Solutions
of
Mechanics Part-II

Lesson 9th to 13th

By DC Pandey

9 Mechanics of Rotational Motion

Introductory Exercise 9.1

1. The mass distribution is at minimum separation from the diagonal passing through centre to opposite corners of the cube, so, moment of inertia is minimum about that axis.

2. $I_1 = \dfrac{ml^2}{12}$

while $I_2 = mr^2 = m\left(\dfrac{l}{2\pi}\right)^2$

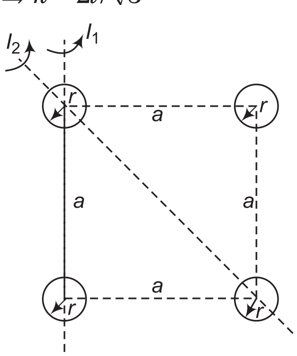
$= \dfrac{ml^2}{4\pi^2}$

$\therefore \quad \dfrac{I_1}{I_2} = \dfrac{4\pi^2}{12} = \dfrac{\pi^2}{3}$

3. $I = \dfrac{m\,(2l)^2}{3} = \dfrac{4ml^2}{3} = mk^2$

$\Rightarrow k = 2l/\sqrt{3}$

4.



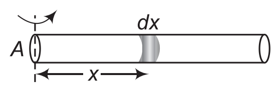
(i) $I_1 = 2 \times \dfrac{2}{5}mr^2 + 2\left(\dfrac{2}{5}mr^2 + ma^2\right)$

$= \dfrac{8}{5}mr^2 + 2ma^2$

(ii) $I_2 = 2 \times \dfrac{2}{5}mr^2 + 2\left(\dfrac{2}{5}mr^2 + m\left(\dfrac{a}{\sqrt{2}}\right)^2\right)$

$= \dfrac{8}{5}mr^2 + ma^2$

5.



(a) $I = \int dI = \int_0^{2l} dm x^2 = m\int_0^{2l} x^3 dx$

$= \dfrac{1}{4}m(2l)^4 = 4\,ml^4$

$M = \int_0^{2l} mx\,dx = \dfrac{1}{2}m\,(2l)^2 = 2ml^2$

$\Rightarrow m = \dfrac{M}{2l^2}$

$\therefore \quad I = 4 \cdot \dfrac{M}{2l^2} \cdot l^4 = 2Ml^2$

(b) $X_{CM} = \dfrac{\int xdm}{\int dm} = \dfrac{\int_0^{2l} x\,mx\,dx}{\int_0^{2l} mx\,dx} = \left.\dfrac{\frac{1}{3}x^3}{\frac{1}{2}x^2}\right|_0^{2l}$

$= \left.\dfrac{2}{3}x\right|_0^{2l} = \dfrac{4}{3}l$

$\therefore I_{CM} = I_A - M\left(\dfrac{4}{3}l\right)^2$

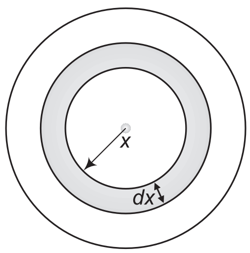
$= 2Ml^2 - \dfrac{16}{9}Ml^2 = \dfrac{2}{9}Ml^2$

$\therefore \quad I_B = I_{CM} + M\left(\dfrac{1}{3}l\right)^2$

$= \dfrac{2}{9}Ml^2 + \dfrac{1}{9}Ml^2$

$= \dfrac{3}{9}Ml^2 = \dfrac{1}{3}Ml^2$

6.



$$I = \int dI = \int dm x^2 = \int_0^a kx^2 \cdot 2\pi x\, dx \cdot x^2$$

$$= 2\pi k \int_0^a x^5 dx = \frac{\pi k}{3} a^6$$

$$M = \int dm = \int kx^2 \cdot 2\pi x\, dx = 2\pi k \int_0^a x^3 dx$$

$$= \frac{\pi k}{2} a^4 \Rightarrow k = \frac{2M}{\pi a^4}$$

$$\therefore I = \frac{\pi}{3} \cdot \frac{2M}{\pi a^4} a^6 = \frac{2}{3} Ma^2$$

7. $I' = \frac{1}{2}Mr^2 - \frac{1}{2} \cdot \frac{M}{6} r^2 = \frac{5}{6} \cdot \frac{1}{2} Mr^2$

$$= \frac{5}{6} \times 0.6 \text{ kg-m}^2 = 0.5 \text{ kg-m}^2$$

8. $I = (1 \times 49^2 + 2 \times 48^2 + 3 \times 47^2 + \ldots + 49 \times 1^2)$

$$+ 50 \times 0^2 + (100 \times 50^2 + 99 \times 49^2 + 98 \times 48^2 + \ldots + 51 \times 1^2)$$

$$= 100 \times 50^2 + 100 \times 49^2 + \ldots + 100 \times 1^2$$

$$= 100(1^2 + 2^2 + 3^2 + \ldots + 50^2)$$

$$= 100 \times \frac{50\,(50+1)\,(100+1)}{6}$$

$$= \frac{50 \times 51 \times 100 \times 101}{6} \text{ g-cm}^2$$

$$= \frac{0.5 \times 5.1 \times 1.01}{6} \text{ kg-m}^2$$

$$= 0.43 \text{ kg-m}^2$$

9. $I = \frac{1}{2} MR^2 + Mx^2 = MR^2$

$$\Rightarrow \quad x^2 = \frac{R^2}{2}$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}$$

10. The disk with smaller density will have larger radius and as $I = \frac{1}{2} Mr^2$, so the disk with larger radius will have higher moment of inertia.

Introductory Exercise 9.2

1. $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$$\Rightarrow \theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$$

$$= \frac{15^2 - 5^2}{2 \times 1} = \frac{5^2 \times 8}{2} = 100 \text{ rad}$$

2. $\theta = \theta_1 + \theta_2 + \theta_3 = \left(\frac{1}{2}\alpha t^2\right) + (\alpha t \cdot t)$

$$+ \left(\alpha t \cdot t - \frac{1}{2}\alpha t^2\right) = 2\alpha t^2$$

$$= 2 \times 4 \times 10^2 = 800 \text{ rad}$$

3. $0 = \omega - \alpha t \Rightarrow \alpha = \frac{\omega}{t} = \frac{10}{10} = 1 \text{ rad/s}^2$

$$\Rightarrow \tau = I\alpha = 5 \text{ kg-m}^2 \times 1 \text{ rad/s}^2$$

$$= 5 \text{ N-m}$$

4. $0^2 = \omega^2 - 2\alpha\theta \Rightarrow \alpha = \frac{\omega^2}{2\theta}$

$$= \frac{\left(100 \times \frac{2\pi}{60}\right)^2}{2 \times 10 \times 2\pi}$$

$$= \frac{100\pi^2}{9 \times 40\pi}$$

$$= \frac{5\pi}{18} \text{ rad/s}^2$$

$$F = \frac{\tau}{r} = \frac{I\alpha}{r} = \frac{\frac{1}{2}mr^2\alpha}{r}$$

$$= \frac{1}{2}mr\alpha = \frac{1}{2} \times 10 \times 0.2 \times \frac{5\pi}{18}$$

$$= 0.87 \text{ N}$$

5. $\theta = 6t - 2t^3,\ \omega = 6 - 6t^2$

$$\Rightarrow \omega = 0 \text{ at } t = 1 \text{ s}$$

(a) $\omega_{av} = \frac{\int_0^1 \omega dt}{\int_0^1 dt} = \int_0^1 (6 - 6t^2)\, dt$

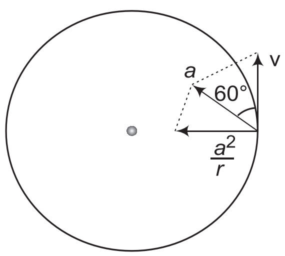
$$= 6t - 2t^2 \Big|_0^1 = 4 \text{ rad/s}$$

$$\alpha_{av} = \frac{\int_0^1 \alpha dt}{\int_0^1 dt} = \int_0^1 -12\, t\, dt$$

$$= -12 \times \frac{1}{2} t^2 \Big|_0^1 = -6 \text{ rad/s}^2$$

(b) $\alpha\,(1s) = -12t = -12 \text{ rad/s}^2$

6.



$$\tan 60° = \frac{v^2/r}{r\alpha} = \frac{v^2}{r^2\alpha} = \frac{\omega^2}{\alpha}$$

$$d\omega = \alpha dt$$

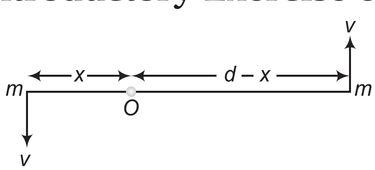
$$\Rightarrow \quad \omega = \int 2 \times 10^{-2}\, t\, dt = 10^{-2} t^2$$

$$\therefore \tan 60° = \frac{10^{-4}\, t^4}{2 \times 10^{-2}\, t} = \sqrt{3}$$

$$\Rightarrow t^3 = 200\sqrt{3} \Rightarrow t = 7 \text{ s}$$

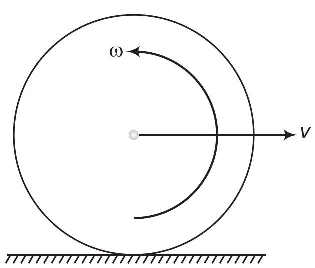
Introductory Exercise 9.3

1.



$L_0 = mvx + mv\,(d - x) = mvd$ is a constant and is independent of x, *i.e.*, position of O.

2.



$$\vec{\mathbf{L}} = \vec{\mathbf{L}}_{CM} + m\,(\vec{\mathbf{r}}_0 \times \vec{\mathbf{v}}_0)$$

$$= I\omega + (-mvR)$$

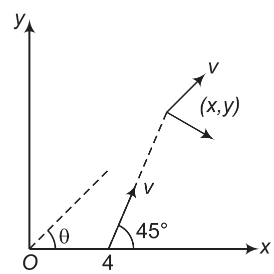
$$= \frac{1}{2} mR^2 \frac{v}{R} - mvR$$

$$= \frac{1}{2} mvR - mvR = -\frac{1}{2} mvR$$

$$\Rightarrow L = \frac{1}{2}\, mvR$$

3. $\vec{\mathbf{L}} = m\,(\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = m\,(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}})$

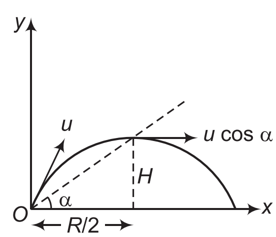
$$\times (v \cos 45°\, \hat{\mathbf{i}} + v \sin 45°\, \hat{\mathbf{j}})$$



$$= m\left(\frac{vx}{\sqrt{2}}\,\hat{\mathbf{k}} - \frac{vy}{\sqrt{2}}\,\hat{\mathbf{k}}\right) = \frac{mv}{\sqrt{2}}\,(x - y)\,\hat{\mathbf{k}}$$

$$= \frac{mv}{\sqrt{2}}\,(x - x + 4)\,\hat{\mathbf{k}} = 2\sqrt{2}\, mv\, \hat{\mathbf{k}}$$

4. $\vec{\mathbf{L}} = m\left(\frac{R}{2}\,\hat{\mathbf{i}} + H\,\hat{\mathbf{j}}\right) \times (u \cos \alpha\, \hat{\mathbf{i}})$



$$= -mHu \cos \alpha\, \hat{\mathbf{k}}$$

$$= -m \cdot \frac{u^2 \sin^2 \alpha}{2g} \times u \cos \alpha\, \hat{\mathbf{k}}$$

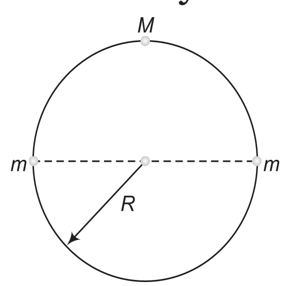
$$= -\frac{mu^3 \cos \alpha \sin^2 \alpha}{2g}\, \hat{\mathbf{k}}$$

$$\therefore \quad L = \frac{mu^3 \cos \alpha \sin^2 \alpha}{2g}$$

5. It depends upon the distance, velocity and angle from the axis, which may not given zero from a different point.

Introductory Exercise 9.4

1.



$$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$$

$$= \frac{MR^2\omega_0}{MR^2 + 2mR^2} = \frac{M\omega_0}{M + 2m}$$

2. As ice melts at pole and flows to equator, it gets distributed away from centre (as equitorial radius is greater than polar radius) such that moment of inertia about axis increases which results in decrease in angular speed (as $I\omega$ = constant), this leads in increase in time period $\left(\text{as } T = \frac{2\pi}{\omega}\right)$ or duration of day and night.

3. True : (Reference previous problem).

Introductory Exercise 9.5

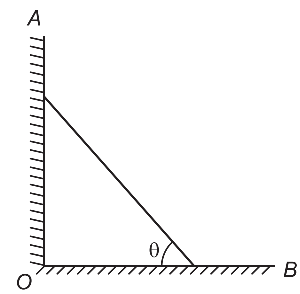
1. $x = \frac{1}{2}a_0t^2 \Rightarrow t = \sqrt{\frac{2x}{a_0}}$

$$\therefore\ y = \frac{v}{\omega} = \frac{a_0 t}{\omega} = \frac{a_0}{\omega}\sqrt{\frac{2x}{a_0}} = \frac{\sqrt{2xa_0}}{\omega}$$

$$\Rightarrow y^2 = \frac{2a_0}{\omega^2}x$$

2. $mgh = \frac{1}{2}I\ \omega^2$

$$mg\,\frac{l}{2}(1 - \sin\theta) = \frac{1}{2}\cdot\frac{ml^2}{3}\cdot\omega^2$$



$$\omega = \sqrt{\frac{3g}{l}(1 - \sin\theta)}$$

Introductory Exercise 9.6

1. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

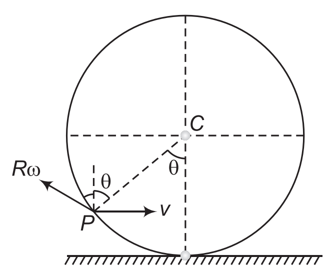
$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mv^2$$

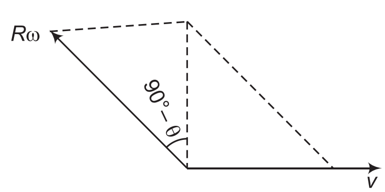
$$= \frac{7}{10}mv^2 = \frac{7}{5}\cdot\frac{1}{2}mv^2$$

$$\therefore \text{ Rotational KE} = mgh - \frac{1}{2}mv^2$$

$$= mgh - \frac{5}{7}mgh = \frac{2}{7}mgh$$

2.





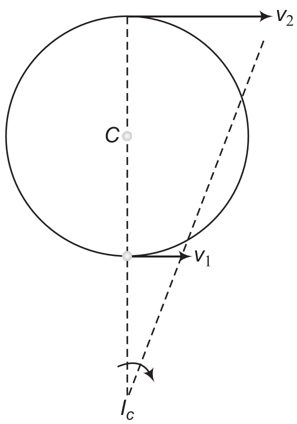
$$\tan(90° - \theta) = \frac{v}{R\omega\cos(90° - \theta)}$$

$$\sin(90° - \theta) = \frac{v}{R\omega}$$

$$\cos\theta = \frac{v}{R\omega}$$

$$\theta = \pm\cos^{-1}\left(\frac{v}{R\omega}\right)$$

3.



$$\omega x = v_1 \quad \text{and} \quad \omega(x + 2R) = v_2$$

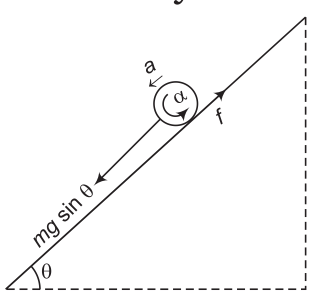
$$\therefore \quad \omega\left[\frac{v_1}{\omega} + 2R\right] = v_2$$

$$2R\omega = v_2 - v_1$$

$$\omega = \frac{v_2 - v_1}{2R}$$

Introductory Exercise 9.7

1.



$$f = f_2 = \mu N = \mu mg \cos\theta$$

(a) $ma = mg \sin\theta - \mu\, mg \cos\theta$

$$a = g(\sin\theta - \mu\cos\theta)$$

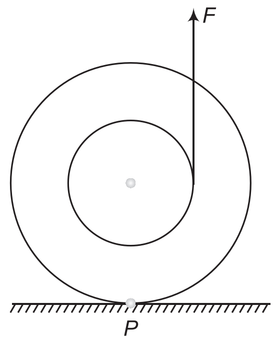
(b) $Rf = I\alpha$

$$\mu Rmg \cos\theta = \frac{2}{5} mR^2 \alpha$$

$$\therefore \quad \alpha = \frac{5\mu g \cos\theta}{2R}$$

2. Work done by friction is zero only in uniform pure rolling but not in accelerated pure rolling. So, the statement is false.

3.



As the torque due to applied force is anti-clockwise, so the point of contact tries to slip rightwards and friction tries to prevent it and so acts leftwards.

4. As during rolling down, friction acts upward which exerts a torque, thus the angular momentum of the system is not conserved. Even if we take axis of rotation at point of contact, then component of weight exerts torque, so, statement is false.

5. Force of friction will be zero, only when there is uniform pure rolling, *i.e.*, there is no external unbalanced torque.

Thus, $a = R\alpha$

$$a = \frac{F_1 + F_2}{M} \quad \text{and} \quad \alpha = \frac{2rF_1 - rF_2}{I}$$

as, $\quad a = 2r\alpha$

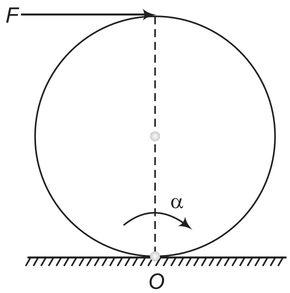
$$\therefore \quad \frac{F_1 + F_2}{M} = \frac{2r^2}{I}(2F_1 - F_2)$$

$$\Rightarrow \quad F_1\left(1 - \frac{4Mr^2}{I}\right) = -F_2\left(1 + \frac{2Mr^2}{I}\right)$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1 + \frac{2Mr^2}{I}}{\frac{4Mr}{I} - 1}$$

$$= \frac{2Mr^2 + I}{4Mr^2 - I}$$

6.



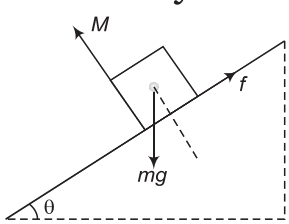
$\tau = 2RF = I\alpha \Rightarrow \alpha = \frac{2R}{I}F$; $\alpha \geq 0$ for $F \geq 0$

i.e., even a slightest amount of force can initiate motion of the disk.

7. Acceleration of point of contact is zero only when there is uniform pure rolling and no slipping.

■ Introductory Exercise 9.8

1.



For sliding,

$$mg \sin\theta > \mu\, mg \cos\theta$$

$\therefore \tan\theta > \mu$

And for toppling,

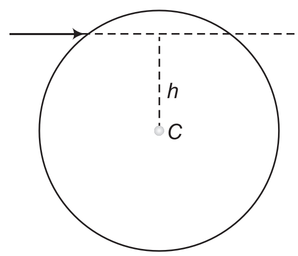
$$mg \sin\theta \frac{a}{2} > mg \cos\theta \frac{a}{2}$$

$\therefore \tan\theta > 1$

(a) Cube will slide before toppling if $\tan\theta < 1$, *i.e.*, $\mu < 1$ and

(b) Cube will topple before sliding if $\tan\theta > 1$, *i.e.*, $\mu > 1$

2.



For rolling without slipping on horizontal plane, $f = 0$

Impulse, $I_m = \Delta p = mv$

Angular Impulse, $I_m h = \Delta L$

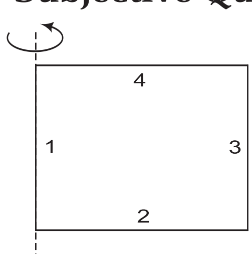
$$= I\omega = \frac{2}{5}mR^2\omega = \frac{2}{5}mRv$$

$$\therefore \quad mvh = \frac{2}{5}mRv \quad \Rightarrow \quad h = \frac{2R}{5}$$

AIEEE Corner

■ Subjective Questions (Level 1)

1.



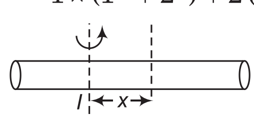
$$I = I_1 + I_2 + I_3 + I_4$$

$$= 0 + \frac{ml^2}{3} + ml^2 + \frac{ml^2}{3} = \frac{5ml^2}{3}$$

2. $I = I_1 + I_2$

$$= 1 \times (1^2 + 2^2) + 2(3^2 + 4^2) = 55 \text{ kg-m}^2$$

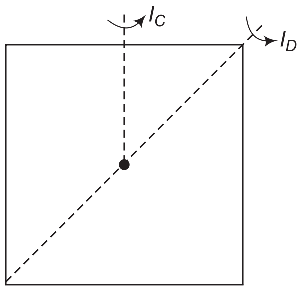
3.



$$I = \frac{ml^2}{12} + mx^2 = \frac{7}{12}ml^2$$

$$\therefore \quad x^2 = \left(\frac{7}{12} - \frac{1}{12}\right)l^2 = \frac{l^2}{2} \Rightarrow x = \frac{l}{\sqrt{2}}$$

4.



$$I_C = \frac{Ma^2}{6} \quad \text{and} \quad I_D = \frac{1}{2} I_C = \frac{Ma^2}{12}$$

5.
$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{I_{CM} + Mx^2}{M}}$$
$$\Rightarrow \quad k^2 = \frac{I_{CM}}{M} + x^2$$
$$\therefore \quad \frac{I_{CM}}{M} = k^2 - x^2$$
$$\Rightarrow \quad k_{CM} = \sqrt{\frac{I_{CM}}{M}} = \sqrt{k^2 - x^2}$$
$$= \sqrt{10^2 - 6^2} = 8 \text{ cm}$$

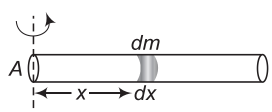
6. $r_1 = \left(\frac{m_2}{m_1 + m_2}\right) r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$

$$I = m_1 r_1^2 + m_1 r_2^2$$

Substituting the values we get,

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2$$

7.



$$dI = dmx^2 = x^2 \lambda \, dx = x^2 \, (\alpha x + \beta) \, dx$$
$$= (\alpha x^3 + \beta x^2) \, dx$$
$$\therefore \; I = \int_0^l (\alpha x^3 + \beta x^2) \, dx$$
$$= \frac{1}{4} \alpha l^4 + \frac{1}{3} \beta l^3$$

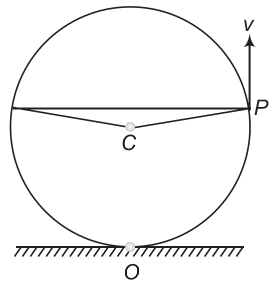
8. In second's clock, it takes 60 s by the seconds hand to rotate by 2π.

So, $\omega = \frac{2\pi}{60}$ rad/s $= \frac{\pi}{30}$ rad/s.

9. $\omega = \frac{1}{r^2} (\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = \frac{1}{25} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & 0 \\ 4 & -3 & 0 \end{vmatrix}$

$$= \frac{1}{25} (-9 - 16) \, \hat{\mathbf{k}} = -\hat{\mathbf{k}} \text{ rad/s}$$

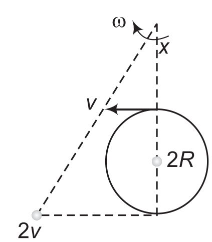
10.



$$\omega_c = 2\omega_o$$
$$\Rightarrow \; \omega_o = \frac{1}{2} \omega_c$$
$$= \frac{1}{2} \cdot \frac{v}{R} = \frac{1}{2} \times \frac{2}{0.1} = 10 \text{ rad/s}$$

11.

$$\omega = \frac{v}{x} = \frac{2v}{x + 2R}$$
$$\Rightarrow \; x + 2R = 2x$$
$$\Rightarrow \; x = 2R$$
$$\therefore \; \omega = \frac{v}{2R}$$



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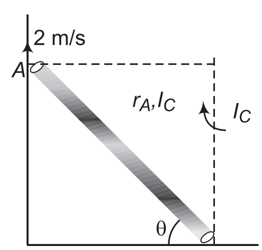


$$\omega = \frac{v_A}{r_{A,I_C}} = \frac{2}{2 \cos 60°} = 2 \text{ rad/s}$$

13. $\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 4 & -2 \\ 2 & 3 & -2 \end{vmatrix}$

$$= \hat{\mathbf{i}} \, (-8 + 6) - \hat{\mathbf{j}} \, (-4 + 4) + \hat{\mathbf{k}} \, (6 - 8)$$
$$= -2 \, \hat{\mathbf{i}} - 2 \, \hat{\mathbf{k}} = -2 \, (\hat{\mathbf{i}} + \hat{\mathbf{k}}) \text{ Nm}$$

14. $\tau = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \frac{R}{2} \cdot mg = \frac{u^2 \sin 2\theta}{2g} \cdot mg$

$= mu^2 \sin\theta \cos\theta = \frac{1}{2} mu^2$

$= \frac{1}{2} \times 1 \times (20\sqrt{2})^2 = 400\,\text{Nm}$

15. $\tau = \tau_{10} + \tau_{20} + \tau_{30}$

$= 0 + 20 \sin 45° \times 0.1 + 30 \sin 60° \times 0.05$

$= \sqrt{2} + 1.5 \times \frac{\sqrt{3}}{2} = \sqrt{2} + \frac{3\sqrt{3}}{4} = 2.71\,\text{Nm}$

16. $\tau = 12 \cos 60° \times 0.1 - 10 \times 0.25 - 9 \times 0.25$

$= 0.6 - \frac{19}{4} = \frac{2.4 - 19}{4} = \frac{16.6}{4} = \frac{83}{20}\,\text{Nm}$

17. $\theta = \frac{1}{2}\alpha t^2 \Rightarrow \alpha = \frac{2\theta}{t^2} = \frac{2 \times 50 \times 2\pi}{5^2}$

$= 8\pi\,\text{rad/s}^2$

$\therefore \omega = \alpha t = 8\pi\,\text{rad/s}^2 \times 5s = 40\,\text{rad/s}$

18. $\theta = \theta_1 + \theta_2 + \theta_3$

$= \frac{1}{2}\alpha t_1^2 + \omega t_2 + \omega t_1 - \frac{1}{2}\alpha t_1^2$

$= \alpha t_1 (t_1 + t_2) = 2 \times 5\,(5 + 2) = 70\,\text{rad}$

19. $\omega = \alpha_1 t_1$

$\Rightarrow \alpha_1 = \frac{\omega}{t_1} = \frac{20}{5} = 4\,\text{rad/s}^2$

$\alpha_2 = \frac{\omega}{t_2} = \frac{20}{60} = \frac{1}{3}\,\text{rad/s}^2,$

$\tau_2 = I\alpha_2 = 0.03 \times \frac{1}{3} = 0.01\,\text{Nm}$

is the torque due to friction. Again,

$\tau_1 - \tau_2 = I\alpha_1$

$\Rightarrow \quad \tau_1 = \tau_2 + I\alpha_1$

or $\quad \tau_1 = 0.01 + 0.03 \times 4 = 0.13\,\text{Nm}$

20. $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad -20\,\text{rad/s}$

$= 20\,\text{rad/s} - 2\,\text{rad/s}^2 \times t$

$\Rightarrow \quad t = \frac{40}{2}\,\text{s} = 20\,\text{s}$

(As KE $= \frac{1}{2} I\omega^2$ is same for $\pm\,\omega$)

21. $\tau = I\alpha = \frac{1}{2} MR^2 \cdot \frac{\omega}{t}$

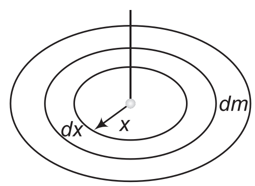
$= \frac{1}{2} \times 20 \times \left(\frac{1}{2}\right)^2 \times \frac{\frac{240}{\pi} \times \frac{2\pi}{60}}{3}$

$= \frac{5}{2} \times \frac{8}{3} = \frac{20}{3}\,\text{Nm}$

$\tau' = I\alpha' = \frac{1}{2} MR^2 \frac{\omega}{t'} = RF$

$\Rightarrow t' = \frac{MR\omega}{2F} = \frac{20 \times \frac{1}{2} \times 8}{2 \times 10} = 4\,\text{s}$

22.



(a) $d\tau = \mu \cdot dm \cdot g \cdot x = \mu g x \cdot \frac{M}{\pi R^2} \cdot 2\pi x\, dx$

$= \frac{2\mu Mg}{R^2} x^2 dx$

$\tau = \frac{2\mu Mg}{R^2} \int_0^R x^2 dx = \frac{2}{3} \mu MgR$

$= I\alpha = \frac{1}{2} MR^2 \alpha$

$\Rightarrow \alpha = \frac{4}{3} \frac{\mu g}{R}, 0 = \omega_0 - \alpha t$

$\Rightarrow t = \frac{\omega_0}{\alpha} = \frac{3R\omega_0}{4\mu g}$

(b) $\theta = \omega_0 t - \frac{1}{2} \alpha t^2$

$= \frac{3R\omega_0^2}{4\mu g} - \frac{1}{2} \cdot \frac{4\mu g}{3R} \cdot \frac{9R^2 \omega_0^2}{16\mu^2 g^2}$

$= \frac{3R\omega_0^2}{4\mu g} - \frac{3R\omega_0^2}{8\mu g} = \frac{3R\omega_0^2}{8\mu g}$

23. (a) $\tau = I\alpha$ and $0 = \omega_0 - \alpha t$

$\Rightarrow t = \frac{\omega_0}{\alpha} = \frac{I\omega_0}{\tau} = \frac{16 \times 9}{4} = 36\,\text{s}$

(b) $\tau = kt = I\frac{d\omega}{dt} \Rightarrow d\omega = \frac{k}{I} t\, dt$

$\Rightarrow \quad \int_{\omega_0}^{0} d\omega = \int_0^t \frac{k}{I} t\, dt$

$\therefore\ \omega_0 = \frac{k}{2I} t^2 \ \Rightarrow\ t = \sqrt{\frac{2\omega_0 I}{k}}$

$= \sqrt{\frac{2 \times 9 \times 16}{k}} = 12\sqrt{\frac{2}{k}}$

24. $\frac{d\omega}{dt} = \alpha = -10 - 5t$

$\Rightarrow \omega = \int (-10 - 5t)\, dt = -10\,t - \frac{5}{2} t^2 + c$

at $t = 0, \omega = c = 65$ rad/s

$\therefore\ \omega = 65 - 10\,t - \frac{5}{2} t^2$

(a) $\omega(3\,\text{s}) = 65 - 10 \times 3 - \frac{5}{2} \times 3^2$

$= 35 - \frac{45}{2} = \frac{25}{2}$ rad/s

= 12.5 rad/s

(b) $\frac{d\theta}{dt} = \omega \ \Rightarrow\ d\theta = \omega dt$

$\Rightarrow\ \theta = 65t - 5t^2 - \frac{5}{6} t^3$

$\therefore\ \theta = 65 \times 3 - 5 \times 3^2 - \frac{5}{6} \times 3^3$

$= 195 - 45 - \frac{45}{2} = 127.5$ rad.

25. $\Delta\theta = \int_0^3 \omega dt = \int_0^3 (12 - 3t^2)\, dt$

$= 12t - t^3\Big|_0^3 = 36 - 9 = 27$ rad

$N = \frac{\Delta\theta}{2\pi} = \frac{27\text{ rad}}{2\pi\text{ rad}} = 4.3;$

$\Delta\theta\,(2\text{ s}) = 12 \times 2 - 2^3 = 16$ rad

$\therefore\ \theta_3 - \theta_2 = 27 - 16 = 9$ rad

So, in third second, $N' = \frac{9\text{ rad}}{2\pi\text{ rad}} = 1.43$

26. $\theta = 6t - 2t^3 \ \Rightarrow\ \omega = \frac{d\theta}{dt} = 6 - 6t^2$

$\Rightarrow\ \omega = 0$ at $t = 1$ s

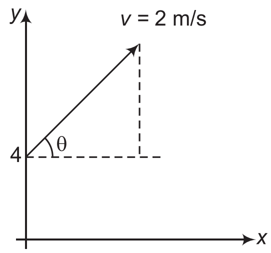
$\therefore\ \theta(0) = 0$ and $\theta(1) = 6 - 2 = 4$ rad

$\therefore\ \omega_{av} = \frac{\theta(1) - \theta(0)}{t} = \frac{4 - 0}{1} = 4$ rad/s

$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{0 - (6 - 6 \times 0)}{1 - 0}$

$= -6$ rad/s^2

27.



$L = m \cdot v \cos\theta \cdot r,\ \tan\theta = \frac{dy}{dx} = 1$

$= 1 \times 2 \times \frac{1}{\sqrt{2}} \times 4 \ \Rightarrow\ \cos\theta = \frac{1}{\sqrt{2}}$

$= 4\sqrt{2}$ kg-m^2/s

28. $I = I_1 + I_2 = \frac{\frac{m}{3} \times l^2}{3} + \frac{\frac{2m}{3} \times 4l^2}{3}$

$= \frac{ml^2}{9} + \frac{8ml^2}{9} = ml^2$

$L = I\omega = ml^2\omega$

29. $L = mvr + I\omega$

$= -\left(mvR + \frac{2}{5} mR^2 \omega\right)\hat{\mathbf{k}}$

$= -\frac{7}{5} mvR\,\hat{\mathbf{k}}$

30. $L = mvr + I\omega$

$= -\left[2mvR + \left(mR^2 + \frac{1}{3} mR^2\right)\omega\right]\hat{\mathbf{k}}$

$= -\left[2mvR + \frac{4}{3} mvR\right]\hat{\mathbf{k}}$

$= -\frac{10}{3} mvR\,\hat{\mathbf{k}}$

31. As $I\omega =$ constant $\Rightarrow T \propto I$.

With increase in R, I increases and so is T. So, length of day will increase.

32. $I_1\omega_1 = I_2\omega_2 \ \Rightarrow\ \omega_2 = \frac{I_1}{I_2}\omega_1$

$= \frac{\frac{1}{2} MR^2}{\frac{1}{2} MR^2 + mR^2 \sin^2\theta} \cdot \omega_1$

$= \frac{M}{M + 2m\sin^2\theta}\,\omega_1$

$$\therefore\ \omega_2 = \frac{1}{1 + 2\times 0.2\times\frac{1}{2}}\times 5\,\text{rad/s}$$

$$= \frac{5}{1.2}\,\text{rad/s} = \frac{25}{6}\,\text{rad/s}$$

33. $I_1\omega_1 = (I_1 + mr^2)\,\omega_2$

$$\Rightarrow\quad I_1 = \frac{mr^2\omega_2}{\omega_1 - \omega_2}$$

$$= \frac{10\times 10^{-3}\times 81\times 10^{-4}\times 90}{100 - 90}$$

$$= 7.29\times 10^{-4}\ \text{kg-m}^2$$

34. (a) $\omega_2 = \dfrac{I_1}{I_2}\,\omega_1$

$$= \frac{I_0 + 2mr_1^2}{I_0 + 2mr_2^2}\cdot\omega_1$$

$$= \frac{1.6 + 8\times 0.81}{1.6 + 8\times 0.0225}\times 0.5\,\text{rev/s}$$

$$= \frac{8.08}{1.78}\times 0.5\,\text{rev/s}$$

$$= 2.27\ \text{rev/s}$$

(b) $E_i = \dfrac{1}{2}I_1\omega_1^2$

$$= \frac{1}{2}(1.6 + 6.48)(\pi)^2 = 39.9\ \text{J}$$

$$E_f = \frac{1}{2}I_2\omega_2^2$$

$$= \frac{1}{2}(1.6 + 0.18)(2.27\times 2\pi)^2 = 181\ \text{J}$$

(c) $\Delta W = E_f - E_i = 181\,\text{J} - 39.9\,\text{J}$

$$= 141.1\,\text{J}$$

35. (a) $\left(\dfrac{1}{2}MR^2 + mR^2\right)\omega_0 = \dfrac{1}{2}MR^2\omega$

$$\Rightarrow\quad \omega = \frac{\frac{1}{2}M + m}{\frac{1}{2}M}\,\omega_0$$

$$= \frac{M + 2m}{M}\,\omega_0$$

(b) $E_i = \dfrac{1}{2}\left(\dfrac{1}{2}MR^2 + mR^2\right)\omega_0^2$

and $\quad E_f = \dfrac{1}{2}\cdot\dfrac{1}{2}MR^2\left(1 + \dfrac{2m}{M}\right)^2\omega_0^2$

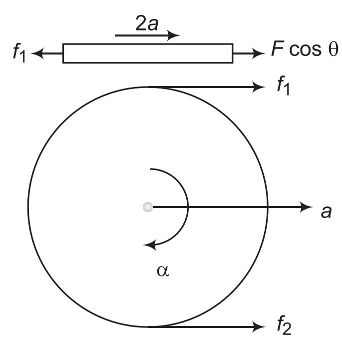
$$\therefore\quad W = E_f - E_i$$

$$= \frac{1}{2}\,\omega_0^2\left[\frac{1}{2}MR^2\left(1 + \frac{4m}{M} + \frac{4m^2}{M^2}\right) - \frac{1}{2}MR^2 - mR^2\right]$$

$$= \frac{1}{2}\,\omega_0^2\left[mR^2 + \frac{2m^2R^2}{M}\right]$$

$$= \frac{1}{2}m\omega_0^2R^2\left(1 + \frac{2m}{M}\right)$$

36.



$F\cos\theta - f_1 = m\cdot 2a$ ...(i)

$(f_1 - f_2)\,R = I\alpha = \dfrac{1}{2}MR^2\cdot\dfrac{a}{R} = \dfrac{1}{2}MRa$

or $\quad f_1 - f_2 = \dfrac{1}{2}Ma$ ...(ii)

While, $\quad f_1 + f_2 = Ma$ ...(iii)

From Eqs. (ii) and (iii),

$$f_1 = \frac{3}{4}Ma \quad\text{and}\quad f_2 = \frac{1}{4}Ma$$

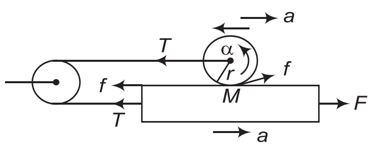
$$\therefore\ F\cos\theta - \frac{3}{4}Ma = 2ma$$

$$\Rightarrow a = \frac{4F\cos\theta}{3M + 8m}$$

$$f_1 = \frac{3M}{4}\cdot\frac{4F\cos\theta}{3M + 8m} = \frac{3MF\cos\theta}{3M + 8m}$$

and $f_2 = \dfrac{M}{4}\cdot\dfrac{4F\cos\theta}{3M + 8m} = \dfrac{MF\cos\theta}{3M + 8m}$

37.



$$\left.\begin{aligned} F - T - f &= Ma \\ T - f &= ma \end{aligned}\right\} \Rightarrow F - 2f = (M + m)\, a$$

$$fR = I\alpha = \frac{1}{2} mR^2 \alpha \Rightarrow f = \frac{1}{2} mR\alpha$$

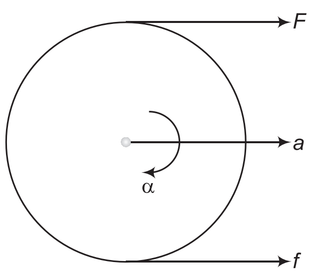
$$R\alpha - a = a \Rightarrow R\alpha = 2a \Rightarrow f = ma$$

$$\therefore F - 2ma = (M + m)\, a$$

$$\Rightarrow F = (M + 3m)\, a$$

$$\therefore a = \frac{F}{M + 3m}$$

38.



$$F + f = ma \qquad \ldots\text{(i)}$$

$$(F - f)\, R = I\alpha = \frac{1}{2} mR^2 \alpha = \frac{1}{2} mRa$$

$$\therefore \quad F - f = \frac{1}{2} ma \qquad \ldots\text{(ii)}$$

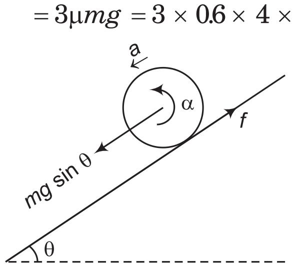
$$\therefore F = \frac{3}{4} ma \text{ and } f = \frac{1}{4} ma$$

$$f_{\max} = \mu mg = \frac{1}{4} ma \Rightarrow a = 4\, \mu g$$

$$\Rightarrow F_{\max} = \frac{3}{4} ma = \frac{3}{4} m 4 \mu g$$

$$= 3\mu mg = 3 \times 0.6 \times 4 \times 10 = 72\,\text{N}$$

39.



$$fR = I\alpha = \frac{2}{5} mR^2 \alpha = \frac{2}{5} mRa$$

$$\Rightarrow f = \frac{2}{5} ma$$

$$mg \sin\theta - f = ma \Rightarrow mg \sin\theta = \frac{7}{5} ma$$

$$\therefore a = \frac{5}{7} g \sin\theta \text{ and } f = \frac{2}{7} mg \sin\theta$$

(a) For minimum value of μ,

$$f = \mu mg \cos\theta = \frac{2}{7} mg \sin\theta$$

$$\text{or } \mu = \frac{2}{7} \tan\theta = \frac{2}{7} \tan 30° = \frac{2}{7\sqrt{3}} = \mu_{\min}$$

(b) $a = \frac{5}{7} \times 10 \times \sin 30° = \frac{25}{7}\ \text{m/s}^2$

(c) For, $\mu = \frac{\mu_{\min}}{2} = \frac{1}{7\sqrt{3}}$

$$\Rightarrow \quad f = \mu mg \cos\theta$$

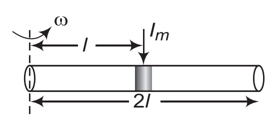
$$\therefore \quad a = g \sin\theta - \mu g \cos\theta$$

$$= g\,(\sin\theta - \mu \cos\theta)$$

$$= g\left(\frac{1}{2} - \frac{1}{7\sqrt{3}} \frac{\sqrt{3}}{2}\right)$$

$$= 5\left(1 - \frac{1}{7}\right) = \frac{30}{7}\ \text{m/s}^2$$

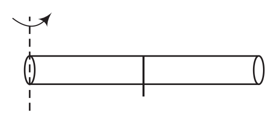
40.



$$I_m l = I\omega = \frac{m(2l)^2}{3} \omega = \frac{4ml^2}{3} \omega$$

$$\therefore I_m = \frac{4}{3} ml\omega$$

41.



(a) $mv \frac{L}{2} = \left(\frac{1}{3} ML^2 + m\frac{L^2}{4}\right) \omega$

$$\therefore \quad \omega = \frac{mv \frac{L}{2}}{\frac{1}{3} ML^2 + \frac{1}{4} mL^2}$$

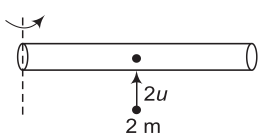
$$= \frac{\frac{M}{6} v \times \frac{L}{2}}{\frac{1}{3}ML^2 + \frac{1}{24}ML^2}$$

$$= \frac{\frac{v}{12}}{\frac{9L}{24}} = \frac{2v}{9L}$$

(b) $$\frac{K_f}{K_i} = \frac{\frac{1}{2}\left(\frac{1}{3}ML^2 + m\frac{L^2}{4}\right)\omega^2}{\frac{1}{2}mv^2}$$

$$= \frac{\frac{1}{2}\left(\frac{ML^2}{3} + \frac{ML^2}{24}\right)\cdot\frac{4v^2}{81L^2}}{\frac{1}{12}Mv^2}$$

$$= \frac{9 \times 12 \times 2}{24 \times 81} = \frac{1}{9}$$

42. 

$3m, 2l$

(a) $\frac{3m \cdot 4l^2}{3}\omega = 2m(2u + v)\,l$

where, $2u = l\omega + v \Rightarrow v = -l\omega + 2u$

$\therefore \quad 4ml^2\omega = 2m(4u - l\omega)\,l$

$6ml^2\omega = 8mul \Rightarrow \omega = \frac{8u}{6l} = \frac{4u}{3l}$

$\therefore \quad v = -\frac{4u}{3} + 2u = \frac{2u}{3}$

(b) $\frac{3m \cdot 4l^2}{3}\omega = 2m(\sqrt{3}u + v)\,l$

where, $\sqrt{3}u = l\omega + v$

$\therefore \quad 4ml^2\omega = 2m(2\sqrt{3}u - l\omega)\,l$

$\Rightarrow \quad 6ml^2\omega = 4\sqrt{3}mul$

$\Rightarrow \quad \omega = \frac{\sqrt{3}\cdot 4u}{6l}$

$$= \frac{\sqrt{3}\cdot 2u}{3l} = \frac{2u}{\sqrt{3}l}$$

$\therefore \quad v = \sqrt{3}u - l\omega = \sqrt{3}u - \frac{2u}{\sqrt{3}} = \frac{u}{\sqrt{3}}$

Objective Questions (Level 1)

1. $I = Mk^2$, which depends upon, mass and its distribution about an axis.

2. $k = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{1}{2}MR^2}{M}} = \frac{R}{\sqrt{2}} = \frac{25}{\sqrt{2}} = 18$ cm

3. $N = \frac{\theta}{2\pi} = \frac{\omega t - \frac{1}{2}\alpha t^2}{2\pi}$

$$= \frac{\omega t - \frac{1}{2}\omega t}{2\pi} = \frac{\omega t}{4\pi}$$

$$= \frac{1725 \times \frac{2\pi}{60} \times 20}{4\pi} = \frac{1725}{6} = 287$$

4. In pulling hands closes, external torque is zero, so angular momentum remains constant. Due to decrease in moment of inertia, angular velocity increases and KE increases.

5. $I = \frac{1}{2}Mr^2$

6. Masses cannot be compared as distribution is not given.

7. As the displacement of point of contact is zero, so work done by friction is zero.

8. Due to melting of ice, water spreads on outer side of the pan, increasing moment of inertia, which leads to a decrease in angular speed due to conservation of angular momentum.

9. $\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2}$

$$= \frac{\frac{1}{2}\cdot MR^2 \cdot \frac{v^2}{r^2}}{\frac{1}{2}Mv^2} = \frac{R^2}{r^2}$$

10. As, $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(m + \frac{I}{r^2}\right)v^2$

$\Rightarrow v =$ constant, irrespective of the inclination, but time is different because of different acceleration due to different inclination.

11. Moment of inertia is maximum, when masses are at maximum distance from axis.

12. $mg\sin\theta - f = ma$ and $fR = I\alpha = \frac{I}{R}a$

$$\Rightarrow f = \frac{I}{R^2}a$$

$$\therefore\ mg\sin\theta = f + \frac{fmR^2}{I} = \left(1 + \frac{mR^2}{I}\right)f$$

$$\Rightarrow f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}}$$

$$\frac{f_D}{f_S} = \frac{1 + \frac{mR^2}{I_S}}{1 + \frac{mR^2}{I_D}}$$

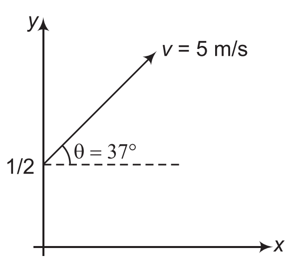
$$= \frac{1 + \frac{5}{2}}{1 + 2} = \frac{7}{6}$$

13. As, there is no external torque, so angular momentum is conserved.

14. As torque due to friction about any point on horizontal surface is zero, so angular moment about that point is conserved.

15. $y = \frac{3}{4}x + \frac{1}{2}$, $\tan\theta = \frac{3}{4}$

$$\Rightarrow \theta = 37° \Rightarrow \cos\theta = \frac{4}{5}$$



$$L = mvr = 3 \times \left(5 \times \frac{4}{5}\right) \times \frac{1}{2} = 6\ \text{kg-m}^2/\text{s}$$

16. Due to elastic collision, velocity will reverse and not angular velocity, so it will take some time to again settle to uniform rolling. During this time there will be sliping and its rotation will stop for a moment.

17. Moment of inertia about an axis perpendicular to the plane and passing through centre of a square plate is $\frac{Ma^2}{6}$ and about an axis passing through its plane is $\frac{1}{2}\cdot\frac{Ma^2}{6} = \frac{Ma^2}{12} = I_0$ is same for AA' and BB'.

18. Due to clockwise torque, the spool will move rightward and so, will the centre of mass.

19. $I\omega_0 = 2I\omega \Rightarrow \omega = \frac{\omega_0}{2}$

$$K_i = \frac{1}{2}I\omega_0^2 = K_0 \text{ and}$$

$$K_f = \frac{1}{2} \times 2I \times \omega^2$$

$$= \frac{1}{4}I\omega_0^2$$

$$\therefore\ \Delta K = K_f - K_i = \frac{1}{4}I\omega_0^2 - \frac{1}{2}I\omega_0^2$$

$$= \frac{1}{4}I\ \omega_0^2$$

$$= -\frac{1}{2}\cdot\frac{1}{2}I\ \omega_0^2 = -\frac{K_0}{2}$$

20. $I = \int x^2 dm = \int_R^{2R} x^2 \cdot 4\pi x^2 dx\ \rho$

$$= 4\pi\rho\int_R^{2R} x^4 dx = \frac{4\pi}{5}\rho x^4\Big|_R^{2R}$$

$$= \frac{4}{5}\pi\rho\,(32R^5 - R^5) = \frac{124}{5}\pi\rho R^5$$

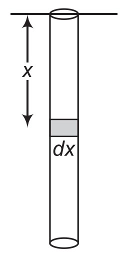
$$M = \int_R^{2R} \rho\cdot 4\pi x^2 dx = 4\pi\rho\frac{1}{3}x^3\Big|_R^{2R}$$

$$= \frac{4}{3}\pi\rho\,(8R^3 - R^3) = \frac{28}{3}\pi\rho R^3$$

$$\therefore\ \rho = \frac{3M}{28\pi R^3} \Rightarrow I = \frac{124}{5}\pi\cdot\frac{3M}{28\pi R^3}\cdot R^5$$

$$= \frac{91}{35}MR^2$$

21. $\lambda(x) = \lambda + \frac{\lambda}{l}x = \lambda\left(1 + \frac{x}{l}\right)$



$$I = \int x^2 dm = \int_0^l x^2 \lambda\left(1 + \frac{x}{l}\right) dx$$

$$= \lambda \int_0^l \left(x^2 + \frac{x^3}{l}\right) dx$$

$$= \lambda\left(\frac{1}{3}x^3 + \frac{x^4}{4l}\right)\Bigg|_0^l$$

$$= \lambda\left(\frac{1}{3}l^3 + \frac{1}{4}l^3\right) = \frac{7}{12}\lambda l^3$$

$$M = \int_0^l \lambda\left(1 + \frac{x}{l}\right) dx = \lambda\left(x + \frac{x^2}{2l}\right)\Bigg|_0^L$$

$$= \frac{3}{2}\lambda L \Rightarrow \lambda = \frac{2M}{3L}$$

$$\therefore\ I = \frac{7}{12}\cdot\frac{2M}{3L}\cdot L^3 = \frac{7}{18}ML^2$$

22. $I_1 = \frac{1}{2}\cdot\frac{Ma^2}{6} = \frac{Ma^2}{12}$

and $I_2 = I_1 + M\left(\frac{a}{\sqrt{2}}\right)^2 = I_1 + \frac{Ma^2}{2}$

$$= I_1 + 6\,\frac{Ma^2}{12} = 7I_1$$

$$\therefore\ I_1 : I_2 = I_1 : 7I_1 = 1 : 7$$

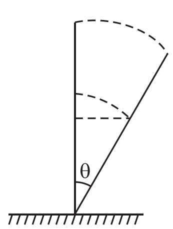
23. $I = \frac{ML^2}{12} + M\left(\frac{L}{2} - \frac{L}{4}\right)^2$

$$= \frac{ML^2}{12} + \frac{ML^2}{16}$$

$$= \frac{4+3}{48}ML^2$$

$$= \frac{7}{48}ML^2$$

24.



$$mg\frac{L}{2}(1 - \cos\theta) = \frac{1}{2}\cdot\frac{mL^2}{3}\cdot\omega^2$$

$$g\cdot 2\sin^2\frac{\theta}{2} = \frac{1}{3}L\omega^2$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{6g}{L}}\cdot\sin\frac{\theta}{2}$$

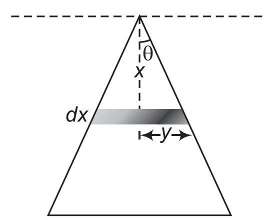
25. $r_1 = \left(\frac{m_1}{m_1 + m_2}\right)r_2$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r_1$

$I = m_1 r_1^2 + m_2 r_2^2$

Substituting the values we get,

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2$$

26.



$$I = \int dI = \int x^2 dm = \int x^2\cdot\sigma\cdot 2y\ dx$$

$$= 2\sigma\int x^2\cdot x\tan\theta\cdot dx$$

$$= 2\sigma\tan\theta\cdot\int x^3 dx = 2\sigma\tan\theta\cdot\frac{1}{4}x^4\Bigg|_0^{L\cos\theta}$$

$$= \frac{1}{2}\sigma L^4\tan\theta\cos^4\theta$$

$$= \frac{1}{2}\sigma L^4\cdot\frac{1}{\sqrt{3}}\cdot\frac{9}{16} = \frac{3\sqrt{3}}{32}\sigma l^4$$

$$M = \int dm = \int\sigma\cdot 2y\ dx$$

$$= 2\sigma\tan\theta\int x dx = 2\sigma\tan\theta\cdot\frac{1}{2}x^2\Bigg|_0^{L\cos\theta}$$

$$= \sigma\tan\theta\cdot L^2\cos^2\theta$$

$$= \sigma L^2\cdot\frac{1}{\sqrt{3}}\cdot\frac{3}{4} = \sigma L^2\cdot\frac{\sqrt{3}}{4}$$

$$\therefore\ \sigma = \frac{4M}{\sqrt{3}L^2}$$

$$\Rightarrow \qquad I = \frac{3\sqrt{3}}{32}\times\frac{4M}{\sqrt{3}L^2}\cdot L^4 = \frac{3}{8}ML^2$$

27. $I = 2\cdot\frac{M(L\sin 45^\circ)^2}{3}$

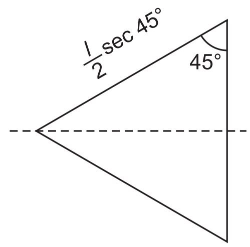
$$+ 2\left[\frac{M(L\sin 45^\circ)^2}{12} + M\left(\frac{3L}{2}\sin 45^\circ\right)^2\right]$$

$$= \frac{ML^2}{3} + 2\left[\frac{ML^2}{24} + \frac{9}{8}ML^2\right]$$
$$= ML^2\left(\frac{1}{3} + \frac{1}{12} + \frac{9}{4}\right)$$
$$= \frac{4+1+27}{12}ML^2$$
$$= \frac{32}{12}ML^2$$
$$= \frac{8}{3}ML^2$$

28. $$I = 2\cdot\frac{Ml^2}{3} + 2\left[\frac{Ml^2}{12} + M\left(\sqrt{l^2 + \frac{l^2}{4}}\right)^2\right]$$
$$= \frac{2}{3}Ml^2 + \frac{1}{6}Ml^2 + \frac{5}{2}Ml^2$$
$$= \frac{4+1+15}{6}Ml^2 = \frac{10}{3}Ml^2$$

29. Is both have same mass and same radius, *i.e.*, same distribution of mass, that is why both of them have same moment of inertia. $\left(\text{As } I = \frac{3}{10}MR^2\right)$

30.



$$I = 2\times\frac{1}{3}M\left(\frac{l}{2}\sec 45°\right)^2\cdot\sin^2 45°$$
$$= \frac{2}{3}Ml^2\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{Ml^2}{6}$$
$$M = x\cdot\frac{l}{2}\sec 45° = \frac{xl}{\sqrt{2}}$$
$$\therefore\ I = \frac{xl}{\sqrt{2}}\cdot\frac{l^2}{6} = \frac{xl^3}{6\sqrt{2}}$$

JEE Corner

Assertion and Reason

1. It depends upon the axis, whether it is parallel or perpendicular to the plane of rotation. So, assertion is not always true.
2. Due to anti-clockwise rotation, friction will start acting leftward, for which translational speed will increase and angular speed will decrease till the moment, when pure rolling starts. So, assertion and reason are both true but not complete explanation.
3. Hollow sphere has larger moment of inertia, such that it has lesser rotational kinetic energy, so assertion is false.

$$R_S + K_S = R_H + K_H$$
$$\frac{1}{2}\times\frac{2}{5}MR^2\omega^2 + \frac{1}{2}Mv^2$$
$$= \frac{1}{2}\times\frac{2}{3}mr^2\omega_1^2 + \frac{1}{2}mv_1^2$$
$$\frac{7}{5}Mv^2 = \frac{5}{3}mv_1^2$$
$$Mv^2 = \frac{25}{21}mv_1^2$$
$$\frac{1}{2}Mv^2 = \frac{25}{42}\cdot mv_1^2$$
$$K = \frac{25}{21}\cdot\frac{1}{2}mv_1^2$$
$$\therefore\ \frac{1}{2}mv_1^2 = \frac{21}{25}K$$
$$\Rightarrow\quad \frac{1}{2}mv_1^2 < K$$

4. Reason is true explanation of assertion.
5. Reason is true explanation of assertion.
6. Reason is true explanation of assertion.
7. As one goes from A to D, distance of CM first decrease and then increase, so moment of inertia also first decrease and then increase by applying parallel axes theorem. So, assertion is true but reason

is false, as perpendicular axis theorem is also applicable.

8. If linear momentum is constant, then angular momentum from a particular axis remains constant as the projection of distance remains constant. So, assertion and reason are both true but not correct explanation.

9. $v_C = v + R\omega, v_B = \sqrt{v^2 + R^2\omega^2}$

and $\quad v_A = v - R\omega$

$\Rightarrow \quad v_C > v_B > v_A$

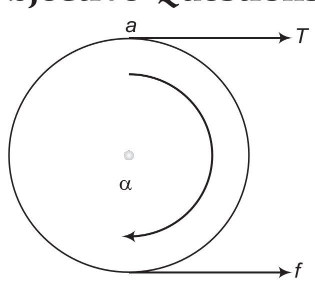
So, assertion and reason are both true but not correct explanation.

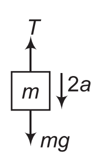
10. Reason is correct explanation of assertion.

11. Here, reason is false as when plank was at rest then work done by friction was not zero.

Objective Questions (Level 2)

1.





$T + f = ma \Rightarrow T = ma,\ f = 0,$

$T - f = \frac{I}{r}\alpha = ma$

$mg - T = m \cdot 2a$

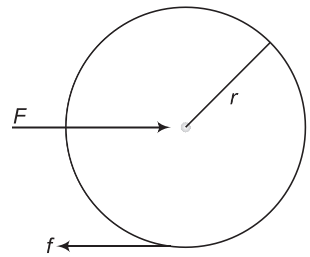
$\Rightarrow \quad mg = 3ma$

$\Rightarrow \quad a = \frac{g}{3}$

i.e., acceleration of the ring is $\frac{g}{3}$ while that of block is $\frac{2g}{3}$.

Here, $f = 0$

2.



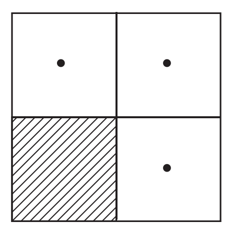
$F - f = ma$

$f = \frac{2}{5}ma \Rightarrow F = \frac{7}{5}ma$

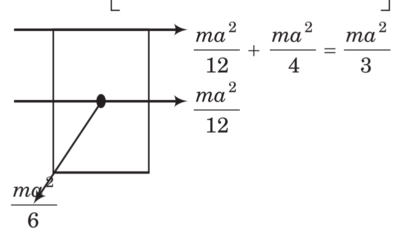
$\Rightarrow ma = \frac{5}{7}F = 5\text{ N}$

$\therefore \ f = \frac{2}{5}ma = \frac{2}{5} \times 5\text{ N} = 2\text{ N}$

3.



$$I = 2 \times \frac{1}{3} \cdot \frac{m}{4} \cdot \left(\frac{a}{2}\right)^2 + \left[\frac{\frac{m}{4} \cdot \left(\frac{a}{2}\right)^2}{12} + \frac{m}{4} \cdot \left(\frac{3a}{4}\right)^2\right]$$



$$= ma^2\left(\frac{2}{3} \times \frac{1}{16} + \frac{1}{12} \times \frac{1}{16} + \frac{9}{4} \times \frac{1}{16}\right)$$

$$= \frac{ma^2}{16}\left[\frac{2}{3} + \frac{1}{12} + \frac{9}{4}\right]$$

$$= \frac{ma^2}{16} \times \frac{8 + 1 + 27}{12}$$

$$= \frac{3ma^2}{16}$$

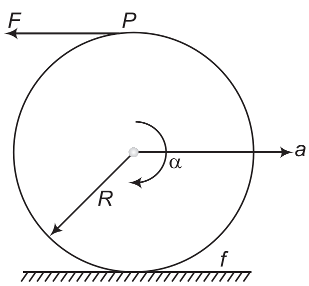
4. $mgR = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mv^2 = \frac{7}{10}mv^2$

$\Rightarrow \frac{mv^2}{R} = \frac{10}{7}mg$

$N = mg + \frac{mv^2}{R} = mg + \frac{10}{7}mg = \frac{17}{7}mg$

5. $F + f = Ma$

$F - f = \frac{I}{R}\alpha = \frac{1}{2}Ma$

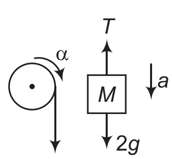


$\Rightarrow 2F = \frac{3}{2}Ma \Rightarrow a = \frac{4F}{3M}$

6. $f = F - \frac{1}{2}Ma = F - \frac{2}{3}F = \frac{1}{3}F$, towards right.

7. $2g - T = 2a$ and $TR = I\alpha = I\frac{a}{R}$

$\Rightarrow T = \frac{Ia}{R^2}$



$2g = \left(2 + \frac{I}{R^2}\right)a$

$\Rightarrow a = \frac{2g}{2 + \frac{I}{R^2}}$

$= \frac{2 \times 10}{2 + \frac{0.32}{0.04}} = 2\ \text{m/s}^2$

8. $p(h+R) = \left(\frac{3}{2}mR^2\right)\cdot\omega$

$\therefore \quad \omega = \frac{2p(h+R)}{3mR^2}$

Further, $p = mv = m(R\omega) = \frac{2p(h+R)}{3R}$

Solving we get, $h = \frac{R}{2}$.

9. $T_i = \frac{mg}{2}$ and $T_f = \frac{mg}{4}$

$\therefore\ T_f < T_i$

10. $Mg - T = Ma_1$

While, $MgR = \frac{1}{2}MR^2\alpha_2$

$\Rightarrow \quad \alpha_2 = \frac{2g}{R}$

$TR = \frac{1}{2}MR^2\alpha_1$

$\therefore\ Mg = \frac{3}{2}Ma_1$

$a_1 = \frac{2}{3}g$

$\alpha_1 = \frac{2g}{3R}$

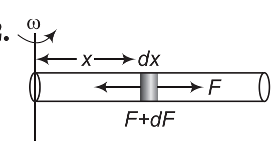
$\therefore\ \alpha_1 < \alpha_2$

or $\alpha_1 \neq \alpha_2$

11. $\frac{1}{2}kx^2 = Mgx\sin\theta$

$\Rightarrow x = \frac{2Mg\sin\theta}{k}$

12.



$\therefore\ F + dF - F = dm\omega^2 x = \frac{m}{l}dx\cdot\omega^2 x$

$\therefore\ dF = \frac{m\omega^2}{l}x\,dx$

$F = \frac{m\omega^2}{l}\int_x^l x\,dx = \frac{1}{2}\frac{m\omega^2}{l}(l^2 - x^2)$

$= \frac{1}{2}m\omega^2 l\left(1 - \frac{x^2}{l^2}\right)$

13. $v = r\omega = l(a + bt) = la + lbt$

$v(t = 0) = la = 1\ \text{m} \times 10\ \text{rad/s} = 10\ \text{m/s}$,

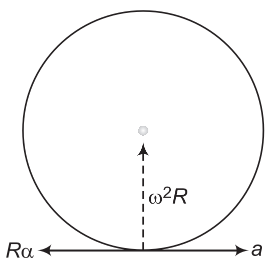
$\vec{\mathbf{v}} = 10\,\hat{\mathbf{j}}\ \text{m/s}$

$\vec{\mathbf{a}} = a_n \hat{\mathbf{j}} + a_n (-\hat{\mathbf{i}}) = l\alpha\,\hat{\mathbf{j}} - l\omega^2\,\hat{\mathbf{i}}$

$= lb\,\hat{\mathbf{j}} - l(a + bt)^2\,\hat{\mathbf{i}}$

$= 5\,\text{m/s}^2\,\hat{\mathbf{j}} - 100\,\text{m/s}^2\,\hat{\mathbf{i}}$

14.



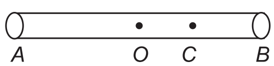
For pure rolling, $\alpha = \dfrac{a}{R}$

Here, $\vec{\mathbf{a}}_{\text{net}} = a\,\hat{\mathbf{i}} - R\alpha\,\hat{\mathbf{i}} + \omega^2 R\,\hat{\mathbf{j}}$

$= \omega^2 R\,\hat{\mathbf{j}}$

$\therefore\ a_{\text{net}} = \omega^2 R$

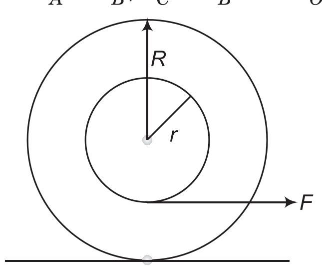
15.



Here, $I_C < I_O < I_B < I_A$

$\Rightarrow I_A > I_B, I_C < I_B$ and $I_O > I_C$

16.



Due to anti-clockwise torque by F and rightward F, the point of contact has the tendency to move rightward, so, frictional force acts leftward.

For, no rotation of the spool, $fR = Fr$

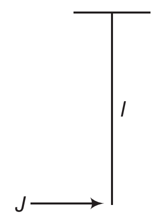
$\Rightarrow f = \dfrac{r}{R}F \Rightarrow f < F$

17. From conservation of angular momentum,

$mvR = 3mR^2\omega \Rightarrow \omega = \dfrac{v}{3R}$

18. As N provides clockwise torque about instantaneous centre of rotation and rod also rotates in clockwise sense with increasing angular speed, so, normal reaction does positive rotational work.

19. $L = Jl = I\omega = \dfrac{1}{3}ml^2\omega$



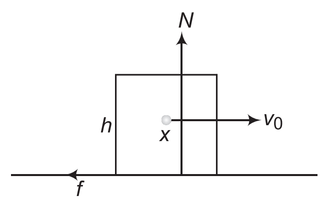
$\Rightarrow \omega = 3\,\text{J}$

$K = \dfrac{1}{2}I\omega^2$

$= \dfrac{1}{2} \times \dfrac{1}{3}ml^2 \times \dfrac{9J^2}{m^2 l^2}$

$= \dfrac{3J^2}{2m}$

20.

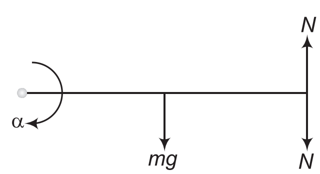


$f_L = \mu mg$

$\therefore\ \tau = Nx - f\dfrac{h}{2} = 0$, as the block is not toppling till, $x > b$ *i.e.*, $\dfrac{\mu h}{2} > b$

or $\mu > \dfrac{2b}{h}$

21.



Immediately after release, the free end is in free fall and so, is the coin, thus reaction on the coin is zero.

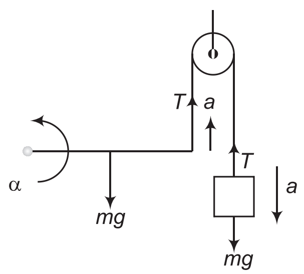
22. $Fr = fR$ and $F\cos\theta = f$

$F\cos\theta = F\dfrac{r}{R}$

$\Rightarrow\ \cos\theta = \dfrac{r}{R}\ \Rightarrow\ \sin\theta = \sqrt{1 - \dfrac{r^2}{R^2}}$

$\Rightarrow\ \theta = \sin^{-1}\sqrt{1 - \dfrac{r^2}{R^2}}$

23.



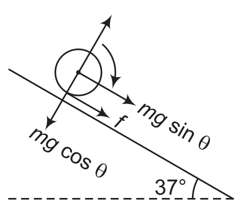
$$mg - T = ma \qquad \ldots(\text{i})$$

$$Tl - mg\frac{l}{2} = \frac{1}{3}ml^2\alpha$$

$$T - \frac{1}{2}mg = \frac{1}{3}ma \qquad \ldots(\text{ii})$$

$$\frac{1}{2}mg = \frac{4}{3}ma \Rightarrow a = \frac{3g}{8}$$

24.



$$\alpha = \frac{fr}{I} = \frac{fr}{\frac{1}{2}mr^2} = \frac{2f}{mr}$$

and $\quad a = g\sin\theta + \mu g\cos\theta$

$$v = at = r\omega = r(\omega_0 - \alpha t)$$

$$g(\sin\theta + \mu\cos\theta)\,t + r\,\alpha\,t = r\omega_0$$

$$\therefore\ t = \frac{r\omega_0}{r\alpha + g(\sin\theta + \mu\cos\theta)}$$

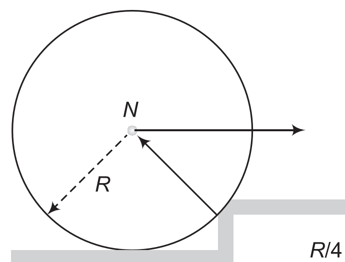
$$= \frac{0.4 \times 54}{0.4 \times \dfrac{2 \times \frac{1}{2} \times g \times \cos 37^\circ}{0.4} + g\left(\sin 37^\circ + \frac{1}{2}\cos 37^\circ\right)}$$

$$= \frac{0.4 \times 54}{10\left(\frac{4}{5} + \frac{3}{5} + \frac{2}{5}\right)}$$

$$= \frac{0.4 \times 54}{2 \times 9}$$

$$= 1.2\text{ s}$$

25.



26. $a = \dfrac{f}{m} = \dfrac{\mu mg}{m} = \mu g$ and

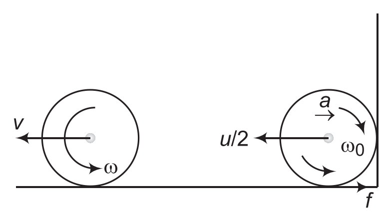
$$v = \frac{u}{2} - at = \frac{u}{2} - \mu g t \qquad \ldots(\text{i})$$

$$\alpha = \frac{\tau}{I} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5\mu g}{2R}$$

$$\omega = -\omega_0 + \alpha t$$

$$v = -\omega_0 R + R\alpha t$$

$$v = -u + \frac{5}{2}\mu g t \qquad \ldots(\text{ii})$$



$$\therefore\ \frac{u}{2} - \mu g t = -u + \frac{5}{2}\mu g t$$

$$\frac{3u}{2} = \frac{7}{2}\mu g t$$

$$\Rightarrow \qquad t = \frac{3u}{7\mu g}$$

27. $\text{KE} = \dfrac{1}{2}\cdot 2mv^2 + \dfrac{1}{2}\cdot\dfrac{2}{3}mr^2\omega^2$

$$= mv^2 + \frac{1}{3}mv^2$$

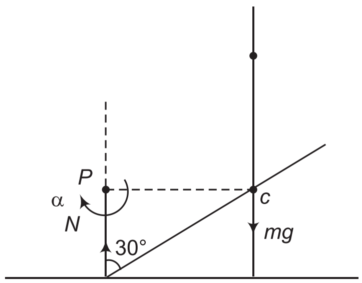
$$= \frac{4}{3}mv^2$$

28. $\tau = I\,\alpha$

$$\Rightarrow f\cdot R = \frac{1}{2}mR^2\cdot\alpha$$

$$\Rightarrow \qquad f = \frac{1}{2}mR\,\alpha = \frac{1}{2}ma$$

29.



$PC = \frac{l}{2}\sin 30° = \frac{l}{4}$

Instantaneous axis of rotation is at points P.

$$\alpha = \frac{\tau}{I} = \frac{(mg)(PC)}{I}$$

Further,

$$mg - N = ma = m(PC)\alpha$$

$$\therefore \quad N = mg - m(PC)\alpha$$

$$= mg\left[1 - \frac{m(PC)^2}{I}\right]$$

$$= mg\left[1 - \frac{m(l/4)^2}{\left(\frac{ml^2}{12}\right) + m\left(\frac{l}{4}\right)^2}\right]$$

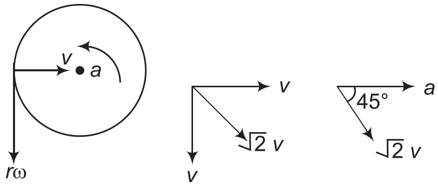
$$= \frac{4}{7}mg$$

30. $a = r\alpha = \left(\frac{l}{2}\cos 30°\right)\left[\frac{mg(PC)}{I}\right]$

$$= \left(\frac{l}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left[\frac{mg\left(\frac{l}{4}\right)}{\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2}\right]$$

$$= \frac{3\sqrt{3g}}{7}$$

31.



So, acceleration is towards centre and resultant velocity is at an angle of 45° with it.

32. $F = ma$ and $F\frac{L}{2} = \frac{mL^2}{12}\alpha$

or $ma\frac{L}{2} = \frac{mL^2}{12}\alpha$

or $L\alpha = 6a$

$\therefore \; a_B = \frac{L\alpha}{2} - a = 3a - a = 2a = 2\frac{F}{m}$

33. Instantaneous centre of rotation forms an equilateral triangle, so, $IA = L$

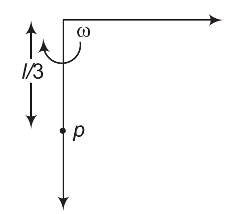
$\therefore \; L\omega = v \quad$ or $\quad \omega = \frac{v}{L}$

34. $L = mvL\cos\theta = \text{constant} = I\omega$

$\Rightarrow \omega = \frac{mvL\cos\theta}{I}$

or $\omega \propto \frac{1}{I}$, with increase of distance of particle from origin, moment of inertia increases, so, angular velocity decreases.

35. $Mg \cdot \frac{L}{2} = \frac{1}{2}\cdot\frac{ML^2}{3}\cdot\omega^2$



$\Rightarrow \omega = \sqrt{\frac{3g}{L}}$

$$dT = gdm + \omega^2 x dm$$

$$= \left(g + \frac{3g}{L}x\right)\frac{M}{L}dx$$

$$= \frac{Mg}{L}\left(1 + \frac{3x}{L}\right)dx$$

$$T = \int_{L/3}^{L}\frac{Mg}{L}\left(1 + \frac{3x}{L}\right)dx$$

$$= \frac{Mg}{L}\left(x + \frac{3x^2}{2L}\right)\Bigg|_{L/3}^{L}$$

$$= \frac{Mg}{L}\left[\left(L - \frac{L}{3}\right) + \frac{3}{2L}\left(L^2 - \frac{L^2}{9}\right)\right]$$

$$= \frac{Mg}{L}\left[\frac{2L}{3} + \frac{3}{2L}\cdot\frac{8L^2}{9}\right]$$

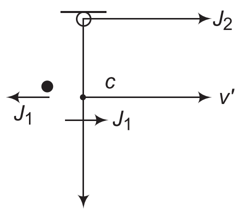
$$= 2\,Mg$$

36. $mv\frac{L}{2} = \frac{mL^2}{3}\omega$

$\therefore \quad \omega = \frac{3}{2}\cdot\frac{v}{L}$

$J_1 = mv$

$v' = \frac{L}{2}\cdot\omega = \frac{3}{4}v$



$J_1 + J_2 = mv'$

$\therefore \; J_2 + mv = \frac{3}{4}mv$

or $\quad J_2 = -\frac{1}{4}mv$

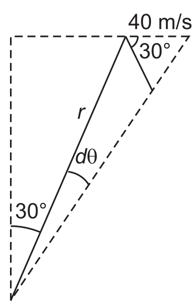
37. $L = I\omega + m\cdot R\omega\cdot R = I\omega' + mvR$

$\therefore \; \omega' = \frac{I\omega + mR^2\omega - mvR}{I}$

$= \frac{(I + mR^2)\,\omega - mvR}{I}$

38. As the point of contact has a net forward velocity, friction acts in backward direction, which tries to bring back the disk at its initial position.

39. $r\frac{d\theta}{dt} = v\cos 30°$



$d\sec 30°\,\omega = v\cos 30°$

$\Rightarrow \omega = \frac{v}{d}\cos^2 30°$

$= \frac{40\text{ m/s}}{30\text{ m}}\cdot\frac{3}{4}$

$= 1\text{ rad/s}$

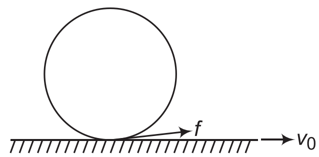
40. Locus of all other points is an ellipse. But locus of a point P such that, $OP = \frac{l}{4}$ is a circle.

41. When A hits ground, velocity of all points except O is vertically downward with no horizontal component. And O_4 at rest.

42. $mg\frac{l}{2} = \frac{1}{2}\cdot\frac{ml^2}{12}\cdot\omega^2 \Rightarrow \quad \omega = 2\sqrt{\frac{3g}{l}}$,

$v_A = \omega\frac{l}{2} = \frac{l}{2}\cdot 2\sqrt{\frac{3g}{l}} = \sqrt{3gl}$

43.



$ft = mv$ ...(i)

and $\quad fRt = I\omega = \frac{2}{5}mR^2\omega$ ...(ii)

Friction will stop acting when,

$v + R\omega = v_0$ ...(iii)

From Eqs. (i) and (ii),

$mvR = \frac{2}{5}mR^2\omega \Rightarrow R\omega = \frac{5}{2}v$

From Eq. (iii),

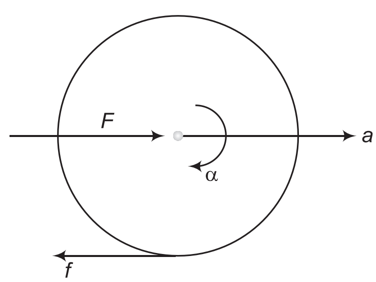
$v + \frac{5}{2}v = v_0 \Rightarrow v = \frac{2}{7}v_0$

$a = \frac{f}{m} = \frac{\mu mg}{m} = \mu g = \frac{2}{7}g$

As, $v^2 = 2as \Rightarrow s = \frac{v^2}{2a} = \frac{\left(\frac{2}{7}v_0\right)^2}{2\left(\frac{2}{7}g\right)} = \frac{v_0^2}{7g}$

48. In case of pure rolling, displacement of point contact is zero. So, work done by friction is also zero.

49.



$F - f = ma$

$fR = I\,\alpha = \frac{1}{2}mR^2\alpha$

$= \frac{1}{2}mRa \Rightarrow f = \frac{1}{2}ma$

50. $f = \frac{1}{2}ma \quad$ and $\quad F = ma + f = \frac{3}{2}ma.$

Angular momentum is conserved about a point which net torque is zero. Let this point is at a distance x above point of contact, so

$$f(x) = F\,(x - R)$$

or $\frac{1}{2} ma \cdot x = \frac{3}{2} ma(x - R)$

or $x = 3\,(x - R)$

or $3R = 2x$

or $x = \frac{3R}{2}$

51. $mgh = \frac{1}{2}\left(\frac{2}{3} mr^2\right)\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{3} mv^2 + \frac{1}{2} mv^2 = \frac{5}{6} mv^2$

$\therefore\ v = \sqrt{\frac{6gh}{5}}$

52. $H = \frac{v^2 \sin^2\theta}{2g}$

$= \frac{\frac{6gh}{5} \times \frac{9}{25}}{2g} = \frac{27h}{125}$

53. $x = \frac{v^2 \sin^2\theta}{g}$

$= \frac{\frac{6gh}{5} \times 2 \times \frac{3}{5} \times \frac{4}{5}}{g} = \frac{144h}{125}$

More than One Correct Options

1. $\frac{1}{2} mv^2 + \frac{1}{2} \cdot \frac{I}{R^2} v^2$

$= mg \cdot \frac{3}{4} \cdot \frac{v^2}{g}$

$\Rightarrow \frac{I}{2R^2} = \left(\frac{3}{4} - \frac{1}{2}\right) m$

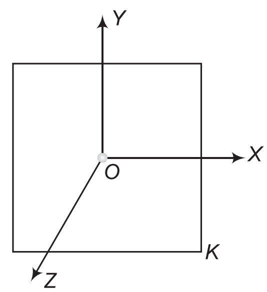
$= \frac{1}{4} m$

$\Rightarrow\ I_{CM} = \frac{1}{2} mR^2 \Rightarrow$ Disc or cylinder

$I_0 = I_{CM} + mR^2$

$= \frac{3}{2} mR^2$

2.



$I_{XX'} = 2 \cdot \frac{ml^2}{12} + 2 \cdot m\left(\frac{l}{2}\right)^2$

$= ml^2 \left(\frac{1}{6} + \frac{1}{2}\right)$

$= \frac{4}{6} ml^2 = \frac{2}{3} ml^2$

$I_{ZZ'} = 4 \cdot \left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]$

$= ml^2 \left(1 + \frac{1}{3}\right) = \frac{4}{3} ml^2$

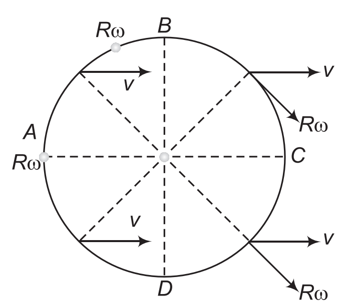
$I_{KZZ'} = I_{ZZ'} + 4m\left(\frac{l}{\sqrt{2}}\right)^2$

$= \frac{4}{3} ml^2 + 2ml^2 = \frac{10}{3} ml^2$

$I_{KXX'} = I_{XX'} + 4m\left(\frac{l}{2}\right)^2$

$= \frac{2}{3} ml^2 + ml^2 = \frac{5}{3} ml^2$

3.



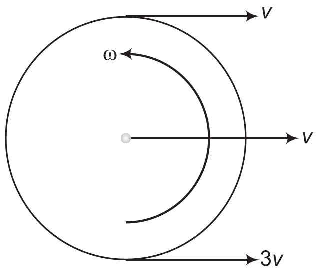
Resultant velocity of particles in region ABC is greater than that in CDA.

So, $(KE)_{ABC} > (KE)_{CDA}$

Similarly, $v_{BC} > v_{CD} \Rightarrow K_{BC} > K_{CD}$

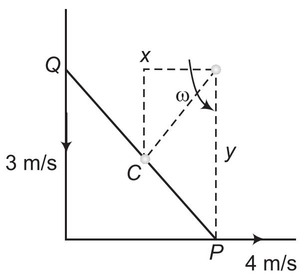
and $v_{CD} = v_{DA} \Rightarrow K_{CD} = K_{DA}$

4.



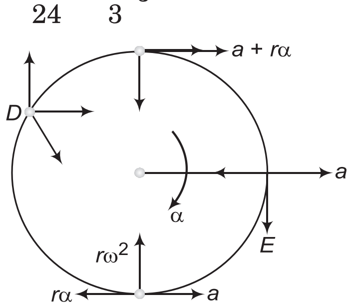
$$u - R\omega = v$$
$$u + R\omega = 3v$$
$$\Rightarrow \quad u = 2v \text{ and } R\omega = v$$

5.



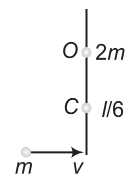
$$\frac{3}{x} = \frac{4}{y} \text{ and } l = \sqrt{x^2 + y^2} = x\sqrt{1 + \frac{16}{9}}$$
$$= \frac{5}{3}x \Rightarrow x = \frac{3}{5}l = 0.3 \text{ m}$$
$$\omega = \frac{v}{r} = \frac{3 \text{ m/s}}{0.3 \text{ m}} = 10 \text{ rad/s}$$
$$v_c = \frac{l}{2}\omega = \frac{1}{4} \times 10 \text{ m/s} = 2.5 \text{ m/s}$$
$$\text{KE} = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]\omega^2$$
$$= \frac{4}{24}ml^2\omega^2 = \frac{4}{24} \times 2 \times \frac{1}{4} \times 100$$
$$= \frac{200}{24} = \frac{25}{3} \text{ J}$$

6.



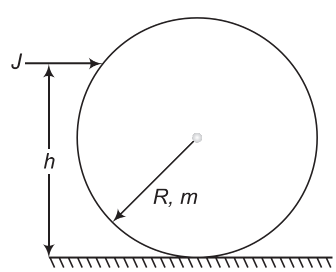
Net acceleration at A is vertical, at B it may be vertical, at C it cannot be horizontal and at E it may be horizontal leftward.

7. $$mv\left(\frac{l}{2} - \frac{l}{6}\right) = \left[\left(\frac{2ml^2}{12} + \frac{2ml^2}{36}\right) + m\left(\frac{l}{2} - \frac{l}{6}\right)^2\right]\omega$$



$$mv\frac{l}{3} = ml^2\left(\frac{1}{6} + \frac{2}{36} + \frac{4}{36}\right)\omega$$
$$= ml^2\omega\frac{6 + 2 + 4}{36} = \frac{12}{36}ml^2\omega$$
$$= \frac{ml^2\omega}{3}$$
$$\Rightarrow \quad l\omega = v$$
$$\Rightarrow \quad \omega = \frac{v}{l}$$
$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}I\omega^2 - \frac{1}{2}\cdot 3mu^2;$$
$$mv = 3mu$$
$$\Rightarrow u = \frac{v}{3}$$
$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{v^2}{l^2}\left[\frac{1}{3}ml^2\right] - \frac{1}{6}mv^2$$
$$= \left(\frac{1}{2} - \frac{1}{3}\right)mv^2$$
$$= \frac{1}{6}mv^2$$

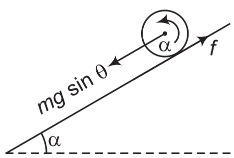
8. $$I = mk^2 = \frac{1}{4}mR^2$$
$$J = mv \text{ and } J\,(h - R) = I\omega$$
$$\therefore \; mv\,(h - R) = \frac{1}{4}mR^2\omega$$



For, $v = R\omega$, $h - R = \frac{1}{4}R \Rightarrow h = \frac{5}{4}R$ for pure rolling.

If impulse is through centre of mass, $h = R$, then there is no angular impulse and no rotation.

9.



$$fR = I\alpha = \frac{2}{3}mR^2\alpha$$
$$= \frac{2}{3}R\,ma \Rightarrow f = \frac{2}{3}ma \quad \ldots\text{(i)}$$

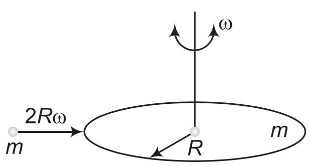
$mg\sin\alpha - f = ma \Rightarrow mg\sin\alpha$
$$= \frac{5}{3}ma \Rightarrow a = \frac{3}{5}g\sin\alpha \quad \ldots\text{(ii)}$$

$f \le \mu mg\cos\alpha$; $f = \frac{2}{3}ma = \frac{2}{3}m\cdot\frac{3}{5}g\sin\alpha$

$$\mu \ge \frac{2mg\sin\alpha}{5mg\cos\alpha}$$

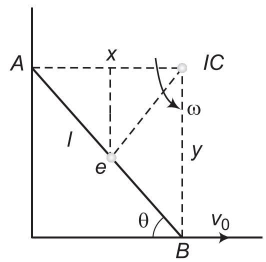
$$\Rightarrow \mu \ge \frac{2}{5}\tan\alpha$$

10.



$$\frac{1}{2}mR^2\omega = \left[\frac{1}{2}mR^2 + mR^2\right]\omega'$$
$$= \frac{3}{2}mR^2\omega'$$
$$\Rightarrow \omega' = \frac{\omega}{3}$$
$$\vec{\mathbf{v}}_f = R\omega'\,\hat{\mathbf{j}},\ \vec{\mathbf{v}}_i = 2R\omega'\,\hat{\mathbf{i}} = \frac{1}{3}R\omega\,\hat{\mathbf{j}}$$
$$\therefore\ \Delta v = R\omega\sqrt{\frac{1}{9} + 4} = \frac{\sqrt{37}}{3}R\omega$$
$$\Rightarrow \Delta p = m\Delta v = \frac{\sqrt{37}}{3}mR\omega$$
$$= \text{Impulse}$$

11.



$$\frac{v_A}{x} = \frac{v_0}{y}$$
$$\frac{v_A}{l\cos\theta} = \frac{v_0}{l\sin\theta} \Rightarrow v_A = v_0\cot\theta$$
$$= v_0\cot 37^\circ = \frac{4}{3}v_0$$
$$\omega = \frac{v_0}{y} = \frac{v_0}{l\sin 37^\circ} = \frac{5v}{3l}$$

Match the Columns

1. $I_S = \frac{2}{5}mr^2$, $I_H = \frac{2}{3}mr^2$, $I_D = \frac{1}{2}mr^2$

(a) As I_H is maximum for hollow sphere, it takes maximum time to reach at the bottom. $\rightarrow q$

(b) As fall in height is same for all, so is their KE $\rightarrow s$

(c) Rotational KE is also maximum for I_H. $\rightarrow q$

(d) As rotational KE is minimum for I_S, so, its translational KE is maximum. $\rightarrow p$

2. (a) As D is at maximum distance from point of contact, so, angular impulse is maximum for $A \rightarrow p$

(b) Linear speed acquired will be same for any point $\rightarrow s$

(c) There can be pure rolling, if angular impulse is such that velocity of point of contact is zero. It is possible when impulse is applied at $A \rightarrow p$

(d) Forward slipping will be there for sure, if impulse is provided below E *i.e.*, at $C \rightarrow r$

3. (a) As the point of contact tries to move downward, so, friction acts upward. And the value of friction will be self adjusting $\rightarrow p, s$

(b) As the point of contact is slipping downward, so, limiting friction is acting upward $\rightarrow p, r$

(c) As the sphere is slipping upward, so, limiting friction is acting downward $\rightarrow q, r$

(d) As the sphere is slipping downward, so limiting friction is acting upward $\rightarrow p, r$

4. Radius of gyration $K = \sqrt{\frac{I}{m}}$

$$K_1 = \sqrt{\frac{m(2a)^2/3}{m}} = \frac{2a}{\sqrt{3}}$$

$$K_2 = \sqrt{\frac{m(a)^2/12}{m}} = \frac{a}{\sqrt{3}}$$

$$K_3 = \sqrt{\frac{ma^2/3}{m}} = \frac{a}{\sqrt{3}}$$

$$K_4 = \sqrt{\frac{ma^2/12}{m}} = \frac{a}{2\sqrt{3}}$$

5. $mgh = \frac{1}{2}\cdot\frac{2}{5}mr^2\omega^2 + \frac{1}{2}mv^2$

$$= \frac{7}{10}mv^2 = \frac{7}{5}\cdot\frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}mv^2 = \frac{5}{7}mgh = (\text{Translational KE})_2$$

$$\frac{1}{5}mv^2 = \frac{2}{5}\cdot\frac{1}{2}mv^2 = \frac{2}{5}\cdot\frac{5}{7}mgh$$

$$= \frac{2}{7}mgh = (\text{Rotational KE})_2 \Rightarrow a \rightarrow q$$

$$mg\frac{h}{2} = \frac{1}{2}\cdot\frac{2}{5}mr^2\omega^2 + \frac{1}{2}mu^2$$

$$= \frac{7}{10}mu^2 = \frac{7}{5}\cdot\frac{1}{2}mu^2$$

$$\frac{1}{2}mu^2 = \frac{5}{7}\cdot mg\frac{h}{2}$$

$$= \frac{5}{14}mgh = (\text{Translational KE})_3$$

$\Rightarrow b \rightarrow s$

$$(\text{Rotational KE})_4 = (\text{Rotational KE})_3$$

$$= \frac{1}{5}mr^2\omega'^2$$

$$= \frac{1}{5}mu^2 = \frac{2}{5}\cdot\frac{1}{2}mu^2$$

$$= \frac{2}{5}\cdot\frac{5}{14}mgh = \frac{1}{7}mgh \Rightarrow c \rightarrow p$$

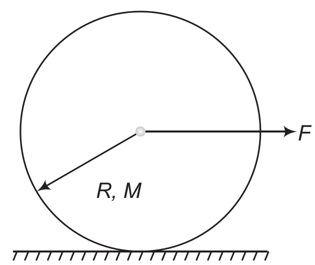
$(\text{Translational KE})_4$

$$= mgh - \frac{1}{7}mgh = \frac{6}{7}mgh$$

$\Rightarrow d \rightarrow s$

6. $F = \frac{80}{8}t = 10t$

$f_L = \mu Mg = 0.2 \times 10 \times 10 = 20$ N



$$F - f = ma,\ fR = \frac{1}{2}mR^2\alpha$$

$$f = \frac{1}{2}mR\alpha$$

For, $R\alpha = a$, $F = \frac{3}{2}ma$ and $f = \frac{1}{2}ma$

$$f_{\max} = \frac{1}{2}ma_{\max} = 20 \text{ N}$$

$$\Rightarrow F = 60 \text{ N} = 10\frac{N}{5}\cdot t$$

$\Rightarrow t = 6$ s, *i.e.*, there will be pure rolling till

$t = 6$ s $\Rightarrow a \rightarrow q, r$

$b \rightarrow p$

$c \rightarrow s$

$$f = \frac{1}{2}ma = \frac{F}{3} = \frac{10t}{3} = 10 \text{ N} \Rightarrow t = 3 \text{ s}$$

$d \rightarrow q$

7. (a) $I = \frac{1}{2}\cdot\frac{1}{2}mr^2 + mr^2 = \frac{5}{4}mr^2 \rightarrow r$

(b) $I = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2 \rightarrow q$

(c) $I = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2 \rightarrow s$

(d) $I = \frac{1}{2}mr^2 \rightarrow p$

10 Gravitation

Introductory Exercise 10.1

1. $g' = g - R\omega^2 \cos^2 \phi$

$$= g - 6.37 \times 10^6 \times \frac{4\pi^2}{(8.64 \times 10^4)^2} \times \cos^2 45°$$

$$= g - \frac{2\pi^2 \times 6.37 \times 10^{-2}}{(8.64)^2}$$

$$= g - 0.0168 \text{ m/s}^2$$

$$\therefore \Delta g = g - g' = 0.0168 \text{ m/s}^2$$

2. $g' = g - R\omega^2 = \frac{3}{5} g$

$$\Rightarrow \frac{2}{5} g = R\omega^2$$

$$\text{or } \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$$

$$= \sqrt{\frac{4}{6400 \times 10^3}} = \frac{1}{\sqrt{1600 \times 10^3}}$$

$$= \frac{1}{40 \times 10\sqrt{10}} = 7.9 \times 10^{-4} \text{ rad/s}$$

3. $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = 0.64 g$

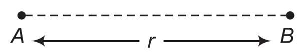
$$\Rightarrow 1 = 0.64\left(1 + \frac{h}{R}\right)^2$$

$$\text{or} \quad 0.8\left(1 + \frac{h}{R}\right) = 1$$

$$\Rightarrow 1 + \frac{h}{R} = \frac{5}{4}$$

$$\Rightarrow h = \frac{R}{4} = 1600 \text{ km}$$

4.



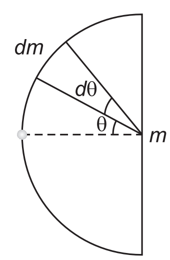
$$F = \frac{GM_A M_B}{r^2}$$

$$\Rightarrow a = \frac{F}{M_A} = \frac{GM_B}{r^2}$$

$$\text{For } F' = \frac{GM_A M_B}{r^4}$$

$$\Rightarrow a_{cc} = \frac{F'}{m_A} = \frac{GM_B}{r^4} = \frac{a}{r^2}$$

5. $F = \int dF \cos\theta = \int \frac{GMdm}{r^2} \cos\theta$



$$= \frac{GM}{r^2} \int_{-\pi/2}^{\pi/2} \frac{m}{\pi r} . r d\theta . \cos\theta$$

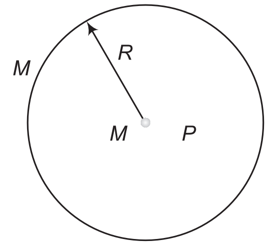
$$= \frac{GMm}{\pi r^2} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{2GMm}{\pi r^2}$$

$$= \frac{2GMm}{\pi\left(\frac{L}{\pi}\right)^2} = \frac{2\pi GMm}{L^2}$$

Introductory Exercise 10.2

1.



$$V = V_e + V_s = -\frac{GM}{R/2} - \frac{GM}{R} = -\frac{3GM}{R}$$

2. $\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m} = \frac{4\hat{\mathbf{i}}}{20 \times 10^{-3}} = 200 \text{ m/s}^2$

$= 200 \text{ N/kg } \hat{\mathbf{i}}$

3. $\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$

$= -(6xy\hat{\mathbf{i}} + 3x^2\hat{\mathbf{j}} + 3y^2 z\hat{\mathbf{j}} + y^3\hat{\mathbf{k}})$

4. Only the variation is given along x-axis, nothing is about y and z axis, so, the statement is false.

5. $\vec{\mathbf{E}} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right)$

$20(x + y) = -20[\hat{\mathbf{i}} + \hat{\mathbf{j}}]$

$\vec{\mathbf{F}} = M\vec{\mathbf{E}} = \frac{1}{2}[-20(\hat{\mathbf{i}} + \hat{\mathbf{j}})]$

$= -10(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$\Rightarrow \quad F = 10\sqrt{2}$ N

6. $W = \int dW = \int \vec{\mathbf{F}}.\vec{\mathbf{ds}} = \int m\vec{\mathbf{E}}.(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}})$

$= m\int E_x dx + m\int E_y dy$

$= 1\int_1^{-2} 2dx + 1\int_1^3 3dy$

$= 0$

Introductory Exercise 10.3

1. Escape velocity is given to overcome the potential barrier and just free the particle from a system, such that its total mechanical energy is just zero at infinity. So, the statement is true.

2. $K = \frac{GMm}{R} = \frac{GM}{R^2}.Rm = gmR.$

3. $Gm_1 m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2.$

$= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2}{m_2}v_1^2$

$= \frac{1}{2}m_1 v_1^2\left(1 + \frac{m_1}{m_2}\right)$

$\therefore \quad Gm_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}v_1^2\left(1 + \frac{m_1}{m_2}\right)$

or $\quad v_1 = \sqrt{\frac{2Gm_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}{1 + \frac{m_1}{m_2}}}$

$= m_2\sqrt{\frac{2G\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}{m_1 + m_2}}$

$= 10\sqrt{\frac{2 \times 6.67 \times 10^{-11}(2 - 1)}{20 + 30}}$

$= 10 \times 10^{-6}\sqrt{\frac{2 \times 6.67}{5}}$

$= \sqrt{\frac{8}{3}} \times 10^{-5}$ m/s

$= 1.63 \times 10^{-5}$ m/s

$v_2 = \frac{m_1}{m_2}v_1 = 2v_1$

$= 3.3 \times 10^{-5}$ m/s

4. $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - \frac{GMm}{R}$

$\Rightarrow \quad v = \sqrt{u^2 - \frac{2GM}{R}}$

$\therefore \quad v = \sqrt{u^2 - v_e^2}$

$= \sqrt{15^2 - (11.2)^2}$ km/s

$= 10$ km/s

5. (i) $\Delta U = \frac{GMm}{R} - \frac{GMm}{(1 + n)R}$

$= \frac{GMm}{R}\left(1 - \frac{1}{1 + n}\right)$

$= \frac{n}{n + 1}.\frac{GM}{R^2}.mR = \left(\frac{n}{n + 1}\right)mgR$

(ii) $K = \Delta U = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2ngR}{n + 1}}$

Introductory Exercise 10.4

1. $\Delta K = \Delta U$

$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{R} - 0 = \frac{GM}{R^2}Rm = mgR$

$\Rightarrow v = \sqrt{2Rg}$

$\therefore\ v = \sqrt{2 \times 6400 \times 10^3 \times 10}$

$= 8\sqrt{2} \times 10^3$ m/s $= 8\sqrt{2}$ km/s $= 11.2$ km/s

2. In planetary motion areal velocity, *i.e.*, angular momentum and total mechanical energy is conserved.

3. $\frac{mv_1^2}{2R} = \frac{GMm}{(2R)^2}$

$\Rightarrow \quad \frac{1}{2}mv_1^2 = \frac{GMm}{4R}$

and $\quad \frac{mv_2^2}{4R} = \frac{GMm}{(4R)^2}$

$\Rightarrow \quad \frac{1}{2}mv_2^2 = \frac{GMm}{8R}$

$\Rightarrow \quad \frac{K_1}{K_2} = \frac{1/4}{1/8} = \frac{2}{1} \Rightarrow 2:1$

$$\frac{U_1}{U_2} = \frac{\frac{-GMm}{2R}}{\frac{-GMm}{4R}} = \frac{2}{1} \Rightarrow 2:1$$

4. $T = \frac{2\pi r}{v}; \frac{mv^2}{r} = \frac{GMm}{r^2}$

$\Rightarrow v = \sqrt{\frac{GM}{r}}$

$\therefore\ T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = \frac{2\pi r\sqrt{r}}{\sqrt{G\rho \frac{4}{3}\pi R^3}}$

$\Rightarrow \quad T^2 = \frac{4\pi^2 r^3}{G\rho \frac{4}{3}\pi R^3}$

$\Rightarrow \quad \rho T^2 = \frac{3\pi r^3}{GR^3}$

For $r \simeq R, \rho T^2 = \frac{3\pi}{G} =$ constant

5. $\frac{m^2 v_o^2}{r} = \frac{GMm}{r^2}$

$\Rightarrow \quad v_o = \sqrt{\frac{GM}{r}}$

(a) $v = \sqrt{1.5}\,v_o = 1.22\,v_o$

While, $v_e = \sqrt{2}\,v_o = 1.41 v_o$, so, the satellite will not escape from the planet, rather it will revolve in elliptical orbit.

(b) As, $v_e = \sqrt{2}\,v_o$, while, $v = 2\,v_o$, *i.e.*, the satellite will escape.

AIEEE Corner

Subjective Questions (Level 1)

1. $a_1 = \frac{F}{m_1} = \frac{GM_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 2}{(1/2)^2}$

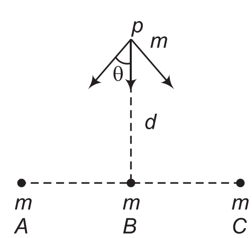
$= 8 \times \frac{20}{3} \times 10^{-11}$ m/s^2

$= 5.3 \times 10^{-10}$ m/s^2

$a_2 = \frac{F}{m_2} = \frac{GM_1}{r^2} = \frac{1}{2}.a_1$

$= 2.65 \times 10^{-10}$ m/s^2

2.

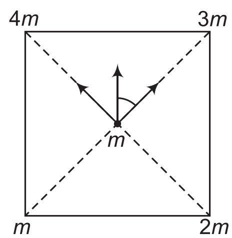


$F = \frac{Gm^2}{d^2} + 2.\frac{Gm^2}{(\sqrt{2}d)^2}.\cos 45°$

$= \frac{Gm^2}{d^2}\left(1 + \frac{1}{\sqrt{2}}\right)$

And the net force is directed along *PB*.

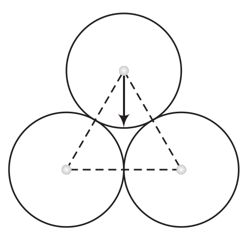
3.



$$F = 2 \cdot \frac{G.m.2m}{(a/\sqrt{2})^2} \cos 45°$$

$$= \frac{4\sqrt{2}\,Gm^2}{a^2}$$

4.



$$F = 2\,\frac{G.MM}{(2a)^2}\cos 30° = \frac{\sqrt{3}GM^2}{4a^2}$$

5. $F = \int_d^{d+l} \frac{G.M.\lambda dx}{x^2} = GM\lambda\left(-\frac{1}{x}\right)_d^{d+l}$

$$= GM\lambda\left(\frac{1}{d} - \frac{1}{d+l}\right) = \frac{GM\lambda l}{d(d+l)}$$

6. $g = \frac{GM}{R^2}$,

$$g' = \frac{G2M}{4R^2} = \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2} = 4.9 \text{ m/s}^2$$

7. (a) $g = \frac{GM}{R^2}$, $g' = \frac{GM}{(R+R)^2} = \frac{1}{4}\frac{GM}{R^2}$

$$= \frac{1}{4}g = 2.45 \text{ m/s}^2$$

(b) $g = \frac{GM}{R^2}$ and

$$g' = \frac{GM}{R^3}.r = \frac{GM}{R^3}(R - R/2)$$

$$= \frac{1}{2}\frac{GM}{R^2} = \frac{g}{2} = 4.9 \text{ m/s}^2$$

8. $\frac{GM}{R^3}(R - x) = \frac{GM}{(R+x)^2}$

$$\Rightarrow (R-x)(R+x)^2 = R^3$$

$$\Rightarrow (R^2 - x^2)(R+x) = R^3$$

or $R^3 + R^2x - x^2R - x^3 = R^3$

or $x^3 + x^2R - xR^2 = 0$

or $x^2 + Rx - R^2 = 0$

$$\therefore\ x = \frac{1}{2}[-R \pm \sqrt{R^2 + 5R^2}]$$

$$= \frac{1}{2}[-R + \sqrt{5}R]$$

$$\therefore\ x = \frac{(\sqrt{5}-1)}{2}R$$

9. $w_p = mg$ and $w_e = mg - m\omega^2 R$

$$\therefore\ w_e = mg\left[1 - \frac{\omega^2 R}{g}\right] = mg\left[1 - \frac{4\pi^2 R}{T^2 g}\right]$$

$$= 1000\left[1 - \frac{4 \times 6400 \times 10^3}{(86400)^2}\right]$$

$$= 1000\left[1 - \frac{4 \times 6.4 \times 10^{-2}}{(8.64)^2}\right]$$

$$= 1000 \times 0.997 = 997 \text{ N}$$

10. $g_{\text{apparent}} = g - \omega^2 R = 0$

$$\Rightarrow \omega = \sqrt{g/R} = \frac{2\pi}{T}$$

$$\therefore\ T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$$= 16\pi \times 10^2$$

$$= 5026.5 \text{ s} = 1.4 \text{ h}$$

and $\omega = \sqrt{g/R} = \sqrt{\frac{10}{6400 \times 10^3}}$

$$= \frac{1}{8 \times 10^2} = 1.25 \times 10^{-3} \text{ rad/s}$$

11. $g(h) = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = g - \omega^2 R$

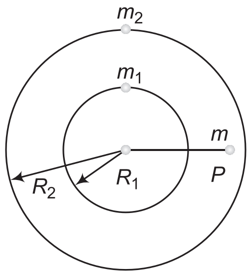
$$\therefore \omega^2 R = g\left[1 - \left(1 + \frac{h}{R}\right)^{-2}\right] = g\left[\frac{2h}{R}\right]$$

or $h = \frac{\omega^2 R^2}{2g} = \frac{4\pi^2 R^2}{2T^2 g} = \frac{2R^2}{T^2}$

$$= 2\left(\frac{6400 \times 10^3}{86400}\right)^2$$

$$= 2\left(\frac{640}{8.64}\right)^2 = 10.97 \times 10^3 \text{m} \simeq 11 \text{ km}$$

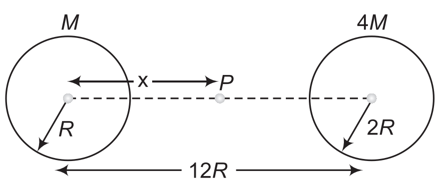
12.



There is no gravitational field at P due to outer shell, so it has no effect. Only m_1 has an effect at P. Thus,

$$F = \frac{Gm_1 m}{\left(\frac{R_1 + R_2}{2}\right)^2} = \frac{4Gm_1 m}{(R_1 + R_2)^2}$$

13.



(a) $\frac{GM}{x^2} = \frac{G.4M}{(12R - x)^2}$ gives field to be zero at P.

$$\therefore \quad \frac{12R - x}{x} = 2$$

$$\Rightarrow 12R = 3x \Rightarrow x = 4R$$

(b) At the surface of second sphere,

$$V = -\frac{G.4M}{2R} - \frac{GM}{10R}$$

$$= -\frac{GM}{R}\left(2 + \frac{1}{10}\right)$$

$$= -\frac{2.1\, GM}{R}$$

At a distance y from the smaller sphere,

$$V' = -\frac{GM}{y} - \frac{G4M}{(12R - y)}$$

$$= -\frac{GM}{y}\left(1 + \frac{4y}{12R - y}\right)$$

as, $V' = \frac{V}{2}$

$$\Rightarrow -\frac{GM}{y}\left[1 + \frac{4y}{12R - y}\right]$$

$$= \frac{1}{2}\left(-\frac{2.1\, GM}{R}\right)$$

$$\frac{12R + 3y}{y(12R - y)} = \frac{1.05}{R}$$

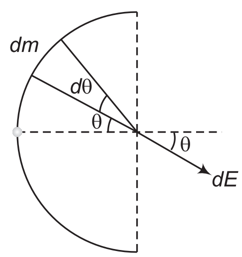
$$\therefore \quad 12R^2 + 3yR = 12.6yR - 1.05y^2$$

or $\quad 1.05y^2 - 9.6Ry + 12R^2 = 0$

$$\therefore \quad y = \frac{R}{2.1}[9.6 \pm \sqrt{(9.6)^2 - 4 \times 1.05 \times 12}]$$

$$= \frac{R}{2.1}[9.6 \pm 6.5] = 7.7R,\ 1.5\, R$$

14.



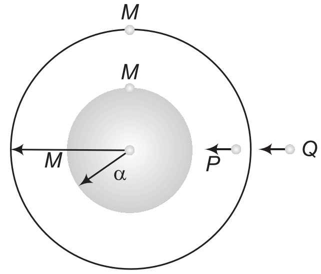
$$E = \int dE \cos\theta$$

$$= \int \frac{Gdm}{r^2}\cos\theta = \int_{-\pi/2}^{\pi/2} \frac{G}{r^2}.\lambda r d\theta.\cos\theta$$

$$= \frac{2G\lambda}{r} = \frac{2G}{r}.\frac{M}{\pi r} = \frac{2GM}{\pi r^2} = \frac{2GM}{\pi(L/\pi)^2}$$

$$= \frac{2\pi GM}{L^2}$$

15.



(a) $E_P = \dfrac{GM}{\left(\dfrac{3}{2}a\right)^2} = \dfrac{4GM}{9a^2}$ and

(b) $E_Q = \dfrac{G2M}{\left(\dfrac{5}{2}a\right)^2} = \dfrac{8GM}{25a^2}$

16. $M = \int dm = \int_0^a \rho . 4\pi r^2 dr = \int_0^a \dfrac{\rho_0 a}{r} . 4\pi r^2 dr$

$= 4\pi\rho_0 a \int_0^a r dr$

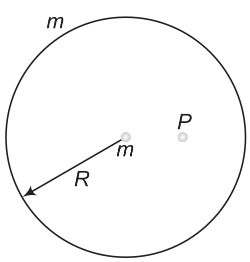
$= 4\pi\rho_0 \; a \times \dfrac{1}{2}a^2$

$= 2\pi\rho_0 a^3$

$E = \dfrac{GM}{(2a)^2} = \dfrac{G 2\pi\rho_0 a^3}{4a^2}$

$= \dfrac{\pi G\rho_0 a}{2}$

17.



$V_P = V_{\text{point}} + V_{\text{shell}}$

$= -\dfrac{Gm}{R/2} - \dfrac{Gm}{R}$

$= -\dfrac{3Gm}{R}$

18. $\dfrac{1}{2}mv^2 = \dfrac{1}{2}m\,(2\sqrt{gR})^2 - \dfrac{GMm}{R}$

$= \dfrac{1}{2}m\,4gR - gRm$

$= mgR$

$v = \sqrt{2Rg}$

19. (a) $2 \times \dfrac{1}{2}mu^2 = GM^2\left(\dfrac{2}{r} - \dfrac{1}{r}\right) = \dfrac{GM^2}{r}$

$u = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{6.67 \times 10^{-11} \times 10^{30}}{10^{10}}}$

$= 10^4\sqrt{66.7}$ m/s $= 8.17 \times 10^4$ m/s

$= 82$ km/s

(b) $2 \times \dfrac{1}{2}Mv^2 = GM^2\left(\dfrac{1}{2 \times 10^5} - \dfrac{1}{10^{10}}\right)$

$v = \sqrt{GM\dfrac{10^{10} - 2 \times 10^5}{2 \times 10^{15}}}$

$\simeq \sqrt{\dfrac{6.67 \times 10^{-11} \times 10^{30}}{2 \times 10^5}}$

$= 10^7\sqrt{\dfrac{6.67}{2}}$

$= 1.83 \times 10^7$ m/s $= 1.8 \times 10^4$ km/s

20. $h = \dfrac{v^2}{2g - v^2 / R}$

$= \dfrac{(10 \times 10^3)^2}{2 \times 9.81 - \dfrac{(10 \times 10^3)^2}{(6400 \times 10^3)}} = 2.5 \times 10^7$ m

$= 2.5 \times 10^4$ km

21. $\dfrac{1}{2}mv^2 = \dfrac{GMm}{R + R}$

$\Rightarrow v = \sqrt{\dfrac{GM}{R}} = \sqrt{Rg} = \sqrt{64 \times 10^5 \times 10}$

$= 8 \times 10^3$ m/s = 8 km/s

22. (a)

$U_A - U_B = \left(\dfrac{G \times 400 \times 1}{2} + \dfrac{G \times 100 \times 1}{8}\right)$

$-\left(\dfrac{G \times 400 \times 1}{8} + \dfrac{G \times 100 \times 1}{2}\right)$

$= 100G\left(2 + \dfrac{1}{8}\right) - 100G\left(\dfrac{1}{2} + \dfrac{1}{2}\right)$

$= 100G \times \dfrac{9}{8} = \dfrac{900}{8} \times 6.67 \times 10^{-11}$ J

$= 7.5 \times 10^{-9}$ J

(b) For $E = 0$,

$\dfrac{G \times 400}{x^2} = \dfrac{G \times 100}{(10 - x)^2}$

$\Rightarrow \dfrac{x}{10 - x} = 2 \Rightarrow x = \dfrac{20}{3}$ m, *i.e.*, the particle has to be given such a velocity, that it can travel from $2m(A)$ to $\frac{20}{3}$ m, then after it will reach at B

$$\therefore K = \Delta U = \left(\frac{G \times 400 \times 1}{2} + \frac{G \times 100 \times 1}{8}\right) - \left(\frac{G \times 400 \times 1}{20/3} + \frac{G \times 100 \times 1}{10/3}\right)$$

$$= 100\,G\left(2 + \frac{1}{8}\right) - 100\,G\left(\frac{6}{10} + \frac{3}{10}\right)$$

$$= 122.5\,G = 8.17 \times 10^{-9}\text{ J}$$

23. (a)

$$\text{TE} = \Delta U + K = \frac{GMm}{R} - \frac{GMm}{2R} + \frac{1}{2}mv^2$$

$$= \frac{GMm}{2R} + \frac{1}{2}\cdot\frac{GMm}{2R} = \frac{3}{4}\cdot\frac{GMm}{R} = \frac{3}{4}mgR$$

$$= \frac{3}{4} \times 2 \times 10^3 \times 10 \times 6400 \times 10^3$$

$$= \frac{3}{2} \times 6.4 \times 10^{10}\text{ J} = 9.6 \times 10^{10}\text{ J}$$

(b) $\text{TE}' = \Delta U' + K'$

$$= \frac{GMm}{R} - \frac{GMm}{3R} + \frac{1}{2}\frac{GMm}{3R}$$

$$= \frac{GMm}{R}\left(1 - \frac{1}{3} + \frac{1}{6}\right)$$

$$= \frac{5}{6}mgR = \frac{5}{6} \times 2 \times 10^3 \times 10 \times 6400 \times 10^3$$

$$= \frac{5}{3} \times 6.4 \times 10^{10}\text{ J} = 10.7 \times 10^{10}\text{ J}$$

$\therefore$ Energy needed

$$= (10.7 - 9.6) \times 10^{10}\text{ J} = 1.1 \times 10^{10}\text{ J}$$

24. (a) $F = \dfrac{GM^2}{4R^2} = \dfrac{mv^2}{R}$

(b) $v = \sqrt{\dfrac{GM}{4R}}$,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM/4R}} = \frac{4\pi R\sqrt{R}}{\sqrt{GM}}$$

(c) $E = \dfrac{GMM}{2R} = \dfrac{GM^2}{2R}$

25. (a) $\text{KE} = \dfrac{1}{2}mv^2 = \dfrac{r}{2}\dfrac{mv^2}{r}$

$$= \frac{r}{2}\cdot\frac{GMm}{r^2} = \frac{GMm}{2r}$$

$$\text{PE} = -\frac{GMm}{r}$$

$$\text{TE} = 2\,(\text{KE} + \text{PE}) = 2\left(\frac{GMm}{2r} - \frac{GMm}{r}\right)$$

$$= -\frac{GMm}{r}$$

(b) $\text{PE} = -\dfrac{GM\,2m}{r}$

$\therefore$ $\text{TE} = -\dfrac{2GMm}{r}$ and the satellites will start falling towards the centre of earth.

26. $T = \dfrac{2\pi R}{v}$

where, $\dfrac{mv^2}{R} = \dfrac{GMm}{R^2} \Rightarrow v = \sqrt{\dfrac{GM}{R}}$

and $\quad T = \dfrac{2\pi R^{3/2}}{\sqrt{GM}}$ as, $T \propto R^{3/2}$

$$\therefore \quad \frac{T_B}{T_A} = \left(\frac{R_B}{R_A}\right)^{3/2} = 2^{3/2} = 2\sqrt{2}$$

$$\Rightarrow \quad T_B = 56\sqrt{2}\text{ h}$$

$$\frac{v_B}{v_A} = \sqrt{\frac{R_A}{R_B}} = \frac{1}{\sqrt{2}}$$

$$T = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$$

$$\Rightarrow \quad \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T} = v$$

$$\therefore v_A = \frac{2\pi \times 10^4}{28}\text{ km/h} = 2244\text{ km/h}$$

$$v_B = \frac{v_A}{\sqrt{2}} = 1587\text{ km/h}$$

$$v_{\text{rel}} = v_A + v_B = 3831\text{ km/h}$$

27. (a) $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$, $v = \sqrt{\dfrac{GM}{r}}$

$$= \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{8400 \times 10^3}}$$

$$= 6.9 \times 10^3\text{ m/s} = 6.9\text{ km/s}$$

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 10^3 \times (6.9)^2 \times 10^6\text{ J}$

$$= 2.38 \times 10^{10}\text{ J}$$

(c) $U = -\frac{GMm}{r}$

$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^3}{8400 \times 10^3}$

$= -4.76 \times 10^{10}$ J

(d) $T = \frac{2\pi r}{v} = \frac{2\pi \times 8400}{6.9}$ s $= 2.12$ h

28. $\frac{Mv^2}{R/2} = \frac{GMm}{R^2}$

$\Rightarrow v = \sqrt{\frac{GM}{2R}}, T = \frac{2\pi R/2}{v} = \frac{\sqrt{2}\pi R\sqrt{R}}{\sqrt{GM}}$ ...(i)

For earth-sun system, $\frac{mv_0^2}{R} = \frac{GMm}{R^2}$

$\Rightarrow$ $v_0 = \sqrt{\frac{GM}{R}}$ and

$T_0 = \frac{2\pi R}{v_0} = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$ ...(ii)

From Eqs. (i) and (ii)

$T = \frac{1}{\sqrt{2}} \frac{2\pi R\sqrt{R}}{\sqrt{GM}} = \frac{T_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ yr $= 0.71$ yr

29. (a)

$E_1 = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GMm}{R}\left(1 - \frac{R}{R+h}\right)$

$= \frac{GMm}{R}\left(1 - \frac{6400}{7900}\right) = \frac{15}{79} \cdot \frac{GMm}{R}$

$E_2 = \frac{1}{2}mv^2 = \frac{r}{2} \cdot \frac{mv^2}{r} = \frac{r}{2} \cdot \frac{GMm}{r^2}$

$= \frac{GMm}{2(R+h)}$

$= \frac{GMm}{R}\left(\frac{R}{2(R+h)}\right) = \frac{GMm}{R}\left(\frac{6400}{2 \times 7900}\right)$

$= \frac{32}{79} \cdot \frac{GMm}{R}$

$\Rightarrow E_2 > E_1$

So, it requires less energy to take it to 1500 km above, than to put it in circular orbit.

(b) For, $E_1 = E_2 . \frac{GMm}{R}$

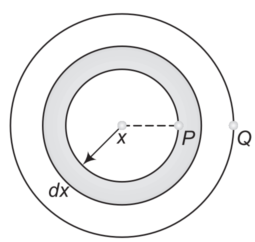
$\frac{h}{R+h} = \frac{GMm}{2(R+h)}$

$\Rightarrow h = R/2 = 6370/2 = 3185$ km. Thus for 3185 km height, energy required to lift and that required to put in orbit are equal.

(c) Similarly for 4500 km, energy required to lift will be more than that to put it in orbit.

Objective Questions (Level 1)

1. Centripetal force is required for circular motion, which is provided gravitational pull by earth on satellite.
2. In planetary motion total mechanical energy and angular momentum (areal velocity) are conserved.
3. Due to rotation and earth, $g_{\text{eff}} = g - R\omega^2 \cos^2 \phi$, where $\phi = 90°$ at poles, such that at poles $g_{\text{eff}} - g$, which do not depend on rotation of earth.
4. The particle has to be fired with escape velocity, which is independent of angle of projection.
5.



$V_Q = -\frac{GM}{R}, dV = -\frac{G.\frac{M}{\frac{4}{3}\pi R^3} . \frac{4}{3}\pi x^3}{x^2} dx$

$= -\frac{GM}{R^3} . x\,dx$

$\Delta V = +\frac{GM}{R^3}\int_R^r x dx = -\frac{MG}{R^3}\frac{1}{2}(R^2 - r^2)$

$\therefore V_P = V_Q + \Delta V = -\frac{GM}{R} - \frac{GM}{2R} + \frac{GM}{2R^3} . r^2$

$= -\frac{3GM}{2R} + \frac{GM}{2R^3} . r^2,$

is the equation of a parabola

(assuming earth to be of uniform density)

6. In electrostatics, $\oint \vec{\mathbf{E}} \cdot \vec{\mathbf{ds}} = \frac{q}{\varepsilon_0}$ and in gravitation, $-\oint \vec{\mathbf{g}} . \vec{\mathbf{ds}} = 4\pi Gm$

as $q = m$ and $\frac{1}{4\pi\varepsilon_0} = G$.

7. In centre of mass system, $\omega_1 = \omega_2$

$\Rightarrow \quad T_1 = T_2$

8. Field inside a shell is zero, so, a mass placed inside it do not experience any force.

9. $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GM}}$

$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}}$

$\therefore \quad T_2 = \frac{365}{2\sqrt{2}} = 129$ day

10. In planetary motion, $\frac{dA}{dt} = \text{constant}$

$\Rightarrow \quad \Delta A \propto \Delta T$

$\frac{\Delta A_{AB}}{\Delta A_{CD}} = \frac{T_{AB}}{T_{CD}}$

$\Rightarrow \quad \frac{A}{2A} = \frac{T_1}{T_2}$

$\Rightarrow \quad T_1 = 0.5\, T_2$

11. $T \propto \frac{1}{\sqrt{g}}$, $g_{\text{eff}} = g\left(1 - \frac{d}{R}\right)$

$\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\frac{g}{g\left(1 - \frac{d}{R}\right)}}$

$= \left(1 - \frac{d}{R}\right)^{-1/2} \simeq 1 + \frac{d}{2R}$

$1 + \frac{0.5}{100} = 1 + \frac{d}{2R}$

$\Rightarrow \quad d = \frac{R}{100} = 64$ km.

12. $g = \frac{GM}{R^2}$

$\Rightarrow \quad \frac{g}{G} = \frac{M}{R^2}$

13. $g_{\text{eff}} = g\left(1 + \frac{h}{R}\right)^{-2} = \frac{g}{100}$

$\Rightarrow \quad \left(1 + \frac{h}{R}\right)^2 = 100$

$\Rightarrow \quad 1 + \frac{h}{R} = 10$

or $\quad h = 9R$

14. $T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$, $\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{4R}{R}\right)^{3/2}$

$= 2^3 = 8$

$\Rightarrow \quad T_2 = 8T_1$

$\therefore \quad T_2 = (84) \times 8$ min

15. $g_{\text{eff}} = g - \omega^2 R = g/2$

$\Rightarrow \quad \omega^2 R = g/2$

$\Rightarrow \quad \omega = \sqrt{\frac{g}{2R}}$

$\therefore \; \omega = \sqrt{\frac{10}{2 \times 6400 \times 10^3}}$

$= \frac{1}{8\sqrt{2} \times 10^{+2}}$ rad/s

$= 8.8 \times 10^{-4}$ rad/s

16. At centre, $V_c = -\frac{3GM}{2R}$ and $V_h = -\frac{GM}{R + h}$

$V_h = \frac{1}{2} V_c$

$\frac{GM}{R + h} = \frac{3}{4} \cdot \frac{GM}{R}$

$\Rightarrow \quad 4R = 3\,(R + h)$

$\Rightarrow \quad h = R/3$

17. $U_c = -\frac{GMm}{R}$, $U_h = -\frac{GMm}{4R}$

$\Rightarrow \Delta U = \frac{GMm}{R} - \frac{GMm}{4R} = \frac{3}{4} . \frac{GMm}{R}$

$= \frac{3}{4} mgR$

18. $T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$ is independent of the radius of planet.

19. $v_e = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.7 \times 10^6}}$ m/s

$= 10^3 \sqrt{\frac{2 \times 6.67 \times 7.34}{17}}$ m/s

$= 2.4$ km/s $= 2.4 \times 10^3$ m/s

20. $g = \frac{GM}{R^2} = \frac{G \frac{4}{3} \pi R^3 \rho}{R^2}$

$= \frac{4}{3} \pi G \rho R \propto \rho R$

$\therefore \rho_1 R_1 = \rho_2 R_2 = 2\rho_1 R_2$

$\Rightarrow \quad R_2 = \frac{R_1}{2} = R/2$

21. $g_{\text{eff}} = g - \omega^2 R$ for $g_{\text{eff}} = 0$,

$\omega = \sqrt{g/R} = \sqrt{\frac{10}{6400 \times 10^3}}$ rad/s

$\omega = \frac{1}{800}$ rad/s, presently,

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{86400}$ rad/s

$\frac{\omega}{\omega_0} = \frac{86400}{1600\pi} = 17.2$

$\Rightarrow \quad \omega = 17.2\,\omega_0 = x\omega_0$

$\Rightarrow \quad x = 17.2$

22. $\omega_{\text{rel}} = \omega_s + \omega_e$

$\frac{1}{T_{\text{rel}}} = \frac{1}{T_s} + \frac{1}{T_e}$

$\Rightarrow \quad \frac{1}{T_s} = \frac{1}{T_{\text{rel}}} - \frac{1}{T_e} = \frac{1}{6} - \frac{1}{24}$

$= \frac{3}{24} = \frac{1}{8} \Rightarrow T_s = 8$ h

$\therefore \quad \omega_s = \frac{2\pi}{T_s} = \frac{2\pi}{8} = \frac{\pi}{4}$

23. According to Keplar's law, $T^2 \propto a^3$

24. External shell do not produce any field inside, so, $E = \frac{GM_1}{a^2}$

25. $t = \frac{T}{2} = \frac{84 \text{ min}}{2} = 42$ min.

26. $U = U_1 + U_2 + U_3$

$= 0 - \frac{Gm^2}{a} - \frac{2Gm^2}{a} = -\frac{3Gm^2}{a}$

$\Delta U = \frac{3Gm^2}{l} - \frac{3Gm^2}{2l} = \frac{3Gm^2}{2l}$

27. $v_e = \sqrt{\frac{2GM}{R}}, \frac{1}{2} mv^2$

$= \frac{GMm}{R} - \frac{GMm}{2R}$

$= \frac{GMm}{2R}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$

$\therefore \quad \frac{v}{v_e} = \frac{1}{\sqrt{2}}$

28. $U = -\frac{GMm}{r}, g_{2r} = -\frac{GM}{4r^2}$

$= -\frac{1}{4mr} \cdot \frac{GMm}{r} = \frac{U}{4mr}$

$\therefore \quad w = mg_{2r} = \frac{U}{4r}$

JEE Corner

Assertion and Reason

1. As PE is proportional to negative of inverse of distance that is why PE decreases with decrease in separation. Thus assertion and reason are true but not the correct explanation.
2. Same explanation as Answer 1
3. While going from A to B, PE first increases, becomes maximum at middle and again decreases. Assertion and reason are true but not correct explanation.
4. $E = -\frac{dV}{dx}$, at zero, slope of V - x graph E is zero if E is a function of x only, otherwise $\vec{\mathbf{E}} = -\vec{\nabla} V$. So, assertion is not always true.
5. $\frac{1}{2}mv^2 = \frac{GMm}{R}\left(1 - \frac{R}{R+h'}\right)$ and

$$\frac{1}{2}m4v^2 = \frac{GMm}{R}\left(1 - \frac{R}{R+h'}\right)$$

$$\therefore \quad 4 = \frac{1 - \frac{R}{R+h'}}{1 - \frac{R}{R+h}} = \frac{h'(R+h)}{h(R+h')}$$

$$\Rightarrow \quad 4hR + 4hh' = Rh' + hh'$$

$$\Rightarrow 4\,hR = Rh' - 3hh' = (R-3h)h'$$

$$\Rightarrow h' = 4h.\frac{R}{R-3h} \neq 4h$$

So, assertion is false.

6. In planetary motion, angular momentum of the planet is constant but it is not true for the system. So, assertion is false.
7. Only geo-stationary satellites are on equatorial plane, not all. So, assertion is false.
8. Reason is true for explanation of assertion.
9. Assertion is true but reason is false (and even Moscow is not at equator).
10. Both assertion and reason are true but not correct explanation.

As, $\quad U = -\frac{GMm}{r}$, $K = \frac{1}{2}mv^2$ and

$$E = U + K = -\frac{GMm}{2r}$$

11. $g = \frac{GM}{R^2}$, so as R is decreased, g increase, thus assertion is false.

Objective Questions (Level 2)

1. $E_1 = -\frac{GMm}{2R}, \; \frac{mv^2}{2R} = \frac{GMm}{4R^2}$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{2R}}$$

Finally, $E_2 = -\frac{GMm}{2R} + \frac{1}{2}.\frac{m}{2}.4v^2$

$$= -\frac{GMm}{2R} + mv^2$$

$$\therefore \quad \Delta E = E_2 - E_1 = -\frac{GMm}{2R} + mv^2 + \frac{GMm}{2R} = mv^2$$

$$= m.\frac{GM}{2R} = \frac{1}{2}.\,m\frac{GM}{R^2}\,R = \frac{1}{2}mgR$$

2. **See : Q. 5, P-92**
3. **See : Q. 5, P-123**
4. $F = \int dF\cos\theta = \int \frac{G.Mdm}{(\sqrt{3a^2 + a^2})^2}\cdot\frac{\sqrt{3}a}{2a}$

$$= \frac{\sqrt{3}}{8}\cdot\frac{GMm}{a^2}$$

5. $\frac{Mv^2}{R} = 2\,.\,\frac{GM^2}{(\sqrt{2}R)^2}\cos 45^\circ + \frac{GM^2}{4R^2}$

$$= \frac{GM^2}{R^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right)$$

$$v = \sqrt{\frac{GM}{R}\,\frac{4+\sqrt{2}}{4\sqrt{2}}} = \sqrt{\frac{GM}{R}\,\frac{2\sqrt{2}+1}{4}}$$

6. $\frac{1}{2}mv^2 = \frac{GMm}{R}\left(1-\frac{R}{r}\right)$

or $\frac{1}{2}mh^2 \frac{2GM}{R} = \frac{GMm}{R}\left(1-\frac{R}{r}\right)$

$\frac{R}{r} = 1-h^2$

or $r = \frac{R}{1-h^2}$

7. $g = \frac{GM'}{y^2}$

$= \frac{G}{y^2} \cdot \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi y^3 = \frac{4}{3}\pi\rho G y$

8. $N = mg_{\text{eff}} - \frac{mv^2}{R}$

$= m(g - \omega'^2 R) - \frac{mv^2}{R}$

$= mg - mg\frac{(\omega')^2 R}{g} - mg\frac{v^2}{Rg}$

$= mg\left[1 - \frac{\omega'^2 R}{g} - \frac{v^2}{Rg}\right];$

$(\omega')^2 R = \omega^2 R - 2\frac{v}{R}.\omega R$

$= \omega^2 R - 2v\omega$

$= mg\left[1 - \frac{(\omega R - 2v)\omega}{g} - \frac{v^2}{Rg}\right]$

9. According to Gauss' law field inside the cavity is uniform and depends upon the mass of sphere of radius a.

10. $U_c = -\frac{3GMm}{2R}$

$\therefore \quad \frac{1}{2}mv_e'^2 = \frac{3GMm}{2R}$

or $v_e' = \sqrt{\frac{3GM}{R}} = \sqrt{3\left(\frac{v_e^2}{2}\right)} = \sqrt{\frac{3}{2}}v_e$

11. $g = -\frac{GM}{r^{2.5}} = \frac{-dV}{dr}$

$\Rightarrow \quad v = GM\int_{\infty}^{r}\frac{dr}{r^{2.5}}$

$= +GM\left.\frac{r^{-2.5+1}}{-2.5+1}\right|_{\infty}^{r}$

$= \frac{-GM}{1.5r^{1.5}} = \frac{-2GM}{3r^{1.5}}$

12. $\frac{Mv^2}{R} = 2\cos 30°\frac{GM^2}{(\sqrt{3}R)^2}$

$= \frac{GM^2}{\sqrt{3}R^2}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{\sqrt{3}R}}$

13. $T^2 \propto R^3$ or $T \propto R^{3/2}$ or $T^{2/3} \propto R$ or $T^2 \propto \frac{1}{1/R^3}$

So, graph in (a), (b) and (c) are correct.

14. $\Delta W = \Delta U + \Delta K$

$\Rightarrow \Delta U = \Delta W - \Delta K$

$= -5.5 - \frac{1}{2} \times 1 \times 3^2$

$= -10$ J

$\Delta V = \frac{\Delta U}{m} = -10$ J/kg

15. $\frac{1}{2}mv^2 = 2.\frac{GMm}{\sqrt{2}R}$

$\Rightarrow v = \sqrt{\frac{2\sqrt{2}GM}{R}}$, it is independent of angle of projection.

16. $U_1 = 0 - \frac{Gm2m}{a} - \frac{Gm3m}{a} - \frac{G3m2m}{a}$

$= -\frac{5Gm^2}{a} - \frac{3\sqrt{2}m^2}{a}$

$= -\frac{Gm^2}{a}(5+3\sqrt{2})$

$U_2 = 0 - \frac{Gm3m}{a} - \frac{Gm2m}{a} - \frac{G3m2m}{a}$

$= -\frac{11Gm^2}{a}$

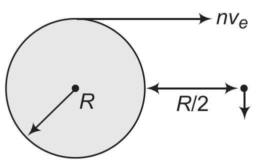
$\Delta W = \Delta U = \frac{Gm^2}{a}(11-5-3\sqrt{2})$

$= \frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$

17. $\frac{1}{2}mv^2 = -\frac{GMm}{R} + \frac{3GMm}{2R} = \frac{GMm}{2R}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$

18.



$Mnv_eR = mv\left(R + \frac{R}{2}\right)$

$\Rightarrow \quad v = \frac{2}{3}nv_e ; v_e = \sqrt{\frac{2GM}{R}}$

$\frac{GMm}{R} - \frac{GMm}{\frac{3}{2}R} = \frac{1}{2}m(nv_e)^2 - \frac{1}{2}mv^2$

or $\quad \frac{1}{3}.\frac{GMm}{R} = \frac{1}{2}mn^2v_e{}^2\left(1 - \frac{4}{9}\right)$

$\Rightarrow \quad \frac{1}{3}\frac{GMm}{R} = \frac{1}{2}mn^2\frac{2GM}{R}.\frac{5}{9}$

or $\quad n = \sqrt{\frac{3}{5}} = \sqrt{0.6}$

19. $2mu - mu = 3mv$

$\Rightarrow v = u/3 ; \frac{1}{2}mu^2 = \frac{3GMm}{2R} - \frac{GMm}{R}$

$u = \sqrt{\frac{Gm}{R}}$

$\Delta U(r) = +\frac{GM \times 3m}{2R^3}A^2 = \frac{1}{2} \times 3m \cdot v^2$

or $\frac{GM}{2R^3}A^2 = \frac{1}{2}.\frac{1}{9}.\frac{GM}{R}$

$\therefore \quad \frac{A^2}{R^2} = \frac{1}{9} \Rightarrow A = \frac{R}{3}$

20. $v_e = \sqrt{2gR}$ and

$\frac{v_{e_1}}{v_{e_2}} = \sqrt{\frac{R_1 g_1}{R_2 g_2}} = \sqrt{kg} = (kg)^{1/2}$

21. $F = -\frac{dU}{dr} = -6r^2 = -\frac{mv^2}{r}$

$mv^2 = 6r^3$ and $\frac{1}{2}mv^2 = 3r^3$

$\therefore \quad E = U + K = 2r^3 + 3r^3 = 5r^3$

$= 5 \times 5^3 = 625\text{J}$

22. $g_{\text{eff}} = g\left(1 + \frac{h}{R}\right)^{-2} = g\left(1 + \frac{1}{2}\right)^{-2} = \frac{4g}{9};$

$F = mg_{\text{eff}} = 200 \times \frac{4}{9} \times 10 = 889\text{ N}$

23. $mg\left(1 + \frac{x}{R}\right)^{-2} = \frac{mv^2}{R + x}$

$\Rightarrow \quad v = \sqrt{g(R + x)\frac{R^2}{(R + x)^2}}$

$= \sqrt{\frac{gR^2}{R + x}}$

24. $F_1 = \frac{G.\frac{4}{3}\pi R^3 \rho m}{4R^2} = \frac{1}{3}\pi\rho GmR$

$F_2 = \frac{G.\frac{4}{3}\pi R^3 \rho m}{4R^2} - \frac{G\frac{4}{3}\pi\frac{R^3}{8}\rho m}{\frac{9}{4}R^2}$

$= \frac{1}{3}\pi\rho GmR - \frac{2}{27}\pi\rho GmR$

$\therefore \frac{F_1}{F_2} = \frac{1/3}{\frac{1}{3} - \frac{2}{27}} = \frac{1}{1 - \frac{2}{9}} = \frac{9}{7}$

$\Rightarrow \quad \frac{F_2}{F_1} = \frac{7}{9}$

More than One Correct Options

1. $g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4}{3}\pi\rho GR$

$\Rightarrow g \propto R$

$\therefore g_1 : g_2 : g_3 = R_1 : R_2 : R_3 = 1 : \frac{1}{2} : \frac{1}{3}$

$\Rightarrow g_1/g_2 = \frac{1}{1/2} = 2$

$g_1/g_3 = \frac{1}{1/3} = 3$

$v_e = \sqrt{2gR}$

$\Rightarrow v_1 : v_2 : v_3 = \sqrt{g_1 R_1} : \sqrt{g_2 R_2} : \sqrt{g_3 R_3}$

$$= 1 : \frac{1}{2} : \frac{1}{3}$$

$$\therefore \frac{v_1}{v_2} = \frac{1}{1/2} = 2 \text{ and } \frac{v_1}{v_3} = \frac{1}{1/3} = 3$$

2. A geo-stationary satellite revolves at a height 36,000 km on equatorial plane with a time period of 24 h.

3. Particle performs SHM inside tunnel but not outside surface, so, if just performs periodic motion with amplitude, $(R + h)$

then

$$\frac{1}{2} mv^2 = \frac{3GMm}{2R} - \frac{GMm}{2R} = \frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}}$$

4. $E(x) = \dfrac{G2m}{(r-x)^2} - \dfrac{Gm}{x^2}$

$$= Gm\left[\frac{2}{(r-x)^2} - \frac{1}{x^2}\right] \text{ at } x = 0 \text{ and } r, E = \infty$$

So, field first decreases and then increases.

$$V(x) = -\left(\frac{G2m}{r-x} + \frac{Gm}{x}\right)$$

$$= -Gm\left[\frac{2}{r-x} + \frac{1}{x}\right]$$

So, V first increases and then decreases.

5. As there is no mass inside B, So. field between A to B is zero, *i.e.*, potential between A to B is constant.

$$\frac{Gm}{x^2} = \frac{G3m}{y^2}$$

Potential between B and C is not zero at any point.

6. On exchanging masses, potential at centre remains unchanged, but field strength increases and its direction also changes.

7. 1 will stop at minimum height while 2 will have a tangential velocity. (Reference : Q. 18; P 132); and particle 2 will rise lesser height than 1, as 2 do not looses its entire KE. And particle 1 returns earlier.

8. While going from A to B, KE increases and value of PE decreases, where speed is minimum at A. Angular momentum remains unchanged.

9. $E_1 = -\dfrac{GMm}{R} = U_1,$

$$U_2 = -\frac{GMm}{2R}$$

$$= \frac{E_1}{2}; E_2 = U_2 + K_2 = \frac{1}{2}U_2$$

$$= -\frac{GMm}{4R} = \frac{E_1}{4}$$

$$K_2 = -\frac{1}{2}U_2 = -E_2$$

10. $v \propto \sqrt{m}$, so, $v_2 = 2v_1$, $T \propto \dfrac{1}{v}$

$$\Rightarrow T_2 = \frac{1}{2} T_1 \text{ as } v \propto \sqrt{G} \text{ so with increase}$$

in G, v increases and T decreases.

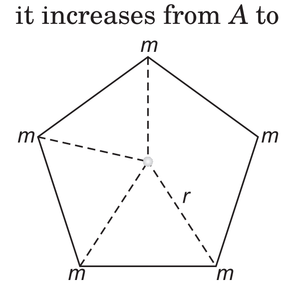
Match the Columns

1. $U(r) = -\dfrac{GMm}{R}$ for $r \geq R$, *i.e.*, PE decreases continuously from A to B. $a \to q$

$U(r) =$ *i.e.*, From B to C, PE remains constant $= -\dfrac{GMm}{R}$ for $r < R \Rightarrow b \to s$

Speed from B to C remains constant. $c \to s$ As between B to C there is no field, So acceleration becomes zero and it increases from A to B. $d \to r$

2.



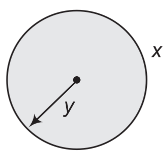
Gravitational field at centre is zero,
$a \to r$. Gravitational potential at centre
$= -\frac{5\,Gm}{r} \Rightarrow b \to s$

With four masses, $E = \frac{Gm}{r^2}$

$\Rightarrow c \to p$ and $V = -\frac{4Gm}{r}$

$\Rightarrow d \to s$

3.



$x = -\frac{GM}{y}$

(a) $g = -\frac{GM}{4y^2} = -\frac{-xy}{4y^2} = \frac{x}{4y} \to r$

(b) $V(y/2) = -\frac{3GM}{2y} + \frac{GM}{2y^3} \cdot \frac{y^2}{4}$

$= -\frac{GM}{y}\left(\frac{3}{2} - \frac{1}{8}\right) = -\frac{GM}{y} \cdot \frac{11}{8}$

$= \frac{11}{8}\, x \to s$

(c) $g(y/2) = g\left(1 - \frac{d}{R}\right) = g\,\frac{1}{2} = \frac{x}{8y} \to s$

(d) $V(2y) = -\frac{GM}{2y} = \frac{xy}{2y} = \frac{1}{2}\, x \to q$

4. $\Delta W = \Delta U = \frac{GMm}{R} - \frac{GMm}{2R}$

$= \frac{1}{2} \cdot \frac{GM}{R^2} . mR$

$= \frac{1}{2} mgR \Rightarrow a \to r$

$\frac{mv^2}{2R} = \frac{GMm}{4R^2} \Rightarrow \frac{1}{2} mv^2 = \frac{1}{4} \frac{GMm}{R}$

$= \frac{1}{4} mgR \Rightarrow b \to p$

$\Delta E = \frac{1}{2} \Delta U = \frac{1}{2}\left(\frac{GMm}{2R} - \frac{GMm}{3R}\right)$

$= \frac{1}{12} \frac{GMm}{R} = \frac{1}{12}\, mgR \Rightarrow c \to s$

$\Delta K = \Delta U = \frac{GMm}{R} - \frac{GMm}{2R}$

$= \frac{1}{12} \frac{GM}{R^2} . mR = \frac{1}{2}\, mgR \Rightarrow d \to r$

5. g = maximum at $r = R \Rightarrow a \to q$

$g = 0$ at $r = 0$ and $r = \infty \Rightarrow b \to p$

V = minimum at $r = 0 \Rightarrow c \to p$

$= -\frac{3GM}{2R}$

$V = 0 \text{ x } r = \infty \Rightarrow d \to s$

11 Simple Harmonic Motion

Introductory Exercise 11.1

1. $F(x) = x^2 - 6x$

$$F_r = F(x + \Delta x) - F(x)$$
$$= (x + \Delta x)^2 - 6(x + \Delta x) - x^2 + 6x$$
$$= 2x\,\Delta x - 6\Delta x$$
$$= -(6 - 2x)\,\Delta x$$
$$= -6\Delta x \quad \text{for} \quad x = 0$$
$$= -k\Delta x \quad \Rightarrow \quad k = 6\text{ N/m}$$

2. $v = \omega\sqrt{A^2 - x^2}$

$$= \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2}A\omega$$
$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\cdot m\cdot\frac{3}{4}A^2\omega^2$$
$$= \frac{3}{4}\cdot\frac{1}{2}mA^2\omega^2$$
$$= \frac{3}{4}\text{ (Translational energy)}$$
$$\text{PE} = \text{Translational energy} - \text{KE}$$
$$= \text{Translational energy} - \frac{3}{4}\text{(TE)}$$
$$= \frac{1}{4}\text{(Translational energy)}$$

3. $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \quad A = \sqrt{\frac{v^2}{\omega^2} + x^2}$$
$$= \sqrt{\left(\frac{2.18 \times 0.25}{2\pi}\right)^2 + (0.05)^2}$$
$$= \sqrt{0.0075 + 0.0025}$$
$$= \sqrt{0.01} = 0.1\text{ m}$$
$$= 10\text{ cm}$$

$$x = A\sin(\omega t + \theta)$$
$$\Rightarrow \quad \frac{x}{A} = \frac{5}{10} = \frac{1}{2}$$
$$= \sin(\omega t + \theta) = \sin\frac{\pi}{6}$$
$$\therefore \quad \phi = \omega t + \theta = \frac{\pi}{6}$$

4. (a) Maximum displacement

$= 15\text{ cm} =$ Amplitude.

(b) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{2}{150}} = \frac{2\pi}{5\sqrt{3}} = 0.73\text{ s}$$

(c) $\nu = \frac{1}{T} = 1.38\text{ Hz}$

(d) $\text{TE} = \frac{1}{2}m\,\omega^2A^2$

$$= \frac{1}{2} \times 2 \times \frac{150}{2} \times (0.15)^2 = 1.69\text{ J}$$

(e) $v_{\max} = A\omega = 0.15 \times \sqrt{\frac{150}{2}}$

$$= 0.15 \times 5\sqrt{3} = 0.75\sqrt{3} = 1.3\text{ m/s}$$

5. $A = 10\text{ cm}$

$8\text{ cm} = 10\sin\omega t_1$ and $6\text{ cm} = 10\sin\omega t_2$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1}{4 \times 10^3}} = \frac{\pi}{100}\text{ s}$$
$$\therefore \quad \omega t_1 = \sin^{-1}\left(\frac{4}{5}\right) = 53^\circ = \frac{53\pi}{180}$$
$$\text{and} \quad \omega t_2 = \sin^{-1}\left(\frac{3}{5}\right) = 37^\circ = \frac{37\pi}{180}$$
$$\therefore \quad t_1 - t_2 = \frac{1}{\omega}\left(\frac{53\pi}{180} - \frac{37\pi}{180}\right)$$

$$= \frac{T}{2\pi} \times \frac{16\pi}{180} = \frac{8T}{180} = \frac{2T}{45} = \frac{2\pi}{4500} \text{ s}$$

$$= 1.4 \times 10^{-3} \text{ s}$$

6. (a) $\text{TE} = \frac{1}{2} m\,\omega^2 A^2$

$$= \frac{1}{2} \times 2 \times \left(\frac{\pi}{4}\right)^2 \cdot (1.5)^2$$

$$= \frac{\pi^2}{16} \times \frac{9}{4} = \frac{9\pi^2}{64} = 1.4 \text{ J}$$

(b) $0.5 = 1.5 \sin\left(\frac{\pi t_1}{4} + \frac{\pi}{6}\right)$

$$\Rightarrow \quad \frac{\pi t_1}{4} + \frac{\pi}{6} = \sin^{-1}\left(\frac{1}{3}\right)$$

$$0.75 = 1.5 \sin\left(\frac{\pi t_2}{4} + \frac{\pi}{6}\right)$$

$$\Rightarrow \quad \frac{\pi t_2}{4} + \frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \quad t_2 = 0$$

7. (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{100}}$

$$= \frac{2\pi}{10}\sqrt{0.2} = 0.28 \text{ s}$$

(b) $(F_{\text{actual}})_{\text{max.}} = -100 \times \frac{4}{100} + 10 \times \left(\frac{4}{100}\right)^2 = -4 + \frac{16}{1000}$

$(F_{\text{approx.}})_{\text{max.}} = -4$

$$\% \text{ Error} = \frac{-4 + \frac{16}{1000} + 4}{4} \times 100\% = \frac{4}{10}\%$$

$$= 0.4\%$$

Introductory Exercise 11.2

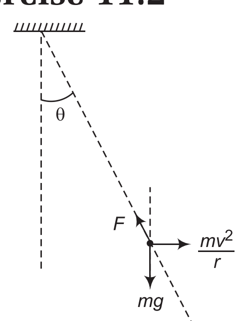
1. At equilibrium,

$$F \sin\theta = \frac{mv^2}{r}$$

$$F \cos\theta = mg$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

or $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$



$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{r^2}\right)^{1/2}}}$$

2. (a) $g_{\text{eff}} = g + a$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g + a}}$$

(b) $g_{\text{eff}} = g - a$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g - a}}$$

(c) $g_{\text{eff}} = 0 \Rightarrow T \to \infty$

(d) $g_{\text{eff}} = \sqrt{g^2 + a^2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}}$$

3. $T \propto l^{1/2} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

$$\Rightarrow \quad \Delta t = \frac{1}{2}\alpha t \Delta\theta$$

∴ With increase in temperature, pendulum clock becomes slow.

$$\Delta t = \frac{1}{2} \times 12 \times 10^{-6} \times 86400 \times 20$$

$$= 1.2 \times 8.64 \text{ s}$$

$$= 10.37 \text{ s}$$

4. $g_{\text{eff}} = \frac{W - B}{m} = \frac{\rho - \sigma}{\rho} g$

$$= \left(1 - \frac{\sigma}{\rho}\right) g = \left(1 - \frac{1}{10}\right) g = \frac{9}{10} g$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$\Rightarrow T_2 = T_1\sqrt{\frac{g_1}{g_2}}$$

$$= T\sqrt{\frac{g}{9/10\, g}}$$

$$= T\frac{\sqrt{10}}{3} = \sqrt{\frac{10}{9}}\, T$$

Introductory Exercise 11.3

1. $\dfrac{1}{k_{eff}} = \dfrac{1}{2k} + \dfrac{1}{k+k} = \dfrac{2}{2k} = \dfrac{1}{k}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}}$

2. $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.2}{80}}$

$= \dfrac{2\pi}{\sqrt{400}} = \dfrac{2\pi}{20} = \dfrac{\pi}{10} = 0.314\ \text{s}$

3. $v = \dfrac{mv_o}{M+m} \Rightarrow \dfrac{1}{2}(M+m)v^2 = \dfrac{1}{2}kA^2$

$\Rightarrow \quad A = v\sqrt{\dfrac{M+m}{k}}$

$\therefore \quad A = \dfrac{mv_o}{M+m}\sqrt{\dfrac{M+m}{k}}$

$= \dfrac{mv_o}{\sqrt{k(M+m)}}$

4. $T_1 = 2\pi\sqrt{\dfrac{m}{k}}; \dfrac{1}{k_{eff}} = \dfrac{1}{3k} + \dfrac{1}{3k+3k}$

$= \dfrac{1}{3k} + \dfrac{1}{6k}$

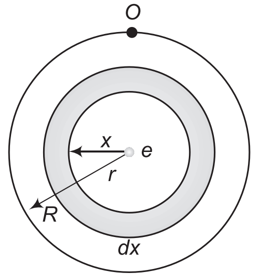
$= \dfrac{2+1}{6k} = \dfrac{1}{2k}$

$T_2 = 2\pi\sqrt{\dfrac{m}{2k}} = \dfrac{1}{\sqrt{2}}T_1,$

i.e., time period becomes $\dfrac{1}{\sqrt{2}}$ times.

Introductory Exercise 11.4

1.



$I_e = \int dI_c = \int x^2 dm = \int x^2 \cdot \sigma\, 2\pi x dx$

$= 2\pi\sigma\int_r^R x^3 dx = \dfrac{\pi\sigma}{2}(R^4 - r^4)$

$m = \int dm = \int \sigma \cdot 2\pi x dx = 2\pi\sigma\int_r^R x dx$

$= \pi\sigma(R^2 - r^2)$

$\therefore\ I_0 = I_c + mR^2 = \dfrac{\pi\sigma}{2}(R^4 - r^4) + \pi\sigma R^2(R^2 - r^2)$

$\therefore\ T = 2\pi\sqrt{\dfrac{I}{mgl}}$

$= 2\pi\sqrt{\dfrac{\dfrac{\pi\sigma}{2}(R^4 - r^4) + \pi\sigma R^2(R^2 - r^2)}{\pi\sigma(R^2 - r^2)g \cdot R}}$

$= 2\pi\sqrt{\dfrac{\dfrac{1}{2}(R^2 + r^2) + R^2}{Rg}}$

$= 2\pi\sqrt{\dfrac{\dfrac{3R}{2} + \dfrac{r^2}{2R}}{g}}$

$\therefore\ l_{eff} = \dfrac{3R}{2} + \dfrac{r^2}{2R},$

For $r \to 0 \Rightarrow l_{eff} = \dfrac{3R}{2}$

and for $r \to R \Rightarrow l_{eff} = 2R$

2. Here, $l_{eff} = 35\ \text{cm}$

and $l = 20\ \text{cm}$

As, $T = 2\pi\sqrt{\dfrac{I}{mgl}} = 2\pi\sqrt{\dfrac{l_{eff}}{g}}$

$\Rightarrow \dfrac{I}{mgl} = \dfrac{l_{eff}}{g} \Rightarrow I = ml\, l_{eff}$

$\therefore\ 200\ \text{g} \times 20\ \text{cm} \times 35\ \text{cm}$

$= 1.4 \times 10^5\ \text{g-cm}^2$

Introductory Exercise 11.5

1. Here, $a_1 = 4.0$ cm and $a_2 = 3.0$ cm

(a) For, $\Delta\phi = 0$, $a = a_{max} = a_1 + a_2$

$= 4.0 + 3.0 = 7.0\ \text{cm}$

(b) For, $\Delta\phi = 60°$,

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi}$

$= \sqrt{4^2 + 3^2 + 12} = \sqrt{37} = 6.1\ \text{cm}$

(c) For, $\Delta\phi = 90°$, $a = \sqrt{a_1^2 + a_2^2}$
$= \sqrt{4^2 + 3^2} = 5.0\,\text{cm}$

(d) For, $\Delta\phi = 180°$, $a = a_{min} = a_1 - a_2$
$= 4.0 - 3.0 = 1.0\,\text{cm}$

2. $x = x_1 + x_2 = 4\sin(100\pi t) + 3\sin\left(100\pi t + \frac{\pi}{3}\right)$

$$= \sqrt{4^2 + 3^2 + 2\times 4\times 3\cos\frac{\pi}{3}}\ \sin\left[100\pi t + \frac{3\sin\frac{\pi}{3}}{4 + 3\cos\frac{\pi}{3}}\right]$$

$$= \sqrt{37}\sin\left(100\pi t + \frac{3\sqrt{3}}{11}\right)$$

(a) At $t = 0$, $x = 4\sin 0 + 3\sin\frac{\pi}{3}$
$= \frac{3\sqrt{3}}{2}$ units

(b) $v_{max} = a\omega = 100\pi\sqrt{37}$ unit

(c) $a_{max} = \omega^2 a = (100\pi)^2\sqrt{37}$ unit

AIEEE Corner

Subjective Questions (Level 1)

1. (a) $20\times 10^{-3} g = k\times 7\times 10^{-2}$

$\Rightarrow k = \frac{2g}{7} = \frac{20}{7} = 2.8$ N/m as $\Delta mg = k\Delta x$

(b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50\times 10^{-3}}{20/7}}$
$= 2\pi\sqrt{17.5\times 10^{-3}} = 0.84$ s

2. $k = \frac{\Delta m\cdot g}{\Delta x} = \frac{\Delta F}{\Delta x}$
$= \frac{9}{0.05} = 180$ N/m

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{F}{k\cdot g}}$
$= 2\pi\sqrt{\frac{27}{1800}} = 0.78$ s

3. $v_{max} = A\omega = 2\pi A\nu$
$= 2\pi\times 8\times 10^{-3}\times 2$ m/s
$= 3.2\,\pi$ cm/s $= 0.101$ m/s

$a_{max} = A\omega^2 = 4\pi^2\nu^2 A$
$= 4\times\pi^2\times 4\times 8\times 10^{-3}$
$= 1.26\,\text{m/s}^2$

$F_{max} = kA = ma_{max} = 0.5\times 1.264$
$= 0.632$ N

4. $\omega = \frac{A\omega^2}{A\omega} = \frac{a_{max}}{v_{max}} = \frac{8\pi}{1.6} = 5\pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi\times 1.6}{8\pi} = 0.4$ s

$A = \frac{v_{max}}{\omega} = \frac{1.6}{5\,\pi}\pi = 0.102$ m

5. $v = \omega\sqrt{a^2 - x^2} = 2\sqrt{(0.2)^2 - (0.05)^2}$
$= 2\sqrt{0.04 - 0.0025}$
$= 0.39$ m/s

$a = -\omega^2 x = -2^2\times 0.05 = -0.2\,\text{m/s}^2$

For, $x = 0$, $v = v_{max} = A\omega = 0.2\times 2$
$= 0.4$ m/s

and $a = a_{min} = 0$

6. $x = A\sin(\omega t + \delta)$

$\Rightarrow \frac{A}{2} = A\sin(0 + \delta)$

$\Rightarrow \sin\delta = \frac{1}{2}$

or $\delta = \frac{\pi}{6}$

7. $\theta = \frac{\pi}{10}\sin\left(\frac{2\pi}{0.05}t + \delta\right)$

$\Rightarrow \frac{\pi}{10} = \frac{\pi}{10}\sin(0 + \delta)$

$\Rightarrow \sin\delta = 1$

or $\delta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10} \sin\left(\frac{2\pi}{0.05}t + \frac{\pi}{2}\right)$

$= \frac{\pi}{10} \cos(40\pi t)$

8. $x = 5 \sin\left(20t + \frac{\pi}{3}\right)$

(a) $v = 0 \Rightarrow \cos\left(20t + \frac{\pi}{3}\right) = 0$

$\Rightarrow \quad 20t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{120}$ s

(b) $a = 0 = \sin\left(20t + \frac{\pi}{3}\right) = \sin \pi$

$\Rightarrow \quad 20t = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, t = \frac{\pi}{30}$ s

(c) $v = v_{max}$ for, $a = 0$ *i.e.*, at $t = \frac{\pi}{30}$

9. $x = A \sin\left(\frac{2\pi}{T}t\right)$

and $\quad v = A \cdot \frac{2\pi}{T} \cos\left(\frac{2\pi}{T}t\right)$

$2 = A \cdot \frac{2\pi}{16} \cos\left(\frac{2\pi}{16} \cdot 2\right)$

or $\frac{16}{\pi} = A \cos\frac{\pi}{4} \Rightarrow A = \frac{16\sqrt{2}}{\pi} = 7.2$ m

10. $x = A \sin \omega t$

(a) $x(4) = A \sin\frac{2\pi}{8} \times 4 = 0,$

So, distance travelled is $2A$. In the next 4s it again travels $2A$ but from other side.

(b) $x(2) = A \sin\frac{2\pi}{8} \times 2 = A,$

i.e., distance is travelled A in first 2s and again A in next 2s but from other side.

11. (a) $A = 0.08$ m

(b) $T = 4$s, $\omega = \frac{2\pi}{T} = \frac{\pi}{2}$ s^{-1} $= 1.57$ rad/s

(c) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$= \frac{4 \times \pi^2 \times 0.8}{4^2} = 1.97$ N/m

(d) $v = A\omega\cos\omega t = 0.08 \times 1.57 \cos\left(\frac{\pi}{2} \times 1\right)$

$= 0$

(e) $a = \omega^2 x = \omega^2 A = (1.57)^2 \times 0.08$ m/s^2

$= 0.197$ m/s^2

12. (a) $u = \omega\sqrt{A^2 - x_1^2}$ and $v = \omega\sqrt{A^2 - x_2^2}$

$\Rightarrow \frac{u}{v} = \sqrt{\frac{A^2 - x_1^2}{A^2 - x_2^2}}$

$\frac{u^2}{v^2} = \frac{A^2 - x_1^2}{A^2 - x_2^2}$

$\Rightarrow \quad u^2(A^2 - x_2^2) = v^2(A^2 - x_1^2)$

$\Rightarrow \quad (u^2 - v^2) A^2 = (u^2 x_2^2 - v^2 x_1^2)$

$\Rightarrow \quad A = \left[\frac{v^2 x_1^2 - u^2 x_2^2}{v^2 - u^2}\right]^{\frac{1}{2}}$

(b) $u_1 = \omega\sqrt{A^2 - x_1^2}; u_2 = \omega\sqrt{A^2 - x_2^2}$

$\Rightarrow \quad A = \left[\frac{u_2^2 x_1^2 - u_1^2 x_2^2}{u_2^2 - u_1^2}\right]^{\frac{1}{2}}$

$\Rightarrow \quad \omega = \frac{u_1}{\sqrt{A^2 - x_1^2}}$

$= \frac{u_1}{\left[\frac{u_2^2 x_1^2 - u_1^2 x_2^2}{u_2^2 - u_1^2} - x_1^2\right]^{1/2}}$

$= \frac{u_1\sqrt{u_2^2 - u_1^2}}{[u_1^2 x_1^2 - u_1^2 x_2^2]^{\frac{1}{2}}}$

$= \sqrt{\frac{u_2^2 - u_1^2}{x_1^2 - x_2^2}}$

$\Rightarrow \quad T = \frac{2\pi}{\omega}$

$= 2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}}$

13. $E_0 = \frac{1}{2}kx_0^2$

(a) $U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2$

$$= \frac{1}{4} \cdot \frac{1}{2} k x_0^2 = \frac{1}{4} E_0$$

$$k = E_0 - U = E_0 - \frac{1}{4} E_0 = \frac{3}{4} E_0$$

(b) $k = U = \frac{1}{2} E_0 = \frac{1}{2} \cdot \frac{1}{2} k x_0^2$

$$= \frac{1}{2} k \left(\frac{x_0}{\sqrt{2}} \right)^2 = \frac{1}{2} k x^2$$

$$\Rightarrow \quad x = \frac{x_0}{\sqrt{2}}$$

14. At equilibrium, $mg = kx_0$

$$\Rightarrow \quad x_0 = \frac{mg}{k}$$

At a distance y below equilibrium position, spring energy and gravitational energy, is given by

$$U = U_s + U_g = \frac{1}{2} k (y + x_0)^2 - mgy$$

$$= \frac{1}{2} k y^2 + k y x_0 + \frac{1}{2} k x_0^2 - mgy$$

$$= \frac{1}{2} k y^2 + k \cdot y \cdot \frac{mg}{k} + \frac{1}{2} k x_0^2 - mgy$$

$$= \frac{1}{2} k y^2 + \frac{1}{2} k x_0^2$$

If equilibrium position is taken as reference frame, then $\frac{1}{2} k x_0^2$ can be considered to be zero, such that

$$U = \frac{1}{2} k y^2$$

15. At mean position,

$$\frac{1}{2} k d^2 = \frac{1}{2} (m_1 + m_2) v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k d^2}{m_1 + m_2}} = v_{max}$$

$$\therefore \quad v_{max} = A\omega = A \sqrt{\frac{k}{m_1}}$$

$$\Rightarrow \quad A = \sqrt{\frac{m_1}{k}} \cdot v_{max}$$

$$= \sqrt{\frac{m_1}{k}} \sqrt{\frac{k d^2}{m_1 + m_2}}$$

or $$A = d \sqrt{\frac{m_1}{m_1 + m_2}}$$

16. (a) At equilibrium, $F = kx_0$

$$\Rightarrow \quad x_0 = \frac{F}{k} = A \text{ and } T = 2\pi \sqrt{\frac{M}{k}}$$

(b) $W = \int dW = \int_0^{x_0} (F - kx) \, dx$

$$= F x_0 - \frac{1}{2} k x_0^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k} = U$$

(c) $k = F x_0 - W = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$

17. (a) $x_0 = \frac{F}{k} = \frac{10}{100} = 0.1 \text{ m} = 10 \text{ cm}$

(b) $E = U + K = \frac{1}{2} k x_0^2 + \frac{1}{2mv^2}$

$$= \frac{1}{2} \times 100 \times (0.1)^2 + \frac{1}{2} \times 1 \times 2^2 = 2.5 \text{ J}$$

(c) $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{1}{100}} = \frac{\pi}{5} \text{ s}$

(d) $\frac{1}{2} k (A)^2 = \frac{1}{2} m v^2$

$$\Rightarrow \quad A = v \sqrt{\frac{k}{m}}$$

$$= 2 \sqrt{\frac{100}{1}} = 20 \text{ cm}$$

(e) $U = \frac{1}{2} k (x_0 + A)^2$

$$= \frac{1}{2} \times 100 \times (0.3)^2 = 4.5 \text{ J}$$

(f) $U = \frac{1}{2} k (A - x_0)^2$

$$= \frac{1}{2} \times 100 \times (0.1)^2 = 0.5 \text{ J}$$

18. $K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2m} \omega^2 A^2$

$$\Rightarrow \omega = \sqrt{\frac{2k}{mA^2}} = \sqrt{\frac{2 \times 8 \times 10^{-3}}{0.1 \times (0.1)^2}}$$

$$= 4 \text{ rad/s}$$

$$x = A \sin \left(\omega t + \frac{\pi}{4} \right)$$

$$= 0.1 \sin \left(4t + \frac{\pi}{4} \right)$$

19. $U = 10 + (x - 2)^2$

$\Rightarrow \quad F = -\dfrac{dU}{dx} = -2(x-2) = -2x + 4$

$F_r = -2\Delta x$

$\Rightarrow \quad k = 2$

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2}{2}} = 1 \text{ rad/s}$

(b) At mean position

$U = 10 \text{ J}$ and $K = 26 - 10 = 16 \text{ J}$

At extreme position

$U = 26 \text{ J}$ and $K = 0$

(c) $v_{\max} = \sqrt{\dfrac{2K}{m}}$

$= \sqrt{\dfrac{2 \times 16}{2}} = 4 \text{ m/s}$

$= A\omega$

$\Rightarrow \quad A = 4 \text{ m}$

(d) $U_{\max} = 26 = 10 + (x_1 - 2)^2$

$\Rightarrow \quad x_1 = 4 + 2 = 6 \text{ m}$

$U_{\max}, \dfrac{dU}{dx} = 0 = -2(x_2 - 2)$

$\Rightarrow \quad x_2 = 2 \text{ m}$

$\therefore$ particle oscillates between

$x = 2 \text{ m to } 6\text{m}$

20. $T' = \dfrac{T_1}{2} + \dfrac{T_2}{2}$

$= \dfrac{1}{2} \times 2\pi\sqrt{\dfrac{l/4}{g}} + \dfrac{1}{2} \times 2\pi\sqrt{\dfrac{l}{g}}$

$= \left(\dfrac{\pi}{2} + \pi\right)\sqrt{\dfrac{l}{g}}$

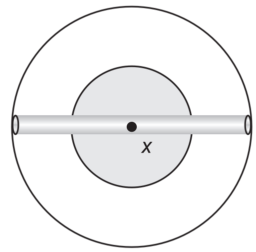
$= \dfrac{3\pi}{2}\sqrt{\dfrac{l}{g}} = \dfrac{3}{4} \cdot 2\pi\sqrt{\dfrac{l}{g}} = \dfrac{3}{4}\, T$

21. $K_{eq} = 2K + 2K = 4K$ and $T \propto \dfrac{1}{\sqrt{K}}$

$\therefore \quad \dfrac{T'}{T} = \sqrt{\dfrac{K}{4K}} = \dfrac{1}{2}$

$\Rightarrow \quad T' = \dfrac{T}{2}$

22.



$g(x) = \dfrac{G \cdot \frac{4}{3}\pi x^3 \rho m}{m x^2} = \dfrac{4}{3}\pi\rho G x$

$F(x) = -mg(x) = -\dfrac{4}{3}\pi\rho G m x$

$= -kx \Rightarrow k = \dfrac{4}{3}\pi\rho G m$

$\Rightarrow F \propto -x$

i.e., the particle will execute SHM

$T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{3}{4\pi\rho G}}$

$= 2\pi\sqrt{\dfrac{1}{\frac{4}{3}\pi \cdot \dfrac{M}{\frac{4}{3}\pi R^3} G}} = 2\pi\sqrt{\dfrac{R^3}{MG}}$

23. $g'(2R) = \dfrac{GM}{4R^2} = \dfrac{g}{4}$;

$T = 2\pi\sqrt{\dfrac{l}{g}}$

$= 2\pi\sqrt{\dfrac{4l}{g}} = 2\pi\sqrt{\dfrac{4 \times 1}{\pi^2}} = 2\pi \times \dfrac{2}{\pi} = 4 \text{ s}$

24. $mg = kx_0$; $F_r = F(x + \Delta x) - F(x)$

$= k(x + \Delta x) - kx = k\Delta x$

$\Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}}$

25. $F_r = -(k\Delta x + A\Delta x \sigma g)$

$= -(k + A\sigma g)\,\Delta x$

$T = 2\pi\sqrt{\dfrac{m}{k + A\sigma g}}$

$= 2\pi\sqrt{\dfrac{10}{100 + 20 \times 10^{-4} \times 10^3 \times 10}}$

$= \dfrac{2\pi}{\sqrt{12}} = \dfrac{\pi}{\sqrt{3}} = 1.8\text{s}$

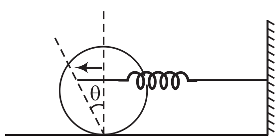
26. $T_A = 2\pi\sqrt{\dfrac{I}{mgl}} = 2\pi\sqrt{\dfrac{\frac{1}{3}md^2}{mg\,\frac{d}{2}}}$

$= 2\pi\sqrt{\dfrac{2d}{3g}}$

and $T_B = 2\pi\sqrt{\dfrac{d}{g}}$

$\Rightarrow \quad \dfrac{T_A}{T_B} = \sqrt{\dfrac{2}{3}} = 0.816$

27.



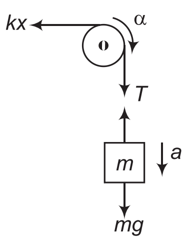
$\tau = I\alpha$

$k \cdot r\theta \cdot r = -\left(\dfrac{1}{2}mr^2 + mr^2\right)\alpha$

$\alpha = -\dfrac{2k}{3m}\theta = -\omega^2\theta$

$\Rightarrow \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{3m}{2k}}$

28.



$(kx - T)\,r = I\alpha \Rightarrow kx - T = \dfrac{I}{r}\cdot\dfrac{a}{r}$

and $\quad T - mg = ma$

$kx - mg = \left(\dfrac{I}{r^2} + m\right)a$

$\therefore \; -k\Delta x = \left(\dfrac{I}{r^2} + m\right)a$

$\nu = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m + \dfrac{I}{r^2}}}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{20}{0.5 + 0.6 \times 5}}$

$= \dfrac{1}{2\pi}\sqrt{\dfrac{20}{3.5}} = \dfrac{1}{\pi\sqrt{0.7}} = 0.38 \text{ Hz}$

29. $x = A\sin\omega t$ and $y = A\sin\left(2\,\omega t + \dfrac{\pi}{2}\right)$

$= A\cos 2\,\omega t$

$= A\,(1 - 2\sin^2\omega t) = A\left(1 - \dfrac{2x^2}{A^2}\right)$

$\therefore \; y = A - \dfrac{2}{A}x^2 \rightarrow$ equation of parabola.

30. $x = A\sin\left(\omega t - \dfrac{\pi}{3}\right) + A\sin\omega t$

$+ A\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$= A\sin\omega t + 2A\sin\omega t\cdot\cos\dfrac{\pi}{3}$

$= 2A\sin\omega t$

So, resultant amplitude is 2 A.

31. $A = \sqrt{A^2 + A^2 + 2AA\cos\theta}$

$= 2A\cos\dfrac{\theta}{2} \Rightarrow \cos\dfrac{\theta}{2} = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\therefore \; \Delta\phi = \theta = \dfrac{2\pi}{3}$

32. $x = x_1 + x_2 = 2\sin 100\pi t$

$+ 2\sin\left(120\pi t + \dfrac{\pi}{3}\right)$

(a) $x = 2\sin 1.25\,\pi + 2\sin\left(1.5\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\left(\pi + \dfrac{\pi}{4}\right) + 2\sin\left(2\pi - \dfrac{\pi}{6}\right)$

$= -2\sin\dfrac{\pi}{4} - 2\sin\dfrac{\pi}{6}$

$= -\sqrt{2} - 1 = -2.414$ cm

(b) $x = 2\sin 2.5\,\pi + 2\sin\left(3\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\left(2\pi + \dfrac{\pi}{2}\right) + 2\sin\left(3\pi + \dfrac{\pi}{3}\right)$

$= 2\sin\dfrac{\pi}{2} - 2\sin\dfrac{\pi}{3}$

$= 2 - \sqrt{3}$ cm

$= 0.27$ cm

Objective Questions (Level 1)

1. $x = A\sin\omega t \Rightarrow \frac{A}{2} = A\sin\omega t$

$\Rightarrow \sin\frac{2\pi}{T}\cdot t = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \frac{2\pi}{T}\cdot t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$

$\therefore \Delta t = \frac{T}{4} - \frac{T}{12} = \frac{3T - T}{12} = \frac{T}{6}$

2. Potential energy becomes maximum to maximum in time $\frac{T}{2}$, whereas oscillation takes a time T.

So, frequency of oscillation is half of frequency of potential energy.

i.e., $\frac{f}{2}$ as $f \propto \frac{1}{T} \Rightarrow \frac{f_P}{f_U} = \frac{T_U}{T_p} = \frac{T/2}{T} = \frac{1}{2}$

3. $v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow v^2 + \omega^2 x^2 = \omega^2 A^2$

or $\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$, is an equation of ellipse.

4. As initial and final distance of centre of mass from the point of suspension is same that is why the time period remains same.

5. $\Delta\phi = \frac{2\pi}{3} + \phi - \frac{\pi}{2} - \phi$

$= \frac{4\pi - 3\pi}{6} = \frac{\pi}{6}$

6. $v = \omega\sqrt{A^2 - x^2} = \frac{1}{2}\omega A$

$\Rightarrow \quad A^2 - x^2 = \frac{A^2}{4}$

$\Rightarrow \quad x^2 = \frac{3}{4}A^2$

or $x = \frac{\sqrt{3}}{2}A$

7. $T \propto \sqrt{l} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$

$= \sqrt{\frac{0.79\, l_1}{l_1}} = 0.89$

$\Rightarrow \quad T_2 = 0.89\, T_1$

i.e., T_2 is 11% lesser than T_1

8. As in (b) two parts of the function has different frequencies their addition will lead to periodic motion Lissajous figures but not simple harmonic motion.

9. $T = 2\pi\sqrt{\frac{M}{k}} = 2$ and $2 + 1 = 2\pi\sqrt{\frac{M+4}{k}}$

$\Rightarrow \frac{3}{2} = \sqrt{\frac{M+4}{M}} \Rightarrow \frac{M+4}{M} = \frac{9}{4}$

$\Rightarrow 16 = 5M$

$\therefore M = \frac{16}{5} = 3.2\text{ kg}$

10. $T_1 = 2\pi\sqrt{\frac{400}{k}}$ and $T_2 = 2\pi\sqrt{\frac{100}{k}}$

$\Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{100}{400}} = \frac{1}{2}$

$T_2 = \frac{T_1}{2} = 1\text{ s}$

11. $y = 4\cos\pi t + 4\sin\pi t$

$= \sqrt{4^2 + 4^2 + 2\times 4\times 4\cos\frac{\pi}{2}}\sin(\pi t + \theta)$

$= 4\sqrt{2}\sin(\pi t + \theta)$

12. $T_1 = 2\pi\sqrt{\frac{m}{k_1}}, T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

and $\quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

$\frac{1}{T_1^2} = \frac{k_1}{4\pi^2 m}; \frac{1}{T_2^2} = \frac{k_2}{4\pi^2 m}$ and

$\frac{1}{T^2} = \frac{k_1 + k_2}{4\pi^2 m}$

$\therefore \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} \Rightarrow T^{-2} = T_1^{-2} + T_2^{-2}$

13. $a = -px = -\omega^2 x$

$\Rightarrow \omega = \sqrt{p}$

14. Let the time in which X completes n oscillations is t, then during that time Y completes $(n-1)$ oscillations to come in same phase.

Therefore, $t = 4n = 4.2\,(n-1)$

$\Rightarrow 4.2 = 0.2\,n \Rightarrow n = 21$

15. $kx = mg \Rightarrow k = \frac{mg}{x}$

$$T = 2\pi\sqrt{\frac{M+m}{k}} = 2\pi\sqrt{\frac{M+m}{\frac{mg}{x}}}$$

$$= 2\pi\sqrt{\frac{(M+m)\,x}{mg}}$$

16. $v = \omega_1 A_1 = \omega_2 A_2$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{\sqrt{\frac{k_2}{m}}}{\sqrt{\frac{k_1}{m}}} = \sqrt{\frac{k_2}{k_1}}$$

17. $|a| = \omega^2 x \Rightarrow \omega = \sqrt{\frac{|a|}{x}} = \sqrt{\frac{1.25}{5\times 10^{-2}}}$

$= \sqrt{25} = 5$ rad/s

$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$ s

18. $T = 2\pi\sqrt{\frac{I}{mgl}}$

$$= 2\pi\sqrt{\frac{\frac{3}{2}mr^2}{mgr}} = 2\pi\sqrt{\frac{3r}{2g}}$$

19. $U = \frac{1}{2}kx^2 \Rightarrow y = mx$, is a straight line passing through origin. The answer will be (d) as x^2 cannot have negative value (for real x).

20. $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$

$$= 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{R}\right)}}$$

$$= 2\pi\sqrt{\frac{R}{2g}}$$

21. At positive maximum displacement force is also maximum but negative, as $F = -m\omega^2 x$, so, the $F-t$ graph will be as in (b).

22. $T = 2\pi\sqrt{\frac{M}{k}}$ and $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

$$\Rightarrow \frac{5}{3} = \sqrt{\frac{M+m}{M}}$$

$$\Rightarrow = \frac{M+m}{M} \Rightarrow \frac{25}{9}$$

$$\Rightarrow 9\,m = 16\,M \Rightarrow \frac{m}{M} = \frac{16}{9}$$

23. $T = 2\pi\sqrt{\frac{m}{k}}$; $mg \sin\theta = kx_0$

$$\Rightarrow x_0 = \frac{mg\sin\theta}{k}$$

24. $\frac{d^2x}{dt^2} = -\pi^2 x = -\omega^2 x$

$\Rightarrow \omega = \pi$;

$\nu = \frac{\omega}{2\pi} = \frac{1}{2}$ Hz

25. $\frac{1}{m_{eq}} = \frac{1}{M} + \frac{1}{m}$

$$\Rightarrow m_{eq} = \frac{mM}{M+m},$$

$$\omega = \sqrt{\frac{k}{m_{eq}}} = \sqrt{\frac{k\,(M+m)}{mM}}$$

$$\Rightarrow \nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k\,(M+m)}{mM}}$$

26. $v = \omega\sqrt{2^2 - 1^2} = \omega\sqrt{3}$;

$|a| = \omega^2 x = \omega^2$

$\Rightarrow \omega\sqrt{3} = \omega^2 \Rightarrow \omega = \sqrt{3}$, $\nu = \frac{\omega}{2\pi} = \frac{\sqrt{3}}{2\pi}$

27. $g = \frac{GM}{R^2}$ and $g' = \frac{G\cdot 2M}{4R^2}$

$= \frac{1}{2}g$

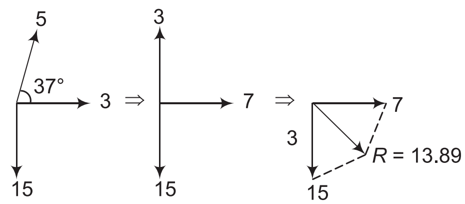
$$T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$$

$$T = \sqrt{\frac{g}{g/2}} = \sqrt{2}$$

$\therefore\ T_2 = \sqrt{2}T_1 = 2\sqrt{2}$ s

28. $x = 3\sin\omega t + 5\sin(\omega t + 37°)$

$$= 3\sin\omega t + \underbrace{15\sin\left(\omega t - \frac{\pi}{2}\right)}_{-15\cos\omega t} + 5\sin(\omega t + 37°)$$



29. When the two SHM are in same direction, the resultant will also be SHM and when they are in perpendicular directions, they will form Lissajous figures performing periodic motion but not SHM. But at 37°, particle will not perform SHM as well as not travel in straight line.

30. $K_{max} = U_{max} = \frac{1}{2}k \times 10^2 = 50\,k$

$$K = K_{max} - U = 50\,k - \frac{1}{2} \times k \times 6^2$$

$$= 32\,k$$

$$= \frac{32}{50}K_{max} = 0.64\,K_{max}$$

$$v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - \frac{A^2}{4}}$$

$$= \frac{\sqrt{3}}{2}\omega A = \frac{\sqrt{3}}{2}v_{max}$$

(a) is correct while (b) is wrong.

31. $k \times 4 \times 10^{-2} = m \times 0.5$

$$\Rightarrow \frac{k}{m} = \frac{0.5}{4 \times 10^{-2}}$$

$$= \frac{50}{4} = \frac{25}{2}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$$

$$= 3.53 \text{ rad/s}$$

32. $Ma_0 = -kx$

$$x = \frac{Ma_0}{k} = \frac{1 \times 2}{100} = 0.02 \text{ m}$$

JEE Corner

Assertion and Reason

1. For, $x = A\cos\omega t$, particle is at $x = A$ at $t = 0$, but the displacement is measured from mean position. So, assertion is false.

2. Assertion and reason are both true but reason is not correct explanation.

3. Assertion is true while time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$, so, the reason is false.

4. $\frac{\sqrt{3}}{2}A = A\sin\omega t$

$$\Rightarrow \quad \sin\omega t = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{T}{6}$$

So, for A to $\frac{\sqrt{3}}{2}A$, $\Delta t_1 = \frac{T}{4} - \frac{T}{6} = \frac{T}{12}$

For, $\frac{A}{2} = A\sin\omega t'$

$$\Rightarrow \quad \frac{2\pi}{T}t' = \frac{\pi}{6}$$

$$\Rightarrow \quad t' = \frac{T}{12}$$

So, for $\frac{\sqrt{3}}{2}A$ to $\frac{1}{2}A$

$$\Rightarrow \quad \Delta t_2 = \frac{T}{6} - \frac{T}{12} = \frac{T}{12}$$

Reason correctly explains assertion.

5. In uniform circular motion also particle performs SHM, so assertion is false.

6. If spring is halved, then spring constant becomes twice and as mass is halved, then $\omega' = \sqrt{\frac{2k}{\frac{m}{2}}} = 2\sqrt{\frac{k}{m}} = 2\omega$, *i.e.*, angular frequency gets double. Thus, assertion is false.
7. Assertion is false as if $F \not\propto -x$, then motion will not be SHM.
8. For $x = A\cos\omega t$ between $t = 0$ to $\frac{\pi}{2\omega}$, particle moves from one extreme to mean position such that velocity and acceleration are in same direction, *i.e.*, angle between them is zero. Such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{v}} = -\omega^2 A\cos\omega t \times (-\omega A\sin\omega t)\cos\theta$ $\cdot\cos\theta = \omega^3 A^2 \sin\omega t \quad \cos\omega t\cos\theta$ as $\cos\theta = \cos 0° = +1$, then $\vec{\mathbf{a}}\cdot\vec{\mathbf{v}}$ = positive. Thus, reason is true explanation of assertion.
9. In SHM, $a = -\omega^2 x$,

   $$i.e., \omega = \sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|} \text{ and } T = \frac{2\pi}{\omega}$$

   So, reason is correct explanation of assertion.
10. Only uniform circular motion can be called SHM, so, assertion is false.

Objective Questions (Level 2)

1. $U = \frac{1}{2}kx^2$

$$\Rightarrow \quad k = \frac{2U}{x^2} = \frac{2\times 1}{(0.4)^2} = \frac{200}{16} = 12.5$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{12.5}}$$

$$= 2\pi\sqrt{\frac{4}{25}} = 2\pi\frac{2}{5} = \frac{4\pi}{5} \text{ s}$$

2. $T = \frac{1}{2}\cdot 2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$, and $\frac{1}{2}ka^2 = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = a\sqrt{\frac{k}{m}}$$

$$F = \frac{dP}{dt} = \frac{2m\cdot v}{T} = \frac{2ma\sqrt{\frac{k}{m}}}{\pi\sqrt{\frac{m}{k}}}$$

$$= \frac{2ma}{\pi}\cdot\frac{k}{m} = \frac{2ak}{\pi}$$

3. $y_2 = 10\sin\omega t + 10C\cos\omega t$

$$= \sqrt{10^2 + 10^2C^2}\sin(\omega t + \theta)$$

$$= 10\sqrt{1 + C^2}\sin(\omega t + \theta)$$

So, $10\sqrt{1 + C^2} = 40$

$\Rightarrow 1 + C^2 = 16$

$\Rightarrow C = \sqrt{15}$

4. $x = 2\sin(5\pi t)$

$\Rightarrow x\,(0.3) = 2\sin(1.5\,\pi)$

$$= 2\sin\left(\pi + \frac{\pi}{2}\right) = -2$$

$$v = \omega\sqrt{A^2 - x^2} = 5\pi\sqrt{2^2 - 2^2} = 0$$

5. $x = A\sin\left(\frac{2\pi}{16}t\right) = A\sin\left(\frac{\pi t}{8}\right)$;

$$x\,(2) = A\sin\left(\frac{\pi\times 2}{8}\right) = \frac{A}{\sqrt{2}}$$

$$v = \omega\sqrt{A^2 - x^2} = \frac{2\pi}{16}\sqrt{A^2 - \frac{A^2}{2}}$$

$$= \frac{\pi A}{8\sqrt{2}} = 1$$

$$\Rightarrow A = \frac{8\sqrt{2}}{\pi}\text{ m}$$

6. $T' = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{g}}\cdot\sqrt{\frac{g}{g_{\text{eff}}}}$

$$T = \sqrt{\frac{10}{\sqrt{10^2 + 4^2}}} = 2\sqrt{\frac{10}{\sqrt{116}}}\text{ s}$$

$$= 1.93 \text{ s}$$

7. $x = A\sin\omega t \Rightarrow v = A\omega\cos\omega t$ and $a = -\omega^2 A\sin\omega t$

$$\frac{v^2}{a^2} = \frac{A^2\omega^2\cos^2\omega t}{A^2\omega^4\sin^2\omega t}$$

or $\frac{v^2}{a^2} = \frac{1}{\omega^2} \cdot \frac{\frac{v^2}{A^2\omega^2}}{1 - \frac{v^2}{A^2\omega^2}}$

$\frac{v^2}{a^2} = \frac{1}{\omega^2} \cdot \frac{v^2}{A^2\omega^2 - v^2}$

or $v^2\omega^2 (A^2\omega^2 - v^2) = a^2\omega^2 v^2$

or $A^2\omega^2 - v^2 = a^2$

or $v^2 = -a^2 + A^2\omega^2$

or $y = -mx + C$

i.e., a straight line with negative slope and positive intercept.

8. $l = \frac{1}{2} \times \frac{1000}{20}\text{ cm} = 25\text{ cm}$

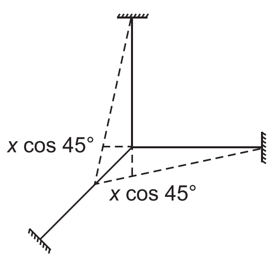
$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.25}{10}} = \frac{\pi}{\sqrt{10}} \simeq 1\text{ s}$

9. $F_r = -A\rho g x$

$\Rightarrow a_{cc} = -\frac{A\rho g}{\sqrt{\rho_0}} x = -\omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{V\rho_0}{A\rho g}} = 2\pi\sqrt{\frac{a\rho_0}{\rho g}}$

10. $\therefore F_r = -[kx + (2kx\cos 45°)\cos 45°]$



$= -2kx$

$a = -\frac{2k}{m}x = -\omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

11. $I = \frac{ml^2}{12} + md^2$,

$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{l^2}{12} + d^2}{gd}}$

For T = minimum, $\frac{\partial T}{\partial d} = 0 = \frac{\partial}{\partial d}\left[\frac{\frac{l^2}{12} + d^2}{d}\right]^{\frac{1}{2}}$

or $= \frac{\partial}{\partial d}\left[\frac{l^2}{12d} + d\right]^{\frac{1}{2}} = \frac{l^2}{12} \cdot \left(-\frac{1}{d^2}\right) + 1$

or $\frac{l^2}{d^2} = 12 \Rightarrow \frac{d}{l} = \frac{1}{\sqrt{12}}$

12. $T_A = 2\pi\sqrt{\frac{m}{k}}$; $T_B = 2\pi\sqrt{\frac{m}{\frac{k}{2}}}$; $T_C = 2\pi\sqrt{\frac{m}{2k}}$

$\therefore T_A : T_B : T_C = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

$\Rightarrow T_B > T_A > T_C \Rightarrow T_2 > T_1 > T_3$

13. $T_B = 2\pi\sqrt{\frac{m}{k}}$,

$T_A = 2\pi\sqrt{\frac{\frac{m}{3}}{k}} = 2\pi\sqrt{\frac{m}{3k}}$;

$T_C = 2\pi\sqrt{\frac{\frac{m}{2} + \frac{\frac{m}{2}}{3}}{k}} = 2\pi\sqrt{\frac{4m}{6k}}$

$\therefore T_A : T_B : T_C = \frac{1}{\sqrt{3}} : 1 : \sqrt{\frac{2}{3}}$

$\Rightarrow T_B > T_C > T_A \Rightarrow T_2 > T_3 > T_1$

or $T_1 < T_3 < T_2$

14. $T = \frac{1}{2}(T_1 + T_2) = \frac{1}{2}\left(2\pi\sqrt{\frac{M}{k}} + 2\pi\sqrt{\frac{M}{4k}}\right)$

$= \frac{1}{2} \cdot 2\pi\sqrt{\frac{M}{k}}\left(1 + \frac{1}{2}\right) = \frac{3\pi}{2}\sqrt{\frac{M}{k}}$

15. $U = 2 - 20x + 5x^2$; $F = -\frac{\partial U}{\partial x} = 20 - 10x$

$F = 0$ at $x = 2$, *i.e.*, $x = 2$ is the equilibrium position.

As particle is released from $x = -3$, so amplitude is 5 m.

Thus, maximum x coordinate is

$2\text{ m} + 5\text{ m} = 7\text{ m}$.

16. $T = 2\pi\sqrt{\frac{m}{k}} = \frac{1}{f}$;

$T_1 = 2\pi\sqrt{\frac{m(1-n)}{\frac{k}{n}}}$ and $T_2 = 2\pi\sqrt{\frac{nm}{\frac{k}{(1-n)}}}$

$\frac{f_1}{f_2} = \frac{T_2}{T_1} = \sqrt{\frac{nm(1-n)}{k}} \cdot \sqrt{\frac{k}{nm(1-n)}} = 1$

17. $x = R \sin \omega t$

$U_B = U_D = \frac{3}{4} \times \frac{1}{2} kR^2 = \frac{1}{2} kx^2$

$\Rightarrow x = \frac{\sqrt{3}}{2} R$

$\therefore\ BD = 2x = \sqrt{3}\ R$

18. $F_{\text{elastic}} = \frac{Y\Delta LA}{l}$

$\Rightarrow F_Q = \frac{-Y \cdot \Delta LA}{L}$

$\Rightarrow a_Q = \frac{-YA}{Lm} \cdot \Delta L$

$\frac{1}{2} T_Q = \frac{1}{2} \cdot 2\pi\sqrt{\frac{Lm}{YA}}$

Similarly, $F_P = -\frac{2Y \cdot \frac{A}{2}}{L} \Delta L = -\frac{YA}{L} \Delta L$

$a_P = -\frac{YA}{Lm} \Delta L$

$\therefore\ \frac{1}{2} T_P = \frac{1}{2} 2\pi\sqrt{\frac{Lm}{YA}}$

$\therefore\ T = t + \frac{1}{2} T_P + \frac{1}{2} T_Q$

$= \frac{2L}{v} + 2\pi\sqrt{\frac{Lm}{YA}}$

19. $5 = 10 \sin \omega t$

$\Rightarrow \sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$

$\Rightarrow \frac{2\pi}{T} \cdot t = \frac{\pi}{6}$

$\Rightarrow t = \frac{T}{12}$

$\therefore\ T' = \frac{T}{2} + 2 \times \frac{T}{12} = \left(\frac{1}{2} + \frac{1}{6}\right) T$

$= \frac{4}{6} T = \frac{2}{3} T$

$= \frac{2}{3} \times 2\pi\sqrt{\frac{0.1}{100}}$

$= \frac{4\pi}{30\sqrt{10}} = 0.133$ s

20. $x = A \cos \omega t$

$\Rightarrow\quad 5 \text{ cm} = 8 \cos \omega t$

$\Rightarrow\quad \omega t = 0.9$

$\frac{2\pi}{1.2} t = 0.9$

$\Rightarrow\quad t = \frac{1.08}{2\pi}$ s= 0.17 s

21. $I = \frac{m}{3} \cdot \frac{a^2}{3} + \frac{m}{3}\frac{a^2}{3}$

$+ \left(\frac{ma^2}{3 \times 12} + \frac{m}{3}\left(\frac{\sqrt{3}}{2} a\right)^2\right)$

$= \frac{2ma^2}{9} + \frac{ma^2}{36} + \frac{ma^2}{4}$

$\therefore\ I = \frac{8 + 1 + 9}{36} ma^2 = \frac{1}{2} ma^2$

$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{2}ma^2}{mg\frac{a}{\sqrt{3}}}}$

$= 2\pi\sqrt{\frac{\sqrt{3}\ a}{2g}} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$ s

22. $t = \frac{2.5\ \pi}{\omega} = 1.25\left(\frac{2\pi}{\omega}\right) = 1.25T = T + \frac{T}{4}$

In one time period distance travelled $d_1 = 4A$. In one time period distance travelled $d_2 = 4A$.

In the remaining time $\frac{T}{4}$, distance travelled $d_2 = A$.

$\therefore$Total distance travelled = 5 A

23. $T = 2\pi\sqrt{\frac{l}{g}}$,

$T' = \frac{T}{2} + \frac{T_1}{2}$

$$= \frac{1}{2} \times 2\pi\sqrt{\frac{l}{g}} + \frac{1}{2} \times 2\pi\sqrt{\frac{\frac{l}{4}}{g}}$$

$$= \left(\frac{1}{2} + \frac{1}{4}\right) 2\pi\sqrt{\frac{l}{g}} = \frac{3}{4}\,T$$

24. $A = \frac{d}{2} = 0.4$ m, $\omega = 30$ rev./min

$$= \frac{30 \times 2\pi}{60} = \pi \text{ rad/s}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2 \text{ s}$$

25. $y = -2\cos\omega t = 2\sin\left(\omega t - \frac{\pi}{2}\right)$

$$= 2\sin\left(\frac{2\pi}{0.6\,\pi}\,t - \frac{\pi}{2}\right) = 2\sin\left(\frac{10}{3}\,t - \frac{\pi}{2}\right)$$

26. $x = A\cos\omega t \Rightarrow \frac{A}{2} = A\cos\omega t$

$$\Rightarrow \quad \cos\omega t = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\frac{2\pi}{T}\,t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{T}{6}$$

$$v_{\text{av}} = \frac{a/2}{t} = \frac{a/2}{T/6} = \frac{3a}{T}$$

27. $|a| = \omega^2 A = g$ for weightlessness at highest point

$$\omega = \sqrt{\frac{g}{A}} = 2\pi f$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{A}} = \frac{1}{2\pi}\sqrt{\frac{g}{0.5}} = \frac{\sqrt{2g}}{2\pi}$$

28. $U = K = \frac{1}{2}U_{\text{max}}$

$$= \frac{1}{2}\cdot\frac{1}{2}KA^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

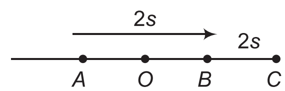
$$\frac{A}{\sqrt{2}} = A\sin\frac{2\pi}{T}\cdot t$$

$$\Rightarrow \quad \frac{2\pi}{T}\cdot t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{T}{8}$$

$$\therefore\ v_{\text{av}} = \frac{2x}{2t} = \frac{x}{t} = \frac{A\sqrt{2}}{T/8} = \frac{4\sqrt{2}A}{T}$$

29.



$\Rightarrow T = 8$ s

$$OB = x_B = A\sin\frac{2\pi}{T}\cdot 1\text{ s}$$

$$= A\sin\frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

$$\therefore\ OB : A = \frac{A}{\sqrt{2}} : A = 1 : \sqrt{2}$$

30. $x = A\cos\omega t$

At $t = 0$, $x = A$ and $t = \frac{\pi}{6\omega}$,

$$x = A\cos\omega\cdot\frac{\pi}{6\omega} = \frac{\sqrt{3}A}{2}$$

$$\therefore \quad v_{\text{av}} = \frac{A - \frac{\sqrt{3}}{2}A}{\frac{\pi}{6}\omega}$$

$$= \frac{6\omega A}{\pi}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{3\omega A}{\pi}(2 - \sqrt{3})$$

31. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{2}} = \sqrt{200}$

$$= 10\sqrt{2} \text{ rad/s}$$

32. $ma = kx_0$

$$\Rightarrow x_0 = \frac{ma}{k} = \frac{2 \times 5}{400} \text{ m}$$

$$= \frac{10}{4} \text{ cm} = 2.5 \text{ cm}$$

More than One Correct Options

1. At equilibrium, $T\sin\theta = ma$ and $T\cos\theta = mg$

$$\Rightarrow \qquad \tan\theta = \frac{a}{g}$$

$$T = \sqrt{T^2 \sin^2\theta + T^2\cos^2\theta}$$

$$= m\sqrt{g^2 + a^2}$$

2. $x = A\cos(\omega t + \theta)$

and $v = -\omega A \sin(\omega t + \theta)$

at $t = 0, x = A\cos\theta$ and $v = -\omega A\sin\theta$

$$\therefore \qquad \frac{A}{2} = A\cos\theta$$

$$\Rightarrow \qquad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\therefore \qquad x = A\cos\left(\omega t + \frac{\pi}{3}\right)$$

$$= A\cos\left(\frac{2\pi}{T}\cdot t + \frac{\pi}{3}\right)$$

$$= A\sin\left(\frac{2\pi}{T}t + \frac{\pi}{2} + \frac{\pi}{3}\right)$$

$$= A\sin\left(\frac{2\pi}{T}t + \frac{5\pi}{6}\right)$$

3. At equilibrium, $Mg = kx_0$

$$\Rightarrow x_0 = \frac{Mg}{k} = \frac{1\times 10}{500}\text{ m} = 2\text{ cm}$$

$$\Rightarrow A = x - x_0 = 45 - 42 = 3\text{ cm}$$

$$\frac{1}{2}\times 1\times v^2 = \frac{1}{2}\times 500\,(0.05^2 - 0.02^2) - 1\times 10\times 0.03$$

$$v = \sqrt{500\,(0.0025 - 0.0004 - 0.6)}$$

$$= \sqrt{1.05 - 0.6}\text{ m/s}$$

$$= \sqrt{0.45}\text{ m/s}$$

$$= 10\sqrt{45}\text{ cm/s}$$

$$= 30\sqrt{5}\text{ cm/s}$$

$$a_{max} = \omega^2 A = \frac{K}{m}\cdot A = \frac{500}{1}\times 0.03\text{ m/s}^2$$

$$= 15\text{ m/s}^2$$

At natural length, $U = 0$

4. $T = 2\pi\sqrt{\dfrac{m_{eff}}{k}} = 2\pi\sqrt{\dfrac{2}{800}}$

$$= \frac{2\pi}{20} = \frac{\pi}{10}\text{ s};\ \frac{1}{m_{eff}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$3x_1 = 6x_2 \Rightarrow x_1 = 2x_2$ and $x_1 + x_2 = 6$ cm

$$\Rightarrow \qquad 3x_2 = 6\text{ cm}$$

$$\Rightarrow \qquad x_2 = 2\text{ cm}$$

and $\qquad x_1 = 4$ cm

$$\frac{1}{2}3v_1^2 + \frac{1}{2}6\,v_2^2 = \frac{1}{2}k\times(0.06)^2$$

$$3v_1^2 + 6v_2^2 = 1.8$$

$$\Rightarrow \qquad v_1^2 + 2v_2^2 = 0.6$$

Again, $\qquad 3v_1 = 6v_2$

$$\Rightarrow \qquad v_1 = 2v_2;$$

$$\therefore\ 6v_2^2 = 0.6, v_2 = \sqrt{0.1}$$

$$\therefore\ P_2 = m_2v_2 = 6\sqrt{0.1}$$

$$= \frac{6}{\sqrt{10}} = 1.9\text{ kg-m/s}$$

$$T_1 : T_2 = 1:1$$

5. At, $t = T$, $y =$ maximum $\Rightarrow a =$ maximum

At, $t = \dfrac{3T}{4}$, $y = 0 \Rightarrow a = 0 \Rightarrow F = 0$

6. $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow v^2 = \omega^2A^2 - \omega^2x^2$$

$$\Rightarrow v^2 + \omega^2x^2 = \omega^2A^2$$

i.e., v-x graph is an ellipse.

$$a = -\omega^2x$$

$\Rightarrow y = -mx \rightarrow$ straight line

7. $a = 0$ at $x = \dfrac{1}{2}$, so, $A = 2 - \dfrac{1}{2} = 1.5$ m

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{100}} = \frac{2\pi}{10} = \frac{\pi}{5}\text{ s} = 0.63\text{ s}$$

$$v_{max} = A\omega = 1.5\sqrt{100} = 15\text{ m/s}$$

8. Same phase means a phase difference of $2n\pi$, *i.e.*, their displacement, velocity, acceleration are equal.

9. $y = 3\sin 100\pi t + 4\,(1 - \cos 100\pi t) - 6$

$$= 3\sin 100\pi t - 4\cos 100\pi t - 2$$

$$= 5\sin(100\pi t + \theta) - 2$$

Match the Columns

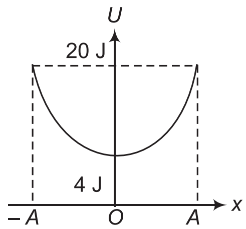
1. $x = 2\cos\omega t + 2$

$\therefore$ Mean position is $x = 2$,

Amplitude is $x = 2$

Extreme positions are, $x = 4$ and $x = 0$ where, PE is also maximum.

While, PE is not zero between $x = 0$ to $x = 4$.

2. 

$U = 4 + ax^2$, where, $20 = 4 + aA^2$

$\Rightarrow \quad a = \frac{16}{A^2}$

$\therefore \quad U = 4 + \frac{16}{A^2}x^2$

$U\left(\frac{A}{2}\right) = 4 + \frac{16}{A^2}\cdot\frac{A^2}{4} = 4 + 4 = 8\text{ J}$

$U\left(\frac{A}{4}\right) = 4 + \frac{16}{A^2}\cdot\frac{A^2}{16} = 5\text{ J}$

$K\left(\frac{A}{4}\right) = 20 - U\left(\frac{A}{4}\right) = 15\text{ J}$

$U(0) = 4\text{ J} \Rightarrow K(0) = 20 - 4 = 16\text{ J}$

$K\left(\frac{A}{2}\right) = 20 - U\left(\frac{A}{2}\right) = 20 - 8 = 12\text{ J}$

3. At t_1, $a \neq 0$, $v \neq 0$ and $x \neq 0$ but

a = positive

$\Rightarrow x = -$ve and $v = -$ve

At t_2, $a = 0, v \neq 0$ and $x = 0$ at t_2, area is –ve, so, $v =$ –ve

and at mean position velocity is maximum.

4. $x = 2\sin 4\pi t, \qquad v = 8\pi\cos 4\pi t$ and

$a = -32\pi^2 \sin 4\pi t$

$v_{\max} = A\omega = 8\pi \text{ m/s} = 25.1\text{ m/s}.$

So, velocity never becomes 30 m/s

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}\text{ s},$$

So, in 1 s particle completes 2 oscillations.

So, speed becomes 10 m/s, 8 times in 2 oscillations and velocity 4 times.

$K_{\max} = \frac{1}{2}m v_{\max}^2 = \frac{1}{2}\times 2\times 64\pi^2 = 640\text{ J}$

So, it becomes 400 J, 8 times in two oscillations.

$a_{\max} = 32\pi^2 = 320$ m/s, in two oscillations it becomes – 100, 4 times.

5. $x = 4 + 6\sin \pi t.$

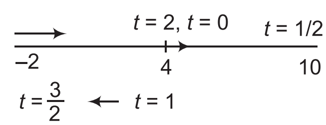
$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\text{ s}.$

$A = 6\text{ m}$ and $-2 \le x \le 10$

at $t = 0, x = 4$ m,

at $t = \frac{1}{2}$ s, $x = 10$ m.

at $t = 1$ s, $x = 4$ m.



at $t = \frac{3}{2}$ s, $x = -2$ m.

and $t = 2$ s, $x = 4$ m.

So,

(a) 10 m → 4 m $\quad \frac{1}{2} \le t \le 1 \rightarrow$ q

(b) 10 m → 7 m $\quad \frac{1}{2} \le t < 1 \rightarrow$ q

(c) 7 m → 1 m $\quad \frac{1}{2} < t < \frac{3}{2} \rightarrow$ r

(d) 10 m → – 2m $\quad \frac{1}{2} \le t \le \frac{3}{2} \rightarrow$ r

12 Elasticity

Introductory Exercise 12.1

1. $Y = \dfrac{F/A}{\Delta l / l} = \dfrac{Fl}{A\,\Delta l}$

$\Rightarrow \quad \Delta l = \dfrac{Fl}{YA}$

$\Rightarrow \quad \Delta l \propto \dfrac{1}{Y}$ as Young's modulus of A is twice of B, so elongation of A will be half of B.

2. $\Delta l = \dfrac{Fl}{YA} = \dfrac{10^4 \times 1}{2 \times 10^{10} \times 4 \times 10^{-4}} = \dfrac{1}{800}$ m

$= \dfrac{1}{8}$ cm $= 0.125$ cm

3. $\dfrac{F}{A} = Y \dfrac{\Delta l}{l} = 90 \times 10^6\ \text{N/m}^2$

$\Rightarrow A = \dfrac{1.6 \times 10^6}{90 \times 10^6} = \dfrac{1.6}{90} = \dfrac{\pi}{4}(d_2^2 - d_1^2)$

$d_2 = \sqrt{\dfrac{4 \times 1.6}{90\,\pi} + (0.2)^2} = \sqrt{0.063}$

$= 0.250 = 250$ mm

AIEEE Corner

Subjective Question (Level 1)

1. $Y = \dfrac{Fl}{A\Delta l} \Rightarrow A = \dfrac{Fl}{Y\,\Delta l} \Rightarrow \pi\dfrac{d^2}{4} = \dfrac{Fl}{Y\,\Delta l}$

$\Rightarrow \quad d = \sqrt{\dfrac{4Fl}{\pi\, Y \Delta l}}$

$\therefore \quad d = \sqrt{\dfrac{4 \times 400 \times 3}{2.1 \times 10^{11} \times 3.14 \times 2 \times 10^{-3}}}$ m

$= 10^{-3}\sqrt{\dfrac{4.8}{4.2 \times 3.14}}$ m

$= 1.9 \times 10^{-3}$ m $= 1.9$ mm

2. $\left(\dfrac{F}{A}\right)_{max} = \dfrac{1}{3} \times 3 \times 10^8\ \text{N/m}^2$

$= \dfrac{900\,(g + a)}{4 \times 10^{-4}}$

$\Rightarrow \quad g + a = \dfrac{4 \times 10^4}{900} = \dfrac{400}{9}$

$\Rightarrow a_{max} = \dfrac{400}{9} - g = 34.64\ \text{m/s}^2$

3. $\left(\dfrac{F}{A}\right)_{max} = 1.5 \times 10^8\ \text{N/m}^2$

$\Rightarrow \pi\dfrac{d^2}{4} = \dfrac{F}{1.5 \times 10^8}$

$\Rightarrow d_{min} = \sqrt{\dfrac{4F}{\pi \times 1.5 \times 10^8}}$

$$= \sqrt{\frac{4 \times 100}{3.14 \times 15 \times 10^8}}$$

$$= 10^{-3} \sqrt{\frac{4}{3.14 \times 1.5}} = 0.92 \text{ mm}$$

4. $dl = \dfrac{\mu\ x\ g + dx}{YA} = \dfrac{\rho g}{Y} x\ dx$

$$\Rightarrow \Delta l = \frac{\rho g}{Y} \int_0^l x\ dx = \frac{\rho g l^2}{2Y}$$

$$= \frac{8 \times 10^3 \times 9.8 \times 25}{2 \times 2 \times 10^{11}} = 5 \times 9.8 \times 10^{-7} \text{ m}$$

$$= 4.9 \times 10^{-6} \text{ m}$$

5. $\left(\dfrac{F}{A}\right)_0 = \dfrac{(m + m_1 + m_2)\ g}{A_0} = 8 \times 10^8$

$$\Rightarrow m = \frac{8 \times 10^8 \times 6 \times 10^{-7}}{10} - 30$$

$$= 48 - 30 = 18 \text{ kg}$$

$$\left(\frac{F}{A}\right)_L = \frac{(m + m_1)\ g}{A_L} = 8 \times 10^8$$

$$\Rightarrow m = \frac{8 \times 10^8 \times 3 \times 10^{-7}}{10} - 10 = 14 \text{ kg}$$

So, lower wire will break if $m > 14$ kg. *i.e.*, maximum load which can be put on hanger without breaking wire is 14 kg

6. $\dfrac{F}{A}$ is same as both have same cross-section area and subjected under same tension.

$$\frac{\Delta l}{l} = \frac{F}{YA} \Rightarrow \frac{\Delta l}{l} \propto \frac{1}{Y}$$

$$\therefore \left(\frac{\Delta l}{l}\right)_{St} \Big/ \left(\frac{\Delta l}{l}\right)_{Cu} = \frac{Y_{Cu}}{Y_{St}} = \frac{1.3}{2} = \frac{13}{20}$$

7. $B = \dfrac{\Delta p}{\dfrac{\Delta l}{V}} = \dfrac{-\Delta p}{\dfrac{\Delta p}{\rho}}$

$$\Rightarrow \Delta\rho = -\frac{\rho \Delta p}{B} = \frac{\rho\ h\ \rho\ g}{B}$$

$$\Delta\rho = \frac{400 \times (1030)^2 \times 10}{2 \times 10^9}$$

$$= \frac{4 \times (1.03)^2}{2} = 2.12 \text{ kg/m}^3$$

8. $\Delta l_1 = \dfrac{Mgl}{YA} = \dfrac{V\rho gl}{YA} \therefore \Delta l_2 = \dfrac{V(\rho - \sigma)\ gl}{YA}$

$$\Rightarrow \Delta l = \Delta l_2 - \Delta l_1 = -\frac{V\sigma gl}{YA}$$

$$= -\frac{10^{-3} \times 800 \times 10 \times 3}{\pi \times (2 \times 10^{-4})^2 \times 8 \times 10^{10}}$$

$$= -\frac{30}{4\pi} \times 10^{-3} \text{ m}$$

$$= -2.39 \times 10^{-3} \text{ m}$$

9. $B = -\dfrac{\Delta p}{\Delta V / V} = \dfrac{-h\rho g}{\Delta V / V} = \dfrac{180 \times 10^3 \times 9.8}{10^{-3}}$

$$= 1.76 \times 10^9 \text{ N/m}^2$$

10. $Y = \dfrac{F / A}{\Delta l / l}$

$$\Rightarrow F = \frac{YA\ \Delta l}{l} = mg + \frac{mv^2}{l}$$

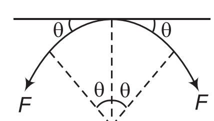
$$\Rightarrow v = \sqrt{\frac{l}{m}\left(\frac{YA\Delta l}{l} - mg\right)}$$

$$v = \sqrt{\frac{5}{25}\left(\frac{2 \times 10^{11} \times \pi \times 4 \times 10^{-6} \times 10^{-2}}{5} - 250\right)}$$

$$= \sqrt{\frac{1}{5}\left(\frac{8\pi}{5} \times 10^3 - 250\right)} = 30.9 \text{ m/s}$$

11. $2F \sin\theta = \dfrac{mv^2}{R} = \dfrac{\mu R \cdot 2\theta \cdot v^2}{R}$

$\sin\theta \approx \theta$



$$\Rightarrow F = \mu v^2$$

$$F = \frac{m}{2\pi R} R^2\ \omega^2$$

$$= \frac{4\pi^2 \nu^2\ mR^2}{2\ \pi\ R}$$

$$= 2\pi\ \nu^2\ mR$$

$$Y = \frac{Fl}{A\ \Delta l} = \frac{2\pi\ \nu^2\ mR \times 2\ \pi R}{\pi r^2 \Delta l}$$

$$\Rightarrow \Delta l = \frac{4\pi^2\ \nu^2\ mR^2}{Y\ \pi r^2}$$

$$l = 2\pi R$$

$$\therefore \Delta R = \frac{\Delta l}{2\pi} = \frac{2\pi \nu^2 mR^2}{\pi Y r^2}$$

But $m = (2\pi R)(\pi r^2)(\rho)$

Substituting the values we get,

$$\Delta R = \frac{4\pi^2 \nu^2 \rho R^3}{Y}$$

12. $\Delta l = \frac{Fl}{YA} = \frac{(mg + m\omega^2\ l)\ l}{YA}$

$$= \frac{(60 + 6 \times 4 \times 4\pi^2 \times 0.6) \times 0.6}{2 \times 10^{11} \times 5 \times 10^{-6}}$$

$$= 3.8 \times 10^{-4}\ \text{m}$$

13. $2T_c + T_s = mg$ ...(i)

$\Delta l_s = \Delta l_c$

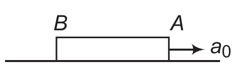
$\frac{T_s l}{AY_s} = \frac{T_c l}{AY_c}$

$T_c = \frac{T_s}{2}$ ...(ii)

Solving these two equations we get,

$T_s = \frac{mg}{2}$ and $T_c = \frac{mg}{4}$

14.



(a) **At centre :**

$T = \frac{m}{2} \cdot a_0 = \left(\frac{L}{2} s\rho\right) a_0$

$\therefore$ Stress $= \frac{T}{S} = \frac{L}{2}\rho a_0$

(b) **At distance *x* from end *B* :**

$T_x = m_x a_0 = (xs\rho) a_0$

$\Delta l = \int_0^L \frac{T_x dx}{SY} = \frac{\rho_0 a_0 L^2}{2Y}$

15. Energy stored in stretched wire = $ms\Delta Q$

■ Objective Questions (Level-1)

1. $\theta = -\frac{\Delta p}{\frac{\Delta V}{V}}$ as $\Delta V = \Delta \Rightarrow B = \infty$

2. Young's modulus do not depend upon length of radius of wire. It is constant for a material.

3. $F = \frac{YA\ \Delta l}{l} \Rightarrow F \propto \frac{1}{l}$

$\Rightarrow F_2 = F_1 \frac{l_1}{l_2} = 2F_1 = 2W$

4. During free fall, stress developed in the rod is zero, so there is no elongation.

5. $B = -\frac{\Delta p}{\Delta V / V} = \frac{1.2 \times 10^5 \times 10^{-3}}{0.3 \times 10^{-3}}$

$= 4 \times 10^5\ \text{N/m}^2$

6. $\frac{F}{A} = Y \frac{\Delta l}{l} = \frac{mv^2}{lA}$

$\Rightarrow v = \sqrt{\frac{l}{m} \frac{F}{A} A} = \sqrt{\frac{0.3}{10} \times 4.8 \times 10^7 \times 10^{-6}}$

$= \sqrt{3 \times 48 \times 10^4 \times 10^{-6}} = 1.2\ \text{m/s}$

$\omega = \frac{v}{l} = \frac{1.2}{0.3} = 4$ rad/s

7. $dl = \frac{\mu g x\ dx}{YA} \Rightarrow \frac{F}{A} = \frac{\mu g x}{A} = \rho g x$ for $x = \frac{1}{2}$

$\Rightarrow \frac{F}{A} = \frac{1}{2} pgL$

8. $\frac{\Delta V}{V} = -\frac{\Delta p}{p} = -10^{-3}$

$\Rightarrow \Delta p = -B \frac{\Delta V}{V} = 2 \times 10^9 \times 10^{-3}$

$= 2 \times 10^6\ \text{N/m}^2$

9. System is bound or force is attractive when potential energy is negative.

10. $Y = \frac{Fl}{A\ \Delta l} = \frac{T_1 l}{A\ (l_1 - l)} = \frac{T_2 l}{A\ (l_2 - l)}$

$\Rightarrow T_1\ (l_2 - l) = T_2\ (l_1 - l)$

$\Rightarrow T_1 l_2 - T_2 l_1 = (T_1 - T_2)\ l$

$\Rightarrow l = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$

11. $\Delta l = \frac{Fl}{YA} = \frac{mgl}{YA} = \frac{mgl\ .\ l}{YAl} = \frac{\rho g l^2}{Y}$

$\frac{\Delta l_2}{\Delta l_1} = \left(\frac{l_2}{l_1}\right)^2 = 4 \Rightarrow \Delta l_2 - 4\Delta l_1 = 4\Delta l$

12. $\Delta l = l\alpha\,\Delta Q, U = \frac{1}{2}\times F \times \Delta l = \frac{1}{2} YA\,\frac{\Delta l}{l}\cdot \Delta l$

$$= \frac{YA\,(\Delta l)^2}{2l} = \frac{YAl^2\;d^2\;(\Delta l)^2}{2l}$$

$$\therefore\; u = \frac{U}{V} = \frac{\frac{1}{2}\,YAl\;\alpha^2\;(\Delta\theta)^2}{Al}$$

$$= \frac{1}{2}\,Y\;\alpha^2\;(\Delta\theta)^2$$

$$= \frac{1}{2}\times 10^{11}\times 144\times 10^{-12}\times 400$$

$$= 72\times 40\text{ J} = 2880\text{ J}$$

13. $\Delta l_t = l\;\alpha\;\Delta\theta = 1\times 10^{-4}\times 20\text{ m} = 2\text{ m}$

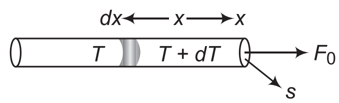
$\Rightarrow\quad \Delta l_e = 1\text{ mm}$

$$\frac{F}{A} = Y\,\frac{\Delta l}{l}$$

$$= 10^{11}\times\frac{10^{-3}}{1} = 10^{8}\;\text{N/m}^2$$

14. $dT = \mu\;dx\cdot a = \rho S\;dx\cdot\frac{F_0}{m} = \frac{\rho SF_0}{PSL}\,dx$

$$= \frac{F_0}{L}\,dx$$



$$dl = \frac{T\cdot dx}{YS} = \frac{F_0\;x\;dx}{YSL}$$

$$\Rightarrow\;\Delta l = \frac{F_0 L^2}{2YSL}$$

$$= \frac{1}{2}\cdot\frac{F_0}{YS}\cdot L$$

$$\therefore\;\frac{\Delta l}{L} = \frac{F_0}{2YS}$$

JEE Corner

Assertion and Reason

1. Elasticity depends upon Young's modulus, which is more for steel as more stress is required to produce same strain, *i.e.*, reason is true explanation of assertion.
2. Assertion and reason are both false as Bulk modulus of gas depends upon thermodynamic process, while the relation between pressure and temperature also depends upon thermodynamic process.
3. Assertion is false as Young's modulus do not depend upon length while reason is true.
4. Assertion and reason are both true but not correct explanation.
5. Reason truly explains assertion.
6. Assertion is not true as it is true only with in elastic limit.
7. $\frac{B_{\text{iso}}}{B_{\text{adio}}} = \frac{p}{rp} = \frac{1}{r} = \frac{1}{5/9} = \frac{3}{5}$ is true and reason is also true but not correct explanation.

More than one Correct Options

1. $dW = F\cdot dx = kx\;dx = \frac{YA}{L}\cdot x\;dx$

$$W = \frac{YA}{l}\int_0^l x\;dx = \frac{YA}{L}\cdot\frac{1}{2}\,l^2 = \frac{YAl^2}{2L} = U$$

2. $F = k\;\Delta x = \frac{YA}{L}\;\Delta x = m\Delta x$

From graph, $m_A < m_B$

$$\Rightarrow\quad \left(\frac{YA}{L}\right)_A < \left(\frac{YA}{L}\right)_B$$

it is possible when, $Y_A < Y_B$ or $A_A < A_B$ or $L_A > L_B$ but as $Y_A = Y_B$ and $L_A = L_B \Rightarrow A_A < A_B$

3. $W_g = Mg \cdot l, \qquad U = \frac{1}{2} kB^2 = \frac{1}{2} mgl,$

$Q = W_g - U = \frac{1}{2} Mgl$

Match the Columns

1. (a) $[\text{stress}] = [F]/[A] = [\text{MLT}^{-2}\text{L}^{-2}]$

$= [\text{ML}^{-1}\text{T}^{-2}]$

$u = \frac{U}{V} = \frac{[\text{MLT}^{-2}\text{L}^{-1}]}{[\text{L}]^3}$

$= [\text{ML}^{-1}\text{T}^{-2}] \longrightarrow r^2$

(b) $[\text{strain}] = \Delta l / l = [\text{M}^0\text{L}^0\text{T}^0] \rightarrow p, q$

(c) $[Y] = \frac{[F/A]}{[\Delta l / l]} = [\text{stress}]$

$= [\text{ML}^{-1}\text{T}^{-2}] \rightarrow r$

(d) $F = kx$

$\Rightarrow [h] = \frac{[F]}{[x]}$

$= [\text{MLT}^{-2}\text{L}^{-1}]$

$= [\text{MT}^{-2}] \rightarrow s$

2. $Y = \frac{Fl}{A\Delta l}$ or $Fl = YA\Delta l$

(a) $F \uparrow \Rightarrow \frac{F}{A} \uparrow \Rightarrow \Delta l \uparrow \rightarrow p, q$

(b) $l \uparrow \Rightarrow \Delta l \uparrow \rightarrow p$

(c) $A \uparrow \Rightarrow \frac{F}{A} \downarrow \Rightarrow \Delta l \downarrow \rightarrow r, s$

(d) $Y \uparrow \Rightarrow \Delta l \downarrow \rightarrow q, r$

13 Fluid Mechanics

Introductory Exercise 13.1

1. $F_A = p_A A = h_A \rho g A$ and $F_B = h_B \rho g A$
   as $h_A < h_B \Rightarrow F_A < F_B$

2. $p = \dfrac{F}{A} = \dfrac{3 \times 10^4 \times 9.8}{425 \times 10^{-4}}$ N/m^2

   $= 6.92 \times 10^5$ N/m^2

3. $h_1 \rho_1 g = h_2 \rho_2 g$

   $\Rightarrow \dfrac{\rho_2}{\rho_1} = \dfrac{h_1}{h_2} = \dfrac{100}{12.5} = \dfrac{4}{5} = 0.8$

4. $(10 + 15) \rho_1 g + l_1 \rho g = (12.5 + 15) \rho_2 g + l_2 \rho g$

   $\Rightarrow (l_2 - l_1) \rho = 25 \rho_1 - 27.5 \rho_2$

   $\therefore \quad l_2 - l_1 = \dfrac{25 \times 1}{13.6} - \dfrac{27.5 \times 0.8}{13.6}$

   $= \dfrac{25 - 22}{13.6}$ cm $= 0.22$ cm

5. (i) $p_a = pxr_0 + h_1 \rho g = (76 + 20) \rho g$

   $= 96$ cm of Hg and $p_a - p_0 = 20$ cm of Hg
   $p_b = p_0 - h_2 \rho g = (76 - 18) \rho g = 58$ cm of Hg and $p_0 - p_b = 18$ cm of Hg

   (ii) 13.6 cm of water $\equiv$ 1 cm of Hg, *i.e.*, Hg in left limb will rise by 0.5 cm and on right limb, it will fall by 0.5 cm. Thus difference in Hg level will become 19 cm.

Introductory Exercise 13.2

1. $V \rho_s g = \dfrac{4}{5} V \rho_w g + \dfrac{1}{5} V \rho_l g$

   $\Rightarrow \rho_s = \dfrac{4}{5} \rho_w + \dfrac{1}{5} \rho_l$

   $= \dfrac{4}{5} \times 10^3 + \dfrac{1}{5} \times 13.5 \times 10^3$

   $\rho_s = \dfrac{17.5}{5} \times 10^3$ kg/m^3 $= 3.5 \times 10^3$ kg/m^3

2. $V \rho_s = 210$ g, $V(\rho_s - \rho_w) = 180$ g and $V(\rho_s - \rho_l) = 120$ g

   $\dfrac{\rho_s - \rho_l}{\rho_s} = \dfrac{180}{210} = \dfrac{6}{7}$

   $\Rightarrow 1 - \dfrac{6}{7} = \dfrac{\rho_w}{\rho_s}$

   $\Rightarrow \rho_s = 7 \rho_w = 7 \times 10^3$ kg/m^3

   $\dfrac{\rho_s - \rho_l}{\rho_s} = \dfrac{120}{210} = \dfrac{4}{7}$

   $\Rightarrow \quad 1 - \dfrac{4}{7} = \dfrac{\rho_l}{\rho_s}$

   $\Rightarrow \quad \rho_l = \dfrac{3}{7} \rho_s = 3 \times 10^3$ kg/m^3

3. As in equilibrium buoyant force and weight balance, so in accelerating lift both buoyant force and weight increase such that there is no change in volume of submerged wood.

4. As $w_{ap} = w - B$ as water is placed in water, so, apparent weight is zero.

5. If a body is placed on water, then it cannot displace water of more weight than its own weight as in such case, buoyant force will be more than weight and the case buoyant jump up the liquid, which is not possible.

6. $w + V\rho g = V\sigma g$

$$m = V(\sigma - \rho)$$

$$\Rightarrow \frac{m'}{\rho}(\sigma - \rho) = 120\left(\frac{\sigma}{\rho} - 1\right)$$

$$= 120 \text{ kg}\left(\frac{1}{0.6} - 1\right) = 120 \times \frac{0.4}{0.6} \text{ kg} = 80 \text{ kg}$$

7. The amount of water drink and the volume of water displaced due to weight of drank water remains same such that water level do not change.

$$V\rho g + mg = V\sigma g$$

$$\Rightarrow V(\sigma - \rho) = m$$

$$\Rightarrow a^3(\sigma - \rho) = m$$

$$\therefore \quad a = \left[\frac{m}{\sigma - \rho}\right]^{1/3}$$

$$= \left[\frac{0.5}{0.1 \times 10^3}\right]^{1/3}$$

$$= 10^{-1} \times 5^{1/3}$$

$$= 10 \times 5^{1/3} \text{ cm} = 17 \text{ cm}$$

9. $V\rho g = 0.6 V \sigma_1 g$

$$\Rightarrow \rho = 0.6\,\sigma_1 = 600 \text{ kg/m}^3$$

$$V\rho g = 0.85 V \sigma_2 g$$

$$\Rightarrow \sigma_2 = \frac{\rho}{0.85} = \frac{600}{0.85} \text{ kg/m}^3$$

$$= 705.88 \text{ kg/m}^3$$

10. $V\sigma g = mg$

$$\Rightarrow m = V\sigma = A h \sigma = \pi r^2 h \sigma$$

$$= \pi \times (0.8 \times 10^{-2})^2 \times 3 \times 10^{-2} \times 10^3 \text{ kg}$$

$$= \pi \times 0.64 \times 3 \text{ g}$$

$$= 6.03 \text{ g}$$

Introductory Exercise 13.3

1. $\Delta W = F \times s = 2\,Tl \times 0.5$

$$= 2 \times 7.2 \times 10^{-2} \times 10^{-1} \times 10^{-3}$$

$$= 1.44 \times 10^{-5} \text{ J}$$

2. $p = \frac{4T}{r}$, $dW = p\,dV = \frac{4T}{r} \cdot 4\pi r^2 dr$

$$= 16\pi\, Tr\, dr$$

$$W = \int_0^R 16\pi\, T\, r\, dr = 8\pi\, TR^2$$

3. $\frac{4}{3}\pi r^3 = 27 \times \frac{4}{3}\pi R^3$

$$\Rightarrow r = 3R$$

$$\Rightarrow R = r/3$$

$$\therefore \Delta S = 27 \times 4\pi R^2 - 4\pi r^2$$

$$= 4\pi\left(27 \times \frac{r^2}{9} - r^2\right) = 8\pi r^2$$

$$\Delta W = T\,\Delta S = 8\,\pi r^2 T$$

4. As wax seals thread capillaries which sucks water, that's why water cannot spread over cloth and cloth becomes waterproof.

5. (a) $v = \frac{p_1 - p_2}{4\,\pi\,L}(R^2 - r^2)$

$$v_0 = \frac{(p_1 - p_2)\,R^2}{4\,\eta\,l} = 3 \text{ m/s}$$

$$= v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$= 3\left(1 - \frac{100}{400}\right) = \frac{9}{4} = 2.25 \text{ m/s}$$

(b) At $r = R \Rightarrow v = 0$

6. (a) $\frac{dV}{dt} = \frac{\pi}{8}\left(\frac{R^4}{\eta}\right)\left(\frac{p_1 - p_2}{L}\right)$

$$= \frac{\pi}{8} \times \frac{4^4 \times 10^{-8}}{1.005 \times 10^{-1}} \times \frac{1400}{20}$$

$$= \frac{32\pi \times 70 \times 10^{-7}}{1.005}$$

$$= 7.005 \times 10^{-4} \text{ m}^3/\text{s}$$

(b) $\Delta p \propto \frac{1}{R^4}$

$$\Rightarrow \frac{\Delta p_2}{\Delta p_1} = \frac{R_1^4}{R_2^4} = 2^4$$

$$= 16$$

$$\Rightarrow \Delta p_2 = 16\Delta p_1 = 2.24 \times 10^4 \text{ Pa}$$

(c) $\frac{dV}{dt} \propto \frac{1}{\eta}$

$\therefore$ For new value of η.

$$\frac{dV}{dt} = \left(\frac{1.005}{0.469}\right)(7.0 \times 10^{-4})\ \text{m}^3/\text{s}$$

$$= 1.5 \times 10^{-3}\ \text{m}^3/\text{s}$$

AIEEE Corner

Subjective Questions (Level 1)

1. $mg = V \sigma g$

$\Rightarrow V \rho_1 g = \frac{3}{4} V \rho_2 g$

$\Rightarrow \rho_2 = \frac{4}{3} \rho_1$

2. $V\rho g = 0.095$ N and $V(\rho - \sigma) g = 0.071$

$\Rightarrow \frac{\rho - \sigma}{\rho} = \frac{71}{96}$

$\frac{\sigma}{\rho} = 1 - \frac{71}{96} = \frac{25}{96}$

$\Rightarrow \rho = \frac{96}{25} \sigma = 3840\ \text{kg/m}^3$

3. $(V + 2) \sigma g = m_1 g + m_2 g$

$\Rightarrow V = \frac{m_1 + m_2}{\sigma} - 2 = \frac{30\ \text{g}}{10/\text{cm}^3} - 2\ \text{cm}^3$

$= 28\ \text{cm}^3$

4. $T = B - W = V(\sigma - \rho) g = mg\left(\frac{\sigma}{\rho} - 1\right)$

$= 71.2\left(\frac{1}{0.75} - 1\right)$ N

$= \frac{71.2}{3}$ N $= 23.73$ N

5. $V \sigma g = V \rho g + mg$

$\Rightarrow V = \frac{m}{\sigma - \rho} = \frac{50}{1000 - 850}\ \text{m}^3$

$= \frac{1}{3}\ \text{m}^3 = 0.3\ \text{m}^3$

6. (a) $\rho V = 10$ g, $(\rho - \sigma) V = 8$ g

$\Rightarrow 1 - \frac{\sigma}{\rho} = 0.8 \Rightarrow \frac{\sigma}{\rho} = 0.2$

or $\rho = \frac{\sigma}{0.2} = 5\sigma$

$= 5000\ \text{kg/m}^3$

$= 6\ \text{g/cm}^3$

$\therefore V = \frac{10\ \text{g}}{p} = 2\ \text{cm}^3 = 2 \times 10^{-6}\ \text{m}^3$

(b) $(\rho - \sigma') V = 8.5\ \text{g} \Rightarrow 1 - \frac{\sigma'}{\rho} = 0.85$

$\Rightarrow \frac{\sigma'}{\rho} = 0.15$

$\Rightarrow \sigma' = 0.15 \rho = 750\ \text{kg/m}^3$

7. $B = V \sigma g = 1\ \text{cm}^3 \times 1\ \text{g/cm}^3 \times 980\ \text{cm/s}^2 = 980$ dyne

$W_{\text{ap}} = 20 \times 980$ dyne $+ 980$ dyne

$= 20580$ dyne $= 0.206$ N

8. $a^3 \rho_i g = a^2 x \sigma_m g + a^2 (a - x) \sigma_w g$

$a\rho_i = x\sigma_m + (a - x)\sigma_w$

$\Rightarrow a(\rho_i - \sigma_w) = x(\sigma_m - \sigma_w)$

$\Rightarrow \frac{x}{a} = \frac{\rho_i - \sigma_w}{\sigma_m - \sigma_w} = \frac{7.7 - 1}{13.6 - 1} = \frac{6.7}{12.6} = 0.532$

$\Rightarrow x = 31.9$ mm is in mercury and 28.1 mm in water.

9. $a = \frac{F}{m} = \frac{B - W}{m}$

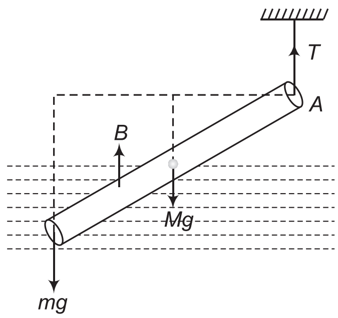
$= \frac{V(\sigma - \rho) g}{m}$

$= \left(\frac{\sigma}{\rho} - 1\right) g = \left(\frac{1}{0.4} - 1\right) g$

$= \frac{3}{2} g = 14.7\ \text{m/s}^2$

$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2 \times 2.9}{14.7}} = 0.63$ s

10. $B + T = (M + m)\,g \Rightarrow T = 18g - B$ ...(i)



Equating the torques above point A, we have

$$(12\,g)\left(\frac{l}{2}\right) + (6g)(l) = B\left(\frac{3l}{4}\right)$$

or $B = 16g$ ...(ii)

Substituting in Eq. (i) we have

$$T = 2g = 19.6\,\text{N}$$

$$B = \frac{V}{2} \times (1000)(9.8) = 16 \times 9.8$$

$\therefore\ V = 32 \times 10^{-3}\ \text{m}^3$

11. (a) $T = B - W = V\,(\sigma - \rho)\,(g + a)$

$$= \frac{m}{\rho}\,(\sigma - \rho)\,(g + a)$$

$$= M\left(\frac{\sigma}{\rho} - 1\right)(g + a)$$

$$= 2\,(2 - 1)\,(10 + 2) = 24\ \text{N}$$

(b) $a = \frac{F}{m} = \frac{B - W}{m} = \frac{24\,\text{N}}{2\ \text{kg}} = 12\ \text{m/s}^2$

12. $F = kx \Rightarrow p = \frac{kx}{A}$ or $x = \frac{pA}{k}$

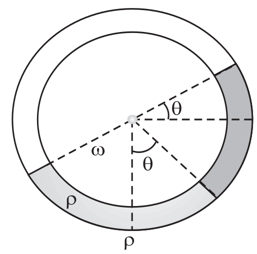
(a) $x_1 = \frac{131 \times 10^3 \times 0.5 \times 10^{-4}}{60}$ m

$$= \frac{131 \times 0.5}{6}\,\text{cm} = 10.9\ \text{cm}$$

(b) $x_2 = \frac{30 \times 10^3 \times 0.5 \times 10^{-4}}{60}$ m $= 2.5$ cm

13. $(r - r\sin\theta)\,\rho g = (r - r\cos\theta)\,\rho g$

$+ (r\sin\theta + r\cos\theta)\,\sigma\,g$



$$(\cos\theta - \sin\theta)\,\rho = (\cos\theta + \sin\theta)\,\sigma$$

$$\cos\theta\,(\rho - \sigma) = \sin\theta\,(\rho + \sigma)$$

$$\therefore\quad \tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$$

14. $p_L = p_R \Rightarrow \frac{600\,g}{800 \times 10^{-4}}$

$$= 8\sigma\,g + \frac{F}{25 \times 10^{-4}}$$

$$\therefore F = 25 \times 10^{-4}\left[\frac{600}{800 \times 10^{-4}} - 8\,\sigma\right]g$$

$$= \left(\frac{600}{32} - 2 \times 10^{-2} \times 780\right)g$$

$$= (18.75 - 15.6)\,g = 31\,\text{N}$$

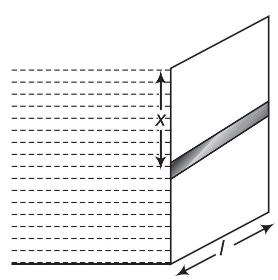
15. $(h_1 + l + h_2)\rho_w g = l\,\rho_w\,g + h_2\,\rho_m\,g$

$$h_1\rho_w = h_2\,(\rho_m - \rho_w)$$

$$\Rightarrow\quad h_1 = \frac{\rho_m - \rho_w}{\rho_w}\,h_2$$

$$= \frac{13.6 - 1}{1} \times 1\ \text{cm} = 12.6\ \text{cm}$$

16.



$$p(x) = x\,\rho g$$

(a) $dF\,(x) = \rho g\,x\,.\,l dx = \rho g l x\,dx$

$$F = \rho g l \int_0^h x\,dx = \frac{1}{2}\rho g\,l h^2$$

(b) $d\tau(x) = dF(x) \cdot (h - x)$

$$= \rho gl\,(h - x)\,x\,dx$$

$$= \rho gl\,(hx - x^2)\,dx$$

$$\tau = \rho gl \int_0^h (hx - x^2)\,dx$$

$$= \rho gl\left(\frac{1}{2}h^3 - \frac{1}{3}h^3\right)$$

$$= \frac{1}{6}\rho glh^3$$

(c) $\frac{1}{2}\rho glh^2 \times y = \frac{1}{6}\rho glh^3$

$$\Rightarrow \quad y = \frac{2}{6}h = \frac{1}{3}h$$

17. $15\rho_w g = x\rho_m g$

$$\Rightarrow \quad x = \frac{15\rho_w}{\rho_m} = \frac{15}{13.6}\text{ cm} = 1.10\text{ cm}$$

$$h = (15 - 1.10)\text{ cm} = 13.90\text{ cm}$$

(a) $\Delta p = 15\rho_w g$

$$= 15 \times 10^{-2}\text{ m} \times 10^3\text{ kg/m}^3 \times 9.8\text{ m/s}^2$$

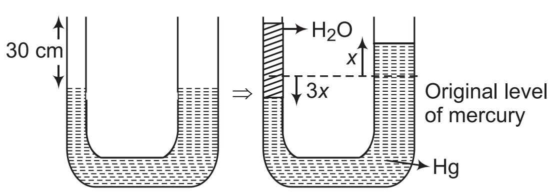
$$= 1470\text{ N/m}^2 = 1470\text{ Pa}$$

18. $h_1\rho_w g = h_2\rho_0 g$

$$\Rightarrow \quad h_1 = h_2\frac{\rho_0}{\rho_w}$$

$$= 20 \times \frac{0.7}{1} = 18\text{ cm}$$

19.



$h_1\rho_1 = h_2\rho_2$

$\therefore \quad (30 + 3x)(1) = (4x)(13.6)$

Solving we get, $x = 0.584$ cm.

20. $p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

$$20\text{kPa} + 0 + \frac{1}{2}\rho \times 3^2 = p + \rho g \times 1 + \frac{1}{2}\rho \times 4^2$$

$$p = 20\text{kPa} - \rho g + \frac{1}{2}\rho\,(3^2 - 4^2)$$

$$= 20 \times 10^3 - 9.8 \times 10^3 + \frac{1}{2} \times 10^3\,(-7)$$

$$= (20 - 13.3) \times 10^3\text{ Pa}$$

$$= 6.7\text{ kPa}$$

$$18\text{kPa} + 0 + \frac{1}{2}\rho \times 0^2$$

$$= p' + pg \times 1 + \frac{1}{2}\rho \times 0^2$$

$$p' = 18\text{ kPa} - 9.8\text{ kPa} = 8.2\text{ kPa}$$

21. $v = \sqrt{2gd}$, $R = vt = t\sqrt{2gd}$ and $h = \frac{1}{2}gt^2$

$$\therefore \; h = \frac{1}{2}g\frac{R^2}{2gd} = \frac{R^2}{4d}$$

$$\Rightarrow d = \frac{R^2}{4h}$$

22. $Au = av$ and $Fu\,dt = \frac{1}{2}mv^2$

$$= \frac{1}{2}\rho \,.\, avdt\,.\,v^2$$

$$Fu = \frac{1}{2}\rho a\,v^3$$

$$F\frac{av}{A} = \frac{1}{2}\rho\,a\,v^3 \Rightarrow v^2 = \frac{2F}{\rho A}$$

$$\Rightarrow \; v = \sqrt{\frac{2F}{\rho A}}$$

23. $p_0 = p_0 + \rho_w gh - \frac{1}{2}\rho_a\,v^2$

$$\Rightarrow v = \sqrt{\frac{2\rho_N gh}{\rho_a}}$$

$$\therefore \; v = \sqrt{\frac{2 \times 10^3 \times 10^{-2} \times 9.8}{1.3}} = 12.28\text{ m/s}$$

24. $\frac{1}{2}\rho v^2 = \frac{F}{A} + \rho gh + \frac{1}{2}\rho u^2 = \frac{F}{A} + \rho gh + \frac{1}{2}\rho\frac{a^2}{A^2}v^2$

$$\frac{1}{2}\rho\left(1 - \frac{a^2}{A^2}\right)v^2 = \frac{F}{A} + \rho gh$$

$$v = \sqrt{\frac{2}{\rho\left(1 - \frac{a^2}{A^2}\right)} - \left(\frac{F}{A} + \rho gh\right)}$$

$$= \sqrt{\frac{2}{10^3\left(1-\frac{10^{-4}}{0.5}\right)}\left(\frac{20}{0.5}+10^3\times10\times0.5\right)}$$

$$= \sqrt{\frac{2\times5.04}{1-0.002}} = \sqrt{\frac{2\times5.04}{0.998}} = 3.18 \text{ m/s}$$

25. $\frac{1}{2}\rho v_A^2 + \rho g h_A = \frac{1}{2}\rho v_B^2 + h_B \rho g$

$v_A . A_A = v_B . A_B$ ...(i)

$v_A^2 - v_B^2 = 2(h_B - h_A)g$ ...(ii)

$v_A^2\left(1-\frac{A_B^2}{A_A^2}\right) = 2(h_D - h_A)\,g$

$$v_A = \sqrt{\frac{2\Delta hg}{1-\frac{A_B^2}{A_B^2}}} = \sqrt{\frac{2\times2\times10^{-2}\times10}{1-\frac{2^2}{4^2}}}$$

$$= \sqrt{\frac{0.4}{3/4}} = \sqrt{\frac{1.6}{3}}$$

$= 0.73$ m/s

$v_B = v_A \cdot \frac{A_A}{A_B} = 0.73 \text{ m/s} \times \frac{4}{2} = 1.46 \text{ m/s}$

26. $\frac{dV}{dt} = Av = 500 \text{ cm}^3\text{/s}$

$\therefore v_1 = 100 \text{ cm/s} = 1 \text{ m/s}$

$v_2 = 250$ cm/s

$= 2.5$ m/s

$\rho_m \ \Delta h = \frac{1}{2}\rho_w\,(v_2^2 - v_1^2)$

$\Delta h = \frac{v_2^2 - v_1^2}{2g\rho_m\rho_w}$

$= \frac{6.25-1}{2\times9.8\times13.6} = 1.97$ cm

27. $v_t = \frac{2}{9}\cdot\frac{r^2\,(\rho-\sigma)\,g}{\eta}$

$= \frac{2}{9}\cdot\frac{4\times10^{-10}\ (2-1)\times10^3\times10}{1\times10^{-3}}$

$= \frac{8}{9}\times10^{-3}$ m/s $= 0.89$ mm/s

28. $Q = \frac{\pi}{8}\cdot\frac{R^4}{\eta}\ \frac{\Delta p}{L} \Rightarrow \Delta p = \frac{8\eta\,LQ}{\pi R^4}$

$= \frac{8\times4\times10^{-3}\times10^{-3}\times0.66\times10^{-3}}{\pi\times16\times10^{-24}}$

$h\rho g = \frac{2\times0.66}{\pi}\times10^{+15}$

$\Rightarrow h = 3.15\times10^9 \text{ m} = 3.15\times10^{12} \text{ cm}$

29. $v_t \propto r^2$

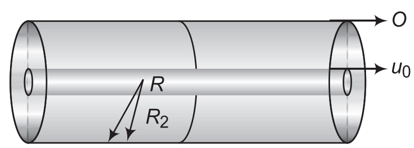
$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}\ r$

$\therefore \quad \frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2$

$= \left(\frac{R}{r}\right)^2 = 2^{2/3}$

$\Rightarrow \quad v_2 = 2^{2/3}\ v$

30. $v = \frac{\Delta p}{4\eta\,L}(R_2^2 - r^2)$



$u_0 = \frac{\Delta p}{4\eta L}(R_2^2 - R^2)$

$\therefore\ v = \frac{u_0}{R_2^2 - R^2}\cdot(R_2^2 - r^2)$

31. $F = -\eta A\,.\,\frac{dv}{dx} = +\,10^{-2}\times0.1\times10\times\frac{2}{1}$

$= 0.02$ N

32. Stress $= \frac{F}{A} = \eta\,\frac{dv}{dx} = 10^{-2}\times0.1\times\frac{5}{5}$

$= 10^{-3}\ \text{N/m}^2$

33. $h = \frac{2T\cos\theta}{r\,\rho g}$

$\Rightarrow \frac{T_2}{T_1} = \frac{h_2\rho_2/\cos\theta_2}{h_1\rho_1/\cos\theta_1}$

$= \frac{3.4\times13.6\times6.71}{9\times1/1}$

$= \frac{3.4\times13.6}{9\times0.71} = 7.24$

34. $p_a = p_0 + \frac{4T}{a}$, $p_b = p_0 + \frac{4T}{b}$

$p_a V_a = nRT$ and $p_b V_b = 2nRT$

$$\left(p + \frac{4T}{a}\right) \times \frac{4}{3} \pi a^3 \times 2 = \left(p + \frac{4T}{b}\right) \times \frac{4}{3} \pi b^3$$

$$2a^3 \left(p + \frac{4T}{a}\right) = b^3 \left(p + \frac{4T}{b}\right)$$

$$p(2a^3 - b^3) = 4T(b^2 - 2a^2)$$

$$\therefore \quad T = \frac{p(2a^3 - b^3)}{4(b^2 - 2a^2)}$$

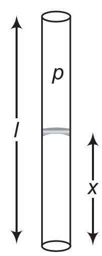
35. $p_0 = p + \frac{2T}{R} = p + h\rho g$

$$\Rightarrow R = \frac{2T}{h\rho g} = \frac{2 \times 0.07}{10^{-2} \times 10^3 \times 10} \text{ m}$$

$$= 1.4 \times 10^{-3} \text{ m} = 1.4 \text{ mm}$$

36. $p = p_0 + \frac{2T}{R}$ and $p_0\, l\, A = p(l - x)\, A$

$$\therefore \quad p = p_0 \frac{l}{l - x}$$



$$\therefore \ p_0 \frac{l}{l - x} - p_0 = \frac{2T}{R}$$

$$p_0 \left[\frac{l - l + x}{l - x}\right] = \frac{2T}{R}$$

$$\frac{p_0 R}{2T} = \frac{l - x}{x} = \frac{l}{x} - 1$$

$$\text{or} \quad x = \frac{l}{\frac{p_0 R}{2T} + 1} = \frac{0.11 \text{ m}}{\frac{10^5 \times 1 \times 10^{-5}}{2 \times 5.06 \times 10^{-2}} + 1}$$

$$= \frac{0.11}{\frac{100}{10.12} + 1} = 1.01 \text{ cm}$$

If seal is broken then water will start rising in the capillary and will go upto a height, $h = \frac{2T \cos\theta}{r\rho g}$

$$h = \frac{2 \times 5.06 \times 10^{-2} \cos\theta}{10^{-5} \times 10^3 \times 10} = 1.01 \cos\theta,$$

above *i.e.*, it will make convex meniscus as the tube is of insufficient length.

Objective Questions (level 1)

1. Net force is zero, when a body moves with constant velocity.

2. $[x] = \frac{[\eta]}{[P]} = \frac{[MLT^{-2}][L^{-2}][T]}{[ML^{-3}]}$

$$= [L^2T^{-1}] = [M^0L^2T^{-1}]$$

3. $\eta \propto \frac{1}{T}$ as with increase in temperature fluidity increases.

4. $$\left(p_0 + \frac{4T}{a}\right) \times \frac{4}{3} \pi a^3 + \left(p_0 + \frac{4T}{b}\right) \times \frac{4}{3} \pi b^3 = \left(p_0 + \frac{4T}{c}\right) \times \frac{4}{3} \pi c^3$$

$$p_0(a^3 + b^3) + 4T(a^2 + b^2) = p_0 c^3 + 4Tc^2$$

(a) In vacuum, $p_0 = 0$

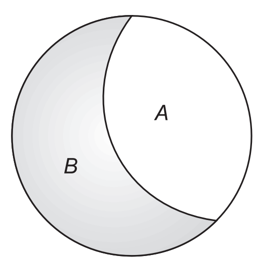
$$\Rightarrow c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = 5 \text{ cm}$$

5. $p_B < p_0$ while, $p_A = p_C = p_0$

6. $ku + V\sigma g = V\rho g \Rightarrow ku = V(\rho - \sigma) g$

$$\therefore \ ku = m\left(1 - \frac{\sigma}{\rho}\right) g \Rightarrow u = \frac{mg}{k}\left(1 - \frac{\sigma}{\rho}\right)$$

7. Because of the pull due to surface lenses of liquid at side B, the film will take a concave shape in side A.



8. $3p_0 = p_a + \frac{1}{2}\rho v^2$

$$\Rightarrow v = \sqrt{\frac{4p_0}{\rho}} = \sqrt{\frac{4 \times 10^3}{10^3}} = \sqrt{400} \text{ m/s}$$

9. $v \propto r^2$ and $m \propto r^3 \Rightarrow v \propto m^{2/3}$

$$\therefore \ \frac{v_2}{v_1} = \left(\frac{m_2}{m_1}\right)^{2/3} = \left(\frac{8m}{m}\right)^{2/3} = 4 \Rightarrow v_2 = 4v$$

10. $F = -\eta A \frac{dv}{dy}$

11. $W = T \Delta A = \sigma (n\, 4\, \pi r^2 - 4\pi R^2)$

$= 4\pi\sigma (nr^2 - R^2)$

Again $R^3 = nr^3$,

$W = 4\pi \sigma (n \,.\, n^{-2/3} R^2 - R^2)$

$\therefore \quad W = 4\pi R^2 (N^{1/3} - 1) \sigma$

12. $p_1 = p_0 + \frac{4T}{R_1}$ and $p_2 = p_0 + \frac{4T}{R_2}$

$\Rightarrow p_2 - p_1 = 4T\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{4T}{R}$

$\therefore \quad \frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$

13. Pressure decreases along the direction of flow for viscous fluid even in case of uniform cross-section and here as velocity increases with decreasing cross-section presses further decreases. So, correct graph is (c).

14. $mg = 6 \pi \eta r v_t$

$\frac{4}{3} \pi r^3 \rho g = 6\pi \eta r v_t$

$r = \sqrt{\frac{9\eta v_t}{2 \rho g}} = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 30}{2 \times 10^3 \times 10}}$

$= \sqrt{27 \times 0.9 \times 10^{-8}} = 4.93 \times 10^{-4}$ m

$\simeq 0.5$ mm

15. $h = \frac{2T \cos \theta}{r \rho g} = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = 90°$

16. $v_t \propto r^2 \Rightarrow \frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow v_2 = \frac{v_1}{4} = 5$ cm/s

17. $p_0 = h \rho g$

$\Rightarrow p_x + \frac{h}{5} \rho g = p_0 = h\rho g$

$\Rightarrow p_x = \frac{4}{5} h \rho g = 0.8\, p_0 = 0.8 \times 10^5$ Pa

18. $V \rho G = \frac{2}{3} V \sigma_w g \Rightarrow \rho = \frac{2}{3} \sigma_w$ and $V\rho g = \frac{1}{3} V \sigma_l g \Rightarrow \sigma_l = 3\rho$ or $\sigma_l = 3 \times \frac{2}{3} \sigma_w = 2\sigma_w$

19. In sugar solution ice was less immersed due to high density of liquid. But with melting ice, density of sugar solution gets decreased and ice gets more immersed, displacing more liquid. So, liquid level will increase.

20. As apparent weight of the downward accelerating body decreases. So, that of water also decreases and this leads to decrease in magnitude of buoyant force.

21. Two limbs will have liquids at same level only when their densities are equal *i.e.*, $\rho = \rho_w$

22. As $F_1 > F_2$ and $F_1 = F_2 + C$, again, $f_2 = F_2 + B$ and $f_1 = F_1$ thus $f_1 > f_2$ only when, $C > B$ but not in all other cases. Thus, $f_1 > f_2$ is not always correct.

23. $p_1 = p_2 + \frac{1}{2} \rho v^2 \Rightarrow v = \sqrt{\frac{2(p_1 - p_2)}{\rho}}$

$v = \sqrt{\frac{2 \times \frac{1}{2} \times 10^5}{10^3}} = 10$ m/s

24. $Mg - V \sigma g = Ma$,

$V \sigma g - (M - m) g = (M - m) a_0$

$M (g - a_0) = V \sigma g = (M - m)(g + a_0)$

$M (g - a_0) - M(g + a_0) = -m (g + a_0)$

$-2Ma_0 = -m (g + a_0)$

$\Rightarrow \quad m = \frac{2Ma_0}{g + a_0}$

25. $W = Mgh$

$\Rightarrow M = \frac{W}{gh} = \frac{500}{10 \times 0.5 \times 10^{-2}} = 10^4$ kg

26. $V\sigma_1 + V\sigma_2 = 2V \times 4 \times \sigma_w$

$\Rightarrow \sigma_1 + \sigma_2 = 8\sigma_N$ ...(i)

$V_1\sigma_1 = V_2\sigma_2$; $V_1\sigma_1 + V_2\sigma_2$

$= (V_1 + V_2) \,.\, 3\sigma_w$

$\Rightarrow 2V_1\sigma_1 = \left(V_1 + \frac{V_1\sigma_1}{\sigma_2}\right) 3\sigma_w$

$$2\sigma_1 = 3\sigma_w \left(1 + \frac{\sigma_1}{\sigma_2}\right) \quad \ldots\text{(ii)}$$

$$2(8\sigma_w - \sigma_2) = 3\sigma_w \left(1 + \frac{(8\sigma_w - \sigma_2)}{\sigma_2}\right)$$

$$16\sigma_w - 2\sigma_2 = 3\sigma_w \left(8\frac{\sigma_w}{\sigma_2}\right)$$

$$\Rightarrow 8\sigma_w \sigma_2 - \sigma_2^2 = 12\sigma_w^2$$

$$\therefore \sigma_2^2 - 8\sigma_w\sigma_2 + 12\sigma_w^2 = 0$$

$$\sigma_2 = \frac{1}{2}[8\sigma_w \pm \sqrt{64\sigma_w^2 - 48\sigma_w^2}]$$

$$= \frac{1}{2}[8\sigma_w \pm 4\sigma_w]$$

$= 6\sigma_w$ or $2\sigma_w$ Similarly for, $\sigma_1 = 2\sigma_w$ or $6\sigma_w$

$\therefore$ Specific gravity of them could be 6 and 2.

27. $A \times 0.5 \times 0.9 \times 10^3 \times g + 100g$

$$= A \times 0.5 \times 10^3 \times g$$

$$\Rightarrow 100 = A \times 0.5 \times 0.1 \times 10^3$$

$$\Rightarrow A = \frac{100}{50} = 2\text{ m}^2$$

28. $(V - v)\rho_g = 38.2$

while $V\sigma = 38.2 - 36.2 = 2$ g

$$\frac{2}{\sigma}\rho - V\rho = 38.2$$

or $\frac{2 \times 19.3}{1} - 38.2 = V \times 19.3$

or $V = \frac{0.4}{19.3}\text{ cm}^3 = 0.02\text{ cm}^3$

29. $F = \frac{1}{2}\rho\, gbh^2 \Rightarrow \frac{F}{b} = \frac{1}{2}\rho g h^2 = \frac{1}{2}\rho g D^2$

30. $av =$ constant and $v = \sqrt{\frac{2gh}{\frac{p^2}{a^2} - 1}}$. So v do not depend upon p.

31. $W = F\,.\,v = p\,.\,A\,.\,\frac{s}{t} = \frac{pV}{t}$

$$= \frac{100 \times 10^{-3} \times 13.6 \times 10^3 \times 10 \times 60 \times 10^{-6}}{60/72}$$

$= 1.36 \times 0.72\text{ W} = 0.98\text{ W}$

32. Accordings to continuity, $Q_{\text{in}} = Q_{\text{out}}$

$$4 \times 10^{-6} = 6 \times 10^{-6} + 5 \times 10^{-6} + 2 \times 10^{-6} + 4 \times 10^{-6} + 8 \times 10^{-6} + Q$$

$$Q = -21 \times 10^{-6}\text{ m}^3\text{/s}$$

33. $Mg = a^2 \times \frac{3}{4}a \times p \times g$

$$\Rightarrow a^3 = 4M/3\rho$$

$$\Rightarrow a = (4M/3\rho)^{1/3}$$

34. $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \frac{T_w}{T_m} = \frac{rh\rho g/2\cos\theta}{rh'\rho' g/2\cos\theta'} = \frac{h\,\rho\cos\theta'}{h'\rho'\cos\theta}$$

$$= \frac{10 \times 1 \times \cos 135^\circ}{-3.42 \times 13.6 \times \cos 0^\circ}$$

$$= \frac{5r_2}{3.42 \times 13.6} = \frac{1}{6.5}$$

35. $h = \frac{2T\cos\theta}{r\rho g} = \frac{2T\cos\theta}{\sqrt{A/\pi}\,\rho g}$

$$\Rightarrow h \propto \frac{1}{\sqrt{A}}$$

$$\Rightarrow \frac{h_2}{h_1} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{A_1}{A/2}} = \sqrt{2}$$

$$\Rightarrow h_2 = \sqrt{2}h_1 = 20\sqrt{2}\text{ cm}$$

36. Volume flow rate $= av = a\sqrt{2gh}$

$\therefore$ In steady state,

$$10^4 = (1)\sqrt{2 \times 1000 \times h}$$

$\therefore \quad h = 5$ cm

37. $v_2 = \frac{A_1v_1}{A_2} = \frac{10^{-2} \times 2}{0.5 \times 10^{-2}} = 4$ m/s

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow p_2 = 8000\text{Pa} + \frac{1}{2} \times 10^3 \times 4 - \frac{1}{2} \times 10^3 \times 16$$

$$p_2 = 8000\text{ Pa} + 2000\text{ Pa} - 8000\text{ Pa}$$

$$= 2000\text{ Pa}$$

38. $8 \times 4/3\,\pi r^3 = 4/3\,\pi R^3$

$$\Rightarrow R = 2r, v_r \propto r^2$$

$$\Rightarrow v_2 = 4V_1 = 24\text{ cm/s}$$

JEE Corner

Assertion and Reason

1. The magnitude of normal force per unit area is defined as pressure and so it is scalar quantity, not a vector, so assertion is false.

2. Assertion and reason are true with correct explanation.

3. Assertion is false as $p(h) = p_0 + h\rho g$ while $p(2h) = p_0 + 2h\rho y$
i.e., $p(2h) \neq 2p(h)$

4. $V \rho g = W$ while, $V(\rho - \sigma) g = \frac{2W}{3}$

$$\Rightarrow \frac{\rho - \sigma}{\rho} = \frac{2}{3}$$

or $1 - \frac{2}{3} = \frac{\sigma}{\rho}$

$\Rightarrow \rho = 3\sigma$. Assertion is true and reason is correct explanation as

$$\frac{\rho}{\sigma} = \frac{W}{W - \frac{2}{3}W} = 3$$

5. Assertion is false as due to different cross-sectional area and height of oil columns will be different producing different pressures which lead to different water levels in the limbs of U-tube.

6. Assertion is not always true as it also depends upon the height from ground.

7. Assertion and reason are both true but not correct explanation as v depends upon P while R depends upon v.

8. Assertion is true and reason is correct explanation as acceleration or retardation depends upon force of buoyancy which in turn depends upon densities of ball and liquid.

9. Assertion is false as barometer measures air pressure but on moon, there is no atmosphere to exert pressure, the barometer reading will be zero.

10. At P, liquid is in motion such that at that point pressure is less than atmospheric pressure. So, assertion is false.

11. Assertion and reason are both false as force of buoyancy can be negligible only when relative density of the body in very high in comparison to that of air, it is not true.

Objective Questions (Level 2)

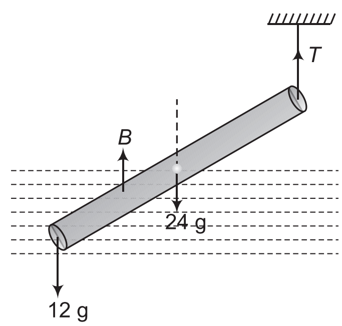
1. $\tan\theta = \frac{a}{g} = \frac{h}{h/2} = 2$

$\Rightarrow a = 2g$

2. As ice melts, its volume decreases, so oil level will decrease, and as volume of water increases, so level of interface will rise.

3. As water is under free fall such that it do not exert pressure on side wall, so water do not come out of the hole.

4. For translational equilibrium
$T + B = 12\text{ g} + 24\text{ g}$



$\therefore T = 36g - B \Rightarrow T < 36g$

$$24g \times 6 + 12g \times 12 - B \times 9 = 0$$

$$\Rightarrow B = \frac{2 \times 24 \times 6g}{9} = 32\,g \Rightarrow T = 4\text{ g}$$

$$B = \frac{V}{2}\sigma g = 32g$$

$$\Rightarrow V = \frac{64}{\sigma} = 6.4 \times 10^{-2}\text{ m}^3$$

5. $Q = Av \Rightarrow v = \frac{Q}{A} = \frac{120 \times 10^3 \text{ cm}^3 / 120 \text{ s}}{5 \text{ m}^2}$

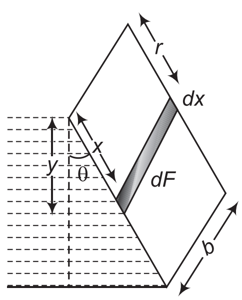
$= 200 \text{ cm/s} = 2 \text{ m/s}$

$v_2 = \frac{A_1 v_1}{A_2} = \frac{5 \times 2}{1} = 10 \text{ m/s}$

$t = \sqrt{\frac{2h}{g}}$ and $R = vt = 10 \text{ m/s}$

$\sqrt{\frac{2 \times 1}{10}} \text{ s} = 2\sqrt{5} \text{ m} = 4.47 \text{ m}$

6. $dF = \rho g y \, b \, dx$



$F = \int_0^{h \sec\theta} \rho g \, x \cos\theta \, b \, dx$

$= \rho \, g \, b \cos\theta \, \frac{1}{2} x^2 \Big|_0^{h \sec\theta}$

$= \frac{1}{2} \rho \, g \, b \, h^2 \sec\theta$

7. $v^2 = 2 \frac{(\sigma - \rho) \, g \, d}{\rho} = 2gh$

$\Rightarrow \quad d = \frac{\rho h}{\sigma - \rho}$

8. $\tan\theta = \frac{a}{g} = \frac{\Delta h}{L}$

$\Delta h = \frac{aL}{g}$

9. $p_A = p_{atm} - \frac{2\sigma}{r} = p_{atm} - \frac{2\sigma \cos\theta}{R}$ and

$p_B = p_A + h \rho g = p_{atm}$

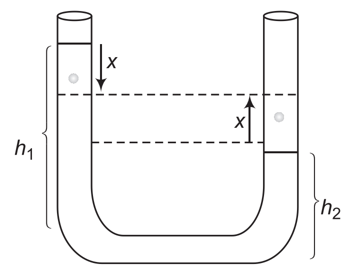
10. $Q = A_1 v_1 = A_2 v_2$

$\Rightarrow L^2 \sqrt{2gh} = \pi R^2 \sqrt{2gh4h}$

$\Rightarrow \quad L^2 = 2\pi R^2$

$\therefore \quad R = L / \sqrt{2\pi}$

11. $x = \frac{h_1 - h_2}{2}$



$W_g = Ax\rho g \, . \, x = A\rho g x^2$

$= \frac{1}{4} A\rho g \, (h_1 - h_2)^2$

12. $\Delta p = h_1 \rho_1 g + h_2 \rho_2 g$

$= 0.1 \times 0.6 \times 10^3 \times 10 + 0.02 \times 10^3 \times 10$

$= 600 + 200 = 800 \text{ Pa}$

13. $\Delta p = \frac{1}{2} \rho v^2$

$\Rightarrow v = \sqrt{\frac{2\Delta p}{p}} = \sqrt{\frac{2 \times 5 \times 10^5}{10^3}}$

$= 10\sqrt{10} = 32 \text{ m/s}$

14. $v_A = v_B = v_0$

$\Rightarrow p_A = p_B$

$\Rightarrow h = 0$

15. $F = \frac{dp}{dt} = 2v \frac{dm}{dt} = 2v \frac{d}{dt} (\rho V)$

$= 2v \, \rho v \, A = 2\rho A \, v^2$

16. $p_A = p_0 + \rho g h$

$\Rightarrow \Delta p = \rho g h$

$p_A - p_0 = \rho a l$

$\Rightarrow \Delta p = \rho a l$

17. $\pi \frac{d^2}{4} (2L - 2 \text{ cm/h } t) \rho g = B = \pi \frac{d^2}{4} \, . \, y \, \sigma \, g$

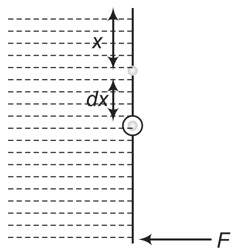
$\Rightarrow y = \frac{\rho}{\sigma} (2L - 2 \text{ cm/h } t)$

$= \frac{1}{2} (2L - 2 \text{ cm/h } t)$

$= L - 1 \text{ cm/h } t$

$\Rightarrow \frac{dy}{dt} = -1 \text{ cm/h}$

18. $dF = \rho gy \cdot bdy = \rho gb\ ydy$



$$\Rightarrow F(0 \to h/2) = \rho gb \int_0^{h/2} ydy = \frac{1}{8}\rho g\ bh^2$$

$$\tau(0 \to h/2) = \int dF > (h/2 - y)$$

$$= \int_0^{h/2} \left(\frac{h}{2} - y\right) \cdot \rho gby\ dy$$

$$= \rho gb \int_a^{h/2} \left(\frac{h}{2}y - y^2\right) dy$$

$$= \rho gb \left[\frac{h}{2}\frac{y^2}{2} - \frac{y^3}{3}\right]_0^{h/2}$$

$$= \rho gbh^3 \left(\frac{1}{16} - \frac{1}{24}\right) = \frac{1}{48}\rho gbh^3$$

$$\tau(h/2 - h) = \int_{h/2}^{h} (y - h/2)\rho gby\ dy$$

$$= \rho gb \left[\frac{y^3}{3} - \frac{h}{2} \cdot \frac{y^2}{2}\right]_{h/2}^{h}$$

$$= \rho gb \left[\frac{1}{3}\left(h^3 - \frac{h^3}{8}\right) - \frac{h}{4}\left(h^2 - \frac{h^2}{4}\right)\right]$$

$$= \rho gbh^3 \left(\frac{1}{3} \cdot \frac{7}{8} - \frac{3}{16}\right)$$

$$= \rho gbh^3\ \frac{14 - 9}{48} = \frac{5\rho gbh^3}{48}$$

$$\therefore \left(\frac{5}{48} - 1\right)\rho gbh^3 = F \times \frac{h}{2}$$

$$\frac{1}{12}\rho gbh^3 = \frac{1}{2}Fh \Rightarrow F = \frac{1}{6}\rho gbh^2 = \frac{1}{6}\rho g$$

19. $(r - r\cos 60°)\,\rho_1 g = (r - r\cos 30°)\ 0$

$(\cos 30° - \cos 60°)\rho_1 = (\cos 30° + \cos 60°)\rho_2$

$$\frac{\rho_1}{\rho_2} = \frac{\cos 30° + \cos 60°}{\cos 30° - \cos 60°} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

20. $F = \dfrac{dp}{dt} = \dfrac{d}{dt}(mv) = (v_1 + v_2)\dfrac{dm}{dt}$

$= (v_1 + v_2)\rho V$

21. $v_t = \dfrac{2r^2\ (\rho - \sigma)\ g}{9\eta}$

$$= \frac{2 \times 9 \times 10^{-6}\ (2 \times 1260 - 1260) \times 10}{9 \times 1.26}$$

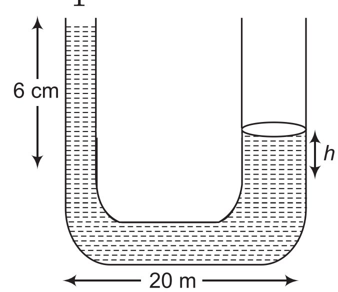
$= 2 \times 10^{-2}$ m/s $= 2$ cm/s

$t = \dfrac{d}{v_1} = \dfrac{(20 - 10)\ \text{cm}}{2\ \text{cm/s}} = 5$ s

22. $(2\rho)(g)\left(\dfrac{h^2}{2}\right) = (3\rho)(g)\left(\dfrac{R^2}{2}\right)$

$\therefore\quad h = \sqrt{\dfrac{3}{2}}R$

23. $l_w = \dfrac{60}{1^2} = 60$ cm



$60\rho_w g = h\rho_l g \Rightarrow h = \dfrac{60 \times 1}{4} = 15$ cm

$\therefore\quad l_1 = 15$ cm $+ 20$ cm $= 35$ cm

$V_L = liA = 35$ cc

24. $mg\sin 37° = \eta A \cdot \dfrac{v}{t}$

$a^3\rho g\ \dfrac{3}{5} = \eta a^2\ \dfrac{v}{t} \Rightarrow \eta = \dfrac{3a\rho gt}{5v}$

25. $\sqrt{2gH}\ t = R$ and $\dfrac{1}{2}gt^2 = h$

$\sqrt{2gH} \cdot \sqrt{\dfrac{2h}{g}} = R$ or $4hH = R^2 \Rightarrow H = \dfrac{R^2}{4h}$

26. $w' + w + B$

$\Rightarrow\ m' = m + \dfrac{B}{g} = m + \dfrac{V}{2}\sigma = m + \dfrac{m_0}{2\rho}\sigma$

$\therefore\ m' = 10\ \text{kg} + \dfrac{7.2}{2 \times 7.2} = 10.5\ \text{kg}$

27.

	P	v	H
A	2	3	1
B	1	2	2
C	4	1	2

For A, $2 + \frac{1}{2} \times 1 \times 3^2 + 1 \times 1 \times 10 = 16.5$

For B, $1 + \frac{1}{2} \times 1 \times 2^2 + 2 \times 1 \times 10 = 23$

For C, $4 + \frac{1}{2} \times 1 \times 1^2 + 2 \times 1 \times 10 = 24.5$

So, Bernoulli's equation gives different values for A, B and C. Thus, they do not lie on same stream line.

28. $a = \frac{F}{m} = \frac{W - B}{m} = \frac{V\rho g - V\sigma g}{VP}$

$= \left(1 - \frac{\sigma}{\rho}\right) g$, downward

$= -\left(\frac{\sigma}{\rho} - 1\right) g$, upward

29. $v = \sqrt{2g(3H - x)}$, $R = v \cdot t$, and $x = \frac{1}{2} gt^2$

$\therefore\ R = \sqrt{2g(3H - x)} \sqrt{\frac{2x}{g}} = 2\sqrt{(3Hx - x^2)}$

For $R = \max$, $\frac{dR}{dx} = 0$

$= \frac{d}{dx} \sqrt{3Hx - x^2}$

$= \frac{1}{2\sqrt{3Hx - x^2}} (3H - 2x)$

$\Rightarrow x = \frac{3}{2} H = 1.5\, H$

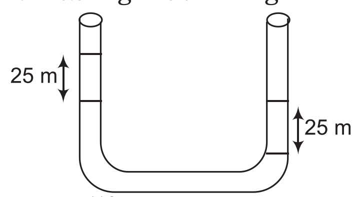
30. $T = \eta A \cdot \frac{v}{H}$ and $F_0 - T = \eta A_2 \frac{v}{H}$

$\Rightarrow F_0 = \eta (A_1 + A_2) \frac{v}{H}$

$\Rightarrow \quad \eta \frac{v}{H} = \frac{F_0}{A_2 + A_2}$

$\therefore\ T = \frac{A_1 F_0}{A_1 + A_2}$

31. $h \times 0.8 \times g = 50 \times 1 \times g$



$\Rightarrow h = \frac{50}{0.8} = 62.5$ cm

$\Delta h = (62.5 - 50)$ cm $= 12.5$ cm

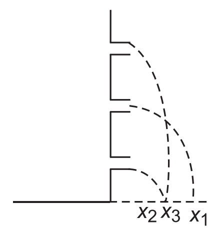
32. $v = \sqrt{2gh}$

33. $R = 2\sqrt{h(H - h)}$

$\Rightarrow x_3 = 2\sqrt{a(4a - a)} = 2\sqrt{3}a$

34. $x_1 = 2\sqrt{3a(4a - 3a)} = 2\sqrt{3}a = x_3$ and

$x_2 = 2\sqrt{2a(4a - 2a)} = 4a$



$x_1 = x_3 < x_2$

35. $L \cdot \frac{A}{5} \rho g = \frac{L}{4} \cdot \frac{A}{5} \cdot 2d \cdot g + \frac{3L}{4} \cdot \frac{A}{5} \cdot d \cdot g$

$\rho = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$

36. $p = p_0 + \left(\frac{H}{2} + \frac{\frac{A}{5} \times \frac{L}{4}}{A}\right) \cdot 2d \cdot g$

$+ \left(\frac{H}{2} + \frac{\frac{A}{5} \times \frac{3L}{4}}{A}\right) \cdot dg$

$= p_0 + \left(\frac{H}{2} \times 2 + \frac{1}{20} \times 2 + \frac{H}{2} + \frac{3L}{20}\right) dg$

$= p_0 + \left(\frac{3H}{2} + \frac{L}{4}\right) dg$

$= p_0 + \frac{(6H + L)}{4} dg$

37. $p_0 + \frac{H}{2} dg + \left(\frac{H}{2} - h\right) 2dg = p_0 + \frac{1}{2} \times 2dv^2$

$\left(\frac{H}{2} + H - 2h\right) g = v^2$

or $v = \sqrt{\frac{g}{2}(3H - 4h)}$

38. $R = vt = \sqrt{\frac{g}{2}(3H - 4h)} \cdot \sqrt{\frac{2h}{g}} = \sqrt{h(3H - 4h)}$

More than One Correct Options

1. $\frac{dv}{dx} = \frac{2}{1} = 2$ m/s

$F = \eta A \frac{dv}{dx} = 10^{-3} \times 10 \times 2 = 0.02$ N

2. For liquids viscosity decreaese with increase in temperature as adhesive and coheive forces decrease while for gases due to in random motion with temperature viscosity increases. Surface tension decreases due to decrease in adhesive force with increase in temperature.

3. In opposite direction to the displacement and it is possible only vertical direction.

As, $F_r = -\Delta B = -\sigma Ax\, g$

$\Rightarrow F_r \propto -x$ irrespective of the value of x.

4. $A_1 v_1 = A_2 v_2$, as $A_2 < A_1 \Rightarrow v_2 > v_1$

$Q = \frac{dv}{dt} = -\frac{dv}{dt}$

So, $p + \rho gh + \frac{1}{2}\rho pv^2 =$ constant

$\Rightarrow \quad p_2 < p_1$

5. $a^3 \rho g = a^2 lg\sigma \Rightarrow \frac{l}{a} = \frac{\rho}{\sigma} = f$

f is independent of atmospheric pressure, f increases with decrease in σ and f decreases with decreases in ρ

6. $\frac{v_1}{v_2} = \frac{\sqrt{2gh}}{\sqrt{2g4h}} = \frac{1}{2}, \frac{t_1}{t_2} = \frac{\sqrt{\frac{2 \times 4h}{g}}}{\sqrt{\frac{2 \times h}{g}}} = \frac{2}{1}$

$\frac{R_1}{R_2} = \frac{2\sqrt{h(5h - h)}}{2\sqrt{4h(5h - h)}} = \frac{\sqrt{h \cdot 4h}}{\sqrt{4h \cdot h}} = 1$

7. $P_A > P_B$ and $P_D > P_C$

8. $W = mg = V\rho g$, $B = V\,.\,2\rho g = 2W$

$a_{\text{air}} = g \downarrow$,

$a_{\text{liquid}} = \frac{B - W}{m} = \frac{2W - W}{m} = g \uparrow$

a, c and d are correct.

9. $A_1 v_1 = \pi 4r^2 \,.\, \sqrt{2gh}$ and

$A_2 v_2 = \pi r^2 \sqrt{2g16h} = 4\pi r^2 \sqrt{2gh} = A_1 v_1$

$A_1 v_1 = A_2 v_2 = Q$

After some time $Q_1 = 0$ and $Q_2 \neq 0$

10. $V\rho g = V\sigma g \Rightarrow \frac{V'}{V} = \frac{\rho}{\sigma} = f =$ constant

Respective of shape.

$\therefore \quad f_1 = f_2 = f_3$

as for 3rd area is uniform, h_3 is minimum.

for 2nd area is decreasing faster, h_2 is Moderate.

And for 1st area is slowly, h_1 is maximum

$\therefore \quad h_3 < h_1$ and $h_3 < h_2$

Match the Columns

1. $v = \sqrt{2gh_{\text{Top}}}\ \therefore v \propto \sqrt{h_{\text{Top}}}$.

R is maximum when $h = \frac{H}{2}$

2. With increase in temperature, density of liquid decreases. Furhter,

Upthrust $F = V\rho_e g$

or $\quad F \propto \rho_e$

3. Upthrust $F = V\rho_e g = V(\propto h)g$

$\therefore \quad F \propto h$

4. No solution is required.

5. $F_2 - F_1 =$ Net upthrust

$= V_1 \rho_1 g + V_2 \rho_2 g$

= Weight of cylinder in equilibrium.