us choose the potential energy to be zero when the particle is at the centre of oscillation $x = 0$. Then

$$U(0) = 0 \text{ and } U(x) = \frac{1}{2}kx^2$$

This expression for potential energy is same as that for a spring and has been used so far in this chapter.

As $\omega = \sqrt{\frac{k}{m}}$, $k = m\omega^2$

We can write

$$U(x) = \frac{1}{2}m\omega^2 x^2 \qquad \text{(i)}$$

The displacement and the velocity of a particle executing a simple harmonic motion are given by $x = A\sin(\omega t)$

and $v = A\omega\cos(\omega t)$

The potential energy at time t is, therefore,

$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t)$$

or $$U = \frac{m\omega^2 A^2}{4}(1 - \cos 2\omega t)$$

and the kinetic energy at time t is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t)$$

or $$K = \frac{m\omega^2 A^2}{4}(1 + \sin 2\omega t)$$

The total mechanical energy at time t is

$$E = U + K = \frac{1}{2}m\omega^2 A^2\left[\sin^2(\omega t) + \cos^2(\omega t)\right] = \frac{1}{2}m\omega^2 A^2 \qquad \text{(ii)}$$

We see that the total mechanical energy at time t is independent of t. Thus, the mechanical energy remains constant as expected.

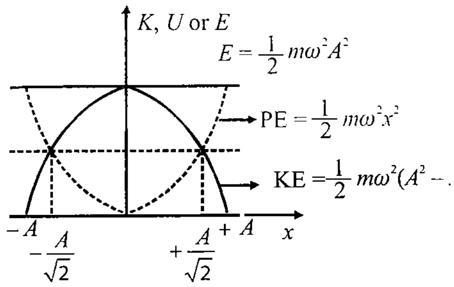

Fig. 4.17

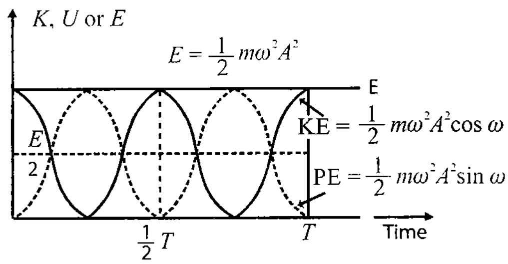

Fig. 4.18

Note:

- ***In SHM U-x graph must be parabolic.***
- ***Energy (both kinetic and potential alternates with angular frequency 2w, where w is the angular frequency of the particle.***

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Average Value of PE and KE

By Eq. (i), PE at distance x is given by

$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t+\phi)$$

{since at time t, $x = A\sin(\omega t+\phi)$}

The average value of PE of complete vibration is given by

$$U_{\text{average}} = \frac{1}{T}\int_0^T U dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t+\phi)$$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T 2\sin^2(\omega t+\phi)dt = \frac{1}{4}m\omega^2 A^2$$

because the average value of sine or of cosine function for the complete cycle is equal to zero.

Now KE at x is given by

$$\text{KE} = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left[\frac{d}{dt}\{A\sin(\omega t+\phi)\}\right]^2$$

$$= \frac{1}{2}m\omega^2 A^2\cos^2(\omega t+\phi)$$

The average value of KE for complete cycle,

$$\text{KE}_{\text{average}} = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t+\phi)dt$$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T \{1+\cos 2(\omega t+\phi)\}dt$$

$$= \frac{m\omega^2 A^2}{4T}T = \frac{1}{4}m\omega^2 A^2$$

Thus average values of KE and PE of harmonic oscillator are equal to half of the total energy.

Note:

We can write total mechanical energy of a particle/body performing SHM as
KE + PE = constant

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \textit{constant} \text{ or } v^2 + \frac{k}{m}x^2 = \textit{constant} \quad (i)$$

Here k is the equivalent force constnat of oscillation ($k = m\omega^2$)

We have differential equation of SHM as

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \textit{or} \quad v^2 = \omega^2 x^2 = \textit{constant} \quad (ii)$$

If we compare Eqs. (i) and (ii), both equations are same.

Illustration 4.9 **A particle of mass 0.50 kg executes a simple harmonic motion under a force $F = -(50\text{ N/m})x$. If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of motion.**

Sol. The kinetic energy of the particle when it is mean position, it is also its total energy as the potential enegy is zero here.

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(0.50\text{ kg})(10\text{ m/s})^2 = 25\text{ J}$$

At the maximum displacement $x = A$, the speed is zero and hence the kinetic energy is zero. The potential energy here is $1/2\,kA^2$. At this position it will also its total energy. Hence

$$\frac{1}{2}kA^2 = 25\text{ J} \quad (i)$$

The force on the particle is given by, $F = -(50\text{ N/m})x$
Thus, the spring constant is $k = 50$ N/m.

Equation (i) gives

$$\frac{1}{2}(50\text{ N/m})A^2 = 25\text{ J} \Rightarrow A = 1\text{ m}$$

Illustration 4.10 **A particle executes SHM with an amplitude of 10 cm and frequency 2 Hz. At $t = 0$, the particle is at a point where potential energy and kinetic energy are same. Find the equation of displacement of particle.**

Sol. Let $x = A\sin(\omega t+\phi)$, therefore

$$v = \frac{dx}{dt} = A\omega\cos(\omega t+\phi)$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$

$$(\text{KE})_{\max} = \frac{1}{2}mA^2\omega^2 [\text{For } \cos^2(\omega t+\phi) = 1]$$

$$\text{PE} = \frac{1}{2}mA^2 - \text{KE} = \frac{1}{2}mA^2\omega^2 - \frac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$

$$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi)$$

According to the problem, KE = PE, therefore

$$\frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi) = \frac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$

$$\tan^2(\omega t+\phi) = 1 \Rightarrow \tan^2(\omega t+\phi) = \tan^2\frac{\pi}{4}$$

$$\Rightarrow \quad \omega t+\phi = \pi/4 \quad \Rightarrow \phi = \pi/4 \ (t=0)$$

$$x = A\sin(\omega t+\phi)$$

Here, $A = 10\text{ cm} = 0.1\text{ m}$

$\omega = 2\pi f = 2\pi \times 2 = 4\pi$ rad/s

$\phi = \pi/4$

Hence, the equation of SHM is

$$x = 0.1\sin\left(4\pi t + \frac{\pi}{4}\right)$$

Illustration 4.11 **A particle of mass 0.2 kg undergoes SHM according to the equation $x(t) = 3\sin(\pi t + \pi/4)$.**

i. What is the total energy of the particle if potential energy is zero at mean position?

ii. What are the kinetic and potential energies of the particle at time $t = 1$ s?

iii. At what time instants is the particle's energy purely kinetic?

Sol. Comparing the given equation with $x(t) = A \sin(\omega t + \phi_0)$,
we get $A = 3$ m $\omega = \pi$ rad/s $\phi_0 = \pi/4$

$$T = \frac{2\pi}{\omega} = 2 \text{ s}$$

$$x(0) = 3 \sin \pi/4 = 1.5\sqrt{2} \text{ m}$$

$$dx/dt = v(t) = 3\pi \cos(\pi t + \pi/4)$$

$$\Rightarrow \quad v(0) = 3\pi/\sqrt{2} \text{ m/s}$$

i. Total energy $= \frac{1}{2}KA^2 = \frac{1}{2}m\omega^2A^2 = 1/2\,(0.2)\,(\pi)^2\,(3)^2 = 0.9\pi^2$

ii. At $t = 1$, $x(t) = 3 \sin(\pi + \pi/4) = -\frac{3}{\sqrt{2}}$ m

$$v(t) = 3\pi \cos(\pi + \pi/4) - \frac{3\pi}{\sqrt{2}} \text{ m/s}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.2)\left(\frac{9\pi^2}{2}\right) = \frac{9\pi^2}{20} \text{ J}$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

$$= \frac{1}{2}(0.2)\,(\pi)^2\left(\frac{9}{2}\right) = \frac{9}{20}\pi^2 \text{ J}$$

iii. Energy is purely kinetic at mean position, i.e., when $x = 0$

Using $x(t) = 3 \sin(\pi t + \pi/4)$, we have

$$0 = 3 \sin(\pi t + \pi/4)$$

$$\pi t + \pi/4 = 0, \pi, 2\pi, 3\pi, \ldots$$

$$\Rightarrow \quad t = \frac{3}{4}\text{ s}, \frac{7}{4}\text{ s}, \frac{11}{4}\text{ s} \ldots$$

At these time instants, particle crosses origin and hence its energy is purely kinetic.

Illustration 4.12 **A particle of mass 0.2 kg executes simple harmonic motion along a path of length 0.2 m at the rate of 600 oscillations per minute. Assume at $t = 0$ the particle starts SHM in positive direction. Find the kinetic and potential energies in joules when the displacement is $x = A/2$ where A stands for the amplitude.**

Sol. Given that $f = 600$ oscillations/min, $m = 0.2$ kg

$$A = \text{amplitude} = 1/2 \times \text{Length of path} = 0.1 \text{ m} = \frac{1}{10} \text{ m}$$

As $\quad v = \frac{1}{T} = \frac{600}{60} = 10 \text{ Hz} \Rightarrow \omega = 2\pi T = 20\pi$ rad/s

The magnitude of velocity of a particle performing SHM is given as $v = \omega\sqrt{A^2 - x^2}$.

At $x = A/2 = \frac{1}{20}$ m, the velocity at this position

$$v_{x=A/2} = 20\pi\sqrt{\left(\frac{1}{10}\right)^2 - \left(\frac{1}{20}\right)^2} = \sqrt{3}\pi \text{ m/s}$$

Hence, kinetic energy at this position

$$k_{x=A/2} = \frac{1}{2}mv^2_{x=A/2} = \frac{1}{2}(0.2)(\sqrt{3}\pi)^2 = \frac{3\pi^2}{10} \text{ J}$$

We can write potential energy at any position x as

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2$$

Hence,

$$U_{x=A/2} = \frac{1}{2}(0.2)(20\pi)^2\left(\frac{1}{20}\right)^2 = \frac{\pi^2}{10} \text{ J}$$

Illustration 4.13 **A particle of mass 0.1 kg is executing SHM with amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.**

Sol. Given that amplitude $a = 0.1$ m; $m = 0.1$ kg, $\phi = 45° = (\pi/4)$ rad, so the equation of SHM will be

$$y = 0.1 \sin[\omega t + (\pi/4)] \quad \text{(i)}$$

Now as in SHM, KE is given by

$$K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

which according to the given problem is 8×10^{-3} J for $y = 0$.

So $8 \times 10^{-3} = \frac{1}{2} \times 0.1 \times \omega^2(0.1^2 - 0^2)$, i.e.,

$$\omega = 4 \text{ rad/s} \quad \text{(ii)}$$

Substituting the value of ω from Eq. (ii) in Eq. (i), we get

$$y = 0.1 \sin[4t + (\pi/4)]$$

Illustration 4.14 **A particle of mass m is located in a unidimensional potential field where potential energy of the particle depends on the coordinates x as: $U(x) = U_0(1 - \cos Ax)$; U_0 and A are constants.**

Find the period of small oscillations that the particle performs about the equilibrium position.

Sol. The equation of potential energy of the particle as a function of position is given as

$$U_x = U_0(1 - \cos Ax) \quad \text{(i)}$$

Hence, force acting on the particle is given as

$$F_x = -\frac{dU_x}{dx} = U_0 A \sin Ax \quad \text{(ii)}$$

So, for the equilibrium of the body the force given by the above equation must be zero and d^2U/dx^2 should be negative. So, $F = 0$ situations are given by $0 = -U_0 A \sin Ax$

$$x = 0, \quad x = \frac{\pi}{A} \quad \text{(iii)}$$

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Since $\dfrac{d^2U}{dx^2} = -U_0A^2\cos Ax$

So, at $x = 0 \Rightarrow \dfrac{d^2U}{dx^2} = -U_0A^2\cos 0° = -U_0A^2$

i.e., negative

So, $x = 0$ satisfies the condition for stable equilibrium situation. Now, if the particle is given a very small displacement from $x = 0$ position, then the small force arising due to this small displacement is given as: $\Delta F = -U_0A\sin(A\Delta x)$

$$\Delta F = -U_0A^2\Delta x \qquad (\because \sin(A\Delta x) = A\Delta x)$$

If the acceleration of the particle is a, then $ma = -U_0A^2\Delta x$

$$a = -\left(\frac{U_0A^2}{m}\right)\Delta x \qquad \text{(iv)}$$

$$a \propto -\Delta x$$

So, from the above relation it is clear that the acceleration is proportional and opposite to the displacement. So the particle performs SHM. But for any SHM,

$$a = -\omega^2\Delta x \qquad \text{(v)}$$

So, from Eqs. (iv) and (v), we get

$$\omega^2 = \frac{U_0A^2}{m} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{U_0A^2}}$$

SUPERPOSITION OF TWO SIMPLE HARMONIC MOTION

In Same Direction and of Same Frequency

Let two particles are preforming SHM with same angular frequency and in same direction.

$$x_1 = A_1\sin\omega t$$

$$x_2 = A_2\sin(\omega t + \phi)$$

Then resultant displacement

$$x = x_1 + x_2 = A_1\sin\omega t + A_2\sin(\omega t + \phi) = A\sin(\omega t + \phi')$$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\phi' = \tan^{-1}\left[\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right]$

If $\phi = 0$, both SHMs are in phase and $A = A_1 + A_2$

If $\phi = \pi$, both SHMs are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHMs of this case can also be found by phasor diagram.

In Same Direction but Are of Different Frequencies

Let in this case angular frequencies are ω_1 and ω_2.

$$x_1 = A_1\sin\omega_1 t \Rightarrow x_2 = A_2\sin\omega_2 t$$

then resultant displacement

$$x = x_1 + x_2 = A_1\sin\omega_1 t + A_2\sin\omega_2 t$$

This resultant motion is not SHM.

Superposition of SHMs Along the Same Direction (Using Phasor Diagram)

If two or more SHMs are along the same line, their resultant can be obtained by vector addition by making a phasor diagram.

1. Amplitude of SHM is taken as length (magnitude) of vector.
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant of vector gives resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

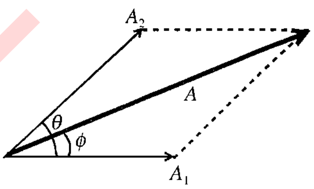

Fig. 4.19

For example, $x_1 = A_1\sin\omega t$

$x_2 = A_2\sin(\omega t + \theta)$

If equation of resultant SHM is taken as $x = A\sin(\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\tan\phi = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$

Illustration 4.15 **Find the amplitude of the simple harmonic motion obtained by combining the motions**

$x_1 = (2.0\text{ cm})\sin\omega t$ and $x_2 = (2.0\text{ cm})\sin(\omega t + \pi/3)$

Sol. The two equations given represent simple harmonic motions along the X-axis with amplitudes $A_1 = 2.0$ cm and $A_2 = 2.0$ cm. The phase difference between the two simple harmonic motions is $\pi/3$. The resultant simple harmonic motion will have an amplitude A given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

$$= \sqrt{(2.0\text{ cm})^2 + (2.0\text{ cm})^2 + 2(2.0\text{ cm})^2\cos\frac{\pi}{3}} = 3.5\text{ cm}$$

Illustration 4.16 **$x_1 = 3\sin\omega t \Rightarrow x_2 = 4\cos\omega t$ Find (i) amplitude of resultant SHM, (ii) equation of the resultant SHM.**

Sol. First, write all SHMs in terms of sine functions with positive amplitude. Keep 'ωt' with positive sign.

$\therefore \quad x_1 = 3\sin\omega t, \quad x_2 = 4\sin(\omega t + \pi/2)$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3}, \quad \phi = 53°$$

Resultant equation, $x = 5\sin(\omega t + 53°)$

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Illustration 4.17 **Two particles A and B execute simple harmonic motion according to the equations $y_1 = 3\sin \omega t$ and $y_2 = 4\sin[\omega t + (\pi/2)] + 3\sin \omega t$. Find the phase difference between them.**

Sol. Representing a quantity with phase (ωt) along the X-axis, quantity with phase $(\omega t + \phi)$ can be represented by a vector making an angle ϕ with the X-axis in the anticlockwise sense and any quantity with a phase $(\omega t - \phi)$ can be represented by a vector making an angle an ϕ with the X-axis in the clockwise sense [as shown in Fig. 4.20(b)].

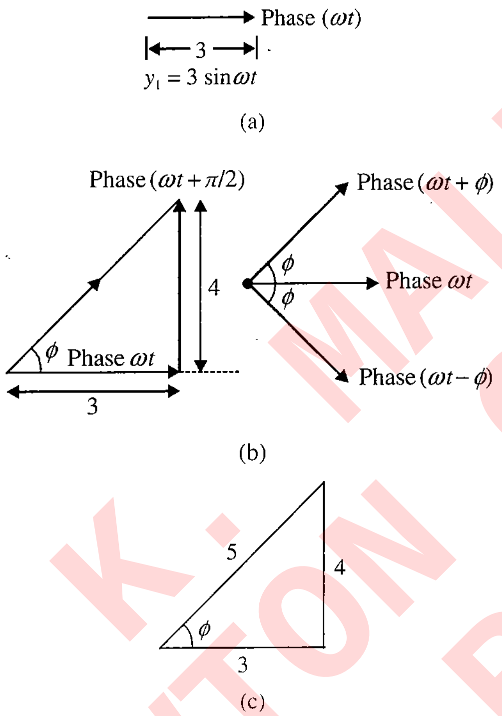


Fig. 4.20

$\therefore$ Phase difference $= \phi = \tan^{-1}(4/3)$

Alternatively, given

$$y_1 = 3\sin \omega t \qquad \text{(i)}$$

and $y_2 = 4\sin(\omega t + \pi/2) + 3\sin\omega t$

$= 4\cos\omega t + 3\sin\omega t$

$= 5\,[(4/5)\cos\omega t + (3/5)\sin\omega t]$

$= 5\,[\sin\phi\cos\omega t + \cos\phi\sin\omega t] = 5\sin(\omega t + \phi)$ (ii)

$\cos\phi = 3/5$; $\sin\phi = 4/5$ and $\tan\phi = 4/3$

From Eqs. (i) and (ii), the results follow.

Illustration 4.18 **If the displacement of a moving point at any time is given by an equation of the form $y(t) = a\cos\omega t + b\sin\omega t$, show that the motion is simple harmonic. If $a = 3$m, $b = 4$m and $\omega = 2$; determine the period, amplitude, maximum velocity and maximum acceleration.**

Sol. The particle is moving along the y-axis.

$$y(t) = a\cos\omega t + b\sin\omega t$$

$$y = \sqrt{a^2+b^2}\sin(\omega t + \phi_0)$$

where $\tan\phi_0 = a/b \Rightarrow y = \sqrt{a^2+b^2}\sin\left(\omega t + \tan^{-1} a/b\right)$

Comparing with $y = a\sin(\omega t + \phi)$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ s}$$

$$A = \sqrt{a^2+b^2} = \sqrt{3^2+4^2} = 5 \text{ m}$$

$$v_{max} = \omega A = 2\times 5 = 10 \text{ m/s}$$

$$\Rightarrow \quad a_{max} = \omega^2 A = 4\times 5 = 20 \text{ m/s}^2$$

Illustration 4.19 **If two SHMs are represented by $y_1 = 10\sin(4\pi t + \pi/2)$ and $y_2 = 5[\sin 2\pi t + \sqrt{8}\cos 2\pi t]$, compare their amplitudes.**

Sol. For the equation $y_1 = 10\sin(4\pi t + \pi/2)$, the amplitude $a_1 = 10$ units

For equation $y_2 = 5\left[\sin 2\pi t + \sqrt{8}\cos 2\pi t\right]$,

Multiplying and dividing by $\sqrt{1+8} = 3$

$$y_2 = \left[15\sin(2\pi t)\times\frac{1}{3} + \cos(2\pi t)\frac{\sqrt{8}}{3}\right]$$

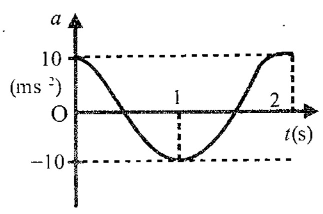
$$= 15\sin(2\pi t + \phi)$$

where $\tan\phi = \sqrt{8}$ whose amplitude a_2 is 15 units.

Therefore, the required ratio $= \dfrac{a_1}{a_2} = \dfrac{10}{15} = \dfrac{2}{3}$

Concept Application Exercise 4.1

1. i. The acceleration versus time graph of a particle executing SHM is shown in the following figure. Plot the displacement versus time graph.



(A)

Fig. 4.21

ii. The frequency of oscillation is ________.

iii. The displacement amplitude is ________.

iv. At $t = 0$, the velocity of the particle is ________.

v. The kinetic energy of the particle is maximum at $t =$ ________ and $t =$ ________.

vi. The potential energy is maximum at $t =$ ________; $t =$ ________ and $t =$ ________.

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2. A particle slides back and forth between two inclined frictionless planes.

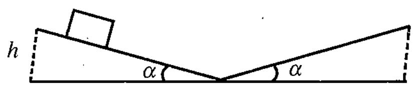

Fig. 4.22

a. If h is the initial height of the particle, the period of oscillation ________.

b. Is the motion oscillatory? Is it SHM?

3. If two SHMs are represented by equations

$y_1 = 10\sin[3\pi t + \pi/4]$ and

$y_2 = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$, the ratio of their amplitudes is ________.

4. Suppose a tunnel is dug along the diameter of earth. A particle is dropped from a point at a distance h directly above the tunnel. The motion of the particle as seen from the earth is ________.

5. The equation of motion of a particle started at $t = 0$ is given by $x = 5\sin(20t + \pi/3)$ where x is in centimetres and t is in second. When does the particle

a. first come to rest ________.

b. first have zero acceleration ________.

c. first have maximum speed ________.

6. A particle starts SHM from mean position O executing SHM. A and B are the two points at which its velocity is zero. It passes through a certain point P at time $t_1 = 0.5$ s and $t_2 = 1.5$ s with a speed of 3 m/s.

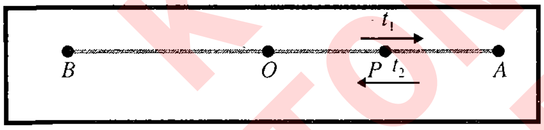

Fig. 4.23

i. The maximum speed ________.

ii. Ratio AP/PB ________.

7. If the maximum speed and acceleration of a particle executing SHM is 20 cm/s and 100π cm/s², find the time period of oscillation.

8. A particle is performing SHM of amplitude 'A' and time period 'T'. Find the time taken by the particle to go from 0 to $A/2$.

9. A particle of mass 2 kg is moving on a straight line under the action force $F = (8 - 2x)$ N. It is released at rest from $x = 6$ m.

a. Is the particle moving simple harmonically.

b. Find the equilibrium position of the particle.

c. Write the equation of motion of the particle.

d. Find the time period of SHM.

10. A particle executing simple harmonic motion has amplitude of 1 m and time period 2 s. At $t = 0$, net force on the particle is zero. Find the equation of displacement of the particle.

11. In the previous question, find maximum velocity and maximum acceleration.

12. A particle in SHM has a period of 4 s. It takes time t_1 to start from mean position and reach half the amplitude. In another case it takes a time t_2 to start from extreme position and reach half the amplitude. Find the ratio t_1/t_2.

13. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

14. A particle executes SHM of period 1.2 s and amplitude 8 cm. Find the time it takes to travel 3 cm from the positive extremity of its oscillation.

15. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to $x - y$ plane) with its axis fixed along the y-axis w.r.t. ground frame and free to rotate about this axis. The platform is given a motion in the x-direction given by $X = A\cos\omega t$. There is sufficient friction present on the surface of contact that can prevent the slipping between the cylinder and platform. What is the maximum torque, in N-m, acting on the cylinder during its motion?

[take $M = 4$ kg, $R = 1$ m, $A = 3$ m, $\omega = 1$ rad/s]

THE SPRING–MASS SYSTEM

Case 1: Spring–Mass System Oscillating on a Smooth Horizontal Surface

Method 1: Force method

Let us find out the time period of a spring–mass system oscillating on a smooth horizontal surface as shown in the figure. The SHM will be about relaxed position of spring.

At the equilibrium position the spring is relaxed. When the block is displaced through a distance x towards right, it experiences a net restoring force $F = -kx$ towards left.

The negative sign shows that the restoring force is always opposite to the displacement. That is, when x is positive, F is negative, the force is directed to the left. When x is negative, F is positive, the force always tends to restore the block to its equilibrium position $x = 0$.

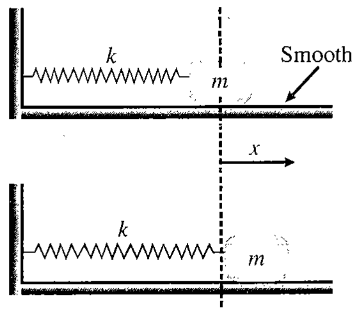

Fig. 4.24

$$F = -kx$$

Applying Newton's second law,

$$F = m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

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Comparing the above equation with $a = d^2x/dt^2 = -\omega^2 x$, we get

$$\omega^2 = \frac{k}{m} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Note: *Time period is independent of the amplitude. For a given spring constant, the period increases with the mass of the block that means more massive block oscillates more slowly. For a given block, the period decreases as k increases. A stiffer spring produces quicker oscillations.*

Method 2: Energy method

The mechanical energy of the system will always remain conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} \qquad \text{(i)}$$

Differentiating Eq. (i) w.r.t. time $\frac{1}{2}m2va + \frac{1}{2}k2xv = 0$

$$a = -\left(\frac{k}{m}\right)x \quad \Rightarrow \omega^2 = \frac{k}{m}$$

and $\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$ H

Case 2: Spring–Mass System Oscillating in a Vertical Plane

A block of mass m attached to a spring of stiffness k oscillates in a vertical plane. Let us find the period of oscillation of a vertical spring–mass system.

Method 1: Force method

Let x_0 be the deformation in the spring in equilibrium. Then $kx_0 = mg$. When the block is further displaced by x, the net restoring force is given by $F = -[k(x + x_0) - mg]$

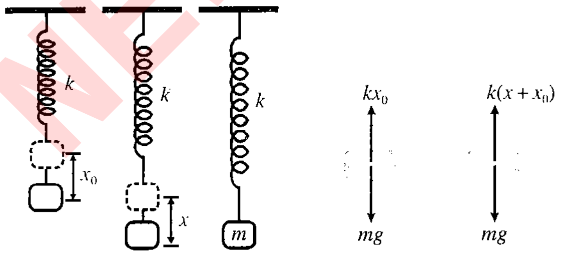


Fig. 4.25

Using second law of motion,

$$m\frac{d^2x}{dt^2} = -kx \quad \Rightarrow \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Thus, $\quad \omega^2 = \frac{k}{m} \quad \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

Note: *Gravity does not influence the time period of the spring–mass system; it merely changes the equilibrium position.*

Method 2: Energy method

The mechanical energy of the system will always remain conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x_0 + x)^2 - mgx = \text{constant} \qquad \text{(iii)}$$

(considering equilibrium position as datum)

Differentiating Eq. (iii) w.r.t. time

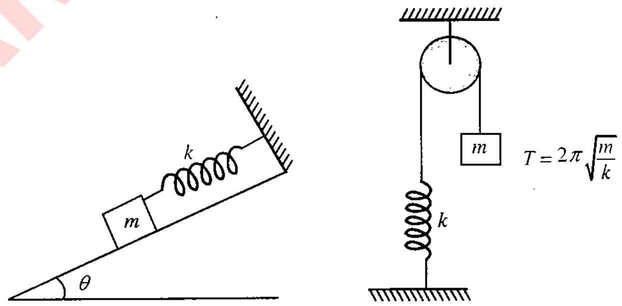
$$\frac{1}{2}m2va + \frac{1}{2}k2(x_0 + x)v - mgv = 0 \qquad \text{(iv)}$$

From Eqs. (i) and (iv), we get

$$a = -\left(\frac{k}{m}\right)x \quad \Rightarrow \omega = \sqrt{\frac{k}{m}} \quad \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Note: *As we have seen that the time period of block–spring system depends only on the mass of the block and force constant of the spring, so the time period in the following cases is*

$$T = 2\pi\sqrt{\frac{l}{g}}$$



(a) (b)

Fig. 4.26

Series and Parallel Combination of Springs

a. Series combination of springs: When two springs are joined in series, the equivalent stiffness of the combination may be obtained as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

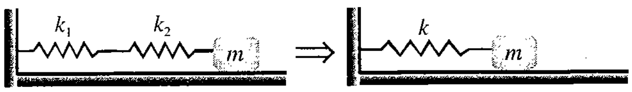


Fig. 4.27

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b. Parallel combination of springs: When two springs are joined in parallel, the equivalent stiffness of the combination is given by

$$k = k_1 + k_2$$

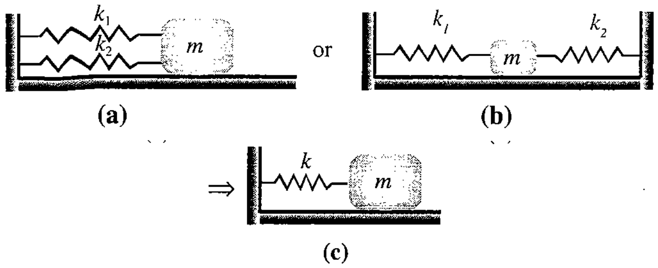

Fig. 4.28

Illustration 4.20 **A spring of stiffness constant k and natural length l is cut into two parts of length $3l/4$ and $l/4$, respectively, and an arrangement is made as shown in Fig. 4.29. If the mass is slightly displaced, find the time period of oscillation.**

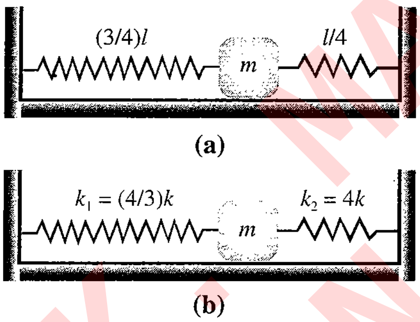

Fig. 4.29

Sol. The stiffness of a spring is inversely proportional to its length. Therefore the stiffness of each part is

$$k_1 = \frac{4}{3}k \quad \text{and} \quad k_2 = 4k$$

Time period, $$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

or $$T = 2\pi\sqrt{\frac{3m}{16k}} = \frac{\pi}{2}\sqrt{\frac{3m}{k}}$$

Illustration 4.21 **A particle of mass m placed on smooth horizontal surface is attached to three identical springs A, B and C each of force constant k as shown in Fig. 4.30. If the particle of mass m is pushed slightly against spring A and released, find the time period of oscillations.**

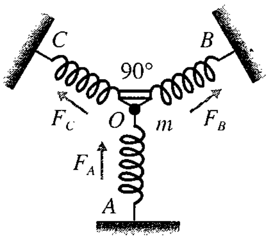

Fig. 4.30

Sol. When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y while B and C will be stretched by $y' = y\cos 45°$; so the total restoring force on mass m along AO,

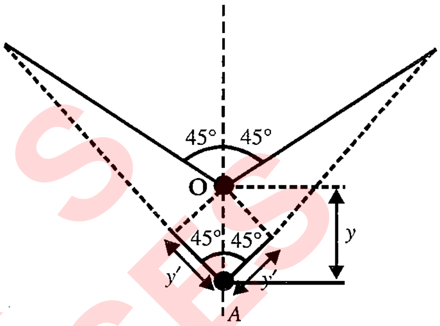

Fig. 4.31

$$RF = F_A + F_B\cos 45° + F_C\cos 45°$$

$$RF = ky + 2(ky')\cos 45°$$

$$RF = ky + 2k(y\cos 45°)\cos 45°$$

$$F = (2k)y \Rightarrow ma = -(2k)y$$

Hence $a = -\left(\dfrac{2k}{m}\right)y$ as compared with $a = -\omega^2 y$

We get $\omega^2 = \dfrac{2k}{m}$ which gives $T = 2\pi\sqrt{\dfrac{m}{2k}}$

Illustration 4.22 **Two light springs of force constants k_1 and k_2 and a block of mass m are in the line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in Fig. 4.32.**

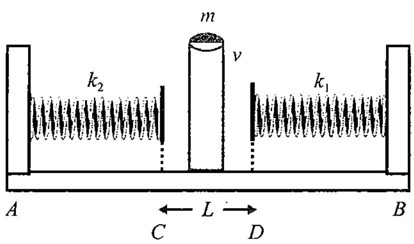

Fig. 4.32

The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/s, in between the springs, calculate the period of oscillation of the block ($k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m and $m = 200$ g). Is the motion simple harmonic?

Sol. When the block touches D, it will compress the spring and its KE will be converted into elastic energy of the spring. The compressed spring will push the block to D with same speed; so time taken by the block to move from D towards B and back to D will be

$$t_1 = \frac{T_1}{2} = \pi\sqrt{\frac{m}{k_1}} = \pi\sqrt{\frac{0.2}{1.8}} = \frac{\pi}{3}\text{ s}$$

Similarly, time t_2 taken by block in contact with spring between A and C.

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$$t_2 = \frac{T_2}{2} = \pi\sqrt{\frac{m}{k_2}} = \pi\sqrt{\frac{0.2}{3.2}} = \frac{\pi}{4}\text{ s}$$

Moreover during complete oscillation between A and B, the block moves the distance CD twice with uniform velocity v, once from C to D and again from D to C. So

$$T_3 = \frac{2L}{v} = \frac{2\times 0.6}{1.2} = 1\text{ s}$$

$$T = t_1 + t_2 + t_3 = \pi\left(\frac{1}{3}+\frac{1}{4}\right)+1 = 2.82\text{ s}$$

Now a motion is simple harmonic only and only if throughout the motion $F = -kx$. Here between C and D, $F = 0$ (as v = constant); the motion is not simple harmonic but oscillatory.

Illustration 4.23 **Figure 4.33 shows a particle mass m = 100 g, attached with four identical springs, each of length l = 10 cm. Initial tension in each spring is F_0 = 25 N. Neglecting gravity, calculate the period of small oscillations of the article along a line perpendicular to the plane of the figure.**

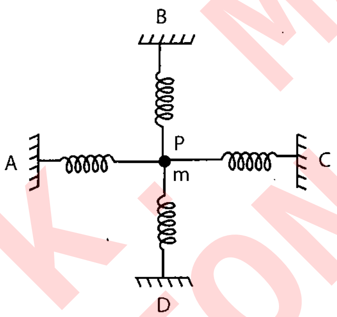

Fig. 4.33

Sol. Let the particle be displaced slightly through x along a line normal to the plane of the figure. Then each spring is further elongated. Since, springs are identical, therefore increase in tension of each spring will be the same. Let this increase be dF_0.

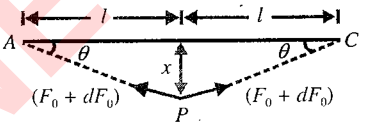

Fig. 4.34

First, consider forces exerted by springs AP and CP only as shown in Fig. 4.34.

Restoring force produced by these two springs = $(F_0 + dF_0)2\sin\theta$.

Since x is very small, therefore, $\sin\theta \approx x/l$.

Neglecting product of very small quantities, restoring force produced by these two springs $= 2F_0\, x/l$.

Similarly, restoring force produced by two remaining springs BP and DP will also be equal to $(2F_0x/l)$.

Resultant restoring force,

$$F = 2\times\left(\frac{2F_0x}{l}\right) = \frac{4F_0}{l}x$$

Restoring acceleration is directly proportional to displacement x, therefore, the particle executes SHM.

Its period $\quad T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

$$T = 2\pi\sqrt{\frac{ml}{4F_0}} = \pi\sqrt{\frac{ml}{F_0}} = 0.02\pi\text{ s}$$

Illustration 4.24 **Find the time period of oscillation of m if pulley P is light and small.**

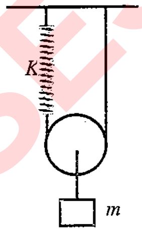

Fig. 4.35

Sol. Let the displacement of block at any time is x and stretch of spring is x', then

$$\frac{x'+0}{2} = x \quad x' = 2x$$

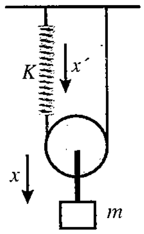

Fig. 4.36

Method I: Force method

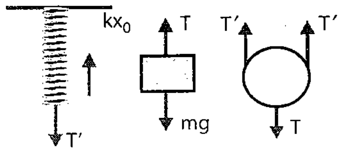

Fig. 4.37

At equilibrium

$$\text{Block,}\quad T = mg \tag{i}$$

$$\text{Pulley,}\quad 2T' = T \tag{ii}$$

$$\text{Spring,}\quad kx_0 = T'$$

Hence at equilibrium, $2Kx_0 = mg$

Let the block is further displaced by small amount and released.

Equation of motion at any time

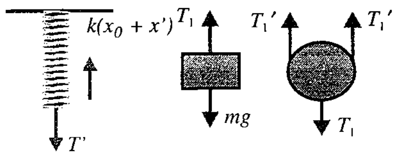

Fig. 4.38

$$T_1 - mg = ma$$

$$2T_1' - mg = ma \tag{iii}$$

$$2K(x_0 + x') - mg = ma \tag{iv}$$

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From Eqs. (iii) and (iv)

$$2Kx' = ma \quad \Rightarrow a = \frac{2kx'}{m} = \left(\frac{4k}{m}\right)x \quad \Rightarrow a = -\left(\frac{4k}{m}\right)x$$

$$\omega^2 = \frac{4k}{m} \quad \Rightarrow \omega = \sqrt{\frac{4k}{m}}$$

Method II: Energy method

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x_0 + x')^2 - mgx = \text{constant}$$

$$\frac{1}{2}m2va + \frac{1}{2}k2(x_0 + x')\frac{dx'}{dt} - mg\frac{dx}{dt} = 0$$

$$mva + k(x_0 + x')\frac{dx'}{dt} - mgv = 0$$

$$mva + k(x_0 + 2x)2v - mgv = 0$$

$$ma + 2k(x_0 + 2x) - mg = 0$$

$$ma + 4kx = 0 \qquad (\text{as } mg = 2kx_0)$$

$$a = -\left(\frac{4k}{m}\right)x \quad \Rightarrow \omega = \sqrt{\frac{4k}{m}}$$

Method III:

Net restoring force on 'm' = Net increase in tension in string connecting m and pulley = ΔT

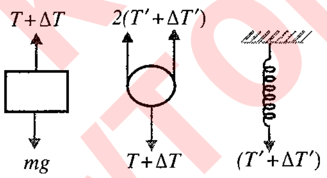

Fig. 4.39

- If increase in the tension connecting the block and the pulley is ΔT, then increase in the tension connecting spring and pulley will be $\Delta T' = \Delta T/2$.
- If x is the displacement of the block beyond equilibrium position, then aditional stretch of spring will be $x' = 2x$.

$$\Delta T = ma \Rightarrow \Delta T = 2\Delta T' = ma$$

$$ma = 2kx' = 4kx$$

$$a = -\left(\frac{4k}{m}\right)x \quad \Rightarrow \omega = \sqrt{\frac{4k}{m}}$$

Illustration 4.25 **A solid cylinder is attached to a massless spring so that it can roll without slipping along a horizontal surface. Calculate the period of oscillation made by the cylinder if m = mass of cylinder and k = spring constant.**

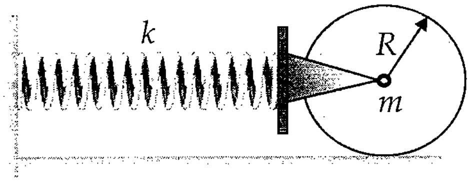

Fig. 4.40

Sol. In the mean position of the cylinder, spring will be in its original length. Let us rotate the cylinder clockwise through an angle θ. This causes a linear displacement $R\theta$ of the centre of mass of the cylinder towards right as shown. Hence the spring is elongated by $R\theta$.

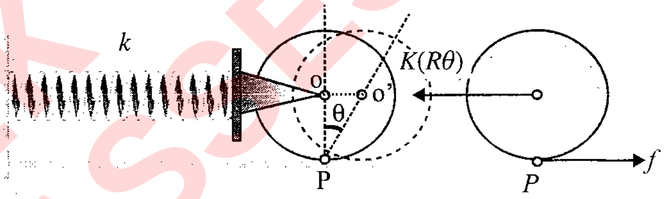

Fig. 4.41

The pull of the spring towards left creates a static friction 'f' at the point of contact towards right. The net force on the cylinder is $(kR\theta - f)$ towards the mean position.

Applying $\tau = I_{CM}\alpha$ and $F = ma$, we have

$$fR = I\alpha \quad \text{and} \quad kR\theta - f = ma$$

where a is the linear acceleration of the centre of mass and α is the angular acceleration of the cylinder.

For rolling without slipping, we have $a = R\alpha$

Eliminating f and a, we have $(kR\theta - mR\alpha)R = I_{CM}\alpha$

$$\alpha = \frac{-kR^2\theta}{I_{CM} + mR^2} \text{ [with proper vector sign]}$$

The magnitude of the angular acceleration is proportional to θ. Hence the cylinder performs angular SHM.

Comparing with $\alpha = -\omega^2\theta$, we have $T = 2\pi/\omega$

$$T = 2\pi\sqrt{\frac{I_{CM} + mR^2}{kR^2}} = \sqrt{\frac{\frac{3}{2}mR^2}{kR^2}} = 2\pi\sqrt{\frac{3m}{2k}}$$

Alternative Method 1

Taking torques about the point P which is the instantaneous centre of rotation ($v_p = 0$) for pure rolling, $\tau = (kR\theta)R = I_P\alpha$

$$kR^2\theta = (I_{CM} + MR^2)\alpha$$

$$\alpha = \frac{-kR^2\theta}{I_{CM} + mR^2} \text{ [with proper vector sign]}$$

and α (clockwise) is opposite to θ (anticlockwise)

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_{CM} + mR^2}{kR^2}} = 2\pi\sqrt{\frac{3m}{2k}}$$

Alternative Method 2

The total energy in the cylinder–spring system at any instant when v is the speed of centre of mass, ω is the angular velocity and x is the spring elongation:

$$\text{Total energy} = \frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}kx$$

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as this remains constant with time and $\omega = v/R$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}I_{\text{CM}}\frac{v^2}{R^2} + \frac{1}{2}kx^2\right) = 0$$

$$\frac{1}{2}m(2v)\frac{dv}{dt} + \frac{1}{2R^2}I_{\text{CM}}\left(2v\frac{dv}{dt}\right) + \frac{1}{2}k(2x)\frac{dx}{dt} = 0$$

Substituting $\frac{dv}{dt} = a;\quad \frac{dx}{dt} = v;\quad I_{\text{CM}} = \frac{1}{2}mR^2,$

$$mva + \frac{m}{2}va + kxv = 0 \qquad \Rightarrow a = \frac{-2k}{3m}x$$

Comparing with $a = -\omega^2 x;\quad \omega = \sqrt{\frac{2k}{3m}} \quad \Rightarrow T = 2\pi\sqrt{\frac{3m}{2k}}$

Alternative Method 3

As the cylinder is rolling, we can take motion of cylinder as pure rotation about point of contact. The total energy associated with oscillation

$$\frac{1}{2}I_P\omega^2 + \frac{1}{2}kx^2 = \text{constant} \qquad \text{(i)}$$

But $I_P = \frac{3}{2}mR^2$ and $\omega = \frac{v}{R}$

Substituting I_P and ω in Eq. (i) we get

$$\frac{1}{2}\left(\frac{3}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}kx^2 = \text{constant}$$

$$\frac{3}{4}mv^2 + \frac{1}{2}kx^2 = \text{constant} \Rightarrow v^2 + \left(\frac{2k}{3m}\right)x^2 = \text{constant}$$

Comparing with equation $v^2 + \omega^2 x = \text{constant} \Rightarrow \omega = \sqrt{\frac{2k}{3m}}$

OSCILLATION OF A TWO-PARTICLE SYSTEM TWO-BODY PROBLEM

Two blocks of masses m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a frictionless horizontal surface. Initially spring is compressed by a distance x_0 as shown in Fig. 4.20.

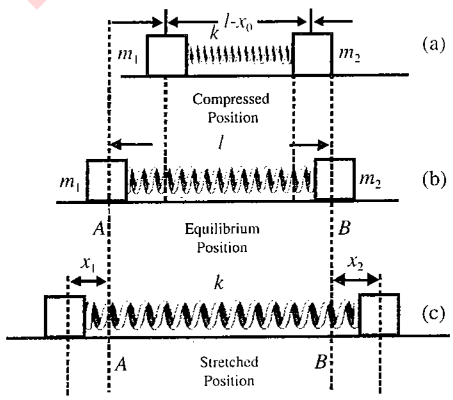


Fig. 4.42

The two blocks will perform SHM about their equilibrium position. We will discuss in this section about the oscillation of the system. If we release the blocks.

a. Time period of the blocks: Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let A and B be equilibrium positions of blocks m_1 and m_2 as shown in Fig. 4.42.

Let at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions (A and B) [Fig. 4.42(c)].

As no external force is acting on the spring–block system, the displacement of centre of mass of the system, $\Delta x_{\text{CM}} = 0$

$$\therefore \quad (m_1 + m_2)\Delta x_{\text{CM}} = m_1x_1 - m_2x_2 \Rightarrow m_1x_1 = m_2x_2$$

For first particle, force equation can be written as

$$k(x_1 + x_2) = m_1\frac{d^2x_1}{dt^2} \quad \text{or,} \quad k(x_1\frac{m_1}{m_2} + x_1) = m_1a_1$$

or, $$a_1 = \frac{k(m_1+m_2)}{m_1m_2}x_1 \qquad \because \omega^2 = \frac{k(m_1+m_2)}{m_1m_2}$$

Using vector we can write $\vec{a}_1 = -\frac{k(m_1+m_2)}{m_1m_2}\vec{x}_1$

'–' sign is included as $\vec{a}_1$ and $\vec{x}_1$ are opposite to each other.

Hence, $T = 2\pi\sqrt{\frac{m_1m_2}{k(m_1+m_2)}} = 2\pi\sqrt{\frac{\mu}{K}}$

where $\mu = m_1m_2/(m_1+m_2)$ which is known as reduced mass.

Similarly time period of 2nd particle can be found. Both will have the same time period.

b. Amplitude of the particles: Let the amplitude of blocks be A_1 and A_2.

$$m_1A_1 = m_2A_2$$

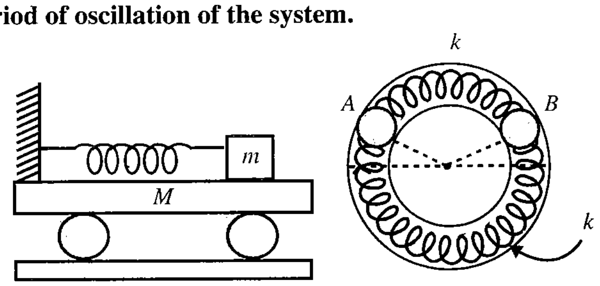
By energy conservation, $\frac{1}{2}k(A_1 + A_2)^2 = \frac{1}{2}kx_0^2$

or, $A_1 + A_2 = x_0$ or, $A_1 + \frac{m_1}{m_2}aA_1 = x_0$

or, $$A_1 = \frac{m_2x_0}{m_1+m_2}$$

Similarly, $$A_2 = \frac{m_1x_0}{m_1+m_2}$$

Illustration 4.26 **Figures 4.43(a) and (b) represent spring–block system. If m is displaced slightly, find the time period of oscillation of the system.**



(a) **(b)**

Fig. 4.43

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Sol. Both the cases are as follows:

Reduced mass of the system **Reduced mass of the system**

$$\mu = \left(\frac{mM}{m+m}\right) \qquad \mu = \frac{mm}{m+m} = \frac{m}{2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\mu}{k}} \qquad \text{and} \quad k_e = k + k = 2k$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\mu}{k_e}}$$

ANALYSIS OF ANGULAR SHM

In case of a simple pendulum a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support.

When the particle is at its extreme position, its angular displacement θ_0 can be regarded as the angular amplitude of oscillations. As the displacement of bob from its mean position is given as

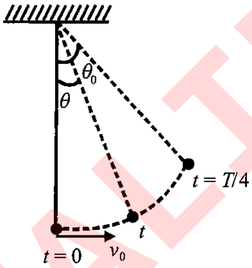

Fig. 4.44

$$x = A\sin(\omega t + \alpha) \quad \text{[general equation of SHM]} \qquad \text{(i)}$$

If θ and θ_0 are angular displacement and angular amplitude of bob, respectively, we have $\theta = \frac{x}{l}$ and $\theta_0 = \frac{A}{l}$

Thus general equation of SHM of bob in angular form can be given by substituting the values of x and in Eq. (i) as

$$\theta = \theta_0 \sin(\omega t + \phi_0) \qquad \text{(ii)}$$

Using the above equation we can find the angular velocity of the body which in angular SHM is

$$\dot{\theta} = \frac{d\theta}{dt} = \theta_0\omega\cos(\omega t + \phi_0) \qquad \text{(iii)}$$

or

$$\dot{\theta} = \omega\sqrt{\theta_0^2 - \theta^2} \qquad \text{(iv)}$$

Note: ***Here we represent angular velocity $\frac{d\theta}{dt}$ by $\dot{\theta}$, not ω, as the notation ω is already being used for angular frequency of body in SHM.***

Similarly angular acceleration of body is given as

$$\alpha = \frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t + \phi_0) \qquad \text{(v)}$$

$$\tau_R = -I\alpha = -I\omega^2\theta \qquad \text{(vi)}$$

Thus restoring torque on body is given as

$$\tau_R = -I\ddot{\alpha} = -I\omega^2\theta \qquad \text{(vii)}$$

Thus we can state that in angular SHM, angular acceleration of the body and the restoring torque on the body are directly proportional to the angular displacement of body from its mean position and are directed toward mean position. Similarly basic differential equation for angular SHM can be written as

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \text{(viii)}$$

In terms of angular velocity and angular displacement we can write above equation as $(\dot{\theta})^2 + \omega^2(\theta)^2 = \text{const.}$

$\dot{\theta}$ = angular velocity

θ = angular displacement

Oscillation of Simple Pendulum

As Angular SHM

Length of the simple pendulum is the distance between the point of suspension and the centre of mass of the suspended mass. Consider the bob when string deflects through a small angle θ.

Forces acting on the bob are tension T in the string and weight mg of the bob.

Torque on the bob about point O is

$$\tau = \tau_{mg} + \tau_T = mgl\sin\theta + 0$$

$$\tau = mgl\theta \quad (\text{as } \theta \text{ is very small}) \qquad \text{(i)}$$

Since MI of the bob about point O is $I = ml^2$,

$$\tau = ml^2\frac{d^2\theta}{dt^2} \qquad \text{(ii)}$$

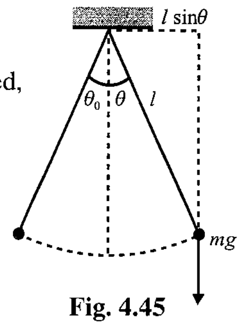

Fig. 4.45

As torque τ and θ are oppositely directed, hence from Eqs. (i) and (ii), we get

$$ml^2\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\frac{d^2\theta}{dt^2} = -(g/l)\theta$$

Comparing with the equation $d^2x/dt^2 = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{g}{l}}$$

Since $T = 2\pi/\omega \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

As Linear SHM

At equilibrium position the bob of pendulum is at lowest position, i.e., the string is in vertical position.

Let us pull the bob from its mean (lowest) position O, through a small distance x.

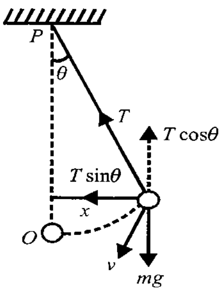

Fig. 4.46

The pendulum bob oscillates simple harmonically experiencing a restoring force $mg\sin\theta$ with small linear and angular displacements.

Referring to the FBD (Fig. 4.45), the net horizontal force acting on the bob is $F_{net} = T\sin\theta$.

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Since θ is very small, $\sin\theta = \theta$; $\theta \approx \frac{x}{\ell}$.

Then, we have $F_{net} = \frac{T}{l}x$.

As the net force, that is, restoring force acts (pushes the bob) towards the mean position, it opposes the displacement x from the mean position. Then, we can write vectorially as

$$\vec{F}_{net} = -\left(\frac{T}{\ell}\right)\vec{x}$$

Since, $T = mv^2/l + mg\cos\theta$ neglecting v^2/l for small speed of the bob (as it falls from a very small angle) and substituting $\cos\theta = 1$, we have

Equivalent force constant (k_{eq}) for simple pendulum: Finally substituting $T = mg$ from Eq. (ii) in Eq. (i), we have

$$\vec{F}_{net} = -\left(\frac{T}{\ell}\right)x = \frac{-mg}{\ell}x$$

Frequency and time period: Comparing the above equation with $F_{net} = -kx$, the effective spring constant is $k_{eff} = mg/l$.

As we know $\quad k_{eq} = m\omega^2$

This gives $\quad \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{g}{l}}$

Then the time period of the simple pendulum is $T = 2\pi/\omega$ where $\omega = \sqrt{g/l}$

This gives $\quad T = 2\pi\sqrt{\frac{l}{g}}$

Note:

- ***In simple pendulum we can find the value of equivalent force constant of oscillation $k_{eq} = T/\ell$; where T is the tension in the string at equilibrium position of the bob and l is the length of the string.***
- ***Even in case of simple pendulum in accelerated frame we can use the same approach for calculating the equivalent force of oscillation. In this case T is the tension in the string at equilibrium position with respect to accelerated frame***

Time Period of Simple Pendulum in Accelerating Reference Frame: Time Period of Simple Pendulum When Point of Suspension Is Accelerating

1. **Acceleration up or retardation down**

from free body diagram of bob

$T - mg = ma$, i.e., $T = m(g + a)$

$$k_{eq} = \frac{T}{\ell} = \frac{m(g+a)}{\ell}$$

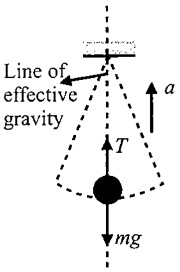

Fig. 4.47

Hence $\quad \omega = \sqrt{\frac{k_{eq}}{m}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{(g+a)}}$

2. **Acceleration down or retardation up**

$$K_{eq} = \frac{T}{\ell} = \frac{m(g-a)}{\ell}$$

$$\omega = \sqrt{\frac{k_{eq}}{m}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{(g-a)}}$$

3. **Pendulum is accelerating horizontally**

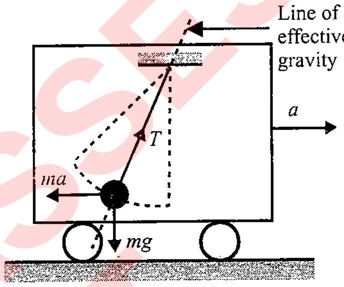

Fig. 4.48

$$k_q = \frac{T}{\ell} = \frac{m\sqrt{a^2+g^2}}{\ell}$$

$$\omega = \sqrt{\frac{k_q}{m}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g^2+a^2}}$$

4. **Pendulum accelerating down an incline**

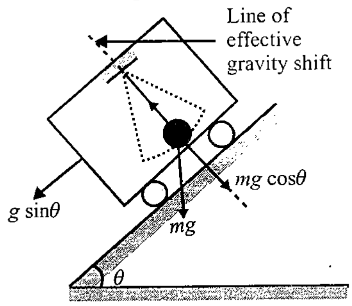

Fig. 4.49

$$T = m\sqrt{g^2 + (g\sin\theta)^2 + 2g(g\sin\theta)\cos(90° + \theta)}$$

$$= mg\cos\theta; \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$

$$k_{eq} = \frac{T}{\ell} = \frac{mg\cos\theta}{\ell}$$

Illustration 4.27 **A simple pendulum of length 40 cm oscillates with an angular amplitude of 0.04 rad. Find (a) the time period, (b) the linear amplitude of the bob, (c) the speed of the bob when the string makes 0.02 rad with the vertical and (d) the angular acceleration when the bob is in momentary rest. Take $g = 10$ m/s².**

Sol. a. The angular frequency is $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10\ \text{m/s}^2}{0.4\ \text{m}}} = 5\ \text{s}^{-1}$

The time period is $\frac{2\pi}{\omega} = \frac{2\pi}{5\,s^{-1}} = 1.26$ s

b. Linear amplitude = 40 cm × 0.04 = 1.6 cm

c. Angular speed $\dot{\theta} = \omega\sqrt{\theta_0^2 - \theta^2}$

$$\dot{\theta} = (5\,s^{-1})\sqrt{(0.04)^2 - (0.02)^2}\ \text{rad} = 0.17\ \text{rad/s}$$

where speed of the bob at this instant

$$V = \dot{\theta}A = (40\ \text{cm}) \times (0.17\,\text{s})$$

$$= 6.8\ \text{cm/s}$$

d. At momentary rest, the bob is in extreme position.
Thus, the angular acceleration $|\alpha| = \omega^2\theta$.

$$|\alpha| = (0.04\ \text{rad})\,(25\ \text{s}^{-2}) = 1\ \text{rad/s}^2$$

Illustration 4.28 **A ball is suspended by a thread of length l at the point O on an incline wall as shown. The inclination of the wall with the vertical is (a) the thread is displaced through a small angle α away from the vertical and (b) the ball is released. Find the period of oscillation of pendulum. Consider both cases.**

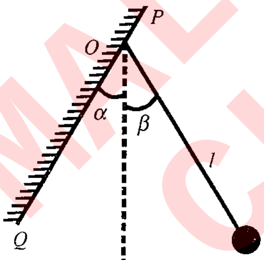

Fig. 4.50

a. $\alpha > \beta$
b. $\alpha < \beta$

Assume that any impact between the wall and the ball is elastic.

Sol. a. If $\alpha > \beta$, the ball does not collide with the wall and it performs full oscillations like simple pendulum.

$$\Rightarrow \text{period} = 2\pi\sqrt{\frac{l}{g}}$$

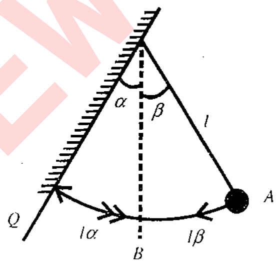

Fig. 4.51

b. If $\alpha < \beta$, the ball collides with the wall and rebounds with same speed; the motion of ball from A to Q is one part of a simple pendulum time period of ball = 2 (t_{AQ}).

Consider A as the starting point ($t = 0$), equation of motion is $x(t) = A\cos\omega t$.

$x(t) = l\beta\cos\omega t$, because amplitude = $A = l\beta$.

Time from A to Q is the time t when x becomes $-l\alpha$.

$$\Rightarrow -l\alpha = l\beta\cos\omega t \quad \Rightarrow t = t_{AQ} = 1/\omega\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

The return path from Q to A will involve the same time interval.

Hence time period of ball = $2t_{AQ}$

$$= \frac{2}{\omega}\cos^{-1}\left(-\frac{\alpha}{\beta}\right) = 2\sqrt{\frac{l}{g}}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

$$= 2\pi\sqrt{\frac{l}{g}} - 2\sqrt{\frac{l}{g}}\cos^{-1}\left(\frac{\alpha}{\beta}\right)$$

Pendulum of Large Length but Small Amplitude

Consider a pendulum of length of the string l which is not negligible compared with the radius R of earth and assume that the bob is oscillating simple harmonically very close to earth's surface. Let us derive an expression for the angular frequency of oscillation.

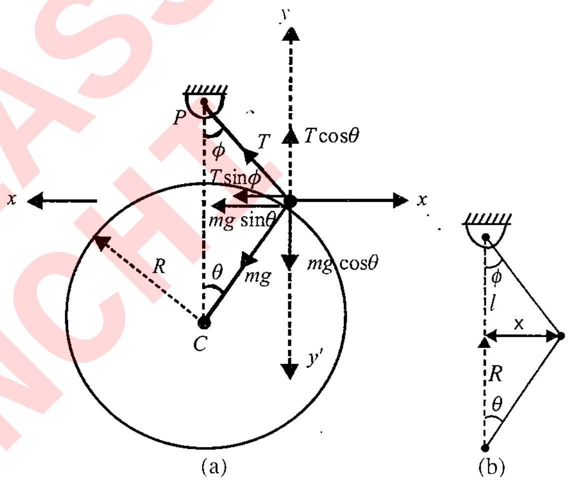

Fig. 4.52

At any angular position ϕ of the string of the simple pendulum, relative to the point of suspension P, the net force acting on the bob in x-direction is

$$F_{\text{net}} = T\sin\phi + mg\sin\theta \qquad \text{(i)}$$

The net force acting on the bob in y-direction is

$$F_y = T\cos\phi - mg\cos\theta \qquad \text{(ii)}$$

Since, the bob has a small speed and θ and ϕ are very small, we can neglect its acceleration in y-direction. That gives $T\cos\phi - mg\cos\theta = 0$. Hence, we have $T = mg$.

Substituting $T = mg$ in Eq. (i) and writing $\sin\phi = \phi$ and $\sin\theta = \theta$ for small angle ϕ and θ, we have

$$F_{\text{net}} = mg(\phi + \theta)$$

where $\phi = \frac{x}{l}$ and $\theta = \frac{R}{l}$

This gives $F_{\text{net}} = mg\frac{x}{l} + mg\frac{x}{R} = mg\left(\frac{1}{l} + \frac{1}{R}\right)x$

The direction of F_{net} opposes the displacement x.

Then, $k_{\text{eff}} = \frac{F_{\text{net}}}{x} = mg\left(\frac{1}{\ell} + \frac{1}{R}\right)$

Substituting $k_{eff} = mg(1/l + 1/R)$ in the formula $\omega = \sqrt{k_{eff}/m}$,

we have $\omega = \sqrt{g[(1/l) + (1/R)]}$

Hence $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{1}{g\left[\dfrac{1}{\ell} + \dfrac{1}{R}\right]}}$

Special cases

i. For $l = R$, $T = 2\pi\sqrt{R/2g}$

On substituting the value of R and g, we get $T = 59.8$ min

ii. For $l \longrightarrow \infty$ $\dfrac{R}{l} \longrightarrow 0$,

Therefore $T = 2\pi\sqrt{\dfrac{R}{g}} = 84.6$ min

Simple Pendulum in a Liquid

If we immerse a simple pendulum in a liquid, the bob of the pendulum will experience buoyant force in upward direction in addition to the other forces such as gravity and tension. To find the k_{eff}, we need to find the net force on the bob near (or at) the equilibrium position. Dividing F_{net} by the small displacement x of the bob from the equilibrium position, we can find $k_{eff}\left(= \dfrac{F_{net}}{x} = \dfrac{T}{\ell}\right)$. After finding k_{eff}, substitute k_{eff} in the formula $\omega = \sqrt{k_{eff}/m}$ to find frequency (or time period). We will explain the above idea through the following illustration.

Illustration 4.29 **Derive an expression for the angular frequency of small oscillation of the bob of a simple pendulum when it is immersed in a liquid of density ρ. Assume the density of the bob as σ and length of the string as l.**

Sol. Method 1

The bob of the pendulum experiences buoyant force due to liquid $= V\rho g$, in addition to gravitational force. Thus net force on the bob $= (mg - V\rho g)$

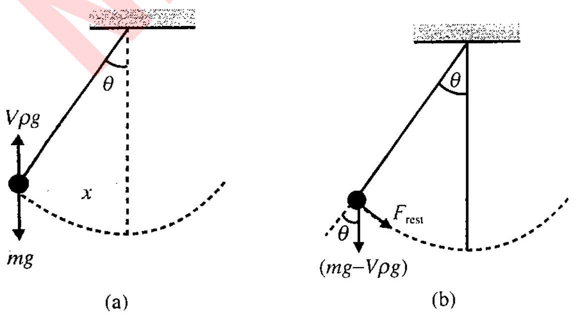

Fig. 4.53

For small displacement x of the bob, restoring force

$$F_{rest} = (mg - V\rho g)\sin\theta = -(mg - V\rho g)\frac{x}{l}$$

and acceleration $= -\left(g - \dfrac{V\rho g}{m}\right)\dfrac{x}{l}$

On comparing with standard equation of SHM, $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{\left(g - \dfrac{V\rho g}{m}\right)}{l}} = \sqrt{\frac{g}{l}\left(1 - \frac{\rho}{\sigma}\right)}$$

and $T = 2\pi\sqrt{\dfrac{\ell}{g\left(1 - \dfrac{\rho}{\sigma}\right)}}$

Method 2

At equilibrium position the pendulum should be vertical. Free body diagram of the bob at equilibrium position

$$T + B = mg$$

$$T = mg - B = V\sigma g - V\rho g = Vg(\sigma - \rho)$$

Equivalent force constnat of oscillation

$$k_{eq} = \frac{T}{\ell} = \frac{Vg(\sigma - \rho)}{\ell}$$

But we know $k_{eq} = m\omega^2$

i.e., $\omega = \sqrt{\dfrac{k_{eq}}{m}} = \sqrt{\dfrac{Vg(\sigma - \rho)}{V\sigma \cdot \ell}} = \sqrt{\dfrac{g}{\ell}\left(1 - \dfrac{\rho}{\sigma}\right)}$

Hence $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\ell}{g\left(1 - \dfrac{\rho}{\sigma}\right)}}$

PHYSICAL PENDULUM (COMPOUND PENDULUM)

Any rigid body suspended from a fixed support constitutes a physical pendulum. Consider the situation when a body is displaced through a small angle θ. Torque on the body about O is given by

$$\tau = mgl\sin\theta \quad \text{(i)}$$

where l = distance between point of suspension and centre of mass of the body.

If I be the MI of the body about O, then

$$\tau = I\alpha \quad \text{(ii)}$$

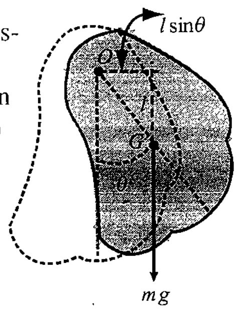

Fig. 4.54

From Eqs. (i) and (ii), we get

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

as θ and $d^2\theta/dt^2$ are oppositely directed.

$\Rightarrow$ $\dfrac{d^2\theta}{dt^2} = \dfrac{-mgl}{I}\theta$, since θ is very small.

Comparing with the equation $d^2\theta/dt^2 = -\omega^2\theta$, we get

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$$\omega = \sqrt{\frac{mgl}{I}} \qquad \Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

Illustration 4.30 **What is the period of a pendulum formed by pivoting a metre stick so that it is free to rotate about a horizontal axis passing through the 75 cm mark?**

Sol. Let m be the mass and l be the length of the stick.

$l = 100$ cm

The distance of the point of suspension from centre of gravity is $d = 25$ cm

Moment of inertia about a horizontal axis through O is

$$I = I_C + md^2 \quad \Rightarrow I = \frac{ml^2}{12} + md^2$$

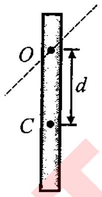

Fig. 4.55

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{m\ell^2}{12} + md^2}{mgd}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l^2 + 12d^2}{12\,gd}} = 2\pi\sqrt{\frac{\ell^2 + 12(0.25)^2}{12 \times 9.8 \times 0.25}} = 153\,s$$

TORSIONAL PENDULUM

Figure 4.56 shows a rigid object suspended by a wire attached at the top to a fixed support. When the object is twisted through some angle θ, the twisted wire exerts on the object a restoring torque that is proportional to the angular position. That is, $\tau = -k\theta$, where k (Greek letter kappa) is called the torsion constant of the support wire. The value of k can be obtained by applying a known torque to twist the wire through a measureable angle θ. Applying Newton's second law for rotational motion, we find that

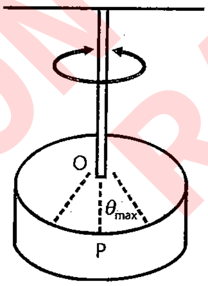

Fig. 4.56

$$\tau = -k\theta = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{I}\theta$$

Again, this result is the equation of motion for a simple harmonic oscillator with $\omega = \sqrt{k/I}$ and a period $T = 2\pi\sqrt{I/k}$.

This system is called a torsional pendulum. There is no small angle restriction in this situation as long as the elastic limit of the wire is not exceeded.

Illustration 4.31 **A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.**

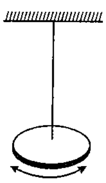

Fig. 4.57

Sol. The situation is shown in Fig. 4.57. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\text{ kg})(5.0 \times 10^{-2}\text{ m})^2}{2}$$

$= 2.5 \times 10^{-4}$ kg-m^2

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{k}} \quad \text{or} \quad k = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4\pi^2(2.5 \times 10^{-4}\text{ kg-m}^2)}{(0.20\text{ s})^2} = 0.25\text{ kg-m}^2/\text{s}^2$$

MOTION OF A BALL IN A TUNNEL THROUGH THE EARTH

Case I: If the tunnel is along a diameter and the ball is released from the surface. If the ball at any time is at a distance y from the centre of the earth, the restoring force will act on the ball due to gravitation between ball and earth.

Acceleration of the particle at the distance y from the centre of earth

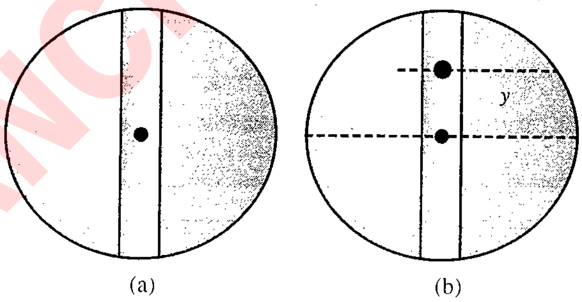

Fig. 4.58

$$a = -GMy/R^3 = -(GM)y/R^3$$

Furthermore as $g = GM/R^2$

$$a = \frac{-(gR^2)y}{R^3} \Rightarrow a = -\frac{g}{R}y$$

Comparing with $a = -\omega^2 y$

$$\omega^2 = \frac{g}{R} \quad \omega = \sqrt{\frac{g}{R}} \quad \Rightarrow T = 2\pi\sqrt{\left(\frac{R}{g}\right)} = 84.6\text{ min}$$

Case II: If the tunnel is along a chord and ball is released from the surface. If the ball at any time is at a distance x from the centre of tunnel, acceleration of the particle at the distance y from the centre of earth

$$a = -GMy/R^3 = -(GM)y/R^3$$

Furthermore, as $g = GM/R^2$

$$a = -(gR^2)y/R^3$$

This acceleration will be towards the centre of earth. The component of acceleration towards the centre of earth

$$a' = a\sin\theta = \left(-\frac{g}{R}y\right)\left(\frac{x}{y}\right) = -\frac{g}{R}x$$

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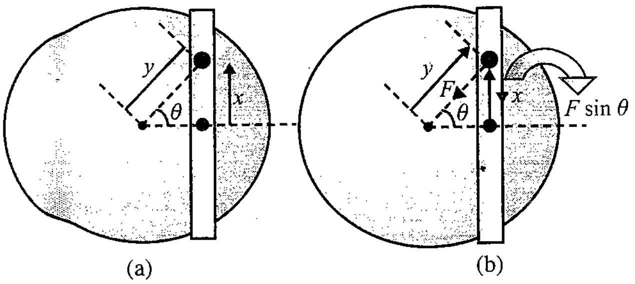

Fig. 4.59

Comparing with $a = -\omega^2 y$

$$\omega^2 = \frac{g}{R} \quad \Rightarrow \omega = \sqrt{\frac{g}{R}} \quad \Rightarrow T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

Note: *Time period of oscillation is same in both the cases wheather the tunnel is along a distance or along the chord.*

Illustration 4.32 **Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to cover the length of the tunnel if**

a. it is projected into the tunnel with a speed of $\sqrt{gR}$

b. it is released from a height R above the tunnel

c. it is thrown vertically upward along the length of tunnel with a speed of $\sqrt{gR}$

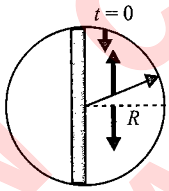

Fig. 4.60

Sol. Let $y = A\sin(\omega t + \phi)$

At $t = 0$,

$$R = A\sin\phi \qquad \text{(i)}$$

and

$$\frac{dy}{dt} = A\omega\cos\theta(\omega t + \phi)$$

At $t = 0$, the velocity of the particle is downward direction i.e., $v = -\sqrt{gR}$.

$$\frac{dy}{dt} = -\sqrt{gR} = A\sqrt{\frac{g}{R}}\cos\phi \qquad \left[as \quad \omega = \sqrt{\frac{g}{R}}\right]$$

$$A\cos\phi = -R \qquad \text{(ii)}$$

$$\tan\phi = -1 \quad \Rightarrow \phi = \frac{3\pi}{4}$$

Squaring and adding Eqs. (i) and (ii), $A = \sqrt{2}R$

$$y = \sqrt{2}R\sin\left(\frac{\sqrt{g}}{R}t + \frac{3\pi}{4}\right)$$

At $y = 0$, $\sin\left(\frac{\sqrt{g}}{R}t + \frac{3\pi}{4}\right) = \pi \;\Rightarrow\; t = \frac{\pi}{4}\sqrt{\frac{R}{g}}$

Hence time to cross the tunnel $= \frac{\pi}{2}\sqrt{\frac{R}{g}}$

OSCILLATION OF A FLOATING BODY IN A LIQUID

A floating body is in a stable equilibrium. When it is displaced up and released, it accelerates down and when it is pushed down and released, it accelerates up. It means, a floating body experiences a net (resting) force towards its stable equilibrium position. Hence, a floating body oscillates when displaced up or down from its mean position.

Illustration 4.33 **Consider a solid cylinder of density ρ_s, cross-sectional area A and height h floating in a liquid of density ρ_l, as shown in Fig. 4.61 ($\rho_l > \rho_s$). It is depressed slightly and allowed to oscillate vertically. Find the frequency of small oscillations.**

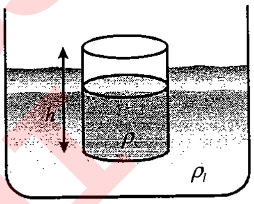

Fig. 4.61

Sol. At equilibrium the net force on the cylinder is zero in the vertical direction.

$F_{net} = B - W = 0$; B = the buoyancy and W = the weight of the cylinder.

When the cylinder is depressed slightly by x, the buoyancy increases from B to $B + \delta B$, where $\delta B = |x|\rho_l A g$

The weight W remains the same.

Therefore the net force, $F'_{net} = B + \delta B - W = \delta B = |x|\rho_l A g$

The equation of motion is, therefore,

$$\rho_s A h\frac{d^2x}{dt^2} = -x\rho_l A g$$

The minus sign takes into account the fact that x and restoring force are in opposite directions. Therefore

$$\frac{d^2x}{dt^2} = -x\frac{\rho_l g}{\rho_s h}$$

and the angular frequency, ω, is

$$\omega = \sqrt{\frac{g\rho_l}{h\rho_s}}$$

OSCILLATION OF A LIQUID COLUMN IN *U*-TUBE

Illustration 4.34 **A V-shaped glass tube of uniform cross section is kept in a vertical plane as shown. A liquid is poured in the tube. In equilibrum the level of liquid in both limbs of tube are equal. Find the angular frequency of small oscillations of liquid.**

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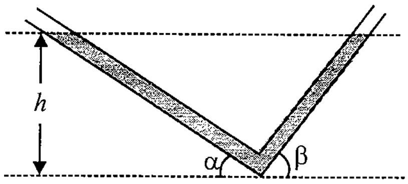


Fig. 4.62

Sol. Let us displace the liquid of left limb by a distance 'x', the liquid of right limb will rise relative to equilibrum line by same distance x.

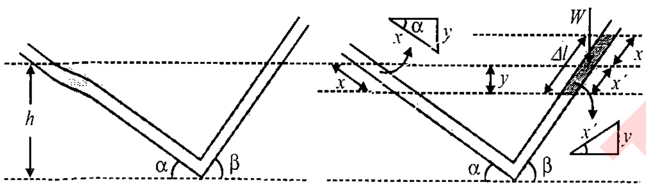


Fig. 4.63

Excess length of the liquid on the right limb

$$\Delta \ell = x + x' \qquad \text{(i)}$$

from the figure

$$y = x \sin\alpha = x' \sin\beta \qquad \text{(ii)}$$

from Eq. (i)

$$x' = \frac{x \sin\alpha}{\sin\beta} \qquad \text{(iii)}$$

from Eqs. (i) and (iii), $\Delta \ell = \left(\dfrac{\sin\alpha + \sin\beta}{\sin\beta}\right)\alpha$

This excess length $\Delta \ell$ in right limb will provide restoring force to the liquid in tube.

Weight of the excess liquid column

$$W = \rho A \cdot \Delta \ell g = \left(\frac{\sin\alpha + \sin\beta}{\sin\beta}\right)\rho\, Axg$$

ρ is the density of liquid and A is the area of cross section of tube. The component of W along the tube will be equal to restoring force

$$F = W \sin\beta = \rho(\sin\alpha + \sin\beta)\, Axg$$

This force helps the entire liquid to restore its original position by moving it with an acceleration $a = F/m$; m = mass of total liquids

$$a = \frac{\rho(\sin\alpha + \sin\beta)Axg}{m}$$

Here m = mass of liquid in the tube = $A\rho\ell$

ℓ = length of total liquid column

$$\ell = \frac{h}{\sin\alpha} + \frac{h}{\sin\beta} = h\left[\frac{\sin\alpha + \sin\beta}{\sin\alpha \cdot \sin\beta}\right]$$

Hence $a = \dfrac{\rho(\sin\alpha + \sin\beta)Axg}{A\rho\left[h\dfrac{(\sin\alpha + \sin\beta)}{\sin\alpha \cdot \sin\beta}\right]}$

$$\vec{a} = -\left(\frac{g\sin\alpha \cdot \sin\beta}{h}\right)\cdot \vec{x} \quad \begin{bmatrix}\text{with proper}\\ \text{vector sign}\end{bmatrix}$$

Compare with $\vec{a} = -\omega^2 \vec{x}$

which gives

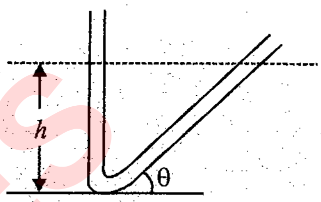
$$\omega = \sqrt{\frac{g\sin\alpha \cdot \sin\beta}{h}} \Rightarrow T = 2\pi\sqrt{\frac{h}{g\sin\alpha \cdot \sin\beta}}$$

Note: ***If we modify the tube as shown in the figure, the different time periods will be***

Here $\sin\alpha = 1$,

$\sin\beta = \sin\theta$

Hence $T = 2\pi\sqrt{\dfrac{h}{g\sin\theta}}$



a.

Here $\alpha = \beta = 90°$

Hence $T = 2\pi\sqrt{\dfrac{h}{g}}$

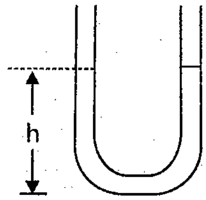


Fig. 4.64

For an arbitary tube if α ***and*** β ***are the angles made by the targets (with horizontal) drawn at the tube at the free surface of the liquid in both limbs. The time period is given by***

$$T = 2\pi\frac{\sqrt{h}}{g\sin\alpha \cdot \sin\beta}$$

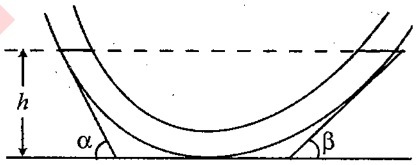


Fig. 4.65

Concept Application Exercise 4.2

1. A block of mass m is suspended from the ceiling of a stationary elevator through a spring of spring constant k; it is in equilibrium. Suddenly, the cable breaks and the elevator starts falling freely. Show that the block now executes a simple harmonic motion of amplitude mg/k in the elevator.

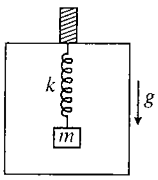


Fig. 4.66

2. The left block in Fig. 4.67 collides inelastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.

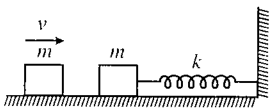


Fig. 4.67

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3. A ball of mass m is connected to two rubber bands of length L, each under tension T as shown in Fig. 4.68. The ball is displaced by a small distance y perpendicular to the length of the rubber bands. Assuming the tension does not change, show that (a) the restoring force is $-(2T/L)y$ and (b) the system exhibits simple harmonic motion with an angular frequency $\omega = \sqrt{2T/mL}s$.

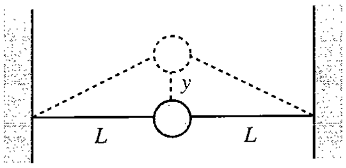

Fig. 4.68

4. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increases by 1 s, find the initial mass m assuming that Hooke's law is obeyed.

5. A uniform pole of length $l = 2L$ is laid on a smooth horizontal table as shown in Fig. 4.69. The mass of pole is M and it is connected to a frictionless axis at O. A spring with force constant k is connected to the other end. The pole is displaced by a small angle θ_0 from equilibrium position and released such that it performs small oscillations. Find its angular frequency.

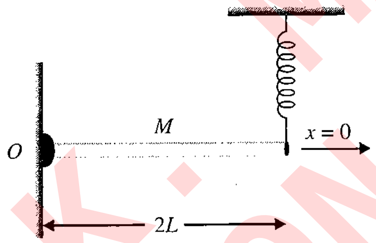

Fig. 4.69

6. A pendulum has a period T for small oscillations. An obstacle is placed directly below the pivot, so that only the lowest one-fourth of the string can follow the pendulum bob when it swings to the right of its resting position as shown.
The pendulum is released from rest at a certain point. Assuming that the angle between the moving string and the vertical stays small throughout the motion, find the time it takes to return to that point.

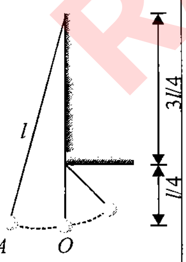

Fig. 4.70

7. A horizontal string–mass system of mass M executes oscillatory motion of amplitude a_0 and time period T_0. When mass M is passing through its equilibrium position another mass m is placed on it such that both move together. Find the new amplitude and time period.

8. A spring of spring constant 200 N/m has a block of mass 1 kg hanging at its one end and from the other end the spring is attached to a ceiling of an elevator. The elevator rises upwards with an acceleration of $g/3$.
When acceleration is suddenly ceased, then what should be the angular frequency and elongation during the time when the elevator is accelerating?

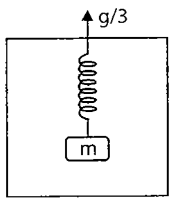

Fig. 4.71

9. Two blocks lie on each other and are connected to a spring as shown in Fig. 4.72. What should be the mass of block A placed on block B of mass 6 kg so that the system is period is 0.75 s. Assuming no slipping, what should be the minimum value of coefficient of static friction μ_s for which block A will not slip relative to block B, if block B is displaced 50 mm from equilibrium position and released?

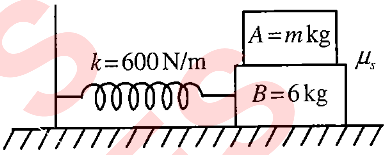

Fig. 4.72

10. Find the time period of small oscillations of the spring–mass system shown in Fig. 4.73. Find the range of values of h such that simple harmonic motion is possible.

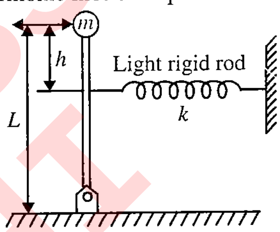

Fig. 4.73

11. A simple pendulum of length l swings from a small angle θ. Its swinging is constrained by the smooth inclined planes OP and PC. Assuming elastic collision of the bob with the plane PC, find

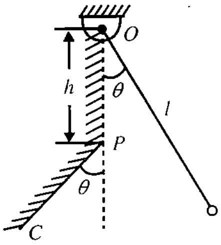

Fig. 4.74

a. angular amplitude for the motion of the bob in the left hand side of its mean position.

b. time period for a complete cycle of motion of the bob.

12. A uniform rod of length l is pivoted at a distance x from the top of the rod. Neglecting friction, find the (a) value of x for minimum period of oscillation, (b) minimum period of oscillation of the rod.

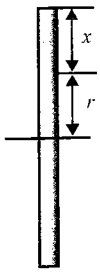

Fig. 4.75

13. The period of oscillation of a spring pendulum is T. If the spring is cut into four equal parts, then find the time period corresponding to each part.

14. A uniform stick of length l is hinged so as to rotated about a horizontal axis perpendicular to the stick, at a distance x from the centre. Find the value of x, for which the time period is minimum.

15. A ball is released in a smooth dimetrical tunnel of earth.

a. After how much time will it pass through the centre of earth?

b. With what speed will the ball pass the centre of earth?

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Solved Examples

Example 4.1 **A uniform horizontal plank is resting symmetrically in a horizontal position on two cylindrical drums, which are spinning in opposite direction about their horizontal axes with equal angular velocity. The distance between the axes is $2L$ and the coefficient of friction between the plank and cylinder is μ. If the plank is displaced slightly from the equilibrium position along its length and released, show that it performs simple harmonic motion. Calculate also the time period of motion.**

Sol. When the plank is situated symmetrically on the drums, the reactions on the plank from the drums will be equal and so the force of friction will be equal in magnitude but opposite in direction and hence, the plank will be in equilibrium along vertical as well as horizontal direction.

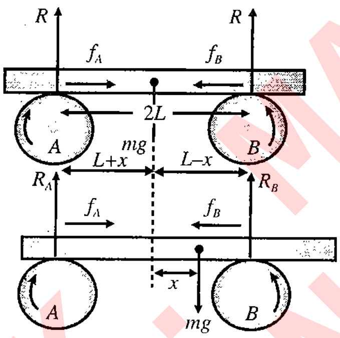

Fig. 4.76

Now if the plank is displaced by x to the right, the reaction will not be equal. For vertical equilibrium of the plank

$$R_A + R_B = mg \quad \text{(i)}$$

And for rotational of plank, taking moment about centre of mass, we have

$$R_A(L+x) = R_B(L-x) \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get $R_A = mg\left(\dfrac{L-x}{2L}\right)$

and $\quad R_B = mg\left(\dfrac{L+x}{2L}\right)$

Now as $f = \mu R$, so friction at B will be more than that at A and will bring the plank back, i.e., restoring force here is

$$F = -(f_B - f_A) = -\mu(R_B - R_A) = -\mu\frac{mg}{L}x$$

As the restoring force is linear, the motion will be simple harmonic motion with force constant $k = \dfrac{\mu mg}{L}$

So $\quad T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{L}{\mu g}}$

Example 4.2 **A uniform plank of mass m, free to move in the horizontal direction only, is placed at the top of a solid cylinder of mass $2m$ and radius R. The plank is attached to a fixed wall by means of a light spring of spring constant k. There is no slipping between the cylinder and the plank, and between the cylinder and the ground. Find the time period of small oscillations of the system.**

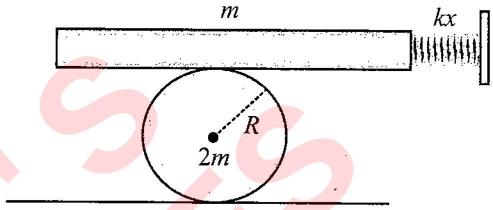

Fig. 4.77

Sol. Method 1: When the plank is displaced slightly (say x) from equilibrium, as there is no sliding at any surfaces, the friction will be of static nature. As cylinder is rolling we can take the motion of the cylinder as pure rotation about point of contact.

Free body diagrams of plank and cylinder

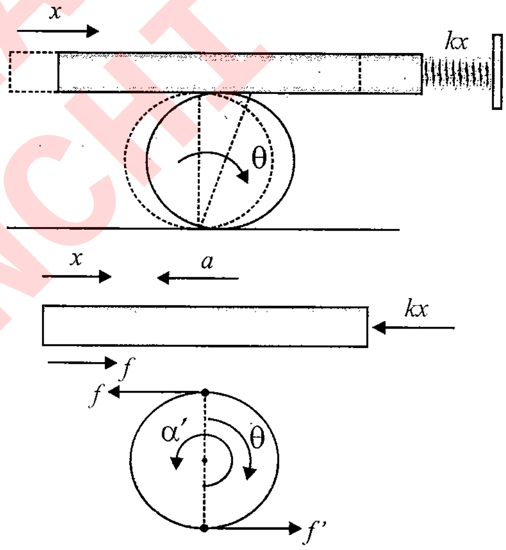

Fig. 4.78

Equation of motion for plank

$$kx - f = m(a) \quad \text{(i)}$$

Writing torque equation to cylinder about point of contact

$$f \cdot (2R) = I_P \cdot \alpha = \left[\frac{1}{2}(2m)R^2 + 2mR^2\right]\alpha$$

$$f(2R) = 3mR^2\alpha$$

$$\Rightarrow \quad f = \frac{3}{2}mR\alpha \quad \text{(ii)}$$

As cylinder is rolling we can write, the acceleration of topmost point of cylinder (or plank)

$$a = \alpha(2R) \;\Rightarrow\; \alpha R = \frac{a}{2} \quad \text{(iii)}$$

Substituting the value of (iii) and (ii) we get

$$f = \frac{3}{4}ma \quad \text{(iv)}$$

Now, from Eqs. (i) and (iv)

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$$kx - \left(\frac{3}{4}ma\right) = ma$$

$$\Rightarrow \quad a = \frac{4k}{7m}x$$

Writing above equation with proper vector relation

$$\vec{a} = -\left(\frac{4k}{7m}\right)\vec{x} \qquad \text{(iv)}$$

Comparing with standard equation $\vec{a} = -\omega^2\vec{x}$

which gives $\omega^2 = \dfrac{4k}{7m}$ or $T = \pi\sqrt{\dfrac{7m}{k}}$

Method 2: Suppose that the plank is displaced from its equilibrium position by x at time t, the centre of the cylinder is, therefore, displaced by $x/2$.

Therefore the mechanical energy of the system is given by,

$$E = \text{KE (Plank)} + \text{PE (spring)} + \text{KE (cylinder)}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + \frac{1}{2}I_P w^2 \qquad \text{(i)}$$

$$I_P = I_G + (2m)R^2 = \frac{1}{2}(2m)R^2 + (2m)R^2$$

$$I_P = 3mR^2$$

As cylinder is rolling velocity of centre of cylinder will be half of the velocity of the plank (top most point of the cylinder)

$$v_{\text{cylinder}} = \frac{v}{2} = \omega R \quad \Rightarrow \quad \omega = \frac{v}{2R}$$

Substiuting the value of ω in Eq. (i), we get

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + \frac{1}{2}(3mR^2)\left(\frac{v}{2R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + \frac{3}{8}mv^2$$

$$E = \frac{7}{8}mv^2 + \frac{1}{2}kx^2 \qquad \text{(ii)}$$

Differentiating Eq. (ii) w.r.t. time

$$\frac{dE}{dt} = 0 = \frac{7}{8}m \cdot 2v\left(\frac{dv}{dt}\right) + \frac{1}{2}k \cdot 2x \cdot \frac{dx}{dt}$$

$$= \frac{7}{4}mv \cdot a + kx \cdot v$$

Here $\dfrac{7}{4}ma + kx = 0$ or $a = -\dfrac{4k}{7m}x$

Comparing with $a = -\omega^2 x \Rightarrow T = 2\pi\sqrt{\dfrac{7m}{4k}} = \pi\sqrt{\dfrac{7m}{k}}$

Method 3: We can write Eq. (ii) as

$$E = v^2 + \frac{4k}{7m}x^2 = \text{constant}$$

as total energy of system performing SHM is constant. Comparing with standard equation $v^2 + \omega^2x^2 = \text{constant}$.

which gives $\omega^2 = \dfrac{4k}{7m}$ or $T = \pi\sqrt{\dfrac{7m}{k}}$

Example 4.3 **A block of mass m hangs by means of a string which goes over a pulley of mass m and moment of inertia I, as shown in the diagram. The string does not slip relative to the pulley. Find the frequency of small oscillations.**

Fig. 4.79

Sol. Suppose the block is depressed by x. The pulley (owing to the constraint) is depressed by $x/2$. Suppose the tension in the strings are T and T' on both sides. We can write

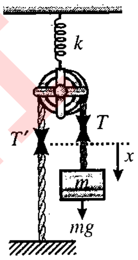

Fig. 4.80

For block, $\quad mg - T = mx \qquad \text{(i)}$

For pulley, $\quad T + T' + mg - k(x + x_0) = m\dfrac{x}{2} \qquad \text{(ii)}$

The angular acceleration of the pulley,

$$\alpha = \frac{x/2}{R} \qquad \text{(iii)}$$

$$(T - T')R = I\frac{x}{2R} \qquad \text{(iv)}$$

From Eqs. (i), (ii), (iii) and (iv), we get

$$3mg - k(x + x_0) = \left(\frac{5m}{2} + \frac{I}{2R^2}\right)x \qquad \text{(v)}$$

The frequency of small oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{5m}{2} + \frac{I}{2R^2}}}$$

Example 4.4 **The pulley shown in Fig. 4.81 has a radius r, moment of inertia I about its axis and mass m. Find the time period of vertical oscillations of its centre of mass. The spring constant of spring is K and the spring does not slip over the pulley.**

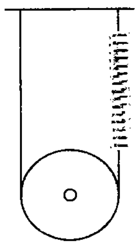

Fig. 4.81

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Sol. For rotational equilibrium the tension in string on both sides must be same.

Let T be the tension in the string in equilibrium position and l_0 the extension of spring. Then for equilibrium of any part of string on RHS.

$$2T = mg \Rightarrow 2K\ell_0 = mg \text{ or } \ell_0 = \frac{mg}{2K}$$

For translational equation of pulley $\ell_0 = \dfrac{mg}{2K}$

Thus when pulley attains equilibrium, the spring is stretched by a distance

$$\ell_0 = \frac{mg}{2K} \qquad \text{(i)}$$

Now suppose that the pulley is pulled down a little and released. The pulley starts up and down oscillations. Let y be instantaneous displacement of centre of pulley from equilibrium. Then total increase in length of (string + spring) is $2y$ (y to the left of pulley and y to the right).

As string is inextensible, total extension is caused in spring, i.e., extension of spring = $2y$.

As pulley also rotates, the total energy of system is

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - mgy + \frac{1}{2}K(\ell_0 + 2y)^2$$

But $\omega = \dfrac{v}{r}$

$$E = \frac{1}{2}I \cdot \frac{v^2}{r^2} + \frac{1}{2}mv^2 - mgy + \frac{1}{2}K(\ell_0 + 2y)^2$$

$$= \frac{1}{2}\left(\frac{I}{r^2} + m\right)v^2 - mgy + \frac{1}{2}K(\ell_0 + 2y)^2$$

As total energy is constant (system is conservative)

$$\frac{dE}{dt} = 0$$

$$\frac{1}{2}\left(\frac{I}{r^2} + m\right)2v\frac{dv}{dt} - mg\frac{dy}{dt} + \frac{1}{2}K\,2(\ell_0 + 2y)\cdot 2\frac{dy}{dt} = 0$$

and $\dfrac{dy}{dt} = v$ and $\dfrac{dv}{dt}$ = acceleration a

and $\ell_0 = \dfrac{mg}{2K}$

we get $\left(\dfrac{I}{r^2} + m\right)a - mg + 2K\left(\dfrac{mg}{2K} + 2y\right) = 0$

$$a \propto -y \qquad \text{(ii)}$$

As $a \propto -y$, motion of pulley is SHM.

Standard equation of SHM is

$$a = -\omega^2 y \qquad \text{(iii)}$$

Comparing Eqs. (ii) and (iii), $\omega^2 = \dfrac{4K}{\left(\dfrac{I}{r^2} + m\right)}$

Time period,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\left\{\frac{\frac{I}{r^2} + m}{4K}\right\}} = \pi\sqrt{\left\{\frac{m + \frac{I}{r^2}}{K}\right\}}$$

Example 4.5 **An L-shaped bar of mass M is pivoted at one of its end so that it can freely rotate in a vertical plane, as shown in the Fig. 4.82.**

a. Find the value of θ_0 at equilibrium.
b. If it is slightly displaced from its equilibrium position, find the frequency of oscillation.

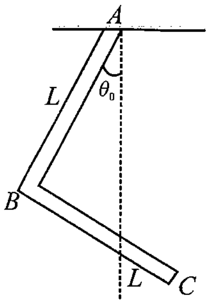

Fig. 4.82

Sol. a. Taking B as the origin, the coordinate of its c and m are

$$x_C = \frac{\frac{M}{2}\frac{L}{2}}{\frac{M}{2} + \frac{M}{2}} = \frac{L}{4}$$

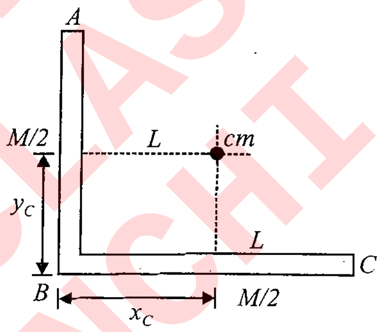

Fig. 4.83

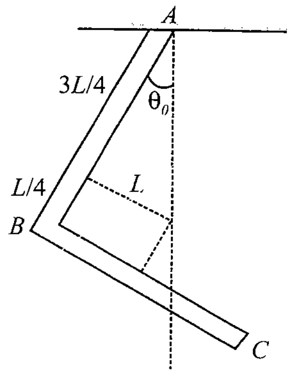

Fig. 4.84

and $\quad y_C = \dfrac{\frac{M}{2}\frac{L}{2}}{M} = \dfrac{L}{4} \quad \Rightarrow \quad \tan\theta_0 = \dfrac{L/4}{3L/4} = \dfrac{1}{3}$

$$\theta_0 = \tan^{-1}\left(\frac{1}{3}\right)$$

b. The frequency of oscillation for a compound pendulum is

$$f = \frac{1}{2\pi}\sqrt{\frac{mg\ell}{I}}$$

where d = distance of the cm from the point of suspension.
I = moment of inertia about the point of suspension.

$$d = \sqrt{\left(\frac{3L}{4}\right)^2 + \left(\frac{L}{4}\right)^2} = \frac{L}{4}\sqrt{10}$$

$$I = \left(\frac{M}{2}\right)\frac{L^2}{3} + \left(\frac{M}{2}\right)\frac{L^2}{12} + \left(\frac{M}{2}\right)\left[L^2 + \left(\frac{L}{2}\right)^2\right] = \frac{ML^2}{3}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{Mg\frac{L}{4}\sqrt{10}}{\frac{ML^2}{3}}} \text{ or } f = \frac{1}{2\pi}\sqrt{(2.37)\frac{g}{L}}$$

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Example 4.6 **A certain mass of a perfect gas is enclosed in a cylinder of volume V_0 fitted with a smooth heavy piston of mass m and area a. The piston is displaced through a small distance downwards so as to compress the gas isothermally, and then it is let go. Show that the resultant motion is SHM and find its period. Take the atmospheric pressure as P_{atm}.**

Sol. Let A be the area of cross section of the cylinder. When the piston is in equilibrium, force on it must balance.

$$P_0A = P_{atm}A + mg \quad \text{(i)}$$

Let us now displace the piston through a small distance x in downward direction. As the compression of the gas is isothermal, the final pressure P is given by

$$P_0V_0 = P(V_0 - xA)$$

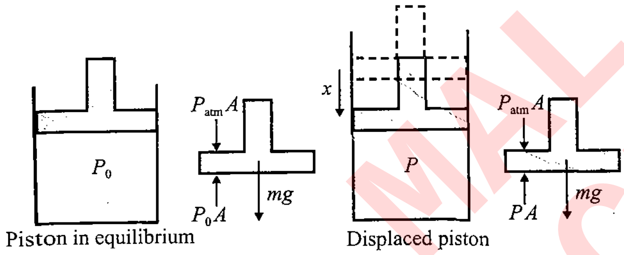

Fig. 4.85

$$P = \frac{P_0V_0}{V_0 - xA} \Rightarrow P = P_0\left(1 - \frac{xA}{V_0}\right)^{-1}$$

$$P_0 = \left(1 + \frac{xA}{V_0}\right) \quad [(1+t)^n = 1 + nt \text{ for } t \ll 1]$$

Considering the forces on the displaced piston, the thrust of the gas pressure now exceeds its weight, and hence, magnitude of net upward force is $F = PA - mg - P_{atm}A$

$$F = P_0\left(1 + \frac{xA}{V_0}\right)A - mg - P_{atm}A$$

$$F = \frac{P_0A^2}{V_0}x \quad \text{[using Eq. (i)]}$$

Hence the magnitude of force is proportional to x and its direction is towards equilibrium position.

Comparing with $F = kx$, we get $k = \frac{P_0A^2}{V_0}$

Time period $\quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mV_0}{P_oA^2}}$

$$T = 2\pi\sqrt{\frac{mV_0}{\left(P_{atm} + \frac{mg}{A}\right)A^2}} \Rightarrow T = 2\pi\sqrt{\frac{mV_0}{P_{atm}A^2 + mgA}}$$

Example 4.7 **A sphere of mass m and radius r rolls without slipping on a rough concave surface of large radius R. It makes small oscillations about the lowest point. Find the time period.**

Sol. Let at any instant, the body is at angular position θ with respect to the vertical line drawn from the centre of mirror. If ϕ is the angular displacement of the ball about its centre, then

$$(R - r)\theta = r\phi \quad \therefore \theta = \left(\frac{r}{R-r}\right)\phi$$

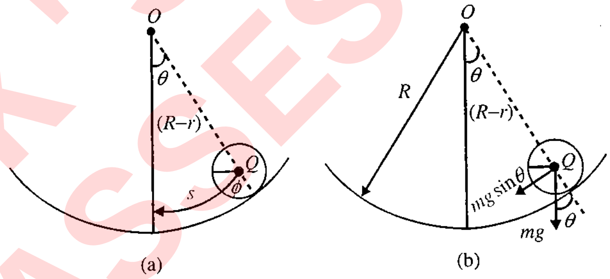

Fig. 4.86

Restoring torque acting on the ball $\tau = -mg\sin\theta \times r$

For small θ, $\sin\theta = \theta \quad \therefore \tau = -mg\,\theta \times r$

or $\quad \tau = -mg\left[\frac{r}{R-r}\right]\phi r = mg\left(\frac{r^2}{R-r}\right)(-\phi)$

Angular acceleration $\alpha = \frac{mg}{I}\left(\frac{r^2}{R-r}\right)(-\phi)$

Now comparing above equation with standard equation of SHM, $\alpha = -\omega^2\phi$, we get $\omega = \sqrt{\frac{mg}{I}\left(\frac{r^2}{R-r}\right)}$

Here I is the moment of inertia of the rolling ball about the point of contact which is $I = 7/5\, mr^2$

$$\therefore \quad \omega = \sqrt{\frac{mg}{\frac{7}{5}mr^2}\left(\frac{r^2}{R-r}\right)} = \sqrt{\frac{5}{7}\frac{g}{(R-r)}}$$

and $\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7(R-r)}{5g}}$

Alternate method: We can imagine that when the CM of the body is shifted slightly by an angle θ relative to the centre of curvature of the surface, the magnitude of acceleration of the CM of the body can be given as

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{r^2}}$$

(because the body is accelerating down the inclined plane of instantaneous angle θ)

Substituting $\quad \sin\theta = \theta = \frac{x}{R-r}$, we have

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$$a = \frac{-gx}{(R-r)\left(1+\frac{k^2}{r^2}\right)}$$

(negative sign is given because $\vec{a}$ and $\vec{x}$ are oppositely directed)

Comparing the above equation with $a = -\omega^2 x$, we have

$$\omega = \sqrt{\frac{g}{(R-r)\left(1+\frac{k^2}{r^2}\right)}}$$

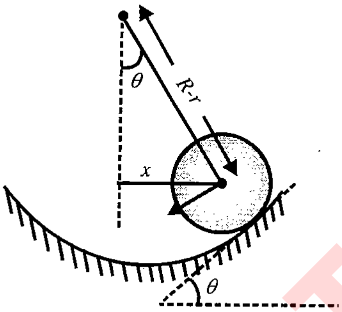

Fig. 4.87

For solid sphere $k^2 = 2/5\ r^2$.

Hence $\omega = \sqrt{\frac{5}{7}\frac{g}{(R-r)}}$

Example 4.8 **A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension. Find the frequency of vibration of the system for small values of the amplitude (small θ). Assume the vertical suspension rod of length L is rigid, but ignore its mass.**

Sol. The frequency of vibration should be greater than that of a simple pendulum since the spring adds an additional restoring force:

$$f > \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

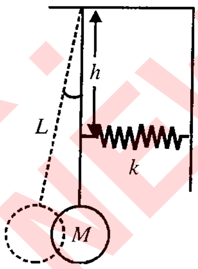

Fig. 4.88

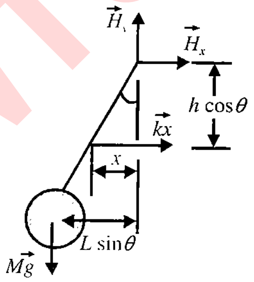

Fig. 4.89

We can find the frequency, ω, which is found in the equation for angular SHM: $d^2\theta/dt^2 = -\omega^2\theta$. The angular acceleration can be found from analysing the torques acting on the pendulum.

For the pendulum (see sketch)

$$\Sigma\tau = I\alpha \text{ and } d^2\theta/dt^2 = -\alpha$$

The negative sign appears because positive θ is measured clockwise in the picture. We take torque around the point of suspension:

$$\Sigma\tau = MgL\sin\theta + kxh\cos\theta = I\alpha$$

For small amplitude vibrations, we use the approximations:

$$\sin\theta \approx \cos\theta \approx 1, \quad \text{and} \quad x = h\tan\theta \approx h\theta$$

Therefore, with $I = mL^2$,

$$\frac{d^2\theta}{dt^2} = -\left[\frac{MgL+kh^2}{I}\right]\theta = -\left[\frac{MgL+kh^2}{ML^2}\right]\theta$$

This is of the SHM form, $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

With angular frequency $\omega = \sqrt{\frac{MgL+kh^2}{ML^2}} = 2\pi f$

The frequency is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MgL+kh^2}{ML^2}}$

Example 4.9 **One end of an ideal spring is fixed to a wall at origin O and axis of spring is parallel to the x-axis. A block of mass $m = 1$ kg is attached to the free end of the spring and it is performing SHM. Equation of position of the block in coordinate system shown in Fig. 4.90 is $x = 10 + 3\sin(10t)$, where t is in seconds and x in centimetres.**

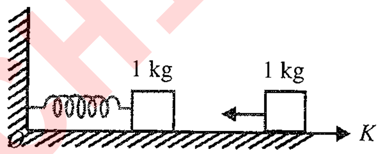

Fig. 4.90

i. Calculate force constant of the spring.

Another identical block, moving towards origin with velocity 0.6 m/s collides elastically with the block performing SHM at $t = 0$. Calculate

ii. new amplitude of oscillations

iii. equation of position of block performing SHM

iv. percentage increase in oscillation energy (neglect friction)

Sol. $x = 10 + 3\sin(10t)$

Since the block is oscillating in horizontal direction, its equilibrium position corresponds to natural length of the spring. Hence, natural length of spring is, $l_0 = 10$ cm.

Displacement of block from equilibrium position is

$$s = x - l_0$$

$$s = 3\sin(10t)\text{ cm} \quad \text{or} \quad s = 0.03\sin(10t)\text{ m} \qquad \text{(i)}$$

But displacement of a block performing SHM is given by $s = a\sin(\omega t)$.

Comparing it with Eq. (i), $A = 0.03$ m and $\omega = 10$ rad/s

But angular frequency of block, performing SHM and tied with an ideal spring is $\omega = \sqrt{\frac{K}{m}}$

Spring constant, $K = \omega^2 m = (10)^2 \times 1 = 100$ N/m

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Velocity of the block (performing SHM) is given by

$$v=\frac{dx}{dt}=30\cos(10t)\text{ cm}$$

Its velocity at time $t=0$ is $v=30$ cm/s or 0.30 m/s

Velocity of the block (just before collision) is 0.6 m/s (leftward); therefore it is to be taken (–0.6 m/s).

Since masses of two colliding blocks are equal and collision is elastic, therefore, velocity of oscillating block (just after collision) becomes equal to (–6.0 m/s).

Note: ***Whenever head-on elastic collision between two bodies having equal masses takes place, their velocities get interchanged.***

It means now oscillating block starts to move leftwards with speed –0.6 m/s. But at instant $t=0$, it is in equilibrium position; therefore, speed –0.6 m/s $=a'\omega$, where a' = new amplitude of oscillation and $\omega=10$ rad/s (angular frequency), which remains unchanged.

$$a'=0.06\text{ m}=6\text{ cm}$$

Since at $t=0$, now block is in equilibrium position and is moving towards negative x-direction, therefore just after collision, phase of the block is π.

Hence, equation of its position x is given by, $x=l_0+\sin(\omega t+\pi)=10+6\sin(10t+\pi)$ cm.

Initially oscillation energy was

$$U_1=\frac{1}{2}ma^2\omega^2=\frac{1}{2}\times1\times(0.03)^2(10)^2=0.045\text{ J}$$

Oscillation energy just after collision

$$U_2=\frac{1}{2}m(a')^2\omega^2=0.18\text{ J}$$

Percentage increase in oscillation energy

$$=\frac{U_2-U_1}{U_1}\times100\%=300\%$$

Example 4.10 **A circular spring of natural length l_0 is cut and welded with two beads of masses m_1 and m_2 such that the ratio of the lengths of the springs between the beads is 4:1. If the stiffness of the original spring is k, find the frequency of oscillation of the beads in a smooth horizontal rigid tube. Assume $m_1=m$ and $m_2=3m$.**

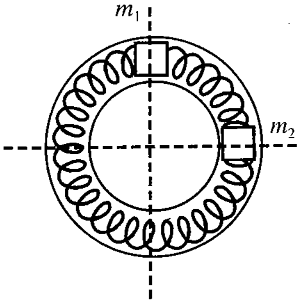

Fig. 4.91

Sol. When m_1 is displaced relative to m_2 by a distance x, each spring will be deformed by same amount. Hence, the springs are connected in parallel. The equivalent spring constant is

$$k_{eq}=k_1+k_2$$

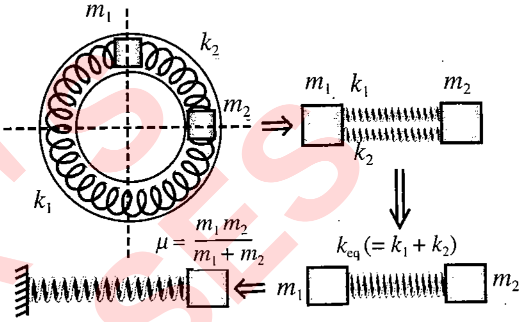

Fig. 4.92

If the spring is cut, the force constant of spring

$$k\propto\frac{1}{\ell}\Rightarrow k_1\ell_1=k_2\ell_2=k\ell$$

Substituting $l_1=l/5$ and $l_2=4l/5$, we have

$$k_1=5k\quad\text{and}\quad k_2=\frac{5}{4}k$$

Then $$k_{eq}=\frac{25}{4}k$$

Now we have two particles of masses m_1 and m_2 and one spring of stiffness

$$k_{eq}=\frac{25}{4}k$$

The reduced mass is $\mu=\dfrac{m_1m_2}{m_1+m_2}$

where $m_1=m$ and $m_2=3m$

This gives $\mu=3/4\,m$

Substituting $\mu=3/4\,m$ and $k_{eq}=25/4k$ in the formula

$$\omega=\sqrt{\frac{k_{eq}}{\mu}}\qquad\Rightarrow\omega=\sqrt{\frac{\frac{25}{4}k}{\frac{3}{4}m}}=\sqrt{\frac{25k}{3m}}=5\sqrt{\frac{k}{3m}}$$

Example 4.11 **A uniform cylinder of mass m and radius R is in equilibrium on an inclined plane by the action of a light spring of stiffness k, gravity and reaction force acting on it. If the angle of inclination of the plane is ϕ, find the angular frequency of small oscillation of the cylinder.**

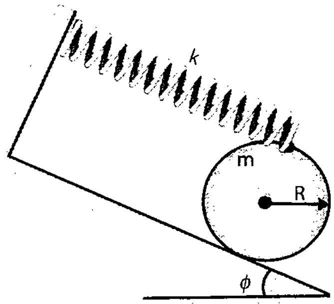

Fig. 4.93

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Sol. If the CM is pushed (shifted) down by the additional distance x, the additional elongation of the spring will be $2x$

Let x' be the equilibirum stretch of spring.

The total elongation of the spring $x'' = (x' + 2x)$

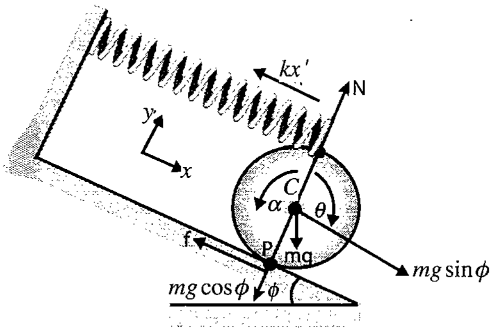

Fig. 4.94

The spring force $= F_S' = kx' = k(x' + 2x)$

The torque acting on the cylinder about P by the spring force is $\tau_S = F_S'(2R) = 2k(x' + 2x)R$

The gravitational torque about P is $\tau_g = -mgR\sin\phi$

The net torque about P is $\tau_{net} = \tau_S + \tau_g$

$$= 2kx'R + 4kxR - mgR\sin\phi$$

Since, $2kx' = mg\sin\phi$, from Eq. (ii), we have $\tau_{net} = 4kxR$

Substituting $x = R\theta$, we have the vector equation,

$$\tau_{net} = 4kR^2\phi$$

Torque equation: The torque rotates the cylinder with angular acceleration α at the angular position θ. Applying Newton's second law of rotation, we have $\alpha = \dfrac{\tau_{net}}{I_P}$

where $\tau_{net} = -4kR^2\theta$

This gives $\alpha = \dfrac{-4kR^2}{I_P}\theta$

Comparing the above equation with $\alpha = -\omega^2\theta$,

we have $\omega = \sqrt{\dfrac{4kR^2}{I_P}}$, where $I_P = \dfrac{3mR^2}{2}$

Then, $\omega = 2\sqrt{\dfrac{2k}{3m}}$

EXERCISES

Subjective Type

Solutions on page 4.58

1. A rigid rod of mass m with a ball of mass M attached to the free end is restrained to oscillate in a vertical plane as shown in the Fig. 4.95. Find the natural frequency of oscillation.

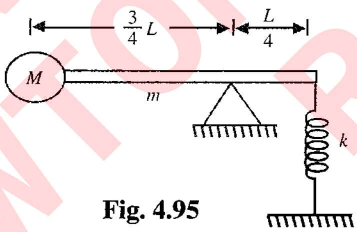

Fig. 4.95

2. A rectangular tank having base 15 cm × 20 cm is filled with water (density ρ = 1000 kg/m³) up to 20 cm height. One end of an ideal spring of natural length h_0 = 20 cm and force constant K = 280 N/m is fixed to the bottom of the tank so that the spring remains vertical.
This system is in an elevator moving downwards with acceleration a = 2 m/s². A cubical block of side l = 10 cm and mass m = 2 kg is gently placed over the spring and released gradually, as shown in Fig. 4.96.

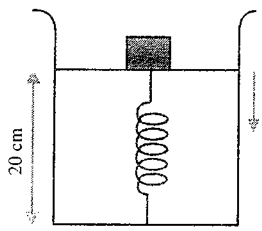

Fig. 4.96

a. Calculate compression of the spring in equilibrium position.

b. If the block is slightly pushed down from equilibrium position and released, calculate frequency of its vertical oscillations.

3. A body A of mass m_1 = 1 kg and body B of mass m_2 = 4.1 kg are interconnected by a spring as shown in Fig. 4.97. The body A performs free vertical harmonic oscillations with the amplitude 1.6 cm and frequency 25 Hz. Neglecting the mass of the spring, find the maximum and minimum value of force that the system exerts on the bearing surface.

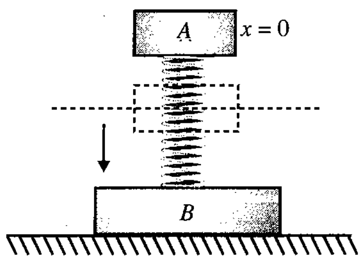

Fig. 4.97

4. In the arrangement shown in Fig. 4.98, the sleeve of mass M is fixed between two identical spring whose combined force constant is k. The sleeve can slide without friction over a horizontal bar AB. The arrangement rotates with a constant angular velocity ω about a vertical axis passing through the middle of the bar. Find the period of small oscillations of the sleeve.

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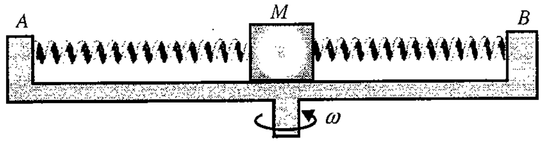

Fig. 4.98

5. A vertical pole of length l, density ρ, area of cross section A floats in two immiscible liquids of densities ρ_1 and ρ_2. In equilibrium position the bottom end is at the interface of the liquids. When the cylinder is displaced vertically, find its time period of oscillation.

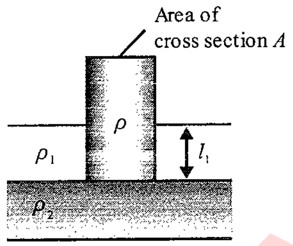

Fig. 4.99

6. In the shown arrangement, both the springs are in their natural lengths. The coefficient of friction between m_2 and m_1 is μ. There is no friction between m_1 and the surface. If the blocks are displaced slightly, they together perform simple harmonic motion.

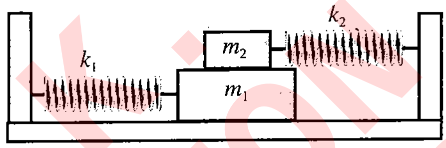

Fig. 4.100

Obtain

a. the frequency of such oscillations.

b. the condition if the frictional force on block m_2 is to act in the direction of its displacement from mean position.

c. the if the condition obtained in (b) is met, what can be the maximum amplitude of their oscillations?

7. A uniform disc of mass m and radius R is connected with two light springs 1 and 2. The springs are connected at the highest point M and the CM 'N' of the disc. The other ends of the springs are rigidly attached with vertical walls. If we shift the CM in horizontal by a small distance, the disc oscillates simple harmonically. Assuming a perfect rolling of the disc on the horizontal surface, find the angular frequency of oscillation.

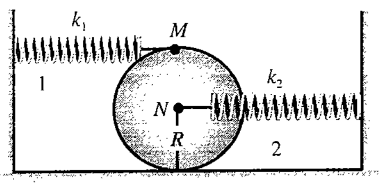

Fig. 4.101

8. Consider a liquid which fills a uniform U-tube, as shown in Fig. 4.102, up to a height h. Find the frequency of small oscillations of the liquid in the U-tube.

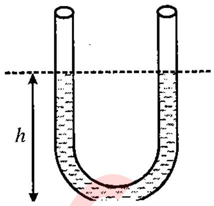

Fig. 4.102

9. A particle of mass m is located in a unidimensional potential field where potential energy of the particle depends on the coordinate x as $U(x) = \frac{A}{x^2} - \frac{B}{x}$ where A and B are positive contacts. Find the time period of small oscillations that the particle perform about equilibrium position.

10. A uniform rod of mass m and length l remains in equilibrium inside a smooth hemisphere of radius R. Find the period of small oscillation of the rod.

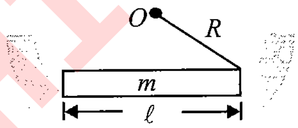

Fig. 4.103

Objective Type

Solutions on page 4.61

1. While a particle executes linear simple harmonic motion

a. its linear velocity and acceleration pass through their maximum and minimum values once in each oscillation.

b. its linear velocity and acceleration pass through their maximum and minimum values twice in each oscillation.

c. its linear velocity and acceleration pass through their maximum and minimum values four times in each oscillation.

d. its linear velocity and acceleration always attain their peak values simultaneously.

2. While a particle executes simple harmonic motion, the rate of change of acceleration is maximum and minimum, respectively at

a. the mean position and extreme positions

b. the extreme positions and mean position

c. the mean position alternatively

d. the extreme positions alternatively

3. A hollow sphere is filled with water. It is hung by a long thread to make it a simple pendulum. As the water flows out of a hole at the bottom of the sphere, the frequency of oscillation will

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a. go on increasing
b. go on decreasing
c. first increases and then decreases
d. first decreases and then increases

4. A simple pendulum oscillates slightly above a large horizontal metal plate. The bob is given a charge. The time period
a. has no effect, whatever be the nature of charge
b. always decreases, whatever be the nature of charge
c. always increases, whatever be the nature of charge
d. a increases or decreases depending upon the nature of charge

5. A block is resting on a piston which executes simple harmonic motion in vertical plain with a period of 2.0 s in vertical plane at an amplitude just sufficient for the block to separate from the piston. The maximum velocity of the piston is

a. $\frac{5}{\pi}$ m/s b. $\frac{10}{\pi}$ m/s
c. $\frac{\pi}{2}$ m/s d. $\frac{20}{\pi}$ m/s

6. The number of independent constituent simple harmonic motions yielding a resultant displacement equation of the periodic motion as $y = 8\sin^2(t/2)\sin(10t)$ is
a. 8 b. 6 c. 4 d. 3

7 The diagram below shows a sinusoidal curve. The equation of the curve will be

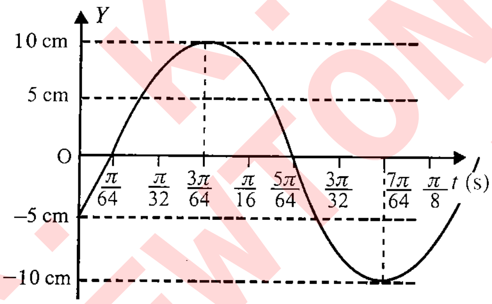

Fig. 4.104

a. $y = 10\sin\left(16t + \frac{\pi}{4}\right)$ cm b. $y = 10\sin\left(16t + \frac{\pi}{3}\right)$ cm
c. $y = 10\sin\left(16t - \frac{\pi}{4}\right)$ cm d. $y = 10\cos\left(16t + \frac{\pi}{4}\right)$ cm

8. The following figure shows the displacement versus time graph for two particles A and B executing simple harmonic motions. The ratio of their maximum velocities is

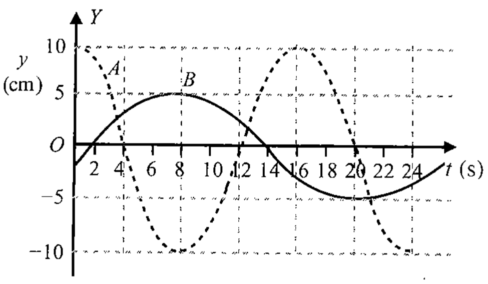

Fig. 4.105

a. 3:1 b. 1:3 c. 1:9 d. 9:1

9. The variation of velocity of a particle executing SHM with time is shown in Fig. 4.106. The velocity of the particle when a phase change of $\pi/6$ takes place from the instant it is at one of the extreme positions will be

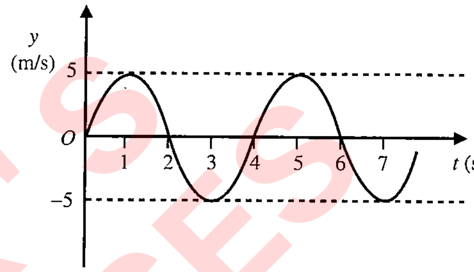

Fig. 4.106

a. 3.53 m/s b. 2.5 m/s
c. 4.330 m/s d. None of these

10. In the previous problem, the displacement of the particle from the mean position corresponding to the instant mentioned is

a. $\frac{5}{\pi}$ m b. $\frac{5\sqrt{3}}{\pi}$ m
c. $\frac{10\sqrt{3}}{\pi}$ m d. $\frac{5\sqrt{3}}{2\pi}$ m

11. In problem 9, the acceleration of the particle is
a. $\frac{5\sqrt{3}\pi}{2}$ m/s^2 b. $\frac{5\pi^2}{2}$ m/s^2
c. $\frac{5\sqrt{3}\pi}{4}$ m/s^2 d. $5\sqrt{3}\pi$ m/s^2

12. In problem 9, the maximum displacement and acceleration of the particle are respectively:
a. $\frac{10}{\pi}$ m and 5π m/s^2 b. $\frac{5}{\pi}$ m and $\frac{5\pi}{2}$ m/s^2
c. $\frac{10}{\pi}$ m and $\frac{5\pi}{2}$ m/s^2 d. $\frac{5}{\pi}$ m and $\frac{5\pi}{4}$ m/s^2

13. Figure 4.107 shows the variation of force acting on a particle of mass 400 g executing simple harmonic motion. The frequency of oscillation of the particle is

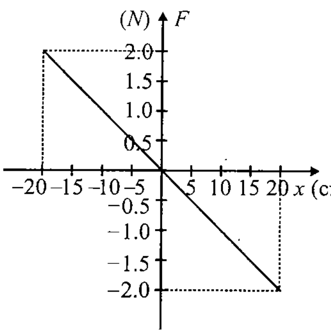

Fig. 4.107

a. 4 s^{-1} b. $(5/2\pi)$ s^{-1}
c. $(1/8\pi)$ s^{-1} d. $(1/2\pi)$ s^{-1}

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14. A block of mass 1 kg hangs without vibrating at the end of a spring whose force constant is 200 N/m and which is attached to the ceiling of an elevator. The elevator is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases. The angular frequency of the block after the acceleration ceases is

a. 13 rad/s **b.** 14 rad/s
c. 15 rad/s **d.** None of these

15. A vertical spring carries a 5 kg body and is hanging in equilibrium, an additional force is applied so that the spring is further stretched. When released from this position, it performs 50 complete oscillations in 25 s, with an amplitude of 5 cm. The additional force applied is

a. 80 N **b.** $80\pi^2$ N **c.** $4\pi^2$ N **d.** 4 N

16. A particle moves with a simple harmonic motion in a straight line. In the first second starting from rest it travels a distance a and in the next second it travels a distance b in the same direction. The amplitude of the motion is

a. $\frac{2a^2}{3b-a}$ **b.** $\frac{3a^2}{3a-b}$
c. $\frac{2a^2}{3a-b}$ **d.** $\frac{3a^2}{3b-a}$

17. A particle, free to move along the x-axis, has potential energy given by $U_{(x)} = K[1 - \exp(-x^2)]$ for $-\infty < x < +\infty$ where k is a positive constant of appropriate dimensions. Then

a. for small displacement from $x = 0$, the motion is simple harmonic
b. if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
c. for any finite non-zero value of x, there is a force directed away from the origin
d. at points away from the origin, the particle is in unstable equilibrium

18. Two simple harmonic motions are represented by equations

$$y_1 = 4\sin(10t + \phi)$$
$$y_2 = 5\cos 10t$$

What is the phase difference between their velocities?

a. ϕ **b.** $-\phi$
c. $\left(\phi + \frac{\pi}{2}\right)$ **d.** $\left(\phi - \frac{\pi}{2}\right)$

19. A simple pendulum is making oscillations with its bob immersed in a liquid of density n times less than the density of the bob. What is its period?

a. $2\pi\sqrt{\frac{l}{ng}}$ **b.** $2\pi\sqrt{\frac{l}{\left(1-\frac{1}{n}\right)g}}$
c. $2\pi\sqrt{\frac{ln}{g}}$ **d.** $2\pi\sqrt{\frac{l}{(n-1)g}}$

20. Two particles move parallel to the x-axis about the origin with same amplitude 'a' and frequency ω. At a certain instant they are found at a distance $a/3$ from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two?

a. $\cos^{-1}\frac{7}{9}$ **b.** $\cos^{-1}\frac{5}{9}$
c. $\cos^{-1}\frac{4}{9}$ **d.** $\cos^{-1}\frac{1}{9}$

21 The potential energy of a particle executing SHM along the x-axis is given by $U = U_0 - U_0\cos ax$. What is the period of oscillation?

a. $2\pi\sqrt{\frac{ma}{U_0}}$ **b.** $2\pi\sqrt{\frac{U_0}{ma}}$
c. $\frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$ **d.** $2\pi\sqrt{\frac{m}{aU_0}}$

22. A particle executing SHM of amplitude 'a' has a displacement $a/2$ at $t = T/4$ and a negative velocity. The epoch of the particle is

a. $\frac{\pi}{3}$ **b.** $\frac{2\pi}{3}$ **c.** π **d.** $\frac{5\pi}{3}$

23. A block of mass 4 kg hangs from a spring of spring constant $k = 400$ N/m. The block is pulled down through 15 cm below and released. What is its kinetic energy when the block is 10 cm above the equilibrium position?

a. 5 J **b.** 2.5 J
c. 1 J **d.** 1.9 J

24. A body of mass 100 g attached to a spring executes SHM of period 2 s and amplitude 10 cm. How long a time is required for it to move from a point 5 cm below its equilibrium position to a point 5 cm above it, when it makes simple harmonic vertical oscillations (take $g = 10$ m/s^2)?

a. 0.6 s **b.** 1/3 s **c.** 1.5 s **d.** 2.2 s

25. A particle executing SHM has velocities u and v and accelerations a and b in two of its positions. Find the distance between these two positions.

a. $\frac{u^2-v^2}{a+b}$ **b.** $\frac{v^2+u^2}{a-b}$
c. $\frac{v^2+u^2}{a+b}$ **d.** $\frac{v^2-u^2}{a-b}$

26. Two particles are executing identical simple harmonic motions described by the equations, $x_1 = a\cos(\omega t + (\pi/6))$ and $x_2 = a\cos(\omega t + \pi/3)$. The minimum interval of time between the particles crossing the respective mean positions is

a. $\frac{\pi}{2\omega}$ **b.** $\frac{\pi}{3\omega}$ **c.** $\frac{\pi}{4\omega}$ **d.** $\frac{\pi}{6\omega}$

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27. A particle of mass 4 kg moves between two points A and B on a smooth horizontal surface under the action of two forces such that when it is at a point P, the forces are $\vec{F_1} = 2(\vec{PA})N$ and $\vec{F_2} = 2(\vec{PB})N$. If the particle is released from rest from A, the time period of motion is

a. π s　　**b.** 2π s
c. 3π s　　**d.** $\sqrt{2}\pi$ s

28. The K.E. and P.E. of a particle executing SHM with amplitude A will be equal when its displacement is:

a. $A\sqrt{2}$　　**b.** $A/2$
c. $A/\sqrt{2}$　　**d.** $A\sqrt{2/3}$

29. A body is performing simple harmonic motion with amplitude a and time period T. Variation of its acceleration (f) with time (t) is shown in Fig. 4.108. If at time t, velocity of the body is v, which of the following graphs is correct ?

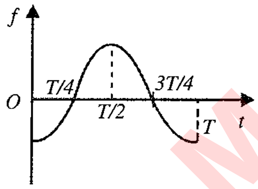
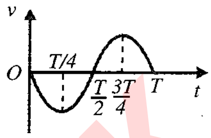
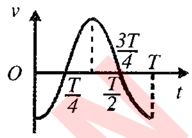
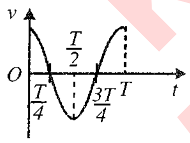
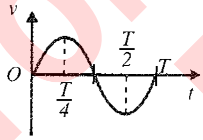


Fig. 4.108

a. 
b. 
c. 
d. 

30. A particle is performing SHM with amplitude a and time period T. Its acceleration f varies with time as shown in Fig. 4.109. If at time t, kinetic energy of the particle is K, which of the following graphs is correct?

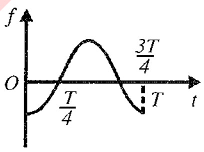
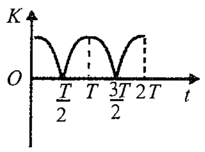
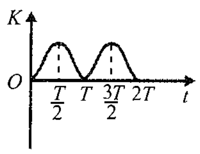
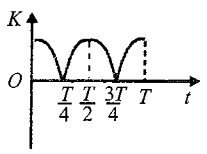
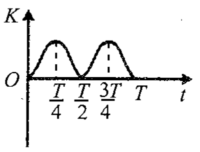


Fig. 4.109

a. 
b. 
c. 
d. 

31. A particle is performing SHM. Its kinetic energy K varies with time t as shown in the figure. Then

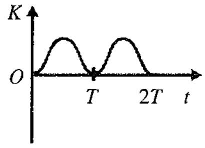
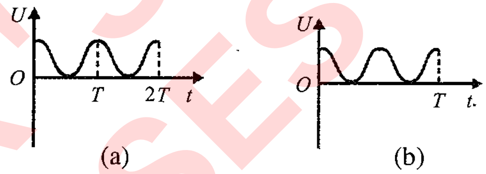


Fig. 4.110

a. period of oscillations of the particle is equal to T.



(a)　　(b)

Fig. 4.111

b. excess potential energy U of the particle varies with time t as shown in Fig. 4.111(a).
c. excess potential energy U of the particle varies with time t as shown in Fig. 4.111(b).
d. none of these.

32 Two particles P and Q describe SHM of same amplitude a and frequency ν along the same straight line. The maximum distance between the two particles is $\sqrt{2}\,a$. The initial phase difference between them is

a. zero　　**b.** $\pi/2$
c. $\pi/6$　　**d.** $\pi/3$

33 Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system; the amplitude of vibration is:

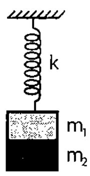


Fig. 4.112

a. m_1g/k　　**b.** m_2g/k
c. $\dfrac{(m_1+m_2)g}{k}$　　**d.** $\dfrac{(m_2-m_1)g}{k}$

34. A body of mass m is released from a height h to a scale pan hung from a spring. The spring constant of the spring is k, the mass of the scale pan is negligible and the body does not bounce relative to the pan; then the amplitude of vibration is

a. $\dfrac{mg}{k}\sqrt{1-\dfrac{2hk}{mg}}$　　**b.** $\dfrac{mg}{k}$
c. $\dfrac{mg}{k}+\dfrac{mg}{k}\sqrt{1+\dfrac{2hk}{mg}}$　　**d.** $\dfrac{mg}{k}-\dfrac{mg}{k}\sqrt{1-\dfrac{2hk}{mg}}$

35. Frequency of a particle executing SHM is 10 Hz. The particle is suspended from a vertical spring. At the highest point of its oscillation the spring is unstretched. Maximum speed of the particle is (g = 10 m/s^2)

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a. 2π m/s b. π m/s
c. $1/\pi$ m/s d. $1/2\pi$ m/s

36. The potential energy of a particle of mass 1 kg in motion along the x-axis is given by: $U = 4(1 - \cos 2x)$, where x is in metres. The period of small oscillation (in seconds) is

a. 2π b. π c. $\pi/2$ d. $\sqrt{2}\pi$

37. An object of mass 0.2 kg executes simple harmonic motion along the x-axis with a frequency of $25/\pi$ Hz. At the position $x = 0.04$ m, the object has a kinetic energy of 0.5 J and potential energy of 0.4 J. The amplitude of oscillation is

a. 0.05 m b. 0.06 m
c. 0.01 m d. none of these

38. The string of a simple pendulum is replaced by a uniform rod of length L and mass M while the bob has a mass m. It is allowed to make small oscillations. Its time period is

a. $2\pi\sqrt{\left(\frac{2M}{3m}\right)\frac{L}{g}}$ b. $2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$

c. $2\pi\sqrt{\left(\frac{M+m}{M+3m}\right)\frac{L}{g}}$ d. $2\pi\sqrt{\left(\frac{2m+M}{3(M+2m)}\right)\frac{L}{g}}$

39. A uniform semicircular ring having mass m and radius r is hanging at one of its ends freely as shown in Fig. 4.113. The ring is slightly disturbed so that it oscillates in its own plane. The time period of oscillation of the ring is

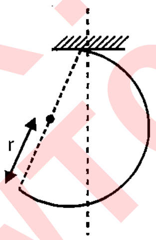

Fig. 4.113

a. $2\pi\sqrt{\frac{r}{g\left(1+\frac{1}{\pi^2}\right)}}$ b. $2\pi\sqrt{\frac{r}{g\left(1-\frac{4}{\pi^2}\right)^{1/2}}}$

c. $2\pi\sqrt{\frac{r}{g\left(1-\frac{2}{\pi^2}\right)^{1/2}}}$ d. $2\pi\sqrt{\frac{2r}{g\left(1+\frac{4}{\pi}\right)^{1/2}}}$

40. Two springs with negligible masses and force constants $k_1 = 200$ N/m and $k_2 = 160$ N/m are attached to the block of mass $m = 10$ kg as shown in the Fig. 4.114. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time $t = 0$, sharp impulse of 50 Ns is given to the block with a hammer along the spring.

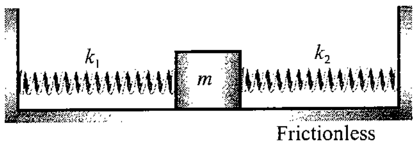

Fig. 4.114

a. Period of oscillations for the mass m is $\pi/6$ s.
b. Maximum velocity of the mass m during its oscillation is 10 m/s.
c. Data are insufficient to determine maximum velocity.
d. Amplitude of oscillation is 0.83 m.

41. A thin uniform vertical rod of mass m and length l pivoted at point O is shown in Fig. 4.115. The combined stiffness of the springs is equal to k. The mass of the spring is negligible. The frequency of small oscillation is

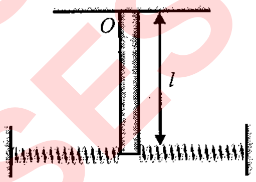

Fig. 4.115

a. $\sqrt{\frac{3k}{2m}+\frac{g}{l}}$ b. $\sqrt{\frac{3k}{2m}+\frac{3g}{l}}$

c. $\sqrt{\frac{3k}{m}+\frac{3g}{2l}}$ d. $\sqrt{\frac{3k}{m}+\frac{2g}{3l}}$

42. A particle executes SHM with time period 8 s. Initially, it is at its mean position. The ratio of distance travelled by it in the 1st second to that in the 2nd second is

a. $\sqrt{2}:1$ b. $1:(\sqrt{2}-1)$
c. $(\sqrt{2}+1):\sqrt{2}$ d. $(\sqrt{2}-1):1$

43. A particle executes S. H. M. starting from its mean position at $t = 0$. If its velocity is $\sqrt{3}b\omega$, when it is at a distance b from the mean position, when $\omega = 2\pi/T$, the time taken by the particle to move from b to the extreme position on the same side is

a. $\frac{5\pi}{6\omega}$ b. $\frac{\pi}{3\omega}$ c. $\frac{\pi}{2\omega}$ d. $\frac{\pi}{4\omega}$

44. In a certain oscillatory system (particle is performing SHM), the amplitude of motion is 5 m and the time period is 4 s. The minimum time taken by the particle for passing between points, which are at distances of 4 m and 3 m from the centre and on the same side of it will approximately be

a. $\frac{16}{45}$ s b. $\frac{7}{45}$ s

c. $\frac{8}{45}$ s d. $\frac{13}{45}$ s

45. A particle of mass m moving along the x-axis has a potential energy $U(x) = a + bx^2$ where a and b are positive constants. It will execute simple harmonic motion with a frequency determined by the value of

a. b alone b. b and a alone
c. b and m alone d. b, a and m alone

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46. Two simple pendulums of lengths l and $4l$ are suspended from same point and brought aside together and released at the same time. If the time period of smaller pendulum is T then after how much time will they be together again and moving in the same direction?

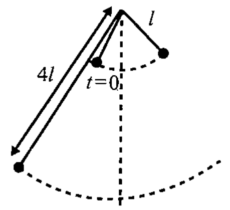

Fig. 4.116

a. $T/2$ **b.** T

c. $2T$ **d.** none of these

47. The instantaneous displacement x of a particle executing simple harmonic motion is given by $x = a_1 \sin \omega t + a_2 \cos (\omega t + \pi/6)$. The amplitude A of oscillation is given by

a. $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\frac{\pi}{6}}$ **b.** $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\frac{\pi}{3}}$

c. $\sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos\frac{\pi}{6}}$ **d.** $\sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos\frac{\pi}{3}}$

48. A simple harmonic motion along the x-axis has the following properties: amplitude = 0.5 m, the time to go from one extreme position to other is 2 s and $x = 0.3$ m at $t = 0.5$ s. The general equation of the simple harmonic motion is

a. $x = (0.5\text{ m}) \sin\left[\frac{\pi t}{2} + 8°\right]$

b. $x = (0.5\text{ m}) \sin\left[\frac{\pi t}{2} - 8°\right]$

c. $x = (0.5\text{ m}) \cos\left[\frac{\pi t}{2} + 8°\right]$

d. $x = (0.5\text{ m}) \cos\left[\frac{\pi t}{2} - 8°\right]$

49. A spring balance has a scale that can read from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance when displaced and released oscillates harmonically with a time period of 0.6 s. The mass of the body is (take $g = 10$ m/s^2)

a. 10 kg **b.** 25 kg

c. 18 kg **d.** 22.8 kg

50. A soil cylinder of mass M and radius R is connected to a spring as shown in Fig. 4.117. The cylinder is placed on a rough horizontal surface. All the parts except the cylinder shown in the figure are light. If the cylinder is displaced slightly from its mean position and released, so that it performs pure rolling back and forth about its equilibrium position, determine the time period of oscillation?

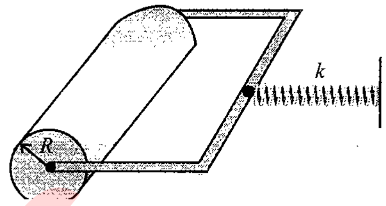

Fig. 4.117

a. $2\pi\sqrt{\frac{M}{k}}$ **b.** $2\pi\sqrt{\frac{3M}{2k}}$

c. $2\pi\sqrt{\frac{3M}{k}}$ **d.** None of these

51. A block A is connected to spring and performs simple harmonic motion with a time period of 2 s. Another block B rests on a. The coefficient of static friction between A and B is $\mu_s = 0.6$. the maximum amplitude of oscillation which the system can have so that there is no relative motion between A and B is (take $\pi^2 = g = 10$)

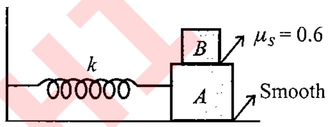

Fig. 4.118

a. 0.3 m **b.** 0.6 m

c. 0.4 m **d.** 0.52 m

52. A block of mass 10 kg is in equilibrium as shown in Fig. 4.119. Initially the springs have same stretch. If the block is displaced in vertical direction by a small amount, then the angular frequency of resulting motion is (assume strings are never slack)

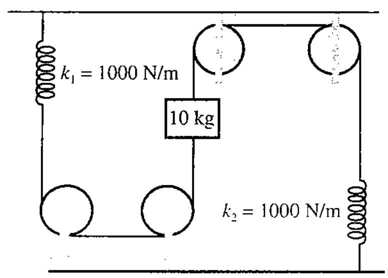

Fig. 4.119

a. $10\sqrt{2}$ **b.** $10\sqrt{5}$

c. $5\sqrt{2}$ **d.** $5\sqrt{5}$

53. A block of mass m is suspended from the ceiling of an elevator (at rest) through a light spring of spring constant k. Suddenly, the elevator starts falling down with acceleration g. Then

a. the block executes simple harmonic motion with time period $2\pi\sqrt{\frac{m}{k}}$.

b. the block executes simple harmonic motion with amplitude $\frac{mg}{k}$.

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c. the block executes simple harmonic motion about its mean position and the mean position is the position when the spring acquires its natural length.

d. All of the above.

54. A mass m attached to a spring of spring constant k is stretched a distance x_0 from its equilibrium position and released with no initial velocity. The maximum speed attained by mass in its subsequent motion and the time at which this speed would be attained are, respectively,

a. $\sqrt{\frac{k}{m}}x_0, \pi\sqrt{\frac{m}{k}}$ **b.** $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \frac{\pi}{2}\sqrt{\frac{m}{k}}$

c. $\sqrt{\frac{k}{m}}x_0, \frac{\pi}{2}\sqrt{\frac{m}{k}}$ **d.** $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \pi\sqrt{\frac{m}{k}}$

55. A plank of mass 12 kg is supported by two identical springs as shown in Fig. 4.120. The plank always remains horizontal. When the plank is pressed down and released, it performs simple harmonic motion with time period 3 s. When a block of mass m is attached to the plank the time period changes to 6 s. The mass of the block is

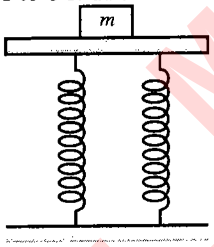

Fig. 4.120

a. 48 kg **b.** 36 kg **c.** 24 kg **d.** 12 kg

56. The time taken by a particle executing simple harmonic motion to pass from point A to B where its velocities are same, is 2 s. After another 2 s, it returns to (b). The time period of oscillation is

a. 2 s **b.** 4 s **c.** 6 s **d.** 8 s

57. Two springs are made to oscillate simple harmonically due to the same mass individually. The time periods obtained are T_1 and T_2. If both the springs are connected in series and then made to oscillate by the same mass, the resulting time period will be

a. $T_1 + T_2$ **b.** $\frac{T_1T_2}{T_1+T_2}$

c. $\sqrt{T_1^2+T_2^2}$ **d.** $\frac{T_1+T_2}{2}$

58. A thin-walled tube of mass m and radius R has a rod of mass m and very small cross section soldered on its inner surface. The side-view of the arrangement is as shown in the following figure.

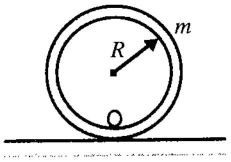

Fig. 4.121

The entire arrangement is placed on a rough horizontal surface. The system is given a small angular displacement from its equilibrium position, as a result, the system performs oscillations. The time period of resulting oscillations if the tube rolls without slipping is

a. $2\pi\sqrt{\frac{4R}{g}}$ **b.** $2\pi\sqrt{\frac{2R}{g}}$

c. $2\pi\sqrt{\frac{R}{g}}$ **d.** None of these

59. A thin uniform rod of mass 1 kg and length 12 cm is suspended by a wire that passes through its centre and is perpendicular to its length. The wire is twisted and the rod is set oscillating. Time period of oscillation is found to be 3 s. When a flat triangular plate is suspended in same way through its centre of mass, the time period is found to be 6 s. The moment of inertia of the triangular plate about this axis is

a. 0.12 kg-m² **b.** 0.24 kg-m²

c. 0.48 kg-m² **d.** Information insufficient

60. A particle performs SHM about $x = 0$ such that at $t = 0$ it is at $x = 0$ and moving towards positive extreme. The time taken by it to go from $x = 0$ to $x = A/2$ is .... times the time taken to go from $x = A/2$ to a. The most suitable option for the blank space is

a. 2 **b.** $\frac{1}{2}$ **c.** $\frac{11}{12}$ **d.** $\frac{12}{11}$

61. A particle performs simple harmonic motion with amplitude A and time period T. The mean velocity of the particle over the time interval during which it travels a distance of $A/2$ starting from executing position is

a. $\frac{A}{T}$ **b.** $\frac{2A}{T}$ **c.** $\frac{3A}{T}$ **d.** $\frac{A}{2T}$

62. A particle performs simple harmonic motion about O with amplitude A and time period T. The magnitude of its acceleration at $t = T/8$ s after the particle reaches the extreme position would be

a. $\frac{4\pi^2A}{\sqrt{2}T^2}$ **b.** $\frac{4\pi^2A}{T^2}$

c. $\frac{2\pi^2A}{\sqrt{2}T^2}$ **d.** None of these

63. In the previous question, the magnitude of velocity of particle at the mentioned instant is

a. $\frac{\pi A}{T}$ **b.** $\frac{\sqrt{2}\pi A}{T}$ **c.** zero **d.** $\sqrt{\frac{7}{8}}\times\frac{2\pi A}{T}$

64. An object of mass 4 kg is attached to a spring having spring constant 100 N/m. It performs simple harmonic motion on a smooth horizontal surface with an amplitude of 2 m. A 6 kg object is dropped vertically onto the 4 kg object when it crosses the mean position, and sticks to it. The change in amplitude of oscillation due to collision is

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a. 1 m b. zero

c. $2\left[1-\sqrt{\frac{2}{5}}\right]$ d. $2\left[1-\frac{1}{\sqrt{5}}\right]$

65. A cork floating on the pond water executes a simple harmonic motion, moving up and down over a range of 4 cm. The time period of the motion is 1 s. At $t = 0$, the cork is at its lowest position of oscillation, the position and velocity of the cork at $t = 10.5$ s, would be

a. 2 cm above the mean position, 0 m/s
b. 2 cm below the mean position, 0 m/s
c. 1 cm above the mean position, $2\sqrt{3}\pi$ m/s up
d. 1 cm below the mean position, $2\sqrt{3}\pi$ m/s up

66. A spring is placed in vertical position by suspending it from a hook at its top. A similar hook on the bottom of the spring is at 11 cm above a table top. A mass of 75 g and of negligible size is then suspended from the bottom hook, which is measured to be 4.5 cm above the table top. The mass is then pulled down a distance of 4 cm and released. Find the approximate. position of the bottom hook after 5 s? Take $g = 10$ m/s^2 and hook's mass to be negligible.

a. 5 cm above the table top
b. 4.5 cm above the table top
c. 9 cm above the table top
d. 0.5 cm above the table top

67. A particle is performing SHM according to the equation $x = (3\text{ cm})\sin\left(\frac{2\pi}{18}+\frac{\pi}{6}\right)$, where t is in seconds. The distance travelled by the particle in 39 s is

a. 24 cm b. 1.5 cm
c. 25.5 cm d. None of these

68. Two springs, each of unstretched length 20 cm but having different spring constants k_1 = 1000 N/m and k_2 = 3000 N/m, are attached to two opposite faces of a small block of mass m = 100 g kept on a smooth horizontal surface as shown in the Fig. 4.122.

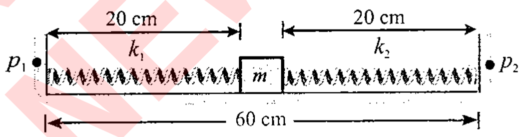

Fig. 4.122

The outer ends of the two springs are now attached to two pins P_1 and P_2 whose locations are shown in the figure. As a result of this, the block acquires a new equilibrium position. The block has been displaced by small amount from its equilibrium position and released to perform simple harmonic motion; then

a. new equilibrium position is at 35 cm from P_1 and time period of simple harmonic motion is $\pi/100$ s.
b. new equilibrium position is at 20 cm from P_1 and time period of simple harmonic motion is $\pi/100$ s.
c. new equilibrium position is at 35 cm from P_1 and time period of simple harmonic motion is $\pi/26$ s.
d. new equilibrium position is at 30 cm from P_1 and time period of simple harmonic motion is $\pi/26$ s.

69. A particle of mass m is present in a region where the potential energy of the particle depends on the x-coordinate according to the expression $U = \frac{a}{x^2} - \frac{b}{x}$, where a and b are positive constants. The particle will perform

a. oscillatory motion but not simple harmonic motion about its mean position for small displacements
b. simple harmonic motion with time period $2\pi\sqrt{\frac{8a^2m}{b^4}}$ about its mean position for small displacements
c. neither simple harmonic motion nor oscillatory about its mean position for small displacements
d. none of the above

70. A particle performing simple harmonic motion having time period 3 s is in phase with another particle which also undergoes simple harmonic motion at $t = 0$. The time period of second particle is T (less than 3 s). If they are again in the same phase for the third time after 45 s, then the value of T will be

a. 2.8 s b. 2.7 s
c. 2.5 s d. None of these

71. A particle performs SHM on the x-axis with amplitude A and time period T. The time taken by the particle to travel a distance $A/5$ starting from rest is

a. $\frac{T}{20}$ b. $\frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$

c. $\frac{T}{2\pi}\cos^{-1}\left(\frac{1}{5}\right)$ d. $\frac{T}{2\pi}\sin^{-1}\left(\frac{1}{5}\right)$

72. A rod of length l is in motion such that its ends A and B are moving along the x-axis and the y-axis, respectively. It is given that $d\theta/dt$ = 2 rad/s always. P is a fixed point on the rod. Let M be the projection of P on the x-axis. For the time interval in which θ changes from 0 to $\pi/2$, choose the correct statement.

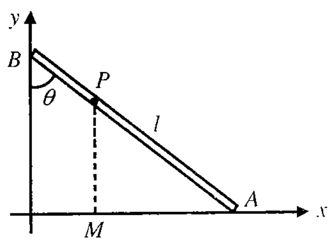

Fig. 4.123

a. The acceleration of M is always directed towards right.
b. M executes SHM.
c. M moves with constant speed
d. M moves with constant acceleration.

73. The coefficient of friction between block of mass m and $2m$ is $\mu = 2\tan\theta$. There is no friction between block of mass $2m$ and inclined plane. The maximum amplitude of the two block system for which there is no relative motion between both the blocks is

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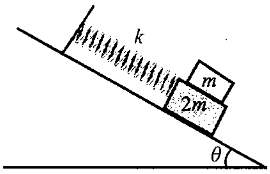

Fig. 4.124

a. $g\sin\theta\sqrt{\dfrac{k}{m}}$ b. $\dfrac{mg\sin\theta}{k}$

c. $\dfrac{3mg\sin\theta}{k}$ d. None of these

74. A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in Fig. 4.125. The amplitude of the SHM is A and spring is never in compressed state during the oscillation. The magnitude of minimum force exerted by spring on the block is

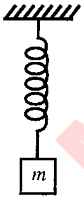

Fig. 4.125

a. $mg-\dfrac{4\pi^2}{T^2}mA$ b. $mg+\dfrac{4\pi^2}{T^2}mA$

c. $mg-\dfrac{\pi^2}{T^2}mA$ d. $mg+\dfrac{\pi^2}{T^2}mA$

75. A particle performs SHM of amplitude A along a straight line. When it is at a distance $\sqrt{3}/2\,A$ from mean position, its kinetic energy gets increased by an amount $1/2\, m\omega^2A^2$ due to an impulsive force. Then its new amplitude becomes

a. $\dfrac{\sqrt{5}}{2}A$ b. $\dfrac{\sqrt{3}}{2}A$ c. $\sqrt{2}A$ d. $\sqrt{5}A$

76. A horizontal spring–block system of mass 2 kg executes SHM. When the block is passing through its equilibrium position, an object of mass 1 kg is put on it and the two move together. The new amplitude of vibration is (A being its initial amplitude)

a. $\sqrt{\dfrac{2}{3}}A$ b. $\sqrt{\dfrac{3}{2}}A$ c. $\sqrt{2}\,A$ d. $\dfrac{A}{\sqrt{2}}$

77. A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick were cut off, then its new frequency of small oscillation would become

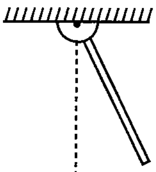

Fig. 4.126

a. f_0 b. $\sqrt{2}\,f_0$ c. $2f_0$ d. $2\sqrt{2}\,f_0$

78. A particle of mass $m = 2$ kg executes SHM in xy plane between points A and B under the action of force $\vec{F} = F_x\hat{i} + F_y\hat{j}$. Minimum time taken by the particle to move from A to B is 1 s. At $t = 0$ the particle is at $x = 2$ and $y = 2$. Then F_x as function of time t is

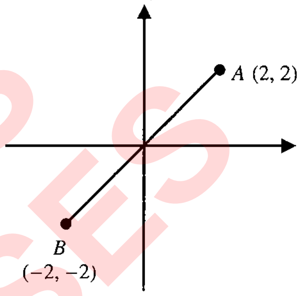

Fig. 4.127

a. $-4\pi^2\sin\pi t$ b. $-4\pi^2\cos\pi t$

c. $4\pi^2\cos\pi t$ d. None of these

79. A physical pendulum is positioned so that its centre of gravity is above the suspension point. When the pendulum is released it passes the point of stable equilibrium with an angular velocity ω. The period of small oscillations of the pendulum is

a. $\dfrac{4\pi}{\omega}$ b. $\dfrac{2\pi}{\omega}$ c. $\dfrac{\pi}{\omega}$ d. $\dfrac{\pi}{2\omega}$

80. A particle executing harmonic motion is having velocities v_1 and v_2 at distances is x_1 and x_2 from the equilibrium position. The amplitude of the motion is

a. $\sqrt{\dfrac{v_1^2x_2 - v_2^2x_1}{v_1^2+v_2^2}}$ b. $\sqrt{\dfrac{v_1^2x_1^2 - v_2^2x_2^2}{v_1^2+v_2^2}}$

c. $\sqrt{\dfrac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2-v_2^2}}$ d. $\sqrt{\dfrac{v_1^2x_2^2 + v_2^2x_1^2}{v_1^2+v_2^2}}$

81. A wire is bent at an angle θ. A rod of mass m can slide along the bended wire without friction as shown in Fig. 4.128. A soap film is maintained in the frame kept in a vertical position and the rod is in equilibrium as shown in the figure. If rod is displaced slightly in vertical direction, then the time period of small oscillation of the rod is

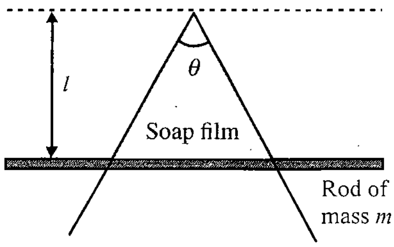

Fig. 4.128

a. $2\pi\sqrt{\dfrac{l}{g}}$ b. $2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

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c. $2\pi\sqrt{\dfrac{l}{g\cos\theta}}$ d. $2\pi\sqrt{\dfrac{l}{g\tan\theta}}$

82. A solid right circular cylinder of weight 10 kg and cross-sectional area 100 cm^2 is suspended by a spring, where k = 1 kg/cm, and hangs partially submerged in water of density 1000 kg/m^3 as shown in Fig. 4.129. What is its period when it makes simple harmonic vertical oscillations? (Take g = 10 m/s^2)

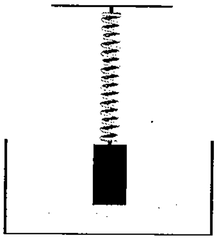

Fig. 4.129

a. 0.6 s b. 1 s
c. 1.5 s d. 2.2 s

83. A block 'A' of mass m is placed on a smooth horizontal platform P and between two elastic massless springs S_1 and S_2 fixed horizontally to two fixed vertical walls. The elastic constants of the two springs are equal to k and the equilibrium distance between the two springs both in relaxed states is d. The block is given a velocity v_0 initially towards one of the springs and it then oscillates between the springs. The time period T of oscillations and the minimum separation d_m of the springs will be

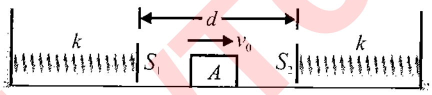

Fig. 4.130

a. $T=2\left(\dfrac{d}{v}+\pi\sqrt{\dfrac{m}{k}}\right)$, $d_m=d$

b. $T=2\left(\dfrac{d}{v}+2\pi\sqrt{\dfrac{m}{k}}\right)$, $d_m=d-v\sqrt{\dfrac{m}{k}}$

c. $T=2\left(\dfrac{d}{v}+2\pi\sqrt{\dfrac{m}{k}}\right)$, $d_m=d-2v\sqrt{\dfrac{m}{k}}$

d. $T=2\pi\sqrt{\dfrac{m}{k}}$, $d_m=d$

84. A certain simple harmonic vibrator of mass 0.1 kg has a total energy of 10 J. Its displacement from the mean position is 1 cm when it has equal kinetic and potential energies. The amplitude A and frequency n of vibration of the vibrator are

a. $A=\sqrt{2}$ cm, $n=\dfrac{500}{\pi}$ Hz b. $A=\sqrt{2}$ cm, $n=\dfrac{1000}{\pi}$ Hz

c. $A=\dfrac{1}{\sqrt{2}}$ cm, $n=\dfrac{500}{\pi}$ Hz d. $A=\dfrac{1}{\sqrt{2}}$ cm, $n=\dfrac{1000}{\pi}$ Hz

85. A simple pendulum of length l and a mass m of the bob is suspended in a car that is travelling with a constant speed v around a circle of radius R. If the pendulum undergoes small oscillations about its equilibrium position, the frequency of its oscillation will be

a. $\dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$ b. $\dfrac{1}{2\pi}\sqrt{\dfrac{g}{R}}$

c. $\dfrac{1}{2\pi}\sqrt{\dfrac{\left(g^2+\dfrac{v^4}{R^2}\right)^{1/2}}{l}}$ d. $\dfrac{1}{2\pi}\sqrt{\dfrac{v^2}{Rl}}$

86. One end of a spring of force constant K is fixed to a vertical wall and the other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $3x_0$ and released. The time taken to strike the wall from the instant of release is (given $\sin^{-1}(1/3)=(\pi/9)$)

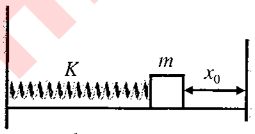

Fig. 4.131

a. $\dfrac{\pi}{6}\sqrt{\dfrac{m}{K}}$ b. $\dfrac{2\pi}{3}\sqrt{\dfrac{m}{K}}$

c. $\dfrac{\pi}{4}\sqrt{\dfrac{m}{K}}$ d. $\dfrac{11\pi}{9}\sqrt{\dfrac{m}{K}}$

87. A block P of mass m is placed on a smooth horizontal surface. A block Q of same mass is placed over the block P and the coefficient of static friction between them is μ_s. A spring of spring constant K is attached to block Q. The blocks are displaced together to a distance A and released. The upper block oscillates without slipping over the lower block. The maximum frictional force between the block is

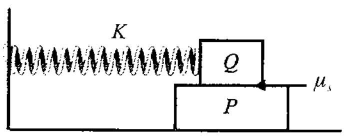

Fig. 4.132

a. zero b. K c. $KA/2$ d. μg

88. A uniform stick of mass M and length L is pivoted at its centre. Its ends are tied to two springs each of force constant K. In the position shown in figure, the strings are in their natural length. When the string is displaced through a small angle θ and released, the stick

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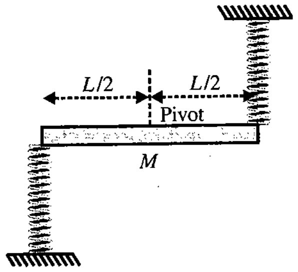

Fig. 4.133

a. executes non-periodic motion.
b. executes periodic motion which is not simple harmonic.
c. executes SHM of frequency $\frac{1}{2\pi}\sqrt{\frac{6K}{M}}$.
d. executes SHM of frequency $\frac{1}{2\pi}\sqrt{\frac{K}{2M}}$.

89. A block of mass m, attached to a fixed position O on a smooth inclined wedge of mass M, oscillates with amplitude A and linear frequency f. The wedge is located on a rough horizontal surface. If the angle of the wedge is 60°, then the force of friction acting on the wedge is given by (coefficient of static friction $=\mu$)

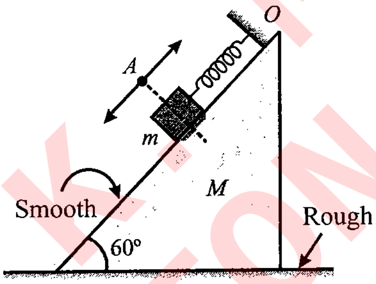

Fig. 4.134

a. $\mu(M+m)g$
b. $\frac{1}{2}m\omega^2 A\sin\omega t$
c. $\mu\left[(M+m)g+\frac{\sqrt{3}}{2}m\omega^2 A\sin\omega t\right]$
d. $\mu(M+m)\omega^2 A\sin\omega t$

90. A mass m is suspended from a spring of force constant k and just touches another identical spring fixed to the floor as shown in the Fig. 4.135. The time period of small oscillations is

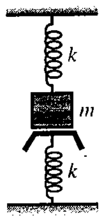

Fig. 4.135

a. $2\pi\sqrt{\frac{m}{k}}$
b. $\pi\sqrt{\frac{m}{k}}+\pi\sqrt{\frac{m}{k/2}}$
c. $\pi\sqrt{\frac{m}{3k/2}}$
d. $\pi\sqrt{\frac{m}{k}}+\pi\sqrt{\frac{m}{2k}}$

Multiple Correct Answers Type

Solutions on page 4.73

1. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
a. at the highest position of the platform
b. at the mean position of the platform
c. for an amplitude of g/ω^2
d. for an amplitude of $\sqrt{g/\omega}$

2. For a simple harmonic motion with given angular frequency ω, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be
a. initial position and initial velocity
b. amplitude and initial phase
c. total energy of oscillation and amplitude
d. total energy of oscillation and initial phase

3. The potential energy U of a body of unit mass moving in one dimensional conservative force field is given by $U = x^2 - 4x + 3$. All units are in SI. For this situation mark out the correct statement(s).
a. The body will perform simple harmonic motion about $x = 2$ units.
b. The body will perform oscillatory motion but not simple harmonic motion.
c. The body will perform simple harmonic motion with time period $\sqrt{2}\pi$ s.
d. If speed of the body at equilibrium position is 4 m/s, then the amplitude of oscillation would be $2\sqrt{2}$ m.

4. For the spring pendulum shown in Fig. 4.136, the value of spring constant is 3×10^4 N/m and amplitude of oscillation is 0.1 m. The total mechanical energy of oscillating system is 200 J. Mark out the correct option(s).

Fig. 4.136

a. Minimum PE of the oscillating system is 50 J
b. Maximum PE of the oscillating system is 200 J.
c. Maximum KE of the oscillating system is 200 J.
d. Minimum KE of the oscillating system is 150 J.

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5. An object of mass m is performing simple harmonic motion on a smooth horizontal surface as shown in Fig. 4.137. Just as the oscillating object reaches its extreme position, another object of mass $2m$ is dropped on to oscillating object, which sticks to it. For this situation mark out the correct statement(s).

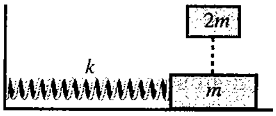

Fig. 4.137

a. Amplitude of oscillation remains unchanged.
b. Time period of oscillation remains unchanged
c. The total mechanical energy of the system does not change.
d. The maximum speed of the oscillating object changes.

6. A simple pendulum consists of a bob of mass m and a light string of length l as shown in the Fig. 4.138. Another identical ball moving with the small velocity v_0 collides with the pendulum's bob and sticks to it. For this new pendulum of mass $2m$, mark out the correct statement(s).

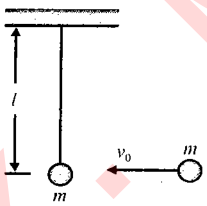

Fig. 4.138

a. Time period of the pendulum is $2\pi\sqrt{\frac{l}{g}}$.

b. The equation of motion for this pendulum is
$\theta = \frac{v_0}{2\sqrt{gl}}\sin\left[\sqrt{\frac{g}{l}}t\right]$.

c. The equation of motion for this pendulum is
$\theta = \frac{v_0}{2\sqrt{gl}}\cos\left[\sqrt{\frac{g}{l}}t\right]$.

d. Time period of the pendulum is $2\pi\sqrt{\frac{2l}{g}}$.

7. A particle performing simple harmonic motion undergoes initial displacement of $A/2$ (where A is the amplitude of simple harmonic motion) in 1 s. At $t = 0$, the particle may be at the extreme position or mean position. The time period of the simple harmonic motion can be

a. 6 s **b.** 2.4 s **c.** 12 s **d.** 1.2 s

8. A particle is subjected to two simple harmonic motions along x and y directions according to $x = 3\sin 100\pi t$; $y = 4\sin 100\pi t$.
a. Motion of particle will be on ellipse travelling in clockwise direction.
b. Motion of particle will be on a straight line with slope 4/3.
c. Motion will be simple harmonic motion with amplitude 5.
d. Phase difference between two motions is $\pi/2$.

9. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by $v^2 = 108 - 9x^2$ (all equations are in CGS units):
a. the motion is uniformly accelerated along the straight line
b. the magnitude of the acceleration at a distance 3 cm from the point is 27 cm/s^2
c. the motion is simple harmonic about the given fixed point
d. the maximum displacement from the fixed point is 4 cm

10. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest.Then
a. the period of oscillation is $\left(\frac{2\pi}{5}\right)$
b. the block weighs double its actual weight, then the plank is at one of the positions of momentary rest.
c. the block weighs 0.5 times its weight on the plank halfway up
d. the block weighs 1.5 times its weight on the plank halfway down
e. the block weighs its true weight on the plank when the later moves fastest

11. A 20 g particle is subjected to two simple harmonic motions $x_1 = 2\sin 10t$, $x_2 = 4\sin\left(10t + \frac{\pi}{3}\right)$, where x_1 and x_2 are in metres and t is in seconds.
a. The displacement of the particle at $t = 0$ will be $2\sqrt{3}$ m.
b. Maximum speed of the particle will be $20\sqrt{7}$ m/s.
c. Magnitude of maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2.
d. Energy of the resultant motion will be 28 J.

12. A coin is placed on a horizontal platform AB which undergoes SHM about mean point O in a vertical plane. The coin does not slip on the platform. The force of friction acting on the coin is F. Then
a. F is always directed towards one end of platform.
b. F is directed towards A when the coin tends to move away from A and away from B when the coin tends to move towards (b)
c. F is minimum when the coin and the platform come to rest momentarily at the one extreme position of SHM.
d. F is maximum when the coin and the platform come to rest momentarily at the other extreme position of SHM.

13. A spring block system undergoes SHM on a smooth horizontal surface, the block is now given some charge and a uniform horizontal electric field E is switched on as shown in Fig. 4.139. As a result

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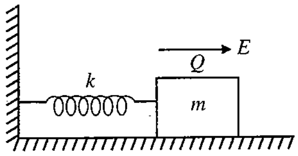

Fig. 4.139

a. time period of oscillation will increase

b. time period of oscillation will decrease

c. time period of oscillation will remain unaffected

d. the mean position of SHM will shift to the right

14. The potential energy of a particle of mass 0.1 kg, moving along the x–axis, is given by $U = 5x(x - 4)$ J, where x is in meter. It can be concluded that

a. the particle is acted upon by a constant force

b. the speed of the particle is maximum at $x = 2$ m

c. the particle executes SHM

d. the period of oscillation of the particle is $(\pi/5)$ s

15. The time period of a particle in simple harmonic motion is T. Assume potential energy at mean position to be zero. After a time of $T/6$ it passes its mean position, its

a. velocity will be half its maximum velocity

b. displacement will be half its amplitude

c. acceleration will be nearly 86% of its maximum acceleration

d. KE = PE

16. Figure 4.140(a) shows a spring of force constant k fixed at one end and carrying a mass m at the other end placed on a horizontal frictionless surface. The spring is stretched by a force F. Figure 4.140(b) shows the same spring with both ends free and a mass m fixed at each free end. Each of the spring is stretched by the same force F. The mass in case (a) and the masses in case (b) are then released. Which of the following statements are true?

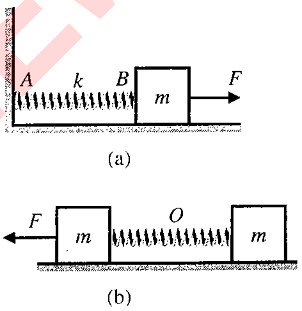

Fig. 4.140

a. While oscillating, the maximum extension of the spring is more in case (a) than in case (b).

b. The maximum extension of the spring is same in both cases.

c. The time period of oscillation is the same in both cases.

d. The time period of oscillation in case (a) is $\sqrt{2}$ times that in case (b).

17. When the point of suspension of pendulum is moved, its period of oscillation

a. decreases when it moves vertically upwards with an acceleration a

b. decreases when it moves vertically downwards with acceleration greater than 2 g

c. increases when it moves horizontally with acceleration a

d. all of the above

18. The displacement–time relation for a particle can be expressed as $y = 0.5[\cos^2(n\pi t) - \sin^2(n\pi t)]$
This relation shows that

a. the particle is executing a SHM with amplitude 0.5 m

b. the particle is executing a SHM with a frequency n times that of a second's pendulum

c. the particle is executing a SHM and the velocity in its mean position is $(n\pi)$ m/s

d. the particle is not executing a SHM at all

19. At two particular closest instants of time t_1 and t_2 the displacements of a particle performing SHM are equal. At these instants

a. instantaneous speeds are equal

b. instantaneous accelerations are equal

c. phase of the motion are unequal

d. kinetic energies are equal

20. Two blocks connected by a spring rest on a smooth horizontal plane as shown in Fig. 4.141. A constant force F starts acting on block m_2 as shown in the figure. Which of the following statements are not correct?

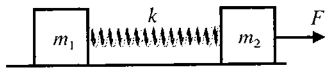

Fig. 4.141

a. Length of the spring increases continuously if $m_1 > m_2$.

b. Blocks start performing SHM about centre of mass of the system, which moves rectilinearly with constant acceleration.

c. Blocks start performing oscillations about centre of mass of the system with increasing amplitude.

d. Acceleration of m_2 is maximum at initial moment of time only.

21. A block of mass m is suspended by a rubber cord of natural length l = mg/k, where k is force constant of the cord. The block is lifted upwards so that the cord becomes just tight and then block is released suddenly. Which of the following will not be true?

a. Block performs periodic motion with amplitude greater than l.

b. Block performs SHM with amplitude equal to l.

c. Block will never return to the position from where it was released.

d. Angular frequency ω is equal to 1 rad/s.

22. The displacement (x) of a particle as a function of time (t) is given by

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$$x = a \sin (bt + c)$$

where a, b and c are constants of motion. Choose the correct statements from the following.

a. The motion repeats itself in a time interval of $2\pi/b$

b. The energy of the particle remains constant.

c. The velocity of the particle is zero at $x = \pm a$

d. The acceleration of the particle is zero at $x = \pm a$

23. A simple pendulum is oscillating between extreme positions P and Q about the mean position O. Which of the following statements are true about the motion of pendulum?

a. At point O, the acceleration of the bob is different from zero.

b. The acceleration of the bob is constant throughout the oscillation.

c. The tension in the string is constant throughout the oscillation.

d. The tension is maximum at O and minimum at P or Q

24. A cylindrical block of density d stays fully immersed in a beaker filled with two immiscible liquids of different densities d_1 and d_2. The block is in equilibrium with half of it in liquid 1 and the other half in liquid 2 as shown in the Fig. 4.142. If the block is given a displacement downwards and released, then neglecting friction study the following statements.

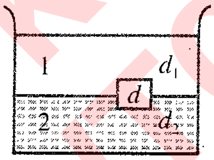

Fig. 4.142

a. It executes simple harmonic motion.

b. Its motion is periodic but not simple harmonic.

c. The frequency of oscillation is independent of the size of the cylinder.

d. The displacement of the centre of the cylinder is symmetric about its equilibrium position.

25. A mass of 0.2 kg is attached to the lower end of a massless spring of force constant 200 N/m, the upper end of which is fixed to a rigid support. Study the following statements.

a. In equilibrium the spring will be stretched by 1 cm.

b. If the mass is raised till the spring becomes unstretched and then released, it will go down by 2 cm before moving upwards.

c. The frequency of oscillation will be nearly 5 Hz.

d. If the system is taken to the moon, the frequency of oscillation will be the same as that on the earth.

26. A spring of spring constant k stores 5 J of energy when stretched by 25 cm. It is kept vertical with one end fixed. A mass m is attached to the other end. It makes 5 oscillations per second. Then

a. $m = 0.16$ kg b. $m = 0.32$ kg

c. $k = 160$ N/m d. $k = 320$ N/m

27. A particle is subjected to SHM as given by equations $x_1 = A_1 \sin\omega t$ and $x_2 = A_2 \sin(\omega t + \pi/3)$. The maximum acceleration and amplitude of the resultant motion are a_{max} and A, respectively. Then

a. $a_{max} = \omega^2 \sqrt{A_1^2 + A_2^2 + A_1 A_2}$

b. $a_{max} = \omega^2 \sqrt{A_1 A_2}$

c. $A = A_1 + A_2$

d. $A = \sqrt{A_1^2 + A_2^2 + A_1 A_2}$

28. Three simple harmonic motions in the same direction, each of amplitude 'a' and periodic time 'T', are superposed. The first and second, and the second and third differ in phase from each other by $\pi/4$, with the first and third not being identical. Then

a. the resultant motion is not simple harmonic

b. the resultant amplitude is $(\sqrt{2}+1)a$

c. the phase difference between the second SHM and the resultant motion is zero

d. the energy in the resultant motion is three times the energy in each separate SHM

Assertion-Reasoning Type

Solutions on page 4.78

In the following questions, each question contains Statement I (Assertion) and Statement II (reason). Each question has four choices (a), (b), (c) and (d) out of which *only one* is correct.

a. **Statement I** is true, Statement II is true; Statement II is a correct explanation for Statement I.

b. **Statement I** is true, Statement II is true; Statement II is NOT a correct explanation for Statement I.

c. **Statement I** is true; Statement II is false.

d. **Statement I** is false; Statement II is true.

1. Statement I: The total energy of a particle performing simple harmonic motion could be negative.
Statement II: Potential energy of a system could be magnetic.

2. Statement I: Two cubical blocks of same material and of sides a and $2a$, respectively are attached rigidly and symmetrically to each other as shown. The system of two blocks is floating in water in such a way that upper surface of bigger block is just submerged in the water. If the system of blocks is displaced slightly in vertical directions, then the amplitude of oscillation on either side of equilibrium position would be different.

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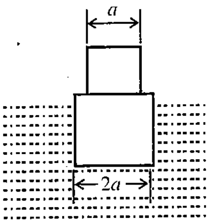

Fig. 4.143

Statement II: The force constant on two sides of equilibrium position in the above-described situation is different.

3. **Statement I:** Three pendulums are suspended from ceiling as shown in the Fig. 4.144.

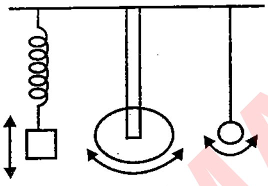

Fig. 4.144

These three pendulums are set to oscillate as shown by arrows, and it is found that all three have same time period. Now, all three are taken to a place where acceleration due to gravity changes to 4/9th of its value at the first place. If spring pendulum makes 60 cycles in a given time at this place, then torsion pendulum and simple pendulum will also make 60 oscillations in same (given) time interval.

Statement II: Time period of torsion pendulum is independent of acceleration due to gravity.

4. **Statement I:** A circular metal hoop is suspended on the edge by a hook. The hoop can oscillate side to side in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. The time period of oscillation would be more when oscillations are carried out in the plane of hoop.

Statement II: Time period of physical pendulum is more if moment of inertia of the rigid body about corresponding axis passing through the pivoted point is more.

5. **Statement I:** In a simple pendulum performing SHM, net acceleration is always between tangential and radial acceleration except at lowest point.

Statement II: At lowest point tangential acceleration is zero.

6. **Statement I:** If the amplitude of a simple harmonic oscillator is doubled, its total energy becomes four times.

Statement II: The total energy is directly proportional to the square of the amplitude of vibration of the harmonic oscillator.

7. **Statement I:** The spring constant of a spring is K. When it is divided into n equal parts, then spring constant of one piece is K/n.

Statement II: The spring constant is independent of material used for the spring.

8 **Statement I:** During the oscillations of simple pendulum, the direction of its acceleration at the mean position is directed towards the point of suspension and at extreme position it is directed towards the mean position.

Statement II: The direction of acceleration of a simple pendulum at the mean position or at the extreme position is decided by the tangential and radial components of force by gravity.

9. **Statement I:** A particle is moving along the x-axis. The resultant force F acting on it is given by $F = -ax - b$, where a and b are both positive constants. The motion of this particle is not SHM.

Statement II: In SHM resultant force must be proportional to the displacement from mean position.

10. **Statement I:** For a particle of mass 1 kg executing simple harmonic motion, if slope of restoring force vs. displacement graph is -1, then the time period of oscillation will be 6.28 s.

Statement II: If 1 kg mass is replaced by 2 kg mass and rest of the information remains same as in Statement I, then the time period of oscillation will remain 6.28 s.

Comprehension Type

Solutions on page 4.78

For Problems 1–3

One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to the x-axis. A block of mass $m = 1$ kg is attached to free end of the spring and it is performing SHM. Equation of position of the block in coordinate system shown in Fig. 4.145 is $x = 10 + 3 \sin (10\,t)$, where t is in second and x in cm.

Another block of mass $M = 3$ kg, moving towards the origin with velocity 30 cm/s collides with the block performing SHM at $t = 0$ and gets stuck to it.

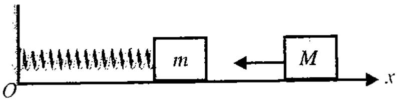

Fig. 4.145

1. Angular frequency of oscillation after collision is

a. 20 rad/s	**b.** 5 rad/s
c. 100 rad/s	**d.** 50 rad/s

2. New amplitude of oscillation is

a. 3 cm	**b.** 20 cm
c. 10 cm	**d.** 100 cm

3. New equation for position of the combined body is

a. $(10 + 3 \sin 5t)$ cm	**b.** $(10 - 3 \sin 5t)$ cm
c. $(10 + 3 \cos 10t)$ cm	**d.** $(10 - 3 \cos 10t)$ cm

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For Problems 4–6

A block of mass m is connected to a spring of spring constant k and is at rest in equilibrium as shown. Now, the block is displaced by h below its equilibrium position and imparted a speed v_0 towards down as shown in the Fig. 4.146. As a result of the jerk, the block executes simple harmonic motion about its equilibrium position. Based on this information, answer the following questions:

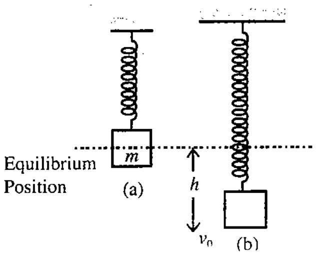

Fig. 4.146

4. The amplitude of oscillation is

 a. h
 b. $\sqrt{\frac{mv_0^2}{k}+h^2}$
 c. $\sqrt{\frac{m}{k}}v_0+h$
 d. None of these

5. The equation for the simple harmonic motion is

 a. $y=-A\sin\left[\sqrt{\frac{k}{m}}t+\sin^{-1}\left(\frac{h}{A}\right)\right]$
 b. $y=-A\cos\left[\sqrt{\frac{k}{m}}t+\sin^{-1}\left(\frac{h}{A}\right)\right]$
 c. $y=A\sin\left[\sqrt{\frac{k}{m}}t+\cos^{-1}\left(\frac{h}{A}\right)+\frac{\pi}{2}\right]$
 d. $y=A\sin\left[\sqrt{\frac{k}{m}}t+\cos^{-1}\left(\frac{h}{A}\right)+\frac{\pi}{4}\right]$

6. Find the time taken by the block to cross the mean position for the first time.

 a. $\dfrac{2\pi-\cos^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$
 b. $\dfrac{\frac{\pi}{2}-\cos^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$
 c. $\dfrac{\pi-\sin^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$
 d. $\dfrac{\pi-\sin^{-1}\left(\frac{h}{A}\right)}{2\sqrt{\frac{k}{m}}}$

For Problems 7–8

A block of mass m is connected to a spring of spring constant k as shown in Fig. 4.147. The block is found at its equilibrium position at $t = 1$ s and it has a velocity of +0.25 m/s at $t = 2$ s. The time period of oscillation is 6 s.

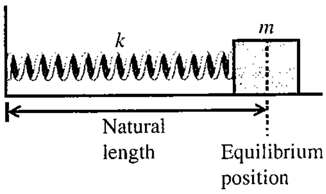

Fig. 4.147

Based on the given information, answer the following questions:

7. The amplitude of oscillation is

 a. $\frac{3}{2\pi}$ m
 b. 3 m
 c. $\frac{1}{\pi}$ m
 d. 1.5 m

8. Determine the velocity of particle at $t = 5$ s.

 a. −0.4 m/s
 b. 0.5 m/s
 c. −0.25 m/s
 d. none of these

For Problems 9–11

In a physical pendulum, the time period for small oscillation is given by, $T=2\pi\sqrt{I/Mgd}$ where I is the moment of inertia of the body about an axis passing through a pivoted point O and perpendicular to the plane of oscillation and d is the separation point between centre of gravity and the pivoted point.

In the physical pendulum, a special point exists where if we concentrate the entire mass of body the resulting simple pendulum (w.r.t. pivot point O) will have the same time period as that of physical pendulum. This point is termed centre of oscillation.

$$T=2\pi\sqrt{\frac{I}{Mgd}}=2\pi\sqrt{\frac{L}{g}}$$

Moreover, this point possesses two other important remarkable properties:

Property I: Time period of physical pendulum about the centre of oscillation (if it would be pivoted) is same as in the original case.

Property II: If an impulse is applied at the centre of oscillation in the plane of oscillation, the effect of this impulse at pivoted point is zero. Because of this property, this point is also known as the centre of percussion.

From the given information answer the following questions:

9. A uniform rod of mass M and length L is pivoted about point O as shown in Fig. 4.148. It is slightly rotated from its mean position so that it performs angular simple harmonic motion. For this physical pendulum, determine the time period of oscillation.

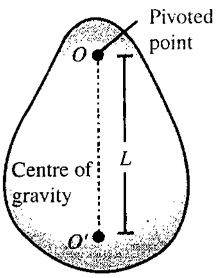

Fig. 4.148

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a. $2\pi\sqrt{\frac{L}{g}}$ b. $\pi\sqrt{\frac{7L}{3g}}$

c. $2\pi\sqrt{\frac{2L}{3g}}$ d. none of these

10. For the above question, locate the centre of oscillation.

a. $\frac{L}{4}$ from O (down) b. $\frac{L}{4}$ from O (up)

c. $\frac{2L}{3}$ from O (down) (d) $\frac{7L}{12}$ from O (down)

11. If an impulse J is applied at the centre of oscillation in the plane of oscillation, then angular velocity of the rod will be

a. $\frac{4J}{ML}$ b. $\frac{2J}{ML}$

c. $\frac{3J}{2ML}$ d. $\frac{J}{ML}$

For Problems 12–14

A block of mass m is suspended from one end of a light spring as shown. The origin O is considered at distance equal to natural length of the spring from the ceiling and vertical downward direction as positive y-axis. When the system is in equilibrium, a bullet of mass $m/3$ moving in vertical upward direction with velocity v_0 strikes the block and embeds into it. As a result, the block (with bullet embedded into it) moves up and starts oscillating.

Based on the given information, answer the following questions:

12. Mark out the correct statement(s).

a. The block–bullet system performs SHM about $y = mg/k$.

b. The block–bullet system performs oscillatory motion but not SHM about $y = mg/k$.

c. The block–bullet system performs SHM about $y = 4mg/3k$.

d. The block-bullet system performs oscillatory motion but not SHM about $y = 4mg/3k$.

13. The amplitude of oscillation would be

a. $\sqrt{\left(\frac{4mg}{3k}\right)^2 + \frac{mv_0^2}{12k}}$ b. $\sqrt{\frac{mv_0^2}{12k} + \left(\frac{mg}{3k}\right)^2}$

c. $\sqrt{\frac{mv_0^2}{6k} + \left(\frac{mg}{k}\right)^2}$ d. $\sqrt{\frac{mv_0^2}{6k} + \left(\frac{4mg}{3k}\right)^2}$

14. The time taken by the block–bullet system to move from $y = mg/k$ (initial equilibrium position) to $y = 0$ (natural length of spring) is (A represents the amplitude of motion)

a. $\sqrt{\frac{4m}{3k}}\left[\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)\right]$

b. $\sqrt{\frac{3k}{4m}}\left[\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)\right]$

c. $\sqrt{\frac{4m}{6k}}\left[\sin^{-1}\left(\frac{4mg}{3kA}\right) - \sin^{-1}\left(\frac{mg}{3kA}\right)\right]$

d. None of the above

For Problems 15–17

Two identical blocks A and B, each of mass $m = 3$ kg, are connected with the help of an ideal spring and placed on a smooth horizontal surface as shown in Fig. 4.149. Another identical block C moving with velocity $v_0 = 0.6$ m/s collides with A and sticks to it, as a result, the motion of system taks place in some way.

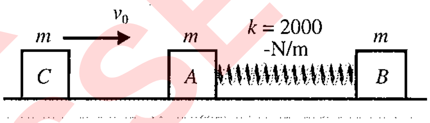

Fig. 4.149

Based on this information, answer the following questions:

15. After the collision of C and A, the combined body and block B would

a. oscillate about centre of mass of system and centre of mass is at rest.

b. oscillate about centre of mass of system and centre of mass is moving.

c. oscillate but about different locations other than the centre of mass.

d. not oscillate.

16. Oscillation energy of the system, i.e., part of the energy which is oscillating (changing) between potential and kinetic forms, is

a. 0.27 J b. 0.09 J

c. 0.18 J d. 0.45 J

17. The maximum compression of the spring is

a. $3\sqrt{30}$ mm b. $3\sqrt{20}$ mm

c. $3\sqrt{10}$ mm d. $3\sqrt{50}$ mm

For Problems 18–20

A small block of mass m is fixed at upper end of a massive vertical spring of spring constant $k = 4mg/L$ and natural length '$10L$'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown. The spring is initially unstressed and the spring–block system is released from rest in the shown position.

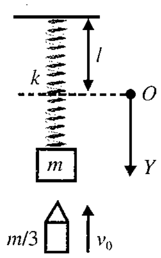

Fig. 4.150

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18. At the instant the speed of block is maximum, the magnitude of force exerted by the spring on the block is

a. $\dfrac{mg}{2}$ **b.** mg

c. zero **d.** None of these

19. As the block is coming down, the maximum speed attained by the block is

a. $\sqrt{gL}$ **b.** $\sqrt{3gL}$ **c.** $\dfrac{3}{2}\sqrt{gL}$ **d.** $\sqrt{\dfrac{3}{2}gL}$

20. Till the block reaches its lowest position for the first time, the time duration for which the spring remains compressed is

a. $\pi\sqrt{\dfrac{L}{2g}}+\sqrt{\dfrac{L}{4g}}\sin^{-1}\dfrac{1}{3}$

b. $\dfrac{\pi}{4}\sqrt{\dfrac{L}{g}}+\sqrt{\dfrac{L}{4g}}\sin^{-1}\dfrac{1}{3}$

c. $\pi\sqrt{\dfrac{L}{2g}}+\sqrt{\dfrac{L}{4g}}\sin^{-1}\dfrac{2}{3}$

d. $\dfrac{\pi}{2}\sqrt{\dfrac{L}{2g}}+\sqrt{\dfrac{L}{4g}}\sin^{-1}\dfrac{2}{3}$

For Problems 21–25

A 100 g block is connected to a horizontal massless spring of force constant 25.6 N/m. As shown in Fig. 4.151(a), the block is free to oscillate on a horizontal frictionless surface. The block is displaced 3 cm from the equilibrium position and, at $t = 0$, it is released from rest at $x = 0$. It executes simple harmonic motion with the positive x-direction indicated in Fig. 4.151(a).

The position–time $(x - t)$ graph of motion of the block is as shown in Fig. 4.151(b).

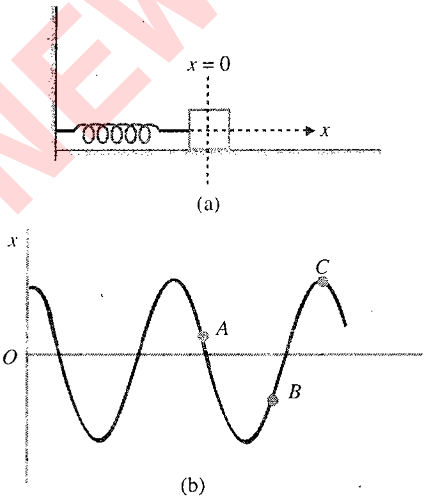

Fig. 4.151

21. When the block is at position A on the graph, its

a. position and velocity both are negative

b. position is positive and velocity is negative

c. position is negative and velocity is positive

d. position and velocity both the positive

22. When the block is at position B on the graph, its

a. position and velocity are positive

b. position is positive and velocity is negative

c. position in negative and velocity is positive

d. osition and velocity are negative

23. When the block is at position C on the graph, its

a. velocity is maximum and acceleration is zero

b. velocity is minimum and acceleration is zero

c. velocity is zero and acceleration is negative

d. velocity is zero and acceleration is positive

Let us now make a slight change to the initial conditions. At $t = 0$, let the block be released from the same position, i.e., from a displacement 3 cm along positive x-direction but with an initial velocity $v_i = -16\sqrt{3}$ cm/s.

24. Position of the block as a function of time can now be expressed as

a. $x = 3\cos\left(16t+\dfrac{\pi}{2}\right)$ cm

b. $x = 3\cos\left(16t+\dfrac{\pi}{3}\right)$ cm

c. $x = 3.5\cos\left(16t+\dfrac{\pi}{6}\right)$ cm

d. $x = 3.2\cos\left(16t+\dfrac{\pi}{4}\right)$ cm

25. Velocity of the block as a function of time can be expressed as

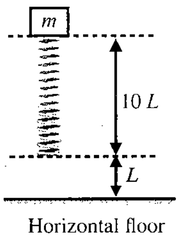

Fig. 4.152

a. $v = -48\sin\left(16t\dfrac{\pi}{2}\right)$ cm/s **b.** $v = -48\sin\left(16t\dfrac{\pi}{3}\right)$ cm/s

c. $v = -56\sin\left(16t\dfrac{\pi}{4}\right)$ cm/s **d.** $v = -56\sin\left(16t\dfrac{\pi}{6}\right)$ cm/s

For Problems 26–27

A spring having a spring constant k is fixed to a vertical wall as shown in Fig. 4.153. A block of mass m moves with velocity v towards the spring from a parallel wall opposite to this wall. The mass hits the free end of the spring compressing it and is decelerated by the spring force and comes to rest and then turns back till the spring acquires its natural length and contact with the spring is broken. In this process, it regains its angular speed

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in the opposite direction and makes a perfect elastic collision on the opposite left wall and starts moving with same speed as before towards right. The above processes are repeated and there is periodic oscillations.

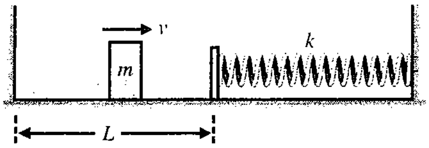

Fig. 4.153

26. What is the maximum compression produced in the spring?

a. $v\sqrt{\frac{m}{k}}$ **b.** $\sqrt{\frac{m}{k}}$ **c.** $v\sqrt{mk}$ **d.** $v\sqrt{\frac{k}{m}}$

27. What is the time period of oscillations?

a. $\pi\sqrt{\frac{m}{k}}$ **b.** $\sqrt{\frac{\pi m}{k}}+\frac{L}{v}$

c. $\pi\sqrt{\frac{m}{k}}+\frac{2L}{v}$ **d.** $\pi\sqrt{\frac{m}{k}}+\frac{L}{v}$

For Problems 28–30

A and B are two fixed points at a distance $3l$ apart. A particle of mass m placed at a point P experiences the force $2(mg/l)\overrightarrow{PA}$ and the force $(mg/l)\overrightarrow{PB}$ simultaneously. Initially at $t = 0$, the particle is projected from A towards B with speed $3\sqrt{gl}$.

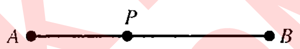

Fig. 4.154

28. The particle moves simple harmonically with period T and amplitude A.

a. $A = 2l,\ T = 2\pi\sqrt{\frac{l}{g}}$ **b.** $A = 3l,\ T = 2\pi\sqrt{\frac{l}{2g}}$

c. $A = 2l,\ T = 2\pi\sqrt{\frac{l}{3g}}$ **d.** $A = l,\ T = 2\pi\sqrt{\frac{l}{g}}$

29. The instant 't' at which the particle arrives at B in terms of the periodic time T will be

a. $t=\frac{T}{2}$ **b.** $t=\frac{T}{3}$ **c.** $t=\frac{T}{4}$ **d.** $t=\frac{2T}{3}$

30. The velocity of the particle when it reaches B will be

a. zero **b.** $3\sqrt{gl}$ **c.** $2\sqrt{gl}$ **d.** $\sqrt{gl}$

For Problems 31–33

A body of mass m is attached by an inelastic string to a suspended spring of spring constant k. Both the string and the spring have negligible mass and the string is inextensible and of length L. Initially, the mass m is at rest.

31. If the mass m is now raised up to point A (the top end of the string, see Fig. 4.155 and allowed to fall from rest, the maximum extension of the spring in the subsequent motion will be

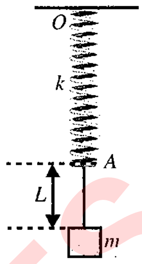

Fig. 4.155

a. L **b.** $\frac{mg}{k}$

c. $\frac{mg}{k}\sqrt{1+\frac{2kL}{mg}}$ **d.** $\frac{mg}{k}\left[1+\sqrt{1+\frac{2kL}{mg}}\right]$

32. If the mass m, from the initial position of rest, is pulled down a distance A and then released, assuming that the string remains taut throughout the motion, the maximum (downward) acceleration of the oscillating body will be

a. $\frac{kA}{m}$ **b.** $\frac{kA}{2m}$ **c.** $\frac{g}{2}$ **d.** g

33. The largest amplitude A_{max}, for which the string will remain taut throughout the motion is

a. $\frac{mg}{2k}$ **b.** $\frac{mg}{k}$ **c.** $\frac{2mg}{3k}$ **d.** L

Matching Column Type

Solutions on page 4.82

1. In Column I equations describing the motion of a particle are given and in Column II possible nature of the motions. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. $y = Ae^{i(\omega t+\phi)}$	**a.** SHM
ii. $y = B\sin\omega t + C\cos\omega t$	**b.** Periodic
iii. $y = A\sin(\omega t + kx)$	**c.** Oscillatory
iv. $y = kx$	**d.** Rectilinear

2. A spring pendulum executes SHM in such a way that the block is having velocity v when it crosses the mean position. Now the changes have been made in such a way that the velocity while crossing the mean position gets doubled without changing mass of the block. In Column I some statements (incomplete) are given and corresponding completions are given in Column II. Match the entries of Column I with the entries of Column II. Assume the horizontal configuration of pendulum.

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Column I	Column II
i. The frequency of oscillation will change by a factor of	**a.** 2
ii. The amplitude of oscillation will change by a factor of	**b.** $\sqrt{2}$
iii. The magnitude of maximum acceleration will change by a factor of	**c.** 1
iv. Maximum PE increases by a factor of	**d.** 4

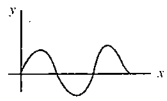
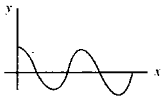
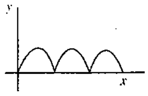
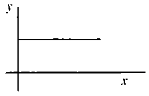
3. Match the following:

Column I	Column II
i. Linear combination of two SHMs	**a.** $T = \sqrt{\frac{R}{g}}$ (R is radius of the earth)
ii. $y = A \sin\omega_1 t + A \sin(\omega_2 t + \phi)$	**b.** SHM for equal frequencies and amplitude
iii. Time period of a pendulum of infinite length.	**c.** Superposition may not always be an SHM
iv. Maximum value of time period of an oscillating pendulum.	**d.** Amplitude will be $\sqrt{2}A$ for $\omega_1 = \omega_2$ and phase difference of $\pi/2$

4. Match the following:

Column I	Column II
i. A constant force acting along the line of SHM affects	**a.** the time period
ii. A constant torque acting along the arc of angular SHM affects	**b.** the frequency
iii. A particle falling on the block executing SHM when the later crosses the mean position affects	**c.** the mean position
iv. A particle executing SHM when kept on a uniformly accelerated car affects	**d.** the amplitude

5. In simple harmonic motion, match the following graphs:

Column I	Column II
i. Position (y) vs. time (x)	**a.**  Fig. 4.156
ii. Velocity (y) vs. time (x)	**b.**  Fig. 4.157
iii. Potential energy (y) vs. time (x)	**c.**  Fig. 4.158
iv. Total energy (y) vs. time (x)	**d.**  Fig. 4.159

6. Two particles 'A' and 'B' start SHM at $t = 0$. Their positions as function of time are given by
$X_A = A \sin \omega t$
$X_B = A \sin (\omega t + \pi/3)$

Column I	Column II
i. Minimum time when x is same	**a.** $\frac{5\pi}{6\omega}$
ii. Minimum time when velocity is same	**b.** $\frac{\pi}{3\omega}$
iii. Minimum time after which $v_A < 0$ and $v_B < 0$	**c.** $\frac{\pi}{\omega}$
iv. Minimum time after which $x_A < 0$ and $x_B < 0$	**d.** $\frac{\pi}{2\omega}$

7. A particle of mass 2 kg is moving on a straight line under the action of force $F = (8 - 2x)$ N. The particle is released from rest at $x = 6$ m. For the subsequent motion, match the following (all the values in the Column II are in their SI units):

Column I	Column II
i. Equilibrium position is at x	**a.** $\pi/4$
ii. Amplitude of SHM is	**b.** $\pi/2$
iii. Time taken to go directly from $x = 2$ to $x = 4$	**c.** 4
iv. Energy of SHM is	**d.** 6
v. Phase constant of SHM assuming equation of the form $A \sin(\omega t + \phi)$	**e.** 2

8. A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless horizontal surface, with equilibrium point $x = 0$. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in Column I with the values given in Column II (use $\pi^2 = 10$):

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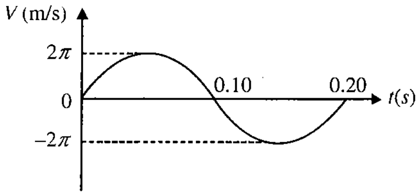
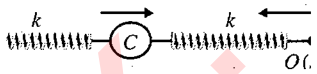
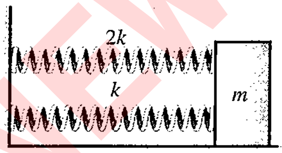
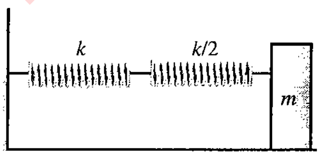


Fig. 4.160

Column I	Column II
i. The block's mass in kg	**a.** −0.20
ii. The block's displacement at $t = 0$ in metres	**b.** −200
iii. The block's acceleration at $t = 0.10$ s in m/s^2	**c.** 0.20
iv. The block's maximum kinetic energy in joules	**d.** 4.0

9. Column I lists the various modes of oscillations of masses connected to springs. Column II lists the corresponding frequencies of oscillations when executing SHM.

Column I	Column II
i.  Fig. 4.161	**a.** $\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$
ii.  Fig. 4.162	**b.** $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$
iii.  Fig. 4.163	**c.** $\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$
iv.  Fig. 4.164	**d.** $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Integer Answer Type

Solutions on page 4.84

1. Two uniform ropes having linear mass densities m and $4m$ are joined to form a closed loop. The loop is hanging over a fixed frictionless small pulley with the lighter rope above as shown in the Fig. 4.165 (in the figure equilibrium position is shown). Now if point A (joint) is slightly displaced in downward direction and released, It is found that the loop performs SHM with the period of oscillation equal to N. Find the value of N (take $l = \frac{150m}{4\pi^2}$, $g = 10$ m/s^2).

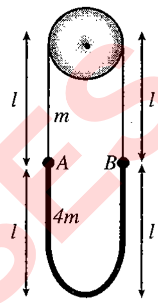


Fig. 4.165

2. In the figure shown a plate of mass 60 g is at rest and in equilibrium. A particle of mass $m = 30$ g is released from height 4.5 mg/k from the plate. The particle sticks to the plate. Neglecting the duration of collision find the time from the collision of the particle and plate to the moment when the spring has maximum compression. Spring has force constant 1 N/m. Calculate the value of time in the form π/x and find the value of X.

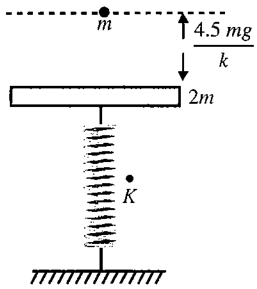


Fig. 4.166

3. Two simple pendulums A and B having lengths l and $l/4$, respectively are released from the position as shown in Fig. 4.167. Calculate the time (in seconds) after which the two strings become parallel for the first time. (Take $\ell = \frac{90}{\pi^2}m$ and $g = 10$ m/s^2)

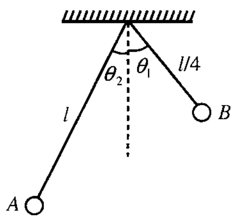


Fig. 4.167

4. A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at

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its midpoint. The bob is displaced slightly perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$ s and fill the value of X.

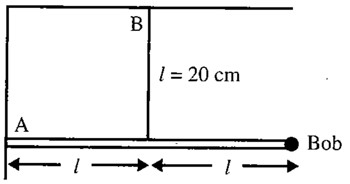

Fig. 4.168

5. A rod of mass m and length l hinged at one end is connected by two springs of spring constants k_1 and k_2 so that it is horizontal at equilibrium. What is the angular frequency of the system? (in rad/s) (Take ℓ = 1 m, b = 1/4 m, K_1 = 16 N/m, K_2 = 61 N/m.)

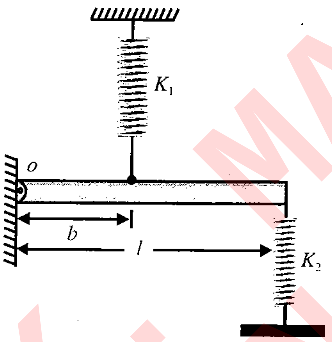

Fig. 4.169

6. A block of mass m is tied to one end of a spring which passes over a smooth fixed pulley A and under a light smooth movable pulley B. The other end of the string is attached to the lower end of a spring of spring constant K_2. Find the period of small oscillation of mass m about its equilibrium position (in second). (Take $m = \pi^2$ kg, $K_2 = 4K_1$ and K_1 = 17 N/m.)

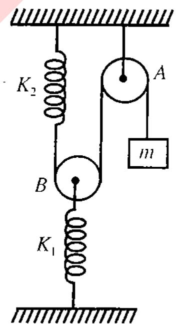

Fig. 4.170

7. A uniform disc of mass m and radius R is pivoted smoothly at its centre of mass. A light spring of stiffness k is attached with the disc tangentially as shown in the Fig. 4.171. Find the angular frequency in rad/s of torsional oscillations of the disc. (Take m = 5 kg and K = 10 N/m.)

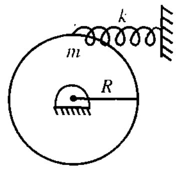

Fig. 4.171

8. A uniform disc of mass m and radius $R = \frac{80}{23\pi^2} m$ is pivoted smoothly at P. If a uniform disc of mass m and radius R is welded at the lowest point of the disc, find the period of SHM of the system (disc + ring). (in seconds)

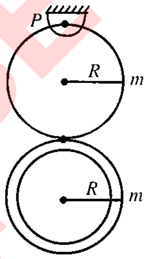

Fig. 4.172

9. In the arrangement shown in Fig. 4.173, pulleys are small and light and springs are ideal and $K_1 = 25\ \pi^2$ N/m, $K_2 = 2K_1$, $K_3 = 3K_1$ and $K_4 = 4K_1$ are the force constants of the springs. Calculate the period of small vertical oscillations of block of mass m = 3 kg.

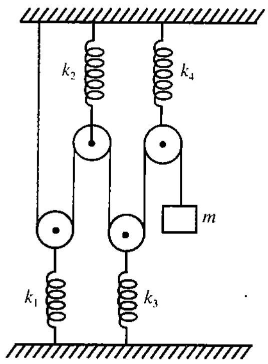

Fig. 4.173

10. A small body of mass m is connected to two horizontal springs of elastic constant k, natural length $3d/4$. In the equilibrium position both springs are stretched to length d, as shown in Fig. 4.174. What wil be the ratio of period of the motion (T_b/T_a) if the body is displaced horizontally by a small distance where T_a is the time period when the particle oscillates along the line of springs and T_b is time period when the particle oscillates perpendicular to the plane of the figure? Neglect effects of gravity.

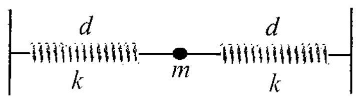

Fig. 4.174

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ANSWERS AND SOLUTIONS

Subjective Type

1. At equilibrium position deformation of the string is x_0.

$$kx_0\frac{l}{4}=Mg\left(\frac{3}{4}l\right)+mg\frac{l}{4}$$

When the rod is further rotated through an angle θ from equilibrium position, the restoring torque,

$$\tau=-\left[k(x+x_0)\frac{l}{4}\cos\theta-Mg\frac{3}{4}L\cos\theta\right]-mg\frac{L}{4}\cos\theta$$

$$=-\left[k(x+x_0)\frac{l}{4}-Mg\frac{3}{4}L-mg\frac{l}{4}\right]\cos\theta$$

For small θ, $\cos\theta\approx 1$

$$\tau=-\frac{kl}{4}x \quad\Rightarrow\quad I\alpha=-\frac{kl^2}{4}\theta$$

$$I=M\left(\frac{3}{4}L\right)^2+\frac{mL^2}{12}+m\left(\frac{L}{4}\right)^2$$

$$f=\frac{1}{2\pi}\sqrt{\frac{3k}{27M+7m}}$$

2. Let, in equilibrium position, compression of spring be x. Liquid of volume l^2x is displaced from its original position and level of liquid in tank rises as shown in the Fig. 4.175.

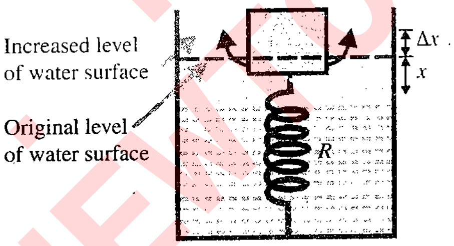

Fig. 4.175

This rise in level, $\Delta x=\dfrac{l^2x}{A-l^2}$

where A = 15 cm × 20 cm (base area of tank),

$$\Delta x=0.5x$$

Mass of water displaced by the block,

$$=l^2(x+\Delta x)\rho\quad=15x\text{ kg}$$

Upthrust exerted by water = apparent weight of water displaced.

Upthrust F_1 = 1.5 x $(g-a)$ = 120 x N

Upward force exerted by spring, $F_2=Kx=280x$

Considering free body diagram of the block,

$$mg-(F_1+F_2)=ma$$

Substituting values of F_1 and F_2, $x=0.04\ m=4$ cm

If the block is slightly pushed downward by dx, both F_1 and F_2, increase.

Increase in F_1 is $dF_1=120\,dx$

Increase in F_2 is $dF_2=280\,dx$

Restoring force on block = increase in F_1 + increase in F_2

$$dF_1+dF_2=(120\,dx+280\,dx)=400\,dx$$

Restoring acceleration $=\dfrac{400dx}{m}=200dx$

Since restoring acceleration α displacement (dx), block performs SHM.

Hence, frequency

$$f=\frac{1}{2\pi}\sqrt{\frac{\text{displacement}}{\text{acceleration}}}=\frac{1}{2\pi}\sqrt{200}=\frac{5\sqrt{2}}{\pi}\text{ per second}$$

3. As A oscillates up and down, the normal reaction between B and the surface is maximum. when A is at lowest position and it is minimum when A is at topmost position.

Case I: Lowest position of A

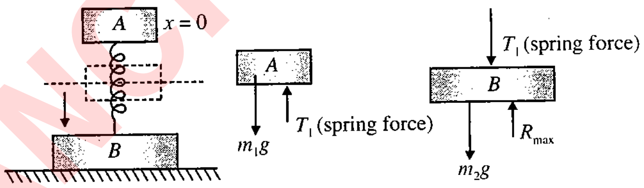

Fig. 4.176

Let T_1 be the force in the compressed spring.

As B is at rest, $T_1=m_2g=R_{max}$

As acceleration of A is ω^2A towards mean position (up),

$$T_1-m_1g=m_1\omega^2$$

Combining, $R_{max}=(m_1+m_2)g+m_2\omega^2A$

Case II: Topmost position of A

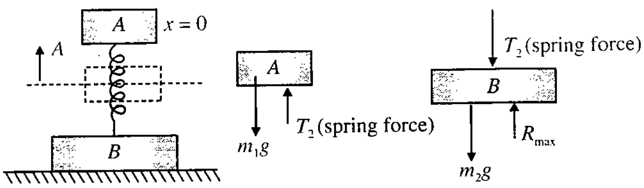

Fig. 4.177

Let T_2 be the tension in the elongated spring. As B is at rest,

$$T_2+R_{min}=m_2g$$

Acceleration of A is ω^2A towards mean position (down)

$$\Rightarrow T_2+m_1g=m_1\omega^2A$$

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Combining, we get $R_{\min} = (m_1 + m_2)g - m_1\omega^2 A$

Note: B will leave the surface if $m_1\omega^2 A > (m_1 + m_2)g$.

4. We will analyse the problem relative to the rotating bar AB. As the acceleration of bar is centripetal, a pseudo force will act on sleeve away from the centre and will be of magnitude $m(\omega^2 x)$.

If the sleeve is displaced by x, net force towards centre

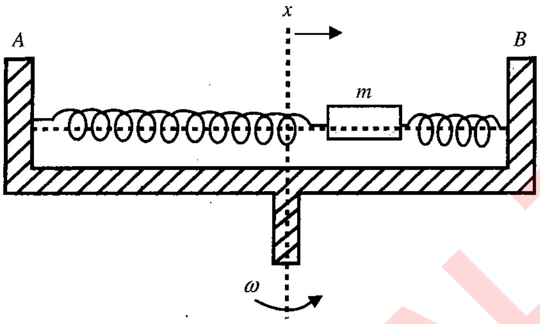

Fig. 4.178

$$F = kx - m\omega^2 x = (m - m\omega^2)x$$

$$\Rightarrow \quad \text{period} = 2\pi\sqrt{\frac{m}{k - m\omega^2}}$$

Note that if $k < m\omega^2$, there will be no oscillations of the sleeve. It will rush to the point B if it is displaced slightly (for $k < m\omega^2$) or will remain in the displaced position (for $k = m\omega^2$).

5. In equilibrium position, net force on pole is zero.

$$\rho A l g = \rho_1 A l_1 g \quad \text{(i)}$$

In displaced position, net force,

$$F_1 = \rho A l g - \rho_1 A l_1 g - \rho_2 A y g = -\rho_2 A g y$$

Acceleration of the pole, $a_1 = -\dfrac{\rho_2 A g}{\rho A l} y$

Hence in the liquid of density r_2,

$$\omega_1 = \sqrt{\frac{\rho_2 g}{\rho l}} \quad \Rightarrow T_1 = 2\pi\sqrt{\frac{\rho l}{\rho_2 g}}$$

The pole system is in the liquid of density ρ_2 for half time period only. When it moves in the liquid of density ρ_1, the net force is $F_2 = -\rho A l g + (l_1 - y')A\rho_1 g = -\rho_1 A g y'$

$$a_2 = -\frac{\rho_1 A g}{\rho A l} y' \quad \Rightarrow \quad \omega_2 = \sqrt{\frac{\rho_1 g}{\rho l}}$$

$$T_2 = 2\pi\sqrt{\frac{\rho l}{\rho_1 g}}$$

Total time period of oscillation of the cylinder is

$$T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\left[\sqrt{\frac{\rho l}{\rho_1 g}} + \sqrt{\frac{\rho l}{\rho_2 g}}\right]$$

6. **a.** For the oscillations of the blocks together, the equivalent force constant equals $K_1 + K_2$ and hence the frequency

$$A = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m_1 + m_2}}$$

b. Assuming that the blocks $(m_1 + m_2)$ are displaced towards right, the various forces (except the frictional force) on m_2, in the reference frame of m_1 are shown in the following free body diagram.

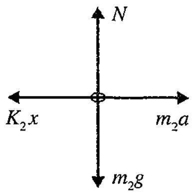

Fig. 4.179

where $a = \dfrac{(K_1 + K_2)x}{m_1 + m_2}$

For the frictional force to act on m_2 in the direction of its displacement

$$K_2 x > m_2\left[\frac{K_1 + K_2}{m_1 + m_2}\right]x$$

$$m_1 K_2 + m_2 K_2 - m_2 K_1 - m_2 K_2 > 0$$

$$m_1 K_2 - m_2 K_1 > 0$$

$$\frac{m_1}{m_2} > \frac{K_1}{K_2}$$

The above inequality is the desired condition.

c. Assuming $\dfrac{m_1}{m_2} > \dfrac{K_1}{K_2}$ the FBD of m_2 with m_1 and m_2 together displaced towards right can be shown as

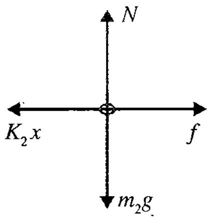

Fig. 4.180

If A be the amplitude of oscillations of m_2, $K_2 A - f = m_2\omega^2 A$ (for the extreme position)

$f = K_2 A - m_2\omega^2 A \quad \Rightarrow \quad f = A(K_2 - m_2\omega^2)$

$f \le \mu\, m_2 g \quad \Rightarrow \quad \mu\, m_2 g \ge A(K_2 - m_2\omega^2)$

$A \le \dfrac{\mu\, m_2 g}{(K_2 - m_2\omega^2)} \quad \Rightarrow \quad A_{\max} = \dfrac{\mu m_2 g}{(K_2 - m_2\omega^2)}$

Here $\omega^2 = \dfrac{(K_1 + K_2)}{(m_1 + m_2)}$; $\quad A_{\max} = \dfrac{\mu m_2 g(m_1 + m_2)}{m_1 K_2 - m_2 K_1}$

7. Let us displace the CM by a small distance x towards right. As a result the body rolls on the horizontal surface. Hence spring 1 is elongated by $x_1 = 2R\theta$ and spring 2 is compressed by $x_2 = R\theta$, where θ = angle of rotation of the rolling body.

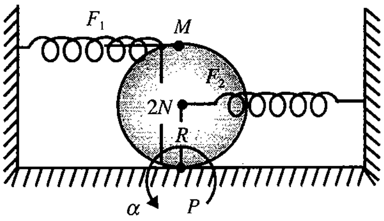

Fig. 4.181

The spring forces F_1 and F_2 produce anticlockwise torques $F_1(2R)$ and $F_2(R)$, respectively about P. Summing up the spring torques about P, we have

$$\tau_{net} = F_1(2R') + F_2(R)$$

where $\qquad F_1 = k_1x_1 = k_1(2R\theta)$

and $\qquad F_2 = k_2x_2 = k_2R\theta$

This gives $\tau_{net} = -(4R^2k_1 + R^2k_2)\theta = -(4k_1 + k_2)R^2\theta$

This torque produces anticlockwise acceleration α for clockwise displacement θ, which can be given as

$$\alpha = \frac{\tau_{net}}{I_P} = \frac{-(4k_1+k_2)R^2\theta}{I_P}$$

Substituting $\qquad I_P = mR^2 + I_C = mR^2 + \frac{mR^2}{2} = \frac{3mR^2}{2}$

We have $\qquad \alpha = \frac{-2(4k_1+k_2)}{3m}\theta$

Comparing the above equation with $\alpha = -\omega^2\theta$, we have

$$\omega = \sqrt{\frac{2(4k_1+k_2)}{3m}}$$

8. Suppose that the liquid is displaced slightly from equilibrium so that its level rises in one arm of the tube, while it is depressed in the second arm by the same amount, x.

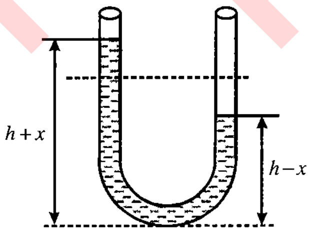

Fig. 4.182

If the density of the liquid is ρ, then the total mechanical energy of the liquid column is

$$E = \frac{1}{2}\{A(h+x)\rho + A(h-x)\rho\}\left(\frac{dx}{dt}\right)^2$$

$$+\left[A(h+x)\rho g\frac{h+x}{2} + A(h-x)\rho g\frac{h-x}{2}\right]$$

$$= \frac{1}{2}(2Ah\rho)\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}(2A\rho g)(h^2+x^2) \qquad \text{(i)}$$

After differentiating the total energy and equating it to zero, one finds acceleration $= -\omega^2 x$.

The angular frequency of small oscillations, ω, is

$$\omega = \sqrt{\frac{2A\rho g}{2Ah\rho}} = \sqrt{\frac{g}{h}} \qquad \text{(ii)}$$

9. The potential energy function for the mass m in a unidimensional potential field is given as

$$U(x) = \frac{A}{x^2} - \frac{B}{x} \qquad \text{(i)}$$

Since this is the potential energy of mass m in a potential field, so the force acting on the mass is obtained by the relation:

$$F = -\frac{dU}{dx} = \frac{2A}{x^3} - \frac{B}{x^2} \qquad \text{(ii)}$$

Now, the equilibrium position is given by

$$F = 0, \quad \text{or} \quad \frac{2A}{x^3} - \frac{B}{x^2} = 0$$

$$x^2(2A - Bx) = 0$$

So, $\qquad x = 0 \quad \text{or} \quad x = \frac{2A}{B}$

Since $x = 2A/B$ gives more stable situation

(because energy is minimum here; d^2U/dx^2 is positive), so stable equilibrium position is given as $x = \frac{2A}{B}$

Now if the mass is displaced slightly away from the equilibrium position, the force acting on mass m is given as

$$F = \frac{2A}{[(2A/B)+\Delta x]^3} - \frac{B}{[(2A/B)+\Delta x]^2}$$

$$= \frac{2A}{\left(\frac{2A}{B}\right)^3\left(1+\frac{B\Delta x}{2A}\right)^3} - \frac{B}{\left(\frac{2A}{B}\right)^2\left(1+\frac{B\Delta x}{2A}\right)^2}$$

$$= \frac{2A}{8A^3/B^3}\left[1+\frac{B\Delta x}{2A}\right]^{-3} - \frac{B}{4A^2/B^2}\left[1+\frac{B\Delta x}{2A}\right]^{-2}$$

So, using bionomial expansion for small Δx, we get

$$F = \frac{B^3}{4A^2}\left[1-\frac{3B\Delta x}{2A}\right] - \frac{B^3}{4A^2}\left[1-\frac{2B\Delta x}{2A}\right]$$

$$= \frac{B^3}{4A^2}\left[1-\frac{3B\Delta x}{2A} - 1 + \frac{2B\Delta x}{2A}\right] = -\left(\frac{B^4}{8A^3}\right)\Delta x$$

We can write

$$m\frac{d^2x}{dt^2} = -\frac{B^4}{8A^3}\Delta x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{B^4}{8mA^3}\right)\Delta x \qquad \text{(iii)}$$

$$\frac{d^2x}{dt^2} \propto -\Delta x$$

So from the above relations, it is clear that particle will perform SHM. But for any SHM,

$$\frac{d^2x}{dt^2} = -\omega^2\Delta x \qquad \text{(iv)}$$

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So, from Eqs. (iii) and (iv), we have $\omega^2 = \dfrac{B^4}{8A^2 m}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8A^3 m}{B^4}} = 4\pi A\sqrt{\frac{mA}{B^4}}$$

10. $t_P = mgx = mgrq$

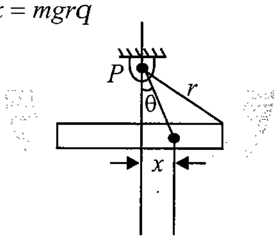

Fig. 4.183

$I_P w^2 q = mgrq$

$$\omega = \sqrt{\frac{mgr}{I_P}}, \text{ where } I_P = \left(\frac{m\ell^2}{12} + mr^2\right)$$

$$\omega = \sqrt{\frac{gr}{\frac{\ell^2}{12} + r^2}}, \text{where } \quad r^2 + \frac{\ell^2}{4} = R^2$$

Then, obtain T.

Objective Type

1. **b.** The velocity is minimum (zero) at the extreme position and maximum ($\pm\omega A$) at the mean position.
The acceleration is maximum ($\pm\omega^2 A$) at the extreme positions and minimum (zero) at the mean position. Since the particle crosses the mean and extreme positions twice, during each oscillation, hence the result.

2. **a.** $a = -\omega^2 x = \omega^2 x$ (numerically)
Rate of change of acceleration

$$\frac{da}{dt} = \omega^2\frac{dx}{dt} = \omega^2 v$$

$$\frac{da}{dt} = \omega^3\sqrt{A^2 - x^2}$$

For da/dt to be maximum, x^2 should be minimum i.e., x = 0 and For da/dt to be minimum x^2 should be maximum i.e., $x = \pm a$.

3. **d.** Frequency of a simple pendulum

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Evidently, $f \propto \dfrac{1}{\sqrt{l}}$ at any place.

The effective length l is the distance of the point of suspension O' from the centre of gravity (CG) of the bob. As, water flows out of the hole at the bottom, the CG descends from centre towards the bottom, increasing the effective length, and consequently f decreases.

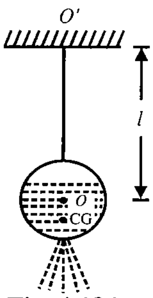

Fig. 4.184

However, when all the water has flows out, the CG of a hollow sphere is once again at its centre and hence the effective length would decrease, thereby increasing the frequency.

4. **b.** When any charge is given to the bob of the pendulum, it induces opposite charge on the metal plate, and hence a net force of attraction acts on the bob. Thus, the effective value of g increases.

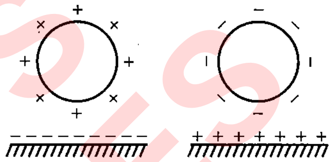

Fig. 4.185

So, from $T = 2\pi\sqrt{l/g_{eff}}$; the value of T decreases.

5. **b.** The situation when the block is just below the mean position is shown in Fig. 4.186, the restoring forces acting on the piston cause a normal reaction F to act on the block. For the block to separate $F \geq mg$
i.e., $m\omega^2 A \geq mg$
(where ω = angular frequency and A = amplitude)

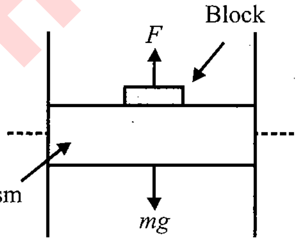

Fig. 4.186

or $\quad A \geq \dfrac{g}{\omega^2} = \dfrac{g}{\left(\dfrac{2\pi}{T}\right)^2} \quad$ or $\quad A \geq \dfrac{gT^2}{4\pi^2}$

Now, the maximum velocity v_{max} at that instant $= \omega A$

$$v_{max} = \left(\frac{2\pi}{T}\right)\left(\frac{gT^2}{4\pi^2}\right) = \frac{10}{\pi}\text{m/s}^2$$

6. **d.** $y = 8\sin^2\left(\dfrac{t}{2}\right)\sin(10t)$

$= 4\,[1 - \cos t]\sin(10t)$ (using $2\sin^2\dfrac{\theta}{2} = 1 - \cos\theta$)

$= 4\sin(10t) - 4\sin(10t)\cos t$

$= 4\sin(10t) - 2\,[\sin 11t + \sin 9t]$

(using $2\sin C\cos D = \sin(C + D) + \sin(C - D)$)

$= 4\sin(10t) - 2\sin(11t) - 2\sin(9t)$

Evidently, y is obtained as the superimposition of three independent (i.e., having different angular frequency ω) SHMs.

7. **c.** Let T be the time period then $\dfrac{T}{2} = \dfrac{5\pi}{64} - \dfrac{\pi}{64} = \dfrac{4\pi}{64}$

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$$\Rightarrow \quad T = \frac{\pi}{8}\,\text{s}$$

$$\therefore \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{(\pi/8)} = 16 \text{ rad/s}$$

Also, A = 10 cm (from the graph)
The equation of the sinusoidal wave can be written as y = 10 sin $(16t + \phi)$ cm, where ϕ is the initial phase.
From the graph, corresponding to the crest = 10 cm; when $t = 3\pi/64$.

$$10\text{ cm} = 10\sin\left[16\left(\frac{3\pi}{64}\right)+\phi\right]\text{cm}$$

$$\sin\left[\frac{3\pi}{4}+\phi\right]=1 \quad \text{or} \quad \frac{3\pi}{4}+\phi=\frac{\pi}{2} \quad \Rightarrow \quad \phi = -\frac{\pi}{4}$$

$$\therefore \quad y = 10\sin\left(16t - \frac{\pi}{4}\right)$$

8. a. For A, time period T_A = 16 s (distance between two adjacent crests)
For B, time period T_B = 2 (20 – 8) = 24 s (length between the crest and trough shown = 20 s – 8 s = 12 s)
Also, amplitudes a_A = 10 cm, a_B = 5 cm

Now, $$\frac{(V_{\max})_A}{(V_{\min})_B} = \frac{\omega_A a_A}{\omega_B a_B}$$

$$= \frac{\left(\frac{2\pi}{T_A}\right)a_A}{\left(\frac{2\pi}{T_B}\right)a_B} = \frac{T_B a_A}{T_A a_B} = \frac{24\times 10}{16\times 5} = \frac{3}{1}$$

Sol. 9. b., 10. b., 11. c., 12. c.

From the graph $T = (5-1) = 4$ s
(distance between the two adjacent crests shown in the figure)
and $\quad v_{\max} = 5$ m/s; $\quad \omega A = 5$ m/s

$$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}\text{ m}$$

Also, $$\omega = \frac{2\pi}{4} = \frac{\pi}{2}\text{ rad/s}$$

The equation of velocity can be written as

$$v = 5\sin\left(\frac{\pi}{2}t\right)\text{ m/s}$$

At extreme position, $v = 0$; $\sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$ s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$.
Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$.

$$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$$

$= 2.5$ m/s (numerically)

$$\frac{dy}{dt} = v \quad \Rightarrow dy = v\,dt$$

$$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right]+C$$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0$; $y = -\frac{10}{\pi}$

$$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta + C \quad \Rightarrow C = 0$$

$$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}\text{ s}$$

$$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5\sqrt{3}}{\pi}\text{ m} \quad \text{(numerically)}$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right) = \frac{5\pi}{2}\frac{\cos \pi t}{2}$$

$$a \text{ at } t = \frac{1}{3}\text{ s} = \frac{5\pi}{2}\cos\frac{\pi}{6} = \frac{5\pi\sqrt{3}}{4}\text{ m/s}^2$$

Maximum displacement, $x_{\max} = A = \frac{10}{\pi}$ m

and maximum acceleration, $a_{\max} = \omega^2 A$

$$= \left(\frac{\pi}{2}\right)^2 \times \frac{10}{\pi} = \frac{5\pi}{2}\text{ m/s}^2$$

13. b. The slope of the length

$$= \frac{F}{x} = -\frac{0.5}{5} = -0.1\text{ N/cm} = -10\text{ N/m}$$

But $\quad F = -m\omega^2 x \quad$ or, $\quad F/x = -m\omega^2$

so, $\quad -m\omega^2 = -10 \quad$ or, $\quad m\omega^2 = 10$

or, $\omega^2 = 10$ /m

$$\therefore \quad \omega^2 = \frac{10}{4\times 10^{-1}} \quad \Rightarrow \omega = \frac{10}{2} = 5$$

$$\therefore \quad f = \frac{\omega}{2\pi} = \frac{5}{2\pi}/\text{s}$$

14. b. The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{1}} \approx 14\text{ rad/s}$$

15. c.

$$T = \frac{25}{50} = \frac{1}{2}\text{ s} \quad \Rightarrow \omega = \frac{2\pi}{T} = 4\pi\text{ rad/s}$$

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Spring constant $k = m\omega^2 = 5 \times (4\pi)^2 = 80\pi^2$ N/m

Force required to stretch the spring by 5 cm is

$$F = kx = 80\pi^2 \times 0.05 \text{ N} = 4\pi^2$$

16. c. Let the acceleration be f, $f = -\omega^2 x$

Therefore, distance of the particle from the centre at any time t is given by

$x = r\cos(\omega t)$, where r is the amplitude

when $t = 1$ s, $x = r - a$

$\therefore$ $(r - a) = r\cos\omega$

$$\cos\omega = \frac{r-a}{r} \qquad \text{(i)}$$

When $t = 2$ s, $x = r - a - b$,

therefore $r - a - b = \cos 2\omega$

$\therefore$ $r - a - b = r(2\cos^2\omega - 1)$ (ii)

Substituting the value of $\cos\omega$ from Eq. (i) in Eq. (ii), we get

$$r - a - b = r\left[2\frac{(r-a)^2}{r^2} - 1\right]$$

$$r - a - b = \frac{2(r-a)^2}{r} - r$$

$$\therefore \quad r(3a - b) = 2a^2 \Rightarrow r = \frac{2a^2}{3a-b}$$

17. a. Since $F = -\dfrac{dU}{dx} = 2kx\exp(-x^2)$

$F = 0$ (at equilibrium as $x = 0$)

U is minimum at $x = 0$ and $U_{min} = 0$

U is maximum at $x \to \pm\infty$ and $U_{max} = k$

The particle would oscillate about $x = 0$ for small displacement from the origin and it is in stable equilibrium at the origin.

18. d. $y_1 = 4\sin(10t + \phi)$, $y_2 = 5\cos 10t$

$$v_1 = \frac{dy_1}{dt} = 40\cos(10t + \phi)$$

$$v_2 = \frac{dy_2}{dt} = -50\sin 10t = 50\cos\left(10t + \frac{\pi}{2}\right)$$

Phase difference between v_1 and $v_2 = \left(\phi - \dfrac{\pi}{2}\right)$

19. b. As discussed in theory part

$$T = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{\rho}{\sigma}\right)}};$$

ρ = density of liquid

σ = density of bob material

Given $\rho = \dfrac{\sigma}{n}$

Hence period is $2\pi\sqrt{\dfrac{\ell}{(1-(1/n)g}}$

20. a. Let $x_1 = a\sin\omega t$ and $x_2 = a\sin(\omega t + \delta)$ be two SHMs.

$$\frac{a}{3} = a\sin\omega t \quad \text{and} \quad -\frac{a}{3} = a\sin(\omega t + \delta)$$

$$\sin\omega t = 1/3 \quad \text{and} \quad \sin(\omega t + \delta) = -1/3$$

Eliminating t, $\dfrac{1}{3}\cos\delta + \sqrt{1 - \dfrac{1}{9}}\sin\delta = -\dfrac{1}{3}$

$$9\cos^2\delta + 2\cos\delta - 7 = 0$$

$$\cos\delta = -1 \quad \text{or} \quad \frac{7}{9}$$

$$\text{i.e., } \delta = 180° \quad \text{or} \quad \cos^{-1}\left(\frac{7}{9}\right)$$

If we put 180°, we find that v_1 and v_2 are of opposite signs. Hence $\delta = 180°$ is not applicable.

$$\therefore \quad \delta = \cos^{-1}\left(\frac{7}{9}\right)$$

21. c. $U = U_0 - U_0\cos ax$

$$F = -\frac{dU}{dx} = -U_0\sin ax = m\frac{d^2x}{dt^2}$$

This is the equation of SHM $\Rightarrow T = \dfrac{2\pi}{a}\sqrt{\dfrac{m}{U_0}}$

22. a. Equation of SHM = $y\sin(\omega t + \phi)$

When $y = \dfrac{a}{2}$, $t = \dfrac{T}{4} = \dfrac{2\pi}{4\omega} = \dfrac{\pi}{2\omega}$

$v = a\omega\cos(\omega t + \phi)$, velocity is negative

$$\frac{\pi}{2} < (\omega t + \phi) < \frac{3\pi}{2}$$

$$\frac{a}{2} = a\sin(\omega t + \phi) \quad \Rightarrow \sin(\omega t + \phi) = \frac{1}{2}$$

$$\left(\frac{\pi}{2} + \phi\right) = \frac{5\pi}{6}$$

Substituting in the above equation, we get $\phi = \pi/3$.

23. b. Amplitude = 0.15 m

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{4}} = 10 \text{ rad/s} \Rightarrow f = \frac{\omega}{2\pi} = \frac{5}{\pi} \text{ Hz}$$

Energy of the particle executing SHM = KE + PE

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Therefore, $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}k(A^2 - x^2)$

$$= \frac{1}{2} \times 400(0.15^2 - 0.1^2) = 2.5 \text{ J}$$

24. b. Period $= 2\dfrac{\pi}{\omega} = 2$ s (or) $\omega = \pi$ rad/s,

amplitude = 10 cm

If the system is released, the equation of motion is

$y = A\cos\omega t$

$5 = 10\cos\pi t_1$ when $y = 5$ cm

$-5 = 10\cos\pi t_2$ when $y = -5$ cm

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$$\pi t_1 = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \quad \text{or} \quad t_1 = \frac{1}{3}\text{ s}$$

$$\pi t_2 = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \quad \text{or} \quad t_2 = \frac{2}{3}\text{ s}$$

$\therefore$ Time interval $= t_2 - t_1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ s

25. a. Let x_1 and x_2 be the distances of the two positions from centre. Then with usual notations

$$u^2 = \omega^2(A^2 - x_1^2) \quad \text{(i)}$$
$$v^2 = \omega^2(A^2 - x_2^2) \quad \text{(ii)}$$
$$a = \omega^2 x_1 \quad \text{(iii)}$$
$$b = \omega^2 x_2 \quad \text{(iv)}$$

Subtracting Eq. (ii) from Eq. (i),

$$u^2 - v^2 = \omega^2\left(x_2^2 - x_1^2\right) \quad \text{(v)}$$

Adding Eqs. (iii) and (iv),

$$a + b = \omega^2(x_1 + x_2) \quad \text{(vi)}$$

Dividing Eq. (v) by Eq. (vi) $\frac{u^2 - v^2}{a+b} = x_2 - x_1$

26. d. Equations are $x_1 = a\cos\left(\omega t + \frac{\pi}{6}\right)$

and $x_2 = a\cos\left(\omega t + \frac{\pi}{3}\right)$

The first will pass through the mean position when $x_1 = 0$

i.e., for instants t for which $\left(\omega t + \frac{\pi}{6}\right) = \frac{n\pi}{2}$, where n is an integer.

The smallest value of t is $n = 1$, $\omega t_1 = (\pi/2) - (\pi/6) = \pi/3$.

The second will pass through the mean position when $x_2 = 0$, i.e., for instants t for which $\left(\omega t + \frac{\pi}{3}\right) = \frac{m\pi}{2}$

where m is an integer.

The smallest value of t is $m = 1$, $= (\pi/2) - (\pi/3) = \pi/6$.

The smallest interval between the instants $x_1 = 0$ and $x_2 = 0$ is therefore,

$$\omega(t_1 : t_2) = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6} \quad \Rightarrow \quad t_1 : t_2 = \frac{\pi}{6\omega}$$

27. b. $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$, $\vec{F}_{net}$ is zero when $PA = PB$

Let $AB = l$

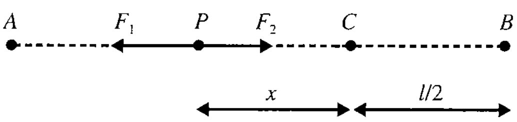

Fig. 4.187

$$F_1 = 2\left(\frac{1}{2} \times x\right) \text{ and } F_2 = 2\left(\frac{1}{2} \times x\right)$$

$$F_{net} = -4x$$

Therefore, motion in SHM will have $k = 4$.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \Rightarrow \quad T = 2\pi$$

28. c. $K = \frac{1}{2}m\omega^2(A^2 - y^2)$

$$U = \frac{1}{2}m\omega^2 y^2$$

$$K + U = \frac{1}{2}m\omega^2(A^2 - y^2) + \frac{1}{2}m\omega^2 y^2$$

i.e., $2y^2 = A^2$ or $y = \frac{A}{\sqrt{2}}$

29. a. From graph of acceleration and time graph

$f = -f_{max}\cos\omega t = -(\omega^2 A)\cos\omega t$

and we have relationship between acceleration and position as $f = -\omega^2 x \Rightarrow x = A\cos\omega t$

which gives $v = \frac{dx}{dt} = -A\omega\sin\omega t = -v_{max}\sin\omega t$

Hence option (a) is correct.

30. d. Since acceleration of the particle at initial moment is maximum possible and is negative; therefore, the particle is at right extreme position at this moment.

When the particle is released, it starts to move to the left. It means, velocity starts to increase from zero initial value to negative value and its magnitude becomes maximum possible at mean position (at $t = T/4$). It means at $t = T/4$, kinetic energy is equal to maximum possible.

At $t = T/2$, the particle comes to instantaneous rest at left extreme position. It means at $t = T/2$, v is equal to zero. Hence kinetic energy is equal to zero. At $t = 3T/4$, particle comes back to mean position and now moves to the right. Therefore, velocity is positive and has maximum possible magnitude. Therefore, kinetic energy is maximum possible.

At $t = T$, particle comes back to initial position (extreme right position). Velocity and kinetic energy become equal to zero.

31. b. When a particle performs SHM, its total energy remains constant. It means, kinetic energy plus excess potential energy is a constant, which is equal to maximum possible kinetic energy.

Hence, excess potential energy will be maximum when KE is equal to zero and zero when KE is maximum possible.

Hence option (b) is correct.

If frequency of oscillations of a particle is equal to n, then frequency of variation of its KE is equal to $2n$. It means, if time period of variations of KE is equal to T, then time period of oscillations of the particle will be equal to $2T$.

Hence option (a) is wrong.

32. b. $y_1 = a\sin\omega t$ and $y_2 = \sin(\omega t + \phi)$

$$y_2 - y_1 = a\sqrt{2} = a\sin(\omega t + \phi) - a\sin\omega t$$

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or $\sqrt{2}a = 2a\cos\left(\frac{\omega t+\phi+\omega t}{2}\right) = \sin\left(\frac{\omega t+\phi-\omega t}{2}\right)$

$$= 2a\cos\left(\omega t+\frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

For maximum value, $\cos(\omega t+\phi)=1$, therefore

$$2\sin\frac{\phi}{2}=\sqrt{2} \Rightarrow \sin\frac{\phi}{2}=\frac{1}{\sqrt{2}}$$

or $\frac{\phi}{2}=\frac{\pi}{4}$ or $\phi=\frac{\pi}{2}$

33. **a.** With mass m_2 alone, the extension of the spring l is given as

$$m_2 g = kl \qquad \text{(i)}$$

With mass (m_1+m_2), the extension l' is given by

$$(m_1+m_2)g = kl' = k(l+\Delta l) \qquad \text{(ii)}$$

Hence Δl is the amplitude of vibration.
Subtracting Eq. (i) from Eq. (ii), we get $m_1 g = k\,\Delta l$

or $$\Delta l = \frac{m_1 g}{k}$$

34. **c.** Decrease in potential energy of the mass when the pan gets lowered by distance y (due to mass hitting on the pan) $= mg(h+y)$, where h is the height through which the mass falls on the pan. Increases in elastic potential of the spring $= 1/2\ ky^2$ (according to law of conservation of energy)

or $$mg(h+y)=\frac{1}{2}ky^2$$

or $$ky^2-2mgy-2mgh=0$$

$\therefore$ $$y=\frac{2mg\pm\sqrt{4m^2g^2+8mghk}}{2k}$$

$$=\frac{mg}{k}\pm\frac{mg}{k}\sqrt{\left(1+\frac{2hk}{mg}\right)}$$

Velocity of the pan will be maximum at the time of collision and will be zero at the lowest position.
Hence y should be the amplitude of oscillation.

So, amplitude of vibration $=\left[\frac{mg}{k}+\frac{mg}{k}\sqrt{\left(1+\frac{2hk}{mg}\right)}\right]$

35. **d.** Mean position of the particle is mg/k distance below the unstretched position of spring. Therefore, amplitude of oscillation is $A=\frac{mg}{k}$

$$\omega=\sqrt{\frac{k}{m}}=2\pi f=20\pi \ (f=10\text{ Hz})$$

$$\frac{m}{k}=\frac{1}{400\pi^2}$$

$$v_{\max}=A\omega=\frac{g}{400\pi^2}\times 20\pi=\frac{1}{2\pi}\text{ m/s}$$

36. **c.** $F=-\frac{dU}{dx}=-8\sin 2x$

For small oscillations, $\sin 2x = 2x$
i.e., $$a=-16x$$

Since $a\ \mu\ -x$, the oscillations are simple harmonic in nature.

$$T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{1}{16}}=\frac{\pi}{2}\text{ s}$$

37. **b.** $E=\frac{1}{2}m\omega^2A^2=\frac{1}{2}m(2\pi f)^2A^2$

$$A=\frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$$

Putting $E=K+U$, we get

$$A=\frac{1}{2\pi(25/\pi)}\sqrt{\frac{2\times(0.5+0.4)}{0.2}}=0.06\text{ m}$$

38. **b.** When the rod is replaced by the string the simple pendulum will act as compound pendulum for which time period is given by

$$T=2\pi\sqrt{\frac{I_0}{Mgd}} \qquad \text{(i)}$$

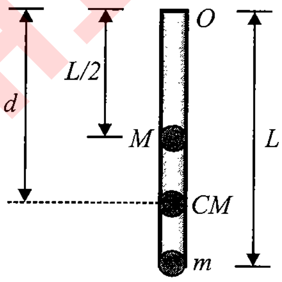

Fig. 4.188

I_0 = moment of inertia about point of suspension

$$=\frac{ML^2}{3}+mL^2=\frac{(M+3m)L^2}{3}$$

M' = total mass = $(M+m)$
d = separation between point of suspension and centre of mass of the pendulum

$$=\frac{M(L/2)+m(L)}{(M+m)}=\frac{(M+2m)L}{2(M+m)}$$

Substituting the values in Eq. (i), we get

$$\Rightarrow T=2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$$

39. **d.** $T=2\pi\sqrt{\frac{I_0}{mgd}}$; $I_0'=I_C+mr'^2 \Rightarrow mr^2=I_C+mr'^2$

$\Rightarrow$ $$I_C=m(r^2-r'^2)$$

$$I_0=I_C+md^2m(r^2-r'^2)+m(r^2+r'^2)=2mr^2$$

$$d=\sqrt{r^2+\left(\frac{2r}{\pi}\right)^2}=r\sqrt{1+\frac{4}{\pi^2}}$$

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$$\Rightarrow \qquad T = 2\pi\sqrt{\frac{2r}{g\left(1+\frac{4}{\pi}\right)^{1/2}}}$$

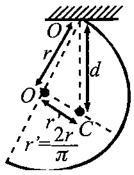

Fig. 4.189

40 d. $T = 2\pi\sqrt{\frac{m}{K_1+K_2}} = 2\pi\sqrt{\frac{10}{360}} = \frac{\pi}{3}$ s

The maximum velocity is always at equilibrium position since at any other point there will be a restoring force attempting to slow the mass.

$$\therefore V_{max} = \frac{\text{impulse}}{\text{mass}} = \frac{50}{10} = 5 \text{ m/s} \Rightarrow \omega = \frac{2\pi}{T} = 6 \text{ rad/s}$$

$$\Rightarrow A = \text{amplitude} = \frac{V_{max}}{\omega} = \frac{5}{6} = 0.83 \text{ m}$$

41. c. Let the bar be rotated through a small angle θ.
The restoring torque of the forces mg, k_1x and k_2x about O can be given as

$$\tau = -\left[mg\left(\frac{l}{2}\right)\sin\theta + k_1x(l\cos\theta) + k_2x(l\cos\theta)\right]$$

Since θ is small, $\sin\theta \cong \theta$, $x = l\,\theta$ and $\cos\theta \cong 1$
Putting $k_1 + k_2 = k$, we obtain

$$\tau = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$$

or $$I\alpha = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$$

$$\Rightarrow \omega_{osc.} = \sqrt{\left(\frac{kl^2 + mg(l/2)}{(ml^2/3}\right)} = \sqrt{\frac{3k}{m} + \frac{3g}{2l}}$$

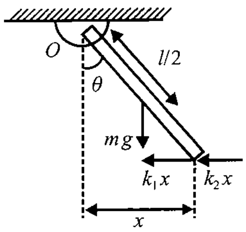

Fig. 4.190

42. b. $x = a\sin\omega t$ with $\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s

$$x = a\sin\frac{\pi t}{4}$$

In 2 s (which is equal to $T/4$), one amplitude will be covered. In 1st second

$$x = a\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}}$$

$$\text{Required ratio} = \frac{\frac{a}{\sqrt{2}}}{a - \frac{a}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1}$$

43. b. $v^2 = \omega^2(A^2 - b^2) = 3\omega^2b^2 \Rightarrow A^2 = 4b^2$

$$b = \frac{A}{2} = A\sin\omega t \quad \Rightarrow t = \frac{\pi}{6\omega}$$

$$\text{Required time, } t_1 = \frac{T}{4} - t = \frac{2\pi}{4\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$$

44. c. $x = A\sin\omega t$, where $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

$$3 = 5\sin\omega t_1 \Rightarrow t_1 = \frac{1}{\omega}\sin^{-1}\left(\frac{3}{5}\right) = \frac{2\times 37}{180}$$

$$4 = 5\sin\omega t_2 \Rightarrow t_2 = \frac{1}{\omega}\sin^{-1}\left(\frac{4}{5}\right) = \frac{2\times 53}{180}$$

$$\text{Hence time interval } (t_2 - t_1) = \frac{1}{90}(53-37) = \frac{8}{45}\text{ s}$$

45. c. $U = a + bx^2$

$$F = -\frac{dU}{dx} = -[0 + 2bx]$$

$$F = ma = -2bx \Rightarrow a = -\left(\frac{2b}{m}\right)x$$

$$\omega = 2\pi f = \left(\frac{2b}{m}\right)^{1/2}$$

Hence frequency depends upon b and m

46. c. Time period $T = 2\pi\sqrt{\frac{l}{g}}$

$$T_1 = 2\pi\sqrt{\frac{l}{g}} = T \Rightarrow T_2 = 2\pi\sqrt{\frac{4l}{g}} = 2T_1 = 2T$$

Hence, in time T_2 ($2T$) small pendulum will perform two oscillations and again at initial position.

47. d. $$x = a_1\sin\omega t + a_2\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$x = a_1\sin\omega t + a_2\sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

or, $$x = a_1\sin\omega t + a_2\sin\left(\omega t + \frac{2\pi}{3}\right)$$

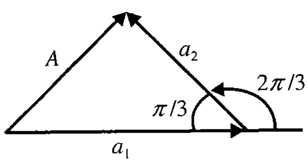

Fig. 4.191

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$$\therefore \quad A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\frac{2\pi}{3}$$

$$= a_1^2 + a_2^2 - 2a_1a_2\cos\frac{\pi}{3}$$

$$\therefore \quad A = \sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos\frac{\pi}{3}}$$

48. **b.** Let equation of simple harmonic motion is

$$x = A\sin(\omega t + \delta)$$

It is given, $A = 0.5$ m and $\omega = \frac{2\pi}{T} = \frac{2\pi}{4}\text{s}^{-1} = \frac{\pi}{2}\text{s}^{-1}$

At $t = 0.5$ s, $x = 0.3$m, so $0.3 = 0.5\sin(\omega t + \delta)$

$$\Rightarrow \sin\left(\frac{\pi}{2}\times\frac{1}{2}+\delta\right) = \frac{3}{5} \quad \Rightarrow \quad \frac{\pi}{4}+\delta = 37^\circ$$

$$\Rightarrow \delta = 37^\circ - 45^\circ = -8^\circ$$

So, equation of motion is $x = (0.5 \text{ m})\sin\left[\frac{\pi t}{2} - 8^\circ\right]$

49. **d.** As the scale can read maximum 50 kg, for a length of 20 cm, let the spring constant be k, then

$kx_0 = mg$ [for $m = 50$ kg, $x_0 = 20$ cm]

$\Rightarrow k \Rightarrow 0.2 = 50 \Rightarrow 10 = 2500$ N/m

Let mass of the body be m_0, then from $T = 2\pi\sqrt{\frac{m_0}{k}}$

$$\Rightarrow \quad 0.6 = 2\pi\sqrt{\frac{m_0}{2500}} \quad \Rightarrow m_0 = 22.8 \text{ kg}$$

50. **b.** Let us say in displaced position, the axis of cylinder is at a distance x from its mean position and its velocity of centre of mass is v and angular velocity is ω. Then, as cylinder is not slipping, $v = R\omega$. In this position, the spring elongates by x.

Using energy method we can find frequency of oscillation very easily.

Total energy of oscillation is

$$E = \frac{I\omega^2}{2} + \frac{Mv^2}{2} + \frac{kx^2}{2}$$

We have $I = \frac{MR^2}{2}$, so $E = \frac{3}{4}Mv^2 + \frac{kx^2}{2}$

$$E = v^2 + \left(\frac{2}{3}\frac{k}{M}\right)x^2 = \text{constant}$$

Comparing with $v^2 + \omega^2x^2 = \text{constant}$

$$\text{So, } \omega = \sqrt{\frac{2k}{3M}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{3M}{2k}}$$

51. **b.** Let A be the maximum amplitude of oscillation for the required situation. Then $f = m\omega^2x$ (from free body diagram of blocks)

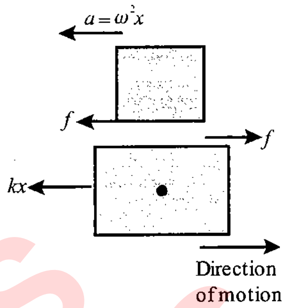

Fig. 4.192

For required condition, $f = m\omega^2 A$

for no slipping $f < f_L$

i.e., $m\omega^2 A < \mu_s mg$

$$\Rightarrow A < \frac{0.6\times g}{\frac{4\pi^2}{4}} = 0.6 \text{ m} \quad \Rightarrow \quad A_{max} = 0.6 \text{ m}$$

52. **a.** In equilibrium, let us say deformations (elongations) in springs are x_{01} and x_{02}. Then

$$mg + k_1x_{01} = k_2x_{02}$$

Let the block be displaced down by x; then elongation in spring 1 reduces by x and in spring 2 it increases by x. In this situation, the net force acting on the block towards equilibrium position is

$$F = k_2(x + x_{02}) - mg - k_1(x_{01} - x)$$

$$= (k_2 + k_1)x \quad \text{(using equilibrium equation)}$$

So, the angular frequency of SHM is

$$\sqrt{\frac{k_1 + k_2}{m}} = 10\sqrt{2} \text{ rad/s}$$

53. **d.** When the elevator is at rest. the elongation in spring is given by $ky_0 = mg$

$$\text{or} \qquad y_0 = \frac{mg}{k}$$

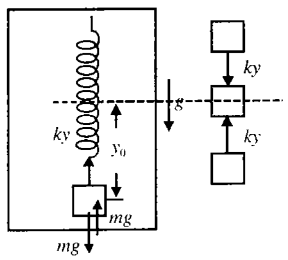

Fig. 4.193

At the instant the elevator starts falling down with acceleration g, the block is at rest w.r.t. elevator and the net force acting on it is ky_0 in the upward direction w.r.t. lift frame of reference. Due to this force, the block moves up and as a result elongation in the spring decreases, and the force experienced by the block becomes zero when spring and stops momentarily till compression in the spring becomes mg/k.

Hence the block will always have net force towards relaxed position of spring and the block will perform simple

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harmonic notion with time period $T = 2\pi\sqrt{\frac{m}{k}}$ and with amplitude mg/k.

54. **c.** At mean position, the speed will be maximum

$$\frac{kx_0^2}{2} = \frac{mv^2}{2} \Rightarrow v_{\max} = \sqrt{\frac{k}{m}}\, x_0$$

and this is attained at $t = T/4$.

Time period of motion is $T = 2\pi\sqrt{\frac{m}{k}}$

So required time is $t = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

55. **b.** Let spring constant of each spring be k, then the equivalent spring constant of the two spring system in parallel is $2k$.

Without mass, $T = 3\text{ s} = 2\pi\sqrt{\frac{12}{2k}}$

With mass, $T_1 = 65 = 2\pi\sqrt{\frac{12+m}{2k}}$

$$\frac{T}{T_1} = \frac{1}{2} = \sqrt{\frac{12}{12+m}}$$

$$m = 36\text{ kg}$$

56. **d.** The situation is illustrated in the following figure,

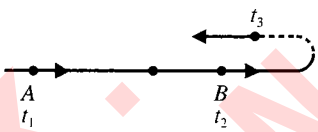

Fig. 4.194

$t_2 - t_1 = 2$ s, $t_3 - t_2 = 2$ s

i.e., $t_3 - t_1 = 4$ s which is half of the time period.

So, $T = 8$ s

57. **c.** Let spring constant of two springs are k_1 and k_2, respectively, then

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$k_1 = \frac{4\pi^2 m}{T_1^2} \quad \text{and} \quad k_2 = \frac{4\pi^2 m}{T_2^2}$$

When the two springs are connected in series, then

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} \quad \text{where} \quad k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = \sqrt{T_1^2 + T_2^2}$$

58. **a.** Rolling can be considered as pure rotation about point of contact. We can consider this system as a compound pendulum oscillating about P.

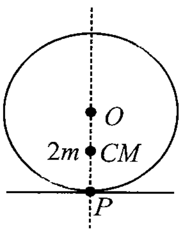

Fig. 4.195

Time period $T = 2\pi\sqrt{\frac{I_P}{mgd}}$

I_P = moment of inertia about rotation axis $= I_0 + mR^2$
$= mR^2 + mR^2 = 2mR^2$

The centre of mass of tube and rod will be at a height $R/2$ from P, hence $d = R/2$

$$\Rightarrow T = 2\pi\sqrt{\frac{2mR^2}{mgR/2}} = 2\pi\sqrt{\frac{4R}{g}}$$

59. **c.** For torsional pendulum, time period is given by

$$T = 2\pi\sqrt{\frac{I}{k}}$$

where k is torsional constant of string and it is same for both the cases.

$$I_{\text{rod}} = \frac{mL^2}{12} = \frac{1\times 1.2^2}{12} = 0.12\text{ kg-m}^2$$

$$\frac{T_{\text{rod}}}{T_{\text{plate}}} = \left(\frac{I_{\text{rod}}}{I_{\text{plate}}}\right)^{1/2} \Rightarrow \left(\frac{3}{6}\right)^2 = \frac{0.12}{I_{\text{plate}}}$$

$$I_{\text{plate}} = 0.48\text{ kg-m}^2$$

60. **b.** From circular motion representation, we can represent SHM by uniform circular motion.

Let t_1 is the time taken by particle to go from $x = 0$ to $x = A/2$

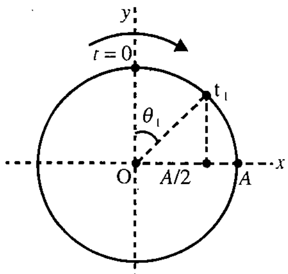

Fig. 4.196

$$\sin\theta_1 = \frac{A/2}{A} = \frac{1}{2}$$

$$\theta_1 = \frac{\pi}{6} \Rightarrow \omega t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{\pi\times T}{6\times 2\pi} = \frac{T}{12}$$

So, time taken to go from $x = A/2$ to A is

$$t_2 = \frac{T}{4} - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$$

Hence, $\frac{t_1}{t_2} = \frac{T/12}{T/6} = \frac{1}{2}$

61. **c.** Time interval is $T/6$

Let the equation of simple harmonic motion be $x = A\cos\omega t$, $v = -A\sin\omega t$, where $\omega = 2\pi/T$

Mean velocity over the required interval is

$$\langle v\rangle = \frac{\int_0^{T/6} v\,dt}{T/6}$$

$$= \frac{\int_0^{T/6} -A\omega\sin\omega t\,dt}{T/6} = \frac{6A}{T}\cos\omega t\Big|_0^{T/6} = \frac{-3A}{T}$$

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62. a. Let at $t = 0$, the particle is at extreme position, then the equation of SHM can be written as

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right)$$

At $\quad t = T/8,$

$$x = A\cos\frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

$$\text{Acceleration} = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 \times \frac{A}{\sqrt{2}}$$

$$\text{Magnitude of acceleration } = \frac{4\pi^2 A}{\sqrt{2}T^2}$$

63. b. Velocity at any displacement x is given by

$$v = \omega\sqrt{A^2 - x^2}$$

$$\text{So, the required velocity } = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{2}} = \frac{\sqrt{2}\pi A}{T}$$

64. c. Time period of the system (object of mass 4 kg) before collision is $T_1 = 2\pi\sqrt{\frac{4}{100}}$

After collision, time period of the combined mass is

$$T_2 = 2\pi\sqrt{\frac{10}{100}}$$

We can apply momentum conservation for just before the collision and just after the collision in the horizontal direction.

$$4A_1\omega_1 = 10A_2\omega_2 \Rightarrow A\frac{4\times 2\times\sqrt{\frac{100}{4}}}{10\times\sqrt{\frac{100}{10}}} = 2\sqrt{\frac{2}{5}}\text{ m}$$

$$\text{So, change in amplitude, } \Delta A = A_1 - A_2 = 2\left[1 - \sqrt{\frac{2}{5}}\right]\text{m}$$

65. a. As the range of motion is 4 cm, the amplitude of motion is +2 cm. 10.5 s is equal to 10.5 time period of simple harmonic motion; so we have to find the height and position of cork after one-half of time period as $T/2 = 0.5$ s. As at $t = 10$ s, the particle is at its lowest position, after half a time period the cork would be at its maximum height and velocity of cork at extreme position is zero.

66. d. From equilibrium position of mass, $mg = ky_0$ where $y_0 = (11 - 4.5)\text{ cm} = 6.5\text{ cm}$

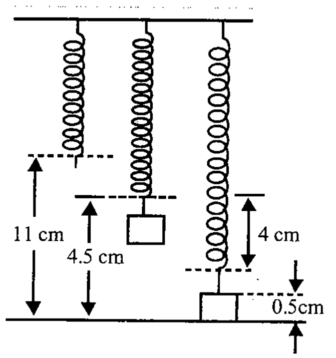

Fig. 4.197

So, time period of simple harmonic motion is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

$$T = 2\pi\sqrt{\frac{5.6\times 10^{-2}}{10}} = 0.5065\text{ s} = 0.5\text{ s}$$

5 s is equivalent to 10 complete time period; so the mass is at its initial position at $t = 5$ s, i.e., it is at 0.5 cm above the table top at $t = 5$ s.

67. c. The time period of simple harmonic motion is $T = 18$ s.

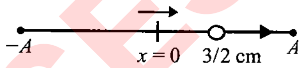

Fig. 4.198

At $t = 0$, the particle is at $x = 3/2$ cm and approaching positive extreme.

$$39\text{ s} = 2\times 18 + 3 = 2T + 3 = 2T + T/6$$

Distance travelled by the particle in $2T$ is $8A = 24$ cm.
Distance travelled by the particle in further $T/6$ is $A/2 = 1.5$ cm.
Total distance travelled = 25.5 cm

68. a. In equilibrium position, net force acting on the object (block of mass m) is zero. Let spring of spring constant k_1 is stretched by x_1 and spring of spring constant k_2 is stretched by x_2, then free body diagram of the block is as shown in Fig. 4.199.

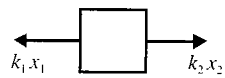

Fig. 4.199

Now, $\quad x_1 + x_2 = 20$ cm $\quad$ and $\quad k_1x_1 = k_2x_2$
$\quad x_1 = 15$ cm $\quad$ and $\quad x_2 = 5$ cm

So, new equilibrium position is at $x = (20 + 15)$ cm from P_1, and time period of oscillation of the block is given by

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} = 2\pi\sqrt{\frac{0.1}{4000}} = \frac{\pi}{100}\text{ s}$$

69. b. $U = \frac{a}{x^2} - \frac{b}{x}$; $F = -\frac{dU}{dx} = -\left[\frac{-2a}{x^3} + \frac{b}{x^2}\right]$

At equilibrium position, $F = 0$

$$x = x_0 = \frac{2a}{b}$$

At $x = x_0 + \Delta x$, i.e., at a displacement of Δx ($<< x_0$) from mean position,

$$F = -\left[\frac{bx - 2a}{x^3}\right] = -\left[\frac{b(x_0 + \Delta x) - 2a}{(x_0 + \Delta x)^3}\right] = -\frac{b\Delta x}{x_0^3}$$

$$= -\frac{b^4}{8a^3}\Delta x$$

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As F µ Δx, so particle performs simple harmonic motion with time period

$$T = 2\pi\sqrt{\frac{8a^3m}{b^4}}$$

70. c. Let ω_1 and ω_2 be the angular frequencies of first and second particle, respectively. Then, the phase by which they will proceed in time t is $\omega_1 t$ and $\omega_2 t$, respectively. According to the given situation,

$$\omega_2 t - \omega_1 t = 3\times 2\pi \quad \text{for} \quad t = 45 \text{ s}$$

$$\frac{2\pi}{T} - \frac{2\pi}{3} = \frac{3\times 2\pi}{45}$$

$$\frac{1}{T} = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} \qquad \Rightarrow T = 2.5 \text{ s}$$

71. b. Method 1: Particle is starting from rest, i.e., from one of its extreme position.
As particle moves a distance $A/5$, we can represent it on a circle as shown.

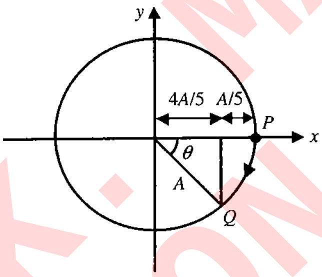

Fig. 4.200

$$\cos\theta = \frac{4A/5}{A} = \frac{4}{5} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\omega t = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

Method 2: As the particle starts from rest, i.e., from extreme position $x = A\sin(\omega t - \phi)$

At $\qquad t = 0; \quad x = A \quad \Rightarrow \phi = \frac{\pi}{2}$

$$A - \frac{A}{5} = A\cos\omega t$$

$$\frac{4}{5} = \cos\omega t \quad \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

72. b. $\frac{d\theta}{dt} = 2 \quad \therefore \theta = 2t$

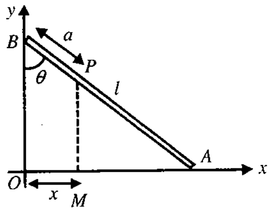

Fig. 4.201

Let $BP = a$; therefore, $x = OM = a\sin\theta = a\sin(2t)$

Hence M executes SHM within the given time period and its acceleration is opposite to 'x' that means towards left.

73. c. Here $\omega = \sqrt{\frac{k}{3m}}$. The maximum static frictional force is

$$f_{max} = \mu mg\cos\theta = 2\tan\theta\, mg\cos\theta = 2\, mg\sin\theta$$

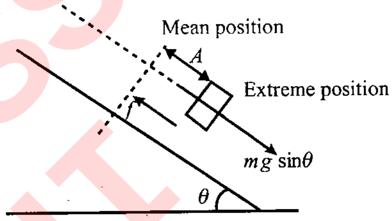

Fig. 4.202

Applying Newton's second law on the block at lower extreme position,

$$f - mg\sin\theta = m\omega^2 A \quad \Rightarrow f = m\omega^2 A + mg\sin\theta$$

$$\text{As } f \le f_{max}, \quad \omega^2 A = g\sin\theta \quad \text{or} \quad A = \frac{3mg\sin\theta}{k}$$

74. a. The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position.

At equilibrium position $kx_0 = mg \Rightarrow x_0 = mg/k$.

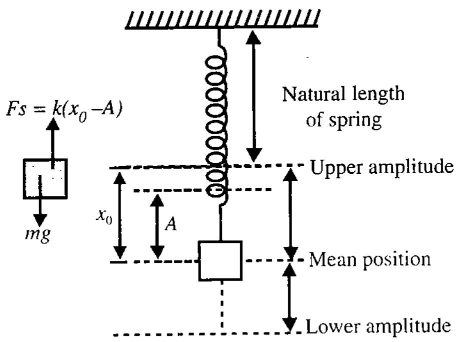

Fig. 4.203

$$F_{max} = k(x_0 - A) = k\left[\frac{mg}{k} - A\right]$$

$$= mg - 4\frac{\pi^2}{T^2}mA = mg - kA = mg - m\omega^2 A$$

75. c. Due to impulse, the total energy of the particle becomes

$$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = m\omega^2A^2$$

Let A' be the new amplitude,

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$$\frac{1}{2}m\omega^2(A')^2 = m\omega^2 A^2 \Rightarrow A' = \sqrt{2}A$$

76. a. Conserving momentum, $2V = 3V'$

$$V' = \frac{2}{3}V$$

$$E_i = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}\times 2V^2 = V^2 = \frac{1}{2}KA^2$$

$$E_f = \frac{1}{2}\times m_2 V_2^2 = \frac{1}{2}\times 3\times\frac{2}{3}\times\frac{2}{3}V^2 = \frac{2}{3}V^2$$

$$\frac{1}{2}KA'^2 = \frac{2}{3}V^2 = \frac{2}{3}E_i \qquad (\therefore E_i = V^2 \text{ from above})$$

$$\frac{1}{2}KA'^2 = \frac{2}{3}\left(\frac{1}{2}KA^2\right) \Rightarrow A' = \sqrt{\frac{2}{3}}A$$

77. b. $f_0 = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$

where l is the distance between point of suspension and centre of mass of the body. Thus, for the stick of length L and mass m,

$$f_0' = \frac{1}{2\pi}\sqrt{\frac{mg\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{12g}{L}} = \sqrt{2}f_0$$

78. b. Let the line joining AB represents axis 'r'. By the conditions given 'r' coordinate of the particle at time t is

$$r = 2\sqrt{2}\cos\omega t$$

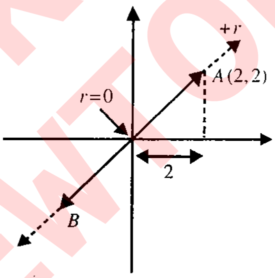

Fig. 4.204

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow r = 2\sqrt{2}\cos\omega t$$

$$x = r\cos 45° = \frac{r}{\sqrt{2}} = 2\cos\pi t$$

$$a_x = -\omega^2 x = -\pi^2 2\cos\pi t$$

$$F_x = ma_x = -4\pi^2\cos\pi t$$

79. a. $\frac{1}{2}I\omega^2 = mg(2l) \Rightarrow \frac{I}{mgl} = \frac{4}{\omega^2}$

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{4}{\omega^2}} = \frac{4\pi}{\omega}$$

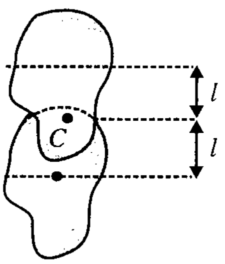

Fig. 4.205

80. c. $v_1 = \omega\sqrt{a^2 - x_1^2}$, $\quad v_2 = \omega\sqrt{a^2 - x_2^2}$

We get, $a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

81. a. Let S be the surface tension of the soap film. For equilibrium of rod

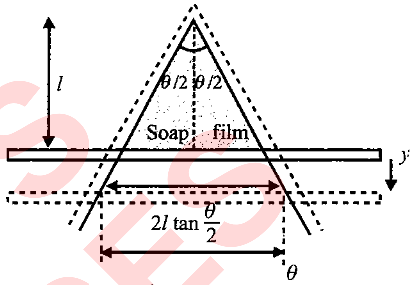

Fig. 4.206

$$2(l+y)\tan\frac{\theta}{2}$$

$$mg = (F_{\text{surface}})_1$$

$$mg = \left(2l\tan\frac{\theta}{2}\right)S\times 2; \quad mg = 4Sl\tan\frac{\theta}{2}$$

If the rod is displaced from its mean position by small displacement y, then restoring force on the rod is

$$F_{\text{rest}} = -\left[(F_{\text{surface}})_2 - mg\right] - (F_{\text{surface}})_1$$

$$F_{\text{rest}} = -\left[4S(l+y)\tan\frac{\theta}{2} - mg\right] - \left[4S\tan\frac{\theta}{2}y\right.$$

$$a = -\frac{4S\tan\frac{\theta}{2}}{m}y = \frac{-4S\tan\frac{\theta}{2}}{\left(\frac{4Sl\tan\frac{\theta}{2}}{g}\right)}y$$

$$\frac{d^2y}{dt^2} = -\left(\frac{g}{l}\right)y \qquad \therefore T = 2\pi\sqrt{\frac{l}{g}}$$

82. a. Let the cylinder be depressed a distance x metres into water from the position of rest.

Increase in tension of spring = $100x$ metres into water from the position of rest

Increase in tension of spring = $100x$ kg-wt

$(A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2)$

Increase in buoyancy $= \frac{100}{100^2}x\times 1000$ kg-wt

$K = 1$ kg-wt/cm = 100 kg-wt/m = $10x$ kg-wt

Total unbalanced vertical force on the cylinder

$= 100x + 10x = 110x$ kg-wt

Acceleration of cylinder $= \frac{110x\times 10}{10}\text{ m/s}^2 = 110x\text{ m/s}^2$

Period of one oscillation $= \frac{2\pi}{\sqrt{110}} = 0.6$ s

83. a. The oscillation of the block will be periodic but not simple harmonic. The springs are only compressed but not extended since the block loses contact with either spring just in its relaxed state. The minimum distance between the springs will therefore be when both the springs are relaxed, i.e., during the interval when the block moves between the springs.

Time taken by the block to move from one spring to the other is $t_1 = d/v$.

Time taken by the block in contact with either spring is

$$t_2 = \frac{1}{2}\left[2\pi\sqrt{\frac{m}{K}}\right] = \pi\sqrt{\frac{m}{K}}$$

Hence the periodic time T of the oscillation of the block is $T = 2(t_1 + t_2)$

$$= 2\left[\frac{d}{v} + \pi\sqrt{\frac{m}{K}}\right]$$

Also minimum distance between the springs $= d$.

84. a. For a displacement x, the kinetic and potential energies are $KE = \frac{1}{2}m(A^2 - x^2)\omega^2$ and $PE = \frac{1}{2}mx^2\omega^2$

Each of these = 10/2 = 5 J when $x = 1$ cm

Hence $\frac{1}{2}mx^2\omega^2 = \frac{1}{2}\times 0.1\times(1\times 10^{-2})^2\omega^2 = 5$

This gives $\omega^2 = \frac{10}{0.1\times 10^{-4}} = 10^6$

Giving $\omega = 10^3 = 1000$ rad/s

Hence $T = \frac{2\pi}{\omega} = \frac{\pi}{500}$ s

And frequency $f = \frac{500}{\pi}$ s

Also, $KE = 5 = \frac{1}{2}\times 0.1[A^2 - (1\times 10^{-2})^2](10^6)$

$$A^2 - 10^{-4} = 10^{-4}$$

$$A^2 = 2\times 10^{-4} \Rightarrow A = \sqrt{2}\times 10^{-2}\text{m} = \sqrt{2}\text{ cm}$$

85. c. The centripetal acceleration on the bob as it oscillates (acting along the radius of circle) $= v^2/R$. This will act horizontally towards the centre of circular path.

The total acceleration acting on the pendulum bob is therefore $a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$

The frequency of oscillation will therefore be

$$n = \frac{1}{2\pi}\sqrt{\frac{a}{l}} = \frac{1}{2\pi}\sqrt{\frac{\left(g^2 + \frac{v^4}{R^2}\right)^{1/2}}{l}}$$

86. d. When spring is compressed by $3x_0$. Amplitude, $A = 3x_0$. The time taken from extreme compressed position to mean position, $t_1 = T/4$.

If time taken (t_2) from mean position to $x = x_0$ is given by

$$x = A\sin\frac{2\pi t_2}{T} \Rightarrow x_0 = 3x_0\sin\frac{2\pi t_2}{T}$$

$$\sin\frac{2\pi t_2}{T} = \frac{1}{3} \Rightarrow \frac{2\pi t_2}{T} = \frac{\pi}{9} \Rightarrow t_2 = \frac{T}{18}$$

$$t_1 + t_2 = \frac{T}{4} + \frac{T}{18} = \frac{11}{18}T = \frac{11}{18}2\pi\sqrt{\frac{m}{K}} = \frac{11}{9}\pi\sqrt{\frac{m}{K}}$$

87. c. Angular frequency of system,

$$\omega = \sqrt{\frac{K}{m+m}} = \sqrt{\frac{K}{2m}}$$

Maximum acceleration, $a_{max} = \omega^2 A$

Frictional force between P and Q

= force exerted on lower block

$$= m\omega^2 A = m\left(\frac{K}{2m}\right)A = \frac{KA}{2}$$

88. c. If the stick is rotated through a small angle θ, the spring is stretched by a distance $L\theta/2$. Therefore restoring force in spring $= (KL\theta/2)$. Each spring causes a torque

$$\tau\ (= Fd) = \left(\frac{KL\theta}{2}\right)\frac{L}{2}$$

in the same direction. Therefore equation of rotational motion is $\tau = I\alpha$.

$$-2\left(\frac{KL\theta}{2}\right)\frac{L}{2} = I\alpha \quad \text{with} \quad I = \frac{ML^2}{12}$$

$$\alpha = -\left(\frac{6K}{M}\right)\theta$$

Standard equation of angular SHM is $\alpha = -\omega^2\theta$

$$\omega^2 = \frac{6K}{M} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

89. b. The small block oscillates along the inclined plane with an amplitude A. As a result the centre of mass of the system undergoes SHM along the horizontal direction

$$x_{CM} = \frac{[m(A\sin\omega t\cdot\cos 60°) + MO]}{m+M} = \frac{1}{2}\frac{m}{m+M}A\sin\omega t$$

The acceleration of CM is $a_{CM} = -\omega^2 x_{CM}$, along the horizontal while the net horizontal force is $= (M+m)a_{CM}$, which is equal to the force of friction acting on it.

90. d. When the spring undergoes displacement in the downward direction it completes one-half oscillation while it completes another half oscillation in the upward direction. The total time period is

$$T = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$$

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Multiple Correct Answers Type

1. **a., c.** Let O be the mean position and a be the acceleration at a displacement x from O.

At position I, $N - mg = ma$

$$\therefore \quad N \neq 0$$

At position II, $mg - N = ma$

For $N = 0$ (loss of contact), $g = a = \omega^2 x$.

Loss of contact will occur for amplitude $x_{max} = g/\omega^2$ at the highest point of the motion.

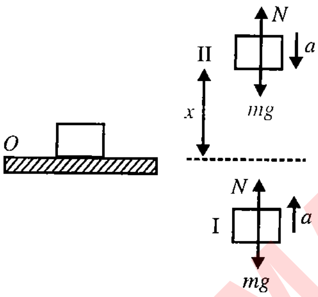

Fig. 4.207

2. **a., b., d.** Description of motion is completely specified if we know the variation of x as a function of time. For simple harmonic motion, the general equation of motion is $x = A\ (\omega t + \delta)$. As ω is given, to describe the motion completely, we need the values of A and δ.

From options (b) and (d), we can have the values of A and δ directly.

For option (a), we can find A and δ if we know initial velocity and initial position. Option (c) cannot give the values of A and δ so it is not the correct condition.

3. **a., c., d.**

$$U = x^2 - 4x + 3 \text{ and } F = -\frac{dU}{dx} = -(2x - 4)$$

At equilibrium position $F = 0$, so $x = 2$ m.

Let the particle is displaced by Δx from equilibrium position, i.e., from $x = 2$, then restoring force on body is,

$$F = -2\ (2 + \Delta x) + 4 = -2\ \Delta x$$

i.e., $F\alpha\ -\Delta x$, so performs simple harmonic motion about $x = 2$ m.

$$\text{Time period, } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{2}} = \sqrt{2}\pi \text{ s}$$

$$\text{From energy conservation, } \frac{mv_{max}^2}{2} + U_{min} = U_{max}$$

$$\frac{1 \times 4^2}{2} + (2^2 - 4 \times 2 + 3) = (A + 2)^2 - 4(A + 2) + 3$$

where A is amplitude. Solving the above equation, we get $A = 2\sqrt{2}$ m.

4. **a., b., d.** Total mechanical energy of the oscillating system is, $E = K_{max} + U_{min} + K_{min}$, $K_{min} = 0$ at extreme position.

So, $\quad U_{max} = E = 200$ J

$$K_{max} = \frac{mv_{max}^2}{2} = \frac{m \times A^2\omega^2}{2} = \frac{KA^2}{2} = 150 \text{ J}$$

So, $\quad U_{min} = 50$ J

5. **a., b., c., d.** Period of oscillation changes as it depends on mass and becomes three times. The amplitude of oscillation does not change, because the new object is attached when the original object is at rest. Total energy does not change as at extreme position the energy is in the form of potential energy stored in spring which is independent of mass, and hence maximum; KE also does not change but as mass changes the maximum speed changes.

6. **a., b.** The time period of simple harmonic pendulum is independent of mass, so it would be same as that $T = 2\pi\sqrt{1/g}$. After collision, the combined mass acquires a velocity of $v_0/2$, as a result of this velocity, the mass ($2m$) moves up and at an angel θ_0 (say) with vertical, it stops, this is the extreme position of bob.

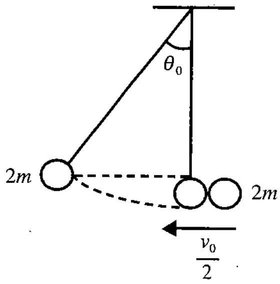

Fig. 4.208

From work–energy theorem, $\Delta K = W_{total}$

$$0 - \frac{2m}{2}\left(\frac{v_0}{2}\right)^2 = -2mgl(1 - \cos\theta_0)$$

$$\frac{v_0^2}{8gl} = 1 - \cos\theta = 2\sin^2\frac{\theta_0}{2}$$

$$\sin\frac{\theta_0}{2} = \frac{v_0}{4\sqrt{gl}}$$

$$\text{If } \theta_0 \text{ is small, } \sin\frac{\theta_0}{2} \simeq \frac{\theta_0}{2} \quad \Rightarrow \quad \theta_0 = \frac{v_0}{2\sqrt{g\ell}}$$

So, the equation of simple harmonic motion is $\theta = \theta_0 \sin(\omega t)$

7. **a., c.** At $t = 0$ when particle is at extreme position, the situation is as shown in Fig. 4.209.

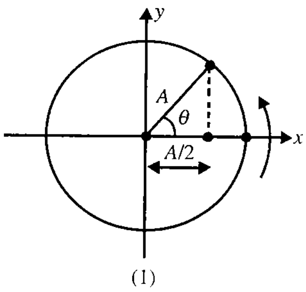

Fig. 4.209

From the figure, $\cos\theta = \dfrac{A/2}{A} = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3} \Rightarrow \dfrac{\pi}{3} = \dfrac{2\pi}{T} \times 1 \Rightarrow T = 6$ s

At $t = 0$ when particle is at mean position, the situation is as shown in Fig. 4.210.

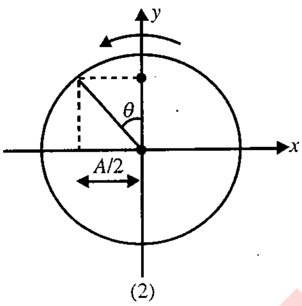

Fig. 4.210

$$\theta = \omega t$$

From the figure, $\sin\theta = \dfrac{A/2}{A}$

$\theta = \dfrac{\pi}{6}$, but $\theta = \omega t \Rightarrow \dfrac{\pi}{6} = \dfrac{2\pi}{T} \times 1 \Rightarrow T = 12$ s

If initially the particle is located somewhere else, then time period comes out to be different. A reverse question can also be formed on the same concept.

8. **b., c.** $x = 3 \sin 100\,\pi t$

$$y = 4 \sin 100\,\pi t$$

Equation of path is $\dfrac{y}{x} = \dfrac{4}{3}$

i.e., $t = \dfrac{4}{3} x$

which is equation of a straight line having slope 4/3.

Equation of resulting motion is

$\vec{r} = x\hat{i} + y\hat{j} = (3\hat{i} + 4\hat{j}) \sin 100\pi t$

Amplitude is $\sqrt{3^2 + 4^2} = 5$

9. **b., c.** $v^2 = 108 - 9x^2$ or $v^2 = 9(12 - x^2)$

We can compare the above expression with $v = \omega\sqrt{A^2 - x^2}$, which is the expression of velocity for SHM.

From this, we will get

$$\omega = 3 \quad \text{and} \quad A = \sqrt{12}$$

SHM is not a uniformly accelerated motion.

Acceleration at a distance 3 cm from the mean position,

$$a = \omega^2 (3 \text{ cm}) = 27 \text{ cm/s}^2$$

Maximum displacement from the mean position $= A = \sqrt{12}$ cm.

10. **a, b., c., d.** and **e.** The position of momentary rest in SHM is the extreme position where velocity of particle is zero.

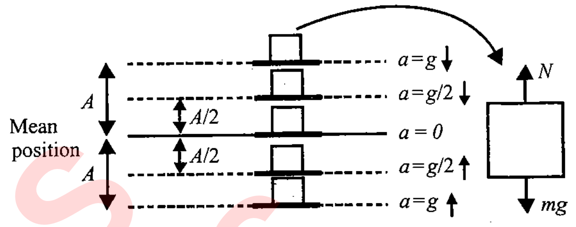

Fig. 4.211

As the block loses contact with the plank at this position, i.e., normal force becomes zero, it has to be the upper extreme where acceleration of the block will be g downwards.

$\omega^2 A = g \qquad \Rightarrow \omega^2 = \dfrac{10}{0.4} = 25$

$\omega = 5$ rad/s $\qquad \Rightarrow T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5}$ s

Acceleration in SHM is given as $a = \omega^2 x$.

From the figure, we can see that at lower extreme, acceleration is g upwards

$$N - mg = ma$$

or $\qquad N = m(a + g) = 2mg$

At halfway up acceleration is $g/2$ downwards.

$$mg - N = ma \quad \text{or} \quad N = m(g - g/2) = \frac{1}{2}mg$$

At halfway down, acceleration is $g/2$ upwards.

$$N - mg = ma \quad \text{or} \quad N = m(g + g/2) = \frac{3}{2}mg$$

At mean position, velocity is maximum and acceleration is zero.

$\therefore \qquad N = mg$

11. **a., b., c., d.** At $t = 0$

Displacement, $x = x_1 + x_2 = 4\sin\dfrac{\pi}{3} = 2\sqrt{3}$ m

Resulting amplitude, $A = \sqrt{2^2 + 4^2 + 2(2)(4)\cos\pi/3}$

$= 2\sqrt{7}$ m

Maximum speed, $A\omega = 20\sqrt{7}$ m/s

Maximum acceleration, $A\omega^2 = 200\sqrt{7}$ m/s^2

Energy of the motion $= \dfrac{1}{2}m\omega^2 A^2 = 28$ J

12. **b., c., d.** The only horizontal force acting on the coin is the force of friction F. Hence its horizontal acceleration is always in the direction of F and its magnitude is F/m. The

magnitude and direction of F can thus be obtained from the magnitude and direction of acceleration.
At the highest point, the normal reaction has the minimum value,

$$N_H = mg - m\omega^2 A \quad \Rightarrow F_{\min} = \mu N_H$$

At the lowest point, the normal reaction has the maximum value,

$$N_L = mg + m\omega^2 A \quad \Rightarrow F_{\max} = \mu N_L$$

13. c., d. When a constant force is superimposed on a system which undergoes SHM along the line of SHM, the time period does not change as it depends on mass of the block and force constant of spring.
The mean position changes as this is the point where net force on the particle is zero.

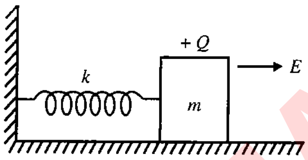

Fig. 4.212

14. b., c., d. $U = 5x(x-4) = 5x^2 - 20x$

$$F = -\frac{dU}{dx} = -10x + 20$$

i.e., force is not constant.
KE or speed of the particle will be maximum at the mean position where force becomes zero.

$$F = 0 \quad \text{or} \quad x = 2\text{ m}$$

Acceleration experienced by the particle is

$$a = \frac{F}{m} = \frac{-10x+20}{0.1} = -(100x - 200)$$

i.e., particle executes SHM.

As $\quad \omega^2 = 100$

$\therefore \quad \omega = 10$

Hence $\quad T = \dfrac{2\pi}{\omega} = \dfrac{\pi}{5}$ s

15. a., c. $y = a \sin \omega t = a \sin \dfrac{2\pi t}{T}$

$$v = \frac{dy}{dt} = \omega a \cos \frac{2\pi t}{T}$$

At $\quad t = \dfrac{T}{6}, \quad v = \omega a \cos\left(\dfrac{2\pi}{T}\dfrac{T}{6}\right) = \dfrac{1}{2}\omega a$

or, $\quad v = (v_{\max}/2)$

$$y = a \sin \frac{2\pi}{T} \times \frac{T}{6} = a \sin \frac{\pi}{3}$$

It is not half of a.

$$\text{Acceleration} = \frac{d^2y}{dt^2} = \frac{dv}{dt} = \omega^2 a \sin \frac{2\pi t}{T}$$

$$= \omega^2 a \sin \frac{\pi}{3} = 0.86(AC)_{\max}$$

At this instant,

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_{\max}}{2}\right)^2 = \frac{(\text{KE})_{\max}}{4} = \frac{1}{4}(\text{TE})$$

$\therefore \quad \text{PE} = \text{TE} - \text{KE} = \dfrac{3}{4}(\text{TE}),$

i.e., $\quad \text{KE} \neq \text{PE}$

16. b., d. The maximum extension x produced in the spring in case (a) is given by

$$F = kx \quad \text{or} \quad x = \frac{F}{k}$$

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{\text{mass}}{\text{force constant}}} = 2\pi\sqrt{\frac{m}{k}}$$

In case (a) one end A of the spring is fixed to the wall. When a force F is applied to the free end B in the direction shown in Fig. 4.140(a), the spring is stretched exerting a force on the wall which in turn exerts an equal and opposite reaction force on the spring, as a result of which every coil of the spring is elongated producing a total extension x.
In case (b), shown in Fig. 4.140(b), both ends of the spring are free equal forces are applied at ends. By application of forces both the cases are same.
Thus, the maximum extension produced in the spring in cases (a) and (b) is the same. In case (b) the mid-point of spring will not move, we can say the blocks are connected with the springs whose lengths are half the original length of spring. Now, the force constant of half the spring is twice of complete spring. In case (b) the force constant = $2k$. Hence, the time period of oscillation will be

$$T' = 2\pi\sqrt{\frac{m}{2k}}$$

$$\frac{T'}{T} = \sqrt{2}$$

Hence, the correct choices are (b) and (d).

17. a., b. When point of suspension of pendulum is moved upwards, $g_{\text{eff}} = g + a$, $g_{\text{eff}} > g$ and as $T \propto 1/\sqrt{g_{\text{eff}}}$, hence T decreases, i.e., choice (a) is correct.
When point of suspension of pendulum is moved downwards and $a > 2g$, then T decreases, i.e., choice (b) is also correct.
In case of horizontal acceleration

$$g_{\text{eff}} = \sqrt{g^2 + a^2}, \quad \text{i.e.,} \quad g_{\text{eff}} > g$$

i.e., again T decreases.

18. **a., c.** $y = 0.5[\cos^2(n\pi t) - \sin^2(n\pi t)] = 0.5 \cos 2n\pi t$

$$\frac{dy}{dt} = -0.5 \times 2n\pi \times \sin 2n\pi t$$

$$\frac{d^2y}{dt^2} = -(0.5)(2n\pi)^2 \cos 2n\pi t = -4n^2\pi^2 y \quad \text{(i)}$$

i.e., $\frac{d^2y}{dt^2} \propto -y$

i.e., particle is executing SHM with amplitude 0.5 m, i.e., choice (a) is correct.

As standard equation of SHM is

$$\frac{d^2y}{dt^2} = -\omega^2 y \quad \text{(ii)}$$

Hence, $\omega = 2n\pi$

For a second's pendulum, $T = 2$ s

Hence, $\omega' = \frac{2\pi}{T} = \pi = 2n$

i.e., choice (b) is not correct.

$$\left(\frac{dy}{dt}\right)_{max} = n\pi$$

i.e., choice (c) is also correct.

19. **a., b., c., d.** If a particle is performing SHM with amplitude A and angular frequency ω and if its initial phase is equal to zero, then its displacement from the mean position will be given by $x = A \sin \omega t$.

Value of $\sin \omega t$ is same at two different values of the phase. If one is ωt, then the other is $(\pi - \omega t)$. Hence at these two instants, the phases are unequal. Therefore, option (c) is correct.

Velocity of the particle will be equal to $\omega\sqrt{(A^2 - x^2)}$. Since x is same at these two instants, therefore the magnitude of velocity will be same or the speeds are equal. However, velocities are unequal because at one instant direction of motion of the particle will be towards the extreme position or away from mean position and at the other instant, it will be towards the mean position. Hence (a) is also correct.

Since speeds are equal, kinetic energies are also equal. Hence, option (d) is also correct. Acceleration of the particle is $a = -\omega^2 x$. Since x is same at these two instants, accelerations are also equal. Therefore, option (b) is also correct.

20. **a., c., d.** The only external horizontal force acting on the system of the two blocks and the spring is F. Therefore, acceleration of the centre of mass of the system is equal to $F/m_1 + m^2$.

Hence, centre of mass of the system moves with a constant acceleration. Initially, there is no tension in the spring, therefore at initial moment m_2 has an acceleration F/m_2 and it starts to move to the right. Due to its motion, the spring elongates and a tension is developed. Therefore, acceleration of m_2 decreases while that of m_1 increases from zero initial value.

The blocks start to perform SHM about their centre of mass and the centre of mass moves with the acceleration calculated above. Hence, option (b) is correct.

Since the blocks perform SHM about centre of mass, therefore the length of the spring varies periodically. Hence, option (a) is wrong.

Since magnitude of the force F remains constant, therefore amplitude of oscillations also remains constant. So, option (c) is also wrong.

Acceleration of m_2 is maximum at the instant when the spring is in its minimum possible length, which is equal to its natural length. Hence, at initial moments, acceleration of m_2 is maximum possible.

The spring is in its natural length, not only at initial moment but at time $t = T, 2T, 3T, \ldots$ also, where T is the period of oscillation. Hence, option (d) is wrong.

21. **a., c., d.** When the block is released suddenly, it starts to move down. During its downward motion the rubber cord elongates. Hence, a tension is developed in it but the block continues to accelerate downwards till tension becomes equal to weight mg of the block.

After this moment, the block continues to move down due to its velocity and rubber cord further elongates. Therefore, tension becomes greater than the weight; hence, the block now retards and comes to an instantaneous rest.

At lowest position of the block, strain energy in the cord equals loss of potential energy of the block. Suppose the block comes to an instantaneous rest when elongation of the rubber cord is equal to y. Then

$$\frac{1}{2}ky^2 = mgy \Rightarrow y = \frac{2mg}{k} \quad \text{and} \quad 0$$

Hence, block will be instantaneously at rest, at $y = 0$ and at $y = 2mg/k$.

In fact, the block oscillates between these two values.

Since the rubber cord is elastic, tension in it is directly proportional to elongation. Therefore, the block will perform SHM.

Its amplitude will be equal to half of the distance between these extreme positions of the block or amplitude

$$= \frac{1}{2} \times \frac{2mg}{k} = \frac{mg}{k} = l$$

Hence, option(b) is correct.

The angular frequency of its SHM will be equal to

$$\omega = \sqrt{\frac{k}{m}}$$

Since k and m are not given in the question, it cannot be calculated. Hence option (d) is not correct.

22 **a., b., c.** The motion of the particle is simple harmonic.

The displacement at time t is: $x = a \sin(bt + c)$

Therefore, displacement at time $(t + (2\pi/b))$ is

$$x \text{ at}\left(t + \frac{2\pi}{b}\right) = a \sin\left[b\left(t + \frac{2\pi}{b}\right) + c\right]$$

$= a\sin[bt + c + 2\pi] = a\sin(bt + c) = x$ (at time t)
Hence, statement (a) is correct.
Statement (b) is also correct since the same displacement is recovered after a time interval of $(2\pi/b)$. Statement (c) is correct because the velocity is zero when the displacement $= \pm$the amplitude, i.e., at the extreme ends of the motion. Statement (d) is incorrect, the acceleration is maximum (in magnitude) at $x = \pm A$.

23. **a., d.** Statement (a) is correct. At any position O and P or between O and Q, there are two accelerations—a tangential acceleration $g\sin\alpha$ and a centripetal acceleration v^2/l (because the pendulum moves along the arc of a circle or radius l), where l is the length of the pendulum and v its speed at that position. When the bob is at the mean position O, the angle $\alpha = 0$, therefore $\sin\alpha = 0$; hence, the tangential acceleration is zero. But at O, speed v is maximum and the centripetal acceleration v^2/l is directed radially towards the point of support. When the bob is at the end points P and Q, the speed v is zero, hence the centripetal acceleration is zero at the end points, but the tangential acceleration is maximum and is directed along the tangent to the curve at P and Q. The tension in the string is not constant throughout the oscillation. At any position between O and end point P or Q, the tension in the string is given by $T = mg\cos\alpha$.

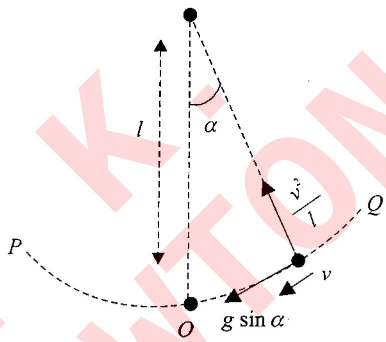

Fig. 4.213

At the end point P and Q, the value of α is the greatest, hence the tension is the least. At the mean position O, $\alpha = 0$ and $\alpha = 1$ which is the greatest; hence tension is greatest at the mean position.

24. **a., d.** Since liquid 2 is below liquid 1, liquid 2 is denser than liquid 1. Let area of cross section of the cylindrical block be A and it be displaced downward by y. Then volume of liquid 2 displaced will get increased by Ay and that of liquid 1 will get decreased by the same amount Ay. Hence, net increase in upthrust on the block will be equal to $(Ayd_2g - Ayd_1g)$. This additional upthrust tries to restore the block in original position.
It means, the block experiences a restoring force $Ay(d_2 - d_1)g$. Since this force is restoring and directly proportional to displacement y, it will execute SHM along a vertical line.
Hence, option (a) is correct and option (b) is wrong.
If mass of the block is equal to m, then its acceleration will be equal to

$$\frac{Ayg(d_2 - d_1)}{m}$$

Since its acceleration depends on mass m, frequency of oscillations will depend on size of the cylinder.
Hence option (c) is wrong.
If the cylinder is displaced upward through y from equilibrium position, then it will experience a net downward force, equal to calculated above. This shows that its motion will be symmetric about its equilibrium position.

25. **c., d.** If in equilibrium position, elongation of the spring is equal to x_0, then

$$Kx_0 = mg \quad \text{or} \quad x_0 = \frac{mg}{K} = 1\text{ cm}$$

If the block is raised till spring becomes unstretched and then released, then during subsequent motion maximum elongation of the spring (y) will be calculated by energy conservation law.

$$\text{Loss of PE of block } (mgy) = \text{strain energy}\left(\frac{1}{2}Ky^2\right)$$

$$y = \frac{2mg}{K} = 2\text{ cm}$$

Hence option (b) is correct.

Frequency of oscillations will be, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}} = 5$ Hz
Hence option (c) is correct.
Since frequency, f, does not depend upon gravitational acceleration, therefore frequency will remain unchanged, even if the system is taken to moon. Hence option (d) is also correct.

26. **a., c.** Energy stored $= \dfrac{1}{2}kx^2 = \dfrac{1}{2}k \times (0.25)^2 = 5$ J

$$k = 160\text{ N/m}$$

Period $\quad T = 2\pi\sqrt{\dfrac{m}{k}}$

$T = \dfrac{1}{5}$ s $\quad \therefore T^2 = \dfrac{4\pi^2 m}{k}$

$$\frac{1}{25} = \frac{4\pi^2 m}{160}$$

$\therefore \quad m = \dfrac{4}{25}$ kg (As $\pi^2 = 10$), hence $m = 0.16$ kg

27. **a., d.** The resultant of two motions is simple harmonic of same angular frequency ω.
The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\frac{\pi}{3}} = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

$$\text{Maximum acceleration} = \omega^2 A = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

28. **b., c.** Let the simple harmonic motions be given by

$$x_1 = a\sin\left(2\pi\frac{t}{T}\right) \quad \text{(i)}$$

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$$x_2 = a\sin\left(2\pi\frac{t}{T}+\frac{\pi}{4}\right) \quad \text{(ii)}$$

and

$$x_3 = a\sin\left(2\pi\frac{t}{T}+\frac{\pi}{2}\right) \quad \text{(iii)}$$

Then the resultant periodic motion, by the principle of superposition is given by

$$x = x_1 + x_2 + x_3$$

$$x = a\left(\sin\frac{2\pi t}{T}\right)+a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)+a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{2}\right)$$

$$x = a\left[\sin\left(\frac{2\pi t}{T}\right)+a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{2}\right)\right]+a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

$$x = 2a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)\cos\frac{\pi}{4}+a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

$$= a\left(\sqrt{2}+1\right)\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

which is a simple harmonic motion with an amplitude $a\left(\sqrt{2}+1\right)$ and phase angle $\pi/4$ and the same period; it has the same phase as second SHM.

The energy of resultant motion is proportional to $\left[a\left(\sqrt{2}+1\right)\right]^2$ (i.e.) $\left[a^2\left(2+1+2\sqrt{2}\right)\right]=(3+2\sqrt{2})a^2$ which is greater than three times the energy of each separate SHM.

Assertion-Reasoning Type

1. **a.** Total energy of the particle performing simple harmonic motion is $E = K + U = k_{max} + U_{min}$. K is always positive, while U could be +ve, –ve or zero. If U_{min} is –ve and its value is greater than K_{max}, then E would be –ve.

2. **a.** Let us assume that density of material of cubes is ρ_0 and density of liquid is ρ. Then from equilibrium condition, $(8a^3 + a^3)\rho_0 g = (8a^3)\rho g$

$$\Rightarrow 9\rho_0 = 8\rho$$

When the block is displaced down by x, the restoring force is $F_1 = (8a^3 + a^2 x)\rho g - 9a^3{}_0 g = \rho a^2 g \times x$

When the block is displaced by x above the mean position, the restoring force is

$$F_2 = 9a^3\rho_0 g - (8a^3 - 4a^2 x)\rho g = 4a^2\rho g x$$

As force constants on two sides of equilibrium position are not same, amplitudes of oscillations on the two sides are different.

3. **d.** Time period of the spring pendulum is $T = 2\pi\sqrt{m/k}$, where k is the force constant.

Time period of the simple pendulum is $T = 2\pi\sqrt{l/g}$.

Time period of the torsional pendulum is $T = 2\pi\sqrt{I/k}$, where k is the torsion constant.

If frequencies of all three are same at one place, then at some other place frequency of torsional and spring pendulums would be same and that of simple pendulum may differ (depending upon g).

So, Statement I is wrong and Statement II is correct.

4. **a.** When the hoop oscillates in its plane, moment of inertia is $I_1 = mR^2 + mR^2$, i.e., $I_1 = 2mR^2$.

While when hoop oscillates in a direction perpendicular to plane of hoop, moment of inertia is

$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

The time period of physical pendulum is, $T = 2\pi\sqrt{\frac{I}{mgd}}$, d is same in both the cases.

5. **d.**

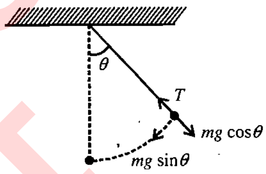

Fig. 4.214

$$T - mg\cos\theta = \frac{mv^2}{r} \qquad \Rightarrow mg\sin\theta = F_T$$

6. (a) Total energy, $E = \frac{1}{2}m\omega^2 a^2$

i.e., $E \propto a^2$.

$$\frac{E'}{E} = \left(\frac{2a}{a}\right)^2 \qquad \Rightarrow E' = 4E$$

7. **d.** Spring constant $\propto \frac{1}{\text{length of spring}} \qquad \Rightarrow K' = \frac{K}{n}$

Also, spring constant depends on material properties of the spring.

8. **a.** Tangent component of weight = $mg\sin\theta$ and radial component of weight = $mg\cos\theta$

at mean position, $\theta = 0$, ⇒ tangent component = 0

Therefore direction of acceleration is along radial component of weight.

At extreme position, tangent component is maximum. Hence direction of acceleration is along tangent component.

9. **d.** The mean position of the particle in Statement I is $x = -b/a$ and the force is always proportional to displacement from this mean position. The particle executes SHM about this mean position. Hence, Statement I is false.

10. **c.** As $F = -m\omega^2 y \Rightarrow$ slope of $F - y$ graph is $-m\omega^2$

$-1 = -m\omega^2 = -\omega^2 \Rightarrow \omega = 1$

$T = 2\pi = 6.28$ s

If mass is changed but slope remains same, the time period will change.

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Comprehension Type

For Problems 1–3

Sol. 1. b.

$$\omega = \sqrt{\frac{K}{m}}, \quad K = \omega^2 m = (10)^2 = 100 \text{ N/m}$$

Angular frequency of oscillation of combined body is

$$\omega' = \sqrt{\frac{K}{m+M}} = \sqrt{\frac{100}{4}} = 5 \text{ rad/s}$$

2. a. Conserving linear momentum.

$$(1+3)\,v = 1 \times 0.3 + 3\,(-0.3)$$
$$v = -0.15 \text{ m/s}$$

Negative sign indicates that combined body starts to move leftward. But at the instant of collision, spring is in its natural length or combined body is in equilibrium position. Hence, at $t = 0$, phase of combined body becomes equal to π.

$\therefore$ New amplitude of oscillation is

$$a' = \frac{|v|}{\omega} = \frac{0.15}{5} = 0.03 \text{ m} = 3 \text{ cm}$$

3. b. Equation for position x of combined body is given by

$$x = l_0 + a' \sin(\omega' t + \pi)$$

or $\quad x = \{10 + 3\sin(5t + \pi)\}$ cm

or $\quad x = 10 - 3\sin(5t)$ cm

For Problems 4–6

Sol. 4. b. The angular frequency of simple harmonic motion is given by

$$\omega = \sqrt{\frac{k}{m}}$$

The velocity of block when it is a displacement of y from mean position is given by $v = \omega\sqrt{A^2 - y^2}$,

From given initial condition, $v_0 = \sqrt{\frac{k}{m}}\sqrt{A^2 - h^2}$

$$\Rightarrow A^2 = \frac{mv_0^2}{k} + h^2 \quad \Rightarrow A = \sqrt{\frac{mv_0^2}{k} + h^2}$$

5. a. To have the equilibrium of simple harmonic motion, it is best to represent simple harmonic motion as uniform circular motion.

At $t = 0$, let particle is making an angle δ with –ve x-axis as shown, then

$$\sin\delta = \frac{h}{A}$$

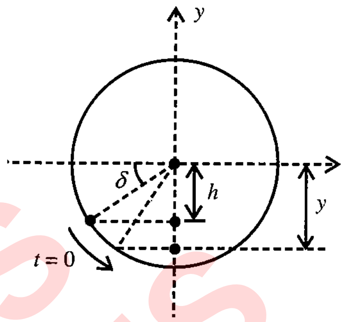

Fig. 4.215

$$\Rightarrow \delta = \sin^{-1}\left(\frac{h}{A}\right)$$

At time t, $\quad y = -A\sin(\omega t + \delta)$

S,o the equation of simple harmonic motion is

$$y = -\sqrt{\frac{mv_0^2}{k} + h^2}\left\{\sin\left[\sqrt{\frac{k}{m}}t + \sin^{-1}\left(\frac{h}{A}\right)\right]\right\}$$

6. c. To compute the time taken by the block to cross mean position for the first time, we can make use of circular motion representation.

$$t = \frac{\pi - \delta}{\omega} = \frac{\pi - \sin^{-1}\left(\frac{h}{A}\right)}{\sqrt{\frac{k}{m}}}$$

For Problems 7–8

Sol. 7. a. Let the equation of simple harmonic motion be

$$x = A\sin(\omega t + \delta)$$

Then, $\quad$ at $t = 1$ s, $x = 0$, so

$$0 = A\sin(\delta + \omega) = \sin(\omega + \delta) = 0$$

At $\quad t = 2$ s, $\quad v = +0.25$ m/s,

so $\quad 0.25 = A\omega\cos(2\omega + \delta)$

Solving above equation, we get, $A = \frac{3}{2\pi}$ m and $\delta = -\frac{\pi}{3}$

Hence equation of SHM will be $x = \frac{3}{2\pi}\sin\left(\frac{\pi}{3}t - \frac{\pi}{3}\right)$

For velocity at $t = 5$ s, i.e., after half a period of $t = 2$ s, the velocity is having same magnitude but opposite direction.

8. c. The equation of simple harmonic motion is

$$x = \frac{3}{2\pi}\sin\frac{\pi}{3}(t-1)$$

and velocity $v = \frac{1}{2}\cos\frac{\pi}{3}(t-1)$

Hence at $t = 5$ s, $v = -\frac{1}{4}$ m/s

For Problems 9–11

Sol. 9. b. In the present case, $d = \frac{L}{4}$

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$$I = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7}{48}ML^2$$

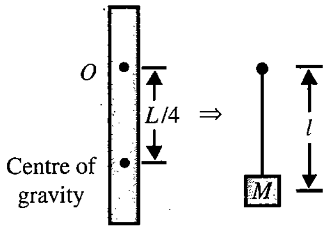

Fig. 4.216

So, time period of the physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{Mgd}}$

$$= 2\pi\sqrt{\frac{7/48ML^2}{Mg \times L/4}} = \pi\sqrt{\frac{7L}{3g}}$$

10. d. Let the location of centre of oscillation is at a distance l from the point of pivot, then

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad l = \frac{7L}{12}$$

11. a. Due to the impulse, let us say the rod acquires an angular velocity ω.

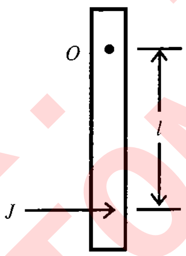

Fig. 4.217

Now, $J \times l = I\omega \quad \Rightarrow \omega = \dfrac{4J}{ML}$

For Problems 12–14

12. c., 13. b., 14. a.

Sol. Initially in equilibrium let the elongation in spring be y_0, then mg $= ky_0$.

$$y_0 = \frac{mg}{k}$$

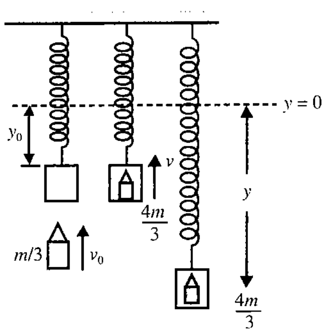

Fig. 4.218

As the bullet strikes the block with velocity v_0 and gets embedded into it, the velocity of the combined mass can be computed by using the principle of moment conservation.

$$\frac{m}{3}v_0 = \frac{4m}{3}v \quad \Rightarrow \quad v = \frac{v_0}{4}$$

Let new mean position is at distance y from the origin, then

$$ky = \frac{4m}{3}g \quad \Rightarrow \quad y = \frac{4mg}{3k}$$

Now, the block executes SHM about mean position defined by $y = 4mg/3k$ with time period, $T = 2\pi\sqrt{4m/3k}$. At $t = 0$, the combined mass is at a displacement of $(y - y_0)$ from mean position and is moving with velocity v, then by using $v = \omega\sqrt{A^2 - x^2}$, we can find the amplitude of motion.

$$\left(\frac{v_0}{4}\right)^2 = \frac{3k}{4m}[A^2 - (y-y_0)^2] = \frac{3k}{4m}\left[A^2 - \left(\frac{mg}{3k}\right)^2\right]$$

$$\Rightarrow \quad A = \sqrt{\frac{mv_0^2}{12k} + \left(\frac{mg}{3k}\right)^2}$$

To compute the time taken by the combined mass from $y = mg/k$ to $y = 0$, we can either go for equation method or circular motion projection method.

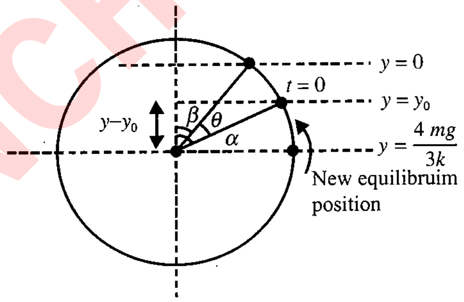

Fig. 4.219

Required time, $\quad t = \dfrac{\theta}{\omega} = \dfrac{\alpha - \beta}{\omega}$

$$\cos\alpha = \frac{y - y_0}{A} = \left(\frac{\frac{4mg}{3k} - \frac{4mg}{k}}{A}\right) = \frac{mg}{3kA}$$

$$\cos\beta = \frac{y}{A} = \frac{4mg}{3kA}$$

So, $$t = \frac{\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)}{\omega}$$

$$= \sqrt{\frac{4m}{3k}}\left[\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)\right]$$

For Problems 15–17

15. b., 16. b., 17. c.

Sol. As C collides with A and sticks to it, the combined mass moves rightward to compress the spring and hence B moves

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rightwards due to the spring force, i.e., B accelerates and the combined mass decelerates. The deformation in the spring is changing and the centre of mass of the system continues to move rightwards with constant speed, while both the blocks oscillate about centre of mass of the system.

The velocity of the combined mass just after collision is $mv_0 = 2mv$

$$v = \frac{v_0}{2} = 0.3 \text{ m/s}$$

Velocity of centre of mass of system,

$$v_{CM} = \frac{2mv + 0}{3m} = \frac{2v}{3} = \frac{v_0}{3} = 0.2 \text{ m/s}$$

Time period with which block B and the combined mass oscillate about centre of mass could be computed by using the reduced mass concept.

$$T = 2\pi\sqrt{\frac{2m}{3k}} = \frac{\pi}{5\sqrt{10}} \text{ s}$$

Oscillation energy of the system is

$$E = \frac{2mv^2}{2} = 0.27 \text{ J}$$

The translational kinetic energy of the centre of mass of system is

$$E_{CM} = \frac{3mv_{CM}^2}{2} = 0.18 \text{ J}$$

The remaining energy is oscillating between kinetic and potential energy during the motion of blocks.

For maximum compression in spring either we can use centre of mass approach or energy approach, here we are using the second method.

Oscillation energy = Maximum elastic potential energy

$$0.09 = \frac{1}{2}kx_m^2 \Rightarrow x_m = 3\sqrt{10} \text{ mm}$$

For Problems 18–20

Sol. 18. b. When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on the block.

19. c. At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by

$$kx = mg \quad \text{(i)}$$

From conservation of energy,

$$mg(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv_{max}^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $v_{max} = \frac{3}{2}\sqrt{gL}$

20. b. $V_{max} = \frac{3}{2}\sqrt{gL}$ and $\omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$

$$A = \frac{V_{max}}{\omega} = \frac{3}{4}L$$

Hence time taken, t, from start of compression till the block reaches mean position is given by

$$x = A\sin\omega t_0, \quad \text{where} \quad x = L/4$$

$$t_0 = \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$$

Time taken by the block to reach from mean position to bottom most position is $\frac{2\pi}{4\omega} = \frac{\pi}{4}\sqrt{\frac{L}{g}}$

Hence the required time $= \frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

For Problems 21–25

Sol. 21. b. At position A, block is in positive region and slope is negative, so velocity is negative.

22. c. At position B, block is in negative region and velocity is positive, so slope is positive.

23. c. At position C, block is in positive region and velocity is zero, so slope is zero.

24. c. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25600}{100}} = 16$ rad/s

Using energy conservation, $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA'^2$

$$A = \sqrt{x^2 + \frac{m}{k}v^2} = \sqrt{9+3} = 2\sqrt{3}$$

$$x = A\cos(\omega t + \phi) = 2\sqrt{3}\cos(16t + \phi)$$

From boundary conditions $t = 0$, ϕ can be obtained which is $\pi/6$.

$$x = 2\sqrt{3}\cos\left(16t + \frac{\pi}{6}\right) \text{cm}$$

25. d. Differentiating previous result

$$v = -32\sqrt{3}\sin\left(16t + \frac{\pi}{6}\right) \text{cm/s}$$

For Problems 26–27

Sol 26. a. Using conservation of mechanical energy

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x^2 = \frac{m}{k} \Rightarrow x = v\sqrt{\frac{m}{k}}$$

27. c. The time taken by the mass to regain its original speed towards left wall is half the period of oscillation $= \pi\sqrt{m/k}$.
The ball strikes the left wall after a time $= L/v$ and since the collision is elastic, it again reaches the spring with same speed v and in a further time L/v.
The block thus undergoes periodic motion with time period

$$\pi\sqrt{\frac{m}{k}} + \frac{2L}{v}$$

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For Problems 28–30

Sol. 28. c. Take x axis along AB and origin at A.
Let $AP = x$, then $PB = (3l - x)$

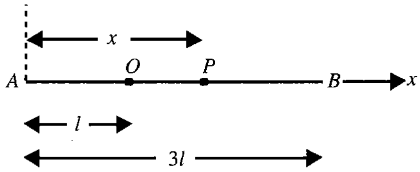

Fig. 4.220

The force on the particle of mass m at P is $\vec{F}$ given by

$$\vec{F} = 2\left(\frac{mg}{l}\right)(-\vec{x}) + \frac{mg}{l}\left(\frac{3l-x}{x}\right)\vec{x}$$

$$\vec{F} = \left(3mg - \frac{3mg}{l}x\right) \quad \text{directed towards } A$$

$$= \frac{3mg}{l}(l - x) \quad \text{directed towards } A$$

Hence the equation of motion of mass m is

$$m\frac{d^2x}{dt^2} = -\frac{3mg}{l}(l-x) = m\omega^2(l-x)$$

where $\omega^2 = 3g/l$

This is a simple harmonic motion of period $T = 2\pi\sqrt{\dfrac{l}{2g}}$.

The mean position is at the point $x = l$ (i.e.) at O, where $AO = l$.
Now for point A, displacement $= -l$ and hence

$$v_A^2 = \omega^2(a^2 - (-l)^2) = \omega^2(a^2 - l^2)$$

where $v_A^2 = \omega^2(a^2 - (-l)^2) = \omega^2(a^2 - l^2)$ and a is the amplitude.

Now $v_A = 3\sqrt{gl}$ (given) and hence $v_A^2 = 9gl = \dfrac{3g}{l}(a^2 - l^2)$

$$a^2 = 4l^2 \quad \Rightarrow a = 2l$$

29. b. $OB = 2l$. O is the mean position, B is the amplitude point. Also $|AP| = l = \dfrac{a}{2}$.

Hence the time t from A to O is given by $\dfrac{a}{2} = a\sin\omega t$

Giving $\omega t = \dfrac{\pi}{6}$, $\Rightarrow$ $t = \dfrac{\pi}{6\omega} = \dfrac{\pi}{6} \times \dfrac{T}{2\pi} = \dfrac{T}{12}$

Hence time from A to B $= \dfrac{T}{12} + \dfrac{T}{4} = \dfrac{T}{3}$

30. a. Since B is the amplitude point, the instantaneous velocity at $B = vB = 0$.

For Problems 31–33

Sol. 31. d. Let x be the maximum extension of the spring. The mass falls from rest through the vertical distance $(L + x)$ and so the energy given by it to the spring is $E_k = mg\ (L + x)$. This must be stored in the spring as elastic energy at its maximum extension. Hence $E_K = \dfrac{1}{2}kx^2 = mg(L + x)$

This is quadratic, $x^2 - \dfrac{2mg}{k}x - \dfrac{2mgL}{k} = 0$

Hence, $x = \dfrac{\dfrac{2mg}{k} \pm \sqrt{\left(\dfrac{2mg}{k}\right)^2 + \dfrac{8mgL}{k}}}{2}$

Maximum value of $x = \dfrac{mg}{k} + \dfrac{mg}{k}\sqrt{1 + \dfrac{2kL}{mg}}$

$$= \frac{mg}{k}\left(1 + \sqrt{1 + \frac{2kL}{mg}}\right)$$

32. a. If the mass is pulled down by distance A from its equilibrium position and then released, it will perform SHM of amplitude A. Assuming that the string remains taut, the maximum acceleration will be at the amplitude points and given by $f_m = A\omega^2$

Now $\omega^2 = \dfrac{k}{m}$; hence, $f_m = \dfrac{Ak}{m}$

33. b. In the position of equilibrium, when the tension in the string $= mg$, the extension in the spring is $x_0 = mg/k$. If the amplitude exceeds this value, then in its upward motion, the body will rise to a height at which the string will become loose and $T = 0$. Hence maximum amplitude is $x_m = mg/k$

It also follows from the fact that for $T \geq 0$, the maximum downward acceleration that the body can have is g. Since $f(t) = g - T(t)/m, f(t) \leq g$. Hence maximum amplitude is

$$A_m = \frac{g}{\omega^2} = \frac{mg}{k}$$

Matching Column Type

1. i. → a., b., c.; ii. → a., b., c.; iii. → a.; iv. → d.

For A, $y = Ae^{i(\omega t + \phi)}$

$$\frac{dy}{dt} = A \times i\omega e^{i(\omega t+\phi)}$$

$$\frac{d^2y}{dt^2} = Ai\omega \times i\omega e^{i(\omega t+\phi)} = -\omega^2 \times Ae^{i(\omega t+\phi)}$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

which represents the standard equation of motion of SHM.
For B, $y = B\sin\omega t + C\cos\omega t$ can be written as

$$y = \sqrt{B^2 + C^2}\sin(\omega t + \delta)$$

where $\tan\delta = \dfrac{C}{B}$

So, B also represents an SHM.

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For C, it is a standard equation of harmonic travelling wave in which the particle performs an SHM.

For D, y/x = constant, it represents the equation of the translatory motion.

2. **i → c.; ii. → a.; iii. → a.; iv. → d.**
Frequency depends only on the mass and the spring constant, so it will change. Due to increase in KE by a factor of 4, the amplitude gets doubled and hence the magnitude of maximum acceleration also gets doubled.
As maximum KE increases by a factor of 4, the maximum PE also increases by the same factor.

3. **i. → b. c.; ii. → b., c., d.; iii. → a.; iv. → a.**
i. Linear combination of two SHMs will be an SHM if the individual SHMs have equal frequencies, their magnitudes may be different.
ii. $y = A\sin\omega_1 t + A\sin(\omega_2 t + \phi)$

If $\qquad \omega_1 = \omega = \omega, \quad \phi = 17/2$

$$y = A\sin\omega t + A\cos\omega t$$

Its amplitude will be $\sqrt{2}A$.
iii. Time period of a pendulum of infinite length is

$$T = 2\pi\sqrt{\frac{R}{g}}$$

iv. The above will be the maximum value of time period of an oscillating pendulum.

4. **i. → c., d.; ii. → c., d.; iii. → a., b., d.; iv. → c., d.**
A constant force and a constant torque affect only the mean position.
In third case, as the block falls on mean position, the mean position is not affected.
In a car a constant pseudo force will act which will affect only the mean position.

5. **i. → a., b.; ii. → a., b.; iii. → c.; iv. → d.**
Position and velocity both can be positive and negative. If at mean position, PE is zero, then at any other position, PE is positive. Total energy is always constant.

6. **i. → b.; ii. → a.; iii. → d.; iv. → c.**

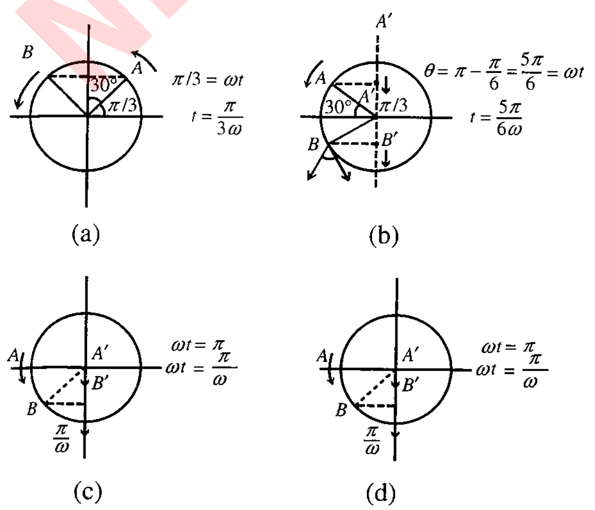

Fig. 4.221

7. **i. → c.; ii. → e.; iii. → b.; iv. → c.; v. → b.**

$$F = 8 - 2x = -2(x-4)$$

At equilibrium position, $F = 0$

$$x = 4 \text{ m}$$

As particle is released at rest from $x = 6$ m, i.e., it is one of the extreme position, amplitude $A = 2$ m.

Hence, force constant $k = 2$ N/m

$$m\omega^2 = 2 \quad \text{or} \quad \omega = 1$$

Time period $T = \frac{2\pi}{\omega} = 2\pi$

Time taken to go from $x = 2$ m to $x = 4$ m (i.e., from extreme position to mean position)

$$= \frac{T}{4} = \frac{\pi}{2}$$

Energy of SHM $= \frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 4$ N-m $= 4$ J

As the particle has started its motion from positive extreme,

Phase constant $= \frac{\pi}{2}$

8. **i. → c.; ii. → a.; iii. → b.; iv. → d.**

$$V_m = A\omega$$

$$A = \frac{V_m}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20 \text{ m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2 \text{ kg}$$

At $t = 0.1$ s, acceleration is maximum $= -\omega^2 A = -200$ m/s²

Maximum energy $= \frac{1}{2}mv_m^2 = 4$ J

$$\frac{1}{2}kA^2 = E_{\max} = \frac{1}{2} \times 200 \times 0.04 = 4 \text{ J}$$

9. **i. → b.; ii. → a.; iii. → d.; iv. → c.**
i. Motion is simple harmonic.

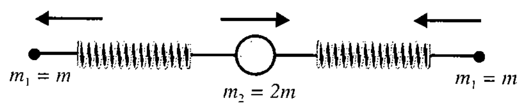

Fig. 4.222

$$\frac{d^2x}{dt^2} = \frac{-k}{m_1 m_2}(m_2 + 2m_1)x$$

$$\omega^2 = k\left(\frac{1}{m_1} + \frac{2}{m_2}\right) = k\left(\frac{1}{m} + \frac{2}{2m}\right)x$$

$$\omega = \sqrt{\frac{2k}{m}}$$

$\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

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ii. Angular frequency $\omega = \sqrt{\dfrac{k(m+2m)}{2m^2}} = \sqrt{\dfrac{3k}{2m}}$

$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{2m}}$

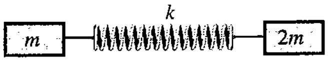

Fig. 4.223

iii. Here effective spring constant = $3k$.

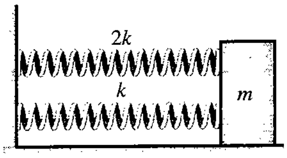

Fig. 4.224

$\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{m}}$

iv. Effective spring constant (K) is given by

$$\frac{1}{k} = \frac{1}{k} + \frac{1}{\frac{k}{2}} = \frac{3}{k}$$

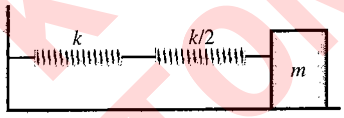

Fig. 4.225

$k = \dfrac{k}{3}$ $\quad\therefore$ Frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{k}{3m}}$

Integer Answer Type

1. (5)

Let point B is displace in the downward direction by x.
Net imbalance forces = $m_{\text{total}} \cdot a$

$$[4mg(\ell+x)+mg(\ell-x)-\{4m(\ell-x)g+m(\ell+x)g\}]$$

$$=(4m\times 2l+m\times 2l)a$$

$$a=-\left(\frac{3g}{5l}\right)x$$

$$\omega=\sqrt{\frac{3g}{5l}} \quad \Rightarrow T=2\pi\sqrt{\frac{5l}{3g}}=5\text{ s}$$

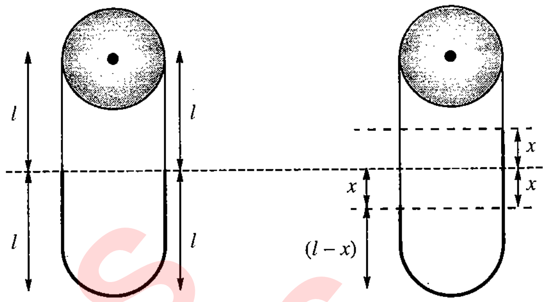

Fig. 4.226

2. Velocity of the particle just before collision,

$$u=\sqrt{2g\times\frac{4.5mg}{K}} \Rightarrow u=3g\sqrt{\frac{m}{K}}$$

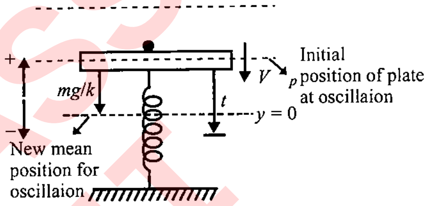

Fig. 4.227

Now it collides with the plate.
Now just after collision velocity (V) of the system of 'plate + particle'

$$mu = 3mV \Rightarrow V=\frac{u}{3}=g\sqrt{\frac{m}{K}}$$

Now the system performs SHM with time period $T=2\pi\sqrt{3m/K}$ $\left(\omega=\sqrt{\dfrac{K}{3m}}\right)$ and mean position as mg/K distance below the point of collision.

Let the equation of motion is

$y = A\sin(\omega t+\phi)$ (i)

$v=\dfrac{dy}{dt}=A\omega\cos(\omega t+\phi)$ (ii)

At $t=0$, $y=\dfrac{mg}{K}$ and $v=g\sqrt{\dfrac{m}{K}}$

from Eqs. (i) and (ii) $\dfrac{mg}{K}=A\sin\phi$ (iii)

$A=\dfrac{2mg}{K}$ (iv)

from Eqs. (iii) and (iv) $\Rightarrow$ $\phi=\dfrac{5\pi}{6}$

$$y=\frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t+\frac{5\pi}{6}\right)$$

Hence equation of SHM should be

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$$y = -A = -\frac{2mg}{K} = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}\,t + \frac{5\pi}{6}\right)$$

The plate will be at rest again when

$$y = -A = -\frac{2mg}{K} = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}\,t + \frac{5\pi}{6}\right)$$

$$\Rightarrow \sin\left(\sqrt{\frac{K}{3m}}\,t + \frac{5\pi}{6}\right) = -1 = \sin\frac{3\pi}{2}$$

$$\Rightarrow \sqrt{\frac{K}{3m}}\,t + \frac{5\pi}{6} = \frac{3\pi}{2} \Rightarrow t = \frac{2\pi}{3}\sqrt{\frac{3m}{K}}$$

Using values, $t = \pi/5$ s.

3. **(1)** The angular positions of pendulums 1 and 2 are (taking angles to the right of reference line xx' to be positive)

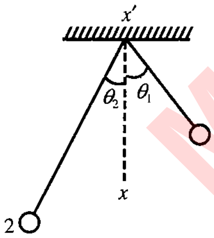

Fig. 4.228

$$\theta_1 = \theta\cos\left(\frac{4\pi}{T}t\right) \quad \left(\text{where } T = 2\pi\sqrt{\frac{l}{g}}\right)$$

$$\theta_2 = -\theta\cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

For the strings to be parallel for the first time

$$\theta_1 = \theta_2$$

$$\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

$$\therefore \quad \frac{4\pi}{T}t = 2n\,\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$$

for $\quad n = 0,\ t = T/2$

for $\quad n = 1,\ t = T/6,\ 3T/2$

Both the pendulums are parallel to each other for the first time after

$$t = \frac{T}{6} = \frac{\pi}{3}\sqrt{\frac{l}{g}} = 1\text{ s}$$

4. **(4)** The bob will execute SHM about the stationary axis passing through AB. If its effective length is l' then

$$T = 2\pi\sqrt{\frac{l'}{g'}}$$

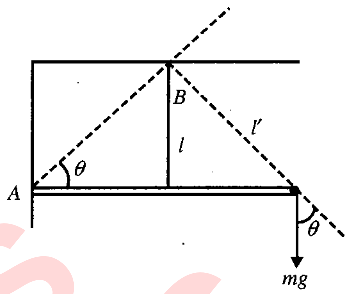

Fig. 4.229

$$l' = \frac{l}{\sin\theta} = \sqrt{2}\,l \qquad [\text{because } \theta = 45°]$$

$$g' = g\cos\theta = \frac{g}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{2l}{g}} = 2\pi\sqrt{\frac{2\times 0.2}{10}} = \frac{2\pi}{5}\text{ s}$$

$$X = 4$$

5. **(8)** Applying torque equation about

$$\tau_0 = I_0\alpha$$

$$k_1 b\theta \times b\cos\theta + \frac{k_2 l_\theta}{\theta}\times l\cos\theta = -\frac{Id^2\theta}{dt^2}$$

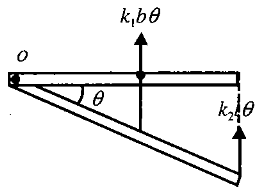

Fig. 4.230

Here, $I = \dfrac{ml^2}{3}$, and as θ is small, $\cos\theta = 1$

$$\frac{ml^2 d^2\theta}{3dt^2} + (k_1b^2 + k_2l^2)\theta = 0$$

$$\text{Hence,} \quad \omega = \sqrt{\frac{3k_1b^2 + k_2l^2}{ml}}$$

On substituting the values we get $\omega = 8$ rad/s

6. **(1)** Net increment in the tension in string connecting the block will provide acceleration to the block. We can write

$$\Delta T = ma \qquad \text{(i)}$$

When the block m is displaced by a distnce x beyond equilibrium position, the addition stretch of springs 1 and 2 are x_1 and x_2 respectively.

We can write

$$x_1 = \frac{x - x_2}{2} \Rightarrow x = 2x_1 + x_2 \qquad \text{(ii)}$$

As $\Delta T = k_2x_2$ and $2\Delta T = k_1x_1$ then $2(k_2x_2) = k_1x_1$

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$$\Rightarrow \quad x_1 = \frac{2k_2x_2}{k_1} \qquad \text{(iii)}$$

from Eqs. (ii) and (iii) $x = 2\left[\frac{2k_2x_2}{k_1}\right] + x_2 = \frac{(4k_2+k_1)}{k_1}x_2$

$$\Rightarrow \quad x_2 = \frac{k_1 x}{(k_1+4k_2)}$$

and $\vec{a} = \frac{-k_1k_2}{(k_1+4k_2)m}\cdot x \quad \Rightarrow \quad \vec{a} = \frac{-k_1k_2}{(k_1+4k_2)m}\cdot x$

$$\omega^2 = \frac{k_1k_2}{(k_1+4k_2)m}$$

Hence $R = \frac{80}{23\pi^2}m$

After substituting the values, we get $T = 1$ s.

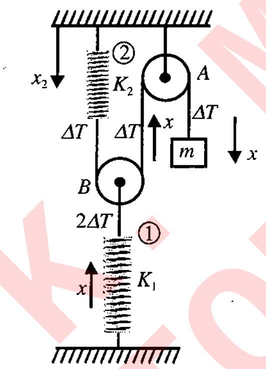

Fig. 4.231

7. **(2)** If we twist (rotate) the disc through a small clockwise angle θ, the spring will be deformed (compressed) by a distance $x = R\theta$. Hence, the spring force $F_S = kx = k(R\theta)$ will produce a restoring torque.

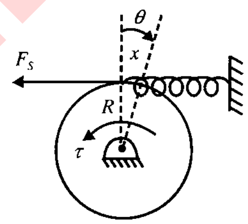

Fig. 4.232

Restoring torque: $\tau = -F_S R$ where $F_S = kR\theta$

This gives $\tau = -kR^2\theta$

It means after removing the external (applied) torque, the restoring torque rotates the disc with an angular acceleration α which will bring the spring–disc system back to its original state.

Newton's law of rotation (or torque equation): Applying Newton's second law of rotation, we have

$$\tau = I_C\alpha$$

where

$$\tau = -kR^2\theta$$

This gives $\quad \alpha = -\frac{kR^2\theta}{I_C} \quad$ where $\quad I_C = \frac{mR^2}{2}$

Then $\quad a = -\frac{2k}{m}\theta$

Comparing the above equation with $\alpha = -\omega^2\theta$, we have

$$\omega = \sqrt{\frac{2k}{m}}$$

After substituting the values we get $\omega = 2$ rad/s.

8. **(2)** The time period of a physical pendulum is

$$T = 2\pi\sqrt{\frac{I_P}{Mgr}}$$

Here we have three quantities I_P, m and r.

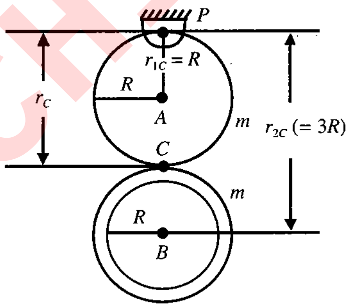

Fig. 4.233

Let us calculate the quantities one by one as follows:

Finding I_P: $I_P = (I_P)_{\text{disc}} + (I_P)_{\text{ring}}$

where $(I_P)_{\text{disc}} = I_A + m(PA)^2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$

and $\quad (I_P)_{\text{ring}} = I_B + m(PB)^2 = mR^2 + m(3R)^2 = 10\,mR^2$

Then, we have $I_P = \frac{3}{2}mR^2 = 10\,mR^2 = \frac{23}{2}mR^2$

Finding r:

$$r = \vec{r}_C = \frac{m_1\vec{r}_{1C} + m_2\vec{r}_{2C}}{m_1+m_2},$$

where $\quad m_1 = m,\ m_2 = m$

$$\vec{r}_{1C} = -R \quad \text{and} \quad r_{2C} = -3R$$

This gives $r_C = 2R$

Finding M:

$$M = (m)_{\text{disc}} + (m)_{\text{ring}} = m + m = 2m$$

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Substituting $I_P = 23/2\, mR^2$, $M = 2m$ and $r = 2R$ in the expression

$$T = 2\pi\sqrt{\frac{I_P}{Mgr}} \quad \text{we have} \quad T = \pi\sqrt{\frac{23R}{2g}}$$

After substituting the values we get $t = 2$ s.

9. **(2)** In static equilibrium of block, tension in the string is exactly equal to its weight. Let a vertically downward force F be applied on the block to pull it downward. Equilibrium is again restored when tension in the string is increased by the same amount F. Hence, total tension in the string becomes equal to $(mg + F)$.

Strings are further elongated due to the extra tension F. Due to this extra tension F in strings, tension in each spring increases by $2F$. Hence increase in elongation of springs is $2F/K_1$ $2F/K_2$ $2F/K_3$ and $2F/K_4$, respectively.

According to geometry of the arrangement, downward displacement of the block from its equilibrium position is

$$y = 2\left(\frac{2F}{K_1}+\frac{2F}{K_2}+\frac{2F}{K_3}+\frac{2F}{K_4}\right) \qquad \text{(i)}$$

If the block is released now, it starts to accelerate upwards due to extra tension F in the strings. It means restoring force on the block is equal to F. From Eq. (i),

$$F = \frac{y}{4\left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+\frac{1}{K_4}\right)}$$

Restoring acceleration of block

$$\frac{F}{m} = \frac{y}{4m\left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+\frac{1}{K_4}\right)}$$

Since acceleration of block is restoring and is directly proportional to displacement y, the block performs SHM.

Its period, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

$$T = 2\pi\sqrt{4m\left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+\frac{1}{K_4}\right)}$$

$$T = 4\pi\sqrt{m\left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}+\frac{1}{K_4}\right)}$$

After substituting the values, we get $T = 2$ s.

10. **(2)** Initial stretch in both springs $= d - \frac{3d}{4} = \frac{d}{4}$

$$F_{\text{restering}} = k\left(\frac{d}{4}+x\right) - k\left(\frac{d}{4}-x\right) = 2kx$$

$$\Rightarrow T_a = 2\pi\sqrt{\frac{m}{2k}}$$

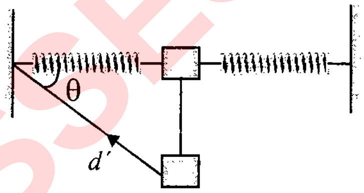

Fig. 4.234

$d' = d\sec\theta$

$$x' = d\sec\theta - \frac{3d}{4} = d\left(\frac{1}{\cos\theta} - \frac{3}{4}\right)$$

force towards equilibrium position $(kx' \sin\theta)$

$= dk\left(\tan\theta - \frac{3\sin\theta}{4}\right)$ due to one spring and

net $= 2dk\left(\tan\theta - \frac{3\sin\theta}{4}\right)$ for small θ, force

$$= 2dk\left[\theta - \frac{3\theta}{4}\right] = k\left(\frac{d\theta}{2}\right)$$

$d\theta$ = displacement from mean position

$$\Rightarrow F - \frac{kx}{2} \Rightarrow T_B = 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \frac{T_B}{T_A} = 2\pi\sqrt{\frac{m}{2k}} \Big/ 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \frac{T_B}{T_A} = 2$$

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CHAPTER

5

Travelling Waves

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PROPAGATION OF DISTURBANCE

The introduction to this chapter alluded to the essence of wave motion: *the transfer of energy through space without the accompanying transfer of matter.*

All mechanical waves require (i) some source of disturbance, (ii) a medium containing elements that can be disturbed and (iii) some physical mechanism through which elements of the medium can influence each other.

Progressive Waves

Wave motion is defined as a form of disturbance transferred from one point to another involving transfer of energy but no transfer of matter. Mechanical waves require a medium for propagation, whereas electromagnetic waves can travel in vacuum also (do not require any medium also called non-mechanical waves).

Mechanical Waves

A mechanical wave can be produced and propagated only in those material media which possess **elasticity** and **inertia**. A wave originates due to the displacement of some portion of an elastic medium from its normal position, causing it to oscillate about an equilibrium position, so the medium should posses elasticity in order that it has a tendency to come back to its original position which is a necessary condition for disturbance to be transmitted from one layer to next and hence the wave to travel. The medium should also have inertia in order that it can store energy and transport it further.

A practical example of wave motion is the disturbance in water on dropping a pebble in it. A disturbance is created initially at the centre, water particles (here the medium) acquire mechanical energy (KE + PE). This energy is transmitted layerwise outwards by water particles in one layer transmitting it to the particles in the next layer. Thus, starting from centre, a disturbance is created throughout the water in the form of concentric circles which go on expanding with no net motion of water particles. The water particles oscillate for a while about their mean position hence transmitting energy to the next layer.

Pulse and Wave

One way to demonstrate wave motion is to flick one end of a long string that is under tension and has its opposite end fixed as shown in Fig. 5.1. In this manner, a single bump (called a pulse) is formed and travels along the string. Figure 5.1 represents four consecutive 'snapshots' of the creation and propagation of the travelling pulse. The string is the medium through which the pulse travels. The pulse has a defined height and a definite speed of propagation along the medium (the string). The shape of the pulse changes very little as it travels along the string.

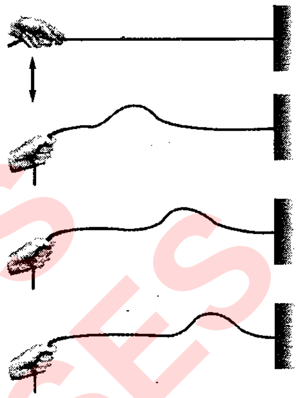

Fig. 5.1 A pulse travelling down a stretched rope. The shape of the pulse is approximately unchanged as it travels along the rope

We shall first focus on a pulse travelling through a medium. Once we have explored the behaviour of a pulse, we will then turn our attention to a wave, which is a *periodic* disturbance travelling through a medium. We create a pulse on our string by flicking the end of the string once as shown in Fig. 5.2. If we were to move the end of the string up and down repeatedly, we should have created a travelling wave, which has the characteristics a pulse does not have.

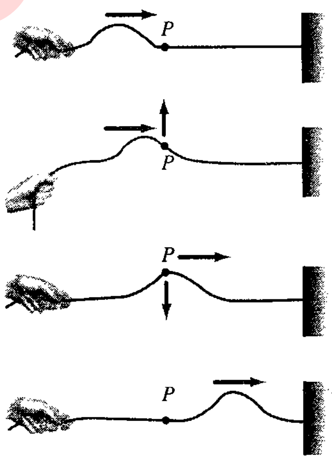

Fig. 5.2

Type of Waves

Transverse Waves

As the pulse in Fig. 5.1 travels, each disturbed element of the string moves in a direction perpendicular to the direction of propagation. Figure 5.2 illustrates this point for one particular element, labelled *P*. Notice that no part of the string ever moves in the direction of the propagation. A travelling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a **transverse wave.**

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For transverse wave propagating in a taut string, when the displacement of a particle is maximum above the line of mean position, the particle is said to be at crest of the wave, and when the displacement of a particle is maximum below the mean position it is said to be at the trough of the wave.

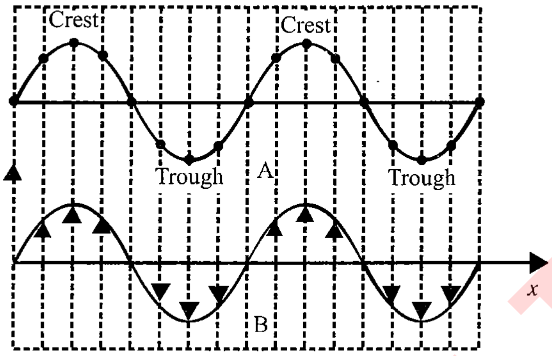

Fig. 5.3 The displacement-position graph of a longitudinal wave at an instant

A particle either at the crest or at the trough has a tendency to move towards the mean position.

- A particle at the crest or the trough has zero velocity, and the distance of the particle from the mean position is termed as amplitude of the wave.
- Distance between two consecutive crests/troughs is equal to the wavelength of the wave.
- Distance between a consecutive pair of crest and trough is half the wavelength of the wave.

Longitudinal Waves

It is a kind of wave motion in which the individual particles of a medium execute periodic motion about their mean position along the direction of propagation of wave. Let us consider a pulse moving down a long stretched spring as shown in Fig. 5.4. The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression in a region of the coils. The compressed region travels along the spring (to the right in figure). Notice that the direction of the displacement of the coils is parallel to the direction of propagation of the compressed region.

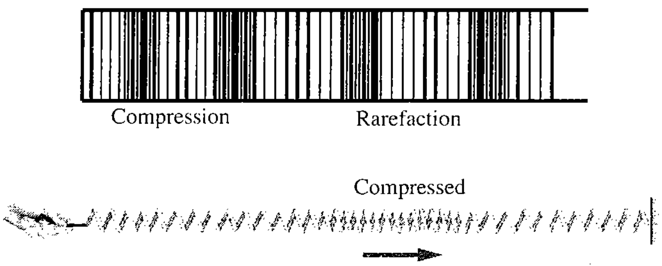

Fig. 5.4

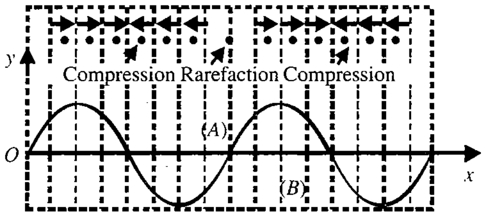

Fig. 5.5 The displacemnt-poition graph of a longitudinal wave at an instant.

Sound waves are example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travels through air.

A longitudinal wave moves by the phenomena of compression and rarefaction in the medium.

In compression, distance between any two consecutive particles of medium is less than their normal distance. Therefore, density is more than the normal density. In a rarefaction, distance between any two consecutive particles of medium is more than the normal distance, therefore density is less than the normal density.

At an instant of time a particle in compression phase and another in rarefaction phase are exactly out of phase from each other.

All particles in compression phase are in phase (at a given instant of time).

All particles in rarefaction phase are in phase (at a given instant of time).

Distance between two consecutive compressions or rarefactions is equal to the wavelength of the wave. Longitudinal waves can propagate through any state of matter.

Wave Parameters

Phase

Phase defines the position (in terms of distance from mean position) and velocity of a particle oscillating under the influence of a wave. The particles of the medium which are in same state of motion (at the same displacement from their respective mean positions moving in the same direction) are said to be in phase or differing in phase by $2n\pi$ where $n = 1, 2, 3, \ldots$ and the particles of which state of motion are exactly opposite (displacements from the mean position and velocities are exactly opposite) are said to be out of phase or differing in phase by $n\pi$ where $n = 1, 3, 5, \ldots$

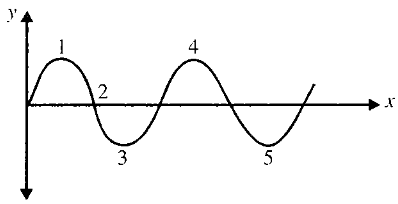

Fig. 5.6

- Oscillations of the particles 1 and 4 are in phase.
- Oscillations of the particles 1 and 2 are not in phase.

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- Oscillations of the particles 3 and 5 are in phase.
- Oscillations of the particles 4 and 5 are out of phase.

Wave Speed

Wave speed is the distance travelled by the wave in unit time.

Amplitude

Amplitude A of oscillations of a particle of the medium propagating the wave is the maximum displacement of the particle from its mean position on either side.

Wavelength

Wavelength λ is the distance between two nearest particles (along the direction of propagation of wave) which are in same phase of vibration. It can also be defined as the distance travelled by the wave in one time period of oscillation.

Wave Frequency

Wave frequency f is the number of times an oscillating particle is at its maximum displacement on one side, during 1 s of motion. It is expressed in Hertz (1 Hz = 1 cycle/s).

Time Period

Time period T is the time taken by an oscillating particle to move from mean position to one extreme, then move on to other extreme position and come back to mean position. (It is the time the oscillating particle takes to make one complete cycle.) The time after which the particle repeats its motion.

Intensity of the Wave

Intensity of the wave is the energy transmitted per unit area per second in the form of the wave in the direction of the propagation of the wave by the source. This energy is carried forward by the medium particles which while oscillating transfer the energy to the next particles.

In our further discussion, we assume that as a wave proceeds forward, it does so without any dissipation of energy, i.e., neglecting the effects of air drag, internal resistances, etc. which cause loss of energy as wave progresses. In other words the amplitude of a simple progressive wave remains same as it progresses forward.

Illustration 5.1 **A sinusoidal wave is travelling along a rope. The oscillator that generates wave completes 40.0 vibrations in 30.0 s. Also, a given 'maximum' travels 425 cm along the rope in 10.0 s. What is the wavelength of the wave?**

Sol. The given information comprises things that can be measured directly in laboratory. A high speed photograph would reveal the wavelength which can be computed from $v = f\lambda$.

The frequency is

$$f = \frac{40.0 \text{ waves}}{30.0 \text{ s}} = 1.33 \text{ s}^{-1}$$

And the wave speed is

$$v = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

Since $v = \lambda f$, the wavelength is

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm}}{1.33 \text{ s}^{-1}} = 0.319 \text{ m}$$

If we turn up the oscillator to a higher frequency, the wavelength would get shorter because speed is a constant factor. It is determined just by the properties of the string itself.

Concept Application Exercise 5.1

1. Transverse waves are possible in solids but not in fluids. Why?
2. How can one create plane waves and spherical waves?
3. Is an oscillation a wave? Explain.
4. Which parts of the curve in the figure shown below represent compression and rarefaction for a longitudinal wave?

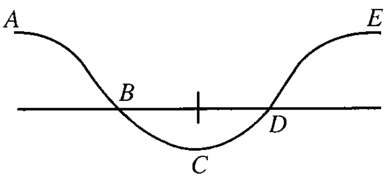

Fig. 5.7

5. How would you create a longitudinal wave in a stretched spring? Would it be possible to create a transverse wave in a spring?
6. A longitudinal wave is produced on a toy slinky. The wave travels at a speed of 30 cm/s and the frequency of the wave is 20 Hz. What is the minimum separation between two consecutive compressions of the slinky?
7. A narrow pulse (for example, a short pip by a whistle) is sent across a medium. If the pulse rate is 1 after every 20 s (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?

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8. Waves are generated on a water surface. Calculate the phase difference between two points A and B, when
   (i) A and B lie on the same wavefront at a distance of 2λ between them
   (ii) A and B lie on successive crests separated by 1 m
   (iii) A and B lie on successive troughs separated by 1.5 m
9. Transverse waves are not produced in liquids and gases. Why?

WAVE FUNCTION

A function of one variable is such as $x(t)$ is sufficient to describe the motion of a point mass moving along a line. To give a mathematical description of an extended moving object such as a moving pulse, functions which depend on two variables such as x and t are required. Functions which can mathematically represent a moving wave pulse are called wave function.

Consider a pulse travelling to the right of a long string as shown in Fig. 5.8. Figure 5.8(a) represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function that we will write as $y(x, 0) = f(x)$. This function describes the transverse position y of the element of the string located at various values of x at time $t = 0$. Because the speed of the pulse is v, the pulse has travelled to the right at a distance vt at the time t (Fig. 5.8 (b)). We assume the shape of the pulse does not change with time. Therefore, at time t, the shape of the pulse is the same as it was at time $t = 0$ as in Fig. 5.8(a). Consequently, an element of the string at x at this time has the same y position as an element located at $x - vt$ had at time $t = 0$.

$$y(x, t) = y(x - vt, 0)$$

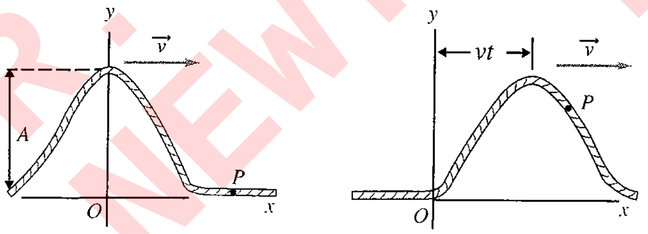

Fig. 5.8 A one-dimensional pulse travelling to the right with a speed v, (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$, (b) At some later time t, the shape remains unchanged and the vertical position of any element of the medium is given by $y = f(x - vt)$

In general, then we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt) \qquad \text{(i)}$$

Similarly, if the pulse travels to the left, the transverse position of elements of the string is described by

$$y(x, t) = f(x + vt) \qquad \text{(ii)}$$

The function y, sometimes called the **wave function**, depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read 'y as a function of x and t.'

It is important to understand the meaning of y. Consider an element of the string at point P, identified by a particular value of its x coordinate. As the pulse passes through P, the y coordinate of this element increases, reaches a maximum and then decrease to zero. The wave function $y(x, t)$ represents the y coordinate – the transverse position – of any element located at position x at any time t. Furthermore, if t is fixed (as, for example, in the case of taking a snapshot of the pulse), the wave function $y(x)$, sometimes called the waveform, defines a curve representing the geometric shape of the pulse at that time.

Illustration 5.2 **A wave pulse is travelling along $+x$ direction on a string at 2 m/s. Displacement y (in cm) of the particle at $x = 0$ at any time t is given by $2/(t^2+1)$. Find**

(i) Expression of the function $y = (x, t)$, i.e., displacement of a particle at position x and time t.
(ii) Draw the shape of the pulse at $t = 0$ and $t = 1$ s.

Sol.

(i) By replacing t by $t-(x/v)$, we can get the desired wave function, i.e.,

$$y = \frac{2}{\left(t - \frac{x}{2}\right)^2 + 1}$$

(ii) We can use wave function at a particular instant to find the shape of the wave pulse using different values of x.

$$\text{at} \quad t = 0, \quad y = \frac{2}{\frac{x^2}{4} + 1}$$

at $x = 0$, $y = 2$ cm
$x = 2$ cm, $y = 1$ cm
$x = -2$ cm, $y = 1$ cm
$x = 4$ cm, $y = 0.4$ cm
$x = -4$ cm, $y = 0.4$ cm

Using these values, shape of the pulse is drawn.

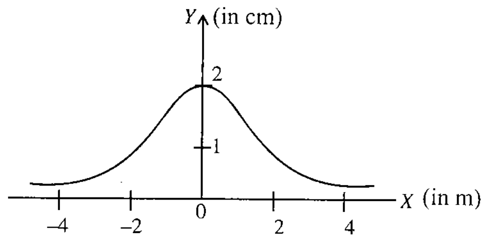

Fig. 5.9

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Similarly for $t = 1$ s, shape can be drawn. What do you conclude about direction of motion of the wave from the graphs? Also check how much the pulse has moved in 1 s time interval. This is equal to wave speed. Here is the procedure:

$$\text{at } t = 1 \text{ s}, \quad y = \frac{2}{\left(1-\frac{x}{2}\right)^2 + 1}$$

at $x = 2$ m $y = 2$ cm (maximum value)
at $x = 0$ $y = 1$ cm
at $x = 4$ m $y = 1$ cm

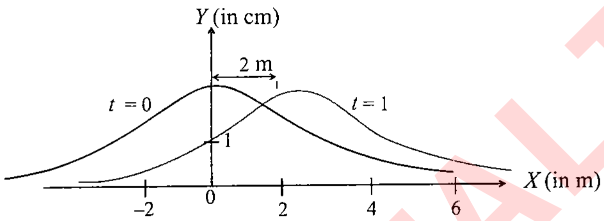


Fig. 5.10

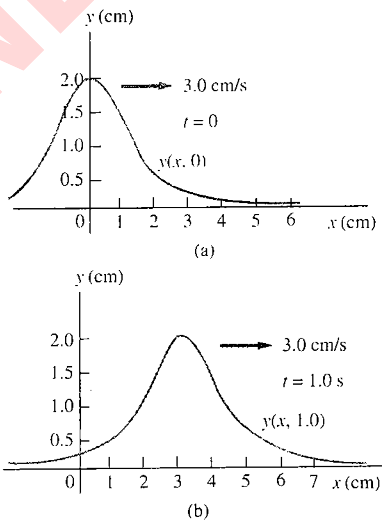
The pulse has moved to the right by 2 in 1 s interval.

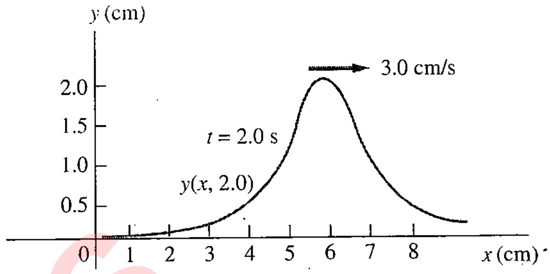
Illustration 5.3 **A pulse moving to the right along the x-axis is represented by the wave function**

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimetres and t is measured in seconds. Find expression for the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s and plot the shape of pulse at these lines.

Sol. Figure 5.11(a) shows the pulse represented by this wave function at $t = 0$. Imagine this pulse moving to the right and maintaining its shape as suggested by Figs. 5.11(b) and (c).





(c)

Fig. 5.11

We categorize this example as a relatively simple analysis problem in which we interpret the mathematical representation of a pulse.

The wave function is of the form $y = f(x - vt)$. Inspection of the expression for $y(x, t)$ reveals that the wave speed is v = 3.0 cm/s. Furthermore, by letting $x - 3.0t = 0$, we find that the maximum value of y given by A = 2.0 cm.

Write the wave function expression at $t = 0$:

$$y(x, 0) = \frac{2}{x^2 + 1}$$

Write the wave function expression at $t = 1.0$ s:

$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1}$$

Write the wave function expression at $t = 2.0$ s:

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1}$$

For each of these expressions we can have various values of x and plot the wave function. This procedure yields the wave functions shown in the three parts of the Fig. 5.11.

These snapshots show that the pulse move to the right without changing its shape and that it has a constant speed of 3.0 cm/s. What if the wave function were

$$y(x, t) = \frac{4}{(x + 3.0t)^2 + 1}$$

How would that change the situation?

Answer: One new feature in this expression is the plus sign in the denominator rather than the minus sign. The new expression represents a pulse with the same shape as that in the previous figure. But moving to the left as time progresses. Another new feature here is the numerator 4 rather than 2. Therefore, the new expression represents a pulse with twice the height of that in figure.

Illustration 5.4 **At $t = 0$, transverse pulse in a wire is described by the function**

$$y = \frac{6}{x^2 + 3}$$

where x and y are in metres. Write the function $y(x, t)$ that describes this pulse if it is travelling in the positive x-direction with a speed of 4.50 m s.

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Sol. At $t = 0$, the wave pulse looks like a bump centred at $x = 0$. As time goes on, the wave function . . . will be function of t as well as x. The point about which the bump is centred will be $x_0 = 4.5t$.

We obtain a function of the same shape by writing

$$y(x, t) = 6/[(x - x_0)^2+3]$$

where the centre of the pulse is at $x_0 = 4.5\ t$. Thus we have

$$y(x, t) = \frac{6}{(x-4.5t)^2+3}$$

Note that for y to stay constant as t increases, x must increase by 4.5 t, as it describes the wave moving at 4.5 m/s.

In general, we can cause any waveform to move along the x-axis at a velocity v, by substituting $(x\ v_x t)$ for x in the wave function y(x) at t = 0. A wave function that depends on t through $(x + v_x t)$ describes a wave moving in the negative x direction.

EQUATION OF A PLANE PROGRESSIVE WAVE (TRAVELLING WAVE)

Definition of a Progressive Wave

A wave which travels in a given direction with constant amplitude (in other words without attenuation) is known as a progressive wave.

TRAVELLING WAVE MODEL

In this section, we introduce an important wave function whose shape is shown in Fig. 5.12. The wave represented by this curve is called a sinusoidal wave because the curve is the same as that of the function $\sin\theta$ plotted against θ. A sinusoidal wave could be established on a rope by shaking the end of the rope up and down in simple harmonic motion.

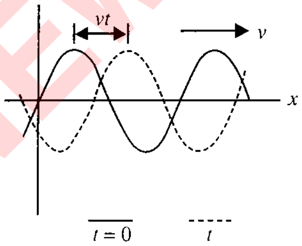

Fig. 5.12

The sinusoidal wave is the simplest example of a periodic continuous wave and can be used to build more complex waves. The bold curve in Fig. 5.13 represents a snapshot of a travelling sinusoidal wave at $t = 0$, and the dotted curve represents a snapshot of the wave at some later time t. Imagine two types of motion that can occur. First, the entire waveform in figure moves to the right so that the bold curve moves toward the right and eventually reaches the position of the dotted curve. This movement is the motion of the wave. If we focus on one element of the medium, such as the element at $x = 0$, we see that each element moves up and down along the y-axis in simple harmonic motion. This movement is the motion of the *elements of the medium*. It is important to differentiate between the motion of the wave and the motion of the elements of the medium.

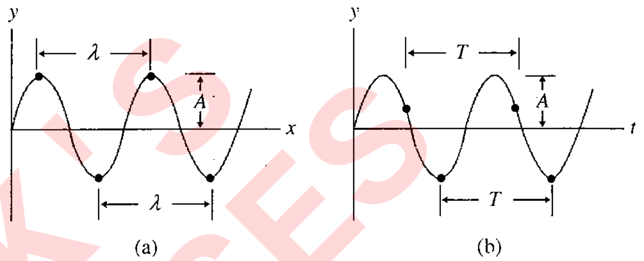

Fig. 5.13

Consider the sinusoidal wave in Fig. 5.13(a), which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Fig. 5.13(a). The next value of x for which y is zero is $x = \lambda/2$. Therefore

$$y\left(\frac{\lambda}{2}, 0\right) = A\sin\left(a\frac{\lambda}{2}\right) = 0 \qquad \text{(iii)}$$

For this equation to be true, we must have $a\lambda/2 = \pi$, or $a = 2\pi/\lambda$. Therefore, the function describing the positions of the elements of the medium through which the sinusoidal wave is travelling can be written as

$$y\,(x, 0) = A\sin\left(\frac{2\pi}{\lambda}x\right) \qquad \text{(iv)}$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. Notice that the vertical position of an element of the medium is the same whenever x is increased by an integral multiple of λ. If the wave moves to the right with a speed v, the wave function at some later time t is

$$y\,(x, t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad \text{(v)}$$

The wave function has the form $f(x - vt)$. If the wave were travelling to the left, the quantity $x - vt$ would have been replaced by $x + vt$ as we learned when we developed Eqs. (i) and (ii).

By definition, the wave travels through a displacement Δx equal to one wave length λ in the time interval of one period T. Therefore, the wave speed, wave length and period are related by the expression

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} \qquad \text{(vi)}$$

Substituting this expression for v into Eq. (v) gives

$$y = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \qquad \text{(vii)}$$

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This form of the wave function shows the periodic nature of y. Note that we will often use y rather than $y\ (x,\ t)$ as a shorthand notation. At any given time t, y has the same value at the positions x, $x + \lambda$, $x + 2\lambda$ and so on. Furthermore, at any given position x, the value of y is the same at times t, $t + T$, $t + 2T$ and so on.

We can express the wave function in convenient form by defining two other quantities, the angular wave number k (usually called wave number) and the angular frequency ω:

Angular wave number:

$$k \equiv \frac{2\pi}{\lambda} \quad \text{(viii)}$$

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \quad \text{(ix)}$$

where

λ is the wavelength (measured in metres),
f is the ordinary frequency (measured in hertz),
T is the period (measured in seconds).

Using these definitions, Eq. (vii) can be written in a more compact form or,

$$y = A\sin(kx - \omega t)$$

Speed of Sinusoidal Wave

Consider a wave travelling along positive x-direction. Figure 5.14 shows two snapshots of the wave at a small interval of time Δt. Let x is the movement of entire wave pattern in time Δt, then wave speed is defined as

$$v = \frac{\Delta x}{\Delta t} \quad \text{(x)}$$

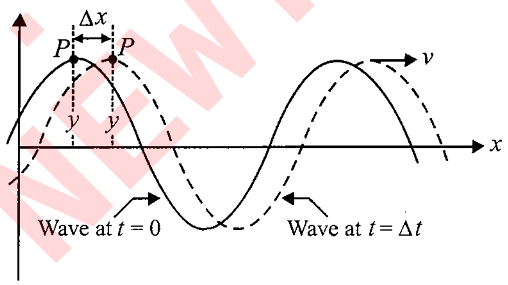


Fig. 5.14

$\therefore$ $\quad \sin(kx - \omega t) = \text{constant}$

or $\quad (kx - \omega t) = \text{a constant} \quad$ (xi)

Differentiating Eq. (ii) w.r.t. time, we get $k\dfrac{dx}{dt} - \omega = 0$

$$\frac{dx}{dt} = v = \frac{\omega}{k} \quad \text{(xii)}$$

General Expression for a Sinusoidal Wave

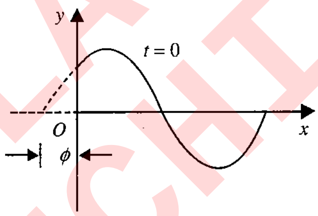
The general equation of simple harmonic progressive wave is given by

$$y = A\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$$

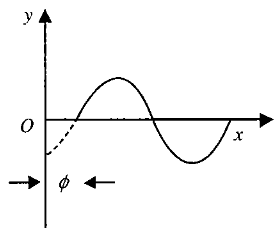
If an initial phase ϕ exists at origination point of wave, equation is changed to $y = A\sin\left[2\pi\left(\dfrac{t}{T} \pm \dfrac{x}{\lambda}\right) + \phi\right]$

Positive and Negative Initial Phase Constants

In general, the equation of a harmonic wave travelling along the positive x-axis is expressed as $y = A\sin(kx - \omega t \pm \phi)$, Where ϕ is called the initial phase constant. It determines the initial displacement of the particle at $x = 0$ when $t = 0$.



Positive initial phase constant
$y = A\sin(kx - \omega t + \phi)$
The sine curve starts from the left of the origin.



Negative initial phase constant
$y = A\sin(kx - \omega t - \phi)$
The sine curve starts from the right of the origin.

Fig. 5.15

Change in Phase with Time for a Constant x, i.e., at a Fixed Point in the Medium

$$[\phi]_{t_1} = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right) + \phi;\ [\phi]_{t_2} = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right) + \phi$$

(For the wave travelling in positive x-direction)

$$\Delta\phi = \phi_{t_2} - \phi_{t_1} = \frac{2\pi}{T}\times(t_2 - t_1) = \frac{2\pi}{T}\times\Delta t \Rightarrow \Delta\phi = \frac{2\pi\times\Delta t}{T}$$

$$\text{Phase difference} = \frac{2\pi}{T}\times\text{Time difference}$$

For $\Delta t = T$, i.e., after one time period,

$$\Delta\phi = \frac{2\pi}{T}\times T = 2\pi$$

A phase change of 2π ($2n\pi$ in general) gives the points oscillating in phase.

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Variation of Phase with Distance

At a given instant of time $t = t$, phase at $x = x_1$,

$$[\phi]_{x_1} = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) + \phi$$

For the wave travelling in positive x-direction and phase at $x = x_2$,

$$[\phi]_{x_2} = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right) + \phi$$

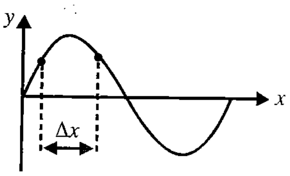

Fig. 5.16

$$\Rightarrow \qquad \Delta\phi = [\phi]_{x_1} - [\phi]_{x_2} = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

i.e., Phase difference $= \dfrac{2\pi}{\lambda} \times$ Path difference

For $\Delta x = n\lambda$, $\Delta\phi = n2\pi$; where $n = 1, 2, 3, \ldots$

Points separated by distance $n\lambda$ where n is an integer are in same phase (at any instant of time).

For $\Delta x = m\dfrac{\lambda}{2}$, where $m = 1, 3, 5, 7, \ldots$

$$\Delta\phi = m \times \pi$$

Points separated by distance m(λ 2), where m is an odd integer are out of phase (at any instant of time).

Illustration 5.5 **Two sinusoidal waves in a string are defined by the functions y_1 = (2.00 cm) sin (20.0x – 32.0t) and y_2 = (2.00 cm) sin (25.0x – 40.0t) where y_1, y_2 and x are in centimetres and t is in seconds.**

a. What is the phase difference between these two waves at the point x = 5.00 cm at t = 2.00 s?

b. What is the positive x value closest to the origin for which the two phases differ by $\pm\pi$ at t = 2.00 s? (That is a location where the two waves add to zero.)

Sol: Think of the two waves as moving sinusoidal graphs with different wavelengths and frequencies. The phase difference between them is a function of the variables x and t. It controls how the waves add up at each point.

Looking at the wave function let us pick out the expressions for the phase as the 'angles we take the sines of'—then arguments of the sine functions. Then algebra will give particular answers.

At any time and place, the phase shift between the waves is found by subtracting the phases of the two waves, $\Delta\phi = \phi_1 - \phi_2$.

$\Delta\phi$ = (20.0 rad/cm)x – (32.0 rad/s)t – [(25.0 rad/cm)x – (40.0 rad/s)t]

Collecting terms, $\Delta\phi$ = –(5.00 rad/cm)x + (8.00 rad/s)t

a. At x = 5.00 cm and t = 2.00 s, the phase difference is

$\Delta\phi$ = (–5.00 rad/cm) (5.00 cm) + (8.00 rad/s)(2.00 s)

$|\Delta\phi|$ = 9.00 rad = 516°

b. The sine functions repeat whenever their arguments change by an integer number of cycles, an integer multiple of 2π radians. Then the phase shift equals $\pm\pi$ whenever $\Delta\phi = \pi + 2n\pi$, for all integer values of n.

Substituting this into the phase equation, we have

$\pi + 2n\pi$ = –(5.00 rad/cm)x + (8.00 rad/s)t

At t = 2.00 s,

$\pi + 2n\pi$ = –(5.00 rad/cm)x + (8.00 rad/s)(2.00 s)

or (5.00 rad/cm)x = (16.0 – π – $2n\pi$)rad

The smallest positive value of x is found when n = 2

$$x = \frac{(16.0 - 5\pi)\text{ rad}}{5.00\text{ rad/cm}} = 0.0584\text{ cm}$$

Illustration 5.6 ***A*, *B* and *C* are the three particles of a medium which are equally separated and lie along the *x*-axis. When a sinusoidal transverse wave of wavelength κ propagates along the *x*-axis, the following observations are made:**

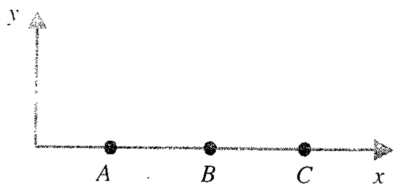

Fig. 5.17

a. *A* and *B* have the same speed.

b. *A* and *C* have the same velocity.

Find (i) the minimum distance between *A* and *B* and (ii) the minimum distance between *A* and *C*.

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Sol.

i. Points A and B have the same speed. Point A goes downward and the point B goes upward. The distance between the points A and B is $\lambda/2$.

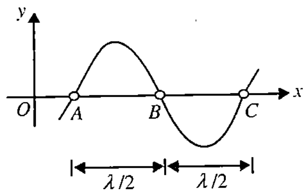

Fig. 5.18

ii. Points A and C have the same velocity. Both the points move downward. The distance between A and C is λ.

Illustration 5.7 **A sinusoidal wave travelling in the positive x-direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15 cm, as shown in Fig. 5.19.**

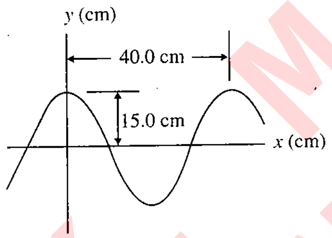

Fig. 5.19

a. **Find the angular wave number, period, angular frequency and speed of the wave.**

b. **Determine the phase constant ϕ, and write a general expression for the wave function.**

Sol.

a. Wave number $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{40 \text{ cm}} = \dfrac{\pi}{20}$ rad/cm

Time period $T = \dfrac{1}{f} = \dfrac{1}{8}$ s,

Angular frequency $\omega = 2\pi f = 16\pi$ rad/s

Speed of wave $v = f\lambda = 320$ cm/s

b. It is given that $A = 15$ cm

and also $y = 15$ cm at $x = 0$ and $t = 0$

then using $y = A \sin(\omega t - kx + \phi)$

$15 = 15 \sin\phi \Rightarrow \sin\phi = 1$

or $\phi = \dfrac{\pi}{2}$ rad

Therefore, the wave function is

$$y = A \sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

$$= (15 \text{ cm}) \sin\left[(16\pi \text{ rad s}^{-1})t - \left(\frac{\pi \text{ rad}}{20 \text{ cm}}\right)x + \frac{\pi}{2}\right]$$

Illustration 5.8 **A wave is described by $y = (2.00 \text{ cm}) \sin(kx - \omega t)$, where $k = 2.11$ rad m, $\omega = 3.62$ rad s, x is in metres, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.**

Sol. The wave function is a moving graph. The position of a particle of the medium, represented by y, is varying all the time and from every point to the next point at each instant. But we can pick out the parameters that characterize the whole wave and have constant values.

We compare the given wave function with the general sinusoidal wave equation

$$y = A \sin(kx - \omega t + \phi)$$

Its functional equality to $y = (2.00 \text{ cm}) \sin(kx - \omega t)$ reveals that the amplitude is $A = 2.00$ cm

The angular wave number is $k = 2.11$ rad/m so that

$$\lambda = 2\pi/k = 2.98 \text{ m}$$

The angular frequency is $\omega = 3.62$ rad/s so that

$$f = \omega/2\pi = 0.576 \text{ Hz}$$

The speed is $v = f\lambda = (0.576 \text{ s}^{-1})(2.98 \text{ m}) = 1.72$ m/s

It is not important to the dynamics of the wave, but we can also identify the phase constant as $\phi = 0$. We could write the wave function to explicitly display the constant parameters as

$$y(x, t) = (2.00 \text{ cm}) \sin\left(\frac{2\pi}{2.98 \text{ m}} x - 2\pi(0.576)/\text{s})t\right)$$

SINUSOIDAL WAVES ON STRINGS

In Fig. 5.20, we demonstrate how to create a pulse by jerking a taut string up and down once. To create a series of such pulses —a wave—let us replace the hand with an oscillating blade vibrating in simple harmonic motion. The figure represents snapshots of the wave created in this way at intervals of $T/4$. Because the end of the blade oscillates in simple harmonic motion, each element of the string, such as that P, also oscillates vertically with simple harmonic motion. That must be the case because each element follows the simple harmonic motion of the blade. Therefore, every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of oscillation of the blade. Notice that although each element oscillates in the y direction, the wave travels in the x-direction with a speed v. Of course, that is the definition of a transverse wave.

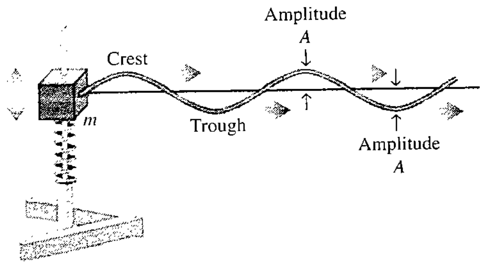

Fig. 5.20

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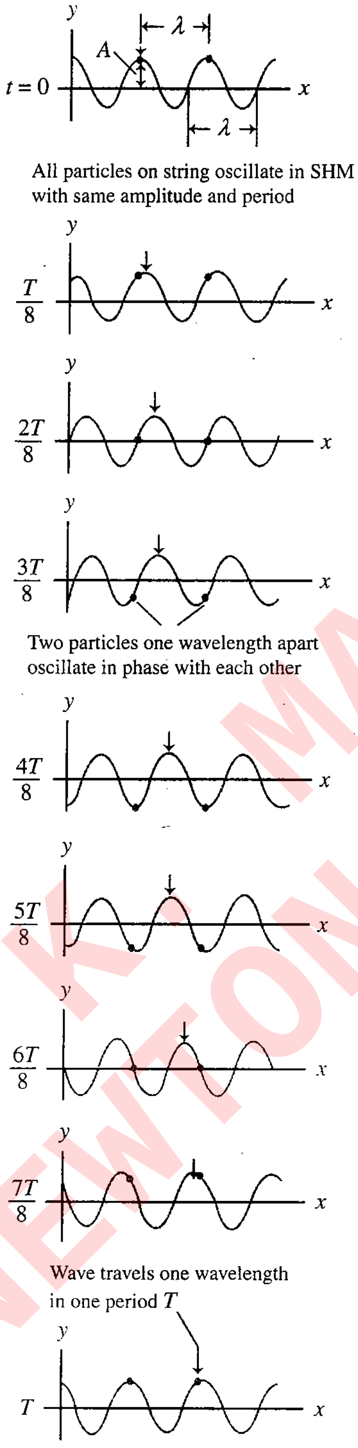

Fig. 5.21

If the wave at $t = 0$ is as described in by the wave function can be written as

$$y = A \sin (kx - \omega t) \qquad \text{(xiii)}$$

We can use this expression to describe the motion of any element of the string. An element at point P (or any other element of the string) moves only vertically, and so its x coordinate remains constant. Therefore, the transverse speed v_y (not to be confused with the wave speed v) and the transverse acceleration a_y of elements of the string are

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A \cos (kx - \omega t) \qquad \text{(xiv)}$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A \sin (kx - \omega t) \qquad \text{(xv)}$$

These expressions incorporate partial derivatives because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\max} = \omega A \qquad \text{(xvi)}$$

$$a_{y,\max} = \omega^2 A \qquad \text{(xvii)}$$

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches to its maximum value (ωA) when $y = 0$, whereas the magnitude of the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = \pm A$. Finally, Eqs. (xvi) and (xvii) are identical in mathematical form to the corresponding equations for simple harmonic motion,

Differentiating Eq. (xiii) with respect to displacement of wave (x) then we will get the slope of displacement curve of wave as

$$\frac{\partial y}{\partial x} = Ak \cos (kx - \omega t) \qquad \text{(xviii)}$$

From Eqs. (xiv) and (xviii) we have

$$\frac{\partial y}{\partial x} = -\frac{k}{\omega}\frac{\partial y}{\partial t} = -\frac{1}{v}\frac{\partial y}{\partial t}$$

$$\frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x} \quad \Rightarrow \boldsymbol{v_p = -v\frac{dy}{dx}}$$

Note:

Particle velocity = – (wave velocity) ± slope of wave curve

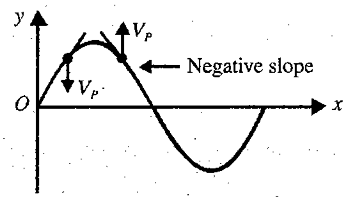

Fig. 5.22

Hence in a propagating wave the speed of medium particle at a given position at a given instant is the negative of the product of wave velocity and the slope of the displacement curve of wave at that point and at that instant.

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Again differentiating Eq. (xviii) w.r.t. x

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(kx - \omega t) \qquad \text{(xix)}$$

From Eqs. (xv) and (xix) we get

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \qquad \text{(xx)}$$

This expression is the linear wave equation as it applies to waves on a string.

Linear wave equation in general

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \text{(xxi)}$$

Equation (xxi) applies in general to various type of travelling waves. For waves on strings, y represents the vertical position of elements of the string. For sound waves, y corresponds to longitudinal position of elements of air from equilibrium or variations in either the pressure or the density of the gas through which the sound waves are propagating. In the case of electromagnetic waves, y corresponds to electric or magnetic field components.

We have shown that the sinusoidal wave function is one solution of the linear wave. Although we do not prove it here, the linear wave equation is satisfied by any wave function having the form $y = f(x \pm vt)$. Furthermore, we have seen that the linear wave equation is a direct consequence of Newton's second law applied to any element of a string carrying a travelling wave.

Illustration 5.9 **Verify that wave function**

$$y = \frac{2}{(x-3t)^2 + 1}$$

is a solution to the linear wave equation, x and y are in centimetres.

Sol. By taking partial derivatives of this function w.r.t. x and t

$$\frac{\partial^2 y}{\partial x^2} = \frac{12(x-3t)^2 - 4}{[(x-3t)^2+1]^3} \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = \frac{108(x-3t)^2 - 36}{[(x-3t)^2+1]^3}$$

or $$\frac{\partial^2 y}{\partial x^2} = \frac{1}{9}\frac{\partial^2 y}{\partial t^2}$$

Comparing with linear wave equation, we see that the wave function is a solution to the linear wave equation if the speed at which the pulse moves is 3 cm/s. It is apparent from wave function therefore it is a solution to the linear wave equation.

Illustration 5.10 **The wave function for a travelling wave on a taut string is**

$$y(x, t) = (0.350 \text{ m})\sin(10\pi t - 3\pi x \quad \pi/4). \quad \text{(SI units)}$$

a. What is the speed and direction of travel of the wave?

b. What is the vertical position of an element of the string at $t = 0$, $x = 0.100$ m?

c. What is the wavelength and frequency of the wave?

d. What is the maximum transverse speed of an element of the string?

Sol. Let us compare the given equation with $y = A\sin(\omega t - kx + \phi)$.

We find that $k = 3\pi$ rad/m and $\omega = 10\pi$ rad/s

a. The speed and direction of the wave are both specified by the vector wave velocity:

$$\vec{v} = f\lambda\hat{i} = \frac{\omega}{k}\hat{i} = \frac{10\pi \text{ rad/s}}{3\pi \text{ rad/m}}\hat{i} = 3.33\hat{i} \text{ m/s}$$

b. Substituting $t = 0$ and $x = 0.100$ m, we have

$y = (0.350 \text{ m}) \sin(-0.300\pi + 0.250\pi)$

$= (0.350 \text{ m}) \sin(-0.157)$

$= (0.350 \text{ m})(-0.156) = -0.0548 \text{ m} = -5.48 \text{ cm}$

Note that when you take the sine of a quantity with no units, the quantity is not in degrees, but in radians.

c. The wavelength is

$$\lambda = \frac{2\pi \text{ rad}}{k} = \frac{2\pi \text{ rad}}{3\pi \text{ rad/m}} = 0.667 \text{ m}$$

and the frequency is

$$f = \frac{\omega}{2\pi \text{ rad}} = \frac{10\pi \text{ rad/s}}{2\pi \text{ rad}} = 5.00 \text{ Hz}$$

d. The particle speed is $v_y = \partial y/\partial t = (0.350 \text{ m})(10\pi \text{ rad}) \cos(10\pi t - 3\pi x + \pi/4)$

The maximum occurs when the cosine term is 1:

$$v_{y,\max} = (10\pi \text{ rad/s})(0.350 \text{ m/s}) = 11.0 \text{ m/s}$$

Illustration 5.11 **(a) Write the expression for y as a function of x and t for a sinusoidal wave travelling along a rope in the negative x direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, t) = 0$ at $t = 0$. (b) Write an expression for y as a function of x and t for the wave in part (a) assuming that $y(x, 0) = 0$ at the point $x = 10.0$ cm.**

Sol. Think about the graph of y as a function of x at one instant as a smooth succession of identical crests and troughs, with the length in space of each cycle being 80.0 cm. The distance from the top of a crest to the bottom of a trough is 16.0 cm. Now think of the whole graph moving towards the left at 240 cm/s.

Using the travelling wave model, we can put constants with the right values into $y = A\sin(kx + \omega t + \phi)$ to have the mathematical representation of the wave. We have the same (positive) signs for both kx and ωt so that a point of constant phase will be at a decreasing value of x as t increases that is, so that the wave will move to the left.

The amplitude is $A = y_{\max} = 8.00 \text{ cm} = 0.0800$ m.

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The wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.800\text{ m}} = \frac{5\pi}{2}$$

The angular frequency, $\omega = 2\pi f = 2\pi(3.00\text{ s}^{-1}) = 6.00\pi$ rad/s

a. We have $y = A\sin(kx + \omega t + \phi)$, choosing $\phi = 0$ will make it true that $y(0, 0) = 0$. Then the wave function becomes upon substitution of the constant values for this wave

$$y = (0.080\text{ m})\sin\left(\frac{5\pi}{2}x + 6.00\pi t\right)$$

b. In general,

$$y = (0.0800\text{ m})\sin\left(\frac{5\pi}{2}x + 6.00\pi t + \phi\right)$$

If $y(x, 0) = 0$ at $x = 0.100$ m, we require

$$0 = (0.0800\text{ m})\sin\left(\frac{5}{2}\pi \times 0.1 + \phi\right)$$

$$\frac{5\pi}{2} \times 0.1 + \phi = 0$$

So we must have the phase constant $\phi = -\frac{\pi}{4}$ rad

Therefore, the wave function for all values of the variable x and t is

$$y = (0.0800\text{ m})\sin\left(\frac{5}{2}\pi x + 6.00\pi t - \frac{\pi}{4}\right)$$

Illustration 5.12 **Given the equation for a wave on a string**

$$y = 0.03\sin(3x - 2t)$$

where y and x are in metres and t is in seconds.

a. **At $t = 0$, what are the values of the displacement at $x = 0$, 0.1 m, 0.2 m, and 0.3 m?**

b. **At $x = 0.1$ m what are the values of the displacements at $t = 0$, 0.1 s, and 0.2 s?**

c. **What is the equation for the velocity of oscillation of the particles of the string?**

d. **What is the maximum velocity of oscillation?**

e. **What is the velocity of propagation of the wave?**

Sol.

a. At $t = 0$, $y = (0.03\sin 3x)$ m

for $x = 0$, $y = 0.03\sin(0) = 0$

$x = 0.1$ m $y = 0.03\sin(0.3\text{ rad})$

$$= 0.03\sin\left(\frac{0.3 \times 180°}{\pi}\right) = 0.03 \times \sin(17.2°)$$

$$= 0.03 \times 0.2957 = 8.87 \times 10^{-3}\text{ m}$$

Similarly for

$x = 0.2$ m, $y = 0.03\sin(0.6\text{ rad}) = 1.69 \times 10^{-2}$ m

and for $x = 0.3$ m, $y = 0.03\sin(0.9\text{ rad}) = 2.35 \times 10^{-2}$ m

b. At $x = 0.1$ m, $y = 0.03\sin(0.3 - 2t)$

At $t = 0$, $y = 0.03\sin(0.3\text{ rad}) = 8.87$ ë 10^{-3} m

$t = 0.1$ s, $y = 0.03\sin(0.3 - 0.2) = 0.03\sin(0.1\text{ rad})$

$$= 2.99 \times 10^{-3}\text{ m}$$

$t = 0.2$ s, $y = 0.03\sin(0.3 - 0.4) = -2.99 \times 10^{-3}$ m

c. Velocity of particle,

$$\frac{dy}{dt} = \frac{d}{dt}[0.03\sin(3x - 2t)] = -0.06\cos(3x - 2t)$$

$$= -6 \times 10^{-2}\cos(3x - 2t)\text{ m/s}$$

d. Maximum velocity or the velocity amplitude $= 6 \times 10^{-2}$ m/s

e. Comparing $y = 0.03\sin(3x - 2t)$ with the wave equation $y = A\sin(kx - \omega t)$,

we have $k = 2\pi/\lambda = 3$, and $\omega = 2\pi f = 2$

Therefore, $v = f\mu = \frac{\xi}{k} = \frac{2}{3} = 0.667$ m/s

Illustration 5.13 **A plane progressive wave is given by $x = (40\text{ cm})\cos(50\pi t - 0.02\pi y)$ where y is in centimetres and t in seconds. What will be the particle velocity at $y = 25$ cm in time $t = 1/200$ s?**

Sol. Given that $x = 40\cos(50\pi t - 0.02\pi y)$

Therefore, particle velocity can be given as

$$v_p = \frac{dx}{dt} = (40 \times 50\pi)\{-\sin(50\pi t - 0.02\pi y)\}$$

Putting $x = 25$ and $t = \frac{1}{200}$ s,

$$v_p = -(2000\pi\text{ cm/s})\sin\left[50\pi\left(\frac{1}{200}\right) - 0.02\pi(25)\right]$$

$$= 10\pi\sqrt{2}\text{ m/s}$$

Illustration 5.14 **Figure 5.23 shows the shape of a progressive wave at time $t = 0$. After a time $t = 1$ 80, the particle at the origin has its maximum negative displacement. If the wave speed is 80 units, then find the equation of the progressive wave.**

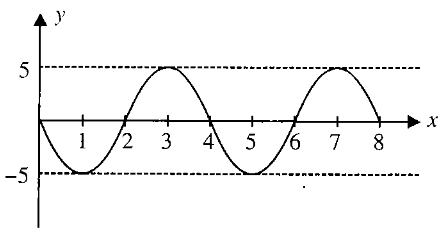

Fig. 5.23

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Sol. The wavelength $\lambda = 4$

We know, $T = \dfrac{\lambda}{v} = \dfrac{4}{80} = \dfrac{1}{20}$

Since after $t = \dfrac{1}{80} = \dfrac{T}{4}$, the particle at the origin is at its maximum negative, the entire wave pattern would have shifted by a distance of 1 unit (λ /4) to the left.

The wave moves in the negative x-axis direction; from this we get

$$y = A\sin\left[2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)+\phi\right]$$

We have initially $y = 0$ for $t = 0$ and $x = 0$

$\therefore \quad \phi = 0, \pi, 2\pi$ etc. (i)

Also, $y = -A$; when $t = T/4$ and $x = 0$

$$\therefore \quad -A = A\sin\left[2\pi\left(\frac{1}{4}\right)+\phi\right]$$

$$\Rightarrow \frac{\pi}{2}+\phi = \frac{3\pi}{2}, \frac{7\pi}{2} \text{ etc.}$$

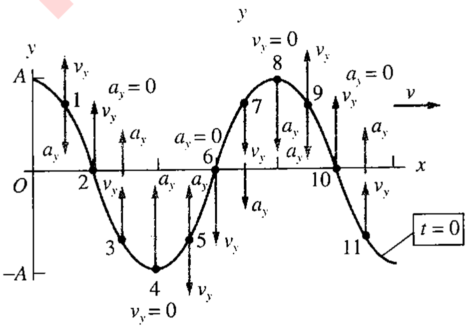
or $\quad \phi = \pi, 3\pi,$ etc. (ii)

From Eqs. (i) and (ii), $\phi = \pi$

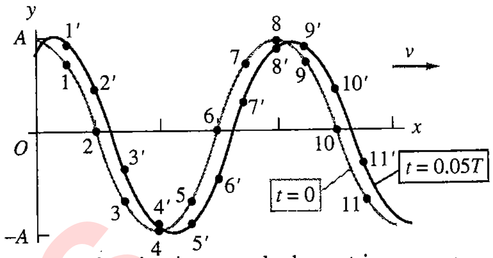
Hence $\quad y = 5\sin\left[2\pi\left(\dfrac{t}{(1/20)}+\dfrac{x}{(4)}\right)+\pi\right]$

SPEED OF WAVES ON STRING

In this section, we determine the speed of a transverse pulse travelling on taut string. Let us first conceptually predict the parameters that determine the speed. If a string under tension is pulled sideways and then released, the force of tension is responsible for accelerating a particular element of the string back towards its equilibrium position. According to Newton's second law of motion the acceleration of an element increases with inasing tension. If the element returns to equilibrium more rapidly due to this increased acceleration, we would intuitively argue that the wave speed is greater. Therefore, we expect the wave speed to increase with increasing tension.



(a) Acceleration a_y at each point is proportional to displacement y at that point



(b) Acceleration is upward where string curves upward, downward where string curves downward

Fig. 5.24

Likewise, because it is more difficult to accelerate a massive element of the string than a light element, the wave speed should decrease as the mass per unit length of the string increases. If the tension in the string is T and its mass per unit length is μ (Greek letter mu), the wave speed, as we shall show, is

$$v = \sqrt{\frac{T}{\mu}} \quad \text{(xxii)}$$

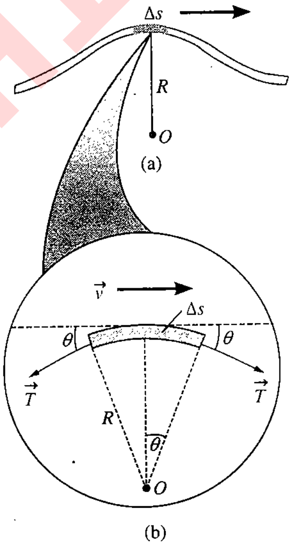


Fig. 5.25

Let us use a mechanical analysis to derive Eq. (xxii). Consider a pulse moving on a taut string to the right with a uniform speed v measured relative to a stationary frame of reference. Instead of staying in this reference frame, it is more convenient to choose a different inertial reference frame that moves along with the pulse with the same speed as the pulse so that the pulse is at rest within the frame. This change of reference frame is permitted because Newton's laws are valid in either a stationary frame, or one that moves with constant velocity. In our new reference frame, all elements of the string move to the left: a given element of the string initially to the right of the pulse moves to the left, rises up and follows the shape of the pulse, and then continues to move to the left. Figure 5.25(a) shows such an element at the instant it is located at the top of the pulse.

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The small element of the string of length Δs, shown in Fig. 5.25(a) and magnified in Fig. 5.25(b), forms an approximate arc of a circle of radius R. In the moving frame of reference (which moves to the right at a speed v along with the pulse), the shaded elements moves to the left with a speed v. This element has a centripetal acceleration equal to v^2/R, which is supplied by components of the force $\vec{T}$ whose magnitude is the tension in the string. The force $\vec{T}$ acts on both sides of the element and is tangent to the arc as shown in Fig. 5.25 (b). The horizontal components of $\vec{T}$ cancel, and each vertical component $T \sin\theta$ acts on radially towards the arc's centre. Hence, the total radial force on the element is $2T\sin\theta$, because the element is small, θ is small, and we can therefore use the small-angle approximation $\sin\theta \approx \theta$. So the total radial force is $F_r = 2T\sin\theta \approx 2T\theta$.

The element has a mass $m = \mu\Delta s$. Because the element forms part of a circle and subtends an angle 2θ at the centre, $\Delta s = R(2\theta)$, and $m = \mu\Delta s = 2\mu R\theta$.

Applying Newton's second law to this element in the radial direction gives

$$F_r = \frac{mv^2}{R}$$

$$2T\theta = \frac{2\mu R\theta v^2}{R} \rightarrow v = \sqrt{\frac{T}{\mu}}$$

This expression for v is Eq. (xxii).

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation $\sin\theta \approx \theta$. Furthermore, the model assumes the tension T is not affected by the presence of the pulse; therefore, T is the same at all points on the string. Finally, this proof does not assume any particular shape for the pulse. Therefore, a pulse of *any shape* travels along the string with speed $v = \sqrt{T/\mu}$ without any change in pulse shape.

Illustration 5.15 **A taut string having tension 100 N and linear mass density 0.25 kg m is used inside a cart to generate a wave pulse starting at the left end, as shown. What should be the velocity of the cart so that pulse remains stationary w.r.t. ground.**

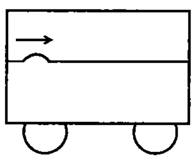

Fig. 5.26

Sol. The expression $v = \sqrt{\frac{T}{\mu}}$ gives the velocity of the wave in the string. Here the string is moving with cart.

Velocity of pulse $\quad v = \sqrt{T/\mu} = 20\,\text{m/s}$

This velocity is w.r.t. car.

Velocity of the pulse can be given as

Now
$$\vec{v}_P = \vec{v}_{PC} + \vec{v}_{CG}$$
$$0 = 20i + \vec{v}_{CG}$$
$$\vec{v}_{CG} = -20i\ \text{m/s}$$

Hence the cart should move with velocity 20 m/s in the direction opposite to the pulse.

Illustration 5.16 **Transverse waves travel with a speed of 20.0 m s in a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m s in the same string?**

Sol. The two wave speeds can be written as:

$$v_1 = \sqrt{T_1/\mu} \quad \text{and} \quad T_2 = \sqrt{T_2/\mu}$$

Dividing the equations

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{so that} \quad T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1$$

$$T_1 = \left(\frac{30.0}{20.0}\right)^2 (6.00\text{ N}) = 13.5\text{ N}$$

Illustration 5.17 **An 80.0 kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. a sling of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the sling, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter?**

Sol. Imagine the effect of the acceleration of the helicopter on the cable. The greater the upward acceleration, the larger the tension in the cable. In turn, the larger the tension, the higher the speed of pulses on the cable.

This problem is a combination of one involving the speed of pulses on a string and one in which the hiker and sling are modelled as a particle under a net force.

Use the time interval for the pulse to travel from the hiker to the helicopter to find the speed of the pulses on the cable:

$$v = \frac{\Delta x}{\Delta t} = \frac{15.0\text{ m}}{0.25\text{ s}} = 60.0\text{ m/s}$$

The tension in the cable: Using $v = \sqrt{T/\mu}$

so, $\quad T = \mu v^2$

Model the hiker and sling as a particle under net force, noting that the acceleration of this particle of mass m is the same as the acceleration of the helicopter:

$$\Sigma F = ma \rightarrow T - mg = ma$$

Solve for the acceleration

$$a = \frac{T}{m} - g = \frac{\mu v^2}{m} - g = \frac{m_{\text{cable}} v^2}{l_{\text{cable}} m} - g$$

Substitute numerical values:

$$a = \frac{(8.00 \text{ kg})(60.0 \text{ m/s})^2}{(15.0 \text{ m})(150.0 \text{ kg})} - 9.80 \text{ m/s}^2 = 3.00 \text{ m/s}^2$$

A real cable has stiffness in addition to tension. Stiffness tends to return a wire to its original straight-line shape even when it is not under tension. For example, a piano wire straightens if released from a curved shape; package-wrapping string does not.

Stiffness represents a restoring force in addition to tension and increase the wave speed. Consequently, for a real cable, the speed of 60.0 m/s that we determined is most likely associated with a smaller acceleration of the helicopter.

Illustration 5.18 **A rope of total mass m and length L is suspended vertically. Show that a transverse pulse travels the length of the rope in a time interval $\Delta = 2\sqrt{L/g}$. Suggestion: first find an expression for the wave speed at any point a distance x from the lower end by considering the rope's tension as resulting from the weight of the segment below that point.**

Sol. The tension will increase as elevation x increases, because the upper part of the hanging rope must support the weight of the lower part. Then the wave speed increase with height.

We will need to do an integral based on $v = dx/dt$ to find the travel time of the variable-speed pulse.

We define $x = 0$ at the bottom of the rope and $x = L$ at the top of the rope. The tension in the rope at any point is the weight of the rope below that point. We can thus write the tension in the rope at each point x as $T = \mu xg$, where μ is the mass per unit length of the rope, which we assume uniform. The speed of the wave pulse at each point along the rope's length is therefore

$$v = \sqrt{\frac{T}{\mu}} \quad \text{or} \quad v = \sqrt{gx}$$

But at each point, the wave propagates at the rate of

$$v = \frac{dx}{dt}$$

So we substitute for v and generate the differential equation:

$$\frac{dx}{dt} = \sqrt{gx} \quad \text{or} \quad dt = \frac{dx}{\sqrt{gx}}$$

Integrating both sides,

$$\Delta t = \frac{1}{\sqrt{g}} \int_0^1 \frac{dx}{\sqrt{x}} = \frac{\left[2\sqrt{x}\right]_0^L}{\sqrt{g}} = 2\sqrt{\frac{L}{g}}$$

What you have learned in mathematic class is another operation that can be done to both sides of an equation: Integrating from one physical point to another. Here the integral extends in time and space from the situation of the pulse being at the bottom on the string to the situation with the pulse reaching the top.

Illustration 5.19 **It is stated in the previous problem that a pulse travels from the bottom to the top of a hanging rope of length L in the time interval $\Delta = 2\sqrt{L/g}$. Use this result to answer the following questions. (It is not necessary to set up any new integrations.) (a) Over what time interval does a pulse travel halfway up the rope? Give your answer as a fraction of the quantity $2\sqrt{L/g}$. (b) A pulse starts travelling up the rope. How far has it travelled after a time interval $\sqrt{L/g}$?**

Sol. The wave pulse travels faster as it goes up the rope because the tension higher in the rope is greater (to support the weight of the rope below it). Therefore it should take more than half the total time Δt for the wave to travel halfway up the rope. Likewise, the pulse should travel less than halfway up the rope in time $\Delta t/2$.

By using the time relationship given in the problem and making suitable substitutions, we can find the required time and distance.

a. From the equation given, the time for a pulse to travel any distance, d, up from the bottom of the rope is $\Delta t_d = 2\sqrt{d/g}$. So the time for a pulse to travel a distance $L/2$ from the bottom is

$$\Delta t_{1/2} = 2\sqrt{\frac{L}{2g}} = 0.707\left(2\sqrt{\frac{L}{g}}\right)$$

b. Likewise, the distance a pulse travels from the bottom of a rope in a time Δt_d is $d = \frac{g\Delta t_d^2}{4}$. So the distance travelled by a pulse after a time

$$\Delta t_d = \sqrt{\frac{L}{g}} \text{ is } d = \frac{g(L/g)}{4} = \frac{L}{4}$$

Illustration 5.20 **An aluminium wire is clamped at each end and under zero tension at room temperature. Reducing the temperature, which results in a decrease in the wire's equilibrium length, increases the tension in the wire. What strain ($\Delta L/L$) results in a transverse wave speed of 100 m s? Take the cross-sectional area of the wire to be 5.00×10^{-6} m², the density to be 2.70×10^3 kg m³, and Young's modulus to be 7.00×10^{10} N m².**

Sol. We expect some small fraction like 10^{-3} as an answer. The situation might remind you of fitting a hot iron rim to a wagon wheel or a shrink-fit plastic seal to a medicine bottle.

We must review the relationship of strain to stress, and algebraically combine it with the equation for the speed of a string wave.

The expression for the elastic modulus

$$Y = \frac{F/A}{\Delta L/L}$$

Becomes an equation for strain

$$\frac{\Delta L}{L} = \frac{F/A}{Y} \quad \text{(i)}$$

We substitute into the equation for the wave speed

$$v = \sqrt{T/\mu} = \sqrt{F/\mu}$$

where tension T means the same as stretching force F.

$$\mu = \frac{m}{L} = \frac{\rho(AL)}{L} = \rho A$$

We substitute and, solve,

$$v^2 = \frac{F}{\mu} = \frac{1}{\rho}\left(\frac{F}{A}\right) \quad \text{or} \quad \left(\frac{F}{A}\right) = \rho v^2 \quad \text{(ii)}$$

and substitute (ii) and (i) to obtain

$$\frac{\Delta L}{L} = \frac{\rho v^2}{Y} = \frac{(2.70\times10^3 \text{ kg/m}^3)(100 \text{ m/s})^2}{7.00\times10^{10} \text{ N/m}^2} = 3.86\times10^{-4}$$

The Young's modulus equation describes stretching that does not really happen, being caused and cancelled out simultaneously by changes in tension and temperature. This wire could be used as a thermometer. Pluck it, pour liquid nitrogen onto it, pluck it again, and listen to the frequency go up.

Illustration 5.21 **A transverse wave of wavelength 50 cm is travelling towards ve x-axis along a string whose linear density is 0.05 g cm. The tension in the string is 450 N. At $t = 0$, the particle at $x = 0$ is passing through its mean position with an upward velocity. Form an equation describing the wave. The amplitude of the wave is 2.5 cm.**

Sol. Let the wave be described by:

$$y(x,t) = A\sin(kx - \omega t + \phi_0) \quad \text{and} \quad A = 2.5 \text{ cm}$$

where $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.5} = 4\pi$$

$$y(0,0) = 0 \quad \Rightarrow A \sin\phi_0 = 0$$

$$\phi_0 = 0, \pi \quad \text{(i)}$$

we also have

$$\left(\frac{dy}{dt}\right)_{0,0} > 0 \ \Rightarrow \ -A\omega\cos\phi_0 > 0 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $\phi_0 = \pi$

Velocity of transverse wave in the string is given by

$$\sqrt{\frac{T}{\mu}} = \sqrt{\frac{450}{0.05\times10^{-1}}} = 300 \text{ m/s}$$

$$\Rightarrow \omega = 2\pi f = \frac{2\pi c}{\lambda} = \frac{2\pi(300)}{0.5} = 1200\pi \text{ rad/s}$$

Using all the quantities, the equation is

$$y = 2.5\sin(4\pi x - 1200\pi t + \pi) \text{ cm}$$

RATE OF ENERGY TRANSFER BY SINUSOIDAL WAVES ON STRING

Waves transport energy through medium as they propagate. For example, suppose an object is hanging on a stretched string and a pulse is sent down the string as in Fig. 5.27(a) when the pulse meets the suspended object, the object is momentarily displaced upward as in Fig. 5.27(b). In the process, energy is transferred to the object and appears as an increase in the gravitational potential energy of the object–Earth system. This section examines the rate at which energy is transported along a string. We shall assume a one-dimensional sinusoidal wave in the calculation of the energy transferred.

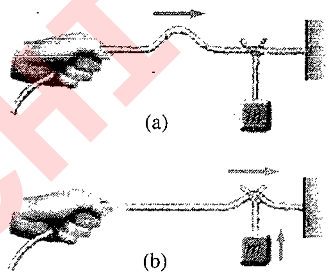

Fig. 5.27

Consider a sinusoidal wave travelling on a string. The source of the energy is some external agent at the left end of the string, which does work in producing the oscillations. We can consider the string to be a non-isolated system. As the external agent performs work on the end of the string, moving it up and down, energy enters the system of the string and propagates along its length. Let us focus our attention on an infinitesimal element of the string of length dx and mass dm. Each such element moves vertically with simple harmonic motion. Therefore, we can model each element of the string as a simple harmonic oscillator, with the oscillation in the y direction. All elements have the same angular frequency ω and the same amplitude A.

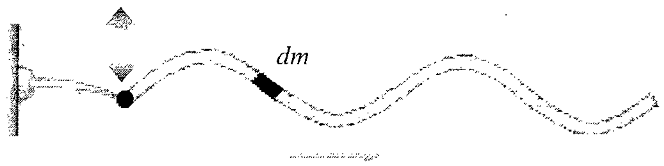

Fig. 5.28 A sinusoidal wave travelling along the x-axis on a stretched string. Every element, such as the one labelled with its mass dm, moves vertically. The average power transmitted by the wave equals the energy contained in one wavelength divided by the period of the wave

The kinetic energy K associated with a moving particle is $K = 1/2mv^2$. If we apply this equation to the infinitesimal element, the kinetic energy dK of this element is

$$dK = \frac{1}{2}(dm)v_y^2$$

where v_y is the transverse speed of the element. If μ is the mass per unit length of the string, the mass dm of the element of length dx is equal to $\mu\, dx$.

Substituting for the general transverse speed of a simple harmonic oscillator

$$dK = \frac{1}{2}\mu[-\omega A \cos(kx-\omega t)]^2 dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx-\omega t)dx$$

If we take a snapshot of the wave at time $t = 0$, the kinetic energy of a given element is

$$dK = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx)dx$$

Integrating this expression over all the string elements in a wavelength of the wave gives the total kinetic energy K_λ in one wavelength.

$$K_\lambda = \int dK = \int_0^\lambda \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx)dx = \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \cos^2(kx)dx$$

$$= \frac{1}{2}\mu\omega^2 A^2\left[\frac{1}{2}x + \frac{1}{4k}\sin 2kx\right]_0^\lambda = \frac{1}{2}\mu\omega^2 A^2\left[\frac{1}{2}\lambda\right] = \frac{1}{4}\mu\omega^2 A^2\lambda$$

In addition to kinetic energy, there is potential energy associated with each element of the string due to its displacement from the equilibrium position and the restoring forces from neighbouring elements. A similar analysis to that above for the total potential energy U_λ in one wavelength gives exactly the same result

$$U_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda \qquad \text{(xxiii)}$$

As the wave moves along the string, this amount of energy passes through a point on the string during a time interval of one period of the oscillation. Therefore, the average power P, or rate of energy transfer T_{MW} associated with the mechanical wave, is

$$P = \frac{E_\lambda}{T} = \frac{1/2\mu\omega^2 A^2\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2\left(\frac{\lambda}{T}\right)$$

$$P = \frac{1}{2}\mu\omega^2 A^2 v \qquad \text{(xxiv)}$$

Power of a Wave

Equation (xxiv) shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to (a) the square of the frequency, (b) the square of the amplitude and (c) the wave speed. In fact, the rate of energy transfer in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.

Intensity of the Wave

Flow of energy per unit area of cross section of the string in the unit time is known as the intensity of the wave. Thus,

$$I = \frac{\text{power}}{\text{area of cross section}} = \frac{(1/2)\mu\omega^2 A^2 v}{s} = \frac{(1/2)\rho s\omega^2 A^2 v}{s}$$

$$I = \frac{1}{2}\rho\omega^2 A^2 v$$

This is the average intensity transmitted through the string.

Illustration 5.22 **Power Supplied to a Vibrating String:** **A string with linear mass density $\mu = 5.00 \times 10^{-2}$ kg m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?**

Sol. The wave speed on the string is

$$v = \sqrt{\frac{T}{\mu}} = \left(\frac{80.0\,\text{N}}{5.00\times10^{-2}\,\text{kg/m}}\right)^{1/2} = 40.0 \text{ m/s}$$

Because $f = 60$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$$

Using these values in Eq. (iii) for the power, with amplitude $A = 6.00 \times 10^{-2}$ m, gives

$$p = \frac{1}{2}\mu\omega^2 A^2 v$$

$$= \frac{1}{2}(5.00\times10^{-2}\text{ kg/m})(377\text{ s}^{-1})^2$$

$$\times (6.00\times10^{-2}\text{ m})^2(40.0\text{ m/s}) = 512\text{ W}$$

Illustration 5.23 **Two waves in the same medium are represented by y–t curves in the figure. Find the ratio of their average intensities?**

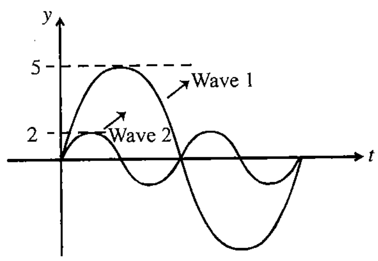

Fig. 5.29

Sol.

$$\frac{I_1}{I_2}=\frac{\omega_1^2A_1^2}{\omega_2^2A_2^2}=\frac{f_1^2\times A_1^2}{f_2^2\times A_2^2}=\frac{1\times 25}{4\times 4}=\frac{25}{16}$$

Illustration 5.24 **Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density of 4.00 × 10⁻² kg-m. The source can deliver a maximum power of 300 W and the string is under a tension of 100 N. What is the highest frequency at which the source can operate?**

Sol. Turning up the source, frequency will increase the power carried by the wave. Perhaps on the order of a hundred hertz will be the frequency at which the energy per second is 300 J/s.

We will use the expression for power carried by a wave on a string.

The wave speed is

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{100\text{ N}}{4.00\times10^{-2}\text{ kg/m}}}=50.0\text{ m/s}$$

From $P=\frac{1}{2}\mu\omega^2A^2v$, we have

$$\omega^2=\frac{2P}{\mu A^2v}=\frac{2(3.00\text{ N m/s})}{(4.00\times10^{-2}\text{ kg/m})(5.00\times10^{-2}\text{ m})^2(50.0\text{ m/s})}$$

Computing, $\omega=346.4$ rad/s and $f=\frac{\omega}{2\pi}=55.1$ Hz

This string wave would softly broadcast sound into the surrounding air, at the frequency of the second-lowest note called A on piano. If we tried to turn the source to a higher frequency, it might just vibrate with smaller amplitude. The power is generally proportional to the squares of both the frequency and the amplitude.

Illustration 5.25 **A sinusoidal wave on a string is described by the wave function**

$$y=(0.15\text{ m})\sin(0.80x-50t)$$

where x and y are in metres and t is in seconds. The mass per unit length of this string is 12.0 g-m. Determine (a) the speed of the wave, (b) the wavelength, (c) the frequency and (d) the power transmitted to the wave.

Sol. Comparing the given wave function,

$$y=(0.15\text{ m})\sin(0.80x-50t)$$

with the general wave function,

$$y=A\sin(kx-\omega t)$$

we have $k=0.80$ rad/m and $\omega=50$ rad/s and $A=0.15$ m

a. The wave speed is then

$$v=f\lambda=\frac{\omega}{k}=\frac{50.0\text{ rad/s}}{0.80\text{ rad/m}}=62.5\text{ m/s}$$

b. The wavelength is

$$\lambda=\frac{2\pi}{k}=\frac{2\pi\text{ rad}}{0.80\text{ rad/m}}=7.85\text{ m}$$

c. The frequency is

$$f=\frac{\omega}{2\pi}=\frac{50\text{ rad/s}}{2\pi\text{ rad}}=7.96\text{ Hz}$$

d. The wave carries power

$$P=\frac{1}{2}\mu\omega^2A^2v$$
$$=\frac{1}{2}(0.0120\text{ kg/m})(50.0\text{ s}^{-1})^2(0.150\text{ m})^2(62.5\text{ m/s})$$
$$=21.1\text{ W}$$

INTERPRETATION OF dy/dx IN LONGITUDINAL WAVES AND TRANSVERSE WAVE

Consider a longitudinal wave travelling along the positive x-axis. Let us choose two layers of medium PQ and RS with planes normal to the direction of wave propagation, at distance x and $x+dx$ from the origin O. After the wave passes through these sections, the layers get displaced from their mean positions, say by y and $y+dy$ respectively. (as shown in Fig. 5.30) if 'a' be the cross-sectional area of the layer, then, the initial volume of the cylinder $PQRS=a\,(dx)$

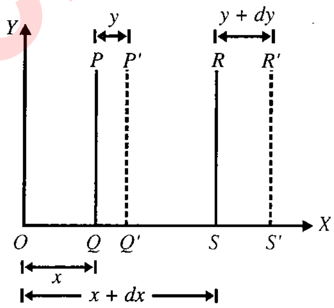

Fig. 5.30

and final volume $=$ volume of cylinder $P'Q'R'S'$

$= a\,[x+dx+y+dy)-(x+y)]$

$= a\,(dx+dy)$

$\therefore$ Change in volume $= a\,dy$

$\therefore$ Volume strain $=\dfrac{\text{Change in volume}}{\text{Original volume}}=\dfrac{a\,dy}{a\,dx}=\dfrac{dy}{dx}$

Hence, In case of longitudinal waves dy/dx represents the volume strain.

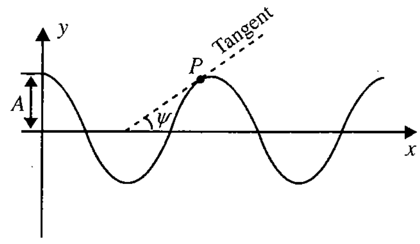

Fig. 5.31

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In case of transverse waves, the term dy/dx has its usual geometrical meaning. If a 'snapshot' be taken, of the particles of a medium at any instant, in which a simple harmonic wave is travelling, then the shape of the waveform will be a sine curve (as shown in Fig. 5.31) dy/dx, at the instant for any particle say P) represents the slope of the curve at that point. Thus, if ψ is be the angle made by the geometrical tangent drawn to the curve at that point, with the positive direction of the x-axis, then $dy/dx = \tan\psi$.

Concept Application Exercise 5.2

1. A transverse wave is travelling along a string in the positive x-axis. The figure shows the photograph of the wave at an instant. Find

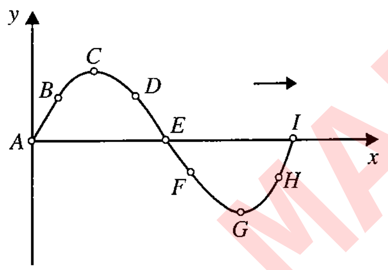

Fig. 5.32

(a) the points moving upward ________
(b) the points moving downwards ________
(c) the points which have zero velocity ____
(d) the points which have maximum velocity ____

2. Does the wave function $y = A_0\cos^2(2\pi f_0 t - 2\pi x/\lambda_0)$ represent a wave? If yes, then determine its amplitude, frequency, and wavelength.

3. Does the equation

$$y = +\sqrt{16-(2x-t)^2},$$
$$(2x-t) < 4 \quad \text{or} \quad (2x-t) > -4;$$
$$= 0, \text{ otherwise}$$

represent a wave? If yes, then find the amplitude and the phase velocity.

4. The equation of a travelling wave is given by

$$y = +\frac{b}{a}\sqrt{a^2-(x-ct)^2} \quad \text{where} \quad -a < x < a$$
$$= 0, \text{ otherwise}$$

Find the amplitude and the wave velocity for the wave. What is the initial particle velocity at the position $x = a/2$?

5. Does a travelling wave in one dimension represented by a function of a linear combination of x and t, i.e., $y = f(ax \pm bt)$ represents a travelling wave?

6. A wave is propagating along the length of a string taken as positive x-axis. The wave equation is given by

$$y = Ae^{-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2}$$

where $A = 5$ mm, $T = 1.0$ s and $\lambda = 8.0$ cm
(a) Find the velocity of the wave.
(b) Find the function $f(t)$ representing the displacement of particle at $x = 0$.
(c) Find the function $g(x)$ representing the shape of the string at $t = 0$
(d) Plot the function $g(x)$ of the string at $t = 0$ and $t = 5$ s

7. The equation of transverse wave travelling on a rope is given by

$$y = 5\sin(4.0t - 0.02x)$$

where y and x are in centimetres and time t in seconds
(i) Calculate the amplitude, frequency, velocity, and wavelength.
(ii) Calculate the maximum transverse speed of a particle in the rope.

8. A uniform wire passes over two pulleys and 9 kg weight is suspended at each end. The length of wire between the pulleys is 1.5 m and its mass is 12.0 g. Find the frequency of vibration with which the wire vibrates in two loops, the middle point of wire being at rest.

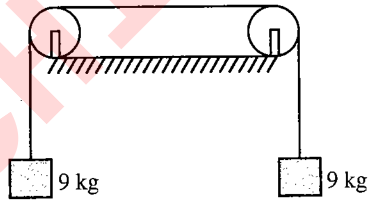

Fig. 5.33

9. The following equation gives the displacement y at time t for a particle at a distance x:

$$y = 0.01\sin 500\pi(t - x/30)$$

where all are in SI units.
Find (i) the wavelength, (ii) the speed of the wave, (iii) the velocity amplitude of the particles of the medium and (iv) the acceleration amplitude of the particles of the medium.

10. The equation of a travelling plane sound wave has the form $y = 60\cos(1800t - 5.3x)$, where y is expressed in micrometres, t is in seconds and x in mm. Find (a) the ratio of the displacement amplitude to the wavelength, (b) the velocity oscillation amplitude of particles of the medium and its ratio to the wave propagation velocity.

11. Spherical waves are emitted from a 1.00-W source in an isotropic non-absorbing medium. What is the wave intensity 1.0 m from the source?

12. A wave travels out in all directions from a point source. Justify the expression $y = (a_0/r)\sin k(r - vt)$, at a distance r from the source. Find the speed, periodicity and intensity of the wave. What are the dimensions of a_0?

13. For plane waves in the air of frequency 1000 Hz and the displacement amplitude 2×10^{-8} m, deduce (i) the velocity amplitude, and (ii) the intensity. (Take ρ = 1.3 kg/m^3, c = 340 m/s)

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14. A heavy uniform rope is held vertically and is tensioned by clamping it to a rigid support at the lower end. A wave of a certain frequency is set up at the lower end. Will the wave travel up the rope with the same speed?

15. What is the phase difference between the particles 1 and 2 located as shown in figure?

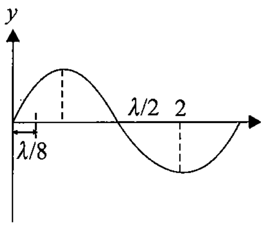

Fig. 5.34

16. Show that (i) $y = (x + vt)^2$, (ii) $y = (x + t)^2$, (iii) $y = (x - vt)^2$, and (iv) $y = 2 \sin x \cos vt$ are each a solution of one-dimensional wave equation but not (v) $y = x^2 - v^2t^2$ and (vi) $y = \sin 2x \cos vt$.

17. For a travelling harmonic wave $y = 2.0 \cos (10t - 0.0080x + 0.35)$, where x and y are in centimetres and t in seconds. What is the phase difference between oscillatory motion of two points separated by a distance of (i) 4 cm (ii) 0.5 cm (iii) $\lambda/2$ (iv) $3\lambda/4$?

18. The figure shows two snapshots, each of a wave travelling along a particular string. The phase for the waves are given by

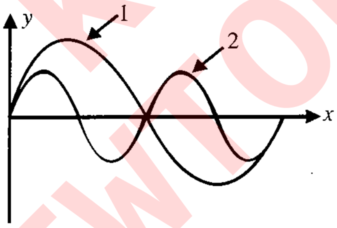

Fig. 5.35

(a) $4x - 8t$
(b) $8x - 16t$. Which phase corresponds to which wave in the figure?

19. A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?

20. A sinusoidal wave is propagating along a stretched string that lies along the x-axis. The displacement of the string as a function of time is graphed in figure for particles at $x = 0$ and at $x = 0.0900$ m. (a) What is the amplitude of the wave? (b) What is the period of the wave? (c) You are told that the two points $x = 0$ and $x = 0.0900$ m are within one wavelength of each other. If the wave is moving in the $+x$-direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the $-x$-direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitely the wavelength in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

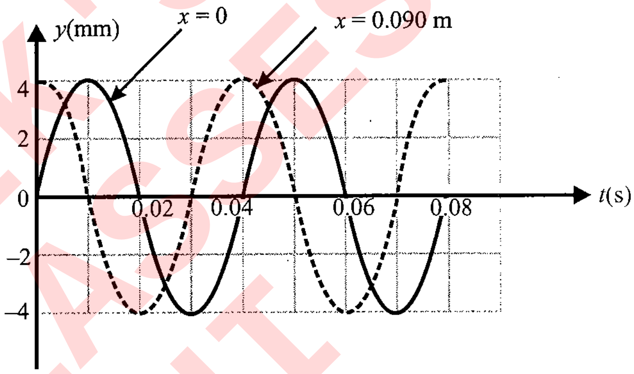

Fig. 5.36

21. A simple harmonic oscillator at the point $x = 0$ generates a wave on a rope. The oscillator operates at a frequency of 40.0 Hz and with an amplitude of 3.00 cm. The rope has a linear mass density of 50.0 g/m and is stretched with a tension of 5.00 N. (a) Determine the speed of the wave. (b) Find the wavelength. (c) Write the wave function $y(x, t)$ for the wave. Assume that the oscillator has its maximum upward displacement at time $t = 0$. (d) Find the maximum transverse acceleration of points on the rope. (e) In the discussion of transverse waves in this chapter, the force of gravity was ignored. Is that a reasonable assumption for this wave? Explain.

22. A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

23. A wave on a string is described by $y(x, t) = A \cos (kx - \omega t)$. (a) Graph y, v_y, and a_y as functions of x for time $t = 0$. (b) Consider the following points on the string: (i) $x = 0$; (ii) $x = \pi/4k$; (iii) $x = \pi/2k$; (iv) $x = 3\pi/4k$; (v) $x = \pi/k$; (vi) $x = 5\pi/4k$; (vii) $x = 3\pi/2k$; (viii) $x = 7\pi/4k$. For a particle at each of these points at $t = 0$, described in words whether the particle is moving and in what direction, and whether the particle is speeding up, slowing down, or instantaneously not accelerating.

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Solved Examples

Example 5.1 **A transverse mechanical harmonic wave is travelling on a string. Maximum velocity and maximum acceleration of a particle on the string are 3 m-s and 90 m-s^2, respectively. If the wave is travelling with a speed of 20 m-s on the string, write wave function describing the wave.**

Sol. Maximum particle velocity,

$$u_{max} = \omega A \quad \text{(i)}$$

maximum particle acceleration,

$$a_{max} = \omega^2 A \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$\therefore \text{Angular frequency, } \omega = \frac{a_{max}}{u_{max}} = \frac{90}{3} = 30 \text{ rad/s}$$

$$\text{From Eq. (i), amplitude, } A = \frac{u_{max}}{\omega} = \frac{3}{30} = 0.1 \text{ m}$$

$$\text{Propagation constant, } k = \frac{\omega}{v} = \frac{30}{20} = 1.5 \text{ m}^{-1}$$

Equation of wave is $y = A \sin(\omega t \pm kx)$
or wave function $y = 0.1 \sin(30t \pm 1.5x)$
where x is in metres and t in seconds.

Positive sign is for wave propagation along negative x-axis and negative sign for wave propagating along positive x-axis.

Example 5.2 **The equation of a travelling plane sound wave has the form $y = 60 \cos(1800t - 5.3x)$, where y is in micrometres, t in seconds and x in metres. Find**

(i) the ratio of the displacement amplitude with which the particles of the medium oscillate to the wavelength,
(ii) the velocity oscillation amplitude of particles of the medium and its ratio to the wave propagation velocity,
(iii) the oscillation amplitude of relative deformation of the medium and its relation to the velocity oscillation amplitude of particles of the medium,
(iv) the particle acceleration amplitude.

Sol. Comparing the given equation $y = 60\cos(1800t - 5.3x)$ with the wave equation $y = a\cos(\omega t - kx)$;

$\omega = 1800$ rad/s, $a = 60 \times 10^{-6}$ m, $k = 5.3$ per metre.

$$\text{As} \quad k = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{5.3} \text{ m}$$

i. Displacement amplitude of particles of the medium = Amplitude of the wave = 60×10^{-6} m.

$$\text{Required Ratio} = \frac{\text{Displacement Amplitude}}{\text{Wavelength}}$$

$$= \frac{60 \times 10^{-6} \text{ m}}{\left(\frac{2\pi}{5.3}\right) \text{m}}$$

$$= \frac{60}{2\pi} \times 10^{-6} \times 5.3 = 5.06 \times 10^{-5}$$

ii. Velocity oscillation amplitude = Magnitude of the velocity of a particle of medium when passes through its equilibrium position $= a\omega$.
Velocity of propagation of the wave

$$= f\lambda = \frac{\omega}{2\pi} \times \lambda = \frac{\omega}{\left(\frac{2\pi}{\lambda}\right)} = \frac{\omega}{k}$$

Required ratio

$$\frac{a\omega}{\left(\frac{\omega}{K}\right)} = ka = 5.3 \times 60 \times 10^{-6} = 3.18 \times 10^{-4}$$

iii. Deriving expression for relative deformation :
Let at $x = x$ at any instant $t = t$, displacement of particle is $y = y$.
and at $x = x + dx$ displacement of particle be $y = y + dy$.
(The displacement verses x graph at time t is continuous.)
Relative deformation, which is given as change in deformation with change in $x = dy/dx$ at an instant of time

$$\left(\frac{dy}{dx}\right)_t = -60\sin(1800t - 5.3x)(-5.3 \times 10^{-6})$$

$$= 318 \times 10^{-6} \sin(1800t - 5.3x)$$

For maximum relative deformation

$$\left(\frac{dy}{dx}\right)_{max} = 318 \times 10^{-6}$$

One should note that $(dy/dx)_t$ is nothing but the slope of the curve at (x, y).

$$\text{Now,} \quad \left(\frac{dy}{dx}\right)_t = \left(\frac{dy}{dt} \times \frac{dt}{dx}\right), \left(\frac{dy}{dt}\right)_t = \left(\frac{dy}{dx} \times \frac{dx}{dt}\right)$$

$$= \frac{dy}{dx} \times \text{wave velocity}$$

dx/dt = velocity of the wave = constant

$$\Rightarrow \left[\frac{dy}{dt}\right]_{max} = \left[\frac{dy}{dx}\right]_{max} \times \text{velocity of wave}$$

velocity oscillation amplitude = velocity of the wave × oscillation amplitude of relative deformation

iv. Velocity of particle $dy/dt = a\omega\cos(\omega t - kx)$
Acceleration of the particle

$$\frac{d^2y}{dt^2} = a\omega \times [-\sin(\omega t - kx)] \times \omega = -a\omega^2 \sin(\omega t - kx)$$

Maximum value of $|\sin(\omega t - kx)| = 1$

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Thus maximum acceleration of the particle = particle acceleration amplitude $= a\omega^2 = 60 \times 10^{-6}\,(1800)^2$
$= 194.4\ \text{m/s}^2$

Example 5.3 **One end of each of two identical springs, each of force constant 0.5 N/m are attached on the opposite sides of wooden block of mass 0.01 kg. The other ends of the springs are connected to separate rigid supports such that the springs are unstretched and are collinear in a horizontal plane. To the wooden piece is fixed a pointer which touches a vertically moving plane paper. The wooden piece kept on a smooth horizontal table is now displaced by 0.02 m along the line of springs and released. If the speed of the paper, perpendicular to the springs' length, is 0.1 m s, find the equation of the path traced by the pointer on the paper and the distance between two consecutive maxima on this path.**

Sol. The effective force constant of the spring system is $2k$ (since they constitute a parallel combination). The angular frequency of simple harmonic oscillation is

$$\omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 0.5}{0.01}} = 10\ \text{rad/s}$$

The amplitude $A = 0.02$ m

The speed of the paper may be assumed as the speed of wave-propagators and the curve traced on the paper can be represented by the equation

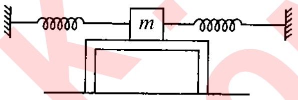

Fig. 5.37

$$y = A\sin(\omega t - kx) \qquad \text{(i)}$$

The wavelength

$$\lambda = \frac{v}{f} = \frac{v}{\omega/2\pi} = \frac{2\pi v}{\omega} = \frac{2\pi\times 0.1}{10} = \frac{\pi}{50}\,\text{m} \qquad \text{(ii)}$$

$$\therefore \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \frac{10}{0.1} = 100$$

Substituting the value in (i), we get the required equation of the path

$$y = A\sin kx = 0.02\sin(100x)$$

Example 5.4 **A wave pulse starts propagating in the +x-direction along a non-uniform wire of length 10 m with mass per unit length given by $\mu = \mu_0 + ax$ and under a tension of 100 N. Find the time taken by a pulse to travel from the lighter end ($x = 0$) to the heavier end. ($\mu_0 = 10^{-2}$ kg/m and $a = 9\times 10^{-3}$ kg/m²).**

Sol. The speed of the wave pulse

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\mu_0 + ax}} = \frac{dx}{dt}$$

$$\therefore t = \int_0^t dt = \int_0^L \sqrt{\frac{\mu_0 + ax}{T}}\,dx = \frac{2}{3}\frac{1}{\alpha\sqrt{T}}[(\mu_0 + aL)^{3/2} - (\mu_0)^{3/2}]$$

substituting the values, we get

$$t = \frac{2}{3}\times\frac{1}{9\times 10^{-3}\times\sqrt{100}}[(10^{-2} + 9\times 10^{-3}\times 10^{-2})^{3/2} - (10^{-2})^{3/2}]$$

$$= \frac{2\times 100}{27}[(10)^{-3/2} - (10)^{-3}] = 0.227\ \text{s}$$

Example 5.5 **When a train of plane wave traverses a medium, individual particles execute periodic motion given by the equation**

$$y = 4\sin\frac{\pi}{2}\left(2t + \frac{x}{8}\right)$$

where the lengths are expressed in centimetres and time in seconds. Calculate the amplitude, wavelength, (i) the phase difference for two positions of the same particle which are occupied at time interval 0.4 s apart and (ii) the phase difference at any given instant of two particles 12 cm apart.

Sol. The equation of a wave motion is given by

$$y = A\sin\frac{2\pi}{\lambda}(vt + x) \qquad \text{(i)}$$

Here,

$$y = 4\sin\frac{\pi}{2}\left(2t + \frac{x}{8}\right)$$

This equation can be written as

$$y = 4\sin\frac{2\pi}{32}(16t + x) \qquad \text{(ii)}$$

Comparing Eq. (i) with Eq. (ii), we get

Amplitude $A = 4$ cm; wavelength $\lambda = 32$ cm; wave velocity $v = 16$ cm/s

Here frequency is given as

$$f = \frac{v}{\lambda} = \frac{16}{32} = \frac{1}{2} = 0.5\ \text{Hz}$$

i. Phase of a particle at instant t_1 is given by

$$\phi_1 = \frac{\pi}{2}\left(2t_1 + \frac{x}{8}\right)$$

The phase at instant t_2 is given by

$$\phi_2 = \frac{\pi}{2}\left(2t_2 + \frac{x}{8}\right)$$

The phase difference is given as

$$\phi_1 - \phi_2 = \frac{\pi}{2}\left[\left(2t_1 + \frac{x}{8}\right) - \left(2t_2 + \frac{x}{8}\right)\right]$$

$$= \pi(t_1 - t_2) = \pi(0.4) \qquad (\text{As } t_1 - t_2 = 0.4)$$

$$= 180\times 0.4 = 72° \qquad (\pi\ \text{rad} = 180°)$$

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ii. Phase difference at an instant between two particles with path difference Δ is

$$\phi = \frac{2\pi}{\lambda}\times\Delta \quad = \frac{2\pi}{32}\times 12 \qquad (\text{As } \Delta = 12 \text{ cm})$$

$$= \frac{3\pi}{4}$$

Example 5.6 **How long will it take sound waves to travel the distance l between the points A and B if the air temperature between them varies linearly from T_1 to T_2? The velocity of sound propagation in air is equal to $v = \alpha\sqrt{T}$, where α is a constant.**

Sol. For linear variation of temperature, we can write temperature at a distance x from point A is

$$T_x = T_1 + \frac{T_2 - T_1}{l}x$$

Thus velocity of sound at this point is given as

$$v = \alpha\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x}$$

$$\frac{dx}{dt} = \alpha\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x} \Rightarrow \frac{dx}{\alpha\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x}} = dt$$

Integrating the above expression within proper limits, we get

$$\int_0^l \frac{dx}{\alpha\sqrt{T_1 + \left(\frac{T_2 - T_1}{l}\right)x}} = \int_0^t dt$$

$$\frac{2l}{\alpha(T_2 - T_1)}\left[\sqrt{T_1 + \frac{T_2 - T_1}{l}x}\right]_0^l = t$$

$$\Rightarrow \quad t = \frac{2l}{\alpha(T_2 - T_1)}\left[\sqrt{T_2} - \sqrt{T_1}\right]$$

$$t = \frac{2l}{\alpha\left[\sqrt{T_2} + \sqrt{T_1}\right]}$$

Example 5.7 **A wave of frequency f = 1000 Hz, propagates at a velocity v = 700 m s along the x-axis.**

i. What is the wavelength of the wave?

ii. Find all the points at a given time, at which the phase of the wave exceeds the phase at the origin by $\pi/3$ radian.

iii. Find the phase gained at a given point x during a time interval $\Delta t = 0.5\times 10^{-3}$ s.

Sol.

i. The function form of the wave is

$$\varphi = A\sin(kx - \omega t) \qquad \text{(i)}$$

where $\omega = 2\pi f = 2000\,\pi$ Hz and $k = \frac{2\pi}{\lambda}$, as λ is given by

$$\lambda = \frac{v}{f} = \frac{700\text{ m/s}}{1000\text{ Hz}} = 0.7\text{ m}$$

ii. Let the plane at x_0 be shifted by $\pi/3$ relative to the phase at x = 0. Equation (i) then yields

$$A\sin\left(\frac{\pi}{3} - \omega t\right) = A\sin\left(\frac{2\pi}{0.7\text{m}}x_0 - \omega t\right) \qquad \text{(ii)}$$

Hence,

$$-\omega t + \frac{\pi}{3} + 2\pi n = \frac{2\pi}{0.7\text{ m}}x_0 - \omega t,\ n = 0, \pm 1, \ldots$$

That is

$$x_0 = \left(\frac{0.7}{6} + 0.7n\right)\text{m} \approx (0.11 + 0.7n)\text{m}, n = 0, \pm 1, \ldots \qquad \text{(iii)}$$

iii. The phase at a point x, at a moment t, is given by

$$\phi(x,t) = kx - \omega t$$

$$\Delta\phi = -\omega\Delta t \qquad \text{(iv)}$$

$$\Rightarrow \Delta\phi = -2\pi\times 1000\text{ Hz}\times 0.5\times 10^{-3}\text{ s}$$

i.e., $\quad \Delta\phi = -\pi$ rad

which corresponds to one half cycle

Example 5.8 **The equation of a progressive wave is given by $y = 0.20\sin 2\pi(60t - x/5)$ where x and y are in metres and t is in seconds. Find the phase difference (i) between two particles separated by a distance of Δx=125 cm, and (ii) between the two instants 1/120 s and 1/40 s, for any particle.**

Sol. The given equation $y = 0.20\sin 2\pi(60t - x/5)$ when compared with the standard one, $y = A\sin 2\pi(ft - x/\lambda)$ where the symbols have their usual meaning. We get the frequency f as 60 s^{-1}, so that the time period $T = 1/60$ s. Also, the wavelength λ = 5 m.

i. Phase difference

$$\Delta\phi = \left(\frac{\Delta x}{\lambda}\right)2\pi = \left(\frac{125\text{ cm}}{500\text{ cm}}\right)2\pi = \frac{\pi}{2}$$

ii. Phase difference

$$\Delta\phi = \left(\frac{\Delta t}{T}\right)2\pi = \frac{\left(\frac{1}{40} - \frac{1}{120}\right)\text{s}}{\frac{1}{60}\text{s}}\times 2\pi = 2\pi$$

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Example 5.9 **Figure 5.38 shows a snapshot of a sinusoidal travelling wave taken at $t = 0.3$ s. The wavelength is 7.5 cm and amplitude is 2 cm. If the crest P was at $x = 0$ at $t = 0$, write the equation of travelling wave.**

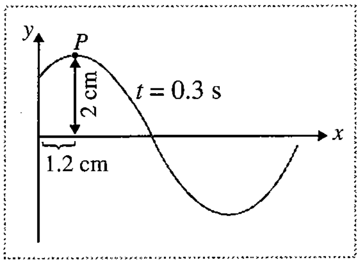

Fig. 5.38

Sol: Given, $A = 2$ cm, $\lambda = 7.5$ cm

$$k = \frac{2\pi}{\lambda} = \frac{4\pi}{15}\ \text{cm}^{-1}$$

The wave has travelled a distance of 1.2 cm in 0.3 s. Hence, speed of the wave, $v = 1.2/0.3 = 4$ cm/s

Therefore, angular frequency $\omega = (v)(k)$

$$= \frac{16\pi}{15}\ \text{rad/s}$$

Since the wave is travelling along positive x-direction and crest (maximum displacement) is at $x = 0$ at $t = 0$, we can write the wave equation as,

$$y(x, t) = 2\cos(kx - \omega t)$$

$$= 2\cos\left(\frac{4\pi}{15}x - \frac{16\pi}{15}t\right)$$

EXERCISES

Subjective Type

Solution on page 5.40

1. A 100 Hz sinusoidal wave is travelling in the positive x-direction along a string with a linear mass density of 3.5×10^{-3} kg/m and a tension of 35 N. At time $t = 0$, the point $x = 0$ has maximum displacement in the positive y-direction. Next, when this point has zero displacement the slope of the string is $\pi/20$. Find (i) amplitude (ii) wave speed (iii) the expression which represents the displacement of string as a function of x (in metres) and t (in seconds).
2. One end of a long string of linear mass density 10^{-2} kg m^{-1} is connected to an electrically driven tuning fork of frequency 150 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves from this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string is at $x = 0$ has a transverse displacement of 2.5 cm and is moving along positive y-direction. The amplitude of the wave is 5 cm. Write down the transverse displacement y (in centimetres) as function of x (in metres) and t (in seconds) that describes the wave on the string.

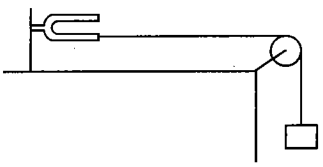

Fig. 5.39

3. An harmonic wave has been set up on a very long string which travels along the length of the string. The wave has a frequency of 50 Hz, amplitude 1 cm and wavelength 0.5 m. Find
   a. the time taken by the wave to travel a distance of 8 m along the length of string
   b. the time taken by a point on the string to travel a distance of 8 m, once the wave has reached the point and sets it into motion
   c. also, consider the above case when the amplitude gets doubled
4. A harmonic wave is travelling in a stationary medium whose equation is given by $y = A\sin(\omega t - kx)$. Find the equation of this wave w.r.t. a frame which is moving along $-$ve x-axis with a constant speed v w.r.t. stationary medium. Also, find speed of wave in moving frame.
5. A plane undamped harmonic wave propagates in a medium. Find the mean space density of total energy, if at any point, the space density of energy becomes equal to W_0 at an instant $t = t_0 + T/6$, where t_0 is the instant when amplitude is maximum at this location and T is the time period of oscillation.
6. A steel wire has a mass of 50 g/m and is under tension 450 N.
   i. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.
   ii. The change in maximum average power if the mass per unit length of the wire is doubled. (Use $\pi^2 \cong 10$.)
7. A wave is propagating on a long stretched string along its length taken as the positive x-axis. The wave equation is given as $y = y_0 e^{-\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2}$ where $y_0 = 4$ mm, $T = 1.0$ s and $\lambda = 4$ cm.
   a. Find the velocity of the wave.
   b. Find the function $f(t)$ giving the displacement of the particle at $x = 0$.
   c. Find the function $g(x)$ giving the shape of the string at $t = 0$.
   d. Plot the shape $g(x)$ of the string at $t = 0$.
   e. Plot the shape of the string at $t = 5$ s.

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8. A travelling wave pulse is given by

$$y = \frac{10}{5+(x+2t)^2}$$

In which direction and with what velocity is the pulse propagating? What is the amplitude of the pulse ?

9. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 19.2×10^{-3} kg/m. Find the speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerate up at the rate of 2.0 m/s^2.

10. The speed of a transverse wave, going on a wire having a length 50 cm and mass 5.0 g is 80 m/s. The area of cross section of the wire is 1.0 mm^2 and its Young's modulus is 1.6×10^{11}. Find the extension of the wire over its natural length.

11. Two blocks each having a mass of 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB. The linear mass density of the wire AB is 10 g/m and that of CD is 8 g/m. Find the speed of a transverse wave pulse produced in AB and CD.

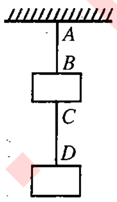

Fig. 5.40

12. A transverse harmonic wave of amplitude 0.01 m is generated at one end (x = 0) of a long horizontal string by a tuning fork of frequency 500 Hz. At a given instant of time the displacement of the particle at x = 0.1 m is –0.005 m and that of the particle at x = 0.2 m is + 0.005 m. Calculate the wavelength and the wave velocity. Obtain the equation of the wave assuming that the wave is travelling along the + x-direction and that the end x = 0 is at the equilibrium position at t = 0.

13. Figure 5.41 shows the position of a medium particles at t = 0, supporting a simple harmonic wave travelling either along or opposite to the positive x-axis.

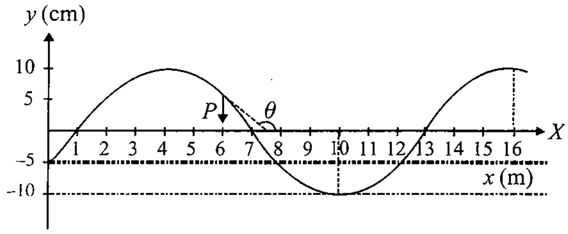

Fig. 5.41

a. Write down the equation of the curve.
b. Find the angle 'θ' made by the tangent at point P with the x-axis.
c. If the particle at P has a velocity v_p m/s, in the negative y-direction, as shown in figure, then determine the speed and direction of the wave.
d. Find the frequency of the waves.
e. Find the displacement equation of the particle at the origin as a function of time.
f. Find the displacement equation of the wave.

14. Figure 5.42 shows two snapshots of medium particles within a time interval of 1/60 s. Find the possible time periods of the wave

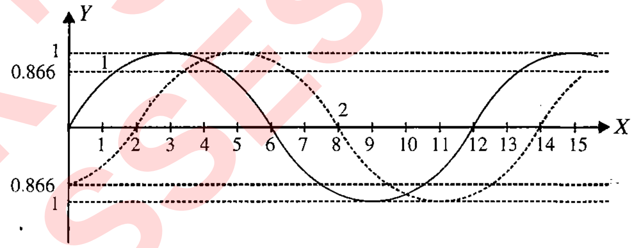

Fig. 5.42

15. The equation of a progressive wave travelling along a string is given by

$$y = 10 \sin\pi\,(0.01x - 2.00t)$$

where x and y are in centimetres and t in seconds.
Find the (i) velocity of a particle at x = 2 m and t = 5/6 s. (ii) acceleration of a particle at x = 1 m and t = 1/4 s. Also, find the velocity amplitude and acceleration amplitude for the wave.

16. A travelling wave pulse is given by

$$y = \frac{0.8}{(3x^2+12xt+12t^2+4)}$$

where x and y are in m and t is in s. Find the velocity and amplitude of the wave.

Objective Type

Solution on page 5.44

1. The speed of a wave in a certain medium is 960 m/s. If 3600 waves pass over a certain point of the medium in 1 min, the wavelength is
   a. 2 m b. 4 m
   c. 8 m d. 16 m

2. A simple harmonic progressive wave is represented by the equation y = 8 sin 2π (0.1x – 2t) where x and y are in centimetres and t is in seconds. At any instant the phase difference between two particles separated by 2.0 cm along the x-direction is
   a. 18° b. 36° c. 54° d. 72°

3. The equation of a transverse wave travelling on a rope is given by y = 10 sin π(0.01x – 2.00t) where y and x are in centimetres and t in seconds. The maximum transverse speed of a particle in the rope is about
   a. 63 cm/s b. 75 cm/s c. 100 cm/s d. 121 cm/s

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4. A wave is represented by the equation

$$y = 7\sin\left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$$

x is in metres and t is in seconds. The speed of the wave is

a. 175 m/s b. 49π m/s
c. $49/\pi$ m/s d. 0.28π m/s

5. The path difference between the two waves

$$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \text{ and } y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right) \text{ is}$$

a. $\frac{\lambda}{2\pi}\phi$ b. $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$
c. $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ d. $\frac{2\pi}{\lambda}(\phi)$

6. A transverse wave is described by the equation

$$Y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

The maximum particle velocity is four times the wave velocity if

a. $\lambda = \frac{\pi y_0}{4}$ b. $\lambda = \frac{\pi y_0}{2}$
c. $\lambda = \pi y_0$ d. $\lambda = 2\pi y_0$

7. The equation of a wave travelling on a string is

$$y = 4\sin\frac{\pi}{2}\left(8t - \frac{x}{8}\right)$$

if x and y are in centimetres, then velocity of wave is

a. 64 cm/s in –ve x-direction
b. 32 cm/s in –ve x-direction
c. 32 cm/s in +ve x-direction
d. 64 cm/s in +ve x-direction

8. If $x = a\sin[\omega t + \pi/6]$ and $x' = a\cos\omega t$, then what is the phase difference between the two waves?

a. $\frac{\pi}{3}$ b. $\frac{\pi}{6}$
c. $\frac{\pi}{2}$ d. π

9. A simple harmonic wave is represented by the relation

$$y(x,t) = a_0 \sin 2\pi\left(vt - \frac{x}{\lambda}\right)$$

If the maximum particle velocity is three times the wave velocity, the wavelength λ of the wave is

a. $\pi a_0/3$ b. $2\pi a_0/3$
c. πa_0 d. $\pi a_0/2$

10. The equation of a travelling wave is

$$y = 60\cos(1800t - 6x)$$

where y is in microns, t in seconds and x in metres. The ratio of maximum particle velocity to velocity of wave propagation is

a. 3.6 b. 3.6×10^{-6}
c. 36×10^{-11} d. 3.6×10^{-4}

11. The equation of a progressive wave is

$$y = 0.02\sin 2\pi\left[\frac{t}{0.01} - \frac{x}{0.30}\right]$$

here x and y are in metres and t is in seconds. The velocity of propagation of the wave is

a. 300 m/s b. 30 m/s
c. 400 m/s d. 40 m/s

12. The amplitude of a wave disturbance propagating in the positive y-direction is given by

$$y = \frac{1}{1+x^2} \text{ at } t = 0 \text{ and } y = \frac{1}{[1+(x-1)^2]} \text{ at } t = 2\text{s}$$

The wave speed is

a. 1 m/s b. 1.5 m/s
c. 0.5 m/s d. 2 m/s

13. A thin plane membrane separates hdrogen at 7°C from hydrogen at 47°C, both being at the same pressure. If a collimated sound beam travelling from cooler gas makes an angle of incidence of 30° at the membrane, the angle of refraction is

a. $\sin^{-1}\sqrt{\frac{7}{32}}$ b. $\sin^{-1}\sqrt{\frac{2}{7}}$
c. $\sin^{-1}\sqrt{\frac{4}{7}}$ d. $\sin^{-1}\sqrt{\frac{7}{4}}$

14. In a medium in which a transverse progressive wave is travelling, the phase difference between two points with a separation of 1.25 cm is $(\pi/4)$. If the frequency of wave is 1000 Hz. Its velocity will be

a. 10^4 m/s b. 125 m/s
c. 100 m/s d. 10 m/s

15. A plane sound wave is travelling in a medium. In reference to a frame A, its equation is $y = a\cos(\omega t - kx)$. Which reference to a frame B, moving with a constant velocity v in the direction of propagation of the wave, equation of the wave will be

a. $y = a\cos[(\omega t + kv)t - kx]$
b. $y = -a\cos[(\omega t - kv)t - kx]$
c. $y = a\cos[(\omega t - kv)t - kx]$
d. $y = a\cos[(\omega t + kv)t + kx]$

16. Small amplitude progressive wave in a stretched string has a speed of 100 cm/s, and frequency 100 Hz. The phase difference between two points 2.75 cm apart on the string, in radians, is

a. 0 b. $11\pi/2$
c. $\pi/4$ d. $3\pi/8$

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17. The linear density of a vibrating string is 10^{-4} kg/m. A transverse wave is propagating on the string, which is described by the equation $y = 0.02 \sin (x + 30\,t)$, where x and y are in metres and time t in seconds. Then tension in the string is

a. 0.09 N **b.** 0.36 N **c.** 0.9 N **d.** 3.6 N

18. Two blocks of masses 40 kg and 20 kg are connected by a wire that has a linear mass density of 1 g/m. These blocks are being pulled across horizontal frictionless floor by a horizontal force F that is applied to 20 kg block. A transverse wave travels on the wave between the blocks with a speed of 400 m/s (relative to the wire). The mass of the wire is negligible compared to the mass of the blocks. The magnitude of F is

a. 160 N **b.** 240 N **c.** 320 N **d.** 400 N

19. At $t = 0$, the shape of a travelling pulse is given by

$$y(x,0)=\frac{4\times10^{-3}}{8-(x)^2}$$

where x and y are in metres. The wave function for the travelling pulse if the velocity of propagation is 5 m/s in the x direction is given by

a. $y(x,t)=\dfrac{4\times10^{-3}}{8-(x^2-5t)}$

b. $y(x,t)=\dfrac{4\times10^{-3}}{8-(x-5t)^2}$

c. $y(x,t)=\dfrac{4\times10^{-3}}{8-(x+5t)^2}$

d. $y(x,t)=\dfrac{4\times10^{-3}}{8-(x^2+5t)}$

20. The amplitude of a wave represented by displacement equation

$$y=\frac{1}{\sqrt{a}}\sin \omega t \pm \frac{1}{\sqrt{b}}\cos \omega t$$

will be

a. $\dfrac{a+b}{ab}$ **b.** $\dfrac{\sqrt{a}+\sqrt{b}}{ab}$

c. $\dfrac{\sqrt{a}\pm\sqrt{b}}{ab}$ **d.** $\sqrt{\dfrac{a+b}{ab}}$

21. Two particles of medium disturbed by the wave propagation are at $x_1 = 0$ and $x_2 = 1$ cm. The respective displacements (in cm) of the particles can be given by the equations:

$$y_1 = 2 \sin 3\pi t, \quad y_2 = 2 \sin (3\pi t - \pi/8)$$

The wave velocity is

a. 16 cm/s **b.** 24 cm/s **c.** 12 cm/s **d.** 8 cm/s

22. The displacement *vs* time graph for two waves A and B which travel along the same string are shown in the figure. Their intensity ratio I_A/I_B is

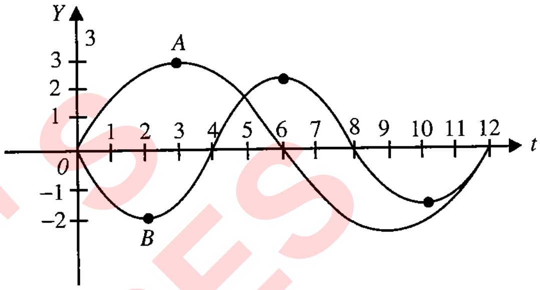

Fig. 5.43

a. $\dfrac{9}{4}$ **b.** 1 **c.** $\dfrac{81}{16}$ **d.** $\dfrac{3}{2}$

23. At $t = 0$, a transverse wave pulse travelling in the positive x direction with a speed of 2 m/s in a wire is described by the function $y = 6/x^2$ given that $x \neq 0$. Transverse velocity of a particle at $x = 2$ m and $t = 2$ s is

a. 3 m/s **b.** −3 m/s **c.** 8 m/s **d.** −8 m/s

24. Wave pulse on a string shown in figure is moving to the right without changing shape. Consider two particles at positions $x_1 = 1.5$ m and $x_2 = 2.5$ m. Their transverse velocities at the moment shown in figure are along directions

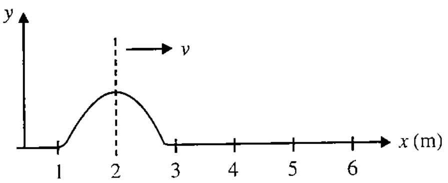

Fig. 5.44

a. positive y-axis and positive y-axis respectively
b. negative y-axis and positive y-axis respectively
c. positive y-axis and negative y-axis respectively
d. negative y-axis and negative y-axis respectively

25. A wave pulse is generated in a string that lies along x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratios of magnitudes of wave speed to the particle speed, then

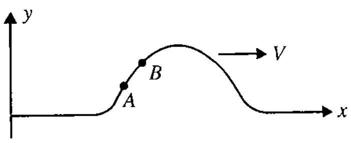

Fig. 5.45

a. $R_A > R_B$
b. $R_B > R_A$
c. $R_B = R_A$
d. Information is not sufficient

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26. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a maximum power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate it, is (take $\pi^2 = 10$)

a. 45.3 Hz **b.** 50 Hz
c. 30 Hz **d.** 62.3 Hz

27. The figure shows three progressive waves A, B and C. What can be concluded from the figure that with respect to wave A?

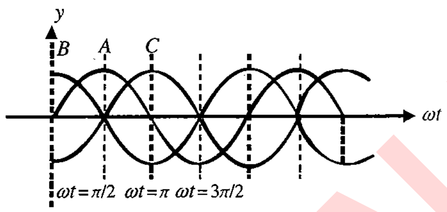


Fig. 5.46

a. The wave C is ahead by a phase angle of $\pi/2$ and the wave B lags behind by a phase angle $\pi/2$.
b. The wave C is lag behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle $\pi/2$.
c. The wave C is ahead by a phase angle of π and the wave B lags behind by a phase angle π.
d. The wave C lags behind by a phase angle of π and the wave B is ahead by a phase angle π.

28. Adjoining figure shows the snapshot of two waves A and B at any time t. The equation for A is $y = A \sin(kx - \omega t - \phi)$, and for B it is $y = A \sin(kx - \omega t)$. It is clearly shown in the figure that wave A is ahead of B by a distance ϕ/k.

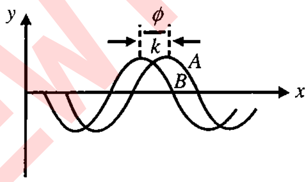
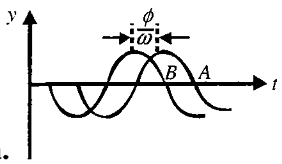
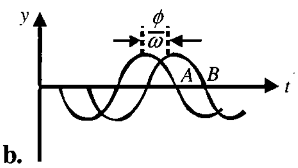
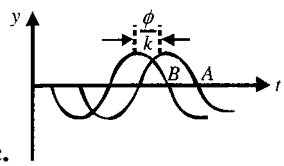


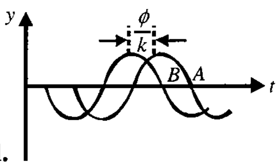
Fig. 5.47

The motion of a single point in time, i.e., y versus t for two waves is best represented by

a.


b.


c.


d.


29. A transverse sinusoidal wave is generated at one end of a long horizontal string by a bar that moves the end up and down through a distance by 2.0 cm. The motion of bar is continuous and is repeated regularly 125 times per second. If the distance between adjacent wave crests is observed to be 15.6 cm and the wave is moving along positive x-direction, and at $t = 0$ the element of the string at $x = 0$ is at mean position $y = 0$ and is moving downward, the equation of the wave is best described by

a. $y = (1 \text{ cm}) \sin [(40.3 \text{ rad/m})\ x - (786 \text{ rad/s})\ t]$
b. $y = (2 \text{ cm}) \sin [(40.3 \text{ rad/m})\ x - (786 \text{ rad/s})\ t]$
c. $y = (1 \text{cm}) \cos [(40.3 \text{ rad/m})\ x - (786 \text{ rad/s})\ t]$
d. $y = (2 \text{ cm}) \cos [(40.3 \text{ rad/m})\ x - (786 \text{ rad/s})\ t]$

30. A travelling wave is having wavelength of 3 cm. At any instant the two particles at a distance of 16.5 cm apart have a phase difference of

a. $\frac{\pi}{2}$ **b.** 5π
c. 10.5π **d.** 11.5π

31. If the maximum speed of a particle on a travelling wave is v_0, then find the speed of a particle when the displacement is half of the maximum value.

a. $\frac{v_0}{2}$ **b.** $\frac{\sqrt{3}\, v_0}{4}$
c. $\frac{\sqrt{3}\, v_0}{2}$ **d.** v_0

32. A sinusoidal wave is generated by moving the end of a string up and down, periodically. The generator must apply the energy at maximum rate when the end of the string attached to generator has .... X and least power when the end of the string attached to generator has .... Y. The most suitable option which correctly fills blanks X and Y, is

a. maximum displacement, least acceleration
b. maximum displacement, maximum acceleration
c. least displacement, maximum acceleration
d. least displacement, least acceleration

33. A point source of sound is placed in a non-absorbing medium. Two points A and B are at the distances of 1 m and 2 m, respectively, from the source. The ratio of amplitudes of waves at A to B is

a. 1:1 **b.** 1:4
c. 1:2 **d.** 2:1

34. Two canoes are 10 m apart on a lake. Each bobs up and down with a period of 4.0 s. When one canoe is at its highest point, the other canoe is at its lowest point. Both canoes are always within a single cycle of the waves. Determine the speed of the wave.

a. 2.5 m/s **b.** 5 m/s
c. 40 m/s **d.** 4 m/s

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35. The mathematical form of three travelling waves are given by

$$Y_1 = (2\text{ cm}) \sin (3x - 6t)$$
$$Y_2 = (3\text{ cm}) \sin (4x - 12t)$$

And $$Y_3 = (4\text{ cm}) \sin (5x - 11t)$$

of these waves,

a. Wave 1 has greatest wave speed and greatest maximum transverse string speed
b. Wave 2 has greatest wave speed and wave 1 has greatest maximum transverse string speed
c. Wave 3 has greatest wave speed and wave 1 has greatest maximum transverse string speed
d. Wave 2 has greatest wave speed and wave 3 has greatest maximum transverse string speed.

36. A transverse wave on a string travelling along +ve x-axis has been shown in the figure below:

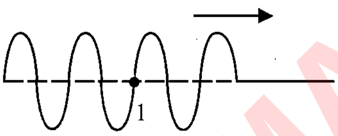

Fig. 5.48

The mathematical form of the shown wave is

$$y = (3.0\text{ cm}) \sin \left[2\pi \times 0.1\,t - \frac{2\pi}{100}x\right]$$

where t is in seconds and x is in centimetres. Find the total distance travelled by the particle at (1) in 10 min 15 s, measured from the instant shown in the figure and direction of its motion at the end of this time.

a. 6 cm, in upward direction
b. 6 cm, in downward direction
c. 738 cm, in upward direction
d. 732 cm, in upward direction

37. A water surfer is moving at a speed of 15 m/s. When he is surfing in the direction of wave, he swings upwards every 0.8 s because of wave crests. While surfing in opposite direction to that of wave motion, he swings upwards every 0.6 s. Determine the wavelength of transverse component of the water wave.

a. 15 m
b. 10.3 m
c. 21.6 m
d. Information insufficient

38. A transverse wave on a string has an amplitude of 0.2 m and a frequency of 175 Hz. Consider a particle of the string at $x = 0$. It begins with a displacement $y = 0$, at $t = 0$, according to equation $y = 0.2 \sin (kx \pm \omega t)$. How much time passes between the first two instant when this particle has a displacement of $y = 0.1$ m?

a. 1.9 ms
b. 3.9 ms
c. 2.4 ms
d. 0.5 ms

39. If a wave is going from one medium to another, then

a. its frequency changes
b. its wavelength does not change
c. its speed does not change
d. its amplitude may change

40. At $t = 0$, a transverse wave pulse travelling in the +ve x-direction with a speed of 2 m/s in a wire is described by $y = 6/x^2$, given that $x \neq 0$. Transverse velocity of a particle at $x = 2$ m and $t = 2$ s is

a. 3 m/s
b. −3 m/s
c. 8 m/s
d. −8 m/s

41. A harmonic wave has been set up on a very long string which travels along the length of string. The wave has frequency of 50 Hz, amplitude 1 cm and wavelength 0.5 m. For the above described wave.

Statement I: Time taken by wave to travel a distance of 8 m along the length of string is 0.32 s.

Statement II: Time taken by a point on the string to travel a distance of 8 m, once the wave has reached at that point and sets it into motion is 0.32 s.

a. Both the statements are correct.
b. Statement I is correct but Statement II is incorrect.
c. Statement I is incorrect but Statement II is correct.
d. Both the statements are incorrect.

42. A point source of sound is placed in a non-absorbing medium. Two points A and B are at the distances of 1 m and 2 m, respectively, from the source. The ratio of amplitudes of waves at A to B is

a. 1:1
b. 1:4
c. 1:2
d. 2:1

43. A wave is represented by the equation

$$y = y_0 \sin [10\pi x - 15\pi t + (\pi/3)]$$

where x is in metres and t in seconds. The equation represents a travelling wave:

a. in the positive direction with a velocity 1.5 m/s and wavelength 0.2 m.
b. in the negative direction with a velocity 1.5 m/s and wavelength 0.2 m.
c. in the positive direction with a velocity 2 m/s and wavelength 0.2 m.
d. in the negative direction with a velocity 2 m/s and wavelength 1.5 m.

44. A progressive wave is given by

$$y = 3 \sin 2\pi [(t/0.04) - (x/0.01)]$$

where x, y are in cm and t in s. The frequency of wave and maximum acceleration will be:

a. 100 Hz, 4.7×10^3 m/s^2
b. 50 Hz, 7.5×10^3 m/s^2
c. 25 Hz, 4.7×10^4 m/s^2
d. 25 Hz, 7.5×10^4 m/s^2

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45. A transverse wave is travelling in a string. Study following statements.

i. Equation of the wave is equal to the shape of the string at an instant t.

ii. Equation of the wave is general equation for displacement of a particle of the string

iii. Equation of the wave must be sinusoidal equation

iv. Equation of the wave is an equation for displacement of the particle at one end only.

Correct statements are

a. (i) and (ii) **b.** (ii) and (iii)
c. (i) and (iii) **d.** (ii) and (iv)

46. The equation of a wave is given by

$$y = 0.5 \sin (100\,t + 25x)$$

The ratio of maximum particle velocity to wave velocity is:

a. 12.5 **b.** 25
c. 4 **d.** 1/8

47. The two waves are represented by

$$y_1 = 10^{-6} \sin\left(100t + \frac{x}{50} + 0.5\right)\text{m}$$

$$y_2 = 10^{-2} \cos\left(100t + \frac{x}{50}\right)\text{m}$$

where x is in metres and t in seconds. The phase difference between the waves is approximately:

a. 1.07 rad **b.** 2.07 rad
c. 0.5 rad **d.** 1.5 rad

48. Which of the following is not true for the progressive wave

$$y = 4 \sin 2\pi\left(\frac{t}{0.02} - \frac{x}{100}\right)$$

where x and y are in cm and t in seconds.

a. The amplitude is 4 cm
b. The wavelength is 100 cm
c. The frequency is 50 Hz
d. The velocity of propagation is 2 cm/s

49. The amplitude of a wave disturbance propagating along positive X-axis is given by $y = 1/(1 + x^2)$ at $t = 0$ and $y = 1/[1 + (x - 2)^2]$ at $t = 4$ s where x and y are in metre. The shape of wave disturbance does not change with time. The velocity of the wave is

a. 0.5 m/s **b.** 1 m/s
c. 2 m/s **d.** 4 m/s

50. Consider a wave represented by $y = a \cos^2 (\omega t - kx)$ where symbols have their usual meanings. This wave has

a. an amplitude a, frequency ω, and wavelength λ.
b. an amplitude a, frequency 2ω, and wavelength 2λ.
c. an amplitude $a/2$, frequency 2ω, and wavelength $\lambda/2$.
d. am amplitude $a/2$, frequency 2ω, and wavelength λ.

51. At $t = 0$, a transverse wave pulse in a wire is described by the function $y = 6/(x^2 - 3)$ where x and y are in metres. The function $y\,(x, t)$ that describes this wave equation if it is travelling in the positive x direction with a speed of 4.5 m/s is

a. $y = \dfrac{6}{(x+4.5t)^2 - 3}$ **b.** $y = \dfrac{6}{(x-4.5t)^2 + 3}$

c. $y = \dfrac{6}{(x+4.5t)^2 + 3}$ **d.** $y = \dfrac{6}{(x-4.5t)^2 - 3}$

52. A stretched rope having linear mass density 5×10^{-2} kg/m is under a tension of 80 N. The power that has to be supplied to the rope to generate harmonic waves at a frequency of 60 Hz and an amplitude of $\dfrac{2\sqrt{2}}{15\pi}$ m is

a. 215 W **b.** 251 W **c.** 512 W **d.** 521 W

53. A string of length $2L$, obeying Hooke's law, is stretched so that its extension is L. The speed of the transverse wave travelling on the string is v. If the string is further stretched so that the extension in the string becomes $4L$. The speed of transverse wave travelling on the string will be

a. $\dfrac{1}{\sqrt{2}}v$ **b.** $\sqrt{2}\,v$ **c.** $\dfrac{1}{2}v$ **d.** $2v$

(n is an integer)

54. A sinusoidal wave travelling in the positive direction on stretched string has amplitude 20 cm, wavelength 1.0 m and wave velocity 5.0 m/s. At $x = 0$ and $t = 0$ it is given that $y = 0$ and $\partial y/\partial t < 0$. Find the wave function $y(x, t)$.

a. $y(x,t) = (0.02\text{ m})\sin\left[(2\pi\text{ m}^{-1})x + (10\pi\text{ s}^{-1})t\right]\text{m}$

b. $y(x,t) = (0.02\text{ m})\cos\left[(10\pi\text{ s}^{-1})t + (2\pi\text{ m}^{-1})x\right]\text{m}$

c. $y(x,t) = (0.02\text{ m})\sin\left[(2\pi\text{ m}^{-1})x - (10\pi\text{ s}^{-1})t\right]\text{m}$

d. $y(x,t) = (0.02\text{ m})\sin\left[(\pi\text{ m}^{-1})x + (5\pi\text{ s}^{-1})t\right]\text{m}$

55. For the wave shown in figure, write the equation of this wave if its position is shown at $t = 0$. Speed of wave is $v = 300$ m/s.

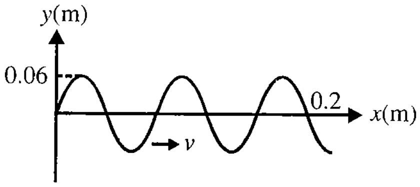

Fig. 5.49

a. $y = (0.06\text{ m}) \cos [(78.5\text{ m}^{-1})x + (23562\text{ s}^{-1})t]$ m
b. $y = (0.06\text{ m}) \sin [(78.5\text{ m}^{-1})x - (23562\text{ s}^{-1})t]$ m
c. $y = (0.06\text{ m}) \sin [(78.5\text{ m}^{-1})x + (23562\text{ s}^{-1})t]$ m
d. $y = (0.06\text{ m}) \cos [(78.5\text{ m}^{-1})x - (28562\text{ s}^{-1})t]$ m

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Multiple Correct Answers Type

Solutions on page 5.49

1. A wave equation which gives the displacement along Y-direction is given by

$$y = 10^{-4} \sin (60t + 2x)$$

where x and y are in metres and t is time in seconds. This represents a wave

a. travelling with a velocity of 30 m/s in the negative x-direction
b. of wavelength π metres
c. of frequency $30/\pi$ Hertz
d. of amplitude 10^{-4} m travelling along the negative x-direction.

2. Consider a wave represented by $y = \cos(500t - 70x)$ where y is in millimetres, x in meters and t in second. Which of following are true?

a. The wave is a standing wave.
b. The speed of the wave is 50/7 m/s
c. The frequency of oscillations is $500 \times 2\pi$ Hz.
d. Two nearest points in the same phase have separation $20\pi/7$ cm.

3 A simple harmonic progressive wave in a gas has a particle displacement of $y = a$ at time $t = T/4$ at the origin of the wave and a particle velocity of $\dot{y} = v$ at the same instant but at a distance $x = \lambda/4$ from the origin where T and λ are the periodic time and wavelength of the wave respectively. Then for this wave,

a. the amplitude A of the wave is $A = 2a$
b. the amplitude A of the wave is $A = a$
c. The equation of the wave can be represented by $y = a \sin \frac{v}{a}\left[t - \frac{x}{V}\right]$
d. The equation of the wave can be represented by $y = 2a \cos \frac{v}{a}\left[t - \frac{x}{V}\right]$

4. The equation to a transverse wave travelling in a rope is given by

$$y = A\cos\frac{\pi}{2}[kx - \omega t - \alpha]$$

where $A = 0.6$ m, $k = 0.005\ \text{cm}^{-1}$, $\omega = 8.0\ \text{s}^{-1}$ and α is a non-vanishing constant. Then for this wave,

a. the wavelength of the wave is $\lambda = 8$ m
b. the maximum velocity v_m of a particle of the rope will be, $v_m = 7.53$ m/s.
c. the equation of a wave which, when superposed with the given wave can produce standing waves in the rope is $y = A\cos\frac{\pi}{2}(kx - \omega t + \alpha)$
d. the equation of a wave which, when superposed with the given wave can produce standing waves in the rope is $y = A\cos\frac{\pi}{2}(kx + \omega t - \alpha)$

5. Mark out the correct statement(s) w.r.t. wave speed and particle velocity for a transverse travelling mechanical wave on a string.

a. The wave speed is same for the entire wave, while particle velocity is different for different points at a particular instant.
b. Wave speed depends upon property of the medium but not on the wave properties.
c. Wave speed depends upon both the properties of the medium and on the properties of wave.
d. Particle velocity depends upon properties of the wave and not on medium properties.

6. Mark out the correct statement(s).

a. For a travelling wave on a string, oscillation energy of an elemental length remains constant.
b. For a sinusoidal travelling wave on a string, oscillation energy of an elemental length varies periodically.
c. For a travelling wave on a string, oscillation energy of all elemental parts having equal lengths are the same.
d. For a stationary wave on a string, oscillation energy of any elemental part is constant.

7. For a transverse wave on a string, the string displacement is described by

$$y(x, t) = f(x - at)$$

where f represents a function and a is a negative constant. Then which of the following is/are correct statement(s)?

a. Shape of the string at time $t = 0$ is given by $f(x)$
b. The shape of wave form does not change as it moves along the string
c. Waveform moves in +ve x-direction
d. The speed of waveform is a

8. A wave is going from one medium to another; then which of its property may/ must change?

a. Frequency
b. Wavelength
c. Velocity
d. Amplitude

9. A wave moves at a constant speed along a stretched string. Mark the incorrect statement out of the following:

a. Particle speed is constant and equal to the wave speed .
b. Particle speed is independent of amplitude of the periodic motion of the source.
c. Particle speed is independent of frequency of periodic motion of the source.
d. Particle speed is dependent on tension and linear mass density the string.

10. A harmonic wave is travelling along +ve x-axis, on a stretched string. If wavelength of the wave gets doubled, then

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a. Frequency of wave may change
b. Wave speed may change
c. Both frequency and speed of wave may change
d. Only frequency will change

11. Mark the correct option(s) out of the following:
a. Mechanical waves can be transverse in liquids.
b. In some medium, the speed of a longitudinal mechanical wave is greater than the speed of transverse mechanical wave.
c. Transverse waves are possible in bulk of a liquid.
d. Non-mechanical waves are transverse in nature.

12. Mark out the correct statement(s) concerning waves.
a. A wave can have both transverse and longitudinal components.
b. A wave does not result in the bulk flow of the materials of its medium.
c. A wave is a travelling disturbance.
d. A wave can be there even in the absence of an elastic medium.

13. Two particles P and Q have a phase difference of π when a sine wave passes through the region:
a. P oscillates at half the frequency of Q.
b. P and Q move in opposite directions.
c. P and Q must be separated by half of the wavelength.
d. The displacements of P and Q have equal magnitudes.

14. As a wave propagates
a. the wave intensity remains constant for a plane wave
b. the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
c. the wave intensity decreases as the inverse of the distance from a line source
d. total power of the spherical wave over the spherical surface centred at the source remains constant at all the times

15. Let a disturbance y be propagated as a plane wave along the x-axis. The wave profiles at the instants $t = t_1$ and $t = t_2$ are represented respectively as: $y_1 = f(x_1 - vt_1)$ and $y_2 = f(x_2 - vt_2)$. The wave is propagating without change of shape.
a. The velocity of the wave is v.
b. The velocity of the wave is $v = (x_2 + x_1)/(t_2 + t_1)$.
c. The particle velocity is $v_P = -vf'(x - vt)$.
d. The phase velocity of the wave is v.

16. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed at any point on the string is $(v/10)$ where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by:
a. $\lambda = 2\pi \times 10^{-2}$ m
b. $\lambda = 10^{-3}$ m
c. $f = 10^3/(2\pi)$ Hz
d. $f = 10^4$ Hz

17. A wave is travelling along a string. At an instant shape of the string is as shown in the enclosed figure. At this instant, point A is moving upwards. Which of the following statements are correct?

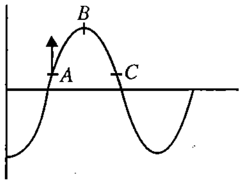

Fig. 5.50

a. The wave is travelling to the right
b. Displacement amplitude of the wave is equal to the displacement of B at this instant
c. At this instant velocity of C is also directed upwards
d. Phase difference between A and C may be equal to $\pi/2$.

18. Which of the following functions represent a travelling wave? Here a, b and c are constants.
a. $y = a \cos(bx) \sin(ct)$
b. $y = a \sin(bx + ct)$
c. $y = a \sin(bx + ct) + a \sin(bx - ct)$
d. $y = a \sin(bx - ct)$

19. A wave is represented by the equation

$$y = A \sin 314 \left[\frac{t}{0.5\,\text{s}} - \frac{x}{100\,\text{m}}\right]$$

The frequency is n and the wavelength is λ. Then:
a. $n = 2$ Hz
b. $n = 100$ Hz
c. $\lambda = 2$m
d. $\lambda = 100$ m

20. Energy density E (energy per unit volume) of the medium at a distance r from a sound source varies according to the curve shown in figure. Which of the following are possible?

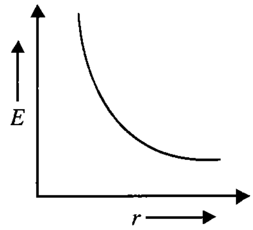

Fig. 5.51

a. The source may be a point isotropic source.
b. If the source is a plane source then the medium particles have damped oscillations.
c. If the source is a plane source then power of the source is decreasing with time.
d. Density of the medium decreases with distance r from the source.

21. Equation of a wave travelling in a medium is: $y = a \sin(bt - cx)$. Which of the following are correct?
a. Ratio of the displacement amplitude, with which the particles of the medium oscillate, to the wavelength is equal to $ac/2\pi$.
b. Ratio of the velocity oscillation amplitude of medium particles to the wave propagation velocity is equal to ac.

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c. Oscillation amplitude of relative deformation of the medium is directly proportional to velocity oscillation amplitude of medium particles.
d. None of the above

22. The equation of a wave is

$$y = 4\sin\left[\frac{\pi}{2}\left(2t + \frac{1}{8}x\right)\right]$$

where y and x are in centimetres and t is in seconds.

a. The amplitude, wavelength, velocity, and frequency of wave are 4 cm, 16 cm, 32 cm/s and 1 Hz, respectively, with wave propagating along $+x$ direction.
b. The amplitude, wavelength, velocity, and frequency of wave are 4 cm, 32 cm, 16 cm/s, and 0.5 Hz, respectively, with wave propagating along $-x$ direction.
c. Two positions occupied by the particle at time interval of 0.4 s have a phase difference of 0.4π radian.
d. Two positions occupied by the particle at separation of 12 cm have a phase difference of 135°.

23. A wire of 9.8×10^{-3} kg/m passes over a frictionless light pulley fixed on the top of a frictionless inclined plane which makes an angle of 30° with the horizontal. Masses m and M are tied at the two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 m/s.

a. $m = 20$ kg
b. $M = 5$ kg
c. $\frac{m}{M} = \frac{1}{2}$
d. $\frac{m}{M} = 2$

24. $y(x,t) = 0.8/[(4x+5t)^2+5]$ represents a moving pulse, where x and y are in metres and t is in seconds, then

a. pulse is moving in $+x$ direction
b. in 2 s it will travel a distance of 2.5 m
c. its maximum displacement is 0.16 m
d. it is a symmetric pulse

Assertion-Reasoning Type

Solutions on page 5.52

In the following questions, a statement of assertion (Statement I) is given which is followed by a corresponding statement of reason (Statement II). Examine the statements carefully and choose the correct option according to the following options.

a. **Statement I** is true, Statement II is true and Statement II is the correct explanation for Statement I.
b. **Statement I** is true, Statement II is true and Statement II is NOT the correct explanation for Statement I.
c. **Statement I** is true, Statement II is false.
d. **Statement I** is false, Statement II is true.

1. **Statement I:** Pressure and density changes do not occur in a transverse stationary wave.
Statement II: the average distance between any two particles of the wave remains the same.

2. **Statement I:** In a progressive longitudinal wave, the amplitude of the wave will not be the same at all points of the medium along the direction of motion of the wave.
Statement II: There is a continuous change of the phase angle of the wave as it progresses in the direction of motion.

3. **Statement I:** In a small segment of string carrying sinusoidal wave, total energy is conserved.
Statement II: Every small part moves in SHM and in SHM total energy is conserved.

4. **Statement I:** Two waves moving in a uniform string having uniform tension cannot have different velocities.
Statement II: Elastic and inertial properties of string are same for all waves in same string. Moreover speed of wave in a string depends on its elastic and inertial properties only.

5. **Statement I:** Waves generated in a metal piece can be transverse or longitudinal.
Statement II: Waves generated depend upon the method of creating waves in the metal.

6. **Statement I:**The intensity of a plane progressive wave does not change with change in distance from the source.
Statement II: The wavefronts associated with a plane progressive wave are planar.

7. **Statement I:** The more the velocity of a simple harmonic wave in a string, the more is the maximum velocity of the particles of string.
Statement II:

$$v_{max} = \omega A$$

8. **Statement I:** For a travelling wave in a string, for small amplitudes the instantaneous values of kinetic and potential energies of any element are equal.
Statement II:

$$dU = \frac{1}{2}T\,dx\left(\frac{\partial y}{\partial x}\right)^2$$

$$d(\text{KE}) = \frac{1}{2}(\mu\,dx)\left(\frac{\partial y}{\partial t}\right)^2$$

where T is the tension and μ is mass per unit length of the string.

9. **Statement I:** A plane progressive harmonic wave is propagating in a string. If tension in the string is made two times then average power transmitted through the string becomes two times.
Statement II: Average power transmission in a string is given by $P = \frac{\omega^2 A^2 F}{2V}$

10. **Statement I:** Compressions and rarefactions involve changes in density and pressure.
Statement II: When particles are compressed, density of medium increases and when they are rarefied, density of medium decreases.

Comprehension Type

Solutions on page 5.52

For Problems 1–5

The figure represents two snaps of a travelling wave on a string of mass per unit length $\mu = 0.25$ kg/m. The first snap is taken at $t = 0$ and the second is taken at $t = 0.05$ s.

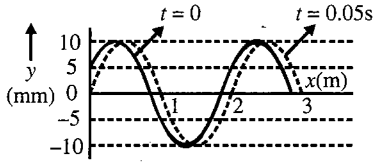

Fig. 5.52

1. Determine the speed of the wave.
   a. 20/3 m/s b. 10/3 m/s
   c. 20 m/s d. 10 m/s
2. Determine the frequency of the wave.
   a. 5/3 Hz b. 10/3 Hz
   c. 5 Hz d. 10 Hz
3. Determine the maximum speed of the particle.
   a $\frac{7}{20}\pi$ m/s b. $\frac{5}{13}\pi$ m/s
   c. $\frac{\pi}{30}$ m/s d. $\frac{7\pi}{20}$ m/s
4. Determine the tension in the string.
   a. 25/3 N b. 25/7 N
   c. 5 N d. 25/9 N
5. Determine the equation of the wave.
   a. $y=10\sin[2\pi x-(10/3)\pi t+(\pi/6)]$
   b. $y=10\sin[\pi x-(10/3)\pi t+(\pi/3)]$
   c. $y=10\sin[\pi x-(10/3)\pi t+(\pi/6)]$
   d. $y=10\sin[2\pi x-(5/3)\pi t+(\pi/3)]$

For Problems 6–9

A long string having a cross-sectional area 0.80 mm^2 and density 12.5 g/cm^3 is subjected to a tension of 64 N along the x-axis. One end (at x = 0) of this string is attached to a vibrator moving in transverse direction at a frequency of 20 Hz. At t = 0, the source is at a maximum displacement y = 1.0 cm.

6. Find the speed of the wave travelling on the string.
   a. 20 m/s b. 10 m/s
   c. 80 m/s d. 40 m/s
7. Write the equation for the wave.
   a. $y=(1.0\text{ cm})\cos\left[(40\pi s^{-1})t-\{(\pi/2\text{ m}^{-1})x\}\right]$
   b. $y=(1.0\text{ cm})\cos\left[(40\pi s^{-1})t+\{(\pi/2\text{ m}^{-1})x\}\right]$
   c. $y=(1.0\text{ cm})\cos\left[(40\pi s^{-1})t-\{(\pi/4\text{ m}^{-1})x\}\right]$
   d. $y=(1.0\text{ cm})\cos\left[(40\pi s^{-1})t+\{(\pi/4\text{ m}^{-1})x\}\right]$
8. What is the displacement of the particle of the string at x = 50 cm at time t = 0.05 s?
   a. $\frac{1}{\sqrt{2}}$ cm b. $\sqrt{2}$ cm
   c. $\frac{\sqrt{3}}{2}$ cm d. $\frac{2}{\sqrt{3}}$ cm
9. What is the velocity of this particle at this instant?
   a. $10\sqrt{2}\pi$ cm/s b. $40\sqrt{2}\pi$ cm/s
   c. $30\sqrt{2}\pi$ cm/s d. $20\sqrt{2}\pi$ cm/s

For Problems 10–13

Consider a sinusoidal travelling wave shown in Fig. 5.53. The wave velocity is +40 cm/s.

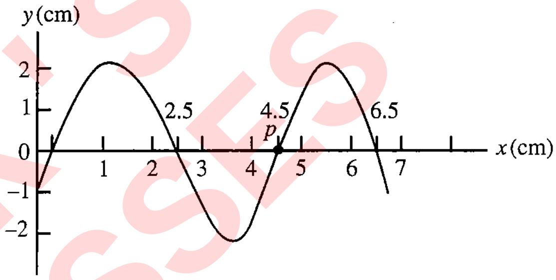

Fig. 5.53

10. Find the frequency of the wave.
    a. 20 Hz b. 30 Hz
    c. 25 Hz d. 10 Hz
11. Find the phase difference between points 2.5 cm apart.
    a. $3\pi/4$ rad b. $5\pi/4$ rad
    c. $7\pi/4$ rad d. $9\pi/4$ rad
12. How long does it take for the phase at a given position to change by 60°?
    a. 1/30 s b. 1/60 s
    c. 1/20 s d. 1/40 s
13. Find the velocity of a particle at point P at the instant shown.
    a. 1.26 m/s upward b. 1.26 m/s downward
    c. 3.52 m/s upward d. None of these

For Problems 14–16

A plane wave propagates along positive x-direction in a homogeneous medium of density $\rho=200\text{ kg/m}^3$. Due to propagation of the wave medium particles oscillate. Space density of their oscillation energy is $E=0.16\pi^2$ J/m^3 and maximum shear strain produced in the medium is $\phi_0=8\pi\times10^{-5}$. If at an instant, phase difference between two particles located at points (1m, 1m, 1m) and (2m, 2m, 2m) is $\Delta\theta=144°$, assuming at t = 0 phase of particles at x = 0 to be zero,

14. Wave velocity is
    a. 300 m/s b. 400 m/s
    c. 500 m/s d. 100 m/s
15. Wave length is
    a. 2.5 m b. 5 m
    c. 10 m d. 6 m
16. Equation of wave is
    a. $y=10^{-4}\sin\pi(2000t-0.8x)$
    b. $y=10^{-4}\sin\pi(400t-0.8x)$

c. $y = 10^{-4}\sin\pi(100t - 8x)$
d. $y = 10^{-4}\sin\pi(100t - 2x)$

For Problems 17–19

A sinusoidal wave is propagating in negative x-direction in a string stretched along x-axis. A particle of string at $x = 2$ cm is found at its mean position and it is moving in positive y-direction at $t = 1$ s. The amplitude of the wave, the wavelength and the angular frequency of the wave are 0.1 m, $\pi/4$ m and 4π rad/s, respectively.

17. The equation of the wave is
a. $y = 0.1 \sin(4\pi(t-1) + 8(x-2))$
b. $y = 0.1 \sin((t-1) - (x-2))$
c. $y = 0.1 \sin(4\pi(t-1) - 8(x-2))$
d. none of these

18. The speed of particle at $x = 2$ m and $t = 1$ s is
a. 0.2π m/s b. 0.6π m/s
c. 0.4π m/s d. 0

19. The instantaneous power transfer through $x = 2$ m and $t = 1.125$ s is
a. 10 J/s b. $4\pi/3$ J/s c. $2\pi/3$ J/s d. 0

For Problems 20–21

Four pieces of string each of length L are joined end to end to make a long string of length $4L$. The linear mass density of the strings are μ, 4μ, 9μ and 16μ, respectively. One end of the combined string is tied to a fixed support and a transverse wave has been generated at the other end having frequency f (ignore any reflection and absorptions). String has been stretched under a tension F.

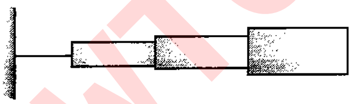

Fig. 5.54

20. Find the time taken by wave to reach from source end to fixed end.
a. $\frac{25}{12} \times \frac{L}{\sqrt{F/\mu}}$ b. $\frac{10L}{\sqrt{F/\mu}}$
c. $\frac{4L}{\sqrt{F/\mu}}$ d. $\frac{L}{\sqrt{F/\mu}}$

21. Find the ratio of wavelengths of the waves on four strings, starting from right hand side.
a. 12:6:4:3 b. 4:3:2:1
c. 3:4:6:12 d. 1:2:3:4

For Problems 22–24

Figure 5.55 shows a student setting up wave on a long stretched string. The student's hand makes one complete up and down movement in 0.4 s and in each up and down movement the hand moves by a height of 0.3 m. The wavelength of the waves on the string is 0.8 m.

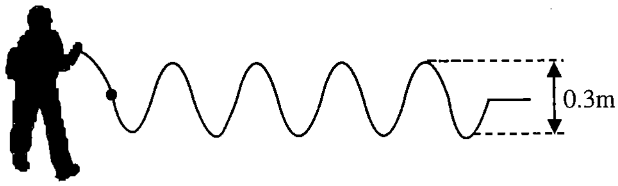

Fig. 5.55

22. The frequency of the wave is
a. 2.5 Hz b. 5 Hz
c. 1.25 Hz d. Cannot be predicted

23. The amplitude of the wave is
a. 0.15 m b. 0.3 m
c. 0.075 m d. Cannot be predicted

24. The wave speed is
a. 2 m/s b. 4 m/s
c. 1 m/s d. Cannot be predicted

For Problems 25–27

A child playing with a long rope ties one end and holds the other. The rope is stretched taut along the horizontal. The child shakes the end he is holding, up and down, in a sinusoidal manner with amplitude 10 cm and frequency 3 Hz. Speed of the wave is 15 m/s and, at $t = 0$, displacement at the child's end is maximum positive. Assuming that there is no wave reflected from the fixed end, so that the waves in the rope are plane progressive waves, answer the following questions.

(Also assume that the wave propagates along the positive x-direction.)

25. A wave function that describes the wave in the given situation is
a. $y = (0.1\text{ m})\cos[(2\text{ rad/s})x - (12.5\text{ rad/s})t]$
b. $y = (0.1\text{ m})\cos[(1.26\text{ rad/m})x - 18.8\text{ rad/s})t]$
c. $y = (0.1\text{ m})\sin[(1.5\text{ rad/m})x - (10\text{ rad/s})t]$
d. $y = (0.1\text{ m})\sin[(1.5\text{ rad/m})x - (4\text{ rad/s})t]$

26. Equation of displacement of a point 2.5 m from the child's end can be expressed as
a. $y = -(0.1\text{ m})\cos(18.8\text{ rad/s})t$
b. $y = 0.1\text{ m}\cos(12.5\text{ rad/s})t$
c. $y = (0.1\text{ m})\sin(4\text{ rad/s})t$
d. $y = -(0.1\text{ m})\sin(10\text{ rad/s})t$

27. Phase difference between the child's end and a point 2.5 m from the child's end will be
a. $\pi/2$ b. $3\pi/2$ c. $\pi/4$ d. π

For Problems 28–32

One end of a long rope is tied to a fixed vertical pole. The rope is stretched horizontally with a tension 8 N. Let us consider the length of the rope to be along x-axis. A simple harmonic oscillator at $x = 0$ generates a transverse wave of frequency 100 Hz and amplitude 2 cm along the rope. Mass of a unit length of the rope is 20 g/m. Ignoring the effect of gravity, answer the following questions.

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28. Wavelength of the wave is

a. 50 cm **b.** 20 cm
c. 8 cm **d.** 32 cm

29. Assuming that the oscillator has its maximum negative displacement at $t = 0$, wave equation function) for the wave can be expressed as $y =$

a. $-(0.02 \text{ m}) \cos [8\pi \text{ (rad/m)}x - 100\pi \text{ (rad/s)}t]$
b. $(0.02 \text{ m}) \cos [10\pi \text{ (rad/m)}x - 200\pi \text{ (rad/s)}t]$
c. $-(0.02 \text{ m}) \cos [10\pi \text{ (rad/m)}x - 200\pi \text{ (rad/s)}t]$
d. $(0.02 \text{ m}) \sin [8\pi \text{ (rad/m)}x - 100\pi \text{ (rad/s)}t]$

30. Which of the following is correct?

a. The wave propagates with a fixed speed and any particle of the medium vibrates with the same fixed speed.
b. The wave propagates with a fixed speed but any particle of the medium vibrates with a variable speed.
c. The wave propagates with a variable speed but any particle of the medium vibrates with some fixed speed.
d. The wave propagates with a variable speed and any particle of the medium also vibrates with a variable speed.

31. Maximum magnitude of transverse acceleration of any point on the rope will be nearly

a. 7888 m/s^2 **b.** 8244 m/s^2
c. 9277 m/s^2 **d.** 3333 m/s^2

32. Tension in the given rope remaining the same, if a simple harmonic oscillator of frequency 200 Hz is used instead of the earlier oscillator of frequency 100 Hz

a. Speed of transverse waves in the rope will be doubled; wavelength will not change
b. Speed of transverse waves in the rope will become half; wavelength will become one-fourth
c. Speed of transverse waves in the rope will become four times; wavelength will be doubled
d. Speed of transverse waves in the rope will not change; wavelength will become half

For Problems 33–36

A rope is attached at one end to a fixed vertical pole. It is stretched horizontally with a fixed value of tension T. Suppose at $t = 0$, a pulse is generated by moving the free end of the rope up and down once with your hand. The pulse arrives at the pole at instant t.

Ignoring the effect of gravity, answer the following questions.

33. If you move your hand up and down once by the same amount but do it more rapidly, say, twice as fast as in the earlier case,

a. Time taken for the pulse to reach the pole will increase and it will be doubled
b. Time taken for the pulse to reach the pole will decrease and it will become half
c. Time taken for the pulse to reach the pole will not change
d. Cannot say

34. If you move your hand up and down once but to a greater distance and in the same amount of time

a. Time taken for the pulse to reach the pole will increase
b. Time taken for the pulse to reach the pole will not change
c. Time taken for the pulse to reach the pole will decrease
d. Time taken for the pulse to reach the pole may increase or decrease

35. If you use a string of same length but of greater mass

a. Time taken for the pulse to reach the pole will not change
b. Time taken for the pulse to reach the pole will increase
c. Time taken for the pulse to reach the pole will decrease
d. Time taken for the pulse to reach the pole may increase or decrease

36. In all the above, questions we have considered a fixed value of tension. However, if tension in a given rope is increased and a pulse is generated as described,

a. Time taken for the pulse to reach the pole may increase to decrease
b. Time taken for the pulse to reach the pole will not change
c. Time taken for the pulse to reach the pole will increase
d. Time taken for the pulse to reach the pole will decrease.

For Problems 37–38

A simple harmonic plane wave propagates along x-axis in a medium. The displacement of the particles as a function of time is shown in figure, for $x = 0$ (curve 1) and $x = 7$ (curve 2).

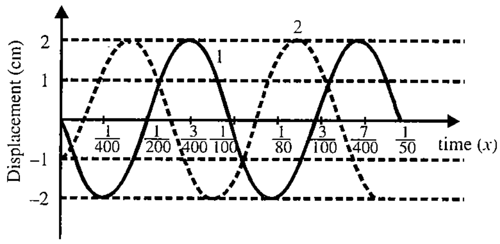

Fig. 5.56

The two particles are within a span of one wavelength.

37. The wavelength of the wave is

a. 6 cm **b.** 24 cm **c.** 12 cm **d.** 16 cm

38. The speed of the wave is

a. 12 m/s **b.** 24 m/s **c.** 8 m/s **d.** 16 m/s

For Problems 39–40

Figure 5.57 shows two 'snapshots' of medium particle supporting a plane progressive wave travelling along positive x-axis, corresponding to instants $t = 0.002$ s and $t = 0.008$ s, respectively shown by curves numbered 1 and 2.

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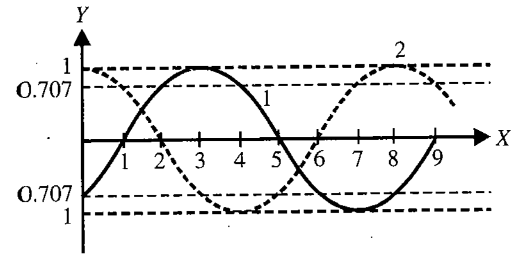


Fig. 5.57

Assume that the interval between the two snapshots is less than the time period.

39. Velocity of the wave is

a. $\frac{1700}{3}$ m/s **b.** $\frac{1700}{5}$ m/s

c. $\frac{2500}{7}$ m/s **d.** $\frac{2500}{3}$ m/s

40. The frequency of the wave

a. 52 s^{-1} **b.** 205 s^{-1}

c. 104 s^{-1} **d.** 84 s^{-1}

For Problems 41–42

The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity $20\sqrt{3}$ cm/s. The tangent at P makes an angle 60° with x-axis.

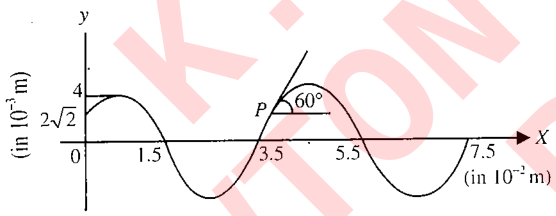


Fig. 5.58

41. Find the wave speed and direction in which the wave is moving.

a. 40 cm/s **b.** 60 cm/s

c. 80 cm/s **d.** 20 cm/s

42. Find the equation of wave.

a. $y=(5\times10^{-3})\sin\left(10\pi t-50\pi x+\frac{\pi}{8}\right)$

b. $y=(4\times10^{-3})\sin\left(10\pi t-50\pi x+\frac{\pi}{8}\right)$

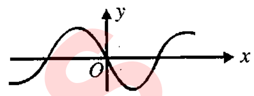
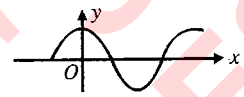
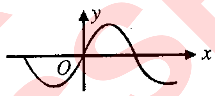
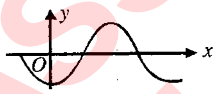
c. $y=(5\times10^{-3})\sin\left(10\pi t+50\pi x+\frac{\pi}{8}\right)$

d. $y=(4\times10^{-3})\sin\left(10\pi t+50\pi x+\frac{\pi}{8}\right)$

Matching Column Type

Solutions on page 5.56

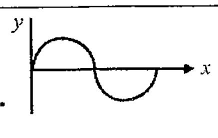
1. For four sine waves, moving on a string along positive x-direction, displacement distance curves (y–x curves) as shown at time t = 0. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have their usual meanings. Correctly match y-x curves with corresponding equations.

Column I	Column II
i. 	**a.** $y = A\cos(\omega t - kx)$
ii. 	**b.** $y = -A\cos(kx - \omega t)$
iii. 	**c.** $y = A\sin(\omega t - kx)$
iv. 	**d.** $y = A\sin(kx - \omega t)$

2. Suppose a wave pulse has been created at free end of a taut string by moving the hand up and down once. The string is attached at its other end to a distant wall.

Column I	Column II
i. Moving hand more quickly but still up and down once by the same amount indifferent time,	**a.** the amplitude changes
ii. Moving hand more quickly but still up and down once by more amount in same time,	**b.** the width of the pulse changes
iii. Moving hand at same speed, but still up and down once by same more amount,	**c.** the wave speed changes
iv. Moving hand more quickly, but still up and down once by less amount.	**d.** the particle speed changes

3. Three travelling sinusoidal waves are on identical strings having same tension. The mathematical form of the waves are $y_1 = A\sin(3x - 6t)$, $y_2 = A\sin(4x - 8t)$ and $y_3 = A\sin(6x - 12t)$.

Column I	Column II
i. Speed of each wave is	**a.** 

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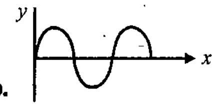
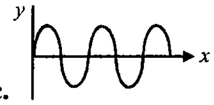
ii. y_1 is best represented by	**b.** 
iii. y_2 is best represented by	**c.** 
iv. y_3 is best represented by	**d.** 2 m/s

4. In case of mechanical wave a particle oscillates and during oscillation its kinetic energy and potential energy changes.

Column I	Column II
i. When particle of travelling wave is passing through mean position.	**a.** Kinetic energy is maximum
ii. When particle of travelling wave is at extreme position.	**b.** Potential energy is maximum
iii. When particle between node and antinode in standing wave passing through mean position.	**c.** Kinetic energy is minimum
iv. When particle between node and antinode in standing wave is at extreme position.	**d.** Potential energy is minimum

5. For transverse wave on a string

Column I	Column II
i. If amplitude increases	**a.** maximum instantaneous power increases
ii. If frequency increases	**b.** average power increases
iii. If amplitude decreases	**c.** maximum instantaneous power decreases
iv. If frequency decreases	**d.** average power decreases

Integer Answer Type

Solutions on page 5.57

1. The speed of a transverse wave, going on a wire having a length 50 cm and mass 5 g is 80 m/s. The area of cross section of the wire is 1.0 mm^2 and its Young's modulus is 8×10^{11} N/m^2. Find the extension (in $\times$ 10^{-2} mm) of the wire over its natural length.

2. A wave pulse passing on a string with a speed of 40 cm/s in the negative x-direction has its maximum at x = 0 at t = 0. Where will this maximum be located at t = 5 s? If the coordinate of required maximum is x = – μm. What is the value to be filled in box.

3. A particle on a stretched string supporting a travelling wave, takes 5.0 ms to move from its mean position to the extreme position. The distance between two consecutive particles, which are at their mean positions, is 3.0 cm. Find the wave speed (in m/s).

4. A string of length 20 cm and linear mass density 0.40 g/cm is fixed at both ends and is kept under a tension of 16 N. A wave pulse is produced at t = 0 near an end as shown in figure which travels towards the other end.

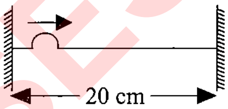


Fig. 5.59

when will the string have the shape shown in the figure again? (in $\times$ 10^{-2} s)

5. A string of length 40 cm and weighing 10 g is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 160 N/m and is stretched by 1.0 cm. If a wave pulse is produced on the sting near the wall. How much time will it take to reach the spring ? (in $\times$ 10^{-2} s)

6. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 19.2×10^{-3} kg/m. The speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 2.0 m/s^2 is $12.5n$. What is the value of n. Take = g = 10 m/s^2

7. A plane progressive wave is given by x = (40 cm) cos $(50\pi t - 0.02\pi y)$ where y is in cm and t in s. The particle velocity at y = 25 m in time $t = \dfrac{1}{100}$ s will be $10\pi\sqrt{n}$ m/s. What is the value of n.

8. A travelling wave is given by

$$y = \frac{0.8}{(3x^2 + 24xt + 48t^2 + 4)}$$

where x and y are in metres and t is in seconds. Find the velocity in m/s.

9. An ant with mass m is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length μ and is under tension F. Without warning, a student starts a sinusoidal transverse wave of wavelength λ propagating along the rope. The motion of the rope is in a vertical plane. What minimum wave amplitude (in mm) will make the ant feel weightless momentarily? Assume that m is so small that the presence of the ant has no effect on the propagation of the wave.
[Given: λ = 0.5 m, μ = 0.1 kg/m, F = 3.125 N, take $g = \pi^2$]

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ANSWERS AND SOLUTIONS

Subjective Type

1. The speed of wave in string is

$$V=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{35}{3.5\times10^{-3}}}=100 \text{ m/s}$$

$$\lambda=\frac{v}{f}=\frac{100}{100}=1 \text{ m}$$

The shape of the string at $t = 0$ s is as shown in Fig. 5.60.

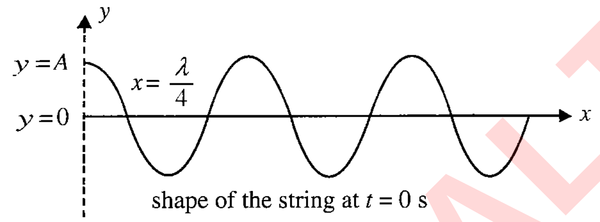

Fig. 5.60

Let us assume that

$$y=A\sin(\omega t-kx+\theta)$$

putting $t = 0$, $x = 0$ and $y = +A$

gives $\theta=\pi/2$

also $\omega=2\pi f=2\pi\times100=200\pi$

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{1}=2\pi$$

Hence $$y=A\sin\left(200\pi t-2\pi x+\frac{\pi}{2}\right)$$

It is given that

$$\frac{\partial y}{\partial x}=\frac{\pi}{20} \quad \text{at} \quad t=\frac{T}{4}=\frac{1}{400} \quad \left[\because T=\frac{1}{f}=\frac{1}{100}\right]$$

$$\left.\frac{\partial y}{\partial x}\right]_{t=\frac{1}{400}}=A2\pi\cos\left(200t-2\pi x+\frac{\pi}{2}\right)=-2A\pi=\frac{\pi}{20}$$

$$A=\frac{1}{40}=0.025$$

i. Amplitude $A = 0.25$ m

ii. Wave speed $= 100$ m/s

iii. Wave equation is

$$y=0.025\sin\left(200\pi t-2\pi x+\frac{\pi}{2}\right)$$

2. $\phi=\pi/6$

$$v=\sqrt{T/\mu}=\sqrt{\frac{900}{0.01}}=300 \text{ m/s}$$

frequency $= 150$ Hz

$$\therefore \quad \lambda=\frac{v}{f}=\frac{300}{150}=2\text{ m}$$

Then equation will be

$$y=A\sin\left\{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)+\phi\right\}$$

$$=5\sin\left\{2\pi\left(150t-\frac{x}{2}\right)+\frac{\pi}{6}\right\}$$

$$=5\sin\left\{\pi(300t-x)+\frac{\pi}{6}\right\}$$

3. The wave is travelling along the length of string, while points on the string are moving perpendicular to the length of the string.

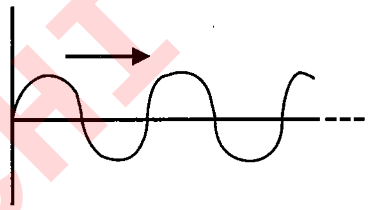

Fig. 5.61

In one time period (cycle), the wave moves forward by one wavelength while the particle on string travels a distance of 4 times the amplitude. Here, time period.

$$T=\frac{1}{f}=\frac{1}{50}\text{ s}=0.02\text{ s}$$

And wave speed, $v=f\lambda=50\times0.5=25$ m/s

a. Time taken by wave to travel a distance of 8 m, $t_1 = 8/25$ s

b. Time taken by a point on string to travel a distance of 8 m,

$$t_2=\frac{8}{4A}\times T=\frac{8}{4\times10^{-2}}\times0.02=4\text{ s}$$

When the amplitude gets doubled, the answer (c) for (a). does not change.

For (b),

$$t_2'=\frac{8}{4A'}\times T=\frac{8}{4\times2A}\times0.02=2\text{ s}$$

4. Let in time t, wave gets displaced by x w.r.t. stationary medium, the displacement of wave w.r.t. moving frame is $x'=x+vt$.

So, equation of wave in moving frame is

$$Y=A\sin[\omega t-k(x'-vt)] \quad [\because x=x'-vt]$$

$$Y=A\sin[(\omega+kv)t-kx']$$

Speed of wave in moving frame

$$V'=\frac{dx'}{dt}=\frac{\omega+kv}{k}=\left(\frac{\omega}{k}+v\right)$$

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5. Let us consider the wave

$$Y = A\cos(\omega t - kx),$$

then its energy density (energy per unit volume) is given by $W = \rho A^2\omega^2 \sin^2(\omega t - kx)$ [where ρ is of medium, for string waves, $\mu = \rho S$].

Let us consider $x = 0$, $t_0 = 0$ at which amplitude is maximum.

At $\qquad t = t_0 + \dfrac{T}{6}$

And $\qquad t = A\cos\left[\dfrac{\omega T}{6}\right]$

and the energy density is

$$W = \rho A^2\omega^2 \sin^2\left[\frac{\omega T}{6}\right] = \rho A^2\omega^2 \sin^2\frac{\pi}{3}$$

$$W = (\rho A^2\omega^2)\frac{3}{4}$$

From given data, $W = W_0$

$$\Rightarrow \rho A^2\omega^2 = \frac{4}{3}W_0$$

And the mean space density of total energy

$$= \frac{\rho A^2\omega^2}{2} = \frac{1}{2}\times\frac{4}{3}W_0 = \frac{2W_0}{3}$$

6. $v = \sqrt{\dfrac{450}{50\times10^{-3}}} = 30\sqrt{10}$ m/s

i. The average power that can be carried by the transverse wave in wire is

$$P_{av} = 2\pi^2\mu VA^2f^2$$

If the amplitude A equals to 20% of wavelength λ, then

$$A = 0.2\,\lambda$$

The maximum power

$$P_A = 2\pi^2\mu V(0.2\lambda)^2 f^2 = 0.8\mu v^3 = 1080\sqrt{10}\text{ W}$$

(ii) $\qquad \rho\,\alpha\,\mu\, v = \mu\sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{\mu}}$

$$\Rightarrow \rho \propto \sqrt{\mu}$$

So if μ is doubled, power becomes $\sqrt{2}$ times.

7. a. The wave equation may be written as $y = y_0 e^{-\frac{1}{T^2}\left(t-\frac{x}{\lambda/T}\right)^2}$.

Comparing with the general equation $y = f(t - x/v)$, we see that

$$v = \frac{\lambda}{T} = \frac{4\text{ cm}}{1.0\text{ s}} = 4\text{ cm/s}$$

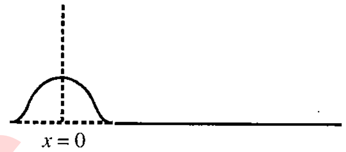
b. putting $x = 0$ in the given equation,

$$f(t) = y_0e^{-(t/T)^2} \qquad \text{(i)}$$

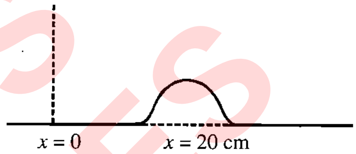
c. Putting $t = 0$ in the given equation

$$g(x) = y_0e^{-(x/\lambda)^2} \qquad \text{(ii)}$$

d.



e.



8. A wave pulse is a disturbance localized only in a small part of space at a given instant (as shown in Fig. 5.62) and its shape does not change during propagation. Though a pulse can be represented by exponential or trigonometric functions also, it is usually expressed by the form

$$y = \frac{a}{b + (x \pm vt)^2}$$

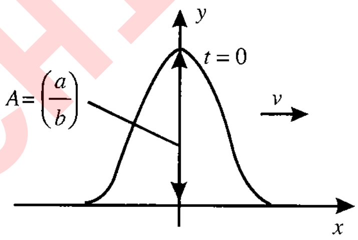


Fig. 5.62

Comparing the above with the given pulse we find that

$$f(x \pm vt) = (x + 2t)^2$$

i.e., the pulse is travelling along negative x-axis with velocity 2 m/s.

Further amplitude is the maximum value of wave function which will be when

$$(x + 2t)^2 = 0$$

So, $\qquad A = y_{max} = \dfrac{10}{5} = 2$

9. As elevator accelerates up at 2 m/s^2, tension in the string is $T = m(g + a) = 4(10 + 2) = 48$ N.

Linear mass density of string is $\mu = 19.2\times10^{-3}$ kg/m

Speed of transverse waves on string is

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{48}{19.2\times10^{-3}}} = 50\text{ m/s}$$

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Objective Type

1. **d.** $v = 960$ m/s; $n = \dfrac{3600}{60}$ Hz.

So $$\lambda = \frac{v}{n} = \frac{960}{60} = 16\text{ m}$$

2. **d.** $y = 8\sin 2\pi\left(\dfrac{x}{10} - 2t\right)$ given by comparing with standard equation

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$\lambda = 10\text{ cm}$$

So phase difference = $(2\pi/\lambda) \times$ path difference

$$= \frac{2\pi}{10} \times 2 = \frac{2}{5} \times 180° = 72°$$

3. **a.** Standard equation of travelling wave $y = A\sin(kx - \omega t)$
By comparing with the given equation

$$y = 10\sin(0.01\pi x - 2\pi t)$$

$$A = 10\text{ cm}, \omega = 2\pi$$

Maximum particle velocity $= A\omega = 2\pi \times 10 = 63$ cm/s

4. **a.** Standard equation

$$y = A\sin(\omega t - kx + \phi_0)$$

In a given equation $\omega = 7\pi$, $k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{0.04\pi} = 175\text{ m/s}$$

5. **b.**

$$y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

$$y_2 = a_2\sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$$

Phase difference

$$= \left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right) - \left(\omega t - \frac{2\pi x}{\lambda}\right) = \left(\phi + \frac{\pi}{2}\right)$$

$$\text{Path difference} = \frac{\lambda}{2\pi} \times \text{Phase difference} = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$$

6. **b.** Maximum particle velocity = 4 wave velocity

$$A\omega = 4f\lambda$$

$$y_0 2\pi f = 4f\lambda$$

$$\lambda = \frac{\pi y_0}{2}$$

7. **d.** $$y = 4\sin\left(4\pi t - \frac{\pi}{16}x\right)$$

$$\omega = 4\pi,\ k = \pi/16$$

$$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64\text{ cm/s}$$

in positive x-direction.

8. **a.**

$$x = a\sin\left(\omega t + \frac{\pi}{6}\right)$$

$$x' = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$$

Therefore, phase difference $= (\pi/2) - \pi/6$
$= (\pi/3)$

9. **b.** Maximum particle velocity $= a_0\omega = 2\pi a_0 v$
Wave velocity $= v\lambda$

Given that $2\pi a_0 v = 3v\lambda$

or $\lambda = (2\pi a_0/3)$

10. **d.** Maximum particle velocity $= \omega A$

Wave velocity $= \dfrac{\omega}{K}$

Therefore, the required ratio

$$= \frac{\omega A}{\omega/K}$$

$$= AK$$

$$= 60 \times 10^{-6} \times 6$$

$$= 3.6 \times 10^{-4}$$

11. **b.** $\omega = \dfrac{2\pi}{0.01}$ and $k = \dfrac{2\pi}{0.30}$

$$v = \frac{\omega}{k} = \frac{2\pi}{0.01} \times \frac{0.30}{2\pi} = 30\text{ m/s}$$

12. **c.** At $t = 2$ s

$$y = \frac{1}{[1 + (x-1)^2]}$$

or $x - vt = x - 1 \Rightarrow 1 = vt$

$$\Rightarrow 1 = v \times 2$$

$$\Rightarrow v = 0.5\text{ m/s}$$

13. **b.** $$v_7 = \sqrt{\frac{3R(273+7)}{M}}$$

$$v_{47} = \sqrt{\frac{3R(273+47)}{M}}$$

$$\frac{v_7}{v_{47}} = \sqrt{\frac{280}{320}} = \sqrt{\frac{7}{8}}$$

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Now $$\frac{\sin i}{\sin r} = \frac{v_7}{v_{47}} = \sqrt{\frac{7}{8}}$$

$$\sin r = \sin 30° \times \sqrt{\frac{8}{7}} = \sqrt{\frac{2}{7}}$$

or $$r = \sin^{-1}\sqrt{\frac{2}{7}}$$

14. c. $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\frac{\pi}{4} = \frac{2\pi}{\lambda} \times \frac{1.25}{100}$$

On solving, we get

$$\lambda = \frac{1}{10}\text{ m/s}$$

$$u = n\lambda = 1000 \times \frac{1}{10} = 100 \text{ m/s}$$

15. c. Suppose at an instant t, the x-coordinate of a point with reference to moving frame is x_0. Since, at this moment, origin of moving frame is at distance vt from origin of the fixed reference frame, therefore, putting this value of x in the given equation, we get

$$y = a\cos[\omega t - k(vt + x_0)]$$
$$y = a\cos[(\omega - kv)t - x_0)]$$

Hence, option (c) is correct.

16. b. $v = 1$ m/s, $\nu = 100$ Hz

$$\lambda = \frac{v}{\nu} = \frac{1}{100}\text{ m} = 1 \text{ cm}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times 2.75 \text{ rad} = 5.5\,\pi \text{ rad} = \frac{11\pi}{2}\text{ rad}$$

17. a. $y = 0.02 \sin(x + 30t)$

Comparing with standard equation

$$y = A\sin(Kx + \omega t),\ \omega = 30,\ K = 1$$

Velocity of wave,

$$v = \frac{\omega}{K} = \frac{30}{1} = 30 \text{ m/s}$$

Expression $v = \sqrt{\frac{T}{m}}$ gives

Tension $$T = v^2 m = (30)^2 \times 10^{-4}$$
$$= 0.09 \text{ N}$$

18. b. Tension T in the wire $= v^2\rho = (400)^2 \times 10^{-3} = 160$ N

Force applied

$$F = \frac{T(m_1 + m_2)}{m_1}$$

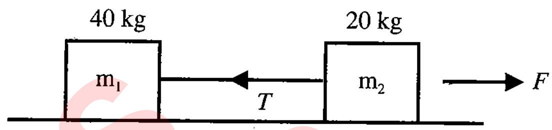

Fig. 5.63

$$= 160 \times \frac{(40+20)}{40} = 240 \text{ N}$$

19. b. $$y(x, t) = f(x - vt)$$

$$y = (x, 0) = \frac{4\times 10^{-3}}{8 - x^2}$$

For a travelling wave in the x-direction

$$y(x, t) = \frac{4\times 10^{-3}}{8-(x-5t)^2}$$

20. d. Assume $1/\sqrt{a} = A\cos\theta$ (i)

$$\frac{1}{\sqrt{b}} = A\sin\theta \quad \text{(ii)}$$

On simplifying, we get $y = A\sin(\omega t + \theta)$

Squaring and adding Eqs. (i) and (ii) $A = \sqrt{\frac{a+b}{ab}}$

21. b. Given $\omega = 3\pi$

$$f = \frac{\omega}{2\pi} = 1.5$$

Also $\Delta x = 1.0$ cm

Given, $$\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$$

$$\lambda = 16 \text{ cm}$$

$$v = f\lambda = 16 \times 1.5 = 24 \text{ cm/s}$$

22. b.

$$\frac{I_1}{I_2} = \frac{a_1^2 f_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$$

23. b.

$$y(x, t=0) = \frac{6}{x^2} \quad \text{then} \quad y(x,t) = \frac{6}{(x-2t)^2}$$

$$\frac{\partial y}{\partial t} = \frac{24}{(x-2t)^3} \quad \text{at} \quad x = 2, \quad t = 2$$

$$V_y = \frac{24}{(-2)^3} = -3 \text{ m/s}$$

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24. b.

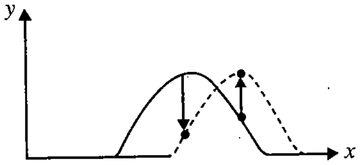

Fig. 5.64

Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particle.

25. a. Slope at any point on the string in wave motion represents the ratio of particle speed to wave speed.
Therefore, slope B < slope A
Hence $R_A > R_B$.

26. c.

$$P=\frac{1}{2}\mu\omega^2A^2V \quad \text{using} \quad V=\sqrt{\frac{T}{\mu}}$$

$$P=\frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

$$\omega=\sqrt{\frac{2P}{A^2\sqrt{T\mu}}},\ f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$$

Using the given data, we get f = 30 Hz.

27. b. In the figure, 'C' reaches the position where 'A' already reaches after $\omega t = \pi/2$ and 'A' reaches the position where 'B' already reaches after $\omega t = \pi/2$.

28. b. The equation for wave A can be rewritten as

$$y=A\sin[kx-\omega t-\phi]$$
$$=A\sin[k(x-\phi/k)-\omega t]$$
$$=A\sin\left[kx-\omega\left(t+\frac{\phi}{k}\right)\right]$$

While equation of wave B is $y = A\sin(kx - \omega t)$.

Comparing above equations, we can easily conclude that A is at a distance ahead of ϕ/k from B or wave A is ahead of B by a time difference of ϕ/ω. So, (b) is the correct option.

Remember! In y versus t 'ahead of means to the left of' while in y versus x 'ahead of means to the right of' if the wave travels in positive x-direction and vice versa.

29. a. Amplitude of wave,

$$A=\frac{2.0\,\text{cm}}{2}=1\,\text{cm}$$

Frequency of wave, f = 125 Hz

Wavelength of wave, λ = 15.6 cm = 0.156 m

Let equation of wave be, $y = A\sin(kx - \omega t + \phi)$ where $k = 2\pi/\lambda = 40.3$ rad/m and $\omega = 2\pi f = 786$ rad/s

Using initial conditions,

$$y(0,0)=0=A\sin\phi$$

and $$\frac{\partial y}{\partial t}(0,0)=-A\omega\cos\phi<0$$

We get, $\phi = 0$

So, the equation of wave is

$$y = (1\text{ cm})\sin[(40.3\text{ rad/m})x-(786\text{ rad/s})t]$$

30. b. Phase difference, $\Delta\phi = k\Delta x$

$$\Delta\phi=\frac{2\pi}{\lambda}\times\Delta x=\frac{2\pi}{3\,\text{cm}}\times16.5\text{ cm}=11\pi$$

So, the phase difference between two waves among the given option is 5π.

31. c. For the wave $y = A\sin(\omega t - kx)$, $v_0 = A\omega$ where A is, the maximum displacement.
For the given condition,

$$\frac{A}{2}=A\sin(\omega t-kx)$$

$$\sin(\omega t-kx)=\frac{1}{2}$$

And $$\frac{\partial y}{\partial t}=A\omega\cos(\omega t-kx)=A\omega\frac{\sqrt{3}}{2}=\frac{\sqrt{3}v_0}{2}$$

32. c. Power for a travelling wave on a string is given by

$$P = pv A^2\omega^2\cos^2(kx-\omega t)$$

For the displacement wave,

$$y = A\sin(kx-\omega t)$$

Power delivered is maximum when $\cos^2(kx - \omega t)$ is maximum, which would be the case when $\sin(kx - \omega t)$ is the least, i.e., displacement is minimum (acceleration is minimum). Power delivered is minimum when $\cos^2(kx - \omega t)$ is minimum, which would be when $\sin(kx - \omega t)$ is maximum, i.e., displacement is maximum (acceleration is maximum).

33. d. Let the power of source be P and it is placed at O. Then intensity at A and B would be given by

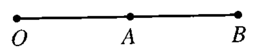

Fig. 5.65

$$I_A=\frac{P}{4\pi\times1^2} \quad \text{and} \quad I_B=\frac{P}{4\pi\times2^2}$$

Since, intensity ∝ (Amplitude)² × (Frequency)² (here, amplitude means displacement amplitudes), the frequency is same at both the points.

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$$\Rightarrow \frac{(\text{Amp})_A}{(\text{Amp})_B} = \sqrt{\frac{I_A}{I_B}} = \sqrt{\frac{2^2}{1^2}} = 2:1$$

34. **b.**

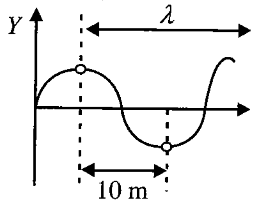

Fig. 5.66

Frequency of wave = 1/4 Hz

Wavelength of wave = λ = 2 × 10 = 20 m

Velocity of wave = $f\lambda$ = 5 m/s

35. **d.** For the wave, $y = A\sin(kx - \omega t)$, the wave speed is ω/k and the maximum transverse string speed is $A\omega$.

36. **c.** At the moment shown in the figure, particle at 1 is moving in the downward direction.

We have, $T = 1/0.1$ s = 10 s.

In one complete cycle, particle travels a distance, 4 times the amplitude. So, in time 10 min 15 s, i.e., 615 s which means 61 full + 1 half cycles, the distance travelled

= (4 × 3) × 61+ (2 × 3) × 1 = 732 + 6 = 738 cm

At time instant, the particle is moving in the upward direction.

37. **b.** Let the speed of wave be v, for crossing one wave crests to the other while travelling in the same direction, the surfing speed has to be greater than speed of the wave, i.e., v < 15 m/s.

Let wavelength of wave be λ m

While surfing in the same direction,

$$\lambda = (15 - v) \times 0.8$$

While surfing in the direction opposite to the wave motion,

$$\lambda = (15 + v) \times 0.6$$

$$(15 - v)\,0.8 = (15 + v)\,0.6$$

$$v = 15/7 \text{ m/s} = 2.143 \text{ m/s}$$

So, $\lambda = (15 - 2.143) \times 0.8 = 10.3$ m

38. **a.**

$$y = (0.2 \text{ m}) \sin[kx \pm \omega t]$$

For $x = 0, y = 0.1$ m

$$0.1 = 0.2 \sin(\omega t)$$

$$\Rightarrow \omega t = \pi/6 \quad \text{or} \quad 5\pi/6$$

So, $t_2 = 5\pi/6\omega$ and $t_1 = \pi/6\omega$

$$t_2 - t_1 = 2\pi/3\omega = 1/3f = 1.9 \text{ ms}$$

39. **d.** On going for one medium to another, frequency remains the same while wavelength and wave speed, both change. Amplitude may decrease or remain same depending on the fact that whether there is some absorption of energy at the boundary or not.

40. **b.**

$$y(x,t) = \frac{6}{(x-2t)^2}$$

$$\Rightarrow v_p = \frac{\partial y}{\partial t} = \frac{24}{(x-2t)^3}$$

$$v_p[x=2, t=2] = \frac{24}{-2^3} = -3 \text{ m/s}$$

41. **b.** The wave is travelling along the length of a string, while particles constituting the string are oscillation in a direction perpendicular to the length to string. In one time period (cycle), the wave moves forward by one wavelength while the particle on string travels a distance of 4 times the amplitude.

Here, $T = 1/f = 0.02$ s

Wave speed, $v = f\lambda = 25$ m/s

Time taken by wave to travel a distance of 8 m,

$$t_1 = 8/25 \text{ s} = 0.32 \text{ s}.$$

Time taken by particle on string to travel a distance of 8 m,

$$t_2 = \frac{8 \times T}{4 \text{ times amplitude}} = \frac{8}{4 \times 0.01} \times 0.02 = 4 \text{ s}$$

42. **d.** Let the power of source be P and it is placed at O.

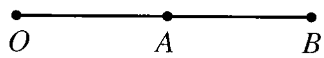

Fig. 5.67

Then, intensity at A and B would be given by

$$I_A = \frac{P}{4\pi \times 1^2}$$

And $$I_B = \frac{P}{4\pi \times 2^2}$$

Since, intensity ∝ (Amplitude) × (Frequency)² (here, amplitude means displacement amplitude)

The frequency is same at both the points.

$$\frac{(\text{Amp})_A}{(\text{Amp})_B} = \sqrt{\frac{I_A}{I_B}}$$

$$= \sqrt{\frac{2^2}{1^2}} = 2:1$$

43. **a.** Negative sign with 'ω' indicates that wave is propagating along positive x-axis.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2 \text{ m}$$

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And $$v = \frac{\omega}{k} = \frac{15\pi}{10\pi} = 1.5 \text{ m/s}$$

44. d. We know $\omega = 2\pi f = \frac{2\pi}{0.04}$

$$\Rightarrow f = 25 \text{ Hz}$$

Differentiating y w.r.t. twice, we have

$$y'' = \frac{-3\times 4\pi^2}{(0.004)^2}\sin\left(2\pi\left[\left(\frac{t}{0.04}\right)-\left(\frac{x}{0.01}\right)\right]\right)$$

For maximum acceleration

$$y_0'' = \frac{3\times 4\pi^2}{(0.004)^2} = 7.5\times 10^4 \text{ m/s}^2$$

45. a. At any instant t, the wave equation will express the variation of y with x which is equal to the shape of the string at an instant t.

46. a.

$$V_P(\max) = \left(\frac{dy}{dt}\right)_{\max} = 50 \text{ units}$$

$$V_\omega = \frac{\omega}{k} = \frac{100}{25} = 4 \text{ units}$$

$$\frac{V_P(\max)}{V_\omega} = 12.5$$

47. a.

$$y_1 = 10^{-6}\sin\left(100t + \frac{x}{50} + 0.5\right) \text{ m}$$

$$y_2 = 10^{-2}\cos\left(100t + \frac{x}{50}\right) \text{ m}$$

$$\Rightarrow y_2 = 10^{-2}\sin\left(100t + \frac{x}{50} + \frac{\pi}{2}\right)$$

Phase difference $= \frac{\pi}{2} - 0.5$

$= 1.07$ rad

48. d. $V = \frac{\omega}{k} = \frac{100}{0.02} = 5000 \text{ cm/s}$

49. a.

$$y = \frac{1}{1+x^2} \quad \text{at} \quad t = 0$$

and $$y = \frac{1}{1+(x-2)^2} \quad \text{at} \quad t = 4 \text{ s}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{x-(x-2)}{4-0} = \frac{2}{4} = 0.5 \text{ m/s}$$

50. c.

$$y = a\left[\frac{1+\cos(2\omega t - 2kx)}{2}\right]$$

$$y = \frac{a}{2} + \frac{a}{2}\cos(2\omega t - 2kx)$$

51. d.

$$y(x,t) = \frac{a}{(x\pm vt)^2 + b}$$

is another form of progressive wave equation propagating with a speed v.

Negative sign to be taken for propagation along $+x$-axis and positive sign to be taken to propagation along $-x$-axis.

52. c. $P = \frac{1}{2}\mu\omega^2 A^2 v$ where $v = \sqrt{\frac{T}{\mu}}$

53. d. $v = \sqrt{\frac{T}{\mu}}$

T can be calculated by using Hooke's law and on stretching μ also changes.

54. c. We start with a general form for a rightward moving wave,

$$y(x, t) = A\sin(kx - \omega t + \phi)$$

The amplitude given is $A = 2.0$ cm $= 0.02$ m.
The wavelength is given as,

$$\lambda = 1.0 \text{ m}$$

Wave number $= k = 2\pi/\lambda = 2\pi \text{ m}^{-1}$

Angular frequency,

$$\omega = vk = 10\pi \text{ rad/s}$$

$$y(x, t) = (0.02)\sin[2\pi(x - 5.0t) + \phi]$$

We are told that for $x = 0$, $t = 0$,

$$y = 0 \quad \text{and} \quad \frac{\partial y}{\partial t} < 0$$

i.e., $0.02\sin\phi = 0$ (as $y = 0$)

and $-0.2\pi\cos\phi < 0$

From these conditions, we may conclude that

$$\phi = 2n\pi \quad \text{where} \quad n = 0, 2, 4, 6, \ldots$$

Therefore,

$$y(x,t) = (0.02 \text{ m})\sin[(2\pi \text{ m}^{-1})x - (10\pi \text{ s}^{-1})t] \text{ m}$$

55. b. The amplitude, $A = 0.06$ m

$$\frac{5}{2}\lambda = 0.2 \text{ m}$$

$$\therefore \quad \lambda = 0.08 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{300}{0.08} = 3750 \text{ Hz}$$

$$k = \frac{2\pi}{\lambda} = 78.5 \text{ m}^{-1} \quad \text{and} \quad \omega = 2\pi f = 23562 \text{ rad/s}$$

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At $t = 0, x = 0, \quad \frac{dy}{dx} = \text{positive}$

and the given curve is a sine curve.
Hence, equation of wave travelling in positive x-direction should have the form,

$$y(x, t) = A \sin(kx - \omega t)$$

Substituting the values, we have

$$y = (0.06 \text{ m}) \sin[(78.5 \text{ m}^{-1})x - (23562 \text{ s}^{-1})t] \text{ m}$$

Multiple Correct Answers Type

1. **a., b., c., d.**

$$y = 10^{-4} \sin(60t + 2x)$$

$$y = a \sin(\omega t + kx)$$

Now, $k = 2, \quad 2\pi/\lambda = 2 \quad \text{or} \quad \lambda = \pi$ metre

Again, $\omega = 60$

or $2\pi f = 60 \quad \text{or} \quad f = \frac{60}{2\pi} \quad \text{or} \quad v = \frac{30}{\pi}\text{Hz}$

Again, $v = \frac{\omega}{k} = \frac{60}{2} \text{ m/s} = 30 \text{ m/s}$

2. **b., d.** Comparing with $y = a\cos(\omega t - kx)$

$$\omega = 500, k = 70$$

Speed of wave

$$= \frac{\omega}{k} = \frac{500}{70} = \frac{50}{7} \text{m/s}$$

$$k = 70$$

$$\frac{2\pi}{\lambda} = 70$$

or $\lambda = \frac{2\pi}{70} \text{m} = \frac{2\pi}{70} \times 100 \text{ cm} = \frac{20\pi}{7} \text{cm}$

3. **b., c.** Let the equation to the wave be

$$y = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right] \qquad \text{(i)}$$

where A is the amplitude of the wave and ϕ, phase angle. It is given that $y = a$, when $x = 0$ and $t = T/4$ and also that $\dot{y} = v$ when $x = \lambda/4$ and $t = T/4$
Substituting in (i),

$$y = a = A \sin\left(\frac{\pi}{2} + \phi\right)$$

$$\dot{y} = v = \frac{2\pi A}{T} \cos\left[2\pi\left(\frac{1}{4} - \frac{1}{4}\right) + \phi\right]$$

$$v = \frac{2\pi A}{T} \cos\phi$$

Putting $\phi = 0, y = a = A$, so that amplitude $A = a$

Also, $v = \frac{2\pi A}{T}[\cos 0] = \frac{2\pi a}{T}$

$$\frac{2\pi}{T} = \frac{v}{a}$$

Hence the equation to the wave is

$$y = a \sin\frac{v}{a}\left[t - \frac{Tx}{\lambda}\right]$$

$$y = a \sin\frac{v}{a}\left[t - \frac{x}{V}\right]$$

where $V = \frac{\lambda}{T}$ is the velocity of the wave in the gas.

4. **a., b., d.**

Given wave is $y = A\cos\frac{\pi}{2}[kx - \omega t - \alpha]$

Here wave number, $k \times \frac{\pi}{2} = \frac{2\pi}{\lambda}$ giving $\lambda = \frac{4}{k}$

Here $k = 0.005 \text{ cm}^{-1}$. Hence

$$\lambda = \frac{4}{0.005} \text{cm} = 8 \text{m}$$

Maximum velocity $V_m = A \times$ angular velocity.
Here angular velocity

$$= \frac{\pi\omega}{2} = \frac{3.14 \times 8}{2} = 12.56 \text{ rad/s}$$

Hence $V_m = 0.6 \times 12.56 \text{ m/s} = 7.53 \text{ m/s}$

Also, to produce stationary waves, the two waves should travel in opposite directions and have same frequency. The wave given by $y = A\cos\frac{\pi}{2}(kx + \omega t - \alpha)$ fulfils this condition.

5. **a., b., d.** It is a known fact as well as experimentally and analytically verified that wave speed depends on the properties of the medium and is same for the entire wave. The particle velocity is given by

$$v_P = \frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

where symbols have their usual meanings. It is clear from above expression that v_P depends upon amplitude and frequency of wave which are wave properties and are having different values for different particles at a particular instant.

6. **b., d.** For a travelling wave on a string, oscillation energy of an elemental length does not remain constant as the force exerted by neighbouring elements, i.e., tension is doing work on any element of string. Oscillation energy

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takes periodically. Oscillation energy of different elements of same length are not the same, it can be easily shown by taking two elements of same length on string.

7. **a., b.** $y(x,0) = f(x)$. So, shape of string at $t = 0$ is given by $y = f(x)$

As velocity of wave $dx/dt = +a$, is constant, so shape of string does not change, or we can say $(x - at)$ is constant. Thus, the shape of string remains the same.

As a is –ve and constant, so $dx/dt =$ –ve and hence, wave is moving along –ve x-direction.

Speed $= -a$ and not a as speed cannot be –ve

8. **b., c., d.** Frequency is the property of source while velocity is the property of medium and wavelength is the property of both medium and source. So, wavelength and velocity of wave must change as the medium change, while frequency remains same. On the boundary some absorption can be there, as a result the amplitude (and hence intensity) can decrease as the medium changes.

Amplitude will either decrease or remain the same but it can never increase due to change in medium (assuming no external source is providing energy).

9. **a., b., c., d.** $v_p = v \times \partial y/\partial x$ for $y = A \sin(\omega t - kx)$, we have

$$v_p = kv \times A\cos(\omega t - kx) = \omega A\cos(\omega t - kx)$$

i.e., it is varying

Also, $\quad v_p \propto \omega \quad$ and $\quad v_p \propto A$

10. **a., b., c.** Wavelength of a wave is a property of source and medium both. So, wavelength can change if either frequency or speed of wave or both change. Here, medium property (like tension in string) can change freq. may change which causes the change in the speed of wave, or source frequency may change.

11. **a., b., d.** Mechanical waves can be transverse on a liquid surface and this is possible only because of surface tension.

In solids, $\quad v_{\text{longitudinal}} > v_{\text{transverse}}$

Transverse waves are possible only on the surface of a liquid because they require the property of rigidity. All non-mechanical waves found till now are transverse in nature.

12. **a., b., c., d.** The statement (a) is supported by water waves. An elastic medium is required for mechanical waves only.

So, option (d) is also correct.

The other two options (b) and (c) are also correct.

13. **b., c., d.**

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta x = \frac{\lambda}{2\pi}\Delta\phi$$

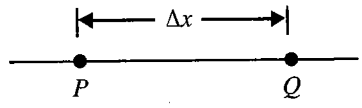

Fig. 5.68

$$= \frac{\lambda}{2\pi}\pi = \frac{\lambda}{2}$$

Let for P: $\quad y_1 = A\sin\omega t$, then

For Q: $\quad y_2 = A\sin(\omega t - \pi) = -A\sin\omega t$

We see that $\quad y_1 = -y_2$

and $\quad |y_1| = |y_2|$

Frequency of both particles should be same because the same wave passes through them.

14. **a., b., c., d.** For a point source

$$I \propto \frac{1}{r^2}$$

For a line source

$$I \propto \frac{1}{r}$$

For a plane wave, intensity remains same because there is no spreading of wave.

15. **a., c., d.**

$$y = f(x - vt)$$

Particle velocity.

$$v_P = \frac{dy}{dt} = -v\,f'(x - vt)$$

To find velocity of wave

$$\frac{d}{dt}(x - vt) = 0$$

$$\frac{dx}{dt} = v$$

16. **a., c.**

$$v_{\max} = a\omega = a(2\pi f)$$

Given that $\quad a(2\pi f) = \frac{v}{10} = \frac{10}{10} = 1$

$$f = \frac{1}{2\pi a} = \frac{10^3}{2\pi}\text{Hz}$$

As $\quad v = f\lambda$

or $\quad \lambda = \frac{v}{f} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\,\text{m}$

17. **b., d.** Since A is moving upwards, after an elemental time interval, the wave will be as shown dotted in Fig. 5.69. It

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means, the wave is travelling leftwards. Therefore, option (a) is wrong.

Displacement amplitude of the wave means maximum possible displacement of medium particles, due to propagation of the wave which is equal to the displacement at B at the instant shown in the figure. Hence, option (b) is correct.

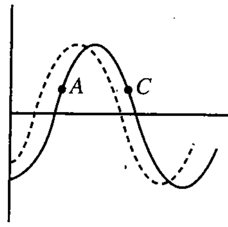

Fig. 5.69

From the figure, it is clear that C is moving downwards at this instant. Hence, option (c) is wrong.

The phase difference between two points will be equal to $\pi/2$ if distance between them is equal to $\lambda/4$. Between A and C, the distance is less than $\lambda/2$. It may be equal to $\lambda/4$. Hence, phase difference between these two points may be equal to $\pi/2$. Therefore, option (d) is correct.

18. **b., d.** A travelling wave is characterized by wave functions of the type $y = f(vt + x)$ or $y = f(vt - x)$. The function $y = a \sin(bx + ct)$ represents a wave travelling in the negative x-direction and the function $y = a \sin(bx - ct)$ a wave in positive x-direction. Hence, the correct choices are (b) and (d).

19. **b., c.** The equation has to be reduced to the form

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$= A\sin 314\left(\frac{t}{0.5\,\text{s}} - \frac{x}{100\,\text{m}}\right)$$

$$= A\sin 2\pi\left(\frac{50t}{0.5\,\text{s}} - \frac{x50}{100\,\text{m}}\right)$$

$$= A\sin 2\pi\left(\frac{t}{0.01\,\text{s}} - \frac{x}{2\,\text{m}}\right)$$

$$n = \frac{1}{T} = \frac{1}{0.01} = 100\,\text{Hz}$$

and $\qquad \lambda = 2\text{ m}$

20. **a., b., d.** Due to propagation of a wave the energy density at a point is given by

$$E = I/v$$

where I is intensity at that point and v is wave propagation velocity.

It means energy density E is directly proportional to intensity I.

If power emitted by a point source is P then intensity at a distance r from it is equal to

$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$

Hence, the shape of the curve between I and r will also be same as that given in figure of the question.

Hence, option (a) is correct.

If the source is a plane sound source then intensity at every point in front of the source will be same if damping does not take place. But if damping takes place then the amplitude of oscillation of medium particles decreases with distance. Hence, the intensity decreases with the distance from the source. In that case, the curve between I and r may have the same shape as shown in the figure given in the question. Hence, option (b) is also correct.

If the source is a plane source, intensity at every point of the source will be the same. But if power of the source is decreasing with time then intensity will also decrease with time. But at an instant, intensity at every point in front of source will be same. Therefore, the energy density at every point in front of source will also be same, though it will decrease with time. Hence, option (c) is wrong.

Intensity, $\qquad I = 2\pi^2 n^2 a^2 \rho v$

Since, intensity $I \propto \rho$ (density of medium) and density I is decreasing with distance, therefore, the density ρ also decreases with distance from the source. Hence, option (d) is also correct.

21. **a., b., c.** Comparing the given equation of travelling wave with

$$y = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$

Amplitude $= a$, angular frequency $\omega = b$ and $2\pi/\lambda = c$ or wavelength $\lambda = 2\pi/c$

$$\frac{a}{\lambda} = \frac{ac}{2\pi}$$

Therefore, option (a) is correct.

Velocity oscillation amplitude of medium particles is $a\omega = ab$.

Wave propagation velocity $v = \omega\lambda/2\pi = b/c$

$$\frac{a\omega}{v} = ac$$

Hence, option (b) is also correct.

Relative deformation or strain produced in the medium is $\varepsilon = u/v$, where u is particle's velocity and v is wave velocity. Since v is property of the medium, therefore, ε is directly proportional to velocity of the medium particles (u).

ε will be maximum possible when u is maximum. Hence, option (c) is also correct.

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22. b., c., d. $y = 4\sin\left[\frac{\pi}{16}(16t + x)\right]$

Compared with $y = a\sin\left[\frac{2\pi}{\lambda}(vt + x)\right]$

Also, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(v\Delta t)$

23. a., d. $v = \sqrt{\frac{T}{\mu}}$

For equilibrium $Mg = mg \sin 30 = T$

$$M = m/2$$

$$100 = \sqrt{\frac{Mg}{9.8 \times 10^{-3}}} = \sqrt{\frac{M(9.8)}{9.8 \times 10^{-3}}}$$

$$100 = \sqrt{M(1000)}$$

$$M = 10 \text{ kg} \quad \text{and} \quad m = 20 \text{ kg}$$

24. b., c., d.

$$y(x,t) = \frac{0.8}{16\left[\left(x + \frac{5}{4}t\right)^2 + \frac{5}{16}\right]}$$

$$y(x,t) = \frac{0.05}{\left(x + \frac{5}{4}t\right)^2 + \frac{5}{16}}$$

where wave velocity = 5/4 m/s

Distance travelled by the wave with this velocity in 2 s is

$$5/4\ (2) = 2.5 \text{ m}$$

$y(x, t)$ will be maximum when $4x + 5t = 0$

$$y_{max} = \frac{0.8}{5} = 0.16 \text{ m}$$

Assertion-Reasoning Type

1. **b.** In a transverse vibration, the mean distance between the successive vibrating particles remains constant. Only crests and troughs are formed.
2. **d.** Amplitude of a progressive longitudinal wave is the same at all points of a medium, assuming there is no attenuation. It is the instantaneous displacement of a particle from the mean position that differs and depends upon the phase angle of the wave.
3. **d.** Every small segment is acted upon by forces from both sides of it hence energy is not conserved, rather it is transmitted by the element.
4. **a.** Two waves moving in uniform string with uniform tension shall have same speed and may be moving in opposite directions.
5. **a.** In the first case the waves produced are transverse and in the second case the waves generated are longitudinal.

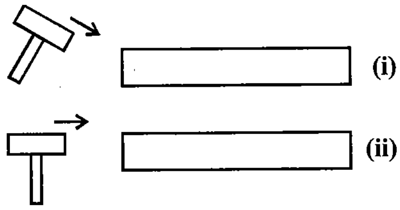

Fig. 5.70

6. **a.** Since the wavefronts are plane, the amount of energy passing per unit time per unit area remains same.
7. **d.** For a given velocity v_{max} depends on the frequency of the wave.
8. **a.** The potential energy of the element is the work done to stretch it from dx to dl.

$$dU = F(dl - dx)$$

$$= F\left(\sqrt{(dx)^2 + (dy)^2} - dx\right)$$

$$= F\,dx\left[\left(1 + \frac{dy}{dx}\right)^{\frac{1}{2}} - 1\right]$$

$$= \frac{1}{2}F\,dx\left(\frac{\partial y}{\partial x}\right)^2$$

Assuming that the disturbance is small.

9. d. $P = \frac{\omega^2 A^2 F}{2V}$

But $V = \sqrt{\frac{F}{\mu}}$

$$\Rightarrow P = \frac{\omega^2 A^2 F}{2\sqrt{F}/\sqrt{\mu}} = \omega^2 A^2\sqrt{\mu F}$$

$$P \propto \sqrt{F}$$

10. a. A compression is a region of medium in which particles come closer means distance between the particles become less than the normal distance between them. Thus there is a temporary decrease in volume and a consequent increase in density of medium.

Similarly, in rarefaction particles get farther apart and a consequent decrease in density.

Comprehension Type

For Problems 1–5

Sol. 1. b., 2. a., 3. c., 4. d., 5. c.

The wave is travelling along the positive x-axis

$\therefore \quad y = A\sin[kx - \omega t + \phi]$

at $\quad x = 0, y = A\sin[-\omega t + \phi]$

Also, at $\quad t = 0, y = A\sin\phi = A/2,$

$\sin\phi = 1/2$

$\Rightarrow \phi = (\pi/6), (5\pi/6), \ldots$ (i)

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and at $t = 0.05$ s, $y = A\sin\left(\frac{-\omega}{20}+\phi\right)=0$

or, $\frac{-\omega}{20}+\phi=0,\ \pi,\ 2\pi,\ \ldots$ (ii)

For $t = 0.05$ s and $x = 1$ m,

$$y = A\sin\left(k-\frac{\omega}{20}+\phi\right)=0$$

Since, $\lambda = 2$ m

$\therefore$ $\pi-\frac{\omega}{20}+\phi=0,\ \pi,\ 2\pi,\ \ldots$ (iii)

From Eqs. (i), (ii) and (iii), we get

$$\phi = \pi/6$$

$$\phi-\frac{\omega}{20}=0$$

$$\omega=\frac{10\pi}{3}$$

$$\therefore \quad f=\frac{\omega}{2\pi}=\frac{5}{3}\text{ Hz}$$

Velocity of wave is

$$V = \lambda f = (2)(5/3) = 10/3 \text{ m/s}$$

Maximum velocity of the particle is

$$V_{max} = \omega A = (10\pi/3)(10 \times 10^{-3}) = \pi/30 \text{ m/s}$$

Tension in the string is

$$T = \mu V^2 = (0.25)(10/3)^2 = 25/9 \text{ N}$$

The equation of the wave is

$$y = 10\sin[\pi x - (10/3)\pi t + (\pi/6)]$$

For Problems 6–9

Sol. 6. c., 7. a., 8. a., 9. d.

Mass per unit length of the string is

$$\mu = Ad = (0.80\,\text{mm}^2)\times(12.5\text{ g/cm}^3)$$

$$=(0.80\times10^{-6}\,\text{m}^2)\times(12.5\times10^3\,\text{kg/m}^3)=0.01\text{ kg/m}$$

Speed of transverse waves produced in the string

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{64}{0.01\text{ kg/m}}}=80\text{ m/s}$$

The amplitude of the source is $a = 1.0$ cm and the frequency is $f = 20$ Hz. The angular frequency is $\omega = 2\pi f = 40\ \pi$/s. Also at $t = 0$, the displacement is equal to its amplitude, i.e., at $t = 0$, $y = a$. The equation of motion of the source is, therefore,

$$y=(1.0\text{ cm})\cos[(40\pi\text{s}^{-1})t] \qquad \text{(i)}$$

The equation of the wave travelling on the string along the positive X-axis is obtained by replacing t by $[t - (x/v)]$ in Eq. (i). It is, therefore,

$$y=(1.0\text{ cm})\cos[(40\pi\text{s}^{-1})\{t-(x/v)\}]$$

$$=(1.0\text{ cm})\cos[(40\pi\text{s}^{-1})t-\{(\pi/2)\text{m}^{-1}\}x] \qquad \text{(ii)}$$

The displacement of the particle at $x = 50$ cm at time $t = 0.05$ s is obtained from Eq. (ii).

$$y=(1.0\text{ cm})\cos[(40\pi\text{ s}^{-1})(0.05\text{ s})$$

$$-\{(\pi/2)\text{m}^{-1}\}(0.5\text{ m})]$$

$$=(1.0\text{ cm})\cos[2\pi-(\pi/4)]$$

$$=1.0\text{ cm}/\sqrt{2}=0.71\text{ cm}$$

The velocity of the particle at position x at time t is also obtained from Eq. (ii).

$$V=\frac{\partial y}{\partial t}=-(1.0\text{ cm})(40\pi\text{ s}^{-1})\sin[(40\pi\text{ s}^{-1})t-\{(\pi/2)\text{m}^{-1}\}x]$$

$$=-\left(40\pi\frac{\text{cm}}{\text{s}}\right)\sin\left(2\pi-\frac{\pi}{4}\right)$$

$$=-\frac{40\pi}{\sqrt{2}}\text{cm/s}=-89\text{ cm/s}$$

For Problems 10–13

Sol. 10. d., 11. b., 12. b., 13. b.

$$\lambda=2(4.5-2.5)=4\text{ cm};\quad v=n\lambda$$

$$\Rightarrow n=\frac{40}{4}=10\,\text{Hz}$$

$$\phi=\frac{2\pi}{\lambda}\Delta x=\frac{5\pi}{4}\text{rad}\quad\Rightarrow\ \phi=\frac{2\pi}{T}\Delta t$$

$$\Delta t=\frac{\phi}{2\pi n}=\frac{\pi/3}{2\pi 10}=\frac{1}{60}\text{s}$$

Velocity of p should be maximum, as it is at mean position.

$$v_p=\omega A=2\pi fA=2\pi\times10\times\frac{2}{100}$$

$$=1.26\text{ m/s}$$

This velocity should be –ve, because slope at p is +ve.

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For Problems 14–16

Sol. 14. c., 15. a., 16. b.

Since, the wave is a plane travelling wave, intensity at every point will be the same.

Since, initial phase of particle at $x = 0$ is zero and the wave is travelling along positive x-direction equation of the wave will be of the form

$$\delta = a \sin\omega\left(t - \frac{x}{v}\right) \quad \text{(i)}$$

Let intensity of the wave be I, then space density of oscillation energy of medium particles will be equal to

$$E = \frac{I}{v}$$

But, $\quad I = 2\pi^2 n^2 a^2 \rho v$

Therefore $\quad E = 2\pi^2 n^2 a^2 \rho = 0.16\pi^2 \text{ J/m}^3$

$$a^2 n^2 = 4\times10^{-4}$$

or, $\quad an = 0.02 \quad$ (ii)

Shear strain of the medium is

$$\phi = \frac{d}{dx}\delta$$

Differentiating Eq. (i),

$$\phi = -\frac{a\omega}{v}\cos\omega\left(t - \frac{x}{v}\right)$$

Modulus of shear strain f will be maximum when

$$\cos\omega\left(t - \frac{x}{v}\right) = \pm1$$

$\therefore$ Maximum shear strain $8\pi\times10^{-5}$

$$\phi_0 = \frac{a\omega}{v}$$

but it is equal to

$$\frac{a\omega}{v} = 8\pi\times10^{-5}$$

where

$$\omega = 2\pi n$$

$$an = 4v\times10^{-5} \quad \text{(iii)}$$

Solving Eqs. (ii) and (iii), $v = 500$ m/s

Since, the wave is travelling along positive x-direction, therefore, phase difference between particles at points (1 m, 1 m, 1 m) and (2 m, 2 m, 2 m) is due to difference between their x coordinates only.

The phase difference is given by

$$\Delta\theta = 2\pi\frac{\Delta x}{\lambda}$$

$$\Delta x = (x_2 - x_1) = (2-1)\text{m} = 1 \text{ m}$$

$$\lambda = \frac{2\pi\Delta x}{\Delta\theta} = 2.5 \text{ m}$$

But $v = n\lambda$, therefore,

$$n = \frac{v}{\lambda} = 200 \text{ Hz}$$

Substituting $n = 200$ Hz in Eq. (ii),

$$a = 1\times10^{-4} \text{ m}$$

Angular frequency, $\omega = 2\pi n = 400\,\pi$ rad/s. Substituting all these values in Eq. (i),

$$\delta = 10^{-4}\sin\pi(400t - 0.8x)\text{m}$$

Since, due to propagation of the wave, shear strain is produced in the medium, the wave is a plane transverse wave.

For Problems 17–19

17. a. The equation of wave moving in negative x-direction, assuming origin of position at $x = 2$ and origin of time (i.e., initial time) at $t = 1$ s.

$$y = 0.1\sin(4\pi t + 8x)$$

Shifting the origin of position to left by 2 m, to $x = 0$. Also shifting the origin of time backwards by 1 s, that is to $t = 0$ s.

$$y = 0.1\,[4\pi(t-1) + 8(x-2)]$$

18. c. As given the particle at $x = 2$ is at mean position at $t = 1$ s.
$\therefore$ Its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\pi$ m/s.

19. d. Time period of oscillation $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$s

Hence at $t = 1.125$ s, that is, at $T/4$ seconds after $t = 1$ s, the particle is at rest at extreme position. Hence instantaneous power at $x = 2$ at $t = 1.125$ s is zero.

For Problems 20–21

20. b. $v_1 = \sqrt{\dfrac{F}{\mu}}$

$$v_2 = \sqrt{\frac{F}{4\mu}} = \frac{1}{2}\sqrt{\frac{F}{\mu}}$$

$$v_3 = \sqrt{\frac{F}{9\mu}} = \frac{1}{3}\sqrt{\frac{F}{\mu}}$$

$$v_4 = \sqrt{\frac{F}{16\mu}} = \frac{1}{4}\sqrt{\frac{F}{\mu}}$$

Total time taken

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$$= \frac{L}{v_1} + \frac{L}{v_2} + \frac{L}{v_3} + \frac{L}{v_4} = \frac{10L}{\sqrt{F/\mu}}$$

21. c. Frequency of wave is same on all the four strings. So,

$$\lambda_1 = \frac{v_1}{f}, \qquad \lambda_2 = \frac{v_2}{f}$$

$$\lambda_3 = \frac{v_3}{f} \qquad \lambda_4 = \frac{v_4}{f}$$

$$\lambda_4 : \lambda_3 : \lambda_2 : \lambda_1 = \frac{1}{4} : \frac{1}{3} : \frac{1}{2} : 1 = 3 : 4 : 6 : 12$$

For Problems 22–24

Sol. 22. a., 23. a., 24. a.

Frequency $= \frac{1}{\text{Time period}} = \frac{1}{0.4} = 2.5 \text{ Hz}$

Amplitude $= \frac{1}{2} \times 0.3 \text{ m} = 0.15 \text{ m}$

Wave speed $= f\lambda = 2.5 \times 0.8 \text{ m} = 2 \text{ m/s}$

For Problems 25–27

Sol. 25. b., 26. a., 27. d.

$$\omega = 2\pi f = 6\pi \text{ rad/s}$$

and $$k = \frac{\omega}{v} = \frac{6\pi}{15} = 1.26$$

$$y = 0.1 \cos(1.26x - 18.8t)$$

At point 2.5 m from child and equation of displacement

$$y = 0.1 \cos(3.15 - 18.8t)$$

$$= -0.1 \cos(18.8)t$$

At $\lambda = 5$, time taken to reach 2.5 m $= T/2$

$$\Delta\phi = \frac{2\pi \, \Delta t}{T} = \pi$$

For Problems 28–32

Sol. 28. b., 29. c., 30. b., 31. a., 32. d.

$$v = \sqrt{\frac{T}{\mu}} = 20 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{20}{100} = 0.2 \text{ m} = 20 \text{ cm}$$

and $$k = \frac{2\pi}{\lambda} = 10\pi, \quad \omega = 2\pi f = 200\pi$$

So $$y = -0.02 \cos(10\pi x - 200\pi t)$$

–ve sign is because at $t = 0$ and $x = 0$, y is –ve

Wave velocity is constant for a medium but particle velocity keeps changing.

as $$v = y' = 4\pi \sin(10\pi x - 200\pi t)$$

$$\frac{d^2y}{dt^2} = -0.02\, x(200\pi)^2 \cos(10\pi x - 200\pi t)$$

For $$a_{\max} = -a\omega^2 = -7888 \text{ m/s}^2$$

$$|a_{\max}| = 7888 \text{ m/s}^2$$

As frequency doubles, λ becomes half, speed of wave remains same.

For Problems 33–36

Sol. 33. c., 34. b., 35. b., 36. d.

Time taken to reach other end is independent of frequency and amplitude $v = \sqrt{\frac{T}{m}}$

As m increases, velocity decreases.
So time taken will be more or will increase.
As T increases, velocity also increases.
So time taken will be less or it will decrease.

For Problems 37–38

Sol. 37. c., 38. a.

Let general wave equation is $y = A \sin(\omega t - kx + \phi)$

$$v = \frac{dy}{dt} = A\omega \cos(\omega t - kx + \phi)$$

for curve (1), $x = 0$

at $t = 0, x = 0$, we have $y = 0$

$\Rightarrow \quad 0 = A \sin[\phi] \Rightarrow \sin\phi = 0$

$\Rightarrow \quad \phi = 0$ or π

here $\phi = \pi$ (because velocity is negative)

for curve (2), $x = 7$ cm

at $t = 0, x = 7$ cm, $y = -1$

$-1 = \sin(-k \times 7 + \pi)$

$\Rightarrow \quad \sin(-7k + \pi) = -1/2$

$$\Rightarrow -7k + \pi = 2n\pi + \frac{7\pi}{6} \text{ or } 2n\pi + \frac{11\pi}{6}$$

here $$\Rightarrow -7k + \pi = 2n\pi + \frac{11\pi}{6}$$

(because at $t = 0$, velocity is positive)

$$\Rightarrow -7\left(\frac{2\pi}{\lambda}\right) = 2n\pi + \frac{5\pi}{6}$$

$$\Rightarrow \lambda = \frac{-14\pi}{\frac{5\pi}{6} + 2n\pi}$$

$$\Rightarrow \lambda = \frac{-84}{12n+5}$$

for $\quad n = -1, \lambda = 12$ cm

for $\quad n = -2, \lambda = \frac{84}{19}$ cm (not possible)

because $\lambda > 7$ cm

$$v = f\lambda = 100 \times \frac{12}{100} = 12 \text{ m/s}$$

For Problems 39–40

Sol. 39. d., 40. c.

From the graph it is clear

Wave velocity $v = \frac{(8-3)}{0.006}$

$$v = \frac{5\times 10^3}{6} \text{ m/s}$$

And wave velocity $v = f\lambda$

$$\frac{5\times 10^3}{6} = f(9-1) \Rightarrow f = \frac{5\times 10^3}{6\times 8} = 104 \text{ s}^{-1}$$

Alternate Method For 39–40

Let $y = A\sin(\omega t - kx + \phi)$

For curve (1):

$$-\frac{1}{\sqrt{2}} = 1\sin(\omega \times 0.002 + \phi)$$

$$\Rightarrow \sin(0.002\omega + \phi) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow 0.002\omega + \phi = \frac{5\pi}{4} \text{ or } \frac{7\pi}{4}$$

but as the velocity is downward

So $0.002\omega + \phi = \frac{5\pi}{4}$ (i)

For curve (2):

$1 = 1\sin(\omega \times 0.008 + \phi)$

$\Rightarrow 0.008\omega + \phi = 2\pi + \frac{\pi}{2}$ (ii)

from Eqs. (i) and (ii)

$6\omega = 1250\pi$

$$\Rightarrow f = \frac{1250\pi}{12\pi} = 104 \text{ Hz}$$

$$v = f\lambda = \frac{1250}{12} \times 8 = \frac{2500}{3} \text{ m/s}$$

For Problems 41–42

Sol. 41. d., 42. d.

a. At P: Slope of tangent

$$= \frac{dy}{dx} = \tan 60° = \sqrt{3}$$

Particle velocity

$$v_p = -v\frac{dy}{dx} \Rightarrow 20\sqrt{3} = -v\sqrt{3}$$

$$\Rightarrow |v| = 20 \text{ cm/s} = \frac{1}{5} \text{ m/s}$$

Hence the wave is travelling in negative x-direction with velocity 20 cm/s.

b. From graph, amplitude $A = 4 \times 10^{-3}$ m

Wave length $\lambda = (5.5 - 1.5) = 4 \times 10^{-2}$ m

Wave number

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{4\times 10^{-2}} = 50\pi \text{ m}^{-1}$$

Angular frequency $\omega = kv$

$$= 50\pi \times \frac{1}{5} = 10\pi$$

Hence equation can be written as

$$y = A\sin(\omega t + kx + \phi)$$

$$y = (4\times 10^{-3})\sin(10\pi t + 50\pi x + \phi) \quad \text{(i)}$$

at $t = 0, x = 0$

$$2\sqrt{2}\times 10^{-3} = 4\times 10^{-3}\sin(\phi)$$

$$\Rightarrow \sin\phi = \frac{1}{\sqrt{2}}, \quad \phi = \frac{\pi}{4}, \frac{3\pi}{4}$$

Particle is moving up at $t = 0, x = 0$

Hence, $\phi = \frac{\pi}{4}$

Hence equation is

$$y = (4\times 10^{-3})\sin\left(10\pi t + 50\pi x + \frac{\pi}{4}\right)$$

Matching Column Type

1. i.→c.; ii.→a.; iii.→d.; iv.→b.

Use $x = 0$; $t = 0$ for y and particle velocity $\frac{\partial y}{\partial x}$. Like for **i.**, $y = 0$ at $x = 0$ and $t = 0$. $\frac{\partial y}{\partial t} > 0$, i.e., positive therefore it matches with (c).

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2. i. → b., d.; ii. → a., d.; iii. → a., b.; iv. → a., b., d.

If we change the speed of hand, then particle speed changes. If the amount by which hand movement changes, then amplitude of pulse changes. If time in which hand comes to its original position changes, the width of pulse changes.

3. i. → d.; ii. → a.; iii. → b.; → iv. → c.

As it is clear from the equation that speed of each wave is same and equal to 2 m/s.

From the graphs it is clear that wavelength is maximum for graph in **p.** and the least for **r.** As wavelength is the property of both source and medium and here medium is the same, so we can conclude that the relation among three wavelengths, is determined by source property, i.e., frequency. From equation it is clear that frequency is maximum for y_3 and least for y_1. From $\lambda = v/f$, we can conclude that wavelength is maximum for the wave having least frequency.

4. i. → a., b.; ii. → c., d.; iii. → a., d.; iv. → b., c.

i. While passing through mean position the deformation is maximum ⇒ KE maximum and PE maximum.

ii. Minimum speed and minimum deformation ⇒ KE minimum and PE minimum.

iii. Speed is maximum and minimum deformation maximum ⇒ KE maximum and PE minimum.

iv. Speed minimum and deformation maximum ⇒ KE minimum and PE maximum.

5. i. → a., b.; ii. → a., b.; iii. → c., d.; iv. → c., d.

Power $\propto f^2A^2$

Integer Answer Type

1. (4) The linear mass density is

$$\mu = \frac{5\times10^{-3}\,\text{kg}}{50\times10^{-3}\,\text{m}} = 1.0\times10^{-2}\,\frac{\text{kg}}{\text{m}}$$

The wave speed is $v = \sqrt{F/\mu}$

Thus, the tension is $F = \mu v^2$

$$= \left(1.0\times10^{-2}\,\frac{\text{kg}}{\text{m}}\right)\times6400\,\frac{\text{m}^2}{\text{s}^2} = 64\,\text{N}$$

The Young's modulus is given by $Y = \dfrac{F/A}{\Delta L/L}$.

The extension is, therefore

$$\Delta L = \frac{FL}{AY} = \frac{64\times0.50}{(1.0\times10^{-6})\times(8\times10^{11})} = 0.04\text{ mm}$$

2. (2) $v = 40$ cm/s

As velocity of a wave is constant, location of maximum after 5 s is given by $40 \times 5 = 200$ cm along the negative x-axis at $x = -2$ m

3. (3) Time period

$T = 4 \times 5\text{ ms} = 20\times10^{-3} = 2\times10^{-2}$ s

Frequency, $f = \dfrac{1}{T} = \dfrac{1}{(2\times10^{-2})} = 50$ Hz

$\lambda = 2 \times 3\text{ cm} = 6$ cm

Wave speed: $v = \lambda f = 0.06 \times 50 = 3$ m/s

4. (2) Velocity of the wave,

$$V = \sqrt{\left(\frac{T}{\mu}\right)} = \sqrt{\frac{(16\times10^5)}{0.4}} = 2000\text{ cm/s}$$

Time taken to reach to the other end $= \dfrac{20}{200} = 0.01$ s.

Time taken to see the pulse again in the original position $= 0.01 \times 2 = 0.02$ s.

5. (5) $L = 40$ cm, mass = 10 g

mass per unit length

$$\mu = \frac{10}{40} = \frac{1}{4}(\text{g/cm})$$

Spring constant $k = 160$ N/m

Deflection, $x = 1\text{ cm} = 0.01$ m

Tension in the string:

$T = kx = 160 \times 0.01 = 1.6$ N

$= 16 \times 10^4$ dyne

Wave velocity is given by

$$v = \sqrt{\left(\frac{T}{\mu}\right)} = \sqrt{\left(\frac{(16\times10^4)}{\frac{1}{4}}\right)} = 800\text{ cm/s}$$

Time taken by the pulse to reach the spring

$$t = \frac{40}{800} = \frac{1}{20} = 0.05\text{ s} = 5\times10^{-2}\text{ s.}$$

6. (4) $\mu = 19.2 \times 10^{-3}$ kg/m

From the free body diagram

$T - 4g - 4a = 0$

$T = 4(a + g) = 4(2 + 10) = 48$ N

Wave speed:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48}{19.2\times10^{-3}}} = 50\text{ m/s}$$

So $n = 4$

7. (2) Given that

$x = 40\cos(50\pi t - 0.02\pi y)$

∴ particle velocity

$$v_p = \frac{dx}{dt} = (40\times50\pi)\{-\sin(50\pi t - 0.02\pi y)\}$$

Putting $x = 25$ and $t = \dfrac{1}{200}$ s,

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$$v_P = -(2000\pi \text{ cm/s})\sin\left[50\pi\left(\frac{1}{200}\right) - 0.02\pi(25)\right]$$

$$= 10\pi\sqrt{2} \text{ m/s}$$

8. **(4)**

$$y = \frac{0.8}{(3x^2 + 24xt + 48t^2 + 4)} = \frac{0.8}{3[x^2 + 8xt + 16t^2] + 4}$$

$$= \frac{0.8}{3(x+4t)^2 + 4}$$

$\therefore x + 4t = x + vt \qquad \therefore v = 4$ m/s

9. **(2)**

$$a_{max} = \omega^2 A = g$$

$$\omega = \frac{2\pi v}{\lambda}, \quad v = \sqrt{\frac{F}{\mu}}$$

$$A_{min} = \frac{g\lambda^2\mu}{4\pi^2 F} = \frac{\lambda^2\mu}{4F} = 2\times10^{-3}\text{ m} = 2 \text{ mm}$$

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CHAPTER

6

Sound Waves and Doppler Effect

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SOUND WAVES

Sound waves travel through any material medium with a speed that depends on the properties of the medium. As sound waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidal sound waves is very similar to that of sinusoidal waves on strings.

PROPAGATION OF SOUND WAVES

Sound is a mechanical three-dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves. When prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a *compression pulse* and it travels away from the prong with the speed of sound.

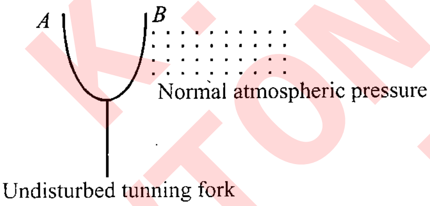

Fig. 6.1

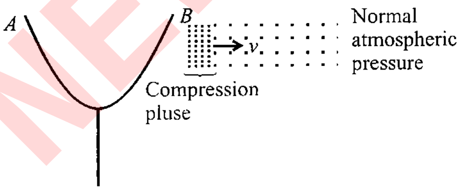

Fig. 6.2

After producing the compression pulse, prong B reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called a *rarefaction pulse*. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.

If the prongs vibrate in SHM, the pressure variation in the layer close to the prong also varies simple harmonically and hence increase in pressure above normal value can be written as

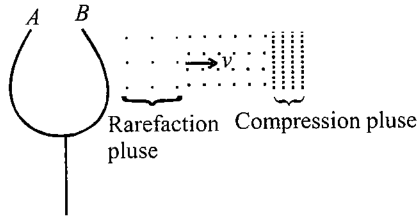

Fig. 6.3

$$\delta P = \delta P_0 \sin \omega t$$

where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travels towards right with wave velocity v, the excess pressure at any position x at time t will be given by

$$\delta P = \delta P_0 \sin [\omega(t - x/v)]$$

Using $\Delta p = \delta P$, $\Delta p_{max} = \delta P_0$, the above equation of sound wave can be written as

$$\Delta p = \Delta p_{max} \sin [\omega(t - x/v)]$$

Illustration 6.1 **The equation of a sound wave in air is given by $\Delta p = (0.02) \sin [(3000)\, t - (9.0)x]$, where all variables are in SI units. (a) Find the frequency, wavelength and the speed of sound wave in air. (b) If the equilibrium pressure of air is 1.01×10^5 N/m², what are the maximum and minimum pressures at a point as the wave passes through that point?**

Sol.

a. Comparing with the standard form of a travelling wave,

$$\Delta p = \Delta p_{max} \sin [\omega\ (t - x/v)]$$

we see that $\omega = 3000\ s^{-1}$. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi}\ \text{Hz}$$

Also from the same comparison,

$$\omega / v = 9.0\ m^{-1}$$

or
$$v = \frac{\omega}{9.0\ m^{-1}} = \frac{3000\ s^{-1}}{9.0\ m^{-1}} = \frac{1000}{3}\ \text{m/s}$$

The wavelength is

$$\lambda = \frac{v}{f} = \frac{1000/3\ \text{m/s}}{3000/2\pi\ \text{Hz}} = \frac{2\pi}{9}\ \text{m}$$

b. The pressure amplitude is $\Delta p_{max} = 0.02$ N/m². Hence, the maximum and minimum pressures at a point in the wave motion will be $1.01 \times 10^5 \pm 0.02$ N/m².

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SPEED OF SOUND WAVES

Let us describe pictorially the motion of a one-dimensional longitudinal pulse moving through a long tube containing a compressible gas as shown in Fig. 6.4. A piston in the left end can be moved to the right to compress the gas and create the pulse. Before the piston is moved to the right to compress the gas and create the pulse, the gas is undisturbed and of uniform density as represented by the uniformly shaded region in Fig. 6.4(a). When the piston is suddenly pushed to the right [Fig. 6.4(b)], the gas just in front of it is compressed (as represented by the more heavily shaded region); the pressure and density in this region are now higher than they were before the piston moved. When the piston comes to rest [Fig. 6.4(c)], the compressed region of the gas continues to move to the right, corresponding to a longitudinal pulse travelling through the tube with speed v.

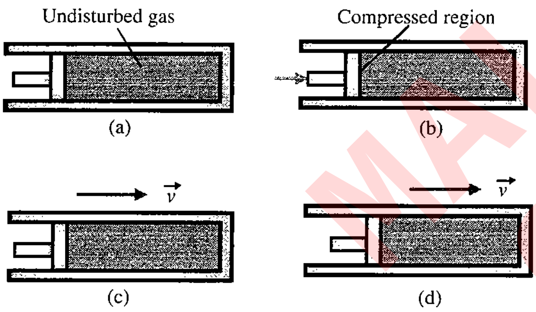

Fig. 6.4 Motion of a longitudinal pulse through a compressible gas. The compression (darker region) is product by the moving pistion

The speed of sound waves in a medium depends on the compressibility and density of the medium. If the medium is a liquid or a gas and has bulk modulus B and density ρ, the speed of sound waves in that medium is

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{(i)}$$

It is interesting to compare this expression with the equation for the speed of transverse waves on a string, $v = \sqrt{T/\mu}$. In both cases, the wave speed depends on an elastic property of the medium (bulk modulus B or string tension T) and on an inertial property of the medium (ρ or μ). In fact, the speed of all mechanical waves follows an expression of the general form

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

For longitudinal sound waves in solid rod of material, for example, the speed of sound depends on Young's modulus Y and the density ρ.

Velocity of Sound/Longitudinal Waves in Solids

Consider a section AB of medium as shown in Fig. 6.5(a) of cross-sectional area S. Let A and B be two cross-sections as shown. Let in this medium sound propagation be from left to right. If wave source is at origin O and when it oscillates, the oscillations at that point propagate along the rod.

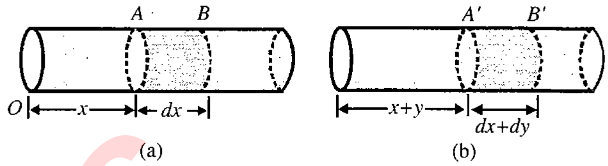

Fig. 6.5

Here we say an elastic wave has propagated along the rod with a velocity determined by the physical properties of the medium. Due to oscillations, say a force F is developed at every point of medium which produces a stress in rod and is the cause of strain or propagation of disturbance along the rod. This stress at any cross-sectional area can be given as

$$\delta_l = \frac{F}{S} \qquad \text{(i)}$$

Let us consider a section AB of medium at a general instant of time t. The end A is at a distance x from O and B is at a distance $x + dx$ from O. Let in time dt due to oscillations, medium particles at A be displaced along the length of medium by y and those at B by $y + dy$. The resulting positions of section are A' and B' as shown in Fig. 6.5(b). Here we can say that the section AB is deformed (elongated) by a length dy. Thus strain produced in it is

$$E = \frac{dy}{dx} \qquad \text{(ii)}$$

If Young's modulus of the material of medium is Y, we have

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\delta_l}{E}$$

From Eqs. (i) and (ii), we have

$$Y = \frac{F/S}{dy/dx}$$

or

$$F = YS\frac{dy}{dx} \qquad \text{(iii)}$$

If net force acting on section AB is dF, then it is given by

$$dF = dm\, a \qquad \text{(iv)}$$

where dm is the mass of section AB and a is its acceleration, which can be given as for a medium of density ρ as

$$dm = \rho S dx$$

and

$$a = \frac{d^2y}{dt^2}$$

From Eq. (iv), we have

$$dF = (\rho S dx)\frac{d^2y}{dt^2}$$

or

$$\frac{dF}{dx} = \rho S \frac{d^2y}{dt^2} \qquad \text{(v)}$$

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From Eq. (iii) on differentiating w.r.t. to x, we can write

$$\frac{dF}{dx} = YS\frac{d^2y}{dx^2} \qquad \text{(vi)}$$

From Eqs. (v) and (vi), we get

$$\frac{d^2y}{dt^2} = \left(\frac{Y}{\rho}\right)\frac{d^2y}{dx^2} \qquad \text{(vii)}$$

Equation (vii) is the different form of wave equation. Comparing it with equation

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

we get the wave velocity in the medium as

$$v = \sqrt{\frac{Y}{\rho}} \qquad \text{(viii)}$$

SPEED OF SOUND: NEWTON'S FORMULA

On the basis of observations, Newton obtained a formula for speed of sound in air as

$$v = \sqrt{\frac{P}{\rho}} \qquad \text{(i)}$$

where P is the isothermal elasticity of the air. He argued that when sound propagates through air, the temperature of air remains constant. By Newton's formula the speed of sound at one atmosphere is

$$V = \sqrt{\frac{1.013\times10^5}{1.29}} \simeq 280 \text{ m/s}$$

This value is less than experimental value of 332 m/s. Hence Newton's formula requires some correction, which was made by Laplace in 1816.

LAPLACE'S CORRECTION

French scientist Laplace pointed out that when sound propagates in air the heat of the medium remains constant instead of its temperature. So he replaced isothermal elasticity by adiabatic elasticity B_{ad}. The corrected formula is

$$v = \sqrt{\frac{B_{ad}}{\rho}}$$

For adiabatic change,

$$PV^\gamma = \text{constant}$$

Differentiating both sides, we get

$$P(\gamma V^{\gamma-1})\,dV + V^\gamma dP = 0$$

or $$\gamma P dV + V dP = 0$$

or $$\frac{dP}{\left(\frac{-dV}{V}\right)} = \gamma P$$

We have $$\frac{dP}{\left(\frac{-dV}{V}\right)} = B_{ad}$$

$\therefore$ $$B_{ad} = \gamma P \qquad \text{(i)}$$

where $\gamma = C_p/C_v$ is the ratio of specific heats. Hence Laplace's formula for the speed of sound in air (gas) is

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{(ii)}$$

For air $\gamma = 7/4$, so the speed of sound in air at STP will be

$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 = 332 \text{ m/s}$$

This value is in very close agreement with the experimental value.

FACTORS AFFECTING SPEED OF SOUND IN GAS

Effect of Pressure

The speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

We know that

$$PV = nRT = \frac{m}{M}RT$$

At constant temperature,

$$P\Delta V = \frac{\Delta m}{M}RT$$

$\therefore$ $$P = \frac{\Delta m}{\Delta V}\frac{RT}{M}$$

or $$P = \rho\frac{RT}{M}$$

or $$\frac{P}{\rho} = \text{constant}$$

Therefore, with the change in pressure, the density also changes in such proportion, so that p/ρ remains constant. Hence pressure has no effect on the speed of sound in a gas.

Effect of Density

For two gases of densities ρ_1 and ρ_2 at same pressure with ratios of specific heats γ_1 and γ_2,

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{\rho_2}{\rho_1}}$$

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Effect of Temperature

We have,

$$\frac{P}{\rho} = \frac{RT}{M}$$

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}}$$

Clearly, $\quad v \propto \sqrt{T}$

Hence the speed of sound in a gas is proportional to the square root of its absolute temperature. If v_0 and v_t are the velocities of sound in gas at 0°C and t°C, respectively, then

$$v_0 = \sqrt{\frac{\gamma R(273+0)}{M}}$$

and

$$v_t = \sqrt{\frac{\gamma R(273+t)}{M}}$$

$$\therefore \quad \frac{v_t}{v_0} = \left(\frac{273+t}{273}\right)^{1/2} = \left(1+\frac{t}{273}\right)^{1/2}$$

For small value of t,

$$v_t = v_0\left(1+\frac{1}{2}\frac{t}{273}\right)$$

or $\quad v_t = v_0 + \dfrac{v_0 t}{546}$

But $\quad v_0 = 332$ m/s.

$$\therefore \quad v_t - v_0 = \frac{332t}{546} = 0.61t$$

When $t = 1$°C,

$$v_t - v_0 = 0.61 \text{ m/s}$$

Hence the velocity of sound in air increases by 0.61 m/s for every 1°C rise in temperature.

Effect of Humidity

With the increase in humidity, the density of air decreases. Now, the speed of sound in air is given by

$$v \propto \frac{1}{\sqrt{\rho}}$$

Hence, the speed of sound will increase.

Effect of Frequency

With the change in frequency of the sound wave, wavelength also changes, so that

$$f\lambda = v \text{ (constant)}$$

Thus, the speed of sound is independent of its frequency.

Effect of Wind

As the sound is carried by air, so its speed is affected by the wind velocity. Suppose the wind is blowing with a velocity v_w at an angle θ with the direction of propagation of the sound. Clearly, the component of wind velocity in the direction of sound is $v_w \cos\theta$. Therefore, the Resultant velocity of sound is $v + v_w \cos\theta$

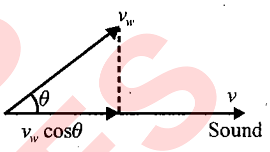

Fig. 6.6

When wind blows in the direction of sound ($\theta = 0$), the resultant velocity is $v + v_w$.

When wind blows in the opposite direction of sound ($\theta = 180°$), the resultant velocity is $= v - v_w$.

Illustration 6.2 **Calculate the velocity of sound in air at NTP. The density of air at NTP is 1.29 g/L. Assume air to be diatomic with $\gamma = 1.4$. Also calculate the velocity of sound in air at 27°C.**

Sol. Velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4\times 1.013\times 10^5 \text{ N/m}^2}{1.29 \text{ kg/m}^3}} = 331.6 \text{ m/s}$$

$$\left(\text{using } \frac{P}{\rho} = \frac{R}{M}T\right)$$

We can see that the velocity of sound is proportional to the square root of absolute temperature. Hence,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow v_2 = v_1\sqrt{\frac{T_2}{T_1}} = 331.6\sqrt{\frac{273+27}{273}} = 347.6 \text{ m/s}$$

Illustration 6.3 **Calculate the stress in a tight wire of a material whose Young's modulus is 19.6×10^{11} dyne/cm² so that the speed of the longitudinal waves is 10 times the speed of transverse waves.**

Sol. The speed of transverse waves in string is

$$c_1 = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{Ad}}$$

where A is cross-sectional area and d is the density.

The speed of longitudinal waves in string is $c_2 = \sqrt{Y/d}$. We have,

$$c_2 = 10c_1$$

$$\Rightarrow \sqrt{\frac{Y}{d}} = 10\sqrt{\frac{T}{Ad}}$$

$$\Rightarrow \text{Stress} = \frac{T}{A} = \frac{Y}{100} = 19.6\times 10^9 \text{ dyne/cm}^2$$

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Illustration 6.4 **Taking the composition of air to be 75% of nitrogen and 25% of oxygen by weight, calculate the velocity of sound through air.**

Sol. The molecular weight of a mixture is given by

$$\frac{m_1 + m_2 + m_3 + \cdots}{M} = \frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3} + \cdots$$

$$\therefore \quad \frac{75+25}{M} = \frac{75}{28} + \frac{25}{32}$$

or $$M = 28.9 \text{ g}$$

$$\therefore \quad c = \sqrt{\gamma \frac{RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 1000 \times 273}{28.9}} = 331.3 \text{ m/s}$$

here $\gamma = 1.4 \rightarrow$ as both the gases are diatomic.

Illustration 6.5 **The velocity of sound in hydrogen at 0°C is 1200 m/s. When some amount of oxygen is mixed with hydrogen, the velocity decreases to 500 m/s. Determine the ratio of H_2 to O_2 by volume in this mixture, given that the density of oxygen is 16 times that of hydrogen.**

Sol. Given that

$$1200 = \sqrt{\frac{\gamma P}{\rho_H}} \quad \text{(i)}$$

Let there be x volume of H_2 and y volume of O_2. Then,

$$(x+y)\rho_{\text{mix}} = x\rho_H + y\rho_0 = x\rho_H + 16\,y\rho_H$$

$$\Rightarrow \rho_{\text{mix}} = \frac{(x+16y)}{x+y}\rho_H$$

$$\therefore \quad 500 = \sqrt{\frac{\gamma P(x+y)}{(x+16y)\rho_H}} \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i),

$$\frac{12}{5} = \sqrt{\frac{x+16y}{x+y}}$$

or $$\frac{x}{y} = \frac{2.2}{1}$$

PRESSURE WAVES

When a longitudinal wave propagates in a gaseous medium, it produces compression and rarefaction in the medium periodically. The region where compression occurs, the pressure is more than the normal pressure of the medium and the region where rarefaction occurs, the pressure is lesser than the normal pressure of the medium. Thus we can also describe longitudinal waves in a gaseous medium as pressure waves and these are also termed as compressional waves in which the pressures at different points of medium also vary periodically with their displacements. Let us discuss the propagation of excess pressure in a medium in longitudinal wave analytically.

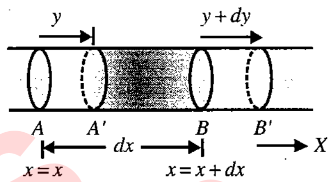

Fig. 6.7

Consider a longitudinal wave propagating in positive x direction as shown in Fig. 6.7. The figure shows a segment AB of the medium of width dx. In this medium let a longitudinal wave is propagating whose equation is given as

$$y = A\sin(kx - \omega t) \quad \text{(i)}$$

where y is the displacement of a medium particle situated at a distance x from the origin along the direction of propagation of wave. In Fig. 6.7, AB is the medium segment such that A is at position $x = x$ and B is at $x = x + dx$ at an instant. If after some time t medium particle at A reaches to a point A' which is displaced by y and the medium particle at B reaches to point B' which is at a displacement $y + dy$ from B. Here dy is given by equation as

$$dy = Ak\cos(kx - \omega t)\,dx$$

Here due to displacement of section AB to $A'B'$ the change in volume of its section is given by

$$dV = Sdy = -SAk\cos(kx - \omega t)\,dx$$

where S is area of cross-section. The volume of section AB is

$$V = Sdx$$

Thus volume strain in section AB is

$$\frac{dV}{V} = \frac{dy}{dx} = \frac{SAk\cos(kx - \omega t)\,dx}{S\,dx}$$

or $$\frac{dV}{V} = Ak\cos(kx - \omega t)$$

If B is the bulk modulus of the medium, then the excess pressure in section AB can be given as

$$\Delta P = -B\left(\frac{dV}{V}\right) = -B\left(\frac{dy}{dx}\right) \quad \text{(ii)}$$

$$\Delta P = -BAk\cos(kx - \omega t)$$

$$\Delta P = -\Delta P_{\max}\cos(kx - \omega t) \quad \text{(iii)}$$

Here $\Delta P_{\max}$ is the pressure amplitude at a medium particle at position x from origin and ΔP is the excess pressure at that point. Equation (iii) shows that excess pressure varies periodically at every point of the medium with pressure amplitude $\Delta P_{\max}$, which is given by

$$\Delta P_{max} = BAk = \frac{2\pi}{\lambda} AB \qquad \text{(iv)}$$

This equation is also termed as the equation of pressure wave in a gaseous medium.

We can also see that the pressure wave differs in phase by $\pi/2$ from the displacement wave and displacement maxima occur where the pressure is at its normal level. Remember that pressure maximum implies that the pressure at a point is pressure amplitude more or less than the normal pressure level of the medium.

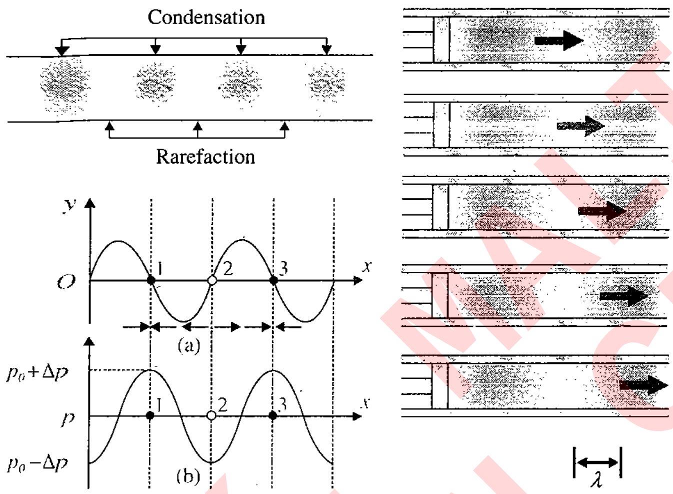

Fig. 6.8

Illustration 6.6 **The figure shows an instantaneous displacement–position graph of a sound wave travelling along the positive x-axis. Identify the points of**

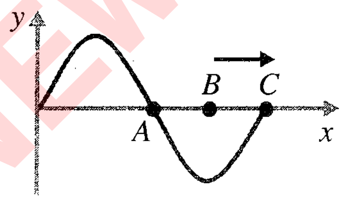

Fig. 6.9

i. maximum pressure;

ii. minimum pressure and

iii. atmospheric pressure (or normal pressure).

Sol.

Method 1:

i. Maximum pressure occurs at A because air particles are displaced towards it.

ii. Minimum pressure occurs at C because air particles are displaced away from it.

iii. Pressure remains atmospheric at B.

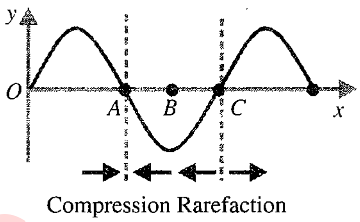

Fig. 6.10

Method 2: Pressure change in the medium is given by

$$\Delta p = B\left(-\frac{dy}{dx}\right)$$

The pressure change will be positive where dy/dx is negative and pressure change will be negative where dy/dx is positive.

i. Maximum pressure occurs at the points of maximum negative slope at point A.

ii. Minimum pressure occurs at the points of maximum positive slope at point C.

iii. Atmospheric pressure remains at the positions of zero slope at point B.

INTENSITY OF PERIODIC SOUND WAVES

We define the intensity I of a wave, or the power per unit area, as the rate at which the energy transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave.

$$I = \frac{P}{A} \qquad \text{(i)}$$

In this case, the intensity is therefore

$$I = \frac{1}{2}\rho v(\omega A)^2$$

Hence, the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency. For sound wave,

$$\Delta P_m = ABk$$

$$\therefore \qquad A = \frac{\Delta P_m}{Bk}$$

Substituting this value in Eq. (i), we get

$$I = \frac{1}{2}\rho v\omega^2\left(\frac{\Delta P_m}{Bk}\right)^2 = \frac{1}{2}\rho v\omega^2 \frac{\Delta P_m^2}{B^2k^2}$$

As $\qquad k = \omega/v \quad$ and $\quad B = v^2\rho,$

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$$\therefore I = \frac{1}{2}\rho v \omega^2 \frac{\Delta P_m^2}{B^2 \frac{\omega^2}{v^2}} = \frac{v\Delta P_m^2}{2B} = \frac{\Delta P_m^2}{2\rho v}$$

Now consider a point source emitting sound wave equally in all directions. From day-to-day experience, we know that the intensity of sound decreases as we move farther from the source. When a source emits sound equally in all directions, the result is a spherical wave. Figure 6.11 shows these spherical waves as a series of circular arcs concentric with the source. Each arc represents a surface over which the phase of the wave is constant. We call such a surface of constant phase a wavefront. The distance between adjacent wavefronts that have the same phase is called the wavelength λ of the wave. The radial lines pointing outward from the source are called rays.

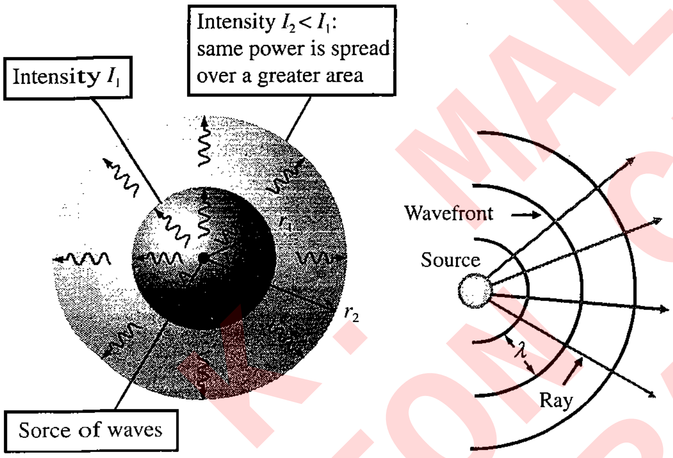

Fig. 6.11

The average power P_{avg} emitted by the source must be distributed uniformly over each spherical wavefront of area $4\pi r^2$. Hence, the wave intensity at a distance r from the source is

$$I = \frac{P_{avg}}{A} = \frac{P_{avg}}{4\pi r^2} \qquad \text{(ii)}$$

The inverse square law, which is reminiscent of the behaviour of gravity, states that the intensity decreases in proportion to the square of the distance from the source.

Illustration 6.7 **A point source emits sound waves with an average power output of 80.0 W. (a) Find the intensity 3.00 m from the source. (b) Find the distance at which the intensity of the sound is 1.00×10^{-8} W/m².**

Sol.

a. Imagine a small loudspeaker sending sound out at an average rate of 80.0 W uniformly in all directions. You are standing 3.00 m away from the speakers. As the sound propagates, the energy of the sound waves is spread out over an ever-expanding sphere. We evaluate the intensity from a given equation, so we categorize this example as substitution problem. Because a point source emits energy in the form of spherical waves, use Eq. (ii) to find the intensity:

$$I = \frac{P_{avg}}{4\pi r^2} = \frac{80.0 \text{ W}}{4\pi(3.00 \text{ m})^2} = 0.707 \text{ W/m}^2$$

This intensity is close to the threshold of pain.

b. Solve for r from above equation and use the given value for I:

$$r = \sqrt{\frac{P_{avg}}{4\pi I}} = \sqrt{\frac{80.0 \text{ W}}{4\pi(1.00\times10^{-8} \text{ W/m}^2)}} = 2.52\times10^4 \text{ m}$$

Illustration 6.8 **The faintest sounds the human ear can detect at a frequency of 1000 Hz correspond to an intensity of about 1.00×10^{-12} W/m², which is called threshold of hearing. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about 1.00 W/m², the threshold of pain. Determine the pressure amplitude and displacement amplitude associated with these two limits. Take speed of sound = 343 m/s and density of air = 1.20 kg/m³.**

Sol. Think about the quietest environment you have ever experienced. It is likely that the intensity of sound in even this quietest environment is higher than the threshold of hearing. Because we are given intensities and asked to calculate pressure and displacement amplitudes, this problem requires the concepts discussed in this section.

To find the pressure amplitude at the threshold of hearing,

$$\Delta P_{max} = \sqrt{2\rho v I}$$

$$= \sqrt{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})(1.00\times10^{-12} \text{ W/m}^2)}$$

$$= 2.87 \times 10^{-5} \text{ N/m}^2$$

Calculate the corresponding displacement amplitude using

$$s_{max} = \frac{\Delta P_{max}}{\rho v \omega} = \frac{2.87\times10^{-5} \text{ N/m}^2}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi\times1000 \text{ Hz})}$$

$$= 1.11 \times 10^{-11} \text{ m} \qquad [\because \omega = 2\pi f]$$

In a similar manner, one finds that the loudest sounds the human ear can tolerate correspond to a pressure amplitude of 28.7 N/m² and a displacement amplitude equal to 1.11×10^{-5} m.

Because atmospheric pressure is about 10^5 N/m², the result for the pressure amplitude tells us that the ear is sensitive to pressure fluctuations as small as 3 parts in 10^{10}. The displacement amplitude is also a remarkably small number. If we compare this result for s_{max} to the size of an atom (about 10^{-10} m), we see that the ear is an extremely sensitive detector of sound waves.

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FREQUENCY AND PITCH OF SOUND WAVES

Each cycle of a sound wave includes one compression and one rarefaction, and frequency is the number of cycles per second that pass through a given location. This is normally equal to the frequency of vibration of the (tuning fork) source producing sound. If the source vibrates in SHM of a single frequency, sound produced has a single frequency and is called a pure tone.

However, a sound source may not always vibrate in SHM (this is the case with most of the common sound sources, e.g. guitar string, human vocal card, surface of drum, etc.), and hence the pulse generated by it may not have the shape of a sine wave. But even such a pulse can be obtained by superposition of a large number of sine waves of different frequency and amplitudes. (This is possible for a pulse of any shape, according to an important theorem of mathematics, the Fourier theorem, the description of which is outside the scope of this book.) We say that the pulse contains all these frequencies.

A normal person hears all frequencies between 20 Hz and 20 kHz. This is a subjective range (obtained experimentally) which may vary slightly from person to person. The ability to hear the high frequencies decreases with age and a middle-age person can hear only up to 12 Hz to 14 kHz.

Sound can be generated with frequency below 20 Hz called **infrasonic sound** and above 20 kHz called **ultrasonic sound**. Even through human cannot hear these frequencies, other animals may. For instance, rhinos communicate through infrasonic frequencies as low as 5 Hz, and bats use ultrasonic frequencies as high as 100 kHz for navigating.

Frequency as we have discussed till now is an objective property measured in units of Hz and which can be assigned a unique value. However, a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called **pitch**. A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound.

LOUDNESS AND FREQUENCY

The discussion of sound level in decibels relates to a physical measurement of the strength of sound. Let us now extend our discussion concerning the psychological 'measurement' of the strength of sound.

Of course, we do not have any instrument in our bodies that can display numerical values of our reactions to stimuli. We have to 'calibrate' our reactions somehow by comparing different sounds to a reference sound, but that is not easy to accomplish. For example, earlier we mentioned that the threshold intensity is 10^{-12} W/m^2, corresponding to an intensity level of 0 dB. In reality, this value is the threshold only for a sound of frequency 1000 Hz, which is a standard reference frequency in acoustics. If we perform an experiment to measure the threshold intensity at other frequencies, we find a distinct variation of this threshold as a function of frequency. For example, at 100 Hz, a barely audible sound must have an intensity level of about 30 dB! Unfortunately, there is no simple relationship between physical measurements and psychological 'measurement'. The 100-Hz 30-dB sound is psychologically 'equal' to the 1000-Hz, 0-dB sound (both are just barely audible), but they are not physically equal (30 dB $\neq$ 0 dB).

SOUND LEVEL IN DECIBELS

There are a wide range of intensities the human ear can detect. Because this range is so wide, it is convenient to use a logarithmic scale, where the sound level β (Greek letter beta) is defined by the equation

$$\beta = 10\log\left(\frac{I}{I_0}\right) \qquad \text{(i)}$$

The constant I_0 is the reference intensity, taken to be at the threshold of hearing ($I_0 = 1.00 \times 10^{-12}$ W/m^2), and I is the intensity in watts per square metre to which the sound level β corresponds, where β is measured in decibels (dB). On this scale, the threshold of pain (I = 1.00 W/m^2) corresponds to a sound level of β = 10 log [(1 W/m^2)/(10^{-12} W/m^2)] = 10 log (10^{12}) = 120 dB, and the threshold of hearing corresponds to β = 10 log [10^{-12} W/m^2)/(10^{-12} W/m^2)] = 0 dB.

Prolonged exposure to high sound levels may seriously damage the human ear. Ear plugs are recommended whenever sound levels exceed 90 dB. Recent evidence suggests that 'noise pollution' may be a contributing factor to high blood pressure, anxiety and nervousness. Let β_1 and β_2 be the sound levels corresponding to sound intensities I_1 and I_2 respectively. Then,

$$\beta_1 = 10\log\frac{I_1}{I_0}$$

$$\beta_2 = 10\log\frac{I_2}{I_0}$$

$$\therefore \quad \beta_2 - \beta_1 = 10\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right) \qquad \text{(ii)}$$

or $$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$$

Illustration 6.9 **Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each operating machine at the worker's location is 2.0×10^{-7} W/m^2. (a) Find the sound level heard by the worker when one machine is operating. (b) Find the sound level heard by the worker when both the machines are operating.**

Sol. Imagine a situation in which one source of sound is active, and is then joined by a second identical source. Such as one person speaking and then a second person speaking at the same time or one musical instrument playing, and then being joined by a second instrument.

a. The sound level at the worker's location with one machine operating is given by

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$$\beta_1 = 10 \log\left(\frac{2.0\times10^{-7}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 10 \log (2.0 \times 10^5) = 53 \text{ dB}$$

b. The sound level at the worker's location with double the intensity is given by

$$\beta_2 = 10 \log\left(\frac{4.0\times10^{-7}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right)$$

$$= 10 \log (4.0 \times 10^5) = 56 \text{ dB}$$

These results show that when the intensity is doubled, the sound level increases by only 3 dB.

Illustration 6.10 **Loudness is a psychological response to a sound. It depends on both the intensity and the frequency of the sound. As a rule of thumb, a doubling in loudness is approximately associated with an increase in sound level of 10 dB. (This rule of thumb is relatively inaccurate at very low or very high frequencies.) If the loudness of the machines in illustration 6.10 is to be doubled, how many machines at the same distance from the worker must be running?**

Sol. Using the rule of thumb, a doubling of loudness corresponds to a sound level increase of 10 dB. Therefore,

$$\beta_2 - \beta_1 = 10 \text{ dB} = 10 \log\left(\frac{I_2}{I_0}\right) - 10 \log\left(\frac{I_1}{I_0}\right) = 10 \log\left(\frac{I_2}{I_1}\right)$$

$$\log\left(\frac{I_2}{I_1}\right) = 1 \quad \Rightarrow I_2 = 10 I_1$$

Therefore, 10 machines must be operating to double the loudness.

Illustration 6.11 **Calculate the sound level (in decibels) of a sound wave that has an intensity of 4.00 μ W/m².**

Sol. We expect about 60 dB. We use the definition of the decibel scale. We use the equation

$$\beta = 10 \text{ dB} \log (I/I_0)$$

where $I_0 = 10^{-12}$ W/m²

$$\therefore \qquad \beta = 10 \log\left(\frac{4.00\times10^{-6}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 66.0 \text{ dB}$$

This could be the sound level of a chamber music concert, heard from a few rows back in the audience.

Illustration 6.12 **A family ice show is held at an enclosed arena. The skaters perform to music with level 80.0 dB. This level is too loud for your baby, who yells at 75.0 dB. (a) What total sound intensity engulfs you? (b) What is the combined sound level?**

Sol. Resist the temptation to say 155 dB. The decibel scale is logarithmic, so addition does not work that way. We must figure out the intensity of each sound, add the intensities, and then translate back to a level. We have,

$$\beta = 10 \log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

$$\therefore \qquad I = \left[10^{\beta/10}\right] 10^{-12}\ \text{W/m}^2$$

a. For your baby,

$$I_b = (10^{75.0/10})(10^{-12}\ \text{W/m}^2) = \sqrt{10}\times10^{-5}\ \text{W/m}^2$$

For the music, $I_m = (10^{80.0/10})(10^{-12}\ \text{W/m}^2) = 10.0 \times 10^{-5}\ \text{W/m}^2$

The combined intensity is

$$I_{\text{total}} = I_m + I_b$$

$$= 10.0 \times 10^{-5}\ \text{W/m}^2 + 3.16 \times 10^{-5}\ \text{W/m}^2$$

$$= 13.2 \times 10^{-5}\ \text{W/m}^2$$

b. The combined sound level is then

$$\beta_{\text{total}} = 10 \log\left(\frac{I_{\text{total}}}{10^{-12}\ \text{W/m}^2}\right)$$

$$= 10 \log\left(\frac{1.32\times10^{-4}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 81.2 \text{ dB}$$

The result is only a little louder than 80 dB.

Illustration 6.13 **A firework charge is detonated many metres above the ground. At a distance of 400 m from the explosion, the acoustic pressure reaches a maximum of 10.0 N/m². Assume that the speed of sound is constant at 343 m/s throughout the atmosphere over the region considered, the ground absorbs all the sound falling on it, and the air absorbs sound energy at the rate of 7.00 dB/km. What is the sound level (in decibels) at 4.00 km from the explosion?**

Sol: At a distance of 4 km, an explosion should be audible, but probably not extremely loud. So based on the data given, we might expect the sound level to be somewhere between 40 and 80 dB. From the sound pressure data given in the problem, we can find the intensity, which is used to find the sound level in dB. The sound intensity will decrease with increased distance from the source and from the absorption of the sound by the air.

At a distance of 400 m from the explosion, $\Delta P_{\max} = 10.0$ Pa. At this point the intensity is

$$I = \frac{\Delta P_{\max}^2}{2\rho v} = \frac{(10.0\ \text{N/m})^2}{2(1.20\ \text{kg/m}^3)(343\ \text{m/s})} = 0.121\ \text{W/m}^2$$

From the inverse square law, we can calculate the intensity and decibel level (due to distance alone) 4 km away

$$I' = I(400\ \text{m})^2/(4000\ \text{m})^2 = 1.21 \times 10^{-3}\ \text{W/m}^2$$

and

$$\beta = 10 \log\left(\frac{I'}{I_0}\right) = 10 \log\left(\frac{1.21\times10^{-3}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 90.8 \text{ dB}$$

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At a distance of 4 km from the explosion, absorption from the air will decrease the sound level by an additional amount,

$$\Delta\beta = (7.00 \text{ dB/km})\,(3.60 \text{ km}) = 25.2 \text{ dB}$$

So at 4 km, the sound level will be

$$\beta_f = \beta - \Delta\beta = 90.8 \text{ dB} - 25.2 \text{ dB} = 65.6 \text{ dB}$$

This sound level falls within our expected range. Evidently, this explosion is rather loud (about the same as a vacuum cleaner) even at a distance of 4 km from the source. It is interesting to note that the distance and absorption effect each reduces the sound level by about the same amount (~25 dB). If the explosion were at ground level, the sound level would be further reduced by reflection and absorption form obstacles between the source and observer, and the calculation could be much more complicated.

Concept Application Exercise 6.1

1. Do displacement, particle velocity and pressure variation in a longitudinal wave vary with the same phase?
2. Why is sound wave of intensity 10^{-12} W/m^2 and frequency 1000 Hz taken as the standard for expressing the intensity level of all other sound waves?
3. What experimental evidence can be cited to show that the speed of sound is the same for all wavelengths?
4. Explain why the speed of sound through a gas cannot be greater than the rm.s. speed of the molecules of the gas.
5. Sound is more clearly heard with the wind. How?
6. Explain: If an observer places his ear to one end of a long iron pipeline, he can distinctly hear two sounds when a workman hammers the other end of the pipeline.
7. Does the velocity of sound in a solid increase significantly on heating the solid?
8. A man stands on the ground at a fixed distance from a siren which emits a clear sound during night than during day. Explain why.
9. Two sound waves from two different sources interfere at a point to yield a sound of varying intensity. The intensity level between the maximum and minimum is 20 dB. What is the ratio of the intensities of the individual waves?
10. A sound differs by 6 dB from a sound of intensity equal to 10 nW/cm^2. Find the absolute value of intensity of the sound.
11. Find the molecular weight for a gas in which the velocity of sound is 1260 m/s at 0°C and whose γ is 1.4.
12. Seven grams of nitrogen is mixed with 12 g of oxygen in a tube and then sealed. Calculate the velocity of sound through the tube at 27°C.
13. Calculate the increase in velocity of sound for 1°C rise of temperature, if the velocity of sound at 0°C is 332 m/s.
14. If the sound level in a room is increased from 50 dB to 60 dB, by that factor is the pressure amplitude increased?

DOPPLER EFFECT

When a car at rest on a road sounds its high frequency horn and you are also standing on the road nearby, you will hear the sound of same frequency it is sounding but when the car approaches you with its horn sounding, the pitch (frequency) of its sound seams to drop as the car passes. This phenomenon was first described by the Austrian scientist Christien Doppler and is called the Doppler effect. He explained that when a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. Let us discuss the Doppler effect in detail for different cases.

Figure 6.12 shows a stationary source of frequency n_0 which produces sound waves in air of wavelength λ_0 given as

$$\lambda_0 = \frac{v}{n_0}$$

where v is speed of sound in air.

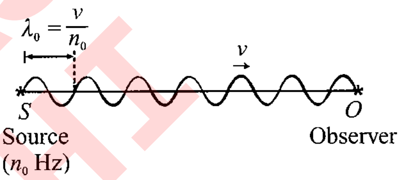

Fig. 6.12

Although sound waves are longitudinal, here we represent sound waves by the transverse displacement curve as shown in Fig. 6.12 to understand the concept in a better way. As source produces waves, these waves travel towards the stationary observer O in the medium (air) with speed v and wavelength λ_0, so the observer will hear the frequency n given by

$$n = \frac{v}{\lambda_0} = n_0 \text{ (same as that of source)} \qquad \text{(i)}$$

This is why when a stationary observer listens the sound from a stationary source of sound, it detects the same frequency of sound which the source is producing. Thus, no Doppler effect takes place if there is no relative motion between source and observer.

Stationary Source and Moving Observer

Figure 6.13 shows the case when a stationary source of frequency n_0 produces sound waves which have wavelength in air given as

$$\lambda_0 = \frac{v}{n_0}$$

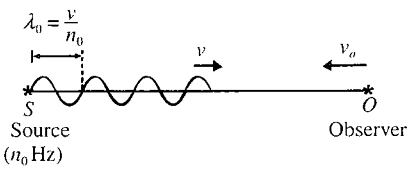

Fig. 6.13

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These waves travel towards the moving observer which moves with velocity v_o towards the source. When sound waves approach the observer, it will receive the waves of wavelength λ_0 with speed $v + v_o$ (relative speed). Thus the frequency of sound heard by the observer (apparent frequency) can be given as

$$n_{ap} = \frac{v+v_o}{\lambda_0}$$

$$= \frac{v+v_0}{\left(\frac{v}{n_0}\right)} = n_0\left(\frac{v+v_0}{v}\right) \quad \text{(ii)}$$

Similarly, we can say that if the observer is receding away from the source the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{v-v_0}{v}\right) \quad \text{(iii)}$$

Illustration 6.14 **A police siren emits a sinusoidal wave with frequency $f_s = 300$ Hz. The speed of sound is 340 m/s. (a) Find the wavelength of waves if the siren is at rest in the air. (b) If the siren is moving at 30 m/s, then find the wavelengths of the waves in front of and behind the source.**

Sol. The Doppler effect is not involved in part (a), since neither the source nor the listener is moving. In part (b), the source is in motion and we must involve the Doppler effect. Figure 6.14 shows the situation. We use the relationship $v = \lambda f$ to determine the wavelength when the police siren is at rest. When it is in motion, we find the wavelength on either side of the siren.

a. When the source is at rest,

$$\lambda = \frac{v}{f_s} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

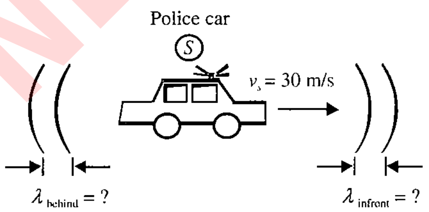

Fig. 6.14

b. The situation is shown in Fig. 6.15. Now, in front of the siren,

$$\lambda_{\text{in front}} = \frac{v-v_s}{f_s} = \frac{340 \text{ m/s} - 30 \text{ m/s}}{300 \text{ Hz}} = 1.03 \text{ m}$$

Behind the siren,

$$\lambda_{\text{behind}} = \frac{v+v_s}{f_s} = \frac{340 \text{ m/s} + 30 \text{ m/s}}{300 \text{ Hz}} = 1.23 \text{ m}$$

The wavelength is less in front of the siren and greater behind the siren, as it should be.

Illustration 6.15 **If a listener L is at rest and the siren in previous illustration is moving away from L at a speed of 30 m/s, what frequency does the listener hear?**

Sol. Our target variable is the frequency f_L heard by the listener, who is behind the moving source. Figure 6.15 shows the situation. We know that $v_L = 0$ and $v_s = 30$ m/s. (The source velocity v_s is positive because the siren is moving in the same direction as the direction from listener to source.)

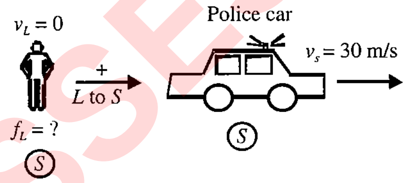

Fig. 6.15

Now,

$$f_L = \frac{v}{v+v_s} f_s = \frac{340 \text{ m/s}}{340 \text{ m/s} + 30 \text{ m/s}}(300 \text{ Hz}) = 276 \text{ Hz}$$

The source and listener are moving apart, so the frequency f_L heard by the listener is less than the frequency f_S emitted by the source.

Here is an alternative approach we can use to check our result. From previous illustration, the wavelength behind the source (which is where the listener in figure is located) is 1.23 m, so

$$f_L = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ m}$$

Even though the source is moving, the wave speed v relative to the stationary listener is unchanged.

Moving Source and Stationary Observer

Figure 6.16 shows the situation when a moving source S of frequency n_0 produces sound waves in medium (air) and the waves travel towards observer with velocity v.

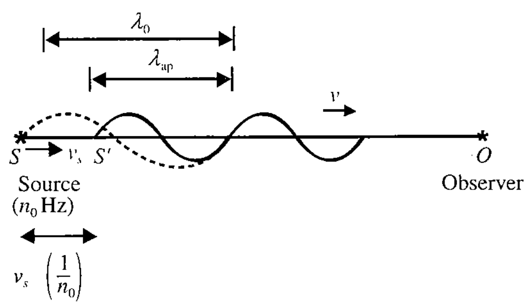

Fig. 6.16

Here let us carefully look at the initial situation when the source starts moving with velocity v_s as well as it starts producing waves. The period of one oscillation is $(1/n_0)$

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seconds and in this duration the source emits one wavelength λ_0 in the direction of propagation of waves with speed v, but in this duration the source will also move forward by a distance v_s $(1/n_0)$. Thus the effective wavelength of emitted sound in air is slightly compressed by this distance as shown in Fig. 6.16. This is known as the apparent wavelength of sound in medium (air) by the moving source. This is given as

$$\lambda_{ap} = \lambda_0 - v_s\left(\frac{1}{n_0}\right) \qquad \text{(iv)}$$

$$= \frac{\lambda_0 n_0 - v_s}{n_0} = \frac{v - v_s}{n_0}$$

Now, this wavelength will approach observer with speed v (as O is at rest). Thus, the frequency of sound heard by the observer can be given as

$$n_{ap} = \frac{v}{\lambda_{ap}}$$

$$= \frac{v}{(v - v_s)/n_0} = n_0\left(\frac{v}{v - v_s}\right) \qquad \text{(v)}$$

Similarly, if the source is receding away from observer, the apparent wavelength emitted by source in air towards the observer will be slightly expanded and the apparent frequency heard by the stationary observer can be given as

$$n_{ap} = n_0\left(\frac{v}{v + v_s}\right) \qquad \text{(vi)}$$

Illustration 6.16 **Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching ambulance. After the ambulance passes, the observed frequency of the siren is 480 Hz. Determine the ambulance's speed from these observations. Speed of sound = 343 m/s.**

Sol. We can assume that an ambulance with its siren on is in a hurry to get somewhere, and is probably travelling between 30 and 150 km/h (~10 m/s to 50 m/s), depending on the driving conditions. We can use the equation for the Doppler effect to find the speed of the vehicle. Let v_a be the magnitude of the velocity of the ambulance. As it approaches, you hear the frequency,

$$f' = \left(\frac{v}{v - v_a}\right) f = 560 \text{ Hz}$$

The negative sign appears because the source is moving towards the observer. The opposite sign with source velocity magnitude describes the ambulance moving away. As it recedes, the Doppler-shifted frequency is

$$f'' = \left(\frac{v}{v + v_a}\right) f = 480 \text{ Hz}$$

Solving the second of these equations for f and substituting into the other gives

$$f' = \left(\frac{v}{v - v_a}\right)\left(\frac{v + v_a}{v}\right) f''$$

or $\qquad f'v - f'v_a = vf'' + v_a f''$

So the speed of the source is

$$v_a = \frac{v(f' - f'')}{f' + f''} = \frac{(343 \text{ m/s})(560 \text{ Hz} - 480 \text{ Hz})}{560 \text{ Hz} + 480 \text{ Hz}} = 26.4 \text{ m/s}$$

This seems like a reasonable speed (about 85 km/h) for an ambulance, unless the street is crowded or the vehicle is travelling on an open highway.

Moving Source and Moving Observer

Let us consider the situation when both source and observer are moving in same direction as shown in Fig. 6.17 at speeds v_s and v_0, respectively.

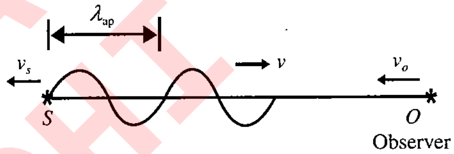

Fig. 6.17

In this case, the apparent wavelength emitted by the source behind it is given as

$$\lambda_{ap} = \frac{v + v_S}{n_0}$$

Now, this wavelength will approach the observer at relative speed $v + v_0$. Thus the apparent frequency of sound heard by the observer is given as

$$n_{ap} = \frac{v + v_o}{\lambda_{ap}} = n_0\left(\frac{v + v_o}{v + v_s}\right) \qquad \text{(vii)}$$

By looking at the expression of apparent frequency given by Eq. (vii), we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to moving source as

$$n_{ap} = n_0\left[\frac{v \pm v_o}{v \pm v_s}\right] \qquad \text{(viii)}$$

Here + and – signs are chosen according to the direction of motion of source and observer. The sign convention related to the direction of motion can be stated as follows:

i. For both source and observer, v_0 and v_s are taken in Eq. (viii) with –ve sign if they are moving in the direction of $\vec{v}$, i.e., the direction of propagation of sound from source to observer.

ii. For both source and observer, v_o and v_s are taken in Eq. (viii) with +ve sign if they are moving in the direction opposite to $\vec{v}$, i.e., opposite to the direction of propagation of sound from source to observer.

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Some Important Cases of Doppler Effect

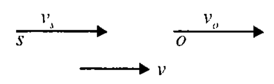
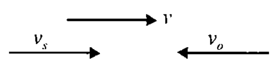
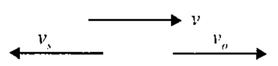
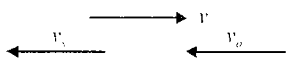
Situation	Apparent frequency	Apparent wavelength
	$f'=f\left(\dfrac{v-v_o}{v-v_s}\right)$	$\lambda'=\dfrac{(v-v_s)}{f}=\lambda\left(\dfrac{v-v_s}{v}\right)$
	$f'=f\left(\dfrac{v+v_o}{v-v_s}\right)$	$\lambda'=\lambda\left(\dfrac{v-v_s}{v}\right)$
	$f'=f\left(\dfrac{v-v_o}{v+v_s}\right)$	$\lambda'=\lambda\left(\dfrac{v+v_s}{v}\right)$
	$f'=f\left(\dfrac{v+v_o}{v+v_s}\right)$	$\lambda'=\lambda\left(\dfrac{v+v_s}{v}\right)$

Illustration 6.17 **Two ships are moving along a line due east. The trailing ship has a speed relative to land-based observation point of 64.0 km/h, and the leading ship has a speed of 45.0 km/h relative to that point. The two ships are in a region of the ocean where the current is moving uniformly due west at 10.0 km/h. The trailing ship transmits a sonar signal at a frequency of 1200.0 Hz. What frequency is monitored by the leading ship? Use 1520 m/s as the speed of sound in ocean water.**

Sol. The speeds of source and observer are small fractions of the speed of sound in water. The Doppler shift should be a small fraction of the radiated frequency. The gap between the ships is closing, so the frequency received will be higher than 1.20 kHz. We guess 1.25 kHz. When the observer is moving in front of and in the same direction as the source, the Doppler equation becomes

$$f'=\left(\frac{v+(-v_l)}{v-v_t}\right)f$$

where v_l and v_t are the speeds of the leading and trailing ships measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite to the direction of travel of the ships and

$$v_l = 45.0\text{ km/h} - (-10.0\text{ km/h}) = 55.0\text{ km/h} = 15.3\text{ m/s}$$

$$v_t = 64.0\text{ km/h} - (-10.0\text{ km/h}) = 74.0\text{ km/h} = 20.6\text{ m/s}$$

Therefore,

$$f'=(1200\text{ Hz})\left(\frac{1520\text{ m/s}-15.3\text{ m/s}}{1520\text{ m/s}-20.6\text{ m/s}}\right)=1204\text{ Hz}$$

The Doppler shift is much smaller than we guessed. Note well that sound moves much faster in water than in air. The difference in speed of the ships is only 19 km/h, and this is only 5.3 m/s.

DOPPLER EFFECT IN REFLECTED SOUND

When a car is moving towards a stationary wall as shown in Fig. 6.18. If the car sounds a horn, wave travels towards the wall and is reflected form the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch. If we wish to measure the frequency of reflected sound then the problem must be handled in two steps.

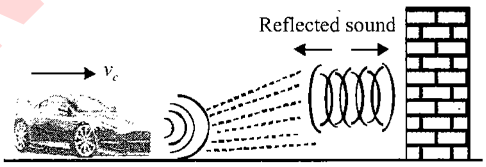


Fig. 6.18

First, we treat the stationary wall as a stationary observer and car as a moving source of sound of frequency n_0. In this case, the frequency received by the wall is given as

$$n_1=n_0\left(\frac{v}{v-v_c}\right) \qquad \text{(ix)}$$

Now, the wall reflects this frequency and behaves like a stationary source of sound of frequency n_1 and car (driver) behaves like a moving observer moving with velocity v_c. Here the apparent frequency heard by the car driver can be given as

$$n_{ap}=n_1\left(\frac{v+v_c}{v}\right)$$

or
$$=n_0\left(\frac{v}{v-v_c}\right)\times\left(\frac{v+v_c}{v}\right)=n_0\left(\frac{v+v_c}{v-v_c}\right) \qquad \text{(x)}$$

Same problem can also be solved in a different manner by using method of sound images. In this procedure, we assume

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the image of the sound source behind the reflector. In previous example, we can explain this by a situation shown in Fig. 6.19.

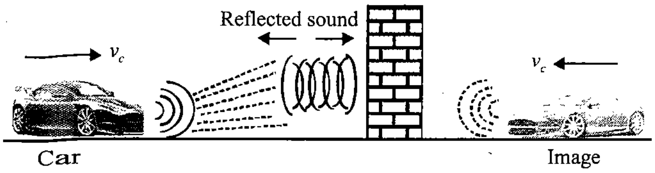

Fig. 6.19

Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming towards it with velocity v_c. Now the frequency of sound heard by car driver can directly be given as

$$n_{ap} = n_0\left[\frac{v+v_c}{v-v_c}\right] \quad \text{(xi)}$$

This method of images for solving problem of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequently when the reflector of sound is moving.

Illustration 6.18 **To permit measurement of her speed, a sky diver carries a buzzer emitting a steady tone at 1800 Hz. A friend on the ground at the landing site directly below listens to the amplified sound he receives. Assume the air is calm and the sound speed is 343 m/s independent of altitude. While the sky diver is falling at terminal speed, his friend on the ground receives waves of frequency 2150 Hz. (a) What is the sky diver's speed of descent? (b) Suppose the sky diver can hear the sound of the buzzer reflected from the ground. What frequency does she receive?**

Sol. Sky divers typically reach a terminal speed of about 150 mi/h (~75 m/s), so this sky diver should also fall near this rate. Since her friend receives a higher frequency as a result of the Doppler shift, the sky diver should detect a frequency with twice the Doppler shift, at approximately

$$f' = 1800\text{ Hz} + 2(2\,150\text{ Hz} - 1800\text{ Hz}) = 2500\text{ Hz}$$

We can use the equation for the Doppler effect to answer both (a) and (b). Let f_e = 1800 Hz represent the emitted frequency, v_e be the speed of the sky diver and f_g = 2150 Hz be the frequency of the wave crests reaching the ground.

a. The sky diver source is moving towards the stationary ground, so we rearrange the equation as follows:

$$f_g = f_e\left(\frac{v}{v-v_e}\right)$$

$$\Rightarrow v_e = v\left(1-\frac{f_e}{f_g}\right) = (343\text{ m/s})\left(1-\frac{1800\text{ Hz}}{2150\text{ Hz}}\right) = 55.8\text{ m/s}$$

b. The ground now becomes a stationary source, reflecting crests with the 2150 Hz frequency at which they reach the ground, and sending them to a moving observer, who receives them at the rate

$$f_{e2} = f_g\left(\frac{v+v_e}{v}\right) = (2150\text{ Hz})\left(\frac{343\text{ m/s}+55.8\text{ m/s}}{343\text{ m/s}}\right) = 2500\text{ Hz}$$

The answers appear to be consistent with our predictions, although the sky diver is falling somewhat slower than expected. The Doppler effect can be used to find the speed of many different types of moving objects, like raindrops (with Doppler radar) and cars (with police radar).

DOPPLER EFFECT FOR ACCELERATED MOTION

For the case of moving source and a moving observer, we know that the apparent frequency can be given as

$$n_{ap} = n_0\left[\frac{v \pm v_o}{v \pm v_S}\right] \quad \text{(xii)}$$

Here v is the velocity of sound and v_o and v_S are the velocity of observer and source, respectively. When a source or observer has accelerated or retarded motion, then in Eq. (xii) we use that value of v_o at which the observer receives the sound and for source, we use that value of v_s at which it has emitted the wave.

The alternative method of solving this case is by the traditional method of compressing or expending wavelength of sound by motion of source and using relative velocity of sound with respect to observer.

Illustration 6.19 **Two trains A and B simultaneously start moving along parallel tracks from a station along same direction. A starts with constant acceleration 2 m/s^2 from rest, while B with the same acceleration but with initial velocity of 40 m/s. Twenty seconds after the start, passenger of A hears whistle of B. If frequency of whistle is 1194 Hz and velocity of sound in air is 322 m/s, calculate frequency observed by the passenger.**

Sol. Since both the trains move in same direction with same acceleration, therefore, their relative acceleration is zero. Initial velocity of train A is zero while that of train B is 40 m/s. Therefore, train A or observer is behind the train B or source when whistle is heard by passenger of A. Let this sound be produced at time t. Considering motion of train B (sound source) up to this instant,

$$u = 40\text{ m/s},\quad a = 2\text{ m/s}^2,\quad \text{time} = t$$

$$v = v_B = ?\quad s = s_B = ? \quad \text{(i)}$$

We have, $\quad v = u + at,$

$$\therefore \quad v_B = 40 + 2t \quad \text{(ii)}$$

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad s_B = 40t + t^2$$

But this sound is heard by the observer at t = 20 s. Considering his motion (motion of train A) up to this instant,

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$$u=0,\quad a=2\quad \text{m/s}^2,\quad t=20\text{ s}$$

$$v=v_A=?\ s=s_A=?$$

We have, $v=u+at,$

$\therefore$ $v_A=40$ m/s

$$s=ut+\frac{1}{2}at^2$$

$\therefore$ $s_A=400$ m

When sound was produced, train B (source) was at a distance s_B from initial point while observer receives this sound at $t=20$ s and at a distance s_A from initial point. It means that sound waves travel a distance (s_B-s_A) in air and take time $(20-t)$ to travel it.

$$s_B-s_A=(20-t)v \qquad \text{(iii)}$$

where v = 322 m/s (speed of sound). Substituting values of s_A and s_B in Eq. (iii),

$$t=18\text{ s}$$

When sound was produced, source was moving with velocity

$$v_B=(40+2t)=76\text{ m/s}$$

(away from observer) while observer receives these sound waves at t = 20 s, when he was moving with velocity v_A = 40 m/s (towards the source). Hence, the observed frequency is

$$n=n_0\frac{v+v_A}{v+v_B}$$

where n_0 = 1194 Hz (natural frequency of source) and so $n=1086$ Hz.

Doppler's Effect When Source and Observer Are Not in Same Line of Motion

Consider the situation shown in Fig. 6.20. Two cars 1 and 2 are moving along perpendicular roads at speeds v_1 and v_2. When car 1 sounds a horn of frequency n_0, it emits sound in all directions and say car 2 is at the position, shown in Fig. 6.20 when it receives the sound. In such cases, we use velocity components of the cars along the line joining the source and observer. Thus, the apparent frequency of sound heard by car 2 can be given as

$$n_{ap}=n_0\left[\frac{v+v_2\cos\theta_2}{v-v_1\cos\theta_1}\right] \qquad \text{(xiii)}$$

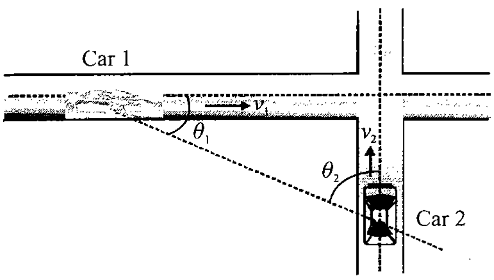

Fig. 6.20

Illustration 6.20 **A train approaching a railway crossing at a speed of 120 km/h sounds a short whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 m/s. What will be the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing?**

Sol. The observer A is at rest with respect to the air and the source is travelling at a velocity of 120 km/h, i.e., (100/3) m/s. As is clear from the figure, the person receives the sound of the whistle in a direction BA making an angle θ with the track where $\cos\theta$ = 300/500 = 3/5. The component of the velocity of the source (i.e., of the train) along the direction AB is (100/3) × (3/5) m/s = 20 m/s. As the source is approaching the person with this component, the frequency heard by the observer is

$$n'=\frac{v}{v-u\cos\theta}n=\frac{340}{340-20}\times 640\text{ Hz}=680\text{ Hz}$$

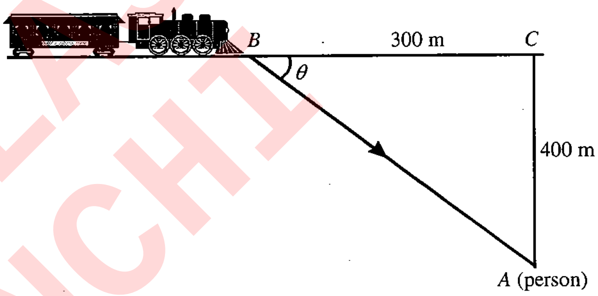

Fig. 6.21

Illustration 6.21 **Your clock radio awakens you with a steady and irritating sound of frequency 600 Hz. On morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, 15.0 m from the ground. Assume the speed of sound is 343 m/s. As you listen to the falling clock radio, what frequency do you hear just before you hear it striking the ground?**

Sol. The speed of clock radio increases as it falls. Therefore, it is a source of sound moving away from you with an increasing speed, so the frequency you hear should be less than 600 Hz. We categorize this problem as one in which we must combine our understanding of falling objects with that of the frequency shift due to Doppler effect. Because the clock radio is modelled as a particle under constant acceleration due to gravity the speed of the source of sound is

$$v_s=gt$$

Now to determine the Doppler-shifted frequency heard from the falling clock radio, We have

$$f'=\left(\frac{v}{v+v_s}\right)f=\left(\frac{v}{v+gt}\right)f \qquad \text{(i)}$$

Find the time at which the clock radio strikes the ground.

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$$\Rightarrow -15.0 \text{ m} = 0 + 0 - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

$$\Rightarrow t = 1.75 \text{ s}$$

From Eq. (i), evaluate the Doppler-shifted frequency just as the clock radio strikes the ground:

$$f' = \left[\frac{343 \text{ m/s}}{343 \text{ m/s} + (9.80 \text{ m/s}^2)(1.75 \text{ s})}\right](600 \text{ Hz}) = 571 \text{ Hz}$$

The frequency is lower than the actual frequency of 600 Hz because the clock radio is moving away from you. If it were to fall from a higher floor so that it passes below $y = -15.0$ m, the clock radio would continue to accelerate and the frequency would continue to drop.

Illustration 6.22 **A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are travelling directly towards each other. The second submarine is moving at 9.00 m/s.**

a. What frequency is detected by an observer riding on sub B as the submarines approach each other?

b. The submarines barely miss each other and pass. What frequency is detected by an observer riding on sub B as the submarines recede from each other?

Sol.

a. Even through the problem involves subs moving in water, there is a Doppler effect just like there is when you are in a moving car and listening to a sound moving through the air from another car. Because both subs are moving, we categorize this problem as one involving the Doppler effect for both a moving source and a moving observer.

We can find the Doppler-shifted frequency heard by the observer in sub B, being careful with the signs of the source and observer velocities:

$$f' = \left(\frac{v + v_0}{v - v_s}\right)f$$

$$= \left[\frac{1533 \text{ m/s} + (+9.00 \text{ m/s})}{1533 \text{ m/s} - (+8.00 \text{ m/s})}\right](1400 \text{ Hz}) = 1416 \text{ Hz}$$

b. In this case, the Doppler-shifted frequency heard by the observer in sub B, again being careful with the signs of the source and observer velocities, is given by

$$f' = \left(\frac{v + v_0}{v - v_s}\right)f$$

$$= \left[\frac{1533 \text{ m/s} + (-9.00 \text{ m/s})}{1533 \text{ m/s} + (+8.00 \text{ m/s})}\right] (1400 \text{ Hz}) = 1385 \text{ Hz}$$

This technique is used by police officers to measure the speed of a moving car. Microwaves are emitted from the police car and reflected by the moving car. By detecting the Doppler-shifted frequency of the reflected microwaves, the police officer can determine the speed of the moving car.

Note: ***Suppose a source of sound moves towards a stationary observer with a speed v'. Then the observed frequency is***

$$f' = f\frac{v}{v - v'}$$

Now if the observer moves towards the stationary source with the same speed v', then the observed frequency is

$$f'' = f'\frac{v + v'}{v}$$

Clearly, $v' = \frac{v}{\quad} v$. That is, the observed frequency in two cases is different, although the relative speed between them is same. For this season, the Doppler effect in sound is said to be asymmetric. However, the Doppler effect in light is symmetric. This is because the observed frequency or wavelength depends only on relative speed between source and observer.

Concept Application Exercise 6.2

1. A person riding on a merry-go-round emits a sound wave of a certain frequency. Does a person at the centre observe the Doppler effect?
2. Does the Doppler effect increase the intensity of wave when its source approaches the observer?
3. The Doppler effect is a wave characteristic. Light and sound are both wave motion. Is there any difference in the Doppler effect in light and sound?
4. Is there a Doppler effect in the case of sound when the observer or the source moves at right angles to the line joining them? How then can we determine the Doppler effect when the motion has a component at right angles to this line?
5. A source moves away from an observer with a certain speed, and the ratio of actual to the apparent frequency as heard by the observer is η. If the two approach each other with the same speed, then find the ratio.
6. An engine blowing a whistle of frequency 133 Hz moves with a velocity of 60 m/s towards a hill from which an echo is heard. Calculate the frequency of the echo heard by the driver. (Velocity of sound in air is 340 m/s.)
7. A source of sound of frequency 1000 Hz moves to the right with a speed of 32 m/s relative to the ground. To its right there is a reflecting surface moving to the left with a speed of 64 m/s relative to the ground. Take the speed of sound in air to be 332 m/s and find
   (a) the wavelength of the sound emitted in air by the source,
   (b) the number of waves per second arriving at the reflecting surface,

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(c) the speed of the reflected waves and
(d) the wavelength of the reflected waves.

8. A stationary source emits sound of frequency $v = 1200$ Hz. If a wind blows at the speed of $0.1c$, deduce (i) the percentage change in the wavelength and (ii) the change in the frequency for a stationary observer on the wind-side of the source. What happens when there is no wind, but the observer moves at speed $0.1c$ towards the source?

9. A stationary observer receives sound waves from two tuning forks, one of which approaches and the other recedes with the same velocity. As this takes place, the observer hears beats of frequency $v = 2$ Hz. Find the velocity of each tuning fork if their oscillation frequency is $v_0 = 680$ Hz and the velocity of sound in air is $v_s = 340$ m/s.

10. A stationary source sends forth monochromatic sound. A wall approaches it with velocity 33 cm/s. The propagation velocity of sound in the medium is $c = 330$ m/s. How much, in per cent, does the wavelength of sound change on reflection from the wall?

11. A source of sound with frequency 1000 Hz moves at right angles to a wall with a velocity $u = 17$ cm/s. Two stationary receivers R_1 and R_2 are located on a straight line coinciding with the path of the source in the following succession: $R_1 \to$ source $\to R_2 \to$ wall. Which receiver registers beating and what is the beat frequency? The velocity of sound is $c = 340$ m/s.

12. A whistle of frequency 540 Hz rotates in a horizontal circle of radius 2 m at an angular speed of 15 rad/s. What is the lowest and the highest frequency heard by a listener a long distance away at rest with respect to the centre of the circle? (Velocity of sound in air is $c = 330$ m/s.)

13. A bat flies perpendicularly towards a wall with a speed of 6 m/s, emitting sound of frequency 450 kHz. What is the frequency of the wave reflected from the wall that it will hear? Given, $c = 340$ m/s.

14. The ratio of the apparent frequencies of a car when approaching and receding a stationary observer is 11:9. What is the speed of the car, if the velocity of sound in air is 330 m/s?

15. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle (speed of sound is 330 m/s).

16. A siren emitting a sound of frequency 2000 Hz moves away from you towards a cliff at a speed of 8 m/s.
(a) What is the frequency of the sound you hear coming directly from the siren.
(b) What is the frequency of sound you hear reflected off the cliff. Speed of sound in air is 330 m/s.

17. A railroad train is travelling at 30 m/s in still air. The frequency of the note emitted by locomotive whistle is 500 Hz. What is the frequency of the sound waves heard by a stationary listener (a) in front of the train and (b) behind the train? (speed of sound is 345 m/s)

18. Two tuning forks A and B having a frequency of 500 Hz each are placed with B to the right of A. An observer in between the forks is moving towards B with a speed of 25 m/s. The speed of sound is 345 m/s and the wind speed is 5 m/s from A to B. Calculate the difference in the two frequencies heard by observer.

Solved Examples

Example 6.1 **The sound level at a distance of 3.00 m from a source is 120 dB. At what distance will the sound level be (a) 100 dB and (b) 10.0 dB?**

Sol. A reduction of 20 dB means reducing the intensity by a factor of 10^2, so we expect the radial distance to be 10 times larger, namely 30 m. A further reduction of 90 dB may correspond to an extra factor of $10^{4.5}$ in distance, to about 30×30000 m, or about 1000 km. We use the definition of the decibel scale and the inverse square law for sound intensity. From the definition of sound level,

$$\beta = 10 \log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

We can compute the intensities corresponding to each of the levels mentioned as

$$I = [10^{\beta/10}]10^{-12}\ \text{W/m}^2$$

They are $I_{120} = 1$ W/m^2, $I_{100} = 10^{-2}$ W/m^2 and $I_{10} = 10^{-11}$ W/m^2.

a. The power passing through any sphere around the source is

$$P = 4\pi r^2 I$$

If we ignore absorption of sound by the medium, conservation of energy requires that

$$r_{120}^2 I_{120} = r_{100}^2 I_{100} = r_{10}^2 I_{10}$$

Then,

$$r_{100} = r_{120}\sqrt{\frac{I_{120}}{I_{100}}} = (3.00\ \text{m})\sqrt{\frac{1\ \text{W/m}^2}{10^{-2}\ \text{W/m}^2}} = 30.0\ \text{m}$$

b. $$r_{10} = r_{120}\sqrt{\frac{I_{120}}{I_{10}}} = (3.00\ \text{m})\sqrt{\frac{1\ \text{W/m}^2}{10^{-11}\ \text{W/m}^2}} = 9.49 \times 10^5\ \text{m}$$

At 949 km away, the faint 10 dB sound would not be identifiable among many other soft and loud sounds produced by other sources across the continent. And 949 km of air is such a thick screen that it may do a significant amount of sound absorption.

Example 6.2 **A student holds a tuning fork oscillating at 2456 Hz. He walks towards a wall at a constant speed of 1.33 m/s. (a) What beat frequency does the student observes between the tuning fork and its echo? (b) How fast must he**

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walk away from the wall to observe a beat frequency of 5.00 Hz?

Sol. An electronic burglar detector may listen for beats between the signal it broadcasts and the Doppler-shifted reflection from a moving surface. The experiment described here is more fun. The student hears beats at a rate that is controlled through his walking speed. We must think of a double Doppler shift, for the wave reaching the wall from the moving tuning fork and the wave reaching the moving auditor.

For this echo,

$$f_{\text{received back}} = f_{\text{radiated}} \frac{v+v_e}{v-v_e}$$

where v_e is the magnitude of the experimenter's velocity. The beat frequency is

$$f_b = |f_{\text{receivedback}} - f_{\text{radiated}}|$$

Substituting gives

$$f_b = f_{\text{radiated}} \frac{2v_e}{v-v_e}$$

when approaching the wall.

a. $f_b = (256\text{ Hz})\dfrac{2(1.33\text{ m/s})}{343\text{ m/s} - 1.33\text{ m/s}} = 1.99\text{ Hz}$

b. When moving away from wall, v_e appears with opposite signs. Solving for v_e gives

$$v_e = f_b \frac{v}{2f_{\text{radiated}} - f_b}$$

$$= (5.00\text{ Hz})\left[\frac{343\text{ m/s}}{2(256\text{ Hz}) - 5.00\text{ Hz}}\right] = 3.38\text{ m/s}$$

Example 6.3 **Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station and the other is moving nearby, a passenger standing on the station platform hears beats with a frequency of 2.00 beats/s when the whistles operate together. What are the two possible speeds and directions the moving train can have? Speed of sound is 343 m/s.**

Sol. A Doppler shift in the frequency received from the moving train is the source of the beats. We consider velocities of approach and of recession separately in the Doppler equation, after we observes from our beat equation $f_b = |f_1 - f_2| = |f - f'|$ that the moving train must have an apparent frequency of either $f' = 182$ Hz or $f' = 178$ Hz. We let v_t represent the magnitude of the train's velocity. If the train is moving away from the station, the apparent frequency is 178 Hz lower, as described by

$$f' = \frac{v}{v+v_t} f$$

and the train is moving away at

$$v_t = \left(\frac{f}{f'} - 1\right)v = (343\text{ m/s})\left(\frac{180\text{ Hz}}{178\text{ Hz}} - 1\right) = 3.85\text{ m/s}$$

If the train is pulling into the station, then the apparent frequency is 182 Hz. Again from the Doppler shift,

$$f' = f\frac{v}{v-v_s}$$

The train is approaching at

$$v_s = v\left(1 - \frac{f}{f'}\right) = (343\text{ m/s})\left(1 - \frac{180\text{ Hz}}{182\text{ Hz}}\right)$$

$$= 3.77\text{ m/s}$$

The Doppler effect does not measure distance, but it can be a remarkably sensitive measure of speed.

Example 6.4 **When a train is approaching the observer, the frequency of the whistle is 100 cps. When it has passed the observer, it is 50 cps. Calculate the frequency when the observer moves with the train.**

Sol. When the source (the train) moves towards the observer, the apparent frequency is

$$f' = \frac{v}{v-v_s} f \qquad \text{(i)}$$

When the source is moving away, the apparent frequency is

$$f'' = \frac{v}{v+v_s} f \qquad \text{(ii)}$$

$$\therefore \quad \frac{f'}{f''} = \frac{v+v_s}{v-v_s}$$

or $$\frac{100}{50} = \frac{2}{1} = \frac{\left(\dfrac{v}{v_s}+1\right)}{\left(\dfrac{v}{v_s}-1\right)}$$

or $$\frac{v}{v_s} = 3 \qquad \text{(iii)}$$

When the observer moves with the train, there is no relative motion between the source and the observer and hence he will listen the true frequency f. Substituting (iii) in (i), we get

$$100 = \frac{3}{2} f$$

or $$f = \frac{200}{3} = 66.6\text{ Hz}$$

Example 6.5 **A source S emitting sound of 300 Hz is fixed on block A which is attached to free end of a spring S_A as shown in the figure. The detector D fixed on block B attached to the free end of spring S_B detects this sound.**

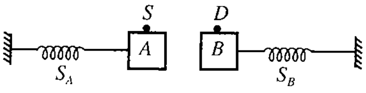

Fig. 6.22

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The blocks A and B are simultaneously displaced towards each other through a distance of 1.0 m and then left to vibrate. Find the maximum and minimum frequencies of sound detected by D if the vibrational frequency of each block is 2 Hz (velocity of sound is 340 m/s).

Sol. The motion of A and B is synchronized. Hence the maximum frequencies detected by D will be

$$f_{\max}=\left(\frac{v+v_D}{v-v_S}\right)f$$

(when S and D are approaching each other while at their equilibrium position.)

$$=\left(\frac{340+A\omega}{340-A\omega}\right)f=\left(\frac{340+12.56}{340-12.56}\right)\times 300=323\text{ Hz}$$

Similarly, the apparent frequency will be minimum when the source and the detector recede from each other with maximum speed $(=A\omega)$

$$f_{\min}=\left(\frac{v-A\omega}{v+A\omega}\right)f=\left(\frac{340-12.56}{340+12.56}\right)\times 300=278.6\text{ Hz}$$

Example 6.6 **The frequency of sound produced by a bell is 500 Hz. The velocity of the source relative to still air is 60 m/s. An observer moves at 30 m/s along the same line as the source. Calculate the frequency of sound wave measured by the observer. Consider all the possible cases (speed of sound $v = 340$ m/s).**

Sol. $\xrightarrow{v_s}$ Source $\xrightarrow{v_o}$ Observer

Case I

Let the observer be at right side of the source. Both source and observer are moving in right direction. There will be apparent increase in frequency because $V_{\text{source}} > V_{\text{observer}}$.

$$f'=f_0\left[\frac{V_s-V_{\text{observer}}}{V_s-V_{\text{source}}}\right]\text{Hz}=500\left[\frac{340-30}{340-60}\right]\text{Hz}=553\text{ Hz}$$

Case II

Source is moving right and observer is moving left.

Source $\rightarrow$ $\leftarrow$ Observer

There will be apparent increase in frequency since both are moving towards each other.

$$f'=f_0\left[\frac{V_s+V_{\text{observer}}}{V_s-V_{\text{source}}}\right]\text{Hz}=500\left[\frac{340+30}{340-60}\right]\text{Hz}$$

$= 600$ Hz (more than that in case i)

Case III

Both source and observer are receding from each other.

$\leftarrow$ Source Observer $\rightarrow$

There will be an apparent decrease in frequency.

$$f'=f_0\left[\frac{V_s-V_{\text{observer}}}{V_s-V_{\text{source}}}\right]\text{Hz}=500\left[\frac{340-30}{340+60}\right]\text{Hz}=387.5\text{ Hz}$$

Case IV

Both source and observer are moving towards left.

$\leftarrow$ Source $\leftarrow$ Observer

$$f'=f_0\left[\frac{V_s+V_{\text{observer}}}{V_s+V_{\text{source}}}\right]=500\left[\frac{340+30}{340+60}\right]\text{Hz}=462.5\text{ Hz}$$

You should repeat the example assuming that the observer is to the left of the source. What conclusions can be derived from this example? Do you get a different set of four apparent frequencies.

Example 6.7 **An observer standing on a railway crossing receives frequencies 2.2 kHz and 1.8 kHz when the train approaches and recedes from the observer. Find the velocity of the train (speed of sound in air is 300 m/s).**

Sol. Let $v = 300$ m/s be the velocity of sound and u the velocity of train. When the train approaches the observer, the frequency heard is

$$n_1=\frac{v}{v-u}n \qquad \text{(i)}$$

When the train recedes from the observer, the frequency heard is

$$n_2=\frac{v}{v+u}n \qquad \text{(ii)}$$

Dividing Eqs. (i) and (ii), we get

$$\frac{n_1}{n_2}=\frac{v+u}{v-u} \Rightarrow u=\frac{n_1-n_2}{n_1+n_2}v$$

Given $n_1 = 2.2$ kHz $= 2.2 \times 10^3$ Hz.

$$\therefore \quad u=\frac{(2.2-1.8)\times 10^3}{(2.2+1.8)\times 10^3}\times 300\text{ m/s}=\frac{0.4\times 300}{4}=30\text{ m/s}$$

Example 6.8 **A source of sound is moving along a circular orbit of radius 3 m with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude $BC = CD = 6$ m. The frequency of oscillation of the detector is $5/\pi$ s^{-1}. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector.**

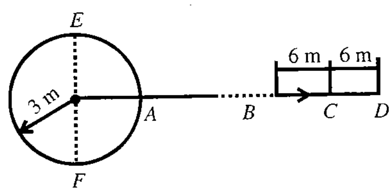

Fig. 6.23

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Sol. Speed of source,

$$v_s = r\omega = 3 \times 10 = 30 \text{ m/s}$$

Maximum speed of detector,

$$v_{\max} = v_d = A\omega' = A\,2\pi f' = 6 \times 2\pi \times \frac{5}{\pi} = 60 \text{ m/s}$$

The speed of sound is $v = 330$ m/s

The frequency recorded by detector is maximum when source and detector approach each other (i.e. source at F and detector at C after times $3T/4$, and $3T'/4$, respectively, T and T' being time periods of source and detector). Then maximum frequency is given by

$$n_{\max} = \frac{v + v_d}{v - v_s} n$$

$$= \frac{330 + 60}{330 - 30} \times 340 = \frac{390}{300} \times 340 = 442 \text{ Hz}$$

The frequency recorded by detector is minimum when source and detector recede from each other (i.e., source at E and detector at C after time $T/4$), so that the minimum frequency is

$$n_{\min} = \frac{v - v_d}{v + v_s} n$$

$$= \frac{330 - 60}{330 + 30} \times 340 = \frac{270}{360} \times 340 = 255 \text{ Hz}$$

Example 6.9 **Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards the observer at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning forks (speed of sound is 340 m/s).**

Sol. Let v be the speed of sound and v_s the speed of forks. The apparent frequency of fork which moves towards the observer is

$$n_1 = \frac{v}{v - v_s} n$$

The apparent frequency of fork which moves away from observer is

$$n_2 = \frac{v}{v + v_s} n$$

If x is the number of beats heard per second, then

$$x = n_1 - n_2 \quad \Rightarrow x = \frac{v}{v - v_s} n - \frac{v}{v + v_s} n$$

or $$x = \frac{v + v_s - (v - v_s)}{v^2 - v_s^2} (vn)$$

or $$\frac{2vv_s n}{v^2 - v_s^2} = x$$

$$2\left(\frac{v_s}{v}\right) n = x \qquad [\because \text{ if } v_s \ll v]$$

or $$v_s = \frac{vx}{2n}$$

Given $v = 340$ m/s, $x = 3$, $n = 340$ Hz.

$\therefore$ $$v_s = \frac{340 \times 3}{2 \times 340} = 1.5 \text{ m/s}$$

Example 6.10 **A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist.**

Sol. Actual frequency of band music is f. The direct frequency heard by motorist is

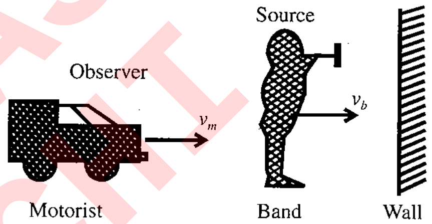

Fig. 6.24

$$f_1 = \frac{v + v_m}{v + v_b} f \qquad \text{(i)}$$

The frequency heard by observer at wall is

$$f' = \frac{v}{v - v_b} f$$

The reflection does not cause any change in frequency. Therefore, frequency f' is reflected as such and the motorist is approaching the wall. Hence, the frequency of reflected sound heard by motorist is

$$f_2 = \frac{v + v_m}{v} f'$$

$$= \frac{v + v_m}{v} \times \frac{v}{v - v_b} f = \frac{v + v_m}{v - v_b} f \qquad \text{(ii)}$$

Therefore, the beat frequency is

$$x = f_2 - f_1 = \frac{v + v_m}{v - v_b} f - \frac{v + v_m}{v + v_b} f$$

$$= \frac{(v + v_b) - (v - v_b)}{v^2 - v_b^2} (v + v_m) f$$

$\therefore$ $$x = \frac{2v_b (v + v_m) f}{v^2 - v_b^2}$$

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Example 6.11 **A boat is travelling in a river with a speed of 10 m/s along the stream flowing with a speed of 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.**

a. What will be the frequency detected by a receiver kept inside downstream?

b. The transmitter and the receiver are now pulled up into air. The air is blowing with a speed of 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water is 20°C; density of river water is 10^3 kg/m³; bulk modulus of the water is 2.088×10^9 Pa; gas constant $R = 8.31$ J/mol-K; mean molecular mass of air is 28.8×10^{-3} kg/mol; C_P/C_V for air is 1.4.)

Sol.

a. Speed of boat $u = 10$ m/s.

Speed of river stream $w = 2$ m/s.

Wavelength of sound wave inside water is 14.45 mm.

Speed of sound inside water is

$$v = \sqrt{\frac{B}{d}} = \sqrt{\left\{\frac{2.088 \times 10^9}{10^3}\right\}}$$

$$= \sqrt{2.088 \times 10^3}\ \text{m/s} = 1445\ \text{m/s}$$

Frequency of wave inside water is

$$n = \frac{v}{\lambda} = \frac{1445}{14.45 \times 10^{-3}} = 10^5\ \text{Hz}$$

In this case source and medium both are in motion.

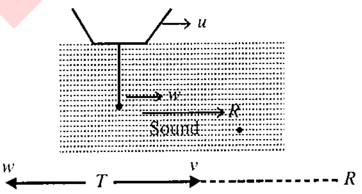

Fig. 6.25

Apparent frequency is given by

$$n' = \frac{(v+w)}{(v+w)-u} n = \frac{(1445+2)}{(1445+2)-10} \times 10^5$$

$$= \frac{1447}{1437} \times 10^5\ \text{Hz} = 1.00696 \times 10^5\ \text{Hz}$$

b. Speed of sound in air,

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 293}{28.8 \times 10^{-3}}} \equiv 344\ \text{m/s}$$

Now speed of air $w = 5$ m/s

$$n' = \frac{(v-w)}{(v-w)-u} n = \frac{344-5}{344-5-10} \times 10^5$$

$$= \frac{339}{329} \times 10^5\ \text{Hz} = 1.03040 \times 10^5\ \text{Hz}$$

Example 6.12 **A sonometer wire under tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of that sonometer wire has a length of 10 cm and a mass of 1 g. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s.**

Sol. The fundamental frequency of wire is equal to the frequency of the stationary fork, which is given by

$$n = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$$

Here, $m = \dfrac{1\ \text{g}}{10\ \text{cm}} = \dfrac{1 \times 10^{-3}\ \text{kg}}{10 \times 10^{-2}\ \text{m}} = 10^{-2}$ kg/m,

$T = 64$ N, $l = 10$ cm $= 0.1$ m.

$$\therefore \quad n = \frac{1}{2 \times 0.1} \times \sqrt{\left(\frac{64}{10^{-2}}\right)} = 400\ \text{Hz}$$

When the fork moves away from the source, its apparent frequency decreases. As the number of beats heard per second is 1, apparent frequency of moving fork is 400 – 1 = 399. The apparent frequency due to receding source is given by

$$n' = \frac{v}{v+v_s} n$$

or $$399 = \frac{300}{300+v_s} \times 400$$

or $$399(300 + v_s) = 300 \times 400$$

or $$399\, v_s = 300$$

or $$v_S = \frac{300}{399} = 0.752\ \text{m/s}$$

Example 6.13 **A train approaching a hill at a speed of 40 km/h sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from the hill. A wind with a speed of 40 km/h is blowing in the direction of motion of the train. Find (i) the frequency of the whistle as heard by an observer on the hill. (ii) the distance from the hill at which the echo from the hill is heard by the driver. Also find the**

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frequency heard by the driver (velocity of sound in air is 1200 km/h).

Sol.

i. According to Doppler's effect, the apparent frequency when both source and observer move along the same direction is

$$n' = \frac{(v+w)-v_0}{(v+w)-v_s}n$$

where w is speed of wind.

According to problem, velocity of observer is $v_0 = 0$

$$\therefore \quad n' = \frac{v+w}{v+w-v_s}n$$

Here $v = 1200$ km/h, $w = 40$ km/h, $v_s = 40$ km/h

and $n = 580$ Hz.

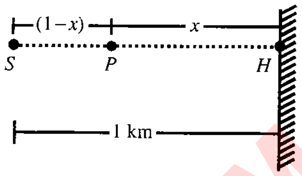

Fig. 6.26

$$\therefore \quad n' = \frac{(1200+40)}{(1200+40)-40} \times 580 = 599.33 \text{ Hz}$$

ii. Let echo be heard by the driver when the train is at P at a distance x km from the hill. Time taken by train to reach at P is

$$\frac{\text{Distance SP}}{\text{Velocity of train}} = \frac{(1-x)\text{ km}}{40 \text{ km/h}} = \frac{1-x}{40}\text{ h} \quad \text{(i)}$$

The time taken by echo to reach at P is

$$\frac{\text{Distance SH}}{v+w} + \frac{\text{Distance PH}}{v-w}$$

$$= \left[\frac{1}{1200+40} + \frac{x}{1200-40}\right]\text{ h} \quad \text{(ii)}$$

Comparing Eqs. (i) and (ii),

$$\frac{1-x}{40} = \frac{1}{1240} + \frac{x}{1160}$$

Solving, we get

$$x = \frac{29}{31}\text{ km} = 935.5 \text{ m}$$

If n' is frequency of reflected sound, then $n' = 599.33$ Hz. The frequency heard by driver is

$$n'' = \frac{(v-w)+v_o}{(v-w)}n' = \frac{1200-40+40}{1200-40} \times 599.3$$

$$= \frac{1200}{1160} \times 599.33 = 620 \text{ Hz}$$

Example 6.14 **A train A crosses a station with a speed of 40 m/s and whistles a short pulse of natural frequency $n_0 = 596$ Hz. Another train B is approaching towards the same station with the same speed along a parallel track. Two tracks are $d = 99$ m apart. When train A whistles, train B is 152 m away from the station as shown in Fig. 6.27.**

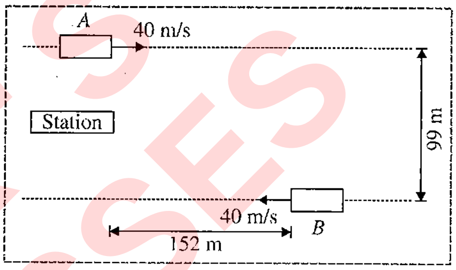

Fig. 6.27

If velocity of sound in air is $v = 330$ m/s, calculate frequency of the pulse heard by driver of train B.

Sol. When train A whistles, sound pulse starts to travel in air from train A to train B. During this interval train B moves some distance towards the station. Let sound pulse take time t to travel from train A to train B. Distance moved by train B during this interval is $40\,t$. Therefore, the distance of train B from station when its driver hears the pulse is $152 - 40t$. Hence, the distance travelled by the pulse is $\sqrt{(152-40t)^2+(99)^2}$. But it is equal to $vt = 330t$.

$$\therefore \quad \sqrt{(152-40t)^2+(99)^2} = 330t$$

or $\quad t = 0.5$ s

Therefore, driver of train B hears the pulse when his train is $152 - 40t = 132$ m from the station. Hence, path of pulse will be as shown in the figure. Its inclination θ with track is given by

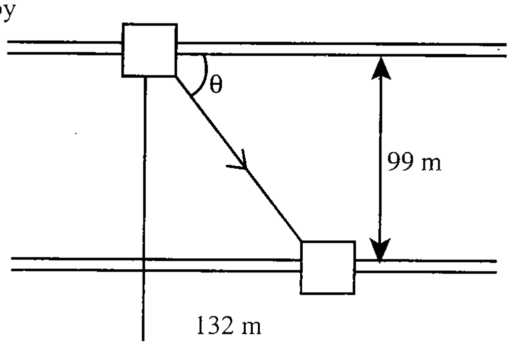

Fig. 6.28

$$\tan\theta = \frac{99}{132} \quad \Rightarrow \theta = 37°$$

Velocity component of train A along path of the pulse, $v_S = 40 \cos 37° = 32$ m/s

Velocity component of train B along path of the pulse, $v_0 = 40 \cos 37° = 32$ m/s

Hence, the frequency of pulse heard by driver of train B is

$$n = n_0\left(\frac{v+v_0}{v-v_s}\right) = 724 \text{ Hz}$$

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EXERCISES

Subjective Type

Solutions on page 6.36

1. In a car race sound signals emitted by two cars are detected by the detector on the straight track at the end point of the race. Frequency observed is 330 Hz and 360 Hz and the original frequency is 300 Hz of both cars. Race ends with the separation of 100 m between the cars. Assume both cars move with constant velocity and velocity of sound is 330 m/s. Find the time taken by wining car.
2. Airport authority has made the regulations that maximum allowable intensity level detected by a microphone situated at the end of 1630 m long runway can be 100 dB. An aeroplane when flying at a height of 200 m produces an intensity level of 100 dB on ground. While taking off, this aeroplane makes an angle of 30° with horizontal. Find the maximum distance this aeroplane can cover on the runway, so that the regulations are not violated (assume no reflection).
3. (a) The power of sound from the speaker of a radio is 20 mW. By turning the knob of volume control the power of sound is increased to 400 mW. What is the power increase in dB as compared to original power? (b) How much more intense is an 80 dB sound than a 20 dB whisper?
4. The sound level at a point is increased by 30 dB. What factor is the pressure amplitude increased?
5. What is the maximum possible sound level in dB of sound waves in air? Given that density of air is 1.3 kg/m^3, $v = 332$ m/s and atmospheric pressure $P = 1.01 \times 10^5$ N/m^2.
6. A window whose area is 2 m^2 opens on street where the street noise result in an intensity level at the window of 60 dB. How much 'acoustic power' enters the window via sound waves. Now if an acoustic absorber is fitted at the window, how much energy from street will it collect in 5 h?
7. Two tuning forks A and B are vibrating at the same frequency 256 Hz. A listener is standing midway between the forks. If both tuning forks move to the right with a velocity of 5 m/s, find the number of beats heard per second by the listener (speed of sound in air is 330 m/s).

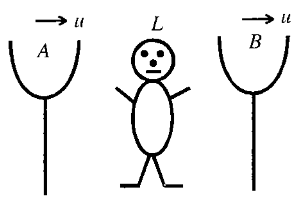

Fig. 6.29

8. A driver in a stationary car horns which produces monochromatic sound waves of frequency $n = 1000$ Hz, normally towards a reflecting wall. If the wall approaches the car will a velocity $u = 3.3$ m/s, calculate the frequency of sound reflected from wall and heard by the driver. What is the percentage change of sound frequency on reflection from the wall?

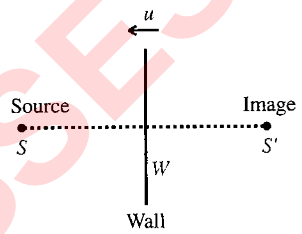

Fig. 6.30

9. The speed of sound in hydrogen gas at certain temperature is v m/s. Find the speed of sound in a gaseous mixture containing 2 moles of oxygen and 1 mole of hydrogen gas, at the same temperature. Assume the gases do not react at the ordinary temperature.
10. A source of sound of frequency 256 Hz is moving towards a wall with a velocity of 5 m/s. How many beats per second will be heard by an observer O standing in such a position that the source S is between O and wall? ($c = 330$ m/s)
11. A vibrating tuning fork tied to the end of a string 1.988 m long is whirled round a circle. If it makes two revolutions in a second, calculate the ratio of the frequencies of the highest and the lowest notes heard by an observer situated in the plane of the tuning fork. Velocity of sound is 350 m/s.
12. A source of sonic oscillations with frequency $n = 1700$ Hz and a receiver are located on the same normal to a wall. Both the source and receiver are stationary, and the wall recedes from the source with velocity $u = 6.0$ cm/s. Find the beat frequency registered by the receiver. The velocity of sound is $v = 340$ m/s.
13. A locomotive approaching a crossing at a speed of 80 mi/h sounds a whistle of frequency 400 Hz when 1 mi from the crossing. There is no wind, and the speed of sound in air is 0.200 mi/s. What frequency is heard by an observer 0.60 mi from the crossing on the straight road which crosses the railroad at right angles?
14. A whistle of frequency 540 Hz is moving in a circle of radius 2 ft. at a constant angular speed of 15 rad/s. What are the lowest and highest frequencies heard by a listener standing at rest, a long distance away from the centre of the circle? (Velocity of sound in air is 1100 ft/s)

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Objective Type

Solutions on page 6.38

1. A man is watching two trains, one leaving and the other coming in with equal speed of 4 m/s. If they sound their whistles, each of frequency 240 Hz, the number of beats heard by the man (velocity of sound in air is 320 m/s) will be equal to
 a. 6 b. 3
 c. 0 d. 12

2. The intensity of a sound wave gets reduced by 20% on passing through a slab. The reduction in intensity on passage through two such consecutive slabs is
 a. 40% b. 36 % c. 30 % d. 50%

3. Two factories are sounding their sirens at 800 Hz. A man goes from one factory to the other at a speed of 2 m/s. The velocity of sound is 320 m/s. The number of beats heard by the person in 1 s will be
 a. 2 b. 4 c. 8 d. 10

4. Two sources A and B are sounding notes of frequency 680 Hz. A listener moves from A to B with a constant velocity u. If the speed of sound is 340 m/s, what must be the value of u so that he hears 10 beats per second?
 a. 2.0 m/s b. 2.5 m/s
 c. 30 m/s d. 3.5 m/s

5. A train has just completed a U-curve in a track which is a semi-circle. The engine is at the forward end of the semi-circular part of the track while the last carriage is at the rear end of the semi-circular track. The driver blows a whistle of frequency 200 Hz. Velocity of sound is 340 m/s. Then the apparent frequency as observed by a passenger in the middle of the train, when the speed of the train is 30 m/s, is
 a. 219 Hz b. 188 Hz
 c. 200 Hz d. 181 Hz

6. One train is approaching an observer at rest and another train is receding from him with the same velocity 4 m/s. Both the trains blow whistles of same frequency of 243 Hz. The beat frequency in Hz as heard by the observer is (speed of sound in air is 320 m/s)
 a. 10 b. 6 c. 4 d. 1

7. Two sound sources are moving in opposite directions with velocities v_1 and v_2 ($v_1 > v_2$). Both are moving away from a stationary observer. The frequency of both the sources is 900 Hz. What is the value of $v_1 - v_2$ so that the beat frequency observed by the observer is 6 Hz? Speed of sound $v = 300$ m/s. Given that v_1 and $v_2 \ll v$.
 a. 1 m/s b. 2 m/s
 c. 3 m/s d. 4 m/s

8. The frequency changes by 10% as the source approaches a stationary observer with constant speed v_s. What should be the percentage change in frequency as the source recedes from the observer with the same speed? Given that $v_s \ll v$ (v is the speed of sound in air).
 a. 14.3 % b. 20%
 c. 16.7 % d. 10%

9. Speed of sound wave is v. If a reflector moves towards a stationary source emitting waves of frequency f with speed u, the wavelength of reflected waves will be
 a. $\frac{v-u}{v+u}f$ b. $\frac{v+u}{v}f$
 c. $\frac{v+u}{v-u}f$ d. $\frac{v-u}{v}f$

10. An observer moves towards a stationary source of sound with a speed (1/5)th of the speed of sound. The wavelength and frequency of the source emitted are λ and f, respectively. The apparent frequency and wavelength recorded by the observer are, respectively,
 a. $1.2f$ and λ b. f and 1.2λ
 c. $0.8f$ and 0.8λ d. $1.2f$ and 1.2λ

11. A source of sound S is moving with a velocity 50 m/s towards a stationary observer. He measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the sound when it is moving away from the observer after crossing him? The velocity of the sound in the medium is 350 m/s.
 a. 750 Hz b. 857 Hz
 c. 1143 Hz d. 1333 Hz

12. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Then the range of frequencies heard by an observer stationed at a large distance from the whistle will be ($v = 330$ m/s)
 a. 400.0 Hz to 484.0 Hz
 b. 403.3 Hz to 480.0 Hz
 c. 400.0 Hz to 480.0 Hz
 d. 403.3 Hz to 484.0 Hz

13. A band playing music at frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, the expression for the beat frequency heard by the motorist is
 a. $\frac{v+v_m}{v+v_b}f$ b. $\frac{v+v_m}{v-v_b}f$
 c. $\frac{2v_b(v+v_m)}{v^2-v_b^2}f$ d. $\frac{2v_m(v+v_b)}{v^2-v_m^2}f$

14. A man standing on a platform hears the sound of frequency 605 Hz coming from a frequency 550 Hz from a train whistle moving towards the platform. If the velocity of sound is 330 m/s, then what is the speed of train?
 a. 30 m/s b. 35 m/s
 c. 40 m/s d. 45 m/s

15. A vehicle, with a horn of frequency n, is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $n + n_1$. Then n_1 is equal to (take velocity of sound in air as 330 m/s)
a. $n_1 = 10n$ **b.** $n_1 = -n$
c. $n_1 = 0.1n$ **d.** $n_1 = 0$

16. An isotropic stationary source is emitting waves of frequency n and wind is blowing due north. An observer A is on north of the source while observer B is on south the source. If both the observers are stationary, then
a. frequency received by A is greater than n
b. frequency received by B is less than n
c. frequency received by A equals to that received by B
d. frequencies received by A and B cannot be calculated unless velocity of waves in still air and velocity of wind are known

17. A train is moving with a constant speed along a circular track. The engine of the train emits a sound of frequency f. The frequency heard by the guard at the rear end of the train
a. is less than f
b. is greater than f
c. is equal to f
d. may be greater than, less than or equal to f depending on factors like speed of train, length of train and radius of circular track

18. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
a. 5% **b.** 20% **c.** 0% **d.** 0.5%

19. An increase in intensity level of 1 dB implies an increase in density of (given $\text{antilog}_{10}\, 0.1 = 1.2589$)
a. 1% **b.** 3.01% **c.** 26% **d.** 0.1%

20. In expressing sound intensity, we take 10^{-12} W/m² as the reference level. For ordinary conversation, the intensity level is about 10^{-6} W/m². Expressed in decibel, this is
a. 10^6 **b.** 6 **c.** 60 **d.** $\log_e(10^6)$

21. The intensity level of two sounds are 100 dB and 50 dB. What is the ratio of their intensities?
a. 10^1 **b.** 10^3 **c.** 10^5 **d.** 10^{10}

22. An engine running at speed $v/10$ sounds a whistle of frequency 600 Hz. A passenger in a train coming from the opposite side at speed $v/15$ experiences this whistle to be of frequency f. If v is speed of sound in air and there is no wind, f is nearest to
a. 711 Hz **b.** 630 Hz
c. 580 Hz **d.** 510 Hz

23. A source of sound produces waves of wavelength 60 cm when it is stationary. If the speed of sound in air is 320 m/s and source moves with speed 20 m/s, the wavelength of sound in the forward direction will be nearest to
a. 56 cm **b.** 60 cm
c. 64 cm **d.** 68 cm

24. The apparent frequency of the whistle of an engine changes in the ratio of 6:5 as the engine passes a stationary observer. If the velocity of sound is 330 m/s, then the velocity of the engine is
a. 3 m/s **b.** 30 m/s
c. 0.33 m/s **d.** 660 m/s

25. A source of sound S is travelling at 100/3 m/s along a road, towards a point A. When the source is 3 m away from A, a person standing at a point O on a road perpendicular to AS hears a sound of frequency ν'. The distance of O from A at that time is 4 m. If the original frequency is 640 Hz, then the value of ν' is (velocity of sound is 340 m/s)

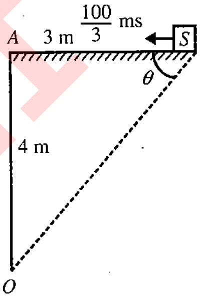

Fig. 6.31

a. 620 Hz **b.** 680 Hz
c. 720 Hz **d.** 840 Hz

26. A source of sound is travelling with a velocity of 30 m/s towards a stationary observer. If actual frequency of source is 1000 Hz and the wind is blowing with velocity 20 m/s in a direction at 60° with the direction of motion of source, then the apparent frequency heard by observer is (speed of sound is 340 m/s)
a. 1011 Hz **b.** 1000 Hz
c. 1094 Hz **d.** 1086 Hz

27. A source of sound emits 200p W power which is uniformly distributed over a sphere of radius 10 m. What is the loudness of sound on the surface of the sphere?
a. 70 dB **b.** 107 dB
c. 80 dB **d.** 117 dB

28. Two cars are moving on two perpendicular roads towards a crossing with uniform speeds of 72 km/h and 36 km/h. If second car blows horn of frequency 280 Hz, then the frequency of horn heard by the driver of first car when the line joining the cars makes angle of 45° with the roads, will be (velocity of sound is 330 m/s)
a. 321 Hz **b.** 298 Hz
c. 289 Hz **d.** 280 Hz

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29. A tuning fork of frequency 380 Hz is moving towards a wall with a velocity of 4 m/s. Then the number of beats heard by a stationary listener between direct and reflected sounds will be (velocity of sound in air is 340 m/s)

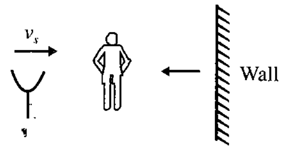

Fig. 6.32

a. 0 **b.** 5
c. 7 **d.** 10

30. A sound wave of frequency n travels horizontally to the right with speed c. It is reflected from a broad wall moving to the left with speed v. The number of beats heard by a stationary observer to the left of the wall is

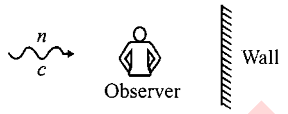

Fig. 6.33

a. zero **b.** $\frac{n(c+v)}{c-v}$ **c.** $\frac{nv}{c-v}$ **d.** $\frac{2nv}{c-v}$

31. A boy is walking away from a wall at a speed of 1.0 m/s in a direction at right angles to the wall. The boy blows a whistle steadily. An observer towards whom the boy is moving hears 4 beats/s. If the speed of sound is 340 m/s, the frequency of whistle is

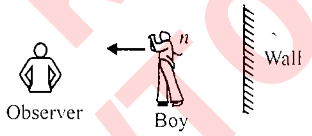

Fig. 6.34

a. 480 Hz **b.** 680 Hz
c. 840 Hz **d.** 1000 Hz

32. A source emitting a sound of frequency f is placed at a large distance from an observer. The source starts moving towards the observer with a uniform acceleration 'a'. Find frequency heard by the observer corresponding to the wave emitted just after the source starts. The speed of sound in medium is v.

a. $\frac{vf^2}{2vf-a}$ **b.** $\frac{2vf^2}{2vf+a}$
c. $\frac{2vf^2}{3vf-a}$ **d.** $\frac{2vf^2}{2vf-a}$

33. Due to a point isotropic sonic source, loudness at a point is $L = 60$ dB. If density of air is $\rho = (15/11)$ kg/m^3 and velocity of sound in air is $v = 33$ m/s, the pressure oscillation amplitude at the point of observation is [$I_0 = 10^{-12}$ W/m^2]

a. 0.3 N/m^2 **b.** 0.03 N/m^2
c. 3×10^{-3} N/m^2 **d.** 3×10^{-4} N/m^2

34. The frequency of a car horn is 400 Hz. If the horn is honked as the car moves with a speed $u_s = 34$ m/s through still air towards a stationary receiver, the wavelength of the sound passing the receiver is [velocity of sound is 340 m/s]

a. 0.765 m **b.** 0.850 m
c. 0.935 m **d.** 0.425 m

35 Spherical sound waves are emitted uniformly in all directions from a point source. The variation in sound level SL as a function of distance 'r' from the source can be written as

a. SL $= -b \log r^a$ **b.** SL $= a - b (\log r)^2$
c. SL $= a - b \log r$ **d.** SL $= a - b/r^2$

where a and b are positive constants.

36. The pressure variation that corresponds to pain threshold (i.e., the ear can tolerate in loud sound) is about 30 Pa. Velocity of sound in water is $\sqrt{2}\times 10^3$ m/s. The intensity of sound wave produced in water corresponding to loud sound is

a. 1 W/m^2 **b.** 0.3×10^{-3} W/m^2
c. 10^3 W/m^2 **d.** 10^{-12} W/m^2

37. When a person wears a hearing aid, the sound intensity level increases by 30 dB. The sound intensity increases by

a. e^3 **b.** 10^3
c. 30 **d.** 10^2

38. A motorcycle starts from rest and accelerates along a straight line at 2.2 m/s^2. The speed of sound is 330 m/s. A siren at the starting point remains stationary. When the driver hears the frequency of the siren at 90% of when the motorcycle is stationary, the distance travelled by the motorcyclist is

a. 123.75 m **b.** 247.5 m
c. 495 m **d.** 990 m

39. A plane longitudinal wave of angular frequency 10^3 rad/s is travelling along negative x-direction in a homogenous gaseous medium of density $\rho = 1$ kg/m^3. Intensity of the wave is $I = 10^{-10}$ W/m^2 and maximum pressure change is $(\Delta P)_m = 2\times 10^{-4}$ N/m^2. Assuming at $x = 0$, initial phase of medium particles to be $\pi/2$, the equation of the wave is

a. $y = 10^{-9}\sin\left(1000t - 5x + \frac{\pi}{2}\right)$

b. $y = 10^{-9}\cos(1000t + 5x)$

c. $y = 10^{-9}\tan(1000t - 5x)$

d. $y = 10^{-9}\cos(1000t - 5x)$

40. The difference in the speeds of sound in air at –5°C, 60 cm pressure of mercury and 30°C, 75 cm pressure of mercury is (velocity of sound in air at 0°C is 332 m/s)

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a. 15.25 m/s b. 21.35 m/s
c. 18.3 m/s d. 3.05 m/s

41. A train is moving in an elliptical orbit in anticlockwise sense with a speed of 110 m/s. Guard is also moving in the given direction with same speed as that of train. The ratio of the length of major and minor axes is 4/3. Driver blows a whistle of 1900 Hz at P, which is received by guard at S. The frequency received by guard is (velocity of sound $v = 330$ m/s)

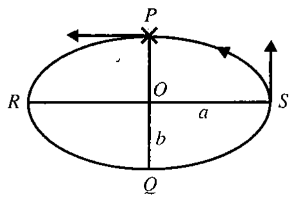

Fig. 6.35

a. 1900 Hz b. 1800 Hz
c. 2000 Hz d. 1500 Hz

42. A stationary observer receives a sound from a sound of frequency v_0 moving with a constant velocity $v_s = 30$ m/s. The apparent frequency varies with time as shown in figure. Velocity of sound $v = 300$ m/s. Then which of the following is incorrect?

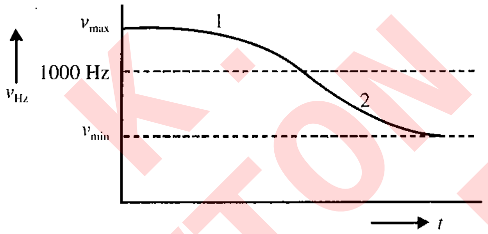

Fig. 6.36

a. The minimum value of apparent frequency is 889 Hz.
b. The natural frequency of source is 1000 Hz.
c. The frequency–time curve corresponds to a source moving at an angle to the stationary observer.
d. The maximum value of apparent frequency is 1111 Hz.

43. The sound from a very high burst of fireworks takes 5 s to arrive at the observer. The burst occurs 1662 m above the observer and travels vertically through two stratifier layers of air, the top one of thickness d_1 at 0°C and the bottom one of thickness d_2 at 20°C. Then (assume velocity of sound at 0°C is 330 m/s)

a. $d_1 = 342$ m b. $d_2 = 1320$ m
c. $d_1 = 1485$ m d. $d_2 = 342$ m

44. A 40 dB sound wave strikes an eardrum whose area is 10^{-6} m^2. To receive a total energy of 1 J, time received is ($I_0 = 10^{-2}$ W/m^2)

a. 10^{-8} s b. 10^{10} s
c. 10^{6} s d. 10^{14} s

45. A car emitting sound of frequency 500 Hz speeds towards a fixed wall at 4 m/s. An observer in the car hears both the source frequency as well as the frequency of sound reflected from the wall. If he hears 10 beats per second between the two sounds, the velocity of sound in air will be

a. 330 m/s b. 387 m/s
c. 404 m/s d. 340 m/s

46. In the figure shown, a source of sound of frequency 510 Hz moves with constant velocity $v_s = 20$ m/s in the direction shown. The wind is blowing at a constant velocity $v_w = 20$ m/s towards an observer who is at rest at point B. Corresponding to the sound emitted by the source at initial position A, the frequency detected by the observer is equal to (speed of sound relative to air is 330 m/s)

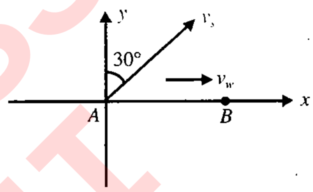

Fig. 6.37

a. 510 Hz b. 500 Hz
c. 525 Hz d. 550 Hz

47. A wall is moving with velocity u and a source of sound moves with velocity u/2 in the same direction as shown in the figure. Assuming that the sound travels with velocity 10u, the ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall is equal to

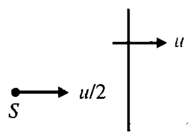

Fig. 6.38

a. 9:11 b. 11:9
c. 4:5 d. 5:4

48. For a sound wave travelling towards $+x$ direction, sinusoidal longitudinal displacement ξ at a certain time is given as a function of x (Fig. 6.39). If bulk modulus of air is $B = 5 \times 10^5$ N/m^2, the variation of pressure excess will be

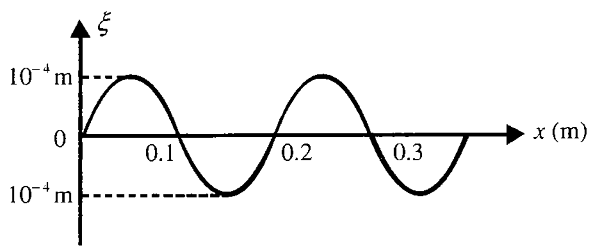

Fig. 6.39

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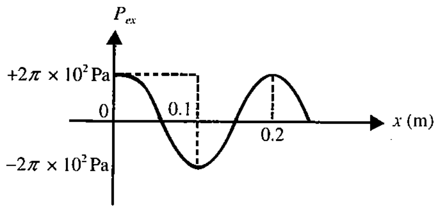
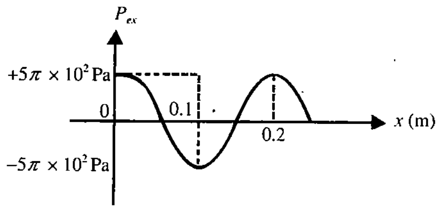
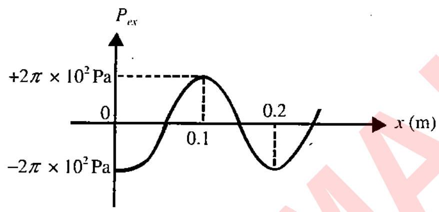


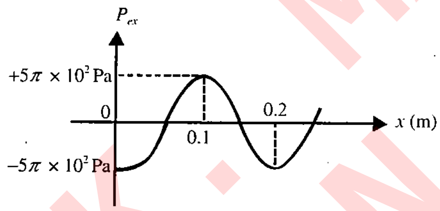
a.



b.



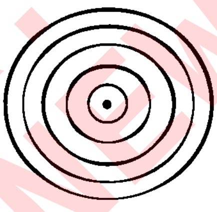
c.



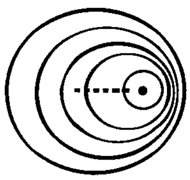
d.

49. If the source is moving towards right, wavefront of sound waves get modified to

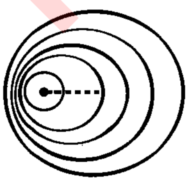
a.



b.



c.



d. None of these

50. Consider a source of sound S, and an observer/detector D. The source emits a sound wave of frequency f_0. The frequency observed by D is found to be
(i) f_1, if D approaches S and S is stationary
(ii) f_2, if S approaches D and D is stationary
(iii) f_3, if both S and D approach each other with the same speed.
In all three cases, relative velocity of S wrt D is the same. For this situation which is incorrect?

a. $f_1 \neq f_2 \neq f_3$ **b.** $f_1 < f_2$
c. $f_3 < f_0$ **d.** $f_1 < f_3 < f_2$

51. When source and detector are stationary but the wind is blowing at speed v_w, the apparent wavelength λ' on the wind side is related to actual wavelength λ by [take speed of sound in air as v]

a. $\lambda' = \lambda$ **b.** $\lambda' = \frac{v_w}{v}\lambda$
c. $\lambda' = \frac{v_w + v}{v}\lambda$ **d.** $\lambda' = \frac{v}{v - v_w}\lambda$

52. Figure 6.40 represents the displacement y versus distance x along the direction of propagation of a longitudinal wave. The pressure is maximum at position marked

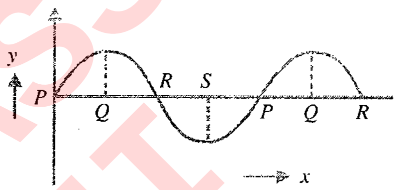


Fig. 6.40

a. P **b.** Q
c. R **d.** S

53. The driver of a car approaching a vertical wall notices that the frequency of the horn of his car changes from 400 Hz to 450 Hz after being reflected from the wall. Assuming speed of sound to be 340 m/s, the speed of approach of car towards the wall is

a. 10 m/s **b.** 20 m/s
c. 30 m/s **d.** 40 m/s

54. The difference between the apparent frequencies of a source of sound as perceived by a stationary observer during its approach and recession is 2% of the actual frequency of the source. If the speed of sound is 300 m/s, the speed of source is

a. 1.5 m/s **b.** 3 m/s **c.** 6 m/s **d.** 12 m/s

55. The frequency of a radar is 780 MHz. After getting reflected from an approaching aeroplane, the apparent frequency is more than the actual frequency by 2.6 kHz. The aeroplane has a speed of

a. 2 km/s **b.** 1 km/s
c. 0.5 km/s **d.** 0.25 km/s

56. A train moves towards a stationary observer with a speed of 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is

a. 18/19 **b.** 1/2
c. 2 **d.** 19/18

57. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he

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records a frequency of 6.0 kHz while approaching the same siren. The ratio of velocity of train B to that of train A is

a. 242/252 b. 2
c. 5/6 d. 11/6

58. A person speaking normally produces a sound of intensity 40 dB at a distance of 1 m. If the threshold intensity for reasonable audibility is 20 dB, the maximum distance at which he can be heard clearly is

a. 4 m b. 5 m
c. 10 m d. 20 m

59. A police car moving at 22 m/s chases a motorcyclist. The police man sounds his horn of frequency 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of motorcyclist if it is given that he does not hear any beat (speed of sound in air is 330 m/s).

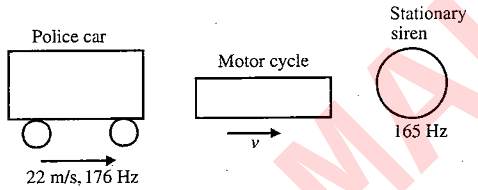

Fig. 6.41

a. 33 m/s b. 22 m/s
c. 11 m/s d. zero

60. In sports meet the timing of a 200 m straight dash is recorded at the finish point by starting an accurate stop watch on hearing the sound of starting gun fired at the starting point. The time recorded will be more accurate

a. in winter b. in summer
c. in all seasons d. none of these

61. When a source moves away from a stationary observer, the frequency is 6/7 times the original frequency. Given: speed of sound = 330 m/s. The speed of the source is

a. 40 m/s b. 55 m/s
c. 330 m/s d. 165 m/s

Multiple Correct Answers Type

Solutions on page 6.46

1. Which of the following statements are incorrect?
   a. Wave pulses in strings are transverse waves.
   b. Sound waves in air are transverse waves of compression and rarefaction.
   c. The speed of sound in air at 20°C is twice that at 5°C.
   d. A 60 dB sound has twice the intensity of a 30 dB sound.

2. A point sound source is situated in a medium of bulk modulus 1.6×10^5 N/m². The equation for the wave emitted from it is given by $y = A \sin (7.5 \pi x - 3000 \pi t)$. Velocity of wave is v and the displacement amplitude of the waves received by the observer standing at a distance 5 m from the source is A. The density of medium is ρ. The pressure amplitude at the observers ear is 30 Pa. The intensity of wave received by the observer is I. Then
   a. $\rho = 1$ kg/m³ b. $v = 400$ m/s
   c. $A = \dfrac{10^{-4}}{4\pi}$ d. $I = 1$ W/m²

3. A source S of sound wave of fixed frequency N and an observer O are located in air initially at the space points A and B, a fixed distance apart. State in which of the following cases, the observer will NOT see any Doppler effect and will receive the same frequency N as produced by the source.
   a. Both the source S and observer O remain stationary but a wind blows with a constant speed in an arbitrary direction.
   b. The observer remains stationary but the source S moves parallel to and in the same direction and with the same speed as the wind.
   c. The source remains stationary but the observer and the wind have the same speed away from the source.
   d. The source and the observer move directly against the wind but both with the same speed.

4. A vibrating tuning fork is first held in the hand and then its end is brought in contact with a table. Which of the following statement(s) is/are correct in respect of this situation?
   a. The sound is louder when the tuning fork is held in hand.
   b. The sound is louder when the tuning fork is in contact with table.
   c. The sound dies away sooner when tuning fork is brought in contact with the table.
   d. The sound remains for a longer duration when turning fork is held in hand.

5. A source of sound and detector are moving as shown in Fig. 6.42 at $t = 0$. Take velocity of sound wave to be v.

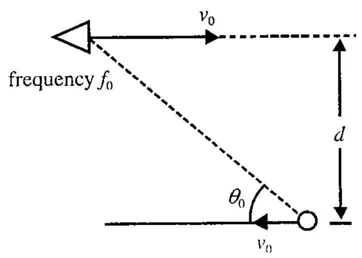

Fig. 6.42

For this situation mark out the correct statement(s).
   a. The frequency received by the detector is always greater than f_0.
   b. Initially, frequency received by the detector is greater than f_0, becomes equal to f_0, and then decreases with the time.
   c. Frequency received by the detector is equal to f_0 at $t = d \cot \theta_0/(2 v_0)$.
   d. Frequency received by the detector can never be equal to f_0.

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6. Which of the following statements are correct?

a. Changes in air temperature have no effect on the speed of sound.

b. Changes in air pressure have no effect on the speed of sound.

c. The speed of sound in water is higher than in air.

d. The speed of sound in water is lower than in air.

7. Consider a source of sound S and an observer P. The sound source is of frequency n_0. The frequency observed by P is found to be n_1 if P approaches S at speed v and S is stationary; n_2 if S approaches P at a speed v and P is stationary and n_3 if each of P and S has speed $v/2$ towards one another. Now,

a. $n_1 = n_2 = n_3$

b. $n_1 < n_2$

c. $n_3 > n_0$

d. n_3 lies between n_1 and n_2

8. An observer A is moving directly towards a stationary sound source while another observer B is moving away from the source with the same velocity. Which of the following statements are correct?

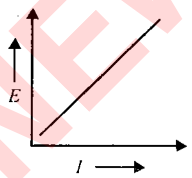
a. Average of frequencies recorded by A and B is equal to natural frequency of the source.

b. Wavelength of wave received by A is less than that of waves received by B.

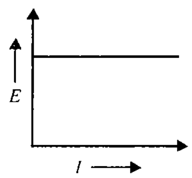
c. Wavelength of waves received by two observers will be same.

d. Both the observers will observe the wave travelling with same speed.

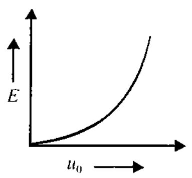
9. A sonic source, located in a uniform medium, emits waves of frequency n. If intensity, energy density (energy per unit volume of the medium) and maximum speed of oscillations of medium particle are, respectively, I, E and u_0 at a point, then which of the following graphs are correct?



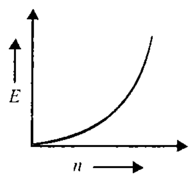
a.



b.



c.



d.

10. Plane harmonic waves of frequency 500 Hz are produced in air with displacement amplitude of 10 μm. Given that density of air is 1.29 kg/m^3 and speed of sound in air is 340 m/s. Then

a. the pressure amplitude is 13.8 N/m^2

b. the energy density is 6.4×10^{-4} J/m^3

c. the energy flux is 0.22 J/(m^2 s)

d. only (a) and (c) are correct

11. A driver in a stationary car blows a horn which produces monochromatic sound waves of frequency 1000 Hz normally towards a reflecting wall. The wall approaches the car with a speed of 3.3 m/s.

a. The frequency of sound reflected from wall and heard by the driver is 1020 Hz.

b. The frequency of sound reflected from wall and heard by the driver is 980 Hz.

c. The percentage increase in frequency of sound after reflection from wall is 2%.

d. The percentage decrease in frequency of sound after reflection from wall is 2%.

12. Mark out the correct statement(s).

a. When a sound wave strikes a wall, the compression pulse is reflected as compression pulse.

b. When a sound wave strikes a wall, the compression pulse is reflected as a rarefaction pulse.

c. When a sound wave is coming out after passing through a narrow pipe, then reflection would be there at the open end.

d. When a sound wave is coming out after passing through a narrow pipe, then compression pulse is reflected as a rarefaction pulse.

Assertion-Reasoning Type

Solutions on page 6.48

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which *only one* is correct.

a. Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

b. Statement I is true, Statement II is true; Statement II is NOT a correct explanation for Statement I.

c. Statement I is true; Statement II is false.

d. Statement I is false; Statement II is true.

1. **Statement I:** A tuning fork is considered as a source of an acoustic wave of a single frequency as marked on its body.

Statement II: The tuning fork cannot produce any of its harmonics due to its special nature of construction.

2. **Statement I:** The apparent frequency which is the frequency as noted by an observer or an observing detection device of the acoustic wave that moves from the source to the observer propagating in a medium may be different from its true frequency.

Statement II: A source in motion relative to an observer sends out less or more number of waves per metre distance in the medium and an observer in motion collects less or more number of waves per second than when both of them remain at rest relatively.

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3. **Statement I:** If two people talk simultaneously and each creates an intensity level of 60 dB at a point P, then total intensity level at the point P is 120 dB.
Statement II: Sound level is defined on a non-linear scale.
4. **Statement I:** Intensity of sound wave changes when the listener moves towards or away from the stationary source.
Statement II: The motion of listener causes the apparent change in wavelength.
5. **Statement I:** A 80 dB sound has twice the intensity of a 40 dB sound.
Statement II: Loudness of a sound of a certain intensity I is defined as

$$L(\text{in dB}) = 10\ \log_{10}\frac{I}{I_0}$$

6. **Statement I:** A person is standing near a railway track. A train is moving on the track. As the train is approaching the person, apparent frequency keeps on increasing and when the train has passed the person, then apparent frequency keeps on decreasing.
Statement II: When train is approaching the person then,

$$f = f_0\left[\frac{c}{c-u}\right]$$

and when train is moving away from person

$$f = f_0\left[\frac{c}{c+u}\right]$$

Here, c is velocity of sound, u is velocity of train and f_0 is original frequency of whistle.
7. **Statement I:** Sound travels faster in solids than gases.
Statement II: Solids possess greater density than gases.
8. **Statement I:** The speed of sound in solids is maximum though their density is large.
Statement II: This is because their coefficient of elasticity is large.
9. **Statement I:** When there is no relative velocity between source and observer, then observed frequency is the same as emitted.
Statement II: Velocity of sound when there is no relative velocity between source and observer is zero.
10. **Statement I:** The fundamental frequency of an organ pipe increases as the temperature increases.
Statement II: As the temperature increases, the velocity of sound increases more rapidly than length of the pipe.
11. **Statement I:** Sound would travel faster on a hot summer day than a cold winter day.
Statement II: Velocity of sound is directly proportional to the square of its absolute temperature.

Comprehension Type

Solutions on page 6.49

For Problems 1–2

A railroad train is travelling at 30 m/s in still air. The frequency of the note emitted by locomotive whistle is 500 Hz. Speed of sound is 345 m/s.

1. What is the frequency of the sound waves heard by a stationary listener in front of the train?
a. 547.6 Hz **b.** 690.6 Hz
c. 590.9 Hz **d.** 520.3 Hz
2. What is the frequency of the sound waves heard by a stationary listener behind the train?
a. 420 Hz **b.** 460 Hz
c. 480 Hz **d.** 430 Hz

For Problems 3–5

A source of sonic oscillations with frequency n_0 = 600 Hz moves away and at right angles to a wall with velocity u = 30 m/s. A stationary receiver is located on the line of source in succession wall → source → receiver. If velocity of sound propagation is v = 330 m/s, then

3. The beat frequency recorded by the receiver is
a. 110 Hz **b.** 210 Hz
c. 150 Hz **d.** 220 Hz
4. The wavelength of direct waves received by the receiver is
a. 50 cm **b.** 100 cm
c. 150 cm **d.** 90 cm
5. The wavelength of reflected waves received by the receiver is
a. 120 cm **b.** 50 cm
c. 90 cm **d.** 60 cm

For Problems 6–7

A source S of acoustic wave of the frequency v_0 = 1700 Hz and a receiver R are located at the same point. At the instant t = 0, the source starts from rest to move away from the receiver with a constant acceleration ω. The velocity of sound in air is v = 340 m/s.

6. If ω = 10 m/s², the apparent frequency that will be recorded by the stationary receiver at t = 10 s will be
a. 1700 Hz **b.** 1.35 Hz
c. 850 Hz **d.** 1.27 Hz
7. If ω = 10 m/s² for 0 < t £ 10 s and then ω = 0 for t > 10 s, the apparent frequency recorded by the receiver at t = 15 s
a. 1700 Hz **b.** 1313 Hz
c. 850 Hz **d.** 1.23 kHz

For Problems 8–10

A small source of sound vibrating at frequency 500 Hz is rotated in a circle of radius $100/\pi$ cm at a constant angular speed of 5.0 revolutions per second. The speed of sound in air is 330 m/s.

8. For an observer situated at a great distance on a straight line perpendicular to the plane of the circle, through its centre, the apparent frequency of the source will be
a. greater than 500 Hz
b. smaller than 500 Hz
c. always remain 500 Hz
d. greater for half the circle and smaller during the other half
9. For an observer who is at rest at a great distance from the centre of the circle but nearly in the same plane, the minimum f_{min} and the maximum f_{max} of the range of values of the apparent frequency heard by him will be

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a. $f_{min} = 455$ Hz, $f_{max} = 535$ Hz
b. $f_{min} = 485$ Hz, $f_{max} = 515$ Hz
c. $f_{min} = 485$ Hz, $f_{max} = 500$ Hz
d. $f_{min} = 500$ Hz, $f_{max} = 515$ Hz

10. If the observer moves towards the source with a constant speed of 20 m/s, along the radial line to the centre, the fractional change in the apparent frequency over the frequency that the source will have if considered at rest at the centre will be

a. 6% **b.** 3%
c. 2% **d.** 9%

For Problems 11–14

An Indian submarine is moving in the Arabian Sea with a constant velocity. To detect enemy it sends out sonar waves which travel with velocity 1050 m/s in water. Initially the waves are getting reflected from a fixed island and the reflected waves are coming back to submarine. The frequency of reflected waves are detected by the submarine and found to be 10% greater than the sent waves.

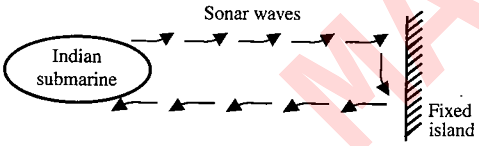

Fig. 6.43

Now an enemy ship comes in front, due to which the frequency of reflected waves detected by submarine becomes 21% greater than the sent waves.

11. The speed of Indian submarine is

a. 10 m/s **b.** 50 m/s **c.** 100 m/s **d.** 20 m/s

12. The velocity of enemy ship should be

a. 50 m/s towards Indian submarine
b. 50 m/s away from Indian submarine
c. 100 m/s towards Indian submarine
d. 100 m/s away from Indian submarine

13. If the wavelength received by enemy ship is λ' and wavelength of reflected waves received by submarine is λ'', then (λ'/λ'') equals

a. 1 **b.** 1.1 **c.** 1.2 **d.** 2

14. Bulk modulus of sea water should be approximately $(\rho_{water} = 1000 \text{ kg/m}^3)$

a. 10^8 N/m^2 **b.** 10^9 N/m^2 **c.** 10^{10} N/m^2 **d.** 10^{11} N/m^2

For Problems 15–16

Due to a point isotropic sound source, the intensity at a point is observed as 40 dB. The density of air is $\rho = (15/11)$ kg/m^3 and velocity of sound in air is 330 m/s. Based on this information answer the following questions.

15. The pressure amplitude at the observation point is

a. 3 N/m^2 **b.** 3×10^3 N/m^2
c. 3×10^{-3} N/m^2 **d.** 6×10^{-2} N/m^2

16. The ratio of displacement amplitude of wave at observation point to wavelength of sound waves is

a. 3.22×10^{-6} **b.** 3.22×10^{-12}
c. 3.22×10^{-9} **d.** 1.07×10^{-10}

For Problems 17–19

In the figure shown below, a source of sound having power 12×10^{-6} W is kept at O, which is emitting sound waves in the directions as shown. Two surfaces are labelled as 1 and 2 having areas $A_1 = 2 \times 10^3$ m^2 and $A_2 = 4 \times 10^3$ m^2, respectively.

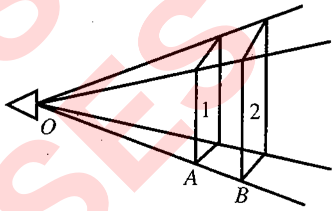

Fig. 6.44

17. Find the intensity at both the surfaces.

a. $I_1 = 12 \times 10^{-6}$ W/m^2, $I_2 = 12 \times 10^{-6}$ W/m^2
b. $I_1 = 6 \times 10^{-9}$ W/m^2, $I_2 = 12 \times 10^{-9}$ W/m^2
c. $I_1 = 6 \times 10^{-9}$ W/m^2, $I_2 = 3 \times 10^{-9}$ W/m^2
d. $I_1 = 12 \times 10^{-9}$ W/m^2, $I_2 = 3 \times 10^{-9}$ W/m^2

18. If two persons (having almost same physique) A and B are standing at the location of surfaces 1 and 2, respectively, then who will hear a quieter sound?

a. Both will hear same sound.
b. A will hear a quieter sound.
c. B will hear a quieter sound.
d. Information is not sufficient.

19. Let the areas of the eardrums of persons A and B be $A_A = 2$ mm^2 and $A_B = 4$ mm^2, respectively. Then, who will hear a quieter sound?

a. A will hear a quieter sound.
b. B will hear a quieter sound.
c. Both will hear the same sound.
d. Cannot say anything.

For Problems 20–24

When a sound wave enters the ear, it sets the eardrum into oscillation, which in turn causes oscillation of 3 tiny bones in the middle ear called ossicles. This oscillation is finally transmitted to the fluid filled in inner portion of the ear termed as inner ear, the motion of the fluid disturbs hair cells within the inner ear which transmit nerve impulses to the brain with the information that a sound is present. The three bones present in the middle ear are named as hammer, anvil and stirrup. Out of these the stirrup is the smallest one and this only connects the middle ear to inner ear as shown in the figure below. The area of stirrup and its extent of connection with the inner ear limits the sensitivity of the human ear. Consider a person's ear whose moving part of the eardrum has an area of about 50 mm^2 and the area of stirrup is about 5 mm^2. The mass of ossicles is negligible. As a result, force exerted by sound wave in air on eardrum and ossicles is same as the force exerted by ossicles on the inner ear. Consider a sound wave having maximum pressure fluctuation of 4×10^{-2} Pa from its normal equilibrium pressure value which is equal to 10^5 Pa. Frequency of sound wave is 1200 Hz.

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Data: Velocity of sound wave in air is 332 m/s. Velocity of sound wave in fluid (present in inner ear) is 1500 m/s. Bulk modulus of air is 1.42×10^5 Pa. Bulk modulus of fluid is 2.18×10^9 Pa.

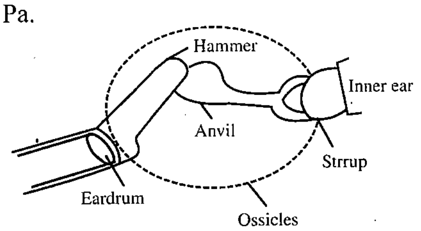

Fig. 6.45

20. Find the pressure amplitude of given sound wave in the fluid of inner ear.

a. 0.03 Pa **b.** 0.04 Pa **c.** 0.3 Pa **d.** 0.4 Pa

21. Find the displacement amplitude of given sound wave in the fluid of inner ear.

a. 4.4×10^{-11} m **b.** 8×10^{11} m
c. 3.65×10^{-11} m **d.** 8.1×10^{-12} m

22. If the person is using an hearing aid, which increase the sound intensity level by 30 dB, then by what factor the intensity of given sound wave change as perceived by inner ear?

a. 1000 **b.** 100
c. 10,000 **d.** None of these

23. This person (without hearing aid machine) is sitting inside a busy restaurant where average sound intensity is 3.2×10^{-5} W/m^2. How much energy in the form of sound is taken up by the person in his meal time of 1 h?

a. 1.2×10^{-5} J **b.** 1.8×10^{-4} J
c. 2.4×10^{-5} J **d.** 3.6×10^{-4} J

24. With respect to information provided above, mark the correct statement.

a. The person will hear more intense sound, if area of stirrup is reduced.
b. The person will hear more intense sound, if area of stirrup is increased.
c. If mass of ossicles is not negligible, then intensity of sound heard by the person increases.
d. If mass of ossicles is not negligible, then intensity of sound heard by the person remains same.

For Problems 25–27

A source of sound and detector are arranged as shown in Fig. 6.46. The detector is moving along a circle with constant angular speed ω. It starts from the shown location in anti-clockwise direction at $t = 0$.

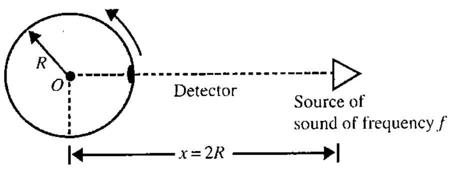

Fig. 6.46

(Take velocity of sound in air as v.)

Based on this information answer the following questions

25. What is the frequency as received by detector, when it rotates by an angle $\pi/2$?

a. f **b.** $\frac{v-\omega R}{v} \times f$
c. $\frac{v-\omega R/2}{v} \times f$ **d.** $\frac{v-\omega R \times 2/\sqrt{5}}{v} \times f$

26. Find the time at which the detector will hear the maximum frequency for the first time.

a. $\pi/(3\omega)$ **b.** $5\pi/(3\omega)$
c. $4\pi/(3\omega)$ **d.** π/ω

27. Find the time interval between minimum and maximum frequency as received by the detector.

a. $\pi/(3\omega)$ **b.** $5\pi/(3\omega)$
c. $4\pi/(3\omega)$ **d.** π/ω

For Problems 28–31

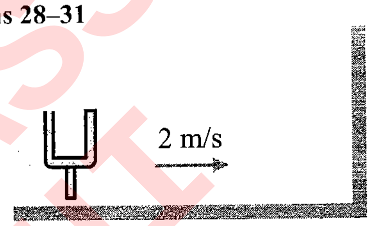

Fig. 6.47

As shown in Fig. 6.47 a vibrating tuning fork of frequency 512 Hz is moving towards the wall with a speed 2 m/s. Take speed of sound as v = 340 m/s and answer the following questions.

28. Suppose that a listener is located at rest between the tuning fork and the wall. Number of beats heard by the listener per second will be

a. 4 **b.** 3 **c.** 0 **d.** 1

29. If the listener is at rest and located such that the tuning fork is moving between the listener and the wall, number of beats heard by the listener per second will be nearly

a. 0 **b.** 6 **c.** 8 **d.** 4

30. If the listener, along with the source, is moving towards the wall with the same speed, i.e., 2 m/s, such that the source remains between the listener and the wall, number of beats heard by the listener per second will be

a. 4 **b.** 8 **c.** 0 **d.** 6

31. If the listener, along with the source, is moving towards the wall with the same speed, i.e., 2 m/s, such that he (listener) remains between the source and the wall, number of beats heard by him will be

a. 2 **b.** 6 **c.** 8 **d.** 4

For Problems 32–33

A source of sound with natural frequency f_0 = 1800 Hz moves uniformly along a straight line separated from a stationary observer by a distance l = 250 m. The velocity of the source is equal to η = 0.80 fraction of the velocity of the sound.

32. Find the frequency of sound received by the observer at the moment when the source gets closest to him.

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a. 2000 Hz **b.** 6000 Hz **c.** 3000 Hz **d.** 5000 Hz

33. The distance between the source and the observer at the moment when the observer receives a frequency $f = f_0$ is

a. 640 m **b.** 420 m **c.** 320 m **d.** 250 m

Matching Column Type

Solutions on page 6.53

1. A loudspeaker diaphragm 0.2 m in diameter is vibrating at 1 kHz with an amplitude of 0.01×10^{-3} m. Assume that the air molecules in the vicinity have the same amplitude of vibration. Density of air is 1.29 kg/m^3. Then match the item given in column I to that in column II. Take velocity of sound = 340 m/s.

Column I	Column II
i. Pressure amplitude immediately in front of the diaphragm (in N/m^2)	**a.** 2.7×10^{-2}
ii. Sound intensity in front of the diaphragm (in W/m^2)	**b.** 2.15×10^{-5}
iii. The acoustic power radiated (in W)	**c.** 27.55
iv. Intensity at 10 m from the loud speaker (in W/m^2)	**d.** 0.865

2. The diagram below shows the apparatus which could be used to demonstrate that the transmission of sound wave requires a material medium. The electric bell in the figure consists of a striker and is a steel hemispherical type structure. Vacuum pump is used to create the vacuum in vessel and when the tap to atmosphere is opened the jar will be filled with air. In column I some operations carried out are mentioned, while in column II, the effect, i.e., what is observed and heard along with the conclusions are mentioned. Match the entries of column I with the entries of column II.

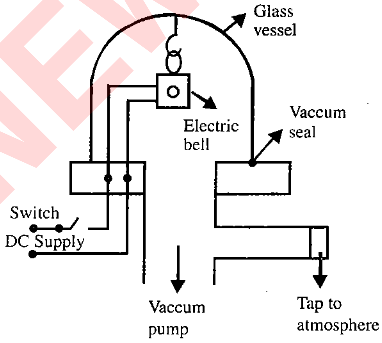

Fig. 6.48

Column I	Column II
i. Switch is closed and tap is opened	**a.** sound heard
ii. Switch is closed, tap is closed and vacuum pump is ON	**b.** striker seen vibrating
iii. Switch is closed and vacuum pump is OFF and tap opened	**c.** sound is not heard
iv. Switch is opened and tap is closed	**d.** light passes through vacuum but sound cannot

3. For transverse wave on a string,

Column I	Column II
i. if amplitude increases,	**a.** maximum instantaneous power increases
ii. if frequency increases,	**b.** average power increases
iii. if amplitude decreases,	**c.** maximum instantaneous power decreases
iv. if frequency decreases,	**d.** average power decreases

Integer Answer Type

Solutions on page 6.54

1. The average power transmitted across a cross-section by two sound waves moving in the same direction are equal. The wavelengths of two sound waves are in the ratio of 1 : 2, then find the ratio of their pressure amplitudes.

2. Loudness of sound from an isotropic point source at a distance of 70 cm is 20 dB. What is the distance (in m) at which it is not heard.

3. Two sound sources are moving away from a stationary observer in opposite directions with velocities V_1 and V_2 ($V_1 > V_2$). The frequency of both the sources is 900 Hz. V_1 and V_2 are both quite less than speed of sound, V = 300 m/s. Find the value of ($V_1 - V_2$) so that beat frequency observed by observer is 9 Hz. (in m/s.).

4. The resultant loudness at a point P is n dB higher than the loudness of S_1 which is one of the two identical sound sources S_1 and S_2 reaching at that point in phase. Find the value of n.

5. An ambulance sounding a horn of frequency 264 Hz is moving towards a vertical wall with a velocity of 5 ms^{-1}. If the speed of the sound is 330 ms^{-1}, how many beats per second will be heard by an observer standing a few meters behind the ambulance?

6. The intensity of sound from a point source is 1.0×10^{-8} W/m^2 at a distance of 5.0 m from the source. What will be the intensity at a distance of 25 m from the source? (in $\times 10^{-10}$ W/m^2)

7. If the intensity of sound is doubled, by how many decibels does the sound level increase? (in dB)

8. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at x = – 8 m and x = + 2.0 m. A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners?

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ANSWERS AND SOLUTIONS

Subjective Type

1. Let the velocities of car 1 and car 2 be V_1 and V_2, respectively.
Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$$f_1 = f_0 \frac{V}{V - V_1},\quad f_2 = f_0 \frac{V}{V - V_2}$$

$$330 = 300\frac{330}{330 - V_1},\quad 360 = 300\frac{330}{330 - V_2}$$

$\therefore \quad V_1 = 30$ m/s and $V_2 = 55$ m/s

The distance between both the cars just when the car 2 reaches end point B (as shown in Fig. 6.49) is

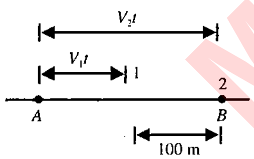

Fig. 6.49

$$100\,m = V_2t - V_1t \quad \Rightarrow t = 4 \text{ s}$$

2. Since aeroplane is producing intensity level of 100 dB at a distance of 200 m from it, and this is also the maximum allowable sound level so as not to violate the regulations, the minimum distance of the plane from microphone has to be 200 m.

The diagram that follows shows the situation exactly. Let the aeroplane leave the runway, before x metres of the location of microphone. Then, PM is the shortest distance between the microphone and the aeroplane.

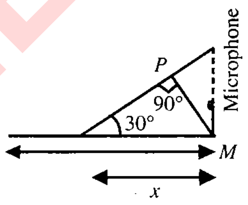

Fig. 6.50

Now,

$$PM = x \sin 30° = \frac{x}{2}$$

$$\Rightarrow PM = 200 = \frac{x}{2}$$

$$\Rightarrow x = 400 \text{ m}$$

So, the required distance is 1630 – 400 = 1230 m.

3. a. As intensity is power per unit area, for a given source $P \propto I$. If L_1 and L_2 are the initial and final loudness levels, then we have

$$L_2 - L_1 = 10\log(I_2/I_1)$$

The increase in loudness level is

$$\Delta L = 10\log\frac{P_2}{P_1} = 10\log\frac{400}{20}$$

$$\Rightarrow \quad \Delta L = 10[\log 20] = 13 \text{ dB}$$

b. Increase in loudness level of sound is

$$L_2 - L_1 = 10\log(I_2/I_1)$$

$$\therefore \quad 80 - 20 = 10\log(I_2/I_1)$$

$$\Rightarrow \quad 6 = \log(I_2/I_1)$$

$$\Rightarrow \quad \left(\frac{I_2}{I_1}\right) = 10^6$$

4. The sound level in dB is

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

If L_1 and L_2 are the sound levels and I_1 and I_2 are the corresponding intensities in the two cases, then

$$L_2 - L_1 = 10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

$$\Rightarrow \quad 30 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$

$$\Rightarrow \quad \frac{I_2}{I_1} = 10^3$$

As the intensity is proportional to the square of the pressure amplitude, thus we have

$$\frac{\Delta p_2}{\Delta p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{1000} \approx 32$$

5. For maximum possible sound intensity, maximum pressure amplitude of wave can be at most equal to atmospheric pressure. So we have,

$$\Delta p_0 = p = 1.01 \times 10^5 \text{ N/m}^2$$

$$I = \frac{\Delta p_0^2}{2\rho v} = \frac{(1.01 \times 10^5)^2}{2 \times 1.3 \times 332} = 1.18 \times 10^7 \text{ W/m}^2$$

Thus loudness level is

$$L = 10\log\frac{I}{I_0} \approx \log_{10}\frac{10^7}{10^{-12}} = 190 \text{ dB}$$

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6. Loudness level is given as

$$L = 10\log\left(\frac{I}{I_0}\right)$$

Thus,

$$10\log\frac{I}{I_0} = 60$$

$$\Rightarrow \quad \frac{I}{I_0} = 10^6$$

$$\Rightarrow \quad I = (10^{-12}\times 10^6) = 10^{-6}\ \text{W/m}^2 = 1\ \mu\text{W/m}^2$$

And the power of sound is given by

$$P = IS$$

$$= 1\times 10^{-6}\times 2 = 2\mu\text{W}$$

Thus energy is given by

$$E = P\times t$$

$$= 2\times 10^{-6}\times 5\times 60\times 60 = 36\times 10^{-3}\ \text{J}$$

7. Tuning fork A is approaching the listener. Therefore apparent frequency of sound heard by listener is

$$n'_A = \frac{v}{v - v_s}n_A = \frac{330}{330-5}\times 256 = 260\ \text{Hz}$$

Tuning fork B is receding away from the listener. Therefore apparent frequency of sound of B heard by listener is

$$n'_B = \frac{v}{v + v_s}n_B = \frac{330}{330+5}\times 256 = 252\ \text{Hz}$$

Therefore the number of beats heard by listener per second is

$$n'_A = n'_B = 260 - 252 = 8$$

8. Let S be the stationary car and W the reflecting wall approaching with velocity u. S' is the image formed of S by the reflecting wall. If reflecting surface moves with velocity u, the relative velocity of source and image is $2u$. Therefore, it may be assumed that image S' approaches the stationary driver S with velocity $2u$. Hence the apparent frequency is

$$n' = \frac{v}{v-2u}n = \frac{v}{v\left(1-\frac{2u}{v}\right)}n = \left(1-\frac{2u}{v}\right)^{-1}n$$

If $u << v$, then

$$n' = \left(1+\frac{2u}{v}\right)n$$

Here $\quad u = 3.3$ m/s, $v = 330$ m/s, $n = 1000$.

$$\therefore \quad n' = \left(1+\frac{2\times 3.3}{330}\right)\times 1000 = 1020\ \text{Hz}$$

percentage change in frequency is

$$\frac{n'-n}{n}\times 100\% = \frac{1020-1000}{1000}\times 100\% = 2\%$$

9. For a mixture of non-reacting gases,

$$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_{\text{mix}} - 1}$$

For given problem,

$$n_1 = 2, n_2 = 2, \gamma_1 = \frac{7}{5}, \gamma_2 = \frac{7}{5}, \gamma_{\text{mix}} = 1.4\,.$$

$$\therefore \quad M_{\text{mix}} = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$$

Here, $\quad m_1 = 32$, $m_2 = 2$, $M_{\text{mix}} = 22$.

$$\therefore \quad V_{\text{mix}} = \sqrt{\frac{\gamma_{\text{mix}}RT}{M_{\text{mix}}}} \quad \text{and} \quad V_H = \sqrt{\frac{\gamma_{H_2}RT}{M_{H_2}}}$$

$$V_{\text{mix}} = \sqrt{\frac{\gamma_{\text{mix}}}{M_{\text{mix}}}\times\frac{M_H}{\gamma_H}}\times V_H = \frac{V}{\sqrt{11}}\ \text{m/s}$$

10. Direct sound: $V_s = -5$ m/s because positive direction is from S to O.

$$f = f_0\left(\frac{c - V_0}{c - V_s}\right) = 256\left(\frac{330-0}{330+5}\right) = 252.2\ \text{Hz}$$

i. Frequency received at wall (f_1) is given by

$(V_s = +5\ \text{m/s})$

$$f = \frac{f_0(c - V_{\text{wall}})}{c - V_s} = \frac{f_0(c-0)}{c - V_s} = \frac{256\times 330}{330-5}\text{Hz}$$

ii. Frequency received by observer (f_2) is given by

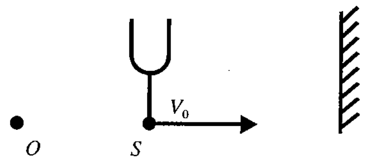

Fig. 6.51

$$f_2 = \frac{f_1(c - V_0)}{c - V_{\text{wall}}} = \frac{f_1(c-0)}{c-0} = f_1$$

$$\Rightarrow \quad f_2 = \frac{256\times 330}{325} = 259.9\ \text{Hz}$$

Beats per second is $f_2 - f = (259.9 - 252.2)$ Hz $= 7.7$ Hz.

11. The number of revolutions per second is 2. Radius of the circle is 1.988 m.

The linear velocity of the tuning fork is

$$v = 2\times 2\pi r = 4\times\frac{22}{7}\times 1.988 = 25\ \text{m/s}$$

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i. Apparent frequency when the tuning fork is approaching the listener is

$$n_1 = \frac{vn}{v - v_s} = \frac{350\,n}{350 - 25} = \frac{14}{13}\,n \text{ [highest note]}$$

ii. Apparent frequency when the tuning fork is moving away from the listener is

$$n_2 = \frac{vn}{v + v_s} = \frac{350n}{(350 + 25)} = \frac{14}{15}n \text{ [lowest note]}$$

The ratio of highest note to the lowest note is given by

$$\frac{n_1}{n_2} = \frac{14\,n}{13} \times \frac{15}{14\,n} = \frac{15}{13} = 1.154$$

12.

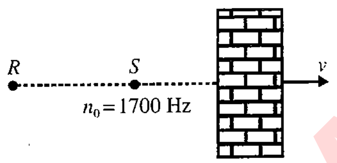

Fig. 6.52

As both source and receiver are at rest, the frequency of sound waves which directly reach receiver will be 1700 Hz and the frequency of sound which the wall will receive as a moving observer is

$$n_1 = n_0\left[\frac{v}{v+u}\right]$$

The wall now behaves as a moving source of frequency n_1 which when received by receiver, the frequency observed is

$$n_2 = n_1\left[\frac{v-u}{v}\right] = n_0\left(\frac{v-u}{v+u}\right)$$

$$= 1700\left[\frac{340 - 0.06}{340 + 0.06}\right]$$

$$= 1700 \times \frac{339.94}{340.06} = 1699.4 \text{ Hz}$$

Thus beat frequency received by detector is Δn = 1700 – 1699.4 = 0.6 Hz.

13. The situation is shown in Fig. 6.53.

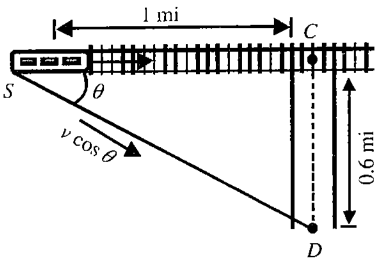

Fig. 6.53

$$SD = \sqrt{1^2 + 0.6^2} = 1.166 \text{ mi}$$

$$\cos\theta = \frac{SC}{SD} = \frac{1}{1.166} = 0.857$$

Thus speed of source along the line of sight is

$$v_S = v\cos\theta = \frac{80}{60\times 60}\times 0.857 = 0.019 \text{ mi/s}$$

Thus the apparent frequency observed is

$$n = n_0\left[\frac{v_{\text{sound}}}{v_{\text{sound}} - v_S}\right] = 400\left[\frac{0.2}{0.2 - 0.019}\right] = 442 \text{ Hz}$$

14. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$$v_S = \omega R$$

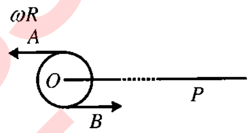

Fig. 6.54

where R is radius of the circle.

At points A and B, the velocity v_S of whistle is parallel to line OP; i.e., with respect to observer at P, the whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, the whistle may be assumed to be moving along line OP.) The observer, therefore, listens maximum frequency when the source is at B moving towards the observer:

$$f_{\max} = f\frac{v}{v - v_s}$$

where v is speed of sound in air. Similarly, the observer listens minimum frequency when the source is at A, moving away from the observer:

$$f_{\min} = \frac{f\omega}{v + v_s}$$

For f = 540 Hz, v_s = 2 ft × 15 rad/s = 30 ft/s and v = 1100 ft/s, we get (approx.) $f_{\max}$ = 555 Hz and $f_{\min}$ = 525 Hz.

Objective Type

1. a. Apparent frequency due to train which is coming in is

$$n_1 = \frac{v}{v - v_s}n$$

Apparent frequency due to train which is leaving is

$$n_2 = \frac{v}{v + v_s}n$$

So the number of beats is

$$n_1 - n_2 = \left(\frac{1}{316} - \frac{1}{324}\right)320 \times 240 \quad \Rightarrow n_1 - n_2 = 6$$

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2. **b.** Intensity after passing through one slab

$$I' = \left[I - \frac{20}{100}\times I\right] = \left[I - \frac{I}{5}\right] = \frac{4I}{5}$$

So, intensity after passing through two slabs

$$I'' = \left[I' - \frac{20}{100}\times I'\right] = \frac{4I'}{5} = \frac{16I}{25}$$

$$\therefore \quad \%\text{ decrease } = \left[\frac{\left(I - \frac{16I}{25}\right)}{I}\right]\times 100 = 36\%$$

3. **d.** When the man is approaching the factory,

$$n' = \left(\frac{v+v_0}{v}\right)n = \left(\frac{320+2}{320}\right)800 = \left(\frac{322}{320}\right)800$$

When the man is going away from the factory,

$$n'' = \left(\frac{v-v_0}{v}\right)n = \left(\frac{320-2}{320}\right)800 = \left(\frac{318}{320}\right)800$$

$$\therefore \quad n' - n'' = \left(\frac{322-318}{320}\right)800 = 10\text{ Hz}$$

4. **b.** Apparent frequency due to source A is

$$n' = \frac{v-u}{v}\times n$$

Apparent frequency due to source B is

$$n'' = \frac{v+u}{v}\times n$$

$$\therefore \quad n'' - n' = \frac{2u}{v}\times n = 10$$

$$\therefore \quad u = \frac{10v}{2n} = \frac{10\times 340}{2\times 680} = 2.5\text{ m/s}$$

5. **c.** The Doppler formula holds for non-collinear motion if v_s and v_o are taken to be the resolved component along the line of slight. In this case, we have

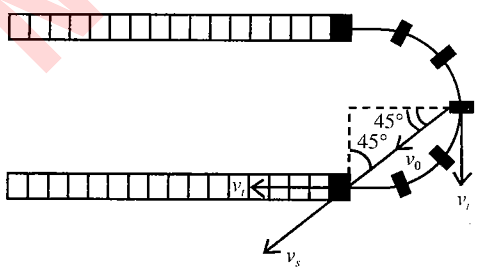

Fig. 6.55

$$v_o = -v_t \sin 45° = -\frac{30}{\sqrt{2}}\text{ m/s}$$

$$v_s = -v_t \sin 45° = -\frac{30}{\sqrt{2}}\text{ m/s}$$

We have, $v = 340$ m/s, $n = 200$ Hz. The apparent frequency n' is given by

$$n' = n\left[\frac{v-v_o}{v-v_s}\right] = 200\left[\frac{340+(30/\sqrt{2})}{340+(30/\sqrt{2})}\right] = 200\text{ Hz}$$

6. **b.** When the train is approaching,

$$n_1 = \frac{v}{v-v_s}\times n = \frac{320}{320-4}\times 243 = \frac{80}{79}\times 243$$

When the train is receding,

$$n_2 = \frac{v}{v+v_s}\times n = \frac{320}{324}\times 243 = \frac{80}{81}\times 243$$

Beat frequency is

$$n = n_1 - n_2 = 80\times 243\left(\frac{1}{79}-\frac{1}{81}\right) = 6\text{ Hz}$$

7. **b.** $f_1 = 900\left(\frac{300}{300+v_1}\right)$

$$\cong 900\left(1+\frac{v_1}{300}\right)^{-1}$$

$$= 900 - 3v_1$$

Similarly,

$$f_2 = 900\left(\frac{300}{300+v_2}\right) = 900 - 3v_2$$

$$f_2 - f_1 = 6$$

$$\therefore \quad 3(v_1 - v_2) = 6$$

$$\therefore \quad 3(v_1 - v_2) = 6$$

$$\text{or} \quad v_1 - v_2 = 2\text{ m/s}$$

8. **d.** When the source approaches the observer,

$$f_1 = f\left(\frac{v}{v-v_s}\right) = f\left(1-\frac{v_s}{v}\right)^{-1} \approx f\left(1+\frac{v_s}{v}\right)$$

$$\text{or} \quad \left(\frac{f_1 - f}{f}\right)\times 100 = \frac{v_s}{v}\times 100 = 10 \qquad \text{(i)}$$

In the second case, when the source recedes from the observer

$$f_2 = f\left(\frac{v}{v+v_s}\right) = f\left(1+\frac{v_s}{v}\right)^{-1} = f\left(1-\frac{v_s}{v}\right)$$

$$\therefore \quad \left(\frac{f_2 - f}{f}\right)\times 100 = -\frac{v_s}{v}\times 100 = -10$$

[from Eq. (i)]

In the first case, observed frequency increases by 10% while in the second case, observed frequency decreases by 10%.

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9. c. Apparent frequency for reflector (which will act here as an observer) would be $f_1 = \left(\frac{v+u}{v}\right)f$

where f is the actual frequency of source. The reflector will now behave as a source. The apparent frequency will now become

$$f_2 = \left(\frac{v}{v-u}\right)f_1$$

Substituting the value of f_1, we get

$$f_2 = \left(\frac{v+u}{v-u}\right)f$$

10. a. Given that velocity of source $v_s = 0$ (because it is stationary). Velocity of observer $v_o = (1/5)v = 0.2v$ (where v is the velocity of sound). Actual frequency of source is f and actual wavelength of source is λ. We know from the Doppler's effect that the apparent frequency recorded, when the observer is moving towards the stationary source, is given by

$$n' = \left(\frac{v+v_o}{v-v_s}\right)\times n$$

$$= \left(\frac{v+0.2v}{v-0}\right)\times n = \frac{1.2v}{v}\times n = 1.2n = 1.2f$$

Since the source is stationary, therefore the apparent wavelength remains unchanged, i.e., λ.

11. a. When the source is coming to stationary observer,

$$n' = \left(\frac{v}{v-v_s}\right)n$$

or $$1000 = \left(\frac{350}{350-50}\right)n$$

or $$n = (1000 \times 300/350) \text{ Hz}$$

When the source is moving away from the stationary observer,

$$n'' = \left(\frac{v}{v+v_s}\right)n$$

$$= \left(\frac{350}{350+50}\right)\left(\frac{1000\times 300}{350}\right) = 750 \text{ Hz}$$

12. d. Frequency heard by the observes will be maximum when the source is in position D. In this case, source will be approaching towards the stationary observer, almost along the line of sight (as observer is stationed at a larger distance).

$$n_{\max} = \frac{v}{v-v_s}n$$

$$= \frac{330\times 440}{330-1.5\times 20}$$

$$= 484 \text{ Hz}$$

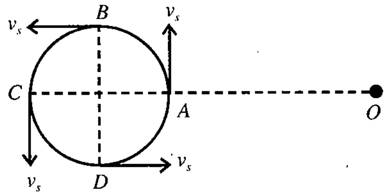

Fig. 6.56

Similarly, frequency heard by the observer will be minimum when the source reaches at position B. Now, the source will be moving away from the observer.

$$n_{\min} = \frac{v}{v+v_s}\times n = \frac{330}{330+1.5\times 20}\times 440$$

$$= \frac{330\times 440}{360} = 403.3 \text{ Hz}$$

13. c. The motorist receives two sound waves: direct one and that reflected from the wall.

$$f' = \frac{v+v_m}{v+v_b}f$$

For reflected sound waves:

Frequency of sound wave reflected from the wall is

$$f'' = \frac{v}{v-v_b}\times f$$

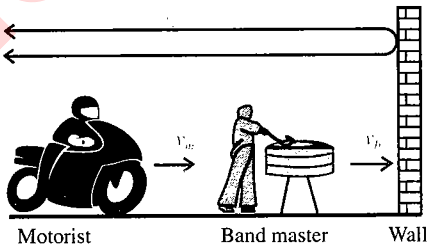

Fig. 6.57

Frequency of the reflected waves as received by the moving motorist is

$$f''' = \frac{v+v_m}{v}\times f'' = \frac{v+v_m}{v-v_b}\times f$$

Therefore, the beat frequency is

$$f''' - f' = \frac{v+v_m}{v-v_b}\times f - \frac{v+v_m}{v+v_b}f$$

$$= \frac{2v_b(v+v_m)}{v^2-v_b^2}f$$

14. a.

$$n' = n\left(\frac{v-v_o}{v-v_s}\right)$$

$\Rightarrow$ $$605 = 550\left(\frac{330-0}{330-v_s}\right)$$

$\therefore$ $$v_s = 30 \text{ m/s}$$

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15. d. No doppler effect, because velocity is perpendicular to line joining vehicle and observer.

16. c. Since source and both the observers are stationary, therefore no change will be observed by the two observers. It means both the observers will receive waves with natural frequency, which is equal to n.

17. c. Let v be the speed of sound, u be the speed of train.

Then, $\quad v_s = v_o = u$

and $\quad f' = f\left(\dfrac{v + u\cos\theta}{v + u\cos\theta}\right) = f$

18. b.

$$f = f_o\left(\frac{v_s + v_o}{v_s}\right)$$

$$= f_0\left[\frac{v + \frac{v}{5}}{v}\right]$$

$$= \frac{6}{5} f_o$$

Hence, percentage increase is

$$\left[\frac{\frac{6}{5}f_0 - f_0}{f_0}\right] \times 100 = 20\%$$

19. c. Intensity level is decibel is given by

$$L = 10\log_{10}\frac{I}{I_0}$$

$$L + 1 = 10\log_{10}\frac{I_1}{I_0}$$

Subtracting, $\quad 1 = 10\log_{10}\dfrac{I_1}{I_0} - 10\log_{10}\dfrac{I}{I_0}$

or $\quad \dfrac{1}{10} = \log_{10}\dfrac{I_1}{I}$

or $\quad 0.1 = \log_{10}\dfrac{I_1}{I}$

or $\quad \dfrac{I_1}{I} = 1.26$

20. c. $B = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

$$= 10\log_{10}\left(\frac{10^{-6}}{10^{-12}}\right)$$

$$= 60 \text{ dB}$$

21. c.

$$100 = 10\log_{10}\frac{I_1}{I_0}$$

$$50 = 10\log_{10}\frac{I_2}{I_0}$$

or $\quad \dfrac{I_1}{I_0} = 10^{10}$ and $\dfrac{I_2}{I_0} = 10^5$

Dividing, $\quad \dfrac{I_1}{I_2} = 10^5$

22. a. As the source and the observer are approaching one another, so n' would be larger.

$$f = \left(\frac{v + v/15}{v - v/10}\right)600 = 711 \text{ Hz}$$

23. a. $\lambda' = \left(\dfrac{v - v_s}{v}\right)\lambda = \left(\dfrac{320 - 20}{320}\right)60$

$= 56.25$ cm

24. b.

$$v' = \frac{v}{v - v_s}v,\quad v'' = \frac{v}{v + v_s}v$$

$$\frac{v'}{v''} = \frac{v + v_s}{v - v_s} \quad \text{or} \quad \frac{6}{5} = \frac{330 + v}{330 - v}$$

$$11v_s = 330 \quad \text{or} \quad v_s = 30 \text{ m/s}$$

25. b. Effective value of velocity of source is

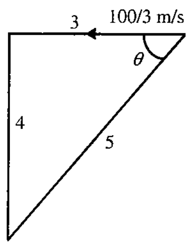

Fig. 6.58

$$v_s = \frac{100}{3}\cos\theta$$

$$= \frac{100}{3} \times \frac{3}{5} = 20 \text{ m/s}$$

$$v' = \frac{v}{v - v_s}v$$

$$v' = \frac{340}{340 - 20} \times 640 \text{ Hz} = 680 \text{ Hz}$$

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26 c.

$$f=\left(\frac{v+v_m}{v+v_m-v_{\text{source}}}\right)1000$$

$$=\left(\frac{340+20\cos 60^\circ}{340+20\cos 60^\circ-30}\right)1000$$

$$=1094 \text{ Hz}$$

27. d. Intensity,

$$I=\frac{P}{A}=\frac{200\pi}{4\pi\times(10)^2}=0.5 \text{ W/m}^2$$

No. of decibels is given by

$$10\log_{10}\frac{I}{I_0}=10\log_{10}\frac{0.5}{10^{-12}}$$

$$=10\log_{10}(5\times 10^{11})$$

$$=10\log_{10}\left(\frac{10^{12}}{2}\right)$$

$$=117 \text{ dB}$$

28. b. The component of velocity of source along line joining the car is

$$v_S=v_1\cos 45^\circ=36\times\frac{1}{\sqrt{2}} \text{ km/h}$$

$$=5\sqrt{2} \text{ m/s}$$

Component of velocity of observer (second car) along the line joining the car is

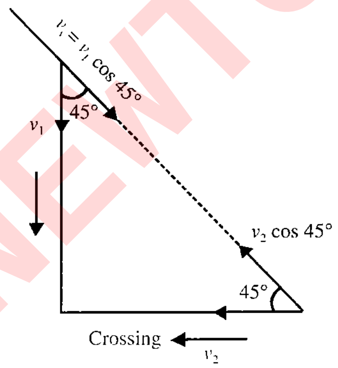

Fig. 6.59

$$v_0=v_2\cos 45^\circ=72\times\frac{1}{\sqrt{2}} \text{ km/h}$$

$$=10\sqrt{2} \text{ m/s}$$

$$n'=\frac{v+v_0}{v-v_S}n=\frac{330+10\sqrt{2}}{330-5\sqrt{2}}\times 280$$

$$=\frac{344}{323}\times 280 \text{ Hz}=298 \text{ Hz}$$

29. a. The frequency of direct and reflected sound is same.

30. d. Initially wall behaves as an approaching observer, so frequency of sound reaching the wall is

$$n_1=\frac{c+v}{c}n$$

While reflecting, the wall behaves as an approaching source, so frequency received by stationary observer is

$$n_2=\frac{c}{c-v}n_1=\frac{c}{c-v}\times\frac{c+v}{c}n=\frac{c+v}{c-v}n$$

Direct frequency received by observer is n. the number of beats is

$$x=n_2-n=\frac{c+v}{c-v}n-n=\frac{2nv}{c-v}$$

31. b. The frequency of direct sound of whistle heard by observer is

$$n_1=\frac{v}{v-v_s}n=\frac{340}{340-1}\times n=\frac{340}{339}n \quad \text{(i)}$$

Frequency of sound of whistle reflected by wall is

$$n_2=\frac{v}{v+v_s}n=\frac{340}{341}n \quad \text{(ii)}$$

Given, $n_1-n_2=4$

Therefore, $\frac{340}{339}n-\frac{340}{341}n=4$

$\Rightarrow$ $n=680$ Hz

32. d. Suppose at $t=0$, distance between source and observer is l. First, wave pulse (say p_1) is emitted at this instant. This pulse will reach the observer after a time

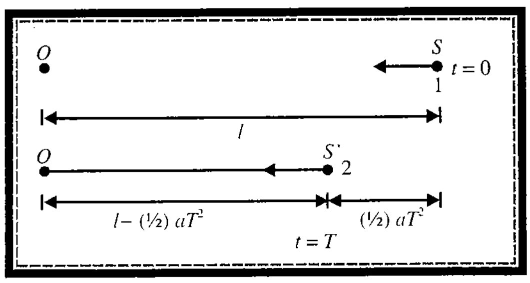

Fig. 6.60

$$t_1=\frac{l}{v} \quad \text{(i)}$$

Source will emit the next pulse (say p_2) after a time $T(=1/f)$.

During this time the source will move a distance $(1/2)aT^2$ towards the observer. This pulse p_2 will reach the observer in a time

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$$t_2 = T + \frac{l - \frac{1}{2}aT^2}{v} \qquad \text{(ii)}$$

The changed time period as observed by the observer is

$$T' = t_2 - t_1 = T + \frac{l}{v} - \frac{1}{2}\frac{aT^2}{v} - \frac{l}{v}$$

Substituting $T' = 1/f'$ and $T = 1/f$ in the above equation, we get

$$f' = \frac{2vf^2}{2vf - a}$$

33. **b.** $60 \text{ dB} = 10 \text{ dB} \log \frac{I}{I_0}$

$\Rightarrow \quad I = (10^6 \times 10^{-12}) \text{ W/m}^2 = 10^{-6} \text{ W/m}^2$
$[I_0 = 10^{-12} \text{ W/m}^2]$

$$I = \frac{(\Delta P_m)^2}{2\rho v}$$

where $\rho = 15/11 \text{kg/m}^3$, $v = 330$ m/s

$\therefore \ (\Delta P_m)^2 = 2\rho v I = 2 \times \frac{15}{11} \times 330 \times 10^{-6}$

$\Rightarrow \Delta P_m = 0.03 \text{ N/m}^2$

34. **a.** The wavelength in front of the car is

$$\lambda = \frac{v - u_S}{f_S} = \frac{340 \text{ m/s} - 34 \text{ m/s}}{400 \text{ Hz}}$$
$$= 0.765 \text{ m}$$

35. **c.**

$$SL = 10 \log \frac{I}{I_0}$$
$$= 10 \log \frac{k}{I_0 r^2}$$
$$= 10 \log K - 10 \log (I_0 r^2)$$
$$= 10 \log k - 10 \log I_0 - 20 \log r$$
$$= a - b \log r$$

36. **b.** Intensity of sound wave,

$$I = \frac{P_0^2}{2\rho v}$$
$$= \frac{30 \times 30}{2 \times 10^3 \times \sqrt{2} \times 10^3} = 0.3 \times 10^{-3} \text{ W/m}^2$$

37. **b.** $L_2 - L_1 = 30$ dB

$$10 \text{ dB} \log \frac{I_2}{I_0} - 10 \text{ dB} \log \frac{I_1}{I_0} = 30 \text{ dB}$$

$$\log_{10} \frac{I_2}{I_1} = 3 \quad \Rightarrow \frac{I_2}{I_1} = 10^3$$

Hence, the sound intensity increases by 10^3.

38. **b.** Let v_m be the velocity of motorcyclist and v be the velocity of sound.

$$v' = \frac{90}{100} v_0 = \frac{v_0(v - v_m)}{v} \Rightarrow 9v = 10v - 10v_m$$

$$v_m = \frac{v}{10}$$

$$v_m^2 - 0 = 2as = \frac{v^2}{100}$$

$$\therefore \quad S = \frac{v^2}{200a} = \frac{(330)^2}{200 \times 2.2} = 247.5 \text{ m}$$

39. **b.** Intensity of wave is given by

$$I = \frac{(\Delta P)_m^2}{2\rho v}$$

$$v = \frac{(\Delta P)_m^2}{2\rho I} = \frac{(2 \times 10^{-4})^2}{2 \times 1 \times 10^{-10}} = 200 \text{ m/s}$$

Amplitude of wave,

$$A = \frac{(\Delta P)_m}{\omega \rho v} = \frac{2 \times 10^{-4}}{10^3 \times 1 \times 200} = 10^{-9} \text{m}$$

Here, $\omega = 10^3$ rad/s, $k = \frac{\omega}{v} = \frac{10^3}{200} = 5 \text{ m}^{-1}$

Initial phase $\phi = \pi/2$.
The equation of the wave travelling in the negative x-axis is

$$y = A \sin(\omega t + kx + \phi)$$
$$= 10^{-9} \sin\left(1000t + 5x + \frac{\pi}{2}\right)$$
$$= 10^{-9} \cos(1000t + 5x)$$

40. **b.** Velocity of sound is not affected by the change in pressure of air. Velocity of sound at 1°C,

$$v_1 = (332 + 0.61\, t) \text{ m/s}$$

At $\quad -5°\text{C}, \ v_{-5°\text{C}} = (332 - 0.61 \times 5)$ m/s

At $\quad 30°\text{C}, \ v_{30°\text{C}} = (332 + 0.61 \times 30)$ m/s

$$\therefore \quad v_{30°\text{C}} - v_{-5°\text{C}} = (0.61 \times 35) \text{ m/s}$$
$$= 21.35 \text{ m/s}$$

41. **b.** Frequency received by guard is

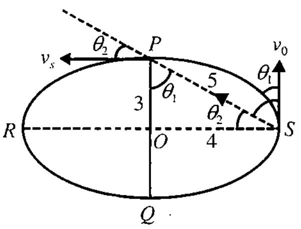

Fig. 6.61

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$$n_0 = n_0 \frac{v_0(v + v_0 \cos\theta_1)}{(v + v_s \cos\theta_2)}$$

$$v = 330 \text{ m/s}$$

Here, $v_0 = v_s = v/3, \cos\theta_1 = 3/5, \cos\theta_2 = 4/5.$

$$\therefore \quad n = n_0 \frac{\left(v + \frac{v}{3} \times \frac{3}{5}\right)}{\left(v + \frac{v}{3} \times \frac{4}{5}\right)}$$

$$= \left(\frac{6}{5} \times \frac{15}{19}\right) n_0 = \frac{18 n_0}{19} = 1800 \text{ Hz}$$

42. a. This frequency–time curve corresponds to a source moving at an angle to a stationary observer.

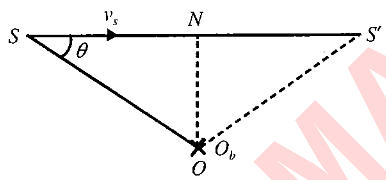

Fig. 6.62

In the region SN, the source is moving towards the observer, i.e., the apparent frequency

$$n' = n_0 \left(\frac{v}{v - v_s \cos\theta}\right)$$

$$n' = n_0 \left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = \pi/2$, i.e., at N,

$n' = n_0 = 1000$ Hz , i.e., natural frequency of source.

In the region NS' the source is moving away from the observer, i.e., apparent frequency

$$n' = n_0 \left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = 0$, i.e., $\cos\theta = 1$,

$$n_{\max} = n_0 \frac{v}{v - v_s} = \frac{(1000 \text{ Hz})(300 \text{ m/s})}{(300 \text{ m/s} - 30 \text{ m/s})}$$

$$= \frac{10}{9} \times 1000 \text{ Hz} = 1111 \text{ Hz}$$

$$n_{\min} = n_0 \frac{v}{v + v_s} = \frac{1000 \times 300}{330} = 909 \text{ Hz}$$

43. d. Time taken is given by

$$T = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

$$v_1 = v_{0°C} = 330 \text{ m/s}$$

$$v_2 = (330 + 0.6t) = 342 \text{ m/s}$$

$$d = 1662 \text{ m}$$

$$\therefore \quad T = \frac{d_1}{330} + \frac{(d - d_1)}{342} = 5 \text{ s}$$

$$\frac{d_1(342 - 330)}{330 \times 342} + \frac{d}{342} = 5 \text{ s}$$

$$12 d_1 = 5(342 \times 330) - 330 \times 1662$$

$$d_1 = 1320 \text{ m}$$

$$d_2 = 342 \text{ m}$$

44. d. Intensity level is given by

$$40 \text{ dB} = 10 \log \frac{I}{I_0}$$

$$\Rightarrow \frac{I}{I_0} = 10^4 \quad \Rightarrow I = 10^{-12} \times 10^4 = 10^{-8} \text{ W/m}^2$$

Energy received by eardrum per second is

$$10^{-8} \times 10^{-6} = 10^{-14} \text{ W}$$

To receive a total energy of 1 J, time required is

$$\frac{1}{10^{-14}} = 10^{14} \text{ s}$$

45. c. The frequency that the observer receives directly from the source has frequency $n_1 = 500$ Hz. As the observer and source both move towards the fixed wall with velocity u, the apparent frequency of the reflected wave coming from the wall to the observer will have frequency

$$n_2 = \left(\frac{V}{V - u}\right) 500 \text{ Hz}$$

where V is the velocity of sound wave in air. The apparent frequency of this reflected wave as heard by the observer will then be

$$n_3 = \left(\frac{V + u}{V}\right) n_2 = \left(\frac{V + u}{V}\right)\left(\frac{V}{V - u}\right) 500 = \left(\frac{V + u}{V - u}\right) 500$$

It is given, that the number of beat per second is $n_3 - n_1 = 10$

$$\therefore \quad (n_3 - n_1) = 10 = \left(\frac{V + u}{V - u}\right) 500 - 500$$

$$= 500\left[\frac{V + u}{V - u} - 1\right]$$

$$10 = \frac{2 \times u \times 500}{V - u}$$

Hence,

$$10V = 1000u + 10u = 1010u$$

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Putting $u = 4$ m/s,

we have $V = \frac{1}{10}[4040] = 404$ m/s

46. c. Apparent frequency is given by

$$n' = n\frac{(u + v_w)}{(u + v_w - v_s\cos 60°)}$$

$$= \frac{510(330+20)}{330+20-20\cos 60°}$$

$$= 510 \times \frac{350}{340} = 525 \text{ Hz}$$

47. a. Wavelength of the incident sound is

$$\lambda_i = \frac{10u - \frac{u}{2}}{f} = \frac{19u}{2f}$$

Frequency of the incident sound is

$$F_i = \frac{10u - u}{10u - \frac{u}{2}} f = \frac{18}{19} f = f_r$$

when f_r is the frequency of the reflected sound.
Wavelength of the reflected sound is

$$\lambda_r = \frac{10u + u}{f_r} = \frac{11u}{18f} \times 19 = \frac{11 \times 19}{18}\frac{u}{f}$$

$$\therefore \quad \frac{\lambda_i}{\lambda_r} = \frac{19u}{2f} \times \frac{18f}{11 \times 19u} = \frac{9}{11}$$

48. d. $\xi = A\sin(kx - \omega t)$

$$P_{ex} = -B\frac{d\xi}{dx} = -BAk\cos(kx - \omega t)$$

Amplitude of P_{ex} is

$$BAk = (5 \times 10^5)(10^{-4})\left(\frac{2\pi}{0.2}\right)$$

$$= 5\pi \times 10^2 \text{ Pa}$$

49. b. Towards right wavelength gets compressed and towards left wavelength gets expanded.

50. c. Let relative velocity be v and the speed of sound be v_0. Then,

$$f_1 = \frac{v_0 - (-v)}{v_0} \times f_0 = \frac{v_0 + v}{v_0} f_0$$

$$f_2 = \frac{v_0}{v_0 - v} \times f_0$$

$$f_3 = \frac{v_0 + v/2}{v_0 - v/2} \times f_0$$

It is clear from above that $f_1 \neq f_2 \neq f_3$, $f_3 > f_0$ and we can prove that $f_2 > f_3 > f_1$.

51. c. $\lambda' = \frac{\text{Wave speed relative to listener}}{f}$

$$\Rightarrow \lambda' = \frac{v + v_\omega}{f} = \frac{v + v_\omega}{v}\lambda$$

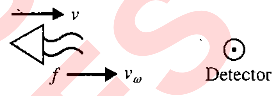

Fig. 6.63

52. c.

$$P = -B\frac{dy}{dx}$$

At R, dy/dx is most negative. So pressure is maximum.

53. b.

$$450 = 400\left(\frac{340 + v_s}{340 - v_s}\right)$$

$$\Rightarrow \quad \frac{9}{8} = \frac{340 + v_s}{340 - v_s}$$

$$\Rightarrow \quad 9(340) - 9v_s = 8(340) + 8v_s$$

$$\Rightarrow \quad 17v_s = 340$$

$$\Rightarrow \quad v_s = 20 \text{ m/s}$$

54. b. $\frac{f_{\text{approach}} - f_{\text{recede}}}{f} = \frac{\Delta f}{f} = \frac{v}{v - v_s} - \frac{v}{v + v_s}$

$$\therefore \quad \frac{\Delta f}{f} = \frac{v(v + v_s - v + v_s)}{v^2 - v_s^2} = \frac{2v\,v_s}{v^2 - v_s^2}$$

But $\frac{\Delta f}{f} \times 100 = 2\%$

$$\Rightarrow \quad 0.02 = \frac{2(300)v_s}{(300)^2 - v_s^2}$$

$$\Rightarrow \quad 0.02 = \frac{2(300)v_s}{(300)^2} = \frac{2}{300}v_s$$

$$\therefore \quad v_s = (0.01)\,300$$

$$= 3 \text{ m/s}$$

55. c. $f' = \left(\frac{c + v_a}{c - v_a}\right)f$

where c is the velocity of the radio wave, an electromagnetic wave, i.e., $c = 3 \times 10^8$ m/s and v_s is velocity of aeroplane.

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$$f' - f = \left[\frac{c + v_a}{c - v_a} - 1\right] f$$

$$\Rightarrow \quad \Delta f = \frac{2v_a f}{c - v_a}$$

Since approaching aeroplane cannot have a speed comparable to the speed of electromagnetic wave, so $v_s << c$.

$$\therefore \quad \Delta f = \frac{2v_a f}{c}$$

$$\Rightarrow \quad 2.6 \times 10^3 = \frac{2v_A(780 \times 10^6)}{3 \times 10^8}$$

$$\Rightarrow \quad v_A = 0.5 \times 10^3 \text{ m/s}$$

$$= 0.5 \text{ km/s}$$

56. d. $f_1 = \left(\frac{340}{340 - 34}\right) f = \frac{10}{9} f$

$$f_2 = \left(\frac{340}{340 - 17}\right) f = \frac{20}{19} f$$

$$\therefore \quad \frac{f_1}{f_2} = \frac{\frac{10}{9}}{\frac{20}{19}} = \frac{19}{18}$$

57. b. $(f_{\text{approach}})_A = 5.5 \text{ kHz} = \left(\frac{v + v_A}{v}\right) 5 \qquad \text{(i)}$

$$(f_{\text{approach}})_B = 6 \text{ kHz} = \left(\frac{v + v_B}{v}\right) 5 \qquad \text{(ii)}$$

where v is the velocity of sound. Now,

$$5.5 = \left(1 + \frac{v_A}{v}\right) 5 \qquad \text{(iii)}$$

$$\Rightarrow \quad \frac{v_A}{v} = 0.1$$

Similarly, $6 = \left(1 + \frac{v_B}{v}\right) 5 \qquad \text{(iv)}$

$$\Rightarrow \quad \frac{v_B}{v} = 0.2$$

$$\Rightarrow \quad \frac{v_B}{v_A} = 2$$

58. c. $40 = 10 \log_{10}\left(\frac{I_1}{I_0}\right) \qquad \text{(i)}$

$$\therefore \quad \frac{I_1}{I_0} = 10^4$$

Also, $20 = 10 \log_{10}\left(\frac{I_2}{I_0}\right)$

$$\Rightarrow \quad \frac{I_2}{I_0} = 10^2 \qquad \text{(ii)}$$

$$\therefore \quad \frac{I_2}{I_1} = 10^{-2} = \frac{r_1^2}{r_2^2}$$

$$\therefore \quad r_2 = 100\, r_1^2 \quad \Rightarrow r_2 = 10 \text{ m} \quad (\because r_1 = 1 \text{ m})$$

59. b. $n_1 = n_2 \quad \Rightarrow \frac{v - v_m}{v - v_C} n' = \frac{v + v_m}{v} n$

$$\Rightarrow \quad \frac{v - v_m}{v - 22} \times 176 = \frac{v + v_m}{v} \times 165$$

$$\Rightarrow \quad n_m = 22 \text{ m/s}$$

60. b. Time recorded in summer is more accurate. The velocity of sound is directly proportional to the square root of absolute temperature. Hence, the sound of the gun fired at the starting point will reach the finishing point quicker in summer than in winter. The lapse of time due to the time taken by the sound in reaching the finish point will be less in summer and hence the time recorded will be more accurate in summer than in winter.

61. b. $v' = \frac{v}{v + v_s} v$

or $\frac{6}{7} v = \frac{330}{330 + v_s} v$

or $6 \times 330 + 6v_s = 7 \times 330$

or $6v_s = 330$ or $v_s = 55$ m/s

Multiple Correct Answers Type

1. b., c., d.

$$v \propto \sqrt{T}$$

$$60 \text{ dB} = 10 \log \frac{I_1}{I_0}$$

$$30 \text{ dB} = 10 \log \frac{I_2}{I_0}$$

$$30 = 10 \log \frac{I_1}{I_2} \quad \Rightarrow \frac{I_1}{I_2} \neq 2$$

2. a., b., c.

$$y = A \sin(7.5\pi x - 3000\pi t)$$

$$k = \frac{2\pi}{\lambda} = 7.5\pi \quad \Rightarrow \lambda = \frac{2}{7.5} \text{ m}$$

$$\omega = 2\pi f = 3000\pi \quad \Rightarrow f = 1500 \text{ Hz}$$

$$v = \frac{2}{7.5} \times 1500 = 400 \text{ m/s}$$

Density

$$\rho = B / v^2 = 1.6 \times 10^5 / (400)^2 = 1 \text{ kg/m}^3$$

$$(\Delta p)_{\max} = BAk$$

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The maximum amplitude of the wave is

$$A = \frac{(\Delta p)_{max}}{Bk} = \frac{30}{1.6\times10^5\times7.5\pi}$$

$$= \frac{10\times10^{-5}}{4\pi} = \frac{10^{-4}}{4\pi}\ \text{m}$$

Intensity of wave at a distance 5 m from the source is

$$I = \frac{(\Delta p)^2_{max}}{2\rho v} = \frac{30^2}{2\times1\times400} = 1.125\ \text{W/m}^2$$

3. **a., d.** In both cases (a) and (d) the source and observer are relatively at rest, thus neither of them is approaching or separating from each other. Effectively, it is the medium that moves in each of these cases. The received (apparent) frequency differs from the emitted frequency if and only if the time required for the wave to travel from the source to observer is different for different wavefronts. With a uniform steady motion of the medium, past the observer and source, the transit time from source to observer is the same for all wavefronts. Hence it follows that apparent frequency is equal to the true emitted frequency. Thus there is no Doppler effect. In cases (b) and (c), Doppler effect will be observed as the source and observer have a relative speed and so they will approach or recede from each other.

4. **b., c., d.** When the vibrating tuning fork is brought in contact with the table, the vibrations of the tuning fork are being transmitted to the surface of table whose surface area is very large as compared to the surface area of tuning fork and hence sound becomes louder and due to the energy transmitted over the table, the sound dies sooner.

5. **b., c.** As time increases, the source and detector are relatively approaching each other up to $t = t_0$, where t_0 is the instant when the source and detector are located perpendicular to direction of motion.

$$v_0\times t_0 = \frac{d\cot\theta_0}{2}$$

$$t_0 = \frac{d\cot\theta_0}{2v_0}$$

For $\quad t < t_0$,

$$f_{ap} > f_0$$

For $\quad t > t_0$,

$$f_{ap} < f_0$$

6. **b., c.**

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Change in temperature affects the velocity of sound in air but as long as temperature remains same change in pressure has no effect.

$$v = \sqrt{\frac{B}{\rho}}$$

Bulk modulus of water is very high, so velocity of sound in water is higher than that in air.

7. **b., c., d.** When observer P approaches the stationary source at speed v,

$$n_1 = \frac{V+n}{V}\times n_0 \qquad \text{(i)}$$

(V is speed of sound)
When source S approaches the stationary observer P at speed v,

$$n_2 = \frac{V}{V-v}\times n_0 \qquad \text{(ii)}$$

Thus, $n_2 > n_1$, i.e., choice (b) is correct when both S and P approach each other with speed $v/2$

$$n_3 = \frac{V+(v/2)}{V-(v/2)}n_0 \qquad \text{(iii)}$$

Hence, $n_3 > n_0$ and n_3 lies between n_1 and n_2.

8. **a., c.** Let velocity of each observer be u as shown in the figure.

Fig. 6.64

Then frequency received by A will be

$$n_1 = n_0\left(\frac{v+u}{v}\right)$$

where n_0 is natural frequency of the source and v is sound propagation velocity. The frequency received by B will be

$$n_2 = n_0\left(\frac{v-u}{v}\right)$$

Since $(n_1+n_2)/2 = n_0$, therefore, option (a) is correct.

9. **a., c., d.** If intensity at a point is I, then energy density at that point is $E = I/v$, where v is wave propagation velocity.

It means that $E \propto I$. Hence, the graph between E and I will be a straight line passing through the origin. Therefore, (a) is correct and (b) is wrong. Intensity is given by

$$I = 2\pi^2 n^2 a^2 \rho v$$

Hence,

$$E = 2\pi^2 n^2 a^2 \rho$$

It means that $E \propto n^2$
Hence, the graph between E and n will be a parabola passing through origin, having increasing slope and symmetric about E-axis. Hence, option (d) is correct.

Particle maximum velocity is

$$u_0 = a\omega = 2\pi na$$

$$\Rightarrow \qquad \pi na = \frac{u_0}{2}$$

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acceleration ω. Consider the wave which is received by the observer at instant $t=\tau$. It will have left the source at an earlier instant of time, say t ($<\tau$), when the distance of source was r (say). If u be velocity of source at instant t, then $r=(1/2)\omega t^2$ and $u=\omega t$. We then have the relation between τ and t,

$$\tau = t+\frac{r}{V}=t+\frac{\omega t^2}{2V}$$

This is a quadratic equation in t, giving the solution

$$\omega t=\frac{-2V+\sqrt{4V^2+8V\omega\tau}}{2}$$

$$u=\omega t=V\left[\sqrt{1+\frac{2\omega\tau}{V}}-1\right]=340$$

$$\times\left[\sqrt{1+\frac{2\times10\times10}{340}}-1\right]=340\left[\sqrt{\frac{27}{17}}-1\right]$$

Then apparent frequency is given by

$$n_a=\left(\frac{V}{V+u}\right)n_0$$

Putting values $V=340$ m/s, $\tau=10$ s, $\omega=10$ m/s^2, we have

$$n_a=\left(\frac{340}{340+u}\right)1700$$

$$=1700\times\sqrt{\frac{17}{27}}=1.35 \text{ kHz}$$

7. **b.** Since $\omega=0$ for $t>10$ s, the source will move at constant speed of $u=10\times10=100$ m/s after $t=10$ s. At $t=10$ s, its distance r from the observer is $r=(1/2)\omega t^2=(1/2)\times10\times100=500$ m. Since the wave will take a time $\tau=10+500/340$ which is less than 15 s, the wave that is received by the observer at $\tau=15$ s would have left the source after 10 s, i.e., when its speed was constant at 100 m/s. Hence apparent frequency is

$$n_a=\left(\frac{340}{340+100}\right)1700$$

$$=\left(\frac{17}{22}\times1700\right)=1313 \text{ Hz}$$

For Problems 8–10
8.c., 9.b., 10.a.

Sol.

8. **c.** Since the source is moving on a small circle in a plane perpendicular to the direction of the wave moving towards the observer located on the axis of the circle, there would be no change in the observed frequency which will be the same as the real frequency, i.e., 500 Hz.

9. **b.** Since the observer is in the same plane as the circle, at one instant of time, the source will move directly towards the observer when the apparent frequency will be maximum given by

$$f_{max}=\left(\frac{V}{V-u}\right)500=\left(\frac{330}{330-u}\right)500$$

$$\text{Now, } u=r\omega=\left(\frac{100}{\pi}\right)5\times2\pi\times10^{-2} \text{ m/s}$$

$$=10 \text{ m/s}$$

$$\therefore \quad f_{max}=\left(\frac{330}{330-10}\right)500=\frac{330}{320}\times500=515 \text{ Hz}$$

Similarly, at another instant, the source will move directly away from the observer with velocity $u=10$ m/s. At this instant, the apparent frequency will be minimum given by

$$f_{min}=\left(\frac{330}{330+10}\right)500=\frac{33}{34}\times500=485 \text{ Hz}$$

10. **a.** The frequency of the source considered stationary at the centre is $n_0=500$ Hz when the observer moves towards the centre with constant speed u, the apparent frequency is

$$n_a=\left(\frac{V+u}{V}\right)500=\frac{330+20}{330}\times500=\frac{35}{33}\times500$$

Change in frequency is given by

$$n_a-n_0=\left(\frac{35}{33}\times500-500\right)=\frac{2\times500}{33} \text{ Hz}$$

Fractional change of frequency is

$$\left(\frac{n_a-n_0}{n_0}\right)=\left(\frac{2\times500}{33}\right)\times\frac{1}{500}$$

$$=\frac{2}{33}=0.06$$

Hence, percentage change is 6%.

For Problems 11–14
11.b., 12.a., 13.b., 14.b.

Sol.
11. b.

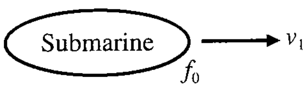

Fig. 6.71

$$f'=f_0\left(\frac{v+v_1}{v-v_1}\right),\ v=1050$$

$$f'=\frac{110f_0}{100}$$

Solve to get: $v_1=50$ m/s

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12. a.

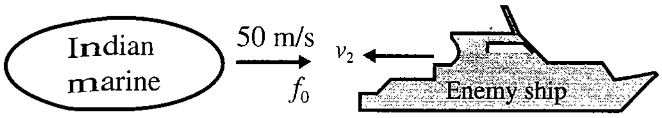

Fig. 6.72

$$f' = f_0\left(\frac{v+v_2}{v-50}\right),\ f'' = f'\left(\frac{v+50}{v-v_2}\right)$$

$$f'' = f_0\left(\frac{(v+v_2)(v+50)}{(v-v_2)(v-50)}\right) = 1.21\ f_0$$

[21% greater than sent waves]
We get $v_2 = 50$ m/s towards Indian submarine.

13. b.

$$\lambda' = \frac{v \text{ w.r.t. observer}}{f'}$$

$$= \frac{v+v_2}{f_0\dfrac{(v+v_2)}{(v-50)}} = \frac{v-50}{f_0}$$

$$\lambda'' = \frac{v+50}{f_0\dfrac{(v+v_2)(v+50)}{(v-v_2)(v-50)}} = \frac{(v-v_2)(v-50)}{f_0(v+v_2)}$$

$$\therefore \quad \frac{\lambda'}{\lambda''} = \frac{v+v_2}{v-v_2} = \frac{1050+50}{1050-50} = 1.1$$

14. b. $v = \sqrt{\dfrac{B}{\rho}} \Rightarrow 1050 = \sqrt{\dfrac{B}{1000}}$

$\therefore \quad B \approx 10^9$ N/m^2

For Problems 15–16

15.c., 16.c.

Sol.

15. c. In the propagation of sound waves, let pressure amplitude be Δp_0 and displacement amplitude be A. Then,

$$\Delta p_0 = BAK$$

where symbols have their usual meanings. We have,

$$SL = 10 \log \frac{I}{I_0}$$

$$\Rightarrow \quad 40 = 10 \log \frac{I}{10^{-12}}$$

$$\Rightarrow \quad I = 10^{-8} \text{ W/m}^2$$

$$I = \frac{\Delta p_0^2}{2\rho v}$$

$$\Rightarrow \quad \Delta p_0 = \sqrt{I \times 2\rho v}$$

$$= \sqrt{10^{-8} \times 2 \times \frac{15}{11} \times 330} \text{ N/m}^2$$

$$= 3 \times 10^{-3} \text{ N/m}^2$$

16. c. We have,

$$\Delta p_0 = BAK$$

$$\Rightarrow \quad \Delta p_0 = \rho v^2 A \times k \quad (\because B = Pv^2)$$

$$\Rightarrow \quad \Delta p_0 = \frac{\rho v^2 A \times 2\pi}{\lambda}$$

$$\therefore \quad \frac{A}{\lambda} = \frac{\Delta p_0}{\rho v^2 \times 2\pi} = 3.22 \times 10^{-9}$$

For Problems 17–19

17.c., 18.c., 19.c.

Sol.

17. c. $I = \dfrac{P}{A}$

so, $I_1 = \dfrac{P}{A_1}$ and $I_1 = I_2 = \dfrac{P}{A_1}$

18. c. The intensity of sound received by listener B is less, so he hears a quieter sound.

19. c. Power received by A's ear is $I_1 \times A_A = 12 \times 10^{-15}$ W
Power received by B's ear is $= I_1 \times A_A = 12 \times 10^{-15}$ W. So, both hear the same sound.

For Problems 20–24

20.d., 21.c., 22.a., 23.a., 24.a.

Sol.

20. d. The force exerted on inner ear is same as that of the force exerted on eardrum, due to negligible mass of ossicles.

$$P_{max} = \frac{F_{max}}{\text{area of stirrup}}$$

$$= \frac{P_{\text{max on eardrum}} \times A_{\text{eardrum}}}{A_{\text{stirrup}}}$$

Pressure amplitude is given by

$\Delta P_0 = P_{max} - P_{\text{normal value}}$

$$= \frac{(P_0 + (\Delta P_0)_{\text{on eardrum}}) \times A_{\text{eardrum}}}{A_{\text{stirrup}}} - \frac{P_0 \times A_{\text{eardrum}}}{A_{\text{stirrup}}}$$

$$= \frac{(\Delta P_0)_{\text{on eardrum}} \times A_{\text{eardru}}}{A_{\text{stirrup}}}$$

$$= \frac{4 \times 10^{-2} \times 50 \times 10^{-6}}{5 \times 10^{-6}} = 0.4 \text{ Pa}$$

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21. **c.** Displacement amplitude,

$$A = \left(\frac{\Delta p_0}{BK}\right)_{\text{inner ear}}$$

$$K_{\text{inner ear}} = \frac{\omega}{v_{\text{inner ear}}} = \frac{2\pi \times 1200}{1500} = 5.03 \text{ rad/m}$$

$$A = \frac{0.4}{5.03 \times 2.18 \times 10^9} = 3.65 \times 10^{-11} \text{ m}$$

22. **a.** For eardrum,

$$SL_2 - SL_1 = 10 \log \frac{I_2}{I_1}$$

$$\Rightarrow \quad 30 = 10 \log \frac{I_2}{I_1}$$

$$\Rightarrow \quad I_2 = 10^3 I_1$$

So, for outer ear, i.e., eardrum due to hearing aid the intensity increases by a factor of 1000, i.e., pressure amplitude at eardrum increases by a factor of $10\sqrt{10}$. By the same factor, the pressure amplitude at inner ear also increases. So, the intensity as perceived by inner ear will increase by the same factor, i.e., 1000.

23. **a.** Energy taken up by the person in 1 h is [(3.2 × 10^{-5}) × 50 × 10^{-6} × 3600] × 2 [through two ears] = 1.152 × 10^{-5} J » 1.2 × 10^{-5} J

24. **a.** If A_{stirrup} is reduced, Dp_o increases and hence intensity increases. If mass of ossicles is not negligible, then, force transferred is less.

For Problems 25–27

25. d., 26. b., 27. c.

Sol.

25. **d.** The location of detector at required instant is shown in Fig. 6.73.

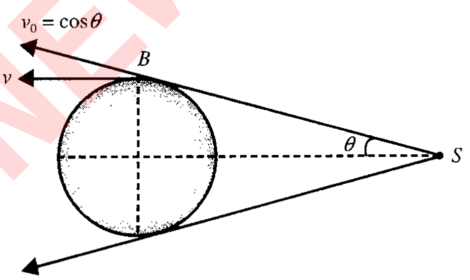

Fig. 6.73

We have, $v_0 = R\omega$ [speed of detector]

$$f_{ap} = \frac{v - v_0 \cos\theta}{v} \times f$$

$$\cos\theta = \frac{2R}{\sqrt{5}R}$$

$$f_{ap} = \frac{v - \omega R \times \frac{2}{\sqrt{5}}}{v} \times f$$

26. **b.** The instants at which the detector hears maximum and minimum frequencies are shown in Fig. 6.74. A corresponds to minimum frequency while B corresponds to maximum frequency.

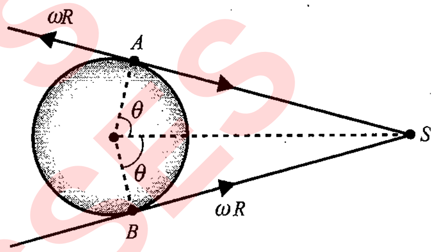

Fig. 6.74

Now,

$$\cos\theta = \frac{R}{2R} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Let the detector reaches A in time t_0 (measured from the starting instant). Then

$$\theta = \omega t_0$$

$$\Rightarrow \quad \frac{\pi}{3} = \omega t_0$$

$$\Rightarrow \quad t_0 = \frac{\pi}{3\omega}$$

Let the detector reaches B in time t_{01} (measured from A). Then,

$$2\pi - 2\theta = \omega \times t_{01}$$

$$\Rightarrow \quad t_{01} = \frac{4\pi}{3\omega}$$

The time taken after which the detector hears the maximum frequency for the first time is

$$t_0 + t_{01} = \frac{5\pi}{3\omega}$$

The interval between maximum and minimum frequencies as received by the detector for the first time is

$$t_{01} = \frac{4\pi}{3\omega}$$

For Problems 28–31

28.c., 29.b., 30.d., 31. b.

Sol.

28. **c.** The frequency heard directly from source is given by

$$f_1 = \left(\frac{v}{v - v_S}\right) f$$

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Here, $v = 340$ m/s, $v_S = 2$ m/s, $f = 512$.

$$f_1 = \frac{340}{338} \times 512 = 515 \text{ Hz}$$

The frequency of the wave reflected from wall will be same (no relative motion between wall and listener, so no change in frequency). Hence no beats are observed.

29. **b.** As the source is moving away from the listener, hence frequency observed by listener is

$$f_1 = \frac{v}{v + v_S} f = \frac{340}{340 + 2} \times 512$$

$$= \frac{340}{342} \times 512 = 509 \text{ Hz}$$

The frequency reflected from wall (we can assume an observer at rest) is

$$f_2 = \frac{v}{v - v_S} \times f$$

$$= \frac{340}{338} \times 512 = 515 \text{ Hz}$$

Therefore, beats heard by observer (L) is $515 - 509 = 6$.

30. **d.** As no relative motion is there between observer and listener, hence frequency heard by observer is 512 Hz. He will observe frequency reflected from wall, $f_1 = 515$ Hz.

Hence, the wave reflected from wall will act as another source of sound of frequency 515 Hz.

Therefore, the frequency received by the observer from wall is

$$f_2 = \frac{v + v_0}{v} f_1$$

$$= \frac{342}{340} \times 515 = 518 \text{ Hz}$$

Hence, beats observed is $f_2 - f = 518 - 512 = 6$.

31. **b.** Frequency received from source directly by observer will remain same. Hence frequency received by observer is 512 Hz. Let f_1 be the frequency reflected by wall. Then,

$$f_1 = \frac{v}{v - v_S} \times f = \frac{340}{338} \times 512 = 515 \text{ Hz}$$

The frequency received by observer (reflected from wall) is

$$f_2 = \left(\frac{v + v_0}{v}\right) f = \frac{342}{340} \times 515 = 518 \text{ Hz}$$

Hence, beats heard is $f_2 - f_1 = 518 - 512 = 6$.

For Problems 32–33

32.d., 33.c.

Sol.

32. **d.** Let the source be moving along the straight line AC and observer be located at O, as shown. Let the velocity of sound in air be v. The velocity of source is ηv.

Let the sound wave received by the observer at the moment when the source is closest to the observer (at C) be emitted by the source when it was at point A.

Therefore, by the time source travels from A to C, the sound wave travels from A to O. If this time interval is t, $AC = \eta vt$ and $AO = vt$. Velocity of approach of source when it is at A,

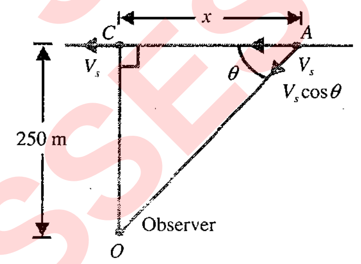

Fig. 6.75

$$v_s = (\eta v)\cos\theta = (\eta v)\left(\frac{AC}{AO}\right) = \eta v\left(\frac{\eta vt}{vt}\right) = \eta^2 v$$

When the sound wave emitted by the source at A reaches the stationary observer at O, it will receive the frequency

$$f = f_0\left(\frac{v}{v - v_s}\right) = f_0\left(\frac{v}{v - n^2 v}\right)$$

$$= \frac{f_0}{1 - \eta^2} = \frac{1800}{1 - (0.8)^2} = 5000 \text{ Hz}$$

33. **c.** When the source is at point C, its velocity of approach towards the stationary observer is zero. Therefore, when the sound wave emitted by the source from C reaches the observer (l/v time later) the observed frequency will be f_0.

In the same time interval (l/v) the source will have moved a distance, say $CD = (\eta v)(l/v) = \eta l$. At this moment, the distance between the source and observer is

$$OD = \sqrt{OC^2 + CD^2}$$

$$= \sqrt{l^2 + \eta^2 l^2} = l\sqrt{1 + \eta^2}$$

$$= 250\sqrt{1 + (0.8)^2} = 320 \text{ m}$$

Matching Column Type

1. **i.→c.; ii.→d.; iii.→a.; iv.→b.**

Pressure amplitude is given by

$P_0 = \rho\omega v A_0 = 1.29 \times 2\pi \times 10^3 \times 340 \times (0.01 \times 10^{-3})$
$= 27.55$ N/m²

Intensity is given by

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$$I=\frac{1}{2}\rho\omega^2A^2v=\frac{1}{2}\times 1.29\times(2\pi\times10^3)^2\times(10^{-5})^2(340)$$

$= 0.865$ W/m^2

Power,

$P = IA = (0.865)\,\pi\,(0.1)^2 = 0.027$ W $= 2.7 \times 10^{-2}$ W

Intensity at $r = 10$ m is $I=\frac{P_{av}}{A}=\frac{2.7\times10^{-2}}{4\pi\times10^2}$

$= 2.15 \times 10^{-5}$ W/m^2

2. **i.→a., b.; ii.→b., c., d.; iii.→a., b.; iv.→c., d.**
To solve this question the only concept required is that sound is a mechanical wave and requires some medium for its propagation while light can travel through vacuum also.

3. **i.→a., b.; ii.→a., b.; iii.→c., d.; iv.→c., d.**
Power transferred in a string wave is given by

$$P=\mu vA^2\omega^2\cos^2(\omega t-kx)$$

and $\quad P_{av}=\frac{\mu vA^2\omega^2}{2}$

Integer Answer Type

1. **(1)** Intensity is given by $I=\frac{p_0^2}{2\rho v}$.

Here v and ρ are same for both. And also given that I is same for both. So pressure amplitude is also same for both.

2. **(7)** Intensity from a point source varies with distance as

$I\propto\frac{1}{r^2}$

Let at distance $r_1 = 10$ m, intensity is I_1,

Then given $\quad 20=10\log\frac{I_1}{I_0}$ (i)

Let for $r = r_2$, sound level be zero. Then intensity at that point should be $I_2 = I_0$.

And $\quad \frac{I_1}{I_2}=\left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{I_1}{I_0}=\left(\frac{r_2}{r_1}\right)^2$ (ii)

From Eqs. (i) and (ii), we get

$$20=10\log\left(\frac{r_2}{r_1}\right)^2 \Rightarrow 20=20\log\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow \frac{r_2}{r_1}=10 \Rightarrow r_2=10r_1=7\text{ m}$$

3. **(3)** $\quad f_1=900\left(\frac{300}{300+V_1}\right)$

Or $\quad f_1=900\left[1+\frac{V_1}{300}\right]^{-1}=900-3V_1$

Likewise, $f_2 = 900 - 3V_2$

Given $f_2-f_1=9$

$3(V_1-V_2)=9 \quad\Rightarrow\quad V_1-V_2=3$ m/s

4. **(6)** Loudness due to $S_1 = I_1 = ka^2$ where a is the amplitude and loudness due to S_1 and S_2 both

$= I_2 = k(2a)^2 = 4I_1$

$n = 10\log_{10}(4I_1/I_1) = 10\log_{10}(4) = 10(0.6) = 6$

5. **(8)** The observer will hear a sound of the source moving away from him and another sound after reflection from the wall. The apparent frequencies of these sounds are

$$f_1=\left(\frac{v}{v+u}\right)f,\ f_2=\left(\frac{v}{v-u}\right)f$$

Number of beats $= f_2 - f_1$

$$\left(\frac{v}{v-u}-\frac{v}{v+u}\right)f=\frac{2uvf}{v^2-u^2}\approx\frac{2uf}{v}=8$$

6. **(4)** Here $I_1 = 1.0 \times 10^{-8}$ W/m^2

$r_1 = 5.0$ m, $I_2 = ?$, $r_2 = 25$ m

We know that $I\propto\left(\frac{1}{r^2}\right)$

$I_1r_1^2=I_2r_2^2$

$$I_2=\frac{I_1r_1^2}{r_2^2}=\frac{1.0\times10^{-8}\times25}{625}=4.0\times10^{-10}\text{ W/m}^2$$

7. **(3)** We know that $\beta=10\log_{10}\frac{I}{I_0}$

According to the problems $\beta_A=10\log_{10}\frac{I}{I_0}$

$\beta_B=10\log_{10}\left(\frac{2I}{I_0}\right)$

$\beta_B-\beta_A=10\log\left(\frac{2I}{I}\right)=10\times0.3010=3$ dB

8. **(4)** Source should be located midway $x = -8$ m and $x = 2$ m. That is at $x = -3$ m. For the same wavefront to reach at C, $SC = SA = SB$

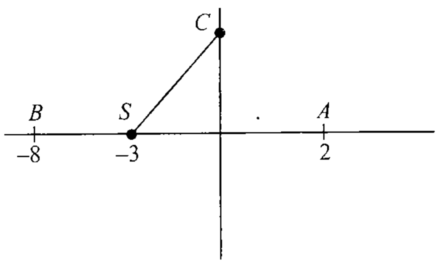

Fig. 6.76

$5=\sqrt{3^2+y^2} \quad\Rightarrow\quad y=4\text{m}$

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CHAPTER

7

Superposition and Standing Waves

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SUPERPOSITION AND INTERFERENCE

Many interesting wave phenomena in nature cannot be described by a single travelling wave. Instead, one must analyse these phenomena in terms of a combination of travelling waves. To analyse such wave combinations, we make use of the superposition principle.

If two or more travelling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave function of the individual waves.

In other words, the wave function $y(x, t)$ that describes the resulting motion in this situation is obtained by adding the two wave functions for the two separate waves:

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

Waves that obey this principle are called linear waves. In the case of mechanical wave, linear waves are generally characterized by having amplitudes much smaller than their wavelengths. Waves that violate the superposition principle are called non-linear waves and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that two travelling waves can pass through each other without being destroyed or even altered. For instance, when two pebbles are thrown into a pond and hit the surface at different locations, the expanding circular surface waves from the two locations do not destroy each other but rather pass through each other. The resulting complex pattern can be viewed as two independent sets of expanding circles.

Figure 7.1 is a pictorial representation of the superposition of two pulses. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving to the left is y_2. The pulses have the same speed but different shapes, and the displacement of the elements of the medium is in the positive y direction for both pulses. When the waves begin to overlap (Fig. 7.1(b)), the wave function for the resulting complex wave is given by $y_1 + y_2$. When the crests of the pulses coincide (Fig. 7.1(c)) the resulting wave given by $y_1 + y_2$ has a larger amplitude than that of the individual pulses. The two pulses finally separate and continue moving in their original directions (Fig. 7.1(d)). Notice that the pulse shapes remain unchanged after the interaction, as if the two pulses had never met!

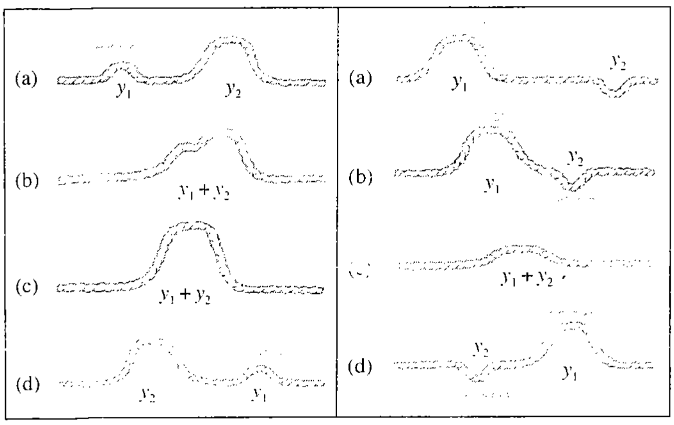

Fig. 7.1

The combination of separate waves in the same region of space to produce a resultant wave is called interference. For the two pulses shown in the figure the displacement of the elements of the medium is in the positive y direction for both pulses, and the resultant pulse (created when the individual pulses overlap) exhibits an amplitude greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as constructive interference.

Now consider two pulses travelling in opposite directions on a taut string where one pulse is inverted relative to the other as illustrated in figure. When these pulses begin to overlap, the resultant pulse is given by $y_1 + y_2$, but the values of the function y_2 are negative. Again, the two pulses pass through each other; because the displacements caused by the two pulses are in opposite directions; however, we refer to their superposition as destructive interference.

The superposition principle is the centre point of the waves in interference model. In many situations, both in acoustics and optics, waves combine according to this the principle and exhibit interesting phenomena with practical applications.

Illustration 7.1 **Two pulses travelling on the same string are described by**

$$y_1 = \frac{5}{(3x - 4t)^2 + 2} \quad \text{and} \quad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

a. In which direction does each pulse travel?
b. At what instant do the two cancel everywhere?
c. At what point do the two pulses always cancel?

Sol.

a. At constant phase, $\phi = 3x - 4t$ will be constant. Then $x = (\phi + 4t)/3$ will change: the wave moves. As t increases in this equation, x increases, so the first wave moves to the right.

In the same way, in the second case $x = (\phi - 4t + 6)/3$. As t increases, x must decrease, so the second wave moves to the left.

b. We require that $y_1 + y_2 = 0$.

$$\frac{5}{(3x - 4t)^2 + 2} + \frac{-5}{(3x + 4t - 6)^2 + 2} = 0$$

This can be written as $(3x - 4t)^2 = (3x + 4t - 6)^2$

Solving for the positive root, $t = 0.750$ s

c. The negative root yields $(3x - 4t) = -(3x + 4t - 6)$

The time terms cancel, leaving $x = 1.00$ m. At this point, the waves always cancel.

The total wave is not a standing wave, but we could call the point at $x = 1.00$ m a node of the superposition.

SUPERPOSITION OF SINUSOIDAL WAVES

Let us now apply the principle of superposition to two sinusoidal waves travelling in the same direction in a linear medium. If the two waves are travelling to the right and have the same

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frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1 = A\sin(kx-\omega t),\quad y_2 = A\sin(kx-\omega t+\phi)$$

where, as usual $k = 2\pi/\lambda$, $\omega = 2\pi f$, and ϕ is the phase constant. Hence, the resultant wave function y is

$$y = y_1 + y_2 = A[\sin(kx-\omega t)+\sin(kx-\omega t+\phi)]$$

To simplify this expression, we use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

Letting $a = kx-\omega t$ and $b = kx-\omega t+\phi$, we find that the resultant wave function y reduces to

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)$$

The result has several important features.

The resultant wave function y is also sinusoidal and has the same frequency and wavelength as the individual waves because the sine function incorporates the same values of k and ω that appear in the original wave functions.

The amplitude of the resultant wave is $2A\cos(\phi/2)$, and its phase is $\phi/2$. If the phase constant ϕ equals 0, then $\cos(\phi/2) = \cos 0 = 1$ and the amplitude of the resultant wave $2A$, twice the amplitude of either individual wave. In this case, the waves are said to be everywhere in phase and therefore interfere constructively. That is, the crests and troughs of the individual waves y_1 and y_2 occur at the same positions and combine to form the bold curve y of amplitude $2A$ shown in Fig. 7.2(a) in which they appear as a single curve. In general, constructive interference occurs when $\cos(\phi/2) = \pm 1$. That is true, for example, when $\phi = 0, 2\pi, 4\pi, \ldots$ rad, that is, when ϕ is an even multiple of π.

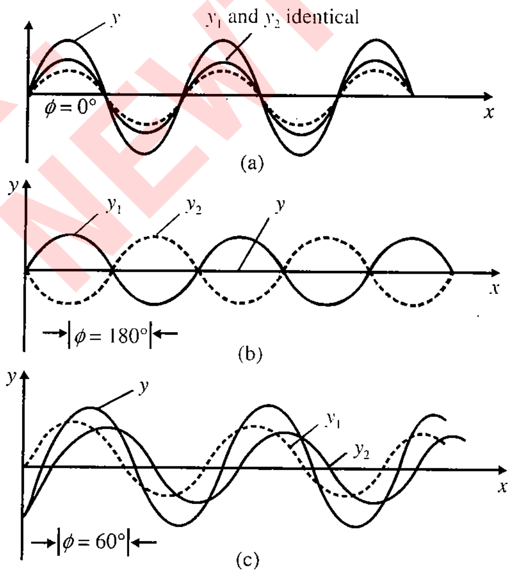

Fig. 7.2

When ϕ is equal to π rad or to any odd multiple of π, then $\cos(\phi/2) = \cos(\pi/2) = 0$ and the crests of one wave occur at the same positions as the troughs of the second wave. Therefore, as a consequence of destructive interference, the resultant wave has zero amplitude everywhere. Finally, when the phase constant has n arbitrary values other than 0 or an integer multiple of π rad (Fig. 7.2(c)) the resultant wave has an amplitude whose value is somewhere between 0 and $2A$.

Illustration 7.2 **Two waves passing through a region are represented by**

$$y = (1.0\text{ m})\sin[(\pi\text{ cm}^{-1})x - (50\pi\text{ s}^{-1})t]$$

and $$y = (1.5\text{ cm})\sin[(\pi/2\text{ cm}^{-1})x - (100\pi\text{ s}^{-1})t].$$

Find the displacement of the particle at $x = 4.5$ cm at time $t = 5.0$ ms.

Sol. According to the principle of superposition, each wave produces its disturbance independent of the other and the resultant disturbance is equal to the vector sum of the individual disturbance. The displacements of the particle at $x = 4.5$ cm at time $t = 5.0$ ms due to the two waves are,

$$y_1 = (1.0\text{ cm})\sin[(\pi\text{ cm}^{-1})(4.5\text{ cm}) - (50\pi\text{ s}^{-1})(5.0\times10^{-3}\text{ s})]$$

$$= (1.0\text{ cm})\sin\left[4.5\pi - \frac{\pi}{4}\right]$$

$$= (1.0\text{ cm})\sin\left[4\pi + \frac{\pi}{4}\right] = \frac{1.0\text{ cm}}{\sqrt{2}}$$

and

$$y_2 = (1.5\text{ cm})\sin[(\pi/2\text{ cm}^{-1})(4.5\text{ cm}) - (100\pi\text{ s}^{-1})(5.0\times10^{-3}\text{ s})]$$

$$= (1.5\text{ cm})\sin\left[2.25\pi - \frac{\pi}{2}\right]$$

$$= (1.5\text{ cm})\sin\left[2\pi - \frac{\pi}{4}\right] = -(1.5\text{ cm})\sin\frac{\pi}{4} = -\frac{1.5\text{ cm}}{\sqrt{2}}$$

The net displacement is $y = y_1 + y_2 = -0.5/\sqrt{2}$ cm

$$= -0.35\text{ cm}$$

Illustration 7.3 **Two travelling sinusoidal waves described by the wave functions**

$$y_1 = (5.00\text{ m})\sin[\pi(4.00x - 1200t)]$$

and $$y_2 = (5.00\text{ m})\sin[\pi(4.00x - 1200t - 0.250)]$$

where x, y_1 and y_2 are in metres and t is in seconds. (a) what is the amplitude of the resultant wave? (b) What is the frequency of resultant wave?

Sol. Graphs for these wave functions are like continuous sine curves. Think of water waves produced by continuously vibrating pencils on the water surface and seen through the side wall of an aquarium.

The pair of waves travelling in the same direction, perhaps separately created, combine to give a single travelling wave.

We can represent the waves symbolically as

$$y_1 = A_0\sin(kx-\omega t)\quad\text{and}\quad y_2 = A_0\sin(kx-\omega t-\phi)$$

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with $A_0 = 5.00$ m, $\omega = 1200\pi$ s^{-1} and $\phi = 0.250\pi$ rad

According to the principle of superposition, the resultant wave function has the form

$$y = y_1 + y_2 = 2A_0 \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t - \frac{\phi}{2}\right)$$

a. with amplitude $A = 2A_0 \cos\left(\frac{\phi}{2}\right)$

$$= 2(5.00)\cos\left(\frac{\pi}{8.00}\right) = 9.24 \text{ m}$$

b. and frequency $f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = 600$ Hz

INTERFERENCE OF THE WAVES

One simple device for demonstrating interference of sound waves is illustrated in Fig. 7.3. Sound from a loudspeaker S is sent into a tube at point P, where there is a T-shaped junction. Half the sound energy travels in one direction, and the other half travels in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of the two paths. The distance along any path from speaker to receiver is called the path length r. The lower path length r_1 is fixed, but the upper path length r_2 can be varied by sliding the U-shaped tube, which is similar to that on a slide trombone. When the difference in the path lengths $\Delta r = |r_2 - r_1|$ is either zero or some integer multiple of the wavelength λ (that I, $\Delta r = n\lambda$, where $n = 0, 1, 2, 3, \ldots$) the two waves reaching the receiver at any instant are in phase and interfere constructively as shown for this case, a maximum in the sound intensity is detected at the receiver. If the path length r_2 is adjusted such that the path difference $\Delta r = \lambda/2, 3\lambda/2, \ldots n\lambda/2$ (for n odd), of the two waves is exactly π rad, or 180°, the two waves are out of phase at the receiver and hence cancel each other. This is the case of destructive interference, no sound is detected at the receiver. This simple experiment demonstrates that a phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths. This important phenomenon will be indispensable in our investigation of the interference of light waves.

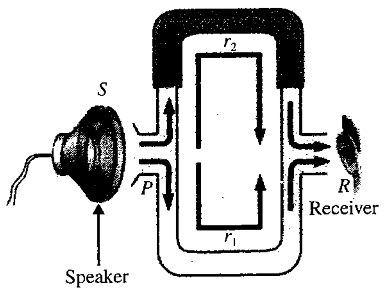

Fig. 7.3

Mathematical Analysis of Interference

Consider two harmonic waves of same frequency (coherent waves). Suppose A_1 and A_2 be the amplitudes of the waves and ϕ is the phase difference between them. It is assumed that the waves are plane and move almost along a line. Thus wave equations are

$$y_1 = A_1 \sin(kx - \omega t) \quad \text{(i)}$$

or $$y_2 = A_2 \sin(kx - \omega t + \phi) \quad \text{(ii)}$$

here $$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$$

When both the waves travel simultaneously, the resultant wave at P can be obtained by principle of superposition.

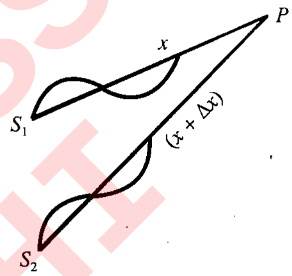

Fig. 7.4

$$y = y_1 + y_2$$
$$= A_1 \sin(kx - \omega t)$$
$$+ [A_2 \sin(kx - \omega t)\cos\phi + A_2 \cos(kx - \omega t)\sin\phi]$$
$$= (A_1 + A_2\cos\phi)\sin(kx - \omega t) + A_2 \sin\phi \cos(kx - \omega t)$$

Let $$A_1 + A_2\cos\phi = R\cos\theta \quad \text{(iii)}$$

and $$A_2 \sin\phi = R\sin\theta \quad \text{(iv)}$$

Squaring and adding Eqs. (iii) and (iv), we get

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \quad \text{(v)}$$

As $$I \propto A^2, \quad \therefore I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \text{(vi)}$$

Also $$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi} \quad \text{(vii)}$$

The resultant wave becomes

$$y = R\sin[(kx - \omega t) + \theta] \quad \text{(viii)}$$

clearly the resultant wave has the same frequency and speed as the interfering waves

Two Types of Interference

i. Constructive interference: If two waves are moving to the medium such that they are in phase (Fig. 7.5). The resulting amplitude due to the superposition of the waves will be summation of the individual amplitudes of the waves.

For constructive interference the resultant intensity will be greater than the sum of the individual intensities of the waves. For maximum intensity using Eq. (vi), we get

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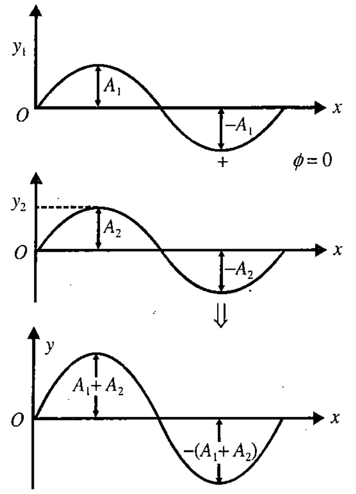


Constructive interference

Fig. 7.5

$$\cos\phi = 1$$

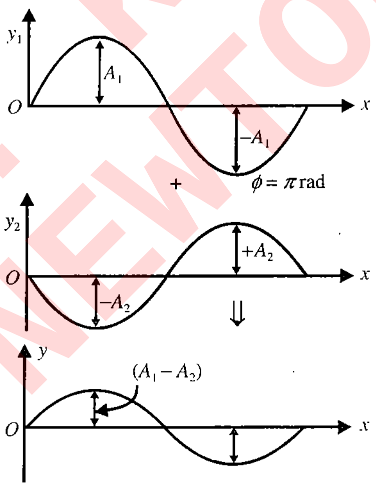
or $\quad \phi = 2\pi n, \quad$ when $\quad n = 0, 1, 2, 3, \ldots$

As 2π phase difference is equal to a path difference λ, so

$$\Delta x = n\lambda$$

Thus from Eq. (v) $\quad R_{max} = A_1 + A_2$

ii Destructive interference: If two waves are moving to the medium such that they are out of phase (Fig. 7.6). The resultant amplitude of the wave may be less than the sum of individual amplitudes of the waves. For minimum amplitude using Eq. (v)



Destructive interference

Fig. 7.6

$$\cos\phi = -1$$

or $\quad \phi = (2n-1)\pi$, where $\quad n = 1, 2, 3, \ldots$

and $\quad \Delta x = (2n-1)\,\lambda/2$

Thus from Eq. (v)$R_{min} = A_1 - A_2$

The ratio of maximum to minimum intensities

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} \qquad \text{(ix)}$$

Note: ***The distance between maximum and next minima = $\lambda/2$.***

Conditions of sustained interference:

1. Mathematically, interference phenomenon can take place between two waves of same frequency and different amplitudes. But for observable interference, the amplitude of the waves should be equal. In this case,

$$A_1 = A_2 = A$$

$$\therefore R^2 = A^2 + A^2 + 2AA\cos\phi$$

or $\quad R^2 = 2A^2(1 + \cos\phi) = 2A^2 \times 2\cos^2\frac{\phi}{2}$

$$R^2 = 4A^2\cos^2\frac{\phi}{2} \qquad \text{(x)}$$

Write $\quad R^2 = I \quad$ and $\quad 4A^2 = I_0$

$$\therefore I = I_0\cos^2\frac{\phi}{2} \qquad \text{(xi)}$$

From Eq. (xi), the maximum intensity is $4A^2$ and minimum intensity is zero. In the phenomenon of interference, the energy is not destroyed but is only redistributed from the positions of minimum intensity to those of maximum intensity. At the maximum intensity positions the intensity due to the two waves should be $2A^2$ but it actually $4A^2$. As shown in Fig. 7.7, the intensity varies from 0 to $4A^2$ and the average is still $2A^2$. It is equal to uniform intensity of $2A^2$ which will be present in the absence of interference phenomenon between two waves. Hence the formation of maxima and minima is due to interference of waves in accordance with the law of conservation of energy.

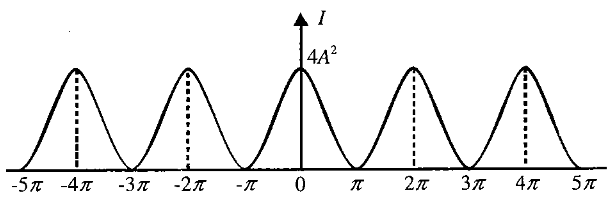


Fig. 7.7 Intensity distribution in interference of two waves

For sustained interference, the phase difference between the two waves must remain constant, it should not change with time. If the phase difference between the waves changes continuously, then the positions of the maximum and minimum intensities do not remain fix.

Illustration 7.4 **Two sound sources of same frequency produce sound intensities I_0 and $4I_0$ at a point P when**

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used separately. Now, they are used together so that the sound waves from them reach P with a phase difference ϕ. Determine the resultant intensity at P for

(i) $\phi = 0$ (ii) $\phi = 2\pi/3$ (iii) $\phi = \pi$

Sol. We know that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Thus,

i. for $\phi = 0$; $I = I_0 + 4I_0 + 2\sqrt{(I_0)(4I_0)} = 9I_0$

ii. for $\phi = 2\pi/3$; $I = I_0 + 4I_0 + 2\sqrt{(I_0)(4I_0)}\cos\dfrac{2\pi}{3}$

or $I = 3I_0$

iii. For $\phi = \pi$; $I = I_0 + 4I_0 + 2\sqrt{(I_0)(4I_0)}\cos\pi$ or $I = I_0$

Illustration 7.5 **Two identical sources of sound S_1 and S_2 produce intensity I_0 at a point P equidistant from each source.**

i. Determine the intensity of each source at the point P.

ii. If the power of S_1 is reduced to 64% and phase difference between the two sources is varied continuously, then determine the maximum and minimum intensities at the point P.

iii. If the power of S_1 is recued by 64%, then determine the maximum and minimum intensities at the point P.

Sol.

i. Both the sources produce maximum at the point P.

Thus, $I_{max} = I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Since the sources are identical, therefore, $I_1 = I_2 = I$.

$$I_0 = 4I \quad \text{or} \quad I = I_0/4$$

ii. Now $I_1 = 0.64\,I = 0.16\,I_0$

And $I_2 = I = 0.25\,I_0$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.81\,I_0$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.01\,I_0$$

iii. Now $I_1 = (1 - 0.64)I_0 = 0.36\,I = 0.09\,I_0$

And $I_2 = I = 0.25\,I_0$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.64\,I_0$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.04\,I_0$$

Illustration 7.6 **Two stereo speakers S_1 and S_2 are separated by a distance of 2.40 m. A person (P) is at a distance of 3.20 m directly in front of one of the speakers as shown in Fig. 7.8. Find the frequencies in the audible range (20–20,000 Hz) for which the listener will hear a minimum sound intensity. Speed of sound in air = 320 m/s.**

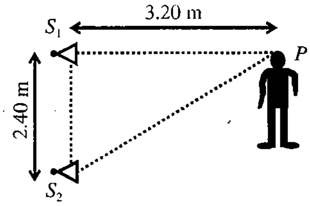

Fig. 7.8

Sol. Waves from S_1 to S_2 reach at point P.
Therefore, the path difference between waves reaching from S_1 to S_2 will be $\Delta = S_2P - S_1P$

Given $S_1P = 3.20$ m

$$S_2P = \sqrt{(2.40)^2 + (3.20)^2} = 4.0 \text{ m}$$

$$\therefore \quad \Delta = 4.0 - 3.20 = 0.80$$

For minima, path difference

$$\Delta = (2r-1)\,\frac{\lambda}{2};\ r = 1, 2, 3, \ldots$$

We have $\lambda = v/n$, where n is the frequency

$$\Delta = (2r-1)\,\frac{v}{2n}$$

$$0.80 = (2r-1)\,\frac{v}{2n}$$

Frequency $n = \dfrac{(2r-1)v}{2\times 0.80} = (2r-1)\dfrac{320}{1.6}\text{ Hz} = (2r-1)200\text{ Hz}$

Therefore, the required frequencies are $n = (2r-1)\times 200$ with $r = 1, 2, \ldots 50$.

Putting the values of r in the above equation, we get

$n = 200$ Hz, 600 Hz, 1000 Hz, 1400 Hz, 1800 Hz, …, 19800 Hz

Illustration 7.7 **Two speakers S_1 and S_2 derived by the same amplifier and placed at $y = 1.0$ m and $y = -1.0$ m (Fig. 7.9) The speakers vibrate in phase at 600 Hz. A man stands at a point on the x-axis at a very large distance from the origin and starts moving parallel to the Y-axis. The speed of sound in air is 330 m/s.**

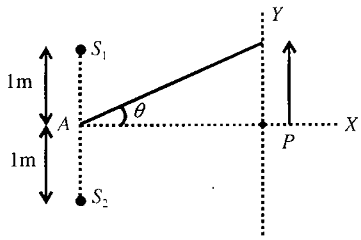

Fig. 7.9

a. At what angle will the intensity of sound drop to a minimum for the first time?

b. At what angle will the sound intensity be maximum for the first time?

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c. If he continues to walk along the line, how many more maxima can he hear?

Sol. When the man is at a large distance from the speaker, the path differences between waves starting from S_1 and S_2 and reaching at Q will be $\Delta = S_2Q - S_1Q = S_1S_2 \sin\theta = 2\sin\theta \approx 2\tan\theta = 2\theta$

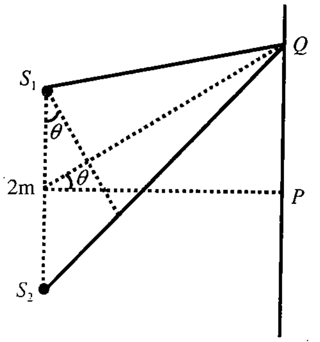

Fig. 7.10

Also $$\lambda = \frac{v}{n} = \frac{330}{600} = 0.55 \text{ m}$$

a. For the first minima

Path difference $$\Delta = \frac{\lambda}{2}$$

$$2\theta = \frac{0.55}{2}$$

$$\theta = \frac{0.55}{4} = 0.1375 \text{ rad} = 7.9°$$

b. For the first maxima, $\Delta = \lambda \quad \Rightarrow 2\theta = 0.55$

$$\theta = \frac{0.55}{2} \text{ rad} = 15.8°$$

c. Maximum path difference = 2 m
If order of maxima is r, then $2 = r\lambda$

$$r = \frac{2}{\lambda} = \frac{2}{0.55} = 3.6 \approx 3$$

Number of more maxima = 3 – 1 = 2

QUINCK'S TUBE

This is an apparatus used to demonstrate the phenomenon of interference and also used to measure velocity of sound in air. This is made up of two U-tubes A and B as shown in Fig. 7.11. Here the tube B can slide in and out from the tube A. There are two openings P and Q in the tube A. At opening P, a tuning fork or a sound source of known frequency n_0 is placed and at the other opening a detector is placed to detect the resultant sound of interference occurred due to superposition of two sound waves coming from the tubes A and B.

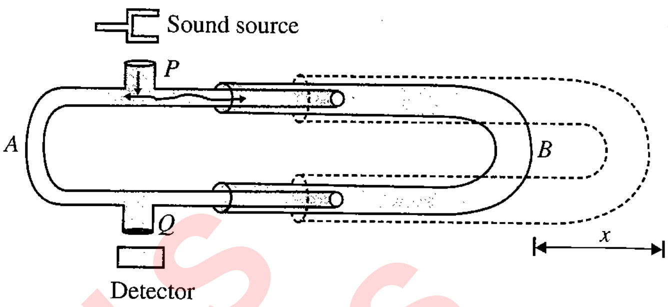

Fig. 7.11

Initially tube B is adjusted so that detector detects a maximum. At this instant if length of paths covered by the two waves from P to Q from the side of A and side of B are l_1 and l_2, respectively then for constructive interference we must have

$$l_2 - l_1 = N\lambda \quad \text{(i)}$$

If now tube B is further pulled out by a distance x so that next maximum is obtained and the length of path from the side of B is l_2' then we have

$$l_2' = l_2 + 2x \quad \text{(ii)}$$

where x is the displacement of the tube. For next constructive interference of sound at point Q, we have

$$l_2' - l_1 = (N+1)\lambda \quad \text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get $l_2' - l_2 = \lambda$

or $$2x = \lambda \quad \text{or} \quad x = \frac{\lambda}{2} \quad \text{(iv)}$$

Thus, by experiment we get the wavelength of sound as for two successive points of constructive interference, the path difference must be λ. As the tube B is pulled out by x, this introduces a path difference $2x$ in the path of sound wave through tube B. If the frequency of the source is n_0, the velocity of sound in the air filled in tube can be given as

$$V = n_0\lambda = 2n_0x \quad \text{(v)}$$

Illustration 7.8 **In a Quinck's experiment, the sound intensity being detected at an appropriate point, changes from minimum to maximum for the second time, when the slidable tube is drawn apart by 9.0 cm. If the speed of sound in air be 336 m/s, then what is the frequency of this sounding source?**

Sol. As the sliding tube is pulled out by a distance of 9.0 cm, the path difference effected is 2 × 9.0 cm = 18.0 cm.

Initially, the position of the tube corresponds to the minimum intensity. When maximum intensity is heard for the first time, the path difference increased by $\lambda/2$ and when heard for the second time, there is an additional path difference of λ.

Therefore, the total path difference

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$$= \frac{\lambda}{2} + \lambda = \frac{3\lambda}{2}$$

$$\frac{3\lambda}{2} = 18.0 \text{ cm}$$

$$\lambda = 12.0 \text{ cm}$$

The frequency of sound emitted by the source will be

$$f = \frac{v}{\lambda} = \frac{336 \text{ m/s}}{12.0 \times 10^{-2} \text{ m}} = 2.8 \text{ kHz}$$

Illustration 7.9 **In an experiment related to interference, Quinck's tube was employed to determine the speed of sound in air. A tuning fork of frequency 1328 Hz was used as the sounding source. Initially, the apparatus, yielded a maximum sound intensity. Later, when the slidable tube was drawn by a distance of 12.5 cm, the intensity was found to be maximum for the first time. Determine the speed of sound in air.**

Sol. A distance of 12.5 cm drawn aside implies a double path difference of 25 cm.

Since the shortest path difference, which enables the intensity to become maximum once again is, obviously, λ.

so $\lambda = 25$ cm

$$v = 1328 \text{ s}^{-1} \times 25 \text{ cm} = 332 \text{ m/s}$$

Therefore, the speed of sound in air is 332 m/s.

Illustration 7.10 **Two identical loudspeakers placed 3.00 m apart are driven by the same oscillator as shown in Fig. 7.12. A listener is originally at point O, located 8.00 m from the centre of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O, and she experiences the first minimum sound intensity. What is the frequency of the oscillator?**

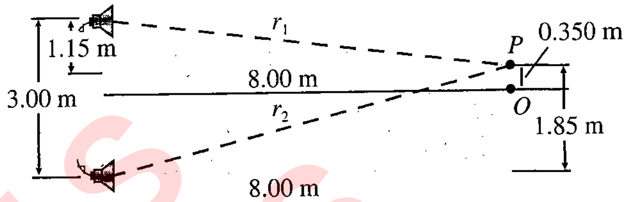

Fig. 7.12

Sol. In this example, signal representing the sound is electrically split and sent to two different loudspeakers. After leaving the speakers, the sound waves recombine at the position of the listener. Despite the difference in how the splitting occurs, the path difference discussion related to Fig. 7.12 can be applied here.

Because the sound waves from two separate sources combine, we apply the waves in interference analysis model.

Figure 7.12 shows the physical arrangement of the speakers, along with two shaded right triangles that can be drawn on the basis of the lengths described in the problem. The first minimum occurs when the two waves reaching the listener at point P are 180° out of phase, in other words, when their path difference Δr equals $\lambda/2$.

From the shaded triangles, find the path lengths from the speakers to the listener:

$$r_1 = \sqrt{(8.00 \text{ m})^2 + (1.15 \text{ m})^2} = 8.08 \text{ m}$$

$$r_2 = \sqrt{(8.00 \text{ m})^2 + (1.85 \text{ m})^2} = 8.21 \text{ m}$$

Hence, the path difference is $r_2 - r_1 = 0.132$ m. Because this path difference must equal $\lambda/2$ for the first minimum, $\lambda = 0.26$ m. To obtain the oscillator frequency, use equation $v = \lambda f$, where v is the speed of sound in air, 343 m/s.

REFLECTION OF WAVES AT FIXED END AND FREE END

Reflection of Waves—Fixed End	Reflection of Waves—Free End
• When the wave reaches the fixed end it exerts an upward pull on the end. According to Newton's third law, the fixed point exerts an equal and opposite force downward on the string. Thus, there is phase change of π for the reflected wave, the wave is inverted as shown in the Fig. 7.13. • Whenever a travelling wave reaches a boundary, some or all of the wave is reflected. • When it is reflected from a fixed end, the wave is inverted.	• If the end of the string is free to move vertically, the free end overshoots twice the amplitude as shown in Fig. 7.14. • When a travelling wave reaches a boundary, all or part of it is reflected. • When reflected from a free end, the pulse is not inverted.

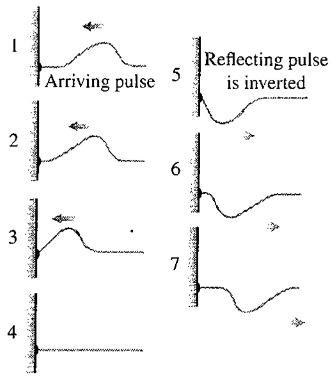

Fig. 7.13

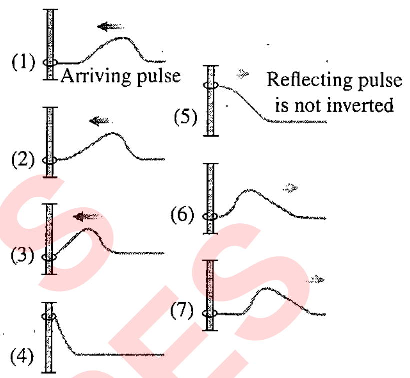

Fig. 7.14

The formation of the reflected pulse is similar to the overlap of two pulses travelling in opposite direction. Figure 7.15 shows two pulses with the same shape, one inverted with respect to the other, travelling in opposite directions. As the pulses overlap and pass each other, the total displacement of the string is the algebraic sum of the displacements at that point in the individual pulses. Because these two pulses have the same shape, the total displacement at point O in the middle of the figure is zero at all times. Thus the motion of the left half of the string would be the same if we cut the string at point O, threw away the right side, and held the end at O fixed. The two pulses on the left side and on the right side then correspond to the incident and reflected pulses, combining so that the total displacement at O is always zero. For this to occur, the reflected pulse must be inverted relative to the incident pulse.

Figure 7.16 shows two pulses with the same shape, travelling in opposite directions but not inverted relative to each other. The displacement at point O in the middle of the figure is not zero, but the slope of the string at this point is always zero. According to equation, this corresponds to the absence of any transverse force at this point. In this case the motion of the left half of the string would be the same as if we cut the string at point O and attach the end to a frictionless sliding ring that maintains tension without exerting any transverse force. In other words, this situation corresponds to reflection of a pulse at a free end of a string at point O. In this case the reflected pulse is not inverted.

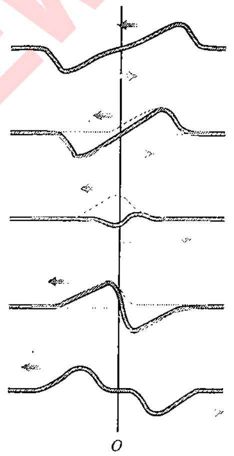

Fig. 7.15

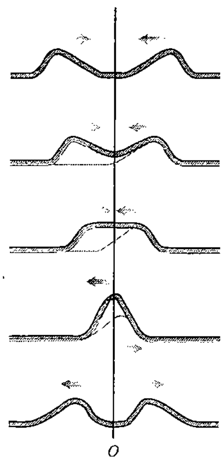

Fig. 7.16

Illustration 7.11 A string of length 20 cm and linear mass density 0.40 g/cm is fixed at both ends and is kept under a tension of 16 N. A wave pulse is produced at $t = 0$ near an end as shown in Fig. 7.17(b), which travels towards the other end. When will the string have the shape shown in the Fig. 7.17(c).

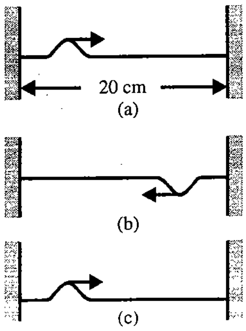

Fig. 7.17

Sol. Given $\mu = 0.40$ g/cm $= \frac{0.40}{1000} \times 100 = 0.040$ kg/m

Wave speed in the stretched string,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{16}{0.040}} = 20 \text{ m/s}$$

The string will have the same shape after the pulse travels a distance = 20 + 20 = 40 cm

Thus time period $t = 0.40/20 = 0.02$ s

Illustration 7.12 Consider a string fixed at one end. A travelling wave given by the wave equation $y = A \sin(\omega t - kx)$ is incident on it.

Show that at the fixed end of a string the wave suffers a phase change of π, i.e., as it travels back as if the wave is inverted.

Sol. Wave equation of incident wave is

$$y_1 = A \sin(\omega t - kx) \text{ (positive } x\text{–direction)}$$

When the wave strikes the fixed end it must be reflected. The wave will now be travelling in the negative x-direction, and so its equation is

$$y_2 = A \sin(\omega t + kx + \phi) \text{ (negative } x\text{-direction)}$$

The phase constant has been added to account for any phase change after reflection. Let us take $x = 0$ at the fixed end. The behaviour of a wave at particular positions is governed by appropriate boundary conditions. For fixed ends, the boundary condition is that the end point is a node.

The resultant motion due to incident and reflected wave is $y = y_1 + y_2$

$$y = A[\sin(\omega t - kx) + \sin(\omega t + kx + \phi)] \quad \text{(i)}$$

The boundary condition that must be satisfied by the resultant wave on string is $y = 0$ at $x = 0$

On substituting these values in Eq. (i), we obtain

$$\sin\omega t + \sin(\omega t + \phi) = 0$$

$$\sin\omega t = -\sin(\omega t + \phi) \quad \text{(ii)}$$

Equation (ii) must be satisfied at all times. For the sake of convenience, we take $\omega t = \pi/2$.

$$\sin\left(\frac{\pi}{2} + \phi\right) = -1$$

$\sin\theta = -1$ implies θ can have any of the following values.

$$3\pi/2, 7\pi/2, 11\pi/2, \ldots$$

Therefore ϕ can have any of the values π, 3π, 5π, etc. All these values are physically possible and distinguishable. We choose the simplest one, so $\phi = \pi$.

Illustration 7.13 **Consider the following wave functions:**

a. $y = A\sin(\omega t - kx)$, **b.** $y = A\sin(kx - \omega t)$

c. $y = A\cos(\omega t - kx)$, **d.** $y = A\cos(kx - \omega t)$

e. $y = A\sin(\omega t + kx)$, **f.** $y = A\cos(\omega t + kx)$

Write the equations of reflected wave after reflection from a free and a fixed boundary. Also find the resulting stationary waves formed by the superposition of its reflected wave.

Sol.

	Incident wave $\omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}$	Reflected wave from free boundary, $\phi = 0$	Reflected wave from fixed boundary, $\phi = \pi$
a.	$y = A\sin(\omega t - kx)$	$y = A\sin(\omega t + kx)$	$y = A\sin(\omega t + kx + \pi) = -A\sin(\omega t + kx)$
b.	$y = A\sin(kx - \omega t)$	$y = A\sin(-kx - \omega t) = -A\sin(kx + \omega t)$	$y = A\sin(-kx - \omega t + \pi) = A\sin(kx + \omega t)$
c.	$y = A\cos(\omega t - kx)$	$y = A\cos(\omega t + kx)$	$y = A\cos(\omega t + kx + \pi) = -A\cos(\omega t + kx)$
d.	$y = A\cos(kx - \omega t)$	$y = A\cos(-kx - \omega t) = A\cos(kx + \omega t)$	$y = A\cos(-kx - \omega t + \pi) = -A\cos(kx + \omega t)$
e.	$y = A\sin(\omega t + kx)$	$y = A\sin(\omega t - kx)$	$y = A\sin(\omega t - kx + \pi) = -A\sin(\omega t - kx)$
f.	$y = A\cos(\omega t + kx)$	$y = A\cos(\omega t - kx)$	$y = A\cos(\omega t - kx + \pi) = -A\cos(\omega t - kx)$

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Stationary wave by superposition of reflected wave

$$y_1 = A\sin(\omega t - kx) \quad \text{and} \quad y_2 = A\sin(\omega t + kx)$$

$$y = y_1 + y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx)] = 2A\cos kx \sin \omega t$$

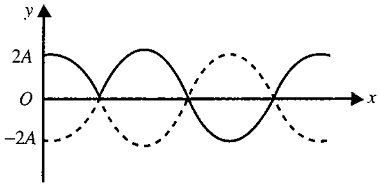

Fig. 7.18

The similar treatment can be done for other combination of wave to get stationary wave.

REFLECTION AND REFRACTION OF WAVE

Consider a plane progressive wave travelling in a certain medium (say 1) having the equation

$$y_1 = A_i \sin(\omega t - k_1 x) \qquad \text{(i)}$$

where A_i is the amplitude.

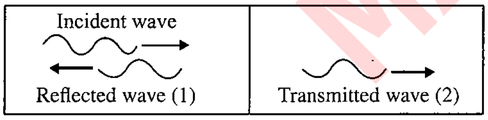

Fig. 7.19

If A_r and A_t be the amplitudes of the reflected and transmitted waves, then their respective equations can be written as

$$y_2 = A_r \sin(\omega t + k_1 x) \qquad \text{(ii)}$$

$$y_3 = A_t \sin(\omega t - k_2 x) \qquad \text{(iii)}$$

For simplicity let us assume that the wave gets reflected at $x = 0$.

Since the wave is continuous, the resultant displacement at the two sides of the interface should be equal, i.e.,

$$y_1 + y_2 = y_3 \quad \text{at} \quad x = 0$$

From Eqs. (i), (ii) and (iii), we get on substituting $x = 0$

$$A_i + A_r = A_t \qquad \text{(iv)}$$

From the condition of continuity of slope at the interface (i.e., $x = 0$), we have,

$$\frac{d(y_1)}{dx} + \frac{d(y_2)}{dx} = \frac{d(y_3)}{dx} \qquad \text{(v)}$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{d(y_1)}{dx} = -A_i k_1 \cos(\omega t - k_1 x)$$

$$\frac{d(y_2)}{dx} = A_r k_1 \cos(\omega t + k_1 x)$$

$$\frac{d(y_3)}{dx} = -A_t k_2 \cos(\omega t - k_2 x)$$

Substituting these in Eq. (v) and remembering that $x = 0$, we get, $-A_i k_1 + A_r k_1 = -A_t k_2$

$$A_i\left(\frac{2\pi}{\lambda_1}\right) - A_r\left(\frac{2\pi}{\lambda_1}\right) = A_t\left(\frac{2\pi}{\lambda_2}\right) \qquad \text{(vi)}$$

Since $v = f\lambda$ and f is constant, so,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad \text{(vii)}$$

From Eqs. (vi) and (vii),

$$A_i - A_r = \frac{v_1}{v_2} A_t \qquad \text{(viii)}$$

Solving Eqs. (iv) and (viii) for A_r and A_t, we have

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i \quad \text{and} \quad A_t = \left(\frac{2v_2}{v_2 + v_1}\right) A_i \qquad \text{(ix)}$$

Thus, we get the amplitudes of the reflected and transmitted waves, in terms of the amplitude of the incident wave.

When $v_1 > v_2$, i.e., medium 1 is rare and 2 is denser, A_r = negative and $|A_r|$ and $|A_t|$ both are individually less than $|A_i|$. Negative value of A_r indicates that the reflected wave suffers a phase change of π.

If opposite is the case, i.e., $v_1 < v_2$, both A_r and A_t are positive. Also $A_t > A_i$.

Fixed end of a string: Since the fixed end is equivalent to a string of infinite linear mass density, $v_2 = \sqrt{(T/\mu_2)} = 0$, and we obtain,

$$A_t = 0 \quad \text{and} \quad A_r = -A_i$$

Free end of a string: In this case $\mu_2 \to 0$, so $v_2 \to \infty$ and we can show that,

$$A_t = 2A_i \quad \text{and} \quad A_r = A_i$$

Illustration 7.14 **A progressive wave gets reflected at a boundary such that the ratio of amplitudes of the reflected and incident wave is 1:2. Find the percentage of energy transmitted.**

Sol. Given $\dfrac{A_r}{A_i} = \dfrac{1}{2}$ $\Rightarrow$ Ratio of intensities is $\dfrac{I_1}{I_2} = \left(\dfrac{1}{2}\right)^2$

Hence the fraction of energy transmitted

$$E_t = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$$

The percentage of energy transmitted (of course per unit time) is $3/4 \times 100 = 75\%$.

Illustration 7.15 **A progressive wave travels in a medium M_1 and enters into another medium M_2 in which its speed decreases to 75%. What is the ratio of the amplitude of the**

a. Reflected and the incident waves, and
b. Transmitted and the incident waves?

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Sol. Let A_i, A_r and A_t be the amplitudes of the incidents, reflected, and transmitted waves.

Given that, velocity in the medium refracted is 75% of that in the initial medium.

$$v_2 = \frac{3}{4}v_1$$

From Eqs. (viii) and (ix)

a. $$\frac{A_r}{A_i} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{\frac{v_2}{v_1} - 1}{\frac{v_2}{v_1} + 1} = \frac{\frac{3}{4} - 1}{\frac{3}{4} + 1} = -\frac{1}{7}$$

i.e., the required ratio is $\left|\frac{A_r}{A_i}\right| = 1:7$

b. $$\frac{A_t}{A_i} = \frac{2v_2}{v_2 + v_1} = \frac{2v_2/v_1}{\frac{v_2}{v_1} + 1} = \frac{2(3/4)}{\left(\frac{3}{4}\right) + 1} = \frac{6}{7}$$

i.e., the required ratio is $\left|\frac{A_t}{A_i}\right| = 6:7$

Illustration 7.16 **A long wire *PQR* is made by joining two wires *PQ* and *QR* of equal radii. *PQ* has length 4.8 m and mass 0.06 kg. *QR* has length 2.56 m and mass 0.2 kg. The wire *PQR* is under a tension of 80 N. A sinusoidal wave pulse of amplitude 3.5 cm is sent along the wire *PQ* from the end? No power is dissipated during the propagation of the wave-pulse. Calculate**

a. The time taken by the wave pulse to reach the other end *R* of the wire, and

b. The amplitude of the reflected and transmitted wave pulses after the incident wave pulse crosses the joint *Q*.

Sol. Given that $m_1 = 0.06$ kg, $m_2 = 0.2$ kg

Let m' be the mass per unit length then $m'_1 = 0.0125$ kg/m, $m'_2 = 0.078125$ kg/m

Wire *PQR* is under a tension of 80 N = T_0. A sinusoidal wave pulse is sent from *P*.

a. v_1 = velocity of wave on *PQ*

$$= \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{0.0125}} = 80 \text{ m/s}$$

v_2 = velocity of wave on *QR*

$$= \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{0.078125}} = 32 \text{ m/s}$$

Total time taken for wave pulse to reach from *P* to *R*

$$= \frac{PQ}{v_1} + \frac{QR}{v_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right) \text{s} = 0.06 + 0.08 = 0.14 \text{ s}$$

b. Amplitude of reflected wave: $A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_i$

$$= \left(\frac{32 - 80}{32 + 80}\right)3.5 = -1.5 \text{ cm}$$

A_r is –ve, so reflected wave is inverted

Amplitude of transmitted wave:

$$A_t = \left(\frac{2v_2}{v_2 + v_1}\right)A_i = \left(\frac{2 \times 32}{32 + 80}\right)3.5 = 2 \text{ cm}$$

Illustration 7.17 **A harmonic wave is travelling on string 1. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at $x = 0$. If the expression for the incident wave is $y_i = A_i \cos (k_1 x - \omega_1 t)$.**

a. What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1 and ω_1?

b. Show that the average power carried by the incident wave is equal to the sum of the average power carried by the transmitted and reflected waves.

Sol:

a. Since $v = \sqrt{T/\mu}$, $T_2 = T_1$ and $\mu_2 = 4\mu_1$

We have,

$$v_2 = \frac{v_1}{2} \quad \text{(i)}$$

Since the frequency does not change, that is,

$$\omega_1 = \omega_2 \quad \text{(ii)}$$

Also, because $k = (\omega/v)$, the numbers of the harmonic waves in the two strings are related by

$$k_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2\frac{\omega_1}{v_1} = 2k_1 \quad \text{(iii)}$$

The amplitudes are $A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$

$$= \left[\frac{2(v_1/2)}{v_1 + (v_1/2)}\right]A_i = \frac{2}{3}A_i \quad \text{(iv)}$$

and $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)A_i$

$$= \left[\frac{(v_1/2) - v_1}{v_1 + (v_1/2)}\right]A_i = \frac{-A_i}{3} \quad \text{(v)}$$

Now with Eqs. (ii), (iii) and (iv), the transmitted wave can be written as

$$y_t = (2/3)A_i \cos(2k_1 x - \omega_1 t)$$

Similarly the reflected wave can be expressed as

$$y_r = -\frac{A_i}{3}\cos(k_1 x + \omega_1 t)$$

$$= \frac{A_i}{3}\cos(k_1 x + \omega_1 t + \pi)$$

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b. The average power of a harmonic wave on a string is given by

$$P=\frac{1}{2}\rho A^2\omega^2 sv=\frac{1}{2}A^2\omega^2\mu v \qquad (\text{as } \rho s=\mu)$$

Now $P_i=\frac{1}{2}\omega_1^2A_i^2\mu_1 v_1$ (vi)

$$P_t=\frac{1}{2}\omega_1^2\left(\frac{2}{3}A_i\right)^2(4\mu_1)\left(\frac{v_1}{2}\right)=\frac{4}{9}\omega_1^2A_i^2\mu_1v_1 \qquad \text{(vii)}$$

$$P_r=\frac{1}{2}\omega_2^2\left(-\frac{A_i}{3}\right)^2(\mu_1)(v_1)=\frac{1}{18}\omega_1^2A_i^2\mu_1v_1 \qquad \text{(viii)}$$

From Eqs. (vi), (vii) and (viii), we can show that $P_i=P_t+P_r$.

Illustration 7.18 **In a stationary wave pattern that forms as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode and a node is b = 1.5. What percentage of the energy passes across the obstacle?**

Sol. As we have studied that when incident wave and reflected wave superimpose to produce stationary wave, the ratio of amplitudes at antinode and at node is given by

$$\frac{A_{\max}}{A_{\min}}=\frac{A_i+A_r}{A_i-A_r}$$

This ratio is given as 1.5 or 3/2.

$$\frac{A_i+A_r}{A_i-A_r}=\frac{3}{2} \quad \text{or} \quad \frac{1+\frac{A_r}{A_i}}{1-\frac{A_r}{A_i}}=\frac{3}{2}$$

Solving this equation, we get

$$\frac{A_r}{A_i}=\frac{1}{5}\Rightarrow\frac{I_r}{I_i}=\left(\frac{A_r}{A_i}\right)^2=\frac{1}{25} \quad \text{or,} \quad I_r=0.04\,I_i$$

this means 4% of the incident energy is reflected or 96% energy passes across the obstacle.

Concept Application Exercise 7.1

1. Two sound waves with amplitude 4 cm and 3 cm interfere with a phase difference of
   a. 0 **b.** $\pi/3$ **c.** $\pi/2$ **d.** π
   Find the resultant amplitude in each case.
2. Find the resultant amplitude and the phase difference between the resultant wave and the first wave, in the event the following waves interfere at a point; $y_1=(3\text{ cm})\sin\omega t$,
   $$y_2=(4\text{ cm})\sin\left(\omega t+\frac{\pi}{2}\right);\; y_3=(5\text{cm})\sin(\omega t+\pi)$$
3. When two waves interfere, does one alter the progress of the other?
4. When waves interfere, is there any loss of energy?
5. A travelling wave has speeds 50 m/s and 200 m/s in two different media A and B. Such a wave travelling through A, gets incident normally on a plane boundary, separating A and B. Find the ratio of amplitudes of the reflected and transmitted waves.
6. Figure 7.20 shows a tube structure in which signal is sent from one end and is received at the other end. The frequency of the sound source can be varied electronically between 2000 and 5000 Hz. Find the frequencies at which maxima of intensity are detected. The speed of sound in air 340 m/s.

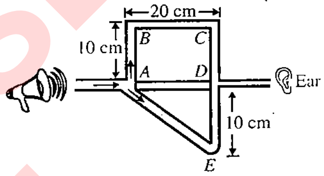

Fig. 7.20

7. Two small loudspeakers A, B (1 m apart) are connected to the same oscillator so that both emit sound waves of frequency 1700 Hz in phase. A sensitive detector, moving parallel to the line AB along PQ 2.40 m away, detects a maximum wave at P on the perpendicular bisector MP of AB and another maximum wave when it first reaches a point Q directly opposite to B. Calculate the speed c of the sound waves in air.

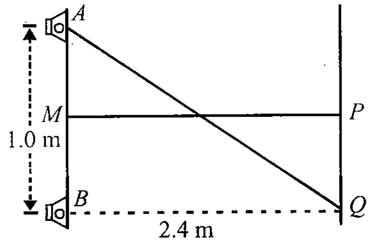

Fig. 7.21

8. A source and a detector D of high frequency waves are a distance d apart on the ground. Maximum signal is received at D when the reflecting layer is at a height H. When the layer rises a distance h, no signal is detected at D. Neglecting absorption in the atmosphere, find the relation between d, h, H, and the wavelength λ of the waves.

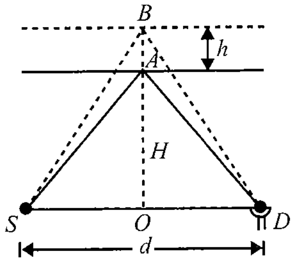

Fig. 7.22

9. Two waves have the same frequency. The first has intensity I_0. The second has intensity $4I_0$ and lags

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behind the first in phase by $\pi/2$. When they meet, find the resultant intensity, and the phase relationship of the resultant wave with the first wave.

10. Determine the amplitude of the resultant motion when two sinusoidal waves of same frequency, travelling in the same direction are combined. Their amplitudes are 3.0 cm and 4.0 cm and they differ in phase by $\pi/2$ radians.

11. In a large room, a person receives direct sound waves from a source 120 m away. He also receives waves from the same source which reach him after being reflected from the 5-m high ceiling at a point halfway between them. For which wavelengths will these two sound waves interfere constructively?

12. Sound waves from a tuning fork placed at a point P reach another point Q, by two separate paths PRQ and PSQ. When PSQ is greater than PRQ by 11.5 cm, there is silence at Q. When the difference is 23 cm, the sound becomes loudest at Q, and when 34.5 cm, there is silence, and so on. Explain this effect and calculate the frequency of the fork if the velocity of sound is taken to be 331.2 m/s.

13. Show that when reflection takes place from a boundary separating two media and the velocity in the second medium is infinitely large, the amplitude of the reflected wave is equal to the amplitude of the incident wave and there is a phase change of π in the displacement wave.

14. Stationary waves are produced in a length of wire fixed between two points. Compare the amplitudes at an antinode for the fundamental and the first overtone. Assume that the total energy of the initial waves is, on an average, equally divided between the two modes.

15. Two speakers connected to the same source of fixed frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their midpoint along the perpendicular bisector shows maximum response. The box is slowly rotated till the speakers are in a line with the microphone. The distance between the midpoint of the speakers and the microphone remains unchanged. Exactly 5 maximum responses are observed in the microphone in doing this. Calculate the wavelength of sound wave.

16. A wave pulse on a string has the dimensions shown in Fig. 7.23, at t = 0. The wave speed is 40 cm/s. (a) If point O is fixed end, draw the total wave on the string at t = 15 ms, 20 ms, 25 ms, 35 ms, 40 ms and 45 ms. (b) Repeat part (a) for the case in which point O is a free end.

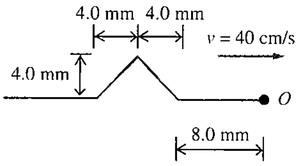

Fig. 7.23

17. A wave pulse on a string has the dimensions shown in Fig. 7.24 at t = 0. The wave speed is 5.0 m/s. (a) If point O is fixed end, draw the total wave on the string at t = 1.0 ms, 2.0 ms, 3.0 ms, 4.0 ms, 5.0 ms, 6.0 ms and 70 ms. (b) Repeat part (a) for the case in which point O is a free end.

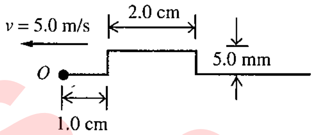

Fig. 7.24

18. Two triangular wave pulses are travelling toward each other on a stretched string as shown in Fig. 7.25. Both pulses are identical to each other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at t = 0. Sketch the shape of the string at t = 0.250 s, t = 0.500 s, t = 0.750 s, t = 1.000 s and t = 1.250 s.

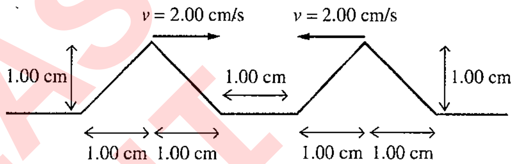

Fig. 7.25

19. A sound wave may be considered either as a displacement wave or as a pressure wave. When reflection takes place from as a rigid wall, what phase change do you expect in its displacement representation and in its pressure representation?

STANDING WAVES

In this situation, two identical waves travel in opposite directions in the same medium as shown in Fig. 7.26. These waves combine in accordance with the wave in interference model.

Fig. 7.26

We can analyse such a situation by considering wave functions for two transverse sinusoidal waves having the same amplitude, frequency, and wavelength but travelling in opposite directions in the same medium. Consider two waves

$$y_1 = A\sin(kx - \omega t) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

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where y_1 represents a wave travelling in the $+x$-direction and y_2 represents one travelling in the $-x$-direction. Adding these two functions gives the resultant wave function y.

$$y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

When we use the trigonometric identity $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$, this expression reduces to

$$y = (2A \sin kx)\cos \omega t \qquad \text{(i)}$$

Notice that Eq. (i) does not contain a function $kx - \omega t$. Therefore, it is not an expression for a single travelling wave. When you observe a standing wave, there is no sense of motion in the direction of propagation of either original wave. comparing Eq. (i) with equation $x(t) = A\cos(\omega t + \phi)$, we see that it describes a special kind of simple harmonic motion. Every element of the medium oscillates in simple harmonic motion with the same angular frequency ω (according to the $\cos \omega t$ factor in the equation). The amplitude of the simple harmonic motion of a given element (given by the factor $2A \sin kx$, the coefficient of the cosine function) depends on the location x of the element in the medium. However, the amplitude of the simple harmonic motion of an element of the medium has a minimum value of zero when x satisfies the condition $\sin kx = 0$, that is, when

$$kx = 0, \pi, 2\pi, 3\pi$$

because $k = 2\pi/\lambda$, these values for kx give

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$

$$= \frac{n\lambda}{2}$$

$$n = 0, 1, 2, 3, \ldots \qquad \text{(ii)}$$

These points of zero amplitude are called nodes.

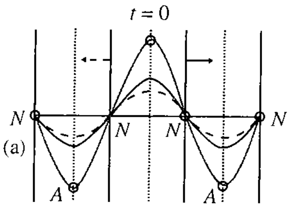
The element of the medium with the greatest possible displacement from equilibrium has an amplitude of $2A$. The positions in the medium at which this maximum displacement occurs are called antinodes. The antinodes are located at positions for which the coordinate x satisfies the condition $\sin kx = \pm 1$, that is, when

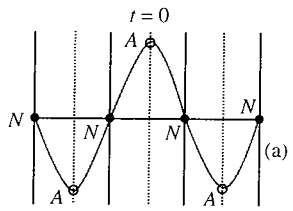
$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Therefore, the positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4}$$

$$n = 1, 3, 5, \ldots \qquad \text{(iii)}$$





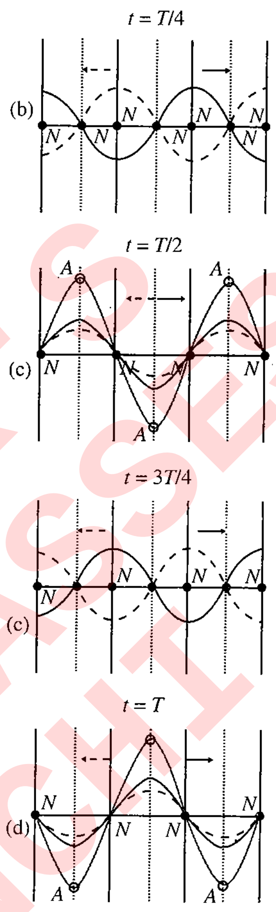


Fig. 7.27

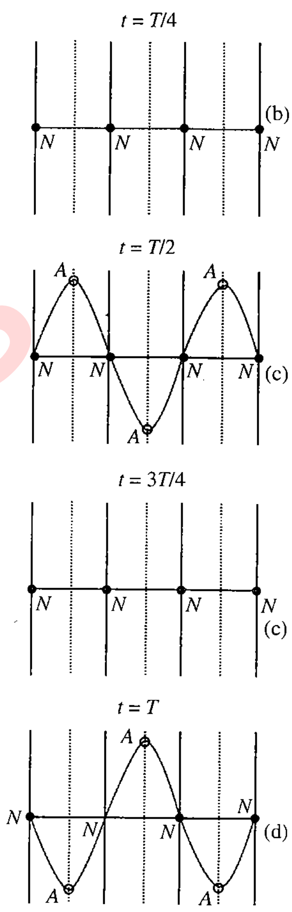


Fig. 7.28

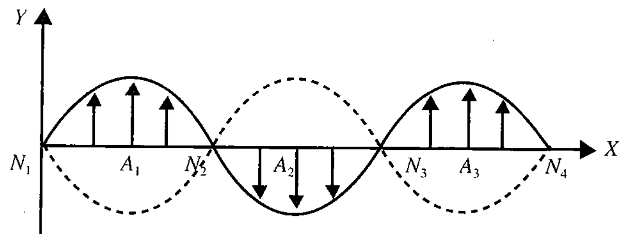
CHARACTERISTICS OF STATIONARY WAVES

i. In a stationary wave, the disturbance does not move in any direction. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite conditions after every $T/2$.

ii. All the particles of the medium, except those at nodes, execute simple harmonic motion with the particle of the wave about their mean position.

iii. During the formation of a stationary wave, the medium is broken into loops between equally spaced points called nodes which remain at rest and in between them are points of maximum displacement called antinodes.

iv. The amplitude of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.



Opposite phases of the particles in consecutive segments

Fig. 7.29

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v. The maximum velocity is different at different points. Its value is zero at the nodes and gradually increases towards the antinodes. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.

vi. All the particles in a particular segment between two nodes vibrate in the same phase but the particles in the neighbouring segments vibrate in opposite phases, as shown in Fig. 7.29.

vii. The energy becomes alternately wholly potential and wholly kinetic twice in each cycle. It is wholly potential when particles are at their positions of maximum displacement and wholly kinetic when the particle pass through the mean positions.

viii. A stationary wave has the same wavelength and time period as the component waves.

ix. The distance between two consecutive nodes and antinodes is $\lambda/2$. The distance between nodes and next antinode is $\lambda/4$.

Illustration 7.19 **Can two waves of the same frequency and amplitude travelling in the same direction give rise to a stationary wave after superposition?**

Sol. Let the two superposing waves be

$$y_1 = A\sin(\omega t - kx)$$

$$y_2 = A\sin(\omega t - kx + \phi)$$

where ϕ is the phase difference between the two waves.

From principle of superposition the resultant wave function is

$$y = y_1 + y_2 = A\sin(\omega t - kx) + A\sin(\omega t - kx + \phi)$$

$$= 2A\cos\frac{\phi}{2}\sin\left(\omega t - kx + \frac{\phi}{2}\right)$$

This is the expression for a travelling wave with amplitude $2A\cos\phi/2$. Hence superposition of two waves travelling in the same direction give rise to another travelling wave with different amplitude, which depends on the phase difference between the two waves.

Illustration 7.20 **Two travelling waves of equal amplitudes and equal frequencies move in opposite direction along a string. They interfere to produce a standing wave having the equation.**

$$y = A\cos kx \sin\omega t$$

in which A = 1.0 mm, k = 1.57 cm^{-1} and ω = 78.5 s^{-1}. (a) Find the velocity and amplitude of the component travelling waves. (b) Find the node closest to the origin in the region $x > 0$. (c) Find the antinode closest to the origin in the region $x > 0$. (d) Find the amplitude of the particle at x = 2.33 cm.

Sol.

a. The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2}\sin(\omega t - kx) \quad \text{and} \quad y_2 = \frac{A}{2}\sin(\omega t + kx)$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5\ \text{s}^{-1}}{1.57\ \text{cm}^{-1}} = 50\ \text{cm/s}; \quad \text{Amplitude} = 0.5\ \text{mm}$$

b. For a node, $\cos kx = 0$.
The smallest positive x satisfying this relation is given by

$$kx = \pi/2$$

or, $$x = \frac{\pi}{2k} = \frac{3.14}{2 \times 1.57\ \text{cm}^{-1}} = 1\ \text{cm}$$

c. For an antinode, $|\cos kx| = 1$.
The smallest positive x satisfying this relation is given by

$$kx = \pi$$

or, $$x = \frac{\pi}{k} = 2\ \text{cm}$$

d. The amplitude of vibration of the particle at x is given by $|A\cos kx|$. For the given point,

$$kx = (1.57\ \text{cm}^{-1})(2.33\ \text{cm}) = 1.57\left[2 + \frac{1}{3}\right]$$

$$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Thus, the amplitude will be

$$(1.0\ \text{mm})\,|\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{2}\ \text{mm} = 0.86\ \text{mm}$$

Illustration 7.21 **Two identical loudspeakers are driven in phase by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along the line joining the two speakers where relative minima of sound pressure amplitude would be expected. (Use v = 343 m/s.)**

Sol. Let two loudspeakers (1) and (2) be separated by a distance 1.25 m, and observer moves between the two speakers along the line joining two as shown in Fig. 7.30.

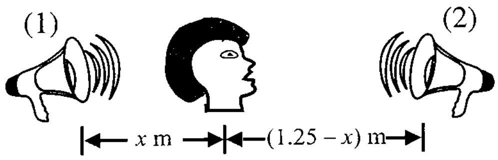

Fig. 7.30

Two identical waves moving in opposite directions constitute a standing wave. We must find the nodes.

The wavelength is

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{800\ \text{Hz}} = 0.429\ \text{m}$$

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The two waves moving in opposite directions along the line between the two speakers will add to produce a standing wave with this distance between nodes.

Distance node to node = $\lambda/2 = 0.214$ m

Because the speakers vibrate in phase, air compressions from each will simultaneously reach the point halfway between the speakers, to produce antinode of pressure here. A node of pressure will be located at this distance on either side of the midpoint.

Distance node to antinode = $\lambda/4 = 0.107$ m

Therefore nodes of sound pressure will appear at these distances from either speaker:

$$[1/2(1.25 \text{ m}) + 0.107 \text{ m} = 0.732 \text{ m}$$

$$\text{and } 1/2(1.25 \text{ m}) - 0.107 \text{ m} = 0.518 \text{ m}]$$

The standing wave contains a chain of equally spaced nodes at distances from either of the speakers

$$0.732 \text{ m} + 0.214 \text{ m} = 0.946 \text{ m}$$

$$0.947 \text{ m} + 0.214 \text{ m} = 1.16 \text{ m}$$

also at $\quad 0.518 \text{ m} - 0.214 \text{ m} = 0.303 \text{ m}$

$$0.303 \text{ m} - 0.214 \text{ m} = 0.089 \text{ m}$$

The standing wave exists only along the line segment between the speakers. No nodes or antinodes appear at distances greater than 1.25 m or less than 0, because waves add to give standing wave only if they are travelling in opposite directions and not in the same direction. Thus, the distances from either of the speakers to the nodes of pressure between the speakers are 0.089 m, 0.303 m, 0.518 m, 0.732 m, 0.946 m and 1.16 m

Illustration 7.22 **Two sinusoidal waves combining in a medium are described by the wave functions**

$$y_1 = (3.0 \text{ cm}) \sin\pi(x + 0.60t)$$

$$y_2 = (3.0 \text{ cm}) \sin\pi(x - 0.60t)$$

where x is in centimetres and t is in seconds. Determine the maximum transverse position of an element of the medium at (a) $x = 0.250$ cm, (b) $x = 0.500$ cm and (c) $x = 1.50$ cm. (d) Find the three smallest values of x corresponding to antinodes.

Sol. The answers for maximum transverse position must be between 6 cm and 0. The antinodes are separated by half a wavelength, which we expect to be a couple of centimetres.

According to the waves in interference model, we write the function $y_1 + y_2$ and start evaluating things.

We get $\quad y = y_1 + y_2 = (6.0 \text{ cm}) \sin(\pi x) \cos(0.60\pi t)$

Since $\cos(0) = 1$, we can find the maximum value of y by setting $t = 0$.

$$y_{max}(x) = y_1 + y_2 = (6.0 \text{ cm}) \sin(\pi x)$$

a. At $x = 0.250$ cm, $y_{max} = (6.0 \text{ cm}) \sin(0.250\pi) = 4.24$ cm

b. At $x = 0.50$cm, $y_{max} = (6.0 \text{ cm}) \sin(0.500\pi) = 6.00$cm

c. At $x = 1.50$ cm, $y_{max} = |(6.0 \text{ cm}) \sin(1.50\pi)| = +6.00$cm

d. The antinodes occur when $x = n\lambda/4$ for $n = 1, 3, 5, \ldots$

But $\quad k = 2\pi/\lambda = \pi$, so $\lambda = 2.00$ cm

and $\quad x_1 = \lambda/4 = (2.00 \text{ cm})/4 = 0.500$ cm

$$x_2 = 3\lambda/4 = 3(2.00 \text{ cm})/4 = 1.50 \text{ cm}$$

$$x_3 = 5\lambda/4 = 5(2.00 \text{ cm})/4 = 2.50 \text{ cm}$$

Two of our answers in (d) can be read from the way the amplitude had its largest possible value in parts (b) and (c). Note again that an amplitude is defined to be always positive, as the maximum absolute value of wave function.

STANDING WAVES IN A STRING FIXED AT BOTH ENDS

Consider a string of length L fixed at both ends as shown in Fig. 7.31. We will use this system as a model for a guitar string or piano string. Standing wave can be set up in the string by a continuous superposition of wave incident on and reflected from the ends. Notice that there is a boundary condition for the waves on the string. Because the ends of the string are fixed, they must necessarily have zero displacement and are therefore nodes by definition. This boundary condition results in the string having a number of discrete natural patterns of oscillation, called normal modes, each of which has a characteristic frequency that can easily be calculated.

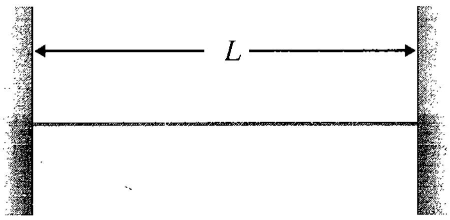

Fig. 7.31

The normal modes of oscillation for the string in Fig. 7.31 can be described by imposing the boundary conditions that the ends be nodes and that the nodes and antinodes be separated by one-fourth of a wavelength. The first normal mode that is consistent with these requirements, shown in Fig. 7.32a, has nodes at its ends and one antinode in the middle. This normal mode is the longest wavelength mode that is consistent with our boundary conditions. The first normal mode occurs when the wavelength $\lambda_1 = 2L$. The section of a standing wave from one node to the next node is called a loop. In the first normal mode, the string is vibrating in one loop. In the second normal mode (Fig. 7.32b) the string vibrates in two loops. In this case, the wavelength λ_2 is equal to the length of the string, as expressed by $\lambda_2 = L$. The third normal mode (Fig. 7.32c) corresponds to the case in which $\lambda_3 = 2L/3$ and our string vibrates in three loops. In general, the wavelengths of the various normal modes for a string of length L fixed at both ends are

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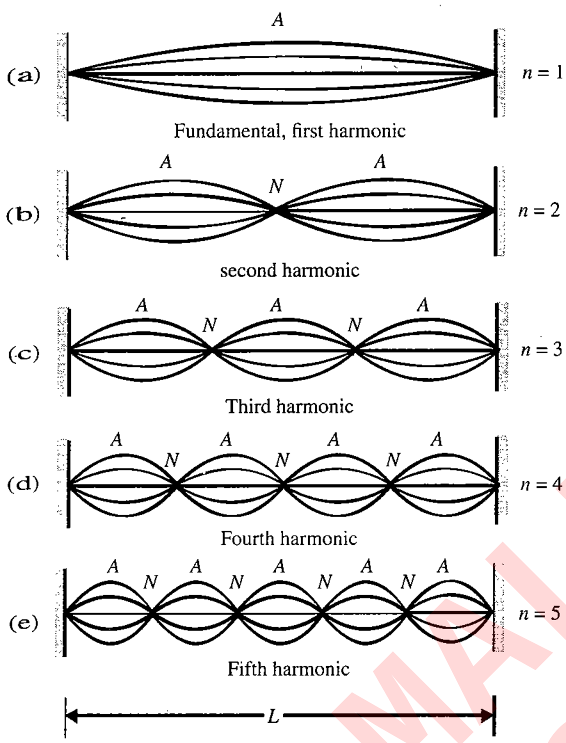

Fig. 7.32

$$\lambda_n = \frac{2L}{n}; \quad n = 1, 2, 3, \ldots \qquad \text{(iv)}$$

where the index n refers to the nth normal mode of oscillation. These modes are the possible modes of oscillation for the string. The actual modes that are excited on a string are discussed shortly.

The natural frequencies associated with the modes of oscillation are obtained from the relationship $f = v/\lambda$, where the wave speed v is same for all frequencies. Using Eq. (iv), we find that the natural frequencies f_n of the normal modes are

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}; \quad n = 1, 2, 3 \qquad \text{(v)}$$

These natural frequencies are also called the quantized frequencies associated with the vibrating string fixed at both ends.

Because $v = \sqrt{T/\mu}$ for waves on a string, where T is the tension in the string and μ is its linear mass density, we can also express the natural frequencies of a taut string as

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}; \quad n = 1, 2, 3, \ldots \qquad \text{(vi)}$$

The lowest frequency f_1, which corresponds to $n = 1$, is called either the fundamental frequency or the first harmonic and is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}; \quad n = 1, 2, 3, \ldots \qquad \text{(vii)}$$

The frequencies of the remaining normal modes are integer multiples of the fundamental frequencies of normal modes that exhibit an integer multiple relationship such as this form a harmonic series, and the normal modes are called harmonics. The fundamental frequency f_1 is the frequency of the first harmonic, the frequency $f_2 = 2f_1$ is the frequency of the second harmonic, and the frequency $f_n = nf_1$ is the frequency of the nth harmonic.

Illustration 7.23 **A string on a cello vibrates in its first normal mode with a frequency of 220 Hz. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of vibration when the string vibrates in three segments.**

Sol: The tension should be less than 500 N (~100 lb) since excessive force on the four cello strings would break the neck of the instrument. If a string vibrates in three segments, there will be three antinodes (instead of one for the fundamental mode), so the frequency should be three times greater than the fundamental.

From the string's length, we can find the wavelength. We then can use the wavelength with the fundamental frequency to find the wave speed. Finally, we can find the tension from the wave speed and the linear mass density of the string.

When the string vibrates in the lowest frequency mode, the length of sting forms a standing wave where $L = \lambda/2$ so the fundamental harmonic wavelength is $\lambda = 2L = 2(0.700\text{ m}) = 1.40$ m and the speed is

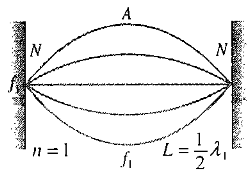

Fig. 7.33

$$v = f\lambda = (220\text{ s}^{-1})(1.40\text{ m}) = 308\text{ m/s}$$

a. From the tension equation

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}$$

We get $T = v^2 m/L$

or $$T = \frac{(308\text{ m/s})^2(1.20\times10^{-3}\text{ kg})}{0.700\text{ m}} = 163\text{ N}$$

b. For the third harmonic, the tension, linear density and speed are the same, but the string vibrates in three segments. Thus, the wavelength is one-third of the fundamental wavelength.

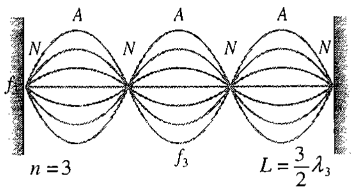

Fig. 7.34

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$$\lambda_3 = \lambda_1/3$$

From the equation $v = f\lambda$, we find that the frequency is three times as high.

$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{\lambda_1} = 3f_1 = 660 \text{ Hz}$$

The tension seems reasonable, and the third harmonic is three times the fundamental frequency as expected. Related to part (b), some stringed instrument players use a technique to double the frequency of a note by playing a natural harmonic, in effect cutting a vibrating string in half. When the string is lightly touched at its midpoint to form a node there, the second harmonic is formed, and the resulting note is one octave higher (twice the original fundamental frequency).

STRING FIXED AT ONE END AND FREE FROM OTHER END

Figure 7.35 shows a string that has one end fixed and the other end free. In the fundamental mode the free end is an antinode; the length of string $L = \lambda/4$. In the next mode

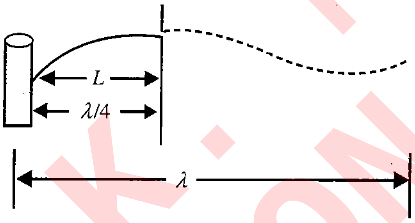
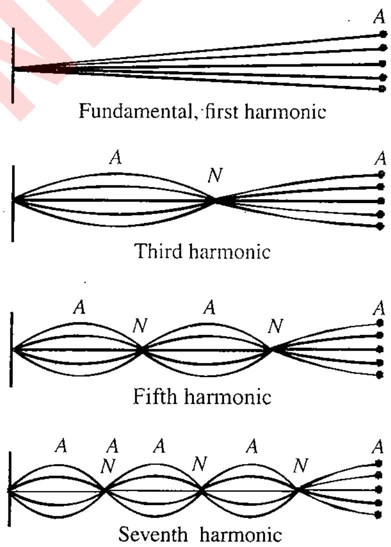


Fig. 7.35

$$L = \frac{3\lambda_3}{4}$$

In general,

$$L = n\frac{\lambda n}{4}, \quad n = 1, 3, 5, \ldots$$



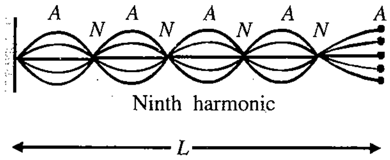


Fig. 7.36

The resonance frequencies are given by

$$f_n = n\frac{v}{4L} = nf_1; \quad n = 1, 3, 5, \ldots$$

where $\qquad f_1 = v/4L \quad$ (fundamental frequency)

The natural frequencies occur in the ratios: 1:3:5:7: ...

Illustration [illegible] **A string 120 cm in length sustains a standing wave with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0 cm. Find the maximum displacement amplitude. To which overtone do these oscillations correspond?**

Sol. When a string fixed at both the ends sustains a standing wave, there are n (a whole number) segments of the length of the string, where particles in each segment of the string oscillate in phase with each other and are out of phase with the particles of the adjacent segment, the string contains n loops between its ends. Under this condition the amplitude of the oscillation varies from point to point on the string as a harmonic function (a sine function or a cosine function). A typical equation for the standing wave can be given as $y = y_{max} \sin kx \cos \omega t$.

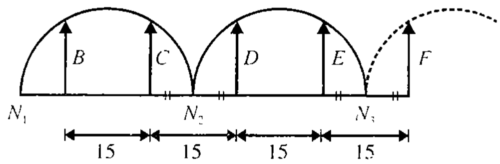


Fig. 7.37

Let the string be lying along the x-axis between $x = 0$ and $x = 120$ cm. Let the amplitude variation along the length of the string be given by $a = a_{max} \sin kx$, where k is the propagation constant.

First Method: $\left(k = \frac{\omega}{v} = \frac{2\pi}{\lambda}\right)$

Let the string be oscillating in its nth harmonic. The string will contain n loops with end points as node. The points in a loop, where the displacement amplitudes are 3.5 mm will be located symmetrically about antinode, as shown in the Fig. 7.37.

Now $\quad BC = DE = \cdots = 15$ cm $\quad$ and $\quad CD = EF = \cdots = 15$ cm

(A sine function given by $y = \sin x (0 \le x \le \pi)$ is symmetrical about $x = \pi/2$, etc.)

$$CD = CN_2 + N_2D$$

$$= \left(\frac{120}{2n} - 7.5\right) + \left(\frac{120}{2n} - 7.5\right) = 2\left(\frac{120}{2n} - 7.5\right)$$

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Thus, $2\left(\frac{120}{2n} - 7.5\right) = 15$ or $n = 4$

The oscillations correspond to the 4th harmonic (the 3rd overtone). The string will oscillate in 4 loops. There are a total of 5 nodes (including the nodes on the end of the string) on the string. Recalling that from a node of the next node the distance is $\lambda/2$, the length of the string $l = 4\lambda/2$.

or $\lambda/2 = l/4 = 30$ cm or $\lambda = 60$ cm

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{30}$$

Amplitude as function of x is

$$a = a_{max} \sin kx = a_{max} \sin\left(\frac{\pi x}{30}\right)$$

Now at $x = 7.5$, 12.5 cm, $a = 3.5$ mm

or $$3.5 = a_{max} \sin\left(7.5\frac{\pi}{30}\right)$$

or $$3.5 = a_{max} \sin\frac{\pi}{4} = \frac{a_{max}}{\sqrt{2}}$$

$$a_{max} = 3.5\sqrt{2} = 4.935 = 5 \text{ mm}$$

Second Method: Suppose that the points shown in Fig. 7.38 represent the points which have equal amplitudes and are equally spaced. Now, distance between two consecutive nodes is $\lambda/2$. Therefore,

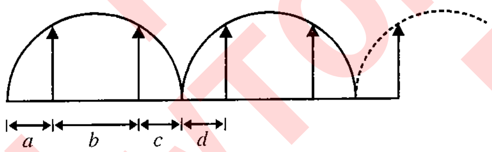

Fig. 7.38

$$a + b + c = \lambda/2 \quad \text{(i)}$$

From symmetry considerations

$$a = c = d \quad \text{(ii)}$$

Now, for such points to be equidistant throughout the string,

$$b = c + d \quad \text{(iii)}$$

From Eqs. (ii) and (iii),

$$b = 2a$$

From Eq. (i), we get

$$a + 2a + a = \lambda/2$$

or $$a = \lambda/8 \quad \text{(iv)}$$

$b = 2a = \lambda/4$. The points where displacement amplitudes are 3.5 mm are $\lambda/4$ apart.

$\therefore$ $\lambda/4 = 15$ or $\lambda = 60$ cm

Hence, $a = \lambda/8 = 7.5$ cm

[Note that only one value for b, i.e., distance between two such consecutive points exist, hence only one such set of points exist on the string and such points are $\lambda/4$ apart]

Now length of string = 120 cm

$$\text{No. of loops} = \frac{2L}{\lambda} = \frac{2 \times 120}{60} = 4$$

The string is in the 4th harmonic or 3rd overtone.

Mathematical analysis

Let the amplitude variation along the string be given by

$$A = A_{max} \sin\left(\frac{2\pi}{\lambda} \times x\right)$$

For amplitude to be same at two points x and x',

$$\sin\left(\frac{2\pi}{\lambda} \times x\right) = \sin\left(\frac{2\pi}{\lambda} \times x'\right)$$

or $$\frac{2\pi}{\lambda} \times x = n\pi + (-1)^n \times \frac{2\pi}{\lambda} x'$$

or $$\frac{2\pi}{60} \times x = n\pi + (-1)^n \times \frac{2\pi}{60} x'$$ where $\lambda = 60$ cm (calculated earlier)

or $$\frac{\pi x}{30} = n\pi + (-1)^n \times \frac{\pi}{30} x'$$

Now to find the first and second such points do following analysis:

for $n = 0$, you get $x = x'$, as we are not considering $x < 0$, leave this.

Put $n = 1$,

$$\frac{\pi x}{30} = \pi - \frac{\pi x'}{30} \quad \text{or} \quad x' + x = 30$$

Also, $x' - x = 15$ $\therefore x' = 22.5x$ and $x = 7.5$

[Here by putting (i) $n = 1$, (ii) $x' - x = 15$, you ensure that you are analysing the equation for the very first point and the point next to it satisfying the required conditions.]

This proves that the solution is correct.

This also proves that the symmetry elements assumed by you in the solution to the problem stand true.

Exercise : Taking $\lambda = 60$ cm and first such point being at $x = 7.5$ cm prove that (i) all such points are equidistant and (ii) such points are 15 cm apart from each other (consecutive points).

SONOMETER

It is a device used to measure velocity of transverse mechanical wave in a stretched metal wire. The principle of sonometer is based on resonance of string vibrations. Working oscillations are induced in a clamped string by an external source like a tuning fork or an oscillator and the corresponding oscillations

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in string will become stronger when resonance takes place, i.e., the frequency of oscillation of source matches with any of the harmonic of the string vibration.

Figure 7.39 shows basic structure and setup of a sonometer. It consists of a wooden box M on which a wire AB is stretched, by a hanging weight as shown in Fig. 7.39. On sonometer box there are two clamps C_1 and C_2 placed which can slide under the wire to change the length of wire between the clamps.

When an oscillating turning fork is placed in contact with the sonometer wire as shown in Fig. 7.39b some oscillations are transferred to the wire. If tension in wire is T and n_0 is the frequency of turning fork, the wavelength of wave in wire is

$$\lambda = \frac{v}{n_0} = \frac{1}{n_0}\sqrt{\frac{T}{\mu}} \qquad \text{(i)}$$

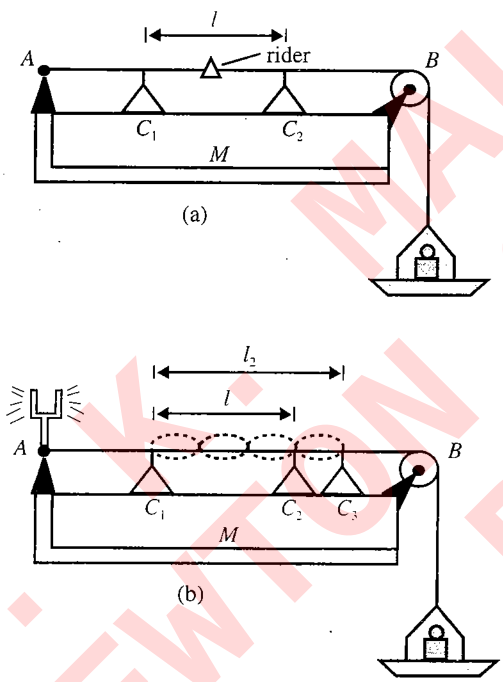

Fig. 7.39

When the length between clamps is an integral multiple of $\lambda/2$ then stationary waves are established in the portion of wire between C_1 and C_2. To adjust this, clamp C_1 is fixed and C_2 is displaced so that a small rider (a piece of paper) on wire start jumping violently on wire and falls indicating that the oscillation amplitude of wire is increasing and stationary waves are established. Let in this situation the length between clamps be l_1. Now again C_2 is displaced away from C_1 so that again resonance is obtained. This will happen again when the clamp reaches the position C_3 and when next node of stationary waves is present as shown in Fig. 7.39(b). Let this length be l_2.

So we can say that if l_1 and l_2 are the two successive resonance lengths then we have

$$l_2 - l_1 = \frac{\lambda}{2}$$

So wavelength of wave is $\lambda = 2(l_2 - l_1)$.

As frequency n_0 of oscillating source is known, we can find the velocity of wave in wire as

$$v = n_0\lambda$$

$$= 2n_0(l_2 - l_1) \qquad \text{(ii)}$$

Equation (ii) gives the partially measured value of velocity of transverse waves in a stretched wire. This can be compared with the theoretical value of v given by $\sqrt{T/\mu}$.

Illustration 7.25 **The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44%, keeping the length constant. Find the change in the fundamental frequency of the sonometer wire when the length of the wire is increased by 20%, keeping the original tension in the wire constant.**

Sol. Fundamental frequency of vibrations of string

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

At constant length, $n \propto \sqrt{T}$ or $\frac{n}{\sqrt{T}} = \text{constant}$ (i)

i. New tension, $T' = T + \frac{44}{100}T = 1.44T$

If n' is new frequency, then $\frac{n'}{\sqrt{T'}} = \frac{n}{\sqrt{T}}$

$$n' = \left(\sqrt{\left(\frac{T'}{T}\right)}\right)n \qquad \text{(ii)}$$

$\therefore$ $n' = n + 6$

From Eq. (iii)

$$n + 6 = \left(\sqrt{\left(\frac{1.44T}{T}\right)}\right)n$$

or $n + 6 = 1.2n$

$$0.2n = 6 \quad \text{or} \quad n = \frac{6}{0.2} = 30 \text{ Hz}$$

ii. As tension is constant

$$n \propto \frac{1}{l} \quad \text{or} \quad nl = \text{constant} \qquad \text{(iii)}$$

When length increase by 20%

New length $l' = l + \frac{20}{100}l = 1.2l$

As $nl = \text{constant}$

Therefore, $nl = n'l'$

$$n' = \frac{l}{l'}n = \frac{l}{1.2l} \times 30 = 25 \text{ Hz}$$

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Change in fundamental frequency

$$\Delta n = n' - n = 25 - 30 = -5 \text{ Hz}$$

Therefore, $\Delta n = 5$ Hz (decrease)

Illustration 7.26 **The length of a sonometer wire between two fixed ends is 1.10 m. Where should the two bridges be placed to divide the wire into three segments whose fundamental frequencies are in the ratio of 1:2:3?**

Sol. Let l_1, l_2, l_3 be the lengths of three segments.

Given $\quad l_1 + l_2 + l_3 = 1.10 \quad$ (i)

From relation $\quad n = \dfrac{1}{2l}\sqrt{\left(\dfrac{T}{\mu}\right)}$

If tension T and mass per unit length μ are fixed, then $n \propto 1/l$

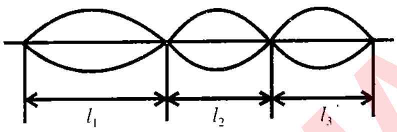

Fig. 7.40

so nl = constant

$$n_1 l_1 = n_2 l_2 = n_3 l_3$$

$$\therefore \quad l_1 : l_2 : l_3 \equiv \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3} \equiv \frac{1}{1} : \frac{1}{2} : \frac{1}{3} \equiv \frac{6:3:2}{6} \equiv 6:3:2$$

$\therefore l_1 = 6k,\ l_2 = 3k,\ l_3 = 2k,\ k$ being a constant.

From Eq. (i)

$$6k + 3k + 2k = 1.10 \quad \text{or} \quad 11k = 1.10$$

$$k = 1.10/11 = 0.1$$

$$\therefore \quad l_1 = 6 \times 0.1 = 0.6 \text{ m}$$

$$l_2 = 3 \times 0.1 = 0.3 \text{ m}$$

$$l_3 = 2 \times 0.1 = 0.2 \text{ m}$$

Therefore the bridges must be placed at distance 0.6 m and $(0.6 + 0.3) = 0.9$ m

Illustration 7.27 **Two tuning forks A and B produce 4 beats per second when sounded simultaneously. The frequency of A is known to be 256 Hz. When B is loaded with a little wax 4 beats per second are again produced. Find the frequency of B before and after loading.**

Sol. Since the beat frequency is 4 per second, the possible frequencies of fork B before loading are

$$(256 + 4) \text{ Hz} \quad \text{and} \quad (256 - 4) \text{ Hz}$$

or $$260 \text{ Hz} \quad \text{and} \quad 252 \text{ Hz}$$

After loading B with wax, again 4 beats/second are heard. Therefore, after loading, the possible frequencies are

$$(256 + 4) \text{ Hz} \quad \text{and} \quad (256 - 4) \text{ Hz}$$

or $$260 \text{ Hz} \quad \text{and} \quad 252 \text{ Hz}$$

But, by loading the fork its frequency can only decrease. Hence, before loading fork B, its frequency can only be 260 Hz, so that after loading it decreases to 252 Hz.

Vibrations of Composite Strings

We have discussed the vibration of a clamped string in previous section, now we discuss the stationary waves in a string composed of two different strings as shown in Fig. 7.41. Here two strings S_1 and S_2 of different material and lengths are joined end to end and tied between two clamps as shown. Now when we induce oscillations in this composite string, stationary waves are established only at those frequencies which match with any one harmonic of both the independent string S_1 and S_2.

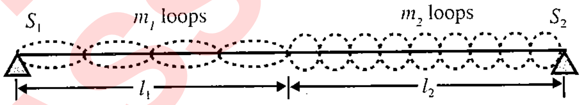

Fig. 7.41

Oscillation of two strings

Let two strings of lengths l_1 and l_2 having mass per unit length μ_1 and μ_2, respectively.

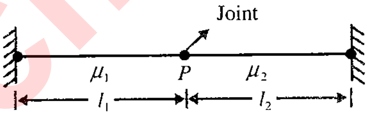

Fig. 7.42

- The combined string is fixed at the ends.
- Hence the end conditions will be 'node' and 'node' (N. N).

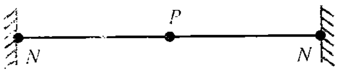

Fig. 7.43

Here two cases are possible,

- If joint P is acting as a node (N).
- If joint P is acting as an antinode (A).

Case I

Joint P acting as a node (N):

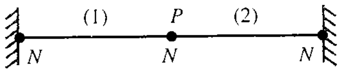

Fig. 7.44

- The end condition of string (1); (N, N).
- Hence the number of loops in string (1) should be an integer, say n_1 loops.

One loop length in terms of wavelength is $\lambda/2$.

Hence, for string (1) $\quad n_1 \dfrac{\lambda_1}{2} = l_1 \quad$ (i)

Similarly for string (2) $\quad n_2 \dfrac{\lambda_2}{2} = l_2 \quad$ (ii)

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From Eqs. (i) and (ii), (i)/(ii)

$$\frac{n_1\lambda_1}{n_2\lambda_2}=\frac{l_1}{l_2}$$

$$\Rightarrow \qquad \frac{n_1}{n_2}=\frac{\lambda_2}{\lambda_1}\frac{l_1}{l_2} \qquad \text{(iii)}$$

Velocity of wave (v) is given by

$$v=f\lambda$$

where f is the frequency of oscillation. If velocity of wave in wire (1) and (2) is v_1 and v_2, respectively then

$$\frac{v_1}{v_2}=\frac{f\lambda_1}{f\lambda_2}$$

$$\Rightarrow \qquad \frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2} \qquad \text{(iv)}$$

But $\qquad v=\sqrt{\frac{T}{\mu}}$

where T is the tension in the strings and μ is the mass per unit length of string

then $\qquad \frac{v_1}{v_2}=\sqrt{\frac{\mu_2}{\mu_1}} \qquad$ (v)

From Eqs. (iv) and (v),

$$\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{\mu_2}{\mu_1}} \qquad \text{(vi)}$$

From Eqs. (iii) and (vi), $\qquad \frac{n_1}{n_2}=\frac{l_1}{l_2}\sqrt{\frac{\mu_1}{\mu_2}}$

- The values of λ_1 and λ_2, μ_1 and μ_2 will be given.
- The ratio n_1/n_2 can be calculated.
- If we know the value of n_1 or n_2 from Eqs. (i) and (ii), we can get the value of λ_1 and λ_2.
- After knowing the value of λ_1 or λ_2, we can calculate the fundamental frequency of oscillation using $v=f\lambda$.
- Then next higher frequency = $2f$ and so on.

Case II

If joint P acts as antinode

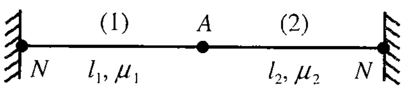

Fig. 7.45

For string (1), $\qquad \frac{(2n_1-1)}{2}$ loops

For string (2), $\qquad \frac{(2n_2-1)}{2}$ loops

Hence, $\qquad \frac{(2n_1-1)}{2}\frac{\lambda_1}{2}=l_1 \qquad$ (vii)

and $\qquad \frac{(2n_2-1)}{2}\frac{\lambda_2}{2}=l_2 \qquad$ (viii)

From Eqs. (vii) and (viii)

$$\frac{(2n_1-1)}{(2n_2-1)}=\frac{\lambda_2}{\lambda_1}\frac{l_1}{l_2}=\frac{l_1}{l_2}\sqrt{\frac{\mu_1}{\mu_2}} \qquad \text{(ix)}$$

- From Eq. (ix) we can calculate the value of n_1 or n_2.
- Once the value of n_1 or n_2 is calculated we can calculate the value of λ_1 or λ_2.
- After getting the value of λ_1 or λ_2 we can calculate the value of 'f' using the expression $v=f\lambda$.

Illustration 7.28 **Two metallic strings A and B of different materials are connected in series forming a joint. The strings have similar cross-sectional area. The length of A is l_A = 0.3 m and that B is l_B = 0.75 m. One end of the combined string is tied with a support rigidly and the other end is loaded with a block of mass m passing over a frictionless pulley. Transverse waves are set up in the combined string using an external source of variable frequency, calculate**

a. The lowest frequency for which standing waves are observed such that the joint is a node and

b. The total number of antinodes at this frequency. The densities of A and B are 6.3×10^3 kg/m³ and 2.8×10^3 kg/m³, respectively.

Sol: Given that l_A = 0.3 m, l_B = 0.75 m, T = mg, $A_A = A_B = A$, $d_A = 6.3\times10^3$ kg/m³.

$$d_B=2.8\times10^3 \text{ kg/m}^3$$

Let p = number of loops formed in string A and q = number of loops formed in string B.

a. Both p and q will be integer numbers, as the one end is fixed rigidly, the other end is loaded and the joint P is a node.

For string A, $\qquad n_A=\frac{p}{2l_A}\sqrt{\frac{T}{Ad_A}}$

For string B, $\qquad n_B=\frac{q}{2l_B}\sqrt{\frac{T}{Ad_B}}$

As both the strings are to be exerted by the same source of variable frequency, hence $n_A=n_B$.

$$\frac{p}{2l_A}\sqrt{\frac{T}{Ad_A}}=\frac{q}{2l_B}\sqrt{\frac{T}{Ad_B}}$$

$$\frac{p}{q}=\frac{l_A}{l_B}\sqrt{\frac{d_A}{d_B}}=\frac{0.3}{0.75}\sqrt{\frac{6.3\times10^3}{2.8\times10^3}}=\frac{2}{5}\sqrt{\frac{9}{4}}=\frac{2}{5}\times\frac{3}{2}=\frac{3}{5}$$

or $\qquad \frac{6}{10}$ or $\frac{9}{15}$

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As we have to determine the lowest frequency hence the mode of variation will be as shown in Fig. 7.46.

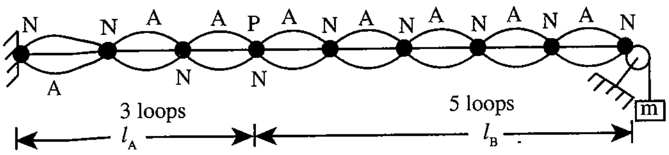


Fig. 7.46

$$n_{\min} = \frac{3}{2l_A}\sqrt{\frac{T}{Ad_A}}$$

$$= \frac{3}{2\times 0.3}\sqrt{\frac{m\times 10}{A\times 6.3\times 10^3}}$$

$$= \frac{3}{2\times 0.3\times 10}\sqrt{\frac{m}{6.3A}} = \sqrt{\frac{m}{25.2A}}$$

b. The total number of antinodes at this frequency = 3 + 5 = 8.

MELDE'S EXPERIMENT

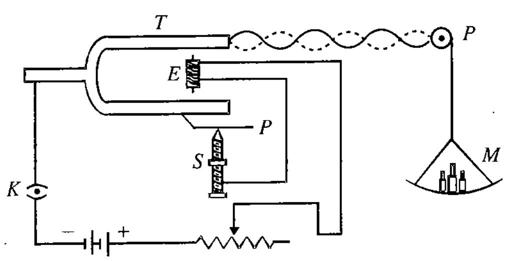
This is an experiment for demonstration of transverse stationary wave in stretched string.

In Melde's experiment, the one end of the string is connected to the prong of an electrically oscillated tuning fork. The other end of the string is connected to the scale pan. The string passes over a smooth frictionless pulley. The distance between tuning fork and pulley can be adjusted. There are two different ways in which oscillations can be established in the string.

Case I: Transverse Mode of Vibration

As shown in Fig. 7.47, tuning fork vibrates right angle to the length of the string. In this case the frequency of vibration of string is equal to the frequency of the tuning fork. First, we adjust the length of string so that stationary waves are formed in string. In this case the vibrations of string are enough so that the loop can be seen in the string. If string vibrates in p loops as shown in Fig. 7.47(a), the frequency of the oscillating string can be given as

$$n_s = \frac{p}{2l}\sqrt{\frac{T}{\mu}} \qquad \text{(i)}$$



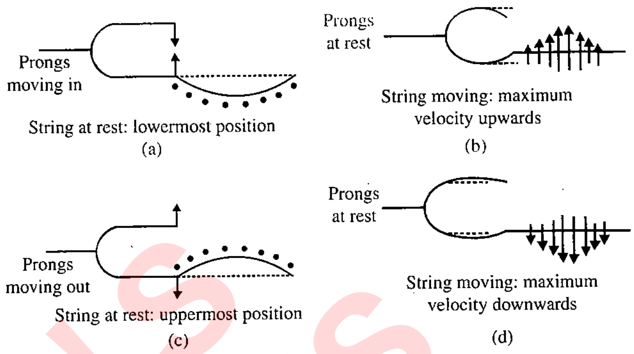


Fig. 7.47

If n_f be the frequency of oscillation of tuning fork then we have

$$n_f = n_s = \frac{p}{2l}\sqrt{\frac{T}{\mu}} \qquad \text{(ii)}$$

Case II: Longitudinal Mode of Vibrations

In this case the vibrations of tuning fork are along the length of the string. The orientation of tuning fork is shown in Fig. 7.48. In this case for one complete vibration of the tuning fork, the string completes only half of its vibrations so the frequency of vibration of string is half of that of the oscillation frequency of tuning fork.

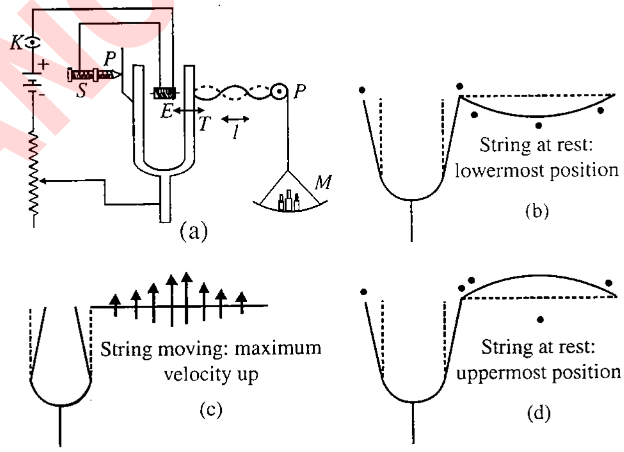


Fig. 7.48

We can see if the frequency of tuning fork remains constant from Eqs. (i) and (ii) we can write

$$p\sqrt{T} = \text{constant} \qquad [\text{As } \mu = \text{constant}] \qquad \text{(iii)}$$

If the tension in the string is changed then the number of loops in the stationary wave formed varies according to Eq. (iii). This is called as Melde's law.

Thus, if in Melde's experiment, stationary waves are formed at two different values of tension T_1 and T_2 at which p_1 and p_2 loops are formed in the string then we can write

$$\frac{P_1}{P_2} = \sqrt{\frac{T_2}{T_1}}$$

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Illustration 7.29 **In Melde's experiment, when a string is stretched by a piece of glass it vibrates with 7 loops. When the glass piece is completely immersed in water the string vibrates in 9 loops. What is the specific gravity of glass?**

Sol. If F_1 and F_2 are the tensions in the string in the two cases, then

$$F_1P_1^2 = F_2P_2^2$$

$$\frac{F_1}{F_2} = \frac{P_2^2}{P_1^2} = \frac{9^2}{7^2} = \frac{81}{49}$$

If F_b is the buoyant force on the glass piece, then $F_b = F_1 - F_2$

$$\text{Specific gravity of glass} = \frac{\text{Weight in air}}{\text{Loss in weight in water}}$$

$$= \frac{F_1}{F_b} = \frac{F_1}{F_1 - F_2} = \frac{1}{\left(1 - \dfrac{F_2}{F_1}\right)}$$

$$= \frac{1}{\left(1 - \dfrac{49}{81}\right)} = 2.53$$

Illustration 7.30 **Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.**

a. Calculate the frequencies of the next two harmonics of the C string.

b. If A and C strings have the same linear mass density μ and length L, determine the ratio of tensions in the two strings.

Sol.

a. Remember that the harmonics of a vibrating string have frequencies that are related by integer multiples of the fundamental.

This first part of the example is a simple substitution problem.

Knowing that the fundamental frequency is $f_1 = 262$ Hz, find the frequencies of the next harmonics by multiplying by integers:

$$f_2 = 2f_1 = 524 \text{ Hz}$$
$$f_3 = 3f_1 = 786 \text{ Hz}$$

b. This part of the example is more of an analysis problem than is part (a).

Use Eq. (iii) to write expression for the fundamental frequencies of the two string.

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}} \quad \text{and} \quad f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

Divide the first equation by the second and solve for the ratio of tensions

$$\frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440}{262}\right)^2 = 2.82$$

If the frequencies of piano strings were determined solely by tension. This result suggests that the ratio of tensions from the lowest string to the highest string on the piano would be enormous. Such large tensions would make it difficult to design a frame to support the strings. In reality, the frequencies of piano strings vary due to additional parameters, including the mass per unit length and the length of the string.

Illustration 7.31 **If look inside a real piano, you'll see that the assumption made in part (b) of Illustration 7.31 is only partially true. The strings are not likely to have the same length. The string densities are equal, but suppose the length of the A string is only 64% of the length of the C string. What is the ratio of their tensions?**

Sol. The ratio of frequencies:

$$\frac{f_{1A}}{f_{1C}} = \frac{L_C}{L_A}\sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = \left(\frac{L_A}{L_C}\right)^2\left(\frac{f_{1A}}{f_{1C}}\right)^2$$

$$\frac{T_A}{T_C} = (0.64)^2\left(\frac{440}{262}\right)^2 = 1.16$$

Notice that this result represents only a 16% increase in tension, compared with the 18.2% increase in part (b) of Illustration 7.31.

Illustration 7.32 **One end of a horizontal string is attached to a vibrating blade, and the other end passes over a pulley as in Fig. 7.49(a). A sphere of mass 2.00 kg hangs at the end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. In this configuration, the string vibrates in its fifth harmonic as shown in Fig. 7.49(b). What is the radius of the sphere?**

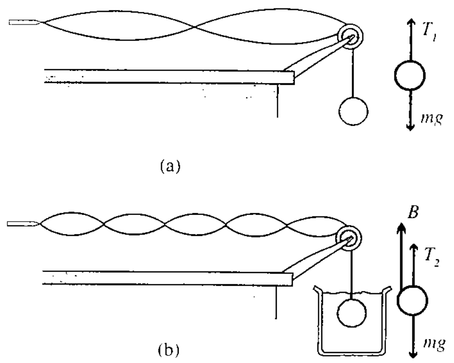

Fig. 7.49

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Sol. Imagine what happens when the sphere is immersed in the water. The buoyant force acts upward on the sphere, reducing the tension in the string. The change in tension causes a change in the speed of waves on the string, which in turn causes a change in the wavelength. This altered wavelength results in the string vibrating in its fifth normal mode rather than the second.

The hanging sphere is modelled as a particle in equilibrium. One of the forces acting on it is the buoyant force from the water. We also apply the waves under boundary conditions model to the string.

Apply the particle in equilibrium model to the sphere in Fig. 7.49(a), identifying T_1 as the tension in the string as the sphere hangs in air.

$$T_l = mg \qquad \text{(i)}$$

Apply the particle in equilibrium model to the sphere in Fig. 7.49(b), where T_2 is the tension in the string as the sphere is immersed in water.

$$B + T_2 = mg \quad \Rightarrow B = mg - T_2 \qquad \text{(ii)}$$

The desired quantity, the radius of the sphere, will appear in the expression for the buoyant force B. Before proceeding in this direction, however, we must evaluate T_2 from the information about the standing wave.

Write the equation for the frequency of a standing wave on a string twice, once before the sphere is immersed and once after. Notice that the frequency f is same in both the cases because it is determined by the vibrating blade. In addition, the linear mass density μ and the length L of the vibrating portion of the string are same in both cases. Divide the two equations.

$$f = \frac{n_1}{2L}\sqrt{\frac{T_1}{\mu}} \qquad \text{(iii)}$$

$$f = \frac{n_2}{2L}\sqrt{\frac{T_2}{\mu}} \qquad \text{(iv)}$$

$$1 = \frac{n_1}{n_2}\sqrt{\frac{T_1}{T_2}} \qquad \text{(v)}$$

Solve for T_2:

$$T_2 = \left(\frac{n_1}{n_2}\right)^2 T_1 = \left(\frac{2}{5}\right)^2 (19.6\ \text{N}) = 3.14\ \text{N}$$

Substituting this result into Eq. (ii)

$$B = mg - T_2 = 19.6\ \text{N} - 3.14\ \text{N} = 16.5\ \text{N}$$

Now $$B = \rho_{\text{water}} g V_{\text{sphere}} = \rho_{\text{water}}\, g\left(\frac{4}{3}\pi r^3\right)$$

Solve for the radius of the sphere:

$$r = \left(\frac{3B}{4\pi\rho_{\text{water}}\ g}\right)^{1/3}$$

$$= \left(\frac{3(16.5\ \text{N})}{4\pi(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}\right)^{1/3}$$

$$= 7.38 \times 10^{-2}\ \text{m} = 7.38\ \text{cm}$$

Notice that only certain radii of the sphere will result in the string vibrating in a normal mode; the speed of waves on the string must be changed to a value such that the length of the string is integer multiple of half wavelengths. The radii of the sphere that cause the string to vibrate in a normal mode are quantized.

RESONANCE

We have seen that a system such as a taut string is capable of oscillating in one or more normal modes of oscillation. If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system. This phenomenon, known as resonance, was discussed in previous section. Although a block spring system or a simple pendulum has only one natural frequency, standing wave systems have a whole set of natural frequencies.

Consider a taut string fixed at one end and connected at the opposite end to an oscillating blade as illustrated in Fig. 7.49. The fixed end is a node, and the end connected to the blade is very nearly a node because the amplitude of the blade's motion is small compared with that of the elements of the string. As the blade oscillates, transverse wave sent down the string are reflected from the fixed end.

As we learned in previous section, the string has natural frequencies that are determined by its lengths, tension and linear mass density. When the frequency of the blade equals one of the natural frequencies of the string, standing waves are produced and the string oscillates with a large amplitude. In this resonance case, the wave generated by the oscillating blade is in phase with the reflected wave and the string absorbs energy from the blade. If the string is driven at a frequency that is not one of its natural frequencies, the oscillations are of low amplitude and exhibit no stable pattern. Resonance is very important in the excitation of musical instruments based on air columns.

STANDING WAVES IN AIR COLUMNS

The waves under boundary conditions model can also be applied to sound waves in a column of air such as that inside an organ pipe. Standing waves are the result of interference between longitudinal sound waves travelling in opposite directions.

In a pipe closed at one end, the closed end is a displacement node because the rigid barrier at this end does not allow longitudinal motion of the air. Because the pressure wave is 90° out of phase with the displacement wave, the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation).

The open end of an air column is approximately a displacement antinode and a pressure node. We can understand

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why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; therefore, the pressure at this end must remain constant at atmospheric pressure.

You may wonder how a sound wave can be reflected from an open end as it may appear that there is no change in the medium at this point: the medium through which the sound wave moves is air both inside and outside the pipe. Sound is a pressure wave; however, a compression region of the sound wave is constrained by the sides of the pipe as long as the region is inside the pipe. Since the compression region exits at the open end of the pipe, the constraint of the pipe is removed and the compressed air is free to expand into the atmosphere. Therefore, there is a change in the character of the medium between the inside of the pipe and the outside even though there is no change in the material of the medium. This change in character is sufficient to allow some reflection.

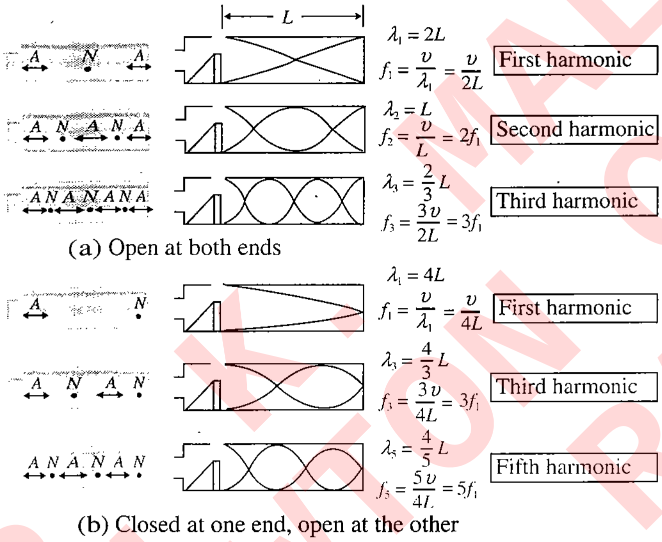

Fig. 7.50

With the boundary conditions of nodes or antinodes at the ends of the air column, we have a set of normal modes of oscillation as is the case for the string fixed at both ends. Therefore, the air column has quantized frequencies.

The first three normal modes of oscillation of a pipe open at both ends are shown in Fig. 7.50(a). Notice that both ends are displacement antinodes (approximately). In the first normal mode, the standing wave extends between two adjacent antinodes, which is distance of half a wavelength. Therefore, the wavelength is twice the length of the pipe, and the fundamental frequency is $f_1 = v/2L$. As Fig. 7.50(a) shows the frequencies of the higher harmonics are $2f_1, 3f_1, \ldots$

In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.

Because all harmonics are present and the fundamental frequency is given by the same expression as that for a string, we can express the natural frequencies of oscillation as

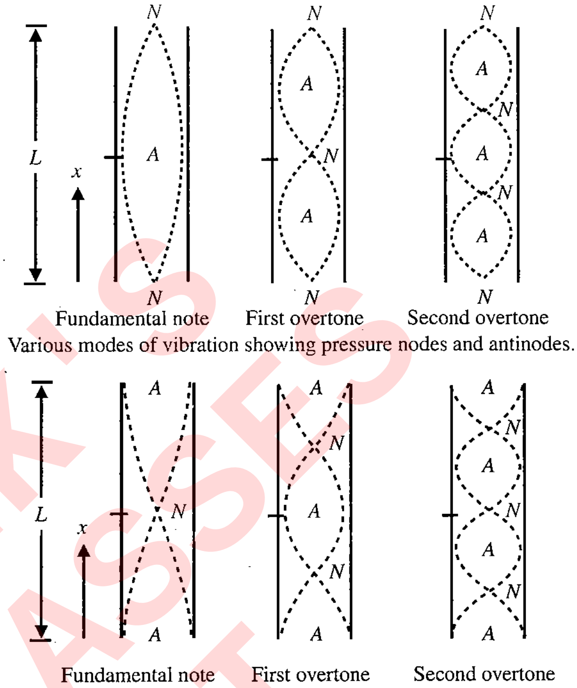

Fig. 7.51 Various modes of vibration showing displacement nodes and antinodes

$$f_n = n\frac{v}{2L} \quad \text{where } n = 1, 2, 3, \ldots \qquad \text{(i)}$$

Analytical treatment:

Consider an open organ pipe of length L lying along the x-axis, with its ends at $x = 0$ and $x = L$. The sound wave travelling along the pipe can be represented as

$$\Delta P_1 = \Delta P_m \sin(kx - \omega t)$$

The reflected sound wave from open end (rigid boundary) is represented by

$$\Delta P_2 = \Delta P_m \sin(-kx - \omega t + \pi) = \Delta P_m \sin(kx + \omega t)$$

The resultant stationary wave is given by

$$\Delta P = \Delta P_1 + \Delta P_2 = \Delta P_m \sin(kx - \omega t) + \Delta P_m \sin(kx + \omega t)$$

or $\quad \Delta P = 2\Delta P_m \sin(kx)\cos(\omega t)$

for all values of t, the resultant pressure variation is zero for which

$$\sin kx = 0$$

or $\quad kx = n\pi \quad$ or $\quad \frac{2\pi}{\lambda}x = n\pi$

$$x = \frac{n\lambda}{2}, \quad \text{when} \quad n = 0, 1, 2, 3, \ldots$$

$\therefore \quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$

These points of zero pressure variation are called pressure nodes.

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On the other hand, the pressure variation is maximum for all values of t, for which

$$\sin kx = \pm 1$$

or $$kx = (2n+1)\frac{\pi}{2}$$

or $$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\lambda}{4}, \quad \text{where } n = 0, 1, 2, 3, \ldots$$

Therefore, $$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

These points of maximum pressure variation are called pressure antinodes

As ΔP is maximum ($\pm 2\Delta P_m$) at the antinodes, so the strain is positive or negative at the pressure antinodes and zero at the pressure nodes.

Fig. 7.52(a) Positions of maximum strain

At antinodes due to the compressions or the rarefactions of the oppositely travelling waves, the strain becomes maximum. At nodes, strain becomes zero due to the compression of one wave coming across the rarefaction of the other, as shown in Fig.10.52.

Fig. 7.52(b) Positions of zero strain

If a pipe is closed at one end and open at the other, the closed end is a displacement node [see Fig. 7.52(b)]. In this case, the standing wave for the fundamental mode extends from an antinode to the adjacent node, which is one-fourth of a wavelength. Hence, the wavelength for the first normal mode is $\lambda = 4L$, and the fundamental frequency is $f_1 = v/4L$. As Fig. 7.50(b) shows, the higher frequency waves that satisfy these conditions are those that have a node at the closed end and an antinode at the open end; hence, the higher harmonics have frequencies $3f_1$, $5f_1$, ...

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency.

We express this result mathematically as

$$f_n = n\frac{v}{4L}; \quad n = 1, 3, 5, \ldots \qquad \text{(ii)}$$

It is interesting to investigate what happens to the frequencies of instruments based on air columns and strings during a concert as the temperature rises. The sound emitted by a flute, for example, becomes sharp (increase in frequency) as the flute warms up because the speed of sound increases in the increasingly warmer air inside the flute (consider Eq. (i)). The sound produced by a violin becomes flat (decrease in frequency) as the string thermally expand and the expansion causes their tension to decrease.

Musical instruments based on air columns are generally excited by resonance. The air column is presented with a sound wave that is rich in many frequency. The air column then responds with a large amplitude oscillation to the frequencies that match the quantized frequencies in its set of harmonics. In many woodwind instruments, the initial rich sound is provided by a vibrating reed. In brass instruments, this excitation is provided by the sound coming from the vibration of the player's lips. In a flute, the initial excitation comes from blowing over an edge at the mouthpiece of the instruments in a manner similar to blowing across the opening of a bottle with narrow neck. The sound of the air rushing across the edge has many frequencies, including one that sets the air cavity in the bottle into resonance.

Analytical treatment: Consider a cylindrical pipe of length L lying along the x-axis with its closed end at $x = 0$ and open end at $x = L$. The sound wave sent along the pipe can be represented as

$$\Delta P_1 = \Delta P_m \sin(kx - \omega t)$$

The reflected waves from the closed end is represented by (sound wave suffers no phase change due to the reflection from the closed end).

$$\Delta P_2 = -\Delta P_m \sin(kx + \omega t)$$

The resultant wave is given by

$$\Delta P = \Delta P_1 + \Delta P_2 = \Delta P_m \sin(kx - \omega t) - \Delta P_m \sin(kx + \omega t)$$

or $$\Delta P = -2\Delta P_m \cos(kx)\sin(\omega t)$$

For all values of t the resultant pressure variation is zero for which

$$\cos(kx) = 0 \quad \text{or} \quad kx = (2n+1)\frac{\pi}{2}$$

$$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\lambda}{4} \quad \text{where} \quad n = 0, 1, 2, 3, \ldots$$

$$\therefore \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

These points of zero pressure variation are called pressure nodes. On the other hand, the pressure variation is maximum for all values of t for which

$$\cos(kx) = \pm 1$$

or $$kx = n\pi \quad \text{or} \quad \frac{2\pi}{\lambda}x = n\pi$$

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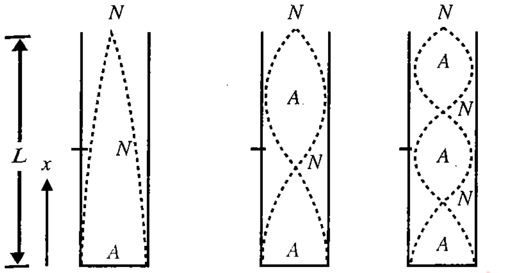
$$x = n\frac{\lambda}{2}, \quad \text{where} \quad n = 0, 1, 2, 3, \ldots$$

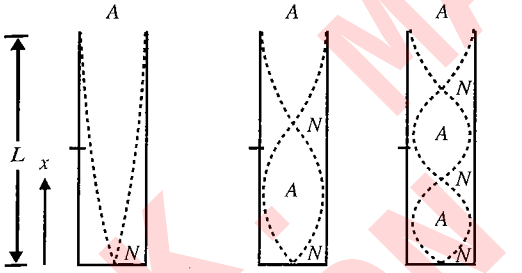
$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$

These points of maximum pressure variation are called pressure antinodes (see Fig. 7.53). The pressure at these points varies from $(P_0 - 2\Delta P_m)$ to $(P_0 + 2\Delta P_m)$



Fundamental note   First overtone   Second overtone

Various modes of vibration of closed organ pipe showing pressure nodes and antinodes.



Fundamental tone   First overtone   Second overtone

Various modes of vibration of closed organ pipe showing displacement nodes and antinodes

Fig. 7.53

Note

- ***If P_0 is the normal pressure in the pipe, then at the position of pressure nodes, the pressure will be P_0 and at the position of pressure antinodes, it will be $P_0 \pm 2\Delta P_m$ or $P_0 + 2ABk$. Thus pressure at antinodes varies from $P_0 - 2ABk$ to $P_0 + 2ABk$.***
- ***The loud sound is heard at pressure antinode or displacement node.***
- ***We know that bulk modulus of medium (air) is***

$$B = \frac{dP}{\left(-\frac{dV}{V}\right)} = \frac{dP}{(-dy/dx)}$$

$$\text{Strain} = \frac{dy}{dx} = -\frac{dP}{B} = -\frac{\Delta P}{B}$$

Illustration 7.33 **Two adjacent natural frequencies of an organ pipe are found to be 550 Hz and 650 Hz. Calculate the fundamental frequency and length of this pipe. (Use v = 340 m/s.)**

Sol. These will be fairly high harmonics of the pipe, which we expect to be long pipe for a bass note.

We use the wave under boundary conditions model. We are not told in advance whether the pipe is open at one end or at both ends, so we must think about both possibilities.

Because harmonic frequencies are given by $f_1 n$ for open pipe, and $f_1(2n - 1)$ for closed pipes, the difference between adjacent harmonics is constant in both cases. Therefore, we can find each harmonic below 650 Hz by subtracting $\Delta f_{\text{Harmonic}}$ = (650 Hz – 550 Hz) = 100 Hz from the previous value.

The harmonic frequencies below 650 Hz by subtracting $\Delta f_{\text{Harmonic}}$ = (650 Hz – 550 Hz) = 100 Hz from the previous value.

The harmonic frequencies are the set of the following values:

650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 150 Hz and 50 Hz

a. So the fundamental frequency, is 50 Hz.

b. The wavelength of the fundamental vibration can be calculated from the speed of sound as

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{50.0 \text{ Hz}} = 6.80 \text{ m}$$

Because the step size Δf is twice the fundamental frequency, we know the pipe is closed, with an antinode at the open end, and a node at the closed end. The wavelength in this situation is four times the pipe length, so

$$L = \frac{\lambda}{4} = 1.70 \text{ m}$$

The frequency 50 Hz is a G in low register. The harmonics measured in the problem are the 11th and the 13th, which can be perfectly observable as resonances using a function generator, a small loudspeaker, and your ear as detector. For some ranks of organ pipes, higher harmonics like these can be important contributors to a brassy or ringing sound quality.

Illustration 7.34 **A shower stall has dimensions 86.0 cm × 86.0 cm × 210 cm. If you singing in this shower, which frequencies would sound the richest (because of resonance)? Assume the stall acts as a pipe closed at both ends, with nodes at opposite sides. Assume the voices of various singers range from 130 Hz to 2000 Hz. Let the speed of sound in the hot air be 355 m/s.**

Sol: We expect one set of equally spaced resonance frequencies for sound moving horizontally, and another set with narrower spacing for sound moving vertically.

Standing waves must fit between the side walls, with nodes at both surfaces, or else between floor and ceiling.

For a closed box, the resonant air vibrations will have nodes at both sides, so the permitted wavelengths will be defined by

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$$L = nd_{NN} = \frac{n\lambda}{2} = \frac{nv}{2f}, \quad \text{with} \quad n = 1, 2, 3, \ldots$$

Rearranging, and substituting $L = 0.860$ m, the side to side resonant frequencies are

$$f_n = n\frac{v}{2L} = n\frac{355 \text{ m/s}}{2(0.860 \text{ m})}$$

$$= n(206 \text{ Hz}), \text{ for each } n \text{ from 1 to 9.}$$

With $L' = 2.10$ m, the top to bottom resonance frequencies are

$$f_n = n\frac{355 \text{ m/s}}{2(2.10 \text{ m})} = n(84.5 \text{ Hz}), \text{ for each } n \text{ from 2 to 23}$$

we found the allowed values for n by checking which resonant frequencies lie in the vocal range between 130 Hz and 2000 Hz. If the equations in this problem look like formulas for a pipe open at both ends or a string fixed at both ends, you should draw pictures of the vibrations instead of just thinking about formulas. The shower stall is different from any organ pipe, because it has nodes at both walls instead of antinodes. The shower stall is different from a string because the air wave is longitudinal. An equation like $d_{NN} = \lambda/2$ is true for all one-dimensional standing waves, so the best solution is based on diagrams and on this equation, with $f = v/\lambda$. A function generator connected to a loudspeaker, with your ear as detector, is all you need to do a nice experiment to observe this phenomenon in any small room with hard parallel walls. Listen at different distances from one wall, and you may even discover whether your ear responds more to air displacement or to air pressure.

Illustration 7.35 **A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows across its open ends.**

a. Determine the frequencies of the first harmonics of the culvert if it is cylindrical in shape and open at both ends. Take $v = 343$ m/s as the speed of sound in air.

b. What are the three lowest natural frequencies of the culvert if it is blocked at one end?

Sol. The sound of the wind blowing across the end of the pipe contains many frequencies, and the culvert responds to the sound by vibrating at the natural frequencies of the air column.

This example is a relatively simple substitution problem.

a. Let us first find the frequency of the first harmonic of the culvert, modelling it as an air column open at both ends.

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}$$

Find the next harmonics by multiplying by integers.

$$f_2 = 2f_1 = 278 \text{ Hz}$$

$$f_3 = 3f_1 = 417 \text{ Hz}$$

b. Find the frequency of the first harmonic of the culvert, modelling it as an air column closed at one end:

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz}$$

Find the next two harmonics by multiplying by odd integers.

$$f_3 = 3f_1 = 209 \text{ Hz}$$

$$f_5 = 5f_1 = 349 \text{ Hz}$$

Illustration 7.36 **Find the number of possible natural oscillations of air column in a pipe frequencies of which lie below $v_0 = 1250$ Hz. The length of the pipe is $l = 85$ cm. The velocity of sound is $v = 340$ m/s. Consider two cases**

i. the pipe is closed from one end,

ii. the pipe is open from both ends.

First Method

Sol.

i. Pipe is closed from one end: An air column in a pipe closed from one end oscillates only odd harmonics [1st harmonic (fundamental mode), 3rd harmonic (1st overtone), 5th harmonic (2nd overtone), 7th harmonic (3rd overtone) etc.]

Fundamental frequency

$$= \frac{v}{4l} = \frac{340}{4 \times \frac{85}{100}} = 100 \text{ Hz}$$

Other modes of oscillation are:

3rd harmonic frequency = 3 × 100 = 300 Hz

5th harmonic frequency = 5 × 100 = 500 Hz

7th harmonic frequency = 7 × 100 = 700 Hz

9th harmonic frequency = 9 × 100 = 900 Hz

11th harmonic frequency = 11 × 100 = 1100 Hz

13th harmonic frequency = 13 × 100 = 1300 Hz

Only those natural oscillations are to be counted frequencies of which lie below $v_0 = 1250$ Hz, the harmonics till 11th harmonic are to be counted.

Since number of possible natural oscillations

= 1 (1st harmonic) + 1 (3rd harmonic) + 1 (5th harmonic) + 1(7th harmonic) + 1(9th harmonic) + 1 (11th harmonic) = 6

Second Method

All the frequencies possible are integral multiples of fundamental frequency which is 100 Hz. Using the fact that integer which is multiplied by fundamental frequency is the number of harmonic itself you get, highest harmonic predicted = [12.50/100] where [x] represents greatest integer less than or equal to x = [12.5] = 12.

Now for closed pipe, only odd harmonics are possible and the highest harmonic possible = 11th. The possible harmonics are 1, 3, 5, 7, 9, 11 which are six in number.

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ii. **Pipe opened from both ends:** Fundamental frequency

$$= \frac{V}{2l} = \frac{340}{2 \times 85} \times 100 = 200 \text{ Hz}$$

Frequency of the other natural modes of oscillational are:
2nd harmonic frequency = 2 × 200 = 400 Hz
3rd harmonic frequency = 3 × 200 = 600 Hz
4th harmonic frequency = 4 × 200= 800 Hz
5th harmonic frequency = 5 × 200 = 1000 Hz
6th harmonic frequency = 6 × 200 = 1200 Hz
7th harmonic frequency = 7 × 200 = 1400 Hz

You have to count only those harmonics whose frequencies are below 1250 Hz. All the harmonics till 6th harmonic are possible, and obviously they are six in number.

Third Method

Fundamental frequency = 200 Hz

Frequencies possible = n × fundamental frequency

$$= n \times 200 \qquad [n \text{ is } 1, 2, ...]$$

Maximum value of n = [12.50/200] = 6 ([x] represents greater than or equal to x)

Now n is also equal to the number of harmonic for which frequency is being calculated, highest harmonic possible = 6th.

As all harmonics are possible in case of open tube, harmonics possible are 1st, 2nd, 3rd, 4th, 5th and 6th.

Number of harmonics possible in this case = 6.

KUNDT'S TUBE

This is an apparatus used to find velocity of sound in a gaseous medium or in different materials. It consists of a glass tube as shown in Fig. 7.54 one end of which is fitted with piston B which is attached to a wooden handle H which can be moved inside and outside the tube, and a rod M of the required material is fixed at clamp C in which the velocity of sound is required, at one end of rod a disc A is fixed as shown below.

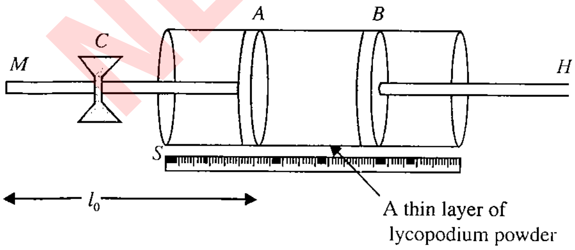

Fig. 7.54

In the tube, air is filled at room temperature and a thin layer of lycopodium powder is put along the length of the tube. It is a very fine powder particles which can be displaced by the air particles also.

When rod M is gently rubbed with a resin cloth or hit gently, it starts oscillating in fundamental mode as shown in Fig. 7.55, frequency of which can be given as

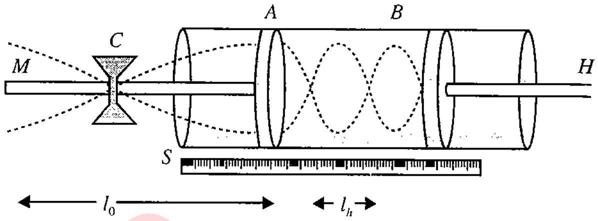

Fig. 7.55

$$n_{\text{rod}} = \frac{v}{\lambda} = \frac{1}{2l_0}\sqrt{\frac{Y}{\rho}} \qquad [\text{As } l_0 = \frac{\lambda}{2}] \qquad \text{(i)}$$

In fundamental mode, the free ends of the rod behave as antinodes and the clamped point C acts as a node, which we have discussed earlier. When these vibrations are transferred to the air column in the tube through the disc A, the air column starts oscillating at the same frequency. Now the piston B is adjusted so that the air column resonates with the vibrations of rod. In resonance condition, the lycopodium powder sets itself in the form of heaps at the position of nodes because at antinodes the air particles vibrate with maximum amplitude and displace the powder to the adjacent nodes. We can measure the length between successive heaps of powder with the help of a scale S attached to the tube, let it be l_h then we can write.

$$l_h = \frac{\lambda_a}{2} \qquad [\lambda_a = \text{wavelength of sound in air}]$$

and we know that frequency of sound in air and rod is equal, thus

$$n_{\text{rod}} = n_{\text{air}} \quad \text{or} \quad \frac{v_{\text{rod}}}{\lambda_{\text{rod}}} = \frac{v_{\text{air}}}{\lambda_{\text{air}}}$$

or
$$v_{\text{rod}} = \left(\frac{\lambda_{\text{rod}}}{\lambda_{\text{air}}}\right) v_{\text{air}} = \frac{2l_0}{2l_h} \times v_{\text{air}} = \left(\frac{l_0}{l_h}\right) \times v_{\text{air}} \qquad \text{(ii)}$$

If the Young's modulus and density of material of rod is known then using Eqs. (i) and (ii) we can find velocity of sound in air or in a gas which is filled in the tube, as

$$n_{\text{rod}} = n_{\text{gas}}$$

$$\frac{1}{2l_0}\sqrt{\frac{Y}{\rho}} = v_{\text{gas}}/2l_h \quad \text{or} \quad v_{\text{gas}} = \left(\frac{l_h}{l_0}\right)\sqrt{\frac{Y}{\rho}} \qquad \text{(iii)}$$

Illustration 7.37 **In a Kundt's tube experiment, with a wooden rod 170 cm long, clamped at the middle, the lycopodium powder gets heated up at regular intervals of 13.4 cm, the experiment being performed with air. If the frequency of vibrations be 1270 Hz, find the velocity of sound in air and in the wooden rod.**

Sol. The wavelengths of sound waves in air and in rod will be respectively,

$$\lambda_a = 2 \times 13.4 \text{ cm} = 26.8 \text{ cm, and}$$
$$\lambda_r = 2 \times 170 \text{ cm} = 340 \text{ cm}$$

Since velocity $v = \lambda f$

$\therefore$ Velocity of sound in air $v_a = l_a f$

$$= 26.8 \text{ cm} \times 1270 \text{ s}^{-1} = 340 \text{ m/s}$$

$\therefore$ Velocity of sound in wooden rod $v_r = \lambda_r f$

$$= 340 \text{ cm} \times 1270 \text{ s}^{-1}$$
$$= 4318 \text{ m/s}$$

Illustration 7.38 **A Kundt's tube experiment is conducted with a 1 m long glass rod, twice, one with air and the other with hydrogen gas filled in the tube. In the first case, there were 11 heaps of lycopodium powder within a length of 64.4 cm between the first and the last. The corresponding parameters in the second case are 5 nodal heaps within 99.7 cm length.**

Find the velocity of sound in glass and in hydrogen, if that in air be 335 m/s.

Sol. In case of air, distance between two consecutive nodes (heaps of powder)

$$= \frac{64.4 \text{ cm}}{10} = 6.44 \text{ cm}$$

If λ_a and λ_r be the respective wavelengths of sound in air and in rod, then

$$\lambda_a = 2 \times 6.44 \text{ cm and } \lambda_r = 2 \times 100 \text{ cm}$$

Since the frequency remains the same, so

$$f = \frac{v_a}{\lambda_a} = \frac{v_r}{\lambda_r}$$

$$v_r = \frac{\lambda_r}{\lambda_a} v_a = \frac{(2 \times 100 \text{ cm})}{(2 \times 6.44 \text{ cm})} \times 335 \text{ m/s} = 5.2 \text{ km/s}$$

If v_h and λ_h correspond to the velocity and wavelength of sound waves in hydrogen respectively, then

$$\frac{v_a}{\lambda_a} = \frac{v_h}{\lambda_h} \quad \text{or} \quad v_h = \frac{\lambda_h}{\lambda_a} \times v_a$$

$$v_h = \frac{(99.7 \text{ cm}/4)}{6.44 \text{ cm}} \times 335 \text{ m/s} = 1.3 \text{ km/s}$$

RESONANCE TUBE

It is used to determine the speed of sound in air with the help of tuning fork of known frequency. It is a close pipe whose length can be changed by changing the level of liquid in the tube. When a vibrating tuning fork is brought over its mouth, its air column vibrates longitudinally. If the length of the air column varies until its natural frequency becomes equal to the frequency of fork, then resonance will occur and loud sound is heard.

Figure 7.56 shows the setup of a resonance tube experiment. There is a long tube T in which initially water is filled up to the top and the water level can be changed by moving reservoir R up and down.

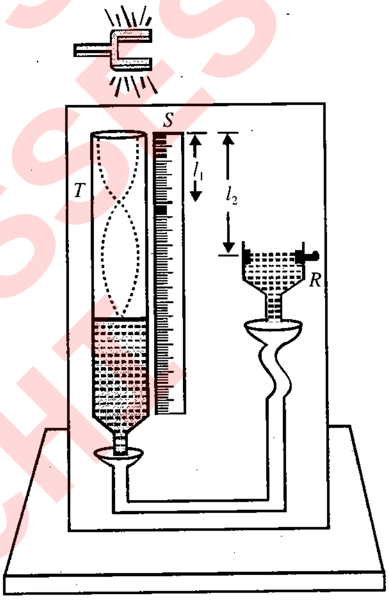

Fig. 7.56

A tuning fork of known frequency n_0 is struck gently on a rubber pad and brought near the open end of tube T due to which oscillations are transferred to the air column in the tube above water level. Now the water level is gradually decreased. This air column behaves like a closed organ pipe and the water level as closed end of pipe. As soon as water level reaches a position where there is a node of corresponding stationary wave, in air column, resonance takes place and maximum sound intensity is detected. Let at this position, length of air column be l_1. If the water level is further decreased, again maximum sound intensity is observed when water level is at another node, i.e., at a length l_2 as shown in Fig. 7.56. Here if we find two successive resonance lengths l_1 and l_2, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2} \quad \text{or} \quad \lambda = 2(l_2 - l_1)$$

Thus, sound velocity in air can be given as $v = n_0\lambda = 2n_0(l_2 - l_1)$

We have discussed that in an organ pipe the incident and reflected waves superpose and give rise to establishment of stationary waves at harmonic frequencies. First, we discuss how a wave is reflected from the open end of an organ pipe.

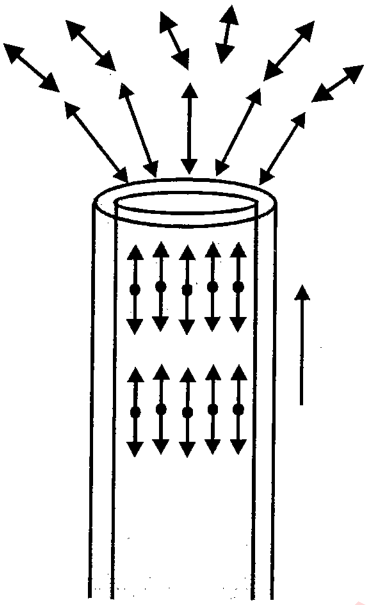

Fig. 7.57

Consider an organ pipe shown in Fig. 7.58. Here we consider a wave is propagating towards its open end. Due to longitudinal wave medium (air), particles oscillate along the length of pipe as shown in Fig. 7.57. But the oscillations are along the length of the pipe within the boundaries of the pipe. When wave reaches the open end, due to collisions the medium particles outside the pipe scatters in the direction away from pipe and due to this, medium (air) density reduces outside the pipe and from the region of this rarer medium the wave is reflected.

Here we can see that when a wave reaches the open end of pipe it penetrates atmosphere up to a small depth where the density is decreased and then it is reflected back into the pipe. Thus, the wave is not exactly reflected from the open end of the pipe. Hence in the formation of stationary waves in organ pipe we say that an antinode is formed always a little above the open end as shown in Fig. 7.58. The distance above the open end where an antinode formed is called the end correction and is represented by e.

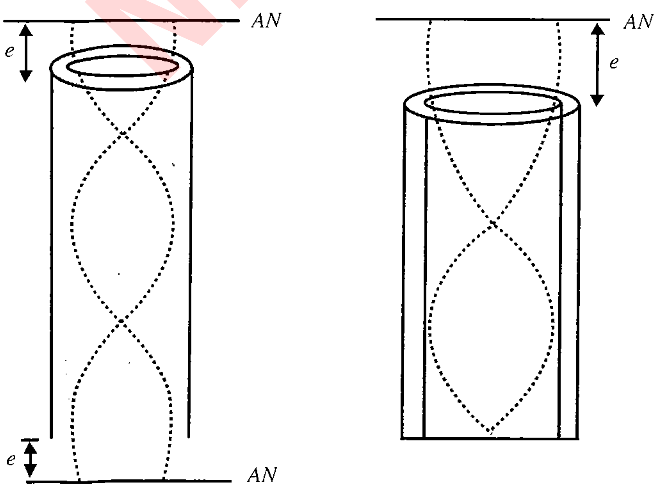

Fig. 7.58

It is observed that the end correction depends on the radius of organ pipe and is experimentally determined and expressed as

$$e = 0.6r \qquad \text{(i)}$$

Thus, for a broad pipe the end correction is more than a narrow pipe. When we find the different harmonic frequencies of oscillations of air column in organ pipe, we must account the end corrections. Now taking into account the end correction the fundamental frequency of a closed pipe of length l can be given as

$$n_0 = \frac{v}{4(l+e)} \quad \text{[One end open]} \qquad \text{(ii)}$$

and fundamental frequency of an open pipe of length l is taken as

$$n_0 = \frac{v}{2(l+2e)} \quad \text{[Both ends open]} \qquad \text{(iii)}$$

Illustration 7.39 **A tuning fork of frequency 340 Hz is vibrated just above a cylindrical tube. The length of the tube is L = 120 cm. Water is slowly poured into the tube. Determine the minimum height of water required for resonance. (Take velocity of sound in air v = 340 m/s)**

Sol. Here, the tuning fork vibrates in resonance with the air column in the pipe closed at one end. The resonant frequency of the air column is given by

$$f = \frac{nv}{4l} \quad \text{where} \quad n = 1, 2, 3, \ldots$$

The length of the air column is given by

$$l = \frac{nv}{4f} = \frac{n(340)}{4(340)} = 0.25n \text{ m} = 25n \text{ cm}$$

$$l = 25 \text{ cm}, 75 \text{ cm}, 125 \text{ cm}$$

Since the length of the tube is 120 cm, therefore, the possible lengths of air column are

$$l = 25 \text{ cm}$$

or

$$l = 75 \text{ cm}$$

If h is the height of water filled in the tube, then

$$l + h = 120 \text{ cm}$$

$$h = 120 - l$$

For minimum value of h, l is maximum, thus

$$h_{min} = 120 - l_{max} = 120 - 75 = 45 \text{ cm}$$

Illustration 7.40 **The first two lengths of an air column, in a resonance column method, were found to be 32.1 cm and 99.2 cm, respectively. Determine the end correction for the tube. If it is known that velocity of sound in the laboratory is 332 m/s, then find the frequency of the vibrating tuning fork.**

Sol. Here, $l_1 = 32.1$ cm and $l_2 = 99.2$ cm

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$\therefore$ from $e = \dfrac{l_2 - 3l_1}{2}$, the end correction is given by

$$e = \frac{99.2\text{ cm} - 3 \times 32.1\text{ cm}}{2} = 1.45\text{ cm}$$

Also, from $v = 2f(l_2 - l_1)$, the frequency of the vibrating tuning fork is given by

$$f = \frac{v}{2(l_2 - l_1)} = \frac{332\text{ m/s}}{2(99.2 - 32.1) \times 10^{-2}\text{ m}} = 247.4\text{ Hz}$$

Illustration 7.41 **A simple apparatus for demonstrating resonance in an air column is depicted in Fig. 7.59. A vertical pipe open at both ends is partially submerge in water, and a tuning fork vibration at an unknown frequency is placed near the top of the pipe. The length L of the air column can be adjusted by moving the pipe vertically. The sound waves generated by the fork are reinforced of the pipe. For a certain pipe, the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm.**

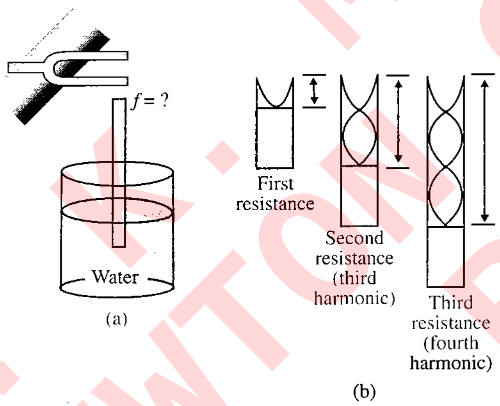

Fig. 7.59

a. **What is the frequency of the tuning fork?**
b. **What are the values of L for the next two resonance conditions?**

Sol. Consider how this problem differs from the preceding problem. In the culvert, the length was fixed and the air column was presented with a mixture of many frequencies. The pipe in this example is presented with one single frequency from the tuning fork, and the length of the pipe is varied until resonance is achieved.

This example is a simple substitution problem. Although the pipe is open at its lower end to allow the water to enter, the water's surface acts like a barrier. Therefore, this setup can be modelled as an air column closed at one end.

The fundamental frequency for $L = 0.090$ m:

$$f_1 = \frac{v}{4L} = \frac{343\text{ m/s}}{4(0.090\text{ m})} = 953\text{ Hz}$$

Because the tuning fork causes the air column to resonate at this frequency, this frequency must also be that of the tuning fork.

b. Use equation $v = \lambda f$ to find the wavelength of the sound wave from the tuning fork.

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{953\text{ Hz}} = 0.360\text{ m}$$

Notice from Fig. 7.59(b) that the length of the air column for the second resonance is $3\lambda/4$.

$$L = 3\lambda/4 = 0.270\text{ m}$$

Notice from Fig. 7.59(b) that the length of the air column for the third resonance is $5\lambda/4$.

$$L = 5\lambda/4 = 0.450\text{ m}$$

Illustration 7.42 **An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature, and a 384 Hz tuning fork is held at the open end. Resonance is heard when the piston is 22.8 cm from the open end and again when it is 68.3 cm from the open end. (a) What speed of sound is implied by these data? (b) How far from the open end will the piston be when the next resonance is heard?**

Sol. The resonance is heard as amplification, when all the air in the tube vibrates in a standing-wave pattern along with the tuning fork.

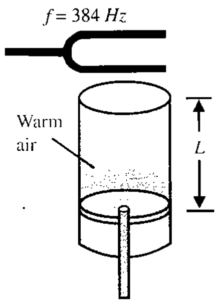

Fig. 7.60

The air vibration has a node at the piston and an antinode at the top end. For an air column closed at one end, resonances will occur when the length of the column is equal to $\lambda/4$, $3\lambda/4$, $5\lambda/4$ and so on. Thus, the change in the length of the pipe from one resonance to the next is $d_{NN} = \lambda/2$. In this case,

$$\lambda/2 = (0.683 - 0.228)\text{ m} = 0.455\text{ m}$$

and $\lambda = 0.910$ m

a. $v = f\lambda = (384\text{ Hz})(0.910\text{ m}) = 349\text{ m/s}$

b. $L = 0.683\text{ m} + 0.455\text{ m} = 1.14\text{ m}$

Illustration 7.43 **An open pipe 40 cm long and a closed pipe 31 cm long, both having same diameter, are producing their first overtone, and these are in unison. Determine the end correction of these pipes.**

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Sol. Let v be the velocity of sound and f be the frequency of the note. Thus, the wavelength of sound produced is

$$\lambda = \frac{v}{f}$$

The wavelength of first overtone of open pipe is

$$\lambda_1 = l + 2e = 40 + 2e$$

$$\frac{v}{f} = 40 + 2e \qquad (\because \lambda_1 = \lambda)$$

The wavelength of first overtone of closed pipe is

$$\frac{3}{4}\lambda_1 = l + e = 31 + e$$

$$\frac{3}{4}\left(\frac{v}{f}\right) = 31 + e \qquad (\because \lambda_1 = \lambda)$$

By eliminating v/f from the above two equations, we get

$$\frac{3}{4}(40 + 2e) = 31 + e$$

$$\Rightarrow \qquad e = 2\ \text{cm}$$

STANDING WAVES IN RODS

Standing waves can also be set up in rods. A rod clamped in the middle and stroked parallel to the rod at one end oscillates as depicted in Fig. 7.61(a). The oscillations of the elements of the rod are longitudinal, and so the curves in Fig. 7.61 represents longitudinal displacements of various parts of the rod. For clarity, the displacements have been drawn in the transverse direction as they were for air columns. The midpoint is a displacement node because it is fixed by the clamp, whereas the ends are displacement antinodes because they are free to oscillate. The oscillations in this setup are analogous to those in a pipe open at both ends. The lines in Fig. 7.61(a) represent the first normal mode, for which the wavelength is $2L$ and the frequency is $f = v/2L$, where v is the speed of longitudinal waves in the rod. Other normal mode may be excited by clamping the rod at different points. For example, the second normal mode (Fig. 7.61b) is excited by clamping the rod a distance $L/4$ away from one end.

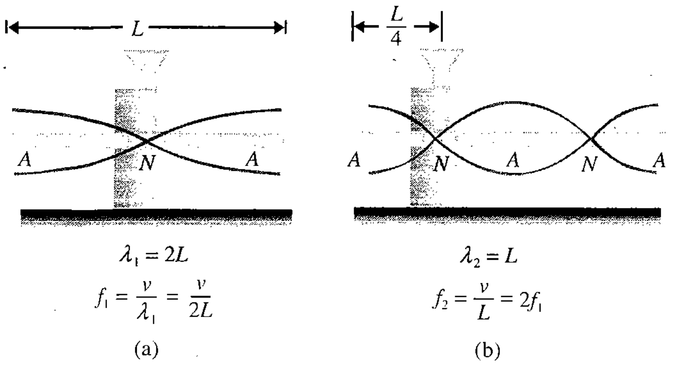

Fig. 7.61

It is also possible to set up transverse standing waves in rods included triangles, marimbas, xylophones, glockenspiels, chimes, and vibraphones. Other devices that make sounds from vibrating bars include music boxes and wind chimes.

Illustration 7.44 **An aluminium rod 1.60 m long is held at its centre. It is stroked with a rosin-coated cloth to set up a longitudinal vibration. The speed of sound in thin rod of aluminium is 5100 m/s. (a) What is the fundamental frequency of the waves established in the rod? (b) What harmonics are set up in the rod held in this manner? (c) What would be the fundamental frequency if the rod were copper, in which the speed of sound is 3560 m/s?**

Sol. Standing waves are important because any wave confined to a restricted region of space will be reflected back onto itself by the boundaries of the region. Then the travelling waves moving in opposite directions constitute a standing wave. The rod can sing at a few hundred hertz and at integer-multiple higher harmonics. The frequency will be proportionately lower with copper.

We must identify where nodes and antinodes are and use the fact that antinodes are separated by half a wavelength with $v = f\lambda$.

a. Since the central clamp establishes a node at the centre, the fundamental node of vibration will be ANA. Thus the rod length is $L = d_{AA} = \lambda/2$.

Our first harmonic frequency is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{5100\ \text{m/s}}{3.20\ \text{m}} = 1.59\ \text{kHz}$$

b. Since the rod is free at each end, the ends will be antinodes. The next vibration state will not have just one more node and one more antinode, reading ANANA, as shown in the diagram, with a wavelength and frequency of

$$\lambda = \frac{2L}{3} \quad \text{and} \quad f = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_1$$

Since $f_2 = 2f_1$ was bypassed as having an antinode at the centre rather than the node required, we know that we get only odd harmonics.

$$f = \frac{nv}{2L} = n(1.59\ \text{kHz})$$

for $\qquad n = 1, 3, 5, \ldots$

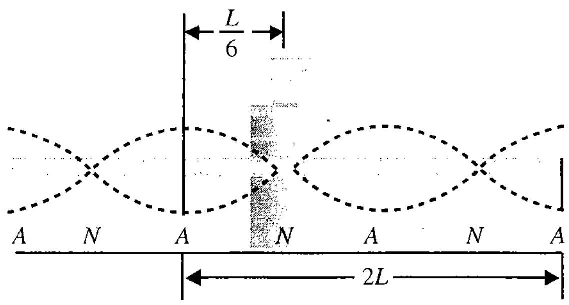

Fig. 7.62

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c. For a copper rod, the density is higher, so the speed of sound is lower, and the fundamental frequency is lower.

$$f_1 = \frac{v}{2L} = \frac{3560 \text{ m/s}}{3.20 \text{ m}} = 1.11 \text{ kHz}$$

The sound is not at just a few hundred hertz, but is a squeak or a squeal at over a thousand hertz. Only a few higher harmonics are in the audible range. Sound really moves fast in materials that are stiff against compression. For a thin rod, it is Young's modulus that determines the speed of a longitudinal wave.

Illustration 7.45 **A metallic rod of length 1 m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the midpoint and those of constituent waves in the rod. ($Y = 2 \times 10^{11}$ N/m² and $\rho = 8 \times 10^3$ kg/m³)**

Sol. As found in case of strings, in case of rods also the clamped point behaves as a node while the free end antinode. The situation is shown in Fig. 7.63.

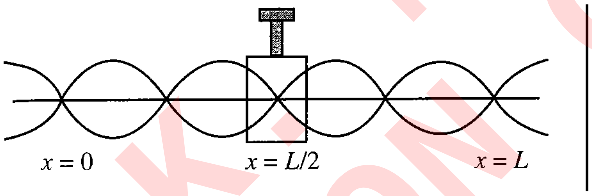

Fig. 7.63

Because the distance between two consecutive nodes is $(\lambda/2)$ while between a node and antinode is $\lambda/4$, hence

$$4 \times \left[\frac{\lambda}{2}\right] + 2\left[\frac{\lambda}{4}\right] = L \quad \text{or} \quad \lambda = \frac{2 \times 1}{5} = 0.4 \text{ m}$$

Further, it is given that

$Y = 2 \times 10^{11}$ N/m² and $\rho = 8 \times 10^3$ kg/m³

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8 \times 10^3}} = 5000 \text{ m/s}$$

Hence, from $v = n\lambda$,

$$n = \frac{v}{\lambda} = \frac{5000}{0.4} = 12500 \text{ Hz}$$

Now if incident and reflected waves along the rod are

$$y_1 = A \sin(\omega t - kx) \quad \text{and} \quad y_2 = A \sin(\omega t + kx + \phi)$$

The resultant wave will be

$$y = y_1 + y_2 = A\,[\sin(\omega t - kx) + \sin(\omega t + kx + \phi)]$$

$$= 2A \cos\left(kx + \frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$

Because there is an antinode at the free end of the rod, hence amplitude is maximum at $x = 0$. So

$$\cos\left(k \times 0 + \frac{\phi}{2}\right) = \text{Maximum} = 1 \quad \text{i.e.,} \quad \phi = 0$$

And

$$A_{max} = 2A = 2 \times 10^{-6} \text{ m (given)}$$

$$y = 2 \times 10^{-6} \cos kx \sin \omega t$$

$$y = 2 \times 10^{-6} \cos\left[\frac{2\pi x}{\lambda}\right] \sin(2\pi nt)$$

Putting values of λ and n, we get

$$y = 2 \times 10^{-6} \cos 5\pi x \sin 25000\pi t$$

Now, because for a point 2 cm from the midpoint $x = (0.50 \pm 0.02)$, hence

$$y = 2 \times 10^{-6} \cos 5\pi(0.5 \pm 0.02) \sin 25000\pi t$$

Illustration 7.46 **A metal rod of length l = 100 cm is clamped at two points A and B as shown in Fig. 7.64. Distance of each clamp from nearer end is a = 30 cm.**

If density and Young's modulus of elasticity of rod material are ρ = 9000 kg/m³ and Y = 144 GPa, respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

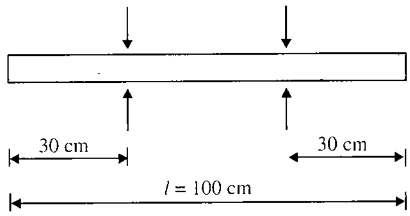

Fig. 7.64

Sol. Speed of longitudinal waves in the rod is

$$v = \sqrt{\frac{Y}{\rho}} = 4000 \text{ m/s}$$

Since points A and B are clamped, therefore, nodes are formed at these points or rod oscillates with integer number of loops in the middle part. Let number of these loops be m.

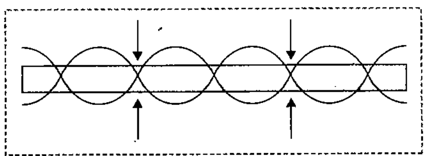

Fig. 7.65

Since, length of each loop is $\lambda/2$, therefore,

$$m\lambda/2 = (l - 2a)$$

or $$m\lambda = 80 \text{ cm} \tag{i}$$

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Since the ends of the rod are free, therefore, antinodes are formed at each end of the rod or at one end of each end part is an antinode and at the other end is a node. It means that number of loops in each end part will be an odd multiple of half.

Let these be $(2n-1)/2$ where n is an integer.

Then,

$$\left(\frac{2n-1}{2}\right)\frac{\lambda}{2}=a \quad \text{or} \quad (2n-1)\lambda=120 \text{ cm} \qquad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii),

$$\frac{m}{(2n-1)}=\frac{2}{3} \qquad \text{(iii)}$$

Minimum possible frequency corresponds to maximum possible wavelength, hence, minimum number of loops.

Hence, from Eq. (iii), for minimum frequency m should be equal to 2 and $(2n-1)$ should be equal to 3 or $n=2$.

Substituting $m=2$ in Eq. (i),

Maximum wavelength $\lambda_0 = 40$ cm

Minimum frequency,

$$f_0=\frac{v}{\lambda_0}=10 \text{ kHz}$$

Next higher frequency corresponds to next higher integer values of m and n which satisfy Eq. (iii). Hence, for this case $m=6$ and $(2n-1)=9$ or $n=5$.

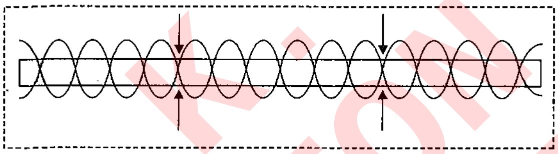

Fig. 7.66

Substituting $m=6$ in Eq. (i),

$$\lambda=\frac{80}{6} \text{ cm} \quad \text{or} \quad \frac{40}{3} \text{ cm}$$

Therefore, next higher frequency,

$$f_1=\frac{v}{\lambda}=30 \text{ kHz}$$

(It means rod oscillates with odd harmonics.)

BEATS: INTERFERENCE IN TIME

The interference phenomena we have studied so far involve the superposition of two or more waves having the same frequency. Because the amplitude of the oscillation of elements of the medium varies with the position in space of the element in such a wave refer to the phenomenon as spatial interference. Standing waves in string and pipes are common examples of spatial interference.

Now let's consider another type of interference, one that results from the superposition of two waves having slightly different frequencies. In this case, when the two waves are observed at a point in space, they are periodically in and out of phase. That is, there is a temporal (time) alternation between constructive and destructive interference. As a consequence, we refer to this phenomenon as interference in time or temporal interference. For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of periodically varying amplitude. This phenomenon is called beating.

Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies.

The number of amplitude maxima one hears per second, or the beat frequency, equals the difference in frequency between the two sources as we shall show below.

Consider two sound waves of equal amplitude travelling through a medium with slightly different frequencies f_1 and f_2. We use equations similar to equation $y = A \sin (kx - \omega t)$ to represent the wave functions for these two waves at a point that we choose so that $kx = \pi/2$:

$$y_1 = A\sin\left(\frac{\pi}{2}-\omega_2 t\right) = A\cos(2\pi f_1 t)$$

$$y_2 = A\sin\left(\frac{\pi}{2}-\omega_2 t\right) = A\cos(2\pi f_2 t)$$

Using the superposition principle, we find that the resultant wave function at this point is

$$y = y_1 + y_2 = A(\cos 2\pi f_1 + \cos 2\pi f_2 t)$$

The trigonometric identity

$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

allows us to write the expression for y as

$$y=\left[2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t\right]\cos 2\pi\left(\frac{f_1+f_2}{2}\right)t \qquad \text{(i)}$$

Graphs of the individual waves and the resultant wave are shown in Fig. 7.67. From the factors in Eq. (i), we see that the resultant wave has an effective frequency equal to the average frequency $(f_1+f_2)/2$. This wave is multiplied by an envelope wave given by the expression in the square brackets:

$$y_{\text{envelope}} = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t \qquad \text{(ii)}$$

That is, the amplitude and therefore the intensity of the resultant sound vary in time. The dashed line in Fig. 7.67(b) is a graphical representation of the envelope wave in Eq. (ii) and is a sine wave varying with frequency $(f_1-f_2)/2$.

A maximum in the amplitude of the resultant sound wave is detected whenever

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t=\pm 1$$

Hence, there are two maxima in each period of the envelope wave. Because the amplitude varies with frequency as $(f_1-f_2)/2$, the number of beats per second, or the beat frequency f_{beat}, is twice this value. That is,

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$$f_{beat} = |f_1 - f_2| \quad \text{(iii)}$$

For instance, if one tuning fork vibrates at 438 Hz and a second one vibrates at 442 Hz, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note *A*) and a beat frequency of 4 Hz. A listener would hear a 440 Hz sound wave go through an intensity maximum four times every second.

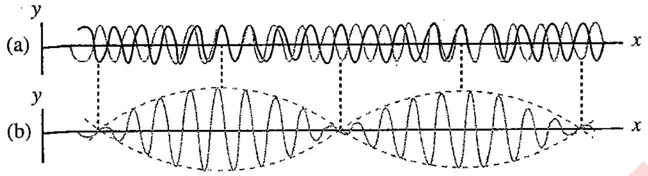

Fig. 7.67

Illustration 7.47 **Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamental of the two strings?**

Sol. As the tension in one of the strings is changed, its fundamental frequency changes. Therefore, when both strings are played, they will have different frequencies and beats will be heard.

We must combine our understanding of the waves under boundary conditions model for strings with our new knowledge of beats.

Set up a ratio of the fundamental frequencies of the two strings using equation

$$\frac{f_2}{f_1} = \frac{(v_2/2L)}{(v_1/2L)} = \frac{v_2}{v_1}$$

Use equation $v = \sqrt{T/\mu}$ to substitute for the wave speeds on the strings.

$$\frac{f_2}{f_1} = \frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}} = \sqrt{\frac{T_2}{T_1}}$$

Incorporate that the tension in one string is 1.0% larger than the other; that is, $T_2 = 1.010\,T_1$.

$$\frac{f_2}{f_1} = \sqrt{\frac{1.010T_1}{T_1}} = 1.005$$

Solve for the frequency of the tightened string:

$$f_2 = 1.005 f_1 = 1.005\ (440\text{ Hz}) = 442\text{ Hz}$$

Find the beat frequency using Eq. (iii)

$$f_{beat} = 442\text{ Hz} - 440\text{ Hz} = 2\text{ Hz}$$

Notice that a 1.0% mistuning in tension leads to an easily audible beat frequency of 2 Hz. A piano tuner can use beats to tune a stringed instrument by 'beating' a note against a reference tone of known frequency. The tuner can then adjust the string tension until the frequency of the sound it emits equals the frequency of the reference tone. The tuner does so by tightening or loosening the string until the beats produced by it and the reference source become too infrequent to notice.

Illustration 7.48 **Wavelength of two notes in air is (90/175) m and (90/173) m, respectively. Each of these notes produces 4 beats/s with a third note of a fixed frequency. Calculate the velocity of sound in air.**

Sol. Given: $\lambda_1 = 90/175$ m and $\lambda_2 = 90/173$ m

If f_1 and f_2 are the corresponding frequencies and v is the velocity of sound in air, we have

$$v = f_1\lambda_1 \quad \text{and} \quad v = f_2\lambda_2$$

$$f_1 = \frac{v}{\lambda_1} \quad \text{and} \quad f_2 = \frac{v}{\lambda_2}$$

Since, $\lambda_1 < \lambda_2$, we must have $f_1 > f_2$.

If f is the frequency of the third note, then

$$f_1 - f = 4 \quad \text{and} \quad f - f_2 = 4$$

$$\Rightarrow \quad f_1 - f_2 = 8$$

$$\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$$

$$v\left[\frac{175}{90} - \frac{173}{90}\right] = 8$$

$$v = 360\text{ m/s}$$

Illustration 7.49 **In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strikes the two strings simultaneously?**

Sol. Directly noticeable beat frequencies are usually only a few hertz, so we should not expect a frequency much greater than this.

Combining the velocity and the tension equations $v = f\lambda$ and $v = \sqrt{T/\mu}$, we find that the frequency is

$$f = \sqrt{\frac{T}{\mu\lambda^2}}$$

Since μ and λ are constant, we can apply that equation to both frequencies, and then divide the two equations to get the proportion.

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$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

With $f_1 = 110$ Hz, $T_1 = 600$ N, and $T_2 = 540$ N, we have $f_2 = (110 \text{ Hz})\sqrt{\frac{540N}{600N}} = 104.4$ Hz

The beat frequency is $f_b = |f_1 - f_2| = 110 \text{ Hz} - 104.4 \text{ Hz} = 5.6$ Hz

As expected, the beat frequency is only a few cycles per second.

Illustration 7.50 **Two wires are welded together end to end. The wires are made of the same material, but the diameter of one is twice that of the other. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present, with the node between them being precisely at the weld. (a) What is the frequency of vibration? (b) Find the length of the thick wire.**

Sol. The mass per volume density, the tension, and the frequency must be the same for the two wires. The linear density, wave speed, wavelength and node-to-node distance are different.

We know enough about the thin wire to find the frequency. The thick wire will have a predictably higher linear density, which will tell us the node-to-node distance for it.

a. Since the first node is at the weld, the wavelength in the thin wire is

$$\lambda = 2L = 80.0 \text{ cm}$$

The frequency and tension are the same in both sections, so

$$f = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400 \text{ m})}\sqrt{\frac{4.60 \text{ N}}{0.00200 \text{ kg/m}}} = 59.9 \text{ Hz}$$

b. Since the thick wire is twice the diameter, it will have four times the cross-sectional area, and a linear density μ' that is four times that of the thin wire. $\mu' = 4(2.00 \text{ g/m}) = 0.008\ 00$ kg/m.

$$L' = \frac{\lambda'}{2} = \frac{v'}{2f} = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$$

$$= \frac{1}{2(59.9 \text{ Hz})}\sqrt{\frac{4.60 \text{ N}}{0.00800 \text{ kg/m}}} = 20.0 \text{ cm}$$

Note that the thick wire is half the length of the thin wire. We could have reasoned the answer by noting that the wave speed on the thick wire is half as large, so the wavelength should be half as large for the same frequency.

Concept Application Exercise 7.2

1. If two waves of the same frequency differ in amplitude and are propagated in opposite directions through a medium, will they produce standing waves? Is energy transported? Are there any nodes?
2. If two sound waves of frequencies 500 Hz and 550 Hz superimpose, will they produce beats? Would you hear the beats?
3. All harmonics are overtones but all overtones are not harmonics. Explain.
4. Resonance produces louder sound than that produced by the forced vibrations of a body. Why then is resonance purposely avoided in many instruments?
5. If f_1 and f_2 be the fundamental frequencies of the two segments into which a stretched string is divided by means of a bridge, then find the original fundamental frequency f of the complete string.
6. Two tuning forks A and B produce 4 beats/s when sounded together. A resonates to 32.4 cm of stretched wire and B is in resonance with 32 cm of the same wire. Determine the frequencies of the two tuning forks.
7. A glass tube of length 1.5 m is filled completely with water; the water can be drained out slowly at the bottom of the tube. Find the total number of resonance obtained, when a tuning fork of frequency 606 Hz is put at the upper open end of the tube, $v_{sound} = 340$ m/s.
8. Calculate the speed of sound in a gas in which two waves of wavelengths 50 cm and 50.5 cm produce 6 beats/s.
9. A stationary wave is given by
$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t$$
where x and y are in cm and t is in second.
   a. What are the amplitude and velocity of the component wave whose superposition can give rise to this vibration?
   b. What is the distance between the nodes?
   c. What is the velocity of a particle of the string at the position $x = 1.5$ cm when $t = 9/8$ s?
10. In Quinck's acoustic interferometer, it is found that the sound intensity has a minimum value of 100 units at one position of the sliding tube, and continuously climbs to a maximum of 900 units at a second position 1.65 cm from the first. Find (a) the frequency of the sound emitted by the source and (b) the relative amplitudes of the two waves arriving at the detector. Velocity of sound in air = 340 m/s.
11. Two tuning forks produce 5 beats when sounded together. A is in unison with 40 cm length of a sonometer wire under a constant tension and B is in unison with the same wire of length 40.5 cm under the same tension. Calculate the frequencies of the forks.

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12. Find the ratio of the fundamental frequencies of two identical strings after one of them is stretched by 2% and the other by 4%.
13. A wire of density 9000 kg/m^3 is stretched between two clamps 100 cm apart while subjected to an extension of 0.05 cm. What is the lowest frequency of transverse vibrations in the wire, assuming Young's modulus of the material to be 9×10^{10} N/m^2?
14. An open organ pipe has a fundamental frequency of 300 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. Find length of each pipe. The velocity of sound in air = 350 m/s.
15. A wire of uniform cross-section is suspended vertically from a fixed point, with a load attached at the lower end. Calculate the fractional change in frequency of the wire due to rise in temperature by t°C. The coefficient of expansion of the wire is α.
16. A tuning fork is found to give 20 beats in 12 s when sounded in conjunction with a stretched string vibrating under a tension of 10.2 or 9.9 kgf. Calculate the frequency of the fork.
17. A wire is in unison with a tuning fork when stretched by a weight of density 9000 kg/m^3 in a sonometer experiment. When the weight is immersed in water, the same wire produces 5 beats/s with the same fork. Find the frequency of the fork.
18. A tuning fork of frequency 256 Hz and an open organ pipe of slightly lower frequency are at 17°C. When sounded together, they produce 4 beats/s. On altering the temperature of the air in the pipe, it is observed that the number of beats per second first diminishes to zero and then increases again to 4. By how much and in what direction has the temperature of the air in the pipe been altered?
19. Show that the period of the fundamental mode of a stretched string is equal to double the time the component waves forming stationary waves in the string take in traversing the distance between the fixed ends.
20. A pipe is closed at one end by a membrane which may be considered a seat of displacement node and provided with a piston at the other end. The membrane is set to sonic oscillations of frequency 2000 Hz. Find the velocity of sound if on moving the piston, resonance occurs at the interval of 8.5 cm.
21. An open organ pipe of length 11 cm in its fundamental mode vibrates in resonance with the first overtone of a closed organ pipe of length 13.2 cm filled with some gas. If the velocity of sound in the air is 330 m/s, calculate the velocity of sound in the unknown gas.
22. A string of length 25 cm is stretched by a load of 10 kg. What is the highest overtone that a man of normal hearing capacity can detect? The mass of the string is
23. A tuning fork A is in resonance with an air column 32 cm long and closed at one end. When the length of this column is increased by 1 cm, it is in resonance with another fork B. When A and B are sounded together, they produce 40 beats in 5 s. Find their frequencies.
24. Find the fundamental frequency and the first four overtones of a 15 cm pipe (a) if the pipe is closed at one end, and (b) if the pipe is open at both ends. (c) How many overtones may be heard by a person of normal hearing in each of the above cases? Velocity of sound in air = 330 m/s.
25. A steel wire of length 1 m and density 8000 kg/m^3 is stretched tightly between two rigid supports. When vibrating in its fundamental mode, its frequency is 200 Hz.
    a. What is the velocity of transverse wave along this wire?
    b. What is the longitudinal stress in the wire?
    c. If the maximum acceleration of the wire is 800 m/s^2, what is the amplitude of vibration at the midpoint?
26. A wire of diameter 0.04 cm and made of steel of density 8000 kg/m^3 is under a tension of 80 N. A fixed length of 50 cm is set into transverse vibrations. How would you cause vibrations of frequency 840 Hz to predominate in intensity?
27. A tube closed at one end has a vibrating diaphragm at the other end, which may be assumed to be a displacement node. It is found that when the frequency of the diaphragm is 2000 Hz, a stationary wave pattern is set up in which the distance between adjacent nodes is 8 cm. When the frequency is gradually reduced, the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate
    i. the speed of sound in air,
    ii. the distance between adjacent nodes at a frequency of 1600 Hz,
    iii. the distance between the diaphragm and the closed end,
    iv. the next lower frequencies at which stationary wave patterns will be obtained.
28. Two sonometer wires of the same material and cross-section are of lengths 50 cm and 60 cm and are stretched by tensions of 4.5 kg and 5.12 kg, respectively. If the number of beats heard (when the two wires are vibrating) be 2 per second, find the mass per unit length of the wires. Take g = 10 m/s^2.
29. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum vibration. The note emitted has a frequency of 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum vibration?

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30. A piano string 1.5 m long is made of steel of density $7.7 \times 10^3\ \text{kg/m}^3$ and $Y = 2 \times 10^{11}\ \text{N/m}^2$. It is maintained at a tension which produces an elastic strain of 1% in the string. What is the fundamental frequency of transverse vibration of the string?

31. Two sound waves travelling in the same direction are superposed. Their frequencies are 300 and 302 Hz and their amplitudes are 0.2 and 0.3 mm, respectively
   a. What is the number of beats per second?
   b. What are the maximum and minimum values of resultant amplitude during the formation of beats?
   c. Calculate the ratio of maximum and minimum intensities of the resultant sound.

32. Two tuning forks A and B when sounded together produce 3 beats/s. What are the possible frequencies of B, if the frequency of A is 400 cycle/s? how can you verify which of the possible values is correct?

Solved Examples

Example 7.1 **A standing wave is set up in a string of variable length and tension by a vibrator of variable frequency. Both ends of the string are fixed. When the vibrator has a frequency f, in a string of length L and under tension T, n antinodes are set up in the string. (a) If the length of the string is doubled, by what factor should the frequency be changed so that the same number of antinodes is produced? (b) If the frequency and length are held constant, what tension will produce $n + 1$ antinodes? (c) If the frequency is tripled and the length of the string is halved, by what factor should the tension be changed so that twice as many antinodes are produced?**

Sol. In (a), we expect a lower frequency to go with a longer wavelength. In (b), lower tension should go with lower wave speed for shorter wavelength at constant frequency. In (c), we will just have to divide it out.

a. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad \text{(i)}$$

Keeping n, T and μ constant, we can create two equations.

$$f_n L = \frac{n}{2}\sqrt{\frac{T}{\mu}} \quad \text{and} \quad f_n' L' = \frac{n}{2}\sqrt{\frac{T}{\mu}}$$

Dividing the equations gives

$$\frac{f_n}{f_n'} = \frac{L'}{L}$$

If $L' = 2L$, then $f_n' = 1/2 f_n$

Therefore, in order to double the length but keep the same number of antinodes, the frequency should be halved.

b. From Eq. (i), we can hold L and f_n constant to get

$$\frac{n'}{n} = \sqrt{\frac{T}{T'}}$$

From this relation, we see that the tension must be decreased to

$$T' = T\left(\frac{n}{n+1}\right)^2 \text{ to produce } n+1 \text{ antinodes}$$

c. The time, we rearrange Eq. (i) to produce

$$\frac{2f_nL}{n} = \sqrt{\frac{T}{\mu}} \quad \text{and} \quad \frac{2f_n'L'}{n'} = \sqrt{\frac{T'}{\mu}}$$

Then dividing gives

$$\frac{T'}{T} = \left(\frac{f_n'}{f_n} \times \frac{n}{n'} \times \frac{L'}{L}\right)^2 = \left(\frac{3f_n}{f_n}\right)^2 \times \left(\frac{n}{2n}\right)^2 \times \left(\frac{L/2}{L}\right)^2 = \frac{9}{16}$$

Example 7.2 **The water level in a vertical glass tube 1.0 m long can be adjusted to any position in the tube. A tuning fork vibrating at 660 Hz is held just over the open top end of the tube. At what positions of the water level will there be resonance. Speed of sound is 330 m/s.**

Sol. Resonance corresponds to a pressure antinode at closed end and pressure node at open end. Further, the distance between a pressure node and a pressure antinode is $\lambda/4$, the condition of resonance would be,

length of air column

$$l = n\frac{\lambda}{4} = n\left(\frac{v}{4f}\right)$$

Here, $n = 1, 3, 5, \ldots$

$$l_1 = (1)\left(\frac{330}{4 \times 660}\right) = 0.125 \text{ m}$$

and $l_2 = 3l_1 = 0.375$ m

$l_3 = 5l_1 = 0.625$ m

and $l_4 = 7l_1 = 0.875$ m

$l_5 = 9l_1 = 1.125$ m

Since $l_5 > 1$ m (the length of tube), the length of air columns can have the values from l_1 to l_4 only. Therefore, level of water at resonance will be

$(1.0 - 0.125)$ m = 0.875 m

$(1.0 - 0.375)$ m = 0.625 m

$(1.0 - 0.625)$ m = 0.375 m

and $(1.0 - 0.875)$ m = 0.125 m

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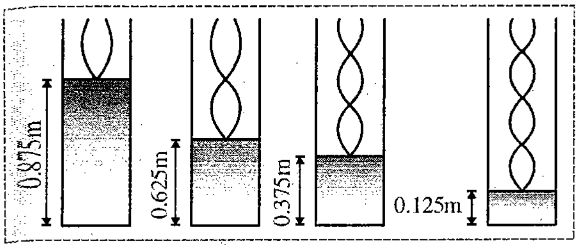

Fig. 7.68

In all the four cases shown in Fig. 7.68, the resonance frequency is 660 Hz but the first one is the fundamental tone or first harmonic. The second is first overtone or the third harmonic and so on.

Example 7.3 **Two radio stations broadcast their programmes at the same amplitude A, and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_2 - \omega_1 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can emit signals only of intensity $\geq 2A^2$.**

i. Find the time intervals between successive maxima of the intensity of the signal received by the detector.

ii. Find the time for which the detector remains idle in each cycle of the intensity of the signal.

Sol.

i. Let the signal waves be given by

$$y_1 = A \sin 2\pi\omega_1 t, \quad y_2 = A \sin 2\pi\omega_2 t.$$

The resultant disturbance is given by

$$y = y_1 + y_2 = A \sin 2\pi\omega_1 t + A \sin 2\pi\omega_2 t$$

$$= 2A \sin\frac{2\pi(\omega_1+\omega_2)t}{2}\cos\frac{2\pi(\omega_2-\omega_1)t}{2}$$

$$= 2A \cos\pi(\omega_2-\omega_1)t \sin 2\pi\frac{(\omega_1+\omega_2)t}{2}$$

Let $\omega_1 = \omega, \omega_2 = \omega + \Delta\omega$

Therefore, $\omega_1 + \omega_2 \approx 2\omega$

$$y = 2A \cos\pi(\omega_2 - \omega_1)t \sin 2\pi\omega t$$

Thus, the resultant disturbance has amplitude $2A \cos \pi(\omega_2 - \omega_1)t$

For maxima: $\cos\pi(\omega_2 - \omega_1)t = \pm 1$

or $\quad \pi(\omega_2 - \omega_1)t = r\pi,\ r = 0, 1, 2, 3$

$$t = \frac{r}{\omega_2-\omega_1} = 0, \frac{1}{\omega_2-\omega_1}, \frac{2}{\omega_2-\omega_1}, \ldots$$

Clearly time interval between successive maxima

$$= \frac{1}{\omega_2-\omega_1} = \frac{1}{10^3} = 10^{-3} \text{ s}$$

ii. The resultant intensity is given by

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

When $\delta = 0$, intensity is maximum $I_{max} = 4A^2$

When $\Delta = \pi/2$, intensity $I = 2A^2$

When $\Delta = \pi$, intensity $I_{min} = 0$

When $\Delta = 3\pi/2$, intensity $I = 2A^2$

When $\Delta = 2\pi$, intensity $I_{max} = 4A^2$

The detector remaining idle from $\theta = \pi/2$ to $3\pi/2$ or in each half cycle. Hence the required time

$$t = \frac{T}{2} = \frac{10^{-3}}{2} = 5 \times 10^{-4} \text{ s}$$

Example 7.4 **A metal wire of diameter 1 mm is held on two knife edges by a distance 50 cm. The tension in the wire is 100 N. The wire vibrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats/s. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. Calculate**

i. the frequency of the fork and

ii. the density of material of wire

Sol. Let the frequency of the tuning fork be n. When tension in the string is decreased, the number of beats remains unchanged. This means initially the frequency of metal wire is higher than that of fork. As number of beats = 5 per second.

Therefore initial frequency of wire $n_1 = n + 5$

So, $\quad n_1 = n + 5 = \frac{1}{2l}\sqrt{\left(\frac{T_1}{m}\right)} = \frac{1}{2\times 0.5}\sqrt{\left(\frac{100}{m}\right)} \quad$ (i)

[Since T = 100 N, l = 50 cm = 0.50 m]

When tension is reduced to T_2 = 81 N, the frequency of wire becomes $n_2 = n - 5$.

Therefore, $\quad n_2 = n - 5 = \frac{1}{2l}\sqrt{\left(\frac{T_2}{m}\right)} = \frac{1}{2\times 0.5}\sqrt{\left(\frac{81}{m}\right)} \quad$ (ii)

i. Dividing Eq. (i) by Eq. (ii), we get

$$\frac{n+5}{n-5} = \sqrt{\left(\frac{100}{81}\right)} = \frac{10}{9}$$

Solving frequency of fork, n = 95 cycles/s.

ii. We have m = mass per unit length $= Ad = \pi r^2 d$.

Substituting this in Eq. (i), we get

$$n + 5 = \frac{1}{2\times 0.5}\sqrt{\left(\frac{100}{\pi r^2 d}\right)}$$

here $\quad n = 95,\ r = \frac{1 \text{ mm}}{2} = \frac{1}{2}\times 10^{-3} \text{ m} = 5\times 10^{-4} \text{ m}$

Putting all values $\quad 95 + 5 = \frac{1}{1.0}\times\frac{10}{\left[\sqrt{3.14\times(5\times10^{-4})^2 d}\right]}$

$$d = 12.7 \times 10^3 \text{ kg/m}^3$$

Example 7.5 **An aluminium wire of cross-sectional area 1×10^{-6} m² is joined to a steel wire of the same cross-sectional area. This compound wire is stretched on a sonometer, pulled by a weight of 10 kg. The total length**

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of the compound wire between the bridges is 1.5 m of which the aluminium wire is 0.6 m and the rest is steel wire. Transverse vibrations are set up in the wire by using an external source of variable frequency. Find the lowest frequency of excitation for which standing waves are formed, such that the joint in the wire is a node. What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire? The density of aluminium is 2.6×10^3 kg/m³ and that of steel is 1.04×10^4 kg/m³

Sol. Let l_1 and l_2 be the lengths of aluminum and steel wires, respectively.

Given $\quad l_1 = 0.6$ m, $l_1 + l_2 = 1.5$ m

$\therefore \quad l_2 = 1.5 - 0.6 = 0.9$ m

If aluminium wire vibrates in p_1 loops and steel wire in p_2 loops, then frequency n is give by

$$n = n_1 = \frac{p_1}{2l_1}\sqrt{\left(\frac{T}{m_1}\right)} = \frac{p_1}{2l_1}\sqrt{\left(\frac{T}{A\rho_1}\right)}$$

Also,
$$n = n_2 = \frac{p_2}{2l_2}\sqrt{\left(\frac{T}{m_2}\right)} = \frac{p_2}{2l_2}\sqrt{\left(\frac{T}{A\rho_2}\right)}$$

We have $\quad n_1 = n_2 = n$, so

$$\frac{p_1}{2l_1}\sqrt{\left(\frac{T}{A\rho_1}\right)} = \frac{p_2}{2l_2}\sqrt{\left(\frac{T}{A\rho_2}\right)}$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{l_1}{l_2}\sqrt{\left(\frac{\rho_1}{\rho_2}\right)} = \frac{0.6}{0.9}\sqrt{\left(\frac{2.6\times10^3}{1.04\times10^4}\right)}$$

$$= \frac{0.6}{0.9}\sqrt{\left(\frac{1}{4}\right)} = \frac{6}{9}\times\frac{1}{2} = \frac{3}{9} = \frac{1}{3}$$

$\therefore \quad p_1 : p_2 \equiv 1 : 3$

The minimum number of loops in aluminium wire = 1 and minimum number of loops in steel wire = 3.

The stationary vibrations in composite wire as shown in Fig. 7.69. Obviously total number of nodes = 5.

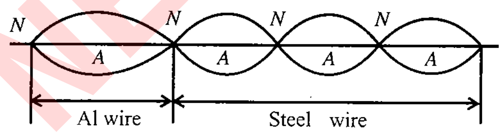

Fig. 7.69

The number of nodes excluding the two at the ends = 5 – 2 = 3.
Thus, the lowest frequency

$$n = \frac{1}{2l_1}\sqrt{\left(\frac{T}{m_1}\right)} = \frac{1}{2\times0.6}\sqrt{\left(\frac{10\times9.8}{1\times10^{-6}\times2.6\times10^3}\right)} = 162 \text{ Hz}$$

Example 7.6 **A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelengths 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?**

Sol. Tension at the lower end of the rope,

$$T_1 = 2g = 2 \times 9.8 = 19.6 \text{ N}$$

Tension at the upper end of rope,

$$T_2 = (2 + 6)\, g = 8 \times 9.8 = 78.4 \text{ N}$$

Let v_1 and v_2 be the speeds of pulse at the lower and upper end, respectively. So

$$v_1 = \sqrt{\left(\frac{T_1}{m}\right)}, \quad v_2 = \sqrt{\left(\frac{T_2}{m}\right)}$$

On dividing, we get

$$\frac{v_2}{v_1} = \sqrt{\left(\frac{T_2}{T_1}\right)} = \sqrt{\left(\frac{78.4}{19.6}\right)} = \sqrt{4} = 2$$

As frequency is independent of medium, therefore if λ_1 and λ_2 are wavelengths at lower and upper ends respectively, Then

$$v_1 = n\lambda_1 \quad \text{and} \quad v_2 = n\lambda_2$$

So, $$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = 2$$

Therefore, the wavelength of pulse at upper end = $2\lambda_1$

$$= 2 \times 0.06 = 0.12 \text{ m}$$

Example 7.7 **The vibrations of a string of length 60 cm fixed at both ends are represented by the equation**

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where x and y are in cm and t in second.

i. What is the maximum displacement at $x = 5$ cm?
ii. Where are the nodes located along the string?
iii. What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s?
iv. Write down the equations of component waves whose superposition gives the above waves.

Sol. The given equation for standing waves in the string is

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t) \qquad \text{(i)}$$

i. The amplitude of the waves is given by

$$A = 4\sin\frac{\pi x}{15} \qquad \text{(ii)}$$

Therefore, the maximum displacement or amplitude at $x = 5$ cm is

$$A = 4\sin\frac{\pi\times5}{15} = 4\sin\frac{\pi}{3}$$

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$$= 4\sin 60° = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3} = 2\times 1.732 = 3.464 \text{ cm}$$

ii. The position of zero displacement or nodes are given by

$$\sin\frac{\pi x}{15} = 0 \quad\text{or}\quad \frac{\pi x}{15} = r\pi \quad (\text{where } r = 0, 1, 2, 3, \ldots)$$

$\Rightarrow x = 15\,r \Rightarrow x = 0, 0.15$ cm, 0.30 cm, ...

iii. Differentiating Eq. (i) with respect to t, we get velocity of particle

$$u = \frac{dy}{dt} = -4\times 96\pi \sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

Substituting $x = 7.5$ cm and $t = 0.25$ s.

$$u = -384\pi \sin\left(\frac{\pi\times 7.5}{15}\right)\sin(96\pi\times 0.25)$$

$$= -384\sin(\pi/2)\sin(24\pi) = 0$$

iv. Using the relation

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

Equation (i) may be expressed as

$$y = 2\left[\sin\left\{\frac{\pi x}{15} + (96\pi t)\right\} + 2\sin\left\{\frac{\pi x}{15} - (96\pi t)\right\}\right]$$

$$= 2\sin\left\{\frac{\pi x}{15} + 96\pi t\right\} + 2\sin\left\{\frac{\pi x}{15} - 96\pi t\right\} = y_1 + y_2$$

Therefore, the component waves are given by

$$\left.\begin{aligned} y_1 &= 2\sin\left(96\pi t + \frac{\pi x}{15}\right) \\ \text{and}\quad y_2 &= -2\sin\left(96\pi t - \frac{\pi x}{15}\right)\end{aligned}\right\}$$

Example 7.8 **The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz, find the lengths of the pipes. Take velocity of sound = 330 m/s.**

Sol. Let l_1 and l_2 be lengths of open organ pipe and closed organ pipe respectively.

First overtone of open organ pipe $= 2n_1 = 2\times\frac{v}{2l_1} = \frac{v}{l_1}$

First overtone of closed organ pipe $= 3n_2 = 3\times\frac{v}{4l_2}$

According to question, $\frac{v}{l_1} - \frac{3v}{4l_2} = \pm 2.2$ (i)

As n_2 is the fundamental frequency of closed organ pipe

$$n_2 = \frac{v}{4l_2}$$

$$l_2 = \frac{v}{4n_2} = \frac{330}{4\times 110} = 0.75 \text{ m}$$

From Eq. (i),

$$\frac{v}{l_1} = \frac{3v}{4l_2} \pm 2.2$$

$$\frac{v}{l_1} = 3n_2 \pm 2.2$$

Taking positive sign

$$\frac{v}{l_1} = 3\times 110 + 2.2 = 332.2$$

$$\therefore\quad l_1 = \frac{v}{332.2} = \frac{330}{332.2} \text{ m} = 0.993 \text{ m} = 99.3 \text{ cm}$$

Taking negative sign

$$\frac{v}{l_1} = 3n_2 - 2.2 = 3\times 110 - 2.2 = 327.8$$

$$\therefore\quad l_1 = \frac{v}{327.8} = \frac{330}{327.8} = 1.006 \text{ m} = 100.6 \text{ cm}$$

Example 7.9 **The air in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air 330 m/s. End correction may be neglected. Let P_0 denotes the mean pressure of any point in the pipe and ΔP_0 the maximum amplitudes of pressure variation.**

a. Find the length L of the air column.

b. What is the amplitude of pressure variation at the middle of the column

c. What are maximum and minimum pressures at the open end of the pipe?

Sol.

a. The fundamental frequency of the closed organ pipe $= v/4l$. In closed organ pipe only odd harmonics are present.

Second overtone of pipe $= 5v/4L$

Given $5v/4L = 440$

On solving, we get

$$L = \frac{5v}{4\times 440} = \frac{5\times 330}{4\times 440} = \frac{15}{16} \text{ m} = 0.9375 \text{ m} = 93.75 \text{ cm}$$

b. The equation of variation of pressure amplitude at any distance x from the node is

$$\Delta P = \Delta P_0 \cos kx$$

Pressure variation is maximum at a node and minimum (zero) at antinode.

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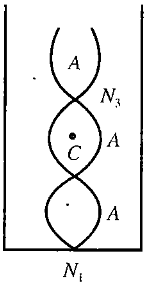

Fig. 7.70

Distance of centre C from N_2 is $\lambda/8$

$$\therefore \quad \Delta P = \Delta P_0 \cos\frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \Delta P_0\frac{\pi}{4} = \frac{\Delta P_0}{\sqrt{2}}$$

c. At antinode, the pressure variation is minimum (zero), therefore at antinode pressure remains equal to P_0 (always).

Therefore, at antinode $P_{max} = P_{min} = P_0$.

Example 7.10 **AB is a cylinder of length 1.0 m fitted with a thin flexible diaphragm C at the middle and two others thin flexible diaphragms A and B at the ends. The portions AC and BC contains hydrogen and oxygen gases, respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which the diaphragm C is a node? Under the conditions of the experiment, the velocity of sound in hydrogen is 1100 m/s and in oxygen is 300 m/s.**

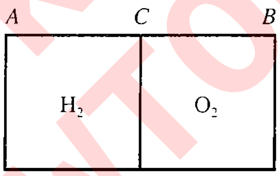

Fig. 7.71

Sol. When diaphragms A and B are set in oscillations, antinodes are formed at A and B while a node is formed at C (given)

Also $\quad AB = 1.0$ m

and $\quad AC = CB = l$ (say) $= 1/2 = 0.5$ m

The portions AC and BC behave as closed pipes.

In a closed pipe, the modes of vibration are given by

$$l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

i.e., $$l = (2r+1)\frac{\lambda}{4}, \quad r = 0, 1, 2, 3, \ldots$$

or $$\lambda = \frac{4l}{2r+1} \quad r = 0, 1, 2, 3, \ldots.$$

For hydrogen $\quad \lambda_1 = 4l/(2r_1+1)$

For oxygen $\quad \lambda_2 = 4l/(2r_2+1)$

In both gases the frequency is same

As $$n_1 = n_2$$

or $$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda} = \frac{l_1}{l_2}\times\frac{(2r_2+1)}{(2r_1+1)}$$

As $$l_1 = l_2 = 0.5 \text{ m}$$

i.e., $$\frac{v_1}{v_2} = \frac{2r_2+1}{2r_1+1}$$

$$= \frac{1100}{300} = \frac{2r_2+1}{2r_1+1}$$

i.e., $$\frac{2r_1+1}{2r_2+1} = \frac{3}{11}$$

For minimum frequency, the integers r_1 and r_2 should be least. Therefore by inspection

$$r_1 = 1 \quad \text{and} \quad r_2 = 5$$

Therefore, the frequency of oscillations is given by

$$n_{min} = \frac{v_1}{\lambda_1} = (2r_1+1)\frac{v_1}{4l}$$

$$= (2\times1+1)\times\frac{1100}{4\times0.5} = \frac{3\times1100}{2} = 1650 \text{ Hz}$$

Example 7.11 **In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz, when minimum length of air column in 16 cm. Find the speed of sound in air at room temperature.**

Sol. Fundamental frequency of air column closed at one end is

$$n = \frac{v}{4(l+e)} = \frac{v}{4(l+0.3D)}$$

Given $\quad n = 480$ Hz, $D = 5$ cm $= 5\times10^{-2}$ m

$l = 16$ cm $= 16\times10^{-2}$ m

$v = 4n\,(l + 0.3D)$

$= 4\times480\,[16\times10^{-2} + 0.3\times5\times10^{-2}]$ m/s

$= 4\times480\times17.5\times10^{-2}$ m/s $= 336$ m/s

Example 7.12 **Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.**

a. If the frequency to the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B; determine the value of M_A/M_B.

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b. Now the open end of pipe B is also closed (so that pipe B is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

Sol.

a. If L is the length of each pipe A and B, then fundamental frequency of pipe A (open at both ends)

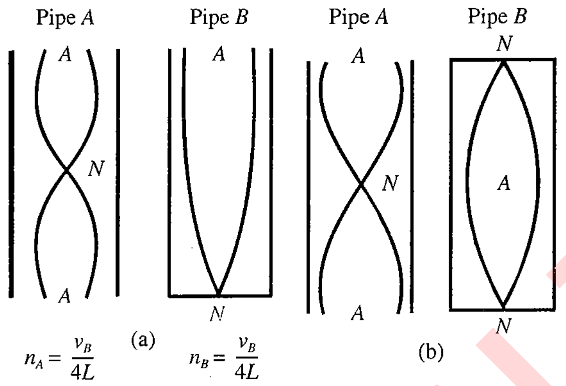

Fig. 7.72

$$n_A = \frac{v_A}{2L} \qquad \text{(i)}$$

Fundamental frequency of pipe B (closed at one end)

$$n_B = \frac{v_B}{4L} \qquad \text{(ii)}$$

Given $\quad 2n_A = 3n_B$

$$2\left(\frac{v_A}{2L}\right) = 3\left(\frac{v_B}{4L}\right)$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{3}{4} \qquad \text{(iii)}$$

But $\quad v_A = \sqrt{\frac{\gamma_A R T_A}{M_A}}; \quad v_B = \sqrt{\frac{\gamma_B R T_B}{M_B}}$

Given $\quad T_A = T_B$

For monoatomic get

$$\gamma_A = \frac{5}{3}$$

For diatomic get

$$\gamma_B = \frac{7}{5}$$

$$\therefore \quad \frac{v_A}{v_B} = \sqrt{\left\{\frac{\gamma_A}{\gamma_B}\frac{M_B}{M_A}\right\}}$$

$$= \sqrt{\frac{(5/3)\, M_B}{(7/5)\, M_A}} =$$

$$= \sqrt{\frac{25\, M_B}{21\, M_A}} \qquad \text{(iv)}$$

From Eqs. (iii) and (iv)

$$\sqrt{\frac{25\, M_B}{21\, M_A}} = \frac{3}{4}$$

$$\frac{M_A}{M_B} = \left(\frac{4}{3}\right)^2 \times \frac{25}{21} = \frac{400}{189}$$

b. When pipe B is closed at both ends, fundamental frequency of pipe B becomes

$$n_B = \frac{v_B}{2L} \qquad \text{(v)}$$

Using Eqs. (i), (iii) and (v), we get

$$\frac{n_A}{n_B} = \frac{v_A}{v_B} = \frac{3}{4}$$

EXERCISES

Subjective Type

Solutions on page 7.69

1. Two wires of different linear mass densities are soldered together end to end and then stretched under a tension F. The wave speed in the first wire is thrice that in the second. If a harmonic wave travelling in the first wire is incident on the junction of the wires and if the amplitude of the incident wave is $A = \sqrt{13}$ cm, find the amplitude of reflected wave.

2. The pulse shown in Fig. 7.73 has a speed of 5 cm/s. If the linear mass density of the right string is 0.5 that of the left string, find the ratio of height of the transmitted pulse to that of incident pulse.

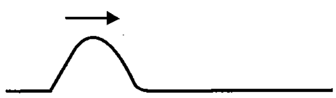

Fig. 7.73

3. A 40 cm long wire having a mass 3.2 g and area of cross-section 1 mm² is stretched between the support 40.05

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cm apart. In its fundamental mode, it vibrates with a frequency 1000/64 Hz. Find the Young's modulus of the wire.

4. A 3 m long organ pipe open at both ends is driven to third harmonic standing wave. If the amplitude of pressure oscillation is 0.1% of the mean atmospheric pressure ($P_0 = 10^5$ N/m²). Find the amplitude of
   i. particle oscillation and
   ii. density oscillation.

   Speed of sound $v = 330$ m/s, density of air $\rho_0 = 1.0$ kg/m³

5. Sound from two coherent sources S_1 and S_2 are sent in phase and detected at point P equidistant from both the sources. Speed of sound in normal air is V_0, but in some part in path S_1, there is a zone of hot air having temperature 4 times, the normal temperature, and width d. What should be minimum frequency of sound, so that minima can be found at P?

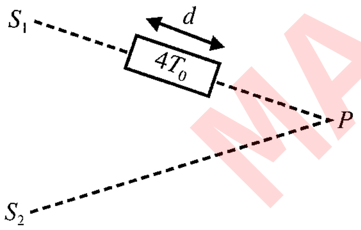

Fig. 7.74

6. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, it gets partially reflected back and partially refracted (transmitted) in water. What Would be the difference of wavelength transmitted to wavelength reflected (speed of sound in air = 330 m/s, Bulk modulus of water = 2.25×10^9, $\rho_{water} = 1000$ kg/m³).

7. Figure 7.75 shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 20.0 cm. The frequency of the sound source can be varied electronically between 1000 and 4000 Hz. Find the frequencies at which maxima of intensity are detected. The speed of sound in air = 340 m/s.

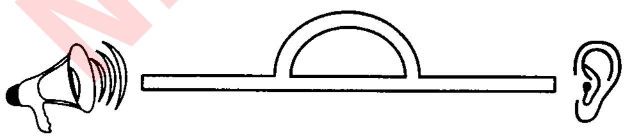

Fig. 7.75

8. A source emitting sound of frequency 180 Hz is placed in front of a wall at distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air = 360 m/s.

9. Two coherent narrow slits emitting wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ, the sound is detected by moving a detector on the screen S at a distance D ($\gg\lambda$) from the slit S_1 as shown in Fig. 7.76. Find the distance x such that the intensity at P is equal to the intensity at O.

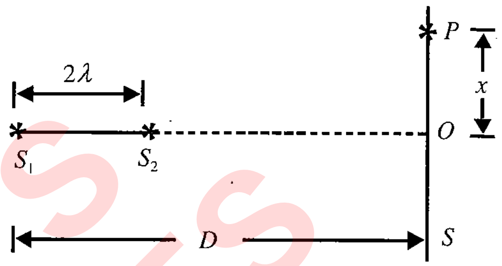

Fig. 7.76

10. The following equation represents standing wave set up in a medium,

$$y = 4\cos\frac{\pi x}{3}\sin 40\pi t$$

where x and y are in cm and t in second. Find out the amplitude and the velocity of the two component waves and calculate the distance between adjacent nodes. What is the velocity of a medium particle at $x = 3$ cm at time 1/8 s?

11. A wave is given by the equation

$$y = 10\sin 2\pi(100t - 0.02x) + 10\sin 2\pi(100t + 0.02x)$$

Find the loop length, frequency, velocity and maximum amplitude of the stationary wave produced.

12. A set of 56 tuning forks is arranged in a sequence of increasing frequencies. If each fork gives 4 beats/s with the preceding one and the last fork is found to be an octave higher of the first, find the frequeny of the first fork.

13. Two tuning forks A and B are sounded together and 8 beats/s are heard. A is in resonance with a column of air 32 cm long in a pipe closed at one end and B is similarly in resonance when the length of the column is increased by one cm. Calculate the frequency of forks.

14. A certain fork is found to give 2 beats/s when sounded in conjunction with a stretched string vibrating transversely under a tension of either 10.2 or 9.9 kg weight. Calculate the frequency of fork.

15. The two parts of a sonometer wire divided by a movable knife edge, differ in length by 2 mm and produce 1 beat/s, when sounded together. Find their frequencies if the whole length of wire is 1.00 m.

16. Two tuning forks A and B give 18 beats in 2 s. A resonates with one end closed air column of 15 cm long and B with both ends open column of 30.5 long. Calculate their frequencies.

17. Six antinodes are observed in the air column when a standing wave forms in a Kundt's tube. What is the length of the air column if steel bar of 1m length is clamped at the middle.

    The velocity of sound in steel is 5250 m/s and in air 343 m/s.

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18. A column of air at 51°C and a tuning fork produce 4 beats/s when sounded together. As the temperature of air column is decreased the number of beats per second tends to decrease and when temperature is 16°C the two produce 1 beat/second. Find the frequency of tuning fork.

19. A uniform horizontal rod of length 0.40 m and mass 1.2 kg is supported by two identical wires as shown in Fig. 7.77. Where should a mass of 4.8 kg be placed on the rod, so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? $g = 10$ m/s²

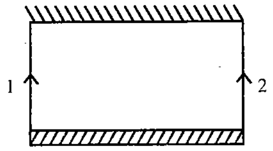

Fig. 7.77

20. A sonometer wire under tension of 128 N vibrates in resonance with a tuning fork. The vibrating portion of sonometer wire has length of 20 cm and mass 1g. The vibrating tuning fork is now moved away from the vibrating wire at constant speed of 0.75 m/s and an observer standing near the sonometer hears 1 beat/s. Find the speed of sound in air.

21. A rod of nickel of length l is clamped at its midpoint. The rod is stuck and vibrations are set up in the rod. Find the general expression for the frequency of the longitudinal vibrations of the rod. Young's modulus and density of the rod is Y and ρ, respectively.

22. A string is stretched by a block going over a pulley. The string vibrates in its fifth harmonic in unison with a particular tuning fork. When a beaker containing a liquid of density ρ is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its seventh harmonic in unison with the same tuning fork. Find the density of the material of the block.

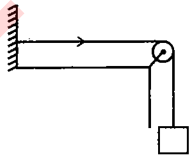

Fig. 7.78

23. An audio oscillator capable of producing notes of frequencies ranging from 500 Hz to 1500 Hz is placed near a 25.0 cm long uniform stretched wire under a constant tension T. The linear mass density of the wire is 0.75 g/m. It is observed that by varying the frequency of the oscillator over the given permissible rang the sonometer wire sets into vibration at frequencies 840 Hz and 1120 Hz.

a. Find the tension in the string.

b. What are the frequencies of the first and fourth overtone produced by the vibrating string?

24 A closed organ pipe of length l_0 is resonating in 5th harmonic mode with rod clamped at two points l and $3l$ from one end. If the length of the rod is $4l$ and it is vibrating in first overtone, find the length of the rod. [Velocity of sound in air = v_s. Young's modulus for the rod Y and density ρ]

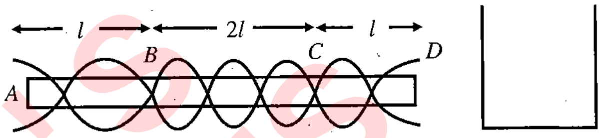

Fig. 7.79

Objective Type

Solutions on page 7.75

1. The displacement of a particle is given by $x = 3\sin(5\pi t) + 4\cos(5\pi t)$. The amplitude of particle is

a. 3 **b.** 4
c. 5 **d.** 7

2. The equation of displacement of two waves are given as

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right); \quad y_2 = 5[\sin 3\pi t + \sqrt{3}\cos 3\pi t]$$

Then what is the ratio of their amplitudes

a. 1 : 2 **b.** 2 : 1
c. 1 : 1 **d.** None of these

3. On sounding tuning fork A with another tuning fork B of frequency 384 Hz, 6 beats are produced per second. After loading the prongs of A with wax and then sounding it again with B, 4 beats are produced per second. What is the frequency of the tuning fork A.

a. 388 Hz **b.** 80 Hz
c. 378 Hz **d.** 390 Hz

4. Two tuning forks A and B give 4 beats/s when sounded together. The frequency of A is 320 Hz. When some wax is added to B and it is sounded with A, 4 beats/s per second are again heard. The frequency of B is

a. 312 Hz **b.** 316 Hz
c. 324 Hz **d.** 328 Hz

5. Forty-one forks are so arranged that each produces 5 beat/s when sounded with its near fork. If the frequency of last fork is double the frequency of first fork, then the frequencies of the first and last fork, respectively are

a. 200, 400 **b.** 205, 410
c. 195, 390 **d.** 100, 200

6. The equation of a stationary wave is $y = 0.8\cos\left(\frac{\pi x}{20}\right)\sin 200\pi t$ where x is in cm and t is in s. The separation between consecutive nodes will be

a. 20 cm **b.** 10 cm
c. 40 cm **d.** 30 cm

7. The following equations represent progressive transverse waves

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$$z_1 = A\cos(\omega t - kx)$$
$$z_2 = A\cos(\omega t + kx)$$
$$z_3 = A\cos(\omega t + ky)$$
$$z_4 = A\cos(2\omega t - 2ky)$$

A stationary wave will be formed by superposing

a. z_1 and z_2 **b.** z_1 and z_4
c. z_2 and z_3 **d.** z_3 and z_4

8. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in Fig. 7.80. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be

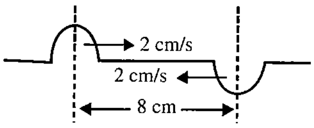

Fig. 7.80

a. zero
b. purely kinetic
c. purely potential
d. party kinetic and partly potential

9. Two identical sounds S_1 and S_2 reach at a point P in phase. The resultant loudness at point P is n dB higher than the loudness of S_1. The value of n is

a. 2 **b.** 4
c. 5 **d.** 6

10. The ratio of intensities between two coherent sound sources is 4:1. The difference of loudness in decibels (dB) between maximum and minimum intensities when they interfere in space is

a. 10 log (2) **b.** 20 log (3)
c. 10 log (3) **d.** 20 log (2)

11. Mark the correct statement:

a In case of stationary waves the maximum pressure change occurs at antinode.
b. Velocity of longitudinal waves in a medium is its physical characteristic.
c. Due to propagation of longitudinal wave in air, the maximum pressure change is equal to $2\pi na/\rho v$.
d. None of the above.

12. Which of the following statements is correct for stationary waves

a. Nodes and antinodes are formed in case of stationary transverse wave only
b. In case of longitudinal stationary wave, compressions and rarefactions are obtained in place of nodes and antinodes respectively
c. Suppose two plane waves, one longitudinal and the other transverse having same frequency and amplitude are travelling in a medium in opposite directions with the same speed, by superposition of these waves, stationary waves cannot be obtained
d. None of the above

13. A sound wave of wavelength λ travels towards the right horizontally with a velocity V. It strikes and reflects from a vertical plane surface, travelling at a speed v towards the left. The number of positive crests striking in a time interval of 3 s on the wall is

a. $3(V+v)/\lambda$ **b.** $3(V-v)/\lambda$
c. $(V+v)/3\lambda$ **d.** $(V-v)/3\lambda$

14. A sonometer wire of length l vibrates in fundamental mode when excited by a tuning fork of frequency 416 Hz. If the length is doubled keeping other things same, the string will

a. vibrate with a frequency of 416 Hz
b. vibrate with a frequency of 208 Hz
c. vibrate with a frequency of 832 Hz
d. stop vibrating

15. Two closed-end pipes, when sounded together produce 5 beats/s. If their lengths are in the ratio 100:101, then fundamental notes (in Hz) produced by them are

a. 245, 250 **b.** 250, 255
c. 495, 500 **d.** 500, 505.

16. Velocity of sound in air is 320 m/s. The resonant pipe shown in Fig. 7.81 cannot vibrate with a sound of frequency.

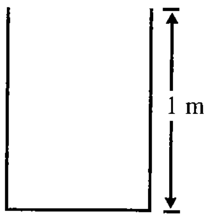

Fig. 7.81

a. 80 Hz **b.** 240 Hz
c. 320 Hz **d.** 400 Hz

17. Waves of frequency 1000 Hz are produced in a Kundt's tube. The total distance between 6 successive nodes is 82.5 cm. the speed of sound in the gas filled in the tube is

a. 33 cm/s **b.** 33 m/s
c. 330 m/s **d.** 660 m/s

18. In a Kundt's tube, the length of the iron rod is 1 m. The stationary waves of frequency 2500 Hz are produced in it. The velocity of sound in iron is

a. 1250 m/s **b.** 2500 m/s
c. 5000 m/s **d.** 10,000 m/s

19. Two strings A and B, made of same material, are stretched by same tension. The radius of string A is double of the radius of B. A transverse wave travels on A with speed v_A and on B with speed v_B. The ratio v_A/v_B is

a. 1/2 **b.** 2
c. 1/4 **d.** 4

20. A closed organ pipe and an open organ pipe have their first overtones identical in frequency. Their lengths are in the ratio

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a. 1:2
b. 2:3
c. 3:4
d. 4:5

21. Two organ pipes, both closed at one end, have lengths l and $l + \Delta l$. Neglect end correction. If the velocity of sound in air is V, then the number of beats / s is

a. $\frac{V}{4l}$
b. $\frac{V}{2l}$
c. $\frac{V}{4l^2}\Delta l$
d. $\frac{V}{2l^2}\Delta l$

22. A closed organ pipe has a frequency 'n'. If its length is doubled and radius is halved, its frequency nearly becomes.

a. halved
b. doubled
c. trebled
d. quadrupled

23. In a resonance tube experiment, the first resonance is obtained for 10 cm of air column and the second for 32 cm. The end correction for this apparatus is

a. 0.5 cm
b. 1.0 cm
c. 1.5 cm
d. 2 cm

24. Two waves having intensity I and $9I$ produce interference. If the resultant intensity at a point is $7I$, what is the phase difference between the two waves?

a. 0°
b. 60°
c. 90°
d. 120°

25. A sonometer wire, 100 cm in length has fundamental frequency of 330 Hz. The velocity of propagation of transverse waves along the wire is

a. 330 m/s
b. 660 m/s
c. 115 m/s
d. 990 m/s

26. In a resonance column experiment, the first resonance is obtained when the level of the water in the tube is at 20 cm from the open end. Resonance will also be obtained when the water level is at a distance of

a. 40 cm from the open end
b. 60 cm from the open end
c. 80 cm from the open end
d. 100 cm from the open end

27. A long glass tube is held vertically in water. A tuning fork is struck and held over the tube. Strong resonances are observed at two successive lengths 0.50 m and 0.84 m above the surface of water. If the velocity of sound is 340 m/s, then the frequency of the tuning fork is

a. 128 Hz
b. 256 Hz
c. 384 Hz
d. 500 Hz

28. A glass tube of 1.0 m length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500 c/s is brought at the upper end of the tube and the velocity of sound is 330 m/s, then the total number of resonances obtained will be

a. 4
b. 3
c. 2
d. 1

29. When the string of a sonometer of length L between the bridges vibrates in the first overtone, the amplitude of vibration is maximum at

a. $L/2$
b. $(L/4)$ and $(3L/4)$
c. $(L/6)$, $(3L/6)$ and $(5L/6)$
d. $\frac{L}{8}, \frac{3L}{8}, \frac{5L}{8}, \frac{7L}{8}$

30. A standard tuning fork of frequency f is used to find the velocity of sound in air by resonance column apparatus. The difference between two resonating lengths is 1.0 m. Then the velocity of sound in air is

a. f m/s
b. $2f$ m/s
c. $f/2$ m/s
d. $3f$ m/s

31. A sufficiently long closed organ pipe has a small hole at its bottom. Initially, the pipe is empty. Water is poured into the pipe at a constant rate. The fundamental frequency of the air column in the pipe

a. continuously increases
b. first increases and then becomes constant
c. continuously decreases
d. first decreases and then becomes constant

32. An open pipe resonates with a tuning fork of frequency 500 Hz. It is observed that two successive notes are formed at distances 16 and 46 cm from the open end. The speed of sound in air in the pipe is

a. 230 m/s
b. 300 m/s
c. 320 m/s
d. 360 m/s

33. If the length of a stretched sting is shortened by 40% and the tension is increased by 44%, then the ratio of the final and initial fundamental frequencies is

a. 3:4
b. 4:3
c. 1:3
d. 2:1

34. Two uniform strings A and B made of steel are made to vibrate under the same tension. If the first overtone of A is equal to the second overtone of B and if the radius of A is twice that of B, the ratio of the lengths of the strings is

a. 2:1
b. 3:2
c. 3:4
d. 1:3

35. A sonometer wire resonates with a given tuning fork forming 5 antinodes when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass m, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

a. 25 kg
b. 5 kg
c. 12.5 kg
d. (1/25) kg

36. In a large room, a person receives direct sound waves from a source 120 m away from him. He also receives waves from the same source which reach, being reflected from the 25 m high ceiling at a point halfway between them. The two waves interfere constructively for a wavelength of

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a. $20, \dfrac{20}{3}, \dfrac{20}{5}$, etc. **b.** 10, 5, 2.5, etc.
c. 10, 20, 30, etc. **d.** 15, 25, 35. etc.

37. Two waves are passing through a region in the same direction at the same time. If the equation of these waves are

$$y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$$

and $$y_2 = b\sin\frac{2\pi}{\lambda}[(vt - x) + x_0]$$

then the amplitude of the resulting wave for $x_0 = (\lambda/2)$ is

a. $|a - b|$ **b.** $a + b$
c. $\sqrt{a^2 + b^2}$ **d.** $\sqrt{a^2 + b^2 + 2ab\cos x}$

38. The vibrations of string of length 60 cm fixed at both ends are represented by the equations

$$y = 4\sin(\pi x/15)\cos(96\pi t)$$

where x and y are in cm and t in s. The maximum displacement at $x = 5$ cm is

a. $2\sqrt{3}$ cm **b.** 4 cm
c. zero **d.** $4\sqrt{2}$ cm

39. Two instruments having stretched strings are being played in unison. When the tension in one of the instruments is increases by 1%, 3 beats are produced in 2 s. The initial frequency of vibration of each wire is

a. 600 Hz **b.** 300 Hz
c. 200 Hz **d.** 150 Hz

40. The displacement ξ in centimetres of a particle is $\xi = 3\sin 314t + 4\cos 314t$. Amplitude and initial phase are

a. 5 cm, $\tan^{-1}\dfrac{4}{3}$ **b.** 3 cm, $\tan^{-1}\dfrac{3}{4}$
c. 4 cm, $\tan^{-1}\dfrac{4}{9}$ **d.** 4 cm, 0

41. A stretched string of length 1 m fixed at both ends, having a mass of 5×10^{-4} kg is under a tension of 20 N. It is plucked at a point situated at 25 cm from one end. The stretched string would vibrate with a frequency of

a. 400 Hz **b.** 100 Hz
c. 200 Hz **d.** 256 Hz

42. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to

a. 1 kg **b.** 2 kg
c. 8 kg **d.** 16 kg

43. A piano wire having a diameter of 0.90 mm is replaced by another wire of the same material but with a diameter of 0.93 mm. If the tension of the wire is kept the same, then the percentage change in the frequency of the fundamental tone is

a. +3% **b.** +3.2%
c. –3.2% **d.** –3%

44. In the sonometer experiment, a tuning fork of frequency 256 Hz is in resonance with 0.4 m length of the wire when the iron load attached to free end of wire is 2 kg. If the load is immersed in water, the length of the wire in resonance would be (specific gravity of iron = 8)

a. 0.37 m **b.** 0.43 m
c. 0.31 m **d.** 0.2 m

45. An air column in a pipe which is closed at one end will be in resonance with a vibrating tuning fork of frequency 264 Hz. The length of the air column in cm is (velocity of sound in air = 330 m/s)

a. 31.25 **b.** 62.5
c. 93.75 **d.** 25

46. If ν_1, ν_2 and ν_3 are the fundamental frequencies of three segments of stretched string, then the fundamental frequency of the overall string is

a. $\nu_1 + \nu_2 + \nu_3$

b. $\left[\dfrac{1}{\nu_1} + \dfrac{1}{\nu_2} + \dfrac{1}{\nu_3}\right]^{-1}$

c. $\nu_1\nu_2\nu_3$

d. $[\nu_1\nu_2\nu_3]^{1/3}$

47. An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at both ends vibrating in third overtone are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

a. 8/3 **b.** 3/8
c. 1/2 **d.** 1/3

48. Two vibrating tuning forks produce progressive waves given by, $y_1 = 4\sin(500\pi t)$ and $y_2 = 2\sin(506\pi t)$. These tuning forks are held near the ear of person. The person will hear

a 3 beats/s with intensity ratio between maxima and minima equal to 2
b. 3 beats/s with intensity ratio between maxima and minima equal to 9
c. 6 beats/s with intensity ratio between maxima and minima equal to 2
d. 6 beats/s with intensity ratio between maxima and minima equal to 9

49. A metal rod 40 cm long is dropped on to a wooden floor and rebounds into air. Compressional waves of many frequencies are thereby set up in the rod. If the speed of compressional waves in the rod in 5500 m/s, what is the lowest frequency of compressional waves to which the rod resonates as it rebounds?

a. 675 Hz **b.** 6875 Hz
c. 16875 Hz **d.** 0 Hz

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50. A wave of frequency 100 Hz travels along a string towards its fixed end. When this wave travels back after reflection, a node is formed at a distance of 10 cm from the fixed end. The speed of the wave (incident and reflected) is

a. 5 m/s b. 10 m/s
c. 20 m/s d. 40 m/s

51. A sound wave starting from source S, follows two paths AOB and ACB to reach the detector D. If ABC is an equilateral triangle, of side l and there is silence at point D, the maximum wavelength (λ) of sound wave must be

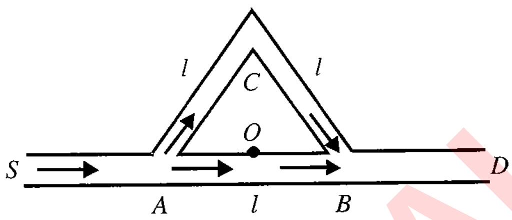


Fig. 7.82

a. l b. $2l$
c. $3l$ d. $4l$

52. Two sounding bodies producing progressive waves are given by

$$y_1 = 4\sin 400\pi t \quad \text{and} \quad y_2 = 3\sin 404\pi t$$

One of these bodies situated very near to the ears of a person who will hear:

a. 2 beats/s with intensity ratio 4/3 between maxima and minima.
b. 2 beats/s with intensity ratio 49/1 between maxima and minima.
c. 4 beats/s with intensity ratio 7/2 between maxima and minima.
d. 4 beats/s with intensity ratio 4/3 between maxima and minima

53. Ten tuning forks are arranged in increasing order of frequency in such a way that any two nearest forks produce 4 beats/s. The highest frequency is twice that of the lowest. Possible lowest and highest frequencies are

a. 40 and 80 b. 50 and 100
c. 22 and 44 d. 36 and 72

54. A long cylindrical tube carries a highly polished piston and has a side opening. A tuning fork of frequency n is sounded at the open end of the tube. The intensity of the sound heard by the listener changes if the piston is moved in or out. At a particular position of the piston he hears a maximum sound. When the piston is moved through a distance of 9 cm, the intensity of sound becomes minimum. If the speed of sound is 360 m/s, the value of n is

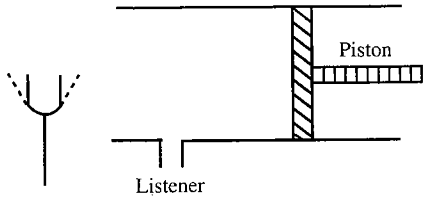


Fig. 7.83

a. 129.6 Hz b. 500 Hz
c. 1000 Hz d. 2000 Hz

55. A sound wave of wavelength 0.40 m enters the tube at S. The smallest radius r of the circular segment to hear minimum at detector D must be

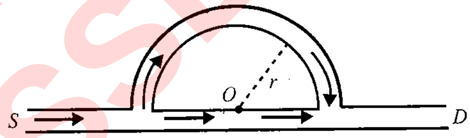


Fig. 7.84

a. 1.75 m b. 0.175 m
c. 0.93 m d. 9.3 m

56. A sound wave starting from source S, follows two paths $SEFD$ and $SEABFD$. If $AB = l$, $AE = BF = 0.6\ l$ and wavelength of wave is $\lambda = 11$ m. If maximum sound is heard at D, then minimum value of length l is

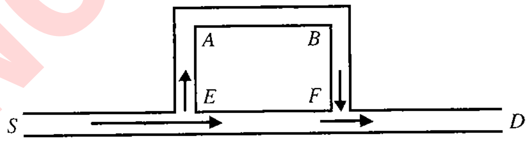


Fig. 7.85

a. 11 m b. 6 m
c. 2.5 m d. 5 m

57. An organ pipe A closed at one end vibrating in its fundamental frequency and another pipe B open at both ends is vibrating in its second overtone are in resonance with a given tuning fork. The ratio of length of pipe A to that of B is

a. 1:2 b. 3:8 c. 2:3 d. 1:6

58. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\frac{t}{2}\sin 1000t$$

How many independent harmonic motions may be considered to superpose to result this expression:

a. two b. three c. four d five

59. Two identical straight wires are stretched so as to produce 6 beats/s when vibrating simultaneously. On changing the tension slightly in one of them, the beats frequency remains unchanged. If T_1 and T_2 are initial tensions in strings such that $T_1 > T_2$ then it may be said while making above changes in tension:

a. T_2 was decreased
b. T_1 was increased
c. both T_1 and T_2 were increased
d. either T_2 was increased or T_1 was decreased

60. An open pipe of length 2 m is dipped in water. To what depth x is to be immersed in water so that it may resonate with a tuning fork of frequency 170 Hz when vibrating in its first overtone. Speed of sound in air is 340 m/s
a. 0.5 m b. 0.75 m
c. 1 m d. 1.5 m

61. A stone is hung in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork of frequency 256 Hz. When the stone is completely immersed in water, the length between the bridges is 22 cm for re-establishing unison. The specific gravity of the material of the stone is
a. $\frac{(40)^2}{(40)^2+(22)^2}$ b. $\frac{(40)^2}{(40)^2-(22)^2}$
c. $256\times\frac{22}{40}$ d. $256\times\frac{40}{22}$

62. A stretched wire of some length under a tension is vibrating with its fundamental frequency. Its length is decreased by 45% and tension is increased by 21%. Now fundamental frequency
a. increases by 50%
b. increases by 100%
c. decreases by 50%
d. decreases by 25%

63. An open and a closed pipe have same length. The ratio of frequency of their nth overtone is
a. $\frac{n+1}{2n+1}$ b. $\frac{2(n+1)}{2n+1}$
c. $\frac{n}{2n+1}$ d. $\frac{n+1}{2n}$

64. A string is under tension so that its length is increased by $1/n$ times its original length. The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be
a. $1:n$ b. $n^2:1$
c. $\sqrt{n}:1$ d. $n:1$

65. A closed organ pipe and an open organ pipe of same length produce 2 beats when they are set into vibration simultaneously in their fundamental mode. The length of the open organ pipe is now halved and of the closed organ pipe is doubled; the number of beats produced will be
a. 8 b. 7
c. 4 d. 2

66. The frequency of a sonometer wire is 10 Hz. When the weights producing the tension are completely immersed in water the frequency becomes 80 Hz and on immersing the weights in a certain liquid the frequency becomes 60 Hz. The specific gravity of the liquid is
a. 1.42 b. 1.77 c. 1.82 d. 1.21

67. An open organ pipe of length l is sounded together with another open organ pipe of length $l+x$ in their fundamental tones. Speed of sound in air is v. The beat frequency heard will be ($x \ll l$):
a. $\frac{vx}{4l^2}$ b. $\frac{vl^2}{2x}$ c $\frac{vx}{2l^2}$ d. $\frac{vx^2}{2l}$

68. n waves are produced on a string in 1 s. When the radius of the string is doubled and the tension is maintained the same, the number of waves produced in 1 s for the same harmonic will be
a. $2n$ b. $\frac{n}{3}$ c. $\frac{n}{2}$ d. $\frac{n}{\sqrt{2}}$

69. The displacement y of a particle executing periodic motion is given by

$$y=4\cos^2\left(\frac{1}{2}t\right)\sin(1000\,t)$$

This expression may be considered as a result of the superposition of
a. two b. three c. four d. five

70. The minimum intensity of audibility of sound is 10^{-12} W/m^{2}s and density of air = 1.293 kg/m^3. If the frequency of sound in 1000 Hz, then the corresponding amplitude of the vibration of the air particles is
[Take velocity of sound = 332 m/s]
a. 1.1×10^{-7} m b. 1.1×10^{-9} m
c. 1.1×10^{-11} m d. 1.1×10^{-14} m

71. The frequency of B is 3% greater than that of A. The frequency of C is 2% less than that of A. If B and C produce 8 beats/s, then the frequency of A is
a. 136 Hz b. 168 Hz
c. 164 Hz d. 160 Hz

72. One end of a 2.4-m string is held fixed and the other end is attached to a weightless ring that can slide along a frictionless rod as shown in Fig. 7.86. The three longest possible wavelength for standing waves in this string are respectively

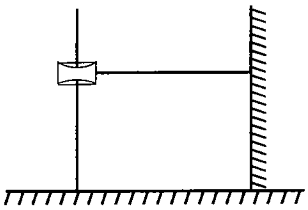


Fig. 7.86

a. 4.8 m, 1.6 m and 0.96 m
b. 9.6 m, 3.2 m and 1.92 m
c. 2.4 m, 0.8 m and 0.48 m
d. 1.2 m, 0.4 m and 0.24 m

73. The strings of a violin are tuned to the tones G, D, A and E which are separated by a fifth from one another. That is $f(D) = 1.5(G)$, $f(A) = 1.5f(D) = 400$ Hz and $f(E) = 1.5f(A)$. The distance between the two fixed points, the bridge at the scroll and over the body of the instrument is 0.25 m. The tension on the string E is 90 N. The mass per unit length of string E is nearly

a. 1 g/m **b.** 2 g/m
c. 3 g/m **d.** 4 g/m

74. Five sinusoidal waves have the same frequency 500 Hz but their amplitudes are in the ratio 2:1/2:1/2:1:1 and their phase angles 0, $\pi/6$, $\pi/3$, $\pi/2$ and π, respectively. The phase angle of resultant wave obtained by the superposition of these five waves is

a. 30° **b.** 45° **c.** 60° **d.** 90°

75. The breaking stress of steel is 7.85×10^8 N/m^2 and density of steel is 7.7×10^3 kg/m^3. The maximum frequency to which a string 1 m long can be tuned is

a. 15.8 Hz **b.** 158 Hz
c. 47.4 Hz **d.** 474 Hz

76. Which of the following travelling wave will produce standing wave, with nodes at $x = 0$, when superimposed on $y = A \sin(\omega t - kx)$

a. $A \sin(\omega t + kx)$ **b.** $A \sin(\omega t + kx + \pi)$
c. $A \cos(\omega t + kx)$ **d.** $A \cos(\omega t + kx + \pi)$

77. A wire of length 'l' having tension T and radius 'r' vibrates with fundamental frequency 'f'. Another wire of the same metal with length '$2l$' having tension $2T$ and radius $2r$ will vibrate with fundamental frequency:

a. f **b.** $2f$ **c.** $\dfrac{f}{2\sqrt{2}}$ **d.** $\dfrac{f}{2}\sqrt{2}$

78. A string of length 1.5 m with its two ends clamped is vibrating in fundamental mode. Amplitude at the centre of the string is 4 mm. Distance between the two points having amplitude 2 mm is

a. 1 m **b.** 75 cm **c.** 60 cm **d.** 50 cm

79 A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is

a. 144 m/s **b.** 216 m/s
c. 108 m/s **d.** 72 m/s

80. A string of length 'L' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $L/3$ from one end is

a. a **b.** 0 **c.** $\dfrac{\sqrt{3}a}{2}$ **d.** $\dfrac{a}{2}$

81. What percentage change in the tension is necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

a. 25% **b.** 50% **c.** 67% **d.** 75%

82. A chord attached about an end to a vibrating fork divides it into 6 loops, when its tension is 36 N. The tension at which it will vibrate in 4 loops is

a. 24 N **b.** 36 N **c** 64 N **d** 81 N

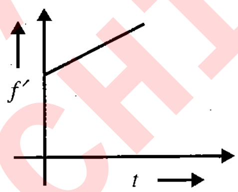
83. A closed organ pipe has length 'l'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. The amplitude at a distance of $l/7$ from closed end of the pipe is equal to

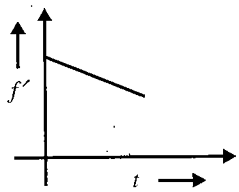
a. a **b.** $a/2$
c. $\dfrac{a\sqrt{3}}{2}$ **d.** zero

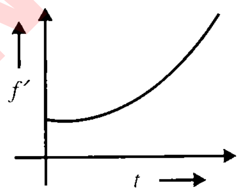
84. When a sound wave is reflected from a wall, the phase difference between the reflected and incident pressure wave is

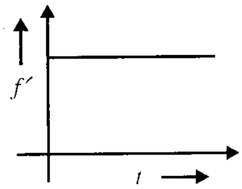
a. 0 **b.** π **c.** $\pi/2$ **d.** $\pi/4$

85. A source of frequency 'f' is stationary and an observer starts moving towards it at $t = 0$ with constant small acceleration. Then the variation of observed frequency 'f'' registered by the observer with time is best represented as

a.


b.


c.


d.


86. A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3 dB is ($\log_e 2 = 0.3$)

a. $\dfrac{1}{2}$ **b.** $\dfrac{1}{\sqrt{2}}$ **c.** $\dfrac{1}{4}$ **d.** $\dfrac{2}{3}$

87. The frequency of a man's voice is 300 Hz and its wavelength is 1 m. If the wavelength of a child's voice is 1.5 m, then the frequency of the child's voice is

a. 200 Hz **b.** 150 Hz
c. 400 Hz **d.** 350 Hz

88. A sound wave of frequency 440 Hz is passing through air. An O_2 molecule (mass = 5.3×10^{-26} kg) is set in oscillation with an amplitude of 10^{-6} m. Its speed at the centre of its oscillation is

a. 1.70×10^{-5} m/s **b.** 17.0×10^{-5} m/s
c. 2.76×10^{-3} m/s **d.** 2.77×10^{-5} m/s

89. S_1 and S_2 are two coherent sources of sound separated by 3 m having no initial phase difference. The velocity of

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sound is 330 m/s. No minima will be formed on the line passing through S_2 and perpendicular to the line joining S_1 and S_2, if the frequency of both the sources is

a. 50 Hz **b.** 60 Hz
c. 70 Hz **d.** 80 Hz

90. Under similar conditions of temperature and pressure, which of the following gases will have the largest velocity of sound.

a. H_2 **b.** N_2 **c.** He **d** CO_2

91. When beats are produced by two progressive waves of nearly the same frequency, which one of the following is correct?

a. The particle vibrate simple harmonically, with the frequency equal to the difference in the component frequencies
b. The amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves
c. The frequency of beats depends upon the position, where the observer is
d. The frequency of beats changes as the time progresses

92. There is a set of four tuning forks, one with the lowest frequency vibrating at 550 Hz. By using any two tuning forks at a time, the following beat frequencies are heard: 1, 2, 3, 5, 7, 8. The possible frequencies of the other three forks are

a. 552, 553, 560 **b.** 557, 558, 560
c. 552, 553, 558 **d.** 551, 553, 558

93. A 100-m long rod of density 10.0×10^4 kg/m^3 and having Young's modulus $Y = 10^{11}$ Pa, is clamped at one end. It is hammered at the other free end. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time the pulse take to go back to initial point.

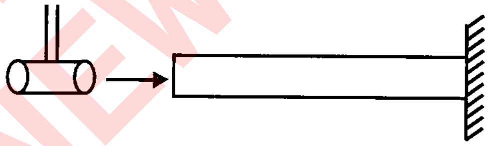


Fig. 7.87

a. 0.1 s **b.** 0.2 s
c. 0.3 s **d.** 2 s

94. Figure 7.88 shows a stretched string of length L and pipes of length L, $2L$, $L/2$ and $L/2$ in options (a), (b), (c) and (d) respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in the air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance?

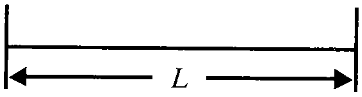
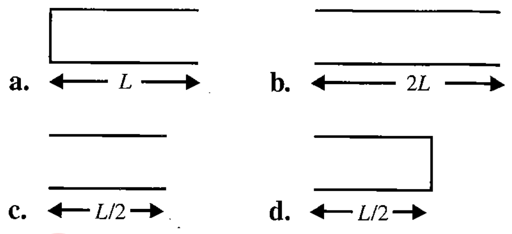


Fig. 7.88



95. Equations of a stationary and a travelling waves are as follows $y_1 = \sin kx \cos \omega t$ and $y_2 = a \sin(\omega t - kx)$. The phase difference between two points $x_1 = \pi/3k$ and $x_2 = 3\pi/2k$ is ϕ_1 in the standing wave (y_1) and is ϕ_2 in travelling wave (y_2) then ratio ϕ_1/ϕ_2 is

a. 1 **b.** 5/6 **c.** 3/4 **d.** 6/7

96. In the resonance tube experiment, the first resonance is heard when length of air column is l_1 and second resonance is heard when length of air column is l_2. What should be the minimum length of the tube so that third resonance can also be heard.

a. $2l_2 - l_1$ **b.** $2l_1$
c. $5l_1$ **d.** $7l_1$

97. Radio waves coming at angle α to vertical are received by a ladder after reflection from a nearby water surface and also directly. What can be height of antenna from water surface so that it records a maximum intensity (a maxima) (wavelength $= \lambda$).

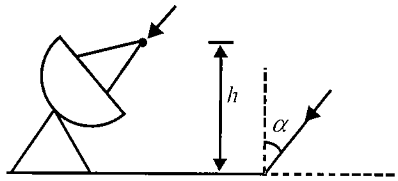


Fig. 7.89

a. $\dfrac{\lambda}{2\cos\alpha}$ **b.** $\dfrac{\lambda}{2\sin\alpha}$
c. $\dfrac{\lambda}{4\sin\alpha}$ **d.** $\dfrac{\lambda}{4\cos\alpha}$

98. Microwaves from a transmitter are directed normally towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance 0.14 m. If the velocity of light is 3×10^8 m/s, find the frequency of the transmitter.

a. 1.5×10^{10} Hz **b.** 10^{10} Hz
c. 3×10^{10} Hz **d.** 6×10^{10} Hz

99. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 m and finds that the echo is again not heard when the drumming rate becomes 60 per minute.

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i. The distance between the mountain and the initial position of the man is

a. 330 m b. 300 m
c. 240 m d. 270 m

ii. The velocity of sound is

a. 330 m/s b. 720 m/s
c. 300 m/s d. 270 m/s

100. Let the two waves $y_1 = A \sin(kx - \omega t)$ and $y_2 = A \sin(kx + \omega t)$ form a standing wave on a string. Now if an additional phase difference of ϕ is created between two waves, then

a. the standing wave will have a different frequency
b. the standing wave will have a different amplitude for a given point
c. the spacing between two consecutive nodes will change
d. none of the above

101. A standing wave on a string is given by $y = (4 \text{ cm}) \cos [x\pi] \sin [50\pi t]$, where x is in metres and t is in seconds. The velocity of the string section at $x = 1/3$ m at $t = 1/5$ s, is

a. zero b. π m/s
c. 840π m/s d. none of these

102. If the velocity of sound in air is 320 m/s, then the maximum and minimum length of a pipe closed at one end, that would produce a just audible sound would be

a. 2.6 m and 3.6 mm b. 4 m and 4.2 mm
c. 3 m and 3 mm d. 4 m and 4 mm

103. Mark out the correct statement(s) regarding standing waves.

a. Standing waves appear to be stationary but transfer of energy from one particle to another continues to take place.
b. A standing wave not only appears to be stationary but net transfer of energy from one particle to the other is also equal to zero.
c. A standing wave does not appear to be stationary and net transfer of energy from one particle to the other is also non-zero.
d. A standing wave does not appear to be stationary, but net transfer of energy from one particle to the other is zero.

104. A harmonic wave is travelling on a stretched string. At any particular instant, the smallest distance between two particles having same displacement, equal to half of amplitude is 8 cm. Find the smallest separation between two particles which have same values of displacement (magnitude only) equal to half of amplitude.

a. 8 cm b. 24 cm
c. 12 cm d. 4 cm

105. Two strings, one thick and other thin are connected as shown in Fig. 7.90.

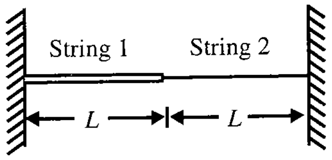

Fig. 7.90

Which of the following statement(s) is correct with regard to above arrangement?

a. If a wave is travelling from string 1 to string 2, then the joint would be treated as free end.
b. If a wave is travelling from string 1 to string 2, then the joint would be treated as a fixed end.
c. If a wave is travelling from string 2 to string 1, then the joint would be treated as a free end.
d. Both (b) and (c) are correct.

106. A string fixed at both ends whose fundamental frequency is 240 Hz is vibrated with the help of a tuning fork having frequency 480 Hz, then

a. The string will vibrate with a frequency of 240 Hz
b. The string will vibrate in resonance with the tuning fork
c. The string will vibrate with a frequency of 480 Hz, but is not a resonance with the tuning fork
d. The string is in resonance with the tuning fork and hence vibrate with a frequency of 240 Hz

107. If a string fixed at both ends having fundamental frequency of 240 Hz is vibrated with the help of a tuning fork having frequency 280 Hz, then the

a. string will vibrate with a frequency of 240 Hz
b. string will be in resonance with the tuning fork
c. string will vibrate with the frequency of tuning fork, but resonance condition will not be achieved
d. string will vibrate with a frequency of 260 Hz

108. A string of length 0.4 m and mass 10^{-2} kg is clamped at one end. The tension in the string is 1.6 N. The identical wave pulses are generated at the free end after regular interval of time, Δt. The minimum value of Δt, so that a constructive interference takes place between successive pulses is

a. 0.1 s b. 0.05 s c. 0.2 s
d. constructive interference cannot take place

109. A train of sound waves is propagated along an organ pipe and gets reflected from an open end. If the displacement amplitude of the waves (incident and reflected) are 0.002 cm, the frequency is 1000 Hz and wavelength is 40 cm. Then, the displacement amplitude of vibration at a point at distance 10 cm from the open end, inside the pipe, is

a. 0.002 cm b. 0.003 cm
c. 0.001 cm d. 0.000 cm

110. An ideal organ pipe resonates at successive frequencies of 50 Hz, 150 Hz, 250 Hz, etc. (speed of sound = 340 m/s) The pipe is

a. open at both ends and of length 3.4 m
b. open at both ends and of length 6.8 m
c. closed at one end, open at the other, and of length 1.7 m
d. closed at one end, open at the other, and of length 3.4 m.

111. A source of sound attached to the bob of a simple pendulum execute SHM. The difference between the apparent frequency of sound as received by an observer during its approach and recession at the mean position

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of the SHM motion is 2% of the natural frequency of the source. The velocity of the source at the mean position is (velocity of sound in the air is 340 m/s)
[Assume velocity of sound source << velocity of sound in air]

a. 1.4 m/s **b.** 3.4 m/s
c. 1.7 m/s **d.** 2.1 m/s

112. A standing wave arises on a string when two waves of equal amplitude, frequency and wavelength travelling in opposite directions superimpose. If the frequency of two component waves is doubled, then the frequency of oscillation of the standing waves

a. gets doubled
b. gets halved
c. remains unchanged
d. changes but not by a factor of 2 or 1/2

113. Two tuning forks of frequency 250 Hz and 256 Hz produce beats. If a maximum is observed just now, after how much time the minimum is observed at the same place?

a. $\frac{1}{18}$ s **b.** $\frac{1}{6}$ s **c.** $\frac{1}{12}$ s **d.** $\frac{1}{24}$ s

114. Two separated sources emit sinusoidal travelling waves but have the same wavelength λ and are in phase at their respective sources. One travels a distance l_1 to get to the observation point while the other travels a distance l_2. The amplitude is minimum at the observation point, if $l_1 - l_2$ is an

a. odd integral multiple of λ
b. even integral multiple of λ
c. odd integral multiple of $\lambda/2$
d. odd integral multiple of $\lambda/4$

115. A standing wave can be produced by combining

a. two longitudinal travelling waves
b. two transverse travelling waves
c. two sinusoidal travelling waves travelling in opposite directions
d. all of the above

116. Regarding an open organ pipe, which of the following is correct?

a. Both the ends are pressure antinodes
b. Both the ends are displacement nodes
c. Both the ends are pressures nodes
d. Both (a) and (b)

117. Two canoes are 10 m apart on a lake. Each bobs up and down with a period of 4.0 s. When one canoe is at its highest point, the other canoe is at its lowest point. Both canoes are always within a single cycle of the waves. The speed of wave is.

a. 2.5 m/s **b.** 5 m/s **c.** 40 m/s **d.** 4 m/s

118. A resonance occurs with a tuning fork and an air column of size 12 cm. The next higher resonance occurs with an air column of 38 cm. What is the frequency of the tuning fork? Assume that the speed of sound is 312 m/s.

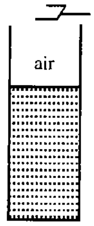

Fig. 7.91

a. 500 Hz **b.** 550 Hz
c. 600 Hz **d.** 650 Hz

119. In a resonance tube experiment, the first two resonances are observed at length 10.5 cm and 29.5 cm. The third resonance is observed at the length .... cm

a. 47.5 **b.** 58.5 **c.** 48.5 **d.** 82.8

120. A sound consists of four frequencies: 300 Hz, 600 Hz. 1200 Hz and 2400 Hz. A sound 'filter' is made by passing this sound through a bifurcate pipe as shown. The sound wave has to travel a distance of 50 cm more in the right branch-pipe than in the straight pipe. The speed of sound in air is 300 m/s. Then, which of the following frequencies will be almost completely muffled or 'silenced' at the outlet?

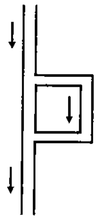

Fig. 7.92

a. 300 Hz **b.** 600 Hz **c.** 1200 Hz **d.** 2400 Hz

121. A sound increases its decibel reading from 20 to 40 dB. This means that the intensity of the sound

a. is doubled
b. is 20 times greater
c. is 100 times greater
d. is the old intensity 20

122. To decrease the fundamental frequency of a stretched string fixed at both ends one might

a. increase its tension
b. increase its wave velocity
c. increase its length
d. decrease its linear mass density

123. If the sound waves produced by the tuning fork can be expressed as $y = 0.2$ (cm) $\sin(kx - \omega t)$, where $K = 2\pi/\lambda$ and $\omega = 2\pi f$ ($f = 512$ Hz), maximum value of amplitude in a beat will be

a. 0.4 cm **b.** 0.6 cm
c. 0.8 cm **d.** 0.2 cm

124. A glass tube of length 1.5 m is filled completely with water; the water can be drained out slowly at the bottom of the tube.

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Find the total number of resonance obtained, when a tuning fork of frequency 606 Hz is put at the upper open end of the tube. Take velocity of sound in air = 340 m/s.

a. 2 b. 3 c. 4 d. 5

125. A wave equation is represented as

$$r = A\sin\left[\alpha\left(\frac{x-y}{2}\right)\right]\cos\left[\omega t-\alpha\left(\frac{x+y}{2}\right)\right],$$

where x and y are in metres and t is in seconds. Then,

a. the wave is a stationary wave.
b. the wave is a progressive wave propagating along $+x$-axis.
c. the wave is a progressive wave propagating at right angle to the $+x$-axis
d. all points lying on line $y = x + (4\pi/\alpha)$ are always at rest.

126. A wave represented by the equation $y = a\cos(kx-\omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for the other wave is

a. $a\sin(kx+\omega t)$ b. $-a\cos(kx-\omega t)$
c. $-a\cos(kx+\omega t)$ d. $-a\sin(kx-\omega t)$

127. A tuning fork A of frequency as given by the anufacturer is 512 Hz is being tested using an accurate oscillator. It is found that they produce 2 beats/s when the oscillator reads 514 Hz and 6 beats/s when it reads 510 Hz. The actual frequency of the fork in Hz is

a. 508 b. 512 c. 516 d. 518

128. A sounding tuning fork whose frequency is 256 Hz is held over an empty measuring cylinder. The sound is faint, but if just the right amount of water is poured into the cylinder, it becomes loud. If the optimal amount of water produce an air column of length 0.31 m, then the speed of sound in air to a first approximation is

a. 317 m/s b. 371 m/s
c. 340 m/s d. 332 m/s

129. A 40 cm long brass rod is dropped one end first onto a hard floor but is caught before it topples over. With an oscilloscope it is determined that the impact produces a 3 kHz tone. The speed of sound in brass is

a. 600 m/s b. 1200 m/s
c. 2400 m/s d. 4800 m/s

130. A metal bar clamped at its centre resonates in its fundamental mode to produce longitudinal waves of frequency 4 kHz. Now the clamp is moved to one end. If f_1 and f_2 be the frequencies of first overtone and second overtone respectively then,

a. $3f_2 = 5f_1$ b. $3f_1 = 5f_2$ c. $f_2 = 2f_1$ d. $2f_2 = f_1$

131. A string under a tension of 100 N, emitting its fundamental mode, gives 5 beats/s with a tuning fork. When the tension is increased to 121 N, again 5 beats/s are heard. The frequency of the fork is

a. 105 Hz b. 95 Hz c. 210 Hz d. 190 Hz

132. The equation for the fundamental standing sound wave in a tube that is closed at both ends if the tube is 80 cm long and speed of the wave is 330 m/s is (assume that amplitude of wave at antinode to be s_0)

a. $y = s_0\cos(3.93t)\sin(1295x)$
b. $y = s_0\sin(7.86t)\cos(1295x)$
c. $y = s_0\cos(7.86t)\sin(1295x)$
d. $y = s_0\cos(1295x)\sin(3.93t)$

133. A cylindrical tube open at both ends has a frequency v in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

a. $v/2$ b. v c. $3v/4$ d. $2v$

134. A stiff wire is bent into a cylinder loop of diameter D. It is clamped by knife edges at two points opposite to each other. A transverse wave is sent around the loop by means of a small vibrator which acts close to one clamp. The resonance frequency (fundamental mode) of the loop in terms of wave speed v and diameter D is

a. $\frac{v}{D}$ b. $\frac{2v}{\pi D}$ c. $\frac{v}{\pi D}$ d. $\frac{v}{2\pi D}$

135. Two wires of radii r and $2r$ are welded together end to end. The combination is used as a sonometer wire and is kept under a tension T. The welded point lies midway between the bridges. The ratio of the number of loops formed in the wires, such that the joint is a node when the stationary waves are set up in the wire is

a. 2/3 b. 1/3 c. 1/4 d. 1/2

136. An air column closed at one end and opened at the other end, resonates with a tuning fork of frequency v when its length is 45 cm and 99 cm and at two other lengths in between these values. The wavelength of sound in air column is

a. 180 cm b 108 cm c. 54 cm d. 36 cm

137. Two identical sonometer wires have a fundamental frequency of 500 Hz when kept under the same tension. The percentage change in tension of one of the wires that would cause an occurrence of 5 beats/s, when both wires vibrate together is

a. 0.5% b. 1% c. 2% d. 4%

138. A long tube open at the top is fixed vertically and water level inside the tube can be moved up or down. A vibrating tuning fork is held above the open end and the water level is pushed down gradually so as to get first and second resonance at 24.1 cm and 74.1 cm, respectively below the open end. The diameter of the tube is

a. 5 cm b. 4 cm c. 3 cm d. 2 cm

139. Two open pipes A and B are sounded together such that beats are heard between the first overtone of A and second overtone of B. If the fundamental frequency of A and B is 256 Hz and 170 Hz respectively, then the beat frequency heard is

a. 4 Hz b. 3 Hz c. 2 Hz d. 1 Hz

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140. S_1 and S_2 are two coherent current sources of radiations separated by distance 100.25λ where λ is the wavelength of radiation. S_1 leads S_2 in phase by $\pi/2$. A and B are two points on the line joining S_1 and S_2. The ratio of amplitude of sources S_1 and S_2 is in ratio 1:2. The ratio of intensity at A to that at B (I_A/I_B) is

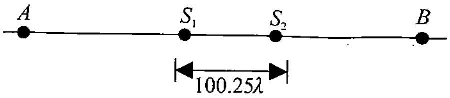

Fig. 7.93

a. ¥ **b.** $\frac{1}{9}$ **c.** 0 **d.** 9

141. A travelling wave $y = A \sin (kx - \omega t + \theta)$ passes from a heavier string to a lighter string. The reflected wave has amplitude $0.5\ A$. The junction of the stings is at $x = 0$. The equation of the reflected wave is

a. $y' = 0.5A \sin (kx + \omega t + \theta)$

b. $y' = -0.5A \sin (kx + \omega t + \theta)$

a. $y' = -0.5A \sin (kx - \omega t - \theta)$

a. $y' = -0.5A \sin (kx + \omega t - \theta)$

Multiple Correct Answers Type

Solutions on page 7.88

1. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting the corrections, the air column in the pipe can resonate for sound of frequency

a. 80 Hz **b.** 240 Hz **c.** 320 Hz **d.** 400 Hz

2. Two identical straight wires are stretched so as to produce 6 beats s^{-1}, when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1 and T_2 the higher and lower initial tensions in the string, then it could be said that while making the above changes in tension

a. T_2 was decreased **b.** T_2 was increased
c. T_1 was increased **d.** T_1 was decreased

3. A loudspeaker that produces signals from 50 to 500 Hz is placed at the open end of a closed tube of length 1.1 m. The lowest and the highest frequency that excites resonance in the tube are f_l and f_h respectively. The velocity of sound is 330 m/s. Then

a. $f_l = 50$ Hz **b.** $f_h = 500$ Hz **c.** $f_l = 75$ Hz **d.** $f_h = 450$ Hz

4. Three simple harmonic waves, identical in frequency n and amplitude A moving in the same direction are superimposed in air in such a way, that the first, second and the third wave have the phase angles ϕ, $\phi + (\pi/2)$ and $(\phi + \pi)$, respectively at a given point P in the superposition Then as the waves progress, the superposition will result in

a. a periodic, non-simple harmonic wave of amplitude 3A

b. a stationary simple harmonic wave of amplitude $3A$

c. a simple harmonic progressive wave of amplitude A

d. the velocity of the superposed resultant wave will be the same as the velocity of each wave

5. A sonometer string AB of length 1m is stretched by a load and the tension T is adjusted so that the string resonates to a frequency of 1 kHz. Any point P of the wire may be held fixed by use of a movable bridge that can slide along the base of sonometer.

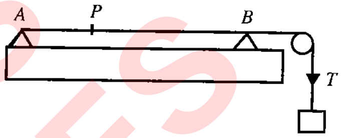

Fig. 7.94

a. If point P is fixed so that $AP:PB::1:4$, then the smallest frequency for which the sonometer wire resonates is 5 kHz.

b. If P be taken at midpoint of AB and fixed, then when the wire vibrates in the third harmonic of its fundamental, the number of nodes in the wire (including A and B) will be totally seven.

c. If the fixed point P divides AB in the ratio 1:2, then the tension needed to make the string vibrate at 1 kHz will be $3T$. (neglecting the terminal effects)

d. The fundamental frequency of the sonometer wire when P divides AB in the ratio $a:b$ will be the same as the fundamental frequency when P divides AB in the ratio $b:a$.

6. A wire of density 9×10^3 kg/m³ is stretched between two clamps 1 m apart and is stretched to an extension of 4.9×10^{-4} m. Young's modulus of material is 9×10^{10} N/m². Then:

a. The lowest frequency of standing wave is 35 Hz

b. The frequency of 1st overtone is 70 Hz

c. The frequency of 1st overtone is 105 Hz

d. The stress in the wire is 4.41×10^7 N/m²

7. For a certain transverse standing wave on a long string, an antinode is formed at $x = 0$ and next to it, a node is formed at $x = 0.10$ m, the displacement $y(t)$ of the string particle at $x = 0$ is shown in Fig. 7.95.

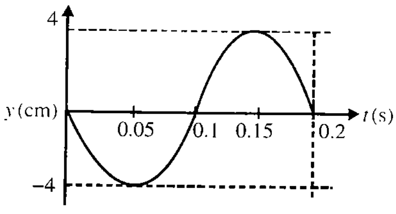

Fig. 7.95

a. Transverse displacement of the particle at $x = 0.05$ m and $t = 0.05$ s is $-2\sqrt{2}$ cm

b Transverse displacement of the particle at $x = 0.04$ m and $t = 0.025$ s is $-2\sqrt{2}$ cm

c. Speed of the travelling waves that interface to produce this standing wave is 2 m/s

d. The transverse velocity of the string particle at $x = 1/15$ m and $t = 0.1$ s is 20π cm/s

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8. Two speakers are placed as shown in Fig. 7.96.

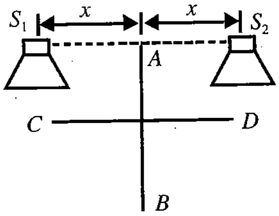

Fig. 7.96

Mark out the correct statement(s)

a. If a person is moving along AB, he will hear the sound as loud, faint, loud and so on

b. If a person moves along CD, he will hear loud, faint, loud and so on

c If a person moves along AB, he will hear uniform intense sound

d If a person moves along CD, he will hear uniform intense sound

9. Two coherent waves represented by $y_1 = A\sin\left(\frac{2\pi}{\lambda}x_1 - \omega t + \frac{\pi}{4}\right)$ and $y_2 = A\sin\left(\frac{2\pi}{\lambda}x_2 - \omega t + \frac{\pi}{6}\right)$ are superposed. The two waves will produce

a. constructive interference at $(x_1 - x_2) = 2\lambda$

b. constructive interference at $(x_1 - x_2) = 23/24\lambda$

c. destructive interference at $(x_1 - x_2) = 1.5\lambda$

d. destructive interference at $(x_1 - x_2) = 11/24\lambda$

10. Two waves travel down the same string. These waves have the same velocity, frequency f and wavelength but having different phase constants ϕ_1 and $\phi_2 (< \phi_1)$ and amplitudes A_1 and A_2 $(< A_1)$. Mark the correct statement(s) for the resultant wave which is produced due to superposition of these two waves.

a. The amplitude of the resultant waves is $A = A_1 + A_2$

b. The amplitude of the resultant wave lies between $A_1 - A_2$ to $A_1 + A_2$

c. The frequency of the resultant wave is f

d. The frequency of the resultant wave is $f/2$

11. A radio transmitter at position A operates at a wavelength of 20 m. A second, identical transmitter is located at a distance x from the first transmitter, at position B. The transmitters are phase locked together such that the second transmitter is lagging $\pi/2$ out of phase with the first. For which of the following values of $BC - CA$ will the intensity at C be maximum.

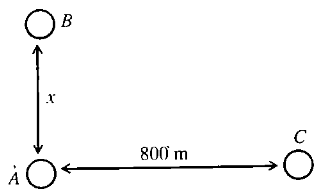

Fig. 7.97

a. $BC - CA = 60$ m **b.** $BC - CA = 65$ m
c. $BC - CA = 55$ m **d.** $BC - CA = 75$m

12. Following are equations of four waves:

(i) $y_1 = a\sin\omega\left(t - \frac{x}{v}\right)$

(ii) $y_2 = a\cos\omega\left(t + \frac{x}{v}\right)$

(iii) $z_1 = a\sin\omega\left(t - \frac{x}{v}\right)$

(iv) $z_2 = a\cos\omega\left(t + \frac{x}{v}\right)$

Which of the following statements are correct?

a. On superposition of waves (i) and (iii), a travelling wave having amplitude $a\sqrt{2}$ will be formed

b. Superposition of waves (ii) and (iii) is not possible

c. On superposition of (i) and (ii), a stationary wave having amplitude $a\sqrt{2}$ will be formed

d. On superposition of (iii) and (iv), a transverse stationary wave will be formed

13. Two waves of equal frequency f and velocity v travel in opposite directions along the same path. The waves have amplitudes A and $3A$. Then:

a. the amplitude of the resulting wave varies with position between maxima of amplitude $4A$ and minima of zero amplitude

b. the distance between a maxima and adjacent minima of amplitude is $v/2f$

c. maximum amplitude is $4A$ and minimum amplitude is $2A$

d. the position of a maxima or minima of amplitude does not change with time

14. A sound wave passes from a medium A to a medium B. The velocity of sound in B is greater than that in A. Assume that there is no absorption or reflection at the boundary. As the wave moves across the boundary:

a. the frequency of sound will not change

b. the wavelength will increase

c. the wavelength will decrease

d. the intensity of sound will not change

15. Mark the correct statements

a. If all the particles of a string are oscillating in same phase, the string is resonating in its fundamental tone

b. To observe interference, two sources of same frequency must be placed some distance apart from each other

c. To observe beats, two sources of same amplitude must be placed some distance apart from each other

d. None of the above

16. Choose the correct statements from the following:

a. Any function of the form $y(x, t) = f(vt + x)$ represents a travelling wave.

b. The velocity, wavelength and frequency of a wave do not undergo any change when it is reflected from the surface.

c. When an ultrasonic wave travels from air into water, it bends towards the normal to air-water interface.

d. The velocity of sound is generally greater in solids than in gases at NTP.

17. Which of the following statements are correct?
   a. The decrease in the speed of sound at high altitudes is due to a fall in pressure.
   b. The standing wave on a string under tension, fixed at its ends, does not have well-defined nodes.
   c. The phenomenon of beats is not observable in the case of visible light waves.
   d. The apparent frequency is f_1 when a source of sound approached a stationary observer with a speed u and is f_2 when the observer approaches the same stationary source with the same speed. Then $f_2 < f_1$, if $u < v$, where v is the speed of sound.

18. Which of the following functions represent a stationary wave? Here a, b and c are constants:
   a. $y = a\cos(bx)\sin(ct)$
   b. $y = a\sin(bx)\cos(ct)$
   c. $y = a\sin(bx + ct)$
   d. $y = a\sin(bx + ct) + a\sin(bx - ct)$

19 The stationary waves set up on a string have the equation:

$$y = (2\text{ mm})\sin\left[(6.28\, m^{-1})x\right]\cos\omega t$$

The stationary wave is created by two identical waves, of amplitude A each, moving in opposite directions along the string. Then:
   a. $A = 2$ mm
   b. $A = 1$ mm
   c. the smallest length of the string is 50 cm
   d. the smallest length of the string is 2 m

20. A plane wave $y = a\sin(kx + ct)$ is incident on a surface. Equation of the reflected wave is: $y' = a'\sin(ct - bx)$. Then which of the following statements are correct?
   a. The wave is incident normally on the surface
   b. Reflecting surface is y–z plane
   c. Medium, in which incident wave is travelling, is denser than the other medium
   d. a' cannot be greater than a

21. A string is fixed at both ends and transverse oscillations with amplitude a_0 are excited. Which of the following statements are correct?
   a. Energy of oscillations in the string is directly proportional to tension in the string
   b Energy of oscillations in nth overtone will be equal to n^2 times of that in first overtone
   c. Average kinetic energy of string (over an oscillation period) is half of the oscillation energy
   d. None of the above

22. Two waves of nearly same amplitude, same frequency travelling with same velocity are superimposing to give phenomenon of interference. If a_1 and a_2 be their respectively amplitudes, ω be the frequency for both, v be the velocity for both and $\Delta\phi$ is the phase difference between the two waves then,
   a. the resultant intensity varies periodically with time and distance.
   b. the resulting intensity with $\dfrac{I_{\min}}{I_{\max}} = \left(\dfrac{a_1 - a_2}{a_1 + a_2}\right)^2$ is obtained.
   c. both the waves must have been travelling in the same direction and must be coherent.
   d. $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$, where constructive interference is obtained for path difference that are odd multiple of $1/2\lambda$ and destructive interference is obtained for path difference that are even multiple of $1/2\lambda$.

23. Two sine waves of slightly different frequencies f_1 and f_2 ($f_1 > f_2$) with zero phase difference, same amplitudes, travelling in the same direction superimpose.
   a. Phenomenon of beats is always observed by human ear.
   b. Intensity of resultant wave is a constant.
   c. Intensity of resultant wave varies periodically with time with maximum intensity $4a^2$ and minimum intensity zero.
   d. A maxima appears at a time $1/[2(f_1 - f_2)]$ later (or earlier) than a minima appears.

24. A sinusoidal wave $y_1 = a\sin(\omega t - kx)$ is reflected from a rigid support and the reflected wave superpose with the incident wave y_1. Assume the rigid support to be at $x = 0$.
   a. Stationary waves are obtained with antinodes at the rigid support.
   b. Stationary waves are obtained with nodes at the rigid support.
   c. Stationary waves are obtained with intensity varying periodically with distance.
   d. Stationary waves are obtained with intensity varying periodically with time.

25. Two waves travelling in opposite directions produce a standing wave. The individual wave functions are given by $y_1 = 4\sin(3x - 2t)$ and $y_2 = 4\sin(3x + 2t)$ cm, where x and y are in cm
   a. The maximum displacement of the motion at $x = 2.3$ cm is 4.63 cm.
   b The maximum displacement of the motion at $t = 2.3$ s is 4.63 cm.
   c. Nodes are formed at x values given by

$$0, \pi/3, 2\pi/3, 4\pi/3, \ldots$$

   d. Antinodes are formed at x values given by

$$\pi/6, \pi/2, 5\pi/6, 7\pi/6, \ldots$$

Assertion-Reasoning Type

Solutions on page 7.92

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which *only one* is correct.

   a. **Statement I** is true, Statement II is true; Statement II is a correct explanation for Statement I.
   b. **Statement I** is true, Statement II is true; Statement II is NOT a correct explanation for Statement I.
   c. **Statement I** is true; Statement II is false.
   d. **Statement I** is false; Statement II is true.

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NEWTON CLASSES

JEE (MAIN & ADV.), MEDICAL
+ BOARD, NDA, FOUNDATION

1. **Statement I:** When a guitar string is plucked, the frequency of the plucked string will not be the same as the wave it produces in air.
Statement II: The speeds of the waves depend on the medium in which they are propagating.
2. **Statement I:** Maximum changes of pressure and density occur at the nodal points of the medium in a stationary transverse wave produced in the medium
Statement II: There will be compressions and rarefractions in a stationary longitudinal wave at the nodal points.
3. **Statement I:** The principle of superposition states that amplitudes, velocities, and, accelerations of the particles of the medium due to the simultaneous operation of two or more progressive simple harmonic waves are the vector sum of the separate amplitude, velocity and acceleration of those particles under the effect of each such wave acting alone in the medium
Statement II: Amplitudes, velocities and accelerations are linear functions of the displacement of the particle and its time derivatives.
4. **Statement I:** A standing wave pattern is formed in a string. The power transfer through a point (other than node and antinode) is zero always.
Statement II: At antinode tension is perpendicular to the velocity.
5. **Statement I:** In a standing wave on a string, the spacing between nodes is Δx. If the tension in string is increased 4 times, keeping the frequency of component wave same as before, then the separation between nearest node and antinode will be Δx.
Statement II: Spacing between nodes (consecutive) in the standing wave is equal to half of the wavelength of component waves.
6. **Statement I:** In standing waves on a string, the medium particles, i.e., different string elements remain at rest.
Statement II: In standing waves all the medium particles attain maximum velocity twice in one cycle.
7. **Statement I:** When a wave goes from one medium to other, then average power transmitted by the wave may change.
Statement II: Due to change in medium, amplitude, speed, wavelength, and frequency of wave may change.
8. **Statement I:** Velocity of particles, while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.
Statement II: Amplitude of vibration at antinodes is maximum and at nodes, the amplitudes is zero and all particles between two successive nodes cross the mean position together.
9. **Statement I:** When two waves interfere, one wave alters the progress of the other wave.
Statement II: In interference there is no loss of energy.
10. **Statement I:** When two vibrating tuning forks have $f_1 = 300$ Hz and $f_2 = 350$ Hz and held close to each other; beats cannot be heard.
Statement II: The principle of superposition is valid only when $f_1 - f_2 < 10$ Hz.
11. **Statement I:** For a closed pipe the first resonance length is 60 cm. The second resonance position will be obtained at 120 cm.
Statement II: In a closed pipe $n_2 = 3n_1$.
12. **Statement I:** The fundamental frequency of an organ pipe increases as the temperature increases.
Statement II: As the temperature increases, the velocity of sound increases more rapidly than length of the pipe.
13. **Statement I:** If two waves of same amplitude, produce a resultant wave of same amplitude, then the phase difference between them will be 120°.
Statement II: Velocity of sound is directly proportional to the square of its absolute temperature.
14. **Statement I:** In a sinusoidal travelling wave on a string potential energy of deformation of string element at extreme position is maximum.
Statement II: The particles in sinusoidal travelling wave perform SHM.
15. **Statement I:** When a closed organ pipe vibrates, the pressure of the gas at the closed end remains constant.
Statement II:. In a stationary–wave system, is placement nodes are pressure antinodes, and displacement antinodes are pressure nodes.

Comprehension Type

Solutions on page 7.92

For Problems 1–3

A steel wire 0.5 m long, of mass 5 g, is stretched with a force of 400 N.

1. What is the minimum possible frequency with which this wire can vibrate?
a. 200 Hz **b.** 300 Hz
c. 250 Hz **d.** 150 Hz
2. Is it possible for the wire to vibrate with a frequency of 1100 Hz?
a. cannot possible **b.** possible
c. cannot say **d.** data insufficient
3. What is the highest overtone that a person can hear if he is capable of hearing up to 10000 Hz?
a. 52nd **b.** 49th
c. 47th **d.** 55th

For Problems 4–5

A closed air column 32 cm long is in resonance with a tuning fork. Another open air column of length 66 cm is in resonance with another tuning fork. If the two forks produce 8 beats/s when sounded together, find

4. the speed of sound in the air
a. 33792 cm/s **b.** 35790 cm/s
c. 31890 cm/s **d.** 40980 cm/s
5. the frequencies of the forks.
a. 230 Hz, 290 Hz **b.** 250 Hz, 300 Hz
c. 264 Hz, 256 Hz **d.** 150 Hz, 300 Hz

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For Problems 6–7

A tube of a certain diameter and length 48 cm is open at both ends. Its fundamental frequency is found to be 320 Hz. The velocity of sound in air is 320 m/s.

6. Estimate the diameter of the tube.
 a. 5.29 cm b. 3.33 cm
 c. 4.78 cm d. 4.29 cm

7. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.
 a. 163.27 Hz b. 205.37 Hz
 c. 153.93 Hz d. 198.88 Hz

For Problems 8–9

Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below f_0 = 1250 Hz. The length of the pipe is l = 85 cm. The velocity of sound is v = 340 m/s. Consider two cases

8. the pipe is closed from one end,
 a. 2 b. 4
 c. 8 d. 6

9. the pipe is opened from both ends.
 a. 3 b. 7
 c. 6 d. 9

For Problems 10–12

In the arrangement shown in Fig. 7.98, a mass can be hung from a string with a linear mass density of 2×10^{-3} kg/m that passes over a light pulley. The string is connected to a vibrator of frequency 700 Hz and the length of the string between the vibrator and the pulley is 1 m.

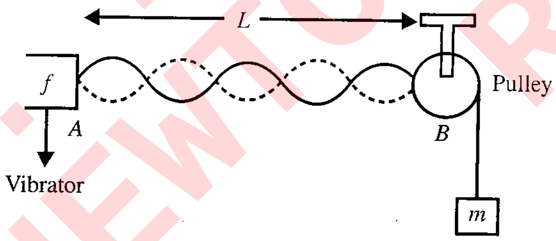

Fig. 7.98

10. If the standing waves are observed, the largest mass to be hung is
 a. 16 kg b. 25 kg
 c. 32 kg d. 400 kg

11. If the mass suspended is 16 kg, then the number of loops formed in the string is
 a. 1 b. 3 c. 5 d. 8

12. The string is set into vibrations and represented by the equation $y = 6\sin\left(\frac{\pi x}{10}\right)\cos(14 \times 10^3 \pi t)$ where x and y are in cm, and t in s, the maximum displacement at $x = 5$ m from the vibrator is
 a. 6 cm b. 3 cm
 c. 5 cm d. 2 cm

For Problems 13–14

Both neon [$M_{Ne} = 20 \times 10^{-3}$ kg] and helium [$M_{He} = 4 \times 10^{-3}$kg] are monoatomic gases and can be assumed to be ideal gases. The fundamental frequency of a tube (open at both ends) of neon is 300 Hz at 270 K $\left(R = (25/3)\text{J/K mol}\right)$

13. The length of the tube is
 a. $\frac{5}{12}$ m b. $\frac{\sqrt{3}}{12}$ m
 c. $\frac{5\sqrt{3}}{12}$ m d. $5\sqrt{3}$ m

14. The fundamental frequency of the tube if the tube is filled with helium, all other factors remaining the same is
 a. 300 Hz b. $\sqrt{2} \times 300$ Hz
 c. $\sqrt{3} \times 300$ Hz d. $\sqrt{5} \times 300$ Hz

For Problems 15–17

A long tube contains air at a pressure of 1 atm and a temperature of 59°C. The tube is open at one end and closed at the other by a movable piston. A tuning fork near the open end is vibrating with a frequency of 500 Hz. Resonance is produced when the piston is at distances 16 cm, 49.2 cm and 82.4 cm from open end. Molar mass of air is 28.8 g/mol.

15. The speed of sound in air at 59°C is
 a. 332 m/s b. 342 m/s
 c. 352 m/s d. 362 m/s

16. Ratio of heat capacities at constant pressure and constant volume for air at 59°C is
 a. 1.4 b. 1.152
 c. 1.60 d. 2

17. Radius of tube is
 a. 1.1 cm b. 1 cm
 c. 1.2 cm d. 2 cm

For Problems 18–20

A turning fork vibrating at 500 Hz falls from rest accelerates at 10 m/s^2.

18. Velocity of the tuning fork when waves with a frequency of 475 Hz reach the release point is (Take the speed of sound in air to be 340 m/s).
 a. 1.79 m/s b. 17.9 m/s
 c. 35.8 m/s d. 3.58 m/s

19. Time taken by the waves with a frequency of 475 Hz to reach the release point is nearly
 a. 1.79 s b 1.84 s
 c. 17.9 s d. 18.4 s

20. How far below the point of release is the tuning fork when wave with a frequency of 475 Hz reach the release point?
 a. 16.9 m b. 16 m c 1.69 m d 1.6 m

For Problems 21–22

A long tube contains air at a pressure of 1 atm and a temperature of 107°C. The tube is open at one end and closed at the other by

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a movable piston. A tuning fork near the open end is vibrating with a frequency of 500 Hz. Resonance is produced when the piston is at distance 19, 58.5 and 98 cm from the open end.

21. The speed of sound at 107°C is

a. 330 m/s **b.** 340 m/s
c. 395 m/s **d.** 495 m/s

22. The molar mass of air is 28.8 g/mol. The ratio of molar heat capacities at constant pressure and constant volume for air at this temperature is nearly

a. 1.66 **b** 1.4
c 1.33 **d** 1.5

For Problems 23–25

A steel rod 2.5 m long is rigidly clamped at its centre C and longitudinal waves are set up on both sides of C by rubbing along the rod. Young's modulus for steel = 2×10^{11} N/m², density of steel = 8000 kg/m³

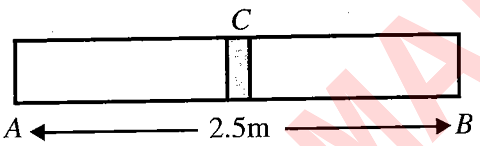


Fig. 7.99

23. If two antinodes are observed on either side of C, the frequency of the mode in which the rod is vibrating will be

a. 1000 Hz **b.** 3000 Hz
c. 7000 Hz **d.** 1500 Hz

24. If the amplitude of the wave at the antinode, when it is vibrating in its fundamental mode is 2×10^{-6} m, the maximum velocity of a steel particle in its vibration is

a. 1.25×10^{-2} m/s b. 1.25×10^{-3} m/s
c. 1 m/s d. 0.12 m/s

25. If the clamp of the rod be shifted to its end A and totally four antinodes are observed in the rod when longitudinal waves are set up in it, the frequency of vibration of the rod in this mode is

a. 500 Hz **b.** 2500 Hz
c. 3500 Hz **d.** 1500 Hz

For Problems 26–28

A longitudinal standing wave $y = a \cos kx \cos \omega t$ is maintained in a homogenous medium of density ρ. Here ω is the angular speed and k, the wave number and a is the amplitude of the standing wave. This standing wave exists all over a given region of space.

26. The space density of the potential energy PE = $E_P(x, t)$ at a point (x, t) in the space is

a. $E_P = \dfrac{\rho a^2\omega^2}{2}$

b. $E_P = \dfrac{\rho a^2\omega^2}{2}\cos^2 kx \sin^2 \omega t$

c. $E_P = \dfrac{\rho a^2\omega^2}{2}\sin^2 kx \cos^2 \omega t$

d. $E_P = \dfrac{\rho a^2\omega^2}{2}\sin^2 kx \sin^2 \omega t$

27. The space density of the kinetic energy, KE = $E_K(x, t)$ at the point (x, t) is given by

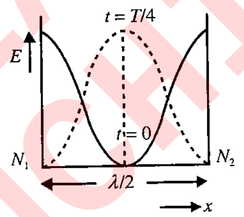
a. $E_K = \dfrac{\rho a^2\omega^2}{2}\cos^2 kx \cos^2 \omega t$

b. $E_K = \dfrac{\rho a^2\omega^2}{2}\sin^2 kx \cos^2 \omega t$

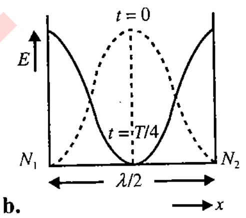
c. $E_K = \dfrac{\rho a^2\omega^2}{2}$

d. $E_K = \dfrac{\rho a^2\omega^2}{2}\cos^2 kx \sin^2 \omega t$

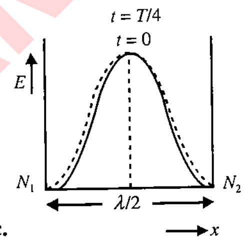
28. If a graph E (= $E_P + E_K$) versus t, i.e., total space energy density versus time were drawn at the instants of time $t = 0$ and $t = T/4$, between two successive nodes separated by distance $\lambda/2$ which of the following graphs correctly shows the total energy (E) distribution at the two instants.



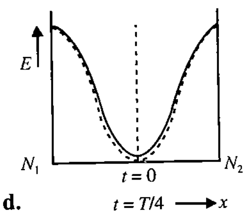
a.



b.



c.



d.

For Problems 29–31

In a standing wave experiment, a 1.2-kg horizontal rope is fixed in place at its two ends ($x = 0$ and $x = 2.0$ m) and made to oscillate up and down in the fundamental mode, at frequency of 5.0 Hz. At $t = 0$, the point at $x = 1.0$ m has zero displacement and is moving upward in the positive direction of y-axis with a transverse velocity 3.14 m/s.

29. Tension in the rope is

a. 60 N **b.** 100 N **c.** 120 N **d** 240 N

30. Speed of the participating travelling wave on the rope is

a. 6 m/s **b.** 15 m/s
c. 20 m/s **d.** 24 m/s

31. What is the correct expression of the standing wave equation?

a. $(0.1) \sin (\pi/2)\, x \sin (10\,\pi)\, t$
b. $(0.1) \sin (\pi)\, x \sin (10\,\pi)\, t$
c. $(0.05) \sin (\pi/2)\, x \cos (10\,\pi)\, t$
d. $(0.04) \sin (\pi/2)\, x \sin (10\,\pi)\, t$

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For Problems 32–35

In an organ pipe (may be closed or open) of 99 cm length standing wave is set up, whose equation is given by longitudinal displacement.

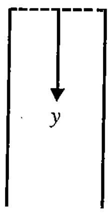

Fig. 7.100

$$\xi = (0.1\,\text{mm})\cos\frac{2\pi}{0.8}(y+1\,\text{cm})\cos(400)\,t$$

where y is measured from the top of the tube in metres and t in seconds. Here 1 cm is the end correction.

32. The upper end and the lower end of the tube are respectively.

a. open–closed **b.** closed–open
c. open–open **d.** closed–closed

33. The air column is vibrating in

a. first overtone
b. second overtone
c. third overtone
d. fundamental mode

34. Equation of the standing wave in terms of excess pressure is (take bulk modulus = 5×10^5 N/m^2)

a. $P_{ex} = (125\pi\ \text{N/m}^2)\sin\frac{2\pi}{0.8}(y+1\,\text{cm})\cos(400t)$

b. $P_{ex} = (125\pi\ \text{N/m}^2)\cos\frac{2\pi}{0.8}(y+1\,\text{cm})\sin(400t)$

c. $P_{ex} = (225\pi\ \text{N/m}^2)\sin\frac{2\pi}{0.8}(y+1\,\text{cm})\cos(200t)$

d. $P_{ex} = (225\pi\ \text{N/m}^2)\cos\frac{2\pi}{0.8}(y+1\,\text{cm})\sin(200t)$

35. Assume the end correction approximately equals to (0.3) × (diameter of tube), estimate the moles of air present inside the tube (Assume tube is at NTP, and at NTP, 22.4 litre contains 1 mole).

a. $\frac{10\pi}{36\times 22.4}$ **b.** $\frac{10\pi}{18\times 22.4}$

c. $\frac{10\pi}{72\times 22.4}$ **d.** $\frac{10\pi}{60\times 22.4}$

For Problems 36–38

A source of sound and a detector are placed at the same place on ground. At $t = 0$, the source S is projected towards reflector with velocity v_0 in vertical upward direction and reflector starts moving down with constant velocity v_0. At $t = 0$, the vertical separation between the reflector and source is $H(>v_0^2/2g)$. The speed of sound in air is v ($\gg v_0$). Take f_0 as the frequency of sound waves emitted by source.

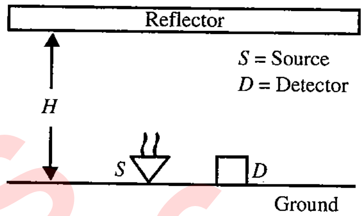

Fig. 7.101

Based on above information answer the following questions

36. Frequency of sound waves emitted by source at $t = v_0/2g$ is

a. f_0 **b.** $f_0\left[\frac{v}{v+\frac{v_0}{2}}\right]$

c. $f_0\left[\frac{v-v_0/2}{v}\right]$ **d.** $f_0\left[\frac{v-v_0/2}{v+v_0/2}\right]$

37. Wavelength of sound waves as received by detector before reflection at $t = v_0/2g$ is

a. $\frac{v}{f_0}$ **b.** $\frac{v+v_0/2}{f_0}$

c. $\frac{(v-v_0/2)^2}{vf_0}$ **d.** None of these

38. Frequency of sound received by detector after being reflected by reflector at $t = v_0/2g$ is

a. $\frac{2f_0(v+v_0)}{2v-v_0}$

b. $\frac{2f_0 v}{v-v_0}$

c. $2f_0\left[\frac{v+v_0}{2v-v_0}\right]\times\left[\frac{v}{v-v_0}\right]$

d. $2f_0\times\frac{v+v_0}{v-v_0}$

For Problems 39–41

Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along the x-axis.

39. Find the number of times intensity is maximum in time interval of 1 s

a. 4 **b.** 6
c. 8 **d.** 10

40. Find wave velocity of louder sound

a. 100 m/s **b.** 192 m/s
c. 200 m/s **d.** 96 m/s

41. Find the number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 s

a. 100 **b.** 46
c. 192 **d.** 96

For Problems 42–45

An oscillator of frequency 680 Hz drives two speakers. The speakers are fixed on a vertical pole at a distance 3 m from each other as shown in Fig. 7.102. A person whose height is almost the same as that of the lower speaker walks towards the lower speaker in a direction perpendicular to the pole. Assuming that there is no reflection of sound from the ground and speed of sound is v = 340 m/s, answer the following questions.

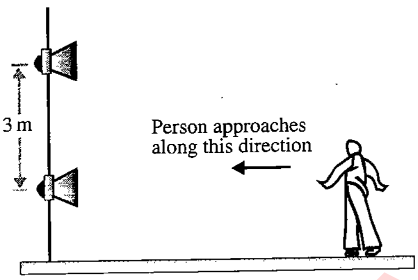

Fig. 7.102

42. As the person walks toward the pole, his distance from the pole when he first hears a minimum in sound intensity is nearly

a. 14.6 m **b.** 17.9 m
c. 10.1 m **d.** 22.4 m

43. How far is the person from the pole when he hears a minimum in sound intensity a second time?

a. 5.6 m **b.** 7.8 m
c. 12.4 m **d.** 17.6 m

44. As the person walks toward the pole, the total number of times that the person hears a minimum in sound intensity will be

a. 2 **b.** 8
c. 4 **d.** 6

45. At some instant, when the person is at a distance 4 m from the pole, the wave function (at the person's location) that describes the waves coming from the lower speaker is $y = A\cos(kx - \omega t)$, where A is the amplitude, $\omega = 2\pi v$ with v = 680 Hz (given) and $k = 2\pi/\lambda$

Wave function (at the person's location) that describes waves coming from the upper speaker can be expressed as:

a. $y = A\cos(kx - \omega t + \pi)$
b. $y = A\cos(kx - \omega t + \pi/2)$
c. $y = A\cos(kx - \omega t + 2\pi)$
d. $y = A\cos\left(KX - \omega t + \dfrac{3\pi}{2}\right)$

For Problems 46–49

Consider a standing wave formed on a string. It results due to the superposition of two waves travelling in opposite directions. The waves are travelling along the length of the string in the x-direction and displacements of elements on the string are along the y-direction. Individual equations of the two waves can be expressed as

$$Y_1 = 6(\text{cm}) \sin [5 (\text{rad/cm})\, x - 4 (\text{rad/s})\, t]$$
$$Y_2 = 6(\text{cm}) \sin [5 (\text{rad/cm})\, x + 4 (\text{rad/s})\, t]$$

Here x and y are is cm.

Answer the following questions.

46. Maximum value of the y-positions coordinate in the simple harmonic motion of an element of the string that is located at an antinode will be

a. ± 6 cm **b.** ± 8 cm
c. ± 12 cm **d.** ± 3 cm

47. If one end of the string is at $x = 0$, positions of the nodes can be described as

a. $x = n\pi/5$ cm, where n = 0, 1, 2, …
b. $x = n\,2\pi/5$ cm, where n = 0, 1, 2, …
c. $x = n\pi/5$ cm, where n = 1, 3, 5, …
d. $x = n\pi/10$ cm, where n = 1, 3, 5, …

48. Amplitude of simple harmonic motion of a point on the string that is located at x = 1.8 cm will be

a. 3.3 cm **b.** 6.7 cm
c. 4.9 cm **d.** 2.6 cm

49. Figure 7.103(c) shows the standing wave pattern at $t = 0$ due to superposition of waves given by y_1 and y_2 in Figs. 7.103(a) and (b). In Fig. 7.103(c), N is a node and A an antinode. At this instant say $t = 0$, instantaneous velocity of points on the string

a. is different for different points
b. is zero for all points
c. changes with position of the point
d. is constant but not equal to zero for all points

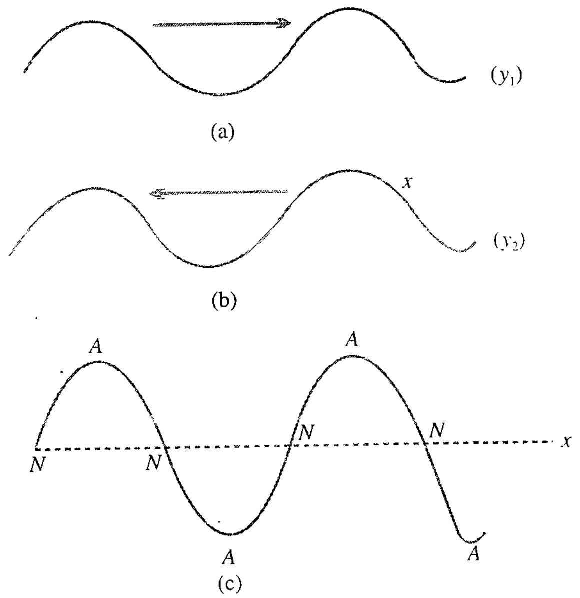

Fig. 7.103

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For Problems 50–54

A vertical pipe open at both ends is partially submerged in water. A tuning fork of unknown frequency is placed near the top of the pipe and made to vibrate. The pipe can be moved up and down and thus length of air column in the pipe can be adjusted. For definite lengths of air column in the pipe, standing waves will be set up as a result of superposition of sound waves travelling in opposite directions. Smallest value of length of air column, for which sound intensity is maximum is 10 cm [take speed of sound, $v = 344$ m/s].

Answer the following questions.

50. The air column here is closed at one end because the surface of water acts as a wall. Which of the following is correct?

a. At the closed end of the air column, there is a displacement node and also a pressure node
b. At the closed end of the air column, there is a displacement node and a pressure antinode
c. At the closed end of the air column, there is a displacement antinode and a pressure node
d At the closed end of the air column, there is a displacement antinode and also a pressure antinode

51. Frequency of the tuning fork is

a. 1072 Hz **b.** 940 Hz
c. 860 Hz **d.** 533 Hz

52. Length of air column for second resonance will be

a. 30 cm **b.** 45 cm
c. 20 cm **d.** 50 cm

53. Length of air column for third resonance will be

a. 30 cm **b.** 45 cm
c. 20 cm **d.** 50 cm

54. Frequency of the second overtone is

a. 3400 Hz **b.** 2500 Hz
c. 4300 Hz **d.** 1720 Hz

Matching Column Type

Solutions on page 7.99

1. Two identical speakers emit sound waves of frequency 10^3 Hz uniformly in all directions. The audio output of each speaker is $9\pi/10$ mW. A point 'P' is at a distance 3 m from the speaker S_1 and 5m from speaker S_2. Resultant intensity at P is I_R. Match the items in Column I with the items in Column II:

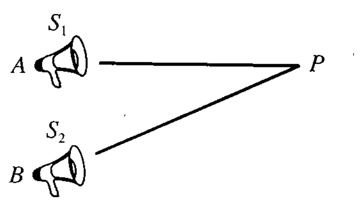

Fig. 7.104

Column I	Column II
i. If the speakers are incoherent, then	**a.** $I_R = 64\ \mu W/m^2$
ii. If the speakers are driven coherently and in phase at P	**b.** $I_R = 25\ \mu W/m^2$
iii. If the speakers are driven coherently and out of phase by 180° at P, then	**c.** $I_R = 34\ \mu W/m^2$
iv. If the speaker S_2 is switched off, then	**d.** $I_R = 4\ \mu$ W/m²

2. Three successive resonance frequencies in an organ pipe are 1310, 1834 and 2358 Hz. Velocity of sound in air is 340 m/s, then match the items given in Column I with that in Column II:

Column I	Column II
i. Length of the pipe in cm	**a.** 262
ii. Fundamental frequency (Hz)	**b.** 786
iii. Frequency of fifth harmonic (Hz)	**c.** 32.4
iv. Frequency of 1 overtone (Hz)	**d.** 1310

3. Match the statements in Column I with the statements in Column II:

Column I	Column II
i. A tight string is fixed at both ends and sustaining standing wave.	**a.** At the middle, antinode is formed in odd harmonic.
ii. A tight string is fixed at one end and free at the other end.	**b.** At the middle, node is formed in even harmonic.
iii. Standing wave is formed in an open organ pipe. End correction is not negligible.	**c.** At the middle, neither node nor antinode is formed.
iv. Standing wave is formed in a closed organ pipe. End correction is not negligible.	**d.** Phase difference between SHMs of any two particles will be either π or zero.

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4 Match the Columns I and II.

Column I	Column II
i. $y = 4\sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$	**a.** Particles at every position are performing SHM
ii. $y = 10\cos\left(t - \frac{x}{330}\right)\sin(100)\left(t - \frac{x}{330}\right)$	**b.** Equation of travelling wave
iii. $y = 10\sin(2\pi x - 120t) + 10\cos(120t + 2\pi x)$	**c.** Equation of standing wave
iv. $y = 10\sin(2\pi x - 120t) + 8\cos(118t - 59/30\pi x)$	**d.** Equation of beats

5. A string fixed at both ends is vibrating in resonance. In Column I some statement(s) are given which can match with one or more entries of Column II. Match these entries.

Column I	Column II
i. All the particles of the string are vibrating in phase.	**a.** Fundamental tone
ii. The particles near both the ends of the string are vibrating in phase.	**b.** Ist harmonic
iii. The particles near the ends of the string are vibrating in opposite phase.	**c.** Even harmonic
iv. All the particles of string cross mean and extreme positions simultaneously twice in one cycle.	**d.** Odd harmonic

6. With respect to various types of strings on piano, match the entries of Column I with that of Column II.

Column I	Column II
i. Bass strings (low frequency)	**a.** Thick
ii. Treble strings (high frequency and small wavelength)	**b.** Thin
iii. To have larger wavelengths, string should be	**c.** Long
iv. To have shorter wavelengths string should be	**d.** Short

7. Two strings are joined as shown in Fig. 7.105 (assume the strings under tension).

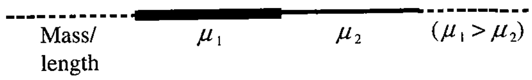
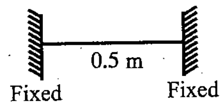
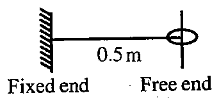
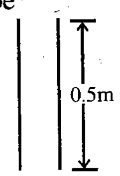
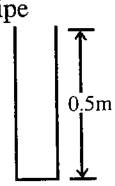


Fig. 7.105

Column I	Column II
i. Wave speed is	**a.** same on both the strings
ii. Wavelength is	**b.** different on both the strings
iii. Frequency is	**c.** more on the Ist string
iv. Power, assuming same amplitude, is	**d.** less on the Ist string

8. In each of the four situations of column I a stretched string or an organ pipe is given along with the required data. In case of strings the tension in string is T = 102.4 N and the mass per unit length of string is 1g/m. Speed of sound in air is 320 m/s. Neglect end corrections. The frequencies of resonance are given in Column II. Match each situation in Column I with the possible resonance frequencies given in Column II.

Column I	Column II
i. String fixed at both ends 	**a.** 320 Hz
ii. String fixed at one end and free at other end 	**b.** 480 Hz
iii. Open organ pipe 	**c.** 640 Hz
iv. Closed organ pipe 	**d.** 800 Hz

9. Select the quantities from Column II which will change with respect to the case when observer, source and air are stationary. Consider all motion along the line joining source and observer:

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Column I	Column II
i. Source moves, observer stationary	**a.** Frequency of sound received by observer
ii. Sound reaches observer after reflection from fixed wall, source and observer stationary	**b.** Speed of sound with respect to medium
iii. Observer moves, source stationary	**c.** Wavelength of wave in medium
iv. Wind blows, source and observer stationary	**d.** None

10. A closed organ pipe of length L vibrating in second overtone, then match the following:

Column I	Column II
i. Displacement node	**a.** Closed end
ii. Displacement antinode	**b.** open end
iii. Pressure node	**c.** $4L/5$ from closed end
iv. Pressure antinode	**d.** $L/5$ from closed end

Integer Answer Type

Solutions on page 7.100

1. For a certain organ pipe, three successive resonance observed are 425, 595 and 765 Hz. Taking the speed of sound to be 340 ms^{-1}, find the length of the pipe, in meter.

2. The standing wave pattern shown in the tube has a wave speed of 5.0 ms^{-1}. What is the frequency of the standing wave [in Hz approx.]?

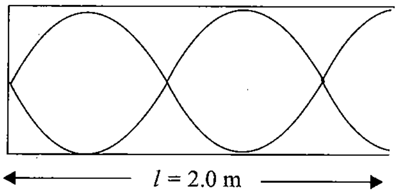

Fig. 7.106

3. A tuning fork of frequency 200 Hz is in unison with a sonometer wire. How many beats are heard in 30 s if the tension is increased by 1% (in terms of × 10]

4. Two identical sinusoidal waves travel in opposite direction in a wire 15 m long and produce a standing wave in the wire. If the speed of the waves is 12 ms^{-1} and there are 6 nodes in the standing wave. Find the frequency.

5. A glass tube of 1.0 m length is filled with water. The water can be drained out slowly at the bottom of the tube. A vibrating tuning fork of frequency 500 Hz is brought at the upper end of the tube and the velocity of sound is 300 ms^{-1}. Find the number of resonances that can be obtained.

6. A tube, opened from both ends is vibrated in its second overtone. At how many points inside the tube maximum pressure variation is observed?

7. nth harmonic of a closed organ pipe is equal to mth harmonic of an open pipe. First overtone frequency of the closed organ pipe is also equal to first overtone frequency of the open organ pipe. Find the value of n, if $m = 6$.

8. A closed and an open organ pipe of same length are set into vibrations simultaneously in their fundamental mode to produce 2 beats. The length of open organ pipe is now halved and of closed organ pipe is doubled. Now find the number of beats produced .

9. The length, radius, tension and density of string A are twice the same parameters of string B. Find the ratio of fundamental frequency of B to the fundamental frequency of A.

ANSWERS AND SOLUTIONS

Subjective Type

1. By conservation of energy, the incident power equals the transmitted power plus the reflected power

$$P_{in} = P_t + P_r$$

$$\frac{1}{2}\mu_1\omega^2 A_{in}^2 v_1 = \frac{1}{2}\mu_2\omega^2 A_t^2 v_2 + \frac{1}{2}\mu_1\omega^2 A_r^2 v_1 \quad \text{(i)}$$

$$\mu_1 = F/v_1^2 \quad \text{and} \quad \mu_2 = F/v_2^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{A_{in}^2}{v_1} = \frac{A_t^2}{v_2} + \frac{A_r^2}{v_1}$$

Given, $v_1 = 3v_2$

$$A_{in}^2 = 3A_t^2 + A_r^2 \quad \text{(iii)}$$

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_{in} = \frac{A_{in}}{2}$$

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_{in} = \frac{-A_{in}}{2}$$

$$= \frac{-\sqrt{13}}{2}\text{ cm}$$

2. Since $v = \sqrt{T/\mu}$, $T_2 = T_1$

Fig. 7.107

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$$\mu_2 = 1/2\mu_1, \quad v_2 = \sqrt{2}v_1$$

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$$

$$\frac{A_t}{A_i} = \frac{2\sqrt{2}v_1}{v_1 + \sqrt{2}v_1}$$

$$= \frac{2\sqrt{2}}{(1+\sqrt{2})}$$

3. For fundamental frequency

$$\mu = \frac{3.2\,\text{g}}{40\,\text{cm}} = \frac{3.2\times10^{-3}}{40\times10^{-2}}$$

$$= \frac{32}{4000}\ \text{kg/m}$$

$$l = \lambda/2$$

$$\lambda = 2l \qquad \text{(i)}$$

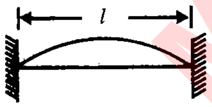

Fig. 7.108

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\frac{1000}{64} = \frac{1}{2\times40\times10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\left[\frac{1000}{64}\times2\times40\times10^{-2}\right]^2\frac{32}{4000} = T$$

$$\Rightarrow \quad T = \frac{10}{8}\ \text{N}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\dfrac{10/8}{10^{-6}}}{\dfrac{0.05\times10^{-2}}{40\times10^{-2}}} = 10^9\ \text{N/m}^2$$

4. $3 = 3\dfrac{\lambda}{2}$

$\lambda = 2$ m

$P_m = 100$ N/m^2, $\quad V = 330$ m/s, $\quad \rho_0 = 1$ kg/m^3

i. $P_m = B s_0 k = \rho_0 v^2 s_0 \dfrac{2\pi}{\lambda}$

$$s_0 = \frac{\lambda P_m}{\rho_0 v^2 2\pi} = \frac{2\times100}{1\times330\times330\times2\pi}$$

$$s_0 = \frac{1}{1089\pi}\text{m}$$

ii. $B = -\dfrac{dp}{dV/V} = \dfrac{dp}{d\rho/\rho}$

$$\left[m = \rho v \Rightarrow 0 = \frac{d\rho}{\rho} + \frac{dv}{v}\right]$$

$$\Rightarrow d\rho = \frac{\rho dp}{B}$$

$$(d\rho)_{\max} = \frac{\rho(d\rho)_{\max}}{B} = \frac{\rho\, p_m}{B}$$

$$(d\rho)_{\max} = \frac{\rho p_m}{\rho v^2} = \frac{100}{108900}\ \text{kg/m}^3 = \frac{1}{1089}\ \text{kg/m}^3$$

5. Path difference = $(\mu - 1)\, d$

$$\mu = \frac{V_0}{V'} = \frac{\sqrt{\dfrac{\gamma R(T_0)}{m_0}}}{\sqrt{\dfrac{\gamma R(4T_0)}{m_0}}} = \frac{1}{2}$$

$$|\text{Path difference}| = \left|\left(\frac{1}{2}-1\right)d\right| = \frac{d}{2}$$

For minima, path difference = $(2n+1)\dfrac{\lambda}{2} = \dfrac{d}{2}$

$$\frac{d}{(2n+1)} = \lambda = \frac{V}{f}$$

$$f = \frac{V(2n+1)}{d}$$

$$f_{\min} = \frac{V}{d}$$

6. $\lambda_{\text{air}} = \dfrac{V_{\text{air}}}{f} = \dfrac{330}{1000} = 0.33\,\text{m}$

$$v_{\text{water}} = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.25\times10^9}{1000}}$$

$$= 1.5\times10^3 = 1500$$

$$\lambda_{\text{water}} = \frac{1500}{1000} = 1.5\ \text{m}$$

$$\lambda_{\text{water}} - \lambda_{\text{air}} = 1.5 - 0.33 = 1.17\ \text{m}$$

7. The sound wave reaches detector by two paths simultaneously by straight as well as semicircular track. The wave through the straight part travels a distance l_1 = 2×20 cm and the wave through the curved part travels a distance $l_2 = \pi$ (20 cm) = 62.8 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore

$$\Delta l = l_2 - l_1 = 62.8\ \text{cm} - 40\,\text{cm} = 22.8\ \text{cm} = 0.228\ \text{m}$$

The wavelength of either wave is

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$$\frac{v}{n} = \frac{340}{n}$$

For constructive interference, $\Delta l = N\lambda$, where N is an integer.

$$0.228 = N\left(\frac{340}{n}\right)$$

or $$n = N\left(\frac{340}{0.228}\right) = N\,(1491.2)\text{ Hz} = N\,(1490)\text{ Hz}.$$

Thus the frequencies within the specified range which cause maxima of intensity are 1490 Hz and 2980 Hz.

8. The situation is shown in Fig. 7.109.

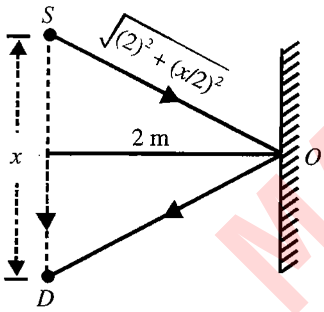

Fig. 7.109

Suppose the detector is placed at a distance of x metre from the source. The wave received from the source travels a distance x metre. The wave reaching the detector after reflection from the wall has travelled a distance of $2[\sqrt{(2)^2 + x^2/4}]$ metre. Thus the path difference between the two waves at detector is

$$\Delta = \left\{2\sqrt{(2)^2 + \frac{x^2}{4}} - x\right\} \qquad \text{(i)}$$

Constructive interference will take place when $\Delta = \lambda$, 2λ, ... Thus the minimum distance x for which a maximum occurs at detector, the path difference will be

$$\Delta = \lambda \qquad \text{(ii)}$$

The wavelength is

$$\lambda = \frac{v}{n} = \frac{360}{180} = 2\text{ m}$$

$$\Delta = 2\sqrt{2^2 + \frac{x^2}{4}} - x = 2$$

$$\sqrt{4 + \frac{x^2}{4}} = 1 + \frac{x}{2}$$

$$4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$$

$$x = 3\text{ m}$$

Thus, the detector should be placed at a distance of 3 m from the source to detect a maximum sound.

9. When detector is at O, we can see that the path difference in the two waves reaching O is $d = 2\lambda$ thus at O detector receives a maximum sound. When it reaches p and again there is a maximum sound detected at P the path difference between two waves must be $\Delta = \lambda$. Thus from the figure the path difference at P can be given as

$$\Delta = S_1P - S_2P \simeq S_1Q$$
$$= 2\lambda \cos\theta$$

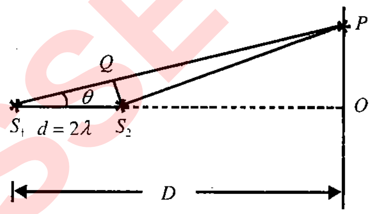

Fig. 7.110

And we have at point P, path difference $\Delta = \lambda$, thus

$$\Delta = 2\lambda \cos\theta = \lambda$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Thus the value of x can be written as $x = D\tan\theta$

$$= D\tan\left(\frac{\pi}{3}\right) = \sqrt{3}\,D$$

10. The given equation of stationary wave is

$$y = 4\cos\frac{\pi x}{3}\sin 40\pi t$$

$$y = 2\times 2\cos\frac{2\pi x}{6}\sin\frac{2\pi\,(120)t}{6} \qquad \text{(i)}$$

We know that

$$y = 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi v t}{\lambda} \qquad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

A =2 cm, $\lambda = 6$ cm and v = 120 cm/s

The component waves are

$$y_1 = A\sin\frac{2\pi}{\lambda}(vt - x)$$

and $$y_2 = A\sin\frac{2\pi}{\lambda}(vt + x)$$

Distance between two adjacent nodes

$$= \frac{\lambda}{2} = \frac{6}{2} = 3\text{ cm}$$

Particle velocity

$$\frac{dy}{dt} = 4\cos\frac{\pi x}{3}\cos(40\pi t)\times 40\pi$$

$$= 160\pi\cos\left(\frac{\pi x}{3}\right)\cos 40\pi t$$

At $x = 3$, $t = 1/8$ the particle velocity is given by

$$= 160\pi\cos\pi\cos\left(40\pi\times\frac{1}{8}\right) = 160\pi \text{ cm/s}$$

11. We know the equation of a stationary wave is given by

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right] + A\sin\left[\frac{2\pi}{\lambda}(vt + x)\right]$$

$$= 2A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

$$= R\sin\frac{2\pi}{\lambda}vt$$

Here, $R = 2A\cos(2\pi x/\lambda)$ is the amplitude of medium particle situated at a distance x.

The given equation can be expressed as

$$y = 10\sin\left[\frac{2\pi}{50}(5000t - x)\right] + 10\sin\left[\frac{2\pi}{50}(5000t + x)\right]$$

$$= 2\times 10\cos\left(\frac{2\pi x}{50}\right)\sin\left(\frac{2\pi}{50}5000t\right)$$

Comparing it with standard equation of stationary wave, we get wavelength $\lambda = 50$ units

Wave velocity $v = 5000$ units

Thus amplitude $R = 2\times 10\cos\frac{2\pi x}{50}$

and maximum amplitude $R_{max} = 2\times 10 = 20$ units

Frequency $= \frac{v}{\lambda} = \frac{5000}{50} = 100$ units

And loop length is

$$\frac{\lambda}{2} = \frac{50}{2} = 25 \text{ units}$$

12. Frequency of first fork $= n$. Frequency of 56th fork $= 2n$

$\therefore \quad n + (56 - 1)4 = 2n$

$\therefore \quad n + 220 = 2n$

$n = 220$ Hz

13. $n_A - n_B = 8$

Also

$$n_A = \frac{v}{4(0.32)}, \quad n_B = \frac{v}{4(0.33)}$$

$$\frac{v}{4(0.32)} - \frac{v}{4(0.33)} = 8$$

$v = 338$ m/s

$$\therefore \quad n_A = \frac{v}{4\times 0.32} = \frac{338}{4\times 0.32} = 264 \text{ Hz.}$$

$$n_B = n_A - 8 = 256 \text{ Hz.}$$

14. $n/\sqrt{T}$ = constant. (i)

If f is frequency of fork, the frequency of string $= f \pm 4$ (ii)

From Eqs. (i) and (ii), we get

$$\frac{f+4}{\sqrt{10.2}} = \frac{f-4}{\sqrt{9.9}}$$

Solving for f, we get $f = 532$ Hz.

15. $l_1 + l_2 = 100$ cm, $l_1 - l_2 = 0.2$ cm

$\therefore \quad l_1 = 50.1$ cm and $l_2 = 49.9$ cm

Also $\quad nl$ = constant

$$n\times 50.1 = (n+1)\times 49.9$$

$$n = 249.5 \text{ Hz}$$

$\therefore \quad n_1 = 249.5$ Hz and $n_2 = n + 1 = 250.5$ Hz

16. $n_A = \frac{v}{4\times 0.15}$, $n_B = \frac{v}{2\times 0.305}$

Clearly $\quad n_A > n_B$

$$n_A - n_B = 9$$

$$\frac{v}{0.60} - \frac{v}{0.61} = 9$$

$$v = 329.4 \text{ m/s}$$

$$n_A = \frac{329.4}{0.60} = 549 \text{ Hz}$$

$$n_B = n_A - 9 = 549 - 9 = 540 \text{ Hz.}$$

17. Kundt's tube is a closed organ pipe. Nodes are formed at ends. The separation between six antinodes is $5\lambda/2$. Length of Kundt's tube

$$= \frac{\lambda}{4} + \frac{5\lambda}{2} + \frac{\lambda}{4} = 3\lambda$$

If steel rod is clamped in the middle, node is formed in the middle and antinodes at ends.

$$\therefore \quad l_{steel} = \frac{\lambda_{steel}}{2}$$

or $\quad \lambda_{steel} = 2l_{steel} = 2$ m

Also $\quad n_{air} = n_{steel}$

$$\therefore \quad \frac{v_{air}}{\lambda_{air}} = \frac{v_{steel}}{\lambda_{steel}}$$

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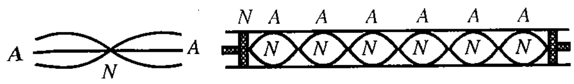

Fig. 7.111

$$\lambda_{air} = \frac{v_{air}}{v_{steel}} \times \lambda_{steel}$$

$$= \frac{343}{5250} \times 2 = 0.1306 \text{ m}$$

$\therefore\ l_{air} = 3\lambda_{air} = 3 \times 0.1306 = 39.2$ cm

18.

$$\frac{v_{51}}{\lambda} - n = 4$$

$$\frac{v_{51}}{\lambda} = n + 4 \qquad \text{(i)}$$

and $$\frac{v_{16}}{\lambda} - n = 1 \qquad \text{(ii)}$$

$$\frac{v_{16}}{\lambda} = n + 1$$

Dividing Eq. (ii) by Eq. (i)

$$\frac{v_{51}}{v_{16}} = \frac{n+4}{n+1}$$

As $$v \propto \sqrt{T}$$

$\Rightarrow$ $$\sqrt{\frac{273+51}{273+16}} = \frac{n+4}{n+1}$$

or $$\sqrt{\frac{324}{289}} = \frac{n+4}{n+1} \quad \text{or} \quad \frac{18}{17} = \frac{n+4}{n+1}$$

$$n = 50 \text{ Hz.}$$

19. Let n_1 be frequency of vibration of left wire and n_2 that of second wire. Then given $n_2 = n_1$. If T_1 and T_2 are tensions in first and second wire, then

$$n_1 = \frac{1}{2l}\sqrt{\frac{T_1}{m}} \quad \text{and} \quad n_2 = \frac{2}{2l}\sqrt{\frac{T_2}{m}}$$

Therefore, $\dfrac{n_2}{n_1} = 2\sqrt{\dfrac{T_2}{T_1}}$

As $$n_1 = n_2 \quad \text{or} \quad \sqrt{T_1} = 2\sqrt{T_2}$$

$\therefore$ $$\sqrt{\frac{T_1}{T_2}} = 2 \quad \text{or} \quad T_1 = 4T_2 \qquad \text{(i)}$$

For vertical equilibrium of rod

$$T_1 + T_2 = 4.8 + 1.2 = 6 \text{ kg wt} \qquad \text{(ii)}$$

From Eqs. (i) and (ii)

$$T_1 = 4.8 \text{ kg}, \quad T_2 = 1.2 \text{ kg}$$

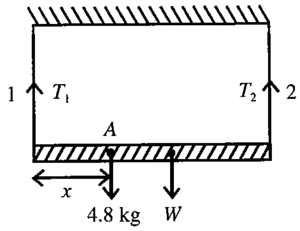

Fig. 7.112

Taking moments about A

$$T_1 x + W(0.2 - x) = T_2(0.4 - x)$$

$$4.8x + 0.24 - 1.2x = 0.48 - 1.2x$$

or $$4.8x = 0.24$$

$$x = \frac{0.24}{4.8} = 0.05 \text{ m} = 5 \text{ cm}$$

20. Frequency of string

$$f_s = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{T}{ML}}$$

$$= \frac{1}{2}\sqrt{\frac{128}{10^{-3} \times 2 \times 10^{-1}}} = 400 \text{ Hz}$$

When tuning fork is moved away from the observer standing near sonometer at constant speed of 0.75 m/s. The apparent frequency of tuning fork.

$$f_1 = f_s\left(\frac{v}{v+0.75}\right) = f_s\left(1 + \frac{0.75}{v}\right)^{-1}$$

$$= f_s\left(1 - \frac{0.75}{v}\right)$$

$$\Delta f = f_s - f_1 = f_s\left(\frac{0.75}{v}\right), \ \Delta f = 1$$

$$v = 300 \text{ m/s (approximately)}$$

21. Case I

$$l = 2\lambda_1/4 \quad \text{and} \quad v = f\lambda_1$$

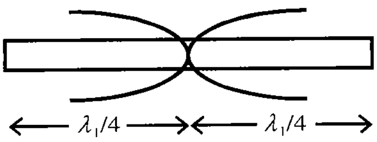

Fig. 7.113

$$V = \sqrt{\frac{Y}{\rho}}, \quad l = \frac{\lambda_1}{2}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

Case II

$$l = 2\left[\frac{\lambda_2}{4} + \frac{\lambda_2}{2}\right]$$

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$$l = \frac{3\lambda_2}{2}$$

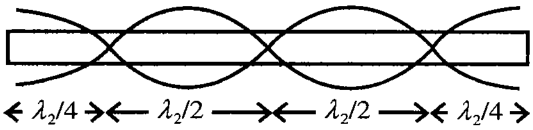

Fig. 7.114

Again $$f_2 = \frac{v}{\lambda_2} = \frac{3}{2l}\sqrt{\frac{Y}{\rho}}$$

Case III

$$l = 2\frac{5\lambda_3}{4}, \quad \lambda_3 = \frac{2l}{5}$$

$$f_3 = \frac{v}{\lambda_3} = \frac{5}{2l}\sqrt{\frac{Y}{\rho}}$$

Comparing expression f_1, f_2, f_3, we have

$$f_n = \frac{2n-1}{2l}\sqrt{\frac{Y}{\rho}}, \text{ where } n = 1, 2, 3, \ldots$$

22. Let σ be the density of the block

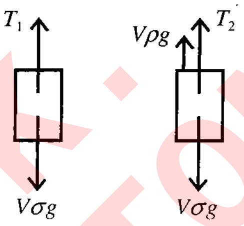

Fig. 7.115

$$n = \frac{5v_1}{2L} = \frac{7v_2}{2L} \quad \text{(i)}$$

$$5v_1 = 7v_2 \quad \text{(ii)}$$

$$5\sqrt{T_1} = 7\sqrt{T_2}$$

where $T_1 = V\sigma g$ and $T_2 = V\sigma g - V\rho g$

squaring both sides of Eq. (ii)

$$25T_1 = 49T_2$$

$$25V\sigma\rho = 49\,(V\sigma g - V\rho g)$$

$$(49 - 25)\,V\sigma g = 49\,V\rho g$$

$$\sigma = \frac{49}{24}\rho$$

23. **a.** $\frac{f}{f'} = \frac{840}{1120} = \frac{3}{4}$

If the frequency of the fundamental note $= f_0$

$\therefore$ $$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

where $$f_0 = \frac{840}{3} = 280 \text{ Hz}$$

or $T = 4l^2\mu f_0^2 = 4(0.25)^2 \times (0.75 \times 10^{-3}) \times (280)^2$

$= 14.7$ N

b. Frequency of the first overtone $= 2f_0 = 2 \times 280 = 560$ Hz and that of fourth overtone $= 5f_0 = 5 \times 280 = 140$ Hz

24. For section AB

$$l = (2n-1)\frac{\lambda}{4}, \quad \text{where } n = 1, 2, 3, \ldots$$

$$f_1 = \frac{v}{\lambda} = \frac{(2n-1)v}{4l}$$

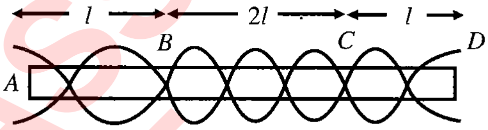

Fig. 7.116

For section BC

$$2l = m\frac{\lambda}{2}, \quad \text{where } m = 1, 2, 3, \ldots$$

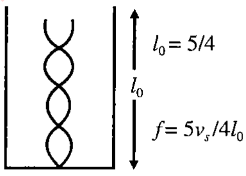

Fig. 7.117

$$f_2 = \frac{vm}{4l}$$

from $$(3) + (4)\; f_1 = f_2$$

$$\frac{(2n-1)}{4l} = \frac{mv}{4l}$$

$$n = \frac{m+1}{2}$$

$m = 1, n = 1$ for fundamental; $m = 3, n = 2$

first overtone $$f_2 = \frac{mv}{4l} = \frac{3v}{4l}$$

Given that $$f = f_2$$

$$\frac{5v_s}{4l_0} = \frac{3v}{4l}$$

$$l = \frac{3vl_0}{5v_s}$$

Therefore, length of the rod $= 4l = \frac{12}{5}\frac{l_0}{v_s}\sqrt{\frac{Y}{\rho}}$

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Objective Type

1. **c.** Standard equation: $x = a\sin\omega t + b\cos\omega t$

$$x=\sqrt{a^2+b^2}\,\sin\left(\omega t+\tan^{-1}(b/a)\right)$$

Given equation $x=3\sin(5\pi t)+4\cos(5\pi t)$

$$x=\sqrt{9+16}\sin\left(5\pi t+\tan^{-1}4/3\right)$$

$$x=5\sin(5\pi t+\tan^{-1}(4/3))$$

2. **c.** $y_2 = 5\ [\sin 3\pi t+\sqrt{3}\cos 3\pi t]$

$$=5\sqrt{1+3}\sin\left(3\pi t+\frac{\pi}{3}\right)$$

$$=10\sin\left(3\pi t+\frac{\pi}{3}\right)$$

So, $A_1 = 10$ and $A_2 = 10$

3. **d.** Probable frequency of A is 390 Hz and 378 Hz and after loading the beats are decreasing from 6 to 4 so the original frequency of A will 390 Hz.

4. **c.** Since there is no change in beats. Therefore the original frequency of B is

$$n_2 = n_1 + x = 320 + 4 = 324$$

5. **a.** Let the frequency of first tuning fork $= n$ and that of last $= 2n$

$n, n+5, n+10, n+15, \ldots 2n$ this forms AP

Formula of AP $l = a + (N-1)\,r$ where l = Last term, a = First term, N = Number of term, r = Common difference

$$2n = n + (41-1)\,5$$

$$2n = n + 200$$

$$n = 200 \text{ and } 2n = 400$$

6. **a.** Standard equation

$$y = A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

By comparing this equation with given equation.

$$\frac{2\pi x}{\lambda}=\frac{\pi x}{20} \Rightarrow \lambda = 40\text{ cm}$$

Distance between two nodes $= \lambda/2 = 20$ cm.

7. **a.** The direction of wave must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

8. **b.** After 2 s, the two pulses will nullify each other. As string now becomes straight, there will be no deformation in the string. In such a situation, the string will not have potential energy at any point. The whole energy will be kinetic.

9. **d.** Let a be the amplitude due to S_1 and S_2 individually.
Intensity due to $S_1 = I_1 = Ka^2$
Intensity due to $S_1 + S_2 = I = K(2a)^2$

$$= 4I_1$$

$$\therefore \quad n = 10\log_{10}\left(\frac{4I_1}{I_1}\right)$$

$$= 10\log_{10}(4) = 6$$

10. **b.** $\frac{I_1}{I_2}=\frac{4}{1}$ or $\sqrt{\frac{I_1}{I_2}}=\frac{2}{1}$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\left[\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right]^2=\left[\frac{2+1}{2-1}\right]^2=9$$

$$\therefore \quad L_1 - L_2 = 10\log\left(\frac{I_{max}}{I_{min}}\right)=10\log 9 = 20\log 3$$

11. **b.** When a stationary wave is established in a medium then maximum deformation of the medium is produced at nodes. Hence, maximum pressure change takes place at nodes and at antinodes, no pressure change takes place. Therefore, option (a) is wrong.

$$v=\sqrt{\frac{\text{Elasticity}}{\text{Density}}}$$

Since, elasticity and density both are the characteristic property of the medium, therefore, velocity of a longitudinal wave in a medium is its physical characteristic. So, option (b) is correct.

Due to propagation of longitudinal wave in a medium pressure change

$$\Delta P = \frac{\gamma P u}{v}$$

where u is the velocity of medium particles.

Pressure change will be maximum possible when medium particles have maximum possible velocity, which is equal to $a\omega = 2\pi na$

Hence, $$\Delta P = \gamma P\frac{2\pi na}{v}$$

But $$\gamma P = \rho v^2$$

$$\therefore \quad \Delta P = 2\pi na\rho v$$

So, option (c) and therefore option (d) is also wrong.

12. **c.** Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence, options (a) and (b) both are wrong. To obtain a stationary wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either

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both of the waves should be longitudinal or both of them should be transverse. Hence, option (c) is correct.

13. **a.** The relative velocity of sound waves w.r.t. the wall is $V + v$. Hence, the apparent frequency of the waves striking the surface of the wall is $(V+v)/\lambda$. The number of positive crests striking per second is same as frequency. In 3 s, the number is $[3(V+v)]/\lambda$.

14. **a.** In both cases, the 'applied frequency' is same. So, the frequency of vibration has to be same. However, the mode of vibration of the string be different.

15. **d.**

$$\frac{f_1}{f_2} = \frac{101}{100}$$

$$f_1 - f_2 = 5$$

$$\frac{101}{100} f_2 - f_2 = 5 \quad \text{or} \quad f_2 = 500 \text{ Hz}$$

and $$f_1 = f_2 + 5 = 505 \text{ Hz}$$

16. **c.** $f = \frac{v}{4l} = \frac{320}{4}\text{Hz} = 80\,\text{Hz}$

Since even harmonics cannot be present therefore 320 Hz $(= 4 \times 80)$ is ruled out.

17. **c.** $\frac{5\lambda}{2} = 82.5 \quad \text{or} \quad \lambda = \frac{2 \times 82.5}{5}\text{ cm} \quad \text{or} \quad \lambda = 33 \text{ cm}$

$$c = 1000 \times \frac{33}{100} \text{ m/s} = 330 \text{ m/s}$$

18. **c.** $\frac{\lambda}{2} = 1 \text{ m} \quad \text{or} \quad \lambda = 2 \text{ m}$

$$v = f\lambda = 2500 \times 2 \text{ m/s} = 5000 \text{ m/s}$$

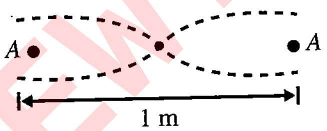

Fig. 7.118

19. **a.** $\frac{v_A}{v_B} = \frac{D_B}{D_A} = \frac{1}{2}$

20. **c.** $3 \times \frac{v}{4l_c} = 2 \times \frac{v}{2l_0} \quad \text{or} \quad \frac{l_c}{l_0} = \frac{3}{4}$

21. **c.** Beats $= \frac{V}{4l} - \frac{V}{4(l+\Delta l)} = \frac{V}{4}\left[\frac{\Delta l}{l(l+\Delta l)}\right]$

$$= \frac{V\Delta l}{4l^2} \qquad (\because \Delta l << l)$$

22. **a.** $v \propto 1/l$

On doubling the length, frequency is halved

The word 'nearly' in the statement has been used keeping is mind 'end correction.'

23. **b.** $l_1 + e = \lambda/4 \quad \text{or} \quad 3l_1 + 3e = 3\lambda/4$

Again $$l_2 + e = \frac{3\lambda}{4}$$

$\therefore$ $$3l_1 + 3e = l_2 + e$$

or $$2e = l_2 - 3l_1$$

or $$e = \frac{1}{2}(l_2 - 3l_1) = \frac{1}{2}(32 - 3 \times 10) = 1 \text{ cm}$$

24. **d.** $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$\Rightarrow 7I = I + 9I + 2\sqrt{I9I}\cos\phi$$

$\cos\phi = -1/2 \quad \text{or} \quad \phi = 120°$

25. **b.** For fundamental mode

$$(\lambda/2) = 100 \text{ cm} \quad \text{or} \quad \lambda = 200 \text{ cm}$$

As $n = 330$ Hz, hence

$$V = n\lambda = 330 \times \frac{200}{100} = 660 \text{ m/s}$$

26. **b.** For second resonance

$$L_2 = \frac{3\lambda}{4} = 3L_1 = 3 \times 20 = 60 \text{ cm}$$

27. **d.** $\lambda = 2(x_2 - x_1) = 2(0.84 - 0.50) = 0.68$

$$n = \frac{v}{\lambda} = \frac{340}{0.68} = 500\,\text{Hz}$$

28. **b.** $\lambda = \frac{330}{500} = 0.66$ m

The resonance occurs at

$$\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots$$

i.e., at 0.165 m, 0.495 m, 0.825 m, 1.115 m. As the length of the tube is only 1.0 m, hence 3 resonances will be observed.

29. **b.** The string vibrates in two segments in the first overtone. Therefore the amplitude of vibration is maximum at $(L/4)$ and $(3L/4)$.

30. **b.** $v = n\lambda$

$$= 2n(l_1 - l_2) = 2f \times 1 = 2f \text{ m/s}$$

31. **b.** Fundamental frequency of a COP is given by

$$f_o = \frac{v}{4l}$$

Length l of air column will first decrease and then becomes constant (when rate or inflow = rate of outflow). Therefore, f_o will first increases and then become constant.

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32. **b.** $\frac{\lambda}{2} = 46 - 16 \Rightarrow \frac{\lambda}{2} = 30\,\text{cm}$

or $\lambda = 60$ cm

$\therefore \quad v = \lambda f = \frac{60}{100} \times 500 = 300$ m/s

33. **d.** $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$$\frac{n_2}{n_1} = \frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$$

$$= \frac{l_1}{\left[l_1 - \frac{40}{100}l_1\right]}\sqrt{\left(\frac{T_1 + \frac{44}{100}T_1}{T_1}\right)}$$

$$= \frac{100}{60} \times \frac{12}{10} = 2:1$$

34. **d.** According to equation

$$2n_1 = 3n_2$$

or $\frac{2}{2l_1}\sqrt{\frac{T}{m_1}} = \frac{3}{2l_2}\sqrt{\frac{T}{m_2}}$

or $\frac{l_1}{l_2} = \frac{2}{3}\sqrt{\frac{m_2}{m_1}} = \frac{2}{3}\sqrt{\frac{a_2\rho}{a_1\rho}}$

or $\frac{l_1}{l_2} = \frac{2}{3}\sqrt{\frac{r_2^2}{r_1^2}} = \frac{2}{3}\sqrt{\left(\frac{1}{2}\right)^2}$

or $\frac{l_1}{l_2} = \frac{1}{3}$

35. **a.** $f_0 = \frac{5}{2l}\sqrt{\frac{9g}{\mu}} = \frac{3}{2l}\sqrt{\frac{Mg}{\mu}}$

$\therefore M = 25$ kg

36. **b.** Length of the path for direct sound = 120 m
Length of the path for reflected sound

$$= 2\sqrt{(60)^2 + (25)^2}] = 130 \text{ m}$$

Geometrical path difference

$$= 130 - 120 = 10 \text{ m}$$

Two waves interfere constructively when $10 = n\lambda$
Putting, $n = 1, 2, 3, \ldots,$ $\quad \lambda = 10, 5, 2.5, \ldots$

37. **a.**

Sol. Let ϕ_1 and ϕ_2 represent angles of the first and second waves, then

$$\phi_2 = \frac{2\pi}{\lambda}[(vt - x) + x_0]$$

and $\quad \phi_1 = \frac{2\pi}{\lambda}(vt - x)$

But $\quad x_0 = \frac{\lambda}{2},$

$$\phi_2 - \phi_1 = \pi$$

Hence, phase difference, $\phi = \pi$. So, amplitude of resultant wave

$$R\sqrt{a^2 + b^2 + 2ab\cos\phi}$$

$$= \sqrt{a^2 + b^2 + 2ab\cos\pi} = \sqrt{(a-b)^2} = a - b$$

or $\quad R = |a - b|$

38. **a.** For $x = 5$, $y = 4\sin\left(\frac{5\pi}{15}\right)\cos(96\pi t)$

$$= 2\sqrt{3}\cos(96\pi t)$$

So, y will be maximum when $\cos(96\pi t) = \max = 1$

$$y_{\text{max.}} = 2\sqrt{3}\,\text{cm at } x = 5$$

39. **b.**

$$v \propto \sqrt{T}, \quad v' \propto \sqrt{T + \frac{1}{100}T}$$

$$\frac{v'}{v} = \left(1 + \frac{1}{100}\right)^{1/2} = 1 + \frac{1}{200}$$

or $\quad \frac{v' - v}{v} = \frac{1}{200}$ or $\frac{3}{2v} = \frac{1}{200}$ or $v = 300\,\text{Hz}$

40. **a.**

$$a = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5\,\text{cm}$$

$$\tan\phi = \frac{4}{3} \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

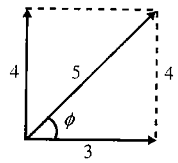

Fig. 7.119

41. **c.** At 25 cm, there will be antinode. So wire will vibrate in two loops.

$$v = \frac{2}{2l}\sqrt{\frac{T \times l}{M}} \quad \text{or} \quad v = \sqrt{\frac{T}{Ml}} = \sqrt{\frac{20}{5 \times 10^{-4} \times 1}}$$

$$= \sqrt{4 \times 10^4}\ \text{Hz} = 200\ \text{Hz}$$

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42. d. For the given problem,

$$\frac{\sqrt{T}}{l} = \text{constant}$$

or $$T \propto l^2$$

If l is to be doubled, T would be quadrupled.

43. c.

$$v = \frac{1}{Dl}\sqrt{\frac{T}{\pi d}} \quad \text{or} \quad v \propto \frac{1}{D}$$

Now, $$\left(\frac{v'}{v}-1\right)\times 100 = \left(\frac{30}{31}-1\right)\times 100$$

$$= -\frac{100}{31} = -3.2.$$

44. a. $\frac{\sqrt{T}}{l}$ = constant; tension decreases by a factor (8 – 1)/8, length decreases by a factor square root of this, i.e., $\sqrt{7/8} = 0.93$

45. a.

$$f = \frac{v}{4l} \quad \text{or} \quad l = \frac{v}{4f} = \frac{330}{4\times 264}\text{m}$$

$$= 0.3125\text{m} = 31.25 \text{ cm}$$

46. b.

$$l = l_1 + l_2 + l_3 \qquad \left[\because v \propto \frac{1}{l}\right]$$

$$\frac{k}{v} = \frac{k}{v_1} + \frac{k}{v_2} + \frac{k}{v_3}$$

$$\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}$$

$$v = \left[\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}\right]^{-1}$$

47. b.

$$3\times\frac{v}{4l_c} = 4\times\frac{v}{2lo} \quad \text{or} \quad \frac{l_c}{lo} = \frac{3v}{4}\times\frac{2}{4v} = \frac{3}{8}$$

48. b.

$$v_1 = 250 \text{ Hz}, v_2 = 253 \text{ Hz}, v_2 - v_1 = 3$$

Now,

$$\frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(4+2)^2}{(4-2)^2} = \frac{36}{4} = 9$$

49. b.

$$\frac{\lambda}{2} = l = \frac{40}{100} = 0.4\ ;\ \lambda = 0.8 \text{ m},$$

$$v = \frac{v}{\lambda} = \frac{5500}{0.8}\text{Hz} = 6875 \text{ Hz}$$

50. c.

$$\frac{\lambda}{2} = 10 \text{ cm} \quad \text{or} \quad \lambda = 20\,\text{cm} = 0.20\,\text{m}$$

$$v = v\lambda = 100\times 0.20 \text{ m/s} = 20 \text{ m/s}$$

51. b. Path difference $= (2l - l) = \lambda/2$ (for minimum)

$$\lambda = 2l$$

52. b.

$$n_1 = \frac{\omega_1}{2\pi} = \frac{400\pi}{2\pi} = 200 \text{ Hz}$$

$$n_2 = \frac{\omega_2}{2\pi} = \frac{404\pi}{2\pi} = 202 \text{ Hz}$$

Therefore, the number of beats $n = n_2 - n_1 = 2$ Hz
Again $A_1 = 4$ and $A_2 = 3$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1+A_2)^2}{(A_1-A_2)^2} = \left(\frac{4+3}{4-3}\right)^2 = \frac{49}{1}$$

This is alternative (b) is correct.

53. d. In ten forks, there are nine intervals

$$n_2 = n_{11} + 9\times 4 \quad (\text{Also given } n_2 = 2n_1)$$

$$2n_1 = n_1 + 36$$

$$n_1 = 36 \text{ Hz}$$

so $$n_2 = 2n_1 = 72 \text{ Hz}$$

54. c. When piston moves a distance x_1, path difference changes by $2x$.
Therefore, the path difference between maxima and consecutive minima $= \lambda/2$

$$2x = \lambda/2$$

or $$\lambda = 4x = 4\times 9 \text{ cm} = 36 \text{ cm} = 0.36 \text{ m}$$

$$n = \frac{v}{\lambda} = \frac{360}{0.36} = 1000 \text{ Hz}$$

55. b. Path difference $= (\pi r - 2r)$

$$= (2n-1)\lambda/2 \text{ for minima}$$

Given $\lambda = 0.40$ m, for smallest radius $n = 1$

$$(3.14-2)r = \lambda/2$$

$$r = \frac{\lambda}{2\times 1.14} = \frac{0.40}{2\times 1.14} = 0.175 \text{ m}$$

56. d. For maximum path difference $\Delta = n\lambda$

$$2\times 0.6\, l = \lambda$$

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$$l = \frac{\lambda}{1.2} = \frac{6}{1.2} = 5\,\text{m}$$

57. d.

$$\frac{v}{4l_1} = 3\left(\frac{v}{2l_2}\right) \Rightarrow \frac{l_1}{l_2} = \frac{1}{6}$$

58. b.

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$$

$$= 2(1+\cos t)\sin 1000\,t$$

$$= 2\sin 1000\,t + 2\cos t \sin 1000 t$$

$$= 2\sin 1000\,t + \sin(1000t + t) + \sin(1000t - t)$$

$$= 2\sin 1000\,t + \sin 1001\,t + \sin 999 t$$

$= y_1 + y_2 + y_3$ = Three waves

59. d. Either, frequency of first wire should decrease or frequency of second wire should increase.

60. a. Length of open organ pipe $l = 2m$.
When it is dipped in water, it becomes closed at one end.
Let l_1 be the length of air column of pipe immersed, then frequency of first overtone of pipe

$$= \frac{3v}{4l_1}$$

Given $\frac{3v}{4l_1} = 170$

$$l_1 = \frac{3v}{4\times 170}$$

$$= \frac{3\times 340}{4\times 170} = 1.5\text{ m}$$

Length immersed

$$x = l - l_1$$

$$= 2 - 1.5 = 0.5\text{ m}$$

61. b. When the stone is suspended in air:

$$n = \frac{1}{2L}\sqrt{\frac{W_a}{m}}$$

When the stone is suspended in water:

$$n = \frac{1}{2L'}\sqrt{\frac{W_w}{m}}$$

Hence,

$$\frac{\sqrt{W_a}}{L} = \frac{\sqrt{W_w}}{L'}$$

or $\frac{W_a}{W_w} = \frac{L^2}{L'^2}$

Now, specific gravity of material of the stone

$$= \frac{W_a}{W_a - W_w} = \frac{1}{1 - \frac{W_w}{W_a}} = \frac{1}{1 - \frac{L'^2}{L^2}}$$

$$= \frac{L^2}{L^2 - L'^2} = \frac{(40)^2}{(40)^2 - (22)^2}$$

62. b.

$$n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}},\ n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$$

Let $l_1 = 100l,\ l_2 = 55l$

$T_1 = 100T,\ T_2 = 121\,T$

$$\therefore \quad \frac{n_2}{n_1} = \frac{100l}{55l}\sqrt{\frac{121T}{100T}}$$

$$= \frac{100}{55}\times\frac{11}{10} = 2$$

$$\therefore \quad n_2 = 2n_1$$

63. b. Let l be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of nth overtone is

$$f_1 = (n+1)\frac{v}{2l}$$

and frequency of closed organ pipe of nth overtone

$$f_2 = (2n+1)\frac{v}{4l}$$

Therefore, the desired ratio is

$$\frac{f_1}{f_2} = \frac{2(n+1)}{(2n+1)}$$

64. c. Velocity of longitudinal waves

$$v_1 = \sqrt{\frac{Y}{\rho}}$$

and velocity of transverse waves

$$v_2 = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\rho s}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{y}{T/s}} = \sqrt{\frac{Y}{Y\left(\frac{\Delta l}{l}\right)}} = \sqrt{n}$$

$$\left[\because \Delta l = \frac{l}{n}\right]$$

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Now $f \propto v$

$\therefore \quad \frac{f_1}{f_2} = \frac{v_1}{v_2} = \sqrt{n}$

In the above expression, ρ = density of string, s = area of cross-section of string, Y = Young's modulus.

65. b.

$$f_o - f_c = 2$$

or $\frac{v}{2l} - \frac{v}{4l} = 2$ or $\frac{v}{4l} = 2$

or $\frac{v}{l} = 8$

When length of OOP is halved and that of COP is doubled, the beat frequency will be

$$f_o' - f_c' = \frac{v}{l} - \frac{v}{8l} = \frac{7}{8}\frac{v}{l} = \frac{7}{8} \times 8 = 7$$

66. b. Frequency $f \propto \sqrt{mg}$

or $f \propto \sqrt{g}$

In water, $f_w = 0.8 f_{air}$

$\therefore \quad \frac{g'}{g} = (0.8)^2 = 0.64$

or $1 - \frac{\rho_w}{\rho_m} = 0.64$

or $\frac{\rho_w}{\rho_m} = 0.36$ (i)

or $-\frac{\rho_L}{\rho_m} = 0.36$

or $\frac{\rho_L}{\rho_m} = 0.64$ (ii)

From Eqs. (i) and (ii),

$$\frac{\rho_L}{\rho_m} = \frac{0.64}{0.36} = 1.77$$

67. c. Beat frequency $= f_1 - f_2 = \frac{v}{2l} - \frac{v}{2(1+x)}$

$$= \frac{v}{2l}\left[1 - \left(1 + \frac{x}{l}\right)^{-1}\right]$$

$$= \frac{v}{2l}\left[1 - 1 + \frac{x}{l}\right]$$

$$= \frac{vx}{2l^2}$$

68. d. Given that the frequency of wave produced in the string is $1/n$.

$\therefore \quad \frac{1}{n} = \frac{1}{2\pi}\sqrt{\frac{T}{m}}$

Now $T' = 2T$

Therefore, new frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{2T}{m}} = \sqrt{2} \times \frac{1}{n}$$

Therefore, the number of waves produced per second is

$$\frac{1}{f} = \frac{n}{\sqrt{2}}$$

69. b.

$$y = \frac{4}{2}\left[2\cos^2\left(\frac{t}{2}\right)\right]\sin(1000t)$$

or $y = 2(1 + \cos t) \sin 1000t$

or $t = 2 \sin 1000t + 2 \sin 1000t \cos t$

or $y = 2 \sin 1000t + \sin(1001t) + \sin(999t)$

So, the given expression is a result of the superposition of three independent harmonic motions.

70. c.

$$I = 2\pi^2 a^2 v^2 \rho v$$

$$a^2 = \frac{I}{2\pi^2 v^2 \rho v} \text{ or } a = \frac{1}{\pi v}\sqrt{\frac{I}{2\rho v}}$$

or $a = \frac{7}{22 \times 1000}\sqrt{\frac{10^{-12}}{2 \times 1.293 \times 332}}$ m

or $a = 1.1 \times 10^{-11}$ m

71. d.

$$v_B = v_A + \frac{3}{100}v_A$$

$$v_C = v_A - \frac{2}{100}v_A$$

$$v_B - v_C = 8$$

$$\frac{3}{100}v_A + \frac{2}{100}v_A = 8$$

or $\quad v_A \times \frac{5}{100} = 8 \quad$ or $\quad v_A = 160\,\text{Hz}$

72. **b.** When the end of the string is free to move, the string being attached to weightless ring that can slide freely along the rod, the phase of reflected pulse is unchanged antinode is formed at the ring.

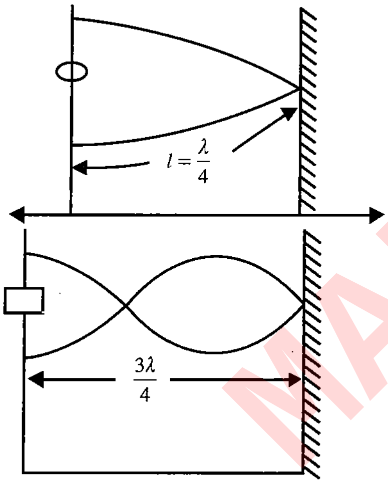

Fig. 7.120

$$l = \frac{\lambda}{4} \Rightarrow \lambda_1 = 4l = 9.6\text{ m}$$

$$\lambda_2 = \frac{4l}{3} = 3.2\text{ m}$$

$$\lambda_3 = \frac{4l}{5} = \frac{9.6}{5} = 1.92\text{ m}$$

73. **a.**

$$f(E) = 1.5 \times 400 = 600\text{ Hz} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2 \times 0.25}\sqrt{\frac{90}{\mu}}$$

$$\mu = \frac{90}{(0.5)^2 \times (600)^2} = 1\text{ g/m}$$

74. **b.**

$$y_1 = 2A\sin\omega t$$

$$y_2 = \frac{A}{2}\sin\left(\omega t + \frac{\pi}{6}\right)$$

$$y_3 = \frac{A}{2}\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_4 = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

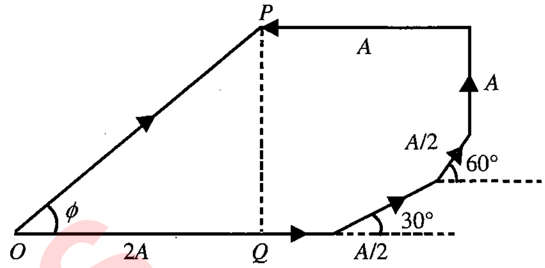

Fig. 7.121

$$y_5 = A\sin(\omega t + \pi)$$

By phaser diagram,

$$\tan\phi = \frac{PQ}{OQ} = 1$$

$$\phi = 45°$$

Alternatively: $\quad y = 2A\sin\omega t +$

$$\frac{A}{2}(\sin\omega t\cos 30° + \cos\omega t \sin 30°)$$

$$+\frac{A}{2}(\sin\omega t\cos 60° + \cos\omega t\sin 60°) + A\cos\omega t - A\sin\omega t$$

$$= A'\cos\phi\sin\omega t + A'\sin\phi\cos\omega t$$

where $\quad A'\cos\phi = \left[A + \frac{A}{4}(\sqrt{3}+1)\right]$

$$A'\sin\phi = \left[A + \frac{A}{4}(\sqrt{3}+1)\right]$$

$$\tan\phi = 1$$

$$\phi = 45°$$

75. **b.** Maximum frequency

$$v = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{A\rho}}$$

$$= \frac{1}{2}\sqrt{\frac{7.85 \times 10^8}{7.7 \times 10^3}} = 158\text{ Hz}$$

76. **b.** Substituting $x = 0$, we have given wave $y = A\sin\omega t$ at $x = 0$ other should have $y = -A\sin\omega t$ equation so displacement may be zero at all the time. Hence, option (b) is correct.

77. **c.**

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

If radius is doubled, mass per unit length will become four times. Hence

$$f' = \frac{1}{2 \times 2l}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$$

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78. a. $\lambda = 2l = 3\text{m}$

Equation of standing wave

(As $x = 0$ is taken as a node)

$$y = 2A \sin kx \cos \omega t,$$

given $2A = 4$ mm

To find value of x for which amplitude is 2 mm, we have

2 mm = (4 mm) sin kx

$$\frac{2\pi}{\lambda}x = \frac{\pi}{6} \quad \Rightarrow x_1 = \frac{1}{4}\text{m}$$

$$\frac{2\pi}{\lambda}x = \frac{\pi}{2} + \frac{\pi}{3} \quad \Rightarrow x_2 = 1.25 \text{ m}$$

$$x_2 - x_1 = 1 \text{ m}$$

79. a. $\Delta v = 384 - 288 = 96$

Thus 288 and 384 (96 × 3; 96 × 4) are third and fourth harmonics.

For fundamental mode:

$$\frac{\lambda}{2} = 0.75$$

$$\lambda = 1.5$$

$$\eta = 96$$

$$v = 96 \times 1.5 = 144 \text{ m/s}$$

80. c. For a string vibrating in its nth overtone $(n + 1)$th harmonic)

$$(n+1)\frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{n+1}$$

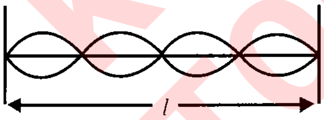

Fig. 7.122

$$kx = \frac{2\pi x}{\lambda} = \frac{\pi(n+1)x}{L}$$

$$y = 2A\sin\left(\frac{(n+1)\pi x}{L}\right)\cos\omega t$$

here $2A = a$ and $n = 3$.

for $x = \frac{L}{3}$, $y = \left[a\sin\left(\frac{4\pi}{L}\times\frac{L}{3}\right)\right]\cos\omega t$

$$= a\sin\frac{4\pi}{3}\cos\omega t = -a\left(\frac{\sqrt{3}}{2}\right)\cos\omega t$$

i.e., at $x = L/3$; the amplitude is $\sqrt{3}a/2$.

81. d. In a sonometer,

$$f \propto \sqrt{T}$$

Thus, $$\frac{f_1}{f_2} = 2 = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = \frac{T_1}{4}$$

So percentage change will be

$$\frac{T_1 - T_2}{T_1}\times 100 = \frac{T_1 - \frac{T_1}{4}}{T_1}\times 100 = 75\%$$

82. d.

$$f = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow p_1\sqrt{T_1} = p_2\sqrt{T_2}$$

$$\Rightarrow 6\sqrt{36} = 4\sqrt{T_2} \Rightarrow T_2 = 81 \text{ N}$$

83. a. Figure 7.123 shows variation of displacement of particle in a closed organ pipe for 3rd overtone.

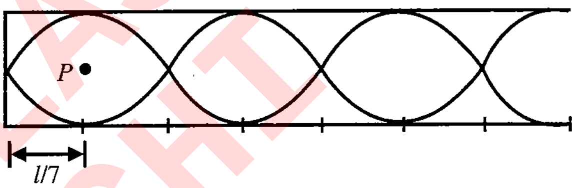

Fig. 7.123

For third overtone

$$l = \frac{7\lambda}{4} \quad \text{or} \quad \lambda = \frac{4l}{7} \quad \text{or} \quad \frac{\lambda}{4} = \frac{l}{7}$$

Hence the amplitude at P at a distance $l/7$ from closed end is 'a' because there is an antinode at that point.

Alternate: Because there is node at $x = 0$ the displacement amplitude as function of x can be written as

$$A = a \sin kx = a\sin\frac{2\pi}{\lambda}x$$

For third overtone

$$l = \frac{7\lambda}{4} \quad \text{or} \quad \lambda = \frac{4l}{7}$$

$$A = a\sin\frac{2\pi 7x}{4l}$$

At $\quad x = \frac{l}{7} \quad A = a$

84. a. When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the oncoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave.

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85. a. After a time t, velocity of observer V_0 = at

$$f_0 = \left(\frac{V+V_0}{V}\right)f_s = \left(\frac{V+at}{V}\right)f_s$$

which is a straight line graph of positive slope.

86. b.

$$dB = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{K/r^2}{I_0}\right)$$

$$= 10\,[\log(K') - 2\log r]$$

$$dB_1 = 10\,(\log K' - 2\log r_1)$$

$$dB_2 = 10\,(\log K' - 2\log r_2)$$

$$3 = dB_1 - dB_2 = 20\,\log\left(\frac{r_2}{r_1}\right)$$

$$(0.3) = \log\left(\frac{r_2}{r_1}\right)^2$$

$$\left(\frac{r_1}{r_2}\right) = \frac{1}{\sqrt{2}}$$

87. a.

$$f_1\lambda_1 = f_2\lambda_2$$

$$(300)\,(1) = (f_2)\,(1.5)$$

$$200\text{ Hz} = f_2$$

88. c.

$$v_{max} = \omega_n A = (2\pi f)A = (2\pi)(440)(10^{-6})$$

$$= 2.76\times10^{-3}\ \text{m/s}$$

89. a. For minimum,

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

The maximum possible path difference = difference between the sources = 3m.
For no minimum

$$\frac{\lambda}{2} > 3 \quad \Rightarrow \lambda > 6$$

$$f = \frac{V}{\lambda} < \frac{330}{6} = 55$$

If $f < 55$ Hz, no minimum will occur.

90. a. The speed of sound in air is

$$v = \sqrt{\frac{\gamma RT}{M}}$$

γ/M of H_2 is least, hence speed of sound in H_2 shall be maximum.

91. b.

As $\quad y = A_b \sin(2\pi n_{av}t)$

where $\quad A_b = 2A\cos(2\pi n_A t)$

where $\quad n_A = \dfrac{n_1 - n_2}{2}$

92. d. As number of beats = Δv
For option (a), the frequencies are

$$v_1 = 550\,\text{Hz}, v_2 = 552\,\text{Hz}, v_3 = 553\,\text{Hz}, v_4 = 560\,\text{Hz}$$

The beats produced will be

$$\Delta v_1 = v_2 - v_1 = 2$$

$$\Delta v_2 = v_3 - v_1 = 3$$

$$\Delta v_3 = v_4 - v_1 = 10$$

$$\Delta v_4 = v_3 - v_2 = 1$$

$$\Delta v_5 = v_4 - v_2 = 8$$

$$\Delta v_6 = v_4 - v_3 = 7$$

which does not match with the given set of beat frequencies. Hence option (a) is not possible.
Similarly options (b) and (c) are also not possible.
For option (d), frequencies are $v_1 = 550$, $v_2 = 551$, $v_3 = 553$, $v_4 = 558$

$$\Delta v_1 = v_2 - v_1 = 1$$

$$\Delta v_2 = v_3 - v_1 = 3$$

$$\Delta v_3 = v_4 - v_1 = 8$$

$$\Delta v_4 = v_3 - v_2 = 2$$

$$\Delta v_5 = v_4 - v_2 = 7$$

$$\Delta v_6 = v_4 - v_3 = 5$$

which matches with the given set of beat frequencies.
Hence option (d).

93. b.

$$V_S = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{11}}{10.0\times10^4}} = 10^3\ \text{m/s}$$

$$t = \frac{2l}{V} = \frac{2\times100}{1000} = 0.2\,\text{s}$$

94. b.

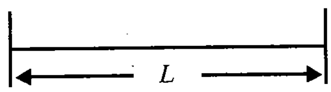

Fig. 7.124

Fundamental frequency of wire $(f_{wire}) = v/2l$

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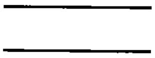
a.

$$f = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l} \text{ cannot match with } f_{\text{wire}}$$

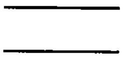
b.



$$f = \frac{v}{2(2l)}, \frac{2v}{2(2l)}, \frac{3v}{2(2l)} \text{ its second harmonic } \frac{2v}{2(2l)}$$

matches with f_{wire}.

c.



$$f = \frac{v}{2(l/2)}, \frac{2v}{2(l/2)} \text{ cannot match with } f_{\text{wire}}$$

d.

$$f = \frac{v}{4(l/2)}, \frac{3v}{4(l/2)}, \cdots \text{ cannot match with } f_{\text{wire}}$$

95. **d.** x_1 and x_2 are in successive loops of stationary waves.

so, $\phi_1 = \pi$

and $\phi_2 = K(\Delta x) = K\left(\frac{3\pi}{2K} - \frac{\pi}{3K}\right) = \frac{7\pi}{6}$

$$= \frac{\phi_1}{\phi_2} = \frac{6}{7}$$

96. **a.**

$$l_1 + \varepsilon = \frac{v}{4f_0}$$

$$l_2 + \varepsilon = \frac{3v}{4f_0}$$

$$l_3 + \varepsilon = \frac{5v}{4f_0}$$

On solving, we get $l_3 = 2l_2 - l_1$

97. **d.** Total path difference $= AB + BC + \lambda/2 = \lambda$ for maxima

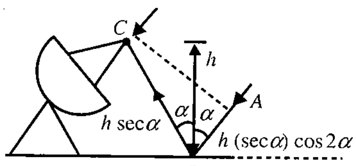


Fig. 7.125

$$h \sec\alpha \cos 2\alpha + h \sec\alpha = \lambda/2$$

$$h \sec\alpha\,(2\cos^2\alpha) = \lambda/2$$

$$h = \frac{\lambda}{4\cos\alpha}$$

98. **a.** If detector moves x distance, distance from direct sound increases by x and distance from reflected sound decreases by x so path difference created $= 2x$

$$2(0.14) = 14\lambda = 14\,c/f$$

$$f = \frac{14 \times 3 \times 10^8}{0.14 \times 2} = 1.5 \times 10^{10}\text{ Hz}$$

99. **i→d.; ii.→b.**

Drumming frequency = 40 cycle/min = 40 cycle/60 s

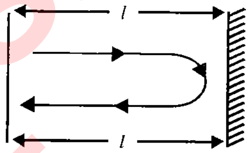


Fig. 7.126

Drumming time period

$$T = \frac{1}{f} = \frac{60\text{ s}}{40\text{ cycle}} = \frac{3}{4}\text{ s/cycle}$$

(time duration between consecutive drumming)

During this time interval, if sound goes to mountain and comes back then echo will not be heard distinctly.

$$\frac{3}{4} = \frac{2l}{v} \qquad \text{(i)}$$

Now if he moves 90 m. This situation arises at t = 60 cycle/min,

$$T = \frac{1}{f} = 1\text{ s/cycle}$$

for this case sound goes to mountain and comes back after time $T/2$:

$$\frac{1}{2} = \frac{2(l-90)}{v} \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii)

so, $l = 270\text{ m}$

$v = 720$ m/s

100. **b.** Initially the standing wave equation is

$$y = 2A\sin kx \cos\omega t$$

If phase difference ϕ is added to one of waves, then resulting standing wave equation is

$$y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$$

Here, frequency does not change and also spacing between two successive nodes does not change as its value for both

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is π/k. But for a particle, in standing wave, amplitude changes.

101. b. Velocity of the string section can be given as

$$v=\frac{\partial y}{\partial t}=4\cos\pi x\times(50\pi)\cos(50\pi t)$$

$$v=4\cos\left[\pi\times\frac{1}{3}\right]\times 50\pi\cos\left[50\pi\times\frac{1}{5}\right]$$

$$=200\pi\times\frac{1}{2}\times 1=100\pi \text{ cm/s } =\pi \text{ m/s}$$

102. d. For closed organ pipe,

$$f=\frac{v}{4L}\times(2n-1)$$

For minimum and maximum length of pipe the fundamental frequency of pipe must be 20 kHz and 20 Hz, respectively.

$$20=\frac{320}{4L_{max}}$$

$$L_{max}=4\text{ m}$$

$$20\times 10^3=\frac{320}{4L_{min}}$$

$$L_{min}=4\text{ mm}$$

103. b. Standing waves form when two waves of equal amplitude, same frequency, same wavelength travelling in opposite directions superimpose, as a result, the net transfer of energy through any cross-section is zero in standing waves.

104. d. Consider the wave as shown in Fig. 7.127. The six particles (1–6) have been shown which all have displacements equal to $\pm A/2$ form their equilibrium positions.

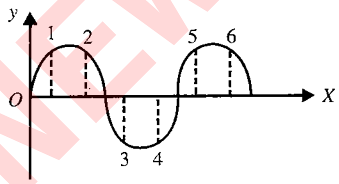

Fig. 7.127

To get the separation between two particles having displacement of amplitude $A/2$, we have

$$\frac{A}{2}=A\sin(kx-\omega t),\text{ at } t=0$$

$$\Rightarrow \quad kx=\frac{\pi}{6},\frac{5\pi}{6},\ldots \quad \text{and} \quad x_2-x_1=\frac{\lambda}{3}$$

separation between particles 1 and 2 comes out to be $\lambda/3$, where λ is the wavelength. Between particles 1 and 3, it is $\lambda/2$. From given information, separation between 1–2, 3–4 or 5–6 is 8 cm.

$$\lambda/3=8\text{cm} \quad \Rightarrow \lambda=24\text{ cm}$$

The separation between 2–3 which is equal to separation between 1–3 minus separation between 1–2

$$=\frac{\lambda}{2}-\frac{\lambda}{3}=\frac{\lambda}{6}=4\text{ cm}$$

105. a. String 1 is heavy so it can easily pull up the lighter string 2, while string 2 being lighter would not be able to displace the point.

106. b. If one of the natural frequencies of the string matches with the source frequency, then resonance condition will arise and the string will vibrate with source frequency.

107. c. If none of the natural frequencies of the string matches with the frequency of the source, then string will finally vibrate with the frequency of tuning fork, but here resonance condition would not be found.

108. c. Velocity of wave on string $=\sqrt{T/\mu}=8$ m/s.

The pulse gets inverted after reflection from the fixed end, so for constructive interference to take place between successive pulses, the first pulse has to undergo two reflections from the fixed end.

So, $$\Delta t=\frac{2\times 0.4+2\times 0.4}{8}=0.2\text{ s}$$

109. d. The equation of stationary wave for open organ pipe can be written as

$$y=2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi ft}{v}\right)$$

where $x=0$ is the open end from where the wave gets reflected.

Amplitude of stationary wave is

$$A_S=2A\cos\left(\frac{2\pi x}{\lambda}\right)$$

For $\quad x=0.1$ m,

$$A_S=2\times 0.002\cos\left[\frac{2\pi\times 0.1}{0.4}\right]=0$$

110. c. The frequencies given are odd multiple of fundamental. Hence close organ pipe

$$50=\frac{1}{4l}\times 340 \quad \Rightarrow l=1.7\text{ m}$$

111. b.

$$f_1=f_0\left(\frac{V_0}{V_0-V}\right) \quad f_2=f_0\left(\frac{V_0}{V_0+V}\right)$$

$$f_1-f_2=f_0V_0\left(\frac{1}{V_0-V}-\frac{1}{V_0+V}\right)$$

$$= f_0V_0\left(\frac{V_0 + V - V_0 + V}{V_0^2 - V^2}\right) = f_0V_0 \times \frac{2V}{V_0^2} = f_0\,\frac{2V}{V_0}$$

given $\frac{2Vf_0}{V_0} = 0.02 \times f_0 \Rightarrow V = 0.01\ V_0 = 3.4$ m/s.

112. a. The frequency of oscillation of the standing wave is same as that of either of the component waves.

113. c. Beat frequency, $\Delta f = 6$ Hz

Time interval between two consecutive maxima is 1/6 s.

So, the required time 1/2 s.

114. c. For destructive interference, path difference has to be equal to an odd integral multiple of $\lambda/2$.

115. d. Sound waves in an organ pipe (which are standing in nature) is an example of superposition of two longitudinal travelling waves. Standing waves on a string is an example of superposition of two transverse travelling waves on a string travelling in opposite directions.

116. c. In an open organ pipe, both the ends are free ends, hence both are displacement antinodes and hence pressure nodes.

117. b. Frequency of wave = 1/4 Hz.

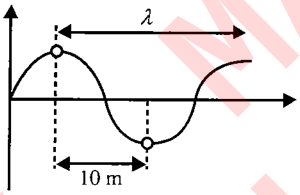

Fig. 7.128

Wavelength of wave, $\lambda = 2 \times 10 = 20$ m

Velocity of wave $f\lambda = 5$ m/s

118. c. Since the standing wave mode has a displacement antinode at the opening, there is a displacement node at the water–air interface. By increasing the height of the air column, to go from one harmonic to the next, an additional length equal to ½ wavelength is required. Hence

$$\frac{\lambda}{2} = (0.38 - 0.12)\text{m} \quad \Rightarrow \lambda = 0.52\text{m}$$

Finally, from $v = f\lambda$, we find that $f = v/\lambda = 312/0.52 =$ 600 Hz. If one checks, this problem deals with the 1st and 3rd harmonics.

119. c.

$$\lambda/2 = 29.5 - 10.5 = 19 \text{ cm}$$

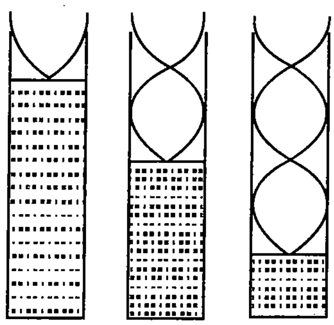

Fig. 7.129

3rd resonance = 19 + 29.5 = 48.5 cm

120. a. For minima,

$$\Delta x = (2n + 1)\,\frac{\lambda}{2} \quad \text{and} \quad \lambda = \frac{v}{f}$$

$$0.5 = \frac{(2n+1)}{2}\,\frac{300}{f}$$

$$f = (2n + 1)\,300$$

Therefore, all odd multiples of 300 are silenced.

121. c. The decibel scale is logarithmic $dB = 10\log(I/I_0)$. Each increase in intensity by a power of ten increases the decibel reading by 10 units. Hence, to increase the decibel reading by 20, there should be an increase in the intensity of 10 × 10 = 100.

122. c.

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

123. a. Waves expressed by tuning fork

$$y = 0.2\sin(kx - \omega t)$$

Maximum value of amplitude of beat is $2A$

$$y = 2 \times 0.2 = 0.4 \text{ cm}$$

124. d. Wavelength of sound

$$= \frac{v}{f} = \frac{340 \text{ m/s}}{606 \text{ s}^{-1}} = 56.1 \text{ cm}$$

Since, closed pipe allows only odd harmonics, so

$$f = (2n+1)\frac{v}{4l} \quad \text{or,} \quad l = (2n+1)\frac{v}{4f};\ n \in I$$

or, $l = (2n + 1) \times 14$ cm

$\therefore$ l = 14 cm, 42 cm, 70 cm, 98 cm, 126 cm, 154 cm, etc.

Since $l > 150$ cm

$\therefore$ No. of resonances = 5

125. d.

For $$y = x + \frac{4\pi}{\alpha}$$

$$\frac{\partial r}{\partial t} = 0$$

i.e., all points lying on the $y = x + \frac{4\pi}{\alpha}$ are always at rest.

126. c. Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\lambda/2$.

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So, If $y_{\text{incident}} = a\cos(kx - \omega t)$, then

$y_{\text{reflected}} = -a\cos(\omega t + kx)$

127. c. For 1st reading of oscillator

$$f_A = (514 \pm 2)\text{ Hz}$$

$$f_A = 516\text{ Hz} \quad \text{or} \quad 512\text{ Hz}$$

For 2nd reading of oscillator

$$f_A = (510 \pm 6)\text{ Hz}$$

$$f_A = 516\text{ Hz or } 504\text{ Hz}$$

A has a frequency 516 Hz.

128. a.

$$f_{\text{closed}} = \frac{v}{4l}$$

$$256 = \frac{v}{4(0.31)}$$

$$v = 317.44\text{ m/s}$$

129. c. The brass rod is open at both ends.

So the longitudinal waves will have a fundamental frequency $f_0 = \frac{v}{2l}$.

$$v = (3000)(2)\left(\frac{40}{100}\right)$$

$$v = 2400\text{ m/s}$$

130. a. With clamp at the centre $L = \lambda/2$ for the fundamental.

So, $$f' = \frac{v}{2L} = 4\text{ kHz}$$

When clamp is moved to one end then

$$L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots, (2n-1)\frac{\lambda}{4}$$

For $n = 1, 2, \ldots$

$$f_n = (2n-1)\frac{v}{4L}$$

$f_0 = 2\text{ kHz}$ (1st harmonic)

$f_1 = 6\text{ kHz}$ (2nd harmonic or 1st overtone)

$f_2 = 10\text{ kHz}$ (3rd harmonic or 2nd overtone)

131. a.

$$f \propto \sqrt{T}$$

$$\frac{f+5}{f-5} = \sqrt{\frac{121}{100}}$$

$$10f + 50 = 11f - 55$$

$$f = 105\text{ Hz}$$

132. d.

Since $$f_n = n\left(\frac{v}{2L}\right) = n\left(\frac{330}{1.6}\right) = 206n$$

$$\lambda_n = \frac{2L}{n} = \frac{1.6}{n} \qquad \left\{L = \frac{n\lambda_n}{2}\right\}$$

And the standing wave equation with nodes at both ends is

$$s = s_0 \sin(3.93\,nx)\cos(1295\,nt)$$

For fundamental mode/frequency $n = 1$

$$s = s_0 \sin(3.93x)\cos(1295t)$$

133. b.

$$f_{\text{open}} = \frac{v}{2l} = v$$

$$f_{\text{closed}} = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} = v$$

134. c. Supports for the loop are reasonable for nodes at two points.

$$\pi r = n\left(\frac{\lambda}{2}\right)$$

$$\pi\frac{D}{2} = n\frac{\lambda}{2}$$

$$f = n\left(\frac{v}{\pi D}\right)$$

Fundamental frequency $= v/\pi D$

135. d.

$$\frac{n_1}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi r^2\rho}}$$

$$= \frac{n_2}{2\left(\frac{l}{2}\right)}\sqrt{\frac{T}{\pi(4r^2)\rho}}$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{1}{2}$$

136. d. 45 cm = 5(9 cm) and 99 cm = 11(9 cm)

So two other lengths between these two values are 7 (9 cm) and 9 (9 cm), i.e., 63 cm and 81 cm respectively.

So the fundamental length is 9 cm

$$9 = \frac{\lambda}{4} \quad \text{(for a closed organ pipe)}$$

$$\lambda = 36\text{ cm}$$

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137. c.

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta T}{T} = 2\left(\frac{\Delta f}{f}\right)$$

$$\frac{\Delta T}{T}\times 100 = \text{percentage change in tension}$$

$$\frac{\Delta T}{T} = 2\left(\frac{5}{500}\right)$$

$$\frac{\Delta T}{T} = \frac{1}{50}$$

$$\frac{\Delta T}{T}\times 100 = 2\%$$

138. c.

$$f_1 = \frac{v}{4(24.1+0.3D)}$$

$$f_3 = \frac{3v}{4(74.1+0.3D)}$$

$$f_1 = f_3$$

$$\Rightarrow \frac{v}{4(24.1+0.3D)} = 3\frac{v}{4(74.1+0.3D)}$$

$$3(24.1 + 0.3D) = 74.1 + 0.3D$$

$$72.3 + 0.9D = 74.1 + 0.3D$$

$$0.6D = 74.1 - 72.3$$

$$0.6D = 1.8$$

$$D = \frac{1.8}{0.6} = 3 \text{ cm}$$

139. c. Beat frequency = 2 (256) – 3(170)

= 512 – 510

= 2 Hz

140. b. For interference at A: S_2 is behind of S_1 by a distance of $100\lambda = \lambda/4$ (equal to phase difference $\pi/2$). Further S_2 lags S_1 by $\pi/2$. Hence the waves from S_1 and S_2 interfere at A with a phase difference of $200.5\pi + 0.5\pi = 201\pi = \pi$. Hence the net amplitude at A is $2a - a = a$. For interference at B: S_2 is ahead of S_1 by a distance of $100\lambda + \lambda/4$ (equal to phase difference $\pi/2$). Further S_2 lags S_1 by $\pi/2$. Hence the waves from S_1 and S_2 interfere at B with a phase difference of $200.5\pi - 0.5\pi = 200\pi = 0\pi$. Hence the net amplitude at A is $2a + a = 3a$.

Hence, $$\left(\frac{I_A}{I_B}\right) = \left(\frac{a}{3a}\right)^2 = \frac{1}{9}$$

141. d. As wave has been reflected from a rare medium; therefore there is no change in phase. Hence equation for the opposite direction can be written as

Multiple Correct Answers Type

1. a., b., d.
Fundamental frequency:

$$v = \frac{320}{4\times 1}\text{ Hz} = 80 \text{ Hz}$$

Now only odd harmonics are present

2. b., d. Since $T_1 > T_2$, $v_1 > v_2$

Now, $v_1 - v_2 = 6$

Beat frequency would remains the same even if $v_2 - v_1 = 6$. To decrease v_1, T_1 needs to be decreased. To increase v_2, T_2 needs to be increased.

3. c., d. For a closed tube

$$f_n = \frac{nv}{4L}$$

$L = 1.1$ m, $v = 330$ m/s

$$f_n = \frac{n\times 330}{4\times 1.1} = 500 \text{ Hz} \qquad n = 6.66$$

highest frequency,

$$f_h = \frac{6\times 330}{4\times 1.1} = 450 \text{ Hz}$$

Lowest frequency,

$$f_l = \frac{1\times 330}{4.4} = 75 \text{ Hz}$$

4. c., d. Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi + \pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity, and acceleration. They in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A. The velocity of this wave is the same as if it were moving alone.

5. a., b., d. If P divides AB in ratio 1:4, then the fundamental frequency corresponds to 5 loops, one loop in AP and 4 loops in PB which corresponds to 5th harmonic of 1 kHz. Hence fundamental = 5 kHz.

If P be taken at midpoint, the third harmonic will have three loops in each half of the wire AB. Hence total number of nodes (including A and B) will be 5 + 2 = 7.

If P divides AB in the ratio 1:2, the fundamental will have three loops, corresponding to the frequency of 3

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kHz. For this string to vibrate with the fundamental of 1 kHz, the tension must be $(T/9)$.

The wire AB will be symmetry, vibrate with the same fundamental frequency when P divides AB in the ratio $a:b$ or in the ratio $b:a$.

6. a., b., d. Speed of wave in wire

$$V=\sqrt{\frac{T}{\rho A}}=\sqrt{\frac{Y\Delta l}{l}A\times\frac{1}{\rho A}}=\sqrt{\frac{Y\Delta l}{l\rho}}$$

Minimum frequency; that means fundamental mode.

$$f=\frac{V}{\lambda}=\frac{V}{2l}=\frac{1}{2l}\sqrt{\frac{Y\Delta l}{l\rho}}=35\text{ Hz}$$

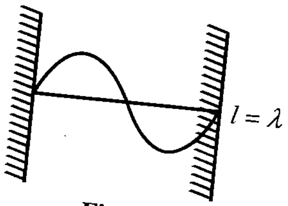

Fig. 7.130

$$\text{Stress}=Y\frac{\Delta l}{l}$$

$$=9\times10^{10}\times\frac{4.9\times10^{-4}}{1}$$

$$=4.41\times10^{7}\text{ N/m}^2$$

and frequency of first overtone = 70 Hz.

7. a., c., d.

$$\frac{\lambda}{4}=0.1\quad\Rightarrow\lambda=0.4$$

From graph $\Rightarrow T=0.2$ s and ampl... wave is $2A=4$ cm.

$$y(x,t)=-2A\cos\left(\frac{2\pi}{0.4}x\right)\sin\left(\frac{2\pi}{0.2}t\right)\text{cm}$$

$$Y(x=0.05,t=0.05)=-2\sqrt{2}\text{ cm}$$

$$Y(x=0.04,t=0.05)=-2\sqrt{2}\cos36^\circ$$

$$\text{Speed}=\frac{\lambda}{T}=2\text{ m/s}$$

$$V_y=\frac{dy}{dt}=-2A\times\frac{2\pi}{0.2}\cos\left(\frac{2\pi x}{0.4}\right)\cos\left(\frac{2\pi t}{0.2}\right)$$

$$V_y\left(x=\frac{1}{15}m,\ t=0.1\right)=20\pi\text{ cm/s}$$

8. b., c. At any point on line AB, the phase difference between two waves is zero and hence waves will interfere constructively.

Along CD, the phase difference changes and waves interfere constructively and destructively and, hence sound will be loud, faint and so on.

9. b., d.

$$\Delta\phi=\frac{2\pi x_1}{\lambda}-\omega t+\frac{\pi}{4}-2\pi\frac{x_2}{\lambda}+\omega t-\frac{\pi}{6}$$

$$=\frac{2\pi}{\lambda}(x_1-x_2)+\frac{\pi}{12}$$

for constructive interference

$$\frac{2\pi}{\lambda}(x_1-x_2)+\frac{\pi}{12}=2n\pi\qquad n=0,1,2,\ldots$$

for destructive interference

$$\frac{2\pi}{\lambda}(x_1-x_2)+\frac{\pi}{12}=(2n-1)\pi$$

10. b., c. When two waves having same frequency superimpose under given conditions the frequency of the resultant wave is the same as that of component waves. From the theory of interference of waves it can be easily understood. The amplitude of resultant wave for the given situation is given by

$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\delta}$ where δ is the phase difference between the waves, so A can be anything between A_1-A_2 to A_1+A_2 depending upon the value of δ.

11. c., d.

$$y_A=A\sin[\omega t-k(AC)]$$

$$y_B=\sin\left[\omega t-\frac{\pi}{2}-k(BC)\right]$$

For maximum intensity at C

$$k(BC-AC)+\frac{\pi}{2}=2n\pi$$

$$BC-AC=\left(n\lambda-\frac{\lambda}{4}\right)=15,35,55,75,\ldots$$

12. a., d.

a. New wave $=y_1\hat{j}+z_1\hat{k}$

$$=(a\hat{j}+a\hat{k})\sin\omega\left(\frac{t-x}{v}\right)$$

$$\text{Amplitude}=|a\hat{j}+a\hat{k}|=a\sqrt{2}$$

(i) and (ii) are travelling in opposite directions, so they will form stationary waves.

Similarly (iii) and (iv) will make the stationary wave.

13. c., d.

$$y=y_1+y_2$$

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… $\cos\omega t \sin kx$

… $kx + 3A\cos\omega t \sin kx$

… $s\, kx + 2A\cos\omega t \sin kx$

… $A\sin\omega t\cos kx + 2A\sin(\omega t + kx)$

It is combination of a stationary and travelling wave.

Maximum amplitude $= 4A$

Minimum amplitude $= 2A$

Distance between points having amplitude $4A$ and $2A$ will be $= \lambda/4 = v/4f$

14. a., b., d.

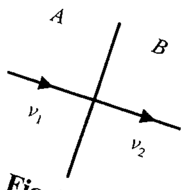

Fig. 7.131

$$v_2 > v_1$$

Frequency does not change on changing the medium.

$$\lambda = \frac{v}{f}$$

As velocity increases, so wavelength increase.

As there is no absorption or reflection of wave, so intensity remain same.

15. a., b. In case of a stationary wave, all the particles lying between two consecutive nodes oscillate in the same phase.

Since all the particles of given string are oscillating in the same phase, therefore, all the parts … string is lie between two consecutive … (a) is correct.

… oscillating in the single … but the intensity varies with … its fundamental … of obtaining constant … of observation. Hence, intensity … point to point. Hence, to observe … two sources having same frequency must be some distance apart. Hence option (b) is correct.

Beats is a phenomenon of obtaining an intensity which varies with time. To obtain beats, two sources having different frequencies are required. Therefore, option (c) is wrong.

a., b., d. Statement (a) is correct. Let us write

$$y(x, t) = f(vt + x) = f(z)$$

Differentiating with respect to time t, we have

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} = v\frac{\partial f}{\partial x}$$

Differentiating again with …

$$\frac{\partial^2 y}{\partial t^2} = v \ldots$$

Similarly, differentiating twice …

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}$$

Hence

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

which is the standard equation (in diff… travelling wave.

Statement (b) is also correct. Becau… reflected back into the same medium, the v… unchanged. The wavelength cannot cha… frequency cannot change by reflection.

Statement (c) is incorrect. The ultrasonic wave bends away from th… because the speed of the wave (being a sound… greater in water than in air.

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Statement (b) is correct. Standing waves are produced due to superposition of the incident wave and the waves reflected from the fixed ends of the string. Since, the ends are never perfectly rigidly fixed, the amplitude of the reflected wave is always less than that of the incident wave. Consequently, the resultant amplitude at nodes is not exactly zero. Thus, the nodes are not well defined.

Statement (c) is also correct. To observe beats, the difference between the two interfering frequencies must be less than about 10–16 Hz. Since, visible light waves have very high frequencies, beats are not observed due to persistence of vision.

Statement (d) is also correct. We know that

$$f_1 = \frac{f}{1 - \frac{u}{v}}$$

And

$$f_2 = f\left(1 + \frac{u}{v}\right) \qquad (i)$$

Expression Eq. (i) may be written as

$$f_1 = f\left(1 - \frac{u}{v}\right)^{-1} \qquad (ii)$$

Expanding binomially and retaining terms up to order u^2/v^2, we have

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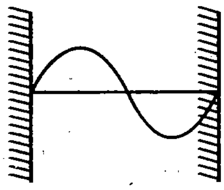

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From graph $\Rightarrow T=0.2$ s and amplitude of standing wave is $2A=4$ cm.

Equation of the standing wave

$$y(x,t)=-2A\cos\left(\frac{2\pi}{0.4}x\right)\sin\left(\frac{2\pi}{0.2}t\right)\text{cm}$$

$$Y(x=0.05,\ t=0.05)=-2\sqrt{2}\text{ cm}$$

$$Y(x=0.04,\ t=0.05)=-2\sqrt{2}\cos 36^\circ$$

$$\text{Speed}=\frac{\lambda}{T}=2\text{ m/s}$$

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$$= A\sin(\omega t - kx) + 3A\sin(\omega t + kx)$$

$$= A\sin\omega t\cos kx - A\cos\omega t\sin kx$$

$$+ 3A\sin\omega t\cos kx + 3A\cos\omega t\sin kx$$

$$= 4A\sin\omega t\cos kx + 2A\cos\omega t\sin kx$$

$$= 2A\sin\omega t\cos kx + 2A\sin(\omega t + kx)$$

It is combination of a stationary and travelling wave.

Maximum amplitude = 4 A

Minimum amplitude = 2 A

Distance between points having amplitude $4A$ and $2A$ will be $= \lambda/4 = v/4f$

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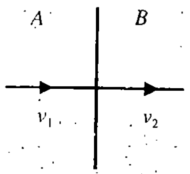

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Since all the particles of given string are oscillating in the same phase, therefore, all the particles of the string lie between two consecutive nodes. Hence, the string is oscillating in the single loop. It means, it is oscillating in its fundamental tone. Hence, option (a) is correct.

Interference is a phenomenon of obtaining constant intensity at a fixed position but the intensity varies with position of the point of observation. Hence, intensity should vary from point to point. Hence, to observe interference, two sources having same frequency must be placed some distance apart. Hence option (b) is correct.

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16. a., b., d. Statement (a) is correct. Let us write

$$y(x, t) = f(vt + x) = f(z)$$

Differentiating with respect to time t, we have

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} = v\frac{\partial f}{\partial x}$$

Differentiating again with respect to time t, we have

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 f}{\partial x^2}$$

Similarly, differentiating twice with respect to x, we have

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}$$

Hence $$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

which is the standard equation (in differential form) of a travelling wave.

Statement (b) is also correct. Because the wave is reflected back into the same medium, the velocity remains unchanged. The wavelength cannot change because frequency cannot change by reflection.

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The ultrasonic wave bends away from the normal because the speed of the wave (being a sound wave) is greater in water than in air.

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Statement (d) is also correct. We know that

$$f_1 = \frac{f}{1 - \frac{u}{v}} \qquad \text{(i)}$$

And $$f_2 = f\left(1 + \frac{u}{v}\right) \qquad \text{(ii)}$$

Expression Eq. (i) may be written as

$$f_1 = f\left(1 - \frac{u}{v}\right)^{-1}$$

Expanding binomially and retaining terms up to order u^2/v^2, we have

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$$f_1 = v\left(1+\frac{u}{v}+\frac{u^2}{v^2}\right) \quad \text{(iii)}$$

Comparing Eqs. (ii) and (iii), we find that $f_1 > f_2$.

18. **a., b., d.** A stationary wave is characterized by a function of type $y = f(t)\, g(x)$. Hence, choices (a) and (b) represent a stationary wave. Choice (d) is superposition of two oppositely travelling waves of the same amplitude and same frequency, which gives rise to a stationary wave. Hence choice (d) also represents a stationary wave.

19. **b., c.** Comparing with the equation

$$y = 2A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t), \text{ we have}$$

$$2A = 2\text{mm} \quad \text{or} \quad A = 1 \text{ mm}$$

and $$\frac{n\pi x}{L} = 6.28x = 2\pi x$$

$$L = \frac{n}{2}\text{m}$$

for $n = 1, L = 0.5$ m

20. **a., b., c., d.** If a wave is incident normally on a surface then it gets reflected back to its original path. The incident wave is travelling along negative x-direction and reflected wave is travelling along positive x-direction. Hence, the wave is incident normally on the surface. Therefore, option (a) is correct.

The equation of the reflected wave will be $y' = a' \sin (ct - bx + \phi)$ only when the reflecting surface is $x = 0$ plane, i.e., $y - z$ plane.

Hence option (b) is correct.

Since, ϕ is equal to zero, it means, no phase change takes place at the reflecting surface. It is possible only when the reflecting surface is boundary of a rarer medium. It means wave is travelling in a denser medium relative to the other medium. Hence, option (c) is also correct.

If the reflecting surface is perfectly elastic then whole of the incident energy gets reflected back. In that case a' will be equal to a. But if a part of wave is refracted into the other medium, then the amplitude of oscillations for the reflected wave will be less than that for incident wave. It implies that a' can never be greater than a. Hence, option (d) is also correct.

21. **a., c.** If a string of length l has cross-sectional area A, density of its material ρ then its oscillation energy is given by

$$E = \pi^2 A\rho a_0^2 l f^2$$

where f is frequency of transverse stationary wave formed in the string.

But $$f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

where λ is wavelength, T is tension in the string and $m = A\rho$.

Since, string has a fixed length, therefore, wavelength of a tone excited in the string is constant. Hence, energy $E \propto T$. Therefore, option (a) is correct.

If the frequency of fundamental tone is f_0, then frequency of nth overtone will be equal to $(n + 1) f_0$.

Hence, oscillation energy of the string will be equal to:

$$E_n = \pi^2 A\rho a_0^2\, l\, f_0^2 (n+1)^2$$

Since, E_n is not directly proportional to n^2, therefore, option (b) is wrong.

Since, every particle of the string performs SHM, therefore, r.m.s. speed of a particle

$$= 1/\sqrt{2} \times \text{its maximum speed}$$

Hence, average KE is half of maximum KE. But maximum KE is equal to oscillation energy of the string. Therefore, option (c) is correct.

22. **b., c.** This is the case of sustained interference in which position of maxima and minima remains fixed all over the screen.

$$\frac{I_{\min}}{I_{\max}} = \left(\frac{a_1 - a_2}{a_1 + a_2}\right)^2$$

And both waves must have been travelling in the same direction with a constant phase difference (condition for coherence)

23. **c., d.**

$$t = 0, \frac{1}{(f_1 - f_2)}, \frac{2}{(f_1 - f_2)}, \frac{3}{(f_1 - f_2)}, \ldots$$

are times at which maxima are obtained

$$t = \frac{\frac{1}{2}}{(f_1 - f_2)}, \frac{\frac{3}{2}}{(f_1 - f_2)}, \frac{\frac{5}{2}}{(f_1 - f_2)}, \ldots$$

are times at which minima are obtained.

24. **b., c.** On reflection from a rigid support the reflected wave suffers an additional phase change of π. When this reflected wave superimposes with incident wave stationary waves are obtained with node at the rigid support and intensity of such stationary waves vary periodically with distance.

25. **a., c., d.**

$$y = y_1 + y_2$$

$$y = 4[\sin(3x - 2t) + \sin(3x + 2t)]$$

$$y = 4[2\sin(3x)\cos(2t)]$$

$$y = 8\sin(3x)\cos(2t)$$

$$y = R\cos(2t)$$

R = Resultant Amplitude = $8 \sin (3x)$

$R = 8 \sin [3\,(2.3)]$

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$$R = 8 \sin (6.9)$$
$$R = 4.63 \text{ cm}$$

Nodes are formed at points of zero intensity, i.e.,

$$I_R = R^2 = 0$$
$$\sin^2(3x) = 0$$
$$\sin(3x) = 0$$
$$3x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$$
$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$$

Antinodes are formed in between.

Assertion-Reasoning Type

1. **d.** The frequency of the plucked string will be same as the wave it produce in air but the speeds of the waves depend on the media in which they are propagating.
2. **d.** Changes of pressure and density occur at nodal points only for a longitudinal standing wave.
3. **c.** Principle of superposition holds true only when the vectors are linear functions of variable and its derivatives.
4. **d.** At node $v = 0$, at antinode tension perpendicular to velocity therefore, at these points power = 0, $(P = \vec{F} \cdot \vec{V})$
At other points $P \neq 0$.
5. **b.** As tension is increased to 4 times, the wave speed (of component waves) increases by a factor of 2 and hence the wavelength.
The spacing between two consecutive nodes in standing waves is equal to half of wavelength of component waves. Let λ be the wavelength of component waves before increasing the tension, then $\Delta x = \lambda/2$.
After increasing the tension in string $\Delta x'$ (spacing between different nodes)
$$= \frac{2\lambda}{2} = 2\Delta x$$
So, spacing between the node and antinode is
$$\frac{\Delta x'}{2} = \Delta x$$
6. **d.** In standing waves the medium particles are oscillating and hence are not at rest, though few particles present at the location of node remain at rest.
7. **c.** $P_{av} = \frac{\rho v \omega^2 A^2}{2}$ when medium changes v, ρ and A can change but frequency remains same.
8. **a.** A node is a place of zero amplitude and an antinode is a place of maximum amplitude.
9. **d.** Each wave continues to move onwards in its respective direction in interference.
10. **c.** Superposition principle is valid for other frequencies also, like standing wave or interference phenomena.
11. **d.** In a closed organ pipe $l_2 = 3l_1$
$$l_2 = 3 \times 60 = 180 \text{ cm}$$
i.e., statement 1 is false and statement 2 is true.
12. **a.** The fundamental frequency of an organ pipe is $n = V/2l$. As temperature increases, both V and l increases but V increases more rapidly than l. Hence fundamental frequency increases as the temperature increases.
13. **c.** The resultant amplitude of two waves is given by
$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta}$$
Here $\quad a_1 = a_2 = A = a \quad$ or $\quad \frac{1}{2} = 1 + \cos\theta$
or $\quad \cos\theta = -\frac{1}{2} \quad$ or $\quad \theta = 120^\circ$
14. **d.**

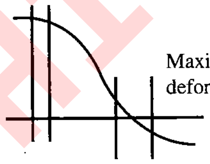

Fig. 7.132

For a travelling wave,
$$y = A \sin(\omega t \pm kx + \theta)$$
at a given position (x):
$$y = A \sin(\omega t + \phi)$$
thus, a particle performs SHM.
At extreme position deformation w.r.t. mean position is minimum, therefore its deformation potential energy is minimum.
15. **d.** Closed end is pressure antinode therefore pressure is not constant. Statement 2 is true.

Comprehension Type

For Problems 1–3

1. a., 2. a., 3. b.

Sol. 1. a. Minimum frequency = fundamental frequency $f = f_0$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.5}\sqrt{\frac{400}{5 \times 10^{-3} \times 2}} = 200 \text{ Hz}$$

2. a. The wire can vibrate with frequencies which are integral multiples of the fundament frequency. The possible frequencies are

200Hz, 400Hz, 600Hz, 800Hz, 1000Hz,...

Now $\qquad 1100/200 = 11/2 \neq$ an integer

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the wire cannot vibrate with 1100 Hz

3. b. 10000 Hz = 50 × 200 Hz = $50f_0$

the frequency of 10,00 Hz is the 50th harmonic and hence 49th overtone. He can hear up to the 49th overtone.

For Problems 4–5

4. a., 5. c.

Sol. Let c be the speed of sound and f_1, f_2 be the frequency of tuning forks.

$$f_1 = \frac{c}{4l_1} = \frac{c}{4\times 32} = \frac{c}{128}$$

$$f_2 = \frac{c}{2l_2} = \frac{c}{2\times 66} = \frac{c}{132}$$

Now $|f_1 - f_2| = 8$

As $f_1 > f_2$ we have $|f_1 - f_2| = 8$

$$\frac{c}{128} - \frac{c}{132} = 8$$

$$c = \frac{128\times 132\times 8}{4} = 33792 \text{ cm/s}$$

$$f_1 = \frac{c}{128} = 264 \text{ Hz}$$

$$f_2 = \frac{c}{132} = 256 \text{Hz}$$

For Problems 6–7

Sol. 6. b. When an air column in a tube vibrates, the antinodes at the open end(s) are located at a small distance outside the open end. This small distance is called as end correction.
Approximate end correction = $0.3d$.
where d is the diameter of the tube.

In case of a tube open at both ends, the effective length of the tube that should be taken in calculation will now be l.

$\Rightarrow l' = l + 2e$ where $e = 0.3d$

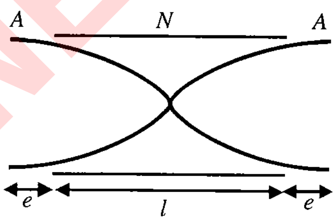

Fig. 7.133

$$v = \frac{c}{2l}$$

$$\Rightarrow 320 = \frac{320}{2(l+2e)}$$

$$l + 2e = 0.5$$

$$0.48 + 2(0.3d) = 0.5$$

$$\Rightarrow d = 1/30 \text{ m} = 3.33 \text{ cm}$$

7. a. If one end is closed, the effective length of the tube will be:

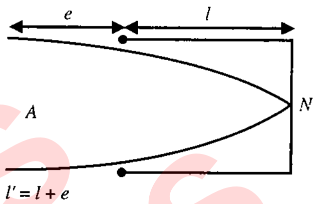

Fig. 7.134

$$l' = l + e$$

$$f = \frac{c}{4l'} = \frac{c}{4(l'+e)}$$

$$v = \frac{320\times 100}{4(48 + 0.3\times 3.33)}$$

$$= \frac{32000}{4\times 49} = 163.27 \text{ Hz}$$

For Problems 8–9

Sol. 8. d. Pipe is closed from one end:

An air column in a pipe closed from one end oscillates only odd harmonics [1st harmonic (fundamental mode), 3rd harmonic (1st overtone), 5th harmonic (2nd overtone), 7th harmonic (3rd overtone) etc.]

$$\text{Fundamental frequency} = \frac{V}{4l} = \frac{340}{4\times \frac{85}{100}} = 100 \text{ Hz}$$

Other modes of oscillation are

3rd harmonic frequency $= 3\times 100 = 300$ Hz

5th harmonic frequency $= 5\times 100 = 500$ Hz

7th harmonic frequency $= 7\times 100 = 700$ Hz

9th harmonic frequency $= 9\times 100 = 900$ Hz

11th harmonic frequency $= 11\times 100 = 1100$ Hz

13th harmonic frequency $= 13\times 100 = 1300$ Hz

Only those natural oscillations are to be counted whose frequencies lie below $f_0 = 1250$ Hz, the harmonics till 11th harmonic are to be counted.

Since, the number of possible natural oscillations = 1 (1st harmonic) + 1 (3rd harmonic) + 1 (5th harmonic) + 1 (7th harmonic) + 1(9th harmonic) + 1 (11th harmonic) = 6.

Second Method

All the frequencies possible are integral multiples of fundamental frequency which is 100 Hz. Using the fact that integer which is multiplied by fundamental frequency is the number of harmonic itself you get, highest harmonic predicted = [12.50/100] where [x] represents greatest integer less than or equal to x = [12.5] = 12.

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For Problems 6–7

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where d is the diameter of the tube.

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$\Rightarrow l' = l + 2e$ where $e = 0.3d$

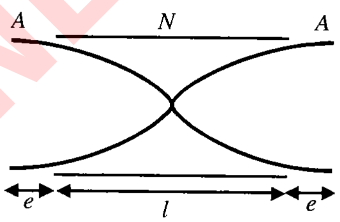

Fig. 7.133

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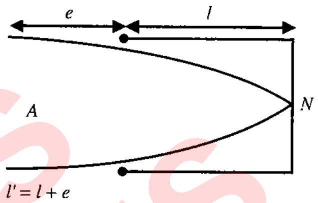

Fig. 7.134

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$$v = \frac{320 \times 100}{4(48 + 0.3 \times 3.33)}$$

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For Problems 8–9

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Other modes of oscillation are

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5th harmonic frequency $= 5 \times 100 = 500$ Hz

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11th harmonic frequency $= 11 \times 100 = 1100$ Hz

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Only those natural oscillations are to be counted whose frequencies lie below $f_0 = 1250$ Hz, the harmonics till 11th harmonic are to be counted.

Since, the number of possible natural oscillations
= 1 (1st harmonic) + 1 (3rd harmonic) + 1 (5th harmonic) + 1 (7th harmonic) + 1(9th harmonic) + 1 (11th harmonic) = 6.

Second Method

All the frequencies possible are integral multiples of fundamental frequency which is 100 Hz. Using the fact that integer which is multiplied by fundamental frequency is the number of harmonic itself you get, highest harmonic predicted = [12.50/100] where [x] represents greatest integer less than or equal to x = [12.5] = 12.

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Now for closed pipe, only odd harmonics are possible, highest harmonic possible = 11th. The possible harmonics are 1, 3, 5, 7, 9, 11 which are six in number.

9. c. Pipe opened from both ends
Fundamental frequency

$$= \frac{V}{2l} = \frac{340}{2\times 85}\times 100 = 200 \text{ Hz}$$

Frequency of the other natural modes of oscillational are:

2nd harmonic frequency $= 2\times 200 = 400$ Hz

3rd harmonic frequency $= 3\times 200 = 600$ Hz

4th harmonic frequency $= 4\times 200 = 800$ Hz

5th harmonic frequency $= 5\times 200 = 1000$ Hz

6th harmonic frequency $= 6\times 200 = 1200$ Hz

7th harmonic frequency $= 7\times 200 = 1400$ Hz

You have to count only those harmonics whose frequencies are below 1250 Hz. All the harmonics till 6th harmonic are possible, and obviously they are six in number.

Second Method
Fundamental frequency = 200 Hz
Frequencies possible

$= n\times$ fundamental frequency

$= n\times 200$ [n is 1, 2, ...]

Maximum value of $n = [12.50/200] = 6$ ([x] represents greatest less than or equal to x).

Now n is also equal to the number of harmonic for which frequency is being calculated, highest harmonic possible = 6th.

As all harmonics are possible in case of open tube, harmonics possible are 1st, 2nd, 3rd, 4th, 5th, 6th.

Number of harmonics possible in this case = 6.

For Problems 10–12

Sol. 10. d. For largest mass, $p = 1$

$$n = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

$$700 = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$

$m = 2\times 10^{-3}$ kg/m, $L = 1$m

$T = [(700)^2\times 4\times 1\times 2\times 10^{-3}] = 3920$ N

Largest mass to be hang, $M_{max} = 3920/9.8 = 400$ kg

11. c. Number of loops formed

$$P = 2nL\sqrt{\frac{\mu}{T}} = 2\times 700\times 1\sqrt{\frac{2\times 10^{-3}}{16\times 9.8}}$$

$$= \frac{2\times 700}{280} = 5$$

12. a. $A(x) = 6\sin\frac{\pi x}{10}$

Maximum displacement at $x = 5$ cm

$$A(x) = 6\sin\left(\frac{\pi}{10}\times 5\right) = 6 \text{ cm}$$

For Problems 13–14

Sol. 13. c. Fundamental frequency

$$n_{Ne} = \frac{1}{2L}\sqrt{\frac{\gamma RT}{M_{Ne}}}$$

$n_{Ne} = 300$Hz, $M_{Ne} = 20\times 10^{-3}$kg

$$\gamma = \frac{5}{3}, R = \frac{20}{3} \text{ J/mol K}$$

$T = 270$ K

$$L = \frac{1}{2\times 300}\sqrt{\frac{\frac{5}{3}\times\frac{25}{3}\times 270}{20\times 10^{-3}}}$$

$$= \frac{250\sqrt{3}}{2\times 300} = \frac{5\sqrt{3}}{12}\text{m}$$

14. d. $M_{He} = 4\times 10^{-3}$ kg

$$\frac{n_{He}}{n_{Ne}} = \sqrt{\frac{M_{ne}}{M_{He}}} = \sqrt{\frac{20}{4}}$$

$$n_{He} = \sqrt{5}\times 300 \text{ Hz}$$

For Problems 15–17

Sol. 15. a. Speed sound in air at $59°C = 2n(l_2 - l_1)$

$$= 2\times 500(49.2-16)\times 10^{-2}$$

$$= 332 \text{ m/s}$$

16. b.

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$M = 28.8$ g/mol $= 28.8\times 10^{-3}$ kg/mol

$T = 332$ K

$R = 8.3$ J/K mol

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$$\gamma = \frac{v^2 M}{RT} = \frac{(332)^2 \times 28.8 \times 10^{-3}}{8.3 \times 332} = 1.152$$

17. b.

$$\frac{\lambda}{4} = l_1 + e$$

$$\frac{3\lambda}{4} = l_2 + e$$

$$e = \frac{l_2 - 3l_1}{2} = \frac{49.2 - 3(16)}{2} = 0.6$$

$\Rightarrow 0.6\, r = 0.6$

$r = 1$ cm

For Problems 18–20

Sol. 18. b. Let t' be the time at which the tuning fork emits a sound wave which reaches the release point at $(t - t')$

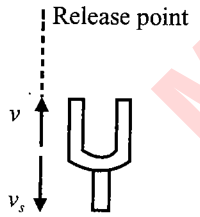

Fig. 7.135

The apparent frequency received at the release point

$$v' = \frac{v_0 v}{v + v_s}$$

$V' = 475$ Hz; $v_0 = 500$ Hz, $v = 340$ m/s

$$475 = \frac{500 \times 340}{340 + v_s} \Rightarrow v_s = 17.9 \text{ m/s}$$

19. b. Time taken $t' = \frac{v_s}{g} = \frac{17.9}{10} = 1.79$ s

Time required to reach the release point

$$t = t' + \frac{\left(\frac{1}{2} g t'^2\right)}{v}$$

$$= 1.79 + \frac{1}{2} \times \frac{10 \times 1.79^2}{340}$$

$= 1.79 + 0.047 = 1.837$ s

20. a. Distance travelled by the tuning fork

$$= \frac{1}{2} g t^2 = \frac{1}{2} \times 10 \times 1.837^2 = 16.875 \text{ m}$$

≈ 16.9 m

For Problems 21–22

Sol. 21. c. Velocity of sound at

$$107°\text{C} = 2n(l_2 - l_1) = 2 \times 500(58.5 - 19) \times 10^{-2}$$
$$= 395 \text{ m/s}$$

22. b. $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v = 395$ m/s

$M = 28.8$ g/mol $= 28.8 \times 10^{-3}$ kg/mol

$T = 107°\text{C} = 380$ K

$R = 8.3$ J/mol K

$$\gamma = \frac{v^2 M}{RT} = \frac{(395)^2 \times 28.8 \times 10^{-3}}{8.3 \times 380} = 1.4$$

For Problems 23–25

Sol. 23. b. Velocity of the longitudinal waves in the rod

$$v = \sqrt{Y/d} = \sqrt{2 \times 10^{11}/8000} = 5000 \text{ m/s}$$

The wavelength of the wave for the mode of vibration in which 2 antinodes occur is

$$\lambda = \frac{1.25}{(3/4)} = \frac{5}{3} \text{m}$$

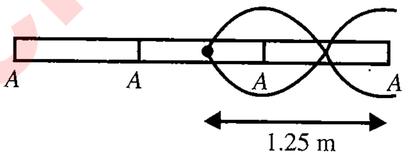

Fig. 7.136

Hence frequency of vibration

$$n = \frac{V}{\lambda} = \frac{5000}{5/3} \text{Hz} = 3000 \text{ Hz}$$

24. a. In its fundamental mode of vibration, the wavelength is

$$\lambda_0 = 4 \times 1.25 \text{ m} = 5 \text{m}$$

And the angular frequency

$$\omega_0 = 2\pi n_0 = \frac{2\pi \times 5000}{5} \text{rad/s}$$

Hence the maximum velocity of the wave at the antinode is

$$v_0 = A\omega_0 = 2 \times 10^{-6} \times \frac{2\pi \times 5000}{5} \text{m/s}$$

$$= 1.25 \times 10^{-2} \text{ m/s}$$

25. c. Since the full length of the rod is set into a single mode of vibration in which four antinodes are seen.

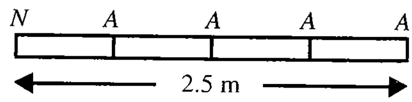

Fig. 7.137

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Hence if λ be the wavelength for this mode of vibration.

We have $\frac{3\lambda}{2}+\frac{\lambda}{4}=2.5$

$$\lambda=\frac{4\times 2.5}{7}=\frac{10}{7}\text{ m}$$

Hence frequency of vibration

$$=\frac{5000}{10/7}=3500\text{ Hz}$$

For Problems 26–28

Sol. 26. c. The given longitudinal standing wave is

$$y=a\cos kx\cos\omega t \quad \text{(i)}$$

The nodes of this wave are located where $\cos kx=0$ (i.e., at the values

$$x=\frac{\lambda}{4},\frac{3\lambda}{4},\ldots$$

and the antinodes are located where $\cos kx=\pm 1$ (i.e.,) at the values

$$x=0,\frac{\lambda}{2},\ldots$$

At the nodes, the space density of kinetic energy (kinetic energy per unit vanishes for the nodes i.e.,

$$x=\frac{\lambda}{4},\frac{3\lambda}{4}\text{ etc.}$$

Also, y is maximum at $t=0$, as we see from Eq. (i). Hence potential energy must be maximum at $t=0$. Hence the time factor in potential energy density must enter as $\cos^2\omega t$. Also, the sum of kinetic and potential energy densities must always be constant for a given x as it represents total energy at that point.

Hence the potential energy density is

$$E_P=\frac{\rho a^2\omega^2}{2}\sin^2 kx\cos^2\omega t \quad \text{(ii)}$$

and the kinetic energy density is

$$E_K=\frac{\rho a^2\omega^2}{2}\cos^2 kx\sin^2\omega t \quad \text{(iii)}$$

27. d. See above.

28. a. The graphs shown are between two successive nodes, say

$x_1=\lambda/4$ at N_1 and $x_2=3\lambda/4$ at N_2.

Total energy density is

$$E=E_P+E_K$$

$$=\frac{\rho a^2\omega^2}{2}[\sin^2 kx\cos^2\omega t+\cos^2 kx\sin^2\omega t]$$

Putting $x=\lambda/2$, (first antinode)

$$(E)_{\lambda/2}=\frac{\rho a^2\omega^2}{2}\left[\sin^2\omega t\right]$$

At $t=0,\ (E)_{\lambda/2}=0$ and at $t=\frac{T}{4}$

$$(E)_{\lambda/2}=\frac{\rho a^2\omega^2}{2}$$

This is truly reflected in the graph (a).

For Problems: 29–31

Sol. 29. d., 30. c., 31. a.

$$\mu=\frac{1.2}{2}=0.6\text{ kg/m}$$

$$n=5\text{ Hz}$$

$$\lambda=2l=4\text{ m}$$

$$V=n\lambda=5\times 4=20\text{ m/s}$$

Using $v=\sqrt{\frac{T}{\mu}}$

$$T=20^2\times 0.6=240\text{ N}$$

$$\left(\frac{\partial y}{\partial t}\right)_{\max}=3.14\text{ m/s}$$

$$(2A)\omega=3.14$$

$$\text{Amplitude } 2A=\frac{3.14}{2\times(3.14)\times 5}=0.1\text{m}$$

Equation of standing wave is

$$y=(0.1)\sin\left(\frac{\pi}{2}\right)x\sin(10\pi)t$$

For Problems: 32–35

32. a.

$$\xi=(0.1\text{ mm})\cos\frac{2\pi}{0.8}(y+1\text{ cm})\cos(400t)$$

End correction is 1 cm, so at $y=-1$ cm

$$\xi=(0.1\text{mm})\cos\frac{2\pi}{0.8}(-1\text{cm}+1\text{cm})=$$

$$=(0.1\text{mm})\cos(0)=\text{Antinode}$$

So upper end is open.

At lower end $y=99\text{ cm}=0.99\text{ m}$

$$\xi=(0.1\text{mm})\cos\frac{2\pi}{0.8}(0.99+0.01)$$

$$=0.01\cos\frac{5\pi}{2}=0 \Rightarrow \text{Node}$$

So tube is open closed.

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33. b.

$$\frac{2\pi}{0.8}=\frac{2\pi}{\lambda} \quad \text{so} \quad \lambda = 0.8$$

And effective length of air column = 0.99 + 0.01 = 1m

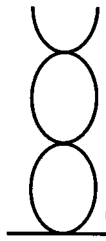

Fig. 7.138

So $$\frac{l}{\lambda}=\frac{5}{4}$$

$$l = 5\frac{\lambda}{4}$$

so five half loops will be formed

$$l = 5\left(\frac{\lambda}{4}\right)$$

so second overtone.

34. a.

$$P_{ex} = -B\frac{d\xi}{dx}$$

$$=(5\times10^5)\times(0.1\times10^{-3})\frac{2\pi}{0.8}\sin\frac{2\pi}{0.8}(y+1\text{ cm})\cos(400)t$$

$$=(125\pi \text{ N/m}^2)\sin\frac{2\pi}{0.8}(y+1\text{ cm})\cos(400t)$$

35. a. End correction = (0.3) d = 1 cm

$$d = \frac{10}{3}\text{cm}$$

Volume of tube

$$=\left(\pi\frac{d^2}{4}\right)l = \frac{\pi}{4}\left(\frac{10}{3}\right)^2 \times 100\,\text{cm}^3$$

(take l = 0.99 m ≈ 1m)

$$=\frac{10\pi}{36}\text{ litres}$$

$$\text{moles} = \frac{10\pi}{36\times22.4}\text{moles}$$

(22.4 l contains 1 mol and $10\pi/36l$ contains $10\pi/36 \times 22.4$ mol)

For Problems: 36–38

36. a. Actual frequency emitted by source does not depend upon the velocity of source but frequency heard may change due to relative motion between the observer and the source.

37. b. As the source is initially projected with velocity v_0 upwards, the velocity of the source after a time $v_0/2g$ is

$$v = v_0 - \frac{v_0}{2g}\times g = \frac{v_0}{2}$$

And the source is moving away from observer, so, frequency heard

$$=\left(\frac{v}{v+\frac{v_0}{2}}\right)f_0$$

So wavelength of sound

$$=\frac{v+\frac{v_0}{2}}{f_0}$$

38. c. As we known, velocity at $t = v_0/2g$ is $v_0/2$.
Number of waves reaching the reflector per second.

$$f_1 = \frac{v+v_0}{v-\frac{v_0}{2}}f_0$$

Now, the wall acts as the source with above frequency and the detector is at rest. So, frequency heard by the detector is

$$f = \frac{v}{v-v_0}f_1 = \frac{v}{v-v_0}\times\frac{v+v_0}{v-\frac{v_0}{2}}f_0$$

$$=\frac{v}{v-v_0}\times\frac{v+v_0}{2v-v_0}2f_0$$

For Problems: 39–41

39. a.

$$y_1 = A\cos(0.5\pi x - 100\pi t)$$

$$y_2 = A\cos(0.46\pi x - 92\pi t)$$

For the first wave angular frequency is $\omega_1 = 100\pi$, $f_1 = 50$ Hz

For the second wave angular frequency is $\omega_2 = 92\pi$, $f_2 = 46$ Hz.

Frequency at which the amplitude of resultant wave varies

$$f_A = \frac{f_1-f_2}{2} = \frac{50-46}{2} = 2$$

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Time interval between this is maximum.

$$\Delta t = \frac{1}{2f_A}$$

$$\Delta t = \frac{1}{4}$$

Therefore, the number of time intensity is maximum in time 1 s is 4.

40. c. Wave velocity of louder sound

$$v = \frac{\omega}{k} = 200 \text{ m/s}$$

41. a. At $x = 0$

$y = y_1 + y_2 = A\cos(100\pi t) + A\cos(92\pi t)$

$\Rightarrow y = 2A\cos(96\pi t)\sin(4\pi t)$

for $y = 0$, we have $\sin(4\pi t) = 0$

$\Rightarrow 4\pi t = n\pi$

$$\Rightarrow t = \frac{n}{4}$$

$$\Rightarrow t = 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4} \quad \text{in 1 s}$$

This is 5 times

also for $y = 0$, we have $\cos(96\pi t) = 0$

$$\Rightarrow 96\pi t = \frac{(2m-1)\pi}{2}$$

$$\Rightarrow t = \frac{2m-1}{192}$$

$$\Rightarrow t = \frac{1}{192}, \frac{3}{192}, \ldots, \frac{191}{192}$$

This is 95 times.

So the total number of times, the value of y is zero is 100.

For Problems: 42–45

42. b.

Frequency of source = 680 Hz

Velocity of sound = 340 m/s

Wave length $= \lambda = v/f = 340/680\text{m} = 1/2\text{m}$

Let the person is at distance D when he observes first minimum intensity.

Hence the path difference between two source $= \lambda/2$

Path difference at that point

$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$$

$$d = 3$$

$$\sqrt{9^2 + D^2} - D = \frac{\lambda}{2}$$

$$D\left(1 + \frac{9}{D^2}\right)^{1/2} - D = \frac{\lambda}{2}$$

Using Binomial theorem

$$D = 18 \text{ m}$$

Hence option (b) closest for second minimum.

43. a.

$$\Delta x = \frac{3}{2}\lambda$$

$$D\left(1 + \frac{9}{D^2}\right)^{1/2} - D = \frac{\lambda}{2}$$

$D = 6$ m closest 5.6 m

44. d. Number of maxima

$$d = n\lambda$$

$$n = \frac{d}{\lambda} = \frac{3}{1/2} = 6$$

Here minima will be also 6.

45. c. When the person is at distance 4 m from pole then the path travelled by wave from $s_2 = 5$ m.

$\therefore$ Path difference $= 5 - 4 = 1$ m

As $\qquad \Delta x = n\lambda, \quad \lambda = \frac{1}{2}\text{m}$

$$n = \frac{1}{\lambda} = 2$$

Therefore, path difference $= 1 \text{ m} = 2\lambda$.

Path difference of 2λ corresponds to 4π

$$y = A\cos(kx - \omega t + 4\pi)$$

It can be expressed as

$$y = A\cos(kx - \omega t + 2\pi)$$

For Problems: 46–49

46. c., 47. a., 48. c., 49. b.

Sol.

$$y = y_1 + y_2 = (12\sin 5x)\cos 4t$$

Maximum value of y-positions in SHM of an element of the string that is located at an antinode $= \pm 12$ cm ($\sin 5x = \pm 1$)

For the position nodes amplitude should be zero.

So, $\qquad \sin 5x = 0 \quad \Rightarrow 5x = n\pi$

$$x = \frac{n\pi}{5}$$

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where $n = 0, 1, 2, 3, \ldots$

Value of amplitude at $x = 1.8$ cm

$$A = 12 \sin (5 \times 1.8) = 4.9 \text{ cm}$$

At any instant say $t = 0$, instantaneous velocity of points on the string is zero for all points as at extreme position velocities of particles are zero.

For Problems: 50–54

50. b., 51. c., 52. a., 53. d., 54. c.

Sol. Displacement node corresponds to pressure antinode.

$$L = \frac{\lambda_0}{4}$$

$$\lambda_0 = 4L = 40 \text{ cm} \quad \text{(First harmonic)}$$

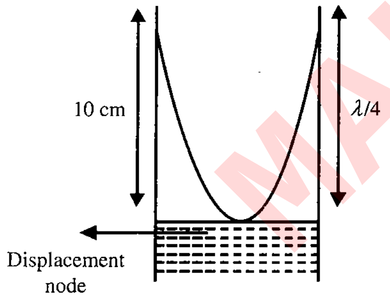

Fig. 7.139

$$v = \lambda f$$

$$f = \frac{v}{\lambda} = \frac{344}{40 \times 10^{-2}} = 860 \text{ Hz}$$

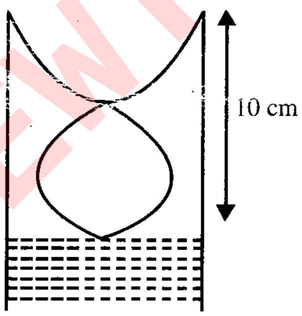

Fig. 7.140

For the second resonance

$$\frac{3\lambda_0}{4} = L$$

$$L = 30 \text{ cm}$$

For the third resonance $5\lambda_0/4 = L$

$$L = 50 \text{ cm}$$

Also, $\nu = v/\lambda$

3rd harmonic is 2nd overtone.

Hence, frequency for 2nd overtone

$$= \frac{5v}{4L} = 4300 \text{ Hz}$$

Matching Column Type

1. i. → c.; ii. → a.; iii. → d.; iv. → b.

Intensity at a distance r from a source of power output P is given by

$$I = \frac{P}{4\pi r^2}$$

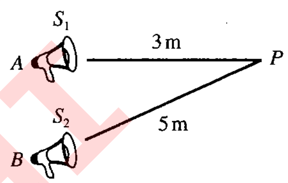

Fig. 7.141

$$I_1 = \frac{0.9\pi \text{ mW}}{4\pi (3)^2} = \frac{1}{40} \text{mW} = 25 \ \mu\text{W/m}^2$$

$$I_2 = \frac{9\pi \text{ mW}}{40\pi (5)^2} = \frac{9}{1000} \text{mW} = 9 \ \mu\text{W/m}^2$$

For incoherent source,

$$I_R = I_1 + I_2 = (25 + 9) = 34 \ \mu\text{W/m}^2$$

For coherent source, $\Delta = 0$

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (5+3)^2 = 64 \ \mu\text{W/m}^2$$

For $\delta = \pm\pi, I_R = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$= (5-3)^2 = 4 \ \mu\text{W/m}^2$$

If the speaker S_2 is switched off, $I_R = I_1 = 25 \ \mu\text{W/m}^2$

2. i. → c.; ii. → a.; iii. → d.; iv. → b.

$$\nu_1 = 1310 \text{ Hz}, \quad \nu_2 = 1834 \text{ Hz}, \quad \nu_3 = 2358 \text{ Hz}$$

$$\frac{\nu_2}{\nu_1} = \frac{7}{5}; \quad \frac{\nu_3}{\nu_1} = \frac{9}{5}$$

$$\nu_1 : \nu_2 : \nu_3 = 5 : 7 : 9$$

(This corresponds to a pipe closed at one end)

Fundamental frequency

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$$n_0 = \frac{2358}{9} = 262 \text{ Hz}$$

Frequency of the first overtone

$$n_1 = 3n_0 = 786 \text{ Hz}$$

Frequency of the fifth overtone

$$= 5n_0 = 1310 \text{ Hz}$$

For fundamental frequency

$$\lambda_0/4 = l$$

$$\lambda_0 = 4l$$

$$n_0 = \frac{v}{4l} \quad \Rightarrow l = \frac{v}{4n_0} = \frac{340}{4 \times 262}$$

$$= 32.4 \times 10^{-2} \text{ m}$$

Length of the pipe = 32.4 cm

3. **i. → a., b., d., ii. → c., d., iii. → d., iv. → c. ,d.**
Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase difference between two particles in same loop will be zero and that between two particles in adjacent loops will be π.

Number of loops will not be integral. Hence neither a node nor an antinode will be formed in the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

4. **i. → a., b.; ii. → d.; iii. → a., c; iv. → d.**

i. $y = 4 \sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$ is super position of two coherent waves, so their equivalent will be an another travelling wave.

ii.

$$y = 10\cos\left(t - \frac{x}{330}\right)\sin(100)\left(t - \frac{x}{330}\right)$$

Let us check at any point, say at $x = 0$.

$$y = (10 \cos t) \sin (100\, t)$$

at any point amplitude is changing sinusoidally, so this is equation of beats.

iii. $y = 10 \sin(2\pi x - 120\, t) + 10\cos(120\, t + 2\pi x)$
= superposition of two coherent waves travelling in opposite direction.

iv. $y = 10 \sin(2\pi x - 120\, t) + 8\cos(118 - 59/30\, \pi x)$
= superposition of two waves whose frequencies are slightly different ($\omega_1 = 120$; $\omega_2 = 118$) ⇒ equation of beats.

5. **i. → a., d.; ii. → a., b., d.; iii. → c.; iv. → a., b., c., d.**
When the string is resonating in 1st harmonic or fundamental tone, all the particles of the string are vibrating in phase. When the string is resonating in even harmonic the particles near the ends of the sting are vibrating out of phase as even number of loops are there, while reverse is the case for odd harmonics. In all modes all the particles of the string are crossing mean position or extreme position simultaneously twice in one cycle.

6. **i. → a., c.; ii. → b., d.; iii. → c.; iv. → d.**
Bass strings have low fundamental frequency and larger wavelengths. For having low frequency the string has to be long according to expression, $f \propto v/L$. For low f, v should be low, i.e., string should be thick. For treble strings also, the same explanation holds true.

7. **i. → b., d.; ii. → b., d.; iii. → a.; iv. → b.; c.**
Velocity of wave on a string is given by $v = \sqrt{T/\mu}$. Frequency is the property of source. Wavelength = v/f

8. **i. → a., c.; ii. → b., d.; iii. → a., c.; iv. → b., d.**

In string $\quad V = \sqrt{T/\mu} = 320$ m/s

Open pipe and string fixed at both ends.

$$f = \frac{nv}{2L} = 320, 640, 960, \ldots$$

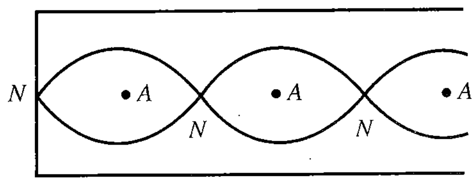
Closed pipe and string free end

$$f = (2n-1)\frac{v}{4L} = 160, 480, 800, \ldots$$

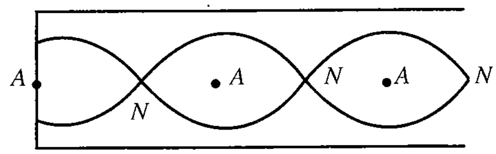
9. **i. → a., c.; ii. → d.; iii. → a.; iv. → c.**
Wavelength of wave in medium changes when there is relative motion between medium and source. Frequency observed by observer is different from source frequency only if there is relative motion between observer and source.

Speed of sound w.r.t. medium will not change until temperature of medium changes.

10. **i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → a., c.**
Second overtone is shown in Fig. 7.142.



Displacement wave



Pressure wave

Fig. 7.142

Distance between nearest node and antinode is 2/5

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Integer Answer Type

1. **(1)** Since frequencies are in odd number ratio, the pipe has to be a closed pipe.
Ratio of 3 frequencies = 425 : 595 : 765
= 5 : 7 : 9

So fundamental frequency = $f = \frac{425}{5} = 85$ Hz
For fundamental frequency

$$l = \frac{v}{4f} = \frac{340}{4 \times 85} = 1 \text{ m}$$

2. **(3)** The pattern corresponds to

$$l = \frac{5\lambda}{4} = 2.0 \text{m} \qquad \lambda = \frac{8}{5} \text{ m}$$

With speed $v = 5.0$ ms^{-1}

$$f = \frac{v}{\lambda} = \frac{5 \times 5}{8} = 3.1 \text{ Hz}$$

3. **(3)** $f \propto \sqrt{T}$ for strings.

On increasing the tension by 1%

$$f' = \sqrt{101T}$$

$$\frac{f'}{f} = \frac{\sqrt{1.01T}}{\sqrt{T}} = (1+0.01)^{\frac{1}{2}} = 1 + \frac{1}{200}$$

$$\text{Beat frequency}, f' - f = f\left(\frac{f'}{f} - 1\right) = 1$$

Number of beats in 3 s = 1 × 30 = 30

4. **(2)** $l = 15.0$ m, $v = 12$ ms^{-1}
Since there are 6 nodes, with the ends as nodes there will be five half wavelength in the string.

So, $\frac{5\lambda}{2} = l = 15 \Rightarrow \lambda = 6.0$ m

Using $f = \frac{v}{\lambda} = \frac{12}{6} = 2.0$ Hz

5. **(2)** $f = 500$ Hz, $v = 300$ ms^{-1}

$$\lambda = \frac{v}{f} = \frac{300}{500} = \frac{3}{5} \text{m}$$

Resonating length $l = \frac{(2n-1)v}{4f}$

$$l = \frac{(2n-1) \times 300}{4 \times 5} \le 1 \text{ m}$$

$n \le 23/6 = 3.83$

Since only odd harmonics are possible there will be only two resonant lengths.

6. **(3)** We have to find the number of pressure antinodes (displacement nodes), which is 3 (from the diagram).

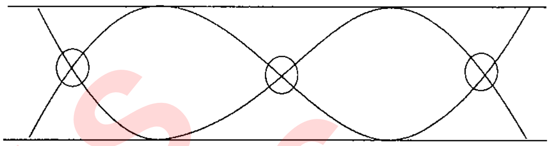

Fig. 7.143

7. **(9)**

$$n\left(\frac{V}{4L_c}\right) = m\left(\frac{V}{2L_0}\right) \qquad \text{(i)}$$

$$\text{Also} \qquad 3\left(\frac{V}{4L_c}\right) = 2\left(\frac{V}{2L_0}\right) \qquad \text{(ii)}$$

From Eq. (ii) $\frac{L_c}{L_0} = \frac{3}{4}$

From Eq. (i) $\frac{n}{m} = 2\left(\frac{L_c}{L_0}\right) = \frac{6}{4} = \frac{3}{2} = \frac{9}{6}$

$n = 9$ if $m = 6$

8. **(7)**

$f_0 - f_c = 2$

$$V\left[\frac{1}{2L} - \frac{1}{4L}\right] = 2 \quad \text{or} \quad V/L = 8$$

In the second case,

$$f_0' - f_c' = \frac{V}{L} - \frac{V}{8L} = \frac{7V}{8L} = \frac{7}{8}(8) = 7$$

9. **(4)**

$$f \propto \frac{(T/\mu)^{1/2}}{L}$$

Where μ = mass per unit length $= \rho a = \rho(\pi r^2)$

So, $f \propto \frac{(T/\rho)^{1/2}}{rL}$

$$\frac{f_2}{f_1} = \left(\frac{T_2}{T_1}\right)^{1/2}\left(\frac{\rho_1}{\rho_2}\right)^{1/2}\left(\frac{r_1 L_1}{r_2 L_2}\right)$$

$$= \left(\frac{1}{\sqrt{2}}\right)(\sqrt{2})(4) = 4$$

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CHAPTER

8

Archives on Chapters 4–7

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EXERCISES

Archives

Solutions on page 8.10

Fill in the Blank Type

1. A travelling wave has the frequency ν and the particle displacement amplitude A. For the wave the particle velocity amplitude is ____ and the particle acceleration amplitude is ____. **(IIT-JEE, 1983)**

2. Sound waves of frequency 660 Hz fall normally on perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ____ metres. **(IIT-JEE, 1984)**

3. Two simple harmonic motions are represented by the equations $y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = (5\sin 3\pi t + \sqrt{3}\cos 3\pi t)$. Their amplitudes are in the ratio of ____. **(IIT-JEE, 1986)**

4. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ____ Hz. **(IIT-JEE, 1987)**

5. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = 1/(1 + x^2)$ at time $t = 0$ and by $y = 1/[1 = (1 = x)^2]$ at $t = 2$ s, where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is ____ m/s. **(IIT-JEE, 1990)**

6. A cylindrical resonance tube open at both ends has fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is ____. **(IIT-JEE, 1992)**

7. A bus is moving towards a huge wall with a velocity of 5 m/s. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be ____ Hz. (Speed of sound in air = 342 m/s.) **(IIT-JEE, 1992)**

8. An object of mass 0.2 kg executes simple harmonic oscillation along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$ m, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ____ m. **(IIT-JEE, 1994)**

9. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is ____, and the corresponding amplitude difference is ____ m. **(IIT-JEE, 1997)**

True/False Type

1. The ratio of the velocity of sound in hydrogen gas (γ = 7/5) to that in helium gas (γ = 5/3) at the same temperature is $\sqrt{21/5}$. **(IIT-JEE, 1980)**

2. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. **(IIT-JEE, 1980)**

3. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **(IIT-JEE, 1984)**

4. A source of sound with frequency 256 Hz is moving with a velocity V towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall, he will hear beats. **(IIT-JEE, 1985)**

Single Correct Answer Type

1. A cylindrical tube, open at both ends, has a fundamental frequency 'f' in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column **(IIT-JEE, 1981)**

   a. $f/2$ **b.** $3f/4$
   c. f **d.** $2f$

2. A transverse wave is described by the equation

$$y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

   The maximum particle velocity is equal to four times the wave velocity if **(IIT-JEE, 1984)**

   a. $\lambda = \pi\dfrac{y_0}{4}$ **b.** $\lambda = \pi\dfrac{y_0}{2}$
   c. $\lambda = \pi y_0$ **d.** $\lambda = 2\pi y_0$

3. A tube, closed at one end and containing air, produces, when excited, the fundamental mode of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz). **(IIT-JEE, 1986)**

   a. 1024 **b.** 512
   c. 256 **d.** 128

4. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is **(IIT-JEE, 1987)**

   a. $f/2$ **b.** f
   c. $2f$ **d.** $4f$

5. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is **(IIT-JEE, 1988)**

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a. $a\sin(kx+\omega t)$ b. $a\sin(kx-\omega t)$
c. $-a\cos(kx+\omega t)$ d. $-a\sin(kx-\omega t)$

6. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both the ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is **(IIT-JEE, 1988)**
a. 8/3 b. 3/8
c. 1/6 d. 1/3

7. Two bodies M and N, of equal masses, are suspended from two separate massless springs of spring constants k_1 and k_2, respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is **(IIT-JEE, 1988)**
a. $\dfrac{k_1}{k_2}$ b. $\sqrt{k_1/k_2}$
c. $\dfrac{k_2}{k_1}$ d. $\sqrt{k_2/k_1}$

8. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is **(IIT-JEE, 1990)**
a. $\dfrac{1}{2\pi}\left(\dfrac{k-A\rho g}{M}\right)^{1/2}$ b. $\dfrac{1}{2\pi}\left(\dfrac{k+A\rho g-j}{M}\right)^{1/2}$
c. $\dfrac{1}{2\pi}\left(\dfrac{k+\rho-gL}{M}\right)^{1/2}$ d. $\dfrac{1}{2\pi}\left(\dfrac{k+A-\rho g}{A\rho g}\right)^{1/2}$

9. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force is applied perpendicular to the side faces of A. after the force is withdrawn, block A executes small oscillations the time period of which is given by **(IIT-JEE, 1992)**
a. $2\pi\sqrt{M\eta L}$ b. $2\pi\sqrt{\dfrac{M-\eta}{L}}$
c. $2\pi\sqrt{\dfrac{M-L}{\eta}}$ d. $2\pi\sqrt{\dfrac{M-N}{\eta L}}$

10. The displacement y of a particle executing periodic motion is given by
$$y=4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$
This expression may be considered to be a result of the superposition of **(IIT-JEE, 1992)**
a. two b. three
c. four d. five

11. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modules of the wire are A and Y, respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to: **(IIT-JEE, 1993)**
a. $2\pi(m/K)^{1/2}$ b. $2\pi\sqrt{\dfrac{m(YA+KL)}{YAK}}$
c. $2\pi[(mYA/KL)]^{1/2}$ d. $2\pi[(mL/YA)]^{1/2}$

12. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **(IIT-JEE, 1995)**
a. $300\left(\dfrac{2\rho-1}{2\rho}\right)^{1/2}$ b. $300\left(\dfrac{2\rho}{2\rho-1}\right)^{1/2}$
c. $300\left(\dfrac{2\rho}{2\rho-1}\right)$ d. $300\left(\dfrac{2\rho-1}{2\rho}\right)$

13. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be **(IIT-JEE, 1996)**
a. 1.22 v b. 0.61 v
c. 1.50 v d. 0.75 v

14. An open pipe is suddenly closed at one end and with the result frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **(IIT-JEE, 1996)**
a. 200 Hz b. 300 Hz
c. 240 Hz d. 480 Hz

15. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed sound = 330 m/s) **(IIT-JEE, 1997)**
a. 409 b. 429
c. 517 d. 500

16. A travelling wave in a stretched string is described by the equation $y = A\sin(kx-\omega t)$. The maximum particle velocity is **(IIT-JEE, 1997)**
a. $A\omega$ b. ω/k
c. $d\omega/dk$ d. x/t

17. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulse is produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interface between successive pulses is **(IIT-JEE, 1998)**

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a. 0.05 s b. 0.10 s
c. 0.20 s d. 0.40 s

18. A particle of mass m is executing oscillations about the origin on the axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is **(IIT-JEE, 1998)**
a. proportional to $1/\sqrt{a}$
b. independent of a
c. proportional to $\sqrt{a}$
d. proportional to $a^{3/2}$

19. A particle free to move along the x-axis has potential energy given by $U(x) = k\,[1-\exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then **(IIT-JEE, 1999)**
a. at points away from the origin, the particle is in unstable equilibrium
b. for any finite non-zero value of x, there is a force directed away from the origin
c. if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
d. for small displacements from $x = 0$, the motion is simple harmonic

20. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300 K is **(IIT-JEE, 1999)**
a. $\sqrt{(2/7)}$ b. $\sqrt{(1/7)}$
c. $(\sqrt{3})/5$ d. $(\sqrt{6})/5$

21. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by **(IIT-JEE, 2000)**
a. $2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$ b. $2\pi\sqrt{\dfrac{L}{g\sin\alpha}}$
c. $2\pi\sqrt{\dfrac{L}{g}}$ d. $2\pi\sqrt{\dfrac{L}{g\tan\alpha}}$

22. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of the sound is 340 m/s, then the ratio f_1/f_2 is **(IIT-JEE, 2000)**
a. 18/19 b. 1/2
c. 2 d. 19/18

23. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r, respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency n_1 and the other with frequency n_2. The ratio n_1/n_2 is given by **(IIT-JEE, 2000)**
a. 2 b. 4
c. 8 d. 1

24. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in Fig. 8.1. The speed of each pulse is 2 cm/s. After 2 s, the total energy of the pulse will be **(IIT-JEE, 2000)**

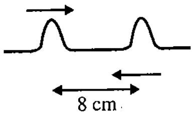


Fig. 8.1

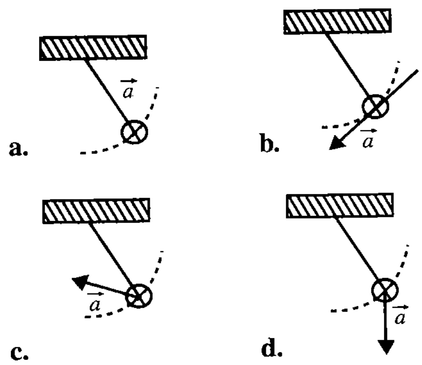
a. zero b. purely kinetic
c. purely potential d. partly kinetic and partly potential

25. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin\omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then **(IIT-JEE, 2001)**
a. $E_2 = E_1$ b. $E_2 = 2E_1$
c. $E_2 = 4E_1$ d. $E_2 = 16E_1$

26. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then **(IIT-JEE, 2001)**
a. $T_1 < T_2$ b. $T_1 > T_2$
c. $T_1 = T_2$ d. $T_1 = 2T_2$

27. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth suface, where R is the radius of the earth. The value of T_2/T_1 is **(IIT-JEE, 2001)**
a. 1 b. $\sqrt{2}$ c. 4 d. 2

28. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in: **(IIT-JEE, 2002)**



29. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is **(IIT-JEE, 2002)**

a. 242/252 **b.** 2
c. 4 **d.** 6

30. A sonometer wire resonates with a tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **(IIT-JEE, 2002)**

a. 25 kg **b.** 5 kg
c. 12.5 kg **d.** 1/25 kg

31. A police car, moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats. **(IIT-JEE, 2003)**

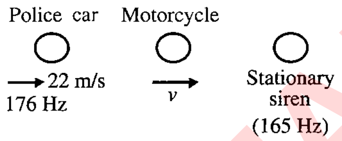


Fig. 8.2

a. 33 m/s **b.** 22 m/s **c.** zero **d.** 11 m/s

32. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. **(IIT-JEE, 2003)**

a. 0.012 m **b.** 0.025 m
c. 0.05 m **d.** 0.024 m

33. For a particle executing SHM, the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential (PE) as a function of time t and displacement x. **(IIT-JEE, 2003)**

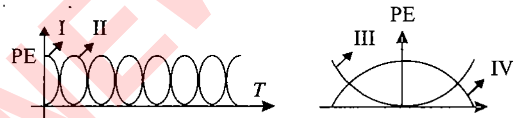


Fig. 8.3

a. I, III **b.** II, IV
c. II, III **d.** I, IV

34. A source of sound of frequency 600 Hz is placed inside water. The speed in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer standing in air is **(IIT-JEE, 2004)**

a. 200 Hz **b.** 3000 Hz
c. 120 Hz **d.** 600 Hz

35. A pipe of length l_1, closed at one end, is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal **(IIT-JEE, 2004)**

a. $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$ **b.** $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$

c. $l_1\sqrt{\frac{\rho_2}{\rho_1}}$ **d.** $l_1\sqrt{\frac{\rho_1}{\rho_2}}$

36. In a resonate tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is **(IIT-JEE, 2005)**

a. 5.12 cm/s **b.** 102.4 cm/s
c. 204.8 cm/s **d.** 153.6 cm/s

37. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option. **(IIT-JEE, 2005)**

a. $n = 3, f_2 = \frac{3}{4}f_1$ **b.** $n = 3, f_2 = \frac{5}{4}f_1$

c. $n = 5, f_2 = \frac{3}{4}f_1$ **d.** $n = 5, f_2 = \frac{5}{4}f_1$

38. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$ ($K = 1$ m/s^2), where y is the vertical displacement. The time period now becomes T_2. The ratio of T_1^2/T_2^2 is ($g = 10$ m/s^2) **(IIT-JEE, 2005)**

a. 5/6 **b.** 6/5
c. 1 **d.** 4/5

39. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD, 'x' is **(IIT-JEE, 2006)**

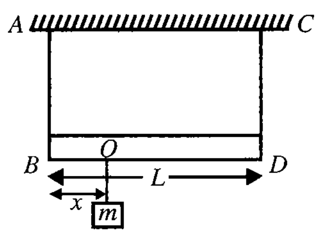


Fig. 8.4

a. $\frac{L}{5}$ **b.** $\frac{4L}{5}$

c. $\frac{3L}{4}$ **d.** $\frac{L}{4}$

40. In the experiment to determine the speed of sound using a resonance column **(IIT-JEE, 2007)**

a. prongs of the tuning fork are kept in a vertical plane
b. prongs of the tuning fork are kept in a horizontal plane

c. in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air

d. in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

41. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in Fig. 8.5. The velocity of point P when its displacement is 5 cm is **(IIT-JEE, 2008)**

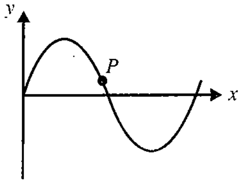


Fig. 8.5

a. $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s **b.** $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

c. $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s **d.** $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

42. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased, the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **(IIT-JEE, 2008)**

a. 344 **b.** 336
c. 117.3 **d.** 109.3

43. A block B is attached to two unstretched springs S1 and S2 with spring constant k and $4k$, respectively [see Fig. 8.6(I)]. The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. Block B is displaced towards wall I by a small distance x [Fig. 8.6 (II)] and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of block B. The ratio y/x is **(IIT-JEE, 2008)**

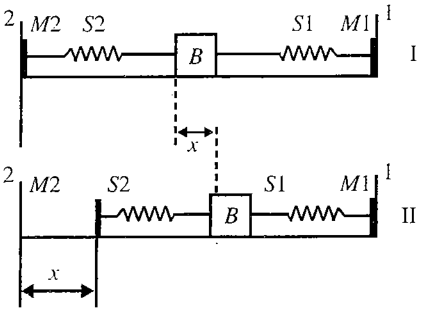


Fig. 8.6

a. 4 **b.** 2
c. 1/2 **d.** 1/4

44. The x–t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **(IIT-JEE, 2009)**

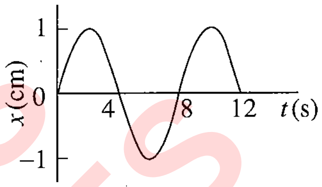


Fig. 8.7

a. $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 **b.** $\frac{-\pi^2}{32}$ cm/s^2

c. $\frac{\pi^2}{32}$ cm/s^2 **d.** $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

45. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in Fig. 8.8, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is **(IIT-JEE, 2009)**

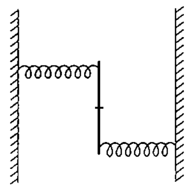


Fig. 8.8

a. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ **b.** $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

c. $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ **d.** $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

independent harmonic motions.

46. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is **(IIT-JEE, 2010)**

a. 5 g **b.** 10 g
c. 20 g **d.** 40 g

47. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2, respectively, are enclosed in separate containers kept at the same temperature. The ration of the speed of sound in gas 1 to that in gas 2 is given by **(IIT-JEE, 2010)**

a. $\sqrt{\frac{m_1}{m_2}}$ **b.** $\sqrt{\frac{m_2}{m_1}}$

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c. $\frac{m_1}{m_2}$ d. $\frac{m_2}{m_1}$

48. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **(IIT-JEE, 2011)**

a. 8.50 kHz b. 8.25 kHz
c. 7.75 kHz d. 7.50 kHz

49. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A\sin(\omega t + 2\pi/3)$. Adding a third sinusoidal displacement $x_3(t) = B \sin (\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **(IIT-JEE, 2011)**

a. $\sqrt{2}A, \frac{3\pi}{4}$ b. $A, \frac{4\pi}{3}$
c. $\sqrt{3}A, \frac{5\pi}{6}$ d. $A, \frac{\pi}{3}$

Multiple Correct Answers Type

1. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin (60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave **(IIT-JEE, 1981)**

a. travelling with a velocity of 30 m/s in the negative x-direction
b. of wavelength π m
c. of frequency $30/\pi$ Hz
d. of amplitude 10^{-4} m travelling along the negative x-direction.

2. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is (Speed of sound = 330 m/s) **(IIT-JEE, 1985)**

a. 31.25 b. 62.50
c. 93.75 d. 125

3. The displacement of particle in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are **(IIT-JEE, 1987)**

a. $\cos kx \sin \omega t$ b. $k^2x^2 - \omega^2t^2$
c. $\cos^2(kx + \omega t)$ d. $\cos(k^2x^2 - \omega^2t^2)$

4. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency **(IIT-JEE, 1989)**

a. 80 Hz b. 240 Hz
c. 320 Hz d. 400 HZ

5. The linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its **(IIT-JEE, 1989)**

a. maximum potential energy is 100 J
b. maximum kinetic energy of 100 J
c. maximum potential energy is 160 J
d. minimum potential energy of zero

6. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in metres and t is in seconds. The expression represents: **(IIT-JEE, 1990)**

a. a wave travelling in the positive x-direction with a velocity 1.5 m/s.
b. a wave travelling in the negative x-direction with a velocity 1.5 m/s.
c. a wave travelling in the negative x-direction having a wavelength 0.2 m.
d. a wave travelling in the positive x-direction having a wavelength 0.2 m.

7. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Let T_1 and T_2 represent the higher and the lower initial tensions in the strings. While making the above change in tension: **(IIT-JEE, 1991)**

a. T_2 was decreased b. T_2 was increased
c. T_1 was increased d. T_1 was decreased

8. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c. **(IIT-JEE, 1995)**

a. The number of waves striking the surface per second is $f\frac{(c+v)}{c}$
b. The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$
c. The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$
d. The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$

9. A wave disturbance in a medium is described by $y(x, t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metre and t is in second. **(IIT-JEE, 1995)**

a. A node occurs at $x = 0.15$ m
b. An antinode occurs at $x = 0.3$ m
c. The speed of wave is 5 m/s
d. The wave length is 0.2 m

10. The (x, y) coordinates of the corners of a square plate are $(0, 0)$, (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant). **(IIT-JEE, 1998)**

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a. $a\cos(\pi x/2L)\cos(\pi y/2L)$
b. $a\sin(\pi x/L)\sin(\pi y/L)$
c. $a\sin(\pi x/L)\sin(2\pi y/L)$
d. $a\cos(2\pi x/L)\sin(\pi y/L)$

11. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by **(IIT-JEE, 1998)**

a. $\lambda = 2\pi \times 10^{-2}$ m b. $\lambda = 10^{-3}$ m
c. $f = \dfrac{10^3}{2\pi}$ Hz d. $f = 10^4$ Hz

12. $y(x, t) = 0.8/[4x + 5t)^2 + 5]$ represents a moving pulse, where x and y are in metre and t in second. Then **(IIT-JEE, 1999)**

a. pulse is moving in $+x$-direction
b. in 2 s it will travel a distance of 2.5 m
c. its maximum displacement is 0.16 m
d. it is a symmetric pulse

13. In a wave motion $y = a\sin(kx - \omega t)$, y can represent **(IIT-JEE, 1999)**

a. electric field b. magnetic field
c. displacement d. pressure

14. Standing waves can be produced **(IIT-JEE, 1999)**

a. on a string clamped at both the ends.
b. on a string clamped at one end and free at the other.
c. when incident wave gets reflected from a wall.
d. when two identical waves with a phase difference of π are moving in the same direction.

15. As a wave propagates, **(IIT-JEE, 1999)**

a. the wave intensity remains constant for a plane wave.
b. the wave intensity decreases as the inverse of the distance from the source for a spherical wave.
c. the wave intensity decreases as the inverse square of the distance from the source for a spherical wave.
d. total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

16. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then **(IIT-JEE, 1999)**

a. the resultant amplitude is $(1+\sqrt{2})a$
b. the phase of the resultant motion relative to the first is 90°
c. the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated any single motion
d. the resulting motion is not simple harmonic

17. The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM for which of the option(s) **(IIT-JEE, 2006)**

a. For all value of A, B and C (except $C = 0$)
b. $A = -B$, $C = 2B$, amplitude $= |B\sqrt{2}|$
c. $A = B$, $C = 0$
d. $A = B$, $C = 2B$, amplitude $= |B|$

18. A student performed an experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then, **(IIT-JEE, 2009)**

a. the intensity of the sound heard at the first resonance was more than that at the second resonance.
b. the prongs of the tuning fork were kept in a horizontal plane above the resonance tube.
c. the amplitude of vibration of the ends of the prongs is typically around 1 cm.
d. the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

19. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' ($<L$) is attached at its centre to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod–disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? **(IIT-JEE, 2011)**

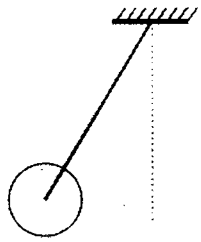


Fig. 8.9

a. Restoring torque in case A = Restoring torque in case B
b. Restoring torque in case A < Restoring torque in case B
c. Angular frequency for case A > Angular frequency for case B
d. Angular frequency for case A < Angular frequency for case B

Comprehension Type

For Problems 1–3

Waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling along the x-axis. (Here x is in metre and t is in second) **(IIT-JEE, 2006)**

1. Find the number of times intensity is maximum in time interval of 1 s.

a. 4 **b.** 6
c. 8 **d.** 10

2. The wave velocity of louder sound is

a. 100 m/s **b.** 192 m/s
c. 200 m/s **d.** 96 m/s

3. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 s is

a. 100 **b.** 46
c. 192 **d.** 96

For Problems 4–6

Two trains A and B are moving with speeds 20 m/s and 30 m/s, respectively in the same direction on the same straight track with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

(IIT-JEE, 2007)

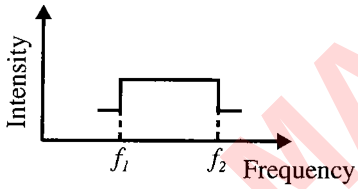


Fig. 8.10

Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

4. The speed of sound of the whistle is

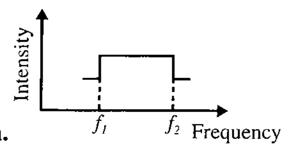
a. 340 m/s for passengers in A and 310 m/s for passengers in B.

b. 360 m/s for passengers in A and 310 m/s for passengers in B.

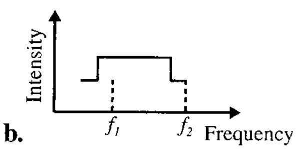
c. 310 m/s for passengers in A and 360 m/s for passengers in B.

d. 340 m/s for passengers in both the trains.

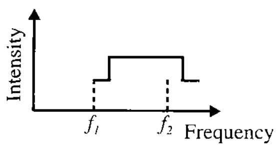
5. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



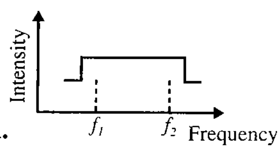
a.



b.



c.



d.

6. The spread of frequency as observed by the passengers in train B is

a. 310 Hz **b.** 330 Hz
c. 350 Hz **d.** 290 Hz

For Problems 7–9

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to m, k as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see Fig. 8.11).

(IIT-JEE, 2010)

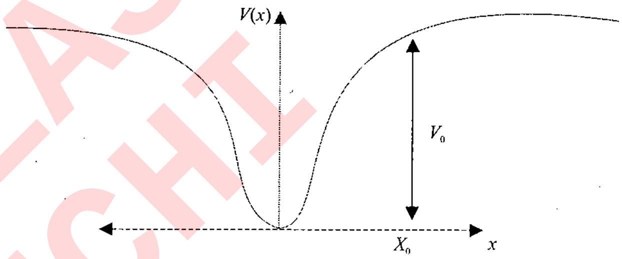


Fig. 8.11

7. If the total energy of the particle is E, it will perform periodic motion only if

a. $E < 0$ **b.** $E > 0$
c. $V_0 > E > 0$ **d.** $E > V_0$

8. For periodic motion of small amplitude A, the time period T of this particle is proportional to

a. $A\sqrt{\dfrac{m}{\alpha}}$ **b.** $\dfrac{1}{A}\sqrt{\dfrac{m}{\alpha}}$
c. $A\sqrt{\dfrac{\alpha}{m}}$ **d.** $A\sqrt{\dfrac{2\alpha}{m}}$

9. The acceleration of this particle for $|x| > X_0$ is

a. proportional to V_0 **b.** proportional to $\dfrac{V_0}{mX_0}$
c. proportional to $\sqrt{\dfrac{V_0}{mX_0}}$ **d.** zero

For Problems 10–12

Phase space diagrams are useful tools in analysing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical the axis.

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The phase space diagram is $x(t)$ vsersus $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in Fig. 8.12. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. **(IIT-JEE, 2011)**

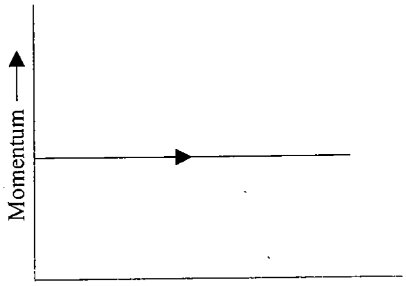
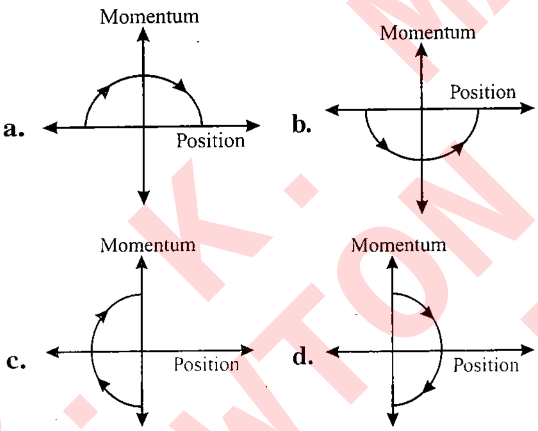


Fig. 8.12

10. The phase space diagram for a ball thrown vertically up from ground is



11. The phase space diagram for simple harmonic motion is a circle centred at the origin. In Fig. 8.13, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

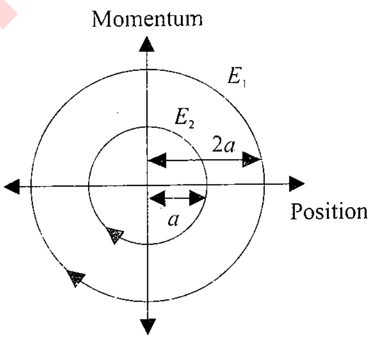


Fig. 8.13

a. $E_1 = \sqrt{2}E_2$ **b.** $E_1 = 2E_2$

c. $E_1 = 4E_2$ **d.** $E_1 = 16E_2$

12. Consider the spring–mass system, with the mass submerged in water, as shown in Fig. 8.14. The phase space diagram for one cycle of this system is:

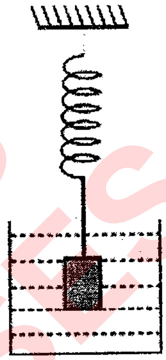
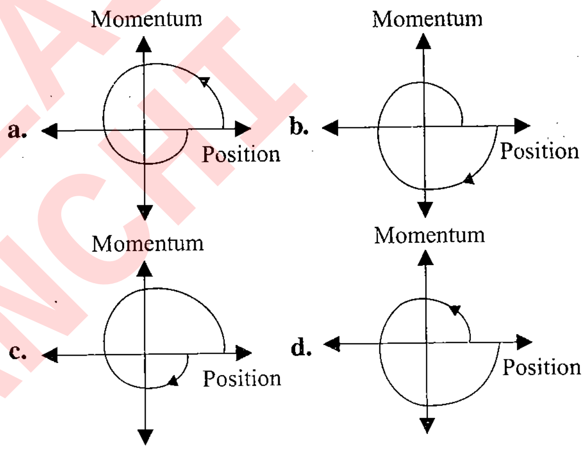


Fig. 8.14



Matching Column Type

1. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match situations in Column I with the characteristics in Column II. **(IIT-JEE, 2007)**

Column I	Column II
i. The object moves on the x-axis under a conservative force in such a way that its 'speed' and 'position' satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	**a.** The object executes a simple harmonic motion.
ii. The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	**b.** The object does not change its direction.

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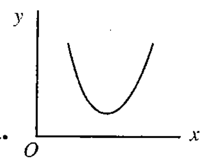
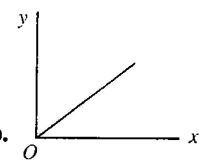
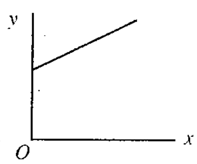
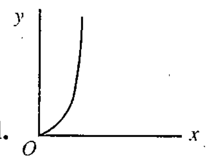
iii. The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	**c.** The kinetic energy of the object keeps on decreasing.
iv. The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	**d.** The object can change its direction only once.

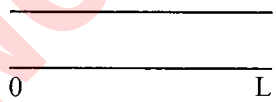
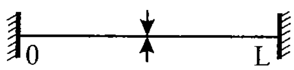
2. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graph given in Column II.

(IIT-JEE, 2008)

Column I	Column II
i. Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis).	**a.** 
ii. Displacement (y-axis) as a function of time (x-axis) for a one-dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	**b.** 
iii. Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle.	**c.** 
iv. The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis).	**d.** 

3. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

(IIT-JEE, 2011)

Column I	Column II
i. Pipe closed at one end 	**a.** Longitudinal waves
ii. Pipe open at both ends 	**b.** Transverse waves
iii. Stretched wire clamped at both ends 	**c.** $\lambda_f = L$
iv. Stretched wire clamped at both ends and at mid-point 	**d.** $\lambda_f = 2L$
	e. $\lambda_f = 4L$

Integer Answer Type

1. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

(IIT-JEE, 2009)

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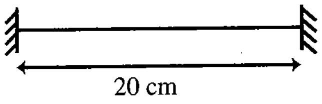

Fig. 8.15

2. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

(IIT-JEE, 2010)

3. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ superimposed, the amplitude of the resultant wave is **(IIT-JEE, 2010)**

4. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

(IIT-JEE, 2010)

ANSWERS AND SOLUTIONS

Archives

Fill in the blank Type

1. If $y = A\sin(\omega t - kx)$, displacement amplitude $= A$ (Maximum displacement)
Particle velocity,
$$v = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$$
Therefore, velocity amplitude $= A\omega = A(2\pi v)$ (Maximum velocity)
Particle acceleration,
$$Acc = \frac{dv}{dt} = -A\omega^2\sin(\omega t - kx)$$
Therefore, acceleration (maximum acceleration) amplitude $= A\omega^2 = A(2\pi v)^2$

2. $c = v\lambda$
$$\therefore \quad \lambda = \frac{c}{v} = \frac{330}{660} = 0.5\,\text{m}$$
The antinode will be at a distance of
$$\frac{\lambda}{4} = \frac{0.5}{4} = 0.125 \text{ m}$$

3. $$y_1 = 10\sin(3\pi t + \pi/4) \qquad \text{(i)}$$
$$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t \qquad \text{(ii)}$$
In Eq. (ii), let
$$5 = a\cos\theta \qquad \text{(iii)}$$
and $$5\sqrt{3} = a\sin\theta \qquad \text{(iv)}$$
$$\therefore \quad y_2 = a\sin 3\pi t\cos\theta + a\sin\theta\cos 3\pi t$$
$$y_2 = a\sin(3\pi t + \theta) \qquad \text{(v)}$$
Squaring and adding Eqs. (iii) and (iv), we get
$$5^2 + (5\sqrt{3})^2 = a^2\cos^2\theta + a^2\sin^2\theta$$
$$\Rightarrow 100 = a^2$$
$$\Rightarrow a = 10$$
Therefore, Eq. (v) can be written as
$$y_2 = 10\sin(3\pi t + \theta) \qquad \text{(vi)}$$
From Eqs. (i) and (vi), the ratio of amplitudes is 10:10, i.e., 1:1.

4. $c = v\lambda$ and $c = \sqrt{T/m}$
$$\therefore \quad \sqrt{\frac{T}{m}} = v\lambda$$
where T = tension in the string and m = mass per unit length of wire
When 50.7 kg mass is suspended for fundamental mode $\lambda = 2l$
$$v_1 \times 2l = \sqrt{\frac{50.7 \times g}{m}} \qquad \text{(i)}$$
when mass is submerged in water, new tension
$$T_2 = \text{weight} - \text{upthrust}$$
$$= 50.7\text{ g} - 0.0075 \times 1000 \times g = 43.2\text{ g}$$
$$\therefore v_2 \times 2l = \sqrt{\frac{43.2g}{m}} \qquad \text{(ii)}$$
On dividing Eqs. (i) and (ii), we get
$$\frac{v_2}{v_1} = \sqrt{\frac{43.2}{50.7}} \Rightarrow v_2 = v_1\sqrt{\frac{43.2}{50.7}}$$
$$= 260\sqrt{\frac{43.2}{50.7}} = 240 \text{ Hz}$$

5. Let the wave velocity is v towards right.
(Displacement at $t = 0$, $x = x$)
= [Displacement at $t = 2$ s, $x = x + v(2)$]

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$$\Rightarrow \quad \frac{1}{1+x^2}=\frac{1}{1+(1+x+2v)^2}$$

$$\Rightarrow \quad v=-0.5\text{ m/s}$$

The negative sign indicates that wave is travelling towards left.

6. In Fig. 8.16(i)

$$\frac{\lambda}{2}=l\Rightarrow\lambda=2l$$

$$f=\frac{c}{\lambda}=\frac{c}{2l}$$

In Fig. 8.16(ii)

$$\frac{\lambda'}{4}=\frac{l}{2}\Rightarrow\lambda'=2l$$

From Figs. 8.16(ii) and (i),

$$f'=\frac{c}{\lambda'}=\frac{c}{2l}=f$$

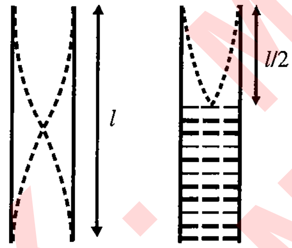

Fig. 8.16

7. The first frequency the driver of bus hears is the original frequency 200 Hz. The second frequency the driver hears is the frequency of sound reflected from the wall. The two frequencies of sound heard by the driver are

i. Original frequency (200 Hz)

ii. Frequency of sound reflected from the wall (v')

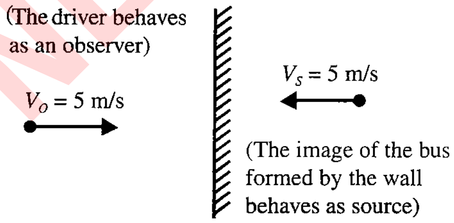

Fig. 8.17

The frequency of sound reflected from the wall

$$v'=v\left[\frac{u+u_0}{u-u_s}\right]$$

$$v'=200\left[\frac{342+5}{342-5}\right]=205.93\text{ Hz}$$

$\therefore$ Frequency of beats

$$=v'-v=205.93-200=5.93=6\text{ Hz}$$

8. $x = 0.04$ m, KE = 0.5 J and PE = 0.4 J

Also $\quad v=\dfrac{25}{\pi}$ Hz

Now, $\quad \text{TE}=\dfrac{1}{2}m\omega^2a^2=\dfrac{1}{2}m\times4\pi^2v^2a^2$

$$\Rightarrow 0.9=\frac{1}{2}\times0.2\times4\pi^2\times\frac{25}{\pi}\times\frac{25}{\pi}\times a^2$$

$$\Rightarrow a=\frac{3}{50}=0.06\text{ m}$$

Alternate Method:

$\frac{1}{2}mv^2=0.5\Rightarrow\frac{1}{2}(0.2)v^2=0.5\Rightarrow v^2=5$

Now $v^2=\omega^2(A^2-x^2)$

$\Rightarrow 5=(2\pi v)^2\,[A^2-(0.04)^2]$

$\Rightarrow 5=4\pi^2\times\frac{25}{\pi}\times\frac{25}{\pi}[A^2-(0.04)^2]$

$\Rightarrow A=0.06$ m

9. $u = 300$ m/s and $n = 25$

$$\therefore \quad \lambda=\frac{u}{n}=\frac{300}{25}=12\text{ m}$$

For a path difference of λ, the phase difference is 2π.
For a path difference of 6 m, the phase difference is

$$\frac{2\pi\times6}{\lambda}=\pi\text{ rad}$$

The amplitude is same at both points, hence amplitude difference is zero.

True or False Type

1. False.

$$v_{\text{sound}}=\sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_{H_2}}{v_{He}}=\frac{\sqrt{\gamma_{H_2}/M_{H_2}}}{\sqrt{\gamma_{He}/M_{He}}}=\frac{\sqrt{(7/5)/2}}{\sqrt{(5/3)/4}}=\sqrt{\frac{42}{25}}$$

2. False. The intensity of sound at a given point is the energy per second received by a unit area perpendicular to the direction of propagation.

$$I=\frac{1}{2}\rho V\omega^2A^2$$

Also intensity varies as distance from the point source as $I\propto 1/r^2$.

None of the parameters are changing in case of a clear night or a clear day. Therefore the intensity will remain the same.

3. True. Speed of sound waves in water is greater than that in air. So water is rare medium for sound waves.

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4. **False.** If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound. Thus, in both the cases, one sound coming directly from the source and the other coming after reflection will have the same frequency (since velocity of source w.r.t. observer is same in both the cases). Therefore, no beats will be heard.

Single Correct Answer Type

1. **c.**

Case I Here $\lambda/2 = l$

$\therefore \quad \lambda = 2l$

Now, $\quad v = f \times \lambda$

$$\therefore \quad f = \frac{v}{\lambda} = \frac{v}{2l} \qquad \text{(i)}$$

Case II Here $\lambda'/4 = l/2$

$\therefore \quad \lambda' = 2l$

Now, $\quad v = f' \times \lambda'$

$$\therefore \quad f' = \frac{v}{\lambda'} = \frac{v}{2l} = f$$

2. **b.**

$$y = y_0 \sin 2\pi\left[ft - \frac{x}{\lambda}\right]$$

$$\therefore \quad \frac{dy}{dt} = \left[y_0 \cos 2\pi\left(ft - \frac{x}{\lambda}\right)\right] \times 2\pi f$$

$$\Rightarrow \left[\frac{dy}{dt}\right]_{max} = y_0 \times 2\pi f$$

Given that the maximum particle velocity is equal to four times the wave velocity ($c = f\lambda$).

$$\therefore \quad y_0 \times 2\pi f = 4 f \times \lambda$$

$$\lambda = \frac{\pi y_0}{2}$$

3. **a.** $\lambda/4 = l$ (Fundamental mode), $\lambda = 4l$, $c = v\lambda$

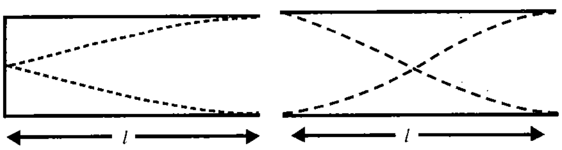

Fig. 8.18

$$\therefore \quad v = \frac{c}{\lambda} = \frac{c}{4l} = 512 \text{ Hz}$$

Given, $\lambda'/2 = l$

Fundamental mode,

$$\therefore \quad \lambda' = 2l \quad \text{but} \quad c = v'\lambda'$$

$$\therefore \quad v' = \frac{c}{\lambda'} = \frac{c}{2l} = 2\left(\frac{c}{4l}\right)$$

$$= 2 \times 512 = 1024 \text{ Hz}$$

4. **c.** During one complete oscillation, the kinetic energy will become maximum twice. Therefore, the frequency of kinetic energy will be $2f$.

5. **c.** Stationary wave is produced when two waves travel in opposite directions. Now,

$$y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$$

$y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.

6. **c.** Given $\dfrac{v}{4l_1} = \dfrac{3v}{2l_2} \Rightarrow \dfrac{l_1}{l_2} = \dfrac{1}{6}$

7. **d.** Both the bodies oscillate in simple harmonic motion for which the maximum velocities will be

$$v_1 = a_1\omega_1 = a_1 \times \frac{2\pi}{T_1}$$

$$v_2 = a_2\omega_2 = a_2 \times \frac{2\pi}{T_2}$$

Given that $v_1 = v_2$

$$a_1 \times \frac{2\pi}{T_1} = a_2 \times \frac{2\pi}{T_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}} = \sqrt{\frac{k_2}{k_1}}$$

8. **b.** When the cylinder is given a small push downwards, say x, then two forces start acting on the cylinder trying to bring it to its mean position. Restoring force = – (upthrust + spring force)

$$= -[\rho Axg + kx]$$

$$= -[\rho Ag + k]x$$

$$M\omega^2 = \rho Ag + k \Rightarrow \omega = \left[\frac{\rho Ag + k}{M}\right]^{1/2}$$

$$\Rightarrow v = \frac{1}{2\pi}\left[\frac{\rho Ag + k}{M}\right]^{1/2}$$

9. **d.** When a force is applied on cubical block A in the horizontal direction, then the lower block B will get distorted as shown by the dotted lines and A will attain a new position (without distortion as A is a rigid body) as shown by the dotted lines.

For cubical block B,

$$\eta = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L} = \frac{F}{L^2} \times \frac{L}{\Delta L} = \frac{F}{L \times \Delta L}$$

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$$\Rightarrow F = \eta L \Delta L$$

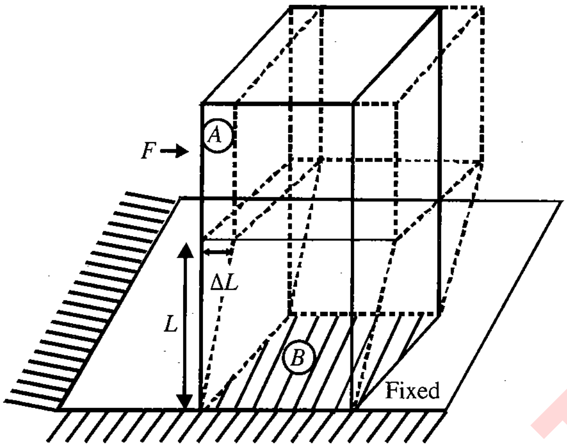

Fig. 8.19

ηL is a constant

$\Rightarrow F \propto \Delta L$ and directed towards the mean position

$\Rightarrow$ oscillation will be simple harmonic in nature. Here, $M\omega^2 = \eta L$

$$\Rightarrow \omega = \sqrt{\frac{\eta L}{M}} = \frac{2\pi}{T}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{M}{\eta L}}$$

10. b.

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$$

$$= 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$$

$= 2[\cos t + 1] \sin 1000t = 2 \cos t \sin 1000t + 2 \sin 1000t$

$= \sin 1001t + \sin 999t + 2 \sin 1000t$

11. b. Let us consider the wire also as a spring. Then the case becomes two springs attached in series. The equivalent spring constant is

$$\frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{K'}$$

where K' is the spring constant of the wire

$$\therefore \quad K_{eq} = \frac{KK'}{K+K'}$$

Now, $$Y = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L}$$

$$\frac{F}{\Delta L} = \frac{YA}{L} = K'$$

We know that time period of the system

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m(K+K')}{KK'}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{K}\left[\frac{K+YA/L}{YA/L}\right]}$$

$$= 2\pi\sqrt{\frac{m(KL+YA)}{KYA}}$$

12. a. We know that

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

In air, $T = mg = \rho Vg$

$$\therefore \quad f = \frac{1}{2l}\sqrt{\frac{\rho Vg}{m}} \quad \text{(i)}$$

In water, $T = mg -$ upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

Therefore,

$$\therefore f' = \frac{1}{2l}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}}$$

$$= \frac{1}{2l}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}} = 300\left[\frac{2\rho-1}{2\rho}\right]^{\frac{1}{2}}$$

$\because \rho_\omega = 1$ g/cc and from Eq. (i)

13. a. According to Hooke's law, $F_g \propto x$ [Restoring force $F_g = T$, tension of spring]

Velocity of sound by a stretched string

$$v = \sqrt{\frac{T}{m}}$$

where m is the mass per unit length

$$\therefore \quad \frac{v}{v'} = \sqrt{\frac{T}{T'}} \Rightarrow v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22v$$

14. a. For both ends open, fundamental frequency

$$\frac{2\lambda_1}{4} = l \quad \Rightarrow \lambda_1 = 2l$$

$$\therefore \quad v_1 = \frac{c}{\lambda_1} = \frac{c}{2l} \quad \text{(i)}$$

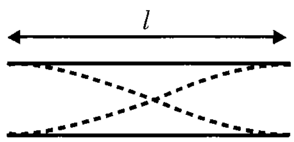

Fig. 8.20

For one end closed the third harmonic

$$\frac{3\lambda_2}{4} = l \quad \Rightarrow \lambda_2 = \frac{4l}{3}$$

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$$v_2 = \frac{c}{\lambda_2} = \frac{3c}{4l} \qquad \text{(ii)}$$

Given $\qquad v_2 - v_1 = 100$

From Eqs. (i) and (ii)

$$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2}$$

On solving, we get $\quad v_1 = 200$ Hz

15. d.

$$f' = f\left[\frac{v}{v - v_s}\right] = 450\left[\frac{330}{330-33}\right] = 500 \text{ Hz}$$

16. a.

$$V = \frac{dy}{dt} = -A\omega\cos(kx - \omega t)$$

$\therefore \qquad V_{\max} = A\omega$

17. b. Velocity of wave: $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{10^{-2}/0.4}} = 8$ m/s

The wave will be in same phase after travelling a distancne of $2l = 2 \times 0.4 = 0.8$ m.

And constructive interference will take place. So time Δt

$$\Delta t = \frac{0.8}{v} = \frac{0.8}{8} = 0.10 \text{ s}$$

18. a. $V(x) = k|x|^3$

$$\therefore \qquad [k] = \frac{[V]}{[x]^3} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Now time period on T and (mass)^x (amplitude)^y $(k)^z$

$$\therefore \qquad [M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z$$

$$= [M^{x+y}L^{y-x}T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$
$$y - z = 0 \quad y = z = -1/2$$

Hence $T \propto$ (amplitude)$^{-1/2}$

or $$T \propto \frac{1}{\sqrt{a}}$$

19. d. Let us plot the graph of the mathematical equation

$$U(x) = K\left[1 - e^{-x^2}\right]$$

$$\therefore \qquad F = -\frac{dU}{dx} = -2kxe^{-x^2}$$

It is clear that the potential energy is minimum at $x = 0$. Therefore, $x = 0$ is the state of stable equilibrium. Now if we displace the particle from $x = 0$, then for small displacements the particle tends to regain the position $x = 0$ with a force $F = 2kx/e^{x^2}$ for x to be small $F \propto x$.

20. b. $\frac{(C)N_2}{(C)He} = \sqrt{\frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{4}{28}} = \sqrt{\frac{1}{7}}$

21. a. Effective gravity $= g\cos\alpha$

$$\therefore \qquad T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

22. d. $n_1 = n_0 \frac{340}{340-34} = \frac{10}{9}n_0$

$$n_2 = n_0\frac{340}{340-17} = \frac{20}{19}n_0$$

$$\frac{n_1}{n_2} = \frac{10}{9} \times \frac{19}{20} = \frac{19}{18}$$

23. d. $$n_1 = \frac{1}{2l}\sqrt{\left[\frac{T}{4\pi r^2\rho}\right]}$$

and $$n_2 = \frac{1}{4l}\sqrt{\left[\frac{T}{\pi r^2\rho}\right]}$$

$$\therefore \qquad \frac{n_1}{n_2} = 2 \times \frac{1}{2} = 1$$

24. b. After 2 s, tubes will overlap each other. According to superposition principle, the string will not have any distortion and will be straight. Hence, there will be no PE. The total energy will be kinetic.

Fig. 8.21

25. c. We know that $E \propto A^2v^2$, where A = amplitude and v = frequency. Also, $\omega = 2\pi v = \omega \propto v$

In case 1: Amplitude $= A$ and $v_1 = v$

In case 2: Amplitude $= A$ and $v_2 = 2v$

$$\therefore \qquad \frac{E_2}{E_1} = \frac{A^2v_2^2}{A^2v_1^2} = 4 \quad \Rightarrow E_2 = 4E_1$$

26. a. Method 1: Qualitative. The velocity of a body executing SHM is maximum at its centre and decreases as the body proceeds to the extremes. Therefore, if the time taken for the body to go from O to $A/2$ is T_1 and to go A is T_2, then obviously $T_1 < T_2$.

Method 2: Quantitative. Any SHM is given by the equation x = sin ωt, where x is the displacement of the body at any instant t. a is the amplitude and ω is the angular frequency.

When $\qquad x = 0, \quad \omega t_1 = 0$

$\therefore \qquad t_1 = 0$

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When $x = a/2, \quad \omega t_2 = \pi/6, \quad t_2 = \pi/6\omega$

When $x = a, \quad \omega t_3 = \pi/2, \quad t_3 = \pi/2\omega$

Time taken from O to $A/2$ will be

$$t_2 - t_1 = \frac{\pi}{6\omega} = T_1$$

Time taken from $A/2$ to A will be

$$t_3 - t_2 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega} = T_2$$

Hence $T_2 > T_1$

27. d. We know that

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and $$T_2 = 2\pi\sqrt{\frac{l}{g'}}$$

$\therefore$ $$\frac{T_2}{T_1} = \sqrt{\frac{g}{g'}} \quad \text{(i)}$$

Also $$g = \frac{GM}{R^2}$$

$\therefore$ $$g' = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$$

$\therefore$ $$\frac{g}{g'} = 4 \Rightarrow \frac{T_2}{T_1} = 2$$

28. c. The components of acceleration are as shown.

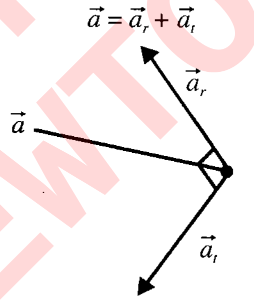

Fig. 8.22

29. b.

$$\frac{V_A + V}{V} = \frac{5.5}{5}$$

$$\frac{V_B + V}{V} = \frac{6}{5}$$

Solve to get $$\frac{V_B}{V_A} = 2$$

30. a. $f_0 = \frac{5}{2l}\sqrt{\frac{9g}{\mu}} = \frac{3}{2l}\sqrt{\frac{Mg}{\mu}} \Rightarrow M = 25$ kg

31. b. f_1 = frequency of the police car heard by motorcyclist, f_2 = frequency of the siren heard by motorcyclist.

$$f_1 = \frac{330 - v}{330 - 22} \times 176, \quad f_2 = \frac{330 + v}{330} \times 165$$

$\therefore$ $$f_1 - f_2 = 0 \Rightarrow v = 22 \text{ m/s}$$

32. b.

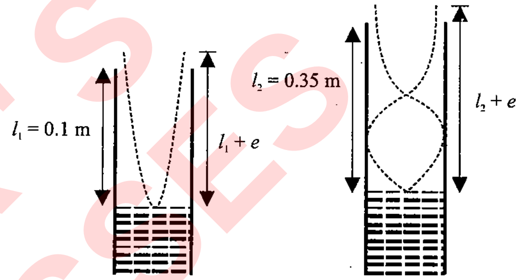

Fig. 8.23

$$l_1 + x = \frac{\lambda}{4} \Rightarrow \lambda = 4(l_1 + x)$$

$$(l_2 + x) = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4}{3}(l_2 + x)$$

$\therefore$ $$v_1 = \frac{v}{\lambda_1} = \frac{v}{4(l_1 + x)}$$

$\therefore$ $$v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(l_2 + x)}$$

Given $v_1 = v_2$

$$\Rightarrow \frac{v}{4(l_1 + x)} = \frac{3v}{4(l_2 + x)} \Rightarrow x = 0.025 \text{ m}$$

33. a. We know that in SHM, at extreme position, PE is maximum when $t = 0$, $x = A$, i.e., at time $t = 0$, the particle executing SHM is at its extreme position. Therefore PE is maximum. Graphs I and III represent the above characteristics.

34. d. The frequency is a characteristic of source. It is independent of the medium. Hence, the correct option is (d)

35. b. Frequency of first overtone in closed pipe,

$$v = \frac{3}{4l_1}\sqrt{\frac{B}{\rho_1}} \quad \text{(i)}$$

Frequency of the first overtone in open pipe,

$$v = \frac{1}{l_2}\sqrt{\frac{B}{\rho_2}} \Rightarrow l_2 = \frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}} \quad \text{(ii)}$$

36. c. For first the resonance $l_1 + e = \frac{\lambda}{4}$

But $v = f\lambda$

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$$\therefore \quad v = f4(l_1 + e)$$

$$\Rightarrow \quad l_1 + e = \frac{v}{4f} \quad \text{(i)}$$

For the second resonance

$$l_2 + e = \frac{3\lambda}{4}$$

$$\therefore \quad v = f\frac{4}{3}(l_2 + e)$$

$$\Rightarrow \quad l_2 + e = \frac{3v}{4f} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v = 2f(l_2 - l_1)$$

$$\therefore \quad \Delta v = 2f(\Delta l_2 + \Delta l_1)$$

$$= 2 \times 512 \times (0.1 + 0.1) \text{ cm/s}$$

$$= 204.8 \text{ cm/s}$$

37. d.

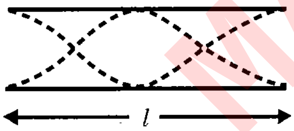

Fig. 8.24

$$\lambda = l$$

$$\therefore \quad f_1 = \frac{v}{\lambda} = \frac{v}{l} \quad \text{(i)}$$

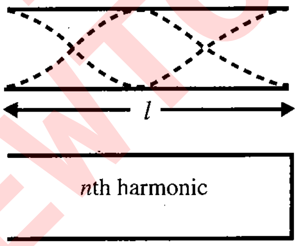

Fig. 8.25

$$\lambda = \frac{4l}{n}$$

$$\therefore \quad f_2 = \frac{v}{\lambda} = \frac{nv}{4l} \quad \text{(ii)}$$

Here n is an odd number

$$f_2 = \frac{n}{4}f_1$$

From Eqs. (i) and (ii), we get
For the first resonance

$$n = 5,$$

$$f_2 = \frac{5}{4}f_1$$

Given $\quad f_2 > f_1$

38. b. $y = kt^2$

$$\therefore \quad \frac{dy}{dt} = 2kt$$

$$\Rightarrow \frac{d^2y}{dt^2} = 2k = 2 \text{ m/s}^2 \quad \text{(i)}$$

$(\because k = 1 \text{ m/s}^2, \text{given})$

We know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g_2}{g_1} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$$

$$\left[\because g_1 = 10 \text{ m/s}^2, g_2 = g + 2 = 12 \text{ m/s}^2\right]$$

39. a. Frequency of the first harmonic of

$$AB = \frac{1}{2l}\sqrt{\frac{T_{AB}}{m}}$$

Frequency of the 2nd harmonic of

$$CD = \frac{1}{l}\sqrt{\frac{T_{CD}}{m}}$$

Given that the two frequencies are equal.

$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_{AB}}{m}} = \frac{1}{l}\sqrt{\frac{T_{CD}}{m}} \Rightarrow \frac{T_{AB}}{4} = T_{CD}$$

$$\Rightarrow T_{AB} = 4T_{CD} \quad \text{(i)}$$

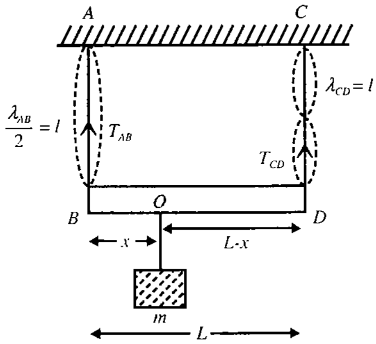

Fig. 8.26

For rotational equilibrium of massless rod taking torque about point O,

$$T_{AB} \times x = T_{CD}(L - x) \quad \text{(ii)}$$

Solve to get $\quad x = \frac{L}{5}$

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40. **a.** As shown in the figure, the prongs of the tuning fork are kept in a vertical plane.

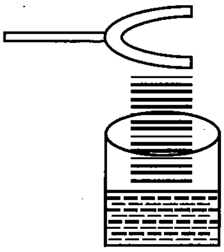

Fig. 8.27

41. **a.** Particle velocity, $v_P = -v$ (slope of $y-x$ graph)
Here, v = +ve, as the wave is travelling in positive direction.
Slope at P is negative.
Therefore, velocity of particle is in negative y (or $\hat{j}$) direction.

42. **a.** With increase in tension, frequency of vibrating string will increase. Since the number of beats are decreasing, therefore frequency of vibrating string or the third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.
Therefore, frequency of tuning fork = third harmonic frequency of closed pipe + 4

$$= 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4\times 0.75}\right) + 4 = 344 \text{ Hz}$$

43. **c.** $\frac{1}{2}kx^2 = \frac{1}{2}4ky^2$

$$\Rightarrow \frac{y}{x} = \frac{1}{2}$$

44. **d.** The given motion is represented by

$$\left(\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\right)$$

$$x = 1\sin\left(\frac{\pi}{4}t\right), \quad \frac{d^2x}{dt^2} = \frac{-\pi^2}{16}\sin(\pi/4)t$$

At $\quad t = 4/3$ s, $\quad \frac{d^2x}{dt^2} = -\frac{\sqrt{3}}{32}\pi^2 \quad \text{cm/s}^2$

45. **c.** Restoring torque $= -2\times k\left(\frac{l}{2}\theta\right)\frac{l}{2} = \frac{Id^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\frac{kl^2}{2}(-\theta)}{\frac{Ml^2}{12}}$$

$$\Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

46. **b.** $\frac{v_s}{4L_P} = \frac{2}{2l_S}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{320}{4\times 0.8} = \frac{1}{0.5}\sqrt{\frac{50}{\mu}}$

$\Rightarrow \mu = 1/50$

Mass of string,

$$m = \mu l_S = \frac{1}{50}\times 0.5 = 0.01 \text{ kg} = 10 \text{ g}$$

47. **b.** We know that

$$C = \sqrt{\left(\frac{\gamma RT}{M}\right)}$$

Hence $\quad C \propto \sqrt{\frac{1}{m}}$

$$\therefore \quad \frac{C_1}{C_2} = \sqrt{\left(\frac{m_2}{m_1}\right)}$$

48. **a.** $f_{\text{incident}} = f_{\text{reflected}} = \frac{320}{320-10}\times 8$ kHz

$$f_{\text{observed}} = \frac{320+10}{320}f_{\text{reflected}} = 8\times\frac{320}{310}$$

$= 8.51$ kHz ≈ 8.5 kHz

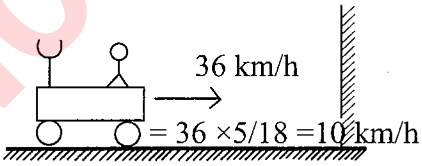

Fig. 8.28

49. **b.** So, $B = A$, $\phi = 240° = \frac{4\pi}{3}$

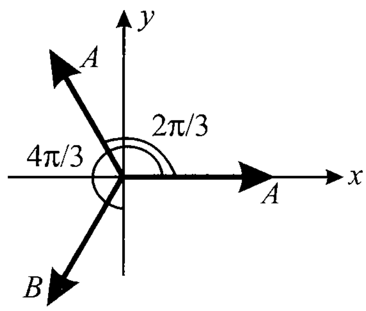

Fig. 8.29

Multiple Correct Answers Type

1. **a., b., c., d.** $y = 10^{-4}\sin(60t + 2x)$. Comparing the given equation with the standard wave equation travelling in negative x-direction,

$$y = a\sin(\omega t + kx)$$

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We get amplitude $a = 10^{-4}$ m

Also, $\omega = 60$

$\therefore \quad 2\pi f = 60 \quad \Rightarrow f = \dfrac{30}{\pi}$ Hz

Also, $k = 2$

$$\Rightarrow \frac{2\pi}{\lambda} = 2 \quad \Rightarrow \lambda = \pi \text{ m}$$

We know that

$$v = f\lambda = \frac{30}{\pi} \times \pi = 30 \text{ m/s}$$

2. **a., c.** The wavelengths possible in an air column in a pipe which has one closed end is

$$\lambda = \frac{4l}{(2n+1)}$$

so, $\quad c = v\lambda \quad \Rightarrow \quad 330 = 264 \times \dfrac{4l}{2n+1}$

as it is in resonance with a vibrating tuning fork of frequency 264 Hz

$$l = \frac{330 \times (2n+1)}{264 \times 4}$$

For $\quad n = 0,\ l = 0.315 \text{ m} = 31.25$ cm

For $\quad n = 1,\ l = 0.9375 \text{ m} = 93.75$ cm

3. **a., c.** $\dfrac{\partial^2 y}{\partial x^2} = (\text{constant})\dfrac{\partial^2 y}{\partial t^2}$

4. **a., b., d.** In general we can write for a closed end pipe

$$v = \frac{(2n-1)c}{4l}$$

where $\quad n = 1, 2, 3, \ldots$

$$\therefore \quad v = \frac{c}{4l}, \frac{3c}{4l}, \frac{5c}{4},$$

$$\cdots = 80, 240, 400, \ldots$$

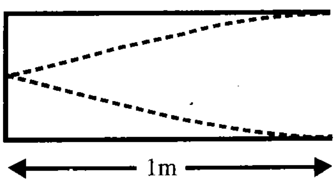

Fig. 8.30

5. **b., c.** $\text{KE}_{\max} = \dfrac{1}{2}kA^2 = \dfrac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100$ J

$$U_{\max} = \text{TE} = 160 \text{ J}$$

6. **b., c.** $y = A \sin(10\pi x = 15\pi t + \pi/3)$
The standard equation of a wave travelling in X-direction is

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right]$$

$$\Rightarrow y = A\sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$$

Comparing it with the given equation we find

$$\frac{2\pi v}{\lambda} = 15\pi$$

and $\quad \dfrac{2\pi}{\lambda} = 10\pi$

$$\lambda = \frac{1}{5} = 0.2 \text{ m}$$

$$\therefore \quad v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5 \text{ m/s}$$

7. **b., d.** T_1 and T_2 are the higher and lowest tensions initially.
Now, frequency $\propto \sqrt{\text{tension}}$.
Therefore, frequency produced in wire with tension T_1 is higher and that with tension T_2 is lower. If we lower the tension T_2 then beat frequency will increase. Therefore, the tension T_1 is decreased. If tension has to be increased then tension T_2 should be increased.

8. **a., b., c.** Moving plane is like a moving observer. Therefore, number of waves encountered by moving plane

$$f_1 = f\left(\frac{v + v_0}{v}\right) = f\left(\frac{c+v}{c}\right)$$

Frequency of reflected wave,

$$f_2 = f_1\left(\frac{v}{v - v_s}\right) = f\left(\frac{c+v}{c-v}\right)$$

Wavelength of reflected wave,

$$\lambda_2 = \frac{v}{f_2} = \frac{c}{f_2} = \frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

Number of beats heard $= f_2 - f = \dfrac{2vf}{c-v}$

9. **a., b., c., d.** It is given that $y(x, t) = 0.02\cos(50\pi t + \pi/2)$ $\cos(10\pi x)$

$$\cong A\cos\left(\omega t + \frac{\pi}{2}\right)\cos kx$$

Node occurs when $kx = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, etc.

$$\Rightarrow 10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$\Rightarrow \quad x = 0.05$ m, 0.15 m $\qquad$ option (a)

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Antinode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow$ $10\pi x = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow$ $x = 0.1$ m, 0.02 m, 0.3 m option (b)

Speed of the wave is given by

$$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s} \quad \text{option (c)}$$

Wavelength is given by

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)m = 0.2m \quad \text{option (d)}$$

10. b., c. Due to the clamping of the square plate at the edges, its displacements along the x- and y-axes will individually be zero at the edges. Only the choices (b) and (c) predict these displacements correctly. This is because $\sin 0 = 0$.

11. a., c. For a transverse sinusoidal wave travelling on a string, the maximum velocity is $a\omega$. Also, the maximum velocity is

$$\frac{v}{10} = \frac{10}{10} = 1 \text{ m/s}$$

$\therefore$ $a\omega = 1 \Rightarrow 10^{-3} \times 2\pi f = 1$

$$\Rightarrow f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

The velocity $v = f\lambda$

$$\therefore \quad \lambda = \frac{v}{f} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} \text{ m}$$

12. b., c., d. Given,

$$y = \frac{0.8}{(4x+5t)^2+5} = \frac{0.8}{16\left[x+\frac{5}{4}t\right]^2+5} \quad \text{(i)}$$

We know that equation of moving pulse is

$$y = f(x + vt) \quad \text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$v = \frac{5}{4} \text{ m/s} = \frac{2.5}{2} \text{ m/s}$$

Wave will travel a distance of 2.5 m in 2 s.

13. a., b., c., d. Factual.

14. a., b., c. Standing waves are produced by two similar waves superposing while travelling in opposite directions. This can happen in options (a) and (c).

15. a., c., d. For a plane wave, intensity (energy crossing per unit area per unit time) is constant at all points. But for a spherical wave, intensity at a distance r from a point source of power (P), is given by

$$I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

But the total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

16. a., c. From superposition principle,

$$y = y_1 + y_2 + y_3$$
$$= a\sin\omega t + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$$
$$= a\{\sin\omega t + \sin(\omega t + 90°)\} + a\sin(\omega t + 45°)$$
$$= 2a\sin(\omega t + 45°)\cos 45° + a\sin(\omega t + 45°)$$
$$= (\sqrt{2}+1)a\sin(\omega t + 45°) = A\sin(\omega t + 45°)$$

Therefore, resultant motion is simple harmonic of amplitude $A = (\sqrt{2}+1)a$ and which differs in phase by 45° relative to the first.

Energy in SHM $\propto$ (amplitude)^2

$$\left[E = \frac{1}{2}mA^2\omega^2\right]$$

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\therefore \quad E_{\text{resultant}} = (3+2\sqrt{2})E_{\text{single}}$$

17. b., d.

For $A = -B$ and $C = 2B$

$X = B\cos 2\omega t + B\sin 2\omega t = \sqrt{2}B\sin\left[2\omega t + \frac{\pi}{4}\right]$

This is equation of SHM of amplitude $\sqrt{2}B$.

If $A = B$ and $C = B$, then $X = B + B\sin 2\omega t$

This is also equation of SHM about the point $X = B$. Function oscillates between $X = 0$ and $X = 2B$ with amplitude B.

18. a., d. Larger the length of air column, feebler is the intensity.

19. a., d. Torque is same for both the cases.

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}}$$

$$I_A = \frac{mL^2}{3} + \left(\frac{MR^2}{2} + ML^2\right)$$

When the disc is free to rotate about its centre, it will not rotate w.r.t. ground. It will act as a point mass, so moment of inertia is given by

$$I_B = \frac{mL^2}{3} + ML^2$$

$$I_A > I_B \Rightarrow T_A > T_B \Rightarrow \omega_A < \omega_B$$

Comprehension Type

For Problems 1–3

1. a., 2. c., 3. a.

1. **a.** Number of maxima in 1 s is called the beat frequency. Hence,

$$f_b = f_1 - f_2$$
$$= \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4 \text{ Hz}$$

2. **c.** Speed of wave,

$$v = \frac{\omega}{k}$$
$$v = \frac{100\pi}{0.5\pi} \text{ or } \frac{92\pi}{0.46\pi} = 200 \text{ m/s}$$

3. **a.**

At $\quad x = 0,\ y = y_1 + y_2$

$$= 2A\cos 96\pi t \cos 4\pi t$$

Frequency of cos ($96\pi t$) function is 48 Hz and that of cos ($4\pi t$) function is 2 Hz. In 1 s, cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with the first. Hence, net y will become zero 100 times in 1 s.

For Problems 4–6

4. b., 5. a., 6. a.

4. **b.** $v_{SA} = 340 + 20 = 360$ m/s
$v_{SB} = 340 - 30 = 310$ m/s

5. **a.** For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities. Therefore, the correct option is (a).

6. **a.** For the passengers in train B, observer is receding with velocity 30 m/s and source is approaching with velocity 20 m/s.

$$f_1' = 800\left(\frac{340-30}{340-20}\right) = 775 \text{ Hz}$$

$$f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085 \text{ Hz}$$

Therefore, spread of frequency, $f'_2 - f'_1 = 310$ Hz

For Problems 7–9

7. c., 8. b., 9. d.

7. **c.** Energy must be less than V_0, so that KE becomes zero before PE becomes maximum and particle returns back.

8. **b.** Dimension of α can be found as
$[\alpha] = \text{ML}^{-2}\text{T}^{-2}$

Only option (b) has dimension of time

Alternatively:

From conservation of energy:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \alpha x^4 = \alpha A^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2\alpha}{m}(A^4 - x^4)$$

$$\int dt = \sqrt{\frac{m}{2\alpha}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}}$$

$$\Rightarrow t = \frac{1}{A}\sqrt{\frac{m}{2\alpha}}\int_0^1 \frac{du}{\sqrt{1-u^4}} \qquad \text{[Substitute } x = Au]$$

$$\Rightarrow t \propto \frac{1}{A}\sqrt{\frac{m}{2\alpha}}$$

9. **d.** As potential energy is constant for $|x| > X_0$, the force on the particle is zero hence acceleration is zero.

10. **d.**

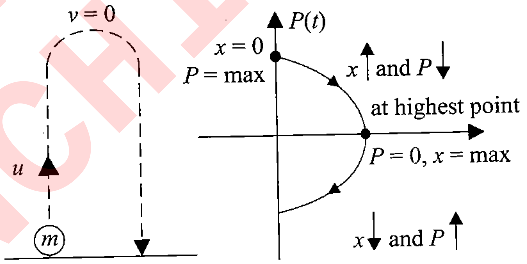

Fig. 8.31

11. **c.** In the 1st case amplitude of SHM is a.
In the 2nd case amplitude of SHM is $2a$
Total energy = 1/2 k (amplitude)^2

$$E_1 = \frac{1}{2}k(2a)^2 \quad \Rightarrow \quad E_2 = \frac{1}{2}k(a)^2$$
$$E_1 = 4E_2$$

12. **b.** Amplitude of oscillation inside liquid will decrease due to viscous force, So radius of circular arcs will decrease as position change. Let initially mass is taken to maximum upward position and released, then its initial momentum is zero and then afterwards momentum will become negative first which is correctly shown in option (b).

Matching Column Type

1. **i.→a.; ii.→b., c.; iii.→a.; iv.→b., c.**

i. Given $v = c_1\sqrt{c_2 - x^2}$. Comparing with $v = \omega\sqrt{A^2 - x^2}$, we find that this is a case of simple harmonic motion. Hence only option (a) is correct.

ii. $v = -kx$

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$$\Rightarrow \frac{dx}{dt} = -kx \Rightarrow \int_{x_0}^{x} \frac{dx}{x} = -\int_0^t k\,dt$$

$$\Rightarrow \ln\left(\frac{x}{x_0}\right) = -kt \quad \Rightarrow \quad x = x_0 e^{-kt}$$

At $\quad t = 0, x = x_0$

At $\quad t = \infty, x = 0$

So x decreases with time and hence v decreases in magnitude always. So KE keeps on decreasing. v always remains negative, so the object does not change its direction of motion.

iii. Here spring–block system will execute simple harmonic motion

iv. Given $v = 2\sqrt{\frac{GM_e}{R_e}} > v_{\text{escape}}$. So the object keeps on moving away from the earth. Its speed goes on decreasing and it never changes its direction of motion.

2. **i.→a., d.; ii.→b., c. d.; iii.→d.; iv.→b.**

i. Potential energy is minimum at mean position.

ii. For $a = 0$, $s = vt$ → option (b).

So, $s = s_0 + vt$ → option (c).

For a = constant, $s = ut + \frac{1}{2}at^2$ → option (d).

iii. $R = \frac{v^2 \sin 2\theta}{g}$

$\therefore R \propto v^2$ → option (d).

iv. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore T^2 \propto l$ → option (b).

3. **i. → a., e.; ii. → a., d.; iii. → b., d.; iv. → b., c.**

i. Sound waves are longitudinal waves

$$\frac{\lambda_f}{4} = L \Rightarrow \lambda_f = 4L$$

ii. Sound waves are longitudinal waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

iii. String waves are transverse waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

iv. String waves are transverse waves

$$\frac{2\lambda_f}{2} = L \Rightarrow \lambda_f = L$$

Integer Answer Type

1. **(5)**

$$v = \sqrt{\frac{T}{\mu}} = 10 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{10}{100} = 10 \text{ cm}$$

Distance between the successive nodes = $\lambda/2$ = 5 cm

2. **(7)**

$f_{\text{app}} = f_0 \frac{c+v}{c-v}$, where c is speed of sound

$$\Rightarrow df = \frac{2f_0c}{(c-v)^2}dv \approx \frac{2f_0c}{(c-v)^2}dv = \frac{2f_0}{c}dv \qquad \text{(i)}$$

Given $df = \frac{1.2}{100}f_0$

From Eqs. (i) and (ii) $dv \approx 2$ m/s = 7 km/h.

3. **(5)**

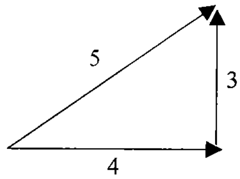

Fig. 8.32

Two waves have phase difference $\pi/2$.

4. **(4)**

We know that $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\gamma A}{mL}}$.

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APPENDIX

Solutions to Concept Application Exercises

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Chapter 1

Exercise 1.1

1. True.
2. True. (Note that the specific latent heat capacity does not depend on mass.)
3. False. The specific heat capacity is the characteristic property of the material of the body and not of the body.
4. Yes, heat can be added to a body without causing the temperature of the body to rise as in the case of latent heat being added to cause a change of state of the body or isothermal process. This does not contradict the concept of heat as 'energy in transit due to temperature difference' because there is a big difference of temperature between the substance and the source from which heat is received by the substance and in this case, the heat supplied increases the intermolecular distance.
5. No, heat cannot be considered to be a form of stored energy. Had it been stored energy it would have been possible to remove all of it.
6. When a 'paddle-wheel' arrangement is dipped in water and the device is worked, the temperature of the water increases without the addition of heat. In such a case, internal energy is increased because some external agent performs work on the system.
7. During a change of state, the latent heat absorbed is equal to the 'difference between the binding energies' of the two states. The difference in binding energies of the gaseous and liquid states is greater than the difference between the binding energies of the solid and liquid states.
8. The external pressure being almost zero, some of the water would evaporate quickly. The evaporating water would take its latent heat of vapourization from the remaining water. Hence, the remaining water would first cool to 0°C and then freeze. Thus, ultimately, a part of the water would evaporate and the rest would freeze.

   This establishes the condition that a liquid cannot exist in an extended vacuum.
9. No, in the process of absorption of latent heat there is no change in temperature though heat is added to a body and absorbed by it.
10. No.

$$m_1s_1(T_1-T)=m_2s_2(T-T_2)$$

$$\frac{T_1-T}{T-T_2}=\frac{m_2s_2}{m_1s_1}$$

If $m_1s_1 \neq m_2s_2$, then, $T_1-T \neq T-T_2$

11. $(m \times 540 + m \times 1 \times 95)$ kcal

$$= (10 \times 10^{-3}) \times 80 + (94 \times 10^{-3}) \times 1 \times 5 \text{ kcal}$$

Hence $m = 2 \times 10^{-3}$ kg or 2 g

12. The final temperature is obviously 0°C.

$$600 \times 10^{-3} \times 0.5 \times 10 + (600-550) \times 10^{-3} \times 80 \text{ kcal} = m \times 0.1 \times 350 \text{ kcal}$$

or $m = 200 \times 10^{-3}$ kg or 200 g

13. The small ice crystal acts as the centre of condensation and so water begins to freeze instantaneously.

$$mL = Mc\,\{0-(-t)\} = Mct$$

or $$m=\frac{Mct}{L}=\frac{1\times4200\times8}{336\times10^3}\text{ kg}=100\text{ g}$$

Now $$ML = Mct \Rightarrow t=\frac{L}{c}=\frac{336\times10^3}{4200}=80°\text{C}$$

14. Energy supplied by heater $= Pt$.

$$= 54 \times 3 \times 60 = 9720 \text{ J}$$

Energy absorbed by water

$$= (650 \times 10^{-3}) \times 4200 \times 3.4 = 9282 \text{ J}$$

Therefore, energy that passes out in the form of radiant heat $= 9720 - 9282 = 438$ J

$\therefore$ Percentage loss $= 438/9720 \times 100 = 4.5\%$

15. Let m kg of water freeze on it.

Then heat lost $= m \times 80 \times 1000$ cal

Heat gained by ice $= 50 \times 10^{-3} \times 500 \times \{0-(-10)\}$ cal

Now,

Heat loss = Heat gain, $m = 3.125\times10^{-3}$ kg $= 3.125$ g

16. Let t be the temperature of the mixture.

Net loss of heat by all

$$= (V\rho_1)\,c_1\,(t_1-t) + (V\rho_2)c_2\,(t_2-t) + (V\rho_3)c_3\,(t_3-t)$$

But net loss $= 0$

$$V\rho_1c_1\,(t_1-t) + V\rho_2c_2\,(t_2-t) + V\rho_3c_3\,(t_3-t) = 0$$

or $$t=\frac{\rho_1c_1t_1+\rho_2c_2t_2+\rho_3c_3t_3}{\rho_1c_1+\rho_2c_2+\rho_3c_3}$$

17. Consider mass m of water falling.

$$mgy = mc\Delta t$$

We express both sides in joules by noting

$$c = 1 \text{ kcal/kg}, K = 4184 \text{ J/kgK}$$

Then $$9.8(122)=4184\Delta t$$

and $$\Delta t = 0.29 \text{ K}$$

18. Let the specific heat capacities of A, B, C be s_1, s_2, s_3.

$$\theta_{A+B}=19=\frac{10s_1+25s_2}{s_1+s_2} \quad \text{or} \quad s_1=\frac{2}{3}s_2$$

$$\theta_{B+c}=35=\frac{25s_2+40s_3}{s_2+s_3} \quad \text{or} \quad s_3=2s_2$$

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$$\theta_{A+B+c} = \frac{10s_1 + 25s_2 + 40s_3}{s_1 + s_2 + s_3}$$

$$= \frac{10 \times \frac{2}{3}s_2 + 25s_2 + 40 \times 2s_2}{\frac{2}{3}s_2 + s_2 + 2s_2} = 30.5°C$$

19. Here water at the surface is evaporated at the cost of the water in the vessel losing heat.

Heat lost by the water in the vessel

$$= (9.5 + 0.5) \times 1000 \times (30 - 20) = 10^5 \text{ cal}$$

Let t be the required time in seconds.

Heat gained by the water at the surface

$$= (t \times 10^{-3}) \times 540 \times 10^3$$

$$(L = 540 \text{ cal/g} = 540 \times 10^3 \text{ cal/kg})$$

$$\therefore \quad 10^5 = 540t \quad \text{or} \quad t = 185\ s = 3 \text{ min } 5 \text{ s}$$

20. In the first interval of 1 min ice is heated to its melting point from its initial temperature which is below the melting point. In the next 4 min there is phase change. In the next 2 minutes there is heating of water

Let Q be the rate of supply of heat in joule per minute

$$\therefore \quad 4Q = mL$$

$$= m \times 336 \times 10^3$$

and $\quad 2Q = m \times 4200 \times \theta$

On dividing $\quad \frac{4Q}{2Q} = \frac{m \times 336 \times 10^3}{m \times 4200 \times \theta}$

$$\theta = \frac{336 \times 10^3}{2 \times 4200} = 40°C$$

21. Heat available on steam (changes into steam to water)

$$= mL = 1 \times 540 = 540 \text{ cal}$$

Heat gained by ice to change into water and then rise its temperature to 100°C

$$= m_{ice} L + m_{wat} c\Delta T$$
$$= 1 \times 80 + 1 \times 1 \times (100 - 0)$$
$$= 180 \text{ cal}$$

The above calculations show that some part of steam will condense to change the ice into water at 100°C. Let m is the mass of steam condensed, then

$$m \times 540 = 180$$

or $\quad m = \frac{80}{540} = \frac{1}{3}$ g

Final contents: ice = 0 g

$$\text{water} = 1 + 1/3 = 4/3 \text{ g}$$

$$\text{steam} = 1 - 1/3 = 2/3 \text{ g}$$

Exercise 1.2

1. No, the change in the volume of a body is always the same whether there is any cavity or not. Suppose we cut off a portion and then place it back in the cavity. On heating the body, the two parts will expand and no gap is created, so the fact that a portion has been cut of f does not affect expansion—the whole body expands, as if it had not been cut.

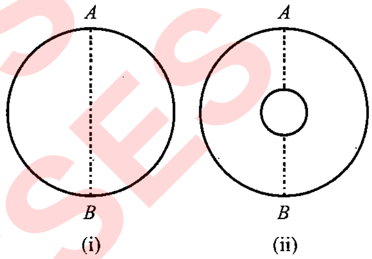

Fig. S-1.1

The distance between A and B remains the same, irrespective of whether matter is (i) continuous from A to B or (ii) discontinuous.

2. Atoms in a solid vibrate longitudinally as well as transversely. The average separation of the planes of atoms increases due to their longitudinal vibrations and decreases due to their transverse vibrations with rise in temperature. Expansion depends on the dominance of one type of vibration over the other. In some crystals, the longitudinal vibrations dominate over the transverse vibrations. Such substances expand on heating. In some other crystals like rubber (consisting of long, interwined and cross-linked chains of atoms in roughly random orientation), transverse vibrations dominate over longitudinal vibrations. Such solids contract on heating.
3. Both diameters will increase. Apply the principle when a body is heated, the distance between any two points must increase.

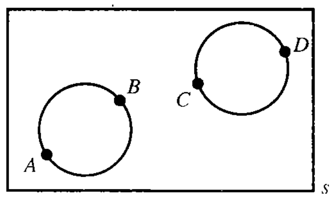

Fig. S-1.2

4. Increase. Remember that on heating, the sheet will expand as a whole. The holes will, therefore, increase in diameter and move outwards. Thus, the distance between them will increase.
5. Increase. Choose two points on the rod on either side of the gap, and apply the above principle.
6. They will show same expansion because expansion of a solid does not depend on its internal construction.
7. $\alpha_{steel} = 12 \times 10^{-6} K^{-1}$ and $\alpha_{brass} = 18 \times 10^{-6} K^{-1}$

Let t be the required common temperature. Then $\Delta T = t - 25$. At the common temperature, both must have the same diameter.

$\therefore \quad D = 3.000(1 + 12 \times 10^{-6}\,\Delta T)$
$= 2.992\,(1 + 18 \times 10^{-6}\,\Delta T)$

$\Rightarrow \quad \Delta T = 448°C \quad \Rightarrow t = 448 + 25$
$= 473°C$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$2\times\frac{86400}{86400+5} = 2\pi\sqrt{\frac{l_{15}}{g}}$$

and $$2\times\frac{86400}{86400-10} = 2\sqrt{\frac{l_{30}}{g}}$$

Dividing $$\sqrt{\frac{l_{30}}{l_{15}}} = \frac{86405}{86390}$$

or $$\sqrt{\frac{l_{15}(1+\alpha\times 15)}{l_{15}}} = 1+\frac{15}{86390}$$

or $$(1+15\alpha)^{\frac{1}{2}} = 1+\frac{15}{86390}$$

or $$1+\frac{1}{2}\times 15\alpha = 1+\frac{15}{86390} \quad (\alpha \text{ is small})$$

or $$\alpha = 23 \times 10^{-6}\,K^{-1}$$

9. Since copper expands more than iron, the length of the iron rod must be great than that of the copper rod. Let their length be l_1 and l_2. Then $l_1 - l_2 = 10$ cm.

Since the difference is always same at all temperatures, their increase in length must be the same whatever the rise in temperature

$\therefore \quad \Delta l = l_1\,\alpha_{Fe}\,\Delta T = l_2\alpha_{Cu}\,\Delta T$

or $$\frac{l_1}{l_2} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11}$$

Also $l_1 - l_2 = 10$ cm

Solving for l_1 and l_2

$l_1 = 28.3$ cm $\quad l_2 = 18.3$ cm

10. $\Delta l = l\alpha\,\Delta T$ or $0.075 = 30 \times \alpha_A \times 100$
or $\alpha_A = 25 \times 10^{-6}\,°C^{-1}$
and $0.045 = 45 \times \alpha_A \times 100$
or $\alpha_B = 10 \times 10^{-6}\,°C^{-1}$

Let l_1 and l_2 be the lengths of the two sections of C.

Then $l_1 + l_2 = 45$ and

$0.04 = l_1 \times 25 \times 10^{-6} \times 50 + l_2 \times 10 \times 10^{-6} \times 50$
or $160 = 5l_1 + 2l_2$
solving $l_1 = 23.3$ cm
$l_2 = 21.7$ cm

11. Since the scale is graduated at 10°C,
1 cm of the scale at 10°C
= exactly 1 cm
$\therefore$ 1 cm of the scale at 30°C
$= \text{exactly } (1 + 18 \times 10^{-6} \times 20)$ cm

$\therefore$ 60 cm of the scale at 30°C
$= \text{exactly } 60.00\,(1 + 36 \times 10^{-5})$
= 60.02 cm

12.
$$\left(\gamma_{mercury} = 18.2\times 10^{-5}K^{-1} \quad \alpha_{glass} = 9\times 10^{-6}K^{-1}\right)$$

Let V_0, V_t = volume of mercury at 0°C and t°C, respectively. A_0, A_t = area of cross-section of capillary tube at 0°C and t°C, respectively.

Then

$v_0 = l_0A_0 \; V_t = l_tA_t \quad V_t = V_0(1 + \gamma_r t)$
$\therefore \quad l_tA_t = l_0A_0(1 + \gamma_r t)$
or $\quad l_tA_0(1 + 2\alpha_g t) = l_0A_0(1 + \gamma_r t)$

or $$l_t = l_0\frac{1+\gamma_r t}{1+2\alpha_g t}$$

expanding and neglecting negligible terms

$l_t = l_0\,[1+(\gamma_r - 2\alpha_g)\,t]$
$\therefore \quad l_{100} = 1\,[1 + (182 \times 10^{-6} - 2 \times 9 \times 10^{-6})\,100]$
$= 1.0164$ m

Let L = required reading of thread, on glass scale, at temperature t. Then this section of the glass (scale) has length L at 0°C and If at t°C.

$It = L\,(1+\alpha g t)$

or $$L = \frac{l_0\left[1+(\gamma_r - 2\alpha_g)t\right]}{1+\alpha_g t}$$

$= l_0\,[1 + (\gamma_r - 30_g)\,t]$
or $L = 1[1 + (182 \times 10^{-6} - 3 \times 9 \times 10^{-6})\,100]$
$= 1.0155$ m

13. Apparent expansion of mercury

$$= \frac{\pi}{4}(6 \times 10^{-4})^2 \times 10 \times 10^{-2}$$

$= 9\pi \times 10^{-9}\,m^3$

Volume of mercury $= \dfrac{43\times 10^{-3}}{13.6\times 1000}$

$= 3.16 \times 10^{-6}\,m^3$

$\therefore\; 9\pi \times 10^{-9} = 3.16 \times 10^{-6} \times \gamma_a \times 50$
$(\Delta V = V\gamma \times \Delta T)$

or $\quad \gamma_a = 18 \times 10^{-5}$
$\gamma_r = \gamma_a + \gamma_g = \gamma_a + 3\alpha_g\;(\gamma_g = 3\alpha_g)$
$\therefore \quad \gamma_r = 18 \times 10^{-5} + 3 \times 9 \times 10^{-6}$
$= 20.7 \times 10^{-5}\,K^{-1}$

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14. By the law of floatation

$$266.5 = \frac{4\pi}{3}\left(\frac{7}{2}\right)^3 \times \rho_{35}$$

where ρ_{35} = density of the liquid at 35°C

$$\rho_{35} = 1.4839 \text{ g/cm}^3$$

But $\rho_0 = \rho_{35}(1 + \gamma \times 35)$

$\therefore$ $1.527 = 1.4839\,(1 + 35\gamma) = 8.3 \times 10^{-4}\,°\text{C}^{-1}$

15. Let V be the required volume.
Apparent increase in volume $= v \times \gamma_a \times 100 \text{ cm}^3$

$$= \pi\,(0.25 \times 10^{-1})^2 \times 20 \text{ cm}^3$$

or $V \times (0.00018 - 0.000027) \times 100$

$$= \pi \times 625 \times 10^{-6} \times 20$$

or $$V = \frac{\pi \times 625 \times 10^{-6} \times 20}{1.53 \times 10^{-4}} = 2.57 \text{ cm}^3$$

16. Apparent increase in volume

$$= V \times (18 \times 10^{-5} - 27 \times 10^{-6}) \times 100$$

Also apparent increase $= 15 \times 10^{-4} \times 30$

$\therefore$ $15 \times 10^{-4} \times 30$

$$= V \times (18 \times 10^{-5} - 27\ 10^{-6}) \times 100$$

or $V = 2.94$ cc

17. Since the scale is correct at 0°C

1 cm at 0°C exactly 1 cm

$\therefore$ 1 cm at t°C

$$= \text{exactly } (1 + \alpha t) \text{ cm}$$

$\therefore$ H_1 cm at t°C $= H_1\,(1 + \alpha t)$ cm

Since pressure is the same, namely, the atmospheric pressure

$$\rho_0 g\, H_0 = \rho_t g H_1\,(1 + \alpha t)$$

or $\rho_1\,(1 + \gamma t)\, H_0 = H_t \rho_1\,(1 + \alpha)$

or $H_0 \approx H_1\,(1 + \alpha)\,(1 - \gamma t)$

18. Volume of the space above the mercury will remain constant if increase in volume of the vessel is equal to the increase in the volume of the mercury for any increase in temperature. Let V' be the volume of mercury and V the volume of the vessel.

Then $V \times 27 \times 10^{-6} \times \Delta T = V' \times 1.8 \times 10^{-4}\,\Delta T$

or $$\frac{V'}{V} = \frac{27 \times 10^{-6}}{1.8 \times 10^{-4}} = \frac{3}{20}$$

19. $\Delta l = l_s\,\alpha_s\,\Delta T + l_c\,\alpha_c \Delta T$

$$0.002 = [1.5\,\alpha_s + 0.5 \times 1.6 \times 10{-}5] \times 100$$

$$\alpha_s = \frac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}\,°\text{C}$$

there is no change in component length

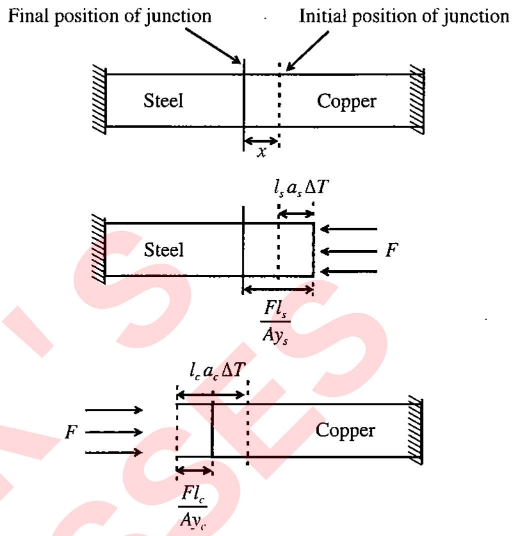

Fig. S-1.3

For steel,

$$x = l_s\,\alpha_s\,\Delta T - \frac{Fl_s}{AY_s} = 0$$

$$\frac{F}{AY_s} = \alpha_s \Delta T \qquad \text{(i)}$$

For copper,

$$x = \frac{Fl_c}{Ay_c} - l_c\,\alpha_c\,\Delta T = 0$$

$$\frac{F}{Ay_c} = \alpha\ \Delta T \qquad \text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{y_s}{y_c} = \frac{\alpha_c}{\alpha_s}$$

$$y_s = y_c \times \frac{\alpha_c}{\alpha_s} = \frac{1.5 \times 10^{13} \times 1.6 \times 10^{-5}}{8 \times 10^{-6}}$$

$$y_s = 3 \times 10^{13} \text{ N/m}^2$$

Exercise 1.3

1. Lakes first freeze on the surface because water has the maximum density at 4°C and the densest part sinks to the bottom. So the water at the bottom of the lake remains at 4°C and that on top is at 0°C, where it freezes by losing heat to the environment.
2. The constant k depends on materials, shape, nature of the surface, and air currents. Its dimensions are T^{-1}.
3. Area of six faces of the box $= 6l^2 = 6 \times (0.30)^2 = 0.54\text{m}^2$

and $L = 5.0 \text{ cm} = 0.05 \text{ m}$

time, $t = 6 \text{ hr} = 6 \times 3600 \text{ s}$

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$$T_1 - T_2 = 45 - 0 = 45°\text{C}$$

Total heat entering into the box in 6 hr

$$Q = \frac{KA(T_1 - T_2)}{L}$$

$$= \frac{0.01 \times 0.54 \times 45 \times 6 \times 3600}{0.05} = 104976 \text{ J}$$

If m be the amount of ice melted, then

$$Q = mL$$

or
$$m = \frac{Q}{L} = \frac{104976}{335 \times 10^3} = 0.313 \text{ kg}$$

Mass of the ice left after 6 h = (4 – 0.313) kg

= 3.687 kg

4. Let T_1 be the temperature of the part of the flame in contact with boiler. The amount of heat flows into water in 1 min

$$Q = KA\frac{T_1 - T_2}{L}t$$

$$= \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} \text{J}$$

Mass of the water boiled in 1 min = 6 kg = 6000 g
Heat required to boil the water $Q = mL = 6000 \times 2256$ J

$$\therefore \quad \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} = 6000 \times 2256$$

or $T_1 - 100 = 138$

or $T_1 = 238°\text{C}$

5 Equivalent thermal conductivity of the wall

$$K = \frac{l_1 + l_2 + l_3}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}}$$

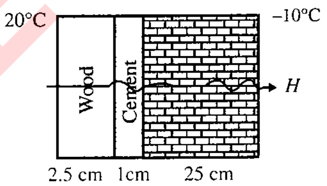

Fig. S-1.4

$$= \frac{0.025 + 0.01 + 0.25}{\left(\frac{0.025}{0.125} + \frac{0.01}{1.5} + \frac{0.25}{1.0}\right)}$$

$$= \frac{0.285}{0.457} = 0.624 \text{ W/m°C}$$

The rate of flow of heat is given by

$$H = KA\frac{T_1 - T_2}{L}$$

$$= 0.624 \times 137 \times \frac{[20 - (-10)]}{0.285}$$

$$= \frac{0.624 \times 137 \times 30}{0.285} = 9000 \text{ W}$$

6. If T is the surface temperature of the filament, then

$$\lambda_m T = b$$

or
$$T = \frac{b}{\lambda_m} = \frac{0.288}{2.16 \times 10^{-5}} = 13333.3 \text{ K}$$

The temperature of surrounding air, $T_0 = 13 + 273 = 286$ K

The net amount of energy radiated per unit area per second

$$E = \sigma(T^4 - T_0^4)$$

$$= 5.77 \times 10^{-5}\left[(13333.3)^4 - (286)^4\right]$$

$$= 1.824 \times 10^{12} \text{ erg s}^{-1} \text{ cm}^{-2}$$

7. Let θ_1 and θ_2 be the temperatures at the ends of the copper bar.

Heat transfer per second through the system is

$$\frac{dH}{dt} = \frac{A(100 - 0)}{\left(\frac{0.001}{K_w} + \frac{100}{K_{Cu}} + \frac{0.01}{K_w}\right)}$$

Heat transfer per second through copper bar

$$= \frac{K_{Cu}A(\theta_1 - \theta_2)}{100}$$

As the rods are in series, heat transfer per second must be same through each part.

$$\frac{A(100 - 0)}{\left(\frac{0.01}{K_W} + \frac{100}{K_{Cu}} + \frac{0.01}{K_W}\right)} = \frac{K_{Cu}A(\theta_1 - \theta_2)}{100}$$

Putting $K_{Cu} = 1.04$ and $K_W = 0.0014$ we get

$$\text{Temperature gradient} = \frac{(\theta_1 - \theta_2)}{100}$$

$$= 0.87°\text{C cm}^{-1}$$

8. i. Thermal resistance is defined as $R = l/kA$

For rod A $\quad R_A = \frac{l}{k_A A} = \frac{l}{3kA}$

For rod B $\quad R_B = \frac{l}{k_B A} = \frac{l}{kA}$

ii. Using electrical analogy, the given problem may be modified as

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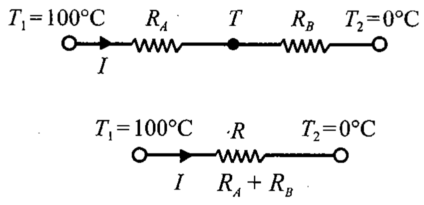

Fig. S-1.5

The equivalent resistance is

$$R = R_A + R_B = \frac{4}{3}\frac{l}{kA}$$

The heat current is given by

$$I = \frac{T_1 - T_2}{R} = \frac{100-0}{\frac{4}{3}\frac{l}{kA}} = 73\left(\frac{kA}{l}\right)$$

iii. The temperature of the junction is given by

$$T = T_1 - IR_A = 100 - \frac{75kA}{l}\left(\frac{l}{3kA}\right) = 75°C$$

iv. The variation of temperature along the length of the conductor is shown in Fig. S-1.6.

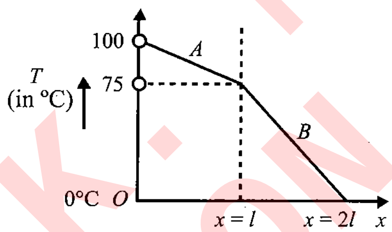

Fig. S-1.6

9. i. The thermal resistance of the two rods are

$$R_A = \frac{l}{3kA}; \quad R_B = \frac{l}{kA}$$

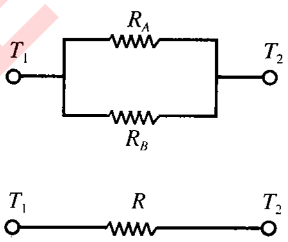

Fig. S-1.7

The equivalent resistance is

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{3kA}{l} + \frac{kA}{l} = \frac{4kA}{l}$$

or
$$R = \frac{l}{4kA}$$

ii. The heat current through each rod is

$$I_A = \frac{T_1 - T_2}{R_A} = \frac{100-0}{l/3kA} = 300\left(\frac{kA}{l}\right)$$

$$I_B = \frac{T_1 - T_2}{R_B} = \frac{100-0}{l/kA} = 100\left(\frac{kA}{l}\right)$$

Total heat current is

$$I = I_A + I_B = 400\frac{kA}{l}$$

10. According to Stefan's law, the rate of cooling of a body is given by

$$\frac{-dT}{dt} = \frac{e\sigma A}{mc}(T^4 - T_0^4)$$

Here e, σ, m, c, T and T_0 are constants for all the three objects, therefore

$$\frac{-dT}{dt} \propto A$$

Since the surface area of the plate is maximum and that of the sphere is minimum for the given mass, therefore, the circular plate cools the fastest, and the sphere cools the slowest.

11. Let us find the amount of heat absorbed by ice per minute,

$A = 1 \times 10^{-4}$ m^2; $\Delta t = 60$ s; $K = 110$ W/m K;
$L = 2$ m; $-\Delta T = 100 - 0 = 100°C$

Using $\Delta Q = KA\Delta T\Delta t/L$ we have

$$\Delta Q = (110)(1\times10^{-4})\frac{(100)(60)}{2} = 33 \text{ J}$$

Now, if m be the mass of ice melting per minute, then, from $\Delta L = mL$,

$$m = \frac{\Delta Q}{L} = \frac{33 \text{ J}}{80\times 4.2 \text{ J/g}} = 0.098 \text{ g}$$

12. Let the thermal conductivities of the rod AB, BC and BD be K, $2K$, and $3K$ respectively. Also, let the length be $2L$, L and L.

If T be the required temperature of the junction B and assuming $T_1 > T > T_2$ and $T > T_3$, we have

$$\left.\frac{\Delta Q}{\Delta t}\right]_{AB} = \left.\frac{\Delta Q}{\Delta t}\right]_{BC} + \left.\frac{\Delta Q}{\Delta t}\right]_{BD} \quad \text{(Point rule)}$$

$$\frac{KA(T_1 - T)}{2L} = \frac{2KA(T - T_2)}{L} + \frac{3KA(T - T_3)}{L}$$

$$\frac{T_1 - T}{2} = 2(T - T_2) + 3(T - T_3)$$

$$T = \frac{1}{11}(T_1 + 4T_2 + 6T_3)$$

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13. Here $A_1 = \pi R^2$; $K_1 = K$

And $A_2 = 4\pi R^2 - \pi R^2 = 3\pi R^2$; $K_2 = 2K$

Using, the expression for the effective thermal conductivity for slabs in parallel,

$$K_{eff} = \frac{A_1K_1 + A_2K_2}{A_1 + A_2}$$

$$= \frac{\pi R^2(K) + 3\pi R^2(2K)}{\pi R^2 + 3\pi R^2} = \frac{7K}{4}$$

14. Let us subdivide the entire cylinder into a number of coaxial cylindrical shells of infinitesimally small thickness. Consider one such shell of radius x and thickness dx. Cross-sectional area of the shell $2\pi x dx$.

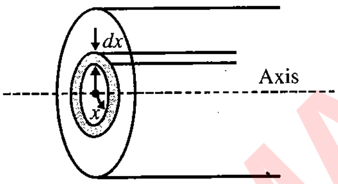

Fig. S-1.8

Using, the expression for the effective thermal conductivity

$$K_{eff} = \frac{\sum A_i K_i}{\sum A_i}$$

We have $$K_{eff} = \frac{1}{\sum A_i}\int_0^R K(2\pi x dx)$$

$$= \frac{1}{\pi R^2}\int_0^R (K_1 x + K_2)2\pi x dx$$

$$= \frac{2}{R^2}\int_0^R (K_1 x^2 + K_2 x)dx$$

$$= \frac{2}{R^2}\left[K_1\frac{x^3}{3} + K_2\frac{x^2}{2}\right]_0^R = \frac{1}{3}(2K_1R + 3K_2)$$

15. From equation, the rate of cooling of a body is given by

$$\frac{-dT}{dt} = \frac{Ae\sigma}{\rho Vs}(T^4 - T_0^4)$$

Since, substance is same for both bodies, so $e/\rho s$ = constant.

Finally, they are allowed to cool under identical conditions, so $(T^4 - T_0^4)$ = constant.

$$\frac{-dT}{dt} \propto \frac{A}{V}$$

Let the edge of the cube or radius of the sphere be a, then, for cube; $A = 6a^2$ and $V = a^3$ so $A/V = 6a$

For the sphere; $A = 4\pi a^2$ and $V = (4/3)\pi a^3$; so $A/V = 3/a$

Evidently, the ratio A/V is more for cube, so, the cube cools at a faster rate. Note the special technique used in this problem.

16. It is given that the temperature of ball is constant. This means that the rate at which it loosing heat by radiation must be equal to the rate at which heat is supplied to this ball externally to keep its temperature constant.

The rate of heat loss by the ball is given as

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_s^4)$$

$= 0.3 \times 4\pi\ (0.01)^2 \times 5.67 \times 10^{-8} \times [(1000)^4 - (300)^4]$
$= 21.1$ W

Thus electrical energy must be supplied to the ball at a rate of 21.1 W.

17. If E_1 and E_2 are the maximum energy radiated by the body in a short wavelength interval at temperature T_1 and T_2 then according to Wein's fifth power law, we have

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^5 \qquad \text{(i)}$$

According to Wein's displacement law, we have

$$\lambda_{m_1}T_1 = \lambda_{m_2}T_2$$

or $$\frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \qquad \text{(ii)}$$

Here λ_{m_1} and λ_{m_2} are the wavelengths at temperatures T_1 and T_2 at which the energy radiated in a small interval of wavelength is maximum

Now from Eqs. (i) and (ii) we have

$$\frac{E_1}{E_2} = \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^5 \quad \text{or} \quad \frac{E_1}{E_2} = \left(\frac{234\times10^{-10}}{4253\times10^{-10}}\right) = 0.0506$$

18. According to Wein's displacement law, we have

$$\lambda_m T = \text{constant}$$

or $$\lambda_{m_1}T_1 = \lambda_{m_2}T_2$$

Thus $20000 \times 10^{-10} \times 1500 = 5500 \times 10^{-10} \times T_2$

or $$T_2 = \frac{200}{55}\times 1500 = 5454.54 \text{ K}$$

Chapter 2

Exercise 2.1

1. True, because average kinetic energy per molecule $= (3/2)kT$.
2. No work is done by the gas and so there is no change in its internal energy which is entirely kinetic. Hence, there is no change in random motion.
3. $P = nkT \Rightarrow n = \dfrac{p}{kT} = \dfrac{10^{-11}\times13.6\times1000\times9.8}{1.38\times10^{-23}\times273}$

   $= 3.5 \times 10^{14}$
4. $$\frac{1}{2}Mv^2 = C_v\Delta T = \frac{R}{\gamma - 1}\Delta T$$

$$\left(C_v = \frac{R}{\gamma - 1}\right) \Rightarrow \Delta T = \frac{(\gamma-1)Mv^2}{2R}$$

Now $pV = RT \Rightarrow p = \frac{RT}{V} \Rightarrow \Delta p = \frac{R}{V}\Delta T$

$$\therefore \quad \frac{\Delta p}{p} = \frac{\left(\frac{7}{5}-1\right)\times 28\times10^{-3}\times100^2}{2\times8.3\times300}$$

$= 0.0225 = 2.25\%$

5. $C_v = \frac{R}{\gamma-1}$

and $C_v = Mc_v$

$\therefore \quad M\times650 = \frac{8.3}{\gamma-1}$ (i)

Further

$\gamma = C_p/C_v = \frac{910}{650}$ (ii)

Solving Eqs. (i) and (ii), $M = 32$ g/mol
$\gamma = 1.4$, so the gas is diatomic and the number of degrees of freedom is 5.

6. $p = \frac{1}{3}\rho v_{\text{rms}}^2 \Rightarrow v_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$

$$\therefore \quad v_{\text{rms}} = \sqrt{\frac{3\times1.00\times10^{-2}\times1.013\times10^5}{1.24\times10^{-2}}}$$

$= 495$ m/s

Again, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

or $M = \frac{3RT}{v_{\text{rms}}^2}$

$\therefore \quad M = \frac{3\times8.3\times400}{(495)^2} = 40.6$ g/mol

This is the molecular weight of argon and so the gas is argon.

7. $C_v = \frac{R}{\gamma-1} = Mc_v$

$\therefore M \times (0.075\times1000)\times42 = \frac{8.3}{\frac{5}{3}-1}$

$\Rightarrow \quad M = 39.65$ g

$\therefore$ Mass of one argon atom

$$m = \frac{M}{N_A} = \frac{39.65}{6\times10^{23}}$$

$\Rightarrow \quad m = 6.58\times10^{-23}$ g

8. U (total energy per mole) $= \frac{RT}{\gamma-1} = \frac{8.3\times293}{\frac{7}{5}-1} = 6079.8$ J

$\therefore E_t$ (transtational energy) $= \frac{3}{5}\times6079.8 = 3647.9$ J

Translation energy per molecule $= \frac{3647.9}{6.0\times10^{23}} = 6.1\times10^{-21}$ J

$$\Delta U = C_v\Delta T = \frac{R}{\gamma-1}\Delta T = \frac{8.3}{\frac{7}{5}-1} = 20.8 \text{ J}$$

9. Number of molecules in 1 g of water $= \frac{6.0\times10^{23}}{18}$

$\therefore$ Number of molecules per unit area

$$= \frac{6.0\times10^{23}}{18\times4\pi(6.4\times10^6)^2}$$

$= 6.6\times10^8$

10. $v = \sqrt{\frac{\gamma p}{\rho}} \Rightarrow \gamma = \frac{v^2\rho}{p}$

If r = degrees of freedom of rotational motion then $\gamma = 1 + \frac{2}{r}$

$$\therefore r = \frac{2}{\frac{v^2\rho}{p}-1} = \frac{2}{\frac{330^2\times1.3}{1.015\times10^5}-1} = 5$$

11. The degrees of freedom of rotational motion = 2

$\therefore \langle\varepsilon\rangle_{\text{rot}} = 2\times\frac{1}{2}kT$ (by equipartition principle)

$$= \left\langle 2\times\frac{1}{2}I\omega^2\right\rangle$$

$\Rightarrow kT = I\langle\omega^2\rangle \Rightarrow \omega_{\text{rms}} = \sqrt{\frac{kT}{I}}$

Therefore, $\omega_{\text{rms}} = \sqrt{\frac{1.38\times10^{-23}\times300}{2\times10^{-40}}} = 4.5\times10^9$ rad/s

12. True. The molar specific heat capacity is given by

$$C = \frac{R}{\gamma-1}$$

R is a universal constant and γ is the same for all monoatomic gases, and hence the molar specific heat capacity of all monoatomic gases is the same.

13. The volume of γ of the mixture is given by

$$\frac{n_1+n_2}{\gamma-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}$$

$$\therefore \quad \frac{1+1}{\gamma-1}=\frac{1}{\frac{5}{3}-1}+\frac{1}{\frac{7}{5}-1} \quad \text{or} \quad \gamma = 1.5$$

In general, $$C_v=\frac{R}{\gamma-1}$$

Therefore, $$C_v=\frac{R}{1.5-1}=2R$$

Hence, the specific heat of the mixture is $2R$.

14. The plot is shown in Fig. S-2.1.

Explanation: Since from $1-2$, V remains constant, the plot of T against V is a straight line parallel to the T-axis. In the process $2-3$, pressure remains constant.

For an ideal gas $pV = RT$. So when p remains constant, $V \propto T$. Hence the plot of T against V is a straight line passing through the origin. The same is repeated from $3-4$ and $4-1$. The plot of p against T is shown in the figure and can be explained similarly.

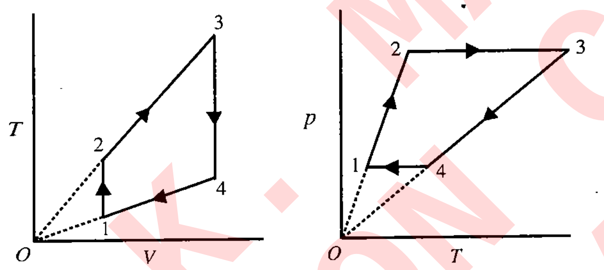

Fig. S-2.1

15. The plot p against V is shown in Fig. S-2.2.

Explanation: Since from $1-2$ temperature remains constant, the plot p against V is a curve (hyperbola) because pV = constant. From $2-3$ volume remains constant, so the plot of p against V is parallel to the p-axis.

The plot of p against T is shown Fig. S-2.2.

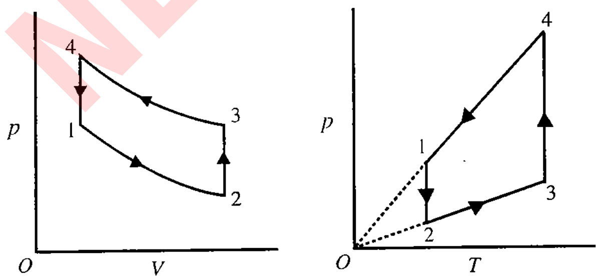

Fig. S-2.2

It can be explained in the same way.

16. The γ of a mixture is given by

$$\frac{n_1+n_2}{\gamma-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$$

Here $n_1 = 3, n_2 = 2, \gamma_1 = 7/5$ (as nitrogen is diatomic)

and $\gamma_2 = 4/3$ (as carbon dioxide is triatomic)

Therefore $$\frac{5}{\gamma-1}=\frac{3}{\frac{7}{5}-1}+\frac{2}{\frac{4}{3}-1} \Rightarrow \gamma=1.37$$

17. Let m be the mass of the neon gas, then mass of argon will be $28 - m$.

Number of moles of neon, $n_1=\frac{m}{20}$

Number of moles of argon, $n_2=\frac{28-m}{40}$

Now by ideal gas equation, we have

$$PV = nRT$$

Here $$n=n_1+n_2=\frac{m}{20}+\frac{28-m}{40},$$

$$P = 1.0 \times 10^5 \text{ N/m}^2$$

$$T = 273 + 27 = 300 \text{ K}, V = 0.02 \text{ m}^3$$

$$\therefore \quad (1.0\times10^5)\times0.02=\left[\frac{m}{20}+\frac{(28-m)}{40}\right]\times8.314\times300$$

After solving, we get $m = 4.07$ g

$\therefore$ Mass of neon gas $= 4.07$ g

Mass of argon gas $= 28 - m$

$= 28 - 4.07 = 23.93$ g

18. As number of moles in the two vessels are equal, we have

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

It is given that $V_1 = V_0$, $V_2 = 2V_0$, $T_1 = 300$ K and $T_2 = 600$ K,

Thus we have $$\frac{P_1V_0}{300}=\frac{P_2(2V_0)}{600}$$

or $$P_1 = P_2$$

or $$\frac{P_1}{P_2}=1$$

19. From gas law, we have for a constant volume container

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}$$

or $$T_2=\left(\frac{P_2}{P_1}\right)T_1$$

Given that $P_1 = 8 \times 10^5$ Pa, $T_1 = 300$ K, $P_2 = 10^6$ Pa, then we have

$$T_2=\frac{10^6}{8\times10^5}\times300 = 375 \text{ K}$$

20. The pressure exerted by a gas is given by

$$P=\frac{2}{3}\text{(KE per unit volume)}$$

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or $$\text{KE} = \frac{3}{2}P$$

Here P = 10 mm of mercury = 1 cm of mercury
$= 1 \times 13.6 \times 980 = 1.33 \times 10^4$ dynes/cm^2

Thus, we have

$$\text{KE} = \frac{3}{2} \times 1.33 \times 10^4 \times 1 = 1.99 \times 10^4 \text{ ergs}$$

As mean kinetic energy per molecule is 4×10^{-14} ergs, the number of molecule will be

$$\frac{1.99\times10^4}{4\times10^{-14}} = 4.9 \times 10^{17} \approx 5 \times 10^{17} \text{ molecules}$$

Exercise 2.2

1. In an adiabatic expansion, a gas does work at the cost of its kinetic energy and so its random motion is reduced. According to the kinetic theory the temperature of a gas is associated with its random motion. This is why the temperature of a gas drops in an adiabatic expansion.
2. On sudden expansion, no heat is supplied to the system. So the gas does work at the cost of its internal energy which is entirely kinetic. Thus there is decrease in temperature.
3. $C = \sqrt{\frac{3RT}{M}}$

Let the rms speed be double at T' then

$$2C = \sqrt{\frac{3RT'}{M}}$$

$$\therefore \quad 4 = \frac{T'}{T} \Rightarrow T' = 4T = 4\times300 = 1200 \text{ K}$$

$$\therefore \quad \Delta T = 1200 - 300 = 900 \text{ K}$$

Here $\Delta Q = \Delta U$ ($\because \Delta W = 0$)

Therefore,

$$\Delta Q = \frac{m}{M}C_v\Delta T = \frac{m}{M}\times\frac{R}{\gamma-1}\times\Delta T$$
$$= \frac{15}{28}\times\frac{8.3}{\frac{7}{5}-1}\times900$$
$$= 10004 \text{ J} \approx 10 \text{ kJ}$$

4. In an adiabatic expansion, $TV^{\gamma-1}$ = constant

Therefore,

$$T_0V^{\gamma-1} = T\left(\frac{V}{\eta}\right)^{\gamma-1} \Rightarrow T = T_0\eta^{\gamma-1} = T_0\eta^{2/r} \quad \left(\because \gamma = 1+\frac{2}{r}\right)$$

$$\langle\varepsilon\rangle_{\text{rot}} = kT = kT_0\eta^{2/r}$$

$$\Rightarrow \langle\varepsilon\rangle_{\text{rot}} = 1.38\times10^{-23}\times273\times5^{2/5}$$
$$= 7\times10^{-21} \text{ J}$$

5. No, because the only external manifestation is a rise in temperature in both cases. Always keep in mind that internal energy is a state function. Any change in internal energy depends only on initial and final state; not on the path followed.
6. **a.** Yes, the temperature of the coffee increases.
   b. No, heat has not been added to the coffee as it is thermally insulated.
   c. Yes, work is done on it by the man who shakes the flask.
7. Work is done by a gas when it expands adiabatically. The source of energy is its own internal energy.
8. This process is irreversible because the same amount of mechanical energy cannot be regained from heat. It follows from the second law of thermodynamics which states that all the heat can never be converted to useful work.
9. In an adiabatic expansion $\Delta Q = 0$. But $\Delta Q = \Delta U + \Delta W$ by the first law of thermodynamics. Therefore, in an adiabatic expansion,

$$\Delta W = -\Delta U$$

Thus, work done by a gas in an adiabatic expansion is associated with a decrease of internal energy, so its temperature decreases.

10. In an isochoric process where $\Delta W = 0$ and, therefore, $\Delta Q = \Delta U$ by the first law of thermodynamics.
11. Yes, since the bulk modulus is γP, an increase of P makes the medium more elastic.
12. For oxygen C_p = 7.03 cal/mol-°C.
$\Delta Q = nC_p\Delta T = 3 \times 7.03 \times (127 - 27) = 2109$ cal (n = 3)
Since $\gamma = C_p/C_v$ and $\gamma = 1.4$ for oxygen as it is diatomic,

$$\Delta U = nC_v\Delta T = 3\times\frac{7.03}{1.4}\times100 = \frac{2109}{1.4} \text{ cal} = 1506 \text{ cal}$$

By the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \Delta W = \Delta Q - \Delta U = 2109 - 1506 = 603 \text{ cal}$$
$$= 603 \times 4.184 = 2523 \text{ J}$$

a. Work done by the gas = 2523 J
b. Change in internal energy = 1506 cal
c. Heat added to the gas = 2109 cal

13. For isobaric process,

$$\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{\gamma}$$

Given,

$$\Delta W = 2 \text{ J}$$

$$\gamma = \frac{7}{5} \text{ for } N_2$$

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$\therefore \quad \frac{2}{\Delta Q} = \frac{2}{7}$

$\therefore \quad \Delta Q = 7$ J

14. By the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$
For 1 mol of an ideal gas $\Delta Q = C_p \Delta T$ and $\Delta U = C_v \Delta T$

$\therefore \Delta W = (C_p - C_v)\,\Delta T = R\Delta T \qquad (\because C_p - C_v = R)$

$= 8.3 \times 72 = -597.6$ J

$\Delta U = \Delta Q - \Delta W = 1.6 \times 1000 - 597.6 = 1002.4$ J

$$\Delta Q = C_p \Delta T$$

$$C_p = \frac{1.6 \times 1000}{72} = 22.2 \ \text{J/kg}^{-k}$$

$$\Delta U = C_v\,\Delta T$$

or $\quad C_v = \frac{\Delta U}{\Delta T} = \frac{1002.4}{72} = 13.9 \ \text{J/kg}^{-k}$

$\therefore \quad \gamma = \frac{C_p}{C_v} = \frac{22.2}{13.9} = 1.6$

15. Yes, because the area between curves 1 and the V-axis is greater than that between curve 2 and the V-axis and the work done is given by the area between the p–V curve and the V-axis.

16. ΔW (work done)

$$= \int p\Delta V = p\frac{m}{M}\frac{R\Delta T}{p} = \frac{m}{M}R\Delta T$$

$\therefore \quad \Delta W = \frac{20}{28}8.3(250-17) = 1381$ J

$$\Delta U = \frac{m}{M}C_v\Delta T = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$$

$\Rightarrow \quad \Delta U = \frac{20}{28} \times \frac{8.3}{\frac{7}{5}-1} \times (250-17) = 3453$ J

$\therefore \quad \Delta Q = \Delta W + \Delta U = 1381 + 3453 = 4834$ J

ΔW is also equal to mgh (see the above problem).

$\therefore \quad 1381 = 75 \times 9.8 \times h$

or $\quad h = 1.88$ m

17. $\Delta W = \int_V^{2V} pdV = p_0 V = RT_0$,

$$\Delta U = \int C_v dT = \int_V^{2V} \frac{R}{\gamma-1} dT$$

At constant pressure fixed at

$p_0 dV = RdT \qquad (\because pV = RT)$

$\therefore \quad \Delta U = \frac{1}{\gamma-1}\int_V^{2V} p_0 dV = \frac{p_0 V}{\gamma-1} = \frac{RT_0}{\gamma-1}$

$\therefore \quad \Delta Q = \Delta U + \Delta W = \frac{RT_0}{\gamma-1} + RT_0 = \frac{\gamma RT_0}{\gamma-1}$

Since oxygen is diatomic, $\gamma = 1.4$

$\therefore \quad \Delta Q = \frac{1.4 \times 8.3 \times 273}{1.4-1} = 7931$ J

18. We have $\Delta U = \int_{T_1}^{T_2} C_v dT = C_v(T_2 - T_1)$

Since the process is adiabatic, $dQ = 0$

$$\Delta Q = \Delta U + \Delta W$$

$\therefore \quad \Delta W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$

19. i. $\Delta U = \int_T^{T+\Delta T} \frac{m}{M} C_v dT = \int_T^{T+\Delta T} \frac{m}{M}\frac{R}{\gamma-1} dT \quad \left(\because C_v = \frac{R}{\gamma-1}\right)$

$\Rightarrow \quad \Delta U = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$

Similarly, $\quad \Delta Q = \frac{m}{M} C\Delta T$

where C is the molar heat capacity in the process.

It is given that $\Delta Q = -\Delta U$

ii. $\quad dQ = dU + dW \quad \Rightarrow 2dQ = dW$

$(\because dQ = -dU$ given)

$\Rightarrow \quad 2CdT = pdV \qquad (\because dQ = CdT)$

$\Rightarrow \quad -\frac{2R}{\gamma-1}dT = pdV$

$\Rightarrow \quad \frac{2R}{\gamma-1}dT + pdV = 0$

$\Rightarrow \quad \frac{2R}{\gamma-1}dT + \frac{RT}{V}dV = 0$

$\Rightarrow \quad \frac{dT}{T} + \frac{\gamma-1}{2}\frac{dV}{V} = 0$

Integrating, $\quad TV^{(\gamma-1)/2} = \text{constant}$

20.

$$\Delta U = \int_T^{T+\Delta T} C_v dT = \frac{R}{\gamma-1}\Delta T = \frac{8.3}{\frac{3}{5}-1}(-26) = -324 \text{ J}$$

and $\quad \Delta W = \int_T^{T+\Delta T} pdV$

Here $\quad pV^{1.5} = \text{constant};\ pV = RT$ (always)

$\therefore \quad V^{0.5} \propto \frac{1}{T} \quad$ or $\quad V \propto \frac{1}{T^2}$

or $\quad V = \frac{a}{T^2}$, where a is a positive constant.

$$\therefore \quad p = \frac{RT}{V} = \frac{RT}{a/T^2} = \frac{R}{a}T^3$$

$$\therefore \quad \Delta W = \int_T^{T+\Delta T} \frac{R}{a}T^3\left(-\frac{2a}{T^3}dT\right) = -\int_T^{T+\Delta T} 2RdT$$

$\Rightarrow \quad \Delta W = -2R\,\Delta T = -2 \times 8.3 \times (-26) = +432$ J

$\Delta Q = \Delta U + \Delta W = -324 + 432 = 108$ J

Chapter 4

Exercise 4.1

1. i.

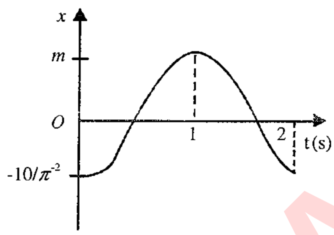

Fig. S-4.1

ii. 0.5

iii. $10/\pi^2$ m

iv. zero

v. 0.5 s, 1.5 s

vi. 0; 1 s, and 2 s

2. Restoring force $F = mg \sin\alpha$

$$F = mg\frac{y}{r}$$

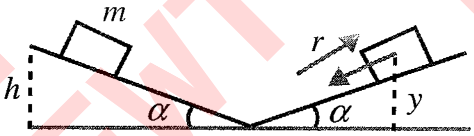

Fig. S-4.2

As far as motion a is constant motion is not SHM. As motion is restricted between two points and separated itself hence oscillatory

Time taken for the block to slide down a height R

$$\frac{h}{\sin\alpha} = \frac{1}{2}g\sin\alpha t^2$$

$$t^2 = \frac{2h}{g}\frac{1}{\sin^2\alpha}$$

$$t = \frac{1}{\sin\alpha}\sqrt{\frac{2h}{g}}$$

Hence period,

$$4t = \frac{4}{\sin\alpha}\sqrt{\frac{2h}{g}}$$

3. $$y_2 = 5\sin(3\pi t) + 5\sqrt{3}\cos(3\pi t)$$

$$A = \sqrt{5^2 + (5\sqrt{3})^2} = 10$$

$$\tan\phi = \sqrt{3},\ \phi = \pi/3$$

$$y_2 = 10\sin\left[3\pi t + \frac{\pi}{3}\right]$$

4. As acceleration $\neq \alpha\ (-y)$, motion is not SHM as seen from earth surface.

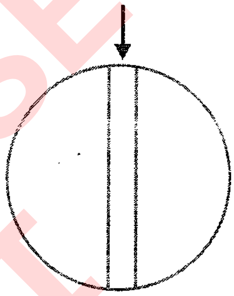

Fig. S-4.3

Note: motion is SHM as seen from earth's centre.

5. a.

$$x = 5\sin\left(20t + \frac{\pi}{3}\right)$$

$$\frac{dx}{dt} = 100\cos\left(20t + \frac{\pi}{3}\right)$$

For particle to come into rest

$$\frac{dx}{dt} = 0$$

$$\cos\left(20t + \frac{\pi}{3}\right) = \cos\frac{\pi}{2}$$

$$20t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

$$t = \frac{\pi}{120}$$

b. The particle will have zero acceleration when it will be at equilibrium position.

As $\quad \omega = 20,\ 2\pi/T = 20 \quad \Rightarrow T = \frac{\pi}{10}$ s

Hence particle will come to equilibrium position.

$$t = \frac{\pi}{120} + \frac{\pi}{40} = \frac{\pi}{40}\left(\frac{1}{3} + 1\right) = \frac{\pi}{30}\text{ s}$$

6. $t_{OP} = 0.5$ s, $\quad t_{OBP} = 1.5$ s

Time taken from P to B

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$$= \frac{1.5-0.5}{2} = 0.5 \text{ s}$$

Time taken from O to $B = 0.5 + 0.5 = 1$ s
Hence time period = 4 s

Let $x = A \sin \omega t$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$v = \frac{dx}{dt} = A\omega \cos \omega t \quad \Rightarrow v_{max} = A\omega$$

Given $\quad v = 3$ m/s $\quad$ at $t = ½$ s,

$$3 = A \times \frac{\pi}{2} \times \cos\left(\frac{\pi}{2} \times \frac{1}{2}\right) \quad \Rightarrow A = \frac{6\sqrt{2}}{\pi} \text{ cm}$$

$$v_{max} = \frac{6\sqrt{2}}{\pi} \times \frac{\pi}{2} = 3\sqrt{2} \text{ m/s}$$

$$x_{OP} = A \sin \omega t = \frac{6\sqrt{2}}{\pi} \sin \frac{\pi}{2} \times \frac{1}{2} = \frac{A}{\sqrt{2}}$$

$$PA = A - \frac{A}{\sqrt{2}} = \frac{A(\sqrt{2}-1)}{\sqrt{2}}$$

$$PB = \frac{A(\sqrt{2}+1)}{\sqrt{2}}$$

7. Given $\quad |v_{max}| = a\omega = 20$ cm/s $\quad$ (i)

and $\quad |f_{max}| = a\omega^2 = 100\pi$ cm/s^2 $\quad$ (ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\omega = 5\pi \text{ rad/s}$$

Therefore, time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5\pi} = 0.4 \text{ s}$$

8. Let equation of SHM be $x = A \sin \omega t$

when $\quad x = 0, t = 0$

when $\quad x = A/2$

$$A/2 = A \sin \omega t$$

or $\quad \sin \omega t = ½$

$$\omega t = \pi/6$$

$$\frac{2\pi}{T} t = \frac{\pi}{6}$$

$$t = T/12$$

Hence, time taken is $T/12$, where T is time period of SHM.

9.

$$F = 8 - 2x$$

or $\quad F = -2(x-4)$

for equilibrium position $F = 0$

$\Rightarrow \quad x = 4$ is equilibrium position

Hence the motion of the particle is SHM with force constant 2 and equilibrium position $x = 4$.

a. Yes, motion is SHM.

b. Equilibrium position is $x = 4$

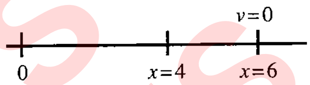

Fig. S-4.4

c. At $x = 6$ m, particle is at rest, i.e., it is one of the extreme position.

Hence amplitude is $A = 2$ m and initially particle is at the extreme position.

∴ Equation of SHM can be written as

$$x - 4 = 2 \cos \omega t,$$

where $\quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1$

i.e., $\quad x = 4 + 2 \cos t$

d. Time period,

$$T = \frac{2\pi}{\omega} = 2\pi \text{ s}$$

10. In the case of SHM, $x = a \sin(\omega t + \phi)$

Here, $\quad a = 1$ m, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s

At $t = 0$, net force on the particle is zero.

Hence, particle is at mean position at $t = 0$.

∴ $\quad x = 0$ at $t = 0$

∴ $\quad 0 = a \sin(\omega \times 0 + \phi)$

$\Rightarrow \quad \phi = 0$

$$x = a \sin \omega t$$

$$x = 1 \times \sin \omega t$$

$$x = \sin \omega t$$

11. $\quad v_{max} = a\omega = 1 \times \pi = \pi$ m/s

$$a_{max} = -\omega^2 a = -\pi^2 \text{ m/s}^2$$

∴ $\quad |a_{max}| = \pi^2$ m/s^2

12. Remember that whenever motion starts from origin or the mean position, then $x = a \sin(\omega t)$ and whenever it starts from extreme position then $x = a \cos(\omega t)$.

Because at $t = 0, x = 0$ for $x = a \sin(\omega t)$

and at $\quad t = 0, x = a$ for $x = a \cos(\omega t)$

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$$\left.\begin{array}{l}\dfrac{a}{2}=a\sin(\omega t_1)\\ \sin(\omega t_1)=\dfrac{1}{2}\\ \omega t_1=\dfrac{\pi}{6}\end{array}\right| \begin{array}{l}\dfrac{a}{2}=a\cos(\omega t_2)\\ \cos(\omega t_2)=\dfrac{1}{2}\\ \omega t_2=\dfrac{\pi}{3}\end{array} \quad \Rightarrow \frac{t_1}{t_2}=\frac{1}{2}$$

13. Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motions is δ,

$$A=\sqrt{A^2+A^2+2AA\cos\delta}$$

or, $$=A\sqrt{2(1+\cos\delta)}=2A\cos\frac{\delta}{2}$$

or, $$\cos\frac{\delta}{2}=\frac{1}{2}$$

or, $$\delta = 2\pi/3$$

14. Given $T = 1.2$ s, $A = 87$ cm
$y = 8 - 3 = 5$ cm

The equation of simple harmonic motion, when motion starts from extreme position.

$$y = A\cos\omega t = A\cos\frac{2\pi t}{T}$$

$$5 = 8\cos\frac{2\pi t}{T}$$

or $$\cos\frac{2\pi t}{T}=\frac{5}{8}=0.625$$

$$\frac{2\pi t}{T}=\cos^{-1}(0.625)=51°\times\frac{\pi}{180}$$

$$t=\frac{51\times\pi}{180}\times\frac{T}{2\pi}=\frac{51}{360}=\frac{51}{360}\times 1.2=0.17\text{ s}$$

15. $$x = A\cos\omega t$$

$$v=\frac{dx}{dt}=-A\omega\sin\omega t$$

$$a=\frac{d^2x}{dt^2}=-A\omega^2\cos\omega t$$

$$a_{max}=A\omega^2$$

$$\tau_{max}=I\alpha_{max}$$

$$\tau_{max}=I\frac{a_{max}}{R}$$

$$=\frac{MR^2}{2}\frac{A\omega^2}{R}=\frac{MRA\omega^2}{2}=6\text{ N-m}$$

Exercise 4.2

1. When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx which should balance the weight of the block.

Thus, $x = mg/k$. As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a psuedo force mg upward on the block. This force will 'balance' the weight. Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by $x = mg/k$, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is mg/k.

2. Assuming the collision to last for a small interval only, we can apply the principle of conservation of momentum. The common velocity after collision is $v/2$. The kinetic energy

$$=\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2=\frac{1}{4}mv^2$$

This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is A, the total energy can also be written as $(1/2)\,kA^2$. Thus,

$$\frac{1}{2}kA^2=\frac{1}{4}mv^2$$

$$A=\sqrt{\frac{m}{2k}}v$$

3. a. If we displace the block upward the net tension towards mean position will provide the restoration force on the block.

$$\sum\vec{F}=-2T\sin\theta,\quad \text{where}\quad \theta=\tan^{-1}(y/L)$$

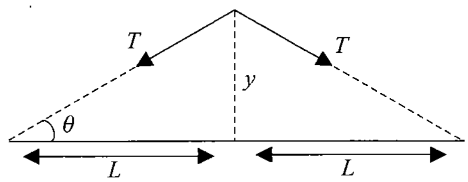

Fig. S-4.5

Since for a small displacement,

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

and the resultant force is

$$\sum\vec{F}=\left(-\frac{2Ty}{L}\right)$$

b. Since there is a restoring force that is proportional to the position, it causes the system to move with simple harmonic motion like a block–spring system. Thus, we can think of

$$\sum F=-\left(\frac{2T}{L}\right)y$$

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as an example of $\Sigma F = -kx$. By comparison, we identify the effective spring constant as $k = 2T/L$. Then the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}$$

Not everything with a stable equilibrium position will execute simple harmonic motion when disturbed and released, but a lot of things do when the disturbance is small.

4. The period of oscillation of a mass M attached to a spring of force constant K is

$$T = 2\pi\sqrt{\left(\frac{m}{K}\right)}$$

In the 1st case, $T = 2$ s

$$2 = 2\pi\sqrt{\left(\frac{M}{K}\right)} \quad \text{(i)}$$

In the 2nd case, $T = 3$ s

$$3 = 2\pi\sqrt{\left(\frac{M+2}{K}\right)} \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{3}{2} = \sqrt{\left(\frac{M+2}{K}\right)}$$

Squaring,

$$\frac{3}{2} = \sqrt{\left(\frac{M+2}{K}\right)}$$

$$\frac{2}{M} = \frac{9}{4} - 1 = \frac{5}{4}$$

$$\therefore \quad M = \frac{4\times 2}{5} = \frac{8}{5} = 1.6 \text{ kg}$$

5. $$I = \frac{1}{3}M(2L)^2 = \frac{4}{3}ML^2$$

Force applied by the spring is $F = -kx$

$$\Rightarrow \quad F = -k(2L\theta)$$

(θ is the angular displacement from the equilibrium position). Further

$$\tau = |\vec{\tau}| = |\vec{r}\times\vec{F}| = -4L^2k\sin\theta = -4L^2k\theta$$

Also, $$\tau = I\alpha = I\ddot{\theta} = -4L^2k\theta$$

$$\Rightarrow \quad \ddot{\theta} + \frac{3k}{M}\theta = 0 \quad \Rightarrow \omega_0 = \sqrt{\frac{3k}{M}}$$

6. When the bob is located to the right of point O, the obstacle acts as the pivot. The effective length of the pendulum is $l/4$ and when it is to the left of obstacle the effective length is l

$$\Rightarrow \quad t = \frac{1}{2}\left(2\pi\sqrt{\frac{l}{4g}} + 2\pi\sqrt{\frac{l}{g}}\right)$$

$$\Rightarrow \quad t = \frac{3T}{4}$$

where $$T = 2\pi\sqrt{\frac{l}{g}}$$

7. $$T_0 = 2\pi\sqrt{\frac{M}{k}} \quad \text{(i)}$$

Let v_0 be the velocity of mass M as it passes the equilibrium position. If a_0 is the initial amplitude, then

$$\frac{1}{2}Mv^2 = \frac{1}{2}ka_0^2$$

$$\Rightarrow \quad v = \left(\sqrt{\frac{k}{M}}\right)a_0$$

If V is the combined velocity of $(M + m)$ system in equilibrium position, then by law of conservation of linear momentum, we have

$$Mv = (M + m)V$$

$$\Rightarrow \quad V = \left(\frac{M}{M+m}\right)v$$

Let a be the new amplitude, then

$$\frac{1}{2}ka^2 = \frac{1}{2}(M+m)V^2$$

$$\Rightarrow \quad \frac{1}{2}ka^2 = \frac{1}{2}(M+m)\left(\frac{M}{M+m}\right)^2 v^2$$

$$\Rightarrow \quad \frac{1}{2}ka^2 = \frac{1}{2}(M+m)\frac{M^2}{(M+m)^2}\left(\frac{k}{M}\right)a_0^2$$

$$\Rightarrow \quad a = a_0\sqrt{\frac{M}{M+m}}$$

New time period is

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\Rightarrow \quad T = \sqrt{\frac{M+m}{M}}T_0$$

$$\left\{\because\ T_0 = 2\pi\sqrt{\frac{M}{k}}\right\}$$

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8. The angular freqency under all circumstances is

$$\omega = \sqrt{(k/m)} = \sqrt{(200/1)} = 14 \text{ rad/s}$$

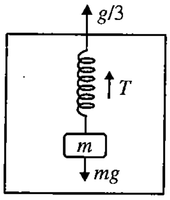

Fig. S-4.6

When elevator is moving up, the equation of motion

$$T - mg = \frac{mg}{3}$$

$$\Rightarrow \quad T = \frac{4mg}{3}$$

This tension elongates the spring by x

$$T = kx$$

$$\Rightarrow \quad x = \frac{4mg}{3k} = 0.07 \text{ m}$$

9. We know,

$$T = 2\pi\sqrt{(m/k)}$$

$$T = 2\pi\sqrt{\left(\frac{m+6}{600}\right)}$$

$$\Rightarrow \quad 0.75 = 2\pi\sqrt{\left(\frac{m+6}{600}\right)}$$

$$m = 2.56 \text{ kg}$$

Maximum acceleration of SHM is $a_{max} = \omega^2 A$ (A = amplitude).
To avoid slipping, maximum force of mass m is $m\omega^2 A$ which is being provided by the force of friction between blocks A and B

$$\mu_s mg \ge m\omega^2 A$$

$$\Rightarrow \quad \mu_s \ge (\omega^2 A)/g$$

$$\Rightarrow \quad \mu_s \ge (2\pi/T)^2 (A/g)$$

$$\Rightarrow \quad \mu_s \ge \left(\frac{2\pi}{0.75}\right)^2 \left(\frac{0.05}{9.8}\right) \quad (A = 50 \text{ mm})$$

$$\Rightarrow \quad \mu_s = 0.358$$

10. From the FBD diagram of the mass with the rigid rod, S (moment of external forces about O)

$$\tau_0 = mg\,(L\sin\theta) - (Kx')\,(L - h)\cos\theta$$

For small θ, $\sin\theta \approx \theta$, $\cos\theta \approx 1$

$$x = L\theta \quad \text{and} \quad x' = L\theta\,(1 - h/L) = mL^2\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = \frac{g\theta}{L} - \frac{k}{m}\left(1 - \frac{h}{L}\right)^2$$

$$= -\left[\frac{k}{m}\left(1-\frac{h}{L}\right)^2 - \frac{g}{L}\right]\theta$$

$$= -\omega^2\theta$$

which represents the SHM

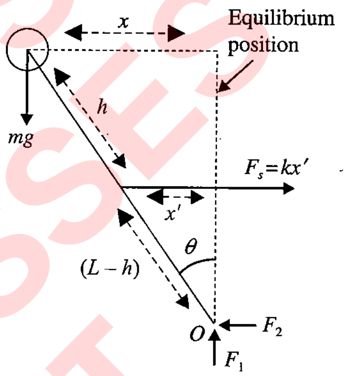

Fig. S-4.7

$$\text{Time period} = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{\sqrt{\left[\frac{k}{m}\left(1-\frac{h}{L}\right)^2 - \frac{g}{L}\right]}}$$

Obviously, for SHM to be positive, $\omega^2 > 0$

$$\Rightarrow \quad \frac{k}{m}\left(1-\frac{h}{L}\right)^2 - \frac{g}{L} > 0$$

$$\Rightarrow \quad h < L - \sqrt{\frac{mgL}{k}}$$

11. a. Let us assume that the bob swings a maximum angle β in the left hand side of the mean position in the absence of the portion PC of the wall.

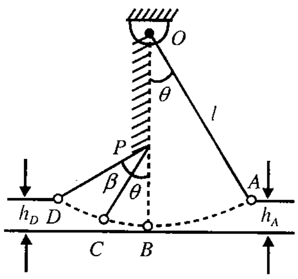

Fig. S-4.8

Conserving energy at A and D, we have

$$U_A + K_A = U_D + K_D$$

where $\quad U_A = mgh_A,\ U_B = mgh_B$

and $\quad K_A = K_D = 0$

because the bob comes to rest at the extreme positions.

Then, we have $h_A = h_D$

Substituting $h_A = OA(1-\cos\theta)$

and $h_D = PA(1-\cos\beta)$, we have

$$OA(1-\cos\theta) = PD(1-\cos\beta)$$

Substituting $OA = l$, $PD = (l-d)$, we have

$$\beta = \left(\sqrt{\frac{l}{l-h}}\right)\theta$$

b. Since the bob swings from A to B,

$$t_{AB} = \frac{T}{4}$$

where $$T = 2\pi\sqrt{\frac{l}{g}}$$

Then, $$t_{AB} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$$

The time of motion from B to C can be found by substituting

$$t = t_{BC},\ \theta = 0$$

and $$\theta = \beta\sqrt{(l/(l-h))}\theta$$

in the equation $\theta = \theta_0 \sin\omega t$,

where $$\omega = \sqrt{g/(l-h)}$$

for the swinging position of the string from B to C. This gives

$$\theta = \left(\sqrt{\frac{l}{l-h}}\right)\theta \sin\sqrt{\frac{g}{l-h}}t_{BC}$$

Then, we have

$$t_{BC} = \sqrt{\frac{l-h}{g}}\sin^{-1}\sqrt{\frac{l-h}{l}}$$

Finally, substituting t_{AB} and t_{BC} in the expression of total time, $T = 2(t_{AB} + t_{BC})$, we have

$$T = \pi\sqrt{\frac{l}{g}} + \sqrt{\frac{l-h}{l}}\sin^{-1}\sqrt{\frac{l-h}{l}}$$

12. **a.** For minimum time period, we have

$$r = k \quad \text{and} \quad k = \sqrt{I_C/m}$$

Substituting $I_C = ml^2/12$ for the rod, we have

$$r = \frac{l}{2\sqrt{3}}$$

Then, $$x = l/2 - r,$$

where $$r = l/2\sqrt{3}$$

This gives $$x = \frac{l}{2}\left(1 - \frac{1}{\sqrt{3}}\right)$$

b.

$$T_{\min} = 2\pi\sqrt{\frac{2k}{g}}$$

where $$k = \frac{l}{2\sqrt{3}}$$

This gives $$T_{\min} = 2\pi\sqrt{\frac{l}{\sqrt{3}g}}$$

13.

$$T \propto 1/\sqrt{K}$$

Now, $$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$$

or, $$\frac{1}{k} = \frac{4}{k'}$$

[Here $k_{\text{eff}} = k$ and $k_1 = k_2 = k_3 = k_4 = k'$]

$$k' = 4k$$

Therefore, time period will be twice the original value.

14. Using time period for a physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgx}} = 2\pi\sqrt{\frac{m\left[(l^2/12) + x^2\right]}{mgx}}$$

or, $$T = K\sqrt{\left(\frac{l^2}{12x} + x\right)}$$

For T to be maximum or minimum

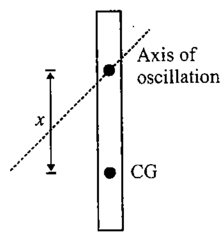

Fig. S-4.9

$$dT/dx = 0$$

$$x = l/\sqrt{12}$$

Further for $$x = l/\sqrt{12}$$

$$\frac{d^2T}{dx^2} = +ve$$

Hence T is minimum.

15. The potential energy function for the mass m in a unidimensional potential field is given as:

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$$U(x) = \frac{A}{x^2} - \frac{B}{x} \quad \text{(i)}$$

Since this is the potential energy of mass m in a potential field, so the force acting on the mass is obtained by the relation:

$$F = -\frac{dU}{dx} = \frac{2A}{x^3} - \frac{B}{x^2} \quad \text{(ii)}$$

Now, the equilibrium position is given by

$$F = 0, \quad \text{or} \quad \frac{2A}{x^3} - \frac{B}{x^2} = 0$$

$$x^2(2x - Bx) = 0$$

So, $x = 0$

or $x = \frac{2A}{B}$

Since $x = 2A/B$ gives more stable situation (because energy is minimum here; d^2U/dx^2 is positive), so stable equilibrium position is given as

$$x = \frac{2A}{B}$$

Now if the mass is displaced slightly away from the equilibrium position, the force acting on mass m is given as:

$$F = \frac{2A}{[(2A/B)+\Delta x]^3} - \frac{B}{[(2A/B)+\Delta x]^2}$$

$$= \frac{2A}{\left(\frac{2A}{B}\right)^3\left(1+\frac{B\Delta x}{2A}\right)^3} - \frac{B}{\left(\frac{2A}{B}\right)^2\left(1+\frac{B\Delta x}{2A}\right)^2}$$

$$= \frac{2A}{8A^3/B^3}\left[1+\frac{B\Delta x}{2A}\right]^{-3} - \frac{B}{4A^2/B^2}\left[1+\frac{B\Delta x}{2A}\right]^{-2}$$

So, using bionomial expansion for small Δx, we get

$$F = \frac{B^3}{4A^2}\left[1-\frac{3B\Delta x}{2A}\right] - \frac{B^3}{4A^2}\left[1-\frac{2B\Delta x}{2A}\right]$$

$$= \frac{B^3}{4A^2}\left[1-\frac{3B\Delta x}{2A}-1+\frac{2B\Delta x}{2A}\right] = -\left(\frac{B^4}{8A^3}\right)\Delta x$$

Using Newton's second law of motion, we get

$$m\frac{d^2x}{dt^2} = -\frac{B^4}{8A^3}\Delta x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{B^4}{8mA^3}\right)\Delta x \quad \text{(iii)}$$

$$\frac{d^2x}{dt^2} \propto -\Delta x$$

So from above relations, it is clear that particle will perform SHM But for any SHM,

$$\frac{d^2x}{dt^2} = -\omega^2\Delta x \quad \text{(iv)}$$

So, from Eqs. (iii) and (iv), we have

$$\omega^2 = \frac{B^4}{8A^3m}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{8A^3m}{B^4}} = 4\pi A\sqrt{\frac{mA}{B^4}}$$

Chapter 5

Exercise 5.1

1. Transverse waves require 'rigidity' in the medium. solids have rigidity but fluids do not. However, in a liquid surface, the surface tension provides some rigidity and hence, partially transverse waves are possible on a liquid surface, e.g., ripples.
2. The wavefront at a large distance from any source is a plane wavefront, so one can create a plane wavefront by keeping a source at a large distance. One can produced a spherical wavefront by using a point source.
3. No, an oscillation is not a wave. The term 'wave' implies the transfer of energy through successive vibration of the particles of the medium. So the oscillations of a body do not constitute a 'wave'.
4. ABC represent compression (negative slope region) and CDE represents rarefaction (positive slope region).
5. A longitudinal wave can be set up in a stretched spring by compressing the coils in a small region, and releasing the compressed region. The disturbance will proceed to propagate as a longitudinal pulse. It is quite possible to set up a transverse wave in a spring, simply by displacing a section of the spring in a direction perpendicular to its length and releasing it.
6. Given, speed of wave, v = 30 cm/s
   And frequency of wave, f = 20 Hz

   $\therefore$ Wavelength of wave, $\lambda = \frac{v}{f} = \frac{30}{20} = 1.5$ cm

   Thus the separation between the consecutive compression = 1.5 cm.
7. The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetition.
8. **i.** On the same wavefront phase difference = 0.
   Linear distance between the points does not matter.

   ii. Between two successive crests, path difference = λ.

   $\therefore$ Phase difference = 2π radian

   iii. Again, path difference between two successive troughs = λ

   $\therefore$ Phase difference between A and B = 2π radian.
9. Transverse waves travel in the form of crests and troughs involving change in shape of the medium. As liquid and gases do not possess the elasticity of shape, therefore mechanical transverse waves cannot be produced in liquids and gases.

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Exercise 5.2

1. **Method 1:** The dotted curved in the figure represents the position of the wave at a later instant. It can be easily obtained from the figure that

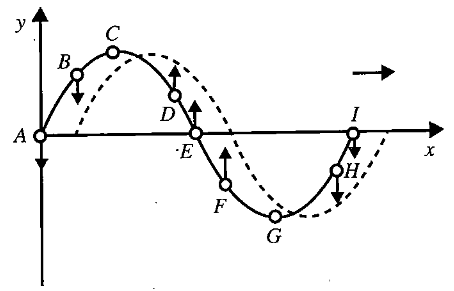

Fig. S-5.1

a. Points D, E, and F move upward.
b. Points A, B, H, and I move downward.
c. Points C, and G have zero velocity.
d. Points A, E, and I have the maximum velocity.

Method 2: Particle velocity

$$V_p = -\text{wave velocity} \times \frac{dy}{dx}$$

As the wave travel in positive x-direction. The points having $dy/dx > 0$ will move down, $dy/dx < 0$ will move up and $dy/dx = 0$ the points having will be at rest. The points having maximum magnitude of $\frac{dy}{dx}$ will have maximum magnitude of speed.

2. Any wave function represents a wave if it satisfies the equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

It can be easily proved that the given wave function satisfies the above equation. Furthermore, it can be easily put in the form $y = f(x - vt)$. Thus, it represents a wave. The amplitude, frequency, and wavelength of the wave are $A_0/2$, $2f_0$ and $\lambda_0/2$, respectively. It is because, the given function can be reduced to

$$y = \frac{A_0}{2}\left[1+\cos\left\{2\left(2\pi f_0 t - \frac{2\pi x}{\lambda_0}\right)\right\}\right]$$

$$y = \frac{A_0}{2}\left[1+\cos\left[2\pi(2f_0)t - \frac{2\pi x}{\lambda_0/2}\right]\right]$$

3. Evidently, the factor, consisting of x and t, appear as a combination of $[(x/v) - t]$ namely $v = 1/2$ unit.

Further, the condition $(2x - t) < 4$ and $(2x - t) > -4$, can be equivalently expressed as

$$|2x-t|^2 < 4^2 \quad \text{or,} \quad 16-(2x-t)^2 > 0$$

Therefore, the function y is defined for the restricted values of x and t. At the same time, the function y is also bounded. Hence, the given equation represents a wave.

The amplitude of the wave will be the maximum displacement y of the oscillating particle.

Therefore, for y to be maximum $(2x - t)^2$ should be minimum; since the minimum value of this square can be zero, so

$$(2x - t)^2_{\text{min.}} = 0 \quad \therefore y_{\max} = 4 \text{ unit}$$

Thus, the amplitude is 4 unit and the phase velocity is 1/2 unit.

4. The amplitude of wave velocity is given by the maximum particle displacement $y_{\max}$ which, evidently, occurs when

$$(x - ct)^2 = 0, \quad \text{i.e.,} \quad y_{\max} = \text{amplitude} = b/a\sqrt{a^2} = b.$$

The combination $(x - ct)$, appearing in the wave equation, when compared with the expression $x - vt$, reveals that, the wave velocity is $v = c$.

Differentiating the displacement equation with respect to time t, we get,

$$\frac{dy}{dt} = \frac{b}{2a}\frac{1}{\sqrt{a^2-(x-ct)^2}}[-2(x-ct)](-c)$$

$$= \frac{bc(x-ct)}{a\sqrt{a^2-(x-ct)^2}}.$$

Putting $x = a/2$ and $t = 0$; we get

Initial particle velocity

$$\frac{dy}{dt} = \frac{bc(a/2)}{a\sqrt{a^2-(a^2/4)}} = \frac{bc\sqrt{3}}{3a}$$

5. No, another obvious condition that this function must satisfy is that it must be finite everywhere and at all times. For example, $1/(x + vt)$ cannot represent a travelling wave whereas $\sin(ax \pm bt)$ does.

6. a. The wave equation is

$$y = Ae^{-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2} \qquad \text{(i)}$$

This may be expressed as

$$y = Ae^{-\frac{1}{\lambda^2}\left(x-\frac{\lambda}{T}t\right)^2}$$

This is of the form $f(x - vt)$

Therefore, the velocity of wave $v = \lambda/T = 8.0/1.0 = 8$ cm/s

b. Substituting $x = 0$ in given Eq. (i), we get

$$f(t) = Ae^{-(t/T)^2}$$

c. Substituting $t = 0$ in given Eq. (i), we get

$$g(x) = Ae^{-(x/\lambda)^2}$$

d. At $t = 0$, the displacement is maximum at the origin while at $t = 5$ s, the displacement is maximum at distance $x = vt = 8 \times 5 = 40$ cm

The function $g(x)$ at $t = 0$ and at 5 s are plotted in Figs. S-5.2(a) and (b).

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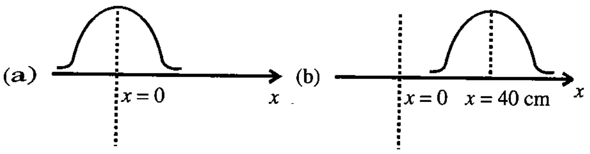

Fig. S-5.2

7. i. Given equation is

$$y = 5 \sin (4.0t - 0.02x)$$

Comparing this with standard equation $y = A \sin (\omega t - kx)$, we get

Amplitude $A = 5$cm

$$\omega = 4.0 \text{ s}^{-1} \quad \text{and} \quad k = 0.02 \text{ cm}^{-1}$$

∴ Frequency

$$f = \frac{\omega}{2\pi} = \frac{4.0}{2\pi} = \frac{2}{3.14} = 0.637 \text{ s}^{-1}$$

Velocity

$$v = \frac{\omega}{k} = \frac{4.0}{0.02} = 200 \text{ cm/s}$$

$$\text{Wavelength } \lambda = \frac{v}{f} = \frac{200}{0.637} = 314 \text{ cm}$$

ii. The maximum velocity, $u_{max} = \omega A = 4 \times 5 = 20$ cm/s.

8. Tension in wire

$$T = 9 \text{ kg wt} = 9 \text{ g} = 9 \times 9.8$$

Mass per unit length $m = M/L$

$$= \frac{12 \times 10^{-3} \text{ kg}}{1.5 \text{ m}} = 8 \times 10^{-3} \text{ kg/m}$$

∴ Frequency of vibration of pth harmonic on string

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{2}{2 \times 1.5}\sqrt{\left[\frac{9 \times 9.8}{8 \times 10^{-3}}\right]} = 70 \text{ Hz}$$

9. Comparing the equation with the standard form of the wave equation

$$y = a \sin (\omega t - kx), \text{ where } \omega = 2\pi f \text{ and } k = 2\pi/\lambda$$

Given that

i. $\omega = 500\pi$ or $f = 250$ Hz

$$\text{and } \frac{2\pi}{\lambda} = \frac{500\pi}{30} \quad \text{or} \quad \lambda = \frac{3}{25} m = 0.12 \text{ m}$$

ii. $\therefore C = f\lambda$ or $C = 250 \times \frac{3}{25} = 30$ m/s

iii. v (particle velocity) $= \frac{dy}{dt} = a\omega \cos(\omega t - kx)$

$$\therefore \quad v_{max} = a\omega = 0.01 \times 500\pi = 15.7 \text{ m/s}$$

iv. f (particle acceleration)

$$= \frac{dv}{dt} = -a\omega^2 \sin(\omega t - kx)$$

$$\therefore \quad f_{max} = a\omega^2 = 0.01 \times (500\pi)^2 = 2.47 \times 10^4 \text{ m/s}^2$$

10. Comparing with the standard form $y = a \cos (\omega t - kx)$.

$$\omega = 1800 \quad \text{or} \quad f = 1800/2\pi$$

$$\text{and} \quad k = 5.3 \quad \text{or} \quad \lambda = 2\pi/5.3 \text{ m}$$

$$a = 60 \times 10^{-6} \text{ m}$$

a. $v = \frac{dy}{dt} = -a\omega \sin(\omega t - kx)$

b. $v = \frac{dy}{dt} = -a\omega \sin(\omega t - kx)$

$$\therefore \quad v_{max} = a\omega$$

$$V_{max} = 60 \times 10^{-6} \times 1800 = 0.108 \text{ m/s}$$

$$c = f\lambda = \frac{1800}{2\pi} \times \frac{2\pi}{5.3} = 339.6 \text{ m/s}$$

$$\therefore \frac{v_{max}}{c} = \frac{0.108}{339.6} = 3.18 \times 10^{-4}$$

11. Area of the sphere $= 4\pi \times 1^2 = 4\pi$

$$I \text{ (intensity)} = \frac{1.00}{4\pi} \text{ W/m}^2 = 8.0 \times 10^{-2} \text{ W/m}^2$$

12. Let P be the power of the source.

$$\text{Then} \quad I = \frac{P}{4\pi r^2}$$

$$\therefore \quad I \propto \frac{1}{r^2} \quad \text{But} \quad I \propto a^2$$

$$\therefore \quad a \propto \frac{1}{r}$$

The equation in the standard form is

$$y = a \sin k (r - vt)$$

Therefore, $y = a_0/r \sin k(r - vt)$, simply replacing a by a_0/r where a_0 is a constant.

Comparing with the usual form $y = a \sin (kr - \omega t)$

$$\omega = kv \quad \text{or} \quad f = kv/2\pi$$

$$\text{and} \quad k = 2\pi/\lambda \quad \text{or} \quad \lambda = 2\pi/k$$

$$\therefore \quad c = f\lambda = \frac{kv}{2\pi} \times \frac{2\pi}{k} = v$$

$$\text{and} \quad T = 1/f = 2\pi/kv$$

$$I = \frac{1}{2}\rho a^2 \omega^2 v = \frac{1}{2}\rho \frac{a_0^2}{r^2} k^2 v^2 v = \frac{1}{2}\rho a_0^2 k^2 v^3 / r^2$$

13. v_{max} (velocity amplitude) $= a\omega$

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$\therefore\ v_{max} = 2 \times 10^{-8} \times (2\pi \times 1000) = 1.3 \times 10^{-4}$ m/s

$$I = \frac{1}{2}\rho a^2\omega^2 c = \frac{1}{2}\rho v_{max}^2 c$$

$$\Rightarrow\ I = \frac{1}{2}\times 1.3\left(1.3\times 10^{-4}\right)^2 \times 340 = 3.7\times 10^{-6}\ \text{W/m}^2$$

14. No. Due to the weight of the rope, the tension will increase along the string from the lower end to the upper end. Hence, the wave will travel with an increasing velocity along the string, since $v \propto \sqrt{T}$

15. The distance between the particles

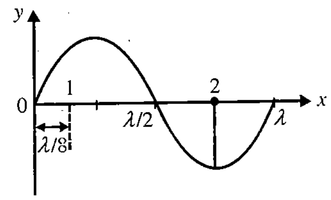

Fig. S-5.3

$$\Delta x = \left(\frac{\lambda}{2} - \frac{\lambda}{8}\right) + \frac{\lambda}{4} = \frac{5\lambda}{8}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times\frac{5\lambda}{8} = \frac{5\pi}{4}$$

16. i. Differentiating expression (i) twice w.r.t. t we have

$$\frac{\partial^2 y}{\partial t^2} = 2v^2$$

And differentiating expression (i) twice w.r.t. x we have,

$$\frac{\partial^2 y}{\partial x^2} = 2$$

Clearly, $$\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

Thus expression (i) is a solution of the one dimensional wave equation.

Similarly, treatment can be done for (ii), (iii) and (iv).

Note: The expression (iv) satisfies differential equation of wave, but it does not represent progressive wave. It represents stationary wave.

v. Differentiating expression (v) twice w.r.t. t we have

$$\frac{\partial^2 y}{\partial t^2} = -2v^2$$

And differentiating expression (v) twice wrt$x$, we have

$$\frac{\partial^2 y}{\partial x^2} = 2$$

Clearly $\dfrac{\partial^2 y}{\partial t^2} \neq v^2\dfrac{\partial^2 y}{\partial x^2}$, so the expression (v) is not a solution of the one dimensional wave equation.

vi. Differentiating expression (vi) twice w.r.t. t, we have

$$\frac{\partial^2 y}{\partial t^2} = -v^2 \sin 2x \cos vt = -v^2 y$$

$$\frac{\partial^2 y}{\partial x^2} = -4\sin 2x \cos vt = -4y$$

Clearly $\dfrac{\partial^2 y}{\partial t^2} \neq v^2\dfrac{\partial^2 y}{\partial x^2}$ and therefore the expression is not a solution of the one dimensional wave equation.

17. Given $y = 2.0\cos(10t - 0.0080x + 0.35)$.

The standard equation of travelling harmonic wave can be written as $y = A\cos(\omega t - kx + \phi)$.

On comparing two equations, we have $\omega = 10$ rad/s

and $\quad k = 0.0080\ \text{cm}^{-1} = 0.80\ \text{m}^{-1}$

$$\lambda = \frac{2\pi}{K} = \frac{2\pi}{0.80}\text{m}$$

Phase difference $\Delta\phi = \dfrac{2\pi}{\lambda}\times\Delta x$

i. When $\Delta x = 4$ m, $\Delta\phi = \dfrac{2\pi}{(2\pi/0.80)}\times 4 = 3.2$ rad

ii. When $\Delta x = 0.5$ m, $\Delta\phi = \dfrac{2\pi}{(2\pi/0.80)}\times 0.5 = 0.40$ rad

iii. When $\Delta x = \dfrac{\pi}{2}$, $\quad \Delta\phi = \dfrac{2\pi}{\lambda}\times\dfrac{\lambda}{2} = \pi$ rad

iv. When $\Delta x = \dfrac{3\lambda}{4}$, $\quad \Delta\phi = \dfrac{2\pi}{\lambda}\times\dfrac{3\lambda}{4} = \dfrac{3\pi}{2}$ rad

18. The standard equation of a plane progressive wave is $y = A\sin(kx - \omega t)$, the phase of the wave is $(kx - \omega t)$.

a. Comparing $4x - 8t$ with $(kx - \omega t)$, we get

$$\omega = 8 \quad \text{and} \quad k = 4$$

$$\therefore \quad \lambda_1 = \frac{2\pi}{k} = \frac{2\pi}{4} = \frac{\pi}{2}$$

b. Comparing $8x - 16t$ with $(kx - \omega t)$, we get $\omega = 16$ and $k = 8$

$$\therefore \quad \lambda_2 = \frac{2\pi}{k} = \frac{2\pi}{8} = \frac{\pi}{4}$$

Clearly $\quad \lambda_1 = 2\lambda_2$

Therefore, snapshots 1 and 2 correspond to a and b respectively.

19. Suppose F is the tension in the string due to its rotation. Choose a small element of the string of length l. If μ is the mass per unit length of the string, then mass of the element,

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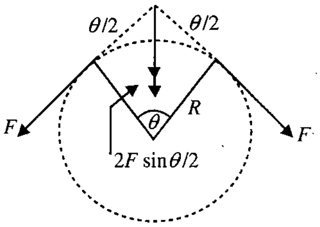

Fig. S-5.4

$$m = \mu l = \mu(R\theta)$$

Using Newton's second law for the element, we have $2F\sin\theta/2 = mv^2/R$

For small θ, $\sin\theta/2 \simeq \theta/2$

$$2F\left(\frac{\theta}{2}\right) = \frac{mv^2}{R}$$

$$F\theta = (\mu R\theta)\frac{v^2}{R}$$

$$F = \mu v^2$$

The speed of the disturbance

$$= \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\mu v^2}{\mu}} = v$$

20. Let equation of wave is $y = A\sin(\omega t - kx + \phi)$

$$v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx + \phi)$$

at $x = 0$, $t = 0$, we have $y = 0$, $v_p > 0$

$\Rightarrow 0 = A\sin\phi \Rightarrow \phi = 0$ or π

But $v_p > 0$, so $\phi = 0$

at $t = 0$, $x = 0.090$ m

$y = A = 4$ mm $\rightarrow$ amplitude

$A = A\sin(-k(0.090))$

$$\Rightarrow \quad -k(0.090) = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow \quad -\frac{2\pi}{\lambda}(0.090) = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow \quad \lambda = \frac{-0.36}{4n+1}$$

for $n = -1$, $\lambda = 0.12$ m

from graph, time period $= T = 0.04$ s

Wave speed: $v = \dfrac{\lambda}{T} = \dfrac{0.12}{0.04} = 3$ m/s

If wave is moving in negative x-direction:

Let wave equation is

$y = A\sin(\omega t + kx + \phi)$

proceeding as above, we will get $\lambda = \dfrac{0.36}{4n+1}$

for $n = 0$, $\lambda = 0.36$ m

Wave speed: $v = \dfrac{\lambda}{T} = \dfrac{0.36}{0.04} = 9$ m/s

21. a. $v = \sqrt{F/\mu} = \sqrt{(5.00\text{ N})/(0.0500\text{ kg/m})} = 10.0$ m/s

b. $\lambda = v/f = (10.0\text{ m/s})/(40.0\text{ Hz}) = 0.250$ m

c. $y(x,t) = A\cos(kx - \omega t)$ (Note: $y(0.0) = A$, as specifed.)

$k = 2\pi/\lambda = 8.00\pi$ rad/m; $\omega = 2\pi f = 80.0\pi$ rad/s

$y(x,t) = (3.00\text{ cm})\cos[\pi(8.00\text{ rad/m})x - (80.0\pi\text{ rad/s})t]$

d. $v_y = +A\omega\sin(kx - \omega t)$ and $a_y = -A\omega^2\cos(kx - \omega t)$

$a_{y,\max} = A\omega^2 = A(2\pi f)^2 = 1890\text{ m/s}^2$

e. $a_{y,\max}$ is much larger than g, so g can be ignored.

22. a. As we know

$$P_{\text{ave}} = \frac{1}{2}\left(\sqrt{\mu F}\right)\omega^2 A^2$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{3.00\times10^{-3}\text{ kg}}{0.80\text{ m}}\right)(25.0\text{ N})}\right][(2\pi(120.0\text{ HZ})]^2(1.6\times10^{-3})^2$$

$= 0.223$ W

b. Halving the amplitude quarters the average power, so 0.056 W.

23. $y = A\cos(kx - \omega t)$

$$v_y = \frac{dy}{dt} = A\omega\sin(kx - \omega t)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y$$

for $t = 0$

$$y = A\cos(kx) = A\cos\left(\frac{2\pi}{\lambda}x\right)$$

$$v_y = A\omega\sin(kx) = A\omega\sin\left(\frac{2\pi}{\lambda}x\right)$$

$$a_y = -\omega^2 A\cos(kx) = -\omega^2 A\cos\left(\frac{2\pi}{\lambda}x\right) = -\omega^2 y$$

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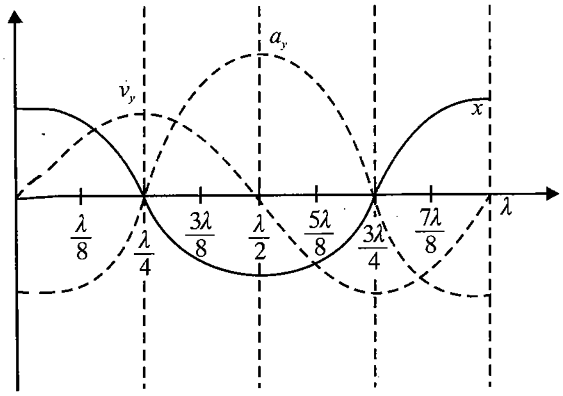

Fig. S-5.5

i. At $x = 0$: $y = A$, $v_y = 0$, $a_y = -\omega^2 A$; the particle is at positive extreme position. It is at rest and accelerating downward at maximum acceleration.

ii. At $x = \frac{\pi}{4k} = \frac{\pi\lambda}{4(2\pi)} = \frac{\lambda}{8}$

$$y = A\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = \frac{A}{\sqrt{2}},\ v_y = \frac{A\omega}{\sqrt{2}},\ ay = \frac{-\omega^2 A}{\sqrt{2}}$$

the particle is above mean position, moving upward and slowing down. Acceleration is in downward direction.

iii. At $x = \frac{\pi}{2k} = \frac{\pi\lambda}{2(2\pi)} = \frac{\lambda}{4}$

here $y = 0$, $v_y = \omega A$, $a_y = 0$

the particle is at mean position and moving upwards with maximum velocity. Acceleratio of particle is zero.

iv. At $x = \frac{3\pi}{4k} = \frac{3\lambda}{8}$

here $y = -\frac{A}{\sqrt{2}}$, $v_y = \frac{A\omega}{\sqrt{2}}$, $a_y = \frac{\omega^2 A}{\sqrt{2}}$

the particle is below mean position, having velocity and aceeleration both in upward direction. Thus particle is speeding up.

v. At $x = \frac{\pi}{k} = \frac{\lambda}{2}$

here $y = -A$, $v_y = 0$, $a_y = \omega^2 A$

the particle is at negative extreme position. It is instantaneously at rest and having acceleration in upward direction.

vi. At $x = \frac{5\pi}{4k} = \frac{5\lambda}{8}$

here $y = \frac{-A}{\sqrt{2}}$, $v_y = -\frac{A\omega}{\sqrt{2}}$, $a_y = \frac{\omega^2 A}{\sqrt{2}}$

the particle is below mean position, moving downward and accelerating upward, thus slowing down.

vii. At $x = \frac{3\pi}{2k} = \frac{3\lambda}{4}$

here $y = 0$, $v_y = -A\omega$, $a_y = 0$

the particle is at mean position and moving downward with maximum velocity. Acceleration of particle is zero.

viii. At $x = \frac{7\pi}{4k} = \frac{7\lambda}{8}$

here $y = \frac{A}{\sqrt{2}}$, $v_y = -\frac{A\omega}{\sqrt{2}}$, $a_y = -\frac{\omega^2 A}{\sqrt{2}}$

the particle is above mean position, having velocity and acceleration both in downward direction. Thus particle is speeding up.

Chapter 6

Exercise 6.1

1. No, the particle velocity is $\pi/2$ out of phase with the displacement, and the pressure variation is out of phase by π with the displacement.
2. The human ear is most sensitive to intensity variation at 1000 Hz. At this frequency, the intensity of just audible sound is 10^{-12} W/m^2. This is why the intensity level of a sound wave of frequency 1000 Hz and intensity 10^{-12} W/m^2 is taken as the reference level for expressing intensity.
3. Sound waves from the different instruments of performers on stage reach the audience simultaneously. This shows that the speed of sound is the same for all wavelength.
4. In gases, the molecules move about at random. The oscillations produced by a sound wave are superimposed on this random thermal motion. An impulse given to one molecule is passed on to another molecule after the first one has moved through the empty space between them and collided with the second. The average of the speeds in the space between molecules is the average speed which is close to the rms speed. Hence, the speed of a sound wave can never be greater than the rms speed of the molecules of a gas.
5. If the wind blows from the source to the listener, the velocity of the wind is added to that of the sound. The upper layer of air possesses a greater velocity of sound than that in the lower layers. The result is that the wavefronts turn downwards and the sound rays curl down. A listener (O) on the ground, thus has a better chance of hearing.
6. Sound travels faster in iron than in air. So there is a difference between the times taken by sound to travel through iron and air. This is why the observer hears two sounds.
7. No, the velocity of sound in a solid does not increase on heating the solid because neither the density ρ nor the modulus of elasticity E change appreciably with change in temperature.
8. During the day, the temperature of air is maximum near the ground and it progressively diminishes upwards. Therefore, the velocity of sound is greater near the ground and it decreases upwards $\left(c \propto \sqrt{T}\right)$. The plane wavefronts, initially vertical, are turned upwards, so, the

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sound rays curl up during the day. At night the conditions are reversed, so sound rays curl down, producing better audibility of distant sounds.

9. If I_{max} and I_{min} are the maximum and minimum intensities of the resultant waves, then the intensity level between them will be

$$L = 10\log\left(\frac{I_{max}}{I_{min}}\right) \text{dB}$$

or, $$20 \text{ dB} = 10\log\left(\frac{I_{max}}{I_{min}}\right) \text{dB}$$

or, $$\frac{I_{max}}{I_{min}} = \frac{100}{1} \quad [\log 100 = 2]$$

If A_1 and A_2 are the amplitudes of the individual waves, intereferring, then

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} \quad [\text{Assuming } A_1 > A_2]$$

$$\therefore \quad \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{100}{1}$$

$$\Rightarrow \quad \frac{A_1 + A_2}{A_1 - A_2} = \frac{10}{1}$$

or, $$\frac{A_1}{A_2} = \frac{11}{9}$$

Therefore, the intensity ratio between the individual waves will be

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \frac{121}{81}$$

10. Intensity level $= \log_{10}\frac{I_2}{I_1}$ bels $= 10\log\frac{I_2}{I_1}$ dB

Here $I_1 = 10 \times 10^{-9} \times 10^4 = 10^{-4}$ W/m^2

$$\therefore \quad 6 = 10\log\frac{I_2}{10^{-4}}$$

or $$I_2 = \frac{10^{0.6}}{10^4} = \frac{3.98}{10^4} = 4.0\times10^{-4} \text{ W/m}^2$$

11. $C = \sqrt{\frac{\gamma P}{\rho}}$

$$1260 = \sqrt{\frac{1.4P}{\rho}}$$

P/ρ of the gas at 0°C $= 11.34 \times 10^5$

$P = \rho/M\ RT$ is the perfect gas equation in terms of density

$$\frac{P}{\rho} = \frac{RT}{M}$$

$$M = \frac{8.3\times273}{11.34\times10^5} = 2\times10^{-3} \text{ kg} = 2 \text{ g}$$

12. $c = \sqrt{\frac{\gamma RT}{M}}$

Since nitrogen and oxygen are both diatomic, $\gamma = 1.4$

$$\frac{m_1 + m_2}{M} = \frac{m_1}{M_1} + \frac{m_2}{M_2}$$

$$\Rightarrow \quad \frac{7+12}{M} = \frac{7}{28} + \frac{12}{32}$$

$$\Rightarrow \quad M = 30.4$$

$$\therefore \quad c = \sqrt{\frac{1.4\times8.3\times300}{30.4\times10^{-3}}} = 338.6 \text{ m/s}$$

13. We know that $\frac{v_t}{v_0} = \sqrt{\left(\frac{T_t}{T_0}\right)} = \sqrt{\left(\frac{237+t}{273}\right)}$

$$v_t = v_0\left(1+\frac{t}{273}\right)^{1/2}$$

$$= v_0\left(1+\frac{t}{2\times273}\right) = v_0\left(1+\frac{t}{546}\right)$$

$$v_t - v_0 = v_0 \times \frac{t}{546}$$

Thus increase in velocity per °C is

$$= \frac{v_0\times1}{546} = \frac{332}{546} = 0.61 \text{ m/s}$$

14. Increase in number of dB,

$$N_2 - N_1 = 60 - 50 = 10 \text{ dB}$$

$\therefore$ Ratio of increase in intensity

$$10 = 10\log_{10}\frac{I_2}{I_1} \Rightarrow \frac{I_2}{I_1} = (10)^1 = 10$$

The relation between pressure amplitude and intensity is $I = p_0^2/2\rho v$.

i.e., $$I \propto p_0^2 \quad \Rightarrow \frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10} = 3.16$$

Exercise 6.2

1. No, because the source moves at right angles to the line joining the source and the observer.

2. When the source approaches the observer, the apparent frequency (n') is greater than the actual frequency (n). Therefore, $T' < T$. If the source emits E units of energy through unit area in one full oscillation of it, then the same amount of energy will pass through unit area in time T'. Therefore,

$$I' = \frac{E}{T'} \quad \text{and} \quad I = \frac{E}{T}$$

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Obviously $I' > I$. Thus, the intensity of a wave is increased by the Doppler effect when the source approaches the listener.

3. Yes, there is a big difference in the Doppler effect for sound and light. The Doppler effect in sound depends on the motion of the medium as well. But light does not require a material medium for its transmission. The velocity of light in vacuum is a universal constant. Hence, the Doppler effect in light is quite different from that for sound. The Doppler effect can be applied to light as a good approximation if v is taken to be the mean relative velocity of the source and observer and if v is very small compared to the velocity of light.

4. No. It is done by considering the component of the velocity of the source along the line joining it with the observer.

5. $f' = f\left(\frac{v}{v+v'}\right)$

where $v' = v_s$

$$\Rightarrow \quad \frac{f}{f'} = \eta \text{ (given)} = 1 + \frac{v'}{v} \qquad \text{(i)}$$

$$f'' = f\left(\frac{v+v'}{v-v'}\right)$$

where $v_0 = v'$

$$\Rightarrow \quad \frac{f}{f''} = \frac{v-v'}{v+v'} = \frac{1-(v'/v)}{1+(v'/v)}$$

$$= \frac{1-(\eta-1)}{1+(\eta-1)} = \frac{2-\eta}{\eta} = \frac{2}{\eta} - 1$$

6. The 'image' of the source approaches the driver at the same speed. Here, the image or echo is the source. Therefore,

$$v_s = +60 \text{ m/s}, \quad v_0 = -60 \text{ m/s}$$

$$n' = \frac{c - v_0}{C - v_s} \times n$$

$$\therefore \quad n' = \frac{340-(-60)}{340-60} \times 133 \quad \Rightarrow n' = 190 \text{ Hz}$$

7. **a.** Due to motion of the source, the wavelength (and hence, the frequency) is actually changed from λ and λ' such that if n = actual frequency,

$$\lambda' = \frac{c - v_s}{n} = \frac{332-32}{1000} = 0.3 \text{ m}$$

b. The number of waves arriving at the reflecting surface is the same as the number of waves received by an observer moving towards the source. This is given by the apparent frequency.

$$n' = \frac{c - v_0}{c - v_s} \times n$$

$$= \frac{332-(-64)}{332-32} \times 1000 = 1320 \text{ Hz}$$

c. Same as that of the incident wave because the speed of wave depends only on the characteristics of the medium.

Therefore, speed of the reflected wave = 332 ms^{-1}

d. Now reflector will act as a source of frequency 1320 Hz. So wavelength emitted by this:

$$\lambda' = \frac{v - v_{ref}}{1320} = \frac{332-64}{1320} = 0.2 \text{ m}$$

8. If there is no wind, n waves emitted by the source in each second will form a train of waves of length equal to speed c of the sound. When the wind blows at speed w, the same number of waves will form a train of length $c + w$ on the windside.

$$\therefore \quad \lambda' = \frac{c+w}{n} = \frac{c+w}{c}\lambda \qquad (\because n\lambda = c)$$

$$\Rightarrow \quad (\lambda' - \lambda)/\lambda = w/c$$

$\therefore$ % increase in wavelength

$$= \frac{w}{c} \times 100 = \frac{0.1c}{c} \times 100 = 10$$

Let n' be the apparent frequency. Then $n'\lambda' = c + w$ (Since speed of sound on the wind-side is $c + w$)

or $$n' = \frac{c+w}{\lambda'} = n$$

so, there is no change in the frequency of the wave.

When the source moves, the wavelength is not changed but the frequency appears to change. The apparent frequency is given by

$$n' = \frac{c - v_0}{c - v_s} \times n$$

Here $v_s = 0$, $v_0 = -0.1\,c$, therefore

$$n' = \frac{c-(-0.1c)}{c} \times n = 1.1n$$

$$\% \text{ increase } = \frac{n'-n}{n} \times 100 = 10$$

9. $n' = \frac{c - v_0}{c - v_s} \times n$

Here

$$n' = \frac{340-0}{340-v} \times 680 = \frac{340}{340-v} \times 680$$

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$$n'' = \frac{340-0}{340+v} \times 680 = \frac{340}{340+v} \times 680$$

Now $$n' - n'' = 2$$

or $$340 \times 680\left(\frac{2v}{340^2 - v^2}\right) = 2$$

or $$v = \frac{340}{680} = 0.5 \text{ m/s}$$

10. When a reflector moves towards the source

$$\lambda' = \frac{c-u}{c+u}\lambda$$

or $$\frac{\lambda' - \lambda}{\lambda} = -\frac{2u}{c+u}$$

The negative sign shows that the wavelength decreases.

$\therefore$ $$\% \text{ decrease} = \frac{2u \times 100}{c+u} = \frac{2 \times 0.33 \times 100}{330 + 0.33}$$

$$= 0.2$$

11. R_2 is approached by the source and the 'image source' at the same speed. So R_2 receives waves of the same frequency and hence, it does not register beats. R_1 is approached by the 'image source' while the source itself recedes from it. Hence, it receives waves of different wavelengths. So it registers beats.

N_1 (apparent frequency of the source wave)

$$= \frac{340-0}{340-(0.17)} \times 1000 = 1000.5 \text{ Hz}$$

N_2 (apparent frequency of the reflected wave)

$$= \frac{340-0}{340+0.17} \times 1000 = 999.5 \text{ Hz}$$

Therefore, beats registered = 1000.5 – 999.5 = 1.0 Hz

12. In general

$$n' = \frac{c - v_0}{c - v_S} \times n$$

here $v_0 = 0$ and $v_s = \omega r = 15 \times 2 = 30$ m/s

When the source moves away from the observer, the apparent frequency is minimum and when it moves towards the observer, the apparent frequency is maximum.

$\therefore$ $$n_{\min} = \frac{330}{330-(-30)} \times 540 = 495 \text{ Hz}$$

$$n_{\max} = \frac{330}{330-30} \times 540 = 594 \text{ Hz}$$

13. Here, the bat is the 'observer' and the 'image' is the source.

$$n' = \frac{c - v_0}{c - v_s} \times n$$

$$= \left[\frac{340-(-6)}{340-6}\right] 450 = 466 \text{ kHz}$$

14. $$n' = \frac{300-0}{300-u} \times 1000$$

$$n'' = \frac{300-0}{300-(-u)} \times 1000$$

Dividing,

$$\frac{n'}{n''} = \frac{300+u}{300-u}$$

It is given that $n'/n'' = 11/9$, therefore

$$\frac{11}{9} = \frac{300+u}{300-u}$$

or $$u = 30 \text{ m/s}$$

15. $v_s = r\omega = 1.5 \times 20 = 30$ m/s

$$n_{\min} = \frac{v}{v+v_s} n = \frac{330}{330+30} \times 440 \text{ Hz} = 403 \text{ Hz}$$

$\therefore$ $$n_{\max} = \frac{330}{330-30} \times 440 = 484 \text{ Hz}$$

16. **a.** The frequency of sound heard directly,

$$f_1 = f_0\left(\frac{v}{v+v_s}\right)$$

$$v_s = 8 \text{ m/s}$$

$\therefore$ $$f_1 = \left(\frac{330}{330+8}\right) \times 2000$$

$$f_1 = \frac{330}{338} \times 2000 = 1953 \text{ Hz}$$

b. The frequency of the reflected sound is given by

$$f_2 = f_0\left(\frac{v}{v-v_s}\right)$$

$\therefore$ $$f_2 = \left(\frac{330}{330-8}\right) \times 2000$$

$$f_2 = \frac{330}{322} \times 2000 = 2050 \text{ Hz}$$

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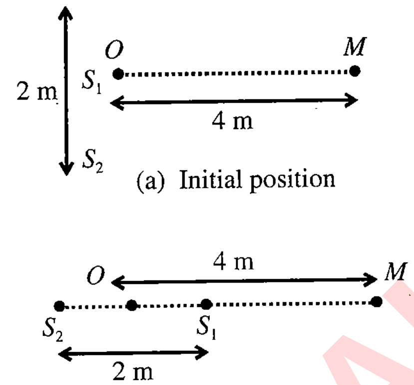
$$\therefore \quad \frac{A_1}{A_0} = \frac{a}{a\sqrt{2}} \quad \text{or} \quad \frac{A_1}{A_0} = \frac{1}{\sqrt{2}}$$

15. The initial and final positions of speakers S_1 and S_2 and microphone M are shown in Figs. S-7.2(a) and (b). Initially the path difference from S_1 to M and S_2 to M is zero.

i.e. $\qquad S_2M - S_1M = 0$



(a) Initial position

(b) Final position

Fig. S-7.2

Finally the path difference

$$\Delta = S_2P - S_1P = 5 - 3 = 2 \text{ m}$$

for nth maxima $\Delta = n\lambda$

$$2 = 5\lambda \quad \text{or} \quad \lambda = 2/5 = 0.4 \text{ m}$$

16. **a.** The wave form for the given times, respectively, is shown.

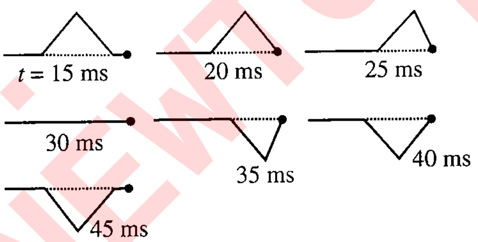


Fig. S-7.3

b.

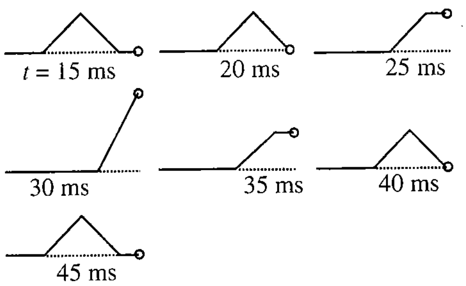


Fig. S-7.4

17. **a.** The wave form for the given times, respectively, is shown.

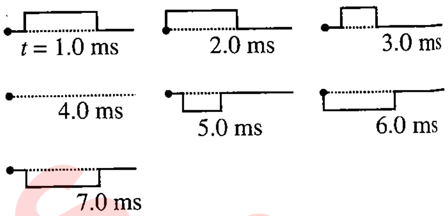


Fig. S-7.5

b.

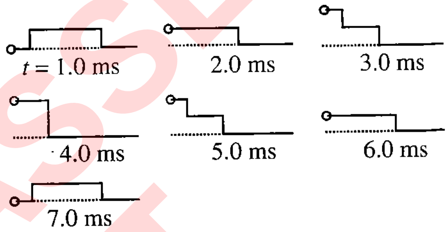


Fig. S-7.6

18.

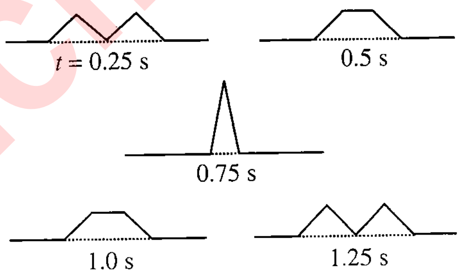


Fig. S-7.7

19. When reflection takes place from the boundary of two media, the ratio of the amplitudes of the reflected wave and the incident wave is given by

$$\frac{a_r}{a_i} = \frac{c_1 - c_2}{c_1 + c_2}$$

When there is a rigid wall, c_2 is infinite. Therefore,

$$\frac{a_r}{a_i} = -1$$

or $\qquad a_r = -a_i$

Thus, the negative sign means that there is a phase change of π in the displacement wave, but there is no change in the amplitude of the wave.

Since a compression is returned as a compression and a rarefaction as a rarefaction from a rigid wall, there is no phase change in the pressure wave.

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Exercise 7.2

1. Yes, they produce standing waves of the form shown here. Energy is transported. No, there are no nodes; instead, there are positions of minimum amplitude.

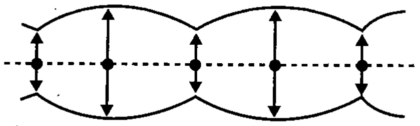

Fig. S-7.8

2. Yes, they will produce beats at the rate of 50 per second. But due to the persistence of hearing, one would not hear those beats. Instead, one would hear a continuous sound of frequency 50 Hz called 'beat tone.'

3. Overtones with frequencies which are integral multiples of the fundamental are called harmonics. Hence, all harmonics are overtones but all overtones are not harmonics.

4. Resonance shortens the duration of sound. That is why it is purposely avoided in many instruments.

5. $f_1 = \dfrac{k}{l_1}$ and $f_2 = \dfrac{k}{l_2}$

$$\therefore f = \frac{k}{(l_1 + l_2)} \quad \Rightarrow l_1 + l_2 = \frac{k}{f}$$

$$\Rightarrow \frac{k}{f_1} + \frac{k}{f_2} = \frac{k}{f} \quad \text{or,} \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

6. Let f_1 and f_2 be the frequencies of the tuning forks A and B respectively.

Since, the beat frequency between A and B is 4

$$\therefore \quad f_1 - f_2 = 4 \text{ or } f_2 - f_1 = 4$$

From, $n = \dfrac{1}{2l}\sqrt{T/\mu}$, we have, $n \propto 1/l$

$\therefore f_1 \propto 1/32.4$ cm and $f_2 \propto 1/32$ cm

$\therefore$ Evidently, $\quad f_2 > f_1$

$$\therefore \quad f_2 - f_1 = 4\ \text{s}^{-1} \qquad \text{(i)}$$

Also, $$\frac{f_2}{f_1} = \frac{324}{320} \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$f_1 = 320 \text{ Hz} \quad \text{and} \quad f_2 = 324 \text{ Hz}$$

7. Wavelength of sound

$$= \frac{v}{f} = \frac{340 \text{ m/s}}{606 \text{ s}^{-1}} = 56.1 \text{ cm}$$

Since, closed pipe allows only odd harmonics, so

$$f = (2n+1)\frac{v}{4l} \quad \text{or,} \quad l = (2n+1)\frac{v}{4f}; \quad n \in I$$

or, $l = (2n + 1) \times 14$ cm

$\therefore l =$ 14 cm, 42 cm, 70 cm, 98 cm, 126 cm, 154 cm, etc.

Since $\qquad l \le 150$ cm

$\therefore$ Number of resonances = 5

8. n_1, frequency of the first wave = $c/0.500$

n_2, frequency of the second wave = $c/0.505$

Obviously $\qquad n_1 > n_2 \quad \therefore n_1 - n_2 = 6$

or $$\frac{c}{0.500} - \frac{c}{0.505} = 6 \quad \text{or} \quad c = 303 \text{ m/s}$$

9. Using the relation $2 \sin C \cos D = \sin(C + D) + \sin(C - D)$

$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t = \frac{5}{2} \times 2\sin\frac{\pi x}{3}\cos 40\pi t$$

$$y = \frac{5}{2}\left[\sin\left(\frac{\pi x}{3} + 40\pi t\right) + \sin\left(\frac{\pi x}{3} - 40\pi t\right)\right]$$

$$= \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right) - \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3}\right)$$

Thus, the given stationary wave is formed by the superposition of the progressive waves

$$y_1 = \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right)$$

and $$y_2 = \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3} + \pi\right)$$

a. Comparing each wave with the standard form of the progressive wave

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x + \alpha\right)$$

$$a = 5/2 = 2.5 \text{ cm}$$

$$\omega = 40\pi \Rightarrow 2\pi f = 40\pi \rightarrow f = 20 \text{ s}^{-1}$$

and $$\frac{2\pi}{\lambda} = \frac{\pi}{3} \quad \text{or} \quad \lambda = 6 \text{ cm} = 0.06 \text{ m}$$

$$\therefore \quad c = f\lambda = 20 \times 0.06 = 1.2 \text{ m/s}$$

b. Distance between the nodes = $\lambda/2 = 0.06/2 = 0.03$ m

c. $$y = 5\sin\frac{\pi x}{3}\cos 40\pi t$$

$$v = \frac{dy}{dt} = -5 \times 40\pi \sin\frac{\pi x}{3}\sin 40\pi t$$

$$\Rightarrow \quad v = -200\pi \sin\frac{\pi x}{3}\sin 40\pi t$$

$\therefore$ At $x = 1.5$ cm and $t = 9/8$ s

d. $v = -200\pi \sin(\pi/2) \sin 45\pi = 0$

10. a. Let d_1 be the length of the fixed path and d_2 be the length of other path that can be varied. Then for minimum sound $d_2 - d_1 = (n + 1/2)\lambda$. When the adjustable tube is moved out by x, the length of the path becomes $d_2 + 2x$. For the next maximum sound

$$(d_2 + 2x) - d_1 = \left(n + \frac{1}{2}\right)\lambda + \frac{\lambda}{2} = n\lambda + \lambda$$

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Subtracting $2x = \frac{\lambda}{2}$ or $x = \frac{\lambda}{4}$

Here x = 1.65 cm.

$\therefore \quad \lambda = 4 \times 1.65 = 6.6 \text{ cm} = 6.6 \times 10^{-2}$ m

$$C = n\lambda \quad \text{or} \quad n = \frac{c}{\lambda} = \frac{340}{6.6\times10^{-2}} = 5152 \text{ Hz}$$

b. If a and b are the amplitudes of the waves, then minimum amplitude = $a - b$ and maximum amplitude = $a + b$.

$$\therefore \frac{I_{\min}}{I_{\max}} = \frac{(a-b)^2}{(a+b)^2} \quad \text{or} \quad \frac{100}{900} = \frac{(a-b)^2}{(a+b)^2}$$

$$\text{or} \quad \frac{a}{b} = \frac{2}{1}$$

11. $n_A = \frac{1}{2\times0.400}\sqrt{\frac{T}{m}} \quad n_B = \frac{1}{2\times0.405}\sqrt{\frac{T}{m}}$

Dividing $\frac{n_A}{n_B} = \frac{405}{400}$ (Obviously $n_A > n_B$)

$\therefore \quad n_A - n_B = 5$ ($\because$ there are 5 beats per second)

$$\text{or} \quad \frac{405}{400}n_B - n_B = 5$$

$$\frac{5}{400}n_B = 5$$

$$n_B = 5\times\frac{400}{5} = 400 \text{ Hz} \quad \text{and } n_A = 405 \text{ Hz}$$

12. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$. If l_0 be the initial length and f be the fractional increase in length, $l = l_0 + fl_0$. Since tension is proportional to the increase in length, $T = k \times fl_0$ where k is a constant.

$$m = \frac{M}{l_0 + fl_0}$$

where M is the mass of the string

$$\therefore \quad n = \frac{1}{2l_0(1+f)}\sqrt{\frac{kfl_0(1+f)}{M}} = \frac{1}{2l_0}\sqrt{\frac{kl_0 f}{M(1+f)}}$$

Since l_0, k and M are constants

$$n \propto \sqrt{\frac{f}{1+f}}$$

$$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{f_1(1+f_2)}{f_2(1+f_1)}} = \sqrt{\frac{0.02(1+0.04)}{0.04(1+0.02)}} = 0.71$$

13. $n = \frac{1}{2l}\sqrt{T/m}$ for the lowest frequency. Here $m = \pi r^2\rho$

T (tension) = $\pi r^2 \times (\gamma \times \text{strain}) = \pi r^2\gamma\varepsilon$

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{\pi r^2\gamma\varepsilon}{\pi r^2\rho}}$$

$$= \frac{1}{2l}\sqrt{\frac{\gamma\varepsilon}{\rho}} = \frac{1}{2\times1}\sqrt{\frac{9\times10^{10}\times\frac{0.05}{100}}{9000}} = 35.4 \text{ Hz}$$

14. For open pipe:

$$f_1 = 300 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{600} = \frac{350}{600} = 0.58 \text{ m}$$

For closed pipe: $\frac{3v}{4l_2} = 2f_1$

$$\Rightarrow \frac{3v}{4l_2} = 600 \Rightarrow l_2 = 0.44 \text{ m}$$

15. With a temperature change, the tension and mass of the wire will not change but length (l) and linear density (m) will change.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{and} \quad m = \frac{M}{l}$$

where M = mass of the wire

$$\therefore \quad n = \frac{1}{2}\sqrt{\frac{T}{Ml}} = \frac{k}{\sqrt{l}}$$

$$\text{where} \quad k = \frac{1}{2}\sqrt{T/M} = a \text{ (constant)}$$

Differentiating both sides

$$\Delta n = k\left(-\frac{1}{2}\right)l^{-3/2}\Delta l$$

$$\therefore \quad \frac{\Delta n}{n}(\text{fractional change}) = \frac{\frac{1}{2}kl^{-3/2}\Delta l}{Kl^{-1/2}}$$

$$\text{or} \quad \frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha t \quad (\Delta l = l \propto t)$$

16. $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$$\therefore \quad n_1 = \frac{1}{2l}\sqrt{\frac{10.2g}{m}} \quad \text{and} \quad n_2 = \frac{1}{2l}\sqrt{\frac{9.9g}{m}}$$

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Dividing $\dfrac{n_1}{n_2} = \sqrt{\dfrac{10.2}{9.9}}$ (Obviously $n_1 > n_2$)

$\therefore \quad n_1 - N = \dfrac{20}{12}$ and $N - n_2 = \dfrac{20}{12}$

or $\quad \dfrac{n_1}{n_2} = \dfrac{N + \frac{20}{12}}{N - \frac{20}{12}}$

$\therefore \quad \dfrac{N + \frac{20}{12}}{N - \frac{20}{12}} = \sqrt{\dfrac{10.2}{9.9}}$ or $N = 223$ Hz

17. $n = \dfrac{1}{2l}\sqrt{\dfrac{W}{m}}$

where $\quad W$ = weight of load in air

$$n' = \frac{1}{2l}\sqrt{\frac{W'}{m}}$$

where $\quad W'$ = weight of load in water

Obviously $\quad n' < n$ or n because $W' < W$

$\therefore \quad n - n' = 5$ or $n' = n - 5$

or $\quad \left(\dfrac{n}{n-5}\right)^2 = \dfrac{W}{W'}$

$W = V(9000)g$

$W' = W - B$

$= V9000g - V(1000)g$

$= V(8000)g$

So $\quad \dfrac{W}{W'} = \dfrac{9}{8}$

$\therefore \quad \dfrac{9}{8} = \left(\dfrac{n}{n-5}\right)^2$ or $n = 87.4$ Hz

18. $n = \dfrac{c_{17}}{2l}$ where l = length of the pipe

$\therefore \quad 256 - \dfrac{c_{17}}{2l} = 4$ or $\dfrac{c_{17}}{2l} = 252$

Since beats decrease first and then increase to 4, the frequency of the pipe increases. This can happen only if the temperature increases.

Let t be the final temperature, in Celsius.

Now $\quad \dfrac{c_t}{2l} - 252 = 4$ or $\dfrac{c_t}{2l} = 260$

Dividing $\quad \dfrac{c_t}{c_{17}} = \dfrac{260}{252}$

or $\quad \sqrt{\dfrac{273+t}{273+17}} = \dfrac{260}{252} \qquad \left(c \propto \sqrt{T}\right)$

or $\quad t = 308.7 - 273 = 35.7$°C

$\therefore$ Rise in temperature = 35.7 – 17 = 18.7°C

19. We have $c = \sqrt{T/m}$. If l = distance between supports, then τ (time to travel the distance between supports) = l/c

The frequency of fundamental mode is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{c}{2l}$$

$\therefore \tau'$ (period of fundamental mode)

$$\tau' = \frac{1}{n} = \frac{2l}{c} = 2\tau$$

20. The two ends of the pipe are the seats of nodes just as in a stretched string. Hence, the air inside emits the full series of harmonics. Resonance occurs when the frequency of the membrane matches with one of the harmonics of the air column in the pipe. Let the nth thrmonic of the column of length l (here l is also the distance of the piston from the membrane) resonate with the tuning fork. Then

$$n \times \frac{c}{2l} = \nu$$

where ν = frequency of the tuning fork

$$nc = 2l\nu$$

Let the air column resonate again when the piston is moved by x away from the membrane, that is, the distance of the piston is $(l + x)$. Now the membrane resonates with $(n + 1)$th harmonic of the new column of air.

Therefore, $\quad (n+1)\dfrac{c}{2(l+x)} = \nu$

or $\quad (n + 1)c = 2\nu\,(l + x)$

Subtracting $\quad c = 2\nu x = 2 \times 2000 \times 8.5 \times 10^{-2}$

$= 340$ m/s

21. The frequency of the fundamental mode of the open pipe filled with air

$$= \frac{c}{2l} = \frac{330}{2 \times 0.11} = 1500 \text{ Hz}$$

The first overtone of the closed pipe

$$= 3 \times \frac{c_{gas}}{4 \times 0.132}$$

$\therefore \quad 3 \times \dfrac{c_{gas}}{4 \times 0.132} = 1500$ or $c_{gas} = 264$ m/s

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22. $f=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2\times0.25}\sqrt{\frac{10\times9.8}{0.5\times10^{-3}}\times\frac{1}{0.25}}$ = 1770.8 Hz

A string emits the full series of harmonics. The highest frequency that a man of normal hearing can detect is 20,000 Hz. Let n be the highest harmonic that can be detected by a man of normal hearing. Then

$n \times 1770.8 = 20000$ or $n = 11.29$

$\therefore$ Acceptable value = 11

$\therefore$ Highest overtone = 11 – 1 = 10

23. $n_A=\frac{c}{4\times0.32}$ and $n_B=\frac{c}{4\times0.33}$

Obviously $n_A > n_B$

$\therefore \quad n_A-n_B=\frac{40}{5}$

$\therefore \quad \frac{c}{4\times0.32}-\frac{c}{4\times0.33}=8$ or $c = 337.92$ m/s

$\therefore \quad n_A=\frac{337.92}{4\times0.32}=264$ Hz

and $\quad n_B = 264 - 8 = 256$ Hz

24. a. $n_0=\frac{c}{4l}$ where n_0 = frequency of the fundamental

$$n_0=\frac{330}{4\times0.15}=550 \text{ Hz}$$

The first four overtones are $3n_0$, $5n_0$, $7n_0$ and $9n_0$.
So, the required frequencies are 550, 1650, 2750, 3850 and 4950 Hz.

b. $n_0=\frac{c}{2l}=\frac{330}{2\times0.15}=1100$ Hz

The first four overtones are $2n_0$, $3n_0$, $4n_0$ and $5n_0$.
So, the required frequencies are 1100, 2200, 3300, 4400 and 5500 Hz.

c. The frequency of the nth overtone is $(2n + 1)n_0$ or $(n + 1)n_0$.

$\therefore (2n + 1)\, n_0 = 20{,}000$ or $(n + 1)1100 = 20{,}000$

$\Rightarrow$ $n = 17.18$ or $n = 17.18$

The acceptable value is 17.

25. a. $n_0=\frac{c}{2l}$ or $c = 2n_0 l$

$\therefore c = 2 \times 200 \times 1 = 400$ m/s

b. $c=\sqrt{\frac{T}{m}}=\sqrt{\frac{\pi r^2\times \text{stress}}{\pi r^2\times \text{density}}}=\sqrt{\frac{\text{stress}}{\text{density}}}$

$\therefore$ Stress = $c^2\rho$ = $400^2 \times 8000 = 1.28 \times 10^9$ N/m^2

c. Acceleration = $-\omega^2 \times$ displacement

$\therefore 800 = (2\pi n)^2 \times a$ where a = amplitude

or $\quad a=\frac{800}{(2\pi\times200)}=5\times10^{-4}$ m

26. n_0 (fundamental frequency) = $\frac{1}{2l}\sqrt{T/m}$

Here $\quad m = \pi r^2\rho = \pi(0.02 \times 10^{-2})^2 \times 8000$

$= 32\pi \times 10^{-5}$ kg/m

$\therefore \quad n_0=\frac{1}{2\times0.5}\sqrt{\frac{80}{32\ \times10^{-5}}}=280$ Hz

Obviously 840 Hz is its 3rd harmonic. The third harmonic needs a node at 1/3 point. So strike the wire at 1/6 point and touch gently 1/3 point to make 840 Hz dominant.

27. Since the node-to-node distance is $\lambda/2$

$\lambda/2 = 0.08$ or $\lambda = 0.16$ m

i. $c = n\lambda$

$\therefore c = 2000 \times 0.16 = 320$ m/s

ii. $320 = 1600 \times \lambda$ or $\lambda = 0.2$ m

$\therefore$ Distance between nodes = 0.2/2 = 0.1 m = 10 cm

iii. Since there are nodes at the ends, the distance between the closed end and the membranes must be exact integrals of $\lambda/2$.

$\therefore \quad l=n\times0.16/2$ and $l=n'\times0.2/2$

$\Rightarrow \quad \frac{n}{n'}=\frac{5}{4}$

When $\quad n = 5, \quad n' = 4$

$l = 5 \times 0.16/2 = 0.4$ m = 40 cm

iv. For the next lower frequency $n = 3$

$\therefore 0.4 = 3\lambda/2$ or $\lambda = 0.8/3$

Since $\quad c = n\lambda, \quad n=\frac{320}{0.8/3}=1200$ Hz

28. $n_1=\frac{1}{2\times0.5}\sqrt{\frac{4.5\times10}{m}}$

$$n_2=\frac{1}{2\times0.6}\sqrt{\frac{5.12\times10}{m}}$$

Dividing $\frac{n_1}{n_2}=\frac{9}{8}$ (Obviously $n_1 > n_2$)

$\therefore \quad n_1-n_2=2 \Rightarrow \frac{9}{8}n_2-n_2=2$

$\Rightarrow \quad n_2 = 16, \; n_1 = 18$

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$$\therefore \quad 16 = \frac{1}{2 \times 0.6}\sqrt{\frac{5.12 \times 10}{m}} \Rightarrow m = 0.14 \text{ kg/m}$$

29. Since the mode has an antinode at 1/4 point

$$l/4 = \lambda/4 \quad \text{or} \quad l = \lambda \quad \text{and} \quad 100 = \frac{c}{l} \qquad (c = n\lambda)$$

In the next mode, between the nearest end and 1/4 point from this end, there must appear one node and antinode.

$\therefore l/4 = \frac{3\lambda'}{4}$ where λ' is the wavelength of the next mode

$$\Rightarrow \quad \lambda' = l/3$$

$$\therefore \quad n' = \frac{c}{l/3} = \frac{3c}{l} = 3 \times 100 = 300 \text{ Hz}$$

30. Stress $= Y \times$ strain $= Y \times \frac{1}{100} = \frac{2 \times 10^{11}}{100} = 2 \times 10^9$

$$T(\text{tension}) = \pi r^2 \times \text{stress} = \pi r^2 \times 2 \times 10^9$$

$$m = \pi r^2 \rho = \pi r^2 \times 7700$$

$$\therefore \quad n_0 = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 1.5}\sqrt{\frac{\pi r^2 \times 2 \times 10^9}{\pi r^2 \times 7700}}$$

$$= 170 \text{ Hz}$$

31. **a.** Number of beats/s $= n_1 = n_2 = 302 - 300 = 2$.

b. Maximum amplitude

$$A_{max} = a_1 + a_2 = 0.2 + 0.3 = 0.5 \text{ mm}$$

Minimum amplitude,

$$A_{min} = a_1 - a_2 = 0.3 - 0.2 = 0.1 \text{ mm}$$

c. $$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(0.5)^2}{(0.1)^2} = \frac{25}{1}$$

$$\therefore \quad I_{max} : I_{min} = 25 : 1$$

32. Frequency of A

$$n_A = 400 \text{ s}^{-1}$$

Number of beats/s = 3.

$\therefore$ Possible frequencies of B,

$$n_B = 400 \pm 3 = 403 \quad \text{or} \quad 397$$

Verification of correct value. A small piece of wax is loaded on B, which decrease the frequency of B. Now the two forks A and B are again sounded, if the number of beats is less than 3, the possible frequency of B is 403, but if number of beats is greater than 3, the possible frequency of B is 397.

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