PHYSICS
Optics & Modern Physics

B.M. Sharma

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Physics for JEE:
Optics & Modern Physics

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Preface

While the paper-setting pattern and assessment methodology have been revised many times over and newer criteria devised to help develop more aspirant-friendly engineering entrance tests, the need to standardize the selection processes and their outcomes at the national level has always been felt. A combined national-level engineering entrance examination has finally been proposed by the Ministry of Human Resource Development, Government of India. The Joint Entrance Examination (JEE) to India's prestigious engineering institutions (IITs, IIITs, NITs, ISM, IISERs, and other engineering colleges) aims to serve as a common national-level engineering entrance test, thereby eliminating the need for aspiring engineers to sit through multiple entrance tests.

While the methodology and scope of an engineering entrance test are prone to change, there are two basic objectives that any test needs to serve:

1. The objective to test an aspirant's caliber, aptitude, and attitude for the engineering field and profession.
2. The need to test an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grassroot level.

Students appearing for various engineering entrance examinations cannot bank solely on conventional shortcut measures to crack the entrance examination. Conventional techniques alone are not enough as most of the questions asked in the examination are based on concepts rather than on just formulae. Hence, it is necessary for students appearing for joint entrance examination to not only gain a thorough knowledge and understanding of the concepts but also develop problem-solving skills to be able to relate their understanding of the subject to real-life applications based on these concepts.

This series of books is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts. It uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. The objective of each book in this series is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time. These books feature all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, comprehension type, or integer type questions. These problems have skillfully been set to help students develop a sound problem-solving methodology.

Not discounting the need for skilled and guided practice, the material in the books has been enriched with a number of fully solved concept application exercises so that every step in learning is ensured for the understanding and application of the subject. This whole series of books adopts a multi-faceted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter.

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which equips the students with the knowledge of the field and paves a confident path for them to accomplish success in the JEE.

With best wishes!

B.M. Sharma

CHAPTER 1

Geometrical Optics

INTRODUCTION

Light is a form of radiant energy, that is, energy emitted by excited atoms or molecules which can cause the sensation of vision in a normal human eye.

The branch of Physics which deals with the phenomena concerning light is called Optics. There are two branches of Optics:

a. Geometrical Optics: This consists the study of light in which light is considered as moving along a straight line as a ray. A ray of light gives the direction of propagation of light. When light meets a surface which separates two media, reflection and refraction take place. An image or an array of images may be formed due to this.

b. Physical Optics: It deals with the theories regarding the nature of light and provides an explanation for the different phenomena in light, such as reflection, refraction, interference, diffraction, polarisation, and rectilinear propagation.

SOME DEFINITIONS

(i) A Ray: The 'path' along which light travels is called a ray. They are represented by straight lines with arrows directed towards the direction of travel of light.

(ii) A Beam: A bundle of rays is called a beam. A beam is parallel when its rays are parallel; divergent when its rays spread out from a point; and convergent when its rays meet at a point.

(iii) A Pencil: A narrow beam is called a pencil of light.

NATURE OF OBJECTS AND IMAGES

Ray Optics primarily deals with determining the position and nature of the image formed when an object is placed in front of an optical element. Before we proceed further, let us clearly define what is an object and what is an image.

Types of Objects

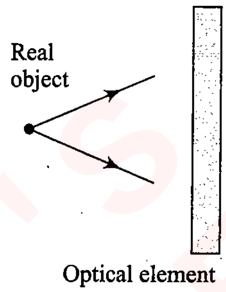
An object is a source of light rays that are incident on an optical element. An object may be a point object or an extended object. Since extended objects can be modeled as a collection of points, we only need to study point objects. Objects are of two kinds: Real objects and Virtual objects.

Real Object

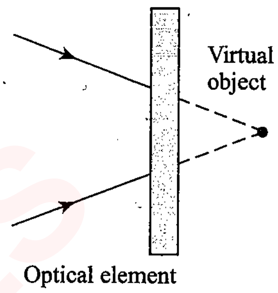
An object is real if two or more incident rays actually emanate or seem to emanate from a point. Fig. 1.1(a) is a typical example of a real object. In this case, two rays emanate from the object and are incident on the optical element and the object is actually present. Hence, it is called a real object.

Virtual Object

Now, consider a converging set of rays as shown in Fig. 1.1(b). If not intercepted the rays will meet at a point. However, if the rays are intercepted by an optical element placed as shown in the figure, then the point of convergence is a virtual point behind the optical element. This point is called the virtual object for the optical element.



A real object placed in front of an optical element
(a)



A virtual object that appears to be behind the optical element
(b)

Fig. 1.1

So, an object is real when two rays emanate or diverge from a point, and an object is virtual when two incident rays seem to converge to that point.

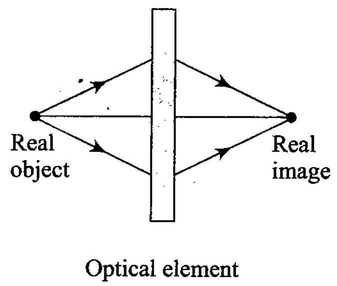
Types of Images

What is an Image?

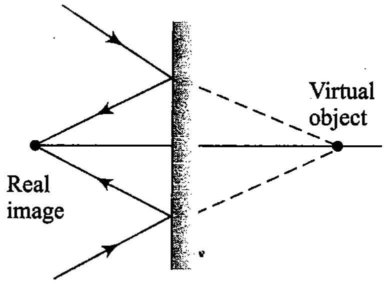
An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image. As in the case of objects, images too can be real or virtual.

Real Image

Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at that point, a bright spot will be visible on the screen. Thus, a real image can be captured on a screen. Examples of real images are shown in Fig. 1.2(a) and (b). Note that in the former the object is real while in the latter the object is virtual. Thus, both real and virtual objects can form real images.

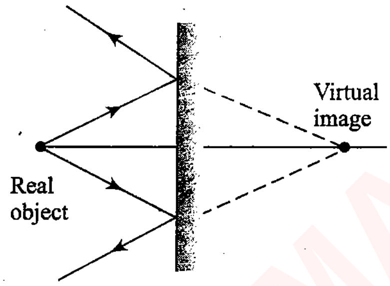


A real image formed by a real object placed in front of the convex lens
(a)



Plane Mirror

A real image formed by a virtual object that appears to be behind the mirror
(b)



Plane Mirror

A virtual image formed by a real object in front of the mirror
(c)

Fig. 1.2

Virtual Image

When light rays, after interacting with the optical element, actually meet at a point the image formed is a real image. However, if the rays do not meet at a point but appear to emanate from a point, then a virtual image is formed. Consider the case of an object placed in front of a plane mirror. Here, the two reflected rays will never meet at any point. However, if we extend the reflected rays backwards, they appear to emanate from a point. This point is the virtual image of the object. As we will see later in the section on Lenses, it is possible for a virtual image to be formed by a virtual object as well. Thus, we can conclude that both virtual and real images can be formed by either real or virtual objects depending on the optical element.

BASIC LAWS

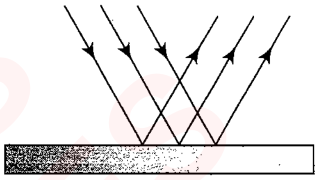
- **Law of Rectilinear Propagation of Light:** It states that light propagates in straight lines in homogeneous media.
- **Law of Independence of Light Rays:** It states that rays do not disturb each other upon intersection.
- **Law of Reversibility of Light Rays:** It states that rays retrace their paths when their direction is reversed.

REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

Regular Reflection

When the reflection takes place from a perfect plane surface it is called *Regular Reflection* (see Fig. 1.3(a)). In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.



(a) Regular reflection

Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface. It is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as ***diffused reflection*** (See Fig. 1.3(b)). For example, reflection from a wall, from a newspaper, etc. This is why you cannot see your face in the newspaper and in the wall.

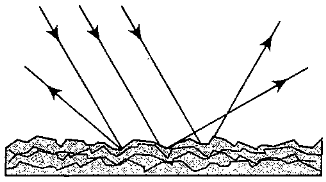


Fig. 1.3(b) Diffused reflection

Laws of Reflection

It has been found experimentally that rays undergoing reflection follow two laws called the Laws of Reflection:

(i) The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. This plane is called the *plane of incidence* (or *plane of reflection*).

(ii) The ***angle of incidence*** (the angle between normal and the incident ray) and the ***angle of reflection*** (the angle between the reflected ray and the normal) are equal, i.e.,

$$\angle i = \angle r$$

Special Cases

Normal Incidence: In case light is incident normally [see Fig. 1.4(b)],

$$i = r = 0$$
$$\delta = 180°$$

Grazing Incidence: In case light strikes the reflecting surface tangentially [see Fig. 1.4(c)],

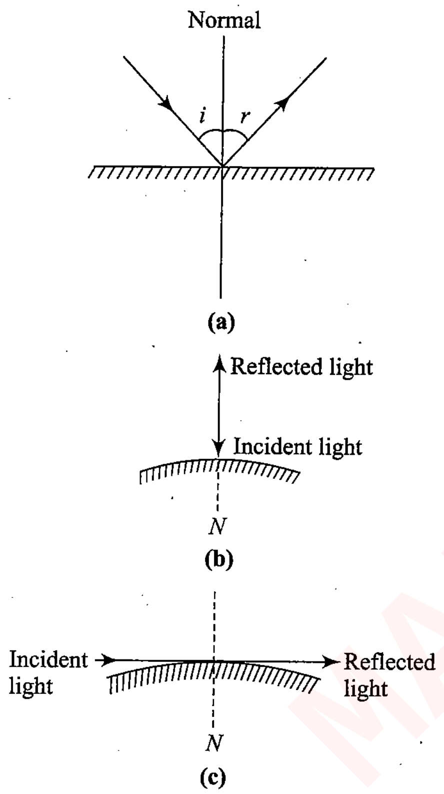


Fig. 1.4

$$i = r = 90°$$
$$\delta = 0° \quad \text{or} \quad 360°$$

Illustration 1.1 **Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.**

Sol. From Fig. 1.5(b), it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 (= $2\pi - \delta_1$) clockwise.

From Fig. 1.5(a), $\quad \delta_1 = \pi - 2i.$

$$\therefore \quad \delta_2 = 2\pi - (\pi - 2i) = \pi + 2i$$

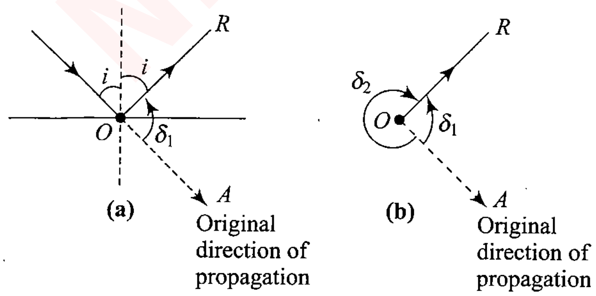


Fig. 1.5

REFLECTION FROM A PLANE SURFACE: PLANE MIRROR

A plane mirror is formed by polishing one surface of a plane thin glass plate (see Fig. 1.6). It is also said to be silvered on one side.

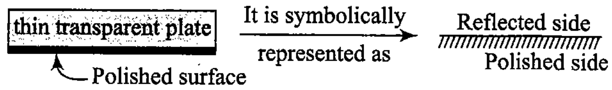


Fig. 1.6 Plane mirror

A beam of parallel rays of light, incident on a plane mirror, will get reflected as a beam of parallel reflected rays.

Illustration 1.2 **For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.**

Sol. In Fig. 1.7, M_1, N_1 and R_1 indicate the initial positions of mirror, normal, and direction of reflected light ray, respectively. M_2, N_2 and R_2 indicate the final position, of mirror, normal, and direction of reflected light ray, respectively.

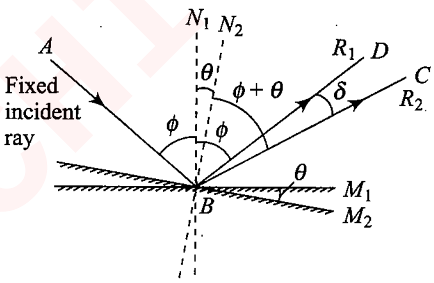


Fig. 1.7

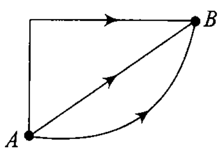
If the mirror is rotated by an angle θ, new angle of incidence $= \angle ABN_2 = \phi + \theta$

Angle of deviation of final ray $= \angle DBC = \delta$

$$\angle DBC = \angle ABC - \angle ABD = 2(\phi + \theta) - 2\phi = 2\theta$$

IMAGE FORMATION FROM PLAIN MIRROR

An explanation for the laws of reflection was provided by Fermat who postulated that a ray of light travels from point A to point B in a path that takes the shortest time. For example, if A and B are two points in the same medium as shown in Fig. 1.8, then the path with the shortest length is the straight line joining A and B. Thus, the light ray will travel in a straight line from A to B.



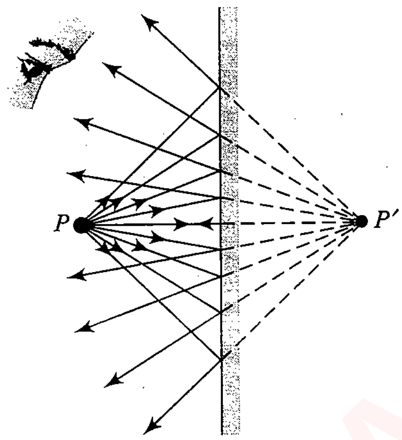
A ray of light can travel from point A to point B in multiple paths. The shortest path is the straight line joining them.

Fig. 1.8

Characteristics of image due to reflection by a plane mirror:

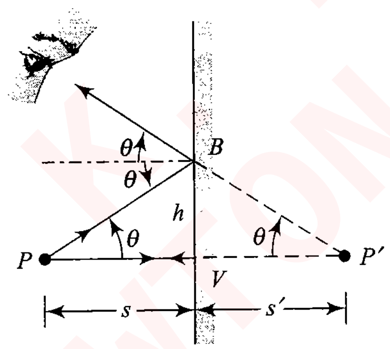
(i) Distance of object from mirror = Distance of image from the mirror.

All the incident rays from a point object after reflection from a plane mirror will meet at a single point which is called image [see Fig. 1.9(a)].



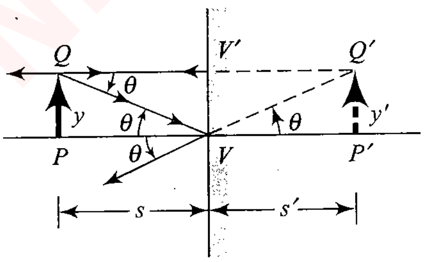
(a)

(ii) The line joining a point object and its image is normal to the reflecting surface [see Fig. 1.9(b)].



(b)

(iii) The size of the image is the same as that of the object [see Fig. 1.9(c)].

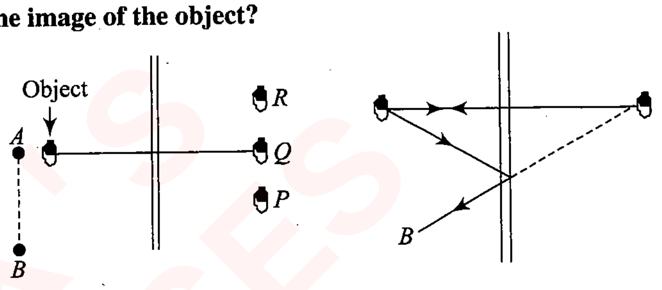


(c)

Fig. 1.9

(iv) For a real object the image is virtual and for a virtual object the image is real.

Illustration 1.3 **Figure 1.10(a) shows an object placed in front of a plane mirror. *P*, *Q* and R are the three positions where the image of object may be seen. Observer *A* is able to see the image at position *Q*. Where does the observer *B* see the image of the object?**



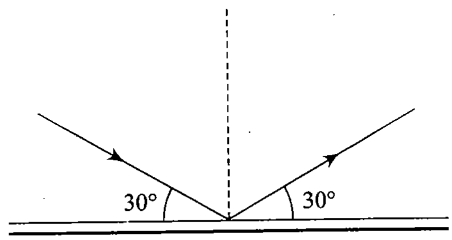
(a) (b)

Fig. 1.10

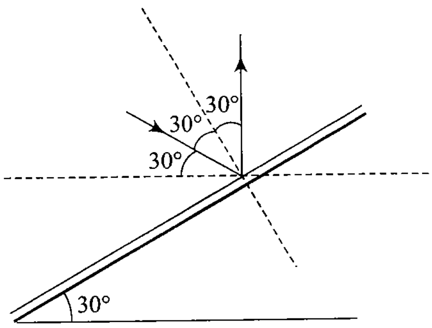
Sol. The observer *B* also observes the image of the object at the position *Q* which is explained by the ray diagram in Fig. 1.10(b). ***The position of image will be independent of the position of the observer.***

Illustration 1.4 **A light ray is incident on a plane mirror at an angle of 30° with the horizontal. At what angle with horizontal must a plane mirror be placed in its path so that it becomes vertically upwards after reflection?**

Sol. To make the reflected ray vertical, it should be rotated through an angle of 60°. So, the mirror should be tilted by 60°/2 = 30°, as shown in Fig. 1.11(b).



(a)



(b)

Fig. 1.11

Illustration 1.5 **Figure 1.12 shows a point object *A* and a plane mirror *MN*. Find the position of the image of object *A*,**

in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called field of view.)

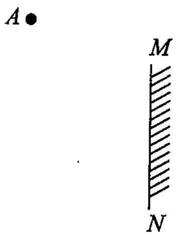
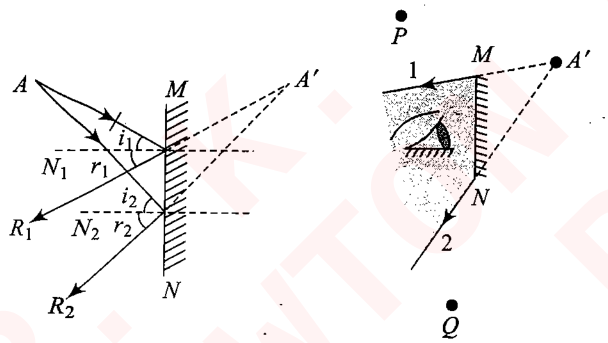


Fig. 1.12

Sol. Consider any two rays emanating from the object (see Fig. 1.13(a)). N_1 and N_2 are normals; $i_1 = r_1$ and $i_2 = r_2$.

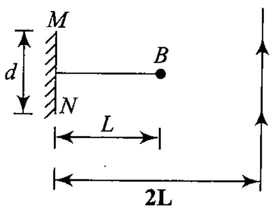
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror MN such that their meeting point is A'. To obtain the region in which reflected rays are present, join A' with the ends of mirror and extend. Fig. 1.13(b) shows this region as shaded. In the figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q will not be able to see the image because they do not receive any reflected ray.



(a) (b)

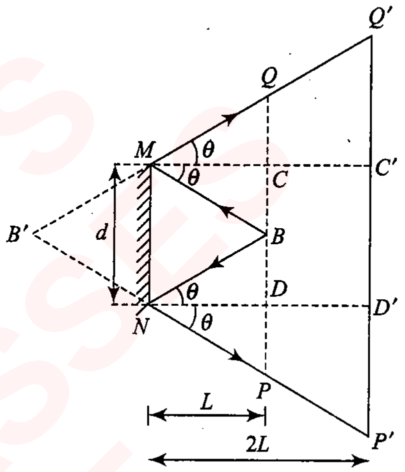
Fig. 1.13

Illustration 1.6 **A point source of light B is placed at a distance L in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it, as shown in Fig. 1.14(a). Find the greatest distance over which he can see the image of the light source from the mirror.**



(a)

Sol. Draw the rays BM and BN incident on the mirror edges, and draw the reflected rays (see Fig. 1.14(b)). From the figure, it is clear that the man can observe in the angular region $P'B'Q'$. The observer can see through the distance $P'Q'$, where reflected ray from B can meet the line.



(b)

Fig. 1.14

From geometry, $CD = d = C'D'$

$BD = DP = d/2;\ CB = QC = d/2$

$P'Q' = C'D' + P'D' + Q'C'$

$$\Rightarrow \quad \frac{ND}{DP} = \frac{ND'}{D'P'} \Rightarrow \frac{L}{d/2} = \frac{2L}{D'P'} \Rightarrow D'P' = d$$

Similarly, $Q'C' = d$

Hence, distance through which the man can observe the image $= Q'C' + C'D' + D'P' = d + d + d = 3d$

IMAGE OF EXTENDED OBJECT FORMED BY PLANE MIRROR

An extended object like AB shown in Fig. 1.15 is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A' will be image of A, C' will be image of C, B' will be image of B, etc. All point images together form an extended image. Thus, extended image is formed of an extended object.

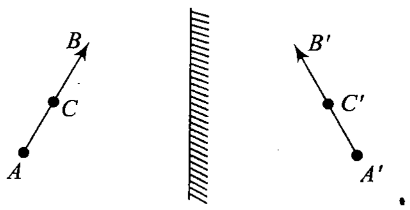
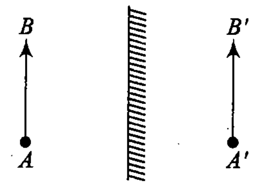


Fig. 1.15

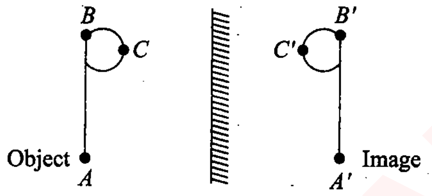
Properties of image of an extended object, formed by a plane mirror:

1. Size of extended object = size of extended image [see Fig. 1.16(a)].



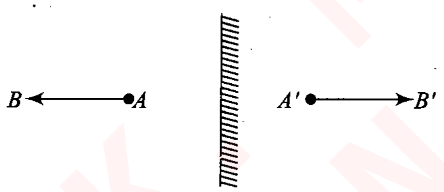
(a)

2. The image is upright, if the extended object is placed parallel to the plane mirror [see Fig. 1.16(b)].



(b)

3. The image is inverted, if the extended object lies perpendicular to the plane mirror [see Fig. 1.16(c)].

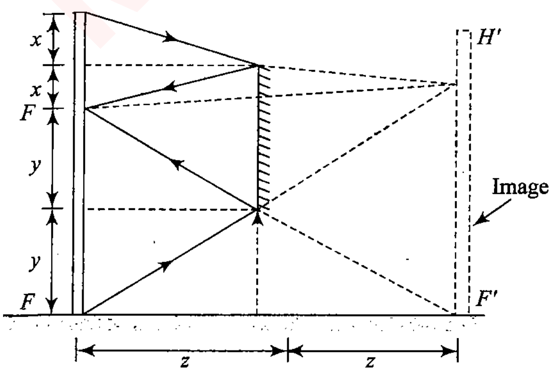


(c)

Fig. 1.16

Illustration 1.7 **Show that the minimum size of a plane mirror required to see the full image of an observer is half the size of that observer.**

Sol. Let HF is the height of the man and 'E' is the eye level. Draw the rays HM_1 and FM_2 incident at the edges of the mirror. Complete the ray diagram as shown in Fig. 1.17. It is self-explanatory if you consider lengths 'x' and 'y' as shown in the figure.



H, Head; F, Feet; E, Eye

Fig. 1.17

Aliter:

ΔEM_1M_2 and $\Delta EH'F'$ are similar

$$\therefore \quad \frac{M_1M_2}{H'F'} = \frac{z}{2z} \quad \text{or} \quad M_1M_2 = H'F'/2 = HF/2$$

Note: The height of the mirror is half the height of eye as shown in the figure.

RELATION BETWEEN VELOCITY OF OBJECT AND IMAGE

In case of plane mirror, distance of the object from the mirror is equal to distance of image from the mirror (see Fig. 1.18).

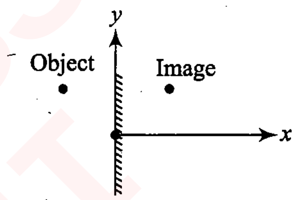


Fig. 1.18

Hence, from the mirror property:

$$x_{im} = -x_{om}, \quad y_{im} = y_{om} \quad \text{and} \quad z_{im} = z_{om}$$

Here, x_{im} means 'x' coordinate of image with respect to mirror. Similarly, others have meaning.

Differentiating w.r.t. time, we get

$$v_{(im)x} = -v_{(om)x}; \quad v_{(im)y} = v_{(om)y}; \quad v_{(im)z} = v_{(om)z},$$

Here, v_{om} = velocity of the object w.r.t. mirror

v_{im} = velocity of the image w.r.t. mirror

$$\Rightarrow \quad v_i - v_m = -(v_o - v_m) \quad \text{(for } x\text{-axis)}$$

In the direction normal to the mirror = | Relative velocity of image w.r.t. mirror | = | Relative velocity of object w.r.t. mirror |

But $\quad v_i - v_m = (v_o - v_m)$

or $\quad v_i = v_o \quad$ for y- and z-axis.

Here, v_i = velocity of image with respect to ground

v_o = velocity of object w.r.t. ground

Velocity of object is equal to velocity of image parallel to mirror surface.

We can write

$$\vec{v}_{om} = \vec{v}_o - \vec{v}_m \quad \text{and} \quad \vec{v}_{im} = \vec{v}_i - \vec{v}_m$$

Illustration 1.8 **Figure 1.19 shows a plane mirror and an object that are moving towards each other. Find the velocity of image.**

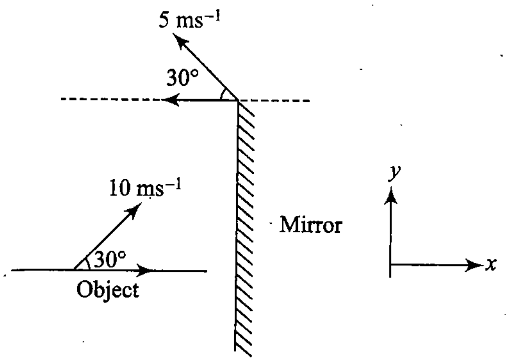


Fig. 1.19

Sol. Along x-direction, applying (relative velocity of image w.r.t. mirror) = – (relative velocity of object w.r.t. mirror).

$\Rightarrow \quad v_i - v_m = -(v_0 - v_m)$

$\Rightarrow \quad v_i - (-5\cos 30°) = -(10\cos 60° - (-5\cos 30°))$

$\therefore \quad v_i = -5(1+\sqrt{3})\ \text{ms}^{-1}$

In the direction parallel to the surface of mirror:

Along y-direction, $v_0 = v_i$

$\therefore \quad v_i = 10\sin 60° = 5\ \text{ms}^{-1}$

$\therefore$ Velocity of the image $= -5(1+\sqrt{3})\,\hat{i} + 5\hat{j}\ \text{ms}^{-1}$.

IMAGES FORMED BY TWO PLANE MIRRORS

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. Let us understand this with the illustration discussed below.

Illustration 1.9 **Figure 1.20 shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.**

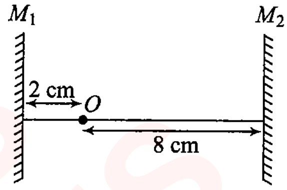


Fig. 1.20

Sol. To understand how images are formed see Fig. 1.21 and the following table. The image formed from the object will act as an object for next reflection. This process will continue again and again upto infinite reflection. You will require to know what symbols like I_{121} stands for. See the following diagram.

Similarly, images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Incident rays	Reflected by	Reflected rays	Object	Image	Object distance (cm)	Image distance (cm)
Rays 1	M_1	Rays 2	O	I_1	$AO = 2$	$AI_1 = 2$
Rays 2	M_2	Rays 3	I_1	I_{12}	$BI_1 = 12$	$BI_{12} = 12$
Rays 3	M_1	Rays 4	I_{12}	I_{121}	$AI_{12} = 22$	$AI_{121} = 22$
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	$BI_{121} = 32$	$BI_{1212} = 32$

And so on ...

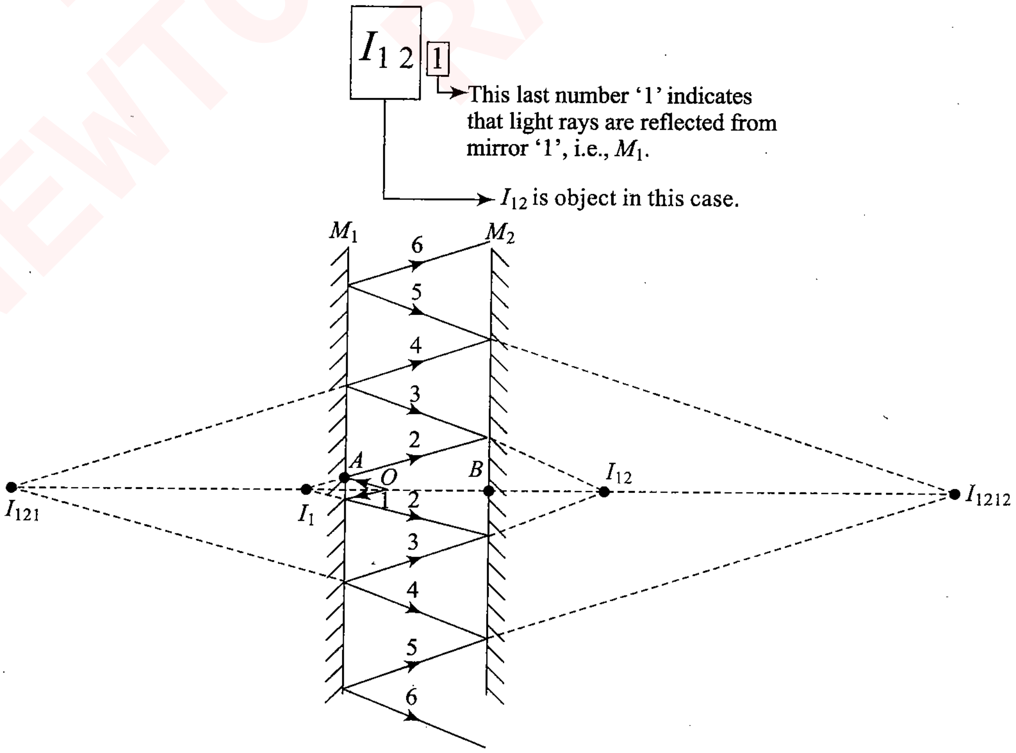


Fig. 1.21

LOCATING ALL THE IMAGES FORMED BY TWO PLANE MIRRORS

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in Fig. 1.22.

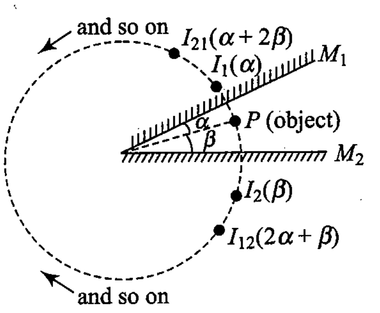


Fig. 1.22

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.

Now, I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21} and so on. Think when will this process stop. [*Hint:* The virtual image formed by a plane mirror must not be in front of the mirror or its extension.]

Number of images formed by two inclined mirrors:

(i) If $\dfrac{360°}{\theta}$ = even number; number of images = $\dfrac{360°}{\theta} - 1$

(ii) If $\dfrac{360°}{\theta}$ = odd number; number of images = $\dfrac{360°}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\dfrac{360°}{\theta}$ = odd number; number of images = $\dfrac{360°}{\theta}$, if the object is not placed on the angle bisector.

(iv) If $\dfrac{360°}{\theta} \neq$ integer, then count the number of images as explained above.

Illustration 1.10 **Consider two perpendicular mirrors M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinates of object as (x, y), find the position and number of images.**

Sol. As shown in Fig. 1.23, rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at $(x, -y)$, such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly, rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at $(-x, y)$, such that O and I_2 are equidistant from mirror M_2.

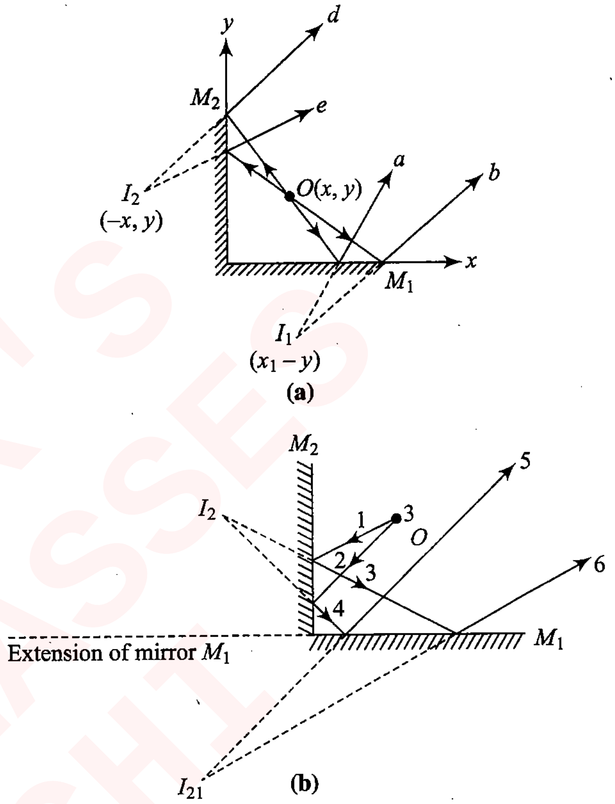


Fig. 1.23

Now, consider those rays which strike mirror M_2 first and then the mirror M_1.

For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.

Now, rays 3, 4 incident on M_1 (object is I_2) reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21} are equidistant from M_1. To summarize, see Fig. 1.24.

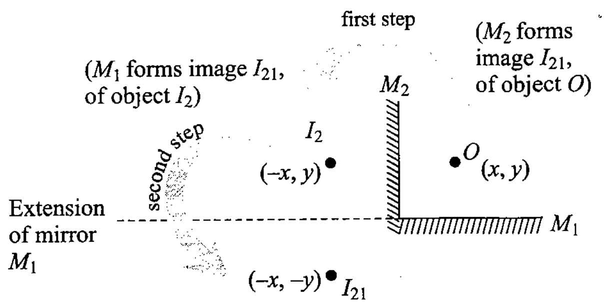


Fig. 1.24

For rays reflecting first from M_1 and then from M_2, first image I_1 (at $(x, -y)$) will be formed. I_1 will act as object for mirror M_2 and then its image I_{12} (at $(-x, -y)$) will be formed. I_{12} and I_{21} coincide.

Hence, ***three images are formed.***

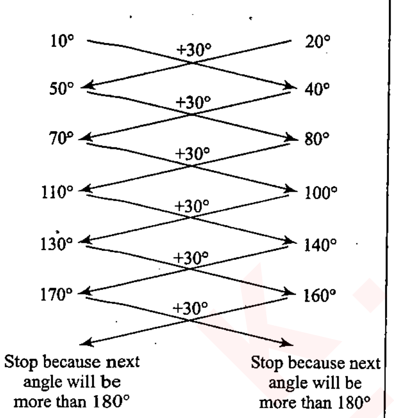
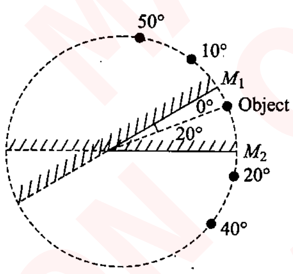
Illustration 1.11 **Two mirrors are inclined at an angle of 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.**

Sol. Number of images:

(i) Using direct formula: 360°/30° = 12 (even number)

Therefore, number of images = 12 – 1 = 11

(ii) By counting: See the following table

Image formed by mirror M_1 (angles are measured from the mirror M_1)	Image formed by mirror M_2 (angles are measured from the mirror M_2)	
		

To check whether the final images formed by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here, last angles made by the mirrors + the angles between the mirrors = 160° + 170° + 30° = 360°. Therefore, in this case the final images coincide.

Therefore, the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 – 1 (as the final images coincide) = 6 + 6 – 1 = 11.

Concept Application Exercise 1.1

1. Two plane mirrors M_1 and M_2 are inclined at angle as shown in Fig. 1.25. A ray of light 1, which is parallel to M_1, strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2. Find the angle θ.

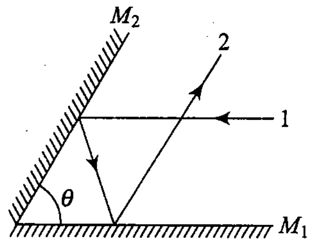


Fig. 1.25

2. Can we project the image formed by a plane mirror on to a screen? Give reasons.

3. Real object means that the object is actually present at the point where the incident ray originates. (True/ False)

4. Two plane mirrors are inclined at an angle of 60° as shown in Fig. 1.26. A ray of light parallel to M_1 strikes M_2. At what angle will the ray finally emerge?

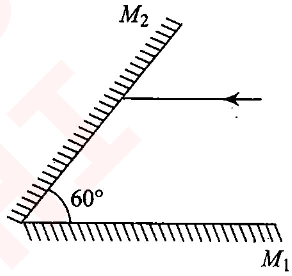


Fig. 1.26

5. Figure 1.27 shows two rays A and B being reflected by a mirror and going as A' and B'. The mirror

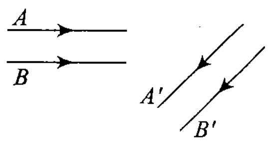


Fig. 1.27

a. is plane **b.** is convex
c. is concave **d.** may be any spherical mirror

6. Two plane mirrors are placed parallel to each other. The distance between the mirrors is 10 cm. An object is placed between the mirrors at a distance of 4 cm from one of them, say M_1. What is the distance between the first image formed at M_1 and the second image formed at M_2?

7. A ray of light travels from a light source S to an observer after reflection from a plane mirror. If the source rotates in the clockwise direction by 10°, by what angle and in what direction must the mirror be rotated so that the light ray still strikes the observer?

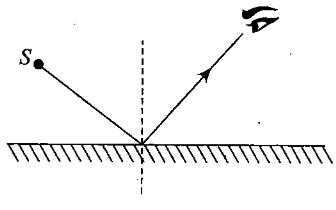


Fig. 1.28

8. Determine image location for the object in Fig. 1.29.

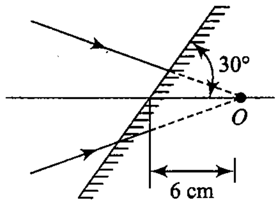


Fig. 1.29

9. Find the region on Y-axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown in Fig. 1.30.

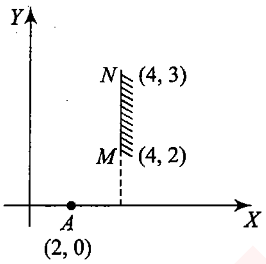


Fig. 1.30

10. An object moves with 5 ms^{-1} toward right while the mirror moves with 1 ms^{-1} toward the left as shown in Fig. 1.33. Find the velocity of image.

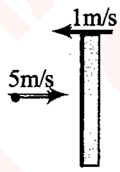


Fig. 1.31

11. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object, find the distance which the image will move.
12. A man is standing at distance x from a plane mirror in front of him. He wants to see the entire wall in mirror which is at distance y behind the man. Find the minimum size of the mirror required.
13. Find the velocity of the image when the object and mirror both are moving towards each other with velocities 2 and 3 ms^{-1}. How are they moving?
14. In Fig. 1.32, a plane mirror is moving with a uniform speed of 5 ms^{-1} along negative x-direction and observer O is moving with a velocity of 10 ms^{-1}. What is the velocity of image of a particle P, moving with a velocity as shown in the figure, as observed by observer O? Also find its direction.

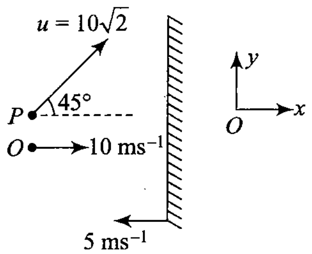
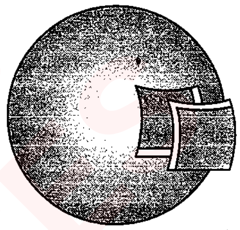


Fig. 1.32

REFLECTION FROM A CURVED SURFACE

Spherical Mirrors

A spherical mirror is formed by polishing one surface of a part of sphere. A spherical mirror is a reflecting surface whose shape is a section of a spherical surface (see Fig. 1.33).

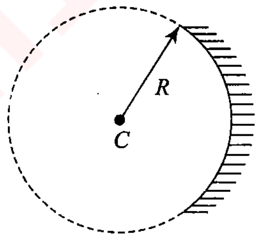


Spherical mirror

Fig. 1.33

Depending upon which part is shining, the spherical mirror is classified as:

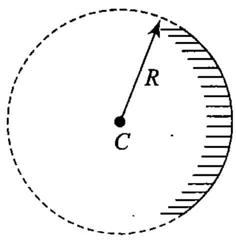
a. **Concave Mirror:** If the inside surface of the mirror is polished, it is a concave mirror (see Fig. 1.34).



Concave mirror

Fig. 1.34

b. **Convex Mirror:** If the outside surface of the mirror is polished, it is a convex mirror (see Fig. 1.35).



Convex mirror

Fig. 1.35

IMPORTANT TERMS

Pole (P): It is the geometrical center of the spherical reflecting surface (see Fig. 1.36).
A point on the surface of the mirror from where the position of the object can be specified easily is called pole. The pole is generally taken at the mid point of reflecting surface.

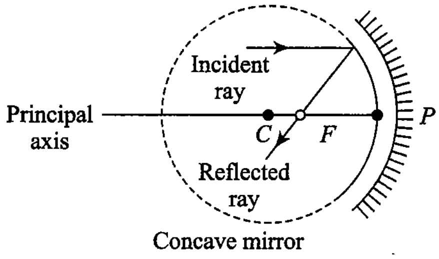


Fig. 1.36

Center of Curvature and Radius of Curvature: The center of the sphere of which the mirror is a part, is called *center of curvature(C)* (see Figs. 1.37, 1.38). The radius of the sphere of which the mirror is a part is called *radius of curvature(R)*. A plane mirror can be treated as a special case of a spherical mirror: one which has an *infinite radius of curvature*.

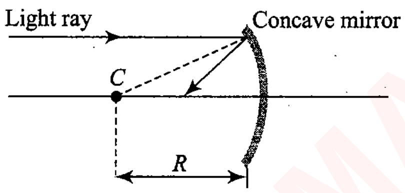


Fig. 1.37

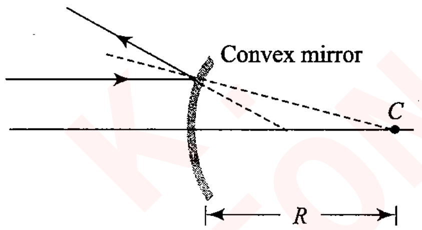


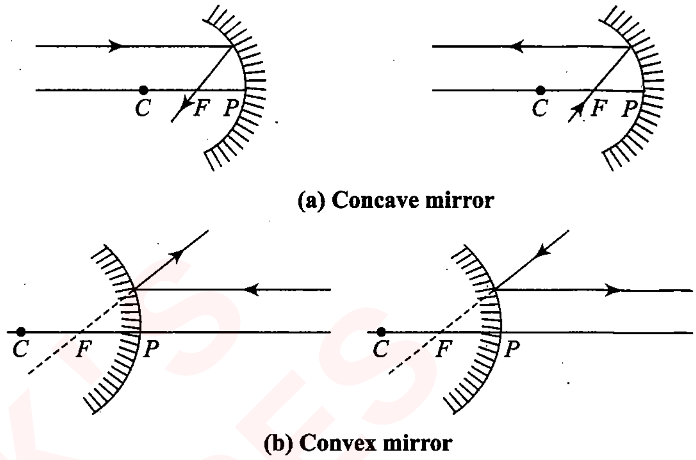
Fig. 1.38

Principal Axis: It is the straight line joining the center of curvature to the pole.

Principal Focus (*F*): It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In a concave mirror it is real and in a convex mirror it is virtual. The distance from pole to focus is called *focal length*.

Aperture (related to the size of mirror): It is the diameter of the mirror.

Focus (*F*): When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror, the reflected beam is found to coverage to or appears to diverge from a point on the principal axis. This point is called the focus (see Fig. 1.39).



(a) Concave mirror

(b) Convex mirror

Fig. 1.39

Focal Length (*f*): It is the distance between the pole and the principal focus. For spherical mirrors, $F = R/2$.

Illustration 1.12 **Find the angle of incidence of the ray shown in Fig. 1.40 for which it passes through the pole, given that $MI \parallel CP$.**

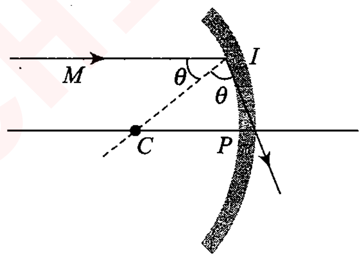


Fig. 1.40

Sol. From the figure, the ray MI is making an angle θ with normal IC. From law of reflection, $\angle MIC = \angle CIP = \theta$

As $\quad MI \parallel CP \Rightarrow \angle MIC = \angle ICP = \theta$

Now, $\quad CI = CP$

$\Rightarrow \quad \angle CIP = \angle CPI = \theta$

$\therefore$ In ΔCIP, all angles are equal, i.e., $3\theta = 180°$

$\Rightarrow \quad \theta = 60°$

Illustration 1.13 **Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also, find the distance CQ if $i \to 0$.**

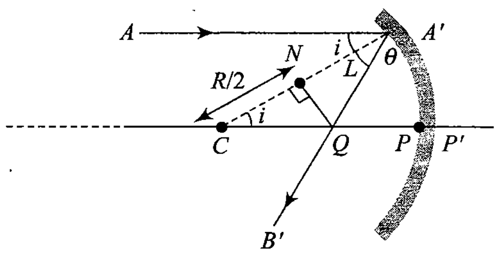


Fig. 1.41

Sol. In triangle CNQ, $\cos i = R/2CQ \Rightarrow CQ = R/2\cos i$

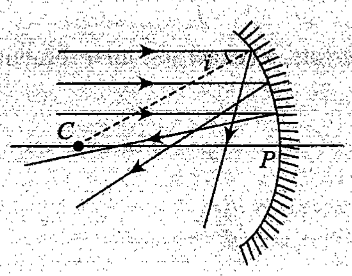
As i increases $\cos i$ decreases. Hence, CQ increases.

If i is a small angle, $\cos i \approx 1$ (see Fig. 1.42(b)).

$$\therefore \qquad CQ = R/2$$

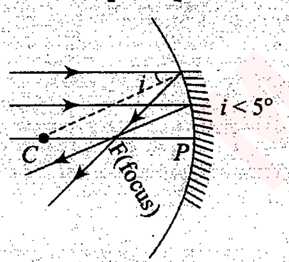
So, paraxial rays meet at a distance equal to $R/2$ from center of curvature, which is called focus.

Note:



(a)

(a) If angle of incident 'i' is more, the rays will focus at different points on the principal axis.



(b)

Fig. 1.42

(b) If angle of incident 'i' is small, the rays will focus at one point on principal axis. This the point is called Focus

SIGN CONVENTION: CARTESIAN CONVENTION

We will use following sign convention for problem solving in case of reflection as well as refraction.

- All distances are measured from the pole.
- Distances measured in the direction of incident rays are taken as positive.
- Distances measured in the direction opposite to that of the incident rays are taken as negative.
- Distances above the principal axis are taken as positive.
- Distances below the principal axis are taken as negative.

Angles measured from the normal in anticlockwise sense are positive, while that in clockwise sense are negative.

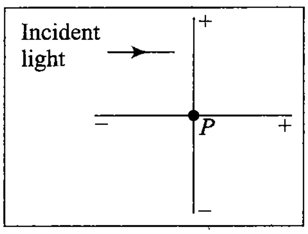
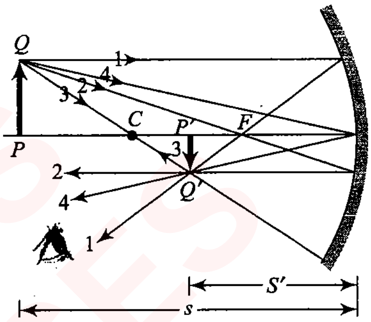


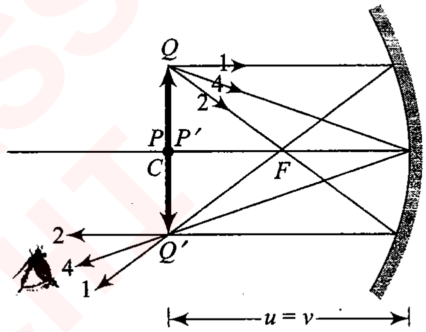
Fig. 1.43

RULES FOR RAY DIAGRAMS

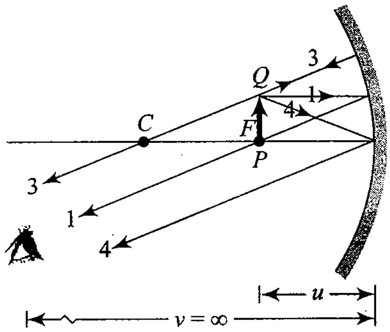
The position, size, and nature of the images formed by mirrors are conventionally expressed by ray diagrams (see Fig. 1.44).



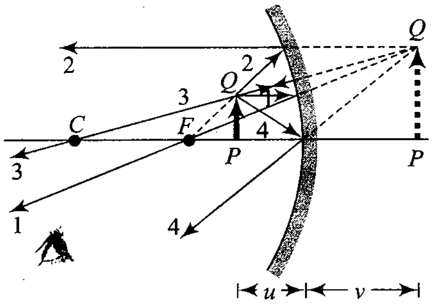
(a)



(b)



(c)



(d)

Fig. 1.44

We can locate the image of any extended object graphically by drawing any two of the following four special rays:

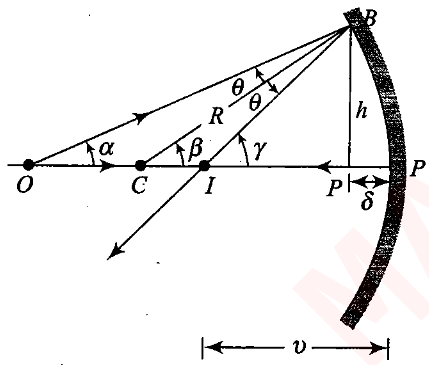
- A ray initially parallel to the principal axis is reflected through the focus of the mirror. (1)

- A ray passing through the center of curvature is reflected back along itself. (3)
- A ray initially passing through the focus is reflected parallel to the principal axis. (2)
- A ray incident at the pole is reflected symmetrically. (4)

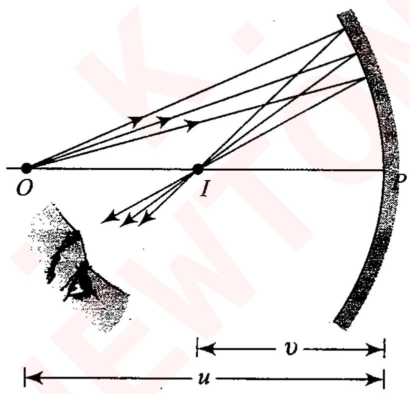
POSITION, SIZE AND NATURE OF IMAGE FORMED BY SPHERICAL MIRRORS

Mirror Formula

Consider Fig. 1.45(a) where O is a point object and I is the corresponding image.



(a)



(b)

Fig. 1.45

CB is normal to the mirror at B. By laws of reflection,

$$\angle OBC = \angle CBI = \theta$$

$$\alpha + \theta = \beta, \quad \beta + \theta = \gamma, \quad \alpha + \gamma = 2\beta$$

For small aperture of the mirror, α, β, γ

$\Rightarrow \quad \alpha \approx \tan \alpha, \quad \beta \approx \tan \beta, \quad \gamma = \tan \gamma, \quad P' \to P$

$\Rightarrow \quad \tan \alpha + \tan \gamma = 2 \tan \beta$

$$\Rightarrow \quad \frac{BP'}{OP'} + \frac{BP'}{IP'} = 2\frac{BP'}{CP'}$$

Applying sign convention,

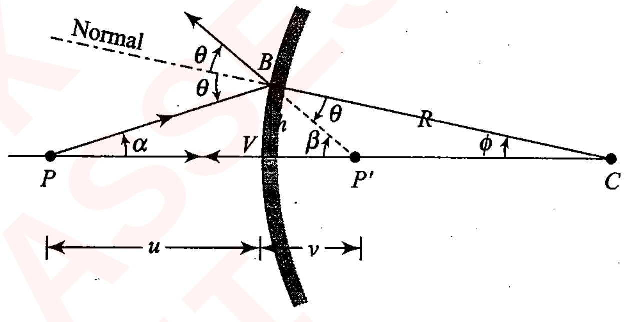
$$u = -OP, v = -IP, R = -CP$$

$$\Rightarrow \quad -\frac{1}{u} + \left(-\frac{1}{v}\right) = -\frac{2}{R}$$

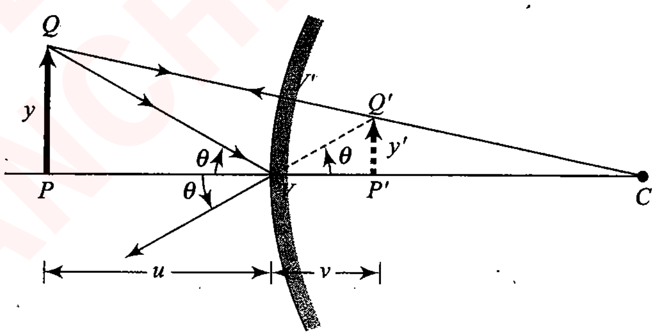
If $\quad u = \infty, \frac{1}{v} = \frac{2}{R}$, but by definition, if $u = \infty$, $v = f$.

Hence, $f = \dfrac{R}{2}$ and $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

For convex mirrors, an exactly similar formula emerges (see Fig. 1.46).



(a)



(b)

Fig. 1.46

Illustration 1.14 **When the position of an object reflected in a concave mirror of 0.25 m focal length is varied, the position of the image varies. Plot the image distance as a function of the object distance, taking the object distance from 0 to + ∞.**

Sol. Figure 1.47 shows the required graph.

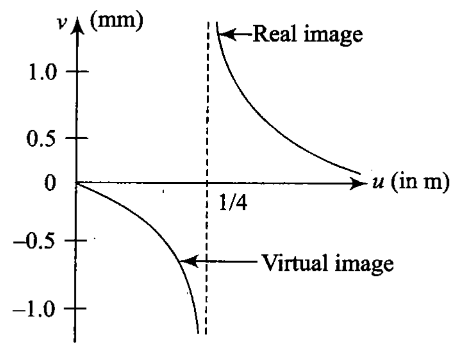


Fig. 1.47

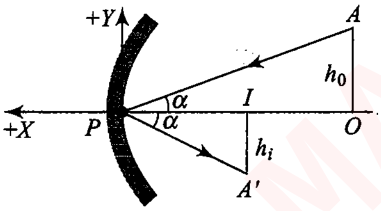
IMAGE FORMATION IN CONVEX MIRROR

Magnification

The lateral magnification is defined as the ratio

$$m_v = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_0}$$

To compute the vertical magnification, consider the extended object OA shown in Fig. 1.48. The base of the object, O, will map on to a point I on the principal axis which can be determined from the equation $(1/u) + (1/v) = (1/f)$. The image of the top of the object, A, will map on to a point A' that will lie on the perpendicular through I. The exact location can be determined by drawing a ray from A passing through the pole and intercepting the line through I at A'.



A real image of an extended object in front of a concave mirror

Fig. 1.48

Consider the triangles APO and $A'PI$ in the figure. As the two triangles are similar, we get

$$\tan\alpha = \frac{AO}{PO} = \frac{A'I}{PI} \quad \text{or} \quad \frac{A'I}{AO} = \frac{PI}{PO}$$

Applying the sign convention, we get, $u = -PO$

$$v = -PI \;\Rightarrow\; h_0 = +AO \;\Rightarrow\; h_i = -A'I$$

Therefore, $$-\frac{h_i}{h_0} = \frac{v}{u} \quad \text{or} \quad m_v = \frac{h_i}{h_0} = -\frac{v}{u}$$

Magnification from a Concave Mirror

Figure 1.49 shows a concave mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P. According to Cartesian sign convention, the mirror formula may be modified as $u = -x; f = -f_0$. Thus,

$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{-f_0} \quad \text{or} \quad v = \frac{xf_0}{f_0 - x}$$

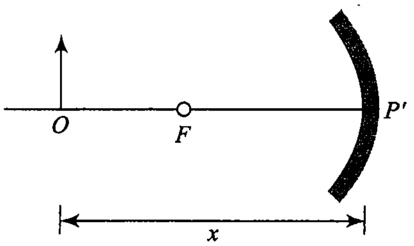


Fig. 1.49

And the magnification formula may be modified as

$$m = \frac{-v}{u} = \frac{y}{x} = \frac{f_0}{f_0 - x}$$

Magnification from a Convex Mirror

Figure 1.50 shows a convex mirror of focal length f_0 in front of which an object O is placed at a distance x from the pole P. According to Cartesian Sign Convention, the formulae may be modified as $u = -x$ and $f = +f_0$.

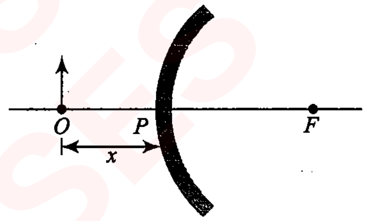


Fig. 1.50

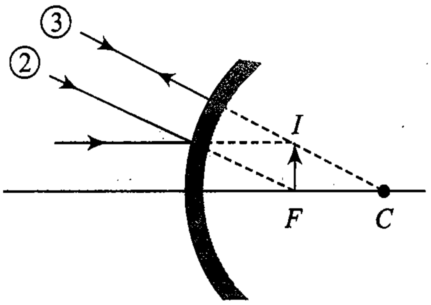
Thus, $$v = \frac{x f_0}{f_0 + x}.$$

The above expression shows that whatever may be the value of $x(|x| > f_0)$, v is always positive and its value is always less than or equal to f_0. The magnification formula may be modified as $m = f_0/(f_0 + x)$. It explains that m always lie between 0 and +1.

Nature of Image Formed by a Convex Mirror

(i) When the object is placed at infinity, a virtual, erect, and very diminished image is formed at the focus.

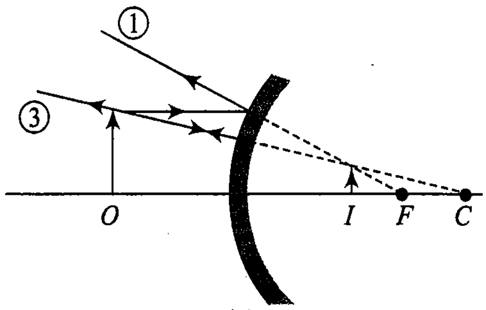
$$x = \infty; \quad v = f_0; \quad m << 1$$



(a)

(ii) When the object is placed in front of the convex mirror, a virtual, erect, and diminished image is formed between F and P.

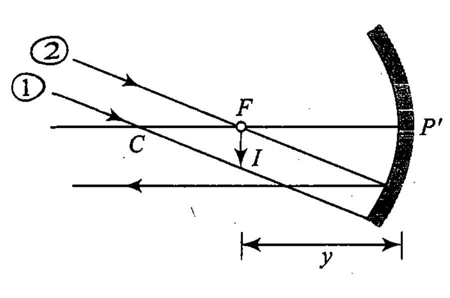
$$0 < x < \infty; \quad v < f_0; \quad m < 1$$



(a)

Fig. 1.51

Nature of Image Formed by a Concave Mirror

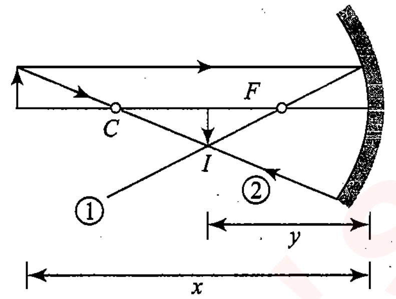


(i) When the object is placed at infinity, a real, inverted, and very small image is formed at the focus.

$x = \infty$

$v = -y,$ where $y = f_0$

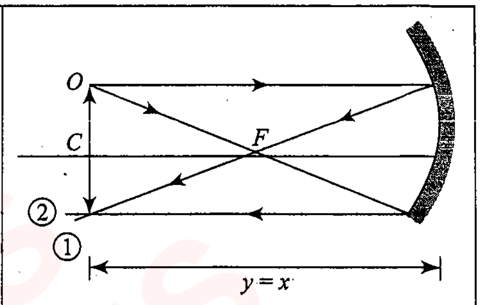
$m = -\delta,$ where $\delta << 1$



(ii) When the object is placed beyond C $(2f_0 < x < \infty)$, a real, inverted and diminished image is formed between F and C.

$v = -y,$ where $f_0 < y < 2f_0$

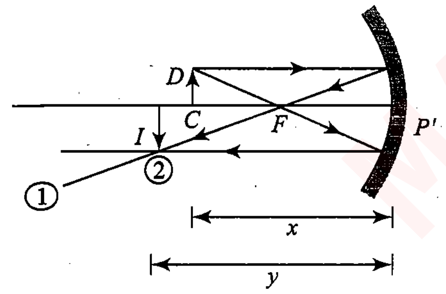
$m = -\delta,$ where $0 < \delta < 1$



(iii) When the object is placed at C $(x = 2f_0)$, a real, inverted, and equal size image is formed at C.

$v = -y,$ where $y = 2f_0$

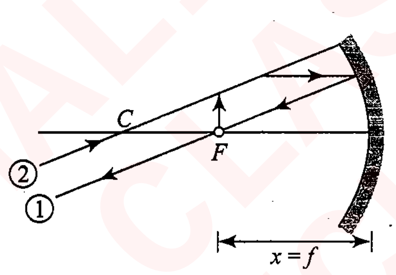
$m = -\delta$ where $\delta = 1$



(iv) When the object is placed between F and C $(f_0 < x < 2f_0)$, an erect, real, inverted, and large image is formed beyond C.

$v = -y,$ where $y > 2f_0$

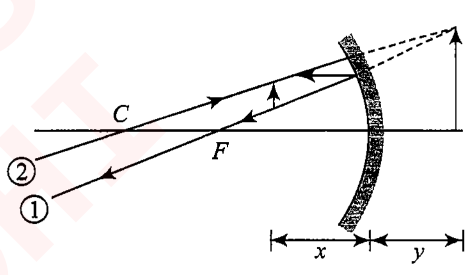
$m = -\delta,$ where $\delta > 1$



(v) When the object is placed at focus F, a real, inverted, and very large image is formed at infinity.

$v = -y,$ where $y = \infty$

$m = -\delta,$ where $\delta >> 1$



(vi) When the object is placed at C $(x = 2f_0)$, a real, inverted, and equal size image is formed at C.

$v = +y$

$m = +\delta,$ where $\delta > 1$

Illustration 1.15 **Can a convex mirror form a real image! Explain.**

Sol. Yes, only when the object is virtual and is placed between F and P. Fig. 1.52 shows a convex mirror exposed to a converging beam which converges to a point that lies between F and P.

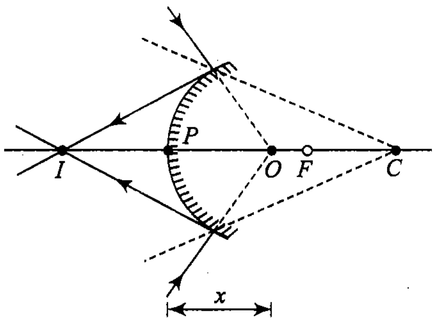


Fig. 1.52

$$v = \frac{-xf}{f_0 - x}$$; v becomes negative (real image) only when $x < f_0$.

Illustration 1.16 **An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from the pole. Find the lateral magnification produced.**

Sol. Given $u = -15$ cm, $f = -10$ cm

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get $v = -30$ cm

$$\therefore \quad m = -\frac{u}{v} = -2$$

Aliter: Using direct formula:

$$m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$$

Illustration 1.17 **A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance of 20 cm, then find the radius of curvature of the mirror.**

Sol. The person will see his face only when the image is virtual. Virtual image of a real object is erect. Therefore,

$$m = 2$$

$$\therefore \quad \frac{-v}{u} = 2 \quad (\text{Here } u = -20 \text{ cm})$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $f = -40$ cm or $R = 80$ cm.

Aliter: $$m = \frac{f}{f-u}$$

$$\Rightarrow \quad 2 = \frac{f}{f-(-20)}$$

$$\Rightarrow \quad f = -40 \text{ cm} \quad \text{or} \quad R = 80 \text{ cm}$$

Illustration 1.18 **An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. Find the nature and radius of curvature of the mirror.**

Sol. Since the image is formed on the screen, it is real. Real object and real image implies concave mirror.

Applying $$m = \frac{f}{f-u} \quad \text{or} \quad -2 = \frac{f}{f-u} \qquad \text{(i)}$$

After shifting, $$-3 = \frac{f}{f-(u+5)} \qquad \text{(ii)}$$

Here, care should be taken that distance of the object becomes $u + 5$ not $u - 5$: In a concave mirror the size of real image will increase only when the real object is brought closer to the mirror. In doing so, its x-coordinate will increase.

From (i) and (ii), we get

$$f = -30 \text{ cm} \quad \text{or} \quad R = 60 \text{ cm}$$

Illustration 1.19 **A point object is placed 60 cm from the pole of a concave mirror of focal length 10 cm on the principal axis.**

Find:

a. **the position of image.**

b. **If the object is shifted 1 mm towards the mirror along principal axis, find the shift in image. Explain the result.**

Sol.

a. $u = -60$ cm

$f = -10$ cm

$$v = \frac{fu}{u-f} = \frac{-10(-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

b. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Differentiating, we get

$$dv = -\frac{v^2}{u^2}\,du = -\left(\frac{-12}{-60}\right)^2 [1 \text{ mm}] = -\frac{1}{25} \text{ mm}$$

[$\because$ $du = 1$ mm; sign of du is +ve because it is shifted in +ve direction defined by sign convention.]

(i) –ve sign of dv indicates that the image will shift towards negative direction.

(ii) The sign of v is negative. Which implies that the image is formed on the negative side of the pole. (i) and (ii) together imply that the image will shift away from the pole.

Note that differentials dv and du denote small changes only.

Illustration 1.20 **The distance between a real object and its image in a convex mirror of focal length 12 cm is 32 cm. Find the size of image if the object size is 1 cm.**

Sol. Let x and y be the magnitudes of object and image distances.

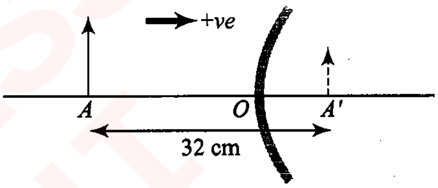


Fig. 1.53

We have $$AA' = 32 \text{ cm}$$

$$\Rightarrow \quad AO + A'O = 32 \text{ cm}$$

$$\Rightarrow \quad (x+y) = 32 \qquad \text{(i)}$$

And also, $$u = -x,\ v = +y$$

$$\frac{1}{-x}+\frac{1}{y} = \frac{1}{+12} \qquad \text{(ii)}$$

Solving (i) and (ii) simultaneously, we can get u and v.

The relevant answers are $u = -24$ cm, $v = +8$ cm

Using $$I = -1\left(\frac{+8}{-24}\right) = +\frac{1}{3}$$

So, the image size is 1/3 cm.

Illustration 1.21 **A plane mirror is placed at a distance of 50 cm from a concave mirror of focal length 16 cm. Where should a short object be placed between the mirrors and facing both the mirrors so that its virtual image in the plane mirror coincides with the real image in concave mirror? What is the ratio of the sizes of the two images?**

Sol. Let the object be at a distance x from the plane mirror.

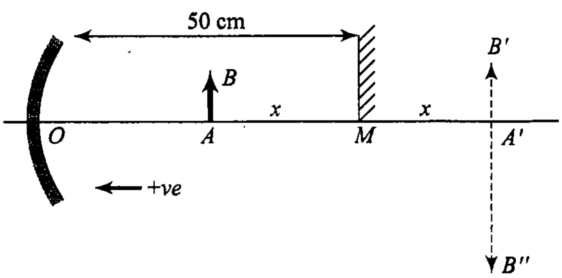


Fig. 1.54

The distance of object from concave mirror = $u = 50 - x$
For the plane mirror, object and image distances are equal.

$$A'M = AM = x$$

$$\Rightarrow \quad OA' = OM + A'M = 50 + x$$

For the concave mirror, $v = 50 + x$

Using $u = -(50 - x)$, $v = -(50 + x)$, $f = -16$ cm

$$\frac{1}{50 - x} + \frac{1}{50 + x} = \frac{1}{16}$$

$$\frac{100}{50^2 - x^2} = \frac{1}{16} \quad \Rightarrow \quad x = 30 \text{ cm}$$

The object must be placed at a distance of 30 cm from the plane mirror.

$$\text{The ratio of image sizes} = \frac{A'B'}{A'B''} = \frac{AB}{A'B''} = \frac{u}{v} = \frac{50 + x}{50 - x} = \frac{1}{4}$$

The real image formed by the concave mirror is 4 times the size of virtual image formed by the plane mirror.

Illustration 1.22 **Two concave mirrors are placed 40 cm apart and are facing each other. A point object lies between them at a distance of 12 cm from the mirror of focal length 10 cm. The other mirror has a focal length of 15 cm. Find the location of final image formed after two reflections— first at the mirror nearer to the object and second at the other mirror.**

Sol.

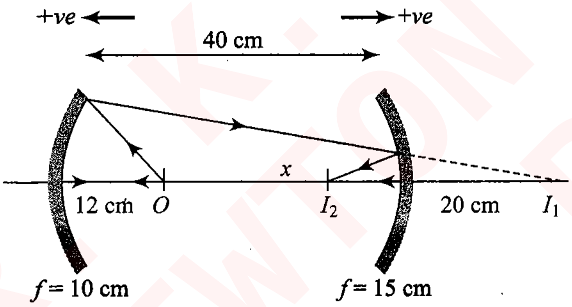


Fig. 1.55

First Reflection:
As incident rays are directed toward left, all quantities to the left of plane are taken positive.

$$u_1 = -12 \text{ cm}, \quad f_1 = -10 \text{ cm}$$

$$v_1 = \frac{u_1 f_1}{u_1 - f_1} = \frac{120}{-12 + 10} = -60 \text{ cm}$$

The rays after reflection are going to converge at a point $I_1 = 60$ cm from the first mirror, i.e., $60 - 40 = 20$ cm behind the other mirror.

Second Reflection:
As incident rays are directed toward right, all quantities towards right will be taken positive.

The converging rays falling at the second mirror create a virtual object for this mirror at I_1 with object distance

$$u_2 = +20 \text{ cm} \quad \text{and} \quad f_2 = -15 \text{ cm},$$

$$v_2 = \frac{u_2 f_2}{u_2 - f_2} = \frac{-20 \times 15}{+20 + 15} = \frac{-300}{35} = -8.57 \text{ cm}$$

As right is positive, image will be formed to the left of mirror. Hence, the final image is formed at a distance of 8.57 cm from the second mirror and is real.

Illustration 1.23 **Figure 1.56 shows a spherical concave mirror with its pole at (0, 0) and principal axis along x-axis. There is a point object at (–40 cm, 1 cm), find the position of image.**

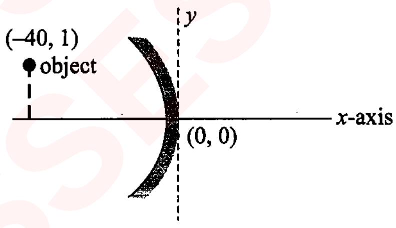


Fig. 1.56

Sol. According to sign convention,

$$u = -40 \text{ cm}$$
$$h_1 = +1 \text{ cm}$$
$$f = -5 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} + \frac{1}{-40} = \frac{1}{-5}; \quad \Rightarrow \quad v = \frac{-40}{7} \text{ cm}$$

$$\frac{h_2}{h_1} = \frac{-v}{u}$$

$$\Rightarrow \quad h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(\frac{-40}{7}\right) \times 1}{-40} = -\frac{1}{7} \text{ cm.}$$

$\therefore$ The position of image is $\left(\frac{-40}{7} \text{ cm}, -\frac{1}{7} \text{ cm}\right)$.

Illustration 1.24 **A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification.** **(IIT-JEE, 1991)**

Sol. As in question, image touches the rod, i.e., image and object coincides, hence one end of the rod should be at the center of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let l be the length of the image.

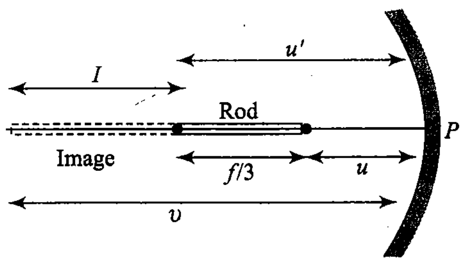


Fig. 1.57

Then, $m = \dfrac{l}{f/3} \Rightarrow l = \dfrac{mf}{3}$

Also, one end of the image coincides with the object, $u' = 2f$.

Now, $u' = u + \dfrac{f}{3} \Rightarrow u = 2f - \dfrac{f}{3} = \dfrac{5f}{3}$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right).$$

Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

Illustration 1.25 **A concave mirror and a convex mirror of focal lengths 10 cm and 15 cm are placed at a distance of 70 cm. An object *AB* of height 2 cm is placed at a distance of 30 cm from the concave mirror. First ray is incident on the concave mirror then on the convex mirror. Find size, position, and nature of the image.**

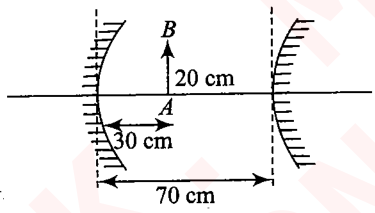


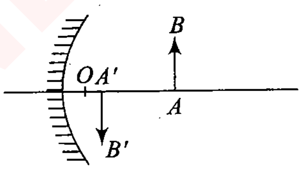
Fig. 1.58

Sol. For concave mirror,

$$u = -30 \text{ cm}, \quad f = -10 \text{ cm}$$

Using $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} - \dfrac{1}{30} = \dfrac{-1}{10} \Rightarrow v = -15$ cm

Now, $\dfrac{A'B'}{AB} = \dfrac{-v}{u} = \dfrac{(-15)}{(-30)} \Rightarrow A'B' = -1$ cm



(a)

Image formed by first reflection will be real, inverted, and diminished.

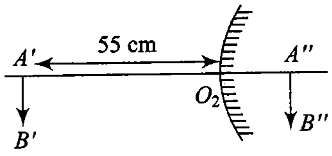
For convex mirror, the image formed by concave mirror will act as object for convex mirror. Now object distance for convex mirror

$$O_2A' = 70 - 15 = 55 \text{ cm}$$

$$u' = -55 \text{ cm}, f' = +15 \text{ cm}$$

Using $\dfrac{1}{v'} + \dfrac{1}{u'} = \dfrac{1}{f'} \Rightarrow \dfrac{1}{v'} - \dfrac{1}{55} = \dfrac{1}{15}$

$$\Rightarrow v' = \frac{165}{14} \text{ cm}$$



(b)

Fig. 1.59

Now, $\dfrac{A''B''}{A'B'} = -\dfrac{v'}{u'} = -\dfrac{\left(+\dfrac{165}{14}\right)}{(-55)}$

$$\Rightarrow A''B'' = \left(-\frac{3}{14}\right)(-1) = -0.2 \text{ cm}$$

Final image will be virtual, inverted, and diminished.

Relation Between Object and Image Velocity

Case (i). Differentiate equation $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{v}$ with respect to time.

$$\Rightarrow -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow -\frac{1}{v^2}V_{im} - \frac{1}{u^2}V_{OM} = 0$$

$\dfrac{dv}{dt} = v_{im}$ =velocity of image w.r.t. mirror

$$\Rightarrow V_{im} = -\frac{v^2}{u^2}V_{OM};$$

$\dfrac{du}{dt} = v_{OM}$ =velocity of object w.r.t. mirror

$$V_{Im} = -m^2 V_{OM}$$

The negative sign shows that if u is decreasing, v will increase, i.e., if real object approaches the mirror, its real image will recede from the mirror.

In this case, $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| < \left|-\dfrac{du}{dt}\right|$

When the object is at center of curvature, $\left|\dfrac{dv}{dt}\right| = \left|-\dfrac{du}{dt}\right|$

Case (ii). Object moves between center of curvature and focus.

In this case $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| > \left|-\dfrac{du}{dt}\right|$.

Speed of image is more than speed of object.

Case (iii). Object moves between focus and pole of the mirror.

In this case image is virtual, hence $\frac{1}{(+v)}+\frac{1}{(-u)}=\frac{1}{(-f)}$

$$\Rightarrow \quad \frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$$

If u is decreasing, v will also decrease, ie., if real object approaches mirror, image will also do so.

As $|m|>1$, speed of image will be greater than speed of object.

Size of image for a small size object placed along principal axis.

Differentiating $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ w.r.t. t, we get

$$\frac{dv}{dt}=-m^2\frac{du}{dt} \Rightarrow dv=-m^2du$$

Note: ***du/dt and dv/dt and are the velocities with respect to mirror not w.r.t. ground. When the mirror is at rest, then velocity of object or image w.r.t. mirror is same as velocity of object or mirror w.r.t. ground.***

Illustration 1.26 **A mirror of radius of curvature 20 cm and an object which is placed at a distance of 15 cm are both moving with velocities 1 ms^{-1} and 10 ms^{-1} as shown in Fig. 1.60. Find the velocity of image at this situation.**

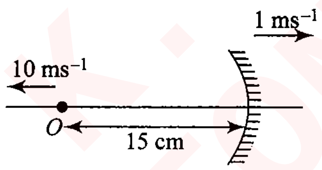


Fig. 1.60

Sol. Using $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$, we get

$$\frac{1}{v}-\frac{1}{15}=-\frac{1}{10} \Rightarrow v=-30 \text{ cm}$$

Now, using $V_{im}=-\frac{v^2}{u^2}V_{OM}$

$$(V_i-v_m)=-\frac{v^2}{u^2}(V_o-V_m)$$

$$\Rightarrow \quad V_i-(1)=-\frac{(-30)^2}{(-15)^2}[(-10)-(1)]$$

$$\Rightarrow \quad V_i=45 \text{ cm s}^{-1}$$

So, the image will move with velocity 45 cm s^{-1}.

Illustration 1.27 **An object AB is placed on the axis of a concave mirror of focal length 10 cm. End A of the object is at 30 cm from the mirror. Find the length of the image.**

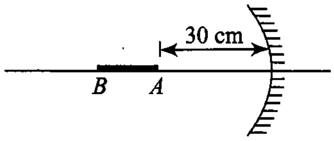


Fig. 1.61

(a) If length of object is 5 cm.

(b) If length of object is 1 mm.

Sol. (a) For point A, $u=-30$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $\frac{1}{v'}-\frac{1}{30}=-\frac{1}{10} \Rightarrow v=-15$ cm

Similarly, for point B, $u=-35$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$v'=-14 \text{ cm}$$

Now, size of image $|A'B'|=|v-v'|=|(-15)-(-14)|=1$ cm

(b) Here, $u=-30$ cm, $f=-10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{(-30)}=\frac{1}{(-10)} \Rightarrow v=-15 \text{ cm}$$

Now, $\frac{dv}{du}=-\frac{(-15)^2}{(-30)^2} \Rightarrow |dv|=\frac{(15)^2}{(30)^2}|du|$

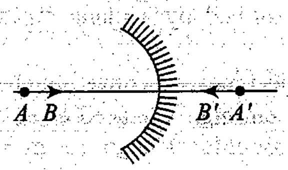
$$\Rightarrow \quad |dv|=\left(\frac{225}{900}\right)(10^{-3})=2.5\times10^{-4} \text{ m}$$

So the length of the image is 2.5×10^{-4} m.

Note: ***Some important points about curved mirror:***

- ***Lateral magnification (or transverse magnification) denoted by m is defined as $m=h_2/h_1$ and is related as $m=v/u$. From the definition of m, positive sign of m indicates erect image and negative sign indicates inverted image.***
  In case of successive reflections from mirrors, the overall lateral magnification is given by $m_1\times m_2\times m_3\times\ldots$, where m_1, m_2, etc. are lateral magnifications produced by individual mirrors.
- ***On differentiating the mirror formula, we get $=dv/du=v^2/u^2$.***
  Mathematically, du' implies small change in position of object and dv' implies corresponding small change in position of image. If a small object lies along principal

axis, du may indicate the size of object and dv the size of its image along principal axis. (Note that the focus should not lie in between the initial and final points of object.) In this case, dv/du is called longitudinal magnification. Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.



- ***Velocity of image***
  - ***Object moving perpendicular to the principal axis:***

    We have $h_2/h_1 = v/u$ or $h_2 = (-v/u)h_1$

    If a point object moves perpendicular to the principal axis, x-coordinate of both the object and the image remains constant. On differentiating the above relation w.r.t. time, we get

    $$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$

    Here, dh_1/dt denotes velocity of object perpendicular to the principal axis and dh_2/dt denotes velocity of image perpendicular to the principal axis.
  - ***Object moving along the principal axis:*** *On differentiating the mirror formula with respect to time, we get $dv/dt = -v^2/u^2(du/dt)$, where dv/dt is the velocity of image along the principal axis and du/dt is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of the object. This discussion is for velocity with respect to mirror and along the x-axis.*
  - ***Object moving at an angle with the principal axis:*** *Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the reusltant.*
- ***Newton's formula:*** *$XY = f^2$*

  X and Y are the distances (along the principal axis) of the object and image, respectively, from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.
- ***Optical power of a mirror (in diopters): $P = -1/f$***

  f = focal length with sign and is in meters.
- ***If object lying along the principal axis is not of very small size, the longitudinal magnification = $(v_2 - v_1)/(u_2 - u_1)$ (it will always be inverted)***

Some Experiments with Curved Mirror

Graphical Method of Determining the Focal Length of a Concave Mirror

It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Using Cartesian sign conversion, we have $u = -x$, $v = -y$ and $f = -f$

or $$\frac{1}{-y} + \frac{1}{-x} = \frac{1}{-f}$$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a straight line, as shown in Fig. 1.62(a).

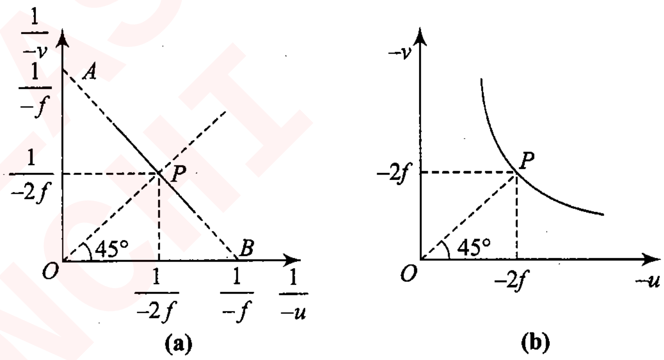


Fig. 1.62

Note that the slope of the straight line is –1 and the intercepts on the horizontal and vertical axes are equal. It is equal to $\frac{1}{-f}$.

A straight line OP at an angle of 45° with the horizontal axis is drawn which intersects the line AB at P. The coordinates of the point P are $\left(\frac{1}{-2f}, \frac{1}{-2f}\right)$.

The focal length of the mirror can be calculated by measuring the coordinates of either of the points A, B or P.

Alternatively, a graph between $-v$ and $-u$ can also be plotted, which is a curve as shown in Fig. 1.62 (b). A line drawn at an angle of 45° from the origin intersects it at the point P whose coordinates are $(-2f, -2f)$. By measuring the coordinates of this point, the focal length of the mirror can also be measured.

Measurement of Refractive Index of a Liquid by a Concave Mirror

A concave mirror of large radius of curvature is placed on a table with its principle axis vertical, as shown in Fig. 1.63. A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the center of curvature of the mirror.

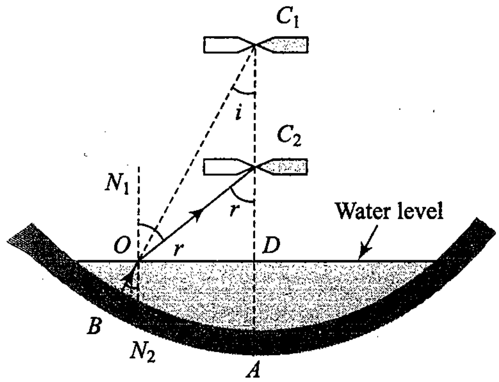


Fig. 1.63

A small quantity of liquid whose refractive index is to be measured is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.

In the figure, C_1 is the position of the pin when its image coincides with itself without liquid, and C_2 is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray BO that is normal to the mirror passes through C_1 before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through C_2.

In Fig. 1.63, $\sin i = \dfrac{OD}{OC_1}$, $\sin r = \dfrac{OD}{OC_2}$

From Snell's law, $\mu \sin i = 1 \sin r$

$$\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$$

Taking paraxial ray assumption, $OC_1 \simeq DC_1$; $OC_2 \simeq DC_2$;

Thus, $\mu = \dfrac{DC_1}{DC_2}$

If we neglect the depth of the liquid, we have $\mu = \dfrac{AC_1}{AC_2}$

Concept Application Exercise 1.2

1. State the following statements as TRUE or FALSE.
   a. A convex mirror cannot form a real image for a real object.
   b. The image formed by a convex mirror is always diminished and erect.
   c. Virtual image formed by a concave mirror is always enlarged.
   d. Only in the case of a convex mirror, it may happen that the object and its image move in same direction.
   e. In the case of a concave mirror, the image always move faster than the object.
   f. If an object is placed in front of a diverging mirror at a distance equal to its focal length, then the height of image formed is half of the height of object.
   g. For two positions of an object, a concave mirror can form enlarged image.
   h. Concave mirror is used as a rear view mirror in motor vehicles.
   i. If some portion of the mirror is covered, then complete image will be formed but of reduced brightness.
   j. A plane mirror always forms a real and erect image of same size as the object.
   k. The image formed by a plane mirror has left-right reversal.
   l. A virtual object means a converging beam.
2. a. An object 1 cm high is placed at 10 cm in front of a concave mirror of focal length 15 cm. Find the position, height, and nature of the image.
   b. A point source S is placed midway between two converging mirrors having equal focal length f as shown in Fig. 1.64. Find the value of d for which only one image is formed.

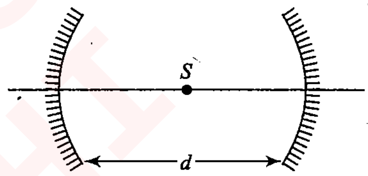


Fig. 1.64

3. Point S' is the image of a point source of light S in a spherical mirror whose optical axis is N_1N_2 (shown in Fig. 1.65). Find by construction the position of the center of the mirror and its focus.

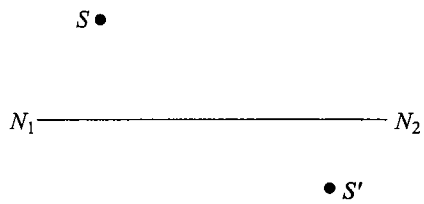


Fig. 1.65

4. The positions of optical axis N_1N_2 of a spherical mirror, the source and the image are known (as shown in Fig. 1.66). Find by construction the positions of the center of the mirror, its focus, and the pole for the cases
   a. A– source, B– image;
   b. B– source, A– image.

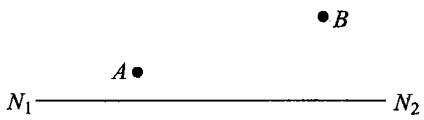


Fig. 1.66

5. An object is placed midway between a concave mirror of focal length f and a convex mirror of focal length f. The distance between the two mirrors is $6f$. Trace the ray that is first incident on the concave mirror and then the convex mirror.

6. A particle moves in a circular path of radius 5 cm in a plane perpendicular to the principal axis of a convex mirror with radius of curvature 20 cm. The object is 15 cm in front of the mirror. Calculate the radius of the circular path of the image.

7. An object is placed between a plane mirror and a concave mirror of focal length 15 cm as shown in Fig. 1.67. Find the positions of the images after two reflections.

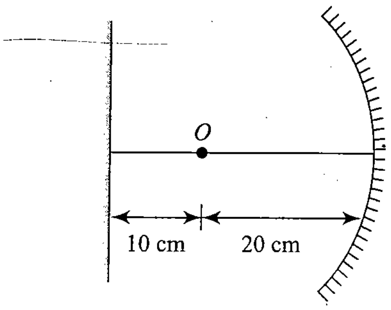


Fig. 1.67

8. A body of length 6 cm is placed 10 cm from a concave mirror of focal length 20 cm. Find the position, size, and nature of the image.

9. An object is placed 15 cm from a mirror and an image is captured on the screen with magnification 2. Calculate the focal length of the mirror and determine if it is concave or convex.

10. An object is placed 15 cm from a mirror and an erect image of size 5 cm is seen. Determine the focal length and the nature of the mirror. Assume the object size is 15 cm.

11. A concave mirror of focal length 10 cm is placed in front of a convex mirror of focal length 20 cm. The distance between the two mirrors is 20 cm. A point object is placed 5 cm from the concave mirror. Discuss the formation of image.

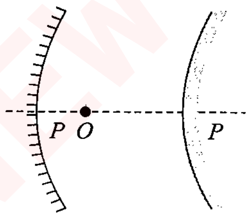


Fig. 1.68

12. A beam of light converges to a point on a screen S. A mirror is placed in front of the screen at a distance of 10 cm from the screen. It is found that the beam now converges at a point 20 cm in front of the mirror. Find the focal length of the mirror.

13. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

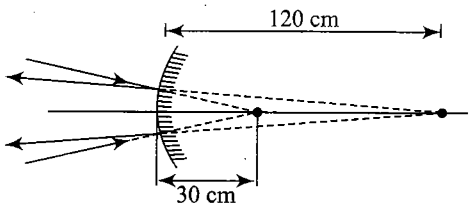


Fig. 1.69

14. Find the position of final image after three successive reflections taking first reflection on m_1.

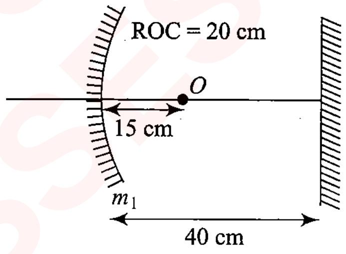


Fig. 1.70

15. A concave mirror gives a real image magnified 4 times. When the object is moved 3 cm the magnification of the real image is 3 times. Find the focal length of mirror.

16. The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.

17. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $m_1/m_2 = 4$, then calculate the focal length of the mirror.

18. A short linear object is placed at a distance u along the axis of a spherical mirror of focal length f.
   a. Obtain an expression for the longitudinal magnification.
   b. Also, obtain an expression for the ratio of the velocity of image (v) to the velocity of object (u).

19. A convex mirror of focal length 10 cm is shown in Fig. 1.71. A linear object AB = 5 cm is placed along the optical axis. Point B is at distance 25 cm from the pole of mirror. Calculate the size of the image of AB.

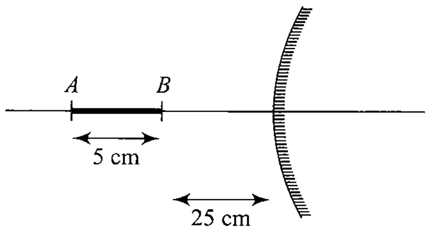


Fig. 1.71

20. A concave mirror forms a real image three times larger than the object on a screen. The object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm, find the shift of screen.

REFRACTION OF LIGHT

Deviation or bending of light rays from their original path while passing from one medium to another is called *refraction*. It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

When a light ray passes from one medium to another such that it undergoes a change in velocity, refraction takes place. Hence, wavelength of light changes, but frequency remains same.

Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

Illustration 1.28 **Determine the refractive index of glass with respect to water. Given that $\mu_g = 3/2$; $\mu_w = 4/3$.**

Sol. Refractive index of glass with respect to water or relative refractive index of glass w.r.t. water:

$$\frac{\text{R.I. of glass}}{\text{R.I. of water}} = \mu_{gw} = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} = 1.125$$

Laws of Refraction

a. The incident ray, the normal to any refracting surface at the point of incidence, and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

b. $\frac{\sin i}{\sin r}$ = constant for any pair of media and for light of a given wavelength (Fig. 1.72). This is known as ***Snell's law***.

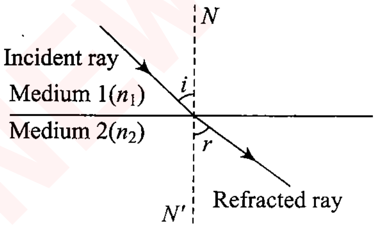


Fig. 1.72

Also, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

$n_1 \sin i = n_2 \sin r$

$\frac{n_2}{n_1} = {}_1n_2$ = refractive index of the second medium with respect to the first medium.

c = speed of light in air (or vacuum) = 3×10^8 ms^{-1}.

Special Cases

Case 1. Normal incidence: $i = 0$ (see Fig. 1.73(a))

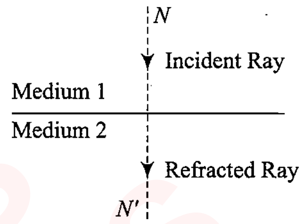


Fig. 1.73(a)

From Snell's law: $r = 0$

Case 2. When light moves from denser to rarer medium, it bends away from the normal (See Fig. 1.73(b))

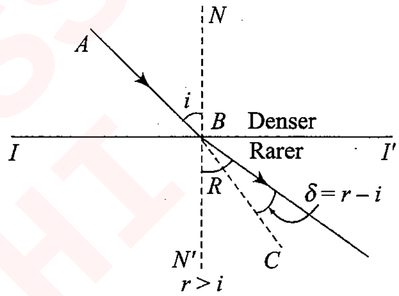


Fig. 1.73(b)

Case 3. When light moves from rarer to denser medium, it bends towards the normal (See Fig. 1.74)

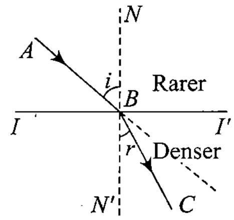


Fig. 1.74

Note:

- ***Higher the value of R.I., denser (optically) is the medium.***
- ***Frequency of light does not change during refraction.***
- ***Refractive index of the medium relative to vacuum*** $= \sqrt{\mu_r \epsilon_r}$

$n_{vacuum} = 1$; $n_{air} \geq 1$; n_{water} ***(average value)*** $= 4/3$; n_{glass} ***(average value)*** $= 3/2$

Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle i$ and refracted at $\angle r$ is given by $\delta = |i - r|$ (see Fig. 1.75)

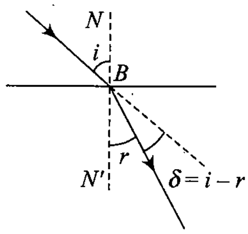


Fig. 1.75

Principle of Reversibility of Light Rays

a. A ray traveling along the path of the refracted ray is refracted along the path of the incident ray.

b. A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident and refracted rays are mutually reversible.

c. According to this principle, ${}_1n_2 = \dfrac{1}{{}_2n_1}$.

Illustration 1.29 **Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in Fig. 1.76.**

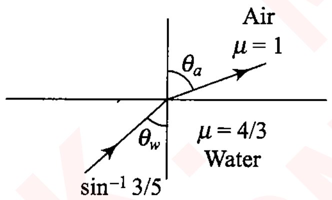


Fig. 1.76

Sol. Snell's law: $\mu_w \sin\theta_w = \mu_a \sin\theta_a$

$$\frac{4}{3} \times \frac{3}{5} = 1 \sin\theta_a$$

$$\sin\theta_a = \frac{4}{5} \quad \theta_a = \sin^{-1}\frac{4}{5}$$

Illustration 1.30 **Find the speed of light in medium '*a*' if speed of light in medium '*b*' is *c*/3, where *c* = speed of light in vacuum and light refracts from medium '*a*' to medium '*b*' making 45° and 60°, respectively, with the normal.**

Sol. Snell's law:

$$\mu_a \sin\theta_a = \mu_b \sin\theta_b \Rightarrow \frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$$

$$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ \Rightarrow v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

Illustration 1.31 **A ray of light is incident on a transparent glass slab of refractive index $\sqrt{3}$. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence?**

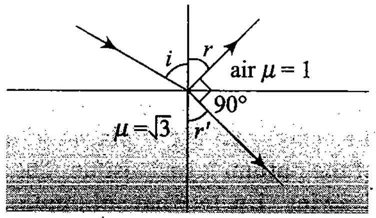


Fig. 1.77

Sol. Let the angle of incidence, angle of reflection, and angle of refraction be *i*, *r* and *r*′, respectively.

Now, as per the equation $(90^\circ - r) + (90^\circ - r') = 90^\circ$

$r' = (90^\circ - i)$ (because $i = r$, in case of reflection according to Snell's law, $1 \sin i = \mu \sin r'$)

or $\qquad \sin i = \mu \sin r'$

or $\qquad \sin i = \mu \sin(90^\circ - i) \Rightarrow \tan i = \mu$

or $\qquad i = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60^\circ$

Illustration 1.32 **A light ray is incident on a glass sphere of reflective index $\mu = \sqrt{3}$ at an angle of incidence 60° as shown in Fig. 1.78. Find the angles *r*, *r*′, *e* and the total deviation after two refractions.**

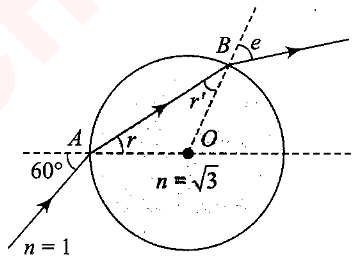


Fig. 1.78

Sol. At point '*A*': Applying Snell's law $1 \sin 60^\circ = \sqrt{3} \sin r \Rightarrow r = 30^\circ$

From symmetry $r' = r = 30^\circ$.

Again applying Snell's law at second surface (at point '*B*') $1 \sin e = \sqrt{3} \sin r$.

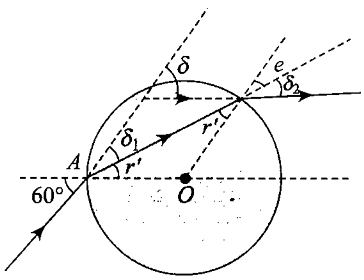


Fig. 1.79

$\Rightarrow \qquad e = 60^\circ$

Deviation at first surface, $\delta_1 = i - r = 60^\circ - 30^\circ = 30^\circ$

Deviation at second surface, $\delta_2 = e - r' = 60^\circ - 30^\circ = 30^\circ$

Therefore, total deviation = 60°.

Illustration 1.33 **A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the center. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible ?**

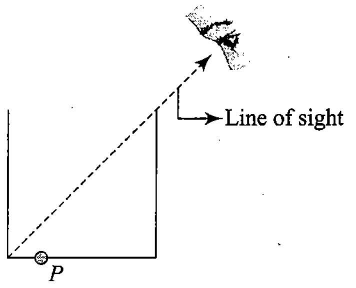


Fig. 1.80

Sol. If we pour water in vessel, refraction will take place at air and water interface.

Applying Snell's law at A, we get 1. $\sin 45° = \frac{4}{3}\sin\theta$

Here, θ is the angle which the incidence ray of light makes with the normal.

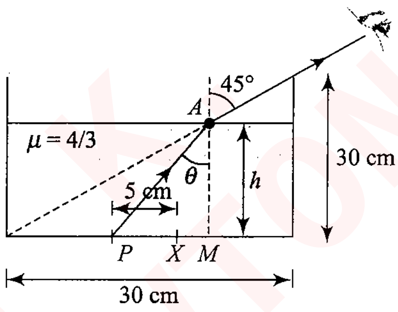


Fig. 1.81

$$\Rightarrow \quad 1\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{3}\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{3}{4\sqrt{2}}$$

$$\tan\theta = \frac{3}{\sqrt{16\times 2-9}} = \frac{3}{\sqrt{23}}$$

$\Rightarrow$ In ΔAPM,

$$\tan\theta = \frac{3}{\sqrt{23}} = \frac{PM}{AM} = \frac{5+x}{h} = \frac{5+h-15}{h}$$

$$\frac{3}{\sqrt{23}} = \frac{h-10}{h} \quad \Rightarrow \quad 3h = h\sqrt{23} - 10\sqrt{23}$$

$$h = \left(\frac{10\sqrt{23}}{\sqrt{23}-3}\right)\text{cm}$$

Hence, water should be poured upto height $\frac{10\sqrt{23}}{\sqrt{23}-3}$ cm to make the particle 'P' visible.

VECTOR REPRESENTATION OF A LIGHT RAY

The angle of incidence of a light ray on an interface is usually determined geometrically by simply drawing the ray diagram. However, in some situations, the vector representation of the line along which the light ray travels is given. The ray may then reflect or refract at an interface and emerge. How do we find the vector representation of the line along which the emergent ray travels? Following are the steps for finding this:

- Determine the unit vector representation of the normal to the interface where the incident ray strikes the surface, say $\hat{e}_n$.
- Find the component of the incident ray along the normal.
- Subtract this component from the original ray to compute the plane in which the incident ray travels. Calculate the unit vector that represents the plane of the incident ray and the normal, say $\hat{e}_p$.
- Using the vector dot product, calculate the angle between the incident ray and the normal.
- From the governing equation calculate the angle of reflection/refraction, say r.
- We know that the emergent ray will be in the same plane as that of the incident ray and the normal. Thus the unit vector representing the direction of the emergent beam is simply $\cos(r)\,\hat{e}_n + \sin(r)\,\hat{e}_p$.

Let us learn to apply these steps through following illustrations.

Illustration 1.34 **The XY plane is the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium 2.** **(IIT-JEE, 1999)**

Sol. Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$.

Component of the incident ray along the normal is $-10\hat{k}$.

The unit vector that represents the plane of the incident ray and the normal

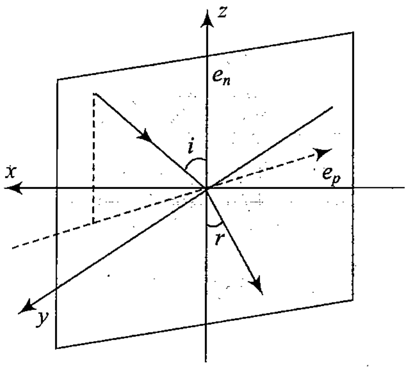
$$\hat{e}_p = \frac{(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j})}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}} = 0.6\hat{i} + 0.8\hat{j}$$

Angle between the incident ray and the normal is given by

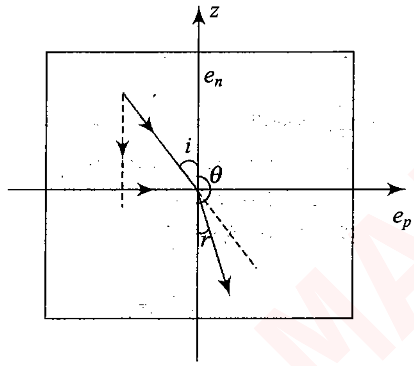
$$\cos\theta = (6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k})\cdot\hat{k}\Big/\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 10^2}$$

or $\quad \cos\theta = -0.5$

Therefore, the angle $\theta = 120°$



(a)



(b)

Fig. 1.82

The angle of incidence is $i = 180° - 120° = 60°$

The angle of the refracted beam is given by $\sqrt{2}\sin(i) = \sqrt{3}\sin(r)$ or $r = 45°$

The equation of the emergent ray is $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

$$= \cos(45°)(-\hat{k}) + \sin(45°)\cdot(0.6\hat{i} + 0.8\hat{j})$$

$$= \frac{1}{\sqrt{2}}(0.6\hat{i} + 0.8\hat{j} - \hat{k})$$

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Consider a ray of light that travels from a denser medium to rarer medium. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases (see Fig. 1.83). The angle of incidence for which the angle of refraction becomes 90° is called *critical angle*.

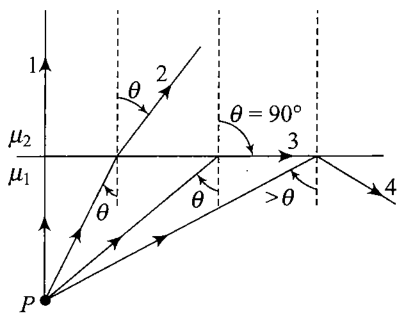


Fig. 1.83

$$\frac{\sin c}{\sin 90°} = {}_2\mu_1 = \frac{\mu_1}{\mu_2} \Rightarrow \sin c = \frac{1}{{}_2\mu_1} = \frac{\mu_2}{\mu_1}$$

or $$\sin C = \frac{\text{R.I. of rarer medium}}{\text{R.I. of denser medium}}$$

When the angle of incidence of a ray traveling from a denser medium to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is totally reflected back into the same medium. Here, the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.

Graph between Angle of Deviation (δ) and Angle of Incidence (i)

(I) For light ray going from denser medium to rarer medium (see Fig. 1.84):

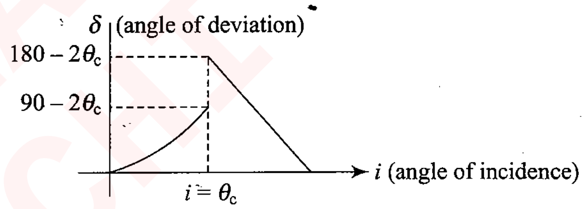


Fig. 1.84

(II) For light ray going from rarer medium to denser medium (see Fig. 1.85):

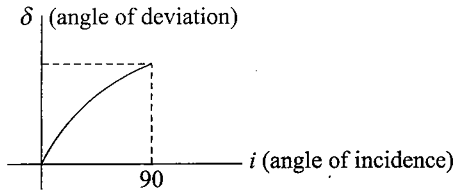


Fig. 1.85

Conditions of Total Internal Reflection

a. Light is incident on the interface from denser medium.

b. Angle of incidence should be greater than the critical angle ($i > c$). Figure 1.86 shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r and d stand for rarer and denser media, respectively.

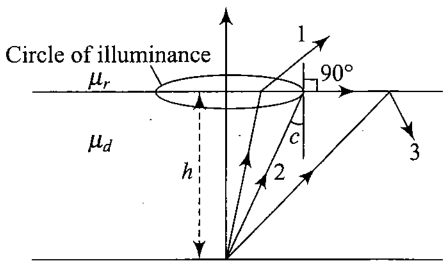


Fig. 1.86

In the figure, ray 1 strikes the surface at an angle less than critical angle c and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called *circle of illuminance* (C.O.I). All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium. Radius of C.O.I. can be easily found.

Illustration 1.35 **Find the maximum. angle that can be made in glass medium ($\mu = 1.5$) if a light ray is refracted from glass to vacuum.**

Sol. Maximum angle of refraction from denser medium to rarer medium is the critical angle. Hence,

$$1.5 \sin C = 1 \sin 90°, \text{ where } C = \text{critical angle.}$$
$$\sin C = 2/3$$
$$C = \sin^{-1} 2/3$$

Illustration 1.36 **Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vacuum, making an angle equal to twice the critical angle.**

Sol. Since the incident light is in rarer medium, total internal reflection cannot take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30°$$

$\therefore$ $i = 2C = 60°$

Applying Snell's law, $1 \sin 60° = 2 \sin r$

$$\sin r = \frac{\sqrt{3}}{4} \Rightarrow r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

Illustration 1.37 **What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB.**

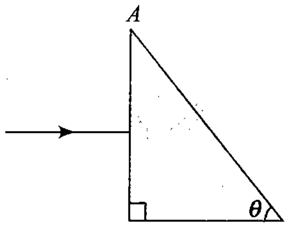


Fig. 1.87

Sol. Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

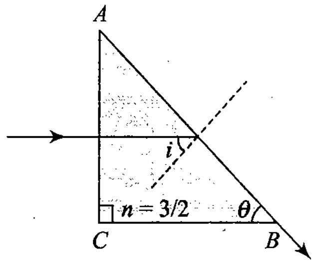


Fig. 1.88

$$i = 90 - \theta$$

For the required condition: $i > C \Rightarrow 90° - \theta > C$

or $\sin(90° - \theta) > \sin C$

or $\cos\theta > \sin C = \dfrac{1}{3/2} = \dfrac{2}{3}$ or $\theta < \cos^{-1}\dfrac{2}{3}$

Illustration 1.38 **A slab of refractive index μ is placed in air and light is incident at maximum angle θ_0 from vertical. Find minimum value of μ for which total internal reflection takes place at the vertical surface.**

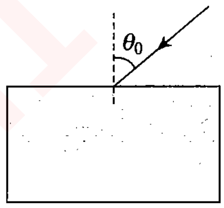


Fig. 1.89

Sol. For vertical surface, $\alpha > C$

$$\Rightarrow \quad \sin\alpha > \sin C \quad \Rightarrow \quad \sin\alpha > \frac{1}{\mu} \quad \text{(i)}$$

For horizontal surface,

$$\sin\theta_0 = \mu\cos\alpha \quad \Rightarrow \quad \sin\alpha = \sqrt{\frac{\mu^2 - \sin^2\theta}{\mu^2}} \quad \text{(ii)}$$

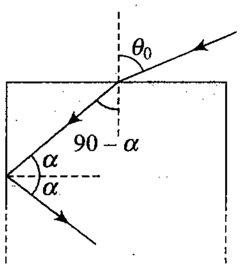


Fig. 1.90

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{\sqrt{\mu^2 - \sin^2\theta}}{\mu} > \frac{1}{\mu} \quad \Rightarrow \quad \mu^2 - \sin^2\theta > 1$$

$$\Rightarrow \quad \mu > \sqrt{1 + \sin^2\theta}$$

So, minimum value of $\mu = \sqrt{1 + \sin^2\theta}$

Illustration 1.39 **A rectangular slab *ABCD*, of refractive index n_1, is immersed in water of refractive index n_2 ($n_1 < n_2$). A ray of light is incident at the surface *AB* of the slab as shown in Fig. 1.91. Find the maximum value of angle of incidence α_{max}, such that the ray comes out only from the other surface *CD*.**

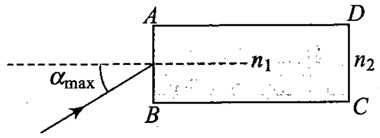


Fig. 1.91

Sol. For a maximum angle of incidence at surface *AB* there will be a maximum angle of incidence at the surface *AD*. A ray to pass through the face *CD* as it should not pass beyond *AD* i.e. it should not refract at *AD*. Hence, the angle θ should be the critical angle.

By Snell's law,

$$\sin\theta = \frac{n_2}{n_1}$$

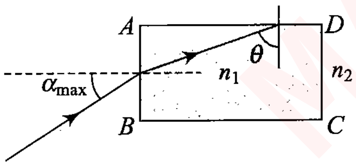


Fig. 1.92

$$n_2 \sin\alpha_{max} = n_1 \sin(90 - \theta)$$

$$\sin\alpha_{max} = \frac{n_1}{n_2}\cos\theta \Rightarrow \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

Illustration 1.40 **A point source of light is placed a distance *h* below the surface of a large and deep lake. What fraction of light will escape through the surface of water?**

Sol. Due to total internal reflection some of the light rays incident at the interface will return back into water. So, only that portion of light will escape for which the angle of incidence at the interface of the medium is less than the critical angle.

If the critical angle is θ_c, then the light rays that reach beyond the base of the cone whose vertical angle is $2\theta_c$ will suffer total internal reflection.

Hence, only the light incident on the base of the cone refracts and escapes.

Method 2:

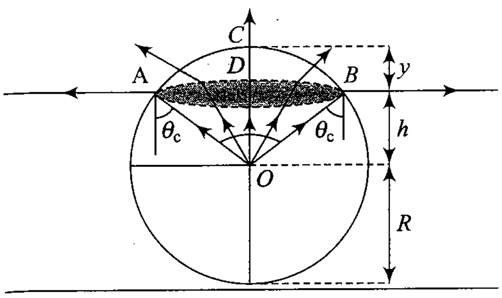


Fig. 1.93

The fraction of light escaping,

$$f = \frac{\text{Area of the cap}}{\text{Area of sphere}} = \frac{2\pi R y}{4\pi R^2}, \text{ i.e., } f = \frac{1}{2}\left[\frac{y}{R}\right] = \frac{1}{2}\left[\frac{R-h}{R}\right]$$

Area of cap *ABCD* can be calculated by using method of integration,

$$\text{i.e., } f = \frac{1}{2}\left[1 - \frac{h}{R}\right] = \frac{1}{2}[1 - \cos\theta_c], \text{ i.e., } f = \frac{1}{2}\left[1 - \sqrt{1 - \sin^2\theta_c}\right]$$

$$\text{i.e., } f = \frac{1}{2}\left[1 - \sqrt{1 - \frac{1}{n^2}}\right]$$

Illustration 1.41 **A spider is on the surface of a glass sphere with a refractive index of 1.5. An insect crawls on the other side of the sphere as shown in Fig. 1.94(a). For what maximum value of θ will the spider be able to still see the insect. Assume the spiders eye is in air.**

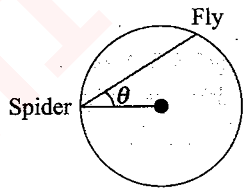


Fig. 1.94(a)

Sol. The spider will be able to see the insect if a ray of light from the insect reaches the spider's eye. If the angle of incidence of the beam is greater than the critical angle for the glass–air interface, the ray will be reflected within the glass sphere and will not emerge from it. Consequently, the spider will not be able to see the insect. Therefore, the angle θ must be greater than the critical angle for the glass–air interface. That is,

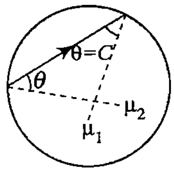


Fig. 1.94(b)

$$\theta > \sin^{-1}\left[\frac{1}{1.5}\right] \Rightarrow \theta > \sin^{-1}\left(\frac{2}{3}\right)$$

Illustration 1.42 **A monochromatic light is incident on the plane interface *AB* between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ as shown in Fig.1.95.**

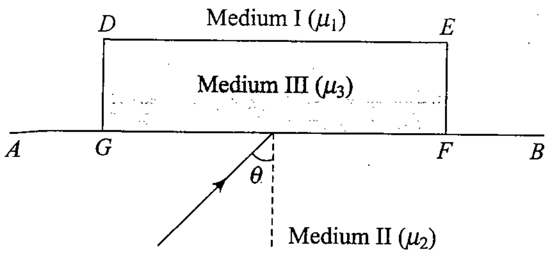


Fig. 1.95

The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab *DEFG* of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.

Sol. We will use the symbol ≤ to mean 'infinitesimally greater than'.

When the slab is not inserted,

$$\theta \leq \theta_c = \sin^{-1}(\mu_1/\mu_2) \text{ or } \sin\theta \geq \mu_1/\mu_2$$

When the slab is inserted, we have two cases

$$\mu_3 \leq \mu_1 \text{ and } \mu_3 > \mu_1.$$

Case I. $\mu_3 < \mu_1$. We have $\sin\theta \geq \mu_1/\mu_2 \geq \mu_3/\mu_2$

Thus, the light is incident on AB at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II. $\mu_3 > \mu_1$

$$\sin\theta \geq \mu_1/\mu_2 < \mu_3/\mu_2$$

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (figure).

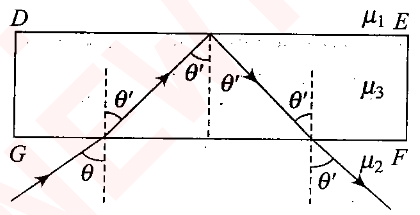


Fig. 1.96

$$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2} \quad \text{(i)}$$

$$\Rightarrow \quad \sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta \leq \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$$

Thus, $\quad \sin\theta' \geq \dfrac{\mu_1}{\mu_3} \Rightarrow \theta' \geq \sin^{-1}\left(\dfrac{\mu_1}{\mu_3}\right) \quad$ (ii)

As the slab has parallel faces, the angle of refraction at the face FG is equal to the angle of incidence at the face DE. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface FG at an angle of incidence θ'.

At this face, it will refract into medium II and the angle of refraction will be θ as shown by Eq. (i). Thus, the total light energy is ultimately reflected back into medium II.

APPARENT SHIFT OF AN OBJECT DUE TO REFRACTION

Due to bending of light at the interface of two different media, the image formed due to refraction appears at a place other than the object position. This image formation due to refraction creates illusion of shifting of the object position.

Locating the position of image formed becomes much simpler if we restrict ourselves to nearly normal incident rays.

Consider an object O in the medium (R.I = μ). After refraction, the ray at the interface bend. When the bent ray falls in our eye our eye perceives it along a straight line and it appears at I. (see Fig. 1.97)

For nearly normal incident rays, θ_1 and θ_2 will be very small.

$$\tan\theta_1 \simeq \sin\theta_1 = \frac{AB}{\text{object distance from the refracting surface}}$$

$$\sin\theta_2 = \frac{AB}{\text{image distance from the refracting surface}}$$

$$\Rightarrow \frac{\text{Image distance from the refracting surface}}{\text{Object distance from the refracting surface}} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \mu_2 = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\tan\theta_1}{\tan\theta_1} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\frac{AB}{OB}}{\frac{BA}{BI}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{BI}{OB} = \frac{\text{Apprant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

So, Shift = Real depth – Apparent depth = Real depth $\left(1 - \dfrac{\mu_2}{\mu_1}\right)$

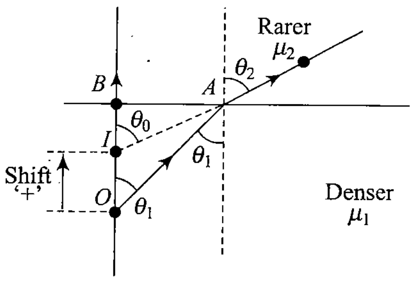


Fig. 1.97

Case I.

If $\mu_1 < \mu_2$, shift becomes negative (in the direction opposite to initial, say travelling). Image distance > object distance, i.e., image is farther from the refracting surface (See Fig. 1.98).

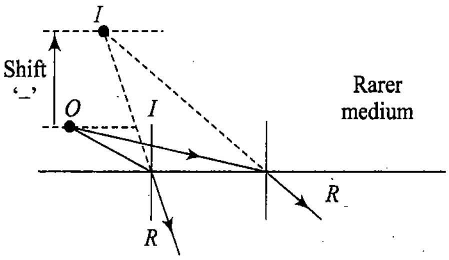


Fig. 1.98

Case II.
If $\mu_1 > \mu_2$, shift becomes positive. Image distance < object distance, i.e., image is closer to the refracting surface (see Fig. 1.99).

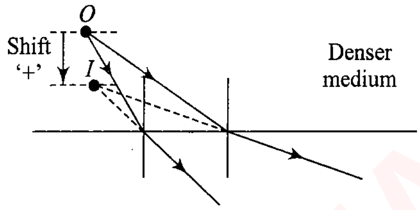


Fig. 1.99

Note: Apparent Depth and Shift of Submerged Object

At near normal incidence (small angle of incidence i), apparent depth (d′) is given by:

$$d' = \frac{d}{n_{relative}}$$

where

$$n_{relative} = \frac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$$

d = distance of object from the interface
= real depth
d′ = distance of image from the interface
= apparent depth
In general, we can write

$$\frac{n_i}{d} = \frac{n_r}{d'}$$

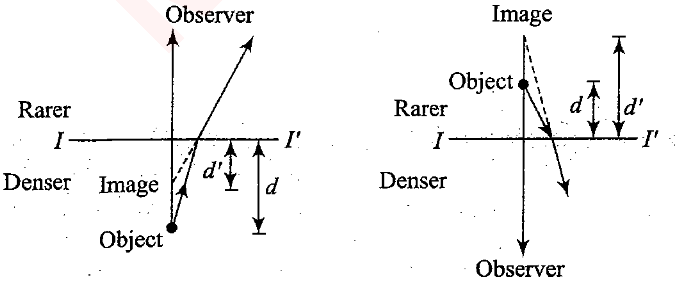


Fig. 1.100

$$\textit{Apparent shift} = d\left(1 - \frac{1}{n_{rel}}\right)$$

Illustration 1.43 **An object lies 100 cm inside water (μ = 4/3). It is viewed from air nearly normally. Find the apparent depth of the object.**

Sol: Here, object is placed inside water and the observer is situated in air. Hence, apparent depth

$$d' = \frac{d}{(n_i/n_r)} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} = d' = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

Illustration 1.44 **See Fig. 1.101 and answer the following questions.**
(i) Find apparent height of the bird.
(ii) Find apparent depth of the fish.
(iii) At what distance will the bird appear to the fish?
(iv) At what distance will the fish appear to the bird?

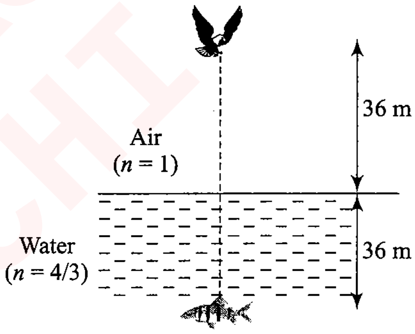


Fig. 1.101

Sol.
(i) Here, bird is an object and fish is an observer. Hence, apparent height observed by the fish

$$d'_B = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{air}}{n_{water}}\right)} \Rightarrow d'_B = \frac{36}{\frac{1}{\left(\frac{4}{3}\right)}} = \frac{36}{3/4} = 48 \text{ m}$$

(ii) Here, the fish is an object and the bird is an observer. Hence, apparent height observed by the bird

$$d'_F = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} \Rightarrow d'_F = \frac{36}{4/3} = 27 \text{ m}$$

(iii) For the fish, the bird will be observed at a distance d'_B from the fish: $d_B = 36 + 48 = 84$ m

(iv) For the bird, the fish will be observed at a distance 'd_F' from the bird: $d_F = 27 + 36 = 63$ m

Illustration 1.45 **Consider the situation in Fig. 1.102. The bottom of the pot is a reflecting plane mirror, S is a small fish, and T is a human eye. Refractive index of water is μ.**

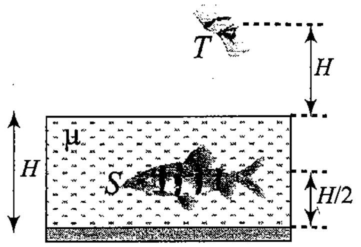


Fig. 1.102

(a) At what distance(s) from itself will the fish see the image(s) of the eye?

(b) At what distance(s) from itself will the eye see the image(s) of the fish?

Sol.

(a) The fish will observe the images of eye one from direct observation and the other reflected image from the plane mirror.

(i) Direct observation of eye from fish

Apparent height, $H' = \dfrac{H}{n_{rel}} = \dfrac{H}{\left(\dfrac{n_{air}}{n_{water}}\right)} = \dfrac{H}{\left(\dfrac{1}{\mu}\right)}$

Hence, $H' = \mu H$

Distance of image of eye from fish

$$d = \frac{H}{2} + \mu H = H\left(\frac{1}{2} + \mu\right)$$

(ii) Observation of reflected image of the eye from the fish

For mirror, the distance of eye from it will be $(H + \mu H)$. Hence, the image of eye from mirror will be $(H + \mu H)$ behind the mirror. Hence, distance of image of eye from the fish

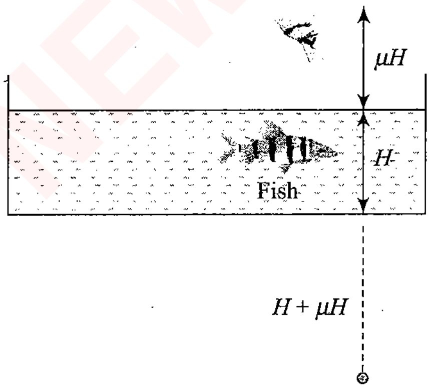


Fig. 1.103

$$d' = \frac{H}{2} + H + \mu H = \frac{3}{2}H + \mu H \quad \Rightarrow \quad d' = H\left(\frac{3}{2} + \mu\right)$$

(b) The eye will also observe two images of the fish, one from direct observation and the other reflected image from the mirror.

(i) Direct observation of fish from eye:

Apparent depth of the fish observed by the eye

$$H' = \frac{H/2}{n_r} = \frac{H/2}{\left(\dfrac{n_{water}}{n_{air}}\right)} = \frac{H/2}{\mu} = \frac{H}{2\mu}$$

Distance of image of the fish from the eye,

$$d = H + \frac{H}{2\mu} = H\left(1 + \frac{1}{2\mu}\right)$$

(ii) Eye observing image of fish:

The eye will observe the image of fish reflected from the mirror.

Apparent depth of image of the fish from air and water interface.

$$H' = \frac{\text{Real depth}}{n_{rel}} = \frac{\frac{3}{2}H}{\left(\dfrac{n_{water}}{n_{air}}\right)}$$

Here real depth from top surface of water $= H + \dfrac{H}{2} = \dfrac{3}{2}H$

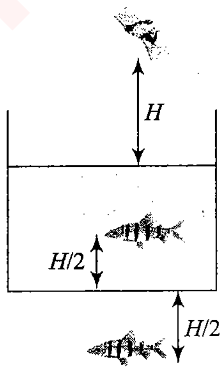


Fig. 1.104

$$H' = \frac{3H}{2\mu}$$

Hence, distance between this image and the eye,

$$d' = H + \frac{3H}{2\mu} = H\left(1 + \frac{3}{2\mu}\right)$$

Illustration 1.46 **A fish in an aquarium approaches the left wall at a rate of 2.5 ms^{-1} observes a fly approaching it at 8 ms^{-1}. If the refractive index of water is (4/3), find the actual velocity of the fly.**

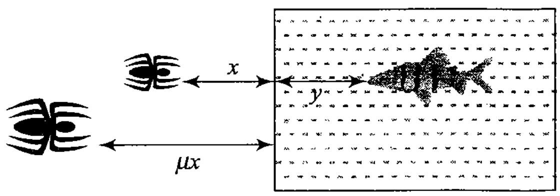


Fig. 1.105

Sol. For the fish, the apparent distance of the fly from the wall of the aquarium is μx, if x is the actual distance.

Then apparent velocity will be $d(\mu x)/dt \Rightarrow (v_{app})_{fly} = \mu v_{fly}$

Now, the fish observes the velocity of the fly to be 8 ms^{-1}

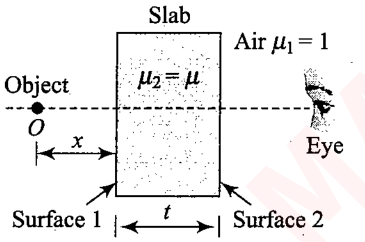
$\Rightarrow$ Apparent relative velocity will be = 8 ms^{-1}

$\Rightarrow$ $v_{fish} + (v_{app})_{fly} = 8$

$\Rightarrow$ $3 + \mu v_{fly} = 8 \Rightarrow v_{fly} = 5 \times \frac{3}{4} = 3.75$ ms^{-1}

REFRACTION THROUGH A PARALLEL SLAB

A slab is formed when a medium is isolated from its surroundings by two plane surfaces parallel to each other. In this section, we will determine the position and nature of the image formed when a slab is placed in front of an object.



All objects viewed through a transparent glass slab

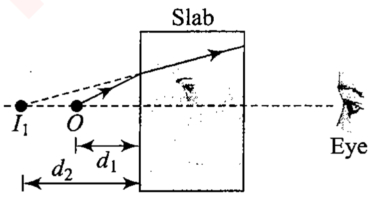
Fig. 1.106

Consider an object O placed a distance x in front of a glass slab of thickness "t" and refractive index μ. The observer is on the other side of the slab. A ray of light from the object first refracts at surface (1) then refracts at surface (2) before reaching the observer (Fig. 1.106). Let us analyse the location of image as sum by observer taking one step at a time.

Refraction at surface 1:

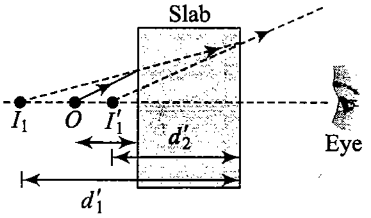
Here, $\mu_1 = 1$, $\mu_2 = \mu$, d_1 = real depth, d_2 = apparent depth

Apparent depth, $d_2 = \dfrac{d_{real}}{n_{relative}} = \dfrac{d_1}{\left(\dfrac{n_{incident}}{n_{refraction}}\right)} = \dfrac{d_1}{\left(\dfrac{\mu_1}{\mu_2}\right)}$



The apparent position of the object after refraction at the first surface.

Fig. 1.107(a)



Final position of the object after refraction at both surfaces

Fig. 1.107(b)

$$d_2 = \frac{x}{(1/\mu)}$$

$$d_2 = \mu x \qquad \text{(i)}$$

Thus, the first image is formed behind the object at the point I_1. I_1 now serves as the object for the second surface [see Fig. 1.107 (a)].

Refraction at surface 2:

Here, $n_{incident} = \mu_2 = \mu$

$n_{refraction} = \mu_1 = 1$

Apparent depth, $d_2' = \dfrac{d_{real}}{n_{real}}$

$$d_2' = \frac{d_1'}{\left(\dfrac{\mu}{1}\right)} \quad \text{but } d_1' = (d_2 + t)$$

$$d_2' = \frac{(\mu x + t)}{\mu} \qquad \text{(ii)}$$

Thus, the final image I_1' is at a distance $x + (t/\mu)$ behind the second interface. The original object was at a distance $x + t$ behind the second interface. Therefore, the image appears shifted by

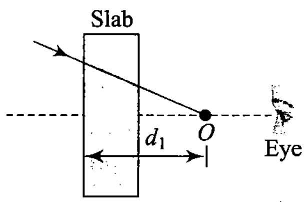
$$s = \left[x + \frac{t}{\mu}\right] - [(x + t)] = t\left[1 - \frac{1}{\mu}\right] \qquad \text{(iii)}$$

We can say a slab appears to shift the object along the perpendicular to the slab by a distance $t[1 - (1/\mu)]$ in the direction of the travelling ray [see Fig. 1.107(b)].

How about the direction of the shift?

In Fig. 1.107, the object is shifted towards the glass slab. Is it always true?

Consider a converging set of rays incident on a glass slab as shown in Fig. 1.108(a). Here, we have a virtual object to the right of the slab. Following the same procedure as that in earlier, we can show that the net shift is same as given by Eq. (iii). However, let us look a little closely at the direction of the shift.



(a)

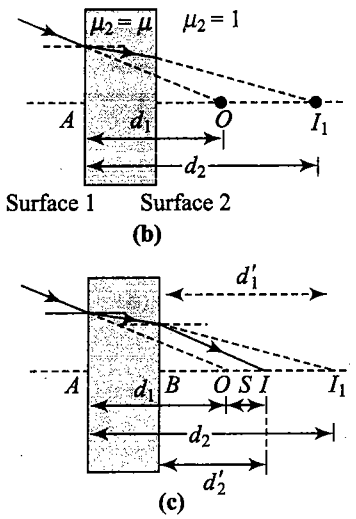


Fig. 1.108

Here, μ is the refractive index of the slab with respect to surrounding medium ($\mu = \mu_2/\mu_1$). If the refractive index of medium is greater than the refractive index of slab, then shifting (s) will be negative. In this case, the shifting will be in the direction opposite to the traveling ray.

Refraction at surface 1:

The converging rays will be focused at O. The point O will act as virtual object for refraction through surface 1.

For surface 1: Real depth for surface 1, $AO = d_1$

Here, $n_{\text{incident}} = \mu_1 = 1$

$n_{\text{refraction}} = \mu_2 = \mu$

$$\text{Apparent depth, } d_2 = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$$

$$d_2 = \frac{d_1}{\left(\dfrac{1}{\mu}\right)} = \mu d_1$$

Refracion at surface 2:

After refraction from surface 1, the image of 'O' will form at I_1. For surface 2, I_1 will act as an object.

For surface 2: Real depth for surface $BI_1 = d_1' = (d_2 - t)$

$$= (\mu d_1 - t)$$

Here, $n_{\text{incident}} = \mu_2 = \mu$

$n_{\text{refraction}} = \mu_1 = 1$

$$\text{Apparent depth, } d_2' = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$$

$$= \frac{d_1'}{\left(\dfrac{\mu}{1}\right)} = \frac{(\mu d_1 - t)}{\mu}$$

$$d_2' = \frac{(\mu d_1 - t)}{\mu} = \left(d_1 - \frac{t}{\mu}\right)$$

Hence, shifting of object position from slab

$$OI = s = BI - BO = d_2' - (d_1 - t)$$

$$s = \left(d_1 - \frac{t}{\mu}\right) - (d_1 - t) = \left(t - \frac{t}{\mu}\right)$$

$$s = t\left(1 - \frac{1}{\mu}\right)$$

Image formation by a slab:

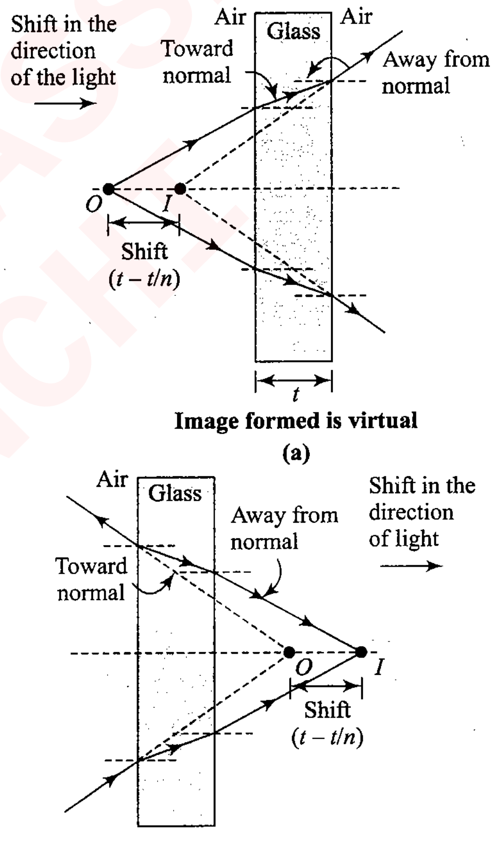


Fig. 1.109

In both cases, shifting is in the direction of the traveling ray.

Lateral Displacement of Emergent Beam Through a Glass Slab

Figure 1.110 represents the refraction of a ray AO incident on the slab at an angle of incidence i through the glass slab $EFGH$. At face EF, the incident ray AO is refracted along OP, the angle of refraction being r.

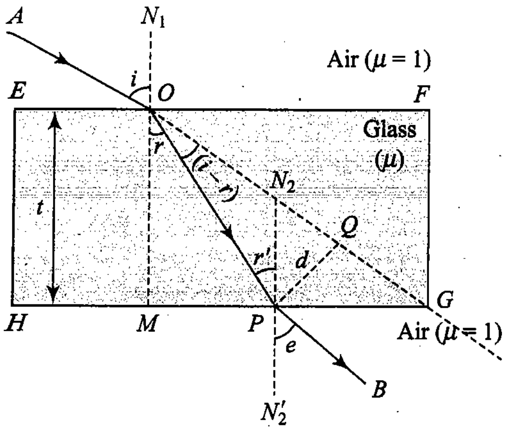


Fig. 1.110

(i) If e is angle of emergence, then from Snell's law at faces EF and HG,

$$\mu_a \sin i = \mu \sin r \quad \text{and} \quad \mu \sin r' = \mu_a \sin e$$

$$r' = r \quad \text{and} \quad \mu_a = 1, \text{ we have}$$

$$\sin i = \sin e \text{ or } e = i.$$

That is the emergent ray is parallel to the incident ray.

(ii) Let PQ be the perpendicular dropped from P on incident ray produced.

Then, the lateral displacement caused by the plate,

$$d = PQ = OP \sin(i - r) = \frac{OM}{\cos r}\sin(i - r)$$

$$= \frac{t \sin(i - r)}{\cos r}$$

Special Case

If i is very small, r is also very small, then $\sin i \to i$, $\sin r \to r$ and $\cos r \to 1$ so that

$\dfrac{\sin i}{\sin r} = \mu$ takes the form $\dfrac{i}{r} = \mu$.

$\therefore$ The expression for lateral displacement takes the form

$$d = \frac{t(i - r)}{1} = ti\left(1 - \frac{r}{i}\right) = ti\left(1 - \frac{1}{\mu}\right)$$

$$d = \left(1 - \frac{1}{\mu}\right)ti$$

Illustration 1.47 **Find the lateral shift of a light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.**

Sol. $$d = \frac{t\sin(i - r)}{\cos r} = \frac{10\sin(60° - 45°)}{\cos 45°}$$

$$= \frac{10\sin 15°}{\cos 45°} = 10\sqrt{5}\ \sin 15°.$$

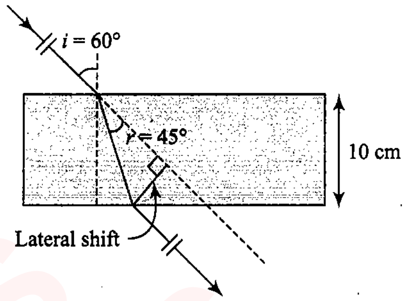
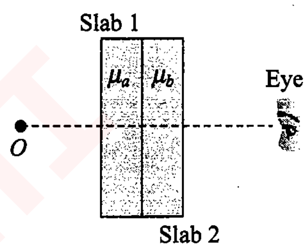


Fig. 1.111

REFRACTION ACROSS MULTIPLE SLABS

In Fig. 1.112, an object is placed in front of two slabs in contact. The thickness and refractive indices of the slabs are t_1, μ_a and t_2, μ_b, respectively. Where will the final image of the object appear to be?



A object placed in front of two glass slabs in contact

Fig. 1.112

A light ray emerging from O now refracts at three surfaces. The first is between air and μ_a the second between μ_a and μ_b while the third is between μ_b and air. Let us solve the problem taking one step at a time.

1st Interface (See Fig. 1.113): Here, $\mu_1 = 1$, $\mu_2 = \mu_a$

$$d_1 = x,\ d_2 = ?$$

$$d_2 = \frac{d_1}{\mu_{\text{relative}}} = \frac{d_1}{\mu_1/\mu_2}$$

$$\Rightarrow \quad \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \quad \text{or} \quad d_2 = \frac{\mu_2}{\mu_1}d_1 = \mu_a d_1$$

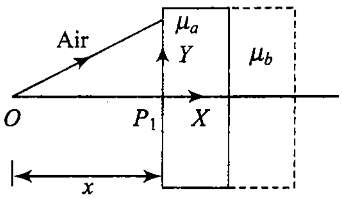


Fig. 1.113

Therefore, the image distance $d_2 = -\mu_a x$ (i)

2nd Interface (See Fig. 1.114): Here, $\mu_1 = \mu_a$, $\mu_2 = \mu_b$

$$d_1 = -(\mu_a x + t_1),\ d_2 = ?$$

Since $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

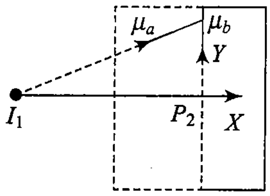


Fig. 1.114

The image distance $d_2 = -\mu_b\left(x + \dfrac{t_1}{\mu_a}\right)$ (ii)

3rd Interface (See Fig. 1.115): Here, $\mu_1 = \mu_b$, $\mu_2 = 1$

$$d_1 = -\mu_b\left(x + \frac{t_1}{\mu_a}\right) + t_2,\ d_2 = ?$$

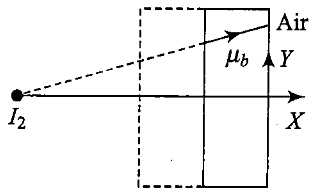


Fig. 1.115

and the final image distance from the 3rd interface is

$$d_2 = -\left[x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right] \quad \text{(iii)}$$

Therefore, the net shift in the position of the image is

$$s = -\left(x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right) - [-(x + t_1 + t_2)]$$

or $$s = t_1\left(1 - \frac{t}{\mu_a}\right) + t_2\left(1 - \frac{t}{\mu_b}\right) \quad \text{(iv)}$$

Looking at the above result, we realize that the net shift in the position of the image is simply the sum of the individual shifts at each of the slabs if they were independently placed in air.

Thus, the simple problem of refraction in a glass slab can be tackled in two ways:

1. By the method of interfaces: Here, the refraction formula, equation ($\mu_1/d_1 = \mu_2/d_2$), is applied at each interface.
2. By the method of elements: Here, the slab itself is an element with a governing equation $t\left[1 - \dfrac{1}{\mu}\right]$.

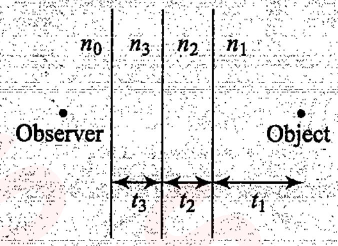
Most problems in ray optics can be solved by either of the two methods. In the following problems, we shall solve problems in both ways and highlight the method that gives a quicker solution.

Note: Refraction Through A Composite Slab

(or Refraction through a number of parallel media, as seen from a medium of R.I. n_0)

- ***Apparent depth (distance of final image from final surface)***

$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \cdots + \frac{t_n}{n_{nrel}}$$



- ***Apparent shift***

$$= t_1\left[1 - \frac{1}{n_{1rel}}\right] + t_2\left[1 - \frac{1}{n_{2rel}}\right] + \cdots + t_n\left[1 - \frac{n}{n_{nrel}}\right]$$

where 't' represents thickness and 'n' represents the R.I. of the respective media, relative to the medium of observer (i.e. $n_{1rel} = n_1/n_0$, $n_{2rel} = n_2/n_0$ etc.)

Illustration 1.48 **In Fig. 1.116, find the apparent depth of the object seen below surface AB.**

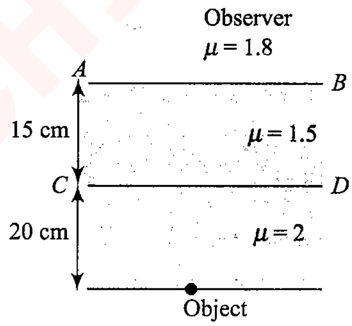


Fig. 1.116

Sol. $D_{app} = \sum \dfrac{d}{\mu} = \dfrac{20}{\left(\dfrac{2}{1.8}\right)} + \dfrac{15}{\left(\dfrac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm

Illustration 1.49 **Light is incident from air on an oil layer at an incident angle of 30°. After moving through the oil 1, oil 2, and glass it enters water. If the refraction index of glass and water are 1.5 and 1.3, respectively. Find the angle which the ray makes with the normal in water.**

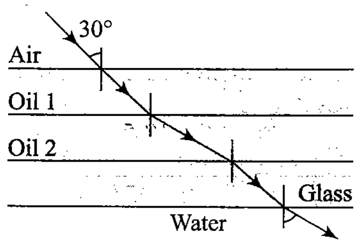


Fig. 1.117

Sol. As we know that $\mu \sin i$ = constant

$\Rightarrow \quad \mu_{air} \sin i_{(air)} = \mu_{glass} \sin r_{(glass)}$

$$\sin i_{(glass)} = \frac{\mu_{air}}{\mu_{glass}} \sin i_{air} \qquad \text{(i)}$$

Again $\quad \mu_{glass} \sin i_{glass} = \mu_{water} \sin r_{water} \qquad$ (ii)

From Eqs. (i) and (ii), $\sin 30° = 1.3 \sin r$

$$\Rightarrow \quad \sin r = \frac{1}{2 \times 1.3} = \frac{1}{2.6};\ r = \sin^{-1}\frac{1}{2.6}$$

Note: ***For parallel layers, we can apply Snell's law directly at initial and final parts.***

Illustration 1.50 **A layer of oil 3 cm thick is floating on a layer of coloured water 5 cm thick. Refractive index of coloured water is 5/3 and the apparent depth of the two liquids appears to be 36/7 cm. Find the refractive index of oil.**

Sol. Apparent depth (AI) $= \dfrac{t_1}{\mu_1} + \dfrac{t_2}{\mu_2}$

$$\therefore \quad \frac{36}{7} = \frac{5}{5/3} + \frac{3}{\mu_2} \Rightarrow \frac{3}{\mu_2} = \frac{36}{7} - 3 = \frac{15}{7} \Rightarrow \mu_2 = \frac{7}{5} = 1.4$$

Illustration 1.51 **In Fig. 1.118, determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.**

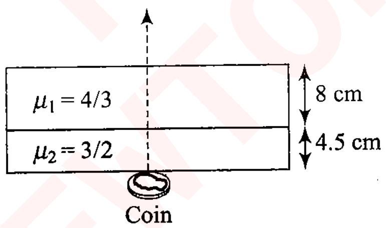


Fig. 1.118

Sol. Total apparent shift is

$$s = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

or
$$s = 8\left(1 - \frac{1}{\frac{4}{3}}\right) + 4.5\left(1 - \frac{1}{\frac{3}{2}}\right)$$

or
$$s = 2 + 1.5 = 3.5 \text{ cm}$$

The apparent depth of the coin from the top is $t = (8 + 4.5) - 3.5 = 9$ cm and, the real depth of the coin is

$$t_1 + t_2 = 8 + 4.5 = 12.5$$

Therefore, the effective refractive index is $\mu_{eff} = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

$$= \frac{t_1 + t_2}{t} = \frac{12.5}{9}$$
$$= 1.39$$

SLAB AND MIRROR COMBINED

Let us observe what happens if one surface of the slab is silvered?

Consider the silvered slab shown in the Fig. 1.119. An object is placed in front of a silvered glass slab.

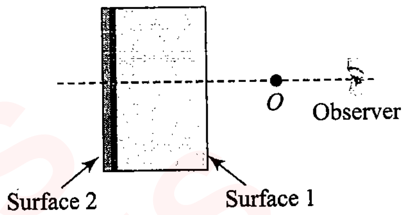


Fig. 1.119

Here, a ray of light from the object first refracts at surface 1. It is then reflected from surface 2 before refracting again at surface 1 and emerging (See Fig. 1.120). So, we can consider a silvered slab as a combination of

1. A refracting surface,
2. A reflecting surface, and
3. A refracting surface again.

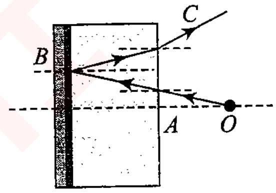


Fig. 1.120

The above situation can be considered as a combination of a slab and a plane mirror placed together. Thus, a silvered slab is a combination of

1. A glass slab,
2. A plane mirror, and
3. A glass slab again.

To learn the concept, we will discuss the situation through an illustration.

Illustration 1.52 **An object is placed in front of a slab (μ= 1.5) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.**

Sol. Method of interface: A ray of light from the object O undergoes refraction, reflection and then refraction.

Refraction at surface 1:

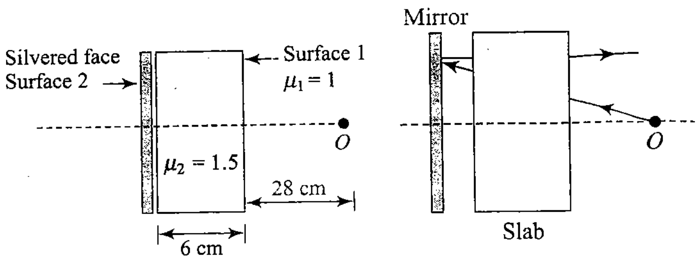


Fig. 1.121

Here, $\quad \mu_1 = 1,\ \mu_2 = 1.5$

$\quad d_1 = 28$ cm, $d_2 = ?$

Since $d_2 = \dfrac{d_1}{n_{\text{relative}}} = \dfrac{d_1}{(1/\mu)}$; $\left[n_{\text{rel}} = \dfrac{n_{\text{incident}}}{n_{\text{refracted}}} = \dfrac{1}{\mu}\right]$

$\Rightarrow \quad d_2 = \mu d_1 = 1.5 \times 28 = 42$ cm

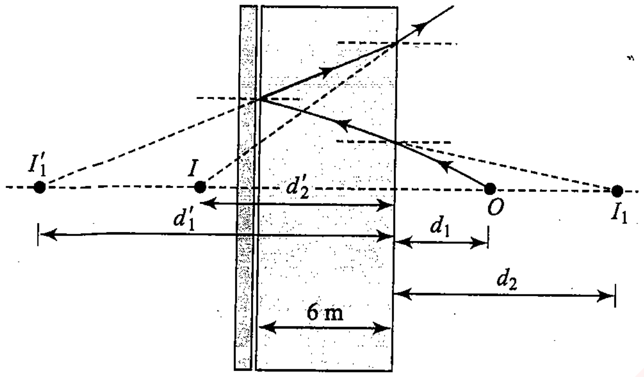


Fig. 1.122

Therefore, $d_2 = 42$ cm from the first interface. The first image I_1 is formed 42 cm in front of the slab.

Reflection at surface 2:

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is = 42 + 6 = 48 cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front of it. Therefore, the second image I_1' is formed 48 cm behind the mirror.

Second refraction at surface 1:

Object distance from surface 1,

$d_1 = 48 + 6 = 54$ cm

$$d_2' = \frac{d_1'}{n_{\text{relative}}} = \frac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{54}{(1.5/1)} = 36 \text{ cm}$$

So, the final image is at a distance $d_2 = 36$ cm behind the first interface.

Hence, final image is formed 36 cm behind surface 1 or 30 cm behind surface 2.

Method 2: Shifting of object

A ray of light from the object first encounters a glass slab, then a mirror, and finally a glass slab again.

Glass slab: A slab simply shifts the object along the axis by a distance $s_1 = t\left(1 - \dfrac{1}{\mu}\right) = 2$ cm

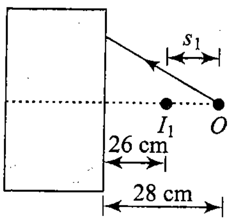


Fig. 1.123

Direction of shift of object is towards left. Therefore, the object appears to be at I_1 which is 28 – 2 = 26 cm from the slab.

For mirror, the object for the mirror is the image I_1 formed after shift due to the slab.

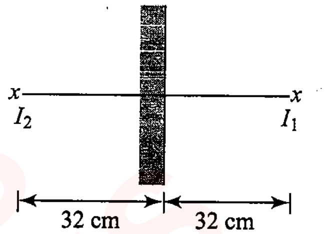


Fig. 1.124

Therefore, object distance from the mirror is 26 + 6 = 32 cm. The image will now be formed 32 cm behind the mirror. Now, reflected rays are travelling from left to right.

The ray now travels through the slab again but this time from right to left. Therefore, it is shifted again by a distance of 2 cm, but towards the right. Thus, final position of the image is 32 – 2 = 30 cm behind the mirror.

Method 3: Shifting of mirror

By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift 2 m towards right for observer in front of the slab.

The position of the object from shifted mirror = 32 cm.

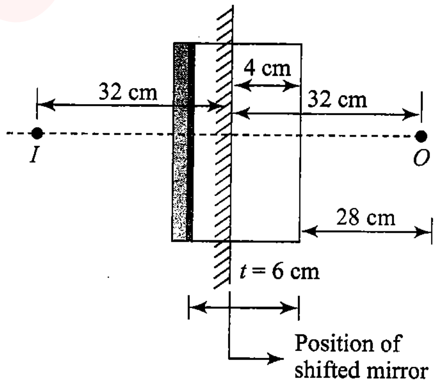


Fig. 1.125

So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

Let us learn the combination of the slab and mirror through some more illustrations.

Illustration 1.53 **A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 cm from the mirror.**

Sol. The rays from O will first pass through slab and produce shifting towards right. The glass slab will form an image of O at I_1 such that

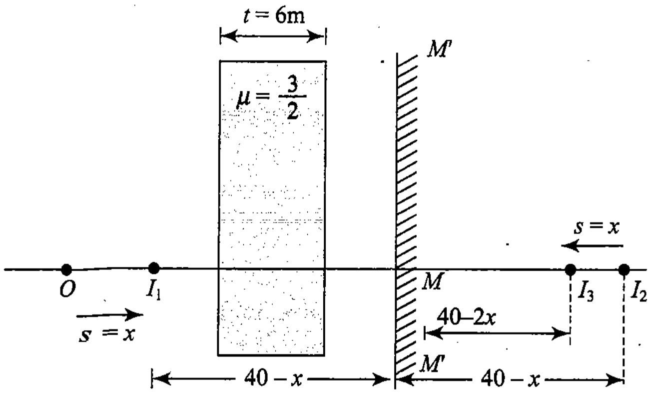


Fig. 1.126

$$OI_1 = x = d\left[1 - \frac{1}{\mu}\right] = 20\left[1 - \frac{2}{3}\right] = \frac{20}{3}\text{ cm} \qquad \text{(i)}$$

Now, the rays after passing through slab will (towards right)

So, the distance of I_1 from the mirror MM',

$$I_1M = (40 - x).$$

This image I_1 will act as an object for the mirror and the mirror will form an image I_2, such that

$$MI_2 = MI_1 = 40 - x \qquad \text{[with } x \text{ given by Eq. (i)]}$$

Now, image I_2 will act as an object again for the glass slab which by producing a shift of x forms the final image I_3 such that $I_2I_3 = x$. Hence, the distance of final image I_3 from the mirror will be

$$MI_3 = MI_2 - I_3I_2 = (40 - x) - x = 40 - 2x$$

$$MI_3 = 40 - 2 \times \left[\frac{20}{3}\right] = \left[\frac{80}{3}\right]\text{ cm}$$

[as from Eq. (i), $x = 20/3$ cm]

Alternative solution: As thickness of glass slab is 20 cm and $\mu = (3/2)$, so shift produced by it will be

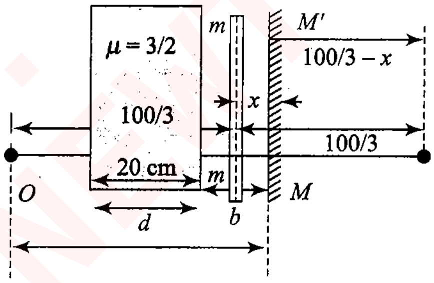


Fig. 1.127

$$x = d\left[1 - \frac{d}{\mu}\right] = 20\left[1 - \frac{2}{3}\right] = \frac{20}{3}\text{ cm}$$

So, the glass slab will shift the mirror from MM' to mm' as shown in.

The distance of object from this virtual mirror will be

$$40 - x = 40 - (2/3) = (100/3)\text{ cm}$$

This virtual mirror will form the image of object O at a distance (100/3) behind it and so the distance of image from actual mirror MM' will be $\frac{100}{3} - \frac{20}{3} = \frac{80}{3}$ cm [as mm' is 20/3 cm in front of MM']

Illustration 1.54 **A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index μ = 3/2 and thickness 6 cm is inserted between the object and mirror.**

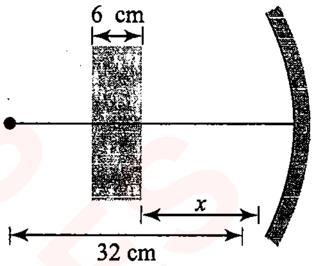


Fig. 1.128

Find the position of final image when the distance x shown in Fig. 1.128 is, (a) 5 cm and (b) 20 cm.

Sol. The normal shift produced by a glass slab is,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2\text{ cm}$$

i.e., for the mirror the object is placed at a distanced $(32 - S) = 30$ cm from it.

Applying mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} + \frac{1}{30} = -\frac{1}{10}$

or $v = -15$ cm

a. When x = 5 cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance S = 2 cm. Hence, the final real image is formed at a distance (15 + 2) = 17 cm from the mirror.

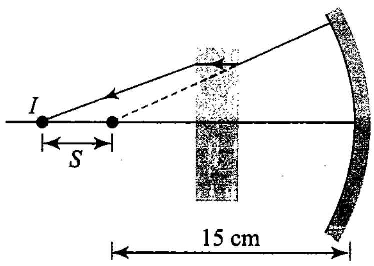


Fig. 1.129

b. When x = 20 cm: This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

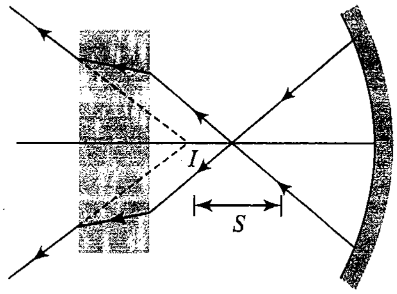


Fig. 1.130

Illustration 1.55 **A vessel having perfectly reflecting plane bottom is filled with water (μ = 4/3) to a depth d. A point source of light is placed at a height h above the surface of water. Find the distance of final image from water surface.**

Sol. As shown in Fig. 1.131, water will form the image of object O at I_1 such that $OI_1 = y = d[1 - (1/\mu)]$, so that the distance of image I_1 from water surface will be $I_1A = h - y = h - d[1 - (1/\mu)]$. Hence, the distance of this image I_1 from mirror MM',

$$I_1M = I_1A + AM = \left[h - d\left(1 - \frac{1}{\mu}\right)\right] + d = h + \left[\frac{d}{\mu}\right]$$

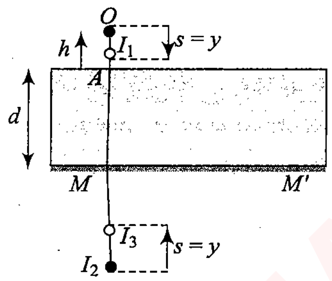


Fig. 1.131

Now, image I_1 will act as object for mirror MM'. As a plane mirror forms image at same distance behind the mirror as the object is in front of it, the image of I_1 formed by the mirror MM' will be I_2 such that $I_1M = I_2M = h + (d/\mu)$.

Now, this image I_2 will act as object for water again and water will produce image I_3 such that $I_2I_3 = y = d\left(1 - \frac{1}{\mu}\right)$. So, the distance of image I_3 from the surface of water AC will be

$$AI_3 = AM + MI_2 - I_2I_3 = d + \left[h + \frac{d}{\mu}\right] - d\left[1 - \frac{1}{\mu}\right]$$

$$AI_3 = h + 2\frac{d}{\mu} = h + \frac{3}{2}d$$

Alternative solution:

As shown in Fig. 1.132, water will form the image of bottom, i.e., mirror MM' at a depth (d/μ) from its surface. So, the distance of object O from virtual mirror mm' will be $h + (d/\mu)$.

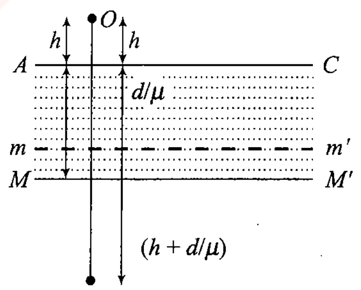


Fig. 1.132

Now, as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image I from mm' will be $h + (d/\mu)$. Also, as the distance of virtual mirror from the surface of water is (d/μ), the distance of image I from the surface of water will be

$$\left[h + \frac{d}{\mu}\right] + \frac{d}{\mu} = h + \frac{2d}{\mu} = h + \frac{3}{2}d \qquad \left[\text{as } \mu = \frac{4}{3}\right]$$

Illustration 1.56 **A concave mirror of focal length 20 cm is placed inside water with its shining surface upwards and principal axis vertical as shown in Fig. 1.133. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.**

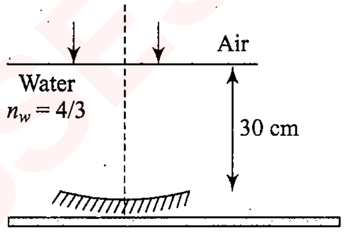


Fig. 1.133

Sol. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A (in figure Fig. 1.134) will be formed at focus which is 20 cm from the mirror. Now, for the interface between water and air. For observer at air, the image formed by mirror will act as an object for observer with real depth $d = 30 - 20 = 10$ cm.

Apparent depth observed by observer,

$$d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm}$$

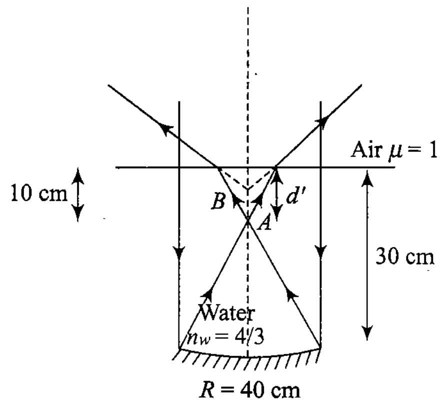


Fig. 1.134

REFRACTION IN A MEDIUM WITH VARIABLE REFRACTIVE INDEX

So far, we have assumed that the refractive index of the slab is a constant. This need not be necessarily true. An example of such a situation is the atmosphere. The atmosphere becomes thinner as we go up. Hence, the refractive index of air is highest close to surface of earth and decreases as we move upward. How do we analyse such a problem?

Divide the medium into different layers. This indicates that each layer has different refractive index having value according to a given expression. If μ is a function of x, then a differential layer will be a thin layer of thickness dx. If μ is a function of y, then a differential layer will be a thin layer of thickness dy.

At a given layer, let the angle of incidence be i. Relate this angle of incidence to the initial condition and the refractive index at that point using the relation

$$\mu \sin(i) = \text{constant} \quad \text{(i)}$$

From the given ray diagram (Fig. 1.135), draw a tangent to the path taken by the ray of light. Geometrically, relate the slope of this tangent to the angle of incidence.

$$\tan\theta = \frac{dy}{dx} = \tan(90 - i) \quad \text{(ii)}$$

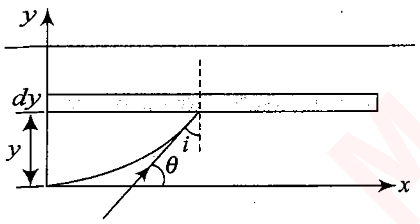


Fig. 1.135

Substitute for i from Eq. (i) and determine dy/dx as a function of x and y.

Integrate and obtain an expression for y as a function of x.

Let us discuss a case where refractive index is changing in y-direction.

As 1: $\mu = f(y)$

Consider a slab with a medium where the refractive index varies from μ_1 to μ_2 in relation with y. Assume that the outside medium is air. Let light be incident on the glass slab at an angle θ. The light ray strikes the glass slab at the point A. The normal at this point is NN' as shown in Fig. 1.136. Once the ray enters the medium, the refractive index varies continuously as if there are infinite number of successive glass slabs. Therefore, the angle of incidence also varies continuously and the path of the ray is curved. The ray finally reaches the second surface at the point B before exiting the slab, once again refracting at the exit point.

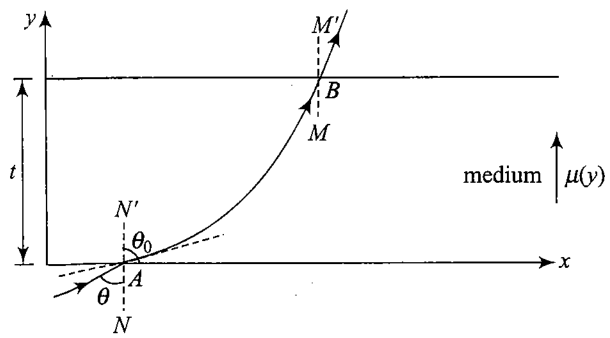


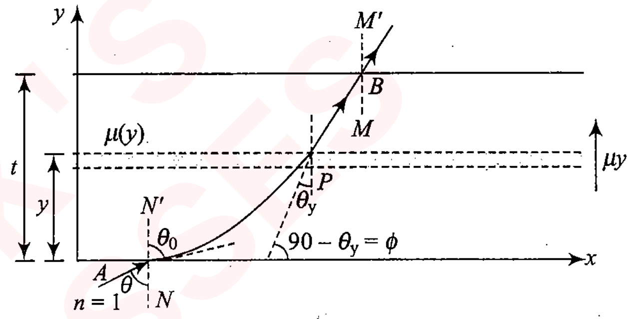
Fig. 1.136

How can we derive a mathematical expression for the equation of the ray in the medium?

Consider refraction at some height y (Fig. 1.137). Here, the angle of incidence is θ_y and the refractive index of the medium is μ_y.

The product of the refractive index and the sine of the angle of incidence is constant. Therefore, at the height y we can say that

$$\mu_y \sin\theta_y = \text{constant}. \quad \text{(i)}$$



Path of a ray of light being refracted in a slab with refractive index as a function of y and y ranges from 0 to 1.

Fig. 1.137

Furthermore, applying the law of refraction at point A which is the interface between air and the medium, we have

$$1 \times \sin(\theta) = \mu_y \sin\theta_y \quad \text{(ii)}$$

Therefore, $$\sin\theta_y = \frac{\sin\theta}{\mu_y}$$

$$\cos\theta_y = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\mu_y}$$

Hence, $$\cot\theta_y = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$$

Now, the slope of the curve at the point P is given by

$$\tan\phi = \frac{dy}{dx} \quad \text{(iii)}$$

But $$\phi = 90 - \theta_y$$

Therefore, $$\frac{dy}{dx} = \cot(\theta_y) = \frac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta} \quad \text{(iv)}$$

$$\frac{\sin\theta\, dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = dx$$

Integrating both side with proper limits.

If the functional dependence of "μ_y" is known, we can easily solve for the equation of the curve.

$$\sin\theta \int_{y_1}^{y_1} \frac{dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = \int_{x_1}^{x_2} dx$$

Illustration 1.57 **A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is $d = 2$ m, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.**

Sol. From the equation, we have $\dfrac{dy}{dx} = \cot(\theta_y) = \dfrac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$

Here, $\mu = \sqrt{(1+y)}$ and $\theta = 90°$. Therefore, $\dfrac{dy}{dt} = y^{1/2}$

Integrating with the boundary condition that $y = 0$ at $x = 0$, we get $y = x^2/4$ to be the equation of the path of the ray through the slab. The ray will obviously exit at the point $(2\sqrt{2}\text{ m}, 2\text{ m})$.

Illustration 1.58 **Due to a vertical temperature gradient in the atmosphere, the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1+ay}$, where n_0 is the index of refraction at the surface and $a = 2.0 \times 10^{-6}$ m^{-1}. A person of height h = 2.0 m stands on a level surface. Beyond what distance will he not see the runway?**

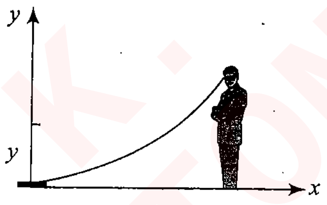


Fig. 1.138

Sol. As refractive index is changing along y-direction, we can assume a number of thin layers of air placed parallel to x-axis. Let O be the distant object just visible to the man. Consider a layer of air at a distance y from the ground. Let P be a point on the trajectory of the ray. From Fig. 1.139, $\theta = 90 - i$.

The slope of tangent at point P is $\tan\theta = dy/dx = \cot i$.

From Snell's law, $n \sin i$ = constant

At the surface, $n = n_0$ and $i = 90°$.

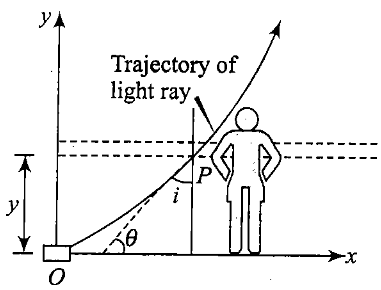


Fig. 1.139

$n_0 \sin 90° = n \sin i = (n_0\sqrt{1+ay}) \sin i$

$\sin i = \dfrac{1}{\sqrt{1+ay}} \Rightarrow \cot i = \dfrac{dy}{dx} = \sqrt{ay}$

$\displaystyle\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx \Rightarrow x = 2\sqrt{\frac{y}{a}}$

On substituting $y = 2.0$ m and $a = 2 \times 10^{-6}$ m^{-1}, we have

$$x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000 \text{ m}$$

MEASUREMENT OF REFRACTIVE INDEX OF A LIQUID BY A TRAVELLING MICROSCOPE

Using the fact that $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

We may determine the refractive index of a glass slab. (Fig. 1.140)

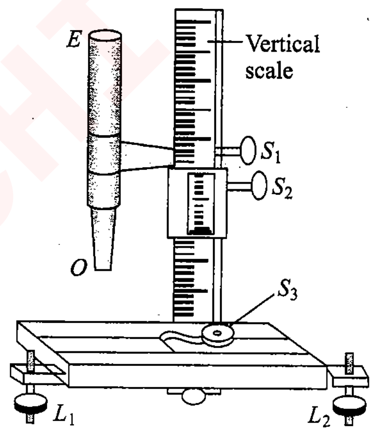


Fig. 1.140

The traveling microscope is first focused on a fine mark on a piece of paper and the reading on the scale is noted as Y_1. Then, the glass slab is placed on the paper.

The microscope is now shifted up and is focused again on the mark so that it is distinctly visible. The reading of the microscope is noted as Y_2.

Now, a little lycopodium powder is scattered on the upper surface of the glass block. The microscope is focused on the powder and the third reading of the scale is noted as Y_3.

The real thickness of the glass slab is $Y_3 - Y_1$ and apparent thickness of the glass slab is $Y_3 - Y_2$. Thus,

$$\mu = \frac{Y_3 - Y_1}{Y_3 - Y_2}$$

The same method can also be used to find the refractive index of a liquid. Any mark on the bottom of a glass vessel is first focused and then liquid is poured into it and the same mark is again focused. Finally, a little lycopodium powder is sprinkled on the surface of the liquid and is focused as before.

Concept Application Exercise 1.3

1. Identify the True and False statements.

(i) A glass slab cannot deviate the light.

(ii) A glass slab can produce lateral displacement.

(iii) The shift produced by a slab depends on the converging and diverging nature of beam.

(iv) Apparent shift in case of a slab always occurs in the direction of light ray travelling.

(v) The shift produced by a slab can never exceed its thickness.

2. In figure, a point source S is placed at a height h above the plane mirror in a medium of refractive index μ.

(i) Find the number of images seen for normal view.

(ii) Find the distance between the images.

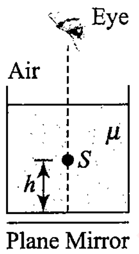


Fig. 1.141

3. A beam of width t is incident at 45° on an air–water boundary. The width of the beam in water is ______.

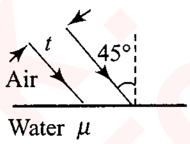


Fig. 1.142

4. A fish is 60 cm under water (μ = 4/3). A bird directly overhead looks at the fish. If the bird is at a distance of 120 cm from the water surface, (i) what is the apparent position of the fish as seen by the bird and (ii) what is the apparent position of the bird as seen by the fish?

5. A converging set of rays, traveling from water to air, is incident on a plane interface. In the absence of the interface, the rays would have converged to a point O, 60 cm above the interface. However, due to refraction the rays will bend. At what distance above the interface will the rays actually converge?

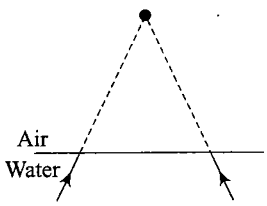


Fig. 1.143

6. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is of oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.

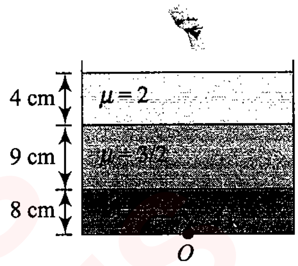


Fig. 1.144

7. A convergent beam is incident on two slabs placed in contact as shown in Fig. 1.145. Where will the rays finally converge?

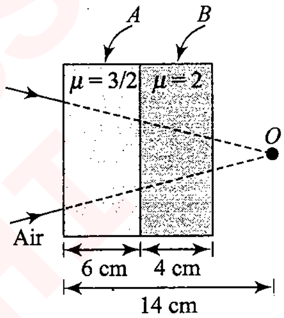


Fig. 1.145

8. A slab of water is on the top of a glass slab of refractive index 2. At what angle to the normal must a ray be incident on the top surface of the glass slab so that it is reflected from the bottom surface as shown in Fig. 1.146?

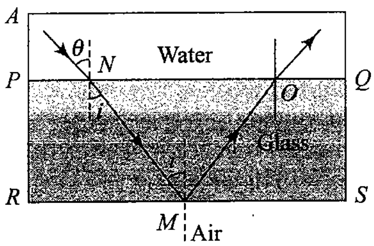


Fig. 1.146

9. A ray of light travels from a liquid of refractive index μ to air. If the incident beam is rotating at a rate ω, what is the angular speed of the refracted beam at the instant the angle of incidence is 30°?

(Given $\mu = \sqrt{2}, \omega = 1/\sqrt{6}$ rad/sec.)

10. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

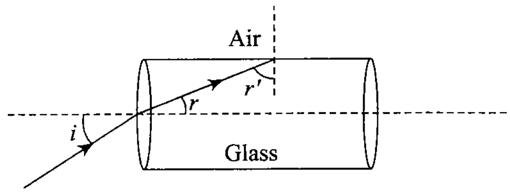


Fig. 1.147

11. An object is placed on the principle axis of a concave mirror of focal length 10 cm at a distance of 21 cm from it. A glass slab is placed between the mirror and the object as shown in Fig. 1.148.

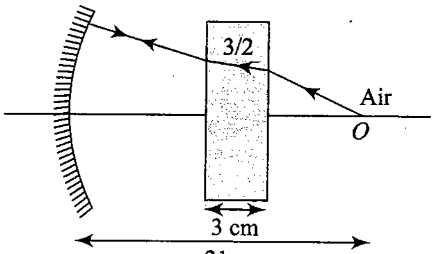


Fig. 1.148

Find the distance of final image formed by the mirror.

12. The image of an object kept at a distance of 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab (μ = 1.5) of thickness 3 cm is introduced between the mirror and the object, then

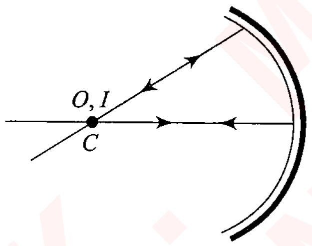


Fig. 1.149

a. Identify, in which direction the mirror should be displaced so that the final image may again coincide with the object itself.

b. Find the magnitude of displacement.

13. In Fig. 1.150, a fish watcher watches a fish through a 3.0 cm thick glass wall of a fish tank. The watcher is in level with the fish; the index of refraction of the glass is 8/5 and that of the water is 4/3.

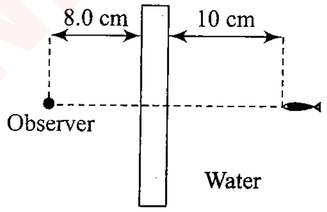


Fig. 1.150

a. To the fish, how far away does the watcher appear to be?

b. To the watcher, how far away does the fish appear to be?

14. A observer can see through a pin hole, the top of a thin rod of height h, placed as shown in Fig. 1.151. The beaker's height is $3h$ and its radius is h. When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Find the refractive index of the liquid.

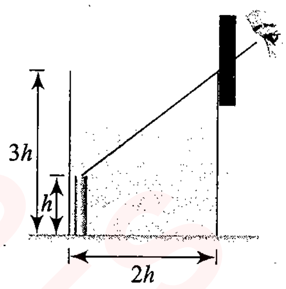


Fig. 1.151

15. A vessel contains a slab of glass 8 cm thick and of refractive index 1.6. Over the slab, the vessel is filled by oil of refractive index μ upto height 4.5 cm and then by another liquid, i.e., water of refractive index 4/3 and height 6 cm as shown in Fig. 1.152. An observer looking down from above observes that a mark at the bottom of glass slab appears to be raised up to a position 6 cm from bottom of the slab. Find refractive index of oil (μ).

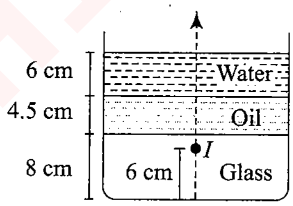


Fig. 1.152

16. An object O is placed at 8 cm in front of a glass slab, whose one face is silvered as shown in Fig. 1.153. The thickness of the slab is 6 cm. If the image formed 10 cm behind the silvered face, find the refractive index of glass.

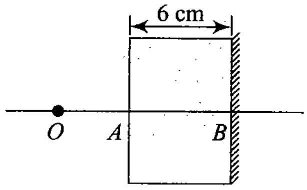


Fig. 1.153

17. x-y plane separates two media, $z \geq 0$ contains a medium of refractive index 1 and $z \leq 0$ contains a medium of refractive index 2. A ray of light is incident from first medium along a vector $\hat{i} + \hat{j} - \hat{k}$. Find the unit vector along the refracted ray.

18. The n transparent slabs of refractive index 1.5 each having thicknesses 1 cm, 2 cm, ... to n cm are arranged one over another. A point object is seen through this combination with near perpendicular light. If the shift of object by the combination is 1 cm, then find the value of n.

19. A concave mirror with its optic axis vertical and mirror facing upward is placed at the bottom of the water tank. The radius of curvature of the mirror is 40 cm and refractive index for water μ = 4/3. The tank is 20 cm deep and if a bird is flying over the tank at a height of 60 cm above the surface of water, find the position of image of the bird.

20. Consider the situation shown in Fig. 1.154. A plane mirror is fixed at a height h above the bottom of a beaker containing water (refractive index μ) upto a height d. Find the position of the image of the bottom formed by the mirror.

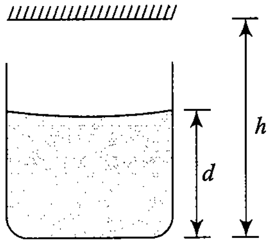


Fig. 1.154

21. A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm. A small dust particles floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.

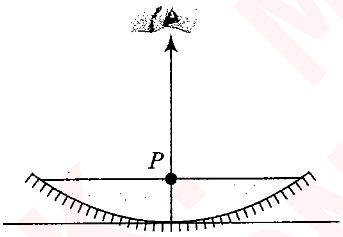


Fig. 1.155

22. A concave mirror of radius R is kept on a horizontal table. Water (refractive index = μ) is poured into it upto a height h. Where should an object be placed so that its image is formed on itself ?

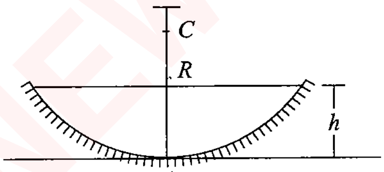


Fig. 1.156

23. The refractive index of an anisotropic medium varies as $\mu = \mu_0 = \sqrt{(x+1)}$, where $0 \le x \le a$. A ray of light is incident at the origin just along y-axis (shown in figure). Find the equation of ray in the medium.

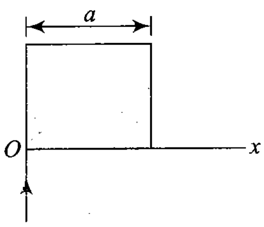


Fig. 1.157

PRISM

A prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other (Fig. 1.158).

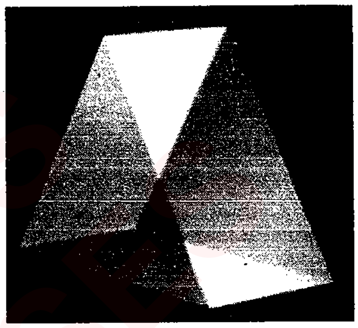


Fig. 1.158

Prisms have the property to change the direction of light. Fig. 1.159 (a) to (e) present some of the useful applications.

The right-angled prism in figure (a) turns the light through 90°. A poro prism shown in figure (b) is a right-angled prism, it turns light through 180°. Figure (c) shows two right-angled prisms used in binoculars. Figures (d) and (e) illustrate image formation with and without deviation; in both the cases the image is laterally inverted.

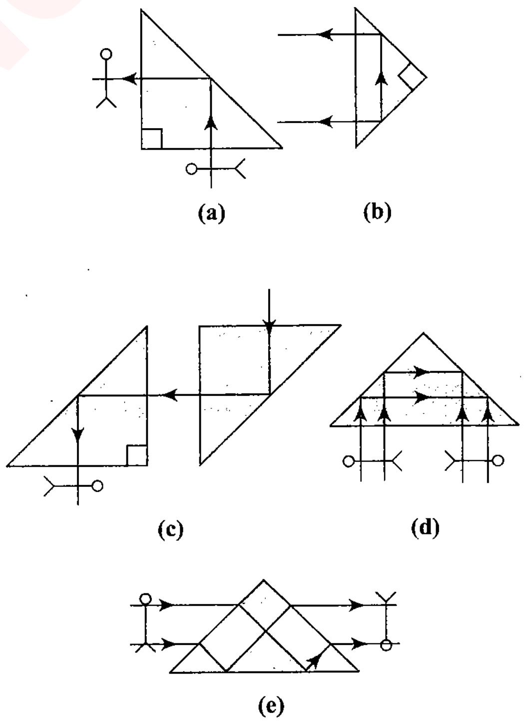


Fig. 1.159

Basic Terms

(i) Angle of prism or reflecting angle (A):

The angle between the faces on which light is incident and from which it emerges (Fig. 1.160).

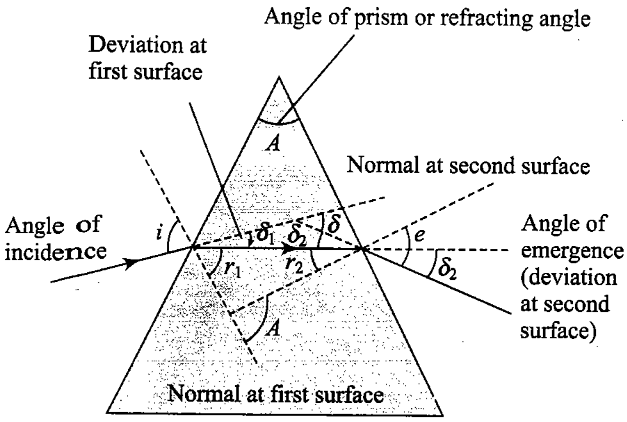


Fig. 1.160

(ii) Angle of deviation (δ):
It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2) \quad \text{or} \quad \delta = i + e - (r_1 + r_2)$$

or $$\delta = i + e - A$$

Condition of No Emergence

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle, i.e., $\mu > 1/[\sin(A/2)]$. (see Fig. 1.161).

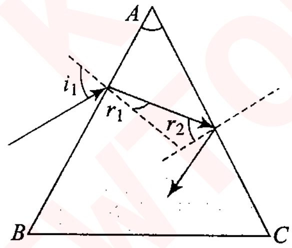


Fig. 1.161

Illustration 1.59 **What should be the minimum value of refractive index of a prism, refracting angle A, so that there is no emergent ray irrespective of the angle of incidence?**

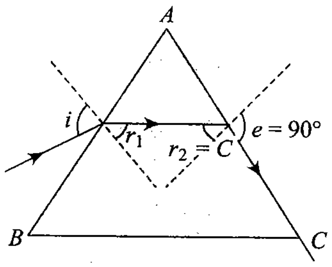


Fig. 1.162

Sol. If the ray just emerges from face AC,

$$e = 90° \quad \text{and} \quad r_2 = C \qquad \text{(i)}$$

From Snell's law at face AB, we have

$$1 \sin i = n \sin r_1 \qquad \text{(ii)}$$

$$A = r_1 + r_2 = r_1 + C \qquad \text{(iii)}$$

From Eq. (ii), n is minimum when r_1 is maximum, i.e., $r_2 = C$. In this case, $i = 90°$

From Eq. (iii), $A = 2C$ or $C = A/2$

As $$\sin C = \frac{1}{n} \Rightarrow \sin\frac{A}{2} = \frac{1}{n}$$

$$n = \operatorname{cosec}\frac{A}{2}$$

Condition of Grazing Emergence

By the condition of grazing emergence, we mean the angle of incidence i at which the angle of emergence becomes 90° (see Fig. 1.163).

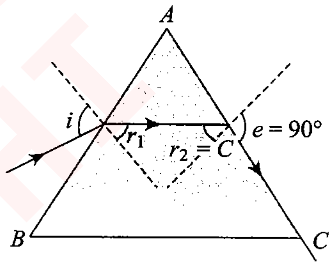


Fig. 1.163

Consider a prism with refracting angle A and refractive index n.

The ray grazes face AC.

So, $$e = 90°, r_2 = C \qquad \text{(i)}$$

and $$A = r_1 + r_2 = r_1 + C \qquad \text{(ii)}$$

Also, $$\sin C = 1/n$$

From Snell's law at face AB, $1 \sin i = n \sin r_1$

$$\sin i = n \sin(A - C)$$
$$= n[\sin A \cos C - \cos A \sin C]$$
$$= n\left[\sin A\sqrt{1 - \sin^2 C} - \cos A \sin C\right]$$
$$= \sqrt{n^2 - 1}\sin A - \cos A$$
$$i = \sin^{-1}\left[\sqrt{n^2 - 1}\sin A - \cos A\right]$$

Condition of Maximum Deviation

Maximum deviation occurs when the angle of incidence is 90° (Fig. 1.164).

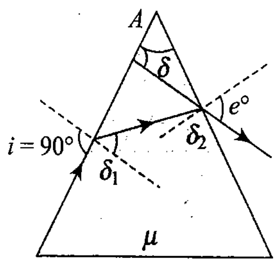


Fig. 1.164

$$\delta_{max} = 90° + e - A$$

where $\quad e = \sin^{-1}[\mu \sin (A - \theta_c)]$

Condition of Minimum Deviation

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence (Fig. 1.165),

i.e, $\quad i = e$; $\delta_{min} = 2i - A$.

Using Snell's law, we get

$$\mu = \frac{\sin[(\delta_{min} + A)/2]}{\sin[A/2]}$$

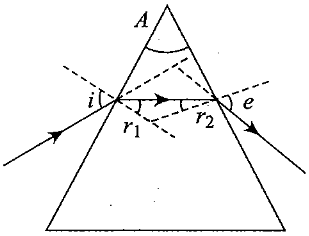


Fig. 1.165

Note that in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

(i) Variation of δ versus i (shown in diagram).
For each δ (except δ_{min}), there are two values of angle of incidence. If i and e are interchanged, then we get the same value of δ because of reversibility principle of light.

(ii) There is one and only one angle of incidence for which the angle of deviation is minimum.

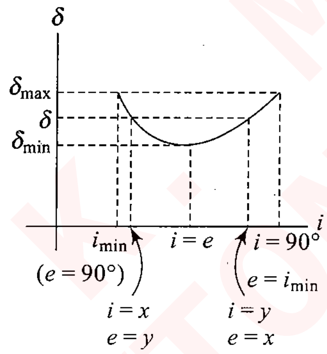


Fig. 1.166

Illustration 1.60 **An isosceles prism has one of the refracting surfaces silvered. A ray of light is incident normally on the refracting face *AB*. After two reflections, the ray emerges from the base of the prism perpendicular to it.**
Find the angle of the prism.

Sol. The incident ray passes without deviation from face *AB*. It suffers reflections at *P* and *Q*. From Fig. 1.167, incident ray and normal at *Q* are parallel; therefore,

$$a = 2A \qquad \text{(i)}$$

Also, $\quad 2\alpha + A = 180° \qquad \text{(ii)}$

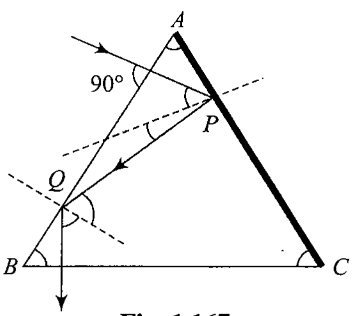


Fig. 1.167

On solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$

Illustration 1.61 **Figure 1.168 shows a triangular prism of refracting angle 90°. A ray of light incident at face *AB* at an angle θ_1 refracts at point *Q* with an angle of refraction 90°.**

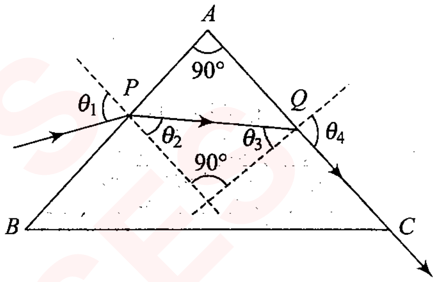


Fig. 1.168

a. What is the refractive index of the prism in terms of θ_1?
b. What is the maximum value that the refractive index can have?
What happens to the light at *Q* if the incident angle at *Q* is
c. increased slightly, and
d. decreased slightly?

Sol. a. Let the ray be incident at an angle θ_1 at the face *AB*. It refracts at an angle θ_2 and is incident at an angle θ_3 at face *AC*. Finally, the ray comes out at an angle $\theta_4 = 90°$.
From Fig. 1.169, the normals at faces *AB* and *AC* make an angle of 90° with each other, $\theta_3 = 90° - \theta_2$

$$\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1 - \sin^2\theta_2} \qquad \text{(i)}$$

From Snell's law at face *AC*, $n \sin\theta_3 = 1$

$$n\sqrt{1 - \sin^2\theta_2} = 1 \qquad \text{(ii)}$$

From Snell's law at face *AB*, $1 \sin\theta_1 = n \sin\theta_2$

$$\sin\theta_2 = \frac{\sin\theta_1}{n} \qquad \text{(iii)}$$

From Eqs. (ii) and (iii), we have

$$n\sqrt{1 - \frac{\sin^2\theta_1}{n^2}} = 1 \qquad \text{(iv)}$$

On squaring Eq. (iv) and solving for *n*, we get

$$n = \sqrt{1 + \sin^2\theta_1}$$

b. The greatest possible value of $\sin^2\theta_1$ is 1, hence the greatest possible value of *n* is $n_{max} = \sqrt{2} = 1.41$.

c. For a given *n*, if θ_1 is increased the angle of refraction θ_2 increases. As $\theta_3 = 90° - \theta_2$, the angle θ decreases, i.e., the angle of incidence at face *AC* is less than the critical angle for total reflection; hence light emerges into air.

d. If the angle of incidence is decreased, the angle of refraction θ_2 decreases. So, the angle θ_3 increases. The angle of incidence at the second surface is greater than the critical angle; so light is reflected at *Q*.

Note: *In the condition of minimum deviation, the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.*

Illustration 1.62 **A prism has refracting angle equal to $\pi/2$. It is given that γ is the angle of minimum deviation and β is the deviation of the ray entering at grazing incidence. Prove that $\sin\gamma = \sin^2\beta$.**

Sol. Applying condition of minimum deviation,

$$\mu = \frac{\sin\frac{(A+\gamma)}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2}\cos\frac{\gamma}{2}+\cos\frac{A}{2}\sin\frac{\gamma}{2}}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad \mu = \cos\frac{\gamma}{2}+\cot\frac{A}{2}\sin\frac{\gamma}{2}$$

Using $A = 90°$, $\mu = \cos\frac{\gamma}{2}+\cot 45°\sin\frac{\gamma}{2}$

$$\Rightarrow \quad \cos\frac{\gamma}{2}+\sin\frac{\gamma}{2} = \mu$$

Squaring, $\cos^2\frac{r}{2}+\sin^2\frac{\gamma}{2}+\gamma = \mu^2 \Rightarrow \sin\gamma = \mu^2-1$ (i)

Deviation at grazing incidence,

$$\beta = \delta_1+\delta_2$$

$$\beta = \left(\frac{\pi}{2}-C\right)+(e-r_2)$$

$$\Rightarrow \quad \beta = \left(\frac{\pi}{2}-C\right)+\left[e-\left(\frac{\pi}{2}-C\right)\right]$$

$$\Rightarrow \quad \beta = e$$

or $\quad \sin\beta = \sin e = \mu\sin r_2 = \mu\sin\left(\frac{\pi}{2}-C\right)$

$\Rightarrow \quad \sin\beta = \mu\cos C$ (ii)

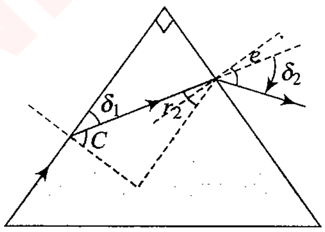


Fig. 1.169

Squaring Eq. (ii),

$$\sin^2\beta = \mu^2\cos^2 C \quad \Rightarrow \quad \sin^2\beta = \mu^2(1-\sin^2 C)$$

Using $\quad \sin C = \frac{1}{\mu}$, $\sin^2\beta = \mu^2\left(1-\frac{1}{\mu^2}\right)$

$\Rightarrow \quad \sin^2\beta = \mu^2-1$ (iii)

From Eqs. (i) and (iii),

$$\sin\gamma = \sin^2\beta$$

Illustration 1.63 **A rectangular block of refractive index μ is placed on a printed page lying on a horizontal surface as shown in Fig. 1.170. Find the minimum value of μ so that the letter L on the page is not visible from any of the vertical sides.**

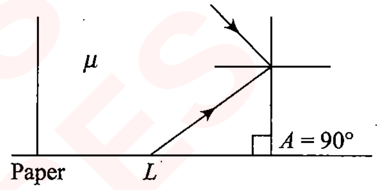


Fig. 1.170

Sol. The letter L will not be visible from the vertical sides if the light ray does not enter through it.

We can apply the condition of no emergence for a prism of angle $A = 90°$.

That is, $\quad \mu > \frac{1}{\sin\frac{A}{2}}$ or $\mu > \frac{1}{\sin\frac{90°}{2}}$ or $\mu > \sqrt{2}$

Illustration 1.64 **The cross section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice, emerges through the base of the prism perpendicular to it. Find the angles of the prism.**

Sol. The incident ray BC at normal incidence is reflected at silvered face, along DE and at E it again suffers reflection along EF. Since the ray emerges normally from the base, therefore the ray EF must fall normally on the base and emerges along EG.

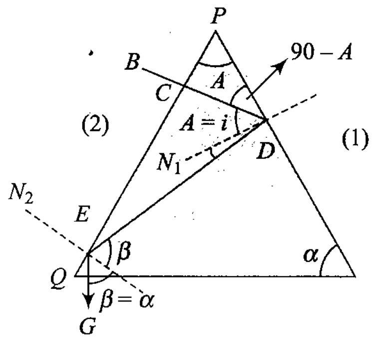


Fig. 1.171

We find $\quad i = A$. Also, $\beta = a$.

Since $\quad EN_2 \parallel CD$, $\beta = 2i$ (alternate angles)

$\therefore \quad \alpha = 2A \quad (\beta = \alpha, i = A)$ (i)

Also, $\quad 2\alpha + A = 180° \quad (\because$ Sum of angles of a triangle $= 180°)$ (ii)

Solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$.

Illustration 1.65 **For a prism, $A = 60°$, $n = \sqrt{7/3}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also, find δ_{max}.**

Sol. In minimum incidence case, the angles will be as shown in Fig. 1.172.

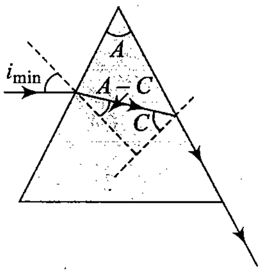


Fig. 1.172

Applying Snell's law, we get

$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin(A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$

$$= \sqrt{\frac{7}{3}} \left(\sin 60 \sqrt{1 - \frac{3}{7}} - \cos 60 \sqrt{\frac{3}{7}} \right) = \frac{1}{2}$$

$\therefore$ $i_{min} = 30°$

$\therefore$ $\delta_{max} = i_{min} + 90° - A = 30° + 90° - 60° = 60°$

THIN PRISMS

In thin prisms, the distance between the refracting surfaces is ineligible and the angle of prism (A) is very small. Since $A = r_1 + r_2$, therefore if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2.

According to Snell's law,

$$\sin i_1 = \mu \sin r_1$$

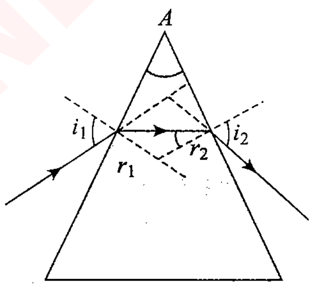


Fig. 1.173

or $\quad i_1 = \mu r_1$

$$\sin i_2 = \mu \sin r_2$$

or $\quad i_2 = \mu r_2$

Therefore, deviation, $\delta = (i_1 - r_1) + (i_2 - r_2) \Rightarrow \delta = (r_1 + r_2)(\mu - 1)$
$\Rightarrow \delta = A\ (\mu - 1)$

Illustration 1.66 **A thin biprism (see Fig. 1.174) of obtuse angle $\alpha = 178°$ is placed at a distance $l = 20$ cm from a slit.**

How many images are formed and what is the separation between them? Refractive index of the material $\mu = 1.6$.

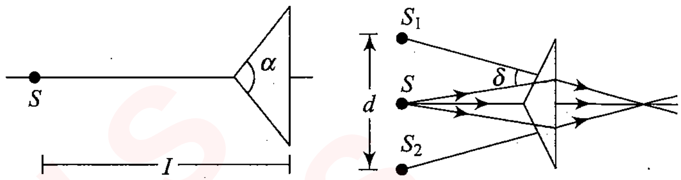


Fig. 1.174

Sol. Two images are formed by the two thin prisms—one above the axis and the other below the axis by the same distance.

The refracting angle of each thin prism is

$$\frac{\pi - \alpha}{2} = \frac{1}{2}(\pi - \alpha)$$

Then, δ (deviation of a ray) $= (\mu - 1)\frac{1}{2}(\pi - \alpha)$

$$\frac{d}{2} = l\delta$$

$$d = 2l(\mu - 1)\frac{1}{2}(\pi - \alpha)$$

$$d = (\mu - 1)l(\pi - \alpha)$$

$$d = (1.6 - 1) \times 0.20 \left(\pi - 178 \times \frac{\pi}{180} \right)$$

$$= 0.6 \times 0.20 \times \pi \times \frac{1}{90} = 0.004 \text{ m} = 4 \text{ mm}$$

Illustration 1.67 **A thin prism of angle $A = 6°$ produces a deviation $d = 3°$. Find the refractive index of the material of prism.**

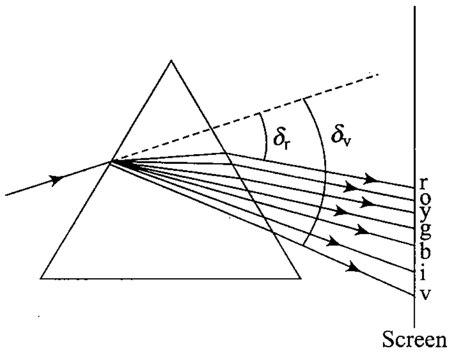


Fig. 1.175

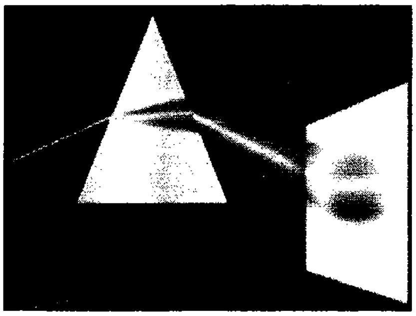


Fig. 1.176

Sol. We know that $d = A(\mu - 1)$ or $\mu = 1 + \dfrac{\delta}{A}$

Here, $A = 6°,\ d_{\min} = 30°$

$$\Rightarrow \quad \mu = \frac{\sin\left(\dfrac{90+30}{2}\right)}{\sin\left(\dfrac{60}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2} = 1.41$$

DISPERSION OF LIGHT

When a ray of light passes through a prism, it splits up into rays of constituent colors or wavelengths. This phenomenon is called dispersion of light.

This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that μ decreases with the increase of wavelength, i.e., $\mu_{\text{blue}} > \mu_{\text{red}}$

Angular dispersion: $\theta = \delta_v - \delta_r$

Dispersive power: Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$

where δ is deviation of mean ray (yellow)

As $\quad d_v = (\mu_v - 1)A,\ d_r = (\mu_r - 1)A$

$$\Rightarrow \quad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \text{where} \quad \mu_y = \frac{\mu_v - \mu_r}{2}$$

Illustration 1.68 **Calculate the dispersive power for crown glass from the given data**

$$\mu_v = \mathbf{1.523}, \quad \textbf{and} \quad \mu_r = \mathbf{1.5145}.$$

Sol. Here, $\mu_v = 1.523$ and $\mu_r = 1.5145$

Mean refractive index,

$$\mu = \frac{1.523 + 1.5145}{2} = 1.51875$$

Dispersive power is given by,

$$\omega = \frac{\mu_v - \mu_r}{(\mu - 1)} = \frac{1.523 - 1.5145}{(1.51875 - 1)} = 0.1639$$

Illustration 1.69 **Find the dispersion produced by a thin prism of 18° having refractive index for red light = 1.56 and for violet light = 1.68.**

Sol. We know that dispersion produced by a thin prism,

$$\theta = (\mu_v - \mu_r)A$$

Here, $\quad \mu_V = 1.68,\ \mu_r = 1.56$ and $A = 18°$

$$\Rightarrow \quad \theta = (1.68 - 1.56) \times 18° = 2.16°$$

Deviation Without Dispersion

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced.

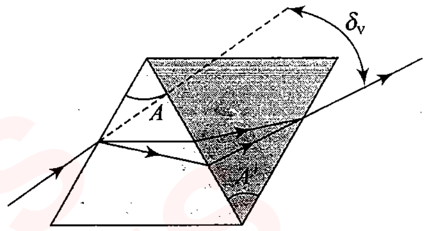


Fig. 1.177

For the two prisms,

$$(\mu_v - \mu_1)A + (\mu_v - \mu_r)A = 0$$

$$\Rightarrow \quad A' = \frac{(\mu_v - \mu_r)A}{\mu_v - \mu_r} \quad \text{and} \quad \omega\delta + \omega'\delta' = 0$$

$$\delta = \delta_1\left[1 - \frac{\omega}{\omega'}\right]$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' their mean deviations.

Dispersion Without Deviation

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism.

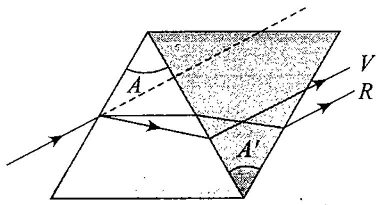


Fig. 1.178

This combination produces dispersion without deviation.

For deviation to be zero, $(\delta + \delta') = 0$

$$\Rightarrow \quad (\mu - 1)A + (\mu' - 1)A = 0$$

$$\Rightarrow \quad A' = \frac{(\mu - 1)A}{(\mu' - 1)}$$

(–ve sign $\Rightarrow$ prism A' has to be kept inverted)

$$\theta = \delta(\omega - \omega')$$

Illustration 1.70 **Two thin prisms are combined to form an achromatic combination. For prism I, $A = 4°$, $\mu_R = 1.35$, $\mu_Y = 1.40$, $\mu_v = 1.42$. For prism II, $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_R = 1.9$. Find the prism angle of prism II and the net mean deviation.**

Sol. Condition for achromatic combination.

$$\theta = \theta'$$

$$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'$$

$$\Rightarrow \quad A = \frac{(1.42 - 1.35)\,4°}{1.9 - 1.7} = 1.4°$$

$$\delta_{\text{Net}} = \delta - \delta' = (\mu_Y - 1)A - (\mu_Y - 1)A'$$

$$= (1.40 - 1)\,4° - (1.8 - 1)\,1.4° = 0.48°.$$

Illustration 1.71 **A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.514, 1.517, and 1.523, respectively, for crown glass and 1.613, 1.620, and 1.632 for flint glass.**

Sol. The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$\delta_{\text{net}} = \delta - \delta' = (\mu - 1)A - (\mu - 1)A'. \qquad \text{(i)}$$

a. If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'$$

or $$A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} \times 5°$$

b. The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is $(\delta_v - \delta_r)A - (\mu'_v - \mu'_r)A$

$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2° = -0.0348°$.

The angular dispersion has magnitude 0.0348°.

a. If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A' \quad \text{or} \quad A = \frac{(\mu' - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} \times 5°$$

b. The angular dispersion produced by the crown prism is

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is

$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A = (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2° = -0.0348°$.

The angular dispersion has magnitude 0.034°.

Concept Application Exercise 1.4

1. The refractive indices of flint glass for red and violet lights are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.
2. Refractive index of glass for red and violet colours are 1.50 and 1.60, respectively. Find:
   a. The refractive index for yellow colour, approximately.
   b. Dispersive power of the medium.
3. An equilateral prism is made of glass of refractive index 1.5. Calculate the angles of minimum and maximum deviations. Given:
   $A = 60°, \mu = 1.5, \sin 48.6° = \frac{3}{4}; \sin 41.8° = \frac{3}{2}; \sin 27.9° = \frac{3}{2} \sin 18.2$
4. A prism is made of glass of refractive index 1.5. If the angle of minimum deviation is equal to the refracting angle of the prism, calculate the angle of the prism.
5. A horizontal ray of light passes through a prism whose apex angle is 4° and then strikes a vertical mirror M as shown in Fig. 1.79. For the ray to become horizontal after reflection, find the angle by which the mirror must be rotated.

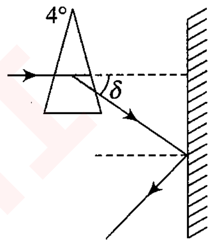


Fig. 1.179

6. Light is incident normally on face AB of a prism as shown in figure (Fig.1.180). A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index 3/2. Find the limits of μ for which total internal reflection takes place on the face AC.

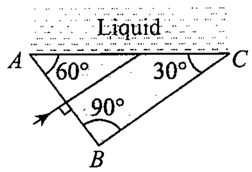


Fig. 1.180

7. A ray of light suffers minimum deviation through a prism of refractive index $\sqrt{2}$. What is the angle of prism if the angle of incidence is double the angle of refraction within the prism?
8. A ray of light undergoes a deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$.

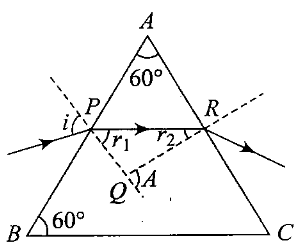


Fig. 1.181

What is the angle subtended by the ray inside the prism with the base of the prism?

9. The path of a ray of light passing through an equilateral glass prism ABC is shown in Fig. 1.182. The ray of light is incident on face BC at the critical angle for just total internal reflection. The total angle of deviation after the refraction at face AC is 108°. Calculate the refractive index of the glass.

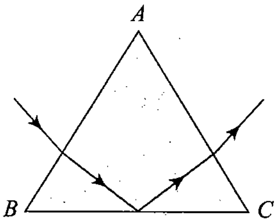


Fig. 1.182

10. In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle, the emergent ray grazes the emergent surface.

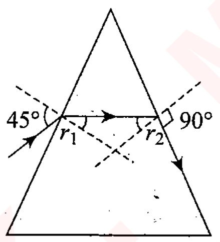


Fig. 1.183

Find the refractive index of the material of the prism. For what angle of incidence, the angle of deviation will be minimum?

11. Refracting angle of a prism $A = 60°$ and its refractive index is $n = 3/2$. What is the angle of incidence i to get minimum deviation. Also, find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).

12. Two identical thin isosceles prisms of refracting angle A and refractive index μ are placed with their bases touching each other and this system can collectively act as a crude converging lens. A parallel beam of light is incident on this system as shown in Fig. 1.184. Find the focal length of this so called converging lens.

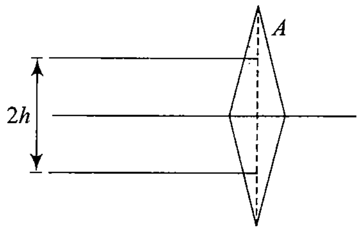


Fig. 1.184

REFRACTION AT SPHERICAL SURFACES

Consider the point object O placed in a medium with refractive index equal to μ_1.

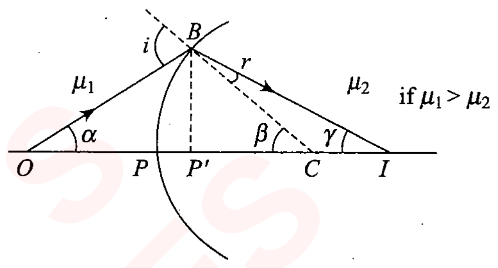


Fig. 1.185

As $\mu_1 \sin i = \mu_2 \sin r$, and for small aperture $i, r \to 0$

$\Rightarrow \quad \mu_1 i = \mu_2 r \quad \Rightarrow \quad i = \alpha + \beta, \beta = \gamma + r$

$\Rightarrow \quad \mu_1(\alpha + \beta) = \mu_2(\beta - \gamma) \quad \Rightarrow \quad \mu_1\alpha + \mu_2\gamma = (\mu_1 - \mu_2)\beta$

As aperture is small, $\alpha \approx \tan\alpha, \beta \approx \tan\beta, \gamma \approx \tan\gamma$

$\Rightarrow \quad \mu_1 \tan\alpha + \mu_2 \tan\gamma = (\mu_2 - \mu_1)\tan\beta$

$$\Rightarrow \quad \tan\alpha = \frac{BP'}{OP'}; \tan\beta = \frac{BP'}{P'C}; \tan\gamma = \frac{BP'}{P'I}$$

$$\Rightarrow \quad \frac{\mu_1}{P'O} + \frac{\mu_2}{P'I} = \frac{\mu_2 - \mu_1}{P'C}$$

Applying sign convention, i.e.,

$$P'O = -u, P'I = v, \text{ and } P'C = R$$

Therefore, for a spherical surface, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

The symbols should be carefully remembered as: μ_2–refractive index of the medium into which light rays are entering; μ_1–refractive index of the medium from which light rays are coming. Care should also be taken while applying the sign convention to R.

Lateral Magnification for Refracting Spherical Surface

$$\text{Lateral magnification, } m = \frac{\text{Image height}}{\text{Object height}} = \frac{-(A'B')}{AB}$$

$$\text{Now, } \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

For small angles of incidence and thus refraction, $\sin i = \tan i$ and $\sin r = \tan r$

$$\Rightarrow \quad \frac{\tan i}{\tan r} = \frac{\mu_2}{\mu_1}$$; in triangles ABP and $A'B'P$, we have

$$\frac{AB/PA}{A'B'/P'A'} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \frac{A'B'}{AB} = -\frac{\mu_1}{\mu_2}\frac{PA'}{PA}$$

$$= -\frac{\mu_1}{\mu_2}\frac{v}{(-u)} = \frac{\mu_1}{\mu_2}\frac{v}{u}$$

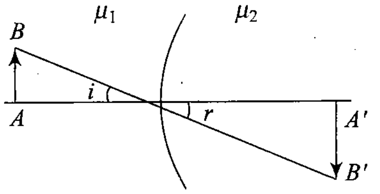


Fig. 1.186

Illustration 1.72 **A ring of radius 1 cm is placed 1 m in front of a spherical glass ball of radius 25 cm with refractive index 1.50. Determine the position of the final image of the ring and its magnification.**

Sol. Light rays from the object are refracted through the glass ball twice; first at surface S_1, from air to glass, and second, at surface S_2, from glass to air. We use paraxial approximation, so that single surface refraction equation can be used.

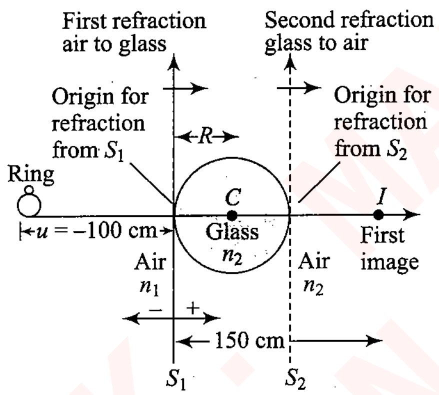


Fig. 1.187

Refraction at first surface

$$n_1 = 1,\ n_2 = 1.5,\ u = -100 \text{ cm},\ R = +25 \text{ cm}$$

The radius of curvature is positive because center of curvature is to the right.

Substituting these values in single surface refraction equation,

$$\frac{1.5}{v} - \frac{1}{(-100)} = \frac{(1.5-1)}{25}$$

On solving for v, we get $v = +150$ cm

The image is located 150 cm to the right of the first refracting surface. The magnification due to refraction at first surface,

$$\mu_1 = \frac{n_1 v}{n_2 u} = \frac{1(150)}{1.5(-100)} = -1$$

Refraction at second surface

For refraction at second surface, the origin of the cartesian coordinate system has to be shifted to the vertex of the second refracting surface.

The object distance for refraction at S_2 is $u' = +(150-50) = 100$ cm.

This is the virtual object for S_2; the light rays converging to I_1 are refracted at S_2 before they can actually converge to form the image.

$$n_1 = 1.5,\ n_2 = 1,\ u = 100 \text{ cm},\ R = -25 \text{ cm}$$

$$\frac{1}{v} - \frac{1.5}{(+100)} = \frac{(1-1.5)}{(-25)} \Rightarrow v = +200/7 \text{ cm}$$

$$\mu_2 = \frac{n_1 v}{n_2 u} = \frac{1.5(200/7)}{1(100)} = \frac{3}{7}$$

$$m = m_1 \times m_2 = -1 \times \frac{3}{7} = -\frac{3}{7}$$

Illustration 1.73 **A glass sphere of radius $2R$, refractive index n has a spherical cavity of radius R, concentric with it. A black spot on the inner surface of the hollow sphere is viewed from the left as well as right. Obtain the shift in position of the object.**

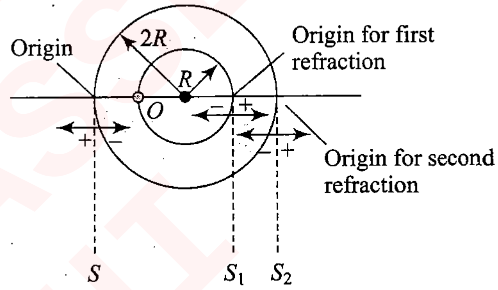


Fig. 1.188

Sol. (i) *Viewer on the left of hollow sphere*: Single refraction takes place at surface S.

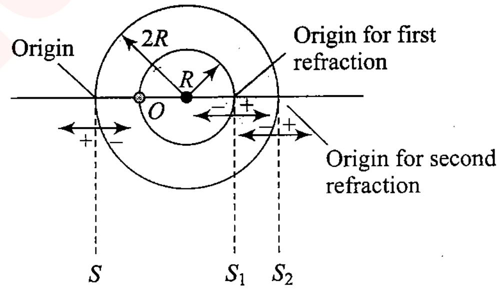


Fig. 1.189

From the single surface refraction equation, we have
$\mu_2 = 1,\ \mu_1 = n,\ u = -R$, and $R = -2R$

$$\frac{1}{v} - \frac{n}{(-R)} = \frac{(1-n)}{(-2R)}$$

which on solving for v yields, $v = -\left(\frac{2R}{n+1}\right)$

Image is on the right of refracting surface S.

Shift = Real depth – Apparent depth

$$= R - \left(\frac{2R}{n+1}\right) = \frac{(n-1)}{(n+1)} R$$

(ii) *When the viewer is on the right*, two refractions take place at surfaces S_1 and S_2.

$\mu_2 = n,\ \mu_1 = 1,\ u = -2R$, and $R = -R$

For refraction at surface S_1: $\dfrac{n}{v_1} - \dfrac{1}{(-2R)} = \dfrac{(n-1)}{(-R)}$

which on solving for v_1 yields, $v_1 = -\dfrac{2nR}{2n-1}$.

The first lies to the left of S_1 and acts as object for refraction at the second surface. We have to shift the origin of cartesian coordinate system to the vertex of S_2. The object distance for the second surface is

$$u_2 = -\left[\frac{2nR}{2n-1} + R\right] = -\left(\frac{4n-1}{2n-1}\right)R$$

Here $\mu_2 = 1$, $\mu_1 = n$, $R = -2R$

$$\Rightarrow \qquad \frac{1}{v_2} - \frac{n}{-\left[\dfrac{4n-1}{2n-1}\right]R} = \frac{1-n}{-2R}$$

On solving for v_2, we get $v_2 = -\dfrac{2(4n-1)}{(3n-1)}R$

The minus sign shows that image is virtual and lies to the left of S_2.

$$\text{Shift} = \text{Real depth} - \text{Apparent depth} = 3R - \frac{2(4n-1)R}{(3n-1)}$$

$$= \frac{(n-1)}{(3n-1)}R$$

Concept Application Exercise 1.5

1. Identify True and False statement.

a. The equation $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{(\mu_2-\mu_1)}{R}$ is applicable to a plane surface for $R = \infty$.

b. In the above equation, μ_1 is the medium in which the object is placed and μ_2 is the medium in which the image is formed.

c. In the figure shown, the real image of object O is formed at a distance $5R$ in the medium.

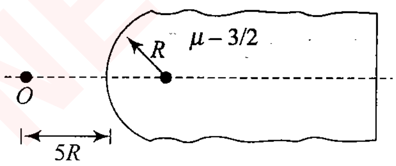


Fig. 1.190

d. In Fig. 1.191, the image of an object O placed on the center face of the sphere is formed at infinity.

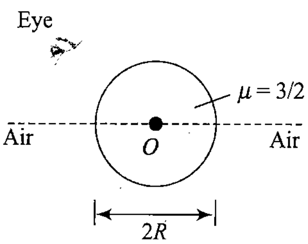


Fig. 1.191

e. In Fig. 1.192, the image of an object O placed on the opposite face of the sphere is formed at infinity.

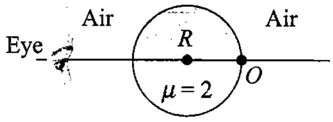


Fig. 1.192

2. One end of a cylindrical glass rod shown in Fig. 1.193 is ground to a hemispherical surface of radius $R = 20$ mm.

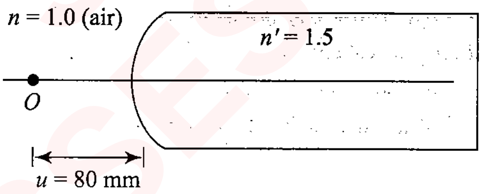


Fig. 1.193

a. Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.

b. Let the same rod be immersed in water of index 4/3, the other quantities having the same values as before. Find the image distance.

3. A lens is 5 cm thick and the radii of curvature of its surface are 10 cm and 25 cm, respectively. A point object is placed at a distance of 12 cm from the surface whose radius of curvature is 10 cm. How far beyond the other surface is the image formed?

4. A spherical surface of radius R separates two media of refractive indices μ_1 and μ_2, as shown in Fig. 1.194. Where should an object be placed in medium 1 so that a real image is formed in medium 2 at the same distance?

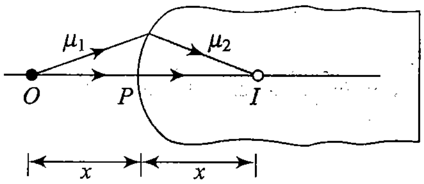


Fig. 1.194

5. A sphere of radius R made of material of refractive index μ_2 is placed in a medium of refractive index μ_1. Where would an object be placed so that a real image is formed at equidistant from the sphere?

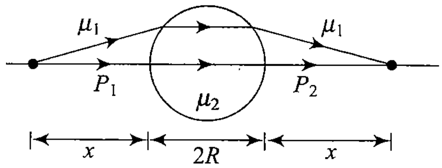


Fig. 1.195

6. A small object of height 0.5 cm is placed in front of a convex surface of glass ($\mu = 1.5$) of radius of curvature 10 cm. Find the height of the image formed in glass.

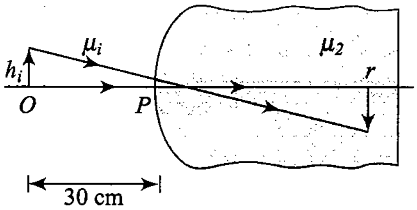


Fig. 1.196

7. An object of height 1 mm is placed inside an sphere of refractive index $\mu = 2$ and radius of curvature 20 cm as shown in the figure. Find the position, size, and nature of image, for the situation shown in Fig. 1.197. Draw ray diagram.

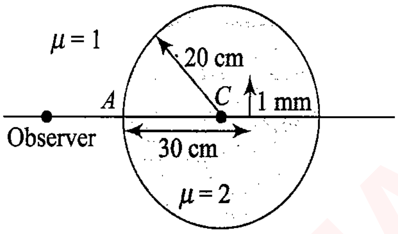


Fig. 1.197

8. A ray of light falls on a transparent sphere with center at C as shown in Fig. 1.198. The ray emerges from the sphere parallel to line AB. Find the refractive index of the sphere.

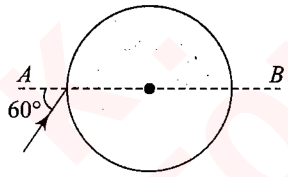


Fig. 1.198

9. A glass sphere, refractive index 1.5 and radius 10 cm, has a spherical cavity of radius 5 cm concentric with it. A narrow beam of parallel light is directed into the sphere. Locate the final image.

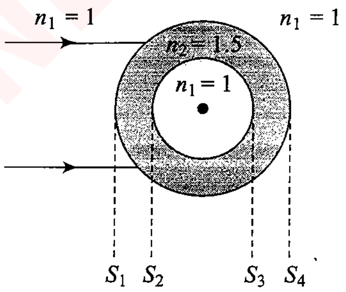


Fig. 1.199

10. One end of a horizontal cylindrical glass rod ($\mu = 1.5$) of radius 5.0 cm is rounded in the shape of a hemisphere. An object 0.5 mm high is placed perpendicular to the axis of the rod at a distance of 20.0 cm from the rounded edge. Locate the image of the object and find its height.

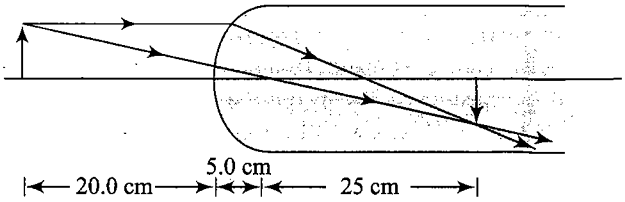


Fig. 1.200

11. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 10 cm. The bubble is 4.0 cm below the surface and is viewed normally from the outside. Find the apparent depth of the bubble.

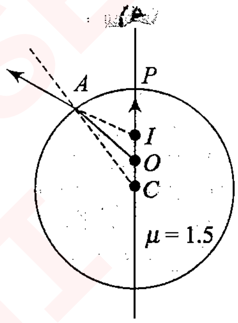


Fig. 1.201

12. Figure 1.202 shows a transparent hemisphere of radius 3.0 cm made of a material of refractive index 2.0.

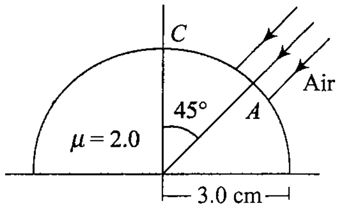


Fig. 1.202

a. A narrow beam of parallel rays is incident on the hemisphere as shown in the figure. Are the rays totally reflected at the plane surface ?
b. Find the image formed by refraction at the first surface.
c. Find the image formed by refraction or by reflection at the plane surface.
d. Trace qualitatively the final rays as they come out of the hemisphere.

13. A small object is embedded in a glass sphere ($\mu = 1.5$) of radius 5.0 cm at a distance 1.5 cm left to the center. Locate the image of the object as seen by an observer standing (a) to the left of the sphere and (b) to the right of the sphere.

14. A paperweight in the form of a hemisphere of radius 3.0 cm is used to hold down a printed page (plain surface down). An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the center appear to the observer ?

15. Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

16. A spherical surface of radius 30 cm separates two transparent media *A* and *B* with refractive indices 1.33 and 1.48, respectively. The medium *A* is on the convex side of the surface. Where should a point object be placed in medium *A* so that the paraxial rays become parallel after refraction at the surface?

17. A point object is above the principal axis in the concave side of a spherical interface separating medium 1 from medium 2. If the object is in medium 1 and $\mu_1 > \mu_2$, determine the position of the image by ray tracing.

18. A solid hemispherical bowl of radius *R*, made of glass, is placed on a flat horizontal surface with its curved surface on the ground. A small object is placed under the hemisphere at the center of the curved surface. If an observer sees the object directly above the plane surface, where will the object appear to be?

19. A small air bubble in a sphere of glass with radius 4 cm appears to be 1 cm from the surface when observed along a diameter. Find the true position of the air bubble.

20. A horizontal ray of light is incident on a solid glass sphere of radius *R* and refractive index μ. What is the net deviation of the beam when it emerges from the other side of the sphere?

21. A converging bundle of rays travel from water (refractive index 4/3) to glass (refractive index 1.5) through a convex interface of radius 16 cm. In the absence of the interface, the rays would have converged to a point 32 cm from the pole of the interface. Where will the rays actually meet after refraction?

22. A parallel incident beam falls on a solid glass sphere at normal incidence. Prove that the distance of the final image after two refractions is at a distance $(2 - \mu)/2(\mu - 1)a$ from the outer edge of the sphere. Refractive index of the sphere is μ and radius of the sphere is *a*.

THIN LENS

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends (Figs. 1.203 and 1.204).

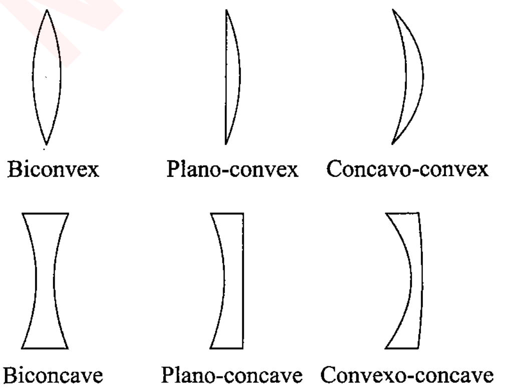


Fig. 1.203

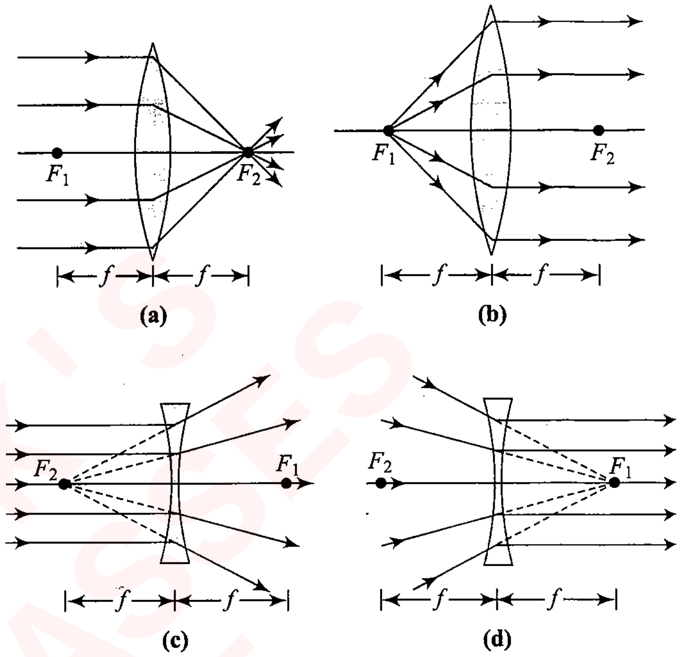


Fig. 1.204

One surface of a convex lens is always convex. Depending on the other surface, a convex lens is categorized as

a. biconvex or convexo-convex, if the other surface is also convex;

b. Plano-convex if the other surface is plane; and

c. Concavo-convex if the other surface is concave.

Similarly, a concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.

Consider Fig. 1.205 with the two refracting surfaces having radii of curvature equal to R_1 and R_2, respectively. The refractive indices of the surrounding medium and of the material of the lens are μ_1 and μ_2, respectively.

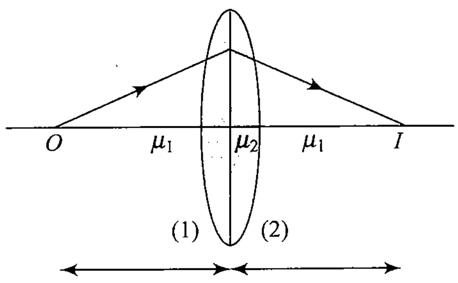


Fig. 1.205

Now, using the result that we obtained for refraction at single spherical surface, we get

For first surface, $$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{(i)}$$

For second surface, $$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \quad \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $$\mu_1\left(\frac{1}{v}-\frac{1}{u}\right)=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\left(\frac{1}{v}-\frac{1}{u}\right)=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{(iii)}$$

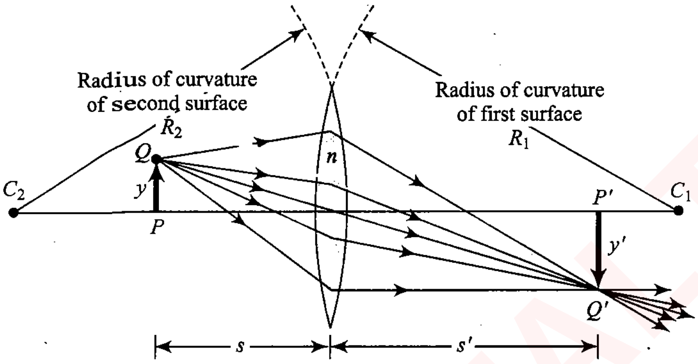


Fig. 1.206

Lensmaker's Formula

In the above equation, if the object is at infinity and image is formed at the focus, then $u = \infty$ and $v = f$.

$$\Rightarrow \quad \frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{(iv)}$$

Thin Lens Formula

Now, comparing Eqs. (iii) and (iv), we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For a spherical thin lens *having the same medium on both sides*:

$$\frac{1}{v}-\frac{1}{u}=(n_{\text{rel}}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{(a)}$$

where $n_{\text{rel}}=\dfrac{n_{\text{lens}}}{n_{\text{medium}}}$ and R_1 and R_2 are x-coordinates of the center of curvature of the first and second surfaces, respectively.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{(Lensmaker's Formula)} \qquad \text{(b)}$$

Lens has two focii:

If $u=\infty$, then $\dfrac{1}{v}-\dfrac{1}{\infty}=\dfrac{1}{f} \Rightarrow v=f$

$\Rightarrow$ If incident rays are parallel to principal axis, then the refracted rays will cut the principal axis at 'f'. It is called second focus. *In case of converging lens, it is positive and in case of diverging lens it is negative.*

If $v=\infty$, that means

$$\frac{1}{\infty}-\frac{1}{u}=\frac{1}{f} \Rightarrow u=-f$$

- *A ray initially parallel to the principal axis will pass (or appear to pass) through focus (Fig. 1.207).*

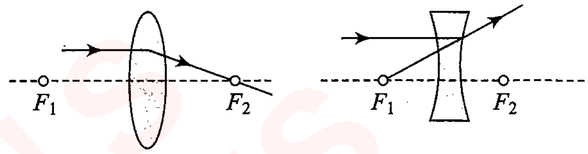


Fig. 1.207

$\Rightarrow$ If incident rays cut principal axis at $-f$, then the refracted rays will become parallel to the principal axis. It is called first focus. In case of converging lens, it is negative (f is positive) and in the case of diverging lens it is positive (f is negative)

- *A ray which initially passes (or appears to pass) through focus will emerge from the lens parallel to the principal axis (Fig. 1.208).*

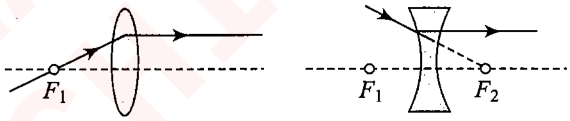


Fig. 1.208

From the relation $\dfrac{1}{f}=(n_{\text{rel}}-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$, it can be seen that the second focal length depends on two factors.

1. The factor $\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ is
   - **a.** *Positive* for all types of *convex* lenses and
   - **b.** *Negative* for all types of *concave* lenses.
2. The factor $(n_{\text{rel}}-1)$ is
   - **a.** *Positive* when *surrounding medium is rarer than the medium of lens.*
   - **b.** *Negative* when *surrounding medium is denser than the medium of lens.*

So, a lens is *converging if f is positive* which happens when *both the factors (A) and (B) are of same sign.*
And a lens is *diverging if f is negative* which happens when the *factors (A) and (B) are of opposite signs.*

METHODS FOR DETERMINING FOCAL LENGTH OF A CONVEX LENS

Graphical Method

A convex lens forms real and inverted images of an object placed between infinity and focus. From lens equation, using Cartesian sign convention, we have

$$u=-x,\ v=+y,\ f=+f$$

$$\frac{1}{+y}-\frac{1}{-x}=\frac{1}{+f} \quad \Rightarrow \quad y-x=\text{constant}$$

If a graph is plotted between $\frac{1}{+v}$ and $\frac{1}{-u}$, it represents a straight line with slope equal to –1, as shown in Fig. 1.209 (a). The intercept with the horizontal axis is $\frac{1}{-f}$ and that with the vertical axis is $\frac{1}{+f}$. If a line OP is drawn at angle 45°, it intersects the line AB at the point P. The coordinates of the point P are $\left(\frac{1}{-2f}, \frac{1}{2f}\right)$.

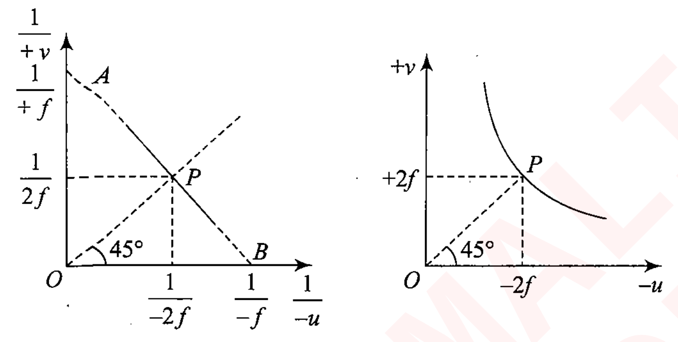
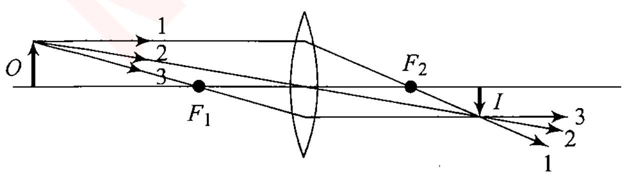


Fig. 1.209

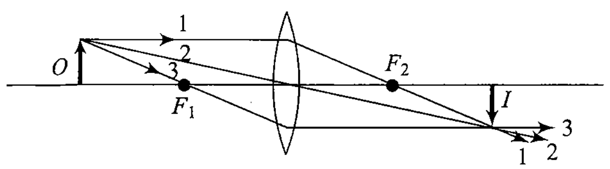
Alternatively, a graph between v and $-u$ can be plotted as shown in Fig. 1.209 (b). A line drawn at angle 45° from the origin intersects the curve at the point P whose coordinates are $(-2f, +2f)$. By measuring the coordinates of this point, the focal length of the lens can also be measured.

The following conclusions may be obtained for a convex lens:

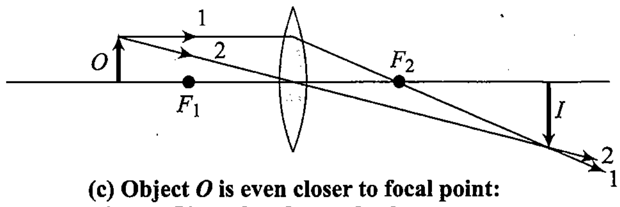
- A convex lens will form a real image for a real object when the object is placed beyond focus ($x > f_0$) (Fig. 1.210a, b, c).
- When the object comes within focus, i.e., $x < f_0$, then a virtual image is formed for the real object. (Fig. 1.210d, e)
- The real image formed is always inverted while the virtual image is always erect.
- Anything (object or image) which is farther from the lens is always larger.



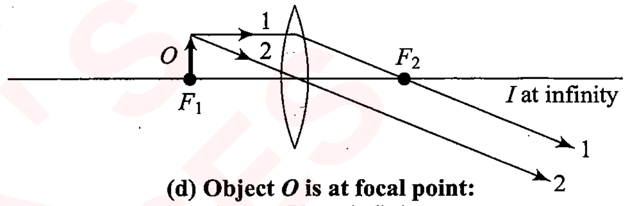
(a) Object O is outside focal point: image I is real



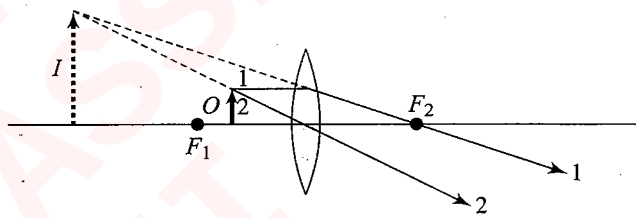
(b) Object O is closer to focal point: image I is real and farther away



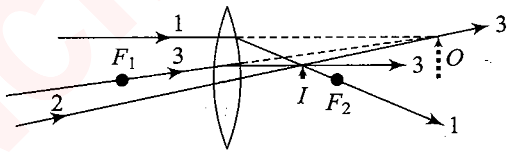
(c) Object O is even closer to focal point: image I is real and even farther away



(d) Object O is at focal point: image I is at infinity



(e) Object O is inside focal point: image I is virtual and larger than object



(f) A virtual object O (light rays are *converging* on lens)

Fig. 1.210

The following conclusions may be obtained for a concave lens:

- A concave lens always forms a virtual image for a real object (Fig. 1.211).

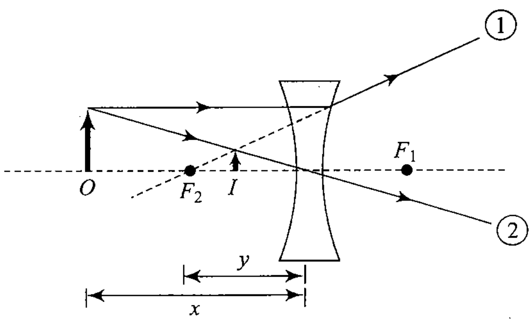


Fig. 1.211

- For real object, the image formed by a concave lens is always erect and diminished in size.
- A concave lens can form a real image if the object is virtual (Fig. 1.212).
  (i) For a real object placed in front of a concave lens, a virtual, erect, and diminished image is formed within focus.

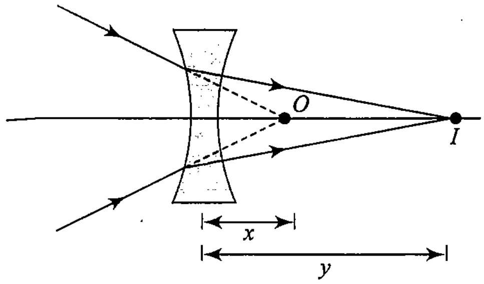


Fig. 1.212

For $u = -x$,
$v = -y$, where $y < f_0$ and $0 < m < 1$

(ii) For a virtual object O placed within focus, a real image I is formed.

For $u = +x; < f_0$
$\Rightarrow$ $v = +y;$

Note: ***Point B, its image B′, and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight.***

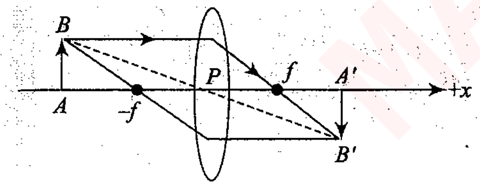


Fig. 1.213

Illustration 1.74 **Find the focal length of the lens shown in the figure.**

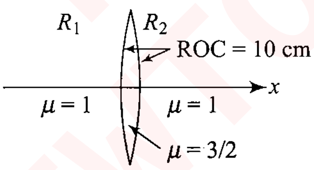


Fig. 1.214

Sol. $\dfrac{1}{f} = (n_{\text{rel}} - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\dfrac{1}{f} = (3/2 - 1)\left(\dfrac{1}{10} - \dfrac{1}{(-10)}\right)$

$\Rightarrow$ $\dfrac{1}{f} = \dfrac{1}{2} \times \dfrac{2}{10}$ $\Rightarrow$ $f = +10$ cm.

Illustration 1.75 **Find the focal length of the lens shown in figure.**

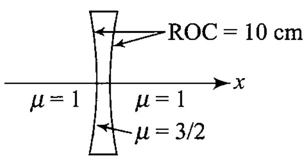


Fig. 1.215

Sol. Here $R_1 = -10$ cm
$R_2 = 10$ cm
$\mu = \dfrac{3}{2}$

$$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$f = -10$ cm

Illustration 1.76 **A point object is placed on the principal axis of a thin lens with parallel curved boundaries, i.e., having same radii of curvature. Discuss about the position of the image formed.**

Sol. $\dfrac{1}{f} = (n_{\text{rel}} - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = 0$ $[\because\ R_1 = R_2]$

$\dfrac{1}{v} - \dfrac{1}{u} = 0$ or $v = u$, i.e., rays pass without appreciable bending.

Illustration 1.77 **Focal length of a thin lens in air is 10 cm. Now, medium on one side of the lens is replaced by a medium of refractive index $\mu = 2$. The radius of curvature of the surface of lens, in contact with the medium, is 20 cm. Find the new focal length.**

Sol. Let radius of surface I be R_1 and refractive index of lens be μ. Let parallel rays be incident on the lens.

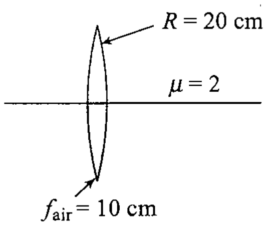


Fig. 1.216

Applying refraction formula at first surface,

$$\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \quad \text{(i)}$$

At II surface $\dfrac{2}{V} - \dfrac{\mu}{V_1} = \dfrac{2 - \mu}{-20}$ (ii)

Adding (i) and (ii), we get

$$\frac{\mu}{V_1} - \frac{1}{\infty} + \frac{2}{V} - \frac{\mu}{V_1} = \frac{\mu - 1}{R_1} + \frac{2 - \mu}{-20} = (\mu - 1)$$

$$\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu - 1}{20} - \frac{2 - \mu}{20} = \frac{1}{f}\ \text{(in air)} + \frac{1}{20} - \frac{2}{20}$$

$\Rightarrow$ $v = 40$ cm $\Rightarrow$ $f = 40$ cm

Illustration 1.78 **A glass or glycerin convex lens of refractive index 3/2 has got a focal length equal to 50 cm. Find the focal length of the lens, if it is immersed in ethyl alcohol of refractive index 1.36.**

Sol. According to lensmakers formula

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(i)}$$

In ethyl alcohol, $\frac{1}{f_{\text{liquid}}} = \left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (ii)

Dividing (i) and (ii), we get

$$\frac{f_{\text{liquid}}}{f} = \frac{(1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\Rightarrow \quad f_{\text{liquid}} = 242.75 \text{ cm}$$

Illustration 1.79 **There are two spherical surfaces of radii $R_1 = 30$ cm and $R_2 = 60$ cm. In how many ways these surfaces may be arranged to get different lenses. If all the lenses are made of glass ($\mu = 1.5$), find the focal length of each lens.**

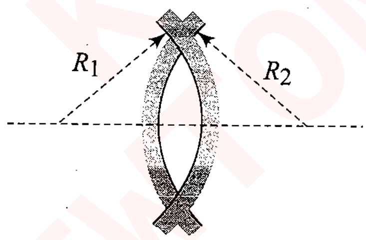


Fig. 1.217

Sol. Two surfaces may be arranged in four different ways. Using lensmaker's formula, we can obtain the focal length of each lens as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(i) $\mu = 1.5$, $R_1 = +30$ cm; $R_2 = -60$ cm

$$\therefore \quad \frac{1}{f_1} = (1.5-1)\left(\frac{1}{30} - \frac{1}{-60}\right) = \frac{1}{40}$$

$$\Rightarrow \quad f_1 = 40 \text{ cm (converging)}$$

(ii) $\mu = 1.5$, $R_1 = +30$ cm; $R_2 = +60$ cm

$$\therefore \quad \frac{1}{f_2} = (1.5-1)\left(\frac{1}{30} - \frac{1}{60}\right) = \frac{1}{120}$$

$$\Rightarrow \quad f_2 = -120 \text{ cm (converging)}$$

(iii) $\mu = 1.5$, $R_1 = -30$ cm; $R_2 = +60$ cm

$$\therefore \quad \frac{1}{f_3} = (1.5-1)\left(\frac{1}{-30} - \frac{1}{60}\right) = \frac{-1}{40}$$

$$\Rightarrow \quad f_3 = -40 \text{ cm (diverging)}$$

(iv) $\mu = 1.5$, $R_1 = -30$ cm; $R_2 = -60$ cm

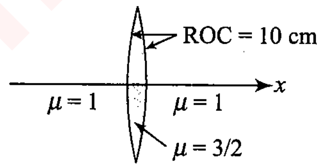
$$\therefore \quad \frac{1}{f_4} = (1.5-1)\left(\frac{1}{-30} - \frac{1}{-60}\right) = \frac{-1}{120}$$

$$\Rightarrow \quad f_4 = -120 \text{ cm (diverging)}$$

Illustration 1.80 **A thin lens made of a material of refractive index μ_0 has a focal length f_0 in air. Find the focal length of this lens if it is immersed in a liquid of refractive index μ.**

Sol. From lensmaker's formula, we know that

$$\frac{1}{f_0} = (\mu_0 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(i)}$$



Converging lens

Fig. 1.218

When the lens is immersed in a liquid of refractive index μ, then

$$\frac{1}{f} = \left(\frac{\mu_0}{\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{f} = \frac{(\mu_0 - \mu)1}{\mu(\mu_0 - 1)f_0} \quad \text{or} \quad f = f_0\left[\frac{\mu(\mu_0 - 1)}{\mu_0 - \mu}\right]$$

POWER OF A LENS

Power of a lens is defined as the reciprocal of focal length, where f is measured in meter.

$$P = \frac{1}{f}$$

The *unit* of power is diopter, 1 D = 1 m^{-1}.

Sign convention: Focal length of a *converging* lens is taken as *positive* and that of a *diverging* lens is taken as *negative*.

In terms of power, the above expression may be written as $P = P_1 + P_2 + P_3 + \cdots + P_n$

Illustration 1.81 **A lens has a power of +5 diopters in air. What will be its power if completely immersed in water? ($_a\mu_w = 4/3$ and $_a\mu_g = 3/2$.)**

Sol. Let f_a and f_w be the focal lengths of the lens in air and water, respectively, then

$$P_a = \frac{1}{f_a} \quad \text{or} \quad +5 = \frac{1}{f_a}$$

$$f_a = 0.2 \text{ m} = 20 \text{ cm}$$

Now,
$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{(i)}$$

and
$$\frac{1}{f_w} = ({}_w\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_w}{f_a} = \left[\frac{{}_a\mu_g - 1}{{}_w\mu_g - 1}\right]$$

Again,
$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\Rightarrow \quad \frac{f_w}{f_a} = \frac{(3/2) - 1}{(9/8) - 1} = \frac{(1/2)}{(1/8)} = 4$$

$$f_w = f_a \times 5 = 20 \times 4 = 80 \text{ cm} = 0.8 \text{ m}$$

$$P_w = \frac{1}{f_w} = \frac{1}{0.8} = 1.25 \text{ diopter.}$$

Concept Application Exercise 1.6

1. Choose the following statements as True or False.
   a. The real image formed by a lens is always inverted.
   b. The image formed by a lens is always inverted.
   c. The image formed by a concave lens is always erect and diminished.
   d. An air bubble inside water acts like a concave lens.
   e. The distance between a real object and its real image formed by a single lens cannot be more than $4f$.
   f. If an object is moved at a constant speed towards a convex lens from infinity to focus, then its image moves slower in the beginning and faster later on, away from the lens.
   g. The focal length of a glass ($\mu = 1.5$) lens is 10 cm in air. When it is completely immersed in water its focal length will become 40 cm.
2. The layered lens shown in Fig. 1.219 is made of two kinds of glasses. How many and what kind of images will be produced by this lens with a point source placed on the optical axis? Neglect the reflection of light at the boundaries between layers.

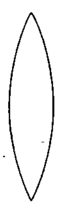


Fig. 1.219

3. What is the relation between the refractive indices μ, μ_1 and μ_2 if the behaviour of light rays is as shown in Fig. 1.220?

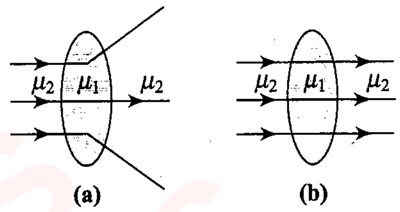


Fig. 1.220

4. Can a single lens ever form a real and erect image?
5. A pencil of height 1 cm is placed 30 cm from an equiconvex lens, refractive index $n = 3/2$, radius of curvature for both the surfaces, $R_1 = R_2 = R = 10$ cm. Find the location of the image and describe its characteristics.

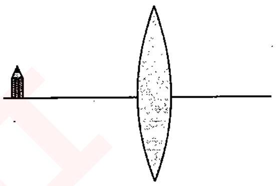


Fig. 1.221

6. If in the previous example, we use a diverging lens with a focal length 10.0 cm to form an image of the pencil kept 15 cm in front of the lens, locate and characterize the image.

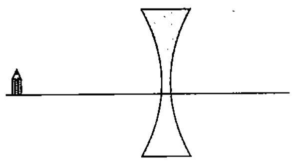


Fig. 1.222

7. **a.** Figure. 1.223(a) shows the optical axis of a lens, the point source of light A, and its virtual image A'. Trace the rays to find the position of the lens and of its focuses. What type of lens is it?

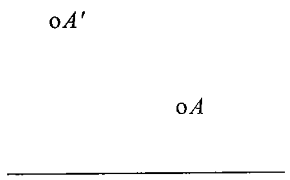


Fig. 1.223(a)

   b. Solve the problem similar to the previous one using Fig. 1.223(b).

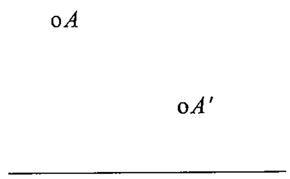


Fig. 1.223(b)

8. A biconvex lens has radii of curvature 20 cm and 40 cm. The refractive index of the material of the lens is 1.5. An object is placed 40 cm in front of the lens. Calculate the position of the image.
9. A converging bundle of rays is intercepted by a biconcave lens. The radii of curvature of both surfaces are 20 cm and the refractive index of the material of the lens is 1.5. If the rays originally converged to a point 10 cm in front of the lens, where will they now converge after passing through the lens.

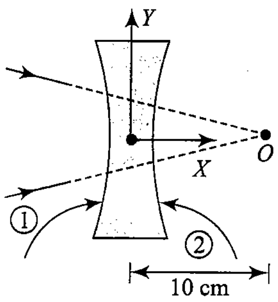


Fig. 1.224

10. A candle is placed 15 cm in front of a lens. If the image of the candle captured on a screen is magnified two times, calculate the focal length and nature of the lens.

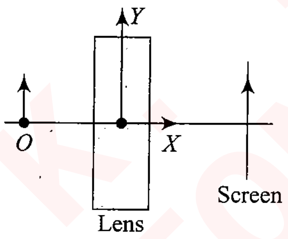


Fig. 1.225

11. A man wishes to view an object through a convex lens of focal length 10 cm. The final image is to be erect and magnified 2 times. How far from the object must he hold the lens?

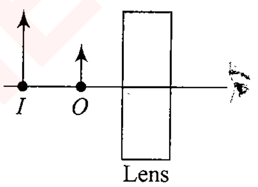


Fig. 1.226

12. An object is placed 12 cm in front of a lens to form an image on a screen. The size of the image is twice that of the object. What is the distance between the object and the image?
13. An object is placed in front of a concave lens. A virtual image of the object is formed on the same side of the lens as the object but is closer to the lens. Will rays from this image refract once again at the lens to form a second image?
14. A biconvex lens of focal length 20 cm, made of glass of refractive index 1.5, has water ($\mu = 1.33$) on one side and air on the other. An object is placed 15 cm from the lens on the side with water. Where is the image formed?
15. A point source is placed on the axis of a symmetrical convex lens of focal length 20 cm at a distance of 40 cm. If the lens is raised by 1 cm, by how much will the image be lifted relative to the previous axis?

LENS DISPLACEMENT METHOD

Consider a convex lens L placed between an object O and a screen S. The distance between the object and the screen is D and the positions of the object and the screen are held fixed. The lens can be moved along the axis of the system and at a position I a sharp image will be formed on the screen. Interestingly, there is another position on the same axis where a sharp image will once again be obtained on the screen.

The position is marked as II in Fig. 1.227.

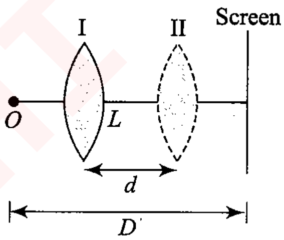


Fig. 1.227

Why are there two and only two positions where a sharp image is formed?

In the figure, let the distance of position I from the object be x_1.

Then, the distance of the screen from the lens is $D - x_1$. Therefore, $u = -x_1$ and $v = +(D - x_1)$.

Substituting in the Lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_1} + \frac{1}{x_1} = \frac{1}{f} \qquad \text{(i)}$$

At position II, let the distance of the lens from the screen be x_2. Then, the distance of the lens from the object is $D - x_2$. Therefore, $u = -x_2$ and $v = +(D - x_2)$.

Substituting in the lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_2} + \frac{1}{x_2} = \frac{1}{f} \qquad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we realize that there are only two solutions;

1. $x_1 = x_2$; or
2. $D - x_1 = x_2$ and $D - x_2 = x_1$

The first solution is trivial. Therefore, if the first position of the lens, for a sharp image, is x_1 from the object, the second position is at $D - x_1$ from the object.

Let the distance between the two positions I and II be d. From the diagram, it is clear that

$$D = x_1 + x_2 \quad \text{and} \quad d = x_2 - x_1 \qquad \text{(iii)}$$

Solving the two equations in (iii), we obtain

$$x_1 = \frac{D-d}{2} \quad \text{and} \quad D - x_1 = \frac{D+d}{2} \qquad \text{(iv)}$$

Substituting Eq. (iv) in Eq. (i), we get

$$\frac{1}{f} = \frac{2}{D-d} + \frac{2}{D+d}$$

or $$f = \frac{D^2 - d^2}{4D} \qquad \text{(v)}$$

Also, $$d = \sqrt{D^2 - 4Df} = \sqrt{D(D-4f)} \qquad \text{(vi)}$$

We notice from Eq. (vi) that a solution for d is possible only when $D \geq 4f$.

When $D < 4f$, there is no position for which a sharp image can be formed.

When $D = 2f$, there is only one position where a sharp image is formed.

When $D > 2f$, there are two positions where a sharp image is formed.

The method is suitable for convex lenses only.

What can we say about the size of the object when we know the size of the images?

The magnifications in the first and second positions are

$$m_1 = \frac{h_{i,1}}{h_0} = \frac{D - x_1}{x_1} = \frac{D+d}{D-d}$$

$$m_2 = \frac{h_{i,2}}{h_0} = \frac{D - x_2}{x_2} = \frac{D-d}{D+d}$$

Thus, in the first position the image is magnified while in the second it is reduced. The product of the two magnifications is equal to unity, i.e.,

$$m_1 \times m_2 = \frac{h_{i,1} \times h_{i,2}}{h_0^2} = 1 \quad \text{or} \quad h_0 = \sqrt{h_{i,1} \times h_{i,2}} \qquad \text{(vii)}$$

The displacement method is not suitable for concave lenses.

Illustration 1.82 **(a) A screen is kept at a distance of 1 m from the object. A converging lens between the object and the screen, when placed at any of the two positions which are 60 cm apart, forms a sharp image of the object on the screen. Find the focal length of the lens.**

(b) In the two positions of the lens, lateral size of the image is 4 cm and 9 cm. Find the size of the object.

Sol. This problem is based on 'Displacement method' which is commonly used to determine the focal length of a converging lens.

(a) As discussed earlier,

$$f = \frac{D^2 - x^2}{4D}$$

where D is the distance between the object and the screen. x is the distance between the two positions of lens for which sharp images of the given object are obtained on the screen.

Here, $D = 1 \text{ m} = 100 \text{ cm}$

$x = 60 \text{ cm}$

$$f = \frac{(100)^2 - (60)^2}{4 \times 100}$$

$f = 16 \text{ cm}$

(b) Size of the object can be obtained from the relation

$$O = \sqrt{I_1 I_2}$$

I_1, I_2 – lateral size of image in the two positions of lens

$$O = \sqrt{4 \times 9} = 6 \text{ cm}$$

Lenses with Different Media on Either Side

Consider a lens made of a material with refractive index μ with a liquid μ_a on the left and a liquid μ_b on the right (Fig. 1.228).

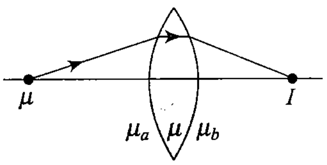


Fig. 1.228

What is the governing equation for this system.

Let an object be placed in medium μ_a. A ray of light from the object undergoes refraction at surface 1 and then refraction at surface 2 before emerging in medium μ_b. Let the radius of curvature of the first surface be R_1 and that of the second surface be R_2.

At surface 1: $\mu_1 = \mu_a, \mu_2 = \mu,$ and $R = R_1$

Therefore, $$\frac{\mu}{v_1} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1}$$

At surface 2: $\mu_1 = \mu, \mu_2 = \mu_b,$ and $R = R_2$

Therefore, $$\frac{\mu_b}{v} - \frac{\mu}{v_1} = \frac{\mu_b - \mu}{R_2}$$

Adding the above two equation, we get

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu}{R_2} \qquad \text{(i)}$$

Illustration 1.83 **A biconvex lens separates two media of refractive indices 1.3 and 1.7. The refractive index of the lens is 1.5 and the radii of curvature of the two sides of the lens are $r_1 = 10$ cm and $r_2 = 60$ cm.**

The medium of refractive index 1.3 extends to 78 cm from the lens and that of refractive index 1.7 extends to 34 cm from the lens. A luminous object O is at a distance of 144 cm from the lens. Find the position of final image from the lens.

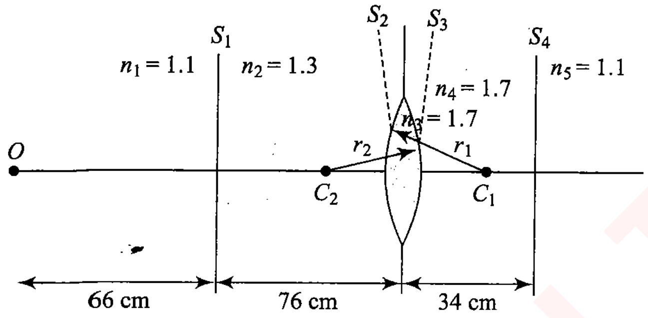


Fig. 1.229

Sol. Once again we have to apply single surface refraction equation to the four surfaces S_1, S_2, S_3, and S_4. But we have learned the general thin lens equation

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

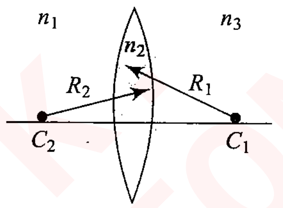


Fig. 1.230

which will take account of the two refractions at S_2 and S_3.

For refraction at surface S_1:

$$\frac{1.3}{v} - \frac{1.1}{(-66)} = \frac{(1.3 - 1.1)}{\infty} = 0 \quad \Rightarrow \quad v = -78 \text{ cm}$$

Image lies to the left of the surface S_1. This acts as an object for the lens.

For refraction through the lens: We apply the general thin lens equation.

Object distance for lens

$$u' = -(78 + 78) \text{ cm}$$

$$\frac{1.7}{v'} - \frac{1.3}{(-156)} = \frac{1.5 - 1.3}{+10} + \frac{1.7 - 1.5}{(-60)}$$

$$v' = 340 \text{ cm}$$

For refraction at surface S_4: Object distance for refraction S_4 is $340 - 34 = 306$ cm

$$\frac{1.1}{v''} - \frac{1.7}{(+306)} = 0$$

$$\Rightarrow \quad v'' = \frac{1.1 \times 3.6}{1.7} = 198 \text{ cm to the right of } S_4$$

Illustration 1.84 **A thin equiconvex lens made of glass of refractive index 3/2 and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\left(\mu = \frac{3}{2}\right)$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis as shown in Fig. 1.231. The separation between the lens and mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to lens) of the image of the object formed by the system.** **(IIT-JEE, 1997)**

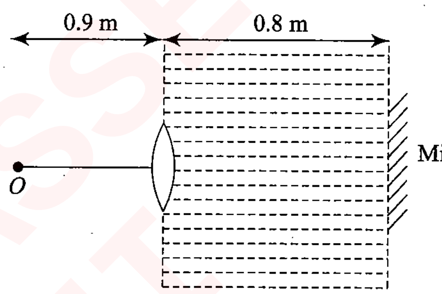


Fig. 1.231

Sol. If R is radius of curvature of each lens surface, then for air on either side of lens, we have

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{R} \Rightarrow f = R \Rightarrow R = f = 0.3 \text{ m}$$

If μ_2 is refractive index of lens material and μ_1, μ_3 are refractive indices on either side of the lens, then the formula is

$$\frac{\mu_3}{v} - \frac{\mu_1}{\mu} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2} \qquad \text{(i)}$$

Here, $R_1 = 0.3$ m, $R_2 = -0.3$ m, $\mu_1 = \mu_{air} = 1$,

$$\mu_2 = \mu_{glass} = \frac{3}{2}, \mu_3 = \mu_{water} = \frac{4}{3}, u = -0.9 \text{ m}, v = ?$$

$$\therefore \quad \frac{4/3}{v} - \frac{1}{(-0.9)} = \frac{\left(\frac{3}{2} - 1\right)}{0.3} + \frac{\left(\frac{4}{3} - \frac{3}{2}\right)}{-0.3}$$

$$\text{or} \quad \frac{4}{3v} + \frac{1}{0.9} = \frac{1}{0.6} + \frac{1}{1.8}$$

$$\text{or} \quad \frac{4}{3v} = \frac{1}{0.6} + \frac{1}{1.8} - \frac{1}{0.9} = \frac{1}{0.9}$$

or $$v = \frac{0.9 \times 4}{3} = 1.2 \text{ m}$$

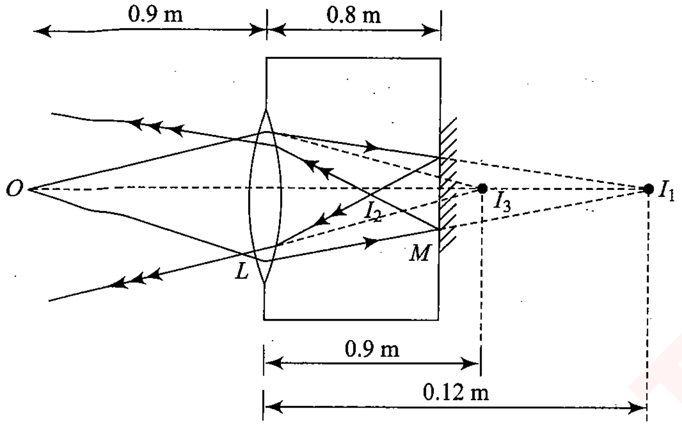


Fig. 1.232

That is image I_1 is at a distance 1.2 m from lens L to the right or at a distance (1.2 – 0.8) = 0.4 m to the right of mirror M. This image (I_1) acts as a virtual source for the plane mirror and forms real image I_2 at a distance 0.4 m on to the left of mirror and hence at a distance (0.8 – 0.4) = 0.4 m to the right of convex lens. This image I_2 acts as an object for the lens.

Again, the formula is

$$\frac{\mu_2'}{v''} - \frac{\mu_1'}{u''} = \frac{\mu_2' - \mu_1'}{R_1} + \frac{\mu_3' - \mu_2'}{R_2}$$

But, now

$$\mu_1' = \mu_{\text{water}} = \frac{4}{3}, \quad \mu_2' = \mu_{\text{glass}} = \frac{3}{2}, \quad \mu_3' = \mu_{\text{air}} = 1$$

$$R_1 = -0.3 \text{ m}, R_2 = +0.3 \text{ m}, u'' = 0.4 \text{ m}, v'' = ?$$

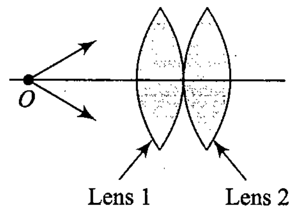
$$\therefore \quad \frac{1}{v''} - \frac{1}{0.3} = -\frac{1}{1.8} - \frac{1}{0.6}$$

or $$\frac{1}{v''} = \frac{1}{0.3} - \frac{1}{1.8} - \frac{1}{0.6} = \frac{1}{0.9} \quad \Rightarrow \quad v'' = 0.9 \text{ m}$$

That is image I_3 is formed to the right at a distance 0.9 m from the lens and is virtual. That is position of final image will be 0.9 m to the right of lens.

Lenses Placed Very Close to Each Other

Consider the arrangement shown in Fig. 1.233. Let us assume that f_1 and f_2 are the focal lengths of the individual lenses, respectively (note that the lenses themselves may be convex or concave and this is captured in the signs of f_1 and f_2). We are now interested in finding out the net focal length of the system and position of image for various positions of the object.



A system of two lense placed close together

Fig. 1.233

Let v_1 be the position of the image formed by the first lens. The lens formula for the first lens gives

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_2} \quad \text{(i)}$$

The image of the first lens now serves as the object for the second lens. If the image after the second lens if formed at v, then for the second lens, we have

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \text{(ii)}$$

Adding both the equations, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{(iii)}$$

We can, therefore, see that the system behaves as a single lens with equivalent focal length "f_e" given by

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{(iv)}$$

And we can use the same lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_e} \quad \text{(v)}$$

What can we say about the magnification of the system?

Let m_1 be the magnification of the first lens and m_2 be the magnification of the second lens. Let the height of the object be h_0, the height of the first image be h_1, and the height of the final image be h_2. Therefore,

$$m_1 = \frac{h_1}{h_0} \quad \text{(vi)}$$

$$m_2 = \frac{h_2}{h_1} \quad \text{(vii)}$$

And the net magnification

$$m = \frac{h_2}{h_0} = \frac{h_2}{h_1} \times \frac{h_1}{h_0} = m_1 m_2 \quad \text{(viii)}$$

To summarize: we can say that if a set of thin lenses are placed next to each other such that the distance between the lenses is zero, then the net effective focal length of the system f_{eff} is given by

$$\frac{1}{f_{\text{eff}}} = \sum \frac{1}{f_i}$$

where f_i is the focal length of every lens and the net magnification is given by

$$m_{\text{eff}} = m_1 \times m_2 \times m_3 \times \cdots = \prod_{i=1}^{n} m_i \qquad \text{(ix)}$$

Cut Lens

Consider a biconvex lens as shown in Fig. 1.234. How will the lens behave if it is cut in half along axis AA' or along axis BB'. Will it behave as a lens? If yes, what is its new focal length?

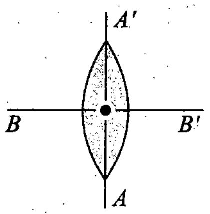


Fig. 1.234

To begin with, a lens which is cut also behaves like a lens simply because a lens is defined only by its two curved surfaces that refract the ray. A cut lens will also have two surfaces and hence will continue to behave as a lens.

Case 1. Lens cut along the principal axis
Consider a lens as shown in Fig. 1.235. We cut the lens along the axis BB'. Now, we have two lenses, both of which are shown in Fig. 1.235.

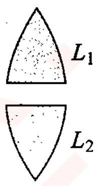


Fig. 1.235

Will the focal length of these two lenses change?

The answer is no as the radii of curvature of the two surfaces of the lens still remain the same. The only difference between this and a normal lens will be the intensity of the image. If the lens is cut at the center, we can say that the image intensity will reduce by 50%.

Case 2. Lens cut perpendicular to the principal axis
Now, let us cut the lens along the perpendicular bisector AA'. We once again have two lenses as shown in Fig. 1.236.

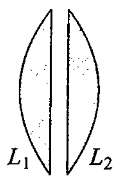


Fig. 1.236

What will be the focal length of the half lenses?

Let "f" be the original focal length of the lens and the new focal length of each half be "f_1". We can now place the two halves close together and recreate our original lens. Therefore, the focal length of the combination must once again be "f". From equation, we get

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_1} = \frac{2}{f_1}$$

Therefore, $f_1 = 2f$ or the focal length is doubled.

Illustration 1.85 **Find the lateral magnification produced by the combination of lenses shown in Fig. 1.237.**

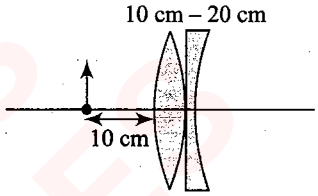


Fig. 1.237

Sol. If lenses are placed in contact, the focal length of single equivalent length can be given as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

$\Rightarrow$ $f = +20$

Now using lens formula considering system of lenses as a single lens, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\therefore$ $$\frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{10} = \frac{-1}{20} \Rightarrow v = -20 \text{ cm}$$

Now using magnification formula, we get

$$m = \frac{-20}{-10} = 2$$

Illustration 1.86 **Find the focal length of equivalent system shown in Fig. 1.238.**

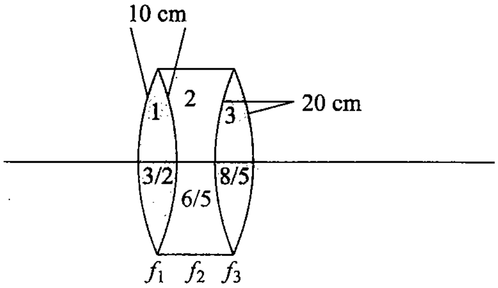


Fig. 1.238

Sol. Here radius of curvatures of different surfaces and refractive index are given, we can calculate focal lengths of lenses by using

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right) = \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{6}{5} - 1\right)\left(\frac{-1}{10} - \frac{1}{20}\right) = \frac{1}{5} \times \left(\frac{-30}{10 \times 20}\right) = \frac{-3}{100}$$

$$\frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{3}{50}$$

Hence, equivalent focal length

$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3} = \frac{1}{10}+\frac{-3}{100}+\frac{3}{50}$$

$\Rightarrow$ $f = 100/13$ cm

Lenses Placed at a Distance from Each Other

Case 1: Lenses placed apart but object at infinity

Consider two lenses placed a distance d apart on the same optical axis. In the special case when the object is placed at infinity, the combination may be replaced by an equivalent single thin lens. This lens should converge the bundle of rays from infinity to the same point as the system of lenses and also do so with the same angle of convergence.

What will be the focal length of the lens?

To answer the question, let us first determine the angle of deviation and the corresponding angle of convergence produced by an equivalent single lens (see Fig. 1.239(b)). Let O be a point object on the axis of the lens. A ray from the object is incident on the lens at a point A, is refracted, and meets the principal axis at B. The angle of deviation produced by the lens is $\angle BAA'$.

From the figure, we see that

$$\angle BAA' = \angle AOB + \angle ABO = \alpha + \beta \quad \text{(x)}$$

Also, from triangle OAP and APB, we have

$$\tan(\alpha) = \frac{AP}{PO}, \tan(\beta) = \frac{AP}{PB}$$

If the rays are very close to the principal axis, we make the paraxial approximation that

$$\tan(\alpha) \approx \alpha, \tan(\beta) \approx \beta \quad \text{(xi)}$$

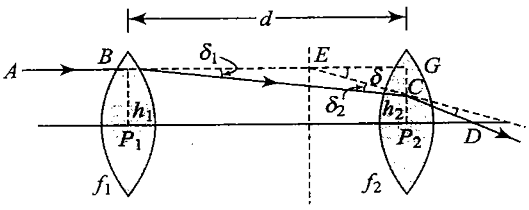
Substituting in Eq. (x), we get the angle of deviation

$$\delta = \frac{AP}{PO}+\frac{AP}{PB}$$

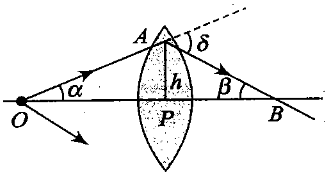
If we now apply the sign convention, we get $u = -PO$, $v = +PB$, $h = +PA$

$$\Rightarrow \quad \delta = h\left[\frac{1}{-u}-\frac{1}{v}\right] = \frac{h}{f} \quad \text{(xii)}$$

Now, consider the combination of two lenses placed a distance d apart as shown in Fig. 1.239(a). The incident ray AB strikes the first lens at B, is deflected by δ_1, meets the second lens at C, is deflected a further angle δ_2, and the emergent ray CD, finally intersects the principal axis at D.



Deviation produced by two lenses a distance apart
(a)



Deviation produced by an equivalent single lens
(b)

Fig. 1.239

When the system is replaced by a single equivalent lens, the incident ray will be along AB while the emergent ray will be along CD and the lens can refract the beam only once. Therefore, the equivalent lens must be placed at the point which is the point of intersection of the incident and the emergent rays. AB and CD (extended) intersect at E. Therefore, the lens is to be placed at E and the effective focal length of the lens is ED.

The total deviation of the beam $\delta = \delta_1 + \delta_2$ (xiii)

But from Eq. (xii),

$$\delta_1 = \frac{h_1}{f_1}, \delta_2 = \frac{h_2}{f_2}, \delta = \frac{h_1}{f_{eff}}$$

$$\frac{h_1}{f_{eff}} = \frac{h_1}{f_1}+\frac{h_2}{f_2} \quad \text{(xiv)}$$

From triangle BGC, we have, $h_1 - h_2 = GC = d\tan(\delta_1) = d\dfrac{h_1}{f_1}$

$$h_2 = h_1\left(1-\frac{d}{f_1}\right) \quad \text{(xv)}$$

Substituting Eq. (xv) in Eq. (xiv), we get

$$\frac{h_1}{f_{eff}} = \frac{h_1}{f_1}+\frac{1}{f_2}h_1\left(1-\frac{d}{f_1}\right)$$

$$\frac{1}{f_{eff}} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \quad \text{(xvi)}$$

This equivalent lens is to be placed at a distance

$$EG = (h_1-h_2)\cot(\delta)$$

$$= \frac{(h_1-h_2)}{\delta} = \frac{\left[\dfrac{h_1 d}{f_1}\right]}{\dfrac{h_1}{f_{eff}}} = \frac{d \cdot f_{eff}}{f_1}$$

before the second lens.

Eqs. (xv) and (xvi) are true only for a parallel incident beam.

Illustration 1.88 **Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. Find the image of a distant object formed by the combination by:**

a. Using thin lens formula separately for the two lenses, and

b. Using the equivalent lens. Note that although the combination forms a real image of a distant object on the

other side, it is equivalent to a diverging lens as far as the location of the final image is concerned.

Sol. **a.** The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at $u = -40$ cm from the second. Using the lens formula for the second lens,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = -\frac{1}{40} + \frac{1}{20} \quad \text{or} \quad v = 40 \text{ cm}$$

The final image is formed 40 cm to the right of the second lens.

b. The equivalent focal length is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{20} + \frac{1}{20} - \frac{60}{(20)^2}$$

or $\qquad F = -20$ cm

It is a divergent lens. It should be kept at a distance

$$D = \frac{dF}{f_1} \text{ behind the second lens.}$$

Here, $\qquad D = \dfrac{60 \times (-20)}{20} = 60$ cm

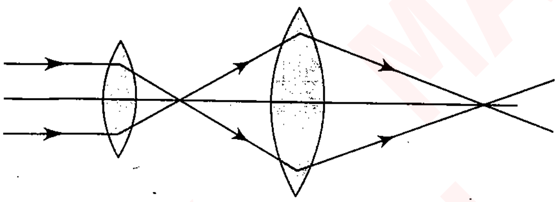


Fig. 1.240

Thus, the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens, i.e., 20 cm of the left of it. It is, therefore, 40 cm to the right of the second lens.

Illustration 1.88 **Consider a coaxial system of two thin convex lenses of focal length f each separated by a distance d. Draw ray diagrams for image formation corresponding to an object at infinity placed on the principal axis in the following cases: (a) $d < f$, (b) $d = f$, (c) $f < d < 2f$, (d) $d = 2f$, and (e) $d > 2f$. Indicate the nature of the combination (concave, convex or plane) in each case.**

Sol. In each of two thin lenses separated by a distance d, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

For $\qquad f_1 = f, f_2 = f$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{d}{f^2}$$

$$F = \frac{f^2}{2f - d} \qquad \text{(i)}$$

So, if $d < f$: F will be positive and $> f$. So, the system will behave as a convex lens of focal length $< f$ as shown in Fig. 1.241.

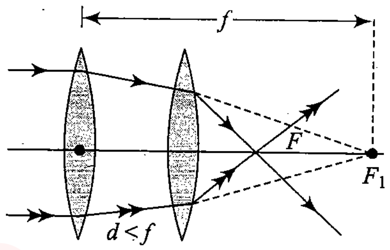


Fig. 1.241

b. **If $d = f$: $F = f$,** i.e., the system will behave as a convex lens of focal length f as shown in Fig. 1.242(a).

c. **If $f < d < 2f$:** F will be positive and $< f$, so the system will behave as a convex lens of focal length $> f$ as shown in Fig. 1.242(b).

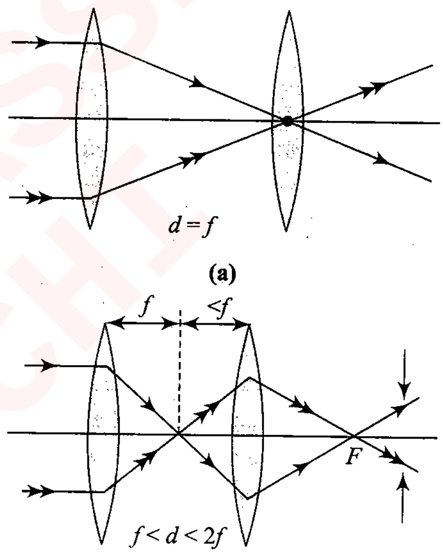


Fig. 1.242

d. **If $d = 2f$:** F will be infinite, i.e, the system will behave as a plane glass plate of infinite focal length. (Fig. 1.243(a)).

e. **If $d > 2f$:** F will become negative, i.e., the system will behave as concave lens as shown in Fig. 1.243(b).

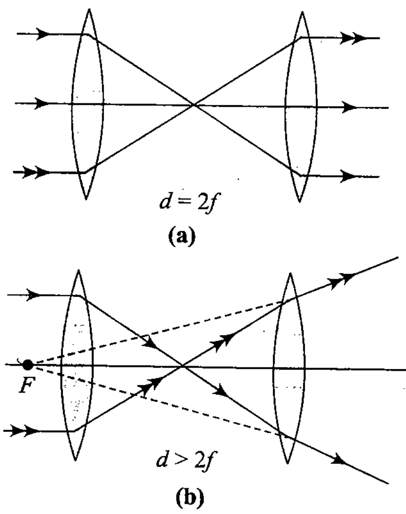
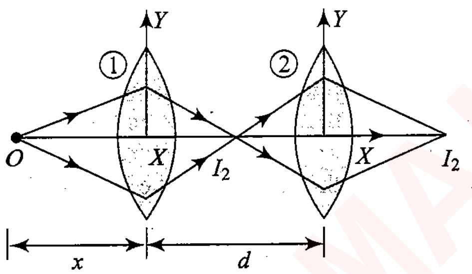


Fig. 1.243

Case 2. Lenses placed apart and object at a finite distance
This system of lenses cannot be reduced to a single equivalent lens. Now, each is to be treated independently. The image formed by the first lens now acts as the object for the second lens. However, the coordinate system for the first lens is at its optical center while the coordinate system for the second lens is centered at the optical center of the second lens. Consequently, care should be taken while determining the object distance for the second lens from the image distance of the first lens.

Consider the two lenses shown in Fig. 1.244. The lenses are placed a distance d apart and an object "O" is placed a distance x in front of lens 1. Let the image be formed at a distance y to the right of lens 1.



An object in front of two lenses a distance apart

Fig. 1.244

Refraction at the first lens:

Here, $u = -x, f = f_1, v = +y$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \quad \text{or} \quad \frac{1}{y} - \frac{1}{-x} = \frac{1}{f_1}$$

from which y can be determined.

Refraction at the second lens:

Here, $u = -(d-y) = y - d, f = f_2$

Let the image be at a distance z to the right of lens 2.

$\therefore \qquad v = +z$

Substituting in the lens equation, we get $\frac{1}{y} - \frac{1}{u} = \frac{1}{f_2}$ or

$\frac{1}{z} - \frac{1}{y-d} = \frac{1}{f_2}$ from which z can be calculated.

Illustration 1.89 **An object is placed at a distance of 10 cm to the left on the axis of a convex lens A of focal length 20 cm. A second convex lens of focal length 10 cm is placed coaxially to the right of the lens A at a distance of 5 cm from A. Find the position of the final image and its magnification. Trace the path of the rays.**

Sol. Here, for 1st lens, $\mu_1 = -10$ cm

$f_1 = 20$ cm

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} - \frac{1}{10} \Rightarrow v_1 = -20 \text{ cm}$$

The image is virtual and hence lies on the same side of the object. This will behave as an object for the second lens.

For 2nd lens:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Here, $u_2 = -(20+5)$

$f_2 = 10$ cm

$$\frac{1}{v_2} + \frac{1}{25} = \frac{1}{10} \Rightarrow v_2 = \frac{50}{3} = 16\frac{2}{3} \text{ cm}$$

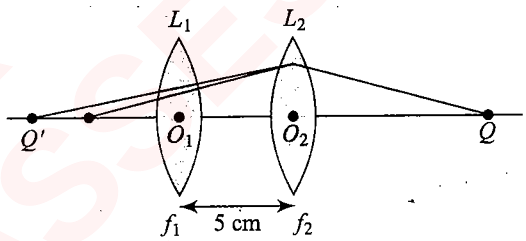


Fig. 1.245

The final image is at a distance of $16\frac{2}{3}$ cm to the right of the second lens.

The magnification of the image is given by,

$$m = \frac{v_1}{u_1} \cdot \frac{v_2}{u_2} = \frac{20}{10} \cdot \frac{50}{3 \times 25} = \frac{4}{3} = 1.33$$

Illustration 1.89 **In Fig. 1.246, length of the object AB is 9 cm. Find the nature and position of the final image and also its length. Assume that each lens is a thin lens.**

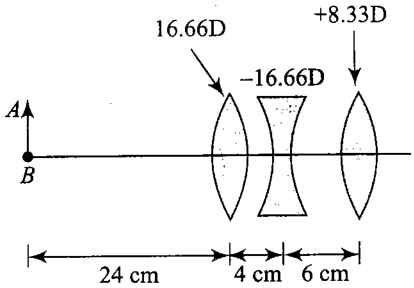


Fig. 1.246

Sol. *Action of first lens:*

$$\text{Focal length} = \frac{1}{16.66} = 0.06 \text{ m} = 6 \text{ cm}$$

$$u = -24 \text{ cm}$$

Using $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{24} = \frac{1}{6}$$

$$\frac{1}{v} = \frac{1}{6} - \frac{1}{24} \quad \text{or} \quad v = 8 \text{ cm}$$

This image would be real and inverted (with respect to object AB).

Its magnification, $m_1 = \dfrac{v}{u} = \dfrac{8}{-24} = -\dfrac{1}{3}$

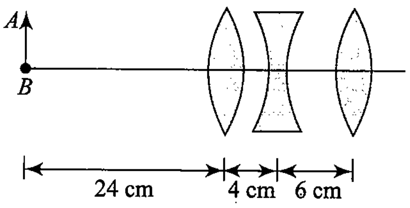


Fig. 1.247

Action of second lens:
Image formed by the first lens, being at distance 8 cm right of first lens, would act as virtual object for the second lens.

$$\text{Focal length} = \frac{1}{-16.66} = -0.06\text{ m} = -6\text{ cm}$$

$u = +4$ cm [distance of image formed by first lens from the second lens]

$$\frac{1}{v} - \frac{1}{4} = -\frac{1}{6}$$

$$\frac{1}{v} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \quad \text{or} \quad v = 12\text{ cm}$$

Thus, the second lens forms an image 12 cm to its right. This image is real and its magnification,

$$m_2 = \frac{v}{u} = \frac{12}{4} = 3$$

Thus, it is also erect relative to the image formed by the first lens.

Action of third lens:
Image formed by the second lens, being 12 cm to its right, will act as a virtual object for the third lens.

$$\text{Focal length} = \frac{1}{8.33} = 0.12\text{ m} = 12\text{ cm}$$

$u = +6$ cm [distance of image formed by second lens from the third lens]

$$\frac{1}{v} - \frac{1}{6} = \frac{1}{12}$$

$$\frac{1}{v} = \frac{1}{12} + \frac{1}{6} \quad \text{or} \quad v = 4\text{ cm}$$

Thus, the final image forms 4 cm to the right of the third lens. The image will be real and its magnification,

$$m_3 = \frac{v}{u} = \frac{4}{6} = \frac{2}{3}$$

This image is erect relative to the image formed by second lens.

Total magnification $m = m_1 \times m_2 \times m_3$

$$= -\frac{1}{3} \times 3 \times \frac{2}{3}$$

$$\Rightarrow \quad m = -\frac{2}{3}$$

Hence, the final image is, relative to the given object, inverted and also reduced in size. It forms 4 cm to the right of the third lens and it is a real image.

To obtain length of the final image,

$$m = \frac{I}{O} = -\frac{2}{3}$$

$$I = \frac{2}{3} \times 9 = -6\text{ cm}$$

SILVERED LENS

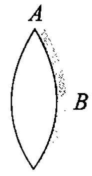
How do we deal with a lens which has one of its surface silvered?

Silvering a surface has the effect of converting the lens into a mirror. In a lens, a ray of light undergoes refraction and emerges on the side opposite to the side of the object. In the case of a silvered lens, after refraction, a ray of light is reflected on the silvered surface and the ray emerges on the same side as the object.

Now, what will be the behavior of the lens?

When silvered, a convex lens acts like a concave mirror and a silvered concave lens acts like a convex mirror.

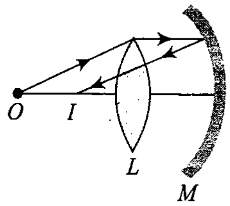
Consider a lens whose side *ABC* is silvered (See Fig. 1.248). We can model a silvered lens in two ways – first by the method of elements and second as a collection of interfaces. We will solve the problem as a combination of elements.



A lens with one surface silvered

Fig. 1.248

Using the method of superposition, we can treat the system as a combination of lenses and a mirror as shown in Fig. 1.249. In this arrangement, a ray of light is first refraction by lens *L*, then it is reflected at the curved mirror *M*, and finally refracted once again at the lens *L*. let the object *O* be located in front of the lens. Let the image from the lens I_1 be formed at v_1. Then, from the lens-makers formula, we have



The silvered lens can be modified as a lens–mirror lens combination

Fig. 1.249

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_{L_1}} \quad \text{(i)}$$

where f_{L_1} is the focal length of the lens L when the ray approaches from the left.

I_1 will now be the object for the mirror M. Let the image I_2 be formed after reflection at v_2 from the pole of the mirror. Then,

$$\frac{1}{v_2}+\frac{1}{v_1}=\frac{1}{f_m} \quad \text{(ii)}$$

where f_m is the focal length of the mirror M.

Now, I_2 will be the object for the final refraction at lens L. If I_3 be the final image formed at v from the center of the lens, then we have $\frac{1}{v}-\frac{1}{v_2}=\frac{1}{f_{L_2}}$ (iii)

where f_{L_2} is the focal length of the lens, L when the ray of light approaches from the right. When a ray of light approaches from the left, the positive X-axis is from left to right. For such a system, the focal length of the convex lens is positive. When a ray is refracted a second time, it once again encounters the same convex lens which means the focal length is once again positive. However, for this lens the coordinate system is in the direction of the incident ray which means the positive X-direction is from right to left. If we use only a single coordinate system, that of the first lens, then the focal length of the second lens will be negative to that of the first lens.

We have $\quad f_{L_1}=f_L \quad$ then $\quad f_{L_2}=f_L$

Substituting the above relation in Eq. (ii), we get

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_L}$$

$$\frac{1}{v_2}+\frac{1}{v_1}=\frac{1}{f_m}$$

$$\frac{1}{v}-\frac{1}{v_2}=-\frac{1}{f_L}$$

Manipulating the above equations, we get

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_m}-\frac{2}{f_L} \quad \text{(iv)}$$

which is the same as the equation for a spherical mirror. Therefore, the silvered lens finally behaves as a mirror with an effective focal length of

$$\frac{1}{f_e}=\frac{1}{f_m}-\frac{2}{f_L} \quad \text{(v)}$$

Here, f_m is the focal length of the mirror and f_L is the focal length of the lens.

Let us solve a few examples to illustrate the principle learnt in this section.

Illustration 1.91 **A biconvex lens is made of glass with refractive index 1.5 and has radii of curvature 20 cm and 30 cm. If the 20 cm surface is silvered, what is the effective focal length of the mirror formed?**

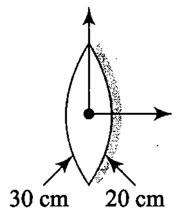


Fig. 1.250

Sol. Here, $R_1=+30$ cm, $R_2=-20$ cm, $\mu_b=1.5$, $\mu_a=1$

Therefore, focal length of the lens $\frac{1}{f_\ell}=(1.5-1)\left[\frac{1}{30}-\frac{1}{-20}\right]$ or $f_\ell=24$ cm.

Therefore, the 20 cm side is silvered.

Focal length of the mirror,

$$f_m=\frac{R_2}{2}=\frac{-20}{2}=-10 \text{ cm} \quad \text{[Note the negative sign]}$$

Effective focal length of the silvered lens is $\frac{1}{f_e}=\frac{1}{f_m}-\frac{2}{f_\ell}$ or $f_e=-\frac{60}{11}$ cm

The silvered convex lens behaves as a concave mirror of focal length 60/11 cm.

Lens with One Silvered Surface

If the back surface of a lens is silvered and an object is placed in front of it then:

1. First, light will pass through the lens and it will form the image I_1.
2. The image I_1 will act as an object for silvered surface which acts as curved mirror and forms an image I_2 of object I_1.
3. The light after reflection from silvered surface will again pass through the lens and lens will form final image I_3 of object I_2.

This is shown in Fig. 1.251. In such a situation, power of the silvered lens will be

$$P=P_L+P_M+P_L$$

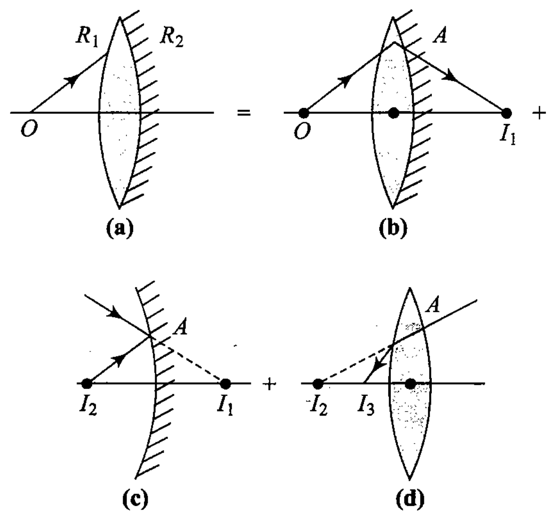


Fig. 1.251

With $\quad P_L = \dfrac{1}{f_L}$, where $\dfrac{1}{f_L} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

And $\quad P_M = -\dfrac{1}{f_M}$, where $f_M = \dfrac{R_2}{2}$

So, the system will behave as a curved mirror of focal length F given by $F = -1/P$.

Illustration 1.91 **The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1.5.**

a. Find the focal length of this lens. The plane surface of the lens is now silvered.

b. At what distance from the lens will parallel rays incident on the convex face converge?

c. Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

d. Calculate the image distance when the object is placed as in (c).

Sol. a. As for a lens, by lensmaker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, $\quad \mu = 1.5;\ R_1 = 12$ cm; and $R_2 = \infty$

So, $\quad \dfrac{1}{f} = (1.5 - 1)\left[\dfrac{1}{12} - \dfrac{1}{-\infty}\right]$ i.e., $f = 24$ cm

i.e., the lens is convergent with focal length 24 cm.

b. As light after passing through the lens will be incident on the mirror which will reflect it back through the lens again, so

$$P = P_L + P_M + P_L = 2P_L + P_M$$

But $\quad P_L = \dfrac{1}{f_L} = \dfrac{1}{0.24}$ and $P_M = -\dfrac{1}{\infty} = 0$

$$\left[\text{as } f_M = \frac{R}{2} = \infty\right]$$

So, $\quad P = 2 \times \dfrac{1}{0.24} + 0 = \dfrac{1}{0.12}\, D$

The system is equivalent to a concave mirror of focal length F.

$$P = -\frac{1}{F}$$

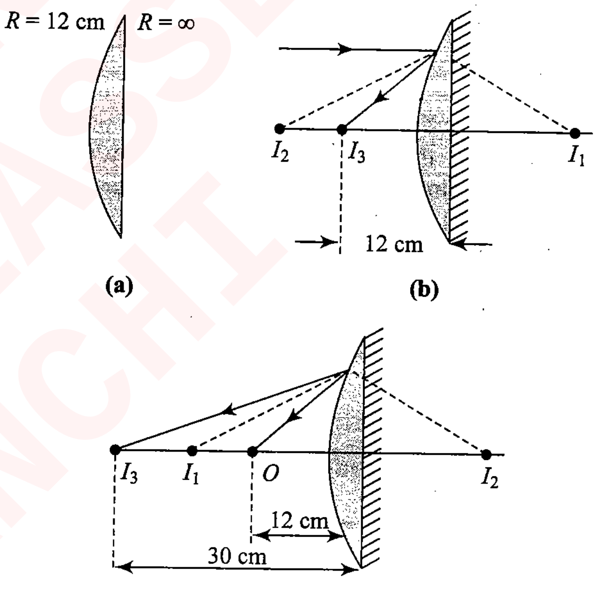
i.e., $\quad F = -\dfrac{1}{P} = -0.12 \text{ m} = -12 \text{ cm}$

i.e., the system will behave as a concave mirror of focal length 12 cm. So, as for parallel incident rays $u = -\infty$, from mirror formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, we have

$$\frac{1}{v} + \frac{1}{-\infty} = \frac{1}{-12} \quad \Rightarrow \quad v = -12 \text{ cm}$$

i.e., parallel incident rays will focus at a distance of 12 cm in front of the lens as shown in Fig. 1.252(b), (c) and (d) When object is at 20 cm in front of the given silvered lens, which behaves as a concave mirror of focal length 12 cm, from mirror formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, we have

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-12} \quad \Rightarrow \quad v = -30 \text{ cm}$$



(a) (b) (c)

Fig. 1.252

i.e., the silvered lens will form an image at a distance of 30 cm in front of it as shown in Fig. 1.252(c).

Illustration 1.92 **An object in placed 30 cm in front of a concave lens that is made of a glass of refractive index 1.5 and has equal radii of curvature of its two surfaces, each 30 cm. The surface of the lens farther away from the object is silvered. Find the nature and position of the final image.**

Sol. Focal length of the concave lens, using the lensmaker's formula, is

$$\frac{1}{f_L} = (1.5 - 1)\left(-\frac{1}{30} - \frac{1}{30}\right) \qquad [R_1 = -30 \text{ cm},\ R_2 = 30 \text{ cm}]$$

$$f_L = -30 \text{ cm} = -0.3 \text{ m}$$

In a silvered lens, light as it enters the lens suffers refraction, then it gets reflected at the silvered surface, and again undergoes refraction as it comes out in air.

In such a situation, power of the silvered lens will be $P = P_L + P_M = 2P_L + P_M$

$$P_L = \frac{1}{f_L} = -\frac{1}{0.3}D$$

$$P_M = -\frac{1}{f_M} = -\frac{1}{0.15}D$$

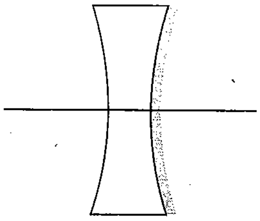


Fig. 1.253

[Silvered surface behaves as a spherical convex mirror of radius of curvature 30 cm so that focal length will be 15 cm = 0.15 m]

$$P = 2P_L + P_M = -\frac{2}{0.3} - \frac{1}{0.15} = -13.33\,D$$

And focal length of the equivalent mirror,

$$F = -\frac{1}{P} - \frac{1}{-13.33} = 0.075 \text{ m} = 7.5 \text{ mm}.$$

i.e., the silverd lens behaves as a convex mirror of focal length 7.5 cm.

For an object placed at a distance of 30 cm from the silvered lens, which behaves as a convex mirror of focal length 7.5 cm,

$u = -30$ cm and $f = 7.5$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{30} = \frac{1}{7.5}$$

$$\frac{1}{v} - \frac{1}{7.5} + \frac{1}{30}$$

$\Rightarrow$ $v = 6$ cm

i.e., the image will be 6 cm to the right of the silvered lens and it will be a virtual image [silvered lens behaves as a convex mirror].

Illustration 1.94 **One face of an equiconvex lens of focal length 60 cm made of glass ($\mu = 1.5$) is silvered. Does it behave like a convex mirror or concave mirror? Determine its focal length.**

Sol. Using lensmaker's formula, we know that, for a glass made of glass ($\mu = 1.5$), the focal length of an equiconvex lens is equal to its radius of curvature.

$$\frac{1}{f} = (1.5-1)\left(\frac{2}{R}\right) \Rightarrow f = R$$

For a lens–mirror combination, we have $\frac{1}{F} = \frac{2}{f_f} + \frac{1}{f_m}$

Here, $f_f = +60$ cm (converging) and $f_m = \frac{+R}{2} = +30$ (converging)

$$\therefore \quad \frac{1}{F} = \frac{2}{60} + \frac{1}{30} = \frac{1}{15} \quad \text{or} \quad F = +15 \text{ cm}.$$

The positive sign indicates that the resulting mirror is converging or concave.

CONCEPT OF IMAGE FORMING AT OBJECT ITSELF

In some unique situations, the image is formed at the same point as the object. For a lens, if we apply the lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{u} - \frac{1}{u} = \frac{1}{f}$$

which is not possible.

On the other hand, for the case of refraction across a single curved surface, the formula is $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

which reduces to $\frac{\mu_2}{u} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

The solution for which is $u = R$. Therefore, the object and image coincide when the object is placed at the center of curvature [Fig. 1.254(a)].

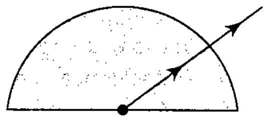


Fig. 1.254(a)

Similarly, in the case of mirrors, the formula is $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$

which simplifies to $\frac{1}{u} + \frac{1}{u} = \frac{2}{R}$

and the solution once again is $u = R$. Therefore, for mirrors and refraction across a single curved surface, we can say that object and image will coincide only when the object is kept at the center of curvature.

We have to be cautious though while dealing with convex mirrors for the center of curvature is behind the mirror [Fig. 1.254(b)]. How can we place an object behind the mirror?

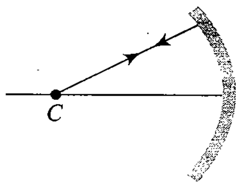


Fig. 1.254(b)

The answer is simply that in the case of convex mirrors, we will have a virtual object, i.e., a bundle of converging rays when incident on the convex mirror will retrace their paths as shown in Fig. 1.255.

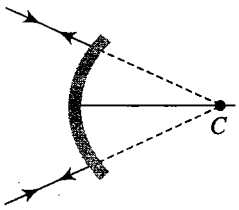


Fig. 1.255

In problems in optics, we will usually have a train of optical elements with the stipulation that the image is formed on the object itself. In such cases, there will have to be a mirror at the end of the optical train and the rays have to be incident normally on the mirror in order to retrace their paths. Three kinds of mirrors are possible.

Case 1. A plane mirror at the end of the optical train
In this situation, the ray of light emerging from the system just before it impinges on the mirror has to be parallel so as to strike the mirror normally. Thus, the image after the last lens must be formed at infinity (See Fig. 1.256).

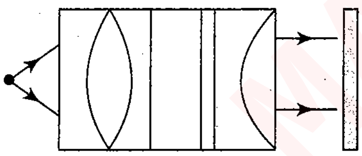


Fig. 1.256

Case 2. A concave mirror of the radius of curvature *R*
Here, the bundle of rays must converge on to the center of curvature which is in front of the concave mirror. If the distance of the lens from the mirror is d, we can say that the image formed from the last lens must be at a distance $d + R$ from the lens (See Fig. 1.257).

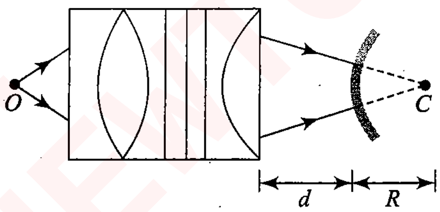


Fig. 1.257

Case 3. A convex mirror of radius of curvature *R*
Here, the bundle of rays must converge on to the center of curvature of the convex mirror. If the distance of the last lens from the mirror is d, we can say that the image formed from the last lens must be at a distance $(d - R)$ from the lens (see Fig. 1.258).

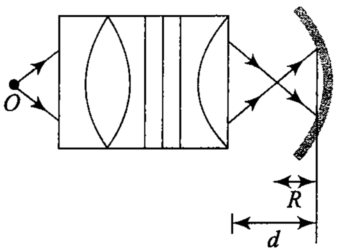


Fig. 1.258

Illustration 1.95 **A glass slab of thickness 3 cm and refractive index 1.5 is placed in front of a concave mirror of focal length 20 cm. Where should a point object be placed if it is to image on to itself?**

Sol. The glass slab and the concave mirror are shown in Fig. 1.259.

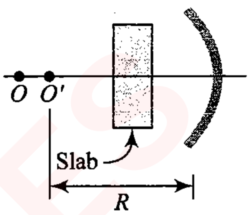


Fig. 1.259

Let the distance of the object from the mirror be x.

We know that the slab simply shifts the object. The shift being equal to $s = t\left[1 - \frac{1}{\mu}\right] = 1$ cm

The direction of shift is toward the concave mirror. Therefore, the apparent distance of the object from the mirror is $x - 1$.

If the rays are to retrace their paths, the object should appear to be at the center of curvature of the mirror.

Therefore, $x - 1 = 2f = 40$ cm or $x = 41$ cm from the mirror.

Illustration 1.96 **A convex lens of focal length 20 cm is placed 10 cm in front of a convex mirror of radius of curvature 15 cm. Where should a point object be placed in front of the lens so that it images on to itself?**

Sol. The convex lens and the convex mirror are shown in Fig. 1.260.

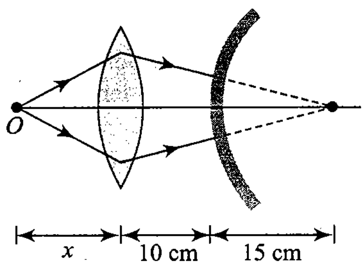


Fig. 1.260

The combination behaves like a concave mirror.

Let the distance of the object from the lens be x.

For the ray to retrace its path, it should be incident normally on the convex mirror, or in other words the rays should pass through the center of curvature of the mirror.

From the diagram, we see that for the lens

$$u = -x, f = +20 \text{ cm}, v = +10 + 15 = +25 \text{ cm}$$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{25} - \frac{1}{-x} = \frac{1}{20} \quad \text{or} \quad x = 100 \text{ cm}$$

Illustration 1.97 **A glass slab of thickness 2 cm and refractive index 2 is placed in contact with a biconvex lens of focal length 20 cm. The refractive index of the material of the lens is 1.5. The far side of the lens is silvered. Where should an object be placed in front of the slab so that it images on to itself?**

Sol. Step 1: Ray diagram: The path of a typical light ray is shown in Fig. 1.261.

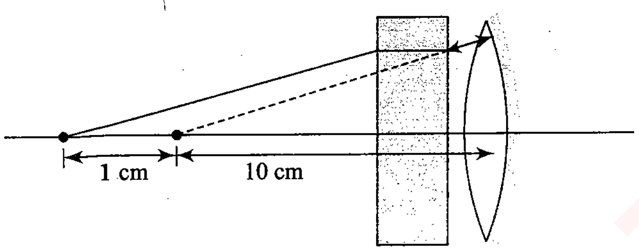


Fig. 1.261

Step 2: The elements encountered are a glass slab, a silvered lens, and a glass slab.

Step 3: The lens is silvered, so it acts as a mirror. Since the image is formed on the object itself, the apparent position of the object after refraction from the glass slab should be at the center of curvature of the mirror.

Step 4: The effective focal length of the silvered lens is

$$\frac{1}{f_e} = \frac{1}{f_m} - \frac{2}{f_L}$$

Now, $f_L = +20$ cm (assuming the +ve X-direction is to the right)

It is a biconvex lens. Therefore,

$$\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) \quad \text{or} \quad R = 20 \text{ cm}$$

Therefore, $f_m = -20/2 = -10$ cm and

$f_e = -5$ cm

For the image to form on the object, the apparent position of the object after refraction at the glass slab must be at $2 \times f_e = -10$ cm, i.e, the object must appear to be 10 cm to the left of the silvered lens.

Step 5: For the glass slab:

Thickness = 2 cm, $\mu = 2$

Shift due to slab $= s = t\left(1 - \frac{1}{\mu}\right) = 1$ cm

Direction of shift is toward the left in the direction of the incident ray.

Therefore, the object should be at 10 + 1 = 11 cm to the left of the mirror so that after refraction at the glass slab it appears to be at 10 cm from the lens which is the center of curvature of the silvered lens. Hence, the rays retrace their paths and the image is on the object itself.

COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used coaxially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens (or mirror) acts as an object for the third, and so on. The total magnification in such situations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \cdots, \quad \text{i.e.,} \quad m = m_1 \times m_2 \times \cdots$$

Here, is it worthy to note that

1. If two thin lenses of equal focal length but of opposite nature (i.e., one convergent and other divergent) are put in contact, the resultant focal length of the combination will be
$$\frac{1}{F} = \frac{1}{f} + \frac{1}{-f} = 0, \quad \text{i.e.,} \quad F = \infty \text{ and } P = 0$$
i.e., the system will behave as a plane glass plate.
2. If two thin lenses of same nature are put in contact, then as
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$
$$\frac{1}{F} > \frac{1}{f_1} \quad \text{and} \quad \frac{1}{F} > \frac{1}{f_2}$$
i.e., $\quad F < f_1 \quad$ and $\quad F < f_2$
i.e., the resultant focal length will be lesser than smallest individual.
3. If two thin lenses of opposite nature with different focal lengths are put in contact, the resultant focal length will be of same nature as that of the lens of shorter focal length but its magnitude will be more than that of the shorter focal length.
4. If a lens of focal length f is divided into two equal parts as shown in Fig. 1.262(a) and each part has a focal length f' then as
$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'}, \quad \text{i.e.,} \quad f' = 2f$$
i.e., each part will have focal length $2f$.
Now, if these parts are put in contact as in Fig. 1.262(b) or (c), the resultant focal length of the combination will be
$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f}, \quad \text{i.e.,} \quad F = f\,(= \text{initial value})$$

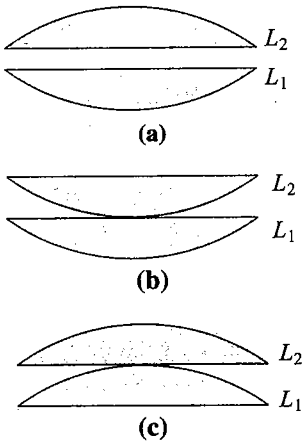


Fig. 1.262

Illustration 1.98 *L* is a lens such that a parallel beam of light incident on it, after refraction, converges to a point at a distance '*x*' from it and *M* is a mirror such that a parallel beam of light incident on it, after reflection, converges to a point at a distance *y* from it. Now the lens *L* and mirror *M* are placed at a distance $2(x+y)$ from each other. An object of size 2 cm is kept infront of lens at a distance $2x$ from it. Find the nature, position, and size of the image that will be seen by an observer looking towards the mirror through the lens.

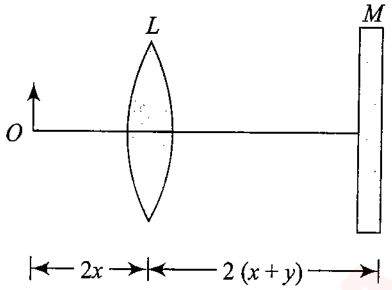


Fig. 1.263

Sol. It is given that a parallel beam incident on the lens, after refraction, converges to a point at a distance x from the lens. This implies that focal length of the lens is 'x'.

It is also given that a parallel beam incident on the mirror, after reflection, converges to a point at a distance 'y' from it. This implies that the mirror is a concave mirror of focal length 'y'.

Obviously, the object O is kept at $2F$ of lens L. Hence, its image I_1 forms on the other side at $2F$, i.e., at distance $2x$ to the right of the lens. The image will be real, inverted, and magnification of this image $m_1 = \frac{v}{u} = \frac{2x}{-2x} = -1$.

Image I_1 of O formed by lens L acts as object for the mirror. Its distance from the pole of mirror will be $2(x+y) - 2x = 2y$. This implies that it will be at the center of curvature of the concave mirror (y being the focal length of mirror). Therefore, the concave mirror forms a real, inverted (relative to I_1) image I_2 at the same position.

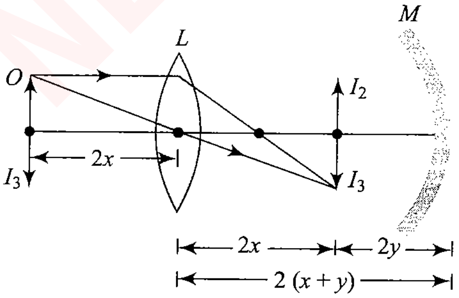


Fig. 1.264

Magnification of I_2 (relative to I_1), $m_2 = -1$. Finally, I_2 acts as object for the lens. Its distance from the lens is $2x$. Hence, it is at $2F$ of the lens (x being the focal length of the lens). Therefore, the lens will form a real and inverted image of I_2 at $2F$ on the other side, i.e., to its left. Obviously, I_3 will be at the same position as of the object.

Magnification of I_3 (relative to I_2) = -1

Total magnification = $m_1 \times m_2 \times m_3 = -1 \times -1 \times -1 = -1$

Thus, the final image is real and inverted and of the same size (2 cm).

Complete ray diagram is shown in Fig. 1.265.

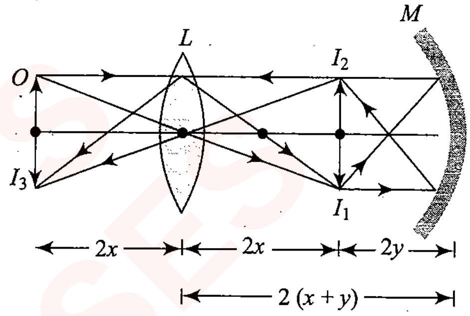


Fig. 1.265

Illustration 1.99 An object of height 2 cm is kept 2 m in front of a convex lens of focal length 1 m. A plane mirror is placed at 3 m from the lens on its other side. Find the nature, position, and magnification of the final image that will be seen by an observer looking toward the mirror through the lens.

Sol. The lens forms image $A'B'$ of object AB, as show in Fig. 1.266.

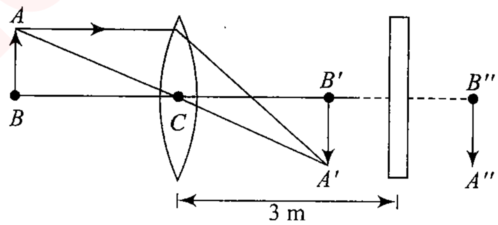


Fig. 1.266

Here, $u = -2$ cm, $f = 1$ cm

Using $(1/v) - (1/u) = (1/f)$, we get

$$\frac{1}{v} + \frac{1}{2} = 1 \quad \Rightarrow \quad v = 2\text{ m}$$

Thus $A'B'$ is formed at distance 2 m from the lens. Image $A'B'$ is real and inverted and has a magnification

$$m_1 = \frac{v}{u} = \frac{2}{-2} = -1$$

[We could also argue that object being at $2F$, image will also form at $2F$ on the other side.]

$A'B'$ acts as an object for the plane mirror. Its distance from the plane mirror = $3 - 2 = 1$ m. The plane mirror forms its virtual image $A''B''$ which will be 1 m behind the plane mirror and with magnification $m_2 = 1$ (relative to its object $A'B'$).

$A''B''$ now acts as a virtual object for the lens at a distance 4 m to its right.

Applying $(1/v) - (1/u) = (1/f)$

$$u = -4 \text{ cm} \quad \Rightarrow \quad \frac{1}{v} + \frac{1}{4} = 1$$

$$\frac{1}{v} = \frac{3}{4} \quad \text{or} \quad v = \frac{4}{3} \text{ m}$$

Since light from $A''B''$ travels from right to left, its image formed by the lens is 4/3 m to the left of lens (v is positive). This image will be real and inverted (relative to $A''B''$). Magnification in this case, $m_3 = (4/3)/-4 = -1/3$

Total magnification, $m = m_1 \times m_2 \times m_3 = (-1) \times (1) \times (-1/3)$

$$\Rightarrow \quad m = \frac{1}{3}$$

Thus, the final image is real, reduced three times in size and erect (relative to the original object AB).

Complete ray diagram is shown in Fig. 1.267.

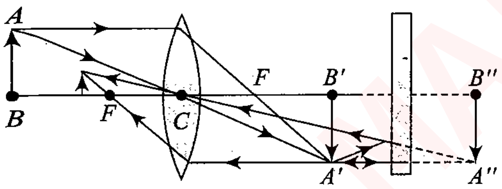


Fig. 1.267

Finding Focal Length of a Convex Lens

Plane Mirror method: A convex lens is placed on a plane mirror (Fig. 1.268). An object pin O is moved along the principal axis of the lens. At a certain position, the image coincides with the object. The distance of object pin O from the lens is the focal length of the lens. When an object is placed at the focus of the lens, the refracted rays become parallel to the principal axis. These parallel rays are incident on the mirror normally, therefore they retrace their path and converge at the focus.

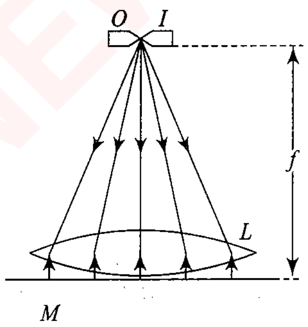


Fig. 1.268

Measurement of Refractive Index of Liquid by a Convex Lens

Figure 1.269 shows an equiconvex lens placed on a plane mirror. An object pin is moved up and down. When the pin lies at the focus of the lens, there is no parallax between the object and the image.

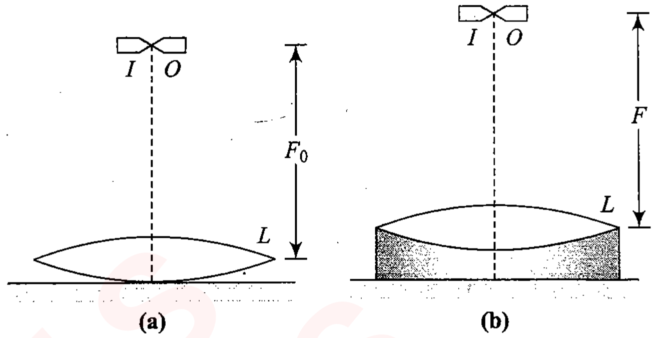


Fig. 1.269

When a liquid whose refractive index is to be obtained is placed between a plane mirror and a convex lens, the object pin O has to be shifted downward so that no parallax exists between it and its image. The position of object O from the lens is now equal to the combined focal length of 'lens and liquid' combination. If f is the focal length of the liquid lens, then the combined focal length is given by

$$\frac{1}{F} = \frac{1}{f_0} + \frac{1}{f}$$

The liquid lens is plano-convex type as its lower surface is the plane surface of the mirror and the upper surface is the curved surface of the convex lens. If R is the radius of curvature of the convex lens, then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \quad \Rightarrow \quad \frac{1}{f} = \frac{(\mu - 1)}{R}$$

Thus, $$\mu = 1 + \frac{R}{f}$$

Concept Application Exercise 1.7

1. In Fig. 1.270, find the position of final image formed.

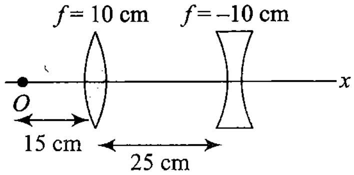


Fig. 1.270

2. Figure 1.271 shows two converging lenses. Incident rays are parallel to the principal axis. What should be the value of d so that final rays are also parallel?

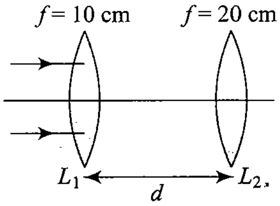


Fig. 1.271

Here, the diameter of ray beam becomes wider.

3. In Fig. 1.272, find the position of final image formed.

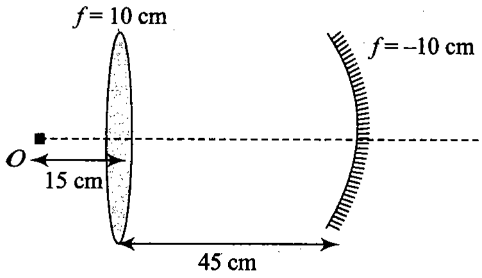


Fig. 1.272

4. In Fig. 1.273, what should be the value of d so that image is formed on the object itself.

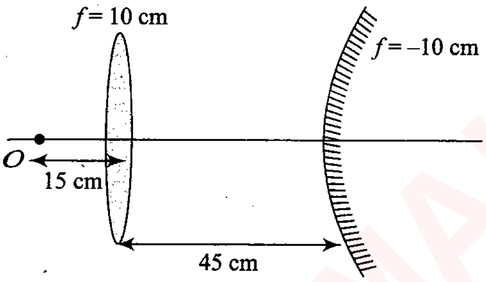


Fig. 1.273

5. In Fig. 1.274, if the image of object O has to coincide with itself, then where object must be placed at a distance from the lens.

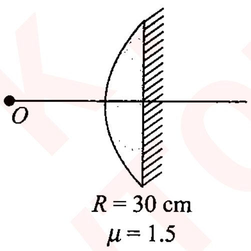


Fig. 1.274

6. A convex lens of focal length 20 cm is placed 10 cm away from a second convex lens of focal length 25 cm. What will be the location of the image of an object at 30 cm in front of the first lens.

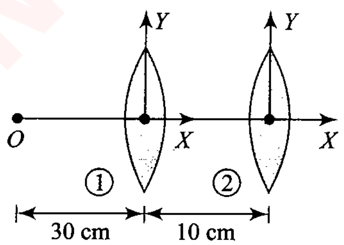


Fig. 1.275

7. When an object is placed at the proper distance to the left of a converging lens, the image is focused in a screen 30 cm to the right of lens. A diverging lens is now placed 15 cm to the right of the converging lens and it is found that the screen must be moved 19.2 cm farther to the right to obtain a sharp image. Find the focal length of the diverging lens.

8. A telephoto combination consists of convex lens of focal length 30 cm and a concave lens of focal length 15 cm, the separation between two lens is 27.5 cm. Where should be the photographic plate placed in order to photograph an object 10 m in front of the first lens?

9. A convex lens is cut in half along its principal axis and the two halves are separated by a distance of 12 cm. An object is placed 6 cm in front of the lens as shown in Fig. 1.276. Two sharp images are formed on the screen placed 80 cm from the object. What is the focal length of the lens?

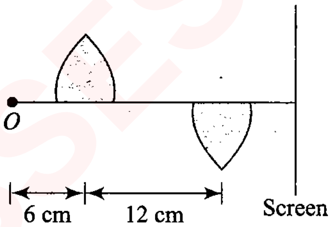


Fig. 1.276

10. A plano-convex lens is silvered on its plane side. The radius of curvature of the other face is 12 cm and the refractive index of the material of the lens is 1.5. An object is placed 24 cm in front of the silvered lens. Where will the image be formed?

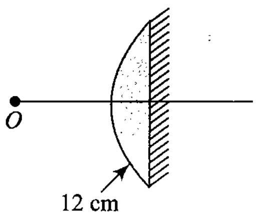


Fig. 1.277

11. A convex lens of focal length 10 cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed after the two lenses. Where should a point object be placed in front of the first lens so that it images on to itself?

12. A concave mirror of focal length 30 cm is placed on the flat horizontal surface with its concave side up. Water with refractive index 1.33 is poured into the lens. Where should an object be placed if its image is to be captured on a screen with a magnification of 2.

13. The convex side of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature of 20 cm. The concave surface has a radius of curvature of 60 cm. What is the focal length of the lens? The convex side is silvered and placed on a horizontal surface. What is the effective focal length of the silvered lens? The concave part is filled with water with refractive index 1.33. What is the effective focal length of the combined glass and water lens? If the convex side is silvered, what is the new effective focal length of the silvered compound lens?

14. The source is placed 30 cm from a convex lens which has a focal length of 20 cm. The source is initially located on

the axis of the lens. The lens is then cut into two halves in a plane along the principal axis. The two halves are separated by a distance of 4 mm. What will be the locations of the image of the source?

15. If final image after two refractions through the lens and one reflection from the mirror forms at the same point O. Refractive index of the material of the lens $\mu = 3/2$. Then find d.

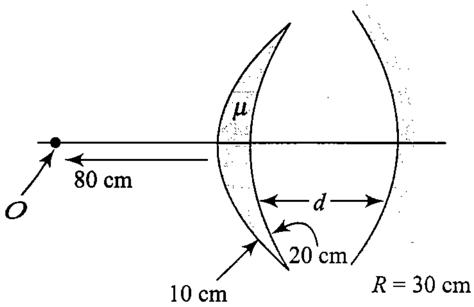


Fig. 1.278

16. A point object O is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in Fig. 1.279. At what distance x from the lens should a concave mirror of focal length 60 cm, be placed so that the final image coincides with the object?

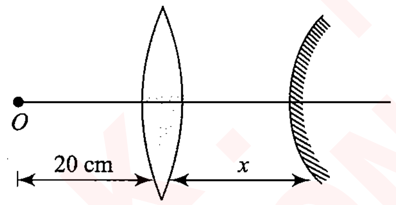


Fig. 1.279

17. Focal length of a thin convex lens is 30 cm. At a distance of 10 cm from the lens there is a plane refracting surface of refractive index 3/2. Where will parallel rays incident on lens converge?

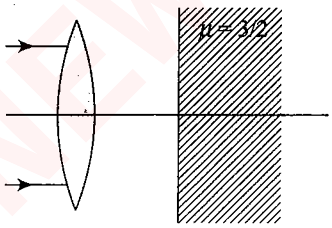


Fig. 1.280

18. A plano-convex glass lens ($\mu_g = 3/2$) of radius of curvature $R = 10$ cm is placed at a distance of 'b' from a concave lens of focal length 20 cm. What should be the distance 'a' of a point object O from the plano-convex lens so that position of final image is independent of 'b'?

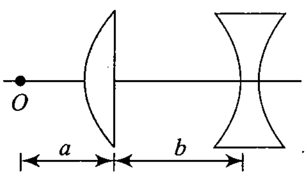


Fig. 1.281

19. On a horizontal plane mirror, a thin equiconvex lens of glass is placed and when the space between the lens and mirror is filled with a liquid, an object held at a distance $D = 30$ cm vertically above the lens is found to coincide with its own image as shown in Fig. 1.282. If equiconvex lens of glass has refractive index $\mu = 1.5$ and radius of curvature $R = 20$ cm, then find refractive index of the liquid.

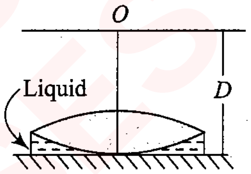


Fig. 1.282

20. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of a material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual?

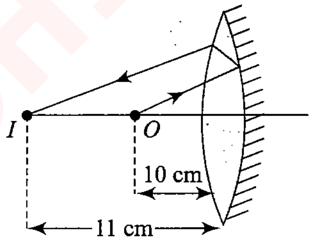


Fig. 1.283

21. A convex lens of glass ($\mu = 1.5$) is formed by combining two surfaces of radii $R_1 = 60$ cm and $R_2 = 30$ cm. It is cut into two parts in two different ways as shown in Fig. 1.284 (a) and (b).

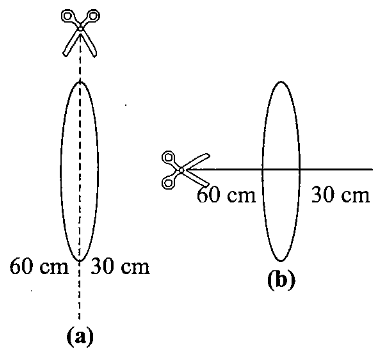


Fig. 1.284

Find:

a. the focal length of the lens.

b. in figure (a), the focal length of left portion, and that of the right portion.

c. in figure (b), the focal length of the upper part and that of the lower part.

OPTICAL INSTRUMENT

Microscopes

The apparent size of an object is determined by the size of its retinal image which, in turn, if the eye is unaided, depends upon the angle subtended by the object at the eye. When one wants to examine a small object in detail, one brings it close to the eye in order that the angle subtended and the retinal image may be as large as possible. Since the eye cannot focus sharply on objects closer than the nearer point, a given object subtends the maximum possible angle at an unaided eye when placed at this point. By placing a converging lens in front of the eye, the accommodation may, in effect, be increased. The object may then be brought closer to the eye than the near point and will subtend a correspondingly larger angle. The optical instrument which is used to increase the visual angle of eye is termed as microscope.

(i) Simple Microscope or Magnifier or Magnifying Glass: A converging lens used for the above mentioned purpose is termed as magnifier or simple microscope. The magnifier forms a virtual image of the object and eye looks at this virtual image, i.e., we can say the object is placed in between the focus and optical center of lens and eye close to it on the other side. Since a normal eye can focus sharply on an object anywhere between the near point and infinity, the image can be seen equally clearly if it is formed anywhere within this range.

Magnifying power or angular magnification of an optical instrument is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when the eye is unaided [i.e., when the object is at the near point].

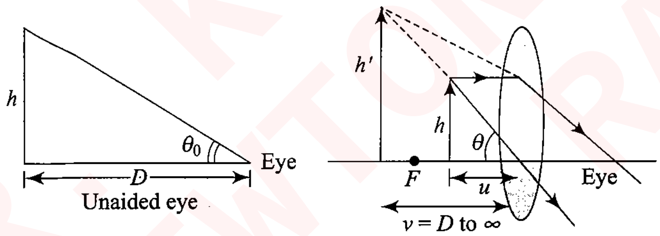


Fig. 1.285

$$\gamma = \text{Magnifying power} = \frac{\tan\theta}{\tan\theta_0}$$

where θ and θ_0 are clearly shown in Fig. 1.285.

$\tan\theta = \frac{h'}{v}$, where $v = D$ to ∞ and D is the least distance of distinct vision.

$$\tan\theta_0 = \frac{h}{D}$$

$\gamma = \frac{h'}{v} \times \frac{D}{h}$, where $\frac{h'}{v} = \frac{h}{u}$, u being the object distance from the lens.

For $v = D$, $\gamma = \gamma = \left[1 + \frac{D}{f}\right]$. In this situation, the magnifying power is maximum and eye is under maximum strain.

For $v = \infty$, $\gamma = \frac{D}{f}$. In this situation, magnifying power is minimum and eye is the least strained.

Thus, angular magnification of a simple magnifier of focal length 10 cm is 2.5 × (as D = 25 cm), i.e., height of retinal image of an object viewed through the magnifier is 2.5 times as great as, when viewed with unaided eye at minimum distance of distinct vision, i.e., the near point. [This is the situation when eye is the least strained, the most general case.]

- It seems that can be increased as much as desired by decreasing f, but actually it is not possible due to aberrations of simple lens.
- Simple magnifier is an essential part of most of the optical instruments (compound microscope or telescopes) in the form of eyepiece or oculars.

(ii) Compound Microscope: When an angular magnification higher than that attainable with a simple magnifier is desired, it is necessary to use a compound microscope, simply termed as a microscope.

It consis of two convergent lenses of short focal lengths and apertures arranged coaxially. One is termed as objective while the other is eyepiece or ocular, object is placed near to objective while eyepiece is arranged in such a way that it is facing the eye. The objective has smaller aperture and focal length as compared to the eyepiece. The separation between the objective and eyepiece can be varied. Construction and ray diagram outlining the basic principle of a microscope is shown in Fig. 1.286.

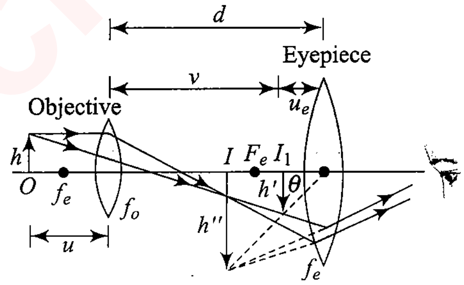


Fig. 1.286

An object O is placed just beyond the focus of the objective, whose image I_1 is formed by the objective within the focus and optical center of eye piece.

This image I_1 is real, inverted, and enlarged image w.r.t. object O. The eyepiece then forms a virtual image of the object between D and ∞. This final image is inverted, virtual, and enlarged w.r.t. the object O.

Angular magnification or magnifying power of the instrument is

$$\gamma = \frac{\tan\theta}{\tan\theta_0}$$

$$\tan\theta = \frac{h'}{u_e} \quad \text{and} \quad \tan\theta_0 = \frac{h}{D}$$

So, $$\gamma = \frac{h'}{h} \times \frac{D}{u_e} = m\gamma_e = -\frac{v}{u} \times \gamma_e$$

where m is the lateral magnification produced by the objective and γ_e is the angular magnification produced by the eyepiece,

- When final image is formed at the least distance of distinct vision (D), the eye is most strained and

$$\gamma = -\frac{v}{u}\left[1+\frac{D}{f_e}\right]$$

with $L = v + \frac{f_e D}{f_e + D}\gamma$ is maximum in this mode.

- In general, as f_0 is small and object is close to objective first focus, i.e., u ; f_0 and L; v as $u_e \ll v$, so for normal adjustment [i.e., when final image is at for point] γ; $\frac{L}{f_0} \times \frac{D}{f_e}$
- In this case, γ is –ve. Hence, the final image is inverted.
- In normal adjustment, if eyepiece and objective lenses are interchanged, the angular magnification remains unchanged.
- Due to small apertures of lenses, spherical aberration is decreased to a great extent.
- F_0 is taken smaller than f_e to increase the field of view and to increase brightness of the image.
- With respect to microscope, the minimum distance between two lines at which they are just distinct is called limit of resolution (Δx) and the reciprocal of limit of resolution is called resolving power.

Resolving power $= \frac{1}{\Delta x} \propto \frac{1}{\lambda}$

Telescopes

(i) Refracting Telescope: The optical system of a refracting telescope is essentially the same as that of a compound microscope. In both instruments, the image formed by the objective is viewed through an ocular (eyepiece), the difference is the telescope is used to examine large objects at large distances while microscope is used to examine small objects close at hands.

(ii) Astronomical Telescope: The working diagram of astronomical telescope is as shown in Fig. 1.287.

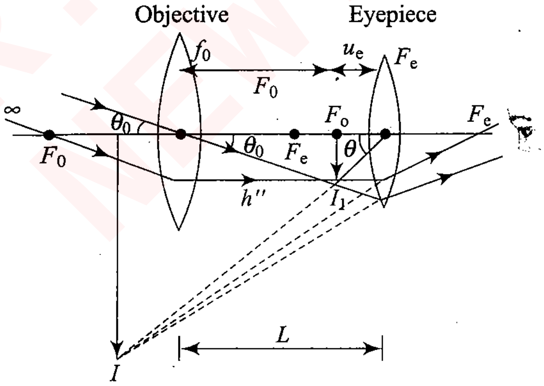


Fig. 1.287

- While seeing the object through telescope, the object is between ∞ and $2F$ of the objective which forms real, inverted, and diminished image within the focus and optical center of eyepiece which then forms its virtual, erect (w.r.t. intermediate image), and magnified image at a distance between D to ∞ from the eye. This means final image is inverted (w.r.t. object), at a distance between D to ∞ from the eye.
- In a telescope, the aperture and focal length of objective are greater than that of eyepiece.
- Magnifying power, $\gamma = \frac{\tan\theta}{\tan\theta_0} = -\frac{y/u_e}{y/f_0} = -\frac{f_0}{u_e}$
- In general case, final image is formed at ∞ and hence $\gamma = -\frac{f_0}{f_e}$. In this case, eye is the least strained and magnification power is minimum with length of tube, $L = f_0 + f_e$.
- In normal adjustment (i.e., final image is at ∞), to have large magnifying power, f_0 must be as large as practically possible while has to be kept small.
- Resolving power of a telescope depends on the aperture of objective and wavelength of light.

$$RP \propto \frac{\text{Aperture of objectve}}{\text{Wavelength}}$$

Terrestrial Telescope

While an inverted image is not a disadvantage if the instrument is to be used for astronomical observation, it is desirable that a terrestrial telescope shall form an erect image. This can be accomplished by using an erecting lens in the astronomical telescope (Fig. 1.288).

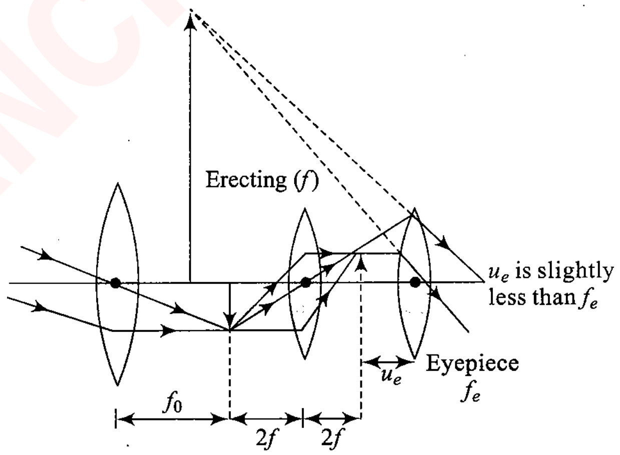


Fig. 1.288

Magnifying power for terrestrial telescope is,

$$\gamma_e = \frac{f_0}{f_e} \qquad \text{[for relaxed eye]}$$

Length of tube, $L = f_0 + 4f + f_e$

Reflecting Telescope

It is an astronomical telescope, in which objective lens is replaced by a concave mirror.

The reflecting telescope is cheap, light, and portable as compared to astronomical telescope.

Lens Camera

In lens camera, the image of the object formed by a converging lens is allowed to fall on a screen (Fig. 1.289). In camera, the aperture of lens and its separation from film/screen can be adjusted. Generally, the object is placed between F and $2F$ and hence the image forms between F an $2F$ which is real (hence can be taken on film), inverted, and diminished.

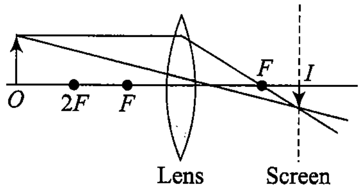


Fig. 1.289

During photographing of the object, first image is focused on the film by adjusting the separation between film and lens. After this, the film is exposed to light through a shutter for a given time. How much energy is incident on the film also depends on the aperture of the lens.

For proper exposure of a particular film, a definite amount of energy is incident on the film. Let the film has been exposed for time t and the intensity of light is I, then $I \times A \times t$ = constant, where A is light transmitting area of the lens.

$$A \propto d^2, \quad \text{where} \quad d \text{ is the aperture of lens.}$$

So, $\quad Id^2t$ = constant

The ratio of focal length to aperture of lens is called f-number of the camera.

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture}}$$

For a given camera, f-number gives us the idea about aperture.

Solved Examples

Example 1.1 **A telescope has an objective of focal length 50 cm and eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 200 cm away from the objective. Calculate:**
(i) the separation between the objective and eyepiece.
(ii) the magnification produced. **(IIT-JEE, 1980)**

Sol. Given $u = -200$ cm, $f = 50$ cm

For image I_1 of object formed by objective lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

We have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{50} + \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$$

$$\Rightarrow \quad v = +\frac{200}{3} \text{ cm}$$

Also, magnification produced by objective lens

$$m_0 = \frac{v}{u} = -\frac{200/3}{200} = \frac{1}{3}$$

The image I_1 acts as an object for eye lens.

Here, $\quad v = -25$ cm, $f = 5$ cm

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{5} = -\frac{1+5}{25}$$

$$\therefore \quad u = -\frac{25}{6} \text{ cm}$$

And magnification produced by eye lens,

$$m_e = \frac{v}{u} = \frac{-25}{(-25/6)} = 6$$

(i) The separation between objective and eyepiece

$$= |V| + |u| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.73 \text{ cm}$$

(ii) Magnification produced, $m = m_0 \times m_e = -\frac{1}{3} \times 6 = -2$

Negative sign shows that the final image is inverted.

Example 1.2 **A plano-convex lens has thickness 4 cm. When placed on a horizontal table with the curved surface in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the center of the plane face of the lens is found to be 25/8 cm. Find the focal length of the lens.**

(IIT-JEE, 1984)

Sol. When the curved surface of the lens (refractive index μ) is in contact with the table, the image of bottom-most point of lens (in glass) is formed due to refraction at plane face. The image of O appears at I_1.

Here, $\quad u_1 = AO = -4$ cm, $v_1 = AI_1 = 3$ cm,
$\mu_1 = \mu$, and $\mu_2 = 1$, and $R_1 = \infty$

$$\therefore \quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \text{ gives } \frac{1}{-3} - \frac{\mu}{-4} = \frac{1-\mu}{\infty} \qquad \text{(i)}$$

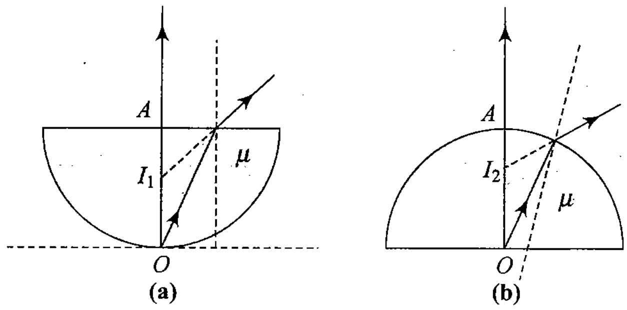


Fig. 1.290

When the plane surface of the lens is in contact with the table, the image of center of the plane face is formed due to refraction at curved surface. The image of O is formed at I_2.

Here, $\quad u = AO = -4$ cm, $v = AI_2 = -25/8$ cm

$\mu_1 = \mu$, $\mu_2 = 1$, and $R_2 = -R$

$$\therefore \quad \frac{\mu_2}{v_2} - \frac{\mu_1}{\mu_2} = \frac{\mu_2 - \mu_1}{R_2}$$

$$\text{Gives} \quad \frac{1}{\left(-\frac{25}{8}\right)} - \frac{\mu}{-4} = \frac{1-\mu}{-R}$$

From (i), $\mu = 4/3$, therefore this equation gives

$$-\frac{8}{25}+\frac{4/3}{4}=-\frac{\left(1-\frac{4}{3}\right)}{R}-\frac{8}{25}+\frac{1}{3}=\frac{1}{3R} \text{ or } \frac{1}{75}=\frac{1}{3R}$$

This gives $R = 25$ cm.

The focal length (f) of plano-convex lens ($R_1 = R$ and $R_2 = \infty$)

is $\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{\mu-1}{R}=\frac{\frac{4}{3}-1}{25}=\frac{1}{75} \Rightarrow f = 75$ cm

Example 1.3 **(a) A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole. What is the size of image? (b)If the object begins to move with speed V_0, what will be the speed of its image? (IIT-JEE, 1987)**

Sol. (a) According to mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{(i)}$$

For a given mirror, focal length f = constant. Therefore, on differentiating, Eq. (i) gives

$$-\frac{1}{v^2}dv-\frac{1}{u^2}du=0 \Rightarrow dv=-\left(\frac{v}{u}\right)^2 du \quad \text{(ii)}$$

Multiplying (i) throughout by u, we get

$$\frac{u}{v}+1=\frac{u}{f} \Rightarrow \frac{u}{v}=\frac{u}{f}-1=\frac{u-f}{f}$$

$$\therefore \quad \frac{v}{u}=\frac{f}{u-f} \quad \text{(iii)}$$

Differential length of the object on the axis, $du = b$ (given).

Therefore, Eq. (ii) gives $dv=-\left(\frac{f}{u-f}\right)^2 b$.

Negative sign shows that the image is longitudinally inverted.

$$\therefore \text{ Size of image } |dv| = b\left(\frac{f}{u-f}\right)^2.$$

(b) From (ii) and (iii), $dv=-\left(\frac{f}{u-f}\right)^2 du$

Dividing both sides by differential time dt, we get

$$\frac{dv}{dt}=-\left(\frac{f}{u-f}\right)^2\frac{du}{dt}$$

Given $\frac{du}{dt}=v_0$ and speed of image $\frac{dv}{dt}=v_1$(say)

$$\Rightarrow v_i=-\left(\frac{f}{u-f}\right)^2 v_0$$

Example 1.4 **A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial,**

(i) Find the position of image due to refraction at first surface and position of the final image.

(ii) Draw a ray diagram showing the position of both images. (IIT-JEE, 1988)

Sol. (i) For refraction at spherical surface,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad \text{(i)}$$

For refraction at first surface,

$\mu_2 = 1$, $\mu_1 = 4/3$, $u = \infty$, $R_1 = +2$ mm, $v = v'$ (say)

Position of image due to refraction at first surface is given by

$$\frac{1}{v'}-\frac{4/3}{\infty}=\frac{1-(4/3)}{2}$$

This given $v' = -6$ mm

That is the image is formed at a distance of 6 mm to the left of first surface.

For refraction at second surface,

$u' = u = -(6+4) = -10$ mm, $\mu_1 = 1$, $\mu_2 = 4/3$

$R_2 = -2$ mm

Substituting these values in (i), we get

$$\frac{(4/3)}{v}-\frac{1}{(-10)}=\frac{\frac{4}{3}-1}{(-2)} \Rightarrow \frac{(4/3)}{v}-\frac{1}{6}-\frac{1}{10}=\frac{-10-6}{60}$$

$$\Rightarrow \quad v=-\frac{60\times\left(\frac{4}{3}\right)}{16}=-5 \text{ mm}$$

The final image I is at a distance of 5 mm to the left of second surface.

(ii) The ray diagram is shown in Fig. 1.291

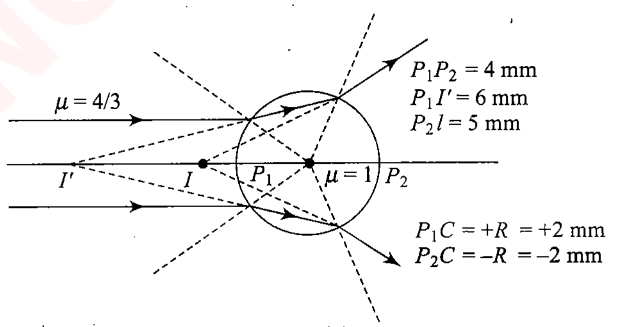


Fig. 1.291

I' is the virtual image formed by first surface and the final image is virtual and is formed at I.

Example 1.5 **Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n, so that the light entering the rod does not emerge from the curved of surface of the rod irrespective of the value of α. (IIT-JEE, 1992)**

Sol. Let α be the angle of incidence on a plane face of the cylindrical rod. If r is the angle of refraction, then according to Snell's law

$$n=\frac{\sin\alpha}{\sin r} \text{ or } \sin r=\frac{\sin\alpha}{n} \quad \text{(i)}$$

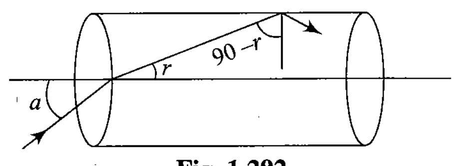


Fig. 1.292

The angle of incidence at curved surface is $(90° - r)$. The ray is now passing from denser to rarer medium, therefore for no $(90° - r) \geq C$, where C is critical angle,

$$\sin(90° - r) \geq \sin C$$
$$\cos r \geq \sin C$$
$$\sqrt{(1-\sin^2 r)} \geq \sin C$$
$$1 - \frac{\sin^2\alpha}{n^2} \geq \sin^2 C \qquad \text{[using (i)]}$$
$$A \sin C = \frac{1}{n} \Rightarrow 1 - \frac{\sin^2\alpha}{n^2} \geq \frac{1}{n^2}$$

This gives

$$\left(\frac{1}{n^2} + \frac{\sin^2\alpha}{n^2}\right) \leq 1$$
$$\frac{1}{n^2}[1+\sin^2\alpha] \leq 1 \quad \text{or} \quad n^2 \geq (1+\sin^2\alpha).$$

Maximum value of $\sin^2\alpha = 1$

$$\therefore \quad n^2 \geq 2 \quad \text{or} \quad n \geq \sqrt{2}$$

Hence, $n_{\min} = \sqrt{2}$

Example 1.6 **An image Y is formed of a point object x by a lens whose optic axis is AB as shown in fig. Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (having the same optic axis AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. (IIT-JEE, 1994)**

Fig. 1.293

Sol. a. When a convex lens is used to form image of X at Y:

1. A ray starting from X parallel to the optic axis passes through the second focus.

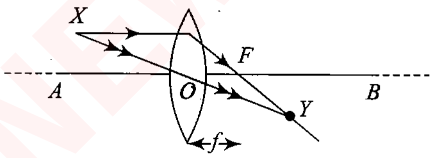


Fig. 1.294

2. A rays starting from X directed toward the optic axis passes undeviated.
   Both the rays meet at Y, which is the real image of X.

b. When a concave mirror is used to form image of X and Y:

1. A ray starting from X parallel to the optic axis passes through principal focus F.

Concave mirror

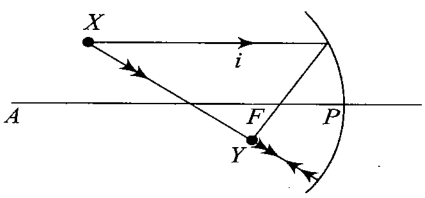


Fig. 1.295

2. A ray starting from X, directed toward the center of curvature C, falls normally on the mirror and retraces its path.
   Both the rays meet at Y, which is the real image of X.

Example 1.7 **A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction $n(y)$ given by.**

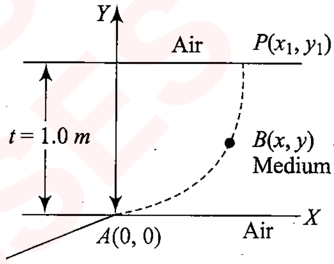


Fig. 1.296

$$n(y) = [ky^{3/2} + 1]^{1/2}$$

where $\quad k = 1.0\ (\text{metre})^{-3/2}$

The refractive index of air is 1.0.

a. Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incident angle at the point.

b. Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

c. Determine the coordinates (x_1, y_1) of the point P, where the ray intersects the upper surface of the slab–air boundary.

d. Indicate the path of the ray subsequently.

(IIT-JEE, 1995)

Sol. a. If i is the angle of incidence at $B(x, y)$, then slope of trajectory at B,

$$dy/dx = \tan\theta = \tan(90° - i) = \cot i \qquad \text{(i)}$$

b. From Snell's law, $n \sin i$ = constant C.

From Snell's law at $A(0, 0)$

$$n \sin i = 1 \times \sin 90° = 1 \quad \Rightarrow \quad n \sin i = 1$$
$$\sin i = 1/n \quad \text{or} \quad i = \sin^{-1}(1/n)$$

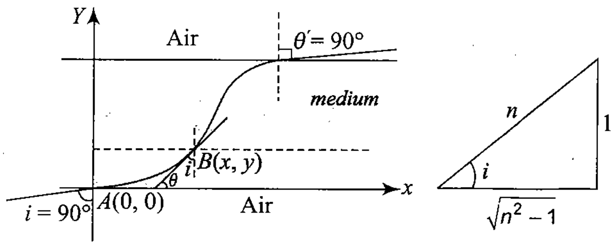


Fig. 1.297

$$\therefore \quad \cot i = \frac{\cos i}{\sin i} = \frac{\sqrt{(1-1/n)^2}}{(1/n)} = \sqrt{n^2 - 1}$$

From (i),

or $n^2 \sin^2 i$ or $n^2 \dfrac{1}{1+\cot^2 i} = 1$

or $\dfrac{n^2}{1+(dy/dx)^2} = 1$

Given $n = [ky^{3/2} + 1]^{1/2} \Rightarrow n^2 = ky^{3/2} + 1$

$\therefore \dfrac{ky^{3/2}+1}{1+(dy/dx)^2} = 1 \Rightarrow ky^{3/2} + 1 = 1 + \left(\dfrac{dy}{dx}\right)^2$

or $(dy/dx)^2 = ky^{3/2} \Rightarrow dy/dx - k^{1/2}y^{3/4}$

or $dy/y^{3/4} = k^{1/2}\, dx.$

Integrating, we get $4y^{1/4} = k^{1/2}\, x + C$

where c is constant of integration.

At $x = 0, y = 0 \Rightarrow C = 0.$

$\therefore 4y^{1/4} = k^{1/2}x$

As $k = 1.0$ (given)

$\therefore y^{1/4} = (1/4)x$

This is the required equation of trajectory.

c. At $y = 1.0$ m, Eq. (ii) gives $x = 4$ m.

$\therefore B(x_1, y_1) = P(4, 1)$

d. The path of the ray subsequently will be the grazing angle of emergence since

$n \sin e = 1$ or $1 \sin e = 1 \Rightarrow e = 90.$

Example 1.8 **A right angled prism (45°, 90°, 45°) of refractive index n has a plate of refractive index ($n_1 < n$) cemented to its diagonal face. The assembly is in air. A ray is incident on AB.** **(IIT-JEE, 1996)**

(i) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

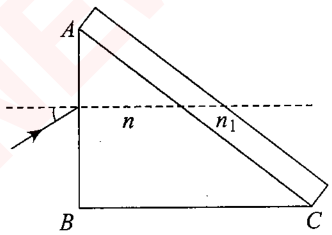


Fig. 1.298

(ii) Assuming n = 1.352, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.

Sol. (i) $\sin C = \dfrac{n_1}{n}$

From Fig. 1.298, $(90° - r_1) + 45° + (90° - C) = 180°$
$\Rightarrow r_1 = 45° - C$

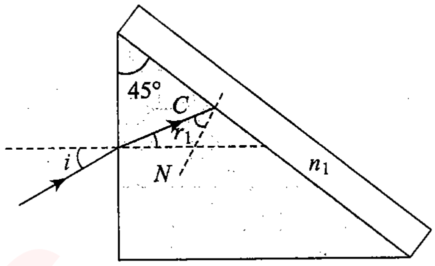


Fig. 1.299

From Snell's law,

$$\frac{\sin i}{\sin r_1} = n$$

$\therefore \sin i = n \sin r_1 = n \sin(45° - C) = n(\sin 45° \cos C - \cos 45° \sin C)$

$$= \frac{n}{\sqrt{2}}(\cos C - \sin C) = \frac{n}{\sqrt{2}}\left[\sqrt{1-\sin^2 C} - \sin C\right]$$

$$= \left[\sqrt{1-\left(\frac{n_1}{n}\right)^2} - \frac{n_1}{n}\right]$$

$$\frac{1}{\sqrt{2}}\left[\sqrt{(n^2 - n_1^2)} - n_1\right] \Rightarrow i = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2}\right) - n_1\right\}$$

(ii) When refracted ray passes through the diagonal face undeflected, the incidence at diagonal face is perpendicular.

$r_2 = 0$ so $r_1 + r_2 = 45° \Rightarrow r_1 = 45°$

Again, $\dfrac{\sin i}{\sin r_1} = n \Rightarrow \sin i = n \sin r_1 = 1.352 \sin 45°$

or $\sin i = 1.3520020 \dfrac{1}{\sqrt{2}} = 0.956$

or $i = \sin^{-1}(0.956) = 72°58'.$

Example 1.9 **A thin plano-convex lens of focal length f is split into two halves, one of the halves is shifted along the optical axis. The separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation.** **(IIT-JEE, 1996)**

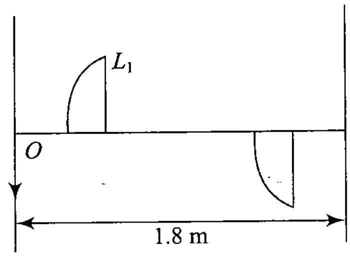


Fig. 1.300

Sol. Let magnification caused by first lens be 2 and distance $OL_1 = x$. The distance v of image from first lens L_1 is given by

$$m = \frac{v}{u} = 2 \quad \Rightarrow \quad v = 2u = 2x.$$

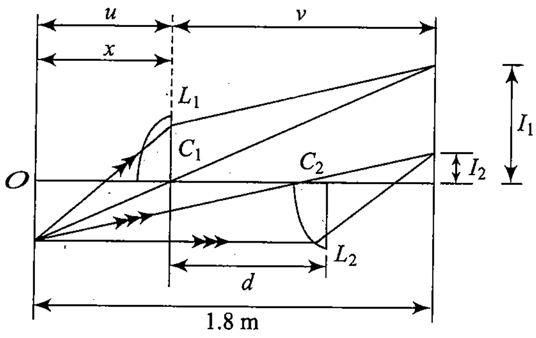


Fig. 1.301

Clearly, $\quad u + v = 1.8 \text{ m} \Rightarrow x + 2x = 1.8 \text{ m}$

or $\quad 3x = 1.8 \text{ m} \Rightarrow x = \frac{1.8}{3} = 0.6 \text{ m}$

By sign convention,

$$u = -x = -0.6 \text{ m}, \; v = 2x = 1.2 \text{ m}$$

Lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ gives

$$\frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.6} = \frac{1+2}{1.2}$$

$\therefore$ Focal length $\quad f = \frac{1.2}{3} = 0.4 \text{ m}$

For real image, lens formula takes the form

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Clearly, u and v are interchangeable. Therefore, for lens L_2

$$u' = v = 1.2 \text{ m} \quad \text{and} \quad v' = 0.6 \text{ m}$$

$$OL_1 = L_2I_2 = x$$

If d is the separation between the lenses, then $x + d + x = 1.8$ m

$\therefore \quad d = 1.8 - 2x = 1.8 - 2 \times 0.6 = 0.6$ m

Example 1.10 **A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in vertical direction by 0.6 cm as shown in Fig. 1.302. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optics axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also, locate positions of A' and B' with respect to the optic axis RS.**

(IIT-JEE, 2000)

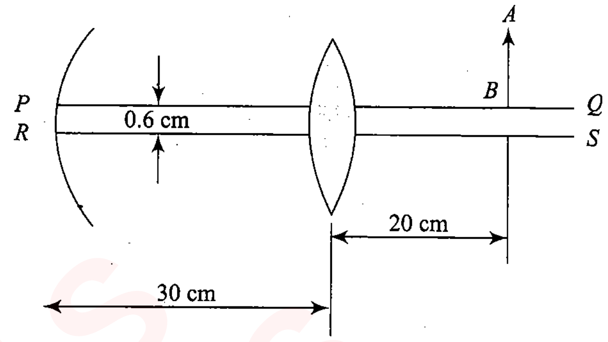


Fig. 1.302

Sol. For convex lens using sign convection of coordinate geometry,

$$u = +20 \text{ cm}, f = -15 \text{ cm}$$

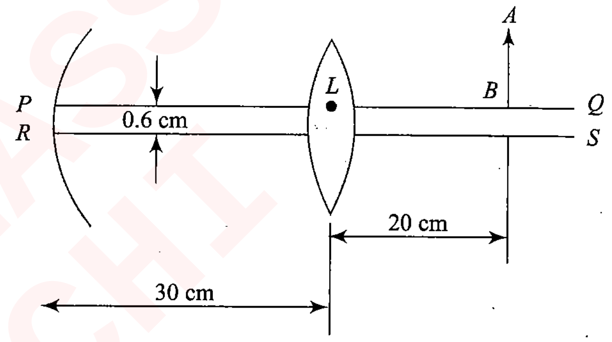


Fig. 1.303

So, $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad -\frac{1}{15} = \frac{1}{v_1} - \frac{1}{20}$

$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{20} - \frac{1}{15} = \frac{3-4}{60} \quad \Rightarrow \quad v_1 = -60$ cm

That is image is formed at a distance 60 cm to the left of lens L.

Magnification, $m_1 = \frac{v_1}{u_1} = -\frac{60}{20} = -3$

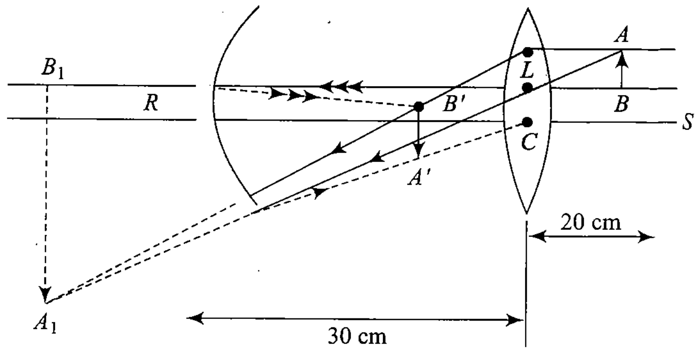


Fig. 1.304

This image is real and inverted. It is intercepted by the mirror.

For concave mirror, $u_2 = -60 + 30 = -30$ cm, $f_2 = +30$ cm

So, $\quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ gives

$$\frac{1}{30} = \frac{1}{v_2} - \frac{1}{30} \Rightarrow \frac{1}{v_2} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30} \Rightarrow v_2 = 15 \text{ cm}$$

Magnification,

$$m_2 = -\frac{v_2}{u_2} = -\frac{15}{-30} = +\frac{1}{2}$$

$\therefore$ Net magnification,

$$m = m_1 \times m_2 = (-3) \times \left(\frac{1}{2}\right) = -1.5$$

Size of image $A'B' = -1.5 \times 1.2$ cm $= -1.8$ cm.

Magnification of mirror is half and image of B formed by convex lens is 0.6 cm above RS, so length of image will be 1.5 cm below RS.

Thus, B' will be 0.3 cm above RS and A' will be 1.5 cm below RS.

Example 1.11 **A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in Fig. 1.305. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On representing with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.** **(IIT-JEE, 2001)**

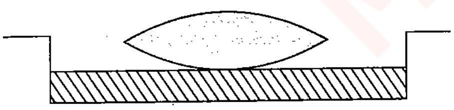


Fig. 1.305

Sol. Let f_1 be the focal length of convex lens; radius of curvature of each curved face is R.

$$\frac{1}{f_1} = (m-1)\left\{\frac{1}{R} - \left(\frac{1}{-R}\right)\right\} = (\mu - 1)\frac{2}{R}$$

$$\Rightarrow \quad f_1 = \frac{R}{2(\mu-1)} = \frac{R}{2\left(\frac{3}{2}-1\right)} = R$$

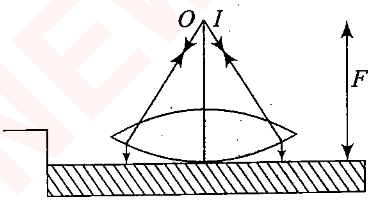


Fig. 1.306

When the space between the lens and mirror is filled by water of refractive index $\mu_1 = 4/3$, then focal length of liquid concave lens f_2 is

$$\frac{1}{f_2} = (\mu_1 - 1)\left(-\frac{1}{R} - \infty\right)$$

$$\Rightarrow \quad f_2 = \frac{-R}{\mu_1 - 1} = -\frac{R}{\left(\frac{4}{3}-1\right)} = -3R$$

The combined focal length of lenses is $F_1 = 15$ cm

$$\therefore \quad \frac{1}{F_1} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad \frac{1}{15} = \frac{1}{R} - \frac{1}{3R} = \frac{3-1}{3R}$$

$$\Rightarrow \quad 3R = 30 \quad \Rightarrow \quad R = 10 \text{ cm}$$

In second case,

$F_2 = 25$ cm. Let $\mu_1 = \mu_2$.

$$\therefore \quad \frac{1}{F_2} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad \frac{1}{25} = \frac{1}{10} + \frac{1}{f_2'}$$

$$\Rightarrow \quad \frac{1}{f_2'} = \frac{1}{25} - \frac{1}{10} = \frac{2-5}{50}$$

$$\therefore \quad f_2' = \frac{-50}{3} \text{ cm}$$

$$f_2' = \frac{R}{\mu_2 - 1} \Rightarrow \mu_2 - 1 = -\frac{R}{f_2'} = \frac{-10}{(-50/3)} = \frac{3}{5} = 0.6$$

$$\Rightarrow \quad \mu_2 = 1 + 0.6 = 1.6$$

Example 1.12 **The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49, respectively and those of the flint glass are 1.77 and 1.73, respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system.** **(IIT-JEE, 2001)**

Sol. Deviation produced by a prism $= (\mu_y - 1)\ \alpha$.

For no deviation caused by combination of two prisms,

$$(\mu_y - 1)\alpha + (\mu_y' - 1)\ \alpha' = 0 \quad \Rightarrow \quad \alpha = -\frac{\mu_y - 1}{\mu_y' - 1}\alpha$$

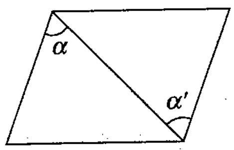


Fig. 1.307

Here $\quad \alpha = 60°$

$$\mu_y' = \frac{\mu_b' + \mu_r'}{2} = \frac{1.77 + 1.73}{2} = 1.75$$

Refracting angle of flint glass prism,

$$\alpha' = \frac{\mu_y - 1}{\mu_y' - 1}\alpha = -\left[\frac{(1.50-1)}{(1.75-1)}\right] \times 6°$$

$$= -\frac{0.50}{0.75} \times 6° = -4°$$

Negative sign shows the bases of prisms are opposite.

Therefore, refracting angle of flint glass prism = 4°

Net dispersion produced

$$\Delta\theta = (\delta_b - \delta_r) + (\delta_b' - \delta_r')$$

$$= (\mu_b - \mu_r)\alpha + (\mu_b' - \mu_r')\alpha'$$

$$= (1.51 - 1.49) \times 6° - (1.77 - 1.73) \times 4°$$

$$= 0.12° - 0.16° = -0.04° = 0.04° \text{ (numerically)}$$

Example 1.13 **Figure 1.308 shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown. After refraction, it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal. (IIT-JEE, 2004)**

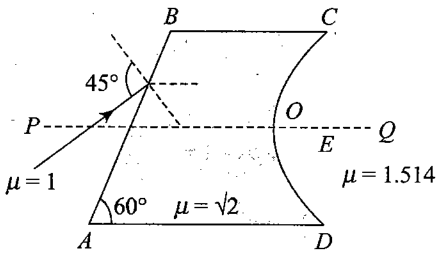


Fig. 1.308

Sol. From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$, we have

$$\sin r = \frac{\mu_1}{\mu_2} \sin i = \frac{1}{\sqrt{2}} \sin 45° = \frac{1}{2} \Rightarrow r = 30°.$$

This means that the ray becomes parallel to side AD inside the slab.

This implies that for second face CD, $u = \infty$

Given $R = 0.4$ m

Now, we have

$$\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R} \Rightarrow \frac{1.514}{v} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{R}$$

$$\Rightarrow \quad v = \frac{1.514 \times 0.4}{1.514 - 1.414} = \frac{1.514 \times 0.4}{0.1}$$

$= 6.056$ m $= 6.06$ m (up to two decimal places).

Example 1.14 **An object is approaching a convex lens of focal length 0.3 m with a speed of 0.01 ms^{-1}. Find the magnitudes of the ratio of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. (IIT-JEE, 2004)**

Sol. Given $f = 0.3$ m, $u = -0.4$ m

From lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \text{(i)}$$

We have,

$$\frac{1}{0.3} = \frac{1}{v} + \frac{1}{0.4} \Rightarrow \frac{1}{v} = \frac{1}{0.3} - \frac{1}{0.4} \text{ or } v = 1.2 \text{ m}$$

Differentiating (i) with respect to time t, we get

$$0 = -\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} \Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} \qquad \text{(ii)}$$

Given $\frac{du}{dt} = 0.01$ ms^{-1}

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1.2}{0.4}\right)^2 \times 0.01 = 0.09 \text{ ms}^{-1}.$$

Therefore, rate of change of position of image = 0.09 ms^{-1}.

Magnification, $m = \frac{v}{u}$

Differentiating with respect to time t,

$$\frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2} = \frac{(-0.4) \times 0.09 - 1.2 \times (0.01)}{(-0.4)^2}$$

$$= \frac{-0.036 - 0.012}{0.16} = -\frac{0.048}{0.016} = -0.3 \text{ per second}$$

Example 1.15 **AB and CD are surfaces of two slabs as shown in Fig. 1.309. The medium between the slabs has refractive index 2. Refractive index of the slab above AB is $\sqrt{2}$ and below CD is $\sqrt{3}$. Find the minimum angle of incidence at Q, so that the ray is totally reflected by both the slabs. (IIT-JEE, 2005)**

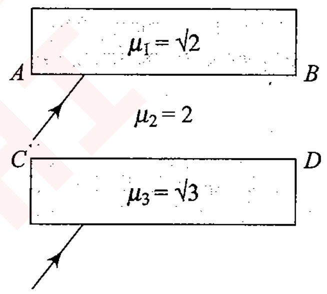


Fig. 1.309

Sol. Let θ be the angle of incidence at face AB, then for total internal reflection at face AB

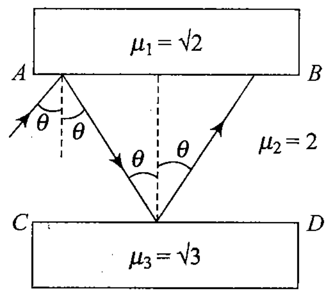


Fig. 1.310

$$\sin \theta > C_1 = \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \sin \theta > \frac{1}{\sqrt{2}} \text{ or } \sin \theta > \sin 45° \Rightarrow \theta > 45°$$

For total internal reflection at face CD, $\sin \theta > \sin C_2$

$$= \frac{\mu_3}{\mu_2} = \frac{\sqrt{3}}{2} \Rightarrow \sin \theta > \frac{\sqrt{3}}{2} \text{ or } \sin \theta > \sin 60° \text{ or } \theta > 60°$$

Therefore, for total internal reflection at both the surface, $\theta_{min} = 60°$.

Example 1.16 **A ray of light strikes a plane mirror at an angle of incidence 45° as shown in Fig. 1.311. After reflection,**

the ray passes through a prism of refractive index 1.5 whose apex angle is 4°. Through what angle must the mirror be rotated if the total deviation of the ray be 90° ?

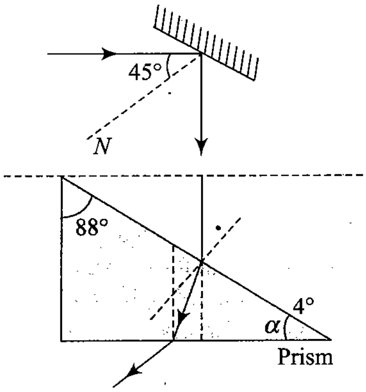


Fig. 1.311

Sol. The deviation produced by small angled prism,

$$\delta_1 = (\mu - 1)\alpha = (1.5 - 1)4° = 2° \text{ (always)}$$

Deviation caused by mirror,

$$\delta_2 = 180° - 2i = 180° - 2 \times 45° = 90°$$

Net deviation produced by system $= \delta_1 + \delta_2 = 2° + 90° = 92°$

This is more than 90°.

Greater is angle of incidence on the mirror, smaller is the deviation.

If β is the angle of rotation of mirror in clockwise direction to increase angle of incidence, then deviation produced by the mirror will be $180° - 2(45° + \beta) = 90° - 2\beta$

Therefore, total deviation produced $= 90° - 2\beta + 2° = 92° - 2\beta$

Given, $92° - 2\beta = 90° \Rightarrow \beta = 1°$

EXERCISES

Subjective Type *Solutions on page 1.130*

1. A ray of light is falling on a glass sphere of $\mu = \sqrt{3}$ such that the incident ray and the emergent ray, when produced, intersect at a point on the surface of the sphere. Find the value of angle of incidence in degrees.
2. There is a spherical glass shell of refractive index 1.5, inner radius 10 cm and outer radius 20 cm. Inside the spherical cavity, there is air. A point object is placed at a point O at a distance of 30 cm from the outer spherical surface. Find the final position of the image as seen by eye.

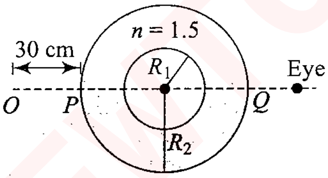


Fig. 1.312

3. A direct-vision prism is made out of three prisms, each with a refracting angle of $\phi = 60°$, attached to each other as shown in Fig. 1.313. Light of a certain wavelength is incident on the first prism. The angle of incidence is 30° and the ray leaves the third prism parallel to the direction of incidence. The refractive index of the glass of the first and third prisms is 1.5. Find the refractive index of the material of the middle prism. ($\sqrt{6} = 2.45$)

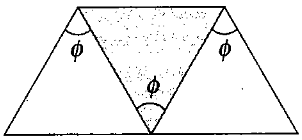


Fig. 1.313

4. A person wants to see AB part of his image (see Fig. 1.314). His eye level is at 1.8 m above ground. If he uses minimum size of mirror required for this, find the height of lowest point of mirror above the ground.

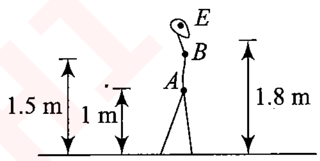


Fig. 1.314

5. The internal surface of the walls of a sphere is specular (i.e., reflecting). The radius of the sphere is R = 36 cm. A point source S is placed at a distance $R/2$ from the center of the sphere and sends light to the remote part of the sphere. Where will the image of the source be after two successive reflections from the remote and then the nearest wall of the sphere? How will the position of the image change if the source sends light to the nearest wall first? Consider paraxial rays.
6. As shown in Fig. 1.315, an object O is at the position (–10, 2) with respect to the origin P. The concave mirror M_1 has radius of curvature 30 cm. A plane mirror M_2 is kept at a distance of 40 cm in front of the concave mirror. Considering first reflection on the concave mirror M_1 and second on the plane mirror M_2. Find the coordinates of the second image w.r.t. the origin P.

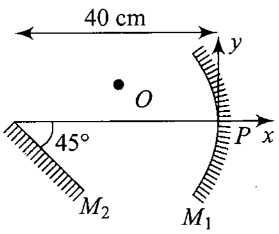


Fig. 1.315

7. In Fig. 1.316, shown AB is the principal axis of the concave mirror. A point object moves on the line PQ which makes small angle 'θ' with the principal axis. Show that image also moves in straight line making same angle θ with principle axis.

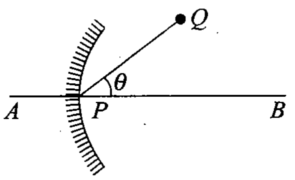


Fig. 1.316

8. A ray of light travelling in air is incident at angle of incidence 30° on one surface of a slab in which refractive index varies with y. The light travels along the curve $y = 4x^2$ (y and x are in meter) in the slab. Find out the refractive index of the slab at $y = 1/2$ m in the slab.

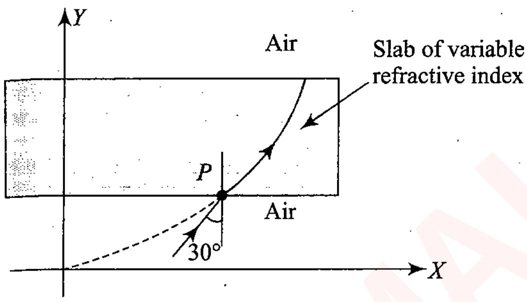


Fig. 1.317

9. If an observer sees the bottom of the vessel shown in Fig. 1.318 at 8 cm, find the refractive index of the medium in which the observer is present.

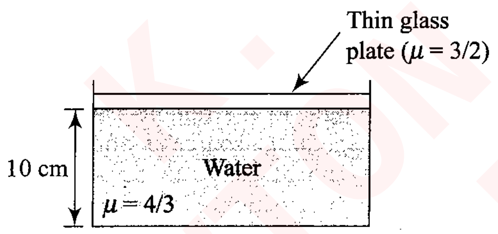


Fig. 1.318

10. O is a point object kept on the principal axis of a concave mirror M of radius of curvature 20 cm. P is a prism of angle 1.8°. Light falling on the prism (at small angle of incidence) gets refracted through the prism and then falls on the mirror. Refractive index of the prism is 3/2. Find the distance between the images formed by the concave mirror due to this light.

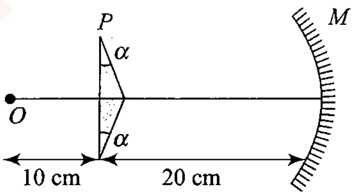


Fig. 1.319

11. In Fig. 1.320, L is a converging lens of focal length 10 cm and M is a concave mirror of radius of curvature 20 cm. A point object O is placed in front of the lens at a distance of 15 cm. AB and CD are optical axes of the lens and mirror, respectively. Find the distance of the final image formed by this system from the optical center of the lens. The distance between CD and AB is 1 cm.

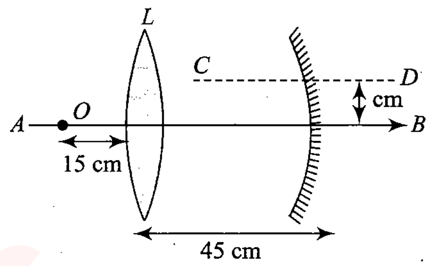


Fig. 1.320

12. A stationary observer O looking at a fish (in water of $\mu = 4/3$) through a converging lens of focal length 90.0 cm. The lens is allowed to fall freely from a height of 62.0 cm with its axis vertical. The fish and the observer are on the principal axis of the lens. The fish moves up with constant velocity 100 cms^{-1}. Initially, it was at a depth of 44.0 cm. Find the velocity with which the fish appears to move to the observer at $t = 0.2$ sec.

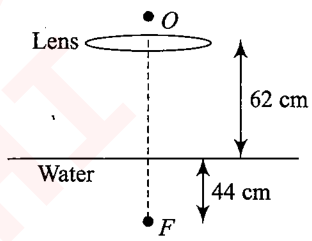


Fig. 1.321

13. The back wall of an aquarium is a mirror that is 30 cm away from the front wall. The sides of the tank are negligibly thin. A fish is swimming midway between the front and back walls.

a. An image of the fish appears behind the mirror. How far does this image appear to be from the front wall of the aquarium?

b. Would the refractive index of the liquid have to be larger or smaller in order for the image of the fish to appear in front of the mirror, rather than behind it? Find the limiting value of this refractive index. Initially, water is there in aquarium having refractive index 4/3.

14. An object is placed 20 cm to the left of a converging lens having focal length 16 cm. A second, identical lens is placed to the right of the first lens, such that the image formed by the combination is of the same size and orientation as the object is. Find the separation between the lenses.

15. A ray of light is incident on the surface of a transparent sphere of refractive index $\mu = \sqrt{7}/2$. After refraction, TIR takes place inside the sphere and light emerges as shown in Fig. 1.322. Find the angle of incidence for which the deviation (total) produced is minimum.

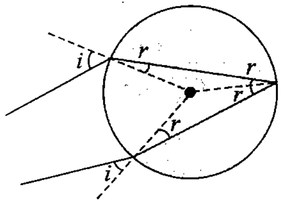


Fig. 1.322

16. A composite slab consisting of different media is placed in front of a concave mirror of radius of curvature 150 cm as shown in Fig. 1.323. The whole arrangement is immersed in water. Locate the final image of point object O.

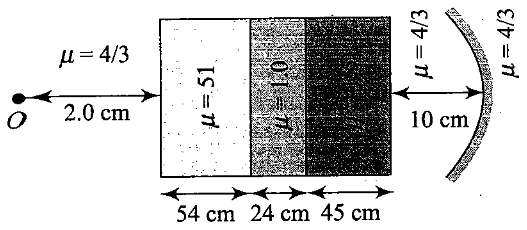


Fig. 1.323

17. A parallel beam of light falls on the surface of a convex lens whose radius of curvature of both sides is 20 cm. The refractive index of the material of the lens varies as $\mu = 1.5 + 0.5r$, where r is the distance of the point on the aperture from the optical centre in cm. Find the length of the region on the axis of the lens where the light will appear. The radius of aperture of the lens is 1 cm.

18. A thin biconvex lens of refractive index 3/2 and radius of curvature 50 cm is placed on a reflecting convex surface of radius of curvature 100 cm. A point object is placed on the principal axis of the system such that its final image coincides with itself. Now, few drops of a transparent liquid is placed between the mirror and lens such that final image of the object is at infinity. Find refractive index of the liquid used. Also, find the position of the object.

19. A convex lens of focal length f_1 is placed in front of a luminous point object. The separation between the object and the lens is $3f_1$. A glass slab of thikness t is placed between the object and the lens. A real image of the object is formed at the shortest possible distance from the object.

a. Find the refractive index of the slab.

b. If a concave lens of very large focal length f_2 is placed in contact with the convex lens, find the shifting of the image.

20. A lens is made of three thin different mediums. Radius of curvature and refractive index of each medium is shown in Fig. 1.324. Surface AB is straight. An object is placed at some distance from the lens by which a real image is formed on the screen placed at a distance of 10 cm from the lens. Find

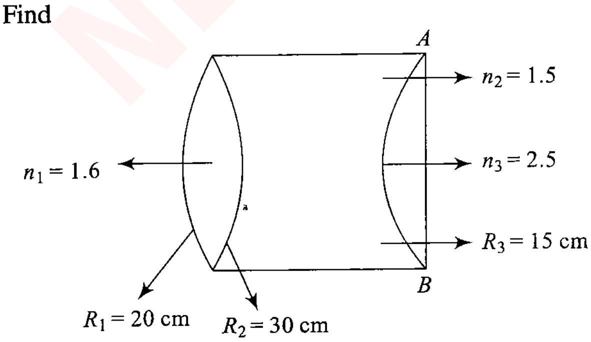


Fig. 1.324

a. The distance of the object from the lens.

b. A slab of thickness 1.5 cm and refractive index 1.5 is introduced between the image and the lens. Find new position of the object so that image is again formed on the screen.

c. Find the position of the image when same slab is placed on the other side of the lens {as in part (b)} with object in the position of part (a).

Objective Type

Solutions on page 1.134

1. A beam of light passes from medium 1 to medium 2 to medium 3 as shown in Fig. 1.325. What may be concluded about the three indices of refraction, n_1, n_2 and n_3?

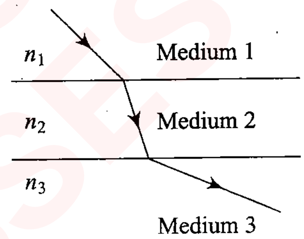
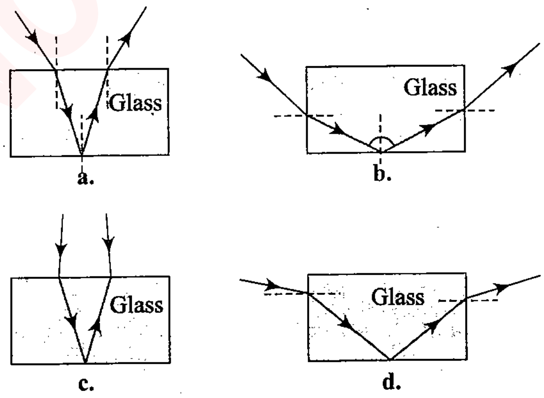


Fig. 1.325

a. $n_3 > n_1 > n_2$
b. $n_1 > n_3 > n_2$
c. $n_2 > n_3 > n_1$
d. $n_2 > n_1 > n_3$

2. A ray of light passes through a rectangular glass block placed in a homogeneous medium. It is refracted and totally reflected. Which diagram shows a possible path of this ray?



3. A ray of light is incident on a medium with angle of incidence i and refracted into a second medium with angle of refraction r. The graph of $\sin(i)$ vs $\sin(r)$ is as shown in Fig. 1.326. Then, the velocity of light in the first medium in n times the velocity of light in the second medium. What should be the value of n?

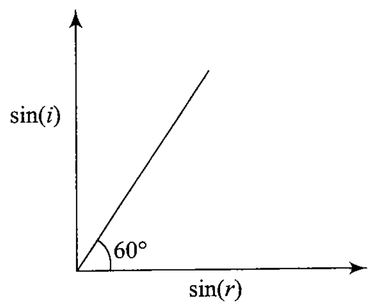


Fig. 1.326

a. $\sqrt{3}$ **b.** $1/\sqrt{3}$

c. $\sqrt{3}/2$ **d.** $2/\sqrt{3}$

4. A beam of light propagates through a medium 1 and falls onto another medium 2, at an angle α_1 as shown in Fig. 1.327. After that, it propagates in medium 2 at an angle α_2 as shown. The light's wavelength in medium 1 is λ_1. What is the wavelength of light in medium 2?

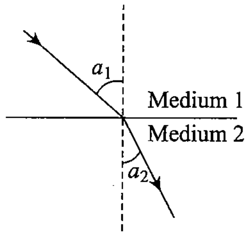


Fig. 1.327

a. $\dfrac{\sin \alpha_1}{\sin \alpha_2}\lambda_1$ **b.** $\dfrac{\sin \alpha_2}{\sin \alpha_1}\lambda_1$

c. $\dfrac{\cos \alpha_1}{\cos \alpha_2}\lambda_1$ **d.** $\dfrac{\cos \alpha_2}{\cos \alpha_1}\lambda_1$

5. You are given two identical plano-convex lenses. When you place an object 20 cm to the left of a single plano-convex lens, the image appears 40 cm to the right of the lens. You then arrange the two plano-convex lenses back to back to form a double convex lens. If the object is 20 cm to the left of this new lens, what is the approximate location of the image?

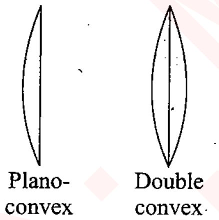


Fig. 1.328

a. 10 cm to the right of the lens
b. 20 cm to the right of the lens
c. 80 cm to the right of the lens
d. 80 cm to the left of the lens

6. Light from a denser medium 1 passes to a rarer medium 2. When the angle of incidence is θ the partially reflected and refracted rays are mutually perpendicular. The critical angle will be

a. $\sin^{-1}(\cot \theta)$ **b.** $\sin^{-1}(\tan \theta)$

c. $\sin^{-1}(\cos \theta)$ **d.** $\sin^{-1}(\sec \theta)$

7. A man is walking under an inclined mirror at a constant velocity V ms^{-1} along the X-axis. If the mirror is inclined at an angle θ with the horizontal, then what is the velocity of the image?

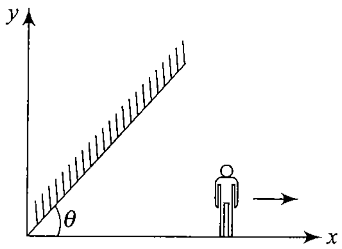


Fig. 1.329

a. $V \sin \theta \hat{i} + V \cos \theta \hat{j}$ **b.** $V \cos \theta \hat{i} + V \sin \theta \hat{j}$

c. $V \sin 2\theta \hat{i} + V \cos 2\theta \hat{j}$ **d.** $V \cos 2\theta \hat{i} + V \sin 2\theta \hat{j}$

8. Figure 1.330 shows the graph of angle of deviation δ versus angle of incidence i for a light ray striking a prism. The prism angle is

a. 30° **b.** 45°

c. 60° **d.** 75°

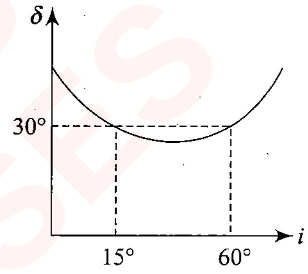


Fig. 1.330

9. Light is incident on a glass block as shown in Fig. 1.331. If θ_1 is increased slightly, what happens to θ_2 ?

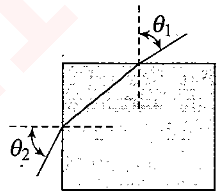


Fig. 1.331

a. θ_2 also increases slightly
b. θ_2 is unchanged
c. θ_2 decreases slightly
d. θ_2 changes abruptly, since the ray experiences total internal reflection

10. A thin equiconvex lens ($\mu = 3/2$) of focal length 10 cm is cut and separated and a material of refractive index 3 is filled between them. What is the focal length of the combination?

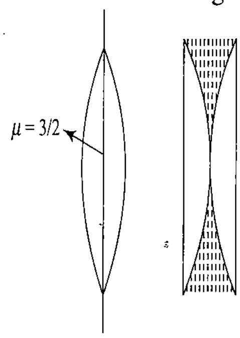


Fig. 1.332

a. –10 cm **b.** –10/4 cm

c. –10/3 cm **d.** None of these

11. Behind a thin converging lens having both the surfaces of the same radius 1 cm, a plane mirror has been placed. The image of an object at a distance of 40 cm from the lens is formed at the same position. What is the refractive index of the lens ?

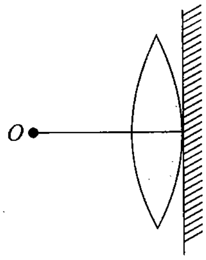


Fig. 1.333

a. 1.5
b. 5/3
c. 9/8
d. None of these

12. What should be the value of distance d so that final image is formed on the object itself. (Focal lengths of the lenses are written on the lenses.)

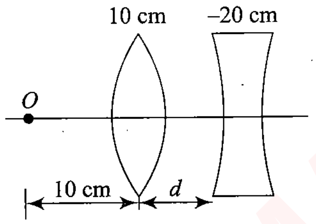


Fig. 1.334

a. 10 cm
b. 20 cm
c. 5 cm
d. None of these

13. A car is fitted with a convex mirror of focal length 20 cm. A second car 2 m broad and 1.6 m high is 6 m away from the first car. The position of the second car as seen in the mirror of the first car is

a. 19. 35 cm
b. 17.45 cm
c. 21.48 cm
d. 5.49 cm

14. In the above question, the breadth and height of the second car seen in the mirror of the first car are, respectively,

a. 5.79 cm and 6.9 cm
b. 6.45 cm and 5.16 cm
c. 2.7 cm and 4.8 cm
d. 0.1 m and 0.3 m

15. In the above question, if the second car is overtaking at a relative speed of 314 ms^{-1}, how fast will the image be moving?

a. $-1\ ms^{-1}$
b. $0.5\ ms^{-1}$
c. $0.3\ ms^{-1}$
d. $-0.032\ ms^{-1}$

16. A ray of light passes from glass, having a refractive index of 1.6, to air. The angle of incidence for which the angle of refraction is twice the angle of incidence is

a. $\sin^{-1}\left(\frac{4}{5}\right)$
b. $\sin^{-1}\left(\frac{3}{5}\right)$
c. $\sin^{-1}\left(\frac{5}{8}\right)$
d. $\sin^{-1}\left(\frac{2}{5}\right)$

17. Consider an equiconvex lens of radius of curvature R and focal length f. If $f > R$, the refractive index μ of the material of the lens

a. is greater than zero but less than 1.5
b. is greater than 1.5 but less than 2.0
c. is greater than 1.0 but less than 1.5
d. none of these

18. A fish is vertically below a flying bird moving vertically down toward water surface. The bird will appear to the fish to be

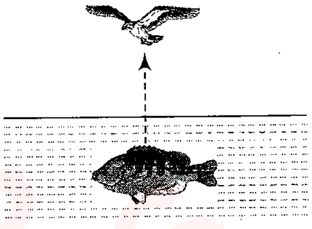


Fig. 1.335

a. moving faster than its speed and also away from the real distance
b. moving faster than its real speed and nearer than its real distance
c. moving slower than its real speed and also nearer than its real distance
d. moving slower than its real speed and away from the real distance

19. What is the angle of incidence for an equilateral prism of refractive index $\sqrt{3}$ so that the ray is parallel to the base inside the prism?

a. 30°
b. 45°
c. 60°
d. Either 30°or 60°

20. A cube of side 2 m is placed in front of a concave mirror of focal length 1 m with its face A at a distance of 3 m and face B at a distance of 5 m form the mirror. The distance between the images of faces A and B and heights of images of A and B are, respectively,

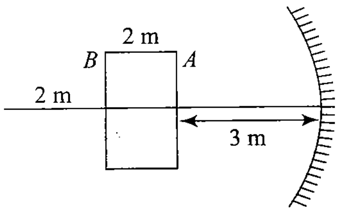


Fig. 1.336

a. 1 m, 0.5 m, 0.25 m
b. 0.5 m, 1 m, 0.25 m
c. 0.5 m, 0.25 m, 1 m
d. 0.25 m, 1 m, 0.5 m

21. A plano-convex lens when silvered on the plane side behaves like a concave mirror of focal length 60 cm. However, when silvered on the convex side, it behaves like a concave mirror of focal length 20 cm. Then, the refractive index of the lens is

a. 3.0
b. 1.5
c. 1.0
d. 2.0

22. Two thin lenses are placed 5 cm apart along the same axis and illuminated with a beam of light parallel to that axis. The first lens in the path of the beam is a converging lens of focal length 10 cm whereas the second is a diverging lens of focal length 5 cm. If the second lens is now moved toward the first, the emergent light

a. remains parallel
b. remains convergent

c. remains divergent
d. changes from parallel to divergent

23. An object is placed 30 cm to the left of a diverging lens whose focal length is of magnitude 20 cm. Which one of the following correctly states the nature and position of the virtual image formed ?

Nature of image	Distance form lens
a. Inverted, enlarged	60 cm to the right
b. Erect, diminished	12 cm to the left
c. Inverted, enlarged	60 cm to the left
d. Erect, diminished	12 cm to the right

24. A lens forms a real image of an object. The distance from the object to the lens is x cm and that from the lens to the image is y cm. The graph (see Fig. 1.337) shows the variation of y with x.

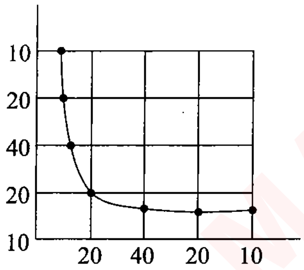


Fig. 1.337

It can be deduced that the lens is
a. converging and of focal length 10 cm
b. converging and of focal length 20 cm
c. converging and of focal length 40 cm
d. diverging and of focal length 20 cm

25. A point source S is placed at the bottom of different layers as shown in Fig. 1.338. The refractive index of the bottom-most layer is μ_0. The refractive index of any other upper layer is

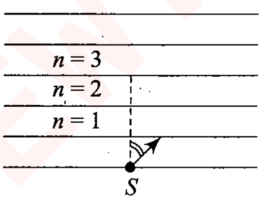


Fig. 1.338

$\mu(n) = \mu_0 - \dfrac{\mu_0}{4n-18}$, where $n = 1, 2, \ldots$

A ray of light starts from the source S as shown. Total internal reflection takes place at the upper surface of a layer having n equal to
a. 3 b. 5
c. 4 d. 6

26. A concave mirror has a focal length of 20 cm. The distance between the two positions of the object for which the image size is double of the object size is
a. 20 cm b. 40 cm
c. 30 cm d. 60 cm

27. A plane mirror is placed at origin parallel to y-axis, facing the positive x-axis. An object starts from (2 m, 0, 0) with a velocity of $(2\hat{i} + 2\hat{j})$ ms^{-1}. The relative velocity of the image with respect to the object is along
a. positive x-axis b. negative x-axis
c. positive y-axis d. negative y-axis

28. A plastic hemisphere has a radius of curvature of 8 cm and an index of refraction of 1.6. On the axis halfway between the plane surface and the spherical one (4 cm from each) is a small object O.

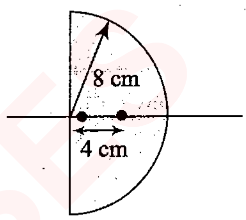


Fig. 1.339

The distance between the two images when viewed along the axis from the two sides of the hemisphere is approximately
a. 1.0 cm b. 1.5 cm
c. 3.75 cm d. 2.5 cm

29. A ray of light falls on a transparent sphere with centre at C as shown in Fig. 1.340.

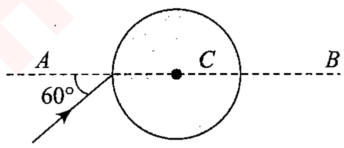


Fig. 1.340

The ray emerges from the sphere parallel to line AB. The refractive index of the sphere is
a. $\sqrt{2}$ b. $\sqrt{3}$
c. 3/2 d. 1/2

30. A circular beam of light of diameter $d = 2$ cm falls on a plane surface of glass. The angle of incidence is 60° and refractive index of glass is $\mu = 3/2$. The diameter of the refracted beam is
a. 4.00 cm b. 3.0 cm
c. 3.26 cm d. 2.52 cm

31. Critical angle of glass is θ_1 and that of water is θ_2. The critical angle for water and glass surface would be ($\mu_g = 3/2$, $\mu_w = 4/3$)
a. less than θ_2 b. between θ_1 and θ_2
c. greater than θ_2 d. less than θ_1

32. Light is incident normally on face AB of a prism as shown in Fig. 1.341. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index 3/2.

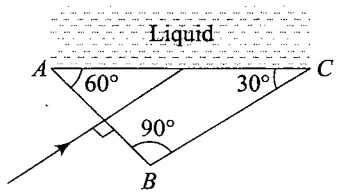


Fig. 1.341

The limit of μ for which total internal reflection takes place on face AC is

a. $\mu > \frac{3}{4}$ b. $\mu < \frac{3\sqrt{3}}{4}$

c. $\mu > \sqrt{3}$ d. $\mu < \frac{\sqrt{3}}{2}$

33. A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere does not come out of the sphere?

a. $\tan^{-1}(2/3)$ b. 60°

c. 90° d. 30°

34. Two identical glass ($\mu_g = 3/2$) equiconvex lenses of focal length f are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is

a. f b. $\frac{f}{2}$

c. $\frac{4f}{3}$ d. $\frac{3f}{4}$

35. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the sizes, of the images formed are equal, the focal length of the lens will be

a. 15 cm b. 17 cm

c. 21 cm d. 11 cm

36. A concave lens forms the image of an object such that the distance between the object and image is 10 cm and the magnification produced is 1/4. The focal length of the lens will be

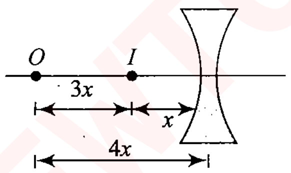


Fig. 1.342

a. 8.6 cm b. 6.2 cm

c. 10 cm d. 4.4 cm

37. A person is looking at the image of his face in a mirror by holding it close to his face. The image is virtual. When he moves the mirror away from his face, the image is inverted. What type of mirror is he using ?

a. Plane mirror

b. Concave mirror

c. Convex mirror

d. Combination of mirror and lenses

38. A diver in a swimming pool wants to signal his distress to a person standing on the edge to the pool by flashing his water proof flash light

a. He must direct the beam vertically upward

b. He has to direct the beam horizontal

c. He has to direct the beam at an angle to the vertical which is slightly less than the critical angle of incidence for total internal reflection

d. He has to direct the beam at an angle to the vertical which is slightly more than the critical angle of incidence for the total internal reflection

39. A, B, and C are three optical media of respective critical angles C_1, C_2, and C_3. Total internal reflection of light can occur from A and B, also from B to C but not from C to A. Then, the correct relation between the critical angles is

a. $C_1 < C_2 < C_3$ b. $C_3 < C_1 < C_2$

c. $C_1 < C_2 < C_3$ d. $C_1 < C_2 < C_3$

40. A point object O is placed on the principal axis of a convex lens of focal length 20 cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. If the eye is placed 60 cm to the right of the lens at a distance h below the principal axis, then the maximum value of h to see the image will be

a. 0 b. 5 cm

c. 2.5 cm d. 10 cm

41. Light of wavelength 500 nm traveling with a speed of $2.0 \times 10^8\ \text{ms}^{-1}$ in a certain medium enters another medium of refractive index 5/4 times that of the first medium. What are the wavelength and speed in the second medium?

	Wavelength (nm)	speed (ms^{-1})
a.	400	1.6×10^8
b.	400	2.5×10^8
c.	500	2.5×10^8
d.	625	1.6×10^8

42. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices n_1 and n_2, respectively ($n_2 > n_1 > 1$). The lens will diverge parallel beam of light if it is filled with

a. air and placed in air b. air and immersed in L_1

c. L_1 and immersed in L_2 d. L_2 and immersed in L_1

43. The velocity of light in a medium is half its velocity in air. If a ray of light emerges from such a medium into air, the angle of incidence, at which it will be totally internally reflected, is

a. 15° b. 30°

c. 45° d. 60°

44. A convex lens of focal length 20 cm and a concave lens of focal length f are mounted coaxially 5 cm apart. Parallel beam of light incident on the convex lens emerges from the concave lens as a parallel beam. Then, f in cm is

a. 35 b. 25

c. 20 d. 15

45. An equiconvex lens is made from glass of refractive index 1.5. If the radius of each surface is changed from 5 cm to 6 cm, then the power

a. remains unchanged b. increases by 3.33 D

c. decreases by 3.33 D d. decreases by 5.5 D

46. An object is put at a distance of 5 cm from the first focus of a convex lens of focal length 10 cm. If a real image is formed, its distance from the lens will be

a. 15 cm b. 20 cm

c. 25 cm d. 30 cm

47. A concave lens made of water ($\mu = 1.33$) is placed inside a glass slab ($\mu = 1.5$) for an object placed between the focus and twice the focus. The image formed is
- **a.** virtual
- **b.** real, inverted, and magnified
- **c.** virtual, inverted, and magnified
- **d.** real, inverted, and diminished

48. A candle is placed 20 cm from the surface of a convex mirror and a plane mirror is also placed so that the virtual images in the two mirrors coincide. If the plane mirror is 12 cm away from the object, what is the focal length of the convex mirror?
- **a.** 20 cm **b.** 15 cm
- **c.** 10 cm **d.** 5 cm

49. A given ray of light suffers minimum deviation in an equilateral prism P.

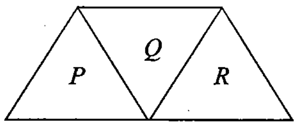


Fig. 1.343

Additional prisms Q and R of identical shape and of the same material as P are now added as shown in Fig. 1.343. The ray will suffer:
- **a.** greater deviation
- **b.** no deviation
- **c.** same deviation as before
- **d.** total internal reflection

50. Consider the situation shown in Fig. 1.344. Water ($\mu = 4/3$) is filled in a beaker upto a height of 10 cm.

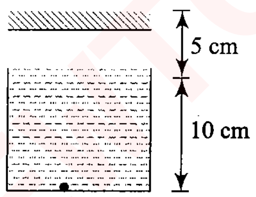


Fig. 1.344

A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object O at the bottom of the beaker is
- **a.** 15 cm **b.** 12.5 cm
- **c.** 7.5 cm **d.** 10 cm

51. Refraction takes place at a concave spherical boundary separating glass–air medium. For the image to be real, the object distance ($\mu_g = 3/2$)
- **a.** should be greater than three times the radius of curvature of the refracting surface
- **b.** should be greater than two times the radius of curvature of the refracting surface
- **c.** should be greater than the radius of curvature of the refracting surface
- **d.** is independent of the radius of curvature of the refracting surface

52. The image of point P when viewed from top of the slabs will be

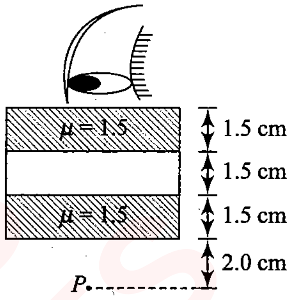


Fig. 1.345

- **a.** 2.0 cm above P **b.** 1.5 cm above P
- **c.** 2.0 cm below P **d.** 1 cm above P

53. The refractive index of a prism is 2. The prism can have a maximum refracting angle of
- **a.** 90° **b.** 60°
- **c.** 45° **d.** 30°

54. One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at angle of 60° retraces its path. The refractive index of the material of prism is
- **a.** $\sqrt{2}$ **b.** $\sqrt{3}$
- **c.** 3/2 **d.** 2

55. Angle of minimum deviation is equal to the angle of prism A of an equilateral glass prism. The angle of incidence at which minimum deviation will be obtained is
- **a.** 60° **b.** 30°
- **c.** 45° **d.** $\sin^{-1}(2/3)$

56. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If the lenses are made of different material of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surface of the lenses, then focal length of the combination is
- **a.** $\dfrac{R}{\mu_1 - \mu_2}$ **b.** $\dfrac{2R}{\mu_2 - \mu_1}$
- **c.** $\dfrac{R}{2(\mu_1 - \mu_2)}$ **d.** $\dfrac{R}{2 - (\mu_1 + \mu_2)}$

57. A parallel beam of light is incident on the system of two convex lenses of focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated?

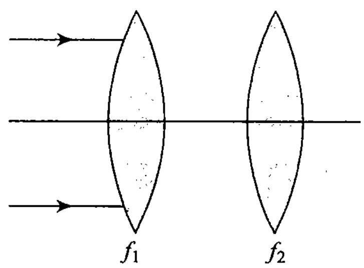


Fig. 1.346

a. 60 cm b. 30 cm
c. 90 cm d. 40 cm

58. A plano-convex glass lens ($\mu_g = 3/2$) of radius of curvature R = 10 cm is placed at a distance of 'b' from a concave lens of focal length 20 cm.

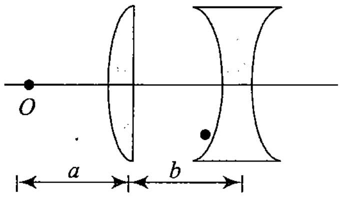


Fig. 1.347

What should be the distance 'a' of a point object O from the plano-convex lens so that the position of final image is independent of 'b'?

a. 40 cm b. 60 cm
c. 30 cm d. 20 cm

59. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness t is

a. 10 cm b. 5 cm
c. 20 cm d. 15 cm

60. A convex lens of focal length 10 cm is painted black at the middle portion as shown in Fig. 1.348. An object is placed at a distance of 20 cm from the lens.

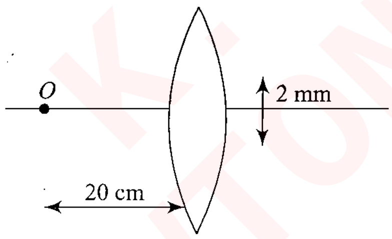
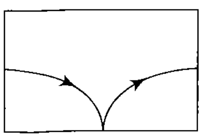


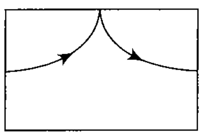
Fig. 1.348

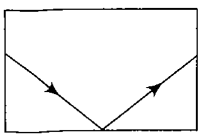
Then,

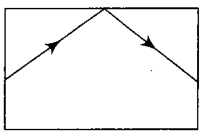
a. only one image will be formed by the lens
b. the distance between the two images formed by such a lens is 6 mm
c. the distance between the images is 4 mm
d. the distance between the images is 2 mm

61. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?

a. 

b. 

c. 

d. 

62. A lens forms a virtual, diminished image of an object placed at 2 m from it. The size of image is half of the object. Which one of the following statements is correct regarding the nature and focal length of the lens?

a. Concave, $|f| = 1$ m b. Convex, $|f| = 1$
c. Concave, $|f| = 2$ m d. Convex, $|f| = 2$ m

63. Consider an equiconvex lens of radius of curvature R and focal length f. If $f > R$, the refractive index μ of the material of the lens

a. is greater than zero but less than 1.5
b. is greater than 1.5 but less than 2.0
c. is greater than 1 but less than 1.5
d. none of these

64. Two convex lenses placed in contact form the image of a distant object at P. If the lens B is moved to the right, the image will

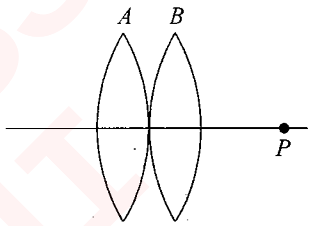


Fig. 1.349

a. move to the left
b. move to the right
c. remain at P
d. move either to the left or right, depending upon focal lengths of the lenses.

65. An equiconvex lens is cut into two halves along (i) XOX' and (ii) YOY'' as shown in Fig. 1.350. Let f, f', f'' be the focal length of the lens, of each half in case (i), and of each half in case (ii), respectively,

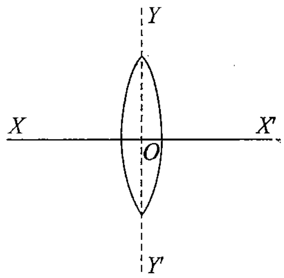


Fig. 1.350

Choose the correct statement from the following:

a. $f' = f,\ f'' = 2f$
b. $f' = 2f,\ f'' = f$
c. $f' = f,\ f'' = f$
d. $f' = 2f',\ f'' = 2f$.

66. A spherical mirror forms an image of magnification 3. The object distance, if focal length of mirror is 24 cm, may be

a. 32 cm, 24 cm b. 32 cm, 16 cm
c. 32 cm only d. 16 cm only

67. The critical angle for light going from medium X into medium Y is θ. The speed of light in medium X is v. The speed of light in medium Y is

a. $v \cos \theta$ b. $v / \cos \theta$
c. $v \sin \theta$ d. $v / \sin \theta$

68. An object 15 cm high is placed 10 cm from the optical center of a thin lens. Its image is formed 25 cm from the optical center on the same side of the lens as the object. The height of the image is
a. 2.5 cm b. 0.2 cm
c. 16.7 cm d. 37.5 cm

69. A convex lens of power +6 dioptre is placed in contact with a concave lens of power –4 dioptre. What will be the nature and focal length of this combination?
a. Concave, 25 cm b. Convex, 50 cm
c. Concave, 20 cm d. Convex, 100 cm

70. A convex lens of focal length 1.0 m and a concave lens of focal length 0.25 m are 0.75 m apart. A parallel beam of light is incident on the convex lens. The beam emerging after refraction from both lenses is
a. parallel to the principal axis
b. convergent
c. divergent
d. none of the above

71. A convex lens A of focal length 20 cm and a concave lens G of focal length 5 cm are kept along the same axis with the distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then distance d in cm will be
a. 25 b. 15
c. 30 d. 50

72. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moves 5 cm toward the lens, the image will be
a. 5 cm toward the lens b. 5 cm away from the lens
c. 10 cm toward the lens d. 10 cm away from the lens

73. With a concave mirror, an object is placed at a distance x_1 from the principal focus, on the principal axis. The image is formed at a distance x_2 from the principal focus. The focal length of the mirror is
a. $x_1 x_2$ b. $(x_1 + x_2)/2$
c. $\sqrt{x_1/x_2}$ d. $\sqrt{x_1 x_2}$

74. A concave mirror is placed on a horizontal table with its axis directed vertically upward. Let O be the pole of the mirror and C its center of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be
a. real and will remain at C
b. real and located at a point between C and ∞
c. real and located at a point between C and O
d. real and located at a point between C and O

75. A cubical room is formed with six plane mirrors. An insect moves along the diagonal of the floor with uniform speed. The velocities of its image in two adjacent walls are $20\sqrt{2}$ cm s^{-1}. Then, the velocity of the image formed by the roof is
a. 20 cm s^{-1} b. $20\sqrt{2}$ cm s^{-1}
c. 40 cm s^{-1} d. $10\sqrt{2}$ cm s^{-1}

76. A plane glass mirror of thickness 3 cm of material of $\mu = 3/2$ is silvered on the back surface. When a point object is placed 9 cm from the front surface of the mirror, then the position of the brightest image from the front surface is
a. 9 cm b. 11 cm
c. 12 cm d. 13 cm

77. A point source of light B is placed at a distance L in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in Fig. 1.351. the greatest distance over which he can see the image of the light source in the mirror is

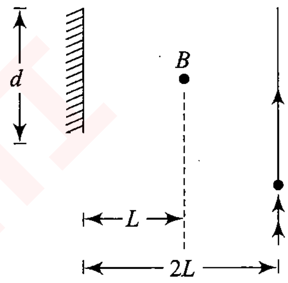


Fig. 1.351

a. $d/2$ b. d
c. $2d$ d. $3d$

78. An object is placed at a distance of 25 cm from the pole of a convex mirror and a plane mirror is set at a distance 5 cm from convex mirror so that the virtual images formed by the two mirrors do not have any parallax. The focal length of the convex mirror is
a. 37.5 cm b. –7.5 cm
c. –37.5 cm d. +7.5 cm

79. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the object is moved with a speed of 9 cm s^{-1} the speed with which the image moves is
a. 0.1 ms^{-1} b. 1 m s^{-1}
c. 3 ms^{-1} d. 9 ms^{-1}

80. A convex mirror of radius of curvature 1.6 m has an object placed at a distance of 1 m from it. The image is formed at a distance of
a. 8/13 m in front of the mirror
b. 8/13 m behind the mirror
c. 4/9 m in front of the mirror
d. 4/9 m behind the mirror

81. In the above question, the magnification is
a. 4/9 b. –4/9
c. 9/4 d. 8/13

82. A convex mirror and a concave mirror of radius 10 cm each are placed 15 cm apart facing each other. An object is placed midway between them. If the reflection first takes place in the concave mirror and then in convex mirror, the position of the final image is

a. on the pole of the convex mirror
b. on the pole of the concave mirror
c. at a distance of 10 cm from the convex mirror
d. at a distance of 5 cm from the concave mirror

83. A small piece of wire bent into an L shape, with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

a. 1 : 2 **b.** 3 : 1
c. 1 : 3 **d.** 2 : 1

84. The image of an object placed on the principal axis of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distance from the mirror than the object. The magnification of the image is

a. 8/3 **b.** 2.5
c. 2 **d.** –1.5

85. A clear transparent glass sphere (μ=1.5) of radius R is immersed in a liquid of refractive index 1.25. A parallel beam of light incident on it will converge to a point. The distance of this point from the center will be

a. –3R **b.** +3R
c. –R **d.** +R

86. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. the reflected and refracted rays make an angle of $\pi/2$ with each other. If the angles of reflection and refraction are r and r', then the critical angle will be

a. $\tan^{-1}(\sin i)$ **b.** $\sin^{-1}(\sin r)$
c. $\sin^{-1}(\tan i)$ **d.** $\sin^{-1}(\tan r)$

87. A ray of light traveling in glass (μ = 3/2) is incident on a horizontal glass–air surface at the critical angle θ_C If a thin layer of water (μ = 4/3) is now poured on the glass–air surface, the angle at which the ray emerges into air at the water–air surface is

a. 60° **b.** 45°
c. 90° **d.** 180°

88. A ray of light enters a rectangular glass slab of refractive index $\sqrt{3}$ at an angle of incidence 60°. It travels a distance of 5 cm inside the slab and emerges out of the slab. The perpendicular distance between the incident and the emergent rays is

a. $5\sqrt{3}$ cm **b.** $\frac{5}{2}$ cm
c. $5\sqrt{3/2}$ cm **d.** 5 cm

89. A ray of monochromatic light is incident on the refracting face of a prism (angle 75° It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the prism is $\sqrt{2}$, then the angle of incidence on the first face of the prism is

a. 15° **b.** 30°
c. 45° **d.** 60°

90. An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is

a. 20 cm **b.** 10 cm
c. 15 cm **d.** 30 cm

91. ACB is right–angled prism with other angles as 60° and 30°. Refractive index of the prism is 1.5. AB has thin layer of liquid on it as shown. Light falls normally on the face AC. For total internal reflections, maximum refractive index of the liquid is

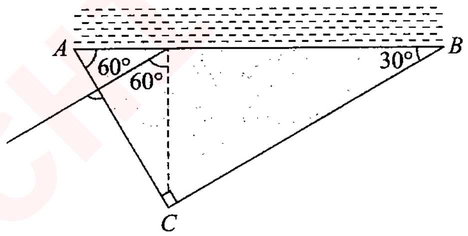


Fig. 1.352

a. 1.4 **b.** 1.3
c. 1.2 **d.** 1.6.

92. The lateral magnification of the lens with an object located at two different positions u_1 and u_2 are m_1 and m_2 respectively. Then the focal length of the lens is

a. $f = \sqrt{m_1 m_2}\,(u_2 - u_1)$ **b.** $f = \sqrt{m_1 m_2}\,(u_2 - u_1)$

c. $\frac{(u_2 - u_1)}{\sqrt{m_2 m_1}}$ **d.** $\frac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$

93. A parallel beam of light falls axially on a thin converging lens of focal length 20 cm. The emergent light falls or a screen placed 30 cm beyond the lens. An opaque plate with a triangular aperture, side 1 cm, is in contact with the lens. (see Figs. 1.353 and 1.354).

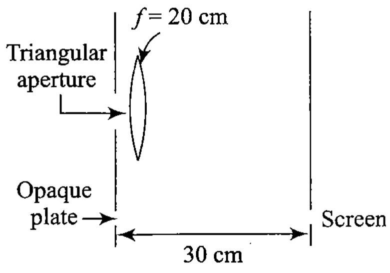


Fig. 1.353

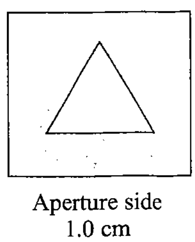


Fig. 1.354

Which one of the following diagrams best shows to appearance of the patch of light seen on the screen?

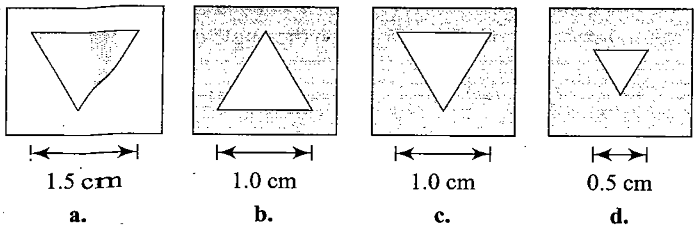


Fig. 1.355

94. A liquid of refractive index 1.6 is contained in the cavity of a glass specimen of refractive index 1.5 as shown in Fig. 1.356, If each of the curved surfaces has a radius of curvature of 0.20 m, the arrangement behaves as a

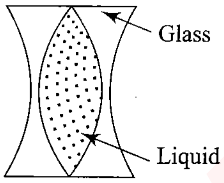


Fig. 1.356

a. converging lens of focal length 0.25 m
b. diverging lens of focal length 0.25 m
c. diverging lens of focal length 0.17 m
d. converging lens of focal length 0.72 m.

95. Figure 1.357 (a) shows two plano-convex lenses in contact as shown. The combination has focal length 24 cm. Figure 1.357 (b) shows the same with a liquid introduced between them. If refractive index of glass of the lenses is 1.50 and that of the liquid is 1.60, the focal length of system in Fig. 1.357 (b) will be

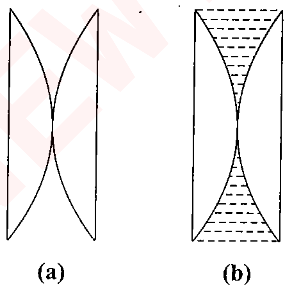


Fig. 1.357

a. –120 cm **b.** 120 cm
c. –24 cm **d.** 24 cm

96. In a slide show programme, the image on the screen has an area 900 times that of the slide. If the distance between the slide and the screen is x times the distance between the slide and the projector lens, then

a. $x = 30$ **b.** $x = 31$
c. $x = 500$ **d.** $x = 1/30$

97. A glass prism has refractive index $\sqrt{2}$ and refracting angle 30°. One of the refracting surface of the prism is silvered. A beam of monochromatic light will retrace it path if its angle of incidence on the unsilvered refracting surface of the prism is

a. 0 **b.** $\pi/6$
c. $\pi/4$ **d.** $\pi/3$

98. A fish rising vertically up toward the surface of water with speed 3 ms^{-1} observes a bird diving vertically down towards it with speed 9 ms^{-1}. The actual velocity of bird is

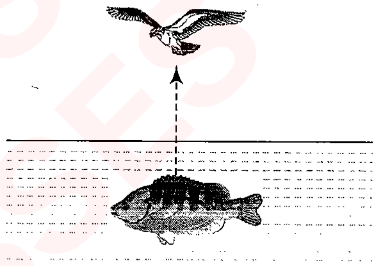


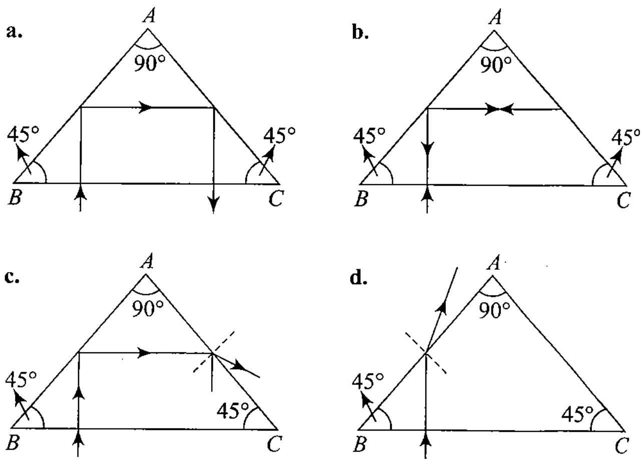
Fig. 1.358

a. 4.5 ms^{-1} **b.** 5.4 ms^{-1}
c. 3.0 ms^{-1} **d.** 3.4 ms^{-1}

99. A concave lens with unequal radii of curvature made of glass ($\mu_g = 1.5$) has a focal length of 40 cm. If it is immersed in a liquid of refractive index $\mu_l = 2$, then

a. it behaves like a convex lens of 80 cm focal length
b. it behaves like a convex lens of 20 cm focal length
c. its focal length becomes 60 cm
d. nothing can be said

100. The refractive index of material of a prism of angles 45°, –45°, and –90° is 1.5. The path of the ray of light incident normally on the hypotenuse side is shown in



101. An object is placed 1 m in front of the curved surface of a plano-convex lens whose plane surface is silvered. A real image is formed in front of the lens at a distance of 120 cm. Then, the focal length of the lens is

a. 100 cm **b.** 120 cm
c. 109.1 cm **d.** 110.0 cm

102. The apparent thickness of a thick plano-convex lens is measured once with the plane face upward and then with the convex face upward. The value will be

a. more in the first case
b. same in the two cases
c. more in the second case
d. any of the above depending on the value of its actual thickness

103. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?

a. 25 cm b. 7 cm
c. 18 cm d. 27 cm

104. A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirrors, on their common axis and 15 cm from the concave mirror. Find the position and nature of the image produced by successive reflections, first at the concave mirror and then at the convex mirror.

a. 12 cm behind convex mirror, real
b. 9 cm behind convex mirror, real
c. 6 cm behind convex mirror, virtual
d. 3 cm behind convex mirror, virtual

105. A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.

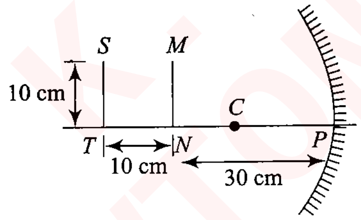


Fig. 1.359

a. 2 cm b. 10 cm
c. 8 cm d. 14 cm

106. An object $ABED$ is placed in front of a concave mirror beyond the center of curvature C as shown in figure. State the shape of the image.

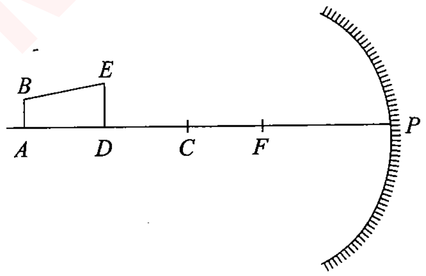


Fig. 1.360

a. $|m_{AB}| < 1$ and $|m_{ED}| < 1$ b. $|m_{AB}| > 1$ and $|m_{ED}| < 1$
c. $|m_{AB}| < 1$ and $|m_{ED}| > 1$ d. $|m_{AB}| > 1$ and $|m_{ED}| > 1$

107. A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0. The gun is fitted with a concave mirror of focal length f facing toward a receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

a. $\left(1+\frac{m_2}{m_1}\right)v_0$ b. $2\left(1-\frac{m_2}{m_1}\right)v_0$

c. $2\left(1+\frac{2m_2}{m_1}\right)v_0$ d. $2\left(1+\frac{m_2}{m_1}\right)v_0$

108. A rod made of glass, refractive index 1.5 and of square cross section, is bent into the shape shown in figure. A parallel beam of light falls normally on the plane flat surface A. Referring to the diagram, d is the width of a side and R is the radius of inner semicircle. Find the maximum value of ratio d/R so that all light entering the glass through surface A emerges from the glass through surface B.

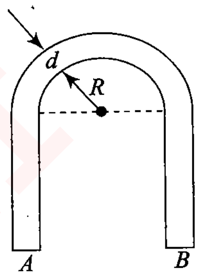


Fig. 1.361

a. 1/2 b. 1/5
c. 1/4 d. 2/3

109. Focal length of a thin convex lens is 30 cm. At a distance of 10 cm from the lens, there is a plane refracting surface of refractive index 3/2. Where will the parallel rays incident on the lens converge?

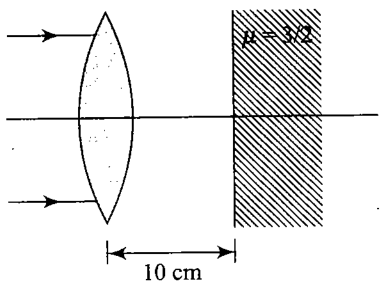


Fig. 1.362

a. At a distance of 27.5 cm from the lens.
b. At a distance of 25 cm from the lens.
c. At a distance of 45 cm from the lens.
d. At a distance of 40 cm from the lens.

110. A point object O is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in Fig. 1.400. At what distance x from the lens should a concave mirror of focal length 60 cm, be placed so that final image coincides with the object?

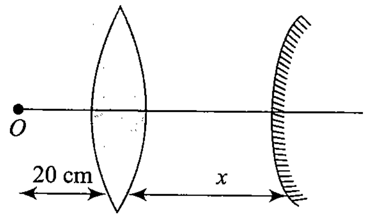


Fig. 1.363

a. 10 cm
b. 40 cm
c. 20 cm
d. Final image can never coincide with the object in the given conditions

111. Two point sources S_1 and S_2 are 24 cm apart. Where should a convex lens of focal length 9 cm be placed in between them so that the images of both sources are formed at the same place?

a. 6 cm from S_1 b. 15 cm from S_1
c. 10 cm from S_1 d. 12 cm from S_1

112. Two thin symmetrical lenses of different nature and of different material have equal radii of curvature $R = 15$ cm. The lenses are put close together and immersed in water ($\mu_w = 4/3$). The focal length of the system in water is 30 cm. The difference between refractive indices of the two lenses is

a. 1/2 b. 1/4
c. 1/3 d. 3/4

113. A glass sphere of radius $R = 10$ cm is kept inside water. A point object O is placed at 20 cm from A as shown in Fig. 1.364. Find the position and nature of the image when seen from other side of the sphere. Given $\mu_g = 3/2$ and $\mu_w = 4/3$.

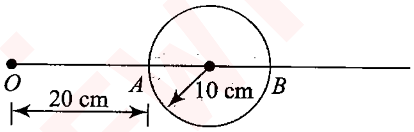


Fig. 1.364

a. 200 cm, virtual b. 100 cm, real
c. 100 cm, virtual d. 300 cm, virtual

114. A spherical convex surface separates object and image spaces of refractive indices 1.0 and 4/3. If radius of curvature of the surface is 10 cm, find its power.

a. 3.5 dioptre b. 2.5 dioptre
c. 25 dioptre d. 1.5 dioptre

115. A transparent sphere of radius 20 cm and refractive index 1.6 is fixed in a hole of the partition separating the two media: A (refractive index $n_1 = 1.2$) and B (refractive index $n_3 = 1.7$). A luminous point object is placed 120 cm from the surface of the sphere in medium A. It is viewed from D in medium B in a direction normal to the sphere. Find the position of the image formed by the rays, from point N.

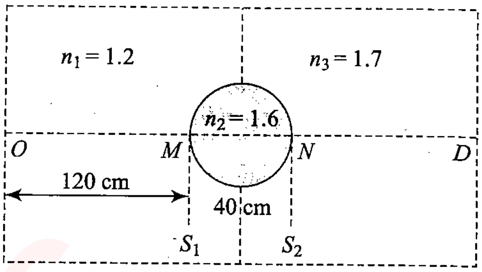


Fig. 1.365

a. 304 cm, left side of N
b. 175 cm, right side of N
c. 204 cm, right side of N
d. 220 cm, left side of N

116. A cubical block of glass, refractive index 1.5, has a spherical cavity of radius $r = 9$ cm inside it as shown in Fig. 1.366. A luminous point object O is at a distance of 18 cm from the cube (see figure). What is the apparent position of O as seen from A?

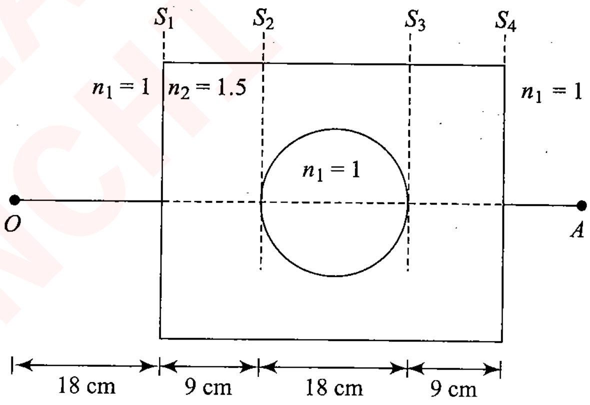


Fig. 1.366

a. 17 cm, left of S_4 b. 25 cm, right of S_4
c. 13 cm, left of S_4 d. 10 cm, right of S_4

117. A glass sphere, refractive index 1.5 and radius 10 cm, has a spherical cavity of radius 5 cm concentric with it. A narrow beam of parallel light is directed into the sphere. Find the final image and its nature.

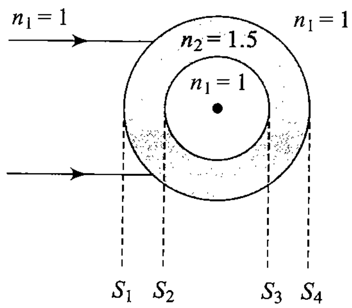


Fig. 1.367

a. 25 cm left of S_4, virtual
b. 25 cm right of S_4, real
c. 15 cm left of S_4, virtual
d. 20 cm right of S_4, virtual

118. A luminous object and a screen are at a fixed distance D apart. A converging lens of focal length f is placed between the object and screen. A real image of the object in formed on the screen for two lens positions if they are separated by a distance d equal to

a. $\sqrt{D(D+4f)}$ **b.** $\sqrt{D(D-4f)}$

c. $\sqrt{2D(D-4f)}$ **d.** $\sqrt{D^2+4f}$

119. For the same statement as above, the ratio of the two image sizes for these two positions of the lens is

a. $\left[\dfrac{D-d}{D+d}\right]^2$ **b.** $\left[\dfrac{D+d}{D-d}\right]^2$

c. $\left[\dfrac{D-2d}{D+2d}\right]^2$ **d.** $\left[\dfrac{D+2d}{D-2d}\right]^2$

120. For statement of question 118, if the heights of the two images are h_1 and h_2, respectively, then the height of the object (h) is

a. h_1+h_2 **b.** h_1h_2

c. $\sqrt{h_1h_2}$ **d.** h_1/h_2

121. The focal length of the lens used in question 118 is

a. $\dfrac{D^2+d^2}{2D}$ **b.** $\dfrac{D^2-d^2}{4D}$

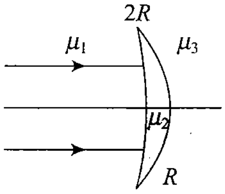
c. $\dfrac{D^2-d^2}{2D}$ **d.** $\dfrac{D^2+d^2}{d}$

122. In question 118, if m_1 and m_2 are the magnifications for two positions of the lens, then

a. $f=\dfrac{d}{m_1+m_2}$ **b.** $f=\dfrac{2d}{m_1+m_2}$

c. $f=\dfrac{3d}{m_1-m_2}$ **d.** $f=\dfrac{d}{m_1-m_2}$

123. The diagram shows a concavo-convex lens μ_2. What is the condition on the refractive indices so that the lens is diverging?



The refractive index of the lens is μ_2

Fig. 1.368

a. $2\mu_3<\mu_1+\mu_2$ **b.** $2\mu_3>\mu_1+\mu_2$

c. $\mu_3>2(\mu_1-\mu_2)$ **d.** None of these

124. The image produced by a concave mirror is one-quarter the size of object. If the object is moved 5 cm closer to the mirror, the image will only be half the size of the object. The focal length of mirror is

a. $f=5.0$ cm **b.** $f=2.5$ cm

c. $f=7.5$ cm **d.** $f=10$ cm

125. Light traveling through three transparent substances follows the path shown in Fig. 1.369. Arrange the indices of refraction in order from smallest to largest. Note that total internal reflection does occur on the bottom surface of medium 2.

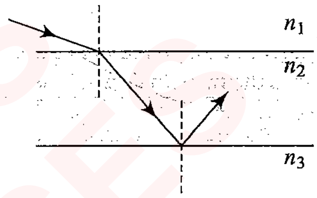


Fig. 1.369

a. $n_1<n_2<n_3$ **b.** $n_2<n_1<n_3$

c. $n_1<n_3<n_2$ **d.** $n_3<n_1<n_2$

126. A linear object AB is placed along the axis of a concave mirror. The object is moving towards the mirror with speed U. The speed of the image of the point A is $4U$ and the speed of the image of B is also $4U$ but in opposite direction. If the center of the line AB is at a distance L from the mirror then find out the length of the object.

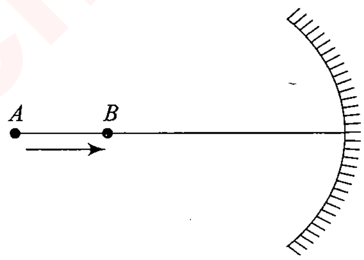


Fig. 1.370

a. $3L/2$ **b.** $5L/3$

c. L **d.** None of these

127. A mango tree is at the bank of a river and one of the branch of tree extends over the river. A tortoise lives in the river. A mango falls just above the tortoise. The acceleration of the mango falling from tree as it appears to the tortoise is (refractive index of water is 4/3 and the tortoise is stationary)

a. g **b.** $3g/4$

c. $4g/3$ **d.** none of these

128. In Fig. 1.371, ABC is the cross section of a right-angled prism and $ACDE$ is the cross section of a glass slab. The value of θ so that light incident normally on the face AB does not cross the face AC is (given $\sin^{-1}(3/5)=37°$).

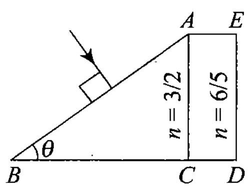


Fig. 1.371

a. $\theta \le 37°$ b. $\theta < 37°$
c. $\theta \le 53°$ d. $\theta < 53°$

129. A bird is flying up at angle $\sin^{-1}$ (3/5) with the horizontal. A fish in a pond looks at that bird. When it is vertically above the fish. The angle at which the bird appears to fly (to the fish) is [$n_{water} = 4/3$]

a. $\sin^{-1}(3/5)$ b. $\sin^{-1}(4/5)$
c. 45° d. $\sin^{-1}(9/16)$

130. A point object 'O' is at the center of curvature of a concave mirror. The mirror starts to move at a speed u, in a direction perpendicular to the principal axis. Then, the initial velocity of the image is

a. $2u$, in the direction opposite to that of mirror's velocity
b. $2u$, in the direction same as that of mirror's velocity
c. zero
d. u, in the direction same as that of mirror's velocity

131. Refractive index of a prism is $\sqrt{7/3}$ and the angle of prism is 60°. The minimum angle of incidence of a ray that will be transmitted through the prism is

a. 30° b. 45°
c. 15° d. 50°

132. For a prism kept in air, it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ', and angle of emergence 'e' become equal. The minimum angle of incidence of a ray that will be transmitted through the prism is

a. 1.73 b. 1.15
c. 1.5 d. 1.33

133. A transparent cylinder has its right half polished so as to act as a mirror. A paraxial light ray incident from left, that is parallel to the principal axis, exits parallel to the incident ray as shown. The refractive index n of the material of the cylinder is

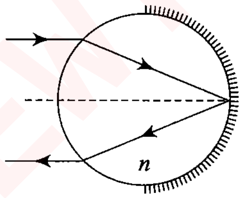


Fig. 1.372

a. 1.2 b. 1.5
c. 1.8 d. 2.0

134. A plano-concave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be ?

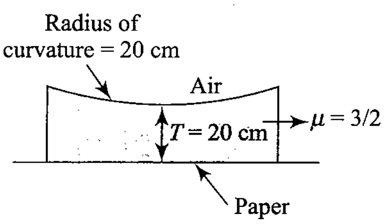


Fig. 1.373

a. 10 cm b. 15 cm
c. 50 cm d. None of these

135. The distance between an object and the screen is 100 cm. A lens produces an image on the screen when the lens is placed at either of the positions 40 cm apart. The power of the lens is nearly

a. 3 diopter b. 5 diopter
c. 2 diopter d. 9 diopter

136. In a thick glass slab of thickness l, and refractive index n_1, a cuboidal cavity of thickness 'm' is carved as shown in the fig and is filled with a liquid of R.I. n_2 ($n_1 > n_2$). The ratio l/m, so that shift produced by this slab is zero when an observer A observes an object B with paraxial rays is

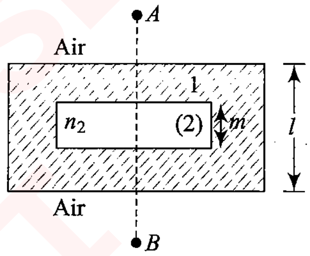


Fig. 1.374

a. $\frac{n_1 - n_2}{n_2 - 1}$ b. $\frac{n_1 - n_2}{n_2(n_1 - 1)}$
c. $\frac{n_1 - n_2}{n_1 - 1}$ d. $\frac{n_1 - n_2}{n_1(n_2 - 1)}$

137. If a prism having refractive index $\sqrt{2}$ has angle of minimum deviation equal to the angle of refraction of the prism, then the angle of refraction of the prism is

a. 30° b. 45°
c. 60° d. 90°

138. In the figure shown, a point object O is placed in air. A spherical boundary separates two media. AB is the principal axis. The refractive index above AB is 1.6 and below AB is 2.0. The separation between the images formed due to refraction at the spherical surface is

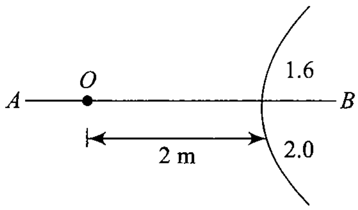


Fig. 1.375

a. 12 m b. 20 m
c. 14 m d. 10 m

139. A square $ABCD$ of side 1 mm is kept at distance 15 cm in front of the concave mirror as shown in fig. The focal length of the mirror is 10 cm. The length of the perimeter of its image will be

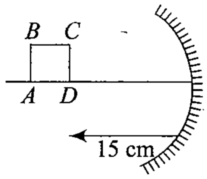


Fig. 1.376

a. 8 mm
b. 2 mm
c. 12 mm
d. 6 mm

140. A point object is kept in front of a plane mirror. The plane mirror is doing SHM of amplitude 2 cm. The plane mirror moves along the x-axis which is normal to the mirror. The amplitude of the mirror is such that the object is always in front of the mirror. The amplitude of SHM of the image is

a. 0
b. 2 cm
c. 4 cm
d. 1 cm

141. In Fig. 1.377, find the total magnification after two successive reflections first on M_1 and then on M_2.

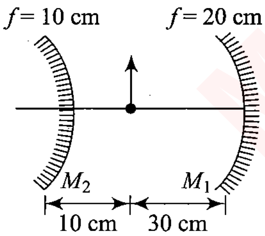


Fig. 1.377

a. +1
b. –2
c. +2
d. –1

142. A particle revolves in clockwise direction (as seen from point A) in a circle C of radius 1 cm and completes one revolution in 2 sec. The axis of the circle and the principal axis of the mirror M coincide, call it AB. The radius of curvature of the mirror is 20 cm. Then, the direction of revolution (as seen from A) of the image of the particle and its speed is

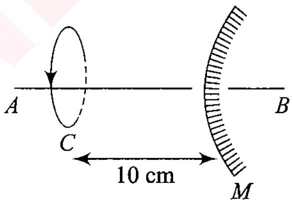


Fig. 1.378

a. clockwise, 1.57 cms^{-1}
b. clockwise, 3.14 cms^{-1}
c. anticlockwise, 1.57 cms^{-1}
d. anticlockwise, 3.14 cms^{-1}

143. The given lens is broken into four parts rearranged as shown. If the initial focal length is f, then after rearrangement the equivalent focal length is

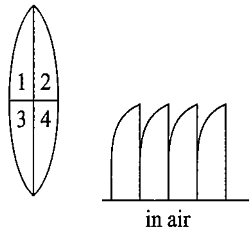


Fig. 1.379

a. f
b. $f/2$
c. $f/4$
d. $4f$

144. Let r and r' denote the angles inside an equilateral prism, as usual, in degrees. Consider that during some time interval from $t = 0$ to $t = t$, r' varies with time as $r' = 10 + t^2$. During this time, r will vary as (assume that r and r' are in degree)

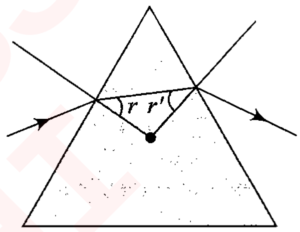


Fig. 1.380

a. $50 - t^2$
b. $50 + t^2$
c. $60 - t^2$
d. $60 + t^2$

145. For a prism kept in air, it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ' and angle of emergence 'e' become equal. Then, the refractive index of the prism is

a. 1.73
b. 1.15
c. 1.5
d. 1.33

146. Choose the correct mirror image of figure given below.

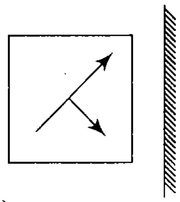
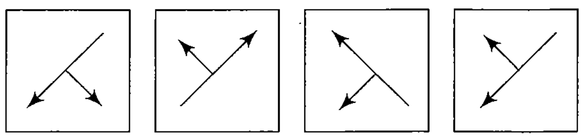


Fig. 1.381



147. A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at frequency $9/\pi$ rps. The light reflected by the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37°, the speed of the spot (a point) on the wall is

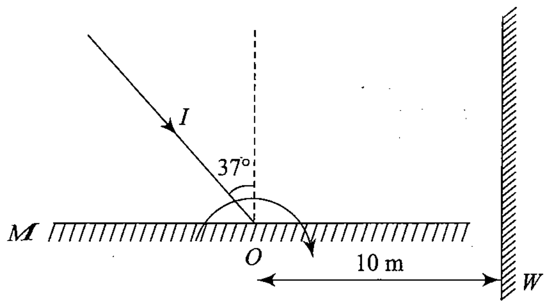


Fig. 1.382

a. $10\ ms^{-1}$ **b.** $1000\ ms^{-1}$
c. $500\ ms^{-1}$ **d.** none of these

148. An object is approaching a fixed plane mirror with velocity $5\ ms^{-1}$ making an angle of 45° with the normal. The speed of image w.r.t. the mirror is

a. $5\ ms^{-1}$ **b.** $5/\sqrt{2}\ ms^{-1}$
c. $5\sqrt{2}\ ms^{-1}$ **d.** $10\ ms^{-1}$

149. A point source of light S is placed in front of a perfectly reflecting mirror as shown in the figure. Σ is a screen. The intensity at the center of screen is found to be I.

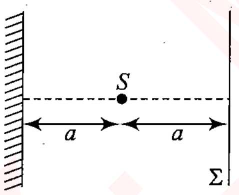


Fig. 1.383

If the mirror is removed, then the intensity at the center of screen would be

a. I **b.** $10I/9$
c. $9I/10$ **d.** $2I$

150. A point source of light is placed in front of a plane mirror as shown in the figure.

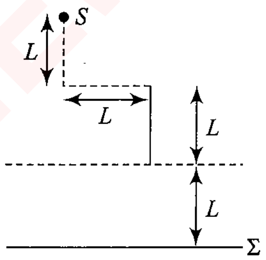


Fig. 1.384

Determine the length of reflected path of light on the screen Σ.

a. L **b.** $2L$
c. $3L/2$ **d.** $L/2$

151. A concave refractive surface of a medium having refractive index μ produces a real image of an object (located outside the medium) irrespective of its location. Choose the correct option from the following.

a. Always
b. May be if refractive index of surrounding medium is greater than μ
c. May be if refractive index of surrounding medium is less than μ
d. None of the above

152. A convex spherical refracting surface with radius R separates a medium having refractive index 5/2 from air. As an object is moved towards the surface from far away from the surface along the principle axis, its image

a. changes from real to virtual when it is at a distance R from the surface
b. changes from virtual to real when it is at a distance R from the surface
c. changes from real to virtual when it is at a distance $2R/3$ from the surface
d. changes from virtual to real when it is at a distance $2R/3$ from the surface

153. A concave spherical refractive surface with radius R separates a medium of refractive index 5/2 from air. As an object is approaching the surface from far away from the surface along the central axis, its image

a. always remains real
b. always remains virtual
c. changes from real to virtual at a distance $2R/3$ from the surface
d. changes from virtual to real at a distance $2R/3$ from the surface

154. Two lenses shown are illuminated by a beam of parallel light from the left. Lens B is then moved slowly toward lens A. The beam emerging from lens B is

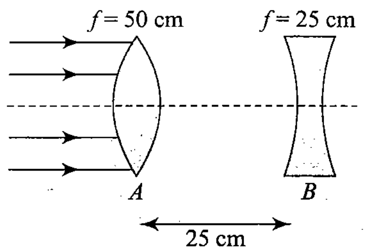


Fig. 1.385

a. always diverging
b. initially parallel and then diverging
c. always parallel
d. initially converging and then parallel

155. In the arrangement shown below, the image of the extended object as seen by the observer is

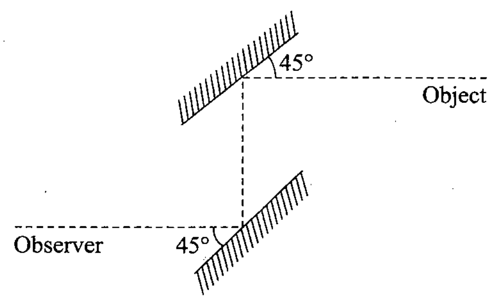


Fig. 1.386

a. real and inverted
b. real and erect
c. virtual and inverted
d. virtual and erect

156. The table below shows object and image distances for four objects placed in front of mirrors. For which one is the image formed by a convex spherical mirror. [Positive and negative signs are used in accordance with standard sign convention].

Object distance	Image distance
a. –7.10 cm	–18.0 cm
b. –25.0 cm	–16.7 cm
c. –5.0 cm	+1.0 cm
d. –20.0 cm	+5.71 cm

157. A fish looks upward at an unobstructed overcast sky. What total angle does the sky appear to subtend? (Take refractive index of water as $\sqrt{2}$.)

a. 180° b. 90°
c. 75° d. 60°

158. For the situations shown in the figure, determine the angle by which the mirror should be rotated, so that the light ray will retrace its path after refraction through the prism and reflection from the mirror?

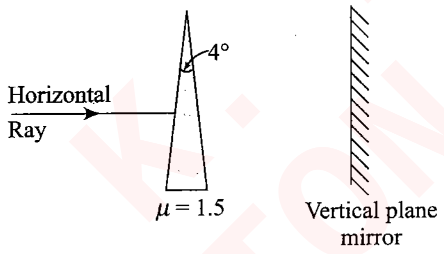


Fig. 1.387

a. 1° ACW b. 1° CW
c. 2° ACW d. 2° CW

159. A real object is placed in front of a convex mirror (fixed). The object is moving toward the mirror. If v_0 is the speed of object and v_i is the speed of image, then

a. $v_i < v_0$ always
b. $v_i > v_0$ always
c. $v_i > v_0$ initially and then $v_0 > v_i$
d. $v_i < v_0$ initially and then $v_i > v_0$

160. A point object is placed in front of a thick plane mirror as shown in figure below. Find the location of final image w.r.t. object.

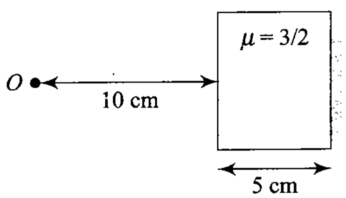


Fig. 1.388

a. 15/2 cm b. 15 cm
c. 40/3 cm d. 80/3 cm

161. Rays from a lens are converging toward a point P, as shown in figure. How much thick glass plate having refractive index 1.6 must be located between the lens and point P, so that the image will be formed at P'?

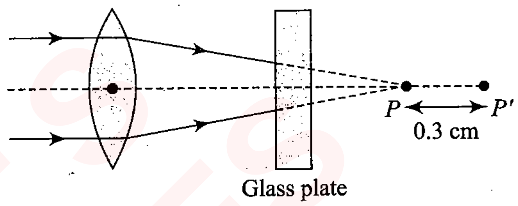


Fig. 1.389

a. 0.8 cm b. 1.6 cm
c. 5 cm d. 2.4 cm

162. A right-angled prism of apex angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. A horizontal ray of light is falling on the prism. Find the total deviation produced in the light ray as it emerges 2nd time from the prism.

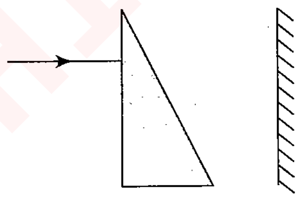


Fig. 1.390

a. 8° CW b. 6° CW
c. 180° CW d. 174°CW

163. Find the net deviation produced in the incident ray for the optical instrument shown in figure below. (Take refractive index of the prism material as 2.)

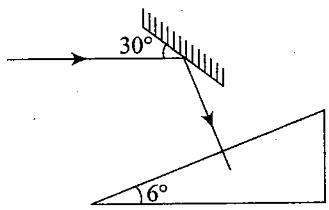


Fig. 1.391

a. 66° CW b. 66° ACW
c. 54° ACW d. 54° CW

164. When an object is placed 15 cm from a lens, a virtual image is formed. Mark the correct statements.

a. The lens may be convex or concave
b. If the lens is diverging, the image distance has to be less than 15 cm
c. If the lens is converging, then its focal length has to be greater than 15 cm
d. All of the above

165. A glass hemisphere of radius R and of material having refractive index 1.5 is silvered on its flat face as shown in figure. A small object of height h is located at a distance $2R$ from the surface of hemisphere as shown in figure. The final image will form

165. A glass hemisphere of radius R and of material having refractive index 1.5 is silvered on its flat face as shown in figure. A small object of height h is located at a distance $2R$ from the surface of hemisphere as shown in figure. The final image will form

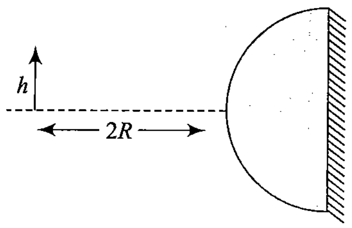


Fig. 1.392

a. at a distance of R from silvered surface, on the right side
b. on the object itself
c. at hemisphere surface
d. at a distance of $2R$ from the silvered surface, on left side

166. The refracting angle of a prism is A and refractive index of the material of prism is $\cot(A/2)$. The angle of minimum deviation will be

a. $180° - 3A$ b. $180° + 3A$
c. $90° - 3A$ d. $180° - 2A$

Multiple Correct Answers Type

Solutions on page 1.157

1. The diagram below shows an object located at point P, 0.25 meter from concave spherical mirror with principal focus F. The focal length of the mirror is 0.10 m.

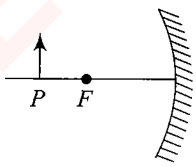


Fig. 1.393

How does the image change if the object is moved from point P toward point F?
a. Its distance from the mirror decreases
b. The size of image decreases
c. Its distance from the mirror increases
d. The size of image increases

2. In Fig. 1.394, light is incident at an angle θ which is slightly greater than the critical angle. Now, keeping the incident angle fixed a parallel slab of refractive index n_3 is placed on surface AB. Which of the following statements are correct?

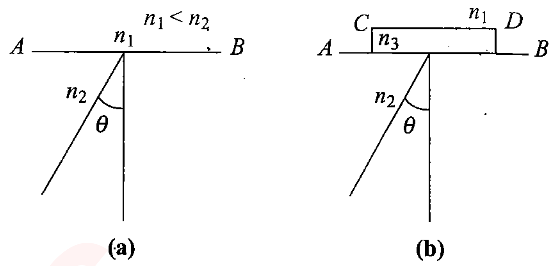


Fig. 1.394

a. Total internal reflection occurs at AB for $n_3 < n_1$
b. Total internal reflection occurs at AB for $n_3 > n_1$
c. The ray will return back to the same medium for all values of n_3
d. Total internal reflection occurs at CD for $n_3 < n_1$

3. In the diagram shown, a light ray is incident on the lower medium boundary at an angle if 45° with the normal. Which of the following statements is/are true?

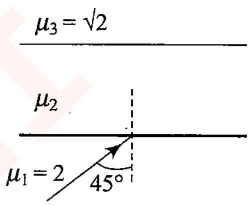


Fig. 1.395

a. If $\mu_2 > \sqrt{2}$, then angle of deviation is 45°
b. If $\mu_2 < \sqrt{2}$, then angle of deviation is 90°
c. If $\mu_2 < \sqrt{2}$, then angle of deviation is 135°
d. If $\mu_2 > \sqrt{2}$, then angle of deviation is 0°

4. In displacement method, the distance between object and screen is 96 cm. The ratio of lengths of two images formed by a converging lens placed between them is 4. Then,
a. ratio of the length of object to the length of shorter image is 2
b. distance between the two positions of the lens is 32 cm
c. focal length of the lens is 64/3 cm
d. when the shorter image is formed on screen, distance of the lens from the screen is 32 cm

5. The distance between an electric lamp and a screen is d = 1 m. A convergent lens of focal length f = 21 cm is placed between the lamp and the lens such that a sharp image of the lamp filament is formed on the screen.
a. The positions of the lens from the lamp for which sharp images are formed on the screen are 35 cm and 65 cm
b. The positions of the lens from the lamp for which sharp images are formed on the screen are 30 cm and 70 cm
c. Magnitude of the difference in magnification is 40/21
d. The size of the lamp filament for which there are two sharp images of 4.5 cm and 2 cm, is 3 cm

6. A lens of focal length 'f' is placed in between an object and screen at a distance 'D'. The lens forms two real images of object on the screen for two of its different positions, a distance 'x' apart. The two real images have magnifications m_1 and m_2, respectively ($m_1 > m_2$). Then,
a. $f = x/(m_1 - m_2)$ **b.** $m_1 m_2 = 1$
c. $f = (D^2 - x^2)/4D$ **d.** $D \geq 4f$.

7. An image of a bright square is obtained on a screen with the aid of a convergent lens. The distance between the square and the lens is 40 cm. The area of the image is nine times larger than that of the square. Select the correct statement(s):
a. Image is formed at a distance of 120 cm from the lens
b. Image is formed at a distance of 360 cm from the lens
c. Focal length of the lens is 30 cm
d. Focal length of the lens is 36 cm

8. Consider the rays shown in the diagram as paraxial. The image of the virtual point object O formed by the lens LL is

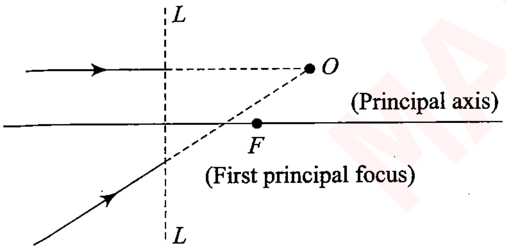


Fig. 1.396

a. virtual
b. real
c. located below the principal axis
d. located to the left of the lens

9. A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in Figs. 1.397 and 1.398. When a ray of white light is incident on the prism parallel to the base

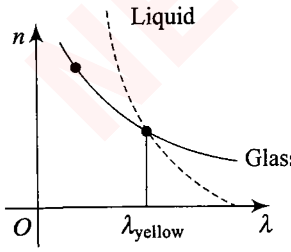


Fig. 1.397

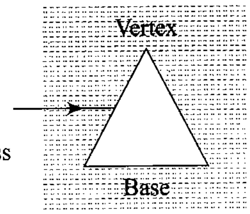


Fig. 1.398

a. yellow ray travels without deviation
b. blue ray is deviated toward the vertex
c. red ray is deviated toward the base
d. there is no dispersion

10. An object AB is placed parallel and close to the optical axis between focus F and center of curvature C of a converging mirror of focal length f as shown in figure. Then,

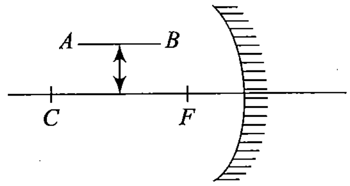


Fig. 1.399

a. image of A will be closer than that of B from the mirror
b. image of AB will be parallel to the optical axis
c. image of AB will be a straight line inclined to the optical axis
d. Image of AB will not be a straight line

11. Which of the following statements is/are correct about the refraction of light from a plane surface when light ray is incident in denser medium. [C is critical angle]
a. The maximum angle of deviation during refraction is $(\pi/2) - C$, it will be at angle of incidence C
b. The maximum angle of deviation for all angles of incidence is $\pi - 2C$, when angle of incidence is slightly greater than C
c. If angle of incidence is less than C, then deviation increases if angle of incidence is also increased
d. If angle of incidence is greater than C, then angle of deviation decreases if angle of incidence is increased

12. A luminous point object is placed at O, whose image is formed at I as shown in the figure. Line AB is the optical axis. Which of the following statement is/are correct?

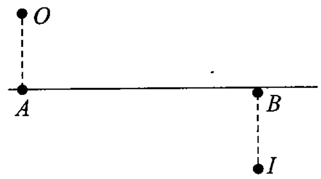


Fig. 1.400

a. If a lens is used to obtain the image, then it must be a converging lens and its optical center will be the intersection point of line AB and OI
b. If a lens is used to obtain the image, then it must be a diverging lens and its optical center will be the intersection point of line AB and OI
c. If a mirror is used to obtain the image, then the mirror must be concave and the object and image subtend equal angles at the pole of the mirror
d. I is a real image

13. Mark the correct statement(s) w.r.t. a concave spherical mirror.
a. For real extended object, it can form a diminished virtual image
b. For real extended object, it can form a magnified virtual image
c. For virtual extended object, it can form a diminished real image
d. For virtual extended object, it can form a magnified real image

14. Mark the correct statement(s) from the following:
- **a.** Image formed by a convex mirror can be real
- **b.** Image formed by a convex mirror can be virtual
- **c.** Image formed by a convex mirror can be magnified
- **d.** Image formed by a convex mirror can be inverted

15. When a real object is placed 25 cm from a lens, a real image is formed. Mark the correct statement(s) from the following:
- **a.** The lens is a converging lens
- **b.** The image may be magnified or diminished
- **c.** The focal length of the lens is less than 25 cm
- **d.** The focal length of the lens may be greater than 25 cm

16. Two converging lenses of focal lengths $f_1 = 10$ cm and $f_2 = 20$ cm are placed at some separation. A parallel beam of light is incident on 1st lens. Then,
- **a.** for emergent beam from 2nd lens to be parallel, the separation between the lenses has to be 30 cm
- **b.** for emergent beam from 2nd lens to be parallel, the separation between the lenses has to be 60 cm
- **c.** if lenses are placed at such a separation that emergent beam from 2nd lens is parallel, then the emergent beam width is 2 cm if original beam has a width of 1 cm
- **d.** if lenses are placed at such a separation that emergent beam from 2nd lens is parallel, then the emergent beam width is 4 cm if original beam width is 1 cm

17. A real object is moving toward a fixed spherical mirror. The image
- **a.** must move away from the mirror
- **b.** may move away from the mirror
- **c.** may move toward the mirror if the mirror is concave
- **d.** must move toward the mirror if the mirror is convex

18. A real point source is 5 cm away from a plane mirror whose reflecting ability is 50%, while the eye of an observer (pupil diameter 5 mm) is 10 cm away from the mirror. Assume that both source and eye are on the same line perpendicular to the surface and refracted rays have no effect on intensity. Then,
- **a.** the area of the mirror used in observing the image of source is $(25\pi/36)$ mm^2
- **b.** the area of the mirror used in observing the image of source is 25π mm^2
- **c.** the ratio of the intensities of light as received by the observer in the presence to that in the absence of mirror is (10/9)
- **d.** the ratio of the intensities of light as received in the presence to that in the absence of mirror is 19/18

19. A plane mirror M is arranged parallel to a wall W at a distance l from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a patch of light on the wall. The mirror moves with velocity v towards the wall.

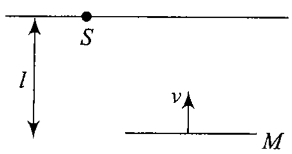


Fig. 1.401

Which of the following statement(s) is/are correct?
- **a.** The patch of light will move with speed v on the wall
- **b.** The patch of light will not move on the wall
- **c.** As the mirror comes closer, the patch of light will become larger and shift away from the wall with speed larger than v
- **d.** The size of the patch of light on the wall remains the same

20. A thin, symmetric double convex lens of power P is cut into three parts A, B, and C as shown. The power of

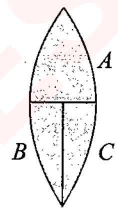


Fig. 1.402

- **a.** A is P
- **b.** A is $2P$
- **c.** B is $P/2$
- **d.** C is $P/4$

21. A diverging lens of focal length f_1 is placed in front of and coaxially with a concave mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement. Then,
- **a.** the beam diameters of the incident and reflected beams must be the same
- **b.** $d = 2|f_2| - |f_1|$
- **c.** $d = |f_2| - |f_1|$
- **d.** if the entire arrangement is immersed in water, the conditions will remain unaltered

22. A converging lens of focal length f_1 is placed in front of and coaxially with a convex mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement, Then,
- **a.** the beam diameters of the incident and reflected beams must be the same
- **b.** $d = f_2 - 2|f_2|$
- **c.** $d = f_1 - |f_2|$
- **d.** if the entire arrangement is immersed in water, the conditions will remain unaltered

23. Which of the following statements are correct?
- **a.** A ray of light is incident on a plane mirror and gets reflected. If the mirror is rotated through an angle θ, then the reflected ray gets deviated through angle 2θ
- **b.** A ray of light gets reflected successively from two mirrors which are mutually inclined. Angular deviation suffered by the ray does not depend upon angle of incidence on first mirror
- **c.** A plane mirror cannot form real image of a real object
- **d.** If an object approaches toward a plane mirror with velocity v, then the image approaches the object with velocity $2v$

24. A fish F, in the pond is at a depth of 0.8 m from the water surface and is moving vertically upward with velocity 2 ms^{-1}. At the same instant, a bird B is at a height of 6 m from the water surface and is moving downward with velocity 3 ms^{-1}. At this instant, both are on the same vertical line as shown in the figure. Which of the following statements are correct?

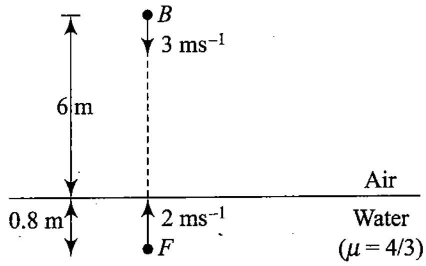


Fig. 1.403

a. Height of B, observed by F (from itself), is equal to 5.30 m
b. Depth of F, observed by B (from itself), is equal to 6.60 m
c. Height of B, observed by F (from itself), is equal to 8.80 m
d. None of the above

25. In the previous question,
a. velocity of B, observed by F (relative to itself), is equal to 4.25 ms^{-1}
b. velocity of B, observed by F (relative to itself), is equal to 6 ms^{-1}
c. velocity of B, observed by F (relative to itself), is equal to 5.50 ms^{-1}
d. velocity of F, observed by B (relative to itself), is equal to 4.50 ms^{-1}

26. Figure 1.404 shows variation of magnification m (produced by a thin convex lens) and distance v of image from pole of the lens. Which of the following statements are correct?

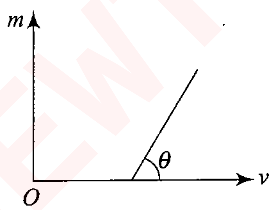


Fig. 1.404

a. Focal length of the lens is equal to intercept on v-axis
b. Focal length of the lens is equal to inverse of slope of the line
c. Magnitude of intercept on m-axis is equal to unity
d. None of the above

Assertion-Reasoning Type

Solutions on page 1.161

Some questions (Assertion-Reason type) are given below. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices a., b., c., and d. out of which **only one** is correct. So select the correct choice

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. Statement I: Although the surfaces of goggle lenses are curved, it does not have any power.
Statement II: In case of goggle, both the curved surfaces have equal radii of curvature and have center of curvature on the same side.

2. Statement I: A beam of white light enters the curved surface of a semicircular piece of glass along the normal. The incoming beam is moved clockwise (so that the angle θ increases), such that the beam always enters along the normal to the curved side. Just before the refracted beam disappears, it becomes predominantly red.

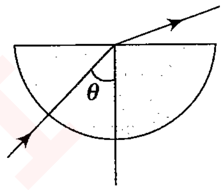


Fig. 1.405

Statement II: The index of refraction for light at the red end of the visible spectrum is more than at the violet end.

3. Statement I: A ray is incident from outside on a glass sphere surrounded by air as shown. This ray may suffer total internal reflection at the second interface.

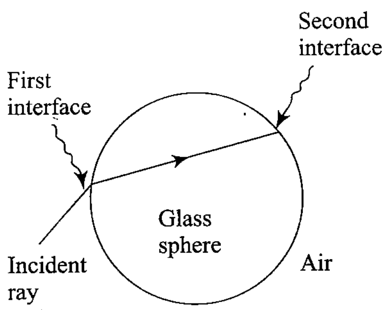


Fig. 1.406

Statement II: For a ray going from a denser to rarer medium, the ray may suffer total internal reflection.

4. Statement I: Keeping a point object fixed, if a plane mirror is moved, the image will also move.
Statement II: In case of a plane mirror, distance of object and its image is equal from any point on the mirror.

5. Statement I: Lights of different colors travel with different speeds in vacuum.
Statement II: Speed of light depends on medium.

6. Statement I: A beam of light rays has been reflected from a rough surface.
Statement II: Amplitude of incident and reflected rays would be different.

7. **Statement I:** A virtual image can be photographed.
Statement II: Only a real image can be formed on a screen.
8. **Statement I:** The focal length of a lens does not depend on the medium in which it is submerged.
Statement II:
$$\frac{1}{f}=\frac{\mu_2-\mu_1}{\mu_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
9. **Statement I:** We cannot produce a real image by plane or convex mirrors under any circumstances.
Statement II: The focal length of a convex mirror is always taken as positive.
10. **Statement I:** A light ray is incident on a glass slab. Some portion of it is reflected and some is refracted. Refracted and reflected rays are always perpendicular to each other.
Statement II: Angle of incidence is equal to angle of reflection.

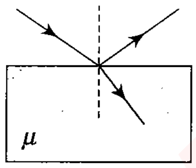


Fig. 1.407

11. **Statement I:** The images formed by total internal reflections are much brighter than those formed by mirrors or lenses.
Statement II: There is no loss of intensity in total internal reflection.
12. **Statement I:** The focal length of a lens does not change when red light is replaced by blue light.
Statement II: The focal length of a lens depends on the color of light used.
13. **Statement I:** A convex lens of focal length f (μ = 1.5) behaves as a diverging lens when immersed in carbon di-sulphide of higher refractive index (μ = 1.65).
Statement II: The focal length of a lens does not depend on the color of light used.
14. **Statement I:** When a light wave travels from a rarer to a denser medium, it loses speed. The reduction in speed implies a reduction in energy carried by the light wave.
Statement II: The energy of a wave is proportional to wave frequency.

Comprehension Type

Solutions on page 1.162

For Problems 1–3

Consider a transparent hemisphere (n = 2) in front of which a small object is placed in air (n = 1) as shown.

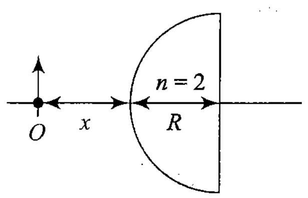


Fig. 1.408

1. For which value of x, of the following, will the final image of the object at O be virtual?
a. $2R$ b. $3R$
c. $R/2$ d. $1.5R$
2. What is the nature of final image of the object when $x = 2R$?
a. Erect and magnified.
b. Inverted and magnified.
c. Erect and same size.
d. Inverted and same size.
3. Consider a ray starting from O which strikes the spherical surface at grazing incidence ($i = 90°$). Taking $x = R$, what will be the angle (from normal) at which the ray may emerge from the plane surface.
a. 90° b. 0°
c. 30° d. 60°

For Problems 4–6

This question concerns a symmetrical lens shown, along with its two focal points. It is made of plastic with n = 1.2 and has focal length f. Four different regions are shown:

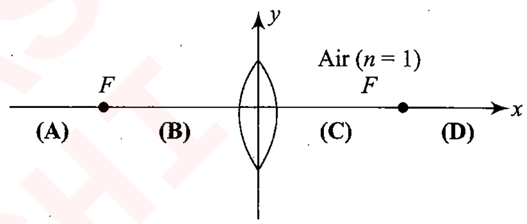


Fig. 1.409

Here,
A. $-\infty < x < -f$ **B.** $-f < x < 0$
C. $0 < x < f$ **D.** $f < x < \infty$

4. If an object is placed somewhere in region (A), in which region does the image appear?
a. A b. B
c. C d. D
5. If incident rays are converging, then in which region does the image appear?
a. A b. B
c. C d. D
6. A second lens is now placed adjacent to the first in contact with it. The new lens is concave, with a focal length $f = -3f$. What is the focal length of the two-lens system? Treat the lenses as thin.
a. A b. B
c. C d. D

For Problems 7–8

A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.

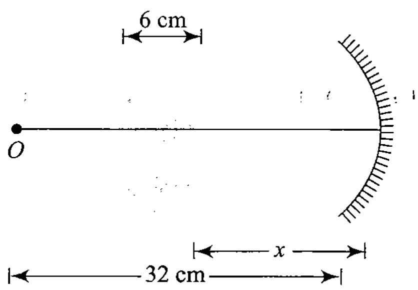


Fig. 1.410

7. Find the position and nature of the final image when the distance x shown in figure, is 5 cm.
   a. 11 cm, virtual   b. 17 cm, real
   c. 14 cm, real   d. 20 cm, virtual

8. Find the position and nature of the final image when the distance x shown in figure, is 20 cm.
   a. 17 cm, virtual   b. 17 cm, real
   c. 12 cm, virtual   d. 15 cm, virtual

For Problems 9–12

Consider the situation in the figure. The bottom of the pot is a reflecting plane mirror, S is a small fish, and T is a human eye. Refractive index of water is μ.

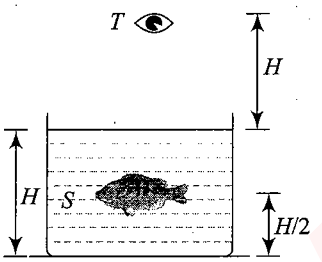


Fig. 1.411

9. At what distance from itself will the fish see the image of the eye by direct observation?
   a. $H\left(\frac{1}{2}+\mu\right)$ b. $H\left(\frac{1}{2}-\mu\right)$
   c. $\frac{H}{2}\left(\frac{1}{2}+\mu\right)$ d. $H\left(\frac{1-\mu}{2}\right)$

10. At what distance from itself will the fish see or observe the image of eye by observing through mirror?
   a. $H\left(\frac{3}{2}+\mu\right)$ b. $H\left(\frac{1}{2}+\mu\right)$
   c. $H\left(\frac{3+\mu}{2}\right)$ d. $2H\left(\frac{3}{2}+\mu\right)$

11. At what distance from itself will the eye see the image of the fish upon direct observation?
   a. $H\left(1+\frac{1}{2\mu}\right)$ b. $2H\left(1+\frac{1}{2\mu}\right)$
   c. $2H\left(1-\frac{1}{2\mu}\right)$ d. $H\left(\frac{\mu+1}{2}\right)$

12. At what distance from itself will the eye see the image of the fish by observing from the mirror?
   a. $2\left(H+\frac{3}{2\mu}\right)$ b. $H+\frac{3}{2\mu}$
   c. $H\left(1+\frac{3}{2\mu}\right)$ d. $H\left(1-\frac{3}{2\mu}\right)$

For Problems 13–14

A glass sphere of radius $2R$ and refractive index n has a spherical cavity of radius R, concentric with it.

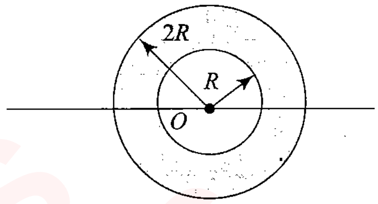


Fig. 1.412

13. When viewer is on left side of the hollow sphere, what will be the shift in position of the object?
   a. $\frac{(n+1)}{(n-1)}R$, right   b. $\frac{(n-1)}{(n+1)}R$, right
   c. $\frac{(2n-1)}{(2n+1)}R$, left   d. $\frac{2(n-1)}{(n+1)}R$, left

14. When viewer is on right side of the hollow sphere, what will be the apparent change in position of the object?
   a. $\frac{(n-1)}{(3n+1)}R$, toward left   b. $\frac{(n+1)}{(3n-1)}R$, toward left
   c. $\frac{(n+1)}{(3n+1)}R$, toward right   d. $\frac{(n-1)}{(3n+1)}R$, toward right

For Problems 15–16

A thin equiconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in figure. The space between the lens and the mirror is filled with a liquid of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image.

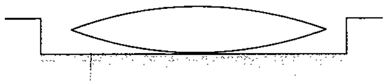


Fig. 1.413

15. The radius of curvature of the convex surface is
   a. 10 cm   b. 15 cm
   c. 20 cm   d. 25 cm

16. If another liquid is filled instead of water, the object and the image coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.
   a. 1.6   b. 2.6
   c. 2.8   d. 3.2

For Problems 17–19

An equiconvex lens, f_1 = 10 cm, is placed 40 cm in front of a concave mirror, f_2 = 7.50 cm, as shown in figure. An object, 2 cm high, is placed 20 cm to the left of the lens.

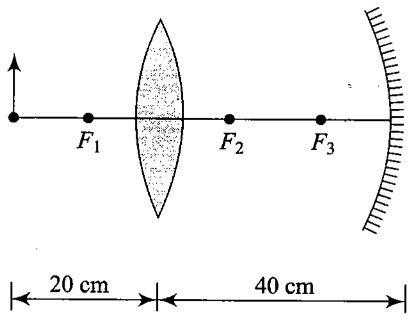


Fig. 1.414

17. The image formed by the lens as rays travel to the right is
a. real and erect
b. virtual and erect
c. real and inverted
d. virtual and inverted

18. The image formed after rays reflect from the mirror is
a. 12 cm to the right of the mirror
b. 12 cm to the left of the mirror
c. 10 cm to the left of the mirror
d. 20 cm to the left of the mirror

19. The final image is formed when leftward traveling rays once again pass through the lens. Find the overall magnification.
a. 0.333
b. 0.52
c. –0.333
d. –0.52

For Problems 20–22

A biconvex lens, $f_1 = 20$ cm, is placed 5 cm in front of a convex mirror, $f_2 = 15$ cm. An object of length 2 cm is placed at a distance of 10 cm from the lens.

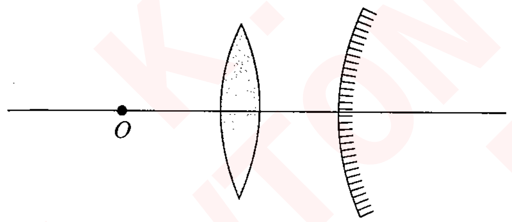


Fig. 1.415

20. Nature of the image formed by the lens as the rays travel to the right is
a. real, inverted, and magnified
b. real, erect, and magnified
c. virtual, inverted, and magnified
d. virtual, erect, and magnified

21. Nature of the image formed after the rays reflect from the mirror is
a. virtual, erect, and magnified
b. virtual, erect, and diminished
c. virtual, inverted, and diminished
d. real, erect, and diminished

22. Location and nature of the final image after the leftward traveling rays once again pass through the lens are
a. 51.1 cm, real, erect, and diminished
b. 51.1 cm, real, inverted, and magnified
c. 51.1 cm, real, erect, and magnified
d. 63 cm, real, erect, and magnified

For Problems 23–24

A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between the object and image planes is 1.8 m. The magnification of the image formed by one of the half lenses is 2.

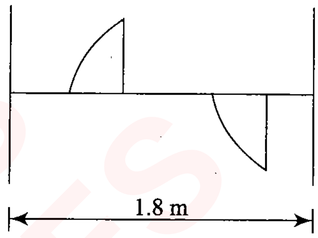


Fig. 1.416

23. Find the focal length of the lens used.
a. 0.4 m
b. 0.6 m
c. 1 m
d. 2 m

24. Find the separation between the two halves of the thin plano-convex lens.
a. 0.4 m
b. 0.6 m
c. 0.2 m
d. 0.8 m

For Problems 25–27

A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in the figure.

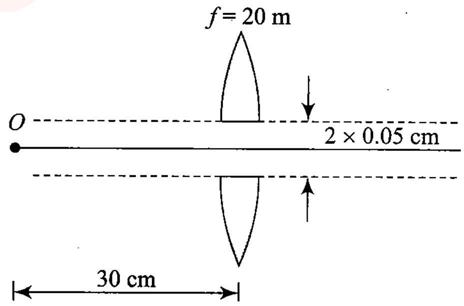


Fig. 1.417

25. What will be the location of the image?
a. 30 cm, right of lens
b. 60 cm, right of lens
c. 70 cm, left of lens
d. 40 cm, left of lens

26. If this arrangement will generate more than one image, then what will be the total number of images?
a. 2
b. 4
c. 6
d. 5

27. Find the spacing between the images so formed.
a. 0.1 cm
b. 0.5 cm
c. 0.3 cm
d. 1 c

For Problems 28–29

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface.

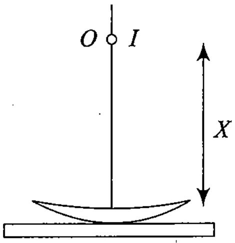


Fig. 1.418

28. Where should a pin be placed on the optic axis such that its image is formed at the same place?

a. $x = 5$ cm b. $x = 20$ cm
c. $x = 15$ cm d. $x = 25$ cm

29. If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin?

a. $\Delta x = 1.15$ cm, up b. $\Delta x = 3.15$ cm, down
c. $\Delta x = 0.05$ cm, up d. $\Delta x = 0.15$ cm, down

For Problems 30–31

Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d ($d < f_1$ and $d < f_2$) and their centers are displaced by a vertical separation as shown in the figure. A parallel beam of rays coming from left. Take the origin of coordinates O at the center of first lens.

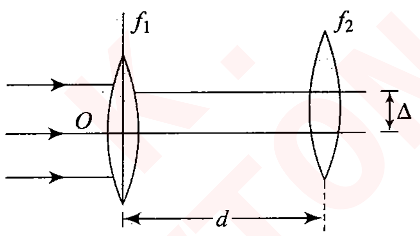


Fig. 1.419

30. Find the y-coordinate of the focal point of this lens system.

a. $\dfrac{(f_1+d)\Delta}{(f_1+f_2-d)}$ b. $\dfrac{2(f_1+d)}{(f_1+f_2-d)}$

c. $\dfrac{2(f_1-d)\Delta}{(f_1+f_2+d)}$ d. $\dfrac{(f_1-d)\Delta}{(f_1+f_2-d)}$

31. Find the x-coordinate in the same problem from the focal point of this lens system.

a. $\dfrac{d(f_1-d)+f_1f_2}{(f_1+f_2-d)}$ b. $\dfrac{f_1f_2}{(f_1+f_2-d)}$

c. $\dfrac{d(f_1-d)}{(f_1+f_2-d)}$ d. $\dfrac{2d(f_1+d)-f_1f_2}{(f_1+f_2-d)}$

For Problems 32–33

A parallel beam of light falls successively on a thin convex lens of focal length 40 cm and then on a thin convex lens of focal length 10 cm as shown in Fig. 1.420(a).

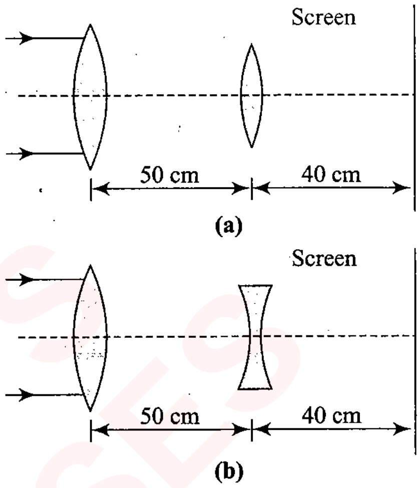


Fig. 1.420

In figure (b) the second lens is an equiconcave lens of focal length 10 cm and made of a material of refractive index 1.5. In both the cases, the second lens has an aperture equal to 1 cm.

32. Compare the area illuminated by the beam of light on the screen, which passes through the second lens in the two cases. The ratio (A_2/A_1) will be

a. 72/5 b. 81/4
c. 56/3 d. 29/2

33. Now, a liquid of refractive index μ is filled to the right of the second lens in case B such that the area illuminated in both the cases is the same. Determine the refractive index of the liquid.

a. 1 b. 2.5
c. 3 d. 1.5

For Problems 34–35

Two identical plano-convex lenses L_1 ($\mu_1 = 1.4$) and L_2 ($\mu_2 = 1.5$) of radii of curvature $R = 20$ cm are placed as shown in the figure.

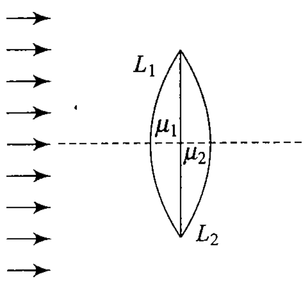


Fig. 1.421

34. Find the position of the image of the parallel beam of light relative to the common principal axis.

a. 100/7 cm b. 200/9 cm
c. 31.2 cm d. 21.8 cm

35. Now, the second lens is shifted vertically downward by a small distance 4.5 mm and the extended parts of L_1 and L_2 are blackened as shown in figure. Find the new position of the image of the parallel beam.

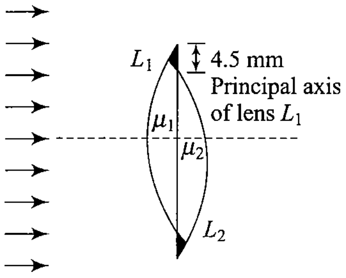


Fig. 1.422

a. 200/9 cm behind the lens 2.5 mm below the principal axis of L_1
b. 100/9 cm in front of the lens 2 mm below the principal axis of L_1
c. 200/9 cm in front of the lens 2.5 mm below the principal axis of L_1
d. 100/9 cm behind the lens 2 mm below the principal axis of L_1

For Problems 36–37

A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm.

36. Find the magnification and nature of the image when point O is 10 cm behind the mirror.
a. 2(virtual, inverted) b. 3 (real, inverted)
c. 5 (real, erect) d. 2 (real, erect)

37. Similarly, as in above question when point O is 30 cm behind the mirror.
a. 2 (virtual, inverted) b. 3 (real, inverted)
c. 3 (virtual, enlarged) d. +1 (real, enlarged)

For Problems 38–39

A ray of light is incident normally at face AB of a glass prism $n = 3/2$. Find the largest value for the angle ϕ so that the ray is totally reflected at face AC if the prism is immersed:

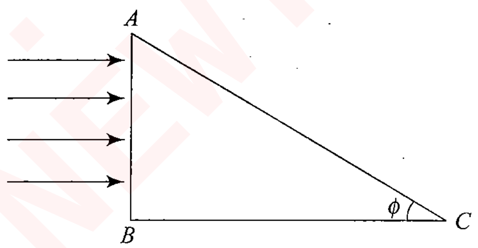


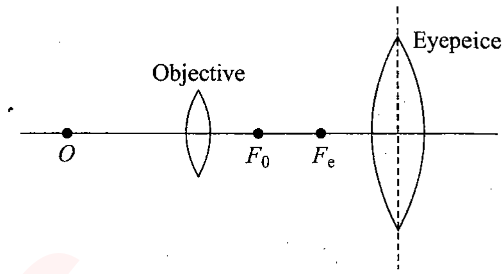
Fig. 1.423

38. in air
a. $\cos^{-1}(2/3)$ b. $\cos^{-1}(3/4)$
c. $\sin^{-1}(3/5)$ d. $\sin^{-1}(5/8)$

39. in water
a. $\cos^{-1}(2/3)$ b. $\sin^{-1}(5/8)$
c. $\cos^{-1}(8/9)$ d. $\sin^{-1}(3/5)$

For Problems 40–42

The schematic diagram of a compound microscope is shown in the adjacent figure. Its main components are two convex lenses: one acts as the main magnifying lens and is referred to as the objective, and another lens called the eyepiece. The two lenses act independently of each other when bending light rays.



F_0 = focal point of objective
F_e = focal point of eyepiece

Fig. 1.424

Light from the object (O) first passes through the objective and an enlarged, inverted first image is formed. The eyepiece then magnifies this image. Usually, the magnification of the eyepiece is fixed (either $\times 10$ or $\times 15$) and three rotating objective lenses are used: $\times 10$, $\times 40$ and $\times 60$. Angular magnification is defined as the angle subtended by the final image at the eye to the angle subtended by the object placed at least distance of distinct vision (≈ 25 cm) when viewed by the naked eye.

40. Based on the passage, what type of image would have to be produced by the objective ?
a. Either virtual or real
b. Virtual
c. Real
d. It depends on the focal length of the lens

41. Where would the first image have to be produced by the objective relative to the eyepiece such that a second, enlarged image would be generated on the same side of the eyepiece as the first image ? (first image distance = d_i from the eyepiece)
a. $d_i < F_e$ b. $d_i = F_e$
c. $2 \times F_e > d_i > F_e$ d. $d_i > 2 \times F_e$.

42. Two compound microscopes A and B were compared. Both had objectives and eyepieces with the same magnification but A gave an overall magnification that was greater than that of B. Which of the following is a possible explanation?
a. The distance between the objective and eyepiece in A is greater than the corresponding distance in B
b. The distance between the objective and eyepiece in A is less than the corresponding distance in B
c. The eyepiece and objective positions were reversed in A
d. The eyepiece and objective positions were reversed in B

For Problems 43–45

A ray of light enters a spherical drop of water of refractive index μ as shown in the figure.

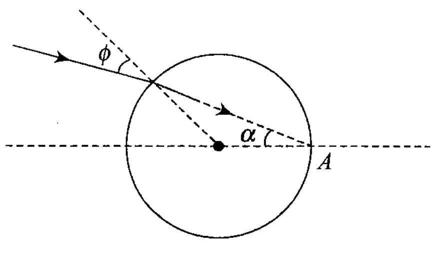


Fig. 1.425

43. Select the correct statement:
a. Incident rays are partially reflected at point A
b. Incident rays are totally reflected at point A
c. Incident rays are totally transmitted through A
d. None of these

44. An expression of the angle between incident ray and emergent ray (angle of deviation) as shown in the figure is

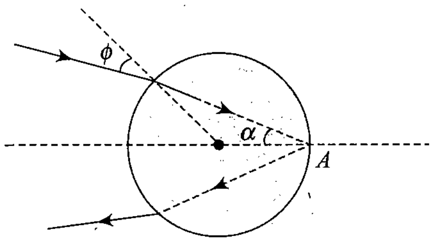


Fig. 1.426

a. 0° b. ϕ
c. $\alpha - \phi$ d. $\pi - 4\alpha + 2\phi$.

45. Consider the figure of question 60, the angle ϕ for which minimum deviation is produced will be given by

a. $\cos^2 \varphi = \frac{\mu^2 + 1}{3}$ b. $\cos^2 \varphi = \frac{\mu^2 - 1}{3}$
c. $\sin^2 \varphi = \frac{\mu^2 + 1}{3}$ d. $\sin^2 \varphi = \frac{\mu^2 - 1}{3}$

For Problems 46–50

The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained, the curvature of lens increases. That means, radius of curvature decreases and focal length decreases. For a clear vision, the image must be on the retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye lens. It is about 2.5 cm for a grown up person.

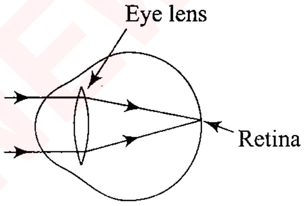


Fig. 1.427

A person can theoretically have clear vision of an object situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown up person, minimum distance of the object should be around 25 cm.

A person suffering from eye defects uses spectacles (eye glass). The function of lens of spectacles is to form the image of the objects within the range in which the person can see clearly. The image of the spectacle lens becomes object for the eye lens and whose image is formed on the retina.

The number of spectacle lens used for the remedy of eye defect is decided by the power of the lens required and the number of spectacle lens is equal to the numerical value of the power of lens with sign. For example, if power of the lens required is +3 D (converging lens of focal length 100/3 cm), then number of lens will be +3.

For all the calculations required, you can use the lens formula and lensmaker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between the eye lens and the spectacle lens.

46. Minimum focal length of eye lens of a normal person is
a. 25 cm b. 2.5 cm
c. 25/9 cm d. 25/11 cm

47. Maximum focal length of a eye lens of a normal person is
a. 25 cm b. 2.5 cm
c. $\frac{25}{9}$ cm d. $\frac{25}{11}$ cm

48. A near sighted man can clearly see objects only upto a distance of 100 cm and not beyond this. The number of the spectacle lenses necessary for the remedy of this defect will be
a. +1 b. 1
c. +3 d. 3

49. A far sighted man cannot see objects clearly unless they are at least 100 c.. from his eyes. The number of the spectacle lenses that will make his range of clear vision equal to an average grown up person will be
a. +1 b. 1
c. +3 d. 3

50. A person can see objects clearly from distance 10 cm to ∞. Then, we can say that the person is
a. normal sighted person
b. near-sighted person
c. far-sighted person
d. a person with exceptional eye having no eye defect

For Problems 51–55

The image of a white object in white light formed by a lens is usually colored and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. As μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in Fig. 1.428.

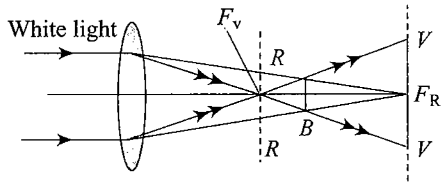


Fig. 1.428

As a result of this, in case of convergent lens if a screen is placed at F_V center of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e. center will be red and focused while sides violet and blurred. The difference between F_V and F_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,

$$\text{L.C.A} = F_R - F_V = -dF \text{ with } dF = F_v - F_R \quad \text{(i)}$$

However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{(ii)}$$

$$\Rightarrow \quad \frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{(iii)}$$

Dividing Eq. (iii) by Eq. (ii), we get

$$-\frac{df}{f} = \frac{d\mu}{(\mu-1)} = \omega$$

$$\left[\omega = \frac{d\mu}{(\mu-1)}\right] = \text{dispersive power} \quad \text{(iv)}$$

And hence, from Eqs. (i) and (iv)

$$\text{L.C.A} = -dF = \omega f$$

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism:

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}; \quad \text{i.e.} \quad -\frac{dF}{F^2} = \frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if $dF = 0$

$$\text{i.e.,} \quad \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0, \quad \text{i.e.,} \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{(v)}$$

The condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition is called achromatic lens. From this condition, i.e., from Eq. (v), it follows that:

(i) the two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$

i.e., the combination will not behave as a lens, but as a plane glass plate.

(ii) As ω_1 and ω_2 are positive quantities, for Eq. (v) to hold, f_1 and f_2 must be of opposite nature, i.e., if one of the lenses is converging, the other must be diverging.

(iii) If the achromatic combination is convergent,

$$f_c < f_D \text{ and as } -\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D} \quad \Rightarrow \quad \omega_C < \omega_D$$

i.e., in a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than the divergent one.

51. Chromatic aberration in the formation of image by a lens arises because:
- **a.** of non-paraxial rays
- **b.** the radii of curvature of the two sides are not same
- **c.** of the defect in grinding
- **d.** the focal length varies with wavelength

52. Chromatic aberration of a lens can be corrected by
- **a.** providing different suitable curvatures of its two surfaces
- **b.** proper polishing of its two surfaces
- **c.** suitably combining it with another lens
- **d.** reducing its aperture

53. A combination is made of two lenses of focal lengths f and f' in contact; the dispersive powers of the materials of the lenses are ω and ω'. The combination is achromatic when:
- **a.** $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = 2f$
- **b.** $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = f/2$
- **c.** $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -f/2$
- **d.** $\omega = \omega_0 \omega' = 2\omega_0$, $f = -2f$

54. The dispersive power of crown and flint glasses are 0.02 and 0.04, respectively. An achromatic converging lens of focal length 40 cm is made by keeping two lenses, one of crown glass and the other of flint glass, in contact with each other. The focal lengths of the two lenses are:
- **a.** 20 cm and 40 cm
- **b.** 20 cm and 40 cm
- **c.** –20 cm and 40 cm
- **d.** 10 cm and –20 cm

55. Chromatic aberration in a spherical concave mirror is proportional to:
- **a.** f
- **b.** f^2
- **c.** $1/f$
- **d.** none of these

For Problems 56–59

The figure is a scaled diagram of an object and a converging lens surrounded by air. F is the focal point of lens as shown.

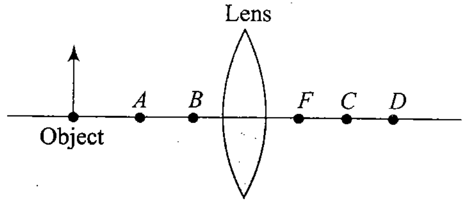


Fig. 1.429

56. At which of the labeled points will image be formed?
- **a.** A
- **b.** B
- **c.** C
- **d.** D

57. Which option described the image most accurately?
- **a.** Real, erect
- **b.** Real, inverted
- **c.** Virtual, erect
- **d.** Virtual, inverted

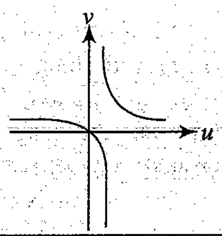
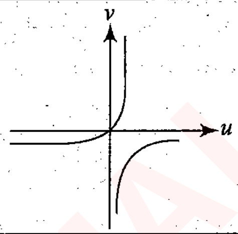
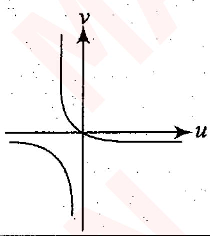
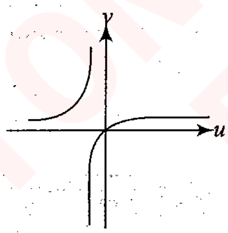
58. If a parallel beam of blue light is focused at F, then parallel beam of red light is focused at
- **a.** F
- **b.** D
- **c.** to the left of and close to F
- **d.** to the right of and close to F

59. The whole system is immersed in a liquid having refractive index greater than R.I. of the lens material. Then, mark the correct option for this new situation.
- **a.** The image will be real
- **b.** The image will be inverted
- **c.** The image will be formed on the same side of the lens as the object is
- **d.** The image will be enlarged relative to the object

Matching Column Type

Solutions on page 1.169

1. Match the correct u-v graph with optical system using Cartesian sign conventions.

Column I	Column II
a. Convex mirror	p. 
b. Convex lens	q. 
c. Concave mirror	r. 
d. Concave lens	s. 

2.

Column I	Column II
a. Object placed between optic center and 1st principal focus in a diverging lens	p. Image is inverted
b. Object placed between optic center and 1st principal focus of a converging lens	q. Image is erect
c. Object placed between optic center and 2nd principal focus of a diverging lens	r. Image is of grater size than the object
d. Object placed between optic center and 2nd principal focus of a converging lens	s. Image is of smaller size than the object

3. Four particles are moving with different velocities in front of a stationary plane mirror (lying in y–z plane). At $t = 0$, velocity of A is $\vec{v}_A = \hat{i}$, velocity of B is $\vec{v}_B = -\hat{i} + 3\hat{j}$, velocity of C is $\vec{v}_C = 5\hat{i} + 6\hat{j}$, velocity of D is $\vec{v}_D = 3i - \hat{j}$. Acceleration of particle A is $\vec{a}_A = 2\hat{i} + \hat{j}$ and acceleration of particle C is $\vec{a}_C = 2t\hat{j}$. The particles B and D move with uniform velocity (Assume no collision to take place till $t = 2$ seconds). All quantities are in S.I. units. Relative velocity of image of object A with respect to object A is denoted by $\vec{v}_{A,A}$. Velocities of images relative to corresponding object are given in column I and their values are given in column II at $t = 2$ second. Match column I with corresponding values in column II.

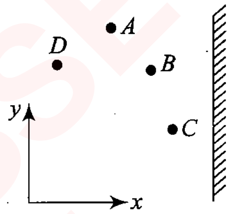


Fig. 1.430

Column I	Column II
a. $\vec{V}_{A,A}$	p. $2\hat{i}$
b. $\vec{V}_{B',B}$	q. $-6\hat{i}$
c. $\vec{V}_{C',C}$	r. $-12\hat{i} + 4\hat{j}$
d. $\vec{V}_{D',D}$	s. $-10\hat{i}$

4. Match the following:

Column I	Column II
a. Real object, Real image	p. Concave mirror
b. Virtual object, Virtual image	q. Convex mirror
c. Real object, virtual image	r. Plane mirror
d. Virtual object, Real image	s. Refraction from a plane surface

5. An object O (real) is placed at focus of an equi-biconvex lens as shown in Fig. 1.431. The refractive index of the lens is $\mu = 1.5$ and the radius of curvature of either surface of lens is R. The lens is surrounded by air. In each statement of column I, some changes are made to situation given above and information regarding final image formed as a result is given in Column II. The distance between lens and object is unchanged in all statements of column I. Match the statements in columnI with resulting image in column II.

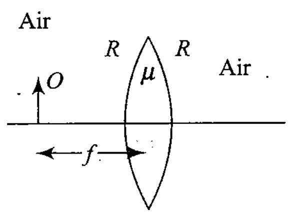
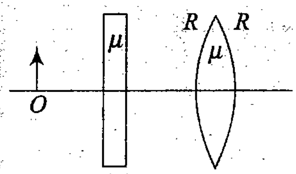
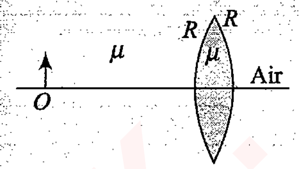


Fig. 1.431

Column I	Column II
a. If the refractive index of the lens is doubled (that is, made 2μ,) then	**p.** Final image is real
b. If the radius of curvature is doubled (that is, made $2R$,) then	**q.** final image is virtual
c. If a glass slab of refractive index $\mu = 1.5$ is introduced between the object and lens as shown, then 	**r.** final image becomes smaller in size in comparison to size of image before the change was made
d. If the left side of lens is filled with a medium of refractive index $\mu = 1.5$ as shown, then 	**s.** final image is of same size as the object

6. Match the following:

Column I	Column II
a. A convex lens in a denser medium will behave like a	**p.** converging lens
b. A concave lens in a rarer medium will behave like a	**q.** diverging lens
c. A plano-convex lens silvered on its curved surface and placed in air will behave like a	**r.** concave mirror
d. A planoconcave lens silvered on its plane surface and placed in air will behave like a	**s.** convex mirror

7. An object O is kept perpendicular to the principal axis of a spherical mirror. Each situation (a, b, c and d) gives object coordinate u in centimeter with sign, the type of mirror, and then the distance (centimeters with sign) between the focal point and the pole of the mirror. On the right side information, regarding the image is given.

Correctly, match the situation on the left side with the images described on the right side.

Situation	u	Mirror	Image
a.	–18	Concave, 12	**p.** Real, erect, enlarged
b.	–12	Concave, 18	**q.** Virtual, erect, diminsihed
c.	–8	Convex, 10	**r.** Real, inverted, enlarged
d.	–10	Convex, 8	**s.** Virtual, erect, enlarged

8. A small particle is placed at the pole of a concave mirror and then moved along the principal axis to a large distance. During the motion, the distance between the pole of the mirror and the image is measured. The procedure is then repeated with a convex mirror, a concave lens, and a convex lens. The graph is plotted between image distance and object distance. Match the curves shown in the graph with the mirror or lens that is corresponding to it. (Curve 1 has two segments.)

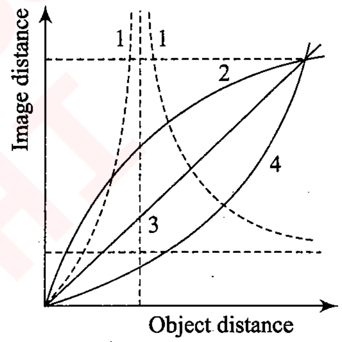


Fig. 1.432

Column I (Lens/Mirror)	Column II (Curve)
a. Converging lens	**p.** 1
b. Converging mirror	**q.** 2
c. Diverging lens	**r.** 3
d. Diverging mirror	**s.** 4

9. A white light ray is incident on a glass prism, and it create four refracted rays A, B, C, and D. Match the refracted rays with the colors given (1 and D are rays due to total internal reflection):

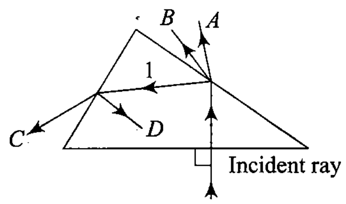
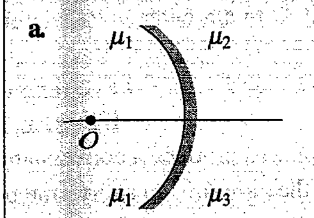
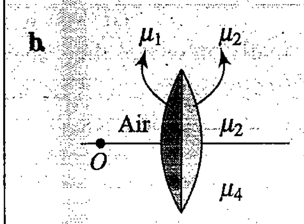
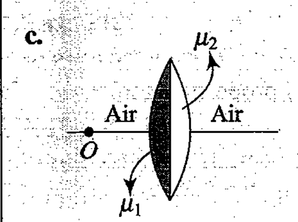
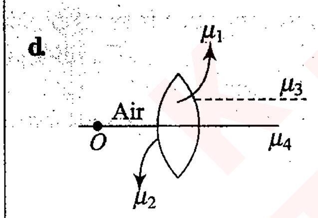


Fig. 1.433

Column I (Ray)	Column II (Colour)
a. A	**p.** red
b. B	**q.** green
c. C	**r.** yellow
d. D	**s.** blue

10. Match the entries of column I with entries of column II:

Column I	Column II
a. 	**p.** Number of images formed is 1.
b. 	**q.** Number of images formed is 2.
c. 	**r.** Number of images formed is 3.
d. 	**s.** Number of images formed is 4.

Integer Answer Type

Solutions on page 1.172

1. The magnification of an object placed in front of a convex lens is +2. The focal length of the lens is 2.0 metres. Find the distance by which object has to be moved to obtain a magnification of –2 (in metres).
2. A point object is placed at a distance of 25 cm from a convex lens. Its focal length is 22 cm. A glass slab of refractive index 1.5 is inserted between the lens and the object, then the image is formed at infinity. Find the thickness of the glass slab (in cm),
3. As shown in the figure, light is incident normally on one face of the prism. A liquid of refractive index μ is placed on the horizontal face AC. The refractive index of prism is 3/2. If total internal reflection taken place on face AC, m should be less than $\frac{I\sqrt{3}}{4}$, where I is an integer. Find the value of I.

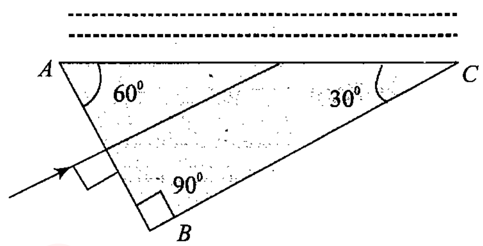


Fig. 1.434

4. Where should a convex lens of focal length 9 cm be placed (in cm) between two point sources S_1 and S_2 which are 24 cm apart, so that images of both the sources are formed at the same place. You have to find distance of lens from S_1 or S_2 whichever is lesser.
5. A ray of light is incident at an angle of 60° on one face of a prism which has refracting angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. If the refractive index of the material of the prism is $\mu = \sqrt{a}$, find the value of a.
6. A container of uniform cross-section has a height of 14 m. Upto what height (in metre) water of refractive index 4/3 should be filled inside the container so that container seems to be half filled for normal viewing.
7. What is the velocity (in cm/s) of image in situation shown below. (O = object, f = focal length). Object moves with velocity 10 cm/s and mirror moves with velocity 2 cm/s as shown.

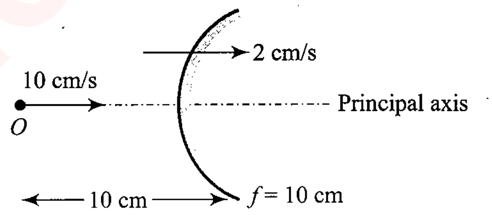


Fig. 1.435

8. An object is placed 50 cm from a screen as shown.

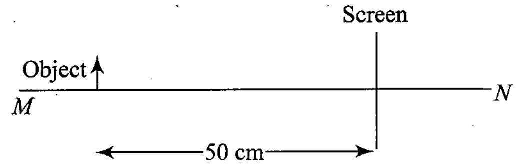


Fig. 1.436

A converging lens is moved such that line MN is its principal axis. Sharp images are formed on the screen in two positions of lens separated by 30 cm. Find the focal length of the lens in cm.

9. An extended object of size 2 mm is placed on the principal axis of a converging lens of focal length 10 cm. It is found that when the object is placed perpendicular to the principal axis the image formed is 4 mm in size. The size of image when it is placed along the principal axis is ________ mm.
10. A point object is placed at a distance 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and object, image is formed at . Thickness t is found to be K times of 5 cm. Find K.

Archives Solutions on page 1.173

Fill in the Blanks Type

1. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then d is equal to ________ cm. **(IIT-JEE, 1985)**

2. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become ________ cm. **(IIT-JEE, 1987)**

3. A slab of a material of refractive index 2 shown in the figure, has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is ________. **(IIT-JEE, 1991)**

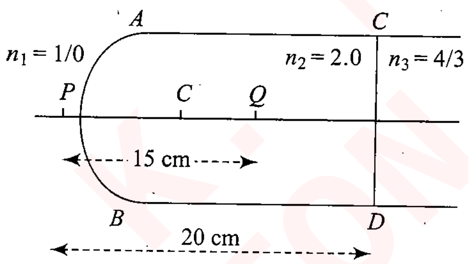


Fig. 1.437

4. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is ________. **(IIT-JEE, 1991)**

5. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is ________. **(IIT-JEE, 1992)**

6. The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is ________ than the wavelength of visible light. **(IIT-JEE, 1992)**

7. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is ________ **(IIT-JEE, 1992)**

8. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7} m. The first diffraction minima on either side of the central diffraction maximum are separated by 2×10^{-3} m. The width d of the slit is ________ m. **(IIT-JEE, 1997)**

9. A light of wavelength 6000 Å, in air, enters a medium with refractive index 1.5. Inside the medium, its frequency is ________ Hz and its wavelength is ________ Å. **(IIT-JEE, 1997)**

10. Two thin lenses, when in contact, produce a combination of power +10 diopters. When they are 0.25 m apart, the power reduces to +6 diopters. The focal lengths of the lenses are m and ________ m. **(IIT-JEE, 1997)**

11. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is ________ degrees. **(IIT-JEE, 1997)**

True or False Type

1. The setting sun appears higher in the sky than it really is. **(IIT-JEE, 1980)**

2. The intensity of light at a distance 'r' from the axis of a long cylindrical source is inversely proportional to 'r'. **(IIT-JEE, 1981)**

3. A convex lens of focal length 1 meter and a concave lens of focal length 0.25 meter are kept 0.75 meter apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **(IIT-JEE, 1983)**

4. A beam of white light passing through a hollow prism gives no spectrum. **(IIT-JEE, 1983)**

5. A parallel beam of white light falls on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm, respectively for the mean wavelength in white light. On the other side of the lens system, one sees colored patterns with violet color at the outer edge. **(IIT-JEE, 1988)**

Multiple Choice Questions with One Correct Answer Type

1. When a ray of light enters a glass slab from air. **(IIT-JEE, 1980)**
   a. its wavelength decreases
   b. its wavelength increases
   c. its frequency increases
   d. neither its wavelength nor its frequency changes

2. In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is **(IIT-JEE, 1981)**
   a. unchanged
   b. halved
   c. doubled
   d. quadrupled

3. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam normally on the face AB is totally reflected to reach on the face BC if **(IIT-JEE, 1981)**

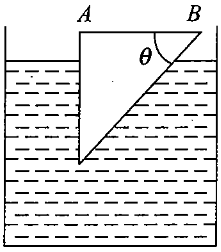


Fig. 1.438

a. $\sin\theta \geq \frac{8}{9}$
b. $\frac{2}{3} < \sin\theta \geq < \frac{8}{9}$
c. $\sin\theta \leq \frac{2}{3}$
d. $\sin\theta \leq \frac{8}{9}$

4. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination is (IIT-JEE, 1982)
a. -1.5 dioptres
b. -6.5 dioptres
c. $+6.5$ dioptres
d. $+6.67$ dioptres

5. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (see figure). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is (IIT-JEE, 1983)

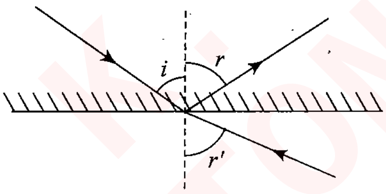


Fig. 1.439

a. $\sin^{-1}(\tan r)$
b. $\sin^{-1}(\tan i)$
c. $\sin^{-1}(\tan r')$
d. $\tan^{-1}(\sin i)$

6. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are (IIT-JEE, 1988)
a. $5I$ and I
b. $5I$ and $3I$
c. $9I$ and I
d. $9I$ and $3I$

7. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to (IIT-JEE, 1988)

a. $b\left(\frac{u-f}{f}\right)^{1/2}$
b. $b\left(\frac{b}{u-f}\right)^{1/2}$
c. $b\left(\frac{u-f}{f}\right)$
d. $b\left(\frac{f}{u-f}\right)^{2}$

8. A beam of light consisting of red, green, and blue colors is incident on a right-angled prism. The refractive indices of the material of the prism for the above red, green, and blue wavelengths are 1.39, 1.44, and 1.47, respectively. The prism will (IIT-JEE, 1989)

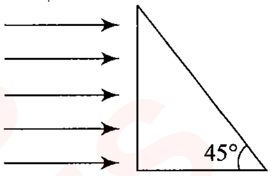


Fig. 1.440

a. separate part of the red color from the green and blue colors
b. separate part of the blue color from the red and green colors
c. separate all the three colors from one another
d. not separate even partially any color from the other two colors

9. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is (IIT-JEE, 1990)
a. 5.33°
b. 4°
c. 3°
d. 2.6°

10. Two thin convex lenses of focal length f_1 and f_2 are separated by a horizontal distance d (where $d < f_1$, $d < f_2$) and their centers are displaced by a vertical separation Δ as shown in the figure. (IIT-JEE, 1993)

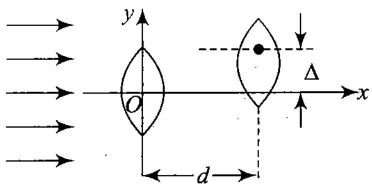


Fig. 1.441

Taking the origin of coordinates O at the center of the first lens, the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by:

a. $x = \frac{f_1 f_2}{f_1 + f_2}$, $y = \Delta$
b. $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}$, $y = \frac{\Delta}{f_1 + f_2}$
c. $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$
d. $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

11. Spherical aberration in a thin lens can be reduced by **(IIT-JEE, 1994)**

a. using a monochromatic light
b. using a doublet combination
c. using a circular annular mark over the lens
d. increasing the size of the lens.

12. A beam of light of wavelength 600 nm from a distance source falls on a single slit 1 mm wide and a resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of central bright fringe is **(IIT-JEE, 1994)**

a. 1.2 cm b. 1.2 mm
c. 2.4 cm d. 2.4 mm

13. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel monochromatic rays enter the prism parallel to each other in air as shown. The rays emerging from the opposite faces **(IIT-JEE, 1995)**

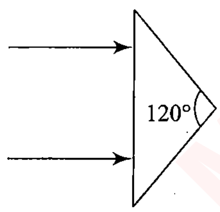


Fig. 1.442

a. are parallel to each other
b. are diverging
c. make an angle of $2[\sin^{-1}(0.72) - 30°]$ with each other
d. make an angle of $2\sin^{-1}(0.72)$ with each other

14. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing **(IIT-JEE, 1995)**

a. a concave mirror of suitable focal length
b. a convex mirror of suitable focal length
c. a convex lens of focal length less than 0.25 m
d. a concave lens of suitable focal length.

15. The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm, respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. The distance, in cm, of the object and the image produced by the objective, measured from the objective lens, are respectively **(IIT-JEE, 1995)**

a. 2.4 and 12.0 b. 2.4 and 15.0
c. 2.0 and 12.0 d. 2.0 and 3.0

16. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is **(IIT-JEE, 1997)**

a. +1.5 b. −1.5
c. +6.67 d. −6.67

17. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration **(IIT-JEE, 1998)**

a. is absent
b. is smaller if the curved surface of the lens faces the object
c. is smaller if the plane surface of the lens faces the object
d. is the same whichever side of the lens faces the object

18. A concave mirror is placed on a horizontal table, with its axis directed vertically upward. Let O be the pole of the mirror and C its center of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be **(IIT-JEE, 1998)**

a. real, and will remain at C
b. real, and located at a point between C and ∞
c. virtual, and located at a point between C and O
d. real, and located at a point between C and O

19. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The center of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O, and $PO = OQ$. The distance PO is equal to **(IIT-JEE, 1998)**

a. $5R$ b. $3R$
c. $2R$ d. $1.5R$

20. A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a **(IIT-JEE, 1999)**

a. convergent lens of focal length 3.5 R
b. convergent lens of focal length 3.0 R
c. divergent lens of focal length 3.5 R
d. divergent lens of focal length 3.0 R

21. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with **(IIT-JEE, 2000)**

a. air and placed in air b. air and immersed in L_1
c. L_1 and immersed in L_2 d. L_2 and immersed in L_1.

22. A point source of light B is placed at a distance L in front of the center of a mirror of width 'd' hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in figure. The greatest distance over which he can see the image of the light source in the mirror is **(IIT-JEE, 2000)**

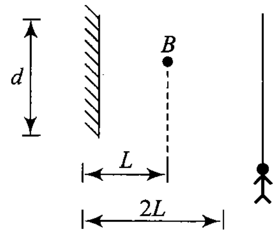


Fig. 1.443

a. $d/2$ b. d
c. $2d$ d. $3d$

23. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is (IIT-JEE, 2000)

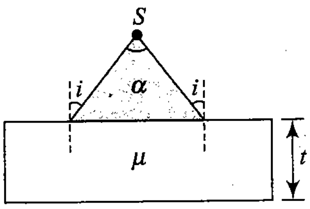


Fig. 1.444

a. zero
b. α
c. $\sin^{-1}\left(\frac{1}{n}\right)$
d. $2\sin^{-1}\left(\frac{1}{n}\right)$

24. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2 (n_1 < n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max} such that the ray comes out from the other surface CD is given by (IIT-JEE, 2000)

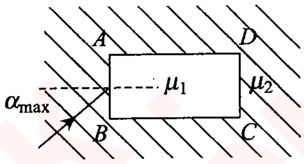


Fig. 1.445

a. $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$
b. $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$
c. $\sin^{-1}\left(\frac{n_1}{n_2}\right)$
d. $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

25. In a compound microscope, the intermediate image is (IIT-JEE, 2000)

a. virtual, erect, and magnified
b. real, erect, and magnified
c. real, inverted, and magnified
d. virtual, erect, and reduced

26. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3, and μ_4 as shown in figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have (IIT-JEE, 2001)

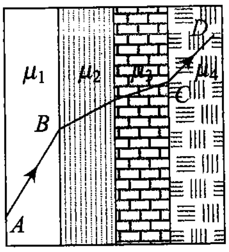


Fig. 1.446

a. $\mu_1 = \mu_2$
b. $\mu_2 = \mu_3$
c. $\mu_3 = \mu_4$
d. $\mu_4 = \mu_1$

27. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will now suffer (IIT-JEE, 2001)

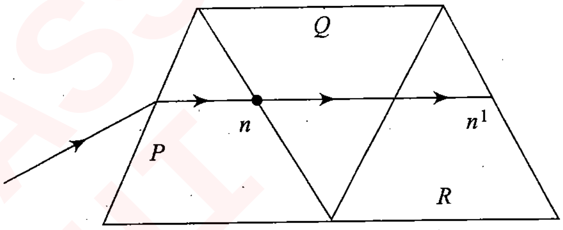


Fig. 1.447

a. greater deviation
b. no deviation
c. same deviation as before
d. total internal reflection

28. An observer can see through a pin-hole the top end of a thin of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is (IIT-JEE, 2002)

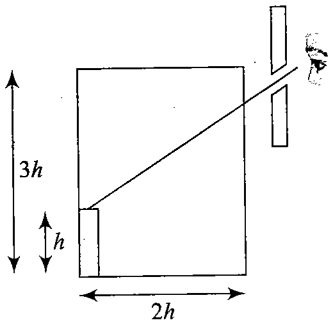
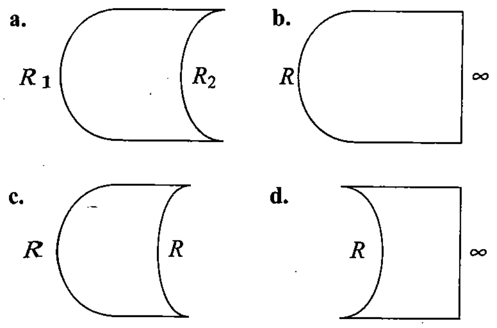


Fig. 1.448

a. $\frac{5}{2}$
b. $\sqrt{\frac{5}{2}}$
c. $\sqrt{\frac{3}{2}}$
d. $\frac{3}{2}$

29. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams. (IIT-JEE, 2002)



$R_1 \neq R_2$

30. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflection (including the first one) before it emerges out is **(IIT-JEE, 2002)**

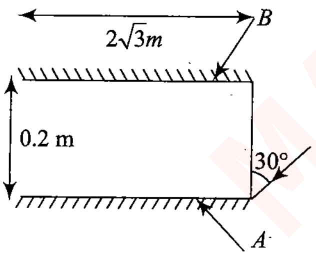


Fig. 1.449

a. 28 b. 30
c. 32 d. 34

31. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **(IIT-JEE, 2003)**

a. 1.25 cm b. 2.5 cm
c. 1.05 cm d. 2 cm

32. A ray of light is incident at the glass–water interface at an angle i. It merges finally parallel to the surface of water. Then, the value of μ_g would be **(IIT-JEE, 2003)**

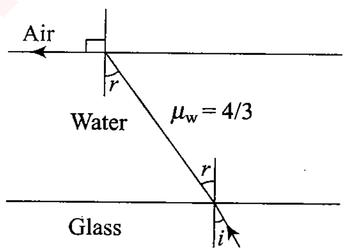


Fig. 1.450

a. (4/3) sin i b. 1/sin i
c. 4/3 d. 1

33. A beam of white light is incident on glass–air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are **(IIT-JEE, 2004)**

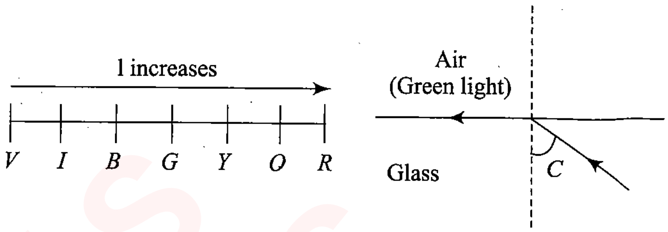


Fig. 1.451

a. Violet, Indigo, Blue b. All colors except green
c. Yellow, Orange, Red d. White light

34. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation,

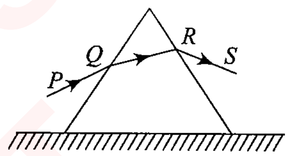


Fig. 1.452

a. PQ is horizontal b. QR is horizontal
c. RS is horizontal d. any one will be horizontal

35. A point object is placed at the center of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is **(IIT-JEE, 2004)**

a. 6 cm b. 4 cm
c. 12 cm d. 9 cm

36. A convex lens is in contact with a concave lens. The magnitude of the ratio of their focal lengths is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths? **(IIT-JEE, 2005)**

a. –15, 10 b. –10, 15
c. 75, 50 d. –75, 50

37. A container is filled with water ($\mu = 1.33$) upto a height of 33.25 cm. A convex mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is **(IIT-JEE, 2005)**

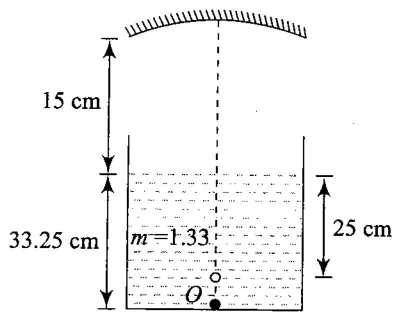


Fig. 1.453

a. 15 cm b. 20 cm
c. –18.31 cm d. 10 cm

38. Focal length of the plano-convex lens is 15 cm. A small object is placed at A as shown in the figure. The plane surface is silvered. The image will form at **(IIT-JEE, 2006)**

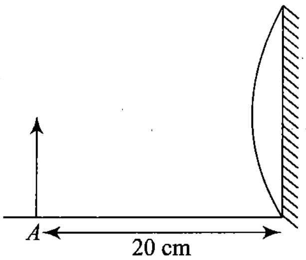


Fig. 1.454(a)

a. 60 cm to the left of lens b. 12 cm to the left of lens
c. 60 cm to the right of lens d. 30 cm to the left of lens

39. The graph shows relationship between object distance and image distance for an equiconvex lens. Then, focal length of the lens is **(IIT-JEE, 2006)**

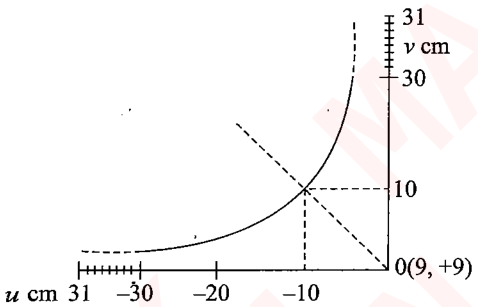


Fig. 1.454(b)

a. 0.50 ± 0.05 cm b. 0.50 ± 0.10 cm
c. 5.00 ± 0.05 cm d. 5.00 ± 0.10 cm

40. A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then, which option is correct? **(IIT-JEE, 2006)**

a. $\pi r^2 \propto f$
b. $\pi r^2 \propto f^2$
c. If lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
d. If f is doubled, intensity will increase

41. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then, there will be **(IIT-JEE, 2007)**

a. only a reflected ray and no refracted ray
b. only a refracted ray and no reflected ray
c. a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
d. a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

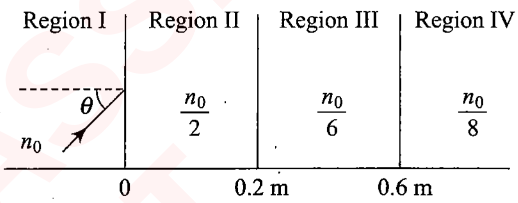
42. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eyes towards left, the image appears to the right of the object pin. Then, **(IIT-JEE, 2007)**

a. $x < f$ b. $f < x < 2f$
c. $x < f$ d. $f > x > 2f$

43. Rays of light from Sun falls on a biconvex lens of focal length f and the circular image of Sun of radius r is formed on the focal plane of the lens. Then, **(IIT-JEE, 2008)**

a. area of image is πr^2 and area is directly proportional of f
b. area of image is πr^2 and area is directly proportional to r
c. intensity of image increases if f is increased
d. If lower half of the lens is covered with black paper, area will become half

44. A light beam is traveling from Region I to Region IV (refer figure). The refractive indices in Regions I, II, III, and IV are n_0, $n_0/2$, $n_0/6$, and $n_0/8$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is **(IIT-JEE, 2008)**



a. $\sin^{-1}(3/4)$ b. $\sin^{-1}(1/8)$
c. $\sin^{-1}(1/4)$ d. $\sin^{-1}(1/3)$

45. Two beams of red and violet colors are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **(IIT-JEE, 2008)**

a. 30° for both the colors
b. greater for the violet color
c. greater for the red color
d. equal but not 30° for both the colors

46. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of ball as **(IIT-JEE, 2009)**

a. 9 ms^{-1} b. 12 ms^{-1}
c. 16 ms^{-1} d. 21.33 ms^{-1}

47. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **(IIT-JEE, 2010)**

a. virtual and at a distance of 16 cm from the mirror
b. real and at a distance of 16 cm from the mirror
c. virtual and at a distance of 20 cm from the mirror
d. real and at a distance of 20 cm from the mirror

Multiple Choice Questions with One or More than One Correct Answer Type

1. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen, **(IIT-JEE, 1986)**

a. half the image will disappear
b. complete image will be formed
c. intensity of the image will increase
d. intensity of the image will decrease

2. In an astronomical telescope, the distance between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and the focal length f_e of the eyepiece are **(IIT-JEE, 1989)**
 a. $f_O = 45$ cm and $f_e = -9$ cm b. $f_0 = 50$ cm and $f_e = 10$ cm
 c. $f_O = 7.2$ cm and $f_e = 5$ cm d. $f_0 = 30$ cm and $f_e = 6$ cm

3. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm. Then, **(IIT-JEE, 1992)**
 a. the distance between the objective and the eyepiece is 16.02 m
 b. the angular magnification of the planet is -800
 c. the image of the planet is inverted
 d. the objective is larger than the eyepiece

4. Which of the following form(s) a virtual and erect image for all positions of the object? **(IIT-JEE, 1996)**
 a. Convex lens b. Concave lens
 c. Convex mirror d. Concave mirror

5. A ray of light traveling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following: **(IIT-JEE, 1998)**
 a. 1.3 b. 1.4
 c. 1.5 d. 1.6

6. In a Young's double slit experiment, the separaton between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s). **(IIT-JEE, 2008)**
 a. If $d = \lambda$, the screen will contain only one maximum
 b. If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
 c. If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed darkand bright fringes will increase
 d. If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed darkand bright fringes will increase

7. A student performed the experiment of determination of focal length of a concave mirror by u–v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), and (78, 39). The data set(s) that ***cannot*** come from experiment and is (are) incorrectly recorded, is (are) **(IIT-JEE, 2009)**
 a. (42, 56) b. (48, 48)
 c. (66, 33) d. (78, 39)

8. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **(IIT-JEE, 2010)**

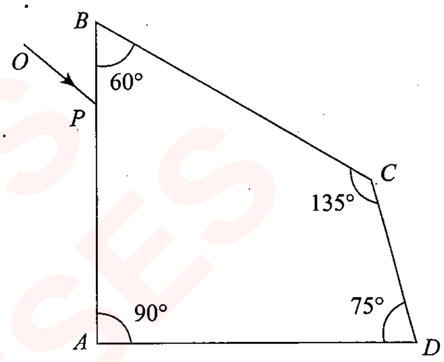


Fig. 1.455

 a. The ray gets totally internally reflected at face CD
 b. The ray comes out through face AD
 c. The angle between the incident ray and the emergent ray is 90°
 d. The angle between the incident ray and the emergent ray is 120°

Assertion and Reasoning Type

In each of the questions, assertion (A) is given by corresponding statement of reason (R) of the statements. Mark the correct answer. (IIT-JEE, 2007)
 a. If both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.
 b. If both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I.
 c. If Statement I is true, but Statement II is false.
 d. If Statement I is false, but Statement II is true.

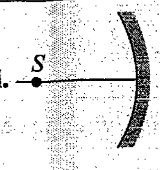
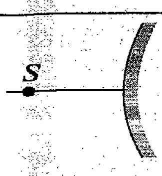
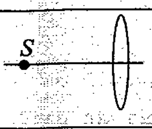
1. **Statement I:** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.
 Statement II: Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
2. **Statement I:** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not energy.
 Statement II: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

Matching Column Type

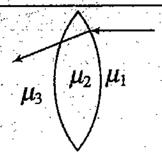
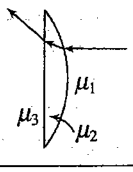
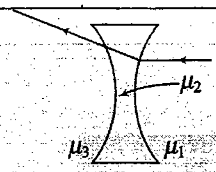
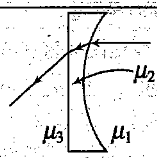
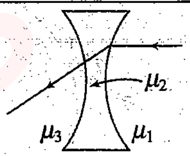
1. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f, respectively. Then, **(IIT-JEE, 2006)**

Column I	Column II
i. Intensity of light received by lens	a. Radius of aperture
ii. Angular magnification	b. Dispersion of lens
iii. Length of telescope	c. Focal length of objective lens and eyepiece lens
iv. Sharpness of image	d. Spherical aberration

2. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. **(IIT-JEE, 2008)**

Column I	Column II
i. 	a. Real image
ii. 	b. Virtual image
iii. 	c. Magnified image
iv. 	d. Image at infinity

3. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II. **(IIT-JEE, 2010)**

Column I	Column II
i. $\mu_1 < \mu_2$	a. 
ii. $\mu_1 > \mu_2$	b. 
iii. $\mu_2 = \mu_3$	c. 
iv. $\mu_2 > \mu_3$	d. 
	e. 

Integer Answer Type

1. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio m_{25}/m_{50} is **(IIT-JEE, 2010)**

2. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **(IIT-JEE, 2010)**

3. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is 25 50 observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour ? **(IIT-JEE, 2010)**

4. Water (with refractive index = 4/3) in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is. **(IIT-JEE, 2011)**

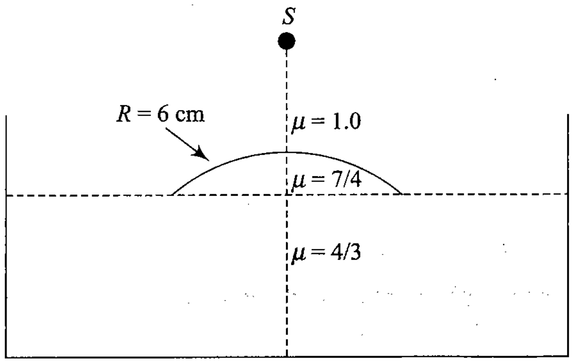


Fig. 1.456

ANSWERS AND SOLUTIONS

Subjective Type

1. From the figure it is clear that

$\angle ABC = \pi - 2(i-r)$

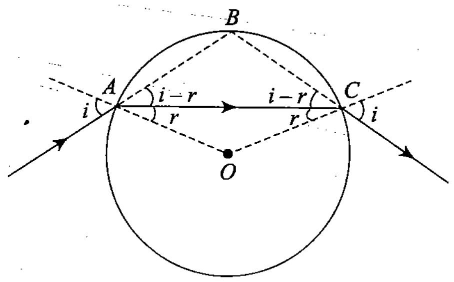


Fig. 1.457

So, $\angle AOC$ (external) $= 2\pi - \{\pi - 2r\}$

$= \pi + 2r$

From the property of circle,

$2\angle ABC = \angle AOC$

$\therefore \quad 2\{\pi - 2(i-r)\} = \pi + 2r$

This gives, $2i - r = \dfrac{\pi}{2} \Rightarrow r = \left(2i - \dfrac{\pi}{2}\right)$

$\therefore \quad \mu = \dfrac{\sin i}{\sin r}$

As $\therefore \mu = \sqrt{3}$ given $\Rightarrow \sqrt{3} = \dfrac{\sin i}{\sin\left[2i - \dfrac{\pi}{2}\right]}$

or $\sqrt{3} = \dfrac{\sin i}{-\cos 2i}$ or $\sqrt{3} = \dfrac{\sin i}{2\sin^2 i - 1}$

$2\sqrt{3}\sin^2 i - \sqrt{3} = \sin i$

Solving the above equation, we get $\sin i = \dfrac{\sqrt{3}}{2}$

2. $\dfrac{1.5}{v} - \dfrac{1}{-30} = \dfrac{1.5-1}{20} \Rightarrow v = -180$ cm

For second surface; $u_1 = -190$ cm

$\Rightarrow \quad \dfrac{1}{v} - \dfrac{1.5}{-190} = \dfrac{1-1.5}{10} \Rightarrow v = \dfrac{-190}{11}$

For third surface, $u = -\left(\dfrac{190}{11} + 20\right) = \dfrac{-410}{11}$

$\Rightarrow \quad \dfrac{1.5}{v} - \dfrac{11}{410} = \dfrac{1.5-1}{-10} = -\dfrac{1}{20}$

$\Rightarrow \quad v = -\dfrac{410}{21}$

For fourth surface, $u = -\left(\dfrac{410}{21} + 10\right) = -\dfrac{620}{21}$ cm

$\mu_1 = 1.5, \mu_2 = 1, R = -20$ cm

$\Rightarrow \quad \dfrac{1}{v} + \dfrac{1.5 \times 21}{620} = \dfrac{1-1.5}{-20}$

$\Rightarrow \quad v = -38.75$ cm

The final image is at a distance of 18.75 cm from the center.

3. For middle prism, $r + r = A$

$2r = 60°$

$\Rightarrow \quad r = 30°$

Refraction from Ist surface

$1 \sin 30° = 1.5 \sin\theta$

$1.5 \sin(60 - \theta) = \mu \sin 30° \quad \Rightarrow \quad \sin\theta = 1/3$ and $\cos\theta = \sqrt{8}/3$

Refraction from interface of Ist & IInd prism,

$1.5 \sin 60° \cos\theta - 1.5 \cos 60° \sin\theta = \mu \sin 30°$

$1.5 \times \dfrac{\sqrt{3}}{2} \times \dfrac{2\sqrt{2}}{3} - 1.5 \times \dfrac{1}{2} \times \dfrac{1}{3} = \mu \times \dfrac{1}{2}$

$\sqrt{\dfrac{3}{2}} - \dfrac{1}{4} = \mu\dfrac{1}{2}$

$\Rightarrow \quad \mu = \sqrt{6} - \dfrac{1}{2} = \dfrac{2\sqrt{6}-1}{2} = 1.95$

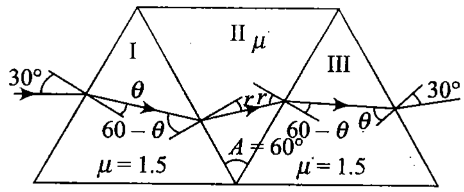


Fig. 1.458

4. Figure 1.459 shows the image of the person and images A' and B' of points A and B, respectively. From the figure,

$h = \dfrac{1+1.8}{2} = 1.4$ m

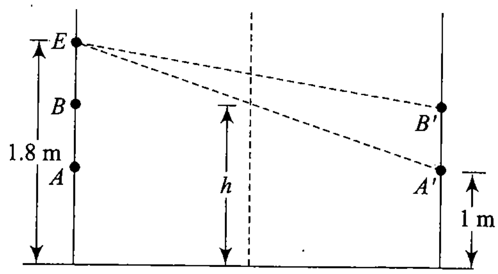


Fig. 1.459

5.

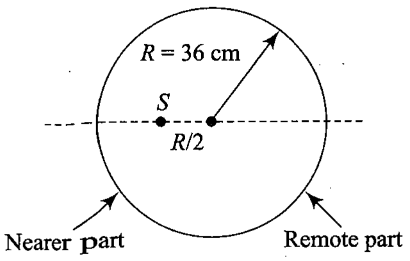


Fig. 1.460

Case I.

Applying mirror formula for remote part $\left(u=-\dfrac{3R}{2},\ f=-\dfrac{R}{2}\right)$,

$$v=\frac{uf}{u-f}=\frac{-3}{4}R$$

Now, consider reflection from the nearer part for nearer part

$$u=\frac{3R}{4}-2R=-\frac{5R}{4}\Rightarrow v=\frac{uf}{u-f}=\infty$$

Reflection from remote part gives $v=-\dfrac{2R}{2}$

Thus, the Image will be at $-\dfrac{5R}{6}$ from near end toward source;

image will be at $+\dfrac{R}{2}$ from remote end.

6.

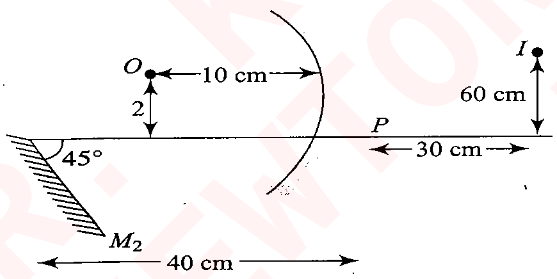


Fig. 1.461

For m_1, $u=-10$ cm, $f=-15$ cm, $h=2$ cm.

Using mirror formula, $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f}$

$$\Rightarrow\quad \frac{1}{v}+\frac{1}{-10}=\frac{1}{-15}$$

$$\Rightarrow\quad \frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{3-2}{30}$$

$$\Rightarrow\quad v=30\text{ cm}$$

and $\dfrac{h_2}{h_1}=\dfrac{v}{u}\Rightarrow h_2=6$ cm

The image formed by the plane mirror is 70 cm below the principal axis and $70+6-30=46$ cm to the left of the mirror. Therefore, oordinates of I_2 w.r.t. P are $(-46,-70)$.

7.

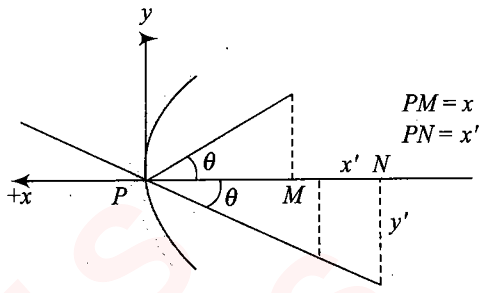


Fig. 1.462

Let $\tan\theta=a$ then equation of the line PQ is

$$y=-ax \qquad \text{(i)}$$

Let $y=y$-coordinate of the image and

$x'=x$-coordinate of image. By using the formula of magnification, $\dfrac{y'}{y}=-\dfrac{x'}{x}$

$$\Rightarrow\quad y'=-\frac{y}{x}x'=-(-a)\,x'=ax'$$

$$\Rightarrow\quad y'=ax'$$

This is equation of a straight line, making angle 'θ' with the principal axis. (Hence, proved.)

8.

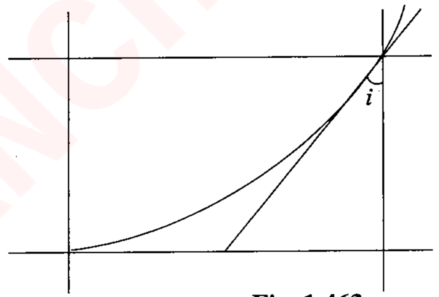


Fig. 1.463

By Snell's law,

$$1\times\sin 30^\circ=n\sin i$$

where n is R.I. at y and i is angle of incidence at y.

$$\tan(90-i)=\frac{dy}{dx}=8x=4\sqrt{y}$$

$$\cot i=4\sqrt{y}=4\sqrt{\frac{1}{2}}=2\sqrt{2}$$

$$\Rightarrow\quad \sin i=\frac{1}{3}$$

$$\therefore\quad n=\frac{\sin 30^\circ}{\sin i}=\frac{1/2}{1/3}=1.5$$

9.

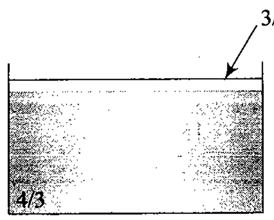


Fig. 1.464

Since width of glass sheet is negligible, so neglecting effect of glass sheet.

$$d_{app} = \frac{d_{act}}{\mu_{rel}} \Rightarrow 8 = \frac{10}{\left(\frac{4}{3\mu}\right)}$$

$$\Rightarrow \quad 8\frac{4}{3\mu} = 10$$

$$\Rightarrow \quad \mu = \frac{32}{30} = \frac{16}{15}$$

10.

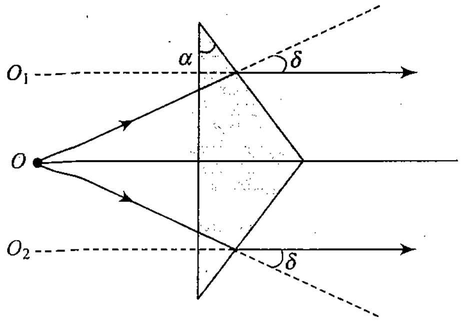


Fig. 1.465

For thin prism $\delta = (\mu - 1)\alpha$

$$= (1.5 - 1)\, 1.8° = (0.5)\, 1.8 \times \frac{\pi}{180} \text{ rad} = \frac{\pi}{200} \text{ rad}$$

$\therefore$ Distance $OO_1 = 10 \times \frac{\pi}{200} = \frac{\pi}{20}$ cm

Similarly, distance $OO_2 = \frac{\pi}{20}$ cm

$\therefore \quad O_1O_2 = \frac{\pi}{10}$ cm

For the mirror, O_1 and O_2 are two point objects, at a distance of 30 cm.

Now, applying mirror formula

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-10} \Rightarrow v = -15 \text{ cm}$$

Latent magnification is

$$-\frac{v}{u} = \frac{(-15)}{-30} = -\frac{1}{2}$$

(–ve sign indicates inverted image)

Therefore, if O_1' and O_2' are the images formed, then distance between them,

$$O_1'O_2' = \frac{1}{2}\, O_1O_2 = \frac{1}{2} \cdot \frac{\pi}{10} = \frac{\pi}{20} \text{ cm}$$

11.

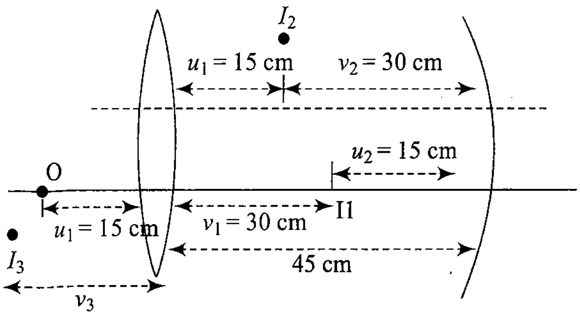


Fig. 1.466

I_1 is the image of object O formed by the lens.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \quad u_1 = -15 \quad \text{and} \quad f_1 = 10$$

Solving, we get $v_1 = 30$ cm

I_1 acts as source for the mirror

$\therefore \quad u_2 = -(45 - v_1) = -15$ cm

I_2 is the image formed by the mirror

$$\therefore \quad \frac{1}{v_1} = \frac{1}{f_m} - \frac{1}{u_2} = -\frac{1}{10} - \frac{1}{15}$$

$\therefore \quad v_2 = -30$ cm

The height of I_2 above principal axis of lens $= \frac{v_2}{u_2} \times 1 + 1$

$= 3$ cm

I_2 acts as a source for the lens

$\therefore \quad u_3 = -(45 - v_2) = -15$ cm

Hence, the lens forms an image I_3 at a distance $v_3 = 30$ cm to the left of the lens and at a distance

The height of I_2 above principal axis of lens $= \frac{v_2}{u_2} \times 1 + 1$

$= 3$ cm

$\therefore \quad$ Required distance $= \sqrt{30^2 + 6^2} = 6\sqrt{26}$ cm

12.

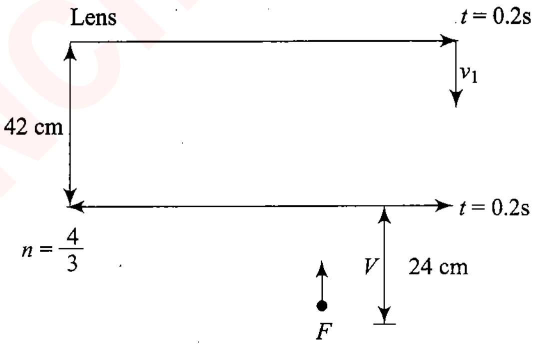


Fig. 1.467

At $\quad t = 0.2$ sec, velocity of lens

$$V_l = gt = 2 \text{ ms}^{-1} \text{ (downward)}$$

For lens, the fish appears to approach with a speed of $2 + \left(1 \times \frac{3}{4}\right) = \frac{11}{4} \text{ ms}^{-1}$

at distance of $\left(42 + \frac{24}{\left(\frac{4}{3}\right)}\right) = 60$ cm

$\therefore$ Image of the fish from the lens,

$$V = \frac{-60 \times 90}{-60 + 90} = -180 \text{ cm}$$

$\therefore$ Velocity of image w.r.t. the lens,

$$V_i = \left(\frac{v^2}{u^2}\right)\frac{du}{dt} = \left(\frac{-180^2}{-60}\right) \times \frac{11}{4} = \frac{99}{4} \text{ ms}^{-1}$$

Velocity of image w.r.t. the observer is

$$V_1 - 2 = \frac{99}{4} - 2 = \frac{91}{4} \text{ ms}^{-1}$$

$$= 22.75 \text{ cms}^{-1} \text{ (upward)}$$

13. **a.** The distance of F_1 from the surface is 45 cm. Apparent depth of F_1 from front wall is,

$$x = \frac{45}{\mu_L} = \frac{45}{4/3} = 33.75\text{cm}$$

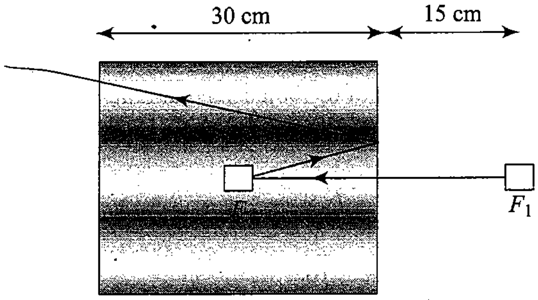


Fig. 1.468

b. For x to decrease, μ_L has to increase.
For limiting case, $x = 30$ cm

So, $\mu_L = \frac{45}{30} = \frac{3}{2}$

14. For L_1, from lens formula,

$$\frac{1}{v} - \frac{1}{-20} = \frac{1}{16}, \Rightarrow \frac{1}{v} = \frac{1}{16} - \frac{1}{20}$$

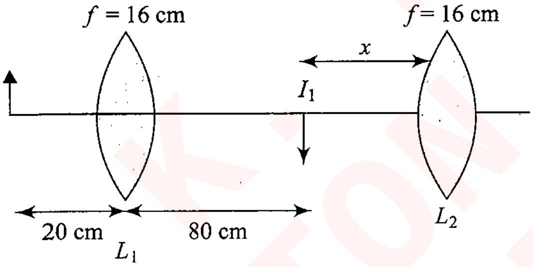


Fig. 1.469

$\Rightarrow \quad v = 20\,\text{cm}$

[Image is real, inverted, and magnified]

$$m_1 = \frac{v}{u} = \frac{80}{-20} = -4$$

Let 2nd lens is at a distance of x cm from I_1. Since final image has to be erect w.r.t. original object, it has to be inverted and diminished w.r.t. I_1.

For L_2, $u = -x$, $v = v$, $m_2 = -\frac{1}{4}$

So, $\quad \frac{v}{-x} = -\frac{1}{4} \Rightarrow v = \frac{x}{4}$

Now, from lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{x/4} - \frac{1}{-x} = \frac{1}{16}$$

$$\frac{5}{x} = \frac{1}{16} \Rightarrow x = 80\text{ cm}$$

Hence, the separation between the lenses is 160 cm.

15. The ray diagram is as shown in the figure below. From geometry and Snell's law, we can verify that various angles are same as shown:

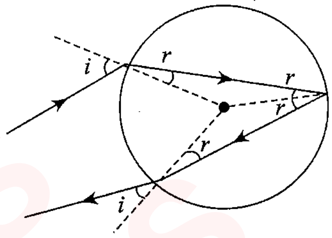


Fig. 1.470

$\delta_1 = i - r$ in CW
$\delta_2 = \pi - r$ in CW
$\delta_3 = i - r$ in CW

Total deviation,

$$\delta = \delta_1 + \delta_2 + \delta_3 = 2i + \pi - 4r$$

From Snell's law,

$$\sin I = \mu \sin r$$

$$\sin r = \frac{\sin r}{\mu}$$

$$r = \sin^{-1}\left[\frac{\sin i}{\mu}\right]$$

So, $\quad \delta = 2i + \pi - 4\left[\sin^{-1}\left(\frac{\sin i}{r}\right)\right]$

Find $d\delta/di$ and equate it to 0 to get the value of i for minimum deviation.

16. The image formed after 4 refractions would be taken as the object. The distance of object (4th image) from mirror's pole is given by

$$x = \left(\frac{20}{4/3} + \frac{54}{1.5} + \frac{24}{1.0} + \frac{45}{1.5}\right) \times \frac{4}{3} + 10$$

$$= 150\,\text{cm}$$

i.e., the center of curvature of mirror. Hence, the ray will retrace its path, i.e., it is formed on the object itself.

17. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$= (1.5 + 0.5r - 1)\frac{2}{20} = 0.5\,(1 + r)\,\frac{1}{10}$$

$$\frac{1}{f} = \frac{1+f}{20} \quad \Rightarrow \quad f = \frac{20}{(1+r)}$$

For $r = 0, f = 20$ cm ;
For $r = 1$ cm, $f = 10$ cm
Light on the axis will be from 10 cm to 20 cm for the lens.

18. The system behaves as a mirror.

$$\text{Power of lens, } P_\ell = \frac{1}{f_\ell} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{50} - \frac{1}{-50}\right) = \frac{1}{50}$$

$\therefore \quad$ Power of equivalent mirror

$$P_{EM} = 2P_\ell + P_m = \frac{2}{50} - \frac{1}{50} = \frac{1}{50}$$

i.e., Focal length of equivalent mirror $f_{cm} = -50$

Since image and object coincides, hence position of the object is at the center of curvature of the equivalent mirror,. i.e., at $2f_{cm} = 100$ cm from the lens.

When the liquid of refractive index be is placed between the lens and convex mirror, it forms a liquid lens of power,

$$P_u = \left(\frac{1}{-50} - \frac{1}{100}\right) = \frac{3(1-\mu)}{100}$$

Now, power of equivalent mirror

$$= 2P_\ell + 2P_u + P_m$$

$$= \frac{2}{50} + \frac{2\times 3(1-\mu)}{100} + \frac{1}{-50} = \frac{3(1+\mu)+1}{50} \quad \text{(i)}$$

Since final image is at infinity, hence the object should be at its focal point

i.e., $f'_{EM} = 100$

$$\Rightarrow \quad = -\frac{1}{f'_{EM}} = \frac{1}{100} \quad \text{(ii)}$$

From Eqs (i) and (ii), we get

$$\frac{3(1-\mu)+1}{50} = \frac{1}{100}$$

$\Rightarrow \quad \mu = 7/6$

19. **a.** For a convex lens, minimum distance between the object and image is $4f$ in which distance between the object and lens is $2f$ and between the lens and screen is $2f$,

Hence, shifting $= 3f_1 - 2f_1 = f_1$

$$s = t[1\ (1/\mu)) \Rightarrow \mu = t(t - f_1)$$

b. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{(-f_2)}; \frac{1}{v'} - \frac{1}{u} = \frac{1}{F}; u = -2f_1$

$$\Rightarrow \quad v' = \frac{2f_1 f_2}{f_2 - 2f_1}$$

Shifting of image,

$$s = v' - v = \frac{2f_1f_2}{f_2 - 2f_1} - 2f_1 = \frac{4f_1^2}{f_2 - 2f_1} = \frac{4f_1^2}{f_2} \quad (\text{as } f_2 >> f_1)$$

20. **a.**

$$\frac{1}{f_1} = (1.6-1)\left[\frac{1}{20} - \frac{1}{-30}\right] = \frac{1}{20}\text{cm}^{-1}$$

$$\frac{1}{f_2} = (1.5-1)\left[\frac{4}{-30} - \frac{1}{15}\right] = \frac{1}{20}\text{cm}^{-1}$$

$$\frac{1}{f_3} = (2.5-1)\left[\frac{1}{15} - \frac{1}{\infty}\right] = \frac{1}{10}\text{cm}^{-1}$$

$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{20} - \frac{1}{20} + \frac{1}{10} = \frac{1}{10}$$

$$\Rightarrow \quad f_v = 10\text{ cm}$$

Since the image is formed on the screen which is 10 cm from the lens, so the object is at infinity.

b. Shift in the image away from the slab due to introduction of slab is

$$\Delta x = \left(1 - \frac{1}{n}\right)t = \left(1 - \frac{1}{1.5}\right)\times 1.5 = 0.5$$

The distance of the object so that image is again formed on the screen

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{9.5} - \frac{1}{u} = \frac{1}{10} \quad \Rightarrow \quad u = 190\text{ cm}$$

c. When the slab is placed on the other side of the lens, there will not be any change in the formation.

Objective Type

1. **d.** Using Snell's law, $n_2 > n_1$ because $i > r$, $n_2 > n_3$ because $e > r$, and $n_1 > n_3$ because $e > i$.

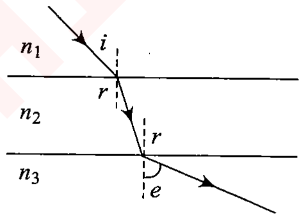


Fig. 1.471

2. **b.** Options (c) and (d) are not possible because ray diagram shows that the R.I. of surrounding of glass is more. a. is wrong because angle of incidence seem to be small as compared to critical angle ($\approx 41°$). Therefore (b) is correct.

3. **a.** From Snell's law,

$$\mu_1 \sin i = \mu_2 \sin r \Rightarrow \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

From the graph,

$$\tan 60° = \frac{\sin i}{\sin r} \Rightarrow \frac{\sin i}{\sin r} = \sqrt{3} = \frac{\mu_2}{\mu_1}$$

$$\mu = \frac{c}{v} \Rightarrow = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

$$v_1 = \left(\frac{\mu_2}{\mu_1}\right)v_2 \Rightarrow v_1 = \sqrt{3}\, v_2$$

4. **b.** $\mu_1 \sin\alpha = \mu_2 \sin\alpha_2$

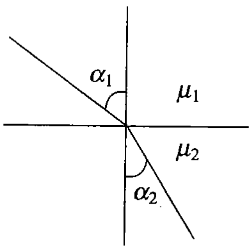


Fig. 1.472

$$\frac{c}{v_1}\sin a_1 = \frac{c}{v_2}\sin a_2$$

$$\frac{\sin a_1}{f\lambda_1} = \frac{\sin a_2}{f\lambda_2}$$

5. a.

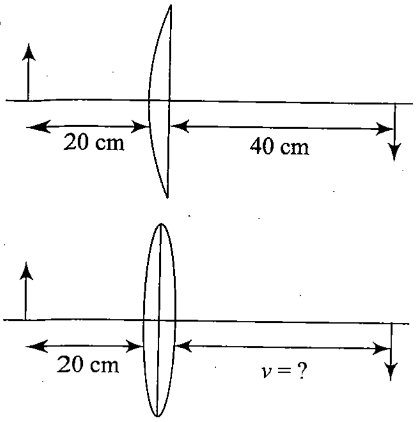


Fig. 1.473

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{40} - \frac{1}{(-20)}$$

$$\Rightarrow \quad f = \frac{40}{3}$$

Focal length for the combination,

$$f_{eq} = \frac{20}{3}$$

$$\frac{1}{f_{eq}} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{30}{20} = \frac{1}{v} + \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{3}{20} - \frac{1}{20}$$

$v = 10$ cm (to the right)

6. b.

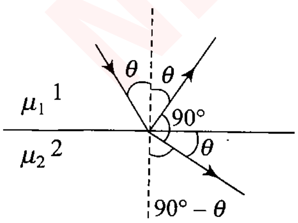


Fig. 1.474

$$\mu_1 \sin\theta = \mu_2 \times \sin(90° - \theta)$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2} = \tan\theta$$

For $\theta_c \Rightarrow \mu_1 \times \sin\theta_C = \mu_2 \times \sin(90°)$

$$\sin\theta_C = \frac{\mu_1}{\mu_2} = \tan\theta$$

$$\Rightarrow \quad \theta_C = \sin^{-1}(\tan\theta)$$

7. d.

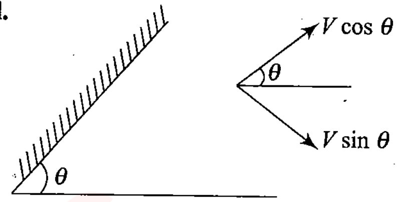


Fig. 1.475

Component of V perpendicular to the mirror gets reversed while that parallel to it remains same. Thus, velocity vector of image becomes

$$V\cos 2\theta\hat{i} + V\sin 2\theta\hat{j}$$

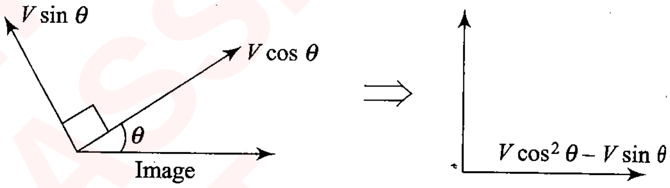


Fig. 1.476

8. b. $\delta = e + i - A$

$30° = 15° + 60° - A$

$\Rightarrow \quad A = 45°$

9. c.

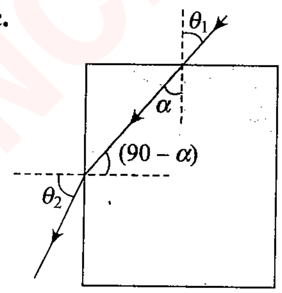


Fig. 1.477

As θ_1 increases, α also increases. So, (90 - α) decreases and hence θ_2 decreases.

10. c. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$R = 10$ cm

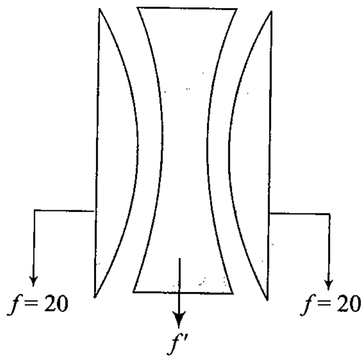


Fig. 1.478

$$f' = (3-1)\left(\frac{1}{-10}-\frac{1}{10}\right)$$

$$f' = \frac{-10}{4}$$

$$\frac{1}{f_{eq}} = \frac{1}{20}-\frac{4}{10}+\frac{1}{20} = \frac{2}{20}-\frac{4}{10}$$

$$f_{eq} = -10/3 \text{ cm}$$

11. c. "O" act as the focal point.

Also, $\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow$ $\frac{1}{40} = (\mu-1)\left(\frac{1}{10}+\frac{1}{10}\right)$

$$\frac{1}{40} = (\mu-1)\left(\frac{2}{10}\right)$$

$\Rightarrow$ $\mu = 9/8$

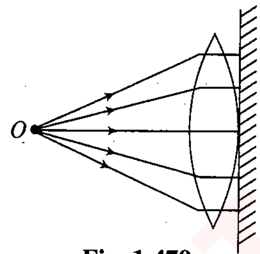


Fig. 1.479

12. a.

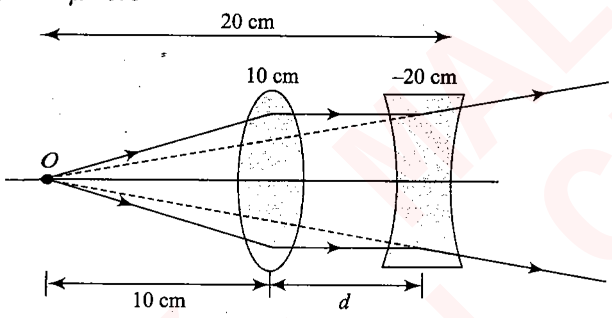


Fig. 1.480

13. a. $\frac{1}{v}+\frac{1}{-600}=\frac{1}{20}$

or $\frac{1}{v}=\frac{31}{600}$

or $v=\frac{600}{31}\text{ cm} = 19.35\text{ cm}$

14. b. $m = -\frac{v}{u} = -\frac{600}{31}\times\frac{1}{-600} = \frac{1}{31}$

Breadth of image $= \frac{1}{31}\times 200\text{ cm} = 6.45\text{ cm}$

Height of image $= \frac{1}{31}\times 160\text{ cm} = 5.16\text{ cm}$.

15. d. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$$-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0$$

or $\frac{du}{dt}=\frac{v^2}{u^2}\frac{du}{dt} = -\frac{1}{31\times 31}\times 31 = -0.032\text{ ms}^{-1}$

16. b. ${}_g\mu_a = \frac{\sin i}{\sin r}$

$$\frac{1}{{}_a\mu_g} = \frac{\sin i}{\sin 2i}$$

$$\frac{1}{16} = \frac{\sin i}{2\sin i\cos i} \text{ or } \frac{1}{\cos i}\frac{1}{10} = \frac{5}{8}$$

or $\frac{1}{\cos i} = \frac{5}{4}$ or $\cos i = \frac{4}{5}$

Now, $\sin i = \sqrt{1-\cos^2 i}$

or $\sin i = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

or $i = \sin^{-1}\left(\frac{3}{5}\right)$

17. c. $\frac{1}{f} = (\mu-1)\left(\frac{2}{R}\right)$

or $f = \frac{R}{2(\mu-1)}$

Now, $f > R$

$\therefore$ $\frac{R}{2(\mu-1)} > R$

or $\frac{1}{2(\mu-1)} > 1$ or $2(\mu-1) < 1$

or $\mu - 1 < \frac{1}{2}$ or $\mu < \left(1+\frac{1}{2}\right)$ or $\mu < 1.5$

18. a. $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

For a plane surface, $R = \infty$

$\therefore$ $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=0$

or $\frac{\mu_2}{v}-\frac{\mu_1}{u}$ or $\frac{\mu}{v}-\frac{1}{u}$

or $v = \mu_2 u$

Clearly, to the fish, the bird appears farther than its actual distance.

Again, $\frac{dv}{dt} = \mu\frac{du}{dt}$

or Apparent speed of bird
= refractive index × actual speed of bird.

19. c. $\sqrt{3} = \frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$

$$\sqrt{3} = \sin\left(\frac{60°+\delta_m}{2}\right)$$

$$\sin 60° = \sin\left(\frac{60°+\delta_m}{2}\right)$$

or $\frac{60°+\delta_m}{2} = 60°$

or $\delta_m = 60° \Rightarrow i = \frac{A+\delta_m}{2} = \frac{60°+60°}{2} = 60°$

20. d. For A:

$u = -3\ m,\ v_1 = ?, f = -1$ m

$$\frac{1}{u_1} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-3} = \frac{1}{3} - 1 = -\frac{2}{3}$$

or $v_1 = -\frac{3}{2}$

For B:

$$\frac{1}{v_2} = \frac{1}{-1} - \frac{1}{-5} \text{ or } \frac{1}{v_2} = \frac{1}{5} - 1 = -\frac{4}{5}$$

or $v_2 = -\frac{5}{4}$ m

Now, $v_1 - v_2 = \frac{3}{2} - \left(-\frac{5}{4}\right)$

$$= -\frac{3}{2} + \frac{5}{4} = -\frac{1}{4}\text{ m} = -0.25\text{ m}$$

Again, $\frac{I_1}{O} = -\frac{v_1}{u}$

or $I_1 = -\frac{v_1}{u}O = -\left(\frac{-3}{2}\right)\left(\frac{-1}{3}\right) = -1$ m

Again, $\frac{I_2}{O} = -\frac{v_2}{u}$

or $I_2 = -\left(-\frac{5}{4}\right)\left(\frac{1}{-5}\right)2 = -0.5$ m

21. b. $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_l} + \frac{1}{f_m}$

$$\frac{1}{F} = \frac{2}{f_l} + \frac{2}{f_m}$$

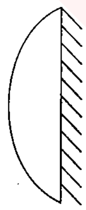


Fig. 1.481

or $\frac{1}{F} = 2(\mu-1)\left(\frac{1}{R}\right) + \frac{1}{\infty}$

or $F = \frac{R}{2\mu(-1)}$

Now, $-60 = \frac{R}{2(\mu-1)}$

or $120(\mu-1) = R$ (i)

Agan, $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_l} + \frac{1}{f_m}$

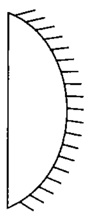


Fig. 1.482

or $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{R/2}$

or $\frac{1}{F} = 2(\mu-1)\left(\frac{1}{R}\right) + \frac{2}{R}$

or $\frac{1}{F} = \frac{2}{R}(\mu - 1 + 1)$

or $F = \frac{R}{2\mu}$

Now, $-20 = \frac{-R}{2\mu}$

or $40\mu = R$ (ii)

Dividing (i) by (ii), we get

$$\frac{120(\mu-1)}{40\mu} = \frac{R}{R} = 1$$

or $120(\mu-1) = 40\mu$

or $120\mu - 40\mu = 120$

or $80\mu = 120$

or $\therefore \mu = \frac{120}{80} = \frac{3}{2} = 1.5$

22. d.

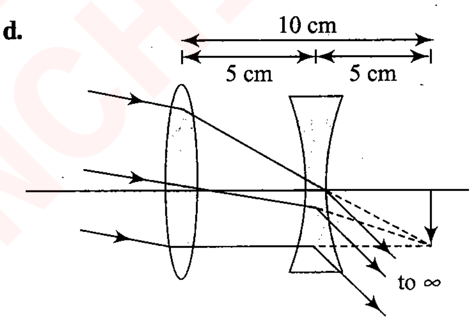


Fig. 1.483

At 5 cm from the lens, the second lens has a virtual object (image of the first lens) at its focal length. The emergent rays are therefore parallel.

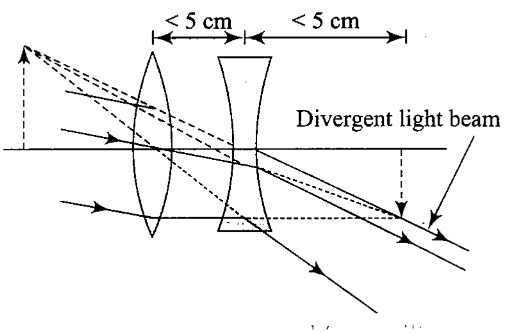


Fig. 1.484

When the second lens is closer than 5 cm to the first lens, its object is outside the focal length of the diverging second lens. This produces a virtual image outside $2f$ of the second lens. The emergent rays are therefore *divergent*.

23. b.

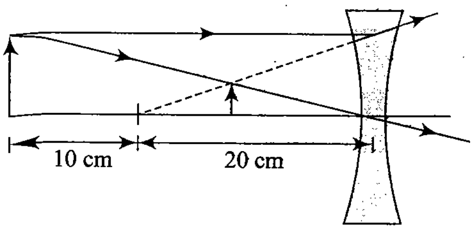


Fig. 1.485

When an object is placed between $2f$ and f (focal length) of the diverging lens, the image is virtual, erect, and diminished as shown in the graph. To compute the distance of the image from the lens, we apply

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{-20}=\frac{1}{v}-\frac{1}{30}$$

$$\Rightarrow \quad v=\frac{(20)(30)}{20+30}$$

$$=-12 \text{ cm (to the left of the diverging lens.)}$$

24. a. A diverging lens is ruled out because both x and y are positive values. Both x and y equal 20 cm at their smallest sum, which occurs when

$$x+y=40\text{ cm}=4f$$

$$\therefore \quad f=10\text{ cm}$$

This indicates a converging lens of focal length = 10 cm

25. c. When $n=1$,

$$\mu(1)=\mu_0-\frac{\mu_0}{4\times1-18}$$

$$\mu(1)>\mu_0$$

When $n=2$

$$\mu(2)=\mu_0-\frac{\mu_0}{4\times2-18}$$

$$\mu(2)>\mu_0$$

When $n=4$,

$$\mu(4)=\mu_0-\frac{\mu_0}{4\times4-18}$$

$$\mu(4)>\mu_0$$

When $n=5$

$$\mu(5)=\mu_0-\frac{\mu_0}{5\times4-18}$$

$$\mu(5)<\mu_0$$

Clearly, the total internal reflection shall take place at the top of a layer having $n=4$.

26. a. For real image:

$$u=-v_1,\ v=-2u_1,\ f=-20\text{ cm}$$

Substituting in $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

we get $\frac{1}{-2u_1}-\frac{1}{u_1}=-\frac{1}{20}$

or $\quad u_1=30$ cm

For virtual image:

$$u=-u_2,\ v=2u_2,\ f=-20\text{ cm}$$

$$\therefore \quad \frac{1}{2u_2}-\frac{1}{u_2}=-\frac{1}{20}$$

or $\quad u_2=10$ cm

Therefore, distance between two positions of the object are u_1-u_2 or 30 cm − 10 cm = 20 cm.

27. b.

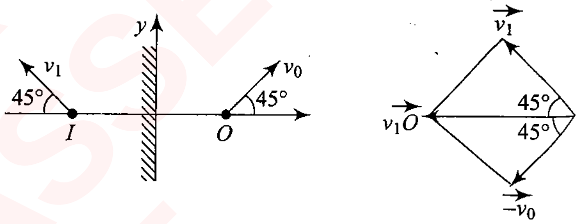


Fig. 1.486

$$|\vec{V}_0|=|\vec{V}_0|=\sqrt{(2)^2+(2)^2}=2\sqrt{2}\text{ ms}^{-1}$$

Relative velocity of image with respect to object is in negative x-direction as shown in figure.

28. d. Distance of image from the plane surface is as follows:

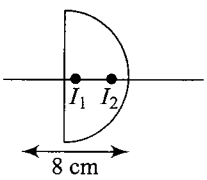


Fig. 1.487

$$x_1=\frac{4}{1.6}=2.5\text{ cm}\left(d_{app}=\frac{d_{actual}}{\mu}\right)$$

For the curved side

$$\frac{1.6}{4}+\frac{1}{x_2}=\frac{1-1.6}{8}$$

$$\therefore \quad x_2\approx-3.0\text{ cm}$$

The minus sign means the image is on the object side.

$$\therefore \quad I_1I_2=(8-2.5-3.0)\text{ cm}=2.5\text{ cm}$$

29. b. Deviation by a sphere is $2(i-r)$

Here, deviation $\delta=60°=2(i-r)$

or $\quad i-r=30°$

$$\therefore \quad r=i-30°=60°-30°=30°$$

$$\therefore \quad \mu=\frac{\sin i}{\sin r}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

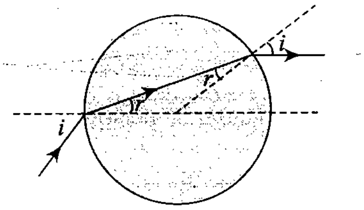


Fig. 1.488

30. c. Let d' be the diameter of refracted beam.

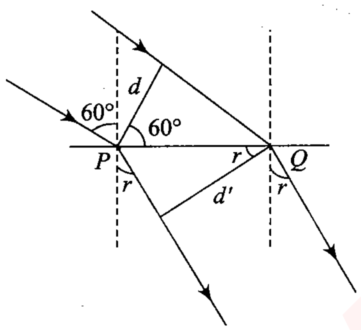


Fig. 1.489

Then,

$d = PQ \cos 60°$

and $d' = PQ \cos r$

i.e. $\frac{d'}{d} = \frac{\cos r}{\cos 60°} = 2 \cos r$

or $d' = 2d \cos r$

sin $r = \frac{\sin i}{\mu} = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$

$\therefore$ $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{\frac{2}{3}}$

$\therefore$ $d' = (2)(2)\sqrt{\frac{2}{3}}$

$= 4\sqrt{\frac{2}{3}}$ cm ≈ 3.26 cm

31. c. $\sin \theta_1 = \frac{1}{\mu_g}$ and $\sin \theta_2 = \frac{1}{\mu_w}$

Since $\mu_g > \mu_w$, $\theta_1 < \theta_2$

Critical angle θ between glass and water will be given by

$\sin \theta = \frac{\mu_w}{\mu_g}$

or $\theta > \theta_2$

Note: *Critical angle increases as the relative refractive index is decreased.*

32. c. Critical angle between glass and liquid face is

$$\sin \theta_c = \frac{3/2}{\mu} = \frac{3}{2\mu} \quad \text{(i)}$$

Angle of incidence at face AC is 60°

i.e., $i = 60°$

For total internal reflection to take place,

$i > \theta_c$

or $\sin i > \sin \theta_c$

or $\sin 60° > \frac{3}{2\mu}$

or $\frac{\sqrt{3}}{2} > \frac{3r}{2\mu}$

or $\mu > \sqrt{3}$

33. c. $\angle ABO = \angle OAB = \theta_C$

$\sin \theta_c = \frac{1}{\mu} = \frac{2}{3}$

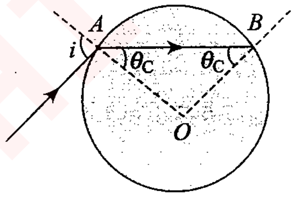


Fig. 1.490

Applying Snell's law at A

$\frac{\sin i}{\sin \theta_c} = \frac{3}{2}$

$\sin i = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$

or $i = 90°$

34. d. Let R be the radius of curvature of each surface. Then

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right).$$

For the water lens,

$$\frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{3}\left(-\frac{2}{f}\right); \frac{1}{f'} = -\frac{2}{3f}$$

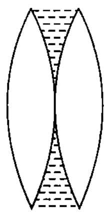


Fig. 1.491

Now, using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$, we have

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f'}=\frac{2}{f}-\frac{2}{3f}=\frac{4}{3f}$$

$$\Rightarrow F=\frac{3f}{4}$$

35. d. Only convex lens can form a real well as virtual image. So, the given lens is a convex lens.

Let f is the focal length of the lens and n the magnitude of magnification.

In the first case, when the image is real:

$$\mu v=-16$$
$$v=+16n$$

So, applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{16n}+\frac{1}{16}=\frac{1}{f}$$

or $$1+\frac{1}{n}=\frac{16}{f} \quad \text{(i)}$$

In the second case, when image is virtual:

$$u=-6$$
$$v=-6n$$
$$f=+f$$

$\therefore$ $$\frac{1}{-6n}+\frac{1}{6}=\frac{1}{f}$$

$\therefore$ $$1-\frac{1}{n}=\frac{6}{f} \quad \text{(ii)}$$

Adding (i) and (ii), we get

$$2=\frac{22}{f} \text{ or } f=11\text{ cm}$$

36. d. Concave lens forms the virtual image of a real object. So, let

$$u=-4x \quad \text{and} \quad v=-x$$

Then, $3x=10\text{cm}$

or $$x=\frac{10}{3}\text{cm}$$

$\therefore$ $$u=-\frac{40}{3}\text{cm and } v=-\frac{10}{3}\text{cm}$$

Substituting in $\frac{1}{f}=\frac{-3}{10}+\frac{3}{40}$

or $$f=-\frac{40}{9}$$

or $$f=-4.4\text{cm}$$

37. b. Clearly, plane mirror and convex mirror cannot produce inverted image.

38. c.

a. is ruled out because the person is standing on the edge of the pool.

b. is ruled out because the signal would not move out of the pool.

d. is ruled out because of total internal reflection.

39. a. Clearly, A is denser than B and B is denser than C.

$\therefore$ $\mu_1>\mu_2>\mu_3$

$\therefore$ $C_1<C_2<C_3$

40. c.

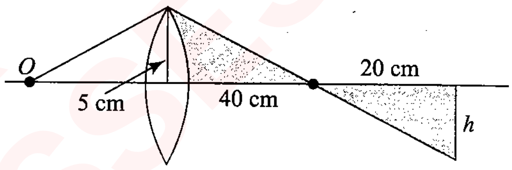


Fig. 1.492

$$\frac{h}{20}=\frac{5}{40}=\frac{1}{8}$$

or $$h=\frac{20}{8}\text{cm}=\frac{5}{2}\text{cm}=2.5\text{cm}$$

41. a. The refractive index, ${}_1n_2$ (from medium 1 to medium 2) for two given media 1 and 2 is given by

$${}_1n_2=\frac{\text{speed of light in medium 1 }(c_1)}{\text{speed of light in medium 2 }(c_2)}$$

$$=\frac{\text{wavelength of light in medium 1 }(\lambda_1)}{\text{wavelength of light in medium 2 }(\lambda_2)}$$

Now, $${}_1n_2=\frac{n_2}{n_1}=\frac{5}{4}$$

$$c_1=2.0\times10^{-8}\text{ms}^{-1}$$
$$\lambda_1=500\text{ nm}$$

$\therefore$ $$\frac{5}{4}=\frac{2.0\times10^{8}}{c_2}=\frac{500}{\lambda_2}$$

Hence, $\lambda_2=400\text{ nm}$

$c_2=1.6\times10^{8}\text{ms}^{-1}$

42. d. If the refractive index of the material of the lens is greater than the refractive index of the surrounding medium, then a concave lens would behave as a concave lens.

43. b. $$v_m=\frac{1}{2}c$$

$$\mu=\frac{c}{V_m}=\frac{c}{\frac{1}{2}c}=2 \text{ or } \frac{1}{\sin i_c}=2$$

or $$\sin i_c=\frac{1}{2} \text{ or } i_c=30°$$

44. d. Clearly, power of system is zero.

$$\therefore \quad 0=\frac{1}{20}+\frac{1}{f}-\frac{5}{20f}$$

or $$-\frac{1}{20}=\frac{15}{20f} \text{ or } f=-15 \text{ cm}$$

45. c. $P=(1.5-1)\left(\frac{200}{5}\right)=20 \text{ D}$

$$P'=(1.5-1)\left(\frac{200}{5}\right)=16.67 \text{ D}$$

Decrease in power $=20 \text{ D}-16.67 \text{ D}=3.33 \text{ D}$

46. d. $$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-15}=\frac{1}{f}$$

or $\frac{1}{v}-\frac{1}{15}+\frac{1}{10}$ or $\frac{1}{v}=\frac{-2+3}{30}$

or $v=30$ cm

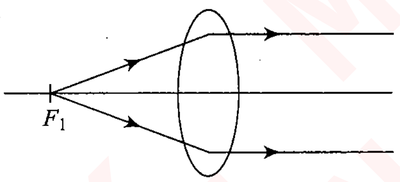


Fig. 1.493

47. b. Since the surrounding medium is denser than the material of the lens, therefore the concave lens shall behave like a convex lens.

48. d. For convex mirror, $u+v=12\times 2=24$ cm (because for plane mirror, distance of object = distance of image). Also, here for the convex mirror $u=20$ cm, therefore $v=4$ cm. Hence, using

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

We find $f=5$ cm

49. c.

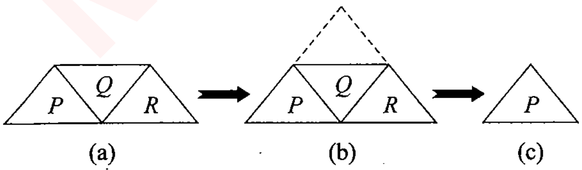


Fig. 1.494

Figure (a) is part of an equilateral prism of figure (b) which is a magnified image of figure (c). Therefore, the ray will suffer the same deviation in figure (a) and figure (c).

50. b. Distance of firs t image (I_1) formed after refraction from the plane surface of water is

$$\frac{10}{4/3}=7.5 \text{ cm}$$

from water surface $\left(d_{app}=\frac{d_{actual}}{\mu}\right)$.

Now, distance of this image is $5+7.5=12.5$ cm from the plane mirror. Therefore, distance of second image (I_2) will also be equal to 12.5 cm from the mirror.

51. a. Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

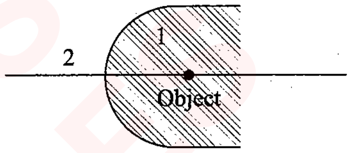


Fig. 1.495

$$\frac{1}{v}-\frac{1.5}{(-u)}=\frac{1-(1.5)}{-R} \quad \text{or} \quad \frac{1}{v}+\frac{3}{2u}=\frac{1}{2R}$$

For v to be positive, $\frac{1}{2R}>\frac{3}{2u}$

or $u>3R$

52. d. The two slabs will shift the image a distance

$$d=2\left(1-\frac{1}{\mu}\right)t=2\left(1-\frac{1}{1.5}\right)(1.5)=1.0 \text{ cm}$$

Therefore, final image will be 1 cm above point P.

53. b. Critical angle $\theta_c=\sin^{-1}\left(\frac{1}{\mu}\right)$

$$=\sin^{-1}\left(\frac{1}{2}\right)=30°$$

If $A>2\theta_c$, the ray does not emerge from the prism. So, maximum angle can be 60°.

54. b. $r_2=0°$

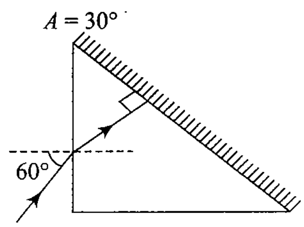


Fig. 1.496

$r_1=A=30°$

and $i_1=60°$

$$\mu=\frac{\sin i_1}{\sin r_1}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

55. a. $A=\delta_m=60°$

At minimum deviation $i=\left(\frac{A+\delta_m}{2}\right)=60°$

56. a. $\dfrac{1}{F}=(\mu-1)\left(\dfrac{1}{\infty}+\dfrac{1}{R}\right)+(\mu_2-1)\left(\dfrac{1}{-R}-\dfrac{1}{\infty}\right)$

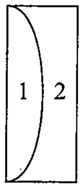


Fig. 1.497

$$=\frac{\mu_1-\mu_2}{R} \text{ or } F=\frac{R}{\mu_1-\mu_2}$$

57. b.

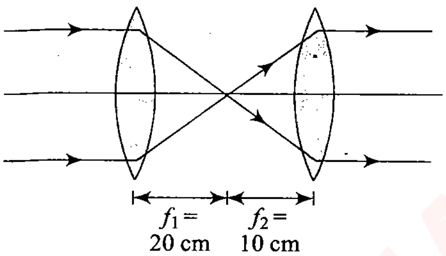


Fig. 1.498

As shown in figure the distance between the lenses should be 30 cm.

58. d. Focal length of plano convex lens is

$$\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{10}-\frac{1}{\infty}\right) \quad \text{or} \quad f=20 \text{ cm}$$

If point object O is placed at a distance of 20 cm from the plano-convex lens, rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of b.

59. d. Image will be formed at infinity if object is placed at focus of the lens, i.e., at 20 cm from the lens. Hence,

$$\text{shift}=25-20=\left(1-\frac{1}{\mu}\right)\mu$$

or $\quad 5=\left(1-\dfrac{1}{1.5}\right)t$ or $t=\dfrac{5\times 1.5}{0.5}=15$ cm

60. a. Only one image will be formed by this lens system because optic axis of both the parts coincide. Two images would have formed if their optic axis were different.

61. a. Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (c) and (d). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.

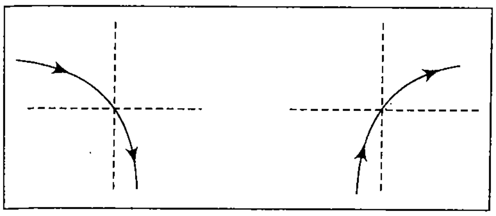


Fig. 1.499

62. c. $m=\dfrac{f}{f+u}, \dfrac{1}{2}=\dfrac{f}{f-2}$; or $2f=f-2$ or $f=-2$ m

$|f|=2$ metre. Since the image is virtual as well as diminished, therefore the lens is concave.

63. c. $\dfrac{1}{f}=(\mu-1)\left(\dfrac{2}{R}\right)$ or $f=\dfrac{R}{2(\mu-1)}$

Now, $\quad f>R$

$\therefore \quad \dfrac{R}{2(\mu-1)}>R$ or $\dfrac{1}{2(\mu-1)}>1$ or $2(\mu-1)<1$

or $\quad \mu-1<\dfrac{1}{2}$; or $\mu<\left(1+\dfrac{1}{2}\right)$; or $\mu<1.5$

64. b. Power of the system decreases due to separation between the lenses. So, the focal length increases.

65. a. In the first case, neither the radii of curvature nor the material of the lens is affected. In the second case,

$$\frac{1}{f''}=(\mu-1)\left(\frac{1}{R}\right) \text{ or}$$

$$\frac{1}{f''}=\frac{1}{2}(\mu-1)\left(\frac{2}{R}\right)=\frac{1}{2f}$$

or $\quad f''=2f.$

66. b. $\quad m=\dfrac{f}{f-u}$

$$3=\frac{-24}{-24-u}$$

or $\quad -24-u=-8$ or $u+24=8$

or $\quad u=(8-24)$ cm $=-16$ cm

If $\quad m=-3$, then

$$-3=\frac{-24}{-24-u}$$

$$u+24=-8$$

or $\quad u=-32$ cm.

Note that the magnification is greater than 1, so mirror cannot be convex.

67. d. Clearly, x is denser medium.

Now, $\quad \dfrac{\sin\theta}{\sin 90^o}=\dfrac{1}{{}_Y\mu_X}={}_X\mu_Y=\dfrac{v}{v'}$

or $\quad v'=\dfrac{v}{\sin\theta}$

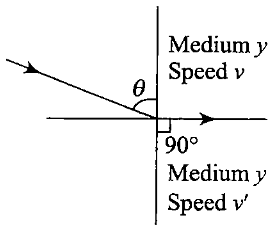


Fig. 1.500

68. d. $\dfrac{I}{O} = \dfrac{v}{u}$

$$\frac{I}{15} = \frac{-25}{-10}$$

$I = 15 \times 2.5 \text{ cm} = 37.5 \text{ cm}$

69. b. Power of combination = (6 – 4)D = 2 *D*

$$\text{Power} = \frac{100}{f \text{ (in cm)}}$$

$$2 = \frac{100}{f} \text{ (in cm)}$$

or f (in cm) = 50

Since the net power is positive, therefore the combination shall behave like a convex lens.

70. a. Power of system

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{1} + \frac{1}{-0.25} - \frac{0.75}{(1)(-0.25)}$$

$$= 1 - 4 + 3 = -3 + 3 = 0$$

Since power of the system is zero, therefore the incident parallel beam of light will remain parallel after emerging from the system.

71. b $P = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$

$$0 = \frac{1}{20} - \frac{1}{5} - \frac{d}{20(-5)}$$

$$\frac{d}{100} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20} = \frac{3}{20}$$

or $d = 15$ cm

72. d. Clearly, $2f = 20$ cm

or $f = 10$ cm

Now, $u = -15$ cm, $v = ?$

$f = 10$ cm

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

or $\dfrac{1}{v} + \dfrac{1}{15} = \dfrac{1}{10}$ or $\dfrac{1}{v} = \dfrac{1}{10} - \dfrac{1}{15}$.

or $\dfrac{1}{v} = \dfrac{3-2}{30} = \dfrac{1}{30}$

or $v = 30$ cm

The change in image distance is (30 – 20) cm, i.e., 10 cm.

73. d. $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

$$\frac{1}{f-x_1} + \frac{1}{f-x_2} = \frac{1}{f} \text{ or } \frac{f-x_2+f-x_1}{(f-x_1)(f-x_2)} = \frac{1}{f}$$

or $f^2 - fx_2 - fx_1 + x_1x_2 = 2f^2 - f(x_1 + x_2)$

or $f^2 = x_1x_2$ or $f = \sqrt{x_1x_2}$

This is Newton's mirror formula.

74. d. When there is no water in the mirror, the rays of light are incident normally on the mirror and retrace their path. So, we get an image coincident with the object as shown in the figure.

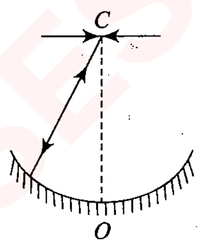


Fig. 1.501

When the mirror is filled with water, then the equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}} + \frac{1}{f_{\text{water lens}}}$$

or $\dfrac{1}{F} = 2 \times \dfrac{1}{f_{\text{water lens}}} + \dfrac{1}{f_{\text{concave mirror}}}$

or $\dfrac{1}{F} = 2(\mu - 1)\left(\dfrac{1}{R}\right) + \dfrac{1}{R}$

or $\dfrac{1}{F} = \dfrac{2(\mu - 1)}{R} + \dfrac{2}{R}$

or $\dfrac{1}{F} = \dfrac{2\mu}{R}$ or $F = \dfrac{R}{2\mu}$

Clearly, focal length of the new optical system in less than the original. So, the object is effectively at a distance greater than twice the focal length. So, the real image will be formed between F and $2F$.

75. c. $v \cos 45° = 20\sqrt{2}$

or $\dfrac{v}{\sqrt{2}} = 20\sqrt{2}$ or $v = 40 \text{ cm s}^{-1}$

The velocity of the image formed by the roof is the same as the velocity of insect.

So, (c) is the correct choice.

76. d. A thick glass mirror produces a number of images.

There is an apparent shift of actual silvered surface toward the unsilvered face.

Effective distance of the reflecting surface from unsilvered face $= \dfrac{d}{\mu} = \dfrac{3}{3/2}$ cm = 2 cm.

Distance of the point object from effective reflecting surface = 9 cm + 2 cm = 11 cm.

Distance of image from the point object = 11 cm + 11 cm = 22 cm

Distance of image from unsilvered face = (22 − 9) cm = 13 cm

77. a. Δs IAB and ICD are similar.

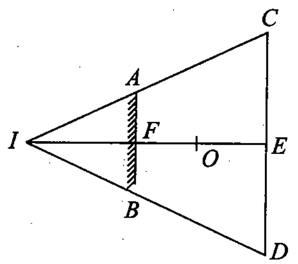


Fig. 1.502

$$\frac{CD}{AB}=\frac{IE}{IF}=\frac{3L}{L}=3$$

$$CD = 3AB = 3d$$

78. a. Clearly, the distance of I from P is 15 cm.

Now, $u = -25$ cm, $v = 15$ cm, $f = ?$

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad \frac{1}{f}=\frac{1}{-25}+\frac{1}{15}$$

or $$\frac{1}{f}=\frac{-3+5}{75} \quad \text{or} \quad f=\frac{75}{2}\text{ cm} = 37.5\text{ cm}$$

79. b. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$$-\frac{du}{u^2}-\frac{dv}{v^2}=0 \quad \text{or} \quad -\frac{dv}{v^2}-\frac{du}{u^2}$$

or $$dv=-\frac{v^2}{u^2}du$$

$$=-\frac{10\times 10}{30\times 30}\times 9\text{ ms}^{-1}=-1\text{ ms}^{-1}$$

80. d. $f=\frac{1.6}{2}\text{ m}=0.8\text{m}, u=-1\text{m}$

$$\frac{1}{v}=\frac{1}{0.8}-\frac{1}{-1}=\frac{10}{8}+1=\frac{18}{8}=\frac{9}{4}$$

or $$v=\frac{4}{9}\text{m}$$

81. a. $m=-\frac{v}{u}=-\frac{\frac{4}{9}}{-1}=\frac{4}{9}$

82. a. For a concave mirror,

$$u=-\frac{15}{2}\text{ cm}, v=?$$

$$f=-\frac{10}{2}\text{ cm}=-5\text{ cm}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-5}-\frac{1}{-15/2}$$

$$=-\frac{1}{5}+\frac{2}{15}=\frac{-1}{15}$$

or $v=-15$ cm

Clearly, the position of the final image is on the pole of the convex mirror.

83. b. For upright portion,

$$m=\frac{f}{f-u}=\frac{\frac{-10}{2}}{\frac{-10}{2}-(-20)}=\frac{-5}{-5+20}$$

$$=\frac{-5}{15}=-\frac{1}{3}.$$

For horizontal portion, magnification is $\left(-\frac{1}{3}\right)^2$ i.e., $\frac{1}{9}$

Required ratio is $\frac{-1/3}{1/9}=-3:1$

84. d. Let $u=-x$

$$\frac{1}{-x}+\frac{1}{-x-10}=-\frac{1}{12}$$

or $$\frac{1}{x}+\frac{1}{x+10}=\frac{1}{12}$$

or $$\frac{x+10+x}{x(x+10)}=\frac{1}{12}$$

or $$\frac{2x+10}{x^2+10x}=\frac{1}{12}$$

or $x^2+10x=24x+120$

or $x^2-14x-120=0$

$x^2-20x+6x-120=0$

or $x(x-20)+6(x-20)=0$

Here $u=-20$ cm

$v=-(20+10)$ cm

$=-30$ cm

Then magnification

$$m=-\frac{u}{v}$$

$$=-\left(\frac{-30}{-20}\right)$$

$$=-1.5$$

85. b. $\dfrac{\mu_1}{-u}+\dfrac{\mu_2}{v}=\dfrac{\mu_2-\mu_1}{R}$

$\dfrac{1.25}{-(-\infty)}+\dfrac{1.5}{v}=\dfrac{1.5-1.25}{R}$ or $\dfrac{1.5}{v}=\dfrac{0.25}{R}$

or $v=\dfrac{1.5R}{0.25}=6R$

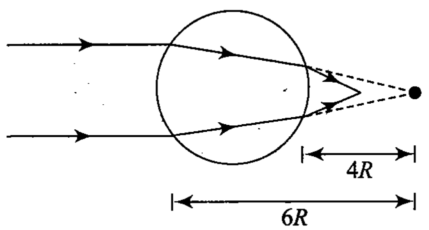


Fig. 1.503

Again, $\dfrac{\mu_2}{-u}+\dfrac{\mu_1}{v}=\dfrac{\mu_1-\mu_2}{R}$

$\dfrac{1.5}{-4R}+\dfrac{12.5}{v}=\dfrac{1.25-1.5}{-R}$

or $\dfrac{1.25}{v}=\dfrac{1}{4R}+\dfrac{1.5}{4R}=\dfrac{2.5}{4R}$

or $=\dfrac{1.25\times 4R}{2.5}=\dfrac{5R}{2.5}$

or $v=2R$.

Distance from the center $=3R$

86. c. $\mu=\dfrac{\sin r'}{\sin i}$

But $r'+r=90°$

or $r'=90°-r$

$r=90°-r$

$\therefore$ $\mu=\dfrac{\cos i}{\sin i}$

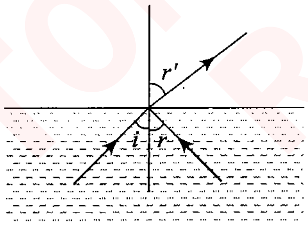


Fig. 1.504

or $\dfrac{1}{\sin i_c}=\dfrac{1}{\tan i}$

or $i_c=\sin^{-1}(\tan i)$

87. c. $\mu_g \sin\theta_c=\mu_1\sin 90°$

or $\mu_g\sin\theta_c=1$

When water is poured,

$\mu_w\sin r=\mu_g\sin\theta_c$

or $\mu_w\sin r=1$

Again, $\mu_a\sin\theta=\mu_w\sin r$

or $\mu_a\sin\theta=1$

or $\sin\theta=1$ or $\theta=90°$

88. b. $\dfrac{\sin 60°}{\sin r_1}=\sqrt{3}$

or $\sin r_1\dfrac{\sin 60°}{\sqrt{3}}$

or $\sin r_1=\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{3}}=\dfrac{1}{2}$

or $r_1=30°$

Now, $\sin(i_1-r_1)=\dfrac{d}{5}$

or $d=5\sin(i_1-r_1)$

or $d=5\sin(60°-30°)=5\sin 30°=\dfrac{5}{2}$ cm

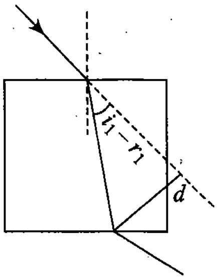


Fig. 1.505

89. c. $\sin i_c=\dfrac{1}{\sqrt{2}}$ $i_c=45°$

Now, $75°=r+45°$

or $r=30°$

Now, $\dfrac{\sin i}{\sin 30°}=\sqrt{2}$

or $\sin i=\sqrt{2}\times\dfrac{1}{2}=\dfrac{1}{\sqrt{2}}$

$\therefore$ $i=45°$

90. b.

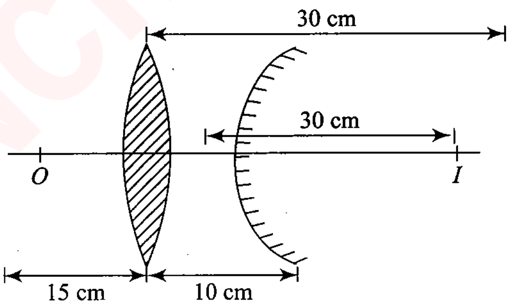


Fig. 1.506

$\dfrac{1}{v}-\dfrac{1}{-15}=\dfrac{1}{10}$

or $\dfrac{1}{v}=\dfrac{1}{10}-\dfrac{1}{15}$ or $\dfrac{1}{v}=\dfrac{3-2}{30}$

or $v=30$ cm

Clearly, the rays coming from the convex lens should fall normally on the convex mirror. In other words, the rays should be directed toward the center of curvature of the convex mirror.

$\therefore$ $2f=20$ cm or $f=10$ cm

91. b.

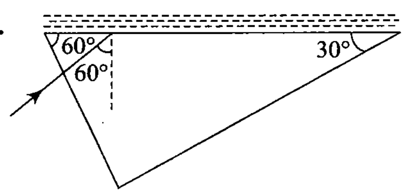


Fig. 1.507

Clearly, $i_c \le 60°$

So, maximum possible value of i_c is 60°.

Now, $${}_1\mu_g = \frac{1}{\sin i_c}$$

$$\frac{\mu_g}{\mu_l} = \frac{1}{\sin i_c}$$

or $$\mu_l = \mu_g \sin i_c = 1.5 \sin 60° = 1.5 \times \frac{\sqrt{3}}{2}$$

$$= 1.5 \times 0.866 = 1.299 = 1.3$$

92. d. For lens $\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$

$$\frac{u_1}{f} = \frac{u_1}{v_1} - 1 \quad \Rightarrow \quad m_1 = \frac{v_1}{u_1} = \frac{f}{u_1 + f}$$

And $m_2 = \frac{f}{u_2 + f}$

$$\frac{1}{m_2} - \frac{1}{m_1} = \frac{(u_2 - u_1)}{f} \quad \Rightarrow \quad f = \frac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$$

93. d.

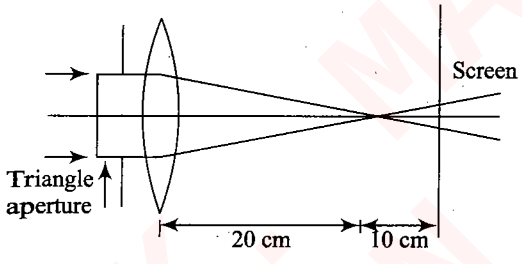


Fig. 1.508

The image on the screen is real and inverted.

The size of the image on the screen has aperture size given by

$$\text{Size} = 1.0\left(\frac{10}{20}\right) = 0.5 \text{ cm}$$

Hence, the path of light on the screen is best represented by diagram (d).

94. b. Power of liquid lens

$$= (1.6 - 1)\left(\frac{2}{0.20}\right) = \frac{6}{10} \times 10 = 6 \text{ D}$$

Power of concave lens

$$= -(1.5 - 1) = -0.5 \times 10 \text{ D}$$

Total power of two concave lenses = –10 D

Power of system

$$= -10 \text{ D} + 6 \text{ D} = -4 \text{ D}$$

Forcal length $= \frac{1}{-4} = -0.25$ m

95. a. Since the refractive index of the liquid is greater than the refractive index of glass, therefore the focal length of the system has to be negative. So, options (b) and (d) are excluded.

Now, $\frac{1}{24} = (1.5 - 1)\frac{2}{R}$

or $R = 24$ cm

Again, for liquid concave lens,

$$\frac{1}{f} = -(1.6 - 1)\left(\frac{2}{24}\right)$$

$$\frac{1}{f} = -\frac{3}{5} \times \frac{1}{12}$$

or $$\frac{1}{f} = -\frac{1}{20} \quad \text{or} \quad f = -20 \text{ cm}$$

Now, $$\frac{1}{F} = \frac{1}{24} - \frac{1}{20} = \frac{5-6}{120}$$

or $F = -120$ cm

96. b. $m = 30$

$v + u = xu$

or $\frac{v}{u} + 1 = x$ or $m + 1 = x$

or $x = 31$

97. c. $A = r_1 + r_2$

$30° = r_1 + 0$

or $r_1 = 30°$

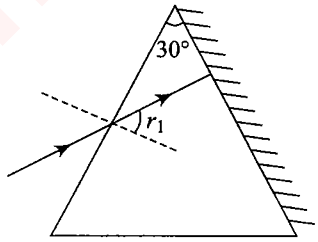


Fig. 1.509

Now, $$\frac{\sin i}{\sin 30°} = \sqrt{2}$$

or $$\sin i = \sqrt{2} \times \frac{1}{2}$$

or $i = 45°$

98. a. Effective height of the bird as seen by the fish,

$$Y = y + \mu y'$$

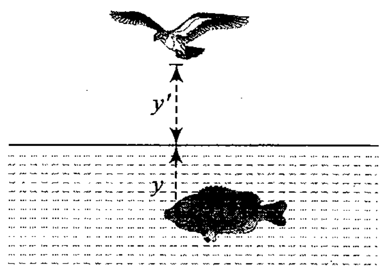


Fig. 1.510

$$\frac{dY}{dt} = \frac{dy}{dt} + \mu\frac{dy'}{dt}$$

$$9 = 3 + \frac{4}{3}\frac{dy'}{dt}$$

or $$\frac{dy'}{dt} = \frac{6\times 3}{4} = \frac{18}{4}$$

$$= \frac{9}{2} = 4.5 \text{ ms}^{-1}$$

Therefore, actual velocity of bird = 4.5 m s^{-1}

99. a. $$-\frac{1}{40} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} - \frac{1}{R_2} = -\frac{1}{20}$$

Now, $$\frac{1}{f} = \left(\frac{1.5}{2} - 1\right)\left(-\frac{1}{20}\right)$$

or $$\frac{1}{f} = -\frac{0.5}{2}\left(-\frac{1}{20}\right)$$

or $$\frac{1}{f} = \frac{1}{80} \text{ or } f = 80 \text{ cm}$$

It behaves like a convex lens of focal length 80 cm.

100. a. Sin $i_c = \frac{1}{1.5} = \frac{2}{3}$

$$i_c = \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}(0.6667) = 41.8^\circ$$

The angle of incidence of face AC is 45° which is clearly greater than 41.8°.

101. c. $u = -100$ cm

$v = -120$ cm

$F = ?$

$$\frac{1}{F} + \frac{1}{u} + \frac{1}{v} = \frac{1}{-100} + \frac{1}{-120}$$

$$\frac{1}{F} = -\frac{6+5}{600} = -\frac{11}{600}$$

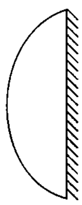


Fig. 1.511

Now, $$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m} = \frac{2}{f_l}$$

or $$f_l = 2F$$

or $$f_l = 2 \times \frac{600}{11}$$

[Negative sign is not to be used here.]

or $$f_l = \frac{1200}{11} = 109.1 \text{ cm}$$

102. c. $$\frac{\mu_2}{-\mu} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R} = 0$$

In the first case,

$$\frac{\mu}{-d} + \frac{1}{-v} = \frac{\mu_1 - \mu_2}{\infty} = 0$$

or $$\frac{1}{-v} = \frac{\mu}{d}$$

or $$v = -\frac{d}{\mu}$$

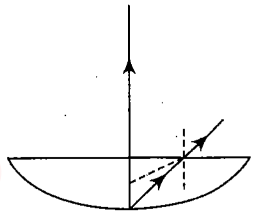


Fig. 1.512

In the second case,

$$\frac{\mu}{-d} + \frac{1}{-v'} = \frac{1-\mu}{-R}$$

or $$\frac{1}{-v'} = \frac{\mu}{-d} - \frac{1-\mu}{-R}$$

or $$\frac{1}{-v'} = \frac{1}{-v} + \frac{1-\mu}{-R}$$

Clearly, $$\frac{1}{-v'} > \frac{1}{-v}$$

or $$\frac{1}{-v'} < \frac{1}{-v}$$

or $$v' > v$$

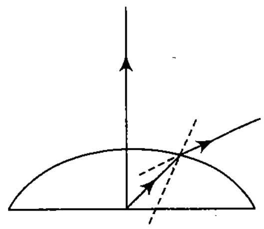


Fig. 1.513

103. a. As shown in figure, the plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it. So, the distance of image formed by plane mirror from convex mirror will be

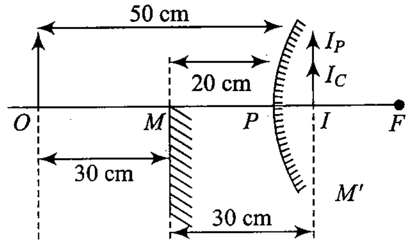


Fig. 1.514

$$PI = MI - MP$$

But as, $MI = MO$

$$PI = MO - MP = 30 - 20 = 10 \text{ cm}$$

Now, as this image coincides with the image formed by convex mirror, therefore for convex mirror,

$u = -50$ cm; $v = +10$ cm

So, $\frac{1}{+10} + \frac{1}{-50} = \frac{1}{f}$, i.e., $f = \frac{50}{4} = 12.5$ cm

So, $R = 2f = 2 \times 12.5 = 25$ cm

104. c.

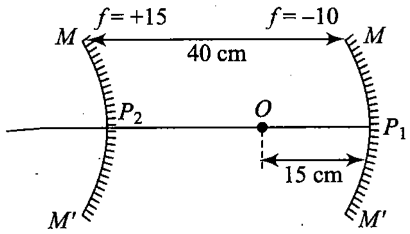


Fig. 1.515

According to given problem, for concave mirror,

$u = -15$ cm and $f = -10$ cm

$$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}, \quad \text{i.e,} \quad v = -30 \text{ cm}$$

i.e., concave mirror will form real, inverted, and enlarged image I_1 of object O at a distance of 30 cm from it, i.e., at a distance $40 - 30 = 10$ cm from the convex mirror.

For convex mirror, the image I_1 will act as an object and so for it $u = -10$ cm and $f = +15$ cm.

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{15}, \quad \text{i.e.,} \quad v = +6 \text{ cm}$$

So, final image I_2 is formed at a distance of 6 cm behind the convex mirror and is virtual as shown in figure.

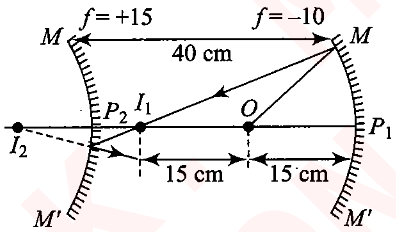


Fig. 1.516

105. b. For part MN,

$u = -30$ cm, $f = -10$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-30} = -\frac{1}{10} + \frac{1}{30}$$

$$\frac{1}{v} = \frac{-3+1}{30} = \frac{-2}{30}$$

$V = -15$ cm

$$m = \frac{I}{O} = -\frac{v}{u}$$

$$\Rightarrow \quad \frac{I}{10} = -\frac{(-15)}{(-30)} = -\frac{1}{2} \Rightarrow I = 5 \text{ cm}$$

For image of ST,

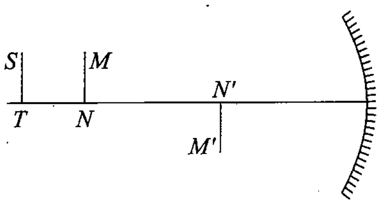


Fig. 1.517

$u = -40$cm

$f = -10$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{(-40)} = \frac{1}{(-10)}$$

$$\frac{1}{v} = -\frac{1}{10} + \frac{1}{40}$$

$$\frac{1}{v} = -\frac{-4+1}{40} = \frac{-3}{40}$$

$$V = -\frac{40}{3} \text{ cm}$$

For length of image of ST,

$$m = \frac{I}{O} = -\frac{v}{u}$$

$$\frac{I}{10} = -\left(\frac{-40/3}{-40}\right) \Rightarrow I = -\frac{10}{3} \text{ cm}$$

Length of image of NT,

$$T'N' = 15 - \frac{40}{3} = \frac{5}{3} \text{ cm}$$

$$\text{Length of wire in image} = 5 + \frac{10}{3} + \frac{5}{3} = 10 \text{ cm}$$

106. a. Object is placed beyond C. Hence, the image will be real and it will lie between C and F. Further u, and v are negative, hence the mirror formula will become

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{u-f}{uf} \quad \text{or} \quad v = \frac{f}{1-(f/u)}$$

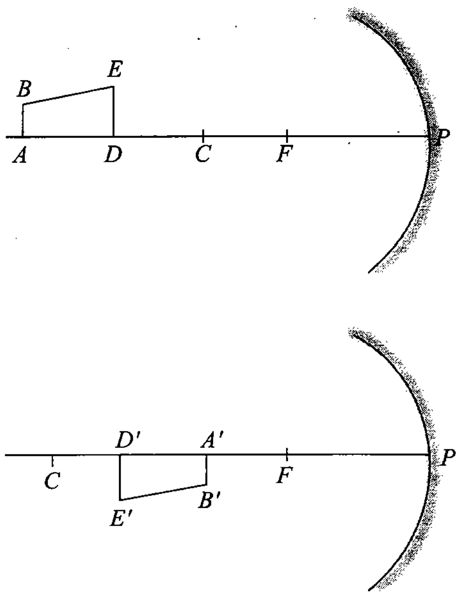


Fig. 1.518

Now $u_{AB} > u_{ED}$

$\therefore \quad v_{AB} < v_{ED}$

$$|m_{AB}| < |m_{ED}| \left(\text{as } m = -\frac{v}{u}\right)$$

Therefore, shape of the image will be as shown in figure.

Also, note that, $v_{AB} < u_{AB}$ and $v_{ED} < u_{ED}$,

So, $|m_{AB}| < 1$ and $|m_{ED}| < 1$

107. d. Let v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum,

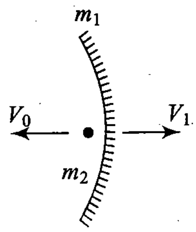


Fig. 1.519

$$m_2 v_0 = m_1 v_1$$

$$v_1 = \frac{m_2 v_0}{m_1} \quad \text{(i)}$$

Now rate at which distance between mirror and bullet is increasing,

$$\frac{du}{dt} = v_1 + v_0 \quad \text{(ii)}$$

$$\frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$

Here, $\frac{v^2}{u^2} = m^2 = 1$

(as at the time of firing, the bullet is at pole).

$$\frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0 \quad \text{(iii)}$$

Here, dv/dt is the rate at which distance between image (of bullet) and mirror is increasing. So, if v_2 is the absolute velocity of image (towards right), then

$$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0$$

or $\quad v_2 = 2v_1 + v_0 \quad$ (iv)

Therefore, speed of separation of the bullet and the image will be

$$v_r = v_2 + v_o = 2\, v_1 + v_o + v_o$$

$$v_r = 2\,(v_1 + v_o)$$

Substituting value of v_1 from Eq. (i), we have

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right)v_0$$

108. a. Figure shows three rays 1, 2, 3 incident on plane face A. We can see that angle of incidence at curved surface is least for ray 3. If ray 3 reflects at the curved surface, then all the rays will reflect as their angle of incidence is greater than angle θ_3.

$$\theta_3 \ge \theta_{\text{critical}}$$

$$\text{Sin}\,\theta_{\text{critical}} = \frac{1}{\mu} = \frac{2}{3}$$

From geometry of figure, we have

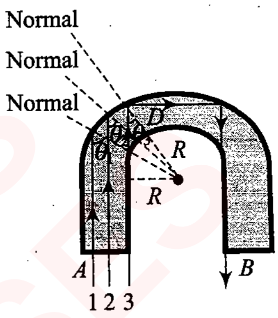


Fig. 1.520

$$\sin\theta_3 = \frac{R}{d+R}$$

$$\sin n\theta_3 \ge \sin\theta_{\text{critical}}$$

$$\frac{R}{d+R} \ge \frac{2}{3}$$

$$\frac{d}{R} \le \frac{1}{2}$$

Therefore, $\left(\frac{d}{R}\right)_{\text{maximum}} = \frac{1}{2}$

109. d. The lens will converge the rays at its focus, i.e., 30 cm from the lens or 20 cm from the refracting surface.

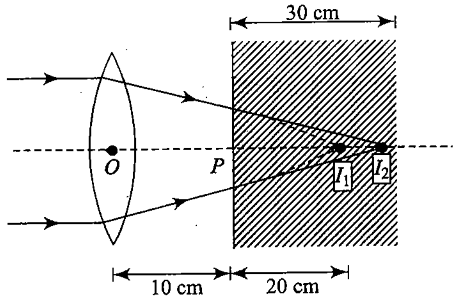


Fig. 1.521

$$PI_1 = 20\,\text{cm}$$

$$\therefore \quad PI_2 = \mu\,(PI_1) = \frac{3}{2} \times 20 = 30\ \text{cm}$$

So, the rays will converge at a distance of 40 cm from the lens.

110. c. Object is placed at a distance of $2f$ from the lens (f = focal length of lens), i.e., the image formed by the lens will be at a distance of $2f$ or 20 cm from the lens. So, if the concave mirror is placed in this position, the first image will be formed at its pole and it will reflect all the rays symmetrically to other side and the final image will coincide with the object.

111. a. In this case, one of the image will be real and the other virtual. Let us assume that image of S_1 is real and that of S_2 is virtual.

Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

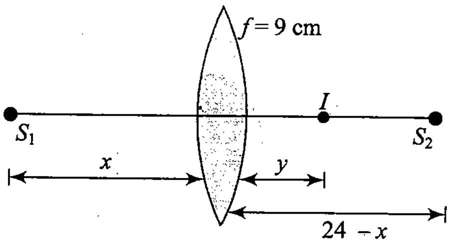


Fig. 1.522

For S_1: $\frac{1}{y} + \frac{1}{x} = \frac{1}{9}$ (i)

For S_2: $\frac{1}{y} + \frac{1}{24-x} = \frac{1}{9}$ (ii)

Solving Eqs. (i) and (ii), we get

$x = 6$ cm

112. c. Let f_1 and f_2 be the focal length in water. Then

$$\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{2}{R}\right) \quad \text{(i)}$$

$$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(\frac{-2}{R}\right) \quad \text{(ii)}$$

Adding (i) and (ii), we get $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$

or $\frac{1}{30} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$

$\therefore$ $(\mu_1 - \mu_2) = \frac{\mu_w R}{60}$

Substituting the values,

$(\mu_1 - \mu_2) = \frac{4 \times 15}{3 \times 60} = \frac{1}{3}$

113. c. A ray of light starting from O gets refracted twice. The ray of light is traveling in a direction from left to right. Hence, the distances measured in this direction are taken positive. Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, twice with proper signs.

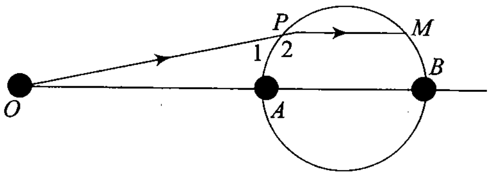


Fig. 1.523

We have, $\frac{3/2}{v} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$

or, $v = -30$ cm

Now, the first image I_1 acts as an object for the second surface, where

$BI_1 = u = -(30 + 20) = -50$ cm

$$\frac{4/3}{v'} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

$\therefore$ $v' = -100$ cm, i.e., the final image I_3 is virtual and is formed at a distance of 100 cm (toward left) from B. The ray diagram is as shown.

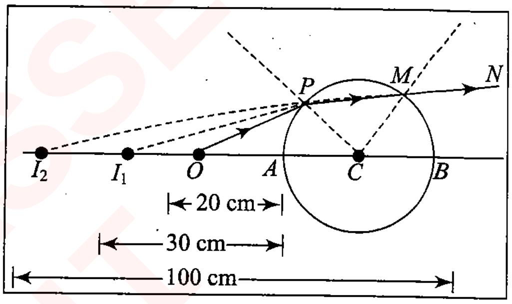


Fig. 1.524

Following points should be noted while drawing the ray diagram.

(i) At P, the ray travels from a rarer to a denser medium. Hence, it will bend toward normal MC.

(ii) PM ray when extended backward meets at I_1 and MN ray when extended meets at I_2.

114. b. Let us see where does the parallel rays converge or diverge on the principal axis. Let us call it the focus and the corresponding length the focal length f. Using

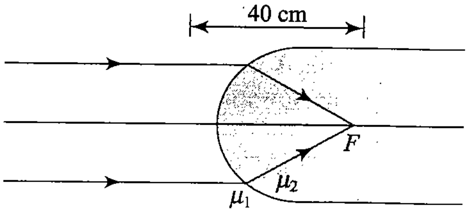


Fig. 1.525

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

With proper values and signs, we have

$$\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/2 - 1.0}{+10}$$

or $f = 40$ cm $= 0.4$ m

Since the rays are converging, its power should be positive. Hence,

$$P(\text{in dioptre}) = \frac{+1}{f\,(\text{meter})} = \frac{1}{0.4}$$

or $\quad P = 2.5$ dioptre

115. c. We will have to consider two refractions: first at surface S_1 and the second at surface S_2.

For refraction at surface S_1 :

$$\frac{1.6}{v_1} - \frac{1.2}{(-120)} = \frac{(1.6-1.2)}{20},\ v_1 = +160 \text{ cm}$$

The positive sign implies that first image is formed to the right of S_1.

For refraction at surface S_2:

The first image is object for refraction at second surface, $v_2 = +(160-40)$cm

$$\frac{1.7}{v_2} - \frac{1.6}{+(160-40)} = \frac{1.6-1.7}{(-20)},\ v_2 = +204 \text{ cm}$$

The final image is formed at 204 cm from vertex and on the right side.

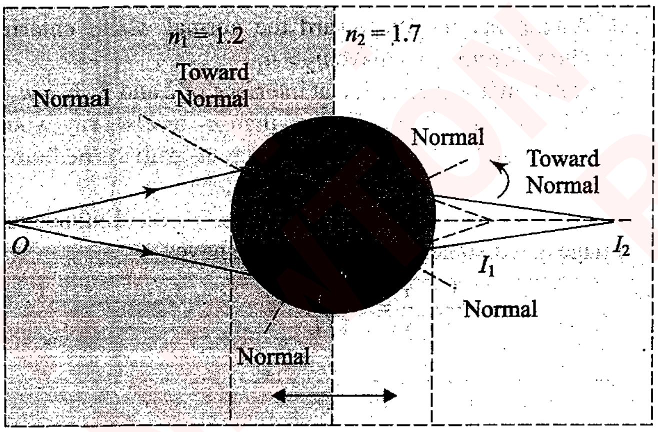


Fig. 1.526

116. a. We have to consider four refractions at S_1, S_2, S_3, and S_4, respectively. At each refraction, we will apply single surface refraction equation.

For refractive at first surface S_1:

$$\frac{3/2}{v_1} - \frac{1}{(-18)} = 0$$

$$v_1 = -27 \text{ cm}$$

First image lies to the left of S_1.

For refractive at second surface S_2:

$$\frac{1}{v_2} - \frac{3/2}{-(27+9)} = \frac{(1-3/2)}{+9}$$

$$v_2 = -\frac{72}{7} \text{ cm}$$

Note that origin of Cartesian coordinate system lies at vertex of surface S_2. The object distance is $(27+9)$ cm. The second image lies to left of S_2.

For refractive at third surface S_3:

$$u_3 = -\left(\frac{72}{7} + 18\right) = -\frac{198}{7}$$

$$\frac{1.5}{v_3} - \frac{1}{(-198/7)} = \frac{(1.5-1)}{(-9)}$$

$$v_3 = -16.5 \text{ cm}$$

For refractive at fourth surface S_4:

$$u_4 = -(16.5 + 9) = -25.5 \text{ cm}$$

$$\frac{1}{v_4} - \frac{3/2}{(-25.5)} = \frac{(1-3/2)}{\infty} = 0$$

$$v_4 = -17 \text{ cm}$$

The final image lies at 17 cm to the left of surface S_4.

117. a. We will have single surface refractions successively at the four surfaces S_1, S_2, S_3, and S_4. Do not forget to shift origin to the vertex of respective surface.

Refractive at first surface S_1: Light travels from air to glass.

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{(1.5-1)}{(+10)}$$

$$v_1 = 30 \text{ cm}$$

First image is object for the refraction at second surface.

For refraction at surface S_2: Light travels from glass to air

$$\frac{1}{v_2} - \frac{1.5}{(+25)} = \frac{1-1.5}{(+5)}$$

$$v_2 = -25 \text{ cm}$$

For refraction at surface S_3: Light travels from air to glass.

$$\frac{1.5}{v_3} - \frac{1}{(-35)} = \frac{(1.5-1)}{(-5)}$$

$$v_3 = -35/3 \text{ cm}$$

For refraction at surface S_4: Light travels from glass to air

$$\frac{1}{v_4} - \frac{1.5}{-(35/3+5)} = \frac{1-1.5}{-10}$$

$$v_4 = -25 \text{ cm}$$

The final image is virtual, formed at 25 cm to the left of the vertex of surface S_4.

118. b. Let the object distance be x. Then, the image distance is $D - x$.

From lens equation,

$$\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f}$$

On algebraic rearrangement, we get

$$x^2 - Dx + Df = 0$$

On solving for x, we get

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2}$$

$$x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}$$

119. b. If the object is at $u = x_1$,

$$m_1 = \frac{I_1}{O} = \frac{D - x_1}{x_1}$$

Now, $\quad x_1 = \frac{1}{2}(D-d),$

where $\quad d = \sqrt{D(D-4f)}$

$$m_1 = \frac{D-(D-d)/2}{(D+d)/2} = \left(\frac{D+d}{D-d}\right)$$

Similarly, when the object is at x_2, the magnification is

$$X_2 = \frac{1}{2}(D+d)$$

$$m_2 = \frac{I_2}{O} = \frac{D-x_2}{x_2} = \frac{D-(D+d)/2}{(D+d)/2} = \frac{D=d}{D+d}$$

The ratio of magnification is

$$\frac{m_2}{m_1} = \frac{(D-d)(D+d)}{(D+d)(D-d)} = \left(\frac{D-d}{D+d}\right)^2$$

120. c. As $\quad m_1 = \frac{I_1}{O} = \frac{v_1}{u_1}$

And $\quad m_2 = \frac{I_1}{O} = \frac{v_2}{u_2}$

$$m_1 \times m_2 = \frac{I_1 I_2}{O^2} = \frac{v_1}{u_1} \times \frac{v_2}{u_2} = 1$$

Hence, $\quad O = \sqrt{I_1 I_2} \Rightarrow \sqrt{h_1 h_2}$

121. b. As $\quad d = \sqrt{D(D-4f)}$

$$d^2 = D^2 - 4Df$$

$$f = \frac{D^2 - d^2}{4D}$$

122. d. As $\quad m_1 - m_2 = \frac{D+d}{D-d} - \frac{D-d}{D+d} = \frac{4Dd}{D^2-d^2}$

Hence, $\quad m_1 - m_2 = \frac{d}{f}$

$$f = \frac{d}{m_1 - m_2}$$

123. b. $\frac{\mu_2}{V} - \frac{\mu_1}{-u_0} = \frac{\mu_2 - \mu_1}{-2R}$

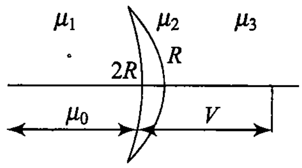


Fig. 1.527

$$\frac{\mu_3}{V_f} - \frac{\mu_2}{V} = \frac{\mu_3 - \mu_2}{-R}$$

$$\frac{\mu_3}{V_f} + \frac{\mu_1}{u_0} = \frac{\mu_1 - \mu_2}{2R} + \frac{\mu_2 - \mu_3}{R}$$

$$\therefore \frac{\mu_1 - \mu_2}{2} < \mu_3 - \mu_2 \Rightarrow \mu_1 - \mu_2 < 2\mu_3 - 2\mu_2 \Rightarrow \mu_1 + \mu_2 < 2\mu_3$$

124. b. In first case, if object distance is x, image distance = $x/4$. While it is 2nd case, object distance becomes (x – 5 cm) and image distance (x – 5 cm)/2. Using mirror formula we get,

In first case, $= \frac{1}{f} = -\frac{5}{x}$

In second case, $\frac{1}{f} = -\frac{3}{(x-5)}$

Solving we get, x = 12.5 cm and f = –2.5 cm and hence $|f| = 2.5$ cm

125. d. As the ray moves toward the normal while entering medium 2 from 1, we have $n_2 > n_1$

For total internal reflection at interface of 2 and 3, $n_2 > n_3$.

Besides n_3 should also be less than n_1 or else ray would have emerged in medium 3, parallel to its path in medium 1.

Hence, $n_3 < n_1 < n_2$ is the correct order.

126. c. For concave mirror, if x and y are object distance and image and distance, respectively, we have

$$-\frac{1}{x} - \frac{1}{y} = -\frac{1}{|f|}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{|f|}$$

$$\Rightarrow \quad -\frac{1}{x^2}\frac{dx}{dt} - \frac{1}{y^2}\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \left|\frac{V_x}{V_y}\right| = \frac{x^2}{y^2}$$

For $\quad \left|\frac{V_x}{V_y}\right| = \frac{1}{4}, \frac{x}{y} = \pm 2$

For $\quad \frac{x}{y} = 2$, we get $x = \frac{3|f|}{2}$ $\quad$ [for point A]

and For $\quad \frac{x}{y} = -2$, we get $x = \frac{|f|}{2}$ $\quad$ [for point B]

As the middle point happens to be focus of the mirror, we get $|f| = L$

127. c. $\frac{x}{1} = \frac{x_{rel}}{\mu} \Rightarrow X_{rel} = \mu x$

$$\frac{d^2x_{rel}}{dt^2} = \mu \frac{d^2x}{dt^2} \Rightarrow a_{rel} = \mu g$$

128. b. $A = 90° - \theta \Rightarrow r_2 = A = 90° - \theta > \theta_c$

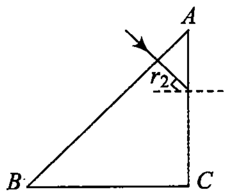


Fig. 1.528

$$\cos\theta > \sin\theta_c = \frac{6/5}{2/3} = \frac{4}{5} \text{ } (\theta_C \text{ is critical angle})$$

$$\theta < \cos^{-1}\frac{4}{5} = 37°$$

129. c. Let y-axis be vertically upward and x-axis be horizontal.

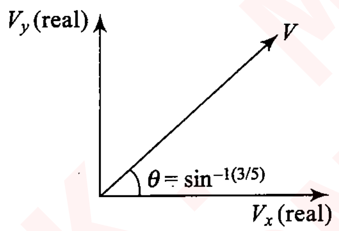


Fig. 1.529

$$V_y(\text{app.}) = \frac{V_y(\text{real})}{\left(\frac{1}{\mu}\right)}$$

$$V_y(\text{app.}) = V_x(\text{real.})$$

$$\tan\phi = \frac{V_y(\text{app})}{V_x(\text{app})} = \frac{4}{3}\tan\theta = \frac{4}{3} \times \frac{3}{4} = 1$$

130. b. Image velocity (w.r.t. mirror) = $-m$ × object velocity (w.r.t. mirror)

Here $m = 1$

131. a. $r_2 < \theta_c$; $A - r_1 < \theta_c$

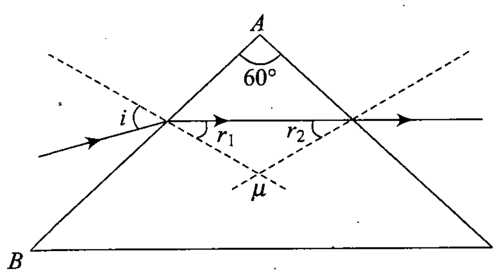


Fig. 1.530

$$r_1 > A - \theta_c$$
$$\sin r_1 > \sin(A - \theta_c)$$
$$\frac{\sin i}{\mu} > \sin(A - \theta_c)$$
$$\sin i > \mu(\sin A\cos\theta_c - \cos A\sin\theta_c)$$
$$\sqrt{\frac{7}{3}}\left(\frac{\sqrt{3}}{2}\sqrt{1 - \frac{3}{7}} - \sqrt{\frac{3}{7}}\cdot\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\sin i > \frac{1}{2} \text{ or } i > 30°$$

132. a. Given $i = 60°$, $A = \delta = e$

$$\delta = i + e - A \quad \Rightarrow \quad \delta = 1$$
$$(\because e = A) \quad \text{and} \quad \delta = i = e$$
$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here, angle of deviation is minimum ($\because i = e$)

$$\mu = \frac{\sin\left(\frac{60^0 + 60^0}{2}\right)}{\sin(60^0/2)} = \sqrt{3}$$

133. d. For spherical surface,

using $\quad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow \quad \frac{n}{R} - \frac{1}{\infty} = \frac{n-1}{R}$

$\Rightarrow \quad n = 2n - 2 \quad \Rightarrow \quad n = 2$

134. a.

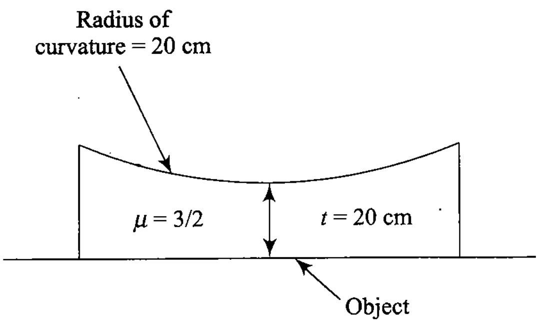


Fig. 1.531

Considering refraction at the curved surface,

$$u = -20 \quad ; \quad \mu_2 = 1$$
$$\mu_1 = 3/2 \quad ; \quad R = +20$$

Applying $\quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v}-\frac{3/2}{-20}=\frac{1-3/2}{20} \Rightarrow v=-10$$

i.e. 10 cm below the curved surface or 10 cm above the actual position of flower.

135. b.

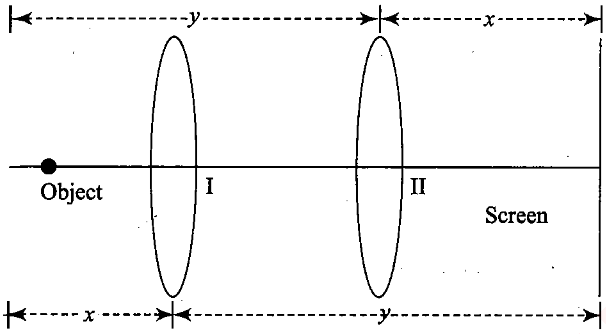


Fig. 1.532

At first position of lens, let the distance of lens from object and screen be x and y, respectively.

$\therefore \quad x+y=100$ (i)

$\therefore$ At second position of lens, the distance of lens from object and screen shall be y and x, respectively.

$\therefore \quad y-x=40$ (ii)

Solving Eqs. (i) and (ii), we get

$$y=70 \text{ cm}=\frac{700}{100} \text{ m and } x=30 \text{ cm}=\frac{30}{100} \text{ m}$$

Therefore, the power of lens is

$$\frac{1}{f}=\frac{1}{y}+\frac{1}{x}=\frac{100}{70}+\left(\frac{100}{30}\right)=\frac{100}{21} \approx 5 \text{ diopter}$$

136. b. Shift $=(\ell-m)\left(1-\frac{1}{n_1}\right)+m\left(1-\frac{1}{n_2}\right)=0$

137. d. Put $A=\delta_{\min}$ and $\mu=\sqrt{2}$

The relation $\quad \mu=\dfrac{\sin\left(\frac{A+\delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

And solve for A.

138. a. Use refraction formulae separately; that is for air and $\mu = 1.6$; and for air and $\mu=2.0$ and positions of the two images.

139. c. $v=-30, m=-\frac{v}{u}=-2$

$\Rightarrow \quad A'B'=CD=2\times 1=2$ mm

Now, $\quad \frac{B'C'}{BC}=\frac{A'D'}{AD}=\frac{v^2}{u^2}=4$

$\Rightarrow \quad B'C'=A'D'=4$ mm

$\therefore \quad$ Length $=2+2+4+4=12$ mm

140. c.

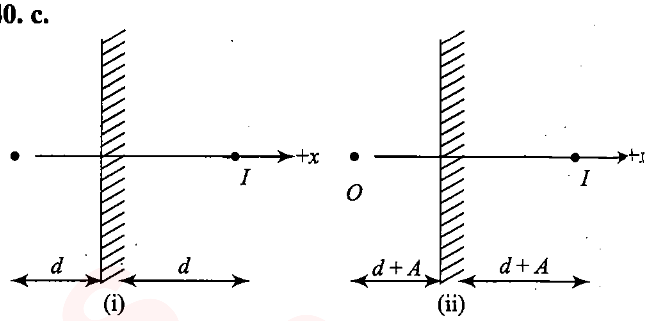


Fig. 1.533

From figure (i) and (ii), it is clear that if the mirror moves distance 'A', then the image moves a distance '$2A$'.

141. c. For M_1: $V=-60$, $m_1=-2$

For $\quad M_1$: $u=+20$, $F=10$

$$\frac{1}{V}-\frac{1}{20}=\frac{1}{10} \quad \Rightarrow \quad V=20$$

$\therefore \quad M_2=-\frac{20}{20}=-1$

$\therefore \quad M=m_1\times m_2=+2$

142. a. By mirror formula: $\frac{1}{v}+\frac{1}{-10}=\frac{1}{10}$

$\Rightarrow \quad v=+5$ cm

$\therefore \quad m=+\frac{1}{2}$

The image revolves in a circle of radius $\frac{1}{2}$ cm. Image of a radius is erect $\Rightarrow$ particle will revolve in the same direction as the particle. The image will complete one revolution in the same time 2 s.

Velocity of image v is

$$\omega r=\frac{2\pi}{2}\times\frac{1}{2}=\frac{\pi}{2} \text{ cms}^{-1}=1.57 \text{ cms}^{-1}$$

143. b. Cutting a lens in transverse direction doubles their focal length, i.e., $2f$.

Using the formula of equivalent focal length,

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}$$

We get equivalent focal length as $\frac{f}{2}$

144. a. In a prism: $r+r'=A \quad \Rightarrow \quad r=A-r'$

$\therefore \quad r=60°-(10+t^2)=50-t^2$

145. a. Given $i=60°$ $A=\delta=e$

$\delta=i+e-A \Rightarrow \delta=1$ ($\because e=A$)

$$\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here, angle of deviation is minimum ($\because i = e$)

$$\mu = \frac{\sin\left(\frac{60^0 + 60^0}{2}\right)}{\sin\left(60^0/2\right)} = 1.73$$

146. c.

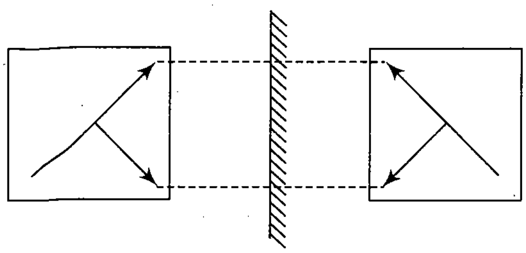


Fig. 1.534

147. b. When mirror is rotated with angular speed ω, the reflected ray rotates with angular speed 2ω (= 36 rads^{-1})

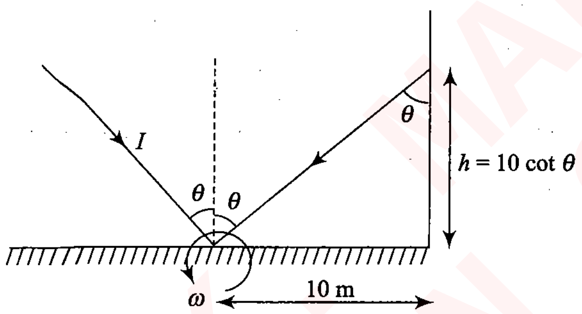


Fig. 1.535

$$\text{Speed of the spot} = \left|\frac{dh}{dt}\right| = \left|\frac{d}{dt}(10\cot\theta)\right|$$

$$= \left|-10\,\text{cosec}^2\theta\frac{d\theta}{dt}\right| = \left|-\frac{10}{(0.6)^2}\times 36\right| = 1000\ \text{ms}^{-1}$$

148. a. Speed of image w.r.t. mirror

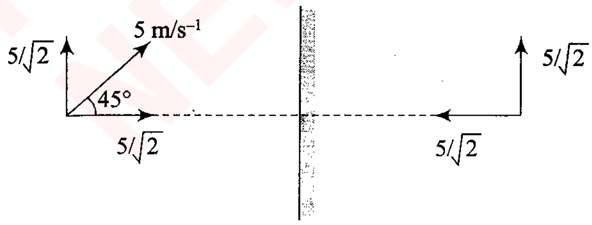


Fig. 1.536

$$= \sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = 5\ \text{m/s}^{-1}$$

149. c. Let power of light source be P, then intensity at any point on the screen is due to light rays directly received from source and that due to light rays after reflection from the mirror.

$$I = \frac{P}{4\pi a^2} + \frac{P}{4\pi \times (3a)^2}$$

When mirror is taken away,

$$I_1 = \frac{P}{4\pi a^2} = \frac{9I}{10}$$

150. c. Ray diagram is as shown in the figure below:

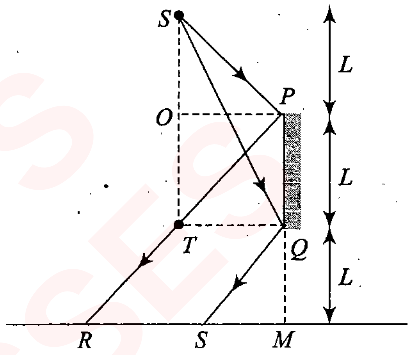


Fig. 1.537

Here, $\frac{SO}{OP} = \frac{PM}{RM} \Rightarrow RM = 2L$

Also, $\frac{ST}{TQ} = \frac{QM}{SM} \Rightarrow SM = \frac{L}{2}$

$$RS = RM - SM = \frac{3L}{2}$$

151. d. We have $\frac{\mu}{v} - \frac{\mu_1}{-u} = \frac{\mu - \mu_1}{-R}$

$$\frac{\mu}{v} = -\frac{\mu_1}{-u} + \frac{\mu_1 - \mu}{R}$$

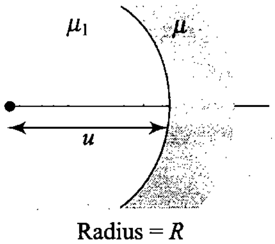


Fig. 1.538

$$\frac{\mu}{v} = \frac{-\mu_1 R + \mu_1 u - \mu u}{uR}$$

$$v = \frac{\mu u R}{(\mu_1 - \mu)u - \mu_1 R}$$

For real image, v should be positive

So, $(\mu_1 - \mu_2)\mu - \mu_1 R > 0$ for real image

i.e., $\mu_1 - \mu > \frac{\mu_1 R}{u}$

or $\mu_1\left(1 - \frac{R}{u}\right) > \mu$

$$\mu_1 > \mu\left[1 - \frac{R}{u}\right]^{-1}$$

which depends upon the location of object.

152. c. $\dfrac{2.5}{v} - \dfrac{1}{-u} = \dfrac{2.5-1}{R}$

$$\frac{2.5}{v} = \frac{1.5}{R} - \frac{1}{u}$$

$$v = \frac{2.5uR}{1.5u - R}$$

$v = \infty$ at $1.5\,u - R = 0$

$$u = \frac{R}{3}$$

For $u < \dfrac{2R}{3}$, $v = 0$, virtual image

$u > \dfrac{2R}{3}$, $v = 0$, real image

So, image is changing from real to virtual at $u = \dfrac{2R}{3}$

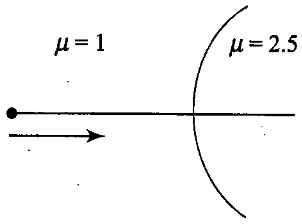


Fig. 1.539

153. b. $\dfrac{2.5}{v} - \dfrac{1}{-u} = \dfrac{1.5}{-R}$

$$v = -\frac{2.5\,uR}{1.5u + R}$$

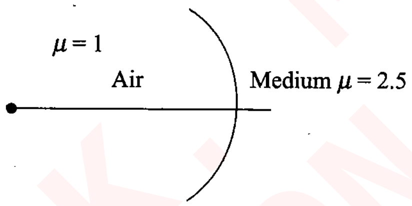


Fig. 1.540

$v < 0$ for all values of u, so the image remains virtual.

154. b. Initially, the object for lens B (the image formed by lens A) is at its focus, so rays are parallel.
As the separation between A and B decreases, the object for B is lying away from focus and hence, the rays are diverged.

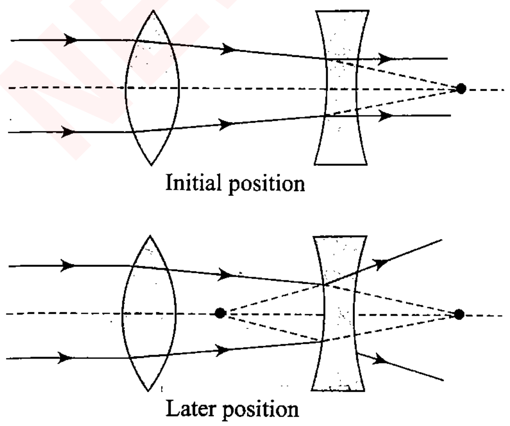


Fig. 1.541

155. d. Ray diagram for image formation is as shown in the figure:

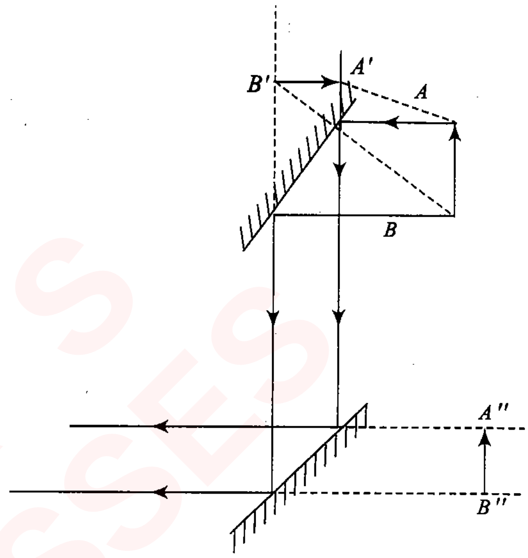


Fig. 1.542

156. d. For (a) and (b), object and image are on the same side, so image is real as the object is real.
For (c), image is virtual but $|m| > 1$. So, concave mirror.
For (d), $|m| < 1$ and image is also virtual, so in (d) image is formed in a convex mirror.

157. b. The fish can observe the sky only if refraction takes place. If TIR takes place, then image of sky cannot be observed.
i.e., $i < i_c$ and $i_c = 45°$

So, angle subtended $= \dfrac{\pi}{2}$

158. d. For light to retrace the path, the light ray falling on the mirror should be along the normal.
Deviation produced by prism,
$\delta_1 = (1.5 - 1)4 = 2°\text{CW}$

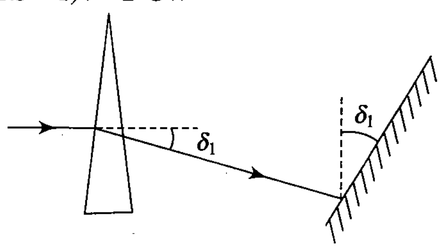


Fig. 1.543

So, mirror has to be rotated by 2° in CW direction.

159. a. As object moves from infinity to the pole of convex mirror, image moves from focus to pole. So, $v_i < v_0$, always.

160. d. Mirror can be shifted to new position $C'D'$. Distances are shown in the figure.

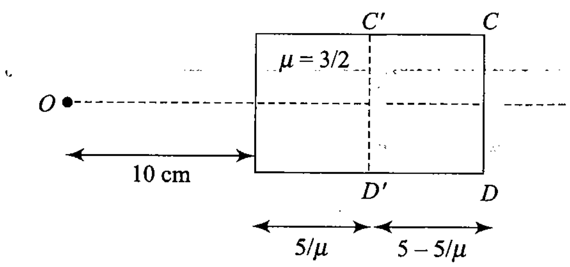


Fig. 1.544

Image will be at equal distance from the mirror $C'D'$ as the object is.

Image distance from $C'D'$ is

$$10+\frac{5}{3/2}=10+\frac{10}{3}=\frac{40}{3}\text{ cm}$$

Separation between object and image is $\frac{80}{3}$ cm .

161. a. Due to the insertion of glass plate, the image has to be shifted by 0.3 cm.

Shift produced by a glass slab of thickness t and refractive index μ is given by $\left(1-\frac{1}{\mu}\right)$.

So, $t\left[1-\frac{1}{1.6}\right]=0.3 \Rightarrow t=0.8$ cm

162. d. Deviation produced by prism is

$$\delta_1=(\mu-1)A=2°\text{CW}$$

Angle of incidence for mirror is δ_1, so deviation produced by the mirror is

$$\delta_2=\pi-2\delta_1=176°\text{CW}$$

Deviation produced by the prism for 2nd refraction is

$$\delta_3=2°\text{ ACW}$$

Net deviation = 174°CW

163. d. Deviation produced by the mirror is

$$\delta_1=\pi-2\times 60=60°\text{CW}$$

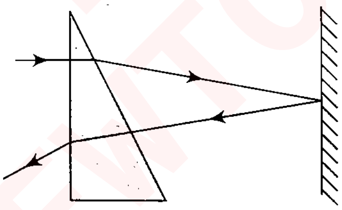


Fig. 1.545

Deviation produced by the prism is

$$\delta_2=(1-2)\times 6=6°\text{ACW}$$

So, net deviation, $\delta=\delta+\delta_2$

$$=(60°-6°)\text{ CW}=54°\text{ CW}$$

164. d. Both the lenses (convex or concave) form virtual image of a real extended object. In both the cases, image would be erect.

For convex lens, virtual image will form when object is in-between focus and optical center.

For concave lens, virtual image will form for any location of object, and corresponding image is lying between focus and optical center.

165. b. Here, three optical phenomena take place—first refraction, then reflection, and finally refraction.

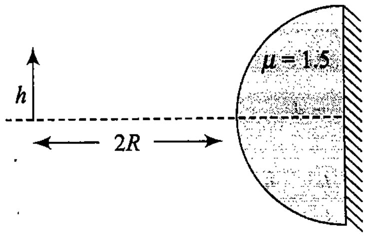


Fig. 1.546

For 1st refraction, $\frac{1.5}{v}-\frac{1}{-2R}=\frac{1.5-1}{R}$

$$\frac{1.5}{v}=0 \Rightarrow v\infty 0$$

i.e., rays after refraction get parallel to the axis and strike the mirror normally, get retraced back, and final image is formed at the same place where the object is and of same size. Image would be real.

166. d. From $\sin\left(\frac{A+\delta_m}{2}\right)=\mu\sin\frac{A}{2}$

$$\sin\left(\frac{A+\delta_m}{2}\right)=\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\times\sin\frac{A}{2}$$

$$\frac{A+\delta_m}{2}=\frac{\pi}{2}-\frac{A}{2}$$

$$\delta_m=\pi-2A=180°-2A$$

Multiple Correct Answers Type

1. c., d. As long as the object moves towards the mirror, image moves away from the mirror (for $u>f$) and $m=-v/u$ ($v>u$), so image size increases.

2. a, c. (a) $\rightarrow n_3<n_1$ so critical angle decreases. Therefore, TIR will take place at AB.

(b) $\rightarrow n_3>n_1$, so critical angle increases. Therefore, TIR will not take place.

(c) $\rightarrow$ If $n_3>n_2$, then TIR will take place at CD and if $n_3>n_1$, then TIR will take place at AB.

(d) $\rightarrow$ TIR will not take place at face CD for $n_3<n_1$ because ray is going from rarer to denser medium.

3. a., b. For $\mu_2>\sqrt{2}$ (TIR will not take place)

$$2\sin 45°=\mu_2\sin r$$

$$\mu_2\sin r=\sqrt{2}\sin e$$

$$e=90°$$

Hence, deviation is 45°

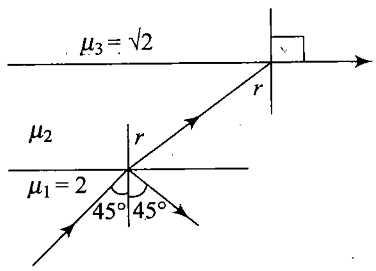


Fig. 1.547

$\mu_2 < \sqrt{2}$. TIR will take place

Deviation is 90°.

4. a, b, c, d. $\dfrac{h_1}{h_2} = 4$

$$\frac{h_1}{h_2} \times \frac{h_2}{h_0} = 1 \Rightarrow \frac{h_2}{h_0} = 0$$

When shorter image is formed, magnification = 2

$\Rightarrow \quad \left|\dfrac{v}{u}\right| = 2$ and $|v| + |u| = 96$

$\Rightarrow \quad |v| = 64$ and $|u| = 32$

Hence, object distance = 32 cm and second lens position can be 96 − 32 = 64 cm

Distance between two positions = 32 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{64} - \frac{1}{(-32)} = \frac{1}{f} \Rightarrow f = \frac{64}{3} \text{ cm}$$

5. b, c. $\dfrac{1}{100-x} = \dfrac{1}{-x} = \dfrac{1}{21}$

$\Rightarrow$ x, The distance of object from the lens is 30 cm, 70 cm

$$m_1 = \frac{70}{-30}, m_2 = \frac{30}{-70} \therefore |m_1 - m_2| = \frac{40}{21}$$

6. a, b, c, d. $v + u = D$

and $v - u = x \Rightarrow v = \dfrac{D+x}{2}$, $u = \dfrac{D-x}{2}$ and $f = \dfrac{D^2 - x^2}{4D}$

$$m_1 = \frac{D+x}{D-x}, m_2 = \frac{D-x}{D+x}$$

7. a, c. $\dfrac{v}{u} = \dfrac{3}{1} \Rightarrow v = 3u$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \frac{1}{3u} + \frac{1}{u} = \frac{1}{f}$$

$f = 30$ cm.

8. a, c, d.

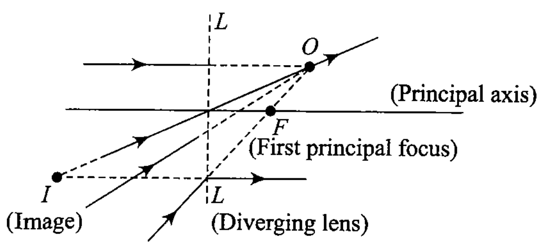


Fig. 1.548

9. a, b, c. n for liquid = n for Glass/yellow light but n for liquid < n for glass (red light), so it will deviated toward base, for blue light n (liquid) > n (glass) so it will deviate towards vertex.

10. a, c. The image of a point closer to the focus will be farther. As the transverse magnification of B will be more than A, the image of AB will be inclined to the optical axis.

11. a, b, c, d.

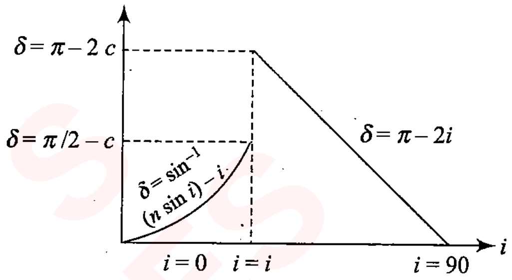
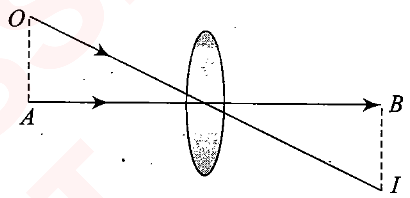


Fig. 1.549

12. a, c, d. a.



c.

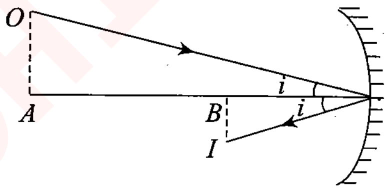
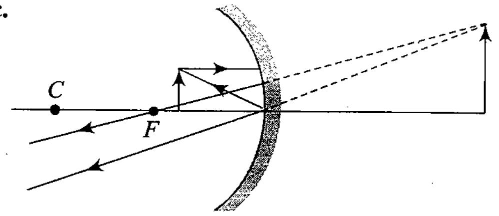


Fig. 1.550

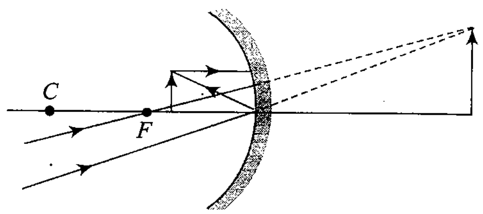
d Image is inverted, so it should be real.

13. b, c.



Real object, Virtual image (magnified)

Fig. 1.551



Virtual object, Diminished real image

Fig. 1.552

14. a, b, c, d. For a real extended object, image formed by convex mirror would be virtual, diminished, erect, and be lying between focus and pole.

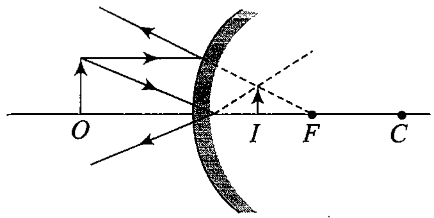


Fig. 1.553

For a virtual object, lying between focus and pole, image formed by a convex mirror would be real, magnified, erect, and lying in front of mirror.

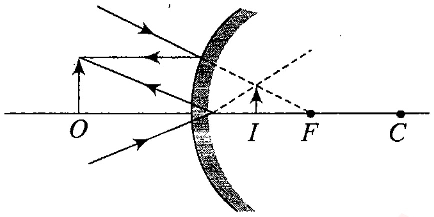


Fig. 1.554

For a virtual object beyond focus, image formed by a convex mirror would be virtual, inverted, magnified or diminished (depending on the location of object), and will lie on the same side of mirror.

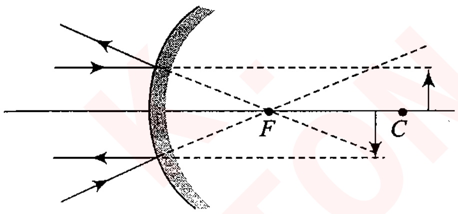


Fig. 1.555

15. a, b, c. For a real object, a real image can be formed by convex lens only (and not by concave lens) and depending on the location of object from infinity to focus, the image can be enlarged or diminished. But when object is in between focus and optical center, image would be virtual. So, $f \le 25$ cm.

16. a, c. Here, $\dfrac{x}{10} = \dfrac{y}{20}$

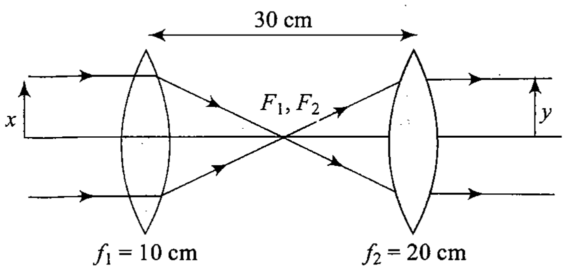


Fig. 1.556

$$y = 2x$$

So, if $\quad x = 1 \text{ cm}, y = 2 \text{ cm}$

17. b, c, d. If the mirror is concave and a real object is approaching it, the image will move away from the mirror for object distance greater than focal length. If object distance is less than the focal length, virtual image will be formed which moves toward the mirror.

If mirror is convex, as object is approaching the mirror, image will move from focus to pole, i.e., toward the mirror.

18. a, d. Here, $\dfrac{OP}{5\text{ cm}} = \dfrac{2.5\text{ mm}}{15\text{ cm}}$

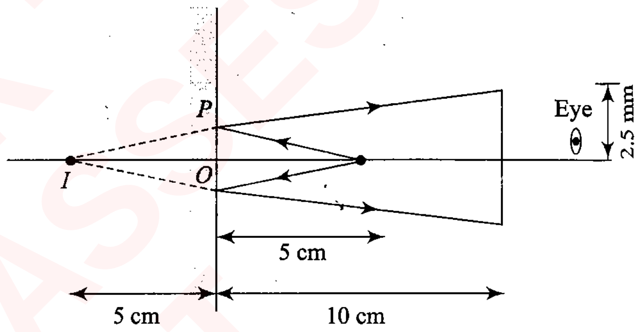


Fig. 1.557

$$OP = \frac{5}{6}\text{ mm}$$

Area of mirror used in reflection is

$$\pi \times OP^2 = \frac{25\pi^2}{36}\text{ mm}^2$$

Let the power of source be P.
Intensity in absence of mirror is

$$I_1 = \frac{P}{4\pi \times (5)^2}$$

Intensity in presence of mirror is

$$I_2 = \frac{P}{4\pi \times (5)^2} + \frac{P/2}{4\pi \times (5)^2}$$

$$\frac{I_2}{I_1} = \frac{19}{18}$$

19. b, d.

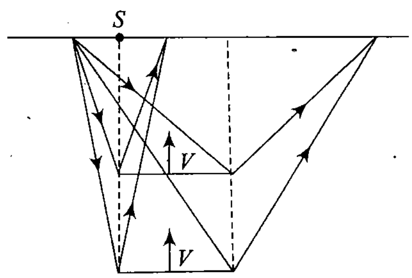


Fig. 1.558

From the above ray diagram, it is clear that the options (b) and (d) are correct.

20. a, c. In case of part A, radii of the two surfaces will remain same. Hence, focal length of part A will be same as that of complete lens, i.e., power of part A will remain unchanged, i.e., power of A is P.
In case of part B, radius of one surface will remain same while that of other plane surface will be ∞. Hence, focal length of part B will be double that of whole lens, i.e., power of part B is $P/2$.

21. a, b.

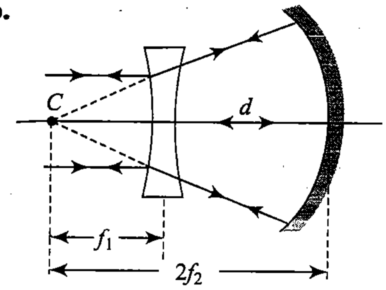


Fig. 1.559

22. a, b.

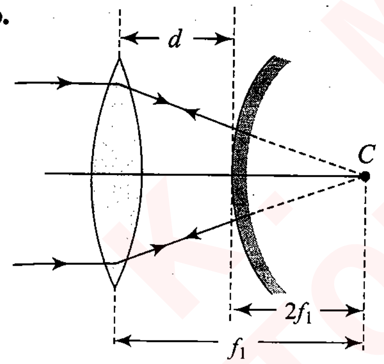


Fig. 1.560

23. a, b, c, d

a. Angle $BOB' = \angle AOB' - \angle AOB'$
$= 2i - (2i - 2\theta) = 2\theta$

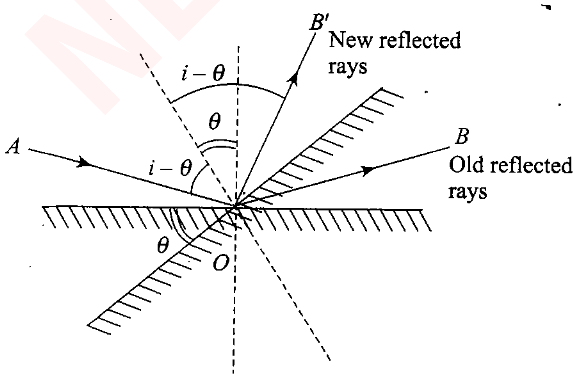


Fig. 1.561

b. Total deviation $\delta = \delta_1 + \delta_2$
$= (180° - 2\theta) + 180° - 2(\alpha - \omega) = 360° - 2\alpha$

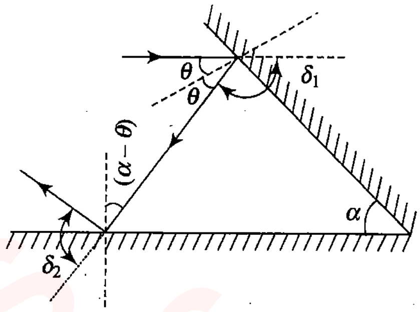


Fig. 1.562

which is independent of angle of incidence.

c. Power of a plane mirror is zero.

d. Velocity of the image toward the object = $v + v = 2v$.

24. b, c. If an object is at a distance x from a plane refracting surface and is viewed normally, then it appears at a distance x/μ from the surface, where μ is refractive index of that medium (in which it is situated) with respect to the medium in which observer is situated.
Suppose height of bird from water surface is x and depth of fish from the surface is y, then depth of fish, as observed by the bird, will be equal to

$$r_1 = x + \frac{y}{\mu} \quad \text{(i)}$$

where μ is refractive index of water with respect to air.

$$r_1 = 6.60 \text{ m}$$

Hence, option (b) is correct.
Height of bird, as observed by the fish, will be equal to

$$r_2 = \frac{x}{\mu} + y \quad \text{(ii)}$$

where μ' is refractive index of air with respect to water.

Now, $$\mu' = \frac{1}{\mu} = \frac{3}{4}$$

Hence, $$r_2 = 8.80 \text{ m}$$

Hence, option (a) is wrong while (c) is correct.

25. b, d. In the previous question, velocity of fish, observed by the bird, will be equal to dr_1/dt. Therefore, differentiating Eq. (i), we get

$$\frac{dr_1}{dt} = \frac{dx}{dt} + \frac{1}{\mu}\frac{dy}{dt} = 4.50 \text{ ms}^{-1}$$

Hence, option (d) is correct. Similarly, velocity of bird, as observed by the fish, will be equal to $\frac{dr_2}{dt}$.

$$\frac{dr_2}{dt} = \frac{1}{\mu'}\frac{dx}{dt} + \frac{dy}{dt} = 6.0 \text{ ms}^{-1}$$

Hence, option (b) is correct.

26. a, b, c. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$1 - \frac{v}{u} = \frac{v}{f} \quad \text{(i)}$$

Since m is positive in the given figure, therefore m represents magnitude of the magnification. In fact, if a real image is formed by a convex len, then the image and object will be on opposite sides of the lens. It means, if v is positive, then u will be negative. Therefore,

$$m = \left|\frac{v}{u}\right| = -\frac{v}{u}$$

Hence, Eq. (i) becomes

$$m = \frac{v}{f} - 1 \quad \text{(ii)}$$

It means, the graph between m and v will be a straight line having intercept -1 on m-axis and slope of the line $\tan\theta$ is equal to $(1/f)$. Hence, options (b) and (c) are correct. Putting $m = 0$ in Eq. (ii), $v = f$. Hence, (a) is also correct. Obviously, (d) is wrong.

Assertion-Reasoning Type

1. **a.** Both statements are true and Statement II is the correct explanation of Statement I.

   The focal length of a lens is given by, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

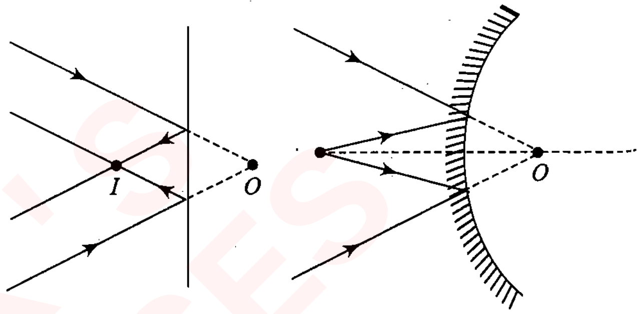
   For goggle, $R_1 = R_2$

   $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$$

   Therefore, power $P = \frac{1}{f} = 0$.

2. **c.** The index of refraction of light at the red end of the visible spectrum is lesser than at the violet end. Statement II is false.
3. **d.** From symmetry, the ray shall not suffer TIR at second interface, because the angle of incidence at first interface equals to angle of emergence at second interface. Hence, Statement I is false.
4. **d.** If the mirror is shifted parallel to itself such that the velocity of the mirror is parallel to its surface, the image shall not shift. Hence, Statement I is false.
5. **d.** Speed of light (for all color) is same in vacuum, equal to $3 \times 10^8\ \text{ms}^{-1}$.

   Speed of light is a property of medium.
6. **b.** Reflection of light rays takes place on rough as well as smooth surfaces. Some light energy would be absorbed by rough surface, so amplitude of reflected ray is less than that of incident ray.
7. **b.** The rays of light are diverging out from a virtual image. These can be easily converged onto the film of a concave lens by convergent action of its lens.
8. **d.** As can be seen from the expression of f, it depends upon the refractive index of the medium in which the lens is submerged.
9. **d.** We can produce a real image by a plane or convex mirror.



(Real image) (Real image)

Fig. 1.563

   Focal length of a convex mirror is taken positive.

10. **d.** $1 \times \sin i = \mu \sin r = \mu \sin(90 - i)$

    $$\sin i = \mu \cos i$$

    $$\Rightarrow \quad \tan i = \mu$$

    So, reflected and refracted rays are perpendicular if $\tan i = \mu$.

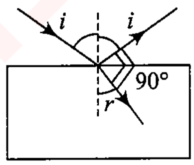


Fig. 1.564

11. **a.** In TIR, 100% of incident light is reflected back into the same medium, and there is no loss of intensity, while in reflection from mirrors and refraction from lenses, there is always some loss of intensity. Therefore, images formed by total internal reflection are much brighter than those formed by mirrors or lenses.
12. **d.** The focal length of a lens is given by formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

    As $\quad \mu_b > \mu_r \Rightarrow f_b < f_r$

    Therefore, focal length of a lens decreases when red light is replaced by blue light.
13. **b.** Since $\mu = \frac{\mu_g}{\mu_{cs}} = \frac{1.5}{1.65} \angle 1$

    From $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow f < 0$

    Therefore, the lens behaves as a diverging lens.

    Hence, (b) is correct option.
14. **d.** When a light wave travels from a rarer to a denser medium it loses speed, but energy carried by the wave does not depend on its speed.

Comprehension Type

For Problems 1–3

1. c, 2. d, 3. a.

Sol. (i) $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$

Taking refraction first at curved surface,

$$\frac{2}{v_1}+\frac{1}{x}=\frac{1}{R}\quad\Rightarrow\quad v_1=\frac{2Rx}{x-R}$$

For plane surface,

$$v'=v_1-R\quad\Rightarrow\quad v'=\frac{xR+R^2}{x-R}$$

$$\Rightarrow\qquad \frac{1}{v}-\frac{2(x-R)}{R(x+R)}=0$$

$$\frac{1}{v}=\frac{2(x-R)}{R(x+R)}$$

For virtual image,

$$\frac{1}{v}<0\quad\Rightarrow\quad\frac{2(x-R)}{R(x+R)}<0$$

$$x<R$$

(ii) For $x=2R$

$$V_1=\frac{4R^2}{R}=4R\quad\Rightarrow\quad u=-2R$$

$$m_1=\frac{\mu_1}{\mu_2}\cdot\frac{v}{u}=\frac{1}{2}\cdot\frac{4R}{(-2R)}=-1$$

$$m_2=1\quad\Rightarrow\quad m_1m_2=-1$$

Image is real, inverted, and of same size.

(iii)

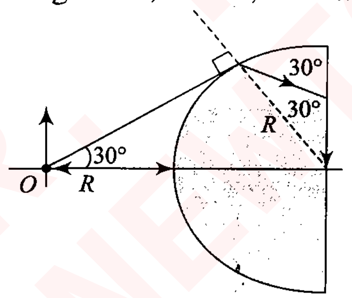


Fig. 1.565

Hence, correct answer is 90°

For Problems 4–6

4. d, 5. c, 6. d.

Sol. From the formula of equivalent focal length of thin lenses in contact, we get

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}-\frac{1}{3f}\quad\text{or}\quad F=\frac{3f}{2}$$

For Problems 7–8

7. b, 8. a.

Sol. The normal shift produced by a glass slab is,

$$\Delta x=\left(1-\frac{1}{\mu}\right)t=\left(1-\frac{2}{3}\right)(6)=2\text{ cm}$$

i.e., for the mirror, the object is placed at a distanced $(32-\Delta x)$ $=30$ cm from it.

Applying mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{30}=-\frac{1}{10}$$

or, $\quad v=-15$ cm

a. When $x=5$ cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance

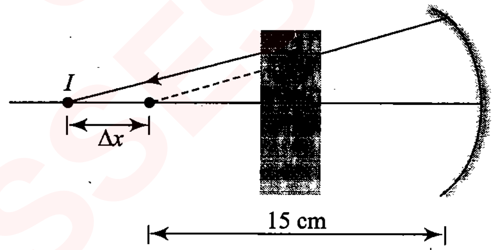


Fig. 1.566

$\Delta x=2$ cm. Hence, the final real image is formed at a distance $(15+2)=17$ cm from the mirror.

b. When $x=20$ cm: This time also the final image is at a distance of 17 cm from the mirror but it is virtual as shown.

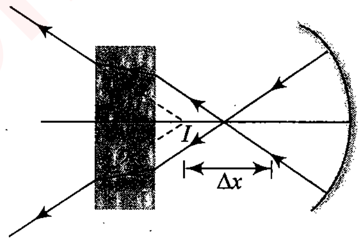


Fig. 1.567

For Problems 9–12

9. a, 10. a, 11. a, 12. b.

Sol. *Fish observing eye:*

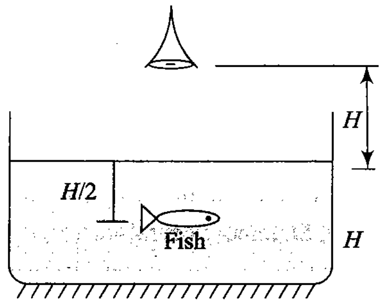


Fig. 1.568

(i) Direct observation

$$H_1=\frac{H}{2}+\mu H$$

$$H_1=H\left(\frac{1}{2}+\mu\right)$$

(ii) Fish observing image of eye by mirror.

Hence, distance of the eye image from fish,

$$H_2 = \mu H + H + \frac{H}{2}$$

$$H_2 = H\left(\frac{3}{2} + \mu\right)$$

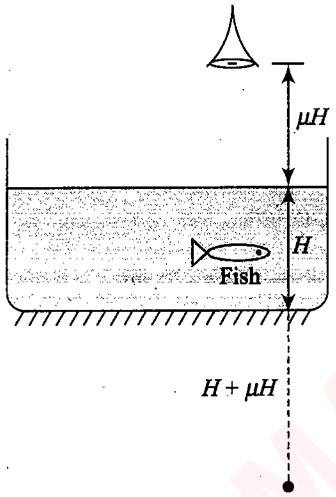


Fig. 1.569

Eye observing fish:

(i) Direct observation

$$H_1 = H + \frac{H}{2\mu} = H\left(1 + \frac{1}{2\mu}\right)$$

(ii) Eye observing image of the fish

$$H_2' = H + \frac{H}{\mu} + \frac{H}{2\mu} = H\left(1 + \frac{2}{2\mu}\right)$$

$$H_2' = H + \frac{3}{2\mu}$$

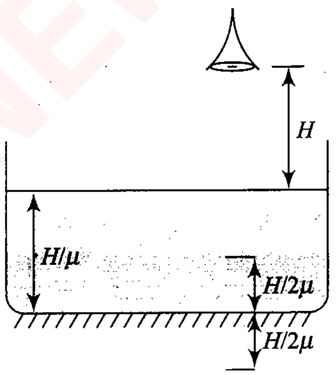


Fig. 1.570

For Problems 13–14

13. b, 14. b.

Sol. (i) Viewer on the left of hollow sphere: Single refraction takes place at surface S. From the single surface refraction equation, we have

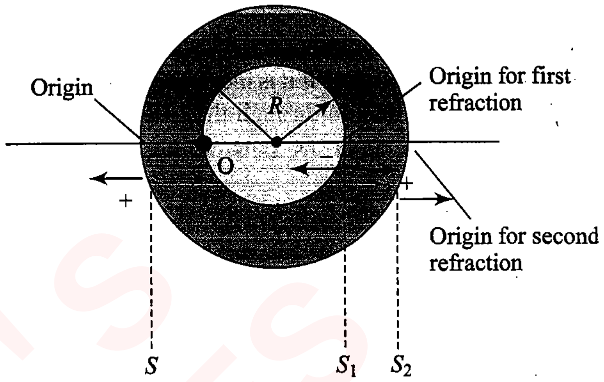


Fig. 1.571

$$\frac{1}{v} - \frac{n}{(-R)} = \frac{(1-n)}{(-2R)}$$

which on solving for v yields

$$v = -\left(\frac{2R}{n+1}\right)$$

Image is on the right of refracting surface S.

Shift = Real depth – Apparent depth

$$= R - \left(\frac{2R}{n+1}\right) = \frac{(n-1)}{(n+1)}R$$

(ii) When the viewer is on the right, two refractions take place at surfaces S_1 and S_2.

For refraction at surface S_1:

$$\frac{n}{v_1} - \frac{1}{(-2R)} = \frac{(n-1)}{(-R)}$$

which on solving for v_1 yields

$$v_1 = -\frac{2nR}{2n-1}$$

The first lies to the left of S_1 and acts as object for refraction at the second surface. We have to shift the origin of Cartesian coordinate system to the vertex of S_2. The object distance for the second surface is

$$u_2 = -\left[\frac{2nR}{2n-1} + R\right] = -\left(\frac{4n-1}{2n-1}\right)R$$

$$\frac{1}{v_2} = -\frac{n}{-\left[\frac{4n-1}{2n-1}\right]R} = \frac{1-n}{-2R}$$

On solving for v_2, we get

$$v_2 = -\frac{2(4n-1)}{(3n-1)}R$$

The minus sign shows that image is virtual and lies to the left of S_2.

Shift = Real depth – Apparent depth

$$= 3R - \frac{2(4n-1)R}{(3n-1)} \Rightarrow = \frac{(n-1)}{(3n-1)}R$$

For Problems 15–16

15. a. 16. a.

Sol. The light retraces its path if it is incident normally on a mirror. The ray after refraction through the lens and the liquid are parallel. We will apply the general thin lens equation with parameters.

$n_1 = 1, n_2 = 3/2, n_3 = 4/3, u = -15$ cm, and $v = \infty$

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R} + \left(\frac{n_3 - n_2}{R}\right)$$

$$\frac{n_3}{\infty} - \frac{1}{(-15)} = \frac{(3/2) - 1}{R} - \frac{[(4/3) - (3/2)]}{R}$$

On solving for R, we get $R = 10$ cm. Similarly, when second liquid is filled, we have

$$\frac{n_3'}{\infty} - \frac{1}{(-25)} = \frac{(3/2) - 1}{10} - \frac{n_3' - (3/2)}{10}$$

On solving for n_3', we get, $n_3' = 1.6$

For Problems 17–19

17. c, 18. b, 19. c.

Sol. *From lens equation,*

$$\frac{1}{v} - \frac{1}{(-20)} = \frac{1}{10} \Rightarrow v = +20 \text{ cm}$$

$$\text{Magnification, } m_1 = \frac{v}{u} = \left(\frac{+20}{-20}\right) = -1$$

Image is real, inverted, and same size as object.

The first image acts as object for concave mirror. Object distance for mirror is (40 – 20) cm.

From mirror equation,

$$\frac{1}{v'} + \frac{1}{(-20)} = \frac{1}{(-7.5)} \Rightarrow v = -12 \text{ cm}$$

$$\text{Magnification, } m_2 = -\left(\frac{u'}{v'}\right) = -\left(\frac{-12}{-20}\right) = -0.6$$

The second image is 12 cm to the left of the mirror, real, and erect (that is reinverted).

The second image acts as object for the lens. The object distance for second refraction at the lens, $u'' = +28$ cm

From lens equation,

$$\frac{1}{v''} - \frac{1}{(+28)} = \frac{1}{-10} \Rightarrow v'' = -15.6 \text{ cm}$$

Note the sign convention for f and u.

$$\text{Magnification, } m_3 = \frac{v''}{u''} = \left(\frac{-15.6}{+28}\right) = -0.556$$

Final image is real, inverted, and lies 15.6 cm to the left of the lens.

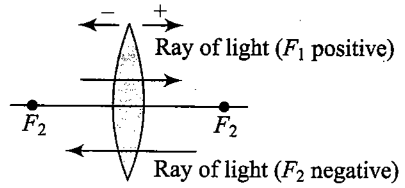


Fig. 1.572

Overall magnification,

$$m = m_1 \times m_2 \times m_3$$
$$= (-1) \times (0.6) \times (-0.556) = -0.333$$

For Problems 20–22

20. a, 21. d, 22. b.

Sol. *From lens equation,*

$$\frac{1}{v} - \frac{1}{(-10)} = -\frac{1}{+20}, \; v = -20 \text{ cm}$$

$$\text{Magnification, } m_1 = \left(\frac{-20}{-10}\right) = +2$$

Image is virtual, erect and magnified.

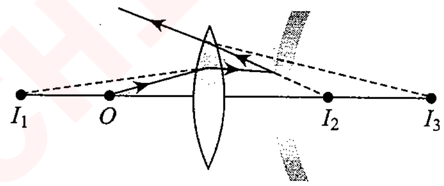


Fig. 1.573

The first image acts as an object for the convex mirror. Object distance for the mirror, $u' = (20 + 5) = 25$ cm.

From lens equation,

$$\frac{1}{v'} + \frac{1}{(-25)} = \frac{1}{+15} \Rightarrow u' = +\frac{75}{8} \text{ cm}$$

$$\text{Magnification, } m_2 = \left(\frac{+75/8}{-25}\right) = \frac{3}{8}$$

Image is virtual (to the right of the mirror), erect and diminished.

The object distance for second refraction at the lens

$$= \frac{75}{8} + 5 = \frac{115}{8}$$

$$\text{From lens equation, } \frac{1}{v''} - \frac{1}{(+115/8)} = \frac{1}{-20},$$

$$v'' = \frac{460}{9} = +51.1 \text{ cm}$$

$$\text{Magnification, } m_3 = \left(\frac{+460/9}{115/8}\right) = \frac{32}{9}$$

Overall magnification, $m = m_1 \times m_2 \times m_3$ is

$$(2) \times \left(\frac{3}{8}\right) \times \left(\frac{32}{9}\right) = \frac{8}{3}$$

Hence, size of image is

$$\left(\frac{8}{3}\times 2\right)\text{cm} = 5.33 \text{ cm}$$

Final image is to the right of the lens at a distance of 51.1 cm from the lens; real, erect and magnified.

For Problems 23–24

23. a, 24. b.

Sol. Splitting of a lens in two parts does not affect the position of the image. Each half forms an image at the same position but of reduced intensity. The previous problem shows that for a fixed object and screen there are two positions of the lens for which the image is formed at the same position.

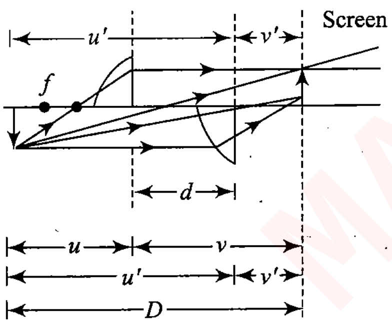


Fig. 1.574

Let the object and image distances for the two lenses be u, v and u', v', respectively. In accordance with principle of reversibility of light,

$$u' = v \quad \text{and} \quad v' = u$$

Hence, $u + d + v' = u + d + u = D$

$$u = \frac{D-d}{2}$$

Also $v = D - u = \frac{D+d}{2}$

As $m = \frac{v}{u} = \frac{D+d}{D-d} = 2$ (given)

$$d = \frac{D}{3} = \frac{1.8}{3} = 0.6 \text{ cm}$$

Hence, $u = \frac{1}{2}(D-d) = 0.6$ cm

And $v = \frac{1}{2}(D+d) = 1.2$ cm

From lens equation,

$$\frac{1}{1.2} - \frac{1}{-(0.6)} = \frac{1}{f}$$

$f = 0.4$ cm,

For Problems 25–27

25. b, 26. a, 27. c

Sol. As explained in the previous problem, each half lens will form an image in the same plane. The optic axes of the lenses are displaced,

$$\frac{1}{v} - \frac{1}{(-30)} = \frac{1}{20},\ v = 60 \text{ cm}$$

From similar triangles OI_1I_2 and OP_1P_2, we have

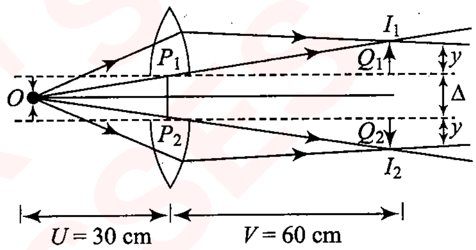


Fig. 1.575

$$\frac{I_1I_2}{P_1P_2} = \frac{u+v}{u}\quad I_1I_2 = \frac{90}{30}(2\times 0.05) = 0.3 \text{ cm}$$

Thus, the two images are 0.3 cm apart.

Alternatively, imagine two arrows(see figure) that act as objects for the lens.

Magnification, $m = \frac{v}{u} = \frac{(+60)}{(-30)} = -2$

Image of height of arrow is

$$y = 2\times(0.05) = 0.10 \text{ cm}$$

Thus, two inverted images are formed whose tips are at I_1 and I_2, respectively.

Thus, $I_1I_2 = 2y + \Delta = (2\times 0.1) + 0.1 = 0.3$ cm

For Problems 28–29

28. c, 29. a

Sol. 28. Method 1: The optical arrangement is equivalent to the concave mirror of focal length F given by

$$\frac{1}{F} = \frac{1}{f_g} + \frac{1}{f_m} + \frac{1}{f_g}$$

where f_x is the focal length of the lens without silvering and f_m is the focal length of the mirror.

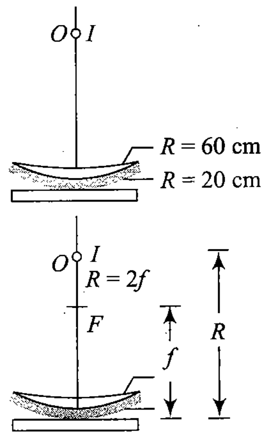


Fig. 1.576

$$\frac{1}{f_g} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$= (1.5-1)\left(\frac{1}{20}-\frac{1}{+60}\right) = \frac{1}{60}$$

$f_g = 60\,\text{cm}$

$f_g = R_1/2 = 20/2 = 10\,\text{cm}$

$$\frac{1}{F} = \frac{1}{60}+\frac{1}{10}+\frac{1}{60} = \frac{8}{60}$$

$$F = \frac{60}{8} = 7.5\,\text{cm}$$

For the image to be formed at the place of the object,

$$X = R = 2F = 7.5\times 2 = 15\,\text{cm}$$

Method 2: We use the relation

$$\frac{n_2}{x_2}-\frac{n_1}{x_1} = \frac{n_2-n_1}{R}$$

For the object and the image to coincide, the rays fall normal on the reflecting surface, i.e., on the silvered face of the lens.

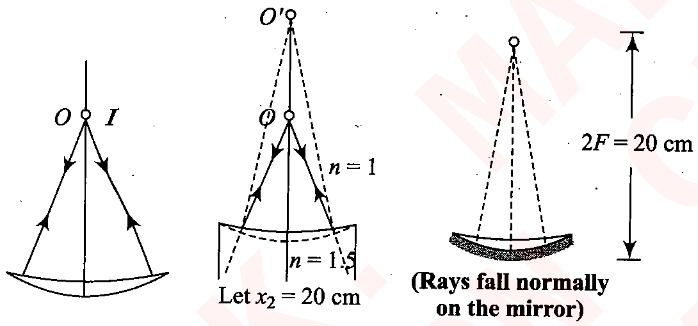


Fig. 1.577

Then, the rays retrace backward and meet at the object point again (optical reversibility).

For the refraction at the upper surface of the lens,

$$n_1 = 1.0, n_2 = 1.5, x_1 = 20, R = +60$$

(x_2 = +20 ensures that the rays fall on the silvered face normally.)

$$\frac{1.5}{20}-\frac{1.0}{x_1} = \frac{1.5-1.0}{+60}$$

$$\frac{1.0}{x_1} = \frac{1.5}{20}-\frac{0.5}{60} = \frac{3.0}{60}$$

$x_1 = 15\,\text{cm}$

Method 3: We use lensmaker's formula and the equation

$$\frac{1}{f} = \frac{1}{x_2}-\frac{1}{x_1}$$

The given optical arrangement can be visualised as a convex lens of focal length 60 cm and a concave mirror of focal length 10 cm kept in contact as shown in the figure.

If the rays fall normally on the mirror after the refraction through the lens, they will retrace backward and meet at the point of the pin again.

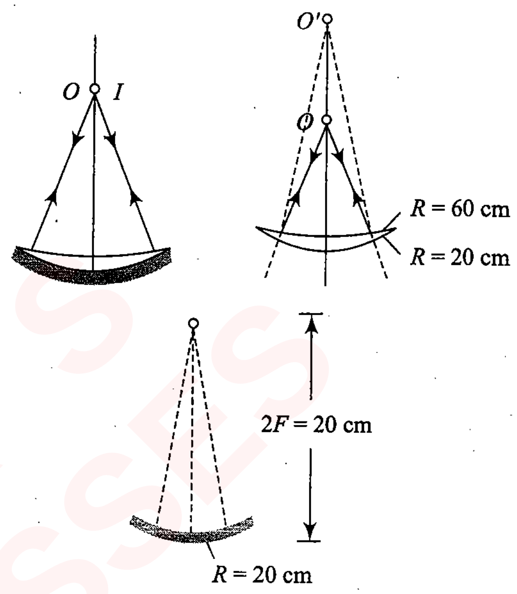


Fig. 1.578

For the lens, $x_1 = ?$

$x_2 = +20$ (for normal incidence on the mirror)

$f = -60$ (using cartesian–coordinate sign convention)

$$\frac{1}{-60} = \frac{1}{+20}-\frac{1}{x_1}$$

29. Method 1: When the concave part is filled with water of refractive index 4/3, the optical arrangement is equivalent to concave mirror of focal length F such that

$$\frac{1}{F} = \frac{1}{f_w}+\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}+\frac{1}{f_w}$$

$$\frac{1}{f_w} = \left(\frac{4}{3}-1\right)\left(\frac{1}{60}-\frac{1}{\infty}\right) = \frac{1}{180}$$

$f_w = 180\,\text{cm}$

$f_g = 60\,\text{cm}$ (calculated earlier)

$$\frac{1}{F} = \frac{1}{180}+\frac{1}{60}+\frac{1}{10}+\frac{1}{60}+\frac{1}{180} = \frac{26}{180}$$

$$F = \frac{180}{26}\,\text{cm}$$

$$X_1 = R = 2F = \frac{180}{26}\times 2 = \frac{180}{13} = 13.85\,\text{cm}$$

$\Delta x = 15.0 - 13.85 = 1.15\,\text{cm}$

Method 2: We use the equation

$$\frac{n_2}{x_2}-\frac{n_1}{x_1} = \frac{n_2-n_1}{R}$$

For refraction at the interface '1' (air water),

$$\frac{4/3}{x_2}-\frac{1}{x_1} = \frac{4/3-1}{\infty} \quad \text{(i)}$$

The image of interface '1' is the object for the interface '2'.

$$\frac{1.5}{+20}-\frac{1}{x_1} = \frac{1.5-4/3}{+60}$$

$$X_1 = \frac{360}{26} = 13.85$$

$\Delta X = 15.0 - 13.85 = 1.15\,\text{cm}$

Method 3: Using lensmaker's formula and the relation

$$\frac{1}{F}=\frac{1}{x_2}-\frac{1}{x_1}$$

$f_m = 180$ cm (using lensmaker's formula)
$f_g = 60$ cm (using lensmaker's formula)

$$\frac{1}{-180}=\frac{1}{x_2}-\frac{1}{x_1} \quad \text{(for the water lens) (i)}$$

$$\frac{1}{-60}=\frac{1}{+20}-\frac{1}{x_1} \quad \text{(for the glass lens) (ii)}$$

(The image by the water lens is the object for the glass lens and if the image by the glass lens is at +20, then the rays will fall normally on the mirror.)

Adding Eqs. (i) and (ii),

$$\frac{1}{-180}+\frac{1}{-60}=\frac{1}{20}-\frac{1}{x_1}$$

$$x_1=\frac{180}{13}=13.85\text{ cm}$$

$$\Delta x = 15.0-13.85=1.15\text{ cm}$$

For Problems 30–31

30. c, 31. a

Sol. The parallel rays will be focussed at the focal point of the first lens. The first image lies at I, at a distance f_1 from the origin. This image I_1 will act as an object for refraction through the second lens. The object distance for the second lens, $u = (f_1 - d)$.

From lens equation, $\frac{1}{v}-\frac{1}{+(f_1-d)}=\frac{1}{f_2}$

$$v=\frac{f_2(f_1-d)}{(f_1+f_2-d)}$$

Hence, the x-coordinate of final image I_2 is

$$x=d+u=\frac{d+f_2(f_1-d)}{(f_1+f_2-d)}=\frac{d(f_1-d)+f_1f_2}{(f_1+f_2-d)}$$

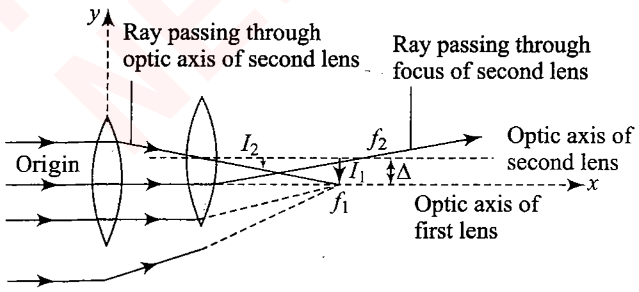


Fig. 1.579

Imagine an arrow tip is at f_1: its image from lens f_1 is the final image.

Magnification, $m=\frac{v}{u}=\frac{I_2}{O}=\frac{I_2}{\Delta}$

$$I_2=\left(\frac{v}{u}\right)\Delta=\frac{f_2}{(f_1+f_2-d)}\Delta$$

Thus, y–coordinate of tip of I_2 is

$$\Delta - I_2 = \Delta\left[1-\frac{f_2}{(f_1+f_2-d)}\right]=\frac{(f_1-d)\Delta}{(f_1+f_2-d)}$$

For Problems 32–33

32. b, 33. c.

Sol. In case (a), the incident parallel beam emerges as a parallel beam. So area illuminated,

$$A_1=\pi(1)^2=\pi\text{ cm}^2$$

In case (b), let x be the diameter of the area illuminated.

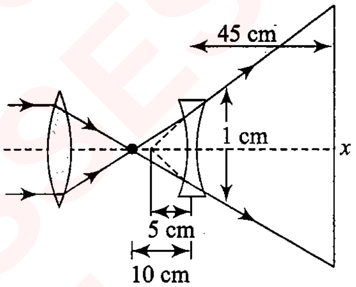


Fig. 1.580

Then,

$$\frac{x}{45}=\frac{1}{5} \Rightarrow x=9\text{ cm}$$

$$A_2=\pi\left(\frac{9}{2}\right)^2=\frac{81}{4}\pi\text{ cm}^2$$

$$\frac{A_2}{A_1}=\frac{81}{4}$$

When liquid of refractive index μ is filled to the right of this lens, the first surface of the lens (radius of curvature = 10 cm) forms the image at the object only. Considering the refraction at the second surface.

$$\frac{\mu}{\infty}-\frac{1.5}{-10}=\frac{\mu-1.5}{10} \quad \text{(therefore, same area} \Rightarrow v\to\infty)$$

$\Rightarrow \mu=3$

For Problems 34–35

34. b, 35. a

Sol. a. Focal lengths of lenses L_1 and L_2 are, respectively, given by

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}-\frac{1}{\infty}\right) \Rightarrow f_1=50\text{ cm}$$

$$\frac{1}{f_2}=(\mu-1)\left[\frac{1}{\infty}-\left(-\frac{1}{R}\right)\right] \Rightarrow f_2=40\text{ cm}$$

The equivalent focal length f of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow f=\frac{200}{9}\text{ cm}$$

Hence, the image of the parallel beam is formed on the common principal axis at a distance of 22.22 cm from the combination on the right side.

b. Image formed by L_1 is at a distance of 50 cm behind the lens. This image lies on the principal axis of L_1 and will act as an object for L_2.

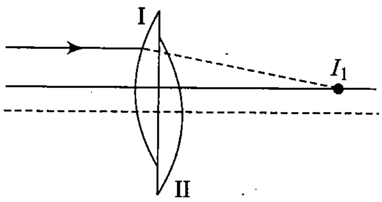


Fig. 1.581

For L_2, object distance, $u = +50$ cm

$$f_2 = +40 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \mu = \frac{200}{9} \text{ cm}$$

Magnification caused by L_2, $m = \frac{v}{u} = \frac{4}{9}$

For L_2, object I_1 is at a distance of 4.5 mm above its principal axis.

Hence, distance of image I_2 of the object (virtual) I_1 is at a distance (4/9)× 4.5 = 2 mm above the principal axis of L_2 [∵ height of image = m × height of object].

Hence, final image is at a distance of 22.22 cm behind the combination at a distance of 2.5 mm below the principal axis of L_1.

For Problems 36–37

36. d, 37. a.

Sol. a. For this situation, object will be virtual as shown in the figure.

Here, $u = +10$ cm and $f = +20$ cm.

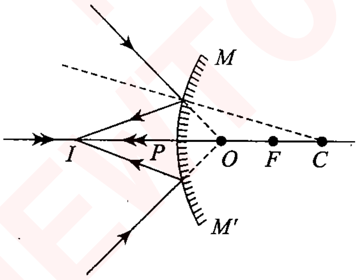


Fig. 1.582

So, $\frac{1}{v} + \frac{1}{+10} = \frac{1}{+20} \Rightarrow$ i.e., $v = -20$ cm

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with $m = -(20/10) = +2$.

b. For this situation also, object will be virtual as shown in the figure.

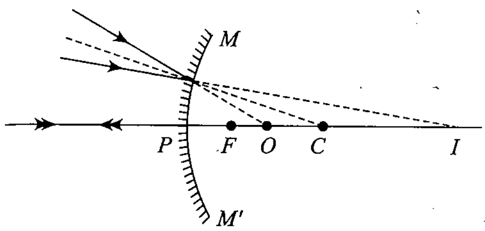


Fig. 1.583

Here, $u = +30$ cm

And $f = +20$ cm

$$\frac{1}{v} + \frac{1}{+30} = \frac{1}{+20} \quad \text{i.e.,} \quad v = +60 \text{ cm}$$

i.e., the image will be at a distance of 60 cm behind the mirror and will be virtual, inverted, and enlarged with $m = -(+60/30) = -2$.

For Problems 38–39

38. a. 39. c.

Sol. a. No deviation occurs at face AC; hence the angle of incidence at surface AC is $90° - f$. For total internal reflection at the second surface,

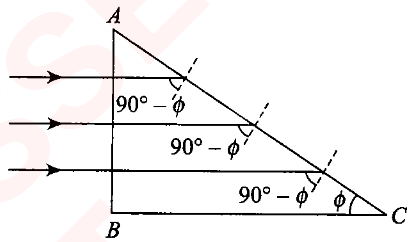


Fig. 1.584

$$n_g \sin(90° - \phi) \geq n_a$$

$$n_g \cos \phi \geq n_a$$

As f increases from zero, $\cos \phi$ decreases. Thus, f has the largest value when

$$n_g \cos \phi = n_a$$

$$\cos \phi = \left(\frac{n_a}{n_g}\right)$$

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{2}{3}\right)$$

b. If the prism is immersed in water,

$$\phi = \cos^{-1} = \left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{8}{9}\right).$$

For Problems 40–42

40. c., 41. a., 42. a.

Sol. 40. c. The objective lens must form a real image for eyepiece to magnify it.

41. a. The image formed by the objective must lie within the focus of the eyepiece.

42. a. To obtain best magnification, the object must be placed just beyond the focus of the objective lens. In this case, the first image distance from the objective is very large.

For Problems 43–45

43. a., 44. d., 45. b.

Sol. 43. a. From Snell's law,

$$\mu \sin \alpha = \sin \phi$$

$$\sin \alpha = \frac{1}{\mu} \sin \varphi < \frac{1}{\mu}$$

α is less than critical angle.

44. d. $\alpha = (\phi - \alpha) + \theta$

$$\delta = \pi - 2\,\theta = \pi - 4\,\alpha + 2\,\phi.$$

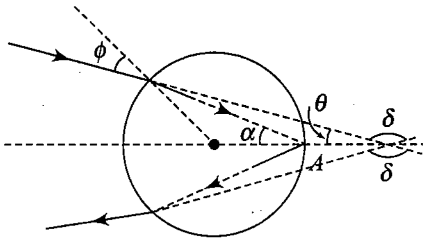


Fig. 1.585

45. b. $\dfrac{d\delta}{d\varphi}=-\dfrac{4d\alpha}{d\varphi}+2$

$$\frac{d\delta}{d\varphi}=0 \quad\Rightarrow\quad \frac{d\alpha}{d\varphi}=\frac{1}{2}$$

$$a=\sin^{-1}\left(\frac{1}{\mu}\sin\varphi\right)$$

$$\cos^2\varphi=\frac{\mu^2-1}{3}$$

For Problems 46–50

46. d., 47. b., 48. b., 49. c., 50. d.

Sol. 46. d. $\dfrac{1}{f}=\dfrac{1}{v}-\dfrac{1}{u}$

Here $v=2.5$ (distance of retina as position of image is fixed)

$u=-x$

$$\frac{1}{f}=\frac{1}{2.5}+\frac{1}{x}$$

For $f_{\min}$: x is minimum $\dfrac{1}{f_{\min}}=\dfrac{1}{2.5}+\dfrac{1}{25}$

47. b. For $f_{\max}$: x is maximum $\dfrac{1}{f_{\max}}=\dfrac{1}{2.5}+\dfrac{1}{\infty}$

48. b. For the near–sighted man, lens should make the image of the object within 100 cm range.

For lens, $u=-\infty$, $v=-100$

$$\frac{1}{f_{\text{lens}}}=\frac{1}{-100}-\frac{1}{-\infty}$$

49. c. For the far-sighted man, lens should make image of the nearby object at distances beyond 100 cm. For grown up person least distance is 25 cm. For lens, $u=-25$, $v=-100$

$$\frac{1}{f}=\frac{1}{-100}-\frac{1}{(-25)} \quad\Rightarrow\quad \frac{1}{f}=\frac{3}{100}$$

$P=+3$ so no. of spectacle is $=+3$

50. d. Since he can see beyond the range of a normal eye [25 cm, ∞] therefore it is an exceptional eye having no defect.

For Problems 51–55

51. d., 52. c., 53. d., 54. b., 55. b.

Sol. 51. d. From passage, (d) is correct

52. c. From passage, (c) is correct

53. d. From points (2) and (3) of passage;

f and f' must be of opposite sign.

Also, $w_C < w_D$ and $f_c < f_D$

Which is satisfied only by (d).

54. b. $\dfrac{\omega_1}{f_1}+\dfrac{\omega_2}{f_2}=0 \quad\Rightarrow\quad \dfrac{\omega_1}{\omega_2}=-\dfrac{f_1}{f_2}=\dfrac{1}{2}$ (i)

$\Rightarrow \quad \dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_2}=\dfrac{1}{40}$ (ii)

After solving (i) and (ii),

$f_1=-40$ cm

55. b. From passage; chromatic aberration in a spherical concave mirror is proportional to f^2. Hence (b).

For Problems 56–59

56. c., 57. b., 58. d., 59. c.

Sol. 56. c. As object is between infinity and $2F$, image will be between F and $2F$ and the point C is lying in this region.

57. b. Real, inverted, diminished

58. d. Wavelength of blue light is smaller than wavelength of red light,

$$\lambda_B<\lambda_R$$

So, $$f_B<f_R$$

[But f_{-B} and f_R differ by very small value]

So, image for red light would be on the right side of and very close to F.

59. c. Now, the lens behaves as a diverging lens, so will form virtual, erect, and diminished image on the same side as the object is.

Matching Column Type

1. a. → p., b. → s., c. → r., d. → q.

a. Concave mirror:

$$v=\frac{uf}{u-f},\ f \text{ is negative let } f=-F_0\ LL$$

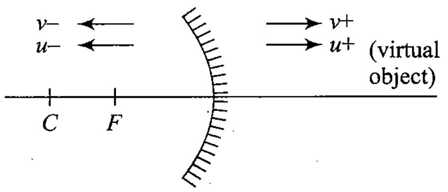


Fig. 1.586

u	$-\infty$	$-2F_0$	$-F_0$	0	F_0	∞
v	$-F_0$	$-2F_0$	$\pm\infty$	0	$\dfrac{-F_0}{2}$	F_0

$$v=\frac{-uF_0}{u+F_0}$$

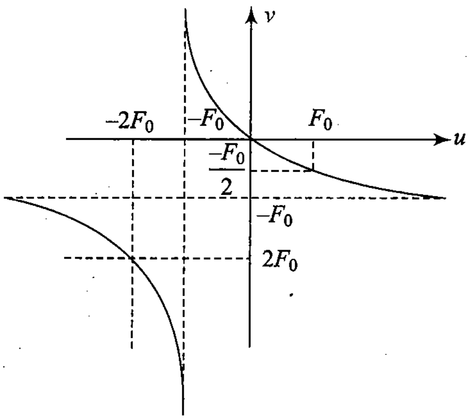


Fig. 1.587

b. Convex lens:

$$v = \frac{uf}{u+f},\ f \text{ is positive, let } f = F_0$$

$$v = \frac{uF_0}{u+F_0}$$

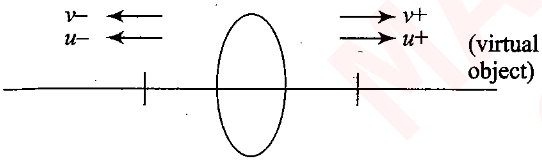


Fig. 1.588

u	$-\infty$	$-2F_0$	$-F_0$	0	F_0	∞
v	F_0	$2F_0$	$\pm\infty$	0	$\frac{-F_0}{2}$	F_0

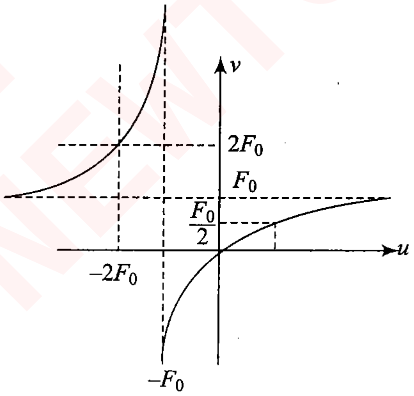


Fig. 1.589

c. Convex mirror:

$$v = \frac{uf}{u-f}$$

f is positive, let $f = F_0$

$$v = \frac{uf_0}{u-f_0}$$

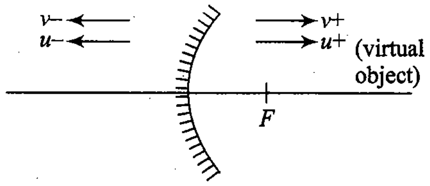


Fig. 1.590

u	$-\infty$	$-F_0$	0	F_0	$2F_0$	∞
v	F_0	$\frac{F_0}{2}$	0	$\pm\infty$	$2F$	F_0

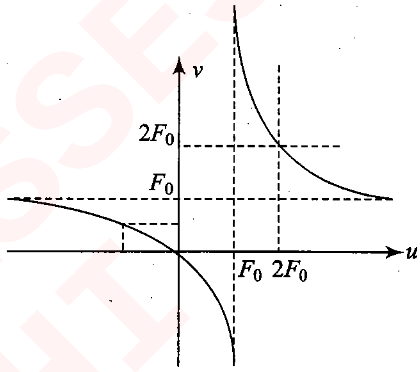


Fig. 1.591

d. Concave lens:

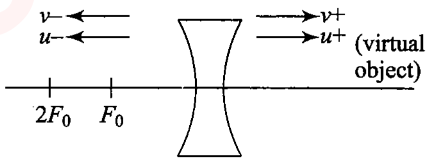


Fig. 1.592

$$v = \frac{uf}{u+f}$$

f is negative, let $f = -F_0$

$$v = \frac{-uf_0}{u-f}$$

u	$-\infty$	$-F_0$	0	F_0	$2F_0$
v	$-F_0$	$\frac{-F_0}{2}$	0	$\pm\infty$	$-2F_0$

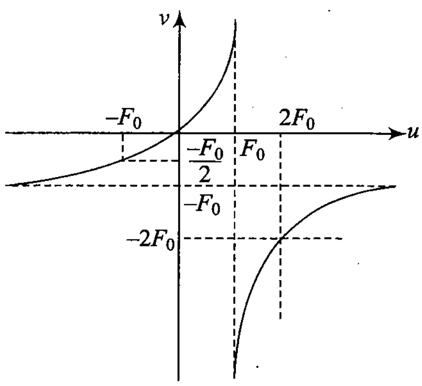


Fig. 1.593

2. a. $\to$ **q., r.; b.** $\to$ **q., r.; c.** $\to$ **q., s.; d.** $\to$ **q., s.**

a. $\dfrac{1}{v}=\dfrac{1}{u}+\dfrac{1}{f}$

$m=\dfrac{f}{u+f}$

Here, f is negative, u is positive and less than f.
So v will also be positive $m>1$, so image is erect and its size is greater than object. Here object is virtual.

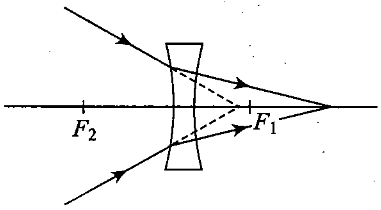


Fig. 1.594

b. $\dfrac{1}{v}=\dfrac{1}{u}+\dfrac{1}{f}$

$m=\dfrac{f}{u+f}$

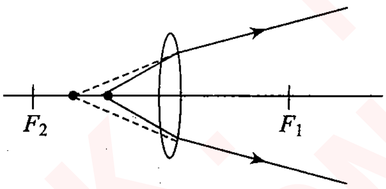


Fig. 1.595

f is positive, u is negative and less than f. So v is negative. $m>1$, so image is erect and its size is greater than object. Here, object is real.

c. u is negative, f is negative u is less than f.

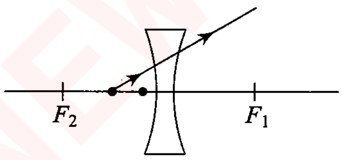


Fig. 1.596

$o<m<1$, object is real

d. u is positive, f is +positive

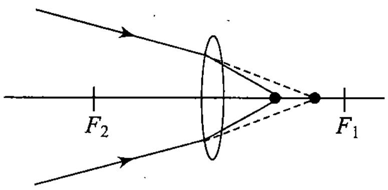


Fig. 1.597

3. a. $\to$ **s., b.** $\to$ **p., c.** $\to$ **s., d.** $\to$ **q.**

$$\vec{v}_A=\hat{i}+at=\hat{i}+(2\hat{i}+\hat{j})(2)=5\hat{i}+2\hat{j}$$

$$\vec{v}_{A'}=-5\hat{i}+2\hat{j}$$

$$\vec{v}_{A',A}=\vec{v}_{A'}-\vec{v}_A=-10\hat{i}$$

$$\vec{v}_B=(-\hat{i}+3\hat{j}),\ \vec{v}+3\hat{j}\ \text{ so }\ \vec{v}_{B',B}=2\hat{i}$$

For particle C:

$$\frac{dv_y}{dt}=21\Rightarrow v_y-6=t^2$$

$$\Rightarrow\quad v_y=6+4=10$$

$$\vec{v}_C=3\hat{i}-\hat{j},\ \vec{v}_{D'}=-3\hat{i}-\hat{j},\ \vec{v}_{D',D}=-6\hat{i}$$

4. a. $\to$ **p.; b.** $\to$ **q.; c.** $\to$ **p., q., r., s.; d.** $\to$ **p., q., r., s.**

A plane surface or a mirror always gives a real object for a point image and vice-versa. For a concave mirror, a virtual image for a virtual object is not possible; for a convex mirror, a real image of a real object is not possible.

5. a. $\to$ **p., r.; b.** $\to$ **q., r.; c.** $\to$ **q., r.; d.** $\to$ **q., r.**

Initially, the image is formed at infinity.

a. As m is increased, the focal length decreases. Hence, the object is at a distance larger than the focal length. Therefore, final image is real. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

b. If the radius of curvature is doubled, the focal length decreases. Hence, the object is at a distance lesser than the focal length. Therefore, final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

c. Due to insertion of slab, the effective object for lens shifts rightward. Hence, final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

d. The object comes to center of curvature of right spherical surface. Hence, the final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

6. a. $\to$ **q.; b.** $\to$ **q.; c.** $\to$ **r.; d.** $\to$ **s.**

a. A convex lens in a denser medium will behave like a concave lens or diverging lens.

b. A concave lens in a rarer medium will behave like a concave lens or diverging lens.

c.

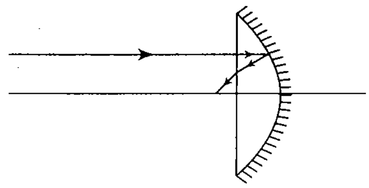


Fig. 1.598

Behaves like concave mirror.

d.

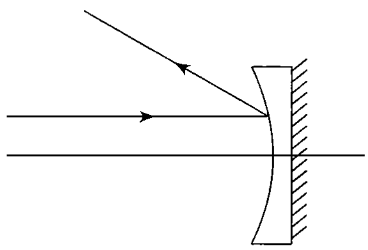


Fig. 1.599

Behaves like convex mirror.

7. a. → r.; b. → s.; c. → q.; d. → q.
Image by a convex mirror is always virtual, erect, and diminished.
In case of concave mirror, see using position of object.

8. a. → p.; b. → p.; c. → q.; d. → q.
a. Converging lens or convex lens,

$$v = \frac{uf}{u+f}$$

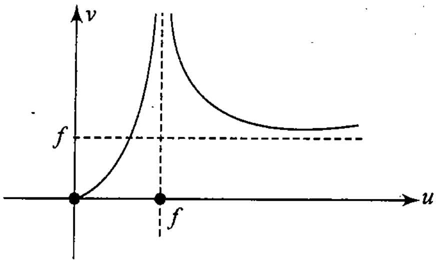


Fig. 1.600

b. Converging mirror or concave mirror,

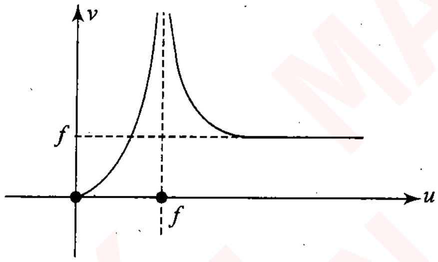


Fig. 1.601

c. Diverging lens or concave lens,

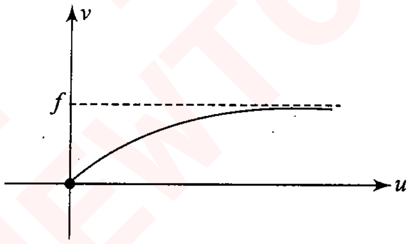


Fig. 1.602

d. Diverging mirror or convex mirror,

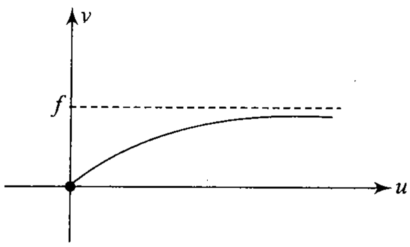


Fig. 1.603

9. a. → p., b. → r., c. → q., d. → s.
$\mu_{blue} > \mu_{green} > \mu_{yellow} > \mu_{red}$

10. a. → p., b. → q., c. → p., d. → r.
a. The focal length of mirror is independent of refractive index of the surrounding medium and hence from mirror formula, only one image can be confirmed.
b. Lens can be considered as two thin plano-convex lenses in contact. Since two media on other side of lens are present, two distinct focal lengths are possible and hence two images.
c. Same reasoning as for (b), but since only one medium is present on the other side, only one image is formed.
d. Lens can be considered as two half lenses having different focal lengths and medium on the other side of upper lens is of two types while for lower lens is of only one type. So, a total of 3 images are possible.

Integer Answer Type

1. (2) For magnification +2, $u = -x$, $v = -2x$ and $f = 2.0$ m

From $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have $\frac{1}{-2x} + \frac{1}{x} = \frac{1}{z}$

$x = 1.0$ m

For magnification of −2, $u = -y$, $v = +2y$, $f = 2.0$ m

we have $\frac{1}{2y} + \frac{1}{y} = \frac{1}{2} \Rightarrow y = 3.0$ m

So distance to be moved $= y - x = 3 - 1 = 2$ m

2. (9) When object is placed at the focus of the lens, i..e at 22cm from the lens, image will be formed at infinity. Shift in the position of object:

$$25 - 22 = \left(1 - \frac{1}{\mu}\right)t \Rightarrow 3 = \left[1 - \frac{1}{1.5}\right]t$$

$$t = \frac{(3)(1.5)}{0.5} = 9 \text{ cm}$$

3. (3) Critical angle between glass and liquid.

$$\sin\theta_C = \frac{\mu}{(3/2)} = \frac{2\mu}{3}$$

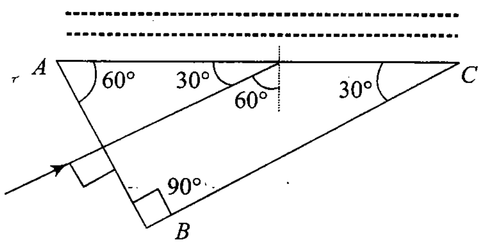


Fig. 1.604

Angle of incidence on $AC = 60°$

For TIR, $i > \theta_C \Rightarrow \frac{2}{3}\mu$

$$\mu < \frac{3\sqrt{3}}{4} = \frac{I\sqrt{3}}{4} \quad \text{(given)} \quad \text{So, } I = 3$$

4. (6) Let both the images are formed at I. For S_1, the image is virtual and for S_2, the image is real.

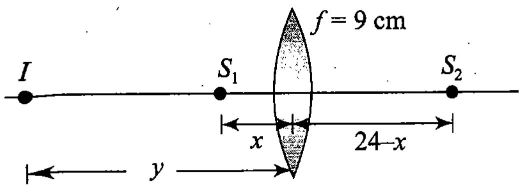


Fig. 1.605

For S_1: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-y}-\frac{1}{-x}=\frac{1}{9}$ (i)

For S_2: $\frac{1}{y}-\frac{1}{-(24-x)}=\frac{1}{9}$ (ii)

From (i) and (ii) $\frac{1}{x}+\frac{1}{24-x}=\frac{2}{9}$

$\Rightarrow \quad x^2-24x+108=0$

$\Rightarrow \quad x=6$ cm and 18 cm

But $x<f$. So answer is 6 cm

5. (3) Given $i=60°$, $\delta=30°$ and $A=30°$.

We have $\quad \delta=i+e-A$

From Eq. (i), we get

$30°=60°+e-30°$ or $e=0$

So r_2 is also zero, then $r_1=A=30°$

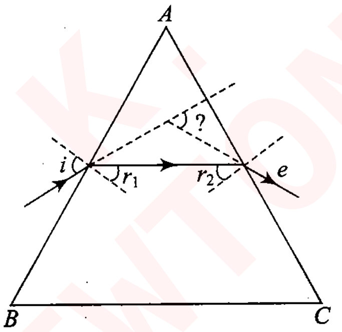


Fig. 1.606

So $\quad \mu=\frac{\sin i}{\sin r_1}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$

Hence the value of $a=3$.

6. (8) $A'B'$ is the apparent position of bottom of container, at a distance h/μ from water surface.

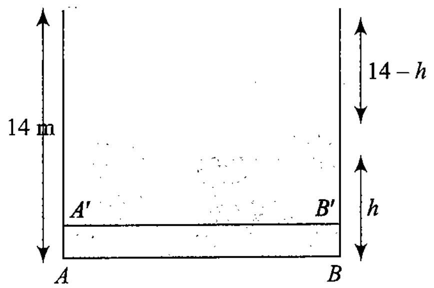


Fig. 1.607

If the container seems to be half filled. Then

$14-h=\frac{h}{\mu} \Rightarrow 14-h=\frac{3h}{4} \Rightarrow h=8$ m

7. (0) $\frac{1}{y}+\frac{1}{-x_1}=\frac{1}{10}$ (i)

$\frac{1}{y}-\frac{1}{10}=\frac{1}{10} \Rightarrow y=5$ cm

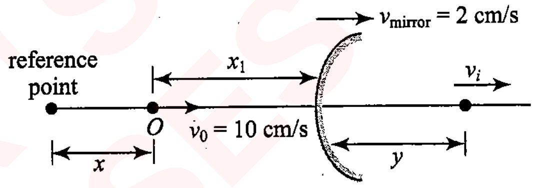


Fig. 1.608

$\frac{dx}{dt}=10$ cm/s, $\quad \frac{d(x+x_1)}{dt}=2$ cm/s

$\frac{dx_1}{dt}=-8$ cm/s

$\frac{dy}{dt}=v_i=2$

From (i): $\frac{dy}{dt}=\left(\frac{y}{x_2}\right)^2\frac{dx_1}{dt}$

$\Rightarrow \quad v_i-2=\left(\frac{5}{10}\right)^2(-8) \Rightarrow v_i=0$

8. (8) $f=\frac{D^2-d^2}{4D}=\frac{50^2-30^2}{4\times 50}=8$ cm

9. (8) When object is placed perpendicular to the principal axis, image size $= m \times$ (object size)

or $\quad 4=m\times 2 \Rightarrow m=2$

When object is placed along principal axis,

image size $= m^2$(object size) $=4\times 2=8$ mm

10. (3) Image is at ∞, so apparent position of object is at focus.

$d_{AC}=25$ cm

Shift $=5$ cm $=\left(\frac{\mu-1}{\mu}\right)t \Rightarrow t=15$ cm

Archives

Fill in the Blanks Type

1. Since the beam leaves B as a parallel beam, therefore the effective focal length of the two lenses combination is ∞. For two lenses placed at some distance d apart, the focal length of the combination is given by the formula

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

$$\frac{1}{\infty}=\frac{1}{+20}+\frac{1}{(-5)}-\frac{d}{(+20)(-5)} \Rightarrow d=+15\text{ cm.}$$

2. According to lensmaker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{(i)}$$

Given refractive index of the lens, i.e.,

$${}^{a}_{g}\mu=1.5=\frac{\mu_g}{\mu_a}$$

Also, given refractive index of medium

$${}^{a}_{m}\mu=\frac{4}{3}\frac{\mu_m}{\mu_a}$$

$$\therefore \quad {}^{m}_{g}\mu=\frac{\mu_g}{\mu_m}=\frac{\mu_g}{\mu_a}\times\frac{\mu_a}{\mu_m}=\frac{1.5}{4/3}=1.125$$

Applying Eq. (i) for the two cases, we get

$$\frac{1}{15}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

and $$\frac{1}{f_2}=\frac{1.5-1}{1.125-1}=\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

On dividing, we get

$$\frac{f_2}{15}=\frac{1.5-1}{1.125-1}=\frac{0.5}{0.125}=4 \Rightarrow f_2=60\text{ cm}$$

3. For refraction at APB,

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}$$

$$\Rightarrow \frac{-2}{-15}+\frac{1}{v}=\frac{1-2}{-10} \Rightarrow v'=-30\text{ cm}$$

$\Rightarrow$ The image O' will be formed at 30 cm to the right at P.

4. Since the image formed is real and elongated, the situation is as shown in the figure. Since the image of B is formed at B' itself, therefore

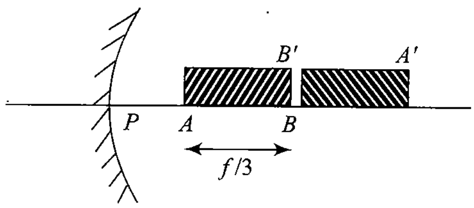


Fig. 1.609

B is situated at the center of curvature, that is at a distance $2f$ from the pole.

$$\therefore \quad PA=2f-\frac{f}{3}=\frac{5f}{3}$$

Let us find the image of A. For point A

$$u=-\frac{5f}{3},\ v=?\ \text{Applying } \frac{1}{u}+\frac{1}{v}=\frac{1}{f},$$

$$\Rightarrow \frac{1}{\frac{-5f}{3}}+\frac{1}{v}=\frac{1}{-f} \Rightarrow \frac{1}{v}=-\frac{1}{f}+\frac{3}{5f}$$

$$\Rightarrow \frac{1}{v}=\frac{-5+3}{5f}=\frac{-2}{5f} \Rightarrow v=-2.5f$$

$$\therefore \quad \text{Image length}=2.5f-2f=0.5f=\frac{f}{2}$$

$$\therefore \quad \text{Magnification}=\frac{\frac{f}{2}}{\frac{f}{3}}=1.5$$

5. For minimum deviation, $\mu=\dfrac{\sin\left(A+\dfrac{\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$=\frac{\sin\left(\dfrac{60+30}{2}\right)}{\sin\left(\dfrac{60}{2}\right)} \Rightarrow \mu=\frac{\sin 45^\circ}{\sin 30^\circ}=\sqrt{2}$$

Since $\mu=\sqrt{2}$ (given for the prism), the condition is for minimum deviation. In this case, the ray inside the prism becomes parallel to the base. Therefore, the angle made by the ray inside the prism with the base of the prism is zero.

6. The resolving power of a magnification device is inversely proportional to the wavelength used. The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is smaller than the wavelength of visible light.

7. We know that the velocity of light in vacuum,

$$c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Also, the velocity of light in a medium, $v=\dfrac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \quad n=\frac{\text{Velocity of light in medium}}{\text{Velocity of light in vacuum}}=\frac{v}{c}$$

$$=\frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}}=\frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$$

8. Given that width of central maxima

$$=2\times10^{-3}\text{ m}$$

$$\therefore \quad \frac{2\lambda D}{d}=2\times10^{-3}$$

$$\Rightarrow \quad d = \frac{2\times 5.89\times 10^{-7}\times 0.5}{2\times 10^{-3}} = 2.945\times 10^{-4}\,\text{m}$$

9. Frequency remains the same, i.e.,

$$f = \frac{c}{\lambda} = \frac{3\times 10^{8}}{6000\times 10^{-10}} = 5\times 10^{14}\,\text{Hz}$$

$$\mu = \frac{V_1}{V_2} = \frac{v\lambda_1}{v\lambda_2} \Rightarrow = \lambda_2 = \frac{\lambda_1}{\mu}$$

The wavelength gets modified to

$$\lambda_2 = \frac{\lambda_1}{\mu} = \frac{6000A^\circ}{1.5} = 4000\mathring{A}$$

10. Let P_1 and P_2 be the powers of the two thin lenses, respectively. Power of the two lenses in contact = $P_1 + P_2$. Power of the two lenses at a distance $x = P_1 + P_2 - xP_1P_2$. From the given data, we get

$$P_1 + P_2 = 10\,\text{m}^{-1} \text{ and } P_1 + P_2 - (0.25)P_1P_2 = 6\,\text{m}^{-1}.$$

From these two expressions, we get

$$P_1P_2 = 16\,\text{m}^{-2} \text{ and } P_1 - P_2 = \sqrt{(P_1+P_2)^2 - 4P_1P_2}$$

$$= \sqrt{(10^{-1})^2 - 4(16^{-1})} = 6\,\text{m}^{-1}$$

Thus, $P_1 + P_2 = 10\,\text{m}^{-1}$ and $P_1 - P_2 = 6\,\text{m}^{-1}$,

we get $P_1 = 8\,\text{m}^{-1}$ and , $P_2 = 2\,\text{m}^{-1}$

Hence, $f_1 = \frac{1}{P_1} = \frac{1}{8}\,\text{m} = 0.125\,\text{m}$

and $f_2 = \frac{2}{P_2} = \frac{1}{2}\,\text{m} = 0.5\,\text{m}$

11. Using Snell's law for the refraction at AC,

we get $\mu \sin i = (1)\sin r$ $\sqrt{2}\sin 30^\circ = \sin r$.

$$\sin r = \frac{1}{\sqrt{2}} \quad r = 45^\circ.$$

Angle of deviation $= 45^\circ - 30^\circ = 15^\circ$

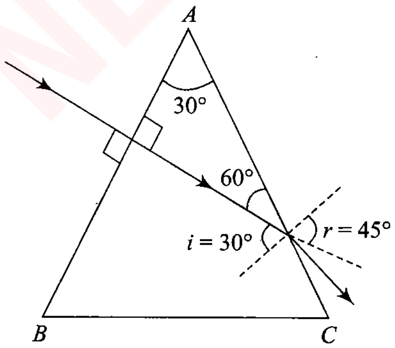


Fig. 1.610

True or False Type

1. True. This is due to atmospheric refraction. The light coming from sun bends toward the normal. Therefore, sun appears higher.

2. True. Intensity of light on a surface is the energy falling per second normally on unit area of the surface. The energy falling/second depends on energy density and the velocity of wave. Energy density is the total energy per unit surface area. As we move away from the cylindrical source of light, the surface area of cylindrical wavefront increases. The surface area of cylinder $\propto r$ (the distance from axis)

$\therefore$ Energy density $\propto \frac{1}{r}$

$\therefore$ Energy falling/second normally on unit area $\propto \frac{1}{r}$

$\therefore$ Intensity $\propto \frac{1}{r}$

3. False. Applying lens formula for convex lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-\infty)} = \frac{1}{1}$$

$\Rightarrow \quad v = 1\,\text{m}$

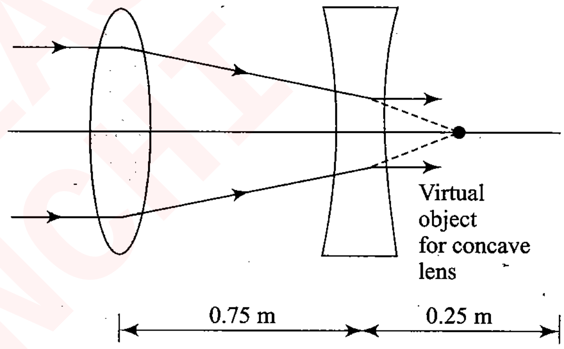


Fig. 1.611

Applying lens formula for concave lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{0.25} = \frac{1}{-0.25}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{-0.25} + \frac{1}{0.25} = 0 \Rightarrow v = \infty$$

The statement is false.

4. True. For the light to split, the material should have refractive index greater than 1 through which the light passes. Since the prism is hollow, we get no spectrum.

5. True. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{F} = \frac{1}{-15} + \frac{1}{30}$

$$= \frac{-2+1}{30} \Rightarrow F = -30\,\text{cm}.$$

This combination behaves as a concave lens of focal length 30 cm. Since $F_v < F$, therefore one sees colored pattern with violet color at the outer edge. The statement is true.

Multiple Choice Questions with One Correct Answer Type

1. a. $\lambda = \frac{v}{f}$

In moving from air to glass, f remains unchanged while v

decreases. Hence, λ should decrease.

2. d. $\omega = \dfrac{\lambda D}{d}$

D is halved and D is doubled.

Therefore, fringe width ω will become four times.

$\therefore$ Correct option is (d).

3. a. The phenomenon of total internal reflection takes place during reflection at P.

$$\sin\theta = \frac{1}{{}_g^w\mu} \quad \text{(i)}$$

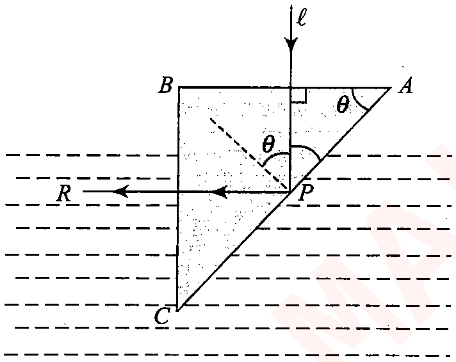


Fig. 1.612

where θ is the angle of incidence at P.

Now, $${}_g^w\mu = \frac{{}_g^a h}{{}_g^a\mu} = \frac{1.5}{4/3} = 1.125$$

Putting in (i),

$$\sin\theta = \frac{1}{1.125} = \frac{8}{9}$$

Therefore, sin θ should be greater than $\frac{8}{9}$

4. a $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.4} + \frac{1}{-0.25} = \frac{1}{0.4} - \frac{1}{0.25}$$

$$= \frac{0.25 - 0.4}{0.4\times 0.25} = \frac{-0.15}{0.4\times 0.25}$$

$$\frac{1}{f} = \frac{-0.15}{0.4\times 0.25} = -1.5$$

$$\Rightarrow \quad P = \frac{1}{f} = -1.5 \text{ dioptre}$$

5. a. $${}_2^1\mu = \frac{\sin 90^\circ}{\sin C} = \frac{1}{\sin C} \quad \text{[For critical angle]}$$

$$\therefore \quad C = \sin^{-1} = \left(\frac{1}{{}_2^1\mu}\right) \quad \text{(i)}$$

Applying Snell's law at P, we get

$${}_2^1\mu = \frac{\sin r'}{\sin i} = \frac{\sin(90 - r)}{\sin r}$$

$[\because i = r; r' + r = 90^\circ]$

$$\therefore \quad {}_2^1\mu = \frac{\cos r}{\sin r} \quad \text{(ii)}$$

From (i) and (ii), $C = \sin^{-1}(\tan r)$

6. c. Let $I_1 = I$ and $I_2 = 4I$

We know that Intensity $\propto$ (amplitude)^2.

Let $I \propto a^2$

$\therefore \quad I_1 \propto a^2$ and $I_2 \propto (2a)^2$

Maximum amplitude $= a + 2a = 3a$

$\therefore \quad I_{max} \propto (3a)^2 \Rightarrow I_{max} \propto 9a^2$

$\Rightarrow \quad I_{max} \propto 9I$

Minimum Amplitude $= 2a - a = a$

$\therefore \quad I_{min} \propto a^2 \Rightarrow I_{min} \propto I.$

c. is the correct option.

7. d. is the correct option.

8. a. For total internal reflection, $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 45^\circ} = 1.414$

i.e., for an angle of incidence of that color will suffer total internal reflection for which the refractive index is less than 1.414.

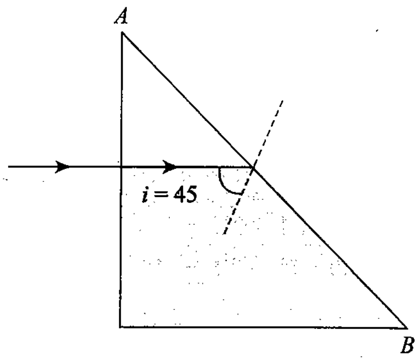


Fig. 1.613

Therefore, red light will be refracted at interface AB whereas blue and green light will be reflected

9. c. The angle of deviation for the first prism P_1, $\delta_1 = (\mu_1 - 1)A_1$.

The angle of deviation for the second prism P_2, $\delta_2 = (\mu_2 - 1)A_2$

Since total deviation is to be zero, therefore

$$\delta_1 + \delta_2 = 0 \Rightarrow (\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$$

$$A_2 = \frac{(1.54 - 1)}{(1.72 - 1)} 4^\circ = 3^\circ$$

c. is the correct option.

10. c. The image I' of parallel rays formed by lens 1 will act as a virtual object.

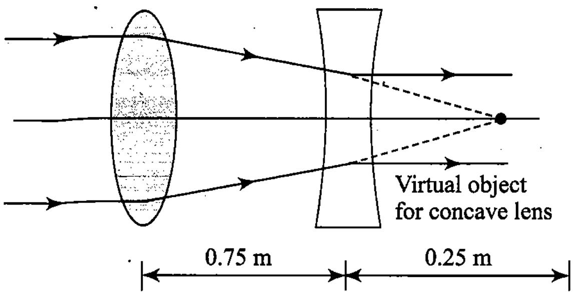


Fig. 1.614

Applying lens formula for lens 2,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{f_1-d}=\frac{1}{f_2}$$

$$\Rightarrow \quad v=\frac{f_2(f_1-d)}{f_2+f_1-d}$$

The horizontal distance of the image I from O is

$$x=d+\frac{f_2(f_1-d)}{f_2+f_1-d}$$

$$=\frac{df_2+df_1-d^2+f_2f_1-df_2}{f_2+f_1-d}$$

$$=\frac{f_1f_2+d(f_1-d)}{f_2+f_1-d}$$

To find the y-coordinate, we use magnification formula for lens 2,

$$m=\frac{v}{u}=\frac{\frac{f_2(f_1-d)}{f_1+f_2-d}}{f_1-d}=\frac{f_2}{f_1+f_2-d}. \text{ Also}$$

$$m=\frac{h^2}{\Delta}\Rightarrow h_2=\frac{\Delta\times f_2}{f_1+f_2-d}$$

Therefore, the y-coordinate,

$$y=\Delta-h_2$$

$$=\Delta-\frac{\Delta f_2}{f_1+f_2-d}$$

$$=\frac{\Delta f_1+\Delta f_2-\Delta d-\Delta f_2}{f_1+f_2-d}==\frac{\Delta(f_1-d)}{f_1+f_2-d}$$

11. c. Spherical abberation occurs due to the inability of a lens to converge marginal rays of the same wavelength to the focus, as it converges the paraxial rays. This defect can be removed by blocking marginal rays. This can be done by using a circular annular mask over the lens.

12. d. The distance between the first dark fringe on either side of the central bright fringe = width of central maximum $=\frac{2D\lambda}{a}$

$$=\frac{2\times2\times600\times10^{-9}}{10^{-3}}=2.4\times10^{-3}\text{ m}=2.4\text{ mm}.$$

d. is the correct option.

13. d. Applying Snell's law at P, $\mu=\frac{\sin r}{\sin 30°}$ $\sin r=\frac{1.44}{2}=0.72$

$\therefore\ \delta=r-30°=\sin^{-1}(0.72)-30°$

Therefore, the rays make an angle of $2\delta=[\sin{-1}(0.72)-30]$ with each other.

d. is the correct option.

14. c. A convex mirror and a concave lens always produce semi image for the objects. Therefore, option (b) and (d) are not correct. The image by a convex lens is diminished when the object is placed beyond $2f$.

Let $u=2f+x$.

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$,

$$\Rightarrow \frac{1}{v}-\frac{1}{-(2f+x)}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{2f+x}$$

$$=\frac{2f+x-f}{f(2f+x)}=\frac{(f+x)}{f(2f+x)}$$

But $u=v=1$ (given) $(2f+x)+\frac{f(2f+x)}{f+x}\le1$

$$\Rightarrow \quad 2f=x\left[1+\frac{f}{f+x}\right]\le1\Rightarrow\frac{(2f+x)^2}{f+x}\le1$$

$\Rightarrow\quad (2f+x)^2\le f+x$

The above is true for $f<0.25$ m

c. is the correct answer.

15. a. Here, $f_v=2$ cm and $f_e=3$ cm.

Using lens formula for eyepiece, $-\frac{1}{u}+\frac{1}{v_1}=\frac{1}{f_e}$

$$\Rightarrow \quad \frac{-1}{u}+\frac{1}{\infty}=\frac{1}{3}\Rightarrow u_1=-3\text{ cm } [\because i=0]$$

But the distance between objective and eyepiece is 15 cm (given).

Therefore, distance of image formed by the objective, $v=15-3=12$ cm. Let u be the object distance from the objective,

then for objective lens $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f_0}$ or $\frac{-1}{u}+\frac{1}{12}=\frac{1}{2}$

$$\Rightarrow \quad \frac{-1}{u}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}\quad u=-\frac{12}{5}=2.4\text{ cm}$$

a. is the correct option.

16. b. $f_1=+40$ cm (for convex lens) $=0.4$ m

$f_2=-25$ (for concave lens) $=-0.25$ m

Therefore, focal length (f) of the combination,

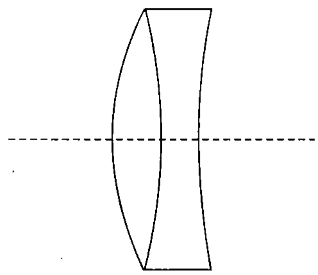


Fig. 1.615

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$=\frac{1}{0.40}-\frac{1}{0.25}=\frac{0.25-0.4}{0.40\times 0.25}$$

$$=-\frac{0.15}{0.1}=-1.5 \text{ dioptre.}$$

$$\Rightarrow \quad P=\frac{1}{f}=-1.5 \text{ dioptre.}$$

b. is the correct option.

17. b. In this case, the total deviation is shared between the two surfaces.

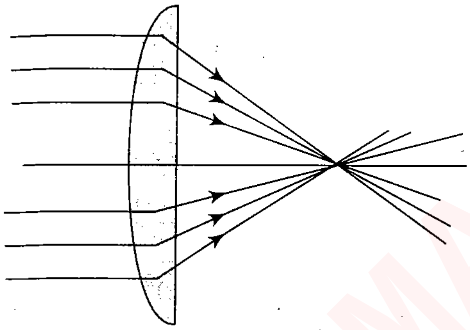


Fig. 1.616

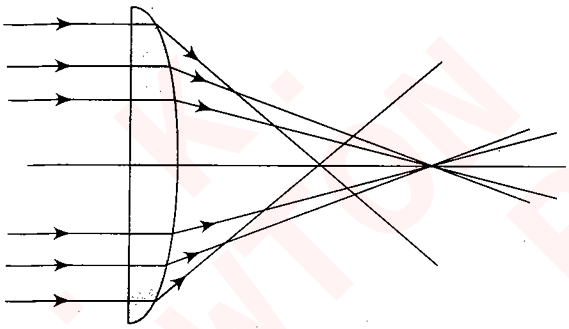


Fig. 1.617

18. d.

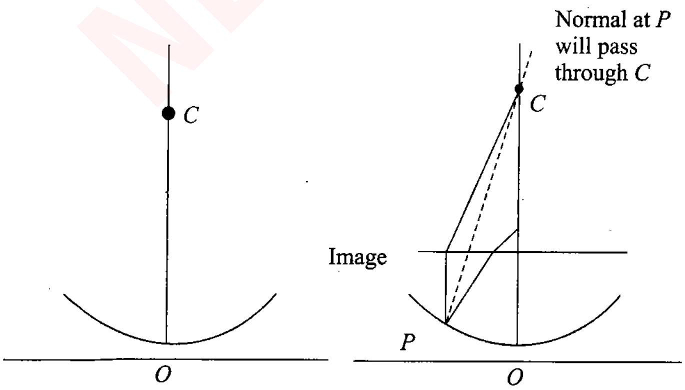


Fig. 1.618

The ray diagram is shown in the figure. Therefore, the image will be real and between C and O.

19. a. The formula for spherical refracting surface is

$$\frac{-\mu_1}{\mu}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

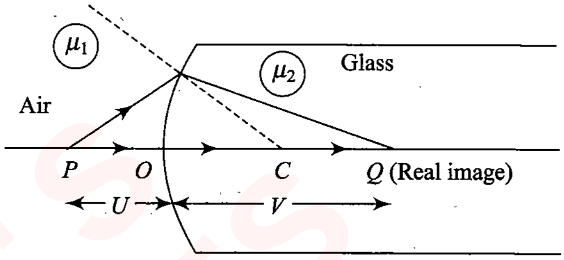


Fig. 1.619

Here, $u=-x, v=+x, R=+R, \mu_1=1, \mu_2=1.5$

$$\frac{-1}{-x}+\frac{1.5}{x}=\frac{1.5-1}{R}$$

$$\Rightarrow \quad x=5R$$

20. a. According to lensmaker's formula,

$$\frac{1}{f}=({}_g^m\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Now, $\quad {}_g^m\mu=\frac{{}_g\mu}{{}_m\mu}=\frac{1.5}{1.75}$

For concave lens, as shown in figure, in this case $R_1=-R$ and $R_2=+R$.

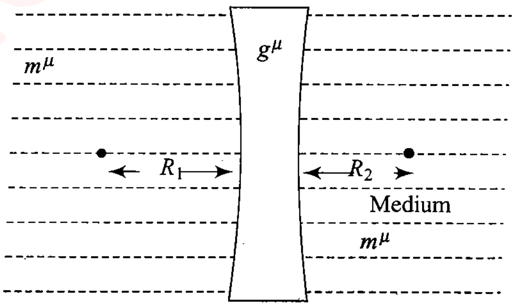


Fig. 1.620

$$\therefore \quad \frac{1}{f}=\left(\frac{1.5}{1.75}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right)=+\frac{0.25\times 2}{1.75R}$$

$$\Rightarrow \quad f=+3.5R$$

The positive sign shows that the lens behaves as a convergent lens.

a. is the correct option.

21. We know that $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

For divergence, $\mu_2>\mu_1$

Here, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is negative.

Therefore, **d.** is the correct option.

22. d. From the ray diagram,

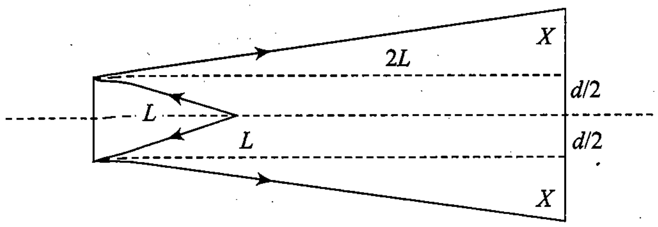


Fig. 1.621

In ΔANM and ΔADC,

$\angle ADC = \angle ANM = 90°$ [$MN \perp AD$]

$\angle MAN = \angle CAN$ (law of reflection)

$\Rightarrow \Delta ANM$ is similar to ΔADC

$\therefore \quad \dfrac{x}{2L} = \dfrac{d/2}{L}$ or $x = d$

So, required distance $= d + d + d = 3d$.

Therefore, **d.** is the correct option.

23. b. The path of rays become parallel to initial direction as they emerge.

Applying Snell's law at P, $\dfrac{1}{2}n = \dfrac{\sin \alpha/2}{\sin r}$ (i)

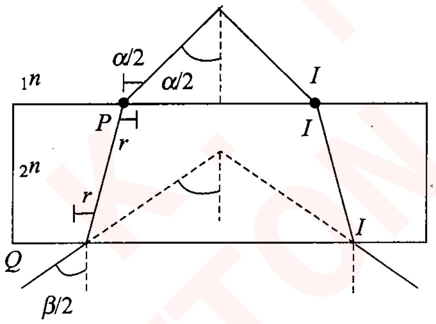


Fig. 1.622

Applying Snell's law at Q, $\dfrac{1}{2}n = \dfrac{\sin \beta/2}{\sin r}$ (ii)

From Eqs. (i) and (ii), $\alpha = \beta$

Therefore, **b.** is the correct answer.

24. a. See figure. The ray will come out form CD if it suffers total internal reflection at surface AD, i.e., it strikes the surface AD at critical angle C (the limiting case). Applying Snell's law at

P, $n_1 \sin C = n_2$ or $\sin C = \dfrac{n_2}{n_1}$

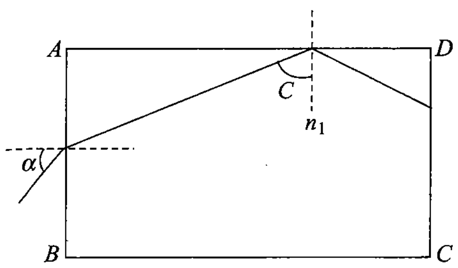


Fig. 1.623

Applying Snell's law at Q, $n_2 \sin \alpha = n_1 \cos C$;

$$\sin \alpha = \frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$$

or $$\alpha = \sin^{-1}\left[\frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$$

$\therefore$ **a.** is the correct option.

25. c. The intermediate image in compound microscope is real, inverted, and magnified.

26. d. Applying Snell's law at P,

$${}^1\mu_2 = \frac{\sin i}{\sin r_1} = \frac{\mu_2}{\mu_1} \quad \text{(i)}$$

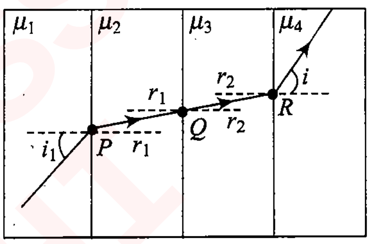


Fig. 1.624

Applying Snell's law at Q,

$${}^2\mu_3 = \frac{\sin r_1}{\sin r_2} = \frac{\mu_3}{\mu_2} \quad \text{(ii)}$$

Again, applying Snell's law at R,

$${}^3\mu_4 = \frac{\sin r_2}{\sin i} = \frac{\mu_4}{\mu_3} \quad \text{(iii)}$$

Multiplying (i), (ii), and (iii), we get

$$\mu_4 = \mu_1$$

Correct option is (d).

27. c. Since there will be no refraction from P to Q and then from Q to R (all being identical). Hence, the ray will now have the same deviation as before the point n, n' being same for the ray. Correct option is (c).

28. b. $\dfrac{\sin i}{\sin r} = \dfrac{1}{n}$ (i)

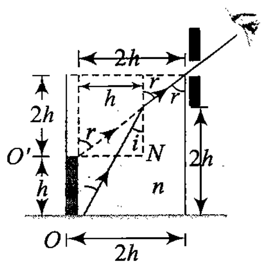


Fig. 1.625

Since $\quad \tan r = \dfrac{2h}{2h} = 1 \Rightarrow r = 45°$

$$\Rightarrow \quad \sin i = \frac{h}{h\sqrt{5}} \Rightarrow \sin i = \frac{1}{\sqrt{5}}$$

$$\therefore \quad \frac{1}{n} = \frac{1/\sqrt{5}}{1/\sqrt{2}} \Rightarrow n = \sqrt{\frac{5}{2}}$$

29. c. Since both surfaces have same radius of curvature on the same side, no dispersion will occur.

30. b. Maximum number of reflections $= \left[\frac{\ell}{x}\right]$, where $x = 0.2 \tan 30°$

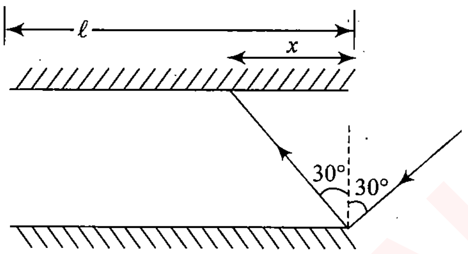


Fig. 1.626

31. b. For the concave lens,

$$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20} \Rightarrow v = 5\text{ cm}; \ \frac{h_2}{h_1} = \left|\frac{v}{u}\right|$$

$$\Rightarrow \quad h_2 = 2x\frac{5}{4} = 2.5\text{ cm}$$

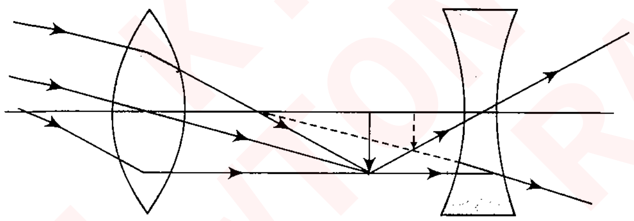


Fig. 1.627

32. b. Applying Snell's law at glass–water surface,

$$_g^\omega\mu = \frac{\sin r}{\sin i} = \frac{_g\mu}{_\omega\mu} \qquad \text{(i)}$$

Applying Snell's law at water–air surface,

$$_g^\omega\mu = \frac{\sin 90°}{\sin r} = \frac{_\omega\mu}{_a\mu} \qquad \text{(ii)}$$

From (i) and (ii),

$$\frac{_g\mu}{_\omega\mu} = \frac{3}{4\sin i} \Rightarrow \frac{3 \times {_g\mu}}{4} = \frac{3}{4\sin i} \Rightarrow {_g\mu} = \frac{1}{\sin i}$$

33. c. We know that $\sin C = \frac{1}{\mu}$ and $\mu \propto \frac{1}{\lambda}$

$\Rightarrow \quad \sin C \propto \lambda$ For more λ, C is more.

34. b. For minimum deviation, incident angle is equal to emerging angle. Therefore, QR is horizontal.

35. a. Here, $u = -6$ cm, $v = ?$, $r = -6$ cm, $\mu_1 = 1$, $\mu = 1.5$

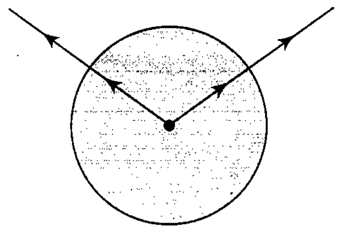


Fig. 1.628

Using the formula of spherical refracting surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

We have $\frac{1.5}{v} - \frac{1}{-6} = \frac{1.5-1}{-6} \Rightarrow v = -6$ cm

Alternative method: The rays coming from the point object fall on the glass–air interface normally and hence pass undeviated. Therefore, if we retrace the path of the refracted rays backward, the image will be formed at the center only.

36. a. $\frac{|P_1|}{|P_2|} = \frac{2}{3} \Rightarrow \frac{f_2}{f_1} = \frac{2}{3}$ (i)

Focal length of their combination, $\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2}$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{f_1} - \frac{1\times 3}{2f_1} \ \text{[from (i)]}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{f_1}\left[1 - \frac{3}{2}\right] = \frac{1}{f_1} \times \left(-\frac{1}{2}\right)$$

$$\therefore \quad f_1 = -15\text{ cm}$$

$$\therefore \quad f_2 = \frac{2}{3} \times f_1 = \frac{2}{3} \times 15 = 10\text{ cm}$$

37. c. The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of $= \frac{33.25}{1.33} = 25$ cm

$$\left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object

$\therefore \quad u = -(15 + 25) = -40$ cm, $f = f$, $v = ?$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{40} \Rightarrow v = \frac{40f}{40+f} \qquad \text{(i)}$$

But $\quad v = -\left[15 + \frac{25}{1.33}\right]$ (ii)

where $\frac{25}{1.33}$ is the real depth of the image.

From Eqs. (i) and (ii),

$$\frac{40f}{40+f} = -\left[15+\frac{25}{1.33}\right] = -33.79$$

$\Rightarrow$ $f = -18.31$ cm

38. b. Refraction from lens: $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$

$$v = 60 \text{ cm} \quad \underset{+\text{ve direction}}{\rightarrow}$$

i.e, first image is formed at 60 cm to the right of lens system.

Reflection from mirror:

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens:

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \leftarrow +\text{ve direction}$$

or $v_3 = 12$ cm

Therefore, the final image is formed at 12 cm to the left of the lens system.

39. c. From lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{ we have}$$

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

or $f = +5$

Further, $\Delta u = 0.1$

and $\Delta v = 0.1$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f^2$$

Substituting the values, we have

$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05$$

$$\therefore f \pm \Delta f = 5 \pm 0.5$$

40. b.

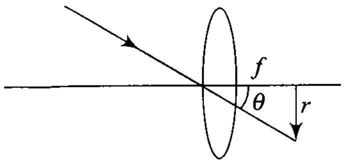


Fig. 1.629

$r = f \tan\theta$

or $r \propto f$

$\therefore$ $\pi r^2 \propto f^2$

41. c. Since $\theta < \theta_c$, both reflection and refraction will take place. From the figure, we can see that angle between reflected and refracted rays, α, is less than $180° - 2\theta$.

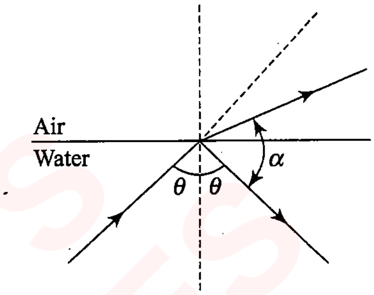


Fig. 1.630

$\therefore$ Option (c) is correct.

42. b. Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and center of curvature, $f < x < 2f$.

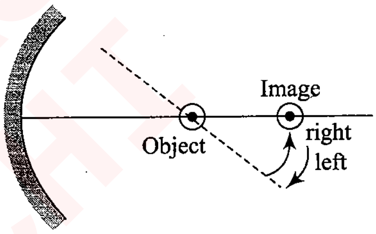


Fig. 1.631

$\therefore$ Correct option is (b).

43. b.

44. b. Critical angle from region III to region IV

$$\sin\theta_c = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0 \sin\theta = \frac{n_0}{6}\sin\theta_c$$

$$\sin\theta = \frac{1}{6}\sin\theta_c = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

$$\theta = \sin^{-1}\left(\frac{1}{8}\right)$$

$\therefore$ Correct option is (b).

45. a. At minimum deviation ($\delta - \delta_m$):

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30 \text{ (for both colors)}$$

$\therefore$ Correct answer is (a).

46. c. $V_{\text{ball}}^2 = 2 \times 10 \times 7.2 \Rightarrow v = 12 \text{ ms}^{-1}$

$$X_{\text{image of ball}} = \frac{4}{3} X_{\text{ball}}$$

$$V_{\text{image of ball}} = \frac{4}{3} V_{\text{ball}} = \frac{4}{3} \times 12 = 16 \text{ ms}^{-1}$$

47. b.

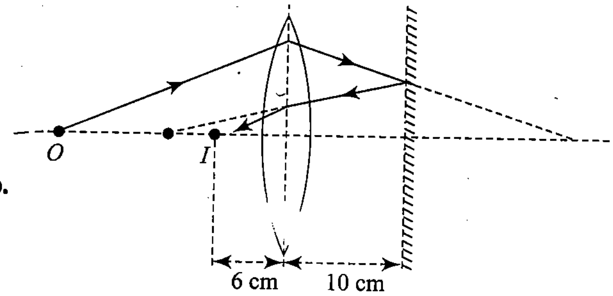


Fig. 1.632

Multiple Choice Questions with One or More than One Correct Answer Type

1. b., d.
The image formed will be complete because light rays from all parts of the object will strike on the lower half. But since the upper half light rays are cut off, the intensity will reduce.
2. a., b.

Using $M=\frac{f_0}{f_e}$ and $L=f_0+f_e$

3. a., b., c., d.
In case of an astronomical telescope, the distance between the objective lens and eyepiece lens $=f_0+f_e=16+0.02$

$= 06.02$ m. The angular magnification $=\frac{f_{\text{objective}}}{f_{\text{eyepiece}}}=\frac{-16}{0.02}$
$=-800$

The image seen by the astronomical telescope is inverted. Also, the objective lens is larger than the eyepiece lens.
4. b., c.
Concave lens and convex mirror are diverging in nature. Therefore, the refracted/reflected rays do not meet and are produced to make them meet. Therefore, the image formed is virtual and erect.
5. c., d.
For total internal reflection to take place: Angle of incidence,

$i>$ critical angle, θ_c $\left[\text{where } \sin\theta_c=\frac{1}{n}\right]$

or $\sin 45^\circ>\frac{1}{n}$ or $\frac{1}{\sqrt{2}}>\frac{1}{n}$ or $>\sqrt{2}$ or $n>1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.
6. a., b.
7. c., d.

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \text{ (mirror formula)}$$
$$f=-24\text{ cm}$$

8. a., b., c.

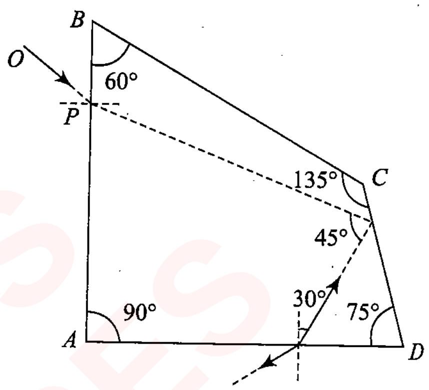


Fig. 1.633

Using Snell's law, $\sin^{-1}\frac{1}{\sqrt{3}}<\sin^{-1}\frac{1}{\sqrt{2}}$

Net deviation is 90°

Assertion and Reasoning Type

1. c. Laws of reflection can be applied to any type of surface.
2. b.

Matching Column Type

1. i. → a.
More the radius of aperture more is the amount of light entering the telescope.
ii. → c.
$M=\frac{f_0}{f_e}$
iii. → c.
$L=f_0+f_e$
iv. → a., b., d.
Depends on dispersion of lens, spherical aberration and radius of aperture.
2. i. → a., b., c., d.
ii. → b.
iii. → a., b., c., d.
iv. → a., b., c., d.
a., c., and d.: In case of concave mirror or convex lens, image can be real, virtual, diminished magnified or of same size.
b.: In case of convex mirror, image is always virtual (for real objects).
3. i. → a., c.
ii. → b., d., e.
iii. → a., c., e.
iv. → b., d.

Integer Answer Type

1. (6) $m = \dfrac{f}{f+u}$

2. (6) $\sin \theta_C = 3/5$

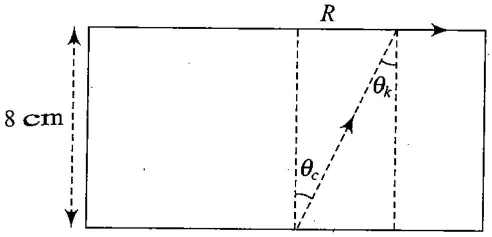


Fig. 1.634

$\therefore \quad R = 6$ cm.

3. (3) For $v_1 = 50/7$ m, $u_1 = -25$ m

$v_2 = 25/3$ m, $u_2 = -50$ m

Speed of object $= \dfrac{25}{30} \times \dfrac{18}{3} = 3$ kmph

4. (2) $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

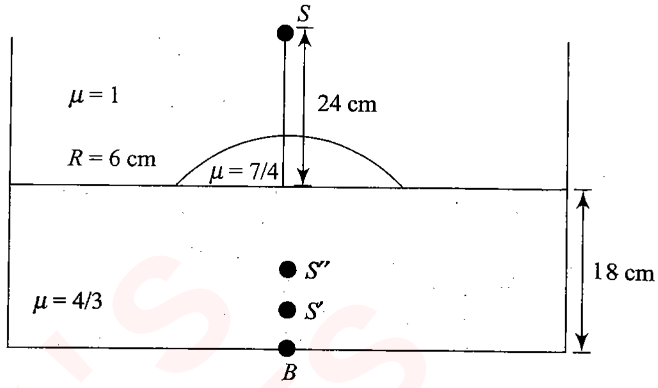


Fig. 1.635

$$\frac{7}{4v} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6}$$

$$\frac{7}{4v} = \frac{3}{24} - \frac{1}{24} = \frac{2}{24} = \frac{1}{12}$$

$$\frac{7 \times 2}{4} = V = 21 \text{ cm}$$

$$\frac{21}{OS''} = \frac{7/4}{4/3}$$

$OS'' = 16$

$\therefore \quad BS'' = 2$ cm

CHAPTER 2

Wave Optics

- Huygens' Wave Theory
- Wavefronts
- Huygens' Construction
- Principle of Linear Superposition
- Conditions of Interference
- Coherent Sources
- Interference
- Thin-Film Interference
- Young's Double-Slit Experiment
- Young's Double-Slit Experiment with White Light
- Different Cases in Young's Double-Slit Experiment
- Geometrical and Optical Paths
- Fresnel's Biprism
- Lloyd's Mirror Experiment

HUYGENS' WAVE THEORY

Huygens assumed that a body emits light in the form of waves. According to him: Each point source of light is a centre of disturbance from which waves spread in all directions.

The locus of all the particles of the medium vibrating in the same phase at a given instant is called the wavefront.

WAVE FRONTS

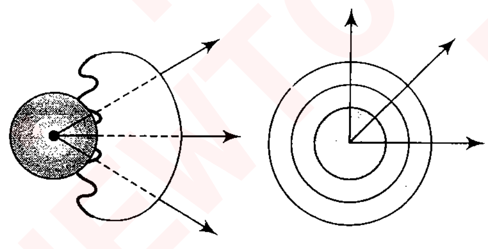
Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outward from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

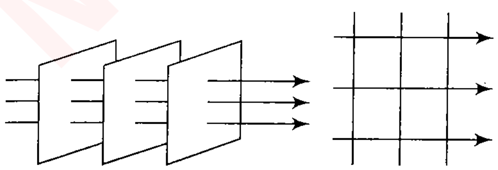
Figure 2.1(a) shows light waves from a point source forming a spherical wavefront in three-dimensional space. The energy travels outward along straight lines emerging from the source, i.e., radii of the spherical wavefront. These lines are the rays. Note that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.



(a)



(b)

Fig. 2.1 **Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave and (b) plane wave. The figure on the left shows a wave (e.g., light) in three dimensions. The figure on the right shows a wave in two dimensions (e.g., a water surface).**

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Fig. 2.1(b).

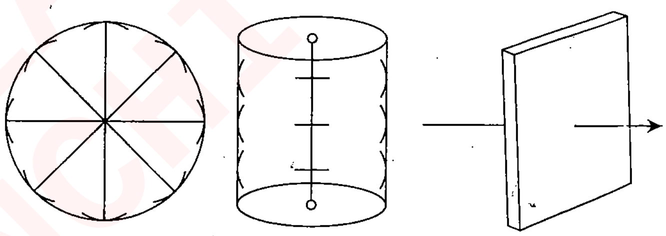
A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at large distance from the source, these wavefronts may be regarded as planar.

A wavefront is a surface joining the points of same phase. For a point source, the wavefronts are spherical which become almost plane at a very large distance. For example, wavefronts of sun light are plane.

Important Points

- Every point on a wavefront is an independent source to produce secondary wavefronts.
- All points on a wavefront vibrate in same phase with same frequency.
- Wavefronts move with the velocity of wave in that medium.

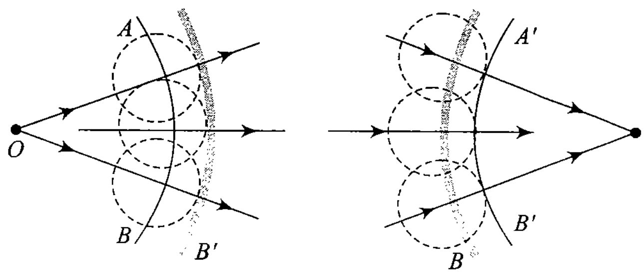
The shape of wavefront depends on the source producing the waves and is usually spherical, cylindrical or plane as shown in Fig. 2.2.



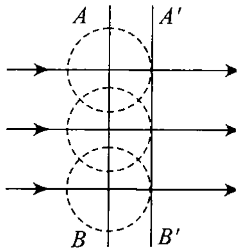
(a) Point source **(b) Line source** **(c) Source at ∞**

Fig. 2.2

Each point on a wavefront is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium. The forward envelope of the secondary wavelets at any instant gives the new wavefront.



(a) **(b)**



(c)

Fig. 2.3

HUYGENS' CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets.

Huygens' principle is illustrated in Fig. 2.4 in the simple case of a plane wave.

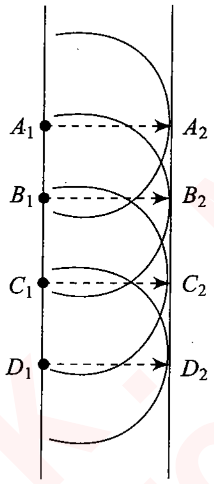


Fig. 2.4

(i) At time $t = 0$, we have a wavefront F_1. F_1 separates these parts of the medium which are undisturbed from those where the wave has already reached.

(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't', each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Note that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, A_1A_2 is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts is the same along any ray. In our example, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbours into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

Behaviour of wavefronts in prisms, lenses and mirrors

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass, this explains the tilt in the emerging wavefront.

(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus.

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig. 2.5). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.

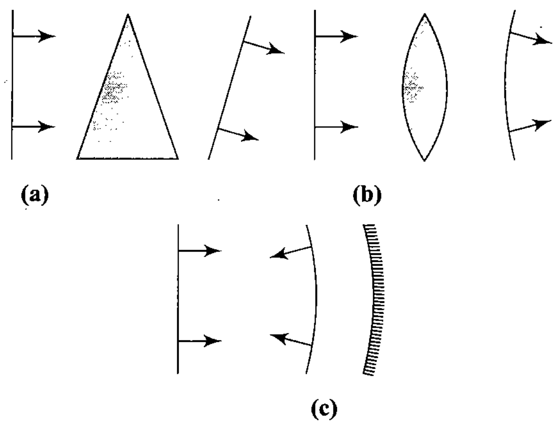


Fig. 2.5

PRINCIPLE OF LINEAR SUPERPOSITION

The pressure disturbance that results is governed by the principle of linear superposition, which states that the resultant disturbance is the sum of the disturbances from the individual waves. Light is also a wave, an electromagnetic wave, and it too obeys the superposition principle. When two or more light waves pass through a given point, their electric fields combine according to the principle of linear superposition and produce a resultant electric field. According to theory of electromagnetic waves, the square of the electric field strength is proportional to the intensity of the light, which, in turn, is related to its brightness. Thus, interference can and does alter the brightness of light, just as it affects the loudness of sound.

Optical wave: The principle of linear superposition explains various interference phenomena associated with light waves (see Fig. 2.6). The beautiful iridescent colors on the wings of tropical butterfly are due to thin-film interference of light. A transparent cuticle-like material on the upper wing surface acts like a thin film.

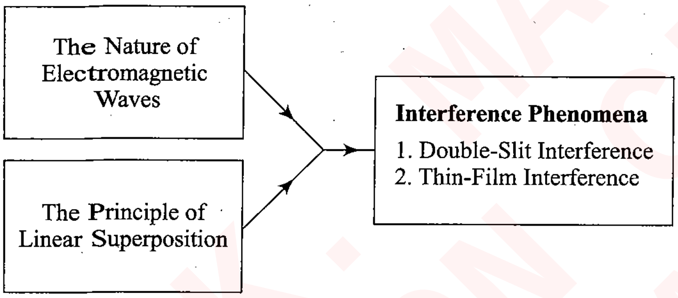


Fig. 2.6

Sound wave: Figure 2.7 indicates that transverse and longitudinal standing waves are related to the principle of linear superposition. Guitars produce sound by utilizing the transverse standing waves that form on each plucked string.

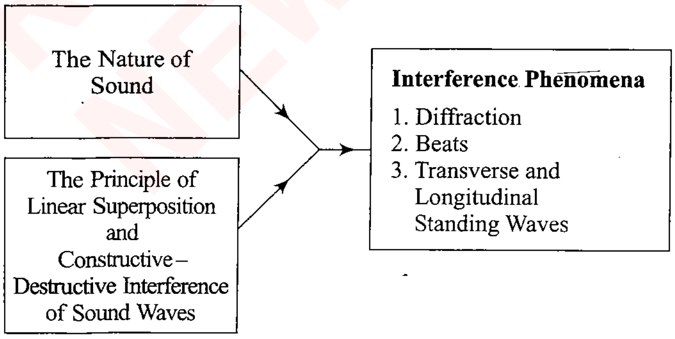


Fig. 2.7

Figure 2.8 illustrates what happens when two identical waves (same wavelength λ and same amplitude) arrive at the point P in phase, that is, crest to crest and trough to trough. According to the principle of linear superposition, the waves reinforce each other and constructive interference occurs. The resulting total wave at P has an amplitude that is twice the amplitude of either individual wave, and in the case of light waves, the brightness at

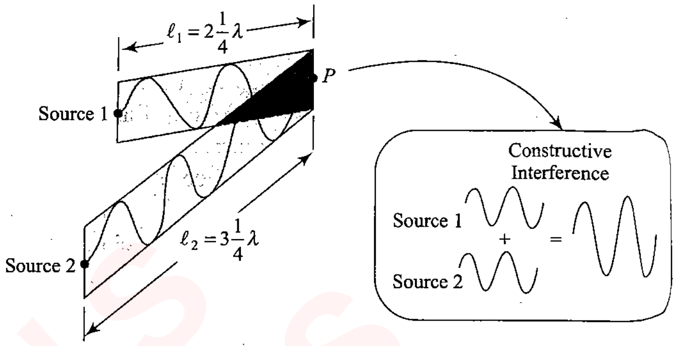


Fig. 2.8 The waves emitted by source 1 and source 2 start out in phase and arrive point at P in phase, leading to constructive interference at that point.

P is greater than that due to either wave alone. The waves start out in phase and are in phase at P because the distances, ℓ_1 and ℓ_2, between this spot and the sources of the waves differ by one wavelength. In general, when the waves start out in phase, constructive interference will result at P whenever the distances are the same or differ by any integer number of wavelengths; that is, in other words, assuming ℓ_2 is the larger distance, whenever $\ell_2 - \ell_1 = m\lambda$, where $m = 0, 1, 2, 3, \ldots$.

Figure 2.9 shows what happens when two identical waves arrive at the point P out of phase with one another, or crest to trough. Now, the waves mutually cancel, according to the principle of linear superposition, and destructive interference results. With light waves, this would mean that there is no brightness. The waves begin with the same phase but are out of phase at P because the distances through which they travel in reaching this spot differ by one-half of a wavelength. For waves that start out in phase, destructive interference will take place at P whenever the distances differ by any odd integer number of half-wavelengths, that is, whenever $\ell_2 - \ell_1 = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \ldots$ where ℓ_2 is the larger distance. This is equivalent to $\ell_2 - \ell_1 = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, 3, \ldots$.

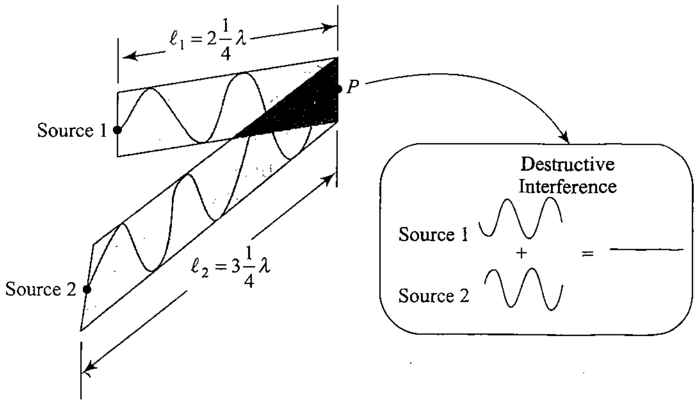


Fig. 2.9 The waves emitted by two sources have the same phase to begin with, but they arrive at point P, out of phase. As a result, destructive interference occurs at P.

CONDITIONS OF INTERFERENCE

We know that the superpositions of two mechanical waves can be constructive or destructive. In constructive interference, the amplitude of the resultant wave at a given position or time is greater than that of either individual wave.

If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference, or some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be incoherent.

In order to observe interference in light waves, the following conditions must be met:

- The sources must be coherent—that is, they must maintain a constant phase with respect to each other.
- The source should be monochromatic—that is, of a single wavelength.

COHERENT SOURCES

Two sources are said to be coherent if they produce waves of same frequency with a constant phase difference. Unlike sound waves, two independent sources of light cannot be coherent. Since sound is a bulk property of matter, therefore two independent sources of sound can be identical in all respects and can produce coherent waves. On the contrary, light is not a bulk property of matter, it is a property of each individual atom. As the individual atoms emit light randomly and independently, therefore two independent sources of light cannot be coherent.

Coherent sources can be obtained by splitting a light beam from a source into two. This can be done in two ways:

- *Division of Wavefront*: It is done in Young's double-slit experiment, Fresnel's biprism, Lloyd's mirror, etc.
- *Division of Amplitude*: It is usually done by partial reflection and transmission at a boundary, as it occurs in thin film interference, Newton's rings, etc.

INTERFERENCE

We have learned that in the superposition of waves there is interference of two sinusoidal waves. Let the two sinusoidal waves be

$$y_1 = a_1 \cos(\omega t + \theta_1) \quad \text{(i)}$$

$$y_2 = a_2 \cos(\omega t + \theta_2) \quad \text{(ii)}$$

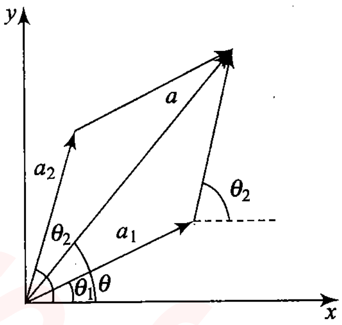


Fig. 2.10

According to the principle of superposition, the resultant displacement would be given by

$$y(t) = a\cos(\omega t + \theta) \quad \text{(iii)}$$

where

$$a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2 \quad \text{(iv)}$$

$$a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2 \quad \text{(v)}$$

The resultant wave is also sinusoidal, with same frequency but different amplitude. On squaring and adding Eqs. (iv) and (v), we get

$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)\right]^{1/2} \quad \text{(vi)}$$

$$\tan\theta = \frac{a_1\sin\theta_1 + a_2\sin\theta_2}{a_1\cos\theta_1 + a_2\cos\theta_2} \quad \text{(vii)}$$

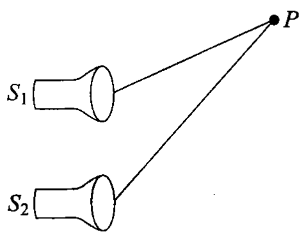
If $\theta_1 = 0$ and $\theta_2 = \theta$, we have

$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos\theta\right]^{1/2} \quad \text{(viii)}$$

$$\therefore \quad \tan\theta = \frac{a_2\sin\theta}{a_1 + a_2\cos\theta} \quad \text{(ix)}$$

From Eq. (viii), we find that if $\theta = 0, 2\pi, 4\pi, \ldots$, then $a = a_1 + a_2$.

When two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes; this is constructive interference.



Path difference = $S_2P - S_1P$

Fig. 2.11

Similarly, if $\theta = 0, 3\pi, 5\pi, \ldots$, then $a = a_1 - a_2$.

When superposing waves are in opposite phase, the resultant amplitude is the difference of the two amplitudes; this is destructive interference.

The difference in the distances from S_1 and S_2 to a given point is called the path difference.

A path difference of one wavelength corresponds to a phase difference of 2π radian because a shift in position of one wavelength along the wave changes its phase by a complete cycle.

For a path difference Δx, the phase difference (in radians) is

$$\delta = \frac{2\pi}{\lambda}\text{(path difference)} = \frac{2\pi}{\lambda}(\Delta x)$$

The factor $\Delta x/\lambda$ is number of wavelengths corresponding to path difference. Each wavelength corresponds to a phase difference of π.

The intensity of a wave is proportional to square of amplitude, therefore Eq. (viii) reduces to

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where $\delta = \frac{2\pi}{\lambda}(\Delta x)$.

$$I_{av} = I_1 + I_2 \left[\text{as} \int_0^{2\pi}(\cos\delta)\,d\delta = 0\right]$$

As the average value of intensity is equal to the sum of individual intensities, in interference energy is neither created nor destroyed but is redistributed and hence in interference energy is conserved.

Illustration 2.1 **Light waves from two coherent sources superimpose at a point. The waves, at this point, can be expressed as $y_1 = a\sin[10^{15}\pi t]$ and $y_2 = 2a\sin[10^{15}\pi t + \phi]$. Find the resultant amplitude if phase difference 'ϕ' is**

a. zero b. $\pi/3$ c. π

Also find the frequency (Hz) of resultant wave in each case.

Sol. Resultant amplitude can be obtained from the relation

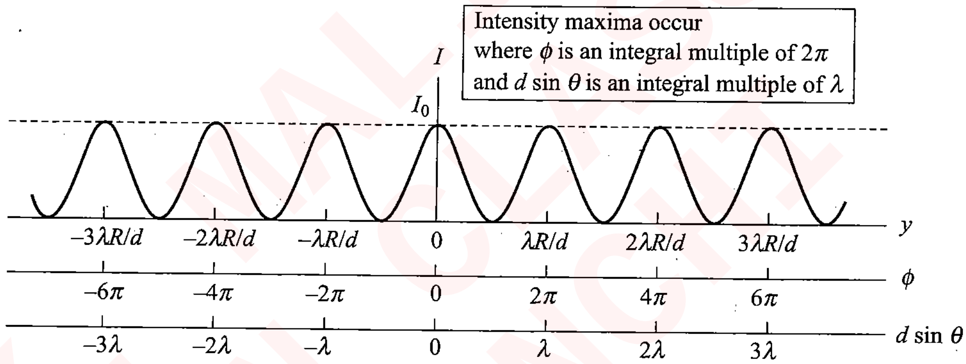


Fig. 2.12

As the maximum and minimum values of $\cos\delta$ are +1 and −1, respectively, the maximum and minimum values of I are given by

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \text{and} \quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

The conditions for maximum intensity are

$$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots \quad \text{or} \quad \Delta x = n\lambda$$

The conditions for minimum intensity are

$$\delta = (2n-1)\pi, \quad n = 0, 1, 2, 3, \ldots$$

or $\Delta x = (n - 1/2)\lambda, \quad n = 0, 1, 2, 3, \ldots$

When $I_1 = I_2 = I_0$, then

$$I = 4I_0\cos^2\delta/2$$

Clearly, if $d = \pm\pi, \pm 3\pi, \ldots$, the resultant intensity will be zero and we will have a minimum. When $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$, the intensity will be maximum ($I = 4I_0$).

Note: ***In interference, energy is neither created nor destroyed but it is conserved.***

In case of interference, intensity at a point is given by

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\delta$$

Now, as at a given point phase difference can have any value between 0 and 2π, the average value of intensity will be

$$I_{av} = \left[\frac{\int_0^{2\pi} I\,d\delta}{\int_0^{2\pi} d\delta}\right] = \frac{1}{2\pi}\int_0^{2\pi}\left(I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta\right)d\delta$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\left[I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi\right]$$

A_1 and A_2 are the amplitudes of interfering waves, ϕ is the phase difference at the given point and A is the resultant amplitude. Here, $A_1 = a$ and $A_2 = 2a$.

a. $\phi = 0$; $A^2 = a^2 + 4a^2 + 2a(2a)\cos 0° = 9a^2 \Rightarrow A = 3a$

b. $\phi = \phi/3$; $A^2 = a^2 + 4a^2 + 2a(2a)\cos\pi/3 = 7a^2 \Rightarrow A = \sqrt{7}a$

c. $\phi = \pi$;

$$A^2 = a^2 + 4a^2 + 2a(2a)\cos\pi = a^2 \Rightarrow A = a$$

Interference results due to superposition of waves of same frequency and frequency of the resultant wave also has the same value. Here, this frequency is

$$\omega = 10^{15}\pi \quad \text{or} \quad 2\pi f = 10^{15}\pi$$

$$\therefore \quad f = 5\times 10^{14}\text{ Hz}$$

Illustration 2.2 **In Young's experiment, the interfering waves have amplitudes in the ratio 3:2. Find the ratio of (a) amplitudes and (b) intensities, between the bright and dark fringes.**

Sol. Here, we have to obtain the ratio

$$\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_1 - A_2}$$

and also the corresponding ratio of intensities

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

But $\quad \frac{A_1}{A_2} = \frac{3}{2}$ (given)

By componendo and dividendo,

$$\frac{A_1 + A_2}{A_1 - A_2} = \frac{3+2}{3-2} = 5$$

Hence,

$$\frac{A_{max}}{A_{min}} = 5 \quad \text{and} \quad \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = 25$$

Illustration 2.3 **Two coherent sources emit light waves which superimpose at a point where these can be expressed as**

$$E_1 = E_0 \sin(\omega t + \pi/4)$$
$$E_2 = 2E_0 \sin(\omega t - \pi/4)$$

Here, E_1 and E_2 are the electric field strengths of the two waves at the given point.

If I is the intensity of wave expressed by field strength E_1, find the resultant intensity.

Sol. Intensity of wave expressed by field strength E_1, $I \propto E_0^2$

[intensity $\propto$ (amplitude)^2]

Intensity of wave expressed by E_2, $I' \propto (2E_0)^2$

$$\frac{I'}{I} = 4 \quad \text{or} \quad I' = 4I$$

Phase difference between the two waves is

$$(\omega t + \pi/4) - (\omega t - \pi/4) = \frac{\pi}{2}$$

Resultant intensity is given by

$$I_R = I + I' + 2\sqrt{II'} \cos\phi$$

$$[I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi]$$

$$I_R = I + 4I + 2\sqrt{I(4I)} \cos\pi/2 \quad \Rightarrow \quad I_R = 5I$$

Thus, resultant intensity is five times the intensity of wave expressed by E_1.

Illustration 2.4 **Determine the resultant of two waves given by $y_1 = 4\sin(200\pi t)$ and $y_2 = 3\sin(200\pi t + \pi/2)$.**

Sol. In general, resultant of two waves given by $y_1 = A_1 \sin\omega t$ and $y_2 = A_2 \sin(\omega t + \phi)$ can be obtained by the principle of superposition, i.e.,

$$y = y_1 + y_2 = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$$
$$y = A\sin(\omega t + \alpha)$$

where

$$A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$$

$$\alpha = \tan^{-1}\left[\frac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right]$$

In the present case,

$$y_1 = 4\sin 200\pi t \quad \text{and} \quad y_2 = 3\sin\left(200\pi t + \frac{\pi}{2}\right)$$

Here, $A_1 = 4$, $A_2 = 3$ and $\phi = \pi/2$.

Resultant wave will be $y = A\sin(200\pi t + \alpha)$ with

$$A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$$

$$\Rightarrow \quad A = [4^2 + 3^2 + 2(4)(3)\cos\pi/2]^{1/2}$$

$$\Rightarrow \quad A = 5$$

$$\tan\alpha = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi} = \frac{3\sin\pi/2}{4 + 3\cos\pi/2} = \frac{3}{4}$$

or $\quad \alpha = 37° \approx 0.2\pi$ rad

Hence, the resultant wave is $y = 5\sin(200\pi t + 0.2\pi)$.

Illustration 2.5 **Two sources S_1 and S_2 emitting light of wavelength 600 nm are placed a distance 1.0×10^{-2} cm apart. A detector can be moved on the line S_1P which is perpendicular to S_1S_2.**

a. What would be the minimum and maximum path difference at the detector as it moves along the line S_1P?

b. Locate the position of the farthest minima detected.

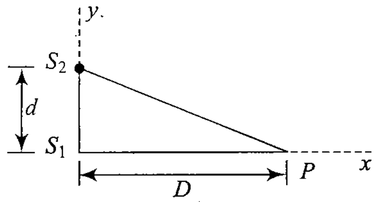


Fig. 2.13

Sol. a. Path difference at P, $\Delta x = S_2P - S_1P$

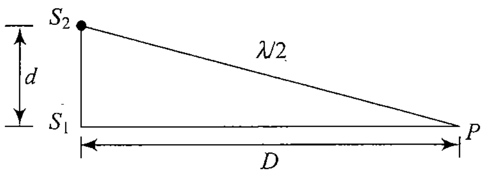


Fig. 2.14

Minimum path difference will occur when the observer will be at a large distance from the sources. The path difference will be maximum when the observer is near source S_1. As observer moves from S_1 towards $+x$-direction, the path difference decreases from 'd' to zero.

$(\Delta x)_{max}$: when $S_1P = 0$, i.e., detector is at position S_1.

$\therefore$ $(\Delta x)_{max} = S_1S_2 = d = 1.0 \times 10^{-2}$ m

$(\Delta x)_{min}$: when detector is at large distance and $(\Delta x)_{min}$ will be approximately zero.

Let the farthest minimum occur at P. As we moves away the path difference decreases, order of maxima or minima increases as we moves towards S_2. At the position of the farthest minimum path difference is

$$S_2P - S_1P = \lambda/2$$

$$(D^2 + d^2)^{1/2} - D = \lambda/2$$

$$\left[D\left(1 + \frac{d^2}{D^2}\right)^{1/2}\right] - D = \lambda/2$$

$$\frac{d^2}{2D} = \frac{\lambda}{2} \Rightarrow D = \frac{d^2}{\lambda} D = \frac{d^2}{\lambda} = \frac{\lambda}{4} = 1.7 \text{ cm}$$

Illustration 2.6 **Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. Consider a line passing through S_2 and perpendicular to the line S_1S_2. Find the position of farthest and nearest minima.**

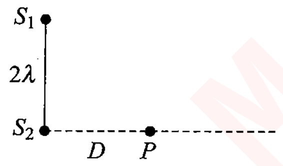


Fig. 2.15

Sol. $\Delta x_{min} = (2n-1)\dfrac{\lambda}{2}$

The farthest minima has path difference $\lambda/2$ while nearest minima has path difference $(3/2)\lambda$. For the nearest minima,

$$S_1P - S_2P = \frac{3}{2}\lambda$$

[as maximum path difference is 2λ]

$$\Rightarrow \sqrt{(2\lambda)^2 + D^2} - D = \frac{3}{2}\lambda$$

$$\Rightarrow (2\lambda)^2 + D^2 = \left(\frac{3}{2}\lambda + D\right)^2$$

$$\Rightarrow 4\lambda^2 + D^2 = \frac{9}{4}\lambda^2 + D^2 \times 2 \times \frac{3}{2}\lambda \times D$$

$$\Rightarrow 3D = 4\lambda - \frac{9\lambda}{4} = \frac{7\lambda}{4} \Rightarrow D = \frac{7}{12}\lambda$$

For the farthest minima,

$$S_1P - S_2P = \frac{\lambda}{2}$$

$$\Rightarrow \sqrt{4\lambda^2 + D^2} - D = \frac{\lambda}{2}$$

$$\Rightarrow 4\lambda^2 + D^2 = \frac{\lambda^2}{4} + D^2 + D\lambda \Rightarrow D = 4\lambda - \lambda/4 = \frac{15\lambda}{4}$$

Illustration 2.7 **Two sources are placed on x-axis at a separation $d = 3\lambda$. An observer starts moving from A on a circular track of radius R ($R \gg d$). How many bright points and dark points will he observe?**

Find the angular positions of maxima and minima.

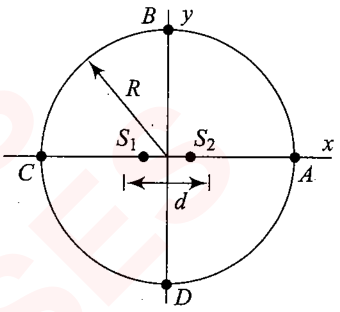


Fig. 2.16

Sol. The observer will have maximum path difference at positions A and C equal to 3λ.

Minimum path difference equals to zero at positions B and D.

In each quarter circle, he will have path difference between 0 and 3λ.

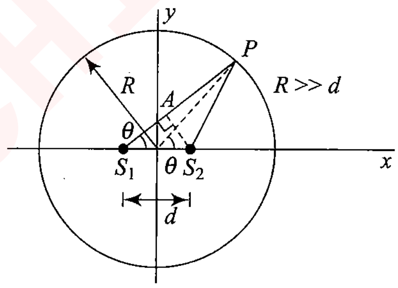


Fig. 2.17

Path difference corresponding to maxima in our given range is $0, \lambda, 2\lambda, 3\lambda$.

Path difference corresponding to minima in our given range is $\lambda/2, 3\lambda/2, 5\lambda/2$.

Hence, total number of bright points in the complete circular path is $4 \times 2 + 4 = 12$.

Total number of dark points on circular path is $4 \times 3 = 12$.

Hence, the observer will find 12 bright points and 12 dark points in the circular track.

Finding angular positions of maxima and minima:

Path difference at point P,

$$S_1P - S_2P = \Delta x$$

$$S_1A = \Delta x$$

$$\Rightarrow d\cos\theta = \Delta x$$

Hence,

$$\cos\theta = \frac{\Delta x}{d}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\Delta x}{d}\right)$$

For maxima, $\Delta x = n\lambda$

For minima, $\Delta x = (2n-1)\dfrac{\lambda}{2}$

Illustration 2.8 **Two point sources are placed on a straight line separated by a distance $d = 3\lambda$. Both the sources are placed at a distance L from a wall which is perpendicular to the straight line. Both the sources are sending waves of equal intensity.**

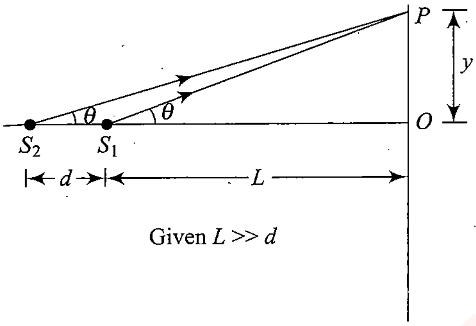


Fig. 2.18

Find:

(i) Locus of the points on the wall having equal intensity.

(ii) Maximum and minimum path difference observed on the wall.

Sol.

(i) As shown in the figure, path difference between the waves is same in the circular path on the wall. Hence, locus of the points of same intensity is a circle.

(ii) Maximum path difference will be at $O = 3\lambda$.
Minimum path difference will be at a large distance from O and it will be zero.

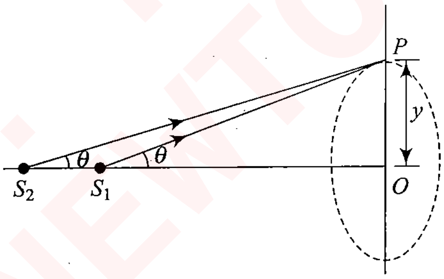


Fig. 2.19

Illustration 2.9 **(i) In Illustration 2.8, how many dark rings will be observed on the wall? (ii) What is the path difference at point P?**

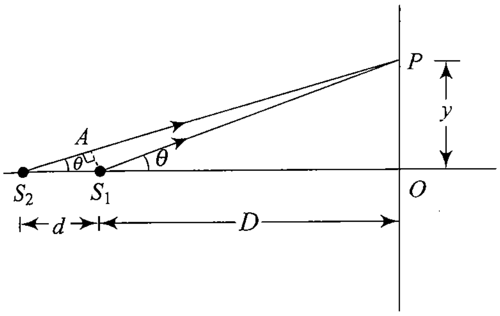


Fig. 2.20

Sol. (i) Path difference corresponding to minima in our given range is $\lambda/2$, $3\lambda/2$, $5\lambda/2$.
Hence, three dark rings will be observed on the wall.
(ii) Path difference between the waves reaching at P,
From Fig. 2.20, $\Delta x = S_2A$

$$S_2P - S_1P = \Delta x \quad \text{or} \quad d\cos\theta = \Delta x \tag{i}$$

$$\cos\theta = \frac{\Delta x}{d} \tag{ii}$$

But

$$\tan\theta = \frac{y}{D}$$

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{D^2 - y^2}}{D} \tag{ii}$$

$$\Rightarrow \quad \cos\theta = \frac{D\sqrt{1-\dfrac{y^2}{D^2}}}{D}$$

$$= \left(1 - \frac{y^2}{D^2}\right)^{1/2} = \left(1 - \frac{y^2}{2D^2}\right)$$

From (i) $\Delta x = d\cos\theta = d\left(1 - \dfrac{y^2}{2D^2}\right)$

From (i) and (ii), we can find the radius of any dark or bright ring.

Illustration 2.10 **A convex lens of focal length 20 cm is cut along its diameter and the two halves are displaced by t = 1 cm along the principal axis. A monochromatic point source of wavelength 476 nm is placed at a distance 40 cm from the first half, as shown in the figure. Find the position of the first maximum.**

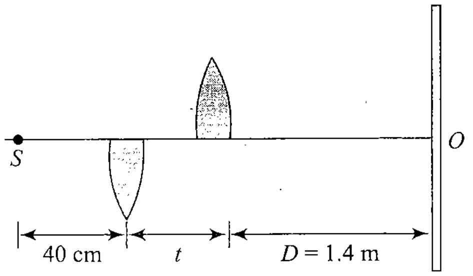


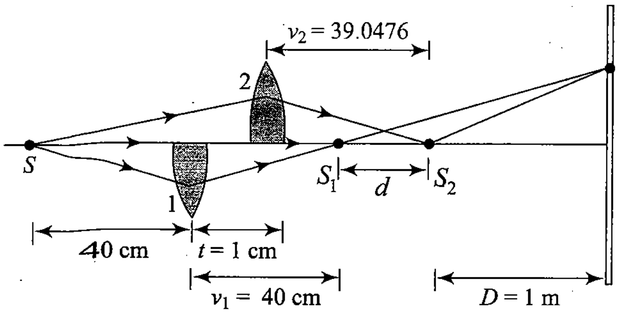
Fig. 2.21

Sol. Using the formula

$$v_1 = \frac{fu_1}{u_1 - f} = \frac{(20)(40)}{(40-20)} = 40 \text{ cm}$$

and $$v_2 = \frac{u_2 f}{u_2 - f} = \frac{(41)(20)}{41-20} = 39.0476 \text{ cm}$$

$$d = v_2 - v_1 = 0.0476 \text{ cm} = 0.476 \text{ mm}$$



The optical path difference at P is

$$\Delta x_p = (SS_2 + S_1P) = S_1S_2 + S_2P - S_1P$$
$$= (SS_2 + S_2P) - (SS_1 + S_1P)$$
$$= (SS_2 - SS_1) - (S_1P - S_2P)$$
$$= d - (S_1P - S_2P)$$

We know that

$$S_1P - S_2P = d\left[1 - \frac{y^2}{2D^2}\right]$$

$$\Delta x_p = d - d\left[1 - \frac{y^2}{2D^2}\right] = \frac{y^2 d}{2D^2}$$

The position of the nth maximum is given by

$$\frac{y^2 d}{2D^2} = n\lambda \quad \text{or} \quad y = D\sqrt{\frac{2n\lambda}{d}}$$

The position of the first maximum is given by

$$y = 10^3\sqrt{\frac{2(1)(0.476 \times 10^{-3})}{0.476}} = 44.72 \text{ mm}$$

Illustration 2.11 **Two parallel beams of light P and Q are incident normally on a prism and the transmitted rays are brought to focus with the help of a convergent lens as shown in Fig. 2.22. If the intensities of the upper and lower beams immediately after transmission from face AC are $4I$ and I, respectively, find the resultant intensity at the focus.**

(IIT-JEE, 1991)

Sol. In case of interference,

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2}) \cos\phi$$

Here, $I_1 = I$ and $I_2 = 4I$. So,

$$I_R = 5I + 4I\cos\phi$$

$$\Rightarrow \quad I_R = I\left[1 + 8\cos^2\left(\frac{\phi}{2}\right)\right] \quad \text{(i)}$$

with

$$\phi = \frac{2\pi}{\lambda}(\Delta x) \quad \text{(ii)}$$

where

$$\Delta x = [DG - EF \text{ (in medium)}] \quad \text{(iii)}$$

Now, from Fig. 2.22,

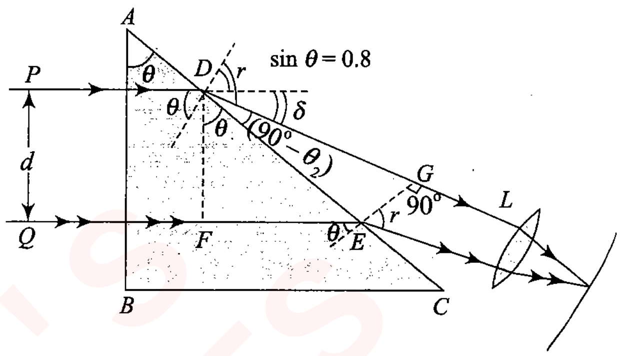


Fig. 2.22

$$FE \text{ (in medium)} = \mu(FE) = \mu d\tan\theta \quad [\text{as } FE = d\tan\theta] \text{ (iv)}$$

$$DG = DE\cos(90° - r) = DE\sin r$$

or $$DG = (d\sin r/\cos\theta) \quad [\text{as } DE = d/\cos\theta]$$

or $$DG = \mu d\tan\theta \quad [\text{as at } D, \mu\sin\theta = 1\sin r] \text{ (v)}$$

Substituting the value of FE and DG from Eqs. (iv) and (v) in Eq. (iii), we get

$$\Delta x = \mu d\tan\theta - \mu d\tan\theta = 0$$

So, Eq. (ii) gives $\phi = 0°$. Hence, from Eq. (i),

$$I_R = I[1 + 8\cos^2 0°] = 9I$$

THIN-FILM INTERFERENCE

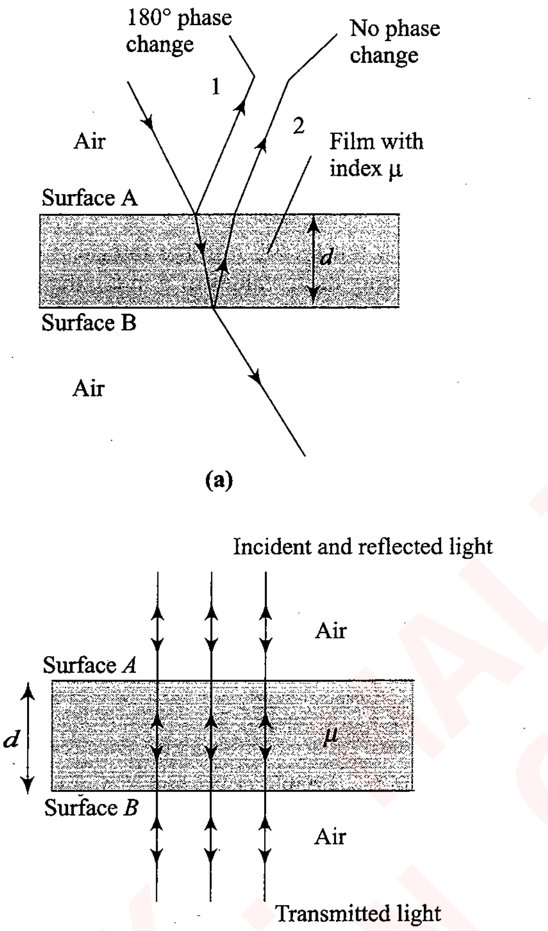
In YDSE, we obtained two coherent sources from a single (incoherent) source by division of wavefront. Here, we do the same by division of amplitude (into reflected and refracted waves).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d, then waves reflected from the upper surface interfere with waves reflected from the lower surface.

Clearly, the waves reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

Further, if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

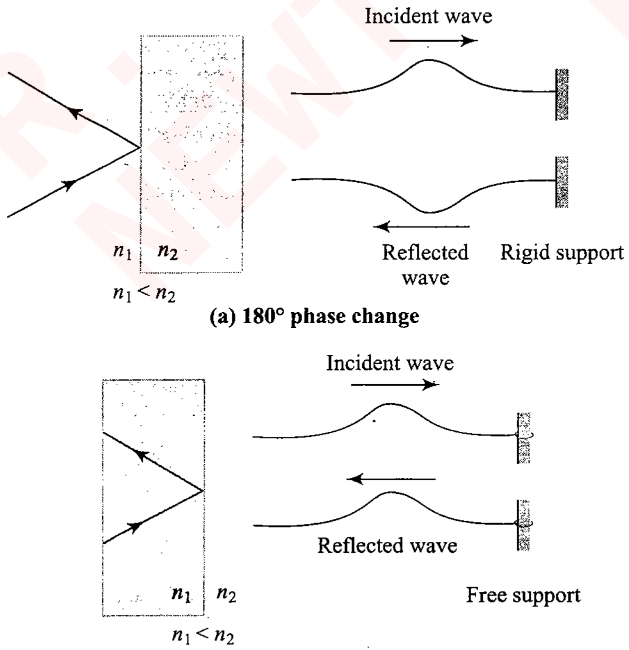
It is useful to draw an analogy between reflected light waves and the reflections of a transverse wave on a stretched string when the wave meets a boundary, as in Fig. 2.23. The reflected pulse on a string undergoes a phase change of 180° when it is reflected from the boundary of denser string, or from a rigid barrier, and no phase change when it is reflected from the boundary of a less dense string. Similarly, an electromagnetic wave undergoes a 180° phase change when reflected from the boundary of a medium of higher index of refraction than the one in which it has been travelling. There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction. The transmitted wave that crosses the boundary also undergoes no phase change.



(a)

(b)

Fig. 2.23



(a) 180° phase change

(b) No phase change

Fig. 2.24

Figure 2.23(a) shows that ray reflecting from a medium of higher refractive index undergoes a 180° phase change. The right side shows the analogy with a reflected pulse on a string.

Figure 2.23(b) shows that a ray reflecting from a medium of lower refractive index undergoes no phase change. Consequently, condition for constructive and destructive interference in the reflected light is given by

$$2\mu d = n\lambda \qquad \text{(for destructive interference)}$$

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(for constructive interference)} \quad \text{(i)}$$

where $n = 0, 1, 2, \ldots$ and λ = wavelength in free space.

Interference will also occur in the transmitted light and here conditions of constructive and destructive interference will be the reverse of (1), i.e.,

$$2\mu d = \begin{cases} n\lambda & \text{(for constructive interference)} \\ \left(n + \frac{1}{2}\lambda\right) & \text{(for destructive interference)} \end{cases} \quad \text{(ii)}$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light). However, the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving Eq. (i), we assumed that the media surrounding the thin film on both sides are rarer compared to the medium of thin film.

If media on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by Eq. (i).

However, if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surfaces.

Illustration 2.12 **White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction $\mu = 1.33$ and thickness $d = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?**

Sol. In this situation, Eq. (i) gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{n + 1/2} = \frac{(2)(1.33)(320\text{ nm})}{n + 1/2} = \frac{851\text{ nm}}{n + 1/2}$$

For $m = 0$, this give us $\lambda = 1700$ nm, which is in the infrared region. For $n = 1$, we find $\lambda = 567$ nm, which is yellow–green light, near the middle of the visible spectrum. For $n = 2$, $\lambda = 340$ nm, which is in the ultraviolet region. So, the wavelength at which the light seen by the observer is brightest is $\lambda = 567$ nm.

Illustration 2.13 **A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (Fig. 2.25). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.**

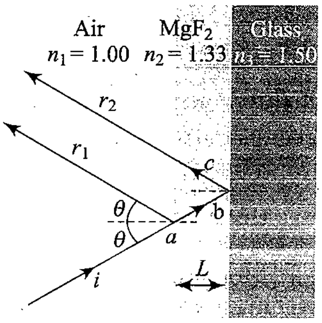


Fig. 2.25

Sol. This situation here is different from previous problem in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see Fig. 2.25). Unwanted reflections from glass can be suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference $2L$ within the film must be equal to an odd number of half wavelengths:

$$2n_2 L = (n + 1/2)\lambda$$

We want the least thickness for the coating, that is, the smallest L. Thus, we choose $n = 0$, the smallest value of n. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550 \text{ nm}}{(4)(1.38)} = 96.6 \text{ nm}$$

Illustration 2.14 **White light is incident normally on a glass plate of thickness 0.50×10^{-6} and index of refraction 1.50. Which wavelengths in the visible region (400 nm–700 nm) are strongly reflected by the plate?**

Sol. The light of wavelength λ is strongly reflected if

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(i)}$$

where n is a non-negative integer.

Here $2\mu d = 2 \times 1.50 \times 0.5 \times 10^{-6}$ m $= 1.5 \times 10^{-6}$ m.

Putting $\lambda = 400$ nm in Eq. (i), we get

$$1.5 \times 10^{-6} \text{ m} = \left(n + \frac{1}{2}\right)(400 \times 10^{-9} \text{ m})$$

or $\qquad n = 3.25$

Putting $\lambda = 700$ nm in Eq. (i), we get

$$1.5 \times 10^{-6} \text{ m} = \left(n + \frac{1}{2}\right)(700 \times 10^{-9} \text{ m})$$

or $\qquad n = 1.66$

Thus, within 400 nm to 700 nm, the integer n can take the values 2 and 3. Putting these values of n in (i), the wavelengths become

$$\lambda = \frac{4\mu d}{2n + 1} = 600 \text{ nm and } 429 \text{ nm}$$

Thus, light of wavelengths 429 nm and 600 nm are strongly reflected.

Concept Application Exercise 2.1

1. Two coherent sources of light of intensity ratio β produce interference pattern. Prove that in the interference pattern

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\beta}}{1 + \beta}$$

where I_{max} and I_{min} are maximum and minimum intensities in the resultant wave.

2. Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the interference is (a) coherent and (b) incoherent.
3. Explain why two flashlights held close together do not produce an interference pattern on a distant screen.
4. Refractive index of a thin soap film of a uniform thickness is 1.34. Find the smallest thickness of the film that gives an interference maximum in the reflected light when light of wavelength 5360 Å falls at normal incidence.
5. A thin transparent film of thickness 3000 Å and refractive index 1.5 is deposited on a sheet made of a metal. Assuming normal incidence of light and also that the film is a plane parallel one, what will be the color of a pot made from this sheet when observed in while light?
6. By an anodizing process, a transparent film of aluminium oxide of thickness $t = 250$ nm and index of refraction $n_2 = 1.80$ is deposited on a sheet of polished aluminium. What is the colour of utensils made from this sheet with observer in white light? Assume normal incidence of the light.
7. As a soap bubble evaporates, it appears black just before it breaks. Explain this phenomenon in terms of the phase changes that occur on reflection from the two surfaces of the soap film.
8. If we are to observe interference in a thin film, why must the film not be very thick (with thickness only on the order of a few wavelengths)?
9. A soap bubble ($n = 1.33$) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?

10. An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the colour of the light in the visible spectrum most strongly reflected and (b) the colour of the light in the spectrum most strongly transmitted. Explain your reasoning.

11. A material having an index of refraction of 1.30 is used as an antireflective coating on a piece of glass (n = 1.50). What should be the minimum thickness of this film in order to minimize refection of 500 nm light ?

12. In solar cells, a silicon solar cell (μ = 3.5) is coated with a thin film of silicon monoxide SiO (μ = 1.45) to minimize reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

YOUNG'S DOUBLE-SLIT EXPERIMENT

In 1801, the English scientist Thomas Young (1773–1829) performed a historic experiment that demonstrated the wave nature of light by showing that two overlapping light waves interfered with each other. His experiment was particularly important because he was also able to determine the wavelength of the light from his measurements, the first such determination of this important property. Fig. 2.26(a) shows one arrangement of Young's experiment, in which light of a single wavelength (monochromatic light) passes through a single narrow slit and falls on two closely spaced narrow slits S_1 and S_2. These two slits act as coherent sources of light waves that interfere constructively and destructively at different points on the screen to produce a pattern of alternating bright and dark fringes. The purpose of the single slit is to ensure that only light from one direction falls on the double slit from different points on the light source would strike the double slit from different directions and cause the pattern on the screen to be washed out. The slits S_1 and S_2 act as coherent sources of light waves because the light from each originates from the same primary source, namely, the single slit.

To help explain the origin of the bright and dark fringes, Fig. 2.26 presents three top views of the double slit and the screen. Part (b) illustrates how a bright fringe arises directly opposite the mid-point between the two slits. In this part of the figure, the waves (identical) from each slit travel to the midpoint on the screen. At this location, the distances ℓ_1 and ℓ_2 to the slits are equal, each containing the same number of wavelengths. Therefore, constructive interference results, leading to the bright fringe. Part (c) indicates that constructive interference produces another bright fringe on one side of the midpoint when the distance ℓ_2 is larger than ℓ_1 by exactly one wavelength. A bright fringe also occurs symmetrically on the other side of the midpoint when the distance ℓ_1 and ℓ_2 by one wavelength; for clarity, however, this bright fringe is not shown. Constructive interference produces additional bright fringes on both sides of the middle wherever the difference between ℓ_1 and ℓ_2 is an integer

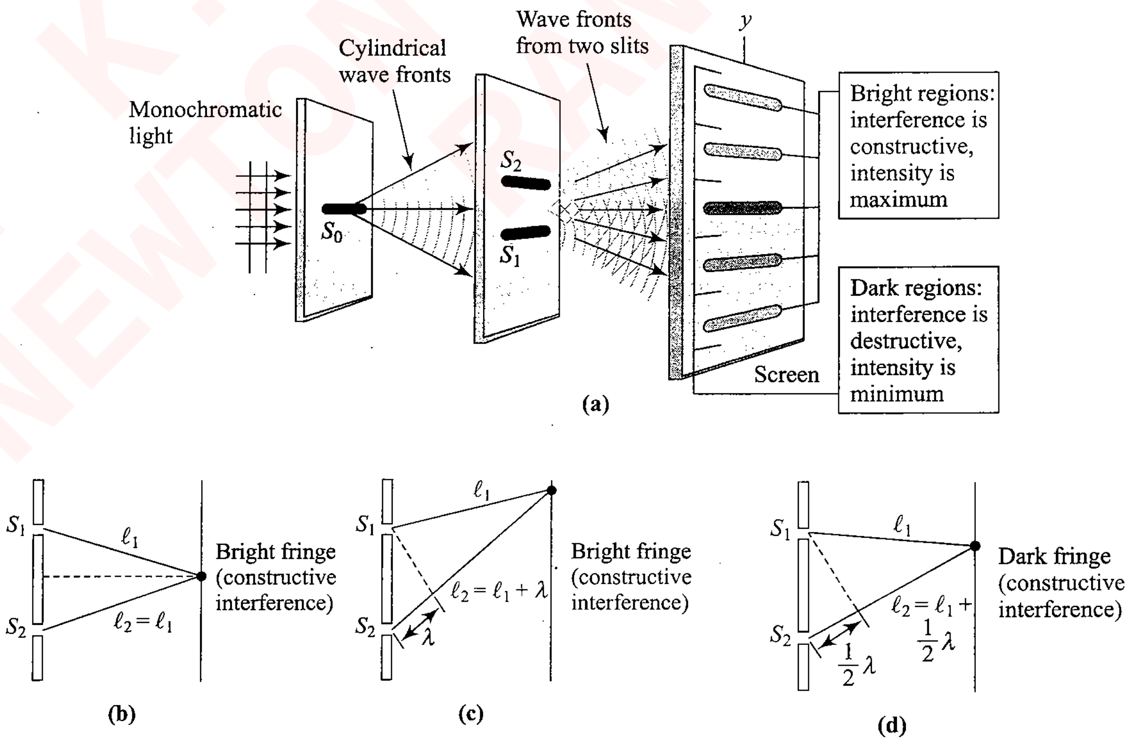
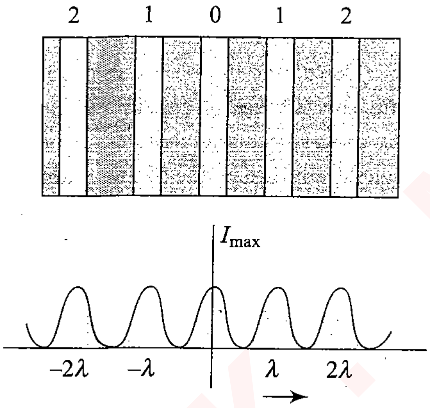


Fig. 2.26 **The waves slits S_1 and S_2 interfere constructively (parts b and c) or destructively (part d) on the screen, depending on the difference in distance between the slits and the screen. The slit widths and the distance between the slits have been exaggerated for clarity.**

number of wavelengths: λ, 2λ, 3λ, etc. Part (d) shows how the first dark fringe arises. Here, the distance l_2 is larger than l_1 by exactly one-half a wavelength, so the waves interfere destructively, giving rise to the dark fringe. Destructive interference creates additional dark fringes on both sides of the center whenever the difference between ℓ_1 and ℓ_2 equals an odd integer number of half-wavelengths: $1\left(\frac{\lambda}{2}\right), 3\left(\frac{\lambda}{2}\right)$, etc.

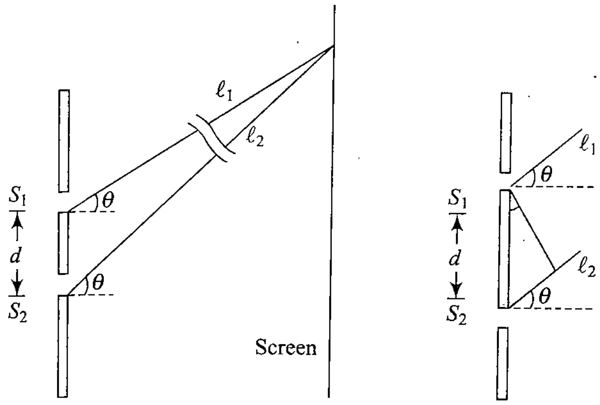
The brightness of the fringes in Young's experiment varies, as the photograph in Fig. 2.27 shows. Below the photograph is a graph to suggest the way in which the intensity varies for the fringe pattern. The central fringe has the greatest intensity. To either side of the center, the intensities of the other fringes decrease symmetrically in a way that depends on how small the slit widths are relative to the wavelength of the light.



Light intensity versus d sin θ for a double slit interference pattern when the screen is far from the two slits ($L >> d$)

Fig. 2.27

The position of the fringes observed on the screen in Young's experiment can be calculated with the aid of Fig. 2.28. If the screen is located far away compared with the separation d of the slits, then the lines labeled ℓ_1 and ℓ_2 in part (a) are nearly parallel. Being nearly parallel, these lines make approximately equal angles θ with the horizontal. The distances ℓ_1 and ℓ_2 differ by an amount $\Delta\ell$, which is the length of the short side of the shaded triangle in part (b) of the figure. Since the triangle is a right triangle, it follows that $\Delta\ell = d \sin\theta$. Constructive interference occurs when the distances differ by an integer number m of wavelengths λ, or $\Delta\ell = d \sin\theta = m\lambda$. Therefore, the angle θ for the interference maxima can be determined from the following expression.



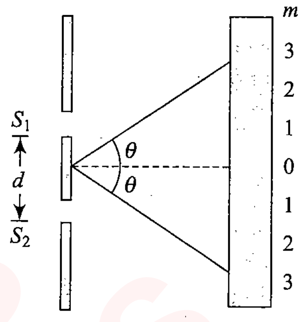


Fig. 2.28 **(a) Rays from slits S_1 and S_2, which make approximately the same angle θ with the horizontal, strike a distant screen at the same spot. (b) The difference in the path lengths of the two rays is $\Delta\ell = d \sin\theta$. (c) The angle θ is the angle at which a bright fringe ($m = 2$, here) occurs on either side of the central bright fringe ($m = 0$).**

Bright fringes of a double slit:

$$\sin\theta = m\frac{\lambda}{d}, \quad m = 0, 1, 2, 3, \ldots \qquad \text{(i)}$$

The value of m specifies the order of the fringe. Thus, $m = 2$ identifies the 'second-order' bright fringe. Part (c) of the figure stresses that the angle θ given by Eq. (i) locates bright fringes on either side of the midpoint between the slits. A similar line of reasoning leads to the conclusion that the dark fringes, which lie between the bright fringes, are located according to the following expression.

Dark fringes of a double slit:

$$\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}, \quad m = 0, 1, 2, 3, \ldots \qquad \text{(ii)}$$

Position of Bright and Dark Fringes in YDSE

Let us consider a point P on the distant screen, at a distance D from the slits with $D \gg d$. The small arc of the circle from P is almost a straight line. From Fig. 2.29, path difference $\Delta x = d \sin\theta$.

The condition for constructive interference is

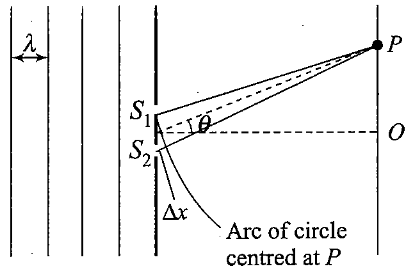
$$\Delta x = S_2P - S_1P = \pm n\lambda, \quad \text{where } n = 0, 1, 2, \ldots \qquad \text{(iii)}$$

$$d \sin\theta = \pm n\lambda \qquad \text{[Condition for maxima]}$$

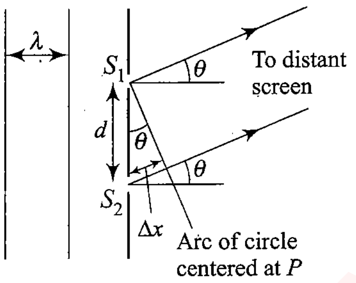
The condition for destructive interference is

$$\Delta x = S_2P - S_1P = \pm\left(n - \frac{1}{2}\right)\lambda, \quad \text{where } n = 1, 2, \ldots$$

$$d \sin\theta = \pm\left(n - \frac{1}{2}\right)\lambda \qquad \text{[Condition for minima] (iv)}$$



(a)



(b)

Fig. 2.29

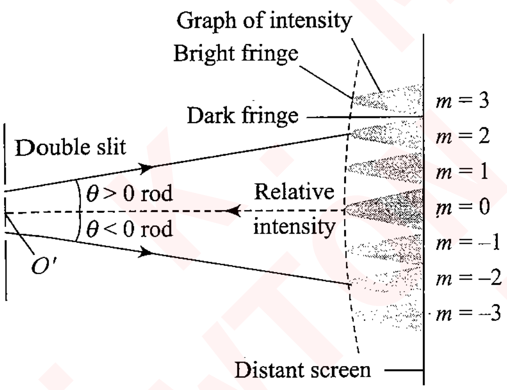


Fig. 2.30

- If the separation between screen and slits is large ($D \gg d$), then we have

$$\sin\theta \approx \tan\theta = \theta = \frac{y}{D}$$

where y is the vertical distance from the center of the pattern. Position of nth bright and dark fringes are, respectively,

$$\frac{y_n d}{D} = n\lambda \quad \text{or} \quad y_n = n\lambda\left(\frac{D}{d}\right)$$

and

$$\frac{y_n d}{D} = \left(n - \frac{1}{2}\right)\lambda \quad \text{or} \quad y_n = \left(n - \frac{1}{2}\right)\lambda\frac{D}{d}$$

Each value of n corresponds to particular bright or dark fringe. The absolute value of n is called the order of interference.

- The nth and $(n + 1)$th maxima are given by

$$y_n = \frac{n\lambda D}{d} \quad \text{or} \quad y_{n+1} = \frac{(n+1)\lambda D}{d}$$

Fringe Width

Separation between adjacent maxima is defined as fringe width β.

$$\therefore \qquad \beta = y_{n-1} - y_n = \frac{\lambda D}{d}$$

This is also the expression for separation between adjacent minima. As long as d and θ are small, the separation between interference fringes is independent of n (order of fringe), i.e., the fringes are evenly spaced. As vertical distance y is related to θ by $\theta = y/D$, so $\Delta\theta = \Delta y/D = \frac{\beta}{D} = \frac{\lambda}{d}$ is referred to as angular fringe width.

Note: ***Fringe width is directly proportional to wavelength and inversely proportional to the distance between the two slits.***

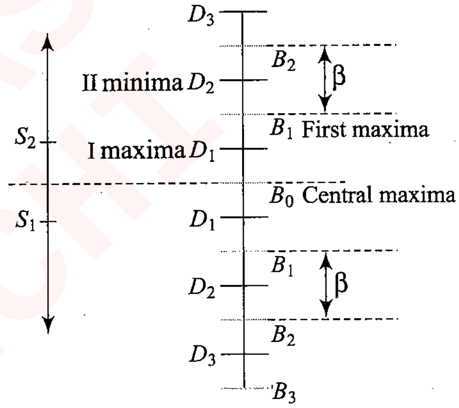


Fig. 2.31 Fringe pattern in YDSE.

From the above equation, following points can be noted.

(i) Fringe width is independent of n, i.e., all the interference fringes have same width in experiments where there is a division of wavefront of the incoming waves.

(ii) Fringe width is directly proportional to the wavelength of light used, i.e., $\beta \propto \lambda$. So, fringes for red light are wider than those for blue light.

(iii) Fringe width is inversely proportional to the separation between the slits, i.e., $\beta \propto (1/d)$. Thus, with increase in separation between the sources, fringe width decreases.

(iv) With increase in distance between screen and plane of slits, fringe width β increases linearly with D. However, with increase in D, intensity of light sources and hence of interfering waves is adversely affected.

(v) If the interference experiment is performed in a medium of refractive index μ (say water) instead of air, the wavelength of light will change from λ to (λ/μ) and so

$$\beta' = \frac{D}{d}\left[\frac{\lambda}{\mu}\right] = \frac{\beta}{\mu} \qquad \text{(vi)}$$

That is, fringe width reduces and becomes $(1/\mu)$ times of its value in air.

If monochromatic light is replaced by white light, due to overlapping of patterns (each corresponding to a single wavelength with fringe width $\beta \propto \lambda$), central band, i.e., principal maxima will be white with red edges.

On either side of it, we shall get a few colored bands and then uniform illumination.

Conditions for observing sustained interference with good contrast:

1. The initial phase difference between the interfering waves must remain constant otherwise the interference will not be sustained.
2. The frequencies and wavelength of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not be sustained.
3. The light must be monochromatic. This eliminates overlapping of pattern as each wavelength corresponds to one interference pattern.
4. The amplitudes of the interfering waves must be equal. This improves constant with $I_{max} = 4I_0$ and $I_{min} = 0$.
5. The sources must be close to each other, otherwise due to small fringe width fringes may be so close to each other that the eye cannot resolve them, resulting in uniform illumination.
6. The sources must be narrow. A board source will be equal to a large number of narrow sources and each set of two sources will give its own pattern and overlapping of patterns will result in uniform illumination.

Maximum Order of Interference Fringes

The position of nth order maxima on the screen,

$y = \dfrac{n\lambda D}{d}$, $n = 0, \pm1, \pm2$ [for interference maxima, but n cannot take infinitely large values, as that would violate the approximation (II)]

i.e., θ is small or $y \ll D \Rightarrow \dfrac{y}{D} = \dfrac{n\lambda}{d} \ll 1$

Hence, the above formula for interference maxima is applicable when $n \ll d/\lambda$.

When n becomes comparable to d/λ, path difference can no longer be given by dy/D. Hence, for maxima,

$$\Delta x = n\lambda \Rightarrow d\sin\theta = n\lambda \Rightarrow n = \frac{d\sin\theta}{\lambda}$$

Hence, highest order of interference maxima,

$$n_{max} = \left[\frac{d}{\lambda}\right] \qquad \text{(vii)}$$

where [ ] represents the greatest integer function.

Similarly, highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] \qquad \text{(viii)}$$

Illustration 2.15 **Red light (λ = 664 nm in vacuum) is used in Young's experiment with the slits separated by a distance from the screen given by D = 2.75 m. Find the distance y on the screen between the central bright fringe and the third-order bright fringe.**

Reasoning: **This problem can be solved by first using Eq. (i) to determine the value of θ that locates the third-order bright fringe. Then, trigonometry can be used to obtain the distance y.**

Sol. According to Eq. (i), we find

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{3(6.64 \times 10^{-9}\text{ m})}{1.20 \times 10^{-4}\text{ m}}\right] = 0.951°$$

According to Fig. 2.32, the distance y can be calculated from $\tan\theta = y/D$:

$$y = D\tan\theta = (2.75\text{ m})\tan 0.951° = 0.0456\text{ m}$$

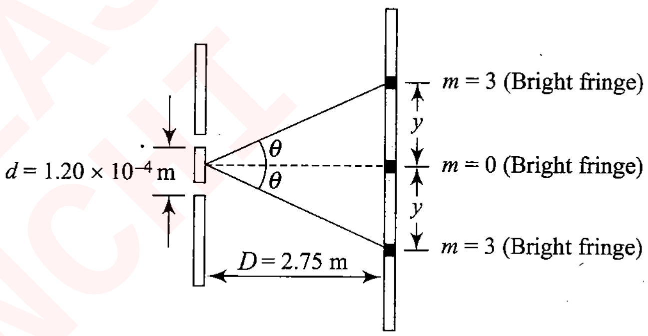


Fig. 2.32 **The third-order bright fringe (m = 3) is observed on the screen at a distance y from the central bright fringe (m = 0).**

Illustration 2.16 **In a YDSE, D = 1 m, d = 1 mm and λ = 1/2 mm.**

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the number of maxima and minima obtained on the screen.

Sol. (i) $D \gg d$

Hence, path difference at any angular position θ on the screen

$$\Delta x = d\sin\theta$$

The path difference for first maxima

$$\Delta x = d\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Hence, distance between central maxima and first maxima

$$y = D\tan\theta = \frac{1}{\sqrt{3}}\text{ m}$$

(ii) Maximum path difference, $\Delta x_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\dfrac{d}{\lambda}\right] = 2$ and highest order minima $n_{min} = \left[\dfrac{d}{\lambda} + \dfrac{1}{2}\right] = 2$

Total number of maxima $= 2n_{max} + 1 = 5$

Total number of minima $= 2n_{min} = 4$

Illustration 2.17 **Monochromatic light of wavelength 5000 Å is used in YDSE, with slit width, $d = 1$ mm, distance between screen and slits, $D = 1$ m. If intensites at the two slits are $I_1 = 4I_0$ and $I_2 = I_0$, find:**

(i) fringe width β;

(ii) distance of 5th minima from the central maxima on the screen;

(iii) intensity at $y = \dfrac{1}{3}$ mm

(iv) distance of the 1000th maxima and

(v) distance of the 5000th maxima.

Sol. (i) $\beta = \dfrac{\lambda D}{d} = \dfrac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n-1)\dfrac{\lambda D}{d}$, $n = 5 \quad \Rightarrow \quad y = 2.25$ mm

(iii) At $y = \dfrac{1}{3}$ mm, $y \ll D \Rightarrow \Delta x = \dfrac{dy}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now, resultant intensity,

$$I = I_1 + I_2 + 2\cos\Delta\phi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi$$

$$= 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(iv) $\dfrac{d}{\lambda} = \dfrac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

$n = 1000$ is not $\ll 2000$

Hence, now $\Delta x = d\sin\theta$ must be used.

$\therefore\ d\sin\theta = n\lambda = 1000\,\lambda \Rightarrow \sin\theta = 1000\dfrac{\lambda}{d} = \dfrac{1}{2} \Rightarrow \theta = 30°$

$y = D\tan\theta = \dfrac{1}{\sqrt{3}}$ m

(v) Highest order maxima, $n_{max} = \left[\dfrac{d}{\lambda}\right] = 2000$

Hence, $n = 5000$ is not possible.

Illustration 2.18 **Young's double slit experiment is performed inside water ($\mu = 4/3$) with light of frequency 6×10^{14} Hz. If the slits are separated by 0.2 mm and the screen kept 1 m from the slits, find the fringe width. Using the same set-up, what will the fringe width be if the experiment is performed in air?**

Sol. Wavelength of given light in air can be obtained from the relation

$$c = f\lambda_{air} \quad \Rightarrow \quad \lambda_{air} = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7} \text{ m}$$

Wavelength of light of same frequency (6×10^{14} Hz) in water will be different and

$$\lambda_{water} = \frac{\lambda_{air}}{\mu_{water}} = \frac{5 \times 10^{-7}}{4/3}$$

$$\Rightarrow \quad \lambda_{water} = 3.75 \times 10^{-7} \text{ m}$$

As the experiment is performed in water, so fringe width,

$$\beta = \frac{\lambda_{water} D}{d}$$

Here, $D = 1$ m and $d = 0.2$ mm $= 0.2 \times 10^{-3}$ m.

$$\therefore \quad \beta = \frac{3.75 \times 10^{-7} \times 1}{0.2 \times 10^{-3}} \text{ m}$$

or $\quad \beta = 1.87$ mm

If the experiment is performed in air, then fringe width,

$$\beta = \frac{\lambda_{air} D}{d} = \frac{5 \times 10^{-7} \times 1}{0.2 \times 10^{-3}} \text{ m}$$

or $\quad \beta = 2.5$ mm

Illustration 2.19 **In Young's experiment performed with light of wavelength 5500 Å, what should be the separation between the two slits so that the highest order of maximum intensity in the interference pattern is 2? You may assume $D \gg d$.**

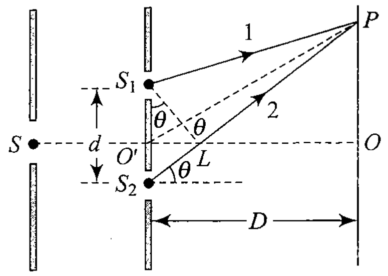


Fig. 2.33

Sol. $S_2L = d\sin\theta \quad$ or $\quad \Delta x = d\sin\theta$

For intensity at P to be maximum,

$$\Delta x = d\sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d}$$

Now,

$$\sin\theta \le 1 \Rightarrow \frac{n\lambda}{d} \le 1 \quad \text{or} \quad n \le \frac{d}{\lambda} \Rightarrow n_{max} = \frac{d}{\lambda}$$

In this problem, $n_{max} = 2$ and $\lambda = 5500$ Å $= 5500 \times 10^{-10}$ m.

$$\therefore \quad d = n_{max}\lambda = 2 \times 5500 \times 10^{-10}$$
$$= 1.1 \times 10^{-6} \text{ m}$$

Illustration 2.20 **In Young's experiment, light of wavelength 600 nm falls on the double slits separated by 0.1 mm. What is the highest order of maximum intensity in the interference pattern obtained on a screen kept 3 m from the slits? How does the highest order change if the distance of screen from the slits is changed?**

Sol. As we know, $n_{max} = \dfrac{d}{\lambda}$

Given, $d = 0.1$ mm $= 0.1 \times 10^{-3}$ m and $\lambda = 600$ nm $= 600 \times 10^{-9}$ m.

$$\therefore \quad n_{max} = \frac{0.1 \times 10^{-3}}{600 \times 10^{-9}} = 166.67 \quad \Rightarrow \quad n_{max} = 166$$

Obviously, $n_{max} = d/\lambda$ does not depend on the distance of screen from the slits.

Hence, it remains 166 if the distance between the slits and the screen is changed.

Illustration 2.21 **At a point on the screen directly in front of one of the slits, find the missing wavelengths.**

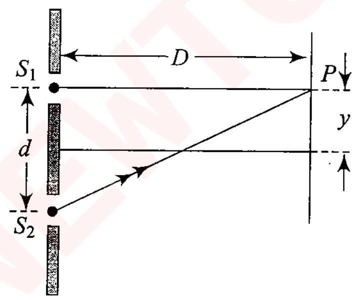


Fig. 2.34

Sol. According to theory of interference, position y of a point on the screen is given by

$$y = \frac{D}{d}(\Delta x)$$

As for missing wavelengths intensity will be minimum (= 0), so

$$\Delta x = (2n - 1)\frac{\lambda}{2}$$

$$\therefore \quad y = D\frac{(2n-1)\lambda}{2d}$$

However, here $d = b$ and $y = (b/2)$. Hence,

$$\lambda = \frac{b^2}{(2n-1)D} \quad \text{with} \quad n = 1, 2, 3, \ldots$$

i.e., wavelengths (b^2/D), $(b^2/3D)$, $(b^2/5D)$, etc., will be absent (or missing) at point P.

Illustration 2.22 **In Young's experiment, the slits, separated by $d = 0.8$ mm, are illuminated with light of wavelength 7200 Å. Interference pattern is obtained on a screen $D = 2$ m from the slits. Find the minimum distance from central maximum at which the average intensity is 50% of the maximum?**

Sol. Resultant intensity at a point due to superposition of coherent waves of intensities I_1 and I_2 and phase difference ϕ can be expressed as $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$. Here,

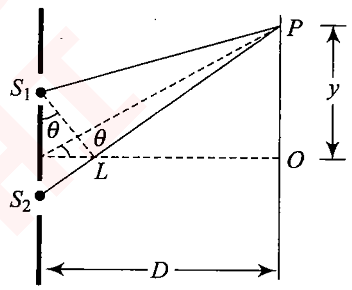


Fig. 2.35

$$I_1 = I_2 = I$$
$$I_R = 2I + 2I\cos\phi = 2I(1 + \cos\phi)$$
$$I_R = 4I\cos 2(\phi/2)$$

At the position of interference maxima,

$$\phi = 2n\pi$$
$$\cos^2(\phi/2) = \cos^2 n\pi = 1 \qquad [n \text{ is integer}]$$
$$I_{max} = 4I \quad \Rightarrow \quad I_R = I_{max}\cos^2(\phi/2)$$

Here, $\quad I_R = \dfrac{I_{max}}{2}$ [50% of maxima]

$$\frac{I_{max}}{2} = I_{max}\cos^2\frac{\phi}{2} \quad \Rightarrow \quad \cos^2\frac{\phi}{2} = \frac{1}{2}$$

$$\cos\frac{\phi}{2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4} \quad \text{or} \quad \phi = \pi/2$$

Therefore, phase difference at a point at which resultant intensity is 50% of maximum is $\pi/2$.

Now, phase difference, $\phi = (2\pi/\lambda)$ (path difference)

$$\therefore \quad \text{Path difference} = \frac{\lambda}{2\pi} \times \frac{\pi}{2} = \frac{\lambda}{4} \qquad \text{(i)}$$

$$\therefore \quad \text{Path difference} = d\frac{y}{D}$$

So, to obtain the desired distance,

$$\frac{dy}{D} = \frac{\lambda}{4} \Rightarrow y = \frac{\lambda D}{4d}$$

Given $\lambda = 7200$ Å $= 7200 \approx 10^{-10}$ m and $D = 2md = 0.8$ mm.

$$\therefore \qquad y = \frac{7200 \times 10^{-10} \times 2}{4 \times 0.8 \times 10^{-3}} \Rightarrow y = 4.5 \Rightarrow 10^{-5} \text{ m}$$

$$\Rightarrow \qquad y = 0.45 \text{ mm}$$

Shape of Interference Fringes in YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$$S_2P - S_1P = \Delta x = \text{constant} \qquad \text{(i)}$$

If $\Delta x = \pm\lambda/2$, the fringe represents 1st minima.

If $\Delta x = \pm 3\lambda/2$, it represents 2nd minima.

If $\Delta x = 0$, it represents central maxima.

If $\Delta x = \pm\lambda$, it represents 1st maxima, etc.

Equation (i) represents a hyperbola with its two foci at S_1 and S_2 (see Fig. 2.36).

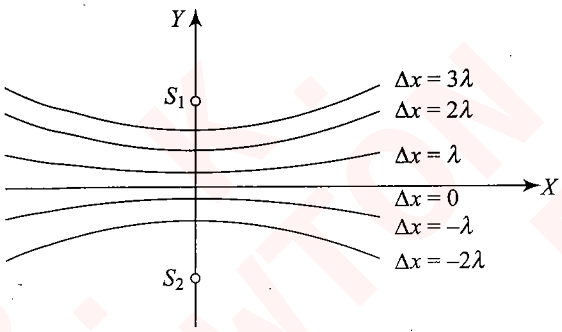


Fig. 2.36

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

If the screen is perpendicular to the X-axis, i.e., in the YZ plane, as is generally the case, firinges are hyperbolic with a straight central section [see Fig. 2.37(a)].

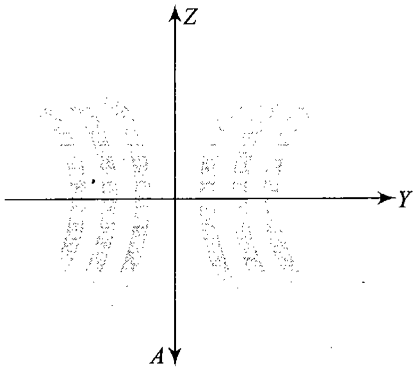


Fig. 2.37(a)

If the screen is in the XY plane, again fringes are hyperbolic [see Fig. 2.37(b)].

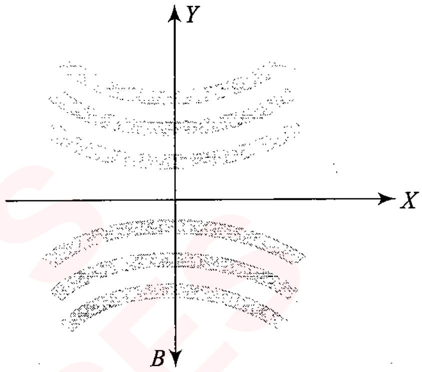


Fig. 2.37(b)

YOUNG'S DOUBLE-SLIT EXPERIMENT WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However, slightly below (or above) the position of central maxima fringes will be coloured. For example, if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{\text{violet}}}{2} = 190 \text{ nm}$$

then completely destructive interference will occur for violet light. Hence, we will have a line devoid of violet color that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{\text{red}}}{2} = 350 \text{ nm}$$

then completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points, there are so many wavelengths which interfere constructively that we obtain a uniform white illumination. For example, if $S_2P - S_1P = 3000$ nm, then constructive interference will occur for wavelengths $\lambda = 3000/n$ nm. In the visible region, these wavelengths are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 430 nm (violet). Clearly, such a light will appear white to the unaided eye.

Thus, with white light we get a white central fringe at the point of zero path difference, followed by a few colored fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Illustration 2.23 **Fig. 2.38 shows a photograph that illustrates the kind of interference fringes that can result when white light, which is a mixture of all colors, is used in Young's experiment. Except for the central fringe, which is white, the bright fringes are a rainbow of colours. Why does**

Young's experiment separate white light into the constituent colours? In any group of coloured fringes, such as the two singled out in the figure, why is red farther out from the central fringe than green is? And finally, why is the central fringe white rather than coloured?

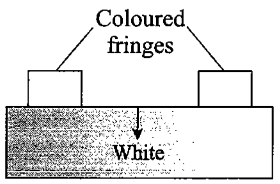


Fig. 2.38 **Results observed on the screen in one version of Young's experiment in which white light (a mixture of all colours) is used.**

Sol. To understand how the colour separation arises, we need to remember that each colour corresponds to a different wavelength λ and that constructive and destructive interference depend on the wavelength. According to equation $\sin\theta = m\lambda/d$, there is a different angle that locates a bright fringe for each value of λ or for each colour. These different angles lead to the separation of colours on the observation screen. In fact, on either side of the central fringe, there is one group of colored fringes for $m = 1$ and another for each value of m.

Now, consider what it means that, within any single group of coloured fringes, red is farther out from the central fringe than green is. It means that, in the equation $\sin\theta = m\lambda/d$, red light has an angle θ greater than green light does. Does this make sense? Yes, because red has the larger wavelength ($\lambda_{red} = 660$ nm and $\lambda_{green} = 550$ nm).

In Fig. 2.38, the central fringe is distinguished from all the other colored fringes by being white. In Eq. (i), the central fringe is different from the other fringes because it is the only one for which $m = 0$. In Eq. (i), a value of $m = 0$ means that $\sin\theta = m\lambda/d = 0$, which reveals that $\theta = 0^\circ$, no matter what the wavelength λ is. In other words, all wavelengths have a zeroth-order bright fringe located at the same place on the screen, so that all colours strike the screen there and mix together to produce the white central fringe.

Illustration 2.24 **White coherent light (4000–7000 Å) is sent through the slits of a YDSE. The separation between the slits is 0.5 mm and screen is 50 cm away from the slits. There is a hole in the screen at a point 1.0 mm away (along the width of the fringe) from the central line.**

a. Which wavelength(s) will be absent in the light coming from the hole?

b. Which wavelength(s) will have a strong intensity?

Sol. a. Order of minima corresponding to 4000 Å,

$$y_n = \frac{(2n-1)D\lambda}{2d} \Rightarrow (2n-1) = \frac{y_n 2d}{D\lambda} \Rightarrow n = \frac{1}{2}\left[\frac{2d\,y_n}{D\lambda} + 1\right]$$

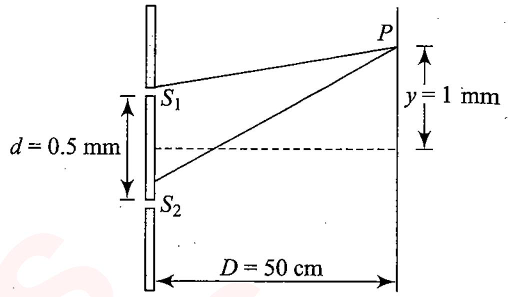


Fig. 2.39

$$n_1 = \frac{1}{2}\left[\frac{0.5\times10^{-3}\times1\times10^{-3}\times2}{50\times10^{-2}\times4000\times10^{-10}} + 1\right] = \frac{1}{2}\left[\frac{2\times10^4}{4000} + 1\right] = 3$$

Order of minima corresponding to 7000 Å,

$$n_2 = \frac{1}{2}\left[\frac{2\times10^4}{7000} + 1\right] = 1.9$$

Number of integers between 1.9 and 3.0 are 2 and 3.

Wavelength corresponding to $n = 2$ is

$$\lambda_{(2)} = \frac{y_n 2d}{(2n-1)D} = \frac{1\times10^{-3}\times2\times0.5\times10^{-3}}{(2\times2-1)\times50\times10^{-2}}$$

$$= \frac{10^{-4}}{(2\times2-1)\times50} = \frac{2\times10^{-6}}{3} = 0.6667\times10^{-6}\ \text{m}$$

Now, wavelength corresponding to $n = 3$ is

$$\lambda_{(3)} = \frac{2\times10^{-6}}{(2\times3-1)} = 400\ \text{nm}$$

Order of maxima corresponding to λ,

$$y_n = \frac{nD\lambda}{d} \Rightarrow n = \frac{y_n d}{D\lambda} = \frac{1\times10^{-3}\times0.5\times10^{-3}}{50\times10^{-2}\times\lambda}$$

$$n_{4000} = \frac{10^{-6}}{4000\times10^{-10}} = \frac{10000}{4000} = 2.5 \text{ and } n_{7000} = \frac{10000}{7000} = 1.4$$

Integer between 1.4 and 2.5 is 2.

$$\therefore \quad \lambda_{(2)} = \frac{y_n d}{nD} = \frac{10^{-6}}{2} = 500\ \text{nm}$$

DIFFERENT CASES IN YOUNG'S DOUBLE-SLIT EXPERIMENT

Rays Not Parallel to Principal Axis

Here, rays reaching at S_1 and S_2 have initial path difference. Path difference between the rays reaching at point P,

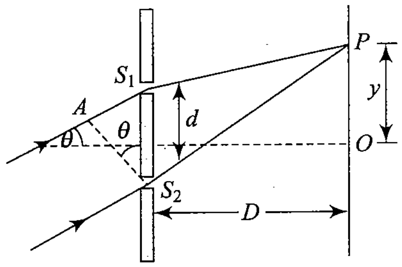


Fig. 2.40

$$\Delta x = (AS_1 + S_1P) = S_2P = AS_1 - (S_2P - S_1P)$$

$$\Rightarrow \quad \Delta x = d \sin\theta - \frac{yd}{D}$$

Illustration 2.25 **A coherent parallel beam of microwaves of wavelength λ = 0.5 mm falls on a Young's double-slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in Fig. 2.41.**

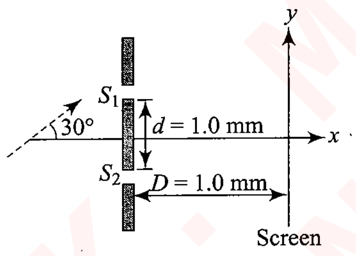


Fig. 2.41

If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in the figure), find the y-coordinates of the first minima on either side of the central maximum. **(IIT-JEE,1998)**

Sol. Path difference before slits = $d \sin\phi$

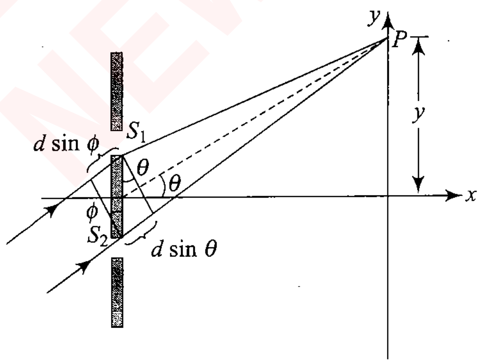


Fig. 2.42

Path difference after slits = $d \sin\theta$

As path of rays before slits is longer at S_1 and $S_2P > S_1P$ after slits, so net path difference for first minima is

$$\Delta x = d\sin\theta - d\sin\phi = \pm\frac{\lambda}{2}$$

$$\Rightarrow \quad \sin\theta = \sin\phi \pm \frac{\lambda}{2d}$$

$$\sin\theta = \sin 30^\circ \pm \frac{0.5}{2\times 1} = \frac{3}{4} \quad \text{or} \quad \frac{1}{4}$$

$$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{7}} \quad \text{and} \quad \frac{1}{\sqrt{15}}$$

So, the position of first minima on either side of the central maxima is

$$y = D\tan\theta = \frac{3}{\sqrt{7}} \quad \text{and} \quad \frac{1}{\sqrt{15}}\ \text{m}$$

Source Placed Beyond the Central Line

Here, the source is placed a little beyond the center line. In this case, the waves interaction with S_1 and S_2 has initial phase difference as shown in Fig. 2.43.

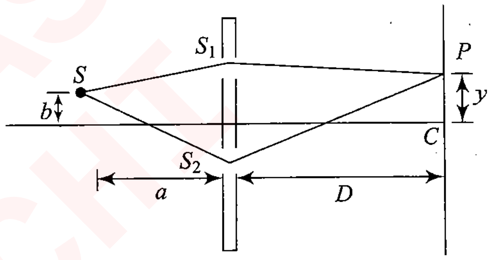


Fig. 2.43

Path difference at point P on the screen,

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P) = (-SS_1 + SS_2) + (S_2P - S_1P)$$

$$\Delta x = \frac{bd}{a} + \frac{yd}{D} \qquad \text{(i)}$$

For maxima: $\quad \Delta x = n\lambda$

For minima: $\quad \Delta x = (2n-1)\dfrac{\lambda}{2}$

If we know the value of Δx from (i), we can calculate different positions of maxima and minima.

Illustration 2.26 **In the Young's double slit experiment, the point source is placed slightly off the central axis as shown in Fig. 2.44.**

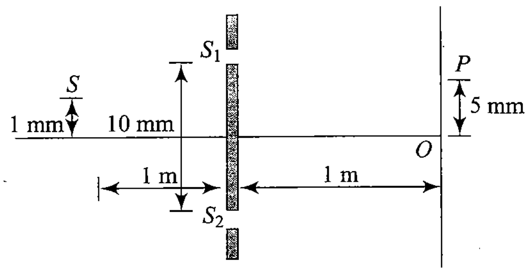


Fig. 2.44

a. Find the nature and order of the interference at point P.
b. Find the nature and order of the interference at point O.

Sol.

a. As discussed in previous case, here $b = 1$ mm, $d = 10$ mm, $a = 1$ m, $D = 1$ m.

Using Eq. (i) for path difference between the waves reaching at point P,

$$\Delta x = \frac{1.10}{10^3} + \frac{5\times 10}{2\times 10^3} = 35\times 10^{-3}\text{ mm}$$

If it is maximum at point P, then $\Delta x/\lambda$ will be an integer, i.e.,

$$\frac{35\times 10^{-3}}{500\times 10^{-9}\times 10^3} = 70$$

A maximum of the order 70 occurs at P.

b. At O, $\Delta x_0' = \frac{1\times 10}{10^3} = 10\times 10^{-3}$ mm

$$\Rightarrow \quad \frac{\Delta x}{\lambda} = 20$$

Hence, maximum of the order 20 occurs at O.

GEOMETRICAL AND OPTICAL PATHS

Optical Path

Consider a light wave travelling in a medium of refractive index μ. Its equation may be written as

$$E = E_0 \sin \omega (t - x/v) = E_0 \sin \omega(t - \mu x/c) = E_0 \sin(\omega t - \delta)$$

If the light wave travels a distance Δx in vaccum, the phase changes by

$$\delta_1 = \mu \frac{\omega}{c}\Delta x \qquad \text{(i)}$$

Instead, if the light wave travels in vaccum, its equation will be

$$E = E_0 \sin \omega(t - x/c)$$

If the light travels through a distance $\mu\Delta x$, the phase changes by

$$\delta_2 = \frac{\omega}{c}(\mu\Delta x) = \mu\frac{\omega}{c}\Delta x \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we see that a wave travelling through a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. In other words, a path length of Δx in a medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum. The quantity $\mu\Delta x$ is called the optical path of the light. In dealing with interference of two waves, we need the difference between the optical paths travelled by the waves. The geometrical path and the optical path are equal only when light travels in vacuum or in air where the refractive index is close to 1.

The concept of optical path may also be introduced in terms of the change in wavelength as the wave changes its medium. The frequency of a wave is determined by the frequency of the source and is not changed when the wave enters in a new medium. If the wavelength of light in vacuum is λ_0 and that in the medium is λ_n, then

$$\lambda_0 = \frac{c}{\nu}, \ \nu = \text{frequency of the wave}$$

and

$$\lambda_n = \frac{u}{\nu} = \frac{c/\mu}{\nu} \quad \text{so that} \quad \lambda_n = \frac{\lambda_0}{\mu}$$

At any given instant, the points differentiated by one wavelength have same phase of vibration. Thus, the points at separation λ_n in the medium have same phase of vibration. On the other hand, in vacuum, points at separation λ_0 will have same phase of vibration. Thus, a path λ_n in a medium is equivalent to a path $\lambda_0 = \mu\lambda_n$ in vacuum. In general, a path Δx, in a medium of refractive index μ, is equivalent to a path $\mu\Delta x$ in vaccum which is called the optical path.

We can also understand the idea of optical path with the help of Fig. 2.45. Suppose a parallel beam of light traveling in vacuum is incident on the surface AC of a medium of refractive index μ. AB is perpendicular to the incident rays and hence represents a wavefront of the incident light. Similarly, CD is perpendicular to the refracted rays and represents a wavefront of the refracted light. Now, phase of the wave has a constant value at different points of a wavefront. Thus, phase at A = phase at B; and phase at C = phase at D.

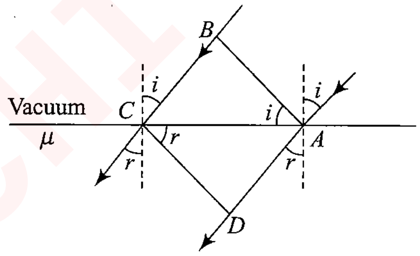


Fig. 2.45

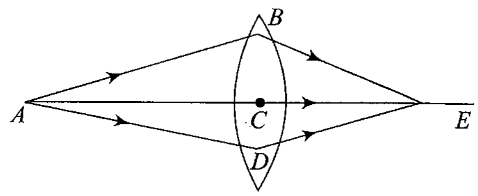
Thus, the phase difference between A and D = phase difference between B and C. From Fig. 245,

$$\mu = \frac{\sin i}{\sin r} = \frac{BC}{AC}\times\frac{AC}{AD} = \frac{BC}{AD}$$

or $\qquad BC = \mu(AD)$

The phase of light wave changes by equal amount whether it covers a distance $BC = \mu(AD)$ in vacuum or AD in the medium. Thus, a path AD in a medium of refractive index μ is equivalent to a path $\mu(AD)$ in vacuum which we call optical path.

Note:



Consider the situation in the figure. The geometrical paths ABE, ACE, and ADE are different, but the optical paths are equal. This is because each path leads to the same phase difference. Note that the ray having longer geometrical path covers less distance in the lens as compared to the ray having shorter geometrical path.

Illustration 2.27 The wavelength of light coming from a sodium source is 589 nm. What will be its wavelength in water? Refractive index of water = 1.33.

Sol. The wavelength in water is $\lambda = \lambda_0/\mu$, where λ_0 is the wavelength in vacuum and μ is the refractive index of water. Thus,

$$\lambda = \frac{589}{1.33} = 443 \text{ nm}$$

Displacement of Fringes

When we introduce a thin transparent plate in front of one of the slits in YDSE, the fringe pattern shifts toward the side where the plate is present.

The path length S_1O' is

$$(S_1O')_{new} = (S_1O' - t)_{air} + t_{plate}$$

$$\Rightarrow \quad (S_1O')_{new} = (S_1O' - t)_{air} + (\mu t)_{air}$$

$$\Rightarrow \quad (S_1O')_{new} = (S_1O') - (t)_{air} + (\mu t)_{air}$$

$$\Rightarrow \quad (S_1O')_{new} = (S_1O') + [\mu t - t]$$

$$\Rightarrow \quad (S_1O')_{new} = (S_1O') + [t(\mu - 1)$$

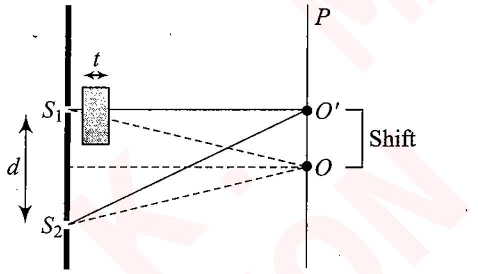


Fig. 2.46

$\Rightarrow$ Path difference, $S_2O' - S_1O'_{new} = \Delta x$

$$S_2O' - (S_1O')_{new} = S_2O' - \{(S_1O') + [t\mu - 1]\}$$

$$\Rightarrow \quad S_2O' - (S_1O')_{new}$$

$$= d\sin\theta - [t(\mu - 1)] = \frac{yd}{D} - [t(\mu - 1)]$$

$$a\,\Delta x = d\sin\theta - [t(\mu - 1] = \frac{yd}{D} - [t(\mu - 1)]$$

$$\Rightarrow \quad y = \frac{\Delta x D}{d} + \frac{D}{d}[t(\mu - 1)]$$

If a transparent sheet of thickness t and refractive index μ is introduced in one of the paths of the interfering waves, then the path increases by $(\mu - 1)t$ and the whole fringe pattern shifts by

$$s = \frac{D}{d}(\mu - 1)t$$

Note: ***This fringe shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upward and if the glass slab is placed before the lower slit the fringe pattern gets shifted downward.***

Illustration 2.28 A Young's double slit apparatus is immersed in a liquid of refractive index μ_1. The slit plane touches the liquid surface. A parallel beam of monochromatic light of wavelength λ (in air) is incident normally on the slits.

a. Find the fringe width.

b. If one of the slits (say S_2) is covered by a transparent slab of refractive index μ_2 and thickness t as shown, find the new position of central maxima.

c. Now, the other slit S_1 is also covered by a slab of same thickness and refractive index μ_3 as shown in figure due to which the central maxima recovers its position find the value of μ_3.

d. Find the ratio of intensities at O in the three conditions (a), (b) and (c).

Sol. a. Fringe width, $w = \dfrac{\lambda_{\mu 1} D}{d} = \dfrac{\lambda D}{\mu_1 d}$

b. Position of central maximum is shifted upward by a distance $\dfrac{(\mu_2 - 1)tD}{d}$.

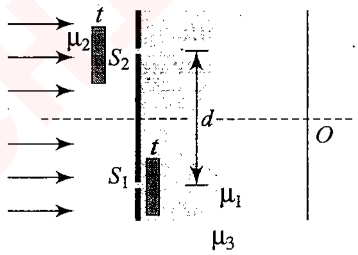


Fig. 2.47

c. $$\frac{(\mu_2 - 1)tD}{d} = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d} \Rightarrow \frac{\mu_3}{\mu_1} = \mu_2 \text{ or } \mu_3 = \mu_1\mu_2$$

d. $I = I_{max}\cos^2\left(\dfrac{\phi}{2}\right)$, where $\phi = \left(\dfrac{2\pi}{\lambda}\right)\Delta x$ or $\dfrac{\phi}{2} = \left(\dfrac{\pi}{\lambda}\right)\Delta x$

$$\Rightarrow \quad I \propto \cos^2\left(\frac{\phi}{2}\right)$$

In the first and third case, $\Delta x = 0$ while in the second case, $\Delta x = (\mu_2 - 1)t$. Therefore, the desired ratio is

$$I_1 : I_2 : I_3 = 1 : \cos^2\left\{\frac{\pi(\mu_2 - 1)t}{\lambda}\right\} : 1$$

Illustration 2.29 A thin paper of thickness 0.02 mm having a refractive index 1.45 is pasted across one of the slits in a YDSE. The paper transmits 4/9 of the light energy falling on it.

a. Find the ratio of maximum intensity to the minimum intensity in interference pattern.

b. How many fringes will cross through the centre if an identical paper piece is pasted on the other slit also? The wavelength of the light used is 600 nm.

Sol. a. $\dfrac{I_{max}}{I_{min}} = \dfrac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \dfrac{(1+2/3)^2}{(1-2/3)^2} = \dfrac{5^2}{1} = 25$

b. Fringe width, $\beta = \dfrac{D\lambda}{d}$

Shifting of fringe pattern due to paper sheet $s = \dfrac{t(\mu-1)D}{d}$

If we paste paper sheet on the other slit, the fringe pattern will shift by same amount on that side.

Hence, number of fringes crossing the centre is

$$n = \frac{s}{\beta} = \frac{t(\mu-1)}{\lambda} = \frac{0.02(1.45-1)}{600\times10^{-9}}\times10^{-3}$$

$$= \frac{0.02\times0.45\times10^6}{600} = \frac{90}{6} = 15$$

Illustration 2.30 **A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and the screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.**

a. Calculate the fringe width.

b. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minima on the axis.

Sol. As fringe width is given by $\beta = D\lambda/d$ and by presence of medium the wavelength becomes λ/μ, so the fringe width in liquid will be

$$\beta' = \frac{D\lambda}{\mu_M d} = \frac{1.33\times6300\times10^{-10}}{1.33\times1\times10^{-3}} = 0.63 \text{ mm}$$

Now, as the distance of a minima from adjacent maxima is $\beta'/2$, so according to given problem, shift

$$y_0 = \frac{D}{d}(\mu-1)t = \frac{\beta'}{2}$$

$$\Rightarrow \quad \frac{D(\mu-1)t}{d} = \frac{1}{2}\left(\frac{D\lambda}{\mu_M d}\right)$$

$$\Rightarrow \quad t = \frac{\lambda}{2\mu_M(\mu-1)} = \frac{6300}{2\times1.33\times(1.53-1)}$$

$$\Rightarrow \quad t = 4468.7 \text{ Å} = 446.87 \text{ μm}$$

Illustration 2.31 **A monochromatic light of $\lambda = 500$ Å is incident on two identical slits separated by a distance of 5×10^{-4}m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness 1.5×10^{-6} m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the screen.**

Sol. In case of interference,

$$I_R = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\phi$$

Now, as for identical slits $I_1 = I_2 = I$, so

$$I_R = 2(1+\cos\phi) = 4I\cos^2(\phi/2)$$

But for central maxima,

$$\phi = 0°, \text{ and here } I_R = I_0 \text{ (given)}$$

$$I_0 = 4I\cos(0°) = 4I$$

Hence,

$$I_R = I_0\cos^2\left(\frac{\phi}{2}\right) \quad \text{(i)}$$

Now, when the glass plate is introduced, path difference between the waves at the position of central maxima will become

$$\Delta x = (\mu-1)t = (1.5-1)1.5\times10^{-6} = 7.5\times10^{-7} \text{ m} \quad \text{(ii)}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{5\times10^{-7}}\times7.5\times10^{-7} = 3\pi$$

So, intensity at central maxima will now be

$$I_R = I_0\cos^2\left(\frac{3\pi}{2}\right) = I_0\times0 = 0$$

Also, from theory of interference, fringe shift

$$y_0 = \frac{D}{d}(\mu-1)t$$

which in the light of Eq. (ii) and given data becomes

$$y_0 = \frac{1}{5\times10^{-4}}\times7.5\times10^{-7} = 1.5\times10^{-3}\text{ m} = 1.5 \text{ mm}$$

Illustration 2.32 **In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å.**

It is found that the point P on the screen where the central maximum ($n = 0$) fell before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the fourth maximum earlier, lies below the point P while the fifth minimum lies above P.

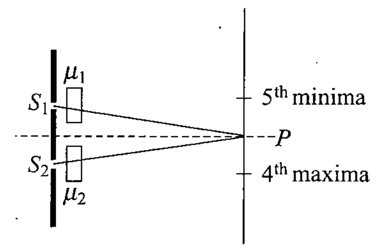


Fig. 2.48

Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected.) (IIT-JEE, 1999)

Sol. At point P (central maxima), Originally it has maximum intensity. If the intensity coming out of each slit is I_0, the intensity at point P should be $4I_0$.

$$I = 2I_0(1 + \cos\phi) \Rightarrow \frac{3}{4}(4I_0) = 2I_0(1 + \cos\phi)$$

$$\cos\phi = \frac{1}{2} \Rightarrow \phi = 2n\pi \pm \frac{\pi}{3} \text{ (} n \text{ is integer)}$$

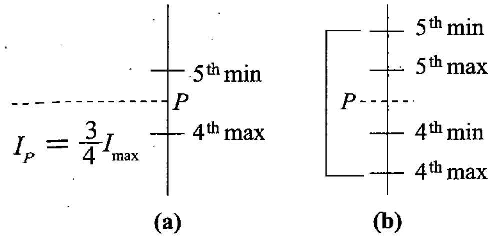


Fig. 2.49

Path difference at P,

$$4\lambda \le \Delta x \le \left(\frac{2\times 5-1}{2}\right)\lambda$$

$$\Rightarrow \quad 4\lambda \le \Delta x \le \frac{9}{2}\lambda \Rightarrow 4\lambda \le \Delta x \le 4.5\lambda \qquad \text{(i)}$$

From Eq. (i),

$$\theta = \frac{2\pi}{\lambda}\Delta x \Rightarrow \Delta x = n\lambda \pm \frac{\lambda}{6} \qquad \text{(ii)}$$

$$\Delta x = \lambda\left(4 \pm \frac{1}{6}\right) \qquad \text{(iii)}$$

From Eqs. (ii) and (iii), $n = 4$. Hence,

$$\Delta x = \lambda\left(4+\frac{1}{6}\right) = \frac{25}{6}\lambda \Rightarrow \Delta x = t(\mu - 1)$$

$$\Rightarrow \quad t = \frac{25\lambda}{6(\mu_2-\mu_1)} = \frac{25\times 5400\times 10}{6\times 0.3\times 10^{-1}} = 7500\times 10^{-9} \text{ m}$$

$$\Rightarrow \quad t = 7.5 \text{ µm}$$

Illustration 2.33 **A screen is at a distance $D = 80$ cm from a diaphragm having two narrow slits S_1 and S_2 which are $d = 2$ mm apart.**

Slit S_1 is covered by a transparent sheet of thickness $t_1 = 2.5$ µm and slit S_2 is covered by another sheet of thickness $t_2 = 1.25$ µm as shown in Fig. 2.50.

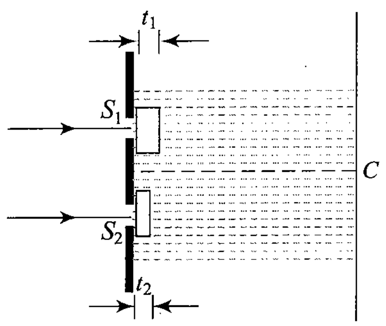


Fig. 2.50

Both sheets are made of same material having refractive index µ = 1.40.

Water is filled in the space between diaphragm and screen. A monodichromatic light beam of wavelength $\lambda = 5000$ Å is incident normally on the diaphragm.

Assuming intensity of beam to be uniform, calculate ratio of intensity of C to maximum intensity of interference pattern obtained on the screen ($\mu_w = 4/3$).

Sol. Path difference at C,

$$\Delta x = t_1(\mu - 1) - t_2(\mu - 1) = \mu(t_1 - t_2) - (t_1 - t_2)$$

$$= (t_1 - t_2)(\mu - 1)$$

$$= (2.5 - 1.25)\left(\frac{1.4\times 3}{4\times 10} - 1\right) = 1.25\times\frac{2}{40} = \frac{2.5}{400}$$

$$\Rightarrow \quad \Delta x = \frac{1}{16} \text{ µm}$$

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi\times 4}{5000\times 3\times 10^{-10}}\times\frac{1}{16}\times 10^{-6} = \frac{\pi}{3}$$

$$\Rightarrow \quad I_{max} = 4I_0$$

Intensity at C,

$$I_C = 2I_0\left(1+\cos\frac{\pi}{3}\right) = 3I_0$$

$$\therefore \text{ Required ratio} = \frac{I_C}{I_{max}} = \frac{3}{4}$$

Concept Application Exercise 2.2

1. In a Young's double slit experiment with wavelength 5890 Å, there are 60 fringes in the field of vision. How many fringes will be observed in the same field of vision if wavelength used is 5460 Å?
2. In a YDSE performed with wavelength $\lambda = 5890$ Å, the angular fringe width is 0.40°. What is the angular fringe width if the entire set-up is immersed in water?
3. In a YDSE for wavelength $\lambda = 589$ nm, the interference fringes have angular separation of 3.50×10^{-3} rad. For what wavelength would the angular separation be 10.0% greater?
4. A beam of light consisting of wavelengths 6000 Å and 4500 Å is used in a YDSE with $D = 1$ m and $d = 1$ mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.
5. White light is used in a YDSE with $D = 1$ m and $d = 0.9$ mm. Light reaching the screen at position $y = 1$ mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.
6. Monochromatic light of wavelength of 600 nm is used in a YDSE. One of the slits is covered by a transparent sheet of thickness 1.8×10^{-5} m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet?
7. Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of

refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, find the thickness of the plate.

8. In a YDSE with $d = 1$ mm and $D = 1$ m, slabs of ($t = 1$ μm, $\mu = 3$) and ($t = 0.5$ μm, $\mu = 2$) are introduced in front of upper and lower slits, respectively. Find the shift in the fringe pattern.

9. In YDSE with $D = 1$ m, $d = 1$ mm, light of wavelength 500 nm is incident at an angle of 0.57° w.r.t. the axis of symmetry of the experimental set-up. If centre of symmetry of screen is O as shown (Fig. 2.51),
 (i) find the position of central maxima,
 (ii) find intensity at point O in terms of intensity of central maxima I_0 and
 (iii) find number of maxima lying between O and the central maxima.

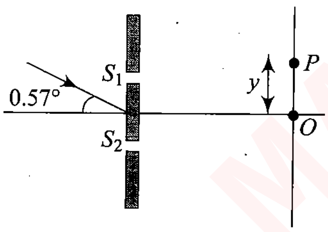


Fig. 2.51

10. In Young's experiment, find the distance between two slits that results in the third minimum for 420 nm violet light at an angle 30°.

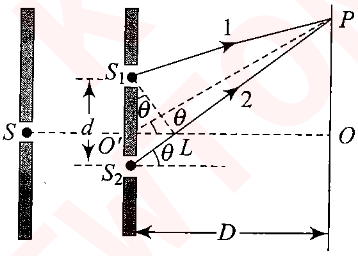


Fig. 2.52

11. In an interference arrangement similar to double-slit experiment, S_1 and S_2 are illuminated with coherent source microwave source each of frequency 1 MHz. The sources are synchronized to have zero phase difference. The slits are separated by distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown in Fig. 2.53. If I_0 is maximum intensity, calculate $I(\theta)$ for:

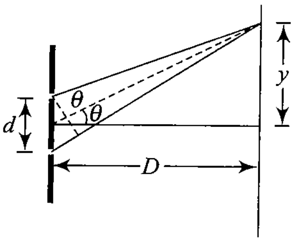


Fig. 2.53

a. $\theta = 0$ **b.** $\theta = 30°$ **c.** $\theta = 90°$

12. If Young's double-slit experiment is performed under water, how would the observed interference pattern be affected?

13. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?

14. A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

15. Two radio antennas separated by 300 m as shown in Fig. 2.54 simultaneously broadcast identical signals at the same wavelength. A radio in a car traveling due north receives the signals. (a) If the car is at the position of the second maximum, what is the wavelength of the signals? (b) How much farther must the car travel to encounter the next minimum in reception? (Note: Do not use the small-angle approximation in this problem.)

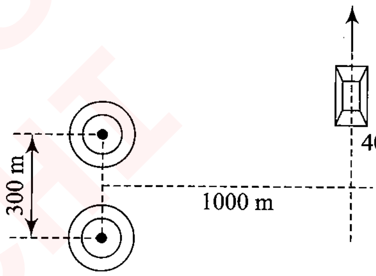


Fig. 2.54

16. Young's double-slit experiment is performed with a 600 nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.5 mm from the central maximum. Determine the spacing of the slits.

17. Two slits are separated by 0.320 mm. A beam of 500 nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range $-30.0° < \theta < 30.0°$.

18. The intensity on the screen at a certain point in a double-slit interference pattern is 25.0% of the maximum value. (a) What minimum phase difference (in radians) between the sources produces this result? (b) Express this phase difference as a path difference for 486.1 nm light.

19. Distance between the slits shown in Fig. 2.55 is $d = 20\lambda$, where λ is the wavelength of light used. Find the angle θ where

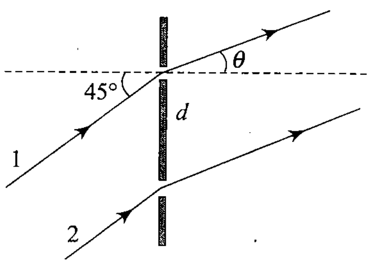


Fig. 2.55

a. central maxima (where path difference is zero) is obtained, and
b. third-order maxima is obtained.

20. The intensity of the light coming from one of the slits in a Young's double slit experiment is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

21. The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width, find the ratio of the maximum to minimum intensity in the interference pattern.

22. Suppose that one of the slits in a Young's experiment is wider than the other, so the amplitude of the light reaching the central part of the screen from one slit, acting alone, is twice that from the other slit, acting alone. Obtain an expression for the light intensity I at the screen as a function of θ, the angular position.

FRESNEL'S BIPRISM

Figure 2.56 shows a schematic diagram of Fresnel's biprism and interference of light using it. Two thin prisms A_1BC and A_2BC are joined at the base to form a biprism. The refracting angles A_1 and A_2 (denoted by α in the figure) are of the order of half a degree each. In fact, it is a simple prism whose base angles are extremely small. A narrow slit S, allowing monochromatic light, is placed parallel to the refracting edge C, The light going through the prisms A_1BC and A_2BC. A screen Σ is placed to intercept the transmitted light. Interference fringes are formed on the portion QR of the screen where the two cones overlap.

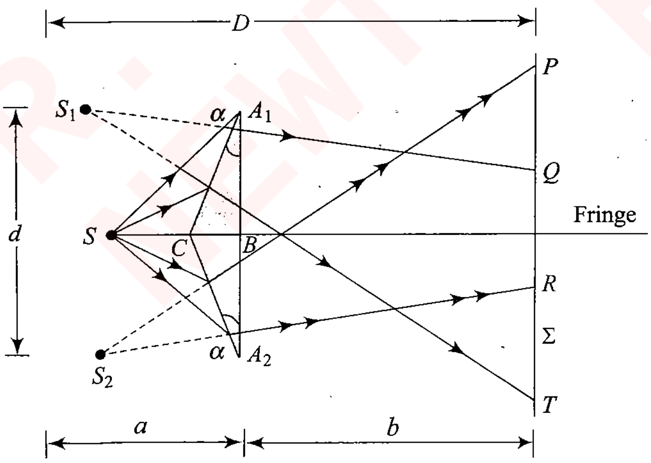


Fig. 2.56

One can treat the points S_1 and S_2 as two coherent sources sending light to the screen. The arrangement is then equivalent to a Young's double-slit experiment with S_1 and S_2 acting as the two slits. Suppose the separation between S_1 and S_2 is d and the separation between the planes of S_1S_2 and Σ is D. Then, the fringe width obtained on the screen is

$$\omega = \frac{D\lambda}{d}$$

Illustration 2.34 **Two small angled transparent prisms (each of refracting angle $A = 1°$) are so placed that their bases coincide, so that common base is BC. This device is called Fresnel's biprism and is used to obtain coherent sources of a point source S illuminated by monochromatic light of wavelength 6000 Å placed at a distance a = 20 cm. Calculate the separation between coherent sources. If a screen is placed at a distance b = 80 cm from the device, what is the fringe width of fringes obtained (Refractive index of material of each prism = 1.5).**

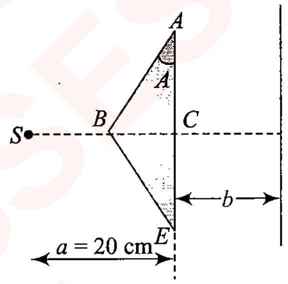


Fig. 2.57

Sol. The virtual images of slit S are formed at S_1 and S_2 by means of prism ABC and BCE, respectively. These images act as coherent sources. The deviation produced by small-angled prism is

$$\delta = (\mu - 1)A$$

Also $\tan\delta = \delta \dfrac{SS_1}{AS_1} = \dfrac{d/2}{a}$

Therefore, separation between coherent sources is

$$d = 2a\delta = 2a(\mu - 1)A$$

Here, a = 20 cm = 0.20 m

$$A = 1° = \frac{\pi}{180} = \frac{3.14}{180} \text{ rad}$$

$$\therefore \quad d = 2 \times 0.20 \times (1.5 - 1)\frac{3.14}{180} \text{ m} = 3.84 \times 10^{-3} \text{ m}$$

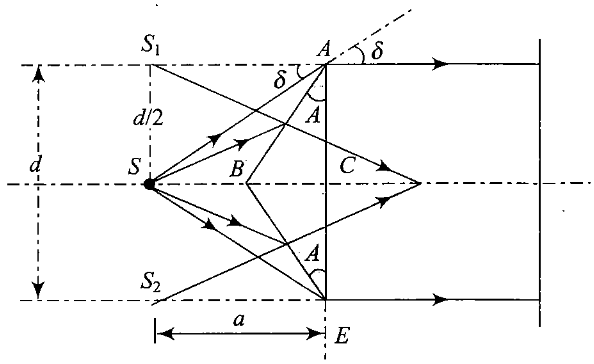


Fig. 2.58

Separation between coherent sources and screen is

$D = a + b = 20 + 80 = 100$ cm = 1 m

$\lambda = 6000$ Å $= 6000 \times 10^{-10}$ cm

Therefore, fringe width $\beta = \dfrac{D\lambda}{d} = \dfrac{1 \times 6000 \times 10^{-10}}{3.48 \times 10^{-3}}$

$= 1.724 \times 10^{-4}\text{ m} = 0.1724\text{ mm}$

LLOYD'S MIRROR EXPERIMENT

Change of Phase Due to Reflection

Young's method of producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, although ingenious, arrangement for producing an interference pattern with a single light source is known as Lloyed's mirror. A light source is placed at point S, close to a mirror, as illustrated in Fig. 2.59. Waves can reach the viewing point P either by the direct path SP or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating at the source S' behind the mirror. Source S', which is the image of S, can be considered a virtual source.

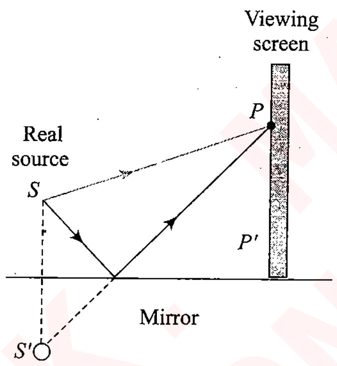


Fig. 2.59 Lloyd's mirror

At points far from the source, we would expect an interference pattern due to waves from S and S', just as is observed for two real coherent sources. An interference pattern is indeed observed. However, the positions of the dark and bright fringes are reversed relative to the pattern of two real coherent sources (Young's experiment). This is because the coherent sources S and S' differ in phase by 180°. This 180° phase change is produced by reflection.

An interference pattern is produced on a screen at P as a result of the combination of the direct ray and the reflected ray.

The reflected ray undergoes a phase change of 180°

To illustrate this further, consider point P', at which the mirror meets the screen. This point is equidistant from S and S'. If path difference alone were responsible for the phase difference, we would expect to see a bright fringe at P' (because the path difference is zero for this point), corresponding to the central fringe of the two-slit interference pattern. Instead, we observe a dark fringe at P' because of the 180° phase change produced by reflection. In general, an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.

In this arrangement, the light reflected from a long mirror and the light coming directly from the source without reflection produce interference on the screen. An important feature of this experiment lies in the fact that when the screen is placed in contact with the end of the mirror, the edge O of the reflecting surface comes at the center of a dark fringe instead of a bright fringe. The direct beam does not suffer any phase change of π radian. Hence, at a point P on the screen the conditions for minima and maxima are

$$S_2P - S_1P = n\lambda,\ n = 0, 1, 2, 3, \ldots \qquad \text{[minima]}$$

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda \qquad \text{[maxima]}$$

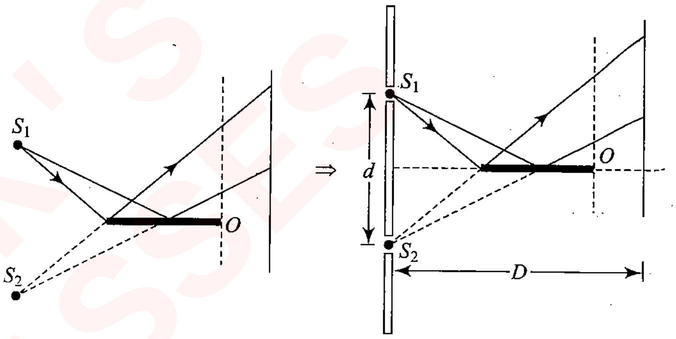


Fig. 2.60

Illustration 2.35 **A Lloyd's mirror of length 5 cm is illuminated with monochromatic light of wavelength λ (= 6000 Å) from a narrow 1 mm slit in its plane and 5 cm plane from its near edge. Find the fringe width on a screen 120 cm from the slit and width of interference pattern on the screen.**

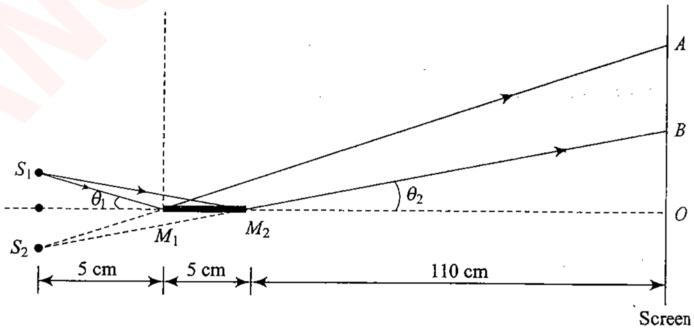


Fig. 2.61

Sol. In a plane mirror, image is formed behind it as the object is in front of it. So,

$$d = 2\text{ mm} = 0.2\text{ cm}$$

$$\lambda = 6000\text{ Å} = 6000 \times 10^{-8}\text{ cm}$$

$$D = 120\text{ cm}$$

Therefore, fringe width,

$$\beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.2} = 0.036\text{ cm}$$

The width of fringe pattern is AB. From Fig. 2.61,

$$\tan\theta_1 = \frac{0.1}{5} \quad \text{and} \quad \tan\theta_2 = \frac{0.1}{10}$$

In right-angled triangles AM_1O and BM_2O,

$$\tan\theta_1 = \frac{0.1}{5} = \frac{OA}{M_1O} \Rightarrow OA = 115 \times \frac{0.1}{5} \text{ cm},$$

$$\tan\theta_2 = \frac{0.1}{10} = \frac{OB}{OM_2} \Rightarrow OB = 110 \times \frac{0.1}{10} \text{ cm}$$

Hence, width of fringe pattern is

$$OA - OB = \frac{115 \times 0.1}{5} - \frac{110 \times 0.1}{10} = 1.2 \text{ cm}$$

Solved Examples

Example 2.1 **In a modified Young's double-slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ W m^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m, respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A (see Fig. 2.62). Calculate the power (in watt) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the aperture. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot. (IIT-JEE, 1989)**

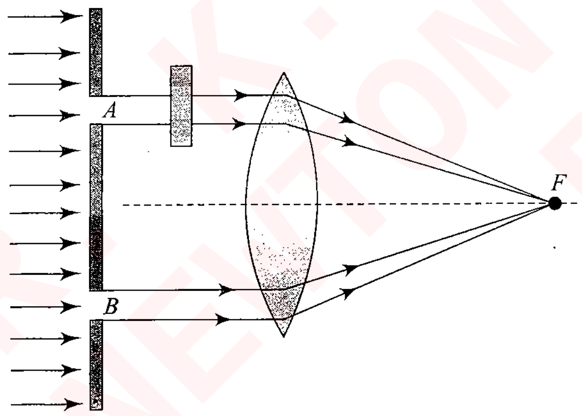


Fig. 2.62

Sol. The intensities of sources A and B are

$$I_A = \left(\frac{10}{\pi}\right) \times \pi r_A^2 = 10 \times (0.001)^2 = 10^{-5} \text{ W}$$

$$I_B = \left(\frac{10}{\pi}\right) \times \pi r_B^2 = 10 \times (0.002)^2 = 4 \times 10^{-5} \text{ W}$$

Intensities of sources A and B received along F are

$$I_A = \frac{10}{100}(1 \times 10^{-5}) = 10^{-6} \text{ W}$$

$$I_B = \frac{10}{100}(4 \times 10^{-5}) = 4 \times 10^{-6}$$

Path difference,

$$\Delta = (\mu - 1)t = (1.5 - 1) \times 2000 \text{ Å} = 1000 \text{ Å}$$

Phase difference,

$$\delta = \frac{2\pi}{\lambda} \times \Delta = \frac{2\pi}{6000} \times 1000 = \frac{\pi}{3}$$

Intensity at point F is given by

$$I_F = a_A^2 + a_B^2 + 2a_A a_B \cos\delta$$
$$= I_A + I_B + 2\sqrt{I_A I_B} \cos\delta$$
$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{10^{-6} \times 4 \times 10^{-6}} \cos\frac{\pi}{3}$$
$$= 10^{-6} + 4 \times 10^{-6} + 2 \times 2 \times 10^{-6} \times \frac{1}{2}$$
$$= 10^{-6} + 4 \times 10^{-6} + 2 \times 10^{-6} = 7 \times 10^{-6} \text{ W}$$

Example 2.2 **A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in Fig. 2.63. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25% of the light incident on it and transmits the remaining. Find the ratio of minimum and maximum intensities in the interference pattern formed by two beams obtained after one reflection at each plate. (IIT-JEE, 1990)**

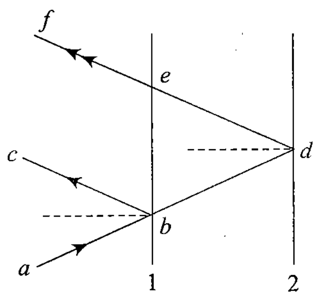


Fig. 2.63

Sol. I is the intensity of incident beam ab. The interfering waves are bc and ef, reflected from surface of plates 1 and 2, respectively.

Reflection coefficient of intensity,

$$r = 25\% = 0.25$$

Transmission coefficient of intensity,

$$t = 75\% = 0.75$$

The intensity of beam bc, $I_1 = 0.25\, I = \frac{1}{4} I$

The intensity of beam $bd = 0.75I$

The intensity of beam $de = 0.25 \times 0.75I$

The intensity of beam ef,

$$I_2 = 0.75 \times 0.25 \times 0.75I = \frac{9}{64} I$$

Ratio of maximum and minimum intensities,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{a_1^2+a_2^2+2a_1a_2}{a_1^2+a_2^2-2a_1a_2}$$

$$= \frac{I_1+I_2+2\sqrt{(I_1I_2)}}{I_1+I_2-2\sqrt{(I_1I_2)}} = \frac{\frac{1}{4}I+\frac{9}{64}I+2\left(\frac{1}{4}I\times\frac{9}{64}I\right)}{\frac{1}{4}I+\frac{9}{64}I-2\sqrt{\left(\frac{1}{4}I\times\frac{9}{64}I\right)}}$$

$$= \frac{(49/64)I}{(1/64)I} = \frac{49}{1}$$

Example 2.3 **In Fig. 2.64, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.01 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m, while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO.**

(i) If the third intensity maximum occurs at point A on the screen, find the distance OA.

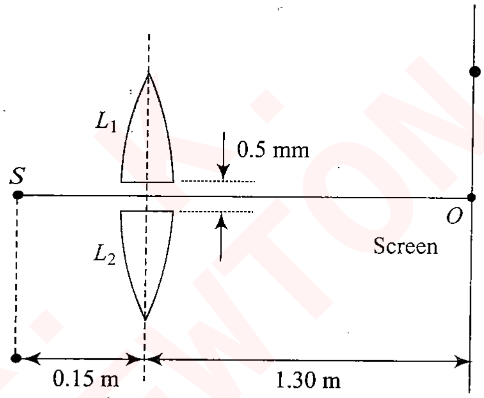


Fig. 2.64

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease or remain the same? (IIT-JEE, 1993)

Sol.

(i) The images of S due to both parts of lenses are formed at S_1 and S_2 whose distance v from the lens is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here, $u = -0.15$ m, $f = 0.10$ m.

$$\therefore \quad \frac{1}{0.10} = \frac{1}{v} - \frac{1}{(-0.15)}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{0.10} - \frac{1}{0.15} = \frac{3-2}{0.30} = \frac{1}{0.30}$$

$$\therefore \quad v = 0.30 \text{ m}$$

As the ray passing through optical center passes undeviated therefore angle θ subtended by lens gap and images S_1 and S_2 must be same. We have,

$$\theta = \frac{O_1O_2}{u} = \frac{S_1S_2}{u+v}$$

$$\therefore \quad S_1S_2 = \frac{u+v}{u}O_1O_2 = \frac{0.30+0.15}{0.15}\times 0.5 \text{ mm} = 1.5 \text{ mm}$$

i.e., $S_1S_2 = d = 1.5$ mm $= 1.5 \times 10^{-3}$ m

Also, $D = 1.30 - 0.30 = 1.00$ m.

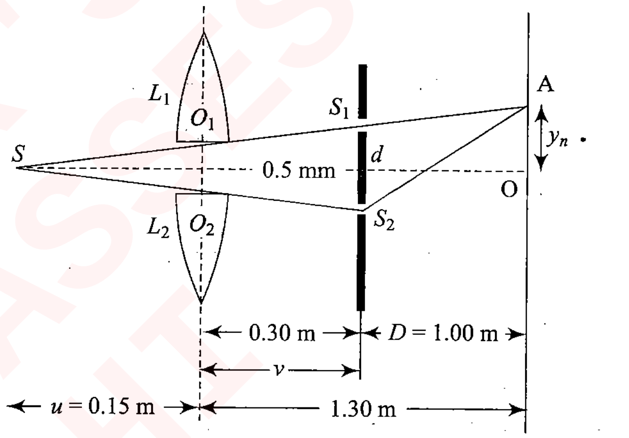


Fig. 2.65

S_1 and S_2 act as coherent sources and produce interference. The positions of maxima are given by

$$y_n = \frac{nD\lambda}{d}$$

As point A is third maxima,

$$\therefore \quad y_n = OA = \frac{3D\lambda}{d} = \frac{3\times 1.00\times 500\times 10^{-9}}{1.5\times 10^{-3}}$$

$$= 1000\times 10^{-6} \text{ m} = 1.0 \text{ mm}$$

(ii) If gap between L_1 and L_2 is reduced, then S_1S_2 or d decreases and hence OA increases.

Example 2.4 **A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.**

(i) Calculate the fringe width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis. (IIT-JEE, 1996)

Sol. Given $\mu_l = 1.33$, $d = 1$ mm $= 10^{-3}$ m, $D = 1.33$ m, $\lambda = 6300$ Å $= 6.3\times 10^{-7}$ m.

When the experiment is performed in liquid, λ changes to

$$\lambda' = \frac{\lambda}{\mu_\ell}$$

(i) Fringe width,

$$\beta = \frac{D\lambda'}{d} = \frac{D(\lambda/\mu_\ell)}{d} = \frac{D\lambda}{\mu_\ell d}$$

$$= \frac{1.33\times 6.3\times 10^{-7}}{1.33\times 10^{-3}} = 6.3\times 10^{-4}\text{m} = 0.63 \text{ mm}$$

(ii) Displacement of fringes,

$$\Delta y = \frac{\beta}{\lambda'}({}_\ell\mu_g - 1)t = \frac{\beta}{\lambda/\mu_\ell}\left(\frac{\mu_g}{\mu_\ell} - 1\right)t$$

$$= \frac{\beta}{\lambda}(\mu_g - \mu_\ell)t \Rightarrow t = \frac{\Delta y\lambda}{\beta.(\mu_g - \mu_\ell)}$$

But Δy = separation between bright and adjacent dark fringe

$$= \frac{\beta}{2}$$

$$\therefore \quad t = \frac{\left(\frac{\beta}{2}\right)\lambda}{\beta(\mu_g - \mu_\ell)} = \frac{\lambda}{2(\mu_g - \mu_\ell)}$$

Given $\mu_g = 1.53, \mu_1 = 1.33$.

$$\therefore \quad t = \frac{6.3\times 10^{-7}}{2(1.53 - 1.33)}$$

$$= \frac{6.3\times 10^{-7}}{2\times 0.20} = 1.575\times 10^{-6}\text{ m} = 1.575 \text{ μm}$$

Example 2.5 **In Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum ($n = 0$) fell before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be fifth maximum earlier, lies below the point P, while the sixth minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected.) (IIT-JEE, 1997)**

Sol. Path difference between two waves starting from S_1 and S_2 and reaching at P,

$$\Delta = (\mu_2 - \mu_1)t$$

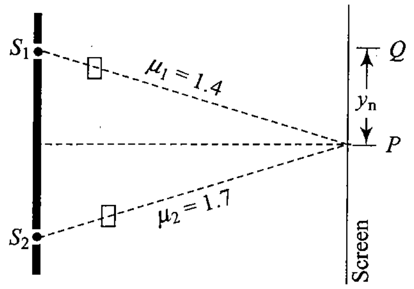


Fig. 2.66

In the absence of plates, phase difference between two waves reaching at P equals zero. When the plates are introduced, let the phase difference between waves at P be δ.

As the waves are of equal amplitude (or intensity), so initial intensity at P,

$$I_0 = (a_1 + a_2)^2 = 4a^2$$

New intensity at P,

$$I_0 = (a_1 + a_2)^2 = (2a)^2 = 4a^2$$

New intensity at P is

$$I = \frac{3}{4}I_0 = \frac{3}{4}\times 4a^2 = 3a^2$$

We have,

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$\Rightarrow \quad 3a^2 = a^2 + a^2 + 2a^2\cos\delta$$

This gives

$$\delta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \ldots = \left(2m\pi \pm \frac{\pi}{3}\right), m = 1, 2, 3, \ldots \quad \text{(i)}$$

Shift of interference pattern upward due to glass plate,

$$1 = \frac{D}{d}(\mu_1 - 1)t = \frac{D}{d}(1.7 - 1)t$$

Net shift downward is

$$\frac{D}{d}(\mu_2 - \mu_1)t = \frac{D}{d}(1.7 - 1.4)t = \frac{D}{d}(0.3t)$$

From given conditions,

$$\frac{D}{d}(0.3t) > \frac{5\lambda D}{d} \text{ and } \frac{D}{d}(0.3t) < \left(6 - \frac{1}{2}\right)\frac{\lambda D}{d}$$

$$\Rightarrow \quad \frac{5\lambda}{0.3} < t < \frac{11\lambda}{2\times 0.3}$$

$$\lambda = 5400 \text{ Å} = 5.4\times 10^{-7}\text{ m} \quad \text{(ii)}$$

Path difference created at P due to both the plates is $(\mu_2 - \mu_1)t$.

Also, path difference,

$$\Delta = \frac{\lambda}{2\pi}\times \text{phase difference}$$

$$\Rightarrow \quad (\mu_2 - \mu_1)t = \frac{\lambda}{2\pi}\delta$$

$$\text{or} \quad 0.3t = \frac{\lambda}{2\pi}\times\left(2m\pi \pm \frac{\pi}{3}\right)$$

$$\text{or} \quad 0.3t = \left(m \pm \frac{1}{6}\right)\lambda$$

$$\text{or} \quad 0.3t = \left(m \pm \frac{1}{6}\right)\times 5.4\times 10^{-7}\text{ m} \quad \text{(iii)}$$

From (ii) and (iii), we have (putting $m = 5$),

$$t = \frac{\left(5 + \frac{1}{6}\right)\times 5.4\times 10^{-7}}{0.3}$$

$$= 93\times 10^{-7}\text{ m taking (+) sign}$$

$$= 87\times 10^{-7}\text{ m taking (–) sign}$$

In view of Eq. (ii),

$$t = 93 \times 10^{-7} \text{ m} = 9.3 \times 10^{-6} \text{ m}$$
$$= 9.3 \ \mu\text{m}$$

Example 2.6 **The Young's double-slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in Fig. 2.67.**

(IIT-JEE, 1999)

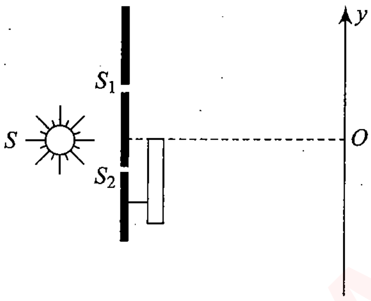


Fig. 2.67

a. Find the location of central maximum (bright fringe with zero path difference) on the y-axis.

b. Find the light intensity of point O relative to the maximum fringe intensity.

c. Now, if 600 nm light is replaced by white light of range 400–700 nm, find the wavelengths of the light that form maxima exactly at point O.

(All wavelengths in the problem are for the given medium of refractive index 4/3. Ignore dispersion.)

Sol. Given $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m, $d = 0.45 \text{ mm} = 0.45 \times 10^{-3}$ m, D = 1.5 m. Thickness of glass sheet, t = 10.4 μm = 10.4 × 10^{-6} m. Refractive index of glass sheet, $\mu_g = 1.5$.

a. Let central maximum is obtained at a distance y below point O. Then,

$$\Delta x_1 = S_1P - S_2P = \frac{yd}{D}$$

Path difference due to glass sheet,

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

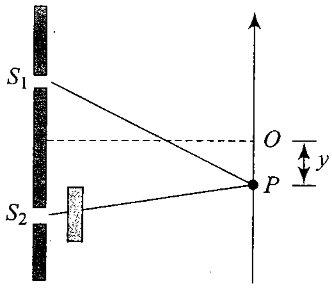


Fig. 2.68

Net path difference will be zero when

$$\Delta x_1 = \Delta x_2$$

$$\Rightarrow \quad \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow \quad y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we have

$$y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

or we can say $y = 4.33$ mm.

b. At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

∴ Net path difference $\Delta x = \Delta x_2$

Corresponding phase difference, $\Delta\phi$ or simply $\phi = \frac{2\pi}{\lambda}\Delta x$.

Substituting the values, we have

$$\phi = \frac{2\pi}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6}) = \left(\frac{13}{3}\right)\pi$$

Now,

$$I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$$I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$= \frac{3}{4}I_{\max}$$

c. At O, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O,

$$\Delta x = n\lambda \qquad (\text{here } n = 1, 2, 3, \ldots)$$

$$\lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3}, \ldots \text{ and so on}$$

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6} \text{ m})$$

$$= \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{3} \text{ nm})$$

$$= 1300 \text{ nm}$$

Maximum intensity will be corresponding to

$$\lambda = 1300 \text{ nm}, \frac{1300}{2}\text{nm}, \frac{1300}{3}\text{nm}, \frac{1300}{4}\text{nm}, \ldots$$

$$= 1300 \text{ nm}, 650 \text{ nm}, 433.33 \text{ nm}, 325 \text{ nm}, \ldots$$

The wavelengths in the range 400 to 700 nm are 650 nm and 433.33 nm.

Example 2.7 **A glass of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces**

of the layer and two reflected rays interfere. Write the condition for their constructive interference. If the λ = 648 nm, obtain the least value of t for which the rays interfere constructively. (IIT-JEE, 2000)

Sol. Path difference between rays reflected from upper and lower faces of layer = $2\mu t$ (for normal incidence)

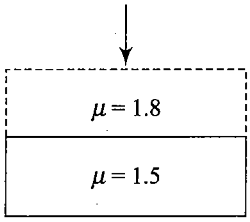


Fig. 2.69

Path difference due to reflection from denser (upper) surface is $\lambda/2$.

Condition of constructive interference (or maxima) is

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$t = \frac{1}{2\mu}(2n-1)\frac{\lambda}{2} \Rightarrow t = \frac{(2n-1)\lambda}{4\mu}, n = 1, 2, 3, \ldots$$

For least thickness, $n = 1$.

$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4 \times 1.8}\text{ nm} = 90\text{ nm}$$

Example 2.8 **A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in Fig. 2.70. Calculate the position of the central bright fringe on the other wall CD with respect of the line OQ. Now, a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid. (IIT-JEE, 2001)**

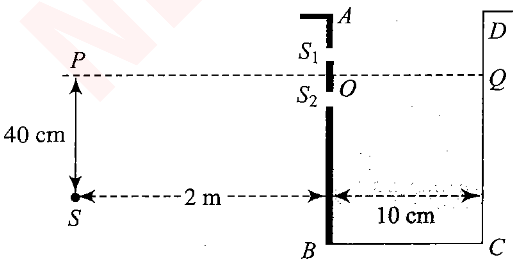


Fig. 2.70

Sol. Given y_1 = 40 cm, D_1 = 2 m = 200 cm, D_2 = 10 cm.

$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$$

$$\sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha \quad \text{[see Fig. 2.71(b)]}$$

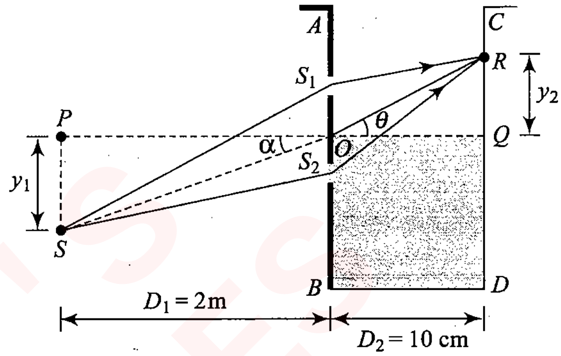


Fig. 2.71(a)

Path difference between SS_1 and SS_2 is

$$\Delta x_1 = d\sin\alpha = (0.8\text{ mm})\left(\frac{1}{5}\right)$$

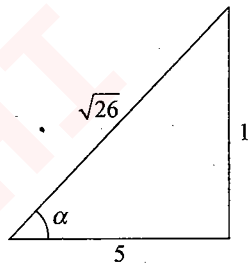


Fig. 2.71(b)

$$\Delta x_1 = 0.16\text{ mm} \quad \text{(i)}$$

Now, let at point R on the screen, central bright fringe is observed (i.e., net path difference = 0).

Path difference between S_2R and S_1R would be

$$\Delta x_2 = S_2R - S_1R$$

$$\Delta x_2 = d\sin\theta \quad \text{(ii)}$$

Central bright fringe will be observed when net path difference is zero.

$$\Delta x_2 - \Delta x_1 = 0$$

$$\Delta x_2 = \Delta x_1$$

$$d\sin\theta = 0.16$$

$$(0.8)\sin\theta = 0.16$$

$$\sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$$

$$\tan\theta = \frac{1}{\sqrt{24}} \approx \sin\theta = \frac{1}{5}$$

Hence,

$$\tan\theta = \frac{y_2}{D_2} = \frac{1}{5}$$

$$y_2 = \frac{D_2}{5} = \frac{10}{5} = 2\text{ cm}$$

Therefore, central bright fringe is observed at 2 cm above point O on side CD.

Alternative solution:

Δx at R will be zero if $\Delta x_1 = \Delta x_2$

$$d \sin \alpha = d \sin \theta$$
$$\alpha = \theta$$
$$\tan \alpha = \tan \theta$$
$$\frac{y_1}{D_1} = \frac{y_2}{D_2}$$
$$y_2 = \frac{D_2}{D_1} y_1 = \left(\frac{10}{200}\right)(40) \text{ cm}$$
$$y_2 = 2 \text{ cm}$$

The central bright fringe will be observed at point Q. If the path difference created by the liquid slab of thickness $t = 10$ cm or 100 mm is equal to Δx, so that the net path difference at Q becomes zero, then

$$(\mu - 1)t = \Delta x_1$$
$$\Rightarrow \quad (\mu - 1)(100) = 0.16$$
$$\Rightarrow \quad \mu - 1 = 0.0016$$
$$\Rightarrow \quad \mu = 1.0016$$

Example 2.9 **A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (see Fig. 2.72). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it. (IIT-JEE, 2002)**

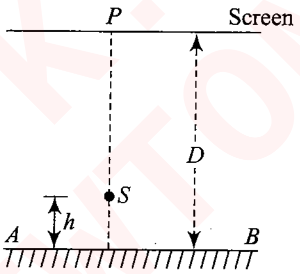


Fig. 2.72

a. What is the shape of the interference fringes on the screen?

b. Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

c. If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

Sol. a. The path difference between the rays 1 and 2 coming from S and S' will be equal on the circular path on the screen, hence fringes will be circular.

b. Let the intensity of light coming from S be $I_1 = I$, then as per the problem, the velocity of the reflected light will be $I_2 = 0.36\, I$.

As we know $\quad I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

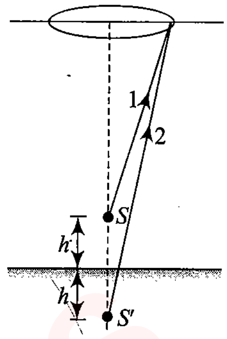


Fig. 2.73

Hence, $\quad \dfrac{I_{max}}{I_{min}} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$$= \frac{(\sqrt{I} + \sqrt{0.36\, I})^2}{(\sqrt{I} - \sqrt{0.36\, I})^2} = \frac{1}{16}$$

c. To have the next maximum at P, the path difference between the interfering waves must change by λ. If AB is moved by a distance x, it will cause an additional path difference $2x$.

$2x = \lambda$ (For minimum value of x)

$$\Rightarrow \quad x = \frac{\lambda}{2} = 300 \text{ mm}$$

Example 2.10 **A prism of refracting angle 30° is coated with a thin film of transparent material of refractive index 2.2 on face AC of the prism. A light ray of wavelength 6600 Å is incident on face AB such that angle of incidence is 60°.**

a. Find the angle of emergence.

b. Find the minimum value of thickness of the coated film on the face AC for which the light emerging from the face has maximum intensity. (Given refractive index of the material of the prism is $\sqrt{3}$.) (IIT-JEE, 2003)

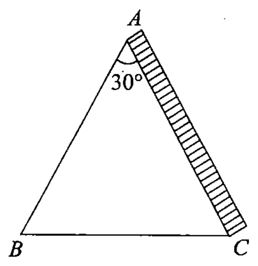


Fig. 2.74

Sol. a. Given, $i_1 = 60°$, $A = 30°$, $\mu_{prism} = \sqrt{3}$.

From Snell's law,

$$\mu_1 \sin i_1 = \frac{1}{\sqrt{3}} \times \sin 60°$$
$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{2} \quad \Rightarrow \quad r_1 = 30°$$

Also, $r_1 + r_2 = A \Rightarrow r_2 = A - r_1 = 30° - 30° = 0°$

That is, the ray is incident normally on face AC and angle of emergence $i_2 = 0°$.

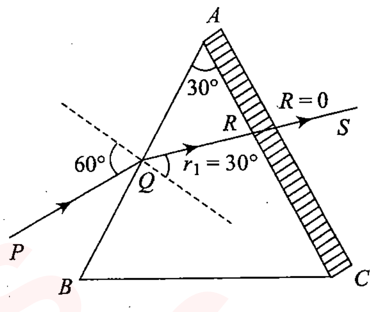


Fig. 2.75

b. For maximum intensity, the thickness of coated film must satisfy the condition of maxima in transmitted light, i.e.,

$$2\mu t \cos r = (2n - 1)\lambda/2$$

$$\Rightarrow \quad t = \frac{(2n-1)\lambda}{4\mu \cos r}$$

For normal incidence, $r = 0$. So, $\cos r = 1$ and for minimum thickness, $n = 1$.

$$\therefore \quad t_{\min} = \frac{\lambda}{4\mu} = \frac{6600 Å}{4 \times 2.2} = 750 Å$$

EXERCISES

Subjective Type *Solutions on page 2.65*

1. A monochromatic beam of light of 6000 Å is used in YDSE set-up. The two slits are covered with two thin films of equal thickness t but of different refractive indices as shown in Fig. 2.76. Considering the intensity of the incident beam on the slits to be I_0, find the point on the screen at which intensity is I_0 and is just above the central maxima. (Assume that there is no change in intensity of the light after passing through the films.)

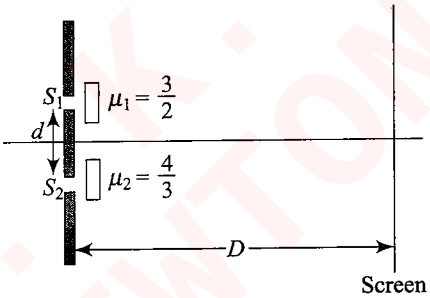


Fig. 2.76

Consider $t = 6$ μm, $d = 1$ mm and $D = 1$ m, where d and D have their usual meaning. Give your answer in mm.

2. A double slit of separation 1.5 mm is illuminated by white light (between 4000 and 8000 Å). On a screen 120 cm away coloured interference pattern is formed. If a pinhole is made on this screen at a distance of 3.0 mm from the central white fringe, some wavelengths will be absent in the transmitted light. Find the second longest wavelength (in Å) which will be absent in the transmitted light.

3. In Fig. 2.78, 'S' is a monochromatic source of light emitting light of wavelength λ (in air). Light falls on slits 'S_1' from 'S' and then reaches is the slit 'S_2' and S_3 through a medium of refractive index 'μ_1'. Light from slits S_2 and S_3 reaches the screen through a medium of refractive index μ_3. A thin transparent film of refractive index μ_2 and thickness 't' is placed in front of 'S_2'. Point 'P' is symmetrical w.r.t. 'S_2' and 'S_3'. Using the values $d = 1$ mm, $D = 1$ m, $\mu_1 = 4/3$,

$\mu_2 = 3/2$, $\mu_3 = 9/5$ and $t = \frac{4}{9} \times 10^{-5}$ m,

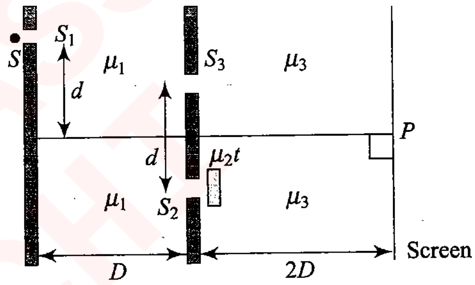


Fig. 2.77

a. find distance of central maxima from P;

b. if the film in front of S_2 is removed, then by what distance and in which direction will the central maxima shift?

4. In a Young's double-slit experiment, the amplitude of source S_1 is three times the amplitude of the source S_2. These sources are covered by perfectly transparent thin plates of same thickness but different refractive indices 1.6 and 1.5, respectively. Now, if the plates are interchanged and the amplitude of the source S_1 is made same as that of S_2, then find the amount by which the intensity is changed at a point where previous central maxima was formed. Take thickness of the plate equal to $(110/3)\lambda$, where λ is the wavelength of light used.

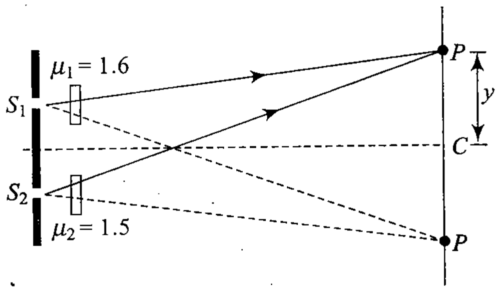


Fig. 2.78

5. Figure 2.79 shows a narrow slit S illuminated by a monochromatic light of wavelength λ in a double-slit experiment. In the path of the rays reaching the upper slit S_1, a tube of length L is interposed in which the index of refraction of the medium varies linearly as shown in Fig. 2.80. The position of the central maximum in the interference pattern on the screen was displaceed by N fringes. Find the value of N in terms of μ_0, L and λ.

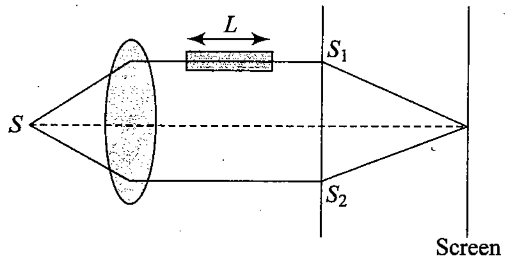


Fig. 2.79

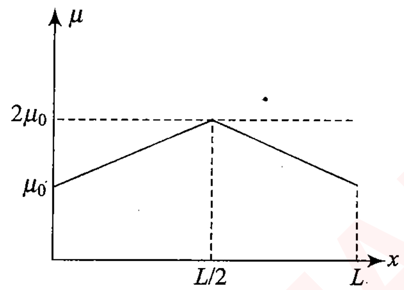


Fig. 2.80

6. In a Young's double-slit experiment, a point source is placed on a solid slab of refractive index 6/5 at a distance of 2 mm from two slits spaced 3 mm apart as shown and at equal distance from both the slits. The screen is at a distance of 1 m from the slits. Wavelength of light used is 500 nm.

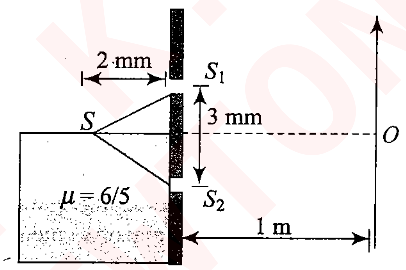


Fig. 2.81

a. Find the position of the central maximum.
b. Find the order of the fringe formed at O.
c. A film of refractive index 1.8 is to be placed in front of S_1 so that central maxima is formed where 200th maxima was formed. Find the thickness of film.

7. In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate.

When the source is now changed to green light of wavelength 5×10^{-7}m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of the glass for the green light. Also, estimate the change in fringe width due to the change in wavelength.

8. Figure 2.82 shows a modified Fresnel biprism experiment with monochromatic source of wavelength 500 nm. The refracting angle of glass prism is 2°. Find the fringe width.

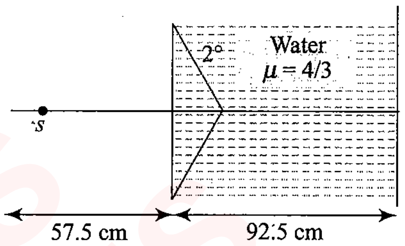


Fig. 2.82

9. Interference fringes are produced by a double-slit experiment and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, find the thickness of the plate.

10. In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit $S1$ so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

11. Bichromatic light is used in YDSE having wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm. Find minimum order of λ_1 which overlaps with λ_2.

Objective Type

Solutions on page 2.67

1. Microwaves from a transmitter are directed normally towards a plane reflector. A detector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima the detector travels a distance 0.14 m. The frequency of the transmitter is ($c = 3 \times 10^8$ m/s)

a. 1.5×10^{10} Hz　　**b.** 10^{10} Hz
c. 3×10^{10} Hz　　**d.** 6×10^{10} Hz

2. In a Young's double-slit experiment, the separation between the slits is d, distance between the slit and screen is D ($D \gg d$). In the interference pattern, there is a maxima exactly in front of each slit. Then, the possible wavelength(s) used in the experiment are

a. d^2/D, $d^2/2D$, $d^2/3D$
b. d^2/D, $d^2/3d$, $d^2/5D$
c. $d^2/2D$, $d^2/4D$, $d^2/6D$
d. none of these

3. In a double-slit experiment, two parallel slits are illuminated first by light of wavelength 400 nm and then by light of unknown wavelength. The fourth-order dark fringe resulting from the known wavelength of light falls in the same place on the screen as the second-order bright fringe from the unknown wavelength. The value of unknown wavelength of the light is

a. 900 nm　　**b.** 700 nm
c. 300 nm　　**d.** none of these

4. Light is incident at an angle ϕ with the normal to a plane containing two slits of separation d. Select the expression that correctly describes the positions of the interference maxima in terms of the incoming angle ϕ and outgoing angle θ.

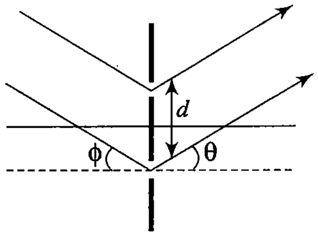


Fig. 2.83

a. $\sin\phi + \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$

b. $d \sin\theta = m\lambda$

c. $\sin\phi - \sin\theta = (m+1)\frac{\lambda}{d}$

d. $\sin\phi + \sin\theta = m\frac{\lambda}{d}$

5. In a Young's double-slit experiment, the slits are illuminated by monochromatic light. The entire set-up is immersed in pure water. Which of the following act *cannot* restore the original fringe width?
   a. Bringing the slits close together.
   b. Moving the screen away from the slit plane.
   c. Replacing the incident light by that of longer wavelength.
   d. Introducing a thin transparent slab in front of one of the slits.

6. Blue light of wavelength 480 nm is most strongly reflected off a thin film of oil on a glass slab when viewed near normal incidence. Assuming that the index of refraction of the oil is 1.2 and that of the glass is 1.6, what is the minimum thickness of the oil film (other than zero)?
   a. 100 nm b. 200 nm
   c. 300 nm d. none

7. The slits in a double-slit interference experiment are illuminated by orange light ($\lambda = 60$ nm). A thin transparent plastic of thickness t is placed in front of one of the slits. The number of fringes shifting on screen is plotted versus the refractive index μ of the plastic in graph shown in Fig. 2.84. The value of t is

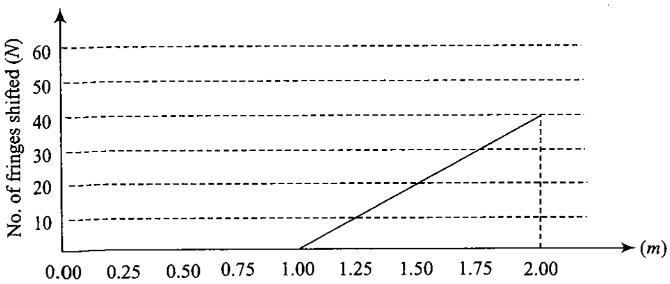


Fig. 2.84

   a. 4.8 mm b. 640 μm
   c. 24 μm d. none of these

8. In a YDSE with identical slits, the intensity of the central bright fringe is I_0. If one of the slits is covered, the intensity at the same point is
   a. $2I_0$ b. I_0
   c. $I_0/2$ d. $I_0/4$

9. The maximum intensity in Young's double-slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = d$?
   a. $\frac{I_0}{2}$ b. $\frac{3}{4}I_0$
   c. I_0 d. $\frac{I_0}{4}$

10. Two identical coherent sources are placed on a diameter of a circle of radius R at separation x ($\ll R$) symmetrical about the center of the circle. The sources emit identical wavelength λ each. The number of points on the circle of maximum intensity is ($x = 5\lambda$)
   a. 20 b. 22
   c. 24 d. 26

11. In Young's double-slit experiment $\frac{d}{D} = 10^{-4}$ = 10^{-4} (d = distance between slits, D = distance of screen from the slits). At a point P on the screen, resulting intensity is equal to the intensity due to the individual slit I_0. Then, the distance of point P from the central maximum is ($\lambda = 6000$ Å)
   a. 2 mm b. 1 mm
   c. 0.5 mm d. 4 mm

12. To produce a minimum reflection of wavelengths near the middle of visible spectrum (550 nm) how thick should a coating of MgF_2 ($\mu = 1.38$) be vacuum-coated on a glass surface?
   a. 10^{-7} b. 10^{-10}
   c. 10^{-9} m d. 10^{-8} m

13. A thin film of refractive index 1.5 and thickness 4×10^{-5} cm is illuminated by light normal to the surface. What wavelength within the visible spectrum will be intensified in the reflected beam?
   a. 4800 Å b. 5800 Å
   c. 6000 Å d. 6800 Å

14. A plane wave of monochromatic light falls normally on a uniform thin film of oil which covers a glass plate. The wavelength of source can be varied continuously. Complete destructive interference is observed for $\lambda = 5000$ Å and $\lambda = 1000$ Å and for no other wavelength in between. If μ of oil is 1.3 and that of glass is 1.5, the thickness of the film will be
   a. 6.738×10^{-5} cm b. 5.7×10^{-5} cm
   c. 4×10^{-5} cm d. 2.8×10^{-5} cm

15. A light ray of frequency ν and wavelength λ enters a liquid of refractive index $\frac{3}{2}$. The ray travels in the liquid with

a. frequency ν and wavelength $\left(\frac{2}{3}\right)\lambda$

b. frequency ν and wavelength $\left(\frac{3}{2}\right)\lambda$

c. frequency ν and wavelength λ

d. frequency $\left(\frac{3}{2}\right)\nu$ and wavelength λ

16. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then, in the interference pattern

a. the intensities of both the maxima and the minima increase

b. the intensity of the maxima increases and the minima has zero intensity

c. the intensity of the maxima decreases and that of the minima increases

d. the intensity of the maxima decreases and the minima has zero intensity

17. Two light waves having the same wavelength λ in vacuum are in phase initially. Then, the first ray travels a path of length L_1 through a medium of refractive index μ_1. The second ray travels a path of length L_2 through a medium of refractive index μ_2. The two waves are then combined to observe interference effects. The phase difference between the two, when they interfere, is

a. $\frac{2\pi}{\lambda}(L_1 - L_2)$ **b.** $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

c. $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$ **d.** $\frac{2\pi}{\lambda}\left[\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right]$

18. Light of wavelength $\lambda = 5890$ Å falls on a double-slit arrangement having separation $d = 0.2$ mm. A thin lens of focal length $f = 1$ m is placed near the slits. The linear separation of fringes on a screen placed in the focal plane of the lens is

a. 3 mm **b.** 4 mm

c. 2 mm **d.** 1 mm

19. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same setup, the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of intensity of light at the mid-point of the screen in the first case to that in the second case is

a. 1:1 **b.** 1:2

c. 2:1 **d.** 4:1

20. In Young's double-slit interference experiment, if the slit separation is made threefold, the fringe width becomes

a. sixfold **b.** threefold

c. 3/6-fold **d.** 1/3-fold

21. Sources 1 and 2 emit lights of different wavelengths whereas 3 and 4 emit lights of different intensities. The coherence

a. can be obtained by using sources 1 and 2

b. can be obtained by using sources 3 and 4

c. cannot be obtained by any of these sources

d. since contrast suffers when sources 3 and 4 are used so coherence cannot be obtained by using sources 3 and 4

22. If one of the two slits of a Young's double-slit experiment is painted so that it transmits half the light intensity as the second slit, then

a. the fringe system will altogether disappear

b. the bright fringes will become brighter and the dark fringes will become darker

c. both dark and bright fringes will become darker

d. dark fringes will become brighter and bright fringes darker

23. A wave front AB passing through a system C emerges as DE. The system C could be

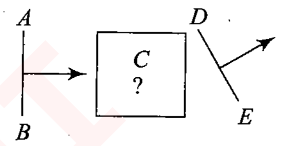


Fig. 2.85

a. a slit **b.** a biprism

c. a prism **d.** a glass slab

24. Fig. 2.86 shows a wavefront P passing through two systems A and B, and emerging as Q and then as R. The systems A and B could, respectively, be

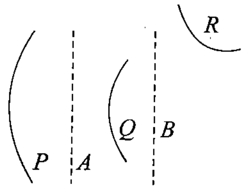


Fig. 2.86

a. a prism and a convergent lens

b. a convergent lens and a prism

c. a divergent lens and a prism

d. a convergent lens and a divergent lens

25. Light waves travel in vacuum along the y-axis. Which of the following may represent the wavefront?

a. x = constant **b.** y = constant

c. z = constant **d.** $x + y + z$ = constant

26. In Young's double-slit experiment, the y-coordinates of central maxima and 10th maxima are 2 cm and 5 cm, respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5, the corresponding y-coordinates will be

a. 2 cm, 7.5 cm **b.** 3 cm, 6 cm

c. 2 cm, 4 cm **d.** 4/3 cm, 103 cm

27. A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in Fig. 2.87. A thin sheet of glass is inserted in front of the lower slit s_2. The central bright fringe (path difference = 0) will be obtained

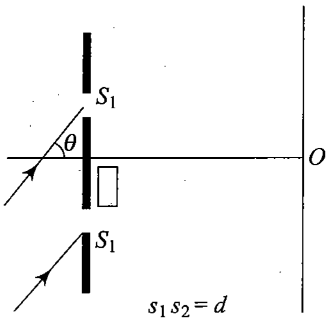


Fig. 2.87

a. at O
b. above O
c. below O
d. anywhere depending on angle θ, thickness of plate t, and refractive index of glass μ

28. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double-slit experiment. What should be the minimum thickness t which will make the intensity at the center of the fringe pattern zero?

a. $(\mu - 1)\dfrac{\lambda}{2}$ **b.** $(\mu - 1)\lambda$
c. $\dfrac{\lambda}{2(\mu - 1)}$ **d.** $\dfrac{\lambda}{(\mu - 1)}$

29. In Young's double-slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe ($\lambda = 2000$ Å)?

a. 12 **b.** 7
c. 18 **d.** 4

30. Young's double-slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

a. 1.8 **b.** 1.54
c. 1.67 **d.** 1.2

31. What happens to the interference pattern if the two slits in Young's experiment are illuminated by two independent sources such as two sodium lamps S and S' as shown in Fig. 2.88?

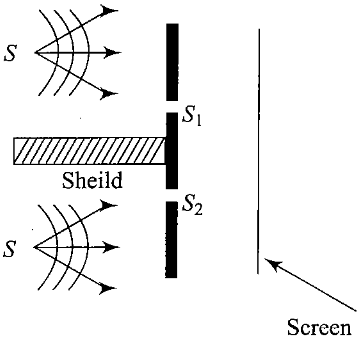


Fig. 2.88

a. Two sets of interference fringes overlap.
b. No fringes are observed.
c. The intensity of the bright fringes is doubled.
d. The intensity of the bright fringes becomes four times.

32. In a Young's double-slit experiment, 30 fringes are obtained in the field of view of the observing telescope, when the wavelength of light used is 4000 Å. If we use monochromatic light of wavelength 6000 Å, the number of fringes obtained in the same field of view is

a. 30 **b.** 45
c. 20 **d.** none of these

33. In a standard Young's double-slit experiment with coherent light of wavelength 600 nm, the fringe width of the fringes in the central region (near the central fringe, P_0) is observed to be 3 mm. An extremely thin glass plate is introduced in front of the first slit, and the fringes are observed to be displaced by 11 mm. Another thin plateis placed before the second slit and it is observed that the fringes are now displaced by and additional 12 mm. If the additional optical path lengths introduced are Δ_1 and Δ_2, then

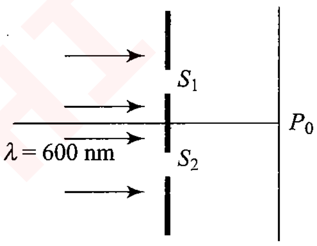


Fig. 2.89

a. $11\Delta_1 = 12\Delta_2$
b. $12\Delta_1 = 11\Delta_2$
c. $11\Delta_1 > 12\Delta_2$
d. none of the above

34. In Young's double-slit experiment, the separation between two coherent sources S_1 and S_2 is d and the distance between the source and screen is D. In the interference pattern, it is found that exactly in front of one slit, there occurs a minimum. Then the possible wavelengths used in the experiment are

a. $\lambda = \dfrac{d^2}{D}, \dfrac{d^2}{3D}, \dfrac{d^2}{5D}$

b. $\lambda = \dfrac{d^2}{D}, \dfrac{d^2}{5D}, \dfrac{d^2}{9D}$

c. $\lambda = \dfrac{d^2}{D}, \dfrac{d^2}{2D}, \dfrac{d^2}{3D}$

d. $\lambda = \dfrac{d^2}{3D}, \dfrac{d^2}{7D}, \dfrac{d^2}{11D}$

35. Let S_1 and S_2 be the two slits in Young's double-slit experiment. If central maxima is observed at P and angle $\angle S_1PS_2 = \theta$, then the fringe width for the light of wavelength λ will be

a. λ/θ **b.** $\lambda\theta$
c. $2\lambda/\theta$ **d.** $\lambda/2\theta$

36. In a two-slit experiment with white light, a white fringe is observed on a screen kept behind the slits. When the screen in moved away by 0.05 m, this white fringe

a. does not move at all
b. gets displaced from its earlier position
c. becomes coloured
d. disappears

37. A Young's double-slit experiment is conducted in water (μ_1) as shown in Fig. 2.90, and a glass plate of thickness t and refractive index μ_2 is placed in the path of S_2. Find the magnitude of the optical path difference at 'O'.

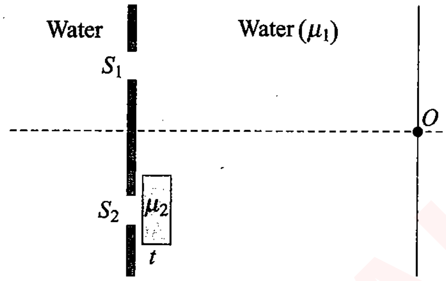


Fig. 2.90

a. $\left|\left(\frac{\mu_2}{\mu_1}-1\right)t\right|$ **b.** $\left|\left(\frac{\mu_1}{\mu_2}-1\right)t\right|$

c. $|(\mu_2-\mu_1)t|$ **d.** $|(\mu_2-1)t|$

38. In a Young's double-slit experiment, first maxima is observed at a fixed point P on the screen. Now, the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (centre of the screen)

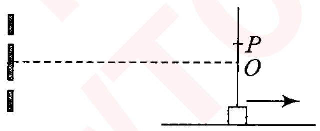


Fig. 2.91

a. remains cosntant
b. keeps on decreasing
c. first decreases and then increases
d. first decreases and then becomes constant

39. Microwaves from a transmitter are directed towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance of 0.14 m. What is the frequency of transmitter?

a. 1.5×10^{10} Hz **b.** 3.0×10^{10} Hz
c. 1.5×10^{9} Hz **d.** 3.0×10^{9} Hz

40. In a double-slit experiment, the slits are separated by a distance d and the screen is at a distance D from the slits. If a maximum is formed just opposite to each slit, then what is the order of the fringe so formed?

a. $\frac{d^2}{2\lambda D}$ **b.** $\frac{2d^2}{\lambda D}$

c. $\frac{d^2}{\lambda D}$ **d.** $\frac{d^2}{4\lambda D}$

41. A parallel beam of white light is incident on a thin film of air of uniform thickness. Wavelengths 7200 Å and 5400 Å are observed to be missing from the spectrum of reflected light viewed normally. The other wavelength in the visible region missing in the reflected spectrum is

a. 6000 Å **b.** 4320 Å
c. 5500 Å **d.** 6500 Å

42. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is

a. $2I$ **b.** $4I$
c. $5I$ **d.** $7I$

43. In a Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen is given by

a. 12 **b.** 18
c. 24 **d.** 30

44. In the ideal double-slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

a. 2λ **b.** $2\lambda/3$
c. $\lambda/3$ **d.** λ

45. A double-slit arrangement produces interference fringes for sodium light ($\lambda = 589$ nm) that have an angular separation of 3.50×0^{-3} rad. For what wavelength would the angular separation be 10% greater?

a. 527 nm **b.** 648 nm
c. 722 nm **d.** 449 nm

46. In Young's double-slit experiment, the intensity of light at a point on the screen, where the path difference is λ, is I. The intensity of light at a point where the path difference becomes $\lambda/3$ is

a. $\frac{I}{4}$ **b.** $\frac{I}{3}$

c. $\frac{I}{2}$ **d.** I

47. Two point sources separated by 2.0 m are radiating in phase with $\lambda = 0.50$ m. A detector moves in a circular path around the two sources in a plane containing them. How many maxima are detected?

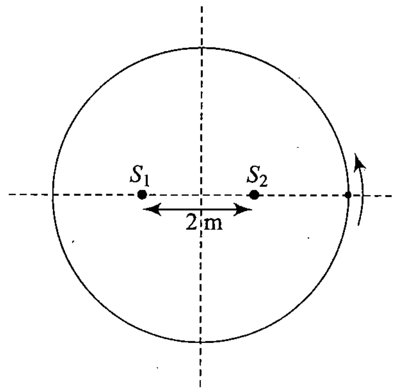


Fig. 2.92(a)

a. 16 b. 20
c. 24 d. 32

48. In a YDSE, light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$ m apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$ m, refractive index $n = 1.17$, is placed over one of the slits. Where does the central maxima of the interference now appear from the center of the screen? (Find the value of y?)

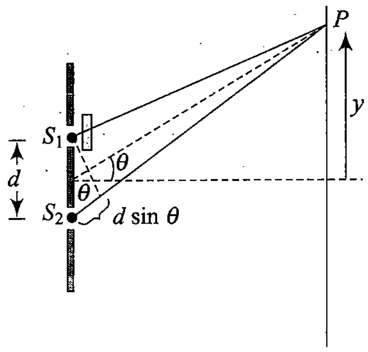


Fig. 2.92(b)

a. $\dfrac{D(\mu-1)t}{2d}$ b. $\dfrac{2D(\mu-1)t}{d}$

c. $\dfrac{D(\mu+1)t}{d}$ d. $\dfrac{D(\mu-1)t}{d}$

49. A double-slit experiment is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6830 Å. Then the fringe width is

a. 6.3×10^{-4} m b. 8.3×10^{-4} m
c. 6.3×10^{-2} m d. 6.3×10^{-5} m

50. In a Young's double-slit experiment, the slit separation is 0.5 mm and the screen is 0.5 m a way from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd maxima from the 2nd minima on the other side of central maxima is

a. 2.75 mm b. 2.5 mm
c. 22.5 mm d. 2.25 mm

51. A light of wavelength 6000 Å shines on two narrow slits separated by a distance 1.0 mm and illuminates a screen at a distance 1.5 m away. When one slit is covered by a thin glass plate of refractive index 1.8 and other slit by a thin glass plate of refractive index μ, the central maxima shifts by 0.1 rad. Both plates have the same thickness of 0.5 mm. The value of refractive index μ of the glass is

a. 1.4 b. 1.5
c. 1.6 d. none of these

52. A plane wavefront traveling in a straight line in vacuum encounters a medium of refractive index m. At P, the shape of the wavefront is

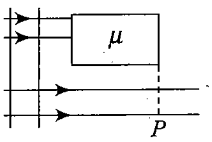
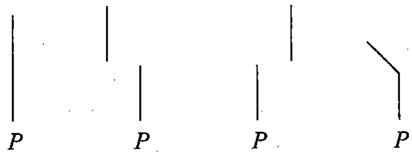


Fig. 2.93



a. b. c. d.

53. In Young's double-slit experiment, the wavelength of light was changed from 7000 Å to 3500 Å. While doubling the separation between the slits, which of the following is not true for this experiment?

a. The width of fringes changes.
b. The colour of bright fringes changes.
c. The separation between successive bright fringes changes.
d. The separation between successive dark fringes remains unchanged.

54. Calculate the wavelength of light used in an interference experiment from the following data: Fringe width = 0.03 cm. Distance between slits and eyepiece through which the interference pattern is observed is 1 m. Distance between the images of the virtual source when a convex lens of focal length 16 cm is used at a distance of 80 cm from the eyepiece is 0.8 cm.

a. 6000 Å b. 0.00006 Å
c. 6000 cm d. 0.00006 m

55. In Young's double-slit experiment, the angular width of a fringe formed on a distant screen is 1°. The wavelength of light used is 6000 Å. What is the spacing between the slits?

a. 344 mm b. 0.1344 mm
c. 0.0344 mm d. 0.034 mm

56. In a double-slit experiment, the distance between the slits is d. The screen is at a distance D from the slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is

a. $\dfrac{d^2}{2\lambda D}$ b. $\dfrac{d}{2\lambda D}$

c. $\dfrac{d^2}{4\lambda D}$ d. 0

57. In Young's double-slit experiment using monochromatic light, the light pattern shifts by a certain distance on the screen when a mica sheet of refractive index μ and thickness t microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of light?

a. $(1/2)t(\mu-1)$ b. $t(\mu-1)$
c. μt d. $3\mu t$

58. In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit S_1 so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

a. 5×10^{-6} m b. 3×10^{-6} m
c. 10×10^{-6} m d. 5×10^{-5} m

59. In Young's double-slit experiment, the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slits. It is found that the ninth bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern. The wavelength of the light used is

a. 5000 Å b. $\frac{5000}{7}$ Å
c. 2500 Å d. $\frac{2500}{7}$ Å

60. Fig. 2.94 shows two coherent sources S_1 and S_2 emitting wavelength λ. The separation $S_1S_2 = 1.5\lambda$ and S_1 is ahead in phase by $\pi/2$ relative to S_2. Then, the maxima occur in direction θ given by $\sin^{-1}$ of

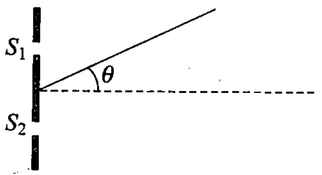


Fig. 2.94

(i) 0 (ii) 1/2
(iii) –1/6 (iv) –5/6

Correct options are
a. (ii), (iii) and (iv) b. (i), (ii) and (iii)
c. (i), (iii) and (iv) d. All the above

61. Two waves of light in air have the same wavelength and are initially in phase. They then travel through plastic layers with thicknesses of L_1 = 3.5 mm and L_2 = 5.0 mm and indices of refraction n_1 = 1.7 and n_2 = 1.25 as shown in the Fig. 2.95. The rays later arrive at a common point. The longest wavelength of light for which constructive interference occurs at the point is

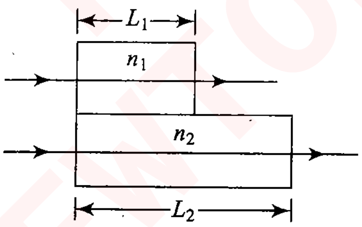


Fig. 2.95

a. 0.8 μm b. 1.2 μm
c. 1.7 μm d. 2.9 μm

62. Light from a source emitting two wavelengths λ_1 and λ_2 is allowed to fall on Young's double-slit apparatus after filtering one of the wavelengths. The position of interference maxima is noted. When the filter is removed both the wavelengths are incident and it is found that maximum intensity is produced where the fourth maxima occured previously. If the other wavelength is filtered, at the same location the third maxima is found. What is the ratio of wavelengths?

a. $\frac{2}{3}$ b. $\frac{3}{2}$
c. $\frac{3}{4}$ d. $\frac{4}{3}$

63. The wavefront of a light beam is given by the equation $x + 2y + 3x = c$ (where c is arbitrary constant), then the angle made by the direction of light with the y-axis is

a. $\cos^{-1}\frac{1}{\sqrt{14}}$ b. $\sin^{-1}\frac{2}{\sqrt{14}}$
c. $\cos^{-1}\frac{2}{\sqrt{14}}$ d. $\sin^{-1}\frac{3}{\sqrt{14}}$

64. As shown in Fig. 2.96, waves with identical wavelengths and amplitudes and which are initially in phase travel through difference media. Ray 1 travels through air and Ray 2 through a transparent medium for equal length L, in four different situations. In each situation, the two rays reach a common point on the screen. The number of wavelengths in length L is N_2 for Ray 2 and N_1 for Ray 1. In the following table, values of N_1 and N_2 are given for all four situations. The order of the situations according to the intensity of the light at the common point in descending order is

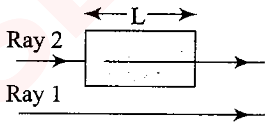


Fig. 2.96

Situations	1	2	3	4
N_1	2.25	1.80	3.00	3.25
N_2	2.75	2.80	3.25	4.00

a. $I_3 = I_4 > I_2 > I_1$ b. $I_1 > I_3 => I_4 > I_2$
c. $I_1 > I_2 > I_3 > I_4$ d. $I_2 > I_3 = I_4 > I_1$

65. If the distance between the first maxima and fifth minima of a double-slit pattern is 7 mm and the slits are separated by 0.15 mm with the screen 50 cm from the slits, then wavelength of the light used is

a. 600 nm b. 525 nm
c. 467 nm d. 420 nm

66. In a YDSE, D = 1 m, d = 1 mm and λ = 5000 nm. The distance of 100th maxima from the central maxima is

a. $\frac{1}{2}$ m b. $\frac{\sqrt{3}}{2}$ m
c. $\frac{1}{\sqrt{3}}$ m d. does not exist

67. Let S_1 and S_2 be the two slits in Young's double-slit experiment. If central maxima is observed at P and angle $\angle S_1PS_2 = \theta$, then the fringe width for the light of wavelength λ will be (assume θ to be a small angle)

a. λ/θ b. $\lambda\theta$
c. $2\lambda/\theta$ d. $\lambda/2\theta$

68. Figure 2.97 shows two coherent sources S_1 and S_2 vibrating in same phase. AB is an irregular wire lying at a far distance from the sources S_1 and S_2. Let $\frac{\lambda}{d} = 10^{-3}$ $\angle BOA = 0.12°$. How many bright spots will be seen on the wire, including points A and B?

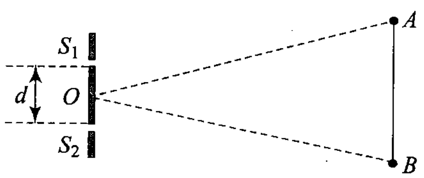


Fig. 2.97

a. 2 **b.** 3
c. 4 **d.** more than 4

69. The path difference between two interfering waves at a point on the screen is $\lambda/6$. The ratio of intensity at this point and that at the central bright fringe will be (assume that intensity due to each slit in same)

a. 0.853 **b.** 8.53
c. 0.75 **d.** 7.5

70. In a YDSE shown in Fig. 2.98, a parallel beam of light is incident on the slits from a medium of refractive index n_1. The wavelength of light in this medium is λ_1. A transparent slab of thickness 't' and refractive index n_3 is put in front of one slit. The medium between the screen and the plane of the slits is n_2. The phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits) is

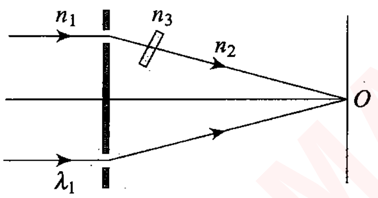


Fig. 2.98

a. $\dfrac{2\pi}{n_1\lambda_1}(n_3 - n_2)t$ **b.** $\dfrac{2\pi}{\lambda_1}(n_3 - n_2)t$

c. $\dfrac{2\pi n_1}{n_2\lambda_1}\left(\dfrac{n_3}{n_2} - 1\right)t$ **d.** $\dfrac{2\pi n_1}{\lambda_1}(n_3 - n_2)t$

71. In Fig. 2.99, if a parallel beam of while light is incident on the plane of the slits, then the distance of the nearest white spot on the screen from O is [assume $d \ll D$, $\lambda \ll d$]

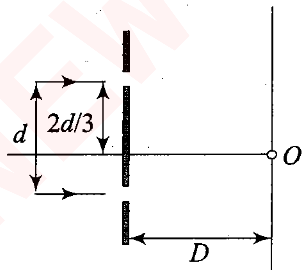


Fig. 2.99

a. 0 **b.** $d/2$
c. $d/3$ **d.** $d/6$

72. In Fig. 2.100, a parallel beam of light is incident on the plane of the slits of a Young's double-slit experiment. Light incident on the slit S_1 passes through a medium of variable refractive index $\mu = 1 + ax$ (where 'x' is the distance from the plane of slits as shown), up to a distance 'l' before falling on S_1. Rest of the space is filled with air. If at 'O' a minima is formed, then the minimum value of the positive constant a (in terms of l and wavelength 'λ' in air) is

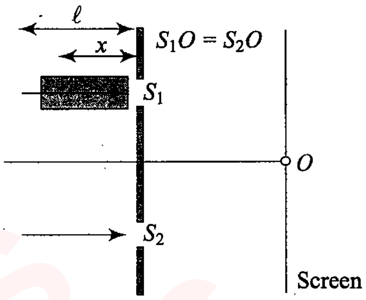


Fig. 2.100

a. $\dfrac{\lambda}{\ell}$ **b.** $\dfrac{\lambda}{\ell^2}$

c. $\dfrac{\ell^2}{\lambda}$ **d.** none of these

73. Interference fringes were produced using light in a double-slit experiment. When a mica sheet of uniform thickness and refractive index 1.6 (relative to air) is placed in the path of light from one of the slits, the central fringe moves through some distance. This distance is equal to the width of 30 interference bands if light of wavelength 4800 is used. The thickness (in μm) of mica is

a. 90 **b.** 12
c. 14 **d.** 24

74. Two coherent light sources, each of wavelength λ, are separated by a distance 3λ. The maximum number of minima formed on line AB, which runs from $-\infty$ to $+\infty$, is

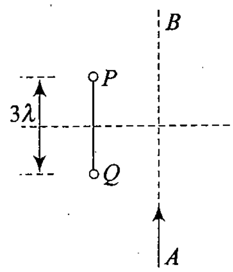


Fig. 2.101

a. 2 **b.** 4
c. 6 **d.** 8

75. M_1 and M_2 are plane mirrors and kept parallel to each other. At point O, there will be a maxima for wavelength λ. Light from a monochromatic source S of wavelength λ is not reaching directly on the screen. Then, λ is

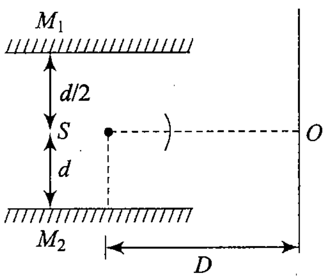


Fig. 2.102

a. $\dfrac{3d^2}{D}$ **b.** $\dfrac{3d^2}{2D}$

c. $\dfrac{d^2}{D}$ **d.** $\dfrac{2d^2}{D}$

76. A ray of light is incident on a thin film. As shown in Fig. 2.103, M and N are two reflected rays while P and Q are two transmitted rays. Rays N and Q undergo a phase change of π. Correct ordering of the refracting indices is

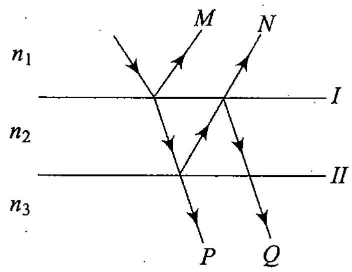


Fig. 2.103

a. $n_2 > n_3 > n_1$ **b.** $n_3 > n_2 > n_1$
c. $n_3 > n_1 > n_2$
d. none of these, the specified changes cannot occur

77. One of the two slits in YDSE is painted over, so that is transmits only light waves having intensity half of the intensity of the light waves through the other slit. As a result of this
a. fringe pattern disappears
b. bright fringes become brighten and dark ones become darker
c. dark and bright fringes get fainter
d. dark fringes get brighter and bright fringes get darker

78. In YDSE, coherent monochromatic light having wavelength 600 nm falls on the slits. First-order bright fringe is at 4.84 mm from central maxima. Determine the wavelength for which the first-order dark fringe will be observed at the same location on screen. (Take $D = 3$ m.)
a. 600 nm **b.** 1200 nm
c. 300 nm **d.** 900 nm

79. The YDSE apparatus is as shown in Fig. 2.104. The condition for point P to be a dark fringe is

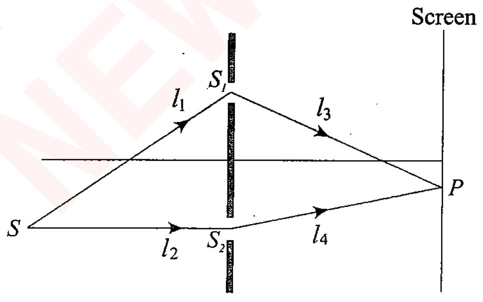


Fig. 2.104

(l = wavelength of light waves)
a. $(l_1 - l_3) + (l_2 - l_4) = n\lambda$
b. $(l_1 - l_2) + (l_3 - l_4) = n\lambda$
c. $(l_1 + l_3) + (l_2 + l_4) = \dfrac{(2n-1)\lambda}{2}$
d. $(l_1 - l_2) + (l_4 - l_3) = \dfrac{(2n-1)\lambda}{2}$

80. Consider the optical system shown in Fig. 2.105. The point source of light S is having wavelength equal to λ. The light is reaching screen only after reflection. For point P to be 2nd maxima, the value of λ would be ($D \gg d$ and $d \gg \lambda$)

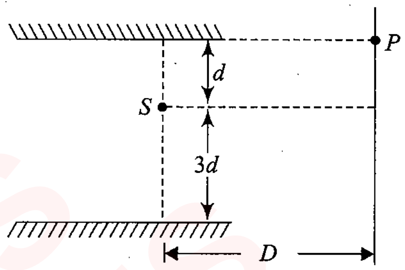


Fig. 2.105

a. $\dfrac{12d^2}{D}$ **b.** $\dfrac{6d^2}{D}$
c. $\dfrac{3d^2}{D}$ **d.** $\dfrac{24d^2}{D}$

81. In YDSE, if a bichromatic light having wavelengths λ_1 and λ_2 is used, then the maxima due to both lights will overlap at a certain distance y of from the central maxima. Take separation between slits as d and distance between screen and slits as D. Then, the value of y will be
a. $\left(\dfrac{\lambda_1 + \lambda_2}{2D}\right)d$ **b.** $\dfrac{\lambda_1 - \lambda_2}{D} \times 2d$
c. LCM of $\dfrac{\lambda_1 D}{d}$ and $\dfrac{\lambda_2 D}{d}$ **d.** HCF of $\dfrac{\lambda_1 D}{d}$ and $\dfrac{\lambda_2 D}{d}$

82. In YDSE, $d = 2$ mm $D = 2$ m and $\lambda = 500$ nm. If intensities of two slits are I_0 and $9I_0$, then find intensity at $y = \dfrac{1}{6}$ mm.
a. $7I_0$ **b.** $10I_0$
c. $16I_0$ **d.** $4I_0$

83. In YDSE, let A and B be two slits. Films of thicknesses t_A and t_B and refractive indices μ_A and μ_B are placed in front of A and B, respectively. If $\mu_A t_A = \mu_A t_B$, then the central maxima will
a. not shift
b. shift towards A
c. shift towards B
d. (b) if $t_B < t_A$ and (c) if $t_B < t_A$

84. In YDSE, find the missing wavelength at $y = d$, where symbols have their usual meaning (take $D \gg d$).
a. $\dfrac{d^2}{D}$ **b.** $\dfrac{2d^2}{7D}$
c. $\dfrac{3d^2}{D}$ **d.** $\dfrac{d^2}{3D}$

85. In YDSE, the amplitude of intensity variation of the two sources is found to be 5% of the average intensity. The ratio of the intensities of two interfering sources is
a. 2564 **b.** 1089
c. 1681 **d.** 869

86. In YDSE, water is filled in the space between the slits and screen. Then,

a. fringe pattern shifts upward but fringe width remains unchanged
b. fringe width decreases and fringe pattern shifts upward
c. fringe width remains unchanged and central fringe does not shift
d. fringe width decreases and fringe pattern does not shift

87. In YDSE, having slits of equal width, let β be the fringe width and I_0 be the maximum intensity. At a distance x from the central bright fringe, the intensity will be

a. $I_0 \cos\left(\frac{x}{\beta}\right)$ b. $I_0 \cos^2 \frac{2\pi x}{\beta}$

c. $I_0 \cos^2 \frac{\pi x}{\beta}$ d. $\frac{I_0}{4} \cos^2 \frac{\pi x}{\beta}$

88. Two identical coherent sources of wavelength λ are placed at $(100\lambda, 0)$ and $(-50\lambda, 0)$, respectively. A detector moves slowly from the origin to $(50\lambda, 0)$ along x-axis. The number of maxima and minima detected are, respectively [include origin and $(50\lambda, 0)$]
a. 51 and 50 b. 101 and 100
c. 49 and 50 d. 50 and 49

89. Two thin films of the same material but different thickness are separated by air. Monochromatic light is incident on the first film. When viewed normally from point A, the second film appears dark.

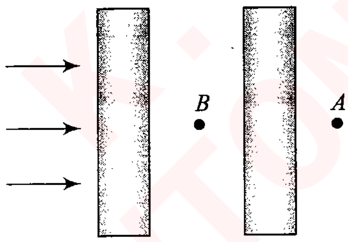


Fig. 2.106

From point B, on normal viewing
a. the first film will appear bright
b. the first film will appear dark
c. the second film will appear bright
d. the second film will appear dark

90. Consider an YDSE that has different slit widths. As a result, amplitude of waves from two slits are A and $2A$, respectively. If I_0 be the maximum intensity of the interference pattern, then intensity of the pattern at a point where phase difference between waves is ϕ is

a. $I_0 \cos^2 \phi$ b. $\frac{I_0}{3} \sin^2 \frac{\phi}{2}$

c. $\frac{I_0}{9}[5 + 4\cos\phi]$ d. $\frac{I_0}{9}[5 + 8\cos\phi]$

91. In YDSE of equal width slits, if intensity at the center of screen is I_0, then intensity at a distance of $\beta/4$ from the central maxima is

a. I_0 b. $\frac{I_0}{2}$

c. $\frac{I_0}{4}$ d. $\frac{I_0}{3}$

92. Two transparent slabs have the same thickness as shown in Fig. 2.107. One is made of material A of refractive index 1.5. The other is made of two materials B and C with thickness in the ratio 1:2. The refractive index of C is 1.6. If a monochromatic parallel beam passing through the slabs has the same number of wavelengths inside both, the refractive index of B is

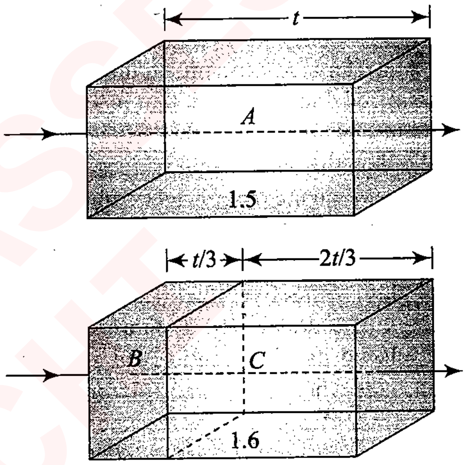


Fig. 2.107

a. 1.1 b. 1.2
c. 1.3 d. 1.4

93. Light of wavelength λ_0 in air enters a medium of refractive index n. If two points A and B in this medium lie along the path of this light at a distance x, then phase difference ϕ_0 between these two points is

a. $\phi_0 = \frac{1}{n}\left(\frac{2\pi}{\lambda_0}\right)x$ b. $\phi_0 = n\left(\frac{2\pi}{\lambda_0}\right)x$

c. $\phi_0 = (n-1)\left(\frac{2\pi}{\lambda_0}\right)x$ d. $\phi_0 = \frac{1}{(n-1)}\left(\frac{2\pi}{\lambda_0}\right)x$

94. In a Young's double-slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen 2 m from the slits, where a bright fringe from one interference pattern coincides with a bright fringe from the other, is
a. 1.5 mm b. 3 mm
c. 4.5 mm d. 6 mm

95. In Young's interference experiment, if the slits are of unequal width, then
a. no fringes will be formed
b. the positions of minimum intensity will not be completely dark

c. bright fringe as displaced from the original central position

d. distance between two consecutive dark fringes will not be equal to the distance between two consecutive bright fringes

96. Two wavelengths of light λ_1 and λ_2 are sent through a Young's double-slit apparatus simultaneously. If the third-order bright fringe coincides with the fourth-order bright fringe, then

a. $\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$ b. $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$

c. $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$ d. $\frac{\lambda_1}{\lambda_2} = \frac{4}{5}$

97. In Young's interference experiment, the central bright fringe can be identified due to the fact that it

a. has greater intensity than other fringes which are bright

b. is wider than the other bright fringes

c. is narrower than the other bright fringes

d. can be obtained by using white light instead of monochromatic light

98. A flake of glass (refractive index 1.5) is placed over one of the openings of a double-slit apparatus. The interference pattern displaced itself through seven successive maxima towards the side where the flake is placed. If wavelength of the light is $\lambda = 600$ nm, then the thickness of the flake is

a. 2100 nm b. 4200 nm

c. 8400 nm d. none of above

99. Two identical sources each of intensity I_0 have a separation $d = \frac{\lambda}{8}$, where λ is the wavelength of the waves emitted by either source. The phase difference of the sources is $\frac{\pi}{4}$. The intensity distribution $I(\theta)$ in the radiation field as a function of θ, which specifies the direction from the sources to the distant observation point P is given by

a. $I(\theta) = I_0 \cos^2 \theta$

b. $I(\theta) = \frac{I_0}{4} \cos^2 \left(\frac{\pi\theta}{8}\right)$

c. $I(\theta) = 4I_0 \cos^2 \left[\frac{\pi}{8}(\sin\theta + 1)\right]$

d. $I(\theta) = I_0 \sin^2 \theta$

100. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then, in the interference pattern

a. the intensities of both the maxima and the minima increases

b. the intensity of the maximum increase and minima has zero intensity

c. the intensity of the maxima decreases and that of minima increases

d. the intensity of the maxima decreases and the minima has zero intensity

101. In Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by

a. 12 b. 18

c. 24 d. 30

102. A certain region of a soap bubble reflects red light of vacuum wavelength $\lambda = 650$ nm. What is the minimum thickness that this region of the soap bubble could have? Take the index of reflection of the soap film to be 1.41.

a. 1.2×10^{-7} m b. 650×10^{-9} m

c. 120×10^{7} m d. 650×10^{5} m

103. In Young's double-slit experiment using monochromatic light of wavelength λ, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 2.0 μm is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.

a. 4500 Å b. 5700 Å

c. 6000 Å d. 4000 Å

104. In YDSE, when a glass plate of refractive index 1.5 and thickness t is placed in the path of one of the interfering beams (wavelength λ), intensity at the position where central maximum occured previously remains unchanged. The minimum thickness of the glass plate is

a. 2λ b. $(2/3)\lambda$

c. $\lambda/3$ d. λ

105. In Young's double-slit experiment, the intensity of light at a point on the screen where path difference is λ is I. If intensity at a point is $I/4$, then possible path difference at this point are

a. $\lambda/2, \lambda/3$ b. $\lambda/3, 2\lambda/3$

c. $\lambda/3, \lambda/4$ d. $2\lambda/3, \lambda/4$

106. Young's double-slit experiment is carried with two thin sheets of thickness 10.4 μm each and refractive index $\mu_1 = 1.52$ and $\mu_2 = 1.40$ covering the slits S_1 and S_2, respectively. If white light of range 400 nm to 780 nm is used, then which wavelength will form maxima exactly at point O, the centre of the screen?

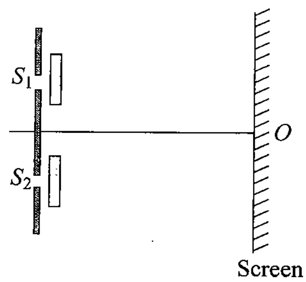


Fig. 2.108

a. fringe pattern shifts upward but fringe width remains unchanged
b. fringe width decreases and fringe pattern shifts upward
c. fringe width remains unchanged and central fringe does not shift
d. fringe width decreases and fringe pattern does not shift

87. In YDSE, having slits of equal width, let β be the fringe width and I_0 be the maximum intensity. At a distance x from the central bright fringe, the intensity will be

a. $I_0 \cos\left(\frac{x}{\beta}\right)$ b. $I_0 \cos^2 \frac{2\pi x}{\beta}$

c. $I_0 \cos^2 \frac{\pi x}{\beta}$ d. $\frac{I_0}{4} \cos^2 \frac{\pi x}{\beta}$

88. Two identical coherent sources of wavelength λ are placed at $(100\lambda, 0)$ and $(-50\lambda, 0)$, respectively. A detector moves slowly from the origin to $(50\lambda, 0)$ along x-axis. The number of maxima and minima detected are, respectively [include origin and $(50\lambda, 0)$]

a. 51 and 50 b. 101 and 100
c. 49 and 50 d. 50 and 49

89. Two thin films of the same material but different thickness are separated by air. Monochromatic light is incident on the first film. When viewed normally from point A, the second film appears dark.

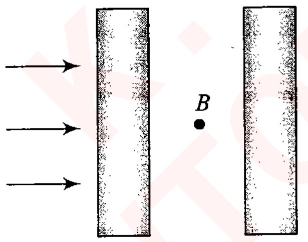


Fig. 2.106

From point B, on normal viewing
a. the first film will appear bright
b. the first film will appear dark
c. the second film will appear bright
d. the second film will appear dark

90. Consider an YDSE that has different slit widths. As a result, amplitude of waves from two slits are A and $2A$, respectively. If I_0 be the maximum intensity of the interference pattern, then intensity of the pattern at a point where phase difference between waves is ϕ is

a. $I_0 \cos^2 \phi$ b. $\frac{I_0}{3} \sin^2 \frac{\phi}{2}$

c. $\frac{I_0}{9}[5 + 4\cos\phi]$ d. $\frac{I_0}{9}[5 + 8\cos\phi]$

91. In YDSE of equal width slits, if intensity at the center of screen is I_0, then intensity at a distance of $\beta/4$ from the central maxima is

a. I_0 b. $\frac{I_0}{2}$

c. $\frac{I_0}{4}$ d. $\frac{I_0}{3}$

92. Two transparent slabs have the same thickness as shown in Fig. 2.107. One is made of material A of refractive index 1.5. The other is made of two materials B and C with thickness in the ratio 1:2. The refractive index of C is 1.6. If a monochromatic parallel beam passing through the slabs has the same number of wavelengths inside both, the refractive index of B is

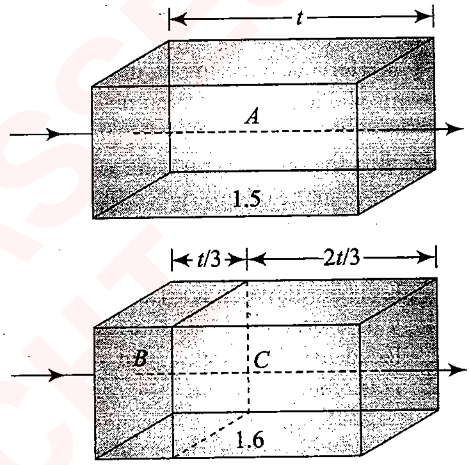


Fig. 2.107

a. 1.1 b. 1.2
c. 1.3 d. 1.4

93. Light of wavelength λ_0 in air enters a medium of refractive index n. If two points A and B in this medium lie along the path of this light at a distance x, then phase difference ϕ_0 between these two points is

a. $\phi_0 = \frac{1}{n}\left(\frac{2\pi}{\lambda_0}\right)x$ b. $\phi_0 = n\left(\frac{2\pi}{\lambda_0}\right)x$

c. $\phi_0 = (n-1)\left(\frac{2\pi}{\lambda_0}\right)x$ d. $\phi_0 = \frac{1}{(n-1)}\left(\frac{2\pi}{\lambda_0}\right)x$

94. In a Young's double-slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen 2 m from the slits, where a bright fringe from one interference pattern coincides with a bright fringe from the other, is

a. 1.5 mm b. 3 mm
c. 4.5 mm d. 6 mm

95. In Young's interference experiment, if the slits are of unequal width, then
a. no fringes will be formed
b. the positions of minimum intensity will not be completely dark

c. bright fringe as displaced from the original central position
d. distance between two consecutive dark fringes will not be equal to the distance between two consecutive bright fringes

96. Two wavelengths of light λ_1 and λ_2 are sent through a Young's double-slit apparatus simultaneously. If the third-order bright fringe coincides with the fourth-order bright fringe, then

a. $\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$ **b.** $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$

c. $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$ **d.** $\frac{\lambda_1}{\lambda_2} = \frac{4}{5}$

97. In Young's interference experiment, the central bright fringe can be identified due to the fact that it
a. has greater intensity than other fringes which are bright
b. is wider than the other bright fringes
c. is narrower than the other bright fringes
d. can be obtained by using white light instead of monochromatic light

98. A flake of glass (refractive index 1.5) is placed over one of the openings of a double-slit apparatus. The interference pattern displaced itself through seven successive maxima towards the side where the flake is placed. If wavelength of the light is $\lambda = 600$ nm, then the thickness of the flake is
a. 2100 nm **b.** 4200 nm
c. 8400 nm **d.** none of above

99. Two identical sources each of intensity I_0 have a separation $d = \frac{\lambda}{8}$, where λ is the wavelength of the waves emitted by either source. The phase difference of the sources is $\frac{\pi}{4}$. The intensity distribution $I(\theta)$ in the radiation field as a function of θ, which specifies the direction from the sources to the distant observation point P is given by
a. $I(\theta) = I_0 \cos^2 \theta$
b. $I(\theta) = \frac{I_0}{4}\cos^2\left(\frac{\pi\theta}{8}\right)$
c. $I(\theta) = 4I_0 \cos^2\left[\frac{\pi}{8}(\sin\theta + 1)\right]$
d. $I(\theta) = I_0 \sin^2 \theta$

100. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then, in the interference pattern
a. the intensities of both the maxima and the minima increases
b. the intensity of the maximum increase and minima has zero intensity
c. the intensity of the maxima decreases and that of minima increases
d. the intensity of the maxima decreases and the minima has zero intensity

101. In Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by
a. 12 **b.** 18
c. 24 **d.** 30

102. A certain region of a soap bubble reflects red light of vacuum wavelength $\lambda = 650$ nm. What is the minimum thickness that this region of the soap bubble could have? Take the index of reflection of the soap film to be 1.41.
a. 1.2×10^{-7} m **b.** 650×10^{-9} m
c. 120×10^{7} m **d.** 650×10^{5} m

103. In Young's double-slit experiment using monochromatic light of wavelength λ, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 2.0 μm is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.
a. 4500 Å **b.** 5700 Å
c. 6000 Å **d.** 4000 Å

104. In YDSE, when a glass plate of refractive index 1.5 and thickness t is placed in the path of one of the interfering beams (wavelength λ), intensity at the position where central maximum occured previously remains unchanged. The minimum thickness of the glass plate is
a. 2λ **b.** $(2/3)\lambda$
c. $\lambda/3$ **d.** λ

105. In Young's double-slit experiment, the intensity of light at a point on the screen where path difference is λ is I. If intensity at a point is $I/4$, then possible path difference at this point are
a. $\lambda/2$, $\lambda/3$ **b.** $\lambda/3$, $2\lambda/3$
c. $\lambda/3$, $\lambda/4$ **d.** $2\lambda/3$, $\lambda/4$

106. Young's double-slit experiment is carried with two thin sheets of thickness 10.4 μm each and refractive index $\mu_1 = 1.52$ and $\mu_2 = 1.40$ covering the slits S_1 and S_2, respectively. If white light of range 400 nm to 780 nm is used, then which wavelength will form maxima exactly at point O, the centre of the screen?

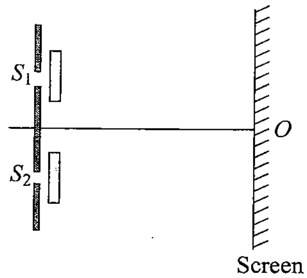


Fig. 2.108

a. 416 nm only **b.** 624 nm only
c. 416 nm and 624 nm only **d.** None of these

107. High-quality camera lenses are often coated to prevent reflection. A lens has an optical index of refraction of 1.72 and a coating with an optical index of refraction 1.31. For near normal incidence, the minimum thickness of the coating to prevent reflection for wavelength of 5.3×10^{-7} m is
a. 0.75 μm **b.** 0.2 mm
c. 0.1 μm **d.** 1.75 mm

108. A long horizontal slit is placed 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1 m away from the slit. If the mirror reflects only 64% of the light falling on it, the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen is
a. 8:1 **b.** 3:1
c. 81:1 **d.** 9:1

109. In a Young's double-slit experiment, $\lambda = 500$ nm, $d = 1$ nm, and $D = 1$ m. The minimum distance from the central maximum for which the intensity is half of the maximum intensity is
a. 2×10^{-4} m **b.** 1.25×10^{-4} m
c. 4×10^{-4} m **d.** 2.5×10^{-4} m

110. In a Young's double-slit experiment setup, source S of wavelength 50 nm illuminates two slits S_1 and S_2 which act as two coherent sources. The source S oscillates about its own position according to the equation $y = 0.5 \sin \pi t$, where y is in nm and t in seconds. The minimum value of time t for which the intensity at point P on the screen exactly in front of the upper slit becomes minimum is

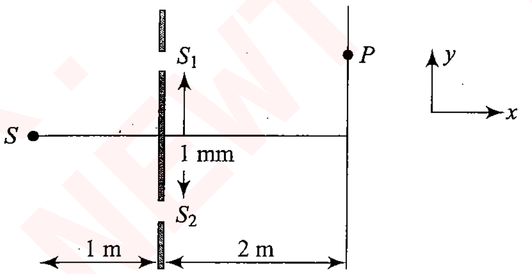


Fig. 2.109

a. 1 s **b.** 2 s
c. 3 s **d.** 1.5 s

111. Intensity observed in an interference pattern is $I = I_0 \sin^2 \theta$. At $\theta = 30°$, intensity $I = 5 \pm 0.002$. The percentage error in angle is
a. $4\sqrt{3} \times 10^{-2}$ % **b.** $\dfrac{4}{\pi} \times 10^{-2}$ %
c. $\dfrac{4\sqrt{3}}{\pi} \times 10^{-2}$ % **d.** $\sqrt{3} \times 10^{-2}$ %

112. A thin uniform film of refractive index 1.75 is placed on a sheet of glass of refractive index 1.5. At room temperature (20°C), this film is just thick enough for light with wavelength 600 nm reflected off the top of the film to be canceled by light reflected from the top of the glass. After the glass is placed in an oven and slowly heated to 170°C, the film concels reflected light with wavelength 606 nm. The coefficient of linear expansion of the film is (Ignore any changes in the refractive index of the film due to the temperature change.)
a. 3.3×10^{-5} °C^{-1}. **b.** 6.6×10^{-5} °C^{-1}
c. 9.9×10^{-5} °C^{-1} **d.** 2.2×10^{-5} °C^{-1}

113. Two slits spaced 0.25 mm apart are placed 0.75 m from a screen and illuminated by coherent light with a wavelength of 650 nm. The intensity at the center of the central maximum ($\theta = 0°$) is I_0. The distance on the screen from the center of the central maximum to the point where the intensity has fallen to $I_0/2$ is nearly
a. 0.1 nmn **b.** .25 mm
c. 0.4 mm **d.** 0.5 mm

114. Two thin parallel slits that are 0.012 mm apart are illuminated by a laser beam of wavelength 650 nm. On a very large distant screen, the total number of bright fringes including the central fringe and those on both sides of it is
a. 38 **b.** 37
c. 40 **d.** 39

115. The index of refraction of a glass plate is 1.48 at $\theta_1 = 30°$C and varies linearly with temperature with a coefficient of 2.5×10^{-5} °C^{-1}. The coefficient of linear expansion of the glass is 5×10^{-5} °C^{-1}. At 30°C, the length of the glass plate is 3 cm. This plate is placed in front of one of the slits in Young's double-slit experiment. If the plate is being heated so that its temperature increases at a rate of 5°C^{-1} min, the light source has wavelength $\lambda = 589$ nm and the glass plate initially is at $\theta = 30°$C. The number of fringes that shift on the screen in each minute is nearly (use approximation)
a. 1 **b.** 11
c. 110 **d.** 1.1×10^3

116. Two thin parallel slits are made in an opaque sheet of film when a monochromatic beam of light is shone through them at normal incidence. The first bright fringes in the transmitted light occur at ±45° with the original direction of the light beam on a distant screen when the apparatus is in air. When the apparatus is immersed in a liquid, the same bright fringes now occur at ±30°. The index of refraction of the liquid is
a. $\sqrt{2}$ **b.** $\sqrt{3}$
c. $\dfrac{4}{3}$ **d.** $\dfrac{3}{2}$

Multiple Correct Answers Type

Solutions on page 2.79

1. Two monochromatic coherent point sources S_1 and S_2 are separated by a distance L. Each source emits light of wavelength λ; where $L \gg \lambda$. The line S_1S_2 when extended meets a screen perpendicular to it at a point A. Then,

a. the interference fringes on the screen are circular in shape
b. the interference fringes on the screen are straight lines perpendicular to the line $S_1 S_2 A$
c. the point A is an intensity maxima if $L = n\lambda$
d. the point A is always an intensity maxima for any separation L

2. In a Young's double-slit experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let β = fringe width. Then, the central maxima will shift
a. towards A
b. towards B
c. by $t(\mu - 1)\dfrac{\beta}{\lambda}$
d. by $\mu t \dfrac{\beta}{\lambda}$

3. If the first minima in a Young's double-slit experiment occurs directly in front of one of the slits (distance between slit and screen $D = 12$ cm and distance between slits $d = 5$ cm), then the wavelength of the radiation used can be
a. 2 cm
b. 4 cm
c. $\dfrac{2}{3}$ cm
d. $\dfrac{4}{3}$ cm

4. If one of the slits of a standard Young's double-slit experiment is covered by a thin parallel sides glass slab so that it transmits only one-half the light intensity of the other, then
a. the fringe pattern will get shifted toward the covered slit
b. the fringe pattern will get shifted away from the covered slit
c. the bright fringes will becomes less bright and the dark ones will becomes more bright
d. the fringe width will remain unchanged

5. A parallel beam of light (λ = 500) is incident at an angle α = 30° with the normal to the slit plane in a Young's double-slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 and S_2. The distance between slits is 1 mm. Then,

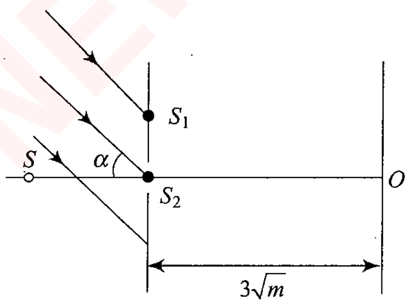


Fig. 2.110

a. the intensity at O is $4I_0$
b. the intensity at O is zero
c. the intensity at a point on the screen 1 m below O is $4I_0$
d. the intensity at a point on the screen 1 m below O is zero

6. Two points monochromatic and coherent sources of light of wavelength λ each are placed as shown in Fig. 2.111. The initial phase difference between the sources is zero O. ($D \gg d$). Mark the correct statement(s).

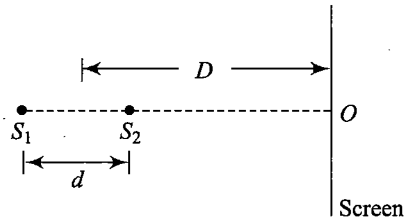


Fig. 2.111

a. If $d = \dfrac{7\lambda}{2}$, O will be a minima.
b. If $d = \lambda$, only one maxima can be observed on the screen.
c. If $d = 4.8\ \lambda$, then total 10 minima would be there on the screen
d. If $d = \dfrac{5\lambda}{2}$, the intensity at O would be minimum.

7. Consider a film of thickness L as shown in four different cases below. Notice the observation of film with perpendicularly falling light. Mark the correct statement(s).

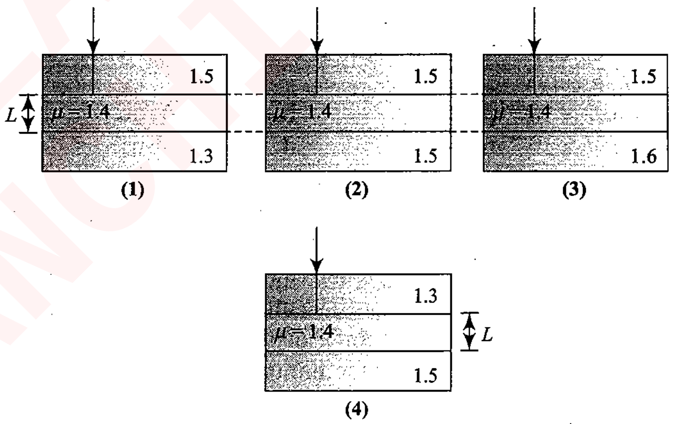


Fig. 2.112

(Take $L = 1.5\ \lambda$)
a. For (1) and (2), the reflection at film interfaces causes zero phase difference for two reflected rays.
b. For (2) and (3), the reflection at film interfaces causes a phase difference of π for two reflected rays.
c. For (1), the film will appear dark, if it is observed through reflected rays from film interfaces.
d. For (3), the film will appear dark, if it is observed through reflected rays from film interfaces.

8. In YDSE, the source is placed symmetrical to the slits. If a transparent slab is placed in front of the upper slit, then (slab can absorb a fraction of light)
a. intensity of central maxima may change
b. intensity of central maxima may not change
c. central maxima will be shifted up
d. intensity of dark fringes will be always zero

9. A transparent slab of thickness t and refractive index μ is inserted in front of upper slit of YDSE apparatus. The wavelength of light used is λ. Assume that there is no absorption of light by the slab. Mark the correct statement(s).

a. The intensity of dark fringes will be 0, if slits are identical.

b. The change in optical path due to insertion of plate is μt.

c. The change in optical path due to insertion of plate is $(\mu - 1)t$.

d. For making intensity zero at center of screen, the thickness can be $\dfrac{5\lambda}{2(\mu - 1)}$.

10. A light wave of wavelength λ_0 propagates from point A to point B. We introduce in its path a glass plate of refractive index n and thickness ℓ. The introduction of the plate alters the phase of the plate at B by an angle ϕ. If λ is the wavelength of light on emerging from the plate, then

a. $\Delta\phi = 0$ **b.** $\Delta\phi = \dfrac{2\pi\ell}{\lambda_0}$

c. $\Delta\phi = 2\pi\ell\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right)$ **d.** $\Delta\phi = \dfrac{2\pi\ell}{\lambda_0}(n-1)$

11. A ray of light of wavelength u_0 and frequency v_0 enters a glass slab of refractive index μ from air. Then,

a. its wavelength increases, frequency decreases

b. its wavelength decreases, frequency remains same

c. its wavelength increases, frequency remains same

d. $\Delta\lambda = \lambda_0\left(\dfrac{1}{\mu} - 1\right)$ and $\Delta v = 0$

12. In the Young's double-slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that

a. the intensities at the screen due to the two slits are 5 units and 4 units, respectively

b. the intensities at the screen due to the two slits are 4 units and 1 units, respectively

c. the amplitude ratio is 3

d. the amplitude ratio is 2

13. In Young's double-slit experiment, two wavelengths of light are used simultaneously where $\lambda_2 = 2\lambda_1$. In the fringe pattern observed on the screen,

a. maxima of wavelength λ_2 can coincide with minima of wavelength λ_1.

b. fringe width of λ_2 will be double that of fringe width of λ_1 and nth order maxima of λ_2 will coincide with 2nd order maxima of λ_1

c. nth order minima of λ_2 will coincide with $2n$th order minima of λ_1

d. none of the above

14. The minimum value of d so that there is a dark fringe at O is d_{min}. For the value of d_{min}, the distance at which the next bright fringe is formed is x. Then,

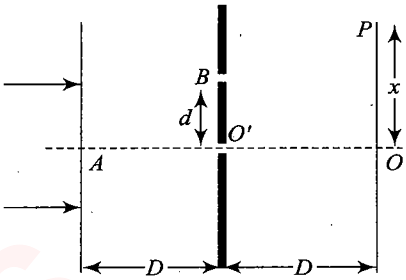


Fig. 2.113

a. $d_{\text{min}} = \sqrt{\lambda D}$ **b.** $d_{\text{min}} = \sqrt{\dfrac{\lambda D}{2}}$

c. $x = \dfrac{d_{\text{min}}}{2}$ **d.** $x = d_{\text{min}}$

15. The slit 1 of a Young's double-slit experiment is wider than slit 2, so that the light from slits are given as $A_1 = A_0 \sin \omega t$ and $A_2 = A_3 A_0 \sin\left(\omega t + \dfrac{\pi}{3}\right)$. The resultant amplitude and intensity, at a point where the path difference between them is zero, are A and I, respectively. Then,

a. $A = \sqrt{13}\,A_0$ **b.** $A = 4\,A_0$

c. $I \propto 16\,A_0^2$ **d.** $I \propto 13\,A_0^2$

16. A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna B is 9 m to the right of antenna A. Consider point P at a horizontal distance x to the right of antenna A as shown in Fig. 2.114. The value of x and order for which the constructive interference will occur at point P are

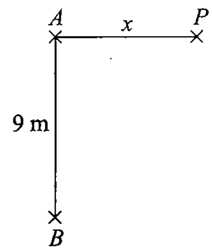


Fig. 2.114

a. $x = 14.95$ m, $n = 1$

b. $x = 5.6$ m, $n = 2$

c. $x = 1.65$ m, $n = 3$

d. $x = 0$, $n = 3.6$

17. A two-slit interference experiment uses coherent light of wavelength 5×10^{-7} m. Intensity in the interference pattern for the following points are I_1, I_2, I_3 and I_4, respectively.

1. A point that is close to one slit than the other by 5×10^{-7} m.

2. A point where the light waves received from the two slits are out of phase by $\dfrac{4\pi}{3}$ rad.

3. A point that is closer to one slit than the other by 7.5×10^{-7} m.

4. A point where the light waves received by the two slits are out of phase by $\frac{\pi}{3}$ rad

Then, which of following statements is/are correct?

a. $I_1 > I_4 > I_2 > I_3$
b. $I_1 > I_4 > I_2 > I_3$
c. $3I_2 = I_4$
d. $I_3 = 0$

Assertion-Reasoning Type

Solutions on page 2.81

Some questions (Assertion-Reason type) are given below. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices a, b, c and d out of which **only one** is correct. So select the correct choice.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** In Young's experiment, the fringe width for dark fringes is different from that for white fringes.
Statement II: In Young's double-slit experiment, when the fringes are observed with a source of white light, then only black and bright fringes are observed.
2. **Statement I:** Thin films such as soap bubble or a thin layer of oil on water show beautiful colours when illuminated by white light.
Statement II: It happens due to the interference of light reflected from the upper surface of thin film.
3. **Statement I:** In Young's double-slit experiment, the two slits are at distance d apart. Interference pattern is observed on a screen at distance D from the slits. At a point on the screen which is directly opposite one of the slits, a dark fringe is observed. Then, the wavelength of wave is proportional to the square of distance between the two slits.
Statement II: For a dark fringe, intensity is zero.
4. **Statement I:** In Young's experiment, for two coherent sources, the resultant intensity is given by $I = 4I_0 \cos^2 \frac{\phi}{2}$.
Statement 2: Ratio of maximum to minimum intensity is
$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}.$$
5. **Statement I:** In calculating the disturbance produced by a pair of superimposed incoherent wave trains, you can add their intensities.
Statement II: $I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$. The average value of $\cos \delta = 0$ for incoherent waves.
6. **Statement I:** Fringe width depends upon refractive index of the medium.
Statement II: Refractive index changes optical path of ray of light forming fringe pattern.
7. **Statement I:** In Young's experiment, the fringe width for dark fringes is same as that of the white fringes.
Statement II: In Young's double-slit experiment, the fringes are performed with a source of white light, then only black and bright fringes are observed.
8. **Statement I:** In interference, all the fringes are of same width.
Statement II: In interference, fringe width is independent of the position of fringe.
9. **Statement I:** Two point coherent sources of light S_1 and S_2 are placed on a line as shown in Fig. 2.115. P and Q are two points on that line. If at point P maximum intensity is observed, then maximum intensity should also be observed at Q.

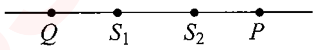


Fig. 2.115

Statement II: In Fig. 2.115, the distance $|S_1P - S_2P|$ is equal to distance $|S_1Q - S_2Q|$.
10. **Statement I:** Two coherent point sources of light having non-zero phase difference are separated by a small distance. Then, on the perpendicular bisector of line segment joining both the point sources, constructive interference cannot be obtained.
Statement II: For two waves from coherent point sources to interfere constructively at a point, the magnitude of their phase difference at that point must be $2m\pi$ (where m is a non-negative integer).
11. **Statement I:** While calculating intensities in interference pattern, we can add the intensities of the individual waves.
Statement II: Principle of superposition is valid for linear waves.
12. **Statement I:** For the situation shown in Fig. 2.116, two identical coherent light sources produce interference pattern on the screen. The intensity of minima nearest to S_1 is not exactly zero.

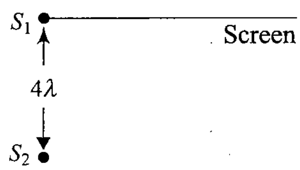


Fig. 2.116

Statement II: Minimum intensity is zero, when interfering waves have same intensity at the location of superposition.
13. **Statement I:** In YDSE, if separation between the slits is less than wavelength of light, then no interference pattern can be observed.
Statement II: For interference pattern to be observed, light sources have to be coherent.
14. **Statement I:** We can hear around corners, but we cannot see around corners.
Statement II: Wavelength of sound is much greater than wavelength of light.

15. **Statement I:** An electron beam is used to obtain interference in a simple Young's double-slit experiment arrangement with appropriate distance between the slits. If the speed of electrons is increased, the fringe width decreases.
Statement II: de Broglie wavelength of electron is inversely proportional to the speed of the electrons.

Comprehension Type

Solutions on page 2.82

For Problems 1–3

A narrow tube is bent in the form of a circle of radius R, as shown in Fig. 2.117. Two small holes S and D are made in the tube at the positions at right angle to each other. A source placed at S generates a wave of intensity I_0 which is equally divided into two parts: one part travels along the longer path, while the other travels along the shorter path. Both the waves meet at the point D where a detector is placed.

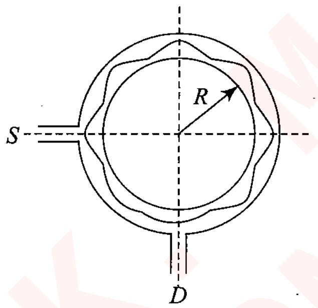


Fig. 2.117

1. If a maxima is formed at a detector, then the magnitude of wavelength λ of the wave produced is given by
   a. nR **b.** $\frac{nR}{2}$
   c. $\frac{nR}{4}$ **d.** all of these
2. If a minima is formed at the detector, then the magnitude of wavelength λ of the wave produced is given by
   a. $2\pi R$ **b.** $\frac{3}{2}\pi R$
   c. $\frac{5}{2}\pi R$ **d.** none of these
3. The maximum intensity produced at D is given by
   a. $4I_0$ **b.** $2I_0$
   c. I_0 **d.** $3I_0$

For Problems 4–6

A thin film of a specific material can be used to decrease the intensity of reflected light. There is destructive interference of waves reflected from upper and lower surfaces of the film. These films are called non-reflecting or anti-reflection coatings. The process of coating the lens or surface with non-reflecting film is called blooming as shown in Fig. 2.118. The refracting index of coating (n_1) is less than that of the glass (n_2).

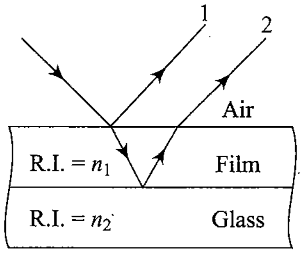


Fig. 2.118

4. If a light of wavelength λ is incident normally and the thickness of film is 't', then optical path difference between waves reflected from upper and lower surface of the film is
   a. $2n_1t$ **b.** $2n_1t - \frac{\lambda}{2}$
   c. $2n_1t + \frac{\lambda}{2}$ **d.** $2t$
5. Magnesium fluoride (MgF_2) is generally used as anti-reflection coating. If refractive index of MgF_2 is 1.25, then minimum thickness of film required is (Take λ = 500 nm)
   a. 125 nm **b.** 75 nm
   c. 100 nm **d.** 225 nm
6. If the thickness of film in above question is not technologically possible to manufacture, then next thickness of film required is (approximately)
   a. 300 nm **b.** 125 nm
   c. 750 nm **d.** 550 nm

For Problems 7–9

In a Young's double-slit experiment setup, source S of wavelength 6000 Å illuminates two slits S_1 and S_2 which act two coherent sources. The source S oscillates about its shown position according to the equation $y = 1 + \cos \pi t$, where y is in millimeter and t in second.

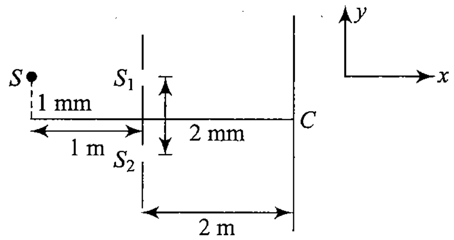


Fig. 2.119

7. At $t = 0$, fringe width is β_1, and at $t = 2$ s, fringe width of figure is β_2. Then
   a. $\beta_1 > \beta_2$ **b.** $\beta_2 > \beta_1$
   c. $\beta_1 = \beta_2$ **d.** data is insufficient
8. At $t = 2$ s, the position of central maxima is
   a. 2 mm above C **b.** 2 mm below C
   c. 4 mm above C **d.** 4 mm below C
9. At $t = 1$ s, a slab of thickness 2×10^{-3} mm and refractive index 1.5 is placed just in front of S_1. The central maximum is formed at
   a. 1 mm above C **b.** 1 mm below C
   c. 2 mm above C **d.** 2 mm below C

For Problems 10–11

Two coherent sources emit light of wavelength λ. Separation between them, $d = 4\lambda$.

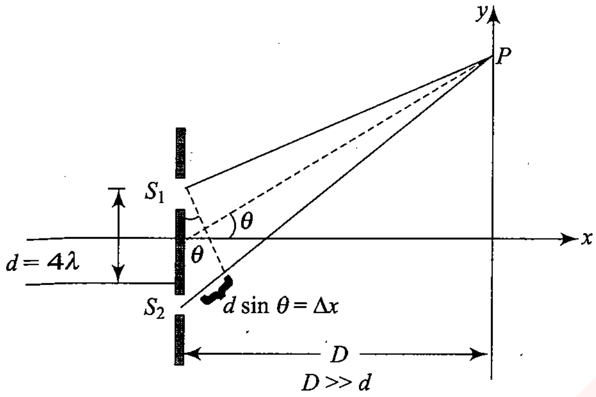


Fig. 2.120

10. If a detector moves along the y-axis, what is the maximum number of minima observed?
a. 6 **b.** 9
c. 5 **d.** 4

11. If the detector moves along the x-axis beginning from S_2, the maximum number of minima observed is
a. 2 **b.** 3
c. 5 **d.** 4

For Problems 12–14

In YDSE, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied by the 5th bright fringe.

12. What is the thickness of the plate?
a. 5 μm **b.** 0.005 μm
c. 7 μm **d.** 0.007 μm

13. If the source is now changed to green light of wavelength 10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. What will be refractive index of glass plate for the 2nd light for changed source of light?
a. 2.6 μm **b.** 1.6 μm
c. 1.2 μm **d.** 2.2 μm

14. Change in fringe width produced due to change in wavelength is
a. -0.57×10^{-4} m **b.** -0.47×10^{-4} m
c. -0.37×10^{-4} m **d.** -0.27×10^{-4} m

For Problems 15–16

A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double-slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in Fig. 2.121.

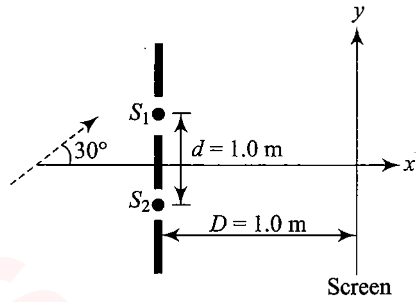


Fig. 2.121

15. If the incident beam falls normally on the double-slit apparatus, find the order of the interference minima on the screen.
a. Only the first order minima are possible
b. Only the first and second order minima are possible
c. Total six minima appear on the screen
d. Total eight minima appear on the screen

16. If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in the figure), find the y-coordinates of the first minima on either side of the central maximum.
a. $\frac{3}{\sqrt{7}}$ and $\frac{1}{\sqrt{15}}$ m **b.** $\frac{3}{\sqrt{7}}$ and $\frac{2}{\sqrt{15}}$ m
c. $\frac{3}{2\sqrt{7}}$ and $\frac{1}{\sqrt{15}}$ m **d.** $\frac{6}{\sqrt{7}}$ and $\frac{3}{\sqrt{15}}$ m

For Problems 17–19

A YDSE is performed in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 nm separation. The lower slit S_2 is covered by a thin glass plate of thickness 10.4 mm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in Fig. 2.122. (All the wavelengths in this problem are for the given medium of refractive index 4/3, ignore absorption.)

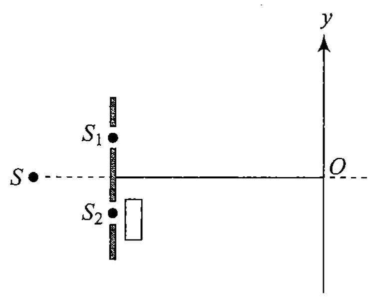


Fig. 2.122

17. The location of the central maximum (bright fringe with zero path difference) on the y-axis will be
a. 2.33 mm **b.** 4.33 mm
c. 6.33 mm **d.** 4.43 mm

18. Find the light intensity at point O relative to maximum fringe intensity.

a. $\frac{1}{4}I_{max}$ **b.** $\frac{5}{4}I_{max}$
c. $\frac{1}{2}I_{max}$ **d.** $\frac{3}{4}I_{max}$

19. Now, if 600 nm light is replaced by white light of range 400 to 800 nm, find the wavelength of the light that forms maxima exactly at point O.

a. 650 nm, 433.3 nm **b.** 550 nm, 750 nm
c. 450 nm, 645.3 nm **d.** 375 nm, 525.3 nm

For Problems 20–22

In a YDSE performed with light of wavelength 600 Å, the screen is placed 1 m from the slits. Fringes formed on the screen are observed by a student sitting close to the slits. The student's eye can distinguish two neighboring fringes. If they subtend an angle more than 1 minute of arc, then

20. In order to have the clear visibility of the fringe, the maximum distance that can be maintained between the slits is

a. 3.06 mm **b.** 2.06 mm
c. 1.31 mm **d.** 3.31 mm

21. Find the location of third bright fringe from center of the screen.

a. 8.74×10^{-4} mm **b.** 6.74×10^{-4} mm
c. 5×10^{-4} mm **d.** 8.74×10^{-7} mm

22. Under similar conditions or arrangement, what will be the position of 5th dark fringe from the center of screen

a. 4.26 μm **b.** 2.93 μm
c. 1.31 μm **d.** 3.14 μm

For Problems 23–24

In a modified YDSE, source S is kept in front of slit S_1. Find the phase difference at a point O that is equidistant from slits S_1 and S_2, and a point P that is in front of slit S_1 in the following situations.

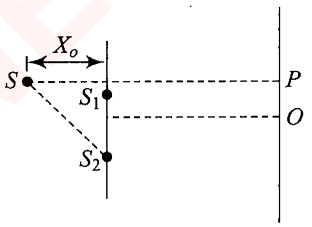


Fig. 2.123

23. A liquid of refractive index μ is filled between the screen and slits.

a. $\frac{2\pi}{\lambda}\left[\left[\sqrt{d^2+x_0^2}+x_0\right]+\frac{\mu d^2}{2D}\right]$

b. $\frac{2\pi}{\lambda}\left[\left[\sqrt{d^2+x_0^2}-x_0\right]+\frac{\mu d^2}{2D}\right]$

c. $\frac{2\pi}{\lambda}\left[\left[\sqrt{d^2-x_0^2}+x_0\right]+\frac{\mu d^2}{2D}\right]$

d. $\frac{2\pi}{\lambda}\left[\left[\sqrt{d^2-x_0^2}-x_0\right]+\frac{\mu d^2}{2D}\right]$

24. Liquid is filled between the slit and the source S.

a. $\frac{2\pi}{\lambda}\left[\mu\sqrt{d^2+x_0^2}-x_0-\frac{d^2}{2D}\right]$

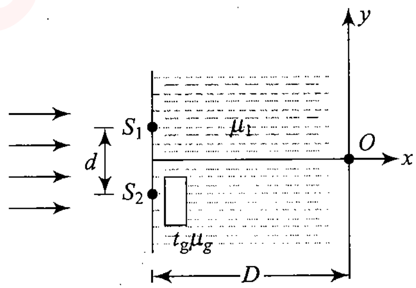
b. $\frac{2\pi}{\lambda}\left[\mu\sqrt{d^2+x_0^2}-x_0+\frac{d^2}{2D}\right]$

c. $\frac{2\pi}{\lambda}\left[\mu\sqrt{d^2-x_0^2}+x_0+\frac{d^2}{2D}\right]$

d. $\frac{2\pi}{\lambda}\left[\mu\sqrt{d^2-x_0^2}-x_0-\frac{d^2}{2D}\right]$

For Problems 25–26

25. In a modified YDSE, the region between the screen and slits is immersed in a liquid whose refractive index varies with time as $\mu_1 = (5/2) - (T/4)$ until it reaches a steady state value of 5/4. A glass plate of thickness 36 μm and refractive index 3/2 is introduced in front of one of the slits.



$D = 1$ m
$d = 2$ mm

Fig. 2.124

25. Find the time when central maxima is at point O, located symmetrically on the x-axis.

a. 2 s **b.** 4 s
c. 6 s **d.** 8 s

26. What is the speed of the central maxima when it is at O?

a. 3×10^{-3} ms^{-1} **b.** 4×10^{-3} ms^{-1}
c. 6×10^{-3} ms^{-1} **d.** 1×10^{-3} ms^{-1}

For Problems 27–28

In a YDSE using monochromatic visible light, the distance between the plane of slits and the screen is 1.7 m. At a point (P) on the screen which is directly in front of the upper slit, maximum path is observed. Now, the screen is moved 50 cm closer to the plane of slits. Point P now lies between third and fourth minima above the central maxima and the intensity at P is one-fourth of the maximum intensity on the screen.

27. Find the value of n.
a. 4 b. 6
c. 2 d. 8

28. Find the wavelength of light if the separation of slits is 2 mm.
a. 2.9×10^{-7} m b. 3.9×10^{-7} m
c. 5.9×10^{-7} m d. 6.9×10^{-7} m

For Problems 29–31

In Fig. 2.125, a screen is placed normal to the line joining the two point coherent sources S_1 and S_2. The interference pattern consists of concentric circles.

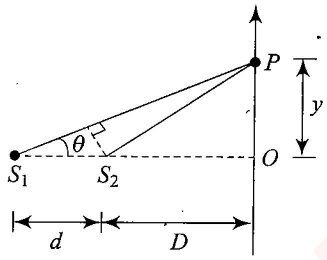


Fig. 2.125

29. Find the radius of the nth bright ring.
a. $D\sqrt{1\left(1-\frac{n\lambda}{d}\right)}$ b. $D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$
c. $2D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$ d. $D\sqrt{2\left(1-\frac{n\lambda}{2d}\right)}$

30. If $d = 0.5$ mm, $\lambda = 5000$ Å and $D = 100$ cm, find the value of n for the closest second bright ring.
a. 888 b. 830
c. 914 d. 998

31. Also, find the value of radius for this ring.
a. 6.32 cm b. 5.52 cm
c. 4.7 cm d. 3.25 cm

For Problems 32–33

In the arrangement shown in Fig. 2.126, light of wavelength 6000 Å is incident on slits S_1 and S_2. Slits S_3 and S_4 have been opened such that S_3 is the position of first maximum above the central maximum and S_4 is the closest position where intensity is same as that of the light used, below the central maximum. The point O is equidistant from S_1 and S_2 and O' is equidistant from S_3 and S_4. The intensity of incident light is I_0.

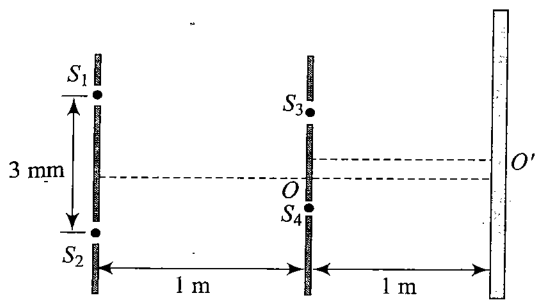


Fig. 2.126

32. Find the intensity at O' (on the screen).
a. $4.5I_0$ b. $3I_0$
c. $2I_0$ d. $5I_0$

33. Find the intensity of the brightest fringe.
a. $9I_0$ b. $5I_0$
c. $3I_0$ d. $7I_0$

For Problems 34–35

A lens of focal length f is cut along the diameter into two identical halves. In this process, a layer of the lens t in thickness is lost, then the halves are put together to form a composite lens. In between the focal plane and the composite lens, a narrow slit is placed near the focal plane. The slit is emitting monochromatic light with wavelength λ. Behind the lens, a screen is located at a distance L from it.

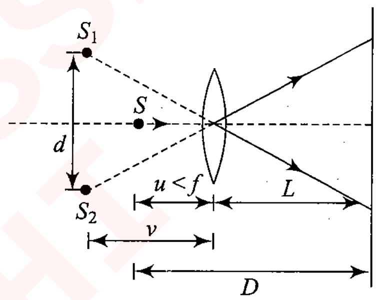


Fig. 2.127(a)

34. Find the fringe width for the pattern obtained under given arrangement on the screen.
a. $\frac{\lambda f}{2t}$ b. $\frac{\lambda f}{t}$
c. $\frac{tf}{\lambda}$ d. $\frac{tf}{2\lambda}$

35. The expression for the number of visible maxima which are obtained through above said arrangement will turn out to be
a. $\frac{Lt^2}{\lambda f^2}$ b. $\frac{2Lt^2}{\lambda f^2}$
c. $\frac{Lt}{2\lambda f^2}$ d. $\frac{Lt^2}{2\lambda f^2}$

For Problems 36–38

In the arrangement shown in Fig. 2.127(b), $D \gg d$.

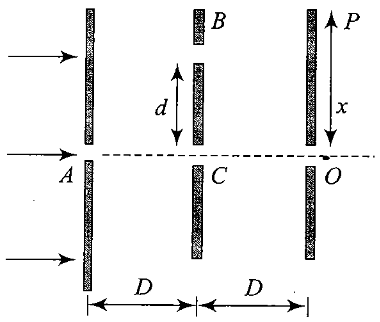


Fig. 2.127(b)

36. For what minimum value of d, is there a dark band at the point O on the screen.

a. $\sqrt{\dfrac{D\lambda}{4}}$ b. $\sqrt{\dfrac{3D\lambda}{4}}$

c. $\sqrt{\dfrac{D\lambda}{8}}$ d. $\sqrt{\dfrac{2D\lambda}{3}}$

37. Find the distance x at which the next bright fringe is formed.

a. $\dfrac{3\lambda}{2}$ b. $\dfrac{\lambda}{4}$

c. $\dfrac{\lambda}{2}$ d. $\dfrac{5\lambda}{2}$

38. Find the fringe width.

a. d b. $2d$
c. $4d$ d. $3d$

For Problems 39–41

Consider the situation shown in Fig. 2.128. The two slits S_1 and S_2 placed symmetrically around the central line are illuminated by monochromatic light of wavelength λ.. The separation between the slits is d. The light transmitted by the slits falls on a screen S_0 placed at a distance D from the slits. The slit S_3 is at the central line and the slit S_4 is at a distance z from S_3. Another screen S_c is placed a further distance D away from S_c. Find the ratio of the maximum to minimum intensity observed on S_c.

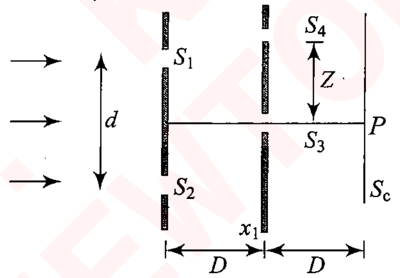


Fig. 2.128

39. If $z = \dfrac{\lambda D}{2d}$

a. 1 b. 1 / 2
c. 3 / 2 d. 2

40. If $z = \dfrac{\lambda D}{d}$

a. 4 b. 2
c. ∞ d. 1

41. If $z = \dfrac{\lambda D}{4d}$

a. $\left[3-2\sqrt{2}\right]^2$ b. $\left[3+\sqrt{2}\right]^2$

c. $\left[3-\sqrt{2}\right]^2$ d. $\left[3+2\sqrt{2}\right]^2$

For Problems 42–44

The arrangement for a mirror experiment is shown in Fig. 2.129. 'S' is a point source of frequency 6×10^{14} Hz. D and C represent the two ends of a mirror placed horizontally and LOM represents the screen.

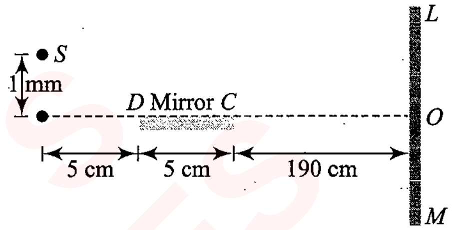


Fig. 2.129

42. Determine the width of the region where the fringes will be visible.

a. 4 cm b. 6 cm
c. 2 cm d. 3 cm

43. Find the fringe width of the fringe pattern.

a. 0.05 cm b. 0.25 cm
c. 0.01 cm d. 0.1 cm

44. Calculate the number of fringes.

a. 10 b. 20
c. 30 d. 40

For Problems 45–47

In Young's double-slit experiment setup with light of wavelength $\lambda = 6000$ Å, distance between the two slits is 2 mm and distance between the plane of slits and the screen is 2 m. The slits are of equal intensity. When a sheet of glass of refractive index 1.5 (which permits only a fraction η of the incident light to pass through) and thickness 8000 Å is placed in front of the lower slit, it is observed that the intensity at a point P, 0.15 mm above the central maxima, does not change.

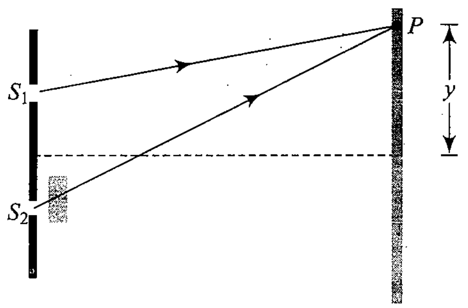


Fig. 2.130

45. The phase difference at point P without inserting the slab is

a. $3\pi/4$ b. $\pi/4$
c. $\pi/2$ d. $\pi/3$

46. Intensity at point P is

a. $3I_0$ b. I_0
c. $2I_0$ d. $8I_0$

47. The value of η is

a. 0.21 b. 0.42
c. 0.12 d. 0.50

For Problems 48–49

An interference is observed due to two coherent sources S_1 placed at origin and S_2 placed at (0, 3l, 0). Here, λ is the wavelength of the sources. Both sources are having equal intensity I_0. A detector D is moved along the positive x-axis.

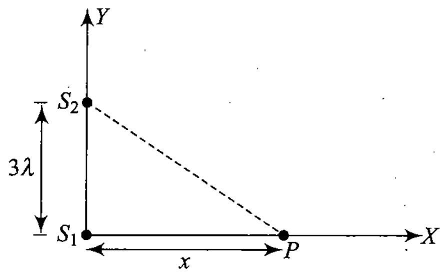


Fig. 2.131

48. Total number of black points observed by the observer on positive x-axis is

a. two　**b.** four
c. three　**d.** five

49. Find x-coordinates on the x-axis (excluding $x = 0$ and $x = \infty$).

a. $x = 4\lambda$　**b.** $x = 7\lambda / 4$
c. $x = 5\lambda / 4$　**d.** $x = 3\lambda$

For Problems 50–52

Figure 2.132 shows the interference pattern obtained in a double-slit experiment using light of wavelength 600 nm. 1, 2, 3, 4 and 5 are marked on five fringes.

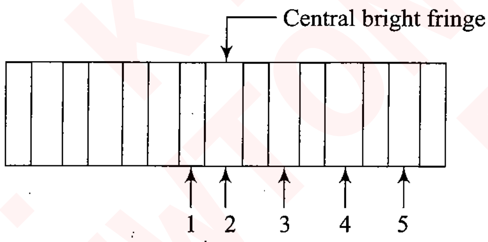


Fig. 2.132

50. The third-order bright fringe is

a. 2　**b.** 3　**c.** 4　**d.** 5

51. Which fringe results from a phase difference of 4π between the light waves incident from the two slits?

a. 2　**b.** 3　**c.** 4　**d.** 5

52. Let ΔX_A and ΔX_C represent path differences between waves interfering at 1 and 3, respectively. Then $(|\Delta X_C| - |\Delta X_A|)$ is equal to

a. 0　**b.** 300 mn
c. 600 nm　**d.** 900 nm

For Problems 53–56

This interference film is used to measure the thickness of slides, paper, etc. The arrangement is as shown in Fig. 2.133.

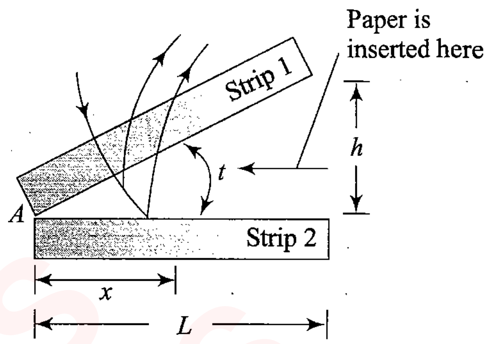


Fig. 2.133

For the sake of clarity, the two strips are shown thick. Consider the wedge formed in between strips 1 and 2. If the interference pattern because of the two waves reflected from wedge surface is observed, then from the observed data we can compute thickness of paper, refractive index of the medium filled in wedge, number of bonds formed, etc.

Consider the strips to be thick as compared to wavelength of light and light is incident normally.

Neglect the effect due to reflection from top surface of strip 1 and bottom surface of strip 2. Take $L = 5$ cm and $\lambda_{air} = 40$ nm.

53. Consider an air wedge is formed by two glass plates having refractive index 1.5 by placing a piece of paper of thickness 20 mm. Determine the number of dark bands formed.

a. 1000　**b.** 500
c. 5000　**d.** 500

54. For strip 1 refractive index is 1.34 and for strip 2 refractive index is 1.6. The wedge is filled with a medium having refractive index 1.5. Then,

a. the band at contact point would be dark
b. the band at contact point would be bright
c. at contact point, maxima or minima occurs
d. at contact point, uniform illumination would be there

55. In question 53, if air wedge has been filled with a medium having refractive index 1.3, then find the number of bright bands.

a. 199　**b.** 99
c. 499　**d.** 130

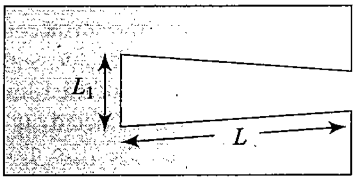
56. For data in question 53, determine the distance of point B from the 20th dark band. Counting of dark points has to start from the contact point.

a. 4 cm　**b.** 3 cm
c. 5 cm　**d.** 2 cm

For Problems 57–59

A block of plastic having a thin air cavity (whose thickness is comparable to wavelength of light waves) is shown in Fig. 2.134. The thickness of air cavity (which can be considered as air wedge for interference pattern) is varying linearly from one end to other as shown.

A broad beam of monochromatic light is incident normally from the top of the plastic box. Some light is reflected back from top and some from the bottom of cavity. The plastic layers above and below the cavity are having thickness much large than wavelength of incident light. An observer when looking down from top sees an interference pattern consisting of eight dark fringes and seven bright fringes along the wedge. Take wavelength of incident light in air as λ_0 and refractive index of plastic as μ.



Front view of plastic box

Fig. 2.134

Assume that the thickness of the ends of air cavity are such that formation of fringes takes place there.

57. Determine the difference $L_1 - L_2 (= \Delta L)$ in terms of λ_0.

a. $\dfrac{4\lambda_0}{\mu}$ **b.** $\dfrac{7\lambda_0}{2\mu}$

c. $\dfrac{3\lambda_0}{\mu}$ **d.** None of the above

58. Determine the distance of 4th dark fringe from the left end of air cavity.

a. $\dfrac{2L}{6} + \lambda_0$ **b.** $L_1 + \dfrac{3L}{4}$

c. $\dfrac{4L}{7}$ **d.** $\dfrac{5L}{7}$

59. Determine the separation between 1st and 2nd dark fringes from the left end of air cavity.

a. $\dfrac{3L}{7} + \dfrac{2\lambda_0}{\mu}$ **b.** $\dfrac{5L}{7}$

c. $\dfrac{4L}{7}$ **d.** $\dfrac{6L}{7}$

For Problems 60–62

In Fig. 2.135, light of wavelength $\lambda = 5000$ Å is incident on the slits (in a horizontally fixed place).

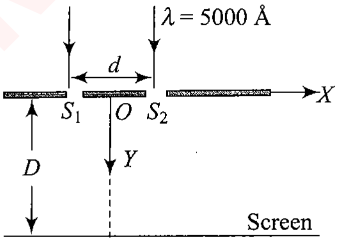


Fig. 2.135

Here, $d = 1$ mm and $D = 1$ m.
Take origin at O and XY plane as shown in the figure. The screen is released from rest from the initial position as shown.

60. The velocity of central maxima at $t = 5$ s is

a. 50 m s^{-1} along Y-axis
b. 50 m s^{-1} along negative Y-axis
c. 25 m s^{-1} along Y-axis
d. 3×10^8 m s^{-1} along Y-axis

61. Velocity of 2nd maxima w.r.t. central maxima at $t = 2$ s is

a. $(8 \text{ cm s}^{-1})\hat{i} + 20 \text{ m s}^{-1}\hat{j}$ **b.** $8 \text{ cm s}^{-1}\hat{i}$

c. $2 \text{ cm s}^{-1}\hat{i}$ **d.** $86 \text{ m s}^{-1}\hat{i}$

62. Acceleration of 3rd maxima w.r.t. 3rd maxima on other side of central maxima at $t = 3$ s is

a. $0.02 \text{ m s}^{-1}\hat{i}$ **b.** $0.03 \text{ m s}^{-1}\hat{i}$

c. $10 \text{ m s}^{-1}\hat{j}$ **d.** $0.6 \text{ m s}^{-2}\hat{i}$

For Problems 63–65

When two coherent sources interact with each other there will be production of alternate bright and dark fringes on the screen. The Young's double-slit experiment demonstrates the idea of making two coherent sources. For better visibility, one has to choose proper amplitude for the sources. The phenomena is good enough to satisfy the conservation of energy principle. The pattern formed in YDSE is of uniform thickness and is nicely placed on a long distance screen.

63. Law of conservation of energy is satisfied because

a. equal loss and gain in intensity is observed
b. all bright fringes are equally bright
c. all dark fringes are of zero brightness
d. the average intensity on screen is equal to the sum of intensities of the two sources

64. For better visibility of fringe pattern

a. amplitudes of the sources are equal
b. the width of the slits should not be equal
c. dark should be the darkest and bright should be the brightest
d. the widths should be same

65. The best combination of independent sources to produce sustained pattern among the following is

$Y_1 = a \sin \omega t$ $\quad$ $Y_2 = a \cos \omega t$

$Y_3 = a \sin\left(\omega t + \dfrac{\pi}{4}\right)$ $\quad$ $Y_4 = 2a \sin(\omega t + \pi)$

a. Y_1, Y_2 only **b.** Y_2, Y_3 only
c. Y_3, Y_4 only **d.** none of these

For Problems 66–68

When a light wave passes from a rarer medium to a denser medium, there will be a phase change of π radians. This difference brings change in the conditions for constructive and destructive interference. This phenomena also reasons the formation of interference pattern in thin films like, oily layer, soap film, etc., but has no reason on the shifting of fringes from the central portion outward. The shift is dependent on the refractive index of the material as per the relation, $\Delta y = (\mu - 1)t$.

66. Thin film interference happens with
   a. point or spherical source
   b. broad source
   c. film thickness of the order of 10,000 Å
   d. very thick transparent slabs

67. On introducing a transparent slab (μ), the central fringe shifts to the point originally occupied by the fifth bright fringe. The thickness of the slab is
   a. $\dfrac{5\lambda}{\mu-1}$
   b. $\dfrac{4\lambda}{\mu-1}$
   c. $\dfrac{\mu-1}{4\lambda}$
   d. $\dfrac{\mu-1}{5\lambda}$

68. The condition for constructive interference in Lloyd's single mirror experiment is the path difference which is equal to
   a. $N\lambda$
   b. $(2N-1)\dfrac{\lambda}{2}$
   c. $(N-1)\dfrac{\lambda}{2}$
   d. $\dfrac{\lambda}{2(2N-1)}$

For Problems 69–71

When light from two sources (say slits S_1 and S_2) interfere, they form alternate dark and bright fringes. Bright fringe is formed at all points where the path difference is an odd multiple of half wavelength. At the condition of equal amplitudes, $A_1 = A_2 = a$, the maximum intensity will be $4a^2$ and the visibility improves. The resultant intensity can also be indicated with phase factor as $I = 2a^2 \cos^2(\phi/2)$. Using this passage, answer the following questions.

69. If the path difference between the slits S_1 and S_2 is $\dfrac{\lambda}{2}$, the central fringe will have an intensity of
   a. 0
   b. a^2
   c. $2a^2$
   d. $4a^2$

70. At a point having a path difference of $\dfrac{\lambda}{4}$ the intensity will be
   a. 0
   b. a^2
   c. $2a^2$
   d. $a^2/2$

71. If the slits S_1 and S_2 are arranged as shown in Fig. 2.136, the ratio of intensity of fringe at P and R is

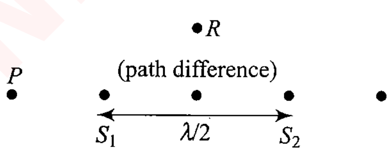


Fig. 2.136

   a. 0
   b. ∞
   c. 1:1
   d. 1:2

For Problems 72–77

Thin films, including soap bubbles and oil show patterns of alternative dark and bright regions resulting from interference among the reflected light waves. If two waves are in phase, their crests and troughs will coincide. The interference will be constructive and the amplitude of resultant wave will be greater than either of constituent waves. If the two waves are out of phase by half a wavelength (180°), the crests of one wave will coincide with the troughs of the other wave. The interference will be destructive and the amplitude of the resultant wave will be less than that of either constituent wave.

At the interface between two transparent media, some light is reflected and some light is refracted.

1. When incident light I, reaches the surface at point a, some of the light is reflected as ray R_a and some is refracted following the path ab to the back of the film.
2. At point b, some of the light is refracted out of the film and part is reflected back through the film along path bc. At point c, some of the light is reflected back into the film and part is reflected out of the film as ray R_c.

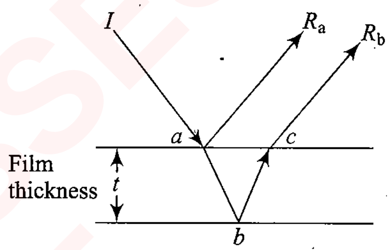


Fig. 2.137

R_a and R_c are parallel. However, R_c has travelled the extra distance within the film of abc. If the angle of incidence is small, then abc is approximately twice the film's thickness.

If R_a and R_c are in phase, they will undergo constructive interference and the region ac will be bright. If R_a and R_c are out of phase, they will undergo destructive interference and the region ac will be dark.

The thickness of the film and the refractive indices of the media at each interface determine the final phase relationship between R_a and R_c.

I. Refraction at an interface never changes the phase of the wave.

II. For reflection at the interface between two media 1 and 2, if $n_1 > n_2$, the reflected wave will change phase. If $n_1 < n_2$, the reflected wave will not undergo a phase change.

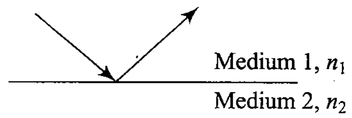


Fig. 2.138

For reference, $n_{air} = 1.00$.

III. If the waves are in phase after reflection at all interfaces, then the effects of path length in the film are:
Constrictive interference occurs when $2t = m\lambda/n$, $m = 0, 1, 2, 3, \ldots$.

Destructive interference occurs when $2t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{n}$, $m = 0, 1, 2, 3, \ldots$.

If the waves are 180° out of the phase after reflection at all interfaces, then the effects of path length in the film are:
Constructive interference occurs when

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}, m = 0, 1, 2, 3, \ldots$$

Destructive interference occurs when

$$2t = \frac{m\lambda}{n}, m = 0, 1, 2, 3, \ldots$$

72. A thin film with index of refraction 1.50 coats a glass lens with index of refraction 1.80. What is the minimum thickness of the thin film that will strongly reflect light with wavelength 600 nm?

a. 150 nm **b.** 200 nm
c. 300 nm **d.** 450 nm

73. A thin film with index of refraction 1.33 coats a glass lens with index of refraction 1.50. Which of the following choices is the smallest film thickness that will not reflect light with wavelength 640 nm?

a. 160 nm **b.** 240 nm
c. 360 nm **d.** 480 nm

74. A soap film of thickness t is surrounded by air and is illuminated at near normal incidence by monochromatic light with wavelength λ in the film. With respect to the wavelength of the monochromatic light in the film, what film thickness will produce maximum constructive interference?

a. $\frac{1}{4}\lambda$ **b.** $\frac{1}{2}\lambda$
c. 1λ **d.** 2λ

75. The average human eye sees colours with wavelength between 430 nm to 680 nm. For what visible wavelength(s) will a 350 nm thick ($n = 1.35$) soap film produce maximum destructive interference?

a. 945 nm **b.** 473 nm
c. 315 nm **d.** None of these

76. A 600 nm light is perpendicularly incident on a soap film suspended in air. The film is 1.00 μm thick with $n = 1.35$. Which statement most accurately describes the interference of light reflected by the two surfaces of the film?

a. The waves are close to destructive interference.
b. The waves are close to constructive interference.
c. The wave show complete destructive interference.
d. The waves show complete constructive interference.

77. A thin film of liquid polymer, $n = 1.25$, coats a slab of Pyrex, $n = 1.50$. White light is incident perpendicularly on the film. In the reflection, full destructive interference occurs for $\lambda = 600$ nm and full constructive interference occurs for $\lambda = 700$ nm. What is the thickness of the polymer film?

a. 120 nm **b.** 280 nm
c. 460 nm **d.** 840 nm

For Problems 78–80

In a YDSE set-up (see Fig. 2.139), the light source executes SHM between P and Q according to the equation $x = A \sin \omega t$, S being the mean position. Assume $d \to 0$ and $A \ll L$. ω is small enough to neglect Doppler effect. If the source were stationary at S, intensity at O would be I_0

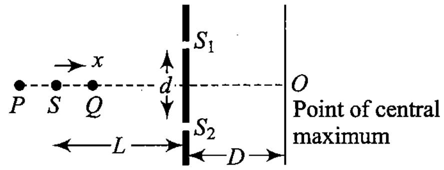


Fig. 2.139

Read the paragraph carefully and answer the following questions.

78. The fractional change in intensity of the central maximum as function of time is

a. $\frac{A \sin \omega t}{L}$ **b.** $\frac{2A \sin \omega t}{L}$
c. $\frac{3A \sin \omega t}{L}$ **d.** $\frac{4A \sin \omega t}{L}$

79. When the source comes towards the point Q,

a. the bright fringes will be less bright.
b. the dark fringes will no longer remain dark
c. the fringe width will increase
d. none of these

80. The fringe width β can be expressed as

a. $\beta = \beta_0 \sin \omega t$ **b.** $\beta = \beta_0 \cos \omega t$
c. $\beta = \beta_0 \sin 2\omega t$ **d.** none of these

For Problems 81–83

In the arrangement shown in Fig. 2.140, slits S_1 and S_4 are having a variable separation Z. Point O on the screen is at the common perpendicular bisector of S_1S_2 and S_3S_4.

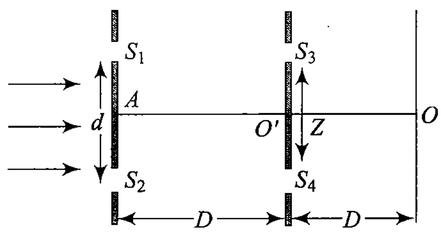


Fig. 2.140

81. When $Z = \frac{\lambda D}{2d}$, the intensity measured at O is I_0. The intensity at O when $Z = \frac{2\lambda D}{d}$ is

a. I_0 **b.** $2I_0$
c. $3I_0$ **d.** $4I_0$

82. The minimum value of Z for which the intensity at O is zero is

a. $\frac{2\lambda D}{d}$ **b.** $\frac{\lambda D}{2d}$
c. $\frac{\lambda D}{3d}$ **d.** $\frac{\lambda D}{d}$

83. If a hole is made at O' on $AO'O$ and the slit S_4 is closed, then the ratio of the maximum to minimum observed on screen at O, if $O'S_3$ is equal to $\frac{\lambda D}{4d}$, is

a. 1 **b.** infinity
c. 34 **d.** 4

Matching Column Type

Solutions on page 2.92

1. A double-slit interference pattern is produced on a screen, as shown in Fig. 2.141, using monochromatic light of wavelength 500 nm. Point P is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips A, B and C of transparent materials with different thicknesses and refractive indices is available, as shown in the table. These are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be shifted across the screen from the original pattern. In Column I, how the strips have been placed, is mentioned whereas in Column II, order of the fringe at point P on the screen that will be produced due to the placement of the strip(s), is shown. Correctly match both the columns.

Film	A	B	C
Thickness (in μm)	5	1.5	0.25
Refractive index	1.5	2.5	2

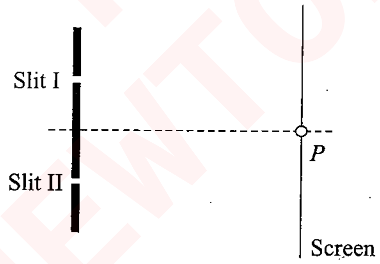


Fig. 2.141

Column I	Column II
a. Only strip B is placed over slit I.	**p.** First bright
b. Strip A is placed over slit I and strip C is placed over slit II.	**q.** Fourth dark
c. Strip A is placed over slit I and strip B and strip C are placed over slit II in conjunction.	**r.** Fifth dark
d. Strip A and strip C are placed placed over slit I (in conjunction) and strip B is placed over slit II.	**s.** Central bright

2. A monochromatic parallel beam of light of wavelength λ is incident normally on the plane containing slits S_1 and S_2. The slits are of unequal width such that intensity only due to one slit on screen is four times that only due to the other slit. The screen is placed along y-axis as shown in Fig. 2.142. The distance between slits is d and that between the screen and slits is D. Match the statements in Column I with results in Column II:

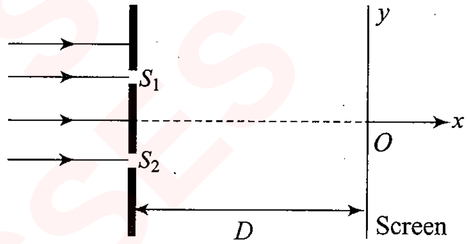


Fig. 2.142

Column I	Column II
a. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{1}{9}$th of maximum intensity.	**p.** $\frac{D\lambda}{3d}$
b. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{3}{9}$th of maximum intensity.	**q.** $\frac{D\lambda}{d}$
c. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{5}{9}$th of maximum intensity.	**r.** $\frac{2D\lambda}{d}$
d. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{7}{9}$th of maximum intensity.	**s.** $\frac{3D\lambda}{d}$

3. For the situation shown in Fig. 2.143, match the entries of Column I with Column II.

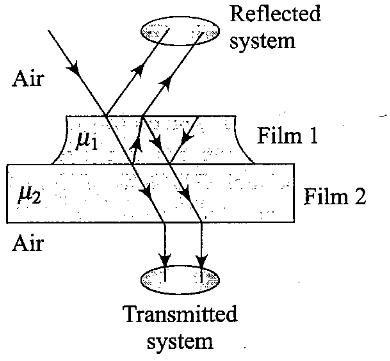
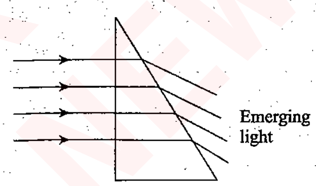


Fig. 2.143

Assume thickness of films to be very small compared to wavelength of incident light.

Column I	Column II
a. $\mu_1 = \mu_2$	**p.** Film 1 appears shiny from the reflected system
b. $\mu_1 > \mu_2$	**q.** Film 1 appears dark from the reflected system
c. $\mu_1 < \mu_2$	**r.** Film 1 appears shiny from the transmitted system
d. $\mu_1 \neq \mu_2$	**s.** Film 1 appears dark from the transmitted system

4.

Column I	Column II
a. Light diverging from a point source	**p.** Plane wavefront
b. Light emerging from a convex lens when a point source is placed at its focus	**q.** Spherical wavefront
c. Light reflected from a concave mirror when a point source is placed at its focus	**r.** Cylindrical wavefront
d.  **Fig. 2.144**	**s.** Concave right wavefront

5. For the situation shown in Fig. 2.145, Column I relates the values of μ_1, μ_2 and μ_3 and Column II shows the possible shapes of wavefronts. Match Column I entries with Column II entries.

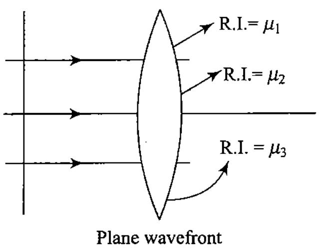
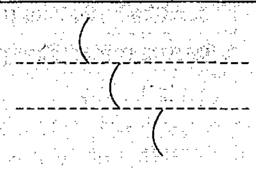
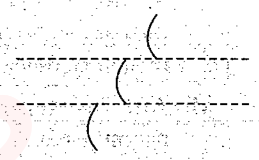
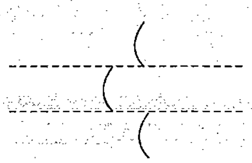
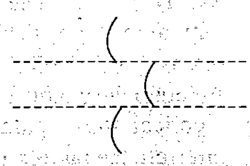


Fig. 2.145

Column I	Column II
a. $\mu_1 = 1.4, \mu_2 = 1.5, \mu_3 = 1.6$	**p.** 
b. $\mu_1 = 1.6, \mu_2 = 1.5, \mu_3 = 1.4$	**q.** 
c. $\mu_1 = 1.7, \mu_2 = 1.5, \mu_3 = 1.7$	**r.** 
d. $\mu_1 = 1.3, \mu_2 = 1.5, \mu_3 = 1.3$	**s.** 

6. If (μ_1, λ_1, v_1) and (μ_2, λ_2, v_2) are refractive indices, wavelengths and speeds of two light waves, respectively, then

Column I	Column II
a. $\mu_1 > \mu_2$	**p.** $v_1 < v_2$
b. $\mu_1 < \mu_2$	**q.** $v_1 > v_2$
c. $\mu_1 \neq \mu_2$	**r.** $\lambda_1 = \lambda_2$
d. $\mu_1 = \mu_2$	**s.** $\lambda_1 < \lambda_2$

7. In Young's double-slit experiment, the point source S is placed slightly off the central axis as shown in Fig. 2.146. If $\lambda = 500$ nm, then match the following.

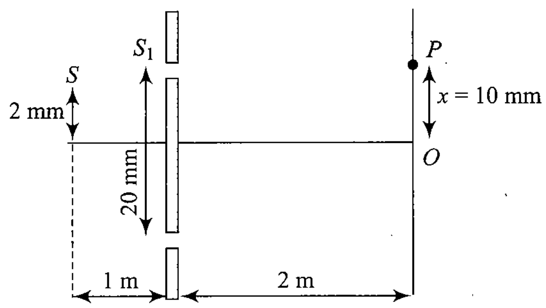


Fig. 2.146

Column I	Column II
a. Nature and order of interference at the point P, $OP = 10$ mm	**p.** Bright fringe of order 80

b. Nature and order of interference at point O	**q.** Bright fringe of order 262
c. If a transparent paper (refractive index μ = 1.45) of thickness t = 0.02 mm is pasted on S_1, i.e., one of the slits, the nature and order of the interference at P	**r.** Bright fringe of order 62
d. After inserting the transparent paper in front of slit S_1, the nature and order of interference at O	**s.** Bright fringe of order 280

Integer Answer Type

Solutions on page 2.94

1. In a Young's double slit experiment $\lambda = 500$ nm, $d = 1$ mm, and $D = 4$ m. The minimum distance from the central maximum for which the intensity is half of the maximum intensity is '*' $\times 10^{-4}$ m. What is the value of '*'?

2. A monochromatic light of $\lambda = 500$ Å is incident on two identical slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness 1.5×10^{-6} m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the slit now.

3. A screen is at a distance $D = 80$ cm from a diaphragm having two narrow slits S_1 and S_2 which are $d = 2$ mm apart. Slit S_1 is covered by a transparent sheet of thickness $t_1 = 2.5$ μm and S_2 by another sheet of thickness $t_2 = 1.25$ μm as shown in the figure. Both sheets are made of same material having refractive index $\mu = 1.40$. Water is filled in space between diaphragm and screen. A monochromatic light beam of wavelength λ = 5000Å is incident normally on the diaphragm. Assuming intensity of beam to be uniform, calculate ratio of intensity of centre of screen to intensity of individual slit, ($\mu_w = 4/3$).

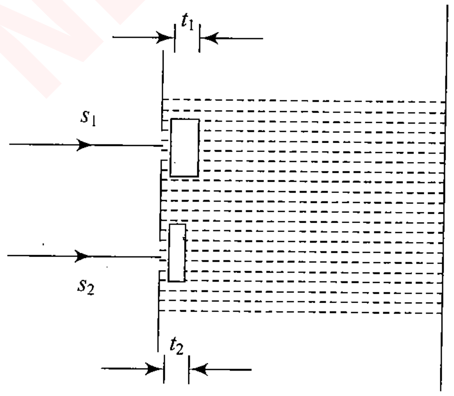


Fig. 2.147

4. In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000Å and intensity $(10/\pi)$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m, respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A (see the figure). Calculate the power (in mW) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the aperture. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.

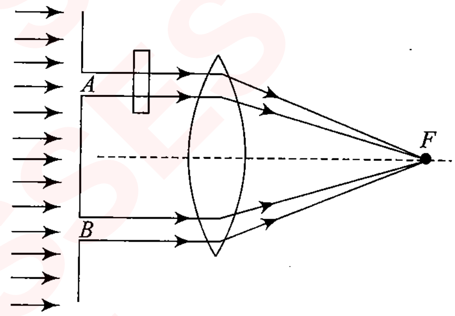


Fig. 2.148

5. A narrow monochromatic beam of light of intensity 1 is incident on a glass plate as shown in the figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25% of the light incident on it and transmits the remaining. Find the ratio $\sqrt{\frac{I_{max}}{I_{mim}}}$ the interference pattern formed by two beams obtained after one reflection at each plate.

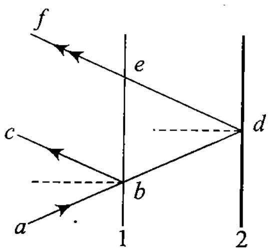


Fig. 2.149

6. A glass of refractive index 1.5 is coated with a thin layer of thickness of t of refractive index 1.8. light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. If λ = 648 nm, obtain the least value of t (in 10^{-8} m) for which the rays interfere constructively.

7. In YDSE find the thickness (in μm) of a glass slab ($\mu = 1.5$) which should be placed before the upper slit S_1 so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

8. A monochromatic beam of light of wavelength 5000 Å is used in Young's double slit experiment. If one of the slits is covered by a transparent sheet of thickness 1.4×10^{-5} m, having refractive index of its medium 1.25. Then the number of fringes shifted is

Archives

Solutions on page 2.95

Fill in the Blanks Type

1. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium, the velocity of the light wave is ____________ and its wavelength is ____________. **(IIT-JEE, 1983)**
2. A monochromatic beam of light of wavelength 6000 Å in a vacuum enters a medium of refractive index 1.5. In the medium, its wavelength is ____________, its frequency is ____________. **(IIT-JEE, 1985)**
3. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude 'A' and of wavelength 'λ'. In another experiment with the same setup, the two slits are sources of equal amplitude 'A' and wavelength 'λ', but are incoherent. The ratio of intensity of light at the mid-point of the screen in the first case to that in the second case is ____________. **(IIT-JEE, 1986)**
4. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meter and 25 meter respectively, from the source. The ratio of amplitudes of the wave at P and Q is ____________. **(IIT-JEE, 1989)**

True or False Type

1. The two slits in a Young's double-slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen. **(IIT-JEE, 1984)**
2. In a Young's double-slit experiment performed with a source of white light, only black and white fringes are observed. **(IIT-JEE, 1987)**

Multiple Choice Questions with One Correct Answer Type

1. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **(IIT-JEE, 1995)**

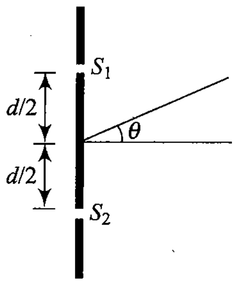


Fig. 2.150

   a. $I(\theta) = I_0/2$ for $\theta = 30°$
   b. $I(\theta) = I_0/4$ for $\theta = 90°$
   c. $I(\theta) = I_0$ for $\theta = 0°$
   d. $I(\theta)$ is constant for all values of θ

2. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown in figure. The observed interference fringes from this combination shall be **(IIT-JEE, 1999)**

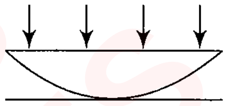


Fig. 2.151

   a. straight
   b. circular
   c. equally spaced
   d. having fringe spacing which increases as we go outwards

3. In a double-slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern **(IIT-JEE, 2000)**
   a. the intensities of both the maxima and the minima increase
   b. the intensity of the maxima increases and the minima has zero intensity
   c. the intensity of the maxima decreases and that of the minima increases
   d. the intensity of the maxima decreases and the minima has zero intensity
4. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is **(IIT-JEE, 2001)**
   a. $2I$ **b.** $4I$
   c. $5I$ **d.** $7I$
5. In a Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **(IIT-JEE, 2001)**
   a. 12 **b.** 18
   c. 24 **d.** 30
6. In the ideal double-slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glassplate is **(IIT-JEE, 2002)**
   a. 2λ **b.** $2\lambda/3$
   c. $\lambda/3$ **d.** λ
7. In Fig. 2.152, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP. **(IIT-JEE, 2003)**

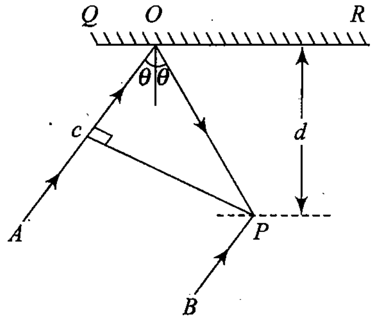


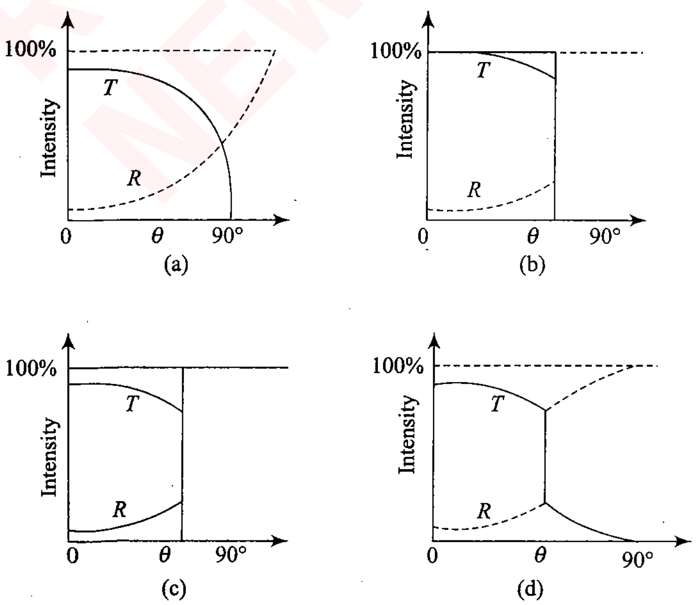
Fig. 2.152

a. $\cos\theta = 3\lambda/2d$ b. $\cos\theta = \lambda/4d$
c. $\sec\theta - \cos\theta = \lambda/d$ d. $\sec\theta - \cos\theta = 4\lambda/2d$

8. Monochromatic lights of wavelength 400 nm and 560 nm are incident simultaneously and normally on a double-slit apparatus whose slit separation is 0.1 mm and screen distance is 1 m. Distance between areas of total darkness will be **(IIT-JEE, 2004)**
a. 4 mm b. 5.6 mm
c. 14 mm d. 28 mm

9. In Young's double-slit experiment, intensity at a point is (1/4) of the maximum intensity. Angular position of this point is **(IIT-JEE, 2005)**
a. $\sin^{-1}(\lambda/d)$ b. $\sin^{-1}(\lambda/2d)$
c. $\sin^{-1}(\lambda/3d)$ d. $\sin^{-1}(\lambda/4d)$

10. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **(IIT-JEE, 2011)**



Multiple Choice Questions with One or More than One Correct Answer Type

1. In the Young's double-slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that **(IIT-JEE, 1988)**
a. the intensities at the screen due to the two slits are 5 units and 4 units, respectively
b. the intensities at the screen due to the twoslits are 4 units and 1 units, respectively
c. the amplitude ratio is 3
d. the amplitude ratio is 2

2. White light is used to illuminate the two slits in a Young's double-slit experiment. The separation between the slits is b and the screen is at a distance d ($>b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are **(IIT-JEE, 1984)**
a. $\lambda = \dfrac{b^2}{d}$ b. $\lambda = \dfrac{2b^2}{d}$
c. $\lambda = \dfrac{b^2}{3d}$ d. $\lambda = \dfrac{2b^2}{3d}$

Comprehension Type

For Problems 1–3

Fig. 2.153 shows a surface XY separating two transparent media, medium 1 and medium 2. The lines ab and cd represent wavefronts of a light wave traveling in medium 1 and incident on XY. The lines ef and gh represent wavefront of the light wave in medium 2 after refraction. **(IIT-JEE, 2007)**

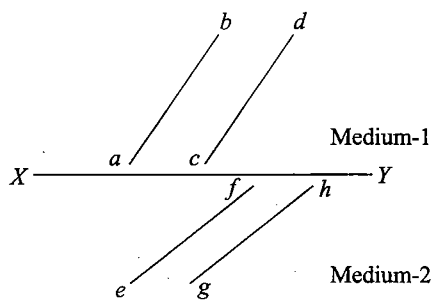


Fig. 2.153

1. Light travels as a
a. parallel beam in each medium
b. convergent beam in each medium
c. divergent beam in each medium
d. divergent beam in one medium and convergent beam in the other medium

2. The phases of the light waves at c, d, e and f are ϕ_c, ϕ_d, ϕ_e, and ϕ_f, respectively. It is given that $\phi_c \neq \phi_f$. Then,
a. ϕ_c cannot be equal to ϕ_d
b. ϕ_d can be equal to ϕ_e
c. $\phi_d - \phi_f$ is equal to $\phi_c - \phi_e$
d. $\phi_d - \phi_c$ is not equal to $\phi_f - \phi_e$

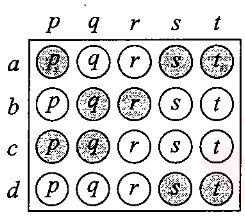
3. Speed of light is
 a. the same in medium 1 and medium 2
 b. larger in medium 1 than in medium 2
 c. larger in medium 2 than in medium 1
 d. different at b and d

Matching Column Type

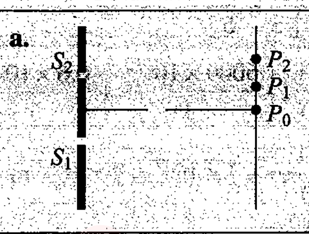
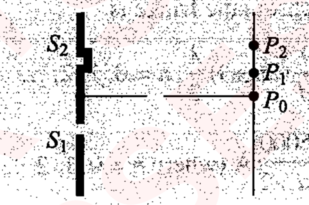
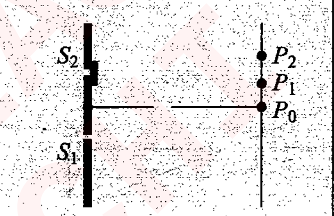
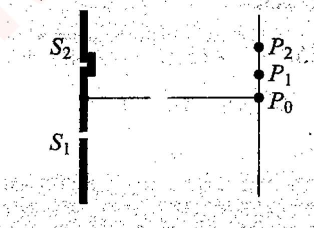
The question given in this section contains statements given in two columns, which have to be matched. The statements in Column I are labelled a, b, c, and d, while the statements in Column II are labeled p, q, r, s, and t. Any given statement in Column I can have correct matching with *One or More* statement(s) in Column II.

These appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are a-p, a-s and a-t; b-q and b-r; c-p and c-q; and d-s and d-t, then the correct darkening of bubbles will look like the following.



1. Column I shows four situations of standard Young's double-slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases, $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases b, c and d, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in the three cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in Column I with the statement(s) in Column II valid for that situation.
(IIT-JEE, 2009)

Column I	Column II
a. 	**p.** $\delta(P_0) = 0$
b. $(\mu - 1)t = \lambda/4$ 	**q.** $\delta(P_1) = 0$
c. $(\mu - 1)t = \lambda/2$ 	**r.** $I(P_1) = 0$
d. $(\mu - 1)t = 3\lambda/4$ 	**s.** $I(P_0) > I(P_1)$ **t.** $I(P_2) > I(P_1)$

ANSWERS AND SOLUTIONS

Subjective Type

1. $I_0 = 4I_0 \cos^2 \dfrac{\phi}{2}$

$$\Rightarrow \quad \cos \frac{\phi}{2} = \frac{1}{2}$$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

Therefore, path difference of the two beams producing intensity I_0 on the screen is $\lambda/3$.

Optical path difference between two beams meeting on the screen is

$$\Delta x = \left\{\left(S_2P - t + \frac{4}{3}t\right) - \left(S_1P - t + \frac{3}{2}t\right)\right\}$$

$$= (S_2P - S_1P) + t\left(\frac{4}{3} - \frac{3}{2}\right)$$

$$\Delta x = d \sin \theta - \frac{t}{6}$$

$$\therefore \quad \frac{dy}{D} - \frac{t}{6} = \frac{\lambda}{3}$$

$$\Rightarrow \quad \frac{dy}{D} = \frac{\lambda}{3} + \frac{t}{6}$$

$$\Rightarrow \quad y = \frac{D}{d}\left[\frac{\lambda}{3}+\frac{t}{6}\right]$$

$$= \frac{1}{10^{-3}}\left(\frac{6000\times10^{-10}}{3}+\frac{0.6\times10^{-6}}{6}\right)$$

$$= \frac{1}{10^{-3}}\times3\times10^{-7} = 3\times10^{-4}\text{ m}$$

$$\therefore \quad y = 30\text{ mm}$$

2. In a double-slit interference pattern, the distance x of a dark fringe from the central achromatic fringe is

$$x = \frac{D}{2d}(2n+1)\frac{\lambda}{2}, n = 0, 1, 2, \ldots$$

$$\lambda = \frac{75000}{2n+1}\text{ Å (where } n = 0, 1, 2, \ldots)$$

Thus, in the range 4000–8000 Å, the absent wavelengths are 6818, 5769, 5800, 4421.

3. a. For the central order bright to be formed at Q,

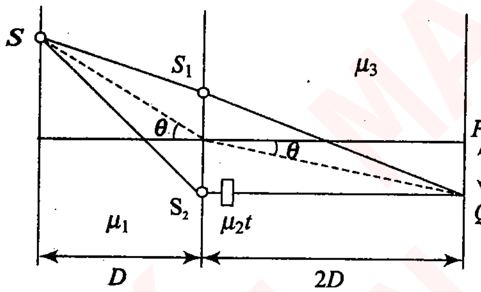


Fig. 2.154

$$(SS_1)\mu_1 + (S_1Q)\mu_3 = (SS_2)\mu_1 + (S_2Q - t)\mu_3 + \mu_2 t$$

$$(S_1Q - S_2Q)\mu_3 = (SS_2 - SS_1)\mu_1 + (\mu_2 - \mu3)t$$

$$(d\sin\phi)\mu_3 = (d\sin\theta)\mu_1 + (\mu_2-\mu_3)t \quad \text{(i)}$$

$$= \frac{d^2}{D}\mu_1 + (\mu_2-\mu_3)t$$

$$= \frac{1^2}{10^3}\times\frac{4}{3}+\left(\frac{3}{2}-\frac{9}{5}\right)\frac{4}{9}\times10^{-2} = 0$$

Hence, $\phi = 0$ or the central order bright is formed at P only.

b. In absence of slab, $t = 0$ and Eq. (i) gives

$$(d\sin\phi)\,\mu_3 = (d\sin\theta)\mu_1$$

$$\frac{x}{2D}\mu_3 = \frac{d}{D}\mu_1 \quad \text{or} \quad x = \frac{2\mu_1 d}{\mu_3} = \frac{40}{27}\text{mm}$$

4. Ratio of intensities of the sources S_1 and S_2,

$$\frac{I_1}{I_2} = 9$$

Let $I_2 = I_0$ and $I_1 = 9I_0$.

Intensity at any point on the screen,

$$I = 9I_0 + I_0 + 2\sqrt{9I_0I_0}\cos\gamma$$

For maxima at P,

$$I = 2I_0[5+2] = 16I_0$$

If transparent plates are interchanged, the central maxima will shift downward by some amount j.

$$j = \frac{Dt(\mu_1-\mu_2)}{d}$$

$$\Delta x_P = \frac{yd}{D} + t(\mu_2-\mu_1) = 0$$

$$\Rightarrow \quad \frac{yd}{D} = t(\mu_1-\mu_2)$$

When μ_2 and μ_1 are interchanged

$$\Delta x_{P_1} = \frac{yd}{D}\;t(\mu_1-\mu_2) = 2t(\mu_1-\mu_3)$$

Phase difference at P,

$$\phi = \frac{2\pi}{\lambda}\;2(\mu_1-\mu_2)t = \frac{44}{3}\pi$$

Now, intensity at P,

$$I' = I + I_0 + 2I_0\cos\frac{44}{3}\pi - I_0$$

Hence, ratio of intensities at P,

$$\frac{I'}{I} = \frac{I_0}{16I_0} = \frac{1}{16}$$

5. Optical path difference,

$$\Delta x = 2\int_0^{L/2}\mu dx - L = 2\mu_0\int_0^{L/2}\left(1+\frac{2x}{L}\right)dx - L$$

$$= \frac{3}{2}\mu_0 L - L = [(1.5)\mu_0 - 1]L$$

Now, $\quad \Delta x = N\lambda$

$$\Rightarrow \quad N = \Delta x/\lambda = [(1.5)\mu_0 - 1]L/\lambda$$

6. a. At P any point on the screen, optical path difference between SS_2P and SS_1P is zero.
$(SS_2 - SS_1P)_{\text{optical}} = 0$

$$y = -SS_2(\mu-1)\frac{D}{d} = -2.5\left(\frac{1}{5}\right)\times\frac{10^3}{3} = \frac{-1}{6}\text{ m}$$

(below 0)

b. Order of fringe at O is

$$\frac{y}{\lambda D/d} = \frac{1\times3\times10^{-3}}{6\times500\times1\times10^{-9}} = 750$$

c. $t(\mu'-1) = 200\lambda \Rightarrow t = 1.25$ mm

7. As due to presence of glass plate path difference changes by $(\mu - 1)t$, so according to given problem

$$(\mu_R - 1)t = 5\lambda_R$$

$$\Rightarrow \quad t = \frac{5 \times 7 \times 10^{-7}}{(1.5 - 1)} = 7\ \mu\text{m}$$

Now, when red light is replaced by green light,

$$(\mu G - 1)t = 6\lambda_R$$

So,

$$\frac{\mu_R - 1}{\mu_G - 1} = \frac{5}{6}$$

$$\Rightarrow \quad \mu_G - 1 = \frac{6}{5}(1.5 - 1)$$

$$\Rightarrow \quad \mu_G = 1.6$$

Further, as $5\beta_R = 10^{-3}$, i.e., $\beta_R = 2 \times 10^{-4}$

$$\therefore \quad \frac{\beta_G}{\beta_R} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$$

$$\Rightarrow \quad \frac{\beta_G - \beta_R}{\beta_R} = \frac{5}{7} - 1 = -\frac{2}{7}$$

$$\Rightarrow \quad \Delta\beta = -\frac{2}{7} \times 2 \times 10^{-4} = -0.57 \times 10^{-4}\ \text{m}$$

Hence, fringe width will decrease by 0.57×10^{-4} m when red light is replaced by green light.

8. Angle of deviation of the prism with water on one side,

$$\delta = A\left[\frac{\mu_g}{\mu_m} - 1\right] = \frac{2}{180} \times \frac{22}{7}\left[\frac{3/2}{4/3} - 1\right]$$

$$\Rightarrow \quad \delta = \frac{2 \times 11}{90 \times 7}\left[\frac{3}{2} \times \frac{3}{4} - 1\right] = \frac{2 \times 11}{90 \times 7} \times \left[\frac{1}{8}\right] = \frac{11}{90 \times 4 \times 7}$$

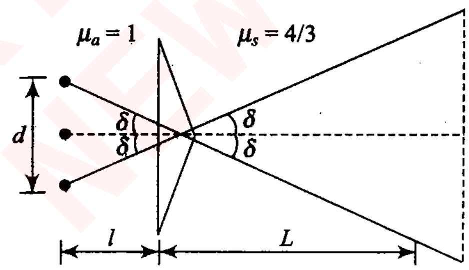


Fig. 2.155

$$d = 2\delta\ell = \frac{2 \times 11 \times 57.5}{90 \times 4 \times 7} = \frac{11 \times 115}{90 \times 4 \times 7}$$

$$D = l + L = 150\ \text{cm}$$

Fringe width,

$$\beta = \frac{11 \times 23}{18 \times 4 \times 7}\ \text{cm}$$

$$= 0.1125\ \text{mm}$$

9. Path difference due to the glass slab,

$$\Delta x = (\mu - 1)t$$

Thirty fringes are displaced due to the slab. Hence,

$$\Delta x = 30\lambda$$

$$(\mu - 1)t = 30\lambda$$

$$t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1}\ 3.6 \times 10^{-3}\ \text{cm}$$

10. According to the question,
shift = 5 (fringe width)

$$\therefore \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000\ \text{Å}$$

11. Let n_1 bright fringes of λ_1 overlaps with n_2 bright fringes of λ_2. Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

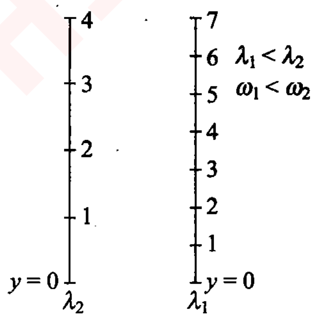


Fig. 2.156

$$\text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{700}{400} = \frac{7}{4}$$

The ratio implies that 7th bright fringe of λ_1 will overlap with 4th bright fringe of λ_2. Similarly, 14th bright fringe of λ_1 will overlap with 8th bright fringe of λ_2 and so on. So, the minimum order of λ_1 which overlaps with λ_2 is 7.

Objective Type

1. a. The detector receives direct as well as reflected waves. The distance moved between two consecutive position of maxima is $\lambda/2$.

$$14 \times \frac{\lambda}{2} = 7l = 0.14\ \text{m}$$

$$\Rightarrow \quad \lambda = 0.02\ \text{m}$$

$$c = n\lambda$$

Putting $c = 3 \times 10^8$ m/s, we have

$$n = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.02} = 1.5 \times 10^{10}\ \text{Hz}$$

2. c. $S_2P - S_1P = \dfrac{dy}{D} = \dfrac{d \times (d/2)}{D} = \dfrac{d^2}{2D}$

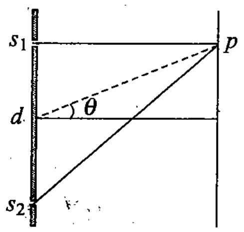


Fig. 2.157

$$\frac{d^2}{2D} = n\lambda$$

$$\lambda = \frac{d^2}{2nD}, n = 1, 2, \ldots$$

$$\lambda = \frac{d^2}{2D}, \frac{d^2}{4D}, \frac{d^2}{6D}$$

3. b. $y_n = \left(n + \dfrac{1}{2}\right)\dfrac{D\lambda}{d}, n = 1, 2, 3$

$$\left(3 + \frac{1}{2}\right)\frac{D\lambda_1}{d} = 2 \times \frac{D\lambda_2}{d}$$

$$\lambda_2 = \frac{7}{4} \times 400 = 700 \text{ nm}$$

4. d. Path difference $= d \sin\phi + d \sin\theta$

For maxima, $\Delta x = m\lambda$

$$\Rightarrow \quad \sin\phi + \sin\theta = \frac{m\lambda}{d}$$

5. d. $\beta_w = \dfrac{\lambda D}{\mu d}$

We need to increase $\beta \Rightarrow D$ increases; d decreases.

6. b.

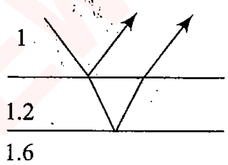


Fig. 2.158

$$2\mu t = n\lambda \Rightarrow t = \frac{n\lambda}{2\mu}$$

$$\Rightarrow \quad t = \frac{\lambda}{2\mu} = 200 \text{ nm}$$

7. c. N = no. of fringes shifted

Shift of central maxima $= n\lambda = (\mu - 1)t$

$\Rightarrow \quad t = 24\ \mu\text{m}$

8. d. Let intensity of one slit be I.

For maxima,

$$\Delta\phi = 0$$

$$\Rightarrow \quad I_0 = I + I + 2\sqrt{II}\cos(0)$$

$$\Rightarrow \quad I = \frac{I_0}{4}$$

9. a. Path difference, $\Delta x = \dfrac{yd}{D}$

Here, $\quad y = \dfrac{5\lambda}{2}$

and $\quad D = 10d = 50\lambda \quad (\text{as } d = 5\lambda)$

So,

$$\Delta x = \left(\frac{5\lambda}{2}\right)\left(\frac{5\lambda}{50\lambda}\right) = \frac{\lambda}{4}$$

Corresponding phase difference will be

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

or $\quad \dfrac{\phi}{2} = \dfrac{\pi}{4}$

$\therefore \quad I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$

$$= I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

10. a. Path difference at P is

$$\Delta x = 2\left(\frac{x}{2}\cos\theta\right) = x\cos\theta$$

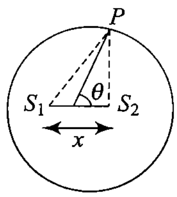


Fig. 2.159

For intensity to be maximum,

$\Delta x = n\lambda \qquad (n = 0, 1, 2, 3, \ldots)$

or $\quad x\cos\theta = n\lambda$

or $\quad \cos\theta = \dfrac{n\lambda}{x} \geq 1$

$\therefore \quad n \geq \dfrac{x}{\lambda}$

Substituting $x = 5\lambda$, we get

$n \geq 5 \quad$ or $\quad n = 1, 2, 3, 4, 5, \ldots$

Therefore, in all four quadrants there can be 20 maxima. There are more maxima at $\theta = 0°$ and $\theta = 180°$. But $n = 5$ corresponds to $\theta = 90°$ and $\theta = 270°$ which are coming only twice while we have multiplied it four times. Therefore, total number of maxima are still 20, i.e., n = 1 to 4 in four quadrants (total 16) plus more at $\theta = 0°, 90°, 180°$ and $270°$.

11. a. $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I_0 = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$$

or $\quad \frac{\phi}{2} = \frac{\pi}{3}$

or $\quad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\Delta x$

or $\quad \frac{1}{3} = \left(\frac{1}{\lambda}\right) y \frac{d}{D} \quad \left(\Delta x = \frac{yd}{D}\right)$

$$\therefore \quad y = \frac{\lambda}{3 \times \frac{d}{D}} = \frac{6 \times 10^{-7}}{3 \times 10^{-4}}$$

$$= 2 \times 10^{-3}\ \text{m} = 2\ \text{mm}$$

12. a. Consider light to be incident at near normal incidence. We wish to cause destructive interference between rays r_1 and r_2 so that maximum energy passes into the glass. A phase change of $\lambda/2$ occurs in each ray because at both the upper and lower surfaces of the MgF_2 film the light is reflected by a medium of greater index of refraction. When striking a medium of lower index of refraction, the light is reflected with no phase change. Since in this problem both rays 1 and 2 experience the same phase shift, no net change of phase is introduced by these two reflections. Hence, the only way a phase change can occur is if the two rays travel through different optical path lengths. The optical path length is the product of the geometric path difference a ray travels through different media and the refractive index of the medium in which it is travelling. For destructive interference, the two rays must be out of phase by an odd number of half wavelengths. Hence, the optical path difference needed for destructive interference is

$$2\mu d = (2n+1)\frac{\lambda}{2}, n = 0, 1, 2, \ldots$$

Note that $2\mu d$ is the total optical path length that the rays traverse when $n = 0$.

$$\therefore \quad d = \frac{\lambda/2}{2\mu} = \frac{\lambda}{4\mu} = \frac{350 \times 10^{-9}}{4 \times 1.38} = 100\ \text{nm} = 1 \times 10^{-7}\text{m}$$

13. a. Condition for observing bright fringe is

$$2nd = \left(m + \frac{1}{2}\right)\lambda$$

$$\therefore \quad \lambda = \frac{2nd}{\left(m + \frac{1}{2}\right)} = \frac{2 \times 1.5 \times 4 \times 10^{-5}}{m + \frac{1}{2}}$$

$$= \frac{12 \times 10^{-5}}{m + \frac{1}{2}}$$

The integer m that gives the wavelength in the visible region (4000 Å to 7000 Å) is $m = 2$. In that case,

$$\lambda = \frac{12 \times 10^{-5}}{2 + \frac{1}{2}} = 4.8 \times 10^{-5} = 4800\ \text{Å}$$

14. a. In this case, both the rays suffer a phase change of 180° and the conditions for destructive interference is

$$2nd = \left(m + \frac{1}{2}\right)\lambda_1$$

$$2nd = \left(m + \frac{3}{2}\right)\lambda_2$$

$$\therefore \quad \frac{m + \frac{1}{2}}{m + \frac{3}{2}} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{7000} = \frac{5}{7}$$

and $\quad d = \frac{\left(m + \frac{1}{2}\right)\lambda_1}{2n} = \frac{2.5 \times 7000}{2 \times 1.3}$

$$= 6738\ \text{Å} = 6.738 \times 10^{-5}\ \text{cm}$$

15. a. $\mu = \frac{c}{v} = \frac{v\lambda}{v\lambda'}$

$$\frac{3}{2} = \frac{\lambda}{\lambda'} \quad \text{or} \quad \lambda' = \frac{2\lambda}{3}$$

Note that the frequency remains unchanged.

16. a. $I_{min} \propto (A_1 - A_2)^2$

$I_{min} \propto (2a - a)^2$

Clearly, the intensity of minima increases. Again,

$$I_{max} \propto (A_1 + A_2)^2$$
$$I_{max} \propto (2a + a)^2$$

Clearly, the intensity of maxima increases.

17. c. Effective path difference is $\mu_1 L_1 - \mu_2 L_2$.

18. a. $\beta = \frac{D\lambda}{d} = \frac{f\lambda}{d} = \frac{1 \times 4890 \times 10^{-10}}{0.2 \times 10^{-3}}$

$$= 0.29 \times 10^{-2}\ \text{m} = 2.9\ \text{mm} \approx 3\ \text{mm}$$

19. c. In the first case,

$$I = I_0 + I_0 + 2I_0 \cos 0°$$

or $\quad I = 4I_0$

In the second case,

$$I' = I_0 + I_0 = 2I_0$$

$$\therefore \quad \frac{I}{I'} = \frac{4I_0}{2I_0} = \frac{2}{1}$$

20. d. $\beta' = \frac{D\lambda}{3d} = \frac{\beta}{3}$

21. b. Different wavelengths would correspond to different frequencies. Lights of different intensities can give coherence even if contrast is poor.

22. d. The contrast between bright and dark fringes is determined by intensity ratio.

23. c. A slit would give divergent; a biprism would give double; a glass slab would give a parallel wavefront. Edge is downward.

24. b. P to Q: convergence increasing; Q to R: direction changing.

25. b. Velocity of light is perpendicular to the wavefront.

26. c. Fringe width, $\beta \propto \lambda$. Therefore, λ and hence β will decrease 1.5 times when immersed in the liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in the liquid, it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y = 4$ cm.

27. d. If $d \sin\theta = (\mu - 1)t$, central fringe is obtained at O.
If $d \sin\theta > (\mu - 1)t$, central fringe is obtained above O.
If $d \sin\theta < (\mu - 1)t$, central fringe is obtained below O.

28. c. Intensity at the centre will be zero if path difference is $\lambda/2$. That is,

$$(\mu - 1)t = \frac{\lambda}{2}$$

or $$t = \frac{\lambda}{2(\mu - 1)}$$

29. b. For maximum intensity on the screen,

$$d \sin\theta = n\lambda$$

or $$\sin\theta = \frac{n\lambda}{d}$$

$$= \frac{(n)(2000)}{(7000)} = \frac{n}{35}$$

$\sin\theta \geq 1 \Rightarrow n = 0, 1, 2, 3$ only

Thus, only seven maxima can be obtained on both sides of the screen.

30. a. Fringe width, $\beta = \lambda D/d$. When the apparatus is immersed in a liquid, λ and hence β is reduced μ (refractive index) times.

$$10\beta' = (5.5)\beta$$

or $$10\lambda'\left(\frac{D}{d}\right) = (5.5)\frac{\lambda D}{d}$$

or $$\frac{\lambda}{\lambda'} = \frac{10}{5.5} = \mu$$

or $$\mu = 1.8$$

31. b. No fringes are observed as two independent sources are not coherent.

32. c. $30\beta = n\beta'$

$$\Rightarrow 30\frac{D \times 4000}{d} = n\frac{D \times 6000}{d} \Rightarrow n = 20$$

33. b. $$S_1 = \frac{\Delta_1 D}{d} = 11 \times 10^{-3}$$

$$S_2 = \frac{\Delta_2 D}{d} = 12 \times 10^{-3}$$

$$\Rightarrow \Delta_1/\Delta_2 = 11/12$$

$$\Rightarrow 12\Delta_1 = 11\Delta_2$$

34. a. $$y = \frac{d}{2} = \frac{2(n-1)D\lambda}{2d}$$

$$\Rightarrow \lambda = \frac{d^2}{D(n-1)}, \quad n = 1, 2, 3$$

35. a. $$\frac{(d/2)}{D} = \frac{\theta}{2} \Rightarrow \theta = \frac{d}{D} \quad \text{and} \quad \beta = \frac{D\lambda}{d} = \frac{\lambda}{\theta}$$

36. a. White fringe is formed at the centre of screen. Position of central fringe will remain unchanged on moving the screen.

37. a. Optical path difference at $O = \left(\frac{\mu_2}{\mu_1} - 1\right)t$

38. c. $$I_p = \frac{I_{max}}{2}[1 + \cos\phi] = \frac{I_{max}}{2}\left(1 + \cos\frac{2\pi y}{\beta}\right)$$

where $\beta = D\lambda/d$.

First maxima is observed at P, i.e., $\cos\frac{2\pi y}{\beta} = 1$. As D increases β will increase and the value of $\cos\frac{2\pi y}{\beta}$ should be negative. Hence, the ratio I_P/I_{max} starts decreasing but starts increasing again as $\cos\frac{2\pi y}{\beta}$ again starts becoming positive.

39. a. Detector receives both the direct as well as the reflected waves. Distance between two consecutive maxima = $\lambda/2$.

For 14 maxima, distance = $14 \times \frac{\lambda}{2} = 0.14$ m

$$\therefore \lambda = 0.02 \text{ m}$$

$$v = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.02} = 1.5 \times 10^{10} \text{ Hz}$$

40. a. $$y_n(\max) = n\frac{D\lambda}{d}$$

Here, $y_n(\max) = d/2$

So, $$n\frac{D\lambda}{d} = \frac{d}{2} \quad \text{or} \quad n = \frac{d^2}{2\lambda D}$$

41. b. The wavelength missing from the reflected spectrum must satisfy the condition, $2\mu t = n\lambda$, where t is thickness of air film.

$$2\mu t = n\lambda_1 = (n + 1)\lambda_2$$

or $$n \times (7200) = (n + 1)\,5400$$

$$\therefore n = 3$$

The next wavelengths must satisfy the condition,

$$n\lambda_1 = (n + 2)\lambda_2$$

or $7200 \times 3 = (3+2)\lambda_2 = 5\lambda_2$

$\Rightarrow$ $\lambda_2 = 4320$ Å

42. b. $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Here,

$I_1 = I$ and $I_2 = 4I$

At point A,

$$\phi = \frac{\pi}{2}$$

$\therefore$ $I_A = I + 4I = 5I$

At point B,

$\phi = \pi$

$\therefore$ $I_B = I + 4I - 4I = I$

$\therefore$ $I_A - I_B = 4I$

43. b. Fringe width, $\beta = \frac{\lambda D}{d}$, i.e., $\beta \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of 4/6 or 2/3 or the number of fringes in the same segment will increase by a factor of 3/2. Therefore, number of fringes observed in the same segment is $12 \times \frac{3}{2} = 18$.

44. a. Path difference due to slab should be integral multiple of λ. Hence,

$\Delta x = n\lambda$

or $(\mu - 1)t = n\lambda$, $n = 1, 2, \ldots$

or $t = \frac{n\lambda}{\mu - 1}$

For minimum value of t, $n = 1$.

$\therefore$ $t = \frac{n\lambda}{\mu - 1} = \frac{\lambda}{1.5 - 1} = 2\lambda$

45. b. Angular separation is λ/d.

For angular separation to be 10% greater, λ should be 10% greater.

$\therefore$ New wavelength $= \left(589 + \frac{589}{10}\right)$nm

$= (589 + 58.9)$ nm $= 647.9$ nm (≈ 648 nm)

46. a. $I \propto 4a^2 \cos^2\frac{\phi}{2}$

In the first case, $\phi = 2\pi$

$\therefore$ $I' \propto 4a^2$

In the second case, $\phi = \frac{2\pi}{3}$

$\therefore$ $I' \propto 4a^2\cos^2\frac{2\pi}{3}$ or $I' \propto a^2$

$\frac{I'}{I} = \frac{1}{4}$ or $I' = \frac{I}{4}$

47. a. Condition for maxima is

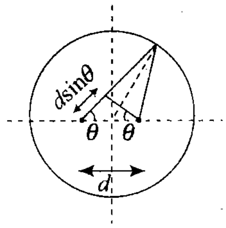


Fig. 2.160

$d\sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{d} = n\left(\frac{0.50}{2.0}\right) = 0.25n$$

As $\sin\theta$ lies between -1 and 1, so we wish to find all values of n for which

$|0.25n| \le 1$

These values are $-4, -3, -2, -1, 0, +1, +2, +3, +4$. For each of these, there are two different values of θ except for -4 and $+4$. A single value of θ, $-90°$ and $+90°$, is associated with $n = -4$ and $n = +4$, respectively. Thus, there are 16 different angles in all and therefore 16 maxima.

48. d. The path difference introduced due to introduction of transparent sheet is given by $\Delta x = (m - 1)t$.

If the central maxima occupies position of nth fringe, then

$(\mu - 1)t = n\lambda = d\sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{(\mu-1)t}{d} = \frac{(1.17-1)\times 1.5\times 10^{-7}}{3\times 10^{-7}}$$

$= 0.085$

Hence, the angular position of central maxima is

$\theta = \sin^{-1}(0.085) = 4.88°$

For small angles,

$\sin\theta = \theta = \tan\theta$

$\Rightarrow$ $\tan\theta = \frac{y}{D}$

$\therefore$ $\frac{y}{D} = \frac{(\mu - 1)t}{d}$

Shift of central maxima is

$$y = \frac{D(\mu - 1)t}{d}$$

This formula can be used if D is given.

49. a. The experimental set-up is in a liquid, therefore the wavelength of light will change

$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{6300}{1.33} = \frac{6300\times 10^{-10}}{1.33}\text{ m}$$

Fringe width,

$$\beta = \frac{\lambda_{\text{liquid}} D}{d} = \frac{\lambda_{\text{air}} D}{\mu d} = \frac{6300\times 10^{-10}}{1.33}\times\frac{1.33}{10^{-3}}$$

$= 6.3 \times 10^{-4}$ m

50. d. $d = 0.5$ mm and $D = 0.5$ m

Separation $= 3\beta + 1.5\beta = 4.5\beta$

$$= 4.5 \times \frac{\lambda D}{d} = 2.22 \text{ mm}$$

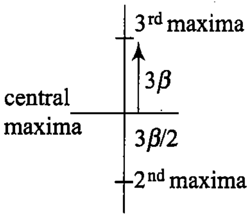


Fig. 2.161

51. c. Change in path difference for any point on screen is $|\mu - 1.8|t$.

For central maxima, phase difference = 0.

Hence,

$$d \sin\theta - |\mu - 1.8|t = 0$$

$$\Rightarrow \quad d\theta = |\mu - 1.8|t \qquad [\theta \text{ is very small and is in radian}]$$

$$\Rightarrow \quad |\mu - 1.8| = \frac{10^{-3} \times 0.1}{0.5 \times 10^{-3}} = 0.2$$

$$\Rightarrow \quad \mu = 2 \quad \text{or} \quad 1.6$$

52. b. Velocity of wave in medium (μ) is less than that in air. Hence, wavefront reaches earlier at P through air.

53. d. Fringe width, $\beta = \dfrac{\lambda D}{d}$

β becomes (1/4th) when λ is halfed and d is doubled. The separation between successive dark fringes reduces. It does not remain unchanged.

54. a. $\beta = 0.03$ cm, $D = 1$ m $= 100$ cm

Distance between images of the source = 0.8 cm

Distance of image from lens, $v = 80$ cm

Distance of slit from lens = u

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} + \frac{1}{u} = \frac{1}{16} \Rightarrow u = 20 \text{ cm}$$

$$\text{Magnification} = \frac{v}{u} = \frac{80}{20} = 4$$

$$\text{Magnification} = \frac{\text{distances between images of slits}}{\text{distance between slits}}$$

$$= \frac{0.8}{d} = \frac{0.8}{d} = 4$$

$$\Rightarrow \quad d = 0.2 \text{ cm} \quad \Rightarrow \quad \beta = \frac{D\lambda}{d} = \frac{100\lambda}{2} = 0.03$$

$$\Rightarrow \quad \lambda = 6000 \text{ Å}$$

55. c. In Young's double slit experiment,

$$\sin\theta = \theta = y/D$$

So, $\Delta y/D$ and hence angular fringe width $\theta_0 = \Delta\theta$ (with $\Delta y = \beta$) will be

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$$\Rightarrow \quad \theta_0 = 1° = \left(\frac{\pi}{180}\right) \text{rad, and } \lambda = 6 \times 10^{-7} \text{ m}$$

$$\text{or} \quad d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times (6 \times 10^{-7}) = 3.44 \times 10^{-5} \text{ m}$$

$$\text{or} \quad d = 0.0344 \text{ mm}$$

56. a. As a bright fringe is formed in front of slit, therefore $d/2$ = integral multiple of fringe width

$$\frac{d}{2} = \frac{n\lambda D}{d} \quad \Rightarrow \quad n = \frac{d^2}{2\lambda D}$$

57. a. When mica sheet of thickness t and refractive index μ is introduced in the path of one of the interferring beams, optical path increases by $(\mu - 1)t$. Therefore, the shift on the screen is given by

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \text{(i)}$$

When the distance between the plane of slits and screen is changed from D to $2D$, then

$$\beta = \frac{2D}{d}\lambda \qquad \text{(ii)}$$

$$\therefore \quad \frac{D}{d}(\mu - 1)t = \frac{2D(\lambda)}{d} \quad \Rightarrow \quad \lambda = \frac{1}{2}(\mu - 1)t$$

58. a. According to the question,

shift = 5 fringe widths

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000 \text{ Å}$$

59. a. y_9 = Position of 9th bright fringe $= 9\left(\dfrac{\lambda D}{d}\right)$

$$y_2 = \text{Position of 2nd dark fringe} = \left(2 - \frac{1}{2}\right)\frac{\lambda D}{d} = \frac{3}{2}\frac{\lambda D}{d}$$

$$y_9 - y_2 = 7.5 \text{ mm} \quad \Rightarrow \quad \frac{\lambda D}{d}\left(9 - \frac{3}{2}\right) = 7.5 \times 10^{-3}$$

$$\therefore \quad \lambda = (7.5 \times 10^{-3})\left(\frac{2}{15}\right)\left(\frac{0.5 \times 10^{-3}}{100 \times 10^{-2}}\right)$$

$$= (75)\left(\frac{2}{15}\right)(5)\,(10^{-4-4}) = 50 \times 10^{-8} \text{ m}$$

$$= 5000 \text{ Å}$$

60. a. δ = phase difference between the waves from S_1 and S_2 at

$$P = \frac{\pi}{2} - \frac{2\pi}{2}(d \sin\theta)$$

For maximum intensity at P, $\delta = n\pi$, where $n = 0, \pm1, \pm2, \ldots$

$$\therefore \frac{2\pi}{2}(1.5\lambda \sin\theta)\frac{\pi}{2} = n\pi$$

$$\Rightarrow n - \frac{1}{2} = 3\sin\theta$$

$$\Rightarrow \sin\theta = \left(\frac{n-\frac{1}{2}}{3}\right)$$

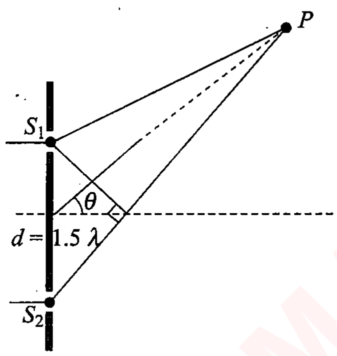


Fig. 2.162

For $n = 0$, $\sin\theta = -\frac{1}{6}$

For $n = \pm1$, $\sin\theta = \frac{1}{6}, -\frac{1}{2}$

For $n = \pm2$, $\sin\theta = \frac{1}{2}, -\frac{5}{6}$

61. b. Δx = path difference between two light waves

$= [n_1L_1 + L_2 - L_1] - [n_2L_2]$

$\Delta\phi$ = phase difference between two waves

For longest wavelengths, $\Delta\phi$ is the smallest.

For constructive interference, $\Delta\phi = 2\pi$

$l_{max} = n_1L_1 - n_2L_2 + (L_2 - L_1)$

$= (1.7)(3.5\times10^{-6}) - (1.25)(5.0\times10^{-6}) + (5.0 - 3.5)\times10^{-6}$

$= (5.95 - 6.25 + 1.5)\times10^{-6} = 1.2\ 5\ 10^{-6}$ m

$= 1.2$ μm

62. c. The maxima in this case is obtained whenever

$$x_{cm} = \frac{n\lambda D}{d}$$

We can write for first wavelength,

$$(x_4)\lambda_1 = \frac{4\lambda_1 D}{d}$$

This must be equal to $(x_3)\lambda_2 = \frac{3\lambda_2 D}{d}$, since $(x_4)\lambda_1 = (x_3)\lambda_2$.

$$\therefore \frac{4\lambda_1 D}{d} = 3\lambda_2\frac{D}{d} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{3}{4}$$

Hence, when both wavelengths are incident simultaneously, the maxima due to two will coincide at a point where the fourth maxima due to λ_1 occured. This point will have maximum intensity and intensity will not be zero at any point.

63. c. Here, direction of light is given by normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Angle made by the $\vec{n}$ with y-axis is given by cos

$$\beta = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$$

64. d. In cases I, II, III and IV, the path differences are respectively, $\frac{\lambda}{2}, \lambda, \frac{\lambda}{4}$ and $\frac{3\lambda}{2}$.

Phase differences are, respectively, π, 2π, $\frac{\pi}{2}, \frac{3\pi}{2}$, and

$$I = I_0\cos^2\left(\frac{\phi}{2}\right)$$

Therefore, the intensities in the four cases are $0, I_0, \frac{I_0}{2}, \frac{I_0}{2}$, respectively.

65. a. There are three and a half fringes from first maxima to fifth minima as shown.

$$\beta = \frac{7\text{ mm}}{3.5} = 2\text{ mm} \Rightarrow \lambda = \frac{\beta D}{d} = 600\text{ nm}$$

66. c. For 100th maximum,

$d\sin\theta = 100\lambda$

$$\Rightarrow \sin\theta = \frac{100\times500\times10^{-9}}{1\times10^{-3}} = \frac{5\times10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$$

$$\therefore y = D\tan\theta = 1\times\tan30 = \frac{1}{\sqrt{3}}$$

67. a. In ΔS_1PO,

$$\tan\frac{\theta}{2} = \frac{d/2}{D}$$

As $D >> d$, θ is very small.

$$\therefore \tan\frac{\theta}{2} = \frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \frac{d}{2D} \Rightarrow \frac{D}{d} = \frac{1}{\theta}$$

$$\therefore \text{Fringe width} = \frac{\lambda D}{d} = \frac{\lambda}{\theta}$$

68. b. Angular width $= \frac{\lambda}{d} = 10^{-3}$ (given)

No. of fringes within 0.12° will be

$$n = \frac{0.12\times2\pi}{360\times10^{-3}} \cong [2.09]$$

The number of bright spots will be three.

69. c. At path difference $\frac{\lambda}{6}$, phase difference is $\frac{\pi}{3}$.

$$I = I_0 + I_0 + 2I_0 \cos\frac{\pi}{3} = 3I_0$$

So, the required ratio is $\frac{3I_0}{4I_0} = 0.75$.

70. a. Optical path difference between the waves $= (n_3 - n_2)t$

$\therefore$ Phase difference $= 2\pi\frac{(n_3 - n_2)t}{\lambda_{(\text{vacuum})}} = 2\pi\frac{(n_3 - n_2)t}{n_1\lambda_1}$

71. d. The nearest white spot will be at P, the central maxima.

$$y = \frac{2d}{3} - \frac{d}{2} = \frac{d}{6}$$

72. b. When light passes through a medium of refractive index μ, the optical path it travels is μt.

Path difference $= (\mu\ell + b) - (\ell + b) = (\mu - 1)\ell$

For a small element 'dx', path difference,

$$\Delta x = [(1 + ax) - 1]\, dx = axdx$$

For the whole length,

$$\Delta x = \int_0^\ell ax\, dx = \frac{a\ell^2}{2}$$

For a minima to be at 'O',

$$\Delta x = (2n + 1)\frac{\lambda}{2}$$

$$\Rightarrow \quad \frac{a\ell^2}{2} = (2n+1)\frac{\lambda}{2}$$

For minimum 'a', $n = 0$.

$$\therefore \quad \frac{a\ell^2}{2} = \frac{\lambda}{2} \quad \Rightarrow \quad a = \frac{\lambda}{\ell^2}$$

73. d. Shift of fringe pattern $= (\mu - 1)\frac{tD}{d}$

$$\Rightarrow \quad \frac{30D(4800 \times 10^{-10})}{d} = (0.6)t\frac{D}{d}$$

$$\Rightarrow \quad 30 \times 4800 \times 10^{-10} = 0.6t$$

$$\therefore \quad t = \frac{30 \times 4800 \times 10^{-10}}{0.6} = \frac{1.44 \times 10^{-5}}{0.6} = 24 \times 10^{-6}\text{ m}$$

74. c. There can be three minima from central point to ∞ corresponding to $\frac{\lambda}{3}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ path differences.

Therefore, total number of minima $= 2n_{max} = 6$.

75. b. The situation can be taken as if there are two sources S_1 and S_2 as shown in the figure in question.

For 'O' to be a maxima:

Path difference $\frac{(3d)(d/2)}{D} = \frac{3d\ d}{2D} = n\lambda$

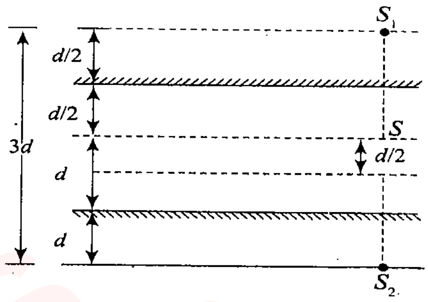


Fig. 2.163

$$\Rightarrow \quad \lambda = \frac{3d^2}{2nD} \quad \text{or} \quad \lambda = \frac{3d^2}{2D}$$

76. b. Ray N undergoes reflection at surface II with phase change of π.

$\Rightarrow \quad n_3 > n_2$

Ray Q undergoes a phase change of π at surface II, but there is no phase change when it is reflected from surface I.

$\Rightarrow \quad n_1 < n_2$

77. d. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\Delta\phi$

Here $I_1 = I_0$

$I_2 = I_0/2$

For maximum intensity, $\cos\Delta\phi = 1$

For minimum intensity, $\cos\Delta\phi = -1$

Before painting,

$I = I_2 = I_0$, $I_{max} = 4I_0$, $I_{min} = 0$

After painting,

$$I_1 = I_0,\ I_{max} = \left(\frac{3 + 2\sqrt{2}}{2}\right)I_0 < 4I_0$$

$$I_2 = \frac{I_0}{2},\ I_{min} = \left(\frac{3 - 2\sqrt{2}}{2}\right)I_0 > 0$$

78. b. $\frac{\lambda_1 D}{d} = 4.84$ mm (i)

Let required wavelength be λ_2. Then according to given information,

$\frac{\lambda_2 D}{2d} = 4.84$ mm (ii)

From (i) and (ii),

$$\frac{\lambda_1}{\frac{l_2}{2}} = 1 \Rightarrow \lambda_2 = 1200\text{ nm}$$

79. b. For dark fringe,

Path difference $= (2n - 1)\frac{\lambda}{2}$

80. a. At P, $\Delta x = \frac{(8d) \times 3d}{D}$

For 2nd maxima,

$\Delta x = 2\lambda$

$$\Rightarrow \quad \frac{24d^2}{D} = 2\lambda$$

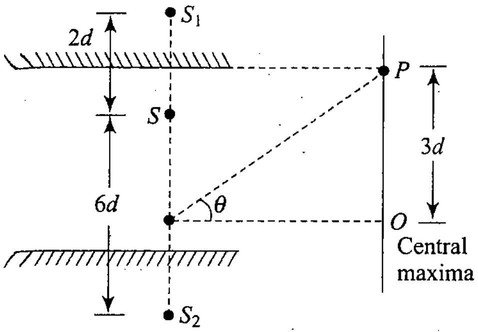


Fig. 2.164

$$\Rightarrow \quad \lambda = \frac{12d^2}{D}$$

81. c. Let n_1^{th} maxima corresponding to λ_1 be overlapping with n_2^{th} maxima corresponding to λ_2. Then, the required distance,

$$y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$= \text{LCM of } \frac{\lambda_1 D}{d} \text{ and } \frac{\lambda_2 D}{d}$$

82. a. Path difference $\Delta x = \frac{dy}{D}$

Here $d = 2 \times 10^{-3}$ m, $y = \frac{1}{6} \times 10^{-3}$ m, $D = 2$ m

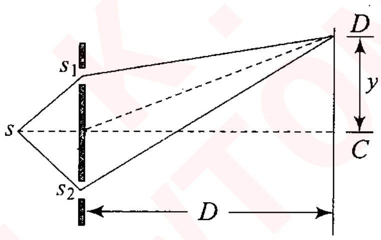


Fig. 2.165

$$\Delta x = \frac{2 \times 10^{-3} \times 1 \times 10^{-3}}{2 \times 6} = \frac{10^{-6}}{5} \text{ m}$$

Phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

$$= \frac{2\pi}{500 \times 10^{-9}} \times \frac{10^{-9}}{6} = -\frac{2\pi}{3}$$

Resultant intensity at P

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\Delta\phi$$

$$= I_0 + 9I_0 + 2 \times 3I_0 \cos\frac{2\pi}{3}$$

$$= 10I_0 - 3I_0 = 7I_0$$

83. d. $\Delta x = (\mu_A - 1)t_A - (\mu_B - 1)t_B$

$= \mu_A t_A - \mu_B t_B - t_B + t_A$

$= t_B - t_A$

If $\Delta x > 0$, the fringe pattern will shift upward.
If $\Delta x < 0$, the fringe pattern will shift downward.

84. b. $\Delta x = d \sin\theta = \frac{dy}{D}$ (approx.)

$$\Delta x = \frac{d \times d}{D} = \frac{d^2}{D}$$

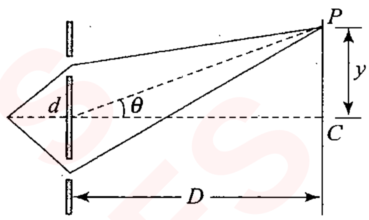


Fig. 2.166

For missing wavelength, destructive interference has to be there at $y = d$.

$$\frac{d^2}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

$$\lambda = \frac{2d^2}{D}, \frac{2d^2}{3D}, \frac{2d^2}{5D}, \ldots$$

85. c. $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$$= \left(\frac{1 + \sqrt{\frac{I_2}{I_1}}}{1 - \sqrt{\frac{I_2}{I_1}}}\right)^2$$

From given information,

$$I_{max} = \frac{105}{100} I$$

$$I_{min} = \left(\frac{95}{100}\right) I$$

where $I = I_1 + I_2$ is average intensity.

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{105}{95} = \left(\frac{1 + \sqrt{\frac{I_2}{I_1}}}{1 - \sqrt{\frac{I_2}{I_1}}}\right)^2$$

$$\Rightarrow \quad \sqrt{\frac{I_1}{I_2}} = 0.0244$$

$$\Rightarrow \quad \frac{I_1}{I_2} = 1681$$

86. d. Fringe width $= \dfrac{\lambda_{\text{medium}} D}{d} = \dfrac{\lambda D}{\mu d}$

As $\mu > 1$, so fringe width decreases. Fringe pattern doesn't shift because same path length change is introduced in both the waves.

87. c. At point P, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$ and $\Delta x = \dfrac{dx}{\Delta}$

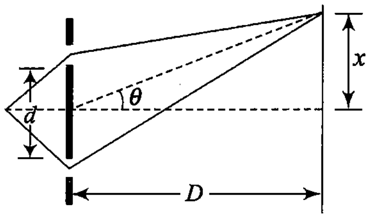


Fig. 2.167

$$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{dx}{D} = \frac{2\pi x}{\beta}$$

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi;\ I_1 = I_2 = I$$

$$I_P = 4I\cos^2\frac{\Delta d}{2}$$

Given $4I = I_0$

$$\Rightarrow \quad I_1 = I_2 = \frac{I_0}{4}$$

$$I_P = \frac{I_0}{2} + \frac{I_0}{2}\cos\Delta\phi$$

$$I_P = I_0\cos^2\frac{\Delta\phi}{2} = I_0\cos^2\frac{\pi x}{\beta}$$

88. b. If the detector move by 0.5λ from origin, it observes 2nd maxima. So, after every 0.5λ, one maxima will be observed.

89. d. Path difference between waves of

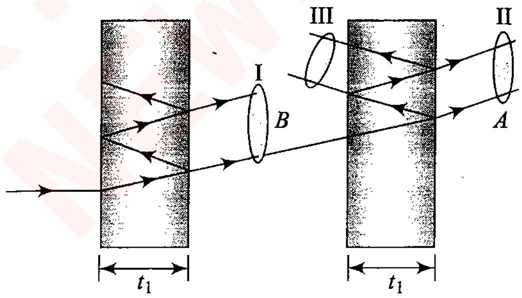


Fig. 2.168

I is $\rightarrow 2\mu t_1$
II is $\rightarrow 2\mu t_2$
III is $\rightarrow 2\mu t_3$

System I is used to observe film 1 from A, system II is used to observe film 2 from B, and system III is used to observe film 2 from A. It is given that $2\mu t_2$ = odd multiple of $\lambda/2$, i.e., dark. So, from A film 2 appears dark while for I, relation between t_1 and t_2 has to be known.

90. c. As amplitudes are A and $2A$, so intensities would be in the ratio 1:4, let us say I and $4I$.

$$I_{\max} = I_0 = I + I_0 + 2\sqrt{4I^2} = 9I$$

$$\Rightarrow \quad I = \frac{I_0}{9}$$

Intensity at any point,

$$I' = I + 4I + 2\sqrt{4I^2}\cos\phi$$

$$\Rightarrow \quad I' = 5I + 4I\cos\phi = \frac{I_0}{9}(5 + 4\cos\phi)$$

91. b. Let the intensity of individual waves be I. Then

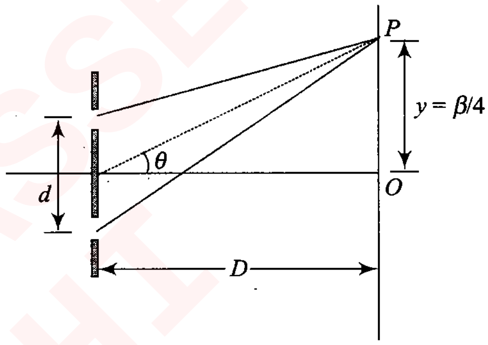


Fig. 2.169

$$I_0 = 4I \Rightarrow I = \frac{I_0}{4}$$

At P,

$$\Delta x = d\sin\theta$$

$$\Rightarrow \quad \Delta x = d\sin\theta = \frac{dy}{D}$$

$$\Rightarrow \quad \Delta x = \frac{d}{D} \times \frac{\beta}{4} = \frac{d}{D} \times \frac{\lambda D}{4d} = \frac{\lambda}{4}$$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\pi}{4} = \frac{\pi}{2} \qquad [\Delta\phi = k\,\Delta x]$$

$$I' = I + I + 2\sqrt{I^2}\cos\frac{\pi}{2} = 2I = \frac{I_0}{2}$$

92. c. Total number of waves $= \dfrac{(1.5)t}{\lambda}$ (i)

$$\begin{pmatrix}\text{Total number}\\ \text{of waves}\end{pmatrix} = \begin{pmatrix}\text{optical path length}\\ \hline \text{wavelength}\end{pmatrix}$$

For B and C:

$$\text{Total number of waves} = \frac{n_0\left(\frac{t}{3}\right)}{\lambda} + \frac{1.6\left(\frac{2t}{3}\right)}{\lambda} \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$n_0 = 1.3$

93. b. $\phi_0 = \dfrac{2\pi}{\lambda_0}$ (optical path)

$$\phi_0 = \frac{2\pi}{\lambda_0}(nx)$$

94. c. $y_n = n\left(\dfrac{D\lambda}{d}\right)$ and $y_n' = n'\left(\dfrac{D\lambda'}{d}\right)$

Equating y_n and y_n', we get

$$\frac{n}{n'} = \frac{\lambda'}{\lambda} = \frac{900}{750} = \frac{6}{5}$$

Hence, the first position at which overlapping occurs is

$$y_6 = y_5' = \frac{6(2)(750 \times 10^{-9})}{2 \times 10^{-3}} = 4.5 \text{ mm}$$

95. b. When slits are of unequal width, then intensity of sources S_1 and S_2 is not equal. So, position of minimum intensity will not be completely dark.

96. a. $y_1 = 3\left(\dfrac{\lambda_1 D}{d}\right)$ and $y_2 = 4\left(\dfrac{\lambda_2 D}{d}\right)$

Since both fall at the same location, so

$$y_1 = y_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

97. d. Because white light will give a general illumination at the central maxima.

98. c. (Shift) = 7 (Position of central maxima)

$$\Rightarrow \quad (\mu - 1)t\frac{D}{d} = 7\left(\frac{\lambda D}{d}\right)$$

$$\therefore \quad t = \frac{7\lambda}{\mu - 1}$$

$$\Rightarrow \quad t = \frac{7(600)}{0.5}$$

$$\Rightarrow \quad t = 8400 \text{ nm}$$

99. c. $\Delta x = d \sin\theta = \dfrac{\lambda}{8}\sin\theta$

$\Delta\phi$ = Phase difference at P

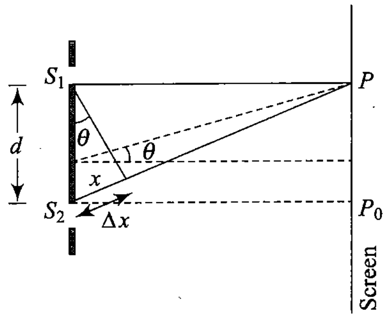


Fig. 2.170

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) + \Delta\phi' = \frac{2\pi}{\lambda}\left(\frac{\lambda}{8}\sin\theta\right) + \frac{\pi}{4}$$

$$\Delta\phi = \frac{\pi}{4}(1 + \sin\theta)$$

$$I(\theta) = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$= 4I_0\cos^2\left[\frac{\pi}{8}(1 + \sin\theta)\right]$$

100. a. When slits of equal width are taken, then intensity at maxima is $4a^2$ and at minima it is zero ($I \propto w$).

When one slit is doubled, then intensity at maxima will increase whereas intensity at minima will not be equal to zero and will be finite.

101. b. $n_1\left(\dfrac{\lambda_1 D}{d}\right) = n_2\left(\dfrac{\lambda_2 D}{d}\right)$

$$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$$

$$\Rightarrow \quad (12)(600) = n_2(400)$$

$$\Rightarrow \quad n_2 = 18$$

102. a. There is air on both sides of the soap film. Therefore, the reflections of the light produces a net 180° phase shift.

The condition for bright fringes is

$$2t = \left(m + \frac{1}{2}\right)\lambda_{\text{film}}$$

$$t = \frac{\left(m + \frac{1}{2}\right)\lambda_{\text{film}}}{2} = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n}$$

$$= \frac{\left(\frac{1}{2}\right)(650 \times 10^{-9}\text{ m})}{2(1.41)} = 1.2 \times 10^{-7}\text{ m}$$

103. c. $\lambda = ?$

When mica sheet of thickness t and refractive index μ is introduced in the path of one of the interfering beams, optical path increases by $(\mu - 1)t$. Hence, the shift on the screen,

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \text{(i)}$$

When the distance between the plane of slits and screen is changed from F to $2D$, then

$$\beta = \frac{2D}{d}\lambda \qquad \text{(ii)}$$

$$\frac{D}{d}(\mu - 1)t = \frac{2D(\lambda)}{d}$$

$$\Rightarrow \quad \lambda = \frac{1}{2}(\mu - 1)t = \frac{1}{2}(1.6 - 1) \times 2.0 \times 10^{-6} \text{ m}$$
$$= 6000 \text{ Å}$$

104. a. If after placing the plate, intensity at the position of central maxima position remains unchanged, then it means first maxima takes position of central maxima. In case of minimum thickness of plate 2, path difference created by the plate should be equal to λ.

i.e., $\quad t(\mu - 1) = \lambda$

$$t\left(\frac{3}{2} - 1\right) = \lambda \Rightarrow t = 2\lambda$$

105. b. $I = I_{max} \cos\frac{2\phi}{2}$

$$\phi = \frac{2\pi}{\lambda}\Delta x$$
$$I = I_{max}\cos^2\frac{\pi}{\lambda}\Delta x$$
$$\frac{I_1}{I_2} = \frac{\cos^2\frac{\pi}{\lambda}\Delta x_1}{\cos^2\frac{\pi}{\lambda}\Delta x_2}$$
$$\frac{I}{I/4} = \frac{\cos^2\pi}{\cos^2\frac{\pi}{\lambda}\Delta x_2}$$
$$\Rightarrow \quad \cos^2\frac{\pi}{\lambda}\Delta x_2 = \frac{1}{4}$$
$$\Rightarrow \quad \cos\frac{\pi}{\lambda}\Delta x = \pm\frac{1}{2}$$
$$\frac{\pi}{\lambda}\Delta x = \frac{\pi}{3}, \frac{2\pi}{3}$$
$$\Delta x = \frac{\lambda}{3}, \frac{2\lambda}{3}$$

106. c. Path difference $= (\mu_2 - \mu_1)t = 12480$ Å

For maxima,

$$n\lambda = 12480 \text{ Å}$$
$$\lambda_1 = 12480 \text{ Å}$$
$$\lambda_2 = 6240 \text{ Å}$$
$$\lambda_3 = 4160 \text{ Å}$$
$$\lambda_4 = 3120 \text{ Å}$$

Therefore, only 6240 Å and 4160 Å exist in the spectrum.

107. c. There is no net change in phase produced by the two reflections. Hence,

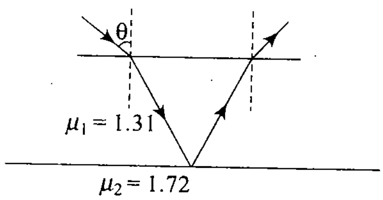


Fig. 2.171

$$\frac{\lambda}{2} = 2\mu_1 d\cos\theta$$

For normal incidence, $\cos\theta = 1$.

$$\therefore \quad d = \frac{\lambda}{4\mu_1} = \frac{5.3 \times 10^{-7}}{4 \times 1.31} = 10^{-7} \text{ m} = 0.1 \text{ µm}$$

108. c. Intensity of direct ray $= I_0 = kA_0^2$

$$\text{Intensity of reflected ray} = \frac{64}{100}I_0 = k\left(\frac{8A_0}{10}\right)^2$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(A_0 + 0.8A_0)^2}{(A_0 - 0.8A_0)^2} = \left(\frac{1.8}{0.2}\right)^2 = \frac{81}{1}$$

109. b. Intensity of central maxima, $I_u = (2A_0)^2 = 4kA_0^2 = 4I_0$

Intensity at distance x from the central maxima is half of the maximum intensity if

$$I = 4I_0\cos^2\frac{\phi}{2} = \frac{4I_0}{2} \quad \Rightarrow \quad \cos^2\frac{\phi}{2} = \frac{1}{2}$$
$$\therefore \quad x = 1.25 \times 10^{-4} \text{ m}$$
$$\frac{\lambda D}{4d} = \frac{500 \times 10^{-9} \times 1}{4 \times 10^{-3}}$$

110. a. $y' = \frac{d}{2}$, at point P exactly in front of S_1

$$\Delta x = \frac{yd}{D} + \frac{d^2}{2D}$$

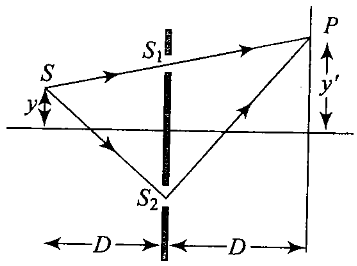


Fig. 2.172

For minimum intensity, $\Delta x = (2n - 1)\frac{\lambda}{2}$ $(n = 1)$

Putting the values, we get

$$(0.5\sin\pi t) \times 10^{-6} + 0.25 \times 10^{-6} = \frac{500}{2} \times 10^{-9}$$
$$0.5\sin\pi t + 0.25 = \frac{0.5}{2}$$
$$\sin\pi t = 0 \quad \Rightarrow \quad \pi t = 0, \pi, 2\pi, \ldots$$
$$\Rightarrow \quad t = 1 \text{ s}$$

111. c. $I = I_0 \sin^2\theta$

Differentiating I with respect to θ,

$$\frac{dI}{d\theta} = 2I_0 \sin\theta \cos\theta$$

$$\frac{dI}{I} = \frac{2I_0 \sin\theta\cos\theta\, d\theta}{I_0 \sin^2\theta} = 2\cos\theta\, d\theta$$

Percentage error in angle is

$$\frac{d\theta}{\theta}\times 100 = \left(\frac{dI}{2I\cos\theta}\right)\frac{1}{\theta}\times 100$$

$$= \frac{0.002}{2\times 5\cos 30^\circ}\times\frac{6\times 100}{\pi}$$

$$= \frac{4}{\pi}\sqrt{3}\times 10^{-2}\,\%$$

112. b. For destructive reflection:

$$\text{At } \theta_1 = 20°\text{C}, \quad \frac{2\mu_1}{\mu_g}t_1 = n\lambda_1 \qquad \text{(i)}$$

$$\text{At } \theta_2 = 170°\text{C}, \quad \frac{2\mu_1}{\mu_g}t_2 = n\lambda_2 \qquad \text{(ii)}$$

$$\frac{t_2}{t_1} = \frac{\lambda_2}{\lambda_1}$$

$$\frac{t_1[1+\alpha(\theta_2-\theta_1)]}{t_1} = \frac{\lambda_2}{\lambda_1}$$

(α is the coefficient of linear expansion of the film)

$$\Rightarrow \quad [1+\alpha(170-20)] = \frac{606}{600}$$

$$\text{or} \quad \alpha = \frac{60}{600\times 150} = 6.6\times 10^{-5}\,°\text{C}^{-1}$$

113. d. $I = I_0\cos^2\dfrac{(\lambda d\sin\theta)}{\lambda} = \dfrac{I_0}{2}$

$$\cos\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$

$$x = \frac{\lambda D}{4d} = \frac{650\times 10^{-9}\times 0.75}{4\times 0.25\times 10^{-3}}$$

$= 487.5\times 10^{-6}$ m

$= 0.4875$ mm ≈ 0.5 mm

114. b. For constructive interference,

$d\sin\theta = n\lambda$

$(\sin\theta)_{max} = 1$

$$n_{max} = \frac{d}{\lambda} = \frac{0.012\times 10^{-3}}{650\times 10^{-9}} = 18.46$$

Therefore, total number of bright fringes including the central fringe is 18 + 18 + 1 = 37.

115. b. Path difference $= \mu t - \mu_0 t_0 = n\lambda_0$

where n is the number of fringes that shift on the screen

$$\Rightarrow \quad \frac{\mu_0(1+\alpha_1\theta)t_0(1+\alpha_2\theta) - \mu_0 t_0}{\lambda_0} = n$$

$$\frac{\mu_0 t_0(\alpha_1+\alpha_2)\theta}{\lambda_0} = n$$

Given, $\mu_0 = 1.48$, $t_0 = 3\times 10^{-2}$ m

$\alpha_1 = 2.5\times 10^{-5}\,°\text{C}^{-1}$

$\alpha_2 = 0.5\times 10^{-8}\,°\text{C}^{-1}$, $\theta = 5°\ \text{min}^{-1}$

$\lambda_0 = 589$ nm

$$\therefore \quad n = \frac{1.48\times 3\times 10^{-2}(3\times 10^{-5})\times 5}{589\times 10^{-9}} = 11$$

116. a. $d\sin\theta_1 = n\lambda_1$, $d\sin\theta_2 = n\lambda_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \mu$$

$$\mu = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Multiple Correct Answers Type

1. a., c. If the screen is perpendicular to y-axis (line joining the sources), i.e., xz plane, the fringes will be circular (i.e., concentric circles with their centers on the point of intersection of the screen with y-axis). In this situation, central fringe will be bright if S_1, $S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\lambda/2$. From all this, it is clear that shape of fringes depends on the nature of sources and direction of observation, i.e., position of screen.

2. a., c. For a certain point P on the screen at a distance x from the center of the screen, path difference $\Delta = \dfrac{xd}{D}$.

Path difference introduced due to sheet $= (\mu-1)t$.

For central maximum at P,

$$\frac{xd}{D} = t(\mu-1)$$

$$\text{or} \quad x = t(\mu-1)\frac{D}{d}$$

$$\text{Now,} \quad \beta = \frac{D\lambda}{d}$$

$$\therefore \quad \frac{D}{d} = \frac{\beta}{\lambda}$$

$$\text{Hence,} \quad x = t(\mu-1)\frac{\beta}{\lambda}$$

3. a., c. Path difference = $\sqrt{D^2+d^2} - D = 1$ m

Also,

$$\left[\sqrt{D^2+d^2} - D\right] = (2n-1)\frac{\lambda}{2}$$

$$\lambda = \frac{2(1)}{2n-1}$$

For $n = 1, 2, 3, \ldots$

$$\lambda = 2 \text{ cm}, \frac{2}{3}, \frac{2}{5} \text{ cm}, \ldots$$

4. a., c., d. $I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2 = \left(\sqrt{I_1}+\sqrt{\frac{I}{2}}\right)^2 < 4I$

$$I_{min} = \left(\sqrt{I_1}-\sqrt{\frac{I}{2}}\right)^2 > 0$$

5. a., c. Path difference at point $O = d \sin \alpha = 0.5$ mm

Corresponding phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$= \frac{2\pi(0.5\times10^{-3})}{5000\times10^{-10}} = 2000\pi = 2\pi \times 1000$$

O is a point corresponding to a maxima with the point at 1 m below O corresponding to central maxima.

6. a., b., c., d. Δx at $O = d$ [path difference is maximum at O]

So, if $d = \frac{7\lambda}{2}$, O will be a minima. If $d = \lambda$, O will be a maxima. If $d = \frac{5\lambda}{2}$, O will be a minima and hence intensity is minimum.

If $d = 4.8\lambda$, then total 10 minima can be observed on the screen, 5 above O and 5 below O, which correspond to

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}, \pm\frac{9\lambda}{2}$$

7. b., d. It is better to make a chart for all the four cases which show the phase difference due to reflection at top and bottom surfaces of film.

	Top surfaces	Bottom surfaces	Due to $2L$	Total
Case I	0	0	6π	6π
Case II	0	π	6π	7π
Case III	0	π	6π	7π
Case IV	π	π	6π	8π

8. a., b., c. Due to absorption of light by the slab, intensity of two sources would be different and hence intensity at central maxima changes. If no absorption takes place, then it will not change. Intensity of dark fringes will not be absolutely zero, if some absorption is there.

9. a., c., d. Δx (at P) after insertion of slab

$$= (S_1P - t)_{air} + t_{medium} - S_2P_{air} = (\mu - 1)t$$

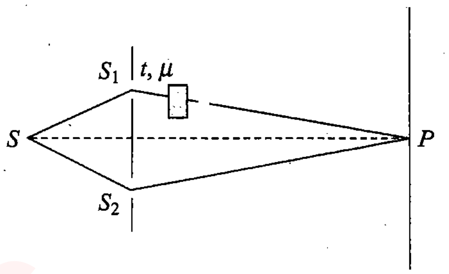


Fig. 2.173

Earlier, Δx (at P) = $S_1P - S_2P = 0$

So, change in optical path due to insertion of slab is $(\mu-1)t$.

For intensity to be zero at P, we have

$$\Delta x = \frac{(2n-1)\lambda}{2} \quad (n = 1, 2, \ldots)$$

$$(\mu-1)t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

10. c., d.

$$\phi_f = \frac{2\pi}{\lambda_0}\ell$$

$$\phi_f = \frac{2\pi}{\lambda}\ell$$

$$\Delta\phi = 2\pi\ell\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Further, by Snell's law,

$$n\lambda = (1)\lambda_0 \Rightarrow \lambda = \frac{l_0}{n}$$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi l}{\lambda_0}(n-1)$$

11. b., d. Whenever a ray goes from one medium to another, its frequency remains same. Its wavelength changes in accordance with $\mu\lambda$ = constant. i.e.,

$$\mu_0\lambda_0 = \mu\lambda$$

$$\lambda_0 = \mu\lambda \qquad (\because \mu_0 = \mu_{air} = 1)$$

$$\lambda = \frac{\lambda_0}{\mu} (<\lambda)$$

So, λ decreases when it goes from rarer to denser medium and vice versa.

$$\Delta\lambda = \lambda - \lambda_0$$

$$\Delta\lambda = \lambda_0\left(\frac{1}{\mu} - 1\right)$$

12. b., d. We know that

$$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2}$$

where $I_1 \propto a^2$ (a is amplitude of wave 1) and $I_2 \propto b^2$.

Hence,

$$\frac{I_{max}}{I_{min}} = \frac{9}{1}$$

$$\frac{a+b}{a-b}=\frac{3}{1} \Rightarrow \frac{a}{b}=\frac{1}{2}$$

$$\frac{I_1^2}{I_2^2}=\frac{a^2}{b^2}=\frac{1}{4}$$

13. b., c. $\beta_2=\dfrac{\lambda_2 D}{d}=2\dfrac{\lambda_1}{d}D$

as $\quad \beta_1=\dfrac{\lambda_1}{d}D$

nth order maxima of λ_2 coincides with $2n$th order maxima of λ_1.

n^{th} order minima of λ_2 does not coincide with $2n$th order maxima of λ_1.

14. b., d. There is a dark fringe at O if the path difference

$$\delta=ABO-AO'O=\frac{\lambda}{2}$$

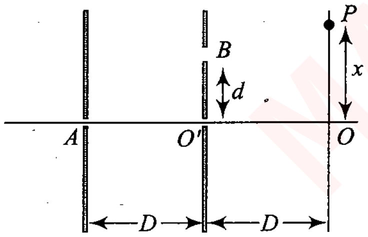


Fig. 2.174

$$\Rightarrow \quad 2\sqrt{D^2-d^2}-2D=\frac{2d^2}{2D}=\frac{d^2}{D}=\frac{\lambda}{2}$$

$$d_{min}=\sqrt{\frac{\lambda D}{2}}$$

The bright fringe is formed at P if the path difference

$$\delta'=AO'P-ABP=\lambda$$

$$=D+\sqrt{D^2+x^2}-\sqrt{D^2+d^2}-\sqrt{D^2+(x-d)^2}=\lambda$$

$$=\frac{x^2}{2D}-\frac{d^2}{2D}-\frac{(s^2+d^2-2xd)}{2D}=\lambda$$

Given $d=d_{min}$

On solving, $x=d_{min}=\sqrt{\dfrac{\lambda D}{2}}$

15. a., d. If the amplitudes due to two individual sources at point P are A_0 and $3A_0$, then the resultant amplitude at P will be

$$A=\sqrt{A_0^2+(3A_0)^2+2(A_0)(3A_0)\cos\pi/3}$$

$$=A_0$$

Resultant intensity, $I \propto 13\,A_0^2$

16. a., b., c. Path difference, $\delta=BP-AP$

$$\sqrt{x^2+9^2}-x=n\lambda$$

$$\Rightarrow \quad x^2+9^2=n^2\lambda^2+x^2+2n\lambda x$$

$$\Rightarrow \quad x=\frac{9^2-n^2\lambda^2}{2n\lambda}$$

$$\lambda=\frac{c}{v}=\frac{3\times10^8}{120\times10^6}=2.5\text{ m}$$

$n=1$, $x=14.95$ m; $n=3$, $x=1.65$ m;
$n=2$, $x=5.6$ m; $n=4$ not possible

17. a. $I=I_0\cos^2\dfrac{(xd\sin\theta)}{\lambda}$

(1) Path difference, $\delta=d\sin\theta=5\times10^{-7}$ m

$$I_1=I_0\cos^2\frac{(\pi\times5\times10^{-7})}{(5\times10^{-7}\text{ m})}=I_0\cos^2(\pi)=I_0$$

(2) $I_2=I_0\cos^2\left(\dfrac{4\pi}{2\times3}\text{rad}\right)$

$$=I_0\cos^2\left(\frac{4\pi}{6}\right)=0.25\,I_0$$

(3) $I_3=I_0\cos^2\left(\dfrac{\pi\times7.5\times10^{-7}\text{m}}{5\times10^{-7}\text{m}}\right)$

$$=I_0\cos^2(1.5\pi\text{ rad})=I_0\cos^2(270°)=0$$

(4) $I_4=I_0\cos^2\left(\dfrac{\pi}{2\times3}\right)\text{rad}=0.75\,I_0$

Assertion-Reasoning Type

1. d. In Young's experiment, fringe width for dark and white fringes is same while in the same experiment. When a white light as a source is used, the central fringe is white around which few colored fringes are observed on either side.

2. c. The beautiful colours are seen on account of interference of light reflected from the upper and the lower surfaces of the thin films. Since condition for constructive and destructive interference depends upon the wavelength of light, therefore coloured interference fringes are observed.

3. b. When dark fringe is obtained at the point opposite to one of the slits, then

$$S_1P=D$$

$$S_2P=\sqrt{D^2+d^2}$$

$$=D\left(1+\frac{d^2}{D^2}\right)^{1/2}=D\left(1+\frac{d^2}{2D^2}\right)$$

Path difference $=S_2P-S_1P$

$$=D\left(1+\frac{d^2}{2D^2}\right)-D=\frac{d^2}{2D}=\frac{\lambda}{2}$$

or $\quad \lambda=\dfrac{d^2}{D} \Rightarrow \lambda\propto d^2$

Now, intensity of a dark fringe is zero.

4. **b.** For two coherent sources,

$$I = I_1 + I_2 + \sqrt{I_1 I_2} \cos\theta$$

Putting $I_1 = I_2 = I_0$, we have

$$I = I_0 + I_0 + 2\sqrt{I_0 \times I_0} \cos\phi$$

Simplifying the above expression,

$$I = 2I_0(1 + \cos\phi)$$

$$= 2I_0\left(1 + 2\cos^2\frac{\phi}{2} - 1\right)$$

$$= 2I_0 \times \cos^2\frac{\phi}{2} = 4I_0\cos^2\frac{\phi}{2}$$

Also,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$I_{min} = (\sqrt{I_1} + \sqrt{I_2})$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})}{(\sqrt{I_1} - \sqrt{I_2})}$$

5. **a.** Average value of $\cos\delta$ is

$$\frac{\int_{t}^{t=T} \cos(\phi_1 - \phi_2)dt}{T} = 0$$

Here, ϕ_1 and ϕ_2 are constantly, randomly fluctuating phases of the two wave trains and integral is taken over a long time (relative to periods of the individual waves).

6. **b.** Wavelength in a medium of refractive index μ is $\lambda' = \lambda/\mu$, where λ is the wavelength in air. Fringe width,

$$\omega = \frac{\lambda D}{d}$$

7. **c.** In Young's experiments, fringe width for dark and white fringes are same while in Young's double-slit experiment when a white light as a source is used, the central fringe is bright around which few coloured fringes are observed on either side.

8. **a.** As given in the expression $\beta = \frac{\lambda D}{d}$, fringe width β is independent of 'n' (position).

9. **d.** If maximum intensity is observed at P, then for maximum intensity to be also observed at Q, S_1 and S_2 must have phase difference of $2m\pi$ (where m is an integer).

10. **d.** Statement I is false because constructive interference can be obtained if phase difference of sources is 2π, 4π, 6π, etc.

11. **d.** Intensity $\propto$ (amplitude)^2 and hence a non-linear physical quantity. So, we cannot add intensities directly, as principle of superposition is valid for linear waves only.

12. **a.** At the location of minima, two waves have different intensities and hence minimum intensity is not exactly zero.

13. **b.** $\Delta x = d\sin\theta \Rightarrow \Delta x_{max} < d$
If $d < \lambda$, then $\Delta x_{max} < \lambda$. So, maxima can be present and interference pattern cannot be observed.
Statement II is true but not explaining statement I.

14. **a.** Diffraction (bending of waves) occurs when obstacle size is comparable to wavelength.

15. **a.** Fringe width, $\beta = \frac{\lambda D}{d}$

de Broglie wavelength, $\lambda = \frac{h}{mv}$

As speed of electrons increases, λ decreases, i.e., β decreases.

Comprehension Type

For Problems 1–3

1. d., 2. a., 3. b.

Sol.

1. Path difference produced is $\Delta x = \frac{3}{2}\pi R - \frac{\pi}{2}R = \pi R$

For maxima: $\Delta x = n\lambda$

$\therefore \quad n\lambda = \pi R$

$$\Rightarrow \quad \lambda = \frac{\pi R}{n}, n = 1, 2, 3, \ldots$$

Thus, the possible values of λ are $\pi R, \frac{\pi R}{2}, \frac{\pi R}{3}, \ldots$

2. Path difference, $\Delta x = (2n - 1)\frac{\lambda}{2}$

3. Maximum intensity, $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

Here, $I_1 = I_2 = \frac{I_0}{2}$ (given)

$$\therefore \quad I_{max} = \left(\sqrt{\frac{I_0}{2}} + \sqrt{\frac{I_0}{2}}\right)^2 = 2I_0$$

For Problems 4–6

4. a., 5. c., 6. a.

Sol.

4. Optical path difference $= 2n_1 t$

5. For destructive interference,

$$2n_1 t_{min} = \frac{\lambda}{2} \quad (\because n_1 < n_2)$$

$$\therefore \quad t_{min} = \frac{500}{4 \times 1.25} = 100 \text{ nm}$$

6. $2n_1 t = \frac{3\lambda}{2} \quad \Rightarrow \quad t = 3t_{min} \cong 300$ nm

For Problems 7–9

7. b., 8. d., 9. a.

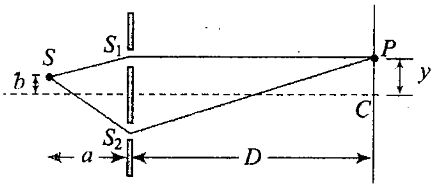


Fig. 2.175

Sol. Path difference at any point on the screen $\Delta x = \frac{bd}{a} + \frac{y'd}{D}$

For central maxima. $\Delta x = 0$

$$\Rightarrow \qquad y' = -\frac{bD}{a} = -\frac{(1+\cos \pi t)2}{1}$$

$$y' = -2(1 + \cos \pi t)$$

Position of central maxima at any time is

$y' = -2\ (1 + \text{con}\ \pi t)$ mm

At $t = 2$ s, $y' = -4$ mm.

At $t = 1$ s, central maxima is at $y' = 0$.

But when a plate is inserted central maxima is at

$$y_1 = \frac{D}{d}(\mu - 1)t = 1 \text{ mm}$$

For Problems 10–11

10. b. 11. d.

Sol. (a) The path difference at any point on the screen is

$$d \sin \theta = n\lambda$$

$$\Rightarrow \qquad n = \frac{d}{\lambda} \sin \theta$$

n is maximum when $\sin \theta$ is maximum; thus maximum value of n is

$$n = \frac{d}{\lambda} = 4$$

Hence, number of minima are four approximately corresponding to path difference

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$$

Thus, apart from central maxima at $\theta = 0$, eight other maxima, 4 on either side of central maxima are registered. Total maxima registered are 9 and 8 minima lie between them.

(b) The path difference at any point Q on the x-axis is

$$\Delta X = AQ - BQ = \frac{AB}{\cos \theta}(1 - \sin \theta)$$

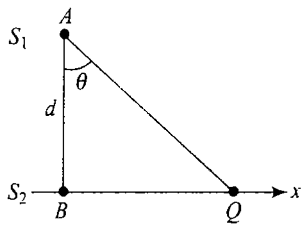


Fig. 2.176

Condition for maxima is

$$\frac{d}{\cos \theta}(1 - \sin \theta) = n\lambda$$

$$n = \frac{4(1 - \sin \theta)}{\cos \theta}$$

At point B, $\theta = 0 \Rightarrow n = 4$, i.e., we get fourth-order maxima.

At $x = \infty$, $\theta = \pi/2$, path difference is zero.

Thus, at $\theta = \pi/2$, we have $n = 0$, i.e., zeroth-order maxima is formed at $x = \infty$.

Hence, along x-axis, beginning from slit B fifth-order maxima are registered.

For Problems 12–14

12. c. 13. b. 14. a.

Sol. Introduction of a glass plate shifts the central maxima by

$$\Delta x = \frac{D(\mu - 1)t}{d} = \frac{\beta(\mu - 1)t}{\lambda}$$

As the central maxima shifts by five fringes, therefore

$$\frac{\beta_R(\mu_R - 1)t}{\lambda_R} = 5\beta_R \qquad \text{(i)}$$

$$t = \frac{5\lambda_R}{(\mu_R - 1)t} = \frac{5 \times 7 \times 10^{-6}}{(1.5 - 1)} = 7\ \mu\text{m}$$

Similarly, when green light is used, the central maxima shifts by six fringes; therefore

$$\frac{\beta_G(\mu_G - 1)t}{\lambda_G} = 6\beta_G \qquad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(\mu_R - 1)}{(\mu_G - 1)} = \frac{5}{6} \quad \Rightarrow \quad \mu_G - 1 = \frac{6}{5}(1.5 - 1)$$

$$\Rightarrow \quad \mu_G = 1.6$$

As given in the problem,

$5\beta_R = 10^{-3} \Rightarrow \beta_R = 2 \times 10^{-4}$

$$\therefore \quad \frac{\beta_G}{\beta_R} = \frac{(\lambda_G D/d)}{(\lambda_R D/d)} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$$

$$\Rightarrow \quad \frac{\beta_G}{\beta_R} - 1 = \frac{5}{7} - 1 = -\frac{2}{7}$$

$$\frac{\beta_G - \beta_R}{\beta_R} = -\frac{2}{7}$$

$$\therefore \quad \Delta\beta = \beta_G - \beta_R = -\frac{2}{7} \times \beta_R = -0.57 \times 10^{-4} \text{ m}$$

The minus sign denotes that fringe width will decrease when green light is used.

For Problems 15–16

15. b., 16. a.

Sol. Position of a point on a screen is

$$y = D \tan \theta = 1 \tan \theta$$

For first minima,

$$n = 1, \sin \theta_1 = \frac{1}{4}, \tan \theta_1 = \frac{1}{\sqrt{15}}$$

For second minima,

$$n = 2,\ \sin\theta_2 = \frac{3}{4},\ \tan\theta_2 = \frac{3}{\sqrt{7}}$$

So, the positions of minima are

$$y_1 = \tan\theta_1 = \frac{1}{\sqrt{15}} = 0.258 \text{ m}$$

$$y_2 = \tan\theta_2 = \frac{3}{\sqrt{7}} = 1.13 \text{ m}.$$

The minima are symmetrically placed on either side of central maxima; therefore there will be 4 minima at positions ± 0.258 m and ± 1.13 m on the screen.

(a) When incident rays are incident normally, the waves arriving at slits are in phase, zero path difference before slits. Path difference after slits, at point P, is $d \sin\theta$.

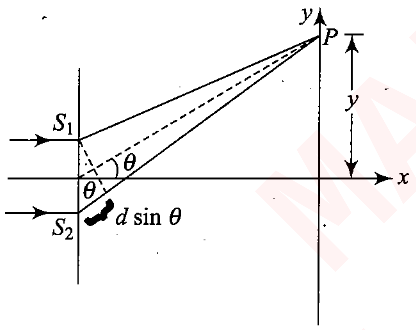


Fig. 2.177

Condition for minima at y-axis is

$$d\sin\theta = (2n-1)\frac{\lambda}{2}$$

$$\sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)(0.5)}{2\times 1} = \frac{(2n-1)}{4}$$

As $\sin\theta \le 1$, $\left(\dfrac{2n-1}{4}\right) \le 1$ or $n \le 2.5$

Hence, only first-order and second-order minima are possible.

(b) Path difference before slits $= d\sin\phi$

Path difference after slits $= d\sin\theta$

As path of rays before slits is longer at S_1 and $S_2P > S_1P$ after slits, so net path difference for first minima is

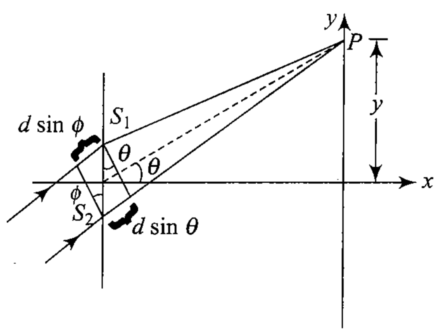


Fig. 2.178

$$d\sin\theta - d\sin\phi = \pm\frac{\lambda}{2}$$

$$\sin\theta = \sin\phi \pm \frac{\lambda}{2}$$

$$= \sin 30^\circ \pm \frac{0.5}{2\times 1} = \frac{3}{4} \quad \text{or} \quad \frac{1}{4}$$

$$\tan\theta = \frac{3}{\sqrt{7}} \quad \text{and} \quad \frac{1}{\sqrt{15}}$$

So, the position of first minima on either side of central maxima is

$$y = D\tan\theta = \frac{3}{\sqrt{7}} \text{ and } \frac{1}{\sqrt{15}} \text{ m}$$

For Problems 17–19

17. b., 18. d., 19. a.

Sol. (a) Path difference at point P on the screen, $\Delta x = \dfrac{yd}{D}$

At the position of central maxima, the optical path lengths S_2P and S_1P are equal.

$$\frac{(S_2P - t)}{c/\mu_1} + \frac{t}{c/\mu_2} = \frac{S_1P}{c/\mu_1}$$

where $\mu_1 = 4/3$, $\mu_2 = 3/2$.

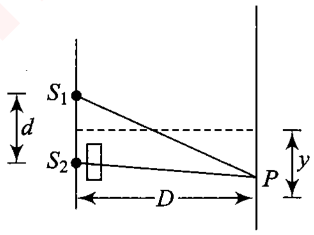


Fig. 2.179

$$\mu_1(S_1P - S_2P) = (\mu_2 - \mu)t$$

$$\mu_1\left(\frac{yd}{D}\right) = (\mu_2 - \mu_1)t$$

$$y = \frac{(\mu_2 - \mu_1)tD}{\mu_1 d}$$

$$= \frac{[(3/2)-(4/3)]\times 10.4\times 1.5}{(4/3)\times 0.45\times 10^{-3}} = 4.33 \text{ mm}$$

(b) At point O, net path difference,

$$\Delta x = \left(\frac{\mu_2}{\mu_1} - 1\right)t$$

Net phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

$$= \frac{2\pi}{6\times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4\times 10^{-6}) = \left(\frac{13}{3}\right)\pi$$

Thus, intensity

$$I = I_{max}\cos^2(\phi/2)$$

$$= I_{max}\cos^2\left(\frac{13\pi}{6}\right) = \frac{3}{4}I_{max}$$

(c) For maximum intensity at point O,

$\Delta x = n\lambda$ (where $n = 1, 2, 3, \ldots$)

Path difference at point O,

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4 \times 10^{-6}) = 1300 \text{ nm}$$

Thus, maximum intensity will correspond to $\frac{1300}{2}$ nm, $\frac{1300}{3}$ nm, ...

In the given range, required values are 650 nm and 433.33 nm.

For Problems 20–22

20. b., 21. a., 22. c.

Sol. The angular fringe width, i.e., the angle subtended by a fringe at the centre of slits is given by

$$\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

According to the given condition,

$$\beta_\theta = \frac{\lambda}{d} \geq \frac{\pi}{180 \times 60}$$

$$d \leq \frac{6 \times 10^{-7} \times 180 \times 60}{\pi}$$

$$d \leq 2.06 \times 10^{-3} \text{ m}$$

Thus, $d_{max} = 2.06 \times 10^{-3}$ m $= 2.06$ mm

Position of 3rd bright fringe,

$$y_3 = \frac{3\lambda D}{d} = \frac{3 \times 6 \times 10^{-7} \times 1}{2.06 \times 10^{-3}} = 8.74 \times 10^{-4} \text{ m}$$

Position of 5th dark fringe,

$$y_5 = \frac{\left(n - \frac{1}{2}\right)\lambda D}{d} = \frac{9\lambda D}{2d}$$

$$= \frac{9 \times 6 \times 10^{-7} \times 1}{2 \times 2.06 \times 10^{-3}} = 13.1 \times 10^{-4} \text{ m} = 1.31 \text{ mm}$$

For Problems 23–24

23. b., 24. b.

Sol. While calculating path difference, it must be remembered that it is equal to difference of optical path lengths from source S to point on the screen.

$$\therefore \quad \Delta x_{total} = [\Delta x]_{\text{before slits}} + [\Delta x]_{\text{after slits}}$$

$$= (SS_2 - SS_1) + (S_2P - S_1P)$$

(a) When liquid is filled between the slits and screen, then

$$[S_2P]_{\text{liquid}} = [\mu S_2P]_{\text{air}}$$

$$[S_1P]_{\text{liquid}} = [\mu S_1P]_{\text{air}}$$

At point O: $\mu S_2O = \mu S_1O$

No path difference is introduced after slits. So,

$$\Delta x_{total} = SS_2 - SS_1 = \sqrt{d^2 + X_0^2} - x_0$$

Thus, phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\sqrt{d^2 + X_0^2} - x_0\right)$$

At point P: $[\Delta x]_{\text{before slits}} = \sqrt{d^2 + X_0^2} - x_0$

$$[\Delta x]_{\text{after slits}} = \mu S_2P - \mu S_1P = \mu(S_2P - S_1P)$$

$$= \frac{\mu y d}{D} = \frac{\mu d^2}{2D} \quad \left(\text{as} \quad y = \frac{d}{2}\right)$$

Therefore,

$$[\Delta x]_{total} = \left[\sqrt{d^2 + x_0^2} - x_0\right] + \frac{\mu d^2}{2D}$$

Thus, phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\left[\left(\sqrt{d^2 + x_0^2} - x_0\right) + \frac{\mu d^2}{2D}\right]$$

(b) When liquid is filled between the source and slits:

At point O: $(\Delta x)_{\text{before slits}} = (SS_2 + SS_1)_{\text{liquid}}$

$$= (\mu SS_2 + \mu SS_1)_{\text{air}}$$

$$= \mu(\sqrt{d^2 + x_0^2} - x_0)$$

$$(\Delta x)_{\text{after slits}} = S_2O + S_1O = 0$$

$$(\Delta x)_{total} = \mu(\sqrt{d^2 + x_0^2} - x_0)$$

Thus, phase difference at P,

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\mu\sqrt{d^2 + x_0^2} - x_0\right)$$

At point P: $(\Delta x)_{\text{before slits}} = (SS_2 + SS_1)_{\text{liquid}} = (\mu SS_2 + \mu SS_1)_{\text{air}}$

$$(\Delta x)_{\text{after slits}} = (S_2P - S_1P)_{\text{air}} = \frac{yd}{D} = \frac{d^2}{2D}$$

$$(\Delta x)_{total} = \left[(\mu\sqrt{d^2 + x_0^2} - x_0) + \frac{d^2}{2D}\right]$$

Thus, phase difference at P,

$$\Delta\phi = \frac{2\pi}{\lambda}\left[\mu\sqrt{d^2 + x_0^2} - x_0 + \frac{d^2}{2D}\right]$$

For Problems 25–26

25. b., 26. a.

Sol. We consider a point P on the screen.

Optical path length,

$$[S_1P]_{\text{liquid}} = [\mu_1 S_1P]_{\text{air}}$$

$$[S_1P - t]_{\text{liquid}} + t_{\text{glass}} = \mu_1[S_2P - t]_{\text{air}} + [\mu_g t]_{\text{air}}$$

$$= [\mu_1 S_2P + (\mu_g - \mu_1)t]_{\text{air}}$$

Hence, optical path difference at P,

$$\Delta x = [\mu_1 S_2P + (\mu_g - \mu_1)t] - \mu_1 S_1P$$

$$= \mu_1(S_2P - S_1P) + (\mu_g + \mu_1)t$$

For a point P at the screen in the absence of liquid,

$$S_2P = S_1P = \frac{yd}{D}$$

If a liquid is filled,

$$[S_2P - S_1P]_{\text{liquid}} = \frac{\mu_1 yd}{D}$$

Thus,

$$\Delta x = \frac{\mu_1 yd}{D} + [\mu_g - \mu_l]t$$

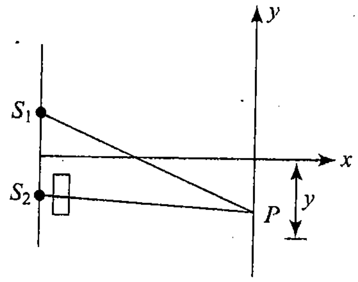


Fig. 2.180

(a) For central maxima,

$$\Delta x = 0$$

$$y = \left[\frac{\mu_g - \mu_l}{\mu_l}\right]\frac{tD}{d} = \frac{(4-T)tD}{(10-T)d}$$

At point O, $y = 0$

Thus, $T = 4$

(b) Speed of central maxima,

$$v = \left|\frac{dy}{dt}\right| = \frac{6Dt}{(10-T)^2 d}$$

Central maxima is at O at time $t = 4$ s.

$$\therefore \quad v = \frac{6Dt}{36d} = \frac{1\times 36\times 10^{-6}}{6\times 2\times 10^{-3}} = 3\times 10^{-3}\ \text{m s}^{-1}$$

For Problems 27–28

27. c., 28. c.

Sol. (a) For point P in front of upper slit, $y = d/2$.

Initially, path difference,

$$\frac{(d/2)d}{D} = n\lambda \qquad \text{(i)}$$

Intensity at any point on the screen is

$$I = I_{\max}\left(\frac{1-\cos\phi}{2}\right)$$

(ϕ being the phase difference)

When the screen is displaced, $I = \dfrac{I_{\max}}{4}$

Hence, $\cos\phi = \dfrac{1}{2} \Rightarrow \phi = 2n\pi \pm \dfrac{\pi}{3}$ (ii)

Also, as P lies between third and fourth minima,

$$5\pi < \phi < 7\pi \qquad \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$\phi = \frac{17\pi}{3} \quad \text{or} \quad \frac{19\pi}{3}$$

$$\phi = \frac{(d/2)d}{D'}\frac{2\pi}{\lambda} = \frac{(n\lambda D)}{D'}\frac{2\pi}{\lambda} = \frac{2n\pi D}{D'}$$

[using Eq. (i)]

$$= \frac{17n\pi}{6} \text{ (putting the values of } D \text{ and } D')$$

$\therefore \quad n = 2$ or (38/17)

But n is an integer, therefore $n = 2$ is the only valid answer.

(b) Putting the value of n and d in Eq. (i), we get $\lambda = 5.9 \times 10^{-7}$ m.

For Problems 29–31

29. b., 30. d., 31. a.

Sol. The optical path difference at P is

$$\Delta x = S_1P - S_2P = d\cos\theta$$

$$\because \quad \cos\theta = 1 - \frac{\theta^2}{2} \text{ for small } \theta$$

$$\therefore \quad \Delta x = d\left(1 - \frac{\theta^2}{2}\right)$$

$$= d\left[1 - \frac{y^2}{2D^2}\right], \text{ where } D + d = D$$

For nth maxima,

$$\Rightarrow \quad \Delta x = n\lambda$$

$$d\left[1 - \frac{y^2}{2D^2}\right] = n\lambda$$

y = radius of the nth bright ring

$$= D\sqrt{2\left(1 - \frac{n\lambda}{d}\right)}$$

At the central maxima, $\theta = 0$.

$$\Delta x = d = n\lambda$$

$$\Rightarrow \quad n = \frac{d}{\lambda} = \frac{0.5}{0.5\times 10^{-3}} = 1000$$

Hence, for the closest second bright ring, $n = 998$.

Putting values, $r = 6.32$ cm.

For Problems 32–33

32. b., 33. a.

Sol. From the given condition,

$$OS_3 = \frac{D\lambda}{d} = \frac{1\times 6\times 10^{-7}}{3\times 10^{-3}} = 2\times 10^{-4}\ \text{m}$$

Let light reaching from S_1 and S_2 to S_4 has phase difference ϕ and intensity of incident light is I_0. Resultant intensity at S_4 is

$$I = 4I_0\cos^2\frac{\phi}{2}$$

As $I = I_0$, hence

$$\frac{I_0}{4I_0} = \cos^2\frac{\phi}{2}$$

$$\Rightarrow \qquad \cos\frac{\phi}{2} = \frac{1}{2} = \cos 60^\circ$$

$$\phi = \frac{2\pi}{3},\ \phi = \frac{2\pi}{3}\Delta x \Rightarrow \Delta x = \frac{\lambda}{3}$$

As $\qquad \Delta x = \frac{\lambda}{3}$, using $\frac{\Delta x}{d} = \frac{y}{D}$

$$y = \frac{\Delta x D}{d} \quad \Rightarrow \quad OS_4 = \frac{D\lambda}{3d}$$

Therefore, $S_3S_4 = OS_3 + OS_4 = \frac{4}{3}\frac{d\lambda}{d} = \frac{8}{3} \times 10^{-4}$ m

Now, resultant wave coming out of S_3 has intensity $4I_0$ and waves coming out of S_4 have intensity I_0.
Phase difference at $S_3 = 2\pi$
Phase difference at $S_4 = 2\pi/3$
These phase differences are relative to the light incident on slits S_1 and S_2.
Now, S_3 and S_4 are secondary sources of light.
Phase difference at $O' = 4\pi/3$, equal to initial phase difference between the light reaching at O', i.e.,

$$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

Let intensity at O' be I'. Then,

$$I' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{4\pi}{3}$$

$$= 5I_0 + 4I_0\cos\left(\pi + \frac{\pi}{3}\right) = 3I_0$$

For the brightest fringe,
phase difference $= 2np$, $n = 0, \pm1, \pm2, \ldots$
Let I'' be the intensity of the brightest fringe.

$$I'' = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\phi \quad (\text{where } \cos\phi = 1)$$
$$= 9I_0$$

For Problems 34–35

34. b., 35. d.

Sol. From lens equation, we have

$$v = \frac{uf}{f-u} \qquad (v \text{ lies on the left side})$$

$$d = 2\left(\frac{1}{2}\right)\left[\frac{v}{u} - 1\right] = t\left[\frac{u}{f-u}\right]$$

$$D = L + v = L + \frac{uf}{f-u}$$

Fringe width,

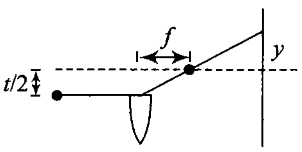


Fig. 2.181

$$\beta = \frac{\lambda D}{d} = \frac{\lambda\left[L + \frac{uf}{f-u}\right]}{t\left[\frac{u}{f-u}\right]}$$

$$\beta = \frac{\lambda}{t}\left[f + \frac{L(f-u)}{u}\right]$$

When $\quad u \to f$, $\beta = \frac{\lambda f}{t}$

From the figure,

$$\frac{y}{L-f} = \frac{t/2}{f} \Rightarrow y = \frac{t/2}{f}(L-f)$$

Distance of the point up to which interference occurs,

$$y_0 = \frac{t/2}{f}(L-f) + \frac{t}{2} = \frac{tL}{2f}$$

Number of visible maxima $= \frac{tLt}{2f \times \lambda f} = \frac{Lt^2}{2\lambda f^2}$

For Problems 36–38

36. a., 37. c., 38. b.

Sol. For dark spot at O,

$$\Delta x = (AB + BO) - AO$$

$$= 2(D^2 + d^2)^{1/2} - 2D = 2D\left[\left(1 + \frac{d^2}{D^2}\right)^{1/2} - 1\right]$$

$$\Rightarrow \quad \Delta x = \frac{d^2}{D}$$

$$\frac{d^2}{D} = \frac{\lambda}{2} \quad \Rightarrow \quad d = \sqrt{\frac{D\lambda}{4}}$$

Δx at P:

$$\Delta x = (AB + BP) - (AC + PC)$$
$$= (AB - AC) + (BP - PC)$$

Δx at O: $\Delta x = \frac{\lambda}{2}$

The path difference at P will be zero if $x = d$. Hence next maxima will form at P.
Fringe width = distance between two consecutive dark or bright fringes
$= 2 \times$ distance between two consecutive bright dark fringes
$= 2 \times d$

For Problems 39–41

39. a., 40. c., 41. d.

Sol. 39. $z = \frac{\lambda D}{2d}$

At S_4: $\frac{\Delta x}{d} = \frac{z}{D}$

$$\Rightarrow \quad \Delta x = \frac{\lambda D}{2d}\frac{d}{D} = \frac{\lambda}{2}$$

Hence, minima at S_4 and maxima at S_3 (intensity $4I_0$).
Hence,

$$\frac{I_{max}}{I_{min}} = 1$$

40. $z = \dfrac{\lambda D}{d}$

$$\Delta x \text{ at } S_4: \Delta x = \frac{\lambda D}{d}\frac{d}{D} = \lambda$$

Hence, maxima at S_4 as well as S_3.
Resultant intensity at S_4, $I = 4I_0$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{[(4I_0)^{1/2} + (4I_0)^{1/2}]^2}{[(4I_0)^{1/2} - (4I_0)^{1/2}]^2} = \infty$$

41. $z = \dfrac{\lambda D}{4d}$

$$\Delta x = y\frac{d}{D} = \frac{\lambda D}{4d}\frac{d}{D} = \frac{\lambda}{4}$$

$$\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{4}$$

Intensity at S_4: $I_4 = 2I_0\left(1 + \cos\dfrac{\pi}{2}\right) = 2I_0$

Intensity at S_3: $I_3 = 4I_0$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{[(4I_0)^{1/2} + (2I_0)^{1/2}]^2}{[(4I_0)^{1/2} - (2I_0)^{1/2}]^2} = \left[\frac{(2+\sqrt{2})}{(2-\sqrt{2})}\right]^2$$

$$= \left[\frac{(2+\sqrt{2})^2}{4-2}\right]^2 = \frac{1}{4}[4+2+4\sqrt{2}]^2$$

$$= \frac{1}{4}[6+4\sqrt{2}]^2 = [3+2\sqrt{2}]^2$$

For Problems 42–44

42. c., **43.** a., **44.** d.

Sol. Fringes will be observed in the region between P_1 and P_2 because the reflected rays lie only in this region.

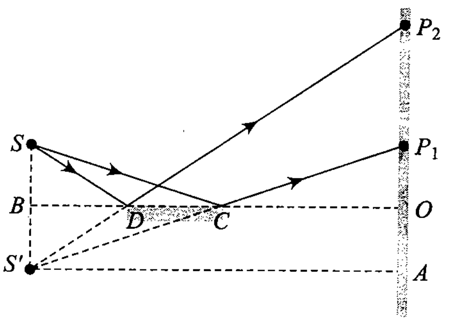


Fig. 2.182

From similar triangles BDS' and $S'P_2A$,

$$\frac{AP_2}{BS'} = \frac{AS'}{BD}$$

$$\therefore \quad AP_2 = \frac{(AS')(BS')}{BD}$$

$$= \frac{(190+5+5)(0.1)}{5} = 4 \text{ cm}$$

Similarly, in triangles BCS' and $S'P_1A$,

$$\frac{AP_1}{BS'} = \frac{AS'}{BD}$$

$$\therefore \quad \frac{(AS')(BS')}{BC} = \frac{(190+5+5)(0.1)}{10} = 2 \text{ cm}$$

$$\therefore \quad P_1P_2 = AP_2 - AP_1 = 2 \text{ cm}$$

Wavelength of the light,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7} \text{ m}$$

Fringe width, $\beta = \dfrac{\lambda D}{d}$

Here, $D = S'A = 190 + 5 + 5 = 200$ cm $= 2.0$ m, $d = SS' = 2$ mm $= 2\times10^{-3}$ m.

$$\therefore \quad \beta = \frac{(5\times10^{-7})(2.0)}{2\times10^{-3}} = 5\times10^{-4} \text{ m}$$

$$= 0.05 \text{ cm}$$

Therefore, number of fringes $= \dfrac{P_1P_2}{\beta} = 40$

For Problems 45–47

45. c., **46.** c., **47.** a.

Sol. Without inserting the slab, path difference at P,

$$\Delta x = \frac{yd}{D} = \frac{0.15\times10^{-3}\times2\times10^{-3}}{2} = 1.5\times10^{-7} \text{ m}$$

Corresponding phase difference at P,

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$$

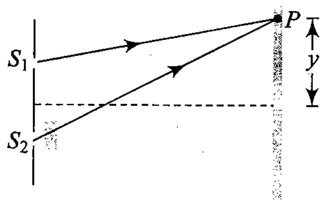


Fig. 2.183

$$= \left(\frac{2\pi}{6000\times10^{-10}}\right)(1.5\times10^{-7}) = \frac{\pi}{2}$$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

Therefore, intensity at P, $I = 4I_0 \cos^2 (\phi/2) = 2I_0$

Phase difference after placing the glass sheet,

$$\phi' = \phi + \frac{2\pi}{\lambda}(\mu - 1)t$$

$$= \frac{\pi}{2} + \frac{2\pi}{6000 \times 10^{-10}}(1.5 - 1)(8000 \times 10^{-19})$$

$$= \frac{11\pi}{6}$$

Now, the intensity at P is,

$$I' = I_0 + \eta I_0 + 2\sqrt{\eta I_0^2} \cos\frac{11\pi}{6} = 2I_0 \text{ (given)}$$

Solving this equation, we get $\eta = 0.21$.

For Problems 48–49

48. b., 49. a., c.

Sol. At $x = 0$, path difference is 3λ. Hence, third-order maxima will be obtained. At $x = \infty$, path difference is zero. Hence, zero-order maxima is obtained. In between, first- and second-order maxima will be obtained.

First-order maxima:

$$S_2P - S_1P = \lambda \quad \text{or} \quad \sqrt{x^2 + 9\lambda^2} - x = \lambda$$

or $\sqrt{x^2 + 9\lambda^2} = x + \lambda$

Squaring this, we get $x^2 + 9\lambda^2 = x^2 + \lambda^2 + 2x\lambda$

Solving this, we get $x = 4\lambda$

Second-order maxima:

$$S_2P - S_1P = 2\lambda \text{ or } \sqrt{x^2 + 9\lambda^2} - x = 2\lambda$$

or $\sqrt{x^2 + 9\lambda^2} = (x + 2\lambda)$

Squaring both sides, we get

$$x^2 + 9\lambda^2 = x^2 + 4\lambda^2 + 4x\lambda$$

Solving this, we get $x = \frac{5}{4}\lambda = 1.25\lambda$

Hence, the desired x-coordinates are

$x = 1.25\,\lambda$ and $x = 4\lambda$

For Problems 50–52

50. d., 51. c., 52. b.

Sol. 50. d. Order of the fringe can be counted on either side of the central maximum. For example, fringe no. 3 is first-order bright fringe.

52. c. See theory

52. b. $\Delta X_C = \lambda$; $\Delta X_A = \frac{\lambda}{2}$

$$\Delta X_C - \Delta X_A = \frac{\lambda}{2} = 300 \text{ nm}$$

For Problems 53–56

53. b., 54. b., 55. d., 56. a.

Sol. 53. For constructive interference, $2t = (2n - 1)\frac{\lambda_{air}}{2}$

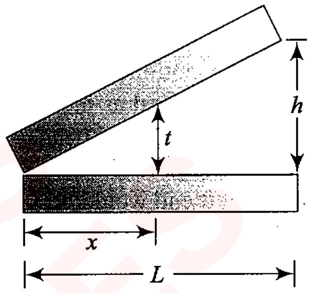


Fig. 2.184

For destructive interference, $2t = n\lambda_{air}$

As due to reflection at the top surface of bottom layer, an additional path difference of $\lambda/2$ occurs, contact point would be dark. Let at distance x, nth dark band forms.

$\therefore \quad 2t = n\lambda_{air}$

$$\frac{t}{x} = \frac{h}{L}$$

$$\frac{2hx}{L} = n\lambda_{air}$$

$$\Rightarrow \quad x = n\left[\frac{\lambda_{air} \times L}{2h}\right]$$

$$\Rightarrow \quad x = n\left[\frac{400 \times 10^{-9} \times 5 \times 10^{-2}}{2 \times 20 \times 10^{-6}}\right]$$

$$= (5 \times 10^{-4}\,n)$$

Maximum value of n would be for $x = L$.

$$\therefore \quad n_{max} = \frac{5 \times 10^{-2}}{5 \times 10^{-4}} = 100$$

54. For reflection from bottom layer of strip 1 as well as for top surface of strip 2, phase shift to π takes place. So, condition for maxima is $2t = n\lambda_{medium}$ and condition for minima is $2t = (2n - 1)\,\lambda_{medium}/2$.

So, contact point would be bright as here path difference is 0.

55. Here λ_{air} has to be replaced by λ_{medium}

$$\lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{400}{1.3} \text{nm}$$

For bright band, $2t = (2n - 1)\frac{\lambda_{meduim}}{2}$

$$\frac{2hx}{L} = (2n - 1)\frac{\lambda_{meduim}}{2}$$

$$x = (2n - 1)\left[\frac{400 \times 10^{-9} \times 5 \times 10^{-2}}{2 \times 1.3 \times 2 \times 20 \times 10^{-6}}\right]$$

For maximum n, $x = L$

$n_{max} = 130.5$

So, 130 maxima would be there.

56. For 20th dark band,

$$x = 20 \times 5 \times 10^{-4}\ \text{m} = 1\ \text{m}$$

So, required separation $= S - 1 = 4$ cm

For Problems 57–59

57. b., 58. c., 59. d.

Sol. 57. From the theory mentioned in passage, at the ends of cavity fringes will form, and as number of dark fringes is greater than the number of bright fringes so the ends will be location of dark fringes. Thickness of the cavity at a distance x from the left end would be

$$t = L_2 + \frac{L_1 - L_2}{L}x$$

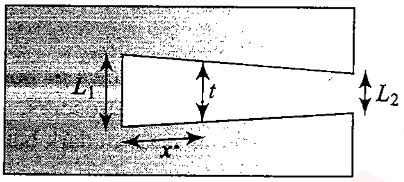


Fig. 2.185

For left end,

$$2\mu L_1 = n\lambda_0 \qquad \text{(for dark fringe)}$$

For right end,

$$2\mu L_2 = (n-7)\lambda_0$$

$$L_1 - L_2 = \frac{7\lambda_0}{2\mu}$$

58. For 4th dark fringe from left end,

$$2\mu t = (n-3)\lambda_0$$

$$2\mu\left(L_2 + \frac{L_1 - L_2}{L}x\right) = 2\mu L_1 - 3\lambda_0$$

$$x = \frac{4L}{7}$$

59. 1st dark fringe is at left end only. So, 2nd dark fringe would be at

$$2\mu\left[L_2 + \frac{L_1 - L_2}{L}x\right] = (n-1)\lambda_0 = (n-1)\lambda_0$$

$$x = \frac{6L}{7}$$

For Problems 60–62

60. a., 61. c., 62. b.

Sol. 60. At any time t, the situation is as shown in the figure below.

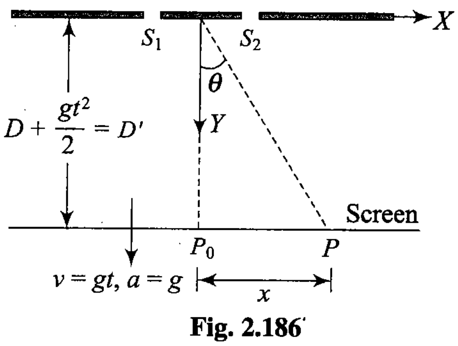


Fig. 2.186

Central maxima is always lying on Y-axis at P_0. Its velocity at any time t is given by $v = gt$ along positive Y-axis. So, required velocity is 50 m s^{-1}.

61. Path difference corresponding to point P,

$$\Delta x = d\sin\theta = d\tan\theta$$

$$\Delta x = \frac{dx}{D'}$$

For 2nd maxima, $\Delta x = 2\lambda$

$$dx = 2\lambda D' \quad \Rightarrow \quad x = \frac{2\lambda D'}{d}$$

Location of central maxima is

$$\left(0, D + \frac{gt^2}{2}\right) \quad \text{or} \quad (0, D')$$

Location of 2nd maxima is

$$\left(\frac{2\lambda D'}{d}, D'\right)$$

Velocity of 2nd maxima w.r.t. central maxima is

$$\frac{\vec{v}_{2\text{nd}}}{D} = \frac{2\lambda}{d}[0 + gt]\hat{i} = \frac{2\lambda}{d} \times gt\,\hat{i} = 2\ \text{cm s}^{-1}\hat{i}$$

62. Location of 3rd maxima is $\left(\frac{3\lambda D'}{d}, D'\right)$

Location of 3$^{\text{rd}}$ maxima on the other side is $\left(-\frac{3\lambda D'}{d}, D'\right)$

$$\frac{\vec{a}_{3\text{rd}}}{-3\text{rd}} = \left[\frac{3\lambda}{d}g - \left(-\frac{3\lambda g}{d}\right)\right]\hat{i} = \frac{6\lambda g}{d}\hat{i} = 0.03\ \text{m s}^{-2}\,\hat{i}$$

For Problems 63–65

63. d.; 64. a., c., d.; 65. d.

Sol. 63. $I_{av} = \dfrac{I_{max} + I_{min}}{2}$

$$= \frac{(A_1 + A_2)^2 + (A_1 - A_2)^2}{2}$$

$$= A_1^2 + A_2^2 = I_1 + I_2$$

$$I_{av} = I_1 + I_2$$

So, choice (d) is correct.

Neither loss nor gain of energy is observed, but only redistribution of energy takes place.

64. If $A_1 = A = a$, $I_{max} = 4a^2$, $I_{min} = 0$. So, visibility is the best. Choice (a) is correct.

Since $I_{min} = 0$, choice (c) is also correct.

Width decides intensity and thereby the amplitude. So, choice (d) is correct making (b) wrong.

65. Sources are independent. They cannot form a coherent source since ϕ cannot be constant with time. So, choices (a), (b) and (c) are wrong. i.e., choice (d) is answer.

For Problems 66–68

66. b., c.; 67. a.; 68. b.

Sol. 66. Broad sources provide wide angular incidence of light. The thickness should be small, since the path difference should be comparable with the wavelength. Thick slabs cannot bring wavelength comparable path differences. So, choices (a) and (d) are wrong. Choices (b) and (c) are correct.

67. Position of nth bright fringe = $Y = D/d(\Delta x)$, where Δx is the path difference.

When a slab of refractive index 'μ' and thickness 't' is introduced, the new position will be

$$Y' = \frac{D}{d}[\Delta x + (\mu + 1)t]$$

$$\therefore \quad \text{Shift} = \frac{D}{d}(\mu - 1)t$$

Shift is given by $5\lambda D/d$.

$$\therefore \quad \frac{5\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow \quad t = \frac{5\lambda}{\mu - 1}$$

Choices (c) and (d) are wrong dimensionally. Choice (b) is wrong numerically. So, choice (a) is correct.

68. b. In Lloyd's single mirror experiment, the interference is between the light reflected by the mirror and the direct light from the source.

The reflected light undergoes a phase difference of π radians or path difference of $\lambda/2$.

So, path difference + $\frac{\lambda}{2}$ = $N\lambda$ for bright fringe. Hence,

Path difference = $(2N - 1)\frac{\lambda}{2}$

So, choice (b) is correct and rest are wrong.

For Problems 69–71

69. a., 70. b., 71. a.

Sol. 69. Net path difference to the central fringe position is $\lambda/2$. Since it is an old multiple of $(\lambda/2)$, the fringe formed is dark. So, choice (a) is correct and the others are wrong.

70. For a path difference of $\lambda/4$, the phase difference will be

$$\phi = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore \quad \text{Intensity} = 2a^2 \cos^2\left(\frac{\phi}{4}\right) = a^2$$

$$= 2a^2 \cos^2\left(\frac{\phi}{4}\right) = a^2$$

So, choice (b) is correct and the others are incorrect.

71. Since the light from slits S_1 and S_2 reach the point P with a path difference of $\lambda/2$, the point P will be dark, $I = 0$. Since path difference is zero for R, there will be a bright fringe at R. So, ratio of intensity will be zero. So, choice (a) is correct.

For Problems 72–77

72. b., 73. c., 74. a., 75. b., 76. d, 77. d.

Sol. 72. $n_{air} < n_{film} < n_{glass}$. Reflection with phase change of $\lambda/2$ occurs for ray a at the air–film interface and for ray b at the film–glass interface. Therefore, reflections keep both rays in phase.

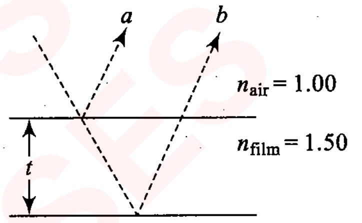


Fig. 2.187

Constructive interference then depends on making the path length difference, $2t$, within the film a multiple of λ.

$2t = m\lambda/nt = m$ (600 nm)/2(1.50) = $m \times 200$ nm

For $m = 1$, $t = 200$.

73. The ratio of the refractive indices is

$n_{air} < n_{film} < n_{glass}$. Reflections at the interfaces do not produce a net phase difference between rays a and b. Destructive interference requires that the optical path length through the film, $2t$, be an odd multiple of $\lambda/2$.

$2t = m + 1/2)\lambda/n$

$t = (m + 1/2)$ (640 nm)/2(1.33)

$t = (m + 1/2)$ 240 nm

For $m = 0$, $t = 120$ nm which is not one of the choices.

For $m = 1$, $t = 360$ nm.

74. It is not necessary to know if

$$n_{air} < n_{film} < n_{glass}$$

Either ray a or ray b will undergo a phase change during reflection. Therefore, the two rays will be out of phase. For constructive interference to occur, the optical path difference must provide a 180° phase change for ray b. This happens if $2t$ is an odd multiple of $\lambda/2$. Since $2t = \lambda/2$, $t = \lambda/4$.

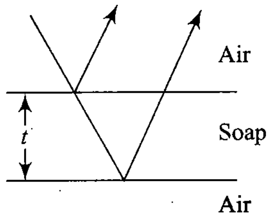


Fig. 2.188

75. b. Choices (a) and (c) are eliminated immediately since they are outside the visible range. For soap, $n_{air} > n_{soap}$. The reflected wave, ray a, undergoes a phase change at the air–soap water interface. Ray R_b does not change phase at the soapy water–air interface. Based on interface reflections, the two rays are out of phase. To maintain this, the optical path difference, $2t$, must not produce a phase change in R_b. Therefore, the path must be an integer multiple of λ.

$2t = m\lambda/n \Rightarrow \lambda = 2tn/m = 2(35 \text{ nm})(1.35)/m = 945/m$

For $m = 1$, $\lambda = 945$ nm
For $m = 2$, $\lambda = 473$ nm [this is choice (b)]
For $m = 3$, $\lambda = 315$ nm
Choice (b), $\lambda = 473$ nm, is the only choice in the visible range.

76. Again, ray R_a is reflected with phase change and ray R_b is not. For the two reflected waves to interact,
$2t = M\lambda/n$, where M is either m or $m + 1/2$
$M = 2tn/\lambda = 2\,(1.00 \times 10^{-6}\text{ m})\,(1.35)/600 \times 10^{-9}\text{ nm} = 4.5$
$M = m + 1/2$ for $m = 4$. This describes the complete constructive interaction.

77. Both rays R_a and R_b are reflected with a change of phase. Therefore, the net change of phase with reflection is zero. For constructive interference, $2t = m\lambda/n$ and for destructive interference $2t = (m + 1/2)\lambda/n$.

$$t = (m + 1/2)\,(600\text{ nm})/2\,(1.25) = m \times 700/2\,(1.25)$$

$$600\,m + 300 = 700\,m \Rightarrow 300 = 100\,m \Rightarrow m = 3$$

Solving either equation for t, we have $t = 3(700\text{ nm})/2\,(1.25)$ $= 840$ nm

For Problems 78–80

78. b., 79. d., 80. d.

Sol. 78. $I \propto \dfrac{1}{(\text{distance})^2}$

$$\therefore \quad \frac{\Delta I}{I_0} = \frac{L^2 - (L-x)^2}{L^2} \approx \frac{2A\sin\omega t}{L}$$

79. Intensity of the fringes will increase.

80. $\beta = \dfrac{\lambda D}{d} = \text{constant}$

For Problems 81–83

81. b., 82. d., 83. c.

Sol. 81. Intensity at O is proportional to intensity at S_3 and at S_4

$I = k\cos^2\dfrac{\delta}{2}$, where k is constant and δ is the phase difference.

$$\delta = \frac{2\pi}{\beta} \times \frac{Z}{2} = \frac{\pi Z}{\beta}, \text{ where } \beta \text{ is fringe width}$$

$$\beta = \frac{\lambda D}{d}$$

When $Z = \lambda D/2d$:

$$\delta = \frac{\pi d}{\lambda D} \times \frac{\lambda D}{2d} = \frac{\pi}{2}$$

$$I = I_0 = k\cos^2\left(\frac{\pi}{4}\right)$$

$$k = 2I_0$$

When $Z' = \dfrac{2\lambda D}{d}$:

$$\text{Path difference } \delta' = \frac{2\pi}{\beta}\frac{Z'}{2} = \frac{2\pi d}{\lambda d} \times \frac{2\pi D}{\lambda d} = 2\pi$$

Required intensity at O is

$$I' = k\cos^2\frac{\delta}{2} = 2I_0\cos^2(\pi) = 2I_0$$

82. Intensity at O is zero if $\delta/2 = \pi/2$. That is,

$$\delta = \frac{2\pi}{\beta}\frac{Z}{2} = \pi \quad \Rightarrow \quad Z = \beta = \frac{\lambda D}{d}.$$

83. Intensity at $S_3 = A_0^2 + A_0^2 + 2A_0^2\cos\phi$

$$\text{where } \phi = \frac{2\pi}{\beta}\frac{\lambda D}{4d} = \frac{2\pi d}{\lambda D} \times \frac{\lambda D}{4d} = \frac{\pi}{2}$$

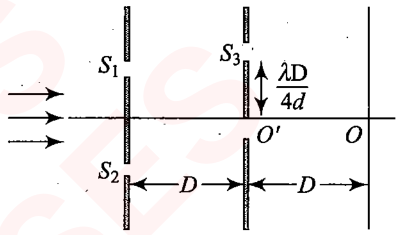


Fig. 2.189

$$I_{S_3} = A_0^2\left(2 + 2\cos\frac{\pi}{2}\right)$$

$$= 2A_0^2 \times 2\cos^2\frac{\pi}{4} = \frac{4}{2}A_0^2 = 2A_0^2$$

Amplitude of wave at S_3, $A_{S_3} = \sqrt{2}A_0$

Maximum intensity at $O' = (A_0 + \sqrt{2}A_0)^2$

Maximum intensity at $O = (\sqrt{2}A_0 - A_0)^2$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = (\sqrt{2}+1)^4 = 34$$

Matching Column Type

1. a → r; b → r; c → s; d → p.

By using $(\mu - 1)\,t = n\lambda$, we can find value of n, that is order of fringe produced at P, if that particular strip has been placed over any of the slit. If two strips are used in conjuction (over each other), path difference due to each is added to get net path difference created. If two strips are used over different slits, their path differences are subtracted to get net path difference. Now,

$$n_1 = \frac{(\mu_1 - 1)t_1}{\lambda} = 5$$

$$n = 4.5$$

$$n_3 = 0.5$$

For (a), order of the fringe is 4.5, i.e., fifth dark.
For (b), net order is $5 - 0.5 = 4.5$, i.e., fifth dark.
For (c), net order is $5 - (0.5 + 4.5) = 0$, i.e., it is central bright again at P.
For (d), net order is $(5 + 0.5) - (4.5) = 1$, i.e., first bright.

2. a → q, r, s; b. → p, q, r, s; c. → q, r, s; d. → p, q, r, s.

Intensity at a distance x from central maxima on screen is

$$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi x}{\beta}$$

where $\beta = \dfrac{D\lambda}{d}$

$I_{max} = 9I_0$ and $I_{min} = I_0$

a. At points where intensity is (1/9)th of maximum intensity, minima is formed. Therefore, distance between such points is β, 2β, 3β, 4β, ...

b. At points where intensity is (3/9)th of maximum intensity,

$$\cos\frac{2\pi x}{\beta} = -\frac{1}{2} \text{ or } x = \frac{\beta}{3}.$$

Hence, distance between such points is

$$\frac{\beta}{3}, \frac{2\beta}{3}, \beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta, \ldots$$

c. $\cos\dfrac{2\pi x}{\beta} = 0$ or $x = \dfrac{\beta}{4}$

Hence, distance between such points is $\dfrac{\beta}{2}, \beta, \beta + \dfrac{\beta}{2}, 2\beta, \ldots$

d. $\cos\dfrac{2\pi x}{\beta} = \dfrac{1}{2}$ or $x = \dfrac{\beta}{6}$

Hence, distance between such points is $\dfrac{\beta}{3}, \dfrac{2\beta}{3}, \beta, \beta + \dfrac{\beta}{3}, \beta + \dfrac{2\beta}{3}, 2\beta, \ldots$

3. a → q, r; b → q, s; c → p, s; d → p, q, r, s.

For $\mu_1 = \mu_2$, the two waves in the reflected system differ by optical phase difference of π and hence due to destructive interference, the film appears to be dark. In transmitted system, the two waves are in phase and hence film appears to be shiny. Similarly, we can go for the other conditions.

4. a → q; b. → p; c. → p; d. → p.

To draw the shape of wavefront, draw the ray diagram. The light will propagate in a direction perpendicular to the wavefronts.

a.

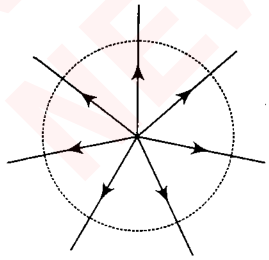


Fig. 2.190

b.

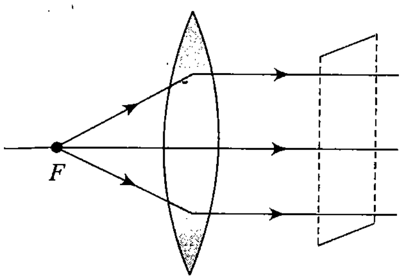


Fig. 2.191

c.

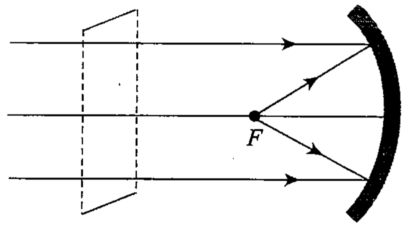


Fig. 2.192

d.

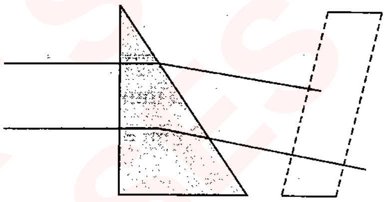


Fig. 2.193

5. a → p; b → q; c → r; d → s.

Depending on the value of refractive index, the path travelled by light would be decided, which in turn decides the shapes of wavefronts.

For more refractive index, speed of light is less and hence wavefront location would be somewhat located back side.

6. a → p, s; b → q; c → p, q, s; d → r.

$$\mu = \frac{c}{v}$$

$$\Rightarrow \quad \mu \propto \frac{1}{v}$$

and

$$\mu \propto \frac{1}{\lambda}$$

7. a → s; b → p; c → q; d → r.

The optical path difference between the two waves arriving at P is

$$\delta = (SS_2 + S_2P) - (SS_1 + S_1P)$$
$$= (SS_2 - SS_1) + (S_2P - S_1P)$$
$$= d\sin\theta_0 + d\sin\theta = \frac{dy_0}{D_1} + \frac{dy}{D_2}$$

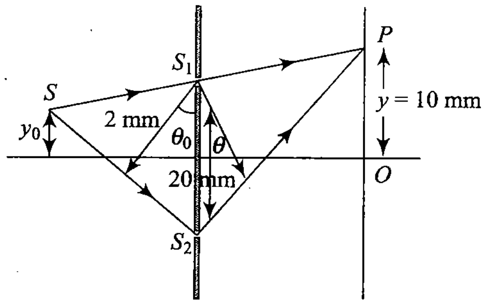


Fig. 2.194

$D = 20$ mm, $y_0 = 2$ mm, $D_2 = 2$ m, $D_1 = 1$ m, $y = 10$ mm

$$\therefore \quad \delta = \frac{20\times 2}{1000} + \frac{20\times 10}{2000} = 0.14 \text{ mm}$$

For a bright fringe, $\delta = n\lambda$

$$\Rightarrow \quad n = \frac{\delta}{\lambda} = \frac{0.14}{0.5 \times 10^{-3}} = 280$$

At the origin O, $\delta' = \dfrac{dy_0}{D_1} = 0.04$ mm

$$n' = \frac{\delta'}{\lambda} = \frac{0.04}{0.5 \times 10^{-3}} = 80$$

Due to transparent paper, the change in optical path is

$$(\mu - 1)\,t = (1.45 - 1)\,(0.02) \text{ mm} = 0.009 \text{ mm}$$

$$\delta'' = 0.14 \text{ mm} - 0.009 \text{ mm} = 0.131 \text{ mm}$$

$$\Rightarrow \quad n = \frac{0.131}{0.5 \times 10^{-3}} = 262$$

Due to transparent paper, the path difference at O,

$$\delta = \delta' - (\mu - 1)\,t = (0.04 - 0.009) \text{ mm}$$
$$= 0.031 \text{ mm}$$

$$\Rightarrow \quad n = \frac{0.031}{0.5 \times 10^{-3}} = 62$$

Integer Answer Type

1. (5) Let intensity of individual slit be I_0, then intensity of central maxima is $4I_0$.
Intensity at distance y from the central maxima is

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \text{ where } \phi = \frac{2\pi}{\lambda}\frac{dy}{D}$$

Given $\quad I = 2I_0 \;\Rightarrow\; 2I_0 = 4I_0 \cos^2\left(\dfrac{\phi}{2}\right)$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}} \;\Rightarrow\; \frac{\phi}{2} = \frac{\pi}{4} \Rightarrow \phi = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}\frac{dy}{D} = \frac{\pi}{2} \;\Rightarrow\; y = \frac{D\lambda}{4d}$$

$$\Rightarrow \quad = \frac{500 \times 10^{-9} \times 4}{4 \times 10^{-3}} = 5 \times 10^{-4} \text{ m}$$

2. (0) Take a point at a distance y from the centre of the slit.
Path difference between waves reaching this point:

$$\Delta x = \frac{dy}{D} - (\mu - 1)\,t$$

For centre of slit: $y = 0$
So $\quad \Delta x = -(\mu - 1)t$

Phase difference: $\phi = \dfrac{2\pi}{\lambda}\,\Delta x = -\dfrac{2\pi}{\lambda}(\mu - 1)\,t$

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = 4I_0 \cos^2\left(\frac{\pi}{\lambda}(\mu - 1)\,t\right)$$

$$= 4I_0 \cos^2\left[\frac{\pi(1.5 - 1)1.5 \times 10^{-6}}{500 \times 10^{-10}}\right] = 4I_0 \cos^2\left(\frac{3\pi}{2}\right) = 0$$

3. (3) Path difference at C,

$$\Delta x = t_1(\mu - 1) - t_2(\mu - 1)$$
$$= \mu(t_1 - t_2) - (t_1 - t_2) = (t_1 - t_2)(\mu - 1)$$
$$= (2.5 - 1.25)\left(\frac{1.4 \times 3}{4 \times 10} - 1\right)$$
$$= 1.25 \times \frac{2}{40} = \frac{2.5}{400} \;\Rightarrow\; \Delta x = \frac{1}{16} \,\mu\text{m}$$

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi \times 4}{5000 \times 3 \times 10^{-10}} \cdot \frac{1}{16} \times 10^{-6}$$

$$\Rightarrow \quad I_{max} = 4I_0$$

I at C, $I_c = 2I_0(1 + \cos\dfrac{\pi}{3}) = 3I_0$

Required ratio $= I_c/I_0 = 3$

4. (7) The powers of sources A and B are

$$P_A = \left(\frac{10}{\pi}\right) \times \pi r_A^2 = 10 \times (0.001)2 = 10^{-5} \text{ W}$$

$$P_B = \left(\frac{10}{\pi}\right) \times \pi r_B^2 = 10 \times (0.002)2 = 4 \times 10^{-5} \text{ W}$$

Powers of sources A and B received along F are

$$P_A = \frac{10}{100}(4 \times 10^{-5}) = 10^{-6} \text{ W}$$

$$P_B = \frac{10}{100}(4 \times 10^{-5}) = 4 \times 10^{-6} \text{ W}$$

Path difference

$$\Delta = (\mu - 1)t = (1.5 - 1) \times 2000\text{Å} = 1000\text{Å}$$

Phase difference

$$\delta = \frac{2\pi}{\lambda} \times \Delta = \frac{2\pi}{6000} \times 1000 = \frac{\pi}{3}$$

Power at point F is given by

$$P_F = P_A + P_B + 2\sqrt{P_A P_B}\cos\delta$$

$$= 10^{-6} + 4 \times 10^{-6} + 2\sqrt{10^{-6} \times 4 \times 10^{-6}}\cos\frac{\pi}{3}$$

$$= 7 \times 10^{-6} \text{ W.}$$

5. (7) I is the intensity of incident beam ab. The interfering waves are bc and ef, reflected from surface of I and II plate, respectively.

Reflection.coefficient of intensity,

$$r = 25\% = 0.25$$

Transmission coefficient of intensity,

$$t = 75\% = 0.75$$

The intensity of beam bc, $I_1 = 0.25\,I = \frac{1}{4}I$

The intensity of beam $bd = 0.75I$.
The intensity of beam $de = 0.25 \times 0.75I$
The intensity of beam ef,

$$I_2 = 0.75 \times 0.25 \times 0.75I = \frac{9}{64}I$$

Ratio of maximum and minimum intensities

$$\frac{\sqrt{I_{max}}}{\sqrt{I_{min}}} = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = 7$$

6. (a) Path difference between rays reflected from upper and lower faces of layer $= 2\mu t \cos r = 2\,\mu t$ (for normal incidence.) But there is abrupt change in path of $\lambda/2$ of light at upper surface. So actual path difference is $2\,\mu t - \lambda/2$

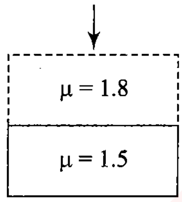


Fig. 2.195

For constructive interference $2\,\mu t - \frac{\lambda}{2} = n\lambda$

$t = \frac{(2n+1)\lambda}{4\mu}$. For least thickness $n = 0$.

$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4 \times 1.8}\text{nm} = 90 \text{ nm}$$

7. (5) According to the question, shift = 5 (fringe width)

$$\therefore \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000 \text{ Å} = 5 \times 10^{-6} \text{ m}$$

8. (7) Number of fringes is

$$\frac{(\mu - 1)D/d}{D\lambda/d} = \frac{(\mu - 1)t}{\lambda} = 7$$

Archives

Fill in the Blanks Type

1. $${}_2^1\mu = \frac{\text{Velocity of light in medium 1}}{\text{Velocity of light in medium 2}}$$

$$\Rightarrow \quad \frac{1}{2\mu} = \frac{v\lambda_1}{v\lambda_2} = \frac{\lambda_1}{\lambda_2} \qquad \text{(i)}$$

In air, $c_1 = v_1\lambda_1$

$$\Rightarrow \quad \lambda_1 = \frac{c_1}{v_1} = \frac{3 \times 10^8}{5 \times 10^{14}} = 0.6 \times 10^{-6} \text{ m}$$

From (i), we have

$$\lambda_2 = \frac{\lambda_1}{\mu} = \frac{0.6 \times 10^{-6}}{1.5} = 0.4 \times 10^{-6} \text{ m}$$

Also, velocity of light in medium 2

$$= \frac{\text{Velocity of light in medium 1}}{\mu}$$

$$= \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}$$

2. $${}_2^1\mu = \frac{\text{Speed of light in medium 1}}{\text{Speed of light in medium 2}} = \frac{v\lambda_1}{v\lambda_2}$$

[$\because$ v does change with the medium]

$${}_m^a\mu = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{{}_m^a\mu} \qquad (a \to \text{air}, m \to \text{medium})$$

$$\therefore \quad \lambda_m = \frac{6000}{1.5} = 4000 \text{ Å}$$

Now,

$$c_a = v_a\lambda_a$$

$$\therefore \quad v_a = \frac{C_a}{\lambda_a} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$$

3. For coherent sources:
Intensity $\propto$ (Amplitude)^2
The amplitude at the mid-point $= A + A = 2A$ [constructive interference]

$$\Rightarrow \quad I' \propto (2A)^2 \Rightarrow I' \propto 4A^2 \qquad \text{(i)}$$

For incoherent sources:
The intensities add up normally (no interference). Therefore, The total intensity $I'' = 2I$.

$$I'' \propto 2A^2 \qquad \text{(ii)}$$

($\because$ $I \propto A^2$)

$$\therefore \quad \frac{I'}{I''} = \frac{4A^2}{2A^2} = 2$$

4. We know that

$$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \qquad \text{(i)}$$

where I_1 is intensity at P where amplitude is A_1 and I_2 is intensity at Q where amplitude is A_2.

But $I \propto \frac{1}{r^2}$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{A_1}{A_2} = \frac{r_1}{r_2} = \frac{25}{9}$$

True or False Type

1. **True.** When the two slits of Young's double-slit experiment are illuminated by two different sodium lamps, then the sources are not coherent and hence sustained interference pattern will not be achieved. It will change so quickly that there will be general illumination and hence interference pattern will not be observed.
2. **False.** In Young's double-slit experiment, if source is of white light then the central fringe is white flanked by colored fringes.

Multiple Choice Questions with One Correct Answer Type

1. **c.** We know that

$$I = I_0 \cos^2 \frac{\delta}{2}$$

where

$$\delta = \frac{2\pi}{\lambda} \times \frac{Dy}{d} = \frac{2\pi \tan\theta}{\lambda}$$

$$I = I_0 \cos^2\left(\frac{\pi d \tan\theta}{\lambda}\right)$$

$$= I_0 \cos^2\left(\frac{\pi \times 150 \times \tan\theta}{3\times 10^8/10^6}\right)$$

$$= I_0 \cos^2\left(\frac{\pi}{2}\tan\theta\right)$$

For $\theta = 30°$, $I = I_0 \cos^2 52$
For $\theta = 90°$, $I = I_0 \cos^2 \pi/2 = 0$
Hence, (c) is the correct option.

2. **a.** Locus of equal thickness are lines running parallel to the axis of the cylinder. Hence straight fringes will be observed.
3. **a.** When slits are of equal width:

$$I_{max} \propto (a+a)^2\ (= 4a^2)$$

$$I_{min} \propto (a-a)^2\ (= 0)$$

When one slit's width is twice that of the other:

$$\frac{I_1}{I_2} = \frac{\omega_1}{\omega_2} = \frac{a^2}{b^2} \Rightarrow \frac{\omega}{2\omega} = \frac{a^2}{b^2} \Rightarrow b = \sqrt{2}a$$

$$\therefore \quad I_{max} \propto (a + \sqrt{2}a)^2\ (= 5.8a^2)$$

$$I_{min} \propto (\sqrt{2}a - a)^2\ (= 0.17a^2)$$

Hence, (a) is the correct option.

4. We know that the resultant amplitude of two interfering waves is given by $R^2 = a^2 + b^2 + 2ab\cos\phi$, where R is the amplitude of resultant wave, a is the amplitude of one wave, b is the amplitude of second wave, and ϕ as the phase difference between the two waves at a point.
Also,

$$I \propto (\text{amplitude})^2$$

$$\therefore \quad I \propto I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\ \cos\phi \qquad \text{(i)}$$

Applying Eq. (i) when phase difference is $\pi/2$,

$$I_{\pi/2} \propto I + 4I$$

$$\Rightarrow \quad I_{\pi/2} \propto 5I$$

Again, applying Eq. (i) when phase difference is π;

$$I_\pi \propto I + 4I + 2\sqrt{I}\sqrt{4I}\ \cos\pi$$

$$\therefore \quad I_\pi \propto I \Rightarrow I_{\pi/2} - I_\pi \propto 4I$$

Correct option is (b).

5. **b.** In Young's double-slit experiment,

$$\text{fringe width} = \frac{\lambda D}{d}$$

Given that

$$x_2 - x_1 = 12\frac{\lambda_1 D}{d} \qquad \text{(i)}$$

where $\lambda_1 = 600$ nm. Also,

$$x_2 - x_1 = k\frac{\lambda_2 D}{d} \qquad \text{(ii)}$$

where $\lambda_2 = 400$ nm and k is the number of fringes.

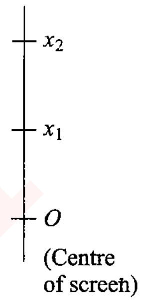


Fig. 2.196

Dividing Eq. (i) by Eq. (ii),

$$1 = \frac{12\lambda_1}{k\lambda_2}$$

$$\therefore \quad k = \frac{12 \times 600}{400} = 18$$

Correct option is (b).

6. **a.** Path difference $(\mu - 1)t = n\lambda$
For minimum t, $n = 1$
$\therefore \quad t = 2\lambda$
7. **b.** In ΔOPM,

$$\frac{OM}{OP} = \cos\theta$$

$$\Rightarrow \quad OP = \frac{d}{\cos\theta}$$

In ΔCOP,

$$\cos 2\theta = \frac{OC}{OP}$$

$$\Rightarrow \quad OC = \frac{d\cos 2\theta}{\cos\theta}$$

Path difference between the two rays reaching P is

$$CO + OP + \frac{\lambda}{2} = \frac{d\cos 2\theta}{\cos\theta} + \frac{d}{\cos\theta} + \frac{\lambda}{2}$$

$$= \frac{d}{\cos\theta}(\cos 2\theta + 1) + \frac{\lambda}{2} \quad 2d\cos\theta + \frac{\lambda}{2}$$

For constructive interference:

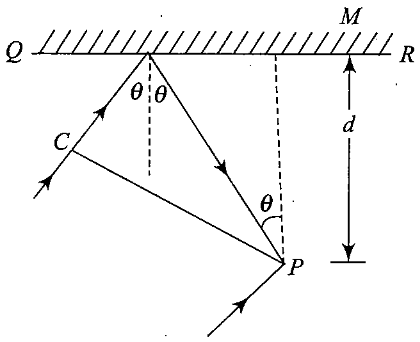


Fig. 2.197

Path difference between the two rays reaching P is $n\lambda$.

$$\therefore \quad 2d\cos\theta + \frac{\lambda}{2} = n\lambda \quad \Rightarrow \quad 2d\cos\theta = \left(n - \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad 2d\cos\theta = \frac{(2n-1)}{2}\lambda \quad \Rightarrow \quad \cos\theta = \frac{(2n-1)}{2}\frac{\lambda}{2d}$$

For $n = 1$, $\cos\theta = \dfrac{\lambda}{4d}$

8. d. At the area of total darkness, minima will occur for both the wavelengths.

$$\therefore \quad \frac{(2n+1)}{2}\lambda_1 = \frac{(2m+1)}{2}\lambda_2$$

$$\Rightarrow \quad (2n+1)\lambda_1 = (2m+1)\lambda_2$$

or $\quad \dfrac{(2n+1)}{(2m+1)} = \dfrac{560}{400} = \dfrac{7}{5}$

or $\quad 10n = 14m + 2$

By inspection: For $m = 2$, $n = 3$. For $m = 7$, $n = 10$. The distance between them will be the distance between such points, i.e.,

$$\Delta s = \frac{D\lambda_1}{d}\left\{\frac{(2n_2+1)-(2n_1+1)}{2}\right\}$$

Putting $n_2 = 10$ and $n_1 = 3$ and on solving, we get $\Delta s = 28$ mm.

9. c. Let P be the point on the central maxima whose intensity is one-fourth of the maximum intensity.

For interference we know that $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, where I is the intensity at P; I_1, I_2 are the intensities of light originating from A and B, respectively; and ϕ is the phase difference at P. In YDSE, $I_1 = I_2 = I$ and $I_{max} = 4I$. We are concentrating at a point where the intensity is one-fourth of the maximum intensity.

$$\therefore \quad I = I + I + 2I\cos\phi$$

$$\Rightarrow \quad -\frac{1}{2} = \cos\phi \quad \Rightarrow \quad \phi = \frac{2\pi}{3}$$

Note that we take the least value of the angle as the point is in central maxima.

For a phase difference of 2π, the path difference is λ.

For a phase difference of $\dfrac{2\pi}{3}$, the path difference is $\dfrac{\lambda}{2\pi} \times \dfrac{2\pi}{3} = \dfrac{\lambda}{3}$.

But the path difference (in terms of P and Q) is θ as shown in figure.

$$\therefore \quad d\sin\theta = \frac{\lambda}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{3d} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

Hence, (c) is the correct option.

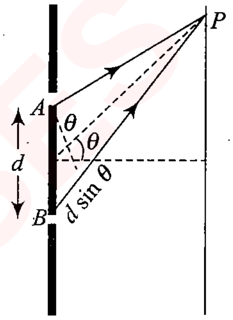


Fig. 2.198

10. c. Initially, most of part will be transmitted. When $\theta > i_C$, all the light rays will be total internal reflected. So transmitted intensity = 0.

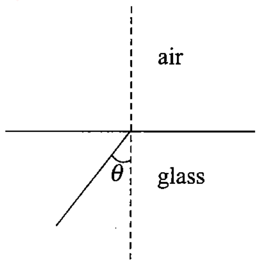


Fig. 2.199

Multiple Choice Questions with One or More than One Correct Answer Type

1. b., d.

We know that

$$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2}$$

where $I_1 \propto a^2$ (a is amplitude of 1 wave) and $I_2 \propto b^2$ (b is amplitude of 2 wave). Here,

$$\frac{I_{max}}{I_{min}} = \frac{9}{1} \quad \Rightarrow \quad \frac{a+b}{a-b} = \frac{3}{1} \quad \Rightarrow \quad \frac{a}{b} = \frac{1}{2}$$

$$\therefore \quad \frac{I_1^2}{I_2^2} = \frac{a^2}{b^2} = \frac{1}{4}$$

2. a., c.

$$y = (2n-1)\frac{\lambda}{2}\frac{D}{d} = (2n-1)\frac{\lambda}{2}\frac{D}{b}$$

($\because d = b$)
But $y = b/2$

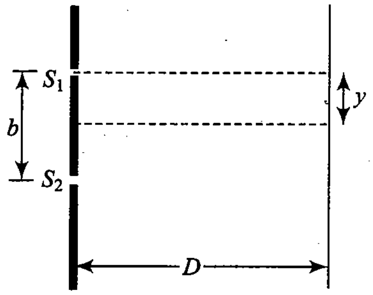


Fig. 2.200

$$\therefore \quad \frac{b}{2} = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$

$$\Rightarrow \quad \lambda = \frac{b^2}{(2n-1)D}$$

When $n = 1, 2,$

$$\lambda = \frac{b^2}{D'}, \frac{b^2}{3D'}$$

Comprehension Type

1. **a.** Wavefronts are parallel in both media. Therefore, light which is perpendicular to the wavefront travels as a parallel beam in each medium.

2. **c.** All points on a wavefront are at the same phase.

$$\phi_d = \phi_c \quad \text{and} \quad \phi_f = \phi_e$$

$$\phi_d - \phi_f = \phi_c - \phi_e$$

Hence, the correct option is (c).

3. **b.** In medium 2, wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends toward the normal in medium 2 during refraction. So, medium 2 is denser or its speed in medium 1 is more.

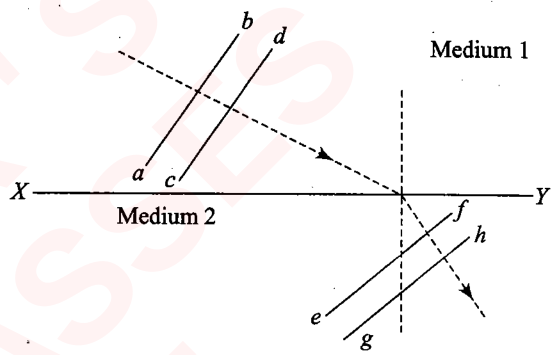


Fig. 2.201

Hence, option (b) is correct.

Matching Column Type

1. a → p, s; b → q; c → t; d → r, s, t.

CHAPTER 3

Photoelectric Effect

QUANTUM THEORY OF LIGHT

In 1900 Planck proposed that electromagnetic radiation (or light) is quantized and exists in elementary amounts (or quanta) that we now call *photons*. According to this proposal, the quantum of light wave of frequency f has the energy:

$$E = hf$$

Here, h is Planck's constant and it has the value, $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s

We can say that the light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence, energy: $Q = NE$, where N (number of photons) $= 1, 2, 3, \ldots$

Here, energy is quantized. $E = hf$ is the quantum of energy, it is a packet of energy called as *photon*.

Also $\quad E = hf = \dfrac{hc}{\lambda}$ and $hc = 12400$ eV

$$\Rightarrow \quad E = \frac{12400}{\lambda(\text{in Å})} \text{eV}$$

So, the least energy a light of frequency f can have is hf. The light cannot have energy like $1.5hf$ or πhf.

Properties of Photons

(i) A source of radiation emits energy in the form of photons and these photons travel in straight line with the speed of light. Photon is not a material particle, it is a packet of energy.

(ii) Energy of a photon depends upon its frequency and it does not change with change in medium.

(iii) With change in medium, the speed and wavelength of the photon change but frequency does not change.

(iv) Photons are electrically neutral. They are not deflected by electric or magnetic fields.

(v) Under suitable conditions, they can show diffraction.

(vi) A photon does not exist at rest. Its rest mass is zero.

Equivalent mass of photon: $E = mc^2 = hf \Rightarrow m = \dfrac{hf}{c^2} = \dfrac{h}{c\lambda}$

(vii) Photons have momentum: $P = mc = \dfrac{h}{c\lambda}c = \dfrac{h}{\lambda}$

(viii) (Intensity of a light beam) $\propto$ (number of photons present in it).

Photon Counts Emitted by a Source Per Second

Consider a light bulb of power P watt as source of light energy (see Fig. 3.1). If the wavelength of light emitted by the bulb is λ, then energy of each photon emitted by the bulb can be given as

$$E = h\nu = \frac{hc}{\lambda} \qquad \text{(i)}$$

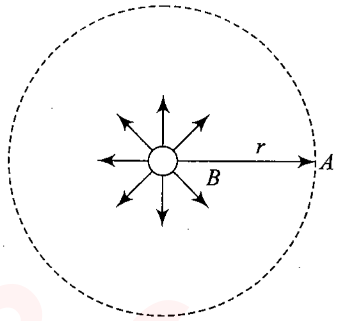


Fig. 3.1

As the power of the source is P watt, we can say that the source is emitting light energy P joule per second in the form of photons. If it is 100% efficient, then the number of photons emitted per second by the source can be given as

$$n = \frac{\text{power of the source}}{\text{energy of a photon}} = \frac{P}{E} = \frac{P\lambda}{hc} \qquad \text{(ii)}$$

These photons are assumed to be emitted uniformly in all directions. Here, we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with center at the source.

Illustration 3.1 **Calculate the number of photons emitted in 10 h by a 60 W sodium lamp ($\lambda = 5893$ Å).**

Sol. The energy of the photon $= \dfrac{hc}{\lambda}$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{5893 \times 10^{-10}} = 3.374 \times 10^{-19} \text{ J}$$

Therefore, number of photons emitted by sodium lamp in 10 h

$$\frac{60 \times 10 \times 3600}{3.374 \times 10^{-19}} = 6.40 \times 10^{24}$$

Intensity of Light due to a Light Source

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.

In Fig. 3.2, a source of light emits a uniform cylindrical light beam of cross-sectional area A. If the source emits a total power

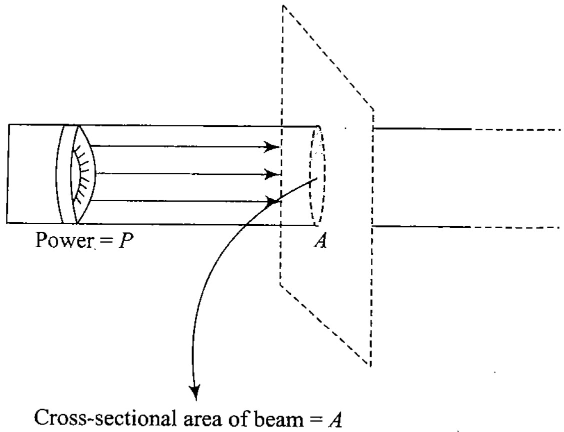


Fig. 3.2

P in the beam, then the intensity of light beam

$$I = \frac{P}{A} \text{ W m}^{-2}$$

As cross-sectional area of the beam is constant throughout, the beam intensity at every point remains constant.

Similarly, we can find the intensity of light due to a point isotropic source as shown in Fig. 3.3.

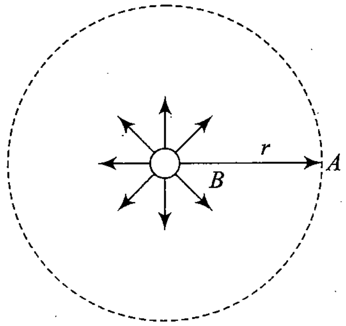


Fig. 3.3

Figure 3.3 shows a point source of light of power P watt emitting light in all directions uniformly. If we want to calculate intensity of light at a point A, at a distance r from the source, then it can be given as

$$I = \frac{P}{4\pi r^2} \text{ W m}^{-2} \quad \text{(iii)}$$

Here, it can be assumed that the whole power P is incident on the normal area of a hypothetical sphere of radius r passing through point A with center at the source as shown. Thus, energy crossing per unit area per second at point A can be given by Eq. (iii).

Photon Flux

It is defined as the number of photons incident on a normal surface per second per unit area. Consider a light beam of intensity I W m^{-2} having wavelength λ incident on a surface. Then, number of photons per second per unit area in the beam can be given as

Photon flux,

$$\phi = \frac{\text{Intensity}}{\text{Energy of a photon}} = \frac{I}{hc/\lambda} = \frac{I\lambda}{hc} \quad \text{(iv)}$$

If we consider a point source of power P watt which emits light in all directions, then it produces photons per second at a rate

$$n = \frac{\text{Power of a source}}{\text{Energy of a photon}} = \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc} \quad \text{(v)}$$

All these photons are distributed in the three-dimensional spherical space around the source. If we find the photon flux at a distance r from the point source, it can be given as

$$\phi = \frac{\text{Number of photons per second}}{\text{Surface area of sphere of radius } r} = \frac{n}{4\pi r^2} \quad \text{(vi)}$$

Illustration 3.2 **The sun delivers about 1.4 kW m^{-2} of electromagnetic flux to the earth's surface. Calculate**

a. the total power incident on a roof of dimensions 8 m × 20 m.
b. the solar energy in joules incident on the roof in 1 h.
c. the radiation pressure and force assuming the roof to be a perfect absorber.

Sol. a. As energy flux $I = [E/(S \times t)] = [P/S]$

$$P = I \times S = (1.4 \times 10^3) \times (8 \times 20)$$
$$= 2.24 \times 105 \text{ W} = 224 \text{ kW}$$

b. As power $P = (E/t)$,

$$E = P \times t = (2.24 \times 10^5) \times (60 \times 60)$$
$$8 \times 10^8 = 800 \text{ MJ}$$

c. For perfectly absorbing surface radiation pressure,

$$P = \frac{I}{c} = \frac{1.4 \times 10^3}{3 \times 10^8} = 4.7 \times 10^{-6} \text{ Nm}^{-2}$$

and as $P = (F/S)$,

$$\Rightarrow \quad F = P \times S = 4.7 \times 10^{-6} \times (8 \times 20)$$
$$= 7.52 \times 10^{-4} \text{ N}$$

Illustration 3.3 **A bulb lamp emits light of mean wavelength of 4500 Å. The lamp is rated at 150 W and 8% of the energy appears as emitted light. How many photons are emitted by the lamp per second?**

Sol. The energy of photon $= \dfrac{hc}{\lambda}$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{4500 \times 10^{-10}}$$

According to the given problem, power of the lamp is 150 W. As 8% of the light energy is utilized for emission of light, hence the utilized energy for emission per second is given by

$$\text{Useful power} = 150 \times \frac{8}{100} = 12 \text{ W}$$
$$= 12 \text{ Js}^{-1}$$

Therefore, number of photons emitted per second by the lamp

$$\frac{\text{useful power}}{\text{energy of photon}} = \frac{12 \times 4500 \times 10^{-10}}{(6.63 \times 10^{-34})(3 \times 10^8)}$$
$$= 27.17 \times 10^{18}$$

Illustration 3.4 **Sun gives light at the rate of 1400 Wm^{-2} of area perpendicular to the direction of light. Assume λ (sunlight) = 6000 Å. Calculate the**

a. number of photons per second arriving at 1 m^2 area at earth
b. number of photons emitted from the sun per second assuming the average radius of Earth's orbit is 1.49 × 10^{11} m.

Sol. $I = 1400\ \text{Wm}^{-2}$; $\lambda = 6000$ Å

a. Energy of the photon, $E = h\nu = \frac{hc}{\lambda}$ $(c = 3 \times 10^8\ \text{ms}^{-1})$

Let n be the number of photons received per second per unit area.

$$\therefore \quad n = \frac{IA}{E} = \frac{(1400 \times 1) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$= 4.22 \times 10^{21}$

b. Total energy emitted per second = power (W)

Number of photons emitted from the sun per second,

$$n = \frac{\text{power of Sun (W)}}{\text{energy of a photon}} = \frac{I \times (4\pi R^2) \times (6000 \times 10^{-10})}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

(R: average radius of Earth's orbit)

$= 1.178 \times 10^{45}$

Photon Density in a Light Beam

Photon density is defined as number of photons per unit volume.

A light source emits photons, which move away from the source with speed of light. If a light source of power P watt is producing a uniform cylindrical light beam of cross-sectional area S (Fig. 3.4), then the intensity of light beam is

$$I = \text{power crossing per unit area} = \frac{P}{S}$$

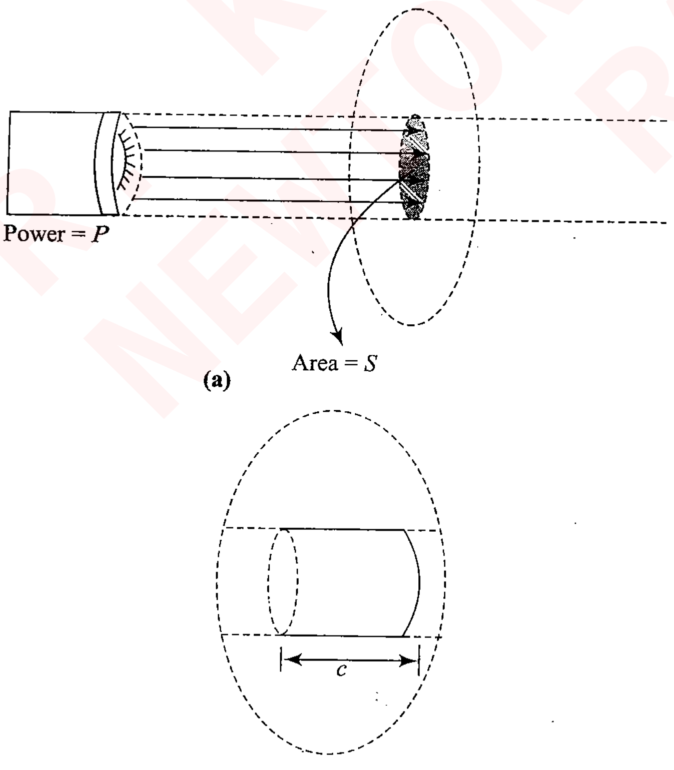


Fig. 3.4

The photon flux, i.e., number of photons crossing per unit area per second at the cross-sectional area S of light beam, is

$$\phi = \frac{I\lambda}{hc} \qquad \text{[where } \lambda \text{ is wavelength of light]} \quad \text{(vii)}$$

In duration of 1s, these photons will cover a distance c. The volume of the region crossed by photons in one second = $S(c \times 1) = Sc$

Thus, total number of photons crossing cross-sectional area S is given as

$$N = \frac{I\lambda}{hc} \times S$$

Thus, photon density in the light beam can be given as

$$\rho_{ph} = \frac{N}{Sc} = \frac{I\lambda}{hc^2} = \frac{\phi}{c} \text{ photons m}^{-2} \quad \text{(viii)}$$

As the beam is uniform and cylindrical, the photon density throughout the beam remains constant and at any point in space, photon density can be given as

$$\rho_{ph} = \frac{\phi}{c} = \frac{\text{photon flux}}{\text{speed of light}} \quad \text{(ix)}$$

Similarly, for a point isotropic source of light, we can say that as the emitted photons move away from the source, the distance between photons increases and the photon density decreases. If we wish to find photon density at a distance r from a point source of light of power P watt, then we first find the photon flux at a distance r from the source which is given as

$$\phi = \frac{P\lambda/hc}{4\pi r^2} \quad \text{(x)}$$

Thus, at a distance r from the source photon density can be given as

$$\rho_{ph} = \frac{\phi}{c} = \frac{P\lambda}{4\pi r^2 hc^2} \quad \text{(xi)}$$

Force Exerted by a Light Beam on a Surface

Figure 3.5 shows a black body of mass M placed on a smooth surface on which a light beam of cross-sectional area S is incident. The beam is produced by a torch of power P watt. If λ is the wavelength of light produced by the torch, then the number of photons emitted per second are

$$N = \frac{P\lambda}{hc} \quad \text{(i)}$$

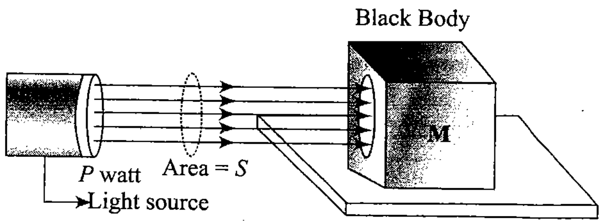


Fig. 3.5

We know that momentum in each photon is

$$p = \frac{h}{\lambda} \quad \text{(ii)}$$

As all the photons incident on the black body will be absorbed by it, here the total momentum absorbed by the body per second or force exerted on the body is

$$F = \frac{P\lambda}{hc} \times \frac{h}{\lambda} = \frac{P}{c} \quad \text{(iii)}$$

In the above case, if the surface of body is perfectly reflecting like a mirror, then the force exerted on the body will become

$$F = \frac{2P}{c} \quad \text{(iv)}$$

Illustration 3.5 **A source of light of power P is shown in Fig. 3.6. Find the force on the block placed in the path of the light rays. The surface of body on which light beam is incident is having a reflection coefficient $a_r = 0.7$ and absorption coefficient $a_a = 0.3$.**

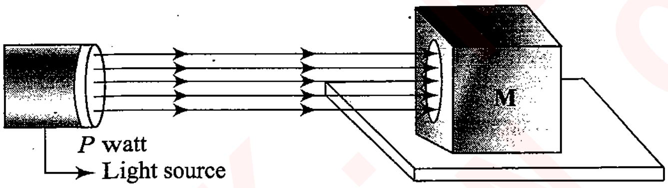


Fig. 3.6

Sol. In this case, 70% of the incident photons are reflected back and 30% are absorbed by the body. Thus, the photon which is absorbed will impart a momentum h/λ to the body and the photon which is reflected will impart the change in momentum $2h/\lambda$ to the body. Thus, net force acting on body can be given as

$$F = \frac{0.7P\lambda}{hc} \times \frac{2h}{\lambda} + \frac{0.3P\lambda}{hc} \times \frac{h}{\lambda}$$

$$F = \frac{1.7P}{c} \quad \text{(v)}$$

RADIATION PRESSURE/FORCE

The force (pressure) experienced by surface exposed to radiation is known as radiation force (pressure). This force can be calculated using Photon theory of radiation.

Let us consider a surface of area A exposed to radiation of intensity I as shown in Fig. 3.7. The radiation is falling normally on the surface. Assume absorption and reflection coefficients of surface are 'a' and 'r', respectively. Assume no transmission.

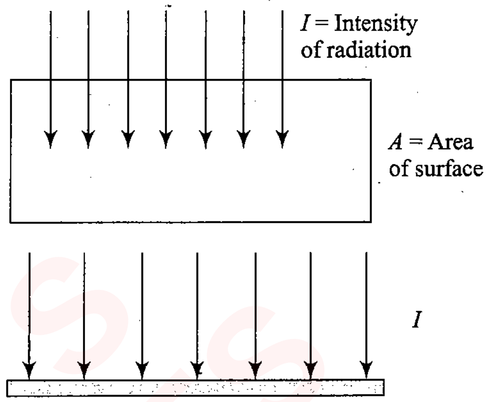


Fig. 3.7

Energy received by the surface per second: $E = IA$
Number of photons received by surface per second:

$$N = \frac{E}{\text{energy of one photon}} = \frac{E}{hc/\lambda} = \frac{IA\lambda}{hc}$$

We know that force is the rate of change of momentum,

i.e., $$\vec{F} = \frac{d\vec{P}}{dt}$$

Let us consider the following cases:

Case I. Whole amount of radiation falling on the surface is absorbed by the surface.

For this, absorption coefficient $a = 1$, reflection coefficient $r = 0$.

Initial momentum of one photon $= \dfrac{h}{\lambda}$ (downward),

Final momentum of the photon = 0

Change in momentum of one photon $= \dfrac{h}{\lambda}$ (upward)

Energy incident per unit time $= IA$
No. of photons incident per unit time:

$$N = \frac{IA}{hf} = \frac{IA\lambda}{hc}$$

Therefore, total change in momentum per unit time

$$= N\frac{h}{\lambda} = \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c} \quad \text{(upward)}$$

Force on photons = total change in momentum per unit time

$$= \frac{IA}{c} \quad \text{(upward)}$$

Therefore, from third law, force on plate due to photons:

$$F = \frac{IA}{c} \quad \text{(downward)}$$

Therefore, pressure $= \dfrac{F}{A} = \dfrac{IA}{cA} = \dfrac{I}{c}$

Case II. Whole amount of radiation falling on the surface is reflected by the surface.

For this, reflection coefficient $r = 1$, absorption coefficient $a = 0$.

Intial momentum of one photon $= \dfrac{h}{\lambda}$ (downward)

Final momentum of the photon $= \dfrac{h}{\lambda}$ (upward)

Change in momentum of one photon $= \dfrac{h}{\lambda} + \dfrac{h}{\lambda} = \dfrac{2h}{\lambda}$ (upward)

Energy incident per unit time $= IA$

No. of photons incident per unit time: $N = \dfrac{IA\lambda}{hc}$

Therefore, total change in momentum per unit time

$$N\frac{2h}{\lambda} = \frac{IA\lambda}{hc}\cdot\frac{2h}{\lambda} = \frac{2IA}{c}$$

Force on photons = total change in momentum per unit time

$$= \frac{2IA}{c} \quad \text{(upward)}$$

Therefore, force on plate due to photons: $F = \dfrac{2IA}{c}$ (downward)

Pressure $P = \dfrac{F}{A} = \dfrac{2IA}{cA} = \dfrac{2I}{c}$

Case III. When some part of radiation is absorbed and remaining part is reflected.

For this,

$0 < r < 1, 0 < a < 1$

But $a + r = 1$, so $a = 1 - r$

Change in momentum of one photon when it is reflected

$= \dfrac{2h}{\lambda}$ (upward)

Change in momentum of one photon when it is absorbed

$= \dfrac{h}{\lambda}$ (upward)

No. of photons incident per unit time: $N = \dfrac{IA\lambda}{hc}$

No. of photons reflected per unit time $= Nr = \dfrac{IA\lambda}{hc}r$

Force on plate due to reflected photons:

$$F_r = \frac{IA\lambda}{hc}r\frac{2h}{\lambda} = \frac{2IA}{c}r \quad \text{(downward)}$$

No. of photons absorbed per unit time

$$= N_a = \frac{IA\lambda}{hc}a = \frac{IA\lambda}{hc}(1-r)$$

Force on plate due to absorbed photons:

$$F_a = \frac{IA\lambda}{hc}(1-r)\frac{h}{\lambda} = \frac{IA}{c}(1-r) \quad \text{(downward)}$$

Total force on plate:

$$F = F_r + F_a = \frac{2IAr}{c} + \frac{IA}{c}(1-r)$$

$$= \frac{IA}{c}(1+r) \quad \text{(downward)}$$

Pressure: $P = \dfrac{F}{A} = \dfrac{I}{c}(1+r)$

Illustration 3.6 **A plate of mass 10 g is in equilibrium in air due to the force exerted by a light beam on the plate. Calculate power of the beam. Assume plate is perfectly absorbing.**

Fig. 3.8

Sol. For equilibrium, force exerted by the light beam should balance the weight of plate.

$F_{\text{photon}} = mg$ $\left(F_{\text{photon}} = \dfrac{IA}{c} = \dfrac{P}{c}\right.$, where power $P = IA)$

$\Rightarrow \quad \dfrac{P}{c} = 10 \times 10^{-3} \times 10$

$\Rightarrow \quad P = 3 \times 10^7$ W

Illustration 3.7 **A radiation of wavelength 200 nm is propagating in the form of a parallel surface. The intensity of the beam is 5 mW and its cross-sectional area is 1.0 mm^2. Find the pressure exerted by radiation on the metallic surface if the radiation is completely reflected.**

Sol. Energy of one photon $E = hc/\lambda$

If power of source is P, the number of photons incident on the metallic surface

$$\frac{P}{E} = \frac{\lambda P}{hc}$$

Momentum of incident photons $= h/\lambda$

Change in momentum due to reflection $= 2h/\lambda$

Total momentum imparted to the surface per second is

$$\frac{2h}{\lambda}\frac{\lambda P}{hc} = \frac{2P}{c}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \left(\frac{2P}{c}\right)\bigg/A = \frac{2P}{cA}$$

$$= \frac{2 \times 5 \times 10^{-3}}{3 \times 10^8 \times 10^{-6}} = 3.33 \times 10^{-5}\ \text{N m}^{-2}$$

Illustration 3.8 **(a) How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of 6.62×10^{-5} N?**
(b) At what rate will temperature of plate rise if its mass is 19.86 kg and specific heat is equal to 2500 J (kg K^{-1}).

Sol. a. If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is $P = n \times \frac{h}{\lambda}$

Since the plate is blackened, all photons are absorbed by it.

$$\therefore \quad \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$$

$$\text{Since} \quad F = \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda} \Rightarrow n = \frac{F.\lambda}{h}$$

$$\text{or} \quad n = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

b. Energy of each photon $= \frac{hc}{\lambda}$

Since n photons fall on the plate per second, total energy absorbed by the plates in one second is $E = n \times \frac{hc}{\lambda}$

$= 1986$ Js^{-1}

$$\text{i.e.,} \quad \frac{dQ}{dt} = 1986 \text{ Js}^{-1}$$

$$mc\frac{dT}{dt} = 1986 \Rightarrow \frac{dT}{dt} = 1986/(19.86 \times 2500)$$

$$= 4 \times 10^{-2} \text{ °C s}^{-1}$$

Illustration 3.9 **A monochromatic beam of light (λ = 4900 Å) incident normally upon a surface produces a pressure of 5×10^{-7} Nm^{-2} on it. Assume that 25% of the light incident is reflected and the rest absorbed. Find the number of photons falling per second on a unit area of thin surface.**

Sol. $P = [2(0.25) + 0.75]\frac{I}{c} = 1.25\frac{I}{c}$

Therefore, intensity of light,

$$I = \frac{cP}{1.25} = \frac{(3 \times 10^{8})(5 \times 10^{-7})}{1.25} = 120 \text{ Wm}^{-2}$$

Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{0.49 \times 10^{-6}} = 4 \times 10^{-19} \text{ J}$$

Therefore, number of photons incident per unit area per second,

$$n = \frac{I}{E} = \frac{120}{4 \times 10^{-19}} = 3 \times 10^{20} \text{ m}^{-2}\text{s}^{-1}$$

Illustration 3.10 **Calculate force exerted by a light beam on a surface if light is incident on the surface at an angle θ as shown in Fig. 3.9. Consider all cases of absorption and reflection.**

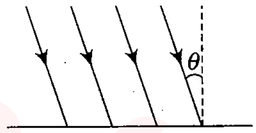
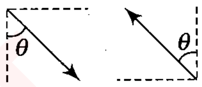


Fig. 3.9

Sol. Case I. Whole amount of radiation falling on the surface is absorbed by the surface.

For this, absorption coefficient $a = 1$, reflection coefficient $r = 0$

Initial momentum of a photon (in downward direction at an angle θ with vertical) $= h/\lambda$



Final momentum of the photon = 0

Change in momentum (in upward direction at an angle λ with vertical) $= h/\lambda$

Intensity = power per unit normal area

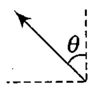
$$\Rightarrow \quad I = \frac{P}{A\cos\theta}$$

Here, $A \cos\theta$ is the projection of area perpendicular to the direction of incident light beam.

Energy incident per unit time: $P = I(A\cos\theta)$

No. of photons incident per unit time: $N = \frac{IA\cos\theta}{hc}\lambda$

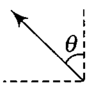
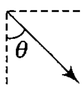
Total change in momentum per unit time (in upward direction at an angle θ with vertical)



$$= N\frac{h}{\lambda} = \frac{IA\cos\theta\,\lambda}{hc}\frac{h}{\lambda} = \frac{IA\cos\theta}{c}$$

Force (F) = total change in momentum per unit time

$$\Rightarrow \quad F = \frac{IA\cos\theta}{c}$$

(direction  on photon and  on the plate)

On plate: Component of force perpendicular to the surface:

$$F\cos\theta = \frac{IA\cos^2\theta}{c}$$

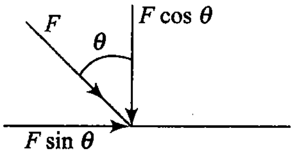


Fig. 3.10

Component of force parallel to the surface:

$$F\sin\theta = \frac{IA\cos\theta\sin\theta}{c}$$

Pressure = normal force per unit area:

$$P = \frac{F\cos\theta}{A} = \frac{IA\cos^2\theta}{cA} = \frac{I}{c}\cos^2\theta$$

Case II. Whole amount of radiation falling on the surface is reflected by the surface.

For this, reflection coefficient $r = 1$, absorption coefficient $a = 0$.

Change in momentum of one photon $= \dfrac{2h}{\lambda}\cos\theta$ (upward)

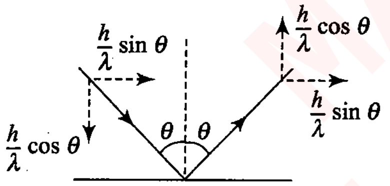


Fig. 3.11

No. of photons incident per unit time: $N = \dfrac{IA\cos\theta\cdot\lambda}{hc}$

Total change in momentum of photons per unit time

$$\frac{IA\cos\theta\cdot\lambda}{hc}\times\frac{2h}{\lambda}\cos\theta = \frac{2IA\cos^2\theta}{c} \quad \text{(upward)}$$

Force on the plate: $F = \dfrac{2IA\cos^2\theta}{c}$ (downward)

Pressure: $P = F/A = \dfrac{2IA\cos^2\theta}{cA} = \dfrac{2I\cos^2\theta}{c}$

Case III. When some part of radiation is absorbed and remaining part is reflected.

For this,

$0 < r < 1, 0 < a < 1$

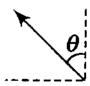
But $a + r = 1$, so $a = 1 - r$

Change in momentum of photon when it is reflected:

$$\Delta P_r = \frac{2h}{\lambda}\cos\theta \quad \text{(upward)}$$

Change in momentum of photon when it is absorbed:

$$\Delta P_a = \frac{h}{\lambda}$$



No. of photons incident per unit time: $N = \dfrac{IA\cos\theta\,\lambda}{hc}$

No. of reflected photons: $N_1 = Nr = \dfrac{IA\cos\theta\,\lambda r}{hc}$

Force on plate due to reflected photons:

$$F_r = N_1\,\Delta P_r = \frac{IA\cos\theta\,\lambda}{hc}r\times\frac{2h}{\lambda}\cos\theta$$

$$= \frac{IA\cos^2\theta}{c}\times 2r \quad \text{(vertically downward)}$$

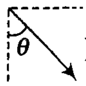
No. of absorbed photons:

$$N_2 = Na = N(1-r) = \frac{IA\cos\theta\,\lambda}{hc}(1-r)$$

Force on plate due to absorbed photons:

$$F_a = N_2\Delta P_a = \frac{IA\cos\theta\,\lambda}{hc}(1-r)\frac{h}{\lambda}$$

$$= \frac{IA\cos\theta}{c}(1-r)$$

(at an angle θ with vertical )

Component of force perpendicular to the surface:

$$F_1 = F_r + F_a\cos\theta = \frac{IA\cos^2\theta}{c}2r + \frac{IA\cos^2\theta}{c}(1-r)$$

$$= \frac{IA\cos^2\theta}{c}(1+r)$$

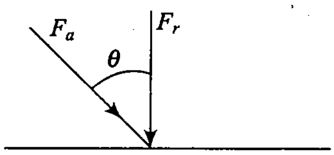


Fig. 3.12

Component of force parallel to the surface:

$$F_2 = F_a\sin\theta = \frac{IA\cos\theta\sin\theta}{c}(1-r)$$

Resultant force:

$$F_R = \sqrt{F_1^2 + F_2^2} = \frac{IA\cos\theta}{c}\sqrt{[(1+r)\cos\theta]^2 + [(1-r)\sin\theta]^2}$$

$$= \frac{IA\cos\theta}{c}\sqrt{1 + r^2 + 2r\cos 2\theta}$$

Resultant force can also be found as:

$$F_R = \sqrt{F_r^2 + F_a^2 + 2F_aF_r\cos\theta}$$

$$= \frac{IA\cos\theta}{c}\sqrt{(1-r)^2 + (2r)^2\cos^2\theta + 4r(1-r)\cos^2\theta}$$

$$= \frac{IA\cos\theta}{c}\sqrt{1 + r^2 + 2r\cos 2\theta}$$

Pressure: $$P = \frac{F_1}{A} = \frac{I\cos^2\theta}{c}(1+r)$$

Illustration 3.11 **A plank of mass 'm' is lying on a rough surface having coefficient of friction as 'μ' in situation as shown in Fig. 3.13. Find the acceleration of the plank assuming that it slips and surface of body exposed to radiation is black body.**

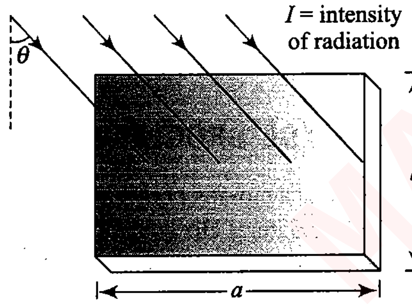


Fig. 3.13

Sol. E = Energy received by surface per second is

$$IA\cos\theta = Iab\cos\theta$$

Number of photons received by the surface per second,

$$N = \frac{E}{hc/\lambda} = \frac{Iab\cos\theta\lambda}{hc}$$

$$\Delta P_{\text{one photon}} = \frac{h}{\lambda}$$

$$\Rightarrow \quad F_{\text{radian}} = \Delta P_{\text{one photon}} \cdot N = \frac{Iab\cos\theta}{c}$$

Let us draw the FBD of the plank (see Fig. 3.14)

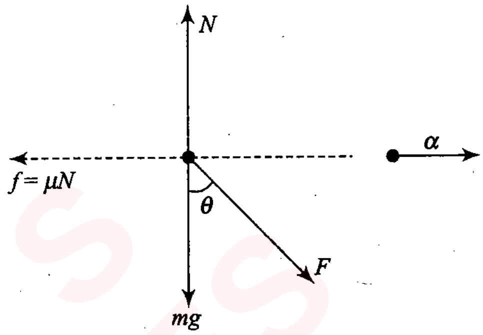


Fig. 3.14

$$F\sin\theta - \mu N = m\alpha \quad \text{(i)}$$

$$N = mg + F\cos\theta \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\alpha = \frac{F\sin\theta - \mu(mg + F\cos\theta)}{m}$$

Illustration 3.12 **In the path of a uniform light beam of large cross-sectional area and intensity I, a solid sphere of radius R which is perfectly reflecting is placed. Find the force exerted on this sphere due to the light beam.**

Sol. The front surface of the sphere will be illuminated by the light as shown in Fig. 3.15, as angle of incidence of light is different at different position of the front face of the sphere.

We can consider a small elemental circular strip (ring) of angular width $d\theta$ on its surface at an angle θ from its horizontal diameter as shown. Let the area of this elemental strip is dS, then

$$dS = 2\theta R\sin\theta\, Rd\theta \quad \text{(i)}$$

$R\sin\theta$ is the radius of this circular strip (ring).

We take the projection of area of this slant elemental circular strip in vertical plane. Let dA is the projection of the slant strip area dS along the cross-sectional plane of the light beam and it is given as

$$dA = dS\cos\theta \quad \text{(ii)}$$

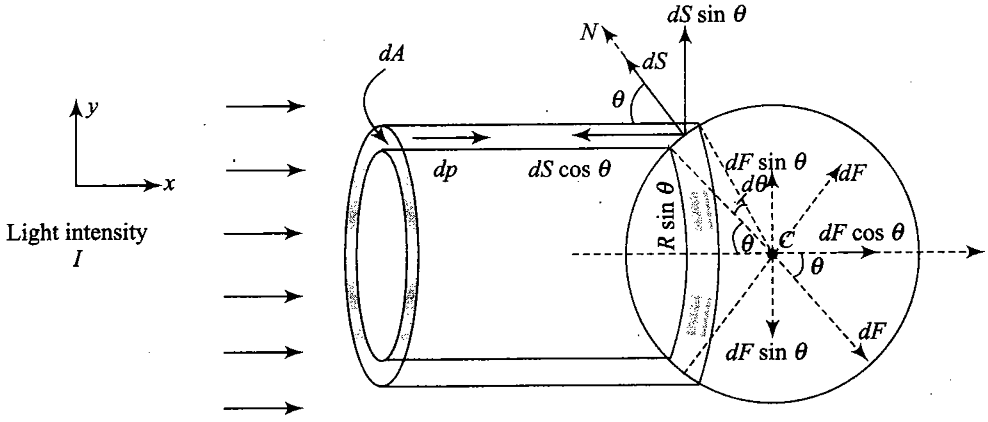


Fig. 3.15

Hence, the power of light incident on this strip is $dP = IdA$

The momentum of photons per second incident on this strip is

$$dp = \frac{dP}{c} = \frac{IdA}{c} \qquad \text{(iii)}$$

These photons are incident at an angle θ to the normal N of this strip. As the surface of the sphere is perfectly reflecting, the photons will be reflected at the same angle θ to N as shown in the figure.

Hence, the change in linear momentum of photons is along the normal and thus force exerted on this strip along the normal is

$$dF = 2dp \cos\theta = \frac{2IdA}{c}\cos\theta \qquad \text{(iv)}$$

Thus, net force on sphere will be along the x-axis only.

Thus, the force on the sphere will be given as

$$F = \int dF \cos\theta = \int \frac{2IdA}{c}\cos^2\theta$$

$$= \int_0^{\pi/2} \frac{2I}{c}(2\pi R \sin\theta \cos\theta \, Rd\theta)\cos^2\theta$$

$$= \frac{4I\pi R^2}{c}\int_0^{\pi/2}\cos^3\theta \sin\theta \, d\theta = \frac{4I\pi R^2}{c}\left[-\frac{\cos^4\theta}{4}\right]_0^{\pi/2}$$

$$= \frac{I\pi R^2}{c}[1-0] = \frac{I\pi R^2}{c} \qquad \text{(v)}$$

Illustration 3.13 **A toy truck has dimensions as shown in Fig. 3.16 and its width, normal to the plane of this paper, is d. Sun rays are incident on it as shown in the figure. If intensity of rays is I and all surfaces of truck are perfectly black, calculate tension in the thread used to keep the truck stationary. Neglect friction.**

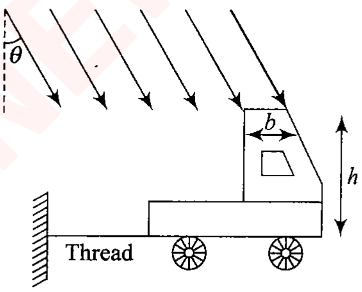


Fig. 3.16

Sol. First, we have to calculate power incident on the truck and then momentum of light photons incident per second. Since truck surfaces are perfectly black, therefore finally momentum reduces to zero. It means that the rate of change of momentum of light is equal to the momentum of photons incident per second.

Area of top of the driving cabin = bd.

Light is incident due to the component $I\cos\theta$ of intensity I.

Power incident on top of the cabin = $bdI\cos\theta$

Similarly, power incident on rear horizontal part of the truck = $adI\cos\theta$ and power incident on rear vertical wall of the driving cabin = $hdI\sin\theta$.

Total power incident on the truck is

$$P = (b\cos\theta + a\cos\theta + h\sin\theta)Id$$

Momentum of these photons is $p = P/c$

(where c is speed of light in vacuum)

But rate of change in momentum of photons,

$$p = (Id/c)(b\cos\theta + a\cos\theta + h\sin\theta)$$

This momentum is inclined at angle θ with the vertical. Its vertical component $p\cos\theta$ is balanced by normal reaction of the floor and horizontal component $p\sin\theta$ is balanced by tension in the thread.

$$\text{Tension} = p\sin\theta = (Id/c)(b\cos\theta + a\cos\theta + h\sin\theta)\sin\theta$$

MATTER WAVES (DE BROGLIE WAVES)

de Broglie suggested that a moving body behaves in certain ways as though it has a wave nature. His conjecture of dual nature of matter was based on following two points:

a. dual nature of radiation, and

b. nature loves symmetry

de Broglie deduced the connection between particle and wave properties from the Einstein–Planck expression for the energy of an electromagnetic wave and the classical result for the momentum of such a wave. The two expressions are

$$E = hf \qquad \text{(i)}$$

and

$$p = \frac{E}{c} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{p} \qquad \text{(iii)}$$

de Broglie said that Eq. (iii) is a completely general one that is applied to material particle as well as to photons. The waves associated with moving particle are called *matter waves or de Broglie waves*. The wavelength associated with a moving particle is known as *de Broglie wavelength* and it is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{(iv)}$$

where p = momentum of the particle, m = mass of the particle, and v = velocity of the particle.

Also, $\quad \text{KE} = \dfrac{p^2}{2m} \Rightarrow \lambda = \dfrac{h}{\sqrt{2m\,\text{KE}}}$

(A) de Broglie wavelength associated with charged particles

- For electrons ($m_e = 9.1 \times 10^{-31}$ kg):

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}} \text{ m}$$

$$= \frac{12.27}{\sqrt{V}} \text{ Å}$$

- For protons ($m_p = 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.286}{\sqrt{V}} \text{ Å}$$

- For deuterons ($m_d = 2 \times 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.202}{\sqrt{V}} \text{ Å}$$

- For α-particles ($m_\alpha = 4 \times 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.101}{\sqrt{V}} \text{ Å}$$

(B) de Broglie wavelength associated with uncharged particles

- For neutrons ($m_n = 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27}\ E}}$$

- For thermal neutrons at ordinary temperatures:

$$E = kT$$

$$\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.835}{\sqrt{T}} \text{ Å}$$

- For gas molecules:

$$\lambda = \frac{h}{m \times C_{rms}}$$

- For gas molecules at TK:

$$E = \frac{3}{2}kT \quad \Rightarrow \quad \lambda = \frac{h}{\sqrt{3mkT}}$$

Properties of Matter Waves

(i) Matter waves are different from electromagnetic waves. This is an obvious fact as matter waves are related to moving particles and independent of the fact whether particle is neutral or charged.

(ii) Matter waves travel with speed more than the speed of light

$$v_{\text{matter waves}} = f\lambda = \frac{E}{h} \times \frac{h}{P} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

As velocity of particle can never exceed speed of light, hence $v_{\text{matter waves}} > C$. As a matter of fact, matter waves travel with the body a group. Group velocity is same as velocity of body, whereas the indivisual waves constituting the group may theoritically move with speed greater than that of light.

(iii) In ordinary situation, de Broglie wavelength is very small and wave nature of matter can be ignored. To appreciate this point, let us calculate de Broglie wavelength of 46 g of golf ball moving with a speed of 30 ms^{-1}.

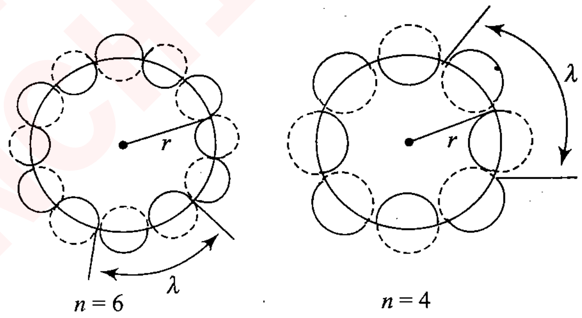
$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.046)(30)} = 4.8 \times 10^{-34} \text{ m}$$

The wavelength of golf ball is so small compared with its dimensions that we would not expect to detect any wave aspects in its behavior using ordinary instrument.

(iv) The wave and particle aspects of moving bodies can never be observed at the same time, i.e., the two nature are mutually exclusive. Which nature dominates in given situation will be directed by how its de Broglie wavelength compares with its dimensions and dimensions of whatever it interacts with.

(v) The square of the amplitude of de Broglie wave at any point is proportional to the probability of finding the particle at that point.

Application of de Broglie Wave Hypothesis



Standing de-Broglie waves of electrons around the circumference of Bohr's orbit

Fig. 3.17

(i) Electron microscope: de Broglie idea of wave associated with a moving material particle had led to the development of a microscope using a beam of fast moving electrons. The microscope is highly suitable for large magnification in the study of atomic structure.

(ii) Quantisation of orbit: The concept of matter waves was used to justify quantisation of angular momentum as proposed in Bohr's atomic model. If we consider standing electron wave, then to maintain a standing wave over the circumference of a circular orbit, the wavelength must be an integral fraction of that circumference.

i.e., $2\pi r = n\lambda = \dfrac{nh}{P} = \dfrac{nh}{mv}$

or $mvr = \dfrac{nh}{2\pi}$

Illustration 3.14 **An electron is accelerated by a potential difference of 50 V. Find the de Broglie wavelength associated with it.**

Sol. For an electron, de Broglie wavelength is given by

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}} = \sqrt{3} = 1.73 \text{ Å}$$

Illustration 3.15 **What voltage must be applied to an electron microscope to produce electrons of wavelength 0.4 Å?**

Sol. Here, $\lambda = 0.4$ Å $= 0.4 \times 10^{-10}$ m

Let V be the required voltage. Then,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

or $$\lambda = \frac{12.27}{\sqrt{V}} \text{ (in Å)}$$

$\therefore$ $$0.4 = \frac{12.27}{\sqrt{V}}$$

or $$\sqrt{V} = 30.675$$

or $$V = (30.675)^2 = 940.96 \text{ V}$$

Illustration 3.16 **An electron and a photon have the same de Broglie wavelength. Which one of these has higher kinetic energy?**

Sol. Let λ be the de Broglie wavelength of the electron and the photon. If m and v are the mass and velocity of the electron, then de Broglie wavelength of the electron

$$\lambda = \frac{h}{mv}$$

The photon has got zero rest mass. Therefore, energy of the photon is totally kinetic in nature. Since the wavelength of the photon is same as that of the electron, the kinetic energy of the photon having wavelength λ,

$$E_1 = \frac{hc}{\lambda} = \frac{hc}{h/mv}$$

or $$E_1 = mvc \quad \text{(i)}$$

Now, the kinetic energy of the electron,

$$E_2 = \frac{1}{2}mv^2 = mv \times \frac{v}{2} \quad \text{(ii)}$$

Since $c > v/2$ (as $c > v$), from the results (i) and (ii), it follows that

$$E_1 > E_2$$

i.e., kinetic energy of the photon is greater than that of the electron. As it moves with the speed c, it is faster than electron.

Illustration 3.17 **A photon and an electron have got the same de Broglie wavelength. Which has greater total energy? Explain.**

Sol. Let λ be the de Broglie wavelength of the electron and also the wavelength of the photon. If m and v are mass and velocity of the electron, then de Broglie wavelength of the electron,

$$\lambda = \frac{h}{mv}$$

Energy of the photon having wavelength λ,

$$E_1 = \frac{hc}{\lambda}$$

Since wavelength of the photon is same as that of the electron,

$$E_1 = hc \times \frac{mv}{h}$$

or $$E_1 = mvc \quad \text{(i)}$$

According to Einstein's mass–energy equivalence relation, the energy of the electron (mass m)

$$E_2 = mc^2 = mc \times (c) \quad \text{(ii)}$$

Since $c > v$, from the results (i) and (ii), it follows that

$$E_2 > E_1$$

i.e., total energy of electron is greater than that of the photon.

Note: ***In previous illustration the kinetic energy of photon and electron is compared while in this problem the total energy is compared.***

Illustration 3.18 **The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which (a) an electron and (b) a neutron would have the same deBroglie wavelength. Given that mass of neutron = 1.66 $\times 10^{-27}$ kg.**

Sol. Here, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m

The de Broglie wavelength of a particle having energy E is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

or $$E = \frac{h^2}{2m\lambda^2}$$

a. For electron: Here, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m and

$m = 9.1 \times 10^{-27}$ kg

$$\therefore \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2}$$

$$= 6.94 \times 10^{-25} \text{ J}$$

b. For neutron: Here, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m and

$m = 1.66 \times 10^{-27}$ kg

$$\therefore \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (589 \times 10^{-9})^2}$$

$$= 3.81 \times 10^{-28} \text{ J}$$

Illustration 3.19 **a. For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?**

b. Also, find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K. Given that mass of neutron = 1.66×10^{-27} kg and $k = 1.38 \times 10^{-23}$ J kg^{-1}

Sol.

a. Here, $\lambda = 1.40\ \lambda \times 10^{-10}$ m; $m = 1.66 \times 10^{-27}$ kg

The energy of neutron is given by

$$E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (1.40 \times 10^{-10})^2}$$

$$= 6.735 \times 10^{-21}\ \text{J}$$

b. Here, $k = 1.38 \times 10^{-23}$ J kg^{-1} and $T = 300$ K

Therefore, energy of neutron

$$E = \frac{3}{2} \times = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}\ \text{J}$$

Now, $$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 6.21 \times 10^{-21}}}$$

$$= 1.46 \times 10^{-10}\ \text{m} = 1.46\ \text{Å}$$

Illustration 3.20 **An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?**

Sol. Here, $V = 50$ kV. Therefore, energy of electrons,

$$E = 50\ \text{keV} = 50 \times 10^3 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-15}$$

Now,

$$\lambda = \frac{h}{\sqrt{2me}}$$

Taking $m = 9.1 \times 10^{-31}$ kg, we have

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 8.0 \times 10^{-15}}}$$

$$= \frac{6.62 \times 10^{-34}}{1.207 \times 10^{-22}} = 5.485 \times 10^{-12}\ \text{m}$$

The resolving power of a microscope is inversely proportional to the wavelength of the radiation used. Since wavelength of the yellow light is 5990 Å, i.e., 5.99×10^{-7} m, power of electron microscope is 10^5 times as large as that of the optical microscope.

Illustration 3.21 **Determine the de Broglie wavelength of a proton, whose kinetic energy is equal to the rest mass energy of an electron. Given that the mass of an electron is 9.1×10^{-31} kg and the mass of a proton is 1837 times as that of the electron.**

Sol. Here, mass of electron, $m_0 = 9.1 \times 10^{-31}$ kg

mass of proton, $m = 1837 \times 9.1 \times 10^{-31} = 1.67 \times 10^{-27}$ kg

Let v be the velocity of the proton. Then,

$$\frac{1}{2}mv^2 = m_0c^2$$

or $$m^2v^2 = 2mm_0c^2$$

or $$mv = \sqrt{2mm_0}c$$

$$= \sqrt{2 \times 1.67 \times 10^{-27} \times 9.1 \times 10^{-31}} \times 3 \times 10^8$$

$$= 1.654 \times 10^{-20}\ \text{kg ms}^{-1}$$

Therefore, de Broglie wavelength of the proton,

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1.654 \times 10^{-20}} = 4 \times 10^{-14}\ \text{m}$$

Illustration 3.22 **The extent of localization of a particle is determined roughly by its de Broglie wave. If an electron is localized within the nucleus (of size about 10^{-14} m) of an atom, what is its energy? Compare this energy with the typical binding energies (of the order of a few MeV) in a nucleus and hence argue why electrons cannot reside in a nucleus.**

Sol. The de Broglie wavelength of electron,

$$\lambda = \text{size of atom} = 10^{-14}\ \text{m}$$

Corresponding to this wavelength, the electron possesses energy,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{10^{-14}} = 1.986 \times 10^{-11}\ \text{J}$$

$$= \frac{1.986 \times 10^{-11}}{1.6 \times 10^{-13}} = 124.1\ \text{MeV}$$

If the electrons possess energy of the order of 124 MeV, they will not be able to stay inside the nucleus. It is because Coulomb's force cannot provide such high value of binding energy to the electrons. Hence, electrons cannot reside inside the nucleus.

Illustration 3.23 **Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C, respectively.**

Sol. de Broglie wavelength is given by $\lambda = h/mv$

Root mean square velocity of a gas particle at the given temperature (T) is given as,

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v = \sqrt{\frac{3kT}{m}}$$

where k = Boltzmann's constant, m = mass of the gas particle, and T = temperature of the gas in K.

$$mv = \sqrt{3mkT}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mkT}}$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{(4)(273+127)}{(2)(273+27)}} = \sqrt{\frac{8}{3}}$$

Concept Application Exercise 3.1

1. Find de Broglie wavelength of 2 MeV proton. Mass of proton = 1.64×10^{-27} kg, $h = 6.625 \times 10^{-34}$ Js.
2. Find the de Broglie wavelength of a neutron at 127°C. Given that Boltzmann's constant $k = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}. Planck's constant = 6.625×10^{-34} Js, mass of neutron = 1.66×10^{-27} kg.
3. The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 kWm^{-2}. If it is completely absorbed, find the corresponding radiation pressure.
4. An electron is accelerated by a potential difference of 25 V. Find the de Broglie wavelength associated with it.
5. Why de Broglie waves associated with a moving football are not apparent to us?
6. The de Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle, if its kinetic energy were $K/4$?
7. The two lines A and B in Fig. 3.18 show the plot of de Broglie wavelength (λ) as a function of $1/\sqrt{V}$ (V is the accelerating potential) for two particles having the same charge. Which of the two represents the particle of heavier mass?

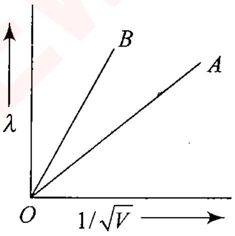


Fig. 3.18

8. Monochromatic light of wavelength 632.8 nm is produced by a helium–neon laser. The power emitted is 9.42 mW.
   a. Find the energy and momentum of each photon in the light beam.
   b. How many photons per second, on an average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross section, which is less than the target area.)
   c. How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon? Given, mass of hydrogen atom = 1.66×10^{-27} kg.
9. The energy flux of sunlight reaching the surface of the earth is 1.388×10^3 W m^{-2}. How many photons (nearly) per square meter are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.
10. An electron and a photon each have a wavelength of 1.0 nm. Find (a) their momenta, (b) the energy of the photon, and (c) the kinetic energy of electron. Take $h = 6.63 \times 10^{-34}$ Js.
11. For a given kinetic energy, which of the following has the smallest de Broglie wavelength: electron, proton and α-particle ?
12. A plate is kept in front of a beam of photons. The plate reflects 40% of the incident photons and absorbs the remainder. If the plate absorbs energy at a rate 1200 J^{-1}s, find the net force acting on it.
13. A sensor is exposed for 0.1 s to a 200 W lamp 10 m away. The sensor has an opening that is 20 mm in diameter. How many photons enter the sensor if the wavelength of light is 600 nm? Assume that all the energy of the lamp is given off as light.
14. Photons of wavelength λ = 662 nm are incident normally on a perfectly reflecting screen. Calculate their number per second falling on the screen such that the exerted force is 1 N.
15. A voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda$ = 26 pm. Find the initial voltage applied to the tube.

ELECTRON EMISSION

The electrons in metals which are free to move within the volume of metal are called free electrons. These electrons do not get ejected out of the surface of metal on their own because of the potential barrier existing all around its surface. The minimum amount of work or energy necessary to take a free electron out of a metal against the attractive forces of positive ions inside a metal is called the *work function* of that metal.

Work functions of some photosensitive metals:

Metal	Work function (eV)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

When a free electron gets extra energy ($\geq$ work function) from an external agent, it is able to overcome potential barrier and is ejected out. There are a number of ways in which energy from outside can be supplied. Hence, there are different ways in which electron emission can take place. These ways are listed as below:

(i) Photoelectric emission: When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface, then electrons are emitted, this phenomenon is called *photoelectric effect.*

(ii) Thermionic emission: In this case, extra energy is given to electrons to overcome potential barrier in the form of heat by passing current through a filament (Joule's heating effect).

(iii) Field emission: In this case, metal is placed in a strong electric field due to which the electrons are accelerated to such a speed that the corresponding kinetic energy is sufficient to overcome potential barrier.

(iv) Secondary emission: Electrons from metal can also be ejected by impinging a beam of high-speed electrons so that through collision, electrons acquire sufficient kinetic energy to overcome potential barrier. This type of electron emission is called secondary emission.

In this chapter, we will discuss in details the photoelectric emission or photoelectric effect.

PHOTOELECTRIC CELL

It is a device which shows photoelectric effect. It converts light energy into electrical energy. It consists of an evacuated bulb B, whose inside is coated with an alkali metal 'P', leaving a clear portion 'W' in the form of a window as shown in figure. The bulb is made up of glass if it is to be used for white light and is made up of quartz if it is to be used in case of ultra violet light. It has an electrode 'C' which is given a positive potential with the help of a battery. Light from a source 'S' is focused into the metal P with the help of a convex lens 'L'. A micrometer connected in the circuit indicates the photoelectric current.

When light of suitable wavelength is allowed to fall on the cathode, photoelectrons are emitted. These are attracted by anode (C) which is kept at positive potential w.r.t. cathode and a current starts flowing in the circuit. The photocurrent obtained is of the order of micro ampere which is very small. So, it is to be amplified first before it is used for some useful purpose. When light is cut off, no photoelectrons are ejected from the cathode and hence there is no current in the external circuit.

Applications of Photoelectric Cell

(i) It plays an important role in television studio. It converts light and shade of the picture into electrical waves which after a proper processing are transmitted to distant stations.

(ii) It is used for reproduction of sound in films. Microphone converts sound into electrical waves which, after amplifications, are fed to an electric lamp. Intensity of light from this lamp records lines of varying transparency on the film. During reproducing, a beam of light falls on a photocell, after crossing through this film. Photocell converts light back into electrical oscillation which produces sound when fed to the receiver.

(iii) It is used for triggering fire alarm. In factories using chemicals or explosive materials, photocells are fitted at selected places. Light from any accidental fire falls on the cell. This produces a current which after amplification is fed to an alarm.

(iv) It is used in operating burglar's alarm. Ultraviolet light from a source is incident constantly on a cell. Any unwanted person entering a room cuts that beam unknowingly, thus stopping the current for a fraction of a second. This triggers an alarm.

(v) It is used for automatic switching of street light. Light from sun, during day, falls constantly on the cell making a current to flow through the circuit continuously. This current after amplification is fed to an electromagnet which keeps the key of the street light circuit open. In the evening, after sunset, intensity of light and hence the current decreases. The electromagnet loses its strength. As a result of this, the key is closed and the street light is automatically switched on.

(vi) A photocell coupled with an electronic counter can be used to count, automatically, the number of persons entering or leaving a hall. Each person will cut a beam of ultraviolet light once, thus producing a kick in the counter.

(vii) It is used to compare the illuminating powers of two sources. Current produced is directly proportional to

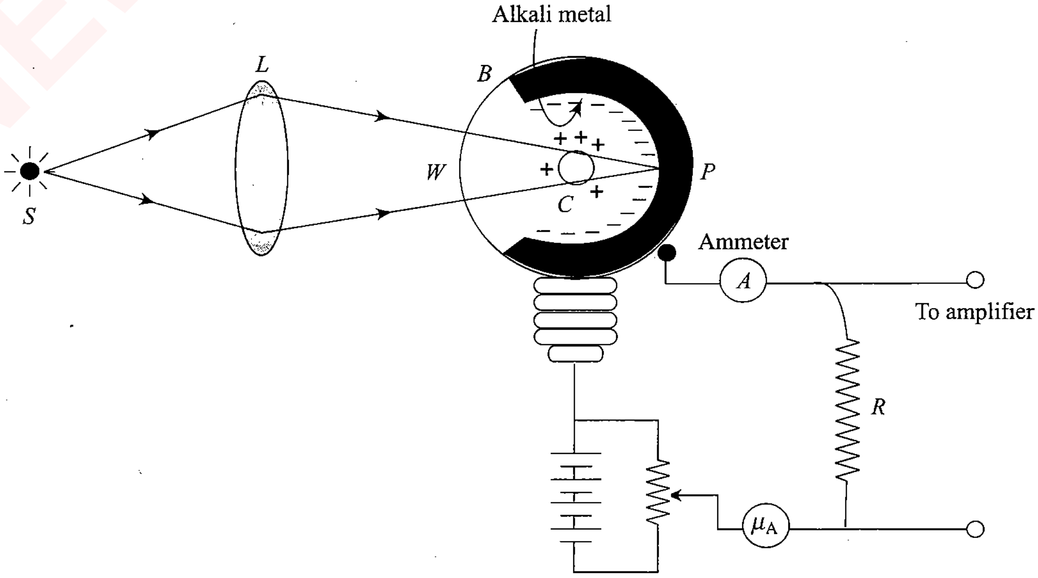


Fig. 3.19

intensity, which, in turn, is proportional to the illuminating power of the source. Ratio of deflection in ammeter with two sources of light gives the ratio of their illuminating powers

PHOTOELECTRIC EFFECT

When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface then electrons are emitted, this phenomenon is called *photoelectric effect*. The electrons so emitted are known as photoelectrons. And the current so obtained is known as photoelectric current.

Alkali metals Li, Na, K, Cs etc., show photoelectric emission with visible light. Zn, Cd, Mg, etc., show photoelectric emission with ultraviolet light

Study of Photoelectric Effect

Figure 3.20 shows the experimental setup to study the photoelectric effect.

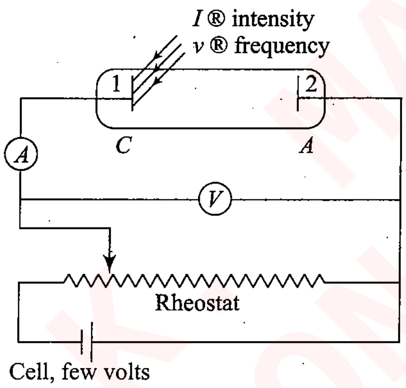


Fig. 3.20

Here, plate 1 is called emitter or cathode and plate 2 is called collector or anode. A suitable potential difference $V = V_A - V_C$ can be applied between them. On cathode, an electromagnetic radiation of intensity I and frequency f is allowed to fall upon. Due to this, electrons are ejected out from plate 1 and they travel to plate 2 as it is at positive potential V w.r.t. plate 1. Thus, a current is established in the circuit known as photoelectric current (PEC).

Observations (by Einstein)

a. If in this experiment the frequency f and potential difference V are kept constant and intensity I is varied (or more number of photons are allowed to fall per unit time), a graph between intensity of light and photoelectric current (PEC) is found to be a straight line as shown in Fig. 3.21.

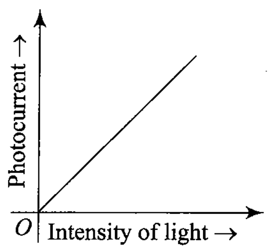


Fig. 3.21

The photoelectric current is directly proportional to the intensity of incident radiation.

b. A graph between photoelectric current and potential difference V is found as shown in Fig. 3.22. Here, I and f are kept constant. If we increase V, then PEC also increases, but after increasing V up to some value there is no increase in PEC. This value of PEC is known as saturation current (I_s). At this value, all the photoelectrons emitted from cathode reach the anode and there is no possibility of increasing the current by increasing V. Hence in case of saturation current, rate of emission of photoelectrons = rate of flow of photoelectrons from cathode to anode.

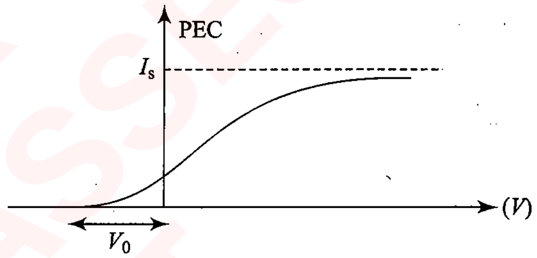


Fig. 3.22

V_0 is known as stopping potential or cut-off potential. It is the minimum negative potential given to plate A w.r.t. C at which PEC just becomes zero.

Einstein's Photoelectric Equation

According to Einstein, photon energy (hf) falling on a metal is utilized for two purposes:

(i) Partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by W_0.

(ii) The balance of the photon energy is used up in giving the electron a kinetic energy KE.

So, we can write:

$$hf = W_0 + \text{KE}$$

$$\Rightarrow \quad \text{KE} = hf - W_0$$

The above equation says that when a single photon carrying an energy hf enters into the surface, there it is absorbed by a single electron. Part of this energy W_0 (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy $(hf - W_0)$ becomes the electron's kinetic energy. If the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus, $(hf - W_0)$ represents the maximum kinetic energy that a photoelectron can have outside the surface.

So, we can write:

$$\text{KE}_{max} = hf - W_0$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from 0 to KE_{max}. The energy distribution of photoelectrons is as shown in Fig. 3.23.

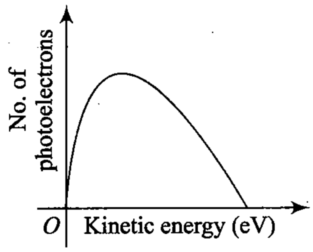


Fig. 3.23

In case the photon energy (say for $f=f_0$) is just sufficient to liberate the electron only, then kinetic energy of the liberated electron will be zero. Then, $hf_0 = W_0$.

Here, f_0 is known as *threshold frequency*. The corresponding wavelength is known as *threshold wavelength*. If the frequency of incident light is less than f_0, no photoelectric emission takes place.

Maximum kinetic energy of photoelectrons is

$$\text{KE}_{\max} = hf - hf_0 = h(f-f_0)$$

$$= hc\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)$$

$$= 124000\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)\text{eV} \qquad \text{(i)}$$

For a given frequency, the stopping potential is related to the maximum KE of photoelectrons which are just stopped from reaching the anode A. For this,

$$eV_0 = \text{KE}_{\max} = \frac{1}{2}mv_{\max}^2 \qquad \text{(ii)}$$

From Eq. (i) and (ii), we get

$$eV_0 = h(f-f_0)$$

$$\Rightarrow \qquad V_0 = \frac{h}{e}f - \frac{hf_0}{e} = \frac{h}{e}f - \frac{W_0}{e}$$

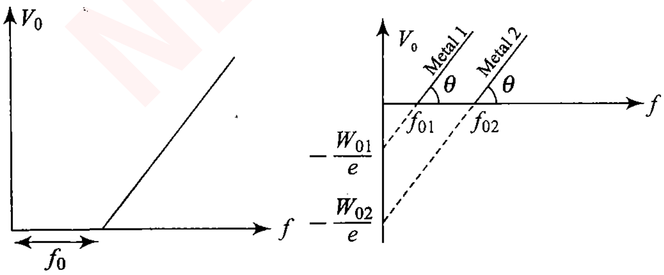


Fig. 3.24

Stopping potential varies linearly with frequency. Slope of the graph (h/e) is same for all metals.

For $f \le f_0$, stopping potential is zero.

c. If intensity is increased (keeping the frequency constant), then saturation current is increased by same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not affected.

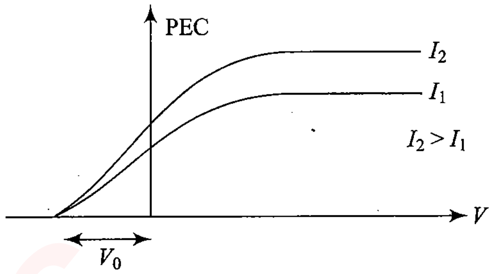


Fig. 3.25

So, maximum KE of photoelectrons emitted is independent of intensity of falling radiation.

d. If light of different frequencies (keeping the intensity same) is used, then obtained plots are shown in Fig. 3.26. It is clear from the graph, as f increases, the magnitude of stopping potential increases. It means, maximum value of kinetic energy increases.

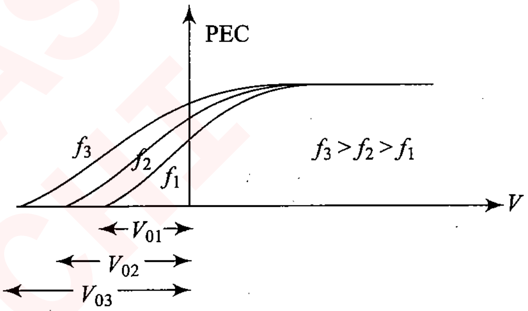


Fig. 3.26

So, maximum KE of photoelectrons emitted depends on the frequency of incident light.

Laws of Photoelectric Effect

(i) The number of photoelectrons emitted per second is proportional to the intensity of the incident light.

(ii) There is a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.

(iii) Above threshold frequency, the maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.

(iv) The emission of photoelectrons is an instantaneous process. It means time difference between incidence of light and emission of photoelectrons is very small, may be even less than 10^{-9} s.

Failure of Classical Wave Theory of Light to Explain the Laws of Photoelectric Effect

(i) The intensity problem: If intensity of light falling on metal is increased, then according to wave theory of light, oscillating electric field vector **E** of the light wave increases in amplitude. The force applied to the electron in the metal will be $e\mathbf{E}$ due to falling radiation and this force will increase on increasing the intensity of light. This suggests that the kinetic energy of the emitted photoelectrons should also increase due to larger force applied in emitting them out as the light beam is made more intense. However, observations show that maximum kinetic energy is independent of the light intensity.

(ii) **The frequency problem:** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided that the light is intense enough to supply the energy needed to eject the photoelectrons. However, observations show that the photoelectric effect does not occur if the frequency of falling radiation is less than the threshold frequency, no matter how intense the light beam is.

(iii) **The time delay problem:** In the classical theory, the light energy is uniformly distributed over the wave front. So, when the light falls on metal, the energy of the incident light will not entirely go to a particular electron in the metal but will be distributed uniformly to other electrons also. So, the electron will take some time to accumulate enough energy to escape from the metal surface. Hence, there should be a measurable time lag between the impinging of the light on the surface and the ejection of the photoelectron. However, no detectable time lag has ever been measured.

Quantum theory solves these problems in providing the correct interpretation of photoelectric effect as follows:

(i) Let a light of intensity I falls on area A. Let N is the number of photons falling per unit time, then

$$IA = Nhf \quad \Rightarrow \quad N = \frac{IA}{hf}$$

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photon. Hence, kinetic energy of emitted electrons does not change on increasing the intensity.

(ii) The second objection (the frequency problem) is met if KE_{max} equals zero. In this case we have $hf_0 = W$, which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If f is reduced below f_0, hf will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

(iii) The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle or in the form of energy packets. It is not spread uniformly over the beam cross section as in the wave theory. Basically, the emission of photoelectron is a single step process in which energy from photon is supplied to electron at once and the electron is ejected immediately.

Illustration 3.24 **What is the energy (in eV) of a photon of wavelength 12400 Å?**

Sol. Using Planck's formula, we have $E = hf = \dfrac{hc}{\lambda}$

where $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}; $\lambda = 12400 \times 10^{-10}$ m

$$\therefore \quad E = \frac{(6.63\times10^{-34})(3\times10^8)}{12400\times10^{-10}} = 1 \text{ eV}$$

Note: ***In general, a photon of wavelength λ (in Å) will have energy E (in eV) as given by $E = 12400/\lambda$.***

Illustration 3.25 **Will photoelectrons be emitted from a copper surface, of work function 4.4 eV, when illuminated by visible light?**

Sol. The threshold wavelength λ_0 corresponding to work function W is given by

$$\lambda_0 = \frac{hc}{W} \quad \text{or} \quad \lambda_0 = \frac{12400}{4.4} = 2820 \text{ Å}$$

Since λ_0 does not lie in the visible range (4000 Å to 7500 Å), therefore it cannot eject photoelectrons from copper.

Illustration 3.26 **A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photocell, the following lines from a mercury source were used:**

λ_1 = 3650 Å, λ_2 = 4047 Å, λ_3 = 4358 Å, λ_4 = 5461 Å, and λ_5 = 6907 Å

The stopping voltages, respectively, were measured to be: V_1 = 1.28 V, V_2 = 0.95 V, V_3 = 0.74 V, V_4 = 0.16 V, and V_5 = 0 V.

a. Determine the value of Planck's constant h.

b. Estimate the threshold frequency and work function for the material.

Sol. a. Let the respective frequencies of the five spectral lines of mercury be V_1, V_2, V_3, V_4, and V_5. Then

$$V_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3650\times10^{-10}} = 8.22\times10^{14} \text{ Hz}$$

$$V_2 = \frac{c}{\lambda_2} = \frac{3\times10^8}{4047\times10^{-10}} = 7.41\times10^{14} \text{ Hz}$$

$$V_3 = \frac{c}{\lambda_3} = \frac{3\times10^8}{4358\times10^{-10}} = 6.88\times10^{14} \text{ Hz}$$

$$V_4 = \frac{c}{\lambda_4} = \frac{3\times10^8}{5461\times10^{-10}} = 5.49\times10^{14} \text{ Hz}$$

and $$V_5 = \frac{c}{\lambda_5} = \frac{3\times10^8}{5461\times10^{-10}} = 4.34\times10^{14} \text{ Hz}$$

From Einstein's photoelectric equation, we have

$$h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2$$

If V_0 is the stopping potential, then

$$eV_0 = \frac{1}{2}mv_{max}^2$$

$\therefore$ $$h\nu = h\nu_0 + eV_0$$

or $$V_0 = \frac{h\nu}{e} - \frac{h\nu_0}{e}$$

It represents the equation of a straight line, whose slope is h/e and makes an intercept $h\upsilon_0/e$ on negative V_0-axis. The plot of graph between υ (along X-axis) and V_0 (along Y-axis) for the given data for the five spectral lines of mercury will be as shown in Fig. 3.27.

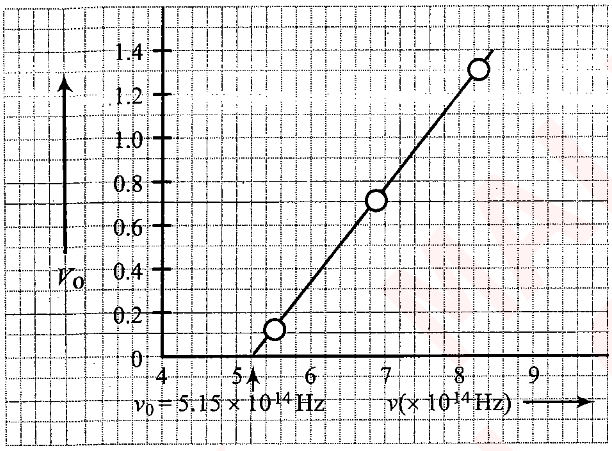


Fig. 3.27

From the graph, slope of the graph is

$$\frac{1.28 - 0.16}{8.22 \times 10^{14} - 5.49 \times 10^{14}} = 4.1 \times 10^{-15} \text{ Js C}^{-1}$$

$\therefore$ $$\frac{h}{e} = 4.1 \times 10^{-15}$$

or $$h = e \times 4.1 \times 10^{-15}$$
$$= 1.6 \times 10^{-19} \times 4.1 \times 10^{-15}$$
$$= 6.56 \times 10^{-34} \text{ Js}$$

b. Also, the intercept made by the graph on ν-axis is equal to ν_0. Therefore, from the graph, we have

$$\nu_0 = 5.15 \times 10^{14} \text{ Hz}$$

Hence, the work function of rubidium,

$$w = h\nu_0$$
$$= 6.456 \times 10^{-34} \times 5.15 \times 10^{14}$$
$$= 3.38 \times 10^{-19} \text{ J} = \frac{3.38 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.11 \text{ eV}$$

Illustration 3.27 **Photoelectric threshold of silver is λ = 3800 Å. Ultraviolet light of λ = 2600 Å is incident on a silver surface. Calculate:**

a. the value of work function in joule and in eV.
b. maximum kinetic energy of the emitted photoelectrons.
c. the maximum velocity of the photoelectrons.
(Mass of the electrons = 9.11×10^{-31} kg)

Sol. a. λ_0 = 3800 Å

$$W = hf_0 = h\frac{c}{\lambda_0} = \frac{6.633 \times 10^{-34} \times 3 \times 10^8}{3800 \times 10^{-10}} \text{ J}$$
$$= 5.23 \times 10^{-19} \text{ J} = 3.27 \text{ eV}$$

b. Incident wavelength λ = 2600 Å. Therefore,

$$f = \text{incident frequency} = \frac{3 \times 10^8}{2600 \times 10^{-10}} \text{ Hz}$$

Then, $KE_{max} = hf - W_0$

$$hf = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2600 \times 10^{-10}}$$
$$= 7.65 \times 10^{-19} \text{ J} = 4.78 \text{ eV}$$
$$KE_{max} = hf - W_0 = 4.78 \text{ eV} - 3.27 \text{ eV} = 1.51 \text{ eV}$$

c. $$KE_{max} = \frac{1}{2}mv_{max}^2$$

$\Rightarrow$ $$v_{max} = \sqrt{\frac{2KE_{max}}{m}} = \sqrt{\frac{2 \times 2.42 \times 10^{-19}}{9.11 \times 10^{-31}}}$$
$$= 0.7289 \times 10^6 \text{ ms}^{-1}$$

Illustration 3.28 **The stopping potential for photoelectrons emitted from a surface illuminated by light wavelength of 5893 Å is 0.36 V. Calculate the maximum kinetic energy of photoelectrons, the work function of the surface, and the threshold frequency.**

Sol. We know that

$$KE_{max} = hf - \phi = \left(\frac{hc}{\lambda}\right) - \phi$$

or $$\phi = \frac{hc}{\lambda} - KE_{max}$$

Also, $$KE_{max} = eV_s = 0.36 \text{ eV}$$

$\Rightarrow$ $$\phi = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{5893 \times 10^{-10}} - 0.36 \times 1.6 \times 10^{-19}$$
$$= 1.746 \text{ eV}$$

The threshold frequency is given by

$$f_0 = \frac{\phi}{h} = \frac{2.794 \times 10^{-19}}{6.62 \times 10^{-34}} = 4.22 \times 10^{14} \text{ Hz}$$

Illustration 3.29 **When a surface is irradiated with light of wavelength 4950 Å, a photocurrent appears which vanishes if a retarding potential greater than 0.6 V is applied across**

the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. Find the work function of the emitting surface and the wavelength of second source.

Sol. We know that

$$hf_1 = \phi + \frac{1}{2}mv_1^2 \quad \text{and} \quad \frac{1}{2}mv_1^2 = eV_1$$

$$\phi = hf_1 - eV_1 = \frac{hc}{\lambda_1} - eV_1 = \frac{(6.6\times10^{-34})\times(3\times10^8)}{4950\times10^{-10}} - (1.6\times10^{-19})(0.6)$$

$$= 3.04\times10^{-19}\ V;\ hf_2 = \phi + eV_2$$

$$\frac{hc}{\lambda_2} = 3.04\times10^{-19} + (1.6\times10^{-19})\times1.1$$

$$= 4.8\times10^{-19}$$

$$\Rightarrow \quad \lambda_2 = \frac{(6.6\times10^{-34})\times(3\times10^8)}{4.8\times10^{-19}} = 4125\ \text{Å}.$$

Illustration 3.30 **Calculate the velocity of a photoelectron if the work function of the target material is 1.24 eV and the wavelength of incident light is 4.36×10^{-7} m. What retarding potential is necessary to stop the emission of the electrons?**

Sol. We know that

$$\frac{1}{2}mv^2 = hf - \phi \text{ or } \frac{1}{2}mv^2 = \frac{hf}{\lambda} - \phi$$

Subsituting the values, we get $v = 7.43\times10^5\ \text{ms}^{-1}$

Let V_S be the retarding potential required to stop the emission of photoelectrons. Then, we have

$$eV_s = \frac{1}{2}mv^2$$

$$\Rightarrow \quad V_s = \frac{\frac{1}{2}mv^2}{e} = \frac{2.511\times10^{-19}}{1.6\times10^{-19}} = 1.58\ \text{V}.$$

Illustration 3.31 **A beam of light has three wavelengths 4144 Å, 4972 Å, and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beams falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2 s.**

Sol. We know that threshold wavelength $(\lambda_0) = \dfrac{hc}{\phi}$

$$\lambda_0 = \frac{(6.63\times10^{-34})(3\times10^8)}{2.3\times(1.6\times10^{-19})} = 5.404\times10^{-7}\ \text{m}$$

$$= 5404\ \text{Å}$$

Thus, wavelengths 4144 Å and 4972 Å will emit electrons from the metal surface.

Energy incident on the surface per unit time for each wavelength

= Intensity of each wavelength × Area of the surface

$$= \frac{3.6\times10^{-3}}{3}\times(1.0\ \text{cm}^2) = 1.2\times10^{-7}\ \text{W}$$

Energy incident on the surface for each wavelength in 2 s,

$$E = (1.2\times10^{-7})\times(2) = 2.4\times10^{-7}\ \text{J}$$

Number of photons n_1 due to wavelength 4144 Å (= 4144 $\times10^{-10}$ m),

$$n_1 = \frac{(2.4\times10^{-7})(4144\times10^{-10})}{(6.63\times10^{-34})(3\times10^8)} = 0.5\times10^{12}$$

Number of photons n_2 due to wavelength 4972 Å,

$$n_2 = \frac{(2.4\times10^{-7})(4972\times10^{-10})}{(6.63\times10^{-34})(3\times10^8)} = 0.575\times10^{12}$$

$$N = n_1 + n_2 = 0.5\times10^{12} + 0.575\times10^{12}$$

$$= 1.075\times10^{12}$$

Illustration 3.32 **A photocell is operating in saturation mode with a photocurrent 4.8 μA when a monochromatic radiation of wavelength 3000 Å and power 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident to be same for both the cases, calculate,**

a. threshold wavelength for the cell

b. efficiency of photoelectron generation. [(No. of photoelectrons emitted per incident photon) × 100]

c. saturation current in second case

Sol. a. $$K_1 = \frac{12400}{3000} - W = 4.13 - W \quad \text{(i)}$$

$$K_2 = \frac{12400}{1650} - W = 7.51 - W \quad \text{(ii)}$$

Since $V_2 = 2V_1$, so $K_2 = 4K_1$ (iii)

Solving above equations, we get

$$W = 3\ \text{eV}$$

Therefore, threshold wavelength

$$\lambda_0 = \frac{12400}{3} = 4133\ \text{Å}$$

b. Energy of a photon in first case $= \dfrac{12400}{3000} = 4.13$ eV

or $\quad E_1 = 6.6\times10^{-19}$ J

Rate of incident photons (number of photons per second)

$$\frac{P_1}{E_1} = \frac{10^{-3}}{6.6\times10^{-19}} = 1.52\times10^{15} \text{ per second}$$

Number of electrons ejected

$$\frac{4.8\times10^{-6}}{1.6\times10^{-19}} \text{ per second} = 3.0\times10^{13} \text{ per second}$$

Therefore, efficiency of photoelectron generation

$$(\gamma) = \frac{3.0\times10^{13}}{1.52\times10^{15}}\times100 = 1.97\%$$

c. Energy of photon in second case,

$$E_2 = \frac{12400}{1650} = 7.51 \text{ eV} = 12\times10^{-19} \text{ J}$$

Therefore, number of photons incident per second,

$$n_2 = \frac{P_2}{E_2} = \frac{5.0\times10^{-3}}{12\times10^{-19}} = 4.17\times10^{15} \text{ per second}$$

Number of electrons emitted per second is

$$\frac{1.97}{100}\times4.7\times10^{15} = 9.27\times10^{13} \text{ per second}$$

Therefore, saturation current in second case

$$i = (9.27\times10^{13})(1.6\times10^{-19}) \text{ A}$$
$$= 14.8\ \mu\text{A}$$

Concept Application Exercise 3.2

1. Explain briefly, how classical theory could not explain the phenomenon of photoelectric effect.

2. The work function for the following metals is given Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 515 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He–Cd laser placed 1 m away from the photocell? What happens, if the laser is brought nearer and placed 50 cm away?

3. One milliwatt of light of wavelength 4560 Å is incident on a cesium surface. Calculate the photoelectric current liberated assuming a quantum efficiency of 0.5%. Given Planck's constant $h = 6.62\times10^{-34}$ J-s and velocity of light $c = 3\times10^8$ ms^{-1}.

4. Light quanta with energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

5. The maximum KE of photoelectrons emitted from a certain metallic surface is 30 eV when monochromatic radiation of wavelength λ falls on it. When the same surface is illuminated with light of wavelength 2λ, the minimum KE of photoelectrons is found to be 10 eV. (a) Calculate the wavelength λ and (b) determine the maximum wavelength of incident radiation for which photoelectric emission is possible

6. A horizontal photoelectric plate whose work function is 1.9 eV is moved vertically downward at a constant speed v in a room full of radiation of wavelength 250 nm and above. What should be the minimum value of v so that the vertically upward component of velocity of ejected photoelectrons is non-positive?

7. A beam of 1.5 mW of 400 nm light is directed at a photoelectric cell. If 0.1% of the incident photons produce electrons, find the current in the photocell. Assume all the photoelectrons to reach the opposite plate.

8. A uniform monochromatic beam of light of wavelength 365×10^{-9} m and intensity 10^{-8} Wm^{-2} falls on a surface having absorption coefficient 0.8 and work function 1.6 eV. Determine the rate of number of electrons emitted per m^2, power absorbed per m^2 and the maximum kinetic energy of emitted photoelectrons.

Solved Examples

Example 3.1 **Find the frequency of light which ejects electrons from a metal surface. Fully stopped by a retarding potential of 3 V, the photoelectric effect begins in this metal at a frequency of 6×10^{14} Hz. Find the work function for this metal. (Given $h = 6.63\times10^{-34}$ Js).**

Sol. According to Einstein's photoelectric equation,

$$E_k = h\nu - W$$

If V_s is retarding or stopping potential and υ_0 the threshold frequency, then above equation becomes

$$eV_s = h\upsilon - h\upsilon_0$$

or $$h\upsilon = eV_s + h\upsilon_0$$

or $$\upsilon = \frac{eV_s}{h} + \nu_0$$

Hence, $e = 1.6\times10^{-10}$ coulomb, $V_s = 3$ V, and

$$\upsilon_0 = 6\times10^{14} \text{ Hz}$$

Required frequency

$$\upsilon = \frac{1.6\times10^{-10}\times3}{6.63\times10^{-34}} + 6\times10^{14}$$
$$= 7.24\times10^{14} + 6\times10^{14}$$
$$= 13.24\times10^{14} \text{ Hz}$$
$$= 1.324\times10^{15} \text{ Hz}$$

Example 3.2 **Light of wavelength of 2000 Å falls on an aluminium surface. In aluminium, 4.2 eV are required to remove an electron from its surface. What is the kinetic energy, in electron volt, of (a) the fastest, and (b) the slowest emitted photo-electrons. (c) What is the stopping potential? (d) What is the cut-off wavelength for aluminium ? (Planck's constant $h = 6.6 \times 10^{-34}$ Js, and speed of light $c = 3 \times 10^8$ ms^{-1}.**

Sol. Energy corresponding to incident photon

$$h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}}$$

$$= 9.9 \times 10^{-19} \text{ J}$$

$$= \frac{9.9 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV} = 6.2 \text{ eV}$$

a. The kinetic energy of fastest electrons,

$$E_k = h\nu - W$$

or $$E_k = 6.2 \text{ eV} - 4.2 \text{ eV} = 2 \text{ eV}$$

b. The kinetic energy of slowest electrons = zero, since the emitted electrons have all possible energies from 0 to certain maximum value E_k.

c. If V_s is the stopping potential, then

$$E_k = eV_s$$

or $$V_S = \frac{E_k}{e}$$

$$= \frac{2eV}{e} = 2\text{V}$$

d. If λ_0 is the cut-off wavelength for aluminium, then

$$W = (hc/\lambda_0)$$

or $$\lambda_0 = (hc/W)$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 1.6 \times 10^{-19}}$$

$$= 3000 \times 10^{-10} \text{ m} = 3000 \text{ Å}$$

Example 3.3 **When a beam of 10.6 eV photons of intensity 2.0 Wm^{-2} falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons, emitted per second, and their minimum and maximum energies (in eV). Take 1 eV = 1.6×10^{-19} J.**

Sol. Energy incident on the surface per second

$$P = IA$$

$$= 2.0 \times 1.0 \times 10^{-4}$$

$$= 2 \times 10^{-4} \text{ Js}^{-1}$$

Energy of each photon

$$= 10.6 \text{ eV}$$

$$= 10.6 \times 1.6 \times 10^{-19} \text{ J}$$

$\therefore$ Number of photons incident on the surface

$$= \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

Number of photoelectrons emitted

$$= \frac{0.53}{100} \times \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{11}$$

According to Einstein's photoelectric equation, maximum KE of photoelectrons,

$$E_k = \varepsilon - W = 10.6 \text{ eV} - 5.6 \text{ eV}$$

$$= 5 \text{ eV}$$

Minimum kinetic energy of photoelectrons = zero.

Example 3.4 **(i) A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Å. For light of wavelength 3000 Å, the stopping potential is 1.85 V. Find the value of Planck's constant. [1 electron volt (eV) = 1.6×10^{-19} J]**

(ii) At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Å, but the intensity of light increased two times, will photoelectric current be obtained? Give reasons for your answer.

Sol. (i) $$\frac{hc}{\lambda_1} = W + eV_1 \qquad \text{(i)}$$

$$\frac{hc}{\lambda_2} = W + eV_2 \qquad \text{(ii)}$$

Subtracting, we get

$$\frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = e(V_2 - V_1)$$

Here, $c = 3 \times 10^8 \text{ ms}^{-1}$,

$$\lambda_1 = 4000 \text{ Å} = 4 \times 10^{-7} \text{ m}, V_1 = 0.82 \text{ V}$$

$$\lambda_2 = 3000 \text{ Å} = 3 \times 10^{-7} \text{ m}, V_2 = 1.85 \text{ V}$$

$$\therefore \quad h \times 3 \times 10^8 \left(\frac{1}{3 \times 10^{-7}} - \frac{1}{4 \times 10^{-7}}\right)$$

$$= 1.6 \times 10^{-19} (1.85 - 0.82)$$

or $$h \times 3 \times 10^8 \frac{1}{12 \times 10^{-7}} = 1.6 \times 10^{-19} \times 1.03$$

Therefore, Planck's constant,

$$h = \frac{1.6 \times 10^{-19} \times 1.03 \times 12 \times 10^{-7}}{3 \times 10^8}$$

$$= 6.592 \times 10^{-34} \text{ J-s}$$

(ii) No, because stopping potential does not depend on intensity of incident light.

Example 3.5 **A 40 W ultraviolet light source of wavelength 2480 Å illuminates a magnesium (Mg) surface placed 2 m away. Determine the number of photons emitted from the surface per second and the number incident on unit area of Mg surface per second. The photoelectric work function for Mg is 3.68 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Determine the maximum wavelength for which the photoelectric effect can be observed with a Mg surface.**

Sol. Energy of each photon,

$$\varepsilon = \frac{hc}{\lambda}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2480\times10^{-10}}\,\text{J}$$

$$= 8.0\times10^{-19}\,\text{J}$$

Number of photons emitted per second,

$$N = \frac{P}{\varepsilon}$$

$$= \frac{40\ \text{Js}^{-1}}{8.0\times10^{-19}\,\text{J}} = 5.0\times10^{19}\ \text{s}^{-1}$$

These photons spread in all directions over surface area $4\pi r^2$, therefore the number of photons incident per unit area per second,

$$N_i = \frac{N}{4\pi r^2}$$

$$= \frac{5.0\times10^{19}}{4\times3.14\times(2)^2}\quad (\text{As } r = 2\text{ m})$$

$$= 9.95\times10^{13}\ \text{s}^{-1}$$

From Einstein's photoelectric equation,

$$E_k = h\nu - W$$

$$= \varepsilon = h\nu = \frac{hc}{\lambda}$$

$$= 8.0\times10^{-19}\,\text{J}$$

$$= \frac{8.0\times10^{-19}}{1.6\times10^{-19}}\,\text{eV} = 5.0\text{ eV}$$

$$\therefore\quad E_k = 5.0\text{ eV} - 3.68\text{ eV} = 1.32\text{ eV}$$

Threshold (or maximum) wavelength for photoelectrons emission,

$$\lambda_0 = \frac{hc}{W}$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{3.68\times1.6\times10^{-19}}\,\text{m}$$

$$= 3.373\times10^{-3}\text{ m}$$

$$= 3373\text{ Å}$$

Example 3.6 **Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground states is found to liberate electrons with kinetic energies 1.8 eV and 4.0 eV, respectively. Find the value of Planck's constant.** **(IIT-JEE, 1973)**

Sol. The energy of incident photon

$$= (hc/\lambda)$$

If W_i is the ionization energy and E_k the kinetic energy of emitted electron, then we have

$$\frac{hc}{\lambda_1} = W_1 + E_{K_1}$$

Therefore, for incident photon of wavelength $\lambda_1 = 800$ Å $= 8\times10^{-8}$ m

$$\frac{hc}{\lambda} = W_1 + E_{K_1} \qquad \text{(i)}$$

And for incident photon of wavelength

$$\lambda_2 = 700\text{ Å} = 7\times10^{-8}\text{ m},$$

$$\frac{hc}{\lambda} = W_1 + E_{K_2} \qquad \text{(ii)}$$

Subtracting Eqs. (ii) from (i), we get

$$hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = E_{K_2} - E_{K_1}$$

or
$$h = \frac{(E_{K_2} - E_{K_1})\lambda_1\lambda_2}{c(\lambda_1 - \lambda_2)}$$

Here, $E_{K_1} = 1.8\text{ eV} = 1.8\times1.6\times10^{-19}\text{ J}$

and $E_{K_2} = 4.0\text{ eV} = 4.0\times1.6\times10^{-19}\text{ J}$

Substituting given values, we get

$$h = \frac{(4.8-1.8)\times1.6\times10^{-19}\times8\times10^{-8}\times7\times10^{-8}}{3\times10^{-8}(8\times10^{-8} - 7\times10^{-8})}$$

$$= 6.57\times10^{-34}\text{ Js}$$

Example 3.7 **Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of a fastest photoelectron emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find:**

a. the energy of the photons causing the photoelectric emission,

b. the quantum numbers of the two levels involved in the emission of these photons,

c. the change in the angular momentum of the electron in the hydrogen atom in the above transition, and

d. **the recoil speed of the emitting atom assuming it to be at rest before the transition. Neglect momentum of photoelectron in comparision to the momentum of recoiling atom.**

(Ionization potential of hydrogen is 13.6 eV)

Sol. $E_k = 0.73$ eV, $W = 1.82$ eV

Ionization energy of H atom = 13.6 eV

a. $h\upsilon = W + E_k = 1.82 \text{ eV} + 0.73 \text{ eV} = 2.55 \text{ eV}$

b. The electronic energy levels of H atoms are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2}\text{eV}$$

For $n = 1$, $E_1 = -13.6$ eV

For $n = 2$, $E_2 = -3.4$ eV

For $n = 3$, $E_3 = -1.51$ eV

For $n = 4$, $E_4 = -0.85$ eV

Clearly, $E_4 - E_2 = -0.85 \text{ eV} - (-3.4 \text{ eV}) = 2.55$ eV

i.e., Quantum numbers involved in the photons of energy 2.55 eV are 2 and 4. The transition is specified by

$$n_1 = 4 \to n_2 = 2.$$

c. The angular momentum of electron in H atom,

$$J = n\frac{h}{2\pi}$$

For $n = 4$, $J_1 = 4\dfrac{h}{2\pi} = \dfrac{2h}{\pi}$

For $n = 2$, $J_2 = 2\dfrac{h}{2\pi} = \dfrac{h}{\pi}$

Therefore, change in angular momentum,

$$\Delta J = J_1 - J_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

d. According to conservation of momentum,

$$\frac{h\upsilon}{c} + mv = 0$$ [since momentum of photoelectron is negligible]

$$\therefore\ v = -\frac{h\nu}{cm}$$

$$= -\frac{2.55 \text{ eV}}{(3\times10^8 \text{ ms}^{-1})\times(1.67\times10^{-27}\text{ kg})}$$

$$= -\frac{2.55\times1.6\times10^{-19}}{3\times10^8\times1.67\times10^{-27}} = -0.814 \text{ ms}^{-1}$$

Recoil speed of H atom = 0.814 ms^{-1}

Example 3.8 **Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α-particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. [Take $h = 4.14 \times 10^{-15}$ eV s]**

(IIT-JEE, 1999)

Sol. From Einstein's photoelectric equation, maximum kinetic energy of emitted electrons

$$E_k = \frac{hc}{\lambda} - W$$

Given $h = 4.14 \times 10^{-15}$ eV-s

$= 4.14 \times 10^{-15} \times 1.6 \times 10^{-19}$ J-s

$= 6.624 \times 10^{-34}$ J-s

$$\therefore\ E = \frac{hc}{\lambda} = \frac{6.624\times10^{-34}\times3\times10^8}{400\times10^{-9}}$$

$= 4.968 \times 10^{-19}$ J

$$= \frac{4.968\times10^{-19}}{1.6\times10^{-19}} = 3.1 \text{ eV}$$

$E_k = 3.1 \text{ eV} - 1.9 \text{ eV}$

$= 1.2$ eV

$$e + He \to He^+ + \text{Photon}$$

Energy of He atom in their fourth ($n = 5$) excited state

$$E_n = -\frac{Z^2Rhc}{n^2} = -\frac{13.6Z^2}{n^2}\text{eV}$$

$$= -\frac{(2)^2\times13.6}{(5)^2} = -2.176 \text{ eV}$$

(for He^+ ion $Z = 2$)

From conservation of energy,

$$1.2 \text{ eV} + 0 = -2.176 \text{ eV} + E_\gamma$$

Energy of photon during combination,

$$E_\gamma = 1.2 + 2.176 = 3.376 \text{ eV}$$

Energy of Helium ion,

$$E_n = -\frac{Z^2Rhc}{n^2}$$

$$= -\frac{4\times13.6}{n^2}$$

$$= -\frac{54.4}{n^2}\text{eV},\quad n = 1, 2, 3, \ldots$$

$= -54.4$ eV, -13.6 eV, -6.04 eV, -3.4 eV, -2.176 eV, -1.51 eV

Difference of energies lying between 2 and 4 eV is

$$-3.4 + 6.04 = 2.64 \text{ eV}$$

$$-2.176 + 6.04 = 3.86 \text{ eV}$$

Energies of photons emitted are 2.64 eV and 3.86 eV.

EXERCISES

Subjective Type

Solutions on page 3.43

1. In Fig. 3.28, electromagnetic radiations of wavelength 200 nm are incident on a metallic plate A. The photoelectrons are accelerated by a potential difference of 10 V.

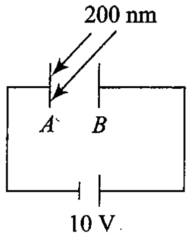


Fig. 3.28

These electrons strike another metal plate B from which electromagnetic radiations are emitted. The minimum wavelength of emitted photons is 100 nm. If the work function of metal A is found to be $(x \times 10^{-1})$ eV, then find the value of x. (Given $hc = 1240$ eV nm)

2. The path of photoelectrons emitted due to electromagnetic radiation incident on a sample of material A, is found to have a maximum bending radius of 0.1 m in a magnetic field of $(\sqrt{2}/3) \times 10^{-4}$ T. When the radiation is incident normally on a double slit having a slit separation of 0.1 mm, it is observed that there are 10 fringes in a width of 3.1 cm on a screen placed at a distance of 1 m from the double slit. Find the work function of the material, and the corresponding threshold wavelength. What should be the wavelength of the incident light so that the bending radius is one-half of what it was before? Given that mass of the electron = 0.5 MeV/c^2, $hc = 12400$ eV-Å.

3. Photons of energy 5 eV are incident on the cathode. Electrons reaching the anode have kinetic energies varying from 6 eV to 8 eV. Find the work function of the metal and state whether the current in the circuit is less than or equal to saturation current.

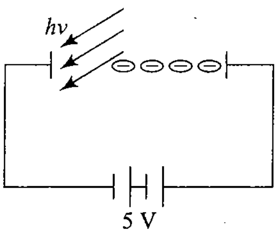


Fig. 3.29

4. A plane light wave of intensity $I = 0.20$ W cm^{-2} falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45°. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.

5. Light of wavelength 180 nm ejects photoelectrons from a plate of a metal whose work function is 2 eV. If a uniform magnetic field of 5×10^{-5} T is applied parallel to the plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

6. When a surface is irradiated with light of wavelength 4950 Å, a photoncurrent appears which vanishes if a retarding potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. Find the work function of the emitting surface and the wavelength of second source. If the photoelectrons (after emission from the surface) are subjected to a magnetic field of 10 T, what changes will be observed in the above two retarding potentials?

7. The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4 V. Find the wavelength of the radiation used. Also, identify the energy levels in hydrogen atom which will emit this wavelength.

8. A vacuum photocell consists of a central cathode and an anode. The internal surface is silver of work function 4.5 eV. The contact potential difference between the electrodes equals to 0.6 V. The photocell is illuminated by light of wavelength 2.3×10^{-7} m.
   a. What retarding potential difference should be applied between electrodes of the photocell for the photocurrent to drop to zero?
   b. If a retarding potential of 1 V is applied between electrodes at what limiting wavelength λ of light incident on the cathode will the photoelectric effect begin?

9. The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit of hydrogen atom falls on a metal to produce photoelectrons. The electrons emitted from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of 1/320 T in a radius of 10^{-3} m. Find:
   (i) the kinetic energy of the electrons,
   (ii) work function of metal, and
   (iii) wavelength of radiation. **(Roorkee, 1996)**
   (Planck's constant $h = 6.62 \times 10^{-34}$ J-s)

10. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find (a) the number of photons absorbed per second by the surface and (b) the force exerted by the light beam on the surface.

11. A uniform monochromatic beam of light of wavelength 365×10^{-9} m and intensity 10^{-8} Wm^{-2} falls on a surface having absorption coefficient 0.8 and work function 1.6 eV. Determine the number of electrons emitted per m^2 per s, power absorbed per m^2, and the maximum kinetic energy of emitted photo electrons.

12. Calculate de Broglie wavelength for an average helium atom in a furnace at 400 K. Given mass of helium = 4.002 amu.

13. What amount of energy should be added to an electron to reduce its de Broglie wavelength from 100 to 50 pm?

14. A surface has light of wavelength $\lambda_1 = 550$ nm incident on it, causing the ejection of photoelectrons for which the stopping potential is $V_{s_1} = 0.19$ V. If the radiation of wavelength $\lambda_2 = 190$ nm is now incident on the surface, calculate the stopping potential V_{s_1}, (b) the work function of the surface, and (c) the threshold frequency for the surface.

15. A cylindrical rod of some laser material 5×10^{-2} m long and 10^{-2} m in diameter contains 2×10^{25} ions per m^3. If on excitation all the ions are in the upper energy level and de-excite simultaneously emitting photons in the same direction, calculate the maximum energy contained in a pulse of radiation of wavelength 6.6×10^{-7} m. If the pulse lasts for 10^{-7} s, calculate the average power of the laser during the pulse.

16. A 100 W sodium lamp is radiating light of wavelength 5890 Å, uniformly in all directions,

(i) At what rate, photons are emitted from the lamp?

(ii) At what distance from the lamp, the average flux is 1 photon (cm^2-s)$^{-1}$?

(iii) What are the photon flux and photon density at 2 m from the lamp?

Objective Type

Solutions on page 3.47

1. The minimum orbital angular momentum of the electron in a hydrogen atom is

a. h **b.** h/2 **c.** $h/2\pi$ **d.** h/λ

2. An electron beam accelerated from rest through a potential difference of 5000 V in vacuum is allowed to impinge on a surface normally. The incident current is 50 μA and if the electrons come to rest on striking the surface the force on it is

a. 1.1924×10^{-8} N **b.** 2.1×10^{-8} N **c.** 1.6×10^{-8} N **d.** 1.6×10^{-6} N

3. 10^{-3} W of 5000 Å light is directed on a photoelectric cell. If the current in the cell is 0.16 μA, the percentage of incident photons which produce photoelectrons, is

a. 40% **b.** 0.04% **c.** 20% **d.** 10%

4. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is

a. 6λ **b.** $4\lambda/3$ **c.** 4λ **d.** 8λ

5. A particle of mass 10^{-31} kg is moving with a velocity equal to 10^5 ms^{-1}. The wavelength of the particle is equal to

a. 0 **b.** 6.6×10^{-8} m **c.** 0.66 m **d.** 1.5×10^7 m

6. Threshold frequency for a certain metal is v_0. When light of frequency $2v_0$ is incident on it, the maximum velocity of photoelectrons is 4×10^8 cms^{-1}. If frequency of incident radiation is increased to $5v_0$, then the maximum velocity of photoelectrons, in cm s^{-1}, will be

a. $(4/5) \times 10^8$ **b.** 2×10^8 **c.** 8×10^8 **d.** 20×10^8

7. Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.04 μm from a mercury vapor lamp, the stopping potential is 1.5 V. Then, the work function [in electron volts] of the photocell surface is

a. 0.75 eV **b.** 1.5 eV **c.** 3 eV **d.** 2.5 eV

8. Ultraviolet light of wavelength 300 nm and intensity 1.0 Wm^{-2} falls on the surface of a photosensitive material. If one per cent of the incident photons produce photoelectrons, then the number of photoelectrons emitted per second from an area of 1.0 cm^2 of the surface is nearly

a. 9.61×10^{14} s^{-1} **b.** 4.12×10^{13} s^{-1} **c.** 1.51×10^{12} s^{-1} **d.** 2.13×10^{11} s^{-1}

9. In a photocell, with excitation wavelength λ, the faster electron has speed v. If the excitation wavelength is changed to $3\lambda/4$, the speed of the fastest electron will be

a. $v(3/4)^{1/2}$ **b.** $v(4/3)^{1/2}$ **c.** less than $v(4/3)^{1/2}$ **d.** greater than $v(4/3)^{1/2}$

10. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are v_1 and v_2, respectively, then

a. $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$ **b.** $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

c. $v_1 + v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$ **d.** $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 - f_2)$

11. If the intensity of radiation incident on a photocell be increased four times, then number of photoelectrons and energy of photoelectrons emitted respectively become

a. four times, doubled
b. doubled, remains unchanged
c. remains unchanged, doubled
d. four times, remains unchanged

12. The work function of a metal is W and λ is the wavelength of the incident radiation. There is no emission of photoelectrons when

a. $\lambda > hc/W$ **b.** $\lambda = hc/W$ **c.** $\lambda < hc/W$ **d.** $\lambda \le hc/W$

13. If a surface has work function of 3.00 eV, the longest wavelength of light which will cause the emission of electrons is

a. 4.8×10^{-7} m **b.** 5.99×10^{-7} m **c.** 4.13×10^{-7} m **d.** 6.84×10^{-7} m

14. In the experiment on photoelectric effect, the graph between E_K(max) and v is found to be a straight line as shown in Fig. 3.30.

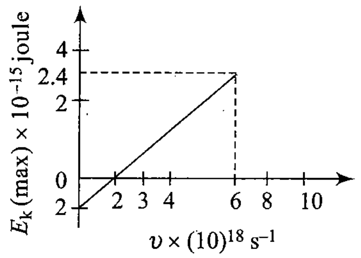


Fig. 3.30

The threshold frequency and the Planck's constant according to this graph are

a. 3.33×10^{18} s^{-1}, 6×10^{-34} J-s
b. 6×10^{18} s^{-1}, 6×10^{-34} J-s
c. 2.66×10^{18} s^{-1}, 4×10^{-34} J-s
d. 4×10^{18} s^{-1}, 3×10^{-34} J-s

15. Monochromatic light incident on a metal surface emits electrons with kinetic energies from zero to 2.6 eV. What is the least energy of the incident photon if the tightly bound electron needs 4.2 eV to remove?

a. 1.6 eV **b.** From 1.6 eV to 6.8 eV
c. 6.8 eV **d.** More than 6.8 eV

16. The work function for sodium surface is 2.0 eV and that for aluminium surface is 4.2 eV. The two metals are illuminated with appropriate radiations so as to cause photoemission. Then

a. the threshold frequency for sodium will be less than that for aluminium
b. the threshold frequency of sodium will be more than that of aluminium
c. both sodium and aluminium will have same threshold frequency
d. none of the above

17. A metal surface is illuminated by a light of given intensity and frequency to cause photoemission. If the intensity of illumination is reduced to one-fourth of its original value, then the maximum KE of emitted photoelectrons will become

a. $(1/16)^{th}$ of original value
b. unchanged
c. twice the original value
d. four times the original value

18. Representing the stopping potential V along y-axis and $(1/\lambda)$ along x-axis for a given photocathode, the curve is a straight line, the slope of which is equal to

a. e/hc **b.** hc/e
c. ec/h **d.** he/c

19. In Q. 18, the intercept on the y-axis is equal to

a. $+W/e$ **b.** $-W/e$
c. $-We$ **d.** e/W

20. Electrons traveling at a velocity of 2.4×10^6 ms^{-1} enter a region of crossed electric and magnetic fields shown in Fig. 3.31. If the electric field is 3.0×10^6 Vm and the flux density of the magnetic field is 1.5 T, the electrons upon entering the region of the crossed fields will

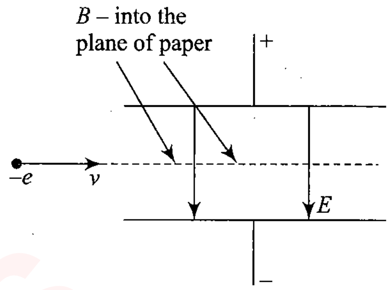


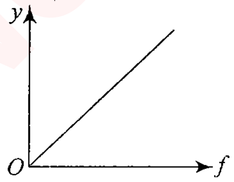
Fig. 3.31

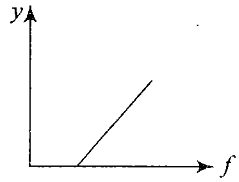
a. continue to travel undeflected in their original direction
b. be deflected upward in the plane of the diagram
c. be deflected downward on the plane of the diagram
d. none of the above

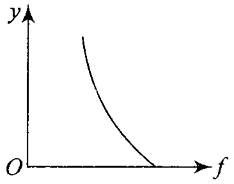
21. A surface irradiated with light λ = 480 nm gives out electrons with maximum velocity v m/s, the cut off wavelength being 600 nm. The same surface would release electrons with maximum velocity $2v$ m/s if it is irradiated by light of wavelength.

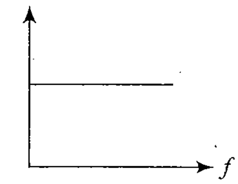
a. 325 nm **b.** 360 nm
c. 384 nm **d.** 300 nm

22. In an experiment on the photoelectric effect, an evacuated photocell with a pure metal cathode is used. Which graph best represents the variation of V, the minimum potential difference need ' to prevent current from flowing, when x, the frequency of the incident light, is varied ?

a.


b.


c.


d.


23. A metal surface in an evacuated tube is illuminated with monochromatic light causing the emission of photoelectrons which are collected at an adjacent electrode. For a given intensity of light, the way in which the photocurrent I depends in the potential difference V between the electrodes is shown by approximate graph in Fig. 3.32.

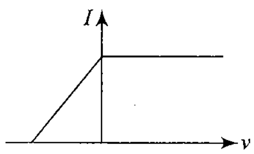
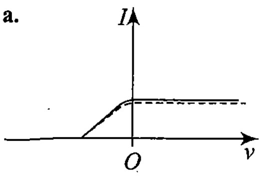
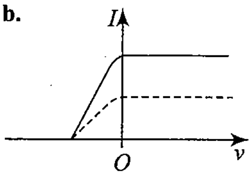
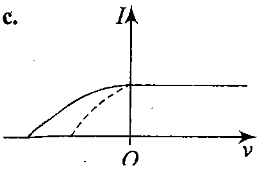


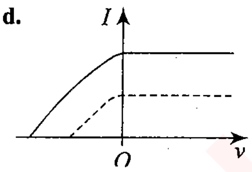
Fig. 3.32

If the experiment were repeated with light of twice the intensity but the same wavelength, which of the graphs below would best represent the new relation between I and V? (In these graphs, the result of the original experiment is indicated by a broken line.)

a.


b.


c.


d.


24. If stopping potentials corresponding to wavelengths 4000 Å and 4500 Å are 1.3 V and 0.9 V, respectively, then the work function of the metal is

a. 0.3 eV **b.** 1.3 eV
c. 2.3 eV **d.** 5 eV

25. The photoelectric threshold of a certain metal is 3000 Å. If the radiation of 2000 Å is incident on the metal

a. electrons will be emitted
b. positrons will be emitted
c. protons will be emitted
d. electrons will not be emitted

26. The frequency and the intensity of a beam of light falling on the surface of a photoelectric material are increased by a factor of two. This will

a. increase the maximum kinetic energy, the photoelectrons, as well as photoelectric current by a factor of 2
b. increase the maximum kinetic energy of the photoelectron and would increase the photoelectric current by a factor of 2
c. increase the maximum kinetic energy of the photoelectrons by a factor of 2 and will have no effect on the magnitude of the photoelectric current produced
d. not produce any effect on the kinetic energy of the emitted electrons but will increase the photoelectric current by a factor of 2

27. The frequency of incident light falling on a photosensitive metal plate is doubled, the KE of the emitted photoelectrons is

a. double the earlier value **b.** unchanged
c. more than doubled **d.** less than doubled

28. Lights of two different frequencies whose photons have energies 1 and 2.5 eV, respectively, successively illuminate a metal whose work function is 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be

a. 1:5 **b.** 1:4
c. 1:2 **d.** 1:1

29. A proton when accelerated through a potential difference of V volt has a wavelength λ associated with it. An α-particle in order to have the same λ must be accelerated through a potential difference of

a. V volt **b.** $4V$ volt
c. $2V$ volt **d.** $(V/8)$ volt

30. Given that a photon of light of wavelength 10,000 Å has an energy equal to 1.23 eV. When light of wavelength 5000 Å and intensity I_0 falls on a photoelectric cell, the saturation current is 0.40×10^{-6} A and the stopping potential is 1.36 V; then the work function is

a. 0.43 eV **b.** 1.10 eV
c. 1.36 eV **d.** 2.47 eV

31. In the previous question, if the intensity of light is made $4I_0$, then the stopping potential will become

a. 1.36×1 V **b.** 1.36×2 V
c. 1.36×3 V **d.** 1.36×4 V

32. In Q. 30, if the intensity of light is made $4I_0$, then the saturation current will become

a. 0.40×1 μA **b.** 0.40×2 μA
c. 0.40×4 μA **d.** 0.40×8 μA

33. In Q. 30, if the cathode and the anode are kept at the same potential, the emitted electrons

a. all have the same KE equal to 1.36 eV
b. all have the average KE equal to (1.36/2) eV
c. all have the maximum KE equal to 1.36 eV
d. all have the minimum KE equal to 1.36 eV

34. In Q. 30, if the wavelength is changed to 4000 Å, then stopping potential will become

a. 1.36 V **b.** 3.40 V
c. 1.60 V **d.** 1.97 V

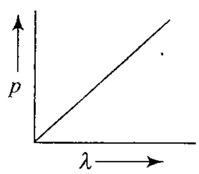
35. A cesium photocell, with a steady potential difference of 60 V across it, is illuminated by a small bright light placed 1 m away. When the same light is placed 2 m away, the electrons crossing the photocell

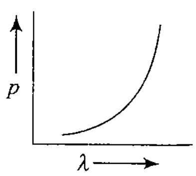
a. each carry one-quarter of their previous momentum
b. each carry one-quarter of their pervious energy
c. are one-quarter as numerous
d. are half as numerous

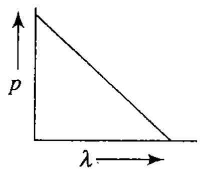
36. If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? Assume wavelength of visible light as 5.6×10^{-5} cm.

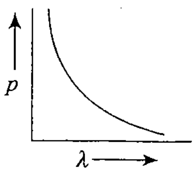
a. 1.4×10^{19} **b.** 3×10^3
c. 1.4×10^{-19} **d.** 3×10^4

37. Which of the following graphs correctly represents the variation of particle momentum with associated de Broglie wavelength ?

a.


b.


c.


d.


38. Five volt of stopping potential is needed for the photoelectrons emitted out of a surface of work function 2.2 eV by the radiation of wavelength

a. 1719 Å **b.** 3444 Å
c. 861 Å **d.** 3000 Å.

39. The work functions for tungsten and sodium are 4.5 eV and 2.3 eV, respectively. If the threshold wavelength λ for sodium is 5460 Å, the value of λ for tungsten is

a. 5893 Å **b.** 10683 Å
c. 2791 Å **d.** 528 Å.

40. In a series of photoelectric emission experiments on a certain metal surface, possible relationships between the following quantities were investigated: threshold frequency f_0, frequency of incident light f, light intensity P, photocurrent I, maximum kinetic energy of photoelectrons T_{max}. Two of these quantities, when plotted as a graph of y against x, give a straight line through the origin.
Which of the following correctly identifies x and y with the photoelectric quantities ?

x	y
a. I	f_0
b. f	f_0
c. P	I
d. P	T_{max}

41. A point source causes photoelectric effect from a small metal plate. Which of the curves in Fig. 3.33 may represent the saturation photo-current as a function of the distance between the source and the metal?

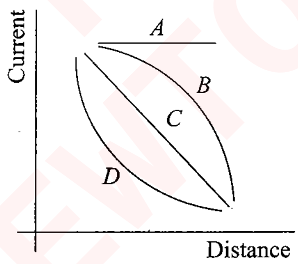


Fig. 3.33

a. A **b.** B
c. C **d.** D

42. Let p and E denote the linear momentum and energy, respectively, of a photon. If the wavelength is decreased,

a. both p and E increase
b. p increases and E decreases
c. p decreases and E increases
d. both p and E decrease

43. Light from a hydrogen discharge tube is incident on the cathode of a photoelectric cell. The work function of the cathode surface is 4.2 eV. In other to reduce the photocurrent to zero, the voltage of the anode relative to the cathode must be made

a. –4.2 V **b.** –9.4 V
c. –17.8 V **d.** +9.4 V

44. The photoelectric threshold for some material is 200 nm. The material is irradiated with radiations of wavelength 40 nm. The maximum kinetic energy of the emitted photoelectrons is

a. 2 eV **b.** 1 eV
c. 0.5 eV **d.** none of these

45. The human eye can barely detect a yellow light ($\lambda = 6000$ Å) that delivers 1.7×10^{-18} W to the retina. The number of photons per second falling on the eye is nearest to

a. 5×10^9 **b.** 5000
c. 50 **d.** 5

46. X-rays are used to irradiate sodium and copper surfaces in two separate experiments and stopping potentials are determined. The stopping potential is

a. equal in both cases **b.** greater for sodium
c. greater for copper **d.** infinite in both cases

47. An electron is accelerated through a potential difference of 200 V. If e/m for the electron be 1.6×10^{11} coulomb kg^{-1}, then the velocity acquired by the electron will be

a. 8×10^6 ms^{-1} **b.** 8×10^5 ms^{-1}
c. 5.9×10^6 ms^{-1} **d.** 5.9×10^5 ms^{-1}.

48. A photoelectric cell is connected to a source of variable potential difference, connected across it and the photoelectric current resulting (μA) is plotted against the applied potential difference (V). The graph in the broken line represents one for a given frequency and intensity of the incident radiation. If the frequency is increased and the intensity is reduced, which of the following graphs of unbroken line represents the new situation?

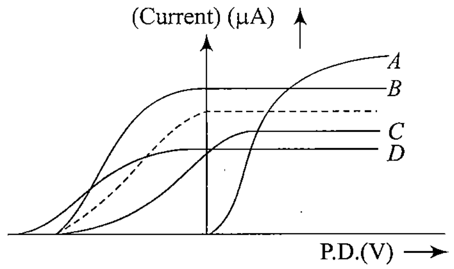


Fig. 3.34

a. A **b.** B
c. C **d.** D

49. Photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photoelectrons come out with velocity

a. 10 ms^{-1} **b.** 10^3 ms^{-1}
c. 10^4 ms^{-1} **d.** 10^6 ms^{-1}

50. If a surface has a work function 4.0 eV, what is the maximum velocity of electrons liberated from the surface when it is irradiated with ultraviolet radiation of wavelength 0.2 μm?

a. 4.4×10^5 m s^{-1} **b.** 8.8×10^7 m s^{-1}
c. 8.8×10^5 m s^{-1} **d.** 4.4×10^7 m s^{-1}

51. An image of the Sun is formed by a lens, of focal length of 30 cm, on the metal surface of a photoelectric cell and a photoelectric current I is produced. The lens forming the image is then replaced by another of the same diameter but of focal length 15 cm. the photoelectric current in this case is

a. $\frac{I}{2}$ b. I
c. $2I$ d. $4I$

52. A modern 200 W sodium street lamp emits yellow light of wavelength 0.6 μm. Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is

a. 62×10^{20} b. 3×10^{19}
c. 1.5×10^{20} d. 6×10^{18}

53. The work function of a metallic surface is 5.01 eV. The photoelectrons are emitted when light of wavelength 2000 Å falls on it. The potential difference applied to stop the fastest photoelectrons is [$h = 4.14 \times 10^{-15}$ eVs]

a. 1.2 V b. 2.24 V
c. 3.6 V d. 4.8 V

54. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de Broglie wavelength associated with an electron to that associated with proton is

a. 1 b. m_p/m_e
c. m_e/m_p d. $\sqrt{m_p/m_e}$

55. A material particle with a rest mass m_o is moving with a velocity of light c. Then, the wavelength of the de Broglie wave associated with it is

a. (h/m_oc) b. zero
c. ∞ d. (m_oc/h)

56. If a photocell is illuminated with a radiation of 1240 Å, then stopping potential is found to be 8 V. The work function of the emitter and the threshold wavelength are

a. 1 eV, 5200 Å b. 2 eV, 6200 Å
c. 3 eV, 7200 Å d. 4 eV, 4200 Å

57. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 mm is incident upon it, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 mm is incident upon silver?

a. 1.5 V b. 3.85 V
c. 2.35 V d. 15.4 V

58. Out of a photon and an electron, the equation $E = pc$, is valid for

a. both b. neither
c. photon only d. electron only

59. When a centimeter thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. Threshold wavelength for the metallic surface is

a. $4\lambda/3$ b. 4λ
c. 6λ d. $8\lambda/3$

60. Light of wavelength λ strikes a photoelectric surface and electrons are ejected with kinetic energy K. If K is to be increased to exactly twice its original value, the wavelength must be changed to λ' such that

a. $\lambda' < \lambda/2$ b. $\lambda' > \lambda/2$
c. $\lambda > \lambda' > \lambda/2$ d. $\lambda' = \lambda/2$

61. The ratio of momenta of an electron and an α-particle which are accelerated from rest by a potential difference of 100 V is

a. 1 b. $\sqrt{2m_e/m_\alpha}$
c. $\sqrt{m_e/m_\alpha}$ d. $\sqrt{m_e/2m_\alpha}$

62. The kinetic energy of an electron is E when the incident wavelength is λ. To increase the KE of the electron to $2E$, the incident wavelength must be

a. 2λ b. $\lambda/2$
c. $(hc\lambda)/(E\lambda + hc)$ d. $(hc\lambda)/(E\lambda + hc)$

63. The KE of the photoelectrons is E when the incident wavelength is $\lambda/2$. The KE becomes $2E$ when the incident wavelength is $\lambda/3$. The work function of the metal is

a. hc/λ b. $2hc/\lambda$
c. $3hc/\lambda$ d. $hc/3\lambda$

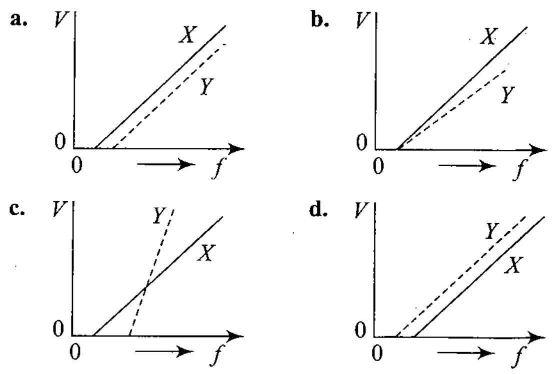
64. If λ_0 stands for mid-wavelength in the visible region, the de Broglie wavelength for 100 V electrons is nearest to

a. $\lambda_0/5$ b. $\lambda_0/50$
c. $\lambda_0/500$ d. $\lambda_0/5000$

65. The threshold frequency for certain metal is v_0. When light of frequency $2v_0$ is incident on it, the maximum velocity of photoelectrons is 4×10^6 ms^{-1}. If the frequency of incident radiation is increased to $5v_0$, then the maximum velocity of photoelectrons will be

a. $4/5 \times 10^6$ m s^{-1} b. 2×10^6 m s^{-1}
c. 8×10^6 m s^{-1} d. 2×10^7 m s^{-1}

66. In a photoelectric emission, electrons are ejected from metals X and Y by light of frequency f. The potential difference V required to stop the electrons is measured for various frequencies. If Y has a greater work function than X, which graph illustrates the expected results ?



67. In a photoelectric cell, the wavelength of incident light is changed from 4000 Å to 3600 Å. The change in stopping potential will be

a. 0.14 V b. 0.24 V
c. 0.35 V d. 0.44 V

68. When a metallic surface is illuminated by a light of frequency 8×10^{14} Hz, photoelectron of maximum energy 0.5 eV is emitted. When the same surface is illuminated by light of frequency 12×10^{14} Hz, photoelectron of maximum energy 2 eV is emitted. The work function is

a. 0.5 eV **b.** 2.85 eV
c. 2.5 eV **d.** 3.5 eV

69. The de Broglie wavelength of neutrons in thermal equilibrium is (Given $m_n = 1.6 \times 10^{-27}$ kg)

a. $30.8/\sqrt{T}$ Å **b.** $3.08/\sqrt{T}$ Å
c. $0.308/\sqrt{T}$ Å **d.** $0.0308/\sqrt{T}$ Å

70. A particle of mass M at rest decays into two masses m_1 and m_2 with non-zero velocities. The ratio λ_1/λ_2 of de Broglie wavelengths of the particles is

a. m_2/m_1 **b.** m_1/m_2
c. $\sqrt{m_1}/\sqrt{m_2}$ **d.** 1 : 1

71. What is the energy of a proton possessing wavelength 0.4 Å?

a. 0.51 eV **b.** 1.51 eV
c. 10.51 eV **d.** 100.51 eV

72. An electron and a photon possess the same de Broglie wavelength. If E_e and E_{ph} are, respectively, the energies of electron and photon while v and c are their respective velocities, then E_e/E_{ph} =

a. v/c **b.** $v/2c$
c. $v/3c$ **d.** $v/4c$

73. In Q. 72, if the velocity of electron is 25% of the velocity of photon, then E_e/E_{ph} equals

a. 1:2 **b.** 1:4
c. 1:8 **d.** 1:16.

74. An electron and a photon, each has a wavelength of 1.2 Å. What is the ratio of their energies ?

a. 1:10 **b.** 1:10^2
c. 1:10^3 **d.** 1:10^4.

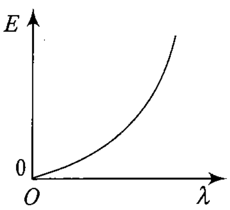
75. What is the wavelength of a photon of energy 1eV ?

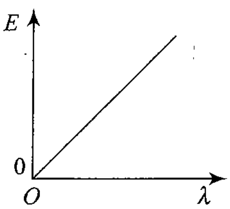
a. 12.4×10^3 Å **b.** 2.4×10^3 Å
c. 0.4×10^2 Å **d.** 1000 Å.

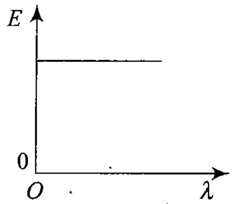
76. If λ_1 and λ_2 denote the wavelengths of de Broglie waves for electrons in the first and second Bohr orbits in a hydrogen atom, then λ_1/λ_2 is equal to

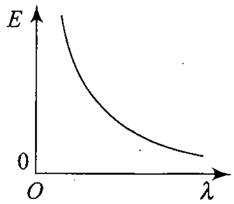
a. 2/1 **b.** 1/2
c. 1/4 **d.** 4/1

77. Which curve shows the relationship between the energy E and the wavelength λ of a photon of electromagnetic radiation ?

a.


b.


c.


d.


78. Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 200 Å is incident on it, electons are emitted. What will be the maximum velocity of emitted electrons ?

a. 3×10^8 m s^{-1} **b.** 6.46×10^5 m s^{-1}
c. 10.36×10^5 m s^{-1} **d.** 8.54×10^6 m s^{-1}

79. An electron is accelerated through a potential difference of V volt. It has a wavelength λ associated with it. Through what potential difference an electron must be accelerated so that its de Broglie wavelength is the same as that of a proton? Take mass of proton to be 1837 times larger than the mass of electron.

a. V volt **b.** 1837 V volt
c. V/1837 volt **d.** $\sqrt{1837}\ V$ volt

80. The kinetic energy of most energetic electrons emitted from a metallic surface is doubled when the wavelength λ of the incident radiation is changed from 400 nm to 310 nm. The work function of the metal is

a. 0.9 eV **b.** 1.7 eV
c. 2.2 eV **d.** 3.1 eV

81. How many photons are emitted per second by a 5 mW laser source operating at 632.8 nm?

a. 1.6×10^{16} **b.** 1.6×10^{13}
c. 1.6×10^{10} **d.** 1.6×10^3

82. Figure 3.35 shows the plot of the stopping potential versus the frequency of the light used in an experiment on photoelectric effect. The ratio h/e is

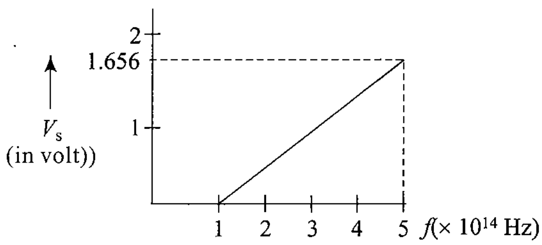


Fig. 3.35

a. 10^{-15} V s^{-15} V s **b.** 2×10^{-15} V s
c. 3×10^{-15} V s **d.** 4.14×10^{-15} V s

83. In the previous question, the work function is

a. 0.212 eV **b.** 0.313 eV
c. 0.414 eV **d.** 0.515 eV

84. Two identical metal plates show photoelectric effect. Light of wavelength λ_A falls on plate A and λ_B falls on plate B, $\lambda_A = 2\lambda_B$. The maximum KE of the photoelectrons are

K_A and K_B, respectively. Which one of the following is true?

a. $2K_A = K_B$ b. $K_A = 2K_B$
c. $K_A < K_B/2$ d. $K_A > 2K_B$

85. The potential difference applied to an X-ray tube is V. The ratio of the de Broglie wavelength of electron to the minimum wavelength of X-ray is directly proportional to

a. V b. $\sqrt{V}$
c. $V^{3/2}$ d. $V^{7/2}$

86. The maximum velocity of electrons emitted from a metal surface is v. What would be the maximum velocity if the frequency of incident light is increased by a factor of 4 ?

a. $2v$ b. $> 2v$
c. $< 2v$ d. between $2v$ and $4v$.

87. A homogeneous ball (mass = m) of ideal black material at rest, is illuminated with a radiation having a set of photons (wavelength = λ) each with same momentum and same energy. The rate at which photons fall on the ball is n. The linear acceleration of the ball is

a. $m\lambda/nh$ b. $nh/m\lambda$
c. $nh/(2\pi)(m\lambda)$ d. $2pm\lambda/nh$

88. The eye can detect 5×10^4 photons $(m^2s)^{-1}$ of green light ($\lambda = 5000$ Å), while ear can detect 10^{-13} W m^{-2}. As a power detector, which is more sensitive and by what factor?

a. Eye is more sensitive and by a factor of 5.00
b. Ear is more sensitive by a facto. of 5.00
c. Both are equally sensitive
d. Eye is more sensitive by a factor of 10^{-1}

89. A photon of wavelength 0.1 Å is emitted by a helium atom as a consequence of the emission of photon. The KE gained by helium atom is

a. 0.05 eV b. 1.05 eV
c. 2.05 eV d. 3.05 eV

90. A monochromatic source of light is placed at a large distance d from a metal surface. Photoelectrons are ejected at rate n, the kinetic energy being E. If the source is brought nearer to distance $d/2$, the rate and kinetic energy per photoelectron become nearly

a. $2n$ and $2E$ b. $4n$ and $4E$
c. $4n$ and E d. n and $4E$

91. How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of 6.62×10^{-5} N?

a. 3×10^{19} b. 5×10^{22}
c. 2×10^{22} d 1.67×10^{18}

92. A nozzle throws a stream of gas against a wall with a velocity v much larger than the thermal agitation of the molecules. The wall deflects the molecules without changing the magnitude of their velocity. Also, assume that the force exerted on the wall by the molecules is perpendicular to the wall. (This is not strictly true for a rough wall). Find the force exerted on the wall.

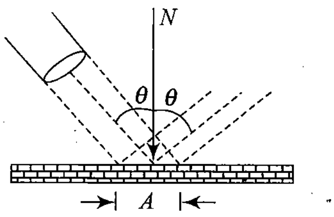


Fig. 3.36

a. $Anmv^2 \cos^2\theta$ b. $2Anmv^2 \cos^2\theta$
c. $2Anmv^2 \sin^2\theta$ d. $Anmv^2 \cos\theta$

93. A plane light wave of intensity $I = 0.20$ W cm^{-2} falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45°. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.

a. 1.2 N cm^{-2} b. 0.2 N cm^{-2}
c. 2.6 N cm^{-2} d. 0.5 N cm^{-2}

94. A plane wave of intensity $I = 0.70$ W cm^{-2} illuminates a sphere with ideal mirror surface. The radius of sphere is $R = 5.0$ cm. From the standpoint of photon theory, find the force that light exerts on the sphere.

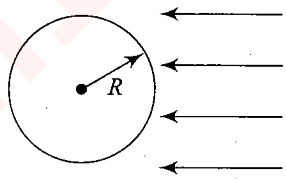


Fig. 3.37

a. 0.8 μN b. 0.2 μN
c. 0.5 μN d. 1.2 μN

95. Light of wavelength λ from a small 0.5 mW He–Ne laser source, used in the school laboratory, shines from a spacecraft of mass 1000 kg. Estimate the time needed for the spacecraft to reach a velocity of 1.0 km s^{-1} from rest. The momentum p of a photon of wavelength λ is given by $p = h/\lambda$, where h is Planck's constant.

a. 6×10^{18} b. 3×10^{17}
c. 6×10^{17} d. 2×10^{15}

96. What is the de Broglie wavelength of the wave associated with an electron that has been accelerated through a potential difference of 50.0 V?

a. 2.7×10^{-10} b. 1.74×10^{-10}
c. 3.6×10^{-9} d. 4.9×10^{-11}

97. An α-particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The α-particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.

a. 2:3 b. 3:4
c. 5:7 d. 1:2

98. Find the ratio of de Broglie wavelength of a proton and an α-particle which have been accelerated through same potential difference.

a. $2\sqrt{2}:1$ b. 3:2

c. $3\sqrt{2}:1$ d. 2:1

99. All electrons ejected from a surface by incident light of wavelength 200 nm can be stopped before traveling 1 m in the direction of a uniform electric field of 4 NC^{-1}. The work function of the surface is

a. 4 eV b. 6.2 eV
c. 2 eV d. 2.2 eV

100. If the short wavelength limit of the continous spectrum coming out of a Coolidge tube is 10 Å, then the de Broglie wavelength of the electrons reaching the target metal in the Coolidge tube is approximately

a. 0.3 Å b. 3 Å
c. 30 Å d. 10 Å

101. In a photoelectric effect, electrons are emitted

a. with a maximum velocity proportional to the frequency of the incident radiation
b. at a rate that is independent of the intensity of the incident radiation
c. only if the frequency of the incident radiation is above a certain threshold value
d. only if the temperature of the emitter is high

102. A particle of mass 'm' is projected from ground with velocity 'u' making angle 'θ' with the vertical. The de Broglie wavelength of the particle at the highest point is

a. ∞ b. $h/mu \sin\theta$
c. $h/mu \cos\theta$ d. h/mu

103. A 60 W bulb is placed at a distance of 4 m from you. The bulb is emitting light of wavelength 600 nm uniformly in all directions. In 0.1 s, how many photons enter your eye if the pupil of the eye is having a diameter of 2 mm? [Take hc = 1240 eV-nm]

a. 2.84×10^{12} b. 2.84×10^{11}
c. 9.37×10^{11} d. 6.48×10^{11}

104. Two identical non-relativistic particles A and B move at right angles to each other, processing de Broglie wavelengths λ_1 and λ_2, respectively. The de Broglie wavelength of each particle in their centre of mass frame of reference is

a. $\lambda_1 + \lambda_2$ b. $2\lambda_1\lambda_2/\left(\sqrt{\lambda_1^2+\lambda_2^2}\right)$

c. $\lambda_1\lambda_2/\left(\sqrt{|\lambda_1^2+\lambda_2^2|}\right)$ d. $(\lambda_1+\lambda_2)/2$

105. A photosensitive material is at 9 m to the left of the origin and the source of light is at 7 m to the right of the origin along x-axis. The photosensitive material and the source of light start from rest and move, respectively, with $8\hat{i}$ ms^{-1} and $4\hat{i}$ ms^{-1}. The ratio of intensities at $t = 0$ to $t = 3$ s as received by the photosensitive material is

a. 16 : 1 b. 1 : 16
c. 2 : 7 d. 7 : 2

106. A sodium metal piece is illuminated with light of wavelength 0.3 μm. The work function of sodium is 2.46 eV. For this situation, mark out the correct statement(s).

a. The maximum kinetic energy of the ejected photoelectrons is 1.68 eV
b. The cut-off wavelength for sodium is 505 nm
c. The minimum photon energy of incident light for photoelectric effect to take place is 2.46 eV
d. All of the above

107. The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5% of the speed of light. The ratio of the photon wavelength to the de Broglie wavelength of the particle is [No need to use relativistic formula for the particle.]

a. 40 b. 4
c. 2 d. 80

108. The resolving power of an electron microscope operated at 16 kV is R. The resolving power of the electron microscope when operated at 4 kV is

a. $R/4$ b. $R/2$
c. $4R$ d. $2R$

109. With respect to Electromagnetic Theory of Light, the photoelectric effect is best explained by statement

a. Light waves carry energy and when light is incident on the metallic surface, the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy when ejected
b. Particles of light (photons) collide with the metal and the electrons take this energy and may eject
c. When light waves fall on a metallic surface, the stability of atoms is disturbed and the electrons come out to make the system stable
d. None of the above

110. Two electrons are moving with same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after some time de Broglie wavelengths of two are λ_1 and λ_2, respectively. Now,

a. $\lambda_1 = \lambda_2$ b. $\lambda_1 > \lambda_2$
c. $\lambda_1 < \lambda_2$
d. λ_1 can be greater than or less than l_2

111. The energy of a photon is equal to the kinetic energy of a proton. The energy of photon is E. Let λ_1 be the de Broglie wavelength of the proton and λ_2 be the wavelength of the photon. Then, λ_1/λ_2 is proportional to

a. E^0 b. $E^{1/2}$
c. E^{-1} d. E^{-2}

112. Upto what potential V can a zinc ball (work function 3.74 eV) removed from other bodies be charged by irradiating it with light of $\lambda = 200$ nm?

a. 2.5 V b. 1.8 V
c. 2.2 V d. 3 V

113. A sensor is exposed for time t to a lamp of power P placed at a distance ℓ. The sensor has an opening that is $4d$ in diameter. Assuming all energy of the lamp is given off as light, the number of photons entering the sensor if the wavelength of light is λ is

a. $N = P\lambda d^2 t/hc\ell^2$ b. $N = 4P\lambda d^2 t/hc\ell^2$

c. $N = P\lambda d^2 t/4hc\ell^2$ d. $N = P\lambda d^2 t/16hc\ell^2$

114. A photon has same wavelength as the de Broglie wavelength of electrons. Given C = speed of light, v = speed of electron. Which of the following relation is correct? [Here E_e = kinetic energy of electron, E_{ph} = energy of photon, P_e = momentum of electron and P_{ph} = momentum of photon]

a. $E_e/E_{ph} = 2C/v$ **b.** $E_e/E_{ph} = v/2C$
c. $P_e/P_{ph} = 2C/v$ **d.** $P_e/P_{ph} = C/v$

115. Light of intensity I is incident perpendicularly on a perfectly reflecting plate of area A kept in a gravity-free space. If the photons strike the plate symmetrically and initially the spring was at its natural length, find the maximum compression in the springs.

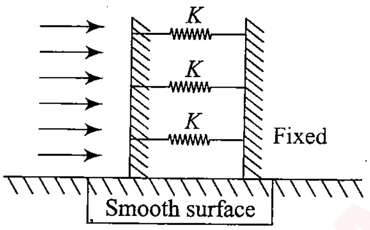


Fig. 3.38

a. IA/Kc **b.** $2Ia/3Kc$
c. $3Ia/Kc$ **d.** $4Ia/3Kc$

116. The de Broglie wavelength of a thermal neutron at 927°C is λ. Its wavelength at 327°C will be

a. $\lambda/2$ **b.** $\lambda/\sqrt{2}$
c. $\lambda\sqrt{2}$ **d.** 2λ

117. A small mirror of mass m is suspended by a light thread of length ℓ. A short pulse of laser falls on the mirror with energy E. Then, which of the following statement is correct?

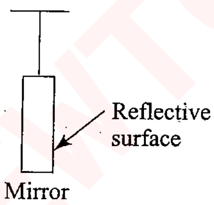


Fig. 3.39

a. If the pulse falls normally on the mirror, it deflects by $\theta = 2E/(mc\sqrt{2g\ell})$
b. If the pulse falls normally on the mirror, it deflects by $\theta = 2E/(mc\sqrt{2g})$
c. Impulse in thread depends on angle at which the pulse falls on the mirror
d. None of the above

118. A particle of mass $3m$ at rest decays into two particles of masses m and $2m$ having non-zero velocities. The ratio of the de Broglie wavelengths of the particles (λ_1/λ_2) is

a. 1/2 **b.** 1/4
c. 2 **d.** none of these

119. The radius of second orbit of an electron in hydrogen atom is 2.116 Å. The de Broglie wavelength associated with this electron in this orbit would be

a. 6.64 Å **b.** 1.058 Å
c. 2.116 Å **c.** 13.28 Å

120. Radiation of wavelength 546 nm falls on a photo cathode and electrons with maximum kinetic energy of 0.18 eV are emitted. When radiation of wavelength 185 nm falls on the same surface, a (negative) stopping potential of 4.6 V has to be applied to the collector cathode to reduce the photoelectric current to zero. Then, the ratio h/e is

a. 6.6×10^{-15} Js C^{-1} **b.** 4.12×10^{-15} Js C^{-1}
c. 6.6×10^{-34} Js C^{-1} **d.** 4.12×10^{-34} Js C^{-1}

121. The human eye is most sensitive to green light of wavelength 505 nm. Experiments have found that when people are kept in a dark room until their eyes adapt to the darkness, a single photon of green light will trigger receptor cells in the rods of the retina. The velocity of typical bacterium of mass 9.5×10^{-12} g, if it had absorbed all energy of photon, is nearly

a. 10^{-6} ms^{-1} **b.** 10^{-8} ms^{-1}
c. 10^{-10} ms^{-1} **d.** 10^{-13} ms^{-1}

122. The light sensitive compound on most photographic films is silver bromide AgBr. A film is exposed when the light energy absorbed dissociates this molecule into its atoms. The energy of dissociation of AgBr is 10^5 J mol^{-1}. For a photon that is just able to dissociate a molecule of AgBr, the photon energy is

a. 1.04 eV **b.** 2.08 eV
c. 3.12 eV **d.** 4.16 eV

Multiple Correct Answers Type

Solutions on page 3.58

1. Photoelectric effect supports the quantum nature of light because

a. there is a minimum frequency of light below which no photoelectrons are emitted
b. the maximum KE of photoelectrons depends only on the frequency of light and not on its intensity
c. even when the metal surface is faintly illuminated by light of wavelength less than the threshold wavelength, the photoelectrons leave the surface immediately
d. electric charge of photoelectrons is quantized

2. A point source of light is taken away from the experimental setup of photoelectric effect. For this situation, mark out the correct statement(s).

a. Saturation photocurrent decreases
b. Saturation photocurrent increases
c. Stopping potential remains the same
d. Stopping potential increases

3. The maximum kinetic energy of the emitted photoelectrons against frequency υ of incident radiation is plotted as shown in Fig. 3.40. This graph help us in determining the following physical quantities

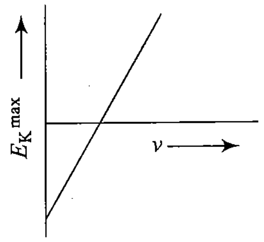


Fig. 3.40

a. work function of the cathode-metal
b. threshold frequency
c. planck's constant
d. charge on an electron

4. In a photoelectric experiment, the wavelength of the incident light is decreased from 6000 Å to 4000 Å . While the intensity of radiations remains the same
a. the cut-off potential will decrease
b. the cut-off potential will increase
c. the photoelectric current will increase
d. the kinetic energy of the emitted electrons will increase

5. When a point light source of power W emitting monochromatic light of wavelength λ is kept at a distance a from a photo-sensitive surface of work function ϕ and area S, we will have
a. number of photons striking the surface per unit time as $W\lambda S/4\pi hca^2$
b. the maximum energy of the emitted photoelectrons as $(1/\lambda)(hc-\lambda\phi)$
c. the stopping potential needed to stop the most energetic emitted photoelectrons as $(e/\lambda)(hc-\lambda\phi)$
d. photo-emission only if λ lies in the range $0 \le \lambda \le (hc/\phi)$

6. A collimated beam of light of flux density 3 k Wm^{-2} is incident normally on a 100 mm^2 completely absorbing screen. If P is the pressure exerted on the screen and Δp is the momentum transferred to the screen during a 1000 s interval, then
a. $P = 10^{-3}\ Nm^{-2}$ **b.** $P = 10^{-4}\ Nm^{-2}$
c. $\Delta p = 10^{-4}\ Kgms^{-1}$ **d.** $\Delta p = 10^{-5}\ Kgms^{-1}$

7. When photons of energy hc/λ fall on a metal surface, photoelectrons are ejected from it. If the work function of the surface is $h\upsilon_0$, then
a. maximum kinetic energy of the electron is $[(hc/\lambda) - h\upsilon_0]$
b. Maximum kinetic energy of the photoelectron is equal to (hc/λ)
c. minimum KE of the photoelectron is zero
d. minimum kinetic energy of the photoelectron is equal to hc/λ

8. Threshold wavelength of certain metal is λ_0. A radiation of wavelength $\lambda < \lambda_0$ is incident on the plate. Then, choose the correct statement from the following.
a. Initially, electrons will come out from the plate
b. The ejected electrons experience retarding force due to development of positive charges on the plate
c. After some time, ejection of electrons stops
d. None of the above

9. When barium is irradiated by a light of $\lambda = 4000$ Å, all the photoelectrons emitted are bent in a circle of radius 50 cm by a magnetic field of flux density 5.26×10^{-6} T acting perpendicular to plane of emission of photoelectrons. Then,
a. the kinetic energy of fastest photoelectron is 0.6 eV
b. work function of the metal is 2.5 eV
c. the maximum velocity of photoelectron is $0.46 \times 10^6\ ms^{-1}$
d. the stopping potential for photoelectric effect is 0.6 V

10. In a photoelectric effect experiment, the maximum kinetic energy of the ejected photoelectrons is measured for various wavelengths of the incident light. Figure 3.41 shows a graph of this maximum kinetic energy K_{max} as a function of the wavelength λ of the light falling on the surface of the metal. Which of the following statement/s is/are correct?

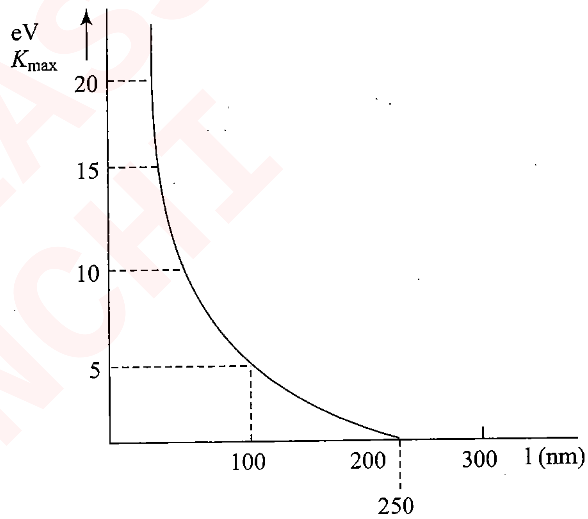


Fig. 3.41

a. Threshold frequency for the metal is 1.2×10^{15} m
b. Work function of the metal is 4.968 eV
c. Maximum kinetic energy of photoelectrons corresponding to light of wavelength 100 nm is nearly 7.4 eV
d. Photoelectric effect takes place with red light

11. A laser used to weld detached retinas emits light with a wavelength of 652 nm in pulses that are 20.0 ms in duration. The average power during each pulse is 0.6 W. Then,
a. the energy of each photon is 3.048×10^{-19} J
b. the energy content in each pulse is 12 mJ
c. the number of photons in each pulse is nearly 4×10^{15}
d. the energy of each photon is nearly 1.9 eV

Assertion-Reasoning Type

Solutions on page 3.59

Some questions (Assertion-Reason type) are given below. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c), and (d) out of which **only one** is correct. So select the correct choice.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. A proton and an electron both have energy 50 eV.
Statement I: Both have different wavelengths.
Statement II: Wavelength depends on energy and not on mass.

2. Statement I: Though light of a single frequency (monochromatic light) is incident on a metal, the energies of emitted photoelectrons are different.
Statement II: The energy of electrons just after they absorb photons incident on the metal surface may be lost in collision with other atoms in the metal before the electron is ejected out of the metal.

3. Statement I: The de Broglie wavelength of a molecule (in a sample of ideal gas) varies inversely as the square root of absolute temperature.
Statement II: The de Broglie wavelength of a molecule (in a sample of ideal gas) depends on temperature.

Comprehension Type

Solutions on page 3.60

For Problems 1–3

Photoelectric threshold of silver is $\lambda = 3800$ Å. Ultraviolet light of $\lambda = 2600$ Å is incident on silver surface. (Mass of the electron 9.11×10^{-31} kg).

1. Calculate the value of work function in eV.
a. 1.77 **b.** 3.27
c. 5.69 **d.** 2.32

2. Calculate the maximum kinetic energy (in eV) of the emitted photoelectrons.
a. 1.51 **b.** 2.36
c. 3.85 **d.** 4.27

3. Calculate the maximum velocity of the photoelectrons.
a. 72.89×10^8 **b.** 57.89×10^8
c. 42.93×10^8 **d.** 68.26×10^8

For Problems 4–6

A 100 W point source emits monochromatic light of wavelength 6000 Å.

4. Calculate the total number of photons emitted by the source per second.
a. 5×10^{20} **b.** 8×10^{20}
c. 6×10^{21} **d.** 3×10^{20}

5. Calculate the photon flux (in SI unit) at a distance of 5 m from the source. Given $h = 6.6 \times 10^{34}$ Js and $c = 3 \times 10^8$ ms^{-1}.
a. 10^{15} **b.** 10^{18}
c. 10^{20} **d.** 10^{22}

6. 1.5 mW of 4000 Å light is directed at a photoelectric cell. If 0.10 per cent of the incident photons produce photoelectrons, find current in the cell. [Given $h = 6.6 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C]
a. 0.59 μA **b.** 1.16 μA
c. 0.48 μA **d.** 0.79 μA

For Problems 7–8

A metallic surface is illuminated alternatively with light of wavelengths 3000 Å and 6000 Å. It is observed that the maximum speeds of the photoelectrons under these illuminations are in the ratio 3:1.

7. The work function of the metal is
a. 1.45 eV **b.** 2.26 eV
c. 1.23 eV **d.** 3.4 eV

8. What will be the maximum speed of the photoelectrons so emitted ?
a. 9×10^5 m/s **b.** 9×10^7 m/s
c. 3.6×10^5 m/s **d.** 4.5×10^7 m/s

For Problems 9–10

A helium–neon laser has a power output of 1 mW of light of wavelength 632.8 nm.

9. Calculate the energy of each photon in electron volt.
a. 2.5 **b.** 1.96
c. 0.53 **d.** 3.3

10. Determine the number of photons emitted by the laser each second.
a. 3.18×10^{15} **b.** 4.5×10^{16}
c. 1.2×10^{15} **d.** 2.9×10^{17}

For Problems 11–13

Photoelectrons are ejected from a surface when light of wavelength $\lambda_1 = 550$ nm is incident on it. The stopping potential for such electrons is $\lambda_{s1} = 0.19$ V. Suppose that radiation of wavelength $\lambda_2 = 190$ nm is incident on the surface.

11. Calculate the stopping potential V_{s2}.
a. 4.47 **b.** 3.16
c. 2.76 **d.** 5.28

12. Calculate the work function of the surface.
a. 3.75 **b.** 2.07
c. 4.20 **d.** 3.60

13. Calculate the threshold frequency for the surface.
a. 500×10^{12} Hz **b.** 480×10^{13} Hz
c. 520×10^{11} Hz **d.** 460×10^{13} Hz

For Problems 14–15

In a photoelectric effect experiment, a metallic surface of work function 2.2 eV is illuminated with a light of wavelength 400 nm. Assume that an electron makes two collisions before being emitted and in each collision 10% additional energy is lost.

14. Find the kinetic energy of this electron as it comes out of the metal.
a. 0.46 eV **b.** 0.31 eV
c. 0.23 eV **d.** none of these

15. After how many collisions the electron will be unable to come out of the metal?
a. 2 **b.** 6
c. 4 **d.** 8

For Problems 16–18

In a photoelectric setup, a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photoelectron emission is 1 for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantaneously swept away after emission.

16. Calculate the number of photoelectrons emitted per second.
a. 10^3 **b.** 10^4
c. 5×10^4 **d.** 10^5

17. It is observed that photoelectron emission stops at a certain time t after the light source is switched on. It is due to the retarding potential developed in the metallic sphere due to left over positive charges. The stopping potential (V) can be represented as
a. $2(KE_{max}/e)$ **b.** (KE_{max}/e)
c. $(KE_{max}/3e)$ **d.** $(KE_{max}/2e)$

18. Evaluate time t (in minutes).
a. 1.85 **b.** 2.36
c. 2.75 **d.** 0.78

For Problems 19–20

The incident intensity on a horizontal surface at sea level from sun is about 1 kW m^{-2}.

19. Assuming that 50 per cent of this intensity is reflected and 50 per cent is absorbed, determine the radiation pressure on this horizontal surface (in pascals).
a. 8.2×10^{-2} **b.** 5×10^{-6}
c. 3×10^{-5} **d.** 6×10^{-5}

20. Find the ratio of this pressure to atmospheric pressure p_0 (about 1×10^5 Pa) at sea level.
a. 5×10^{-11} **b.** 4×10^{-8}
c. 6×10^{-12} **d.** 8×10^{-11}

For Problems 21–22

Light of intensity I falls along the axis on a perfectly reflecting right circular cone having semi-vertical angle θ and base radius R. If E is the energy of one photon and c is the speed of light, then find

21. the number of photons hitting the cone per second
a. $\pi R^2 I/2E$ **b.** $2\pi R^2 I/E$
c. $\pi R^2 I/4E$ **d.** $\pi R^2 I/E$

22. The net force on the cone.
a. $(\pi R^2 I)/c\ (1 - \cos 2\theta)$ **b.** $(\pi R^2 I)/2c\ (1 - \cos 2\theta)$
c. $2(\pi R^2 I)/c\ (1 + \cos 2\theta)$ **d.** $(\pi R^2 I)/2c\ (1 + \cos 2\theta)$

For Problems 23–27

An experimental setup of verification of photoelectric effect is shown in Fig. 3.42. The voltage across the electrodes is measured with the help of an ideal voltmeter, and which can be varied by moving jockey 'J' on the potentiometer wire. The battery used in potentiometer circuit is of 20 V and its internal resistance is 2 Ω. The resistance of 100 cm long potentiometer wire is 8 Ω.

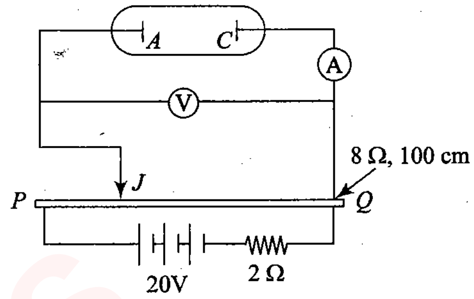


Fig. 3.42

The photocurrent is measured with the help of an ideal ammeter. Two plates of potassium oxide of area 50 cm^2 at separation 0.5 mm are used in the vacuum tube. Photocurrent in the circuit is very small, so we can treat the potentiometer circuit as an independent circuit.

The wavelength of various colors is as follows:

Light	1 Violet	2 Blue	3 Green	4 Yellow	5 Orange	6 Red
λ in Å	4000 – 5000	4500 – 5000	5000 – 5500	55000 – 6000	6000 – 6500	6500 – 7000

23. Calculate the number of electrons that appear on the surface of the cathode plate, when the jockey is connected at the end 'P' of the potentiometer wire. Assume that no radiation is falling on the plates.
a. 8.85×10^6 **b.** 11.0625×10^9
c. 8.85×10^9 **d.** 0

24. When radiation falls on the cathode plate, a current of 2 μA is recorded in the ammeter. Assuming that the vacuum tube setup follows Ohm's law, the equivalent resistance of vacuum tube operating in this case when jockey is at end P is
a. 8×10^8 Ω **b.** 16×10^6 Ω
c. 8×10^6 Ω **d.** 10×10^6 Ω

25. It is found that ammeter current remains unchanged (2 μA) even when the jockey is moved from the end 'P' to the middle point of the potentiometer wire. Assuming that all the incident photons eject electrons and the power of the light incident is 4×10^{-6} W. Then, the color of the incident light is
a. Green **b.** Violet
c. Red **d.** Orange

26. Which of the following colors may not give photoelectric effect for this cathode?
a. Green **b.** Violet
c. Red **d.** Orange

27. When other light falls on the anode plate, the ammeter reading remains zero till jockey is moved from the end P to the middle point of the wire PQ. Thereafter, the deflection is recorded in the ammeter. The maximum kinetic energy of the emitted electron is
a. 16 eV **b.** 8 eV
c. 4 eV **d.** 10 eV

For Problems 28–30

Light having photon energy $h\upsilon$ is incident on a metallic plate having work function ϕ to eject the electrons. The most energetic electrons are then allowed to enter in a region of uniform magnetic field B as shown in Fig. 3.43. The electrons are projected in X-Z plane making an angle θ with X-axis and magnetic field is $\vec{B} = B_0\hat{i}$ along X-axis.

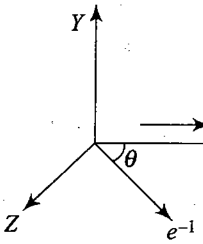


Fig. 3.43

Maximum pitch of the helix described by an electron is found to be p. Take mass of electron as m and charge as q.

Based on above information, answer the following questions:

28. The correct relation between p and B_0 is

a. $qpB_0 = 2\pi\cos\theta\sqrt{2(h\nu-\phi)m}$

b. $qpB_0 = 2\pi\cos\theta\sqrt{\dfrac{2(h\nu-\phi)}{m}}$

c. $pqB_0 = 2\pi\sqrt{2(h\nu-\phi)m}$

d. $p = \dfrac{2\pi m}{qB_0}\times\sqrt{h\nu-\phi}$

29. Considering the instant of crossing origin at $t = 0$, the Z-coordinate of the location of electron as a function of time is

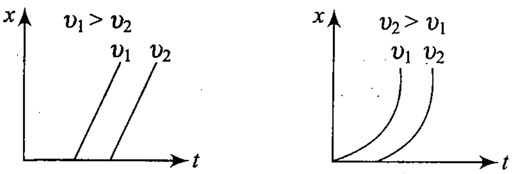
a. $-\dfrac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\left[1-\cos\left(\dfrac{qB_0t}{m}\right)\right]$

b. $\dfrac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\times\sin\left[\dfrac{qB_0t}{m}\right]$

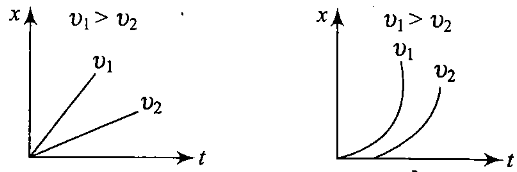
c. $\dfrac{-\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\times\sin\left[\dfrac{qB_0t}{m}\right]$

d. $\dfrac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\left(\dfrac{qB_0t}{m}\right)$

30. The plot between X-coordinate of the location of electron as a function of time for different frequencies υ of the incident light, is



a. b.



c. d.

For Problem 31–33

When light of sufficiently high frequency is incident on a metallic surface, electrons are emitted from the metallic surface. This phenomenon is called photoelectric emission. Kinetic energy of the emitted photoelectrons depends on the wavelength of incident light and is independent of the intensity of light. Number of emitted photoelectrons depends on intensity. $(h\nu - \phi)$ is the maximum kinetic energy of emitted photoelectrons (where ϕ is the work function of metallic surface). Reverse effect of photo emission produces X-ray. X-ray is not deflected by electric and magnetic fields. Wavelength of a continuous X-ray depends on potential difference across the tube. Wavelength of characteristic X-ray depends on the atomic number.

31. If frequency ($\upsilon > \upsilon_0$) of incident light becomes n times the initial frequency (ν), then KE of the emitted photoelectrons becomes (ν_0 threshold frequency).

a. n times of the initial kinetic energy

b. more than n times of the initial kinetic energy

c. less than n times of the initial kinetic energy

d. kinetic energy of the emitted photoelectrons remains unchanged

32. A monochromatic light is used in a photoelectric experiment on photoelectric effect. The stopping potential

a. is related to mean wavelength

b. is related to shortest wavelength

c. is related to the maximum kinetic energy of emitted photoelectrons

d. intensity of incident light

33. If potential difference across the tube is increased then

a. λ_{min} will decrease

b. characteristic wavelength will increase

c. λ_{min} will increase

d. none of these

For Problems 34–36

The energy received from the Sun by earth and surrounding atmosphere is 2 cal cm^{-2} min^{-1} on a surface normal to the rays of sun.

34. What is total energy received, in joule, by the Earth and its atmosphere

a. 10.645×10^{18} J min^{-1}

b. 10.645×10^{15} J min^{-1}

c. 8.645×10^{17} J min^{-1}

d. 9.645×10^{14} J min^{-1}

35. What is the total energy radiated, in J min^{-1}, by Sun to the universe. Distance of Sun from earth is 1.49×10^{11} m

a. 2.3444×10^{28} J min^{-1}
b. 2.33×10^{24} J min^{-1}
c. 2.34×10^{20} J min^{-1}
d. none of these

36. At what rate, in mega gram per minute, must hydrogen be consumed in fusion reaction to provide the Sun with the energy it radiates? (Take mass defect per reaction to be 0.028706 a.m.u)

a. 3.66×10^{14} mega gram min^{-1}
b. 3.66×10^{16} mega gram min^{-1}
c. 3.66×10^{15} mega gram min^{-1}
d. 3.66×10^{10} mega gram min^{-1}

For Problems 37–39

When a high frequency electromagnetic radiation is incident on a metallic surface, electrons are emitted from the surface. Energy of emitted photoelectrons depends only on the frequency of incident electromagnetic radiation and number of emitted electrons depends only on the intensity of incident light.

Einstein photoelectric equation [$K_{max} = h\nu - \phi$] correctly explains the PE, where υ = frequency of incident light and ϕ = work function.

37. Light of wavelength 3300 is incident on two metals A and B, whose work functions are 4 eV and 2 eV, respectively. Then

a. A will emit photoelectrons but B will not
b. B will emit photoelectrons, but A will not
c. both A and B will not emit photoelectrons
d. neither A nor B will emit photoelectrons

38. For photoelectric effect in a metal, the graph of the stopping potential V_0 (in volt) versus frequency υ (in hertz) of the incident radiation is shown in Fig. 3.44. The work function of the metal (in eV) is

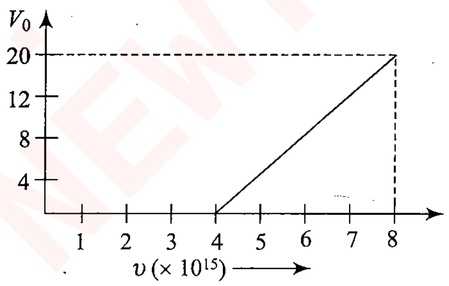


Fig. 3.44

a. 12.5 **b.** 14.5
c. 16.5 **d.** 18.5

39. The slope of the graph shown in above figure [here, h is the plancks constant and e is the charge of an electron] is

a. $\frac{h}{e}$ **b.** eh
c. h **d.** $\frac{e}{h}$

For Problems 40–42

A Cs plate is irradiated with a light of wavelength $\lambda = hc/\phi$, ϕ being the work function of the plate, h the Planck's constant, and c the velocity of light in vacuum. Assume all the photoelectrons are moving perpendicular to the plate toward a YDSE setup when accelerated through a potential difference V. Take charge on a proton = e and mass of an electron = m.

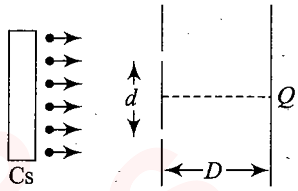


Fig. 3.45

Read the paragraph carefully and answer the following questions:

40. The fringe width due to the electron beam is

a. $\lambda D/d$ **b.** $\lambda D/2d$
c. $hD/(d\sqrt{2emV})$ **d.** none of these

41. If the wavelength of light used in photoemission is less than λ, then the fringe width will

a. increase **b.** decrease
c. remain same **d.** cannot be decided

42. Instead of moving perpendicular to the plate, if the electrons were moving randomly, then the central maximum would shift

a. upward **b.** downward
c. no shift **d.** no fringes will be formed

For Problems 43–45

A pushed dye laser emits light of wavelength 585 nm. Because this wavelength is strongly absorbed by the haemo- globin in the blood, the method is especially effective for removing various types of blemishes due to blood. To get a reasonable estimate of the power required for such laser surgery, we can model the blood as having the same specific heat and heat of vaporization as water. [$S = 4.2 \times 10^3$ J (kg K)$^{-1}$, $L = 2.25 \times 10^6$ J kg]

43. Suppose that each pulse must remove 2 μg of blood by evaporating it starting at 30°C. The energy that each pulse must deliver to the blemish is nearly

a. 5.1 J **b.** 5.1 mJ
c. 5.1 μJ **d.** 5.1 kJ

44. The power output of the laser must be

a. 5.5 W **b.** 11 W
c. 16.5 W **d.** 22 W

45. The number of photons that each pulse delivers to the blemish is

a. 1.5×10^{16} **b.** 1.5×10^8
c. 3×10^{16} **d.** 3×10^8

Matching Column Type

Solutions on page 3.65

1. With respect to photoelectric effect experiment, match the entries of Column I with the entries of Column II.

Column I	Column II
a. If f (frequency) is increased keeping I (intensity) and ϕ (work function) constant	**p.** Stopping potential increases
b. If I is increased keeping f and ϕ constant	**q.** saturation photocurrent increases
c. If the distance between anode and cathode increases	**r.** maximum KE of the photoelectrons increases
d. If ϕ is decreased keeping f and I constant.	**s.** stopping potential remains the same

2. In a photoelectric experimental arrangement, light of frequency f is incident on a metal target whose work function is $\phi = hf/3$ as shown. In Column I, KE of photoelectron is mentioned at various locations/instants and in Column II, the corresponding values. Match the entries of Column I with the entries of Column II.

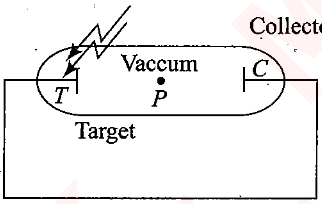


Fig. 3.46

Column I	Column II
a. Maximum KE of photoelectrons just after emission from target	**p.** zero
b. KE of photoelectrons just after emission from target	**q.** $hf/3$
c. KE of photoelectron when they are halfway between the target and collector	**r.** $hf/2$
d. KE of photoelectrons as they reach the collector	**s.** $2hf/3$

3. Related to photoelectric effect, in Column I, some physical quantities change while in Column II effects of these changes are given. Match the entries of column I with the entries of Column II.

Column I	Column II
a. Intensity of incident light changes	**p.** K_{max} of emitted photo-electrons changes
b. Frequency of incident light changes	**q.** Stopping potential changes
c. Target material changes	**r.** Saturation current changes
d. Potential difference between the emitter and collector changes	**s.** Time delay in emission of photoelectrons changes

4. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic light falls on photosensitive electrode. The emf of battery shown is high enough such that all photoelectrons ejected from left electrode will reach the right electrode. Under initial conditions, photoelectrons are emitted. As changes are made in each situation of Column I. Match the statement in Column I with results in Column II.

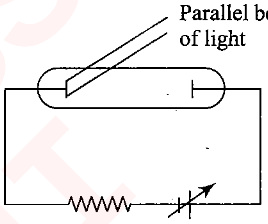


Fig. 3.47

Column I	Column II
a. If frequency of incident light is increased keeping its intensity constant	**p.** magnitude of stopping potential will increase
b. If frequency of incident light is increased and its intensity is decreased	**q.** current through the circuit may stop
c. If work function of photon sensitive electrode is increased	**r.** maximum kinetic energy of ejected photoelectrons will increase
d. If intensity of incident light is increased keeping its frequency constant	**s.** saturation current will increase

5. In Column I, the nature of light is given and in Column II the information about the photons are mentioned. Match the entries of Column I with the entries of Column II.

Column I	Column II
a. A bichromatic light source	**p.** Few photons have same energy and momenta
b. A point source of white light emitting light uniformly in all directions	**q.** Few photons have different energy and different momenta
c. A point source of monochromatic light emitting light uniformly in all directions	**r.** Few photons have same energy and different momenta
d. Laser light source	**s.** Few photons have different energy and same momenta

Integer Answer Type

Solutions on page 3.65

1. The radius of an α-particle moving in a circle in a constant magnetic field is half of the radius of an electron moving in circular path in the same field. The de Broglie wavelength of α-particle is n times that of the electron. Find n (an integer).
2. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^8 ms^{-1} is equal to that of a photon. Find the ratio of the kinetic energy of the photon to that of the electron.
3. An element of atomic number 9 emits K_α X-ray of wavelength λ. Find the atomic number of the element which emits K_α X-ray of wavelength 4λ.
4. A monochromatic sources of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of the light. (in $\times 10^{-7}$m)
5. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}. Calculate the force exerted by light beam on the mirror. (in 10^{-8}N)
6. A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power (in $\times 10^8$W) of the source needed to support the weight of the mirror. Take $g = 10$ m/s^2.

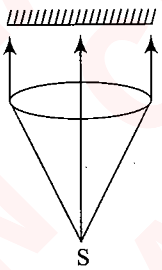


Fig. 3.48(a)

7. A silver ball of radius 4.8 cm is suspended by a thread in a vacuum chamber. Ultraviolet light of wavelength 200 nm is incident on the ball for some time during which a total light energy of 1.0×10^{-7} J falls on the surface. Assuming that on the average, one photon out of ten thousand photons is able to eject a photoelectron, find the electric potential (in $\times 10^{-1}$V) at the surface of the ball assuming zero potential at infinity.
8. In the arrangement shown in the figure, $y = 1.0$ mm, $d = 0.24$ mm and $D = 1.2$ m. The work function of the material of the emitter is 2.2 eV. find the stopping potential V needed to stop the photocurrent. (in $\times 10^{-1}$V)

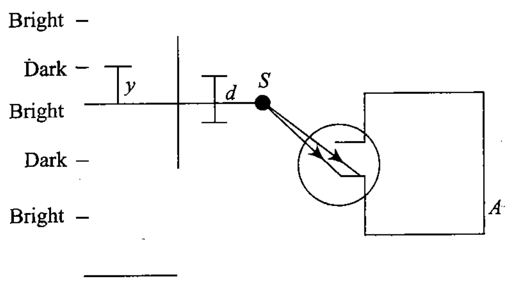


Fig. 3.48(b)

Archives

Solutions on page 3.66

Fill in the Blanks Type

1. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the ____________ of the incident radiation. **(IIT-JEE, 1984)**

True or False Type

1. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. **(IIT-JEE, 1981)**
2. In a photoelectric emission process the maximum energy of the photo-electrons increases with increasing intensity of the incident light. **(IIT-JEE, 1986)**

Multiple Choice Questions with One Correct Answer Type

1. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volts is **(IIT-JEE, 1997)**
   a. 2 b. 4
   c. 6 d. 10
2. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **(IIT-JEE, 1998)**
   a. 540 nm b. 400 nm
   c. 310 nm d. 220 nm
3. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de Broglie wavelengths of the particles λ_1/λ_2 is **(IIT-JEE, 1999)**
   a. m_1/m_2 b. m_2/m_1
   c. 1.0 d. $\sqrt{m_2}/\sqrt{m_1}$
4. A proton has kinetic energy $E = 100$ keV which is equal to that of a proton. The wavelength of photon is λ_2 and that of proton is λ_1. The ratio of λ_2/λ_1 is proportional to **(IIT-JEE, 2004)**
   a. E^2 b. $E^{-1/2}$
   c. E^{-1} d. $E^{1/2}$
5. In a photoelectric experiment, anode potential is plotted against plate current. Select the correct statement from the following. **(IIT-JEE, 2004)**

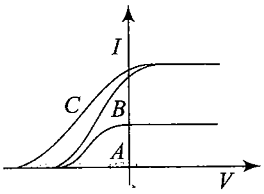


Fig. 3.49

   a. A and B will have different intensities while B and C will have different frequencies
   b. B and C will have different intensities while A and C will have different frequencies

c. A and B will have different intensities while A and C will have equal frequencies
d. A and B will have equal intensities while B and C have different frequencies

6. A beam of electron is used in a YDSE experiment. The slit width is d. When the velocity of electron is increased, then **(IIT-JEE, 2005)**
a. no interference is observed
b. fringe width increases
c. fringe width decreases
d. fringe width remains same

7. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu] **(IIT-JEE, 2005)**
a. 7.6 MeV b. 56.12 MeV
c. 10.24 MeV d. 23.9 MeV

Multiple Choice Questions with One or More than One Correct Answer Type

1. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a **(IIT-JEE, 1982)**
a. 50 W infrared lamp b. 1 W infrared lamp
c. 50 W ultraviolet lamp d. 1 W ultraviolet lamp

2. Photoelectric effect supports quantum nature of light because **(IIT-JEE, 1987)**
a. there is a minimum frequency of light below which no photoelectrons are emitted
b. the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
c. even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
d. electric charge of the photoelectrons is quantized

3. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are, respectively, 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then **(IIT-JEE, 1992)**
a. the stopping potential will be 0.2 V
b. the stopping potential will be 0.6 V
c. the saturation current will be 6.0 mA
d. the saturation current will be 2.0 mA

4. When photons of energy 4.25 eV strike the surface of metal A, the ejected photoelectrons have maximum kinetic energy T_A and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then **(IIT-JEE, 1994)**
a. the work function of A is 2.25 eV
b. the work function of B is 4.20 eV
c. $T_A = 2.00$ eV
d. $T_B = 2.75$ eV

5. The graph between the stopping potential (V_0) and ($1/\lambda$) is shown in the figure. ϕ_1, ϕ_2 and ϕ_3 are work functions. Then which of the following is/are correct? **(IIT-JEE, 2006)**

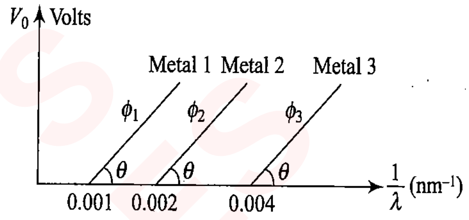


Fig. 3.50

a. $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
b. $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
c. $\tan\theta \propto (hc)/e$
d. Ultraviolet light can be used to emit photoelectrons from metal 2 and metal 3 only

6. In Young's double-slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s). **(IIT-JEE, 2008)**
a. If $d = \lambda$, the screen will contain only one maximum
b. If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
c. If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
d. If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Comprehension Type

For Problems 1–3

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = p^2/2m$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving along the line from $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C. **(IIT-JEE, 2009)**

1. The allowed energy for the particle for a particular value of n is proportional to

a. a^{-2}
b. $a^{-3/2}$
c. a^{-1}
d. a^2

2. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to
a. 0.8 meV
b. 8 meV
c. 80 meV
d. 800 meV

3. The speed of the particle that can take discrete values is proportional to
a. $n^{-3/2}$
b. n^{-1}
c. $n^{1/2}$
d. n

Integer Answer Type

1. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio λ_p/λ_α, to the nearest integer, is **(IIT-JEE, 2010)**

2. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of 'Z' is. **(IIT-JEE, 2011)**

ANSWERS AND SOLUTIONS

Subjective Type

1. Incident energy of photons $= \dfrac{hc}{200} = 6.2$ eV

$$6.2 = K_1 + W_0,\ K_1 = 6.2 - W_0$$
$$K_2 = K_1 + 10e = 16.2 - W_0$$
$$K_2 = \frac{hc}{\lambda_{min}} = 12.4,\ W_0 = 3.8\text{ eV, given } W_0 = \frac{x}{10}$$
$$\therefore \quad x = 38$$

2. The bending radius r of a particle of mass m and charge e moving with a velocity v in a uniform magnetic field is given by

$\dfrac{mv^2}{r} = evB$, e being the charge and B the strength of the field.

Momentum, $\quad mv = Ber \quad$ (i)

When the *em* radiation is incident normally on a double slit.

$$\text{F.W.} = \text{the fringe width}$$
$$= \frac{3.1}{10}\text{cm} = 3.1 \times 10^{-3}\text{ m}$$

The wavelength of the radiation, λ, satisfies, F.W. $= \dfrac{\lambda D}{d}$

$$\lambda = \frac{d}{D} \times \text{F.W.}$$
$$= \frac{0.1 \times 10^{-3} \times 3.1 \times 10^{-3}}{1}\text{ m} = 3100\text{ Å}$$

The kinetic energy of the electron is

$$\text{KE} = \frac{(Ber)^2}{2m} = \frac{(Brec)^2}{2mc^2}$$
$$= \frac{\left(\frac{\sqrt{2}}{3} \times 10^{-4} \times 10^{-1} \times 3 \times 10^8\right)^2}{2 \times 0.5 \times 10^6} = 2\text{ eV}$$

The threshold wavelength is given by

$$\frac{hc}{\lambda} = \text{KE} + \frac{hc}{\lambda_{th}}$$
$$\frac{12400}{3100} = 2 + \frac{hc}{\lambda_{th}}$$
$$\frac{hc}{\lambda_{th}} = 2\text{ eV},\ \lambda_{th} = \frac{12400}{2}\text{ Å} = 6200\text{ Å}$$

The kinetic energy of the most energetic photoelectron is directly proportional to the square of the radius of its track in a magnetic field.

So, if the bending radius is half of what it was before, the kinetic energy of the electron is one-fourth of the previous value.

$$(\text{KE}') = 2 \times \left(\frac{1}{2}\right)^2\text{ eV or } 0.5\text{ eV}$$

The wavelength, λ, is given by

$$\frac{hc}{\lambda} = 2\text{ eV} + 0.5\text{ eV} = 2.5\text{ eV}$$
$$\Rightarrow \quad \lambda = \frac{12400}{2.5} = 4960\text{ Å}$$

3. $\text{KE}_{max} = (5 - \phi)$ eV

When electrons are accelerated through 5 V, they will reach the anode with energy $= (5 - \phi + 5)$ eV

$$\therefore \quad 10 - \phi = 8$$
$$\text{or} \quad \phi = 2\text{ eV}$$

Current is less than saturation current. This is because if the slowest electron also reached the plate it would have 5 eV energy at the anode, but it is given that the minimum energy is 6 eV.

4. Momentum corresponding to incident photons normal to the surface,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c}dA\cos^2\theta$$

Since reflection coefficient is ρ, so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt}\right)_{\text{reflected}} = \frac{I}{c}dA\rho\cos^2\theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt}\right)_{\text{photons}} = -\frac{I}{c}dA(\rho-1)\cos^2\theta$$

From Newton's third law,

$$F=\left(\frac{dp}{dt}\right)_{\text{surface}} = \frac{I}{c}dA(\rho+1)\cos^2\theta$$

Hence, pressure exerted on surface,

$$P=\frac{dF}{dA}=\frac{I}{c}(\rho+1)\cos^2\theta$$

On substituting values, we get

$$P=0.5\,\text{N cm}^{-2}$$

5. Maximum kinetic energy of ejected electrons

$$K_{max}=\frac{1}{2}mv_{max}^2=\frac{hc}{\lambda}-W$$

$$\therefore\quad K_{max}=\frac{(6.62\times10^{-34})(3\times10^8)}{(180\times10^{-9})}-(2\times1.6\times10^{-19})$$

$$=7.8\times10^{-19}\,\text{J}$$

$$\therefore\quad v_{max}=\sqrt{\left(\frac{2K_{max}}{m}\right)}=\sqrt{\left(\frac{2\times(7.8\times10^{-19})}{9.1\times10^{-31}}\right)}$$

$$=1.3\times10^6\,\text{ms}^{-1}$$

Now,

$$Bev_{max}=\frac{mv_{max}^2}{r}$$

$$r=\frac{mv_{max}}{Be}=\frac{(9.1\times10^{-31})(1.3\times10^6)}{(5\times10^{-5})(1.6\times10^{-19})}$$

$$=0.148\,\text{m}$$

6. (i) $h\upsilon_1=W+\frac{1}{2}mv_1^2$

and $\frac{1}{2}mv_1^2=eV_1$

$$\therefore\quad W=h\upsilon_1-eV_1=\frac{hc}{\lambda_1}-eV_1$$

$$=\frac{(6.6\times10^{-34})\times(3\times10^8)}{4950\times10^{-10}}-(1.6\times10^{-19})(0.6)$$

$$=1.9\,\text{eV}=1.9\times(1.6\times10^{-19})\,\text{V}$$

$$=3.04\times10^{-19}\,\text{V}$$

(ii) $h\upsilon_2=W+eV_2$

$$\text{or}\quad\frac{hc}{\lambda_2}=3.04\times10^{-19}+(1.6\times10^{-19})\times1.1$$

$$=4.8\times10^{-19}$$

$$\therefore\quad\lambda_2=\frac{hc}{4.8\times10^{-19}}=\frac{(6.6\times10^{-34})\times(3\times10^8)}{4.8\times10^{-19}}$$

$$=4125\,\text{Å}$$

(iii) Since the magnetic field does not change the speed of ejected electrons, there will be no change in the stopping potential.

7. According to Einstein's equation, the maximum kinetic energy E_k of emitted photoelectrons is given by

$$E_k=\frac{hc}{\lambda}-W$$

where W = work function.

Let V_0 be the stopping potential. Then,

$$E_k=eV_0$$

$$\therefore\quad eV_0=\frac{hc}{\lambda}-W\quad\text{or}\quad\lambda=\frac{hc}{W+eV_0}$$

$$\text{or}\quad\lambda=\frac{(6.62\times10^{-34})(3\times10^8)}{(1.7+10.4)\times(1.6\times10^{-19})}$$

$$=1.026\times10^{-7}\,\text{m}=1026\,\text{Å}$$

This wavelength lies in ultraviolet region. The series lying in ultraviolet region is Lyman series. Hence,

$$\frac{1}{\lambda}=R\left[\frac{1}{(1)^2}-\frac{1}{n^2}\right]$$

$$\Rightarrow\quad\frac{1}{1.026\times10^{-7}}=1.1\times10^7\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\text{or}\quad1-\frac{1}{n^2}=\frac{1}{1.026\times(1.1)}$$

Solving this equation, we get

$$n^2=9\quad\text{or}\quad n=3$$

Hence, the energy levels involved in hydrogen atom are $n=3$ to $n=1$.

8. a. Contact potential difference, $U_0=0.6$ V

The energy acquired by electron on reaching the electrode $=eU_0$

According to Einstein's equation,

$$h\nu=W+\text{KE}-eU_0$$

$$\text{or}\quad\text{KE}=h\upsilon-W+eU_0$$

Maximum $KE = eV_0$, where V_0 = stopping potential.

$$\therefore \quad eV_0 = h\upsilon - W + eU_0 = \frac{hc}{\lambda} - W + eU_0$$

$$V_0 = \frac{hc}{\lambda e} - \frac{W}{e} + U_0$$

$$= \frac{(6.62\times10^{-34})(3\times10^8)}{2.3\times10^{-7}\times1.6\times10^{-19}} - 4.5 + 0.6$$

$$= 5.4 - 4.5 + 0.6 = 1.5\text{V}$$

b. $V_0 = \frac{hc}{\lambda e} - \frac{W}{e} + U_0$

$$1 = \frac{hc}{\lambda e} - 4.5 + 0.6$$

or $\frac{hc}{\lambda e} - 4.5 - 0.6 = 4.9$

$$\lambda = \frac{hc}{4.9e} = \frac{(6.62\times10^{-34})(3\times10^8)}{4.9\times(1.6\times10^{-19})}$$

$$= 2540\times10^{-10}\text{m} = 2540\text{ Å}$$

9. (i) When charged particle moves perpendicular to a magnetic field, then magnetic field provides necessary centripetal force for circular path of radius r given by

$$\frac{mv^2}{r} = qvB \Rightarrow mv = qBr$$

As momentum, $p = mv = \sqrt{2mE_k}$

where E_k is kinetic energy

$$\therefore \quad \sqrt{2mE_k} = qBr$$

$$\therefore \quad E_k = \frac{(qBr)^2}{2m}$$

$$= \frac{\left\{1.6\times10^{-19}\times\left(\frac{1}{320}\right)\times10^{-3}\right\}^2}{2\times9.1\times10^{-31}}\text{ J}$$

$$= 1.374\times10^{-19}\text{ J}$$

$$= \frac{1.374\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 0.86\text{ eV}$$

(ii) Energy of photon released due to transition from $n = 3$ to $n = 2$ in hydrogen atom,

$$\varepsilon = \Delta E$$

$$= Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= (13.6\text{ eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89\text{ eV}$$

Work function of metal,

$$W = \varepsilon - E_k$$

$$= 1.89 - 0.86 = 1.03\text{ eV}$$

(iii) Wavelength of emitted radiation (photon) is given by

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{1.89\times1.6\times10^{-19}}$$

$$= 6.567\times10^{-7}\text{ m} = 6567\text{ Å}$$

10. a. The energy of each photon is

$$E = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\text{ eV}-\text{s})\times(3\times10^8\text{ m/s})}{500\text{ nm}}$$

$$= \frac{1242\text{ eV}-\text{nm}}{500\text{ nm}} = 2.48\text{ eV}$$

In 1 s, 10 J of energy passes through any cross section of the beam. Thus, the number of photons crossing a cross section per unit time is

$$n = \frac{10\text{ J}}{2.48\text{ eV}} = 2.52\times10^{19}$$

This is also the number of photons falling on the surface per second and being absorbed.

b. The linear momentum of each photon is $p = h/\lambda = h\nu/c$

The total momentum of all the photons falling per second on the surface is

$$\frac{nh\nu}{c} = \frac{10\text{ J}}{c} = \frac{10\text{ J}}{3\times10^8\text{ m/s}} = 3.33\times10^{-8}\text{ N-s}$$

As the photons are completely absorbed by the surface, this much momentum is transferred to the surface per second. The rate of change of the momentum of the surface, i.e., the force on it is

$$F = \frac{dp}{dt} = \frac{3.33\times10^{-8}\text{ N-s}}{1\text{ s}} = 3.33\times10^{-8}\text{ N}$$

11. Let N be the number of photoelectrons per second.

$\therefore$ Intensity = $N\times$ energy of one photon

$$I = N\frac{hc}{\lambda}$$

$\therefore$ Number of incident photons,

$$N_i = \text{Incident intensity}\times\frac{\lambda}{hc}$$

$$= \frac{10^{-8}\times365\times10^{-9}}{6.62\times10^{-34}\times3\times10^8} = 18.35\times10^9$$

Number of photon N_{ab} absorbed by the surface per unit area per unit time (i.e., absorbed flux),

$$N_{ab} = 0.8\times18.35\times10^9$$

$$= 1.47\times10^{10}\text{ m}^{-2}\text{s}^{-1}$$

Emitted flux = No. of electrons emitted $\text{m}^{-2}\text{s}^{-1}$

$$= 1.47\times10^{10}\text{ m}^{-2}\text{s}^{-1}$$

Power absorbed per $\text{m}^2 = 0.8\times$ Incident intensity

$$= 0.8\times10^{-8} = 8\times10^{-9}\text{ W m}^{-2}$$

From Einstein's photoelectric equation,

$$(\text{KE})_{max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{365\times10^{-9}} - 1.6\times1.6\times10^{-19}$$

$$= 2.89\times10^{-19}\text{ J} = 1.80\text{ eV}$$

12. de Broglie wavelength, $\lambda = \dfrac{h}{p}$

Mean KE of helium atom $= \dfrac{3kT}{2}$

or $\quad E = \dfrac{3\times(1.38\times10^{-23})\times 400}{2}$

In terms of momentum, the energy is given by

$$E = \frac{p^2}{2m} \quad \text{or} \quad p^2 = 2mE \quad \text{or} \quad p = \sqrt{2mE}$$

$$\therefore \quad p = \sqrt{2\times(4.002\times1.66\times10^{-27})\times3\times(1.38\times10^{-23})\times200}$$

(where mass of helium $= 4.002$ amu $= 4.002\times1.66\times10^{-27}$ kg)

Now, $\quad \lambda = \dfrac{6.625\times10^{-34}}{p}$

Substituting the value of p and solving it, we get

$$\lambda = 0.63\times10^{-10}\text{ m}$$

13. We know that, de Broglie wavelength

$$\lambda = \frac{h}{mv} \quad \text{and} \quad E = \frac{1}{2}mv^2$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}}$$

In first case, $100\times10^{-12} = \dfrac{h}{\sqrt{2mE_1}}$ (i)

In second case, $50\times10^{-12} = \dfrac{h}{\sqrt{2mE_2}}$ (ii)

Dividing Eqs. (i) by (ii), we get

$$2 = \sqrt{\left(\frac{E_2}{E_1}\right)} \quad \text{or} \quad E_2 = 4E_1$$

So, energy to be added $= 4E_1 - E_1 = 3E_1$

Now, $\quad \dfrac{h}{\sqrt{2mE_1}} = 100\times10^{-12}$

or $\quad \sqrt{2mE_1} = \dfrac{6.625\times10^{-34}}{10^{-10}}$

or $\quad \sqrt{2mE_1} = 6.625\times10^{-24}$

or $\quad E_1 = \dfrac{(6.625\times10^{-24})^2}{2\times(9.1\times10^{-31})} = 150\text{ eV}$

Therefore, energy added $= 3E_1 = 450$ eV

14. a. For the first wavelength:

$$eV_{s_1} = h\upsilon - W \quad \text{(i)}$$

$W =$ work function

For the second surface:

$$eV_{s_2} = h\upsilon_2 - W \quad \text{(ii)}$$

Subtracting, we get $V_{s_2} - V_{s_1} = \dfrac{h}{e}(\upsilon_2 - \upsilon_1)$ or

$$V_{s_2} = V_{s_1} + \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= V_{s_1} + \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$$

$$= 0.19 + 1240\left(\frac{550-190}{190\times550}\right)$$

$$= 4.47\text{ V}$$

b. From Eq. (i):

$$W = \frac{hc}{\lambda_1} - eV_{s_1}$$

Work function (in eV)

$$\frac{W}{e} = \frac{hc}{e\lambda_1} - V_{s_1} = \frac{1240}{550} - 0.19 = 2.07\text{ eV}$$

c. Threshold frequency:

$$V_0 = \frac{W}{h} = \frac{2.07\times1.6\times10^{-19}}{6.62\times10^{-34}} = 5\times10^{14}\text{ Hz}$$

15. Total number of ions in the rod

= number of ions per unit volume $\times$ volume of rod

$= (2\times10^{25})\times\{3.14\times(0.005)^2\times(5\times10^{-2})\}$

$= 7.85\times10^{19}$

The number of photons excited in one direction is equal to the total number of ions because all ions are excited. Now, excited energy

= number of excited photons $\times$ energy of photon

$$= (7.85\times10^{19})\times\frac{hc}{\lambda}$$

$$= (7.85\times10^{19})\times\frac{(6.6\times10^{-34})\times(3\times10^8)}{6.6\times10^{-7}}$$

$= 23.55$ W

16. (i) The energy of photon is given by

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{1990\times10^{-28}\text{ J}}{\lambda}$$

$$= \frac{1990\times10^{-28}}{5890\text{ Å}} = 3376\times10^{-22}\text{ J} \quad \text{(i)}$$

Given that the lamp is emitting energy at the rate of 100 Js^{-1} (Power = 100 W). Hence, number of photons N emitted is given by

$$N = \frac{100}{3376\times10^{-22}} \cong 3\times10^{20}\text{ photons s}^{-1} \quad \text{(ii)}$$

(ii) We regard the lamp as a point source. Therefore, at a distance r from the lamp, the light energy is uniformly distributed over the surface of sphere of radius r. So, N photons are crossing area $4\pi r^2$ of spherical surface per second. So, flux at a distance r is given by

$$n=\frac{N}{4\pi r^2}$$

For a flux of $n=1$ photons $cm^{-2}\,s^{-1}$, we have

$$1=\frac{N}{4\pi r^2} \quad \text{or} \quad r=\sqrt{\left(\frac{N}{4\pi}\right)}$$

or $$r=\sqrt{\left(\frac{3\times10^{20}}{4\times3.14}\right)}\text{ cm}=48860\text{ km} \qquad \text{(iii)}$$

So, at this distance, on the average one photon will cross through 1 cm^2 area normal to radial direction.

(iii) Photon flux at $t=2$ m,

$$n=\frac{3\times10^{20}}{4\pi(200\,\text{cm})^2}=5.9\times10^{14}\text{ photons cm}^{-2}$$

Average density of photons at $r=2$ m is given by

$$\rho=\frac{3\times10^{20}}{4\times3.14\times(200)^2\times(3\times10^{10})}$$

$$=2\times10^4\text{ photons cm}^{-2}$$

Objective Type

1. c. Orbital angular momentum $=\dfrac{nh}{2\pi}$ for H-atom

Therefore, minimum value of $L=h/2\pi$ (for $n=1$)

2. a. Energy $=\dfrac{1}{2}mv^2=5000$ eV

$$=5000\times1.6\times10^{-19}\text{ J}$$

$$mv=\sqrt{2\times5000\times(1.6\times10^{-19})m}$$

$$=4\times10^{-8}\times\sqrt{m}$$

Number of electrons striking per second is

$$n=\frac{q}{e}=\frac{It}{e}=\frac{50\times10^{-6}\times1}{1.6\times10^{-19}}=31.25\times10^{13}$$

Force = change of momentum per second

$$=n(mv)=31.25\times10^{13}\times4\times10^{-8}\sqrt{m}$$

$$=125\times10^5\sqrt{9.1\times10^{-31}}$$

$$=1.1924\times10^{-8}\text{ N}$$

3. b. Number of photons falling per second:

$$N_p=\frac{10^{-3}}{\dfrac{6.6\times10^{-34}\times3\times10^8}{5000\times10^{-10}}}=2.5\times10^{15}$$

Let N_e is the number of photoelectrons emitted per second.

$$\therefore\ I=\frac{q}{t}=\frac{N_e e}{1} \Rightarrow N_e=\frac{I}{e}=\frac{0.16\times10^{-6}}{1.6\times10^{-19}}=10^{12}$$

Percentage of photons producing photoelectrons

$$=\frac{N_e}{N_p}\times100=\frac{10^{12}}{2.5\times10^{15}}\times100=0.04\%$$

4. c. Using photoelectric equation,

$$h\nu-h\nu_0=\frac{1}{2}mv^2=eV_s$$

or $$\left(\frac{hc}{\lambda}-\frac{hc}{\lambda_0}\right)=eV_s$$

For the first case, $$\frac{hc}{\lambda}-\frac{hc}{\lambda_0}=e(3V_0) \qquad \text{(i)}$$

For the second case, $$\frac{hc}{2\lambda}-\frac{hc}{\lambda_0}=e(V_0) \qquad \text{(ii)}$$

Solving $\lambda_0=4\lambda$

5. b. $\lambda=\dfrac{h}{p} \Rightarrow \lambda=\dfrac{6.6\times10^{-34}}{10^{-31}\times10^5}$

$$\lambda=6.6\times10^{-8}$$

6. c. Einstein's equation for photoelectric effect is

$$h\upsilon-h\upsilon_0=\frac{1}{2}m\upsilon^2_{max}$$

When $\upsilon=2\upsilon_0$, $\upsilon_{max}=4\times10^8$ cms^{-1}

$$2h\upsilon_0-h\upsilon_0=(1/2)\,m(4\times10^8)^2$$

$$h\upsilon_o=\frac{1}{2}m(4\times10^8)^2 \qquad \text{(i)}$$

When $\quad v=5v_o,\ v_{max}=v'$

$$h(5v_o)-hv_o=\frac{1}{2}mv'^2 \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$v'=8\times10^8\text{ cms}^{-1}$$

7. b. $E-W_0=\dfrac{1}{2}mv^2=eV_s$

or $$\frac{hc}{\lambda}-W_0=eV_s$$

Hence, $$\frac{hc}{0.6\times10^{-6}}-W_0=e(0.5) \qquad \text{(i)}$$

and $$\frac{hc}{0.4\times10^{-6}}-W_0=e(1.5) \qquad \text{(ii)}$$

Solving, we get $W_0=1.5$ eV

8. c. $n = \dfrac{\text{power}}{hc/\lambda} = \dfrac{300 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}}$

$= 1.5 \times 10^{18}\ \text{m}^{-2}\ \text{s}^{-1} = 1.5 \times 10^{14}\ \text{cm}^{-2}\ \text{s}^{-1}$

As only 1 percent of photons cause emission of photo-electrons, number of photo electrons is

$$n_e = 1.5 \times 10^{12}\ \text{s}^{-1}$$

9. d. $\dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - W_0$ (i)

Let the speed of the fastest electron be V_1 when excitation wavelength is changed to $3\lambda/4$.

$\therefore \quad \dfrac{1}{2}mv_1^2 = \dfrac{4hc}{3\lambda} - W_0$

$\Rightarrow \quad \dfrac{1}{2}mv_1^2 = \dfrac{4}{3}\left(\dfrac{hc}{\lambda} - W_0\right) + \dfrac{W_0}{3}$

$\Rightarrow \quad \dfrac{1}{2}mv_1^2 = \dfrac{4}{3}\left(\dfrac{1}{2}mv^2\right) + \dfrac{W_0}{3}$ [using Eq. (i)]

$\Rightarrow \quad v_1^2 = \dfrac{4v^2}{3} + \dfrac{2W_0}{3m}$

$\therefore \quad v_1 > \sqrt{\dfrac{4}{3}}\,v$

10. b. $hv_1 - hv_0 = \dfrac{1}{2}mv_1^2$

$hv_2 - hv_0 = \dfrac{1}{2}mv_2^2$

$\therefore \quad h(v_1 - v_2) = \dfrac{1}{2}m(v_1^2 - v_2^2)$ [$\because v_1 = f_1$ and $v_2 = f_2$]

$\therefore \quad v_1^2 - v_2^2 = \dfrac{2h}{m}(f_1 - f_2)$

11. d. Since the number of photoelectrons emitted is directly proportional to the intensity of incident radiation, the number of photoelectrons emitted becomes four times.
The energy of photoelectrons does not change with the intensity light.

12. a. $\dfrac{hc}{\lambda} < W$ (for no emission) $\Rightarrow \lambda > \dfrac{hc}{W}$

13. c. $\dfrac{hc}{\lambda_{max}} = 3 \times 1.6 \times 10^{-19}\ \text{J}$

$\Rightarrow \quad \lambda_{max} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 1.6 \times 10^{-19}}$

or $\quad \lambda_{max} = 4.125 \times 10^{-7}\ \text{m}$

14. a. $K_{max} = hv - W$

ω is the intercept on y-axis and h is the slope.

$\therefore \quad h = \dfrac{2.4 \times 10^{-15}}{4 \times 10^{18}} = 6 \times 10^{-34}\ \text{Js}$

$W = 2 \times 10^{-15}\ \text{J}$

$\Rightarrow \quad h\nu_0 = 2 \times 10^{-15}$

or $\quad \nu_0 = 3.33 \times 10^{18}$

15. c. According to Einstein's equation,

$E = W_0 + \text{KE}$

$W_{0max} = 4.2\ \text{eV}$

$\text{KE} = 2.6\ \text{eV}$

$\therefore \quad E_{min} = W_{0max} + \text{KE} = (4.2 + 2.6)\ \text{eV} = 6.8\ \text{eV}$

16. a. As work function $W = h\upsilon_0$, where υ_0 is the threshold frequency.
Greater the work function, greater is the threshold frequency. Therefore, the threshold frequency of sodium will be lesser than that for aluminium.

17. b. Maximum KE depends on the frequency of incident radiation, not on intensity.

18. b. The maximum KE of the photoelectron is given by

$$\left(\frac{1}{2}mv^2\right)_{max} = h\upsilon - W$$

Now, $\quad \nu = \dfrac{c}{\lambda}$ and $\left(\dfrac{1}{2}mv^2\right) = eV$

$\therefore \quad eV = \dfrac{hc}{\lambda} - W$

or $\quad V = \left(\dfrac{hc}{e}\right)\dfrac{1}{\lambda} - \dfrac{W}{e}$

Slope of straight line $= hc/e$
Intercept of straight line $= -(W/e)$

19. b. The maximum KE of the photoelectron is given by

$$\left(\frac{1}{2}mv^2\right)_{max} = h\upsilon - W$$

Now, $\quad \upsilon = \dfrac{c}{\lambda}$ and $\left(\dfrac{1}{2}mv^2\right) = eV$

$\therefore \quad eV = \dfrac{hc}{\lambda} - W$ or $V = \left(\dfrac{hc}{e}\right)\dfrac{1}{\lambda} - \dfrac{W}{e}$

Since V is represented along y-axis and $(1/\lambda)$ along x-axis, the above equation represents a straight line.
Slope of straight line $= hc/e$
Intercept of straight line $= -(W/e)$

20. c. The ratio of electric force to magnetic force is:

$$\frac{F_e}{F_B} = \frac{qE}{qvB} = \frac{3 \times 10^6}{2.4 \times 10^6 \times 1.5} = \frac{5}{6}$$

Now, magnetic force on beta particles acts downward, whereas electric force acts upward and both are in the plane of diagram. But since magnetic force is larger, so beta particles are deflected downward.

21. d. By photoelectric equation $= \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ where λ is the wavelength of incident radiation and λ_0 is the threshold wavelength.

$$\frac{1}{2}m(2v)^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda_0}\right)$$

Solving for λ', we get $\lambda' = 300$ nm

22. b. By Einstein's particle (photon) theory, the maximum kinetic energy E_{max} of the emitted electrons from the cathode is proportional to the frequency f of the light. This is expressed in the Einstein's photoelectric equation below

$$E_{max} = hf - \phi = hf - hf_0$$
$$= h(f - f_0) \quad \text{(i)}$$

where h is the Planck's constant, ϕ_0 is the work function of the metal and is related to the threshold frequency f_0 by $\phi_0 = hf_0$. It is the minimum amount of work or energy necessary to take a free electron out of the metal against the attractive forces of surrounding positive ions.

At a particular negative potential difference V applied to the anode A, the current becomes zero. This is value of the negative potential difference which just stops the electrons with maximum energy from reaching A. V is called the stopping potential. Therefore,

$$eV = E_{max} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$eV = E_{max} = h(f - f_0)$$

$$V = \frac{h}{e}(f - f_0) \text{ or } y = \frac{h}{e}(x - x_0)$$

The variation of V (or y) is thus a straight line of gradient h/e, when it is plotted against f (or x) at f_0. It is best represented in graph b.

23. b. On increasing intensity, only saturation current increases, whereas retarding potential remains the same because wavelength of light is unchanged.

24. c. $eV_s = \frac{hc}{\lambda} - \phi_0$ or $eV_s + \phi_0 = \frac{hc}{\lambda}$

or $$\lambda = \frac{hc}{eV_s + \phi_0}$$

$\Rightarrow$ $$\frac{\lambda_2}{\lambda_1} = \frac{eV_{s_1} + \phi_0}{eV_{s_2} + \phi_0}$$

or $$\frac{4500}{4000} = \frac{1.3 + \phi_0}{0.9 + \phi_0}$$

or $$\phi_0 = 851.3 - 9 \times 0.9$$

Solving $$\phi_0 = (10.4 - 8.1) = 2.3 \text{ eV}$$

25. a. The photoelectrons will be emitted because wavelength of incident radiation is less than threshold wavelength ($\lambda < \lambda_0$)

26. b. $$\left(\frac{1}{2}mv^2\right)_{max} = h\nu - W$$

When υ is doubled (W remains same), $\left(\frac{1}{2}mv^2\right)_{max}$

i.e., (KE) is increased. The photoelectric current is directly proportional to the intensity of incident light.

27. c. Let $h\nu_0 - W_0 = K$ (i)

If frequency is doubled, let kinetic energy of photoelectrons be K_1.

$$2h\nu_0 - W_0 = K_1 \quad \text{(i)}$$
$$\Rightarrow \quad 2(h\nu - W_0) + W_0 = K_1$$
$$\Rightarrow \quad 2K + W_0 = K_1$$

i.e., kinetic energy is more than doubled.

28. c. If E is the energy of incident photon and W the work function, then $E - W_0$ = available energy.

$$E - W_0 = \frac{1}{2}mv^2$$

or $$v = \sqrt{\frac{2(E - W_0)}{m}}$$

$\therefore$ $$\frac{v_1}{v_2} = \sqrt{\frac{1 - 0.5}{2.5 - 0.5}} = \sqrt{\frac{0.5}{2}} = \frac{1}{2}$$

29. d. $\lambda_p = \lambda_\alpha$

or $$\frac{h}{\sqrt{2m_pQ_pV}} = \frac{h}{\sqrt{2m_\alpha Q_\alpha V_\alpha}}$$

$\therefore$ $$m_pQ_pV_p = m_\alpha Q_\alpha V_\alpha$$

$\therefore$ $$V_\alpha = \left(\frac{m_p}{m_\alpha}\right)\left(\frac{Q_p}{Q_\alpha}\right)V = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)V = \frac{V}{8}$$

30. b. Since $E \propto \frac{1}{\lambda}$, so energy corresponding to 5000 Å is

$$E = 2.46 \text{ eV}$$

Now, $$h\upsilon - W = eV_s$$

or $$2.46 \text{ eV} - W = 1.36 \text{ eV}$$

$$W = (2.46 - 1.36) \text{ eV} = 1.1 \text{ eV}$$

31. a. Change in intensity from I_0 to $4I_0$ does not affect the stopping potential.

32. c. When intensity is increased from I_0 to $4I_0$, i.e., four times, then the saturation current increases by a factor of 4, i.e., the saturation current becomes

$$= 4\times(0.40\times10^{-6})\ \text{A}$$

33. c. $V_s = 1.36$ V

$$\therefore \quad eV_s = 1.36 \text{ eV}$$

$$\text{or} \quad \frac{1}{2}m(v_{max})^2 = 1.36 \text{ eV}$$

i.e., various electrons have KE between zero and 1.36 eV.

34. d. Let energy corresponding to wavelength of 4000 Å be E. Then,

$$\frac{E}{E'} = \frac{\lambda'}{\lambda} \quad \text{or} \quad \frac{E}{1.23} = \frac{10,000}{4000}$$

$$\therefore \quad E = 1.23\times2.5 = 3.075 \text{ eV}$$

But $\quad h\upsilon - h\upsilon_0 = eV_s$

or $3.075 \text{ eV} - 1.1 \text{ eV} = eV_s$

$$\therefore \quad V_s = 1.975 \text{ V}$$

35. c. $I \propto 1/d^2$

When source is placed 2 m away, then $I' = (I/4)$. The number of electrons emitted $\propto$ intensity. Hence, the number of emitted electrons is reduced to one-fourth.

36. a. Energy radiated as visible light is

$$\frac{5}{100}\times100 = 5 \text{ J/s}$$

Let n be the number of photons emitted per second. Then,

$$\frac{nhv}{\lambda} = 5$$

$$\therefore \quad n = \frac{5\lambda}{hc} = \frac{5\times5.6\times10^{-7}}{(6.62\times10^{-34})(3\times10^8)}$$

$$= 1.4\times10^{19}$$

37. d. $\lambda = \dfrac{h}{p} \Rightarrow \lambda \propto \dfrac{1}{p}$

So, graph between λ and p is a rectangular hyperbola.

38. a. $hv = 5 \text{ eV} + 2.2 \text{ eV} = 7.2 \text{ eV}$

$$7.2 = \frac{12375}{\lambda(\text{in Å})}$$

$$\text{or} \quad \lambda(\text{in Å}) = \frac{12375}{7.2} \approx 1719$$

39. c. $\quad \phi_0 = h\upsilon_0 = \dfrac{hc}{\lambda_0}$

$$\phi_0\lambda_0 = \text{constant}$$

$$4.5\times\lambda = 2.3\times5460$$

$$\text{or} \quad \lambda = \frac{2.3\times5460}{4.5} \text{ Å}$$

$$= 2790.7 \text{ Å} \approx 2791 \text{ Å}$$

40. c.

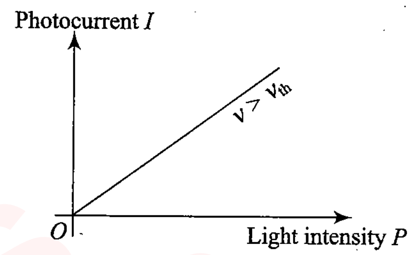


Fig. 3.51

The rate of emission of photoelectrons (i.e., photocurrent) depends linearly on the rate of incident photons.

41. d. Saturation current is inversely proportional to the square of distance of cathode from point source.

42. a. $p = \dfrac{h}{\lambda}$

Also, $E = \dfrac{hc}{\lambda}$

So, if λ is decreased, both p and E increase.

43. b. H discharge tube means max. $h\upsilon = 13.6$ eV. Work function = 4.2 V. So, (13.6 – 4.2) V = 9.4 V. So, required voltage is –9.4 V.

44. d. Work function $W_0 = \dfrac{hc}{\lambda_0}$

$$\Rightarrow \quad W_0 = \frac{2\times10^{-25}}{2\times10^{-7}} = 10^{-18} \text{ J}$$

$$= \frac{10^{-18}}{1.6\times10^{-19}} \text{eV} = 6.25 \text{ eV}$$

Energy of incident ratiation is $E = \dfrac{hc}{\lambda} = 31.25$ eV

$\therefore$ KE of photoelectrons $= E - W_0 = 25$ eV

45. d $n = \dfrac{W}{hc} = \dfrac{1.7\times10^{-18}\times6000\times10^{-10}}{6.6\times10^{-34}\times3\times10^8}$

$$= \frac{10^{-24}}{2\times10^{-25}} \approx 5$$

46. b. Sodium has low work function. So, maximum kinetic energy is more in the case of sodium. Thus, stopping potential is more for sodium.

47. a. $\dfrac{1}{2}mv^2 = eV$ or $v = \sqrt{\dfrac{2eV}{m}}$

$$= \sqrt{2\times1.6\times10^{11}\times200}\ \text{m s}^{-1} = 8\times10^6 \text{ms}^{-1}$$

48. d. Intensity reduced therefore saturation current reduced. Frequency increased, therefore stopping potential increased.

49. d. $\dfrac{1}{2}mv^2_{max} = \dfrac{hc}{\lambda} - \phi_0$

$$\frac{1}{2}mv^2_{max} = \frac{12375\text{ eV}}{3000} - 1\text{ eV}$$

$$\frac{1}{2}mv^2_{max} = 3.125 \times 1.6 \times 10^{-19}$$

$$v_{max} = \sqrt{\frac{2 \times 3.125 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \approx 10^6\text{ m s}^{-1}$$

50. c. Let us calculate energy corresponding to 0.2×10^{-6} m or $0.2 \times 10^{-6} \times 10^{10}$ Å

or $$2000\text{ Å} = \frac{12375}{2000} = 6.1875\text{ eV}$$

$$\frac{1}{2}mv^2_{max} = (6.1875 - 4)\text{ eV}$$

or $$mv^2_{max} = \frac{2 \times 2.1875 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

or $$mv^2_{max} = 0.769 \times 10^{12} = 0.876 \times 10^6$$
$$= 8.76 \times 10^5\text{ m s}^{-1}$$

51. b. In both the cases, the intensity is same.

52. c. Effective power $= \frac{25}{100} \times 200$ W
$= 50$ W

Now, $$50 = nh\nu = \frac{nhc}{\lambda}$$

$$n = \frac{50\lambda}{hc}$$

$$n = \frac{50 \times 0.6 \times 10^{-6}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 1.5 \times 10^{20}$$

53. a. $eV = h\nu - \phi_0$

$$eV = \left(\frac{12375}{2000} - 5.01\right)\text{eV}$$

$V = (6.1875 - 5.01)\text{ V} = 1.18\text{ V} \approx 1.2\text{ V}$

54. d. If q is the charge on the particle and V the potential difference through which it is accelerated, then

$$qV = \frac{1}{2}mv^2$$

or $$mv = \sqrt{2mqV}$$

de Broglie's wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

55. b. We know that mass m in motion and the rest mass m_0 is related through the equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $v = c$, $m = \frac{m_0}{\sqrt{1-1}} = \frac{m_0}{0} = \infty$

$\therefore$ de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{(\infty)(c)} = 0$$

56. b. $W = h\nu - eV_s$

$h\nu =$ energy of incident photon

Here $$h\nu = \frac{12400}{1240}\text{ eV} = 10\text{ eV}$$

$\therefore$ $$W = 10 - 8 = 2\text{ eV}$$

So, $\lambda_0 =$ Threshold wavelength

$$= \frac{12400}{2eV}\text{ Å} = 6200\text{ Å}$$

57. a $$\lambda_1 = \frac{12375}{E_1(eV)}\text{ Å} = 1000\text{ Å}$$

$\therefore$ $$E_1 = 12.375\text{ eV}$$

Similarly, $$\frac{12375}{\lambda_2(\text{Å})}\text{eV} = \frac{12375}{2000} = 6.1875\text{eV}$$

Now, $E_1 - W_0 = eV_s$

and $E_2 - W_0 = eV_S$

Hence, $12.375 - W_0 = 7.7$eV

and $6.1875 - W_0 = eV'_s$

Solving, we get $V'_s = 1.5$ V

58. c. Energy is given by

$$E = \frac{m_0c^2}{\sqrt{1 - (v^2/c^2)}}$$

or $$E^2 = \frac{m^2{}_0c^2}{c^2 - v^2}$$

Momentum p is given by

$$p = \frac{m_0 v}{\sqrt{1(v^2/c^2)}}$$

or $$p^2c^2 = \frac{m_0^2c^4v^2}{c^2 - v^2}$$

$\therefore$ $$E^2 - p^2c^2 = m_0{}^2c^4 \quad \text{or} \quad E^2 = p^2c^2 + m_0{}^2c^4$$

For photon, rest mass

$m_0 = 0$, so $E = pc$

For electron, $m_0 \neq 0$, so $E \neq pc$

59. b. $eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ (i)

$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(ii)}$$

Dividing Eq. (i) and (ii), we get $\lambda_0 = 4\lambda$.

60. c. $\frac{hc}{\lambda} = E + \phi_0$

$$\frac{hc}{\lambda'} = 2E + \phi_0$$

Dividing, we get $\frac{\lambda'}{\lambda} = \left(\frac{E+\phi_0}{2E+\phi_0}\right)$ or $\frac{\lambda'}{\lambda} < 1$

$\therefore \quad \lambda' < \lambda$ or $\lambda > \lambda'$ (i)

Also, $\frac{\lambda'}{\lambda} = \frac{1}{2}\left[\frac{E+\phi_0}{E+\frac{\phi_0}{2}}\right]$

or $\frac{\lambda'}{\lambda} > \frac{1}{2}$ or $\lambda' > \frac{\lambda}{2}$ (ii)

It follows from Eqs. (i) and (ii) that

$$\lambda > \lambda' > \frac{\lambda}{2}$$

61. d $\lambda = \frac{h}{\sqrt{2meV}}$

$$p = \sqrt{2meV}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{2m_e eV}{2m_\alpha(2e)V}}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{m_e}{2m_\alpha}}$$

62. c. $E = \frac{hc}{\lambda} - \phi_0$

$$2E = \frac{hc}{\lambda'} - \phi_0$$

Solving $\lambda' = \frac{hc\lambda}{E\lambda + hc}$

63. a. $E = \frac{hc}{\lambda/2} - \phi_0$

$$2E = \frac{hc}{\lambda/3} - \phi_0$$

or $2\left(\frac{2hc}{\lambda} - \phi_0\right) = \frac{3hc}{\lambda} - \phi_0$

or $\frac{4hc}{\lambda} - 2\phi_0 = \frac{3hc}{\lambda} - \phi_0$ or $\phi_0 = \frac{hc}{\lambda}$

64. d. Order of magnitude calculation is enough.

$$(2m_e eV)^{1/2} = (2\times 9\times 10^{-31}\times 100\times 1.6\times 10^{-19})^{1/2}$$

$$\approx 5\times 10^{-24}\ \text{kg ms}^{-1}$$

and $h \approx 6\times 10^{-34}$ Js

So, $\lambda \approx 10^{-10}$ m.

Mid-wavelength in visible region is

$$\lambda_0 \approx 5000\times 10^{-10}\ \text{m}$$

Thus, $\lambda = \lambda_0/5000$

65. c. In the first case,

$$\frac{1}{2}mv_{max}^2 = 2h\upsilon_0 - h\upsilon_0 = h\upsilon_0$$

In the second case,

$$\frac{1}{2}mv_{max}^2 = 5h\upsilon_0 - h\upsilon_0 = 4h\upsilon_0$$

Clearly, v_{max} is doubled.

66. a. V versus f has a constant slope of h/e, so both lines must be parallel.

Also, work function is equal to intercept on f-axis

67. c. $eV_s = h\upsilon - \phi_0$

$$eV_s' = h\upsilon' - \phi_0$$

$$e(V_s' - V_s) = h\upsilon' - h\upsilon = \left(\frac{12375}{3600} - \frac{12375}{4000}\right)$$

$\therefore V_s' - V_s = 3.44 - 3.09 = 0.35$ V

68. c. $8\times 10^{14}h = \phi_0 + 0.5$

$$12\times 10^{14}h = \phi_0 + 2$$

Dividing, we get $\frac{12}{8} = \frac{\phi_0 + 2}{\phi_0 + 0.5}$

$$\frac{3}{2} = \frac{\phi_0 + 2}{\phi_0 + 0.5}$$

$$3\phi_0 + 1.5 = 2\phi_0 + 4$$

or $\phi_0 = 2.5$ eV

69. a. $\lambda = \frac{h}{\sqrt{2mkT}}$

$$= \frac{6.62\times 10^{-34}}{\sqrt{2\times 1.67\times 10^{-27}\times 1.38\times 10^{-23}\,T}}\ \text{m}$$

$$= \frac{6.62\times 10^{-34}}{2.15\times 10^{-25}\sqrt{T}}\ \text{m} = \frac{3.079}{\sqrt{T}}\times 10^{-9}\ \text{m}$$

$$\frac{30.79}{\sqrt{T}}\ \text{Å} \approx \frac{30.8}{\sqrt{T}}\ \text{Å}$$

70. d. $\lambda = \frac{h}{mv}$

Here, $O \times M = m_1v_1 + m_2v_2$

Clearly, $m_1v_1 = -m_2v_2$

In magnitude,

$$mv = \text{constant}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{1}$$

71. a. $\lambda = \frac{0.286}{\sqrt{E(\text{in eV})}}$ Å

$$\sqrt{E}(\text{in eV}) = \frac{0.286}{0.4} \approx 0.707 \approx \frac{1}{\sqrt{2}}$$

$$\therefore \quad E(\text{in eV}) = 0.51$$

72. b. $\lambda = \frac{h}{\sqrt{2mE_e}} = \frac{hc}{E_{ph}}$ or $2mE_e = \frac{E_{ph}^2}{c^2}$

But $E_e = \frac{1}{2}mv^2$ or $m = \frac{2E_e}{v^2}$

$$\therefore \quad 2\left[\frac{2E_e}{v^2}\right]E_e = \frac{E_{ph}^2}{c^2}$$

or $\frac{4E_e^2}{v^2} = \frac{E_{ph}^2}{c^2}$ or $\frac{E_e^2}{E_{ph}^2} = \frac{v^2}{4c^2}$

or $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

73. c. $\frac{E_e}{E_{ph}} = \frac{\frac{c}{4}}{2c} = \frac{1}{8}$

74. b. $\frac{E_e}{E_{ph}} = \frac{\frac{h^2}{2m\lambda^2}}{\frac{hc}{\lambda}} = \frac{h^2}{2m\lambda^2} \times \frac{\lambda}{hc} = \frac{h}{2m\lambda c}$

$$= \frac{6.6 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 1.2 \times 10^{-10} \times 3 \times 10^8}$$

$$= \frac{1}{100}$$

75. a. $\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1 \times 1.6 \times 10^{-19}} \times 10^{10}$ Å

$= 12.375 \times 10^3$ Å

76. b. Energy of electron in n^{th} orbit, $E_n = -\frac{13.6}{n^2}$ eV

For first Bohr orbit, $n = 1$

$$E_1 = -13.6 \text{ eV}$$

For second Bohr orbit, $n = 2$

$$E_2 = -\frac{13.6}{4}\text{eV} \quad \text{or} \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1}{4}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{1}{2}$$

77. d. Each photon has associated with it an energy wave given by

$$E = hf = \frac{hc}{\lambda}$$

and graph of E vs. λ is a hyperbola.

Thus, $E \propto \frac{1}{\lambda}$

78. b. Energy corresponding to 2000 Å = 12375/2000 eV = 6.2 eV

Maximum kinetic energy is

$$(6.2 - 5.01)\text{ eV} = 1.19 \text{ eV}$$

Now, $\frac{1}{2} \times 9.1 \times 10^{-31} \times v_{max}^2$

$$= 1.19 \times 1.6 \times 10^{-19}$$

or $\quad v_{max}^2 = \frac{1.19 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$

or $\quad v_{max}^2 = 0.418 \times 10^{12} = 41.8 \times 10^{10}$

or $\quad v_{max} = 6.46 \times 10^5 \text{ m s}^{-1}$

79. c. $\lambda = \frac{h}{\sqrt{2mqV}}$

$$mV = \text{constant}$$

$$1837\, V' = 1 \times V$$

or $\quad V' = \frac{V}{1837}$ volt

80. c. $E_k = \frac{12375}{4000} - \phi = 3.1 - \phi_0$

$$2E_k = \frac{12375}{3100} - \phi = 3.99 - \phi_0$$

$$6.2 - 2\phi_0 = 3.99 - \phi_0$$

or $\quad \phi_0 = 6.2 - 3.99 = 2.21$ eV

81. a. $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$ J

Number of photons per second $\quad n = \frac{P}{E}$

$$\Rightarrow n = \frac{5 \times 10^{-3}}{3.14 \times 10^{-19}} = 1.6 \times 10^{16}$$

82. d. $\quad eV_s = h\nu - \phi_0$

$$V_s = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Now, $\frac{h}{e} = \text{slope} = \frac{1.656}{4 \times 10^{14}} = 0.414 \times 10^{-14}$ Vs

$$= 4.14 \times 10^{-15} \text{ Vs}$$

83. c. $\phi_0 = h\nu_0$

or $\phi_0 = e \times 4.14 \times 10^{-15} \times 1 \times 10^{14}$

$= 0.414$ eV

84. c. $K_A = \dfrac{hc}{\lambda_A} - \phi_0$; $K_B = \dfrac{hc}{\lambda_B} - \phi_0$

But $\lambda_A = 2\lambda_B$, therefore

$$\therefore \quad K_A = \frac{hc}{2\lambda_B} - \phi_0$$

$$K_A = \frac{1}{2}[K_B + \phi_0] - \phi_0$$

$$\text{or} \quad K_A = \frac{K_B}{2} - \frac{\phi_0}{2}$$

$$\therefore \quad K_A < \frac{K_B}{2}$$

85. b. $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$\frac{hc}{\lambda_{\min}} = \text{eV}$$

$$\lambda \times \frac{hc}{\lambda_{\min}} = \frac{h}{\sqrt{2meV}}\text{eV} \quad \text{or} \quad \frac{\lambda}{\lambda_{\min}} \propto \sqrt{V}$$

86. b. $\dfrac{1}{2}mv^2 = h\nu - \phi_0$

$$\frac{1}{2}mv'^2 = 4h\nu - \phi_0$$

$$\frac{v'^2}{v^2} = \frac{4h\nu - \phi_0}{h\nu - \phi_0}$$

$$\text{or} \quad \frac{v'^2}{v^2} = \frac{4[h\nu - \phi] + 3\phi_0}{h\nu - \phi_0}$$

Clearly, $v' > 2v$

87. b. Momentum imparted per unit time $= np$

$$\Rightarrow \quad F = \frac{nh}{\lambda}$$

$$\therefore \quad \text{Acceleration} = \frac{nh}{m\lambda}$$

88. a. Energy received by the eye,

$$E = \frac{nhc}{\lambda}$$

$$= \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$$

$$= 0.2 \times 10^{-13} \text{ Wm}^{-2}$$

So, eye is more sensitive by a factor of $\dfrac{1}{0.200} = 5.00$

89. c. $E = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 4 \times 1.67 \times 10^{-27} \times (0.1 \times 10^{-10})^2}$$

$$\times \frac{1}{1.6 \times 10^{-19}} \text{eV}$$

$$= \frac{43.82 \times 10^{-68}}{21.376 \times 10^{-68}} = 2.05 \text{ eV}$$

90. c. Kinetic energy is same, that settles for (c).
Intensity 4-fold, so n 4-fold.

91. b. The momentum of photon $= h/\lambda$

If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is :

$$P = n \times \frac{h}{\lambda}$$

Since the plate is blackened, all photons are absorbed by it.

$$\frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$$

$$\text{Since} \quad F = \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$$

$$\therefore \quad n = \frac{F\lambda}{h}$$

$$n = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

92. a. Since the molecules rebound from the wall, the component of velocity perpendicular to the wall is reversed, while its velocity parallel to the wall does not change. The change in velocity of molecules is parallel to normal N. The magnitude of change is

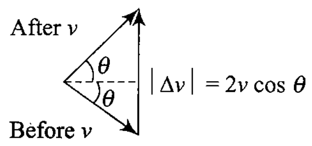


Fig. 3.52

$$|\Delta \vec{v}| = 2v\cos\theta$$

The change in momentum of a molecule is

$$|\Delta \vec{p}| = m|\Delta \vec{v}| = 2mv\cos\theta$$

in the direction of normal N. Let n be the number of molecules per unit volume. The number of molecules arriving at an area A of the wall per unit time is the number in a slanted cylinder whose length is equal to the velocity v and whose cross section is $A \cos\theta$.

Number of molecules $= n\,(Av\cos\theta)$

Each molecule suffers a change of momentum $2mv \cos\theta$.

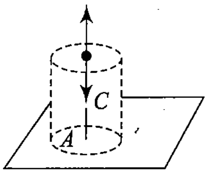
Change of momentum of a stream of gas in a direction perpendicular to the wall is equal to $(nAv\cos\theta) \times (2mv\cos\theta) = 2A\,nmv^2\cos^2\theta$.

Hence, force exerted on stream of gas by the wall,

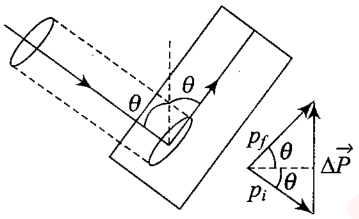
$$F = 2Anmv^2\cos^2\theta$$

This is also the force exerted by gas molecules on the wall.

$$\text{Pressure} = \frac{\text{Normal force}}{\text{Area}} = \frac{F}{A}$$
$$= 2nmv^2 \cos^2\theta$$



(a) Radiation pressure at normal incidence



(b) Radiation pressure at oblique incidence

Fig. 3.53

Remark: For oblique incidence, the change in momentum of the radiation per unit volume at the perfectly reflecting surface is $2p\cos\theta$ and the corresponding radiation pressure is

$$P_{rad} = 2pc\cos^2\theta = 2E\cos^2\theta$$

93. d. Momentum corresponding to incident protons normal to the surface,

$$\left(\frac{dP}{dt}\right)_{\text{incident}} = \frac{I}{c} dA\cos^2\theta$$

Since reflection coefficient is ρ, so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dP}{dt}\right)_{\text{reflected}} = -\frac{I}{c} dA\rho\cos^2\theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dP}{dt}\right)_{\text{photons}} = -\frac{I}{c} dA(\rho+1)\cos^2\theta$$

From Newton's third law,

$$\left(\frac{dP}{dt}\right)_{\text{surface}} = -\frac{I}{c} dA(\rho+1)\cos^2\theta$$

Hence, pressure exerted on surface,

$$P = \frac{dF}{dA} = \frac{I}{c} dA(\rho+1)\cos^2\theta$$

On substituting values, we get $P = 0.5\ \text{N cm}^{-2}$.

94. a. Imagine the sphere to be made of thin circular rings of radius r, thickness $ds = R\,dq$ and subtending an angle of θ at the center.

Momentum per second of incident photons,

$$\left(\frac{dP}{dt}\right)_{\text{incident}} = \frac{I}{c} dA\cos^2\theta$$

Since surface of mirror is considered to be ideal, i.e., reflection coefficient is unity, photons suffer momentum change in normal direction only.

$$\left(\frac{dP}{dt}\right)_{\text{photon}} = \frac{2I}{c} dA\cos^2\theta$$

$$dF_n = \left(\frac{dP}{dt}\right)_{\text{ball}} = +\frac{2I}{c} dA\cos^2\theta$$

This force may be resolved into horizontal and vertical components. The vertical component $dF_n \sin\theta$ is cancelled because every element on the upper half has a symmetrically placed element in the lower half. So, resultant force on the ball,

$$F = \int dF_n \cos\theta = \int \frac{2I}{c} dA\cos^3\theta$$

$$dA = (2\pi R\sin\theta)R\,d\theta$$

$$F = \int_0^{\pi/2} 4\pi\frac{I}{c}R^2\cos^3\theta\sin\theta\,d\theta$$

$$= \frac{4\pi R^2 I}{c}\int_0^{\pi/2}\cos^3\theta\sin\theta\,d\theta = \frac{\pi R^2 I}{c}$$

On substituting values, we get $F = 0.8\ \mu\text{N}$.

95. c. Photons have momentum $(P = h/\lambda)$ which they carry away; the spacecraft will acquire momentum in the opposite direction according to law of conservation of momentum.

No. of photons per second from laser = n

Then, from energy considerations,

$$0.5\times10^{-3} = nh\left(\frac{c}{\lambda}\right)$$

$$n = (0.5\times10^{-3})\lambda/(ch)$$

Rate of change of momentum of spacecraft

$$= np = n\frac{h}{\lambda} = (0.5\times10^{-3})\frac{\lambda}{ch}\left(\frac{h}{\lambda}\right) = \frac{0.5\times10^{-3}}{c}$$

From Newton's second law, $\frac{nh}{\lambda} = ma$

$$1000a = \frac{0.5\times10^{-3}}{3.00\times10^8} = \frac{1}{6}\times10^{-11}$$

$$v = at$$

$$t = \frac{v}{a} = \frac{1000}{\left(\frac{1}{1000}\right)\times\frac{1}{6}\times10^{-11}}\ \text{s} = 6\times10^{17}\ \text{s}$$

96. b. The gain of kinetic energy by an electron is eV.

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19})(50)}{(9.11\times10^{-31})}}$$

$$= 4.19\times10^6\,\text{ms}^{-1}$$

Thus, the electron's de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(4.19\times19^6)} = 1.74\times10^{-10}\,\text{m}$$

97. d. Magnetic force experienced by a charged particle in a magnetic field is given by

$$F_B = q\vec{v}\times\vec{B} = qvB\sin\theta$$

In our case, $F_B = qvB$ [as $\theta = 90°$]

Hence, $Bqv = \frac{mv^2}{r} \Rightarrow mv = qBr$

The de Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{qBr}$$

$$\frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

Since $\frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$$

98. a. Kinetic energy gained by a charge q after being accelerated through a potential difference V volt.

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$mv = \sqrt{2mqV}$$

de Broglie wavelength $= \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

Putting $V_\alpha = V_p$, $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4\times2}{1\times1}} = 2\sqrt{2}$

99. d. The electrons ejected with maximum speed V_{max} are stopped by electric field $E = 4N/C$ after traveling a distance $d = 1$ m.

$$\frac{1}{2}mv_{max}^2 = eEd = 4\,\text{eV}$$

The energy of incident photon $= \frac{1240}{200} = 6.2$ eV

From equation of photo electric effect

$$\frac{1}{2}mv_{max}^2 = h\nu - \phi_0$$

$$\therefore \quad \phi_0 = 6.2 - 4 = 2.2\,\text{eV}$$

100. a. We have $KE = \frac{P^2}{2m_e} = \frac{hc}{\lambda_{min}}$

$$P = \sqrt{\frac{2hcm_e}{\lambda_{min}}}$$

Also, $\lambda_{\text{de Broglie}} = \frac{h}{p} = \sqrt{\frac{h\lambda_{min}}{2m_e c}}$

For $\lambda_{min} = 10$ Å,

$\lambda_{\text{de Broglie}} \cong 0.3$ Å

101. c. $KE_{max} = h\nu - \phi$

$$\Rightarrow \frac{1}{2}mv^2_{max} = h\nu - \phi$$

$$\Rightarrow v_{max} = \sqrt{\frac{2(h\nu - \phi)}{m}}$$

Hence, (a) is incorrect.

Since $n = (IA/h\nu)$, therefore rate of emission of electrons is proportional to the intensity (I).

$$KE_{max} = h\nu - \phi$$

Hence, (c) is true.

102. b. Velocity at highest point $= u\sin\theta$

$$\therefore \quad \lambda_D = \frac{h}{mu\sin\theta}$$

(since θ is angle w.r.t. vertical)

103. b. The intensity of light at the location of your eye is

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi\times4^2}\ \text{W m}^{-2}$$

The energy entering into your eye per second is

$$P_1 = I\times\frac{\pi d^2}{4}$$

where d is the diameter of pupil.

$$P_1 = \frac{60}{4\pi\times4^2}\times\frac{\pi\times(2\times10^{-3})^2}{4}$$

$$= 9.375\times10^{-7}\,\text{Js}^{-1}$$

Let n be the number of photons entering into the eye per second, then

$$P_1 = n \times \frac{hc}{\lambda}$$

$$9.375 \times 10^{-7} = n \times \frac{1240 \times 1.6 \times 10^{-19}}{600}$$

$$n = 2.84 \times 10^{12} \text{ photon s}^{-1}$$

So, the number of photons entering the eye in 0.1 s = 0.1 n $= 2.84 \times 10^{11}$

104. b. Let m be the mass of each particle, then $\lambda_1 = (h/mv_1)$ and $\lambda_2 = (h/mv_2)$, where v_1 and v_2 are the velocities of two particles as shown in the figure.

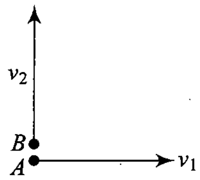


Fig. 3.54

$$\vec{v}_{CM} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Velocity of A w.r.t. C frame is,

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_{CM} = \frac{\vec{v}_1 - \vec{v}_2}{2}$$

$$|\vec{v}_{1c}| = \frac{\sqrt{\vec{v}_1 - \vec{v}_2}}{2} = |\vec{v}_{2c}|$$

So, required wavelength is

$$\lambda = \frac{h}{m|\vec{v}_{1c}|} = \frac{h}{m} \times \frac{2}{\frac{h}{m}\sqrt{\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}}$$

$$\lambda = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

105. b. The separation between source and photosensitive material at $t = 0$ is 16 m. Therefore, intensity received by photosensitive material at $t = 0$ is $I_0 = P/(4\pi \times 16^2)$, where P is the power of source of light.

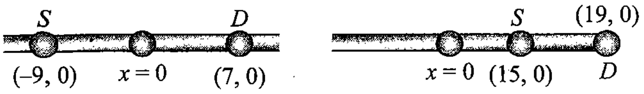


Fig. 3.55

At t = 3 s, the source is at (15, 0) and detector is at (19, 0), so the separation between them is 4 m.

$$I_2 = \frac{P}{4\pi \times 4^2}$$

So, $$\frac{I_1}{I_2} = \frac{1}{16}$$

106. d. $K_{max} = h\nu - \phi$

$$= \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.3 \times 10^{-6} \times 1.6 \times 10^{-19}} - 2.46\right) \text{eV}$$

$$= 1.68 \text{ eV}$$

Cut-off wavelength, $\lambda_0 = \frac{hc}{\phi} = 505$ nm

The minimum energy required to eject the photo-electrons is equal to work function.

107. a. Let m be the mass of particle.

$\frac{mv^2}{2} = \frac{hc}{\lambda_{photon}}$, where symbols have their usual meanings.

$$\frac{p^2}{2m} = \frac{hc}{\lambda_{photon}}$$

and $p = \frac{h}{\lambda_{particle}} \Rightarrow \frac{h^2}{2m\lambda^2_{particle}} = \frac{hc}{\lambda_{photon}}$

$$\Rightarrow \frac{\lambda_{photon}}{\lambda_{particle}} = \frac{2mc}{h} \times \lambda_{particle} = \frac{2mc}{h} \times \frac{h}{mv}$$

$$= \frac{2c}{0.05c} = 40$$

108. b. Resolving power is proportional to inverse of wavelength,

i.e., $R \propto \frac{1}{\lambda}$

and $\lambda \propto \frac{1}{P}$

So, $R \propto p = \sqrt{2mE}$

So, $R \propto \sqrt{E}$

$$\frac{R'}{R} = \sqrt{\frac{4kV}{16kV}} = \frac{1}{2}$$

$$R' = \frac{R}{2}$$

109. a. Option (a) correctly explains the photoelectric effect on the basis of electromagnetic theory. Its correct explanation is given by Quantum theory of light'.

110. d. Speed of electron which enters into electric field may increase or decrease while for 2nd electron, it remains constant.

So, from $\lambda = \frac{h}{mv'}$

$$\lambda_1 > \lambda_2 \text{ or } \lambda_1 < \lambda_2$$

111. b. $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$$\lambda_2 = \frac{hc}{E}$$

So, $$\frac{\lambda_1}{\lambda_2} \propto \frac{E}{\sqrt{E}} = E^{1/2}$$

112. a. When the positive potential of the ball is enough to hold back the most energetic photoelectron, the ball will not emit photoelectrons.

$$\frac{hc}{\lambda} - \phi = Ve$$

$$V = \frac{\frac{hc}{\lambda} - \phi}{e}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - 3.74$$

$$= 6.216 - 3.74 = 2.5 \text{ V}$$

113. a. $E = \frac{hc}{\lambda}$

Number of photons emitted is

$$\frac{Pt}{\left(\frac{hc}{\lambda}\right)} = n_0$$

$$n_0 = \frac{P\lambda t}{hc}$$

Since the radiation is spherically symmetric, so total number of photons entering the sensor is n_0 times the ratio of aperture area to the area of a sphere of radius ℓ.

$$N = n_0 \frac{\pi(2d)^2}{4\pi\ell^2} = \frac{P\lambda t}{hc} \frac{d^2}{\ell^2}$$

114. b. $\lambda_{ph} = \lambda_e$

$$\frac{h}{p_{ph}} = \frac{h}{p_e}$$

$$\frac{E_{ph}}{C} = \frac{2E_e}{v}$$

$$\frac{E_e}{E_{ph}} = \frac{v}{2C}$$

115. d. $\frac{2IA}{c} = F$ and (K_{eq}) parallel $= 3K$

$$\Delta X = \frac{2F}{3K} = \frac{4IA}{3Kc}$$

116. c. $\lambda \propto \frac{1}{V}$ and $V \propto \sqrt{T}$

117. c. Consider energy conservation and concept of impulse.

118. d. From conservation of linear momentum, both the particles will have equal and opposite momentum. The de Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow \lambda_1/\lambda_2 = 1$$

119. a.

$$mvr = n\frac{h}{2\pi}$$

$$= 2\left(\frac{h}{2\pi}\right) \quad (n = 2)$$

or $$mvr = \frac{h}{\pi}$$

$\therefore$ $$\lambda = \frac{h}{mv} = \text{de Broglie wavelength}$$

$$= \pi r = (3.14)(2.116 \text{ Å}) = 6.64 \text{ Å}$$

120. b. $h(v_1 - v_2) = e(V_1 - V_2)$

$$\frac{h}{e} = \frac{1}{c}\frac{(V_1 - V_2)}{\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)} = \frac{10^{-9}}{3 \times 10^8}\left(\frac{4.6 - 0.08}{\frac{1}{185} - \frac{1}{546}}\right)$$

$$= \frac{4.42 \times 185 \times 546 \times 10^{-17}}{361 \times 31}$$

$$= 4.12 \times 10^{-15} \text{ Js C}^{-1}$$

121. d. Momentum of photon of green light,

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{505 \times 10^{-9}} \; 1.3 \times 10^{-27} \text{ kg ms}^{-1}$$

Velocity of bacterium,

$$v = \frac{p}{m} = \frac{1.3 \times 10^{-27}}{9.5 \times 10^{-15}}$$

$$= 1.368 \times 10^{-13} \text{ ms}^{-1}$$

122. a. Energy of dissociation,

$$E_s = 10^5 \text{ J mol}^{-1}$$

Photon energy, $$E_p = \frac{E_S}{N_a} = \frac{10^5}{6.02 \times 10^{23}}$$

$$= 1.66 \times 10^{-19} \text{ J} = 1.04 \text{ eV}$$

Multiple Correct Answers Type

1. a., b., c. Existence of cut-off frequency and photoemission takes place even when intensity is low.

2. a., c. As the source is taken away, the intensity of light reaching the target decreases, and hence the photocurrent decreases.
But as motion of the source does not affect frequency of light, the stopping potential given by $V_0 = (hv/e) - (\phi/e)$ remains the same.

3. a., b., c. Intercept of straight line on negative energy axis gives the value of work function of the cathode metal. The point where the straight line cuts the frequency axis gives the value of threshold frequency while the slope of straight line provides the value of Planck's constant.

4. b., d. $eV_0 = E_K^{max} = \frac{hc}{\lambda} - W$

5. a., b., d. The energy of each photon is hc/λ, so that the number of photons released per unit time is $W/(hc/\lambda)$. These photons are spread out in all directions over an area $4\pi a^2$, so that the 'share' of an area S is a fraction $S/4\pi a^2$ of the total number of photons emitted.

The maximum energy of emitted photoelectrons is

$$E_{max} = hc - \phi = \frac{hc}{\lambda} - \phi = \frac{1}{\lambda}(hc - \lambda\phi)$$

The stopping potential is given by

$$eV_S = E_{max}$$

Hence, $$V_s = \frac{E_{max}}{e} = \frac{1}{e\lambda}(hc - \lambda\phi)$$

Hence, choice (c) is incorrect.
For photoemission to be possible, we have $hc \geq \phi$.

Hence, $$\frac{hc}{\lambda} \geq \phi \quad \text{or} \quad \lambda \leq \frac{hc}{\phi}$$

Thus, the permitted range of values of λ is

$$0 \leq \lambda \leq \frac{hc}{\phi}$$

Hence, the correct choices are (a), (b), and (d).

6. b, d. Since $P = \frac{I}{c} = 10^4\,\text{Nm}^{-2}$

$$P = \frac{E}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t}$$

$$\Delta p = PA\Delta t = 10^{-5}\,\text{kgms}^{-1}$$

7. a, c. Use Einstein's photoelectric equation.

8. a., b. For photoemission, $\lambda < \lambda_0$.

9. a., b., c., d. $\frac{1}{2}mv_{max}^2 = v - W$

Due to magnetic field,

$$\frac{mv_{max}^2}{r} = Bev_{max}$$

$$\Rightarrow \quad v_{max} = \frac{Ber}{m}$$

$$= \frac{5.26 \times 10^{-6} \times 1.6 \times 10^{-19} \times 0.5}{9.1 \times 10^{-31}}$$

$$= 0.46 \times 10^6\,\text{ms}^{-1}$$

$$(\text{KE})_{max} = \frac{1}{2}mv_{max}^2 = \frac{Bev_{max}r}{2}$$

$$= \frac{5.26 \times 10^{-6} \times 1.6 \times 10^{-19} \times 0.46 \times 10^6 \times 0.5}{2}$$

$$= 0.973 \times 10^{-19}\,\text{J} = 0.6\,\text{eV}$$

Energy of proton,

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eVnm}}{400\,\text{nm}} = 3.1\,\text{eV}$$

Work function, $W = 3.1\,\text{eV} - 0.6\,\text{eV} = 2.5\,\text{eV}$

$$(\text{KE})_{max} = eV_0 \quad \Rightarrow \quad V_0 = 0.6\,\text{V}$$

10. a., b., c. Cut-off wavelength, $\lambda_0 = 250$ nm
Threshold frequency,

$$v_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{250 \times 10^{-9}}\,\text{Hz} = 1.2 \times 10^{15}\,\text{Hz}$$

Work function of the metal,

$$W = \frac{hc}{\lambda_0} = \frac{1242\,\text{eV nm}}{250} = 4.968\,\text{eV}$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + K_{max}$$

$$K_{max} = \frac{1242\,\text{eV nm}}{100\,\text{nm}} - 4.968\,\text{eV}$$

$$= 7.432\,\text{eV} \approx 7.4\,\text{eV}$$

Photoelectric effect takes place only for light of wavelength less than 250 nm, whereas $\lambda_{red} \approx 700$ nm.

11. a., b., c., d. Energy of photon, $E_0 = \frac{hc}{\lambda}$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{652 \times 10^{-9}}$$

$$= 3.048 \times 10^{-19}\,\text{J} = 1.905\,\text{eV}$$

Energy content in each pulse is

$$0.6\,\text{W} \times 20 \times 10^{-3}\,\text{s}$$

$$E_p = 12 \times 10^{-3}\,\text{J} = 12\,\text{mJ}$$

The number of photons in each pulse is

$$\frac{E_p}{E_o} = \frac{12 \times 10^{-3}}{3.048 \times 10^{-19}}$$

$$= 3.9 \times 10^{16} \approx 4 \times 10^{16}$$

Assertion-Reasoning Type

1. c. $\lambda = \frac{h}{\sqrt{2mE}}$

2. a. Energy of photoelectrons emitted is different because after absorbing the photon, electrons within metals collide with other atoms before being ejected out of metal. Hence, Statement II is correct explanation of Statement I.

3. b. de Broglie wavelength associated with a gas molecule varies as, $\lambda \propto 1/\sqrt{T}$.

Comprehension Type

For Problems 1–3

1. b., **2.** a., **3.** a.

Sol. $\lambda_0 = 3800$ Å

$$W = hf_0 = h\frac{c}{\lambda_0} = \frac{6.633\times10^{-34}\times3\times10^8}{3800\times10^{-10}}$$

$$= 5.23\times10^{-19}\,\text{J} = 3.27\,\text{eV}$$

Incident wavelength $\lambda = 2600$ Å

$$f = \text{incident frequency} = \frac{3\times10^8}{2600\times10^{-10}}\,\text{Hz}$$

Then,

$$\text{KE}_{max} = hf - W_0$$

$$hf = \frac{6.63\times10^{-34}\times3\times10^8}{2600\times10^{-10}}$$

$$= 7.65\times10^{-19}\,\text{J} = 4.78\,\text{eV}$$

$$\text{KE}_{max} = hf - W_0 = 4.78\,\text{eV} - 3.27\,\text{eV} = 1.51\,\text{eV}$$

$$\text{KE}_{max} = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{2\text{KE}_{max}}{m}}$$

For Problems 4–6

4. d., **5.** b., **6.** c.

Sol.

4. Energy of a photon of wavelength λ,

$$E = h\nu = (hc/\lambda)$$

$$E = \frac{(6.6\times10^{-34})\times(3\times10^8)}{6000\times10^{-10}}$$

$$= 3.3\times10^{-19}\,\text{J}$$

So, if n is the number of photons emitted per second,

$$nE = \frac{\text{energy}}{\text{second}} = \text{power } (P)$$

Hence, $$n = \frac{P}{E} = \frac{100}{3.3\times10^{-19}}$$

$$\approx 3\times10^{20},\ \text{photons s}^{-1}$$

5. Intensity (i.e., energy flux) at a distance of 5 m from the source of power P,

$$I = \frac{P}{4\pi r^2} = \frac{100}{4\pi\times5^2} = \frac{1}{\pi}\frac{W}{m^2}$$

The number of photons passing normally per unit area per second, i.e.,

$$\text{Photon flux} = \frac{I}{E} = \frac{1/\pi}{3.3\times10^{-19}} \approx 10^{18}\ \text{photons m}^{-2}\,\text{s}^{-1}$$

6. Energy of photon

$$E = h\nu\frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{4000\times10^{-10}}$$

$$\approx 5\times10^{-19}\,\text{J}$$

So, number of photons emitted per second by light source,

$$n_P = \frac{P}{E} = \frac{1.5\times10^{-3}}{5\times10^{-19}} = 3\times10^{15}$$

Now, as only 0.1% of the photons emit electrons, number of photoelectrons emitted per second,

$$n_e = \frac{0.1}{100}\times3\times10^{15} = 3\times10^{12}$$

But current is the rate of flow of charge, i.e.,

$$I = \frac{q}{t} = \frac{Ne}{t} = n_e\times e \qquad \left[\text{as } \frac{N}{t} = n_e\right]$$

So, $$I = 3\times10^{12}\times1.6\times10^{-19} = 0.48\,\mu\text{A}$$

For Problems 7–8

7. c., **8.** a.

Sol. From Einstein's photoelectric effect equation,

$$\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$$

For $\lambda_1 = 3000$ Å,

$$\frac{hc}{3000\times10^{-10}} = \phi_0 + \frac{1}{2}m(3v)^2 \qquad \text{(i)}$$

For $\lambda_1 = 6000$ Å,

$$\frac{hc}{6000\times10^{-10}} = \phi_0 + \frac{1}{2}mv^2 \qquad \text{(ii)}$$

Now, on multiplying Eq. (ii) by 9 and subtracting Eq. (i) from it, we get

$$8\phi_0 = 9\frac{hc}{6000\times10^{-10}} - \frac{hc}{3000\times10^{-10}}$$

Substituting the values, we get

$$\phi_0 = 1.23\,\text{eV}$$

Maximum speed of the photoelectrons will be for the incident light of wavelength $\lambda = 3000$ Å. From Eq. (i)

$$\frac{1}{2}m(3v)^2 = \frac{6.62\times10^{-34}\times3\times10^8}{3000\times10^{-10}} - 2.896\times10^{-19}$$

$$= 3.724\times10^{-19}$$

$$\therefore \quad 3v = v_{max}\left(\frac{2\times3.724\times10^{-19}}{9.1\times10^{-31}}\right)^{1/2} = 9\times10^5\ \text{ms}^{-1}$$

For Problems 9–10

9. b., 10. a.

Sol. The energy of each photon is

$$E = hv$$

Since the wavelength and frequency are related to the speed of light by

$$c = v\lambda$$

$$E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34})(3.00\times10^8)}{632.8\times10^{-9}}$$

$$= 3.14\times10^{-19}\ \text{J}$$

$$1\ \text{eV} = 1.602\times10^{-19}\ \text{J}$$

$$E = \frac{3.14\times10^{-19}}{1.602\times10^{-19}} = 1.96\ \text{eV}$$

The number of photons emitted per second is equal to the energy emitted by the laser each second divided by the energy of one photon.

$$N = \frac{1.00\times10^{-3}}{3.14\times10^{-19}} = 3.18\times10^{15}\ \text{photon s}^{-1}$$

$$hc = (6.626\times10^{-34})(3\times10^8)$$

$$= 1.99\times10^{-25}\ \text{J m} = 1.24\times10^3\ \text{eV nm}$$

If a light of wavelength λ nm is incident, energy of photon, in eV, is

$$E = \frac{1.24\times10^3}{\lambda}$$

For Problems 11–13

11. a., 12. b., 13. a.

Sol. From Einstein's relation,

$$eV_s = hv - W$$

As work function is a constant for a surface,

$$e(V_{s_2} - V_{s_1}) = h(v_2 - v_1)$$

$$V_{s_2} = V_{s_1} + \frac{h}{e}(v_2 - v_1)$$

$$= V_{s_1} + \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= 0.19 + 1240\left(\frac{1}{190} - \frac{1}{550}\right) = 4.47\ \text{V}$$

$$W = \frac{hc}{\lambda_1} - eV_{s_1} = \frac{1240}{550} - 0.19 = 2.07\ \text{eV}$$

$$hv_c = W$$

$$v_c = \frac{W}{h} = \frac{(2.07)(1.602\times10^{-19})}{6.626\times10^{-34}} \approx 500\times10^{12}\ \text{Hz}$$

For Problems 14–15

14. d., 15. c.

Sol.

Energy of photon, $E = \dfrac{hc}{\lambda}$

$$= \frac{1.24\times10^3}{400} = 3.1\ \text{eV}$$

Remaining energy $= 3.1 - 0.31 = 2.79$ eV

Energy lost in first collision is

$$(3.1)\times\left(\frac{10}{100}\right) = 0.31\ \text{eV}$$

Remaining energy is

$$3.1 - 0.31 = 2.79\ \text{eV}$$

Energy lost in second collision is

$$(2.79)\times\left(\frac{10}{100}\right) = 0.279\ \text{eV}$$

Total energy lost in two collisions is

$$(0.31) + (0.279)\ \text{eV} = 0.589\ \text{eV}$$

So, from conservation of energy, we have

$$\frac{hc}{\lambda} = \phi + \text{KE}_{max}$$

+ energy lost in two collision

$$3.1 = 2.2 + \text{KE}_{max} + 0.589$$

$$\text{KE}_{max} = 0.31\ \text{eV}$$

Total energy after second collision is

$$(2.79 - 0.279) = 2.511\ \text{eV}$$

Energy lost in third collision is

$$2.511\times\frac{10}{100} = 0.2511\ \text{eV}$$

Remaining energy $= (2.511 - 0.2155)$

$$= 2.2599\ \text{eV}$$

Energy lost in fourth collision

$$= \left(2.2599\times\frac{10}{100}\right) = 0.2259\ \text{eV}$$

Remaining energy = (2.2599 – 0.2259) = 2.034 eV

After the fourth collision, the electron does not have enough energy to overcome the work function, so it cannot come out.

For Problems 16–18

16. d., 17. b., 18. a.

Sol. If P is the power of point source of light, the intensity at a distance r is

$$I = \frac{P}{4\pi r^2}$$

The energy intercepted by the metallic sphere is

$$E = \text{intensity} \times \text{projected area of sphere} = \frac{P}{4\pi r^2} \times \pi R^2$$

If e is the energy of the single photon and η the efficiency of the photon to liberate an electron, the number of ejected electrons is

$$\eta \frac{PR^2}{4r^2 e}$$

$$= \frac{(10^{-6})(3.2\times10^{-3})(8\times10^{-3})^2}{4\times(0.8)^2\times(5\times1.6\times10^{-19})}$$

$$= 10^5 \text{ electron/s}^{-1}$$

The emission of electrons from a metallic sphere leaves it positively charged. As the potential of the charged sphere begins to rise, it attracts emitted electrons. The emission of electrons will stop when the kinetic energy of the electrons is neutralised by the retarding potential of the sphere. So, we have

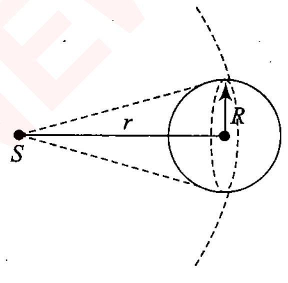


Fig. 3.56

$$eV = KE_{max}$$

$$V = \left(\frac{KE_{max}}{e}\right)$$

From Einstein's photoelectric equation,

$$KE_{max} = h\nu - \phi = (5-3) = 2\text{eV}$$

The potential of a charged sphere is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{R}\right)$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{R}\right) = 2$$

$$n = \frac{4\pi\varepsilon_0 2R}{e}$$

$$= \frac{2\times8\times10^{-3}}{9\times10^9\times1.6\times10^{-19}} = 1.11\times10^7$$

The photoelectric effect will stop when 1.11×10^7 electrons have been emitted.
The time taken by it to emit 1.11×10^7 electrons,

$$t = \frac{1.11\times10^7}{10^5} = 111\text{ s} = 1.85\text{ min}$$

For Problems 19–20

19. b., 20. a.

Sol. Pressure exerted by absorbed light $= \frac{1}{2}\left(\frac{S}{c}\right)$

Pressure exerted by reflected light $= \frac{1}{2}\left(\frac{2S}{c}\right)$

Total radiation pressure on the surface is

$$P_{rad} = \frac{\frac{3}{2}S}{c} = \frac{1.5\times10^3}{3\times10^8} = 5\times10^{-6}\text{ Pa}$$

$$\frac{P_{rad}}{P_0} = \frac{5\times10^{-6}}{1\times10^5} = 5\times10^{-11}$$

For Problems 21–22

21. d., 22. a.

Sol. Power of light received by the cone $= I(\pi R^2)$
Let number of photons hitting the cone per second is n.

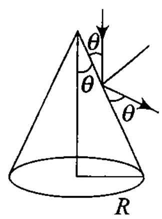


Fig. 3.57

Then, $nE = I\pi R^2 \Rightarrow n = \pi R^2 I/E$

By symmetry, the net force on the cone will be vertically downward.

Force due to one photon:

$$f = 2\frac{h}{\lambda}\sin\theta$$

This force is perpendicular to the surface of cone. Hence, net force on the cone will be

$$F = nf\sin\theta = n\left(2\frac{h}{\lambda}\sin\theta\right)\sin\theta$$

$$= \frac{\pi R^2 I}{c}(1-\cos 2\theta)$$

For Problems 23–27

23. c., **24.** c., **25.** d., **26.** c., **27.** b.

Sol.

23. $Q = CV \Rightarrow \quad ne = \frac{\varepsilon_0 A}{d}V$

$$n = \frac{2.85\times10^{-12}\times10}{0.5\times10^{-3}\times1.6\times10^{-19}}\times16$$

$$n = 8.85\times10^{9}$$

24. Equivalent resistance

$$R = \frac{V}{1} = \frac{16V}{2\times10^{-6}A}$$

$$= 8\times10^{6}\,\Omega$$

25. $P = \frac{nhC}{e\lambda}$

where n = no. of photons incident per unit time.

Also, $\quad I = ne$

$$\Rightarrow \quad P = \frac{IhC}{e\lambda}$$

$$\lambda = \frac{(2\times10^{-6})(6.6\times10^{-34})(3\times10^{8})}{(4\times10^{-6})(1.6\times10^{-19})}$$

$$= \frac{9.9}{1.6}\times10^{-7}\,\text{m} = \frac{9900}{1.6}\text{Å}$$

$$= 6187\,\text{Å}$$

Which is in the range of orange light.

26. The range of wavelength for red light is beyond the wavelength of incident light.

27. Stopping potential, $V_s = 8\text{V}$

and $\quad \text{KE} = eV_s$

$\therefore \quad \text{KE} = 8\text{eV}$

For Problems 28–30

28. a., **29.** b., **30.** c.

a. At any time t the location of electron is shown as P. In two dimensional view of electron in YZ-plane, the situation is more clear.

$$v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2(h\nu-\phi)}{m}}$$

$$p = v\cos\theta\frac{2\pi m}{qB_0}$$

$$pqB_0 = 2\pi\cos\theta\, m\sqrt{\frac{2(h\nu-\phi)}{m}}$$

$$= 2\pi\cos\theta = \sqrt{2m(h\nu-\phi)}$$

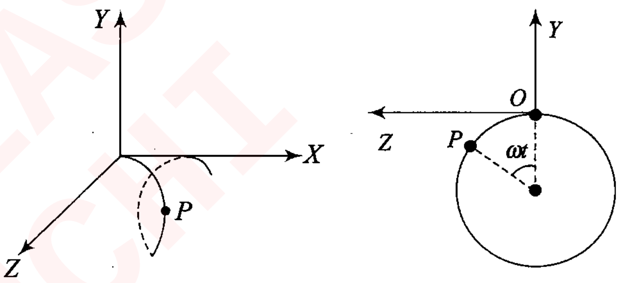


Fig. 3.58

X-coordinate, $x = v\cos\theta\times t$

Y-coordinate, $y = -[R - R\cos\omega t]$

Z-coordinate, $z = R\sin\omega t$

So, $$z = \frac{mv\sin\theta}{qB_0}\times\sin\left[\frac{qB_0}{m}t\right]$$

$$= \frac{\sqrt{2m(h\nu-\phi)}\times\sin\theta}{qB_0}\times\sin\left[\frac{qB_0 t}{m}\right]$$

From $x = v\cos\theta\times t = \frac{\sqrt{2(h\nu-\phi)}}{m}\times\cos\theta\times t$

As v increases, slope of x versus t graph (a straight line) increases.

For Problems 31–33

31. b., **32.** b., c., **33.** a.

Sol.

31. b.

$$\text{KE}_1 = h\nu - \phi$$

$$\text{KE}_2 = nh\nu - \phi$$

$$= n(h\nu-\phi) + (n-1)\phi$$

$$\text{KE}_2 = n\text{KE}_1 + (n-1)\phi$$

$$\text{KE}_2 < n\text{KE}_1$$

32. b, c. Stopping potential is the measurement of maximum kinetic energy of emitted photoelectrons and kinetic energy of emitted photoelectrons is linearly related with the frequency of incident light corresponding (i.e., corresponding to shortest wavelength, KE is maximum).
Stopping potential is independent of intensity.
From above explanation, it is clear that choices (a) and (d) are wrong.

33. $\lambda_{min} = \dfrac{hv}{eV}$

$$\lambda_{min} \propto \frac{1}{V}$$

As λ_{min} decreases, V increases. So, choice (a) is correct and the rest are incorrect.

For Problems 34–36

34. a., 35. a., 36. b.

Sol.

34. The effective area of Earth receiving radiation normally

$$= \pi\left(\frac{D}{2}\right)^2$$

$$= \pi\left(\frac{1.27\times10^4}{4}\right) \text{sq km}$$

Energy received by Earth per minute is

$$\frac{\pi}{4}(1.27)^2\times10^{18}\times2\times4.2$$

$$= 10.645\times10^{18}\,\text{J min}^{-1}$$

35. The area of surface surrounding sun at a distance equal to earth distance $4\pi d^2$
Energy radiated by sun (in J min^{-1}) is

$$2\times4.2\,[4\pi(1.49)^2\times10^{26}]\,\text{J min}^{-1}$$

$$= 2.3444\times10^{28}\,\text{J min}^{-1}$$

36. We know that 4H $\to$ He + Q
(As by fusion of four atoms of H_2, one Helium atom is formed and energy is released)

$\therefore\quad Q\,(\text{in a.m.u.}) = (4\times1.008145) - 4.003874$

$$= 0.028706\,\text{a.m.u.}$$

$$= 0.028706\times931\times1.6\times10^{-19}$$

Mass of hydrogen required is

$$\frac{4.032580\times1.659\times10^{-24}\times2.33\times10^{28}}{0.028706\times931\times1.6\times10^{-19}}$$

$$= 3.6673\times10^{16}\,\text{mega gram min}^{-1}$$

For Problems 37–39

37. b., 38. c., 39. a.

Sol.

37. The energy of the incident photon is

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{3300\times10^{-10}}$$

$$= 3.75\,\text{eV}$$

A will not emit photoelectrons because energy of incident photon is less than work function of A.

38. $eV_0 = h(v - v_0)$
When $V_0 = 0$, $v = v_0$, the threshold frequency.
From the graph it follows that

$$v_0 = 4\times10^{15}\,\text{Hz}$$

Therefore, work function is

$$\phi = hv_0 = 6.6\times10^{-34}\times4\times10^{15}$$

$$= 16.5\,\text{eV}$$

39. $eV_0 = hv - hv_0$

$$V_0 = \frac{h}{e}v - \frac{h}{e}v_0$$

$$Y = mn + e$$

$$\text{Slope} = \frac{h}{e}$$

For Problems 40–42

40. c., 41. b., 42. c.

Sol.

40. $$\frac{1}{2}mv^2 = ev$$

$$v = \sqrt{\frac{2ev}{n}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mev}}$$

Fringe width $\quad B = \dfrac{\lambda D}{d}$

$$= \frac{hD}{d\sqrt{2mev}}$$

41. As v increases, λ decreases.

42. Central maxima will remain at the same position.

For Problems 43–45

43. b., 44. b., 45. a.

Sol.

43. Energy required,

$$E = ms\,\Delta\theta + mL$$

$$= 2\times10^{-9}\times4.2\times10^3\times70 + 2\times10^{-9}\times2.25\times10^6$$

$$= 5.088\times10^{-3}\,\text{J} = 3.18\times10^{16}\,\text{eV}$$

44. Power output of the laser,

$$P = \frac{E}{t} = \frac{5.088 \times 10^{-3}}{450 \times 10^{-6}} = 11.3 \simeq 11 \text{ W}$$

45. $N = \frac{E}{h\nu} = \frac{E\lambda}{hc}$

$$= \frac{(3.18 \times 10^{16} \text{ eV})(585 \text{ nm})}{(1240 \text{ eV nm})} = 1.5 \times 10^{16}$$

Matching Column Type

1. a. → p., r.; b. → q., s.; c. → s.; d. → p., r.

a. From $eV_0 = hf - \phi$ and $K_{max} = hf - \phi$

If f increases keeping ϕ constant, then V_0 and K_{max} increase.

b. If I increases, more photons/time are incident on the metal surface and more photoelectrons would be liberated. Hence, saturation photocurrent increases. Stopping potential and KE_{max} will remain the same.

c. If separation between cathode and anode is increased, then there is no effect on V_0, K_{max} or current.

d. If ϕ decreases keeping f and I constant, then V_0 and K_{max} increase.

2. a. → s.; b. → p., q., r., s.; c. → q., r., s.; d. → p., r., s.

a. Maximum kinetic energy of ejected electron is given by Einstein's photoelectric equation $K_{max} = hf - (hf/3) = 2hf/3$. As no potential difference is applied across target and collector and vaccum is there in the tube, so this maximum KE remains same at all locations.

b, c and **d.** Kinetic energy of ejected photoelectrons can be anything from 0 to K_{max} (as found for a). It remains the same at all locations (reasoning is same as for above).

3. a. → r.; b. → p., q.; c. → p., q.; d. → s.

a. If intensity changes, then number of photons/time incident on the metal surface change and hence number of photoelectrons liberated change, so saturation photocurrent changes. Stopping potential and KE_{max} will remain the same.

b. From $eV_0 = hf - \phi$ and $K_{max} = hf - \phi$

If f changes, then V_0 and K_{max} change.

c. From $eV_0 = hf - \phi$ and $K_{max} = hf - \phi$

If target material changes, then ϕ changes, then V_0 and K_{max} change.

d. If we change the potential difference between emitter and collector, then time taken for electrons to eject changes.

4. a. → p., r.; b. → p., r.; c. → q.; d. → s.

Consider two equations.

$$eV_s = \frac{1}{2} m v^2_{max} = hf - \phi_0 \qquad \text{(i)}$$

No. of photoelectrons ejected per second ∝ intensity (ii)

a. As frequency is increased keeping intensity constant, V_S will increase and hence, $1/2m(v^2_{max})$ will increase.

b. As frequency is increased and intensity is decreased, V_s will increase and hence $1/2m(v^2_{max})$ will increase and saturation current will decrease.

c. It work function is increased, photoemission may stop.

d. If intensity is increased, then saturation current will increase.

5. a. → p., q., r., s.; b. → p., q., r., s.; c. → p, r.; d. → p.

a. Bichromatic light source is having two wavelengths and hence from energy point of view, two types of photons are possible. As light propagates in different directions, the photon can have different or same momenta depending upon the magnitude and direction of photon motion.

b. Same reasoning as for (a).

c. For a monochromatic light source, all photons have same energy but momenta can be different due to different directions.

d. Laser is a very narrow beam of monochromatic light, so all photons have nearly same energy and momenta.

Integer Answer Type

1. (1) $r \propto \frac{P}{q} \quad \left(\text{since, } r = \frac{P}{Bq}\right)$

where P = momentum, Given $r_\alpha = \frac{1}{2} r_e$

$$\frac{P_\alpha}{B2e} = \frac{1}{2}\left(\frac{P_e}{Be}\right) \text{ or } P_\alpha = P_e$$

$$\lambda \propto \frac{1}{P} \quad \left(\text{since, } \lambda = \frac{h}{p}\right)$$

So, $\lambda\alpha = \lambda_e$ or $n = 1$

2. (4) Speed of photon (c) = 3×10^8 ms^{-1}. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron is h/mv.

Given $\lambda = \frac{h}{mv}$. Now

$$\frac{\text{K.E. of photon}}{\text{K.E. of electron}} = \frac{hf}{(1/2)mv^2} = \frac{2hc}{mv^2\lambda} = \frac{2c}{v}$$

$$\left(\therefore \lambda = \frac{h}{mv}\right)$$

$$= \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

3. (5) For K_α X-ray, $(Z-1)^2 \lambda$ = constant. Hence,

$(9-1)^2\lambda = (Z-1)^2(4\lambda)$

$$(Z-1)^2 = \frac{64}{4} = 16$$

$Z-1=4$ or $Z=5$

4. (4) The energy of each photon $=\dfrac{200\text{ J/s}}{4\times10^{20}/\text{s}}=5\times10^{-19}$ J

Wavelength $=\lambda=\dfrac{hc}{E}$

$$=\frac{(6.63\times10^{-34}\text{ J-s})\times(3\times10^{8}\text{ m/s})}{(5\times10^{-19}\text{ J})}$$

$=4.0\times10^{-7}$ m

5. (1) $\lambda=663\times10^{-9}$m, $\theta=60°$, $n=1.0\times10^{19}$

$$P=\frac{h}{\lambda}=\frac{6.63\times10^{-34}}{6.63\times10^{-9}}=10^{-27}$$

Force exerted on the wall is

$$n\times2\times P\cos\theta=2\times1\times10^{19}\times10^{-27}\times\frac{1}{2}$$

$=1\times10^{-8}$N

6. (1) Let n photons (each of frequency f) per second are emitted from source. Then power of source is

$$P=nhf$$

But only 30% of the photons go towards mirrors.
Then force exerted on mirror is

$$F=2\left[\frac{30}{100}n\right]\frac{h}{\lambda}=\frac{3}{5}\frac{nhf}{c}=\frac{3}{5}\frac{P}{c}$$

and this force should be equal to weight of mirror, so

$$\frac{3}{5}\frac{P}{c}=20\times10^{-3}g$$

$$\Rightarrow\quad P=\frac{5\times3\times10^{8}\times20\times10^{-3}\times10}{3}=10^{8}\text{ W}$$

7. (3) Given $\lambda=200$ nm $=2\times10^{-7}$ m
Energy of one photon is

$$\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3\times10^{8}}{2\times10^{-7}}=9.945\times10^{-19}$$

Number of photons is

$$\frac{1\times10^{-7}}{9.945\times10^{-19}}=1\times10^{11}$$

Hence, number of photoelectrons emitted is

$$\frac{1\times10^{11}}{10^{4}}=1\times10^{7}$$

Net amount of +ve charge 'q' developed due to the outgoing electrons $=1\times10^{7}\times1.6\times10^{-19}=1.6\times10^{-12}$C.
Now potential developed at the centre as well as at the surface due to these charges is

$$\frac{Kq}{r}=\frac{9\times10^{9}\times1.6\times10^{-12}}{4.8\times10^{-2}}=3\times10^{-1}\text{ V}=0.3\text{ V}$$

8. (9) Given: Fringe width,
$y=1.0\text{ mm}\times2=2.0$ mm

$d=0.24$ mm, $W_0=2.2$ eV, $D=1.2$ m

$$y=\frac{\lambda D}{d}\quad\text{or}\quad\lambda=\frac{yd}{D}$$

$$=\frac{2\times10^{-3}\times0.24\times10^{-3}}{1.2}=4\times10^{-7}\text{ m}$$

$$E=\frac{hc}{\lambda}=\frac{4.14\times10^{-15}\times3\times10^{8}}{4\times10^{-7}}=3.105\text{ eV}$$

Stopping potential
$eV_0=3.105-2.2=0.905$ eV

$$V_0=\frac{0.905}{1.6\times10^{-19}}\times1.6\times10^{-19}\text{ V}=0.905\text{ V}$$

Archives

Fill in the Blanks Type

1. According to laws of photoelectric effect,

$$\frac{1}{2}mv^2=h\nu=h\nu_0$$

i.e., the maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the frequency of incident radiation.

True or False Type

1. False.

For photoelectric effect:

$$h\nu-h\nu_0=(\text{KE})_{max}$$

where $h=$ Planck's constant,
$\nu=$ frequency of incident radiation, and
$\nu_0=$ threshold frequency.

$\Rightarrow\quad(\text{K.E.})_{max}\propto n$

KE does not depend on the intensity of incident radiation.

2. False. $(\text{KE.})_{max}=h\nu-h\nu_0\Rightarrow(\text{KE})_{max}\propto\nu$
Thus, maximum kinetic energy is proportional to frequency and not intensity.

Multiple Choice Questions with One Correct Answer Type

1. b. Stopping potential is the negative potential applied to stop the electrons having maximum kinetic energy. Therefore, stopping potential will be 4 V.

2. c. $\lambda_{min}=\dfrac{hc}{\omega}=\dfrac{6.63\times10^{-34}\times3\times10^{8}}{4(1.6\times10^{-19})}=310\times10^{-9}$ m

3. c. Applying conservation of linear momentum:

Initial momentum = Final momentum

$$0=m_1v_1-m_2v_2\quad\Rightarrow\quad m_1v_1=m_2v_2$$

Now, $\dfrac{\lambda_1}{\lambda_2} = \dfrac{h/m_1v_1}{h/m_2v_2} = 1$

4. b. For photon: $E = h\nu$

or $E = \dfrac{hc}{\lambda}$

$\Rightarrow \quad \lambda_2 = \dfrac{hc}{E}$ (i)

For proton: $E = \dfrac{1}{2}m_p v_p^2$

$E = \dfrac{1}{2}\dfrac{m_p^2 v_p^2}{m} \Rightarrow P = \sqrt{2mE}$

From de Broglie equation,

$P = \dfrac{h}{\lambda_1}$

$\Rightarrow \quad \lambda_1 = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$ (ii)

$\dfrac{h_2}{\lambda_1} = \dfrac{hc}{E \times \dfrac{h}{\sqrt{2mE}}} \propto E^{-1/2}$

5. d. From the graph, it is clear that A and B have the same stopping potential and therefore the same frequency. Also, B and C have the same intensity.

6. c. Since electron shows wave nature, it will show the phenomenon of interference.

For electron: $\lambda = \dfrac{h}{mv}$

When speed of electron increases, λ will decrease.
The distance between two consecutive fringes

$= \beta = \dfrac{\lambda D}{d}$

As λ decreases, β also decreases.

7. c. $4\,{}_2^4He \rightarrow {}_8^{16}O$

$$\begin{aligned} \text{B.E.} &= \Delta m \times 931.5\ \text{MeV} \\ &= (4 \times 4.0026 - 15.9994) \times 931.5 \\ &= 10.24\ \text{MeV} \end{aligned}$$

Multiple Choice Questions with One or More than One Correct Answer Type

1. c., d. The threshold wavelength is 5200 Å. For ejection of electrons, the wavelength of the light should be less than 5200 Å so that frequency increases and hence the energy of incident photon increases. UV light has less wavelength than 5200 Å.

2. a., b., c. Standard result.

3. b., d. Since the stopping potential depends on the frequency and not on the intensity and the source is same, the stopping potential remains unaffected. The saturation current depends on the intensity of incident light on the cathode of the photocell which in turn depends on the distance of the source from cathode. The intensity of light is inversely proportional to the square of the distance between the light source and photocell.

Intensity, $I \propto 1/r^2$ and saturation current $\propto I$ (Intensity)

$\Rightarrow \quad \text{Saturation current} \propto \dfrac{1}{r^2}$

$\Rightarrow \quad \dfrac{(\text{Saturation current})_{\text{final}}}{(\text{Saturation current})_{\text{initial}}} = \dfrac{r_{\text{initial}}^2}{r_{\text{final}}^2}$

$\Rightarrow \quad (\text{Saturation current})_{\text{final}} = \dfrac{0.2 \times 0.2}{0.6 \times 0.6} \times 18 = 2\ \text{mA}$

4. a., b., c.

For metal A: $4.25 = W_A + T_A$ (i)

Also, $T_A = \dfrac{1}{2}mv_A^2 = \dfrac{1}{2}\dfrac{m^2v_A^2}{m} = \dfrac{p_A^2}{2m} = \dfrac{h^2}{2m\lambda_A^2}$ (ii)

$\left[\because \lambda = \dfrac{h}{p}\right]$

For metal B: $4.7 = (T_A - 1.5) + W_B$ (iii)

Also, $T_B = \dfrac{h^2}{2m\lambda_B^2} \times \dfrac{2m\lambda_A^2}{h^2} = \dfrac{\lambda_A^2}{\lambda_B^2}$

$\Rightarrow \quad \dfrac{T_A - 1.5}{T_A} = \dfrac{\lambda_A^2}{2\lambda_A^2} = \dfrac{\lambda_A^2}{4\lambda_A^2} = \dfrac{1}{4}$ [$\because \lambda_B = 2\lambda_A$ given]

$\Rightarrow \quad 4T_A - 6 = T_A \Rightarrow T_A = 2\text{eV}$

5. a., c. $\phi_1 : \phi_2 : \phi_3 = eV_{0_1} : eV_{0_2} : eV_{0_3}$

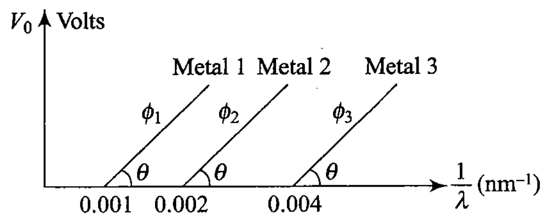


Fig. 3.59

$V_{0_1} : V_{0_2} : V_{0_3} = 0.001 : 0.002 : 0.004 = 1:2:4$

Therefore, option (a) is correct.

By Einstein's photoelectric equation, $\dfrac{hc}{\lambda} - \phi = \text{eV}$

$\Rightarrow \quad V = \dfrac{hc}{e\lambda} - \dfrac{\phi}{e}$ (i)

Comparing Eq. (i) by $y = mx + c$, we get the slope of the line

$m = \dfrac{hc}{e} = \tan\theta$

$\Rightarrow$ Option (c) is correct.

From the graph it is clear that,

$$\frac{1}{\lambda_{0_1}} = 0.001\ \text{nm}^{-1}$$

$$\Rightarrow \quad \lambda_{0_1} = \frac{1}{0.001} = 1000\ \text{nm}$$

Also, $\frac{1}{\lambda_{0_2}} = 0.002\ \text{nm}^{-1} \Rightarrow \lambda_{0_2} = 500\ \text{nm}$

and $\lambda_{0_3} = 250\ \text{nm}$

Violet color light will have wavelength less than 400 nm. Therefore, this light will be unable to show photoelectric effect on plate 3 $\Rightarrow$ Option (d) is wrong.

6. a., b. For $d = \lambda$, there will be only one central maxima. For $\lambda < d < 2\lambda$, there will be three maxima on the screen corresponding to path difference,

$$\Delta x = 0 \quad \text{and} \quad \Delta x = \pm\lambda$$

$\therefore$ Correct options are (a) and (b).

Comprehension Type

1. a. $a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$

$$\lambda_{\text{de Broglie}} = \frac{h}{p}$$

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8a^2m}$$

$$\Rightarrow \quad E \propto 1/a^2$$

2. b. $E = \frac{h^2}{8a^2m} = \frac{(6.6\times10^{-34})^2}{8\times(6.6\times10^{-9})^2\times10^{-30}\times1.6\times10^{-19}}$

$$= 8\ \text{meV}$$

3. d $mv = \frac{nh}{2a}$

or $v = \frac{nh}{2am} \Rightarrow v \propto n$

Integer Answer Type

1. (3) $\frac{1}{2}mv^2 = qV$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \sqrt{8} \simeq 3$$

2. (7) $R = 1$ cm

$f = 4.7$ cm

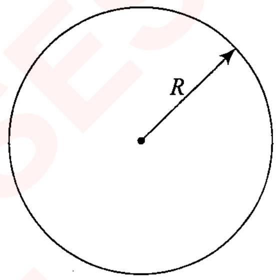


Fig. 3.60

$$\frac{hc}{\lambda} = \phi + eV$$

$$\frac{1240(ev)(nm)}{200(nm)} = 4.7(eV) + eV$$

$$\frac{1240}{200}e = 4.7e + eV$$

$$6.2 - 4.2 = V$$

$\therefore \quad V = 1.5$ Volt

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 1.5$$

$$(9\times10^9)\frac{Ne}{\frac{1}{100}} = 1.5$$

$$9\times10^{11}Ne = 15$$

$$N = \frac{1.5}{9\times10^{11}\times1.6\times10^{-19}} = \frac{15}{16}\times\frac{1}{9}\times10^8$$

$$= \frac{5}{3\times16}\times10^8 = \frac{50}{48}\times10^7$$

$\therefore \quad Z = 7$

CHAPTER 4

Atomic Physics

THOMSON'S ATOMIC MODEL

In 1897, the English physicist Joseph J. Thomson (1856–1940) established the charge-to-mass ratio for electrons. The following year, he suggested a model that describes the atom as a region in which positive charge is spread out in space with electrons embedded throughout the region, much like the seeds in a watermelon or raisins in thick pudding. The atom as a whole would then be electrically neutral.

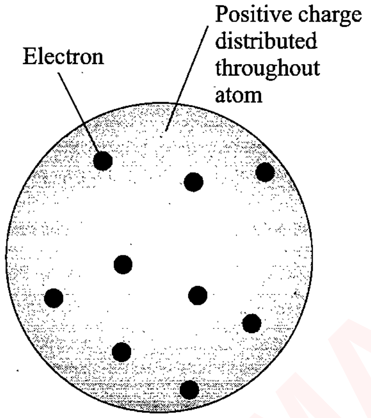


Fig. 4.1 Thomson's model of the atom: negatively charged electrons in a volume of continuous positive charge.

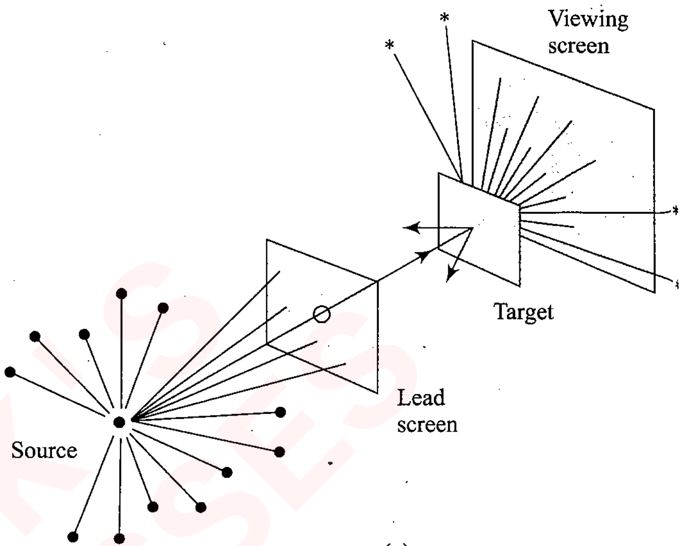
The nuclear atom is a relatively recent idea. In the early part of the twentieth century a widely accepted model, to proposed by Joseph J. Thomson, pictured the atom very differently. In Thomson's view

(i) Atom is a positively charged sphere of radius of the order of 10^{-10} m.

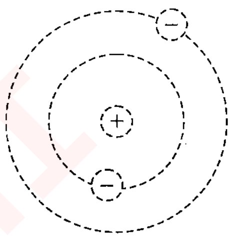
(ii) There was no nucleus at the center of an atom. Instead, the positive charge (and also mass) was assumed to be spread throughout the atom, forming a kind of "paste" or "pudding." The negative electrons were assumed to be suspended like "plums."

(iii) The number of electrons is such that their total negative charge is equal to the total positive charge. Hence, atom is electrically neutral.

The "plum–pudding" model was discredited in 1911 when the New Zealand physicist Ernest Rutherford (1871–1937) published the experimental results that this model could not explain the atom. As Fig. 4.2(a) indicates, Rutherford and his co-workers directed a beam of alpha particles (articles) at a thin metal foil made of gold. Alpha particles are positively charged particles (the nuclei of helium atoms, although this was not recognized at the time) emitted by some radioactive materials. If the "plum–pudding" model were correct, the α particles would be expected to pass nearly straight through the foil. After all, there is nothing in this model to deflect the relatively massive α particles, since the electrons have a comparatively small mass and the positive charge is spread out in a diluted "pudding." Using a zinc sulfide screen, which flashed briefly when struck by



(a)



(b)

Fig. 4.2 (a) Rutherford's technique for observing the scattering of alpha particles from a thin foil target. The source is a naturally occurring radioactive substance, such as radium. (b) Rutherford's planetary model of the atom.

an α particle, Rutherford and co-workers were able to determine that not all the α particles passed straight through the foil. Instead, some particles were deflected at large angles, even backward.

Rutherford concluded that the positive charge, instead of being distributed thinly and uniformly throughout the atom, was concentrated in a small region called the nucleus.

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. This model suffers from two defects:

1. Regarding Stability of Atom: An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should, therefore, emit radiations continuously and thereby lose energy. If this happened, the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of a second. But atoms do not collapse. In 1913, an effort was made by Neil Bohr to overcome this paradox.

2. Regarding Explanation of Line Spectrum: In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be changing. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

BOHR MODEL OF THE HYDROGEN ATOM

In 1913, Bohr presented a model that led to equations such as Balmer's for predicting the specific wavelength that the hydrogen atom radiates. Bohr's theory begins with Rutherford's picture of an atom as a nucleus surrounded by electrons moving in circular orbits, the necessary centripetal force to electrons is provided by the electrostatic force of attraction between nucleus and electrons. In analyzing this picture, Bohr made a number of assumptions in order to combine the new quantum ideas of Planck and Einstein with the traditional description of a particle in uniform circular motion.

1. Adopting Planck's idea of quantized energy levels, Bohr hypothesized that in a hydrogen atom there can be only certain values of the total energy (electron kinetic energy plus potential energy) for the electrons. These allowed energy levels correspond to different orbits for the electron as it moves around the nucleus, the larger orbits being associated with larger total energies. Figure 4.2(b) illustrates two of the orbits. In addition, Bohr assumed that an electron in one of these orbits does not radiate electromagnetic wave. For this reason, the orbits are called stationary orbits or stationary states. Bohr recognized that radiationless orbits violated the laws of physics. But the assumption of such orbits was necessary because the traditional laws indicated that an electron radiates electromagnetic waves as it accelerates around a circular path, and the loss of the energy carried by the waves would lead to the collapse of the orbit.
2. To incorporate Einstein's photon concept, Bohr theorized that a photon is emitted only when the electron changes orbits from a larger one with higher energy to a smaller one with lower energy. But how do electrons get into the higher-energy orbits? They get there by picking up energy when atoms collide, which happens more often when a gas is heated, or by acquiring energy when a high voltage is applied to a gas.

   When an electron in an initial orbit with a large energy E_i changes to a final orbit with a smaller energy E_f, the emitted photon has an energy of $E_i - E_f$, consistent with the law of conservation of energy. But according to Einstein, the energy of a photon is hf, where f is its frequency and h is Planck's constant and since the frequency of an electromagnetic wave is related to the wavelength by $\lambda = c/f$, Bohr could use this equation to determine the wavelengths radiated by a hydrogen atom. First, however, he had to derive expressions for the energies E_i and E_f.
3. Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be an integral multiple of $h/2\pi$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate, $mvr = \dfrac{nh}{2\pi}$ $\quad (n = 1, 2, 3....)$

Radius of Orbit

We have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{(Ze)(e)}{r^2} \qquad \text{(i)}$$

and

$$mvr = \frac{nh}{2\pi} \qquad \text{(ii)}$$

From Eq. (ii), $v = \dfrac{nh}{2\pi mr}$

Substituting the value of v in Eq. (i), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z} \quad \text{or} \quad r_n = (0.53)\frac{n^2}{Z}\ \text{Å}$$

So, for H-like atoms, $r_n \propto \dfrac{n^2}{Z}$.

Stationary orbits are not equally spaced.

Velocity of Electron in n^{th} Orbit

Since $\quad v = \dfrac{nh}{2\pi mr}$ and $r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2Z}$

Put the value of r to get

$$v = \frac{Ze^2}{2\varepsilon_0 nh} \quad \text{or} \quad v = \left(\frac{e^2}{2\varepsilon_0 ch}\right)\left(\frac{cZ}{n}\right)$$

or

$$v = \alpha\left(\frac{cZ}{n}\right)$$

where $\alpha = e^2/2h\varepsilon_0 c$ is the Summerfield's fine structure constant (a pure number) whose value is 1/137.

$$\therefore \quad v = \frac{1}{137}\left(\frac{cZ}{n}\right)$$

i.e., velocity of electron in Bohr's first orbit of hydrogen ($Z = 1$) is $c/137$, in second orbit is $c/274$, and so on.

Orbital Frequency of Electron

$$f = \frac{v}{2\pi r}$$

Put the value of v and r to get: $f = \dfrac{mZ^2e^4}{4\varepsilon_0^2n^3h^3} \Rightarrow f \propto \dfrac{Z^2}{n^3}$

Illustration 4.1 **The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of the two energy states. Assume the Bohr's model to be valid. The time period of the electron in initial state is eight times that in the final state. What are the possible values of n_1 and n_2?**

Sol. Here, $Z = 1$. We can derive the expression for time period of electron in n^{th} orbit as, $T_n = \frac{1}{f_n} \propto n^3$

Hence, $\frac{T_1}{T_2} = \frac{n_1^3}{n_2^3}$. As $T_1 = 8T_2$, therefore we get

$$8 = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow n_1 = 2n_2$$

Thus, the possible values of n_1 and n_2 are $n_1 = 2, n_2 = 1; n_1 = 4, n_2 = 2; n_1 = 6, n_2 = 3$; and so on.

Illustration 4.2 **How many times does the electron go round the first Bohr orbit of hydrogen atoms in 1 s?**

Sol. Here, we have to find the frequency which is given as

$$f = \frac{mZ^2e^4}{4\varepsilon_0^2 n^3 h^3}$$

$$\Rightarrow \quad f = \frac{9.1 \times 10^{-31} \times 1^2 \times (1.6 \times 10^{-19})^4}{4 \times (8.85 \times 10^{-12})^2 \times 1^3 \times (6.6 \times 10^{-34})^3}$$

$$= 6.62 \times 10^{15} \text{ Hz}$$

Illustration 4.3 **What is the angular momentum of an electron in Bohr's hydrogen atom whose energy is –3.4 eV?**

Sol. First, we need to identify the quantum number of the energy level.

As $E = -\frac{13.6}{n^2}$, we have

$$-3.4 = -\frac{13.6}{n^2} \quad \text{or} \quad n = 2$$

The angular momentum quantization gives, $L = mvr = \frac{nh}{2\pi}$

Substituting $n = 2$, we get $L = \frac{2h}{2\pi} = \frac{h}{\pi}$

Illustration 4.4 **If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$ (Bohr radius $a_0 = 5.3 \times 10^{-11}$m)?**

Sol. The angular momentum of an electron in n^{th} orbit $= \frac{nh}{2\pi}$

By Bohr's hypothesis: Again, $mvr = \frac{nh}{2\pi}$

where r is the radius of the orbit.

$$\text{or} \quad v = \frac{nh}{2\pi mr} \quad \text{(i)}$$

The time period to completing an orbit

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh}$$

$$\text{or} \quad T = \frac{4\pi^2 mr^2}{nh} \quad \text{(ii)}$$

Since the radius of the orbit is proportional to n^2, hence

$$r = a_0 n^2$$

$$\therefore \quad T = \frac{4\pi^2 m a_0^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

Number of orbits completed in 10^{-8} s

$$= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3} = \frac{10^{-8} \times (6.6 \times 10^{-34})}{4(3.14)^2(9.1 \times 10^{-31})(5.3 \times 10^{-11})^2(2)^3}$$

$$= 8 \times 10^6$$

Energy of Electron in n^{th} Orbit

Kinetic Energy: Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$\text{or} \quad \text{KE} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

Potential Energy: $U = -\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r}$ or $U = -\frac{Ze^2}{4\pi\varepsilon_0 r}$

Total Energy: $\text{E} = \text{KE} + \text{PE}$ or $E = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$

$$\text{or} \quad E = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

So, we conclude that Total Energy $= -\text{KE} = \frac{1}{2}$(PE)

Using $r = \frac{n^2 h^2 e_0}{pme^2 Z}$, we get

$$E = -\left(\frac{me^4}{8h^2 e_0^2}\right)\frac{Z^2}{n^2} \quad \text{or} \quad E = -(13.6)\frac{Z^2}{n^2}\text{eV}$$

$$\text{Also} \quad E = -\left(\frac{me^4}{8e_0^2 ch^3}\right)ch\frac{Z^2}{n^2} \quad \text{or} \quad E = -(Rch)\frac{Z^2}{n^2}$$

where R = Rydberg's constant $= \frac{me^4}{8e_0^2 ch^3}$

$= 1.097 \times 10^7 \text{ m}^{-1}$ and

Substituting values of m, e, ε_0, and h with $n = 1$, we get the least energy of the atom in first orbit, which is –13.6 eV.

Rch = Rydberg's energy $\approx 2.17 \times 10^{-18}$ J $\approx$ 13.6 eV is the electron energy in first orbit of H atom.

Hence, $$E_1 = -13.6 \text{ eV} \qquad \text{(i)}$$

and $$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}\text{eV} \qquad \text{(ii)}$$

Substituting n = 2, 3, 4, ..., etc., we get energies of atom in different orbits.

The significance of negative sign in equation (ii) is that the electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Giving different values to n, we can calculate the orbital energy or binding energy of the electron in different orbitals.

$E_1 = -13.6$ eV where n = 1 (K-shell)

$E_2 = -3.4$ eV n = 2 (L-shell)

$E_3 = -1.5$ eV n = 3 (M-shell)

.

.

.

$E_\infty = 0$ eV $n = \infty$ (Limiting case)

Now, we shall consider the energy level diagram. This diagram can be drawn in terms of various electron orbits to the scale of their radii but it is customary to draw horizontal lines to the energy scale as shown in Fig. 4.3.

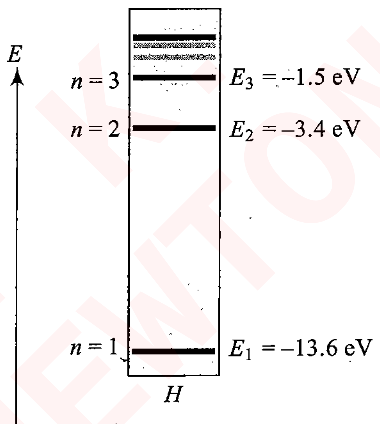


Fig. 4.3

The diagram is known as energy level diagram. The lowest energy level (n = 1) corresponds to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram, the lower energies (more negative) are at the bottom while higher energies (less negative) are at the top. By such a consideration, the various electron jumps between allowed orbits will be vertical arrows between different energy levels. The energy of radiated photon is greater when the length of arrow is greater.

Frequency of Emitted Radiation

If electron makes a transition (jumps) from final state 'n_f' to the initial state 'n_i', then frequency of emitted radiation f is given by

$$hf = E_f - E_i \quad \text{or} \quad f = \frac{E_f - E_i}{h} = -Z^2Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If λ is the wavelength of emitted radiation, then

$$f = \frac{c}{\lambda} = -Z^2Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

So, reciprocal of wavelength or wave number ($\bar{\nu}$) is given by

$$\bar{\nu} = \frac{1}{\lambda} = -Z^2R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

This relation holds for radiations emitted by hydrogen-like atoms, i.e.,

H (Z = 1), He^+(Z = 2), Li^{++}(Z = 3), and Be^{+++}(Z = 4)

If the electron makes a transition from n = 1 to the higher states, then absorption of radiation takes place.

Illustration 4.5 **Consider energy level diagram of a hydrogen atom. How will the kinetic energy and potential energy of electron vary if the electron moves from a lower level to a higher level ?**

Sol. Kinetic energy of an electron in a hydrogen atom is given by

$$K_n = \frac{e^2}{8\pi\varepsilon_0 r_n} = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} = \frac{K_0}{n^2}, \quad \text{where} \quad K = \frac{me^4}{8\varepsilon_0^2 h^2}$$

Potential energy is given by

$$U_n = -\frac{e^2}{4\pi\varepsilon_0 r_n} = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2} = -\frac{2K_0}{n^2}$$

The kinetic energy is always positive and it is inversely proportional to square of n. Hence, as the electron is excited to higher states its kinetic energy decreases.

The potential energy is always negative for a particle in bound state. The magnitude of potential energy is twice that of kinetic energy. As the electron is raised to higher energy level, n increases. The potential energy becomes less negative that means the potential energy actually increases. Potential energy is maximum for $n \to \infty$, which corresponds to infinite separation between nucleus and electron.

Bohr Model to Define Hypothetical Atomic Energy Levels

Originally, Bohr's model was to define only different energy levels for hydrogen atom.

According to Bohr's first postulate: $$\frac{KZe^2}{r_n^2} = \frac{mv_n^2}{r_n} \qquad \text{(i)}$$

and from Bohr's second postulate: $$mv_n r_n = \frac{nh}{2\pi} \qquad \text{(ii)}$$

This equation gives explanation for balancing the inward coulombian force on electron by the outward centrifugal force on it in the rotating frame of reference. There may be some situation in which it is given that electron of an atom is revolving under the influence of a hypothetical potential energy field function given as $U = f(r)$ [non-coulombian field] and using Bohr model, we can develop the properties of the electron in this new atom. For this, first we will develop the first postulate equation for this new atom. As the electron is orbiting in a new potential energy field given as $U = f(r)$, it will experience an inward force toward the center of orbit given as

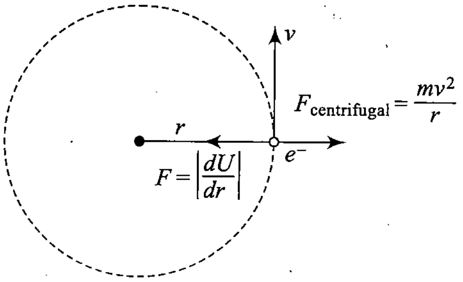


Fig. 4.4

As we know, $$F = -\frac{\partial U}{\partial r}$$

The magnitude of force can be written as

$$F = \left|\frac{dU}{dr}\right| = \left|\frac{d}{dr}[f(r)]\right| \qquad \text{(iii)}$$

Thus, for a stable n^{th} orbit if electron is moving with speed v_n in the orbit of radius r_n, we have

$$\left|\frac{d}{dr}[f(r_n)]\right| = \frac{mv_n^2}{r_n} \qquad \text{(iv)}$$

This, equation (iv), will be the new equation for first postulate and from Bohr model the second postulate is based on quantization of angular momentum of electron, its equation will remain same as

$$mv_n r_n = \frac{nh}{2\pi} \qquad \text{(v)}$$

Now, using Eqs. (iv) and (v) we can derive all the properties for electron motion like radius of n^{th} orbit, velocity of electron in n^{th} orbit, angular velocity, frequency, time period, current, magnetic induction, magnetic moment, and the total energy of energy levels for this hypothetical atom in the same way we have derived these for properties for a general hydrogenic atom. Let us discuss few examples to understand these concepts in a better way.

Illustration 4.6 **Suppose potential energy between electron and proton at separation r is given by $U = k \log r$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of n^{th} Bohr's orbit and its energy levels.**

Sol. For a conservative force field, $F = -\dfrac{dU}{dr} = -\dfrac{k}{r}$

This force $F = -k/r$ provides the centripetal force for the circular motion of electron.

So, $$\frac{mv^2}{r} = \frac{k}{r} \Rightarrow m^2v^2 = mk \Rightarrow mv = \sqrt{mk} \qquad \text{(i)}$$

Applying Bohr's quantization rule, $$mvr = \frac{nh}{2\pi} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $$r = \frac{nh}{2\pi\sqrt{mk}}$$

From Eq. (i), KE of electron $$= \frac{1}{2}mv^2 = \frac{1}{2}k$$

Total energy of electron = KE + PE $$= \frac{1}{2}k + k\log r$$

$$= \frac{k}{2}\left[1 + \log\frac{n^2h^2}{4\pi^2 mk}\right]$$

HYDROGEN-LIKE ATOMS

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}), and so on. Such atoms are called *hydrogen-like atoms*.

The Bohr model of hydrogen can be extended to hydrogen-like atoms, i.e., one electron atoms, the nuclear charge is $+Ze$, where Z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 every where by Ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2} = a_0\frac{n^2}{Z} \quad \text{or} \quad r_n \propto \frac{n^2}{Z} \qquad \text{(i)}$$

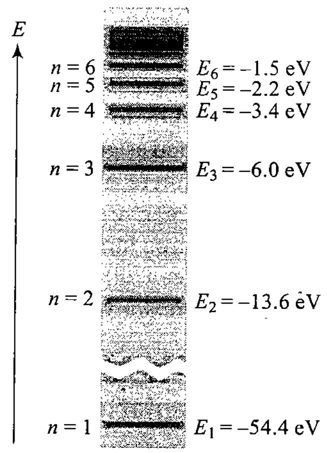
where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh} = \frac{Z}{n}v_1 \quad \text{or} \quad v_n \propto \frac{Z}{n} \qquad \text{(ii)}$$

where $v_1 = 2.19 \times 10^6\ \text{ms}^{-1}$ (speed of electron in first orbit of H)

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{Z^2}{n^2}E_1 \quad \text{or} \quad E_n \propto \frac{Z^2}{n^2} \qquad \text{(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)



He^+

Energy level diagram of He

Fig. 4.5

Important Results

For single-electron system

Ground state: Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = –13.6 eV
Ground state energy of He^+ ion = –54.4 eV
Ground state energy of Li^{++} ion = –122.4 eV

Illustration 4.7 **Determine the minimum wavelength of a photon that can cause ionization of He^+ ion.**

Sol. The ground state energy of He^+ is given by the equation

$$E_n = -(13.6)\left(\frac{Z^2}{n^2}\right)$$

With $n = 1$ and $Z = 2$, we get $E_1 = -(13.6)(4) = -54.4$ eV

Thus, to ionize the He^+ ion, we should require 54.4 eV of energy. The minimum wavelength of photon that can cause ionization will have energy $E = hf = 54.4$ eV and wavelength

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(54.4)(1.6\times10^{-19})} = 22.8 \text{ nm}$$

If the atom absorbed a photon of greater energy (wavelength less than 22.8 nm), the atom could still be ionized and the freed electron would have some kinetic energy when it is away from the influence of He^+ ion.

Illustration 4.8 **Find the quantum number 'n' corresponding to the excited state of He^+ ion if on transition to the ground state that ion emits two photons in succession with wavelengths 1026.7 and 304 Å. (R = 1.096 × 10^7/m)**

Sol. Given $\dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2} = RZ^2\left(1 - \dfrac{1}{n^2}\right)$

$$\frac{1}{n^2} = 1 - \left[\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2} \times \frac{1}{RZ^2}\right]$$

$$= 1 - \frac{1330.7\times10^{-10}}{1026.7\times304\times10^{-20}\times4\times1.096\times10^{7}}$$

Thus, $\dfrac{1}{n^2} = 0.0275 \Rightarrow n = 6.03$

Hence, the quantum number = 6

Illustration 4.9 **A gas of hydrogen-like atoms can absorb radiations of 698 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal to or smaller than that of the absorbed photon.**

a. Determine the initial state of the gas atoms.
b. Identify the gas atoms.
c. Find the minimum wavelength of the emitted radiation.
d. Find the ionization energy and the respective wavelength for the gas atoms.

Sol. a. Since three radiations are emitted, therefore the final excited state of the gas is $n = 3$.
The initial state of the gas atoms is $N = 2$ as all the wavelengths are smaller and the energy will be higher.

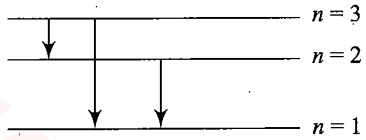


Fig. 4.6

b. $13.6Z^2\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right] = 68$

or $13.6Z^2\left[\dfrac{5}{36}\right] = 68 \Rightarrow Z = 6$

c. The minimum wavelength corresponds to the transition $n = 3$ to $n = 1$

$$\frac{1}{\lambda_{min}} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

or $$\lambda_{min} = \frac{9}{8RZ^2} = \frac{9}{8(1.097\times10^7)(6)^2} = 28.5 \text{ Å}$$

d. The ionization energy of the gas atoms is

$$E = 13.6Z^2 = (13.6)(6)^2 = 489.6 \text{ eV}$$

$$\lambda = \frac{1}{RZ^2} = \frac{1}{(1.097\times10^7)(6)^2} = 25.32 \text{ Å}$$

IONIZATION ENERGY (I.E.) AND IONIZATION POTENTIAL (I.P.)

- **Ionization Energy (I.E.):** Total energy zero of a hydrogen atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n = \infty$ is called ionization energy of the atom or ion.

$$E_{\text{ionization}} = E_\infty - E_n = -E_n = \frac{13.6Z^2}{n^2} \text{ eV}$$

Ionization energy of H atom = 13.6 eV
Ionization energy of He^+ ion = 54.4 eV
Ionization energy of Li^{++} ion = 122.4 eV

- **Ionization Potential (I.P.):** Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{\text{ionization}} = \frac{E_n}{e} = \frac{13.6Z^2}{n^2} \text{ V}$$

I.P. of H atom = 13.6 V
I.P. of He^+ ion = 54.4 V
I.P. of Li^{++} ion = 122.4 V

EXCITATION ENERGY AND EXCITATION POTENTIAL

- **Excitation:** The process of absorption of energy by an electron so as to raise it from a lower energy level to some higher energy level is called *excitation*.
- **Excited State:** The states of an atom other than the ground state are called its excited states.

$n = 2,$ first excited state
$n = 3,$ second excited state
$n = 4,$ third excited state
$n = n_0 + 1,$ n_0^{th} excited state

- **Excitation Energy:** Energy required to move an electron from ground state of the atom to any other excited state of the atom is called excitation energy of that state.

$$E_{\text{excitation}} = E_{\text{higher}} - E_{\text{lower}}$$

Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H atom = –3.4 eV
I^{st} excitation energy = – 3.4 – (– 13.6) = 10.2 eV.

- **Excitation Potential:** Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

$$V_{\text{excitation}} = \frac{E_{\text{excitation}}}{e}$$

I^{st} excitation energy = 10.2 eV, so I^{st} excitation potential = 10.2 V.

$n = 1,$	$E_1 = -13.6$ eV.	This is the ground state energy.
$n = 2,$	$E_2 = -3.4$ eV.	This is the first excited level.
$n = 3,$	$E_3 = -1.51$ eV.	This is the second excited level.
⋮	⋮	⋮
$n = \infty,$	$E_x = 0.$	The atom is said to be ionised.

BINDING ENERGY OR SEPARATION ENERGY

- Energy liberated when constituents of a system are brought from infinity to assemble the system. The binding energy is negative of ionization energy.

$$E_{\text{binding}} = E_n$$

- Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular state n is called binding energy of that state. Binding energy of ground state of H atom = 13.6 eV.

ATOMIC EXCITATION

An atom can be excited to an energy level above its ground state by following two ways:

a. By photon absorption, and
b. By collision

When an atom absorbs photon, it becomes excited and returns to its ground state in an average of 10^{-6} s by emitting one or more photons. An atom will not absorb photon of any arbitrary energy. For a photon to be absorbed by an atom, it must have energy equal to difference of energy of ground state and any excited state. For example, when H atom absorbs a photon of wavelength 121.7 nm (10.2 eV), it will bring up H atom from $n = 1$ state to $n = 2$ state. This explains the origin of absorption spectra. On subsequent de-excitation, a photon of wavelength 121.7 nm is emitted when H atom drops from $n = 2$ to $n = 1$ state.

The second type of mechanism for excitation of atoms is by collision. For atomic excitation, collision has to be inelastic and kinetic energy lost in the process is utilised for atomic excitation. Numerical problems based on atomic excitation by collision are worked out using following principles:

(i) Conservation of linear momentum (COLM), and
(ii) Conservation of energy (COE).

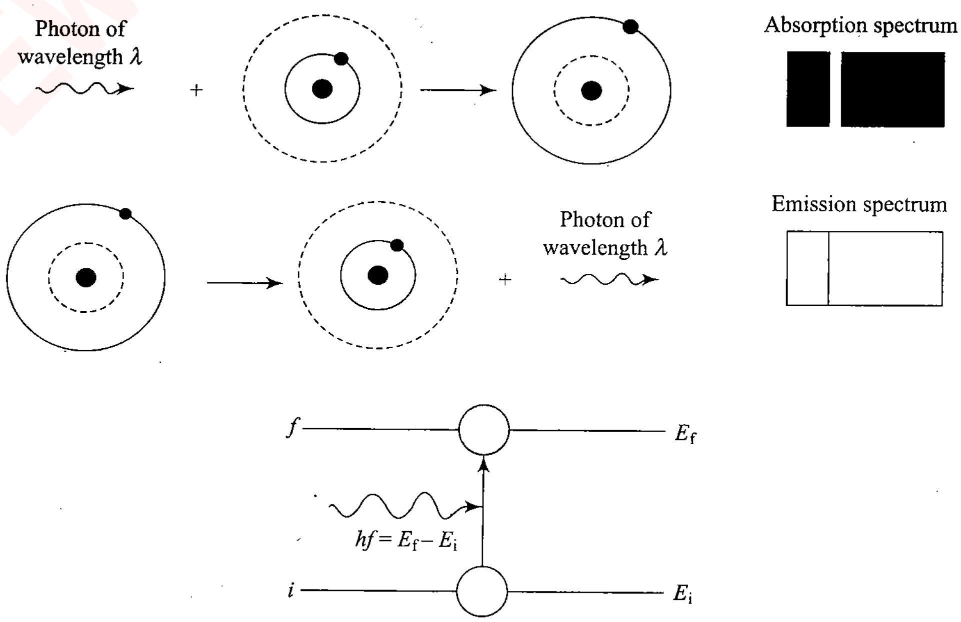


Fig. 4.7

LIMITATIONS OF BOHR'S ATOMIC MODEL

(i) It is applicable for hydrogen-like atoms, i.e., one electron systems only. For example, He^+, Li^{++}, etc.

(ii) Bohr's atomic model could not explain hyperfine structure of spectral lines. As the technology of spectroscopy advanced, wavelengths could be measured with greater accuracy and deviations were observed even in the case of hydrogen spectral lines. On examination, a spectral line was found to consist of very closely spaced lines, which is known as hyperfine structure of spectral line. For example, at least seven components having slightly different wavelengths are revealed in what was previously known as 656.3 nm line.

(iii) Bohr could not explain why electrons are permitted to disobey law of electrodynamics while revolving in stable orbits.

(iv) The model could not account for splitting of spectral line in magnetic and electric fields.

Illustration 4.10 **Find the ratio of ionization energy of Bohr's hydrogen atom and doubly ionized lithium ion (Li^{2+}).**

Sol. The energy of ground state of a Bohr's hydrogen-like atom is $E_n = -(13.6)Z^2$

The ionization energy is equal in magnitude to energy of ground state; so $E_{\text{ionization}} = (13.6)Z^2$

$$\frac{(E_{\text{ionization}})_H}{(E_{\text{ionization}})_{Li^{2+}}} = \frac{(Z_H)^2}{(Z_{Li^{2+}})^2} = \left(\frac{1}{3}\right)^2 + \frac{1}{9}$$

Illustration 4.11 **The Bohr model does not apply when more than one electron orbit the nucleus because it does not account for the electrostatic force that the electrons exert on one another. For instance, an electrically neutral lithium atom (Li) contains three electrons in orbit around a nucleus that includes three protons ($Z = 3$), and Bohr's analysis is not applicable. However, the Bohr model can be used for the doubly charged positive ion of lithium (Li^{2+}) that results when two electrons are removed from the neutral atom, leaving only one electron to orbit the nucleus. Obtain the ionization energy that is needed to remove the remaining electron from Li^{2+}.**

Sol. The lithium ion Li^{2+} contains three times the positive nuclear charge as that of the hydrogen atom. Therefore, the orbiting electron is attracted more strongly to the nucleus in Li^{2+} than in the hydrogen atom. As a result, we expect that more energy is required to ionize Li^{2+} than the 13.6 eV required for atomic hydrogen.

The Bohr energy levels for Li^{2+} are given by

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2}$$

With $Z = 3$ and $n = 1$,

$$E_1 = -(13.6\text{ eV})\frac{3^2}{1^2} = -122.4\text{ eV}$$

To remove the electron from Li^{2+}, 122.4 eV of energy must be supplied. So, the ionization energy for Li^{2+} is 122.4 eV.

Illustration 4.12 **The energy levels of a hypothetical one electron atom are given by**

$$E_n = -\frac{18.0}{n^2}\text{ eV}, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

a. Compute the four lowest energy levels and construct the energy level diagram.

b. What is the excitation potential of the state $n = 2$?

c. What wavelengths (Å) can be emitted when these atoms in the ground state are bombarded by electrons that have been accelerated through a potential difference of 16.2 V?

d. If these atoms are in the ground state, can they absorb radiation having a wavelength of 2000 Å?

e. What is the photoelectric threshold wavelength of this atom?

Sol. a. $E_1 = -\frac{18.0}{1^2} = -18.0\text{ eV}$, $E_2 = -\frac{18.0}{2^2} = -4.5\text{ eV}$,

$E_3 = -\frac{18.0}{3^2} = -2.0\text{ eV}$, $E_4 = -\frac{18.0}{4^2} = -1.125\text{ eV}$

The energy level diagram is shown Fig. 4.8

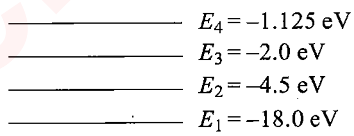


Fig. 4.8

b. The excitation potential of state $n = 2$ is: $18.0 - 4.5 = 13.5$ V

c. Energy of the electron accelerated by a potential difference of 16.2 V is 16.2 eV. With this energy, the electron can excite the atom from $n = 1$ to $n = 3$ as

$$E_4 - E_1 = 1.125 - (-18.0) = 16.875\text{ eV} > 16.2\text{ eV}$$

and $E_3 - E_1 = 2.0 - (-18.0) = 16.0\text{ eV} < 16.2\text{ eV}$

$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{-2.0 - (-4.5)} = 4950\text{ Å},$$

$$\lambda_{31} = \frac{12375}{E_3 - E_2} = \frac{12375}{16} = 773\text{ Å}$$

and $$\lambda_{21} = \frac{12375}{E_2 - E_1} = \frac{12375}{-4.5 - (-18.0)} = 917\text{ Å}$$

d. No. The energy corresponding to $\lambda = 2000$ Å ;

$$E = \frac{12375}{2000} = 6.1875\text{ eV}$$

The minimum excitation energy is 13.5 eV ($n = 1$ to $n = 2$)

e. Threshold wavelength for photoemission to take place from such an atom is, $\lambda_{max} = \frac{12375}{18} = 687.5$ Å

Illustration 4.13 **A doubly ionized lithium atom is hydrogen like with atomic number 3. Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to third Bohr orbit. The ionization energy of the hydrogen atom is 13.6 eV.**

Sol. $E_n = -\dfrac{13.6Z^2}{n^2}$ eV, $E_1 = -\dfrac{13.6\times 3^2}{1^2} = -122.4$ eV,

$$E_3 = -\frac{13.6\times 3^2}{3^2} = -13.6 \text{ eV}$$

The corresponding wavelength is $\lambda = \dfrac{12375}{E_3 - E_1} = 113.74$ Å

Illustration 4.14 **A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third Bohr orbit. Find:**

a. the value of Z.
b. the energy required to excite the electron from the third to the fourth Bohr orbit.
c. the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity.
d. find the KE, PE and angular momentum of electron in the 1st Bohr orbit.
e. the radius of the first Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} m, velocity of light = 3×10^8 ms^{-1}, Planck's constant = 6.6×10^{-34} J-s]

Sol. The energy required to excite the electron from n_1 to n_2 orbit revolving round the nucleus with charge $+Ze$ is given by

$$E_{n_2} - E_{n_1} = Z^2\times 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ electron volt.}$$

a. Since 47.2 eV energy is required to excite the electron from $n_1 = 2$ to $n_2 = 3$ orbit, therefore

$$47.2 = Z^2\times 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad Z^2 = \frac{47.2\times 36}{13.6\times 5} = 25 \quad \text{or} \quad Z = 5$$

b. The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ orbit is given by

$$E_4 - E_3 = 25\times 13.6\times\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25\times 13.6\times 7}{144} = 16.53 \text{ eV}$$

c. The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$$E_\infty - E_1 = 13.6\times Z^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6\times 25 \text{ eV}$$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$$hf = \frac{hc}{\lambda} = 13.6\times 25\times(1.6\times 10^{-19}) \text{ J}$$

or $$\lambda = \frac{(6.6\times 10^{34})\times(3\times 10^8)}{13.6\times 25\times(1.6\times 10^{-19})} = 36.5 \text{ Å}$$

d. KE = – Total energy = $-(-13.6\times 5^2)\times 1.6\times 10^{-19}$ J = 5.44×10^{-17} J

PE = $-2\times$ KE = $-2\times 5.44\times 10^{-17} = 10.88\times 10^{-17}$ J

Angular momentum = $mv_1r_1 = h/2\pi = 1.05\times 10^{-13}$ Js

e. The radius r_1 of the first Bohr's orbit is given by

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2}\frac{1}{Z} = \frac{0.53\times 10^{-10}}{5} = 0.106\times 10^{-10} \text{ m} = 0.106 \text{ Å}$$

$$\left(\because \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53\times 10^{-10}\right)$$

Illustration 4.15 **In a transition from state n to a state of excitation energy 10.19 eV, a hydrogen atom emits a 4890 Å photon. Determine the binding energy of the initial state.**

Sol. The energy of the emitted photon is $hf = \dfrac{12400}{4890} = 2.54$ eV

The excitation energy is the energy to excite the atom to a level above the ground state. Therefore, the energy of the level to which the transition occurs is: $E_x = -13.6 + 10.19 = -3.41$ eV

The photon arises from the transition between energy states (say n to x), then

$$E_n - E_x = hf, \quad \text{hence} \quad E_n - (-3.41) = hf$$

$$\Rightarrow \quad E_n - (-3.41) = 2.54 \quad \text{or} \quad E_n = -0.87 \text{ eV}$$

Therefore, the binding energy of an electron in the state is 0.87 eV.

Note that the transition is from

$$n = \sqrt{\frac{E_1}{E_n}} = \sqrt{\frac{13.6}{0.87}} = 4 \quad \text{to} \quad x = \sqrt{\frac{E_1}{E_x}} = \sqrt{\frac{13.6}{3.41}} = 2$$

Illustration 4.16 **First excitation potential of a hypothetical hydrogen-like atom is 15 V. Find third excitation potential of the atom.**

Sol. Let energy of ground state = $-E_0$, then

$$E_n = -\frac{E_0}{n^2}$$

Given: $E_2 - E_1 = 15$ eV $\Rightarrow -\dfrac{E_0}{4} + E_0 = 15 \Rightarrow \dfrac{3E_0}{4} = 15$

$$\Rightarrow \quad E_0 = 20 \text{ eV}$$

For third excitation potential: $n = 4$, then

$$E_4 - E_1 = -\frac{E_0}{16} + E_0 = \frac{75}{4} \text{ eV}$$

Therefore, third excitation potential is (75/4)V

Illustration 4.17 **The energy level diagram for a hydrogen-like atom is shown in Fig. 4.9.**

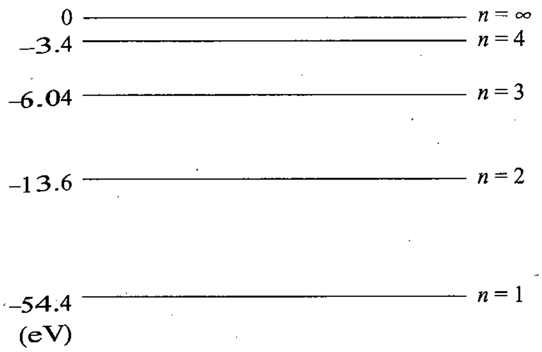


Fig. 4.9

(i) Find the value of Z.
(ii) If initially the atom is in the ground state, then
a. determine its first excitation potential, and
b. determine its ionization potential.
(iii) Can it absorb a photon of 42 eV?
(iv) Can it absorb a photon of 56 eV ?
(v) Calculate the radius of its first Bohr orbit.
(vi) Calculate the kinetic energy and potential energy of an electron in the first orbit.

Sol. (i) We know that $E_n = -13.6\dfrac{Z^2}{n^2}$ eV

Here, for $n = 1, E_1 = -54.4$ eV, therefore $-54.4 = -13.6\dfrac{Z^2}{1^2}$

$$\therefore \quad Z = \sqrt{\frac{54.4}{13.6}} = 2$$

(ii) a. The first excitation energy is required to excite the electron from $n = 1$ to $n = 2$. Thus,

$$\Delta E_{12} = E_2 - E_1 = -13.6 - (54.4) = 40.8 \text{ eV}$$

Therefore, the first excitation potential is 40.8 V.

b. The ionization energy is required to eject the electron from $n = 1$ to $n = \infty$. Thus,

$$\Delta E_{1\infty} = E_\infty - E_1 = 0 - (-54.4) = 54.4 \text{ eV}$$

Therefore, the ionization potential is 54.4 V.

(iii) No.

By absorbing a photon of 42 eV, the energy of electron will become

$$E = -54.4 + 42 = -12.4 \text{ eV}$$

This energy level exists between $n = 1$ and $n = 2$ shells which is not possible.

Hence, an electron in the first orbit cannot absorb it.

(iv) Yes

By absorbing a photon of 56 eV, the electron comes out of the atom where the energy is not quantized. Hence, a photon of 56 eV can be absorbed.

(v) We know that

$$r_n = 0.53\frac{n^2}{Z} \text{ (Å)}$$

Here, $n = 1$; $Z = 2$, therefore $r = 0.53\dfrac{(1)^2}{2} = 0.265$ (Å)

(vi) Since $K = -E$ and $U = 2E$, therefore

$$K = 54.4 \text{ eV} = 8.7 \times 10^{18} \text{ J} \quad \text{and} \quad U = -108.8 \text{ eV}$$
$$= 1.74 \times 10^{17} \text{ J}$$

WAVELENGTH OF PHOTON EMITTED IN DE-EXCITATION

According to Bohr, when an atom makes a transition from a higher energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $hn = hc/\lambda$, then conservation of energy gives,

$$hv = \frac{hc}{\lambda} = E_i - E_f \quad \text{(energy of emitted photon) (i)}$$

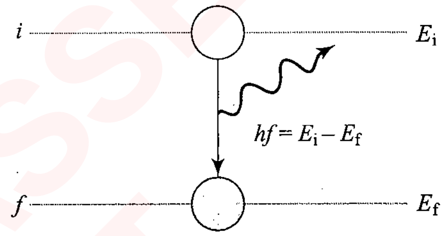


Fig. 4.10

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in the red and is called Hα, the next line, in the blue-green is called H_β, and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wavelengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \text{(ii)}$$

Here, $n = 3, 4, 5, \ldots$, etc.,

R = Rydberg constant = 1.097×10^7 m^{-1}; and

λ is the wavelength of light/photon emitted during transition,

For $n = 3$, we obtain the wavelength of H_α line.

Similarly, for $n = 4$, we obtain the wavelength of H_β line. For $n = \infty$, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation $E = hc/\lambda$, we can find the photon energies corresponding to the wavelengths of the Balmar series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = \frac{Rhc}{n^2}, n = 1, 2, 3, \ldots \quad \text{(iii)}$$

The wavelengths corresponding to other spectral series (Lymen, Paschen, etc.) can be represented by formula similar to Balmer's formula.

Lymen series: $\quad \dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right), n = 2, 3, 4, \ldots$

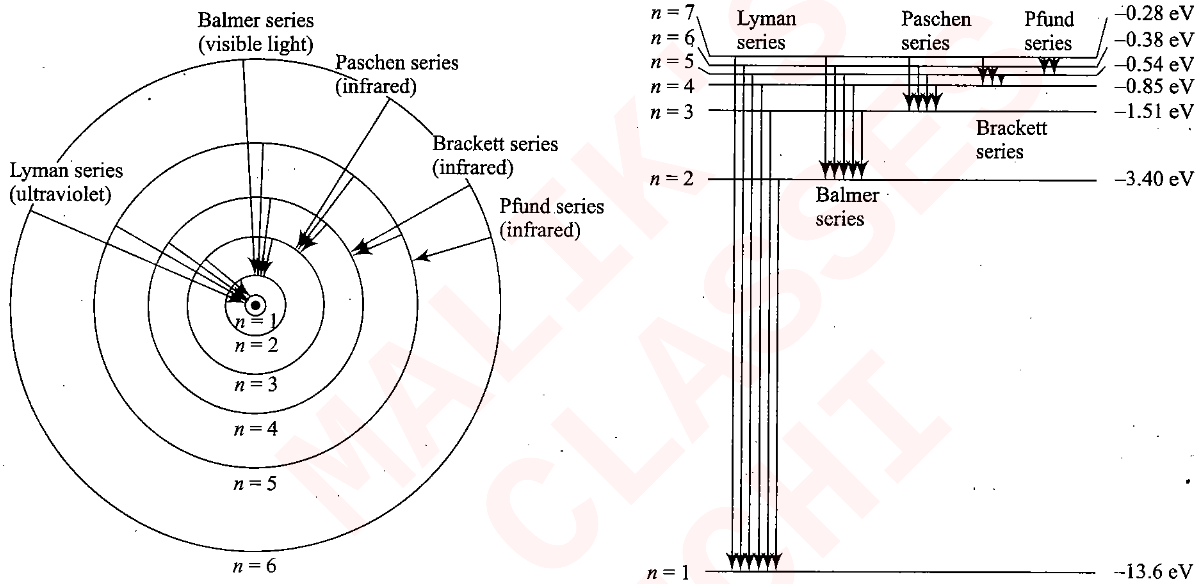
Paschen series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right), n = 4, 5, 6, \ldots$

Brackett series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{4^2} - \dfrac{1}{n^2}\right), n = 5, 6, 7, \ldots$

Pfund series: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{5^2} - \dfrac{1}{n^2}\right), n = 6, 7, 8$

The Lymen series is in the ultraviolet, and the Paschen, Brackett and Pfund series are in the infrared region.

Hydrogen Spectrum



(a) (b)

Fig. 4.11

	Initial State	Final State	Wavelength Formula	First Member-Second Member	Series Limit $n_i \to \infty$ to n_f	Maximum Wavelength $(n_f + 1)$ to n_f	Lines Found in
Lyman	$n_i = 2, 3, 4,$ $5, 6, \ldots$	$n_f = 1$	$\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n_i^2}\right)$	$n_i = 2$ to $n_f = 1$ $n_i = 3$ to $n_f = 1$	From ∞ to 1 $l = \dfrac{4}{R}$ $\lambda = 911$ Å	From 2 to 1 $l = \dfrac{4}{3R}$ $l = 121$ Å	UV region
Balmer	$n_i = 3, 4, 5,$ $6, 7, \ldots$	$n_f = 2$	$\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n_i^2}\right)$	$n_i = 3$ to $n_f = 2$ $n_i = 4$ to $n_f = 2$	From ∞ to 2 $l = \dfrac{4}{R}$ $l = 3646$ Å	From 3 to 2 $l = \dfrac{36}{5R}$ $l = 6563$Å	Visible region
Paschen	$n_i = 4, 5, 6,$ $7, 8, \ldots$	$n_f = 3$	$\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n_i^2}\right)$	$n_f = 4$ to $n_f = 3$ $n_f = 5$ to $n_f = 3$	From ∞ to 3 $l = \dfrac{9}{R}$ $l = 8204$ Å	From 4 to 3 $l = \dfrac{144}{7R}$ $l = 18753$ Å	IR region
Brackett	$n_i = 5, 6,$ $7, 8, 9, \ldots$	$n_f = 4$	$\dfrac{1}{\lambda} = R\left(\dfrac{1}{4^2} - \dfrac{1}{n_i^2}\right)$	$n_i = 5$ to $n_f = 4$ $n_i = 6$ to $n_f = 4$	From ∞ to 4 $l = \dfrac{16}{R}$ $l = 1485$ Å	From 5 to 4 $l = \dfrac{400}{9R}$ $l = 40515$ Å	IR region
Pfund	$n_i = 6, 7,$ $8, 9, 10, \ldots$	$n_f = 5$	$\dfrac{1}{\lambda} = R\left(\dfrac{1}{5^2} - \dfrac{1}{n_i^2}\right)$	$n_i = 6$ to $n_f = 5$ $n_i = 7$ to $n_f = 5$	From ∞ to 5 $l = \dfrac{25}{R}$ $l = 22790$ Å	From 6 to 5 $l = \dfrac{900}{11R}$ $l = 74583$ Å	Far IR region

Illustration 4.18 **Brackett series of lines are produced when electrons excited to high energy levels make transitions to the $n = 4$ level. Determine**

a. the longest wavelength in this series, and

b. the wavelength that corresponds to the transition from $n_f = 6$ to $n_i = 4$.

c. Identify the spectral region in which these lines are found.

Sol. a. The longest wavelength corresponds to transition from $n_f = 5$ to $n_i = 4$; the smallest energy change.

So, $$\frac{1}{\lambda_{max}} = (1.097\times10^7)\left(\frac{1}{4^2}-\frac{1}{5^2}\right) = 2.468\times10^5 \text{m}^{-1}$$

$$\lambda_{max} = \frac{1}{2.468\times10^5} = 4051 \text{ nm}$$

b. The wavelength for transition $n_f = 6$ to $n_i = 4$ is

$$\frac{1}{\lambda} = (1.0947\times10^{-7})\left(\frac{1}{4^2}-\frac{1}{6^2}\right) = 3.801\times10^5$$

$$\Rightarrow \quad \lambda = \frac{1}{3.801\times10^5} = 2630 \text{ nm}$$

Illustration 4.19 **Electrons of energy 12.09 eV can excite hydrogen atoms. To which orbit is the electron in the hydrogen atom raised and what are the wavelengths of the radiations emitted as it drops back to the ground state?**

Sol. The energies of the electron in different state are:

$E_1 = -13.6\text{eV}$ for $n = 1$,

$E_2 = -3.4$ eV for $n = 2$,

and $E_3 = -1.51$ eV for $n = 3$

Evidently, the energy needed by an electron to go to the E_3 level ($n = 3$ or M-level) is $13.6 - 1.51 = 12.09$ eV. Thus, the electron is raised to the third orbit of principal quantum number $n = 3$.

Now, an electron in the $n = 3$ level can return to the ground state making the following possible jumps:

(i) $n = 3$ to $n = 2$ and then from $n = 2$ to $n = 1$.

(ii) $n = 3$ to $n = 1$.

Thus, the corresponding wavelengths emitted are:

a. For $n = 3$ to $n = 2$:

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5R}{36}$$

or $$\lambda_1 = \frac{36}{5R} = \frac{36}{5\times1.097\times10^7} = 6563 \text{ Å}$$

This wavelength belongs to the Balmer series and lies in the visible region.

b. For $n = 2$ to $n = 1$:

$$\frac{1}{\lambda_2} = R\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = \frac{3R}{4}$$

or $$\lambda_2 = \frac{4}{3R} = \frac{4}{3\times1.097\times10^7} = 1215 \text{ Å}$$

λ_2 belongs to the Lyman series and lies in the ultraviolet region.

c. For the direct jump $n = 3$ to $n = 1$:

$$\frac{1}{\lambda_3} = R\left[\frac{1}{1^2}-\frac{1}{3^2}\right] = \frac{8R}{9}$$

or $$\lambda_3 = \frac{9}{8R} = \frac{9}{8\times1.097\times10^7} = 1026 \text{ Å}$$

which also belongs to the Lyman series and lies in the ultraviolet region.

Illustration 4.20 **A hydrogen atom is in third excited state. It makes a transition to a different state and a photon is either absorbed or emitted. Determine the quantum number n_f of the final state and the energy of the photon if it is**

a. emitted with the shortest possible wavelength,

b. emitted with the longest possible wavelength, and

c. absorbed with the longest possible wavelength.

Sol. a. When a photon is emitted, it carries energy with it, so the final energy of the atom is less than its initial energy. So, the final quantum number is less than the initial when a photon is emitted.

An atom gains energy when a photon is absorbed. So, the final quantum number is greater than the initial when a photon is absorbed.

The largest possible energy arises when the electron jumps from the initial state ($n = 4$) to the ground state ($n_l = 1$) as shown by transition A in the figure. Therefore, the quantum number of final state is $n = 1$. The energy of the n^{th} state is given by

$$E_n = \left(\frac{Z^2}{n^2}\right)$$

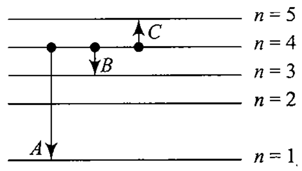


Fig. 4.12

The energy of the photon emitted corresponding to transition from excited state quantum number n_u to lower energy state n_l can be calculated from

$$\frac{hc}{\lambda} = E_u - E_l = (13.6)(1)^2\left[\frac{1}{n_l^2}-\frac{1}{n_u^2}\right]$$

So, the energy of the photon with minimum wavelength is

$$\Delta E = E_4 - E_1$$

$$= (13.6)(1)^2\left(\frac{1}{1^2}-\frac{1}{4^2}\right) = 12.75 \text{ eV}$$

b. The longest possible wavelength photon corresponds to minimum energy. This happens when the electron jumps from the initial state ($n_u = 4$) to the next lower state ($n_l = 3$) as shown by transition B in the figure. Therefore, the quantum number of the final state is $n = 3$. The energy of the photon is

$$\Delta E = E_4 - E_3 = (13.6)(1)^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right) = 0.661 \text{ eV}$$

c. The absorbed photon has the longest possible wavelength when its energy is the smallest, i.e., the electron jumps from the initial state ($n_l = 4$) to the next higher state ($n_u = 5$). This is shown by transition C in the figure. Therefore, the quantum number of the final state is $n = 5$. The energy of the photon is

$$\Delta E = E_5 - E_4 = (13.6)(1)^2\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 0.306 \text{ eV}$$

Illustration 4.21 **The Balmer series for the hydrogen atom corresponds to electron transitions that terminate in the state of quantum number $n = 2$.**

a. **Find the longest wavelength photon emitted and determine its energy.**

b. **Find the shortest wavelength photon emitted in the Balmer series.**

Sol. a. The longest wavelength photon in Balmer series results corresponding to transition from $n = 3$ to $n = 2$. Using the equation

$$\frac{1}{\lambda} = R\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) \Rightarrow \frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R$$

$$\Rightarrow \quad \lambda_{max} = \frac{36}{5R} = \frac{36}{5(1.097 \times 10^7)} = 656.3 \text{ nm}$$

This wavelength lies in the red region of the visible spectrum. The energy of emitted photon is

$$E = \frac{hc}{\lambda_{max}} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(656.3 \times 10^{-9})}$$

$$= 3.03 \times 10^{-19} \text{ J} = 1.89 \text{ eV}$$

b. The shortest wavelength photon in the Balmer series is emitted when the electron makes a transition from $n = \infty$ to $n = 2$.

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\Rightarrow \quad \lambda_{min} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 364.6 \text{ nm}$$

This wavelength lies in the ultra-violet region.

Illustration 4.22 **How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to a state with principal quantum number n?**

Sol. From the n^{th} state, the atom may go to $(n-1)^{th}$ state, ..., 2^{nd} state or 1^{st} state. So, there are $(n-1)$ possible transitions starting from the n^{th} state. The atoms reaching $(n-1)^{th}$ state may make $(n-2)$ different transitions and so on. In general, the total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + \cdots + 2 + 1 = \frac{n(n-1)}{2}$$

ORIGIN OF SPECTRA

The origin of spectra can be successfully explained on the basis of atomic theory. An atom consists of a central positive core called nucleus, which is surrounded by electrons revolving in various orbits according to a definite scheme. The number of the revolving electrons or the number of protons in the nucleus is equal to atomic number of the atom, while the number of neutrons is equal to the difference of its mass number and atomic number. For example, mass number of sodium is 23 and its atomic number is 11. Electrons revolve around the nucleus in various orbits (2 in K-orbit, 8 in L-orbit, and 1 in M-orbit) as shown in Fig. 4.13. A definite amount of energy is associated with each orbit. The energy associated with an electron in an outer orbit is more than that associated with an electron in an inner orbit. Therefore, when energy is given to an electron, it gets raised to some outer orbit. Since the number of electrons in an inner orbit must always be equal to $2n^2$, the electron raised to the outer orbit returns back. When it does so, radiation having energy equal to the difference of energy associated with the two orbits is emitted.

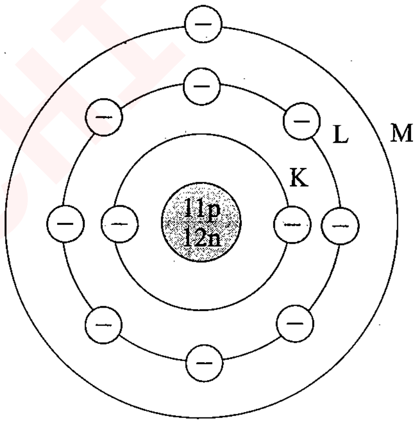


Fig. 4.13

Emission Spectra

The light emitted by a source of light and resolved into its constituent wavelengths is called emission spectrum.

When a solid is heated, its atoms collide with each other randomly. Due to this, the orbital electrons in the atoms of the solid get raised to higher energy levels. When the electrons return back, the radiation of wavelength corresponding to the difference of energies associated with the two levels is emitted. Usually, the radiation emitted has more than one wavelength. When the emitted radiation is studied using a prism spectrometer, the constituent wavelengths of the emitted radiation are observed in order of their wavelengths. Such a spectrum is called emission spectrum. An arc discharge, a spark discharge, the discharge of electricity through a gas, a vapour lamp (such a sodium lamp), etc. produce emission spectra.

Emission spectra are of the following three types:

1. **Continuous Emission Spectrum:** A continuous emission spectrum is one which covers a wide range of wavelengths without any gap or discontinuity. The solar spectrum is an

example of a continuous emission spectrum. In general, solids and liquids emit continuous spectra. The reason is that in solids and liquids, the atoms and molecules are very close to each other. Due to this, energy changes in an atom are influenced by the neighboring atoms so precisely that radiations of all possible wavelengths are emitted. In gases, atoms are comparatively far apart and therefore an atom is not influenced by the neighboring atoms. As a result, the radiation having a wavelength characteristic of the change in energy of the electron in a particular atom is emitted. Since the color of light is characteristic of the wavelength, the color of the spectrum changes from the spectrum of one solid to the other. The intensity is maximum corresponding to a certain wavelength and decreases on its two sides. As the temperature of the solid is increased, the point of maximum intensity shifts toward the violet end of the spectrum.

2. **Line Emission Spectrum:** A line emission spectrum is one which consists of bright lines (wavelengths) separated from each other by dark spaces. The line spectrum is emitted by the atoms of an element. In other words, each line in the spectrum of an element corresponds to a definite wavelength and occupies the same position in the spectrum. The spectra emitted by hydrogen, helium, mercury, sodium, calcium, etc. are line spectra. Figure 4.14 shows the line spectrum of hydrogen in visible region.

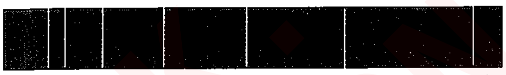


Fig. 4.14

3. **Band Emission Spectrum.** A band emission spectrum is one which consists of a number of bright bands (group of wavelengths) of light, sharp at one end and fading off at the other end.

The band spectrum is emitted by the molecules in gaseous form. These bands are sharply defined at one edge of the spectrum and fade off at the other edge. Thus, intensity of light in a band is not uniform. A band spectrum is discontinuous when a band is found to consist of a large number of fine lines spaced very close to each other. The band spectra are emitted by O_2, N_2, CO_2, NH_3, CaF_2, etc.

Fig. 4.15

Absorption Spectra

When light from a source emitting full spectrum is passed through a substance (a solid or liquid in vapor state or a gas), the dark lines (or bands) appearing in the positions occupied by the bright lines (or bands) in the emission spectrum of the substance constitute the absorption spectrum of the substance.

In general, the emission and absorption spectra are complimentary to each other. Some substances absorb a few selective wavelengths and transmit all other remaining wavelengths of the incident light. Such an absorption is termed as selective absorption. Accordingly, the absorption spectra are usually of the following two types:

1. Absorption line spectrum
2. Absorption band spectrum

To study absorption spectrum. Figure 4.16 shows an experimental arrangement to study the absorption line spectrum of sodium.

Light from an arc lamp is allowed to pass through the sodium vapors and is transmitted by a prism spectrometer. It is found that the continuous spectrum of the arc lamp is crossed by two dark lines in the yellow region. It is called the absorption line spectrum of sodium and can be explained as below:

We know that on heating, sodium vapors emit yellow light of wavelengths 5890 and 5896 Å. When light from the arc lamp emitting continuous spectrum is passed through the sodium vapors, the continuous spectrum is crossed by two dark lines in the yellow region corresponding to the wavelengths of 5890 and 5896 Å. The study of absorption spectra led Kirchhoff to formulate the following law, known as Kirchhoff's law:

If a substance emits light of a certain wavelength at a given temperature, then it will absorb the same wavelength selectively, when light is passed through the substance in vapor state. It follows from the fact that emission and absorption spectra are complimentary.

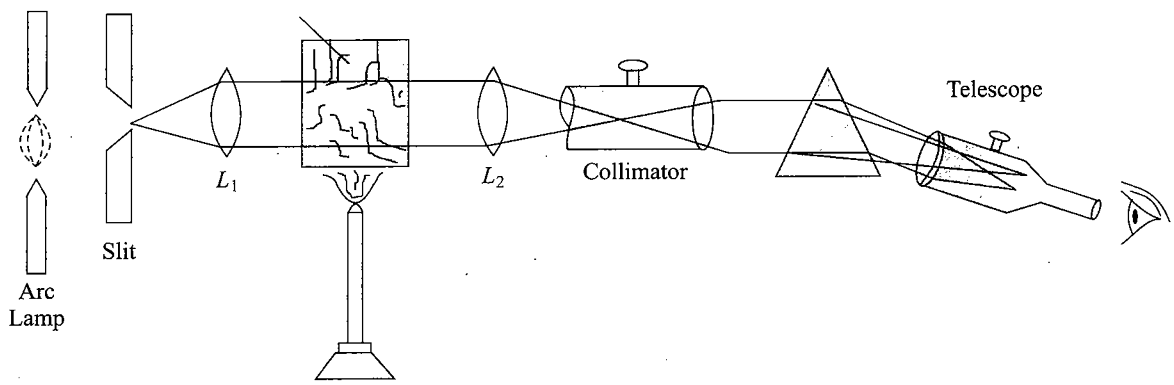


Fig. 4.16

EFFECT OF NUCLEUS MOTION ON ENERGY OF ATOM

Effect of Mass of Nucleus on Bohr Model

Upto this level, we have discussed about the structure of a hydrogenic atom by considering nucleus at rest and electron revolving around the nucleus. In fact, we know that as no external force is acting on a nucleus–electron system, hence the center of mass of the nucleus–electron system must remain at rest. Theoretically, mass of electron is negligible or small compared to that of nucleus and due to this we assume that center of mass of the atom is almost situated at nucleus. That is why in Bohr atomic model it was assumed that in an atom, nucleus remains at rest and electron revolves around it. But practically the situation is a bit different. Actually, center of mass of nucleus–electron system is close to nucleus as it is heavy and to keep center of mass at rest, both electron and nucleus revolve around their center of mass like a double star system as shown in Fig. 4.17. If r is the distance of electron from nucleus, the distances of nucleus and electron from the center of mass, r_1 and r_2, can be given as

$$r_1 = \frac{m_e r}{m_N + m_e}$$

and

$$r_2 = \frac{m_N r}{m_N + m_e}$$

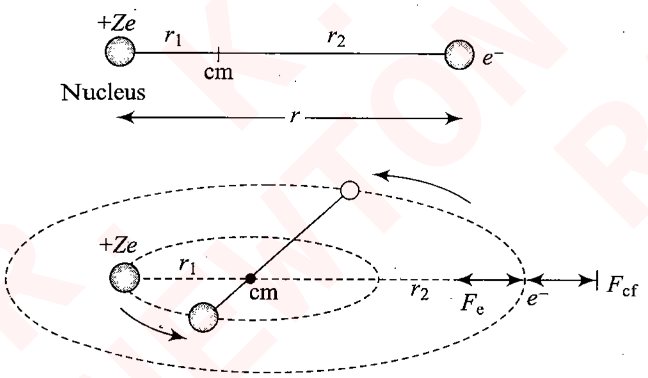


Fig. 4.17

We can see that in the atom, nucleus and electron revolve around their center of mass in concentric circles of radii r_1 and r_2 to keep the center of mass at rest. In above system, we can analyze the motion of electron with respect to nucleus by assuming nucleus to be at rest and the mass of electron replaced by its reduced mass μ_e, given as

$$\mu_e = \frac{m_N m_e}{m_N + m_e}$$

Now, the relative picture of atom will be same what we have considered earlier as shown in Fig. 4.18 but electron mass is replaced by its reduced mass.

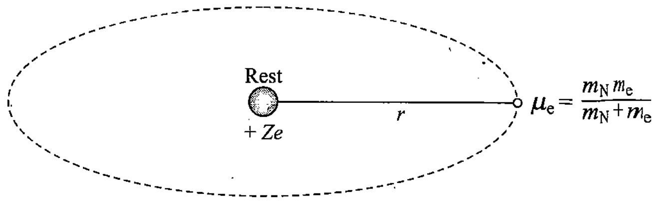


Fig. 4.18

We can use all those relations which we have derived for Bohr model just by replacing m_e by μ_e, such as the radius of electron in n^{th} orbit of Bohr's model is given as

$$r_n = \frac{n^2 h^2}{4\pi^2 KZe^2 m_e} \quad \text{(i)}$$

But if we take the motion of nucleus into account, the radius of n^{th} orbit will be given as

$$r'_n = \frac{n^2 h^2}{4\pi^2 KZe^2 \mu_e} \Rightarrow r'_n = \frac{n^2 h^2 (m_N + m_e)}{4\pi^2 KZe^2 m_e m_N} \quad \text{(ii)}$$

or

$$r'_n = r_n \times \frac{m_e}{\mu_e} \Rightarrow r = (0.529\ \text{Å})\frac{mn^2}{\mu Z} \quad \text{(iii)}$$

Similarly, if we find the speed of electron in n^{th} Bohr orbit, it can be given by

$$V_n = \frac{2\pi KZe^2}{nh} \quad \text{(iv)}$$

Here, we can see that in the above expression of speed the term m_e (mass of electron) is not present. Thus, speed of electron as n^{th} orbit does not depend on electron mass. Here, no change will be there in the electron's speed of revolution if we take the motion of nucleus into account.

Similarly, we know the expression of energy of electron in n^{th} orbit of Bohr model is given as

$$E_n = -\frac{2\pi^2 K^2 Z^2 e^4 m_e}{n^2 h^2} \quad \text{(v)}$$

or

$$E'_n = -\frac{2\pi^2 K^2 Z^2 e^4 m_N m_e}{n^2 h^2 (m_N + m_e)} \quad \text{(vi)}$$

or

$$E'_n = E_n \times \frac{\mu_e}{m_e} \Rightarrow E_n = -(13.6\ \text{eV})\frac{Z^2}{n^2}\left(\frac{\mu}{m}\right) \quad \text{(vii)}$$

Thus, we can say that the energy of electron will be slightly less compared to what we have derived earlier. But for numerical calculations this small change can be neglected unless in a given problem it is asked to consider the effect of motion of nucleus.

Illustration 4.23 **A positronium 'atom' is a system that consists of a positron and an electron that orbit each other. Compare the wavelengths of the spectral lines of positronium with those of ordinary hydrogen.**

Sol. Here, the two particles have the same mass m, so the reduced mass is $\mu = \dfrac{mm}{m+m} = \dfrac{m^2}{2m} = \dfrac{m}{2}$

where m is the electron mass.

We know that $\quad E_n \propto m \quad \Rightarrow \quad \dfrac{E_n'}{E_n} = \dfrac{\mu}{m} = \dfrac{1}{2}$

Hence, energy levels of positronium atom are half of corresponding energy level in a hydrogen atom.

As a result, the wavelengths in the positronium atom spectral lines are all twice those of the corresponding lines in the hydrogen spectrum.

Illustration 4.24 **A muon is an unstable elementary particle whose mass is $207m_e$ and whose charge is either $+e$ or $-e$. A negative muon (μ^-) can be captured by a hydrogen nucleus (or proton) to form a muonic atom.**

a. Find the radius of the first Bohr orbit of this atom.

b. Find the ionization energy of the atom.

Sol. a. Here, $m = 207m_e$ and $M = 1836m_e$, so the reduced mass is

$$\mu = \frac{mM}{m+M} = \frac{(207m_e)(1836m_e)}{207m_e + 1836m_e} = 186\, m_e$$

According to equation $\lambda = h/p$, the orbit radius corresponding to $n = 1$ is $r_1 = \dfrac{h^2\varepsilon_0}{\pi m_e e^2} = 5.29 \times 10^{-11}$ m

Hence, the radius r' that corresponds to the reduced mass μ is

$$r_1' = \left(\frac{m}{\mu}\right) r_1 = \left(\frac{m_e}{1836 m_e}\right) r_1 = 2.85 \times 10^{-13} \text{ m}$$

The muon is 186 times closer to the proton than an electron would be in the same orbit.

b. We have $E_1 = -13.6$ eV for $n = 1$

$$E_1' = \left(\frac{\mu}{m}\right) E_1 = 186 \times (-13.6) \text{ eV}$$

$$\Rightarrow \quad E_1 = -2.53 \times 10^3 \text{ eV} = -2.53 \text{ keV}$$

The ionization energy is, therefore, 2.53 keV, 186 times that for an ordinary hydrogen atom.

Illustration 4.25 **A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of electron (called a μ-meson) moves in a circular orbit around a nucleus of charge +3e. (Take the mass of the nucleus to be infinite.) Assuming that Bohr model of the atom is applicable to this system:**

(i) derive and expression for the radius of the n^{th} Bohr orbit.

(ii) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) find the wavelength of the radiation emitted when the μ-meson jumps from the third orbit to the first orbit. (Rydberg's constant = 10967800 m)

Sol. (i) We have radius of n^{th} orbit of a hydrogen atom as

$$r_n = \frac{n^2h^2}{4\pi^2 Kze^2 m}$$

If electron is replaced by a heavy particle of mass 208 times that of the electron, then the radius is given as

$$r_n = \frac{n^2h^2}{4\pi^2 K(3)e^2(208m)} = \frac{n^2h^2}{2496\pi^2 Kze^2 m} \qquad \text{(i)}$$

Here, we have not used reduced mass because it is given that mass of nucleus is assumed to be infinite and here we take $z = 3$.

(ii) The radius of first Bohr orbit is given as

$$r_1 = \frac{h^2}{4\pi^2 Ke^2 m}$$

From equation (i), we have

$$\frac{n^2h^2}{2496\pi^2 Ke^2 m} = \frac{h^2}{4\pi^2 Ke^2 m}$$

or $\quad n^2 = 624$

or $\quad n = 25$

(iii) Rydberg constant for hydrogen-like atom is

$$R = \frac{2\pi^2 K^2 e^4 m}{ch^3}$$

Now, when electron is replaced by μ-meson, Rydberg constant will change as

$$R' = \frac{2\pi^2 K^2 e^4 (208m)}{ch^3}$$

or $\quad = 208R = 208 \times 1967800 \text{ m}^{-1}$

Now, the wavelength λ of the radiation emitted when μ-meson makes a transition from $n_2 = 3$ to $n = 1$ is

$$\frac{1}{\lambda} = R'Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $\quad \dfrac{1}{\lambda} = R' \times 9\left[\dfrac{1}{1^2} - \dfrac{1}{3^2}\right] \quad$ or $\quad \dfrac{1}{\lambda} = 8R'$

or $\quad \lambda = \dfrac{1}{8R'}$

$$\lambda = \frac{1}{8 \times 208 \times 10967800} \text{m}^{-1}$$

or $\quad \lambda = 0.548$ Å

ATOMIC COLLISION

There are two ways to excite an electron in an atom:

(i) By supplying energy to an electron through electromagnetic photons, which we have already discussed in photoelectric effect. An electron absorbs a photon only when the photon energy is equal to the difference in energies of the two energy levels of atom otherwise the photon will not be absorbed.

(ii) The energy can be supplied to an electron by atomic collisions. In such collisions, assume that the loss in kinetic energy of the system is possible only if it becomes excited or gets ionised.

Collision of a Neutron with an Atom

To understand the energy supplied by collisions, we consider an example of a head on collision of a moving neutron with a stationary hydrogen atom as shown in Fig 4.19. Here, for mathematical analysis, let us assume the masses of neutron and H atom to be same.

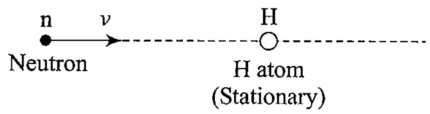


Fig. 4.19

(i) Perfectly elastic collision:
Here, when perfectly elastic collision takes place, then (for equal masses) the neutron will come to rest while the hydrogen atom will move with the same speed and kinetic energy with which the neutron was moving initially as shown in Fig. 4.20.

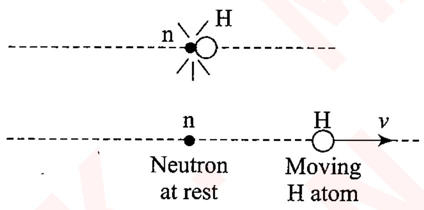


Fig. 4.20

(ii) Perfectly inelastic collision:
Now, if we consider the collision to be perfectly inelastic, then after collision both the neutron and H atom will move together with speed v_1 which can be given by law of conservation of momentum as

$$mv_0 = 2mv_1$$

or $$v_1 = \frac{v}{2} \qquad \text{(i)}$$

In this case, the loss of energy can be given as the difference in initial and final kinetic energies of the neutron and H atom, given as

$$\Delta E = E_i - E_f$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{4}mv^2$$

$$= \frac{1}{4}mv^2 = \frac{1}{2}E_i \qquad \text{(ii)}$$

Thus, half of the initial kinetic energy will be lost in the collision. One important point which we should understand carefully is that in collisions of elementary particles and atoms, no energy can be lost as heat because here we cannot consider the deformation of any lattice in the colliding body. The energy lost can only be absorbed by the atom involved in the collision and may get excited or ionized by this energy loss which takes place in case of inelastic collision.

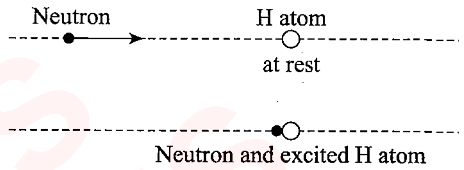


Fig. 4.21

In our case of collision of a neutron with a H atom, the energy loss is $(1/4)mv^2$ (half of initial energy of the neutron). This loss in energy can be absorbed by H atom only. We know that the minimum energy required to excite an H atom is 10.2 eV for $n = 1$ to $n = 2$. Thus, hydrogen atom can absorb only when this energy loss is equal to 10.2 eV. If this energy loss in perfectly inelastic collision is more then 10.2 eV, then the H atom may absorb 10.2 eV energy for its excitation and rest of the energy will remain in the colliding particles (neutron and H atom) as their kinetic energy and the collision will not be perfectly inelastic in this case.

Illustration 4.26 **A neutron which is going to collide head on to a stationary H atom has initial kinetic energy of 24.5 eV. Discuss different possibilities of collision.**

Sol. If perfectly inelastic collision takes place, then the maximum energy loss can be given as

$$\Delta E_{max} = \frac{1}{2}E_i = \frac{1}{2}(24.5)\text{ eV} = 12.25\text{ eV}$$

From this energy, H atom can absorb either 10.2 eV or 12.09 eV for its excitation from $n = 1$ to $n = 2$ or from $n = 1$ to $n = 3$ energy level. Thus, in this case there may be three possibilities for collision. These are:

(i) It may be possible that the collision is perfectly elastic and no energy is absorbed by H atom as shown in Fig. 4.22

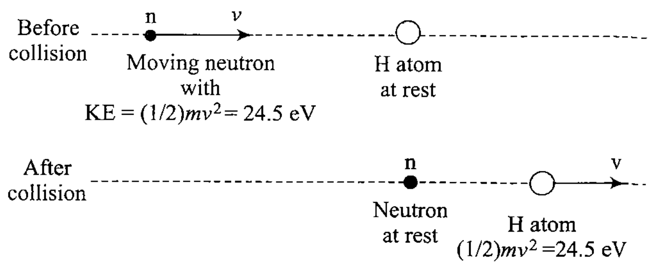


Fig. 4.22

(ii) It may be possible that the H atom will absorb 10.2 eV energy during collision and both the neutron and H atom will be moving with kinetic energy 24.5 − 10.2 = 14.3 eV after collision as shown in Fig. 4.23. In this case, the collision will be partially elastic.

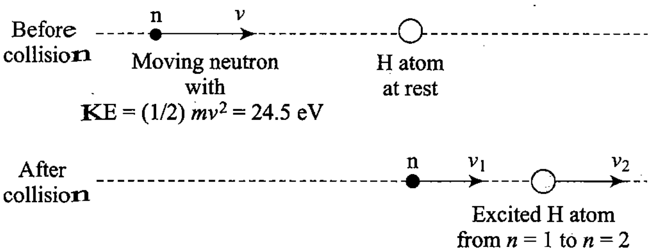


Fig. 4.23

Final kinetic energy,

$$E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 14.3\text{ eV}$$

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2$$

or $$v = v_1 + v_2 \quad \text{(i)}$$

Using energy conservation, we have

$$\frac{1}{2}mv^2 - 10.2\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$24.5\text{ eV} - 10.2\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 14.3\text{ eV} \quad \text{(ii)}$$

Now, solving Eqs. (i) and (ii), we will get the values of v_1 and v_2 which are the possible speeds of neutron and H atom after collision.

(iii) It may be possible that H atom will absorb 12.09 eV energy during collision and both the neutron and H atom will be moving with kinetic energy 24.5 – 12.09 = 12.41 eV after collision as shown in Fig. 4.24. In this case, the collision will be partially elastic.

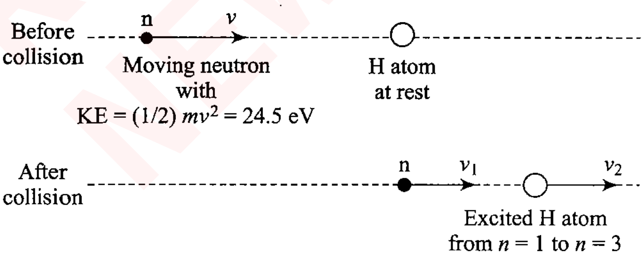


Fig. 4.24

Final kinetic energy,

$$E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 12.41\text{ eV}$$

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2$$

or $$v = v_1 + v_2 \quad \text{(iii)}$$

Using energy conservation, we have

$$\frac{1}{2}mv^2 - 12.09\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$24.5\text{ eV} - 12.09\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 12.41\text{ eV} \quad \text{(iv)}$$

Now, solving Eqs. (iii) and (iv), we will get values of v_1 and v_2 which are the possible speeds of neutron and H atom after collision.

Illustration 4.27 **In the figure shown below, what type of collision can be possible, if K = 14 eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perfectly inelastic).**

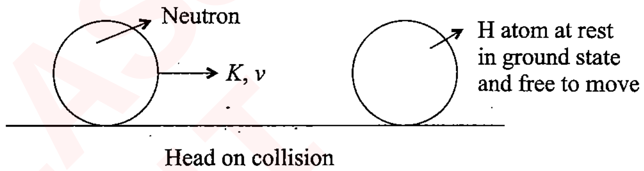


Fig. 4.25

Sol. Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another. According to Bohr's theory, for hydrogen atom,

$$\Delta E = \{0, 10.2\text{ eV}, 12.09\text{ eV}, \ldots, 13.6\text{ eV})$$

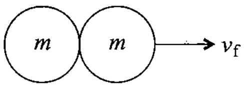


Fig. 4.26

According to Newtonion mechanics, minimum loss = 0 for elastic collision.

For maximum loss, collision will be perfectly inelastic.

If neutron collides perfectly inelastically, then applying momentum conservation:

$$mv_0 = 2mv_f \Rightarrow v_f = \frac{v_0}{2}$$

$$\text{Final KE} = \frac{1}{2} \times 2m \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{K}{2}$$

So, $$\text{maximum loss} = \frac{K}{2}$$

According to classical mechanics, energy loss lies in the range:

$$\Delta E = \left[0, \frac{K}{2}\right]$$

a. K = 14 eV: According to quantum mechanics, ΔE = {0, 10.2 eV, 12.09 eV}
According to classical mechanics, ΔE = [0, 7 eV]
So, possible loss = 0, hence it is an elastic collision.

b. $K = 20.4$ eV: According to quantum mechanics, $\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 10.2 \text{ eV}]$
If possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ perfectly inelastic collision

c. $K = 22$ eV: According to quantum mechanics, $\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 11 \text{ eV}]$
If possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly)

d. $K = 24.18$ eV: According to quantum mechanics, $\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 12.09 \text{ eV}]$
Possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly),
and If loss = 12.09 eV $\Rightarrow$ perfectly inelastic collision

Illustration 4.28 **A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particles.**

Fig. 4.27

Sol. Here, the loss during the collision can only be used to excite the atoms or electrons.

So, according to quantum mechanics, possible losses can be

$$\{0, 40.8 \text{ eV}, 48.36 \text{ eV}, \ldots, 54.4 \text{ eV}\} \quad \text{(i)}$$

from $$E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

Now, according to newtonion mechanics: Minimum loss = 0
Maximum loss will be for perfectly inelastic collision.
Let v_0 be the initial speed of neutron and v_f be the final common speed.
So, by momentum conservation:

$$mv_0 = mv_f + 4mv_f \quad \Rightarrow \quad v_f = \frac{v_0}{5}$$

where m = mass of neutron and mass of He^+ ion = $4m$
Final kinetic energy of system:

$$KE = \frac{1}{2}mv_f^2 + \frac{1}{2}4mv_f^2$$

$$= \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$

$$\text{Maximum loss} = K - \frac{K}{5} = \frac{4K}{5}$$

$$\text{So, loss will be } \left[0, \frac{4K}{5}\right] \quad \text{(ii)}$$

For inelastic collision, there should be at least one common value other than zero in (i) and (ii)

$$\therefore \quad \frac{4K}{5} > 40.8 \text{ eV} \quad \Rightarrow \quad K > 51 \text{ eV}$$

Hence, minimum value of K for an inelastic collision: K_{min} = 51 eV.

Illustration 4.29 **In the previous question find minimum value of K so that all types of collisions are possible.**

Sol. For all three types of collisions, in set (1) and (2),

$$\frac{4K}{5} = 48.36 \text{ eV} \quad \Rightarrow \quad K = 60.45 \text{ eV}$$

Illustration 4.30 **An H atom in ground state is moving with initial kinetic energy K. It collides head on with a He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.**

Sol. Here, energy loss during the collision is used to excite the atom or ion.

Now, according to quantum mechanics loss in energy (ΔE) for H atom $\{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots, 13.06 \text{ eV}\}$
For He^+ ion: $\{0, 40.8 \text{eV}, 48.36 \text{ eV}, \ldots, 54.4 \text{ eV}\}$
To excite the hydrogen atom and He^+ ion in first excited state, minimum energy = 40.8 + 10.2 = 51 eV
Now, according to newtonion mechanics, minimum loss = 0 (for elastic collision). Maximum loss will be when there is perfectly inelastic collision.
Now, let mass of H atom = m, then mass of He^+ ion = $4m$.
Let v_0 be the initial speed of H atom and v_f the final common speed. According to momentum conservation:

$$mv_0 = 4mv_f + mv_f \quad \Rightarrow \quad v_f = \frac{v_0}{5}$$

$$\text{Kinetic energy} = \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$

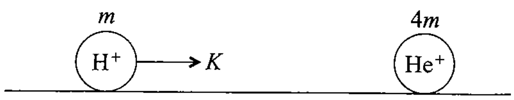


Fig. 4.28

Now, for minimum value of K so that the electron excite to first excited state of H atom and He^+ ion.

$$\frac{4K}{5} = 51 \text{ eV} \quad \text{or} \quad K = \frac{51 \times 5}{4} \text{eV} \quad \text{or} \quad K = 63.75 \text{ eV}$$

Illustration 4.31 **A moving hydrogen atom makes a head-on collision with a stationary hydrogen atom. Before collision, both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.**

Sol. Let K be the kinetic energy of the moving hydrogen atom and K' the kinetic energy of combined mass after collision.

From conservation of linear momentum,

$$p = p' \Rightarrow \sqrt{2Km} = \sqrt{2K'(2m)}$$

or $$K = 2K' \quad \text{(i)}$$

From conservation of energy,

$$K = K' + \Delta E \quad \text{(ii)}$$

Solving equations (i) and (ii), we get $\Delta E = \dfrac{K}{2}$

Now, minimum value of ΔE for hydrogen atom is 10.2 eV.

or $$\Delta E \geq 10.2 \text{ eV}$$

$\therefore$ $$\frac{K}{2} \geq 10.2 \text{ eV}$$

$\therefore$ $$K \geq 20.4 \text{ eV}$$

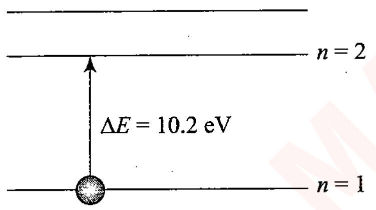


Fig. 4.29

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

Illustration 4.32 **A neutron with an energy of 4.6 MeV collides elastically with protons and is retarded. Assuming that upon each collision the neutron is deflected by 45°, find the number of collisions which will reduce its energy to 0.23 eV.**

Sol. Mass of neutron-mass of proton = m

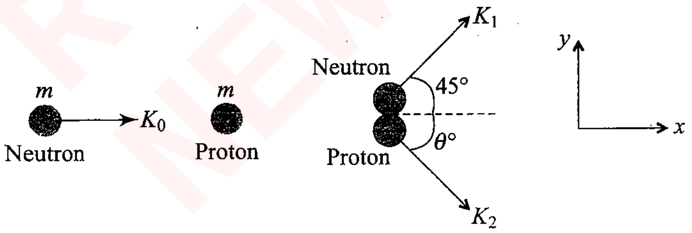


Fig. 4.30

From conservation of momentum in y-direction,

$$\sqrt{2mK_1} \sin 45° = \sqrt{2mK_2} \sin \theta \quad \text{(i)}$$

In x-direction,

$$\sqrt{2mK_0} = \sqrt{2mK_1} \cos 45° + \sqrt{2mK_2} \cos \theta \quad \text{(ii)}$$

Squaring and adding equations (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0 K_1} \quad \text{(iii)}$$

From conservation of energy,

$$K_2 = K_0 - K_1 \quad \text{[as collision is elastic]} \quad \text{(iv)}$$

Solving Eqs. (iii) and (iv), we get

$$K_1 = \frac{K_0}{2}$$

i.e., after each collision energy remains half. Therefore, after n collisions,

$$K_n = K_0 \left(\frac{1}{2}\right)^n$$

$\therefore$ $$0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n \qquad 2^n = \frac{4.6 \times 10^6}{0.23}$$

Taking log and solving, we get

$$n \approx 24$$

Calculation of recoil speed of atom on emission of a photon

Momentum of photon $= mc = \dfrac{h}{\lambda}$

a.

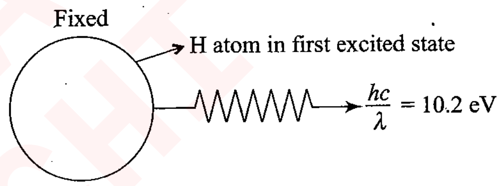


Fig. 4.31

b.

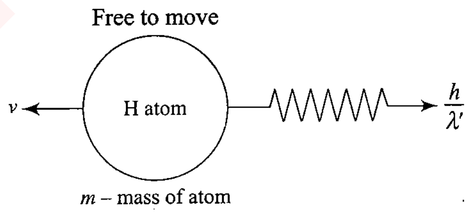


Fig. 4.32

According to momentum conservation, $mv = \dfrac{h}{\lambda'}$ (i)

According to energy conservation, $\dfrac{1}{2}mv^2 + \dfrac{hc}{\lambda'} = 10.2$ eV

Since mass of atom is very large than photon, hence (kinetic energy of atom) can be neglected in comparision to KE of photon.

$$\frac{hc}{\lambda'} = 10.2 \text{ eV} \quad \text{or} \quad \frac{h}{\lambda} = \frac{10.2}{c} \text{eV}$$

$\Rightarrow$ $$mv = \frac{10.2}{c}$$

or $$v = \frac{10.2}{cm}$$

Therefore, recoil speed of atom $= \dfrac{10.2}{cm}$

Illustration 4.33 **Explain why nearly all H atoms are in the ground state at room temperature and hence emit no light.**

Sol. According to kinetic theory, the average kinetic energy of atoms or molecules in a gas is given by $K = (3/2)kT$ where $k = 1.38 \times 10^{-23}$ JK^{-1} is Boltzmann's constant and T is Kelvin (absolute) temperature. Room temperature is about $T = 300$ K. Hence,

$$K = \frac{3}{2}(1.38 \times 10^{-23})(300)$$
$$= 6.2 \times 10^{-21} \text{ J}$$
$$= \frac{6.2 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 0.04 \text{ eV}$$

The energy required to raise an electron from ground state to the next higher state is $13.6 - 3.4 = 10.2$ eV. The average kinetic energy (0.04 eV) of atom is too small to excite an electron from ground state. Any atom in excited state emits photons and eventually falls to the ground state. Once in ground state, collision with other atoms can transfer energy of nearly 0.04 eV. Very high temperature is required to excite electron to upper status.

Illustration 4.34 **Consider a hydrogen-like atom whose energy in n^{th} excited state is given by**

$$E_n = \frac{13.6 Z^2}{n^2}$$

When this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$ eV.

Find the atomic number of atom and the initial state of excitation.

Sol. Maximum energy is liberated for transition $E_n \to 1$ and minimum energy for $E_n \to E_{n-1}$. Hence,

$$\frac{E_1}{n^2} - E_1 = 52.224 \text{ eV}$$

$$\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV}$$

Solving the above equations simultaneously, we get

$$E_1 = -54.4 \text{ eV} \quad \text{and} \quad n = 5$$

Now, $$E_1 = -\frac{13.6 Z^2}{1^2} = -54.4 \text{ eV}$$

Hence, $$Z = 2$$

i.e., the gas is helium, originally excited to $n = 5$ energy state.

Illustration 4.35 **A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to higher energy level by absorbing monochromatic light of photon energy 2.7 eV.**

Subsequently, the atoms emit radiations of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more, and some have less than 2.7 eV.

a. Find the principal quantum number of the initially excited level B.

b. Find the ionization energy for the gas atoms.

c. Find the maximum and the minimum energies of the emitted photons.

Sol. a. Since only six different transitions take place, the final state is $n = 4$. The energy levels of hydrogen atom are given by

$$E_n = -\frac{13.6}{n^2} \text{eV}$$

If n_B is the principal quantum number of the initially excited state B, then

$$E_4 - E_{n_B} = -\frac{13.6}{4^2} - \left(-\frac{13.6}{n_B^2}\right)$$
$$= 13.6\left[\frac{1}{n_B^2} - \frac{1}{16}\right]$$
$$E_4 - E_{n_B} = 2.7 \text{ eV}$$
$$2.7 = 13.6\left[\frac{1}{n_B^2} - \frac{1}{16}\right]$$

which gives $n_B \approx 2$. (Rounding off to nearest integer)

b. The transition energy is numerically equal to the ground state energy E_1 of level A.

$$E_4 = \frac{E_1}{16}, \quad E_2 = \frac{E_1}{4}$$
$$E_4 - E_2 = \frac{E_1}{16} - \frac{E_1}{4}$$
$$2.7 \text{ eV} = -\frac{3}{16}E_1$$
$$E_1 = -14.4 \text{ eV}$$

Thus, the ionization energy of the given atom is 14.4 eV.

c. Maximum energy of the emitted photon is for the electron transition $n = 4$ to $n = 1$, i.e.,

$$E_4 - E_1 = \frac{E_1}{16} - E_1 = -\frac{15}{16}E_1$$
$$= \frac{15}{16} \times (14.4) = 13.5 \text{ eV}$$

Thus, the maximum energy of the emitted photon is 13.5 eV. Minimum energy of the emitted photon corresponds to the transition $n = 4$ to $n = 3$, i.e.,

$$E_4 - E_3 = \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7}{144}E_1$$
$$= -\frac{7}{144} \times (-14.4) = 0.7 \text{ eV}$$

Illustration 4.36 **A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV, respectively.**

Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.954 eV, respectively. Determine the values of n and Z (ionization energy of hydrogen atom = 13.6 eV).

Sol. For the transition from a higher state n to the first excited state $n_1 = 2$, the total energy released is

$$10.2 + 17.0 \text{ eV} \quad \text{or} \quad 27.2 \text{ eV}.$$

Thus, for $\Delta E = 27.2$ eV, $n_1 = 2$, and $n_2 = n$, we have

$$27.2 = 13.6\,Z^2\left[\frac{1}{4} - \frac{1}{n^2}\right]$$

For the eventual transition to the second excited state $n_1 = 3$, the total energy released is (4.25 + 5.95) eV or 10.2 eV. Thus,

$$10.2 = 13.6\,Z^2\left[\frac{1}{9} - \frac{1}{n^2}\right]$$

Dividing the two equations, we get $\dfrac{27.2}{10.2} = \dfrac{9n^2 - 36}{4n^2 - 36}$

Solving, we get $\quad n^2 = 36 \quad \text{or} \quad n = 6$

Substituting $n = 6$ in any of the above equations, we obtain

$$Z^2 = 9 \quad \text{or} \quad Z = 3$$

Thus, $\quad n = 6$ and $Z = 3$

Illustration 4.37 **Electrons in a hydrogen-like atom ($Z = 3$) make transitions from the fourth excited state to the third excited state and from the third excited state to the second excited state. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for photoelectrons ejected by shorter wavelength is 3.95 eV.**

Calculate the work function of the metal and stopping potential for the photoelectrons ejected by the longer wavelength.

Sol. Energy of a photon corresponding to transition from $n = 5$ (fourth excited state) to $n = 4$ (third excited state) is

$$h\nu = 13.6(3)^2\left[\frac{1}{4^2} - \frac{1}{5^2}\right] = 2.75 \text{ eV} \qquad \text{(i)}$$

Similarly, energy of a photon corresponding to transition from $n = 4$ to $n = 3$ is

$$h\nu = 13.6(3)^2\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 5.95 \text{ eV} \qquad \text{(ii)}$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + \text{KE}_{max}$$

$$\text{KE}_{max} = eV_s = h\nu - \phi$$

The shorter wavelength corresponds to greater energy difference between energy levels involved in the transition. So, shorter wavelength photons are emitted for transition $n = 4$ to $n = 3$. Thus, we have

$$3.95 = 5.95 - \phi \quad \Rightarrow \quad \phi = 2 \text{ eV}$$

and for longer wavelength photon, $eV_s = 2.75 - 2 = 0.75$ eV

$$\text{So, stopping potential} = \left(\frac{0.75 \text{ eV}}{e}\right) = 0.75 \text{ V}$$

Illustration 4.38 **The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit in a hydrogen atom falls on a metal to produce photoelectrons. The electrons from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of (1/320) T in a radius of 10^{-3} m. Find (a) the kinetic energy of the electrons, (b) work function of the metal, and (c) wavelength of radiation.**

Sol. a. Radius of the circle traced by a charged particle in a magnetic field is given by

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

Hence, $\quad K = \dfrac{(BqR)^2}{2m} = \dfrac{[10^{-3} \times 1.6 \times 10^{-19} \times (1/320)]^2}{2 \times 9.1 \times 10^{-31}}$

$$= \frac{10^{-17}}{72.8}\text{ J} = \frac{10^{-17}}{72.8} \times \frac{1}{1.6 \times 10^{-19}}\text{ eV} = 0.86 \text{ eV}$$

b. Energy of the photon emitted when the electron makes a transition from $n = 3$ to $n = 2$ energy level is

$$h\nu = E_3 - E_2 = 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]\text{eV} = 1.9 \text{ eV}$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + \text{KE}_{max}$$

So, $\quad \phi = (1.9 - 0.86)\text{eV} = 1.04$ eV

c. Wavelength of radiation is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{1.9 \times 1.6 \times 10^{-19}}\text{m} = 6542 \text{ Å}$$

Concept Application Exercise 4.1

1. State the following statements as TRUE or FALSE.
   a. The radius of n^{th} Bohr orbit is proportional to n^2.
   b. The total energy of electron in the n^{th} orbit is inversely proportional to n^2.
   c. The time period of revolution of electron in the n^{th} orbit is directly proportional to n^3.
   d. The kinetic energy of electron is double of its potential energy with a negative sign.
   e. Velocity of electron in an atom is independent of its mass.

f. Only Balmer series of hydrogen atom lies in the visible range.

g. The angular momentum of an electron revolving in the first orbit of hydrogen is equal to that in the first orbit of helium.

h. The binding energy of deuterium atom is more than that of hydrogen atom.

i. Lyman series of hydrogen atom lies in the infrared region.

2. Three energy levels of an atom are shown in Fig. 4.33. The wavelength corresponding to three possible transitions are λ_1, λ_2, and λ_3. The value of λ_3 in terms of λ_1 and λ_2 is given by

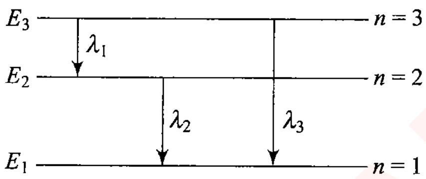


Fig. 4.33

3. A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.

a. What is the initial state of the hydrogen atom?

b. What is the final state of the hydrogen atom?

c. What is the wavelength of the photon emitted?

4. Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.

5. Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

6. Using the known values for hydrogen atom, calculate:

a. radius of third orbit for Li^{2+}.

b. speed of electron in fourth orbit for He^+.

7. Find the kinetic energy, potential energy, and total energy in first and second orbits of hydrogen atom if potential energy in the first orbit is taken to be zero.

8. A small particle of mass m moves in such a way that the potential energy $U = ur^2$, where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

9. An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+4e$. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system:

a. Derive an expression for the radius of n^{th} Bohr orbit.

b. Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to second orbit.

10. Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of H atom becomes −1.7 eV. Now, the photoelectric plate having $W = 2.3$ eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de Broglie wavelength of ejected photoelectrons.

11. Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in percent) do the biding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding values of these quantities?

12. An electron having energy 20 eV collides with a hydrogen atom in the ground state. As a result of the collision, the atom is excited to a higher energy state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with emission of radiation of wavelength 1.216×10^{-7} m. Find the velocity of the scattered electron.

13. According to classical physics, an electron in periodic motion will emit electromagnetic radiation with the same frequency as that of its revolution. Compute this value for hydrogen atom in n^{th} quantum state. Under what conditions does Bohr's quantum theory permit emission of such photons due to transitions between adjoining orbits? Discuss the result obtained.

14. The energy levels of an atom are as shown in Fig. 4.34. Which one of these transitions will result in the emission of a photon of wavelength 275 nm?

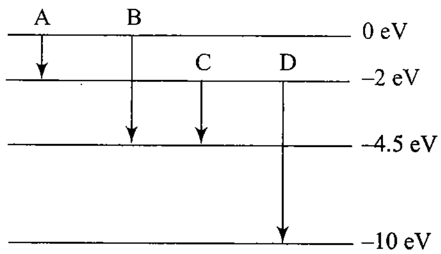


Fig. 4.34

15. The ground state energy of hydrogen atom is −13.6 eV.

a. What is the kinetic energy of an electron in the second excited state?

b. If the electron jumps to the ground state from the second excited state, calculate the wavelength of the spectral line emitted.

16. Suppose that means were available for stripping 28 electrons from $_{29}Cu$ in vapour of this metal.

a. Compute the first three energy levels for the remaining electron.

b. Find the wavelengths of the spectral lines of the series for which $n_1 = 1$, $n_2 = 2, 3, 4$. What is the ionization potential for the last electron?

X-RAYS

X-rays are electromagnetic radiations of very short wavelength (0.1 Å to 100 Å) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ ms^{-1} in vacuum.

Its photons have energy around 1000 times more than the visible light.

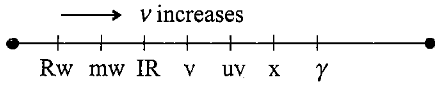


Fig. 4.35

When fast moving electrons having energy of the order of several keV strike the metallic target, then X-rays are produced.

Discovery of X-Rays

X-rays were discovered by Roentgen (1895) who found that a discharge tube, operating at low pressure and high voltage, emitted a radiation that caused a fluorescent screen in the neighborhood to glow brightly. Crystals of barium platinocyanide also showed fluorescence. Results were same, if the discharge tube was wrapped in black paper, to prevent visible light. This indicated that some unknown radiation (X-rays) were responsible for fluorescence. Roentgen then confirmed that X-rays are emitted when cathode rays (electrons) strike the wall of discharge tube.

Production of X-Rays (Coolidge's Tube)

X-rays are produced when energetic (fast moving) electrons strike a target such as a metal piece. When electrons collide with the atoms of solid, they loose their kinetic energy which is converted into radiant energy in the form of X-rays. Figure 4.36 shows the essential features of a modern X-ray tube developed by Coolidge.

Coolidge's X-ray tube consists of a glass bulb exhausted to nearly perfect vacuum. The cathode C is the source of electrons

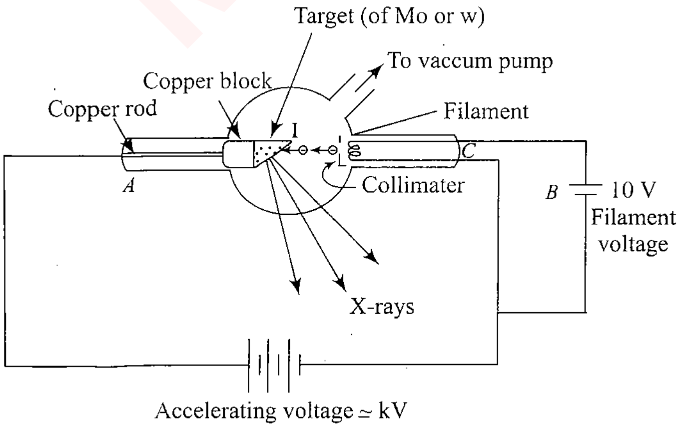


Fig. 4.36

by using a heated filament getting supply from battery B. The anode is made of solid copper bar A. A high melting metal like platinum or tungsten is embedded at the end of copper rod and serves as target T. A high dc voltage V(50 kV) is maintained between cathode and anode.

The melting point, specific heat capacity, and atomic number of the target should be high. When voltage is applied across the filament, then the filament on being heated emits electrons from it. For giving the beam shape to electrons, collimator is used. Now, when the electrons strike the target then X-rays are produced.

When electrons strike with the target, some part of energy is lost and converted into heat. Since the target should not melt or it should absorb the heat, so the melting point and specific heat of the target should be high.

Here, a copper rod is attached so that heat produced can go behind and it can absorb heat and the target does not get heated very high.

For more energetic electrons, accelerating voltage is increased.

For more number of photons, voltage across the filament is increased.

The energetic electrons strike the target and the X-ray are produced. Only about 1–10% of the energy of the electrons is converted to X-rays and the rest is converted into heat. The target T as a result becomes very hot and therefore should have high melting point. The heat generated is dissipated through the copper rod and the anode is cooled by water flowing through the anode.

The nature of emitted X-rays depends on:

(i) the current in the filament F, and

(ii) the voltage between the filament and the anode.

- An increase in the filament current increases the number of electrons it emits. Larger number of electrons means an intense beam of X-rays is produced. This way we can control the quantity of X-rays, i.e., intensity of X-rays.
- An increase in the voltage of the tube increases the kinetic energy of electrons ($eV = (1/2)\, mv^2$). When such highly energetic beam of electrons is suddenly stopped by the target, an energetic beam of X-rays is produced. This way we can control the quality of X-rays, i.e., penetration power of X-rays.
- Based on penetrating power, X-rays are classified into two types: HARD-X-rays and SOFT-X-rays. The ones having high energy and hence high penetration power are HARD-X-rays and the ones with low energy and hence low penetration power are SOFT-X-rays.

Properties of X-Rays

1. These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
2. They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.

3. They cause fluorescence in several materials. A plate coated with barium platinocyanide, ZnS (zinc sulphide), etc. becomes luminous when exposed to X-rays.
4. They affect photographic plates especially designed for the purpose.
5. They are not deflected by electric and magnetic fields, showing that they are not charged particles.
6. They show all the properties of the waves except refraction. They show diffraction patterns when passed through a crystal which behaves like a grating.

Application of X-Rays

X-rays have important and useful applications in surgery, medicine, engineering, and studies of crystal structures.

1. **Scientific Applications:**
   The diffraction of X-rays at crystals opened new dimension to X-rays crystallography. Various diffraction patterns are used in determining internal structure of crystals. The spacing and positions of atoms of a crystal can be precisely determined using Bragg's Law.
2. **Industrial Applications:**
   Since X-rays can penetrate through various materials, they are used in industry to detect defects in metallic structures of big machines, railway tracks and bridges. X-rays are used to analyze the composition of alloys and pearls.
3. **In Radio Therapy:**
   X-rays can cause damage to the tissues of body (cells are ionized and molecules are broken). So, X-rays damage the malignant growths like cancer and tumors which are dangerous to life, when it is used in proper and controlled intensities.
4. **In Medicine and Surgery:**
   X-rays are absorbed more in heavy elements than the lighter ones. Since bones (containing calcium and phosphorus) absorb more X-rays than the surrounding tissues (containing light elements like H, C, O), their shadow is casted on the photographic plate. So, the cracks or fracture in bones can be easily located. Similarly, intestine and digestive system abnormalities are also detected by X-rays.

   Variation of Intensity of X-rays with λ is plotted as shown in Fig. 4.37

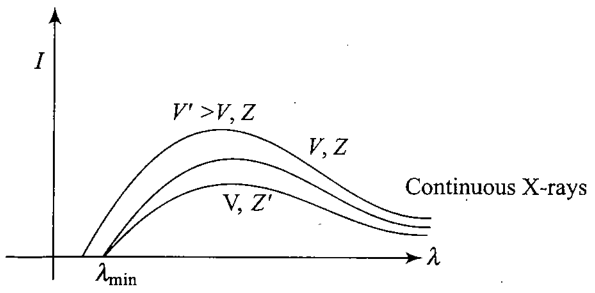


Fig. 4.37

The minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy (eV) of the striking electrons. Thus,

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V_{(\text{in volts})}}\ \text{Å}.$$

We see that cut-off wavelength λ_{min} depends only on accelerating voltage applied between the target and filament. It does not depend upon material of the target, it is same for two different metals (Z and Z').

Illustration 4.39 **An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.**

Sol. Kinetic energy acquired by the electron is

$$K = 20 \times 10^3 \text{ eV}.$$

The energy of the photon is

$$0.05 \times 20 \times 10^3 \text{ eV} = 10^3 \text{ eV}$$

Thus, $$\frac{hc}{\lambda} = 10^3 \text{ eV}$$

$$\Rightarrow \quad \lambda = \frac{(4.14 \times 10^{-15} \text{ eV} - s) \times (3 \times 10^8 \text{ ms}^{-1})}{10^3 \text{ eV}}$$

$$= \frac{1242\, eV - \text{nm}}{10^3\, eV} = 1.24 \text{ nm}$$

Illustration 4.40 **An X-ray tube operates at 20 kV. Find the maximum speed of the electrons striking the anode, given the charge of electron = 1.6×10^{-19} coulomb and mass of electron = 9×10^{-31} kg.**

Sol. When an electron of charge e is accelerated through a potential difference V, it acquires energy eV. If m be the mass of the electron and v_{max} the maximum speed of electron, then

$$\frac{1}{2} m{v^2}_{max} = eV \text{ or } v_{max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{max} = \sqrt{\left(\frac{2 \times (1.6 \times 10^{-19}) \times 20{,}000}{9 \times 10^{-31}}\right)}$$

$$= 8.4 \times 10^7 \text{ ms}^{-1}.$$

Illustration 4.41 **The voltage applied to an X-ray tube being increased η = 1.5 times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda$ = 26 pm. Find the initial voltage applied to the tube.**

Sol. $$\lambda_{min} = \frac{ch}{eV}$$

Here, $\lambda_1 = \dfrac{ch}{eV_1}$ and $\lambda_2 = \dfrac{ch}{eV_2}$

or $$\frac{ch}{e}\left[\frac{1}{V_2} - \frac{1}{V_1}\right] = \lambda_2 - \lambda_1 = \Delta\lambda = 26 \times 10^{-12}$$

or $$\frac{ch}{e}\left[\frac{2}{3V_1} - \frac{1}{V_1}\right] = 26 \times 10^{-12} \qquad (\because \quad V_2 = 1.5\,V_1)$$

Solving for V_1, we get $V_1 = 16000$ V or 16 kV

X-Ray Absorption

The intensity of X-rays at any point may be defined as the energy falling per second per unit area held perpendicular to the direction of energy flow. The intensity of an X-ray beam decreases during its passage through the sheet of any material. The decrease in the intensity of X-rays is due to the absorption of X-rays by the material.

Let I_0 be the intensity of incident beam and I be the intensity of beam after penetrating a thickness x of a material, then $I = I_0 e^{-mx}$; where m is the coefficient of absorption or absorption coefficient of the material. The absorption coefficient depends upon wavelength of X-rays, density of material, and atomic number of material. The elements of high atomic mass and high density absorb X-rays to a higher degree.

X-Ray Spectra and Origin of X-Rays

Experimental observation and studies of spectra of X-rays reveal that X-rays are of two types and so are their respective spectra: Characteristic X-rays and Continuous X-rays.

Characteristic X-rays

These X-rays are called characteristic X-rays because they are characteristic of the element used as target anode. Characteristic X-rays have a line spectral distribution unlike the continuous X-rays. The wavelength spectrum of the X-frequencies corresponding to these lines are the characteristic of the material or the target, i.e., anode material.

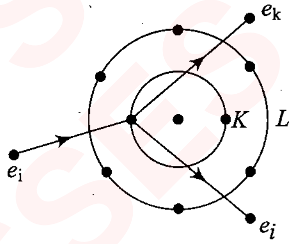
The spectrum of this group consists of several radiations with specific sharp wavelengths and frequency similar to the spectrum (line) of atoms like hydrogen. The wavelengths of this group show characteristic discrete radiations emitted by the atoms of the target material. The characteristic X-rays spectra help us to identify the element of target material.

Origin of Characteristic X-rays

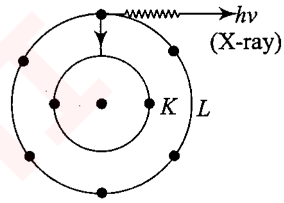
- When the atoms of the target material are bombarded with high energy electrons (or hard X-rays), which possess enough energy to penetrate into the atom, they knockout the electron of inner shell (say K shell, $n = 1$). When an electron is missing in the 'K' shell, an electron from next upper shell makes a quantum jump to fill the vacancy in the 'K' shell. In the transition process, the electron radiates energy whose frequency lies in the X-ray region. The frequency of emitted radiation (i.e., of photon) is given by

$$\nu = RZ_e^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is constant and Z_e is effective atomic number. Generally, Z_e is taken to be equal to $Z - s$, where Z is proton number or atomic number of the element and s is called the screening constant. Due to the presence of other electrons, the charge of the nucleus as seen by the electron will be different in different shells.



(a) Knocking out e' of K shell by incident electron e_i



(b) Emission of X-ray photon K_α series)

Fig. 4.38

- Another vacancy is now created in the 'L' shell which is again filled up by another electron jump from one of the upper shell (M) which results in the emission of another photon, but of different X-ray frequency. This transition continues till outer shells are reached, thus, resulting in the emission of series of spectral lines.
- The transitions of electrons from various outer shells to the inner most 'K' shell produces a group of X-ray lines called as K-series. These radiations are most energetic and most penetrating. K-series is further divided into K_α, K_β, K_g,...depending upon the outer shell from which the transition is made (see Fig. 4.39).

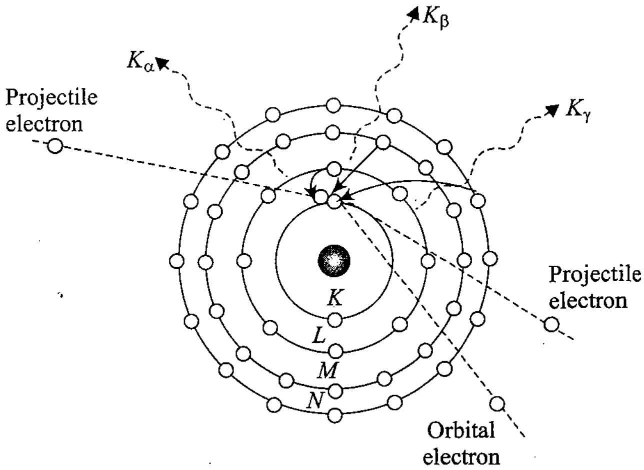
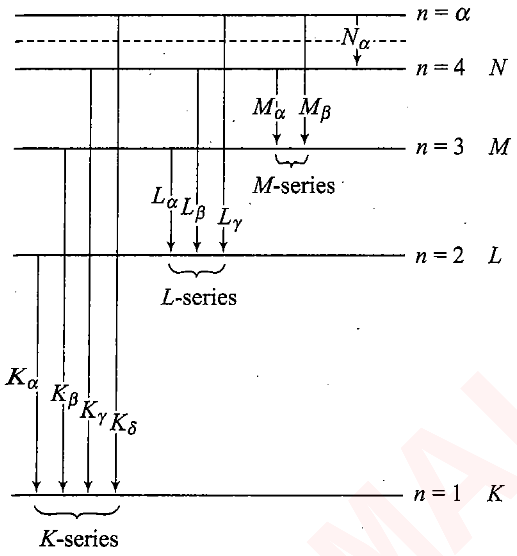
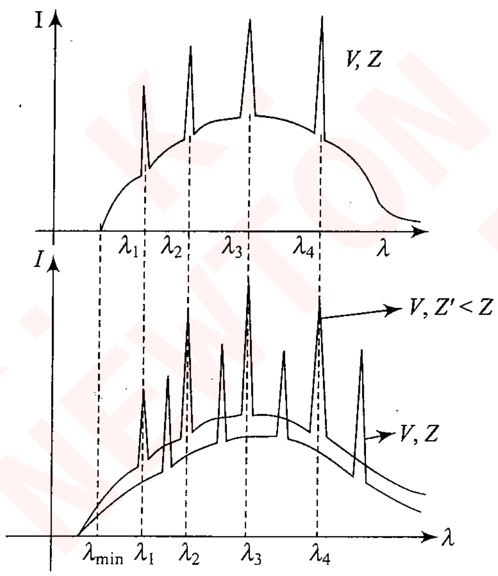


Fig. 4.39

- If cavity is created in L-shell, then according to the transition of electrons from high orbits there will be L_α, L_β, ... lines, and this is called L-series. There may also be M-series as shown in Fig. 4.40(a). Figure 4.40(b) shows the wavelength spectrum for the characteristic X-rays when cavity is created in K-shell.



(a)



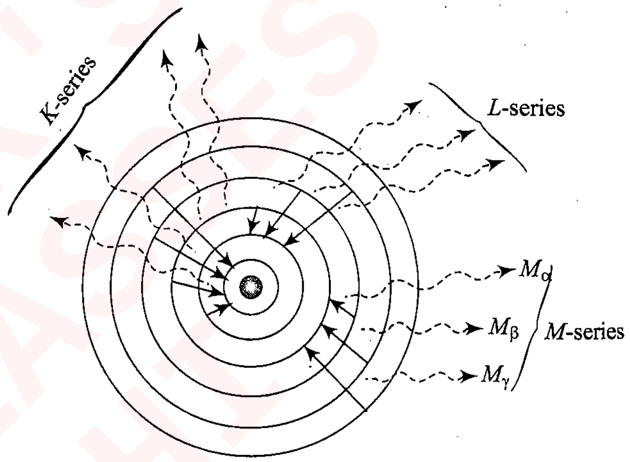
(b)

Fig. 4.40

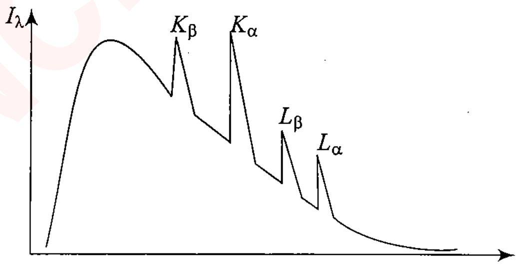
The sharp peaks obtained in graph are known as characteristic X-rays because they are characteristic of the target material.

The characteristic wavelengths of the material having atomic number Z are called *characteristic* X-*rays* and the spectrum obtained is called *characteristic spectrum*. If a target material of atomic number Z' is used, then peaks are shifted.

Thus, the characteristic lines of the characteristic X-rays depend only on the target material and not on the accelerating voltage. One more thing we can see here that the characteristic X-rays are emitted only when the projectile electron makes a collision with another bound electron of the atom of the target material. When cathode rays pass through the target, then the probability of a projectile electron to collide with the bound electron is very less. Majority of the projectile electrons will pass through the anode without making the collision and they produce continuous X-rays. So, always both types of X-rays, continuous and characteristic, will be emitted. X-rays of only a single type cannot be emitted.



(a)



(b)

Fig. 4.41

Illustration 4.42 **In Fig. 4.42, find which is K_α and K_β?**

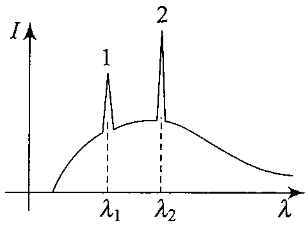


Fig. 4.42

Sol. $\Delta E = \dfrac{hc}{\lambda}$, $\lambda = \dfrac{hc}{\Delta E}$

Since energy difference of K_α is less than K_β,

i.e., $\Delta E_{k_\alpha} < \Delta E_{k_\beta}$, therefore

$$\lambda_{k_\alpha} > \lambda_{k_\beta}$$

So, peak 1 is K_β and peak 2 is K_α

Illustration 4.43 **In Fig. 4.43, find which is K_α and L_α?**

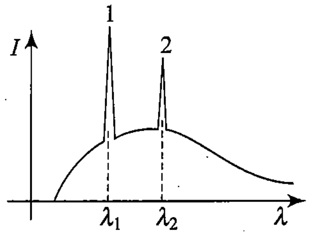


Fig. 4.43

Sol. $\Delta E_{K_\alpha} > \Delta E_{L_\alpha} \Rightarrow \lambda K_\alpha < \lambda L_\alpha$ so peak 1 is K_α and peak 2 is L_α

Illustration 4.44 **The K_α X-ray emission line of tungsten occurs at λ = 0.021 nm. What is the energy difference between K and L levels in this atom?**

Sol. The origin of K_α X-ray can be understood on the basis of Bohr model of atom. If there is only one electron in the K shell, and fill up this vacancy. An electron from higher shell will get de-excited. In this process of de-excitation of electron a photon is emitted during this transition which is the X-ray.

$$\Delta E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9}}$$

$$= 9.46 \times 10^{-15}\text{ J} = 5.9 \times 10^4\text{ eV} = 59\text{ keV}$$

Continuous X-rays

In addition to characteristic X-rays, tubes emit a continuous spectrum also. The characteristic line spectra is superimposed on a continuous X-rays spectra of varying intensities. The wavelengths of the continuous X-rays spectra are independent of the material. One important feature of continuous X-rays is that they end abruptly at a certain lower wavelength for a given voltage.

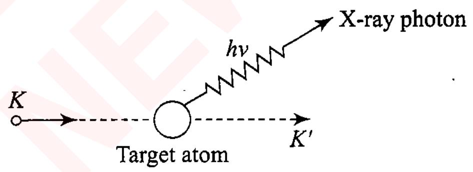


Fig. 4.44

Origin of Continuous X-rays

The mechanism for the production of continuous X-rays can be explained on the basis of Fig. 4.45. It shows an atom of the anode material of high atomic weight with its electron configuration. In the Coolidge tube, an electron is projected toward the anode with an accelerating voltage V. So, the kinetic energy of the projectile electron will be eV. As shown in the figure, when the projectile electron enters into the extremely high electric field of the nucleus of the atom of the anode material, it experiences strong electric force toward the nucleus of the atom and due to this strong attraction the velocity of this electron, when it emerge from the atom, will be highly reduced and negligible compared with the initial velocity of the projectile electron. This electron in the influence of the highly positive nucleus experiences a very high acceleration and according to the classical theory every accelerated charge particle emits the electromagnetic radiations, so this electron will also emit electromagnetic radiations. These electromagnetic radiations are called X-rays. According to the law of conservation of energy, the energy of these electromagnetic radiations will be equal to the decrease in the kinetic energy of the projectile electron. This amount can be calculated.

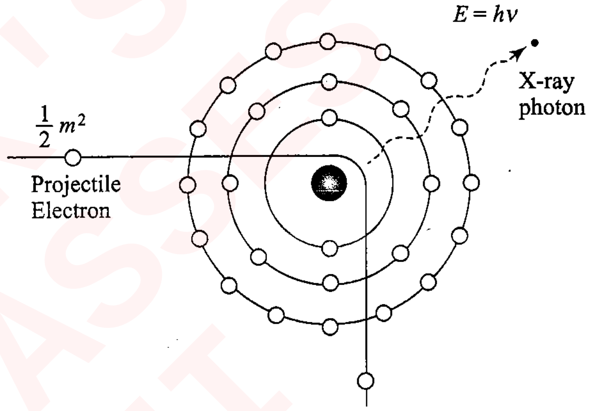


Fig. 4.45

If the initial velocity of the projectile electron is v, then as the velocity is gained due to the accelerating voltage V, we have

$$eV = \frac{1}{2}mv^2 \quad \text{(i)}$$

or
$$v = \sqrt{\frac{2eV}{m}} \quad \text{(ii)}$$

When this electron comes out from the atom of anode material, the speed of this electron will be very less as compared to its initial speed. Thus, the difference of kinetic energy of this electron is emitted in the form of an X-ray photon from the anode atom.

The electrons which pass through the atom very close to the nucleus, will be more accelerated and the energy corresponding to the electrons will be more as compared to those electrons which pass through the atom at relatively large distance from the nucleus. The maximum energy of X-ray photon will be corresponding to that electron which loses almost all of its energy during passing through the atom. The photon corresponding to this electron will have the shortest wavelength among all the photons radiated by other electrons. If this shortest wavelength is λ_c, then we have

$$\Delta E = \frac{1}{2}mv^2 = eV = \frac{hc}{\lambda_c}$$

or
$$\lambda_c = \frac{hc}{eV} = \frac{12431}{V}\text{ Å} \quad \text{(iii)}$$

This is the minimum wavelength of X-rays emitted from an X-ray tube. Thus, from Eq. (iii) we can see that the maximum energy or minimum wavelength of X-rays emitted depends only on the potential difference applied across the discharge tube.

Thus, we can obtain X-rays in any range λ_c to ∞ by applying an appropriate voltage across the discharge tube, which will fix λ_c and other photons emitted from the tube will have wavelengths more than λ_c and ranging up to ∞. That is why these X-rays are called continuous X-rays as shown in Fig. 4.46.

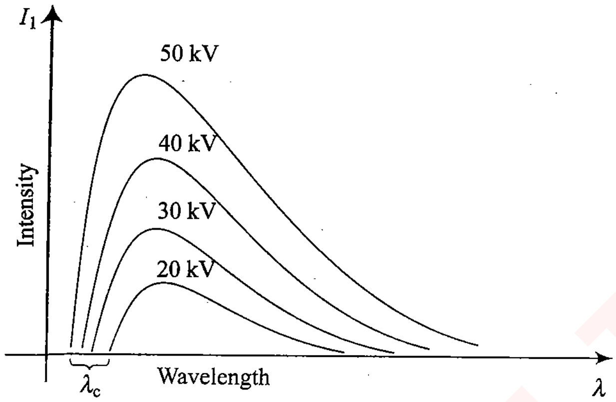


Fig. 4.46

As we can see in the graph, the intensity of emitted X-rays will be maximum (maximum number of photons) for a particular value of wavelength at a particular accelerating voltage across the discharge tube. At a particular voltage the intensity of X-rays can be varied by changing the current in the circuit because the intensity of X-rays (number of photons) is proportional to the number of electrons attacking the anode. The broad continuous spectrum beyond the peak intensity is referred as "Bremsstrahlung".

Illustration 4.45 **When 0.50 Å X-rays strike a material, the photoelectrons from the K shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2} tesla acting perpendicular to direction of emission of photoelectrons. What is the binding energy of K-shell electrons?**

Sol. The velocity of the photoelectrons is found by the relation:

$$evB = m\frac{v^2}{R} \quad \text{or} \quad v = \frac{e}{m}BR$$

The kinetic energy of the photoelectrons is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6\times10^{-19})^2(2\times10^{-2})^2(23\times10^{-3})^2}{(9.1\times10^{-31})}$$

$$= 2.97\times10^{-15}\ \text{J}$$

or $$K = (2.97\times10^{-15})\frac{1}{1.6\times10^{-19}} = 18.36\ \text{KeV}$$

The energy of the incident photon is $E_v = \dfrac{hc}{\lambda} = \dfrac{12.4}{0.50} = 24.8$ KeV

The binding energy is the difference between these two values:

$$\text{BE} = E_v - K = 24.8 - 18.6 = 6.2\ \text{keV}$$

MOSELEY'S LAW

Moseley measured the frequencies of characteristic X-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that the plot is very close to a straight line not passing through origin. The relation of straight line is expressed as $\sqrt{v} = a(Z-b)$, where a and b are constants. This relation is called as *Moseley's Law*. It helps to determine the atomic number Z of an atom. Here, b is the screening constant.

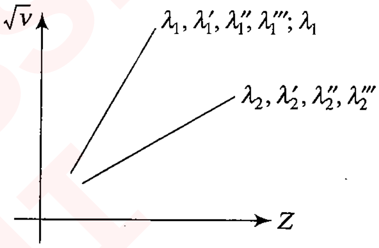


Fig. 4.47

Moseley's observations can be mathematically expressed as $\sqrt{v} = a(Z-b)$, where a and b are positive constants for one type of X-rays and for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom.

Frequency of X-rays is given by

$$\sqrt{v} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}\,(Z-b) \qquad \text{(For multielectron system)}$$

b is known as the screening constant or shielding constant, and $(Z-b)$ is the effective nuclear charge.

For K_a line: $n_1 = 1$, $n_2 = 2$, therefore

$$\sqrt{v} = \sqrt{\frac{3RC}{4}}(Z-b) = a(Z-b)$$

Here $$a = \sqrt{\frac{3RC}{4}}, \qquad [b = 1 \text{ for } K_a \text{ lines}]$$

Illustration 4.46 **If the K_α radiation of Mo (Z = 42) has a wavelength of 0.71 Å, calculate wavelength of the corresponding radiation of Cu, i.e., K_α for Cu (Z = 29) assuming b = 1.**

Sol. According to Moseley's Law: $\sqrt{v} = a(Z-1)$

$\Rightarrow \quad (Z-1)^2 \propto v$ or $(Z-1)^2 \propto 1/\lambda$

$$\Rightarrow \quad \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2} = \frac{\lambda_{Cu}}{\lambda_{Mo}} \quad \text{or} \quad \lambda_{Cu} = \lambda_{Mo}\frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2}$$

$$= 0.71 \times \left(\frac{41}{28}\right)^2 = 1.52\ \text{Å}$$

Illustration 4.47 Compare Z_1 and Z_2.

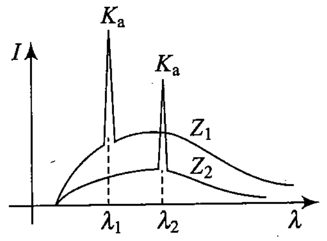


Fig. 4.48

Sol. $\sqrt{v} \equiv \sqrt{cR\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}\cdot(Z-b)$

If Z is greater, then v will be greater and hence λ will be less. Since $\lambda_1 < \lambda_2$, $Z_1 > Z_2$.

Illustration 4.48 **A cobalt target ($Z = 27$) is bombarded with electrons and the wavelengths of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?**

Sol. Using Moseley's law and putting c/λ for v (and assuming $b = 1$), we obtain

$$\sqrt{\frac{c}{\lambda_{co}}} = aZ_{co} - a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

Dividing yields $\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$

Substituting gives us $\sqrt{\frac{178.9\text{ pm}}{143.5\text{ pm}}} = \frac{Z_x - 1}{27 - 1}$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Illustration 4.49 **Find the constants a and b in Moseley's equation $\sqrt{v} = a(Z-b)$ from the following data.**

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Sol. Moseley's equation is $\sqrt{v} = a(Z-b)$

Thus, $$\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \qquad \text{(i)}$$

and $$\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b) \qquad \text{(ii)}$$

From (i) and (ii), $\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$

or $$a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$

$$= \frac{(3\times10^8)^{1/2}}{42-27}\left[\frac{1}{(71\times10^{-12})^{1/2}} - \frac{1}{178.5\times10^{-12})^{1/2}}\right] = 5.0\times10^7\ (\text{Hz})^{1/2}$$

Dividing (i) by (ii), $\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$ or $\sqrt{\frac{178.5}{71}} = \frac{42-b}{27-b}$ or

$b = 1.37$

Illustration 4.50 **An X-ray tube with a copper target is found to be emitting lines other than those due to copper. The K_α line of copper is known to have a wavelength 1.5405 Å and the other two K_α lines observed have wavelengths 0.7090 Å and 1.6578 Å. Identify the impurities.**

Sol. According to Moseley's equation for K_α radiation,

$$\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \ (\lambda = \text{wavelength corresponding to Cu})$$

Let λ_1 and λ_2 be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z-1)^2}{(Z_1-1)^2} = \frac{0.7092}{1.5405}$$

For copper, $Z = 29$, therefore

$$(Z_1 - 1) = 28\sqrt{\frac{1.5405}{0.7092}} = 41$$

or $$Z_1 = 42 \text{ (Molybdenum)}$$

Similarly, $$\frac{\lambda_2}{\lambda} = \frac{(28)^2}{(Z_2-1)^2} = \frac{1.6578}{1.5405}$$

$$(Z_2 - 1) = 28\sqrt{\left(\frac{1.5405}{1.6578}\right)}$$

or $(Z_2 - 1) = 27$ or $Z_2 = 28$ (Nickel)

So, the impurities are molybdenum and nickel.

Illustration 4.51 **The K-absorption edge of an unknown element is 0.171 Å.**

a. Identify the element.
b. Find the average wavelengths of the K_α, K_β and K_γ lines.
c. If a 100 eV electron strikes the target of this element, what is the minimum wavelength of the X-ray emitted?

Sol. From Moseley's law, the wavelength of K series of X-rays is given by taking modified Rydberg's formula given as

$$\frac{1}{\lambda} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right) \text{ for } K \text{ lines, where } n = 2, 3, 4, \ldots$$

a. For K-absorption edge, we put $n = \infty$ in above expression to get

$$(Z-1) = \sqrt{\frac{1}{\lambda R}}$$

or $$Z = \sqrt{\frac{1}{(0.171\times10^{-10})(1.097\times10^{7})}} + 1 = 74$$

The element is Tungsten.

b. For K_α line: $\frac{1}{\lambda_{K_\alpha}} = R(74-1)^2\left[1-\frac{1}{2^2}\right]$

$\Rightarrow \quad \lambda_{K_\alpha} = 0.228$ Å

For K_β line: $\frac{1}{\lambda_{K_\beta}} = R(74-1)^2\left[1-\frac{1}{3^2}\right]$

$\Rightarrow \quad \lambda_{K_\beta} = 0.192$ Å

For K_γ line: $\frac{1}{\lambda_{K_\gamma}} = R(74-1)^2\left[1-\frac{1}{4^2}\right]$

$\Rightarrow \quad \lambda_{K_\gamma} = 0.182$ Å

c. The shortest wavelength corresponding to an electron with kinetic energy 100 eV is given by

$$\lambda_c = \frac{hc}{E} = \frac{12431}{100}\text{ Å} = 124.31\text{Å}$$

Concept Application Exercise 4.2

1. State the following statements as TRUE or FALSE.

(i) X-rays are electromagnetic waves because these are produced by the deceleration of fast moving electrons.

(ii) X-rays are electromagnetic waves because these produce line spectrum.

(iii) The cut-off wavelength depends on the target material.

(iv) The intensity of K_α radiation is more than that of K_β.

(v) The frequency of K_α radiation is more than that of K_β.

2. Calculate the cut-off wavelength of the X-ray emitted by a Coolidge tube operating at 40 kV.

3. Calculate the wavelength of K_α-line for the target made of tungsten ($Z = 74$).

4. Obtain a relation between the frequencies of K_α, K_β and L_α lines for a target material.

5. An X-rays tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode, given the charge of electron = 1.6×10^{-19} coulomb and mass of electron = 9×10^{-31} kg.

6. (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation? (b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

7. The wavelength of characteristic X-ray K_α line emitted from Zinc ($Z = 30$) is 1.415Å. Find the wavelength of the K_α line emitted from molybdenum ($Z = 42$).

8. If the short series limit of the Balmer series for hydrogen is 3644 Å, find the atomic number of the element which gives X-ray wavelengths down to 1 Å. Identify the element.

9. A material whose K-absorption edge is 0.2 Å is irradiated by X-rays of wavelength 0.15 Å. Find the maximum energy of the photoelectrons that are emitted from the K shell.

10. Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from an M shell to a vacancy in the shell.

11. A potential difference of 20 kV is applied to an X-ray tube. Find the minimum wavelength of X-rays generated.

12. The wavelength of X-rays produced by an X-ray tube is 0.76 Å. What is the atomic number of the anode material of the tube?

Solved Examples

Example 4.1 **Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground states is found to liberate electrons with kinetic energies 1.8 eV and 4.0 eV, respectively. Find the value of Planck's constant.** **(IIT-JEE, 1973)**

Sol. The energy of incident photon = (hc/λ)

If W_i is the ionization energy and E_k the kinetic energy of the emitted electrons, then we have

$$\frac{hc}{\lambda_1} = W_i + E_{K_1}$$

For incident photon of wavelength $\lambda_1 = 800$ Å $= 8\times10^{-8}$ m,

$$\frac{hc}{\lambda_1} = W_i + E_{K_1} \quad \text{(i)}$$

And for incident photon of wavelength $\lambda_2 = 700$ Å $= 7\times10^{-8}$ m,

$$\frac{hc}{\lambda_2} = W_i + E_{K_2} \quad \text{(ii)}$$

Subtracting (i) from (ii), we get $\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right) = E_{K_2} - E_{K_1}$

or $$h = \frac{(E_{K_2}-E_{K_1})\lambda_1\lambda_2}{c(\lambda_1-\lambda_2)}$$

Here, $E_{K_1} = 1.8\text{eV} = 1.8\times1.6\times10^{-19}$ J

and $E_{K_2} = 4.0\text{ eV} = 4.0\times1.6\times10^{-19}$ J

Substituting given values, we get

$$h = \frac{(4.8-1.8)\times1.6\times10^{-19}\times8\times10^{-8}\times7\times10^{-8}}{3\times10^{8}(8\times10^{-8}-7\times10^{-8})}$$

$= 6.57\times10^{-34}$ J-s

Example 4.2 **A single electron orbits around a stationary nucleus of charge +Ze, where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find**

(i) the values of Z,

(ii) the energy required to excite the electron from the third to the fourth Bohr orbit,

(iii) the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity,

(iv) the kinetic energy, potential energy, and the angular momentum of the electron in the first Bohr orbit.

(v) the radius of the first Bohr orbit.

The ionization energy of the hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} m, speed of light = 3×10^8 ms^{-1}, Planck's constant = 6.63×10^{-34} J-s (IIT-JEE, 1981)

Sol. The energy of the electron in n^{th} orbit of hydrogen-like atom is given by

$$E_n = -\frac{Z^2 Rhc}{n^2} \quad \text{(i)}$$

(i) The energy required to excite the electron from second ($n = 2$) Bohr orbit to third ($n = 3$) Bohr orbit is given by

$$\Delta E = E_3 - E_2 = -\frac{Z^2 Rhc}{3^2} - \left(-\frac{Z^2 Rhc}{3^2}\right)$$

$$= Z^2 Rhc\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36} Z^2 Rhc$$

Given, ionization energy of hydrogen atom = $Rhc = 13.6$ eV and $\Delta E = E_3 - E_2 = 47.2$ eV

Thus, we have 47.2 eV = $\frac{5}{36} Z^2 \times 13.6$ eV

or $Z^2 = \frac{36}{5} \times \frac{47.2}{13.6} = 25 \Rightarrow Z = 5$

(ii) The energy required to excite the atom from third to fourth orbit is given by

$$E_4 - E_3 = -\frac{Z^2 Rhc}{4^2} - \left(-\frac{Z^2 Rhc}{3^2}\right) = Z^2 Rhc\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 5^2 \times (13.6 \text{ eV}) \times \frac{7}{144} = 16.53 \text{ eV}$$

(iii) The ionization energy of atom or the energy required to remove the electron from first Bohr orbit to ∞ is

$$E_\infty - E_1 = -\frac{Z^2 Rhc}{(\infty)^2} - \left(-\frac{Z^2 Rhc}{1^2}\right) = Z^2 Rhc$$

If λ is the wavelength of corresponding electromagnetic radiation, then

$$\frac{hc}{\lambda} = Z^2 Rhc$$

i.e., $$\frac{hc}{\lambda} = 5^2 \times 13.6 \text{ eV} = 340 \text{ eV}$$

$$\therefore \quad \lambda = \frac{hc}{340 \text{ eV}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{340 \times 1.6 \times 10^{-19}}$$

$$= 36.056 \times 10^{-10} \text{ m} = 36.056 \text{ Å}$$

(iv) Kinetic energy of electron in first Bohr's orbit,

$$E_{K_1} = -E_1 = 340 \text{ eV}$$

Potential energy of electron in first Bohr's orbit,

$$U_1 = 2E_1 = -2 \times 340 \text{ V} = -680 \text{ eV}$$

Angular momentum of electron in first Bohr's orbit

$$= n\frac{h}{2\pi} = 1\frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2\pi}$$

$$= 1.0546 \times 10^{-34} \text{ Js}$$

(v) Radius of first Bohr orbit for given atom

$$= \left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)_{n=1} = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z}$$

$$= \frac{5.3 \times 10^{-11}}{5} \text{m} = 1.06 \times 10^{-11} \text{ m}$$

Example 4.3 **The ionization energy of a hydrogen-like Bohr atom is 4 rydbergs.**

(i) What is the wavelength of radiation emitted when the electron jumps from first excited state to ground state?

(ii) What is the radius of first orbit for this atom?

Given that Bohr radius of hydrogen atom = 5×10^{-11} m and 1 rydberg = 2.2×10^{-18}J. (IIT-JEE, 1984)

Sol. The energy of electron in hydrogen-like atoms in n^{th} orbit is

$$E_n = \frac{Z^2 Rhc}{n^2}$$

We have $Rhc = 1$ rydberg.

The ionization energy

$$E_\infty - E_1 = Z^2 Rhc = 4 \text{ rydberg}$$

$$\therefore \quad Z^2 = \frac{4 \text{ rydberg}}{Rhc} = \frac{4 \text{ rydberg}}{1 \text{ rydberg}} = 4$$

$$\therefore \quad Z = 2$$

(i) The energy required to excite the electron from $n = 1$ to $n = 2$ is given by

$$E_2 - E_1 = -\frac{Z^2 Rhc}{2^2} - \left(-\frac{Z^2 Rhc}{1^2}\right)$$

$$= Z^2 Rhc\left(1 - \frac{1}{4}\right) = \frac{3}{4} Z^2 Rhc$$

$$= \frac{3}{4} \times 4 \text{ rydberg} = 3 \text{ rydberg}$$

If λ is the wavelength of radiation emitted, then

$$\frac{hc}{\lambda} = 3 \text{ rydberg, i.e., } \lambda = \frac{hc}{(3 \text{ rydberg})}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 2.2 \times 10^{-18}}$$

$$= 301.4 \times 10^{-10} \text{ m} = 301.4 \text{ Å}$$

(ii) Radius of first Bohr orbit, $r_1 = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z} = \frac{5 \times 10^{-11}}{2}$$

$$= 2.5 \times 10^{-11} \text{ m}$$

Example 4.4 **A doubly ionized Lithium atom is hydrogen like with atomic number 3.**

(i) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to third Bohr orbit (ionization energy of hydrogen equals 13.6 eV).

(ii) How many spectral lines are observed in the emission spectrum of the above excited system? (IIT-JEE, 1985)

Sol. The energy of electron in n^{th} orbit of hydrogen-like atoms is

$$E_n = -\frac{Z^2 Rhc}{n^2}$$

Here, $Z = 3$

and $Rhc = 13.6$ eV

(i) The energy required to excite the electron in Li^{++} from $n = 1$ to $n = 3$ is

$$E_3 - E_1 = -\frac{Z^2 Rhc}{3^2} - \left(-\frac{Z^2 Rhc}{1^2}\right)$$

$$= Z^2 Rhc\left(1 - \frac{1}{9}\right) = \frac{8}{9} Z^2 Rhc$$

$$= \frac{8}{9} \times (3)^2 \times 13.6 \text{ eV} = 8 \times 13.6 \text{ eV}$$

$$= 8 \times 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

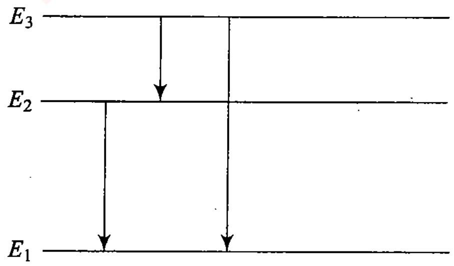


Fig. 4.49

Therefore, the wavelength of incident radiation required for excitation is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$

or $$\lambda = \frac{hc}{E_3 - E_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{8 \times 13.6 \times 1.6 \times 10^{-19}}$$

$$= 114.26 \times 10^{-10} \text{ m} = 114.26 \text{ Å}$$

(ii) The possible lines in the emission spectrum of this excited system are $\left[\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3\right]$

Three in number, represented in Fig. 4.48.

Example 4.5 **The energy of an electron in an excited hydrogen atom is –3.4 eV. Calculate the angular momentum of the electron according to Bohr's theory.**

Given:

Rydberg's constant $R = 1.09737 \times 10^{-7}$ m^{-1}, Planck's constant $h = 6.626176 \times 10^{-34}$ J-s, speed of light $c = 3 \times 10^{8}$ ms^{-1}.

(IIT-JEE, 1986)

Sol. The energy of an electron in n^{th} orbit of hydrogen atom is

$$E_n = -\frac{Rhc}{n^2}$$

$$Rhc = 1.09737 \times 10^{7} \times 6.626176 \times 10^{-34} \times 3 \times 10^{8} \text{ joule}$$

$$\therefore \quad E_n = \frac{1.09737 \times 10^{7} \times 6.626176 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19}} \text{eV}$$

$$= 13.6 \text{ eV.}$$

From Eq. (i), $-3.4\text{eV} = \frac{13.6}{n^2}\text{eV}$

$$\therefore \quad n^2 = \frac{13.6}{3.4} = 4, \text{ i.e., } n = 2$$

According to Bohr's quantum condition,
Angular momentum,

$$mvr = n\frac{h}{2\pi} = 2 \times \frac{6.626176 \times 10^{-34}}{2 \times 3.14} = 2.1102 \times 10^{-34} \text{ Js}$$

Example 4.6 **Light of wavelength 2000 Å falls on an aluminium surface. In aluminium, 4.2 eV of energy is required to remove an electron from its surface. What is the kinetic energy, in electron volt of (a) the fastest and (b) the slowest emitted photo-electrons. (c) What is the stopping potential? (d) What is the cut-off wavelength for aluminium? (Plack's constant $h = 6.6 \times 10^{-34}$ J-s and speed of light $c = 3 \times 10^{8}$ ms^{-1}). (IIT-JEE, 1986)**

Sol. Energy corresponding to incident photon,

$$h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2000 \times 10^{-10}}$$

$$= 9.9 \times 10^{-19} \text{ J} = \frac{9.9 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 6.2 \text{ eV}$$

a. The kinetic energy of fastest electrons,

$$E_k = h\nu - W$$

or $$E_k = 6.2 \text{ eV} - 4.2 \text{ eV} = 2 \text{ eV}$$

b. The kinetic energy of slowest electrons = zero, since the emitted electrons have all possible energies from 0 to certain maximum value E_k.

c. If V_s is the stopping potential, then

$$E_k = eV_s$$

or
$$V_S = \frac{E_K}{e} = \frac{2\text{ eV}}{e} = 2\text{ V}$$

d. If λ_0 is the cut-off wavelength for aluminium, then $W = (hc/\lambda_0)$

or
$$\lambda_0 = (hc/W)$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{4.2\times1.6\times10^{-19}} = 3000\times10^{-10}\text{ m}$$

$$= 3000\text{ Å}$$

Example 4.7 **Find the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photoelectric effect begins in this metal at a frequency of 6×10^{15} Hz. Find the work function for this metal. (Given $h = 6.63\times10^{-34}$ J-s).** **(IIT-JEE, 1987)**

Sol. According to Einstein's photoelectric equation,

$$E_k = h\nu - W$$

If V_s is retarding or stopping potential and ν_0, the threshold frequency, then above equation becomes

$$eV_s = h\nu - h\nu_0$$

or
$$h\nu = eV_s + h\nu_0$$

or
$$\nu = \frac{eV_s}{h} + \nu_0$$

Hence, $e = 1.6\times10^{-10}$ coulomb, $V_s = 3$ V, and $\nu_0 = 6\times10^{-14}$ Hz. Therefore, required frequency

$$\nu = \frac{1.6\times10^{-10}\times3}{6.63\times10^{-34}} + 6\times10^{14}$$

$$= 7.24\times10^{14} + 6\times10^{14} = 13.24\times10^{14}\text{ Hz}$$

$$= 1.324\times10^{15}\text{ Hz}$$

Example 4.8 **A 40 W ultraviolet light source of wavelength 2480 Å illuminates a magnesium (Mg) surface placed 2 m away. Determine the number of photons emitted from the surface per second and the number incident on unit area of Mg surface per second. The photoelectric work function for Mg is 3.68 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Determine the maximum wavelength for which the photoelectric effect can be observed with a Mg surface.** **(IIT-JEE, 1988)**

Sol. Energy of each photon,

$$\varepsilon = \frac{hc}{\lambda}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2480\times10^{-10}}\text{ J} = 8.0\times10^{-19}\text{ J}$$

Number of photons emitted per second,

$$N = \frac{P}{\varepsilon} = \frac{40}{8.0\times10^{-19}} = 5.0\times1019\text{ s}^{-1}$$

These photons spread in all directions over surface area $4\pi r^2$, therefore the number of photons incident per unit area per second

$$N_i = \frac{N}{4\pi r^2} = \frac{5.0\times10^{19}}{4\times3.14\times(2)^2}$$

$$(\text{As } r = 2m) = 9.95\times10^{13}\text{ s}^{-1}$$

From Einstein's photoelectric equation,

$$E_k = h\nu - W$$

$$\varepsilon = h\nu = \frac{hc}{\lambda} = 8.0\times10^{-19}\text{J}$$

$$= \frac{8.0\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 5.0\text{ eV}$$

$\therefore$
$$E_k = 5.0\text{ eV} - 3.68\text{ eV} = 1.32\text{ eV}$$

Threshold (or maximum) wavelength for photoelectrons emission,

$$\lambda_0 = \frac{hc}{W} = \frac{6.62\times10^{-34}\times3\times10^{8}}{3.68\times1.6\times10^{-19}}\text{ m}$$

$$= 3.373\times10^{-7} = 3373\text{ Å}$$

Example 4.9 **Hydrogen atom in its ground state is excited by means of monochromic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.** **(IIT-JEE, 1983)**

Sol. The energy of an electron in n^{th} orbit of hydrogen ($Z = 1$) is given by

$$E_n = \frac{Rhc}{n^2}$$

$\therefore$ Ionization energy $= E_\infty - E_1 = Rhc = 13.6$ eV

$\therefore$
$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

Therefore, energy of electron in ground state ($n = 1$),

$$E_1 = -\frac{13.6}{1} = -13.6\text{ eV}$$

Energy of electron in first excited state ($n = 2$),

$$E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4\text{ eV}$$

The energy of electron in second excited state ($n = 3$),

$$E_3 = -\frac{13.6}{3^3} = \frac{13.6}{9} = -1.511\text{ eV}$$

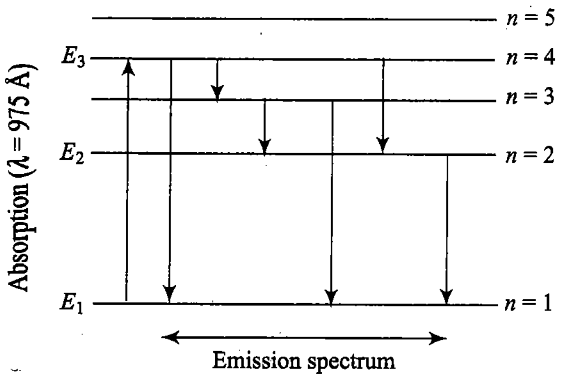


Fig. 4.50

The energy of electron in third excited state ($n = 4$),

$$E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.85 \text{ eV}$$

Energy of incident photon of wavelength

$$\lambda = 975 \text{ Å} = 975 \times 10^{-10} \text{ m}$$

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}} \text{ J}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV} = 12.75 \text{ eV}$$

When the incident photon of this energy is absorbed by hydrogen atom, let its ground state electron occupy $(n-1)^{\text{th}}$ excited state or n^{th} orbit. Then,

$$E = -\frac{Rhc}{n^2} - \left(-\frac{Rhc}{1^2}\right) = Rhc\left(1 - \frac{1}{n^2}\right)$$

i.e., $$12.75 \text{ eV} = 13.6 \text{ eV}\left(1 - \frac{1}{n^2}\right)$$

This gives $n = 4$.

That is the electron is excited to third excited state. The emission spectrum will contain the transitions shown in Fig. 4.50. The longest wavelength emitted corresponds to transition $(4 \to 3)$ for which the energy difference is minimum.

i.e., $$E_{min} = E_4 - E_3 = -0.85 - (-1.511) \text{ eV}$$

$$= 1.511 - 0.85 = 0.661 \text{ eV} = 0.661 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \quad \lambda_{max} = \frac{hc}{E_{min}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.661 \times 1.6 \times 10^{-19}} = 18.807 \times 10^{-7} \text{ m}$$

$$= 18807 \text{ Å}$$

Example 4.10 **A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level *A* and some atoms in a particular upper (excited) energy level *B* and there are no atoms in any other energy level. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV, some have energy more and some have less than 2.7 eV.**

(i) Find the principal quantum number of the initially excited level B.

(ii) Find the ionization energy for the gas atoms.

(iii) Find the maximum and minimum energies of the emitted photons. **(IIT-JEE, 1989)**

Sol. The energy levels of identical hydrogen-like atoms are given by

$$E_n \approx -\frac{B}{n^2}, \; B \text{ being a constant.}$$

(i) When a hydrogen-like atom absorbs energy 2.7 eV. This is only possible if transition of electron is from state *B* to any higher energy state. Six radiations are emitted only if the final state is $n = 4$.

The energy of emitted radiation is also more than 2.7 eV, therefore initial state cannot be $n = 1$. As energy of emitted radiation is also less than 2.7 eV. This shows that initial excited state cannot be $n = 3$. Hence, only possibility is that initially excited state *B* has $n = 2$.

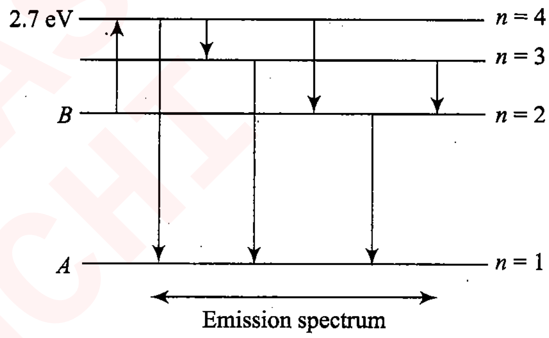


Fig. 4.51

That is, the principal quantum number of initially excited state *B* is $n = 2$.

(ii) Given $E_4 - E_2 = 2.7$ eV

$$\therefore \quad -\frac{B}{4^2} - \left(-\frac{B}{2^2}\right) = 2.7 \text{ eV}$$

Therefore, this gives $B = 14.4$ eV

The transition of electron should be from ground state ($n = 1$) to $n = \infty$.

Ionization energy of identical hydrogen-like gas atoms,

$$\Delta E = E_\infty - E_1 = -\frac{B}{\infty^2} - \left(-\frac{B}{1^2}\right)$$

(iii) Maximum energy of emitted photon is obtained when transition of electron is from $n = 4$ to $n = 1$.

$$\therefore \quad E_{max} = -\frac{B}{4^2} - \left(-\frac{B}{1^2}\right) = B\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$= 14.4\left(1 - \frac{1}{16}\right) \text{eV} = 13.65 \text{ eV}$$

Minimum energy of emitted photon is obtained when electron jumps from $n = 4$ to $n = 3$.

i.e., $$E_{min} = B\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 14.4\left(\frac{1}{9} - \frac{1}{16}\right) = 0.4 \text{ eV}$$

Example 4.11 **The wavelength of the characteristic X-ray K_α line emitted by a hydrogen-like element is 0.32 Å. Calculate the wavelength of K_β line emitted by the same element.** **(IIT-JEE, 1990)**

Sol. For hydrogen-like element

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For K_α line, $\frac{1}{\lambda_\alpha} = Z^2R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ (i)

For K_β line, $\frac{1}{\lambda_\beta} = Z^2R\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$ (i)

Dividing $$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{\frac{1}{1^2} - \frac{1}{2^2}}{\frac{1}{1^2} - \frac{1}{3^2}} = \frac{1-\frac{1}{4}}{1-\frac{1}{9}} = \frac{3/4}{8/9} = \frac{3\times 9}{4\times 8} = \frac{27}{32}$$

$\therefore$ $$\lambda_B = \frac{27}{32}\lambda_\alpha = \frac{27}{32} \times 0.32 \text{ Å} = 0.27 \text{ Å}$$

Example 4.12 **Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find**

a. the energy of the photons causing the photoelectric emission,

b. the quantum numbers of the two levels involved in the emission of these photons,

c. the change in the angular momentum of the electron in the hydrogen atom in the above transition, and

d. the recoil speed of the emitting atom assuming it to be at rest before the transition.
(Ionization potential of hydrogen is 13.6 eV)

(IIT-JEE, 1992)

Sol. $E_k = 0.73$ eV, $W = 1.82$ eV
Ionization energy of H atom = 13.6 eV

a. $h\nu = W + E_k = 1.82 \text{ eV} + 0.73 \text{ eV} = 2.55$ eV

b. The electronic energy levels of H-atoms are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

For $n = 1, E_1 = -13.6$ eV
For $n = 2, E_2 = -3.4$ eV
For $n = 3, E_3 = -1.51$ eV
For $n = 4, E_4 = -0.85$ eV
Clearly, $E_4 - E_2 = -0.85 \text{ eV} - (-3.4 \text{ eV}) = 2.55$ eV
i.e. Quantum numbers involved in the photon of energy 2.55 eV are 2 and 4. The transition is specified by $n_1 = 4 \rightarrow n_2 = 2$.

c. The angular momentum of electron in H atom

$$J = n\frac{h}{2\pi}$$

For $n = 4$, $J_1 = 4\frac{h}{2\pi} = \frac{2h}{\pi}$

For $n = 2$, $J_2 = 2\frac{h}{2\pi} = \frac{h}{\pi}$

Therefore, change in angular momentum,

$$\Delta J = J_1 - J_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

d. According to conservation of momentum,

$$\frac{h\nu}{c} + mv = 0$$

$\therefore$ $$v = -\frac{h\nu}{cm} = -\frac{2.55}{(3\times 10^8)\times(1.67\times 10^{-27})}$$

$\Rightarrow$ $$v = -\frac{2.55 \times 1.6 \times 10^{-19}}{3\times 10^8 \times 1.67 \times 10^{-27}} = -0.814 \text{ ms}^{-1}$$

Therefore, recoil speed of H atom = 0.814 ms^{-1}.

Example 4.13 **Two hydrogen-like atoms A and B are of different masses and each atom contains equal numbers of protons and neutrons. The difference in the energies between the first Balmer lines emitted by A and B, is 5.667 eV. When atoms A and B, moving with the same velocity, strike a heavy target, they rebound back with the same velocity. In the process, atom B imparts twice the momentum to the target than that A imparts. Identify the atoms A and B.**

(IIT-JEE, 1993)

Sol. Let Z_A and Z_B be the atomic numbers and m_A and m_B be the mass numbers of hydrogen-like atoms A and B, respectively.
Energy of n^{th} state of hydrogen-like atom is

$$E_n = -\frac{Z^2Rhc}{n^2} = -\frac{Z^2 \times 13.6}{n^2} \text{ eV}$$

Energy emitted for I line of Balmer series for atom A,

$$\Delta E_1 = -Z_A^2 \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{eV}$$

Energy emitted for I line of Balmer series for atom B,

$$\Delta E_2 = Z_B^2 \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{eV}$$

Given $\Delta E_1 - \Delta E_2 = 5.667$ eV, therefore

$$5.667 \text{ eV} = (Z_B^2 - Z_A^2) \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{eV}$$

$\therefore$ $$Z_B^2 - Z_A^2 = \frac{5.667 \times 36}{13.6 \times 5} = 3$$

Let u be the initial velocity of each from A and B and m the mass of target.

If v_1 and v_2 are velocities of target after collision with A and B, respectively, then according to principle of conservation of momentum for A,

$$m_A u = M v_1 - m_A u$$

or $\quad M v_1 = 2 m_A u \quad$ (ii)

Similarly, for B $M v_2 = 2 m_B u \quad$ (iii)

Given, $\quad M v_2 = 2 m v_1$, therefore

$$2 m_B u = 2(2 m_A u)$$

This gives, $m_B = 2 m_A \quad$ (iv)

As number of protons and neutrons in each of A and B is same separately,

$\therefore \quad m_B = 2 Z_B$ and $m_A = 2 Z_A$

Substituting this in (iv), we get

$$2 Z_B = 2(2 Z_A), \text{ i.e., } Z_B = 2 Z_A \quad \text{(v)}$$

Solving (i) and (v), we get $Z_A = 1$ and $Z_B = 2$
i.e., atom A contains 2 protons and 2 neutrons.
Hence, atom B is singly ionized helium.

Example 4.14 **(i) A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Å. For light of wavelength 3000 Å, the stopping potential is 1.85 volt. Find the value of Planck's constant.**

[1 electron-volt (eV) = 1.6×10^{-19} joule.

(ii) At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Å, but the intensity of light increased two times, will photoelectric current be obtained? Give reasons for your answer. (IIT-JEE, 1993)

Sol. $$\frac{hc}{\lambda_1} = W + eV_1 \quad \text{(i)}$$

$$\frac{hc}{\lambda_2} = W + eV_2 \quad \text{(ii)}$$

Subtracting $\frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = e(V_2 - V_1)$

Here, $\quad c = 3 \times 10^8 \text{ ms}^{-1}$,

$\lambda_1 = 4000 \text{ Å} = 4 \times 10^{-7} \text{ m}, V_1 = 0.82 \text{ V}$

$\lambda_2 = 3000 \text{ Å} = 3 \times 10^{-7} \text{ m}, V_2 = 1.85 \text{ V}$

$$\therefore \quad h \times 3 \times 10^8 \left(\frac{1}{3 \times 10^{-7}} - \frac{1}{4 \times 10^{-7}}\right)$$
$$= 1.6 \times 10^{-19} (1.85 - 0.82)$$

or $$h \times 3 \times 10^8 \frac{1}{12 \times 10^{-7}} = 1.6 \times 10^{-19} \times 1.03$$

$\therefore$ Planck's constant,

$$h = \frac{1.6 \times 10^{-19} \times 1.03 \times 12 \times 10^{-7}}{3 \times 10^8}$$
$$= 6.592 \times 10^{-34} \text{ J-s}$$

(ii) No, because stopping potential does not depend on intensity of incident light.

Example 4.15 **An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius R (Fig. 4.52).**

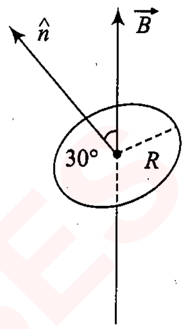


Fig. 4.52

(i) Obtain an expression for the orbital magnetic dipole moment of the electron.

(ii) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane normal to the electron orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron. (IIT-JEE, 1996)

Sol. (i) If f is the frequency of orbital motion, then the orbital magnetic dipole moment of circulating electron

$$M = iA = feA \quad \text{(i)}$$

According to Bohr's condition of stationary orbits,

$$mvR = n\frac{h}{2\pi}$$

For ground state, $n = 1$ and area $A = \pi R^2$

Also, $\quad v = R\omega = R\, 2\pi f$

$$\therefore \quad m(R\, 2\pi f)R = 1\frac{h}{2\pi}$$

$$\therefore \quad f = \frac{h}{4\pi^2 m R^2}$$

$$\therefore \text{ From (i), } M = \left(\frac{h}{4\pi^2 m R^2}\right) e\pi R^2$$

$$\Rightarrow \quad M = \frac{eh}{4\pi m} \text{ A-m}^2$$

(ii) The direction of magnetic moment is along the normal to the plane of electron orbit, i.e., along $\hat{n}$.

Angle between $\vec{M}$ and $\vec{B}$ is 30°.
$\therefore$ Torque on orbiting electron,

$$\tau = MB \sin\theta = \left(\frac{eh}{4\pi m}\right) B \sin 30°$$

i.e., $$\tau = \frac{ehB}{8\pi m} \text{ newton} \times \text{meter}$$

Example 4.16 **The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit of hydrogen atom falls on a metal to produce photoelectrons. The electrons emitted from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of (1/320)T in a radius of 10^{-3} m. Find:**

(i) the kinetic energy of the electrons,
(ii) work function of metal, and
(iii) wavelength of radiation.
(Planck's constant, $h = 6.62 \times 10^{-34}$J-s) (IIT-JEE, 1996)

Sol. (i) When charged particle moves perpendicular to a magnetic field, then magnetic field provides necessary centripetal force for circular path of radius r given by

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \quad mv = qBr$$

As momentum, $p = mv = \sqrt{2mE_K}$, where E_k is kinetic energy, therefore

$$\sqrt{2mE_K} = qBr$$

$$\therefore \quad E_k = \frac{(qBr)^2}{2m}$$

$$= \frac{\left\{1.6 \times 10^{-19} \times \left(\frac{1}{320}\right) \times 10^{-3}\right\}^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= 1.374 \times 10^{-19} \text{ J} = \frac{1.374 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 0.86 \text{ eV}$$

(ii) Energy of photon released due to transition from $n = 3$ to $n = 2$ in hydrogen atom,

$$\varepsilon = \Delta E$$

$$= Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (13.6 \text{ eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.89 \text{ eV}$$

Work function of metal $W = \varepsilon - E_k = 1.89 - 0.86 = 1.03$ eV

(iii) Wavelength of emitted radiation (photon) is given by

$$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{\Delta E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.89 \times 1.6 \times 10^{-19}}$$

$$= 6.567 \times 10^{-7} \text{ m} = 6567 \text{ Å}$$

Example 4.17 **An electron in a hydrogen-like atom is in an excited state. It has a total energy of –3.4 eV. Calculate**
(i) the kinetic energy, and
(ii) the de Broglie wavelength of the electron.
($h = 6.63 \times 10^{-34}$ J-s) (IIT-JEE, 1996)

Sol. (i) Energy of electron in hydrogen-like atom,

$$E_n = -\frac{Z^2 Rhc}{n^2} = -3.4 \text{ eV}$$

Kinetic energy of electron in hydrogen-like atom is equal to negative of total energy,

i.e., $E_k = -E_n = -(-3.4 \text{ eV}) = +3.4$ eV

(ii) The de Broglie wavelength of electron,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{\{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}\}}}$$

$$= 6.66 \text{ Å}$$

Example 4.18 **Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron, volt and the least value of d for which the standing wave of the type described above can form. ($h = 6.63 \times 10^{-34}$ J-s) (IIT-JEE, 1997)**

Sol. For standing wave, the separation between consecutive nodes (or antinodes) is $\lambda/2$.

$$\therefore \quad d = \frac{n\lambda}{2}, \text{ where } n \text{ is integer.}$$

For next node,

$$D' = (n+1)\frac{\lambda}{2}$$

$$\therefore \quad d' - d = \frac{\lambda}{2}$$

$$\Rightarrow \quad \lambda = 2(d' - d)$$

Here, $d = 2$ Å, $d' = 2.5$ Å

$$\therefore \quad \lambda = 2(2.5 \text{ Å} - 2 \text{ Å}) = 1 \text{ Å}$$

$\therefore$ Energy of electrons,

$$E = \frac{p^2}{2m}$$

$$\Rightarrow \quad E = \frac{h^2}{2m\lambda^2}$$

Here $h = 6.63 \times 10^{-34}$ J-s

$m = 9.1 \times 10^{-31}$ kg

$\lambda = 1$ Å $= 10^{-10}$ m.

$$\therefore \quad E = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2} \text{ J}$$

$$= \frac{(6.63)^2}{2 \times 9.1} \times 10^{-17} \text{ joule} = 2.415 \times 10^{-17} \text{ J}$$

$$= \frac{2.415 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} \approx 151 \text{ eV}$$

For least value of d, $n = 1$. Therefore, from (i)

$$d_{min} = \frac{\lambda}{2} = \frac{1}{2} \text{ Å} = 0.5 \text{ Å}$$

Example 4.19 **A gas of hydrogen-like ion is prepared in such a way that the ions are only in the ground state and the first excited state. A monochromatic light of wavelength 1216 Å is absorbed by the ions. The ions are lifted to higher excited states and emit radiation of six wavelengths, some higher and some lower than the incident wavelength. Find the principle quantum number of the excited states. Identify the nuclear charge on the ions. Calculate the values of the maximum and minimum wavelengths. (IIT-JEE, 1997)**

Sol. The energy state of hydrogen-like atoms is given by

$$E_n = -\frac{B}{n^2}$$

where $B = Z^2 Rhc$.

The emission of six wavelengths implies that the higher excited state corresponds to quantum number $n = 4$. As some wavelengths are lower and some wavelengths are higher, it implies that the initial excited state is $n = 2$. Thus, the principal quantum numbers of excited states are $n = 2$, $n = 3$, $n = 4$.

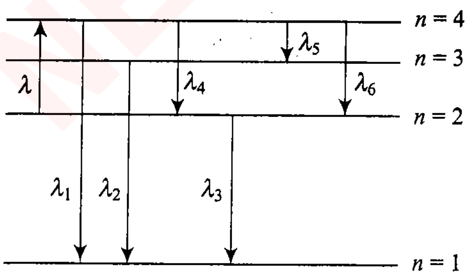


Fig. 4.53

$$\therefore \quad \frac{hc}{\lambda} = B\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow \frac{hc}{\lambda} = \frac{3B}{16}$$

$$\therefore \quad B = \frac{16 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times (1216 \times 10^{-10})}$$

$$= 8.71 \times 10^{-18} \text{ J}$$

If Z is atomic number of nucleus, then $B = Z^2 Rhc$.

$$\therefore \quad Z^2 = \frac{B}{Rhc} = \frac{8.71 \times 10^{-18} \text{ J}}{13.6 \text{ eV}}$$

$$= \frac{8.71 \times 10^{-18}}{13.6 \times 1.6 \times 10^{-19}} = 4$$

$$\therefore \quad Z = 2$$

The maximum wavelength of emitted radiation corresponds to transition $n = 4$ to $n = 3$ and is given by

$$\frac{hc}{\lambda_{max}} = B\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \quad \lambda_{max} = \frac{144hc}{7B} = \frac{144 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{7 \times 8.71 \times 10^{-18}}$$

$$= 4.69 \times 10^{-7} \text{ m}$$

The minimum wavelength of emitted radiation corresponds to transition $n = 4$ to $n = 1$ and is given by

$$\frac{hc}{\lambda_{min}} = B\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \quad \lambda_{min} = \frac{16hc}{15B} = \frac{16 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{15 \times 8.71 \times 10^{-18}}$$

$$= 0.24 \times 10^{-7} \text{ m}$$

Example 4.20 **Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α particles. A maximum energy electron combines with an α particles to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. [Take $h = 4.14 \times 10^{-15}$ eV s] (IIT-JEE, 1999)**

Sol. From Einstein's photoelectric equation, maximum kinetic energy of emitted electrons

$$E_k = \frac{hc}{\lambda} - W$$

Now, $$E = \frac{hc}{\lambda} = \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$$= 4.968 \times 10^{-19} \text{ J-s} = \frac{4.968 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.1 \text{ eV}$$

$$E_k = 3.1 \text{ eV} - 1.9 \text{ eV} = 1.2 \text{ eV}$$

$$e + \alpha \rightarrow He^+ + \text{Photon}$$

Energy of He atom in their fourth excited state ($n = 5$) is

$$E_n = -\frac{Z^2 Rhc}{n^2} = -\frac{(2)^2 \times 13.6}{(5)^2} = -2.176 \text{ eV}$$

From conservation of energy, 1.2 eV + 0 = –2.176 eV + E_γ

Energy of photon during combination,

$$E_\gamma = 1.2 + 2.176 = 3.376 \text{ eV}$$

Energy of helium ion, $E_n = -\dfrac{Z^2Rhc}{n^2} = -\dfrac{4\times 13.6}{n^2}$

$$= -\frac{54.4}{n^2}\text{eV}, \; n = 1, 2, 3, \ldots$$

$$= -54.4 \text{ eV}, -13.6\text{eV}, -6.04 \text{ eV}, -3.4 \text{ eV}, -2.176 \text{ eV}, -1.51 \text{ eV}$$

Difference of energies lying between 2 to 4 eV is

$$-3.4 + 6.04 = 2.64 \text{ eV}$$

$$-2.176 + 6.04 = 3.86 \text{ eV}$$

Energies of photons emitted are 2.64 eV and 3.86 eV.

Example 4.21 **When a beam of 1.6 eV photons of intensity 2.0 Wm^{-2} falls on a platinum surface of area 1.0 × 10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1 eV = 1.6 × 10^{-19} J.** **(IIT-JEE, 2000)**

Sol. Energy incident on surface per second

$$P = IA$$
$$= 2.0 \times 1.0 \times 10^{-4} = 2 \times 10^{-4} \text{ J}$$

Energy of each photon

$$= 10.6 \text{ eV} = 10.6 \times 1.6 \times 10^{-19} \text{ J}$$

Number of photons incident on the surface

$$= \frac{2\times 10^{-4}}{10.6\times 1.6\times 10^{-19}}$$

Number of photoelectrons emitted

$$= \frac{0.53}{100}\times\frac{2\times 10^{-4}}{10.6\times 1.6\times 10^{-19}} = 6.25\times 10^{11}$$

According to Einstein's photoelectric equation, maximum KE of photoelectrons

$$E_k = \varepsilon - W = 10.6 \text{ eV} - 5.6 \text{ eV} = 5 \text{ eV}$$

Minimum kinetic energy of photoelectrons = zero.

Example 4.22 **An electron in hydrogen-like atom makes a transition from n^{th} orbit and emits radiation corresponding to Lyman series. If de Broglie wavelength of electron in n^{th} orbit is equal to the wavelength of radiation emitted, find the value of n. The atomic number of atom is 11.** **(IIT-JEE, 2006)**

Sol. If λ is de Broglie wavelength, then for n^{th} stationary orbit $2\pi r_n = n\lambda$

where r_n is radius of n^{th} orbit $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$

$$\therefore \quad 2\pi\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right) = n\lambda \quad \Rightarrow \quad \frac{1}{\lambda} = \frac{mZe^2}{2\varepsilon_0 h^2 n} \qquad \text{(i)}$$

For Lyman series of hydrogen like atom

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{1^2}-\frac{1}{n^2}\right) \qquad \text{(ii)}$$

From (i) and (ii), $Z^2R\left(1-\dfrac{1}{n^2}\right) = \dfrac{mZe^2}{2\varepsilon_0 h^2 n}$

Ryberg constant,

$$R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$\therefore \quad \frac{Z^2 me^4}{8\varepsilon_0^2 ch^3}\left(1-\frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$$

$$\left(1-\frac{1}{n^2}\right) = \frac{4\varepsilon_0 ch}{ne^2 Z}$$

$$= \frac{4\times(8.85\times 10^{-12})\times(3\times 10^8)\times(6.62\times 10^{-34})}{n\times(1.6\times 10^{-19})^2\times 11} = \frac{25}{n}$$

$$\therefore \quad n^2 - 1 = 25n \quad \text{or} \quad n^2 - 25n - 1 = 0$$

$$n = \frac{25\pm\sqrt{(-25)^2+4\times 1\times 1}}{2} \approx \frac{25\pm\sqrt{625}}{2} \approx 25$$

As negative n is not possible, $n \approx 25$.

Example 4.23 **An X-ray tube operated at 40 kV emits a continuous X-ray spectrum with a short wavelength limit λ_{min} = 0.310 Å. Calculate the Planck's constant.**

Sol. We have,

$$\lambda_{min} = \frac{hc}{eV} \quad \text{or} \quad h = \frac{eV\lambda_{min}}{c}$$

Here, $e = 1.6 \times 10^{-19}$ coulomb, $V = 40$ kV $= 40 \times 10^3$ V

$$\lambda_{min} = 0.310 \text{ Å} = 0.310 \times 10^{-10} \text{ m}$$
$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \quad h = \frac{(1.6\times 10^{-19})\times(40\times 10^3)\times 0.310\times 10^{-10}}{3\times 10^8}$$

$$= 6.61 \times 10^{-34} \text{ J-s}$$

EXERCISES

Subjective Type

Solutions on page 4.65

1. The peak emission from a black body at a certain temperature occurs at a wavelength of 9000 Å. On increasing its temperature, the total radiation emitted is increased to 81 times. At the initial temperature when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between $n = 2$ and $n = 3$ Bohr levels of hydrogen atoms is required. Find the work function of the metal.
2. In a hydrogen-like atom, an electron is orbiting in an orbit having quantum number n. Its frequency of revolution is found to be 13.2×10^{15} Hz. Energy required to move this electron from the atom to the above orbit is 54.4 eV. In a time of 7 nano second the electron jumps back to orbit having quantum number $n/2$. If τ be the average torque acted on the electron during the above process, then find $\tau \times 10^{27}$ in Nm. (given: $h/\lambda = 2.1 \times 10^{-34}$ J-s, frequency of revolution of electron in the ground state of H atom $v_0 = 6.6 \times 10^{15}$ and ionization energy of H atom $E_0 = 13.6$ eV).
3. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantization of angular momentum holds good for this electron, find
   a. the allowed values of the radius 'r' of the orbit.
   b. the kinetic energy of the electron in orbit.
   c. the potential energy of the interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
   d. the total energy of the allowed energy levels.
   e. the total magnetic flux due to the magnetic field B passing through the n^{th} orbit (assume that the charge on the electron is e and the mass of the electron is m).
4. A hydrogen-like gas is kept in a chamber having a slit of width $d = 0.01$ mm. The atoms of the gas are continuously excited to a certain energy state. The excited electrons make transitions to lower levels; from the initial excited state to the second excited state and then from the second excited state to the ground state. In the process of de-excitation, photons are emitted and come out of the container through a slit. The intensity of the photons is observed on a screen placed parallel to the plane of the slit. The ratio of the angular width of the central maximum corresponding to the two transitions is 25/2. The angular width of the central maximum due to first transition is 6.4×10^{-2} radian. Find the atomic number of the gas and the principal quantum number of the initial excited state.
5. A gas containing hydrogen-like ions, with atomic number Z, emits photons in transition $n + 2 \rightarrow n$, where $n = Z$. These photons fall on a metallic plate and eject electrons having minimum de Broglie wavelength λ of 5 Å. Find the value of Z, if the work function of metal is 4.2 eV.
6. A hydrogen atom (mass = 1.66×10^{-27} kg, ionization potential = 13.6 eV), moving with a velocity 6.24×10^4 ms^{-1} makes a completely inelastic head-on collision with another stationary hydrogen atom. Both atoms are in ground state before collision. Up to what state either one atom may be excited?
7. Radiation from hydrogen gas excited to first excited state is used for illuminating certain photoelectric plate. When the radiation from some unknown hydrogen-like gas excited to the same level is used to expose the same plate, it is found that the de Broglie wavelength of the fastest photoelectron has decreased 2.3 times. It is given that the energy corresponding to the longest wavelength of the Lyman series of the unknown gas is 3 times the ionization energy of the hydrogen gas (13.6 eV). Find the work function of photoelectric plate in eV. [Take $(2.3)^2 = 5.25$.]
8. The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen-like ion X. Calculate energies of the first four levels of X. Also, find its ionization potential. (Given: Ground state binding energy of hydrogen atom is 13.6 eV.)
9. When the voltage applied to an X-ray tube is increased fom 10 kV to 20 kV, the wavelength interval between the K_α line and the short wave cut off of the continuous X-ray spectrum increases by a factor 3. Find the atomic number of the element of which the tube anti cathode is made. (Rydberg's constant = 10^7 m^{-1}).
10. X-rays are emitted by a tube containing the element Niobium ($Z = 41$) as anticathode and the K_α X-rays are allowed to be incident on an unknown gas containing hydrogen-like ions. It is found that the X-rays cause the emission of photoelectrons with an energy of 2.7 keV from these ions. Find
    a. the minimum voltage at which the X-ray tube should be operated so that the momentum of the emitted photoelectrons is doubled.
    b. the approximate value of Z for the target of the anticathode after the momentum has been doubled.
    c. the approximate value of the atomic number of the atoms of the gas.
    d. the ionization in the gas caused by X-rays, if the intensity of X-rays is 100 mWm^{-2} and 1% of the X-rays cause ionization.
11. Electrons of energies 10.20 eV and 12.09 eV can cause radiation to be emitted from hydrogen atoms. Calculate, in each case, the principal quantum number of the orbit to which electron in the hydrogen atom is raised and the wavelength of the radiation emitted if it drops back to the ground state.

12. If the short series limit of the Balmer series for hydrogen is 3646 Å, calculate the atomic number of the element which gives X-ray wavelength down to 1.0 Å. Identify the element.
13. Calculate the separation between the particles of a system in the ground state, the corresponding binding energy and wavelength of first line in Lyman series of such a system is positronium consisting of an electron and a positron revolving round their common center.
14. A μ-meson (charge $-e$, mass = 207m, where m is the mass of electron) can be captured by a proton to form a hydrogen-like 'mesic' atom. Calculate the radius of the first Bohr orbit, the binding energy, and the wavelength of the line in the Lyman series for such an atom. The mass of the proton is 1836 times the mass of the electron. The radius of first Bohr orbit and the binding energy of hydrogen are 0.529 Å and 13.6 eV, respectively. Take $R = 109678\ \text{cm}^{-1}$
15. A π-meson hydrogen atom is a bound state of negatively charged pion (denoted by π^-, $m_\pi = 273m_e$) and a proton. Estimate the number of revolutions a π-meson makes (averagely) in the ground state of the atom before it decays (mean life of a π-meson $\cong 10^{-8}$ s, mass of proton $= 1.67 \times 10^{-27}$ kg).
16. Suppose the potential energy between an electron and a proton at a distance r is given by $-ke^2/3r^3$. Use Bohr's theory to obtain energy level of such a hypothetical atom.
17. A 100 eV electron collides with a stationary helium ion (He^+) in its ground state and excites to a higher level. After the collision, He^+ ion emits two photons in succession with wavelengths 1085 Å and 304 Å. Find the principal quantum number of the excited state. Also, calculate the energy of the electron after the collision. Given $h = 6.63 \times 10^{-34}$ Js.

Objective Type

Solutions on page 4.69

1. The wavelength of the second line of Balmer series in the hydrogen spectrum is 4861 Å. The wavelength of the first line is
   a. $\frac{27}{20} \times 4861$ Å
   b. $\frac{20}{27} \times 4861$ Å
   c. 20×4861 Å
   d. 4861Å.
2. Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12.1 eV, 10.2 eV, and 1.9 eV. These photons must come from
   a. a single atom
   b. two atoms
   c. three atoms
   d. either two atoms or three atoms
3. Whenever a hydrogen atom emits a photon in the Balmer series
   a. it need not emit any more photon
   b. it may emit another photon in the Paschen series
   c. it must emit another photon in the Lyman series
   d. it may emit another photon in the Balmer series
4. Two electrons are revolving around a nucleus at distances 'r' and '$4r$'. The ratio of their periods is
   a. 1:4
   b. 4:1
   c. 8:1
   d. 1:8
5. A hydrogen atom in ground state absorbs 10.2 eV of energy. The orbital angular momentum of the electron is increased by
   a. 1.05×10^{-34} Js
   b. 2.11×10^{-34} Js
   c. 3.16×10^{-34} Js
   d. 4.22×10^{-34} Js
6. In terms of Rydberg constant R, the shortest wavelength in Balmer series of hydrogen atom spectrum will have wavelength
   a. $1/R$
   b. $4/R$
   c. $3/2R$
   d. $9/R$
7. The ratio of minimum to maximum wavelength in Balmer series is
   a. 5:9
   b. 5:36
   c. 1:4
   d. 3:4
8. The frequency of revolution of an electron in n^{th} orbit is f_n. If the electron makes a transition from n^{th} orbit to $(n-1)^{th}$ orbit, then the relation between the frequency (ν) of emitted photon and f_n will be
   a. $\nu = f_n^2$
   b. $\nu = \sqrt{f_n}$
   c. $\nu = \frac{1}{f_n}$
   d. $\nu = f_n$
9. If $n >> 1$, then the dependence of frequency of photon emitted as a result of transition of an electron from n^{th} orbit to $(n-1)^{th}$ orbit, on n will be
   a. $\nu \propto \frac{1}{n}$
   b. $\nu \propto \frac{1}{n^2}$
   c. $\nu \propto \frac{1}{n^3}$
   d. $\nu \propto \frac{1}{n^4}$.
10. Figure 4.54 shows five energy levels of an atom, one being much lower than the other four. Five transitions between the levels are indicated, each of which produces a photon of definite energy and frequency.

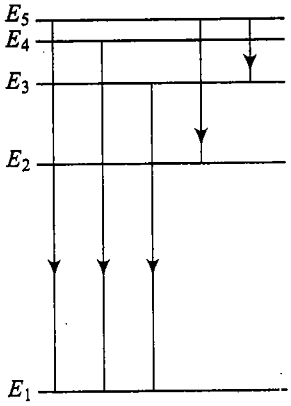
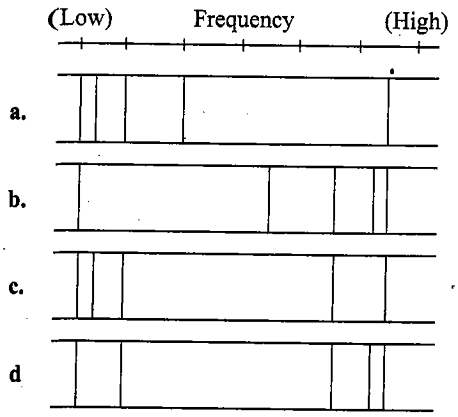


Fig. 4.54

Which one of the spectra below best corresponds to the set of transitions indicated?



11. Figure 4.55 represents some of the lower energy levels of the hydrogen atom in simplified form.

If the transition of an electron from E_4 to E_2 were associated with the emission of blue light, which one of the following transitions could be associated with the emission of red light?

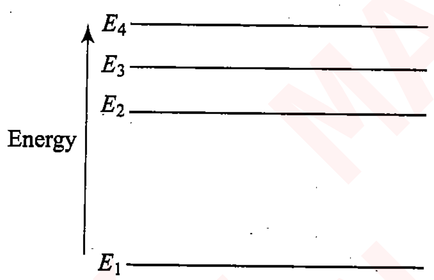


Fig. 4.55

a. E_4 to E_1 **b.** E_3 to E_1
c. E_3 to E_2 **d.** E_1 to E_3

12. The orbiting speed v_n of e^- in the n^{th} orbit in the case of positronium is x-fold compared to that in n^{th} orbit in a hydrogen atom, where x has the value
a. 1 **b.** $\sqrt{2}$
c. $1/\sqrt{2}$ **d.** 2

13. Figure 4.56 shows the electron energy levels, referred to the ground state (the lowest possible energy) as zero, for five different isolated atoms. Which atom can produce radiation of the shortest wavelength when atoms in the ground state are bombarded with electrons of energy W ?

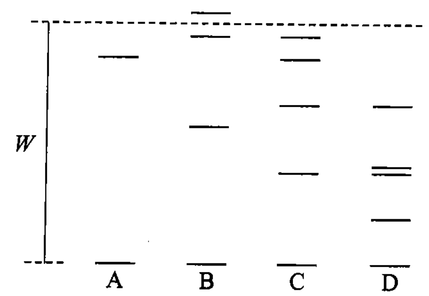


Fig. 4.56

a. A **b.** B
c. C **d.** D

14. When an electron jumps from a level $n = 4$ to $n = 1$, the momentum of the recoiled hydrogen atom will be
a. 6.5×10^{-27} kg m s^{-1} **b.** 12.75×10^{-19} kg m s^{-1}
c. 13.6×10^{-27} kg m s^{-1} **d.** zero

15. Check the correctness of the following statements about Bohr model of hydrogen atom:
(i) The acceleration of the electron in $n = 2$ orbit is more than that in $n = 1$ orbit.
(ii) The angular momentum of the electron in $n = 2$ orbit is more than that in $n = 1$ orbit.
(iii) The KE of the electron in $n = 2$ orbit is less than that in $n = 1$ orbit.
a. All the statements are correct.
b. Only (i) and (ii) are correct.
c. Only (ii) and (iii) are correct.
d. Only (iii) and (i) are correct.

16. When a hydrogen atom is raised from the ground state to fifth state:
a. both KE and PE increase
b. both KE and PE decrease
c. PE increases and KE decreases
d. PE decreases and KE increases

17. As the electron in Bohr orbit of hydrogen atom passes from state $n = 2$ to $n = 1$, the KE (K) and PE (U) change as
a. K two-fold, U also two-fold
b. K four-fold, U also four-fold
c. K four-fold, U two-fold
d. K two-fold, U four-fold

18. Consider a spectral line resulting from the transition $n = 5$ to $n = 1$, in the atoms and ions given below. The shortest wavelength is produced by
a. helium atom
b. deuterium atom
c. singly ionized helium
d. ten times ionized sodium atom

19. Which of the following statement is true regarding Bohr's model of hydrogen atom?
(I) Orbiting speed of an electrons decreases as it falls to discrete orbits away from the nucleus.
(II) Radii of allowed orbits of electrons are proportional to the principal quantum number.
(III) Frequency with which electrons orbit around the nucleus in discrete orbits is inversely proportional to the cube of principal quantum number.
(IV) Binding force with which the electron is bound to the nucleus increases as it shifts to outer orbits.
Select the correct answer using the codes given below:
a. I and III **b.** II and IV
c. I, II and III **d.** II, III and IV

20. In the Bohr model of a hydrogen atom, the centripetal force is furnished by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass, and e is the charge on the electron and ε_0 is the permittivity of vacuum, then the speed of the electron is:

a. 0 b. $\frac{e}{\sqrt{\varepsilon_o a_o m}}$

c. $\frac{e}{\sqrt{4\pi\varepsilon_o a_o m}}$ d. $\sqrt{\frac{4\pi\varepsilon_o a_o m}{e}}$

21. Hydrogen atoms are excited from ground state to the state of principal quantum number 4. Then, the number of spectral lines observed will be
a. 3 b. 6
c. 5 d. 2

22. When an electron jumps from n_1^{th} orbit to n_2^{th} orbit, the energy radiated is given by
a. $h\nu = E_1/E_2$ b. $h\nu = E_2/E_1$
c. $h\nu = E_1 - E_2$ d. $h\nu = E_2 - E_1$.

23. In Bohr's model of hydrogen atom, let PE represent potential energy and TE the total energy. In going to a higher orbit,
a. PE increases, TE decreases
b. PE decreases, TE increases
c. PE increases, TE increases
d. PE decreases, TE decreases

24. If the electron in an hydrogen atom jumps from an orbit with level $n_f = 3$ to an orbit with level $n_f = 2$, the emitted radiation has a wavelength given by
a. $\lambda = \frac{R}{6}$ b. $\lambda = \frac{36}{5R}$
c. $\lambda = \frac{6}{R}$ d. $\lambda = \frac{5R}{36}$

25. The energy change is greatest for a hydrogen atom when its state changes from
a. $n = 2$ to $n = 1$ b. $n = 3$ to $n = 2$
c. $n = 4$ to $n = 3$ d. $n = 5$ to $n = 4$.

26. The electron in a hydrogen atom jumps from ground state to the higher energy state where its velocity is reduced to one-third its initial value. If the radius of the orbit in the ground state is r, the radius of new orbit will be
a. $3r$ b. $9r$
c. $\frac{r}{3}$ d. $\frac{r}{9}$

27. The energy of an electron in an excited hydrogen atom is −3.4 eV. Then, according to Bohr's Theory, the angular momentum of this electron, in Js, is
a. 2.11×10^{-34} b. 3×10^{-34}
c. 1.055×10^{-34} d. 0.5×10^{-34}

28. The wavelength of first line of Lyman series in hydrogen atom is 1216 . The wavelength of first line of Lyman series for 10 times ionized sodium atom will be added
a. 0.1 Å b. 1000 Å
c. 100 Å d. 10 Å

29. The ionization potential of H atoms is 13.6 V. The energy difference between $n = 2$ and $n = 3$ levels is nearest to
a. 1.9 eV b. 2.3 eV
c. 3.4 eV d. 4.5 eV

30. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is
a. 10.2 eV b. 0 eV
c. 3.4 eV d. 6.8 eV

31. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum?
a. Doubly ionized lithium b. Singly ionized helium
c. Deuterium atom d. Hydrogen atom

32. The radius of the Bohr orbit in the ground state of hydrogen atom is 0.5 Å. The radius of the orbit of the electron in the third excited state of He^+ will be
a. 8 Å b. 4 Å
c. 0.5 Å d. 0.25 Å

33. Let ν_1 be the frequency of series limit of Lyman series, ν_2 the frequency of the first line of Lyman series, and ν_3 the frequency of series limit of Balmer series. Then which of the following is correct?
a. $\nu_1 - \nu_2 = \nu_3$ b. $\nu_2 - \nu_1 = \nu_3$
c. $\nu_3 = \frac{1}{2}(\nu_1 + \nu_2)$ d. $\nu_2 + \nu_1 = \nu_3$

34. The speed of electrons in the second orbit of Be^{3+} ion will be
a. $\frac{c}{137}$ b. $\frac{2c}{137}$
c. $\frac{3c}{137}$ d. $\frac{4c}{137}$

35. The potential energy of an electron in the fifth orbit of hydrogen atom is
a. 0.54eV b. −0.54 eV
c. 1.08 eV d. −1.08 eV

36. If the radius of an orbit is r and the velocity of electron in it is v, then the frequency of electron in the orbit will be
a. $2\pi rv$ b. $\frac{2\pi}{vr}$
c. $\frac{vr}{2\pi}$ d. $\frac{v}{2\pi r}$

37. A sample of hydrogen is bombarded by electrons. Through what potential difference should the electrons be accelerated so that third line of Lyman series be emitted?
a. 2.55 V b. 10.2 V
c. 12.09 V d. 12.75 V

38. When the hydrogen atom emits a photon in going from $n = 5$ to $n = 1$ state, its recoil speed is nearly
a. 10^{-4} ms^{-1} b. 2×10^{-2} ms^{-1}
c. 4 ms^{-1} d. 8×10^{2} ms^{-1}

39. If R is the Rydberg constant for hydrogen, then the wave number of the first line in the Lyman series is
a. $\frac{R}{2}$ b. $2R$
c. $\frac{R}{4}$ d. $\frac{3R}{4}$

40. As the quantum number increases, the difference of energy between conservative energy levels:
 a. decreases
 b. increases
 c. first decreases and then increases
 d. remains the same

41. If λ_1 and λ_2 are the wavelengths of the first members of the Lyman and Paschen series, respectively, then λ_1:λ_2 is
 a. 1:3 **b.** 1:30
 c. 7:50 **d.** 7:108

42. In Fig. 4.57, E_1 to E_6 represent some of the energy levels of an electron in the hydrogen atom.

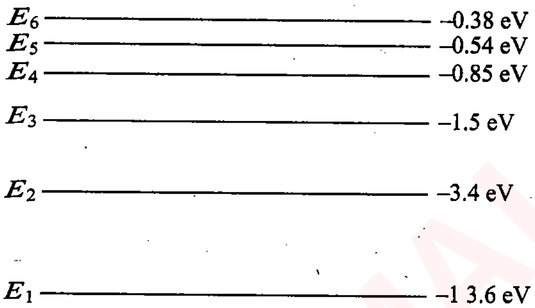


Fig. 4.57

Which one of the following transitions produces a photon of wavelength in the ultraviolet region of the electromagnetic spectrum?
 a. $E_2 - E_1$ **b.** $E_3 - E_2$
 c. $E_4 - E_3$ **d.** $E_6 - E_4$

43. Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states?
 a. Radius of the orbit.
 b. Speed of the electron.
 c. Energy of the atom.
 d. Orbital angular momentum of the electron.

44. An atom emits a spectral line of wavelength λ when an electron makes a transition between levels of energy E_1 and E_2. Which expression correctly relates λ, E_1, and E_2 ?
 a. $\lambda = \dfrac{hc}{E_1 + E_2}$ **b.** $\lambda = \dfrac{2hc}{E_1 + E_2}$
 c. $\lambda = \dfrac{2hc}{E_1 - E_2}$ **d.** $\lambda = \dfrac{hc}{E_1 - E_2}$

45. The frequency of emission line for any transition in positronium atom (consisting of a positron and an electron) is x times the frequency for the corresponding line in the case of H atom, where x is
 a. $\sqrt{2}$ **b.** $1/2\sqrt{2}$
 c. $1/2\sqrt{2}$ **d.** 1/2

46. If wavelength of photon emitted due to transition of an electron from third orbit to first orbit in a hydrogen atom is λ, then the wavelength of photon emitted due to transition of electron from fourth orbit to second orbit will be
 a. $\dfrac{128}{27}\lambda$ **b.** $\dfrac{25}{9}\lambda$
 c. $\dfrac{36}{7}\lambda$ **d.** $\dfrac{125}{11}\lambda$

47. Which of the following is true when Bohr gave his model for hydrogen atom?
 a. It was not known that hydrogen lines could be explained as differences of terms like R/n^2 with R being a constant and n an integer.
 b. It was not known that positive charge is concentrated in a nucleus of small size.
 c. It was not known that radiant energy occurred in energy bundles defined by $h\nu$ with h being a constant and ν a frequency.
 d. Bohr knew terms like R/n^2 and in the process of choosing the allowed orbits to fit them he got "angular momentum $= n_i/2\pi$ as a deduction.

48. If the series limit wavelength of the Lyman series for hydrogen atom is 912 Å, then the series limit wavelength for the Balmer series for the hydrogen atom is
 a. 912 Å /2 **b.** 912 Å
 c. 912×2 Å **d.** 912×4 Å

49. The ratio of maximum to minimum possible radiation energy in Bohr's hypothetical hydrogen atom is equal to
 a. 2 **b.** 4
 c. 4 / 3 **d.** 3 / 2

50. A stationary hydrogen atom of mass M emits a photon corresponding to the first line of Lyman series. If R is the Rydberg's constant, the velocity that the atom acquires is
 a. $\dfrac{3}{4}\dfrac{Rh}{M}$ **b.** $\dfrac{Rh}{4M}$
 c. $\dfrac{Rh}{2M}$ **d.** $\dfrac{Rh}{M}$

51. In interpreting Rutherford's experiments on the scattering of alpha particles by thin foils, one must examine what were the known factors, and what did the experiment conclude. Which of the following are true in this context?
 a. The number of electrons in the target atoms (i.e., Z) was settled by these experiments.
 b. The validity of Coulomb's law for distances as small as 10^{-13} was known before these experiments.
 c. The experiments settled that size of the nucleus could not be larger than a certain value.
 d. The experiments also settled that size of the nucleus could not be smaller than a certain value.

52. An electron jumps from the 4th orbit to the 2nd orbit of hydrogen atom. Given: the Rydberg's constant $R = 10^5$ cm^{-1}. The frequency, in Hz, of the emitted radiation will be
 a. $\dfrac{3}{16} \times 10^5$ **b.** $\dfrac{3}{6} \times 10^{15}$
 c. $\dfrac{9}{16} \times 10^5$ **d.** $\dfrac{9}{16} \times 10^{15}$

53. Given: mass number of gold = 197, density of gold = 19.7 g cm^{-3}, Avogadro's number $= 6 \times 10^{23}$. The radius of the gold atom is approximately:
 a. 1.5×10^{-8} m **b.** 1.5×10^{-9} m
 c. 1.5×10^{-10} m **d.** 1.5×10^{-12} m

54. Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in increasing magnitudes, λ_1, λ_2, and λ_3. Which one of the following equations correctly relates λ_1, λ_2, and λ_3?
 a. $\lambda_1 = \lambda_2 - \lambda_3$
 b. $\lambda_1 = \lambda_3 - \lambda_2$
 c. $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$
 d. $\frac{1}{\lambda_1} = \frac{1}{\lambda_3} - \frac{1}{\lambda_2}$

55. The ratio of the speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where e, h, and c have their usual meanings in cgs system)
 a. $2\pi hc/e^2$
 b. $e^2h/2\pi c$
 c. $e^2c/2\pi h$
 d. $2\pi e^2/hc$

56. Suppose two deuterons must get as close as 10^{-14} m in order for the nuclear force to overcome the repulsive electrostatic force. The height of the electrostatic barrier is nearest to
 a. 0.14 MeV
 b. 2.3 MeV
 c. 1.8 x 10 MeV
 d. 0.56 MeV

57. An electron in H atom makes a transition from $n = 3$ to $n = 1$. The recoil momentum of H atom will be
 a. 6.45×10^{-27} N s
 b. 6.8×10^{-27} N s
 c. 6.45×10^{-24} N s
 d. 6.8×10^{-24} N s

58. A hydrogen-like atom emits radiations of frequency 2.7×10^{15} Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be
 a. 1.8×10^{15} Hz
 b. 3.2×10^{15} Hz
 c. 4.7×10^5 Hz
 d. 6.9×10^{15} Hz

59. The electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are
 a. $n_1 = 4, n_2 = 2$
 b. $n_1 = 8, n_2 = 2$
 c. $n_1 = 8, n_2 = 3$
 d. $n_1 = 6, n_2 = 2$

60. An electron revolving in an orbit of radius 0.5 Å in a hydrogen atom executes 10^{16} revolutions per second. The magnetic moment of electron due to its orbital motion will be
 a. 1.256×10^{-23} A m^2
 b. 653×10^{-26} A m^2
 c. 10^{-3} A m^2
 d. 256×10^{-26} A m^2

61. The total energy of an electron in the ground state of hydrogen atom is –13.6 eV. The potential energy of an electron is the ground state of Li^{2+} ion will be
 a. 122.4 eV
 b. –122.4 eV
 c. 244.8 eV
 d. –244.8 eV

62. The wavelength of radiation required to excite the electron from first orbit to third orbit in a doubly ionized lithium atom will be
 a. 134.25 Å
 c. 125.5 Å
 c. 113.7 Å
 d. 110 Å

63. The force acting on the electron in a hydrogen atom depends on the principal quantum number as
 a. $F \propto n^2$
 c. $F \propto \frac{1}{n^2}$
 c. $F \propto n^4$
 d. $F \propto \frac{1}{n^4}$

64. The minimum energy to ionize an atom is the energy required to
 a. add one electron to the atom
 b. excite the atom from its ground state to its first excited state
 c. remove one outermost electron from the atom
 d. remove one innermost electron from the atom

65. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would have been
 a. 32
 b. 60
 c. 64
 d. 4

66. The orbital velocity of an electron in the ground state is v. If the electron is excited to energy state –0.54 eV, its orbital velocity will be
 a. v
 b. $\frac{v}{3}$
 c. $\frac{v}{5}$
 d. $\frac{v}{7}$

67. If potential energy between a proton and an electron is given by $|U| = ke^2/2R^3$, where e is the charge of electron and R is the radius of atom, then radius of Bohr's orbit is given by (h = Planck's constant, k = constant)
 a. $\frac{ke^2m}{h^2}$
 b. $\frac{6\pi^2}{n^2}\frac{ke^2m}{h^2}$
 c. $\frac{2\pi}{n}\frac{ke^2m}{h^2}$
 d. $\frac{4\pi^2ke^2m}{n^2h^2}$

68. In a hydrogen atom, the transition takes place from $n = 3$ to $n = 2$. If Rydberg's constant is 1.09×10^7 m^{-1}, the wavelength of the line emitted is
 a. 6606 Å
 b. 4861 Å
 c. 4340 Å
 d. 4101 Å

69. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
 a. 10^{-15} cm
 b. 10^{-13} cm
 c. 10^{-12} cm
 d. 10^{-19} cm

70. How many revolutions does an electron complete in one second in the first orbit of hydrogen atom?
 a. 6.62×10^{15}
 b. 100
 c. 1000
 d. 1

71. The radius of hydrogen atom in the ground state is 5.3×10^{-11} m. When struck by an electron, its radius is found to be 21.2×10^{-11} m. The principal quantum number of the final state will be
 a. 1
 b. 2
 c. 3
 d. 4

72. The wavelength of the first line of Balmer series is 6563 Å. The Rydberg's constant is
 a. 1.09×10^5 per m^{-1}
 b. 1.09×10^6 per m^{-1}
 c. 1.097×10^7 per m^{-1}
 d. 1.09×10^8 per m^{-1}

73. A hydrogen atom and a Li^{++} ion are both in the second excited state. If ℓ_H and ℓ_{LI} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then

a. $\ell_H < \ell_{LI}$ and $|E_H| > |E_{LI}|$ **b.** $\ell_H = \ell_{LI}$ and $|E_H| < |E_{LI}|$
c. $\ell_H = \ell_{LI}$ and $|E_H| > |E_{LI}|$ **d.** $\ell_H < \ell_{LI}$ and $|E_H| < |E_{LI}|$

74. Imagine an atom made of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength [given in terms of the Rydberg constant R for the hydrogen atom] equal to

a. $\frac{9}{5R}$ **b.** $\frac{36}{5R}$
c. $\frac{18}{5R}$ **c.** $\frac{4}{R}$

75. The electric potential energy between a proton and an electron is given by $U = U_0 \ln r/r_0$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r with n, n being the principal quantum number.

a. $r_n \propto n$ **b.** $r_n \propto 1/n$
c. $r_n \propto n^2$ **d.** $r_n \propto 1/n^2$

76. In order to break a chemical bond in the molecules of human skin, causing sunburn, a photon energy of about 3.5 eV is required. This corresponds to wavelength in the

a. infrared region **b.** X-ray region
c. visible region **d.** ultraviolet region

77. The longest wavelength that a singly ionised helium atom in its ground state will absorb is

a. 912 Å **b.** 304 Å
c. 606 Å **d.** 1216 Å

78. The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume Bohr model is valid in this case. The frequency of the orbital motion of the electron in the initial state is 1/27 of that in the final state. The possible values of n_1 and n_2 are

a. $n_1 = 6, n_2 = 3$ **b.** $n_1 = 4, n_2 = 2$
c. $n_1 = 8, n_2 = 1$ **d.** $n_1 = 3, n_2 = 1$

79. In which of the following transitions will the wavelength be minimum?

a. $n = 5$ to $n = 4$ **b.** $n = 4$ to $n = 3$
c. $n = 3$ to $n = 2$ **d.** $n = 2$ to $n = 1$

80. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. Almost instantaneously, another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?

a. Two photons of energy 10.2 eV.
b. Two photons of energy 1.4 eV.
c. One photon of energy 10.2 eV and one electron of energy 1.4 eV.
d. One electron having kinetic energy nearly 11.6 eV.

81. The shortest wavelength of Lyman series of hydrogen is equal to the shortest wavelength of Balmer series of a hydrogen-like atom of atomic number Z. The value of Z is

a. 4 **b.** 2
c. 3 **d.** 6

82. In a hypothetical system, a particle of mass m and charge $-3q$ is moving around a very heavy particle charge q. Assume that Bohr's model is applicable to this system, then velocity of mass m in first orbit is

a. $\frac{3q^2}{2\varepsilon_0 h}$ **b.** $\frac{3q^2}{4\varepsilon_0 h}$
c. $\frac{3q}{2\pi\varepsilon_0 h}$ **d.** $\frac{3q}{4\pi\varepsilon_0 h}$

83. The angular momentum of an electron in a hydrogen atomis proportional to:

a. $1/\sqrt{r}$ **b.** $1/r$
c. $\sqrt{r}$ **d.** r^2

84. The ratio (in S.I. units) of magnetic dipole moment to that of the angular momentum of an electron of mass m kg and charge e coulomb in Bohr's orbit of hydrogen atom is

a. $\frac{e}{2m}$ **b.** $\frac{e}{m}$
c. $\frac{2e}{m}$ **d.** none of these

85. If an electron in $n = 3$ orbit of hydrogen atom jumps down to $n = 2$ orbit, the amount of energy released and the wavelength of radiation emitted are

a. 0.85 eV, 6566 Å **b.** 1.89 eV, 1240 Å
c. 1.89 eV, 6566 Å **d.** 1.5 eV, 6566 Å

86. In hydrogen spectrum, the shortest wavelength in Balmer series is λ. The shortest wavelength in Brackett series will be

a. 2λ **b.** 4λ
c. 9λ **d.** 16λ

87. The angular momentum of an electron in first orbit of Li^{++} ion is

a. $\frac{3h}{2\pi}$ **b.** $\frac{9h}{2\pi}$
c. $\frac{h}{2\pi}$ **d.** $\frac{h}{6\pi}$

88. If first excitation potential of a hydrogen-like atom is V electron volt, then the ionization energy of this atom will be

a. V electron volt
b. $\frac{3V}{4}$ electron volt
c. $\frac{4V}{3}$ electron volt
d. cannot be calculated by given information

89. Two hydrogen atoms are in excited state with electrons residing in $n = 2$. The first one is moving toward left and emits a photon of energy E_1 toward right. The second one is moving toward right with same speed and emits a photon of energy E_2 towards left. Taking recoil of nucleus into account, during the emission process

a. $E_1 > E_2$ b. $E_1 < E_2$
c. $E_1 = E_2$ d. information insufficient

90. In a hydrogen atom following the Bohr's postulates, the product of linear momentum and angular momentum is proportional to $(n)^x$ where 'n' is the orbit number. Then 'x' is

a. 0 b. 2
c. –2 d. 1

91. The voltage applied to an X-ray tube is 18 kV. The maximum mass of photon emitted by the X-ray tube will be

a. 2×10^{-13} kg b 3.2×10^{-36} kg
c. 3.2×10^{-32} kg d. 9.1×10^{-31} kg

92. The wavelength of K_α X-rays of two metals 'A' and 'B' are $4/1875R$ and $1/675R$, respectively, where 'R' is Rydberg's constant. The number of elements lying between 'A' and 'B' according to their atomic numbers is

a. 3 b. 6
c. 5 d. 4

93. One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the 2^{nd} line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is

a. $n = 4 \rightarrow n = 2$ b. $n = 8 \rightarrow n = 2$
c. $n = 8 \rightarrow n = 4$ d. $n = 12 \rightarrow n = 6$

94. The photon radiated from hydrogen corresponding to 2^{nd} line of Lyman series is absorbed by a hydrogen-like atom 'X' in 2^{nd} excited state. As a result Then, the hydrogen-like atom 'X' makes a transition of n^{th} orbit.

a. $X = He^+, n = 4$ b. $X = Li^{++}, n = 6$
c. $X = He^+, n = 6$ d. $X = Li^{++}, n = 9$

95. The element which has a K_α X-rays line of wavelength 1.8 Å is

($R = 1.1 \times 10^7\ m^{-1}$, b = 1 and $\sqrt{5/33} = 0.39$)

a. Co, $Z = 27$ b. Iron, $Z = 26$
c. Mn, $Z = 25$ d. Ni, $Z = 28$

96. When an electron accelerated by potential difference U is bombarded on a specific metal, the emitted X-ray spectrum obtained is shown in Fig. 4.58. If the potential difference is reduced to $U/3$, the correct spectrum is

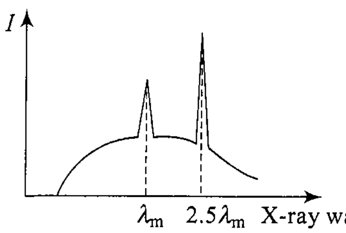
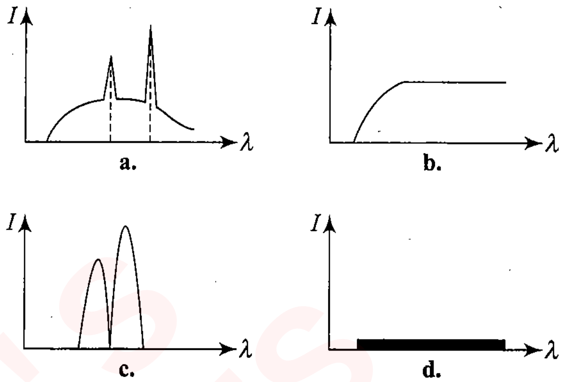


Fig. 4.58



97. In a hydrogen atom, the electron makes a transition from $n = 2$ to $n = 1$. The magnetic field produced by the circulating electron at the nucleus

a. decreases 16 times b. increases 4 times
c. decreases 4 times d. increases 32 times

98. An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron:

a. increases 2 times b. increases 4 times
c. increases 8 times d. remains the same

99. When the voltage applied to an X-ray tube increases from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between K_α line and cut-off wavelength of continuous spectrum increase by a factor of 3. Atomic number of the metallic target is

a. 28 b. 29
c. 65 d. 66

100. Monochromatic radiations of wavelength λ are incident on a hydrogen sample in ground state. Hydrogen atom absorbs the light and subsequently emits radiations of 10 different wavelengths. The value of λ is nearly

a. 203 nm b. 95 nm
c. 80 nm d. 73 nm

101. The power of an X-ray tube is 16 W. If the potential difference applied across the tube is 5 kV, then the number of electrons striking the target per second is

a. 8.4×10^{16} b. 5×10^{17}
c. 2×10^{16} d. 2×10^{19}

102. A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.2 eV and 17.0 eV, respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV, respectively. The values of n and Z are, respectively,

a. 6 and 6 b. 3 and 3
c. 6 and 3 d. 3 and 6

103. Hydrogen atoms in a sample are excited to $n = 5$ state and it is found that photons of all possible wavelengths are present in the emission spactra. The minimum number of hydrogen atoms in the sample would be

a. 5 b. 6
c. 10 d. infinite

104. A sample of hydrogen atoms is in excited state (all the atoms). The photons emitted from this sample are made to pass through a filter through which light having wavelength greater than 800 nm can only pass. Only one type of photons are found to pass the filter. The sample's excited state initially is [Take hc = 1240 eV-nm, ground state energy of hydrogen atom = –13.6 eV.]

a. 5^{th} excited state **b.** 4^{th} excited state
c. 3^{rd} excited state **d.** 2^{nd} excited state

105. Mark out the correct statement regarding X-rays.

a. When fast moving electrons strike the metal target, they enter the metal target and in a very short time span come to rest, and thus an accelerated charged electron produces electromagnetic waves (X-rays).
b. Characteristic X-rays are produced due to transition of an electron from higher energy levels to vacant lower energy levels.
c. X-rays spectrum is a discrete spectra just like hydrogen spectra.
d. Both (a) and (b) are correct.

106. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$. The energy given to hydrogen atom in this inelastic collision is [Neglect the recoiling of hydrogen atom]

a. 10.2 eV **b.** 12.1 eV
c. 12.5 eV **d.** None of these

107. Electrons in a hydrogen-like atom ($Z = 3$) make transitions from 4^{th} excited state to 3^{rd} excited state and from 3^{rd} to 2^{nd} excited state. The resulting radiations are incident on a metal plate to eject photoelectrons. The stopping potential for photoelectrons ejected by the shorter wavelength is 3.95 V. The stopping potential for the photoelectrons ejected by the longer wavelength is

a. 2.0 V **b.** 0.75 V
c. 0.6 V **d.** none of the above

108. In which of the following transitions will the wavelength be minimum?

a. $n_1 = 5$ to $n_2 = 4$ **b.** $n_1 = 4$ to $n_2 = 3$
c. $n_1 = 3$ to $n_2 = 2$ **d.** $n_1 = 2$ to $n_2 = 1$

109. A neutron having kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision

a. must be elastic
b. must be completely inelastic
c. may be partially elastic
d. information is insufficient

110. If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, then the number of revolutions an electron will make when it is in $n = 2$ state before coming to ground state will be [Take $a_0 = 0.53$ Å and all standard data if required]

a. 10^7 **b.** 8×10^6
c. 2×10^5 **d.** none of these

111. An electron of energy 11.2 eV undergoes an inelastic collision with a hydrogen atom in its ground state. [Neglect recoiling of atom as $m_H >> m_e$]. Then in this case

a. the outgoing electron has energy 11.2 eV
b. the entire energy is absorbed by the H atom and the electron stops
c. 10.2 eV of the incident electron energy is absorbed by the H atom and the electron would come out with 1.0 eV energy
d. none of the above

112. The recoil speed of hydrogen atom after it emits a photon in going from $n = 2$ state to $n = 1$ state is nearly [Take $R_\infty = 1.1 \times 10^7$ m^{-1} and h = 6.63×10^{-34} J s]

a. 1.5 ms^{-1} **b.** 3.3 ms^{-1}
c. 4.5 ms^{-1} **d.** 6.6 ms^{-1}

113. A beam of 13.0 eV electrons is used to bombard gaseous hydrogen. The series obtained in emission spectra is/are

a. Lyman series **b.** Balmer series
c. Brackett series **d.** All of these

114. The wavelength of the spectral line that corresponds to a transition in hydrogen atom from n = 10 to ground state would be [In which part of electromagnetic spectrum this line lies?]

a. 92.25 nm, ultraviolet **b.** 92.25 nm, infrared
c. 86.95 nm, ultraviolet **d.** 97.65 nm, ultraviolet

115. The angular momentum of an electron in hydrogen atom is $4h/2\pi$. Kinetic energy of this electron is

a. 4.35 eV **b.** 1.51 eV
c. 0.85 eV **d.** 13.6 eV

116. Difference between n^{th} and $(n + 1)^{th}$ Bohr's radius of hydrogen atom is equal to $(n - 1)^{th}$ Bohr's radius. The value of n is

a. 1 **b.** 2
c. 3 **d.** 4

117. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ. The atomic number for an atom that emits K_α radiation with wavelength 4λ is

a. 6 **b.** 4
c. 11 **d.** 44

118. A proton of mass m moving with a speed v_0 approches a stationary proton that is free to move. Assume impact parameter to be zero, i.e., head-on collision. How close will the incident proton go to other proton?

a. $\dfrac{e^3}{\pi \varepsilon_0 m^2 v_0}$ **b.** $\dfrac{e^3}{\pi \varepsilon_0 m v_0}$

c. $\dfrac{e^2}{\pi \varepsilon_0 m v_0^2}$ **d.** None of the above

119. If an X-ray tube operates at the voltage of 10 kV, find the ratio of the de Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced. The specific charge of electron is 1.8×10^{11} C kg.

a. 1 **b.** 0.1
c. 1.8 **d.** 1.2

120. The potential difference across the Coolidge tube is 20 kV and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The power carried by the X-ray beam is P. Then

a. $P = 0.1$ W   **b.** $P = 1$ W
c. $P = 2$ W   **d.** $P = 10$ W

121. Determine the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smaller wavelength that would work?

a. 133 nm   **b.** 13.3 nm
c. 10.3 nm   **d.** 103 nm

122. In order to determine the value of E_0, a scientist shines photons ("light particles") of various energies at a cloud of atomic hydrogen. Most of the hydrogen atoms occupy the ground state. A detector records the intensity of light transmitted through that cloud; see Fig. 4.59(a). Figure 4.59(b) is a graph of part of the scientist's data, showing the intensity of the transmitted light as a function of the photon energy. A hydrogen atom's electron is likely to absorb a photon only if the photon gives the electron enough energy to knock it into a higher shell.

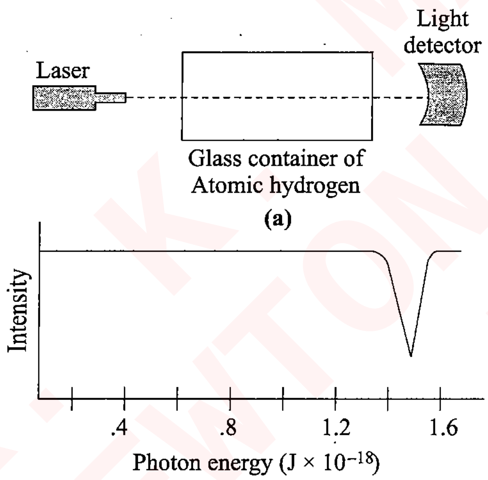


Fig. 4.59

According to this experiment, what is the approximate value of E_0?

a. 1.6×10^{-18} J   **b.** 2.1×10^{-18} J
c. 3.2×10^{-18} J   **d.** 6.4×10^{-18} J

123. In the above question, if the scientist continues taking data at higher photon energies, he will find the next major "dip" in the intensity graph at what photon energy?

a. $\frac{1}{9}E_0$ **b.** $\frac{8}{9}E_0$
c. $3E_0$ **d.** $9E_0$

124. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have –72.5 keV energy. X-rays emitted by the tube contain only

a. a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of –0.155 Å
b. a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths
c. a continuous X-ray spectrum of tungsten
d. a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of 0.155 Å and the characteristic X-ray spectrum of tungsten

125. Consider a hypothetical annihilation of a stationary electron with a stationary positron. What is the wavelength of the resulting radiation?

a. $\lambda = \frac{h}{m_0 c}$ **b.** $\lambda = \frac{2h}{m_0 c^2}$
c. $\lambda = \frac{h}{2m_0 c^2}$ **d.** None of these

126. If 10,000 V are applied across an X-ray tube, find the ratio of wavelength of the incident electrons and the shortest wavelength of X-ray coming out of the X-ray tube, given e/m of electron $= 1.8 \times 10^{11}$ C kg^{-1}

a. 1:10   **b.** 10:1
c. 5:1   **d.** 1:5

127. If the potential difference applied across a Coolidge tube is increased, then

a. wavelength of K_α will increase
b. λ_{min} will increase
c. difference between wavelength of K_α and λ_{min} increases
d. none of these

128. An X-ray tube is operating at 150 kV and 10 mA. If only 1% of the electric power is converted into X-rays, the rate at which the target is heated, in cal s^{-1}, is

a. 3.57   **b.** 35.7
c. 4.57   **d.** 15

129. Figure 4.60 shows Moseley's plot between $\sqrt{f}$ and Z, where f is the frequency and Z is the atomic number. Three lines A, B and C shown in the graph may represent

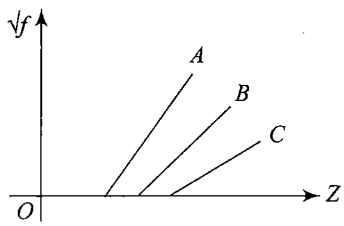


Fig. 4.60

a. K_α, K_β and K_γ lines, respectively
b. K_γ, K_β and K_α lines, respectively
c. K_α, L_α and M_α lines, respectively
d. Nothing

130. An electron in the ground state of hydrogen has an angular momentum L_1 and an electron in the first excited state of lithium has an angular momentum L_2. Then ,

a. $L_1 = L_2$ **b.** $L_1 = 4L_2$
c. $L_2 = 2L_1$ **d.** $L_1 = 2L_2$

131. The approximate value of quantum number n for the circular orbit of hydrogen of 0.0001 mm in diameter is
a. 1000 b. 60
c. 10000 d. 31

132. If elements of quantum number greater than n were not allowed, the number of possible elements in nature would have been
a. $\frac{1}{2}n(n+1)$ b. $\left\{\frac{n(n+1)}{2}\right\}^2$
c. $\frac{1}{6}n(n+1)(2n+1)$ d. $\frac{1}{3}n(n+1)(2n+1)$

133. The velocity of an electron in the first orbit of H atom is v. The velocity of an electron in the 2nd orbit of He^+ is
a. $2v$ b. v
c. $\frac{v}{2}$ d. $\frac{v}{4}$

134. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is
a. 4.718 ms^{-1} b. 7.418 ms^{-1}
c. 4.178 ms^{-1} d. 7.148 ms^{-1}

135. If a hydrogen atom emits a photon of energy 12.1 eV, its orbital angular momentum changes by ΔL. Then, ΔL equals
a. 1.05×10^{-34} Js b. 2.11×10^{-34} Js
c. 3.16×10^{-34} Js d. 4.22×10^{-34}

136. An electron with kinetic energy E eV collides with a hydrogen atom in the ground state. The collision is observed to be elastic for
a. $0 < E < \infty$ b. $0 < E < 10.2$ eV
c. $0 < E < 13.6$ eV d. $0 < E < 3.4$ eV

137. The wavelength K_α of X-rays produced by the X-ray tube is 0.76 Å. The atomic number of the node material of the tube is
a. 30 b. 40
c. 50 d. 60

138. Let the potential energy of a hydrogen atom in the ground state be zero. Then, its energy in the first excited state will be
a. 10.2 eV b. 13.6 eV
c. 23.8 eV d. 27.2 eV

139. When photons of wavelength λ_1 are incident on an isolated sphere suspended by an insulated thread, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice the above value. If light of wavelength λ_3 is used, calculate the stopping potential for this case
a. $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$ b. $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$
c. $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$ d. $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

140. The ionization energy of the ionized sodium atom Na^{10+} is:
a. 13.6 eV b. 13.6×11 eV
c. (13.6/11) eV d. $13.6 \times (11^2)$ eV

141. The shortest wavelength produced in an X-ray tube operating at 0.5 million volt is
a. dependent on the target element
b. about 2.5×10^{-12} m
c. double of the shortest wavelength produced at 1 million volt
d. dependent only on the target material

142. If the K_α radiation of Mo ($Z = 42$) has a wavelength of 0.71 Å find the wavelength of the corresponding radiation of Cu ($Z = 29$)
a. 1 Å b. 2 Å
c. 1.52 Å d. 1.25 Å

143. A beam of electron accelerated by a large potential difference V is made to strike a metal target to produce X-rays. For which of the following values of V, will the resulting X-rays have the lowest minimum wavelength ?
a. 10 kV b. 20 kV
c. 30 kV d. 40 kV

144. The minimum kinetic energy required for ionization of a hydrogen atom is E_1 in case electron is collided with hydrogen atom. It is E_2 if the hydrogen ion is collided and E_3 when helium ion is collided. Then,
a. $E_1 = E_2 = E_3$ b. $E_1 > E_2 > E_3$
c. $E_1 < E_2 < E_3$ d. $E_1 > E_3 > E_2$

145. Magnetic field at the center (at nucleus) of the hydrogen-like atoms (atomic number = z) due to the motion of electron in n^{th} orbit is proportional to
a. $\frac{n^3}{z^5}$ b. $\frac{n^4}{z}$
c. $\frac{z^2}{n^3}$ d. $\frac{z^3}{n^5}$

146. Magnetic moment due to the motion of the electron in n^{th} energy of hydrogen atom is proportional to
a. n b. n^0
c. n^5 d. n^3

147. The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
a. 1 b. 8
c. 4 d. 16

148. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number = z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of z is
a. 2 b. 3
c. 4 d. 6

149. A hydrogen atom is in an excited state of principle quantum number n. It emits a photon of wavelength λ when it returns to the ground state. The value of n is
a. $\sqrt{\lambda R(\lambda R-1)}$ b. $\sqrt{\frac{\lambda(R-1)}{\lambda R}}$
c. $\sqrt{\frac{\lambda R}{\lambda R-1}}$ d. $\sqrt{\lambda(R-1)}$

150. A neutron moving with a speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision will take place is (assume that mass of proton is nearly equal to the mass of neutron)
a. 10.2 eV **b.** 20.4 eV
c. 12.1 eV **d.** 16.8 eV

151. In a hydrogen atom, the electron is in n^{th} excited state. It comes down to first excited state by emitting 10 different wavelengths. The value of n is
a. 6 **b.** 7
c. 8 **d.** 9

152. Angular momentum (L) and radius (r) of a hydrogen atom are related as
a. Lr = constant **b.** Lr^2 = constant
c. Lr^4 = constant **d.** none of these

153. The angular momentum of an electron in an orbit is quantized because it is a necessary condition for the compatibility with
a. wave nature of electron
b. particle nature of electron
c. Pauli's exclusion behaviour
d. none of these

154. The maximum angular speed of the electron of a hydrogen atom in a stationary orbit is
a. 6.2×10^{15} rad s^{-1} **b.** 4.1×10^{16} rad s^{-1}
c. 24×10^{10} rad s^{-1} **d.** 9.2×10^{6} rad s^{-1}

155. In hydrogen and hydrogen-like atoms, the ratio of $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with atomic number z and principal quantum number n as
a. $\frac{z^2}{n^2}$ **b.** $\frac{z^4}{n^4}$
c. $\frac{z}{n}$ **d.** none of these

156. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the n^{th} orbit and its radius in the n^{th} orbit
a. is proportional to n^2
b. is inversely proportional to n^3
c. has a constant value of 10.2 eV-Å
d. has a constant value of 7.2 eV-Å

157. An electron and a photon have same wavelength. If p is the momentum of electron and E the energy of photon, the magnitude of p/E in S.I.unit is
a. 3.0×10^8 **b.** 3.33×10^{-9}
c. 9.1×10^{-31} **d.** 6.64×10^{-34}

158. In X-ray tube, when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum
a. remains constant
b. becomes more than two times
c. becomes half
d. becomes less than two times

159. An electron is in an excited state in a hydrogen like atom. It has a total energy = –3.4 eV. The kinetic energy of electron is E and its de Broglie wavelength is λ
a. $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-10}$ m
b. $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-10}$ m
c. $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-11}$ m
d. $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-11}$ m

160. The circumference of the second Bohr orbit of electron in hydrogen atom is 600 nm. The potential difference that must be applied between the plates so that the electrons have the de Broglie wavelength corresponding in this circumference is
a. 10^{-5} V **b.** $\frac{5}{3} \times 10^{-5}$ V
c. 5×10^{-5} V **d.** 3×10^{-5} V

161. X-rays emitted from a copper target and a molybdenum target are found to contain a line of wavelength 22.85 nm attributed to the K_α line of an impurity element. The K_α lines of copper (Z = 29) and molybdenum (Z = 42) have wavelengths 15.42 nm and 7.12 nm, respectively. The atomic number of the impurity element is
a. 22 **b.** 23
c. 24 **d.** 25

162. An electron in a Bohr orbit of hydrogen atom with the quantum number n_2 has an angular momentum 4.2176 $\times 10^{-34}$ kg m^2 s^{-1}. If the electron drops from this level to the next lower level, the wavelength of this line is
a. 18 nm **b.** 187.6 pm
c. 1876 Å **d.** 1.876×10^4 Å

163. In the Bohr model of a π-mesic atom, a π-meson of mass m_π and of the same charge as the electron is in a circular orbit of radius r about the nucleus with an orbital angular momentum $h/2\pi$. If the radius of a nucleus of atomic number Z is given by $R = 1.6 \times 10^{-15} Z^{\frac{1}{3}}$ m, Then the limit on Z for which ($\varepsilon_0 h^2/\pi m e^2 = 0.53$ Å and $m_\pi/m_e = 264$) π-mesic atoms might exist is
a. < 105 **b.** > 105
c. < 37 **d.** > 37

164. In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron
a. from $n_1 = 6$ to $n_2 = 4$ **b.** from $n_1 = 5$ to $n_2 = 3$
c. from $n_1 = 4$ to $n_2 = 2$ **d.** from $n_1 = 3$ to $n_2 = 2$

165. A K_α X-ray emitted from a sample has an energy of 7.46 keV. Of which element is the sample made?
a. Calcium (Ca, $Z = 20$) **b.** Cobalt (Co $Z = 27$)
c. Cadmium (Cd $Z = 48$) **d.** Nickel (Ni, $Z = 28$)

Multiple Correct Answers Type

Solutions on page 4.82

1. Hydrogen atoms absorb radiation of wavelength λ_0 and consequently emit radiations of 6 different wavelengths of which two wavelengths are shorter than λ_0. Then,
a. the final excited state of the atoms is $n = 4$
b. the initial state of the atoms may be $n = 2$
c. the initial state of the atoms may be $n = 3$
d. there are three transitions belonging to Lyman series

2. Energy liberated in the de-excitation of hydrogen atom from 3^{rd} level to 1^{st} level falls on a photo-cathode. Later when the same photo-cathode is exposed to a spectrum of some unknown hydrogen-like gas, excited to 2^{nd} energy level, it is found that the de Broglie wavelength of the fastest photoelectrons now ejected has decreased by a factor of 3. For this new gas, difference of energies of 2^{nd} Lyman line and 1^{st} Balmer line is found to be 3 times the ionization potential of the hydrogen atom. Select the correct statement(s):
   a. The gas is lithium.
   b. The gas is helium.
   c. The work function of photo-cathode is 8.5 eV.
   d. The work function of photo-cathode is 5.5 eV.
3. Which of the following products, in a hydrogen atom, are independent of the principal quantum number n? The symbols have their usual meanings.
   a. $\omega^2 r$ b. $\frac{E}{v^2}$
   c. $v^2 r$ d. $\frac{E}{r}$
4. According to Bohr's theory of hydrogen atom, for the electron in the n^{th} permissible orbit
   a. linear momentum $\propto \frac{1}{n}$
   b. radius of orbit $\propto n$
   c. kinetic energy $\propto \frac{1}{n^2}$
   d. angular momentum $\propto n$
5. When a hydrogen atom is excited from ground state to first excited state, then
   a. its kinetic energy increases by 20 eV
   b. its kinetic energy decreases by 10.2 eV
   c. its potential energy increases by 20.4 eV
   d. its angular momentum increases by 1.05×10^{-34} Js
6. In an X-ray tube, the voltage applied is 20 kV. The energy required to remove an electron from L shell is 19.9 keV. In the X-rays emitted by the tube,
   a. minimum wavelength will be 62.1 nm.
   b. energy of the characteristic x-rays will be equal to or less than 19.9 keV.
   c. L_α X-ray may be emitted.
   d. L_α X-ray will have energy 19.9 keV.
7. Suppose the potential energy between an electron and a proton at a distance r is given by $Ke^2/3r^3$. Application of Bohr's theory to hydrogen atom in this cases shows that:
   a. energy in the n^{th} orbit is proportional to n^6
   b. energy is proportional to m^{-3} (m = mass of electron)
   c. energy in the n^{th} orbit is proportional to n^{-2}
   d. energy is proportional to m^3 (m = mass of electron)
8. Let A_n be the area enclosed by the n^{th} orbit in a hydrogen atom. The graph of ln (A_n/A_1) against ln(n)
   a. will pass through origin
   b. will be a straight line with slope 4
   c. will be a monotonically increasing nonlinear curve
   d. will be a circle
9. Mark out the correct statement(s).
   a. Line spectra contain information about atoms only.
   b. Line spectra contain information about both atoms and molecules.
   c. Band spectra contain information about both atoms and molecules.
   d. Band spectra contain information about molecules only.
10. A hydrogen atom having kinetic energy E collides with a stationary hydrogen atom. Assume all motions are taking place along line of motion of the moving hydrogen atom. For this situation, mark out the correct statement(s).
   a. For $E \geq 20.4$ eV only, collision would be elastic.
   b. For $E \geq 20.4$ eV only, collision would be inelastic.
   c. For $E = 2.4$ eV, collision would be perfectly inelastic.
   d. For $E = 18$ eV, the KE of initially moving hydrogen atom after collision is zero.
11. In Bohr's model of hydrogen atom:
   a. the radius of n^{th} orbit is proportional to n^2
   b. the total energy of electron in n^{th} orbit is proportional to n
   c. the angular momentum of the electron in an orbit is an integral multiple of $h/2\pi$
   d. the magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy
12. Which of the following are in the ascending order of wavelength?
   a. H_α, H_β, H_γ,… lines in Balmer series of hydrogen atom.
   b. Lyman limit, Balmer limit, and Paschen limit in the hydrogen spectrum
   c. Violet, blue, yellow, and red colors in solar spectrum.
   d. None of the above.
13. Continuous spectrum is produced by
   a. incandescent electric bulb
   b. sun
   c. hydrogen molecules
   d. sodium vapor lamp
14. A gas of monoatomic hydrogen is bombarded with a stream of electrons that have been accelerated from rest through a potential difference of 12.75 V. In the emission spectrum, one can observe lines of
   a. Lyman series   b. Balmer series
   c. Paschen series   d. Pfund series
15. If the potential energy of the electron in the first allowed orbit in hydrogen atom is E; its
   a. ionization potential is $-E/2$ b. kinetic energy is $-E/2$
   c. total energy is $E/2$ d. none of these
16. According to Bohr's theory of the hydrogen atom, for the electron in the n^{th} allowed orbit:
   a. the linear momentum is proportional to $(1/n)$
   b. the radius is proportional to n
   c. the kinetic energy is proportional to $(1/n^2)$
   d. the angular momentum is proportional to n
17. Whenever a hydrogen atom emits a photon in the Balmer series:
   a. it may emit another photon in the Balmer series
   b. it must emit another photon in the Lyman series
   c. the second photon, if emitted, will have a wavelength of about 122 nm
   d. it may emit a second photon, but the wavelength of this photon cannot be predicted

18. Let λ_α, λ_β and λ'_α denote the wavelengths of the X-rays of the K_α, K_β and L_α lines in the characteristic X-rays for a metal. Then,

a. $\lambda'_\alpha > \lambda_\alpha > \lambda_\beta$
b. $\lambda'_\alpha > \lambda_\beta > \lambda_\alpha$
c. $\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$
d. $\frac{1}{\lambda_\alpha} = \frac{1}{\lambda_\beta} + \frac{1}{\lambda'_\alpha}$

19. Which of the following statements are correct for an X-ray tube?

a. On increasing potential difference between the filament and target, photon flux of X-rays increases.
b. On increasing potential difference between the filament and target, frequency of X-rays increases.
c. On increasing filament current, cut-off wavelength increases.
d. On increasing filament current, intensity of X-rays increases.

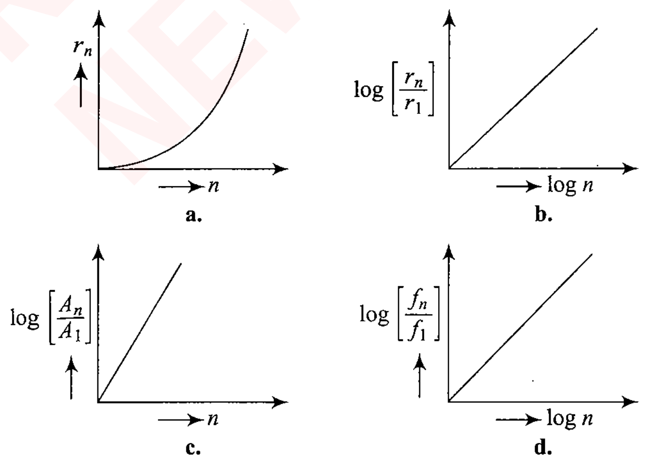
20. Suppose frequency of emitted photon is f_0 when the electron of a stationary hydrogen atom jumps from a higher state m to a lower state n. If the atom is moving with a velocity v ($<< c$) and emits a photon of frequency f during the same transition, then which of the following statements are possible?

a. f may be equal to f_0
b. f may be greater than f_0
c. f may be less than f_0
d. f cannot be equal to f_0

21. Which of the following statements about hydrogen spectrum are correct?

a. All the lines of Lyman series lie in ultraviolet region.
b. All the lines of Balmer series lie in visible region.
c. All the lines of Paschen series lie in infrared region.
d. None of the above.

22. If, in a hydrogen atom, radius of n^{th} Bohr orbit is r_n, frequency of revolution of electron in n^{th} orbit is f_n, and area enclosed by the n^{th} orbit is A_n, then which of the following graphs are correct?



23. Which of the following statements are true?

a. The shortest wavelength of X-rays emitted from an X-ray tube depends on the current in the tube.
b. Characteristic X-ray spectra is simple as compared to optical spectra.
c. X-rays cannot be diffracted by means of an ordinary grating.
d. There exists a sharp limit on the short wavelength side for each continuous X-ray spectrum.

24. An X-ray tube is operated at 6.6 kV. In the continuous spectrum of the emitted X-rays, which of the following frequencies will be missing?

a. 10^{18} Hz
b. 1.5×10^{18} Hz
c. 2×10^{18} Hz
d. 2.5×10^{18} Hz

25. Which of the following statements are correct?

a. If angular momentum of the Earth due to its motion around the Sun were quantized according to the Bohr's relation $L = nh/2\pi$, then the quantum number n would be of the order of 10^{74}.
b. If elements with principal quantum number > 4 were not allowed in nature, then the number of possible elements would be 64.
c. Rydberg's constant varies with mass number of the element.
d. The ratio of the wave number of H_a line of Balmer series for hydrogen and that of H_a line of Balmer series for singly ionized helium is exactly 4.

26. According to Einstein's theory of relativity, mass can be converted into energy and vice-versa. The lightest elementary particle, taken to be the electron, has a mass equivalent to 0.51 MeV of energy. Then, we can say that

a. the minimum amount of energy available through conversion of mass into energy is 1.2 MeV
b. the least energy of a γ-ray photon that can be converted into mass is 1.02 MeV.
c. whereas the minimum energy released by conversion of mass into energy is 1.02 MeV, it is only a γ-ray photon of energy 0.51 MeV and above that can be converted into mass
d. whereas the minimum energy released by conversion of mass into energy is 0.51 MeV, it is only a γ-ray photon of energy 1.02 MeV and above that can be converted into mass

27. X-ray from a tube with a target A of atomic number Z shows strong K lines for target A and weak K lines for impurities. The wavelength of K_α lines is λ_z for target A and λ_1 and λ_2 for two impurities.

$$\frac{\lambda_z}{\lambda_1} = 4 \quad \text{and} \quad \frac{\lambda_z}{\lambda_2} = \frac{1}{4}$$

Assuming the screening constant of K_α lines to be unity, select the correct statement(s).

a. The atomic number of first impurity is $2z - 1$.
b. The atomic number of first impurity is $2z + 1$.
c. The atomic number of second impurity is $\frac{(z+1)}{2}$.
d. The atomic number of second impurity is $\frac{z}{2} + 1$.

28. In Bohr's model of the hydrogen atom, let R, V, T, and E represent the radius of the orbit, speed of the electron, time period of revolution of electron, and the total energy of the electron, respectively. The quantities proportional to the quantum number n are

a. VR
b. RE
c. $\frac{T}{R}$
d. $\frac{V}{E}$

29. An X-ray tube is operating at 50 kV and 20 mA. The target material of the tube has mass of 1 kg and specific heat 495 J kg^{-1} °C^{-1}. One percent of applied electric power is converted into X-rays and the remaining energy goes into heating the target. Then,

a. a suitable target material must have high melting temperature
b. a suitable target material must have low thermal conductivity
c. the average rate of rise of temperature of the target would be 2°C s^{-1}
d. the minimum wavelength of X-rays emitted is about 0.25 $\times 10^{-10}$ m

30. For a certain metal, the K absorption edge is at 0.172 Å. The wavelength of K_α, K_β, and K_γ lines of K series are 0.210 Å, 0.192 Å, and 0.180 Å, respectively. The energies of K, L, and M orbits are E_K, E_L and E_M, respectively. Then

a. $E_K = -13.04$ keV
b. $E_L = -7.52$ keV
c. $E_M = -3.21$ keV
d. $E_K = 13.04$ keV

31. The third line of the Balmer series spectrum of a hydrogen-like ion of atomic number Z equals to 108.5 nm. The binding energy of the electron in the ground state of these ions is E_B. Then

a. $Z = 2$
b. $E_B = 54.4$ eV
c. $Z = 3$
d. $E_B = 122.4$ eV

Assertion-Reasoning Type

Solutions on page 4.86

Some questions (Assertion-Reason type) are given below. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices a., b., c., and d. out of which **only one** is correct. So select the correct choice.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is NOT a corect explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** In a hydrogen atom, energy of emitted photon corresponding to transition from $n = 2$ to $n = 1$ is much greater as compared to transition from $n = \infty$ to $n = 2$.
Statement II: Wavelength of photon is directly proportional to the energy of emitted photon.

2. **Statement I:** In an X-ray tube, if the energy with which an electron strikes the metal target increases, then the wavelength of the characteristic X-rays also changes.
Statement II: Wavelength of characteristic X-rays depends only on the initial and final energy levels.

3. **Statement I:** The energy of a He^+ ion for a given n is almost exactly four times that of H atom for the same n.
Statement II: Photons emitted during transition between corresponding pairs of levels in He^+ and H have the same energy E and the same wavelength $\lambda = hc/E$.

Comprehension Type

Solutions on page 4.87

For Problems 1–3

Hydrogen is the simplest atom of nature. There is one proton in its nucleus and an electron moves around the nucleus in a circular orbit. According to Niels Bohr, this electron moves in a stationary orbit. When this electron is in the stationary orbit, it emits no electromagnetic radiation. The angular momentum of the electron is quantized, i.e., $mvr = (nh/2\pi)$, where m = mass of the electron, v = velocity of the electron in the orbit, r = radius of the orbit, and $n = 1, 2, 3, \ldots$ When transition takes place from K^{th} orbit to J^{th} orbit, energy photon is emitted. If the wavelength of the emitted photon is λ, we find that $\frac{1}{\lambda} = R\left[\frac{1}{J^2} - \frac{1}{K^2}\right]$, where R is Rydberg's constant.

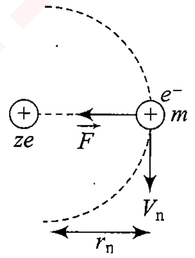


Fig. 4.61

On a different planet, the hydrogen atom's structure was somewhat different from ours. There the angular momentum of electron was $P = 2n(h/2\pi)$, i.e., an even multiple of $(h/2\pi)$.

Answer the following questions regarding the other planet based on above passage:

1. The minimum permissible radius of the orbit will be
a. $\frac{2\varepsilon_0 h^2}{m\pi e^2}$
b. $\frac{4\varepsilon_0 h^2}{m\pi e^2}$
c. $\frac{\varepsilon_0 h^2}{m\pi e^2}$
d. $\frac{\varepsilon_0 h^2}{2m\pi e^2}$

2. In our world, the velocity of electron is v_0 when the hydrogen atom is in the ground state. The velocity of electron in this state on the other planet should be
a. v_0
b. $v_0/2$
c. $v_0/4$
d. $v_0/8$

3. In our world, the ionization potential energy of a hydrogen atom is 13.6 eV. On the other planet, this ionization potential energy will be
a. 13.6 eV
b. 3.4 eV
c. 1.5 eV
d. 0.85 eV

For Problems 4–6

In an ordinary atom, as a first approximation, the motion of the nucleus can be ignored. In a positronium atom, a positron replaces the proton of hydrogen atom. The electron and positron masses are equal and, therefore, the motion of the positron cannot be ignored. One must consider the motion of both electron and positron about their center of mass. A detailed analysis shows that formulae of Bohr model apply to positronium atom provided that we replace m_e by what is known as the reduced mass of the electron. For positronium, the reduced mass is $m_e/2$.

4. The orbital radius of the first excited level of positronium atom is
   a. $4a_0$ b. $a_0/2$
   c. $8a_0$ d. $2a_0$
   where a_0 is the orbital radius of ground state of hydrogen atom.
5. If Rydberg constant for hydrogen atom is R, then Rydberg constant for positronium atom is
   a. $2R$ b. R
   c. $R/2$ d. $4R$
6. When system de-excites from its first excited state to ground state, the wavelength of radiation is
   a. 1217 Å b. 2431 Å
   c. 608 Å d. none of these

For Problems 7–9

The electrons in a H-atom kept at rest, jumps from the m^{th} shell to the n^{th} shell ($m > n$). Suppose instead of emitting electromagnetic wave, the energy released is converted into kinetic energy of the atom. Assume Bohr model and conservation of angular momentum are valid. Now, answer the following questions:

7. What principle is violated here?
   a. Laws of motion b. Energy conservation
   c. Nothing is violated d. Cannot be decided
8. Calculate the angular velocity of the atom about the nucleus if I is the moment of inertia
   a. $\dfrac{(m+n)}{6.28}\dfrac{h}{I}$ b. $\dfrac{(m+n)}{1.57}\dfrac{h}{I}$
   c. $\dfrac{(m-n)}{6.28}\dfrac{h}{I}$ d. $\dfrac{(m-n)}{1.57}\dfrac{h}{I}$
9. If the above comprehension be true, what is not valid here?
   a. $F = ma$ b. $\tau = I\alpha$
   c. $F = dp/dt$ d. All of them

For Problems 10–15

The energy levels of a hypothetical one electron atom are shown in Fig. 4.62.

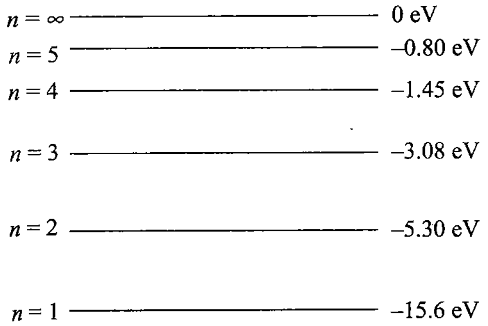


Fig. 4.62

10. Find the ionization potential of the atom.
    a. 11.2 eV b. 13.5 eV
    c. 15.6 eV d. 12.6 eV
11. Find the short wavelength limit of the series terminating at $n = 2$.
    a. 3256 Å b. 2339 Å
    c. 2509 Å d. 3494 Å
12. Find the excitation potential for the state $n = 3$.
    a. 14.64 eV b. 9.93 eV
    c. 12.52 eV d. 10.04 eV
13. Find the wave number of the photon emitted for the transition $n = 3$ to $n = 1$
    a. $2.23 \times 10^7\ \text{m}^{-1}$ b. $1.009 \times 10^7\ \text{m}^{-1}$
    c. $3.005 \times 10^6\ \text{m}^{-1}$ d. $0.432 \times 10^6\ \text{m}^{-1}$
14. If an electron with initial kinetic energy 6 eV is to interact with this hypothetical atom, what minimum energy will this electron carry after interaction?
    a. 2 eV b. 3 eV
    c. 6 eV d. 0 eV
15. The initial kinetic energy of an electron is 11 eV and it interacts with above said hypothetical one electron atom, the minimum energy carried by the electron after interaction is
    a. 0.7 eV b. 0.3 eV
    c. 0.9 eV d. 1 eV

For Problems 16–17

The electron in a hydrogen atom at rest makes a transition from $n = 2$ energy state to the $n = 1$ ground state.

16. Find the energy (eV) of the emitted photon.
    a. 5.8 eV b. 8.3 eV
    c. 10.2 eV d. 12.7 eV
17. Assume that all of the energy corresponding to transition from $n = 2$ to $n = 1$ is carried off by the photon. By setting the momentum of the system (atom + photon) equal to zero after the emission and assuming that the recoil energy of the atom is small compared with the $n = 2$ to $n = 1$ energy level separation, find the energy of the recoiling hydrogen atom.
    a. 2.75×10^{-7} eV b. 5.54×10^{-8} eV
    c. 8.11×10^{-8} eV d. 10.36×10^{-7} eV

For Problems 18–19

Consider a hypothetical hydrogen-like atom. The wavelength, in Å, for the spherical lines for transitions from $n = p$ to $n = 1$ are given by

$$\lambda = \frac{1500p^2}{p^2 - 1} \quad \text{where} \quad p = 1, 2, 3, 4, \ldots$$

18. Find the wavelength of the most energetic photons in this series.
    a. 1800 Å b. 1500 Å
    c. 1300 Å d. 1650 Å

19. What is the ionization potential of this element?
a. 3.96 V
b. 9.23 V
c. 6.34 V
d. 8.28 V

For Problems 20–23

A sample of hydrogen gas in its ground state is irradiated with photons of 10.02 eV energies. The radiation from the above sample is used to irradiate two other samples of excited ionized He^+ and excited ionized Li^{2+}, respectively. Both the ionized samples absorb the incident radiation.

20. How many spectral lines are obtained in the spectra of Li^{2+}?
a. 10
b. 15
c. 20
d. 17

21. Which is the smallest wavelength that will be observed in spectra of He^+ ion?
a. 24.4 nm
b. 28.8 nm
c. 22.2 nm
d. 30.6 nm

22. How many spectral lines are observed in spectra of He^+ ion?
a. 2
b. 4
c. 6
d. 8

23. Which is the smallest wavelength observed in the spectra of Li^{2+}?
a. 8.6 nm
b. 10.4 nm
c. 12.8 nm
d. 14.6 nm

For Problems 24–25

A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle 90° with respect to its original direction.

24. Find the minimum allowed values of energy of the neutron.
a. 0.39 eV
b. 0.32 eV
c. 0.25 eV
d. 0.43 eV

25. Find the maximum allowed value of energy of the He atom?
a. 13.68 eV
b. 19.88 eV
c. 15.26 eV
d. 17.84 eV

For Problems 26–27

An electron and a photon are separated by a distance r so that the potential energy between them is $u = k \log r$, where k is a constant.

26. In such an atom, radius of n^{th} Bohr's orbit is
a. $\dfrac{2nh}{\pi\sqrt{mk}}$
b. $\dfrac{nh}{2\pi\sqrt{2mk}}$
c. $\dfrac{nh}{2\pi\sqrt{mk}}$
d. $\dfrac{nh}{4\pi\sqrt{mk}}$

27. The expression for various energy levels of the above said hypothetical atom is
a. $\dfrac{k}{2}\left[1+\log\dfrac{n^2h^2}{4\pi^2mk}\right]$
b. $2k\left[2+\log\dfrac{n^2h^2}{4\pi^2mk}\right]$
c. $k\left[2+\log\dfrac{n^2h^2}{4\pi^2mk}\right]$
d. $\dfrac{k}{2}\left[1+\log\dfrac{n^2h^2}{2\pi^2mk}\right]$

For Problems 28–30

Pertain to the following statement and Fig. 4.63.

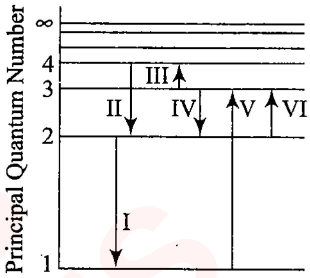


Fig. 4.63

The figure above shows an energy level diagram of the hydrogen atom. Several transitions are marked as I, II, III, ... The diagram is only indicative and not to scale.

28. In which transition is a Balmer series photon absorbed?
a. II
b. III
c. IV
d. VI

29. The wavelength of the radiation involved in transition II is:
a. 291 nm
b. 364 nm
c. 487 nm
d. 652 nm

30. Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 103 nm?
a. I
b. II
c. IV
d. V

For Problems 31–33

A certain species of ionized atoms produces emission line spectrum according to the Bohr model. A group of lines in the spectrum is forming a series in which the shortest wavelength is 22.79 nm and the longest wavelength is 41.02 nm. The atomic number of atom is Z.

Based on above information, answer the following questions:

31. The value of Z is
a. 2
b. 3
c. 4
d. 5

32. The series belongs to
a. Lyman
b. Balmer
c. Paschen
d. Brackett

33. The next to longest wavelength in the series of lines is
a. 35.62 nm
b. 30.47 nm
c. 25.68 nm
d. 12.64 nm

For Problems 34–36

Simplified model of electron energy levels for a certain atom is shown in Fig. 4.64. The atom is bombarded with fast moving electrons. The impact of one of these electrons can cause the removal of electron from K-level, thus creating a vacancy in the K-level. This vacancy in K-level is filled by an electron from L-level and the energy released in this transition can either appear as electromagnetic waves or may all be used to knock out an electron from M-level of the atom.

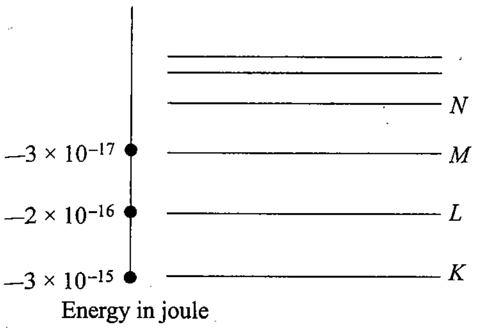


Fig. 4.64

Based on the above information, answer the following questions:

34. The minimum potential difference through which bombarding electron beam must be accelerated from rest to cause the ejection of electron from *K*-level is

a. 18750 V b. 400 kV
c. 2.16 kV d. 21.6 kV

35. Wavelength of the electromagnetic waves emitted due to transition from *L* to *K*-level is

a. 8×10^{-10} m b. 7.104×10^{-11} m
c. 22.46 nm d. 142.6×10^{-8} m

36. The KE of the emitted electron from *M*-level is

a. 260×10^{-17} J b. 2×10^{-18} J
c. 280×10^{-17} J d. 227×10^{-17} J

For Problems 37–39

A monochromatic beam of light having photon energy 12.5 eV is incident on a sample *A* of atomic hydrogen gas in which all atoms are in ground state. The emission spectra obtained from this sample is incident on another sample *B* of atomic hydrogen gas in which all atoms are in 1^{st} excited state.

Based on the above information, answer the following questions:

37. The atoms of sample *A* after passing of light through it

a. may be in 1^{st} excited state
b. may be in 2^{nd} excited state
c. may be in both 1^{st} and 2^{nd} excited states
d. none of the above

38. The emission spectra of sample *A*

a. must have 3 lines b. must have 2 lines
c. may have 2 lines d. it is not formed

39. The atoms of sample *B*

a. will ionize when emission spectra of *A* is incident on *B*
b. may ionize when emission spectra of *A* is incident on *B*
c. will excited to some higher energy state but won't ionize
d. none of the above

For Problems 40–42

An electron orbits a stationary nucleus of charge $+ze$, where z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to third Bohr orbit.

40. The value of z is

a. 5 b. 4
c. 3 d. 2

41. The radius of first Bohr orbit is

a. 0.529×10^{-10} m b. 0.106×10^{-10} m
c. 0.318×10^{-10} m d. none of these

42. Angular momentum of electron in first Bohr orbit is

a. 0.105×10^{-13} Js b. 2.10×10^{-13} Js
c. 3.15×10^{-13} Js d. Can be (a) or (b)

For Problems 43–44

When high energetic electron beam, (i.e., cathode rays) strike the heavier metal, then X-rays are produced. Spectrum of X-rays are classified into two categories: (i) continuous spectrum, and (ii) characteristic spectrum. The wavelength of continuous spectrum depends only on the potential difference across the electrode. But wavelength of characteristic spectrum depends on the atomic number (z).

43. The production of characteristic X-ray is due to the

a. continuous acceleration of incident electrons toward the nucleus
b. continuous retardation of incident electrons toward the nucleus
c. electron transitions between inner shells of the target atom
d. electron transitions between outer shells of the target atom

44. The production of continuous X-ray is due to the

a. acceleration of incident electrons by the nucleus of the target atom
b. electron transitions between inner shells of the target atom
c. electron transitions between outer shells of the target atom
d. annihilation of the mass of incident electrons

For Problems 45–47

Light from a discharge tube containing hydrogen atoms falls on the surface of a plate of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV.

45. The energy of the photons causing the photoelectric emission is

a. 2.55 eV b. 0.73 eV
c. 1.82 eV d. Information insufficient

46. The quantum numbers of two levels involved in the emission of these photons is

a. $4 \to 2$ b. $3 \to 1$
c. $3 \to 2$ d. $4 \to 3$

47. The change in the angular momentum of the electron in the hydrogen atom in above transition is

a. $\dfrac{2h}{\pi}$ b. $\dfrac{h}{2\pi}$
c. $\dfrac{h}{\pi}$ d. $\dfrac{h}{4\pi}$

For Problems 48–50

The electron in a Li^{++} ion is in the n^{th} shell, n being very large. One of the *K*-electrons in another metallic atom has been knocked out. The second metal has four orbits. Now, we take two samples—one of Li^{++} ions and the other of the second metallic ions. Suppose the probability of electronic transition from higher to lower energy levels is directly proportional to the energy difference between the two shells. Take $hc = 1224$ eV nm, where h is Planck's constant and c the velocity of light in vacuum. It is found that major electromagnetic waves emitted from the two samples are identical. Now, answer the following questions:

48. What is the X-ray having least intensity emitted by the second sample?

a. K_α b. L_α
c. M_α d. data insufficient

49. What is the major X-ray emitted by the sample?

a. K_α b. K_β
c. K_γ d. K_δ

50. The wavelength of this major X-ray is

a. 0.90 Å b. 1.0 Å
c. 1.1 Å d. none of these

For Problems 51–52

Two hydrogen-like atoms A and B are of different and each atom has ratio of neutron to proton equal to unity. The difference in the energies between the first Lyman lines emitted by A and B is 81.6 eV. When the atoms A and B moving with the same velocity strike separately a heavy target, they rebound back with half of the speed before collision. However, in this process atom B imparts the target a momentum which is three times the momentum imparted to target by atom A.

51. Atom A is

a. ${}^{1}_{1}H$ b. ${}^{2}_{1}H$
c. ${}^{6}_{3}Li$ d. ${}^{4}_{2}Li$

52. The difference in the energies between the first Balmer lines emitted by A and B is

a. 12.1 eV b. 13.6 eV
c. 14.3 eV d. 15.1 eV

For Problems 53–54

1.8 g of hydrogen is excited by irradiation. The study of spectra indicated that 27% of the atoms are in the first excited state, 15% of the atoms in the second excited state, and the rest in the ground state. The ground state ionization potential energy of hydrogen atom is 21.4×10^{-12} ergs.

53. The number of atoms present in the second excited state is

a. 1.61×10^{23} b. 0.805×10^{23}
c. 2.92×10^{23} d. 1.46×10^{23}

54. The total amount of energy that would be evolved when all the atoms return to the ground state is

a. 782 kJ b. 978.7 kJ
c. 19.63×10^{11} erg d. 97.87×10^{11} erg

For Problems 55–57

In a set of experiments on a hypothetical one-electron atom, the wavelengths of the photons emitted from transitions ending in the ground state ($n = 1$) are shown in the energy level diagram (Fig. 4.65).

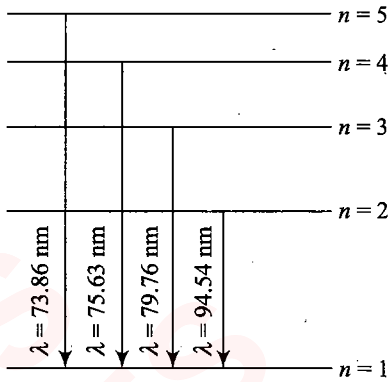


Fig. 4.65

$\lambda_{5\to1} = 73.86$ nm
$\lambda_{4\to1} = 75.63$ nm
$\lambda_{3\to1} = 79.76$ nm
$\lambda_{2\to1} = 94.54$ nm

55. The energy of the atom in level $n = 1$ is nearly

a. 13.14 eV b. 15.57 eV
c. 17.52 eV d. 16.42 eV

56. If an electron made a transition from $n = 4$ to $n = 2$ level, the wavelength of the light that it would emit is nearly

a. 380 nm b. 190 nm
c. 76 nm d. 510 nm

57. The possible energy of the atom in $n = 3$ cannot be

a. –19.5 eV b. –0.4875 eV
c. –0.121 eV d. –7.8 eV

Matching Column Type

Solutions on page 4.92

1. In each situation of Column I, a physical quantity related to orbiting electron in hydrogen-like atom is given. The terms 'Z' and 'n' given in Column II have usual meaning in Bohr's theory. Match the quantities in Column I with the terms they depend on it Column II

Column I	Column II
a. Frequency of orbiting electron	p. is directly proportional to Z^2
b. Angular momentum of orbiting electron	q. is directly proportional to n
c. Magnetic moment of orbiting electron	r. is inversely proportional to n^3
d. The average current due to orbiting of electron	s. is independent of Z

2. Take the usual meanings of the symbols to match the following:

Column I	Column II
a. Average kinetic energy of photoelectrons	p. zero
b. Minimum kinetic energy of photoelectrons	q. $hc/\lambda - \lambda$

c. Maximum wavelength of continuous X-rays	r. hc/eV
d. Minimum wavelength of continuous X-rays	s. not predictable

3. Match the following:

Column I	Column II
a. The voltage applied to X-ray tube is increased	p. Average KE of the electrons decreases
b. In photoelectric effect, work function of the target is increased	q. Average KE of the electrons increases
c. Stopping potential decrease	r. Cut-off wavelength decreased
d. Wavelength of K_α X-ray increased	s. Atomic number of target material decreases

4. Match the following:

Column I	Column II
a. Characteristic X-ray	p. Inverse process of photoelectric effect
b. X-ray production	q. High potential difference
c. Cut-off wavelength	r. Moseley's law
d. Continuous X-ray	s. Emission of radiations

5.

Column I	Column II
a. K_α photon of aluminium	p. will be most energetic among the four
b. K_β photon of aluminium	q. will be least energetic among the four
c. K_α photon from sodium	r. will be more energetic than the K_α photon of lithium
d. K_β photon of beryllium	s. constant speed

6. Match the entries of Column I with the entries of Column II:

Column I	Column II
a. Emission spectra	p. Discrete
b. Absorption spectra	q. Continuous
c. X-ray spectra	r. Electronic Transition
d. Thermal radiation spectra	s. Quantum theory of electromagnetic radiation

7.

Column I	Column II
a. Radius of orbit depends on principal quantum number as	p. increase
b. Due to orbital motion of electron, magnetic field arises at the center of nucleus is proportional to principal quantum number as	q. decrease
c. If electron is going from lower energy level to higher energy level, then velocity of electron will	r. proportional to $\frac{1}{n^2}$
d. If electron is going from lower energy level to higher energy level, then total energy of electron will	s. proportional to n^2
	t. is proportional to $\frac{1}{n^5}$

8.

Column I	Column II
a. Radius of orbit is related with atomic number (Z)	p. is proportional to Z
b. Current associated due to orbital motion of electron with atomic number (Z)	q. is inversely proportion to Z
c. Magnetic field at the center due to orbital motion of electron related with Z	r. is proportional to Z^2
d. Velocity of an electron related with atomic number (Z)	s. is proportional to Z^3

9. The spectral lines of hydrogen-like atom fall within the wavelength range from 950 Å to 1350 Å. Then, match the following:

Column I	Column II
a. If it is atomic hydrogen atom and energy $E = -0.85$ eV	p. $\lambda = 1212$ Å and it corresponds to transition from 2 to 1
b. If it is atomic hydrogen atom and energy $E = -3.4$ eV	q. $\lambda = 134$ Å and it corresponds to transition from 2 to 1
c. If it is doubly ionized lithium atom, then	r. $\lambda = 303$ Å and it corresponds to transition from 2 to 1
d. If it is singly ionized helium atom, then	s. $\lambda = 970$ Å and it corresponds to transition from 4 to 1

Integer Answer Type

Solutions on page 4.93

1. Find recoil speed (approximately in m /sec) when a hydrogen atom emits a photon during the transition from $n = 5$ to $n = 1$.
2. An atom of atomic number $Z = 11$ emits K_α wavelength which is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ (an integer).
3. An electron in nth excited state in a hydrogen atom comes down to first excited state by emitting ten different wavelengths. Find value of n (an integer).
4. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number Z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. Find the value of Z.
5. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays. (in $\times 10^{-2}$A)
6. An electron in an H-atom kept at rest, jumps from the m^{th} shell to the n^{th} shell ($m > n$). Suppose instead of emitting electromagnetic wave, the energy released is converted into the kinetic energy of the atom. Assume Bohr model and conservation of angular momentum are valid. If I is the moment of inertia the angular velocity of the atom about the nucleus is $4(m\text{-}n)h / kI$. Calculate k.
7. In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron from $n_1 = 6$ to n_2 = '*'. What is the value of '*'.
8. A Bohr hydrogen atom undergoes a transition $n = 5$ to $n = 4$ and emits a photon of frequency f. Frequency of circular motion of electron in $n = 4$ orbit is f_4. The ratio f/f_4 is found to be 18/5m. State the value of m.
9. The average lifetime for the $n = 3$ excited state of a hydrogen-like atom is 4.8×10^{-8} sec and that for the $n = 2$ state is 12.8×10^{-8} sec. The ratio of the average number of revolutions made in the $n = 2$ state to the average number of revolutions made in the $n = 3$ state before any transitions can take place from these states is.
10. An electron in hydrogen atom jumps from n_1 state to n_2 state, where n_1 and n_2 represent the quantum number of two states. The time period of revolution of electron in initial state is 8 times that in final state. Then the ratio of n_1 and n_2 is.

Archives

Solutions on page 4.94

Fill in the Blanks Type

1. To produce characteristic X-rays using a tungsten target in an X-ray generator, the accelerating voltage should be greater than ______ volt and the energy of the characteristic radiation is ______ eV.
(The binding energy of the innermost electron in tungsten is 40 keV.) **(IIT-JEE,1983)**
2. When the number of electrons striking the anode of an X-ray tube is increased, the ______ of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased, the cut-off wavelength of the emitted X-rays ______. **(IIT-JEE, 1986)**
3. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen-like element is 0.32 Å. The wavelength of the K_β line emitted by the same element will be ______. **(IIT-JEE, 1990)**
4. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as a dopant in silicon (relative dielectric constant = 12) is ______ Å. **(IIT-JEE, 1991)**
5. In the X-ray tube, electrons accelerated through a potential difference of 15,000 volt strike a copper target. The speed of the emitted x-ray inside the tube is ______ ms^{-1}. **(IIT-JEE, 1992)**
6. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is ______ **(IIT-JEE, 1992)**
7. The wavelength of K_α X-rays produced by an x-ray tube is 0.76 Å. The atomic number of the anode material of the tube is ______. **(IIT-JEE, 1996)**
8. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is $ms^{-1}$is ______. **(IIT-JEE, 1997)**

Multiple Choice Questions with One Correct Answer Type

1. The shortest wavelength of X-rays emitted from an X-ray tube depends on **(IIT-JEE, 1982)**
   a. the current in the tube
   b. the voltage applied to the tube
   c. the nature of the gas in the tube
   d. the atomic number of the target material
2. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be **(IIT-JEE, 1983)**
   a. 60
   b. 32
   c. 4
   d. 64
3. Consider the spectral line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength is produced by **(IIT-JEE, 1983)**
   a. hydrogen atom
   b. deuterium atom
   c. singly ionized helium
   d. doubly ionized lithium
4. The X-ray beam coming from an X-ray tube will be **(IIT-JEE, 1985)**
   a. monochromatic

b. having all wavelengths smaller than a certain maximum wavelength
c. having all wavelengths larger than a certain minimum wavelength
d. having all wavelengths lying between a minimum and a maximum wavelength

5. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.02$ nm. The energy difference between K and L levels in this atom is about **(IIT-JEE, 1997)**
a. 0.51 MeV
b. 1.2 MeV
c. 59 MeV
d. 13.6 MeV

6. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is **(IIT-JEE, 1997)**
a. 1.51
b. 13.6
c. 40.8
d. 122.4

7. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from **(IIT-JEE, 1998)**
a. 0 and ∞
b. λ_{min} to ∞; where $\lambda_{min} > 0$.
c. 0 to λ_{max}; where $\lambda_{max} < \infty$.
d. λ_{min} to λ_{max}; where $0 < \lambda_{min} < \infty$

8. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (giving in terms of the Rydberg constant R for the hydrogen atom) equal to **(IIT-JEE, 2000)**
a. $9/(5R)$
b. $36/(5R)$
c. $18/(5R)$
d. $4/R$

9. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true? **(IIT-JEE, 2000)**
a. Its kinetic energy increases and its potential and total energies decrease.
b. Its kinetic energy decreases, potential energy increases, and its total energyremains the same.
c. Its kinetic and total energies decrease and its potential, energy increases.
d. Its kinetic, potential and total energies decrease.

10. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have -72.5 keV energy. X-rays emitted by the tube contain only **(IIT-JEE, 2000)**
a. a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of -0.155 Å
b. continuous X-ray spectrum (Bremsstrahlung) with all wavelengths
c. the characteristic X-ray spectrum of tungsten
d. a continuous X-ray spectrum (Bremsstrahlung) with a minimumwavelength of -0.155 Å and the characteristic X-ray spectrum of tungsten

11. The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultra violet radiation. Infrared radiation will be obtained in the transition **(IIT-JEE, 2001)**
a. $2 \to 1$
b. $3 \to 2$
c. $4 \to 2$
d. $5 \to$

12. The intensity of X-rays from a Coolidge tube is plotted against wavelength λ as shown in Fig. 4.66. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased, **(IIT-JEE, 2001)**

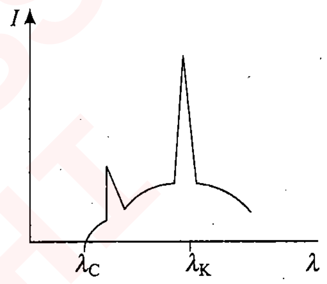


Fig. 4.66

a. $\lambda_K - \lambda_C$ increases
b. $\lambda_K - \lambda_C$ decreases
c. λ_K increases
d. λ_K increases

13. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then, the number of electrons striking the target per second is **(IIT-JEE, 2002)**
a. 2×10^{16}
b. 5×10^{6}
c. 1×10^{17}
d. 4×10^{15}

14. A hydrogen atom and a Li^+ ion are both in second excited state. If ℓ_H and ℓ_{Li} are their respective electronic angular momenta and E_H and E_{Li} their respective energies, then **(IIT-JEE, 2002)**
a. $\ell_H > \ell_{Li}$ and $|E_H| > |E_{Li}|$
b. $\ell_H = \ell_{Li}$ and $|E_H| < |E_{Li}|$
c. $\ell_H = \ell_{Li}$ and $|E_H| > |E_{Li}|$
d. $\ell_H = \ell_{Li}$ and $|E_H| < |E_{Li}|$

15. The electric potential between a proton and an electron is given by $V = V_0 \ln r/ r_0$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number? **(IIT-JEE, 2003)**
a. $r_n \propto n$
b. $r_n \propto 1/n$
c. $r_n \propto n^2$
d. $r_n \propto 1/n^2$

16. If the atom ${}_{100}Fm^{257}$ follows the Bohr model and the radius of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n. **(IIT-JEE, 2003)**
a. 100
b. 200
c. 4
d. 1/4

17. K_α wavelength emitted by an atom, of atomic number $Z = 11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ. **(IIT-JEE, 2005)**
a. $Z = 6$
b. $Z = 4$
c. $Z = 11$
d. $Z = 44$

18. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of a microsecond, another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector? **(IIT-JEE, 2005)**

a. One photon of energy 10.2 eV and an electron of energy 1.4 eV.
b. Two photons of energy of 1.4 eV.
c. Two photons of energy 10.2 eV.
d. One photon of energy 10.2 eV and another photon of 1.4 eV.

19. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **(IIT-JEE, 2007)**

a. 802 nm b. 823 nm
c. 1882 nm d. 1648 nm

20. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **(IIT-JEE, 2007)**

a. $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ b. $\lambda_0 = \dfrac{2h}{mc}$
c. $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ d. $\lambda_0 = \lambda$

21. Which one of the following statement is wrong in the context of X-rays generated from an X-ray tube? **(IIT-JEE, 2008)**

a. Wavelength of characteristic X-raysdecreases when the atomic number of the target increases.
b. Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.
c. Intensity of the characteristic X-rays depends on the electric power given to the X-ray tube
d. Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

22. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is: **(IIT-JEE, 2011)**

a. 1215 Å b. 1640 Å
c. 2430 Å d. 4687 Å

Multiple Choice Questions with One or More than One Correct Answer Type

1. In Bohr's model of the hydrogen atom, **(IIT-JEE, 1984)**

a. the radius of the n^{th} orbit is proportional to n^2
b. the total energy of the electron in n^{th} orbit is inversely proportional to n
c. the angular momentum of the electron in an orbit is an integral multiple of $h/2\lambda$
d. the magnitude of potential energy of the electron in any orbit is greater than its KE

2. The mass number of a nucleus is **(IIT-JEE, 1986)**

a. always less than its atomic number
b. always more than its atomic number
c. sometimes equal to its atomic number
d. sometimes more than and sometimes equal to its atomic number

3. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation **(IIT-JEE, 1986)**

a. the intensity increases
b. the minimum wavelength increases
c. the intensity remains unchanged
d. the minimum wavelength decreases

4. The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are **(IIT-JEE, 1998)**

a. $n_1 = 4, n_2 = 2$ b. $n_1 = 8, n_2 = 2$
c. $n_1 = 8, n_2 = 1$ d. $n_1 = 6, n_2 = 3$

Assertion and Reasoning Type

In each of the questions, assertion (A) is given by corresponding statement of reason (R) of the statements. Mark the correct answer. **(IIT-JEE, 2007)**

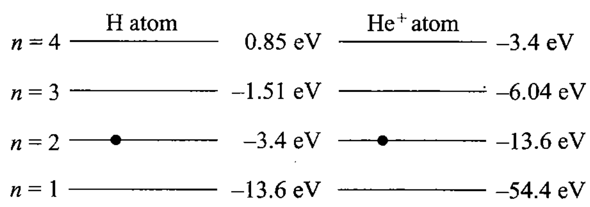
a. If both Statement I and Statement II are true and Statement II is correct explanation of the Statement I.
b. If both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I.
c. If Statement I is true, but Statement II is false.
d. If Statement I is false but Statement II is true.

1. Statement I: If the accelerating potential in an X-ray tube is increased, the wavelength of the characteristic X-rays does not change.
Statement II: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy. **(IIT-JEE, 2007)**

Comprehension Type

For Problems 1–3

In a mixture of H–He$^+$ gas (He$^+$ is singly ionized He atom), H atoms and He$^+$ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He$^+$ ions (by collisions). Assume that the Bohr model of atom is exactly valid. **(IIT-JEE, 2008)**



1. The quantum number n of the state finally populated in He$^+$ ions is

a. 2 b. 3
c. 4 d. 5

2. The wavelength of light emitted in the visible region by He$^+$ ions after collisions with H atoms is

a. 6.5×10^{-7} m b. 5.6×10^{-7} m
c. 4.8×10^{-7} m d. 4.0×10^{-7} m

3. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He$^+$ ion is

a. $\frac{1}{4}$ b. $\frac{1}{2}$
c. 1 d. 2

For Problems 4–6

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **(IIT-JEE, 2010)**

4. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is

a. $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ b. $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

c. $n\left(\frac{h^2}{8\pi^2 I}\right)$ d. $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

5. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ Js)

a. $2.76 \times 10^{-46}\,\text{kg m}^2$ b. $1.87 \times 10^{-46}\,\text{kg m}^2$
c. $4.67 \times 10^{-47}\,\text{kg m}^2$ d. $1.17 \times 10^{-47}\,\text{kg m}^2$

6. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u $5/3 \times 10^{-27}$ kg, is close to

a. $2.4 \times 10^{-10}\,\text{m}$ b. $1.9 \times 10^{-10}\,\text{m}$
c. $1.3 \times 10^{-10}\,\text{m}$ d. $4.4 \times 10^{-11}\,\text{m}$

ANSWERS AND SOLUTIONS

Subjective Type

1. Let T_1 be the initial temperature and T_2 be the increased temperature of the black body.

According to Stefan's law,

$$\left(\frac{T_2}{T_1}\right)^4 = 81 = (3)^4$$

$$T_2 = 3T_1 \quad \text{(i)}$$

Also, $\lambda_1 T_1 = \lambda_2 T_2$

$$\lambda_2 = \frac{\lambda_1 \times T_1}{T_2} = \frac{9000 \times T_1}{3T_1} = 3000\,\text{Å}$$

Now, $\frac{h\nu}{\lambda_2} - W = eV_0$ or $\frac{h\nu}{\lambda_2} - W = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

Solving, we get, $W = 2.25\,\text{eV}$

2. $v = v_0 \frac{Z^2}{n^3} \Rightarrow \frac{Z^2}{n^3} = 2$ (i)

$E = E_0 \frac{Z^2}{n^2} \Rightarrow \frac{Z^2}{n^2} = 4$ (ii)

Solving (i) and (ii), $n = 2, Z = 4$

$$L = mvr = \frac{nh}{2\pi},\ \Delta L = \tau \Delta t = \Delta n \frac{h}{2\pi}$$

$$\tau = \frac{\Delta n}{\Delta t} \times \frac{h}{2\pi} = \frac{1}{7 \times 10^{-9}} \times \frac{1}{2} \times 2.1 \times 10^{-34} = \frac{2.1}{14} \times 10^{-25}$$

$$\tau \times 10^{27} = \frac{2.1}{14} \times 10^{-25} \times 10^{27} = 15$$

3. $\frac{mv^2}{r} = eVB$

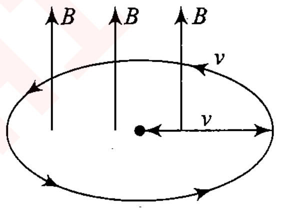


Fig. 4.67

or $\frac{v}{r} = \frac{eB}{m}$ (i)

Using Bohr quantization condition,

$$mvr = \frac{nh}{2\pi} = nh, \quad n \to \text{integer}$$

or $vr = \frac{nh}{m}$ (ii)

a. $r^2 = \frac{nh}{Be}$

or $r = \sqrt{n}\sqrt{\frac{h}{Be}} = \sqrt{n}\, a_0$ (iii)

b. $T\,(\text{kinetic energy}) = \frac{1}{2}mv^2$

$$= \frac{m^2v^2}{2m} = \frac{1}{2m} \times \frac{n^2h^2}{r^2}$$

$$= \frac{1}{2m} \times 2hBe = \frac{1}{2}nh\left(\frac{Be}{m}\right) \quad \text{(iv)}$$

c. The time period of revolution of the electron,

$$\tau = \frac{2\pi r}{v} = \frac{2\pi m}{Be}$$

The current, $i = \frac{e}{\tau} = \frac{e^2 B}{2\pi m}$ the magnetic moment, $\mu = 1$.

$$\text{Area} = \pi r^2 \times \frac{e^2 B}{2\pi m} = \frac{e}{2m} nh$$

The potential energy of interaction

$$U = -\vec{\mu}\vec{B} = \frac{1}{2} nh\left(\frac{eB}{m}\right) \quad \text{(v)}$$

d. The total energy,

$$E = T + U = nh\left(\frac{eB}{m}\right) \text{ which is quantized.}$$

e. The total magnetic flux through the n^{th} orbit,

$$\phi = \pi r^2 B = \pi\left(\frac{nh}{Be}\right)B = \frac{nh}{2e} \quad \text{(vi)}$$

4. $$\frac{1}{\lambda} = RZ\left(\frac{1}{(n_f)^2} - \frac{1}{(n_i)^2}\right)$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{9} - \frac{1}{n^2}\right)}{8/9},$$

where n is the principal quantum number of the initial excited state.

Angular width $\theta = \dfrac{2\lambda}{d}$

$$\frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} = \frac{2}{25} = \frac{n^2 - 9}{8n^2} \Rightarrow n = 5$$

$$\frac{2\lambda_1}{d} = \theta_1, \quad \frac{1}{\lambda_1} = \frac{2}{d \times \theta_1} = \frac{2}{6.4} \times 10^7 \text{ m}^{-1}$$

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{9} - \frac{1}{n^2}\right)$$

$$\frac{2}{6.4} \times 10^7 = 1.097 \times 10^7 z^2\left(\frac{1}{9} - \frac{1}{25}\right)$$

$$z^2 = \frac{2}{6.4} \times \frac{225}{16 \times 1.097} \Rightarrow z = 2$$

5. $$E = -(13.6\text{ eV})Z^2\left(\frac{1}{(Z+2)^2} - \frac{1}{Z^2}\right)$$

$$= -13.6 \times \left(\frac{Z^2 - (Z+2)^2}{(Z+2)^2}\right) = \frac{4(Z+1) \times 13.6}{(Z+2)^2} \text{eV} \quad \text{(i)}$$

[Now energy of electron is $K = \dfrac{h^2}{2\lambda^2 m}$

Solving $K = 6$ eV

So, $$\frac{4(Z+1) \times 13.6}{(Z+2)^2} = 6 + 4.2 = 10.2 \text{ eV}$$

$$\frac{Z+1}{(Z+2)^2} = \frac{3}{16} \Rightarrow (Z-2)(3Z+2) = 0$$

So, the value of $Z = 2$ (neglecting the negative/fractional value)

6. Using COM: $mu = 2mv$

$$v = \frac{u}{2}, \Delta E = \frac{1}{2}mu^2 - 2 \times \frac{1}{2}m\left(\frac{u}{2}\right)^2$$

$$\Rightarrow \quad \Delta E = \frac{1}{4}mu^2 = \frac{1}{4} \times 1.66 \times 10^{-27} \times (6.24 \times 10^4)^2$$

$$= 13.6\left(\frac{1}{l^2} - \frac{1}{n^2}\right)$$

$$= \frac{1.66 \times 10^{-27} \times (6.24 \times 10^4)^2}{4 \times 1.6 \times 10^{-19}} \Rightarrow n = 2$$

7. We have,

$$10.2 = W + K_{\max,1} \quad \text{(i)}$$

and $$10.2\, Z^2 = W + K_{\max,2} \quad \text{(ii)}$$

Also $$\lambda_{\text{de Broglie}} = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = 2.3 \Rightarrow K_2 = 5.25\, K_1 \quad \text{(iii)}$$

Also, $10.2\, Z^2$ = energy corresponding to longest wavelength of the Lyman series $= 3 \times 13.6$

$\Rightarrow \quad Z = 2.$

From Eqs. (i), (ii) and (iii), $W = 3$ eV.

8. We know that

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For first line of Lyman series in hydrogen atom,

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \quad \text{(i)}$$

For second line of Balmer series of hydrogen-like ion X,

$$\frac{1}{\lambda_2} = Z^2 R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3Z^2 R}{16} \quad \text{(ii)}$$

Given that $\lambda_1 = \lambda_2$

$$\therefore \quad \frac{3R}{4} = \frac{3Z^2 R}{16} \quad \text{or} \quad Z = 2$$

Thus, the ion X is singly ionized helium atom.

Energy of n^{th} state of ion X is given by

$$E_X = -\frac{13.4}{n^2} \times Z^2$$

$$\therefore \quad (E_X)_1 = -\frac{13.4 \times 4}{1} = -54.4 \text{ eV}$$

$$(E_X)_2 = -\frac{13.4 \times 4}{4} = -13.6 \text{ eV}$$

$$(E_X)_3 = -\frac{13.4 \times 4}{9} = -6.04 \text{ eV}$$

$$(E_X)_4 = -\frac{13.4 \times 4}{16} = -3.52 \text{ eV}$$

9. $$\lambda_{C_1} = \frac{h}{eV} = \frac{12375}{10 \times 10^3} \text{ Å} = 1.2375 \text{ Å} \quad \text{(i)}$$

$$\lambda_{C_2} = 0.61875 \text{ Å} \quad \text{(ii)}$$

$$\frac{1}{\lambda_{K_\alpha}} = (Z-1)^2\left\{\frac{1}{1} - \frac{1}{4}\right\} \times 10^7 \quad \text{(iii)}$$

It is given

$$3\times(\lambda_{K_\alpha}-1.2375)=\lambda_{K_\alpha}-0.61875$$

$$\Rightarrow \quad \lambda_{K_\alpha}=1.54338\text{ Å} \quad \text{(iv)}$$

Putting this value in (iii),

$$Z-1=29$$

$$\Rightarrow \quad Z=30$$

10. The wavelength of the K_α radiation from Nb is

$$\lambda=\frac{4hc}{3R}\frac{1}{(z-1)^2}=\frac{4hc}{3R}\frac{1}{40^2}$$

$$E_{K_\alpha}=16.3\text{ KeV}$$

$$Z^2_{gas}=1000 \Rightarrow Z_{gas}=32$$

If the momentum of the electrons is doubled, then the KE is increased by 4 times.

Therefore, the minimum energy of the X-rays is $(16.3+3\times 2.7)$ keV $=24.5$ keV.

Since $\quad E_{K_\alpha}\propto(Z-1)^2$

$$\left(\frac{Z-1}{40}\right)^2=\left(\frac{24.5}{16.3}\right)\Rightarrow Z=50$$

The voltage at which the tube should be operated should be greater than 24.5 kV.

The number of X-ray photons incident $=100\text{ mWm}^{-2}$

$$\Rightarrow I=\frac{100\times10^{-3}\times10^{19}}{1.6\times24.5\times10^3}\text{ m}^{-2}$$

Number of ions produced in 1 s $(n)=1\%$ of $\dfrac{I}{C}$

$$=\frac{100\times10^{-3}\times10^{-2}\times10^{19}}{1.6\times24.5\times10^3\times3\times10^8}\text{ m}^{-3}\approx 850\text{ per m}^{-3}$$

11. We know the orbital energy of an electron revolving in n^{th} orbit is given by

$$E_n=-\frac{13.6}{n^2}\text{ eV}$$

where n is the principal quantum number.

When $\quad n=1, \quad E_1=-13.6\text{ eV};$

$n=2, \quad E_2=-3.4\text{ eV};$

$n=3, \quad E_3=-1.51\text{ eV}.$

Energy needed by an electron to go from K to L level is $(13.6-3.4)=10.2$ eV and that required to go from K to M level is $(13.6-1.51)=12.09$ eV. The corresponding quantum numbers are $n=2$ and $n=3$, respectively. Hence, electron will be raised to principal quantum numbers 2 and 3 corresponding to energies 10.20 eV and 12.09 eV, respectively.

$n=2$ and 3

When electrons are coming back from $n=2$ and $n=3$ to the ground state, i.e., $n=1$. That is the case of Lyman series. Hence,

$$\frac{1}{\lambda_1}=R\left[\frac{1}{1^2}-\frac{1}{2^2}\right]=\frac{3R}{4}$$

$$\Rightarrow \quad \lambda_1=\frac{4}{3R}=\frac{4}{3\times(10.97\times10^6)}=1216\text{ Å}$$

and $\quad \dfrac{1}{\lambda_2}=R\left[\dfrac{1}{1^2}-\dfrac{1}{3^2}\right]=\dfrac{8R}{9}$

$$\lambda_2=\frac{9}{8R}=\frac{9}{8\times(10.97\times10^6)}=1020\text{ Å}$$

(Rounding off to nearest integer)

12. The short limit of Balmer series is given by

$$\bar{\nu}=1/\lambda=R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)=R/4$$

$$\therefore \quad R=4/\lambda=(4/3646)\times10^{10}\text{m}^{-1}$$

Further, the wavelengths of the K_α series are given by the relation

$$\bar{\nu}=\frac{1}{\lambda}=R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$$

The maximum wave number corresponds to $n=\infty$ and therefore, we must have

$$\bar{\nu}=\frac{1}{\lambda}=R(Z-1)^2$$

or $\quad (Z-1)^2=\dfrac{1}{R\lambda}=\dfrac{3646\times10^{-10}}{4\times1\times10^{-10}}=911.5$

$$\therefore \quad Z=31.2\cong31$$

Thus, the atomic number of the element concerned is 31. The element having atomic number $Z=31$ is Gallium.

13. The reduced mass of the system (electron and positron) is given by

$$\mu=\frac{mm}{m+m}=\frac{m}{2}$$

where $m=$ mass of electron or positron.

The radius of first Bohr's orbit is given by

$$r_1=\frac{h^2}{4\pi^2Ke^2(m/2)}=2\times\text{radius of first Bohr's orbit of hydrogen}$$

$$=2\times0.529=1.058\text{ Å}$$

Energy of first Bohr's orbit is given as

$$E_1=\frac{2\pi^2K^2z^2e^4(m/2)}{h^2}$$

$$=-\frac{1}{2}\times\text{ground state energy of hydrogen}$$

$$=-\frac{1}{2}\times13.6\text{ eV}$$

$$=-6.8\text{ eV (Binding energy)}$$

The wavelength of Lyman series is given by

$$\frac{1}{\lambda}=R_\mu\left[\frac{1}{1^2}-\frac{1}{n^2}\right]=R_\mu\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\lambda=\frac{4}{3R_\mu}$$

$$R_\mu=\frac{2\pi^2K^2e^4\mu}{3}=\frac{R}{m_e}\mu=\frac{1}{2}R$$

$$=10967800\times\frac{1}{2}=5483900\text{ m}^{-1}$$

or $\quad \lambda=\dfrac{4}{3\times5483900}\text{ m}$

$$=2430\text{ Å}$$

14. The reduced mass of the system is given by

$$\mu = \frac{(207m)(1836m)}{(207m + 1836m)} = 186m$$

The radius of first orbit is given by

$$r_1 = \frac{h^2}{4\pi^2 Ke^2(186m)}$$

$$= \frac{1}{186} \times \text{radius of first Bohr orbit of hydrogen atom}$$

$$= \frac{1}{186} \times 0.529 = 0.002844 \text{ Å} \quad \text{(i)}$$

From Bohr's theory, the ground state energy for hydrogen-like atom with $Z = 1$ is given by

$$E_1 = \frac{2\pi^2K^2e^4\mu}{h^2} = \frac{2\pi^2K^2e^4(186m)}{h^2}$$

$$= -186 \times 13.6 \text{ eV}$$

$$= 2530 \text{ eV} \quad \text{(ii)}$$

Hence, the binding energy is 2530 eV.

The wavelength of the Lyman lines are given by

$$\frac{1}{\lambda} = R_\mu\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad n = 2, 3, 4, \ldots$$

where R_μ = Rydberg constant for mesic atom.

For first line, $\frac{1}{\lambda} = R_\mu\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ or $\lambda = \frac{4}{3R_\mu}$

Now, $R_\mu = \frac{2\pi^2K^2e^4\mu}{ch^3} = R\frac{\mu}{m}$

or $= 186\,R$

or $= 186 \times 10967800 \text{ m}^{-1}$

Substituting the value of R_μ in equation (iii), we get

$$\lambda = \frac{4}{3 \times 186 \times 109687} \text{ m} = 653.6 \text{ Å}$$

15. The frequency of revolution of an electron in hydrogen atom in first orbit is given by

$$f_1 = \frac{4\pi^2K^2e^4m}{h^3}$$

Here, in case of a π-meson the mass of electron is replaced by the reduced mass as

$$\mu = \frac{m_p(273)m}{m_p + 273m}$$

Thus, the frequency of revolution of π-meson will become

$$f_1 = \frac{4\pi^2Ke^4m_p(273m)}{h^3(m_p + 273m)}$$

$$= \frac{4 \times (3.14)^2 \times 9 \times 10^9 (1.6 \times 10^{-19})^4 \times (1.67 \times 10^{-27})(273 \times 9.1 \times 10^{-31})}{(6.63 \times 10^{-34})^3(1.67 \times 10^{-27} + 273 \times 9.1 \times 10^{-31})}$$

$$= 1.77 \times 10^{18} \text{ s}^{-1}$$

Thus, in the life of 10^{-8} s, number of revolutions made by the π-meson is given as

$$N = f_1 \times \Delta t$$

or $= 1.77 \times 10^{18} \times 10^{-8}$

or $= 1.77 \times 10^{10}$ revolutions

16. According to given situation for an electron revolving in an n^{th} orbit, the potential energy is given as

$$U = -\frac{ke^2}{3r_n^3} \quad \text{(i)}$$

The centripetal force on the electron due to this is given as

$$F = -\frac{dU}{dr} = \frac{ke^2}{r_n^4} \quad \text{(ii)}$$

If in n^{th} orbit electron revolves at speed v_n, then we have

$$\frac{mv_H^2}{r_n} = \frac{ke^2}{r_n^4} \quad \text{or} \quad mv_n^2 = \frac{ke^2}{r_n^3} \quad \text{(iii)}$$

From Bohr's second postulate, we have

$$mv_n r_n = \frac{nh}{2\pi} \quad \text{(iv)}$$

From Eqs. (iii) and (iv), we have

$$v_n = \frac{nh}{2\pi m r_n} \quad \text{and} \quad m\left(\frac{nh}{2\pi m r_n}\right)^2 = \frac{ke^2}{r_n^3}$$

or $r_n = \frac{4\pi^2ke^2m}{n^2h^2}$ and $v_n = \frac{n^3h^3}{8\pi^3km^2e^2}$

Now energy in n^{th} orbit is equal to negative of K.E.

$$E_n = \frac{-1}{2}mv_n^2 = -\frac{ke^2}{2r_n^3}$$

or $$E_n = -\frac{1}{2}ke^2\left(\frac{n^2h^2}{4\pi^2ke^2m}\right)^3$$

or $$E_n = \frac{-n^2h^2}{128k^2e^4m^3}$$

17. The energy of the electron in the n^{th} state of He^+ ion of atomic number Z is given by

$$E_n = -(13.6 \text{ eV})\frac{Z^2}{n^2}$$

For He^+ ion, $Z = 2$, therefore

$$E_n = -\frac{(13.6 \text{ eV}) \times (2)^2}{n^2}$$

$$= -\frac{54.4}{n^2} \text{ eV} \quad \text{(i)}$$

The energies E_1 and E_2 of the two emitted photons, in eV, are

$$E_1 = \frac{12431}{1085} \text{ eV} = 11.4 \text{ eV}$$

and $E_2 = \frac{12431}{304}\text{ eV} = 40.9\text{ eV}$

Thus, total energy $E = E_1 + E_2 = 11.4 + 40.9 = 52.3$ eV
Let n be the principal quantum number of the excited state. Using Eq. (i), we have for the transition from $n = n$ to $n = 1$.

$$E = -(54.4\text{ eV})\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

But $E = 52.3$eV, therefore

$$52.3\text{ eV} = 54.4\text{ eV} \times \left(1 - \frac{1}{n^2}\right)$$

or $1 - \frac{1}{n^2} = \frac{52.3}{54.4} = 0.96$

$\Rightarrow$ $n^2 = 25$ or $n = 5$.

The energy of the incident electron = 100 eV (given). The energy supplied to He^+ ion = 52.3 eV. Therefore, the energy of the electron left after the collision = 100 − 52.3 = 47.7 eV.

Objective Type

1. a. $\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

$$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda_2} = R\left[\frac{3}{16}\right] \quad \text{or} \quad \lambda_2 = \frac{16}{3R}$$

Again, $\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$$\frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_1} = \frac{5R}{36} \quad \text{or} \quad \lambda_1 = \frac{36}{5R}$$

$\therefore$ $\frac{\lambda_1}{\lambda_2} = \frac{36}{5R} \times \frac{3R}{16} = \frac{27}{20}$

or $\lambda_1 = \frac{27}{20} \times 4861$ Å

2. d. If the electron jumps from n_3 level to n_1 level, then photon of energy 12.1 eV is emitted. If the electron jumps from n_2 level to n_1 then 10.2 eV photon is emitted. Clearly, these transitions are possible in minimum two atoms and maximum three atoms.

3. c. The electron is still in the state $n = 2$. It has to reach the ground state by emitting a photon.

4. d $T^2 \propto R^3$

$$\frac{T_R}{T_{4R}} = \left(\frac{R}{4R}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$$

5. a. $-13.6 - (-10.2) = -3.4$ eV

$$\frac{-13.6}{n^2} = -3.4 \quad \text{or} \quad n^2 = \frac{13.6}{3.4} = 4$$

or $n = 2$

Increase in angular momentum $= \frac{2h}{2\pi} - \frac{h}{2\pi} = \frac{h}{2\pi}$

$$= \frac{6.625 \times 10^{-34}}{2 \times 3.14}\text{ J s}$$

$$= 1.05 \times 10^{-34}\text{ J s}$$

6. b. Frequency $= R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)c$

Hence, $\lambda = \frac{4}{R}$

7. a. $\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{\lambda_{min.}}{\lambda_{max.}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{\infty}\right)} = \frac{5}{9}$$

8. d $mr\omega^2 = \frac{he^2}{r^2}$ or $\omega^2 = \frac{ke^2}{mr^3}$

or $4\pi^2 f_n^2 = \frac{ke^2}{mr^3}$ or $f_n^2 = \frac{ke^2}{4\pi^2 mr^3}$

But, $r = \frac{1}{k} \times \frac{n^2h^2}{4\pi^2 me^2}$

$\therefore$ $f_n^2 = \frac{ke^2(k \times 4\pi^2 me^2)^2}{4\pi^2 m(n^2h^2)^3}$

or $f_n^2 = \frac{k^4e^8(4\pi^2)^2 m^2}{(n^2h^2)^3}$

or $f_n = \frac{4\pi^2 k^2 me^4}{n^3h^3}$

Again, $h\nu = k^2\frac{2\pi^2 me^4}{h^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

or, $\nu = k^2\frac{2\pi^2 me^4}{h^3}\left[\frac{n^2 - (n-1)^2}{n^2(n-1)^2}\right]$

or, $$v = k^2 \frac{2\pi^2 me^4}{h^3}\left[\frac{(2n-1)}{n^2(n-1)^2}\right]$$

If n is very large, then

$$v = k^2 \frac{2\pi^2 ke^4}{h^3} \times \frac{2n}{n^4}$$

or $$v = \frac{4\pi^2 k^2 me^4}{n^3 h^3} = f_n$$

9. c. $$v = \frac{4\pi^2 k^2 me^4}{n^3 h^3}$$

$$v \propto \frac{1}{n^3}$$

10. d. The first three transitions from the left fall in the Lyman series of the hydrogen spectrum which corresponds to ultraviolet radiation.

The fourth transition falls in the Balmer series of the spectrum which corresponds to the visible light emission.

The last transition falls in the Paschen series which corresponds to the infrared radiation.

Thus, frequencies of the last two transitions are closer to each other on the extreme left of the frequency spectrum whereas the frequencies of the first three transitions are closer to one another and fall on the right corner of the frequency spectrum.

The spectrum of the transitions is thus best represented in diagram (d).

11. c. For emission of a photon with greater wavelength, energy gap should be less.

Blue light falls in the Balmer series and it is obtained when the atom makes a transition from E_4 to E_3. Red light also falls in the Balmer series and it has a lower frequency compared to blue light. By quantum theory of radiation, the energy change E is proportional to the frequency of electromagnetic radiation f by $E = hf$. Thus, red light is associated with a smaller energy change from a lower energy level (compared to E_4) to the first excited state E_2. Hence, the only possible transition that results in the emission of red light is the E_3 to E_2 transition.

12. a. $mv_n^2 = K/r_n$. Since modified m is half and modified r_n is double, v_n remains the same as in H-atom.

Note: ***Positronium is an atom in which an electron (e^-) and a positron (e^+) go around their center of mass. Bohr's conditions hold for it, as used in hydrogen atom, but the mass m_e of the electron is replaced by the modified mass $\mu = \frac{m_e m_p}{m_e + m_p}$, where m_e is the positronium mass, which is equal to m. With this, one may treat the electron going round the positron and apply the equations used for hydrogen atom case.***

13. b. From the graphs given, atom in graph B will absorb most of the energy W from the electron and re-radiate, in all directions, radiation of shortest wavelength when the atom returns to its ground state.

14. a Momentum of the recoiled hydrogen atom

= momentum of the emitted photon

$$= \frac{h}{\lambda} = hR\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$= 6.6 \times 10^{-34} \times 10^{+7}\left(\frac{1}{1} - \frac{1}{16}\right)$$

$$= 6.5 \times 10^{-27}\ \text{kg m s}^{-1}$$

15. c. Centripetal acceleration $= mv^2/r$

Further, as n increases, r also increases. Therefore, centripetal acceleration for $n = 2$ is less than that for $n = 1$. So, statement (i) is wrong. Statements (ii) and (iii) are correct.

16. c. As the number of orbit increases, the velocity decreases. The potential energy becomes less negative, i.e., PE increases while KE decreases.

17. b. KE $\propto \frac{1}{n^2}$ and PE $\propto \frac{1}{n^2}$

18. d. $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{1^2} - \frac{1}{5^2}\right)$

Hence, λ is minimum when Z is maximum.

19. a. In case of Bohr's model of hydrogen atom,

$$\text{Frequency} = \frac{v}{2\pi r}$$

Here, $v \propto \frac{1}{n}$ and $r \propto n^2$

$$\therefore \quad \text{Frequency} \propto \frac{1}{n^3}$$

20. c. $$\frac{mv^2}{a_o} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{a_0^2}$$

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o a_o m}}$$

21. b. Possible transitions are:

$4 \to 3$, $4 \to 2$, $4 \to 1$,
$3 \to 2$, $3 \to 1$, and
$2 \to 1$

22. c. This is Bohr's postulate.

23. c. Potential energy $= -C/r^2$ and total energy $= -Rhc/n^2$. With higher orbit, both r and n increase. So, both become less negative; hence both increase.

24. b $$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36};\ \lambda = \frac{36}{5R}$$

25. a. In this case, there is the widest energy gap.

26. b $v = \frac{1}{137}\frac{c}{n}$ or $v \propto \frac{1}{n}$

Since v is reduced to one-third, therefore

$n = 3$

Now, $r \propto n^2$

27. a $E_n = -3.4\text{ eV}, E_n \propto \frac{1}{n^2}$

$E_1 = -13.6\text{ eV}$

Clearly, $n = 2$

Angular momentum

$$= \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi} = 2.11 \times 10^{-34}\text{ Js}$$

28. d $\lambda \propto \frac{1}{Z^2}$

Now, $\lambda_{Na} = \frac{1216}{11 \times 11} \approx 10\text{ Å}$

29. a. $13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{ eV} = 1.9\text{ eV}$

30. c. $\frac{13.6}{4}\text{ eV} = 3.4\text{ eV}$

31. a. Radius of first orbit, $r \propto 1/Z$. For doubly ionized lithium, Z will be maximum. Hence, for doubly ionized lithium r will be minimum.

32. b. For third excited state, $n = 4$

$$r_n = r_0\frac{n^2}{2}$$

or $r_1 = 0.5 \times \frac{4 \times 4}{2}\text{ Å} = 4\text{ Å}$

33. a. Series limit means the shortest possible wavelength (maximum photon energy) and first line means the largest possible wavelength (minimum photon energy) in the series.

$$v = C\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{(where } C \text{ is a constant)}$$

For series limit of Lyman series:

$n = 1, m = \infty \Rightarrow v_1 = C$

For first line of Lyman series:

$n = 1, m = 2 \Rightarrow v_2 = 3C/4$

For series limit of Balmer series:

$n = 2, m = \infty \Rightarrow v_3 = C/4$

34. b $v = Z\left[\frac{1}{137}\frac{c}{n}\right]$

$\Rightarrow v = 4 \times \frac{1}{137} \times \frac{c}{2}$

or $v = \frac{2c}{137}$

35. d $E = -\frac{13.6}{5^2}\text{ eV}$

$E = -0.544\text{ eV}$

$E_p = -2 \times 0.544\text{ eV} = -1.088\text{ eV}$

36. d $v = 2\pi r f$

$\Rightarrow f = \frac{v}{2\pi r}$

37. d. $13.6 - 0.85 = 12.75\text{ eV}$

So, 12.75 V is the required potential difference.

38. c. Photon energy $= hf = 13.6\left[1 - \frac{1}{25}\right]\text{eV} = 13\text{ eV}$

Photon momentum = momentum of hydrogen atom:

$\Rightarrow p = \frac{hf}{c}$ or $mv = \frac{hf}{c}$

$$v = \frac{hf}{mc} = \frac{13 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 4\text{ ms}^{-1}$$

39. d. $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

or $\frac{1}{\lambda} = R\left[1 - \frac{1}{4}\right]$ or $\frac{1}{\lambda} = \frac{3R}{4}$

Hence, wavenumber $\frac{1}{\lambda} = \frac{3R}{4}$

40. a. Total energy for n^{th} level $= -\frac{13.6}{n^2}\text{ eV}$

$$E_2 - E_1 = -13.6\left(\frac{1}{4} - \frac{1}{1}\right) = \frac{13.6 \times 3}{4}$$

$= 0.75 \times 13.6\text{ eV}$

$$E_3 - E_2 = -13.6\left(\frac{1}{9} - \frac{1}{4}\right) = \frac{13.6 \times 5}{36}$$

$= 0.14 \times 13.6\text{ eV}$

$$E_4 - E_3 = -13.6\left(\frac{1}{16} - \frac{1}{9}\right) = \frac{13.6 \times 7}{144}\text{ eV}$$

$= 0.05 \times 13.6\text{ eV}$

Obviously, the difference of energy between consecutive energy levels decreases.

41. d. For Lyman series, $n_1 = 1$ and $n_2 = 2$ for first line

$$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$$

For Paschen series, $n_1 = 3$ and $n_2 = 4$ for first line

$$\therefore \quad \frac{1}{\lambda_2} = R\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = R\left[\frac{1}{9}-\frac{1}{16}\right] = \frac{7R}{144}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4/3R}{144/7R} = \frac{7}{108}$$

42. a. The wavelengths of the hydrogen spectrum could be arranged in a formula or series named after its discoverer. For ultraviolet spectrum the series is called Lyman series, for visible spectrum the Blamer series, and for infrared region we have the Paschen series.

The ultraviolet series is obtained when the energy of the atom falls from higher states to the energy level corresponding to $n = 1$. Thus, ultraviolet radiation can only be possible with transition from E_2 to E_1 out of the given transitions.

43. d. $$L = \frac{nh}{2\pi}$$

Clearly, L is constant and independent of Z.

44. d. By quantum theory of radiation, the energy change ΔE between energy levels is proportional to the frequency of electromagnetic radiation f and is given by

$$\Delta E = hf = \frac{hc}{\lambda}$$

Hence, $$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_1 - E_2}$$

45. d. Rydberg's constant determines the frequencies. We have $R \propto m$. So, modified R for positronium atom is half of H atom. Hence, frequencies are reduced to half.

46. a. $$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{3^2}\right] \Rightarrow \lambda = \frac{9}{8R}$$

Again, $$\frac{1}{\lambda'} = R\left[\frac{1}{2^2}-\frac{1}{4^2}\right] \Rightarrow \lambda' = \frac{16}{3R}$$

Now, $$\frac{\lambda'}{\lambda} = \frac{16}{3R}\times\frac{8R}{9} \quad \text{or} \quad \lambda' = \frac{128}{27}\lambda$$

47. d.

a. No, since Balmer formula was known.
b. No, since Rutherford scattering experiment was known.
c. No, since Einstein's photon theory was known.
d. Bohr chose 'allowed' energy levels $\propto 1/n^2$ and these led to angular momentum quantized as a derivation.

48. d. For Lyman series, the series limit wavelength is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right] = R \quad \text{or} \quad \lambda = \frac{1}{R}$$

For Balmer series, the series limit wavelength is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right] = \frac{R}{4} \quad \text{or} \quad \lambda' = \frac{4}{R}$$

Clearly,

$$\lambda' = 4\left[\frac{1}{R}\right] \quad \text{or} \quad \lambda' = 4\lambda$$

49. c. $$E_{max} = 13.6\text{ eV}; E_{min} = 31.6\left(1-\frac{1}{2^2}\right) = \frac{3}{4}\times 13.4\text{ eV}$$

$$\Rightarrow \frac{E_{max}}{E_{min}} = \frac{4}{3}$$

50. a. From conservation of momentum:

$$MV = \frac{h}{\lambda} = hR\left(1-\frac{1}{4}\right) \Rightarrow V = \frac{3}{4}\frac{hR}{M}$$

51. c.

a. Z was taken from X-ray scattering experiments.
b. Validity not known earlier; established by Rutherford's experiments.
c. Yes, the experiments said $r < \dfrac{Ze^2}{2\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$.
This sets upper limit for r.
d. Lower limit of r not set.

52. d. $$\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

or $$\frac{f}{c} = R\left[\frac{1}{4}-\frac{1}{16}\right]$$

or $$f = cR\left[\frac{1}{4}-\frac{1}{16}\right]$$

$$= 3\times10^8\times10^7\times\frac{3}{16}$$

$$= \frac{9}{16}\times10^{15}\text{ Hz}$$

53. c. Volume occupied by one mole of gold

$$= \frac{197\text{ g}}{19.7\text{ gm}^{-3}} = 10\text{ cm}^3$$

Volume of one atom

$$= \frac{10}{6\times10^{-23}} = \frac{5}{3}\times10^{23}\text{ cm}^3$$

Let r be the radius of the atom. Therefore,

$$\frac{4}{3}\pi r^3 = \frac{5}{3}\times10^{23} \quad \text{or} \quad r \cong 1.5\times10^{-10}\text{ m}$$

54. c. Let the three energy levels be E_1, E_2, and E_3. The wavelengths λ_1, λ_2, and λ_3 of the spectral lines corresponding to the three energy transitions are depicted as shown in Fig. 4.68.

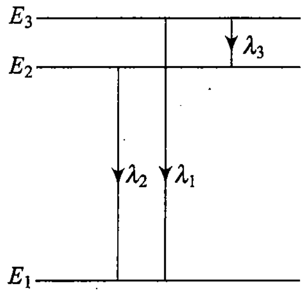


Fig. 4.68

$$E = hf = \frac{h}{\lambda} \quad \text{or} \quad E \propto \frac{1}{\lambda} \qquad (\text{given } \lambda_1 < \lambda_2 < \lambda_3)$$

Thus, for the three wavelengths, we have

$$\begin{cases} E_3 - E_2 = \dfrac{h}{\lambda_3} & \text{(i)} \\ E_2 - E_1 = \dfrac{h}{\lambda_2} & \text{(ii)} \\ E_3 - E_1 = \dfrac{h}{\lambda_1} & \text{(iii)} \end{cases}$$

Now, $\quad E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$

$$\Rightarrow \quad \frac{h}{\lambda_1} = \frac{h}{\lambda_3} + \frac{h}{\lambda_2} \quad \Rightarrow \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

55. d $\quad v_n = k\dfrac{2\pi e^2}{nh}$

We know that in cgs system $k = 1$

$$\therefore \quad v_n = \frac{2\pi e^2}{nh} \quad \Rightarrow \quad v_1 = \frac{2\pi e^2}{h}$$

So $\quad \dfrac{v_1}{c} = \dfrac{2\pi e^2}{ch}$

56. a. Barrier height

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}\,\text{J} = \frac{1}{4\pi\varepsilon_0}\frac{e}{r_e}\,\text{eV}$$

$$= \frac{9\times10^9\times1.6\times10^{-19}}{10^{-14}}\,\text{eV} = 1.44\times10^5\ \text{eV}$$

57. a. The recoil momentum of atom is same as that of photon but in opposite direction.
Hence, recoil momentum:

$$P = \frac{E}{c} = \frac{12.09\times1.6\times10^{-19}}{3\times10^8}\,\text{Ns} = 6.45\times10^{-27}\,\text{Ns}$$

Note that almost whole of the energy will be carried away by the photon because it is very light in comparison to H atom.

58. b $\quad f = cZ^2R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$$\Rightarrow \quad 2.7\times10^{15} = cz^2R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$f' = cZ^2R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

Divide and solve to get: $f = 3.2\times10^{15}$ Hz

59. a. $T^2 \propto r^3$ and $r \propto n^2 \Rightarrow T^2 \propto n^6 \Rightarrow T \propto n^3$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \quad \Rightarrow \quad 8 = \left(\frac{n_1}{n_2}\right)^3 \quad \text{or} \quad \frac{n_1}{n_2} = 2$$

Only a. satisfies the above, hence this is right choice.

60. a. $M = IA = ef\pi r^2$

$= 1.6\times10^{-19}\times10^{16}\times3.14\times(0.5\times10^{-10})^2$ A m^2

$= 1.256\times10^{-23}$ Am^2

61. d $\quad E_p = -\dfrac{ke^2}{r}, E = -\dfrac{ke^2}{2r}$

So, $\quad E_p = 2E = 2(-13.6)\,\text{eV} = -27.2$ eV.

Potential energy of electron in the ground state of Li^{2+} ion is $= -3^2\times27.2$ eV or -244.8 eV.

62. c. Required energy $= \left[\left(\dfrac{-13.6}{9}\right) - \left(\dfrac{-13.6}{1}\right)\right]\times9$

$$= \left[13.6 - \frac{13.6}{9}\right]9 = 8\times13.6\,\text{eV}$$

$$\text{Wavelength} = \frac{12375}{8\times13.6} = 113.7\ \text{Å}$$

63. d $\quad F = \dfrac{mv^2}{r}$

But $\quad v \propto \dfrac{1}{n}$ and $r \propto n^2$

$$\Rightarrow \quad F \propto \frac{1}{n^4}$$

64. c. The minimum energy to ionize an atom is the energy required to remove an outermost electron in the atom.

65. b. $\quad N = \Sigma 2n^2$

$N = 2(1^2 + 2^2 + 3^2 + 4^2) = 60$

66. c. $\quad E_n = -\dfrac{13.6}{n^2} \quad \Rightarrow \quad n^2 = -\dfrac{13.6}{-0.54}$

or $\quad n^2 = 25.2$ or $n = 5$(nearly)

As $v \propto 1/n$, so $v_n = \dfrac{v}{5}$

67. b $\quad U = -\dfrac{ke^2}{2R^3}, F = -\dfrac{dU}{dR} = -\dfrac{3ke^2}{2R^4}$

But, $\quad F = \dfrac{mv^2}{R} \quad \Rightarrow \quad \dfrac{mv^2}{R} = \dfrac{3ke^2}{2R^4}$

Also, $mvR = \frac{nh}{2\pi}$

Solve to get: $R = \frac{6\pi^2 ke^2 m}{n^2 h^2}$

68. a. $\frac{1}{\lambda} = 1.09\times10^7\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$

$\Rightarrow \lambda = 6.606\times10^{-7}\,\text{m} \Rightarrow 6606\,\text{Å}.$

69. c. Use $E = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r_0}$

70. a. Frequency of electron revolution:

$$f = \frac{mZ^2e^4}{4\varepsilon_0^2 n^3 h^3},$$

Put the various values to get

$$f = 6.62\times10^{15}\frac{Z^2}{n^3}$$

Now, put $Z = 1$ and $n = 1$ to get

$$f = 6.62\times10^{15}\,\text{Hz}$$

71. b. $r_n \propto n^2$

$$\frac{n'^2}{n^2} = \frac{21.2\times10^{-11}}{5.3\times10^{-11}} \text{ or } \frac{n'^2}{n^2} = 4$$

or $\frac{n'^2}{1^2} = 4$ or $n' = 2$

72. c. $\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$

or $R = \frac{36}{5\lambda} = \frac{36}{5\times6563\times10^{-10}}\,\text{m}^{-1}$

$$= \frac{36000}{5\times6563}\times10^7\,\text{m}^{-1} = 1.097\times10^7\,\text{m}^{-1}$$

73. b. L will be same for both because it does not depend upon Z. But for energy

$$(E_n)_{\text{Li}} = -\frac{Z^2\times13.6}{n^2} \text{ and } (E_n)_{\text{H}} = -\frac{13.6}{n^2}$$

Clearly, $|E_{\text{H}}| < |E_{\text{Li}}|$

74. c. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For longest wavelength, $n_1 = 2, n_2 = 3$

$$\therefore \frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \text{ or } \frac{1}{\lambda} = R\left[\frac{1}{4}-\frac{1}{9}\right]$$

$\Rightarrow \lambda = \frac{36}{5R}$ for electron,

But $\lambda \propto \frac{1}{m}$

So $\lambda' = \frac{1}{2}\times\frac{36}{5R} = \frac{18}{5R}$

75. a. $|F| = \left|\frac{-dU}{dr}\right| = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{U_0}{m}}$, which is a constant.

So, $mv_n r_n = \frac{nh}{2\pi} \Rightarrow r_n \propto n.$

76. d. $\lambda = \frac{hc}{E} = \frac{1240\text{ eV-nm}}{3.5\text{ eV}} = 354\,\text{nm}$

This wavelength is in the ultraviolet region.

77. b. $\lambda = \frac{912\,\text{Å}}{Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]}$

For singly ionized helium atom $Z = 2$.

For the wavelength to be longest, $n_1 = 1, n_2 = 2$

$$\therefore \lambda = \frac{912\,\text{Å}}{(2^2)\left[1-\frac{1}{4}\right]} = \frac{912}{3} = 304\,\text{Å}$$

78. d. We know that frequency of orbital motion:

$$f \propto \frac{1}{n^3} \text{ and given } f_1 = \frac{1}{27}f_2$$

$$\Rightarrow \left(\frac{n_2}{n_1}\right)^3 = \frac{f_1}{f_2} \Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$$

79. d. λ_{min} is found for $n = 2 \to 1$ since energy gap is maximum.

80. d. Total energy received by the atom will be 25.2 eV. 13.6 eV energy is needed to remove the electron from the attraction of the nucleus. Rest of the energy will be almost available in the form of KE of electron.

81. b. $R\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right] = RZ^2\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]$ or $Z = 2$.

82. a. $\frac{mv^2}{r} = \frac{3q^2}{4\pi\varepsilon_0 r^2} \Rightarrow mvr = \frac{3q^2}{4\pi\varepsilon_0 v}$ (i)

and $\frac{nh}{2\pi} = mvr$ (ii)

Using (i) and (ii) and putting $n = 1$

$$\frac{h}{2\pi} = \frac{3q^2}{4\pi\varepsilon_0 v} \Rightarrow v = \frac{3q^2}{2\varepsilon_0 h}$$

83. c. Using Bohr's theory, $\frac{mv^2}{r} = \frac{ke^2}{r^2}$

$$v^2 = \frac{ke^2}{mr} \quad \Rightarrow \quad L = mvr$$

$$\therefore \quad L = m\sqrt{\frac{ke^2}{mr}}\cdot r \quad \Rightarrow \quad L = \sqrt{mke^2 r}$$

$$\Rightarrow \quad L \propto \sqrt{r}$$

84. a. As magnetic moment, $\mu_B = \frac{1}{2} evr$

$$L = mvr \quad \Rightarrow \quad vr = \frac{L}{m}$$

$$\therefore \quad \mu_B = \frac{1}{2}\frac{eL}{m} \quad \Rightarrow \quad \frac{\mu_B}{L} = \frac{e}{2m}$$

85. c. $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \lambda = \frac{36}{5R} = \frac{36 \times 10^{-7}}{5 \times 1.097} = 6.566 \times 10^{-7}$$

$$\lambda = 6566 \text{ Å}$$

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{6.566 \times 10^{-7}} = 3.03 \times 10^{-19} \text{ J}$$

$$\therefore \quad E = 1.89 \text{ eV}$$

86. b. For shortest wavelength in Balmer series,

$$n_1 = 2; n_2 = \infty$$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{\infty}\right] \quad \text{or} \quad \lambda = \frac{4}{R}$$

For shortest wavelength in Brackett series,

$$n_1 = 4; \; n_2 = \infty$$

$$\therefore \quad \frac{1}{\lambda'} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$$

$$\text{or} \quad \lambda' = \frac{16}{R} = 4 \times \frac{4}{R} = 4\lambda.$$

87. c. Angular momentum, $mvr = n\frac{h}{2\pi} = \frac{h}{2\pi} (n = 1)$ which is independent of Z.

88. c. First excitation energy is

$$Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Rhc\frac{3}{4}$$

$$\frac{3}{4} Rhc = V\text{eV}$$

$$\therefore \quad Rhc = \frac{4V}{3} \text{eV}$$

89. b.

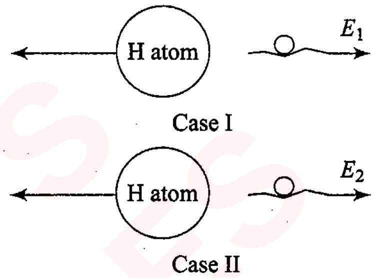


Fig. 4.69

In the first case, KE of H atom increases due to recoil whereas in the second case KE decreases due to recoil.

$$\therefore \quad E_2 > E_1$$

90. a. Linear momentum $\Rightarrow mv \propto \frac{1}{n}$

Angular momentum $\Rightarrow mvr \propto n$

Therefore, product of linear momentum and angular momentum $\propto n^0$

91. c. Energy of photon is given by mc^2. Now, the maximum energy of photon is equal to the maximum energy of electrons $= eV$

Hence, $mc^2 = eV \quad \Rightarrow \quad m = \frac{eV}{c^2}$

$$= \frac{1.6 \times 10^{-19} \times 18 \times 10^3}{(3 \times 10^8)^2} = 3.2 \times 10^{-32} \text{ kg}$$

92. d. Using $\frac{1}{\lambda} = R(z-1)^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

For K_α line; $n_1 = 2, n_2 = 1$

For metal A; $\frac{1875R}{4} = R(z_1 - 1)^2\left(\frac{3}{4}\right)$

$$\Rightarrow \quad z_1 = 26$$

For metal B; $675R = R(z-1)^2\left(\frac{3}{4}\right)$

$$\Rightarrow \quad z_2 = 31$$

Therefore, 4 elements lie between A and B.

93. d. For 2nd line of Balmer series in hydrogen spectrum, corresponds to $4 \to 2$ transition of hydrogen atom.

It is equivalent to $4 \times 3 \to 2 \times 3$ i.e., $12 \to 6$ transition of Li^{2+}

94. d. Energy of n^{th} state in hydrogen is same as energy of $3n^{th}$ state in Li^{++}.

$\therefore$ $3 \to 1$ transition in H would give same energy as the $3 \times 3 \to 1 \times 3$ i.e., $9 \to 3$ transition in Li^{++}.

95. a. $$\frac{1}{\lambda_\alpha} = \frac{3R}{4}(Z-1)^2$$

$$(Z-1) = \sqrt{\frac{4}{3R\lambda_\alpha}} = \sqrt{\frac{4}{3\times1.1\times10^7\times1.8\times10^{-10}}}$$

$$= \frac{200}{3}\sqrt{\frac{5}{33}} = \frac{78}{3} = 26 \Rightarrow Z = 27$$

96. b. λ_m will increase to $3\lambda_m$ due to decrease in the energy of bombarding electrons. Hence, no characteristic X-rays will be visible, only continuous X-ray will be produced.

97. d. $$B = \frac{\mu_0 I}{2r} \quad \text{and} \quad I = \frac{e}{T}$$

$$B = \frac{\mu_0 e}{2rT} \qquad [r \propto n^2,\ T \propto n^3]$$

$$B \propto \frac{1}{n^5}$$

98. c. $i = \dfrac{q}{T}$

Now $T^2 \propto r^3 \propto n^6 \quad \Rightarrow \quad i \propto \dfrac{1}{n^3}$

99. b. The cut-off wavelength when $V = V_1 = 10$ kV is

$$\lambda_1 = \frac{hc}{eV_1} = 1243.125\times10^{-13}\text{ m}$$

The cut-off wavelength when $V = V_2 = 20$ kV is,

$$\lambda_2 = \frac{hc}{eV_2} = 621.56\times10^{-13}\text{ m}$$

The wavelength corresponding to K_α line is,

$$\frac{1}{\lambda} = \frac{3R}{4}(Z-1)^2$$

From given information, $(\lambda-\lambda_2) = 3(\lambda-\lambda_1)$
Solving above equation, we get $Z = 29$

100. b. In the emission spectrum 10 lines are observed, so the energy level (n) to which the sample has been excited after absorbing the radiation is given by

$$\frac{n(n-1)}{2} = 10$$

which gives $\quad n = 5$

So, $$\frac{hc}{\lambda} = 13.6\left(1-\frac{1}{5^2}\right)\text{eV}$$

$$\frac{1242}{\lambda}\text{ eV-nm} = 13.6\times\frac{24}{25}\text{ eV}$$

$$\therefore \quad \lambda = 95\text{ nm}$$

101. c. Let n be the number of electrons per second striking the target. $P = VI$

$$16\text{W} = (5\times10^3\text{V})\times ne$$

$$\therefore \quad n = \frac{16}{5\times10^3\times1.6\times10^{-19}} = 2\times10^{16}$$

102. c. In first case, transition is from n^{th} state to 2^{nd} (1^{st} excited) state.

$$\therefore \quad (10.2 + 17.0)\text{ eV} = 13.6\times Z^2\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

In 2^{nd} case, transition is from n^{th} state to 3^{rd} state

$$\therefore \quad (4.25 + 5.95)\text{ eV} = 13.6\,Z^2\left[\frac{1}{3^2}-\frac{1}{n^2}\right]$$

Solving above equations, we get $n = 6$ and $Z = 3$.

103. b. The wavelengths present in emission spectra are shown in Fig. 4.70:

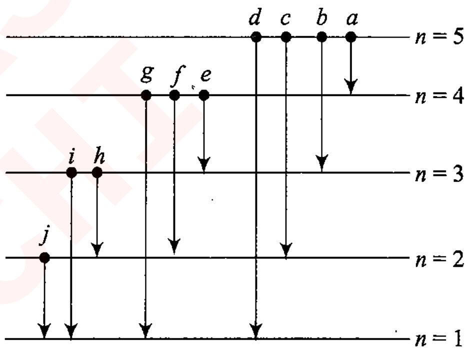


Fig. 4.70

Transitions a, e, h and j can be performed by a single atom also. This is also true about transitions b and i, other transitions require one atom each.

104. c. Through filter, only those photons will pass through whose energy is less than $E = \dfrac{hc}{800\text{ nm}} = 1.55$ eV.

From hydrogen's energy level diagram, we can easily identify that when electron jumps from 3^{rd} excited state (4^{th} energy level) to 2^{nd} excited state (3^{rd} energy level), then photons of 1.55 eV energy have been emitted. So, the required initial energy level is the 3^{rd} excited state.

105. b. Option (a) explains the production of X-rays on the basis of electromagnetic theory of light, which is not able to explain the characteristic X-rays and cut-off wavelength, so option (a) is wrong.

Option (b) correctly explains the production of characteristic X-rays.

Option (c) is wrong as X-ray spectra is a continuous spectra having some peaks representing characteristic X-rays.

106. b. The energy taken by hydrogen atom corresponds to its transition from $n = 1$ to $n = 3$ state.

ΔE (given to hydrogen atom)

$$= 13.6\left(1-\frac{1}{9}\right) = 13.6\times\frac{8}{9} = 12.1\text{ eV}$$

107. b. ΔE_1 (for 4^{th} to 3^{rd} excited states)

$$= 13.6\times 3^2\left[\frac{1}{4^2}-\frac{1}{5^2}\right] = 2.75\text{ eV}$$

ΔE_2 (for 3^{rd} to 2^{nd} excited states)

$$= 13.6\times 3^2\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = 5.95\text{ eV}$$

For shorter wavelength, i.e., for ΔE_2, $V_{01} = 3.95$ volt

From $eV_{01} = hc - \phi$,

$3.95 = 5.95 - \phi$

$\therefore \quad \phi = 2\text{ eV}$

For longer wavelength,

$eV_{02} = 2.75 - 2 = 0.75\text{ eV}$

So, $V_{02} = 0.75\text{ V}$

108. d. In transition of electron from higher energy level to lower energy level, the wavelength is given by $\lambda = hc/\Delta E$, where ΔE is the energy difference between two levels.
For minimum λ, ΔE should be maximum, so (d) is the correct option.

109. a. For a collision of neutron with hydrogen atom in ground state to be inelastic (partial or complete), the minimum KE of striking neutron must be 20.4 eV. [This condition is derived in theory.]
As the energy of the given incident neutron is less than 2.4 eV; the collision must be elastic.

110. b. Time period for n^{th} energy level electron is,

$$T = \frac{2\pi r_n}{v_n} = \frac{4\pi^2 m}{h}\times\frac{r_n^2}{n}$$

$$r_n = n^2 a_0$$

$$T = \frac{4\pi^2 m}{h}\times n^3 a_0^2$$

Required number of revolutions, $N = \dfrac{10^{-8}}{T}$

After substituting $n = 2$, $m = 9.1\times 10^{-31}$ kg, and $h = 6.63 \times 10^{-34}$ J-s, we get $N = 8\times 10^6$.

111. c. As the collision is inelastic, it means a part of kinetic energy is transformed into some other form due to collision. In this case, the kinetic energy of incident electron can be absorbed by H atom and it can absorb only 10.2 eV out of 11.2 eV, so that it can reach to 1^{st} excited state and the electron leaves with remaining energy, i.e., 1.0 eV.

112. b. $E = R_\infty ch\times\left[1-\dfrac{1}{2^2}\right] = \dfrac{3}{4}R_\infty\times hc$

Momentum of photon emitted is,

$$p = \frac{E}{c} = \frac{3R_\infty h}{4}$$

Recoiling speed of hydrogen atom is given by $v = P/m$, where m is the mass of hydrogen atom.

$$v = \frac{3R_\infty h}{4m} = \frac{3\times 1.1\times 10^7\times 6.63\times 10^{-34}}{4\times 1.67\times 10^{-27}} = 3.3\text{ ms}^{-1}$$

113. d. As the electron beam is having energy of 13 eV, it can excite the atom to the states whose energy is less than or equal to 0.6 eV (13.6 – 13). $E_5 = 0.544$ eV and $E_4 = 0.85$ eV. So, the electron beam can excite the hydrogen gas maximum to 4^{th} energy state, hence the transit electron can come back to ground state from either of three excited states, thus emitting Lyman, Balmer and Paschen series.

114. a. $\dfrac{hc}{\lambda} = 13.6\left[\dfrac{1}{1^2}-\dfrac{1}{10^2}\right] = 13.6\times 0.99$

$$\lambda = \frac{1242}{13.6\times 0.99}\text{ nm} = 92.25\text{ nm}$$

The line belongs to UV part of electromagnetic spectrum.

115. c. As angular momentum of electron is $4h/2\pi$, it means electron is in the 4^{th} orbit.
TE of atom in 4^{th} orbit is –0.85 eV
KE of electron = |TE| = 0.85 eV

116. d. We know, $r_n \propto n^2$
So, $(n+1)^2 - n^2 = (n-1)^2 \Rightarrow n = 4$

117. a. From Moseley's law, $\sqrt{\nu} = a(Z-1)$ for K_α X-ray

i.e., $\dfrac{1}{\sqrt{\lambda}} = a(Z-1)$

Given, $\dfrac{1}{\sqrt{\lambda}} = a(11-1)$ and $\dfrac{1}{\sqrt{4\lambda}} = a(Z-1)$

Dividing, we get $Z = 6$.

118. c. The protons move toward each other till their relative velocity becomes equal to zero. At the closest distance of approach, both the protons will be moving with the same velocity.
As coulombian repulsive force is internal for the system of protons, we can apply the law of conservation of momentum.

$\therefore \quad mv_0 = 2mv$

$$\text{Change in KE} = \frac{1}{2}mv_0^2 - 2\times\frac{1}{2}m\left(\frac{v_0}{2}\right)^2$$

This change in energy is equal to the electrical potential energy

$$\frac{mv_0^2}{2} - m\left(\frac{v_0}{2}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\therefore \quad r = \frac{e^2}{\pi\varepsilon_0 m v_0^2}$$

119. b. Kinetic energy gained by a charge q after being accelerated through a potential difference V volt, is given by

$$qV = \frac{1}{2}mv^2$$

$$mV = \sqrt{2MqV}$$

As $$\lambda_b = \frac{h}{mV} = \frac{h}{\sqrt{2qmV}}$$

For cut-off wavelength of X-rays, we have $qV = \frac{hc}{\lambda_m}$

or $$\lambda_m = \frac{hc}{qV}$$

Now, $$\frac{\lambda_b}{\lambda_m} = \frac{\sqrt{\frac{qV}{2m}}}{c}$$

As $$\frac{q}{m} = 1.8 \times 10^{11} \text{ C kg}^{-1} \text{ for electron,}$$

we have $$\frac{\lambda_b}{\lambda_m} = \frac{\sqrt{1.8 \times 10^{11} \times 10 \times 10^3/2}}{3 \times 10^8} = 0.1$$

Therefore, the answer is (b).

120. b. $P = VI$

Therefore, total power drawn by Coolidge tube $P_T = VI = 200$ W.

As 0.5% of the energy is carried by electron,

Power carried by X-rays is

$$0.5\% \text{ of } P_T = \frac{0.5}{100} \times 200 = 1 \text{ W}$$

The answer is (b).

121. d. When wavelength is maximum, the energy is minimum. Hence, this is from the ground state to the first excited state, for which the energy is 13.6 eV – 3.4 eV = 10.2 eV.

Hence, the required wavelength is 122 nm.

The next possibility is to jump from the ground state to the second excited state, which requires = 13.6 – 1.5 = 12.1 eV. Hence, it corresponds to a wavelength

$$\lambda = \frac{c}{v} = \frac{hc}{E_3 - E_1} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(12.1) \times (1.6 \times 10^{-19})} = 103 \text{ nm}$$

Therefore, (d) is the answer.

122. b. From figure (b), photons of energy 1.6×10^{-18} J get absorbed in large numbers, no lower energy photon gets absorbed. And according to the passage, substantial absorption occurs only if the photon bumps the ground state electron into a higher shell.

Therefore, 1.6×10^{-18} J photon knocks a ground state electron ($n = 1$) into the first excited state ($n = 2$). Hence, the difference in energy between the ground state and the first excited state must be 1.6×10^{-18} J.

Using $E_n = \frac{E_0}{n^2}$

$$1.6 \times 10^{-18} = E_2 - E_1 = -\frac{E_0}{4} - \left(-\frac{E_0}{1}\right) = \frac{3}{4}E_0$$

$$\therefore \quad E_0 = \frac{4}{3}(1.6 \times 10^{-18} \text{ J}) = 2.1 \times 10^{-18} \text{ J}$$

Therefore, the answer is (b).

123. b $$E_{photon} = E_3 - E_1 = -\frac{E_0}{3^2} - \left(-\frac{E_0}{1^2}\right) = \frac{8}{9}E_0$$

Therefore, (b) is the answer.

124. d. As $\lambda_0 = \frac{hc}{E} = 1.55 \times 10^{-11}$ m

$$\therefore \quad \lambda_0 = 0.155 \text{ Å}$$

Which is the minimum wavelength of continuous X-rays which carry energy equivalent to energy of incident electrons.

Now, as the energy of incident radiation is more than that of K-shell electrons, the characteristic X-rays appear as peaks on the continuous spectrum.

Therefore, (d) is the answer.

125. a. From conservation of momentum, two identical photons travel in opposite directions with equal magnitude of momentum and energy hc/λ.

From conservation of energy, we have

$$\frac{hc}{\lambda} + \frac{hc}{\lambda} = m_0C^2 + m_0C^2$$

$$\lambda = \frac{h}{m_0C}$$

Therefore, the answer is (a).

126. a. For the incident electron,

$$\frac{1}{2}mv^2 = Ve$$

$$p^2 = 2meV$$

de Broglie wavelength of incident electron,

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mVe}}$$

Shortest X-ray wavelength, $\lambda_2 = \frac{hc}{Ve}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{Ve}{c\sqrt{2mVe}} = \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\frac{e}{m}}$$

$$= \frac{\sqrt{\frac{10^4}{2} \times 1.8 \times 10^{11}}}{3 \times 10^8} = 0.1 = \frac{1}{10}$$

or $$\lambda_1 : \lambda_2 = 1 : 10$$

127. c. The characteristic X-ray depends on the material used.

128. a. $P = VI = 150 \times 10^3 \times 10 \times 10^{-3} = 1500 \text{ Js}^{-1}$

Heating rate of target $= \frac{1500}{4.2} \times \frac{1}{100} = 3.57 \text{ cal s}^{-1}$

129. d. The K, L, and M lines have different intercepts. The intercept of K is more than that of L, which in turn is more than that of M.

130. c. $L_1 = (1)\frac{h}{2\pi}$ (i)

(Using Bohr's Quantization Rule)
In the first excited state of Li,

$$L_2 = (2)\frac{h}{2\pi} \quad \text{(ii)}$$

$$\therefore \quad \frac{L_2}{L_1} = 2$$

131. d. $\frac{0.001}{2} \times 10^{-3} = (0.5 \times 10^{-10})n^2$

$$\left\{\begin{array}{l}\text{Because radius of } n^{\text{th}} \text{ orbit is equal to} \\ r_n = n^2 r_0, \quad \text{where} \quad r_0 = 0.529 \text{ Å}\end{array}\right\}$$

$\therefore \quad n^2 = 1000$

or $\quad n = 31$

132. d. For each principal quantum number n, number of electrons permitted equals the number of elements corresponding to the quantum number.

$$\begin{pmatrix}\text{Total number} \\ \text{of electrons}\end{pmatrix} = \sum 2n^2 = \frac{n(n+1)(2n+1)}{3}$$

133. b. $v_n = \alpha\left(\frac{cZ}{n}\right)$, where $\alpha = \frac{e^2}{2h\varepsilon_0 c}$ is the fine structure constant $\left(\alpha = \frac{1}{137}\right)$

$$v_{He^+} = \alpha\frac{c(2)}{2} = \alpha c$$

and $\quad v_H = \alpha\frac{c(1)}{1} = \alpha c = v_{He^+}$

134. c. $E = R_\infty hc\left(1 - \frac{1}{25}\right)$

Momentum of photon emitted is $p = \frac{E}{c} = R_\infty h\left(\frac{24}{25}\right)$

Recoil momentum of H atom will also be p.

$$mv = p$$

$$v = \frac{p}{m} = \frac{(1.097 \times 10^7)(6.626 \times 10^{-34})24}{(25)(1.67 \times 10^{-27})}$$

$\therefore \quad v = 4.178 \text{ ms}^{-1}$

135. b. Since $E = -\frac{13.6}{n^2}$ eV

$$E_1 = -13.6 \text{ eV}$$
$$E_2 = -3.4 \text{ eV}$$
$$E_3 = -1.50 \text{ eV}$$
$$E_4 = -0.85 \text{ eV}$$

From above, we can see that

$$E_3 - E_1 = 12.1 \text{ eV}$$

i.e., the electron must be making a transition from $n = 3$ to $n = 1$ level.

$$\Delta L = (3-1)\frac{h}{2\pi} = \frac{h}{\pi}$$
$$= 2.11 \times 10^{-34} \text{ Js}$$

136. b. For an elastic collision to take place, there must be no loss in the energy of electron. The hydrogen atom will absorb energy from the colliding electron only if it can go from ground state to first excited state, i.e., from $n = 1$ to $n = 2$ state. For this, hydrogen atom must absorb energy $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV
So, if the electron possesses energy less than 10.2 eV, it would never lose it and hence collision would be elastic.

137. b. $$\frac{1}{\lambda} = Z^2 R_\infty\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For K_α line, $n_1 = 1$ and $n_2 = 2$

$$\frac{1}{\lambda} = Z^2 R_\infty\left(\frac{3}{4}\right)$$

$$Z = \sqrt{\frac{4}{3\lambda R_\infty}}$$

$$= 39.9 \approx 40$$

138. c. Making potential energy zero increases the value of total energy by $13.6 - (-13.6) = 27.2$ eV
Now, actual energy in second orbit $= -3.4$ eV
Hence, new value is $(-3.4 + 27.2)$ eV $= 23.8$ eV

139. b. $$\text{KE}_{\lambda_1} = \frac{hc}{\lambda_1} - \psi = e\Delta V$$

$$\text{KE}_{\lambda_2} = \frac{hc}{\lambda_2} - \psi = 2e\Delta V$$

$$\Rightarrow \quad 3\left(\frac{hc}{\lambda_1} - \psi\right) = \frac{hc}{\lambda_2} - \psi$$

$$\Rightarrow \quad \psi = hc\left(\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right)$$

$$\Rightarrow \quad KE_{\lambda_3} = \frac{hc}{\lambda_3} - hc\left[\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right]$$
$$= hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$
$$e\Delta V = hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$
$$\Delta V = \frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$

140. d $\quad E = \frac{Z^2}{n^2}E_0$

141. c. $\quad \lambda_{min} = \frac{hc}{eV_{max}}$

142. b. Use Moseley's law

143. d $\quad \lambda_{min} = \frac{hc}{eV_{max}}$

144. c. Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv - 0 = (m + m_H)\,u$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle, and u the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was

$$E_0 = \frac{mv_0^2}{2}$$

The total kinetic energy after collision

$$E = \frac{(m + m_H)u^2}{2} = \frac{m^2v_0^2}{2(m + m_H)}$$

The decrease in kinetic energy must be equal to ionization energy. Therefore,

$$E_1 = E_0 - E = \left(\frac{m_H}{m + m_H}\right)E_0$$

i.e., $$\frac{E_1}{E_0} = \frac{1}{1 + \frac{m}{m_H}}$$

i.e., the greater the mass m, the smaller the fraction of initial kinetic energy that be used for ionization.

145. d $\quad B_n = \frac{\mu_0 I_n}{2r_n}$

or $\quad B_n \propto \frac{I_n}{r_n}$

$$\propto \frac{(f_n)}{r_n}$$

$\therefore \quad B_n \propto \frac{(v_n/r_n)}{r_n}$

$$\propto \frac{v_n}{(r_n)^2}$$
$$\propto \frac{(z/n)}{(n^2/z)^2}$$
$$\propto \frac{z^3}{n^5}$$

146. a. $\quad \frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{e}{2m}$

$\therefore$ Magnetic moment $\propto$ angular momentum

$$\propto n \qquad \left(\because L = n\frac{h}{2\pi}\right)$$

147. b $\quad a = \frac{v^2}{r}$

$\therefore \quad a \propto \frac{(z)^2}{(1/z)} \qquad$ (for $n = 1$)

or $\quad a \propto z^3$

$\therefore \quad \frac{a_1}{a_2} = \left(\frac{2}{1}\right)^3 = 8$

148. a. Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So,

$$(z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$$

$\therefore \quad z^2 = 4$

or $\quad z = 2$

149. c. $\quad \frac{hc}{\lambda} = Rhc(1 - 1/n^2)$

or $\quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

150. b. Let $\quad v$ = speed of neutron before collision,

v_1 = speed of neutron after collision,

v_2 = speed of proton or hydrogen atom after collision,

and $\quad \Delta E$ = energy of excitation

From conservation of linear momentum,

$$mv = mv_1 + mv_2 \qquad \text{(i)}$$

From conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \qquad \text{(ii)}$$

From Eq. (i),

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

From Eq. (ii),

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

$$\therefore \quad 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$

$$\Rightarrow \quad (v_1 - v_2)^2 = v^2 - 4\frac{\Delta E}{m}$$

As $v_1 - v_2$ must be real, therefore

$$v^2 - 4\frac{\Delta E}{m} \geq 0$$

or $$\frac{1}{2}mv^2 \geq 2\Delta E$$

The minimum energy that can be absorbed by hydrogen atom in ground state to go into excited state is 10.2. eV. Therefore,

$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 \text{ eV}$$

$$= 20.4 \text{ eV}$$

151. a. Number of possible emission lines are $n(n-1)/2$ when an electron jumps from n^{th} state to ground state. In this question, this value should be $(n-1)(n-2)/2$.

Hence, $10 = \frac{(n-1)(n-2)}{2}$

Solving this, we get $n = 6$.

152. d. $$r = \frac{\varepsilon_0 n^2 h^2}{e^2 \pi m}$$

$$= \frac{\varepsilon_0 (2\pi L)^2}{\varepsilon^2 \pi m}\left(L = n\frac{h}{2\pi} \text{ or } nh = 2\pi L\right)$$

$$\therefore \quad Lr^{-\frac{1}{2}} = \text{constant}$$

153. a. $$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad \frac{h}{mv} = \frac{(2\pi r)}{n}$$

$$\frac{h}{mv} = \text{de Broglie wavelength}$$

154. b. Maximum angular speed will be in its ground state. Hence,

$$\omega_{max} = \frac{v_1}{r_1} = \frac{2.2 \times 10^6}{0.529 \times 10^{-10}}$$

$$= 4.1 \times 10^{16} \text{ rad s}^{-1}$$

155. d. $$\frac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \frac{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}}{\frac{E_1}{4n^2} - \frac{E_1}{n^2}} = \frac{1}{4} = \text{constant}$$

156. d. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$

Therefore, $E_n r_n$ is independent of n.

Hence, $$E_1 r_1 = (13.6 \text{ eV})(0.53 \text{ Å})$$

$$= 7.2 \text{ eV-Å}$$

$$= \text{constant}$$

157. b. $$\lambda = \frac{h}{p} \text{ (for electron)}$$

or $$p = h/\lambda$$

and $$E = \frac{hc}{\lambda} \text{ (for photon)}$$

$$\therefore \quad \frac{p}{E} = \frac{1}{c} = \frac{1}{3 \times 10^8} = 3.33 \times 10^{-9} \text{ s m}^{-1}$$

158. d. $$\Delta\lambda = \lambda_{K_\alpha} - \lambda_{min}$$

When V is halved λ_{min} becomes two times but λ_{K_α} remains the same.

$$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{min}$$

$$= 2(\Delta\lambda) - \lambda_{K_\alpha}$$

$$\therefore \quad \Delta\lambda' < 2(\Delta\lambda)$$

159. b. Potential energy $= -2 \times$ kinetic energy $= -2E$

Total energy $= -2E + E = -3.4$ eV $= -E$

or $E = 3.4$ eV

p = momentum, m = mass of electron

$$E = \frac{p^2}{2m}$$

or $$p = \sqrt{2mE}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}} \approx 10^{-24}$$

de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 6.6 \times 10^{-10}$ m

160. b. de Broglie wavelength of electron in hydrogen atom

$$= \frac{h}{mv} = \frac{2\pi r_n}{n}$$

For second Bohr orbit, $\lambda = \frac{600 \times 10^{-9}}{2} = 3000 \times 10^{-9}$ m

$$\lambda = \sqrt{\frac{150}{V}} \text{ Å} = 300 \text{ Å}$$

$$\therefore \quad V = \frac{150}{(3000)^2} = \frac{5}{3} \times 10^{-5} \text{ V}$$

161. c. Using Moseley's law, $\nu^{1/2} = a(z-b)$

$$\left(\frac{c}{\lambda_1}\right)^{1/2} = a(z_1-b) \text{ and } \left(\frac{c}{\lambda_2}\right)^{1/2} = a(z^2-b)$$

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} = \frac{a(z_1-b)}{a(z_2-b)} \Rightarrow \left(\frac{7.12}{15.42}\right)^{\frac{1}{2}} = \frac{(29-b)}{(42-b)}$$

$(42-b) = 1.47\,(29-b) \Rightarrow b = 1.44$

$$\left(\frac{\lambda_1}{\lambda}\right)^{\frac{1}{2}} = \frac{(z-1.44)}{(z_1-1.44)}$$

$$\left(\frac{15.42}{22.85}\right)^{\frac{1}{2}}(27.56) = z - 1.44 \Rightarrow = 24$$

162. d. Angular momentum, $L = 4.2176 \times 10^{-34} = \frac{n_2 h}{2\pi}$

$\Rightarrow \quad n_2 = 4$

For the transition from $n_2 = 4$ to $n_1 = 3$, the wavelength of spectral line $= \lambda$

$$\frac{1}{\lambda} = \frac{13.6}{hc}\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= \frac{13.6 \text{ eV}}{1240 \text{ eV nm}}\left(\frac{7}{9 \times 16}\right)$$

$$\lambda = \frac{1240 \times 144}{13.6 \times 7} = 1876 \text{ nm} = 18760 \text{ Å}$$

$$= 1.876 \times 10^4 \text{ Å}$$

163. c. The angular momentum is $mvr = \frac{nh}{2\pi} \Rightarrow n = 1$

Centripetal force, $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m_\pi e^2 Z} = \left(\frac{\varepsilon_0 h^2}{\pi m_e e^2}\right)\left(\frac{m_e}{m_\pi}\right)\frac{1}{Z}$$

$$= \frac{0.53 \times 10^{-10}}{264\, Z} = \frac{200 \times 10^{-15}}{Z}$$

$$\left[\because \frac{m_\pi}{m_e} = 264\right]$$

Since r cannot be less than nuclear radius,

$$r > 1.6\, Z^{\frac{1}{3}} \times 10^{-15} \text{ m}$$

or $\frac{200 \times 10^{-15}}{Z} > 1.6 \times 10^{-15}\, Z^{\frac{1}{3}}$

$\Rightarrow \quad Z < \left(\frac{200}{1.6}\right)^{\frac{3}{4}} < 37$

164. a. For the first line of Balmer series of hydrogen,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R}$$

For singly ionized helium ($Z = 2$),

$$\frac{1}{\lambda'} = 4R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Given $\lambda' = \lambda$

For $\quad n_1 = 6$ to $n_2 = 4$

$$\frac{1}{\lambda'} = 4R\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \frac{20\,R}{144} = \frac{5R}{36}$$

It corresponds to transition from $n_1 = 6$ to $n_2 = 4$.

165. d. The energy of the K_α X-ray photon

$$E_{K_\alpha} = E_i - E_f = (Z-1)^2(-3.4 \text{ eV} + 13.6 \text{ eV})$$

$$= (Z-1)^2 (10.2 \text{ eV})$$

Given $\quad E_{K_\alpha} = 7.46$ keV

$\therefore \quad 7.46 \times 10^3 \text{ eV} = (Z-1)^2 (10.2 \text{ eV})$

or $\quad (Z-1)^2 = \frac{7.46 \times 10^3}{10.2} = 731.4$

or $\quad (Z-1) = 27 \Rightarrow Z = 28$

Multiple Correct Answers Type

1. a., b., d. $\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$

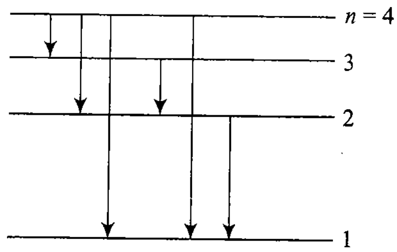


Fig. 4.71

If the initial state were $n = 3$, in the emission spectrum, no wavelengths shorter than λ_0 would have occurred.
This is possible if initial state were $n = 2$.

2. b., c. $E_0 z^2\left(1 - \frac{1}{9}\right) - E_0 z^2\left(\frac{1}{4} - \frac{1}{9}\right) = 3E_0 \Rightarrow \frac{27}{36}E_0 Z^2 = 3E_0$

$\Rightarrow \quad z = 2 \quad \frac{\lambda_1}{\lambda_2}; = 3$

$$KE_1 = E_0\left(1 - \frac{1}{9}\right) - \phi;$$

$$KE_2 = E_0 z^2\left(1 - \frac{1}{4}\right) - \phi$$

$$KE \propto \frac{1}{\lambda^2} = 8.5 \text{ eV}$$

3. b., c. $v \propto \frac{1}{n}$, $E \propto \frac{1}{n^2}$, and $r \propto n^2$

4. a., c., d.

a. As $v \propto 1/n$, so momentum $= mv \propto 1/n$

b. is not true as radius $r \propto n^2$

c. $\text{KE} = \frac{1}{2}mv^2$,

As $v \propto 1/n$, so $\text{KE} \propto \frac{1}{n^2}$

d. is true.

5. b., c., d. Ground state $n = 1$

First excited state $n = 2$

$$\text{KE} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} \quad (z = 1)$$

$$\text{KE} = \frac{14.4 \times 10^{-10}}{2r}\ \text{eV}$$

Now $r = 0.53\, n^2$ Å $(z = 1)$

$$(\text{KE})_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10}}\ \text{eV} = 13.58\ \text{eV}$$

$$(\text{KE})_2 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4}\ \text{eV} = 3.39\ \text{eV}$$

KE decreases by 10.2 eV

Now, $$\text{PE} = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{-14.4 \times 10^{-10}}{r}\ \text{eV}$$

$$(\text{PE})_1 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10}}\ \text{eV} = -27.1\ \text{eV}$$

$$(\text{PE})_2 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10} \times 4} = -6.79\ \text{eV}$$

PE increases by 20.4 eV

Now, angular momentum, $L = mvr = \frac{nh}{2\pi}$

$$L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{6.28} = 1.05 \times 10^{-34}\ \text{Js}$$

6. a., b., c. $\lambda_{\min} = \lambda_{\min} = \frac{12400}{v_0}$ Å

$$= \frac{12400}{20,000} = 62\ \text{Å}$$

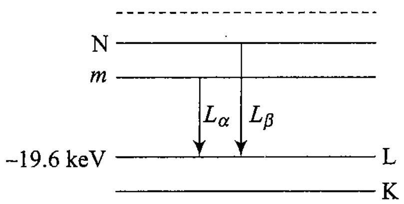


Fig. 4.72

7. a., b. $$|F| = \frac{dU}{dt} = \frac{Ke^2}{r^4} \qquad \text{(i)}$$

$$\frac{Ke^2}{r^4} = \frac{mv^2}{r} \qquad \text{(ii)}$$

and $$mvr = \frac{nh}{2\pi} \qquad \text{(iii)}$$

By (ii) and (iii),

$$r = \frac{Ke^2\, 4\pi^2}{h^2}\frac{m}{n^2} = K_1\frac{m}{n^2} \qquad \text{(iv)}$$

Total energy $= \frac{1}{2}$ (potential energy)

$$= \frac{Ke^2}{6r^3} = \frac{-Ke^2}{6\left(\frac{K_1 m}{n^2}\right)^3} = \frac{-Ke^2 n^6}{6K_1^3 m^3}$$

Total energy $\propto n^6$

Total energy $\propto m^{-3}$

$\therefore$ (a) and (b) are correct.

8. a., b. $r_n = n^2 r_1$

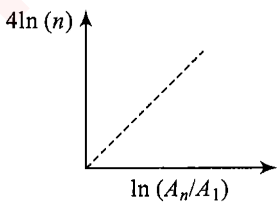


Fig. 4.73

$$\ln\left(\frac{A_n}{A_1}\right) = \ln\left(\frac{\pi r_n^2}{\pi r_1^2}\right) = \ln n^4 = 4\ln(n)$$

9. b., d. Line emission spectra can be obtained for atoms and molecules both. An atom or molecule in an excited state emits photons by making a transition from excited state to ground state thus constituting line emission spectra.

The wavelengths emitted by the molecular energy levels which are generally grouped into several bunches are also grouped and each group is well separated from the other. The spectrum in this case looks like a band spectrum.

10. b., c., d. Let collision between two atoms be an inelastic one. From momentum conservation, $mv_0 = mv_1 + mv_2$

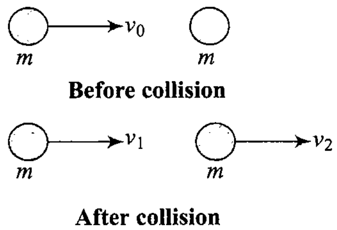


Fig. 4.74

From energy conservation,

$$\frac{mv_1^2}{2}+\frac{mv_2^2}{2}-\frac{mv_0^2}{2}=-\Delta E$$

where ΔE is the energy absorbed by the initially stationary atom to change its state.

Solving above equations, we get

$$(v_1-v_2)^2=v_0^2-\frac{4\Delta E}{m}$$

For collision to be inelastic, $(v_1-v_2)^2 \geq 0$: a real quantity [equal to sign for perfect inelastic collision.]

The minimum value of ΔE is 10.2 eV, so for collision to be inelastic, $E \geq 20.4$ eV.

For perfectly inelastic collision, $v_1 = v_2$ and hence $E = 20.4$ eV.

For $E = 18$ eV, the collision is elastic one and as masses are the same, velocities would be interchanged during collision.

11. a., c., d.

a. $r_n=\frac{n^2h^2}{4\pi^2 mKe^2}$, i.e, $r_n \propto n^2$

c. Bohr's 2nd postulate, $mvr=\frac{nh}{2\pi}$

d. $K_n=\frac{KZe^2}{2r_n}$, $U_n=\frac{KZe^2}{r_n}$

12. b., c. Lyman series lies in the ultraviolet region, Balmer series in visible region, and Paschen series in infrared region.

c. $\lambda_R > \lambda_Y > \lambda_B > \lambda_V$

13. a, b. Continuous spectrum is obtained from the bulk state of matter, it has no relation with the atomic or molecular state. It is produced by thermal vibrations of atoms in the macroscopic matter (not by the transition between the energy states of atoms). Every vibrating atom emits light of frequency of its vibrations. In a white hot matter, atoms vibrating with all frequencies within a definite range are present, so this matter emits frequencies of continuous range.

14. a., b., c. $E_n^H = E_1^H + \Delta E = -13.6\,\text{eV} + 12.75\,\text{eV} = -0.85\,\text{eV}$ i.e., hydrogen atoms are excited to $n = 4$ level, i.e., transitions $4 \to 1$, $3 \to 1$, $2 \to 1$ are possible which correspond to Lyman series, then transitions $4 \to 2$ and $3 \to 2$ are possible which correspond to Balmer series, and then transition $4 \to 3$ is also possible which correspond to Paschen series.

15. a., b., c

a. $U_1 = E$, then total energy in the orbit $=\frac{U_1}{2}=\frac{E}{2}$

b. $\text{IE}=(\text{TE})_{n=\infty}-(\text{TE})_{n=1}=0-(\text{TE})_{n=1}=-\left(\frac{E}{2}\right)$

c. $(\text{KE})_{n=1}=-(\text{TE})_{n=1}=-\left(\frac{E}{2}\right)$

16. a., c., d. $p_n = mv_n$, $p_n = v_n \propto \frac{1}{n}$, $r_n \propto n^2$ and $K_n=\frac{Ke^2}{2r_n}$ i.e., $K_n \propto \frac{1}{n^2}$ and $L \propto n$

17. b., c. Any transition causing a photon to be emitted in the Balmer series must end at $n = 2$. This must be followed by the transition from $n = 2$ to $n = 1$, emitting a photon of energy 10.2 eV, which corresponds to a wavelength of about 122 nm. This belongs to the Lyman series.

18. a., c. $E_K - E_L = \frac{hc}{\lambda_\alpha}$ (i)

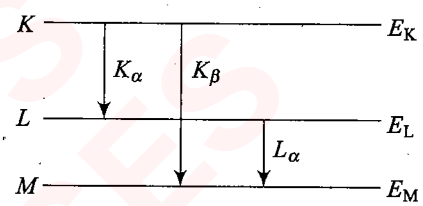


Fig. 4.75

$$E_K - E_M = \frac{hc}{\lambda_\beta} \quad \text{(ii)}$$

$$E_L - E_M = \frac{hc}{\lambda'_\alpha} \quad \text{(iii)}$$

(ii) − (i) ⇒ $E_L - E_M = \frac{hc}{\lambda'_\alpha}=\frac{hc}{\lambda_\beta}-\frac{hc}{\lambda_\alpha}$

$$\frac{1}{\lambda_\beta}=\frac{1}{\lambda_\alpha}+\frac{1}{\lambda'_\alpha}$$

Also, $(E_K - E_M) > (E_K - E_L) > (E_L - E_M)$

$$\frac{hc}{\lambda_\beta} > \frac{hc}{\lambda_\alpha} > \frac{hc}{\lambda'_\alpha}$$

19. b., d. When potential difference between filament and target is increased, then KE of striking electrons gets increased. Since most energetic electrons now strike the target, therefore more energetic electrons are emitted. It means, frequency of X-ray photons increases. It means, penetration power of X-rays gets increased.

To increase the photon flux, rate of collision of electrons with the target must be increased. This can be achieved only when rate of emission of electrons from the filament is increased. To achieve this, filament current must be increased. Therefore, options (b) and (d) are correct.

20. a., b., c. When a stationary hydrogen atom emits a photon, then energy of the emitted photon will be equal to the difference of the energy of the two levels involved in the transition. Hence, energy of emitted photon will be equal to $(E_m - E_n)$.

If a hydrogen atom is moving and a photon is emitted by it along the same direction in which it is moving, due to momentum of the emitted photon, the momentum of hydrogen atom will get decreased. Therefore, energy of the emitted photon will be equal to $(E_m - E_n$ + loss of KE of the hydrogen atom).

But if the photon is emitted in a direction normal to the motion of the hydrogen atom, then the frequency of the emitted photon will be equal to f_0. Hence, option (a) is correct, obviously, option (d) is wrong.

If the photon is emitted by the hydrogen atom in the direction opposite to its motion, then frequency of the emitted photon will be less than f_0. Hence, option (c) is correct.

21. a., c. First line of Lyman series is obtained during transition of hydrogen atom from $n = 2$ to $n = 1$. Hence, its energy is equal to $E_2 - E_1 = (13.6)\text{ eV} - (-3.4\text{ eV}) = -1.2\text{ eV}$.

$\therefore$ Wavelength of the first line of Lyman series is equal to

$$\frac{12375}{10.2}\text{ Å} = 1215\text{ Å}$$

Therefore, it lies in ultraviolet region.

Since energy of all the other lines of Lyman series is greater than that of first line, therefore all the lines of Lyman series lie in ultraviolet region. Hence, option (a) is correct.

First line of Balmer series is obtained during transition of hydrogen atom from $n = 3$ to $n = 2$. Hence, its energy is equal to $E_3 - E_2 = 1.89$ eV.

$\therefore$ Wavelength of first line of Balmer series is equal to 12375/1.89 Å = 6563 Å. It lies in visible region.

Energy of last line of Balmer series is equal to 3.4 eV. Therefore, its wavelength is equal to12375/3.4 Å = 3640 Å.

Since it is less than 4000 Å, therefore it lies in ultraviolet region. Hence, option (b) is wrong. Energy of last line of Paschen series is equal to 1.51 eV. It lies in infrared region. Since energy of all the other lines of Paschen series is less than its energy, therefore all the lines of Paschen series will lie in infrared region. Hence, option (c) is also correct.

22. a., b., c. Since in hydrogen atom $r_n \propto n^2$, therefore graph between r_n and n will be a parabola through origin and having increasing slope. Therefore, option (a) is correct.

Since, $r_n \propto n^2$, therefore $r_n/r_1 = n^2$

Hence, $\log (r_n/r_1) = 2 \log n$

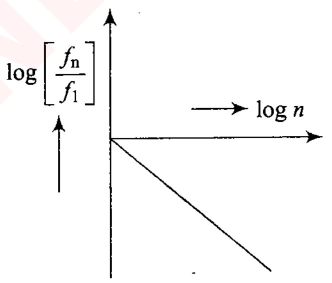


Fig. 4.76

It means, graph between $\log (r_n/r_1)$ and $\log n$ will be a straight line passing through origin and having positive slope ($\tan\theta = 2$). Therefore, option (b) is also correct. If radius of an orbit is equal to r, then area enclosed by it will be equal to $A = \pi r^2$.

Since $r_n \propto n^2$, therefore $A_n \propto n^4$

Hence, $\frac{A_n}{A_1} = n^4$ or $\log\left(\frac{A_n}{A_1}\right) = 4 \log n$

It means, graph between $\log (A_n/A_1)$ and $\log n$ will be a straight line passing through origin and having positive slope ($\tan\theta = 4$). Therefore, option (c) is also correct.

If frequency of revolution of electron is f, then its angular velocity will be equal to $\omega = 2\pi f$. Hence, its angular momentum will be equal to $I\omega = mr^2\omega$. But according to Bohr's theory, it is equal to $nh/2\pi$, therefore,

$$mr^2(2\pi f) = \frac{nh}{2\pi} \quad \text{or} \quad f = \frac{nh}{4\pi^2 mr^2}$$

Since $r \propto n^2$, therefore $f \propto \frac{1}{n^3}$

Hence, $\frac{f_n}{f_1} = \frac{1}{n^3}$ or $\log\left(\frac{f_n}{f_1}\right) = 3 \log n$

It means, graph between $\log (f_n/f_1)$ and $\log n$ will be a straight line passing through origin and having negative slope, $\tan\theta = -3$. Hence, it will be as shown in figure. Hence, the option (d) is wrong.

23. b., c., d. Statement (a) is false. The shortest wavelength of the X-rays emitted depends on the energy of the electrons incident on the target. This, in turn, depends on the potential through which they have fallen. In fact,

$$\lambda_{\min} = \frac{hc}{eV}$$

Statement (b) is true. X-ray spectra of all heavy elements are similar in character.

Statement (c) is also true. The short wavelength of the X-rays (compared to the grating constant of optical grating) makes it difficult to observe X-ray diffraction with ordinary gratings.

Statement (d) is also true. The sharp limit on the short wavelength side is dependent on the voltage applied to the incident electrons and is given by:

$$\lambda_{\min} = \frac{hc}{eV}$$

24. c., d. $V = 6.6\text{ kV} = 6600\text{ V}$

Now, $\nu_{\max} = \frac{eV}{h} = \frac{1.6 \times 10^{-19} \times 6600}{6.6 \times 10^{-34}}$

$= 1.6 \times 10^{18}$ Hz

Thus, the frequency of the X-rays cannot exceed 1.6×10^{18} Hz. Hence, the correct choices are (c) and (d).

25. a., c. Statement (a) is correct. The angular momentum of the earth ($= mr^2\omega$) has to be equal to $nh/2\pi$. This gives, $n = 2\pi mr^2\omega/h$. Putting the numerical values of the Earth's mass, radius, and the angular velocity, we get the given value of n.

Statement (b) is incorrect. The maximum number of electrons allowed in an orbit being $2n^2$, the required number is $2(1^2 + 2^2 + 3^2 + 4^2) = 60$.

Statement (c) is correct. The 'reduced mass' of the electron [$\mu = mM/(m + M)$] being dependent on the mass of the nucleus (M), the Rydberg constant also varies with the mass number of the given element.

Statement (d) is incorrect. The ratio is not exactly equal to 4 but slightly different from 4 because of the dependence of the Rydberg constant on the mass number of the element concerned.

26. a., b. The correct choices are (a) and (b). The last two statements are incorrect because they violate the principle of conservation of charge. We always have either an electron–positron 'pair production' or an electron–positron 'pair annihilation'. It is only then that the total charge remains zero both before and after reaction.

27. a., c. Moseley's law: $\lambda \propto \frac{1}{(z-1)}2$; $\frac{\lambda_z}{\lambda_1} = \frac{(z_1-1)^2}{(z-1)^2}$

$$z_1 - 1 = (z-1)2;$$

$$z_1 = 2z - 1; \ \frac{\lambda_z}{\lambda_2} = \frac{1}{4} = \left(\frac{z_2-1}{z-1}\right)^2$$

28. a., c., d. In Bohr model of hydrogen atom,

$$R \propto n^2$$

$$V \propto \frac{1}{n}$$

$$T \propto n^3 \text{ and } E \propto \frac{1}{n^2}$$

$$VR \propto n$$

$$TE \propto n$$

$$\frac{T}{R} \propto n$$

$$\therefore \quad \frac{V}{E} \propto n$$

29. a., c. Power loss increases the temperature.

30. a., b., c. Energy of K absorption edge

$$E_K = \frac{1242 \text{ eVnm}}{0.0172 \text{ nm}} = 72.21 \times 10^3 \text{ eV} = 72.21 \text{ Ke V}$$

Energy of K_α line is

$$E_{k_\alpha} = \frac{he}{e\lambda_\alpha} = \frac{1242 \text{ eVnm}}{0.021 \text{ nm}} = 59.14 \text{ KeV}$$

Similarly, $$E_{k_\beta} = \frac{1242}{0.0192} = 64.69 \text{ KeV}$$

$$E_{k_\gamma} = \frac{1242}{0.0180} = 69 \text{ KeV}$$

Energy of K shell $= (E_{K_\alpha} - E_K)$

$$= (59.14 - 72.21) \text{ KeV} = -13.04 \text{ keV}$$

Energy of L shell $= E_{k_\beta} - 72.21$ keV

$$= 64.69 \text{ keV} - 72.21 \text{ keV} = -7.52 \text{ keV}$$

Energy of M shell $= E_{k_\gamma} - E_k = \frac{1242 \text{eVnm}}{0.018 \text{ nm}} - 72.21$ keV

$$= 69 \text{ keV} - 72.21 \text{ keV} = -3.21 \text{ keV}$$

31. a., b. For the third line of Balmer series, $n_1 = 2$, $n_2 = 5$

$$\therefore \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = RZ^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \frac{21 RZ^2}{100}$$

$$E = -13.6 \text{ eV}$$

$$Z^2 \times \frac{21}{100} = \frac{hc}{\lambda} = \frac{1242 \text{ eVnm}}{108.5 \text{ nm}}$$

$$Z^2 = \frac{1242 \times 100}{108.5 \times 21 \times 13.6} = 4 \Rightarrow Z = 2$$

Binding energy of an electron in the ground state of hydrogen-like ion $= 13.6Z^2/n^2 = 54.4$ eV ($n = 1$).

Assertion-Reasoning Type

1. c. Lyman series: Its energy is in the ultraviolet region.

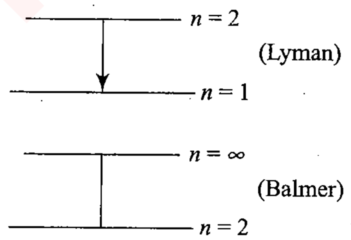


Fig. 4.77

Balmer series: Its energy is in visible region.

Now, frequency of energy of ultraviolet photon is much greater than frequency of visible region. So, statement I is true. Statement II is false.

2. d. As the energy of striking electron is increased, the wavelength of the characteristic X-ray does not change as characteristic X-rays are emitted when electrons are making transition from a higher energy level to a lower energy and energy of characteristic X-ray is given by

$$E = \frac{hc}{\lambda} = |E_f| - |E_i| = E_i - E_f$$

which does not depend at all on the energy of the striking electron. The only dependence is that the striking electron should possess enough energy to knock out an electron from the inner shell.

3. a. A H atom that drops from $n = 2$ level to $n = 1$ level emits a photon of energy 10.2 eV and wavelength 122 nm. A He^+ ion emits a photon of the same energy and wavelength when it drops from $n = 4$ level to $n = 2$ level.

Comprehension Type

For Problems 1–3

1. b., 2. b., 3. b.

Sol.

1. b. On other planet: $mvr = 2n\dfrac{h}{2\pi} \Rightarrow v = \dfrac{nh}{\pi mr}$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \Rightarrow \frac{mn^2h^2}{n^2m^2r^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Putting $n = 1$, we get $r = \dfrac{4h^2\varepsilon_0}{m\pi e^2}$

2. b. On our planet: $v_0 = \dfrac{e^2}{2\varepsilon_0 nh}$

On other planet: $v = \dfrac{e^2}{2\varepsilon_0(2n)h} = \dfrac{v_0}{2}$

3. b. On our planet: $E_n = -\dfrac{13.6}{n^2}$

On other planet: $E'_n = -\dfrac{13.6}{(2n)^2}$

$$\Rightarrow \quad E'_n = \frac{E_n}{4} = -3.4 \text{ eV}$$

For Problems 4–6

4. c., 5. c., 6. b.

Sol.

4. c. $a_0 = \dfrac{h^2\varepsilon_0}{\pi me^2}$ and $r = \dfrac{n^2h^2\varepsilon_0}{\pi m_e e^2} = \dfrac{2^2h^2\varepsilon_0}{\pi(m/2)e^2} = 8a_0$

5. c. $R \propto m$

6. b. $\dfrac{1}{\lambda} = \dfrac{R}{2}\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) \Rightarrow \lambda = \dfrac{8}{3R}$

$$\Rightarrow \lambda = \frac{8}{3 \times 1.097 \times 10^7} = 2431 \text{ Å}$$

For Problems 7–9

7. a., 8. c., 9.

Sol.

7. a. Internal forces act between electron and proton, then how can the atom get an acceleration.

8. c. Change in angular momentum $= I\omega$

$$\Rightarrow \frac{mh}{2\pi} - \frac{nh}{2\pi} = I\omega \Rightarrow \omega = \frac{(m-n)h}{6.8I}$$

9. d. Since no external force and torque act on the atom, still it gets an acceleration and angular acceleration. So, none is true.

For Problems 10–15

10. c., 11. b., 12. c., 13. b., 14. c., 15. a.

Sol.

10. c. Given that $E_1 = -15.6$ eV, $E_\infty = 0$ eV.

Ionization energy of the atom:

$E_\infty - E_1 = 0 - (-15.6 \text{ eV}) = 15.6$ eV

So, ionization potential = 15.6 V

11. b. For short wavelength limit of the series terminating at $n = 2$, a transition must take place from $n = \infty$ state to $n = 2$ state. For this, $\Delta E = 5.30$ eV

$$\lambda = \frac{12400}{\Delta E(\text{eV})} \text{ Å} = \frac{12400}{5.30} \text{ Å} = 2339 \text{ Å}$$

12. c. The excitation energy for the $n = 3$ state is

$\Delta E = E_3 - E_1 = 15.6 - 3.08 = 12.52$ eV

Excitation potential = 12.52 V

13. b. $\lambda = \dfrac{12400}{E_3^Z - E_1^Z} = \dfrac{12400}{12.52} \text{ Å} = 990 \text{ Å}$

Wave number $= \dfrac{1}{\lambda} = \dfrac{1}{990 \times 10^{-10}\text{ m}} = 1.009 \times 10^7 \text{ m}^{-1}$

14. c. Because excitation energy for the second shell is more than 6 eV, hence electron having initial kinetic energy of 6 eV will not interact with the atom. Because it cannot transfer its energy to the electron in the atom.

15. a. As $E_2 - E_1 = 15.6 - 5.3 = 10.3$ eV

hence energy of the electron after interaction is $11 - 10.3 \text{ eV} = 0.7$ eV.

For Problems 16–17

16. c., 17. b.

Sol.

16. c. We can use the equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda = \frac{4}{3R} = \frac{4}{3(1.097 \times 10^7)}$$

$$= 1.215 \times 10^{-7} \text{ m} = 121.5$$

The frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{1.215 \times 10^{-7}} = 2.47 \times 10^{15} \text{ Hz}$$

The energy of the photon is $E = hf$

$= (4.136 \times 10^{-15})(2.47 \times 10^{15}) = 10.2$ eV

17. b. From conservation of momentum, as the total momentum before emission is zero, the total momentum after emission must also be zero. The photon and atom therefore move off in opposite directions, with $mv = E_{\text{photon}}/C$ where m and v are the mass and recoil speed of the hydrogen atom, E_{photon} is the actual energy of the photon (less than 10.2 eV), and c is the speed of light. The energy difference between the $n = 2$ and $n = 1$ levels, E is the source of both the photon energy and the recoil kinetic energy of the atom.

From energy conservation, we have

$$E = E_{\text{photon}} + \frac{1}{2}mv^2$$

Atom being massive, we can assume that its recoil speed v and kinetic energy are so small that $E = E_{\text{photon}}$. Substituting $E_{\text{photon}} = 10.2$ eV into the expression for mv yields.
$mv = 10.2\,\text{eV}c$

The recoil kinetic energy of the hydrogen atom can now be calculated.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = (0.5)\frac{(10.2)^2}{mc^2}$$

$$= \frac{(0.5)(10.2)^2}{938.8\times 10^6} = 5.54\times 10^{-8}\,\text{eV}$$

Thus, the fraction of the energy difference between the $n = 2$ and $n = 1$ levels that goes into atomic recoil energy is very small.

$$\frac{K}{E} = \frac{5.54\times 10^{-8}}{10.2} = 5.43\times 10^{-9}$$

That is why by equating the photon's energy to the atomic energy level separation yields accurate answers because little energy is needed to conserve momentum.

For Problems 18–19

18. b., 19. d.

Sol.

18. b. As we know, energy of a photon is given by $E = \dfrac{hc}{\lambda}$

From the given condition, $\lambda = \dfrac{1500p^2}{p^2-1}$

Hence, $E = \dfrac{hc}{\lambda} = \dfrac{hc}{1500}\left(1-\dfrac{1}{p^2}\right)\times 10^{10}\,\text{J}$

$$= \frac{hc}{(1500)(1.6\times 10^{-19})}\left(1-\frac{1}{p^2}\right)\times 10^{10}\,\text{eV}$$

$$= 8.28\left(1-\frac{1}{p^2}\right)\,\text{eV}$$

Hence, energy of n^{th} state is given by

$$E_n = -\frac{8.28}{n^2}\,\text{eV}$$

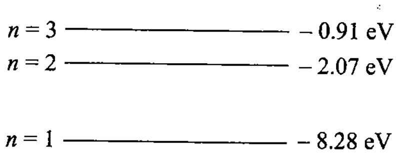


Fig. 4.78

Maximum energy is released for transition from $p = \infty$ to $p = 1$; hence, wavelength of most energetic photon is 1500 Å.

19. d. The ionization potential corresponds to energy required to liberate an electron from its ground state,
i.e. ionization energy = 8.28 eV
Hence, ionisation potential = 8.28 V

For Problems 20–23

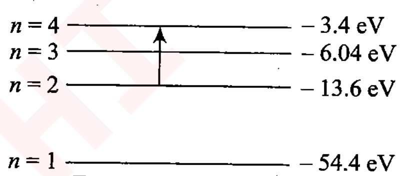
20. b., 21. a., 22. c., 23. b.

Sol. Consider energy level diagram of hydrogen atom. After absorbing photons of energy 10.2 eV, it would reach the first excited state of –3.4 eV, since energy difference corresponding to $n = 1$ and $n = 2$ is 10.2 eV. When this excited hydrogen atom de-excites, it would release 10.2 eV energy, which is absorbed by He^+ and Li^{2+}.

Energy of the n^{th} state of a hydrogen-like atom with atomic number Z is given by

$$E_n = \frac{13.6Z^2}{n^2}\,\text{eV}$$

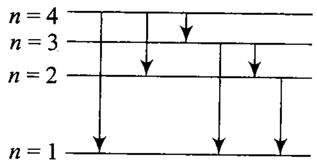
After absorbing 10.2 eV, He^+ electron moves from $n = 2$ to $n = 4$ and Li^{2+} electron moves from $n = 3$ to $n = 6$.



Energy levels of He^+

Fig. 4.79

In the spectrum of He^+, there would be $^4C_2 = 6$ lines.



Transition in He^+

Fig. 4.80

$$\lambda_{\min\ n=4\to n=1} = \frac{hc}{\Delta E} = \frac{1242}{13.6\left[4-\dfrac{4}{16}\right]} = 24.4\,\text{nm}$$

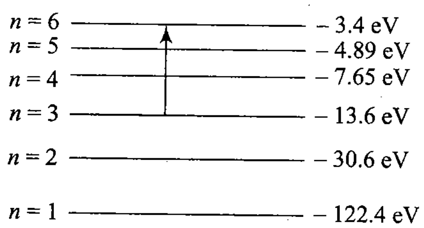


Fig. 4.81

Similarly, in spectrum of Li^{2+} there will be $^6C_2 = 15$ lines.

$$\lambda_{\min\ n=6\to n=1} = \frac{1242}{13.6\left[9-\dfrac{9}{36}\right]} = 10.4\,\text{nm}$$

For Problems 24–25

24. b., 25. d.

Sol.

Let the final speeds of neutron and singly ionized helium atom be v_1 and v_2, respectively. From conservation of momentum, we have

Along x-axis: $mu = 4mv_2 \cos\theta$ (i)

Along y-axis: $mv_1 = 4mv_2 \sin\theta$ (ii)

On squaring and adding equations (i) and (ii), we get $u^2 + v_1^2 = 16v_2^2$

$$\frac{1}{2}mu^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}16\,mv_2^2 \quad \text{(iii)}$$

The initial kinetic energy of neutron = 65 eV.

Let kinetic energies of neutron and helium atom be

$$K_n = \frac{1}{2}mv_1^2,\ K_{He} = \frac{1}{2}(4m)v_2^2$$

So, Eq. (iii) reduces to $65 + K_n = 4K_{He}$

$$4K_{He} - K_n = 65 \quad \text{(iv)}$$

a. Energy required to excite an electron from ground state of singly ionized helium atom (He^+) to n^{th} energy level is $u^2 + v_1^2 = 16v_2^2$

If the neutron has sufficient energy to excite the helium atom, then from conservation of energy, the energy of neutron must be equal to the sum of kinetic energy of neutron, helium atom, and excitation energy. So, we have

$$65 = K_n + K_{He} + 54.4\left(1 = \frac{1}{n^2}\right)$$

$$K_{He} + K_n = 10.6 + \frac{54.4}{n^2} \quad \text{(v)}$$

On solving Eqs. (iv) and (v), we get

$$K_{He} = \left[15.52 + \frac{10.88}{n^2}\right]\text{eV} \quad \text{(vi)}$$

$$K_n = \left[\frac{43.52}{n^2} - 4.52\right]\text{eV} \quad \text{(vii)}$$

The kinetic energy is always positive, so from equation (vii) we have

$$\frac{43.52}{n^2} > 4.52, \quad n < 3.1$$

So, the only possible values of n are 2 and 3.

Possible values of n

2, 3

Allowed values of neutron energy

K_N

6.36 eV, 0.32 eV

Allowed values of He atom energy

K_{He}

17.84 eV, 16.33 eV

b. When the atom de-excites,

$$\nu = \frac{(13.6)2^2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]$$

$$= 13.3\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right] \times 10^{15}\,\text{Hz}$$

The electron excited to $n = 3$ can make three transitions:

$n = 3$ to $n = 1$, $\nu_1 = 11.67 \times 10^{15}$ Hz

$n = 3$ to $n = 2$, $\nu_2 = 9.84 \times 10^{15}$ Hz

$n = 2$ to $n = 1$, $\nu_3 = 1.83 \times 10^{15}$ Hz

For Problems 26–27

26. c., 27. a.

Sol. For a conservative force field,

$$-\frac{dU}{dr} = F$$

Since $U = k\log r$,

$$-\frac{dU}{dr} = -\frac{k}{r} = F$$

This force $F = -k/r$ provides the centripetal force for circular motion of electron.

$$\frac{mv^2}{r} = \frac{k}{r} \quad \text{(i)}$$

Applying Bohr's quantization rule,

$$mvr = \frac{nh}{2\pi} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$r = \frac{nh}{2\pi\sqrt{mk}}$$

From Eq. (i),

$$\text{KE of electron} = \frac{1}{2}mv^2 = \frac{1}{2}k$$

Total energy of electron = KE + PE

PE of electron = $k\log r$

$$= \frac{1}{2}k + k\log r$$

$$E = \frac{K}{2}\left[1 + \log\frac{n^2h^2}{4\pi^2 mk}\right]$$

For Problems 28–30

28. d., 29. c., 30. d.

Sol.

28. d. For Balmer series, $n_1 = 2$; $n_2 = 3, 4, \ldots$

(lower) (higher)

Therefore, in transition (VI), proton of Balmer series is absorbed.

29. c. In transition II,

$$E_2 = -3.4\text{ eV}, E_4 = -0.85\text{ eV}$$

$$\Delta E = 2.55\text{ eV}$$

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

$$\lambda = 487 \text{ nm}$$

30. d. Wavelength of radiation = 103 nm = 1030 Å

$$\Delta E = \frac{12400}{1030 \text{ Å}} = 12.0 \text{ eV}$$

So, difference of energy should be 12.0 eV (approx)

Hence $n_1 = 1$ and $n_2 = 3$
(–13.6) eV (–1.51) eV

Therefore, transition is V.

For Problems 31–33

31. c., 32. b., 33. b.

Sol. Let n be the lowest energy level of the given series of lines, then maximum wavelength is given by

$$\frac{hc}{\lambda_{max}} = 13.6 \times Z^2 \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$\frac{hc}{\lambda_{min}} = 136 \times Z^2 \left[\frac{1}{n^2} - \frac{1}{\infty^2}\right]$$

where $\lambda_{max} = 41.02$ nm and λ_{min} 22.79 nm.

Solving above equations, we get $n = 2$, which corresponds to Balmer series.

$$Z = 4$$

For next to longest wavelength, transition takes place from $n_1 = 4$ to $n_2 = 2$.

So, $$\frac{hc}{\lambda} = 13.6 \times 4^2 \left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\therefore \quad \lambda = 30.47 \text{ nm}$$

For Problems 34–36

34. a., 35. b., 36. d.

Sol.

34. a. The minimum energy required to remove the electron from K-level is 3×10^{-15} J.

Let V be the potential difference required, then

$$eV = 3 \times 10^{-15} \text{ J}$$

$$V = \frac{3 \times 10^{-15}}{1.6 \times 10^{-19}} = 18750 \text{ V}$$

35. b. The energy released due to transition of electron from L-level to K-level is

$$\Delta E = 3 \times 10^{-15} - 2 \times 10^{-16}$$
$$= 28 \times 10^{-16} \text{ J}$$

The wavelength of the electromagnetic wave is,

$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{28 \times 10^{-16}} \text{ m}$$
$$= 7.104 \times 10^{-11} \text{ m}$$

36. d. The minimum energy required to eject the electron from M-level is 3×10^{-17} J. The energy used up in ejecting the electron from M-level is, 28×10^{-16} J. So, the kinetic energy of emitted electrons from M-level is $K = (28 \times 10^{-16} - 3 \times 10^{-17})$ J or $K = 277 \times 10^{-17}$ J

For Problems 37–39

37. d., 38. d., 39. d.

Sol. As photon energy of incident light is not equal to energy difference between two energy states, that is why sample A does not absorb any photon and therefore remains in ground state, and hence, no emission spectra. Thus, sample B remains as it is, but it will de-excite itself to ground state by emitting radiations of energy 10.2 eV.

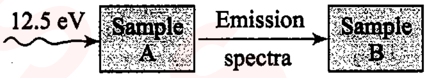


Fig. 4.82

When photon energy is replaced by electron beam, then atoms of sample A can absorb either 10.2 eV (to reach 1st excited state) or 12.1 eV (to reach 2nd excited state). In emission spectra of A, we will have 3 lines corresponding to $n = 2$ to 1, $n = 3$ to 2 and $n = 3$ to 1, having energies equal to 10.2 eV, 1.9 eV, and 12.1 eV, respectively.

As least energy of this emission spectra is corresponding to transition from $n = 2$ to 3 and sample B is in 1st excited state, so B can be excited to some higher energy level and if B absorbs energy corresponding to $n = 1$ to 2 and $n = 1$ to 3, then it may ionize.

For Problems 40–42

40. a., 41. b., 42. a.

Sol.

40. a. $$\Delta E = -13.6 z^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$47.2 = 13.6 z^2 \left[\frac{1}{4} - \frac{1}{9}\right] \Rightarrow z = 5$$

41. b. $$r_n = \frac{0.529}{z} \times 10^{-10} \text{ m}$$
$$= \frac{0.529}{5} \times 10^{-10} = 0.106 \times 10^{-10} \text{ m}$$

42. a. $$L = mvr = \frac{nh}{2\pi}$$

For first Bohr orbit, $L = 1 \times \dfrac{h}{2\pi} = 1.05 \times 10^{-13}$ Js

For Problems 43–44

43. c., 44. a.

Sol.

43. c. Characteristic X-ray arises due to transition of electrons.

44. a. When the accelerated electron beam strikes the nucleus of target atom, continuous X-rays are produced.

For Problems 45–47

45. a., 46. a., 47. c.

Sol.

45. a. In photoelectric effect

$$E = W + K_{max}$$
$$= 0.73 + 1.82$$
$$= 2.55 \text{ eV}$$

46. a. $E_n = \frac{-13.6 z^2}{n^2}$ eV

$E_1 = -13.6$ eV

$E_2 = -3.4$ eV

$E_3 = -1.51$ eV

$E_4 = -0.85$ eV

$E_4 - E_2 = -(0.85) - (-3.4) = 2.55$ eV

47. c. $L_4 - L_2 = \frac{4h}{2\pi} = \frac{2h}{2\pi}$ $\left[L = \frac{nh}{2\pi}\right]$

$= \frac{2h}{2\pi} = \frac{h}{\pi}$

For Problems 48–50

48. a., 49. c., 50. d.

Sol.

48. a. Least intensity $2 \to 1 \Rightarrow K_\alpha$, X-rays

49. c. $4 \to 1 \Rightarrow K_\gamma$, X-rays

50. d. Calculate from energy of Li^{++}.

For Problems 51–52

51. b., 52. d.

Sol.

$$\Delta E = \left\{13.6\, Z_B^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) - 13.6 Z_A^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\right\} \text{eV}$$

$$81.6 \text{ eV} = \frac{1.36 \times 3}{4}(Z_B^2 - Z_A^2) \text{ eV}$$

$$Z_B^2 - Z_A^2 = 8 \quad \text{(i)}$$

Using conservation of momentum,

For A, $m_A u = MV_1 - m_A \frac{u}{2}$

$\frac{3}{2} m_A u = MV_1$

For B, $\frac{3}{2} m_B u = MV_2$

But, $MV_2 = 3MV_1$

$m_B - 3m_A$

Since both A and B carry same number of protons and neutrons, we have

$$Z_B = 3Z_A \quad \text{(ii)}$$

But $Z_B^2 - Z_A^2 = 8$

$9Z_A^2, Z_{BN}^2 = 8$

$Z_A = 1, Z_B = 3$

Hence, A is ${}_1^2$H and B is ${}_4^6$Li

Now, the difference in energy between the first Balmer lines emitted by A and B

$$\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)Z_B^2 - 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)Z_A^2$$

$$= 13.6 \times \frac{5}{36} \times (Z_B^2 - Z_A^2)$$

$$= \frac{13.6 \times 5 \times 8}{36} = 15.1 \text{ eV}$$

For Problems 53–54

53. c., 54. a.

Sol.

53. c. Number of atoms in the second excited state

$$N_2 = \frac{N_{av}}{M} \times m \times \frac{15}{100}$$

$$= \frac{6.023 \times 10^{23}}{1.008} \times 1.8 \times \frac{15}{100} = 1.61 \times 10^{23}$$

In the first excited state,

$$N_1 = \frac{N_{av}}{M} m \times \frac{27}{100} = 2.9 \times 10^{23}$$

54. a. Amount of energy that would be evolved $= U_i - U_f$

$U_1 = N_1E_1 + N_2E_2$

$$= -1.61 \times 10^{23} \times \frac{21 \times 10^{-12}}{3^2}$$

$$- \frac{2.9 \times 10^{23} \times 21.7 \times 10^{-12}}{2^2}$$

$= -3.88 \times 10^{11} - 15.75 \times 10^{11}$

$= -19.63 \times 10^{11}$ erg

$U_f = (N_1 + N_2)E_0$

$= -(1.61 \times 10^{23} + 2.9 \times 10^{23})\, 21.7 \times 10^{-12}$

$= -97.87 \times 10^{11}$ erg

Energy evolved

$= U_i - U_f = (-19.63 \times 10^{11} + 97.87 \times 10^{11})$

$= 78.237 \times 10^{11} = 782$ kJ

For Problems 55–57

55. d., 56. a., 57. d.

Sol.

55. d. $E_2 - E_1 = \left(\frac{E_1}{2^2} - E_1\right)$

$$\frac{hc}{\lambda_{2\to1}} = \frac{1242 \text{ eV nm}}{94.54 \text{ nm}} = 13.14 \text{ eV}$$

$E_1 = -17.52 \text{ eV} = Z^2 E_{01}$

$$E_3 - E_1 = \frac{hc}{\lambda_{3\to1}} = \frac{1242 \text{ eV nm}}{79.76 \text{ nm}} = 15.57 \text{ eV}$$

$E_3 = -1.95$ eV

$$E_4 - E_1 = \frac{hc}{\lambda_{4\to 1}} = \frac{1242\text{ eV nm}}{75.63\text{ nm}} = 16.42\text{ eV}$$

56. a. For the transition $n = 4 \to n = 2$

$$\Delta E_{4\to 2} = (E_4 - E_1) - (E_2 - E_1)$$
$$= 16.42\text{ eV} - 13.14\text{ eV} = 3.28\text{ eV}$$

$$\lambda_{4\to 2} = \frac{hc}{\Delta E_{4\to 2}} = \frac{(1242\text{ eV nm})}{3.28\text{ eV}}$$
$$= 378.65\text{ nm} \approx 380\text{ nm}$$

57. d. The possible energy of the atom in $n = 3$,

$$E_{03} = \frac{E_3}{Z^2}$$

For $Z = 1$, $E_{03} = -1.95$ eV
$Z = 2$, $E_{03} = -0.4875$ eV
$Z = 3$, $E_{03} = -0.2166$ eV
$Z = 4$, $E_{03} = -0.121$ eV

Matching Column Type

1. a. → p., r.; b. → q., s.; c. → q., s.; d. → q., r.

a. $f = \frac{mZ^2e^4}{4\varepsilon_0^2 h^3 n^3} \Rightarrow f \propto \frac{Z^2}{n^3}$

b. $L = \frac{nh}{2\pi} \Rightarrow L \propto n$

c. Magnetic moment: $M = IA = \frac{e}{2\pi r} v\pi r^2$

$$= \frac{e}{2} vr = \frac{e}{2m}(mvr) = \frac{e}{2m} L$$

$$\Rightarrow M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) \Rightarrow \propto n$$

d. $I = \frac{ev}{2\pi r} = \frac{e}{2\pi}\left(\frac{\pi m Z e^2}{n^2 h^2 \varepsilon_0}\right)\frac{Ze^2}{2\varepsilon_0 nh} \Rightarrow I \propto \frac{Z^2}{n^3}$

2. a. → s.; b. → p.; c. → s.; d. → r.

a. Photoelectrons can have KE anywhere from 0 to $KE_{max} = hc/\lambda - \phi$.
So, average KE is unpredictable.

b. Minimum KE can be zero.

c., d. In continuous X-rays, wavelength can vary from $\lambda_{min} = hc/eV$ to ∞.

3. a. → q., r.; b. → p., r.; c. → p., r.; d. → s.

a. KE of electrons striking the target: $KE = eV$.
So, if V increases, KE increases.

Cut-off wavelength: $\lambda_{min} = \frac{12400}{V}$ Å

So, if V increases, λ_{min} decreases.

b. If work function of target is increased in photoelectric effect, the KE of photoelectrons emitted decreases from $KE = hf - W_0$

Now, $hf_0 = W_0 \Rightarrow \frac{hc}{\lambda_0} = W_0 \Rightarrow \lambda_0 = \frac{hc}{W_0}$

If work function (W_0) increases, then cut-off wavelength (λ_0) decreases.

c. $eV_0 = kE$, where V_0 is stopping potential. If V_0 decreases, then KE decreases.

V_0 is also decreased by increasing W_0, hence λ_0 decreases as explained above.

d. $f_{k_\alpha} = \frac{3}{4}Rc(z-1)^2$. If f_{k_α} increases, then f_{k_α} decreases and hence Z decreases.

4. a. → r.; b. → p., q., r., s.; c. → q., d. → s.

a. Moseley's law is about characteristic X-rays.

b. In photoelectric effect, from electromagnetic radiation, electrons are ejected. In X-rays, the process is reverse; here, electromagnetic radiation (X-ray) is produced from fast moving electrons.

Also X-rays are produced using high potential difference.

c. Cut-off wavelength is related to potential difference applied in continuous X-rays.

d. In continuous X-rays, electromagnetic radiations are emitted.

5. a. → r., s.; b. → q., r., s.; c. → r., s.; d. → q., r., s.

From Moseley's law: $f = \frac{1}{\lambda} = R(z-\sigma)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For Al, Z is highest and for K_β, $n_1 = 1$ and $n_2 = 3$; and for K_α, $n_1 = 1$, and $n_2 = 2$

Hence, order of frequency:

$$f_{k_\beta}(\text{Al}) > f_{k_\alpha}(\text{Al}) > f_{k_\alpha}(\text{Na}) > f_{k_\beta}(\text{Be}) > f_{k_\alpha}(\text{Li})$$

Speed will be same (c) for all photons of any frequency.

6. a. → q., r., s.; b. → p., q., r., s.; c. → q., r., s.; d. → q., r., s.

a. Emission spectra is discrete as only those wavelengths are emitted which correspond to energy difference of two energy levels.

It is due to electronic transition, i.e., due to transition of electron from one energy level (higher) to the another energy level (lower). It is explained by quantum theory of light.

b. For energies of incident light less than ionization energy, the absorption spectra is discrete. For energies of incident photon greater than ionization energy, the absorption spectra is continuous. It occurs due to electronic transition which is explained by quantum theory of light.

c. Continuous X-rays constitute continuous spectra and characteristic X-rays arise due to electronic transition.
d. Thermal radiation spectra is a continuous spectra and is explained by atomic transition and quantum theory.

7. a. → s.; b. → t.; c. → q.; d. → p.

$$r_n = \frac{.529 n^2}{Z} \text{ Å}$$

So, (a) → (s)

Magnetic field, $B = \frac{12.5 Z^3}{n^5}$ T

So, (b) → (t)

$$B \propto \frac{1}{n^5}$$

$$V_n = \frac{2.2 \times 10^6 Z}{n}$$

$$V_n \propto \frac{1}{n}$$

$$n \uparrow V_n \uparrow$$

So, (c) → (q)

Total energy, $E_n = \frac{-13.6 Z^2}{n^2}$ eV

$$n \uparrow E_n \uparrow$$

So, d. → p.

8. a. → q.; b. → r.; c. → s.; d. → p.

$$r_n = \frac{.529 n^2}{Z} \text{ Å}$$

So, (a) → (q)

$$V_n = \frac{2.2 \times 10^6 Z}{n} \text{ ms}^{-1}$$

So, (d) → (p)

$$I = \frac{1.06 Z^2}{n^3} \text{ mA}$$

So, (b) → (r)

Magnetic field, $B = \frac{12.5 Z^3}{n^5}$ T

So, c. → s.

9. a. → s.; b. → p.; c. → q.; d. → r.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Wavelength range from 950 Å to 1350 Å corresponds to ultraviolet region of electromagnetic spectrum, i.e., Lyman series.

For transition 4 → 1:

$$\frac{1}{\lambda_1} = 1.1 \times 10^7 \left(\frac{15}{16}\right)$$

$$\therefore \quad \lambda_1 = \frac{16}{15 \times 1.1 \times 10^7} = 970 \text{ Å}$$

For transition 3 → 1:

$$\lambda_2 = \frac{9}{8 \times 1.1 \times 10^7} = 1023 \text{ Å}$$

For transition 2 → 1:

$$\lambda_3 = \frac{4}{3 \times 1.1 \times 10^7} = 1212 \text{ Å}$$

For Li^2 atom ($Z = 3$), for transition 2 → 1:

$$\frac{1}{\lambda'} = RZ^2\left(1 - \frac{1}{2^2}\right)$$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 9 \times 0.75} = 134 \text{ Å}$$

For He^+ atom ($Z = 2$) for transition 2 → 1:

$$\frac{1}{\lambda'} = RZ^2\left(1 - \frac{1}{2^2}\right)$$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 4 \times 0.75} = 303 \text{ Å}$$

Integer Answer Type

1. (4) $E_{photon} = 13.6\left(1 - \frac{1}{25}\right)\text{eV} = 13.0 \text{ eV}$

$E/c = mv$ (momentum conserved)

$$v = \frac{E}{mc} = \frac{(13)(1.6 \times 10^{-19})}{(1.67)(10^{-27})(3)(10^8)} = 4 \text{ m/s}$$

2. (6) $\frac{\lambda_1}{\lambda_2} = \frac{(Z_2 - 1)^2}{(Z_1 - 1)^2}$ $\left(\text{since, } \frac{1}{\lambda} \propto (Z-1)^2\right)$

$$\frac{1}{4} = \frac{(Z_2 - 1)^2}{(11 - 1)^2}; \text{ on solving, } Z_2 = 6$$

3. (6) When electron jumps from n^{th} state to ground state, number of possible emission lines $= n(n-1)/2$. But here transition takes place from n^{th} state to state $n_1 = 2$
Here, number of possible emission lines $= n(n-1)(n-2)/2 = 10$ (given).
On solving, $n = 6$.

4. (2) The shortest wavelength of Brackett series is corresponding to transition of electron between $n_1 = 4$ and $n_2 = \infty$.
Similarly, the shortest wavelength of Balmer series is corresponding to transition of electron between $n_1 = 2$ and $n_2 = \infty$.

$$(Z)^2\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right) \quad \text{or} \quad Z = 2$$

5. (1) Heat produced/sec = 200 W

$$\Rightarrow \quad i = \frac{200}{V} = \frac{200}{20 \times 10^3} = 10 \text{ mA}$$

6. (8) Change in angular momentum $= I\omega$

$$\Rightarrow \quad \frac{mh}{2\pi} - \frac{nh}{2\pi} = I\omega \quad \Rightarrow \quad \omega = \frac{(m-n)h}{6.28\, I}$$

7. (4) For the first line of Balmer series of hydrogen

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}\Rightarrow\lambda=\frac{36}{5R}$$

For singly ionized helium $Z=2$

$$\frac{1}{\lambda'}=4R\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$$

Given $\lambda'=\lambda \Rightarrow 4R\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)=\frac{5R}{36}$

$\Rightarrow n_2=4$

8. (5) $E_n=-\frac{mZ^2e^4}{8\varepsilon_0^2n^2h^2}$, so $hf=+\frac{mZ^2e^4}{8\varepsilon_0^2h^2}\left[\frac{1}{16}-\frac{1}{25}\right]$

$$\therefore \quad f=\frac{mZ^2e^4}{8\varepsilon_0^2h^3}\left[\frac{9}{16\times 25}\right] \quad \text{(i)}$$

and frequency $f_4=\frac{Z^2e^4m}{4\varepsilon_0^2n^3h^3}=\frac{Z^2e^4m}{4\varepsilon_0^2(4)^3h^3}$ (ii)

$\therefore$ $f/f_4=18/25$, so $m=5$

9. (9) Number of revolutions before transition = frequency $\times$time

Also, frequency $\propto \frac{1}{n^3}$

So required ratio $=\frac{(1/2)^3 12.8\times10^{-8}}{(1/3)^3 4.8\times10^{-8}}=9$

10. (2) $\frac{T_1}{T_2}=\frac{n_1^3}{n_2^3} \Rightarrow \frac{8_1}{8_2}=\frac{n_1^3}{n_1^3} \Rightarrow \frac{n_1}{n_2}=\frac{2}{1}$

Archives

Fill in the Blanks Type

1. For minimum accelerating voltage, the electron should jump from $n=2$ to $n=1$ level.

For characteristic X-ray, $\frac{1}{\lambda}=R_\alpha(z-1)^2\left[1=\frac{1}{n^2}\right]$

$$=R_\alpha(z-1)^2$$

But $E=h\frac{c}{\lambda} \Rightarrow \frac{1}{\lambda}=\frac{E}{hc}$

$$\therefore \frac{E}{hc}=R_\alpha=\frac{E}{hc}$$

$$\Rightarrow \frac{E_1}{E_2}=\frac{\left[1-\frac{1}{2^2}\right]}{\left[1-\frac{1}{\alpha^2}\right]}$$

$\Rightarrow E_1=\frac{3}{4}E_2=\frac{3}{4}$ 40,000 eV = 30,000 eV

2. Intensity decreases.
More the number of electrons striking the anode, more is the emitted number of photons of X-ray.

$\left[\because \lambda=\frac{12375}{V}\text{Å}\right]$, where V is in volt.

3. For K_α, $\frac{1}{\lambda}=c\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

$$\Rightarrow \frac{1}{0.32}=c\left[\frac{1}{1^2}-\frac{1}{2^2}\right]=\frac{3c}{4} \quad \text{(i)}$$

For K_β, $1=c\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=\frac{8c}{9}$ (ii)

On dividing, we get $\lambda=0.27$

4. The fifth valance electron of phosphorous is in its third shell, i.e., $n=3$. For phosphorous, $Z=15$. Also, the Bohr's radius for n^{th} orbit

$$r_n=\left(\frac{n^2}{Z}\varepsilon r\right)r_0=\frac{3^2}{15}\times12\times0.529\text{ Å}=3.81\text{ Å}$$

5. The speed of X-rays is always $3\times10^8\text{ ms}^{-1}$ in vacuum. It does not depend on the potential differences through which electrons are accelerated in an X-ray tube.

6. $KE=\frac{KZe^2}{2r}$ and

Total energy, $TE=\frac{-KZe^2}{2r} \Rightarrow \frac{|KE|}{|TE|}=1$

7. $\frac{1}{\lambda}=R(z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

Since for K_α, $n_2=2$ and $n_1=1$

$$\therefore \frac{1}{0.76\times10^{-10}}=1.097(z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$$

$\Rightarrow z-1=40 \Rightarrow z=41$

8. For photon emitted from hydrogen atom, the wavelength is

$$\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \text{(i)}$$

But according to de Broglie concept

$$\lambda=\frac{h}{p}\Rightarrow\frac{1}{\lambda}=\frac{p}{h} \quad \text{(ii)}$$

From (i) and (ii),

$$\frac{p}{n}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \Rightarrow p=Rh\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \text{(iii)}$$

Since the momentum of the hydrogen atom initially was zero, therefore finally the momentum of photon is equal to momentum of hydrogen atom in magnitude (By law of conservation of momentum). Let the momentum of hydrogen atom be $m_H V_H$. Then, from (iii)

$$m_H V_H = R_h\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad V_H = \frac{Rh}{m_H}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$= \frac{1.097\times10^{7}\times6.63\times10^{-34}}{1.67\times10^{-27}}\left(\frac{1}{1^2}-\frac{1}{5^2}\right)$$

$$\Rightarrow \quad V_H = 4.178\ \text{ms}^{-1}$$

Alternatively:

Energy of photon

$$= E = E_5 - E_1 = -13.6\left[\frac{1}{5^2}-\frac{1}{2^2}\right]\text{eV}$$

$$= 2.09\times10^{-18}\ \text{J}$$

According to momentum conservation,

Momentum of recoil hydrogen atom = Momentum of photon

$$\therefore \quad mv = \frac{E}{c}$$

$$\Rightarrow \quad v = \frac{E}{mc} = \frac{2.09\times10^{-18}}{(1.67\times10^{-27})(3\times10^{8})} = 4.17\ \text{ms}^{-1}$$

Multiple Choice Questions with Single Correct Answer Type

1. **b.** Shortest wavelength or cut-off wavelength depends only upon the voltage applied in the Coolidge tube.

2. **a.** The maximum number of electrons in an orbit is $2n^2$. Since $n > 4$ is not possible, therefore the maximum number of electrons that can be in the first four orbits are

$$2(1)^2+2(2)^2+2(3)^2+2(4)^2 = 2+8+18+32 = 60$$

Therefore, possible elements are 60.

3. **d.** We know that

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right] \Rightarrow \frac{1}{\lambda} \propto z^2$$

λ is shortest when $1/\lambda$ is largest, i.e., when z is big. z is highest for lithium.

4. **c.**

5. **c.** $E = \dfrac{hc}{\lambda} = \left[\dfrac{6.63\times10^{-34}\times3\times10^{8}}{0.021\times10^{-9}}\right]$ J

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{0.021\times10^{-9}\times1.6\times10^{-13}}\ \text{MeV}$$

$$= 591.96\times10^{-4}\ \text{MeV} = 59.196\ \text{keV}$$

$\therefore$ (c) is the correct option.

6. **d.** For hydrogen and hydrogen-like atoms,

$$E_n = -13.6\,\frac{(z)^2}{(n)^2}\ \text{eV}$$

Therefore, ground state energy of doubly ionized lithium atom ($z = 3, n = 1$) will be

$$E_1 = (-13.6)\,\frac{(3)^2}{(2)^2} = -122.4\ \text{eV}$$

$\therefore$ Ionization energy of an electron in ground state of doubly ionized lithium atom will be 122.4 eV.

7. **b.** The continuous X-ray spectrum is shown in Fig. 4.83.

All wavelengths $> \lambda_{min}$ are found where $\lambda_{min} = \dfrac{12375}{V}$ Å

Here, V is the applied voltage.

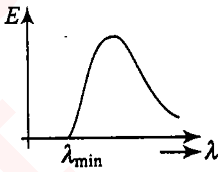


Fig. 4.83

8. **c.** We know that $\lambda \propto \dfrac{1}{m}$

For ordinary hydrogen atom,

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5R}{36} \quad \text{or} \quad \lambda = \frac{36}{5R}$$

With hypothetical particle, required wavelength

$$\lambda' = \frac{1}{2}\times\frac{36}{5R} = \frac{18}{5R}$$

9. **a.** We know that as the electron comes nearer to the nucleus, the potential energy decreases $\left(\dfrac{-KZe^2}{r} = \text{PE and } r \text{ decreases}\right)$.

The KE will increase $\left[\because KE = \dfrac{1}{2}|PE| = \dfrac{1}{2}\dfrac{kZe^2}{r}\right]$.

The total energy decreases $\left[TE = \dfrac{1}{2}\dfrac{kZe^2}{r}\right]$.

10. **d.** $\lambda_{min} = \dfrac{hc}{E} \Rightarrow \lambda_{min} = \dfrac{12400}{80\times10^3}$ Å $= 0.155$

11. **d. Method 1:** Memorisation.

In Lyman series, we get energy in UV region.

In Balmer series, we get energy in visible region.

In Paschen/Brackett/Pfund series, we get energy in IR region.

Method 2:

We know that $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ for hydrogen atom for e^- transition from $n_2 \to n_1$.

For 4 to 3, $E \propto \frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7}{9\times16}R$

For 2 to 1, 3 to 2, and 4 to 2, we get more value of $1/\lambda$, i.e., more energy than $4 \to 3$.
IR radiation has less energy than UV radiation.
Therefore, the correct option is (d).

12. a. In case of coolidge tube,

$$\lambda_{\min} = \frac{hc}{eV} = \lambda_c \text{ (as given here)}$$

Thus, the cut-off wavelength is inversely proportional to accelerating voltage. As V increases, λ_c decreases.

λ_k is the wavelength of K_∞ line which is a characteristic of an atom and does not depend on accelerating voltage of bombarding electron since λ_k always refers to a photon wavelength of transition of e^- from the target element from 2 $\to$ 1.

The above two facts lead to the conclusion that $\lambda_k - \lambda_c$ increases as accelerating voltage is increased.

13. a. No. of electrons striking the target per second

$$= \frac{i}{e} = 2\times10^{16}$$

14. b. $L = \frac{nh}{2\pi}$, $|E| \propto Z^2/n^2$; $n = 3$

$$\Rightarrow \quad L_H = L_{Li} \quad \text{and} \quad |E_H| < |E_{Li}|$$

15. a. Given potential energy between electron and proton is

$$V_0 \log_e \frac{r}{r_0}$$

$$\therefore \quad |F| = \frac{d}{dr}\left[V_0 \log_e \frac{r}{r_0}\right] = \frac{V_0}{r_0} \times \frac{1}{r}$$

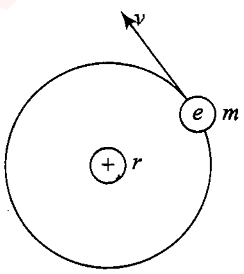


Fig. 4.84

But this force acts as centripetal force

$$\therefore \quad \frac{mv^2}{r} = \frac{V_0}{rr_0} \Rightarrow mv^2 = \frac{V_0}{r_0} \qquad \text{(i)}$$

By Bohr's postulate, $mvr = \frac{nh}{2\pi}$ (ii)

From (i) and (ii),

$$\frac{m^2v^2r^2}{mv^2} = \frac{n^2h^2r_0^2}{4\pi^2V_0^2} \Rightarrow r^2 = \frac{n^2h^2r_0^2}{4\pi V_0^2 m}$$

$$\Rightarrow \quad r \propto n$$

16. d. For an atom following Bohr's model, the radius is given by

$$r_m = \frac{r_0 m^2}{Z}$$

where r_0 = Bohr's radius and
m = orbit number.

For Fm, $m = 5$ (i.e. fifth orbit in which the outermost electron is present)

$$\therefore \quad r_m = \frac{r_0 5^2}{100} = 100n \text{ (given)} \quad \Rightarrow \quad n = \frac{1}{4}$$

17. a. According to Moseley's law,

$$\sqrt{f} = a(z-b) \Rightarrow f = a^2(z-b)^2$$

$$\Rightarrow \quad \frac{c}{\lambda} = a^2(z-b)^2 \qquad \text{(i)}$$

For K_a line, $b = 1$

From (i), $\frac{\lambda_2}{\lambda_1} = \frac{(z_1-1)^2}{(z_2-1)^2} \Rightarrow \frac{4\lambda}{\lambda} = \frac{(11-1)^2}{(z_2-1)^2}$

$$\Rightarrow \quad Z_2 - 1 = \frac{10}{2} \Rightarrow z_2 = 6$$

Therefore, (a) is the correct option.

18. a. Initially, a photon of energy 10.2 eV collides inelastically with a hydrogen atom in ground state.
For hydrogen atom,

$$E_1 = -13.6 \text{ eV}; E_2 = -\frac{13.6}{4} = -3.4 \text{ eV}$$

$$\therefore \quad E_2 - E_1 = 10.2 \text{ eV}$$

The electron of hydrogen atom will jump to second orbit after absorbing the photon of energy 10.2 eV. The electron jumps back to its original state in less than a microsecond and releases a photon of energy 10.2 eV. Another photon of energy 15 eV strikes the hydrogen atom inelastically. This energy is sufficient to knock out the electron from the atom as ionization energy is 13.6 eV. The remaining energy of 1.4 eV is left with the electron as its kinetic energy.

19. b. The smallest frequency and largest wavelength in ultra-violet region will be for transition of electron from orbit 2 to orbit 1. Therefore,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{122\times10^{-9}m} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow \quad R = \frac{4}{3 \times 122 \times 10^{-9}} \text{ m}^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{4}{3 \times 122 \times 10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \quad \lambda = \frac{3 \times 122 \times 9 \times 10^{-9}}{4} = 823.5 \text{ nm}$$

20. a. The cut-off wavelength is given by

$$\lambda_0 = \frac{hc}{eV} \qquad \text{(i)}$$

According to de Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda^2 = \frac{h^2}{2meV}$$

$$\Rightarrow \quad V = \frac{h^2}{2me\lambda^2} \qquad \text{(ii)}$$

From (i) and (ii), $\lambda_0 = \dfrac{hc \times 2me\lambda^2}{eh^2} = \dfrac{2mc\lambda^2}{h}$

21. b. Cut-off wavelength depends on the applied voltage and not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of the target.

22. b $\dfrac{1}{\lambda_{H_2}} = RZ_H^2\left[\dfrac{1}{4} - \dfrac{1}{9}\right] = R(1)^2\left[\dfrac{5}{36}\right]$

$$\frac{1}{\lambda_{He}} = RZ_{He}^2\left[\frac{1}{4} - \frac{1}{16}\right] = R(4)^2\left[\frac{3}{16}\right]$$

$$\frac{\lambda_{H_2}}{\lambda_{He}} = \frac{1}{4}\left[\frac{16}{3} \times \frac{5}{36}\right] = \frac{5}{27}$$

$$\lambda_{He} = \frac{5}{27} \times 6561 = 1215 \text{ Å}$$

Multiple Choice Questions with One or More than One Correct Answer Type

1. a., c., d.

2. c., d. In the case of hydrogen,
atomic number = mass number.
In other atoms, atomic number < mass number.
Therefore, (c) and (d) are the correct options.

3. c., d.

4. a., d.
The time period of the electron in a Bohr orbit is given by
$T = 2\pi r/v$
Since for the n^{th} Bohr orbit, $mvr = n(h/2\pi)$, the time period becomes

$$T = \frac{2\pi r}{nh/(2\pi mr)} = \left(\frac{4\pi^2 m}{nh}\right) r^2$$

Since the radius of the orbit r depends on n, we replace r.

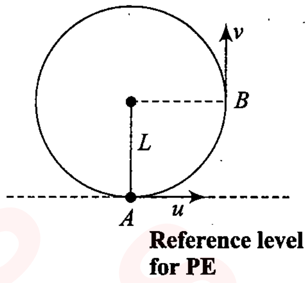


Fig. 4.85

The expression of Bohr radius of a hydrogen atom is $r = n^2\left(\dfrac{h^2\varepsilon_0}{\pi me^2}\right)$

Hence, $T = \left(\dfrac{4\pi^2 m}{nh}\right)\left(\dfrac{n^4h^4\varepsilon_0^2}{\pi^2 m^2 e^4}\right) = n^3\left(\dfrac{4h^3\varepsilon_0^2}{me^4}\right)$

For two orbits, $\dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3$

It is given that $T_1/T_2 = 8$. Hence, $n_1/n_2 = 2$.

Assertion and Reasoning Type

1. b. Cut-off wavelength depends on the accelerating voltage, not on the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilized in producing X-rays. Rest 98% is lost in heat.
Therefore, option (b) is correct.

Comprehension Type

1. c.

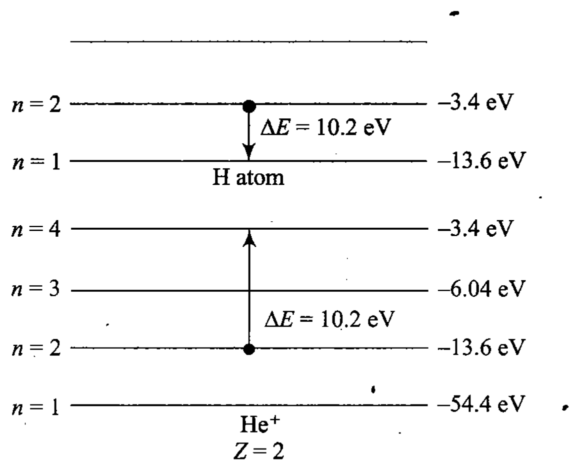


Fig. 4.86

Energy given by H atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

2. c. Visible light lies in the range, $\lambda_1 = 4000$ Å to $\lambda_2 = 7000$ Å. Energy of photons corresponding to these wavelengths (in eV) would be:

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}, \quad \text{and}$$

$$E_2 = \frac{12375}{7000} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04)$$
$$= 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{264} \text{ Å} = 4687.5 \text{ Å}$$
$$= 4.68 \times 10^{-7} \text{ m}$$

3. a. Kinetic energy, $K \propto Z^2$

$$\frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

4. d. $L = \dfrac{nh}{2\pi}$

$$\text{K.E.} = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I}$$

5. a. $h\nu = KE_{n=2} - kE_{n=1}$

$I = 1.87 \times 10^{-46} \text{ kg m}^2$

6. c. $r_1 = \dfrac{m_2 d}{m_1 + m_2}$ and $r_2 = \dfrac{m_1 d}{m_1 + m_2}$

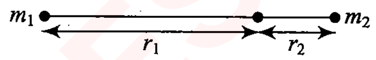


Fig. 4.87

$$I = m_1 r_1^2 + m_2 r_2^2$$

$$\therefore \quad d = 1.3 \times 10^{-10} \text{ m}$$

CHAPTER 5

Nuclear Physics

- Nuclear Structure
- Atomic Mass Number
- Some Definitions
- Size of Nuclei
- Nuclear Binding Energy
- Mass–Energy Equivalence
- Binding Energy Per Nucleon
- Q Values
- Nuclear Stability
- Radioactivity
- Stability and Neutron-Proton Ratio (n/p)
- Radioactive Decay and Activity
- Measurement of Radioactivity
- Fundamental Laws of Radioactivity
- Radioactive Decay Law
- Activity
- Half-Life Time
- Alternative Form of Decay Equation in Terms of Half-Life Time
- Average Life
- Radioactive Dating
- Natural Radioactivity: Radioactive Decay Series
- Nuclear Reactions
- Nuclear Fission
- Nuclear Reactors
- Nuclear Fusion
- Fusion Reactors

Nuclear physics is the branch of physics which deals with the study of nucleus.

NUCLEAR STRUCTURE

Atoms consist of electrons in orbit about a central nucleus. The electron orbits are quantum mechanical in nature and have interesting characteristics. Little has been said about the nucleus, however. Since the nucleus is interesting in its own right, we now consider it in greater detail.

The nucleus of an atom consists of neutrons and protons, collectively referred to as nucleons. The neutron, discovered in 1932 by the English physicist James Chadwick (1891–1974), carries no electrical charge and has a mass slightly larger than that of a proton.

The number of protons in the nucleus is different in different elements and is given by the atomic number Z. In an electrically neutral atom, the number of nuclear protons equals the number of electrons in orbit around the nucleus. The number of neutrons in the nucleus is N. The total number of protons and neutrons is referred to as the atomic mass number A because the total nuclear mass is approximately equal to A times the mass of a single nucleon:

$$\underset{\substack{\text{[Number of proton}\\ \text{and neutrons}\\ \text{(atomic mass}\\ \text{number or nucleon}\\ \text{number)]}}}{A} = \underset{\substack{\text{[Number of}\\ \text{protons (atomic}\\ \text{number)]}}}{A} + \underset{\substack{\text{[Number of}\\ \text{neutrons]}}}{N} \quad \text{(i)}$$

Sometimes A is also called the nucleon number. A shorthand notation is often used to specify Z and A along with the chemical symbol for the element. For instance, the nuclei of all naturally occurring aluminum atoms have $A = 27$, and the atomic number for aluminum is $Z = 13$. In shorthand notation, then, the aluminium nucleus is specified as ${}^{27}_{13}\text{Al}$. The number of neutrons in an aluminium nucleus is $N = A - Z = 14$. In general, for an element whose chemical symbol is X, the symbol for the nucleus is shown in Fig. 5.1.

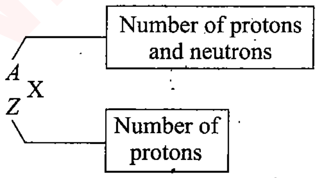


Fig. 5.1

For a proton the symbol is ${}^{1}_{1}\text{H}$, since the proton is the nucleus of a hydrogen atom. A neutron is denoted by ${}^{1}_{0}\text{n}$. In the case of an electron we use ${}^{0}_{-1}\text{e}$, where $A = 0$ because an electron is not composed of protons or neutrons and $Z = -1$ because the electron has a negative charge.

The number of protons in the nucleus characterizes the species of the atom—the element to which the atom belongs. Electrons can be added to or removed form an atom to form an ion, but this does not change its species. For example, removing two electrons from calcium results in calcium ion Ca^{2+}, but its species is still calcium. The number of electrons can vary by ionization, but the proton number cannot vary without changing an atom of a different element, the chemical symbol implies the proton number.

Each nuclear species with a given Z and A is called a nuclide. Each Z characterizes a chemical element. Nucleus mass is roughly the sum of its constituent proton and neutron masses. The nuclear charge is Z (atomic number) times the charge of a proton ($e = 1.6 \times 10^{-19}$ C).

The chemical properties of an atom are determined by its electron configuration. The number of electrons and protons are equal in number in a neutral atom; the chemical properties are essentially determined by Z. The dependence of the chemical properties on N is negligible.

Illustration 5.1 **How many electrons, protons and neutrons are there in 12 g of ${}_6C^{12}$ and in 14 g of ${}_6C^{14}$?**

Sol. Mass number (or atomic weight) of ${}_6C^{12}$ is 12.

Therefore, the number of atoms in 12 g of ${}_6C^{12}$ is

$$\text{Avogadro number} = 6 \times 10^{23}$$

The number of electrons in 12 g of ${}_6C^{12}$ is

$$6 \times 6 \times 10^{23} = 36 \times 10^{23}$$

The number of protons in 12 g of ${}_6C^{12}$ is 36×10^{23}.

The number of neutrons in 12 g of ${}_6C^{12}$ is

$$(A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}$$

Similarly, number of electrons in 14 g of ${}_6C^{14}$ is 36×10^{23}.

Number of protons in 14 g of ${}_6C^{14}$ is 36×10^{23}.

Number of neutrons in 14 g of ${}_6C^{14}$ $= (A - Z) \times 6 \times 10^{23}$

$$= (14 - 6) \times 6 \times 10^{23}$$
$$= 48 \times 10^{23}$$

ATOMIC MASS NUMBER

It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

Atomic masses refer to the masses of neutral atoms. Thus, an atomic mass always includes the mass of its Z electrons. The atomic mass of an atom is measured relative to the mass of an atom of the neutral carbon-12 isotope (the nucleus plus six electrons). Atomic masses are measured in atomic mass units, which are denoted by the symbol u. Atomic mass units are defined in terms of the mass of the isotope ^{12}C, whose atomic mass is defined to be exactly 12 u.

$$1 \text{ a.m.u.} = \frac{1}{12} \times [\text{mass of one atom of } {}_6C^{12} \text{ atom at rest and in ground state}]$$

$$= 1.6603 \times 10^{-27} \text{ kg}$$

Table 5.1 Properties of particles in the atom

Particle	Electric charge (C)	Mass	
		Kilogram (kg)	Atomic mass units (u)
Electron	-1.60×10^{-19}	9.109390×10^{-31}	5.485799×10^{-4}
Proton	$+1.60 \times 10^{-19}$	1.672623×10^{-27}	1.007276
Neutron	0	1.674929×10^{-27}	1.008665

SOME DEFINITIONS

1. **Isotopes:** The nuclei having the same number of protons but different number of neutrons are called isotopes.

   Isotopes of carbon: ${}^{6}_{11}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$, ${}^{14}_{6}C$

   Isotopes of hydrogen: ${}^{1}_{1}H$, ${}^{2}_{1}H$, ${}^{3}_{1}H$

   The isotopes ${}^{1}H$ is called ordinary hydrogen or simply hydrogen; ${}^{2}H$ is called deuterium. Deuterium, sometimes known as heavy hydrogen, can combine with oxygen to form heavy water (D_2O). The third isotope of hydrogen, ${}^{3}H$, is called tritium, which is unstable.
2. **Isotones:** Nuclei with the same neutron number N but different atomic number Z are called isotones.

   $${}^{14}_{6}C,\ {}^{15}_{7}N,\ {}^{16}_{8}C,\ {}^{17}_{9}F$$
3. **Isobars:** The nuclei with the same mass number but different atomic number are called isobars.

   $${}^{16}_{6}C,\ {}^{16}_{7}N,\ {}^{16}_{8}O,\ {}^{16}_{9}F$$

SIZE OF NUCLEI

The size and structure of nuclei were first investigated in the scattering experiments of Rutherford. Using the principle of conservation of energy, Rutherford found an expression for how close an alpha particle moving directly toward the nucleus can come to the nucleus before being turned around by Coulomb repulsion.

In such a head-on collision, the kinetic energy of the incoming alpha particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around (Fig. 5.2). If we equate the initial kinetic energy of the alpha particle to the maximum electrical potential energy of the system (alpha particle + target nucleus), we have

$$\frac{1}{2}mv^2 = k_e\frac{q_1q_2}{r} k_e\frac{(2e)(Ze)}{d}$$

where d is the distance of closest approach. Solving for d, we get

$$d = \frac{4k_eZ_e^2}{mv^2}$$

From this expression, Rutherford found that alpha particles approached to within 3.2×10^{-14} m of a nucleus when the foil was made of gold. Thus, the radius of the gold nucleus must be less than this value. For silver atoms, the distance of closest approach was 2×10^{-14} m. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, whose radius is no greater than about 10^{-14} m. Because such small lengths are common in nuclear physics, a convenient unit of length is the femtometer (fm), sometimes called the Fermi, defined as

$$1 \text{ fm} = 10^{-15} \text{ m}$$

Since the time of Rutherford's scattering experiments, a multitude of other experiments have shown that most nuclei are approximately spherical and have an average radius given by

$$r = r_0A^{1/3} \qquad \text{(i)}$$

where A is the total number of nucleons and r_0 is a constant equal to 1.2×10^{-15} m. Because the volume of a sphere is proportional to the cube of its radius, it follows from Eq. (i) that the volume of a nucleus (assumed to be spherical) is directly proportional to A, the total number of nucleons. This suggests that all nuclei have nearly the same density. Nucleons combine to form a nucleus as though they were tightly packed spheres (Fig. 5.3).

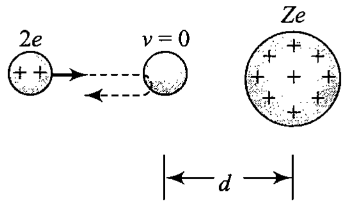


Fig. 5.2 An alpha particle on a head-on collision with a nucleus of charge Ze. Because of the the Coulomb repulsion between charges of the same sign, the alpha particle approaches to a distance d from the target nucleus, called the distance of closest approach.

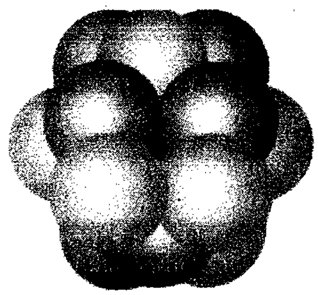


Fig. 5.3 A nucleus can be modelled as a cluster of tightly packed spheres, each of which is a nucleon.

Illustration 5.2 (i) **Find an approximate expression for the mass of a nucleus of mass number A.**

(ii) Find an expression for the volume of this nucleus in terms of the mass number.

(iii) Find a numerical value for its density.

Sol. (i) The mass of the proton is approximately equal to that of neutron. Thus, if the mass of one of these particles is m, the mass of the nucleus is approximately Am.

(ii) Assuming the nucleus is spherical and using Eq. (i), we find that the volume is

$$V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi r_0^3 A$$

(iii) The nuclear density can be found as follows:

$$\rho_n=\frac{\text{mass}}{\text{volume}}=\frac{Am}{\frac{4}{3}\pi r_0^3 A}=\frac{3m}{4\pi r_0^3}$$

The fact that A cancels out of our expression for nuclear density proves our statement above that all nuclei have roughly the same density. Taking $r_0 = 1.2 \times 10^{-15}$ m and $m = 1.67 \times 10^{-27}$ kg, we find that

$$\rho_n=\frac{3(1.67\times10^{-27}\,\text{kg})}{4\pi(1.2\times10^{-15}\,\text{m})^3}=2.3\times10^{17}\ \text{kg m}^{-3}$$

Note that the nuclear density is about 2.3×10^{14} times greater than the density of water (1×10^3 kg m^{-3})!

Illustration 5.3 **Calculate the radius of ^{70}Ge.**

Sol. We have,

$$R = R_0 A^{1/3} = (1.2\ \text{fm})\ (70)^{1/3}$$

$$= (1.2\ \text{fm})\ (4.12) = 4.94\ \text{fm}$$

Illustration 5.4 **The most common kind of iron nucleus has a mass number of 56. Find the radius, approximate mass and approximate density of the nucleus.**

Sol. We use two key ideas. The radius and mass of a nucleus depend on the mass number A and density is mass divided by volume.

We use Eq. (i) to determine the radius of the nucleus. The mass of the nucleus in atomic mass units is approximately equal to the mass number. The radius is

$$R = R_0 A^{1/3} = (1.2 \times 10^{-15}\ \text{m})(56)^{1/3}$$

$$= 4.6 \times 10^{-15}\ \text{m} = 4.6\ \text{fm}$$

Since $A = 56$, the mass of the nucleus is approximately 56 u, or $m \approx (56)\ (1.66 \times 10^{-27}\ \text{kg}) = 9.3 \times 10^{-26}$ kg. The volume is

$$v=\frac{4}{3}\pi R^3=\frac{4}{3}\times3.14\times(4.6\times10^{-15})^3$$

$$\approx 4.1 \times 10^{-13}\ \text{m}^3$$

And the density ρ is approximately

$$\rho=\frac{m}{V}\approx\frac{9.3\times10^{-26}\,\text{kg}}{4.1\times10^{-43}\,\text{m}^3}=2.3\times10^{17}\ \text{kg m}^{-3}$$

The density of solid iron is about 700 kg m^{-3}, so we see that the iron nucleus is more than 10^{13} times as dense as the bulk material. Densities of this magnitude are also found in neutron stars, which are similar to gigantic nuclei made almost entirely of neutrons. A 1 cm cube of material with this density would have a mass of 2.3×10^{11} kg or 230 million metric tons!

Illustration 5.5 **Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at the surface.**

Sol. The radius of a ^{12}C nucleus is

$$R = R_0 A^{1/3}$$

$$= (1.2\ \text{fm})\ (12)^{1/3} = 2.75\ \text{fm}$$

The separation between the centeres of the nuclei is $2R = 5.50$ fm. The potential energy of the pair is

$$U=\frac{q_1q_2}{4\pi\varepsilon_0 r}=(9\times10^9\ \text{N m}^2\ \text{C}^{-2})\ \frac{(6\times1.6\times10^{-19}\,\text{C})^2}{5.50\times10^{-15}\,\text{m}}$$

$$= 1.50 \times 10^{-12}\ \text{J} = 9.39\ \text{MeV}$$

NUCLEAR BINDING ENERGY

We know that in a stable nucleus, because of strong nuclear force of attraction, the nucleons are held tightly together in a small volume. As the system is stable, we can relatively say that the total potential energy of the system is negative and to separate all the nucleons from each other some energy must be supplied to break the nucleus. The more stable the nucleus is, the greater is the energy needed to break it apart. This required energy is called binding energy of the nucleus.

The mass of each element atom is less than the sum of masses of its constituent particles.

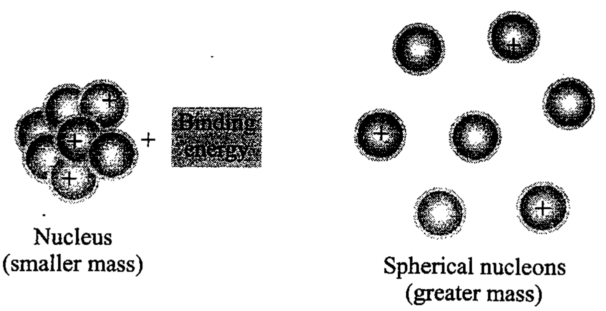


Fig. 5.4

For example, let us discuss the $^4_2$He atom. We compare the mass of atom to that of its constituents. To calculate the mass of the components, we can either add the masses of two protons, two neutrons, and two electrons or we can add the masses of two H atoms and two neutrons. Using the values from Table 5.1, we have

$$2m_H + 2m_H = 2(1.007825) + 2(1.008665) = 4.0329804 \qquad \text{(i)}$$

Now, we can see that in the table mass of $^4_2$He atom is 4.0026034 which is less than the value given in Eq. (i), the sum of masses of constituent particles. Similarly, this can be verified for all the other elements. Also, the reason for this can be explained by Eq. (ii) which is a basic nuclear reaction for formation of a nuclei ${}^{A}_{Z}X$. In this reaction, we can see that when Z protons and $A - Z$ neutrons fuse together to form a nucleus ${}^{A}_{Z}X$, some amount of energy must be released.

$$Zm_p - (A + Z)m_n \rightarrow M_X + \text{Energy} \qquad \text{(ii)}$$

If we think from where this energy comes, we can simply say by Einstein mass–energy relationship some amount of mass from independent nucleons is converted into energy and released when nucleons bind with each other to form a stable nucleus. This is the energy that hold the nucleons together in a nucleus, we call this binding energy of nucleus. So, when this energy is supplied to a nucleus, it splits into its constituent particles.

The binding energy used to disassemble the nucleus appears as extra mass of the separated nucleons. In other words, the sum of the individual masses of the separated protons and neutrons is greater by an amount Δm than the mass of the stable nucleus. The difference in mass Δm is known as the mass defect of the nucleus.

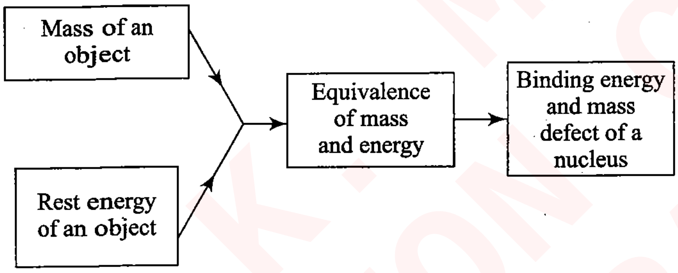


Fig. 5.5

For the reaction given in Eq. (ii), if we calculate the difference in masses of nucleons and that of nucleon X, then it is given as

$$\Delta m = Zm_p + (A - Z)m_n - M_X \qquad \text{(iii)}$$

This difference in masses of independent nucleons and mass of nucleus is called mass defect of the nuclear reaction. Using mass defect Δm, we can find the energy released in a nuclear reaction, e.g., the binding energy of the above nucleus X can be given as

$$\Delta E_{BE} = \Delta mc^2 \qquad \text{(iv)}$$

In a similar way, we can find the binding energy for any nucleus in nature for a known composition.

Mass Defect

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{\text{expected}} = Zm_p + (A - Z)m_n$$
$$M_{\text{observed}} = M_{\text{atom}} - Zm_e$$

It is found that

$$M_{\text{observed}} < M_{\text{expected}}$$

Hence, mass defect is defined as

$$\text{Mass defect} = M_{\text{expected}} - M_{\text{observed}}$$

$$\Rightarrow \quad \Delta m = [Zm_p + (A - Z)m_n] - [M_{\text{atom}} - Zm_e]$$

MASS–ENERGY EQUIVALENCE

We have discussed in previous section that using mass defect of a nuclear reaction, we can find the energy released in the nuclear process. We know generally nuclear masses are given in atomic mass unit where

$$1 \text{ a.m.u.} = 1.656 \times 10^{-27} \text{ kg}$$

If in a reaction 1 a.m.u. mass is converted into energy, then using Einstein's mass–energy relationship the amount of energy released is

$$\Delta E' = \Delta mc^2 = (1.656 \times 10^{-27})\,(3 \times 10^8)^2 \text{ J}$$

$$= \frac{(1.656 \times 10^{-27})(3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931.5 \times 10^6 \text{ eV} = 931.5 \text{ MeV}$$

Thus, we can say that 1 a.m.u. mass is equivalent to 931.5 MeV energy.

Illustration 5.6 **The most abundant isotope of helium has a ${}^{4}_{2}$H nucleus whose mass is 6.6447×10^{-27} kg. For this nucleus, find (a) the mass defect and (b) the binding energy. Given: Mass of the electron: $m_e = 5.485799 \times 10^{-4}$ u, mass of the proton: m_p = 1.007276 u and mass of the neutron: m_n = 1.008 665 u.**

Sol. The symbol ${}^{4}_{2}$He indicates that the helium nucleus contains $Z = 2$ protons and $N = 4 - 2 = 2$ neutrons. To obtain the mass defect Δm, we first determine the sum of the individual masses of the separated protons and neutrons. Then, we subtract from this sum the mass of the nucleus.

a. We find that the sum of the individual masses of the nucleons is

$$\underbrace{2(1.6726 \times 10^{-27} \text{ kg})}_{\text{Two protons}} + \underbrace{2(1.6749 \times 10^{-27} \text{ kg})}_{\text{Two neutrons}}$$

$$= 6.6950 \times 10^{-27} \text{ kg}$$

This value is greater than the mass of the intact He nucleus, and the mass defect is

$$\Delta m = 6.6950 \times 10^{-27} \text{ kg} - 6.6447 \times 10^{-27} \text{ kg}$$
$$= 0.0503 \times 10^{-27} \text{ kg}$$

b. According to Eq. (iv), the binding energy is given by

$$\Delta E_{BE} = (\Delta m)c^2 = (0.0503 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m s}^{-1})^2$$
$$= 4.53 \times 10^{-12} \text{ J}$$

Usually, binding energies are expressed in energy units of electron volt instead of joule ($1 \text{ eV} = 1.60 \times 10^{-19}$ J).

$$\therefore \quad \text{Binding energy} = (4.53 \times 10^{-12} \text{ J})\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)$$

$$= 2.83 \times 10^7 \text{ eV} = 28.3 \text{ MeV}$$

In this result, one million electron volt is denoted by the unit MeV. The value of 28.3 MeV is more than two million times greater than the energy required to remove an orbital electron from an atom.

Illustration 5.7 **The atomic mass of $^4_2$He is 4.0026 u and the atomic mass of $^1_1$H is 1.0078 u. Using atomic mass units instead of kilograms, obtain the binding energy of the $^4_2$He nucleus.**

Sol. To determine the binding energy, we calculate the mass defect in atomic mass units and then use the fact that one atomic mass unit is equivalent to 931.5 MeV of energy. The mass of 4.0026 u for $^4_2$He includes the mass of the two electrons in the neutral helium atom. To calculate the mass defect, we must subtract 4.0026 u from the sum of the individual masses of the nucleons, including the mass of the electrons. As Fig. 5.6 illustrates, the electron mass will be included if the masses of two hydrogen atoms are used in the calculation instead of the masses of two protons. The mass of a hydrogen atom is given as 1.0078 u, and the mass of a neutron is given in Table 5.1 as 1.0087 u.

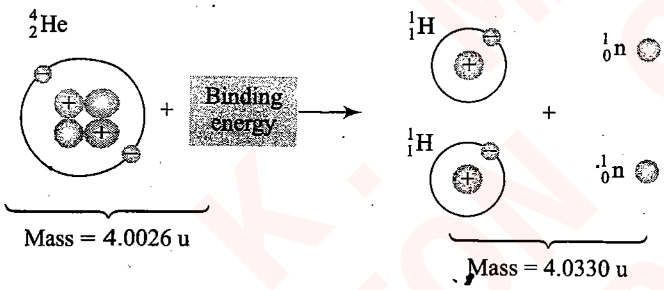


Fig. 5.6

The sum of the individual masses is

$$\underbrace{2(1.0078\text{ u})}_{\text{Two hydrogen atoms}} + \underbrace{2(1.0087\text{ u})}_{\text{Two neutrons}} = 4.0330\text{ u}$$

The mass defect is $\Delta m = 4.0330\text{ u} - 4.0026\text{ u} = 0.0304$ u. Since 1 u is equivalent to 931.5 MeV, the binding energy is 28.3 MeV.

Illustration 5.8 **The nucleus of the deuterium atom, called the deuteron, consists of a proton and a neutron. Calculate the deuteron's binding energy, given atomic mass, i.e., the mass of a deuterium nucleus plus an electron is measured to be 2.014 102 u.**

Sol. We know that the proton and neutron masses are

$$m_p = 1.007\,825\text{ u}, \quad m_n = 1.008\,665\text{ u}$$

Note that the masses used for the proton and neutron in this example are actually those of the neutral atoms. We are able to use atomic masses for these calculations because the electron masses cancel out. That is, when m_d containing one electron mass is subtracted from $(m_p + m_n)$ containing one electron mass, the electron masses cancel out. We have,

$$m_p + m_n = 2.016\,490\text{ u}$$

To calculate the mass difference, we subtract the deuteron mass from this value.

$$\therefore \quad \Delta m = (m_p + m_n) - m_d$$

$$= 2.016\,490\text{ u} - 2.014\,102\text{ u} = 0.002\,388\text{ u}$$

Because 1 u corresponds to an equivalent energy of 931.494 MeV (i.e., 1 u.c^2 = 931.494 MeV), the mass difference corresponds to the binding energy

$$E_b = (0.002\,388\text{ u})\,(931.494\text{ MeV/u}) = 2.224\text{ MeV}$$

This result tells us that to separate a deuteron into a proton and a neutron, it is necessary to add 2.224 MeV of energy to the deuteron to overcome the attractive nuclear force between the proton and neutron. One way of supplying the deuteron with this energy is by bombarding it with energetic particles.

If the binding energy of a nucleus were zero, the nucleus would separate into its constituent protons and neutrons without the addition of any energy, i.e., it would spontaneously break apart.

BINDING ENERGY PER NUCLEON

If we wish to see how nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy per nucleon basis. The binding energy per nucleon for a given element can be given as binding energy divided by the nucleon number A as

$$(\text{BE})_N = \frac{\Delta mc^2}{A} \qquad \text{(v)}$$

This value $(\text{BE})_N$ in Eq. (v) gives the criterion of stability among different elements. We can define binding energy per nucleon theoretically as the amount of energy needed to remove a nucleon from the nucleus of an element. For example, let us consider two elements X and Y with mass numbers A_X and A_Y ($A_X > A_Y$) and binding energies ΔE_X and ΔE_Y such that $\Delta E_X > \Delta E_Y$. Here, one can say that as for element X biding energy ΔE_x is more as compared to that for element Y, nucleus X is more stable than nucleus Y. But if we find the $(\text{BE})_N$ values for both elements, it is given as

$$(\text{BE}_N)_X = \frac{\Delta E_X}{A_X};\ (\text{BE}_N)_Y = \frac{\Delta E_Y}{A_Y}$$

These values are such that $(\text{BE}_N)_X < (\text{BE}_N)_Y$ which implies that to remove a nucleon from element X requires less energy than from element Y. This implies that for the nucleus of Y it is difficult to remove one nucleon from its nucleus, thus structure of Y is more stable than X.

To understand this in a better way, let us consider an example of ^{56}Fe and ^{209}Bi. Their binding energies are given as

$$\Delta E_{Fe} = 492.8\text{ MeV}$$

$$\Delta E_{Bi} = 1640\text{ MeV}$$

From the above values, it seems that to break the nucleus of Bi more energy is required, hence it is more stable than that of Fe. But we should not ignore the bigger size of bismuth nucleus. If we find biding energy per nucleon for both of these nuclei, we get

$$(BE_N)_{Fe} = \frac{492.8}{56} = 8.8 \text{ MeV}$$

$$(BE_N)_{Bi} = \frac{1640}{209} = 7.84 \text{ MeV}$$

Now, we can see that removal of one electron from Fe nucleus is more difficult as compared to that from Bi nucleus. So, Fe nuclei are more stable than Bi. Thus, to judge or compare the stability of different nuclei, we compare the binding energy per nucleon and not the nuclear binding energy.

To see how the nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy for each nucleus on a per-nucleon basis. Figure 5.7 shows a graph in which the binding energy divided by the nucleon number A is plotted against the nucleon number itself. In the graph, the peak for the ${}^4_2\text{He}$ isotope of helium indicates that the ${}^4_2\text{He}$ nucleus is particularly stable. The binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.7 MeV/nucleon for a nucleon number of about A = 60. For greater nucleon numbers, the binding energy per nucleon decreases gradually. Eventually, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleus together. Nuclei more massive than the ${}^{209}_{83}\text{Bi}$ nucleus of bismuth are unstable and hence radioactive.

Variation of Binding Energy Per Nucleon with Mass Number

The binding energy per nucleon is a characteristic property of elements. The graph in Fig. 5.7 shows the variation of binding energy per nucleon with mass number for all the elements of periodic table. In the graph, we can see that the binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.8 MeV/nucleon for iron $\left({}^{56}_{26}\text{Fe}\right)$. That is, nuclei with mass numbers greater or less than 60 are not as strongly bound as those near the middle of the periodic table. As we shall see later, this fact allows energy to be released in fission and fusion reactions. The curve is slowly varying for A > 40, which suggests that the nuclear force saturates. In other words, a particular nucleon can interact with only a limited number of other nucleons, which can be viewed as the 'nearest neighbours' in the close-packed structure illustrated in Fig. 5.3.

For greater nucleon numbers, the binding energy per nucleon decreases gradually. Later, the binding energy per nucleon decrease enough so there is insufficient binding energy to hold the nucleon together in the nucleus. It is observed that nuclei with A > 209 are unstable and hence radioactive.

In the figure, in the beginning there are some fluctuations in the graph. We can see that binding energies per nucleon for ${}^4\text{He}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ are relatively higher as compared to their neighboring elements or these elements have nuclei which are relatively more stable than their neighbours. This is because of the existence of nuclear energy levels in the nucleus. Each nuclear energy level can contain two neutrons of opposite spins or two protons of opposite spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms, to achieve configurations of minimum energy and therefore maximum stability. Similar to the case of atomic orbitals, here also the configuration of opposite spins in nucleons with pairs in nuclear energy levels are more stable. This concept can be used to explain the reason of more stability of ${}^4\text{He}$, ${}^{12}\text{C}$ and ${}^{16}\text{O}$ compared to their neighbouring elements.

Note: ***If binding energy per nucleon is more for a nucleus, then it is more stable. For example, if***

$$\left(\frac{BE_1}{A_1}\right) > \left(\frac{BE_2}{A_2}\right)$$

then nucleus 1 would be more stable.

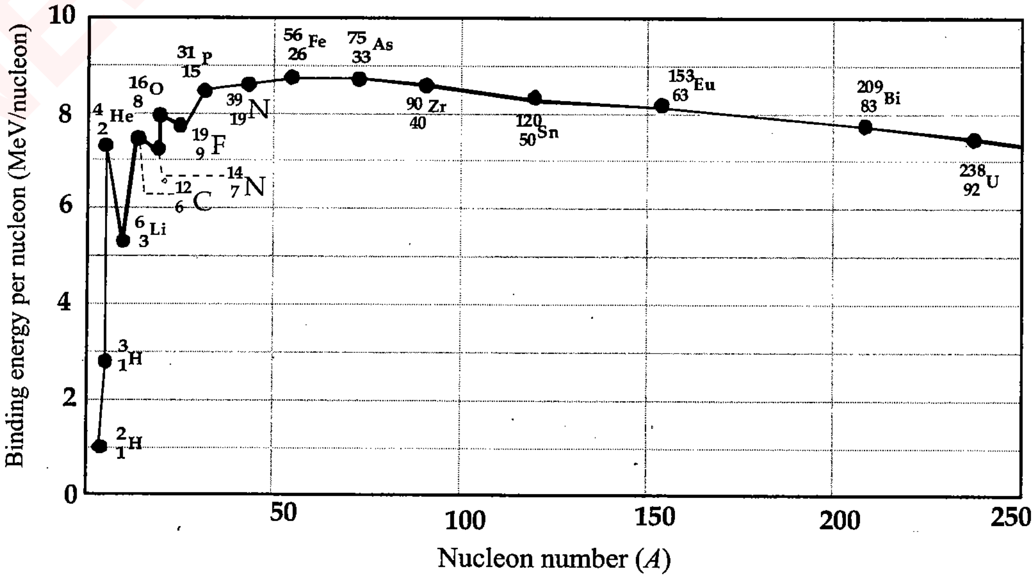


Fig. 5.7 A plot of binding energy per nucleon versus the nucleon number A.

Important observations from the graph

- The maximum binding energy per nucleon is 8.8 MeV and it is for Fe^{56}. Thus, iron nucleus is the most stable and the most tightly bound.
- The fact that (BE/A) varies by less than 10 percent above A = 10 suggests that each nucleon in the nucleus interacts only with the neighbors, independently of the total number of nucleons present in the nucleus.
- A small decrease after $A = 56$ is due to the destabilizing effect of the long-range repulsive Coulomb force.
- Not only the Coulomb force reduces the BE, it also shifts the neutron–proton ratio in heavy nuclei towards the neutron excess, by an amount that will increase with A.

Illustration 5.9 **Calculate the binding energy for nucleon of ${}_6^{12}C$ nucleus, if mass of proton $m_p = 1.0078$ u, mass of neutron $m_n = 1.0087$ u, mass of C_{12}, $m_C = 12.0000$ u; and 1 u = 931.4 MeV.**

Sol. In the nucleus of carbon, there are 6 protons and 6 neutrons.

Now,

$$6m_p = 6(1.0078) = 6.0468 \text{ u}$$
$$6m_n = 6(1.0087) = 6.05224 \text{ u}$$
$$\text{Total mass} = 6m_p + 6m_n = 12.0990 \text{ u}$$
$$m_C = 12.0000 \text{ u}$$
$$\text{Mass defect, } \Delta m = 0.0990 \text{ u}$$
$$\text{BE} = \Delta m \times 931.4 = (0.0990)\,931.4 = 92.2 \text{ MeV}$$

Q VALUES

We have just examined some nuclear reactions for which mass numbers and atomic numbers must be balanced in the equations. We shall now consider the energy involved in these reactions, because energy is another important quantity that must be conserved.

Let us illustrate this procedure by analyzing the following nuclear reaction:

$${}_1^2H + {}_7^{14}N \rightarrow {}_6^{12}C + {}_2^4He \qquad \text{(i)}$$

The total mass on the left side of the equation is the sum of the mass of ${}_1^2H$ (2.014 102 u) and the mass of ${}_7^{14}N$ (14.003 074 u), which equals 16.017 176 u. Similarly, the mass on the right side of the equation is the sum of the mass of ${}_6^{12}C$ (12.000 000 u) plus the mass of ${}_2^4He$ (4.002 602 u), for a total of 16.002 602 u. Thus, the total mass before the reaction is greater than the total mass after the reaction. The mass difference in this reaction is greater than the total mass after the reaction. The mass difference in this reaction is equal to 16.017 176 u – 16.002 602 u = 0.014 574 u. This 'lost' mass is converted to the kinetic energy of the nuclei present after the reaction. In energy units, 0.014 5774 u is equivalent to 13.576 MeV of kinetic energy carried away by the carbon and helium nuclei.

The energy required to balance the equation is called the Q value of the reaction. In Eq. (i), the Q value is 13.576 MeV. Nuclear reactions in which there is a release of energy—that is, positive Q values—are said to exothermic reactions.

The energy balance sheet is not complete; however, we must also consider the kinetic energy of the incident particle before the collision. As an example, let us assume that the deuteron in Eq. (i) has a kinetic energy of 5 MeV. Adding this to our Q value, we find that the carbon and helium nuclei have a total kinetic energy of 18.576 MeV following the reaction. Now, consider the reaction

$${}_2^4He + {}_7^{17}N \rightarrow {}_8^{17}O + {}_1^1H \qquad \text{(ii)}$$

Before the reaction, the total mass is the sum of the masses of the alpha particle and the nitrogen nucleus: 4.002 602 u + 14.003 074 u = 18.005 676 u. After the reaction, the total mass is the sum of the masses of the oxygen nucleus and the proton: 16.999133 u + 1.07 825 u = 18.006 958 u. In this case, the total mass deficit is 0.001 282 u, equivalent to an energy deficit of 1.194 MeV. This deficit is expressed by the negative Q value of the reaction, –1.194 MeV. Reactions with negative Q values are called endothermic reactions. Such reactions will not take place unless the incoming particle has at least enough kinetic energy to overcome the energy deficit.

At first it might appear that the reaction in Eq. (ii) could take place if the incoming alpha particle had a kinetic energy of 1.194 MeV. In practice, however the alpha particle must have more energy than this. If it had an energy of only 1.194 MeV, energy would be conserved but careful analysis would show that momentum was not. This can easily be understood by recognizing that the incoming alpha particle has some momentum before the reaction. However, if its kinetic energy were only 1.194 MeV, the products (oxygen and a proton) would be created with zero kinetic energy and thus zero momentum. It can be shown that, in order to conserve both energy and momentum, the incoming particle must have a minimum kinetic energy given by

$$KE_{min} = \left(1 + \frac{m}{M}\right)|Q| \qquad \text{(iii)}$$

where m is the mass of the incident particle, M is the mass of the target, and the absolute value of the Q value is used. For the reaction given by Eq. (ii), we find

$$KE_{min} = \left(1 + \frac{4.002\,62}{14.003\,074}\right)|-1.194 \text{ MeV}|$$
$$= 1.535 \text{ MeV}$$

This minimum value of the kinetic energy of the incoming particle is called the threshold energy. The nuclear reaction shown in Eq. (ii) will not occur if the incoming alpha particle has a kinetic energy of less than 1.535 MeV, but can occur if its kinetic energy is equal to or greater than 1.535 MeV.

NUCLEAR STABILITY

Given that the nucleus consists of a closely packed collection of protons and neutrons, you might be surprised that it can exist.

The very large repulsive electrostatic forces between protons should cause the nucleus to fly apart. However, nuclei are stable because of the presence of another, short-range (about 2 fm) force, the *nuclear force*. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist. Moreover, the strong nuclear force is nearly independent of charge. In other words, the nuclear forces associated with the proton–proton, proton–neutron and neutron–neutron interactions are approximately the same.

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. A plot of N versus Z for a number of stable nuclei is given in Fig. 5.8. Note that light nuclei are most stable if they contain equal number of protons and neutrons—that is, if $N = Z$—but heavy nuclei are more stable if $N > Z$. This can be partially understood by recognizing that, as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable, because neutrons experience only the attractive nuclear forces. In effect, the additional neutrons 'dilute' the nuclear charge density. Eventually, when $Z = 83$, the repulsive forces between protons cannot be compensated by the addition of more neutrons. Elements that contain more than 83 protons do not have stable nuclei, but decay or disintegrate into other particles in various amounts of time. The masses of several stable particles are given in Table 5.1.

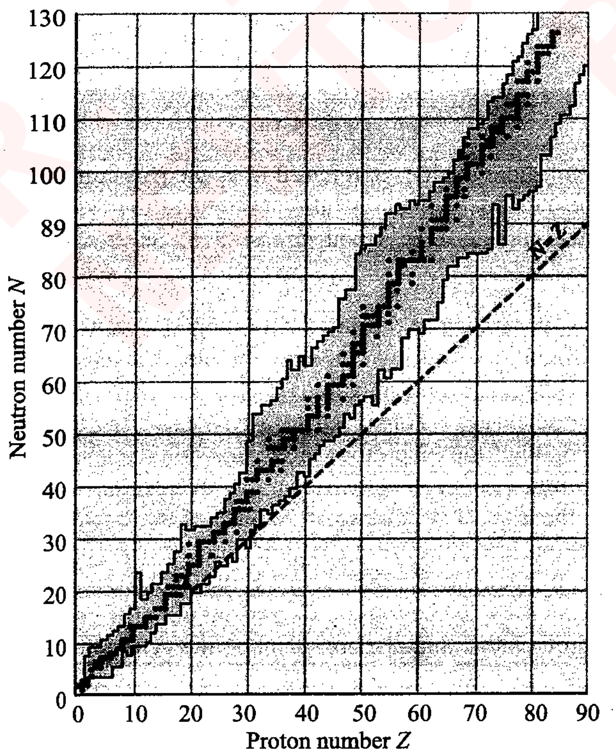


Fig. 5.8

Concept Application Exercise 5.1

1. How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24?
2. Calculate the average binding energy per nucleon of ${}^{93}_{41}Nb$ having mass 92.906 u.
3. Protons and neutrons exist together in an extremely small space within the nucleus. How is this possible when protons repel each other?
4. Following data are available about three nuclei P, Q and R. Arrange them in decreasing order of stability.

	P	Q	R
Atomic mass number (A)	10	5	6
Binding energy (MeV)	100	60	66

5. A nucelus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.
6. A nuclear reaction is given as
$$A + B \rightarrow C + D$$
Binding energies of A, B, C and D are given as B_1, B_2, B_3 and B_4. Find the energy released in the reaction.
7. Calculate the binding energy of an alpha particle from the following data:
$$\text{Mass of } {}^1_1H \text{ atom} = 1.007826 \text{ u}$$
$$\text{Mass of neutron} = 1.008665 \text{ u}$$
$$\text{Mass of } {}^4_2He \text{ atom} = 4.00260 \text{ u}$$
(take $1 \text{ u} = 931 \text{ MeV}/c^2$)
8. Find the binding energy of ${}^{56}_{26}Fe$. Atomic mass of ${}^{56}Fe$ is 55.9349 u and that of ${}^1H$ is 1.00783 u. Mass of neutron is 1.00867 u.
9. Use Avogadro's number to show that the atomic mass unit is $1 \text{ u} = 1.66 \times 10^{-27}$ kg.
10. Take a sample of lead and oxygen. They contain different atoms and the density of solid lead is much greater than that of gaseous oxygen. Decide whether the density of the nucleus in a lead atom is greater than, approximately equal to, or less than that in an oxygen atom.
11. Show that the nuclide ${}^8Be$ has a positive binding energy but is unstable with respect to decay into two alpha particles, where masses of neutron, ${}^1H$ and ${}^8Be$ are 1.008665 u, 1.007825 u and 8.005305 u, respectively.

RADIOACTIVITY

We have discussed that inside a nucleus electrostatic attraction is counter balanced by short-range strong nuclear forces and the nucleus becomes stable. Despite the forces being balanced, many nuclides are unstable because of nuclear size or the limited range of nuclear forces. Due to slight imbalance in small sized nuclides, these nuclides spontaneously disintegrate into other nuclides. This phenomenon of spontaneous disintegration is called radioactivity. Further in this chapter we will discuss the aspects of radioactivity by which unstable nuclides disintegrate to achieve stability.

When an unstable or radioactive nucleus disintegrates spontaneously, certain kinds of particles and/or high-energy photons are released. These particles and photons are collectively called 'rays'. Three kinds of rays are produced by naturally occurring radioactivity: α-rays, β-rays and γ-rays. They are named according to the first three letters of the Greek alphabet, alpha (α), beta (β) and gamma (γ), to indicate the extent of their ability to penetrate matter. α-rays are the least penetrating, being blocked by a thin ($\approx$0.01 mm) sheet of lead, while β-rays penetrate lead to a much greater distance ($\approx$0.1 mm). γ-rays are the most penetrating and can pass through an appreciable thickness ($\approx$100 mm) of lead.

The nuclear disintegration process that produces α-, β- and γ-rays must obey the laws of physics; these laws are called conservation laws because each of them deals with a property (such as mass/energy, electric charge, linear momentum, and angular momentum) that is conserved or does not change during a process. To these conservation laws, we now add a fifth, the conservation of nucleon number. In all radioactive decay processes, it has been observed that the number of nucleons present before the decay is equal to the number of nucleons after the decay. Therefore, the number of nucleons is conserved during a nuclear disintegration. As applied to the disintegration of a nucleus, the conservation laws require that the energy, electric charge, linear momentum, angular momentum, and nucleon number that a nucleus possesses must remain unchanged when it disintegrates into nuclear fragments and accompanying α-, β- or γ-rays.

The three types of radioactivity that occur naturally can be observed in a relatively simple experiment. A piece of radioactive material is placed at the bottom of a narrow hole in a lead cylinder. The cylinder is located within an evacuated chamber, as Fig. 5.9 illustrates. A magnetic field is directed perpendicular to the plane of the paper, and a photographic plate is positioned to the right of the hole. Three nods of spots appear on the developed plate, which are associated with the radioactivity of the nuclei in the material. Since moving particles

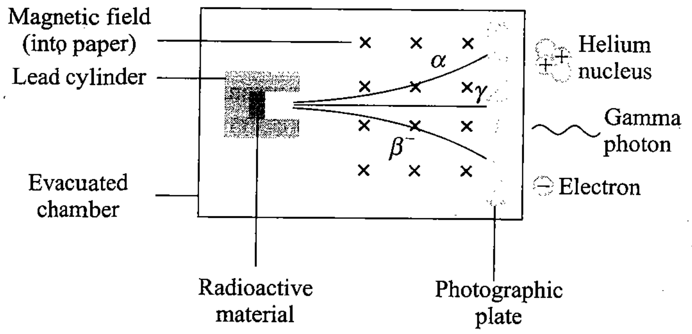


Fig. 5.9

are deflected by a magnetic field only when they are electrically charged, this experiment reveals that two types of radioactivity (α- and β-rays, as it turns out) consist of charged particles, while the third type (γ-rays) does not.

Fig. 5.9: α- and β-ray are deflected by a magnetic field and, therefore, consist of moving charged γ-rays are not deflected by a magnetic field and, consequently, must be uncharged.

α-Decay

When a nucleus disintegrates and produces α-rays, it is said to undergo α-decay. Experimental evidence shows that α-rays consist of positively charged particles, each one being the ${}^{4}_{2}He$ nucleus of helium. Thus, an α-particle has a charge of +2e and a nucleon number of $A = 4$. Since the grouping of 2 protons and 2 neutrons in a ${}^{4}_{2}He$ nucleus is particularly stable, as we have seen in connection with Fig. 5.6, it is not surprising that an α-particle can be ejected as a unit from a more massive unstable nucleus.

Fig. 5.10 shows the disintegration process for one example of α-decay.

$$\underset{\substack{\text{Parent}\\\text{nucleus}\\\text{(uranium)}}}{{}^{238}_{92}U} \rightarrow \underset{\substack{\text{Daughter}\\\text{nucleus}\\\text{(thorium)}}}{{}^{234}_{90}Th} + \underset{\substack{\text{a-particle}\\\text{(helum}\\\text{nucleus)}}}{{}^{4}_{2}He}$$

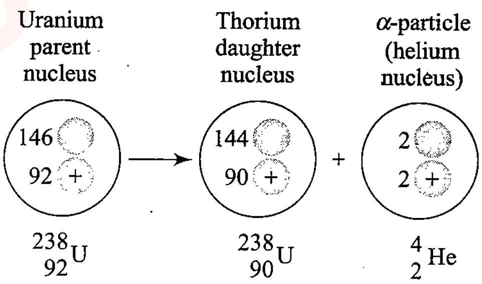


Fig. 5.10

The original nucleus is referred to as the parent nucleus (P), and the nucleus remaining after disintegration is called the daughter nucleus (D). Upon emission of an α-particle, the uranium ${}^{238}_{92}U$ parent is converted into the ${}^{234}_{90}Th$ daughter, which is an isotope of thorium. The parent and daughter nuclei are different, so α-decay converts one element into another, a process known as transmutation.

Electric charge is conserved during α-decay. In Fig. 5.10, for instance, 90 of the 92 protons in the uranium nucleus end up in the thorium nucleus, and the remaining two protons are carried off by the α-particle. The total number of 92, however, is the same before and after disintegration. α-decay also conserves the number of nucleons, for the number is the same before (238) and after (234 + 4) disintegration. Consistent with the conservation of electric charge and nucleon number, the general form for α-decay is

$$\textbf{α-decay:} \quad \underset{\substack{\text{Parent}\\\text{nucleus}}}{{}^{A}_{Z}P} \rightarrow \underset{\substack{\text{Daughter}\\\text{nucleus}}}{{}^{A-4}_{Z-2}D} \rightarrow \underset{\substack{\alpha\text{-particle}\\\text{(helium nucleus)}}}{{}^{4}_{2}He}$$

When a nucleus releases an α-particle, the nucleus also releases energy. In fact, the energy released by radioactive decay is responsible, in part, for keeping the interior of the earth hot and, in some places, even molten. The following example shows how the conservation of mass/energy can be used to determine the amount of energy released in α-decay.

α-Decay:

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

Q value: *It is defined as the energy released during the decay process.*

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in KE of the products. Let us consider the following:

$$M_X = \text{mass of atom } {}_{Z}X^{A}$$
$$M_Y = \text{mass of atom } {}_{Z-2}Y^{A-4}$$
$$M_{He} = \text{mass of atom } {}_{2}He^{4}$$

$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-2)m_e) + (M_{He} - 2m_e)\}]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

Considering actual number of electrons in α-decay

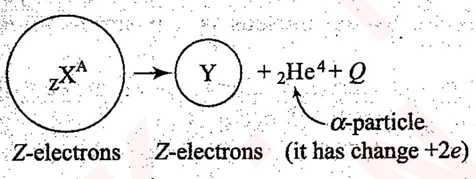


Fig. 5.11

$$Q \text{ value} = [M_X - (M_Y + 2m_e) - (M_{He} - 2m_e)]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

One widely used application of α-decay is in smoke detectors. Figure 5.12 illustrates how a smoke detector operates. Two small and parallel metal plates are separated by a distance of about one centimetre. A tiny amount of radioactive material at the centre of one of the plates emits α-particles, which collide with air molecules. During the collisions, the air molecules are ionized to form positive and negative ions. The voltage from a battery causes one plate to

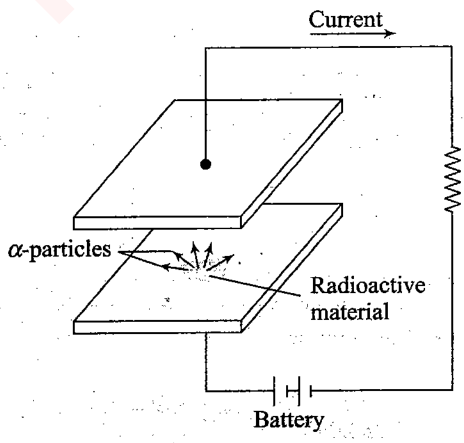


Fig. 5.12 A smoke detector.

be positive and the other negative, so that each plate attracts ions of opposite charge. As a result there is a current in the circuit attached to the plates. The presence of smoke particles between the plates reduces the current, since the ions that collide with a smoke particle are usually neutralized. The drop in current that smoke particles cause is used to trigger an alarm.

Illustration 5.10 **The atomic mass of uranium $^{238}_{92}U$ is 238.0508 u, that of thorium $^{234}_{90}Th$ is 234.0436 u and that of an alpha particle $^{4}_{2}He$ is 4.0026 u. Determine the energy released when α-decay converts $^{238}_{92}U$ into $^{234}_{90}Th$.**

Sol. Since energy is released during the decay, the combined mass of the $^{234}_{90}Th$ daughter nucleus and the α-particle is less than the mass of the $^{238}_{92}U$ parent nucleus. The difference in mass is equivalent to the energy released. We will determine the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV.

The decay and the masses are shown below:

$$\underset{238.0508\text{ u}}{^{238}_{92}U} \rightarrow \underbrace{\underset{234.0436\text{ u}}{^{234}_{90}Th} + \underset{4.0026\text{ u}}{^{4}_{2}He}}_{238.0462\text{ u}}$$

The decrease in mass is 238.0508 u – 238.0462 u = 0.0046 u. As usual, the masses are atomic masses and include the mass of the orbital electrons. But this causes no error here because the same total number of electrons is included for $^{238}_{92}U$, on the one hand, and for $^{234}_{90}Th$ plus $^{4}_{2}He$, on the other. Since 1 u is equivalent to 931.5 MeV, the released energy is 4.3 MeV.

When α-decay occurs as discussed in Illustration 5.10, the energy released appears as kinetic energy of the recoiling $^{234}_{90}Th$ nucleus and the α-particle, except for a small portion carried away as a γ-ray. Illustration 5.11 discusses how the $^{234}_{90}Th$ nucleus and the α-particle share the released energy.

Illustration 5.11 **Refer to illustration 5.10, the energy released by the α-decay of $^{238}_{92}U$ is found to be 4.3 MeV. Since this energy is carried away as kinetic energy of the recoiling $^{234}_{90}Th$ nucleus and the α-particle, it follows that $KE_{Th} + KE_{\alpha}$ = 4.3 MeV. However, KE_{Th} and KE_{α} are not equal. Which particle carries away more kinetic energy, the $^{234}_{90}Th$ nucleus or the α-particle?**

Sol. Kinetic energy depends on the mass m and speed v of a particle, since KE = $\frac{1}{2}mv^2$. The $^{234}_{90}Th$ nucleus has a much greater mass than the α-particle, and since the kinetic energy is proportional to the mass, it is tempting to conclude that the $^{234}_{90}Th$ nucleus has greater kinetic energy. This conclusion is not correct, however, since it does not take into account the fact that the $^{234}_{90}$Th nucleus and the α-particle have different speeds after the decay. In fact, we expect the thorium nucleus to recoil with smaller speed precisely because it has the greater mass. The decaying of $^{238}_{92}$U is like a father and his young daughter on ice skates, pushing off against one another. The more massive father recoils with much less speed than the daughter. We can use the principle of conservation of linear momentum to verify our explanation.

As we know, the conservation principle states that the total linear momentum of an isolated system remains constant. An isolated system is one for which the vector sum of the external forces acting on the system is zero, and the decaying $^{238}_{92}$U nucleus fits this description. It is stationary initially, and since momentum is mass times velocity, its initial momentum is zero. In its final form, the system consists of the $^{234}_{90}$Th nucleus and the α-particle and has a final total momentum of $m_{Th}v_{Th} + m_\alpha v_\alpha$. According to momentum conservation, the initial and final values of the total momentum of the system must be the same, so that $m_{Th}v_{Th} + m_\alpha v_\alpha = 0$. Solving this equation for the velocity of the thorium nucleus, we find that $v_{Th} = -m_\alpha v_\alpha / m_{Th}$. Since m_{Th} is much greater than m_α, we can see that the speed of the thorium nucleus is less than the speed of the α-particle. Moreover, the kinetic energy depends on the square of the speed and only the first power of the mass. As a result of its much greater speed, the α-particle has the greater kinetic energy.

β-Decay

The β-rays are deflected by the magnetic field in a direction opposite to that of the positively charged α-rays. Consequently, these β-rays, which are the most common kind, consist of negatively charged particles or β-particles. Experiment shows that β-particles are electrons. When a radioactive nucleus undergoes β-decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1.

As an illustration of β-decay, consider the thorium $^{234}_{90}$Th nucleus, which decays by emitting a β-particle, as shown in Fig. 5.13. β-decay occurs when a neutron in an unstable parent nucleus decay into a proton and an electron, the electron being emitted as the β-particle. In the process, the parent nucleus is transformed into the daughter nucleus.

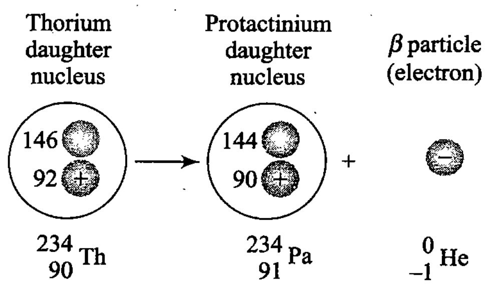


Fig. 5.13

β^- decay, like α-decay, causes a transmutation of one element into another. In this case, thorium $^{234}_{90}$Th is converted into protactinium $^{234}_{91}$Pa. The law of conservation of charge is obeyed, since the net number of positive charges is the same before (90) and after (91 – 1) the β^- emission. The law of conservation of nucleon number is obeyed, since the nucleon number remains at $A = 234$.

β^- decay

$$\underset{\text{Parent nucleus}}{^A_ZP} \rightarrow \underset{\text{Daughter nucleus}}{^{A}_{Z+1}D} \rightarrow \underset{\substack{\beta^-\text{ particle}\\ \text{(electron)}}}{^{0}_{-1}e}$$

The electron emitted in β^- decay does not actually exist within the parent nucleus and is not one of the orbital electrons. Instead, the electron is created when a neutron decays into a proton and an electron; when this occurs, the proton number of the parent nucleus increases from Z to $Z + 1$ and the nucleon number remains unchanged. The electron is usually fast-moving and escapes from the atom, leaving behind a positively charged atom.

β^+ decay

A second kind of β-decay sometimes occurs. In this process, the particle emitted by the nucleus is a positron rather than an electron. A positron, also called a β^+ particle, has the same mass as an electron but carries a charge of $+e$ instead of $-e$. The disintegration process for β^+ decay is

$$\underset{\text{Parent nucleus}}{^A_ZP} \rightarrow \underset{\text{Daughter nucleus}}{^{A}_{Z-1}P} \rightarrow \underset{\substack{\beta^+\text{particle}\\ \text{(positron)}}}{^{0}_{1}e}$$

The emitted positron does not exist within the nucleus but, rather, is created when a nuclear proton is transformed into a neutron. In the process, the proton number of the parent nucleus decreases from Z to $Z - 1$, and the nucleon number remains the same. As with β^- decay, the laws of conservation of charge and nucleon number are obeyed, and there is a transmutation of one element into another.

In the following illustration, we will discuss how energy is released during β^- decay, just as it is during α-decay, and that the conservation of mass/energy applies.

Illustration 5.12 **The atomic mass of thorium $^{234}_{90}$Th is 234.043 59 u, while that of protactinium $^{234}_{91}$Pa is 234.043 30 u. Find the energy released when β^- decay changes $^{234}_{90}$Th into $^{234}_{91}$Pa.**

Sol. To find the energy released, we follow the usual procedure of determining how much the mass has decreased because of the decay and then calculating the equivalent energy. The decay and the masses are shown below:

$$\underset{234.04359\text{ u}}{^{234}_{90}\text{Th}} \rightarrow \underbrace{^{234}_{91}\text{Pa} + {^{0}_{-1}}\text{e}}_{234.04330\text{ u}}$$

When the $^{234}_{90}$Th nucleus of a thorium atom is converted into a $^{234}_{91}$Pa nucleus, the number of orbital electrons remains the same, so the resulting protactinium atom is missing one orbital electron. However, the given mass includes all 91 electrons of a neutral protactinium atom. In effect, then, the value of 234.043 30 u for $^{234}_{91}$Pa already includes the mass of the β^- particle. The mass decrease that accompanies the β^- decay is 234.043 59 u $-$234.043 30 u = 0.000 29 u. The equivalent energy (1 u = 931.5 MeV) is 0.27 MeV. This is the maximum kinetic energy that the emitted electron can have.

Illustration 5.13 **Consider the beta decay**

$$^{198}\text{Au} \rightarrow {}^{198}\text{Hg}^* + \beta^- + \bar{\nu}$$

where ^{198}Hg* represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass of ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

Sol. If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2$$

As ^{198}Hg* has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2 - 1.088 \text{ MeV}$$

$$= (197.968233\,\text{u} - 197.966760\,\text{u})\left(931\frac{\text{MeV}}{\text{u}}\right) - 1.088\,\text{MeV}$$

$$= 1.3686 \text{ MeV} - 1.088 \text{ MeV} = 0.2806 \text{ MeV}$$

This is also the maximum possible kinetic energy of the electron emitted.

Illustration 5.14 **Calculate the Q value in the following decays:**

a. $^{19}\text{O} \rightarrow {}^{19}\text{F} + e^- +$ **b.** $^{25}\text{Al} \rightarrow {}^{25}\text{Mg} + e^+ + n$

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

Sol. a. The Q value of β^--decay is

$$Q = [m(^{19}\text{O}) - m(^{19}\text{F})]c^2$$

$$= [19.003576 \text{ u} - 18.998403 \text{ u}]\ (931 \text{ MeV/u})$$

$$= 4.816 \text{ MeV}$$

b. The Q value of β^+-decay is

$$Q = [m(^{25}\text{Al}) - m(^{25}\text{Mg}) - 2m_e]c^2$$

$$= \left[24.99032\text{ u} - 24.985839\text{ u} - 2\times 0.511\frac{\text{MeV}}{c^2}\right]c^2$$

$$= (0.004593 \text{ u})\ (931 \text{ MeV/u}) - 1.022 \text{ MeV}$$

$$= 4.276 \text{ MeV} - 1.022 \text{ MeV} = 3.254 \text{ MeV}$$

Illustration 5.15 **Find the energy liberated in the beta decay of $^{14}_{6}$C to $^{14}_{7}$N as represented by Eq. (iii). Equation (iii) refers to nuclei. Adding six electrons to both sides of Eq. (iii) gives**

$$^{14}_{6}\text{C atom} \rightarrow {}^{14}_{7}\text{N atom}$$

Sol. We know that $^{14}_{6}$C has a mass of 14.003 242 u and $^{14}_{7}$N has a mass of 14.003 074 u. here, the mass difference between the initial and final states is

$$\Delta m = 14.003\,242 \text{ u} - 14.003\,74 \text{ u} = 0.000\,168 \text{ u}$$

This corresponds to an energy release of

$$E = (0.000\,168 \text{ u})(931.494 \text{ MeV/u}) = 0.156 \text{ MeV}$$

The Neutrino

From illustrations discussed in previous section, we see that the energy released in the beta decay of ^{14}C is approximately 0.16 MeV. As with alpha decay, we expect the electron to carry away virtually all of this as kinetic energy because apparently it is the lightest particle produced in the decay. However, as Fig. 5.14 shows, only a small number of electrons have this maximum kinetic energy, represented as KE_{max} on the graph; most of the electrons emitted have kinetic energies lower than this predicted value. If the daughter nucleus and the electron are not carrying away this liberated energy, then the energy conservation requirement leads to the question, what accounts for the missing energy? As an additional complication, further analysis of beta decay shows that the principles of conservation of both angular momentum and linear momentum appear to be violated!

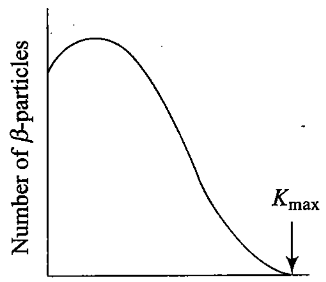


Fig. 5.14

When a-β particle is emitted by a radioactive nucleus, energy is simultaneously released. Experimentally, however, it is found that most β-particles do not have enough kinetic energy to account for all the energy released. If a β-particle carries away only part of the energy, where does the remainder go? In 1930, Pauli proposed that a third particle must be present to carry away the 'missing' energy and to conserve momentum. Enrico Fermi later developed a complete theory of beta decay and named this particle the neutrino ('little neutral one') because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino (ν) was finally detected experimentally in 1956. For instance, the β-decay of thorium $^{234}_{90}Th$ is more correctly written as

$$^{234}_{90}Th \rightarrow\, ^{234}_{91}Pa + \, ^{0}_{-1}e + \bar{v}$$

The bar above the v is included because the neutrino emitted in this particular decay process is an antimatter neutrino or antineutrino. A normal neutrino (v without the bar) is emitted when β^+ decay occurs.

The neutrino has zero electrical charge and is extremely difficult to detect because it interacts very weakly with matter.

The neutrino has the following properties:

- Zero electric charge.
- A mass much smaller than that of the electron, but probably not zero. (Recent experiments suggest that the neutrino definitely has mass, but the value is uncertain—perhaps less than 1 eV/c^2.)
- A spin of $\frac{1}{2}$.
- Very weak interaction with matter, making it quite difficult to detect.

Thus, with the introduction of the neutrino, we are now able to represent the beta decay process of equation in its correct form:

$$^{14}_{6}C = \, ^{14}_{7}N + e^- + \bar{v} \qquad \text{(i)}$$

where the bar in the symbol $\bar{v}$ indicates and antineutrino. To explain what an antineutrino is, let us first consider the following decay:

$$^{12}_{7}N = \, ^{12}_{6}C + e^+ + v \qquad \text{(ii)}$$

Here, we see that when ^{12}N decays into ^{12}C, a particle is produced that is identical to the electron except that it has a positive charge of +e. This particles is called a positron. Because it is like the electron in all respects except charge, the positron is said to be the antiparticle to the electron. We shall discuss antiparticles further in the chapter. For now, suffice it to say that in beta decay, an electron and an antinueutrino are emitted or a positron and a neutrino are emitted.

γ-Decay

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower energy state, perhaps to the ground state, by emitting one or more high-energy photons. The process is very similar to the emission of light by an atom. An atom emits radiation to release some extra energy when an electron 'jumps' from a state of higher energy to a state of lower energy. Likewise, the nucleus uses essentially the same method to release any extra energy it may have following a decay or some other nuclear event. In neutrons in the nucleus s they move from a higher energy level to a lower level. The photons emitted in such a de-excitation process are called gamma rays, which have very high energy relative to the energy of visible light.

A nucleus may reach an excited state as the result of a violent collision with another particle. However, it is more common for a nucleus to be in an excited state as a result of alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:

$$^{12}_{5}B = \, ^{12}_{6}C^* + e^- + \bar{v} \qquad \text{(iii)}$$

$$^{12}_{6}C^* = \, ^{12}_{6}C + \gamma \qquad \text{(iv)}$$

Equation (iii) represents beta decay in which ^{12}B decays to $^{12}C^*$ where the asterisk indicates that the carbon nucleus is left in an excited state following the decay. The excited carbon nucleus then decays to the ground state by emitting a gamma ray, as indicated by Eq. (iv). Note that gamma emission does not result in any change in either Z or A.

Illustration 5.16 **What is the wavelength of the 0.186 MeV γ-ray photon emitted by radium $^{226}_{88}Ra$?**

Sol. The photon energy is the difference between two nuclear energy levels. Equation $E_i - E_f = hf$ gives the relation between the energy level separation ΔE and the frequency f of the photon as $\Delta E = hf$. Since $f\lambda = c$, the wavelength of the photon is $\lambda = hc/\Delta E$.

First, we must convert the photon energy into joule:

$$\Delta E = (0.186 \times 10^6 \text{ eV}) \left(\frac{1.60 \times 10^{-19}}{1 \text{ eV}}\right)$$

$$= 2.98 \times 10^{-14} \text{ J}$$

The wavelength of the photon is

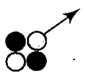
$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J.s})(3.00 \times 10^8 \text{ ms}^{-1})}{2.98 \times 10^{-14} \text{ J}}$$

$$= 6.67 \times 10^{-12} \text{ m}$$

STABILITY AND NEUTRON–PROTON RATIO (n/p)

Stable nuclei have the ratio n/p (neutron/proton) either equal to 1 or more than 1. The ratio is nearly 1 for light nuclei upto calcium $\left(^{40}_{20}Ca\right)$. It increases up to 1.6 for heavy nuclei. The table following shows n/p ratio for some stable nuclei.

(i) When n/p ratio is higher than that required for stability, β-emission takes place. It decrease n/p ratio, e.g., when $^{14}_{6}C$ decays to $^{14}_{7}N$, and n/p ratio decreases from 1.33 to 1.

	Decay event	Reason for instability
Gamma decay	Emission of gamma ray reduces energy of nucleus	Nucleus has excess energy
Alpha decay	 Emission of alpha particle reduces size of nucleus	Nucleus too large

Beta decay	⊙ = ● + • Emission of electron by neutron in nucleus changes the neutron to a proton	Nucleus has too many neutrons relative to number of protons
Electron capture	• + ● = ⊙ Capture of electron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons
Positron emission	● + ○ = ⊙ Emission of positron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons

● Proton (charge = +e) • Electron (charge = –e)

⊙ Neutron (charge = 0) ○ Positron (charge = +e)

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}$$

Isotope	*n*	*p*	*n*/*p*
${}^{12}_{6}C$	6	6	1
${}^{14}_{7}N$	7	7	1
${}^{16}_{8}O$	8	8	1
${}^{20}_{10}Ne$	10	10	1
${}^{40}_{20}Ca$	20	20	1
${}^{64}_{30}zn$	34	30	1.13
${}^{90}_{40}Zn$	50	40	1.40
${}^{202}_{80}Hg$	122	80	1.53

(ii) When n/p ratio is less than that required for stability, the nuclei can either emit α-particle, emit β^+ or undergo K-electron capture.

$$n/p \qquad \underset{146/92}{{}^{238}_{92}U} \rightarrow \underset{144/90}{{}^{234}_{90}Th} + {}^{4}_{2}He$$

$$n/p \qquad \underset{6/7}{{}^{13}_{7}N} \rightarrow \underset{7/6}{{}^{13}_{6}C} + \beta^+$$

$$n/p \qquad \underset{115/79}{{}^{194}_{79}N} + {}^{0}_{-1}e + \underset{116/78}{{}^{194}_{78}P}$$

RADIOACTIVE DECAY AND ACTIVITY

The question of which radioactive nucleus in a group of nuclei disintegrates at a given instant is decided like the drawing of numbers in a state lottery; individual disintegrations occur randomly. As time passes, the number N of parent nuclei decreases, as Fig. 5.15 shows. This graph of N versus time indicates that the decrease occurs in a smooth fashion, with N approaching zero after enough time has passed. To help describe the graph, it is useful to define the half-life $T_{1/2}$ of a radioactive isotope as the time required for one-half of the nuclei present to disintegrate. For example, radium ${}^{226}_{88}Ra$ has a half-life of 1600 years, for it takes this amount of time for one-half of a given quantity of this isotope to disintegrate. In another 1600 years, one-half of the remaining radium atoms will disintegrate, leaving only one-fourth of the original number intact. In the figure, the number of nuclei present at time t = 0 s is $N = N_0$, while the number present at $t = T_{1/2}$ is $N = \frac{1}{2}N_0$. The number present at $t = 2T_{1/2}$ is $N = \frac{1}{4}N_0$ and so forth. The value of the half-life depends on the nature of the radioactive nucleus.

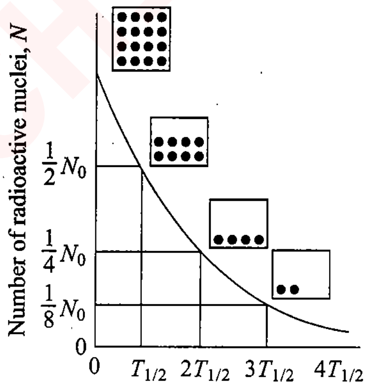


Fig. 5.15 The half-life $T_{1/2}$ of a radioactive decay is the time in which one-half of the radioactive nuclei disintegrate.

MEASUREMENT OF RADIOACTIVITY

Radioactivity of an element is measured in terms of 'activity'. The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms disintegrate. If N are the number of nuclei present in a radioactive sample at an instant, then activity of this sample is given as

$$A_c = -\frac{dN}{dt} \qquad \text{(i)}$$

Here, $\frac{dN}{dt}$ is negative as with time the number of elements always decreases due to disintegration; due to negative sign, A_c is always taken positive. The activity of a substance is measured in terms of 'dps' or disintegrations per second. The SI unit of activity is named after Bequeral, defined as

1 bequeral = 1 Bq = 1 disintegration/second

Generally, activities of radioactive samples in nature are very high. That is why bequeral being a very small unit, in normal practice, more often MBq or GBq is used. For the same traditional units, Curie (Ci) and Rutherford (Ru) are also used. These are defined as

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations s}^{-1}$$

$$1 \text{ Ru} = 10^{6} \text{ disintegrations s}^{-1}$$

Curie was originally defined as roughly the activity of 1 g of $^{226}_{88}\text{Ra}$. Similarly, it was observed that 1 kg of ordinary potassium has an activity of about 1 mCi (10^{-3} Ci) because in ordinary potassium small proportion of radioisotope $^{40}_{19}\text{K}$ is also present.

FUNDAMENTAL LAWS OF RADIOACTIVITY

On the basis of experiments performed by Rutherford and Soddy, some conclusions were made for behaviour radioactive elements and the properties of radioactivity. These conclusions are summarized as fundamental laws of radioactivity. These are discussed as follows.

(i) Radioactivity is purely a nuclear process. It is not concerned in any manner with the electrons orbiting the nucleus.

(ii) As radioactivity is a nuclear process, it is independent from any chemical property of the element. As we have discussed that radioactive property of an element is only the process concerned with nucleus of the element, it does not affect the electronic configuration of the element. If this element takes part in a chemical reaction, the product formed will also have the radioactive property in the same fraction by which the radioactive atom is present in the molecule of product.

(iii) As radioactivity is a random process, its study is only possible by laws of probability. In a group of several radioactive atoms, which one will disintegrate first is just a matter of chance.

(iv) As radioactivity is a random process, the disintegration density throughout the volume of a radioactive element remains constant. If an element X decays to a daughter nuclide Y, then in a given volume of the element, all portions of volume will have same ratio of number of atoms of Y to that of X. Thus, homogenity is maintained.

Thus due to randomness, the amount of disintegrations per unit volume per second (called disintegration density) remains approximately constant in the whole volume of the substance.

RADIOACTIVE DECAY LAW

This law relates the activity of a substance with the number of active or undecayed atoms present in a group of radioactive atoms at an instant of time. This law is stated as follows.

'The activity of a radioactive element at any instant is directly proportional to the number of undecayed active atoms (parent atoms) present at that instant.'

Let us consider that at $t = 0$, there are N_0 parent atoms in a substance and after a time t, N atoms are left undecayed. This implies that in the duration from $t = 0$ to $t = t$, $N_0 - N$ atoms are decayed to their daughter element. If in further time from $t = t$ to $t = t + dt$, dN more atoms will decay then at time $t = t$, we can say that the activity of the element is given as

$$A_c = \left|\frac{dN}{dt}\right| \quad \text{(i)}$$

Now, according to radioactive decay law, we have

$$\left|\frac{dN}{dt}\right| \propto N \quad \text{or} \quad A_c = \left|\frac{dN}{dt}\right| = \lambda N \quad \text{(ii)}$$

Here, λ is the proportionality constant, called decay constant for the decay process. The value of decay constant differs for different elements. From Eq. (ii), we can see that if λ is high the element will have high value of activity and if λ is less, the activity will be relativity less. Thus, we can say that the decay constant for a radioactive element gives a relative criteria of its stability as well as rate of reaction. If the value of λ for an element is more, it is more active or relatively less stable and if for an element λ is less, it is more stable. From Eq. (ii), we can also write

$$\lambda = \frac{\left|\frac{dN}{dt}\right|}{N} \quad \text{(iii)}$$

Thus, decay constant of a process can be given as 'activity per atom' [as given in Eq. (iii)]. This shows that for a given radioactive element the activity per atom always remains constant whereas we have already discussed that with time the overall activity of a substance decreases with time as the number of parent elements continuously decreases with time.

Now, from Eq. (iii), we can write

$$\frac{dN}{dt} = -\lambda N \quad \text{(iv)}$$

Here, negative sign shows that $\left|\frac{dN}{dt}\right|$, the rate at which the activity elements are disintegrating, is negative or number of active elements are decreasing with time. Now, we have from Eq. (iv),

$$\frac{dN}{N} = -\lambda dt$$

Decay constant is different for different radioactive substances, but it does not depend on the amount of substance and time. SI unit of λ is s^{-1}.

If $\lambda_1 > \lambda_2$, then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

$$A \xrightarrow{\lambda} B$$

$$t = 0 \qquad N_0 \qquad 0$$

where N_0 = number of active nuclei of A at $t = 0$.

$$t = t \qquad N \qquad N'$$

where N = number of active nuclei of A at $t = t$.

Integrating the above expression within time limits from $t = 0$ to $t = t$, we have

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda dt \quad \text{or} \quad [\ln N]_{N_0}^{N} = -\lambda t$$

or $$\ln\left(\frac{N}{N_0}\right) = -\lambda t \quad \text{or} \quad N = N_0 e^{-\lambda t} \qquad \text{(v)}$$

Equation (v) gives the number of active parent atoms N present at time t in the mixture. This equation is called radioactive decay equation.

From Eq. (v), we can have

$$\lambda N = \lambda N_0 e^{-\lambda t} \quad \text{or} \quad A_c = A_{c_0} e^{-\lambda t} \qquad \text{(vi)}$$

Here, $A_{c_0} = \lambda N_0$ is the initial activity of the substance at $t = 0$. Equation (vi) is another form of radioactive decay equation. This equation can be used to find the activity of a radioactive substance at any instant of time.

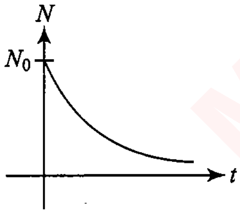


Fig. 5.16

Number of nuclei decayed (i.e., the number of nuclei of B formed),

$$N' = N_0 - N = N_0 - N_0 e^{\lambda t}$$

or $$N' = N_0(1 - e^{\lambda t})$$

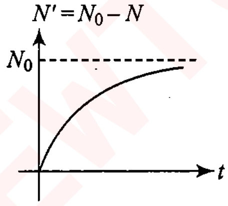


Fig. 5.17

ACTIVITY

The activity of a radioactive sample is the number of disintegrations per second that occur. Each time a disintegration occurs, the number N of radioactive nuclei decreases. As a result, the activity can be obtained by dividing ΔN (the change in the number of nuclei) by Δt (the time interval during which the change take place); the average activity over the time interval Δt is the magnitude of $\Delta N/\Delta t$. Since the decay of any individual nucleus is completely random, the number of disintegrations per second that occur in a sample is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad \text{(iv)}$$

where λ is a proportionality constant referred to as the decay constant. The minus sign is present in this equation because each disintegration decreases the number N of nuclei originally present.

The SI unit for activity is the becquerel (Bq); one becquerel equals one disintegration per second. Activity is also measured in terms of a unit called the curie (Ci), in honour of Marie Curie (1867–1934) and Pierre Curie (1859–1906), the discoverers of radium and polonium. Historically, the curie was chosen as a unit because it is roughly the activity of one gram of pure radium. In terms of becquerels,

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}$$

The activity of the radium put into the dial of a watch to make it glow in the dark is about 4×10^4 Bq, and the activity used in radiation therapy for cancer treatment is approximately a billion times greater, or 4×10^{13} Bq.

HALF-LIFE TIME

In previous section, we have discussed that during decay of a radioactive sample, the amount of radionuclide fall off exponentially with time. Every radioactive sample has a characteristic half-life. Half-life time is defined as the time duration in which half of the total number of nuclei will decay or be left undecayed. Say, for example, at any instant if we look into the quantity of a sample of radioactive element, it is observed that after every 3 hour some half-lives are only a millionth of a second for highly active elements and some less active elements have half-life in billions of years.

In some nuclear power plants, a major problem is disposal of the radioactive wastes since some of the nuclide present in waste have long half-lives.

For a radioactive element in a sample if at $t = 0$, N_0 nuclei are present of active parent element and during observation after $t = T$, $\frac{N_0}{2}$ are left, then this duration T can be taken as half-life of this element. From radioactive decay equation, we have

$$N = N_0 e^{\lambda t} \qquad \text{(i)}$$

Here, at $t = T$, $N = N_0/2$. Thus, we have in the above equation

$$\frac{N_0}{2} = N_0 e^{\lambda T} \quad \text{or} \quad \ln\left(\frac{1}{2}\right) = -\lambda T$$

or $$\ln(2) = \lambda T$$

or $$T = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \qquad \text{(ii)}$$

Number of nuclei present after n half-lives, i.e., after a time $t = n\, t_{1/2}$,

$$N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda n t_{1/2}} = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0\, e^{\ln 2^{(-n)}} = N_0\,(2)^{-n} = N_0(1/2)^n = \frac{N_0}{2^n}$$

$\left[n = \frac{t}{t_{1/2}}\right.$. It may be a fraction, need not to be an integer. $\left.\right]$

or $N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3$

$$\cdots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

Illustration 5.17 **A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?**

Sol. In one half-life, the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$6.0 \times 10^{18} \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = 1.5 \times 10^{18}$$

Illustration 5.18 **The half-life of a radioactive nuclide is 20 h. What fraction of original activity will remain after 40 h?**

Sol. 40 h means 2 half-lives. Thus,

$$A = \frac{A_0}{2^2} = \frac{A_0}{4} \quad \text{or} \quad \frac{A}{A_0} = \frac{1}{4}$$

So, one-fourth of the original activity will remain after 40 h.

Specific activity: The activity per unit mass is called specific activity.

ALTERNATIVE FORM OF DECAY EQUATION IN TERMS OF HALF-LIFE TIME

Let a radioactive element X decays to a daughter nucleus Y with a decay process nuclear reaction written as

$$X \xrightarrow{\lambda} Y$$

If initially N_0 nuclei of element X are present, then after time t number of nuclei present in the sample is given by decay equation given as

$$N = N_0 e^{-\lambda t} \qquad \text{(i)}$$

If we rearrange the equation, we have

$$\ln \frac{N}{N_0} = -\lambda T$$

We know that half-life of a substance is defined as

$$T = \frac{\ln(2)}{\lambda}$$

or we can write

$$\lambda = \frac{\ln(2)}{T}$$

Therefore, we have

$$\ln\left(\frac{N}{N_0}\right) = -\frac{\ln(2)}{T} t \quad \text{or} \quad \ln\left(\frac{N}{N_0}\right) = \ln(2)^{-t/T}$$

Taking antilog on both sides, we get

$$\frac{N}{N_0} = (2)^{-t/T} \quad \text{or} \quad N = N_0(2)^{-t/T}$$

This equation is an alternate form of decay equation useful for numerical applications.

Illustration 5.19 **Suppose 3.0×10^7 radon atoms are trapped in a basement at a given time. The basement is sealed against further entry of the gas. The half-life of radon is 3.83 days. How many radon atoms remain after 31 days?**

Sol. During each half-life, the number of radon atoms is reduced by a factor of two. Thus, we determine the number of half-lives in a period of 31 days and reduce the number of radon atoms by a factor of two for each one.

In a period of 31 days, there are 31 days/3.83 days = 8.1 half-lives. In 8 half-lives, the number of radon atoms is reduced by a factor of $2^8 = 256$. Ignoring the difference between 8 and 8.1 half-lives, we find that the number of atoms remaining is $3.0 \times 10^7/256 = 1.2 \times 10^5$.

Illustration 5.20 **In the above illustration, suppose there are 3.0×10^7 radon atoms ($T_{1/2} = 3.83$ days or 3.31×10^5 s) trapped in a basement. (a) How many radon atoms remain after 31 days? Find the activity (b) just after the basement is sealed against further entry of radon and (c) 31 days later.**

Sol. The number N of radon atoms remaining after a time t is given by $N = N_0 e^{-\lambda t}$, where $N_0 = 3.0 \times 10^7$ is the original number of atoms when $t = 0$ s and λ is the decay constant. The decay constant is related to the half-life $T_{1/2}$ of the radon atoms by $\lambda = 0.693/T_{1/2}$. The activity can be obtained from the equation $\Delta N/\Delta t = -\lambda N$.

a. The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.83 \text{ days}} = 0.181 \text{ days}^{-1} = 0.181 \text{ days}^{-1} \qquad \text{(i)}$$

and the number N of radon atoms remaining after 31 days is

$$N = N_0 e^{\lambda t} = (3.0 \times 10^7) \exp\{-(0.181 \text{days}^{-1})(31 \text{days})\}$$
$$= 1.1 \times 10^5 \qquad \text{(ii)}$$

This value is slightly different from that found in Illustration 5.19 because there we ignored the difference between 8.0 and 8.1 half-lives.

b. The activity can be obtained from equation $\Delta N/\Delta t = -\lambda N$, provided the decay constant is expressed in reciprocal seconds: $\lambda = 0.693/(3.31 \times 10^5 \text{ s}) = 2.09 \times 10^{-6} \text{ s}^{-1}$. Thus, the number of disintegrations per second is

$$\frac{\Delta N}{\Delta t} = -\lambda N = -(2.09 \times 10^{-6} \text{ s}^{-1})(3.0 \times 10^7)$$
$$= -63 \text{ disintegrations s}^{-1}$$

The activity is the magnitude of $\Delta N/\Delta t$, so initially

Activity = 63 Bq

c. From part (a), the number of radioactive nuclei remaining at the end of 31 days is $N = 1.1 \times 10^5$, and reasoning similar to that in part (b) reveals that

$$\text{Activity} = 0.23 \text{ Bq}$$

SI unit of activity is becquerel (Bq) which is same as 1 dps (disintegrations per second).

The popular unit of activity is curie which is defined as

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ dps}$$

(which is activity of 1 g radium)

Illustration 5.21 **The decay constant for the radioactive nuclide ^{64}Cu is 1.516×10^{-5} s^{-1}. Find the activity of a sample containing 1 mg of ^{64}Cu. Atomic weight of copper = 63.5 g mol^{-1}. Neglect the mass difference between the given radioisotope and normal copper.**

Sol. 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 mg of Cu is

$$N = \frac{6 \times 10^{23} \times 1\,\mu\text{g}}{63.5 \text{ g}} = 9.45 \times 10^{15}$$

The activity $= \lambda N$

$$= (1.516 \times 10^{-5} \text{ s}^{-1}) \times (9.45 \times 10^{15})$$

$$= 1.43 \times 10^{11} \text{ disintegrations s}^{-1}$$

$$= \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}} \text{ Ci} = 3.86 \text{ Ci}$$

Activity after n half-lives is $A_0/2^n$.

AVERAGE LIFE

$$T_{avg} = \frac{\text{Sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty \lambda N_0 e^{-\lambda t} dt\, t}{N_0} \frac{1}{\lambda}$$

Important Points

- Time constant or average life is more than the half-life, i.e.,

$$\tau = T_{av} > T$$

- The fractional radioactive decay in one mean life (or time constant) is more than that in half life, as shown in Fig. 5.18.

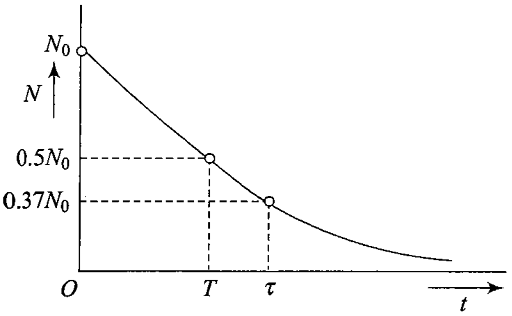


Fig. 5.18

- The number of undecayed unclei present after n mean life is

$$N = (0.37)^n N_0 = \left(\frac{1}{e}\right)^n N_0$$

- If a nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2, then the effective decay constant of the nuclide is

$$\lambda = \lambda_1 + \lambda_2\,;\, N = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

Illustration 5.22 **The half-life of ^{198}Au is 2.7 days. Calculate (a) the decay constant, (b) the average-life and (c) the activity of 1.00 mg of ^{198}Au. Take atomic weight of ^{198}Au to be 198 g mol^{-1}.**

Sol. a. The half-life and the decay constant are related as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad \text{or} \quad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7 \text{ days}}$$

$$= \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}} = 2.9 \times 10^{-6} \text{ s}^{-1}$$

b. The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days.

c. The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms. The number of atoms in 1.00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1.0 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$$

Thus,

$$A = \lambda N = (2.9 \times 10^{-6} \text{ s}^{-1})(3.03 \times 10^{18})$$

$$= 8.8 \times 10^{12} \text{ disintegrations s}^{-1}$$

$$= \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = 240 \text{ Ci}$$

Illustration 5.23 **A 50.0 g sample of carbon is taken from the pelvis bone of a skeleton and is found to have a ^{14}C decay rate of 200.0 decays min^{-1}. It is known that carbon from a living organism has a decay rate of decays/min^{-1} g^{-1} and that ^{14}C has a half-life of 5730 year = 3.01×10^9 min. Find the age of the skeleton.**

Sol. Let us start with the equation $N = N_0 e^{-\lambda t}$ and multiply both sides by λ to get $\lambda N = \lambda N_0 e^{-\lambda t}$ which is equivalent to

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad \frac{R}{R_0} = e^{-\lambda t}$$

where R is the present activity and R_0 was the activity when the skeleton was a part of a living organism. We can solve for the time by taking the natural log of both sides of this equation.

$$\ln\left(\frac{R}{R_0}\right) = \ln(e^{-\lambda t}) = -\lambda t$$

$$t = -\frac{\ln\left(\frac{R}{R_0}\right)}{\lambda}$$

Because we are given the decay rate and mass of the sample, we can find R_0 as

$$R_0 = \left(15.0\frac{\text{decays}}{\text{min g}}\right)(50.0\text{ g}) = 750\frac{\text{decays}}{\text{min}}$$

The decay constant is found as

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.01\times10^9\text{ min}}$$

$$= 2.30\times10^{-10}\text{ min}^{-1}$$

Illustration 5.24 **(a) Determine the number of carbon $^{14}_{6}C$ atoms present for every gram of carbon $^{12}_{6}C$ in a living organism. Find (b) the decay constant and (c) the activity of this sample.**

Sol. The total number of carbon $^{12}_{6}C$ atoms in one gram of carbon $^{12}_{6}C$ is equal to the corresponding number of moles times Avogadro's number. Since there is only one $^{14}_{6}C$ atom for every 8.3×10^{11} atoms of $^{12}_{6}C$, the number of $^{14}_{6}C$ atoms is equal to the total number of $^{12}_{6}C$ atoms divided by 8.3×10^{11}. The decay constant λ for $^{14}_{6}C$ is $\lambda = 0.693/T_{1/2}$, where $T_{1/2}$ is the half-life. The activity is equal to the magnitude of $\Delta N/\Delta t$, which is the decay constant times the number of $^{14}_{6}C$ atoms present.

a. One gram of carbon $^{12}_{6}C$ (atomic mass = 12 u) is equivalent to 1.0/1.2 mol. Since Avogadro's number is 6.02×10^{23} atoms mol^{-1} and since there is one $^{14}_{6}C$ atom for every 8.3×10^{11} atoms of $^{12}_{6}C$, the number of $^{14}_{6}C$ atoms for every 1.0 g of carbon $^{12}_{6}C$ is

$$\left(\frac{1.0}{2}\text{mol}\right)\left(6.02\times10^{23}\frac{\text{atoms}}{\text{mol}}\right)\left(\frac{1}{8.3\times10^{11}}\right)$$

$$= 6.0\times10^{10}\text{ atoms}$$

b. Since the half-life of $^{14}_{6}C$ is 5730 years (1.81×10^{11} s), the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.81\times10^{11}\text{s}}$$

$$= 3.83\times10^{-12}\text{ s}^{-1}$$

c. The activity is the magnitude of $\Delta N/\Delta t$ or λN. Thus, we find activity of $^{14}_{6}C$ for every 1.0 g of carbon $^{12}_{6}C$ in a living organism $= \lambda N = (3.83\times10^{-12}\text{ s}^{-1})(6.0\times10^{10}\text{ atoms}) = 0.23$ Bq.

An organism that lived thousands of years ago presumably had an activity of about 0.23 Bq per gram of carbon. When the organism died, the activity began decreasing. From a sample of the remains, the current activity per gram of carbon can be measured and compared to the value of 0.23 Bq to determine the time that has transpired since death.

RADIOACTIVE DATING

One important application of radioactivity is the determination of the age of archeological or geological samples. If an object contains radioactive nuclei when it is formed, then the decay of these nuclei marks the passage of time like a clock. If the half-life is known, a measurement of the number of nuclei present today relative to the number present initially can give the age of the sample. A more accurate way is to determine the present number of radioactive nuclei with the aid of a mass spectrometer.

The present activity of a sample can be measured, but how is it possible to know what the original activity was, perhaps thousands of years ago? Radioactive dating methods entail certain assumptions that make it possible to estimate the original activity. For instance, the radiocarbon technique utilizes the $^{14}_{6}C$ isotope of carbon, which undergoes β^- decay with a half-life of 5730 years. This isotope is present in the earth's atmosphere at an equilibrium concentration of about one atom for every 8.3×10^{11} atoms of normal carbon $^{12}_{6}C$. It is often assumed that this value has remained constant over the years because $^{14}_{6}C$ is created when cosmic rays interact with the earth's upper atmosphere, a production method that offsets the loss via β^- decay. Moreover, nearly all living organisms ingest the equilibrium concentration of $^{14}_{6}C$. However, once an organism dies, metabolism no longer sustains the input of $^{14}_{6}C$, and β^- decay causes half of the $^{14}_{6}C$ nuclei to disintegrate every 5730 years. Illustration 5.24 illustrates how to determine the $^{14}_{6}C$ activity of one gram of carbon in a living organism.

Illustration 5.25 **The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5 × 10 9 years. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.**

Sol. Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^9 years old.

Illustration 5.26 **A bottle of red wine is thought to have been sealed about 5 years ago. The wine contains a number of different kinds of atoms, including carbon, oxygen and hydrogen. Each of these has a radioactive isotope. The radio-**

active isotope of carbon is the familiar $^{14}_{6}C$, with a half-life of 5730 years. The radioactive isotope of oxygen is $^{15}_{8}O$ and has a half-life of 122.2 s. The radioactive isotope of hydrogen is $^{3}_{1}H$ and is called tritium; its half-life is 12.33 years. The activity of each of these isotopes is known at the time the bottle was sealed. However, only one of the isotopes is useful for determining the age of the wine accurately. Which is it?

Sol. In a dating method that measures the activity of a radioactive isotope, the age of the sample is related to the change in the activity during the time period in question. Here, the expected age is about 5 years. This period is only a small fraction of the 5730-year half-life of $^{14}_{6}C$. As a result, relatively few of the $^{14}_{6}C$ nuclei would decay during the wine's life, and the measured activity would change little from its initial value. To obtain an accurate age from such a small change would require prohibitively precise measurements. Nor is the $^{15}_{8}O$ isotope very useful. The difficulty is its relatively short half-life of 122.2 s. During a 5-year period, so many half-lives of 122.2 s would occur that the activity would decrease to a vanishingly small level. It would not be even possible to measure it. The only remaining option is the tritium isotope of hydrogen. The expected age of 5 years is long enough relative to the half-life of 12.33 years that a measurable change in activity will occur, but not so long that the activity will have completely vanished for all practical purposes.

NATURAL RADIOACTIVITY: RADIOACTIVE DECAY SERIES

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei found in nature, which give rise to what is called natural radioactivity and (2) nuclei produced in the laboratory through nuclear reactions, which exhibit artificial radioactivity.

Three series of naturally occurring radioactive nuclei exist (as shown in the following table). Each series starts with a specific long-lived radioactive isotope whose half-life exceeds

Four radioactive series			
Series	**Starting isotope**	**Half-life (year)**	**Stable end product**
Uranium	$^{238}_{92}U$	4.47×10^9	$^{206}_{82}U$
Actinium	$^{235}_{92}U$	7.04×10^8	$^{207}_{82}Pb$
Thorium	$^{232}_{90}U$	1.41×10^{10}	$^{208}_{82}Pb$
Neptunium	$^{237}_{93}U$	2.14×10^6	$^{209}_{83}Pb$

that of any of its descendants. The fourth series in the table begins with ^{237}Np, a transuranic element (on having an atomic number greater than that of uranium) not found in nature. This element has a half-life of 'only' 2.14×10^6 years.

The two uranium series are somewhat more complex than the ^{232}Th series. Also, several other naturally occurring radioactive elements that would otherwise have disappeared long ago. For example, because the solar system is about 5×10^9 years old, the supply of ^{226}Ra (whose half-life is only 1600 years) would have been depleted by radioactive decay long ago if it were not for the decay series that starts with ^{238}U, with a half-life of 4.47×10^9 years. Figure 5.19 shows the radioactive decay series that begins with uranium $^{238}_{92}U$ and ends with lead $^{206}_{82}Pb$. Half-lives are given in seconds (s), minutes (min), hours (hr), days (d) or years (yr). The inset in the upper left identifies the type of decay that each nucleus undergoes.

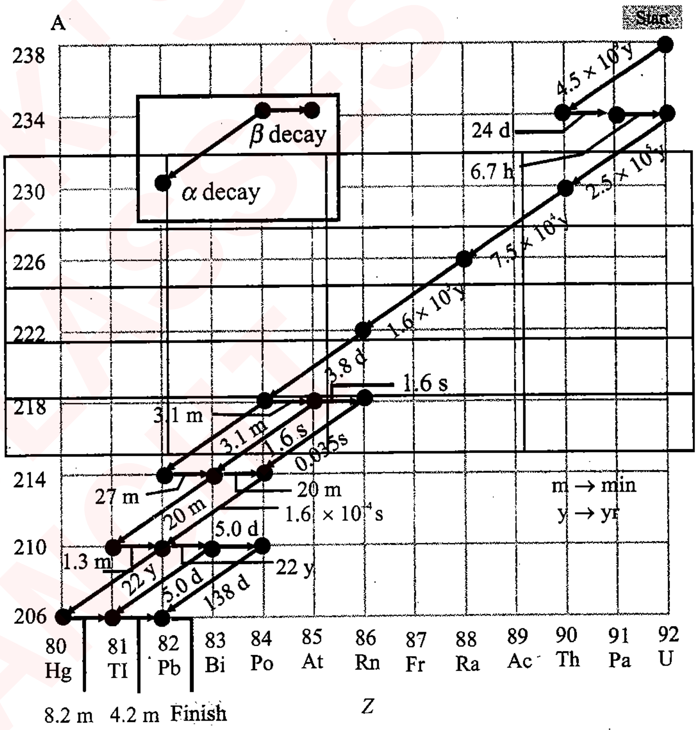


Fig. 5.19

Illustration 5.27 **A radioactive nucleus undergoes a series of decay according to the scheme**

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, what are these numbers for A_4?

Sol. The successive process may be expressed as

$$_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} \xrightarrow{\beta^-} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} \xrightarrow{\gamma} {}_{69}A_4^{172}$$

Radioactive Equilibrium

In a radioactive series, if we talk about an intermediate element, it is produced due to the decay of its previous element and it decays to the next element of the series. In Eq. (i), an intermediate element J decays to K with a decay constant λ_1 and K decays to L with decay constant λ_2.

$$\rightarrow \underset{N_1}{J} \xrightarrow{\lambda_1} \underset{N_2}{K} \xrightarrow{\lambda_2} L \qquad \text{(i)}$$

If at an instant, N_1 nuclei of J are present, its disintegration rate can be given as

$$R_1 = \lambda_1 N_1 \quad \text{(ii)}$$

As J decays to K, the above relation in Eq. (ii) also gives the formation rate of nuclei of K. If at this instant N_2 nuclei of K are present, its decay rate can be given as

$$R_2 = \lambda_2 N_2 \quad \text{(iii)}$$

If at some instant the production rate and decay rate of the element K become equal, then the amount of K appears to be a constant as the number of nuclei of K produced per second are equal to the number of nuclei of K disintegrating per second. This situation for the intermediate element K is called radioactive equilibrium. We can also state that his equilibrium is a dynamic equilibrium in which the amount of K element appears to be a constant along with the process of its continuous formation by decaying element J and its continuous disintegration to element L. Thus, for an element, condition of radioactive equilibrium is

Rate of formation = Rate of disintegration

Here, for element K to be in radioactive equilibrium, we have

$$\lambda_1 N_1 = \lambda_2 N_2 \quad \text{(i)}$$

Simultaneous Decay Modes of Radioactive Element

We know that due to radioactive disintegration a radio nuclide transforms into its daughter nucleus. Depending on the nuclear structure and its unstability, a parent nucleus may undergo either α or β emission. Sometimes a parent nucleus may undergo both types of emission with the probabilities of α-decay or β-decay. The amount of daughter nuclide produced by α- and β-decay will be in the probability ratio of α- and β-decays.

Now, we analyze the decay of such elements which disintegrate with two or more decay modes simultaneously. If an element decays to different daughter nuclei with different decay constants λ_1, λ_2, λ_3, etc., for each decay mode, then the effective decay constant of the parent nuclei can be given as

$$\lambda_{\text{eff}} = \lambda_1 + \lambda_2 + \cdots$$

Similarly, for a radioactive element with decay constant λ which decays by both α- and β-decays given that the probability for an α-emission is P_1 and that for β-emission is P_2, the decay constant of the element can be split for individual decay modes. Like in this case the decay constants for α- and β-decays separately can be given as

$$\lambda_\alpha = P_1\lambda$$

$$\lambda_\beta = P_2\lambda$$

Accumulation of a Radioactive Element in Radioactive Series

In a radioactive series, we have discussed that each element decays into its daughter nuclei until a stable element appears. Consider a radioactive series shown below

$$A_1 \xrightarrow{\lambda_1} A_2 \xrightarrow{\lambda_2} A_3 \xrightarrow{\lambda_3} \cdots$$

To analyze mathematically the above series, we assume initially at $t = 0$, N_0 atoms of parent element A_1 are present which decay to the element A_2 with a decay constant λ_1. Thus, after time t, number of undecayed nuclei of A_1 present at a time instant t can be given be decay law as

$$N_1 = N_0 e^{-\lambda_1 t} \quad \text{(i)}$$

Due to disintegration of A_1, nuclei of A_2 are formed and these start decaying with a decay constant λ_2 to another element A_3. Let at an instant t, N_2 undecayed nuclei of A_2 are present. Then, the decay rate of A_2 at this instant can be given as

$$\text{Decay rate of } A_2 = \lambda_2 N_2 \quad \text{(ii)}$$

Due to disintegration of A_1, A_2 is produced. Thus, the production rate of nuclei of A_2 will be the decay rate of nuclei of A_1. Hence, production rate of A_2 at this instant can be given as

$$\text{Production rate of } A_2 = \lambda_1 N_1 \quad \text{(iii)}$$

Now, in a further time dt, if dN_2 nuclei of element A_2 are accumulated, then the accumulation rate of nuclei of element A_2 can be given as

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

or
$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t} \quad \text{(iv)}$$

Equation (iv) is a simple linear differential equation which on solving gives the number of nuclei of element A_2 as a function of time t. On solving the equation, we get

$$N_2 = \frac{\lambda_1 N_0}{(\lambda_1 - \lambda_2)}(e^{-\lambda_2 t} - e^{-\lambda_1 t}) \quad \text{(v)}$$

Here, we can see that in the beginning as $N_2 = 0$. Due to disintegration of A_1, A_2 is being formed and as the amount of A_1 is decreased and that of A_2 is increased, decay rate of A_2 increases and that of A_1 decreases. After a time when both decay rates becomes equal, the element A_2 will be said to be in radioactive equilibrium when the yield of the radionuclide A_2 is maximum.

Let us take some examples to understand the above phenomenon better.

Illustration 5.28 **Suppose the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.**

Sol. The number of parent nuclei decaying in a short time interval t to $t + dt$ is $\lambda_p N_p dt$. The number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt \quad \text{or} \quad \lambda_p N_p = \lambda_d N_d$$

Illustration 5.29 **A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by**

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

Sol. The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \frac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus,

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or $$\lambda = \lambda_1 + \lambda_2$$

or $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

Illustration 5.30 **A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find (i) the number of nuclei of A and (ii) number of nuclei of B, at any time t assuming the production of A starts at $t = 0$. (iii) Also, find out the maximum number of nuclei of 'A' present at any time during its formation.**

Sol. (i) Factory $\xrightarrow[\text{const. rate}]{R}$ A $\xrightarrow[\text{decay}]{\lambda}$ B

Let N be the number of nuclei of A at any time t.

$$\therefore \quad \frac{dN}{dt} = R - \lambda N \qquad \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving, we will get

$$N = R/\lambda(1 - e^{-\lambda t})$$

(ii) Number of nuclei of B at any time t,

$$N_B = Rt - N_A$$

$$= Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda\,(\lambda t - 1 + e^{-\lambda t})$$

(iii) Maximum number of nuclei of 'A' present at any time during its formation $= R/\lambda$.

Illustration 5.31 **Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.**

a. Calculate the number N of nuclei of A at time t.

b. If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life time of A and also the limiting value of N at $t \to \infty$. **(IIT-JEE, 1998)**

Sol. a. The rate of formation of radioactive nuclei is α.

Rate of decay of radioactive nuclei is λN. Therefore,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha - \lambda N} = dt$$

Integrating,

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + A \qquad \text{(i)}$$

where A is constant of integration.

At $t = 0$, $N = N_0$

$$\therefore \quad \frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = A$$

Therefore, Eq. (i) gives

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - \lambda N)}{-\lambda}$$

$$\Rightarrow \log_e\left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right) = -\lambda t$$

$$\Rightarrow \quad \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t} \qquad \text{(ii)}$$

b. Given, $a = 2N_0\lambda$

$$t = T_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore \quad N = \frac{2N_0\lambda}{\lambda}(1 - e^{-0.693}) + N_0 e^{-0.693}$$

$$= N_0(2 - e^{-0.693}) = N_0(2 - 0.5)$$

$$\therefore \quad N = 1.5\,N_0$$

When $t \to \infty$, Eq. (ii) gives

$$N = \frac{\alpha}{\lambda}$$

Illustration 5.32 **The mean lives of a radioactive substance are 1620 years and 405 years for α-emission and β-emission, respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously.**

Sol. Radioactivity is the statistical phenomenon. In statistics the important theorem is that the probability of a composite event is equal to the product of probabilities of individual and independent events. Let λ_α and λ_B be the decay constant for α- and β-emission, respectively.

According to theorem, the decay constant for composite event is $\lambda_\alpha + \lambda_\beta$.

If T is half-life of composite event and T_α and T_β are the half-lives of α- and β-emissions, then $\lambda = \frac{0.693}{T}$.

$$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} \quad \text{or} \quad \frac{1}{\tau} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta}$$

This gives

$$\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \frac{1620 \times 405}{1620 + 405} = 324 \text{ years}$$

Let t be the time in which the given sample decays three-fourth. Therefore, the fraction of sample undecayed in time t is 1/4. That is,

$$\frac{N}{N_0} = \frac{1}{4}$$

From the relation $N = N_0 e^{-\lambda t}$, we have

$$\frac{N}{N_0} = e^{-\lambda t}$$

or $$\log_e \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \quad t = \frac{1}{\lambda} \log_e \frac{N_0}{N} = \tau \log_e \frac{N_0}{N} \qquad \left(\because \lambda = \frac{1}{\tau}\right)$$

$$= 2.3026\, \tau \log_{10} \frac{N_0}{N} = 2.3026 \times 324 \log_{10} 4$$

$$= 2.3026 \times 324 \times 0.6031$$

Required time, $t = 449.94$ years

Illustration 5.33 **Find the half-life of uranium, given that 3.32×10^{-7} g of radium is found per gram of uranium in old minerals. The atomic weights of uranium and radium are 238 and 226 and half-life of radium is 1600 years (Avogadro number is 6.023×10^{23}/g-atom).**

Sol. In very old minerals, the amount of an element is constant. This implies that the element exists in radioactive equilibrium. Thus, here we can use

$$\lambda_U N_U = \lambda_R N_R$$

or $$\frac{N_U}{T_U} = \frac{N_R}{T_R} \quad \text{or} \quad T_U = \frac{N_U}{N_R} \times T_R$$

or $$T_U = \frac{m_U A_R}{m_R A_U} \times T_R$$

or $$T_U = \frac{1 \times 226}{3.32 \times 10^{-7} \times 238} \times 1600 \text{ years}$$

$$= 4.7 \times 10^9 \text{ years}$$

NUCLEAR REACTIONS

It is possible to change the structure of nuclei by bombarding them with energetic particles. Such changes are called nuclear reactions. Rutherford was the first to observe nuclear reactions, using naturally occurring radioactive source for the bombarding particles. He found that protons were released when alpha particles were allowed to collide with nitrogen atoms. The process can be represented symbolically as

$${}^4_2\text{He} + {}^{14}_7\text{N} \rightarrow \text{X} + {}^1_1\text{H} \qquad \text{(i)}$$

This equation says that an alpha particle $({}^4_2\text{He})$ strikes a nitrogen nucleus and produces are an unknown product nucleus (X) and a proton $({}^1_1\text{H})$. Balancing atomic numbers and mass numbers, as we did for radioactive decay, enables us to conclude that the unkown is characterized as ${}^{17}_8\text{X}$. Because the element with atomic number 8 is oxygen, we see that the reaction is

$${}^4_2\text{He} + {}^{14}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H} \qquad \text{(ii)}$$

This nuclear reaction starts with two stable isotopes, helium and nitrogen, and produces two different stable isotopes, hydrogen and oxygen.

Kinematics of Nuclear Reactions

When two particles approach each other, their mutual interaction changes their motion. Exchange of momentum and energy takes place between particles.

When the final particles or system are the same as initial ones, the term scattering is used. The collision is said to be a reaction if final particles are not identical to initial ones, e.g., in a collision between an electron and a proton, the final product may be a hydrogen atom.

Since only internal force is involved in a collision, both the momentum and total energy are conserved. From conservation of momentum,

$$\underbrace{P_1' + P_2'}_{\text{after}} = \underbrace{P_1 + P_2}_{\text{before}} \qquad \text{(i)}$$

The kinetic energies of particles before and after collision are given by

$$E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \qquad \text{(before) (ii)}$$

$$E_K' = \frac{1}{2} m_1' v_1'^2 + \frac{1}{2} m_2' v_2'^2 = \frac{P_1'^2}{2m_1'} + \frac{P_2'^2}{2m_2} \qquad \text{(after) (iii)}$$

The internal energy of the particles may be different after collision. Hence,

$$\underbrace{E_K' + E_{int}'}_{\text{after}} = \underbrace{E_K + E_{int}}_{\text{before}}$$

If the internal energy changes in the collision, the kinetic energy also changes.

The Q of the reaction or collision is defined by

$$Q = E_K' - E_K = U_{int} - U_{int}$$

$$= (U_{int})_{before} - (U_{int})_{after} = -\Delta U_{int}$$

$$Q = \left(\frac{p_1'^2}{2m_1'} + \frac{p_2'^2}{2m_2'}\right) - \left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}\right) \quad \text{(iv)}$$

When $Q = 0$, there is no change in kinetic energy or internal potential energy and the collision is elastic.

When $Q < 0$, there is a decrease in kinetic energy with a corresponding increase in internal energy of the particles. Such collision is called endothermic. When $Q > 0$, there is an increase in kinetic energy at the expense of internal energy. This is an exothermic reaction.

Figure 5.20 shows the possible stages of nuclear reactions. Reflection of the incident particles as described by Rutherford's theory is called elastic scattering.

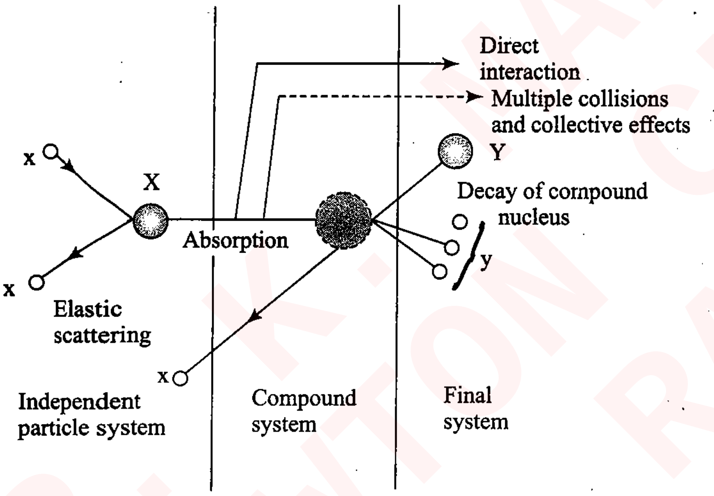


Fig. 5.20

In a direct interaction, the incident particles (which have high energy and can penetrate deeper into the nucleus) interact with a single nucleon in the nucleus so that the nucleon leaves the nucleus. If the nucleon does not leave the nucleus but interacts with several other nucleons, then the nucleus is raised to an excited state called a compound nucleus. The energy carried by the incident particle is shared by many nucleons. The compound nucleus can emit a particle identical to the incident particle and with the same kinetic energy or emit photons or other particles. The decay of the compound nucleus is a statistical process. It does not depend on the process of formation or external conditions applied, similar to radioactivity. The emission of particles or photons cannot be predicted.

A nuclear reaction can occur only if certain conservation laws are followed. These are:

1. Conservation of mass number A.
2. Conservation of charge.
3. Conservation of energy, linear momentum and angular momentum.

Energy Conservation in Nuclear Reactions

Consider a general reaction of particle x that is incident on nucleus X resulting in nucleus Y and particle y. The reaction may be written as

$$x + X \to Y + y + Q$$

Sometimes the reaction is written as X(x, y)Y, e.g., consider the reaction ^{7}Li (p, α) ^{4}He which can be written as

$$^1_1H + ^7_3Li \to ^4_2He + ^4_2He$$

We apply conservation of energy to a reaction to compute total kinetic energy released or absorbed in the reaction. Here, we assume that target particle is at rest, K_x, K_y, K_Y represent kinetic energyies of bombarding particle x and reaction products y and Y, respectively.

From conservation of energy,

$$m_xc^2 + K_x + m_Xc^2 = m_yc^2 + K_y + m_Yc^2 + K_Y$$

The total energy released in the reaction (Q) is equal to the difference in kinetic energies of the final particles and of initial particles. So,

$$Q = K_Y + K_y - K_x = (m_x + m_X - m_y - m_Y)c^2$$

which is the energy released in the reaction and is called the Q value of the reaction.

In exothermic reactions, energy is released by a nuclear reaction. In an exothermic reaction, the total mass of the initial particles is greater than that of the final particles and the Q value is positive. In these reactions, some mass is converted to energy.

In endothermic reactions, energy is absorbed in a nuclear reaction. If the total mass of the initial particles is less than that of the final particles, the Q value is negative and some minimum input kinetic energy from the bombarding particles is required. For example,

$$n + {}^1H \to {}^2H + y + 2.22\text{ MeV} \quad \text{(exothermic)}$$

$$y + {}^2H \to {}^1H + (n) - 2.22\text{ MeV} \quad \text{(endothermic)}$$

The minimum kinetic energy needed to initiate a nuclear reaction is called threshold energy. An endothermic reaction cannot take place unless certain threshold energy is supplied to the system. The threshold energy K_{th} must not only supply $|Q|$ but also some kinetic energy to the products to conserve momentum. In the centre of mass frame, the total momentum is zero; the threshold energy is just $|Q|$.

But for reactions occurring with nucleus X at rest relative to the laboratory (called the laboratory frame), the incident particle X must have energy greater than $|Q|$ because by conservation of momentum, the kinetic energies of y and Y cannot be zero. Consider a particle x, of mass m, incident on X, of mass M. Both particles have momenta of equal magnitude in the centre of mass frame.

The total kinetic energy is

$$E_{CM} = \frac{p^2}{2m} + \frac{p^2}{2M} = \frac{1}{2}p^2\left(\frac{m+M}{mM}\right)$$

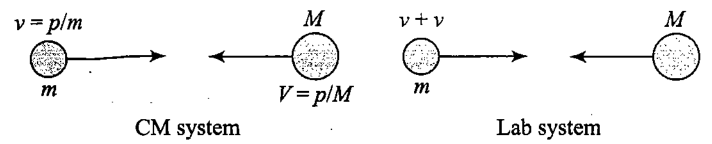


Fig. 5.21

$$p = mv = MV \qquad p_{\text{lab}} = m(v+V)$$

$$E_{\text{CoM}} = p^2/2m + p^2/2M \qquad = mv(1 + m/M)$$

$$= (m+M)_p^2/2mM \qquad = \frac{M+m}{M}p$$

$$E_{\text{lab}} = \frac{p_L^2}{2m}$$

$$= \left(\frac{p^2}{2m}\right)\left(\frac{M+m}{M}\right)^2$$

$$= \frac{M+m}{M}E_{\text{CM}}$$

We can transform the velocity of incident and target particles to the lab frame by adding V to each velocity so that M is at rest and m has velocity $v + V$. The momentum of m in the lab frame is then

$$P_{\text{lab}} = m(v+V) = mv\left(1+\frac{m}{M}\right) = p\left(\frac{m+M}{M}\right)$$

And its energy is

$$E_{\text{lab}} = \frac{p_{\text{lab}}^2}{2m} = \frac{p^2}{2m}\left(\frac{M+m}{M}\right)^2 = \frac{M+m}{M}E_{\text{CM}}$$

Hence, the threshold energy for an endothermic reaction in the lab frame is

$$E = \frac{m+M}{M}|Q|$$

Illustration 5.34 **Find the Q value of the reaction p + ^{7}Li → ^{4}He + ^{4}He. Determine whether the reaction is exothermic or endothermic. The atomic masses of ^{1}H, ^{4}He and ^{7}Li are 1.007825 u, 4.002603 u and 7.016004 u, respectively.**

Sol. The total mass of the initial particles,

$$m_i = 1.007825 + 7.016004 = 8.023829 \text{ u}$$

and the total mass of the final particles

$$m_f = 2 \times 4.002603 = 8.005206 \text{ u}$$

Difference between initial and final mass of particles

$$\Delta m = m_i - m_f = 8.023829 - 8.0065206 = 0.018623 \text{ u}$$

This mass is converted into energy and the reaction is exothermic.

The Q value is positive and given by

$$Q = (\Delta m)c^2 = 0.018623 \times 931.5 = 17.3.5 \text{ MeV}$$

Illustration 5.35 **Consider a collision between two particles one of which is at rest and the other strikes it head on with momentum p_1. Calculate the energy of reaction Q in terms of the kinetic energy of the particles before and after they collide.**

Sol. Initially, m_1 has a momentum p_1 and m_2 is at rest ($p_2 = 0$) in the lab frame. The masses of the particles after the collision are m_1' and m_2'.

The conservation of momentum gives

$$p_1' + p_2' = p_1 \text{ or } p_2' = p_1 - p_1' \qquad \text{(i)}$$

Squaring this equation, we have

$$p_2'^2 = (p_1 - p_1')^2 = p_1^2 + p_1'^2 - 2p_1p_1'$$

$$= p_1^2 + p_1'^2 - 2p_1p_1' \cos\theta$$

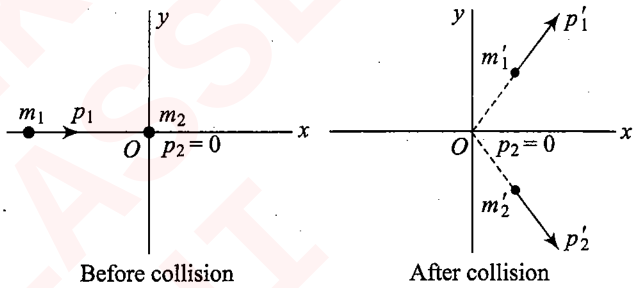


Fig. 5.22

We have

$$Q = \frac{p_1'^2}{2m_1'} + \frac{p_2'^2}{2m_2'} - \frac{p_1^2}{2m_1} + \frac{p_1'^2}{2m_1'}$$

$$+ \frac{1}{2m_2'}(p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta) - \frac{p_1^2}{2m_1}$$

$$\text{or} \quad Q = \frac{1}{2}\left(\frac{1}{m_1'} + \frac{1}{m_2'}\right)p_1'^2 + \frac{1}{2}\left(\frac{1}{m_2'} - \frac{1}{m_1}\right)p_1^2 - \frac{p_1p_1'}{m_2'}\cos\theta \qquad \text{(ii)}$$

Note that the kinetic energy of a particle can be expressed in terms of momentum of particles as $E_k = \dfrac{p^2}{2m}$.

Now, we can express the above results as

$$Q = E_{k,1}'\left(1 - \frac{m_1}{m_2'}\right) = -\frac{2(m_1m_1'E_{k,1}E_{k,1}')^{1/2}}{m_2'}\cos\theta \qquad \text{(iii)}$$

The equation derived above is called Q equation. It is applied in analysis of nuclear collisions.

NUCLEAR FISSION

Nuclear fission occurs when a heavy nucleus, such as ^{235}U, splits, or fissions, into two smaller nuclei. In such a reaction, the total mass of the products is less than the original mass of the heavy nucleus.

Nuclear fission was first observed in 1939 by Otto Hahn and Fritz Strassman, following some basic studies by Fermi. After bombarding uranium ($Z = 92$) with neutrons, Hahn and Strassman discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lisa Meithner and Otto Frisch explained what had happened. The uranium nucleus had split into two nearly equal fragments after absorbing a neutron. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus. But it was to have even more far-reaching consequences. Measurements showed that about 200 MeV of energy is released in each fission event, and this fact was to affect the course of human history.

The fission of ^{235}U by slow (low-energy) neutrons can be represented by the equation

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^* \rightarrow X + Y + \text{neutrons} \qquad \text{(i)}$$

where $^{236}U^*$ is an intermediate state that lasts only for about 10^{-12} s before splitting into X and Y. The resulting nuclei, X and Y, are called fission fragments. Many combinations of X and Y satisfy the requirements of conservation of energy and charge. In the fission of uranium, there are about 90 different daughter nuclei that can be formed. The process also results in the production of several (typically two or three) neutrons per fission event. On the average, 2.47 neutrons are released per event.
A typical reaction of this type is

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n \qquad \text{(ii)}$$

The fission fragments, barium and krypton, and the released neutrons have a great deal of kinetic energy following the fission event.

The breakup of the uranium nucleus can be equated to the case of a drop of water when excess energy is added to it. All of the atoms in the drop have energy, but not enough to break up the drop. However, if enough energy is added to set the drop vibrating, it will undergo elongation and compression until the amplitude of vibration becomes large enough to cause the drop to break apart. In the uranium nucleus, a similar process occurs (Fig. 5.23). The sequence of events is as follows:

1. ^{235}U nucleus captures a thermal (slow-moving) neutron.
2. This capture results in the formation of ^{236}U*, and the excess energy of this nucleus causes it to undergo violent oscillations.
3. The ^{236}U* nucleus becomes highly elongated, and the force of repulsion between protons in the two halves of the dumbbell shape tends to increase the distortion.
4. The nucleus splits into two fragments, emitting several neutrons in the process.

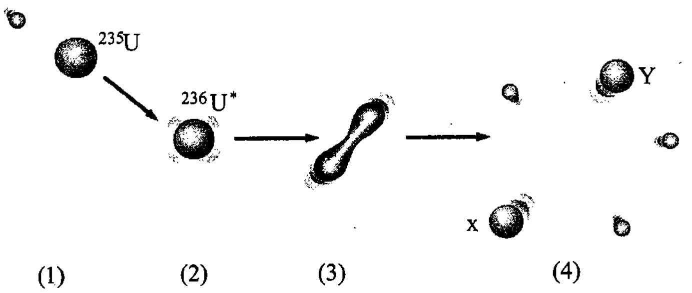


Fig. 5.23

Let us estimate the disintegration energy, Q, released in a typical fission process. From Fig. 5.23, we see that the binding energy per nucleon is about 7.2 MeV for heavy nuclei (those having a mass number of approximately 240) and about 8.2 MeV for nuclei of intermediate mass. This means that the nucleons in the fission fragments are more tightly bound and therefore have less mass than the nucleons in the original heavy nucleus. This decrease in mass per nucleon appears as released energy when fission occurs. The amount of energy released is (8.2–7.2) MeV per nucleon. Assuming a total of 240 nucleons, we find that the energy released per fission event is

$$Q = (240 \text{ nucleons})(8.2 \text{ MeV/nucleon} - 7.2 \text{ MeV/nucleon}) = 240 \text{ MeV}$$

This is a very large amount of energy relative to the amount released in chemical processes.

In nuclear fission, heavy nuclei of A above 200 break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A $_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming $_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$$_{92}U^{236} \rightarrow {}_{53}I^{137} + {}_{39}Y^{97} + 2n$$
$$_{92}U^{236} \rightarrow {}_{53}I^{140} + {}_{36}Y^{94} + 2n$$

and a number of other combinations.

- On an average, 2.5 neutrons are emitted in each fission event.
- Mass lost per reaction ≈ 0.2 a.m.u.
- In nuclear fission, the total BE increases and excess energy is released.
- In each fission event, about 200 MeV of energy is released, a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV. For example,

$$^{235}_{92}U + {}_0n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_0n^1 + \text{energy}$$

$$Q \text{ value} = [(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\,m_e) + (M_{Kr} - 36m_e) + 3m_n\}]c^2$$
$$= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]c^2$$

- A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor, refer your text book.

Illustration 5.36 **Two other possible ways by which ^{235}U can undergo fission when bombarded with a neutron are (1) by the release of ^{140}Xe and ^{94}Sr as fission fragments and (2) by the release of ^{132}Sn and ^{101}Mo as fission fragments. In each case, neutrons are also released. Find the number of neutrons released in each of these events.**

Sol. By balancing mass numbers and atomic numbers, we find that these reactions can be written as

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{132}_{50}Sn + {}^{101}_{42}Mo + 3{}^{1}_{0}n$$

Thus, two neutrons are released in the first event and three in the second.

Illustration 5.37 **Calculate the total energy released if 1.0 kg of ^{235}U undergoes fission, taking the disintegration energy per event to be $Q = 208$ MeV (a more accurate value than the estimate given previously).**

Sol. We need to know the number of nuclei in 1.00 kg of uranium. Because $A = 235$, the number of nuclei is

$$N = \left(\frac{6.02 \times 10^{23} \text{ nuclei mol}^{-1}}{235 \text{ g mol}^{-1}}\right)(1.00 \times 10^3 \text{ g})$$

$$= 2.56 \times 10^{24} \text{ nuclei}$$

Hence, the disintegration energy is

$$E = nQ = (2.56 \times 10^{24} \text{ nuclei})\left(208 \frac{\text{MeV}}{\text{nucleus}}\right)$$

$$= 5.32 \times 10^{26} \text{ MeV}$$

Because 1 MeV is equivalent to 4.45×10^{-20} kWh, $E = 2.3.7 \times 10^7$ kWh. This is enough energy to keep a 100 W light bulb burning for about 30000 years.

NUCLEAR REACTORS

We have seen that neutrons are emitted when ^{235}U undergoes fission. These neutrons can in turn trigger other nuclei to undergo fission, with the possibility of a chain reaction (see Fig. 5.24). Calculations show that if the chain reaction is not controlled (that is, if it does not proceed slowly), it can result in a violent explosion, with the release of an enormous amount of energy. This, of course, was the principle behind the first nuclear bomb, an uncontrolled fission reaction.

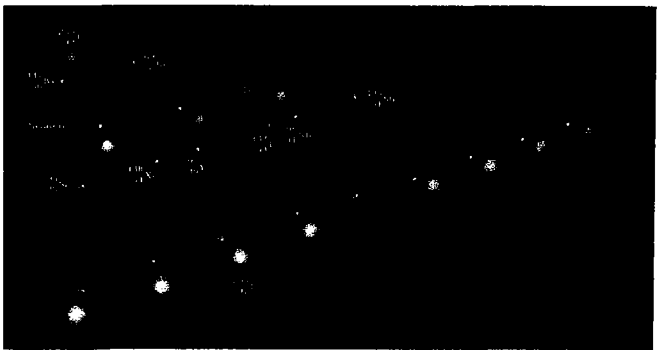


Fig. 5.24

A nuclear reactor is a system designed to maintain what is called a self-sustained chain reaction. This important process was first achieved in 1942 by a group led by Fermi at the University of Chicago, with natural uranium as the fuel. Most reactors in operation today also use uranium as fuel. Natural uranium contains only about 0.7% of the ^{235}U isotope, with the remaining 99.3% being the ^{238}U isotope. This is important to the operation of a reactor because ^{238}U almost never undergoes fission. Instead, it tends to absorb neutrons, producing neptunium and plutonium, For this reason, reactor fuels must be artificially enriched so that percentage of the ^{235}U isotope can be increased.

Earlier, we mentioned that an average of about 2.5 neutrons are emitted in each fission event of ^{235}U. In order to achieve a self-sustained chain reaction, one of these neutrons must be captured by another ^{235}U nucleus and cause it to undergo fission. A useful parameter for describing the level of reactor operation is the reproduction constant K, defined as the average number of neutrons from each fission event that will cause another event. As we have seen, K can have a maximum value of 2.5 in the fission of uranium. However, in practice K is less than this because of several factors, which we shall discuss in a moment.

In a self-sustained chain reaction, $K = 1$ and the reactor is said to be critical. When K is less than unity, the reactor is subcritical and the reaction dies out. When K is greater than unity, the reactor is said to be supercritical, and a runaway reaction occurs. In a nuclear reactor used to furnish power to a utility company, it is necessary to maintain a K value close to unity.

Neutron Leakage

In any reactor, a fraction of the neutrons produced in fission will leak out of the core before inducing other fission events. If the fraction leaking out is too large, the reactor will not operate. The percentage lost is large if the reactor is very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical requirement of reactor design is choosing the correct surface area to volume ratio so that a sustained reaction can be achieved.

Regulating Neutron Energies

The neutrons released in fission events are very energetic, with kinetic energies of about 2 meV. It is found that slow neurtons are far more likely than fast neutrons to produce fission events in ^{235}U. ^{238}U does not absorb slow neurtons. Therefore, in order for the chain reaction to continue, the neutrons must be slowed down. This is accomplished by surrounding the fuel with a moderator substance.

To understand how neutrons are slowed down, consider a collision between a light object and a very massive one. In such an event, the light object rebounds from the collision with most of its original kinetic energy. However, If the collision is between objects with nearly the same masses, the incoming projectile transfers a large percentage of its kinetic energy to the target. In the first nuclear reactor ever constructed, Fermi placed bricks of graphite (carbon) between the fuel elements. Carbon nuclei are about 12 times more massive than neutrons, but after about 100 collisions with carbon nuclei, a neutron is slowed sufficiently to increase its likelihood of fission with ^{235}U. In this design, the carbon is the moderator; most modern reactors use heavy water (D_2O) as the moderator.

Neutron Capture

In the process of being slowed down, the neutrons may be captured by nuclei that do not undergo fission. The most common event of this type is neutron capture by ^{238}U. The probability of neutron capture by ^{238}U is very high when the neutrons have high kinetic energies and very low when they have low kinetic energies. Thus, the slowing down of the neutrons by the moderator serves the dual purpose of making them available for reaction with ^{235}U and decreasing their chances of being captured by ^{238}U.

NUCLEAR FUSION

Figure 5.25 shows that the binding energy for lighter nuclei (those having a mass number lower than 20) is much smaller than the binding energy for heavier nuclei. This suggests a possible process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fussion. Because the mass of the final nucleus is less than the masses of the original nuclei, there is a loss of mass accompanied by a release of energy. Although fusion power plants have not yet been developed, a worldwide effort is under way to harness the energy from fusion reactions in the laboratory. Later, we shall discuss the possibilities and advantages of this process for generating electric power.

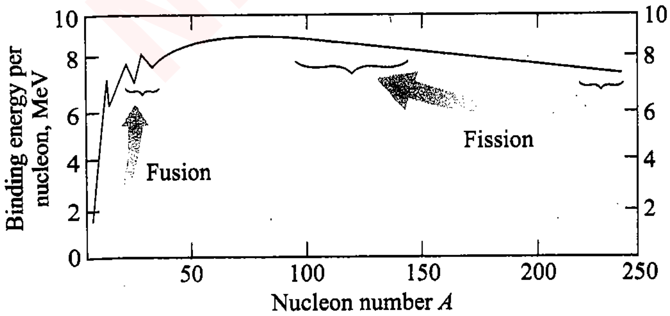


Fig. 5.25

Fusion in The Sun

All stars generate their energy through fusion process. About 90% of the stars, including the Sun, fuse hydrogen, whereas some older stars fuse helium or other heavier elements. Stars are born in regions of space containing vast clouds of dust and gas. Recent mathematical models of these clouds indicate that star formation is triggered by shock waves passing through a cloud. These shock waves are similar to sonic booms and are produced by events such as the explosion of a nearby star, called a supernova explosion. The shock wave compresses certain regions of the cloud, causing these regions to collapse under their own gravity. As the gas falls inward toward the center, the atoms gain speed, which causes the temperature of the gas to rise. Two conditions must be met before fusion reactions in the star can sustain its energy needs.

(1) The temperature must be high enough (about 10^7 K for hydrogen) to allow the kinetic energy of the positively charged hydrogen nuclei to overcome their mutual Coulomb repulsion as they collide.
(2) The density of nuclei must be high enough to ensure a high rate of collision.

When fusion reactions occur at the core of a star, the energy liberated eventually becomes sufficient to prevent further collapse of the star under its own gravity. The star then continues to live out the remainder of its life under a balance between the inward force of gravity pulling it toward collapse and the outward force due to thermal effects and radiation pressure.

The proton–proton cycle is a series of three nuclear reactions that are believed to be the stages in the liberation of energy in the Sum and other stars rich in hydrogen. An overall view of the proton–proton cycle is that four protons combine to form an alpha particle and two positrons, with the release of 25 MeV of energy in the process.

The specific steps in the proton–proton cycle are

$$^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + v$$

$$^1_1H + ^2_1H \rightarrow ^3_2He + \gamma$$

This second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:

$$^1_1H + ^3_2He \rightarrow ^4_2He + e^+ + v$$

or $$^3_2He + ^3_2He \rightarrow ^4_2He + ^1_1H + ^1_1H$$

The energy liberated is carried primarily by gamma rays, positrons, and neutrinos, as can be seen from the reactions. The gamma rays are soon absorbed by the dense gas, thus raising its temperature. The positrons combine with electrons to produce gamma rays, which in turn are also absorbed by the gas within a few centimeters. The neutrinos, however, almost never interact with matter; hence, they escape from the star, carrying about 2% of the generated energy with them. These energy-liberating fusion reactions are called thermonuclear fusion reactions. The hydrogen (fusion) bomb, first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

Note:

- *Some unstable light nuclei of A below 20, fuse together, the BE per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).*

$$_1H^2 + {_1H^2} \rightarrow {_2He^3} + n + 3.3 \text{ MeV (D–D)}$$

$$Q\text{-value} = [2(M_D - m_e) - \{(M_{He\text{-}3} - 2m_e) + m_n\}]c^2$$
$$= [2M_D - (M_{He\text{-}3} + m_n)]c^2$$

$$_1H^2 + {_1H^2} \rightarrow {_1H^3} + {_1H^1} + 4.0 \text{ MeV (D–D)}$$

$$Q\text{-value} = [2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]c^2$$
$$= [2M_D - (M_T + M_H)]c^2$$

$$_1H^2 + {_1H^3} \rightarrow {_2He^4} + n + 17.6 \text{ MeV (D–D)}$$

$$Q\text{-value} = [\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He\text{-}4} - 2m_e) + mn\}]c^2$$
$$= [(M_D + M_T) - (M_{He\text{-}4} + m_n)]c^2$$

In case of fission and fusion, $\Delta m = \Delta m_{atom} = \Delta m_{nucleus}$

- *These reactions take place at ultra-high temperature ($\cong 10^7$–10^9). At high pressure, it can take place at low temperature also. For these reactions to take place, nuclei should be brought upto 1 fermi distance which requires very high kinetic energy.*
- *Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.*

FUSION REACTORS

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes on Earth. A great deal of effort is focused on developing a sustained and controllable thermonuclear reactor—a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the vast availability of its fuel source: water. An additional advantage of fusion reactors is that comparatively few radioactive by products are formed. As noted in Eq. (iii), the end product of the fusion of hydrogen nuclei is safe, nonradioactive helium. Unfortunately, a thermonuclear reactor that can deliver a net power output over a reasonable time interval is not yet a reality, and many difficulties must be solved before a successful device is constructed.

We have seen that the Sun's energy is based, in part, on a set of reactions in which ordinary hydrogen is converted to helium. Unfortunately, the proton–proton interaction is not suitable for use in a fusion reactor because the event requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior. In fact, even at the densities and temperature that exist at the center of the Sun, the average proton takes 14 billion years to react!

The fusion reactions that appear most promising in the construction of a fusion power reactor involve deuterium and tritium, which are isotopes of hydrogen. These reactions are

$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n, \qquad Q = 3.27 \text{ MeV}$$

$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H, \qquad Q = 4.03 \text{ MeV}$$

$${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n, \qquad Q = 17.59 \text{ MeV}$$

where the Q values refer to the amount of energy released per reaction. As noted earlier, deuterium is available in almost unlimited quantities form our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2}$ = 12.3 yr) and undergoes beta decay to ^{3}He. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One of the major problems in obtaining energy from nuclear fusion is the fact that the Coulomb repulsion force between two charged nuclei must be overcome before they can fuse. The fundamental challenge is to give the two nuclei enough kinetic energy to overcome this repulsive force. This can be accomplished by heating the fuel to extremely high temperatures (about 10^8 K, far greater than the interior temperature of the Sun). As you might expect, such high temperatures are not easy to obtain in a laboratory or a power plant. At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a plasma.

Illustration 5.38 **On disintegration of one atom of ^{235}U, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kilo watt. How many atoms are disintegrated per second in the reactor? What is the decay in mass per hour?**

Sol. Power produced in the reactor is

$$P = 1000 \text{ kW} = 1000 \times 10^3 \text{ W}$$
$$= 10^6 \text{ J s}^{-1}$$
$$= \frac{10^6}{1.6 \times 10^{-19}} \text{eV s}^{-1}$$
$$= 6.25 \times 10^{18} \text{ MeV s}^{-1}$$

As in each disintegration 200 MeV energy is released, number of atoms disintegrated per second are

$$N = \frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16} \text{ s}^{-1}$$

Energy released per second is 10^6 J.

Energy released per hour = $10^6 \times 60 \times 60$ J

Thus, mass decay per hour can be given as

$$\Delta m = \frac{\Delta E}{c^2} \qquad \text{(Einstein's mass–energy formula)}$$

$$= \frac{10^6 \times 60 \times 60 \text{ J}}{(3 \times 10^8 \text{ m/s})^2} = 4 \times 10^{-8} \text{ kg}$$

$$= 4 \times 10^{-5} \text{ g}$$

Illustration 5.39 **A deuterium reaction that occurs in an experimental fusion reactor is in two stages:**

(i) Two deuterium ($^2_1$D) nuclei fuse together to form a tritium nucleus, with a proton as a by product written as D(D, p)T.

(ii) A tritium nucleus fuses with another deuterium nucleus to form a helium $^4_2$He nucleus with neutron as a by product, written as T (D, n) $^4_2$He.

Compute (a) the energy released in each of the two stages, (b) the energy released in the combined reaction per deuterium. (c) What percentage of the mass energy of the initial deuterium is released. Given,

$$^2_1\text{D} = 2.014102 \text{ a.m.u.}$$
$$^3_1\text{T} = 3.016049$$
$$^4_2\text{He} = 4.002603 \text{ a.m.u.}$$
$$^1_1\text{H} = 1.007825 \text{ a.m.u.}$$
$$^1_0\text{n} = 1.00665 \text{ a.m.u.}$$

Sol. a. (i) The reaction involved in the process is

$$^2_1\text{D} + ^2_1\text{D} \rightarrow ^3_1\text{T} + ^1_1\text{p} + \text{E}_1$$

Mass defect of the above reaction is

$$\Delta m = [2(2.014102) - (3.016049 + 1.007825)]$$
$$= 0.00433 \text{ a.m.u.}$$

Energy released in the process is

$$E_1 = 0.00433 \times 931.2 \text{ MeV} = 4.033 \text{ MeV}$$

(ii) The reaction involved in the process is

$$^3_1\text{T} + ^2_1\text{D} \rightarrow ^4_2\text{He} + ^1_0\text{n} + \text{E}_2$$

Mass defect of above reaction is

$$\Delta m = [(3.016049 + 2.014102) - (4.002603 + 1.008665)]$$
$$= 0.01888 \text{ a.m.u.}$$

Energy released in the process is

$$E_2 = 0.01888 \times 931 \text{ MeV} = 17.585 \text{ MeV}$$

Total energy released in both processes is

$$17.585 + 4.033 = 21.618 \text{ MeV}$$

b. Energy released per deuterium atom is

$$E_1 = \frac{21.618}{3} = 7.206 \text{ MeV}$$

c. % of rest mass of $^2_1$D released

$$f = \frac{7.206}{2.014102 \times 931} = 0.385\%$$

Illustration 5.40 **In the process of nuclear fission of 1 g uranium, the mass lost is 0.92 mg. The efficiency of power house run by the fission reactor is 10%. To obtain 400 megawatt power from the power house, how much uranium will be required per hour? ($c = 3 \times 10^8$ m s^{-1})**

Sol. Power to be obtained from power house = 400 megawatt

In this case, energy obtained per hour

= 400 megawatt × 1 hour

= $(400 \times 10^6 \text{ W}) \times 3600$ s

= 144×10^{10} J

Here, only 10% of output is utilized. In order to obtain 144×10^{10} joule of useful energy, the output energy from the power house is given by

$$E = \frac{(144 \times 10^{10}) \times 100}{10} = 144 \times 10^{11} \text{ J}$$

Let this energy is obtained from a mass loss of Δm kg. Then,

$$(\Delta m)c^2 = 144 \times 10^{11} \text{ J}$$

or
$$\Delta m = \frac{144 \times 10^{11}}{(3 \times 10^8)^2} = 16 \times 10^{-5} \text{ kg} = 0.16 \text{ g}$$

Since 0.92 mg $(= 0.92 \times 10^{-3}$ g) mass is lost in 1 g uranium, hence for a mass loss of 0.16 g the uranium required is given by

$$\Delta m' = \frac{1 \times 0.16}{0.92 \times 10^{-3}} = 174 \text{ g}$$

Thus, to run the power house, 174 g uranium is required per hour.

Illustration 5.41 **A nuclear reactor using ^{235}U generates 250 MW of electrical power. The efficiency of the reactor (i.e., efficiency of conversion of thermal energy into electrical energy) is 25%. What is the amount of ^{235}U used in the reactor per year? The thermal energy released per fission of ^{235}U is 200 MeV.**

Sol. Rate of electrical energy generation is 250 MW = 250×10^6 W (or J s^{-1}). So, electrical energy generation is 250 MW = 250×10^6 W (or J s^{-1}). Therefore, electrical energy generated in 1 year is

$$(250 \times 10^6 \text{ J s}^{-1}) \times (365 \times 24 \times 60 \times 60 \text{ s}) = 7.884 \times 10^{15} \text{ J}$$

Thermal energy from fission of one ^{235}U nucleus is 200 MeV $= 200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11}$ J. Since the efficiency is 25%, the electrical energy obtained from the fission of one ^{235}U nucleus is

$$E_1 = 3.2 \times 10^{-11} \times \frac{25}{100} = 8.0 \times 10^{-12} \text{ J}$$

Therefore, the number of fissions of ^{235}U required in one year will be

$$N = \frac{7.884 \times 10^{15}}{8.0 \times 10^{-12}} = 9.855 \times 10^{26}$$

Number of moles of ^{235}U required per year is

$$n = \frac{9.855 \times 10^{26}}{6.023 \times 10^{23}} = 1.636 \times 10^3$$

Therefore, mass of ^{235}U required per year is

$$m = 1.636 \times 10^3 \times 235$$
$$= 3.844 \times 10^5 \text{ g} = 384.4 \text{ kg}$$

Illustration 5.42 **What is the power output of a $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy?**

Sol. 235 a.m.u. of Uranium gives 185 MeV energy. Therefore, the energy given by 1 a.m.u. of $_{92}U^{235}$ is

$$\frac{185}{2.35} \text{ MeV} = \frac{185}{235} \times 1.6 \times 10^{-13} \text{ J}$$

But 1 a.m.u. $= 1.66 \times 10^{-27}$ kg. Therefore, energy released by 1.66×10^{-27} kg of $_{92}U^{235}$ is

$$\frac{185 \times 1.6 \times 10^{-13}}{235}$$

Hence, energy released by 2 kg of $_{92}U^{235}$,

$$W = \frac{185 \times 1.6 \times 10^{-13} \times 2}{235 \times 1.66 \times 10^{-27}} = 1.517 \times 10^{14} \text{ J}$$

$$\text{Therefore, power output of reactor} = \frac{W}{t} = \frac{1.517 \times 10^{14}}{(30 \text{ days})}$$

$$= \frac{1.517 \times 10^{14}}{30 \times 24 \times 60 \times 60}$$

$$= 5.85 \times 10^7 \text{ W}$$

Illustration 5.43 **A nuclear explosion is designed to deliver 1 MW of heat energy, how many fission events must be required in a second to attain this power level. If this explosion is designed with a nuclear fuel consisting of uranium 235 to run a reactor at this power level for one year, then calculate the amount of fuel needed. You can assume that the amount of energy released per fission event is 200 MeV.**

Sol. (i) Energy per fission, $\varepsilon = 200$ MeV

$$= 200 \times 1.6 \times 10^{-13} \text{ J}$$
$$= 3.2 \times 10^{-11} \text{ J}$$

Power delivered, $P = 1 \text{ MW} = 10^6 \text{ J s}^{-1}$

If t is the time, then energy delivered $= Pt$

$$\therefore \quad \text{Number of fissions} = \frac{Pt}{\varepsilon} = \frac{10^6 \times t}{3.2 \times 10^{-11}}$$

If $t = 1$ s, number of fissions per second is

$$\frac{10^6}{3.2 \times 10^{-11}} = 3.125 \times 10^{16}$$

(ii) Total energy required for 1 year $= Pt$

$$= 10^6 \times 3.15 \times 10^7 \text{ J}$$
$$= 3.15 \times 10^{13} \text{ J}$$

Hence, number of fissions required is

$$\frac{3.15 \times 10^{13}}{3.2 \times 10^{-11}} = 9.85 \times 10^{23}$$

Mass of uranium required is

$$9.85 \times 10^{23} \times \frac{235}{6.02 \times 10^{23}} \text{ g} = 384.5 \text{ g}$$

Concept Application Exercise 5.2

1. A uranium nucleus (atomic number 92, mass number 231) emits an α-particle and the resultant nucleus emits a β-particle. What are the atomic and mass numbers of the final nucleus?
2. A radioactive nucleus undergoes a series of decay according to the scheme
$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_3 \xrightarrow{\gamma} A_4$$
If the mass number and atomic number of A are 180 and 72, respectively, what are these number for A_4?
3. A nucleus, absorbing a neutron, emits an electron to go over to neptunium which on further emitting an electron goes over to plutonium. How would you represent the resulting plutonium?
4. A uranium nucleus U-238 of atomic number 92 emits two α-particles and two β-particles and transforms into a thorium nucleus. What is the mass number and atomic number of the thorium nucleus so produced?

5. How many electrons, protons, and neutrons are there in 12 g of $_6C^{12}$ and in 14 g of $_6C^{14}$?

6. Determine the product of the reaction:

$$^7_3Li + ^4_2He \rightarrow ? + n$$

What is the Q value of the reaction?

7. The half-life of radon is 3.8 days. After how many days will only one-twentieth of radon sample be left over? **(IIT-JEE, 1981)**

8. How many alpha and beta particles are emitted when uranium ($_{92}U^{238}$) decays to lead $_{82}Pb^{206}$? **(IIT-JEE, 1985)**

9. A radioactive sample has mass m, decay constant λ and molecular weight M. If the Avogadro number is N_A, then

a. find the initial number of nuclei present;

b. find the number of decayed nuclei after a time t;

c. find the activity of the sample after a time t;

10. Calculate the time taken to decay 100 percent of a radioactive sample in terms of

a. half-life T and

b. mean-life T_{av}

11. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Obtain an expression for its mean-life.

12. An α-particle strikes an aluminium atom $^{27}_{13}Al$. As a result, an unknown nucleus ^A_ZX and a neutron are formed.

$$^2_4He + ^{27}_{13}Al \rightarrow ^A_ZX + ^1_0n$$

Identify the nucleus produced, indicating its atomic number Z (the number of protons) and its mass number A (the number of nucleons).

13. A neutron is observed to strike an $^{16}_8O$ nucleus and a deuteron is given off. What is the nucleus that result?

14. A $^{238}_{92}U$ undergoes alpha decay. What is the resulting daughter nucleus?

15. Is the sulphur isotope $^{38}_{16}S$ likely to be stable?

16. Thorium $^{238}_{90}Th$ produces a daughter nucleus that is radioactive. The daughter, in turn, produces its own radioactive daughter and so on. This process continues until bismuth $^{812}_{83}Bi$ is reached.

What are the total number N_α of α-particles and the total number N_β of β-particles that are generated in this series of radioactive decays ?

17. Determine the average ^{14}C activity in decays per minute per gram of natural carbon found in living organisms if the concentration of ^{14}C relative to that of ^{12}C is 1.4×10^{-12} and half-life of ^{14}C is $T_{1/2} = 57.30$ years.

18. Radium 226 is found to have a decay constant of 1.36×10^{-11} Bq.

Determine its half-life in years. If a 200 g sample of radium was taken in 1902, how much of it will remain a hundred years later?

19. A sample of wine is 5 years old; it has different kinds of atoms, including carbon, oxygen, and hydrogen. Each of these has a radioactive isotope. The radioactive isotopes $^{14}_6C$, $^{15}_8O$, ^3_1H have half-lives 5730 years, 122.2 s, 12.33 years, respectively.

The activity of each of these isotope is known at the time the bottle was heated. However, only one of the isotope is useful for determining the age of the wine accurately. Which is it?

20. A radio nuclide A_1 with decay constant λ_1 transforms into a radio nuclide A_2 with decay constant λ_2. Assuming that at the initial moment, the preparation contained only the ratio nuclide A_1.

a. Find the equation describing accumulation of radio nuclide A_2 with time.

b. Find the time interval after which the activity of radio nuclide A_2 reaches its maximum value.

21. Consider the beta decay of an unstable $^{14}_6C$ nucleus initially at rest: $^{14}_6C \rightarrow ^{14}_7N + ^0_{-1}e + v_e$.

Is it possible for the maximum kinetic energy of the emitted beta particle to be exactly equal to Q?

22. The atomic mass of uranium $^{238}_{92}U$ is 238.0508 u, while that of thorium $^{234}_{90}Th$ is 234.1436 u, and that of an alpha particle ^4_2He is 4.0026 u.

Determine the energy released when α-decay converts $^{238}_{92}U$ into $^{234}_{90}Th$.

23. The energy released by the α-decay to $^{238}_{92}U$ is found to be 4.3 MeV. Since this energy is carried away as kinetic energy of the recoiling $^{234}_{90}Th$ nucleus and the α-particle, it follows that $KE_{Th} + KE_\alpha = 4.3$ MeV. However, KE_{Th} and KE_α are not equal.

Which particle carries away more kinetic energy, the $^{234}_{90}Th$ nucleus or the α-particle?

24. Estimate the minimum amount of $^{235}_{92}U$ that needs to undergo fission in order to run a 1000 MW power reactor per year of continuous operation. Assume an efficiency of about 33 percent.

25. The isotope $^{14}_6C$ is radioactive and has a half- life of 5730 years. If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 17,190 years?

26. The half-life of the radioactive nucleus $^{226}_{86}Ra$ is 1.6×10^3 yr. If a sample contains 3.0×10^{16} such nuclei, determine the activity at this time.

27. Radon, $^{222}_{86}Rn$, is a radioactive gas that can be trapped in the basement of homes, and its presence in high concentrations is a known health hazard. Radon has a half-life of 3.83 days. A gas sample contains 4.0×10^8 radon atoms initially.

a. How many atoms will remain after 12days have passed if no more radon leaks in?

b. What is the initial activity of the radon sample?

28. The $^{226}_{88}Ra$ nucleus undergoes alpha decay to $^{226}_{88}Rn$. Calculate the amount of energy liberated in this decay. Take the mass of $^{226}_{88}Ra$ to be 226.025 402 u, that of $^{226}_{88}Rn$ to be 222.017 571 u, and that of $^{4}_{2}He$ to be 4.002 602 u.

29. Calculate the energy released when three alpha particles combine to form a ^{12}C nucleus. The atomic mass of $^{4}_{2}He$ is 4.002603 u.

30. a. Find the energy needed to remove a neutron from the nucleus of the calcium isotope $^{42}_{20}Ca$.

b. Find the energy needed to remove a proton from this nucleus.

c. Why are these energies different?

Given:

Mass of $^{41}_{20}Ca$ = 40.962278 u, mass of neutron = 1.007825 u,

mass of $^{42}_{20}Ca$ = 40.974599 u, mass of proton = 1.007825 u.

Solved Examples

Example 5.1 **A count rate meter is used to measure the activity of a given sample. At one instant, the meter shows 4750 counts per minute. Five minutes later, it shows 2700 counts per minute. Find (a) the decay constant and (b) the half-life of the sample. ($\log_{10} 1.760 = 0.2455$)**

Sol. We have,

$$N = N_0 e^{-\lambda t} \qquad \text{(i)}$$

The activity of the sample is given by

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N \qquad \text{(ii)}$$

i.e., activity is proportional to the number of undecayed nuclei.

At $t = 0$, $$\left(\frac{dN}{dt}\right)_{t=0} = -\lambda N$$

At $t = 5$ min, $$\left(\frac{dN}{dt}\right)_{t=5\,\text{min}} = -\lambda N$$

$$\therefore \quad \frac{N_0}{N} = \frac{\left(\frac{dN}{dt}\right)_{t=0}}{\left(\frac{dN}{dt}\right)_{t=5\,\text{min}}} = \frac{4750}{2700} = 1.760$$

a. From Eq. (i), we have

$$\frac{N}{N_0} = e^{-\lambda t}$$

or $$\frac{N_0}{N} = e^{\lambda t} \quad \text{or} \quad \log_e \frac{N_0}{N} = \lambda t$$

$$\Rightarrow \quad \lambda = \frac{1}{t}\log_e \frac{N_0}{N} = \frac{2.3026}{t} \times \log_{10}\left(\frac{N_0}{N}\right) \qquad \text{(iii)}$$

Substituting the given values, we have

$$\lambda = \frac{2.3026}{5} \times 0.2455 = 0.113\ \text{min}^{-1}$$

b. Half-life of sample, $T = \frac{0.6931}{\lambda} = \frac{0.6931}{0.113} = 6.1\ \text{min}^{-1}$

Example 5.2 **There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? Given mass of neutron = 1.676×10^{-27} kg.**

(IIT-JEE, 1986)

Sol. The velocity v of neutron is given by

$$\frac{1}{2}mv^2 = 0.0327\ \text{eV}$$

or $$\frac{1}{2} \times 1.675 \times 10^{-27} v^2 = 0.0327 \times 1.6 \times 10^{-19}\ \text{J}$$

$$\therefore \quad v^2 = \frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}$$

or $$v = 2.5 \times 10^3\ \text{m s}^{-1}$$

Time taken by neutron to travel a distance of 10 m is

$$t = \frac{\text{disance}}{\text{velocity}} = \frac{10}{2.5 \times 10^3} = 4 \times 10^{-3}\ \text{s}$$

and half-life $T = 700$ s.

The fraction of neutrons decayed in time t is

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{4\times10^{-3}/700} = \left(\frac{1}{2}\right)5.7 \times 10^{-6}$$

$$= 0.999952$$

Fraction of neutrons decayed is 1 – 0.999952 = 0.000048 = 4.8×10^{-5}.

Example 5.3 **The binding energies per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV, respectively. Calculate the energy released when two deuterons fuse to form a helium nucleus ($_2He^4$).**

(IIT-JEE, 1988)

Sol. The equation of fusion is represented as

$$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + \text{Energy}$$

The binding energy per nucleon of deuteron is 1.1 MeV.

Therefore, the net initial binding energy = $4 \times 1.1 = 4.4$ MeV

The binding energy per nucleon of $_2He^4$ is 7.0 MeV.

Therefore, the net final binding energy = $4 \times 7.0 =$ 28.0 MeV

Hence, energy released = 28.0 MeV – 4.4 MeV = 23.6 MeV

Example 5.4 **Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days will the room be safe for occupation?**

Sol. We have,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

Given, half-life T = 10 days.

$$\frac{N}{N_0} = \frac{1}{50} \Rightarrow \frac{1}{50} = \left(\frac{1}{2}\right)^{t/T}$$

Taking log,

$$\log 1 - \log 50 = \frac{t}{T}(\log 1 - \log 2)$$

$$\Rightarrow \quad 0 - \log\frac{100}{2} = \frac{t}{T}(0 - \log 2)$$

$$\Rightarrow \quad \log 100 - \log 2 = \frac{t}{T}(0 - \log 2)$$

or $$\log 100 - \log 2 = \frac{t}{T}\log 2$$

As $\log_{10} 2$ = 0.3010, we have

$$2 - 0.3010 = \frac{t}{T} \times 0.3010$$

or $$t = \frac{1.6990}{0.3010}T = \frac{1699}{301} \times (10 \text{ days}) = 56.45 \text{ days}$$

Example 5.5 **A small quantity of solution containing ^{24}Na radionuclide (half-life 15 h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 h shows an activity of 296 disintegrations min^{-1}. Determine the total volume of blood in the body of the person. Assume that the radioactivity solution mixes uniformly in the blood of the person. (1 curie = 3.7 × 10^{10} disintegrations s^{-1}.)** **(IIT-JEE, 1994)**

Sol. Initial activity of ^{24}Na,

$$A = \frac{dN}{dt} = 1.0\,\mu\text{C}$$

$$= 1.0 \times 10^{-6} \times 3.7 \times 10^{10}$$

$$= 3.7 \times 10^4 \text{ disintegrations s}^{-1}$$

Half-life $T = 15\text{ h} = 15 \times 3600$ s

Initial activity,

$$A = \frac{dN}{dt} = \lambda N_0$$

$$3.7 \times 10^4 = \frac{0.693}{15 \times 3600} N_0$$

$$\therefore \quad N_0 = \frac{3.7 \times 10^4 \times 15 \times 3600}{0.693}$$

$$= 2.883 \times 10^9$$

Let the number of radioactive nuclei present after 5 h be N' in 1 cm^3 of sample of blood. Then,

$$\frac{dN}{dt} = \lambda N' \Rightarrow \frac{296}{60} = \frac{0.693}{15 \times 3600} N'$$

$$\therefore \quad N' = \frac{296 \times 15 \times 3600}{60 \times 0.693} = 3.844 \times 10^5$$

If N_0' is initial number of radioactive nuclei in 1 cm^3 of sample, then

$$\frac{N'}{N_0'} = \left(\frac{1}{2}\right)^{t/T}$$

$$N_0' = (2)^{t/T} N' = (2)^{5/15} N'$$

$$= (2)^{1/3} \times 3.844 \times 10^5$$

Let $y = (2)^{1/3}$.

$$\therefore \quad \log y = \frac{1}{3}\log 2 = \frac{1}{3} \times 0.3010 = 0.1003$$

$$\Rightarrow \quad y = \text{Antilog } 0.1003 = 1.2598$$

$$\therefore \quad N_0' = 1.2598 \times 3.844 \times 10^5$$

$$= 4.843 \times 10^5$$

$$\text{Volume of blood} = \frac{N_0}{N_0'} = \frac{2.883 \times 10^9}{4.843 \times 10^5} = 0.595 \times 10^4 \text{ cm}^3$$

$$= 5.95 \text{ L}$$

Example 5.6 **At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12.5%. Calculate (i) mean life of the nuclei and (ii) the time in which the number of decayed nuclei will further reduce to 6.25% of the reduced number.** **(IIT-JEE, 1996)**

Sol. Half-life of radioactive sample, i.e., the time in which number of undecayed nuclei becomes half (T) is 10 s.

(i) Mean life, $\tau = \dfrac{T}{\log_e 2} = \dfrac{10}{0.693}\text{ s} = 1.443 \times 10 = 14.43$ s

(ii) The reduced number further reduces to 6.25% in n half-lives given by

$$\frac{N}{100} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{6.25}{100} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

Time, $t = 4T = 4 \times 10 = 40$ s

Example 5.7 **In an ore containing uranium, the ratio of U^{238} to Pb^{206} is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of U^{238}. Take the half-life of U^{238} to be 4.5×10^9 years.** **(IIT-JEE, 1997)**

Sol. Given, $\dfrac{M_U}{M_{Pb}} = 3$

Let the initial mass of uranium be M_0.

Final mass of uranium after time t, $M = \dfrac{3}{4}M_0$

According to law of disintegration,

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} \quad \Rightarrow \quad \frac{M_0}{M} = (2)^{t/T}$$

$$\therefore \quad \log_{10}\frac{M_0}{M} = \frac{t}{T}\log_{10}2$$

$$t = T\frac{\log_{10}\dfrac{M_0}{M}}{\log_{10}2} = \frac{4\log_{10}\dfrac{4}{3}}{\log_{10}2} = T\frac{\log_{10}1.3333}{\log_{10}2}$$

$$= 4.5\times10^9\left(\frac{0.1249}{0.3010}\right)$$

$$\therefore \quad t = 1.867\times10^9 \text{ years}$$

Example 5.8 **The element curium $^{218}_{96}Cm$ has a mean-life of 10^{13} s. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV energy. The masses involved in α-decay are as follows: $^{218}_{98}Cm$ = 248.07220 u, $^{244}_{94}Pu$ = 244.064100 u and ^4_2He = 4.002603 u.**

Calculate the power output from a sample of 10^{20} Cm atoms. (1 u = 931 MeV c^{-2}) **(IIT-JEE, 1997)**

Sol. Activity (or rate) of fission,

$$A = \frac{dN}{dt} = \lambda N = \frac{1}{\tau}N = \frac{10^{20}}{10^{13}} = 10^7$$

As probability of fission is 8% only, therefore actual rate of fission is

$$\frac{8}{100}\times10^7 = 8\times10^5 \text{ s}^{-1}$$

Energy released per fission = 200 MeV

Power output of fission = $8\times10^5\times200$ MeV s^{-1}

$= 16\times10^7$ MeV s^{-1}

Probability of α-particle decay is 92%. Therefore rate of decay for α-particle is

$$\frac{92}{100}\times10^7 = 92\times10^5 \text{ s}^{-1}$$

For α-decay, the equation is

$$^{248}_{96}Cm \rightarrow {}^{244}_{94}Pu + {}^4_2He$$

(248.072220)(244.064100) (4.002603)

Mass of decay products is 244.064100 + 4.002603 = 248.06673

Mass defect,

$$\Delta m = 248.072220 - 248.066703 = 0.005517 \text{ u}$$

Energy released per α-decay is

$$92\times10^5\times5.1363 \text{ MeV s}^{-1} = 4.725\times10^7 \text{ MeV s}^{-1}$$

Total power output = $16\times10^7 + 4.725\times10^7$

$= 20.725\times10^7$ MeV s^{-1}

$= 20.725\times10^7\times1.6\times10^{-13}$ J s^{-1}

$= 33.16\times10^{-6}$ W $= 33.16\ \mu$W

Example 5.9 **In a nuclear reactor, ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.** **(IIT-JEE, 2001)**

Sol. Total energy produced by the reactor in time t = 10 years,

$$E = 1000\times10^6\times10\times3.15\times10^7 \text{ J}$$

$$= 3.15\times10^{17} \text{ J}$$

$$\text{Efficiency} = \frac{\text{output energy}}{\text{input energy}}$$

$$\Rightarrow \text{Input energy caused by fission} = \frac{\text{output energy}}{\text{efficiency}}$$

$$= \frac{3.15\times10^{17}}{(10/100)} = 3.15\times10^{18} \text{ J}$$

Energy produced by 1 fission of ^{235}U = 200 MeV

$= 200\times1.6\times10^{-13}$ J

$= 3.2\times10^{-11}$ J

$$\text{Therefore, number of fissions required} = \frac{\text{total energy}}{\text{energy per fission}}$$

$$= \frac{3.15\times10^{18}}{3.2\times10^{-11}} \approx 9.8\times10^{28}$$

Hence, mass of uranium required is given by

$$m = \frac{N}{N_a}\times235 \text{ kg} = \frac{9.8\times10^{28}}{6.02\times10^{26}}$$

$$= 38.2\times10^3 \text{ kg}$$

Example 5.10 **A nucleus at rest undergoes α decay emitting an α particle of de Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 a.m.u. and that of the α-particle is 4.002 a.m.u., determine the total kinetic energy in the final state. Hence, obtain the mass of parent nucleus in a.m.u. (1 a.m.u. = 931.470 MeV c^{-2})**
(IIT-JEE, 2001)

Sol. de Broglie wavelength of α-particle, $\lambda_\alpha = 5.76 \times 10^{-15}$ m

de Broglie wavelength, $\lambda = \frac{h}{p}$

Therefore, momentum of α-particle,

$$p_\alpha = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}} = 1.15 \times 10^{-19} \text{ kg m s}^{-1}$$

If p_d is momentum of daughter nucleus, then from conservation of linear momentum,

$$p_d = -p_\alpha = -1.15 \times 10^{-19} \text{ kg m s}^{-1}$$

Hence, total kinetic energy of final state,

$$E = E_\alpha + E_d = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_\alpha^2}{2m_\alpha} + \frac{p_d^2}{2m_d}$$

$$= \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_d}\right) = \frac{p_\alpha^2 (m_d + m_\alpha)}{2m_\alpha m_d}$$

$$1 \text{ a.m.u.} = 931.470 \text{ MeV}/c^2 = \frac{931.470 \times 1.6 \times 10^{-13}}{(3 \times 10^8)^2} \text{kg}$$

$$= 1.66 \times 10^{-27} \text{ kg}$$

Therefore, total kinetic energy of final state is

$$\frac{(1.15 \times 10^{-19})^2 \times (223.610 + 4.002) \text{ a.m.u.}}{2 \times (4.002 \text{ a.m.u.}) \times (223.610 \text{ a.m.u.})}$$

$$= \frac{(1.15 \times 10^{-19})^2 \times 227.612}{2 \times 4.002 \times 223.610 \times 1.66 \times 10^{-27}} \text{J}$$

$$= \frac{(1.15)^2 \times 227.612}{2 \times 4.002 \times 223.610 \times 1.66} \times 10^{-11} \text{ J}$$

$$= 10^{-12} \text{ J} = \frac{10^{-12}}{1.6 \times 10^{-13}} \text{MeV} = 6.25 \text{ MeV}$$

Mass of parent nucleus $= m_d + m_\alpha - (\text{BE})$

$$= \left(223.610 + 4.002 - \frac{6.25}{931.47}\right) \text{a.m.u.}$$

$$= (227.612 - 0.007) \text{ a.m.u.} = 227.605 \text{ a.m.u.}$$

Example 5.11 **A radioactive element decays by β-emission. A detector records n beta particles in 2 s and in next 2 s it records $0.75n$ beta particles. Find mean life correct to nearest whole number. Given ln 2 = 0.6931, ln 3 = 1.0986.**
(IIT-JEE, 2003)

Sol. Let N_0 be initial number of nuclei at time $t = 0$.
The number of undecayed nuclei in time t is $N = N_0 e^{-\lambda t}$
The number of nuclei decayed in time t is

$$n = N_0 - N = N_0 - N_0 e^{-\lambda t} \quad \text{(i)}$$

The number of undecayed nuclei in next time t,

$$N' = Ne^{-\lambda t} = (N_0 e^{-\lambda t})\, e^{-\lambda t} = N_0 e^{-2\lambda t} \quad \text{(ii)}$$

Number of decayed nuclei in next time t,

$$0.75n = N - N' = N_0 e^{-\lambda t} - N_0 e^{-2\lambda t}$$

$$= N_0 e^{-\lambda t}(1 - e^{-\lambda t}) \quad \text{(iii)}$$

Dividing (ii) by (i), we get

$$0.75 = e^{-\lambda t} \Rightarrow \frac{4}{3} = e^{\lambda t}$$

Taking natural logarithm,

$$\ln\frac{4}{3} = \lambda t \Rightarrow \lambda = \frac{\ln\frac{4}{3}}{t}$$

Given $t = 2$ s.

$$\therefore \lambda = \frac{\ln 4 - \ln 3}{2} = \frac{2\ln 2 - \ln 3}{2} = \frac{2 \times 0.6931 - 1.0986}{2}$$

$$= 0.1438 \text{ s}^{-1}$$

$$\text{Mean life, } \tau = \frac{1}{\lambda} = \frac{1}{0.1438} = 7 \text{ s (whole number)}$$

Example 5.12 **A rock is 1.5 × 10^9 years old. The rock contains ^{238}U which disintegrates to form ^{206}U. Assume that there was no ^{206}Pb in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ^{238}U to that of ^{206}Pb in the rock. Half-life of ^{238}U is 4.5 × 10^9 years. ($2^{1/3} = 1.259$)** **(IIT-JEE, 2004)**

Sol. ^{238}U atoms decay to form ^{206}Pb atoms. Let N_0 be initial number of U atoms. After time t, let N_U be the number of U atoms left. Then

$$\frac{N_U}{N_0} = \left(\frac{1}{2}\right)^n, \text{ where } n \text{ is number of half-lives}$$

$$n = \frac{t}{T} = \frac{1.5 \times 10^9 \text{ years}}{4.5 \times 10^9 \text{ years}} = \frac{1}{3}$$

$$\therefore \quad N_U = \left(\frac{1}{2}\right)^{1/3} N_0 \quad \text{(i)}$$

Number of Pb atoms, $N_{Pb} = N_0 - N_U$

Hence, required ratio is

$$R = \frac{N_U}{N_{Pb}} = \frac{1}{2^{1/3} - 1} = \frac{1}{1.259 - 1} = \frac{1}{0.259} = 3.86$$

Example 5.13 What is the minimum photon energy required to remove the least bound neutron of $^{40}_{20}$Ca and $^{40}_{18}$Ar. The necessary atomic masses (in u) are given below:

$$M\,(^{40}\text{Ca}) = 39.962591 \text{ u}$$
$$M\,(^{39}\text{Ca}) = 38.970719 \text{ u}$$
$$M\,(^{40}\text{Ar}) = 39.962383 \text{ u}$$
$$M\,(^{39}\text{Ar}) = 38.964314 \text{ u}$$
$$m_n = 1.008665 \text{ u}$$

Sol. Total mass (neutron plus product nucleus) after the removal is expected to be more than the initial nucleus mass.

This mass difference when expressed in energy units (MeV) is the binding energy of the least bound neutron.

The mass after break-up,

$$M\,(^{39}_{30}\text{Ca}) = 38.970719 \text{ u}$$
$$m_n = 1.08665 \text{ u}$$
$$\text{Total mass} = 39.979384 \text{ u}$$

The mass difference is

$$\Delta m = [M(^{39}\text{Ca}) + m_n] - M(^{40}\text{Ca})$$
$$= (39.979384 - 39.962591) = 0.016793 \text{ u}$$
$$= (0.016793) \times (931.5) \text{ MeV} = 15.64 \text{ MeV}$$

So, the binding energy of least bound neutron is 15.64 MeV.

Similarly, for $^{39}_{20}$Ar,

$$\Delta m = [M(^{39}_{20}\text{Ar}) + m_n] - M(^{40}_{18}\text{Ar})]$$
$$= (38.964314 + 1.008665) - 39.962383 \text{ u}$$
$$= 0.010596 \text{ u} = (0.010596) \times (931.5) \text{ MeV}$$
$$= 9.870 \text{ MeV}$$

The inert elements are relatively unreactive because their outer shells of electrons are full. Large energies are involved in gaining or losing electrons which is therefore unlikely. An analogous behavior takes place in the nucleus. Experimental evidence does indicate the existence of 'closed nuclear shell' when the number of protons or neutrons is 2, 8, 20, 28, 50, 82 or 126, although the concepts of individual nucleon 'orbits' and filled shells inside the nucleus is hard to believe.

The above mentioned number of protons or neutron are called magic numbers. The elements with magic number of protons have unusually high number of stable isotope. For example, aluminium with 13 protons has just one stable isotope ^{27}Al, but tin with $Z = 50$ (a magic number) has 10 stable isotopes ranging from $N = 62$ to $N = 74$, whereas neighboring indium with $Z = 49$ and antimony with $Z = 51$ have only two.

Thus, the magic numbers are associated with extra binding energies, implying higher stability.

Thus, in the given example it takes about 6 MeV more energy to remove a paired neutron from a pair that completes a closed nuclear shell (a magic number) than from a pair that does not complete a shell.

Example 5.14 Consider a body at rest in the L-frame which explodes into two fragments of masses m_1 and m_2. Calculate energies of the fragments of the body.

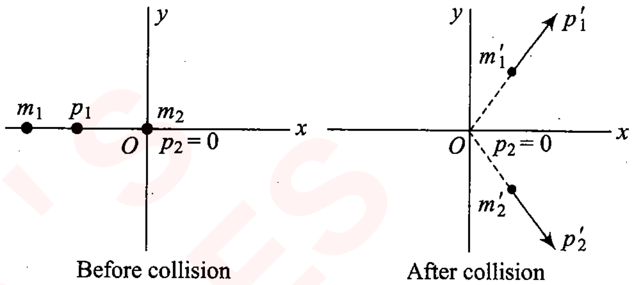


Fig. 5.26

Sol. Since the body is initially at rest, its total momentum is zero. As a result of the explosion, the two fragments will separate in opposite directions with momenta p_1 and p_2 such that $p_1' + p_2'$ = 0; magnitude of momentum will be same, i.e., $p_1' = p_2'$.

On substituting,

$$E_k' = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} = \frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)p_1'$$
$$E_k = 0$$

In Eq. (iv) of kinematics of reaction, we get

$$\frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)p_1'^2 = Q$$

On rearranging this equation, we get

$$p_1' = \left(\frac{2m_1m_2}{m_1 + m_2}Q\right)^{1/2} = (2\mu_{12}Q)^{1/2}$$

where μ_{12} is the reduced mass of the system. The kinetic energies of the fragments are $\frac{p_1'^2}{2m_1}$ and $\frac{p_2'^2}{2m_2}$ with $p_1' = p_2'$. Hence,

$$E_{k,1}' = \frac{p_1'^2}{2m_1} = \frac{m_1 Q}{m_1 + m_2} \quad \text{and} \quad E_{k,2}' = \frac{p_2'^2}{2m_2} = \frac{m_2 Q}{m_1 + m_2}$$

Remarks

1. The kinetic energy of the fragments are inversely proportional to their masses.
2. This analysis applies to the fission of the nucleus into two fragments, to the dissociation of disatomic molecule or to the disintegration of the nucleus.
3. When there are three fragments, several solutions are possible. There are three unknown momenta with two equations: of conservation of energy and of momentum. When two particles are observed in a reaction and the energy and momentum are not conserved, it is suspected that a third particle is present.

   It is possible that the particle is unobserved because it has no electric charge. Missing momentum and energy are assigned to this hypothetical particle, in order to conserve energy and momentum. Neutron and the neutrino were discovered as a result of such an experiment.

Example 5.15 **a. Find the energy needed to remove a neutron from the nucleus of the calcium isotope $^{42}_{20}$Ca.**

b. Find the energy needed to remove a proton from this nucleus.

c. Why are these energies different?

Mass of $^{42}_{20}$Ca = 40.962278 u, mass of proton = 1.007825 u.

Sol. a. $^{41}_{20}$Ca nucleus is formed after removing a neutron from $^{42}_{20}$Ca.

The mass of $^{41}_{20}$Ca plus the mass of a free neutron = 40.962278 u

Mass of neutron = 40.962278 u + 1.008665 u = 41.970943 u

Difference between $^{41}_{20}$Ca plus the mass of a free neutron and the mass of $^{42}_{20}$Ca is 0.012321 u.

So, the binding energy of the missing neutron is

$$(0.012321 \text{ u})\ (931.49 \text{ MeV/u}) = 11.5 \text{ MeV}$$

b. When a proton is removed from $^{42}_{20}$Ca, the resulting nucleus is the postassium isotope $^{41}_{19}$K. On a similar pattern as above, the binding energy for the missing proton can be calculated; result is 10.27 MeV.

c. Neutron and proton have different energies because only attractive nuclear forces act on the neutron whereas the proton was also acted upon by repulsive electric force that decreases its binding energy.

Example 5.16 **Write the decay equations and expressions for the disintegration energy Q of the following decay: β^- decay, β^+ decay, electron capture.**

Sol. In all the cases discussed here, we neglect any neutrino mass.

$$\beta^- \text{ decay: } M_{\text{nucl}}(^A_Z\text{X}) = M_{\text{nucl}}(^{A}_{Z+1}\text{D}) + m_e + Q/c^2$$

where M_{nucl} indicates the nuclear mass.

In order to convert it to atomic mass, we add Zm_e on both sides:

$$M_{\text{nucl}}(^A_Z\text{X}) + Zm_e = M_{\text{nucl}}(^{A}_{Z+1}\text{D}) + (Z+1)m_e + Q/c^2$$

Since atomic binding energies are less than nuclear binding energies, they are neglected on the two sides of the equation.

$$[M(^A_Z\text{X}) = M(^{A}_{Z+1}\text{D})]\,c^2$$

$$Q \text{ for this equation} = [M(^A_Z\text{X}) - M(^{A}_{Z+1}\text{D})]\,c^2$$

$$\beta^+ \text{ decay: } M_{\text{nucl}}(^A_Z\text{X}) = M_{\text{nucl}}(^{A}_{Z-1}\text{D}) + m_e + Q/c^2$$

In this case, only $(Z-1)m$ is needed for the daughter atomic mass which gives us a remaining mass of $2m_e$.

$$[M(^A_Z\text{X}) - M(^{A}_{Z-1}\text{D}) - 2m_e]c^2$$

$$Q \text{ for this equation} = [M(^A_Z\text{X}) - (^{A}_{Z-1}\text{D}) - 2m_e]c^2$$

We add $(Z-1)m_e$ to each side above and obtain

$$M_{\text{nucl}}(^A_Z\text{X}) + Zm_e = M_{\text{nucl}}(^{A}_{Z-1}\text{D}) + (Z-1)m_e + Q/c^2$$

In this case, electron masses are balanced, i.e., cancel out in equation

$$M(^A_Z\text{X}) - M(^{A}_{Z-1}\text{D}) + Q/c^2$$

$$Q \text{ for this equation} = [M(^A_Z\text{X}) - M(^{A}_{Z-1}\text{D})]c^2$$

[Electron capture]

Example 5.17 **Find whether alpha decay or any of the beta decay are allowed for $^{226}_{89}$Ac.**

Sol. Our first step will be to write the reaction, then find the disintegration energy Q. If $Q > 0$, the decay is allowed.

Alpha decay: $^{226}_{89}\text{Ac} \rightarrow {}^{222}_{87}\text{Fr} + \alpha$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{222}_{87}\text{Fr}) - M(^4\text{He})]c^2$$

$= 5.50$ MeV (alpha decay is allowed)

β^- decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{90}\text{Th})]c^2$$

$= 1.12$ MeV (β^- decay is allowed)

β^+ decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{88}\text{Ra} + \beta^+ + \text{v}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra}) - 2m_e]c^2$$

$= -0.38$ MeV (β^+ decay is not allowed)

Electron capture: $^{226}_{89}\text{Ac} + e^- \rightarrow {}^{226}_{88}\text{Ra} + \text{v}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra})]c^2$$

$= 0.64$ MeV (Electron capture is allowed)

EXERCISES

Subjective Type *Solutions on page 5.61*

1. In a certain hypothetical radioactive decay process, species A decays into species B and species B decays into species C according to the reactions

$A \rightarrow 2B$ + particles + energy
$B \rightarrow 2C$ + particles + energy

The decay constant for species A is $\lambda_1 = 1\ \text{s}^{-1}$ and that for species B is $\lambda_2 = 100\ \text{s}^{-1}$. Initially, 10^4 moles of species of A were present while there was none of B and C. It was found that species B reaches its maximum number at a time $t_0 = 2 \ln(10)$ s. Calculate the value of maximum number of moles of B.

2. 'm_1' g of non-radioactive isotopes $_zX^A$ are mixed with 'm_2' g of the radioactive isotope $_zX^{A'}$. How much will the specific activity decrease? Half-life of $_zX^{A'} = T$. Take N_A as Avagadro number.

3. C^{14} disintegrates by β-emission with a reaction energy (Q value) of 0.155 MeV. A β-particle with an energy of 0.025 MeV is emitted in a direction at 135° to the direction of motion of the recoil nucleus. Determine the momenta of the three particles ($\beta^- = \bar{\nu}$, ^{14}N) involved in this disintegration in MeV/c units (where c is speed of light in vacuum). ($M_0 = 0.511$ MeV/c^2)

4. A radionuclide with disintegration constant λ is produced in a reactor at a constant rate α (= 2λ) nuclei per second. At $t = 0$, there are no nuclei present in the reactor. During each decay, energy E_0 is released. 20% of this energy is utilized in increasing the temperature of water. Find the increase in temperature of mass m of water in time $t = T_{1/2}$ if specific heat of water is s. Assume that there is no loss of energy through water surface.

(Given E_0 = 100 times the energy is required to raise the temperature of mass m of water from 0°C to 100°C)

5. A stable nuclei C is formed from radioactive nuclei A and B with decay constants of λ_1 and λ_2, respectively. Initially, the number of nuclei of A is N_0 and that of B is zero. Nuclei B are produced at a constant rate of P. Find the number of nuclei of C after time t.

6. Suppose a nucleus initially at rest undergoes α-decay according to equation

$$^{235}_{92}\text{X} \rightarrow \text{Y} + \alpha$$

At $t = 0$, the emitted α-particles enter a region of space where a uniform magnetic field $\vec{B} = B_0\hat{j}$ and electric field $\vec{E} = E_0\hat{i}$ exist. The α-particle enters in the region with velocity $\vec{v} = v_0\hat{j}$ from $x = 0$. At time $t = \sqrt{3} \times 10^6 \dfrac{m_0}{q_0 E_0}$ s, the particle was observed to have speed twice the initial velocity v_0. Then, find

a. the velocity v_0 of the α-particle;

b. the initial velocity v_0 of the α-particle;

c. the binding energy per nucleon of the α-particle.

[Given that m(Y) = 221.03 u, $m(\alpha)$ = 4.003 u, m(n) = 1.09 u, m(p) = 1.008 u]

7. Natural uranium is a mixture of three isotopes $_{92}U^{234}$, $_{92}U^{235}$ and $_{92}U^{238}$ with the shares of negligible 0.0006%, 0.71% and 99.28%, respectively. The half-life of the three isotopes are 2.5×10^5 years, 7.1×10^8 years and 4.5×10^9 years, respectively. Determine the share of radioactivity of each isotope into the total activity of the natural uranium.

8. A nuclear reaction is given as

$$\text{p} + {}^{15}\text{N} \rightarrow {}^{A}_{Z}\text{X} + \text{n}$$

a. Find, A, Z and identify the nucleus X.

b. Find the Q value of the reaction.

c. If the proton were to collide with the ^{15}N at rest, find the minimum KE needed by the proton to initiate the above reaction.

d. If the proton has twice the energy in (c) and the outgoing neutron emerges at an angle of 90° with the direction of the incident proton, find the momentum of the nucleus X.

[Given, m(p) = 1.007825 u, $m(^{15}$C) = 15.0106 u,
$m(^{16}$N) = 16.0061 u, $m(^{15}$N) = 15.000 u),
$m(^{16}$O) = 15.9949 u, m(n) = 1.008665 u,
$m(^{15}$O) = 15.0031 u, and 1 u ≈ 931.5 MeV.]

9. $^{239}_{235}\text{Pu}_{94}$ is undergoing α-decay according to the equation $^{235}_{94}\text{Pu} \rightarrow {}^{235}_{97}\text{U} + {}^{4}_{2}\text{He}$. The energy released in the process is mostly kinetic energy of the α-particle. However, a part of the energy is released as γ-rays. What is the speed of the emitted α-particle if the γ-rays radiated out have energy of 0.90 MeV? Given: Mass of $^{239}_{94}$Pu = 239.05122 u, mass of $^{235}_{97}$U = 235.04299 u and mass of $^{4}_{1}$He = 4.002602 u (1 u = 931 MeV).

10. A tritium gas target is bombarded with a beam of monoenergetic protons of kinetic energy K_1 = 3 MeV. The KE of the neutron emitted at 30° to the incident beam is K_2? Find the value of K_1/K_2 (approximately in whole number). Atomic masses are H^1 = 1.007276 a.m.u., n^1 = 1.008665 a.m.u., $_1$H^3 = 3.016050 a.m.u., $_2$He^3 = 3.016030 a.m.u.

11. Consider a nuclear reaction $A + B \rightarrow C$. A nucleus 'A' moving with kinetic energy of 5 MeV collides with a nucleus 'B' moving with kinetic energy of 3 MeV and forms a nucleus 'C' in excited state. Find the kinetic energy of nucleus 'C' just after its formation, if it is formed in a state with excitation energy 10 MeV. Take masses of nuclei of A, B and C as 25.0, 10.0, 34.995 a.m.u., respectively.
(1 a.m.u. = 930 MeV/c^2)

12. Find the Q value of the reaction

$$\text{N}^{14} + \alpha \rightarrow \text{O}^{17} + \text{p}$$

The masses of N^{14}, He^4, p and O^{17} are, respectively, 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.
Find the total kinetic energy of the products if the striking α-particle has the minimum kinetic energy required to initiate the reaction.

13. A sample has two isotopes A^{150+} and B having masses 50 g and 30 g, respectively. A is radioactive and B is stable. A decays to A' by emitting α-particles. The half-life of A is 2 h. Find the mass of the sample after 4 h and number of α-particles emitted.

14. The nucleus ^{23}Ne decays by β-emission into the nucleus ^{23}Na. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given, $m(^{23}_{10}\text{Ne})$ = 22.994466 a.m.u. and $m(^{23}_{11}\text{Na})$ = 22.989770 a.m.u. Ignore the mass of antineutrino ($\bar{\nu}$).

15. A radioactive source in the form of a metal sphere of diameter 10^{-3} m emits beta particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0

V, assuming that 80% of the emitted beta particles escape the source?

16. Find whether alpha decay or any of the beta decay are allowed for $^{226}_{89}Ac$. Given masses are $M(^{226}_{89}Ac)$ = 226.028356 a.m.u., $M(^{222}_{87}Fr)$ = 222.017415 a.m.u., $M(^{226}_{90}Th)$ = 226.017388 a.m.u., $M(^{226}_{88}Ra)$ = 226.025406 a.m.u., $M(^{4}_{2}He)$ = 4.002603 a.m.u.

17. Show that $^{55}_{26}Fe$ may electron capture, but not β^+ decay. Masses given are $M(^{55}_{26}Fe)$ = 54.938298 a.m.u., $M(^{55}_{25}Mn)$ = 54.938050 a.m.u., $m(e)$ = 0.000549 a.m.u.

18. A sample of ^{18}F is used internally as a medical diagnostic tool to look for the effects of the positron decay ($T_{1/2}$ = 110 min). How long does it take for 99% of the ^{18}F to decay?

19. Find the binding energy of an α-particle from the following data.
Mass of the helium nucleus = 4.001265 a.m.u.
Mass of proton = 1.007277 a.m.u.
Mass of neutron = 1.00866 a.m.u.
(take 1 a.m.u. = 931.4813 MeV)

20. A neutron breaks into a proton and an electron. Calculate the energy produced in this reaction in MeV. Mass of an electron = 9×10^{-31} kg, mass of proton = 1.6725×10^{-27} kg, mass of neutron = 1.6747×10^{-27} kg and speed of light = 3×10^8 m s^{-1}.

21. The binding energy of $^{35}_{17}Cl$ nucleus is 298 MeV. Find its atomic mass. Given, mass of a proton (m_p) = 1.007825 a.m.u., mass of a neutron (m_n) = 1.008665 a.m.u.

22. Find the density of $^{12}_{6}C$ nucleus. Take atomic mass of $^{12}_{6}C$ as 12.000 a.m.u. Take $R_0 = 1.2 \times 10^{-15}$ m.

23. Calculate the binding energy per nucleon for $^{20}_{10}Ne$, $^{56}_{26}Fe$ and $^{238}_{92}U$. Given that mass of neutron is 1.008665 a.m.u., mass of proton is 1.007825 a.m.u., mass of $^{20}_{10}Ne$ is 19.9924 a.m.u., mass of $^{56}_{26}Fe$ is 55.93492 a.m.u. and mass of $^{238}_{92}U$ is 238.050783 a.m.u.

24. One gram of a radioactive material having a half-life period of 2 years is kept in store for a duration of 4 years. Calculate how much of the material remains unchanged?

25. One gram of a radioactive substance takes 50 s to lose 1 centigram. Find its half-life period.

26. One gram of a radioactive substance disintegrates at the rate of 3.7×10^{10} disintegrations per second. The atomic mass of the substance is 226. Calculate its mean life.

27. There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutron is 700 s, what fraction of neutrons will decay before they travel a distance of 10 km.

28. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.
   a. Calculate the number N of nuclei of A at time t.
   b. If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.

29. The fission type of warhead of some guided missiles is estimated to be equivalent to 30000 tons of TNT. If 3.5×10^8 J of energy is released by one ton of exploding TNT, how many fissions occur and how much ^{235}U would be consumed in the explosion of warhead? An energy of 200 MeV is released by fission of one atom of ^{235}U.

30. In a nuclear reactor, fission is produced in 1 g of ^{235}U (235.0349 a.m.u.) In assuming that $^{92}_{53}Kr$ (91.8673 a.m.u.) and $^{141}_{36}Ba$ (140.9139 a.m.u.) are produced in all reactions and no energy is lost, write the complete reaction and calculate the total energy produced in kilowatt hour. Given: 1 a.m.u. = 931 MeV.

31. In the fission of $^{239}_{94}Pu$ by a thermal neutron, two fission fragments of equal masses and sizes are produced and four neutrons are emitted. Find the force between the two fission fragments at the moment they are produced. Given: R_0 = 1.1 fermi.

32. Calculate the excitation energy of the compound nuclei produced when ^{235}U and ^{238}U absorb thermal neutrons. Given:
$M(^{235}U)$ = 235.0439 a.m.u., $M(n)$ = 1.0087 a.m.u.,
$M(^{238}U)$ = 238.0508 a.m.u., $M(^{236}U)$ = 236.0456 a.m.u.,
$M(^{239}U)$ = 239.0543 a.m.u.

33. Calculate the ground state Q value of the induced fission reaction in the equation
$$n + ^{235}_{92}U \rightarrow ^{236}_{92}U^* \rightarrow ^{99}_{40}Zr + ^{134}_{52}Te + 3n$$
if the neutron is thermal. A thermal neutron is in thermal equilibrium with its environment; it has an average kinetic energy given by (3/2) kT. Given:
$m(n)$ = 1.0087 a.m.u., $M(^{235}U)$ = 235.0439 a.m.u.,
$M(^{99}Zr)$ = 98.916 a.m.u., $M(^{134}Te)$ = 133.9115 a.m.u.

34. Show that $^{230}_{92}U$ does not decay by emitting a neutron or proton. Given:
$M(^{230}_{92}U)$ = 230.033927 a.m.u., $M(^{230}_{92}U)$ = 229.033496 a.m.u., $M(^{229}_{92}Pa)$ = 229.032089 a.m.u., $m(n)$ = 1.008665 a.m.u., $m(p)$ = 1.007825 a.m.u.

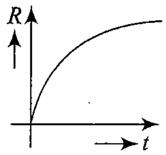
35. The nuclear reaction $n + ^{10}_{5}B \rightarrow ^{7}_{3}Li + ^{4}_{2}He$ is observed to occur even when very slow-moving neutrons (M_n = 1.0087 a.m.u.) strike a boron atom at rest. For a particular reaction in which $K_n = 0$, the helium (M_{He} = 4.0026 a.m.u.) is observed to have a speed of 9.30×10^6 m s^{-1}. Determine (a) the kinetic energy of the lithium (M_{Li} = 7.0160 a.m.u.) and (b) the Q value of the reaction.

Objective Type

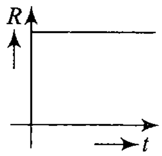
Solutions on page 5.67

1. An element A decays into an element C by a two-step process:
$$A \rightarrow B + He^4_2 \text{ and } B \rightarrow C + 2e^0_{-1}$$
Then
   a. A and C are isotopes
   b. A and C are isobars
   c. B and C are isotopes
   d. A and B are isobars

2. Consider two arbitrary decay equations and mark the correct alternative(s) given below.
   (i) ${}^{230}_{92}\text{U} \rightarrow n + {}^{229}_{92}\text{U}$
   (ii) ${}^{230}_{92}\text{U} \rightarrow p + {}^{229}_{91}\text{Pa}$
   Given: $M({}^{230}_{92}\text{U}) = 230.033927$ u,
   $M({}^{229}_{92}\text{U}) = 229.03349$ u, $m_n = 1.008665$ u,
   $M({}^{229}_{91}\text{Pa}) = 229.032089$, $m_p = 1.007825$, 1 a.m.u. = 931.5 MeV.
   a. Only decay (i) is possible.
   b. Only decay (ii) is possible.
   c. Both the decays are possible.
   d. Neither of the two decays is possible.
3. In a sample of rock, the ratio of ^{206}Pb to ^{238}U nuclei is found to be 0.5. The age of the rock is (given half-life of U^{238} is 4.5×10^9 years)
   a. 2.25×10^9 year
   b. $4.5 \times 10^9 \ln 3$ year
   c. $4.5 \times 10^9 \dfrac{\ln\left(\frac{3}{2}\right)}{\ln 2}$ year
   d. $2.25 \times 10^9 \ln\left(\frac{3}{2}\right)$ year
4. Let E_1 and E_2 be the binding energies of two nuclei A and B. It is observed that two nuclei of A combine together to form a B nucleus. This observation is correct only if
   a. $E_1 > E_2$
   b. $E_2 > E_1$
   c. $E_2 > 2E_1$
   d. nothing can be said
5. A radioactive sample decays by 63% of its initial value in 10 s. It would have decayed by 50% of its initial value in
   a. 7 s
   b. 14 s
   c. 0.7 s
   d. 1.4 s
6. A nucleus moving with velocity $\vec{v}$ emits an α- particle. Let the velocities of the α-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2, then
   a. $\vec{v}, \vec{v}_1$ and $\vec{v}_2$ must be parallel to each other
   b. none of the two of $\vec{v}, \vec{v}_1$ and $\vec{v}_2$ should be parallel to each other
   c. $\vec{v}_1 + \vec{v}_2$ must be parallel to $\vec{v}$
   d. $m_1\vec{v}_1 + m_2\vec{v}_2$ must be parallel to $\vec{v}$
7. Which of the following statements is incorrect for nuclear forces?
   a. These are strongest in magnitude.
   b. They are charge dependent.
   c. They are effective only for short ranges.
   d. They result from interaction of every nucleon with the nearest limited number of nucleons.
8. A certain radioactive material can undergo three different types of decay, each with a different decay constant λ, 2λ and 3λ. Then, the effective decay constant λ_{eff} is
   a. 6λ
   b. 4λ
   c. 2λ
   d. 3λ
9. In an α-decay, the kinetic energy of α-particle is 48 MeV and Q value of the reaction is 50 MeV. The mass number of the mother nucleus is (assume that daughter nucleus is in ground state)
   a. 96
   b. 100
   c. 104
   d. none of these
10. A sample of radioactive material decays simultaneously by two processes A and B with half-lives $\frac{1}{2}$ and $\frac{1}{4}$ h, respectively. For first half hour it decays with the process A, next one hour with the process B, and for further half an hour with both A and B. If originally there were N_0 nuclei, find the number of nuclei after 2 h of such decay.
   a. $\dfrac{N_0}{(2)^8}$
   b. $\dfrac{N_0}{(2)^4}$
   c. $\dfrac{N_0}{(2)^6}$
   d. $\dfrac{N_0}{(2)^5}$
11. In which of the following processes, the number of protons in the nucleus increase?
   a. α-decay
   b. β^--decay
   c. β^+-decay
   d. k-capture
12. A radioactive nucleus 'X' decays to a stable nucleus 'Y'. Then, time graph of rate of formation of 'Y' against time 't' will be:



a.



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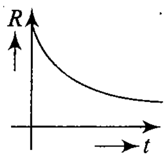
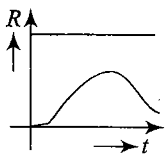


c.



d.

13. A heavy nucleus having mass number 200 gets disintegrated into two small fragments of mass numbers 80 and 120. If binding energy per nucleon for parent atom is 6.5 MeV and for daughter nuclei is 7 MeV and 8 MeV, respectively, then the energy released in the decay will be
   a. 200 MeV
   b. –220 MeV
   c. 220 MeV
   d. 180 MeV
14. An element X decays, first by positron emission and then two α-particles are emitted in successive radioactive decay. If the product nucleus has a mass number 229 and atomic number 89, the mass number and atomic number of element X are
   a. 237, 93
   b. 237, 94
   c. 221, 84
   d. 237, 92

15. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$?
a. 20% b. 19%
c. 40% d. 38%

16. A radioactive element X converts into another stable element Y. Half-life of X is 2 h. Initially, only X is present. After time t, the ratio of atoms of X and Y is found to be 1:4. Then t in hours is
a. 2 b. 4
c. between 4 and 6 d. 6

17. $A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$

$T=O$ N_0 0 0
T N_1 N_2 N_3

The ratio of N_1 to N_2 when N_2 is maximum is
a. at no time this is possible b. 2
c. 1/2 d. $\frac{\ln 2}{2}$

18. The binding energy of an electron in the ground state of He-atom is $E_0 = 24.6$ eV. The energy required to remove both the electrons from the atom is
a. 24.6 eV b. 79.0 eV
c. 54.4 eV d. none of these

19. The mean life time of a radionuclide, if its activity decreases by 4% for every 1 h, would be [product is non-radioactive, i.e., stable]
a. 25 h b. 1.042 h
c. 2 h d. 30 h

20. On an average, a neutron loses half of its energy per collision with a quasi-free proton. To reduce a 2 MeV neutron to a thermal neutron having energy 0.04 eV, the number of collisions required is nearly
a. 50 b. 52
c. 26 d. 15

21. Atomic masses of two isobars $^{64}_{29}$Cu and $^{64}_{30}$Zn are 63.9298 u and 63.9292 u, respectively. It can be concluded from this data that
a. both the isobars are stable
b. ^{64}Zn is radioactive, decaying to ^{64}Cu through β-decay
c. ^{64}Cu is radioactive, decaying to ^{64}Zn through β-decay
d. ^{64}Cu is radioactive, decaying to ^{64}Zn through γ-decay

22. If a nucleus such as ^{226}Ra that is initially at rest undergoes alpha decay, then which of the following statements is true?
a. The alpha particle has more kinetic energy than the daughter nucleus.
b. The alpha particle has less kinetic energy than the daughter nucleus.
c. The alpha particle and daughter nucleus both have same kinetic energy
d. We cannot say anything about kinetic energy of alpha particle and daughter nucleus.

23. If the Q value of an endothermic reaction is 11.32 MeV, then the minimum energy of the reactant nuclei to carry out the reaction is (in laboratory frame of reference)
a. 11.32 MeV
b. less than 11.32 MeV
c. greater than 11.32 MeV
d. Data is insufficient

24. 1.00 kg of ^{235}U undergoes fission process. If energy released per event is 200 MeV, then the total energy released is
a. 5.12×10^{24} MeV b. 6.02×10^{23} MeV
c. 5.12×10^{26} d. 6.02×10^{26} MeV

25. Mark out the incorrect statement.
a. A free neutron can transform itself into photon.
b. A free proton can transform itself into neutron.
c. In beta minus decay, the electron originates from nucleus.
d. All of the above.

26. U-235 can decay by many ways, let us here consider only two ways A and B. In decay of U-235 by means of A, the energy released per fission is 210 MeV while in B it is 186 MeV. Then, the uranium 235 sample is more likely to decay by
a. scheme A
b. scheme B
c. equally likely for both schemes
d. it depends on half-life of schemes A and B

27. $^{49}_{19}$K isotope of potassium has a half-life of 1.4×10^9 yr and decays to form stable argon, $^{40}_{18}$Ar. A sample of rock has been taken which contains both potassium and argon in the ratio 1:7, i.e.,

$$\frac{\text{Number of potassium-40 atoms}}{\text{Number of argon-40 atoms}} = \frac{1}{7}$$

Assuming that when the rock was formed no argon-40 was present in the sample and none has escaped subsequently, determine the age of the rock.
a. 4.2×10^9 years b. 9.8×10^9 years
c. 1.4×10^9 years d. 10×10^9 years

28. The probability of survival of a radioactive nucleus for one mean life is
a. $\frac{1}{e}$ b. $1-\frac{1}{e}$
c. $\frac{\ln 2}{e}$ d. $1-\frac{\ln 2}{e}$

29. Consider one of fission reactions of ^{235}U by thermal neutrons $^{235}_{92}\text{U} + \text{n} \rightarrow {}^{94}_{38}\text{Sr} + {}^{140}_{54}\text{Xe} + 2\text{n}$. The fission fragments are however unstable and they undergo successive β-decay until $^{94}_{38}$Sr becomes $^{94}_{40}$Zr and $^{140}_{54}$Xe becomes $^{140}_{58}$Ce. The energy released in this process is [Given: $m(^{235}\text{U}) = 235.439$ u, m (n) = 1.00866 u, m (^{94}Zr) = 93.9064 u, $m(^{140}\text{Ce}) = 139.9055$ u, 1 u = 931 MeV]
a. 156 MeV b. 208 MeV
c. 456 MeV d. cannot be computed

30. A star initially has 10^{40} deuterons. It produces energy via the processes $^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{He} + \text{p}$ and $^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + \text{n}$.

If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of
[Given: $M(^2H) = 2.014$ u, $M(n) = 1.008$ u, $M(p) = 1.008$ u, and $M(^4He) = 4.001$ u]

a. 10^6 s b. 10^8 s
c. 10^{12} s d. 10^{16} s

31. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

a. $\frac{1}{10\lambda}$ b. $\frac{1}{11\lambda}$
c. $\frac{11}{10\lambda}$ d. $\frac{1}{9\lambda}$

32. A radioactive substance is being consumed at a constant rate of 1 s^{-1}. After what time will the number of radioactive nuclei become 100. Initially, there were 200 nuclei present.

a. 1 s b. $\frac{1}{\ln(2)}$ s
c. ln(2) s d. 2 s

33. A radioactive isotope is being produced at a constant rate X. Half-life of the radioactive substance is Y. After some time, the number of radioactive nuclei become constant. The value of this constant is

a. $\frac{XY}{\ln(2)}$ b. XY
c. (XY) ln (2) d. $\frac{X}{Y}$

34. A radioactive substance X decays into another radioactive substance Y. Initially, only X was present. λ_x and λ_y are the disintegration constants of X and Y. N_y will be maximum when

a. $\frac{N_y}{N_x - N_y} = \frac{\lambda_y}{\lambda_x - \lambda_y}$ b. $\frac{N_x}{N_x - N_y} = \frac{\lambda_x}{\lambda_x - \lambda_y}$
c. $\lambda_y N_y = \lambda_x N_x$ d. $\lambda_y N_x = \lambda_x N_y$

35. There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant?

a. 2:1 b. 1:2
c. e d. e^{-1}

36. Half-life of a radioactive substance A is two times the half-life of another radioactive substance B. Initially, the number of A and B are N_A and N_B, respectively. After three half-lives of A, number of nuclei of both are equal. Then, the ratio N_A/N_B is

a. 1/4 b. 1/8
c. 1/3 d. 1/6

37. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After n half-lives of A, rates of disintegration of both are equal. The value of n is

a. 1 b. 2
c. 4 d. all of these

38. A radioactive nucleus A finally transforms into a stable nucleus B. Then, A and B may be

a. isobars b. isotones
c. isotopes d. none of these

39. If $_{92}U^{238}$ changes to $_{85}At^{210}$ by a series of α- and β-decays, the number of α- and β-decays undergone is

a. 7 and 5 b. 7 and 7
c. 5 and 7 d. 7 and 9

40. Number of nuclei of a radioactive substance are 1000 and 900 at times $t = 0$ and time $t = 2$ s. Then, number of nuclei at time $t = 4$ s will be

a. 800 b. 810
c. 790 d. 700

41. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ. If N_0 are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are

a. $\frac{\alpha}{\lambda}$ b. $N_0 \frac{\alpha}{\lambda}$
c. N_0 d. $\frac{\lambda}{\alpha} + N_0$

42. In a sample of a radioactive substance, what fraction of the initial nuclei will remain undecayed after a time $t = T/2$, where T = half-life of radioactive substance?

a. $\frac{1}{\sqrt{2}}$ b. $\frac{1}{2\sqrt{2}}$
c. $\frac{1}{4}$ d. $\frac{1}{\sqrt{2}-1}$

43. The activity of a radioactive substance is R_1 at time t_1 and R_2 at time t_2 ($> t_1$). Its decay constant is λ. Then

a. $R_1 t_1$ b. $R_2 = R_1 e^{\lambda(t_1 - t_2)}$
c. $\frac{R_1 - R_2}{t_2 - t_1}$ = constant d. $R_2 = R_1 e^{\lambda(t_2 - t_1)}$

44. In problem 43, number of atoms decayed between time interval t_1 and t_2 are

a. $\frac{\ln(2)}{\lambda}(R_1 R_2)$ b. $R_1 e^{-\lambda t_2} - R_2 e^{-\lambda t_2}$
c. $\lambda(R_1 - R_2)$ d. $\left(\frac{R_1 - R_2}{\lambda}\right)$

45. The ratio of molecular mass of two radioactive substances is $\frac{3}{2}$ and the ratio of their decay constants is $\frac{4}{3}$. Then, the ratio of their initial activity per mole will be

a. 2 b. $\frac{8}{9}$

c. $\frac{4}{3}$ d. $\frac{9}{8}$

46. N_1 atoms of a radioactive element emit N_2 beta particles per second. The decay constant of the element is (in s^{-1})

a. $\frac{N_1}{N_2}$ b. $\frac{N_2}{N_1}$
c. $N_1 \ln(2)$ d. $N_2 \ln(2)$

47. The binding energies of nuclei X and Y are E_1 and E_2, respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then,

a. $Q = 2E_1 - E_2$ b. $Q = E_2 - 2E_1$
c. $Q < 2E_1 - E_2$ d. $Q > E_2 - 2E_1$

48. Binding energy per nucleon of $_1H^2$ and $_2He^4$ are 1.1 MeV and 7.0 MeV, respectively. Energy released in the process $_1H^2 + _1H^2 = _2He^4$ is

a. 20.8 MeV b. 16.6 MeV
c. 25.2 MeV d. 23.6 MeV

49. $_{92}U^{238}$ absorbs a neutron. The product emits an electron. This product further emits an electron. The result is

a. $_{94}Pu^{239}$ b. $_{90}Pu^{239}$
c. $_{93}Pu^{237}$ d. $_{94}Pu^{237}$

50. The activity of a radioactive element decreases to one-third of the original activity A_0 in a period of 9 years. After a further lapse of 9 years, its activity will be

a. A_0 b. $\frac{2}{3}A_0$
c. $\frac{A_0}{9}$ d. $\frac{A_0}{6}$

51. The half-life of a radioactive decay is x times its mean life. The value of x is

a. 0.3010 b. 0.6930
c. 0.6020 d. $\frac{1}{0.6930}$

52. Neutron decay in the free space is given as follows: $_0n^1 \to {}_1H^1 + {}_{-1}e^0 + [\ ]$
Then, the parenthesis represents

a. photon b. graviton
c. neutrino d. antineutrino

53. A nucleus ^A_ZX emits an α-particle. The resultant nucleus emits a β^+ particle. The respective atomic and mass numbers of the final nucleus will be

a. $Z-3, A-4$ b. $Z-1, A-4$
c. $Z-2, A-4$ d. $Z, A-2$

54. Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is

a. 32 days b. 8 days
c. 64 days d. 28 day

55. What is the age of an ancient wooden piece if it is known that the specific activity of C^{14} nuclide in it amounts to 3/5 of that in fresh trees? Given: the half of C nuclide is 5570 years and $\log_e(5/3) = 0.5$.

a. 1000 years b. 2000 years
c. 3000 years d. 4000 years

56. A helium atom, a hydrogen atom and a neutron have masses of 4.003 u, 1.008 u and 1.009 u (unified atomic mass units), respectively. Assuming that hydrogen atoms and neutrons can fuse to form helium, what is the binding energy of a helium nucleus?

a. 2.01 u b. 3.031 u
c. 1.017 u d. 0.031 u

57. A certain radioactive element has half-life of 4 days. The fraction of material that decays in 2 days is

a. 1/2 b. $1/\sqrt{2}$
c. $\sqrt{2}$ d. $(\sqrt{2}-1)/\sqrt{2}$

58. In the disintegration series

$$^{238}_{92}U \xrightarrow{\alpha} X \xrightarrow{\beta^-} {}^A_ZY$$

the values of Z and A, respectively, will be

a. 92, 326 b. 88, 230
c. 90, 234 d. 91, 234

59. In the case of thorium ($A = 232$ and $Z = 90$), we obtain an isotope of lead ($A = 208$ and $Z = 82$) after some radioactive disintegrations. The number of α- and β-particles emitted are, respectively,

a. 6, 3 b. 6, 4
c. 5, 5 d. 4, 6

60. The initial activity of a certain radioactive isotope was measured as 16000 counts min^{-1}. Given that the only activity measured was due to this isotope and that its activity after 12 h was 2100 counts min^{-1}, its half-life, in hours, is nearest to [Given $\log_e(7.2) = 2$]

a. 9.0 b. 6.0
c. 4.0 d. 3.0

61. The minimum frequency of a γ-ray that causes a deuteron to disintegrate into a proton and a neutron is ($m_d = 2.0141$ a.m.u., $m_p = 1.0078$ a.m.u., $m_n = 1.0087$ a.m.u.)

a. 2.7×10^{20} Hz b. 5.4×10^{20} Hz
c. 10.8×10^{20} Hz d. 21.6×10^{20} Hz

62. The fission of a heavy nucleus gives, in general, two smaller nuclei, two or three neutrons, some β-particles, and some γ-radiation. It is always true that the nuclei produced

a. have a total rest-mass that is greater than that of the original nucleus
b. have large kinetic energies that carry off the greater part of the energy released
c. travel in exactly opposite directions
d. have neutron-to-proton ratios that are too low for stability.
e. have identical neutron-to-proton ratios

63. The activity of a radioactive sample is 1.6 curie, and its half-life is 2.5 days. Its activity after 10 days will be

a. 0.8 curie b. 0.4 curie
c. 0.1 curie d. 0.16 curie

64. The rest mass of a deuteron is equivalent to an energy of 1876 MeV, that of a proton to 939 MeV and that of a neutron to 940 MeV.
A deuteron may disintegrate to a proton and a neutron if it

a. emits an X-ray photon of energy 2 MeV
b. captures an X-ray photon of energy 2 MeV
c. emits an X-ray photon of energy 3 MeV
d. captures an X-ray photon of energy 3 MeV

65. A newly prepared radioactive nuclide has a decay constant λ of 10^{-6} s^{-1}. What is the approximate half-life of the nuclide?
a. 1 hour **b.** 1 day
c. 1 week **d.** 1 month

66. The half-life of a certain radioactive isotope is 32 h. What fraction of a sample would remain after 16 h?
a. 0.25 **b.** 0.71
c. 0.29 **d.** 0.75

67. Samples of two radioactive nuclides, X and Y, each have equal activity A at time $t = 0$. X has a half-life of 24 years and Y a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture at $t = 48$ years?
a. $\frac{1}{2}A_0$ **b.** $\frac{1}{4}A_0$
c. $\frac{3}{16}A_0$ **d.** $\frac{3}{8}A_0$

68. A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is
a. 1.35 g **b.** 2.50 g
c. 3.70 g **d.** 6.30 g

69. Atomic mass number of an element is 232 and its atomic number is 90. The end product of this radioactive element is an isotope of lead (atomic mass 208 and atomic number 82). The number of α-and β-particles emitted are
a. $\alpha = 3, \beta = 3$ **b.** $\alpha = 6, \beta = 4$
c. $\alpha = 6, \beta = 0$ **d.** $\alpha = 4, \beta = 6$

70. After an interval of one day, 1/16th initial amount of a radioactive material remains in a sample. Then, its half-life is
a. 6 h **b.** 12 h
c. 1.5 h **d.** 3 h

71. The half-life of At is 100 μs. The time taken for the radioactivity of a sample of At to decay to 1/16th of its initial value is
a. 400 μs **b.** 6.3 μs
c. 40 μs **d.** 300 μs

72. A stationary Thorium nucleus ($A = 220$, $Z = 90$) emits an alpha particle with kinetic energy E_α. What is the kinetic energy of the recoiling nucleus?
a. $\frac{E_\alpha}{108}$ **b.** $\frac{E_\alpha}{110}$
c. $\frac{E_\alpha}{55}$ **d.** $\frac{E_\alpha}{54}$

73. The fraction of a radioactive material which remains active after time t is 9/16. The fraction which remains active after time $t/2$ will be
a. $\frac{4}{5}$ **b.** $\frac{7}{8}$
c. $\frac{3}{5}$ **d.** $\frac{3}{4}$

74. The rate of decay of a radioactive element at any instant is 10^3 disintegrations s^{-1}. If the half-life of the elements is 1 s, then the rate of decay after 1 s will be
a. 500 s^{-1} **b.** 1000 s^{-1}
c. 250 s^{-1} **d.** 2000 s^{-1}

75. The percentage of quantity of a radioactive material that remains after 5 half-lives will be
a. 31% **b.** 3.125%
c. 0.3% **d.** 1%

76. ^{238}U decays with a half-life of 4.5×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock sample analysis shows that the ratio of the number of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb is produced by the decay of ^{238}U and that all other half-lives on the chain are negligible, the age of the rock sample is ($\ln 1.0058 = 5.78 \times 10^{-3}$)
a. 38×10^8 years **b.** 38×10^6 years
c. 19×10^8 years **d.** 19×10^6 years

77. A radioactive nucleus undergoes a series of decays according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, then what are these number for A_4?
a. 172 and 69 **b.** 174 and 70
c. 176 and 69 **d.** 176 and 70

78. If 10% of a radioactive substance decays in every 5 years, then the percentage of the substance that will have decayed in 20 years will be
a. 40% **b.** 50%
c. 65.6% **d.** 34.4%

79. Stationary nucleus ^{238}U decays by a emission generating a total kinetic energy T:

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}\alpha$$

What is the kinetic energy of the α-particle?
a. Slightly less than $T/2$ **b.** $T/2$
c. Slightly less than T **d.** Slightly greater than T

80. The activity of a radioactive element decreases to one-third of the original activity I_0 in a period of nine years. After a further lapse of nine years, its activity will be
a. I_0 **b.** $(2/3)I_0$
c. $(I_0/9)$ **d.** $(I_0/6)$

81. The half-life period of RaB($_{82}$Pb^{214}) is 26.8 min. The mass of one curie of RaB is
a. 3.71×10^{10} g **b.** 3.71×10^{-10} g
c. 8.61×10^{10} g **d.** 3.064×10^{-8} g

82. A 5×10^{-4} Å photon produces an electron–positron pair in the vincinity of a heavy nucleus. Rest energy of electron is 0.511 MeV. If they have the same kinetic energies, the energy of each particle is nearly
a. 1.2 MeV **b.** 12 MeV
c. 120 MeV **d.** 1200 MeV

83. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is
a. 6 h **b.** 12 h
c. 24 h **d.** 128 h

84. Uranium ores contain one radium-226 atom for every 2.8×10^6 uranium-238 atoms. Calculate the half-life of $_{92}U^{238}$, given that the half-life of $_{88}Ra^{226}$ is 1600 years ($_{88}Ra^{226}$ is a decay product of $_{92}U^{238}$).

a. 1.75×10^3 years
b. $1600 \times \frac{238}{92}$ years
c. 4.5×10^9 years
d. 1600×238 years

85. Plutonium has atomic mass 210 and a decay constant equal to 5.8×10^{-8} s^{-1}. The number of α-particles emitted per second by 1 mg Plutonium is
(Avogadro's constant $= 6.0 \times 10^{23}$)
a. 1.7×10^9
b. 1.7×10^{11}
c. 2.9×10^{11}
d. 3.4×10^9

86. At any instant, the ratio of the amounts of two radioactive substances is 2:1. If their half-lives be, respectively, 12 h and 16 h, then after two days, what will be the ratio of the substances?
a. 1:1
b. 2:1
c. 1:2
d. 1:4

87. The radioactivity of a sample is R_1 at a time T_1 and R_2 at a time T_2. If the half-life of the specimen is T, the number of atoms that have disintegrated in the time $(T_2 - T_1)$ is proportional to
a. $R_1 T_1 = R_2 T_2$
b. $R_1 - R_2$
c. $\frac{(R_1 - R_2)}{T}$
d. $(R_1 - R_2)T$

88. Half-lives of two radioactive substances A and B are, respectively, 20 min and 40 min. Initially, the samples of A and B have equal number of nuclei. After 80 min, the ratio of the remaining number of A and B nuclei is
a. 1:16
b. 4:1
c. 1:4
d. 1:1

89. A radioactive nucleus can decay by two different processes. The mean value period for the first process is t_1 and that for the second process is t_2. The effective mean value period for the two processes is
a. $\frac{t_1 + t_2}{2}$
b. $t_1 + t_2$
c. $\sqrt{t_1 t_2}$
d. $\frac{t_1 t_2}{t_1 + t_2}$

90. The half-life of radium is 1620 years and its atomic weight is 226. The number of atoms that will decay from its 1 g sample per second will be
a. 3.6×10^{10}
b. 3.6×10^{12}
c. 3.1×10^{15}
d. 31.1×10^{15}

91. The nuclear radius of a nucleus with nucleon number 16 is 3×10^{-15} m. Then, the nuclear radius of a nucleus with nucleon number 128 is
a. 3×10^{-15} m
b. 1.5×10^{-15} m
c. 6×10^{-15} m
d. 4.5×10^{-15} m

92. The nuclear radius of $_8O^{16}$ is 3×10^{-15} m. If an atomic mass unit is 1.67×10^{-27} kg, then the nuclear density is approximately
a. 2.35×10^{17} g cm^{-3}
b. 2.35×10^{17} kg m^{-3}
c. 2.35×10^{17} g m^{-3}
d. 2.35×10^{17} kg mm^{-3}

93. What would be the energy required to dissociate completely 1 g of Ca-40 into its constituent particles?
Given: Mass of proton = 1.007277 a.m.u.,
Mass of neutron = 1.00866 a.m.u.,
Mass of Ca-40 = 39.97545 a.m.u.
(take 1 a.m.u. = 931 MeV)
a. 4.813×10^{24} MeV
b. 4.813×10^{24} eV
c. 4.813×10^{23} MeV
d. None of the above

94. In fission, the percentage of mass converted into energy is about
a. 10%
b. 1%
c. 0.1%
d. 0.01%

95. In the nuclear reaction
$$_1H^2 + _1H^2 \rightarrow _2He^3 + _0n^1$$
if the mass of the deuterium atom = 2.014741 a.m.u., mass of $_2He^3$ atom = 3.016977 a.m.u. and mass of neutron =1.008987 a.m.u., then the Q value of the reaction is nearly
a. 0.00352 MeV
b. 3.27 MeV
c. 0.82 MeV
d. 2.45 MeV

96. Assuming that about 20 MeV of energy is released per fusion reaction
$$_1H^2 + _1H^2 \rightarrow + _2He^4 + E + \text{other particles}$$
then the mass of $_1H^2$ consumed per day in a fusion reactor of power 1 megawatt will approximately be
a. 0.001 g
b. 0.1 g
c. 10.0 g
d. 1000 g

97. Assuming that about 200 MeV of energy is released per fission of $_{92}U^{235}$ nuclei, the mass of U^{235} consumed per day in a fission reactor of power 1 megawatt will be approximately
a. 10^{-2} g
b. 1 g
c. 100 g
d. 10,000 g

98. If mass of U^{235} = 235.12142 a.m.u., mass of U^{236} = 236.1205 a.m.u. and mass of neutron = 1.008665 a.m.u., then the energy required to remove one neutron from the nucleus of U^{236} is nearly about
a. 75 MeV
b. 6.5 MeV
c. 1 eV
d. zero

99. The binding energies per nucleon of deuteron ($_1H^2$) and helium ($_2He^4$) atoms are 1.1 MeV and 7 MeV. If two deuteron atoms react to form a single helium atom, then the energy released is
a. 13.9 MeV
b. 26.9 MeV
c. 23.9 MeV
d. 19.2 MeV

100. In the fusion reaction $^2_1He + ^2_1H \rightarrow ^3_2He + ^1_0n$, the masses of deuteron, helium and neutron expressed in a.m.u. are 2.015, 3.017 and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, find the amount of total energy released. (1 a.m.u. = 931.5 meV/c^2)
a. $\approx 6.02 \times 10^{13}$ J
b. $\approx 5.6 \times 10^{13}$ J
c. $\approx 9.0 \times 10^{13}$ J
d. $\approx 0.9 \times 10^{13}$ J

101. The half-life of radium is 1500 years. In how many years will 1 g of pure radium be reduced to one centigram?
a. 3.927×10^2 years
b. 9.972×10^2 years
c. 99.927×10^2 years
d. 0.927×10^2 years

102. A proton and a neutron are both shot at 100 ms^{-1} towards a ${}^{12}_{6}C$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?
- **a.** The proton.
- **b.** The neutron.
- **c.** Both particles are about equally likely to be absorbed.
- **d.** Neither particle will be absorbed.

103. A container is filled with a radioactive substance for which the half-life is 2 days. A week later, when the container is opened, it contains 5 g of the substance. Approximately how many grams of the substances were initially placed in the container?
- **a.** 40
- **b.** 60
- **c.** 80
- **d.** 100

104. The half-life of ${}^{131}I$ is 8 days. Given a sample of ${}^{131}I$ at time $t = 0$, we can assert that
- **a.** no nucleus will decay before $t = 4$ days
- **b.** no nucleus will decay before $t = 8$ days
- **c.** all nuclei will decay before $t = 16$ days
- **d.** a given nucleus may decay at any time after $t = 0$.

105. When an atom undergoes β^+ decay,
- **a.** a neutron 'changes into' a proton
- **b.** a proton 'changes into' a neutron
- **c.** a neutron 'changes into' an antiproton
- **d.** a proton 'changes into' an antineutron

106. Calculate the binding energy of a deuteron atom, which consists of a proton and a neutron, given that the atomic mass of the deuteron is 2.014102 u.
- **a.** 0.002388 MeV
- **b.** 2.014102 MeV
- **c.** 2.16490 MeV
- **d.** 2.224 MeV

107. The compound unstable nucleus ${}^{236}_{92}U$ often decays in accordance with the following reaction

$${}^{236}_{92}U \to {}^{140}_{54}Xe + {}^{94}_{38}Sr + \text{other particles}$$

In the nuclear reaction presented above, the 'other particle' might be
- **a.** an alpha particle, which consists of two protons and two neutrons
- **b.** two protons
- **c.** one proton and one neutron
- **d.** two neutrons

108. Why is a ${}^{4}_{2}He$ nucleus more stable than a ${}^{4}_{3}Li$ nucleus?
- **a.** The strong nuclear force is larger when the neutron to proton ratio is higher.
- **b.** The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.
- **c.** Forces other than the strong nuclear force make the lithium nucleus less stable.
- **d.** None of the above.

109. What is the power output of ${}_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy? Avogadro's number = 6.02×10^{26} per kilomole.
- **a.** 45 megawatt
- **b.** 58.46 megawatt
- **c.** 72 megawatt
- **d.** 92 megawatt

110. Consider the following reaction

$${}^1H_2 + {}^1H_2 \to {}_1He^4 + Q$$

If $m({}_1H^2) = 2.0141$ u: $m({}_2He^4) = 4.0024$ u, the energy released (in MeV) in this fusion reaction is
- **a.** 12
- **b.** 6
- **c.** 24
- **d.** 48

111. A radioactive nuclide is produced at the constant rate of n per second (say, by bombarding a target with neutrons). The expected number N of nuclei in existence t s after the number is N_0 is given by
- **a.** $N = N_0 e^{-\lambda t}$
- **b.** $N = \frac{n}{\lambda} + N_0 e^{-\lambda t}$
- **c.** $N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right)e^{-\lambda t}$
- **d.** $N = \frac{n}{\lambda} + \left(N_0 + \frac{n}{\lambda}\right)e^{-\lambda t}$

112. A radioactive sample undergoes decay as per the following graph. At time $t = 0$, the number of undecayed nuclei is N_0. Calculate the number of nuclei left after 1 h.

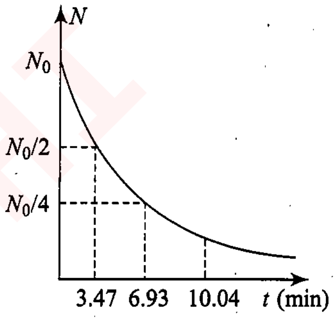


Fig. 5.27

- **a.** N_0/e^8
- **b.** N_0/e^{10}
- **c.** N_0/e^{12}
- **d.** N_0/e^{14}

113. Binding energy per nucleon for C^{12} is 7.68 MeV and for C^{13} is 7.74 MeV. The energy required to remove a neutron from C^{13} is
- **a.** 5.49 MeV
- **b.** 8.46 MeV
- **c.** 9.45 MeV
- **d.** 15.49 MeV

114. A radionuclide A_1 with decay constant λ_1 transform into a radionuclide A_2 with decay constant λ_2. Assuming that at the initial moment the preparation contained only the radionuclide A_1, then the time interval after which the activity of the radionuclide A_2 reaches its maximum value is
- **a.** $\frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$
- **b.** $\frac{\ln(\lambda_1/\lambda_2)}{\lambda_2 - \lambda_1}$
- **c.** $\ln(\lambda_2 - \lambda_1)$
- **d.** none of these

115. A radio isotope 'X' has a half-life of 10 s. Find the number of nuclei in the sample (if initially there are 1000 isotopes which are falling from rest from a height of 3000 m) when it is at a height of 1000 m from the reference plane.
- **a.** 50
- **b.** 250
- **c.** 29
- **d.** 100

116. In the nuclear reaction given by ${}_2He^4 + {}_7N^{14} \to {}_1H^1 + X$, the nucleus X is
- **a.** nitrogen of mass 16
- **b.** nitrogen of mass 17
- **c.** oxygen of mass 16
- **d.** oxygen of mass 17

117. A free nucleus of mass 24 a.m.u. emits a gamma photon (when initially at rest). The energy of the photon is 7 MeV. The recoil energy of the nucleus in keV is
  a. 2.2 **b.** 1.1
  c. 3.1 **d.** 22

118. There are n number of radioactive nuclei in a sample that undergoes beta decay. If from the sample, n' number of β-particles are emitted every 2 s, then half-life of nuclei is
  a. $n'/2$ **b.** $0.693 \times (2n/n')$
  c. $0.693 \ln(2n/n')$ **d.** $0.693 \times n/n'$

119. The luminous dials of watches are usually made by mixing a zinc sulphide phosphor with an α-particle emitter. The mass of radium (mass number 226, half-life 1620 years) that is needed to produce an average of 10 α-particles per second for this purpose is
  a. 2.77 mg **b.** 2.77 g
  c. 2.77×10^{-23} g **d.** 2.77×10^{-13} kg

120. The following deuterium reactions and corresponding reaction energies are found to occur.
$^{14}N(d, p)\,^{15}N$, $Q = 8.53$ MeV
$^{15}N(d, \alpha)\,^{13}C$, $Q = 7.58$ MeV
$^{13}C(d, \alpha)\,^{11}B$, $Q = 5.16$ MeV
The rotation ^{14}N (d, p) ^{15}N represents the reaction
$^{14}N + d \rightarrow \,^{15}N + p$
$^{4}_{2}He = 4.0026$ a.m.u., $^{2}_{1}He = 2.014$ a.m.u., $^{1}_{1}H = 1.0078$ a.m.u., n = 1.0087 a.m.u. (1 a.m.u. = 931 MeV)
The Q values of the reaction $^{11}B(\alpha, n)\,^{14}N$ is
  a. 0.5 eV **b.** 0.5 MeV
  c. 0.05 MeV **d.** 0.05 eV

121. A neutron of energy 1 MeV and mass 1.6×10^{-27} kg passes a proton at such a distance that the angular momentum of the neutron relative to the proton approximately equals 10^{-33} J s. The distance of closest approach neglecting the interaction between particles is
  a. 0.44 nm **b.** 0.44 mm
  c. 0.44 Å **d.** 0.44 fm

122. Rank the following nuclei in order from largest to smallest value of the binding energy per nucleon: (i) $^{4}_{2}He$, (ii) $^{52}_{24}Cr$, (iii) $^{152}_{62}Sm$, (iv) $^{100}_{80}Hg$, (v) $^{252}_{92}Cf$.
  a. $E_{(v)} > E_{(iv)} > E_{(iii)} > E_{(ii)} > E_{(i)}$
  b. $E_{(i)} > E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)}$
  c. $E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)} > E_{(i)}$
  d. $E_{(i)} = E_{(ii)} = E_{(iii)} = E_{(iv)} = E_{(v)}$

123. A nucleus with atomic number Z and neutron number N undergoes two decay processes. The result is a nucleus with atomic number $Z - 3$ and neutron number $N - 1$. Which decay processes took place?
  a. Two β^- decays
  b. Two β^+ decays
  c. An α-decay and a β^- decay
  d. An α-decay and a β^+ decay

124. Gold $^{198}_{79}Au$ undergoes β^- decay to an excited state of $^{198}_{80}Hg$. If the excited state decays by emission of a γ-photon with energy 0.412 MeV, the maximum kinetic energy of the electron emitted in the decay is (This maximum occurs when the antineutrino has negligible energy. The recoil energy of the $^{198}_{80}Hg$ nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for $^{198}_{79}Au$ and 197.966752 u for $^{198}_{79}Hg$.)
  a. 0.412 MeV **b.** 1.371 MeV
  c. 0.959 MeV **d.** 1.473 MeV

Multiple Correct Answers Type

1. For a certain radioactive substance, it is observed that after 4 h, only 6.25% of the original sample is left undecayed. If follows that
  a. the half-life of the sample is 1 h
  b. the mean life of the sample is $\dfrac{1}{\ln 2}$ h
  c. the decay constant of the sample is $\ln(2)\ h^{-1}$
  d. after a further 4 h, the amount of the substance left over would by only 0.39% of the original amount

2. Mark out the correct statement(s).
  a. Higher binding energy per nucleon means the nucleus is more stable.
  b. If the binding energy of nucleus were zero, then it would spontaneously break apart.
  c. Binding energy of a nucleus can be negative.
  d. Binding energy of a nucleus is always positive.

3. Mark out the correct statement(s).
  a. In alpha decay, the energy released is shared between alpha particle and daughter nucleus in the form of kinetic energy and share of alpha particle is more than that of the daughter nucleus.
  b. In beta decay, the energy released is in the form of kinetic energy of beta particles.
  d. In beta minus decay, the energy released is shared between electron and antineutrino.
  d. In gamma decay, the energy released is in the form of energy carried by photons termed as gamma rays.

4. Mark the correct statement(s).
  a. For an exothermic reaction, if Q value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 15.00 MeV.
  b. For an exothermic reaction, if Q value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 12.56 MeV.
  c. For an endothermic reaction, if we give the energy equal to $|Q|$ value of reaction, then the reaction will be carried out.
  d. For an exothermic reaction, the BE per nucleon of products should be greater than the BE per nucleon of reactants.

5. Mark out the correct statement(s).
  a. In both fission and fusion processes, the mass of reactant nuclide is greater than the mass of product nuclide.

b. In fission process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.
c. In fusion process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.
d. In fusion process, BE per nucleon of reactant nuclide is greater than the binding energy per nucleon of product nuclide.

6. During β-decay (beta minus), the emission of antineutrino particle is supported by which of the following statement(s)?
a. Angular momentum conservation holds good in any nuclear reaction.
b. Linear momentum conservation holds good in any nuclear reaction.
c. The KE of emitted β-particle is varying continuously to a maximum value.
d. None of the above.

7. Two samples A and B of same radioactive nuclide are prepared. Sample A has twice the initial activity of sample B. For this situation, mark out the correct statement(s).
a. The half-lives of both the samples would be same.
b. The half-lives of the samples are different.
c. After each has passed through 5 half-lives, the ratio of activity of A to B is 2:1.
d. After each has passed through 5 half-lives, ratio of activities of A to B is 64:1.

8. The decay constant of a radioactive substance is 0.173 year^{-1}. Therefore,
a. nearly 63% of the radioactive substance will decay in (1/0.173) year
b. half-life of the radioactive substance is (1/0.173) year
c. one-fourth of the radioactive substance will be left after 8 years
d. all of the above

9. A nuclide A undergoes α-decay and another nuclide B undergoes β- decay. Then,
a. all the α-particles emitted by A will have almost the same speed
b. the α-particles emitted by B will have widely different speeds
c. all the β-particles emitted by B will have almost the same speed
d. the β-particles emitted by B may have widely different speeds

10. If A, Z, and N denote the mass number, the atomic number, and the neutron number for a given nucleus, we can say that
a. $N = Z + A$
b. isobars have the same A but different Z and N
c. isotopes have the same Z but different N and A
d. isotopes have the same N but different A and Z

11. It has been found that nuclides with 2, 8, 20, 50, 82, and 126 protons or neutrons are exceptionally stable. These numbers are referred to as the magic numbers and their existence has led us to
a. the idea of periodicity in nuclear properties similar to the periodicity of chemical elements in periodic table
b. the so-called 'liquid drop model of the nucleus'
c. the so-called 'shell model of the nucleus'
d. have a convenient explanation of 'nuclear fission'

12. The phenomenon of nuclear fission can be carried out both in a controlled and in an uncontrolled way. Out of the following the correct statements vis-à-vis these phenomena are:
a. The fission energy released per reaction is much more than conventional nuclear reactions and one of the products of the reaction is that very particle which initiates the reaction.
b. It is the 'surface to volume' ratio of the sample of nuclear fuel used which determines whether or not the reaction would sustain itself as a 'chain reaction'.
c. The 'control rods' in a nuclear reactor must be made of a material that absorbs neutrons effectively.
d. The energy released per fission as well as energy released per unit mass of the fuel in nuclear fission are both greater than the corresponding quantities for nuclear fusion.

13. Choose the correct statements from the following:
a. Like other light nuclei, the $_2He^4$ nuclei also have a low value of the binding energy per nucleon.
b. The binding energy per nucleon decreases for nuclei with small as well as large atomic number.
c. The energy required to remove one neutron from $_3Li^7$ to transform it into the isotope $_3Li^6$ is 5.6 MeV, which is the same as the binding energy per nucleon of $_3Li^6$.
d. When two deuterium nuclei fuse together, they give rise to a tritium nucleus accompanied by a release of energy.

14. It is observed that only 0.39% of the original radioactive sample remains undecayed after eight hours. Hence
a. the half-life of that substance is 1 h
b. the mean-life of the substance is [1/(log 2)] h
c. decay constant of the substance is (log 2) h^{-1}
d. if the number of radioactive nuclei of this substance at a given instant is 10, then the number left after 30 min would be 7.5

15. In a nuclear reactor
a. the chain reaction is kept under control by rods of cadmium, which reduces the rate
b. the thick concrete shield is used to slow down the speed of fast neutrons
c. heavy water (or graphite) moderate the activity of the reactor
d. out of U^{238} and U^{235} natural uranium has less than 1% of U^{235}

16. A radioactive sample has initial concentration N_0 of nuclei. Then,
a. the number of undecayed nuclei present in the sample decays exponentially with time
b. the activity (R) of the sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that time
c. the number of decayed nuclei grows exponentially with time
d. the number of decayed nuclei grows linearly with time

17. An O^{16} nucleus is spherical and has a radius R and a volume $V = \frac{4}{3}\pi R^3$. According to the empirical observations, the volume of the $_{54}X^{128}$ nucleus assumed to be spherical is V' and radius is R'. Then

a. $V' = 8V$
b. $V' = 2V$
c. $R' = 2R$
d. $R' = 8R$

Assertion-Reasoning Type

Solutions on page 5.79

1. **Statement I:** Heavy nuclides tend to have more number of neutrons than protons.
Statement II: In heavy nuclei, as there is coloumbic repulsion between protons, so excess of neutrons are preferable.
2. **Statement 1:** $_zX^4$ undergoes 2 α-decays, 2 β-decays (negative β) and 2 γ-decays. As a result, the daughter product is $_{z-2}Y^{A-B}$.
Statement II: In α-decay, the mass number decreases by 4 unit and atomic number decreases by 2 unit. In β-decay (negative β), the mass number remains unchanged and atomic number increases by 1 unit. In γ-decay, mass number and atomic number remain unchanged.
3. **Statement I:** The nucleus $^A_Z X$ is having atomic mass as well as its mass number as A.
Statement II: Mass number of an element is an integer that specifies an isotope and has no units, while atomic mass is generally not an integer.
4. **Statement I:** Light nuclei are most stable if $N = Z$, while heavy nuclei are more stable if $N > Z$. [$N \rightarrow$ number of neutrons, $Z \rightarrow$ number of protons]
Statement II: As the number of protons increases in a nucleus, the Coulomb's repulsive force increases, which tends to break the nucleus apart. So, to keep the nucleus stable, more number of neutrons are needed which are neutral in nature.
5. **Statement I:** In alpha decay of different radioactive nuclides, the energy of alpha particles has been compared. It is found that as the energy of alpha particle increases the half-life of the decay goes on decreasing.
Statement II: More is the energy in any decay process, more is the probability of decaying the nuclide which leads to faster rate of decay.
6. **Statement I:** To determine the age of certain very old organic samples, dating of the sample with radioactive isotopes having larger half-life is a better choice than with radioactive isotopes having smaller half-lives.
Statement II: The activity of a radioactive sample having smaller half-life is negligibly small after a very long time and hence makes it next to impossible to get detected.
7. **Statement I:** The amount of energy required to remove an average nucleon from different nuclei having different mass numbers is approximately the same, while to remove an average electron from atoms having different mass numbers widely varying amounts of energies are required.
Statement II: Nucleons in a nucleus are bounded by short-range nuclear force while electrons in an atom are bounded by long-range Coulomb's forces.
8. **Statement I:** The fission of a heavy nucleus is always accompanied with the neutrons along with two product nuclei.
Statement II: For a lighter stable nuclide, the $\frac{N}{Z}$ ratio has to be slightly greater than 1.

Comprehension Type

Solutions on page 5.79

For Problems 1–3

Nuclei of a radioactive element X are being produced at a constant rate K and this element decays to a stable nucleus Y with a decay constant λ and half-life $T_{1/2}$. At time $t = 0$, there are N_0 nuclei of the element X.

1. The number N_X of nuclei of X at time $t = T_{1/2}$ is

a. $\frac{K + \lambda N_0}{2\lambda}$
b. $(2\lambda N_0 - K)\frac{1}{\lambda}$
c. $\left[\lambda N_0 + \frac{K}{2}\right]\frac{1}{\lambda}$
d. Data insufficient

2. The number N_Y of nuclei of Y at time t is

a. $Kt - \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} + \frac{K - \lambda N_0}{\lambda}$
b. $Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$
c. $Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t}$
d. $Kt - \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t}$

3. The number N_Y of nuclei of Y at $t = T_{1/2}$ is

a. $K\frac{\ln 2}{\lambda} + \frac{3}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$
b. $K\frac{\ln 2}{\lambda} + \frac{1}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$
c. $K\frac{\ln 2}{\lambda} - \frac{1}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$
d. $K\frac{\ln 2}{\lambda} - 2\left(\frac{K - \lambda N_0}{\lambda}\right)$

For Problems 4–6

A radionuclide with decay constant λ is being produced in a nuclear reactor at a rate $q_0 t$ per second, where q_0 is a positive constant and t is the time. During each decay, E_0 energy is released. The production of radionuclide starts at time $t = 0$.

4. Which differential equation correctly represents the above process?

a. $\dfrac{dN}{dt} + \lambda N = q_0 t$ b. $\dfrac{dN}{dt} - \lambda N = q_0 t$

c. $\dfrac{dN}{dt} + q_0 t = \lambda N$ d. $\dfrac{dN}{dt} + q_0 t = -\lambda N$

5. Instantaneous power developed at time 't' due to the decay of the radionuclide is

a. $\left(q_0 t - \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda} e^{-\lambda t}\right) E_0$ b. $\left(q_0 t + \dfrac{q_0}{\lambda} - \dfrac{q_0}{\lambda} e^{-\lambda t}\right) E_0$

c. $\left(q_0 t + \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda} e^{-\lambda t}\right) E_0$ d. $\left(q_0 t - \dfrac{q_0}{\lambda} - \dfrac{q_0}{\lambda} e^{-\lambda t}\right) E_0$

6. Average power developed in time 't' due to the decay of the radionuclide is

a. $\left(\dfrac{q_0 t}{2} - \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda^2 t} - \dfrac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

b. $\left(\dfrac{q_0 t}{2} + \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda^2 t} - \dfrac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

c. $\left(\dfrac{q_0 t}{2} - \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda^2 t} + \dfrac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

d. $\left(\dfrac{q_0 t}{2} + \dfrac{q_0}{\lambda} + \dfrac{q_0}{\lambda^2 t} + \dfrac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

For Problems 7–9

Various rules of thumb have been proposed by the scientific community to explain the mode of radioactive decay by various radioisotopes. One of the major rules is called the *n*/*p* ratio. If all the known isotopes of the elements are plotted on a graph of number of neutrons (n) versus number of protons (p), it is observed that all isotopes lying outside of a 'stable' *n*/*p* ratio region are radioactive as shown in Fig. 5.28.

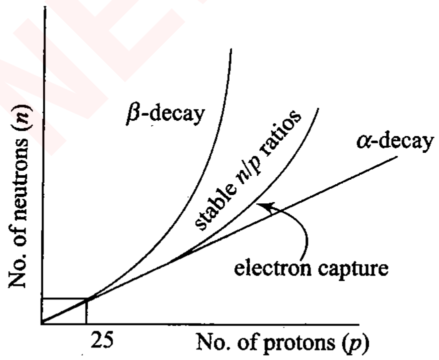


Fig. 5.28

The graph exhibits straight line behaviour with unit slope up to $p = 25$. Above $p = 25$, those isotopes with an *n*/*p* ratio lying below the stable region usually undergo electron capture while those with *n*/*p* ratios lying above the stable region usually undergo beta decay. Very heavy isotopes ($p > 83$) are unstable because of their relatively large nuclei and they undergo alpha decay. Gamma ray emission does not involve the release of a particle. It represents a change in an atom from a higher energy level to a lower energy level.

7. How would the radioisotope of magnesium with atomic mass 27 undergo radioactive decay?

a. electron capture b. alpha decay

c. beta decay d. gamma ray emission

8. Th-230 undergoes a series of radioactive decay processes resulting in Bi-214 being the final product. What was the sequence of the processes that occurred?

a. $\alpha, \alpha, \alpha, g, \beta$ b. $\alpha, \alpha, \alpha, \alpha, \beta$

c. $\alpha, \alpha, \beta, \beta$ d. $\alpha, \beta, \beta, \beta, \gamma$

9. Which of the following represents the relative penetrating power of the three types of radioactive emission in decreasing order?

a. $\beta > \alpha > \gamma$ b. $\beta > \gamma > \alpha$

c. $\gamma > \alpha > \beta$ d. $\gamma > \beta > \alpha$

For Problems 10–12

The radionuclide ^{56}Mn is being produced in a cyclotron at a constant rate P by bombarding a manganese target with deuterons. ^{56}Mn has a half-life of 2.5 h and the target contains large number of only the stable manganese isotopes ^{56}Mn. The reaction that produces ^{56}Mn is

$$^{56}\text{Mn} + \text{d} \rightarrow {}^{56}\text{Mn} + \text{p}$$

After being bombarded for a long time, the activity of ^{56}Mn becomes constant, equal to 13.86×10^{10} s^{-1}. (Use ln 2 = 0.693; Avagardo number = 6×10^2; atomic weight of ^{56}Mn = 56 g mol^{-1}.)

10. At what constant rate P, ^{56}Mn nuclei are being produced in the cyclotron during the bombardment?

a. 2×10^{11} nuclei s^{-1} b. 13.86×10^{10} nuclei s^{-1}

c. 9.6×10^{10} nuclei s^{-1} d. 6.93×10^{10} nuclei s^{-1}

11. After the activity of ^{56}Mn becomes constant, number of ^{56}Mn nuclei present in the target is equal to

a. 5×10^{11} b. 20×10^{11}

c. 1.2×10^{14} d. 1.8×10^{15}

12. After a long time bombardment, number of ^{56}Mn nuclei present in the target depends upon

(i) the number of ^{56}Mn nuclei present at the start of the process

(ii) half-life of the ^{56}Mn

(iii) the constant rate of production P

a. All (i), (ii) and (iii) are correct.

b. Only (i) and (ii) are correct.

c. Only (ii) and (iii) are correct.

d. Only (i) and (iii) are correct.

For Problems 13–15

Many unstable nuclei can decay spontaneously to a nucleus of lower mass but different combination of nucleons. The process of spontaneous emission of radiation is called radioactivity. Three types of radiations are emitted by radioactive substance.

Radioactive decay is a statistical process. Radioactivity is independent of all external conditions.

The number of decays per unit time or decay rate is called activity. Activity exponentially decreases with time. Mean lifetime is always greater than half-life time.

13. Choose the correct statement about radioactivity:
a. Radioactivity is a statistical process.
b. Radioactivity is independent of high temperature and high pressure.
c. When a nucleus undergoes α- or β-decay, its atomic number changes.
d. All of these.

14. If T_H is the half-life and T_M is the mean life. Which of the following statement is correct.
a. $T_M > T_H$
b. $T_M < T_H$.
c. Both are directly proportional to square of the decay constant.
d. $T_M \propto \lambda_0$.

15. 'n' number of α-particles per second are being emitted by B atoms of a radioactive element. The half-life of element will be

a. $\frac{n}{N}$ s b. $\frac{N}{n}$ s
c. $\frac{0.693N}{n}$ s d. $\frac{0.693n}{N}$ s

For Problems 16–18

All nuclei consist of two types of particles—protons and neutrons. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron–proton ratio or mass defect or binding energy per nucleus or packing fraction. Shape of nucleus is calculated by quadrupole moment. Spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centered at the nucleus. Magnetic moment of the nucleus is measured in terms of the nuclear magnetons.

16. The correct statements about nuclear force is/are
a. charge independent b. short-range force
c. non-conservative force d. spin-dependent force

17. Binding energy per nucleon is maximum.
a. for lighter order element (low mass number)
b. for heavier order elements (high mass number)
c. for middle order elements
d. equal for all order elements

18. Volume (V) of the nucleus is related to mass number (A) as
a. $V \propto A^2$ b. $V \propto A^{1/3}$
c. $V \propto A^{2/3}$ d. $V \propto A$

For Problems 19–22

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass 'contributes' to its total energy, in accordance with Einstein's famous equation, $E = mc^2$.

In this equation, E denotes the energy a particle carries because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles.

Consider a neutron at rest, and well separated from other particles. It decays into a proton, an electron, and an undetected third particle:

$$\text{Neutron} \rightarrow \text{proton} + \text{electron} + ???$$

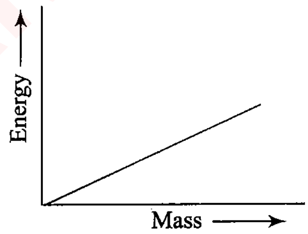
The table below summarizes some data from a single neutron decay. An MeV (mega electron volt) is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	Mass × c^2 (MeV)	Kinetic energy (MeV)
Neutron	940.97	0.00
Proton	939.67	0.01
Electron	0.51	0.39

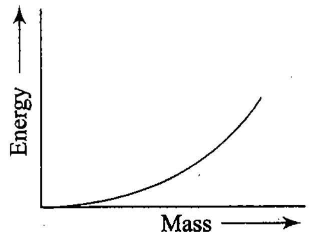
19. Assuming the table contains no major errors, what can we conclude about the (mass × c^2) of the undetected third particle?
a. It is 0.79 MeV.
b. It is 0.39 MeV.
c. It is less than or equal to 0.79 MeV; but we cannot be more precise.
d. It is less than or equal to 0.39 MeV; but we cannot be more precise.

20. From the given table, which properties of the undetected third particle can we calculate?
a. Total energy, but not kinetic energy.
b. Kinetic energy, but not total energy.
c. Both total energy and kinetic energy.
d. Neither total energy nor kinetic energy.

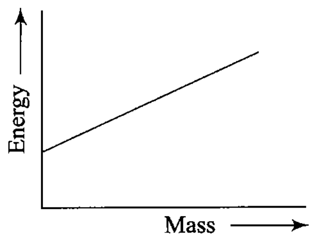
21. Consider an ensemble of particles, all of which have the same positive kinetic energy but different masses. For this ensemble, which graph best represents the relationship between the particle's mass and its total energy?



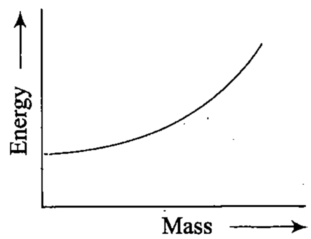
a.



b.



c.



d.

22. Could this reaction occur?
Proton → neutron + other particles
a. Yes, if the other particles have much more kinetic energy than mass energy.
b. Yes, but only if the proton has potential energy (due to interactions with other particles).
c. No, because a neutron is more massive than a proton.
d. No, because a proton is positively charged while a neutron is electrically neutral.

For Problems 23–26

The compound unstable nucleus $^{236}_{92}U$ often decays in accordance with the following reaction:

$$^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + \text{other particles}$$

During the reaction, the uranium nucleus 'fissions' (splits) into the two smaller nuclei. The reaction is energetically favorable because the small nuclei have higher nuclear binding energy per nucleon (although the lighter nuclei have lower total nuclear binding energies, because they contain fewer nucleons).

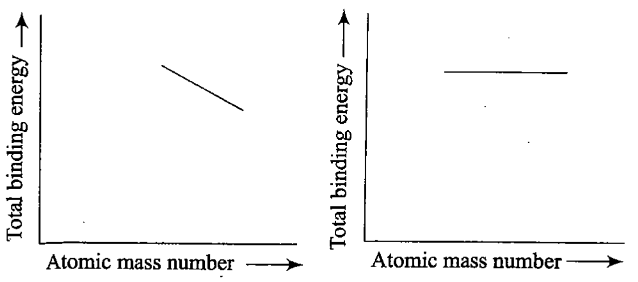
Inside a nucleus, the nucleons (protons and neutrons) attract each other with a 'strong nuclear' force. All nucleons exert approximately the same strong nuclear force on each other. This force holds the nucleus together. Importantly, the strong nuclear force becomes important only when the protons and neutrons are very close together at intranuclear distances.

23. In the nuclear reaction presented above, the 'other particles' might be
- **a.** an alpha particle, which consists of two protons and neutrons
- **b.** two protons
- **c.** one proton and one neutron
- **d.** two neutrons

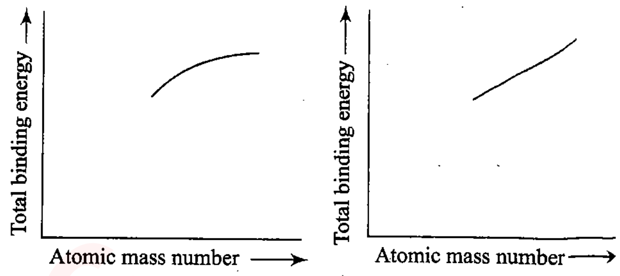
24. Why is a ^4_2He nucleus more stable than a ^4_3Li nucleus?
- **a.** The strong nuclear force is larger when the neutron-to-proton ratio is higher.
- **b.** The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.
- **c.** Forces other than the strong nuclear force make the lithium nucleus less stable.
- **d.** None of the above.

25. A proton and a neutron are both shot at 100 m s^{-1} toward a $^{12}_6C$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?
- **a.** The proton.
- **b.** The neutron.
- **c.** Both particles are about equally likely to be absorbed.
- **d.** Neither particle will be absorbed.

26. Which of the following graphs might represent the relationship between atomic number (i.e., 'atomic weight') and the total binding energy of the nucleus, for nuclei heavier than $^{94}_{38}Sr$?



a. **b.**



c. **d.**

For Problems 27–29

A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in 'head-on' to a particular lead nucleus and stops 6.50×10^{-14} m away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of alpha particle is 6.64×10^{-27} kg.

27. Calculate the electrostatic potential energy at the instant when the alpha particle stops?
- **a.** 36.3 MeV **b.** 45.0 MeV
- **c.** 3.63 MeV **d.** 40.0 MeV

28. What initial kinetic energy (in joule and in MeV) did the alpha particle have?
- **a.** 36.3 **b.** 0.36
- **c.** 3.63 **d.** 2.63

29. What was the initial speed of the alpha particle?
- **a.** $132. \times 10^2$ m s^{-1} **b.** $1.32. \times 10^7$ m s^{-1}
- **c.** $13.2. \times 10^2$ m s^{-1} **d.** $0.13. \times 10^7$ m s^{-1}

For Problems 30–32

A nucleus, kept at rest in free space, break up into two smaller nuclei of masses m and $2m$. Total energy generated in this fission is E. The bigger part is radioactive, emits five gamma ray photons in the direction opposite to its velocity, and finally comes to rest. Now, answer the following questions: (given: $h = 6.6 \times 10^{-34}$ J s, $m = 1.00 \times 10^{-26}$ kg, $E = 3.63 \times 10^{-8}$ mc^2, $c = 3 \times 10^8$ m s^{-1})

30. Fractional loss of mass in the fission is
- **a.** 1.21×10^{-8} **b.** 2.56×10^{-8}
- **c.** 1.73×10^{-8} **d.** 3.52×10^{-8}

31. Velocity of smaller daughter nucleus is
- **a.** 5.6×10^4 m s^{-1} **b.** 6.6×10^4 m s^{-1}
- **c.** 7.6×10^4 m s^{-1} **d.** 8.6×10^4 m s^{-1}

32. The wavelength of the gamma ray is
- **a.** 0.02 Å **b.** 0.03 Å
- **c.** 0.04 Å **d.** 0.05 Å

For Problems 33–35

The results of activity measurements on a radioactive sample are given in the table below.

Time (h)	Decays (s^{-1})
0	20000
0.5	14800
1.0	11000
1.5	8130
2.0	6020

Time (h)	Decays (s^{-1})
2.5	4460
3.0	3300
4.0	1810
5.0	1000
6.0	550
7.0	300

33. The half-life of the radioactive nuclei is nearly ($\ln 2 = 0.693$, $\ln 3 = 1.0986$)
a. 2.5 h **b.** 7 h
c. 5 h **d.** 1.2 h

34. The number of radioactive nuclei that were present in the sample at $t = 0$ is
a. 2×10^4 **b.** 2×10^8
c. 1.25×10^8 **d.** 1.25×10^4

35. The number of radioactive nuclei that were present after 7 h is
a. 2×10^4 **b.** 1.25×10^8
c. 1.875×10^6 **d.** 1.25×10^4

Matching Column Type

Solutions on page 5.82

1. Four physical quantities are given in Column I and their order of values in Column II. Match appropriately.

Column I	Column II
a. Thermal energy of air molecules at room temperature	**p.** 0.02 eV
b. Binding energy of heavy nuclei per nucleon	**q.** 2 eV
c. X-ray photon energy	**r.** 10 KeV
d. Photon energy of visible light	**s.** 7 MeV

2. Match the Column I of properties with Column II of reactions

Column I	Column II
a. Mass of products formed is less than the original mass of the system in	**p.** α-decay
b. Binding energy per nucleon increase in	**q.** β-decay
c. Mass number is conserved in	**r.** Nuclear fission
d. Charge number is conserved in	**s.** Nuclear fusion

3. In Column I consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in Column I with the result they produce in Column II:

Column I	Column II
a. Spontaneous radioactive decay of a uranium nucleus initially at rest as given by reaction ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + \cdots$	**p.** Number of protons is increased
b. Fusion reaction of two hydrogen nuclei as given by reaction ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + \cdots$	**q.** Momentum is conserved
c. Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}K + 3{}^{1}_{0}n + \cdots$	**r.** Mass is converted to energy or vice versa
d. β-decay (negative beta decay)	**s.** Charge is conserved

4.

Column I	Column II
a. Photoelectric effect	**p.** Photon
b. Wave	**q.** Frequency
c. X-rays	**r.** K capture
d. Nucleus	**s.** γ-rays

5.

Column I	Column II
a. Binding energy per nucleon for middle order of element is	**p.** Shell model
b. Nuclear force depends on	**q.** 8.8 MeV
c. For nuclear fission, $\frac{Z^2}{A}$ is	**r.** 2.5 eV
d. Magic numbers 2, 8, 20, 28, 50, 82, 126 are explained by	**s.** Spin of nucleons
	t. Greater than 15

6.

Column I	Column II
a. Stability of nucleus decided by	**p.** –ve
b. Four radioactive substance spontaneously decays because its	**q.** Binding energy per nucleon is minimum
c. For the stable orbit or bound orbit, total energy is	**r.** Neutron–proton ratio
d. Stopping potential	**s.** Packing fraction
	t. Mass defect

7.

Column I	Column II
a. Nuclear fusion	p. Satisfies $E = mc^2$
b. Nuclear fission	q. Generally possible for nuclei with low atomic number
c. β-decay	r. Generally possible for nuclei with higher atomic number and unstable
d. Exothermic nuclear reaction	s. Essentially proceeds by weak nuclear forces
	t. Significant momentum conservation

8. In Column I some of the nuclear reactions are given. Match this with the energy involved in these reactions in Column II.

Column I	Column II
a. ${}_1^2H + {}_1^2H \to {}_1^3H + {}_1^1H + E_1$	p. 3.3 MeV
b. ${}_1^3H + {}_1^2H \to {}_2^4He + {}_0^1n + E_2$	q. 18.3 MeV
c. ${}_1^2H + {}_1^2H \to {}_2^3He + {}_0^1n + E_3$	r. 4 MeV
d. ${}_2^3H + {}_1^2H \to {}_2^4He + {}_1^1H + E_4$	s. 17.6 MeV
	t. 200 MeV

Integer Answer Type

Solutions on page 5.8

1. Nuclei A and B convert into a stable nucleus C. Nucleus A is converted into C by emitting 2 α-particles and 3β-particles. Nucleus B is converted into C by emitting one α-particle and 5 β-particles. At time $t = 0$, nuclei of A are $4N_0$ and nuclei of B are N_0. Initially, number of nuclei of C are zero. Half-life of A (into conversion of C) is 1 min and that of B is 2 min. Find the time (in minutes) at which rate of disintegration of A and B are equal.
2. The half-life of a radioactive nuclide is 20 h. It is found that the fraction $(1/x)$ of original activity remains after 40 hours? What is the value of x?
3. A radioactive sample has 8.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the active state after two half-lives (in $\times 10^{18}$).
4. A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of R (in $\times 10^2\,\Omega$) so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?
5. A radioactive sample decays through two different decay processes α decay and β decay. Half-life time for α decay is 3 h and half-life time for β decay is 6 h. What will be the ratio of number of initial radioactive nuclei to the number of radioactive nuclei present after 6 h.
6. ${}_{92}U^{238}$ changes to ${}_{85}At^{210}$ by a series of α and β decays. Find the number of α - decays undergone (an integer).
7. A certain radioactive material can undergo three different types of decay, each with a different decay constant λ, 2λ and 3λ. Then, the effective decay constant λ_{eff} is equal to $n\lambda$. What is the value of n?
8. The radioactivity of a sample is R_1 at a time T_1 and R_2 at a time T_2. If the half-life of the specimen is T, the number of atoms that have disintegrated in the time $(T_2 - T_1)$ is equal to $\frac{n(R_1 - R_2)T}{\ln 4}$. Here n is some integral number. What is the value of n?

Archives

Solutions on page 5.83

Fill in the Blanks Type

1. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration s^{-1} at a certain time. If the half-life of the element is 1 s, the decay rate after 1 s is ________ and after 3 s the decay rate is ________. **(IIT-JEE, 1983)**
2. In the uranium radioactive series, the initial nucleus is ${}_{92}^{238}U$ and the final nucleus is ${}_{92}^{206}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is ________ and the number of β-particles emitted is ________. **(IIT-JEE, 1985)**
3. When boron nucleus $\left({}_5^{10}B\right)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element and has the mass number ________. **(IIT-JEE, 1986)**
4. Atoms having the same but different ________ are called isotopes. **(IIT-JEE, 1986)**
5. The binding energies per nucleon for deuteron $({}_1H^2)$ and helium $({}_2He^4)$ are 1.1 MeV and 7.0 MeV, respectively. The energy released when two deuterons fuse to form a helium nucleus $({}_2He^4)$ is ________. **(IIT-JEE, 1988)**
6. In the nuclear process ${}_6C^{11} \to {}_5B^{11} + \beta^+ + X$, X stands for ________. **(IIT-JEE, 1992)**
7. Consider the following reaction:
$${}_1^2H + {}_1^2H = {}_2^4He + Q$$
[Mass of the deuterium atom = 2.0141 u and mass of helium atom = 4.0024]
This is a nuclear ________ reaction in which the energy Q released is MeV. **(IIT-JEE, 1996)**

True or False Type

1. The order of magnitude of the density of nuclear matter is 10^4 kg m^{-2}. **(IIT-JEE, 1989)**

Multiple Choice Questions with One Correct Answer Type

1. The half-life of radioactive radon is 3.8 days. The time at the end of which 1/20th of the radon sample will remain undecayed is (given $\log_{10} e = 0.4343$) **(IIT-JEE, 1981)**
   a. 3.8 days
   b. 16.5 days
   c. 33 days
   d. 76 days

2. Beta rays emitted by a radioactive material are (IIT-JEE, 1983)
   a. electromagnetic radiations
   b. the electrons orbiting around the nucleus
   c. charged particles emitted by the nucleus
   d. neutral particles
3. The equation $4{}_1^1H \rightarrow {}_2^4He^2 + 2e^- + 26\text{ MeV}$ represents (IIT-JEE, 1983)
   a. β-decay b. γ-decay
   c. fusion d. fission
4. During a negative beta decay, (IIT-JEE, 1987)
   a. an atomic electron is ejected
   b. an electron which is already present within the nucleus is ejected
   c. a neutron in the nucleus decays emitting an electron
   d. a part of the binding energy of the nucleus is converted into an electron
5. During a nuclear fusion reaction, (IIT-JEE, 1987)
   a. a heavy nucleus breaks into two fragments by itself
   b. a light nucleus bombarded by thermal neutrons breaks up
   c. a heavy nucleus bombarded by thermal neutrons breaks up
   d. two light nuclei combine to give a heavier nucleus and possibly other products
6. Four physical quantities are listed in Column I. Their values are listed in Column II in a random order. (IIT-JEE, 1987)

Column I	Column II
p. Thermal energy of air molecules at room temperature	(i) 0.02 eV
q. Binding energy of heavy nuclei per nucleon	(ii) 2 eV
r. X-ray photon energy	(iii) 10 keV
s. Photon energy of visible light	(iv) 7 MeV

   The correct matching of Column I and Column II is given by
   a. $p \rightarrow i, q \rightarrow iv, r \rightarrow iii, s \rightarrow ii$
   b. $p \rightarrow i, q \rightarrow iii, r \rightarrow ii, s \rightarrow iv$
   c. $p \rightarrow ii, q \rightarrow i, r \rightarrow iii, s \rightarrow iv$
   d. $p \rightarrow ii, q \rightarrow iv, r \rightarrow i, s \rightarrow iii$
7. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is (IIT-JEE, 1988)
   a. 6 h b. 12 h
   c. 24 h d. 28 h
8. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are, respectively, given by (IIT-JEE, 1989)
   a. $1/\lambda$ and $(\ln 2)/\lambda$ b. $(\ln 2)\lambda$ and $1/\lambda$
   c. $\lambda(\ln 2)$ and $1/\lambda$ d. $\lambda/(\ln 2)$ and $1/\lambda$
9. A star initially has 10^{40} deuterons. It produces energy via the processes ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^2 + p$ and ${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$.
   If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of (IIT-JEE, 1993)
   a. 10^6 s b. 10^8 s
   c. 10^{12} s d. 10^{16} s
   [The mass of the nuclei are as follows:
   $M(H^2) = 2.014$ a.m.u., $M(n) = 1.008$ a.m.u.,
   $M(p) = 1.007$ a.m.u., $M(He^4) = 4.001$ a.m.u.]
10. Fast neutrons can easily be slowed down by (IIT-JEE, 1994)
   a. the use of lead shielding
   b. passing them through water
   c. elastic collision with heavy nuclei
   d. applying a strong electric field
11. Consider α-particles, β-particles and γ-rays, each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are: (IIT-JEE, 1994)
   a. α, β, γ b. α, γ, β
   c. β, γ, α d. γ, β, α
12. Masses of two isobars ${}_{29}Cu^{64}$ and ${}_{30}Zn^{64}$ are 63.9298 u and 63.9292 u, respectively. It can be concluded from these data that (IIT-JEE, 1997)
   a. both the isobars are stable
   b. Zn^{64} is radioactive, decaying to Cu^{64} through β-decay
   c. Cu^{64} is radioactive, decaying to Zn^{64} through γ-decay
   d. Cu^{64} is radioactive, decaying to Zn^{64} through β-decay
13. The half-life of ${}^{131}I$ is 8 days. Given a sample of ${}^{131}I$ at time $t = 0$, we can assert that (IIT-JEE, 1998)
   a. no nucleus will decay before $t = 4$ days
   b. no nucleus will decay before $t = 8$ days
   c. all nuclei will decay before $t = 16$ days
   d. a given nucleus may decay at any time after $t = 0$
14. In hydrogen spectrum, the wavelength of $H\alpha$ line is 656 nm, whereas in the spectrum of a distant galaxy, $H\alpha$ line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is (IIT-JEE, 1999)
   a. $2 \times 10^8\ m\ s^{-1}$ b. $2 \times 10^7\ m\ s^{-1}$
   c. $2 \times 10^6\ m\ s^{-1}$ d. $2 \times 10^5\ m\ s^{-1}$
15. Order of magnitude of density of uranium nucleus is $[m_p = 1.67 \times 10^{-27}$ kg] (IIT-JEE, 1999)
   a. $10^{20}\ kg\ m^{-3}$ b. $10^{17}\ kg\ m^{-3}$
   c. $10^{14}\ kg\ m^{-3}$ d. $10^{11}\ kg\ m^{-3}$
16. ${}^{22}Ne$ nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is (IIT-JEE, 1999)
   a. nitrogen b. carbon
   c. boron d. oxygen
17. Binding energy per nucleon vs. mass number curve for nuclei is shown in Fig. 5.29. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is (IIT-JEE, 1999)

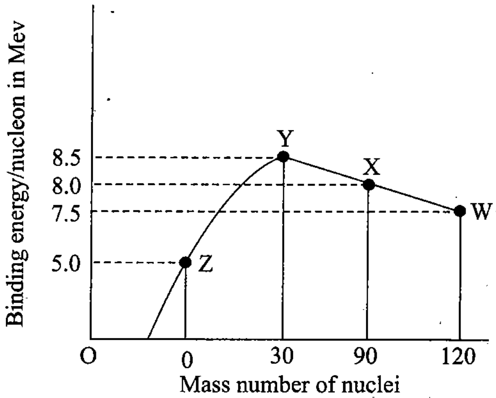


Fig. 5.29

a. Y → 2Z b. W → X + Z
c. W → 2Y d. X → Y + Z

18. The half-life period of a radioactive element X is same as the mean lifetime of another radioactive element Y. Initially, both of them have the same number of atoms. Then, **(IIT-JEE, 1999)**

a. X and Y have the same decay rate initially
b. X and Y decay at the same rate always
c. Y will decay at a faster rate than X
d. X will decay at a faster rate than Y

19. Which of the following is a correct statement? **(IIT-JEE, 1999)**

a. Beta rays are same as cathode rays.
b. Gamma rays are high-energy neutrons.
c. Alpha particles are singly ionized helium atoms.
d. Protons and neutrons have exactly the same mass.

20. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time **(IIT-JEE, 2000)**

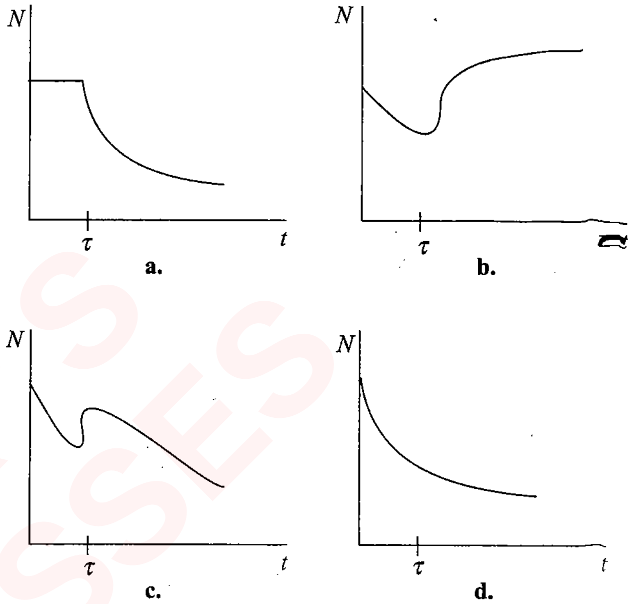
a. $\frac{1}{10\lambda}$ b. $\frac{1}{11\lambda}$

c. $\frac{11}{10\lambda}$ d. $\frac{1}{9\lambda}$

21. The electron emitted in beta radiation originates from **(IIT-JEE, 2001)**

a. inner orbits of atoms
b. free electrons existing in nuclei
c. decay of a neutron in a nucleus
d. photon escaping from the nucleus

22. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean lifetime of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of this plot? **(IIT-JEE, 2001)**



23. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **(IIT-JEE, 2002)**

a. 400 μs b. 6.3 μs
c. 40 μs d. 300 μs

24. Which of the following processes represents a γ-decay? **(IIT-JEE, 2002)**

a. $^{A}X_{Z} + \gamma \rightarrow {}^{A}X_{Z-1} + a + b$
b. $^{A}X_{Z} + {}^{1}n_{0} \rightarrow {}^{A-3}X_{Z-2} + c$
c. $^{A}X_{Z} \rightarrow {}^{A}X_{Z} + f$
d. $^{A}X_{Z} + e_{-1} \rightarrow {}^{A}X_{Z-1} + g$

25. For uranium nucleus, how does its mass vary with volume? **(IIT-JEE, 2003)**

a. $m \propto V$ b. $m \propto 1/V$
c. $m \propto \sqrt{V}$ d. $m \propto V^2$

26. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle. **(IIT-JEE, 2003)**

a. 4.4 MeV b. 5.4 MeV
c. 5.6 MeV d. 6.5 MeV

27. A 280-day-old radioactive substance shows an activity of 6000 dps. 140 days later its activity becomes 3000 dps. What was its initial activity? **(IIT-JEE, 2004)**

a. 20000 dps b. 24000 dps
c. 120000 dps d. 6000 dps

28. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 a.m.u. and mass of oxygen nucleus is 15.9994 a.m.u.] **(IIT-JEE, 2005)**

a. 7.6 MeV b. 56.12 MeV
c. 10.24 MeV d. 23.9 MeV

29. $^{221}_{87}$Ra is a radioactive substance having half-life of 4 days. Find the probability that a nucleus undergoes decay after two half-lives. **(IIT-JEE, 2006)**

a. 1 b. $\frac{1}{2}$

c. $\frac{3}{4}$ d. $\frac{1}{4}$

30. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is **(IIT-JEE, 2007)**

a. $E({}^{236}_{92}\text{U}) > E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(\text{n})$

b. $E({}^{236}_{92}\text{U}) < E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(\text{n})$

c. $E({}^{236}_{92}\text{U}) < E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + 2E(\text{n})$

d. $E({}^{236}_{92}\text{U}) = E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + 2E(\text{n})$

31. A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be **(IIT-JEE, 2008)**

a. 20 years and 5 years, respectively.
b. 20 years and 10 years, respectively.
c. 10 years each
d. 5 years each

Multiple Choice Questions with One or More than One Correct Answer Type

1. From the following equations, pick out the possible nuclear fusion reaction: **(IIT-JEE, 1984)**

a. ${}_6C^{13} + {}_1H^1 \rightarrow {}_6C^{14} + 4.3\text{ MeV}$
b. ${}_6C^{12} + {}_1H^1 \rightarrow {}_7C^{13} + 2\text{ MeV}$
c. ${}_7C^{14} + {}_1H^1 \rightarrow {}_8O^{15} + 7.3\text{ MeV}$
d. ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{104} + {}_{38}Sr^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200\text{ MeV}$

2. Which of the following statement(s) is (are) correct? **(IIT-JEE, 1994)**

a. The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.
b. The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.
c. In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 a.m.u.).
d. In nuclear fission, energy is released by fragmentation of a very heavy nucleus.

3. Let m_p be the mass of proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}\text{Ne}$ nucleus and M_2 the mass of a ${}^{40}_{20}\text{Ca}$ nucleus. Then, **(IIT-JEE, 1998)**

a. $M_2 = 2M_1$ b. $M_2 > 2M_1$
c. $M_2 < 2M_1$ d. $M_1 < 10(m_p + m_p)$

4. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in Fig. 5.30. Use this plot to choose the correct choice(s) given below: **(IIT-JEE, 2008)**

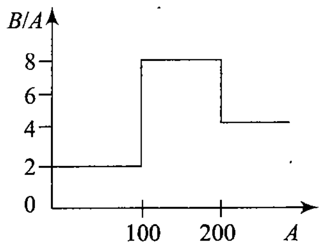


Fig. 5.30

a. Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
b. Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
c. Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into equal fragments.
d. Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into equal fragments.

Comprehension Type

For Problems 1–3

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1$H, known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D–D reaction is ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \text{n} + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1$H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following: Boltzmann constant,

$$k = 8.6 \times 10^{-5}\text{ eV K}^{-1};\ \frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}\text{ eV m}.$$

(IIT-JEE, 2009)

1. In the core of nuclear fusion reactor, the gas becomes plasma because of

a. strong nuclear force acting between the deuterons
b. Coulomb force acting between the deuterons
c. Coulomb force acting between deuteron–electron pairs
d. the high temperature maintained inside the reactor core

2. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving toward each other, each with kinetic energy $1.5kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

a. $1.0 \times 10^9\text{ K} < T < 2.0 \times 10^9\text{ K}$
b. $2.0 \times 10^9\text{ K} < T < 3.0 \times 10^9\text{ K}$
c. $3.0 \times 10^9\text{ K} < T < 4.0 \times 10^9\text{ K}$
d. $4.0 \times 10^9\text{ K} < T < 5.0 \times 10^9\text{ K}$

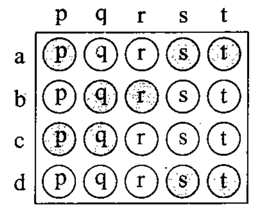
3. Results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is most promising based on Lawson criterion?

a. Deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s
b. Deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s
c. Deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s
d. Deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s

Matching Column Type

Each question in this section contains statements given in two columns, which have to be matched. The statements in Column I are labelled a, b, c, and d, while the statements in Column II are labelled p, q, r, s and t. Any given statement in Column I can have correct matching with one or more statement(s) in Column II.

The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in treh following examples. If the correct matches are a-p, a-s and a-t; b-q and b-r; c-p and c-q; and d-s and d-t; then the correct darkening of bubbles will look like the following.



1. Match the following: **(IIT-JEE, 2006)**

Column I	Column II
a. Nuclear fusion	**p.** Converts some matter into energy
b. Nuclear fission	**q.** Generally possible for nuclei with low atomic number
c. β-decay	**r.** Generally possible for nuclei with higher atomic number
d. Exothermic nuclear reaction	**s.** Essentially proceeds by weak nuclear forces

2. Some laws/processes are given in Column I. Match these with the physical phenomena given in Column II. **(IIT-JEE, 2007)**

Column I	Column II
a. Transition between two atomic energy levels	**p.** Characteristic X-rays
b. Electron emission from a material	**q.** Photoelectric effect
c. Moseley's law	**r.** Hydrogen spectrum
d. Change of photon energy into kinetic energy of electrons	**s.** β-decay

3. Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II. **(IIT-JEE, 2009)**

Column I	Column II
a. The energy of the system is increased.	**p.** System: A capacitor, initially uncharged Process: It is connected to a battery
b. Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	**q.** System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston
c. Internal energy of the system is converted into its mechanical energy.	**r.** System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
d. Mass of the system is decreased.	**s.** System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	t. System: A resistive wire loop Process: The loop is placed in a time-varying magnetic field perpendicular to its plane

Integer Answer Type

1. To determine the half life of a radioactive element, student plots a graph of $\ln|dN(t)/dt|$ versus t. Here $dN(t)/dt$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is ______. **(IIT-JEE, 2010)**

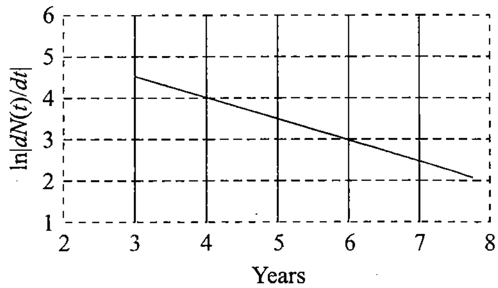


Fig. 5.31

2. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is ______. **(IIT-JEE, 2011)**

ANSWERS AND SOLUTIONS

Subjective Type

1. $\dfrac{dN_A}{dt}=-\lambda_1 N_A,\ \dfrac{dN_B}{dt}=2\lambda_1 N_A-\lambda_2 N_B$

$$N_B=\text{maximum}\Rightarrow\frac{dN_B}{dt}=0$$

$$\Rightarrow\quad 2\lambda_1 N_A=\lambda_2 N_{B_{max}}$$

or $$N_{B_{max}}=\frac{2\lambda_1}{\lambda_2}N_A$$

or $$N_{B_{max}}=\frac{2\lambda_1}{\lambda_2}M_0e^{-\lambda_2 t}=2$$

2. Before mixing the two, the specific activity a_1 is given by

$$a_1=\frac{dN}{m_2dt}=\frac{\lambda N}{m_2}=\frac{\log_e 2}{T}\times\frac{N_A m_2}{A'}\times\frac{1}{m_2}$$

$$=\frac{\log_e 2}{T}\times\frac{N_A}{A'}$$

After mixing, the activity a_2 is given by

$$a_2=\frac{dN}{(m_1+m_2)dt}=\frac{\log_e 2}{T}\frac{N_A m_2}{A'(m_1+m_2)}$$

$\therefore$ Change in activity $=a_1-a_2=\dfrac{(\log_e 2)N_A m_1}{TA'(m_1+m_2)}$

3. We know that the Q value of the reaction,

$$E_0=E_\nu+E_\beta$$

$$E_\nu=E_0-E_\beta=0.155-0.025=0.130\text{ MeV}$$

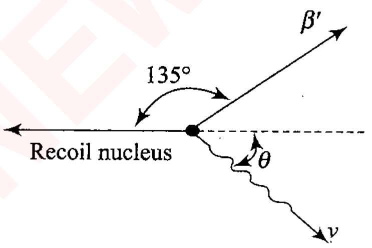


Fig. 5.32

Hence, the momentum of the neutrino

$$p_\nu=E_\nu/c=0.13\text{ MeV}/c$$

Momentum of a β-particle can be obtained from the relation

$$p_\beta=\sqrt{(2M_eE_\beta)}=\sqrt{2\times0.511\times0.025}\text{ MeV}/c$$

$$=0.158\text{ MeV}/c$$

Using conservation of momentum of the system, we have

$$p_\beta\sin45°=p_\nu\sin\theta$$

$$\sin\theta=(p_\beta/p_\nu)\sin45°=0.859$$

$$p_R=p_\beta\cos45°+p_\nu\cos\theta$$

$$=0.158(1/\sqrt{2})+0.13\sqrt{[1-(0.859)^2]}$$

$$=0.179\text{ MeV}/c$$

4. Rate of formation of nuclei at time t is

$$\frac{dN}{dt}=\alpha-\lambda N$$

where N = number of nuclei at that time.

$$N=\frac{\alpha}{\lambda}(1-e^{-\lambda t})$$

Number of nuclei disintegrated in time t,

$$N_d=\alpha t-\frac{\alpha}{\lambda}(1-e^{-\lambda t})$$

Therefore, energy released till time t,

$$E=E_0N_d=E_0\left[\alpha t-\frac{\alpha}{\lambda}(1-e^{-\lambda t})\right]$$

20% of this energy is used for heating water. Hence,

$$0.2E_0\left[\alpha t-\frac{\alpha}{\lambda}(1-e^{-\lambda t})\right]=ms\Delta\theta$$

$$\Rightarrow\quad\Delta\theta=\frac{0.2E_0\left[\alpha t-\frac{\alpha}{\lambda}(1-e^{-\lambda t})\right]}{ms}$$

At $t=T_{1/2}$,

$$\Delta\theta=\frac{0.2E_0\left[2\lambda T_{1/2}-\frac{2\lambda}{\lambda}\left(1-\frac{1}{2}\right)\right]}{ms}$$

$$=\frac{0.2E_0[0.386]}{ms}=772\text{ K}$$

5. $\dfrac{dN_B}{dt}=P-\lambda_2N_B$

$$\int_0^{N_2}\frac{dN_B}{P-\lambda_2N_B}=\int_0^t dt$$

$$\ln\left(\frac{P-\lambda_2N_B}{P}\right)=-\lambda_2t$$

$$N_B=P(1-e^{-\lambda_2t})$$

The number of nuclei of A after time t is $N_A=N_0e^{-\lambda t}$. Thus,

$$\frac{dN_C}{dt}=\lambda_1N_A+\lambda_2N_B$$

$$\frac{dN_C}{dt}=+\lambda_1N_0e^{-\lambda t}+P(1-e^{-\lambda_2t})$$

$$N_C=N_0(1-e^{-\lambda_1t})+P\left(t+\frac{e^{-\lambda_2t}-1}{\lambda_2}\right)$$

6. Charge on α-particle, $q_\alpha=4\times1.6\times10^{-19}$ C and 1 u = 931 MeV/c^2. Force acting on α-particle,

$$\vec{F}_B = q(\vec{v} \times \vec{B}) + q_\alpha E_0 \hat{i}$$

Hence, the particle will move in a circular path in yz plane. In the same time, it will move along x-direction. Hence, resulting path will be a helical spiral with increasing pitch. Thus, velocity of the circulating α-particle after time t.

$$\vec{v}_0 = v_0 \cos\theta \hat{j} - v_0 \sin\theta \hat{k}$$

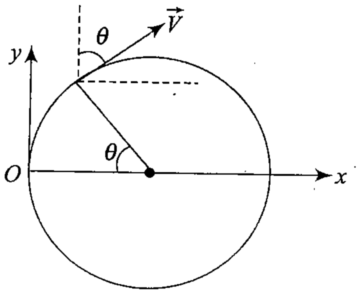


Fig. 5.33

Due to electric field present, the velocity of the α-particle will increase along x-axis. Hence, velocity of α-particle along x-axis,

$$\vec{v}_x = \left(\frac{q_0 E_0}{m_0}\right) t \hat{i}$$

Hence, velocity of α-particle,

$$\vec{v} = \left(\frac{q_0 E_0 t}{m_0}\right)\hat{i} + v_0 \cos\theta \hat{j} - v_0 \sin\theta \hat{k} \quad \text{(i)}$$

$$\theta = \omega t = \frac{2\pi t}{T}$$

where $\quad T = \dfrac{2\pi m}{q_\alpha B}$

Hence,

$$\vec{v} = \left(\frac{q_0 B_0}{m_0}\right) t\hat{i} + v_0 \cos\left(\frac{2\pi t}{T}\right)\hat{j} - v_0 \sin\left(\frac{2\pi}{T}\right)\hat{k} \quad \text{(ii)}$$

Given, $|\vec{v}| = 2v_0$, at $t = \sqrt{3} \times 10^7 \dfrac{m_0}{q_0 E_0}$ s

$$4v_0^2 = (\sqrt{3} \times 10^7)^2 + v_0^2$$

$\therefore \quad v_0 = 10^7 \text{ m s}^{-1}$

When an α-particle is emitted with velocity v_0 from a stationary nucleus X, decay product (nucleus Y) recoils. Hence,

$$m_Y V = m_\alpha v_0$$

$$\Rightarrow \quad V = \frac{m_\alpha v}{m_Y} = 1.811 \times 10^5 \text{ m s}^{-1}$$

7. Let A_1, A_2 and A_3 be activities of $_{92}U^{234}$, $_{92}U^{235}$ and $_{92}U^{238}$, respectively. Total activity,

$$A = A_1 + A_2 + A_3$$

Share of $_{92}U^{234}$ is

$$\frac{A_1}{A} = \frac{\lambda_1 N_1}{\lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3}$$

Let m be the total mass of natural uranium. Then,

$$m_1 = \frac{0.006}{100} m, \; m_2 = \frac{0.71}{100} m, \; m_3 = \frac{99.28}{100} m$$

Now,

$$N_1 = \frac{m_1}{M_1}, \; N_2 = \frac{m_2}{M_2}, \; N_3 = \frac{m_3}{M_3}$$

where M_1, M_2 and M_3 are atomic weights.

$$\therefore \quad \frac{A_1}{A} = \frac{\left(\frac{m_1}{M_1}\right)\frac{1}{T_1}}{\frac{m_1}{M_1 T_1} + \frac{m_2}{M_2 T_2} + \frac{m_3}{M_3 T_3}} = 0.99 \approx 9.9\%$$

$$\frac{A_2}{A} = 0.039 = 3.9\%$$

$$\frac{A_3}{A} = 0.864 = 86.4\%$$

8. **a.** The nucleus is identified by: $Z = 8, A = 15 \Rightarrow X = {_8}O^{15}$

b. $Q = [m(p) + m(N^{15}) - m(O^{15}) - m(n)]c^2 = -3.67$ MeV

c. $K_{th} = -Q\left(1 + \dfrac{m_P}{m_N}\right) = 3.9$ MeV

d. Now, $E_k = 2 \times K_{th} = 2 \times 3.9$ MeV $= 7.8$ MeV and $Q = -3.63$ MeV

(i) Conservation of momentum:

$$p_0 \cos\theta = \sqrt{2m_p E_k}$$

$$p_0 \sin\theta = p_n$$

(ii) Conservation of energy:

$$\frac{p_n^2}{2m_n} + \frac{p_0^2}{2m_0} = E_k + W$$

$$p_n^2 = \frac{E_k\left(1 - \frac{m_p}{m_0}\right) + Q}{\frac{1}{2m_0} + \frac{1}{2m_n}}$$

$\therefore \quad p_n = 79.4$ MeV/c, $p_0 = 145$ MeV/C and $\theta = 33°$

9. $\Delta m = 0.00563$ u $= 5.24$ MeV

KE of α-particle $= 5.24 - 0.9 = 4.34$ MeV

$$\therefore \quad v = \sqrt{\frac{2.434.10.1.6.10^9}{4.1.67.10}} = 1.44.10 \text{ m s}^{-1}$$

10. Nuclear reaction is ${}_1H^1 + {}_1H^3 \rightarrow {}_2He^3 + {}_0n^1 + Q$

$$Q = -1.2745 \text{ MeV}$$

Using conservation of linear momentum, we get

$$K_2 = 1.44 \text{ MeV}, K_1 = 3 \text{ MeV}$$

11. Applying conservation of energy:

$$m_A c^2 + K_A + m_B c^2 + K_B = m_C c^2 + K_C + \text{excitation energy}$$

$$(m_A + m_B + m_C)c^2 + K_A + K_B = K_C + \text{excitation energy}$$

$$4.65 + 5 + 3 = K_C + 10$$

or $\quad K_C = 2.65 \text{ MeV}$

12. $N^{14} + \alpha \rightarrow O^{17} + \text{proton}$

Q value $= (14.00307 + 4.00260 - 1.00783 - 16.99913)\, 931.5$

$= -1.20 \text{ MeV}$

Let m and M be the mass of α-particle and nitrogen nucleus, respectively, and let minimum KE of α-particle be $\frac{1}{2}mu^2$.

From energy equation,

$$\frac{1}{2}mu^2 = |Q| + \text{minimum KE of system}$$

$$= |Q| + |Q| + \frac{1}{2}(m+M)\left[\frac{mu}{m+M}\right]^2$$

$$\frac{1}{2}mu^2\left(\frac{M}{m+M}\right) = |Q|$$

$$\frac{1}{2}mu^2 = |Q|\left(\frac{m+M'}{M}\right)$$

$$\text{KE of products} = \frac{1}{2}(m+M)\left[\frac{mu}{m+M}\right]^2$$

$$= \frac{1}{2}mu^2\left(\frac{m}{m+M}\right)$$

$$= |Q|\frac{m}{M}$$

$$= 1.2 \times \frac{4}{14} = 0.34 \text{ MeV}$$

13. After 4 h, sample will contain 30 g of B, 12.5 g of A and $\left(\frac{146}{150} \times 37.5\right)$ g of A′.

$\therefore$ Total mass $= 30 + 12.5 + \frac{146}{150} \times 37.5 = 79$ g

Number of α-particles emitted is

$$\frac{37.5}{150}N_A = \frac{1}{4}N_A \approx 1.5 \times 10^{23}$$

14. The β-decay of ^{23}Ne is represented by the equation

$${}^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + e^- + \bar{\nu} + Q$$

Mass defect for the above reaction is

$$\Delta m = \left[({}^{23}_{10}\text{Ne}) - M({}^{23}_{11}\text{Na})\right]$$

$$= [22.994466 - 22.989770] \text{ a.m.u.} = 0.004696 \text{ a.m.u.}$$

Now, Q value of the above reaction is given as

$$Q = \Delta m \times 931.5 \text{ MeV} = 4.374 \text{ MeV}$$

The energy Q released is shared by ^{23}Na nucleus and the electron–antineutrino pair. Since ^{23}Na is massive, most of the kinetic energy is carried by the e–$\bar{\nu}$ pair. The electron will carry the maximum energy if the antineutrino carries zero energy. Thus, the maximum energy of the electrons emitted is 4.374 MeV.

15. Let t be the time for potential of metal sphere to rise by 1.0 V. Then, up to this time β-particles emitted from sphere are

$$N = (6.25 \times 10^{10}) \times t$$

Number of β-particles escaped in this time are

$$N_e = (80/100) \times (6.25 \times 10^{10})t$$

$$= 5 \times 10^{10}\, t$$

Thus, charge acquired by the sphere in t s

$$Q = (5 \times 10^{10} t) \times (1.6 \times 10^{-19})$$

$$= 8 \times 10^{-9}\, t \text{ coulomb}$$

(emission of β-particle leads to a charge e on the metal sphere)

The capacitance C of a metal sphere is given by

$$C = 4\pi\varepsilon_0 \times r$$

$$= \left(\frac{1}{9\times 10^9}\right) \times \left(\frac{10^{-3}}{2}\right) = \frac{10^{-12}}{18} \text{ farad}$$

We know that

$$Q = C \times V \qquad \text{(here, } V = 1 \text{ volt)}$$

or $$(8 \times 10^{-9})t = \left(\frac{10^{-12}}{18}\right) \times 1$$

Solving it for t, we get $t = 6.95\ \mu$s.

16. Our first step will be to write the reaction, then to find the disintegration energy Q. If $Q > 0$, the decay is allowed.

Alpha decay: ${}^{226}_{89}\text{Ac} \rightarrow {}^{222}_{87}\text{Fr} + \alpha$

$$Q = \left[M({}^{226}_{89}\text{Ac}) - M({}^{222}_{87}\text{Fr}) - M({}^4\text{He})\right]c^2$$

$= 5.50$ MeV (Alpha decay is allowed)

β decay: ${}^{226}_{89}\text{Ac} = {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$

$$Q = \left[M({}^{226}_{89}\text{Ac}) - M({}^{226}_{90}\text{Th})\right]c^2$$

$= 1.12$ MeV (β^- decay is allowed)

β^- decay: ${}^{226}_{89}\text{Ac} \rightarrow {}^{226}_{88}\text{Ra} + \beta^+ + \nu$

$$Q = \left[M({}^{226}_{89}\text{Ac}) - M({}^{226}_{88}\text{Ra}) - 2m_e\right]c^2$$

$= -0.38$ MeV (β^+ decay is not allowed)

Electron capture: ${}^{226}_{89}\text{Ac} + e^- \rightarrow {}^{226}_{88}\text{Ra} + \nu$

$$Q = \left[M({}^{226}_{89}\text{Ac}) - M({}^{226}_{88}\text{Ra})\right]c^2$$

$= 0.64\,\text{MeV}$ (Electron capture is allowed)

In the above analysis, it is clear that during α-decay, Q-value is maximum. Thus, chances of α-decay are maximum.

17. The two possible reactions for electron capture and β^+ decay by ${}^{55}_{26}\text{Fe}$ are

Electron capture: ${}^{55}_{26}\text{Fe} + \text{e} \rightarrow {}^{55}_{25}\text{Mn} + \text{n}$ (i)

β^+ decay: ${}^{55}_{26}\text{Fe} \rightarrow {}^{55}_{25}\text{Mn} + \beta^+ + \text{v}$ (ii)

First, we will determine the disintegration energy Q of Eqs. (i) and (ii).

$M({}^{55}_{26}\text{Fe}) = 54.938298$ a.m.u.

$M({}^{55}_{25}\text{Mn}) = 54.938050$ a.m.u. and $m_e = 0.000549$ a.m.u.

For β^+ decay, the Q value of the reaction is

$$Q = [54.938298 - 54.938050 - 2\times 0.000549] \times 931.5\,\text{MeV}$$

$$= -0.79\,\text{MeV}$$

Negative value of Q implies that positron decay is not possible spontaneously. For electron capture, the Q value of reaction is

$$Q = [54.938298 - 54.938050] \times 931.5$$
$$= 0.23\,\text{MeV}$$

Positive value of Q implies that electron capture is possible in this case.

18. Radioactive decay equation is

$$N = N_0 e^{-\lambda t} = N_0 e^{-\ln(2)t/T} \qquad \left(\because \lambda = \frac{\ln 2}{T}\right)$$

After decay of 99% of the initial sample, only 1% will be left, i.e., $N/N_0 = 1\%$.

$$\therefore \quad \frac{N}{N_0} = \frac{1}{100} = e^{-\ln(2)t/T}$$

If we take the natural logarithm, we have

$$-\ln 100 = -\frac{\ln 2 \times t}{T}$$

which on solving for t yields

$$t = \frac{\ln 100}{\ln 2} \times T = \frac{\log 100}{\log 2} \times T$$

$$= \frac{2}{0.3010} \times 10 = 731\,\text{min} = 12.2\,\text{h}$$

19. Mass of two protons is $2 \times 1.007277 = 2.014554$ a.m.u.

Mass of two neutrons is 2×1.008666 a.m.u. $= 2.017332$ a.m.u.

Total initial mass of two protons and two neutrons is

$$2.041554 + 2.017332 = 4.031866\,\text{a.m.u.}$$

Mass defect,

$$\Delta m = 4.031816 - 4.001265\,\text{a.m.u.}$$
$$= 0.030621\,\text{a.m.u.}$$

Now, binding energy of α-particle in MeV is given as

$$\Delta E_n = \Delta m \times 931.2\,\text{MeV}$$
$$= 0.030621 \times 931.2 = 28.5142\,\text{MeV}$$

Binding energy per nucleon $= 28.5142/4 = 7.12855$ MeV

20. Mass defect of the process is given by

$\Delta m =$ [Mass of neutron − (mass of proton + mass of electron)]

$$= [1.6747 \times 10^{-27} - (1.6725 \times 10^{-27} + 9 \times 10^{-31})]$$
$$= 0.0013 \times 10^{-27}\,\text{kg}$$

According to mass–energy relationship,

Energy released $= \Delta mc^2$

$$E = (0.0013 \times 10^{-27}) \times (3 \times 10^8)^2$$

$$= \frac{1.17 \times 10^{-3}}{1.6 \times 10^{-19}} = 0.73 \times 10^6\,\text{eV} = 0.73\,\text{MeV}$$

21. The ${}^{35}_{17}\text{Cl}$ nucleus has 17 protons and 18 neutrons.

Therefore, the mass of contents nucleus of ${}^{35}_{17}\text{Cl}$ is

$$M = 17m_p + 18m_n = 17 \times 1.007825 + 18 \times 1.0086645$$
$$= 35.289\,\text{a.m.u.}$$

Now, mass defect for the nucleus is

$$\Delta m = \frac{298\,\text{MeV}}{9312\,\text{MeV/a.m.u.}} = 0.3200\,\text{a.m.u.}$$

Thus, atomic mass of ${}^{35}_{17}\text{Cl}$ = mass of contents nucleus − mass defect

$$= m - \Delta m = 35.289\,\text{a.m.u.} - 0.3200\,\text{a.m.u.}$$
$$= 34.969\,\text{a.m.u.}$$

22. The radius of ${}^{12}_{6}\text{C}$ nucleus can be given as

$$R = R_0 A^{1/3}$$

or $\quad R = 1.2 \times 10^{-15} \times (2)^{1/3} = 2.75\ 10^{-15}\,\text{m}$

The atomic mass of ${}^{12}_{6}\text{C}$ is 12 a.m.u. Neglecting the masses and binding energies of the six electrons.

$$\text{Nuclear density} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{12 \times 1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi\right)(2.7 \times 10^{-15})^3}$$

$$= 2.4 \times 10^{17}\,\text{kg m}^{-3}$$

23. Binding energy of nucleus is given by the equation

$$B({}^{A}_{Z}\text{X}) = [(A - Z)m_n + Zm_p - M({}^{A}_{Z}\text{X})]c^2$$

On dividing binding energy by the mass number, we obtain the binding energy per nucleon.

$$B({}^{20}_{10}\text{Ne}) = [10m_n + 10m_p - M({}^{20}_{10}\text{Ne})]\,c^2$$
$$= [10 \times 1.008665 + 10 \times 1.007825 - 55.93492] \times 931.5$$
$$= 492\,\text{MeV}$$

Hence, binding energy per nucleon $= \dfrac{B({}^{56}_{26}\text{Fe})}{56} = 8.79$ MeV/nucleon

Binding energy for ${}^{238}_{92}\text{U}$ is

$$[146m_n + 92m_p - M({}^{238}_{92}\text{U})]c^2$$

$$= [146 \times 1.008665 + 92 \times 1.007825 - 238.050783] \times 9315$$

$$= 1802 \text{ MeV}$$

Binding energy per nucleon

$$= \frac{B(^{238}_{92}\text{Fe})}{238} = \frac{1802}{238} = 7.57 \text{ MeV}$$

24. According to radioactive decay law, we have

$$N = N_0 2^{-t/T}$$

Here, we have $T = 2$ yr and $t = 4$ yr. Thus, we have

$$N = N_0 2^{-4/2}$$

$$= \frac{N_0}{4} = \frac{1}{4} \text{g}$$

Thus, after 4 years, 0.25 g of the material will be left.

25. Given that after 50 s, the amount remaining is 0.99 g as out of 1 g, 1 centigram is lost. Now, from radioactive decay equation, we have

$$N = N_0 2^{-t/T}$$

or $\quad 0.99 = (1)\, 2^{-50/T} \quad$ or $\quad \log\left(\frac{99}{100}\right) = -\frac{50}{T}$

or $\quad T = \dfrac{50}{\log 2\left(\dfrac{100}{99}\right)}$

or $\quad T = 50 \times \dfrac{\log_{10}(2)}{\log_{10}(100) - \log_2(99)}$

or $\quad T = 50\left[\dfrac{0.301}{2 - 1.9956}\right]$

or $\quad T = 3420 \text{ s} = 57 \text{ min}$

26. Given that activity of substance is

$$A_c = 3.7 \times 10^{10} \text{ dps}$$

The number of atoms in 1 g of substance,

$$N = \frac{1 \times 6.023 \times 10^{23}}{226} = 2.66 \times 10^{21} \text{ atoms}$$

If λ is the decay constant of the substance, we know that activity is given by

$$A_c = \lambda N$$

or $\quad \lambda = \dfrac{A_c}{N}$

$$= \frac{3.7 \times 10^{10}}{2.66 \times 10^{21}} \text{s}^{-1} = 1.39 \times 10^{-11} \text{ s}^{-1}$$

Thus, mean life of the radioactive substance is

$$T_m = \frac{1}{\lambda} = \frac{1}{1.39 \times 10^{-11}} = 7.194 \times 10^{10} \text{ s}$$

27. Given that kinetic energy of neutrons is

$$\frac{1}{2}mv^2 = 0.0327 \times (1.6 \times 10^{-19}) \text{ J}$$

or $\quad v^2 = \dfrac{2 \times 0.0327 \times (1.6 \times 10^{-19})}{1.675 \times 10^{-27}}$

or $\quad v^2 = 625 \times 10^4 \quad$ or $\quad v = 2500 \text{ m s}^{-1}$

Time to travel a distance of 10 km is

$$\frac{10^4 \text{ m}}{2500 \text{ ms}^{-1}} = 4 \text{ s}$$

After 4 s, number of neutrons left can be given as

$$N = N_0 2^{-n}$$

where $\quad n = \dfrac{t}{T} =$ no. of half-lives. Here, $n = \dfrac{4}{700} = \dfrac{1}{175}$

or $\quad \dfrac{N}{N_0} = 2^{-1/175} = 0.996 \quad$ or $\quad N = 0.996 N_0$

Thus, fraction of neutrons decayed is

$$f = \frac{N_0 - N}{N_0} = \frac{0.004 N_0}{N_0} = 0.004$$

28. a. At $t = 0, N = N_0$

Rate of decay $= -\lambda N$, and rate of formation $= \alpha$. Thus accumulation rate of element is

$$\frac{dN}{dt} = \alpha - \lambda N$$

or $\quad \dfrac{dN}{\alpha - \lambda N} = dt$

Integrating this expression, we get

$$\log_e\left[\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right] = -\lambda t$$

or $\quad \dfrac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$

or $\quad \alpha - \lambda N = (\alpha - \lambda N_0)e^{-\lambda t}$

or $\quad N = \dfrac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$

b. If $\alpha = 2N_0\lambda$,

$$N = \frac{1}{\lambda}[2N_0\lambda - (2N_0\lambda - N_0\lambda)e^{-\lambda t}]$$

$$N = N_0(2 - e^{-\lambda t})$$

At the time of half-life, $T = 0.693/\lambda$. So,

$$N = N_0[2 - e^{0.693}] - \frac{3N_0}{2}$$

Limiting value of N (as $t \to \infty$) is

$$N = N_0[2 - e^{-\infty}] = 2N_0$$

29. Energy released by warhead $= 3000$ tons of TNT $= (3.5 \times 10^8)(30000) = 10.5 \times 10^{12}$ J.

Number of fusions in warhead

$$= \frac{10.5 \times 10^{12}}{(200 \times 10^6)(1.6 \times 10^{-19})} \text{g} = 0.1729 \text{ kg}$$

30. The nuclear fission reaction in the above process is

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n}$$

The sum of the masses before reaction is

$$235.0439 + 1.0087 = 236.0526 \text{ a.m.u.}$$

The sum of the masses after reaction is

$$140.9139 + 91.8973 + (1.0087) = 235.8375 \text{ a.m.u.}$$

Mass defect,

$$\Delta m = 236.0526 - 235.8373 = 0.2153 \text{ a.m.u.}$$

Energy released in the fission of ^{235}U nucleus is given by

$$\Delta E = \Delta m \times 931.2 \text{ MeV}$$

$$= 0.2153 \times 931.2 = 200 \text{ MeV}$$

Number of atoms in 1 g of ^{235}U,

$$N = \frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$$

Energy released in fission of 1 g of ^{235}U,

$$E = 200 \times 2.56 \times 10^{21} \text{ MeV}$$

$$= 5.12 \times 10^{23} \text{ MeV} = (5.12 \times 10^{23}) \times (1.6 \times 10^{-13})$$

$$= 8.2 \times 10^{10} \text{ J}$$

$$= \frac{8.2 \times 10^{10}}{3.6 \times 10^{6}} \text{ kWh} = 2.28 \times 10^{4} \text{ kWh}$$

31. Total mass number of $^{239}_{94}$Pu + neutron (thermal) = 239 + 1 = 240. Since 4 neutrons are produced, the mass number of each fragment $(A) = \frac{240 - 4}{2} = 118$. The atomic number of each fragment = 94/2 = 47. Therefore, charge of each fragment is

$$q = 47 \times 1.6 \times 10^{-19} = 7.52 \times 10^{-18} \text{ C}$$

The radius of each nucleus of the fragment is

$$R = R_0(A)^{1/3}$$

$$= 1.1 \times 10^{-15} \times (118)^{1/3} \quad (\text{As } 1 \text{ fermi} = 10^{-15} \text{ cm})$$

$$= 5.395 \times 10^{-15} \text{ m}$$

Distance between the centres of the two fragments at the moment they are produced is

$$r = 2 \times 5.395 \times 10^{-15} = 10.79 \times 10^{-15} \text{ m}$$

The electrostatic force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 9 \times 10^9 \frac{(7.5 \times 10^{-18})^2}{(10.79 \times 10^{-15})^2} = 4.37 \times 10^3 \text{ N}$$

32. The two reactions are

$$\text{n} + ^{235}\text{U} \rightarrow ^{236}\text{U}^*$$

$$\text{n} + ^{238}\text{U} \rightarrow ^{239}\text{U}^*$$

We find the excitation energy from the atomic masses. A thermal neutron has a negligible kinetic energy (about 0.03 eV).

$$E(^{236}\text{U}^*) = [m(\text{n}) + M(^{235}\text{U}) - M(^{236}\text{U})]c^2$$

$$= [1.0087 \text{ a.m.u.} + 235.0439 \text{ a.m.u.} - 236.0456 \text{ a.m.u.}]c^2$$

$$= 0.0070 \times 931.5 = 6.5 \text{ MeV}$$

$$E(^{239}\text{U}^*) = [m(\text{n}) + M(^{238}\text{U}) - M(^{239}\text{U})]c^2$$

$$= [1.0087 \text{ a.m.u.} + 238.0508 \text{ a.m.u.} - 239.0543 \text{ a.m.u.}]c^2$$

$$= 0.0052 \times 931.5 = 4.8 \text{ MeV}$$

Thus, ^{236}U* has more excitation energy than ^{239}U* when both are produced by thermal neutron absorption. This is why ^{235}U more easily undergoes thermal neutron fission.

33. Kinetic energy of a thermal neutron can be neglected; even for a temperature of 10^6 K, the thermal energy is only 130 eV. The Q value of the above reaction is given by the equation

$$Q = \Delta m \times 931.2 \text{ MeV}$$

Here, Δm is the mass defect of the reaction, given by

$$\Delta m = [M(^{235}\text{U}) + m(\text{n})] - [M(^{99}\text{Zr}) + M(^{134}\text{Te}) + 3m(\text{n})]$$

$$= [235.0439 - 98.9165 - 133.9115 - 2(1.0087)] \text{ a.m.u.}$$

$$= 0.1985 \text{ a.m.u.}$$

Thus, energy released is $Q = 0.1985 \times 931.2$. Here, Δm is the mass defect of the reaction, given by

$$\Delta m = [M(^{235}\text{U}) + m(\text{n})] - [M(^{99}\text{Zr}) + M(^{134}\text{Te}) + 3m(\text{n})]$$

Thus, energy released is

$$Q = 0.1985 \times 931.2 = 184.84 \text{ MeV}$$

Even with a thermal neutron of negligible kinetic energy, a tremendous amount of energy is released.

34. The corresponding decay equations would be

(i) $^{230}_{92}\text{U} \rightarrow \text{n} + ^{229}_{92}\text{U}$

(ii) $^{230}_{92}\text{U} \rightarrow \text{n} + ^{229}_{91}\text{U}$

In the first equation, the energy released can be given by

$$Q = [230.033927 - 229.033496 - 1.008665] \times 931.2$$

$$= -7.7 \text{ MeV}$$

Because $Q < 0$, neutron decay is not possible spontaneously.

Similarly, in the second equation, the energy released can be given as

$$Q = [230.033927 - 229.032089 - 1.007825] \times 931.5$$

$$= -5.6 \text{ MeV}$$

As $Q < 0$, proton decay is also not possible spontaneously.

35. a. Since the neutron and boron are both initially at rest, the total momentum before the reaction is zero and afterward also it is zero. Therefore, $M_{\text{Li}}v_{\text{Li}} = M_{\text{He}}v_{\text{He}}$.

We solve this for v_{Li} and substitute it into the equation for kinetic energy. We can use classical kinetic energy with little error, rather than relativistic formulas, because $v_{\text{He}} = 9.30 \times 10^6 \text{ m s}^{-1}$ is not close to the speed of light c and v_{Li} will be even less since $M_{\text{Li}} > M_{\text{He}}$. Thus, we can write

$$K_{\text{Li}} = \frac{1}{2}M_{\text{Li}}v_{\text{Li}}^2 = \frac{1}{2}M_{\text{Li}}\left(\frac{M_{\text{He}}v_{\text{He}}}{M_{\text{Li}}}\right)^2 = \frac{M_{\text{He}}^2 v_{\text{He}}^2}{2M_{\text{Li}}}$$

We put in numbers, changing the mass in u to kg and recalling that 1.60×10^{-13} J = 1 MeV

$$K_{Li}=\frac{(4.0026)^2(1.66\times10^{-27})(9.30\times10^6)^2}{2(7.0160)(1.66\times10^{-27})}$$

$$=1.64\times10^{-13}\text{ J}=1.02\text{ MeV}$$

b. We are given the data $K_\alpha=K_X=0$, so $Q=K_{Li}+K_{He}$, where

$$K_{He}=\frac{1}{2}M_{He}V_{He}^2$$

$$=\frac{1}{2}(4.0026)(1.66\times10^{-27})(9.30\times10^6)^2$$

$$=2.87\times10^{-13}\text{ J}=1.80\text{ MeV}$$

Hence, $Q=1.02\text{ MeV}+1.80\text{ MeV}=2.82\text{ MeV}$

Objective Type

1. a. Isotopes A and C have same number of protons.

2. d. For decay (i):

$$Q=[230.033927-229.033496-1.008665]\times931.5$$
$$=-7.7\text{ MeV}$$

For decay (ii):

$$Q=[230.033927-229.032089-1.007825]\times931.5$$
$$=-5.6\text{ MeV}$$

As Q is negative for both the decays, so none of the decays is allowed.

3. c. Suppose an initial radionuclide I decays to a final product F with a half-life $T_{1/2}$.

At any time, $N_I=N_0e^{-\lambda t}$

Number of product nuclei $=N_F=N_0-N_I$

$$\frac{N_F}{N_I}=\frac{N_0-N_I}{N_I}=\left(\frac{N_0}{N_I}-1\right)$$

$$\frac{N_0}{N_I}=\left(1+\frac{N_F}{N_I}\right)=1+0.5=1.5$$

$$e^{\lambda t}=1.5\quad\Rightarrow\lambda t=\ln 1.5$$

$$\therefore\quad\frac{T_{1/2}\ln(1.5)}{\ln 2}=4.5\times10^9\,\frac{\ln\left(\frac{3}{2}\right)}{\ln 2}\text{ year}$$

4. c. Transformation occurs only when the same net energy is released, which is possible only when $E_2>2E_1$.

5. a. Given, $N_2=\frac{N_0}{e}=N_0e^{-\lambda t}\Rightarrow t=\frac{1}{\lambda}=10\text{ s}$

$$\therefore\quad T_{1/2}=\frac{\ln 2}{\lambda}=0.693\times10\approx7\text{ s}$$

6. d. Since no external force is present, so momentum conservation principle is completely applicable.

$$\therefore\quad m\vec{v}=m_1\vec{v}_1+m_2\vec{v}_2$$

or $\quad(m_1+m_2)\,\vec{v}=m_1\vec{v}_1+m_2\vec{v}_2$

7. b. Nuclear forces are charge independent.

8. a. $\frac{dN_A}{dt}=(-\lambda N_A)+(-2\lambda N_A)+(-2\lambda N_A)=-6\lambda N_A$

9. b. Let, ${}^A_ZX\to{}^{A-4}_{A-2}Y+{}^4_2\text{He}$

$$K_\alpha=\frac{m_y}{m_y+m_\alpha}Q$$

$$\therefore\quad K_\alpha=\frac{A-4}{A}Q$$

or $\quad 48=\frac{A-4}{A}\times50\quad\Rightarrow\quad A=100$

10. a. After first half hours,

$$N=N_0\frac{1}{2}$$

For $t=\frac{1}{2}$ h to $t=1\frac{1}{2}$ h, 1 h = four half-lives

Hence, $N=\left(N_0\frac{1}{2}\right)\left[\frac{1}{2}\right]^4=N_0\left(\frac{1}{2}\right)^5$

For $t=\frac{1}{2}$ to $t=2$ h

$$\left[\text{for both } A \text{ and } B,\ \frac{1}{t_{1/2}}=\frac{1}{t_{1/2}}+\frac{1}{t_{1/4}}=2+4=6\Rightarrow t_{1/2}=\frac{1}{6}\right]$$

$\frac{1}{2}$ h = three half-lives

$$\therefore\quad N=\left[\left(N_0\frac{1}{2}\right)^5\right]\left(\frac{1}{2}\right)^3=N_0\left(\frac{1}{2}\right)^8$$

11. b. For α-decay: ${}_xA^y\to{}_{x-2}B^{y-4}+\alpha$

For β^- decay: ${}_xA^y\to{}_{x+1}B^y+{}_{-1}\beta^0$

For β^+ decay: ${}_xA^y\to{}_{x-1}B^y+{}_{+1}\beta^0$

For k-capture, there will be no change in the number of protons. Hence, only case in which number of protons increases is β^- decay.

12. c. $N=N_0e^{-\lambda t}$, $N_Y=N_0(1-e^{-\lambda t})$

Rate of formation of Y is

$$\frac{dN}{dt}=+\lambda N_0e^{-\lambda t}$$

which decreases exponentially with time.

13. c. Energy released is

$(80\times7+120\times8-200\times6.5)=220$ MeV

14. b. ${}^A_ZX\xrightarrow{\text{Proton}}{}^A_{Z-1}Y\xrightarrow{2\alpha}{}^{A-8}_{A-5}Y$

Given: $A-8=224$ and

$$Z-5=89\quad\Rightarrow\quad A=237,Z=94$$

15. b. 90% of the sample is left undecayed after time t.

$$\therefore\quad\frac{9}{10}N_0=N_0e^{-\lambda t}$$

$$\lambda=\frac{1}{t}\ln\left(\frac{10}{9}\right)\qquad\text{(i)}$$

After time $2t$,

$$N_c = N_0 e^{-\lambda(2t)} = N_0 e^{-\frac{1}{t}\left[\ln\left(\frac{10}{9}\right)\right]2t} \quad \text{(ii)}$$

$$N = N_0 e^{-\ln\left(\frac{10}{9}\right)^2} = N_0\left(\frac{9}{10}\right)^2 \approx 81\% \text{ of } N_0 \quad \text{(iii)}$$

Therefore, 19% of initial value will decay in time $2t$.

16. c. Let N_2 be the number of atoms of X at time $t = 0$. Then, at $t = 4$ h (two half-lives),

$$N_x = \frac{N_0}{4} \quad \text{and} \quad N_y = \frac{3N_0}{4}$$

$$\therefore \quad \frac{N_x}{N_y} = \frac{1}{3} \approx 0.33$$

At $t = 6$ h (three half-lives),

$$N_x = \frac{N_0}{8} \quad \text{and} \quad N_y = \frac{7N_0}{8} \text{ or } \frac{N_x}{N_y} = \frac{1}{7} \approx 0.142$$

The given ratio $\frac{1}{4}$ lies between $\frac{1}{3}$ and $\frac{1}{7}$.

Therefore, t lies between 4 h and 6 h.

17. b. $\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$

For n_2 to be maximum,

$$\frac{dN_2}{dt} = 0$$

$$\Rightarrow \quad \lambda N_1 = 2\lambda N_2 \quad \text{or} \quad \frac{N_1}{N_2} = 2$$

18. b. The total energy required to make the electron free from nucleus is the sum of the energy required to separate the electrons from the influence of each other and the energy required to separate the electrons from the influence of nucleus, i.e.,

Total required energy = BE of electron in He atom + ionization energy of He atom

$$= (24.6 + 13.6 \times 2^2)\text{ eV}$$

$$= (24.6 + 54.4)\text{ eV} = 79\text{ eV}$$

19. a. From given information,

$$\frac{dN}{dt} = \frac{-0.04N}{3600}$$

Comparing above equation with standard decay equation,

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = 1.1 \times 10^{-5}\text{ s}^{-1}$$

$$\therefore \quad \tau = \frac{1}{\lambda} = \frac{3600}{0.04}\text{ s} = 25\text{ h}$$

20. c. Let n collisions are required for the given condition. Then,

$$\left(\frac{1}{2}\right)^n \times 2\text{ MeV} = 0.04 \times 10^{-6}\text{ MeV}$$

$$2^n = \frac{2}{0.04} \times 10^6 = 50 \times 10^6$$

After solving above equation, $n = 26$.

21. c. Expected atomic mass of Cu must be less than that of zinc, but it is not so. So, it means Cu is radioactive and unstable and decays to Zn through β-decay.

22. a. As the alpha particle decays, the daughter nucleus recoils. In such a process, the momentum conservation holds good. So,

$$P_\alpha = P_D = P$$

$$K_\alpha = \frac{P^2}{2M_\alpha} \quad \text{and} \quad K_D = \frac{P^2}{2M_D}$$

As $M_D > M_\alpha$, so, $K_\alpha > K_D$.

23. c. The minimum energy needed to carry out an endothermic reaction is greater than the Q value of the reaction. This is because to conserve the momentum some extra energy has to be provided.

$\text{KE}_{\min} = \left(1 + \frac{m}{M}\right) \times |Q|$, where m is the mass of the incident particle and M is the mass of target.

24. c. The number of nuclei in 1 kg ^{235}U is

$$N = \frac{N_A}{235} \times (1 \times 10^3)$$

$$N = \frac{6.023 \times 10^{23}}{235} \times 10^3 = 2.56 \times 10^{24}\text{ nuclei}$$

Total energy released is

$$E = N \times 200\text{ MeV}$$

$$= 5.12 \times 10^{26}\text{ MeV}$$

25. a. When a free neutron decays to a proton along with an electron and an antineutrino, the Q value of the reaction is positive which means the reaction is possible all by itself, while a free proton cannot convert itself into a neutron due to negative Q value.

In beta minus decay, the electron originates from nucleus only, by the transformation of neutron into a proton, with simultaneous emission of an antineutrino.

26. a. Since scheme A releases more energy than scheme B, scheme A is more likely to occur. This is because the more the energy released, the more stable the daughter nucleus is. A heavy nucleus undergoes fission such that its products will be more stable than the parent nucleus.

27. a. At present,

$$\frac{\text{Number of K atoms}}{\text{Number of Ar atoms}} = \frac{1}{7}$$

Let age of rock be n half-lives of K-nuclide. Then,

$$\left(\frac{1}{2}\right)^n = \frac{\text{Number of K-atoms present now}}{\text{Number of K-atom present initially}} = \frac{1}{1+7}$$

where number of K atoms present initially = number of K atoms + number of Ar atoms present now.

$\therefore n = 3$

So, age of rock is 3 half-lives of K nuclides, i.e., 4.2×10^9 years.

28. a. Probability of survival for any nucleus at time t is

$$P=\frac{N}{N_0}=\frac{N_0e^{-\lambda t}}{N_0}=e^{-\lambda t}$$

So, in one mean life, required probability is

$$e^{-\lambda\times\frac{1}{\lambda}}=\frac{1}{e}$$

29. b. The complete fission reaction is

$${}^{235}_{92}\text{U}+\text{n}\rightarrow{}^{94}_{40}\text{Zr}+{}^{140}_{58}\text{Ce}+2\text{n}+6\text{e}^{-1}$$

$$Q=[m(^{235}\text{U})-m(^{94}\text{Zr})-m(^{140}\text{Ce})-m(\text{n})]c^2$$

$$=208\text{ MeV}$$

30. c. The net reaction is

$$3({}^2_1\text{H})\rightarrow{}^4_2\text{He}+\text{n}+\text{p}$$

$$Q=[3\times m(^2\text{H})-m(^4\text{He})-m(\text{n})-m(\text{p})]\times931\text{ MeV}$$

$$=3.87\times10^{-12}\text{ J}$$

This is the energy produced by the consumption of 3 deuteron atoms. So, the total energy released by 10^{40} deuterons is

$$\frac{3.87\times10^{-12}}{3}\times10^{40}=1.29\times10^{28}\text{ J}$$

Let total supply of deuterons in star be exhausted in t seconds. Then,

$$10^{16}\times t=1.29\times10^{28}$$

$$\Rightarrow\quad t=1.29\times10^{12}\text{ s}$$

31. d. $N_{x_1}=N_0e^{-10\lambda t}$

$$N_{x_2}=N_0e^{-\lambda t}$$

$$\frac{N_{x_1}}{N_{x_2}}=\frac{1}{e}=\frac{e^{-10\lambda t}}{e^{-\lambda t}}=e^{-9\lambda t}$$

$$9\lambda t=1\quad\Rightarrow\quad t=\frac{1}{9\lambda}$$

32. c. Let N be the number of nuclei at any time t. Then,

$$\frac{dN}{dt}=200-\lambda N$$

$$\therefore\quad\int_0^N\frac{dN}{200-\lambda N}=\int_0^t dt$$

$$\text{or}\quad N=\frac{200}{\lambda}(1-e^{-\lambda t})$$

Given: $N=100$ and $\lambda=1\text{ s}^{-1}$

$$\therefore\quad 100=200\,(1-e^{-t})$$

$$\text{or}\quad e^{-t}=\left(\frac{1}{2}\right)\quad\therefore\quad t=\ln(2)\text{ s}$$

33. a. Number of radio nuclei become constant, when rate of production becomes equal to rate of decay.

$$X=\lambda N$$

$$\text{or}\quad N=\frac{X}{\lambda}.\ \text{Given, } y=\frac{\ln 2}{\lambda}$$

$$\Rightarrow\quad N=\frac{XY}{\ln(2)}$$

34. c.

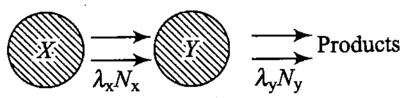


Fig. 5.34

Net rate of formation of Y at any time t is

$$\frac{dN_y}{dt}=\lambda_xN_x-\lambda_yN_y$$

N_y is maximum when

$$\frac{dN_y}{dt}=0$$

$$\text{or}\quad\lambda_xN_x=\lambda_yN_y$$

35. a. $\dfrac{\lambda_A}{\lambda_B}=\dfrac{1}{2}$

Probabilities of getting α- and β-particles are same. Thus, rates of disintegration are equal.

$$\therefore\quad\lambda_AN_A=\lambda_BN_B$$

$$\text{or}\quad\frac{N_A}{N_B}=\frac{\lambda_B}{\lambda_A}=2$$

36. b. Three half-lives of A is equivalent to six half-lives of B. Hence,

$$N_A\left(\frac{1}{2}\right)^3=N_B\left(\frac{1}{2}\right)^6$$

$$\text{or}\quad\frac{N_A}{N_B}=\frac{1}{8}$$

37. a. Let $\lambda_A=\lambda$ and $\lambda_B=2\lambda$. Initially, rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$. After one half-life of A, rate of disntigration of A will become $\dfrac{\lambda N_0}{2}$ (half-life of B = one-half the half-life of A). So, after one half-life of A or two half-lives of B,

$$\left(-\frac{dN}{dt}\right)=\left(-\frac{dN}{dt}\right)_B$$

$$\therefore\quad n=1$$

38. c. A and B can be isotopes if number of β-decays is two times the number of α-decays.

39. b. Let number of α-decays are x and number of β-decays are y. Then,

$$92-2x+y=85$$

$$\text{or}\quad 2x-y=7\qquad\text{(i)}$$

$$\text{and}\quad 238-4x=210$$

$$\therefore\quad x=7$$

Substituting this value in Eq. (1), we get $y=7$.

40. b. In 2 s only 90% nuclei are left behind. Thus, in next 2 s 90% of 900 or 810 nuclei will be left.

41. a. Maximum number of nuclei will be present when rate of decay = rate of formation

or $\lambda N = \alpha$

$\therefore \quad N = \dfrac{\alpha}{\lambda}$

42. a. Fraction of nuclei which remain undecayed is

$$f = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0}$$
$$= e^{-\lambda t}$$
$$= e^{-\left(\frac{\ln 2}{T}\right)\left(\frac{T}{2}\right)}$$
$$= \frac{1}{e^{\ln\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

43. b. Let R_0 be the initial activity. Then,

$$R_1 = R_0 e^{-\lambda t_1}$$

and $$R_2 = R_0 e^{-\lambda t_2}$$

$\therefore$ $$\frac{R_2}{R_1} = e^{\lambda(t_1 - t_2)}$$

or $$R_2 = R_1 e^{\lambda(t_1 - t_2)}$$

44. d. $R_1 = \lambda N_1 \quad \Rightarrow N_1 = \dfrac{R_1}{\lambda}$

and $R_2 = \lambda N_2 \quad \Rightarrow \quad N_2 = \dfrac{R_2}{\lambda}$

Therefore, number of atoms decayed $= N_1 - N_2 = \left(\dfrac{R_1 - R_2}{\lambda}\right)$

45. c. Activity, $R = \lambda N$. Number of nuclei (N) per mole are equal for both the substances.

$\therefore \quad R \propto \lambda$

or $$\frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

46. b. Activity of a radioactive substance,

$$R = \lambda N$$

$\therefore$ $$\lambda = \frac{R}{N}$$

Here, $R = N_2$ particles per second and $N = N_1$.

$\therefore$ $$\lambda = \frac{N_2}{N_1}$$

47. b. During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is released. Hence,

$$Q = E_2 - 2E_1$$

48. d. Energy released would be

$$\Delta E = \text{total binding energy of } {}_2\text{He}^4$$
$$-2 \times (\text{total binding energy of } {}_1\text{He}^4)$$
$$= 4 \times 7.0 - 2\,(1.1)\,(2)$$
$$= 23.6\,\text{MeV}$$

49. a. ${}_{92}\text{U}^{238} + {}_0\text{n}^1 \to {}_{92}\text{U}^{239}$

$\to {}_{-1}e^0 + {}_9\text{Np}^{239} \to {}_{-1}e^0 + {}_{94}\text{Pu}^{239}$

50. c. $$\frac{A_0}{3} = A_0\left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

$$A' = \frac{A_0}{3}\left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$$

Dividing, we get

$$\frac{A' \times 3}{A_0} = \frac{1}{3} \quad \text{or} \quad A' = \frac{A_0}{9}$$

51. b. $T_{1/2} = \dfrac{0.693}{\lambda}$ or $T_{1/2} = 0.693\left[\dfrac{1}{\lambda}\right]$

or $\quad T_{1/2} = 0.693\,\tau$

Clearly, $x = 0.693$.

52. d. The emission of antineutrino is a must for the validity of different laws.

53. a. ${}^{A}_{Z}\text{X} \to {}^4_2\text{He} + {}^{A-4}_{Z-2}\text{Y}$

${}^{A-4}_{Z-2}\text{Y} \to e^+ + {}^{A-4}_{Z-3}\text{Y}'$

During β^+ emission,

$${}_1\text{p}^1 \to {}_0\text{n}^1 + \beta^+$$

The proton changes into neutron. So, charge number decreases by 1 but mass number remains unchanged.

54. b. $\dfrac{N_0}{4} = \dfrac{N_0}{2n} \Rightarrow n = 2$

Thus, 10 days = 2 half-lives

$\therefore$ Half-life = 8 days

55. d. $\dfrac{3}{5}N_0 = N_0 e^{-\lambda t}$

$\Rightarrow$ $$e^{\lambda t} = \frac{5}{3}$$

$$\log_e e^{\lambda t} = \log_e \frac{5}{3} \text{ or } \lambda t = \log_e \frac{5}{3}$$

or $$t = \frac{1}{\lambda}\log_e \frac{5}{3}$$
$$= \frac{T}{0.693} \times 0.5 \quad \left[\because\ T = \frac{0.693}{\lambda}\right]$$
$$= \frac{5570 \times 0.5}{0.693}\text{ years} = 4018.7 \text{ years}$$
$$= 4000 \text{ years}$$

56. d. Refer to the definition of mass defect.

57. d. After n half-lives, the radioactive nuclei remaining is $\dfrac{N_0}{2^n}$.

So, number of nuclei disintegrated in n half-lives is $\left(N_0 - \dfrac{N_0}{2^n}\right)$.

For $n = \dfrac{1}{2}$, the fraction distintegrated is $\left(1 - \dfrac{1}{\sqrt{2}}\right)$.

58. d. α-decay decreases mass number by 4 and reduces charge number by 2. β-decay keeps mass number unchanged and increases charge by 1. Clearly, option (d) is the right choice.

59. b. Decrease in mass number = 232 – 208 = 24

$$\text{Number of } \alpha\text{-particles emitted} = \frac{24}{4} = 6$$

Due to emission of 6 particles, decrease in charge number is 12. But actual decrease in charge number is 8. Clearly, 4 β-particles are emitted.

60. c. $A = A_0 e^{-\lambda t}$; $2100 = 16000\, e^{-12\lambda} \Rightarrow e^{12\lambda} = 7.6$

$$\Rightarrow \quad 12\lambda = \log_e 7.6 = 2 \Rightarrow \lambda = \frac{2}{12} = \frac{1}{6}$$

$$\therefore \quad T = \frac{0.6931 \times 6}{1} = 4$$

61. b. Total mass of the products = 2.0165 a.m.u., which is greater than the mass of the deuteron by 0.0024 a.m.u. The extra mass must be provided by the energy of the photon so that minimum possible frequency must be given by

$$v = \frac{0.0024 \text{ a.m.u.}\, c^2}{h} \quad (1 \text{ a.m.u.} = 1.66 \times 10^{-27} \text{ kg})$$

$$\Rightarrow \quad v = 5.4 \times 10^{20} \text{ Hz}$$

62. b. The nuclear fission differs from other nuclear reactions in three respects.

a. The nucleus is deeply divided into two large fission fragments or nuclei of roughly equal mass. The nuclei or fission fragments fly apart at great speed and thus possess large kinetic energies that carry off the greater part of the energy released.

b. The mass decrease is appreciable and hence large energy is released.

c. Other neutrons, called fission neutrons, are emitted in the process. Small amount of energy is released in the form of radiation.

63. c. Since four half-lives have elapsed.

$$A = \frac{A_0}{2^4} = \frac{A_0}{16} = \frac{1.6}{16} \text{ curie} = 0.1 \text{ curie}$$

64. d. Disintegration of deuteron to a proton and a neutron can be represented by

$$^2_1\text{H} + \text{Q} \rightarrow {}^1_1\text{H} + {}^1_0\text{n}$$

The energy captured is the γ-ray photon E_γ is given by

$$E_\gamma + 1876 = 939 + 940$$

$$\Rightarrow \quad E_\gamma = (939 + 940) - 1876 = 3 \text{ MeV.}$$

65. c. Decay constant, $\lambda = 10^{-6}\ \text{s}^{-1}$. The half-life $T_{1/2}$ is thus given by

$$T_{1/2} = \frac{0.639}{\lambda} = \frac{0.639}{10^{-6}} = 0.639 \times 10^6 \text{ s}$$

$$= 192.5 \text{ h} \approx 8 \text{ days} = 1.14 \text{ week}$$

$$\approx 1 \text{ week}$$

66. b. The radioactive decay constant λ is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{32} \text{h}^{-1}$$

From the equation $N = N_0 e^{-\lambda t}$, the fraction of a sample remaining after 16 h is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-\left(\frac{0.693}{32}\right)} = e^{-0.3465} = 0.71$$

67. d. Given:

X has activity A_0 at $t = 0$ and its half-life is 24 years.

Y has activity A_0 at $t = 0$ and its half-life is 16 years.

At $t = 48$ years, activity of X $= \frac{1}{4} A_0$

(2 half-lives have elapsed)

At $t = 48$ years, activity of Y $= \frac{1}{8} A_0$

(3 half-lives have elapsed)

Thus, total activity of the mixture of X and Y at $t = 48$ years is

$$\frac{1}{4} A_0 + \frac{1}{8} A_0 = \frac{3}{8} A_0$$

68. a. Let $t = 0$, $M_0 = 10$ g

$$t = 2\tau = 2\left(\frac{1}{\lambda}\right)$$

Then,

$$M = M_0 e^{-\lambda t} = 10 e^{-\lambda\left(\frac{2}{\lambda}\right)}$$

$$= 10\left(\frac{1}{e}\right)^2 = 1.35 \text{ g}$$

69. b. Number of α-particles emitted $= \frac{232 - 208}{4} = 6$

Decrease in charge number due to α-emission = 12.
But actual decrease in charge number = 90 – 82 = 8.
Clearly, four β-particles are emitted.

70. a.

$$N = \frac{N_0}{2^{t/T}}$$

$$\frac{N_0}{16} = \frac{N_0}{2^{t/T}}$$

$$2^{t/T} = 16 = 2^4 \quad \text{or} \quad \frac{t}{T} = 4$$

$$\text{or} \quad T = \frac{t}{T} = \frac{24}{4} \text{ h} = 6 \text{ h}$$

71. a. $\dfrac{1}{16} = \dfrac{1}{2^{\frac{t}{100}}}$

or $\frac{1}{2^4} = \frac{1}{2^{t/100}}$ or $4 = \frac{t}{100}$

or $t = 400\ \mu s$

72. d. The α-particle emitting radioactive gas, thoron-220, decays to radium-216 an emits an α-particle. The reaction can be represented by

$$^{220}_{90}\text{Th} \to {}^4_2\text{He} + {}^{216}_{66}\text{Ra}$$

By conservation of momentum, we have momentum of α-particle = momentum of recoiling nucleus Ra

$\Rightarrow \quad m_\alpha v_\alpha = m_R v_R$

$$\Rightarrow \quad \frac{v_R}{v_\alpha} = \frac{m_\alpha}{m_R} = \frac{4}{216} = \frac{1}{54}$$

The kinetic energy of Ra, E_R, is related to the kinetic energy of alpha particle E_α by

$$\frac{E_R}{E_\alpha} = \frac{\frac{1}{2}m_R v_R^2}{\frac{1}{2}m_\alpha v_\alpha^2} = \left(\frac{m_R}{m_\alpha}\right)\left(\frac{v_R}{v_\alpha}\right)^2 = \left(\frac{m_R}{m_\alpha}\right)\left(\frac{m_\alpha}{m_R}\right)^2$$

$$= \frac{m_\alpha}{m_R} = \frac{1}{54}$$

$$\therefore \quad E_R = \frac{E_\alpha}{54}$$

73. d

$$\frac{9}{16} = \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{2T}}$$

$$\left(\frac{N}{N_0}\right)^2 = \left(\frac{1}{2}\right)^{t/T} \text{ or } \left(\frac{N}{N_0}\right)^2 = \frac{9}{16}$$

or $\quad \frac{N}{N_0} = \frac{3}{4}$

Note the special technique used in the problem.

74. a. $\frac{A_2}{A_1} = \frac{N_2}{N_1}$

$$\frac{A_2}{10^3} = \frac{1}{2} \quad \text{or} \quad A_2 = \frac{1000}{2} = 500\ s^{-1}$$

75. b

$$\frac{N}{N_0} = \frac{1}{2^{5T/T}}$$

$$\frac{N}{N_0} = \frac{1}{2^5}$$

$$\therefore \quad \frac{N}{N_0} \times 100 = \frac{100}{32} = 3.125$$

76. b. After a time t, a sample of ^{238}U originally consisting of N atoms will have decayed to $Ne^{-\lambda t}$. The number of ^{206}Pb atoms,

$$N_{Pb} = N(1 - e^{-\lambda t})$$

$$\therefore \quad \frac{N_{Pb}}{N_u} = N\frac{(1-e^{-\lambda t})}{Ne^{-\lambda t}} = 0.0058$$

$$e^{\lambda t} - 1 = 0.0058 \quad \Rightarrow \quad e^{\lambda t} = 1.0058$$

$$\therefore \quad t = \frac{1}{\lambda}\ln(1.0058) = \frac{(4.5\times10^9 \text{ years})}{\ln 2}\ln(1.0058)$$

$$= 0.0376\times10^9 \text{ years} = 38\times10^6 \text{ years}$$

77. a. Two α-particles reduce mass number by 8.

Therefore, new mass number = 180 − 8 = 172

Emission of two α-particles reduces charge number by 4. Emission of β-particles increases charge number by 1. Therefore, the new charge number = 72 − 4 + 1 = 69.

78. d

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \quad \frac{N_0}{N} = e^{\lambda t}$$

$$\lambda t = \log_e \frac{N_0}{N}$$

$$t = \frac{1}{\lambda}\log_e \frac{N_0}{N}$$

$$t \propto \log_e \frac{N_0}{N}$$

$$5 \propto \log_e \frac{100}{90}$$

and $\quad 20 \propto \log_e \frac{N_0}{N}$

Dividing,

$$\frac{5}{20} = \frac{\log_{10}\frac{100}{90}}{\log_{10}\frac{N_0}{N}}$$

or $\quad \log_{10}\frac{N_0}{N} = 4\log_{10}\frac{10}{9}$

or $\quad \frac{N_0}{N} = \left(\frac{10}{9}\right)^4 \Rightarrow \frac{N}{N_0} = 0.6561$

Percentage of substance decayed is

$$(1 - 0.6561)\times 100 = 34.39$$

79. c. Let the kinetic energy of the α-particle be E_α and that of the thorium Th be E_{th}. The ratio of kinetic energies is

$$\frac{E_\alpha}{E_{th}} = \frac{\frac{1}{2}m_\alpha v_\alpha^2}{\frac{1}{2}m_{th}v_{th}^2} = \left(\frac{m_\alpha}{m_{th}}\right)\left(\frac{v_\alpha}{v_{th}}\right)^2 \qquad \text{(i)}$$

By conservation of momentum, the momentum of α-particle and that of the recoiling thorium must be equal. Thus,

$$m_\alpha v_\alpha = m_{th} v_{th}$$

or $$\frac{v_\alpha}{v_{th}} = \frac{m_{th}}{m_\alpha} \quad \text{(ii)}$$

Substituting Eq. (ii) in Eq. (i), we have

$$\frac{E_\alpha}{E_{th}} = \left(\frac{m_\alpha}{m_{th}}\right)\left(\frac{m_{th}}{m_\alpha}\right)^2 = \frac{m_{th}}{m_\alpha} = \frac{234}{4} = 58.5$$

Thus, the kinetic energy of the α-particle expressed as the fraction of the total kinetic energy T is given by

$$E_\alpha = \frac{58.5}{1+58.5}T = \frac{58.5}{59.5}T = 0.98\,T$$

which is slightly less than T.

80. c. $\frac{A}{I_0} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ or $A = \frac{I_0}{9}$

81. d. Here, $T = 26.8\text{ min} = 26.8 \times 60\text{ s}$

$\therefore$ Decay constant,

$$\lambda = \frac{0.693}{T} = \frac{0.693}{26.8 \times 60}$$

$$= 4.32 \times 10^{-4}\text{ s}^{-1}$$

Now, 1 curie is equal to 3.71×10^{10} disintegrations per second $= 3.71 \times 10^{10}$.

If N be the number of atoms in one curie, then

$$-\frac{dN}{dt} = \lambda N$$

or $$3.71 \times 10^{10} = 431 \times 10^{-4}\text{ N}$$

$$\therefore \quad N = \frac{3.71 \times 10^{10}}{4.31 \times 10^{-4}} = 8.607 \times 10^{13}$$

Further, atomic weight of RaB = 214 and Avogadro's number $= 6.025 \times 10^{23}$.

$$\text{Mass of one atom} = \frac{214}{6.025 \times 10^{23}}$$

$$\text{Mass of } N \text{ atoms} = \left(\frac{214}{6.025 \times 10^{23}}\right) \times (8.607 \times 10^{13})$$

$$= 3.064 \times 10^{-8}\text{ g}$$

82. b. If the kinetic energy of each particle is k, then

$$2k + 2(0.511\text{ MeV}) = \frac{hc}{\lambda} = \frac{12.4 \times 10^{-3}\text{ MeV Å}}{5 \times 10^{-4}\text{ Å}} = 24.8\text{ MeV}$$

$$\Rightarrow \quad k = \frac{24.8 - 1.022}{2} = 11.9\text{ MeV}$$

83. b. $\frac{A}{A_0} = \frac{N}{N_0}$

Let safe level activity be A. Initial activity $= 64A$.

Hence,

$$\frac{N}{N_0} = \frac{A}{A_0} = \frac{A}{64A} = \frac{1}{64}$$

or $$\left(\frac{1}{2}\right)^n = \frac{1}{64} \quad \text{or} \quad n = 6$$

Hence,

$$\frac{t}{T} = n = 6$$

$$\because \quad T = 2\text{ h}$$

$$\therefore \quad t = 12\text{ h}$$

84. c. $N_1\lambda_1 = N_2\lambda_2$

$$T = \frac{0.693}{\lambda}$$

Hence,

$$2.8 \times 10^6 \times \frac{0.693}{T_1(\text{U})} = 1 \times \frac{0.693}{T_2(\text{Ra})}$$

$$\therefore \quad T_1(\text{U}) = 1600 \times 2.8 \times 10^6$$

$$= 4.48 \times 10^9 \text{ years.}$$

$$\approx 4.5 \times 10^9 \text{ years}$$

85. b. Number of α-particles per second = activity $= (-dN/dt) = N\lambda$

where

$$N = \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3}$$

$$\lambda = 5.8 \times 10^{-8}\text{ s}^{-1}$$

So,

$$A = N\lambda$$

$$= \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3} \times 5.8 \times 10^{-8}$$

$$= 1.7 \times 10^{11}$$

86. a. Let $\frac{M_1}{M_2}$ (mass ratio) $= \frac{1}{2}$

2 days $= 2 \times 24\text{ h} = 48\text{ h}$

For first substance, 4 half-life periods and for second substance 3 half-life periods are passed; the masses are reduced to

$$M_1' = M_1 \times \left(\frac{1}{2}\right)^4$$

$$M_2' = M_2 \times \left(\frac{1}{2}\right)^3$$

$$\therefore \quad \frac{M_1'}{M_2'} = \frac{M_1}{M_2} \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2} = \frac{1}{1}$$

87. d. $R_1 = N_1\lambda, \quad R_2 = N_2\lambda$

Also,

$$T = \frac{\log_e 2}{\lambda} \quad \text{or} \quad \lambda = \frac{\log_e 2}{T}$$

$$\therefore \quad R_1 - R_2 = (N_1 - N_2)\lambda$$

$$= (N_1 - N_2)\frac{\log_e 2}{T}$$

$$\therefore \quad (N_1 - N_2) = \frac{(R_1 - R_2)T}{\log_e 2}$$

i.e., $(N_1 - N_2) \propto (R_1 - R_2)T$

88. c. We know that

$$N = N_0 \left(\frac{1}{2}\right)^{n_A}$$

For A,

$$N = N_0 \left(\frac{1}{2}\right)^{n_A} = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$\left[\because n_A = \frac{t}{T_A} = \frac{80}{20} = 4\right]$$

For B,

$$N_B = N_0 \left(\frac{1}{2}\right)^{n_B} = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\therefore \quad \frac{N_A}{N_B} = \frac{1}{4} \quad \text{or} \quad N_A : N_B = 1:4$$

89. d. Let the decay constants for the first and second processes be λ_1 and λ_2 and the effective decay constant for the combined process be λ. Then,

$$\lambda_1 = \frac{\log_e 2}{t_1}, \quad \lambda_2 = \frac{\log_e 2}{t_2} \quad \text{and} \quad \lambda = \frac{\log_e 2}{t}$$

Now, the probability for decay through first process in a small time interval dt is $\lambda_1\,dt$ and the probability for decay through second process in the same time interval dt is $\lambda_2\,dt$. The probability for decay by the combined process in the same time interval dt is $\lambda_1\,dt + \lambda_2\,dt$.

But this is also equal to λdt.

$$\therefore \quad \lambda\,dt = \lambda_1\,dt + \lambda_2 dt$$

$$\therefore \quad \lambda = \lambda_1 + \lambda_2$$

$$\text{or} \quad \frac{\log_e 2}{t} = \frac{\log_e 2}{t_1} + \frac{\log_e 2}{t_2}$$

$$\text{or} \quad \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \text{ or } t = \frac{t_1 t_2}{t_1 + t_2}$$

90. a. According to Avogadro's hypothesis,

$$N_0 = \frac{6.02 \times 10^{23}}{226} = 2.66 \times 10^{21}$$

$$\text{Half-life} = T = \frac{0.693}{\lambda} = 1620 \text{ years}$$

$$\therefore \quad \lambda = \frac{0.6931}{1620 \times 3.16 \times 10^7}$$

$$= 1.35 \times 10^{-11}\,\text{s}^{-1}$$

Because half-life is very much large as compared to its time interval, hence $N \approx N_0$. Now,

$$\frac{dN}{dt} = \lambda N \approx \lambda N_0$$

or
$$dN = \lambda N_0\,dt$$
$$= (1.35 \times 10^{-11})(2.66 \times 10^{21}) \times 1$$
$$= 3.61 \times 10^{10}$$

91. c. Radius R of a nucleus changes with the nucleon number A of the nucleus as

$$R = 1.3 \times 10^{-15} \times A^{1/3}\,\text{m}$$

Hence,

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{128}{16}\right)^{1/3} = (8)^{1/3} = 2$$

$$\therefore \quad R_2 = 2R_1 = 2(3 \times 10^{-15})\,\text{m}$$
$$= 6 \times 10^{-15}\,\text{m}$$

92. b. For nucleus of ${}_8O^{16}$:

$$\text{Mass} = (16)(1.67 \times 10\text{–}27)\,\text{kg}$$

$$\text{Volume} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi (3 \times 10^{-15})^3\,\text{m}^3$$

$$= 36\pi \times 10^{-45}\,\text{m}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{16 \times 67 \times 10^{-27}\,\text{kg}}{36\pi \times 10^{-46}\,\text{m}^3}$$

$$= 2.35 \times 10^{17}\,\text{kg m}^{-3}$$

93. a. Mass defect,

$$\Delta m = 20(1.007277 + 1.00866) - 39.97545$$
$$= 40.31874 - 39.97545$$
$$= 0.34329\,\text{a.m.u.}$$

$\therefore$ Binding energy $= 0.34329 \times 931 = 319.6$ MeV

When one atom of Ca-40 completely dissociates, the energy to be supplied $= 319.6$ MeV.

$$1\text{ g of Ca-40 contains } \frac{6.023 \times 10^{23}}{40} = 1.506 \times 10^{22}\text{ atoms}$$

The energy required for the dissociation of 1 g of Ca-40

$$= 319.6 \times 1.506 \times 10^{22}$$
$$= 4.813 \times 10^{24}\,\text{MeV}$$

94. c. Binding energy per nucleon of fission products is 8.5 MeV.

Binding energy per nucleon of reactants = 7.6 MeV

Increase in binding energy per nucleon is $8.5 - 7.6 = 0.9$ MeV.

Energy released per nucleon in fission is 0.9 MeV.

$$\therefore \text{ Fractional energy released} = \frac{0.9}{931} = \frac{1}{1000}$$

Percentage of mass converted into energy during fission

$$= \frac{1}{1000} \times 100 = 0.1\%$$

95. b. $Q = (\Sigma B_r - \Sigma B_p)\,c^2$

where ΣB_r = sum of the masses of reactants

and ΣB_p = sum of the masses of the products

$$\Sigma B_r = 2 \times 2.014741\text{ a.m.u.} = 4.0294892\text{ a.m.u.}$$

$$\Sigma B_p = (3.016977 + 1.008987)\text{ a.m.u.}$$
$$= 4.025964\text{ a.m.u.}$$

$$\Sigma B_r - \Sigma B_p = (4.029482 - 4.025694)\text{ a.m.u.}$$
$$= 0.003518\text{ a.m.u.}$$

Decrease in mass appears as equivalent energy.

$\therefore \quad Q = 0.003518 \times 931$ MeV
$= 3.27$ MeV

96. b. $P = 10^6$ W

Time $= 1$ day $= 24 \times 36 \times 10^2$ s

Energy produced,

$U = Pt = 10^6 \times 24 \times 36 \times 10^2$
$= 24 \times 36 \times 10^8$ J

Energy released per fusion reaction is

$20\text{ MeV} = 32 \times 10^{-13}$ J

Energy released per atom of $_1H^2$ is

32×10^{-13} J

Number of $_1H^2$ atoms used is

$$\frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 22 \times 10^{21}$$

Mass of 6×10^{23} atoms $= 2$ g

Mass of 27×10^{21} atoms $= \frac{2}{6 \times 10^{23}} \times 27 \times 10^{21} = 0.1$ g

97. b. Power P of fission reactor,

$P = 10^6\text{ W} = 10^6\text{ J s}^{-1}$

Time $= t = 1$ day $= 24 \times 36 \times 10^2$ s

Energy produced, $U = Pt$

or $\quad U = 10^6 \times 24 \times 36 \times 10^2$
$= 24 \times 36 \times 10^8$ J

Energy released per fission of U^{235} is

$200\text{ MeV} = 32 \times 10^{-12}$ J

Number of U^{235} atoms used is

$$\frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 27 \times 10^{20}$$

Mass of 6×10^{23} atoms of $U^{235} = 235$ g

Mass of 27×10^{20} atoms of U^{235} is

$$\left(\frac{235}{6 \times 10^{23}}\right)(27 \times 10^{20}) = 1.058\text{ g} = 1\text{ g}$$

98. b. Mass of one atom of U^{235} is

235.121420 a.m.u.

Mass of one neutron $= 1.008665$ a.m.u.

Sum of the masses of U^{235} and neutron

$= 236.130085 = 236.130$ a.m.u.

Mass of one atom of U^{236} is

236.123050 a.m.u. $= 236.123$ a.m.u.

Mass defect $= 236.136 - 236.123$
$= 0.007$ a.m.u.

Therefore, energy required to remove one neutron is

$0.007 \times 931\text{ MeV} = 6.517\text{ MeV} = 6.5$ MeV

99. c. Total binding energy of helium atom ($_2He^4$) is

$4 \times 7 = 28$ MeV

Total binding energy of deuteron $_1H^2$(1p + 1n) is

$2 \times 1.1 = 2.2$ MeV

Hence, binding energy of 2 deuterons is

$2 \times 2.2 = 4.4$ MeV

So, the energy released in forming helium nucleus from two deuterons is

$28 - 4.4\text{ MeV} = 23.6$ MeV

100. c. Mass defect,

$\Delta m = 2(2.015) - (3.017 + 1.009) = 0.004$ a.m.u.

As 1 a.m.u. $= 931.5\text{ MeV}/c^2$, energy released will be $0.004 \times 931.5\text{ MeV} = 3.726$ MeV.

Energy released per deuteron is

$$\frac{3.726}{2} = 1.863\text{ MeV}$$

Number of molecules in 1 kg deuterons is

$$\frac{6.02 \times 10^{26}}{2} = 3.01 \times 10^{26}$$

Therefore, energy released per kg of deuterium fusion

$= (3.01 \times 10^{26} \times 1.863)$
$= 5.6 \times 10^{26}\text{ MeV} \approx 9.0 \times 10^{13}$ J

101. b. Here, half-life of radium, $t = 1500$ years

Disintegration constant $\lambda = \frac{0.693}{T} = \frac{0.693}{1500}$ year^{-1}

$N_0 = 1$ g $\quad N = 10\text{ mg} = 1\text{ centigram} = 10^{-2}$ g

$\therefore \quad N = 10$ mg

Now apply $N = N_0\, e^{-\lambda t}$.

102. b. Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. Same happens with the proton except it is electrostatically repelled by the six protons already inside the carbon nucleus. The repulsion prevents a 100 ms^{-1} proton from getting close enough to the nucleus. Therefore, the answer is (b).

103. c. For this substance 7 days correspond to 3.5 half-lives. Over 3 half-lifes the sample reduces to $\frac{1}{2^3} = \frac{1}{8}$ of its initial mass. After 4 half-lifes, the sample has only $\frac{1}{2^4} = \frac{1}{16}$ of its initial mass. Hence, after 3.5 half-lives the sample must contain somewhere between 1/8 and 1/16 of its initial mass.

Hence, 5 g is somewhere between 1/8 and 1/16 of the initial mass.

So, the initial mass is somewhere between $8 \times 5 = 40$ g and $16 \times 5 = 80$ g.

104. d. As we regard the decay process as a spontaneous and statistical process, therefore the decay can start any time after $t = 0$. Therefore, the answer is (d).

105. b. A nucleus contains protons and neutrons with no antiprotons and antineutrons. Hence, answer can be either (b) or (d). Due to conservation of spin, the answer is (b).

106. d. Atomic mass $M(\text{H})$ of hydrogen and nuclear mass (M_n) are

$M(\text{H}) = 1.007825\text{ u}$ and $M_n = 1.008665\text{ u}$

Mass defect,

$$\Delta m = [M(\text{H}) + M_n - M(\text{D})]$$

$$M(\text{D}) = \text{mass of deuteron} = 2.016490\text{ u} - 2.014102\text{ u} = 0.002388\text{ u}$$

As 1 u corresponds to 931.494 MeV energy, therefore, mass defect corresponds to energy,

$$E_b = 0.002388 \times 931.5 = 2.224\text{ MeV}$$

107. d. Nuclear reactions conserve total charge, and also conserve the total approximate mass. The other particles in the reaction will have mass $= 236 - 140 - 94 = 2$

The other particles are two neutrons. Hence, (a) is not correct.

For nuclei, number of protons tells the charge. So, the other particles must have charge Z such that

$$92 = 54 + 38 + Z$$

$$\therefore \quad Z = 0$$

Therefore, the other particles have a total atomic mass 2 and total charge 0. Hence, only (d) is correct.

108. c. All neutrons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron $\left({}_3^4\text{Li}\right)$ just as vigourously as it holds together two protons and two neutrons $\left({}_2^4\text{He}\right)$. Specifically, protons electrostatically repel other protons. This repulsion tries to make a nucleus fly apart. Since ${}_2^4\text{He}$ contains only two protons, the attractive strong nuclear forces overcome the repulsion of the protons. Hence, the nucleus holds together. But in ${}_3^4\text{Li}$, the mutual repulsion of the three protons overcomes the strong nuclear attractions and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus). Therefore, the answer will be (c).

109. b. Number of atoms in 2 kg fuel

$$\frac{2}{235} = 6.02 \times 10^{26} = 5.12 \times 10^{24}$$

Fission rate = Number of atoms fissioned in one second

$$= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60}$$

$$= 1.975 \times 10^{18}\text{ s}^{-1}$$

Each fission gives 185 MeV. Hence, energy obtained in one second,

$$P = 185 \times 1.975 \times 10^{18}\text{ MeV s}^{-1}$$

$$= 185 \times 1.975 \times 10^{18} \times 1.6 \times 10^{-19}\text{ J s}^{-1}$$

110. c. ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_1\text{He}^4 + \text{Q}$

$$\Delta m = m({}_2\text{He}^4) - 2m({}_1\text{H}^2)$$

$$\Delta m = 4.0024 - 2(2.0141)$$

$$\Delta m = -0.0258\text{ u}$$

Now, $Q = c^2 \Delta m$

or $= (0.0258)(931.5)\text{ MeV}$

or $\approx 24\text{ MeV}$

111. c. $$\frac{dN}{dt} = n - \lambda N$$

Because the population N is simultaneously increasing at rate n and decreasing due to decay at rate λN.

$$\int_{N_0}^{N} \frac{dN}{n - \lambda N} = \int_0^t dt$$

$$\frac{1}{\lambda}\ln\left(\frac{n - \lambda N_0}{n - \lambda N}\right) = t$$

$$N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right)e^{-\lambda t}$$

112. c. $t = 0, N = N_0$

$t = 6.93, N = N_0/4$

$N_0/4$ is the sample left after two half-lives

$$\therefore \quad 2t_{1/2} = 6.93$$

$$\Rightarrow \quad 2 \times \frac{0.693}{\lambda} = 6.93 \Rightarrow \lambda = 0.2\text{ min}^{-1}$$

$$\Rightarrow \quad t = 60\text{ min}$$

$$\therefore \quad N = N_0 e^{-\lambda t} = N_0 e^{-0.2 \times 60} = \frac{N_0}{e^{12}}$$

113. b. The difference in the binding energies is the energy required to add an extra neutron.

114. d. Conserve the number of nucleons.

115. b. Calculate time when it reaches a height of 1000 m, then use $A = \lambda N$.

116. d. Use mass balance and balance of atomic number.

117. b. Use conservation of linear momentum

118. b $$t_{1/2} = \frac{0.639}{\lambda}$$

119. d $$\left|\frac{dN}{dt}\right| = \lambda N$$

Number of radium nuclei in m g $= \dfrac{N_A m}{226}$

Decay constant, $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{1620 \times 3.16 \times 10^7}$

$$\left|\frac{dN}{dt}\right| = 10 = \frac{6.02\times10^{23}m}{226}\times\frac{0.693}{1620\times3.16\times10^7}$$

$$\therefore \quad m = \frac{10\times226\times1620\times3.16\times10^7}{6.02\times10^{23}\times0.693}$$

$$= 2.77\times10^{-10}\,\text{g}$$

$$= 2.77\times10^{-13}\,\text{kg}$$

120. c. The Q value of the first reaction implies that

$$^{14}\text{N} + \text{d} = {}^{15}\text{N} = \text{p} + 8.53\,\text{MeV}$$

where ^{14}N represents the mass of ^{14}N nucleus in energy units. This can be rearranged to give

$$^{14}\text{N} - {}^{15}\text{N} = \text{p} - \text{d} + 8.53\,\text{MeV}$$

The second reaction similarly implies as

$$^{15}\text{N} - {}^{13}\text{C} = \alpha - \text{d} + 7.58\,\text{MeV}$$

and the third reaction gives

$$^{13}\text{C} - {}^{11}\text{B} = \alpha - \text{d} + 5.16\,\text{MeV}$$

Adding these three equations, we have

$$^{14}\text{N} - {}^{11}\text{B} = \text{p} + 2\alpha - 3\text{d} + 21.27\,\text{MeV}$$

$$^{11}\text{B}\,(\alpha,\text{n})^{14}\text{N} = {}^{11}\text{B} - {}^{14}\text{N} + \alpha - \text{n}$$

$$= 3\text{d} - \alpha - \text{p} - \text{n} - 21.27\,\text{MeV}$$

Now,

$$3\text{d} - \alpha - \text{p} - \text{n} = (3\times2.014 - 4.0020 - 1.0078 - 1.0087)\,\text{a.m.u.}$$

$$= 0.0229\,\text{a.m.u.}$$

$$\therefore \quad Q = (0.0229\times931 - 21.27)\,\text{MeV} = 0.05\,\text{MeV}$$

121. d. If d is the distance of closest approach given, then the angular momentum $= mvd = 10^{-33}$ J s

$$E = \frac{1}{2}mv^2 = 1\,\text{MeV} = 1.6\times10^{-13}\,\text{J}$$

Momentum,

$$p = \sqrt{2m_n E} = \sqrt{2\times1.6\times10^{-27}\times1.6\times10^{-13}}$$

$$= 1.6\sqrt{2}\times10^{-20}\,\text{kg m s}^{-1}$$

Distance of closest approach,

$$d = \frac{10^{-33}}{1.6\sqrt{2}\times10^{-20}}$$

$$= \frac{1}{1.6\sqrt{2}}\times10^{-13} = \frac{100}{1.6\sqrt{2}}\,\text{fm} = 0.44\,\text{fm}$$

122. c. The binding energy per nucleon is lowest for very light nuclei such as $^4_2$He, is greatest around A = 60, and then decreases with increasing A.

123. d. Two protons and two neutrons are lost in an α-decays, so Z and N each decrease by 2. A β^+ decay changes a proton to a neutron, so Z decreases by 1 and N increases by 1. The net result is Z decreases by 3 and N decreases by 1.

$$^A_Z\text{X} \xrightarrow{\alpha\text{-decay}} {}^{A-4}_{Z-2}\text{Y} \xrightarrow{\beta\text{-decay}} {}^{A-4}_{Z-3}\text{Z}$$

Initially, number of neutrons $N_i = (A - Z)$

Now, number of neutrons $N_f = A - 4 - Z + 3 = N_i - 1$

124. c. $m(^{198}\text{Au}_{79}) = 197.968225$ u

$$m\left(^{198}_{80}\text{Hg}\right) = 197.966752\,\text{u}$$

Mass defect,

$$\Delta m = 1.473\times10^{-3}\,\text{u} = 1.371\,\text{MeV}$$

Energy of γ-photon = 0.412 MeV

Maximum kinetic energy of the electron emitted in the decay is

$$E_e = 1.371\,\text{MeV} - 0.412\,\text{MeV} = 0.959\,\text{MeV}$$

Multiple Correct Answers Type

1. a., b., c., d. We have, $6.25\% = \frac{6.25}{100} = \frac{1}{16}$

The given time of 4 h thus equals 4 half-lives so that the half-life is 1 h.

Since half-life $= \frac{\ln 2}{\text{decay constant}}$ and mean life $= \frac{1}{\text{decay constant}}$, after further 4 h, the amount left over would be $\frac{1}{2^4}\times\frac{1}{2^4}$, i.e., $\frac{1}{256}$ or $\frac{100}{256}$ or 0.39% of original amount.

2. a., b., d. It has been observed that total mass of nucleus is always less than the sum of the masses of its nucleons. The energy difference between the nucleus and its constituent particles due to their mass difference is termed as the binding energy of the nucleus.

In other words, we can say that to break the nucleus into its constituent particles, some energy is needed to be supplied. This energy is termed as binding energy of the nucleus.

For (a), more is the binding energy per nucleon, more is the energy required to break the nucleus and hence we can say the more stable the nucleus is.

For (b), (c) and (d), in actual the binding energy is always positive but if it were zero, then nucleus will break spontaneously.

3. a., c., d. All the statements are very conceptual statements related to different decays.

4. a, d. For an exothermic reaction

$$X + x \to Y + y$$

If K_i is the kinetic energy of incident particle x, then from energy conservation,

$$K_i + (m_x + M_x)c^2 = K_Y + K_y + (M_Y + m_y)c^2$$
$$K_Y + K_y = K_i + (m_x + M_X - M_Y - m_y)c^2$$
$$K_Y + K_y = K_i + Q$$

In any exothermic reaction, mass of the products is less than the mass of reactants, i.e., in products, the nucleons are more tightly bound and hence have greater BE per nucleon as compared to BE per nucleon of reactants. For endothermic reaction to be carried out, minimum energy given to the reactant must be greater than $|Q|$ value.

5. a., b., c. In general, fission and fusion processes are exothermic reactions, i.e., energy is released. Hence, mass of products must be less than mass of the reactant nuclide, and BE/A of reactants < BE/A of product nuclides.

6. a., b., c. If the nuclear reaction involving β-decay is n $\to$ p + e^{-1}, the spins on two sides are not equal as all the three (neutron, proton and electron) have spins of $+\frac{1}{2}$. So, to conserve angular momentum (spin), some other particle must be emitted.

Through experiments it has been observed that direction of emitted electron and recoiling nuclei are almost never exactly opposite as required for linear momentum to be conserved.

During β-decay, the energy of electron is found to vary continuously from 0 to a maximum value (this maximum value is a characteristic of nuclide). To explain this experimental observation, we also need some other particle.

7. a., c. Half-lives of both the samples would be same as half-life is the property of radioactive material and is independent of number of nuclei present or its activity. Let $R_{0B} = R_0$, then $R_{0A} = 2B_0$, where R_0 denotes initial activity.

Activity of A after 5 half-lives is

$$R_A = \frac{R_{0A}}{2^5} = \frac{2R_0}{2^5}a$$

Activity of B after 5 half-lives is

$$R_B = \frac{R_{0B}}{2^5} = \frac{R_0}{2^5}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{2}{1}$$

8. a., c. $\lambda = (0.173 \text{ year})^{-1}$

$$N = N_0 e^{-\lambda t}$$

As $t = \frac{1}{\lambda}$, hence

$$N = \frac{N_0}{e} = \frac{N_0}{2.178} = 0.37\,N_0$$

$$\Rightarrow \quad T = \frac{0.693}{\lambda} = \frac{0.693}{0.173} = 4 \text{ years}$$

9. a., d. In α-decay, the entire energy is carried away by the α-particles as its kinetic energy. In β^--decay, the energy is shared between the β-particle and the anti-neutrino. Hence the speed of the β-particle will vary, depending on the energy of the anti-neutrino.

10. b., c., d. Statement (a) is incorrect. In fact,

$$A = Z + N$$

Statements (b), (c) and (d) are correct; they are the definitions of isobars, isotopes and isotones.

11. a., c. The idea of 'magic number' has led to the shell model and the nuclides with these number of protons or neutrons have been compared with the 'inert gases' vis-à-vis stability in terms of 'closed shells'.

12. a., b., c. The last statement is incorrect because the amount of energy released per unit mass of the fuel is much more for fusion than for fission. Hence, (a), (b) and (c) are correct.

13. b., c. Statement (a) is incorrect. The $_2He^4$ nucleus (or the α-particle) is exceptionally stable and has a much higher value of BE per nucleon than that for most other light nuclei. Statement (b) is correct but the reason of decrease in binding energy is different for the cases of smaller and larger values of A. The reason for the decrease in the BE per nucleon for nuclei with large A is that with an increase in the number of protons, the Coulomb repulsion increases. On the other hand, the decrease in the BE per nucleon for nuclei with small A is due to a surface effect: the nucleons at the surface being less strongly bound than those in the interior. Statement (c) is also correct. The energy required to remove one neutron (i.e., one nucleon) is the same as the binding energy per nucleon for a given isotope.
Statement (d) is incorrect. To ensure both charge and mass number conservation, a proton must be produced as a by-product of the reaction:

$$_1D^2 + {}_1D^2 \to {}_1T^3 + {}_1P^1 + Q$$

14. a., b., c. Use $\frac{N}{N_0} = e^{-\lambda t}$

15. a., d.

a. True, Cd absorbs neutrons.
b. No, concrete reflects, does not slow down.
c. 'Moderate the activity' is not correct. 'Moderator' in the sense of slowing the neutrons is different.
d. True, it is a fact.

16. a., b., c. We know,

$$N = N_0\, e^{-\lambda t} \qquad \text{(i)}$$

where, N = number of decayed nuclei in the sample at time t, N_0 = initial number of nuclei.
Hence, total number of undecayed nuclei = $(N_0 - N)$
Substituting it in (i), we get

$$N_0 - N = N_0(1 - e^{-\lambda t})$$

This shows that the total number of undecayed nuclei decays exponentially with time and total number of decayed unclei grows exponentially with time. Now,

$$R = -\lambda N = \frac{dN}{dt} \qquad (R = \text{activity})$$

Hence, activity $(R) \propto$ number of undecayed nuclei.
Therefore, (a), (b), (c) are correct answers.

17. a., c. $R = R_0 A^{1/3}$

For O^{16}, $R = R_0(16)^{1/3}$

For ${}_{54}X^{128}$, $R' = R_0(128)^{1/3}$

$$R' = \left(\frac{128}{16}\right)^{1/3} R = 2R$$

$$\therefore \quad V' = \frac{4}{3}\pi R'^3 = 8V$$

Assertion-Reasoning Type

1. **a.** Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
2. **a.** Statement II is true by definition and correctly explains Statement I, namely, ${}_ZX^4$ undergoes 2 α-decays, 2 β-decays (negative β) and 2 γ-decays. As a result, the daughter product is ${}_{Z-2}X^{A-B}$.
3. **d.** Atomic mass and mass number are different. Mass number is simply representing number of nucleons, while atomic mass is the average of the masses of isotopes of a given element and has units of u (atomic mass unit).
4. **a.** Here, both the statements are correct and Statement II correctly explains Statement I.
5. **a.** Statement II is correctly explaining Statement I. More probability of decay means faster decay process and hence shorter half-life.
6. **a.** If the half-life of a radioactive isotope is small as compared to the age of organic sample, then over the age of the sample the activity of radioactive isotope becomes very small and hence is impossible to detect. While this process will not arise if we use radioactive isotope having larger half-life for dating with organic samples.
7. **a.** As in a nucleus, nucleons are bounded by short-range nuclear force, so a given nucleon is in interaction only with neighboring nucleons. So, detaching a nucleon from a nucleus is irrespective of the fact that how many nucleons are present in the nucleus. Moreover, due to short-range nuclear force only, the E_b/A versus A curve is slowly varying for $A > 40$.

   While in atoms electrons are bound with nucleus by Coulomb's force which is a long-range force and depends on the number of protons in the nucleus and electron separation from the nucleus. If we take the average of the energies required to detach all the electrons from the outermost shell to the innermost K shell, then this average increases rapidly with increase in atomic number.
8. **a.** When fission of heavy nucleus takes place, it splits itself into two lighter nuclei which are having too many neutrons and are highly unstable. To attain stability, they decay neutrons and hence try to achieve N/Z ratio somewhat greater than 1.

Comprehension Type

For Problems 1–3

1. a., 2. b., 3. c.

$$\frac{dN_X}{dt} = K - \lambda N_X$$

$$N_X = \frac{1}{\lambda}[K - (K - \lambda N_0)e^{-\lambda t}]$$

$$\frac{dN_Y}{dt} = \lambda N_X$$

$$N_Y = Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$$

For Problems 4–6

4. a., 5. a., 6. a.

4. a. $\dfrac{dN}{dt} = q_0 t - \lambda N; \quad \dfrac{dN}{dt} + \lambda N = q_0 t$

5. a. $N = \dfrac{q_0 t}{\lambda} - \dfrac{q_0}{\lambda^2} + \dfrac{q_0}{\lambda^2}e^{-\lambda t}$

$$P_{\text{inst}} = \lambda N E_0 = \left[q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda}e^{-\lambda t}\right]E_0$$

6. a. $P_{\text{av}} = \dfrac{\int_0^t \left[q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda}e^{-\lambda t}\right]E_0\,dt}{\int_0^t dt}$

$$= \left[\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t}e^{-\lambda t}\right]E_0$$

For Problems 7–9

7. c., 8. b., 9. d.

7. c. From the graph and the fact that the n/p (= no. of neutrons/ no. of protons) ratio for magnesium is 27/12, which is greater than 1 (= unit slope).

8. b. ${}^{230}_{90}\text{Th} \rightarrow {}^{214}_{83}\text{Bi} + x\,{}^{0}_{-1}e^- + y\,{}^{4}_{2}\text{He}^{2+} + z$ (gamma ray)

Since the sum of the atomic numbers and mass numbers on either side of the equation must be equal (matter cannot be created or destroyed), we get

$$90 = 83 - x + 2y \quad \text{and} \quad 230 = 214 + 4y$$

Solving, we get

$$y = 4 \quad \text{and} \quad x = 1$$

The order of the reactions is irrelevant (i.e., alpha, beta,...). Since gamma rays have no atomic number or mass number, the value of z does not affect this particular calculation.

9. d. Alpha radiation consists of the largest particles (helium nuclei with a mass number of 4, thus the greatest inertia) and are the slowest (about 1/3 times the speed of the light). Beta radiation consists of smaller (electrons, 1/1370 times lighter than a proton) and faster particles (about 4/5 times the speed of light). Gamma radiation consists of the smallest particles (photons, no mass) which travel at the greatest speed (at the speed of light)

For Problems 10–12

10. b., 11. d., 12. c.

10. b. In equilibrium,
rate of decay = rate of production

11. d. As rate of decay = Rate of production

$$P = \lambda N \quad \Rightarrow \quad N = \frac{P}{\lambda} = \frac{Pt_{1/2}}{\ln 2} = 1.8 \times 10^{15}$$

12. c. As $N = \dfrac{Pt_{1/2}}{\ln 2}$

it is dependent upon P and $t_{1/2}$. Initial number of ^{56}Mn nuclei will make no difference as in equilibrium, rate of production equals rate of decay. Large initial number will only make equilibrium come sooner.

For Problems 13–15

13. d., 14. a., 15. c.

13. d. Radioactivity is independent of all external conditions. When a nucleus undergoes an α-decay, its atomic number decreases by 2 and in beta decay, atomic number increases by 1.

14. a. $T_M = \dfrac{1}{\lambda}$ and $T_H = \dfrac{0.693}{\lambda}$

Now,

$$\frac{T_M}{T_H} = \frac{1}{\lambda} \times \frac{\lambda}{0.693} = \frac{1}{0.693}$$

i.e., $\quad T_M > T_H$

From the above explanation, it is clear that choice (a) is correct and other choices (b), (c) and (d) are wrong.

15. c. $\quad A = \dfrac{dN}{dt} = \lambda N; \quad n = \lambda N$

$$n = \frac{0.693}{T} N\,; \quad T = \frac{0.693\, N}{n}$$

For Problems 16–18

16. a., b., c., d.; 17. c., 18. d.

16. a., b., c., d. All options are basic properties of nuclear forces. So, all options are correct.

17. c. From graph, it is obvious that binding energy per nucleon $\left(\text{i.e., } \vec{B} = \dfrac{B}{A}\right)$ is maximum for middle order element and its value is around 8.8 MeV per nucleon.

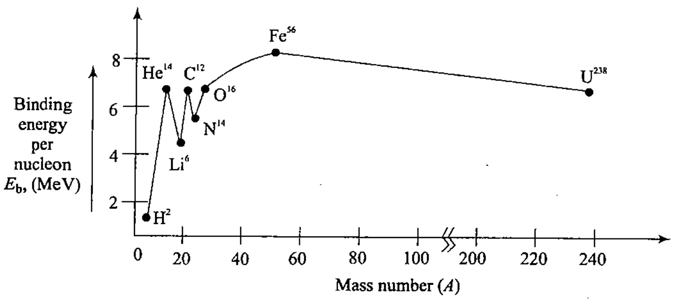


Fig. 5.35

18. d. Radius (R) of nucleus is related with mass number (A) as

$$R \propto A^{1/3}$$

Now, $\quad$ Volume $\propto R^3 \propto A$

For Problems 19–22

19. d., 20. a., 21. c., 22. b.

19. d. According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting m_{neutron} equal to $m_{\text{proton}} + m_{\text{electron}} + m_{\text{third particle}}$. By constrast, the total energy in this case, the sum of 'mass energy' and kinetic energy, is conserved. If E denotes total energy, then

$$E_{\text{neutron}} = E_{\text{proton}} + E_{\text{electron}} + E_{\text{third particle}}$$

The neutron has energy 949.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV = 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy

$$E_{\text{third particle}} = E_{\text{neutron}} - E_{\text{proton}} - \text{Electron}$$

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

20. a. As just shown, energy conservation allows us to calculate the third particle's total energy. But we don not know what percentage of that total is mass energy vs, kinetic energy.

21. c. The term 'c^2' in $E = mc^2$ can mislead you into thinking that E varies with the square of mass. But E varies linearly with m. Indeed, we can write Einstein's equation as E = (constant)m, where the constant happens to equal c^2. By contrast, graphs (b) and (d) represent an equation such as E = (constant) m^2.

Since the particles in this ensemble all have positive kinetic energy, the total energy is positive even when the mass is zero [relativistic physics permits this, because kinetic energy no longer equals $(1/2)mv^2$]. So, the graph does not begin at the origin.

22. b. A proton with only mass energy ($E_{\text{total}} = mc^2$) could never decay into a neutron and other particles, because a neutron has more mass energy than a proton does. Therefore, the reaction products would have more energy than the original proton, in violation of energy conservation.

But suppose the proton has additional energy, such as potential energy from its interactions with other particles in a nucleus. Then, the total energy of the proton—mc^2 plus potential energy—can equal the total energy of the products. In this case, the products (neutron and other particles) have more mass and more kinetic energy than the proton did. Therefore, during the reaction, the increase in mc^2 and the increase in kinetic energy must be offset by an equally big decrease in the system's potential energy. This happens when a nucleus undergoes β^+ decay.

For Problems 23–26

23. d., 24. a., 25. b., 26. c.

23. d. Nuclear reactions conserve total charge and also conserve the total approximate mass (as measured by the atomic mass number). Therefore, since the uranium, xenon, and strontium nuclei have atomic masses 236, 140 and 94, the 'other particles' must have total atomic mass A such that

$$236 = 140 + 94 + A$$

So, $A = 2$. The other particles are two nucleons. This narrows down the answer to options (b), (c) and (d). For nuclei, the atomic number—i.e., the number of protons—tells us the charge. So, the other particles must have total charge Z such that

$$92 = 54 + 38 + Z \quad \text{or} \quad Z = 0$$

In summary, the other particles have total atomic mass 2 and total charge 0. Only option (d) fits this description.

24. a. According to the passage, all nucleons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron ($^4_3$Li) just as vigorously as it holds together two protons and two neutrons ($^4_2$He). But other forces play a role, too. Specifically, protons electrostatically repel other protons. This repulsion 'tries' to make a nucleus fly apart. Since $^4_2$He contains only two protons, the attractive strong nuclear force overcomes the repulsion of the protons. Therefore, the nucleus holds together. But in $^4_3$Li, the mutual repulsion of the three protons overcomes the strong nuclear attraction, and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus).

25. b. Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. The same reasoning would apply to the proton, except for one thing; the proton feels electrostatically repelled by the six protons already inside the carbon nucleus. This repulsion prevents a 100 m s^{-1} proton from getting close enough to the nucleus for the strong force to 'kick in'. Remember, the strong force becomes important only at tiny distances.

26. c. Graph (a) would correctly show the binding energy per nucleons, but this differs from the total binding energy of the nucleus. The total binding energy takes into account not only the binding energy per nucleon, but also the number of nucleons.

To understand this subtle point, compare $^{94}_{38}$Sr to $^{236}_{32}$U.

As the passage states, $^{94}_{38}$Sr has a higher binding energy per nucleon. Here,

$^{94}_{38}$Sr binding energy per nucleon ≈ 9 MeV

$^{236}_{32}$U binding energy per nucleon ≈ 7 MeV

But $^{236}_{32}$U still has a higher total binding energy, simply because it contains more nucleons. Indeed, since total binding energy of a nucleus = binding energy per nucleon × number of nuclei, the total binding energies are

$^{94}_{38}$Sr × (94 nucleons) ≈ 846 MeV

$^{236}_{32}$U × (236 nucleons) ≈ 1650 MeV

So, as atomic mass increases, so does the total binding energy. Uranium has more binding energy, even though it has less binding energy per nucleon.

Critically, however, the total binding energy does not increase linearly with atomic mass. It increases at a smaller and smaller rate, precisely because the binding energy per nucleon decreases. Indeed, since 'atomic mass number' specifies the number of nucleons, the binding energy per nucleon is simply the slope of a binding energy vs. atomic mass number graph. Between strontium and uranium, this slope decreases, as just explained. Thus, the answer must be (c) rather than (d).

For Problems 27–29

27. c., 28. c., 29. b.

27. c. If the particles are treated as point charges,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$Q_1 = 2e \text{ (alpha particle)}, q_2 = 82e \text{ (gold nucleus)},$$

$$r = 6.5 \times 10^{-14}\,\text{m}$$

$$\therefore \quad U = (8.987 \times 10^8\,\text{N m}^2\text{C}^2)$$

$$\times \frac{(2 \times 82)(1.602 \times 10^{-19}\,\text{C})}{6.50 \times 10^{-14}\,\text{m}} = 5.82 \times 10^{-13}\,\text{J}$$

$$\text{or} \quad U = 5.82 \times 10^{-13}\,\text{J}$$

$$\times \left(\frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}}\right) = 3.63 \times 10^6\,\text{eV} = 3.63\,\text{MeV}$$

28. c. Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$

Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$. Conservation of energy thus says

$$K_1 = U_2 = 5.82 \times 10^{-13}\,\text{J} = 3.63\,\text{MeV}$$

29. b. $K = \frac{1}{2}mv^2$

$$\therefore \quad V = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82 \times 10^{-13}\,\text{J})}{6.64 \times 10^{-27}\,\text{kg}}} = 1.32 \times 10^7\,\text{m s}^{-1}$$

For Problems 30–32

30. a., 31. b., 32. b.

30. a. Use conservation of energy and momentum.
Momentum of a photon = h/λ

31. b. Use conservation of energy and momentum.
Momentum of a photon = h/λ

32. d. Use conservation of energy and momentum.
Momentum of a photon = h/λ

For Problems 33–35

33. d., 34. c., 35. c.

33. d. $\frac{dN}{dt} = \lambda N(t)$

From the given data,

$$20000 = \lambda N(0)$$
$$14800 = \lambda N(0.5\text{ h})$$
$$\frac{N}{N_0} = \frac{148}{200}$$
$$N = N_0 e^{-\lambda t}$$

$\therefore$ $$e^{-\lambda t} = \frac{148}{200}$$

or $$\lambda = \frac{\left(\ln\frac{200}{148}\right)}{t} \approx 1.6 \times 10^{-4}\text{ decays s}^{-1}$$

Half-life, $$T = \frac{0.693}{\lambda} = 4340\text{ s} = 1.2\text{ h}$$

34. c. $\frac{dN}{dt} = \lambda N_0$

Number of radioactive nuclei that were present in the sample at $t = 0$,

$$N_0 = \frac{20000}{1.6 \times 10^{-4}} = 1.25 \times 10^8$$

35. c. $N(t) = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = \lambda N(t)$$
$$N(t) = \frac{300}{1.6 \times 10^{-4}} = 1.875 \times 10^6$$

Matching Column Type

1. a → p, b → s, c → r, d → q.

Sol. a. Thermal energy of air molecules at room temperature:
$kT = 1.38 \times 10^{-23} \times 300\text{ J} = 0.025\text{ eV}$

b. Binding energy of heavy nuclei per nucleon ≈ 7 MeV

c. X-ray wavelength ≈ 1 Å

$$E = \frac{hc}{\lambda} \approx 12\text{ KeV}$$

d. For visible light: wavelength s ≈ 6000 Å

$$E = \frac{hc}{\lambda} \approx 2\text{ eV}$$

2. a → p, q, r, s; b → p, q, r, s; c → p, q, r, s; d → p, q, r, s.

Sol. In all the reactions in Column II:

Mass of products will be less than original mass of the system. The mass converts into energy, hence binding energy increases.

Basically, in all four reactions mentioned in Column II, energy is released and hence for all

$$m_{\text{products}} > m_{\text{original system}}$$

As energy is released in all 4 reactions, BE/nucleon increases in all.

Mass number and charge number are conserved in all processes.

3. a → q, r, s; b → q, r, s; c → q, r, s; d → p, r, s.

Sol. a. In the given spontaneous radioactive decay, the number of protons remain constant and all conservation principles are obeyed.

b. In fusion reaction of two hydrogen nuclei, a proton is decreased as a position shall be emitted in the reaction. All the three conservation principles are obeyed.

c. In the given fission reaction, the number of protons remain constant and all conservation principles are obeyed.

d. In beta negative decay, a neutron transforms into a proton within the nucleus and the electron is ejected out.

4. a → p, b → q, c → r, d → s.

Sol. For all types of waves, sound wave, light wave, string wave the term related is frequency, which is given only in one option. Other phenomenon are properly matching.

Photoelectric effect proves photon character of light.

γ-rays can only be produced from nucleus.

In case of k capture x-rays are emitted.

5. a → p, b → s, c → t, d → p.

Sol. Binding energy per nucleon for middle order element is maximum because middle order element is most stable.

So, (a) → (q)

Nuclear force depends only on spin of nucleons.

So, (b) → (s)

For nuclear fission, $\frac{Z^2}{A}$ is greater than 15.

So, (c) → (t)

Magic numbers are explained by Shell model.

So, (d) → (q)

6. a → r, s, t; b → q; c → p; d → p.

Sol. Stability of nucleus is decided by

(i) Mass defect → greater → stability greater

(ii) Neutron–proton ratio, i.e., $e\frac{N}{P} \approx 1 = 1 \rightarrow$ More stable

(iii) Packing fraction = negative → more stable

(iv) Binding energy per nucleon greater → greater stability

For radioactive substance binding energy per nucleon is minimum. So, they are unstable.

For bound orbit, total energy is always negative.

Stopping potential is the particular negative potential when no electron reaches the plate (i.e., anode).

7. a → p, q, t; b → p, r; c → s; d → q, s.

Sol. In nuclear fusion, two lighter nuclei fuse and make big nuclei. In this, mass defect is converted into energy according to $E = mc^2$.

In nuclear fission, heavy nuclei split into two or more than two smaller nuclei. In this process, mass is converted into energy according to $E = mc^2$.

In β-decay, neutron proton ratio decreases, so nucleus becomes more stable.

Both nuclear fission and nuclear fusion are exothermic reactions.

8. a → r; b → s; c → p; d → q.

Sol. $E_1 = [2m({}_1H^2) - m({}_1H^3) - m({}_1H^1)]\,931.5\text{ MeV} = 4\text{ MeV}$

$E_2 = [-m({}_2He^4) - m({}_0n^1) + m({}_1H^3) + m_1({}_1H^2)]$
$\times 931.5\text{ meV} = 17.6\text{ MeV}$

$E_3 = [-m({}_2He^3) - m({}_0n^1) + 2m({}_1H^2)]$
$\times 931.5\text{ meV} = 3.3\text{ MeV}$

$E_4 = [m({}_2He^3) - m({}_1H^2) - m({}_2He^4) - m({}_1H^1)]$
$\times 931.5\text{ MeV} = 18.3\text{ MeV}$

Integer Answer Type

1. (6) We have to find the time at which

$$\lambda_A N_A = \lambda_B N_B$$

$$\left(\frac{\ln 2}{T_A}\right)(4N_0 e^{-\lambda_A t}) = (N_0)\left(\frac{\ln 2}{T_B}\right)(e^{-\lambda_B t})$$

$$e^{(\lambda_A - \lambda_B)t} = 8$$

$$(\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$$

$$\left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3\ln(2) \quad \Rightarrow \quad t = 6 \text{ minutes.}$$

2. (4) We have $\frac{t}{t_{1/2}} = \frac{40\text{ hours}}{20\text{ hours}} = 2$

Thus, $A = \frac{A_0}{2^{t/t_{1/2}}} = \frac{A_0}{2^2} = \frac{A_0}{4}$

So one fourth of the original activity will remain after 40 hours.

3. (2) In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$8.0 \times 10^{18} \times \left(\frac{1}{2}\right) = 2 \times 10^{18}.$$

4. (0) The activity of the sample at time t is given by $A = A_0 e^{-\lambda t}$
Where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time t is given by

$$Q = Q_0 e^{-t/CR}$$

Where Q_0 is the charge at $t = 0$ and $C = 100\mu F$ Thus,

$$\frac{Q}{A} = \frac{Q_0}{A_0}\frac{e^{-t/CR}}{e^{-\lambda t}}$$

It is independent of t if $\lambda = \frac{1}{CR}$

Or $R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20 \times 10^{-3}\text{ s}}{100 \times 10^{-6}\text{ F}} = 200\,\Omega$

5. (8) $x \xrightarrow[\alpha]{t_1}$, $\downarrow t_2\ \beta$

$$6h = 3(t_{eq})$$

$$\Rightarrow \quad N = \frac{N_0}{(2)^3} \quad \Rightarrow \quad \frac{N_0}{N} = 8$$

6. (7) x and y are number of α-decays and β-decays respectively.

$$92 - 2x + y = 85$$

or $2x - y = 7$ (i)

Similarly, $238 - 4x = 210$ (ii)

$x = 7$, put in (i) we get $y = 7$

7. (6) Effective decay constant will be sum of all different decay constants.

So $\lambda_{\text{eff}} = \lambda + 2\lambda + 3\lambda = 6\lambda$, hence $n = 6$

8. (2) $R_1 = \lambda N_1, R_2 = \lambda N_2$,

No of atoms decayed in $(T_1 - T_2)$

$$= N_1 - N_2 = \frac{R_1 - R_2}{\lambda} = \frac{(R_1 - R_2)T}{\ln 2} = \frac{2(R_1 - R_2)T}{\ln 4}$$

Hence $n = 2$

Archives

Fill in the Blanks Type

1. $\frac{dN}{dt} = 1000$ at $t = 0$ s

$$t_{1/2} = 1\text{ s} \quad \Rightarrow \quad \lambda = 0.693\text{ s}^{-1}$$

At $t = 1$ s,

$$1 = \frac{2.303}{0.693}\log 10\frac{N_0}{N}$$

Now,

$$t = \frac{2.303}{\lambda}\log_{10}\frac{N_0}{N}$$

$$\therefore \quad 0.3010 = \log_{10}\frac{N_0}{N} \quad \Rightarrow \quad \log_{10}\frac{N_0}{N} = \log_{10}2$$

$$\therefore \quad \frac{N_0}{N} = 2 \quad \Rightarrow \quad N = \frac{N_0}{N}$$

$$\therefore \quad N = \frac{1000}{2} = 500$$

Here,

$$\frac{dN}{dt} \propto N_0$$

and $\frac{dN}{dt} \propto N$

$\therefore$ $\frac{dN}{dt} = 500$ dps at $t = 1$ s

Similarly, at $t = 3$ s, $\frac{dN}{dt} = 125$ dps.

Alternative solution:

$$A = A_0\left(\frac{1}{2}\right)^n$$

where A_0 = initial activity = 100 dps (given), A = activity after n half-lives.

At $t = 1, n = 1 \Rightarrow A = 1000\left(\frac{1}{2}\right)^1 = 500$ dps

At $t = 3, n = 3 \Rightarrow A = 1000\left(\frac{1}{2}\right)^3 = 125$ dps

2. ${}^{238}_{92}U \to {}^{206}_{82}Pb + x\,{}^4_2He + y\,{}^0_{-1}e$
First, we find the number of α-particles. The change in mass number during the decay from uranium to lead is $238 - 206 = 32$. Therefore, the number of α-particles (with mass no. 4) is $32/4 = 8$.
The change in atomic number (i.e., number of protons) taking place when 8 α-particles are emitted and lead is formed $= 92 - (82 + 16) = 6$ which means six β-particles are emitted.

3. ${}^{10}_{5}B + {}^1_0n \to {}^4_2He + {}^7_3Li$
The resulting nucleus is of element lithium and mass number is 7.

4. Atomic number, mass number

5. ${}^2_1H + {}^2_1H \to {}^4_2He$
Binding energy of two deuterons is
$$2[1.10020 \times 2] = 4.4 \text{ MeV}$$
Binding energy of one helium $= 4 \times 7 = 2.8$ MeV
The energy released $= 28 - 4.4 = 23.6$ MeV

6. ${}^{11}_{6}C \to {}^{11}_{5}B + \beta^+ + X \Rightarrow {}^{11}_{6}C \to {}^{11}_{5}B + {}^{0}_{+1}e + X$ (neutrino)
The balancing of atomic number and mass number is OK. Therefore X stands for energy.

7. This is a nuclear fusion reaction.
Energy released $= (\Delta m)$ [931.5 MeV/u]
$= [4.0024 - 2 \times 2.0141] \times 931.5$ MeV
$= -24.03$ MeV (heat released)

True or False Type

1. False.

$$\text{Density} = \frac{m}{V} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi\left[R_0 A^{1/3}\right]} = 3 \times 10^{17}\ \text{kg m}^{-3}$$

where A = mass number

$$= \frac{1.67 \times 10^{-27}}{1.33 \times 3.14 \times (1.1 \times 10^{-15})}$$

Alternative solution:
Remember that the order of nuclear density is 10^{17} kg m^{-3}.

Multiple Choice Questions with One Correct Answer Type

1. **b.** $T_{1/2} = 3.8$ day
$\therefore$ $\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.8} = 0.182$
If the initial number of atoms is $a = A_0$, then after time t the number of atoms is $a/20 = A$. We have to find t.
$$t = \frac{2.303}{\lambda}\log\frac{A_0}{A} = \frac{2.303}{0.182}\log\frac{a}{a/20} = \frac{2.303}{0.182}\log 20$$
$= 16.46$ day

2. **c.** β-particles are radioactive material emitted by the nucleus.

3. **c.** $4\,{}^1_1H^+ \to {}^4_2He^{2+} + 2e^- + 26$ MeV
represents a fusion reaction.

4. **c.** Following nuclear reaction takes place
${}_0n^{-1} \Rightarrow {}_1H^1 + {}_{-1}e^0 + \overline{\nu}$
$\overline{\nu}$ is antineutrino.

5. **d.**

6. **a.**

7. **b.** From $R = R_0\left(\frac{1}{2}\right)^n$, we have
$$1 = 64\left(\frac{1}{2}\right)^n$$
or $n = 6$ = number of half-lives
$$t = n \times t_{1/2} = 6 \times 2 = 12 \text{ h}$$

8. **b.**

9. **c.** The given reactions are
$${}_1H^2 + {}_1H^2 \to {}_1H^3 + p$$
$${}_1H^2 + {}_1H^3 \to {}_2He^4 + n$$
$$3\,{}_1H^2 \to {}_2He^4 + n + p$$
Mass defect,
$$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008) \text{ a.m.u.}$$
$= 0.026$ a.m.u.
Energy released $= 0.026 \times 931$ MeV
$= 0.026 \times 931 \times 1.6 \times 10^{-13}$ J
$= 3.87 \times 10^{-12}$ J
This is the energy produced by the consumption of three deuteron atoms. Therefore, total energy released by 10^{40} deuterons is

$$\frac{10^{40}}{3}\times 3.87\times 10^{-12}\,\text{J} = 1.29\times 10^{28}\,\text{J}$$

The average power radiated is $P = 10^6$ W or 10^{16} J s^{-1}.
Therefore, total time to exhaust all deuterons of the star will be

$$t = \frac{1.29\times 10^{28}}{10^{16}} = 1.29\times 10^{12}\,\text{s} \approx 10^{12}\,\text{s}$$

10. b. Fast neutrons can be easily slowed down by passing them through water. This is because of comparable masses the energy passed by neutron to water molecule is high.

11. c. The penetrating power is dependent on velocity. For a given energy, the velocity of γ-radiation is highest and α-particle is least.

12. d. The mass defect for ^{64}Zn is more than that for ^{64}Cu. So, Zn is more stable. Therefore, ^{64}Cu is radioactive and will decay to ^{64}Zn through β^--decay as follows

$$^{29}_{64}\text{Cu} \rightarrow {}^{64}_{30}\text{Zn} + {}^{0}_{-1}\text{e}$$

Alternative soltution:

By the conservation of charge and nucleons, only potential is feasible.

13. d. Number of nuclei decreases exponentially,

$$N = N_0 e^{-\lambda t}$$

Rate of decay, $-\dfrac{dN}{dt} = \lambda N$

Therefore, decay process lasts upto $t = \infty$. Therefore, a given nucleus may decay at any time after $t = 0$.

14. b. According to Doppler's effect of light, the wavelength shift is given by

$$\Delta\lambda = \frac{V}{c}\times\lambda$$

$$\Rightarrow \quad V = \frac{\Delta\lambda\times c}{\lambda} = \frac{(706-656)}{656}\times 3\times 10^8$$

$$= 2\times 10^7\,\text{m s}^{-1}$$

15. b. Nuclear density of an atom of mass number A,

$$D = \frac{\text{mass}}{\text{volume}} = \frac{A(1.67\times 10^{-27})}{\frac{4}{3}\pi[1.25\times 10^{-15}\;A^{1/3}]^3}$$

$$\left[\because V = \frac{4}{3}\pi R^3,\; R = R_0A^{1/3}, R_0 = 1.25\times 10^{-15}\right]$$

$$\therefore \quad D = 2\times 10^{17}\,\text{kg m}^{-3}$$

16. b. $^{22}_{10}\text{Ne} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} + {}^{14}_{6}\text{X}$.

The new element X has a atomic number 6. Therefore, the element is carbon.

17. c. Energy will be released when stability increases. This will happen when binding energy per nucleon increases.

	Reactant	Product
Reaction (a)	60×8.5 MeV $= 510$ MeV	$20\times 30\times 5$ $= 300$ MeV
Reaction (b)	120×7.5 $= 900$ MeV	$(90\times 8 + 30\times 5)$ $= 870$ MeV
Reaction (c)	120×7.5 $= 900$ MeV	$2\times 60\times 8.5$ $= 1020$ MeV
Reaction (d)	90×8 $= 720$ MeV	$(60\times 8.5 + 30\times 5)$ $= 600$ MeV

18. c. $(t_{1/2})_x = (t_{\text{mean}})_y$

$$\frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$

$$\lambda_x = 0.693\,\lambda_y$$

$$\lambda_x < \lambda_y$$

or Rate of decay $= \lambda N$

Initially, number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, Y will decay at a faster rate than X.

19. a. Both the beta rays and the cathode rays are made up of electrons. So, only option (a) is correct.
Gamma rays are electromagnetic waves.
Alpha particles are doubly ionized helium atoms.
Protons and neutrons have approximately the same mass.
Therefore, (b), (c) and (d) are wrong options.

20. d. $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = \frac{1}{e^{9\lambda t}}$$

Given,

$$\frac{N_1}{N_2} = \frac{1}{e} \Rightarrow \frac{1}{e^{9\lambda t}} = \frac{1}{e}$$

or $\quad 9\lambda t = 1 \quad$ or $\quad t = \left(\dfrac{1}{9\lambda}\right)$

21. c. We know that in a nucleus, neutron converts into proton as follows:

$$n \rightarrow p^+ + e^-$$

Thus, decay of neutron is responsible for β-radiation orgination.

22. d. $N_1 = N_0 e^{-\frac{t}{\tau}}$ (i) and $\tau = \dfrac{1}{\lambda_1}$

$N_2 = N_0 e^{-\lambda_2 t} = N_0 e^{-\frac{1}{5\tau}}$ (ii) and $5\tau = \dfrac{1}{\lambda_2}$

Adding (i) and (ii), we get

$$N = N_1 + N_2 = N_0\,(e^{-t/\tau} + e^{-t/5\tau})$$

(a) is not the correct option as there is a time τ for which N is constant, which means for time τ there is no process of radioactivity which does not makes sense.
(b) and (c) show intermediate increase in the number of radioactive atoms which is impossible as N will only decrease exponentially. Hence, the correct option is (d).

23. a. $A = A_0(1/2)^n$; n = number of half-lives.

24. c. In γ-decay, the atomic number and mass number do not change.

25. a. We know that radius of the nucleus,

$$R = R_0 A^{1/3}$$

where A is the mass number.

$$\therefore \quad R^3 = R_0^3 A \quad \Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

$\Rightarrow$ Volume $\propto$ mass

26. b. By conservation of linear momentum,

$$MV = mv \quad \Rightarrow \quad 216V = 4v \quad \Rightarrow \quad V = \frac{v}{54} \qquad \text{(i)}$$

By energy conservation,

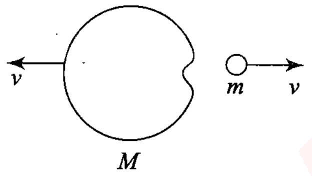


Fig. 5.36

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = 5.5 \times 1.6 \times 10^{-13}\,\text{J}$$

$$\Rightarrow 4 \times 1.67 \times 10^{-27} v^2 + 216 \times 1.67 \times 10^{-27} \times \frac{v^2}{54 \times 54}$$

$$= 2 \times 5.5 \times 1.6 \times 10^{-13}$$

$$\Rightarrow 1.67 \times 10^{-27} v^2 \left[4 + \frac{216}{54 \times 54}\right]$$

$$= 2 \times 5.5 \times 1.6 \times 10^{-13}$$

or $$v^2 = \frac{2 \times 5.5 \times 1.6 \times 54 \times 54 \times 10^{-13}}{(4 \times 54 \times 54 + 216)1.67 \times 10^{-27}}$$

$$= 2.586 \times 10^{14}$$

$$\therefore \text{ KE of } \alpha\text{-particle} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 4 \times 1.67 \times 10^{-27} \times 2.586 \times 10^{14}$$

$$= 8.637 \times 10^{-13}\,\text{J} = \frac{8.637 \times 10^{-13}}{1.6 \times 10^{-13}}\,\text{MeV}$$

$$= 5.4\,\text{MeV}$$

Alternative solution:

By conservation of momentum,

$$P_1 = P$$

$$\Rightarrow \quad \sqrt{2k_2 m_1} = \sqrt{2k_2 m_2}$$

$$\Rightarrow \quad \sqrt{2k_1(216)} = \sqrt{2k_2(4)}$$

$$\Rightarrow \quad k_2 = 54\,k_1 \qquad \text{(i)}$$

Also, $\quad k_1 + k_2 = 5.5\,\text{MeV} \qquad$ (i

Solve Eqs. (i) and (ii).

27. b. We know that $\lambda = \dfrac{2.303}{t}\log\dfrac{A_0}{A}$, where A_0 is the initial activity. A is the activity at time t.

$$\therefore \quad \lambda = \frac{2.303}{280}\log\frac{A_0}{6000} = \frac{2.303}{420}\log\frac{A_0}{3000}$$

On solving, we get

$$A_0 = 24000\,\text{dps}$$

28. c. ${}_2^4\text{He} \rightarrow {}_8^{16}\text{O}$

$$\text{BE} = \Delta m \times 931.5\,\text{MeV}$$

$$= (4 \times 4.0026 - 15.9994) \times 931.5$$

$$= 10.24\,\text{MeV}$$

29. b. Radioactive decay is a random process. Each decay is a completely independent event. Therefore, which particular nucleus will decay at a given instant of time cannot be predicted. In other words, when a particular nucleus will decay cannot be predicted. Each nucleus has same probability of disintergration.

30. a. Iodine and Yttrium are medium sized nuclei and therefore have more binding energy per nucleon as compared to uranium which has a big nuclei and less BE/nucleon. In other words, Iodine and Yttrium are more stable and therefore possess less energy and less rest mass. Also, when uranium nuclei explodes, it will convert into I and Y nuclei having kinetic energies.

31. a. Activity of $S_1 = \dfrac{1}{2}$ (activity of S_2)

or $$\lambda_1 N_1 = \frac{1}{2}(\lambda^2 N^2)$$

$$\frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$$

$$\frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left(T = \text{half-life} = \frac{\ln 2}{\lambda}\right)$$

Given $\quad N_1 = 2N_2$

$$\frac{T_1}{T_2} = 4$$

Multiple Choice Questions with One or More than One Correct Answer Type

1. a., b., c.

2. a., d. In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.

3. c., d. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

$^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of $^{20}_{10}Ne$ nucleus,

$$M_1 < 10(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect.

$$20(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$$

Thus, $10(m_n + m_p) > M_2 - M_1$

or $\quad M_2 < M_1 + 10(m_p + m_n)$

Now, since $\quad M_1 < 10(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

4. b., d. In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus.
In fission, a heavy nucleus beaks into two or more comparatively lighter nuclei.
Further, energy will be released in a nuclei process if total binding energy increases.
Hence, correct options are (b) and (d).

Comprehension Type

1. d

2. a. $2 \times 1.5\,kT = \dfrac{e^2}{4\pi\varepsilon_0 d}$ (conservation of energy)

$\therefore \quad T = 1.4 \times 10^9$ K

3. b. $nt_0 > 5 \times 10^{14}$ (as given)

Matching Column Type

1. a → p, q; b → p, r; c → p, s; d → p, q, r.

Sol. In a nuclear fusion reaction, matter is converted into energy and nuclei low atomic number generally give this reaction.

In a nuclear fission reaction, matter is converted into energy and nuclei of high atomic number generally give this reaction.

2. a → p, r; b → q, s; c → p; d → q.

Sol. Characteristic X-rays are produced due to transition of electrons from one energy level to another.

Similarly, the lines in the hydrogen spectrum are obtained due to transition of electrons from one energy level to another.

In photoelectric effect electrons from the metal surface are emitted out upon the incidence of light of appropriate frequency. In β-decay, electrons are emitted from the nucleus of an atom.

Moseley gave a law which related frequency of emitted X-rays with the atomic number of the target material as $\sqrt{v} = a(z - b)$.

In photoelectric effect, energy of photons of incident ray gets converted into kinetic energy of emitted electrons.

3. a → p, q, t; b → q; c → s; d → s.

Integer Answer Type

1. (8) $N = N_0 e^{-\lambda t}$

$\ln|dN/dt| = \ln(N_0\lambda) - \lambda t$

From graph

$$\lambda = \frac{1}{2} \text{ per year}$$

$$\frac{t_1}{2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

$$4.16 \text{ years} = 3t_{1/2}$$

$$p = 8$$

2. (1) $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = 10^{10} = N_0(\lambda)\, e^{-10^{-9}t}$$

At $(t = 0)$

$10^{10} = N_0 10^{-9}$

Mass of sample $= N_0 = 10^{-25}$

$= N_0$ (mass of the atom)

$= 10^{-6}$ kgm

$= 10^{-6} \times 10^3$ gm

$= 10^{-3}$ gm

$= 1$ mg

Appendix

Solutions to Concept Application Exercises

Chapter 1

Exercise 1.1

1. Different angles are as shown in the figure below.
In triangle ABC,

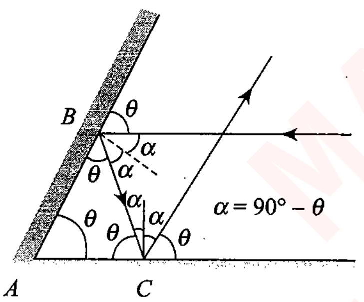


Fig. S-1.1

$$\theta + \theta + \theta = 180°$$

$$\therefore \quad \theta = 60°$$

2. Yes, for a virtual object. Because the plane mirror will form a the real image of a virtual object and real image can be taken on a screen.

3. True. If the object is real, then incident rays will emerge from it.

4. The angle that the ray makes after each reflection is clearly labeled in the following figure. It is clear that final ray makes an angle of 60° with the horizontal, or in other words it emerges parallel to mirror 1.

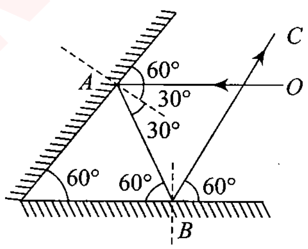


Fig. S-1.2

5. a. It should be a plane mirror, because $u = \infty$ and $v = \infty$.

6. The first image at M_1 is formed at a distance of 4 cm behind it.
The first image at M_2 is 6 cm behind it.
The second image at M_2 is at a distance of 4 + 10 = 14 cm behind M_2.
Therefore, the distance between the first image at M_1 and second image at M_2 is 4 + 14 + 10 = 28 cm.

7.

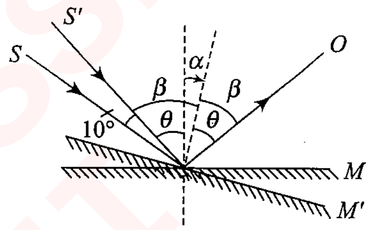


Fig. S-1.3

$$\theta + \alpha - 10° = \beta \quad \text{and} \quad \theta = \beta + \alpha$$

$$\Rightarrow \quad \alpha = 5° \text{ clockwise}$$

8. $OM = 6 \sin 30° = 3$ cm

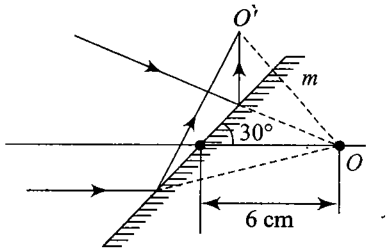


Fig. S-1.4

O is the virtual object and O' is the real image.

So, $\quad O'M^2 = OM = 3$ cm

9. The image of point A in the mirror is at A' (6, 0).
Join $A'M$ and extend to cut Y-axis at M' (ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y-axis at N' (ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').
From Geometry:

$$M' \equiv (0, 6)$$

and $$N' \equiv (0, 9)$$

$M'N'$ is the region on Y-axis in which reflected rays are present.

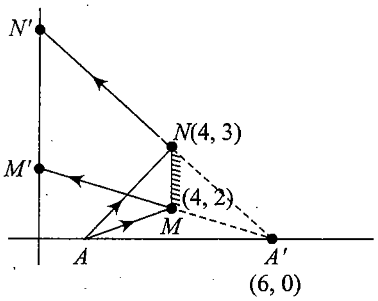


Fig. S-1.5

10.

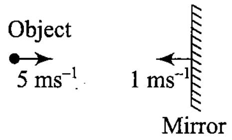


Fig. S-1.6

Take direction toward right as positive.

$$v_i - v_m = v_m - v_0$$

$$v_i - (-1) = (-1) - 5$$

$$\therefore \quad v_i = -7 \text{ m s}^{-1}$$

$\therefore$ The velocity of image is 7 m s^{-1} and direction is toward left.

11. We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$ or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$. Here $\Delta x_o = 0$; $\Delta x_m = 10$ cm. Therefore, $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

or

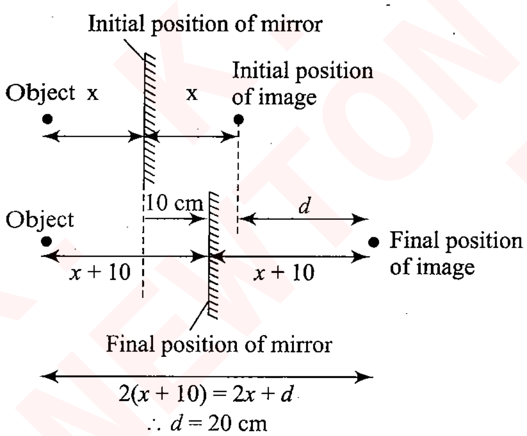


Fig. S-1.7

12.

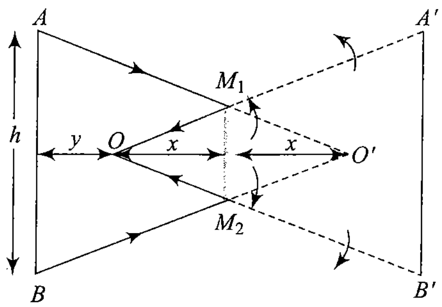


Fig. S-1.8

From triangles $Q'M_1M_2$ and $O'AB$

$$\frac{M_1M_2}{h} = \frac{x}{2x+y}$$

$$\text{Size of mirror, } M_1M_2 = \frac{hx}{y+2x}$$

13. Here, $v_{OM} = -v_{IM}$

$$v_0 - v_M = -0\,(v_1 - v_M)$$

$$(+2 \text{ ms}^{-1}) - (-3 \text{ ms}^{-1}) = -v_1 + (-3)$$

$$\Rightarrow \quad v_I = -8 \text{ ms}^{-1}$$

14. Velocity of point P, $v_p = 10(\hat{i} + \hat{j})$

Velocity of mirror $v_m = -5\hat{i}$

Velocity of particle relative to mirror is

$$\vec{v}_{pm} = -15\hat{i} + 10\hat{j}$$

Velocity of image relative to observer

$$\vec{v}_{IO} = \vec{v}_{pm} - \vec{v}_{Om} = (-15\hat{i} + 10\hat{j}) - (\vec{v}_O - \vec{v}_m)$$

$$= (-30\hat{i} + 10\hat{j})\,\text{ms}^{-1}$$

Angle with X-axis,

$$\alpha = \tan^{-1}(-1/3)$$

Exercise 1.2

1. a. True. According to the magnification formula

$$m = \frac{f_0}{f_0 + x}$$

where x is the distance of a real object. For all positive values of x, m is always positive.

b. False. For a real object, the image formed by a convex mirror is always erect and diminished.

$$m = \frac{f_0}{f_0 + x}$$

For a virtual object lying between pole and focus, the image formed is real, enlarged, and erect.

$$m = \frac{f_0}{f_0 + x}$$

c. True. For a concave mirror, we know that

$$m = \frac{f_0}{f_0 + x}$$

where x is the object distance.

If $x < f_0$, then $m > 1$, which implies a virtual enlarged image.

d. False. In case of mirrors (convex, concave, and plane), the object and its image always move in opposite directions.

That is, $$\frac{V}{U} = -\left(\frac{f}{u-f}\right)^2$$

where V = velocity of image and U = velocity of object.

e. False. When the object is beyond the center of curvature, its image moves slower.

f. True. For a convex mirror, $m = f_0/f_0 + x$

i.e., if $x = f_0$, then $m = 1/2$

g. True. A concave mirror forms an enlarged image in two situations:

i Object is placed between C and F.

ii Object is placed between F and P.

h. False. Convex mirror is used as a rear view mirror.

i. True. Complete image will be formed, but of less brightness because now the incident rays will be reflected from reduced area to form the image.

j. True. $m = -v/u$. For a plane mirror, $v = -u$

$\Rightarrow$ $m = +1$. Hence, image is erect and of same size.

k. True. We see our left hand as right hand and right as left hand in a plane mirror.

l. True. Virtual object is that point where incident rays seem to be converging.

2. a. Mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Here, $u = -10$ cm; $f = -15$ cm

$$\therefore \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{15-10}{(15)(10)} = \frac{1}{30}$$

The positive sign indicates that the image is virtual and erect and it is formed behind the mirror.
Alternatively, we can apply Newton's formula.
Here $x_0 = u - f = -10 - (-15) = 5$ cm

$$x_i = \frac{f^2}{x_0} = \frac{(15)^2}{5} = 45 \text{ cm}$$

Since $x_i = v - f$, therefore

$$v = x_i + f = 45 - 15 = 30 \text{ cm}$$

The magnification,

$$m = -\frac{v}{u} = \frac{h_i}{h_0}$$

$$h_i = h_0\left(-\frac{v}{u}\right) = 1\left(-\frac{30}{-10}\right) = 3 \text{ cm}$$

b. One image is formed when the object is at the centre of curvature of both the mirrors, i.e., $d = 2f + 2f = 4f$; other will be formed when it is at focus of both the mirrors, i.e., $2f$.

3. Since the ray incident on the mirror at its pole is reflected symmetrically w.r.t. the major optical axis, let us plot point S_1 symmetrical to S' and draw ray SS_1 until it intersects the axis at point P (see the following figure). This point will be the pole of the mirror.

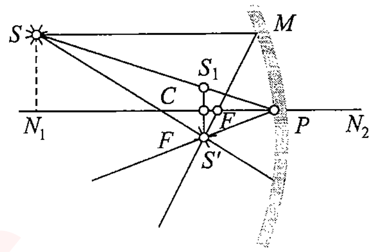


Fig. S-1.9

The optical center C of the mirror can obviously be found as the point of intersection of ray SS' with axis N_1N_2.
The focus can be found by the usual construction of ray SM parallel to the axis. The reflected ray must pass through focus F (lying on the optical axis of the mirror) and through S'.

4. a. Let us construct, as in the previous example, the ray BAC and find point C (optical center of the mirror) (see Fig. S-1.10). Pole P can be found by constructing the path of the ray APA' reflected in the pole with the aid of symmetrical point A'. The position of the mirror focus F is determined by means of the usual construction of ray AMF parallel to the axis.

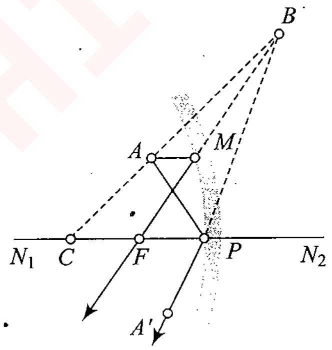


Fig. S-1.10

b. This construction can also be used to find the center C of the mirror and pole P (Fig. S-1.11). The reflected ray BM will pass parallel to the optical axis of the mirror. For this reason, to find the focus, let us first determine point M at which straight line AM, parallel to the optical axis, intersects the mirror, and then extend BM to the point of intersection with the axis at the focus F.

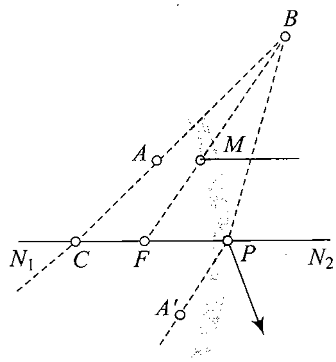


Fig. S-1.11

5. Reflection from concave mirror

$$u = 3f_1,\ f = -f$$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{3}{2}f$

Now reflection from convex mirror

$$u = \left(3f + \frac{3}{2}f\right) = +\frac{9}{2}f,\ f = -f$$

Again using mirror formula, we get $v = \frac{9f}{11}$

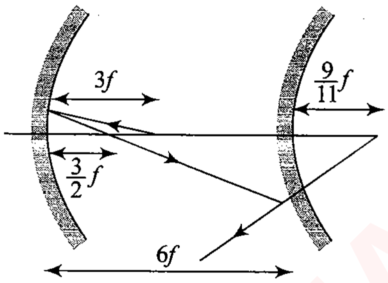


Fig. S-1.12

6. $u = -15\text{ cm}, f = \frac{+20}{2} = 10\text{ cm}$

$$\frac{1}{-15} + \frac{1}{v} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{15} \Rightarrow v = 6\text{ cm}$$

$$m = -\frac{v}{u} = \frac{h_2}{h_1}$$

$$\frac{-6}{-15} = \frac{h_2}{5} \Rightarrow h_2 = 2\text{ cm}$$

So, radius of circular path of the image is 2 cm.

7. **(i)** If reflection is first taken on the plane mirror:

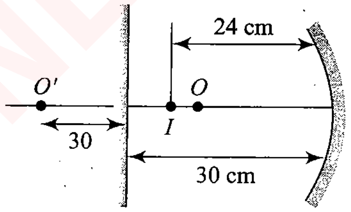


Fig. S-1.13

For second reflection at the concave mirror:

$$u = -40\text{ cm}, f = -15\text{ cm}$$

$$\frac{1}{v} + \frac{1}{-40} = -\frac{1}{15} \Rightarrow v = -24\text{ cm}$$

(ii) If reflection is first taken on the concave mirror:

$$u = -20\text{ cm}$$

$$\frac{1}{v} + \frac{1}{-20} = -\frac{1}{15} \Rightarrow v = -60\text{ cm}$$

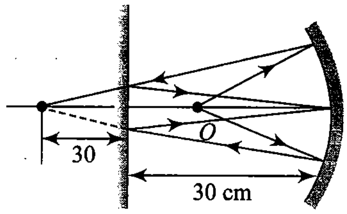
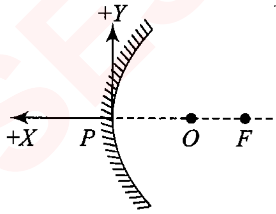


Fig. S-1.14

Final image will be formed at the pole of concave mirror.

8. Consider the situation shown in the figure below which also includes the coordinate system.



Concave mirror

Fig. S-1.15

Here, $u = -10$ cm

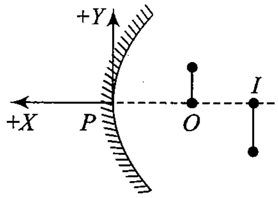
$f = -20$ cm

Substituting in the equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = +20\text{ cm}$$

Thus, the image is formed 20 cm behind the mirror, magnification +2, erect and virtual.

9. Consider the situation shown in the following figure which also indicates the coordinate system.



Concave mirror

Fig. S-1.16

Here, $u = -15$ cm

and $m = -2$

Note the negative sign for m. The fact that the image is captured on the screen implies the image is real. Therefore, the magnification is to be taken negative, though the problem simply states that the magnification is 2.

Substituting in equation

$$m = -\frac{v}{u}, \text{ we get } v = -30\text{ cm}$$

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } f = -10\text{ cm}$$

Conclusion: The focal length of the mirror is 10 cm. The negative sign implies it is concave.

10. Consider the situation shown in the following figure where the optical element is represented by a box as we do not know its nature.

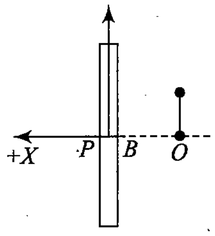


Fig. S-1.17

Here $u = -15$ cm,
$h_i = +5$ cm, and
$h_v = +5$ cm.

Substituting in equation

$$m = \frac{h_i}{h_0} = +\frac{1}{3} \quad \text{and} \quad m = \frac{-v}{u}, \text{ we get } v = +5 \text{ cm}$$

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad f = +75$$

Conclusion: The focal length of the mirror is 7.5 cm. The positive sign implies that the mirror is convex.

11. Let us consider the case of rays first striking the concave mirror and then the convex mirror.

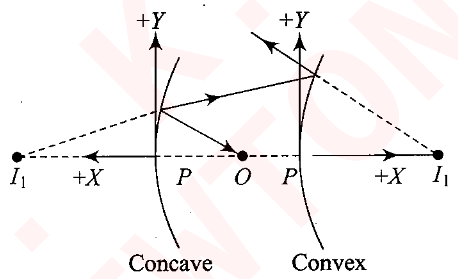


Fig. S-1.18

Concave mirror: Here, $u = -5$ cm and $f = -10$ cm

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = 10 \text{ cm}$$

Thus, the image is formed 10 cm behind the mirror.

This image now serves as the object for the convex mirror. However, keep in mind that for convex mirror, all measurements will now be from the pole of this mirror. Furthermore, the sign convention for the convex mirror is shown in the figure.

Convex mirror: Here, $u = -30$ cm and $f = +20$ cm

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } v = 12 \text{ cm}$$

Thus, the image is formed 12 cm behind the convex mirror. This image will now be the object for the concave mirror and the process can be repeated.

Refer to the following figure. Here, *for the convex mirror:*

$u = -15$ cm and $f = +20$ cm

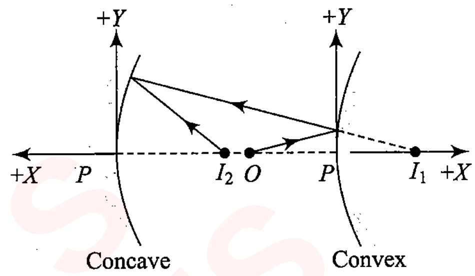


Fig. S-1.19

Substituting in the equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = +(60/7) \text{ cm}$$

Thus, the image is formed (60/7) cm behind the convex mirror. The image now serves as the object for the concave mirror.

For the concave mirror:

$u = -(20 + 60/7) = -200/7$ and $f = -10$ cm

Substituting in the equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = -200/13$$

The image is formed (200/13) cm in front of the concave mirror.

12. The mirror can be concave or convex. Let f be the focal length. Let the incident rays be directed toward right as shown. All quantities toward right will be positive.

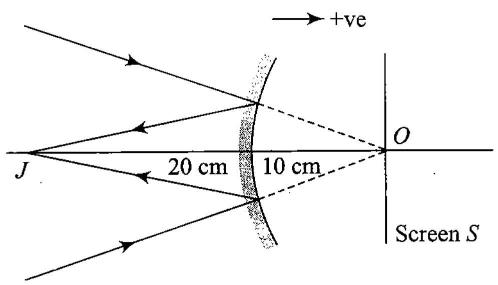


Fig. S-1.20

The initial point of convergence (O) now acts as a virtual object for the mirror.

Hence, $u = -10$ cm and $v = -20$ cm

$$f = \frac{uv}{u+v} = \frac{-200}{10-20} = +20 \text{ cm}$$

As the focal length is positive, the mirror is convex having focal length 20 cm.

13. In this case:

$u = +30$ and

$v = +120$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{120} + \frac{1}{30}$$

$$\Rightarrow \quad f = 24 \text{ cm}$$

14. First reflection:

Focus of mirror = –10 cm

$u = -15$ cm

Applying mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$v = -30 \text{ cm}$$

For second reflection on plane mirror:

$$u = -10 \text{ cm} \Rightarrow v = 10 \text{ cm}$$

For third reflection on curved mirror again:

$$u = -50 \text{ cm} \quad \text{and} \quad f = -10 \text{ cm}$$

Applying mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad v = -12.5 \text{ cm}$$

15. We know that, $m = \frac{v}{u} = \frac{f}{u-f}$

In first case, $\frac{f}{u-f} = 4$

$F = 4u - 4f$ and $5f = 4u$

In 2nd case, $\frac{f}{(u+3)-f} = +3$

$$F = 3u + 9 - 3f$$

$$\therefore \quad 4u - 4f = 3u + 9 - 3f$$

$$u = f + 9 = \frac{4u}{5} + 9$$

$u = 45$ cm and $f = 36$ cm

16. Mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here, $\quad v = +4 \text{ cm}; f = +\frac{R}{2} = +\frac{24}{2} = +12 \text{ cm};$

$$\therefore \quad u = \frac{vf}{v-f} = \frac{(4)(12)}{4-12} = -6 \text{ cm}$$

The negative sign shows that the object is real and it is placed in front of the mirror.

The magnification, $m = -\frac{u}{v} = \frac{-(+4)}{-6} = \frac{2}{3}$

Thus, the image is two-thirds as high as the object.

17. We know that $m = -\frac{v}{u} = \frac{f}{f-u}$

Here, $\quad m_1 = \frac{f}{f-(-25)} = \frac{f}{f+25}$ and

$$m_2 = \frac{f}{f-(-25-15)} = \frac{f}{f+40}$$

Since $\frac{m_1}{m_2} = 4$, therefore $\frac{f+40}{f+25} = 4$. Thus

$$f + 40 = 4f + 100$$

or $\quad f = -20$ cm.

The negative sign shows that the mirror is concave.

18. a. Mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

On differentiating both sides, we get

$$-\frac{dv}{v^2}-\frac{du}{u^2}=0 \quad \text{or} \quad dv = -du\left(\frac{v}{u}\right)^2$$

Since $\frac{v}{u} = \frac{f}{u-f}$, therefore

$$dv = -du\left(\frac{f}{u-f}\right)^2$$

Lateral magnification, $m_2 = \frac{dv}{du} = -\left(\frac{f}{u-f}\right)^2$

The negative sign shows that the image is longitudinally inverted.

b. Since velocity of object, $U = \frac{du}{dt}$

and velocity of image,

$$V = \frac{dv}{dt}$$

$$\therefore \quad V = -U\left(\frac{f}{u-f}\right)^2 \quad \text{or} \quad \frac{V}{U} = -\left(\frac{f}{u-f}\right)^2$$

The negative sign shows that the object and image always move in opposite directions.

$$v = \frac{-xf_0}{f_0 - x}$$

v becomes negative (real image) only when $x < f_0$.

19. For point B, $u = -25$ cm, $f = +10$ cm

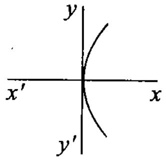


Fig. S-1.21

$$\therefore \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

or $$\frac{1}{v} - \frac{1}{25} = \frac{1}{10}$$

or $$\frac{1}{v} = \frac{1}{10} + \frac{1}{25}$$

or $$\frac{1}{v} = \frac{5+2}{50}$$

$\therefore$ $$v = \frac{50}{7}$$

For point A, $u' = -30$ cm and $f' = 10$ cm

$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'} \quad \text{or} \quad \frac{1}{v'} - \frac{1}{30} = \frac{1}{10}$$

or $$\frac{1}{v'} = \frac{1}{30} + \frac{1}{10} = \frac{1+3}{30} = \frac{4}{30}$$

$\therefore$ $$v' = \frac{30}{4} = \frac{15}{2} \text{ cm}$$

$\therefore$ $$A'B' = \frac{15}{2} - \frac{50}{7} = \frac{5}{14} \text{ cm}$$

20. $$\frac{v}{u} = \frac{I}{o} = 3 \qquad \text{(i)}$$

and $$\frac{v'}{u'} = \frac{I'}{o} = 2 \Rightarrow \frac{v'}{u+6} = 2 \qquad \text{(ii)}$$

From (i) and (ii), we get

$$v' = 2\left(\frac{v}{3} + 6\right) \qquad \text{(iii)}$$

$\Rightarrow$ $$v' - \frac{2}{3}v = 12$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{1}{v'} + \frac{1}{u'}$$

$\Rightarrow$ $$\frac{1}{v} + \frac{3}{u} = \frac{1}{v'} + \frac{2}{u'}$$

$\Rightarrow$ $$4\left(\frac{3}{2} - 1\right)$$

$\Rightarrow$ $$v = \frac{4}{3}v' \qquad \text{(iv)}$$

From (iii) and (iv), $v' - \frac{2}{3} \times \frac{4}{3} v' = 12$

$\Rightarrow$ $$\frac{1}{9}v' = 12 \Rightarrow v' = 108 \text{ cm}$$

$\Rightarrow$ $$v = \frac{4}{3}v' = 144 \text{ cm}$$

Shifting will be 144 – 108 = 36 cm toward the image.

Exercise 1.3

1. **(i)** True. The emergent ray is always parallel to the incident ray.
(ii) True. See the figure below.

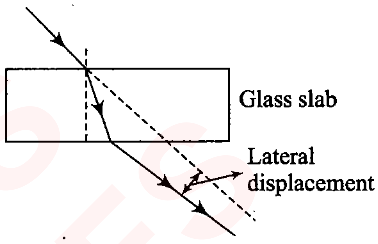


Fig. S-1.22

(iii) False. The shift produced by a glass slab is always in the direction of incident light, irrespective of the converging or diverging nature of beam.

(iv) False. As shifting $s = t\left(1 - \frac{1}{\mu_{rel}}\right)$

$$\mu_{rel} = \frac{\mu_{slab}}{\mu_{medium}}$$

If $\mu_{medium} > \mu_{slab}$, the value of μ_{real} will be less than 1 and shifting will be negative, i.e., opposite to direction of ray tralling

(v) True. As we know shifting $s = t\left(1 - \frac{1}{\mu_{rel}}\right)$

$$\mu_{rel} = \frac{\mu_{slab}}{\mu_{medium}} = \frac{\mu_2}{\mu_1}$$

If $\mu_2 > \mu_1$, then $s < t$
If $\mu_1 > \mu_2$, then shift (s) can be greater than t only if $\mu_1/\mu_2 > 2$.

2. **(i)** To observer in air, there are two objects placed in the medium of refractive index μ. One is the actual object S and the other object is S' which is the image of S in the plane mirror.

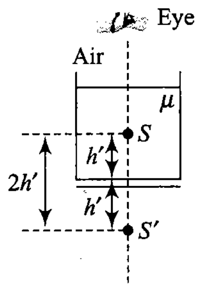


Fig. S-1.23

The observer is able to see two images, one for each object.

(ii) For observer in air, apparent depth $h' = \dfrac{h}{\mu}$

Hence distance between two images will be $2h' = \dfrac{2h}{\mu}$

3. $AB = \dfrac{t}{\cos 45°} = \dfrac{t'}{\cos r} \quad \Rightarrow \quad t' = t\dfrac{\cos r}{\cos 45°}$

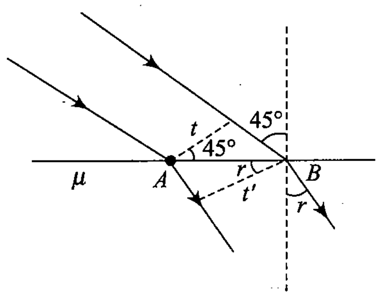


Fig. S-1.24

$$\mu = \frac{\sin 45°}{\sin r} \quad \Rightarrow \quad \sin r = \frac{1}{\sqrt{2}\,\mu}$$

Solving we get, $\quad t' = \dfrac{t\sqrt{2\mu^2 - 1}}{\mu}$

4. *When the bird sees the fish*:
The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the fish, through water, across the interface, through air and reaches the bird. Here, medium 1 is water and medium 2 is air.

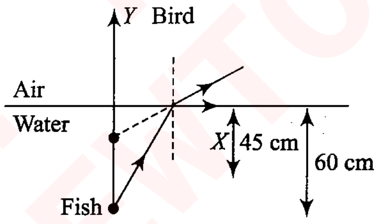


Fig. S-1.25

$$d_1 = -60 \text{ cm}, \mu_1 = \frac{4}{3}, \ \mu_2 = 1,$$

As $\quad \dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2} \quad$ or $\quad d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get

$d_2 = -45\text{cm}$

Thus, the fish appears to be 45 cm below the water–air interface. Image of fish is formed at I.

When the fish sees the bird:
The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the bird, through air, across the interface, through water and reaches the fish. Here, medium 1 is air and medium 2 is water.

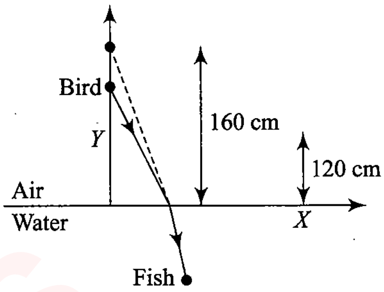


Fig. S-1.26

$$d_1 = +120 \text{ cm}, \mu_1 = 1, \mu_2 = \frac{4}{3}$$

As $\quad \dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2} \quad$ or $\quad d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get $d_2 = 160$ cm

Thus, the bird appears to be 160 cm above the water–air interface. Image of bird is formed at I.

5. A ray of light now travels through water, across the interface through air and converges to form an image at I. Here, medium 1 is water and medium 2 is air.

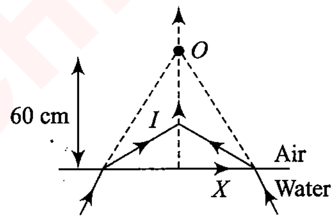


Fig. S-1.27

$$d_1 = +60 \text{ cm}, \mu_1 = \frac{4}{3}, \mu_2 = 1$$

As $\quad \dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2} \quad$ or $\quad d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +45$ cm

Thus, the image is formed 45 cm above the interface.

6. Method of interfaces:
A ray of light from the object undergoes refraction at three interfaces: (1) Water–oil, (2) Oil–glycerine, and (3) Glycerine–air. The coordinate system for each of the interfaces is shown in the figure below.

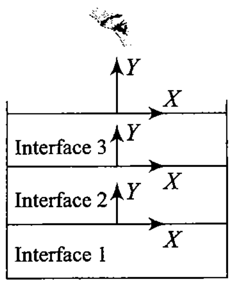


Fig. S-1.28

Water–oil interface:

$$d_1 = -8 \text{ cm}, \mu_1 = \frac{4}{3}, \mu_2 = 1.5$$

$$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = -9 \text{ cm}$$

Oil–glycerine inteface:

$$d_1 = -(9 + 9) = -18 \text{ cm}, \mu_1 = 1.5, \mu_2 = 2$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1$$

$$d_2 = -24 \text{ cm}$$

Glycerine–air interface

$$d_1 = -(4 + 24) = -28 \text{ cm}, \mu_1 = 2, \mu_2 = 1$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get } d_2 = -14 \text{ cm}$$

Thus, the final image is 14 cm below the glycerine–air interface.

Method 2:Method of shifting

Net shifting

$$s = d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + d_3\left(1 - \frac{1}{\mu_3}\right)$$

$$= 8\left(1 - \frac{1}{4/3}\right) + 9\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right)$$

$$= 7 \text{ cm}$$

The shifting will be in the direction of ray travelling, i.e., upwards.

Hence, apparent depth $h' = (21–7) = 14$ cm.

7. A ray of light from the object undergoes refraction at three interfaces: (1) Air–medium A, (2) Medium A–medium B, (3) Medium B–air. The coordinate system for each of the interface is shown in the following figure.

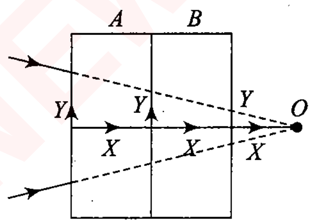


Fig. S-1.29

Air–medium A interface:

$$d_1 = +14 \text{ cm}, \mu_1 = 1, \mu_2 = 1.5$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +21 \text{ cm}$$

Medium A–medium B interface:

$$d_1 = (21 - 6) = 15 \text{ cm}, \mu_1 = 1.5, \mu_2 = 2$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +20 \text{ cm}$$

Medium B–air interface:

$$d_1 = +(20 - 4) = +16 \text{ cm}, \mu_1 = 2, \mu_2 = 1$$

As $$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \text{ or } d_2 = \frac{\mu_2}{\mu_1} d_1, \text{ we get}$$

$$d_2 = +8 \text{ cm}$$

Thus, the final image is 8 cm in front of the medium B–air interface.

Method 2: Method of shifting

$$\text{Net shifting } S = 6\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right)$$

$$= 4 \text{ cm}$$

The shifting will be in the direction of ray travelling, i.e., towards right.

Hence, rays will converge finally = 14 + 4 = 18 cm from left surface or 8 cm from right surface.

8. For the light ray to be reflected from the lower surface *RS*, the angle of incidence must be greater than the critical angle for the glass–air interface (at *N*).

$$i > \sin^{-1}\left(\frac{1}{2}\right) \text{ or } i > 30°$$

At the water–glass interface,

$$\mu_1 = \frac{4}{3}, \mu_2 = 2$$

$$\theta_1 = \theta = ?, \theta_2 = i = 30°$$

Since $\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$, we get

$$\theta = \sin^{-1}\left[\frac{\mu_1}{\mu_2} \sin \theta_1\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{4}\right)$$

9. $\sin r = \mu \sin \theta$ (i)

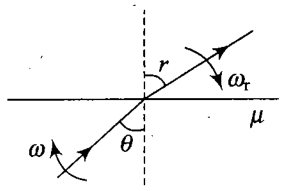


Fig. S-1.30

For $\theta = 30°, \mu = \sqrt{2}$, we get $r = 45°$

Differentiating (i), we get

$$\cos r \left(\frac{dr}{dt}\right) = \mu \cos \theta \left(\frac{d\theta}{dt}\right)$$

$$(\cos 45°)\ \omega_r = \sqrt{2} (\cos 30°) \omega$$

$$\omega_r = \frac{\sqrt{2}(\sqrt{3}/2)}{\left(\frac{1}{\sqrt{2}}\right)}\left(\frac{1}{\sqrt{6}}\right) = \frac{1}{\sqrt{2}}$$

10. $\mu = \dfrac{\sin i}{\sin r} \Rightarrow \sin r = \dfrac{\sin i}{\mu}$

Now, $r' >$ critical angle

$$\sin r' > \frac{1}{\mu}$$

$$\sin[90° - r] > \frac{1}{\mu}$$

$$\cos r > \frac{1}{\mu}$$

$$\sqrt{1-\sin^2 r} > \frac{1}{\mu}$$

$$\sqrt{1-\frac{\sin^2 i}{\mu^2}} > \frac{1}{\mu}$$

$$\mu^2 - \sin^2 i > 1$$

$$\mu > \sqrt{1+\sin^2 i}$$

$i_{max} = 90°$, so $\mu > \sqrt{1+\sin^2 90°}$

$$\mu > \sqrt{2}$$

11. Shift $= 3\left(1-\dfrac{1}{3/2}\right) = 3\left(1-\dfrac{1}{3/2}\right)$

For the mirror, object is at a distance

$$= 21 - 3\left(1-\frac{1}{3/2}\right) = 20 \text{ cm}$$

Therefore, object is at the center of curvature of the mirror. Hence, the light rays will retrace and image will be formed on the object itself.

12. a. Since the apparent shift occurs in the direction of incident light, therefore the mirror should be displaced away from the object.

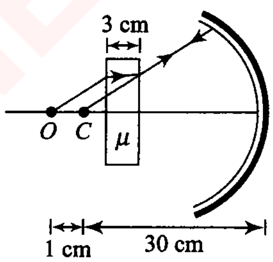


Fig. S-1.31

b. The magnitude of displacement is equal to the apparent shift, i.e.,

$$s = t\left(1-\frac{1}{\mu}\right) = 3\left(1-\frac{1}{3/2}\right) = 1 \text{ cm}$$

13. a. Distance of the watcher as seen from fish

$$8 \times \frac{4}{3} + 3 \times \frac{4/3}{8/5} + 10 = \frac{139}{6} \text{ cm}$$

b. Distance of the fish as seen from watcher is

$$\frac{10}{4/3} + \frac{3}{8/5} + 8 = \frac{139}{8} \text{ cm}$$

14. $\dfrac{\sin i}{\sin r} = \dfrac{1}{n}$

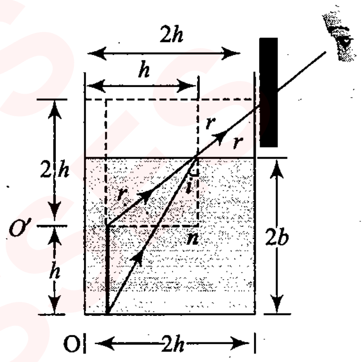


Fig. S-1.32

Since $\tan r = \dfrac{2h}{2h} = 1 \Rightarrow r = 45°$

$\Rightarrow \sin i = \dfrac{h}{h\sqrt{5}} \Rightarrow \sin i = \dfrac{1}{\sqrt{5}}$

$\therefore \dfrac{1}{n} = \dfrac{1/\sqrt{5}}{1/\sqrt{2}} \Rightarrow n = \sqrt{\dfrac{5}{2}}$

15. Point 'O' is at the bottom and boundaries of the medium are parallel.

So, total apparent shift (OI) = shift due to glass + shift due to oil + shift due to water is

$$8\left(1-\frac{1}{1.6}\right) + 4.5\left(1-\frac{1}{\mu}\right) + 6\left(1-\frac{1}{4/3}\right) = 6$$

After solving, we get $\mu = 1.5$

16. At first surface,

$$\frac{\mu_1}{d_1} - \frac{\mu_2}{d_2} \Rightarrow \frac{1}{(-8)} = \frac{\mu_2}{d_2} \Rightarrow d_2 = -8\mu$$

Image I_1 will serve as an object for the mirror and form an image I_2 behind it at a distance of $(8\mu + 6)$ cm.

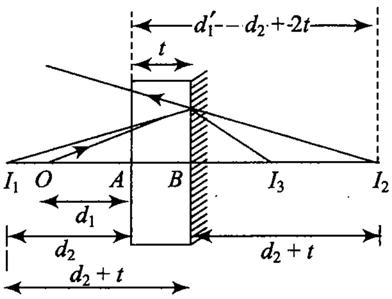


Fig. S-1.33

I_2 will serve as an object for the first surface.
The rays will reflect from the mirror.

Again using $\dfrac{d_1'}{\mu} = \dfrac{d_2'}{1}$

$$d_1' = d_2 + 2t = -(8\mu + 12)$$
$$d_2' = -(10+6) = -16 \text{ cm}$$

After substituting the values, $\mu = 1.5$.

17. The law of refraction in vector form is

$$\mu_1 (\hat{e}_1 \times \hat{n}) = \mu_2 (\hat{e}_2 \times \hat{n})$$

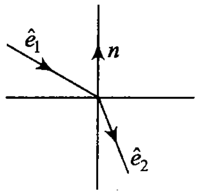


Fig. S-1.34

Here, $\hat{e}_1 = \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

= unit vector along incident ray

$\hat{n} = \hat{k}$ = unit vector along normal or incidence point

$\therefore \quad \mu_1 (\hat{e}_1 \times \hat{n}) = \mu_2 (\hat{e}_2 \times \hat{n})$

$\hat{e}_2 = x\hat{i} + y\hat{j} + z\hat{k}$

= unit vector along refracted ray

or $\quad 1\left\{\left(\dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}\right) \times \hat{k}\right\} = 2\{(x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{k}\}$

or $\quad \dfrac{-\hat{j} + \hat{i}}{\sqrt{3}} = 2\{-x\hat{i} + y\hat{j}\}$

or $\quad x = \dfrac{1}{2\sqrt{3}}, \quad y = \dfrac{1}{2\sqrt{3}}$

As $\hat{e}_2$ is a unit vector, therefore

$|\hat{e}_2| = 1$

$\Rightarrow \quad \sqrt{x^2 + y^2 + z^2} = 1$

or $\quad \sqrt{\left(\dfrac{1}{2\sqrt{3}}\right)^2 + \left(\dfrac{1}{2\sqrt{3}}\right) + z^2} = 1$

or $\quad \sqrt{\left(\dfrac{1}{12} + \dfrac{1}{12} + z^2\right)} = 1$ or $\dfrac{1}{6} + z^2 = 1$

$\therefore \quad z^2 = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

$\therefore \quad z = \pm\sqrt{\left(\dfrac{5}{6}\right)}$

Since refracted ray is in negative z-axis region

$\therefore \quad z = -\sqrt{\dfrac{5}{6}}$

$\therefore \quad \hat{e}_2 = \dfrac{1}{2\sqrt{3}}\hat{i} + \dfrac{1}{2\sqrt{3}}\hat{j} - \sqrt{\left(\dfrac{5}{6}\right)}\hat{k}$

18. x = total shift

$$x = x_1 + x_2 + \cdots + x_n$$
$$= t_1\left(1 - \frac{1}{\mu}\right) + t_2\left(1 - \frac{1}{\mu}\right) + \cdots + t_n\left(1 - \frac{1}{\mu}\right)$$

or $\quad 1 \text{ cm} = \left(1 - \dfrac{2}{3}\right) 1 + 2 + 3 + \cdots + n$

$$1 = \frac{n(n+1)}{6}$$
$$6 = n(n+1) \quad \Rightarrow \quad n = 2$$

19. $\dfrac{4/3}{v} + \dfrac{1}{60} = \dfrac{\left(\frac{4}{3} - 1\right)}{\infty} = 0$

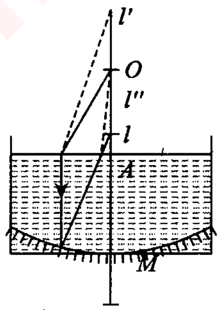


Fig. S-1.35

$v = -80$ cm

I' acts as an object for the mirror.

$\therefore \quad \dfrac{1}{v} + \dfrac{1}{-100} = \dfrac{-1}{20}$

$\Rightarrow \quad v = -25$ cm

I'' will serve as an object for the plane surface.

$\therefore \quad \dfrac{1}{v} - \dfrac{(4/3)}{5} = \dfrac{\left(1 - \frac{4}{3}\right)}{\infty}$

or $\quad v = +3.75$ cm

20. The bottom of the beaker appears to be shifted up by a distance

$$\Delta t = \left(1 - \frac{1}{\mu}\right)d$$

Thus, the apparent distance of the bottom from the mirror is $h - \Delta t = h - [1 - (1/\mu)]^d = h - d + (d/\mu)$. The image is formed behind the mirror at a distance $h - d + (d/\mu)$.

21. The ray diagram is shown in the following figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then, $R = -40$ cm. The object distance is $u = -5$ cm. Using the mirror equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

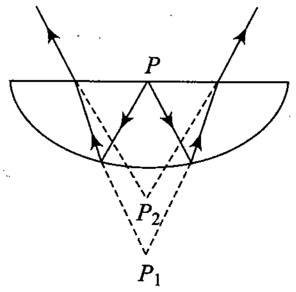


Fig. S-1.36

$$\frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40 \text{ cm}} - \frac{1}{-5 \text{ cm}} = \frac{6}{40} \text{ cm}$$

or $v = 6.67$ cm

The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is 6.67 cm + 5.00 cm = 11.67 cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance (11.67 cm)

$$\left(1 - \frac{1}{1.33}\right) = 2.92 \text{ cm}$$

Thus, the final image is formed at a point (11.67 – 2.92) cm = 8.75 cm below the water surface.

22. Let the object be placed at a height 'x' above the surface of water. The apparent position of the object with respect to the mirror should be at the center of curvature so that the image is formed at the same position.

Since $\dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{1}{\mu}$

(with respect to the mirror), therefore

$$\frac{x}{R-h} = \frac{1}{\mu}$$

or $x = \dfrac{R-h}{\mu}$

23. $\mu = \mu_0\sqrt{x+1}$, at $x = 0 \to \mu = \mu_0$

$\mu \sin\theta = \text{constant}$

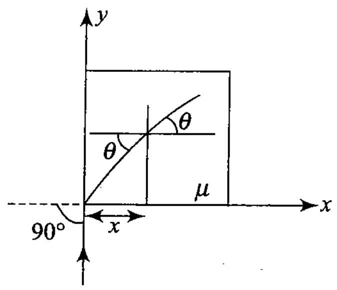


Fig. S-1.37

$$\mu_0 \sin 90° = \mu_0\sqrt{x+1}\,\sin\theta$$

$$\sin\theta = \frac{1}{\sqrt{x+1}} \Rightarrow \cos\theta = \sqrt{\frac{x}{x+1}}$$

$$\frac{dy}{dx} = \tan\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{x}}$$

$$\therefore \quad \int dy = \int \frac{1}{\sqrt{x}}\,dx$$

or $\quad y = 2\sqrt{x}$

Exercise 1.4

1. Angle of dispersion:

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$
$$= (1.632 - 1.613)\,5 = 0.095°$$

2. a. $\mu_y = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.6 + 1.5}{2} = 1.55$

b. Dispersive power:

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.6 - 1.5}{1.55 - 1} = \frac{2}{11}$$

3. At minimum deviation,

$$2r = A \text{ or } r = 30°$$

Also, $\quad \mu = \dfrac{\sin(i)}{\sin(r)}$

Substituting the values, we get

$$1.5 = \frac{\sin(i)}{\sin(30)} \text{ or } i = 48.6°$$

The angle of deviation, $\delta = 2i - A = 37.2°$

At maximum deviation:

$$A = 60°,\ \mu = 1.5 \text{ and } i_1 = 90°$$

Since $\quad \mu = \dfrac{\sin(i_1)}{\sin(r_1)}$, we get $1.5 = \dfrac{\sin(90)}{\sin(r_1)}$

or $\quad r_1 = 41.8°$

But $\quad r_1 + r_2 = A$, therefore, $r_2 = 18.2°$

Once again, $\mu = \dfrac{\sin(i_2)}{\sin(r_2)}$

$$\therefore \quad 1.5 = \frac{\sin(i_2)}{\sin(18.2)} \text{ or } i_2 = 27.9°$$

Therefore, the angle of deviation,

$$\delta = i_1 + i_2 - A = 57.9°$$

4. Given, $\mu = 1.5$ and $A = \delta_{min}$

At minimum deviation,

$$r = A/2 \text{ and } \delta_{min} = 2i - A$$

Substituting for A, we get $i = \delta_{min}$

But $\mu = \dfrac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{\sin A}{\sin\left[\dfrac{A}{2}\right]} = 2\cos\left[\dfrac{A}{2}\right]$

Therefore, $1.5 = 2\cos\left[\dfrac{A}{2}\right]$ or $A = 2\cos^{-1}\left(\dfrac{3}{4}\right)$

5. $\delta = A(\mu - 1)$

$= 4\left(\dfrac{3}{2} - 1\right)$

$= 2°$

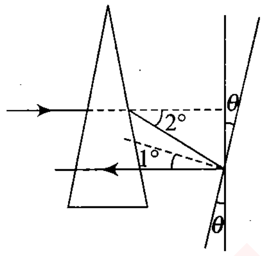


Fig. S-1.38

Hence, angle of rotation of mirror = 1°.

6. Critical angle between glass and liquid face is

$$\sin\theta_c = \frac{3/2}{\mu} = \frac{3}{2\mu} \quad \text{(i)}$$

Angle of incidence at face AC is 60°, i.e., $i = 60°$.
For total internal reflection to take place,

$i = \theta_c$ or $\sin i > \sin\theta_c$

or $\sin 60° > \dfrac{3}{2\mu}$ or $\sqrt{\dfrac{3}{2}} > \dfrac{3}{2\mu}$

or $\mu > \sqrt{3}$

7. In the case of minimum deviation,

$i_1 = i_2,\ r_1 = r_2$

$\delta = 2i - A$

$r = A/2$

According to problem, $i = 2r = A$

$\delta_{min} = 2A - A = A$

$$n = \frac{\sin\left(\dfrac{A + \delta_{min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin A}{\sin\dfrac{A}{2}} = \frac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}}$$

$\cos\dfrac{A}{2} = \dfrac{n}{2} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$\therefore\ A = 90°$

8. The given parameters are $\delta = 30°$ and $A = 60°$. Let us test whether the prism is in the position of minimum deviation.

$$n = \frac{\sin\left(\dfrac{30° + 60°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$= \dfrac{1}{\sqrt{2}} \times 2 = \sqrt{2}$

As $n = \sqrt{2}$, the ray suffers minimum deviation through the prism. Thus,

$r_1 = r_2 = r = \dfrac{A}{2} = 30°$

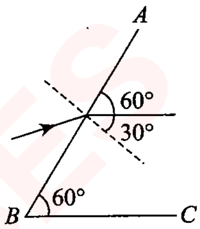


Fig. S-1.39

Inside the prism, the ray makes an angle of 60° with the face AB, so it is parallel to the base.

9. In the figure shown,

$r_2 = C$

$r_1 = 60 - r_2 = 60 - C$

$r_3 = 60 - r_2 = 60 - C$

$r_1 = r_3 = r$ (say)

$i_1 = i_2 = i$ (say)

Net deviation, $\delta = (1 - r) + (180 - 2r_2) + (i - r) = 108°$

$r_2 = r - i = 36°$

$C + 60 - C - i = 36°$

$i = 24°$, $\sin 24° \approx 0.40$

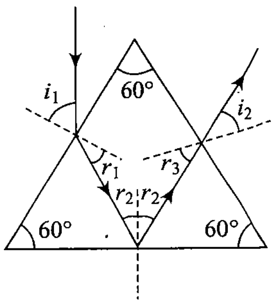


Fig. S-1.40

From Snell's law, we have

$\sin 24° = \mu \sin r$

$0.4 = \mu\sin(60° - C)$

$0.4 = \mu\left[\dfrac{\sqrt{3}}{2}\cos C - \dfrac{1}{2\mu}\right]$

$0.8 = \sqrt{3}\mu\sqrt{1 - \dfrac{1}{\mu^2}} - 1$

$\therefore\ \mu^2 - 1 = 1.08$

$\mu = 1.447$

10. As the ray of light grazes the second surface, r_2 is the critical angle.

$\sin r_2 = \dfrac{1}{\mu}$

$r_2 = (45° - r_1)$

$\sin r_1 = \dfrac{\sin 45°}{\mu} = \dfrac{1}{\sqrt{2}\mu}$

$\sin r_2 = \sin(45° - r_1)$

$= \dfrac{1}{\sqrt{2}}[\cos r_1 - \sin r_1]$

$\dfrac{1}{\mu} = \dfrac{1}{\sqrt{2}}\left[\sqrt{1 - \dfrac{1}{2\mu^2}} - \dfrac{1}{\sqrt{2}\mu}\right]$

$$\frac{1}{\mu}=\frac{1}{\sqrt{2}}\left[\frac{\sqrt{2\mu^2-1}}{\sqrt{2}\mu}-\frac{1}{\sqrt{2}\mu}\right]$$

$$2=\sqrt{2\mu^2-1}-1$$
$$2\mu^2-1=9$$
$$2\mu^2=10$$
$$\Rightarrow \quad \mu^2=5 \quad \text{or} \quad \mu=\sqrt{5}$$

At minimum deviation,

$$r_1=r_2\frac{45°}{2}=22.5°$$

$$\mu=\frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1=(\sqrt{5})\sin(22.5°)$$

$$i_1=58.8°$$

11. For minimum deviation:

$$\mu=\frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$$

$$\frac{3}{2}=\frac{\sin[(60°+\delta_m)/2]}{\sin(60°/2)}=\sin\left(\frac{60+\delta_m}{2}\right)=\frac{3}{4}$$

$$\Rightarrow \quad i=\frac{A+\delta_m}{2}=\sin^{-1}\left(\frac{3}{4}\right)=48.5°$$

12. Since $\tan\delta=\frac{h}{f} \quad \Rightarrow \quad f=\frac{h}{\tan\delta}$

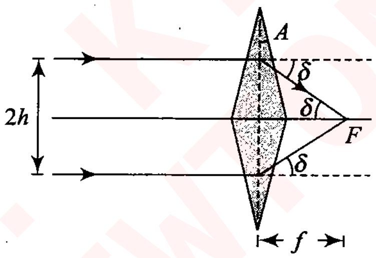


Fig. S-1.41

Further, $\delta=(\mu-1)A \quad \Rightarrow \quad f=\frac{h}{(\mu-1)A}$

Exercise 1.5

1. a. True. For plane surfaces, $\frac{\mu_2}{v}=\frac{\mu_1}{u}$

b. False. In the equation,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

μ_1 is the medium which comes before the boundary and μ_2 is the medium which comes after the boundary.

c. True. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

Here, $\mu_2=1.5$, $\mu_1=1$, $u=5R$, $R=+R$

$$\frac{1.5}{v}-\frac{1}{-5R}=\frac{1.5-1}{+R}$$
$$\frac{1.5}{v}=\frac{1}{2R}-\frac{1}{5R}=\frac{3}{10R}$$
$$v=5R$$

d. False. All the rays coming out from the object are radial. The radial rays are not deviated with the change in medium.

e. True. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

Here, $\mu_2=1$, $\mu_1=2$, $R=-R$

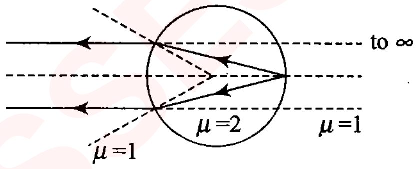


Fig. S-1.42

$$u=-2R$$
$$\frac{1}{v}-\frac{2}{-2R}=\frac{1-2}{-R}=\frac{1}{R}$$
$$\text{or} \quad \frac{1}{v}=\frac{1}{R}-\frac{1}{R}$$
$$\Rightarrow \quad v=\infty$$

2. a. $\mu_1=1$, $\mu_2=1.5$, $R=+20$ mm, $u=+80$ mm

$$\frac{1.5}{v}-\frac{1}{-80}=\frac{1.5-1}{-20}$$
$$v'=+120 \text{ mm}$$

The image is therefore formed at the right of the vertex. (v is positive and at a distance of 120 mm from it.)

b. $\frac{1.5}{v}+\frac{-1.33}{-80}=\frac{1.5-1.33}{+20}$, $v=-180$ mm

The fact that v is negative means that the rays after refraction by the surface are not converging but appear to diverge from a point 180 mm to the left of the vertex. In this illustration, then, the surface forms a virtual image 180 mm to the left of the vertex.

3. We know that $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$u=-12$ cm, $\quad R=10$ cm, $\quad \mu_1=1$, $\quad \mu_2=1.5$

$$\Rightarrow \quad \frac{1.5}{v}-\frac{1}{-12}=\frac{1.5-1}{10} \quad \Rightarrow \quad v=-45 \text{ cm}$$

This image will serve as an object for the second surface.
For the second surface, object distance, $u=5+45=50$ cm
For the second surface again, $u=-50$ cm, $R=-25$, $\mu=1.5$, $\mu_2=1$

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1}{v}-\frac{1.5}{-50}=\frac{1-1.5}{-25}$$

or $\quad v=-100$ cm

Final image will be at a distance of –95 cm from the first surface on the same side as the object.

4. Let x be the distance of the object from the pole in medium 1.
Here we have, $u = -x$, $v = +x$ and $R = +R$

Using the equation, we get $\dfrac{\mu_2}{+x} - \dfrac{\mu_1}{-x} = \dfrac{\mu_2 - \mu_1}{+R}$

or $x = \left(\dfrac{\mu_2 + \mu_1}{\mu_2 - \mu_1}\right)R$

Note that the real image is formed only when $\mu_2 > \mu_1$.

5. Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed at equidistant from the sphere, then the ray must pass symmetrical through the sphere, as shown in the figure in question.
Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{+R}$$

or $x = \left(\dfrac{\mu_1}{\mu_2 - \mu_1}\right)R$

6. According to cartesian sign convention,
$u = -30$ cm, $R = +10$ cm; $\mu_1 = 1$; $\mu_2 = 1.5$

Applying the equation, we get $\dfrac{1.5}{v} = \dfrac{1}{-30} = \dfrac{1.5 - 1}{+10}$

or $v = 90$ cm (real image)

Let h_i be the height of the image, then

$$\frac{h_i}{h_0} = \frac{\mu_1 v}{\mu_2 u} = \frac{(1)(90)}{(1.5)(-30)} = -2$$

$\Rightarrow$ $h_i = -2h_0\,(0.5) = -2(0.5) = -1$ cm

The negative sign shows that the image is inverted.

7. For refraction near point A, $u = -30$; $R = -20$; $n_1 = 2$; $n_2 = 1$.

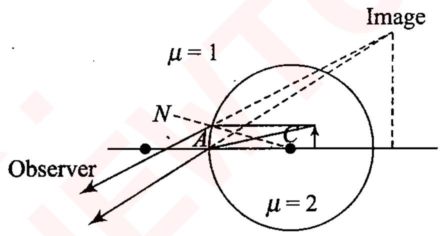


Fig. S-1.43

Applying refraction formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$

$v = -60$ cm

$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

$\therefore$ $h_2 = 4$ mm

8. Deviation by a sphere is $2(i - r)$
Here, deviation $\delta = 60° = 2(i - r)$
or $i - r = 30°$
$\Rightarrow$ $r = i - 30° = 60° - 30° = 30°$

$\therefore$ $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin 60°}{\sin 30°} = \sqrt{3}$

9. We will have single surface refractions successively at the four surfaces S_1, S_2, S_3, and S_4. Do not forget to shift origin to vertex of the respective surface.
Refractive at first surface S_1: Light travel from air to glass.

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{(1.5-1)}{(+10)}$$

$v_1 = 30$ cm

First image is the object for the refraction at second surface.
For refraction at surface S_2: Light travels from glass to air.

$$\frac{1}{v_2} - \frac{1.5}{(+25)} = \frac{1-1.5}{(+5)}$$

$v_2 = -25$ cm

For refraction at surface S_3: Light travels from air to glass.

$$\frac{1.5}{v_3} - \frac{1}{(-35)} = \frac{(1.5-1)}{(-5)}$$

$v_3 = -35/3$ cm.

For refraction at surface S_4: Light travels from glass to air.

$$\frac{1}{v_4} - \frac{1.5}{-(35/3+5)} = \frac{1-1.5}{-10}$$

$v_4 = -25$ cm

The final image is virtual, formed at 25 cm to the left of the vertex of surface S_4.

10. Taking the origin at the vertex, $u = -20.0$ cm and $R = 5.0$ cm.

We have, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{v} = \frac{1}{-20.0\,\text{cm}} + \frac{0.5}{5.0\,\text{cm}} = \frac{1}{20\,\text{cm}}$$

$\Rightarrow$ $v = 30$ cm

The image is formed inside the rod at a distance of 30 cm from the vertex.

11. The observer sees the image formed due to refraction at the spherical surface when the light from the bubble goes from the glass to the air.
Here, $u = -4.0$ cm, $R = -10$ cm, $\mu = 1.5$ and $\mu_2 = 1$.

$$\frac{1}{v} = \frac{0.5}{-4.0\,\text{cm}} = \frac{1-1.5}{-10\,\text{cm}}$$

$$\frac{1}{v} = \frac{0.5}{10\,\text{cm}} - \frac{1.5}{4.0\,\text{cm}}$$

$v = -3.0$ cm

Thus, the bubble will appear 3.0 cm below the surface.

12. a. Critical angle: θ_c

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{2} \Rightarrow \theta_c = 30°$$

Since angle of incidence is 45° (> θ_c) at the plane surface, so incident rays will be reflected from the plane surface.

b. $\dfrac{2}{v} - \dfrac{1}{\infty} = \dfrac{2-1}{3}$ $\Rightarrow$ $v = 6$ cm

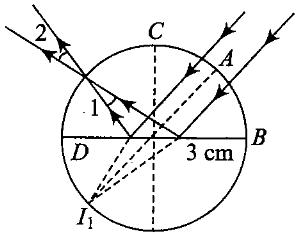


Fig. S-1.44

If the sphere were completed, then image would form at I_1, but due to reflection at DB, final image will be formed at I.

c. Final image is formed at I.

d. See the figure.

13. a. $R = 5$ cm for the observer at left:

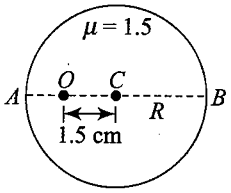


Fig. S-1.45

$$\frac{1}{v} - \frac{\mu}{-AO} = \frac{1-\mu}{-R}, \; AO = 3.5 \text{ cm}$$

$$\Rightarrow \quad v = -\frac{70}{23} = -3 \text{ cm}$$

Hence, image is formed 3 cm to the right of A or 2 cm to the left of C.

b. For observer to right

$$\frac{1}{v} - \frac{\mu}{-BO} = \frac{1-\mu}{-R}$$

$$v = -\frac{130}{17} = -7.65 \text{ cm}$$

Hence, image is formed 7.65 cm left of B or 2.65 cm left of C.

14. $\frac{1}{v} - \frac{1.5}{-3} = \frac{1-1.5}{-3}$

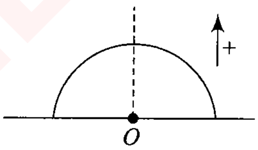


Fig. S-1.46

$v = -3$ cm

Hence, no shifting.

15. $\frac{1}{v} - \frac{1.5}{-3} = \frac{1-1.5}{\infty}$

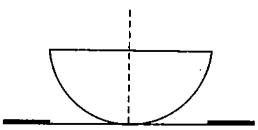


Fig. S-1.47

$v = -2$ cm

$\therefore$ Shifting $= 3 - 2 = 1$ cm

16.

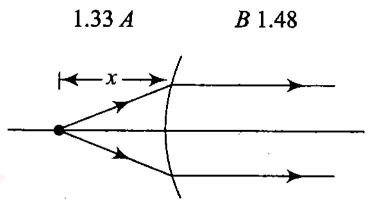


Fig. S-1.48

$$\frac{1.48}{\infty} - \frac{1.33}{-x} = \frac{1.48-1.33}{30}$$

$\therefore \quad x = 266$ cm

17.

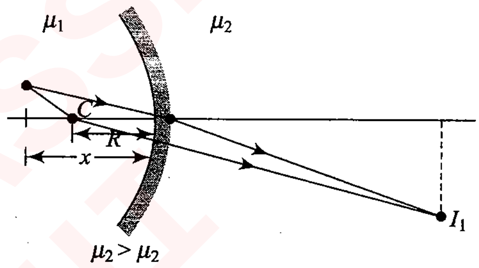


Fig. S-1.49

(i) If $x > R$, then image is at I_1.

(ii) If $R < x < \dfrac{\mu_1 R}{\mu_1 - R_2}$, then image is formed at I_2.

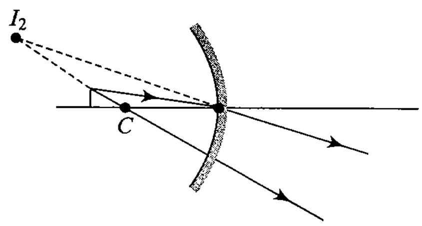


Fig. S-1.50

(iii) If $x < R$, image is formed at I_2.

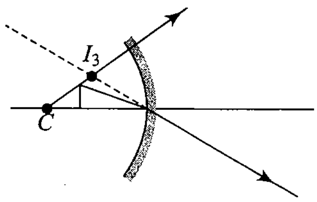


Fig. S-1.51

18. $\frac{1}{v} - \frac{\mu}{-R} = \frac{1-\mu}{\infty} \Rightarrow v = -\frac{R}{\mu}$

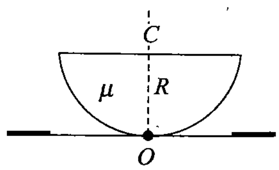


Fig. S-1.52

So, image will be formed at a distance of R/μ below C.

19. $\dfrac{1}{-1}-\dfrac{\mu}{u}=\dfrac{1-\mu}{-R}$

Put R = 4 cm, μ = 1.5 to get u = 4/3 cm. So, the object is 4/3 cm away from the surface.

20. $\dfrac{\mu}{v_1}-\dfrac{1}{\infty}=\dfrac{\mu-1}{R}\Rightarrow v_1=\dfrac{\mu R}{\mu-1}$

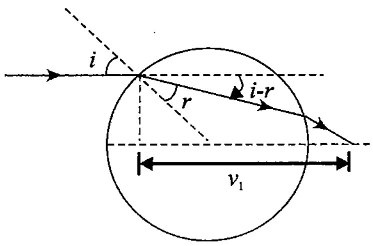


Fig. S-1.53

21. We now have refraction at a single curved surface. The coordinate system for this problem is shown in figure. Fig. S-1.54 see that

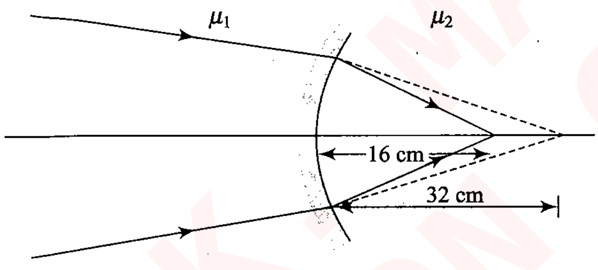


Fig. S-1.54

Refraction: Ray from water to glass

$$\mu_1=\frac{4}{3},\ \mu_2=1.5$$

$$u=+32\text{ cm},\ R=+16\text{ cm}$$

Substituting in the equation

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

we get $\quad v=+(144/5)$ cm

i.e., the object appears to be 144/5 cm to the right of interface.

22. $\dfrac{\mu}{y_1}-\dfrac{1}{\infty}=\dfrac{\mu-1}{a}$ (i)

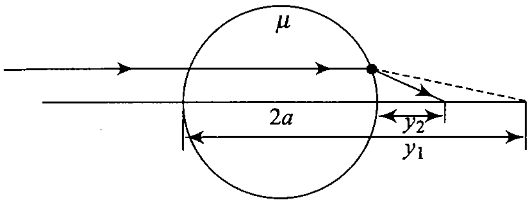


Fig. S-1.55

$$\frac{1}{y_2}-\frac{\mu}{(y_1-2a)}=\frac{1-\mu}{-a} \quad \text{(ii)}$$

Solving (i) and (ii), we get

$$y_2=\frac{(2-\mu)a}{2(\mu-1)}$$

Exercise 1.6

1. a. False. Real images of real object are always inverted. But for virtual object the image may be erect.

b. False. Real images are always inverted.

c. True. From equation, we have

$$m=\frac{f_0}{x+f_0}$$

The value of m is always positive and less than 1.

d. True. The air bubble in water diverges a parallel beam of light.

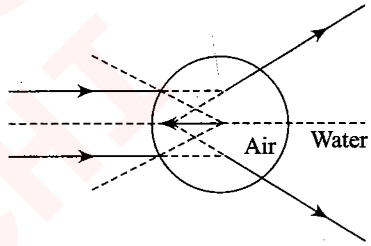


Fig. S-1.56

e. True. The least distance between a real object and its real image is $4f$.

f. True. $\dfrac{1}{y}+\dfrac{1}{x}=\dfrac{1}{f},\ \dfrac{dx}{dt}=-u,\ \dfrac{dy}{dt}=v$

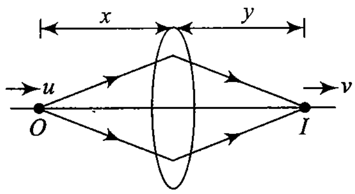


Fig. S-1.57

$$-\frac{1}{y^2}\frac{dy}{dt}-\frac{1}{x^2}\frac{dx}{dt}=0$$

$$v=\frac{y^2}{x^2}u$$

Initially, x is large and y is small. So, initially v is small. But as the object moves from ∞ to focus, x decreases and y increases. So, v also increases.

g. True. Using equation, we have

$$f=f_0\left[\frac{\mu(\mu_0-1)}{\mu_0-\mu}\right]$$

Here, $\quad \mu_0=\dfrac{3}{2},\ \mu=\dfrac{4}{3}$

$\Rightarrow \qquad f=4f_0$

The effective focal length comes out to be positive.

2. Two images will be formed, one from each type of material.
3. a. $\mu_1 < \mu_2$, because it is behaving like a divergence lens.
   b. $\mu_1 = \mu_2$, because no refraction is taking place.
4. For a real object, it is not possible for a single lens to form a real and erect image.
   But for a virtual object, it may be possible.
5. The rays of light diverge from the object; it is a real object.
   Focal length of a convex lens is positive.
$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{2}{R}\right)=\frac{(3/2)-1}{1}\left(\frac{2}{10}\right)=\frac{1}{10}$$
   The given parameters are
$$u=-30 \text{ cm} \quad \text{and} \quad f=+10 \text{ cm}$$
   Substituting these values in thin-lens equation, we have
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{v}-\frac{1}{(-30)}=\frac{1}{10}$$
   which on solving gives $v=+15$ cm.
   The image distance is positive, which implies that the image is real. The lateral magnification is
$$m=\frac{v}{u}=\frac{15}{(-30)}=-\frac{1}{2}$$
   The image is half the size of the object; size of the image is 0.50 cm. The negative sign with magnification shows inverted image.
6. In accordance with cartesian sign convention, the given parameters are
$$f=-10 \text{ cm},\ u=-15 \text{ cm}$$
   From lens equation, we have
$$=\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$
$$=\frac{1}{(-10)}+\frac{1}{(-15)}=-\frac{1}{6},\ v=-6 \text{ cm}$$
   Lateral magnification, $m=\frac{v}{u}=\frac{-6}{-15}=0.40$
   The minus sign with image shows that the image is located on the side of the object. The magnification is positive and $m<1$, which shows that the image is upright and diminished.
7. a. The paths of the rays are shown in the following figure. First, draw the ray AA' until it intersects with the principal optical axis and then find the center of the lens C. Since the virtual image is magnified, the lens is convex.
   Draw the ray AB parallel to the principal optical axis. It is refracted by the lens so that it passes through its focus and its continuation passes through the virtual image.

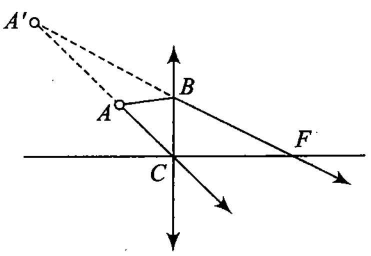


Fig. S-1.58

   The ray $A'B$ intersects the principal optical axis at point F, the focus of the lens.
   b. The paths of the rays are shown in the following figure.

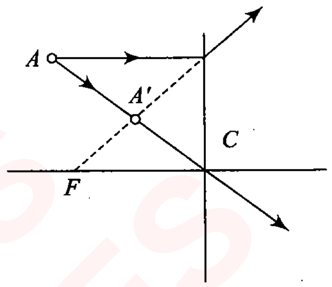


Fig. S-1.59

8. We have, $\mu=1.5$, $R_1=20$ cm, and $R_2=-40$ cm.
   Therefore, the focal length of the lens is
$$\frac{1}{f}=(1.5-1)\left[\frac{1}{20}-\frac{1}{-40}\right]$$
   or $\quad f=\frac{80}{3}$ cm
   Now, $\quad f=\frac{80}{3},\ u=-40$ cm
   Therefore, from equation, we get
$$\frac{1}{v}=\frac{3}{80}-\frac{1}{40} \quad \text{or} \quad v=+80 \text{ cm}$$
   Therefore, the image is formed 80 cm on the other side of the lens.
9. Here, $\mu=1.5$, $R_1=-20$ cm, and $R_2=+20$ cm.
   Substituting in equation, we get $f=-20$ cm
   Now, $u=+10$ cm, $f=-20$cm
   Substituting in equation, we get
$$\frac{1}{v}=\frac{1}{-20}+\frac{1}{10} \quad \text{or} \quad v=+20 \text{ cm}$$
   Thus, the rays now converge to a point 20 cm in front of the lens.
10. Here, $u=-15$ cm, $m=-2$ (Note the negative sign. As the image is on a screen, it is real which implies m must be negative)
   $\therefore \quad v=mu=+30$ cm
   Applying equation, we get
$$\frac{1}{f}=\frac{1}{30}-\frac{1}{-15} \quad \text{or } f=10 \text{ cm}$$
   Therefore, the lens is convex, with focal length of 10 cm.
11. Here, $f=+10$ cm and $m=+2$. (Magnification is now positive as the image is erect.) Let the object distance from the lens be x.
   Then, $u=-x$ and $v=mu=2x$.
   Substituting in equation, we get
$$\frac{1}{10}=\frac{1}{-2x}-\frac{1}{-x} \quad \text{or } x=5 \text{ cm}$$
   Therefore, the lens is to be held 5 cm from the object.

12. $m = 2 = \frac{v}{u} \Rightarrow v = 2u = 2 \times 12 = 24$ cm

Distance between object and image: 12 + 24 = 36 cm.

13. No, the rays will refract only once.

14. Ray diagram: The path of a typical light ray is shown in the following figure.

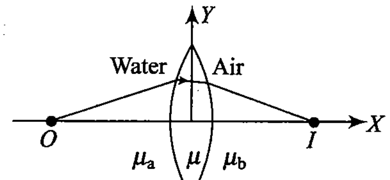


Fig. S-1.60

This problem corresponds to the situation of a different medium on either side of the lens. The governing equation is

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu_a}{R_2} \quad \text{(i)}$$

Here, $\mu = -15$ cm, $\mu_a = 1.33$, $\mu_b = 1$, $\mu = 1.5$

To complete the problem will still need the radius of curvature of the two sides of the lens.

Since it is a biconvex lens, $R_1 = R_2 = R$ and

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

or $\quad R = 20$ cm

Therefore, $R_1 = +20$ cm and $R_2 = -20$ cm.

Substituting in Eq. (i), we get $v = 10.3$ cm

15. Here, $u = -40$ cm, $f = +20$ cm

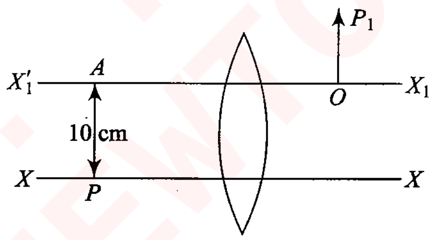


Fig. S-1.61

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $\quad \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40}$

$v = 40$ cm

$$m = \frac{v}{u} = \frac{40}{-40} = -1$$

$\therefore \quad OP_1 = m \times AP = (-1)(-1)$ cm = 1 cm

Therefore, the vertical position of P_1 from P is $(1+1) = 2$ cm.

As we know, when mirror is rotated by θ, reflected ray is rotated by 2θ. So, in this case, final image is below PO produced at 10 cm.

Exercise 1.7

1. For convex lens:

$$v = \frac{uf}{u+f} = \frac{-15 \times 10}{-15 + 10} = 30 \text{ cm}$$

For concave lense:

$u = (30 - 25) = 5$ cm

$$v = \frac{uf}{u+f} = \frac{5(-10)}{5 - 10} = 10 \text{ cm}$$

So, final image is formed 10 cm to the right of the concave lens.

2. $d = 10 + 20 = 30$ cm

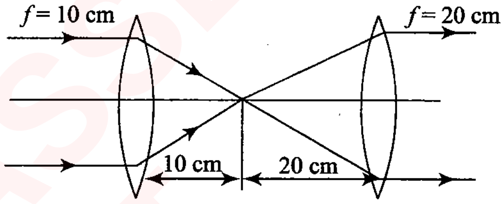


Fig. S-1.62

3. For convex lens: $v_1 = \frac{uf}{(u+f)} = \frac{-15 \times 10}{-15 + 10} = 30$ cm

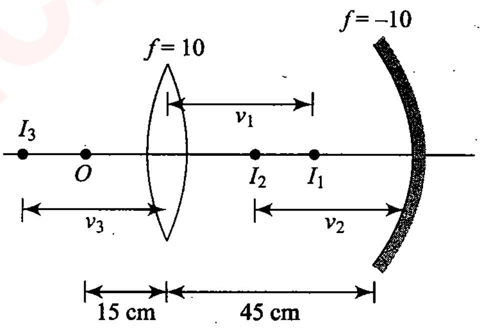


Fig. S-1.63

For concave mirror: $u = -(43 - 30) = -15$ cm

$$\frac{1}{v_2} + \frac{1}{-15} = \frac{1}{-10} \Rightarrow v_2 = -30 \text{ cm}$$

Now, again for convex lens:

$u - (45 - 30) = -15$ cm

$$\Rightarrow \quad v_3 = \frac{-15 \times 10}{-15 + 10} = 30 \text{ cm}$$

So, final image is formed at I_3 at a distance of 30 cm from the lens.

4. For convex lens: $v = \frac{-15 \times 10}{-15 + 10} = 30$ cm

The refracted rays from lens should fall normally on the mirror for the image to coincide with the object.

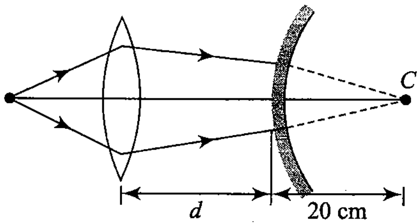


Fig. S-1.64

$d + 20 = v = 30$

$\Rightarrow \quad d = 10$ cm

5. Let F is the effective focal length of this concave mirror. Then,

$$\frac{1}{F} = \frac{1}{f_m} - \frac{2}{f_l}$$

$$f_m = \infty, \ \frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R}\right)$$

$$\frac{1}{F} = (\mu - 1)\frac{2}{R} \quad \Rightarrow \quad F = \frac{-R}{2(\mu - 1)}$$

$$F = \frac{-30}{2(1.5 - 1)} = -30 \text{ cm}$$

So, object should be placed at $2F = 60$ cm from the lens to coincide the image with the object.

6. Let the image from the first lens be formed at a distance y to the right of lens.

Refraction at the first lens:

Here, $u = -3$ cm, $f = +20$ cm, $v = +y$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \quad \text{or} \quad \frac{1}{y} - \frac{1}{-30} = \frac{1}{20} \quad \text{or} \quad y = 60 \text{ cm}$$

Refraction at the second lens:

Here, $u = 60 - 10 = +50$ cm, $f = +25$ cm

Let the image be at a distance z to the right of lens 2. Therefore, $v = +z$.

Substituting in the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2} \quad \text{or} \quad \frac{1}{z} = \frac{1}{25} + \frac{1}{50} \quad \text{or } z = \frac{50}{3} \text{ cm}$$

7. Image formed due to converging lens acts as object for diverging lens

$u = +15$ cm, $\quad v = (19.2 + 15)$ cm $= 34.2$ cm

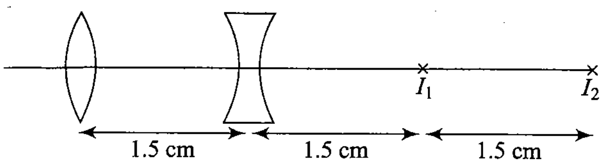


Fig. S-1.65

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f} = \frac{1}{34.2} - \frac{1}{15} = \frac{15 - 34.2}{34.2 \times 15} = \frac{-19.2}{34.2 \times 15}$$

$f = -26.7$ cm

8. For lens f_1

$f_1 = 30$ cm, $u = 10$ m

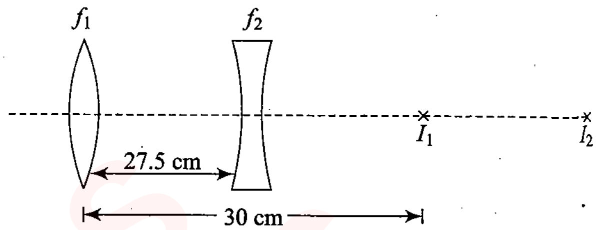


Fig. S-1.66

$\therefore \quad v = 30$ cm

$f_2 = -10$ cm, $u = 2.5$ cm

$$\frac{1}{v} - \frac{1}{25} = \frac{-1}{15}$$

$$\frac{1}{v} = \frac{-1}{15} + \frac{1}{2.5} = \frac{1}{3} \Rightarrow v = 3 \text{ cm}$$

9. Ray diagram: The path of a typical light ray is shown in the following figure.

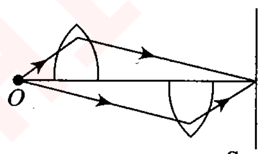


Fig. S-1.67

The element encountered is only a lens in either path of the light rays.

The lens is cut in half along the axis. So, both halves have the same focal length. Since two sharp images are formed on the screen, the result is equivalent to the lens displacement method with $D = 80$ cm and $d = 12$ cm.

For the lens displacement method, the focal length of the lens is given by

$$f = \frac{D^2 - d^2}{4D} \quad \text{or } f = 19.55 \text{ cm}$$

10. Here, $R_1 = +12$ cm, $R_2 = \infty$, $\mu_p = 1.5$, $\mu_a = 1$, therefore

Focal length of the lens, $\dfrac{1}{f_1} = (1.5 - 1)\left[\dfrac{1}{12} - \dfrac{1}{\infty}\right]$

or $\quad f_1 = 24$ cm

The plane side of the lens is silvered; focal length of the mirror

$$f_m = \frac{R_2}{2} = \frac{\infty}{2} = \infty$$

Therefore, effective focal length of the silvered lens is

$$\frac{1}{f_e} = \frac{1}{f_m} - \frac{2}{f_1} \quad \text{or} \quad f_e = -12 \text{ cm}$$

The object is placed 24 cm in front of the lens.

$u = -24$ cm, $f = -12$ cm

And from the mirror Eq. (i), we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \text{(i)}$$

or $v=-24$ cm

The final image is formed on the object itself. The behavior is like that of a concave mirror.

11. The convex lenses and the plane mirror are shown in figure. The combination behaves like a concave mirror.

Let the distance of the object from the first lens be x.

For the ray to retrace its path, it should be incident normally on the plane mirror.

From the diagram, we see that for lens L_2

$v=\infty, f=+10$ cm, $u=?$

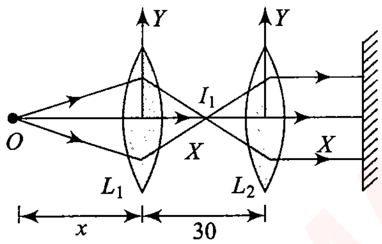


Fig. 1.68

From the lens equation, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad u=-10 \text{ cm}$$

From the diagram, we see that for lens L_1

$v=30-10=20$ cm, $f=+10$ cm, $u=-x$

From the lens equation, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{or} \quad \frac{1}{20}-\frac{1}{-x}=\frac{1}{10} \quad \text{or} \quad x=20 \text{ cm}$$

12. $y=\mu x=\frac{4}{3}x$

For the mirror, $u=-y=\frac{4}{3}x$

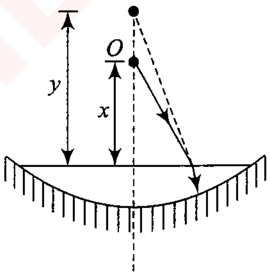


Fig. S-1.69

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{3}{4x}=\frac{1}{-30}$$

$$v=-\left(\frac{60x}{2x-45}\right)$$

Let the final image is formed at a distance z from the mirror,

then $\left(\frac{60x}{2x-45}\right)\frac{1}{z}=\mu=\frac{4}{3}$ and $\frac{z}{x}=2$ (magnification)

Solving, we get $x=33.75$ cm

13. $\frac{1}{f_l}=(1.5-1)\left[\frac{1}{20}-\frac{1}{60}\right]=\frac{1}{60}$

$f_l=60$ cm

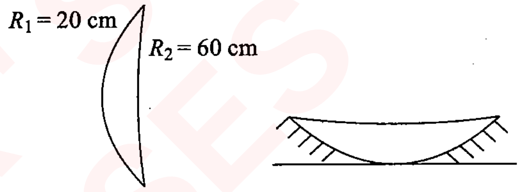


Fig. S-1.70

When convex side is silvered and water is not filled:

$$\frac{1}{F}=\frac{2}{f_l}-\frac{1}{f_m}, f_m=\frac{-R_1}{2}=-10 \text{ cm}$$

$$\frac{1}{F}=\frac{1}{-10}-2\left[\frac{1}{60}\right]$$

$$F=\frac{-30}{4}=-7.5 \text{ cm}$$

After water is filled:

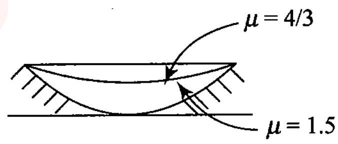


Fig. S-1.71

Let f_{l_1} is the focal length of water lens:

$$\frac{1}{f_{l_1}}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}\right)=\frac{1}{180}$$

$$\frac{1}{F}=\frac{2}{f_{l_1}}-\frac{2}{f_{l_2}}-\frac{1}{f_m}$$

Solving, we get $F=\frac{-90}{13}$ cm

14. $v=\frac{uf}{u+f}=\frac{-30\times 20}{-30+20}=60$ cm

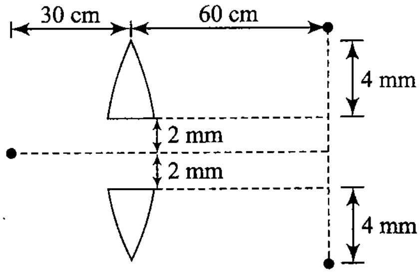


Fig. S-1.72

$$m=\frac{v}{u}=\frac{h_2}{h_1} \Rightarrow h_2=\frac{vh_1}{u}=\frac{60\times 2}{30}=4 \text{ cm}$$

15. $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = \dfrac{1}{40}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{(d-30)} - \frac{1}{(-80)} = \frac{1}{40}$$

$\Rightarrow \quad d = 110$ cm

16. Object is placed at a distance of $2f$ from the lens (f = focal length of the lens), i.e., the image formed by the lens will be at a distance of $2f$ or 20 cm from the lens.

So, if the concave mirror is placed in this position, the first image will be formed at its pole and it will reflect all the rays symmetrically to the other side as shown below and the final image will coincide with the object

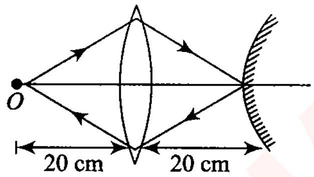


Fig. S-1.73

17. The lens will converge the rays at its focus, i.e., 30 cm from the lens or 20 cm from the refracting surface.

$PI_1 = 20$ cm

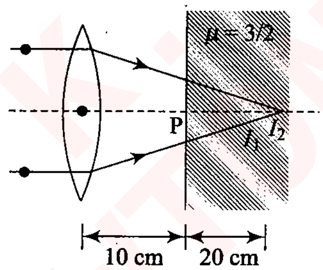


Fig. S-1.74

$$\therefore \quad PI_2 = \mu\,(PI_1) = \frac{3}{2} \times 20 = 30 \text{ cm}$$

So, the rays will converge at a distance of 40 cm from the lens.

18. Focal length of the plano-convex lens is

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$$

or $\quad f = 20$ cm

If point object O is placed at a distance of 20 cm from the plano-convex lens, rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of b.

19. If an object has to coincide with its image, then the ray have to retrace its path; hence, it is concluded that object is at the focus of the combination.

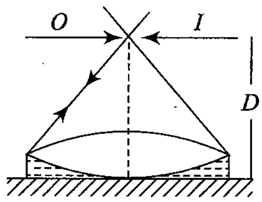


Fig. S-1.75

Focus of equivalent combination.

$$\frac{1}{F} = \frac{1}{f_g} + \frac{1}{f_l}$$

$$\frac{1}{f_g} = (1.5 - 1)\frac{2}{R} = \frac{1}{R}$$

$$\frac{1}{f_l} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{(\mu - 1)}{R}$$

$$\frac{1}{F} = \frac{1}{R} - \left(\frac{\mu - 1}{R}\right) = \left(\frac{2 - \mu}{R}\right)$$

Since, $\quad F = D$

$$\therefore \quad \mu = 2 - \frac{R}{D}$$

20. As radius of curvature of silvered surface is 22 cm, so

$$f_M = \frac{R}{2} = \frac{-22}{2} = -11 \text{ cm} = -0.11 \text{ m}$$

And hence, $P_M = -\dfrac{1}{f_M} = -\dfrac{1}{-0.11} = \dfrac{1}{0.11}$ D

Further as the focal length of lens is 20 cm, i.e., 0.20 m, its power will be given by

$$P_L = \frac{1}{f_L} = \frac{1}{0.20} \text{ D}$$

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $\quad P = \dfrac{2}{0.20} + \dfrac{1}{0.11} = \dfrac{210}{11}$ D

So the focal length of equivalent mirror

$$F = -\frac{1}{P} = -\frac{11}{210} \text{ m} = -\frac{110}{21} \text{ cm}$$

i.e., the silvered lens behaves as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it,

$$\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110} \quad \text{i.e., } v = -11 \text{ cm}$$

i.e., image will be 11 cm in front of the silvered lens and will be real as shown in the figure.

21. a. Using Lensmaker's formula, we get

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{30}-\frac{1}{-60}\right)=\frac{1}{2}\left(\frac{1}{30}+\frac{1}{60}\right)=\frac{1}{40}$$

$\Rightarrow\ f=+40$ cm

b. The focal length of left position is given by

$$\frac{1}{f_1}=(1.5-1)\left(\frac{1}{60}-\frac{1}{\infty}\right)=\frac{1}{120}$$

$\Rightarrow\ f_1=+20$ cm

and, the focal length of the right portion is given by

$$\frac{1}{f_2}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-30}\right)=\frac{1}{60}$$

$\Rightarrow\ f_2=60$ cm

c. Since the refracting surfaces remain unchanged, therefore the focal lengths of the upper part and the lower part are equal, i.e.,

$f_1=f_2=+40$ cm

Chapter 2

Exercise 2.1

1. If a_1, a_2 are amplitudes of the superposing waves and I_1, I_2 are intensities, then

$$\beta=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}\quad\text{or}\quad\frac{a_1}{a_2}=\sqrt{\beta}$$

$$\therefore\quad I_{max}=a_1^2+a_2^2+2a_1a_2=(a_1+a_2)^2$$

$$\text{and}\quad I_{min}=a_1^2+a_2^2+2a_1a_2=(a_1-a_2)^2$$

$$\Rightarrow\quad\frac{I_{max}-I_{min}}{I_{max}+I_{min}}=\frac{(a_1+a_2)^2-(a_1-a_2)^2}{(a_1+a_2)^2+(a_1-a_2)^2}$$

$$=\frac{4a_1a_2}{2(a_1^2+a_2^2)}=\frac{2\left(\frac{a_1}{a_2}\right)}{\left(\frac{a_1^2}{a_2^2}+1\right)}=\frac{2\sqrt{\beta}}{(1+\beta)}$$

2. The resultant intensity is given by

$$I=I_1+I_2+2\sqrt{I_1I_2}\ \cos\phi$$

a. The sources are said to be coherent if they have constant phase difference between them. The intensity will be maximum when $f=2np$; the sources are in same phase.

Thus, $I_{max}\,I_1+I_2+2\sqrt{I_1I_2}=(\sqrt{I_1}+\sqrt{I_2})^2$

Similarly, for n identical waves,

$$I_{max}=(\sqrt{I_0}+\sqrt{I_0}+\cdots)^2=n_2I_0$$

b. The incoherent sources have phase difference that varies randomly with time.

Thus, $[\cos\phi]_{av}=0$

Hence, $I=I_1+I_2$

Hence, for n identical waves, $I=I_0+I_0+\cdots=nI_0$

3. The light from the flashlights consists of many different wavelengths (that is why it is white) with random time difference between the light waves; therefore there is no coherence between the two sources; and no possibility of an interference pattern.

4. Condition for maximum interference in the reflected light, in case of thin film interference, can be expressed as

$2\mu t\cos r=(2n-1)\,\lambda/2$, $n=1,2,\ldots$ [for plane parallel films]

where μ is the refractive index of film relative to the surrounding:

t is the thickness of film; and

r is the angle of refraction.

For normal incidence, $r=0$

$\therefore\quad 2\mu t=(2n-1)\lambda/2$, $n=1,2,3,\ldots$

For minimum thickness , $n=1$

$$\therefore\quad 2\mu t_{min}=\lambda/2\quad\text{or}\quad t_{min}=\frac{\lambda}{4\mu}$$

Given: $\lambda=5360$ Å and $\mu=1.34$

$$\therefore\quad t_{min}=\frac{5360}{4\times1.34}\quad\text{or}\quad t_{min}=1000$$

5. Conditions for maximum and minimum intensity in reflected light, in case of thin film interference, are

Maxima: $2\mu t=(2n-1)\lambda/2$, $n=1,2,\ldots$

Minima: $2\mu t=n\lambda$, $n=0,1,2,3,\ldots$

Given: $\mu=1.5$, $t=3000$ Å $=3000\times10^{-10}$ m (normal incidence)

Maxima: $2\mu t=(2n-1)\lambda/2$

$$\therefore\quad 2\times1.5\times(3000\times10^{-10})=(2n-1)\lambda/2$$

$$\text{or}\quad\lambda=\frac{4\times1.5\times3000\times10^{-10}}{2n-1}$$

$$\therefore\quad\lambda=\frac{18000}{2n-1}\times10^{-10}\text{ m}$$

For $n=1$, $\lambda=18000$ Å

$n=2$, $\lambda=6000$ Å

$n=3$, $\lambda=3600$ Å

and so on

Only $\lambda=6000$ Å for $n=2$ lies in the visible range. Intensity will be maximum for $\lambda=6000$ Å which belongs to the oranges–red part of the visible spectrum.

Minima: $2\mu t\cos r=n\lambda$

$$2\times1.5\times(3000\times10^{-10})=n\lambda$$

$$\therefore\quad\lambda=\frac{9000\times10^{-10}}{n}=\frac{9000}{n}\text{ Å}$$

In this case, only $\lambda=4500$ Å for $n=2$ lies in the visible range. Intensity will be minimum for $\lambda=4500$ Å which falls in the violet–blue part of the visible spectrum.

When observed in white light, the red–orange part will be strongly reflected and the violet–blue part will be quite reduced in intensity. Hence, color of the pot will appear to be orange–red.

6. We must find which colours in the visible region, having vacuum wavelengths from 400 nm (violet) to 700 nm (red), will interfere constructively and which destructively. From the problem, with $n_1 = 1$ (air) maximum constructive interference occurs for

$$\lambda_1 = \frac{2(n_2/n_1)t}{m+\frac{1}{2}} = \frac{(900\text{ nm})}{m+\frac{1}{2}}$$

$(m = 0, 2, \ldots)$

Only the value corresponding to $m = 1$, that is, $\lambda_1 = 450$ nm (violet) is in the visible range.

We infer that the red–orange–yellow end of the spectrum will be strongly reflected, while the violet–blue end will be greatly diminished in intensity as compared with the illuminating white light.

7. As the soap bubble becomes very thin, the thickness of the bubble approaches zero. Since light reflecting off the front of the soap surface is phase-shifted 180° and light reflecting off the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus, the soap film appears black.

8. If the film is more than a few wavelengths thick, the interference fringes are so close together that you cannot resolve them.

9. Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda n$$

where $\lambda_n = \dfrac{\lambda}{n}$ is the wavelength in the material.

Then, $2t = \dfrac{\lambda_n}{2} = \dfrac{\lambda}{2n}$

$\therefore \quad \lambda = 4nt = 4(1.33)\,(115\text{ nm}) = 612\text{ nm}$

10. a. The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

$$2nt = \left(m+\frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m+\frac{1}{2}} = \frac{2(1.45)(280\text{ nm})}{m+\frac{1}{2}}$$

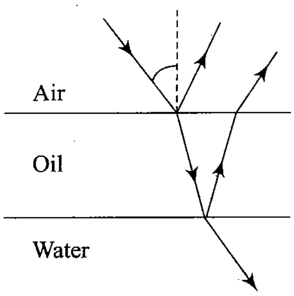


Fig. S-2.1

Substituting for m gives:

$m = 0,\ \lambda_0 = 1620$ nm (infrared)

$m = 1,\ \lambda_1 = 541$ nm (green)

$m = 2,\ \lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet lights are invisible to the human eye, so the dominant color in reflected light is green.

b. The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}$$

Substituting for m gives

$m = 1,\ \lambda_1 = 812$ nm (near infrared)

$m = 2,\ \lambda_2 = 406$ nm (violet)

$m = 3,\ \lambda_3 = 271$ nm (ultraviolet)

Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is violet.

11. Treating the anti-reflectance coating like a camera lens coating,

$$2t = \left(m+\frac{1}{2}\right)\frac{\lambda}{n}$$

Let $m = 0$: $t = \dfrac{\lambda}{4n} = \dfrac{3.00\text{ cm}}{4(1.50)} = 0.500$ cm

This anti-reflectance coating could be easily countered by changing the wavelength of the radar to 1.50 cm—now creating maximum reflection!

12. The reflected light is minimum when rays 1 and 2 (shown in figure) meet the condition of destructive interference.

Note: ***Both rays undergo a 180° phase change upon reflection. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda/\mu/2$.***

Hence,

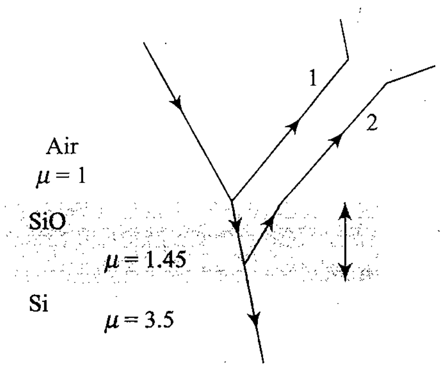


Fig. S-2.2

$$2t = \frac{\lambda}{2\mu}$$

or $$t = \frac{\lambda}{4\mu} = 94.8 \text{ nm}$$

Exercise 2.2

1. The fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

If the extent of field of vision is ℓ.

$$\ell = N_1\beta_1$$

where N_1 is the number of fringes formed with wavelength ℓ.

As the field of vision is fixed, $\ell = N_1\beta_1 = N_2\beta_2$

$$N_1\left(\frac{\lambda_1 D}{d}\right) = N_2\left(\frac{\lambda_2 D}{d}\right)$$

$$N_1\lambda_1 = N_2\lambda_2$$

Thus, $$N_2 = \frac{N_1\lambda_1}{\lambda_2} = \frac{60 \times 5890}{5460} = 64.72 \approx 65$$

2. Angular fringe width is given by

$$\beta\theta = \frac{\lambda}{d}$$

$$\beta_\theta^{\text{air}} = \frac{\lambda_{\text{air}}}{d}, \beta_\theta^{\text{water}} = \frac{\lambda_{\text{water}}}{d}$$

$$\frac{\beta_\theta^{\text{air}}}{\beta_\theta^{\text{water}}} = \frac{\lambda_{\text{air}}}{\lambda_{\text{water}}} = \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{3}{4}$$

$$\therefore \quad \beta_\theta^{\text{water}} = \frac{3}{4}\beta_\theta^{\text{air}} = 0.40° \times \frac{3}{4} = 0.30°$$

3. The angular position of maxima is given by

$$d \sin\theta = n\lambda$$

$$d\theta = n\lambda$$

[For small θ, $\sin\theta = \theta$]

The angular separation of two adjacent maxima is

$$\Delta\theta = \frac{\lambda}{d}$$

Let angular separation be 10% greater for wavelength λ'.

$$\therefore \quad \frac{1.10\lambda}{d} = \frac{\lambda'}{d}$$

Then, $\lambda' = 1.10\lambda = (1.10 \times 589) = 648$ nm

4. $$\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6 \text{ mm}$$

$$\beta_2 = \frac{\lambda 2D}{d} = 0.45 \text{ mm}$$

Let n_1^{th} maxima of λ_1 and n_2^{th} maxima of λ_2 coincide at a position y.

Then, $y = n_1P_1 = n_2P_2 =$ LCM of β_1 and β_2

$\Rightarrow$ $y =$ LCM of 0.6 mm and 0.45 mm

or $y = 1.8$ mm

At this point, 3rd maxima for 6000 Å and 4th maxima for 4500 Å coincide.

5. $$\Delta x = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3} \text{ m} = 900 \text{ nm}$$

For minima, $\Delta x = (2n - 1)\lambda/2$

$$\Rightarrow \quad \lambda = \frac{2\Delta x}{(2n-1)} = \frac{1800}{(2n-1)}$$

$$= \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}, \ldots$$

Of these, 600 nm and 360 nm lie in the visible range. Hence, these will be the missing lines in the visible spectrum.

6. As derived earlier, the total fringe shift $= \frac{w}{\lambda}(\mu - 1)t$

As each fringe width $= w$,

The number of fringes that will shift is

$$\frac{\text{total fringe shift}}{\text{fringe width}}$$

$$= \frac{\frac{w}{\lambda}(\mu-1)t}{w} = \frac{(\mu-1)t}{\lambda}$$

$$= \frac{(1.6-1) \times 1.8 \times 10^{-5}\ m}{600 \times 10^{-9}\ m} = 18$$

7. Path difference due to the glass slab, $\Delta x = (\mu - 1)\, t$. Thirty fringes are displaced due to the slab.

Hence, $\Delta x = 30\lambda \Rightarrow (\mu - 1)t = 30\lambda$

or $$t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1} = 3.6 \times 10^{-3} \text{ cm}$$

8. Optical path for light coming from upper slit S_1 is

$$S_1P + 1\ \mu\text{m} \times (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Similarly, optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu\text{m} \times (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Path difference:

$$\Delta x = (S_2P + 0.5\ \mu\text{m}) - (S_1P + 2\ \mu\text{m})$$

$$= (S_2P - S_1P) - 1.5\ \mu\text{m}$$

$$= \frac{yd}{D} - 1.5\ \mu\text{m}$$

For central bright fringe, $\Delta x = 0$

$$\Rightarrow \quad y = \frac{1.5\ \mu\text{m}}{1\ \text{mm}} \times 1\ \text{m} = 1.5\ \text{mm}$$

The whole pattern is shifted by 1.5 mm upward.

9. (i) $\theta = \theta_0 = 0.57°$

$$\Rightarrow \quad y = -D\tan\theta \simeq D\theta$$

$$= -1\ \text{m} \times \left(\frac{0.57}{57}\ \text{rad}\right) \Rightarrow y = -1\ \text{cm}.$$

(ii) For point O, $\theta = 0$
Hence, $\Delta x = d \sin \theta_0 \simeq d\theta_0$
$= 1 \text{ mm} \times (10^{-2} \text{ rad})$
$= 10{,}000 \text{ nm} = 20 \times (500 \text{ nm})$
$\Rightarrow \quad \Delta x = 20\lambda$
Hence, point O corresponds to 20^{th} maxima
$\Rightarrow$ Intensity at $O = I_0$
(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

10. $S_2L = d \sin \theta$
or $\Delta x = d \sin \theta$
For intensity to be minimum,
$$\Delta x = d \sin \theta$$
$$\Delta x = (2n - 1)\lambda/2$$
$$n = 1, 2, 3, \ldots$$
For third minimum, $n = 3$
$$d \sin \theta = \frac{5\lambda}{2}$$
Given: $\lambda = 420 \text{ nm} = 420 \times 10^{-9}$ m
$$\theta = 30°$$
$$\therefore \quad d \sin 30° = \frac{5}{2} \times 420 \times 10^{-9} \Rightarrow d = 2.1 \times 10^{-6} \text{ m}$$

11. In an interference arrangement similar to double-slit experiment, S_1 and S_2 are illuminated with coherent source microwave source each of frequency 1 MHz.
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^6} = 300 \text{ m}$$
$$\Delta x = d \sin \theta;\ \phi = \frac{2\pi}{\lambda} \text{ d} \sin \theta = \pi \sin \theta$$
$$I_R = 4I \cos^2 \phi/2 = 4I\cos^2\left(\frac{\pi \sin \phi}{2}\right)$$
For $I(\theta)$ to be maximum, $I_0 = 4I \quad \Rightarrow \quad I = \frac{I_0}{4} \quad \Rightarrow \quad I_0 = 4I$
$$I_R = I_0 \cos^2\left(\frac{\pi \sin \phi}{2}\right)$$
a. $\theta = 0° \quad \Rightarrow \quad I_R = I_0$

b. $\theta = 30° \quad \Rightarrow \quad I_R = \dfrac{I_0}{2}$

c. $\theta = 90° \quad \Rightarrow \quad I_R = 0$

12. Underwater, the wavelength of the light would decrease, $\lambda_{UW} = \lambda_{air}/n_{water}$. Since the positions of light and dark bands are proportional to λ, the underwater fringe separation will decrease.

13. Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With several colors, the patterns are superimposed and it is difficult to pick out a single maximum. Using monochromatic light, we can eliminate this problem.

14. $y_{bright} = \dfrac{\lambda L}{d} m$

For $m = 1$, $\lambda = \dfrac{yd}{L} = \dfrac{(3.40 \times 10^{-3} \text{ m})(5.00 \times 10^{-4}) \text{ m}}{3.30 \text{ m}}$
$= 515$ nm

15. Note, with the conditions given, the small angle approximation does not work well. That is, sin θ, tan θ, and θ are significantly different. The approach to be used is outlined below.

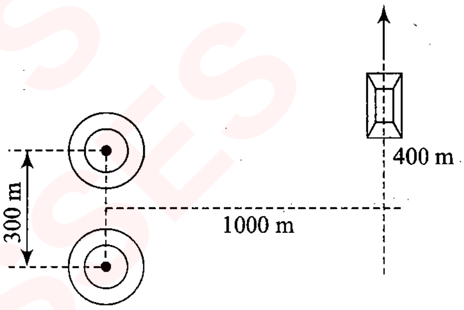


Fig. S-2.3

a. At $m = 2$ maximum,
$$\tan \theta = \frac{400 \text{ m}}{1000 \text{ m}} = 0.400 = \frac{2}{5} \quad \Rightarrow \quad \sin \theta = \frac{2}{\sqrt{29}}$$
$$\text{So,} \quad \lambda = \frac{d \sin \theta}{m} = \frac{(300 \text{ m})\frac{2}{\sqrt{29}}}{2} = \frac{300}{\sqrt{29}} \text{ m} = 55.7 \text{ m}$$
b. The next minimum encountered is the $m = 2$ minimum.

At that point, $d \sin \theta' = \left(m + \frac{1}{2}\right)\lambda$

which becomes $d \sin\theta' = \frac{5}{2}\lambda$
$$\text{or} \quad \sin\theta' = \frac{5}{2}\frac{\lambda}{d} = \frac{5}{2}\left(\frac{\frac{300}{\sqrt{29}} \text{ m}}{300 \text{ m}}\right) = \frac{5}{2\sqrt{29}}$$
$$\Rightarrow \quad \tan \theta' = \frac{5}{\sqrt{141}}$$
$$\text{So,} \quad y = (1000 \text{ m})\frac{5}{\sqrt{141}} = 524 \text{ m}$$
Therefore, the car must travel an additional 124 m.

16. In equation $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$,

The first minimum is described by $m = 0$

And the tenth by $m = 9$ $\sin \theta = \dfrac{\lambda}{d}\left(9 + \dfrac{1}{2}\right)$

Also, $\tan \theta = y/D$
But for small θ, $\sin \theta \approx \tan \theta$
$$\text{Thus,} \quad d = \frac{9.5\lambda}{\sin \theta} = \frac{9.5\lambda D}{y}$$

$$D = \frac{9.5(6000 \times 10^{-10}\ \text{m})(2.00\ \text{m})}{7.5 \times 10^{-3}\ \text{m}}$$

$$= 1.52 \times 10^{-3}\ \text{m} = 1.52\ \text{mm}$$

17. At 30.0°, $d \sin \theta = m\lambda$

$(3.20 \times 10^{-4}\ \text{m}) \sin 30.0^\circ = m(500 \times 10^{-9}\ \text{m})$

So, $m = 320$.

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

Therefore, there are 641 maxima.

18. a. $\dfrac{I}{I_{max}} = \cos^2\left(\dfrac{\phi}{2}\right)$ (i)

Therefore, $\phi = 2\cos^{-1}\sqrt{\dfrac{I}{I_{max}}} = 2\cos^{-1}\sqrt{0.25}$

$$= 2\cos^{-1}\left(\frac{1}{2}\right) = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}\ \text{rad}$$

b. $\delta = \dfrac{\lambda\phi}{2\pi} = \dfrac{(486\ \text{nm})\left(\frac{2}{3}\pi\ \text{rad}\right)}{2\pi} = 162\ \text{nm}$

19. Ray 1 has a longer path than that of ray 2 by a distance d sin 45°, before reaching the slits. Afterward ray 2 has a path longer than ray 1 by a distance $d \sin \theta$. The net path difference is therefore, $d \sin \theta - d \sin 45°$.

a. Central maximum is obtained where net path difference is zero

or $d \sin \theta - d \sin 45° = 0$

or $\theta = 45°$

b. Third order maxima is obtained where net path difference is 3λ, or

$$d \sin \theta - d \sin 45° = 3\lambda$$

$$\sin \theta = \sin 45° + \frac{3\lambda}{d}$$

Putting $d = 20\ \lambda$, we have

$$\sin \theta = \sin 45° + \frac{3\lambda}{20\lambda}$$

or $\theta = \sin^{-1}\left[\dfrac{1}{\sqrt{2}} + \dfrac{3}{20}\right]$

20. $\dfrac{I_{max}}{I_{min}} = \left[\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right]^2$

As $I_1 = 2I_2$

$\Rightarrow \dfrac{I_{max}}{I_{min}} = \left[\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right]^2 = 34$

21. Amplitudes from the slits are A and $2A$.

$A_{max} = A + 2A = 3A$ and $A_{min} = 2A - A$

$\Rightarrow \dfrac{I_{max}}{I_{min}} = \dfrac{A_{max}^2}{A_{min}^2} = 9$

22. The expression for resultant intensity at any point is

$$I = k(A_1^2 + A_2^2 + 2A_1A_2 \cos \phi)$$

where k is constant of proportionality, ϕ is the path difference between interfering waves, A_1 and A_2 are amplitudes of superposing waves. As $A_1 = 2A_2 = 2A$, we have the intensity at the central position.

$$I_0 = k(A_1A_2)^2 = k(2A + A)^2 = 9kA^2$$

So, $k = \dfrac{I_0}{9A^2}$ (i)

The path difference at any point P is $\Delta x = d \sin \theta$

The phase difference $\dfrac{2\pi}{\lambda} d \sin \theta$ (ii)

Thus, Eq. (i) becomes

$$I = \frac{I_0}{9A^2}\left[(2A)^2 + A^2 + 2(2A)(A)\cos\left(\frac{2\pi d \sin \theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9A^2}\left[5A^2 + 4A^2 \cos\left(\frac{2\pi d \sin \theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9}\left[1 + 8\cos^2\frac{\pi d \sin \theta}{\lambda}\right] \text{ as } \cos 2\theta = \left[\cos^2\frac{\theta}{2} - 1\right]$$

Chapter 3

Exercise 3.1

1. Energy of proton, $E = 2$ MeV $= 2 \times 1.6 \times 10^{-13}$ J

$\therefore$ de Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$$\Rightarrow \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.64 \times 10^{-27} \times 2 \times 1.6 \times 10^{-13}}}$$

$\Rightarrow \lambda = 2.044 \times 10^{-14}$ m

2. Given: $k = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}

and $T = 273 + 127 = 400$ K

KE of the neutron

$$E = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$$

$\Rightarrow E = 8.25 \times 10^{-21}$ J

$\therefore$ de Broglie wavelength, $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mE}}$

or $\lambda = \dfrac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 8.28 \times 10^{-21}}}$

or $\lambda = 1.264 \times 10^{-10}$ m

3. For completely absorbing surface,

$$P_{rod} = \frac{I}{c} = \frac{1.4 \times 10^3}{3.0 \times 10^8} = 4.66 \times 10^{-6}\ \text{m}^{-2}$$

4. For an electron, de Broglie wavelength is given by

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{25}} = \sqrt{6}\ \text{Å} \approx 2.5\ \text{Å}$$

5. The value of de Broglie wavelength associated with a moving particle is given by

$$\lambda = \frac{h}{mv}$$

Consider a ball of 0.5 kg moving with a velocity of 10 ms^{-1}. It is an object from our daily observations. The de Broglie wavelength of the ball will be

$$\lambda = \frac{6.625\times10^{-34}}{0.5\times10} = 13.25\times10^{-34}\ \text{m}$$

The value of λ is very small as compared to the size a ball of 0.5 kg would possess. Hence, wave nature of matter is not apparent in our daily observations.

6. If λ is the de Broglie wavelength of the particle of kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Suppose that the de Broglie wavelength of the particle becomes λ', when its kinetic energy is $K/4$, then

$$\lambda' = \frac{h}{\sqrt{2mk/4}} = 2\left(\frac{h}{\sqrt{2mK}}\right) = 2\lambda$$

7. In terms of accelerating potential V, the de Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \text{(i)}$$

where e is charge and m is mass of the particle.

Equation (i) represents a straight line, whose slope is $h/\sqrt{2me}$. The slope of the line is inversely proportional to $\sqrt{m}$. Since the slope of line A is lesser, it represents the particle of heavier mass.

8. Here, $\lambda = 632.8$ nm $= 632.8\times10^{-9}$ m;

Power, $P = 9.42$ mW $= 9.42\times10^{-3}$ W

a. Now, energy of each photon in the light beam,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{632.8\times10^{-9}}$$

$$= 3.14\times10^{-19}\ \text{J} = \frac{3.14\times10^{-19}}{1.6\times10^{-9}} = 1.963\ \text{eV}$$

Also, momentum of each photon in the light beam,

$$p = \frac{h}{\lambda} = \frac{6.62\times10^{-34}}{632.8\times10^{-9}} = 1.046\times10^{-27}\ \text{kg ms}^{-1}$$

b. The number of photons falling on the target per second,

$$N = \frac{P}{E} = \frac{9.42\times10^{-3}}{3.14\times10^{-19}} = 3\times10^{16}\ \text{s}^{-1}$$

c. The required speed of the hydrogen atom,

$$V = \frac{p}{m_H} = \frac{1.046\times10^{-27}}{1.66\times10^{-27}} = 0.63\ \text{ms}^{-1}$$

9. Here, energy flux (energy per second per unit area) of sunlight reaching the surface of the earth,

$\phi = 1.388\times10^{3}$ W m^{-2}

$\lambda = 550$ nm $= 550\times10^{-9}$m

Therefore, energy of a photon of sunlight,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 3.61\times10^{-19}\ \text{J}$$

Therefore, the number of protons emitted per second is

$$\frac{1.388\times10^{3}}{3.61\times10^{-19}} = 3.845\times10^{21}\ \text{s}^{-1}$$

10. Here, $h = 6.63\times10^{-34}$ Js; $\lambda = 1.0$ nm $= 10^{-9}$ m

a. The electron and the proton will possess the same momentum, which is given by

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-9}} = 6.63\times10^{-25}\ \text{kg ms}^{-1}$$

b. The energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-9}} = 1.99\times10^{-16}\ \text{J}$$

$$= \frac{1.99\times10^{-16}}{1.6\times10^{-19}} = 1{,}243.1\ \text{eV}$$

c. The kinetic energy of electron,

$$E = \frac{p^2}{2m} = \frac{(6.63\times10^{-25})^2}{2\times9.1\times10^{-31}} = 2.415\times10^{-19}\ \text{J}$$

$$= \frac{2.415\times10^{-19}}{1.6\times10^{-19}} = 1.51\ \text{eV}$$

11. $\lambda = \dfrac{h}{\sqrt{2mk}}$

The wavelength is smallest for α-particle since kinetic energy (K) is same for all.

12. Energy incident on the plate $= \dfrac{1200\times100}{60} = 2000$ Js^{-1}

$$\therefore \quad nh\nu = 2000 \qquad \text{(i)}$$

Initial momentum of photons $= \dfrac{nh\nu}{c}$

Final momentum of photons

$$= \frac{40}{100}\times\frac{nh\nu}{c} = \frac{0.4nh\nu}{c}$$

Force acting on the plate

= Rate of change of momentum

$$= \frac{0.4nh\nu}{c} - \left(-\frac{nh\nu}{c}\right) = \frac{1.4nh\nu}{3\times10^{8}} \qquad \text{(ii)}$$

Substituting the value of $nh\nu$ from Eq. (i) in Eq. (ii), we get the force acting on the plate.

13. Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} \approx 3.3 \times 10^{-19}\ \text{J}$$

The number of photons emitted per second,

$$n = \frac{200}{3.3 \times 10^{-19}} = 6.1 \times 10^{20}\ \text{s}^{-1}$$

The radiation is spherically symmetrical, the number of photons entering the sensor per second is the product of n and ratio of aperture area to the area of sphere of radius 10 m:

$$(6.1 \times 10^{20}) \frac{\lambda \times (0.01)^2}{4 \times \pi \times 10^2} = 1.53 \times 10^{14}\ \text{photon s}^{-1}$$

Therefore, number of photons entering the sensor in 1 s = $(0.1) \times (1.53 \times 10^{14}) = 1.53 \times 10^{13}$.

14. We know that force is rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t}$$

Since the photons are incident normally on a perfectly reflecting screen, Δp is twice of incident momentum. If n is the number of photons per second.

$$F = \frac{\Delta p}{\Delta t} = n \times 2 \times \frac{h}{\lambda}, \quad \left(\text{since } p = \frac{h}{\lambda}\right)$$

$$= 1\text{N}$$

$$\therefore \quad n = \frac{\lambda}{2h} = \frac{662 \times 10^{-9}}{2 \times (6.62 \times 10^{-34})} = 5 \times 10^{26}\ \text{photons}$$

15. The maximum energy given to X-ray photon is given by

$$h\nu_{max} = eV$$

where V = voltage applied to X-ray tube

$$\therefore \quad \nu_{max} = \frac{eV}{h}$$

We know that $\nu_{max} = \dfrac{c}{\lambda_{min}}$ or $l_{min} = \dfrac{c}{\nu_{max}}$

$$\therefore \quad \lambda_{min} = \frac{ch}{eV}$$

Here, $\lambda_1 = \dfrac{ch}{eV_1}$ and $\lambda_2 = \dfrac{ch}{eV_2}$

$$\text{or} \quad \frac{ch}{e}\left[\frac{1}{V_2} - \frac{1}{V_1}\right] = \lambda_2 - \lambda_1 = \Delta\lambda = 26 \times 10^{-12}$$

$$\text{or} \quad \frac{ch}{e}\left[\frac{2}{3V_1} - \frac{1}{V_1}\right] = 26 \times 10^{-12}\ (\because V_2 = 1.5\ V_1)$$

Solving for V_1, we get $V_1 = 16000$ V or 16 kV

Exercise 3.2

1. To eject an electron from a metal surface, a minimum amount of energy, called the work function of the metal, is required. Classically, if incident radiation is a wave, its energy is shared by all the atoms in the metal surface and calculations show that it will take almost a year, when the requisite amount of energy may be accumulated by an electron. Hence, on classical picture, the photoelectric emission cannot be instantaneous.

When the incident radiation is assumed to be consisting of photons, a photon of frequency ν_0 or more can possess the energy needed to eject an electron. If frequency of the incident photon is less than ν_0, no photoelectric emission will take place. In a way, ν_0 is a cut-off frequency.

2. Here, $\lambda = 3300$ Å $= 3300 \times 10^{-10}$ m

Therefore, energy of a photon of incident light,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3{,}300 \times 10^{-10}}$$

$$= 6.018 \times 10^{-19}\ \text{J} = \frac{6.018 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76\ \text{eV}$$

Since the work functions of Mo and Ni are greater than the energy of the photon of incident light, photoelectric emission will not occur for these metals.

If the laser is brought nearer, the intensity of incident radiation on the metals will increase. It will increase the photoemission in case of Na and K. However, no photoelectric emission will take place in case of Mo and Ni.

3. The energy of each photon of incident light is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{(4560 \times 10^{-10})}$$

$$= 4.35 \times 10^{-19}\ \text{J}$$

Number of photons in one milliwatt source

$$= \frac{10^{-3}}{4.35 \times 10^{-19}} = 2.29 \times 10^{15}\ \text{s}^{-1}$$

As quantum efficiency = 0.5%. Hence, number of electrons liberated per second $= 2.29 \times 10^{15} \times \dfrac{0.5}{100}$

$$= 1.14 \times 10^{13}\ \text{per second}$$

$\therefore$ Photoelectric current $= (1.14 \times 10^{13})\,(1.6 \times 10^{-19})$
$= 1.824 \times 10^{-6}$ A $= 1.824$ μA

4. According to Einstein's photoelectric equation,

$$\frac{1}{2}mv_{max}^2 = h\nu - W$$

$$= 4.9 - 4.5 = 0.4\ \text{eV}$$

If E be the energy, then

$$E = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2E}{m}}$$

Momentum $= mv = \sqrt{2Em}$

We know that change of momentum is impulse

Hence, impulse = $mv - 0$

$\therefore$ Impulse = $mv = \sqrt{2Em}$

Substituting the values, we get

Maximum impulse

$$= \sqrt{\{2 \times (0.4 \times 1.6 \times 10^{-19}) \times 9.1 \times 10^{-31}\}}$$

$$= 3.45 \times 10^{-25} \text{ kg m s}^{-1}$$

5. According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = W + K_{max}$$

Here, $\frac{hc}{\lambda} = W + 30 \text{ eV}$ and $\frac{hc}{2\lambda} = W + 10 \text{ eV}$

$$\therefore \quad \frac{hc}{\lambda} - \frac{hc}{2\lambda} = \frac{hc}{2\lambda} = 20 \text{ eV}$$

$$\lambda = \frac{hc}{40 \text{ eV}} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{40 \times (1.6 \times 10^{-19})} = 310.3 \text{ Å}$$

Now, $W = \frac{hc}{\lambda} - K_{max} = (40 - 30) \text{ eV} = 10 \text{ eV}$

$$\therefore \quad \lambda_0 = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{10 \times (1.6 \times 10^{-19})} = 1241 \text{ Å}$$

6. Minimum energy of the photon falling on photoelectric plate

$$= h\nu = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34})(3 \times 10^8)}{250 \times 10^{-19}}$$

According to Einstein's photoelectric equation,

$$\frac{1}{2} m v_{min}^2 = \frac{hc}{\lambda} - W$$

$$[v_{min}]^2 = \frac{2}{m}\left[\frac{hc}{\lambda} - W\right]$$

$$= \frac{2}{9.1 \times 10^{-31}}\left[\frac{(6.625 \times 10^{-34})(3 \times 10^8)}{250 \times 10^{-9}} - 1.9 \times (1.6 \times 10^{-19})\right]$$

$$= 1.08 \times 10^{12}$$

$$\therefore \quad v_{min} = 1.04 \times 10^6 \text{ m s}^{-1}$$

7. Let the number of photons hitting the photocell per second be n, then

$$nh\nu = 1.5 \text{ mW}$$

or $$n = \frac{(1.5 \times 10^{-3}) \times 400 \times 10^{-9}}{(6.63 \times 10^{-34}) \times (3 \times 10^8)}$$

$$\left[\therefore \ \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{400 \times 10^{-9}}\right]$$

$$\therefore \quad n = 3 \times 10^{15}$$

Number of photoelectrons produced per second is

$$3 \times 10^{15} \times \frac{0.1}{100} = (3 \times 10^{12})$$

So, current in the photocell is

$$(3 \times 10^{12})(1.6 \times 10^{-19}) = 4.8 \times 10^{-7} \text{ A}$$

8. The intensity of radiation, I, is defined as the energy passing per unit time per unit area normal to the direction of the beam. If N be the number of photons crossing unit area per unit time, then

$$I = N \times \text{energy carried by one photon}$$

$$I = N\frac{hc}{\lambda} \text{ or } N = \frac{I\lambda}{hc}$$

If N_i be the number of incident photons per unit area per unit time, then

$$N_i = \frac{I_i\lambda}{hc} = \frac{10^{-8} \times 365 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 18.35 \times 10^9$$

The number of photons absorbed, N_{ab}, by the surface per unit area per unit time is given by

$$N_{ab} = \text{absorption coefficient of surface} \times N_i$$

$$= 0.8 \times 18.35 \times 10^9 = 1.47 \times 10^{10} \text{ m}^{-2}\text{s}^{-1}$$

Now, assuming that each photon ejects only one electron, the rate of electrons emitted per unit area is given by

$$N = N_{ab} = 1.47 \times 10^{10} \text{ m}^{-2}\text{s}^{-1}$$

Power absorbed per m^2 = Absorption coefficient

$$\times \frac{\text{incident power}}{m^2} = 0.8 \times 10^{-8} = 8 \times 10^{-9} \text{ W m}^{-2}$$

From Einstein's equation, maximum kinetic energy is given by

$$\text{KE}_{max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{365 \times 10^{-9}} - 1.6 \times 1.6 \times 10^{19}$$

$$= 2.89 \times 10^{-19} \text{ J} = 1.80 \text{ eV}$$

9. Energy of the photon beam striking the platinum surface in time $t = 1$ s is

$$E_b = \text{intensity of beam} \times \text{area of surface} \times \text{time}$$

$$= (2.0 \text{ Wm}^{-2}) \times (1.0 \times 10^{-4} \text{ m}^2) \times 1 \text{ s}$$

$$= 2.0 \times 10^{-4} \text{ Ws} = 2.0 \times 10^{-4} \text{ J}$$

Energy carried by each photon,

$$E_p = 10.6 \text{ eV} = 10.6 \times 1.6 \times 10^{-19} = 16.96 \times 10^{-19} \text{ J}$$

Thus, the number of photons striking the surface per second is

$$n = \frac{E_b}{E_o} = \frac{2.0 \times 10^{-4}}{16.96 \times 10^{-19}} = 1.18 \times 10^{14}$$

It is given that 0.53% of the incident photons eject photoelectrons. The number of photoelectrons emitted is

$$\frac{1.18\times10^{14}\times0.53}{100} = 6.254\times10^{11}$$

The maximum energy is

$$E_{max} = 10.6\text{ eV} - 5.6\text{ eV} = 5.0\text{ eV}$$

Chapter 4

Exercise 4.1

1. a. True. $r = 0.53\frac{n^2}{Z}$ Å

b. True. $E_n = -13.6\frac{Z^2}{n^2}$ (eV)

c. True. $T = \frac{2\pi r}{v} \Rightarrow T \propto \frac{n^2}{Z}\cdot\frac{n}{Z} \propto \frac{n^3}{Z}$

Since $r \propto n^2$ and $v \propto \frac{1}{n}$, therefore

$$T \propto n^2$$

d. False. $U = -2K$

e. True. $v = \frac{2\pi kZe^2}{nh}$

f. True.

g. True. The expression $mvr = nh/2\pi$ is independent of the atomic number Z.

h. True. The value of Rydberg constant for deuterium is more than that of hydrogen. Therefore, ground state energy level for deuterium is lower than that of hydrogen.

i. False. Lyman series lies in the ultraviolet region.

2. $$\frac{1}{\lambda_1} = R_\infty Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \quad \text{(i)}$$

$$\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \quad \text{(ii)}$$

$$\frac{1}{\lambda_3} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right] \quad \text{(iii)}$$

On adding (i) and (ii), we get

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

Thus, $$\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

3. a. Let n_1 be the initial state of electron. Then,

$$E_1 = -\frac{13.6}{n_1^2}\text{eV}$$

Here, $E_1 = -0.85$ eV

$$\therefore \quad -0.85 = -\frac{13.6}{n_1^2}$$

or $n_1 = 4$

b. Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore

$$\Delta E = 13.6\left[1-\frac{1}{n_2^2}\right]$$

Here, $\Delta E = 10.2$ eV

$$\therefore \quad 10.2 = 13.6\left[1-\frac{1}{n_2^2}\right]$$

or $n_2 = 2$

Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

c. The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right]$$

$$\frac{1}{\lambda} = 1.09\times10^7\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$$\lambda = 4860\text{ Å}$$

4. a. H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097\times10^7)\left(\frac{1}{2^2}-\frac{1}{4^2}\right)$$

$$= 0.2056\times10^7\text{ m}$$

$$\lambda = 4.9\times10^{-7}\text{ m}$$

b. $$f = \frac{c}{\lambda} = \frac{3.0\times10^8}{4.9\times10^{-7}}$$

$$= 6.12\times10^{14}\text{ Hz}$$

5. The transition equation for Lyman series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2}-\frac{1}{n^2}\right), n = 2, 3, \ldots$$

The largest wavelength is corresponding to $n = 2$

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097\times10^7\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$= 0.823\times10^7$$

$$\Rightarrow \quad \lambda_{max} = 1.2154\times10^{-7}\text{ m} = 1215\text{ Å}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{min}} = 1.097\times10^7\left(\frac{1}{1}-\frac{1}{\infty}\right)$$

or $\lambda_{min} = 0.911\times10^{-7}\text{ m} = 911$ Å

Both of these wavelengths lie in the ultraviolet (UV) region of electromagnetic spectrum.

6. a. $z = 3$ for Li^{2+}.
Further, we know that

$$r_n = \frac{n^2}{z} a_0$$

Substituting $n = 3$, $z = 3$, and $a_o = 0.529$ Å, we have

$$r_3 \text{ for } Li^{+2} = \frac{(3)^2}{(3)}(0.529)\text{ Å} = 1.587\text{ Å}$$

b. $z = 2$ for He^+. Also, we know that

$$v_n = \frac{z}{n} v_1$$

Substituting $n = 4$, $z = 2$, and $V_1 = 2.19 \times 10^6$ m s^{-1}, we get

$$V_4 \text{ for } He^+ = \left(\frac{2}{4}\right)(2.19 \times 10^6)\text{ms}^{-1}$$

$$= 1.095 \times 10^6 \text{ ms}^{-1}$$

7. $E_1 = -13.60$ eV $\Rightarrow$ $K_1 = -E_1 = 13.60$ eV
$V_1 = -2K_1 = -27.2$ eV

$$E_2 = \frac{E_1}{(2)^2} = -3.40 \text{ eV}$$

$\Rightarrow$ $K_2 = 3.40$ eV and $U_2 = -6.80$ eV

Now, $U_1 = 0$, i.e., potential energy of each shell has been increased by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U (eV)	E (eV)
First Second	13.6 3.40	0 20.40	13.60 23.80

8. The force at a distance r is $f = -dU/dr = -2ur$.
Suppose r be the radius of nth orbit. Then, the necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ur \quad \text{(i)}$$

Further, the quantization of angular momentum gives,

$$mvr = \frac{nh}{2\pi} \quad \text{(ii)}$$

Solving Eqs. (i) and (ii) for r, we get

$$r = \left(\frac{n^2h^2}{8um\pi^2}\right)^{1/4} \quad \text{(iii)}$$

9. a. We have

$$\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{4e^2}{r_n^2} \quad \text{(i)}$$

The quantization of angular momentum gives

$$m_p v r_n = \frac{nh}{2\pi} \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$r = \frac{n^2h^2\varepsilon_0}{2\pi m_p} e^2$$

Substituting $m_p = 100\,m$, where m = mass of electron, we get

$$r_n = \frac{n^2h^2\varepsilon_0}{400\pi me^2}$$

b. As we know, $E_1 = -13.60$ eV (For H-atom)

and $E_n \propto \left(\frac{z^2}{n^2}\right)m$

For the given particle,

$$E_4 = \frac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360 \text{ eV}$$

and $$E_2 \propto \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440 \text{ eV}$$

$$\Delta E = E_4 - E_2 = 4080 \text{ eV}$$

$$\lambda(\text{in Å}) = \frac{12375}{\Delta E(\text{in eV})} = \frac{12375}{4080} = 3.03\text{ Å}$$

$$= 3.03\text{ Å}$$

10. Given that electrostatic potential energy = –1.7 eV.

$$\therefore \quad KE = \frac{1.7}{2} = 0.85 \text{ eV}$$

So, total energy = –1.7 + 0.85 = –0.85 eV

Now, $E_n = -\frac{13.6}{n^2} = -0.85$

$$\therefore \quad n^2 = \frac{13.6}{0.85} \quad \text{or} \quad n = 4$$

Hence, the atom is excited to state $n = 4$.

$$\Delta E = -0.85 - (-13.6) = 12.75 \text{ eV}$$

Using Einstein's equation, we have

$$h\nu = W + (KE)_{max}$$

or $(KE)_{max} = h\nu - W = 12.75 - 2.3 = 10.45$ eV

The minimum de Broglie wavelength is given by

$$\lambda_{min} = \frac{h}{(p)_{max}} = \frac{h}{\sqrt{2m(KE)_{max}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31})(10.45 \times 1.6 \times 10^{-19})}}$$

$$= 3.8 \times 10^{-10} \text{ m} = 3.8\text{ Å}$$

11. If mass of nucleus is considered (not infinite), then the reduced mass of nucleus–electron system can be taken as

$$\mu = \frac{mM}{m+M}$$

Here, m is the mass of electron and M is that of the nucleus. The binding energy in ground state of hydrogen atom can now be given as

$$E = +\frac{2\pi^2 K^2 e^4 \mu}{h^2} = 13.6 \times \frac{\mu}{m}\text{eV}$$

or $$= \frac{13.6M}{m+M}\text{eV}$$

We have hydrogen atom Rydberg constant given as

$$R = \frac{2\pi^2 K^2 e^4 m}{ch^3}$$

If effect of mass of nucleus is considered, the new value of Rydberg constant can be given as

$$R_\mu = \frac{2\pi^2 K^2 e^4 \mu}{ch^3} \quad \text{or} \quad R_\mu = \frac{RM}{m+M}$$

Percentage difference in the values of R and R_μ is given as

$$\frac{\Delta R}{R}\times 100 = \frac{(R_\mu - R)\times 100}{R_\mu} = \frac{m}{M}\times 100 \simeq 0.055\%$$

12. The energy lost by the electron in exciting the hydrogen atom equals the energy corresponding to $\lambda = 1.216 \times 10^7$ m

$$h\nu = \frac{hc}{\lambda} = \frac{6.63\times 10^{-34}\times 3.0\times 10^8}{1.216\times 10^{-7}}$$

$$= 16.36 \times 10^{-19}\ \text{J}$$

Now, the initial energy of electron = 20 eV = 32×10^{-19} J.

Hence, the kinetic energy of the scattered electron,

$$E = 32\times 10^{-19}\ \text{J} - 16.36\times 10^{-19}\ \text{J} = 15.64\times 10^{-19}\ \text{J}$$

The velocity v of the scattered electron is given by $\frac{1}{2}mv^2 = E$

or $$v = \left(\frac{2E}{m}\right)^{1/2} = \left(\frac{2\times 15.64\times 10^{-19}}{9.11\times 10^{-31}}\right)^{1/2}$$

$$= 1.86 \times 10^6\ \text{ms}^{-1}$$

13. The orbital frequency of n^{th} orbit is given by

$$\nu_n = \frac{\text{electron speed}}{\text{orbital circumference}}$$

$$= \frac{e^2}{2nh\,\varepsilon_0}\times\frac{1}{2\pi}\times\frac{\pi m e^2}{n^2\,\varepsilon_0 h^2} = \frac{me^4}{4\,\varepsilon_0^2 n^3 h^3}$$

$$\left[\text{As } K = \frac{1}{4\pi\varepsilon_0}\right]$$

From the laws of electromagnetic theory, the frequency of the radiation emitted by this electron will also be ν_n.

According to Bohr's theory, the frequency of the radiation for the adjoining orbits is given by

$$\nu_n = \frac{E_n - E_{n-1}}{h} = \frac{me^4}{8\,\varepsilon_0^2 h^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4}{8\,\varepsilon_0^2 h^2}\,\frac{(2n-1)}{(n-1)^2\times n^2}$$

A comparison of this expression with the classical expression above show that the difference in their predictions will be large for small n, i.e., for $n = 2$, $1/n^3 = 1/8$ and $(2n-1)/2n^2(n-1)^2 = 3/8$, so that the frequency given by quantum theory is 3 times that calculated from classical theory. However, for very large n, the quantum expression reduces to that obtained from classical theory because for $n \to \infty$.

$$\frac{2n-1}{2n^2(n-1)^2} \cong \frac{2n}{2n^2 n^2} = \frac{1}{n^3}$$

Consequently, the predictions of Bohr's theory agrees with that of the classical theory in the limit of very large quantum numbers. This correspondence is called as Bohr's correspondence principle.

14. Here, $\lambda = 275$ nm $= 275 \times 10^{-9}$ m

Therefore, energy of the emitting photon,

$$E = \frac{hc}{\lambda} = \frac{6.62\times 10^{-34}\times 3\times 10^8}{275\times 10^{-9}}$$

$$= 7.22\times 10^{-19}\text{J} = \frac{7.22\times 10^{-19}}{1.6\times 10^{-19}} = 4.5\ \text{eV}$$

The photon of energy 4.5 eV (or of wavelength 275 nm) will be emitted, corresponding to the transition B.

15. a. The second excited state corresponds to $n = 3$,

Now, $$E_3 = -\frac{13.6}{n^2} = -\frac{13.6}{3^2} = -\frac{13.6}{9} = 1.51\ \text{eV}$$

The kinetic energy of an electron in the second excited state $= -(-1.51) = 1.51$ eV

b. Energy emitted, when the electron jumps from the second excited state to the ground state,

$$E = -1.51 - (-13.6) = 12.09\ \text{eV}$$
$$= 12.09 \times 1.6 \times 10^{-19}\ \text{J}$$

The wavelength of the spectral line emitted,

$$\lambda = \frac{hc}{E} = \frac{6.62\times 10^{-34}\times 3\times 10^8}{12.09\times 1.6\times 10^{-19}}$$

$$= 1.027 \times 10^{-7}\text{m} = 1.027\ \text{Å}$$

16. a. Since $E_n \propto \frac{Z^2}{n^2}$ in a hydrogen-like atom, the energy levels will be $29^2 = 841$ times the corresponding energies for hydrogen.

$$\therefore \quad E_n = 841\times\left(\frac{-13.6}{n^2}\right) \quad \text{or} \quad E_1 = -11.44\ \text{keV}$$

$$E_2 = -2.86\ \text{keV}$$

$$E_3 = -1.27\ \text{keV}$$

b. Making use of

$$\Delta E = E_n - E_1$$

$$\Rightarrow \quad \lambda_1 = 1.44\ \text{Å},\ \lambda_2 = 1.22\ \text{Å}, \quad \text{and} \quad \lambda_3 = 1.15\ \text{Å}$$

Ionization potential is the potential corresponding to energy $+|E_1|$.

$$V_\infty = \frac{|E_1|}{e} = 11.44\ \text{kV}$$

Exercise 4.2

1. (i) True. According to the theory of electromagnetism, electromagnetic waves are generated by the acceleration or retardation of charges.

(ii) True. Line spectrum of electromagnetic waves are produced due to the transition of electron from the higher level to the lower level.

(iii) False. The cut-off wavelength depends only on the kinetic energy of the striking electrons.
(iv) True. The K_α transition is more probable than the K_β transition.
(v) False. Frequency of K_β is more than that of K_∞ because $\lambda_B > \lambda_A$ as shown:

$$\frac{1}{\lambda_\beta} = R_\infty (Z-1)^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right] = \frac{8}{9} R_\infty (Z-1)^2$$

and $$\frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = \frac{3}{4} R_\infty (Z-1)^2$$

2. We know that

$$\lambda_{\min} = \frac{hc}{eV} = \frac{12400}{V} = \frac{12400}{40\times 10^3} = 0.31 \text{ Å}$$

3. We know that for K_∞-line

$$\frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2\left[1-\frac{1}{2^2}\right] = \frac{3}{4}R_\infty (Z-1)^2$$

or $$\lambda_\alpha = \frac{4}{3R_\infty (Z-1)^2}$$

Here, $R_\infty = 1.096\times 10^7 \text{ m}^{-1}$ and $Z = 74$, therefore

$$\lambda_\alpha = \frac{4}{3(1.096\times 10^7)(74-1)^2} = 2.28\times 10^{-11}\text{ m}$$

or $$\lambda_\alpha = 0.228 \text{ Å}$$

4. From the energy diagram as shown in the following figure, we know that

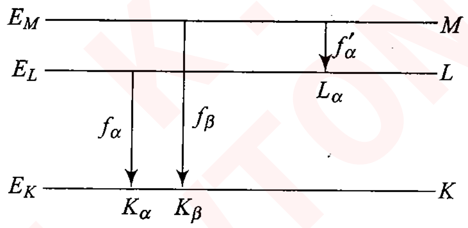


Fig. S-4.1

Frequency of K_α-line is

$$f_\alpha = \frac{E_L - E_K}{h}$$

Frequency of K_β-line is

$$f_\beta = \frac{E_M - E_K}{h}$$

Frequency of L_α-line is

$$f'_\alpha = \frac{E_M - E_L}{h}$$

Since $$E_M - E_K = (E_M - E_L) + (E_L - E_K)$$

$\therefore$ $$f_\beta = f_\alpha + f'_\alpha$$

5. When an electron of charge e is accelerated through a potential difference V, it acquires energy eV. If m be the mass of the electron and $v_{\max}$ the maximum speed of the electron, then

$$\frac{1}{2}mv_{\max}^2 = eV \quad \text{or} \quad v_{\max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{\max} = \sqrt{\left(\frac{2\times(1.6\times10^{-19})\times 20{,}000}{9\times 10^{-31}}\right)}$$
$$= 8.4\times 10^7 \text{ ms}^{-1}$$

6. a. $\lambda_{\min} = 0.45$ Å

$$E_{\max} = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$$= \frac{12431}{0.45} = 27624.44 \text{ eV} = 27.624 \text{ keV}$$

b. The minimum accelerating voltage for electrons is

$$\frac{27.6 \text{ keV}}{e} = 27.6 \text{ kV, i.e., of the order of 30 kV.}$$

7. According to Moseley's law, the frequency for K series is proportional to $V(Z-1)^2$

or $$\frac{c}{\lambda} \propto (Z-1)^2$$

or $$\frac{1}{\lambda} = k(Z-1)^2 \qquad \text{(i)}$$

where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \qquad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\lambda' = \left(\frac{Z-1}{Z-1}\right)^2 \lambda = \left(\frac{30-1}{42-1}\right)^2 \times 1.415 \text{ Å}$$
$$= 0.708 \text{ Å}$$

8. The short series limit of the Balmer series is corresponding to transition $n = \infty$ to $n = 2$ which is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) = \frac{R}{4}$$

or $$R = \frac{4}{\lambda} = \frac{4}{3644}\,(\text{Å})^{-1}$$

The shortest wavelength corresponds to transition from $n = \infty$ to $n = 1$ which is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right]$$

or $$(Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1\text{Å}\times\frac{4}{3644}(\text{Å})^{-1}} = \frac{3644}{4} = 911$$

or $$Z - 1 = 30.2$$

or $$Z = 31.2 \simeq 31.$$

Thus, the atomic number of the element is 31 which is gallium.

9. The binding energy for K shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12431}{0.2}\text{ eV} = 62.155 \text{ keV}$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12431}{0.15} = 82.873 \text{ keV}$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{max} = E - E_k = (82.873 - 62.155) \text{ keV}$$
$$= 20.718 \text{ keV}$$

10. Tungsten is a multielectron atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge Z_{eff}, which is different for different shells.
For an electron in the K shell, $\sigma = 1$.
Thus, $Z_{eff} = Z - \sigma = Z - 1$
Here, as electron drops from M shell ($n = 3$) to K shell ($n = 1$), we call the radiated emission K_β X-ray and from Moseley's law the wavelength emitted of K_β X-ray is given as

$$\frac{1}{\lambda_{k_\beta}} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

or $$\frac{1}{\lambda_{k\beta}} = 10967800 \times (74-1)^2\left[\frac{8}{9}\right]$$

or $$\lambda_{k\beta} = 0.192 \text{ Å}$$

11. The X-rays produced under an accelerating potential V will have varying wavelengths with the minimum due to the entire energy of the accelerated electron being lost in a single collision with the target atoms. Here, the shortest wavelength generated of X-rays can be given by

$$\lambda_c = \frac{12431}{V}\text{Å} = \frac{12431}{20000} = 0.6215 \text{ Å}$$

12. K_α X-rays are produced when an electron makes a transition from $n = 2$ to $n = 1$ to fill a vacancy in K shell. The wavelength of X-ray lines is given by

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R(Z-1)^2$$

or $$(Z-1)^2 = \frac{4}{3R\lambda_{k_\alpha}}$$

$$= \frac{4}{3 \times (1.097 \times 10^7) \times (0.76 \times 10^{-10})}$$
$$= 1599.25$$

or $(Z-1)^2 \simeq 1600$
or $Z - 1 = 40$
or $Z = 41$

Chapter 5

Exercise 5.1

1. Number of protons in nucleus
= atomic number = 11
Number of electrons in an atom
= Number of protons=11
Number of neutrons =
mass number (A) – atomic number (Z) = 24 – 11 = 13

2. In order to compute binding energy, let us first find the total mass of all protons and neutrons in Nb and subtract mass of the Nb:
Given: m_p= 1.007276 u and m_n= 1.008665 u
Number of protons: $N_p = 41$
Number of neutrons: $N_n = 93 - 41 = 52$
Mass difference: $\Delta m = 41m_p + 52m_n - m_{Nb}$
$= 41(1.007825 \text{ u}) + 52(1.008665 \text{ u}) - (92.9063768 \text{ u})$
$= 0.865028$ u
Thus, binding energy per nuclear is

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.865028\,u)(931.5\text{MeV/u})}{93}$$
$$= 8.66 \text{ MeV nucleon}^{-1}$$

3. Nuclei are stable because of the presence of another, short-range (about 2 fm) force, the *nuclear force*. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist.

4. $\left(\frac{\text{BE}}{A}\right)_P = \frac{100}{10} = 10,$

$\left(\frac{\text{BE}}{A}\right)_Q = \frac{60}{5} = 12$, and $\left(\frac{\text{BE}}{A}\right)_R = \frac{66}{6} = 11$

Therefore, stability order is Q > R > P.

5. After releasing 10 MeV, it will become more stable and its binding energy will increase.
New binding energy = 100 + 10 = 110 MeV

6. $(B_3 + B_4) - (B_1 + B_2)$

7. The alpha particle contains 2 protons and 2 neutrons. The binding energy is
$B = (2 \times 1.007826 \text{ u} + 2 \times 1.008665 \text{ u} - 4.00260 \text{ u})c^2$
$= (0.03038 \text{ u})c^2$
$= 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}.$

8. The number of protons in $^{56}_{26}\text{Fe}$ = 26 and the number of neutrons = 56 – 26 = 30. The binding energy of $^{56}_{26}\text{Fe}$
$= [26 \times 1.00783 \text{ u} + 30 \times 1.00867 \text{ u} - 55.9349 \text{ u}]\,c^2$
$= (0.52878 \text{ u})c^2$
$= (0.52878 \text{ u})\,(931 \text{ MeV/u}) = 492 \text{ MeV}.$

9. Mass of Avogadro number of atoms of ^{12}C is exactly 12 g. Avogadro's number, N_A, has the value 6.02×10^{23} atoms mol^{-1}. Thus, the mass of one carbon atom is

$$\frac{0.012 \text{ kg}}{6.02 \times 10^{23} \text{ atom}} = 1.99 \times 10^{-26} \text{ kg}$$

Since one atom of ^{12}C is defined to have a mass of 12 u, we find that

$$1 \text{ u} = \frac{1.99 \times 10^{-26} \text{ kg}}{12} = 1.66 \times 10^{-27} \text{ kg}$$

10. The nuclear density is nearly same in all atoms. The difference in densities between solid lead and gaseous oxygen arises mainly because of the difference in how closely the atoms themselves are packed together in solid and gaseous phase.

11. The binding energy of ^{8}Be is determined by the equation

$$B(^8\text{Be}) = [4m_n + 4M(^1\text{H}) - M(^8\text{Be})]c^2$$

The binding energy

$$B(^8\text{Be}) = [4 \times 1.008665 + 4 \times 1.007825 - 8.005305] \times 931.5 = 56.5 \text{ MeV}$$

Now, we calculate the binding energy (B) of the decay of ^{8}Be into two α-particles.

$^8\text{Be} \to 2\alpha$,

$$\text{B} = [2M(^4\text{He}) - M(^8\text{Be})]c^2$$
$$= [2 \times 4.002603 - 8.005305] \times 931.5$$
$$= -0.092 \text{ MeV}$$

Because B is negative for this reaction, hence ^{8}Be is unstable against decay to two alpha particles.

Exercise 5.2

1. When an α-particle is emitted, we have the product nucleus X:

$$_{92}\text{U}^{238} \to {}_{90}\text{X}^{234} + {}_2\text{He}^4$$

When the product nucleus emits a β-particle, the final nucleus is Y (say) given by $_{90}\text{X}^{234} \to {}_{91}\text{Y}^{234} + {}_{-1}\beta^0$.
Thus, atomic number of final nucleus = 91, mass number of final nucleus = 234.

2. The successive processess may be expressed as

$$_{72}\text{A}^{180} \xrightarrow{\alpha} {}_{70}\text{A}_1^{176} \xrightarrow{\beta} {}_{71}\text{A}_2^{176} \xrightarrow{\alpha} {}_{69}\text{A}_3^{172} \xrightarrow{\gamma} {}_{69}\text{A}_4^{172}$$

Thus, the mass number of A_4 = 172 and the atomic number of A_4 = 69.

3. The successive processess may be expressed as

$$_{92}\text{U}^{238} \xrightarrow{_0n^1} {}_{92}\text{U}^{239} \xrightarrow{\beta(\text{electron})} {}_{93}\text{Np}^{239} \xrightarrow{\beta(\text{electron})} {}_{94}\text{Pu}^{239}$$

Thus, the resulting plutonium has atomic number 94 and mass number 239 and is expressed as $_{94}\text{Pu}^{239}$.

4. The process is expressed as

$$_{92}\text{U}^{238} \to {}_{90}\text{Th}^{230} + 2(_2\text{He}^4) + 2(_{-1}\beta^0).$$

The mass number of thorium nucleus is 230 and atomic number is 90.

5. Mass number (or atomic weight) of $_6\text{C}^{12}$ is 12.

Therefore, the number of atoms in 12 g of $_6\text{C}^{12}$

$$= \text{Avogadro number} = 6 \times 10^3$$

The number of electrons in 12 g of $_6\text{C}^{12}$

$$= 6 \times 6 \times 10^{23} = 36 \times 10^{23}$$

The number of protons in 12 g of $_6\text{C}^{12}$

$$= 36 \times 10^{23}$$

The number of neutrons in 12 g of $_6\text{C}^{12}$

$$= (A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23}$$
$$= 36 \times 10^{23}.$$

Similarly, number of electrons in 14 g of $_6\text{C}^{14}$

$$= 3.6 \times 10^{23}$$

Number of protons in 14 g of $_6\text{C}^{14}$

$$= (A - Z) \times 6 \times 1023$$
$$= (14 - 6) \times 6 \times 1023 = 48 \times 1023.$$

The emitted particles are α, β, β, α and α, respectively.
The processess may be expressed as

$$_{92}\text{U}^{238} \to {}_{90}\text{Th}^{234} + {}_2\text{He}^4 \ (\alpha\text{-particle})$$
$$_{90}\text{Th}^{234} \to {}_{91}\text{Pa}^{234} + {}_{-1}\beta^0 \ (\beta\text{-particle})$$
$$_{91}\text{Pa}^{234} \to {}_{92}\text{U}^{234} + {}_{-1}\beta \ (\beta\text{-particle})$$
$$_{92}\text{U}^{234} \to {}_{90}\text{Th}^{230} + {}_2\text{He} \ (\alpha\text{-particle})$$
$$_{90}\text{Th}^{230} \to {}_{88}\text{Ra}^{226} + {}_2\text{He}^4 (\alpha\text{-particle})$$

6. *Given:* reaction
Find: Q = ?
In order to balance the reaction, the total amount of nucleons (sum of A-numbers) must be the same on both sides. Same for the Z-number.

Number of nucleons (A): $7 + 4 = X + 1 \Rightarrow X = 10$

Number of protons (Z): $3 + 2 = Y + 0 \Rightarrow Y = 5$

Thus, it is B, i.e., $^7_3\text{Li} + {}^4_2\text{He} \to {}^{10}_5\text{B} + {}^1_0\text{n}$

7. We know that $\lambda = 0.693/T_{1/2}$
Here, $T_{1/2} = 3.8$ day

$$\therefore \quad \lambda = \frac{0.693}{3.8} = 0.182 \text{ per day}$$

If initially (at $t = 0$) the number of atoms present be N_0, then the number of atoms N left after a time given by

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t} \text{ or } \frac{1}{20} = e^{-\lambda t}$$

Taking log, $\lambda t = \log_e 20 = 2.3026 \log_{10} 20$

$$\therefore \quad t = \frac{2.3026 \log_{10} 20}{\lambda} = \frac{2.3026 \log_{10} 20}{0.182} = 16.45 \text{ days}$$

8. Let a and b be the number of α-and β-particles emitted when $_{92}\text{U}^{238}$ decays to $_{82}\text{Pb}^{206}$.
We know that
(i) The emission of an α-particle ($_2\text{He}^4$) decreases the charge number by two and mass number by four.
Therefore, emission of a α-particles will reduce the charge number by $2a$ and mass number by $4a$.
(ii) The emission of a β-particle increases the charge number by one and leaves the mass number unchanged.
Therefore, emission of b β-particles will increase the charge number by $b \times 1 = b$.
Thus, $_{92}\text{U}^{238} \to {}_{82}\text{Pb}^{206} + a(_2\text{He}^4) = \text{b}(_{-1}\beta^0)$

Applying the law of conservation of charge number and mass number before and after the decay, we have

$92 = 82 + 2a - b$ (i)

$238 = 206 + 4a$ (ii)

From Eq. (ii), $a = 8$, i.e., number of emitted α-particles = 8

From Eq. (i), $b = 6$, i.e., number of emitted β-particles = 6

9. a. The initial number of nuclei is $N_0 = \dfrac{mN_A}{M}$

b. The number of undecayed nuclei after a time t is

$$N = N_0 e^{-\lambda t} = \left(\frac{mN_A}{M}\right)e^{-\lambda t}$$

Therefore, the number of decayed nuclei is

$$N' = N_0 - N = \frac{mN_A}{M}(1 - e^{-\lambda t})$$

c. The activity of the sample is $A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$

$$A = \left(\frac{mN_A\lambda}{M}\right)e^{-\lambda t}$$

10. We know that $N = N_0 e^{-\lambda t}$, where $\lambda = \dfrac{\ln 2}{T} = \dfrac{1}{T_{av}}$

Here, $N = (1-\eta)N_0$

$(1-\eta)N_0 = N_0 e^{-\lambda t}$

$$t = \frac{1}{\lambda}\ln\left|\frac{1}{(1-\eta)}\right|$$

a. Thus, $t = \dfrac{T}{\ln 2}\left[\ln\left|\dfrac{1}{1-\eta}\right|\right] = \dfrac{T}{0.693}\ln\left|\dfrac{1}{(1-\eta)}\right|$

b. $t = T_{av}\ln\left|\dfrac{1}{(1-\eta)}\right|$

11. $A_1 = A_0 e^{-\lambda t_1}$ and $A_2 = A_0 e^{-\lambda t_2}$

$\therefore \dfrac{A_1}{A_2} = e^{\lambda(t_2 - t_1)}$

$\Rightarrow \lambda(t_2 - t_1) = \ln\left|\dfrac{A_1}{A_2}\right| \Rightarrow T = \dfrac{1}{\lambda} = \dfrac{t_2 - t_1}{\ln\left|\dfrac{A_1}{A_2}\right|}$

12. As the net electric charge is conserved, hence number of protons before the reaction is equal to the total number after the reaction. The total number of nucleons is conserved as well, so we can equate the total number before and after the reaction.

The results are listed in the table as follows:

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	2 + 13	= Z + 0
Total number of nucleons	4 + 27	= A + 1

Thus, we get $Z = 15$ and $A = 30$. Since $Z = 15$ identifies the element as phosphorus, the nucleus produced is ${}^{30}_{15}P$.

13. A deuteron is the nucleus of deuterium, the isotope of hydrogen containing one proton and one neutron, ${}^2_1H$, we have the reaction

$${}^1_0n + {}^{16}_8O \to {}^A_ZX + {}^2_1H$$

Proceeding similar to previous problem,

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	1 + 16	= Z
Total number of nucleons	0 + 8	= A

Hence, the product nucleus must have $Z = 7$ and $A = 15$. From the periodic table we find that it is nitrogen that has $Z = 7$, so the nucleus is ${}^{15}_7N$. The reaction can be written ${}^{16}_8O$ (n, d) ${}^{15}_7N$, where d represents deuterium ${}^2_1H$.

14. The equation for decay is ${}^{238}_{92}U \to {}^A_ZX + {}^4_2He$

Proceed similar to previous problem, to get $Z = 90$ and $A = 234$. Thus, the nucleus is ${}^{234}_{90}Th$.

15. We apply general criteria for nuclear stability.

Criterion 1. Isotopes with $Z > 83$ are unstable. With $Z = 16$, this criterion is satisfied.

Criterion 2. Isotopes with even–even proton and neutron numbers are stable. The isotope ${}^{38}_{16}S_{22}$ is an even–even nucleus. This criterion is satisfied.

Criterion 3. For stability, N/Z ratio is approximately unity. Here, $Z = 16$ and $N = 22$. N/Z is not approximately unity. This criterion is not satisfied.

Therefore, ${}^{38}S$ is likely to be unstable. The nucleus is unstable and decays by β^- emission since it is neutron rich.

16. Each α-particle carries off two protons; the total number of protons carried by the α-particles is $2N_\alpha$.

Each β-particle is an electron ${}^0_{-1}e$ and is emitted when a neutron in the nucleus decays into a proton and an electron. Thus, the total number of protons left behind by the β^--particles is N_β.

Each α-particle carries four nucleons; therefore the total number of nucleons carried off by the α-particles is $4N_\alpha$.

With the emission of each β-particle, a neutron is replaced by a proton. But since each is a nucleon, the number of nucleons is not changed.

The overall decay process is

$${}^{228}_{90}Th \to {}^{212}_{83}Bi + N_\alpha({}^4_2He) + N_\beta({}^0_{-1}e)$$

Now, we can apply conservation of charge and nucleon number.

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	90	$83 + 2N_\alpha + (-1)N_\beta$
Total number of nucleons	228	$212 + 4N_\alpha + (0)N_\beta$

On solving the two equations, we get $N_\alpha = 4$ and $N_\beta = 1$.

17. The decay constant is $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730 \times 5.26 \times 10^5}$

$= 2.30 \times 10^{-10}\ \text{min}^{-1}$

As 1 year = 3.01×10^9 min and for 1.0 g of carbon, the number of moles is $n = 1/12$, so $N = nN_A = \left(\frac{1}{2}\right)(6.02 \times 10^{23})$

$= 5.0 \times 10^{22}$ nuclei g^{-1}.

The given concentration factor is $\frac{^{12}C}{^{12}C} = 1.4 \times 10^{-12}$

Thus, the number of ^{14}C nuclei per gram is $N\left(\frac{^{14}C}{^{12}C}\right) = (5.0 \times 10^{22})(1.4 \times 10^{-12}) = 7.0 \times 10^{10}\ {}^{14}\text{C nuclei g}^{-1}$

The activity, number of decays per gram per minute, is

$$\frac{\Delta N}{\Delta t} = \lambda N = (2.30 \times 10^{-10})(7.0 \times 10^{10})$$

$$= 16\ {}^{14}\text{C decays g}^{-1}\ \text{min}^{-1}$$

18. Given: $\lambda = 1.36 \times 10^{-11}$ decays s^{-1}

The half-life is

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.36 \times 10^{-11}} = 5.096 \times 10^{10}\ \text{s}$$

$$= 1.62 \times 10^3 \text{ years}$$

As initial mass $m_0 = 200$ g,

The mass at time t (= 100 years = 3.15×10^9 s) is

$$m = m_0 e^{-\lambda t} = (200)e^{-(1.36 \times 10^{-11} \times 3.15 \times 10^9)}$$

$$= (200)(0.958) = 192 \text{ g}$$

19. The age of a sample is related to the change in activity during the time period in question. The expected age of 5 years is a small fraction of the 5730 years half-life of $^{14}_{6}$C, so very few nuclei of $^{14}_{6}$C would decay during wine's life; the change in activity would be very small.

Due to short half-life of $^{15}_{8}$O (122.25) so many half-lives of 122.25 would occur that the activity would decrease to a vanishingly small level. The expected age of 5 years is long enough relative to the half-life of $^{3}_{1}$H (12.33 years) that a measurable change in activity will occur; so it is the most appropriate isotope.

⇒ The radioactive isotope is chosen such that the expected age of the sample should be great enough to allow measurements of a change in activity but not great enough so that the activity is undetectable.

If a sample is about 50 years old and it contained hydrogen, one could measure activity of $^{3}_{1}$H (tritium) a beta emitter with a half-life of 12.3 years.

20. a. This part has already been discussed in section on successive disintegrations with the result

$$N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

b. The activity of nuclide A_2 is $\lambda_2 N_2$. This is maximum when N_2 is maximum, i.e., when $dN_2/dt = 0$, activity is maximum.

$$\frac{d[N_2(t)]}{dt} = \frac{d}{dt}\left[\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})\right]$$

$$\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}) = 0$$

$$e^{-(\lambda_1 - \lambda_2)t} = \frac{\lambda_2}{\lambda_1}$$

$$t = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$$

21. In beta decay, total energy and momentum is conserved. The loss of mass shows up as the gain of kinetic energy of the product particles. As there is initially no momentum, there should be no total momentum after the decay. Even if the daughter nucleus does not recoil, the neutrino must move in a direction exactly opposite that of the beta particle to preserve zero momentum.

Hence, the beta particle cannot get all of Q as its kinetic energy. Some amount must go to either the neutrino or the daughter nucleus (or to both) to cancel the momentum of the beta particle. Thus, for the beta particle, $k_{max} < Q$.

22. Since energy is released during the decay, the combined mass of the $^{234}_{90}$Th daughter nucleus and the α-particle is less than the mass of the $^{238}_{90}$U parent nucleus. The difference in mass is equivalent to the energy released. We will determine the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV.

The decay and the masses are shown below:

$$\underset{238.0508}{^{238}_{92}\text{U}} \rightarrow \underbrace{\underset{234.0436}{^{234}_{90}\text{Th}} + \underset{4.0026\,\text{u}}{^{4}_{2}\text{He}}}_{238.0462\ \text{u}}$$

The decrease in mass is 238.0508 u − 238.0462 u = 0.0046 u. Since 1 u is equivalent to 931.5 MeV, the released energy is 4.3 Me V.

When α-decay occurs the energy released appears as kinetic energy of the recoiling $^{234}_{90}$Th nucleus and the α-particle, except for a small portion carried away as a γ-ray.

23. Kinetic energy depends on the mass m and speed v of a particle, since KE = $(1/2)mv^2$. The $^{234}_{90}$Th nucleus has a much greater mass than the α-particle, and since the kinetic energy is proportional to the mass, we may conclude that the $^{234}_{90}$Th nucleus has the greater kinetic energy. This conclusion is not correct, since it does not take into account the fact that the $^{234}_{90}$Th nucleus and the α-particle have different speeds after the decay.

The conservation principle states that the total linear momentum of an isolated system remains constant. An isolated system is one for which the vector sum of the external forces acting on the system is zero and the decaying $^{238}_{92}$U nucleus is stationary initially, and since momentum is mass times velocity, its initial momentum is zero.

In its final form, the system consists of the $^{234}_{90}$Th nucleus and the α-particle and has a final total momentum of $m_{Th}v_{Th} + m_\alpha v_\alpha$. According to momentum conservation, the initial and final values of the total momentum of the system must be the same, so that $m_{Th}v_{Th} + m_\alpha v_\alpha = 0$. Solving this equation for the velocity of the thorium nucleus, we find that $v_{Th} = -m_\alpha v_\alpha/m_{Th}$. Since m_{Th} is much greater than m_α, we can see that the speed of the thorium nucleus is less than the speed of the α-particle.

Moreover, $$\frac{KE_{Th}}{KE_\alpha} = \frac{\frac{1}{2}m_{Th}v_{Th}^2}{\frac{1}{2}m_\alpha v_\alpha^2}$$

$$= \frac{(m_{Th}v_{Th})^2}{m_{Th}} \times \frac{m_\alpha}{(m_\alpha v_\alpha)^2} = \frac{m_\alpha}{m_{Th}}$$

As $|m_{Th}v_{Th}| = |m_\alpha v_\alpha|$, thus we say that kinetic energy is shared in inverse ratio of mass.

24. For 1000 MW output, the total power generation is 3000 MW, of which 2000 MW is rejected as "waste" heat. Thus, the total energy released in 1 year (3×10^7 s) from fission is about $(3 \times 10^9)(3 \times 10^7) \approx 10^{17}$.
If each fission releases 200 MeV of energy, the number of fissions required is

$$\frac{(10^{17})}{(2\times 10^8)(1.6\times 10^{-19})} \approx 3 \approx 10^{27} \text{ fissions}$$

The mass of a single uranium atom is about (235 u) (1.66 $\times 10^{-27}$ kg/u) $\approx 4.5 \times 10^{-25}$ kg, so that mass needed is (4.5 $\times 10^{-25}$ kg) $(3 \times 10^{27}$ fissions) $\approx$ 1000 kg or about a ton. Since $^{235}_{92}U$ is only a fraction of normal uranium, and even when enriched it is never more than 10 per cent of the total, the yearly requirement for uranium is of the order of tens of tons.

25. In 5730 years, half the sample will have decayed, leaving 500 radioactive $^{14}_{6}C$ nuclei. In another 5730 years (for a total elapsed time of 11460 years), the number will be reduced to 250 nuclei. After another 5730 years (total time 17190 years), 125 nuclei remain.

These numbers represent ideal circumstances. Radioactive decay is an averaging process over a very large number of atoms, and the actual outcome depends on statistics. Our original sample in this example contained only 1000 nuclei, certainly not a very large number. Thus, if we were actually to count the number remaining after one half-life for this small sample, it probably would not be exactly 500.

26. First, let us convert the half-life to seconds.

$$T_{1/2} = (1.6 \times 10^3 \text{ years})(3.16 \times 10^7 \text{ s/years})$$
$$= 5.0 \times 10^{10} \text{ s}$$

$$\therefore \quad \lambda = \frac{0.693}{T^{1/2}} = \frac{0.693}{5.0\times 10^{10} s} = 1.4 \times 10^{-11} \text{ s}^{-1}$$

We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$.

$$R_0 = \lambda N_0 = (1.4 \times 10^{-11} \text{s}^{-1})(3.0 \times 10^{16})$$
$$= 4.1 \times 10^5 \text{ decays s}^{-1}$$

Because Ci = 3.7×10^{10} decays s^{-1}, the activity or decay rate at $t = 0$ is

$$R_0 = 11.1 \ \mu\text{Ci}$$

27. a. A more precise answer is obtained by first finding the decay constant from equation

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.83 \text{ days}} = 0.181 \text{ days}^{-1}$$

Taking $N_0 = 4 \times 10^8$ and the value of λ just found to obtain the number N remaining after 12 days as

$$N = N_0 e^{-\lambda t} = (4.0 \times 10^8 \text{ atoms})\, e^{-(0.181 \text{ days-1})(12 \text{ days})}$$
$$= 4.6 \times 10^7 \text{ atoms}$$

This is vey close to our original estimate of 5.0×10^7 atoms.

b. First, we must express the decay constant in units of s^{-1}. Using this value of λ, we find that the initial activity as

$$R = \lambda N_0 = (2.09 \times 10^{-6} \text{ s}^{-1})(4.0 \times 10^8)$$
$$= 840 \text{ decays s}^{-1} = 840 \text{ Bq}$$

28. After decay, the mass of the daughter m_d plus the mass of the alpha particle m_α is

$$m_d + m_\alpha = 222.017\ 571 \text{ u} + 4.002\ 602 \text{ u} = 226.020\ 173 \text{ u}$$

Thus, calling the mass of the parent nucleus M_p, we find that the mass lost during decay is

$$\Delta m = M_p - (m_d + m_\alpha)$$
$$= 226.025\ 402 \text{ u} - 226.020\ 173 \text{ u} = 0.005\ 229 \text{ u}$$

Using the relationship that 1 u corresponds to 931.494 MeV, we find that the energy liberated is

$$E = (0.005\ 229\text{u})(931.494 \text{ MeV/u}) = 4.87 \text{ MeV}$$

29. The mass of a ^{12}C atom is exactly 12 u. The energy released in the reaction is $3(^4_2He) \rightarrow {}^{12}_{6}C$
$[3m(^4_2He) - m(^{12}_{6}C)]\,c^2 = [3 \times 4.002603 \text{ u} - 12 \text{ u}]$ (931 MeV/u) = 7.27 MeV.

30. a. $^{41}_{20}Ca$ nucleus is formed after removing a neutron from $^{42}_{20}Ca$.

The mass of $^{41}_{20}Ca$ plus the mass of a free neutron = 40.962278 + 1.008665 = 41.970943 u

Difference between $^{41}_{20}Ca$ plus the mass of a free neutron and the mass of $^{42}_{20}Ca$ is 0.012321 u; so the binding energy of the missing neutron = (0.012321)(931.49) = 11.4 MeV

b. When a proton is removed from $^{42}_{20}Ca$, the resulting nucleus is the potassium isotope $^{41}_{19}K$. On a similar pattern as above, the binding energy for the missing proton can be calculated; result is 10.27 MeV.

c. Neutron and proton have different energies because only attractive nuclear forces act on the neutron whereas the proton is acted upon by repulsive electric forces that decrease its binding energy.