Arihant Mechanics Part 1 by 1 One of Two (Chapter 1 till 5)

Understanding Physics for IIT JEE

by

D C Pandey

arihant
Inspiring Minds

According to Current Test Pattern
at the Level of Class XI-XII

Leading Edge Texts

Understanding Physics

Mechanics

Part 1

DC Pandey

Contains All Types of
Questions Including Reasoning, Aptitude & Comprehension

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According to Current Test Pattern
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Understanding Physics

Mechanics

Part 1

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Chapter 1 – Basic Maths and Measurement

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Basic Maths & Measurement

Chapter Contents

1.1 Basic Maths

The following formulae are frequently used in Physics. So, the students who have just gone in class XI are advised to remember them first.

(a) Logarithms

(i) $e \approx 2.7183$

(ii) If $e^x = y$, then $x = \log_e y = \ln y$

(iii) If $10^x = y$, then $x = \log_{10} y$

(iv) $\log_{10} y = 0.4343 \log_e y = 0.4343 \ln y$

(v) $\log(ab) = \log(a) + \log(b)$

(vi) $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

(vii) $\log a^n = n \log(a)$

(b) Trigonometry

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(iv) $\sin 2\theta = 2\sin\theta\cos\theta$

(v) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$

(vi) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(vii) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(viii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ix) $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

(x) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(xi) $\cos C - \cos D = 2\sin\frac{D-C}{2}\sin\frac{C+D}{2}$

(xii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(xiii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(xiv) $\sin(90° + \theta) = \cos\theta$

(xv) $\cos(90° + \theta) = -\sin\theta$

(xvi) $\tan(90° + \theta) = -\cot\theta$

(xvii) $\sin(90° - \theta) = \cos\theta$

(xviii) $\cos(90° - \theta) = \sin\theta$

(xix) $\tan(90° - \theta) = \cot\theta$

(xx) $\sin(180° - \theta) = \sin\theta$

(xxi) $\cos(180° - \theta) = -\cos\theta$

(xxii) $\tan(180° - \theta) = -\tan\theta$

(xxiii) $\sin(180° + \theta) = -\sin\theta$

(xxiv) $\cos(180° + \theta) = -\cos\theta$

(xxv) $\tan(180° + \theta) = \tan\theta$

(xxvi) $\sin(-\theta) = -\sin\theta$

(xxvii) $\cos(-\theta) = \cos\theta$

(xxviii) $\tan(-\theta) = -\tan\theta$

(c) Differentiation

(i) $\frac{d}{dx}(\text{constant}) = 0$

(ii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(iii) $\frac{d}{dx}(\log_e x)$ or $\frac{d}{dx}(\ln x) = \frac{1}{x}$

(iv) $\frac{d}{dx}(\sin x) = \cos x$

(v) $\frac{d}{dx}(\cos x) = -\sin x$

(vi) $\frac{d}{dx}(\tan x) = \sec^2 x$

(vii) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$ (viii) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(ix) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$ (x) $\frac{d}{dx}(e^x) = e^x$

(xi) $\frac{d}{dx}\{f_1(x) \cdot f_2(x)\} = f_1(x)\frac{d}{dx} f_2(x) + f_2(x)\frac{d}{dx} f_1(x)$

(xii) $\frac{d}{dx}\frac{f_1(x)}{f_2(x)} = \frac{f_2(x)\frac{d}{dx} f_1(x) - f_1(x)\frac{d}{dx} f_2(x)}{\{f_2(x)\}^2}$

(xiii) $\frac{d}{dx} f(ax+b) = a\frac{d}{dx} f(X)$, where $X = ax + b$

(d) Integration

(i) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$ (ii) $\int \frac{dx}{x} = \log_e x + c$ or $\ln x + c$

(iii) $\int \sin x\, dx = -\cos x + c$ (iv) $\int \cos x\, dx = \sin x + c$

(v) $\int e^x dx = e^x + c$ (vi) $\int \sec^2 x\, dx = \tan x + c$

(vii) $\int \text{cosec}^2 x\, dx = -\cot x + c$ (viii) $\int \sec x \tan x\, dx = \sec x + c$

(ix) $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + c$

(x) $\int f(ax+b)\, dx = \frac{1}{a}\int f(X)\, dX$, where $X = ax + b$

Here, c is constant of integration.

(e) Graphs

Following graphs and their corresponding equations are frequently used in Physics.

(i) $y = mx$, represents a straight line passing through origin. Here, $m = \tan\theta$ is also called the slope of line, where θ is the angle which the line makes with positive x-axis, when drawn in anticlockwise direction from the positive x-axis towards the line.

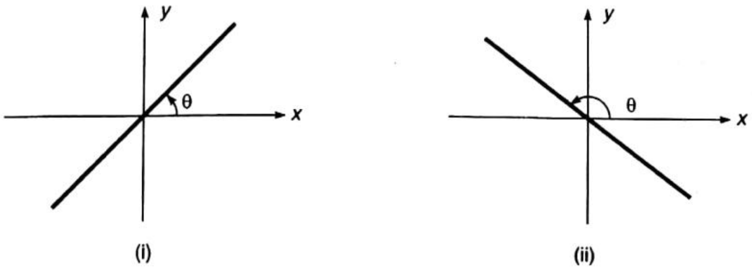


Fig. 1.1

The two possible cases are shown in Fig. 1.1. In Fig. 1.1 (i), $\theta < 90°$. Therefore, $\tan\theta$ or slope of line is positive. In Fig. 1.1 (ii), $90° < \theta < 180°$. Therefore, $\tan\theta$ or slope of line is negative.

Note *That $y = mx$ or $y \propto x$ also means that value of y becomes 2 times if x is doubled. Or it will remain $\frac{1}{4}$ th if x becomes $\frac{1}{4}$ times.*

(ii) $y = mx + c$, represents a straight line not passing through origin. Here, m is the slope of line as discussed above and c the intercept on y-axis.

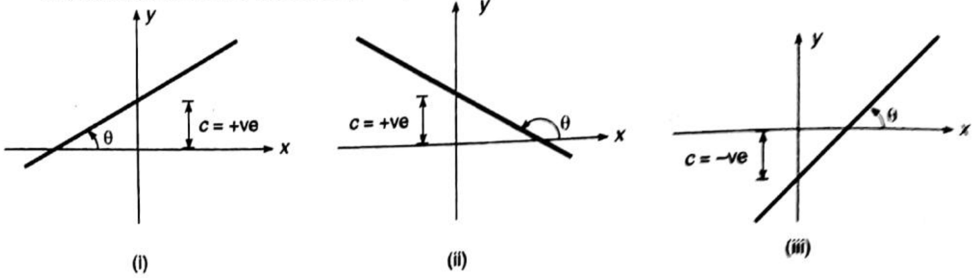


Fig. 1.2

In figure (i) : slope and intercept both are positive.
In figure (ii) : slope is negative but intercept is positive and
In figure (iii) : slope is positive but intercept is negative.

Note *That in $y = mx + c$, y does not become two times if x is doubled.*

(iii) $y \propto \frac{1}{x}$ or $y = \frac{2}{x}$ etc., represents a rectangular hyperbola in first and third quadrants. The shape of rectangular hyperbola is shown in Fig. 1.3(i).

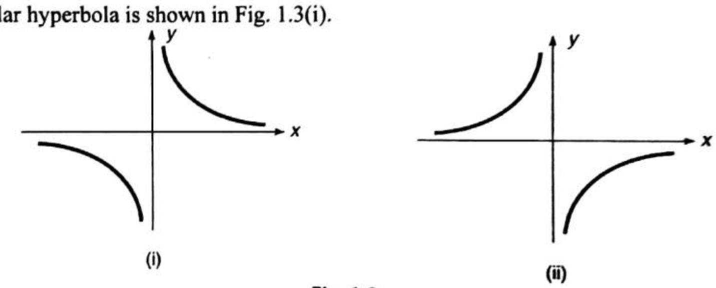


Fig. 1.3

From the figure and from the graph we can see that $y \to 0$ as $x \to \infty$ or $x \to 0$ as $y \to \infty$.
Similarly, $y = -\frac{4}{x}$ represents a rectangular hyperbola in second and fourth quadrants as shown in Fig. 1.3(ii).

Note *That in case of rectangular hyperbola if x is doubled y will become half.*

(iv) $y \propto x^2$ or $y = 2x^2$, etc., represents a parabola passing through origin as shown in Fig. 1.4(i).

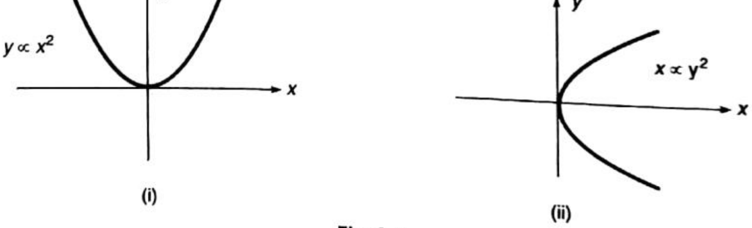


Fig. 1.4

Note *That in the parabola $y = 2x^2$ or $y \propto x^2$, if x is doubled, y will become four times.*

Graph $x \propto y^2$ or $x = 4y^2$ is again a parabola passing through origin as shown in Fig 1.4 (ii). In this case if y is doubled, x will become four times.

(v) $y = x^2 + 4$ or $x = y^2 - 6$ will represent a parabola but not passing through origin. In the first equation ($y = x^2 + 4$), if x is doubled, y will not become four times.

(vi) $y = Ae^{-Kx}$; represents exponentially decreasing graph. Value of y decreases exponentially from A to 0. The graph is shown in Fig. 1.5.

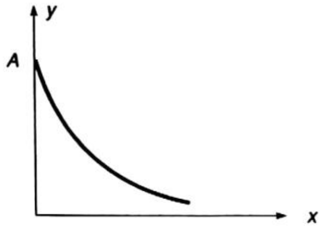


Fig. 1.5

From the graph and the equation, we can see that $y = A$ at $x = 0$ and $y \to 0$ as $x \to \infty$.

(vii) $y = A(1 - e^{-Kx})$, represents an exponentially increasing graph. Value of y increases exponentially from 0 to A. The graph is shown in Fig. 1.6.

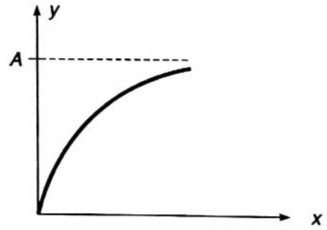


Fig. 1.6

From the graph and the equation we can see that $y = 0$ at $x = 0$ and $y \to A$ as $x \to \infty$.

(f) Maxima and Minima

Suppose y is a function of x. Or $y = f(x)$.

Then we can draw a graph between x and y. Let the graph is as shown in Fig. 1.7.

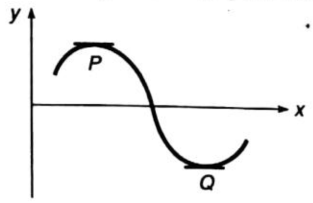


Fig. 1.7

Then from the graph we can see that at maximum or minimum value of y slope $\left(\text{or } \frac{dy}{dx}\right)$ to the graph is zero. Thus,

$$\frac{dy}{dx} = 0 \quad \text{at maximum or minimum value of } y.$$

By putting $\frac{dy}{dx} = 0$ we will get different values of x. At these values of x, value of y is maximum if $\frac{d^2y}{dx^2}$ (double differentiation of y with respect to x) is negative at this value of x. Similarly y is minimum if $\frac{d^2y}{dx^2}$ is positive. Thus,

$$\frac{d^2y}{dx^2} = -\text{ve for maximum value of } y$$

and

$$\frac{d^2y}{dx^2} = +\text{ve for minimum value of } y$$

Note *That at constant value of y also* $\frac{dy}{dx} = 0$ *but in this case* $\frac{d^2y}{dx^2}$ *is zero.*

Sample Example 1.1 *Differentiate the following functions with respect to x*

(a) $x^3 + 5x^2 - 2$ *(b)* $x \sin x$ *(c)* $(2x+3)^6$ *(d)* $\frac{x}{\sin x}$ *(e)* $e^{(5x+2)}$

Solution (a)

$$\frac{d}{dx}(x^3 + 5x^2 - 2) = \frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) - \frac{d}{dx}(2)$$

$$= 3x^2 + 5(2x) - 0$$

$$= 3x^2 + 10x$$

(b)

$$\frac{d}{dx}(x \sin x) = x\frac{d}{dx}(\sin x) + \sin x \,.\, \frac{d}{dx}(x)$$

$$= x\cos x + \sin x\,(1)$$

$$= x\cos x + \sin x$$

(c)

$$\frac{d}{dx}(2x+3)^6 = 2\frac{d}{dX}(X)^6, \quad \text{where} \quad X = 2x+3$$

$$= 2\{6X^5\} = 12X^5$$

$$= 12(2x+3)^5$$

(d)

$$\frac{d}{dx}\left(\frac{x}{\sin x}\right) = \frac{\sin x \frac{d}{dx}(x) - x\frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{(\sin x)(1) - x(\cos x)}{\sin^2 x}$$

$$= \frac{\sin x - x\cos x}{\sin^2 x}$$

(e)

$$\frac{d}{dx}e^{(5x+2)} = 5\frac{d}{dX}e^X, \text{ where } X = 5x+2$$

$$= 5e^X$$

$$= 5e^{5x+2}$$

Sample Example 1.2 *Integrate the following functions with respect to x*

(a) $\int (5x^2 + 3x - 2)\,dx$ *(b)* $\int \left(4 \sin x - \frac{2}{x}\right) dx$ *(c)* $\int \frac{dx}{4x+5}$ *(d)* $\int (6x+2)^3\,dx$

Solution (a)
$$\int (5x^2 + 3x - 2)\,dx = 5\int x^2\,dx + 3\int x\,dx - 2\int dx$$
$$= \frac{5x^3}{3} + \frac{3x^2}{2} - 2x + c$$

(b)
$$\int \left(4 \sin x - \frac{2}{x}\right) dx = 4\int \sin x\,dx - 2\int \frac{dx}{x}$$
$$= -4\cos x - 2\ln x + c$$

(c)
$$\int \frac{dx}{4x+5} = \frac{1}{4}\int \frac{dX}{X}, \quad \text{where } X = 4x+5$$
$$= \frac{1}{4}\ln X + c_1 = \frac{1}{4}\ln(4x+5) + c_2$$

(d)
$$\int (6x+2)^3\,dx = \frac{1}{6}\int X^3\,dX, \quad \text{where } X = 6x+2$$
$$= \frac{1}{6}\left(\frac{X^4}{4}\right) + c_1 = \frac{(6x+2)^4}{24} + c_2$$

Sample Example 1.3 *Draw straight lines corresponding to following equations*

(a) $y = 2x$ *(b)* $y = -6x$ *(c)* $y = 4x + 2$ *(d)* $y = 6x - 4$

Solution (a) In $y = 2x$, slope is 2 and intercept is zero. Hence, the graph is as shown below :

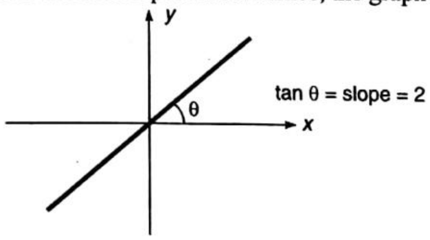


Fig. 1.8

(b) In $y = -6x$, slope is -6 and intercept is zero. Hence, the graph is as shown below :

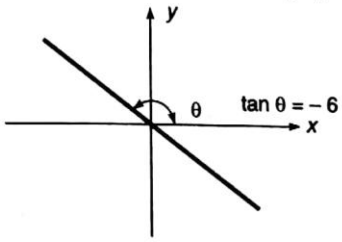


Fig. 1.9

(c) In $y = 4x + 2$, slope is + 4 and intercept is 2. The graph is as shown below :

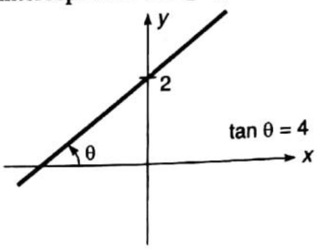


Fig. 1.10

(d) In $y = 6x - 4$, slope is + 6 and intercept is – 4. Hence, the graph is as shown in Fig. 1.11.

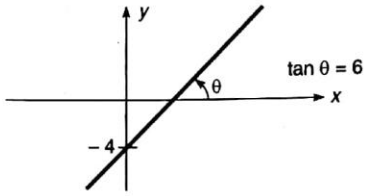


Fig. 1.11

Sample Example 1.4 *Find maximum or minimum values of the functions*

(a) $y = 25x^2 + 5 - 10x$ *(b)* $y = 9 - (x-3)^2$

Solution (a) For maximum and minimum value, we can put $\frac{dy}{dx} = 0$.

or $$\frac{dy}{dx} = 50x - 10 = 0 \qquad \therefore \quad x = \frac{1}{5}$$

Further, $$\frac{d^2y}{dx^2} = 50$$

or $\frac{d^2y}{dx^2}$ has positive value at $x = \frac{1}{5}$. Therefore, y has minimum value at $x = \frac{1}{5}$. Substituting $x = \frac{1}{5}$ in given equation, we get

$$y_{min} = 25\left(\frac{1}{5}\right)^2 + 5 - 10\left(\frac{1}{5}\right) = 4$$

(b) $$y = 9 - (x-3)^2 = 9 - x^2 - 9 + 6x$$

or $$y = 6x - x^2$$

$\therefore$ $$\frac{dy}{dx} = 6 - 2x$$

For minimum or maximum value of y we will substitute $\frac{dy}{dx} = 0$

or $$6 - 2x = 0$$

or $$x = 3$$

To check whether value of y is maximum or minimum at $x = 3$ we will have to check whether $\frac{d^2y}{dx^2}$ is positive or negative.

$$\frac{d^2y}{dx^2} = -2$$

or $\frac{d^2y}{dx^2}$ is negative at $x = 3$. Hence, value of y is maximum. This maximum value of y is,

$$y_{max} = 9 - (3-3)^2 = 9$$

1.2 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

"All accurately known digits in a measurement plus the first uncertain digit together from significant figures."

For example, when we measure the length of a straight line using a metre scale and it lies between 7.4 cm and 7.5 cm, we may estimate it as $l = 7.43$ cm. This expression has three significant figures out of these 7 and 4 are precisely known but the last digit 3 is only approximately known.

Rules for counting significant figures

For counting significant figures, we use the following rules :

Rule 1. All non-zero digits are significant. For example $x = 2567$ has four significant figures.

Rule 2. The zeros appearing between two non-zero digits are counted in significant figures. For example 6.028 has 4 significant figures.

Rule 3. The zeros occurring to the left of last non-zero digit are NOT significant.
For example 0.0042 has two significant figures.

Rule 4. In a number without decimal, zeros to the right of non-zero digit are NOT significant. However when some value is recorded on the basis of actual measurement the zeros to the right of non-zero digit become significant. For example $L = 20$ m has two significant figures but $x = 200$ has only one significant figure.

Rule 5. In a number with decimal, zeros to the right of last non-zero, digit are significant. For example $x = 1.400$ has four significant figures.

Rule 6. The powers of ten are NOT counted as significant digits. For example 1.4×10^{-7} has only two significant figures 1 and 4.

Rule 7. Change in the units of measurement of a quantity does not change the number of significant figures. For example, suppose distance between two stations is 4067 m. It has four significant figures. The same distance can be expressed as 4.067 km or 4.067×10^5 cm. In all these expressions, number of significant figures continues to be four.

Example : **Table 1.1**

Measured value	Number of significant figures	Rule
12376	5	1
6024.7	5	2
0.071	2	3
410 m	3	4
720	2	4
2.40	3	5
1.6×10^{14}	2	6

Rounding off a digit

Following are the rules for rounding off a measurement.

Rule 1. If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However if it is more than 5, then the cut off digit is increased by 1.
For example $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.

Rule 2. If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is increased by 1.
For example $x = 14.252$ is rounded off to $x = 14.3$ to three significant digits.

Rule 3. If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit it left unchanged if it is even. For example $x = 6.250$ or $x = 6.25$ becomes $x = 6.2$ after rounding off to two significant digits.

Rule 4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one if it is odd.
For example $x = 6.350$ or $x = 6.35$ becomes $x = 6.4$ after rounding off to two significant digits.

Example : **Table 1.2**

Measured value	After rounding off to three significant digits	Rule
7.364	7.36	1
7.367	7.37	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4

Algebraic operations with significant figures

In addition, subtraction, multiplication or division inaccuracy in the measurement of any one variable affects the accuracy of the final result. Hence, in general, the final result shall have significant figures corresponding to their number in the least accurate variable involved. To understand this, let us consider a chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus, the strength of the chain cannot be more than the strength of the weakest link in the chain. Hence, we do not imply a greater accuracy in our result than was obtained originally in our measurements.

(i) Addition and subtraction

Suppose in the measured values to be added or subtracted the least number of significant digits after the decimal is *n*. Then in the sum or difference also, the number of significant digits after the decimal should be *n*.

Example : $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

Example : $12.63 - 10.2 = 2.43 \approx 2.4$

(ii) Multiplication or division

Suppose in the measured values to be multiplied or divided the least number of significant digits be *n*. Then in the product or quotient, the number of significant digits should also be *n*.

Example : $1.2 \times 36.72 = 44.064 \approx 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore, the answer is 44.

Example : $\dfrac{1100\ \text{ms}^{-1}}{10.2\ \text{ms}^{-1}} = 107.8431373 \approx 108$

Note *In this case answer becomes* 108. *Think why?*

1.3 Error Analysis

No measurement is perfect, as the errors involved in a measurement cannot be removed completely. Measured value is always somewhat different from the true value. The difference is called an error.

Errors can be classified in two ways. First classification is based on the cause of error. Systematic error and random errors fall in this group. Second classification is based on the magnitude of error. Absolute error, mean absolute error and relative (or fractional) error lie on this group. Now, let us discuss them separately.

(i) Systematic errors

These are the errors whose causes are known to us. Such errors can therefore be minimised. Following are few causes of these errors :

(a) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

(b) Sometimes errors arise on account of ignoring certain facts. For example in measuring time period of simple pendulum error may creap because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

(c) Change in temperature, pressure, humidity, etc., may also sometimes cause errors in the result. Relevant corrections can be made to minimise their effects.

(ii) Random errors

The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons. For example the reading of a sensitive beam balance may change by the vibrations caused in the building due to persons moving in the laboratory or vehicles running nearby. The random errors can be minimised by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus,

$$a_{mean} = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

(iii) Absolute errors

The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

Then by definition, absolute errors in the measured values of the quantity are,

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots \quad \ldots \quad \ldots$$
$$\Delta a_n = a_m - a_n$$

Absolute error may be positive or negative.

(iv) Mean absolute error

Arithmetic mean of the magnitudes of absolute errors in all the measurements is called the mean absolute error. Thus,

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

The final result of measurement can be written as,

$$a = a_m \pm \Delta a_{mean}$$

This implies that value of a is likely to lie between $a_m + \Delta a_{mean}$ and $a_m - \Delta a_{mean}$.

Relative or fractional error

The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error. Thus,

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_m}$$

Relative error expressed in percentage is called as the percentage error, *i.e.*,

$$\text{Percentage error} = \frac{\Delta a_{mean}}{a_m} \times 100$$

Sample Example 1.5 *The diameter of a wire as measured by screw gauge was found to be 2.620, 2.625, 2.628 and 2.626 cm. Calculate:*

(a) mean value of diameter. *(b) absolute error in each measurement.*
(c) mean absolute error. *(d) fractional error*
(e) percentage error. *(f) Express the result in terms of percentage error.*

Solution (a) Mean value of diameter

$$a_m = \frac{2.620 + 2.625 + 2.630 + 2.628 + 2.626}{5}$$
$$= 2.6258 \text{ cm}$$
$$= 2.626 \text{ cm}$$

(rounding off to three decimal places)

(b) Taking a_m as the true value, the absolute errors in different observations are,

$$\Delta a_1 = 2.626 - 2.620 = +0.006\,\text{cm}$$
$$\Delta a_2 = 2.626 - 2.625 = +0.001\,\text{cm}$$
$$\Delta a_3 = 2.626 - 2.630 = -0.004\,\text{cm}$$
$$\Delta a_4 = 2.626 - 2.628 = -0.002\,\text{cm}$$
$$\Delta a_5 = 2.626 - 2.626 = 0.000\,\text{cm}$$

(c) Mean absolute error,

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$
$$= \frac{0.006 + 0.001 + 0.004 + 0.002 + 0.000}{5}$$
$$= 0.0026 = 0.003$$

(rounding off to three decimal places)

(d) Fractional error $= \pm \frac{\Delta a_{mean}}{a_m} = \frac{\pm 0.003}{2.626} = \pm 0.001$

(e) Percentage error $= \pm 0.001 \times 100 = \pm 0.1\%$

(f) Diameter of wire can be written as,

$$d = 2.626 \pm 0.1\%$$

Combination of Errors

(i) Errors in sum or difference

Let $x = a \pm b$

Further, let Δa is the absolute error in the measurement of a, Δb the absolute error in the measurement of b and Δx is the absolute error in the measurement of x.

Then,
$$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b)$$
$$= (a \pm b) \pm (\pm \Delta a \pm \Delta b)$$
$$= x \pm (\pm \Delta a \pm \Delta b)$$

or
$$\Delta x = \pm \Delta a \pm \Delta b$$

The four possible values of Δx are $(\Delta a - \Delta b)$, $(\Delta a + \Delta b)$, $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$.

Therefore, the maximum absolute error in x is,

$$\Delta x = \pm (\Delta a + \Delta b)$$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Sample Example 1.6 *The volumes of two bodies are measured to be* $V_1 = (10.2 \pm 0.02)\,cm^3$ *and* $V_2 = (6.4 \pm 0.01)\,cm^3$. *Calculate sum and difference in volumes with error limits.*

Solution
$$V_1 = (10.2 \pm 0.02)\,\text{cm}^3$$

and
$$V_2 = (6.4 \pm 0.01)\,\text{cm}^3$$

$$\Delta V = \pm(\Delta V_1 + \Delta V_2)$$
$$= \pm(0.02 + 0.01)\,\text{cm}^3$$
$$= \pm 0.03\,\text{cm}^3$$
$$V_1 + V_2 = (10.2 + 6.4)\,\text{cm}^3 = 16.6\,\text{cm}^3$$

and $$V_1 - V_2 = (10.2 - 6.4)\,\text{cm}^3 = 3.8\,\text{cm}^3$$

Hence, sum of volumes $= (16.6 \pm 0.03)\,\text{cm}^3$

and difference of volumes $= (3.8 \pm 0.03)\,\text{cm}^3$

(ii) Errors in a product

Let $x = ab$

Then, $$(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

or $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}\cdot\frac{\Delta b}{b} \qquad \text{(as } x = ab\text{)}$$

or $$\pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\cdot\frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a}\cdot\frac{\Delta b}{b}$ is a small quantity, so can be neglected. Hence,

$$\pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$.

Hence, maximum possible value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

(iii) Errors in division

Let $$x = \frac{a}{b}$$

Then, $$x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

or $$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$$

or $$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1} \qquad \left(\text{as } x = \frac{a}{b}\right)$$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

or
$$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is small quantity, so can be neglected. Therefore,

$$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$. Therefore, the maximum value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

(iv) Error in quantity raised to some power

Let
$$x = \frac{a^n}{b^m}$$

Then,
$$\ln(x) = n\ln(a) - m\ln(b)$$

Differentiating both sides, we get

$$\frac{dx}{x} = n \cdot \frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm \frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore, maximum value of

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$$

Sample Example 1.7 *The mass and density of a solid sphere are measured to be* $(12.4 \pm 0.1)\,kg$ *and* $(4.6 \pm 0.2)\,kg/m^3$. *Calculate the volume of the sphere with error limits.*

Solution Here, $m \pm \Delta m = (12.4 \pm 0.1)$ kg

and
$$\rho \pm \Delta\rho = (4.6 \pm 0.2)\ \text{kg/m}^3$$

Volume
$$V = \frac{m}{\rho} = \frac{12.4}{4.6} = 2.69\ \text{m}^3 = 2.7\ \text{m}^3$$

(rounding off to one decimal place)

Now,
$$\frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$$

or
$$\Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta \rho}{\rho}\right) \times V$$
$$= \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right) \times 2.7 = \pm\, 0.14$$
$\therefore$
$$V \pm \Delta V = (2.7 \pm 0.14)\,\text{m}^3$$

Sample Example 1.8 *Calculate percentage error in determination of time period of a pendulum,*
$$T = 2\pi\sqrt{\frac{l}{g}}$$
where l and g are measured with ± 1% and ± 2% errors.

Solution
$$\frac{\Delta T}{T} \times 100 = \pm\left(\frac{1}{2} \times \frac{\Delta l}{l} \times 100 + \frac{1}{2} \times \frac{\Delta g}{g} \times 100\right)$$
$$= \pm\left(\frac{1}{2} \times 1 + \frac{1}{2} \times 2\right) = \pm\, 1.5\%$$

1.4 Length Measuring Instruments

Length is an elementary physical quantity. The device generally used in everyday life for measurement of the length is a metre scale. It can be used for measurement of length with an accuracy of 1 mm. So, the least count of a metre scale is 1 mm. To measure length accurately upto (1/10)th or $\left(\frac{1}{100}\right)$th of a millimetre, the following instruments are used.

(1) Vernier callipers (2) Micrometer (3) Screw gauge.

(1) Vernier Callipers

It has three parts.

(i) Main scale : It consists of a steel metallic strip *M*, graduated in cm and mm at one edge. It carries two fixed jaws *A* and *C* as shown in figure.

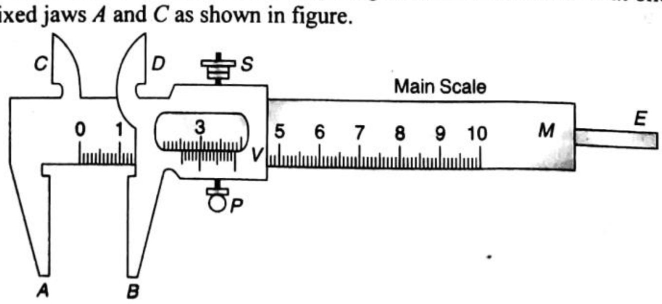


Fig. 1.12

(ii) Vernier scale : Vernier scale *V* slides on metallic strip *M*. It can be fixed in any position by screw *S*. The side of the vernier scale which slide over the mm sides has ten divisions over a length of 9 mm. *B* and *D* two movable jaws are fixed with it. When vernier scale is pushed towards *A* and *C*, then *B* touches *A* and straight side of C will touch straight side of D. In this position, if the instrument is free from error, zeros of vernier scale will coincide with zeros of main scales. To measure the external diameter of an object it is held between the jaws A and B, while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

(iii) Metallic strip : There is a thin metallic strip E attached to the back side of M and connected with vernier scale. When jaws A and B touch each other, the edge of E touches the edge of M. When the jaws A and B are separated E moves outwards. This strip E is used for measuring the depth of a vessel.

Principle (Theory)

In the common form, the divisions on the vernier scale V are smaller in size than the smallest division on the main scale M, but in some special cases the size of the vernier division may be larger than the main scale division.

Let n vernier scale divisions (V.S.D.) coincide with $(n-1)$ main scale divisions (M.S.D.). Then,

$$n\,\text{V.S.D.} = (n-1)\,\text{M.S.D.}$$

or

$$1\,\text{V.S.D.} = \left(\frac{n-1}{n}\right)\text{M.S.D.}$$

$$1\,\text{M.S.D.} - 1\,\text{V.S.D.} = 1\,\text{M.S.D.} - \left(\frac{n-1}{n}\right)\text{M.S.D.} = \frac{1}{n}\,\text{M.S.D.}$$

The difference between the values of one main scale division and one vernier scale division is known as **Vernier constant (V.C.)** or the **Least count (L.C.)**. This is the smallest distance that can be accurately measured with the vernier scale. Thus,

$$\text{V.C.} = \text{L.C.} = 1\,\text{M.S.D.} - 1\,\text{V.S.D.} = \left(\frac{1}{n}\right)\text{M.S.D.} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

In the ordinary vernier callipers one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions.

$$1\,\text{V.S.D.} = \frac{9}{10}\,\text{M.S.D.} = 0.9\text{ mm}$$

$$\text{V.C.} = 1\,\text{M.S.D.} - 1\,\text{V.S.D.} = 1\text{ mm} - 0.9\text{ mm}$$

$$= 0.1\text{ mm} = 0.01\text{ cm}$$

Reading a vernier callipers

If we have to measure a length AB, the end A is coincided with the zero of main scale, suppose the end B lies between 1.0 cm and 1.1 cm on the main scale. Then,

$$1.0\,\text{cm} < AB < 1.1\,\text{cm}$$

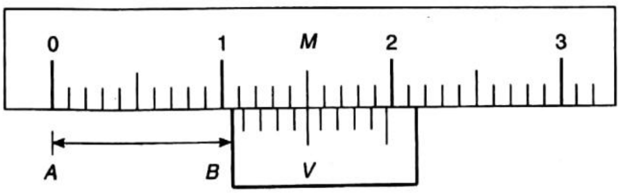


Fig. 1.13

Let 5th division of vernier scale coincides with 1.5 cm of main scale.

Then, $AB = 1.0 + 5 \times \text{V.C.} = (1.0 + 5 \times 0.01)\text{ cm} = 1.05\text{ cm}$

Thus, we can make the following formula,

$$\text{Total reading} = N + n \times \text{V.C.}$$

Here, N = main scale reading before on the left of the zero of the vernier scale.
n = number of vernier division which just coincides with any of the main scale division.

Note *That the main scale reading with which the vernier scale division coincides has no connection with reading.*

Zero error and zero correction

If the zero of the vernier scale does not coincide with the zero of main scale when jaw B touches A and the straight edge of D touches the straight edge of C, then the instrument has an error called **zero error.** Zero error is always algebraically subtracted from measured length.

Zero correction has a magnitude equal to zero error but its sign is opposite to that of the zero error. Zero correction is always algebraically added to measured length.

Zero error ⟶ algebraically subtracted
Zero correction ⟶ algebraically added

Positive and negative zero error

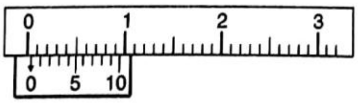
If zero of vernier scale lies to the right of the main scale the zero error is positive and if it lies to the left of the main scale the zero error is negative (when jaws A and B are in contact).

$$\text{Positive zero error} = (N + x \times \text{V.C.})$$

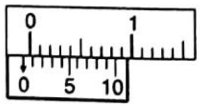
Here, N = main scale reading on the left of zero of vernier scale.
x = vernier scale division which coincides with any main scale division.

When the vernier zero lies before the main scale zero the error is said to be negative zero error. If 5th vernier scale division coincides with the main scale division, then

$$\text{Negative zero error} = -[0.00\text{ cm} + 5 \times \text{V.C.}]$$
$$= -[0.00\text{ cm} + 5 \times 0.01\text{ cm}]$$
$$= -0.05\text{ cm}$$



(A) Positive zero error



(B) Negative zero error

Fig. 1.14 Positive and negative zero error

Summary

(1) $\text{V.C.} = \text{L.C.} = \dfrac{1\text{ M.S.D.}}{n} = \dfrac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$
$= 1\text{ M.S.D.} - 1\text{ V.S.D.}$

(2) In ordinary vernier callipers, 1. M.S.D. = 1 mm and $n = 10$
$\therefore$ $\text{V.C. or L.C.} = \dfrac{1}{10}\text{ mm} = 0.01\text{ cm}$

(3) Total reading = $(N + n \times \text{V.C})$

(4) Zero correction = − zero error

(5) Zero error is algebraically subtracted while the zero correction is algebraically added.

(6) If zero of vernier scale lies to the right of zero of main scale the error is positive. The actual length in this case is less than observed length.

(7) If zero of vernier scale lies to the left of zero of main scale the error is negative and the actual length is more than the observed length.

(8) Positive zero error $= (N + x \times \text{V.C.})$

Sample Example 1.9 *N-divisions on the main scale of a vernier callipers coincide with N + 1 divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument.*

(IIT JEE 2003)

Solution $(N+1)$ divisions on the vernier scale $= N$ divisions on main scale

$\therefore$ 1 division on vernier scale $= \dfrac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

$\therefore$ 1 division on vernier scale $= \left(\dfrac{N}{N+1}\right)$ a units $= a'$ (say)

Least count = 1 main scale division − 1 vernier scale division

$$= a - a' = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$$

Sample Example 1.10 *In the diagram shown in figure, find the magnitude and nature of zero error.*

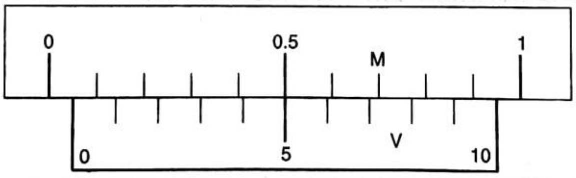


Fig. 1.15

Solution Here, zero of vernier scale lies to the right of zero of main scale, hence, it has positive zero error.

Further, $N = 0,\ x = 5,\ \text{L.C. or V.C.} = 0.01$ cm

Hence, Zero error $= N + x \times \text{V.C.}$

$$= 0 + 5 \times 0.01$$

$$= 0.05 \text{ cm}$$

Zero correction $= -0.05$ cm

$\therefore$ Actual length will be 0.05 cm less than the measured length.

(2) Principle of a Micrometer Screw

The least count of vernier callipers ordinarily available in the laboratory is 0.01 cm. When lengths are to be measured with greater accuracy, say upto 0.001 cm, screw gauge and spherometer are used which are based on the principle of **micrometer screw** discussed below.

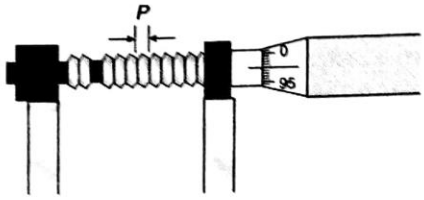


Fig. 1.16

If an accurately cut single threaded screw is rotated in a closely fitted nut, then in addition to the circular motion of the screw there is a linear motion of the screw head in the forward or backward direction, along the axis of the screw. The linear distance moved by the screw, when it is given one complete rotation is called the **pitch (p)** of the screw. This is equal to the distance between two consecutive threads as measured along the axis of the screw. In most of the cases it is either 1 mm or 0.5 mm. A circular cap is fixed on one end of the screw and the circumference of the cap is normally divided into 100 or 50 equal parts. If it is divided into 100 equal parts, then the screw moves forward or backward by $\frac{1}{100}\left(\text{or } \frac{1}{50}\right)$ of the pitch, if the circular scale (we will discuss later about circular scale) is rotated through one circular scale division. It is the minimum distance which can be accurately measured and so called the **least count (L.C.)** of the screw.

Thus, $$\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

If pitch is 1 mm and there are 100 divisions on circular scale then,

$$\text{L.C.} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm} = 0.001\,\text{cm} = 10\,\mu\text{m}$$

Since, L.C. is of the order of 10 μm, the screw is called micrometer screw.

(3) Screw Gauge

Screw gauge works on the principle of micrometer screw. It consists of a *U*-shaped metal frame *M*. At one end of it is fixed a small metal piece *A*. It is called stud and it has a plane face. The other end *N* of *M* carries a cylindrical hub *H*. It is graduated in millimetres and half millimetre depending upon the pitch of the screw. This scale is called **linear scale** or **pitch scale.**

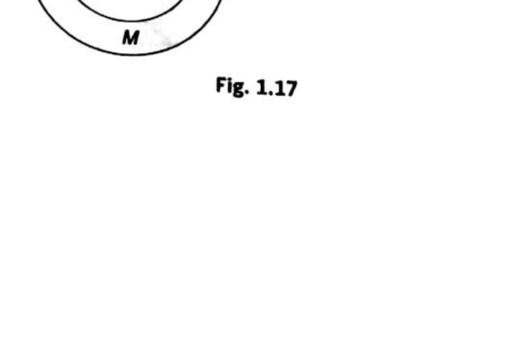


Fig. 1.17

A nut is threaded through the hub and the frame *N*. Through the nut moves a screw *S*. The front face *B* of the screw, facing the plane face *A* is also plane. A hollow cylindrical cap *K* is capable of rotating over the hub when screw is rotated. As the cap is rotated the screw either moves in or out. The surface *E* of the cap *K* is divided into 50 or 100 equal parts. It is called the **circular scale** or **head scale.** In an accurately adjusted instrument when the faces *A* and *B* are just touching each other. Zero of circular scale should coincide with zero of linear scale.

To measure diameter of a given wire using a screw gauge

If with the wire between plane faces *A* and *B*, the edge of the cap lies ahead of *N*th division of linear scale, and *n*th division of circular scale lies over reference line.

Then, Total reading $= N + n \times \text{L.C.}$

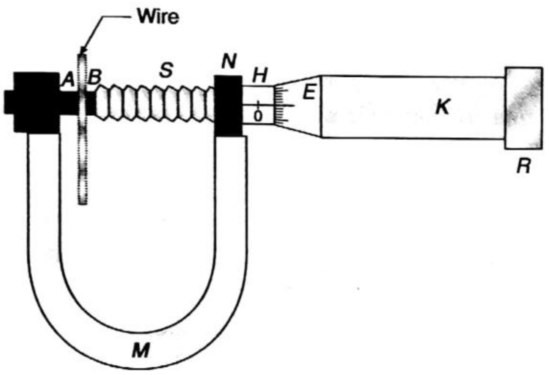


Fig. 1.18

Zero error and zero correction

If zero mark of circular scale does not coincide with the zero of the pitch scale when the faces *A* and *B* are just touching each other, the instrument is said to possess zero error. If the zero of the circular scale advances beyond the reference line the zero error is **negative** and zero correction is **positive.** If it is left behind the reference line the zero is **positive** and zero correction is **negative.** For example if zero of circular scale advances beyond the reference line by 5 divisions, zero correction $= +5 \times (\text{L.C.})$ and if the zero of circular scale is left behind the reference line by 4 divisions, zero correction $= -4 \times (\text{L.C.})$.

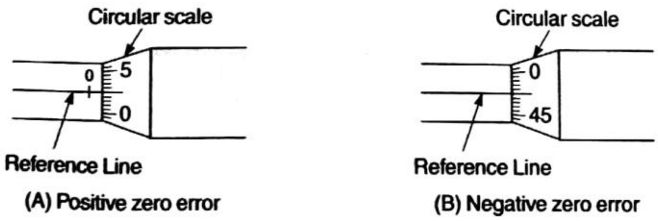


Fig. 1.19

Back lash error

When the sense of rotation of the screw is suddenly changed, the screw head may rotate, but the screw itself may not move forward or backwards. Thus, the scale reading may change even by the actual movement of the screw. This is known as back lash error. This error is due to loose fitting of the screw. This arises due to wear and tear of the threadings due to prolonged use of the screw. To reduce this error the screw must always be rotated in the same direction for a particular set of observations.

Sample Example 1.11 *The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and Bare just touching each without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero of linear scale is not hidden from circular scale when A and B touches each other.*

Solution

$$\text{Least count L. C.} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$= \frac{1}{100}\text{ mm}$$

$$= 0.01\text{ mm}$$

As zero is not hidden from cricular scale when A and B touches each other. Hence, the screw gauge has positive error.

$$e = +n\,(\text{L.C.}) = 32 \times 0.01 = 0.32\text{ mm}$$

$$\text{Linear scale reading} = 4 \times (1\text{ mm}) = 4\text{ mm}$$

$$\text{Circular scale reading} = 16 \times (0.01\text{ mm}) = 0.16\text{ mm}$$

$$\therefore \quad \text{Measured reading} = (4 + 0.16)\text{ mm} = 4.16\text{ mm}$$

$$\therefore \quad \text{Absolute reading} = \text{Measured reading} - e$$

$$= (4.16 - 0.32)\text{ mm} = 3.84\text{ mm}$$

Therefore, thickness of the glass plate is 3.84 mm.

Sample Example 1.12 *The smallest division on main scale of a vernier callipers is 1 mm and 10 vernier divisions coincide with 9 scale divisions. While measuring the length of a line, the zero mark of the vernier scale lies between 10.2 cm and 10.3 cm and the third division of vernier scale coincide with a main scale division.*

(a) Determine the least count of the callipers.

(b) Find the length of the line.

Solution (a)

$$\text{Least count (L.C.)} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

$$= \frac{1}{10}\text{ mm} = 0.1\text{ mm}$$

$$= 0.01\text{ cm}$$

(b)

$$L = N + n\,(\text{L.C}) = (10.2 + 3 \times 0.01)\text{ cm}$$

$$= 10.23\text{ cm}$$

Sample Example 1.13 *The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. Find the diameter of the sphere.*

Solution

$$\text{L.C.} = \frac{1}{100} = 0.01\text{ mm}$$

$$\text{Linear scale reading} = 6\,(\text{pitch}) = 6\text{ mm}$$

$$\text{Circular scale reading} = n\,(\text{L.C.}) = 40 \times 0.01 = 0.4\text{ mm}$$

$$\therefore \quad \text{Total reading} = (6 + 0.4) = 6.4\text{ mm}$$

Extra Points

- **Least count :** The minimum measurement that can be measured accurately by an instrument is called the least count. The least count of a meter scale graduated in milimetre mark is 1 mm. The least count of a watch having second's hand is 1 second.
- Least count of vernier callipers

  = {Value of 1 part of main scale(s)} − {Value of one part of vernier scale (V)}

  or Least count of vernier callipers = 1 MSD − 1 VSD

  where, MSD = Main scale division

  VSD = Vernier scale division
- $\text{Least count} = \dfrac{\text{Value of 1 part of main scale } (s)}{\text{Number of parts on vernier scale } (n)}$
- $\text{Least count of screw gauge} = \dfrac{\text{Pitch } (p)}{\text{Number of parts on circular scale } (n)}$

Solved Examples

Example 1 *Least count of a vernier callipers is* 0.01 *cm. When the two jaws of the instrument touch each other the 5th division of the vernier scale coincide with a main scale division and the zero of the vernier scale lies to the left of the zero of the main scale. Furthermore while measuring the diameter of a sphere, the zero mark of the vernier scale lies between* 2.4 *cm and* 2.5 *cm and the 6th vernier division coincides with a main scale division. Calculate the diameter of the sphere.*

Solution The instrument has a negative error,

$$e = (-5 \times 0.01)\,\text{cm}$$

or

$$e = -\,0.05\,\text{cm}$$

$$\text{Measured reading } = (2.4 + 6 \times 0.01) = 2.46\,\text{cm}$$

$$\text{True reading} = \text{Measured reading} - e$$

$$= 2.46 - (-\,0.05)$$

$$= 2.51\,\text{cm}$$

Therefore, diameter of the sphere is 2.51 cm.

Example 2 *The pitch of a screw gauge is* 1 *mm and there are* 100 *divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies* 6 *divisions below the reference line. When a wire is placed between the jaws,* 2 *linear scale divisions are clearly visible while* 62 *divisions on circular scale coincide with the reference line. Determine the diameter of the wire.*

Solution

$$\text{L.C.} = \frac{p}{N} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}$$

The instrument has a positive zero error,

$$e = +\,n\,(\text{L.C.}) = +\,(6 \times 0.01) = +\,0.06\,\text{mm}$$

$$\text{Linear scale reading} = 2 \times (1\,\text{mm}) = 2\,\text{mm}$$

$$\text{Circular scale reading} = 62 \times (0.01\,\text{mm}) = 0.62\,\text{mm}$$

$\therefore$

$$\text{Measured reading} = 2 + 0.62 = 2.62\,\text{mm}$$

or

$$\text{True reading} = 2.62 - 0.06$$

$$= 2.56\,\text{mm}$$

Example 3 *Write down the number of significant figures in the following*

(a) 6428 *(b)* 62.00 *m* *(c)* 0.00628 *cm* *(d)* 1200 *N*

Solution (a) 6428 has four significant figures.

(b) 62.00 m has four significant figures.

(c) 0.00628 cm has three significant figures.

(d) 1200 N has four significant figures.

Example 4 *Round off to four significant figures:*

(a) 45.689 *(b)* 2.0082

Solution (a) 45.69, (b) 2.008

Example 5 *Add* 6.75×10^3 cm *to* 4.52×10^2 cm *with regard to significant figures.*

Solution

$$a = 6.75\times 10^3 \text{ cm}$$
$$b = 4.52\times 10^2 \text{ cm}$$
$$= 0.452\times 10^3 \text{ cm}$$
$$= 0.45\times 10^3 \text{ cm} \quad \text{(upto 2 places of decimal)}$$
$$\therefore \quad a+b = (6.75\times 10^3 + 0.45\times 10^3)\text{cm}$$
$$= 7.20\times 10^3 \text{ cm}$$

Example 6 *A thin wire has a length of* 21.7 *cm and radius* 0.46 *mm. Calculate the volume of the wire to correct significant figures.*

Solution Given: $l = 21.7$ cm, $r = 0.46$ mm $= 0.046$ cm.

$$\text{Volume of wire } V = \pi r^2 l$$
$$= \frac{22}{7}(0.046)^2\ (21.7)$$
$$= 0.1443 \text{ cm}^3 = 0.14 \text{ cm}^3$$

Note *The result is rounded off to least number of significant figures in the given measurements i.e., 2 (in 0.46 mm).*

Example 7 *The refractive index* (n) *of glass is found to have the values* 1.49, 1.50, 1.52, 1.54 *and* 1.48. *Calculate:*

(a) the mean value of refractive index.
(b) absolute error in each measurement.
(c) mean absolute error.
(d) fractional error and
(e) percentage error.

Solution (a) Mean value of refractive index,

$$n_m = \frac{1.49+1.50+1.52+1.54+1.48}{5}$$
$$= 1.506 = 1.51 \quad \text{(rounded off to two decimal places)}$$

(b) Taking n_m as the true value, the absolute errors in different observations are,

$$\Delta n_1 = 1.51 - 1.49 = +0.02$$
$$\Delta n_2 = 1.51 - 1.50 = +0.01$$
$$\Delta n_3 = 1.51 - 1.52 = -0.01$$
$$\Delta n_4 = 1.51 - 1.54 = -0.03$$
$$\Delta n_5 = 1.51 - 1.48 = +0.03$$

(c) Mean absolute error,

$$\Delta n_{\text{mean}} = \frac{|\Delta n_1| + |\Delta n_2| + |\Delta n_3| + |\Delta n_4| + |\Delta n_5|}{5}$$
$$= \frac{0.02+0.01+0.01+0.03+0.03}{5} = 0.02$$

(d) Fractional error $= \frac{\pm \Delta n_{\text{mean}}}{n_m} = \frac{\pm 0.02}{1.51} = \pm 0.0132$

(e) Percentage error $= (\pm 0.0132 \times 100) = \pm 1.32\%$

Example 8 *The radius of sphere is measured to be* (2.1 ± 0.5) *cm. Calculate its surface area with error limits.*

Solution Surface area,

$$S = 4\pi r^2 = (4)\left(\frac{22}{7}\right)(2.1)^2$$

$$= 55.44 = 55.4 \text{ cm}^2$$

Further,

$$\frac{\Delta S}{S} = 2 \cdot \frac{\Delta r}{r}$$

or

$$\Delta S = 2\left(\frac{\Delta r}{r}\right)(S) = \frac{2 \times 0.5 \times 55.4}{2.1}$$

$$= 26.38 = 26.4 \text{ cm}^2$$

$\therefore$

$$S = (55.4 \pm 26.4) \text{ cm}^2$$

Example 9 *Calculate focal length of a spherical mirror from the following observations. Object distance* $u = (50.1 \pm 0.5)$ *cm and image distance* $v = (20.1 \pm 0.2)$ *cm.*

Solution

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

or

$$f = \frac{uv}{u+v} = \frac{(50.1)(20.1)}{(50.1 + 20.1)} = 14.3 \text{ cm}$$

Also,

$$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u + \Delta v}{u + v}\right]$$

$$= \pm\left[\frac{0.5}{50.1} + \frac{0.2}{20.1} + \frac{0.5 + 0.2}{50.1 + 20.1}\right]$$

$$= [0.00998 + 0.00995 + 0.00997]$$

$$= \pm (0.0299)$$

$\therefore$

$$\Delta f = 0.0299 \times 14.3 = 0.428 = 0.4 \text{ cm}$$

$\therefore$

$$f = (14.3 \pm 0.4) \text{ cm}$$

EXERCISES

Section-I

Single Correct Option

1. Percentage error in the measurement of mass and speed are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and speed will be
   (a) 12% (b) 10% (c) 8% (d) 2%
2. The density of a cube is measured by measuring its mass and length of its sides. If the maximum error in the measurement of mass and length are 4% and 3% respectively, the maximum error in the measurement of density will be
   (a) 7% (b) 9% (c) 12% (d) 13%
3. A force F is applied on a square plate of side L. If the percentage error in the determination of L is 2% and that in F is 4%. What is the permissible error in pressure?
   (a) 8% (b) 6% (c) 4% (d) 2%
4. By what percentage should the pressure of a given mass of gas be increased, so as to decrease its volume by 10% at a constant temperature?
   (a) 11.1% (b) 10.1% (c) 9.1% (d) 8.1%
5. If the error in the measurement of the momentum of a particle is (+ 100%). Then, the error in the measurement of kinetic energy is
   (a) 400% (b) 300% (c) 100% (d) 200%
6. The number of significant figures in 3400 is
   (a) 7 (b) 6 (c) 12 (d) 2
7. The length and breadth of a metal sheet are 3.124 m and 3.002 m respectively. The area of this sheet upto correct significant figure is
   (a) $9.378\ m^2$ (b) $9.37\ m^2$ (c) $9.378248\ m^2$ (d) $9.3782\ m^2$
8. Let g be the acceleration due to gravity at earth's surface and K the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2%. Keeping all other quantities constant, then
   (a) g increases by 2% and K increases by 2% (b) g increases by 4% and K increases by 4%
   (c) g decreases by 4% and K decreases by 2% (d) g decreases by 2% and K decreases by 4%
9. The heat generated in a circuit is dependent upon the resistance, current and time for which the current is flown. If the error in measuring the above are 1%, 2% and 1% respectively. The maximum error in measuring the heat is
   (a) 8% (b) 6% (c) 18% (d) 12%
10. The length, breadth and thickness of a block are given by $l = 12\ cm$, $b = 6\ cm$ and $t = 2.45\ cm$. The volume of the block according to the idea of significant figures should be
    (a) $1 \times 10^2\ cm^3$ (b) $2 \times 10^2\ cm^3$ (c) $1.763 \times 10^2\ cm^3$ (d) None of these

11. If separation between screen and point source is increased by 2%. What would be the effect on the intensity?
(a) increases by 4% (b) increases by 2% (c) decreases by 2% (d) decreases by 4%

12. The significant figures in the number 6.0023 are
(a) 2 (b) 5 (c) 4 (d) 1

13. If error in measurement of radius of a sphere is 1%, what will be the error in measurement of volume?
(a) 1% (b) $\frac{1}{3}$% (c) 3% (d) 10%

14. The volume of a cube in m^3 is equal to the surface area of the cube in m^2. The volume of the cube is
(a) $64\ m^3$ (b) $216\ m^3$ (c) $512\ m^3$ (d) $196\ m^3$

15. The length of a simple pendulum is about 100 cm known to have an accuracy of 1 mm. Its period of oscillation is 2 s determined by measuring the time for 100 oscillations using a clock of 0.1 s resolution. What is the accuracy in the determined value of g?
(a) 0.2% (b) 0.5% (c) 0.1% (d) 2%

16. Charge on the capacitor is given by $Q = I\alpha e^{\frac{-tI}{\Delta V \varepsilon_0 \beta}}$, where α and β are constants. t = time, I = current, ΔV = potential difference. Then dimensions of $\frac{\beta}{\alpha}$ is same as the dimensions of
(a) $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ (b) $\mu_0 \varepsilon_0$ (c) $\sqrt{\varepsilon_0 \mu_0}$ (d) $\frac{1}{\mu_0 \varepsilon_0}$

17. A physical quantity A is dependent on other four physical quantities p, q, r and s as given by $a = \frac{\sqrt{pq}}{r^2 s^3}$. The percentage error of measurement in p, q, r and s are 1%, 3%, 0.5% and 0.33% respectively, then the maximum percentrage error in A is
(a) 2% (b) 0% (c) 4% (d) 3%

18. The distance moved by the screw of a screw gauge is 2 mm in four rotations and there are 50 divisions on its cap. When nothing is put between its jaws, 30th division of circular scale coincides with reference line, with zero of circular scale lying above the reference line. When a plate is placed between the jaws, main scale reads 2 divisions and circular scale reads 20 divisions. Thickness of plate is
(a) 0.9 mm (b) 1.2 mm (c) 1.4 mm (d) 1.5 mm

More than One Correct Options

1. Given, $x = \frac{ab^2}{c^3}$. If the percentage errors in a, b and c are ± 1%, ± 13% and ± 2% respectively.
(a) The percentage error in x can be ±13%
(b) The percentage error in x can be ±7%
(c) The percentage error in x can be ±18%
(d) The percentage error in x can be ±19%

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*
(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*
(c) *If **Assertion** is true, but the **Reason** is false.*
(d) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : A screw gauge having a smaller value of pitch has greater accuracy.
Reason : The least count of screw gauge is directly proportional to the number of divisions on circular scale.

Match the Columns

1. Match the following two columns.

Column-I	Column-II
(a) GM_eM_s	(p) $[M^2L^2T^{-3}]$
(b) $\frac{3RT}{M}$	(q) $[ML^3T^{-2}]$
(c) $\frac{F^2}{q^2B^2}$	(r) $[L^2T^{-2}]$
(d) $\frac{GM_e}{R_e}$	(s) None

Section-II

Subjective Questions

Trigonometry

1. Find the value of
   (a) $\cos 120°$ (b) $\sin 240°$ (c) $\tan(-60°)$ (d) $\cot 300°$
   (e) $\tan 330°$ (f) $\cos(-60°)$ (g) $\sin(-150°)$ (h) $\cos(-120°)$
2. Find the value of
   (a) $\sec^2\theta - \tan^2\theta$ (b) $\text{cosec}^2\theta - \cot^2\theta - 1$
   (c) $2\sin 45°\cos 15°$ (d) $2\sin 15°\cos 45°$

Calculus

3. Differentiate the following functions with respect to x
   (a) $x^4 + 3x^2 - 2x$ (b) $x^2\cos x$ (c) $(6x+7)^4$ (d) $e^x x^5$
   (e) $\frac{(1+x)}{e^x}$
4. Integrate the following functions with respect to t
   (a) $\int(3t^2 - 2t)\,dt$ (b) $\int(4\cos t + t^2)\,dt$ (c) $\int(2t-4)^{-4}\,dt$ (d) $\int\frac{dt}{(6t-1)}$
5. Integrate the following functions
   (a) $\int_0^2 2t\,dt$ (b) $\int_{\pi/6}^{\pi/3}\sin x\,dx$ (c) $\int_4^{10}\frac{dx}{x}$ (d) $\int_0^{\pi}\cos x\,dx$
   (e) $\int_1^2(2t-4)\,dt$
6. Find maximum/minimum value of y in the functions given below
   (a) $y = 5 - (x-1)^2$ (b) $y = 4x^2 - 4x + 7$
   (c) $y = x^3 - 3x$ (d) $y = x^3 - 6x^2 + 9x + 15$
   (e) $y = (\sin 2x - x)$, where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Graphs

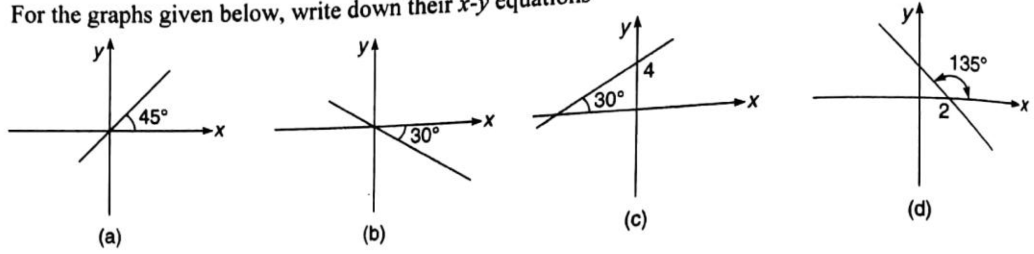
7. Draw the graphs corresponding to the equations

(a) $y = 4x$ (b) $y = -6x$

(c) $y = x + 4$ (d) $y = -2x + 4$

(e) $y = 2x - 4$ (f) $y = -4x - 6$

8. For the graphs given below, write down their x-y equations



9. For the equations given below, tell the nature of graphs.

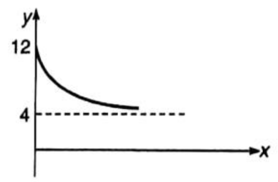
(a) $y = 2x^2$ (b) $y = -4x^2 + 6$ (c) $y = 6e^{-4x}$ (d) $y = 4(1 - e^{-2x})$

(e) $y = \frac{4}{x}$ (f) $y = -\frac{2}{x}$

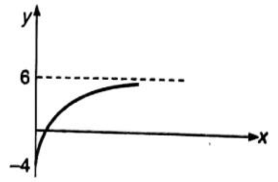
10. Value of y decreases exponentially from $y = 10$ to $y = 6$. Plot *a* x-y graph.

11. Value of y increases exponentially from $y = -4$ to $y = +4$. Plot a x-y graph.

12. The graph shown in figure is exponential. Write down the equation corresponding to the graph.



13. The graph shown in figure is exponential. Write down the equation corresponding to the graph.



Significant Figures

14. Write down the number of significant figures in the following.

(a) 6428 (b) 62.00 m (c) 0.0628 cm (d) 1200 N

15. Round off to four significant figures.

(a) 45.689 (b) 2.0082

16. Add 6.75×10^3 cm and 4.52×10^2 cm with due regards to significant figures.

17. A thin wire has length of 21.7 cm and radius 0.46 mm. Calculate the volume of the wire to correct significant figures.

18. A cube has a side of length 2.342 m. Find volume and surface area in correct significant figures.

19. Find density when a mass of 9.23 kg occupies a volume of $1.1\,\text{m}^3$. Take care of significant figures.

20. Length, breadth and thickness of a rectangular slab are 4.234 m, 1.005 m and 2.01 cm respectively. Find surface area and volume to correct significant figures.

21. Solve with due regards to significant figures

$$(4.0\times 10^{-4} - 2.5\times 10^{-6})$$

Error Analysis

22. The refractive index (n) of glass is found to have the values 1.49, 1.50, 1.52, 1.54 and 1.48. Calculate
    (a) the mean value of refractive index
    (b) absolute error is each measurement
    (c) fractional error and
    (d) percentage error

23. The radius of a sphere is measured to be (2.1 ± 0.5) cm. Calculate its surface area with error limits.

24. Calculate focal length of a spherical mirror from the following observations. Object distance $u = (50.1\pm 0.5)$ cm and image distance $v = (20.1\pm 0.2)$ cm.

25. Find the percentage error in specific resistance given by $\rho = \dfrac{\pi r^2 R}{l}$ where r is the radius having value (0.2 ± 0.02) cm, R is the resistance of (60 ± 2) ohm and l is the length of (150 ± 0.1) cm.

26. A physical quantity ρ is related to four variables α, β, γ and η as

$$\rho = \frac{\alpha^3\beta^2}{\sqrt{\gamma\eta}}$$

The percentage errors of measurements in α, β, γ and η are 1%, 3%, 4% and 2% respectively. Find the percentage error in ρ.

27. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$. Length L is about 10 cm and is known to 1 mm accuracy. The period of oscillation is about 0.5 s. The time of 100 oscillations is measured with wristwatch of 1 s time period. What is accuracy in the determination of g.

Vernier Callipers and Screw Gauge

28. 19 divisions on the main scale of a vernier callipers coincide with 20 divisions on the vernier scale. If each division on the main scale is of 1 cm, determine the least count of instrument.

29. The pitch of a screw guage is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures.

30. The edge of a cube is measured using a vernier callipers. [9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm]. The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g. Calculate the density in g/cm^3 upto correct significant figures.

ANSWERS

Section-I

Single Correct Option

1.(c)	**2.**(d)	**3.**(a)	**4.**(a)	**5.**(b)	**6.**(d)	**7.**(a)	**8.**(b)	**9.**(b)	**10.**(b)
11.(d)	**12.**(b)	**13.**(c)	**14.**(b)	**15.**(a)	**16.**(a)	**17.**(c)	**18.**(d)		

More than One Correct Options

1. (a,b)

Assertion and Reason

1. (c)

Match the Columns

1. (a) → q (b) → r (c) → r (d) → r

Section-II

Subjective Questions

1. (a) $-\frac{1}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$ (e) $-\frac{1}{\sqrt{3}}$ (f) $\frac{1}{2}$ (g) $-\frac{1}{2}$ (h) $-\frac{1}{2}$

2. (a) 1 (b) 0 (c) $\left(\frac{\sqrt{3}+1}{2}\right)$ (d) $\left(\frac{\sqrt{3}-1}{2}\right)$

3. (a) $4x^3 + 6x - 2$ (b) $2x\cos x - x^2 \sin x$ (c) $24(6x+7)^3$ (d) $5e^x x^4 + e^x x^5$ (e) $-xe^{-x}$

4. (a) $t^3 - t^2 + C$ (b) $4\sin t + \frac{t^3}{3} + C$ (c) $-\frac{1}{6(2t-4)^3} + C$ (d) $\frac{1}{6}\ln(6t-1) + C$

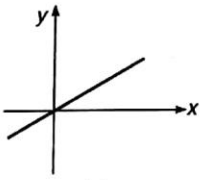
5. (a) 4 (b) $\frac{(\sqrt{3}-1)}{2}$ (c) $\ln(5/2)$ (d) Zero (e) -1

6. (a) $y_{max} = 5$ at $x = 1$ (b) $y_{min} = 6$ at $x = 1/2$ (c) $y_{min} = -2$ at $x = 1$ and $y_{max} = 2$ at $x = -1$

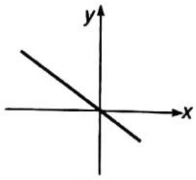
(d) $y_{min} = 15$ at $x = 3$ and $y_{max} = 19$ at $x = 1$ (e) $y_{min} = -\left(\frac{\sqrt{3}}{2} + \frac{\pi}{6}\right)$ at $x = -\pi/6$ and

$y_{max} = \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$ at $x = \pi/6$

7.



(a)



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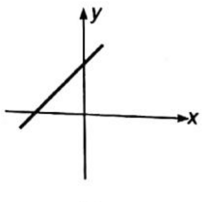

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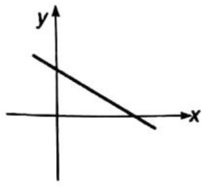

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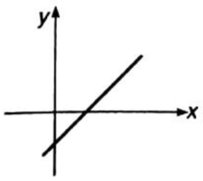
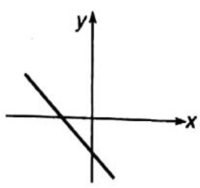


(e)

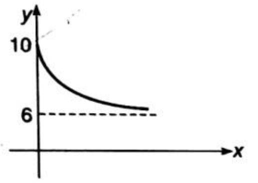


(f)

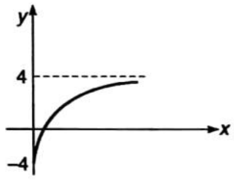
8. (a) $y = x$ (b) $y = -\frac{x}{\sqrt{3}}$ (c) $y = \frac{x}{\sqrt{3}} + 4$ (d) $y = -x + 2$

9. (a) parabola passing through origin (b) parabola not passing through origin
(c) exponentially decreasing graph (d) exponentially increasing graph
(e) Rectangular hyperbola in first and third quadrant
(f) Rectangular hyperbola in second and fourth quadrant

10.



11.



12. $y = 4 + 8e^{-Kx}$ Here, K is a positive constant **13.** $y = -4 + 10(1 - e^{-Kx})$ Here, K is positive constant

14. (a) Four (b) Four (c) Three (d) Four **15.** (a) 45.69 (b) 2.008 **16.** 7.20×10^3 cm

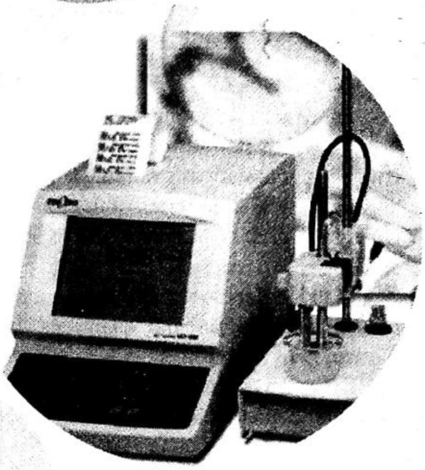
17. 0.14 cm^3 **18.** Area = 5.485 m^2, volume = 12.85 m^3 **19.** Density = 8.4 kg/m^3

20. Area = 4.255 m^2, volume = 8.55 m^3 **21.** 4.0×10^{-4}

22. (a) 1.51 (b) + 0.02, + 0.01, – 0.01, – 0.03, + 0.03 (c) ± 0.0132 (d) ± 1.32% **23.** (55.4 ± 26.4) cm^2

24. (14.3 ± 0.4) cm **25.** 23.4% **26.** 13% **27.** 5% **28.** 0.05 cm **29.** 2.6 cm^2 **30.** 2.66 g/cm^3

Chapter 2 – Units & Dimensions Vectors



2

Units & Dimensions Vectors

Chapter Contents

2.1 Units and Dimensions

To measure a physical quantity we need some standard unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives how many times of the standard unit and the second part gives the name of the unit. Thus, suppose I say that length of this wire is 5 metres. The numeric part 5 says that it is 5 times of the unit of length and the second part metre says that unit chosen here is metre.

Fundamental and Derived Quantities

There are a large number of physical quantities and every quantity needs a unit.

However, not all the quantities are independent. For example, if a unit of length is defined, a unit of volume is automatically obtained. Thus, we can define a set of fundamental quantities and all other quantities may be expressed in terms of the fundamental quantities. Fundamental quantities are only seven in numbers. Unit of all other quantities can be expressed in terms of the units of these seven quantities by multiplication or division.

Many different choices can be made for the fundamental quantities. For example, if we take length and time as the fundamental quantities then speed is a derived quantity and if we take speed and time as fundamental quantities then length is a derived quantity.

Several system of units are in use over the world. The units defined for the fundamental quantities are called fundamental units and those obtained for derived quantities are called the derived units.

SI Units

In 1971 General Conference on Weight and Measures held its meeting and decided a system of units which is known as the International System of Units. It is abbreviated as SI from the French name Le Systeme International *d'* Unites. This system is widely used throughout the world. Table below gives the seven fundamental quantities and their SI units.

Table 2.1 Fundamental quantities and their SI units.

S. No.	Quantity	SI Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Thermodynamic temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

Two **supplementary** units namely plane angle and solid angle are also defined. Their units are radian (rad) and steradian (st) respectively.

(i) **CGS System**: In this system, the units of length, mass and time are centimetre (cm), gram (g) and second (s) respectively. The unit of force is dyne and that of work or energy is erg.

(ii) **FPS System**: In this system, the units of length, mass and time are foot, pound and second. The unit of force in this system is poundal.

Definitions of Some Important SI Units

(i) **Metre**: 1 m = 1,650,763.73 wavelengths in vacuum, of radiation corresponding to orange-red light of krypton-86.

(ii) **Second**: 1 s = 9,192,631,770 time periods of a particular radiation from Cesium-133 atom.

(iii) **Kilogram** : 1 kg = mass of 1 litre volume of water at 4°C.

(iv) **Ampere** : It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of one metre in vacuum produces a force of 2×10^{-7} N/m between them.

(v) **Kelvin** : $1\,K = 1/273.16$ part of the thermodynamic temperature of triple point of water.

(vi) **Mole** : It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12 g of carbon-12.

SI Prefixes

The most commonly used prefixes are given below in tabular form.

Power of 10	Prefix	Symbol
6	mega	M
3	kilo	k
– 2	centi	c
– 3	mili	m
– 6	micro	μ
– 9	nano	n
– 12	pico	p

(vii) **Candela** : It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right)$ m^2 of a black body at the temperature of freezing platinum under a pressure of 1.013×10^5 N/m^2.

(viii) **Radian** : It is the plane angle between two radii of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) **Steradian** : The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

Dimensions

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

For example, $$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$$

or $$\text{density} = (\text{mass})\,(\text{length})^{-3} \qquad \text{...(i)}$$

Thus, the dimensions of density are 1 in mass and – 3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by M, length by L, time by T and electric current by A.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

Thus, Eq. (i) can be written as $[\text{density}] = [ML^{-3}]$

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional formula.

Here, it is worthnoting that constants such as 5, π or trigonometrical functions such as sin θ, cos θ, etc., have no units and dimensions.

$$[\sin\theta] = [\cos\theta] = [\tan\theta] = [\log x] = [e^x] = [M^0L^0T^0]$$

Table 2.2 Dimensional formulae and SI units of some physical quantities frequently used in physics

S. No.	Physical Quantity	SI Units	Dimensional Formula
1.	Velocity = displacement/time	m/s	$[M^0LT^{-1}]$
2.	Acceleration = velocity/time	m/s^2	$[M^0LT^{-2}]$
3.	Force = mass × acceleration	$kg\text{-}m/s^2$ = newton or N	$[MLT^{-2}]$
4.	Work = force × displacement	$kg\text{-}m^2/s^2$ = N-m = joule or J	$[ML^2T^{-2}]$
5.	Energy = Work	J	$[ML^2T^{-2}]$
6.	Torque = force × perpendicular distance	N-m	$[ML^2T^{-2}]$
7.	Power = work/time	J/s or watt	$[ML^2T^{-3}]$
8.	Momentum = mass × velocity	kg-m/s	$[MLT^{-1}]$
9.	Impulse = force × time	N-s	$[MLT^{-1}]$
10.	Angle = arc/radius	radian or rad	$[M^0L^0T^0]$
11.	Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	no units	$[M^0L^0T^0]$
12.	Stress = force/area	N/m^2	$[ML^{-1}T^{-2}]$
13.	Pressure = force/area	N/m^2	$[ML^{-1}T^{-2}]$
14.	Modulus of elasticity = stress/strain	N/m^2	$[ML^{-1}T^{-2}]$
15.	Frequency = 1/time period	per sec or hertz (Hz)	$[M^0L^0T^{-1}]$
16.	Angular velocity = angle/time	rad/s	$[M^0L^0T^{-1}]$
17.	Moment of inertia = (mass) × (distance)^2	$kg\text{-}m^2$	$[ML^2T^0]$
18.	Surface tension = force/length	N/m	$[ML^0T^{-2}]$
19.	Gravitational constant = $\frac{\text{force} \times (\text{distance})^2}{(\text{mass})^2}$	$N\text{-}m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
20.	Angular momentum	$kg\text{-}m^2/s$	$[ML^2T^{-1}]$
21.	Coefficient of viscosity	$N\text{-}s/m^2$	$[ML^{-1}T^{-1}]$
22.	Planck's constant	J-s	$[ML^2T^{-1}]$
23.	Specific heat (s)	J/kg-K	$[L^2T^{-2}\theta^{-1}]$
24.	Coefficient of thermal conductivity (K)	watt/m-K	$[MLT^{-3}\theta^{-1}]$
25.	Gas constant (R)	J/mol-K	$[ML^2T^{-2}\theta^{-1}mol^{-1}]$
26.	Boltzmann constant (k)	J/K	$[ML^2T^{-2}\theta^{-1}]$
27.	Wein's constant (b)	m-K	$[L\theta]$
28.	Stefan's constant (σ)	$watt/m^2\text{-}K^4$	$[MT^{-3}\theta^{-4}]$
29.	Electric charge	C	$[AT]$
30.	Electric intensity	N/C	$[MLT^{-3}A^{-1}]$
31.	Electric potential	volt	$[ML^2T^{-3}A^{-1}]$
32.	Capacitance	farad	$[M^{-1}L^{-2}T^4A^2]$
33.	Permittivity of free space	$C^2N^{-1}m^{-2}$	$[M^{-1}L^{-3}T^4A^2]$
34.	Electric dipole moment	C-m	$[LTA]$
35	Density	kg/m^3	$[ML^{-3}T^0]$

35.	Resistance	ohm	$[ML^2T^{-3}A^{-2}]$
36.	Magnetic field	tesla (T) or weber/m^2 (Wb/m^2)	$[MT^{-2}A^{-1}]$
37.	Coefficient of self induction	henry	$[ML^2T^{-2}A^{-2}]$

Check Points

- Astronomical unit
  1 AU = mean distance of earth from sun $\approx 1.5\times 10^{11}$ m
- Light year
  1 ly = distance travelled by light in vacuum in 1 year $= 9.46 \times 10^{15}$ m
- Parsec
  1 Parsec $= 3.07\times 10^{16}$ m = 3.26 light year
- X-ray unit
  1 U $= 10^{-3}$ m
- 1 shake $= 10^{-8}$ s
- 1 Bar $= 10^5$ N/m^2 $= 10^5$ Pa
- 1 torr = 1 mm of Hg = 133.3 Pa
- 1 barn $= 10^{-28}$ m^2
- 1 horse power = 746 W
- 1 pound = 453.6 g = 0.4536 kg

Sample Example 2.1 *Find the dimensional formulae of*

(a) coefficient of viscosity η *(b) charge* q

(c) potential V *(d) capacitance* C *and*

(e) resistance R

Some of the equations containing these quantities are

$$F = -\eta A\left(\frac{\Delta v}{\Delta l}\right), \quad q = It, \quad U = VIt,$$

$$q = CV \quad and \quad V = IR$$

Where A denotes the area, v the velocity, l is the length, I the electric current, t the time and U the energy.

Solution (a) $\eta = -\frac{F}{A}\frac{\Delta l}{\Delta v}$

$$\therefore \quad [\eta] = \frac{[F][l]}{[A][v]} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$$

(b) $q = It \quad \therefore \quad [q] = [I][t] = [AT]$

(c) $U = VIt \quad \therefore \quad V = \frac{U}{It}$ or $[V] = \frac{[U]}{[I][t]} = \frac{[ML^2T^{-2}]}{[A][T]} = [ML^2T^{-3}A^{-1}]$

(d) $q = CV \quad \therefore \quad C = \frac{q}{V}$ or $[C] = \frac{[q]}{[V]} = \frac{[AT]^2}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$

(e) $V = IR \quad \therefore \quad R = \frac{V}{I}$

or $[R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$

Uses of Dimensions

Theory of dimensions have following main uses:

(i) **Conversion of units** : This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, *i.e.*,

$$n\,[u] = \text{constant}$$

or

$$n_1\,[u_1] = n_2\,[u_2]$$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then, we can write

$$n_1\,[M_1^a\,L_1^b\,T_1^c] = n_2\,[M_2^a\,L_2^b\,T_2^c] \qquad \text{...(i)}$$

Here, n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

Sample Example 2.2 *The value of gravitation constant is $G = 6.67 \times 10^{-11}$ N-m^2/kg^2 in SI units. Convert it into CGS system of units.*

Solution The dimensional formula of G is $[M^{-1}\,L^3\,T^{-2}]$.

Using equation number (i), *i.e.*,

$$n_1[M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

$M_1 = 1\,\text{kg}$, $M_2 = 1\,\text{g} = 10^{-3}\,\text{kg}$ $L_1 = 1\,\text{m}$, $L_2 = 1\,\text{cm} = 10^{-2}\,\text{m}$, $T_1 = T_2 = 1\,\text{s}$

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\,\text{kg}}{10^{-3}\,\text{kg}}\right]^{-1}\left[\frac{1\,\text{m}}{10^{-2}\,\text{m}}\right]^{3}\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2} \quad \text{or} \quad n_2 = 6.67 \times 10^{-8}$$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm^2/g^2.

(ii) **To check the dimensional correctness of a given physical equation** : Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

Sample Example 2.3 *Show that the expression of the time period T of a simple pendulum of length l given by $T = 2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.*

Solution

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Dimensionally } [T] = \sqrt{\frac{[L]}{[LT^{-2}]}} = [T]$$

As in the above equation, the dimensions of both sides are same. The given formula is dimensionally correct.

Principle of Homogeneity of Dimensions

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Note *The physical quantities separated by the symbols +, −, =, >, < etc., have the same dimensions.*

Sample Example 2.4 *The velocity v of a particle depends upon the time t according to the equation* $v = a + bt + \frac{c}{d+t}$. *Write the dimensions of a, b, c and d.*

Solution From principle of homogeneity

$$[a] = [v]$$

or $$[a] = [\mathrm{LT^{-1}}]$$

$$[bt] = [v]$$

or $$[b] = \frac{[v]}{[t]} = \frac{[\mathrm{LT^{-1}}]}{[\mathrm{T}]}$$

or $$[b] = [\mathrm{LT^{-2}}]$$

Similarly, $$[d] = [t] = [\mathrm{T}]$$

Further, $$\frac{[c]}{[d+t]} = [v]$$

or $$[c] = [v][d+t]$$

or $$[c] = [\mathrm{LT^{-1}}][\mathrm{T}] \quad \text{or} \quad [c] = [\mathrm{L}]$$

(iii) **To establish the relation among various physical quantities** : If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors. Let us take an example.

Sample Example 2.5 *The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length l of the string and the mass per unit length μ of string. Derive the formula for frequency.*

Solution Suppose, that the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c. Then,

$$f \propto [F]^a \, [l]^b \, [\mu]^c$$

or $$f = k\,[F]^a \, [l]^b \, [\mu]^c \qquad \text{...(i)}$$

Here, k is a dimensionless constant. Thus,

$$[f] = [F]^a \, [l]^b \, [\mu]^c$$

or $$[\mathrm{M^0L^0T^{-1}}] = [\mathrm{MLT^{-2}}]^a \, [\mathrm{L}]^b \, [\mathrm{ML^{-1}}]^c$$

or $$[\mathrm{M^0L^0T^{-1}}] = [\mathrm{M}^{a+c}\mathrm{L}^{a+b-c}\mathrm{T}^{-2a}]$$

For dimensional balance, the dimensions on both sides should be same.

Thus, $$a + c = 0 \quad \text{...(ii)}$$

$$a + b - c = 0 \quad \text{...(iii)}$$

and $$-2a = -1 \quad \text{...(iv)}$$

Solving these three equations, we get

$$a = \frac{1}{2}, \quad c = -\frac{1}{2} \quad \text{and} \quad b = -1$$

Substituting these values in Eq. (i), we get

$$f = k(F)^{1/2} (l)^{-1} (\mu)^{-1/2} \quad \text{or} \quad f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$

Experimentally, the value of k is found to be $\frac{1}{2}$.

Hence, $$f = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

Limitations of Dimensional Analysis

The method of dimensions has the following limitations :

(i) By this method the value of dimensionless constant cannot be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

2.2 Vector and Scalar Quantities

Any physical quantity is either a scalar or a vector. A scalar quantity can be described completely by its magnitude only. Addition, subtraction, division or multiplication of scalar quantities can be done according to the ordinary rules of algebra. Mass, volume, density, etc., are few examples of scalar quantities. If a physical quantity in addition to magnitude has a specified direction as well as obeys the law of parallelogram of addition, then and then only it is said to be a vector quantity. Displacement, velocity, acceleration, etc., are few examples of vectors. Any vector quantity should have a specified direction but it is not a sufficient condition for a quantity to be a vector. For example, current flowing in a wire is shown by a direction but it is not a vector because it does not obey the law of parallelogram of vector addition. For example, in the figure shown here.

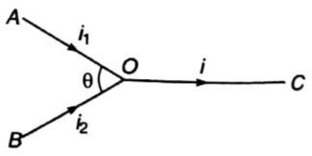


Fig. 2.1

Current flowing in wire OC = current through AO + current through BO

or $$i = i_1 + i_2$$

It would had been $$i^2 = i_1^2 + i_2^2 + 2i_1 i_2 \cos\theta$$

In case, the current would had been a vector quantity.

2.3 General Points Regarding Vectors

Vector Notation

Usually a vector is represented by a bold capital letter with an arrow over it, as $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ etc. The magnitude of a vector $\vec{\mathbf{A}}$ is represented by A or $|\vec{\mathbf{A}}|$.

Graphical Representation of a Vector

Graphically a vector is represented by an arrow drawn to a chosen scale, parallel to the direction of the vector. The length and the direction of the arrow thus represent the magnitude and the direction of the vector respectively.

Thus, the arrow in Fig. 2.2 represents a vector $\vec{A}$ in xy-plane making an angle θ with x-axis.

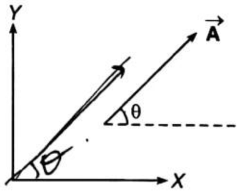


Fig. 2.2

Angle between two Vectors (θ)

To find angle between two vectors both the vectors are drawn from a point in such a manner that arrows of both the vectors are outwards from that point. Now, the smaller angle is called the angle between two vectors.

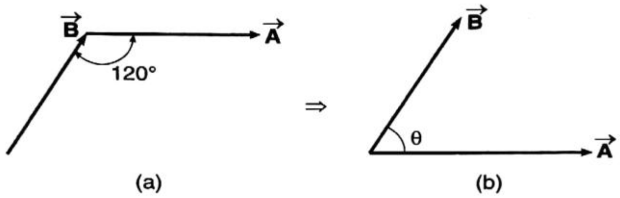


Fig. 2.3

For example in Fig. 2.3 angle between $\vec{A}$ and $\vec{B}$ is 60° not 120°. Because in figure (a) they are wrongly drawn while in figure (b) they are drawn as we desire.

Unit and Zero Vector

A vector of unit magnitude is called a unit vector and the notation for it in the direction of $\vec{A}$ is $\hat{A}$ read as '*A* hat or *A* caret'.

Thus, $\vec{A} = A\hat{A}$

A unit vector merely indicates a direction. Unit vector along x, y and z-directions are $\hat{i}$, $\hat{j}$ and $\hat{k}$.

A vector of zero magnitude is called a zero or a null vector. Its direction is arbitrary.

Negative of a Vector

Means a vector of same magnitude but opposite in direction.

Multiplication and Division of Vectors by Scalars

The product of a vector $\vec{A}$ and a scalar m is a vector $m\vec{A}$ whose magnitude is m times the magnitude of $\vec{A}$ and which is in the direction or opposite to $\vec{A}$ according as the scalar m is positive or negative. Thus,

$$|m\vec{A}| = mA$$

Further, if m and n are two scalars, then

$$(m+n)\vec{A} = m\vec{A} + n\vec{A} \quad \text{and} \quad m(n\vec{A}) = n(m\vec{A}) = (mn)\vec{A}$$

The division of vector $\vec{A}$ by a non-zero scalar m is defined as the multiplication of $\vec{A}$ by $\frac{1}{m}$.

Equality of Vectors

All vectors with the same magnitude and direction are equal despite their different locations in space. Thus, if a vector is displaced parallel to itself, it does not change.

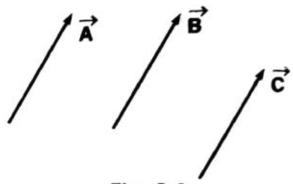


Fig. 2.4

In Fig. 2.4, $\vec{A}, \vec{B}$ and $\vec{C}$ are all equal, since they have the same magnitude and direction even though they are differently located in space.

2.4 Addition and Subtraction of Two Vectors

Addition

(i) **The Parallelogram law** : Let $\vec{R}$ be the resultant of two vectors $\vec{A}$ and $\vec{B}$. According to parallelogram law of vector addition, the resultant $\vec{R}$ is the diagonal of the parallelogram of which $\vec{A}$ and $\vec{B}$ are the adjacent sides as shown in figure. Magnitude of $\vec{R}$ is given by

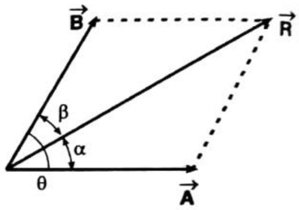


Fig. 2.5

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad \text{...(i)}$$

Here, θ = angle between $\vec{A}$ and $\vec{B}$. The direction of $\vec{R}$ can be found by angle α or β of $\vec{R}$ with $\vec{A}$ or $\vec{B}$.

Here, $$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

and $$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta} \qquad \text{...(ii)}$$

Special cases

If $\theta = 0°$, $R = \text{maximum} = A + B$

$\theta = 180°$, $R = \text{minimum} = A \sim B$

and if $\theta = 90°$, $R = \sqrt{A^2 + B^2}$

In all other cases magnitude and direction of $\vec{R}$ can be calculated by using Eqs. (i) and (ii).

(ii) **The Triangle Law** : According to this law if the tail of one vector be placed at the head of the other, their sum or resultant $\vec{R}$ is drawn from the tail end of the first to the head end of the other.

As is evident from the figure that the resultant $\vec{R}$ is the same irrespective of the order in which the vectors $\vec{A}$ and $\vec{B}$ are taken, Thus, $\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$

Fig. 2.6

Subtraction

Negative of a vector say $-\vec{A}$ is a vector of the same magnitude as vector $\vec{A}$ but pointing in a direction opposite to that of $\vec{A}$.

Fig. 2.7

Thus, $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ can be written as $\vec{\mathbf{A}} + (-\vec{\mathbf{B}})$ or $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is really the vector addition of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$.

Suppose angle between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is θ. Then angle between $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$ will be $180 - \theta$ as shown in Fig. 2.8(b).

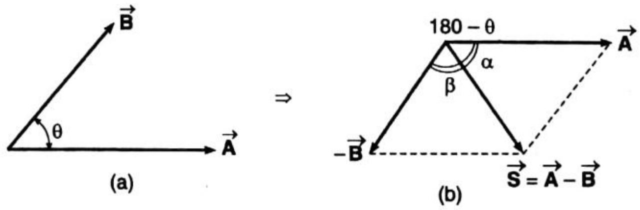


Fig. 2.8

Magnitude of $\vec{\mathbf{S}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ will be thus given by

$$S = |\vec{\mathbf{A}} - \vec{\mathbf{B}}| = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

or $$S = \sqrt{A^2 + B^2 - 2AB\cos\theta} \qquad \text{...(i)}$$

For direction of $\vec{\mathbf{S}}$ we will either calculate angle α or β, where,

$$\tan\alpha = \frac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)} = \frac{B\sin\theta}{A - B\cos\theta} \qquad \text{...(ii)}$$

or $$\tan\beta = \frac{A\sin(180 - \theta)}{B + A\cos(180 - \theta)} = \frac{A\sin\theta}{B - A\cos\theta} \qquad \text{...(iii)}$$

Note $\vec{A} - \vec{B}$ *or* $\vec{B} - \vec{A}$ *can also be found by making triangles as shown in Fig. 2.9 (a) and (b).*

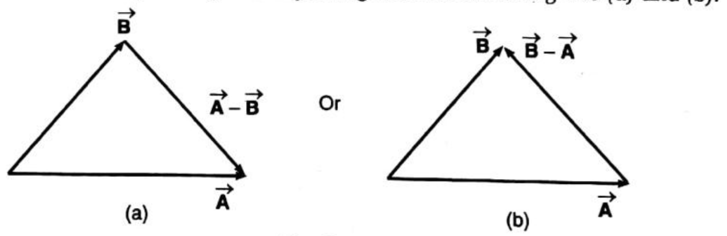


Fig. 2.9

Sample Example 2.6 *Find $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ in the diagram shown in figure. Given $A = 4$ units and $B = 3$ units.*

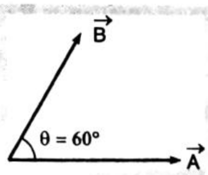


Fig. 2.10

Solution Addition : $$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{16 + 9 + 2 \times 4 \times 3\cos 60^\circ} = \sqrt{37} \text{ units}$$

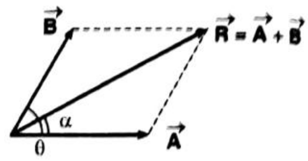


Fig. 2.11

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$= \frac{3\sin 60°}{4 + 3\cos 60°} = 0.472$$

$$\therefore \qquad \alpha = \tan^{-1}(0.472) = 25.3°$$

Thus, resultant of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{\mathbf{A}}$ in the direction shown in figure.

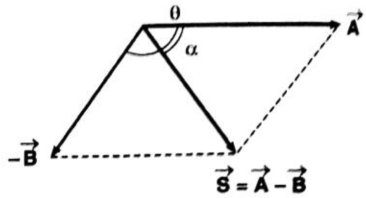


Fig. 2.12

Subtraction :

$$S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{16 + 9 - 2 \times 4 \times 3\cos 60°}$$

$$= \sqrt{13} \text{ units}$$

and

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$= \frac{3\sin 60°}{4 - 3\cos 60°} = 1.04$$

$$\therefore \qquad \alpha = \tan^{-1}(1.04) = 46.1°$$

Thus, $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is $\sqrt{13}$ units at 46.1° from $\vec{\mathbf{A}}$ in the direction shown in figure.

Polygon Law of Vector Addition for more than two Vectors

This law states that if a vector polygon be drawn, placing the tail end of each succeeding vector at the head or the arrow end of the preceding one their resultant $\vec{\mathbf{R}}$ is drawn from the tail end of the first to the head or the arrow end of the last.

Thus, in the figure $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$

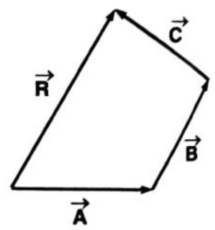


Fig. 2.13

2.5 Components of a Vector

Two or more vectors which, when compounded in accordance with the parallelogram law of vector $\vec{\mathbf{R}}$ are said to be components of vector $\vec{\mathbf{R}}$. The most important components with which we are concerned are mutually perpendicular or rectangular ones along the three co-ordinate axes *ox*, *oy* and *oz* respectively. Thus, a vector $\vec{\mathbf{R}}$ can be written as $\vec{\mathbf{R}} = R_x\,\hat{\mathbf{i}} + R_y\,\hat{\mathbf{j}} + R_z\,\hat{\mathbf{k}}$.

Here, R_x, R_y and R_z are the components of $\vec{\mathbf{R}}$ in *x*, *y* and *z*-axes respectively and $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along these directions. The magnitude of $\vec{\mathbf{R}}$ is given by

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

This vector $\vec{\mathbf{R}}$ makes an angle of $\alpha = \cos^{-1}\left(\frac{R_x}{R}\right)$ with *x*-axis

$$\beta = \cos^{-1}\left(\frac{R_y}{R}\right) \text{ with } y\text{-axis}$$

and $$\gamma = \cos^{-1}\left(\frac{R_z}{R}\right) \text{ with } z\text{-axis}$$

Refer figure (a)

We have resolved a two dimensional vector (in xy plane) $\vec{\mathbf{R}}$ in mutually perpendicular directions x and y.

Component along x-axis $= R_x = R\cos\alpha$ or $R\sin\beta$ and component along y-axis $= R_y = R\cos\beta$ or $R\sin\alpha$.

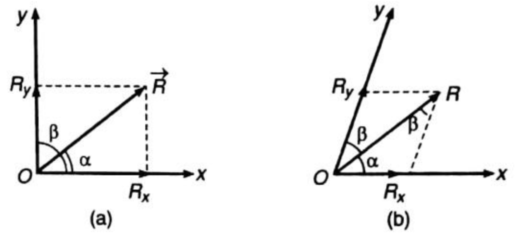


Fig. 2.14

If $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ be the unit vectors along x and y axes respectively, we can write

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

Refer figure (b)

Vector $\vec{\mathbf{R}}$ has been resolved in two axes x and y not perpendicular to each other. Applying sine law in the triangle shown , we have

$$\frac{R}{\sin[180-(\alpha+\beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

or $$R_x = \frac{R\sin\beta}{\sin(\alpha+\beta)} \quad \text{and} \quad R_y = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$

If $\alpha+\beta = 90°$, $R_x = R\sin\beta$ and $R_y = R\sin\alpha$

Sample Example 2.7 *Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.*

Solution Component perpendicular to the plane

$$W_\perp = W\cos 30° = (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

and Component parallel to the plane

$$W_{||} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

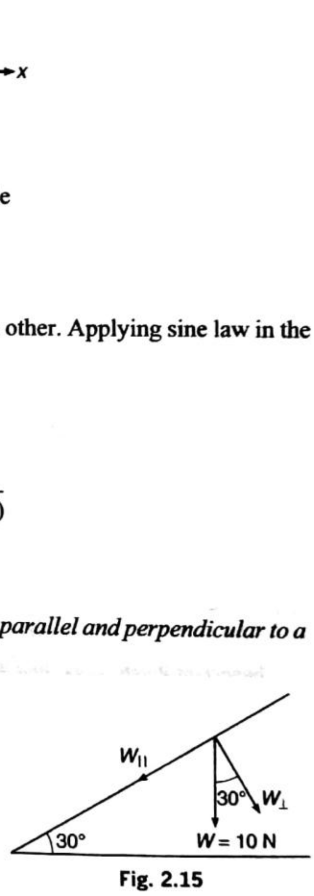


Fig. 2.15

Sample Example 2.8 *Resolve horizontally and vertically a force F = 8 N which makes an angle of 45° with the horizontal.*

Solution Horizontal component of $\vec{\mathbf{F}}$ is

$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\ \text{N}$$

and vertical component of $\vec{\mathbf{F}}$ is

$$F_V = F\sin 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\ \text{N}$$

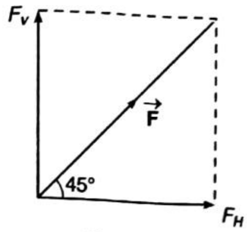


Fig. 2.16

Two vectors in the form of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ can be added, subtracted on multiplied by a scalar directly as done in the following example.

Sample Example 2.9 *Obtain the magnitude of* $2\vec{\mathbf{A}}-3\vec{\mathbf{B}}$ *if*

$$\vec{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}} \quad \textit{and} \quad \vec{\mathbf{B}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}.$$

Solution

$$2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) - 3(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$
$$= -4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$$

$\therefore$ Magnitude of $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} = \sqrt{(-4)^2 + (5)^2 + (-7)^2} = \sqrt{16 + 25 + 49} = \sqrt{90}$

2.6 Product of Two Vectors

The product of two vectors is of two kinds.

(i) a scalar or dot product.

(ii) a vector or a cross product.

Scalar or Dot Product

The scalar or dot product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is denoted by $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}$ and is read as $\vec{\mathbf{A}}$ dot $\vec{\mathbf{B}}$.

It is defined as the product of the magnitudes of the two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and the cosine of their included angle θ

Thus, $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta$ (a scalar quantity)

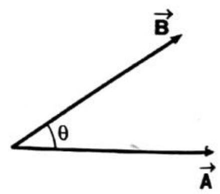


Fig. 2.17

Important Points Regarding Dot Product

The following points should be remembered regarding the dot product.

(i) $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$

(ii) $\vec{\mathbf{A}}\cdot(\vec{\mathbf{B}}+\vec{\mathbf{C}}) = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}} + \vec{\mathbf{A}}\cdot\vec{\mathbf{C}}$

(iii) $\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} = A^2$

(iv) $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A(B\cos\theta) = A$ (Component of $\vec{\mathbf{B}}$ along $\vec{\mathbf{A}}$)

or $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = B(A\cos\theta) = B$ (Component of $\vec{\mathbf{A}}$ along $\vec{\mathbf{B}}$)

(v) $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = (1)(1)\cos 0° = 1$

(vi) $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = (1)(1)\cos 90° = 0$

(vii) $(a_1\hat{\mathbf{i}} + b_1\hat{\mathbf{j}} + c_1\hat{\mathbf{k}})\cdot(a_2\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + c_2\hat{\mathbf{k}}) = a_1a_2 + b_1b_2 + c_1c_2$

(viii) $\cos\theta = \dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}$ (cosine of angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$)

(ix) Two vectors are perpendicular if their dot product is zero. ($\theta = 90°$)

Sample Example 2.10 *Work done by a force $\vec{\mathbf{F}}$ on a body is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}}$, where $\vec{\mathbf{s}}$ is the displacement of body. Given that under a force $\vec{\mathbf{F}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\,N$ a body is displaced from position vector $\vec{\mathbf{r}}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\,m$ to the position vector $\vec{\mathbf{r}}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\,m$. Find the work done by this force.*

Solution The body is displaced from $\vec{\mathbf{r}}_1$ to $\vec{\mathbf{r}}_2$. Therefore, displacement of the body is

$$\vec{\mathbf{s}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) = (-\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\text{m}$$

Now, work done by the force is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}}$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(-\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$$

$$= (2)(-1) + (3)(-2) = -8\text{ J}$$

Sample Example 2.11 *Find the angle between two vectors $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = \hat{\mathbf{i}} - \hat{\mathbf{k}}$.*

Solution

$$A = |\vec{\mathbf{A}}| = \sqrt{(2)^2 + (1)^2\,(-1)^2} = \sqrt{6}$$

$$B = |\vec{\mathbf{B}}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{k}}) = (2)(1) + (-1)(-1) = 3$$

Now, $$\cos\theta = \frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \frac{3}{\sqrt{6}\cdot\sqrt{2}} = \frac{3}{\sqrt{12}} = \frac{\sqrt{3}}{2}$$

$\therefore$ $\theta = 30°$

Sample Example 2.12 *Prove that the vectors $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ are mutually perpendicular.*

Solution

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (2)(1) + (-3)(1) + (1)(1)$$

$$= 0 = AB\cos\theta$$

$\therefore$ $\cos\theta = 0$ (as $A \neq 0, B \neq 0$)

or $\theta = 90°$

or the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are mutually perpendicular.

Vector or Cross Product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$ and read as $\vec{A}$ cross $\vec{B}$. It is defined as a third vector $\vec{C}$ whose magnitude is equal to the product of the magnitudes of the two vectors $\vec{A}$ and $\vec{B}$ and the sine of their included angle θ.

Thus, if $\vec{C} = \vec{A} \times \vec{B}$, then $C = AB \sin\theta$.

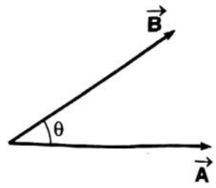


Fig. 2.18

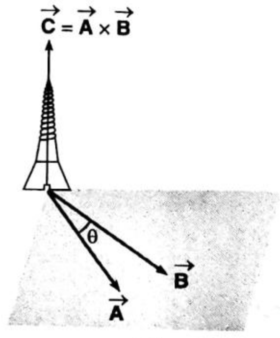
The vector $\vec{C}$ is normal to the plane of $\vec{A}$ and $\vec{B}$ and points in the direction in which a right handed screw would advance when rotated about an axis perpendicular to the plane of the two vectors in the direction from $\vec{A}$ to $\vec{B}$ through the smaller angle θ between them or, alternatively, we might state the rule as :

If the fingers of the right hand be curled in the direction in which vector $\vec{A}$ must be turned through the smaller included angle θ to coincide with the direction of vector $\vec{B}$, the thumb points in the direction of $\vec{C}$ as shown in Fig. 2.19.

Either of these rules is referred to as the right handed screw rule. Thus, if $\hat{n}$ be the unit vector in the direction of $\vec{C}$, we have

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$$

where $$0 \le \theta \le \pi$$



Plane of $\vec{A}$ and $\vec{B}$
Fig. 2.19

Important Points About Vector Product

(i) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

(ii) The cross product of two parallel vectors is zero, as $|\vec{A} \times \vec{B}| = AB \sin\theta$ and $\theta = 0°$ for two parallel vectors. Thus, $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(iii) If two vectors are perpendicular to each other, we have $\theta = 90°$ and therefore, $\sin\theta = 1$. So that $\vec{A} \times \vec{B} = AB\,\hat{n}$. The vectors $\vec{A}$, $\vec{B}$ and $\vec{A} \times \vec{B}$ thus form a right handed system of mutually perpendicular vectors. It follows at once from the above that in case of the orthogonal triad of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ (each perpendicular to each other)

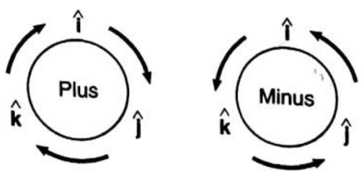


Fig. 2.20

$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i} \quad \text{and} \quad \hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

(iv) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(v) A vector product can be expressed in terms of rectangular components of the two vectors and put in the determinant form as may be seen from the following:

Let $$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

and $$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

Then, $\vec{A}\times\vec{B}=(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})\times(a_2\hat{i}+b_2\hat{j}+c_2\hat{k})$

$$=a_1a_2(\hat{i}\times\hat{i})+a_1b_2(\hat{i}\times\hat{j})+a_1c_2(\hat{i}\times\hat{k})+b_1a_2(\hat{j}\times\hat{i})+b_1b_2(\hat{j}\times\hat{j})$$
$$+b_1c_2(\hat{j}\times\hat{k})+c_1a_2(\hat{k}\times\hat{i})+c_1b_2(\hat{k}\times\hat{j})+c_1c_2(\hat{k}\times\hat{k})$$

Since, $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$ and $\hat{i}\times\hat{j}=\hat{k}$, etc., we have

$$\vec{A}\times\vec{B}=(b_1c_2-c_1b_2)\hat{i}+(c_1a_2-a_1c_2)\hat{j}+(a_1b_2-b_1a_2)\hat{k}$$

or putting it in determinant form, we have

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

It may be noted that the scalar components of the first vector $\vec{A}$ occupy the middle row of the determinant.

Sample Example 2.13 *Find a unit vector perpendicular to $\vec{A}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+\hat{k}$ both.*

Solution As we have read, $\vec{C}=\vec{A}\times\vec{B}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$. Hence, a unit vector $\hat{n}$ perpendicular to $\vec{A}$ and $\vec{B}$ can be written as

$$\hat{n}=\frac{\vec{C}}{C}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

Here, $$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & -1 & 1\end{vmatrix}$$

$$=\hat{i}(3+1)+\hat{j}(1-2)+\hat{k}(-2-3)=4\hat{i}-\hat{j}-5\hat{k}$$

Further, $|\vec{A}\times\vec{B}|=\sqrt{(4)^2+(-1)^2+(-5)^2}=\sqrt{42}$

$\therefore$ The desired unit vector is:

$$\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}\quad\text{or}\quad\hat{n}=\frac{1}{\sqrt{42}}(4\hat{i}-\hat{j}-5\hat{k})$$

Sample Example 2.14 *Show that the vector $\vec{A}=\hat{i}-\hat{j}+2\hat{k}$ is parallel to a vector $\vec{B}=3\hat{i}-3\hat{j}+6\hat{k}$.*

Solution A vector $\vec{A}$ is parallel to an another vector $\vec{B}$ if it can be written as

$$\vec{A}=m\vec{B}$$

Here, $\vec{A}=(\hat{i}-\hat{j}+2\hat{k})=\frac{1}{3}(3\hat{i}-3\hat{j}+6\hat{k})$

or $\vec{A}=\frac{1}{3}\vec{B}$

This implies that $\vec{A}$ is parallel to $\vec{B}$ and magnitude of $\vec{A}$ is $\frac{1}{3}$ times the magnitude of $\vec{B}$.

Note *Two vectors can be shown parallel to one another if*

(i) *The coefficients of* $\hat{i}$, $\hat{j}$ *and* $\hat{k}$ *of both the vectors bear a constant ratio. For example, a vector* $\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ *is parallel to an another vector* $\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ *if:* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(ii) *The cross product of both the vectors is zero. For instance,* $\vec{A}$ *and* $\vec{B}$ *are parallel to each other if*

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Sample Example 2.15 *Let a force* $\vec{\mathbf{F}}$ *be acting on a body free to rotate about a point O and let* $\vec{\mathbf{r}}$ *the position vector of any point P on the line of action of the force. Then torque* $(\vec{\tau})$ *of this force about point O is defined as:*

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

Given, $\vec{\mathbf{F}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\,N$ *and* $\vec{\mathbf{r}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 6\hat{\mathbf{k}})\,m$

Find the torque of this force.

Solution

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 6 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{\mathbf{i}}\,(1-18) + \hat{\mathbf{j}}(12+1) + \hat{\mathbf{k}}(3+2)$$

or

$$\vec{\tau} = (-17\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 5\hat{\mathbf{k}})\,\text{N-m}$$

Extra Points

- Pressure, surface tension and current are not vectors.
- To qualify as a vector, a physical quantity must not only possess magnitude and direction but must also satisfy the parallelogram law of vector addition. For instance, the finite rotation of a rigid body about a given axis has magnitude (the angle of rotation) and also direction (the direction of the axis) but it is not a vector quantity. This is so for the simple reason that the two finite rotations of the body do not add up in accordance with the law of vector addition. However if the rotation be small or infinitesimal, it may be regarded as a vector quantity.
- Area can behave either as a scalar or a vector and how it behaves depends on circumstances.
- Moment of inertia is neither a vector nor a scalar as it has different values about different axes. It is tensor. Although tensor is a generalised term which is characterized by its rank. For example scalars are tensors of rank zero. Vectors are tensors of rank one.
- Area (vector), dipole moment and current density are defined as vectors with specific direction.
- Vectors associated with a linear or directional effect are called polar vectors or, usually, simply as vectors and those associated with rotation about an axis are referred to as axial vectors. Thus force, linear velocity and acceleration area polar vectors and angular velocity, angular acceleration are axial vectors.

- Examples of Dot-product and Examples of Cross-product

Examples of Dot-product	Examples of Cross-product
$W = \vec{F} \cdot \vec{s}$	$\vec{\tau} = \vec{r} \times \vec{F}$
$P = \vec{F} \cdot \vec{v}$	$\vec{L} = \vec{r} \times \vec{P}$
$d\phi_e = \vec{E} \cdot \vec{ds}$	$\vec{v} = \vec{\omega} \times \vec{r}$
$d\phi_B = \vec{B} \cdot \vec{ds}$	$\vec{\tau}_e = \vec{P} \times \vec{E}$
$U_e = \vec{P} \cdot \vec{E}$	$\vec{\tau}_B = \vec{M} \times \vec{B}$
$U_B = \vec{M} \cdot \vec{B}$	$\vec{F}_B = q(\vec{v} \times \vec{B})$
	$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i(\vec{dl} \times \vec{r})}{r^3}$

- Students are often confused over the direction of cross product. Let us discuss a simple method. To find direction of $\vec{A} \times \vec{B}$ move from $\vec{A}$ to $\vec{B}$ through smaller angle. If it is clockwise then $\vec{A} \times \vec{B}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and away from you and if it is anti-clockwise then $\vec{A} \times \vec{B}$ is towards you perpendicular to the plane of $\vec{A}$ and $\vec{B}$.
- The area of triangle bounded by vectors $\vec{A}$ and $\vec{B}$ is $\frac{1}{2}|\vec{A} \times \vec{B}|$.

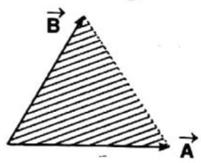


Fig. 2.21

Exercise : Prove the above result.

- **Scalar triple product :** $\vec{A} \cdot (\vec{B} \times \vec{C})$ is called scalar triple product. It is a scalar quantity. We can show that $\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C} = \vec{B} \cdot (\vec{C} \times \vec{A})$.
- The volume of a parallelopiped bounded by vectors $\vec{A}, \vec{B}$ and $\vec{C}$ can be obtained by $(\vec{A} \times \vec{B}) \cdot \vec{C}$.
- If three vectors are coplanar then the volume of the parallelopiped bounded by these three vectors should be zero or we can say their scalar triple product should be zero.
- If $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then $\vec{A} \cdot (\vec{B} \times \vec{C})$ is also written as $[\vec{A}\vec{B}\vec{C}]$ and it has the following value,

$$[\vec{A}\vec{B}\vec{C}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

= Volume of parallelopiped whose adjacent sides are along $\vec{A}, \vec{B}$ and $\vec{C}$.

- Properties of scalar triple product :

(a) $[\vec{A}\vec{B}\vec{C}] = [\vec{B}\vec{C}\vec{A}] = [\vec{C}\vec{A}\vec{B}]$

(b) $[\vec{A}\vec{B}\vec{C}] = -[\vec{B}\vec{A}\vec{C}]$

(c) If vectors are coplanar, then $[\vec{A}\vec{B}\vec{C}] = 0$

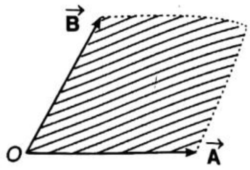


Fig. 2.22

- Area of triangle ABC if position vector of A is $\vec{\mathbf{a}}$ position vector of B is $\vec{\mathbf{b}}$ and position vector of C is $\vec{\mathbf{c}}$

$$\text{Area} = \frac{1}{2}|\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}|$$

Exercise : Prove the above relation.

- Area of parallelogram shown in figure is, Area $= |\vec{\mathbf{A}}\times\vec{\mathbf{B}}|$

Exercise : Prove the above relation.

- If co-ordinates of point A are (x_1, y_1, z_1) and B are (x_2, y_2, z_2). Then, position vector of A

$$= \vec{\mathbf{r}}_A = \overrightarrow{\mathbf{OA}} = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$$

position vector of $B = \vec{\mathbf{r}}_B = \overrightarrow{\mathbf{OB}} = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A$

$$= (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

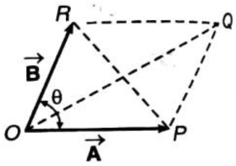


Fig. 2.23

- Suppose $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ represent the two adjacent sides of a parallelogram $OPQR$, then,

Diagonal $OQ = |\vec{\mathbf{A}} + \vec{\mathbf{B}}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

while diagonal $RP = |\vec{\mathbf{A}} - \vec{\mathbf{B}}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

We can see that $OQ = RP$ when $\theta = 90°$.

- $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = ab\cos\theta$. Here, a and b are always positive as these are the magnitudes of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. Hence,

	$0° \le \theta < 90°$	if $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$ is positive
	$90° \le \theta < 180°$	if $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$ is negative.
and	$\theta = 90°$	if $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$ is zero.

Solved Examples

Example 1 *Check the correctness of the relation $s = ut + \frac{1}{2}at^2$, where u is initial velocity, a the acceleration, t the time and s the displacement.*

Solution Writing the dimensions of either side of the given equation.

$$\text{LHS} = s = \text{displacement} = [M^0LT^0]$$

$$\text{RHS} = ut = \text{velocity} \times \text{time} = [M^0LT^{-1}][T] = [M^0LT^0]$$

and $$\frac{1}{2}at^2 = (\text{acceleration}) \times (\text{time})^2 = [M^0LT^{-2}][T]^2 = [M^0LT^0]$$

As LHS = RHS, formula is dimensionally correct.

Example 2 *Write the dimensions of a and b in the relation,*

$$P = \frac{b - x^2}{at}$$

where P is power, x is distance and t is time.

Solution The given equation can be written as, $Pat = b - x^2$

Now, $$[Pat] = [b] = [x^2] \quad \text{or} \quad [b] = [x^2] = [M^0L^2T^0]$$

and $$[a] = \frac{[x^2]}{[Pt]} = \frac{[L^2]}{[ML^2T^{-3}][T]} = [M^{-1}L^0T^2]$$

Example 3 *The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.*

Solution Let $F = k(m)^x (v)^y (r)^z$ ...(i)

Here, k is a dimensionless constant of proportionality. Writing the dimensions of RHS and LHS in Eq. (i), we have

$$[MLT^{-2}] = [M]^x [LT^{-1}]^y [L]^z = [M^x L^{y+z} T^{-y}]$$

Equation the powers of M, L and T of both sides, we have,

$$x = 1, \quad y = 2 \quad \text{and} \quad y + z = 1 \quad \text{or} \quad z = 1 - y = -1$$

Putting the values in Eq. (i), we get

$$F = kmv^2r^{-1} = k\frac{mv^2}{r}$$

$$F = \frac{mv^2}{r} \qquad (\text{where } k = 1)$$

Example 4 *Discuss why an infinitesimal displacement is regarded as a vector quantity.*

Solution Infinitesimal rotations may be regarded as vector quantity because the arc described by the body in a small interval of time is more or less a straight line and is thus representable as a vector.

Note *In some of the books it is written that large angular displacement does not obey the law of commutation ($\vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$) so they cannot be treated as vectors while infinitesimal do. So, they are vectors. As per my opinion the sufficient and necessary condition to be a vector is that is should obey the law of parallelogram of vector. If this is satisfied the law of commutation is automatically satisfied. It can be explained and proved in a better way but that require three dimensional treatment. Which I feel is not necessary for any competition.*

Example 5 *Find component of vector $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ along (i) x-axis, (ii) $\vec{\mathbf{C}}$*

Given $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, $\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ *and* $\vec{\mathbf{C}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Solution $\vec{\mathbf{A}} + \vec{\mathbf{B}} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

(i) Component of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ along x-axis is 3.

(ii) Component of $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{R}}$ (say) along $\vec{\mathbf{C}}$ is

$$R\cos\theta = \frac{\vec{\mathbf{R}} \bullet \vec{\mathbf{C}}}{C} = \frac{(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \bullet (\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{(1)^2 + (1)^2}} = \frac{3-2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Example 6 *Find the angle that the vector $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ makes with y-axis.*

Solution

$$\cos\theta = \frac{A_y}{A} = \frac{3}{\sqrt{(2)^2 + (3)^2 + (-1)^2}} = \frac{3}{\sqrt{14}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

Example 7 *Explain why pressure and surface tension are not vectors?*

Solution Pressure (force per unit area normal to it) or surface tension are scalars. They have direction which is unique so need not to be specified.

Example 8 *If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are the vectors $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{BC}}$ determined by the adjacent sides of a regular hexagon. What are the vectors determined by the other sides taken in order?*

Solution Given $\overrightarrow{\mathbf{AB}} = \vec{\mathbf{a}}$ and $\overrightarrow{\mathbf{BC}} = \vec{\mathbf{b}}$

From the method of vector addition (or subtraction) we can show that,

$$\overrightarrow{\mathbf{CD}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$$

Then $$\overrightarrow{\mathbf{DE}} = -\overrightarrow{\mathbf{AB}} = -\vec{\mathbf{a}}$$

$$\overrightarrow{\mathbf{EF}} = -\overrightarrow{\mathbf{BC}} = -\vec{\mathbf{b}}$$

and $$\overrightarrow{\mathbf{FA}} = -\overrightarrow{\mathbf{CD}} = \vec{\mathbf{a}} - \vec{\mathbf{b}}$$

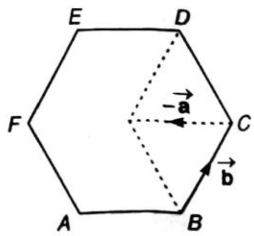


Fig. 2.24

Example 9 *If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$ with $\vec{a} \neq -\vec{c}$ then show that $\vec{a} + \vec{c} = k\,\vec{b}$, where k is scalar.*

Solution

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$$

$$\vec{a} \times \vec{b} = -\vec{c} \times \vec{b}$$

$$\therefore \quad \vec{a} \times \vec{b} + \vec{c} \times \vec{b} = 0$$

$$(\vec{a} + \vec{c}) \times \vec{b} = 0$$

$\therefore$ $\vec{a} \times \vec{b} \neq 0$, $\vec{b} \times \vec{c} \neq 0$, $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors. $(\vec{a} + \vec{c}) \neq \vec{0}$

Hence, $\vec{a} + \vec{c}$ is parallel to $\vec{b}$.

$$\therefore \quad \vec{a} + \vec{c} = k\,\vec{b}$$

Example 10 *If $\vec{A} = 2\hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j}$ and $\vec{C} = \hat{j} - \hat{k}$. Find $\vec{A} \cdot (\vec{B} \times \vec{C})$.*

Solution $\vec{A} \cdot (\vec{B} \times \vec{C}) = [\vec{A}\,\vec{B}\,\vec{C}]$, volume of parallelopiped

$$= \begin{vmatrix} 2 & -3 & 7 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 2(-2-0) + 3(-1-0) + 7(1-0)$$

$$= -4 - 3 + 7 = 0$$

Example 11 *Find the resultant of three vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in figure. Radius of circle is 'R'.*

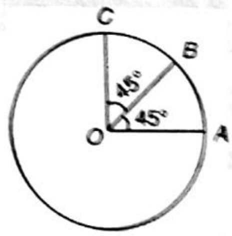


Fig. 2.25

Solution $OA = OC$

$\overrightarrow{OA} + \overrightarrow{OC}$ is along $\overrightarrow{OB}$, (bisector) and its magnitude is $2R \cos 45° = R\sqrt{2}$

$(\overrightarrow{OA} + \overrightarrow{OC}) + \overrightarrow{OB}$ is along $\overrightarrow{OB}$ and its magnitude is $R\sqrt{2} + R = R(1 + \sqrt{2})$

Example 12 *Prove that*

$$|\vec{a} \times \vec{b}|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

Solution Let $|\vec{a}| = a$, $|\vec{b}| = b$

and θ be the angle between them.

$$|\vec{a} \times \vec{b}|^2 = (ab\sin\theta)^2 = a^2b^2\sin^2\theta$$
$$= a^2b^2(1-\cos^2\theta)$$
$$= a^2b^2 - (a \cdot b\cos\theta)^2$$
$$= a^2b^2 - (\vec{a}\cdot\vec{b})^2$$

Proved.

Example 13 *Show that the vectors* $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ *and* $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$ *form a right angled triangle.*

Solution We have $\vec{b} + \vec{c} = (\hat{i} - 3\hat{j} + 5\hat{k}) + (2\hat{i} + \hat{j} - 4\hat{k}) = 3\hat{i} - 2\hat{j} + \hat{k} = \vec{a}$

Hence, $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

Also, we observe that no two of these vectors are parallel, therefore, the given vectors form a triangle.

Further, $\quad \vec{a}\cdot\vec{c} = (3\hat{i} - 2\hat{j} + \hat{k})\cdot(2\hat{i} + \hat{j} - 4\hat{k}) = 0$

Dot product of two non-zero vectors is zero. Hence, they are perpendicular so they form a right angled triangle.

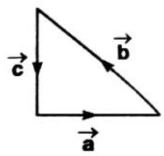


Fig. 2.26

$$|\vec{a}| = \sqrt{9+4+1} = \sqrt{14},$$
$$|\vec{b}| = \sqrt{1+9+25} = \sqrt{35}$$

and $$|\vec{c}| = \sqrt{4+1+16} = \sqrt{21}$$

Example 14 *Let* $\vec{A}, \vec{B}$ *and* $\vec{C}$ *be unit vectors. Suppose that* $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C} = 0$ *and that the angle between* $\vec{B}$ *and* $\vec{C}$ *is* $\frac{\pi}{6}$ *then prove that* $\vec{A} = \pm 2(\vec{B}\times\vec{C})$

Solution Since, $\vec{A}\cdot\vec{B} = 0$, $\vec{A}\cdot\vec{C} = 0$

Hence, $\quad (\vec{B} + \vec{C})\cdot\vec{A} = 0$

So $\vec{A}$ is perpendicular to $(\vec{B} + \vec{C})\cdot\vec{A}$ is a unit vector perpendicular to the plane of vectors $\vec{B}$ and $\vec{C}$.

$$\vec{A} = \frac{\vec{B}\times\vec{C}}{|\vec{B}\times\vec{C}|}$$

$$|\vec{B}\times\vec{C}| = |\vec{B}||\vec{C}|\sin\frac{\pi}{6} = 1\times 1\times\frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad \vec{A} = \frac{\vec{B}\times\vec{C}}{|\vec{B}\times\vec{C}|} = \pm 2(\vec{B}\times\vec{C})$$

Example 15 *A particle moves on a given line with a constant speed v. At a certain time it is at a point P on its straight line path. O is fixed point. Show that $(\overrightarrow{\mathbf{OP}} \times \vec{\mathbf{v}})$ is independent of the position P.*

Solution Let $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$

$$\overrightarrow{\mathbf{OP}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

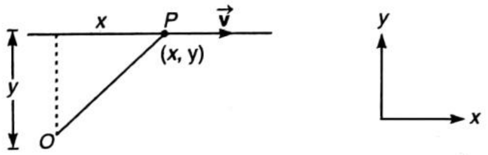


Fig. 2.27

Take $\overrightarrow{\mathbf{OP}} \times \vec{\mathbf{v}} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \times v\hat{\mathbf{i}}$

$$= -yv\hat{\mathbf{k}} \qquad (\because y \text{ is constant})$$

which is independent of position.

Example 16 *Prove that the mid-point of the hypotenuse of right angled triangle is equidistant from its vertices.*

Solution Here, $\angle CAB = 90°$, let D be the mid-point of hypotenuse, we have

$$\overrightarrow{\mathbf{BD}} = \overrightarrow{\mathbf{DC}}$$

$$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{DB}}$$

$$\overrightarrow{\mathbf{AC}} = \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{DC}} = \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BD}} \qquad \ldots\text{(i)}$$

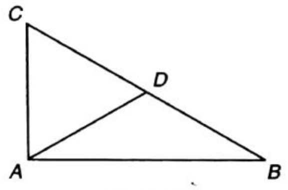


Fig. 2.28

Since, $\angle BAC = 90°$ $\quad \overrightarrow{\mathbf{AB}} \perp \overrightarrow{\mathbf{AC}}$

$$(\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{DB}}) \cdot (\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BD}}) = 0$$

$$(\overrightarrow{\mathbf{AD}} - \overrightarrow{\mathbf{BD}}) \cdot (\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BD}}) = 0$$

$$AD^2 - BD^2 = 0$$

$\therefore$ $\quad AD = BD \quad$ also $\quad BD = DC$

$\because$ D is mid-point of BC.

Thus, $|AD| = |BD| = |DC|$. Hence, the result.

EXERCISES

Section-I

Single Correct Option

1. Which one of the following has the dimensions of pressure?
(a) $[ML^{-2}T^{-2}]$ (b) $[M^{-1}L^{-1}]$ (c) $[MLT^{-2}]$ (d) $[ML^{-1}T^{-2}]$

2. Which of the following will have the dimensions of time
(a) LC (b) $\frac{R}{L}$ (c) $\frac{L}{R}$ (d) $\frac{C}{L}$

3. The force F on a sphere of radius a moving in a medium with velocity v is given by $F = 6\pi\eta a v$. The dimensions of η are
(a) $[ML^{-3}]$ (b) $[MLT^{-2}]$ (c) $[MT^{-1}]$ (d) $[ML^{-1}T^{-1}]$

4. The dimensional formula for magnetic flux is
(a) $[ML^2T^{-2}A^{-1}]$ (b) $[ML^3T^{-2}A^{-2}]$ (c) $[M^0L^{-2}T^{-2}A^{-2}]$ (d) $[ML^2T^{-1}A^2]$

5. Dimensions of linear impulse are
(a) $[ML^{-2}T^{-3}]$ (b) $[ML^{-2}]$ (c) $[MLT^{-1}]$ (d) $[MLT^{-2}]$

6. What is the dimensional formula of gravitational constant?
(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-1}]$ (c) $[M^{-1}L^3T^{-2}]$ (d) None of these

7. Using mass (M), length (L), time (T) and current (A) as fundamental quantities, the dimension of permeability is
(a) $[M^{-1}LT^{-2}A]$ (b) $[ML^{-2}T^{-2}A^{-1}]$ (c) $[MLT^{-2}A^{-2}]$ (d) $[MLT^{-1}A^{-1}]$

8. The equation of a wave is given by
$$y = a \sin \omega\left(\frac{x}{v} - k\right)$$
where ω is angular velocity and v is the linear velocity. The dimensions of k will be
(a) $[T^2]$ (b) $[T^{-1}]$ (c) $[T]$ (d) $[LT]$

9. A force is given by
$$F = at + bt^2$$
where t is the time. The dimensions of a and b are
(a) $[MLT^{-4}]$ and $[MLT^1]$ (b) $[MLT^{-1}]$ and $[MLT^0]$
(c) $[MLT^{-3}]$ and $[MLT^{-4}]$ (d) $[MLT^{-3}]$ and $[MLT^0]$

10. The dimensional formula for Planck's constant and angular momentum are
(a) $[ML^2T^{-2}]$ and $[MLT^{-1}]$
(b) $[ML^2T^{-1}]$ and $[ML^2T^{-1}]$
(c) $[ML^3T^1]$ and $[ML^2T^{-2}]$
(d) $[MLT^{-1}]$ and $[MLT^{-2}]$

11. If the energy (E), velocity (v) and force (F) be taken as fundamental quantities, then the dimensions of mass will be
(a) Fv^{-2} (b) Fv^{-1} (c) Ev^{-2} (d) Ev^2

12. The dimension of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 is the permittivity of free space and E is electric field), is
(a) $[ML^2T^{-1}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[ML^2T^{-2}]$ (d) $[MLT^{-1}]$

13. The dimensions of $\frac{a}{b}$ in the equation $P = \frac{a - t^2}{bx}$, where P is pressure, x is distance and t is time, are
(a) $[M^2LT^{-3}]$ (b) $[MT^{-2}]$ (c) $[LT^{-3}]$ (d) $[ML^3 T^{-1}]$

14. Dimension of velocity gradient is
(a) $[M^0L^0T^{-1}]$ (b) $[ML^{-1}T^{-1}]$ (c) $[M^0LT^{-1}]$ (d) $[ML^0T^{-1}]$

15. If force F, length L and time T are taken as fundamental units, the dimensional formula for mass will be
(a) $[FL^{-1}T^2]$ (b) $[FLT^{-2}]$ (c) $[FL^{-1}T^{-1}]$ (d) $[FL^5T^2]$

16. Which of the following is the dimension of the coefficient of friction?
(a) $[M^2L^2T]$ (b) $[M^0L^0T^0]$ (c) $[ML^2T^{-2}]$ (d) $[M^2L^2T^{-2}]$

17. If C and R denote capacitance and resistance, then dimensions of CR will be
(a) $[M^0L^0TA^0]$ (b) $[ML^0TA^{-2}]$ (c) $[ML^0TA^2]$ (d) $[ML^0T^2A^{-2}]$

18. The unit of permittivity of free space, ε_0 is
(a) coulomb/newton-metre
(b) newton-metre^2/coulomb^2
(c) coulomb^2/newton-metre^2
(d) coulomb^2/(newton-metre)^2

19. The ratio of the dimensions of Planck's constant and that of the moment of inertia is the dimension of
(a) frequency (b) velocity (c) angular momentum (d) time

20. The velocity v of a particle at time t is given by $v = at + \frac{b}{t + c}$, where a, b and c are constants. The dimensions of a, b and c are respectively
(a) $[LT^{-2}]$, $[L]$ and $[T]$
(b) $[L^2]$, $[T]$ and $[LT^2]$
(c) $[LT^2]$, $[LT]$ and $[L]$
(d) $[L]$, $[LT]$ and $[T^2]$

21. Given that $y = A \sin\left[\left(\frac{2\pi}{\lambda}(ct - x)\right)\right]$, where y and x are measured in metres. Which of the following statements is true ?
(a) The unit of λ is same as that of x and A
(b) The unit of λ is same as that of x but not of A
(c) The unit of c is same as that of $\frac{2\pi}{\lambda}$
(d) The unit of $(ct - x)$ is same as that of $\frac{2\pi}{\lambda}$

22. The physical quantity having the dimensions $[M^{-1}L^{-3}T^{3}A^{2}]$ is
(a) resistance (b) resistivity
(c) electrical conductivity (d) electromotive force

23. The torque of force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})$ newton acting at the point $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})$ metre about origin is (in N-m)
(a) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (b) $17\hat{i} - 6\hat{j} - 13\hat{k}$ (c) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (d) $-17\hat{i} + 6\hat{j} + 13\hat{k}$

24. If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ the value of c is
(a) 1 (b) $\sqrt{0.11}$ (c) $\sqrt{0.01}$ (d) 0.39

25. If $\vec{A}$ and $\vec{B}$ are two vectors such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ the angle between vectors $\vec{A}$ and $\vec{B}$ is
(a) 0° (b) 60° (c) 90° (d) 120°

26. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
(a) are not equal to each other in magnitude (b) cannot be predicted
(c) are equal to each other (d) are equal to each other in magnitude

27. Which one of the following is a scalar quantity?
(a) Displacement (b) Electric field (c) Acceleration (d) Work

28. Which one of the following is not the vector quantity?
(a) Torque (b) Displacement (c) Velocity (d) Speed

29. What is the dot product of two vectors of magnitude 3 and 5, if angle between them is 60°?
(a) 5.2 (b) 7.5 (c) 8.4 (d) 8.6

30. Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$.

If θ is the angle between positive direction of $\vec{A}$ and $\vec{B}$ then the correct statement is
(a) $\theta = \pi$ (b) $\theta = \frac{2\pi}{3}$ (c) $\theta = 0$ (d) $\theta = \frac{\pi}{2}$

31. Which one is a vector quantity?
(a) Time (b) Temperature
(c) Flux density (d) Magnetic field intensity

32. Given that $P = 12$, $Q = 5$ and $R = 13$ also $\vec{P} + \vec{Q} = \vec{R}$, then the angle between $\vec{P}$ and $\vec{Q}$ will be
(a) π (b) $\frac{\pi}{2}$ (c) zero (d) $\frac{\pi}{4}$

33. The forces, which meet at one point but their lines of action do not lie in one plane, are called
(a) non-coplanar non-concurrent forces (b) non-coplanar concurrent forces
(c) coplanar concurrent forces (d) coplanar non-concurrent forces

34. Given that $\vec{P} + \vec{Q} + \vec{R} = 0$. Two out of the three vectors are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of the other two. Which of the following can be the angles between these vectors?
(a) 90°, 135°, 135° (b) 45°, 45°, 90° (c) 30°, 60°, 90° (d) 45°, 90°, 135°

35. The angle between $\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$ will be
(a) 90° (b) between 0° and 180°
(c) 180° only (d) None of these

36. Two vectors of equal magnitude have a resultant equal to either of them, then the angle between them will be

(a) 30° (b) 120° (c) 60° (d) 45°

37. A force $(3\hat{i} + 4\hat{j})$ newton acts on a body and displaces it by $(3\hat{i} + 4\hat{j})$ metre. The work done by the force is

(a) 5 J (b) 25 J (c) 10 J (d) 30 J

38. If the vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other then the positive value of a is

(a) zero (b) 1 (c) 2 (d) 3

39. The angles which the vector $\vec{A} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ makes with the co-ordinate axes are

(a) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{6}{7}$ and $\cos^{-1}\frac{2}{7}$ (b) $\cos^{-1}\frac{4}{7}, \cos^{-1}\frac{5}{7}$ and $\cos^{-1}\frac{3}{7}$

(c) $\cos^{-1}\frac{3}{7}, \cos^{-1}\frac{4}{7}$ and $\cos^{-1}\frac{1}{7}$ (d) none of the above

40. Unit vector parallel to the resultant of vectors $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 8\hat{i} + 8\hat{j}$ will be

(a) $\frac{24\hat{i} + 5\hat{j}}{13}$ (b) $\frac{12\hat{i} + 5\hat{j}}{13}$ (c) $\frac{6\hat{i} + 5\hat{j}}{13}$ (d) None of these

41. The value of n so that vectors $2\hat{i} + 3\hat{j} - 2\hat{k}, 5\hat{i} + n\hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ may be coplanar, will be

(a) 18 (b) 28 (c) 9 (d) 36

42. Which one of the following statement is false?

(a) A vector has only magnitude, whereas a scalar has both magnitude and direction
(b) Distance is a scalar quantity but displacement is a vector quantity
(c) Momentum, force, torque are vector quantities
(d) Mass, speed and energy are scalar quantities

43. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ is

(a) $2(\vec{b} \times \vec{a})$ (b) $-2(\vec{b} \times \vec{a})$ (c) $\vec{b} \times \vec{a}$ (d) $\vec{a} \times \vec{b}$

44. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ is

(a) 60° (b) 0° (c) 90° (d) None of these

45. Maximum and minimum values of the resultant of two forces acting at a point are 7 N and 3 N respectively. The smaller force will be equal to

(a) 5 N (b) 4 N (c) 2 N (d) 1 N

46. The component of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the vector $\hat{i} + \hat{j}$ is

(a) $\frac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$ (c) $5\sqrt{2}$ (d) 5

47. The resultant of two forces $3P$ and $2P$ is R. If the first force is doubled then the resultant is also doubled. The angle between the two forces is

(a) 60° (b) 120° (c) 70° (d) 180°

48. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is

(a) 120° (b) 60° (c) 90° (d) 150°

49. Three vectors satisfy the relation $\vec{A}\cdot\vec{B}=0$ and $\vec{A}\cdot\vec{C}=0$, then $\vec{A}$ is parallel to

(a) $\vec{C}$ (b) $\vec{B}$ (c) $\vec{B}\times\vec{C}$ (d) $\vec{B}\cdot\vec{C}$

50. The sum of two forces at a point is 16 N. If their resultant is normal to the smaller force and has a magnitude of 8 N. Then two forces are

(a) 6 N, 10 N (b) 8 N, 8 N (c) 4 N, 12 N (d) 2 N, 14 N

51. If $|\vec{A}\times\vec{B}|=\sqrt{3}\,\vec{A}\cdot\vec{B}$, then the value of $|\vec{A}+\vec{B}|$ is

(a) $(A^2+B^2+AB)^{1/2}$ (b) $\left(A^2+B^2+\dfrac{AB}{\sqrt{3}}\right)^{1/2}$

(c) $(A+B)$ (d) $(A^2+B^2+\sqrt{3}AB)^{1/2}$

52. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B}\times\vec{A})\cdot\vec{A}$ is equal to

(a) $BA^2\cos\theta$ (b) $BA^2\sin\theta$ (c) $BA^2\sin\theta\cos\theta$ (d) zero

53. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is

(a) −1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

54. Minimum number of vectors of unequal magnitudes which can give zero resultant are

(a) two (b) three (c) four (d) more than four

55. The (x, y, z) co-ordinates of two points A and B are given respectively as $(0, 3, -1)$ and $(-2, 6, 4)$. The displacement vector from A to B is given by

(a) $-2\hat{i}+6\hat{j}+4\hat{k}$ (b) $-2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}+3\hat{j}+5\hat{k}$ (d) $2\hat{i}-3\hat{j}-5\hat{k}$

56. The sum of two vectors $\vec{A}$ and $\vec{B}$ is at right angles to their difference. Then

(a) $A=B$ (b) $A=2B$

(c) $B=2A$ (d) $\vec{A}$ and $\vec{B}$ have the same direction

Match the Columns

1. Column-I shows some vector equations. Match coloumn I with the value of angle between $\vec{A}$ and $\vec{B}$ given in column II.

Column I	Column II
(a) $\lvert\vec{A}\times\vec{B}\rvert=\lvert\vec{A}\cdot\vec{B}\rvert$	(p) zero
(b) $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$	(q) $\frac{\pi}{2}$
(c) $\lvert\vec{A}+\vec{B}\rvert=\lvert\vec{A}-\vec{B}\rvert$	(r) $\frac{\pi}{4}$
(d) $\vec{A}+\vec{B}=\vec{C}$ and $A+B=C$	(s) $\frac{3\pi}{4}$

Section-II

Subjective Questions

Units and Dimensions

1. Young's modulus of steel is 2.0×10^{11} N/m^2. Express it in dyne/cm^2.
2. Surface tension of water in the CGS system is 72 dynes/cm. What is its value in SI units?
3. In the expression $y = a\sin(\omega t + \theta)$, y is the displacement and t is the time. Write the dimensions of a, ω and θ.
4. The relation between the energy E and the frequency ν of a photon is expressed by the equation $E = h\nu$, where h is Planck's constant. Write down the SI units of h and its dimensions.
5. Write the dimensions of a and b in the relation.

$$P = \frac{b - x^2}{at}$$

where P is power, x is distance and t is time.

6. Check the correctness of the relation $S_t = u + \frac{a}{2}(2t - 1)$, where u is initial velocity, a is acceleration and S_t is the displacement of by the body in t^{th} second.
7. Let x and a stand for distance. Is $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ dimensionally correct?
8. In the equation

$$\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right).$$

Find the value of n.

9. Show dimensionally that the expression, $Y = \frac{MgL}{\pi r^2 l}$ is dimensionally correct, where Y is Young's modulus of the material of wire, L is length of wire, Mg is the weight applied on the wire and l is the increase in the length of the wire.
10. The energy E of an oscillating body in simple harmonic motion depends on its mass m, frequency n and amplitude a. Using the method of dimensional analysis find the relation between E, m, n and a.
11. The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius r of the circle. Derive the formula for F using the method of dimensions.
12. Taking force F, length L and time T to be the fundamental quantities, find the dimensions of (a) density, (b) pressure, (c) momentum and (d) energy.

Vectors

13. Find the cosine of the angle between the vectors $\vec{A} = (3\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{B} = (2\hat{i} - 2\hat{j} + 4\hat{k})$.
14. Obtain the angle between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ if $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} - 2\hat{j}$.
15. Under what conditions will the vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 5\hat{k}$ and $5\hat{i} - \hat{j} + b\hat{k}$ be perpendicular to each other?

16. Deduce the condition for the vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - a\hat{\mathbf{j}} + b\hat{\mathbf{k}}$ to be parallel.

17. Three vectors which are coplanar with respect to a certain rectangular co-ordinate system are given by

$\vec{\mathbf{a}} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}}, \vec{\mathbf{b}} = -3\mathbf{i} + 2\hat{\mathbf{j}}$ and $\vec{\mathbf{c}} = -3\hat{\mathbf{j}}$

Find

(a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$

(b) $\vec{\mathbf{a}} + \vec{\mathbf{b}} - \vec{\mathbf{c}}$

(c) Find the angle between $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}} - \vec{\mathbf{c}}$

18. Find the components of a vector $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ along the directions of $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} - \hat{\mathbf{j}}$.

19. If vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ be respectively equal to $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$. Find the unit vector parallel to $\vec{\mathbf{A}} + \vec{\mathbf{B}}$.

20. If two vectors are $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$. By calculation, prove that $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

21. Find the area of the parallelogram whose sides are represented by $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{k}}$.

22. The resultant of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is at right angles to $\vec{\mathbf{A}}$ and its magnitude is half of $\vec{\mathbf{B}}$. Find the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

23. The x and y-components of vector $\vec{\mathbf{A}}$ are 4 m and 6 m respectively. The x and y-components of vector $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ are 10 m and 9 m respectively. Calculate for the vector $\vec{\mathbf{B}}$ the following

(a) its x and y-components

(b) its length

(c) the angle it makes with x-axis

24. Prove by the method of vectors that in a triangle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

25. Four forces of magnitude P, $2P$, $3P$ and $4P$ act along the four sides of a square $ABCD$ in cyclic order. Use the vector method to find the resultant force.

26. If $\vec{\mathbf{P}} + \vec{\mathbf{Q}} = \vec{\mathbf{R}}$ and $\vec{\mathbf{P}} - \vec{\mathbf{Q}} = \vec{\mathbf{S}}$, prove that

$$R^2 + S^2 = 2(P^2 + Q^2)$$

ANSWERS

Section-I

Single Correct Option

1.(d)	**2.** (c)	**3.**(d)	**4.**(a)	**5.**(c)	**6.**(c)	**7.**(c)	**8.**(c)	**9.**(c)	**10.**(b)
11.(c)	**12.**(b)	**13.**(b)	**14.**(a)	**15.**(a)	**16.**(b)	**17.**(a)	**18.**(c)	**19.**(a)	**20.**(a)
21.(a)	**22.**(c)	**23.**(b)	**24.**(b)	**25.**(c)	**26.**(d)	**27.**(d)	**28.**(d)	**29.**(b)	**30.**(d)
31.(d)	**32.**(b)	**33.**(b)	**34.**(a)	**35.**(b)	**36.**(b)	**37.**(b)	**38.**(d)	**39.**(a)	**40.**(b)
41.(a)	**42.**(a)	**43.**(a)	**44.**(c)	**45.**(c)	**46.**(a)	**47.**(b)	**48.**(a)	**49.**(c)	**50.**(a)
51.(a)	**52.**(d)	**53.**(b)	**54.**(b)	**55.**(c)	**56.**(a)				

Match the Columns

1. (a) → r,s (b) → p (c) → q (d) → p

Section-II

Subjective Questions

1. 2.0×10^{12} dyne/cm^2 **2.** Surface tension of water = 0.072 N/m

3. $[M^0LT^0]$, $[M^0L^0T^{-1}]$, $[M^0L^0T^0]$, **4.** J-s, $[ML^2T^{-1}]$ **5.** $[M^0L^2T^0]$, $[M^{-1}L^0T^2]$

7. No **8.** Zero **10.** $E = kmn^2a^2$ (k= constant) **11.** $F = \frac{kmv^2}{r}$

12. (a) $[FL^{-4}T^2]$ (b) $[FL^{-2}]$ (c) $[FT]$ (d) $[FL]$ **13.** $\frac{3}{\sqrt{21}}$ **14.** $\cos^{-1}\left(\frac{4}{\sqrt{65}}\right)$ **15.** $b = -4$

16. $a = -4.5$, $b = -6$ **17.** (a) $\hat{i} - 2\hat{j}$ (b) $\hat{i} + 4\hat{j}$ (c) $\cos^{-1}\left(\frac{-7}{\sqrt{85}}\right)$ **18.** $\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

19. $\frac{1}{\sqrt{27}}(5\hat{i} - \hat{j} + \hat{k})$ **21.** Area = 13.4 units **22.** 150°

23. (a) 6 m, 3 m (b) $3\sqrt{5}$ m (c) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ **25.** $2\sqrt{2}\,P$, 225°

Chapter 3 – Motion in One Dimension

3

Motion in One Dimension

Chapter Contents

3.1 Introduction to Mechanics and Kinematics

Mechanics is the branch of physics which deals with the motion of particles or bodies in space and time. Position and motion of a body can be determined only with respect to other bodies. Motion of the body involves position and time. For practical purposes a coordinate system, *e.g.*, the cartesian system is fixed to the reference body and position of the body is determined with respect to this reference body. For calculation of time generally clock is used.

Kinematics is the branch of mechanics which deals with the motion regardless of the causes producing it. The study of causes of motion is called **dynamics.**

3.2 Basic Definitions

(a) Position and Position Vector

In cartesian coordinate system position of any point (say A) is represented by its coordinates (x_A, y_A, z_A) with respect to an orgin O.

Position vector of point A with respect to O will now be:

$$\vec{\mathbf{r}}_A = \overrightarrow{\mathbf{OA}} = x_A\hat{\mathbf{i}} + y_A\hat{\mathbf{j}} + z_A\hat{\mathbf{k}}$$

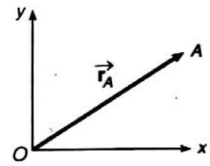


Fig. 3.1

Now, suppose coordinates of two points A and B are known to us and we want to find position vector of B with respect to A, then

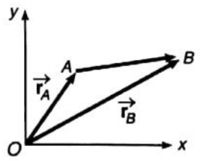


Fig. 3.2

$$\overrightarrow{\mathbf{AB}} = \vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A$$

$$\overrightarrow{\mathbf{AB}} = (x_B - x_A)\hat{\mathbf{i}} + (y_B - y_A)\hat{\mathbf{j}} + (z_B - z_A)\hat{\mathbf{k}}$$

(b) Distance and Displacement

Distance is the actual path length covered by a moving particle or body in a given time interval, while displacement is the change in position vector, *i.e.*, a vector joining initial to final positions. If a particle moves from A to C (Fig. 3.3) through a path ABC. Then distance (Δs) travelled is the actual path length ABC, while the displacement is

Fig. 3.3

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_C - \vec{\mathbf{r}}_A$$

If a particle moves in a straight line without change in direction, the magnitude of displacement is equal to the distance travelled, otherwise, it is always less than it. Thus,

$$|\text{displacement}| \le \text{distance}$$

(c) Average Speed and Velocity

The average speed of a particle in a given interval of time is defined as the ratio of distance travelled to the time taken while average velocity is defined as the ratio of displacement to time taken. Thus, if the distance travelled is Δs and displacement of a particle is $\Delta\vec{\mathbf{r}}$ in a given time interval Δt, then

$$v_{\text{av}} = \text{Average speed} = \frac{\Delta s}{\Delta t}$$ total distance / total time.

and

$$\vec{\mathbf{v}}_{\text{av}} = \text{Average velocity} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$ total displ / total time.

(d) Instantaneous Speed and Velocity

Instantaneous speed and velocity are defined at a particular instant and are given by

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \quad \text{and} \quad \vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

(e) Average and Instantaneous Acceleration

Average acceleration is defined as the ratio of change in velocity, *i.e.*, $\Delta\vec{\mathbf{v}}$ to the time interval Δt in which this change occures. Hence,

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$ (*) no limits.

The instantaneous acceleration is defined at a particular instant and is given by

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

Sample Example 3.1 *In one second, a particle goes from point A to point B moving in a semicircle (Fig. 3.4). Find the magnitude of the average velocity.*

Fig. 3.4

Solution

$$|\vec{v}_{av}| = \frac{AB}{\Delta t} = \frac{2.0}{1.0} \text{ m/s} = 2 \text{ m/s}$$ **Ans.**

Sample Example 3.2 *A table is given below of a particle moving along x-axis. In the table speed of particle at different time intervals is shown.*

Table 3.1

Time interval (in sec)	Speed of particle (in m/s)
0 – 2	2
2 – 5	3
5 – 10	4
10 – 15	2

Find total distance travelled by the particle and its average speed.

Solution Distance = speed × time speed x time

$\therefore$ Total distance $= (2 \times 2) + (3)(3) + (5)(4) + (5)(2) = 43$ m

Total time taken is 15 s Hence,

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{43}{15} = 2.87 \text{ m/s}$$

Sample Example 3.3 *(a) What does* $\left|\frac{d\vec{v}}{dt}\right|$ *and* $\frac{d|\vec{v}|}{dt}$ *represent?*

(b) Can these be equal?

(c) Can $\frac{d|\vec{v}|}{dt} = 0$ *while* $\left|\frac{d\vec{v}}{dt}\right| \neq 0$

(d) $\frac{d|\vec{v}|}{dt} \neq 0$ *while* $\left|\frac{d\vec{v}}{dt}\right| = 0$

Solution (a) $\left|\frac{d\vec{v}}{dt}\right|$ is the magnitude of total acceleration. While $\frac{d|\vec{v}|}{dt}$ represents the time rate of change of speed (called the tangential acceleration, a component of total acceleration) as $|\vec{v}| = v$.

(b) These two are equal in case of one dimensional motion (without change in direction).

(c) In case of uniform circular motion speed remains constant while velocity changes.

Hence, $$\frac{d|\vec{v}|}{dt} = 0 \quad \text{while} \quad \left|\frac{d\vec{v}}{dt}\right| \neq 0$$

(d) $\frac{d|\vec{v}|}{dt} \neq 0$ implies that speed of particle is not constant. Velocity cannot remain constant, if speed is changing. Hence, $\left|\frac{d\vec{v}}{dt}\right|$ cannot be zero in this case. So, it is not possible to have $\left|\frac{d\vec{v}}{dt}\right| = 0$ while $\frac{d|\vec{v}|}{dt} \neq 0$.

Sample Example 3.4 *Give examples where*

(a) the velocity is in opposite direction to the acceleration

(b) the velocity of the particle is zero but its acceleration is not zero

(c) the velocity is perpendicular to the acceleration.

Solution (a) A particle thrown upwards has its velocity in opposite direction to its acceleration (g, downwards).

(b) When the particle is released from rest from a certain height, its velocity is zero, while acceleration is g downwards. Similarly, at the extreme position of a pendulum velocity is zero, while acceleration is not zero.

(c) In uniform circular motion velocity is perpendicular to its radial or centripetal acceleration.

Introductory Exercise 3.1

1. When a particle moves with constant velocity its average velocity, its instantaneous velocity and its speed all are equal. Is this statement true or false?
2. A stone is released from an elevator going up with an acceleration of $g/2$. What is the acceleration of the stone just after release?
3. A clock has its second hand 2.0 cm long. Find the average speed and modulus of average velocity of the tip of the second hand in 15 s.
4. (a) Is it possible to be accelerating if you are travelling at constant speed?
   (b) Is it possible to move on a curved path with zero acceleration, constant acceleration, variable acceleration?
5. A particle is moving in a circle of radius 4 cm with constant speed of 1 cm/s. Find:
   (a) Time period of the particle.
   (b) Average speed, average velocity and average acceleration in a time interval from $t = 0$ to $t = T/4$. Here, T is the time period of the particle. Give only their magnitudes.
6. A particle moves in a straight line with constant speed of 4 m/s for 2 s, then with 6 m/s for 3 s. Find the average speed of the particle in the given time interval.

3.3 Motion in One, Two and Three Dimensions

Motion of a block in a straight line is one dimensional (1-D) motion. Motion of a particle in a straight line can be described by only one component of its velocity or acceleration. The motion of a particle thrown in vertical plane at some angle with horizontal ($\neq 90°$) is an example of two dimensional (2-D) motion. This is called a projectile motion. Similarly a circular motion is also an example of 2-D motion. A 2-D motion takes place in a plane and its velocity (or acceleration) can be described by two components in any two mutually perpendicular direction (v_x and v_y).

Motion of a bird (or a monkey) in space is a three dimensional (3-D) motion. In a 3-D motion velocity and acceleration of a particle can be resolved in three components (v_x, v_y, v_z, a_x, a_y and a_z). Here x, y and z are any three mutually perpendicular axes.

The position of a particle in one dimensional motion is described by one variable (say x). In a 2-D motion it involves two variables (normally x and y) and in a 3-D motion three variables are x, y and z.

3.4 Uniformly Accelerated Motion

Equations of motion for uniformly accelerated motion ($\vec{a}$ = constant) are as under,

$$\vec{v} = \vec{u} + \vec{a}\,t,\ \vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2,\ \vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\,\vec{a}\cdot\vec{s}$$

Note *The above formulae for one dimensional motion has been derived in article* 3.5.

Here, $\vec{u}$ = initial velocity of particle, $\vec{v}$ = velocity of particle at time t and

$\vec{s}$ = displacement of particle in time t

Note *If initial position vector of a particle is* $\vec{r}_0$, *then position vector at time t can be written as*

$$\vec{r} = \vec{r}_0 + \vec{s} = \vec{r}_0 + \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$$

One-dimensional Uniformly Accelerated Motion

If the motion of a particle is taking place in a straight line, there is no need of using vector addition (or subtraction) in equations of motion. We can directly use the equations.

$$v = u + at,\ s = ut + \frac{1}{2}at^2 \text{ and } v^2 = u^2 + 2as$$

Just by taking one direction as the positive (and opposite to it as negative) and then substituting u, a, etc., with sign. Normally we take vertically upward direction positive (and downward negative) and horizontally rightwards positive (or leftwards negative).

Sign convention for (a) motion in vertical direction (b) motion in horizontal direction is shown in Fig. 3.5.

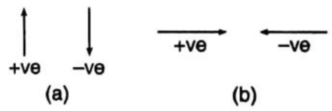


Fig. 3.5

Sample Example 3.5 *A ball is thrown upwards from the top of a tower* 40 *m high with a velocity of* 10 *m/s. Find the time when it strikes the ground. Take* $g = 10\ m/s^2$.

Solution In the problem

$$u = +10\ \text{m/s},\quad a = -10\ \text{m/s}^2$$

and $s = -40$ m

(at the point where stone strikes the ground)

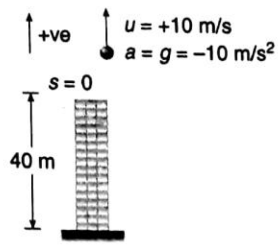


Fig. 3.6

Substituting in $s = ut + \frac{1}{2}at^2$, we have

$$-40 = 10t - 5t^2$$

or $5t^2 - 10t - 40 = 0$

or $t^2 - 2t - 8 = 0$

Solving this, we have $t = 4$ s and -2 s. Taking the positive value $t = 4$ s.

Note *The significance of* $t = -2$ s *can be understood by following figure:*

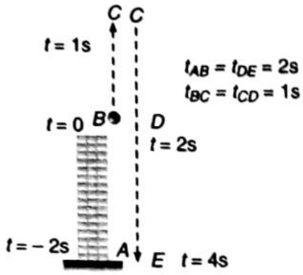


Fig. 3.7

Sample Example 3.6 *A ball is thrown upwards from the ground with an initial speed of u. The ball is at a height of 80 m at two times, the time interval being 6 s. Find u. Take* $g = 10\ m/s^2$.

Solution Here, $u = u$ m/s, $a = g = -10\,\text{m/s}^2$ and $s = 80\,\text{m}$.

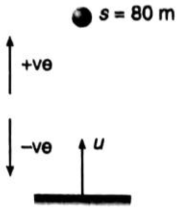


Fig. 3.8

Substituting the values in $s = ut + \frac{1}{2}at^2$, we have

$$80 = ut - 5t^2$$

or $$5t^2 - ut + 80 = 0$$

or $$t = \frac{u + \sqrt{u^2 - 1600}}{10} \quad \text{and} \quad \frac{u - \sqrt{u^2 - 1600}}{10}$$

Now, it is given that

$$\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6$$

or $$\frac{\sqrt{u^2 - 1600}}{5} = 6 \quad \text{or} \quad \sqrt{u^2 - 1600} = 30$$

or $$u^2 - 1600 = 900$$

$\therefore$ $$u^2 = 2500$$

or $$u = \pm\, 50\,\text{m/s}$$

Ignoring the negative sign, we have

$$u = 50\,\text{m/s}$$

Sample Example 3.7 *A particle of mass 1 kg has a velocity of 2 m/s. A constant force of 2 N acts on the particle for 1 s in a direction perpendicular to its initial velocity. Find the velocity and displacement of the particle at the end of 1 second.*

Solution Force acting on the particle is constant. Hence, acceleration of the particle will also remain constant.

$$a = \frac{F}{m} = \frac{2}{1} = 2\,\text{m/s}^2$$

Since, acceleration is constant. We can apply

$$\vec{v} = \vec{u} + \vec{a}\,t \quad \text{and} \quad \vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$$

Refer Fig. 3.9(a) $\qquad \vec{v} = \vec{u} + \vec{a}\,t$

Here, $\vec{u}$ and $\vec{a}\,t$ are two mutually perpendicular vectors. So,

$$|\vec{v}| = \sqrt{(|\vec{u}|)^2 + (|\vec{a}\,t|)^2} = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} \text{ m/s}$$

$$\alpha = \tan^{-1}\frac{|\vec{a}\,t|}{|\vec{u}|} = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = 45°$$

Thus, velocity of the particle at the end of 1s is $2\sqrt{2}$ m/s at an angle of 45° with its initial velocity.

Refer Fig. 3.9(b) $\qquad \vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$

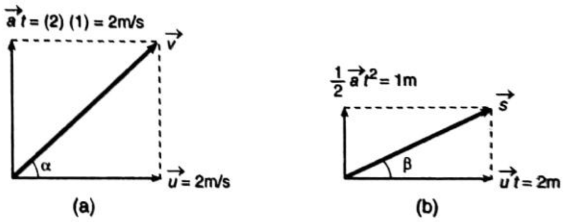


Fig. 3.9

Here, $\vec{u}\,t$ and $\frac{1}{2}\vec{a}\,t^2$ are also two mutually perpendicular vectors. So,

$$|\vec{s}| = \sqrt{(|\vec{u}\,t|)^2 + (|\tfrac{1}{2}\vec{a}\,t^2|)^2}$$

$$= \sqrt{(2)^2 + (1)^2} = \sqrt{5} \text{ m}$$

and

$$\beta = \tan^{-1}\frac{|\frac{1}{2}\vec{a}\,t^2|}{|\vec{u}\,t|} = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus, displacement of the particle at the end of 1 s is $\sqrt{5}$ m at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ from its initial velocity.

Sample Example 3.8 *Velocity and acceleration of a particle at time $t = 0$ are $\vec{u} = (2\hat{i} + 3\hat{j})$ m/s and $\vec{a} = (4\hat{i} + 2\hat{j})\,m/s^2$ respectively. Find the velocity and displacement of particle at $t = 2$ s.*

Solution Here, acceleration $\vec{a} = (4\hat{i} + 2\hat{j})\,\text{m/s}^2$ is constant. So, we can apply

$$\vec{v} = \vec{u} + \vec{a}\,t \quad \text{and} \quad \vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$$

Substituting the proper values, we get

$$\vec{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j})$$

$$= (10\hat{i} + 7\hat{j})\ \text{m/s}$$

and
$$\vec{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j})$$
$$= (12\hat{i} + 10\hat{j})\ \text{m}$$

Therefore, velocity and displacement of particle at $t = 2\text{s}$ are $(10\hat{i} + 7\hat{j})$ m/s and $(12\hat{i} + 10\hat{j})$ m respectively.

Following points are worthnoting in case of one dimensional motion with constant acceleration.

(i) This type of motion can be observed when either $u = 0$, $\vec{u} \uparrow\uparrow \vec{a}$ or $\vec{u} \uparrow\downarrow \vec{a}$

(ii) In the first two cases when either $u = 0$ or $\vec{u} \uparrow\uparrow \vec{a}$ motion is only accelerated.

(iii) When $\vec{u} \uparrow\downarrow \vec{a}$ motion is first retarded (till the velocity becomes zero) and then accelerated in opposite direction.

(iv) As per our convention (vertically upward positive) acceleration due to gravity g is always negative whether the particle is moving upwards or downwards. We are now left with the sign of u and s. Displacement s is measured from the point of projection.

(v) For fast calculation in objective problems, remember the following results.

(a) Maximum height attained by a particle, thrown upwards from ground

$$h = \frac{u^2}{2g}$$

(b) Velocity of particle at the time of striking the ground when released ($u = 0$) from a height h is,

$$v = \sqrt{2gh}$$

(c) In (b) time of collision with ground

$$t = \sqrt{\frac{2h}{g}}$$

(d) Displacement of particle in t^{th} second of its motion,

$$s_t = u + at - \frac{1}{2}a$$

EXERCISE : *Derive the above four relations for h, v, t and s_t.*

Difference between distance (d) and displacement (s)

The s in equations of motion

$$(s = ut + \frac{1}{2}at^2 \quad \text{and} \quad v^2 = u^2 + 2as)$$

is really the displacement not the distance. They have different values only when u and a are of opposite sign or $u \uparrow\downarrow a$.

Let us take the following two cases.

Case 1. When u is either zero or parallel to a, then motion is simply accelerated and in this case distance is equal to displacement. So, we can write

$$d = s = ut + \frac{1}{2}at^2$$

Case 2. When u is antiparallel to a, the motion is first retarded then accelerated in opposite direction. So distance is either greater than or equal to displacement ($d \geq |s|$). In this case first find the time when velocity becomes zero. Say it is t_0.

$$0 = u - at_0$$

$$\therefore \qquad t_0 = \left|\frac{u}{a}\right|$$

Now, if the given time $t \leq t_0$: distance and displacement are equal. So

$$d = s = ut + \frac{1}{2}at^2$$

For $t \leq t_0$ (with u positive and a negative)

For $t > t_0$: distance is greater than displacement.

$$d = d_1 + d_2$$

Here, $\quad d_1$ = distance travelled before coming to rest $= \left|\frac{u^2}{2a}\right|$

d_2 = distance travelled in remaining time $t - t_0$

$$= \frac{1}{2}|a(t - t_0)^2|$$

$$\therefore \qquad d = \left|\frac{u^2}{2a}\right| + \frac{1}{2}|a(t - t_0)^2|$$

Note *The displacement is still* $\quad s = ut + \frac{1}{2}at^2 \quad$ *with u positive and a negative.*

Sample Example 3.9 *A particle is projected vertically upwards with velocity* 40 *m/s. Find the displacement and distance travelled by the particle in*

(a) 2 *s* *(b)* 4 *s* *(c)* 6 *s*

Take $g = 10\,m/s^2$.

Solution Here, u is positive (upwards) and a is negative (downwards). So, first we will find t_0, the time when velocity becomes zero.

$$t_0 = \left|\frac{u}{a}\right| = \frac{40}{10} = 4\text{ s}$$

(a) $t < t_0$. Therefore, distance and displacement are equal.

$$d = s = ut + \frac{1}{2}at^2$$

$$= 40 \times 2 - \frac{1}{2} \times 10 \times 4 = 60\text{ m}$$

(b) $t = t_0$. So, again distance and displacement are equal.

$$d = s = 40 \times 4 - \frac{1}{2} \times 10 \times 16 = 80\text{ m}$$

(c) $t > t_0$. Hence, $d > s$

$$s = 40 \times 6 - \frac{1}{2} \times 10 \times 36 = 60\,\text{m}$$

While
$$d = \left|\frac{u^2}{2a}\right| + \frac{1}{2}|a(t - t_0)^2|$$

$$= \frac{(40)^2}{2 \times 10} + \frac{1}{2} \times 10 \times (6-4)^2 = 100\,\text{m}$$

3.5 Non-Uniformly Accelerated Motion

When acceleration of particle is not constant, we go for basic equations of velocity and acceleration, *i.e.*,

(i) $\vec{\mathbf{v}} = \frac{d\vec{\mathbf{s}}}{dt}$ or sometimes $\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt}$ (ii) $\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt}$ (iii) $d\vec{\mathbf{s}} = \vec{\mathbf{v}}\,dt$ (iv) $d\vec{\mathbf{v}} = \vec{\mathbf{a}}\,dt$

For one dimensional motion, above relations can be written as under.

(i) $v = \frac{ds}{dt}$ (ii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$ (iii) $ds = v\,dt$ and (iv) $dv = a dt$ or $v\,dv = a\,ds$

Such, problems can be solved either by differentiation or integration (with some boundary conditions).

$s - t \longrightarrow v - t \longrightarrow a - t$ (Differentiation)

$a - t \longrightarrow v - t \longrightarrow s - t$ (Integration with boundary conditions)

Note (*i*) *By boundary condition we mean that velocity or displacement at some time (usually at $t = 0$) should be known to us. Otherwise we cannot find constant of integration.*

(*ii*) *Equation $a = v\frac{dv}{ds}$ or $v\,dv = a\,ds$ is useful when acceleration displacement equation is known and velocity displacement equation is required.*

Derivation of Equation of Motion ($\mathbf{v} = \mathbf{u} + \mathbf{at}$ etc.)

For one dimensional motion with $a =$ constant.

We can write,

$$dv = a\,dt \qquad \left(\text{as } a = \frac{dv}{dt}\right)$$

Integrating both sides, we have

$$\int dv = a \int dt \qquad (\text{as } a = \text{constant})$$

At $t = 0$, velocity is u and at $t = t$ velocity is v. Hence,

$$\int_u^v dv = a \int_0^t dt$$

$\therefore$
$$[v]_u^v = a[t]_0^t$$

or
$$v - u = at$$

$\therefore$
$$v = u + at$$

Hence proved.

Further, we can write

$$ds = v\,dt \qquad \left(\text{as } v = \frac{ds}{dt}\right)$$

$$= (u + at)\,dt \qquad (\text{as } v = u + at)$$

At time $t = 0$ suppose $s = 0$ and at $t = t$, displacement is s, then

$$\int_0^s ds = \int_0^t (u + at)\,dt$$

$\therefore$ $$[s]_0^s = \left[ut + \frac{1}{2}at^2\right]_0^t$$

or $$s = ut + \frac{1}{2}at^2$$ **Hence proved.**

We can also write, $$v \cdot dv = a \cdot ds \qquad \left(\text{as } a = v \cdot \frac{dv}{ds}\right)$$

When $s = 0$, v is u and at $s = s$, velocity is v. Therefore,

$$\int_u^v v \cdot dv = a \int_0^s ds \qquad (\text{as } a = \text{constant})$$

or $$\left[\frac{v^2}{2}\right]_u^v = a\,[s]_0^s$$

$\therefore$ $$\frac{v^2}{2} - \frac{u^2}{2} = as$$

or $$v^2 = u^2 + 2as$$ **Hence proved.**

Sample Example 3.10 *Displacement time equation of a particle moving along x-axis is*

$$x = 20 + t^3 - 12t \qquad \textit{(SI Units)}$$

(a) Find, position and velocity of particle at time t = 0.
(b) State whether the motion is uniformly accelerated or not.
(c) Find position of particle when velocity of particle is zero.

Solution (a) $$x = 20 + t^3 - 12t \qquad \ldots\text{(i)}$$

At $t = 0$, $$x = 20 + 0 - 0 = 20\text{ m}$$

Velocity of particle at time t can be obtained by differentiating Eq. (i) w.r.t. time *i.e.*,

$$v = \frac{dx}{dt} = 3t^2 - 12 \qquad \ldots\text{(ii)}$$

At $t = 0$, $$v = 0 - 12 = -12\text{ m/s}$$

(b) Differentiating Eq. (ii) w.r.t. time t, we get the acceleration

$$a = \frac{dv}{dt} = 6t$$

As acceleration is a function of time, the motion is non-uniformly accelerated.

(c) Substituting $v = 0$ in Eq. (ii), we have

$$0 = 3t^2 - 12$$

Positive value of t comes out to be 2 second from this equation. Substituting $t = 2$ second in Eq. (i), we have

$$x = 20 + (2)^3 - 12(2) \quad \text{or} \quad x = 4 \text{ m}$$

Sample Example 3.11 *Velocity-time equation of a particle moving in a straight line is,*

$$v = (10 + 2t + 3t^2) \qquad \text{(SI Units)}$$

Find :

(a) displacement of particle from the mean position at time $t = 1$ second, if it is given that displacement is 20 m at time $t = 0$

(b) acceleration-time equation.

Solution (a) The given equation can be written as,

$$v = \frac{ds}{dt} = (10 + 2t + 3t^2)$$

or
$$ds = (10 + 2t + 3t^2)\,dt$$

or
$$\int_{20}^{s} ds = \int_0^1 (10 + 2t + 3t^2)\,dt$$

or
$$s - 20 = [10t + t^2 + t^3]_0^1$$

or
$$s = 20 + 12 = 32 \text{ m}$$

(b) Acceleration-time equation can be obtained by differentiating the given equation w.r.t. time. Thus,

$$a = \frac{dv}{dt} = \frac{d}{dt}(10 + 2t + 3t^2)$$

or
$$a = 2 + 6t$$

Introductory Exercise 3.2

1. A ball is thrown vertically upwards. Which quantity remains constant among, speed, kinetic energy, velocity and acceleration?
2. Equation $s_t = u + at - \frac{1}{2}a$ does not seem dimensionally correct, why?
3. Can the speed of a particle increase as its acceleration decreases? If yes give an example.
4. The velocity of a particle moving in a straight line is directly proportional to 3/4th power of time elapsed. How does its displacement and acceleration depend on time?
5. A particle is projected vertically upwards with an initial velocity of 40 m/s. Find the displacement and distance covered by the particle in 6 seconds. Take $g = 10 \text{ m/s}^2$.
6. Velocity of a particle moving along positive x-direction is $v = (40 - 10t)$ m/s. Here, t is in seconds. At time $t = 0$, the x coordinate of particle is zero. Find the time when the particle is at a distance of 60 m from origin.
7. A particle moves rectilinearly with initial velocity u and a constant acceleration a. Find the average velocity of the particle in a time interval from $t = 0$ to $t = t$ second of its motion.
8. A particle moves in a straight line with uniform acceleration. Its velocity at time $t = 0$ is v_1 and at time $t = t$ is v_2. The average velocity of the particle in this time interval is $\frac{v_1 + v_2}{2}$. Is this statement true or false?
9. Find the average velocity of a particle released from rest from a height of 125 m over a time interval till it strikes the ground. $g = 10 \text{ m/s}^2$.

10. Velocity of a particle moving along x-axis varies with time as, $v = (10 + 5t - t^2)$
At time $t = 0$, $x = 0$. Find
(a) acceleration of particle at $t = 2$ s (b) x-coordinate of particle at $t = 3$ s

11. Velocity of a particle at time $t = 0$ is 2 m/s. A constant acceleration of 2 m/s^2 acts on the particle for 2 seconds at an angle of 60° with its initial velocity. Find the magnitude of velocity and displacement of particle at the end of $t = 2$ s.

12. Velocity of a particle at any time t is $\vec{\mathbf{v}} = (2\hat{\mathbf{i}} + 2t\,\hat{\mathbf{j}})$ m/s. Find acceleration and displacement of particle at $t = 1$ s. Can we apply $\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}t$ or not?

13. The coordinates of a particle moving in x-y plane at any time t are $(2t, t^2)$.. Find: (a) the trajectory of the particle, (b) velocity of particle at time t and (c) acceleration of particle at any time t.

3.6 Graphs

The theory of graphs can be generalised and summarised in following six points :

(i) A linear equation between x and y represents a straight line, *e.g.*, $y = 4x - 2$, $y = 5x + 3$, $3x = y - 2$ equations represent straight line on x-y graph.

(ii) $x \propto y$ or $y = kx$ represents a straight line passing through origin.

(iii) $x \propto \frac{1}{y}$ represents a rectangular hyperbola in x-y graph. Shape of rectangular hyperbola is as shown in Fig. 3.10.

e.g., $P \propto \frac{1}{V}$ in an isothermal process (T = constant). Hence, P-V graph in isothermal process is a rectangular hyperbola as shown in Fig 3.11.

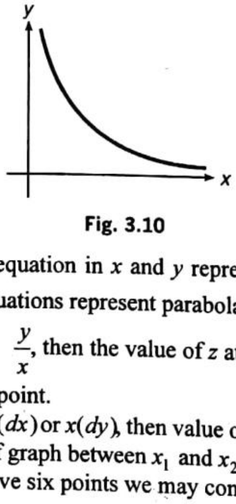


Fig. 3.10

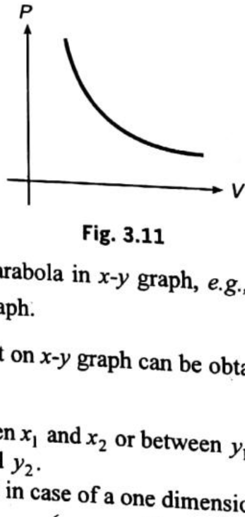


Fig. 3.11

(iv) A quadratic equation in x and y represents a parabola in x-y graph, *e.g.*, $y = 3x^2 + 2$, $y^2 = 4x$, $x^2 = y - 2$ equations represent parabola in x-y graph.

(v) If $z = \frac{dy}{dx}$ or $\frac{y}{x}$, then the value of z at any point on x-y graph can be obtained by the slope of the graph at that point.

(vi) If $z = yx$ or $y(dx)$ or $x(dy)$, then value of z between x_1 and x_2 or between y_1 and y_2 can be obtained by the area of graph between x_1 and x_2 or y_1 and y_2.

From the above six points we may conclude that in case of a one dimensional motion :

(a) slope of displacement-time graph gives velocity $\left(\text{as } v = \frac{ds}{dt}\right)$.

(b) slope of velocity-time graph gives acceleration $\left(\text{as } a = \frac{dv}{dt}\right)$.

(c) area under velocity-time graph gives displacement (as $ds = v\,dt$).

(d) area under acceleration-time graph gives change in velocity (as $dv = a\,dt$).

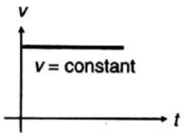
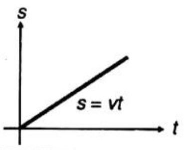
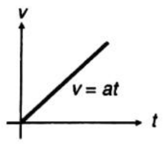
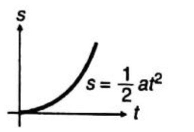
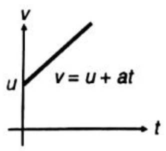
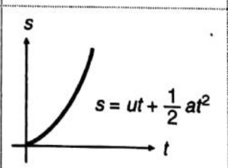
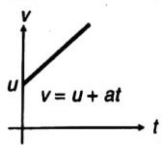
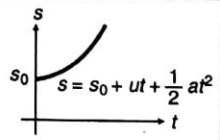
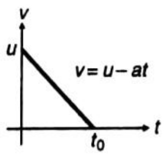
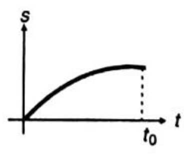
(e) displacement-time graph in uniform motion is a straight line passing through origin, if displacement is zero at time $t = 0$ (as $s = vt$).

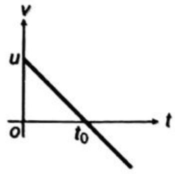
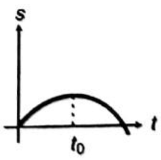
(f) velocity-time graph is a straight line passing through origin in a uniformly accelerated motion if initial velocity $u = 0$ and a straight line not passing through origin if initial velocity $u \neq 0$ (as $v = u + at$).

(g) displacement-time graph in uniformly accelerated or retarded motion is a parabola $\left(\text{as } s = ut \pm \frac{1}{2}at^2\right)$.

Now, we can plot v-t and s-t graphs of some standard results in tabular form as under. But note that all the following graphs are drawn for one-dimensional motion with uniform velocity or with constant acceleration.

Table 3.2

S. No.	Different Cases	v-t Graph	s-t Graph	Important Points
1.	Uniform motion			(i) Slope of v-t graph $= v =$ constant (ii) In s-t graph $s = 0$ at $t = 0$
2.	Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$			(i) $u = 0$, *i.e.*, $v = 0$ at $t = 0$ (ii) a or slope of v-t graph is constant (iii) $u = 0$, *i.e.*, slope of s-t graph at $t = 0$, should be zero
3.	Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$			(i) $u \neq 0$, *i.e.*, v or slope of v-t graph at $t = 0$ is not zero (ii) s or slope of s-t graph gradually goes on increasing
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$			(i) $v = u$ at $t = 0$ (ii) $s = s_0$ at $t = 0$
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s-t graph at $t = 0$ gives u (ii) Slope of s-t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero

6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, $v = 0$ or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

Important Points in Graphs

- Slopes of v-t or s-t graphs can never be infinite at any point, because infinite slope of v-t graph means infinite acceleration. Similarly, infinite slope of s-t graph means infinite velocity. Hence, the following graphs are not possible :

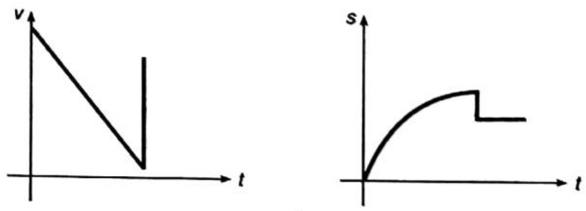


Fig. 3.12

- At one time, two values of velocity or displacement are not possible. Hence, the following graphs are not acceptable :

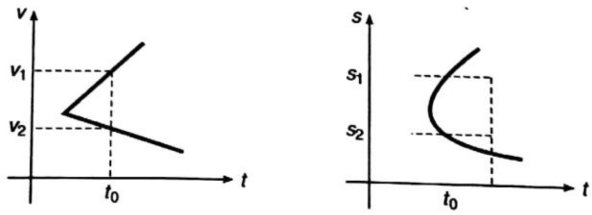


Fig. 3.13

- Different values of displacements in s-t graph corresponding to given v-t graph can be calculated just by calculating areas under v-t graph. There is no need of using equations like $v = u + at$, etc.

Sample Example 3.12 *Displacement-time graph of a particle moving in a straight line is as shown in figure. State whether the motion is accelerated or not. Describe the motion in detail. Given $s_0 = 20$ m and $t_0 = 4$ second.*

Fig. 3.14

Solution Slope of s-t graph is constant. Hence, velocity of particle is constant. Further at time $t = 0$, displacement of the particle from the mean position is $-s_0$ or -20 m. Velocity of particle,

$$v = \text{slope} = \frac{s_0}{t_0} = \frac{20}{4} = 5 \text{ m/s}$$

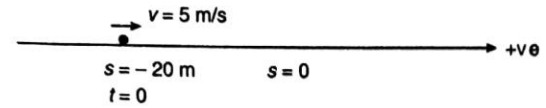


Fig. 3.15

Motion of the particle is as shown in figure. At $t = 0$ particle is at -20 m and has a constant velocity of 5 m/s. At $t_0 = 4$ second particle will pass through its mean position.

Sample Example 3.13 *A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t second evaluate (a) the maximum velocity reached and (b) the total distance travelled.*

Solution (a) Let the car accelerates for time t_1 and decelerates for time t_2. Then,

$$t = t_1 + t_2 \quad \text{...(i)}$$

and corresponding velocity-time graph will be as shown in Fig. 3.16.

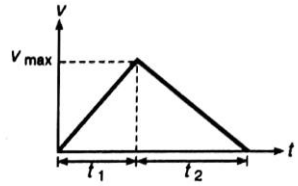


Fig. 3.16

From the graph,

$$\alpha = \text{slope of line } OA = \frac{v_{max}}{t_1}$$

or $$t_1 = \frac{v_{max}}{\alpha} \quad \text{...(ii)}$$

and $$\beta = - \text{ slope of line } AB = \frac{v_{max}}{t_2}$$

or $$t_2 = \frac{v_{max}}{\beta} \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \quad \text{or} \quad v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$$

or $$v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) $$\text{Total distance} = \text{displacement} = \text{area under } v\text{-}t \text{ graph}$$

$$= \frac{1}{2} \times t \times v_{max}$$

$$= \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$$

or $$\text{Distance} = \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$$

Note *This problem can also be solved by using equations of motion ($v = u + at$ etc.). Try it yourself.*

Sample Example 3.14 *A rocket is fired vertically upwards with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 s its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity-time and displacement-time graphs for the complete journey. Take g = 10 m/s^2.*

Solution In the graphs,

$$v_A = at_{OA} = (4)(5) = 20\ m/s$$

$$v_B = 0 = v_A - gt_{AB}$$

$\therefore$ $$t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2\text{s}$$

$\therefore$ $$t_{OAB} = (5+2)\text{s} = 7\text{s}$$

Now, $$s_{OAB} = \text{area under } v\text{-}t \text{ graph between 0 to 7 s}$$

$$= \frac{1}{2}(7)(20) = 70\text{ m}$$

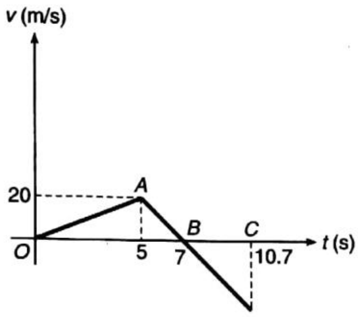


Fig. 3.17

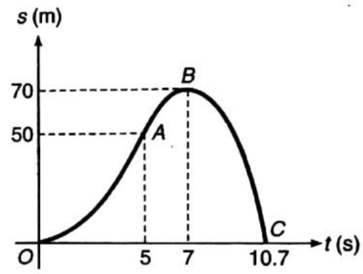


Fig. 3.18

Now, $$|s_{OAB}| = |s_{BC}| = \frac{1}{2} g t_{BC}^2$$

$\therefore$ $$70 = \frac{1}{2}(10)\,t_{BC}^2$$

$\therefore$ $$t_{BC} = \sqrt{14} = 3.7\text{ s}$$

$\therefore$ $$t_{OABC} = 7 + 3.7 = 10.7\text{ s}$$

Also, $$s_{OA} = \text{area under } v\text{-}t \text{ graph between } OA$$

$$= \frac{1}{2}(5)(20) = 50\text{ m}$$

Sample Example 3.15 *Acceleration-time graph of a particle moving in a straight line is shown in Fig. 3.19. Velocity of particle at time t = 0 is 2 m/s. Find velocity at the end of fourth second.*

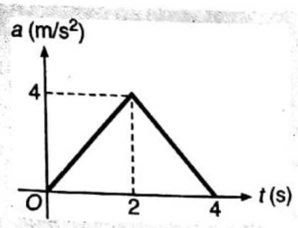


Fig. 3.19

Solution $$dv = a\,dt$$

or change in velocity = area under a-t graph

Hence, $$v_f - v_i = \frac{1}{2}(4)(4) = 8\text{ m/s}$$

$\therefore$ $$v_f = v_i + 8 = (2+8)\text{ m/s} = 10\text{ m/s}$$

Sample Example 3.16 *Velocity-time graph of a particle moving in a straight line is shown in Fig. 3.20.*

Plot the corresponding displacement-time graph of the particle if at time $t = 0$, displacement $s = 0$.

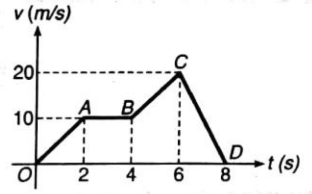


Fig. 3.20

Solution Displacement = Area under velocity-time graph

Hence, $$s_{OA} = \frac{1}{2} \times 2 \times 10 = 10\text{ m}$$

$$s_{AB} = 2 \times 10 = 20\text{ m}$$

or $$s_{OAB} = 10 + 20 = 30\text{ m}$$

$$s_{BC} = \frac{1}{2} \times 2(10 + 20) = 30\text{ m}$$

or $$s_{OABC} = 30 + 30 = 60\text{ m}$$

and $$s_{CD} = \frac{1}{2} \times 2 \times 20 = 20\text{ m}$$

or $$s_{OABCD} = 60 + 20 = 80\text{ m}$$

Between 0 to 2 s and 4 to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape. At the end of 8 s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in Fig. 3.21.

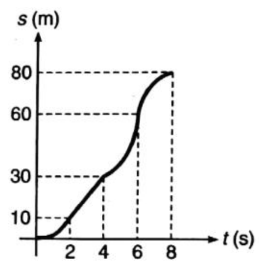


Fig. 3.21

Introductory Exercise 3.3

1. Figure shows the displacement-time graph of a particle moving in a straight line. Find the signs of velocity and acceleration of particle at time $t = t_1$ and $t = t_2$.

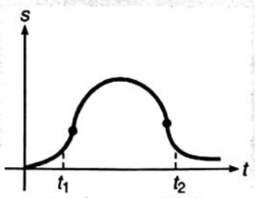


Fig. 3.22

2. A particle of mass m is released from a certain height h with zero initial velocity. It strikes the ground elastically (direction of its velocity is reversed but magnitude remains the same). Plot the graph between its kinetic energy and time till it returns to its initial position.
3. A ball is dropped from a height of 80 m on a floor. At each collision, the ball loses half of its speed. Plot the speed-time graph and velocity-time graph of its motion till two collisions with the floor. [Take $g = 10\text{ m/s}^2$]

4. Figure shows the acceleration-time graph of a particle moving along a straight line. After what time the particle acquires its initial velocity?

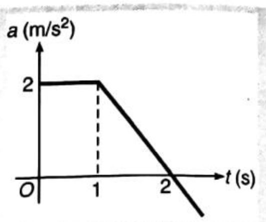


Fig. 3.23

3.7 Relative Motion

The word 'relative' is a very general term, which can be applied to physical, nonphysical, scalar or vector quantities. For example, my height is five feet and six inches while my wife's height is five feet and four inches. If I ask you how high I am relative to my wife, your answer will be two inches. What you did? You simply subtracted my wife's height from my height. The same concept is applied everywhere, whether it is a relative velocity, relative acceleration or anything else. So, from the above discussion we may now conclude that relative velocity of A with respect to B (written as $\vec{v}_{AB}$) is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Similarly, relative acceleration of A with respect to B is

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

If it is a one dimensional motion we can treat the vectors as scalars just by assigning the positive sign to one direction and negative to the other. So, in case of a one dimensional motion the above equations can be written as

$$v_{AB} = v_A - v_B$$

and $$a_{AB} = a_A - a_B$$

Further, we can see that

$$\vec{v}_{AB} = -\vec{v}_{BA} \quad \text{or} \quad \vec{a}_{BA} = -\vec{a}_{AB}$$

Sample Example 3.17 *Seeta is moving due east with a velocity of 1 m/s and Geeta is moving due west with a velocity of 2 m/s. What is the velocity of Seeta with respect to Geeta?*

Solution It is a one dimensional motion. So, let us choose the east direction as positive and the west as negative. Now, given that

$$v_S = \text{velocity of Seeta} = 1\ \text{m/s}$$

and

v_G = velocity of Geeta = -2 m/s

Thus, $$v_{SG} = \text{velocity of Seeta with respect to Geeta}$$

$$= v_S - v_G = 1 - (-2) = 3\ \text{m/s}$$

Hence, velocity of Seeta with respect to Geeta is 3 m/s due east.

Sample Example 3.18 *Car A has an acceleration of 2 m/s^2 due east and car B, 4 m/s^2 due north. What is the acceleration of car B with respect to car A?*

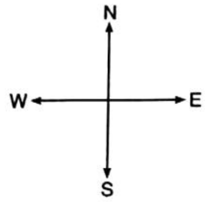


Fig. 3.24

Solution It is a two dimensional motion. Therefore,

$\vec{a}_{BA}$ = acceleration of car B with respect to car A

$$= \vec{a}_B - \vec{a}_A$$

Here, $\vec{a}_B$ = acceleration of car $B = 4 \text{ m/s}^2$ (due north)

and $\vec{a}_A$ = acceleration of car $A = 2 \text{ m/s}^2$ (due east)

$$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5} \text{ m/s}^2$$

and $$\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$$

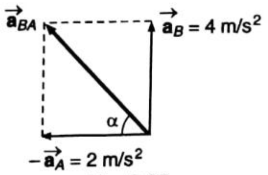


Fig. 3.25

Thus, $\vec{a}_{BA}$ is $2\sqrt{5}$ m/s^2 at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

The topic 'relative motion' is very useful in two and three dimensional motion. Questions based on relative motion are usually of following four types :

(a) Minimum distance between two bodies in motion
(b) River-boat problems
(c) Aircraft-wind problems
(d) Rain problems

(a) Minimum distance between two bodies in motion

When two bodies are in motion, the questions like, the minimum distance between them or the time when one body overtakes the other can be solved easily by the principle of relative motion. In these type of problems one body is assumed to be at rest and the relative motion of the other body is considered. By assuming so two body problem is converted into one body problem and the solution becomes easy. Following example will illustrate the statement.

Sample Example 3.19 *Car A and car B start moving simultaneously in the same direction along the line joining them. Car A with a constant acceleration a = 4 m/s^2, while car B moves with a constant velocity v = 1 m/s. At time t = 0, car A is 10 m behind car B. Find the time when car A overtakes car B.*

Solution Given : $u_A = 0$, $u_B = 1 \text{m/s}$, $a_A = 4 \text{ m/s}^2$ and $a_B = 0$

Assuming car B to be at rest, we have

$$u_{AB} = u_A - u_B = 0 - 1 = -1 \text{ m/s}$$
$$a_{AB} = a_A - a_B = 4 - 0 = 4 \text{ m/s}^2$$

Now, the problem can be assumed in simplified form as follows :

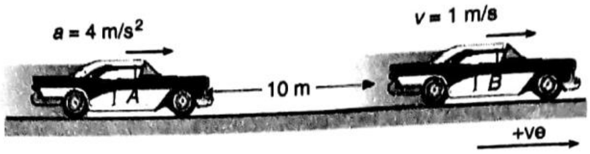


Fig. 3.26

Substituting the proper values in equation

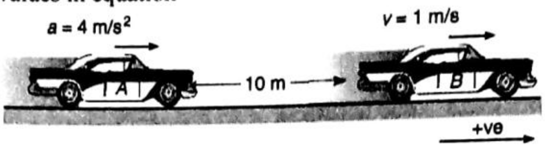


Fig. 3.27

$$s = ut + \frac{1}{2}at^2$$

we get $$10 = -t + \frac{1}{2}(4)(t^2)$$

or $$2t^2 - t - 10 = 0$$

or $$t = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4}$$

$$= \frac{1 \pm 9}{4} \quad \text{or} \quad t = 2.5\,\text{s} \quad \text{and} \quad -2\,\text{s}$$

Ignoring the negative value, the desired time is 2.5 s.

Note *The above problem can also be solved without using the concept of relative motion as under. At the time when A overtakes B,*

$$s_A = s_B + 10$$

$\therefore$ $$\frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$$

or $$2t^2 - t - 10 = 0$$

Which on solving gives $t = 2.5$ s and -2 s, the same as we found above.
As per my opinion, this approach (by taking absolute values) is more suitable in case of two body problem in one dimensional motion. Let us see one more example in support of it.

Sample Example 3.20 *An open lift is moving upwards with velocity 10 m/s. It has an upward acceleration of 2 m/s^2. A ball is projected upwards with velocity 20 m/s relative to ground. Find :*

(a) time when ball again meets the lift.

(b) displacement of lift and ball at that instant.

(c) distance travelled by the ball upto that instant. Take $g = 10\,m/s^2$

Solution (a) At the time when ball again meets the lift,

$$s_L = s_B$$

$\therefore$ $$10t + \frac{1}{2} \times 2 \times t^2 = 20t - \frac{1}{2} \times 10t^2$$

Solving this equation, we get

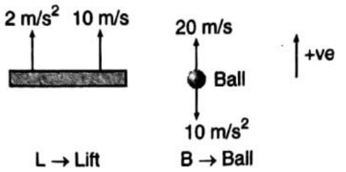


Fig. 3.28

$$t = 0 \quad \text{and} \quad t = \frac{5}{3}\text{ s}$$

$\therefore$ Ball will again meet the lift after $\frac{5}{3}$ s.

(b) At this instant

$$s_L = s_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 = \frac{175}{9}\text{ m} = 19.4\text{ m}$$

(c) For the ball $u \uparrow\downarrow a$. Therefore, we will first find t_0, the time when its velocity becomes zero.

$$t_0 = \left|\frac{u}{a}\right| = \frac{20}{10} = 2\text{ s}$$

As $t\left(= \frac{5}{3}\text{ s}\right) < t_0$, distance and displacement are equal

or $$d = 19.4\text{ m}$$

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Sample Example 3.21 *Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 km/h and ship B is streaming north at 20 km/h. What is their distance of closest approach and how long do they take to reach it?*

Solution Ships A and B are moving with same speed 20 km/h in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{v}_{BA}$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Here, $$|\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2}$$
$$= 20\sqrt{2}\text{ km/h}$$

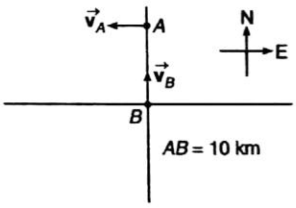


Fig. 3.29

i.e., $\vec{v}_{BA}$ is $20\sqrt{2}$ km/h at an angle of 45° from east towards north. Thus, the given problem can be simplified as :

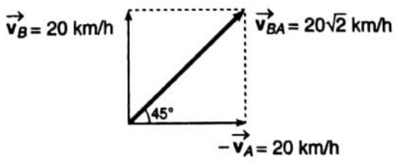


Fig. 3.30

A is at rest and B is moving with $\vec{v}_{BA}$ in the direction shown in Fig. 3.31. Therefore, the minimum distance between the two is

$$s_{min} = AC = AB \sin 45°$$
$$= 10\left(\frac{1}{\sqrt{2}}\right) \text{km} = 5\sqrt{2} \text{ km}$$

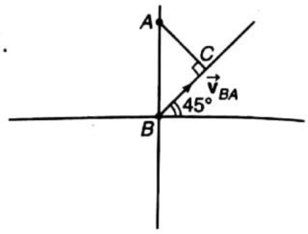


Fig. 3.31

$(BC = AC = 5\sqrt{2} \text{ km})$

and the desired time is

$$t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}}$$
$$= \frac{1}{4} \text{ h} = 15 \text{ min}$$

(b) River-Boat Problems

In river-boat problems we come across the following three terms:

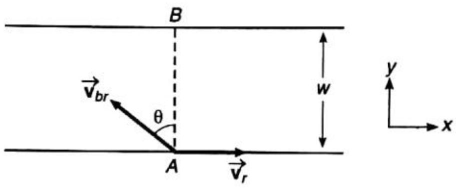


Fig. 3.32

$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

and $\vec{v}_b$ = absolute velocity of boatman.

Here, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.

Further, $$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in Fig. 3.32. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is w, then

$$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$$

Therefore, $$v_{bx} = v_{rx} + v_{brx} = v_r - v_{br} \sin \theta$$

and $$v_{by} = v_{ry} + v_{bry}$$
$$= 0 + v_{br} \cos \theta = v_{br} \cos \theta$$

Now, time taken by the boatman to cross the river is :

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br} \cos \theta}$$

or $$t=\frac{w}{v_{br}\cos\theta} \quad \text{...(i)}$$

Further, displacement along x-axis when he reaches on the other bank (also called drift) is:

$$x=v_{bx}\,t=(v_r-v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta}$$

or $$x=(v_r-v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \quad \text{...(ii)}$$

Three special cases are:

(i) Condition when the boatman crosses the river in shortest interval of time

From Eq. (i) we can see that time (t) will be minimum when $\theta=0°$, *i.e.*, the boatman should steer his boat perpendicular to the river current.

Also, $$t_{min}=\frac{w}{v_{br}} \quad \text{as} \quad \cos\theta=1$$

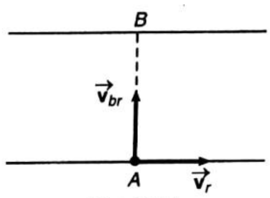


Fig. 3.33

(ii) Condition when the boatman wants to reach point B, *i.e.*, at a point just opposite from where he started

In this case, the drift (x) should be zero.

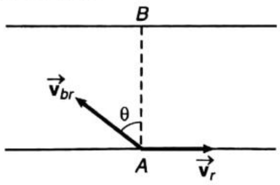


Fig. 3.34

$\therefore$ $$x=0$$

or $$(v_r-v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta}=0$$

or $$v_r=v_{br}\sin\theta$$

or $$\sin\theta=\frac{v_r}{v_{br}} \quad \text{or} \quad \theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

Hence, to reach point B the boatman should row at an angle $\theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

Further, since $\sin\theta \not> 1$.

So, if $v_r \ge v_{br}$, the boatman can never reach at point B. Because if $v_r=v_{br}$, $\sin\theta=1$ or $\theta=90°$ and it is just impossible to reach at B if $\theta=90°$. Moreover it can be seen that $v_b=0$ if $v_r=v_{br}$ and $\theta=90°$. Similarly, if $v_r>v_{br}$, $\sin\theta>1$, *i.e.*, no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

(iii) Shortest path

Path length travelled by the boatman when he reaches the opposite shore is

$$s=\sqrt{w^2+x^2}$$

Here, w = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

When $v_r < v_{br}$: In this case $x = 0$, when $\theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

or $$s_{\min}=w \quad \text{at} \quad \theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

When $v_r > v_{br}$: In this case x is minimum, where $\frac{dx}{d\theta}=0$

or $$\frac{d}{d\theta}\left\{\frac{w}{v_{br}\cos\theta}(v_r-v_{br}\sin\theta)\right\}=0$$

or $$-v_{br}\cos^2\theta-(v_r-v_{br}\sin\theta)(-\sin\theta)=0$$

or $$-v_{br}+v_r\sin\theta=0$$

or $$\theta=\sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Now, at this angle we can find $x_{\min}$ and then $s_{\min}$ which comes out to be

$$s_{\min}=w\left(\frac{v_r}{v_{br}}\right) \quad \text{at} \quad \theta=\sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Sample Example 3.22 *A man can row a boat with 4 km/h in still water. If he is crossing a river where the current is 2 km/h.*

(a) In what direction will his boat be headed, if he wants to reach a point on the other bank, directly opposite to starting point?

(b) If width of the river is 4 km, how long will the man take to cross the river, with the condition in part (a)?

(c) In what direction should he head the boat if he wants to cross the river in shortest time and what is this minimum time?

(d) How long will it take him to row 2 km up the stream and then back to his starting point?

Solution (a) Given, that $v_{br}=4$ km/h and $v_r=2$ km/h

$$\therefore \quad \theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)=\sin^{-1}\left(\frac{2}{4}\right)=\sin^{-1}\left(\frac{1}{2}\right)=30^\circ$$

Hence, to reach the point directly opposite to starting point he should head the boat at an angle of 30° with AB or 90° + 30° = 120° with the river flow.

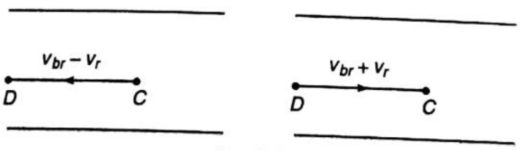


Fig. 3.35

(b) Time taken by the boatman to cross the river

$$w = \text{width of river} = 4 \text{ km}$$

$$v_{br} = 4 \text{ km/h} \quad \text{and} \quad \theta = 30°$$

$$\therefore \quad t = \frac{4}{4\cos 30°} = \frac{2}{\sqrt{3}} \text{ h}$$

(c) For shortest time $\theta = 0°$

and $$t_{min} = \frac{w}{v_{br}\cos 0°} = \frac{4}{4} = 1 \text{ h}$$

Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time is 1 h.

(d) $t = t_{CD} + t_{DC}$

or $$t = \frac{CD}{v_{br} - v_r} + \frac{DC}{v_{br} + v_r}$$

$$= \frac{2}{4-2} + \frac{2}{4+2} = 1 + \frac{1}{3} = \frac{4}{3} \text{ h}$$

(c) Aircraft Wind Problems

This is similar to river boat problem. The only difference is that $\vec{v}_{br}$ is replaced by $\vec{v}_{aw}$ (velocity of aircraft with respect to wind or velocity of aircraft in still air), $\vec{v}_r$ is replaced by $\vec{v}_w$ (velocity of wind) and $\vec{v}_b$ is replaced by $\vec{v}_a$ (absolute velocity of aircraft). Further, $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$. The following example will illustrate the theory.

Sample Example 3.23 *An aircraft flies at 400 km/h in still air. A wind of $200\sqrt{2}$ km/h is blowing from the south. The pilot wishes to travel from A to a point B north east of A. Find the direction he must steer and time of his journey if AB = 1000 km.*

Solution Given that $v_w = 200\sqrt{2}$ km/h

$v_{aw} = 400$ km/h and $\vec{v}_a$ should be along AB or in north-east direction. Thus, the direction of $\vec{v}_{aw}$ should be such as the resultant of $\vec{v}_w$ and $\vec{v}_{aw}$ is along AB or in north-east direction.

Let $\vec{v}_{aw}$ makes an angle α with AB as shown in Fig. 3.36.

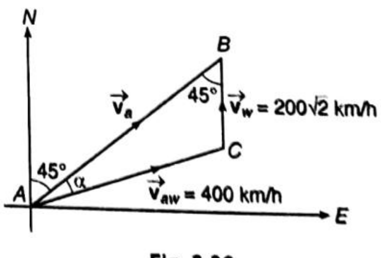


Fig. 3.36

Applying sine law in triangle ABC, we get

$$\frac{AC}{\sin 45°} = \frac{BC}{\sin \alpha}$$

or $$\sin \alpha = \left(\frac{BC}{AC}\right)\sin 45° = \left(\frac{200\sqrt{2}}{400}\right)\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\therefore \quad \alpha = 30°$$

Therefore, the pilot should steer in a direction at an angle of $(45° + \alpha)$ or 75° from north towards east.

Further, $$\frac{|\vec{v}_a|}{\sin(180^\circ - 45^\circ - 30^\circ)} = \frac{400}{\sin 45^\circ}$$

or $$|\vec{v}_a| = \frac{\sin 105^\circ}{\sin 45^\circ} \times (400)\frac{\text{km}}{\text{h}}$$

$$= \left(\frac{\cos 15^\circ}{\sin 45^\circ}\right)(400)\frac{\text{km}}{\text{h}} = \left(\frac{0.9659}{0.707}\right)(400)\frac{\text{km}}{\text{h}}$$

$$= 546.47 \text{ km/h}$$

$\therefore$ The time of journey from A to B is

$$t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47} \text{ h}$$

$$t = 1.83 \text{ h}$$

(d) Rain Problems

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_{rm}$, Here,

$\vec{v}_r$ = velocity of rain

$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man.

Here, $\vec{v}_{rm}$ is the velocity of rain which appears to the man. Now, let us take one example of this.

Sample Example 3.24 *To a man walking at the rate of 3 km/h the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.*

Solution Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain

$$\vec{v}_r = a\hat{i} + b\hat{j} \quad ...(i)$$

Then speed of rain will be

$$|\vec{v}_r| = \sqrt{a^2 + b^2} \quad ...(ii)$$

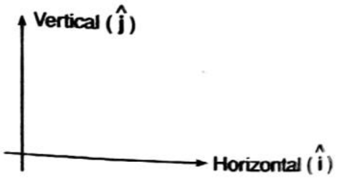


Fig. 3.37

In the first case

$$\vec{v}_m = \text{velocity of man} = 3\hat{i}$$

$\therefore$ $$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$$

It seems to be in vertical direction. Hence,

$$a - 3 = 0 \quad \text{or} \quad a = 3$$

In the second case $$\vec{v}_m = 6\hat{i}$$

$\therefore$ $$\vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$$

This seems to be at 45° with vertical.

Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is

$$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}\ \text{km/h}$$

Introductory Exercise 3.4

1. Two balls A and B are projected vertically upwards with different velocities. What is the relative acceleration between them?
2. In the above problem what is the shape of the graph between distance between the balls and time before either of the two collide with ground?
3. A river 400 m wide is flowing at a rate of 2.0 m/s. A boat is sailing at a velocity of 10.0 m/s with respect to the water in a direction perpendicular to the river.
   (a) Find the time taken by the boat to reach the opposite bank.
   (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank?
4. An aeroplane has to go from a point A to another point B, 500 km away due 30° east of north. Wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (a) Find the direction in which the pilot should head the plane to reach the point B. (b) Find the time taken by the plane to go from A to B.
5. Two particles A and B start moving simultaneously along the line joining them in the same direction with acceleration of 1 m/s^2 and 2 m/s^2 and speeds 3 m/s and 1 m/s respectively. Initially A is 10 m behind B. What is the minimum distance between them?

Extra Points

- If y (may be velocity, acceleration etc.) is a function of time or $y = f(t)$ and we want to find the average value of y between a time interval of t_1 and t_2. Then,

$$\langle y \rangle_{t_1 \text{ to } t_2} = \text{average value of } y \text{ between } t_1 \text{ and } t_2$$

$$= \frac{\int_{t_1}^{t_2} f(t)\,dt}{t_2 - t_1}$$

or

$$\langle y \rangle_{t_1 \text{ to } t_2} = \frac{\int_{t_1}^{t_2} f(t)\,dt}{t_2 - t_1}$$

But if $f(t)$ is a linear function of t then

$$y_{av} = \frac{y_f + y_i}{2}$$

Here, y_f = final value of y and y_i = initial value of y

At the same time we should not forget that

$$v_{av} = \frac{\text{total displacement}}{\text{total time}}$$

and

$$a_{av} = \frac{\text{change in velocity}}{\text{total time}}$$

Example : In one dimensional uniformly accelerated motion. Find average velocity from $t = 0$ to $t = t$.

Solution : We can solve this problem by three methods.

Method 1. $v = u + at$

$$\therefore \quad \langle v \rangle_{0-t} = \frac{\int_0^t (u + at)\,dt}{t - 0}$$

$$= u + \frac{1}{2}at$$

Method 2. Since v is a linear function of time, we can write

$$v_{av} = \frac{v_f + v_i}{2} = \frac{(u + at) + u}{2}$$

$$= u + \frac{1}{2}at$$

Method 3.

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{1}{2}at$$

- A particle is thrown upwards with velocity u. Suppose it takes time t to reach its highest point, then distance travelled in last second is independent of u.
  This is because this distance is equal to the distance travelled in first second of a freely falling object. Thus,

$$s = \frac{1}{2}g \times (1)^2$$

$$= \frac{1}{2} \times 10 \times 1 = 5 \text{ m}$$

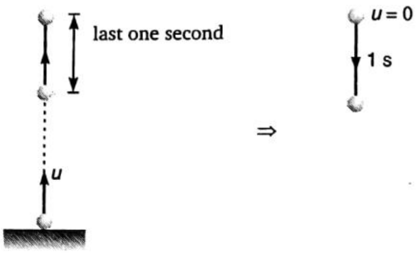


Fig. 3.38

Exercise : A particle is thrown upwards with velocity u (>20 m/s). Prove that distance travelled in last 2 second is 20 m.

- Suppose we have given velocity-time v-t graph. We want to plot corresponding displacement-time s-t graph then values of displacements at different times can be found just by adding the corresponding areas under v-t graph.
- The modulus of velocity is really the speed or

$$|\vec{\mathbf{v}}| = v$$

- Rate of change of velocity is acceleration, while rate of change of speed is the tangential acceleration (component of acceleration along velocity). Thus,

$$\frac{d\vec{\mathbf{v}}}{dt} = \vec{\mathbf{a}}$$

while

$$\frac{dv}{dt} = \frac{d|\vec{\mathbf{v}}|}{dt} = a_t$$

- Angle between velocity vector $\vec{\mathbf{v}}$ and acceleration vector $\vec{\mathbf{a}}$ decides whether the speed of particle is increasing, decreasing or constant.

Speed increases, if	$0° \le \theta < 90°$
Speed decreases, if	$90° < \theta \le 180°$
Speed is constant, if	$\theta = 90°$

The angle θ between $\vec{\mathbf{v}}$ and $\vec{\mathbf{a}}$ can be obtained by the relation,

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}}{va}\right)$$

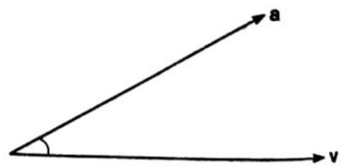


Fig. 3.39

Exercise : Prove that speed of a particle increases if dot product of $\vec{v}$ and $\vec{a}$ is positive, speed decreases, if the dot product is negative and speed remains constant if dot product is zero.

- It is found that problems of two (or three) dimensional motion become easy by writing the different vectors in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$ rather by using graphical method of vector addition or subtraction. For details you can refer Sample Example 3.24 in article 3.7.
- The magnitude of instantaneous velocity is called the instantaneous speed, *i.e.*,

$$v = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$$

Speed is not equal to $\frac{dr}{dt}$, *i.e.*, $$v \neq \frac{dr}{dt}$$

where r is the modulus of radius vector $\vec{r}$ because in general case $|d\vec{r}| \neq dr$. For example when $\vec{r}$ changes only in direction, *i.e.*, if a point moves in a circle, then r = constant, $dr = 0$ but $|d\vec{r}| \neq 0$.

- I have personally felt that the concept which I am going to present here becomes very useful while dealing with addition or subtraction of two vectors. Suppose a vector $\vec{C}$ is a vector sum of two vectors $\vec{A}$ and $\vec{B}$ and the direction of $\vec{C}$ is given to us. Say the vector $\vec{C}$ has to be along a line PQ. Then $\vec{A} + \vec{B}$ should be along PQ or sum of components of $\vec{A}$ and $\vec{B}$ perpendicular to line PQ should be zero. Similarly, if $\vec{C} = \vec{A} - \vec{B}$ and $\vec{C}$ has to be along the line PQ, then the sum of components of $\vec{A}$ and $-\vec{B}$ perpendicular to line PQ should be zero. For instance, in example 2.29, $\vec{v}_a$ has to be along AB and we know that $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$. Therefore, sum of components of $\vec{v}_{aw}$ and $\vec{v}_w$ perpendicular to line AB (shown as dotted) should be zero.

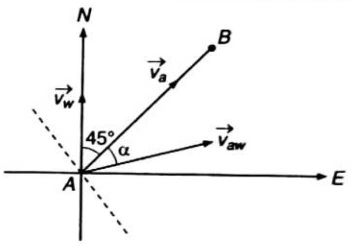


Fig. 3.40

or $$|\vec{v}_{aw}| \sin\alpha = |\vec{v}_w| \sin 45°$$

or $$\sin\alpha = \frac{|\vec{v}_w|}{|\vec{v}_{aw}|} \sin 45°$$

$$= \left(\frac{200\sqrt{2}}{400}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$$

∴ $$\alpha = 30°$$

Now, $$|\vec{v}_a| = |\vec{v}_{aw}| \cos\alpha + |\vec{v}_w| \cos 45°$$

$$= (400)\cos 30° + (200\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = (400)\frac{\sqrt{3}}{2} + 200$$

$$= 346.47 + 200 = 546.47 \text{ km/h}$$

∴ Time of journey from A to B will be

$$t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47} = 1.83 \text{ h.}$$

- I have found students often confused over the sign of 'g'. As per our sign convention (positive upwards and negative downwards) it is always negative, whether the particle is moving upwards or downwards. Now if u is upwards (*i.e.*, $u \uparrow \downarrow g$) motion is retarded and if u is either zero or downwards ($u \uparrow \uparrow g$) motion is accelerated.

Solved Examples

Level 1

Example 1 *A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 minutes to reach the field.*

(a) What distance he has to walk to reach the field ?

(b) What is the displacement from his house to the field ?

(c) What is the average speed of farmer during the walk ?

(d) What is the average velocity of farmer during the walk ?

Solution

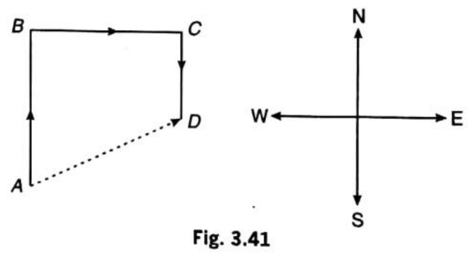


Fig. 3.41

(a) $$\text{Distance} = AB + BC + CD = (500 + 400 + 200) = 1100\text{ m}$$

(b) $$\text{Displacement} = AD = \sqrt{(AB - CD)^2 + BC^2}$$
$$= \sqrt{(500 - 200)^2 + (400)^2}$$
$$= 500\text{ m}$$

(c) $$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{1100}{20} = 55\text{ m/min}$$

(d) $$\text{Average velocity} = \frac{AD}{t} = \frac{500}{20} = 25\text{ m/min (along } AD)$$

Example 2 *A particle starts with an initial velocity and passes successively over the two halves of a given distance with accelerations a_1 and a_2 respectively. Show that the final velocity is the same as if the whole distance is covered with a uniform acceleration $\frac{(a_1 + a_2)}{2}$.*

Solution In the first case,

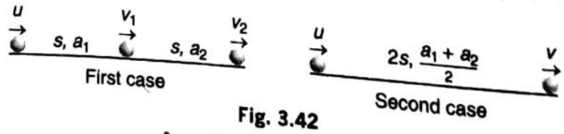


Fig. 3.42

$$v_1^2 = u^2 + 2a_1 s \quad \text{...(i)}$$

$$v_2^2 = v_1^2 + 2a_2 s \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have

$$v_2^2 = u^2 + 2\left(\frac{a_1 + a_2}{2}\right)(2s) \qquad \ldots\text{(iii)}$$

In the second case

$$v^2 = u^2 + 2\left(\frac{a_1 + a_2}{2}\right)(2s) \qquad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we can see that

$$v_2 = v$$

Hence proved.

Example 3 *In a car race, car A takes a time t less than car B at the finish and passes the finishing point with speed v more than that of the car B. Assuming that both the cars starts from rest and travel with constant acceleration a_1 and a_2 respectively. Show that $v = \sqrt{a_1 a_2}\, t$.*

Solution Let A takes t_1 second, then according to the given problem B will take $(t_1 + t)$ seconds. Further, let v_1 be the velocity of B at finishing point, then velocity of A will be $(v_1 + v)$. Writing equations of motion for A and B.

$$v_1 + v = a_1 t_1 \qquad \ldots\text{(i)}$$

$$v_1 = a_2 (t_1 + t) \qquad \ldots\text{(ii)}$$

From these two equations, we get

$$v = (a_1 - a_2) t_1 - a_2 t \qquad \ldots\text{(iii)}$$

Total distance travelled by both the cars is equal.

or $$s_A = s_B$$

or $$\frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 (t_1 + t)^2$$

or $$t_1 = \frac{\sqrt{a_2}\, t}{\sqrt{a_1} - \sqrt{a_2}}$$

Substituting this value of t_1 in Eq. (iii), we get the desired result

or $$v = (\sqrt{a_1 a_2})\, t$$

Example 4 *A particle is moving with a velocity of $v = (3 + 6t + 9t^2)$ cm/s. Find out :*

(a) the acceleration of the particle at $t = 3$ s.

(b) the displacement of the particle in the interval $t = 5$ s to $t = 8$ s.

Solution (a) Acceleration of particle,

$$a = \frac{dv}{dt} = (6 + 18t)\ \text{cm/s}^2$$

At $t = 3$ s,

$$a = (6 + 18 \times 3)\ \text{cm/s}^2$$

$$= 60\ \text{cm/s}^2$$

(b) Given, $$v = (3 + 6t + 9t^2)\ \text{cm/s}$$

or $$\frac{ds}{dt} = (3 + 6t + 9t^2)$$

or $$ds = (3 + 6t + 9t^2)\, dt$$

$\therefore \qquad \int_0^s ds = \int_5^8 (3 + 6t + 9t^2)\, dt$

$\therefore \qquad s = [3t + 3t^2 + 3t^3]_5^8$

or $\qquad s = 1287\,\text{cm}$

Example 5 *The motion of a particle along a straight line is described by the function $x = (2t - 3)^2$ where x is in metres and t is in seconds.*

(a) Find the position, velocity and acceleration at $t = 2\,s$.

(b) Find velocity of the particle at origin.

Solution (a) Position, $x = (2t - 3)^2$

Velocity, $\qquad v = \dfrac{dx}{dt} = 4(2t - 3)\,\text{m/s}$

and acceleration, $\qquad a = \dfrac{dv}{dt} = 8\,\text{m/s}^2$

At $t = 2\,\text{s}$, $\qquad x = (2 \times 2 - 3)^2 = 1.0\,\text{m}$

$v = 4(2 \times 2 - 3) = 4\,\text{m/s}$

and $\qquad a = 8\,\text{m/s}^2$

At origin, $x = 0$

or $\qquad (2t - 3) = 0$

$\therefore \qquad v = 4 \times 0 = 0$

Example 6 *An open elevator is ascending with zero acceleration. The speed $v = 10\,m/s$. A ball is thrown vertically up by a boy when he is at a height $h = 10\,m$ from the ground. The velocity of projection is $v = 30\,m/s$ with respect to elevator. Find:*

(a) the maximum height attained by the ball.

(b) the time taken by the ball to meet the elevator again.

(c) time taken by the ball to reach the ground after crossing the elevator.

Solution (a) Absolute velocity of ball = 40 m/s (upwards)

$\therefore \qquad h_{max} = h_i + h_f$

Here, h_i = initial height = 10 m

and h_f = further height attained by ball

$$= \frac{u^2}{2g} = \frac{(40)^2}{2 \times 10} = 80\,\text{m}$$

$\therefore \qquad h_{max} = (10 + 80)\,\text{m} = 90\,\text{m}$

(b) The ball will meet the elevator again when displacement of lift = displacement of ball

or $\qquad 10 \times t = 40 \times t - \dfrac{1}{2} \times 10 \times t^2$

or $\qquad t = 6\,\text{s}$

(c) Let t_0 be the total time taken by the ball to reach the ground. Then,

$$-10 = 40 \times t_0 - \frac{1}{2} \times 10 \times t_0^2$$

Therefore, time taken by the ball to reach the ground after crossing the elevator,

$$= (t_0 - t) = 2.24\,\text{s}$$

Example 7 *From an elevated point A, a stone is projected vertically upwards. When the stone reaches a distance h below A, its velocity is double of what it was at a height h above A. Show that the greatest height attained by the stone is* $\frac{5}{3}h$.

Solution Let u be the velocity with which the stone is projected vertically upwards.

Given that,
$$v_{-h} = 2v_h$$
or
$$(v_{-h})^2 = 4v_h^2$$
$\therefore$
$$u^2 - 2g(-h) = 4(u^2 - 2gh)$$
$\therefore$
$$u^2 = \frac{10gh}{3}$$
Now,
$$h_{max} = \frac{u^2}{2g} = \frac{5h}{3}$$
Hence proved.

Example 8 *A man crosses a river in a boat. If he cross the river in minimum time he takes* 10 *min with a drift* 120 m. *If he crosses the river taking shortest path, he takes* 12.5 *min, find :*

(a) width of the river.

(b) velocity of the boat with respect to water.

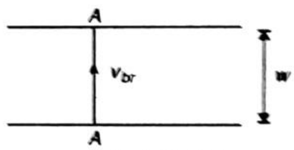
(c) speed of the current.

Solution Let v_r = velocity of river

v_{br} = velocity of river in still water and

w = width of river

Given,
$$t_{min} = 10\text{ min}$$
or
$$\frac{w}{v_{br}} = 10 \quad \text{...(i)}$$

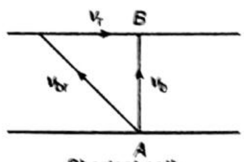


For minimum time

Fig. 3.43

Drift in this case will be,
$$x = v_r t$$
$\therefore$
$$120 = 10v_r \quad \text{...(ii)}$$

Shortest path is taken when v_b is along AB. In this case,
$$v_b = \sqrt{v_{br}^2 - v_r^2}$$
Now,
$$12.5 = \frac{w}{v_b} = \frac{w}{\sqrt{v_{br}^2 - v_r^2}} \quad \text{...(iii)}$$



Shortest path

Fig. 3.44

Solving these three equations, we get
$$v_{br} = 20\text{ m/min},$$
$$v_r = 12\text{ m/min}$$
and
$$w = 200\text{ m}$$

Example 9 *The acceleration versus time graph of a particle moving along a straight line is shown in the figure. Draw the respective velocity-time graph. Given $v = 0$ at $t = 0$.*

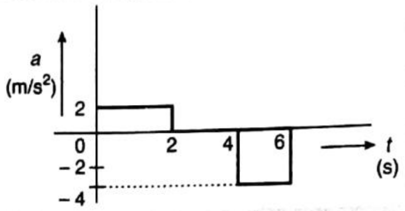


Fig. 3.45

Solution From $t = 0$ to $t = 2\,s$, $a = +2\,m/s^2$

$\therefore$
$$v = at = 2t$$

or v-t graph is a straight line passing through origin with slope $2\ m/s^2$.
At the end of 2 s,

$$v = 2 \times 2 = 4\ m/s$$

From $t = 2$ to 4 s, $a = 0$. Hence, $v = 4$ m/s will remain constant.
From $t = 4$ to 6 s, $a = -4\ m/s^2$. Hence,

$$v = u - at = 4 - 4t \qquad \text{(with } t = 0 \text{ at 4 s)}$$

$v = 0$ at $t = 1$ s or at 5 s from origin.
At the end of 6 s (or $t = 2$ s) $v = -4$ m/s. Corresponding v-t graph is as shown below :

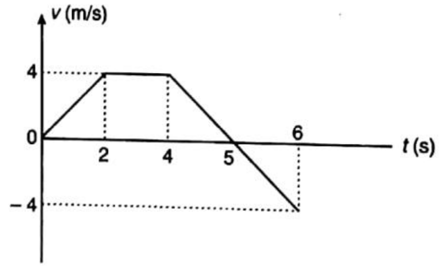


Fig. 3.46

Example 10 *A ball is thrown upward with an initial velocity of $100\ ms^{-1}$. After how much time will it return? Draw velocity-time graph for the ball and find from the graph (i) the maximum height attained by the ball and (ii) height of the ball after 15 s. Take $g = 10\ ms^{-2}$.*

Solution Here, $u = 100\ ms^{-1}$, $g = -10\ ms^{-2}$

At highest point, $v = 0$

As $v = u + gt \quad \therefore\ 0 = 100 - 10 \times t$

$\therefore$ Time taken to reach highest point,

$$t = \frac{100}{10} = 10\,s$$

The ball will return to the ground at $t = 20\,s$.

Corresponding velocity-time graph of the ball is shown in Fig. 3.47.

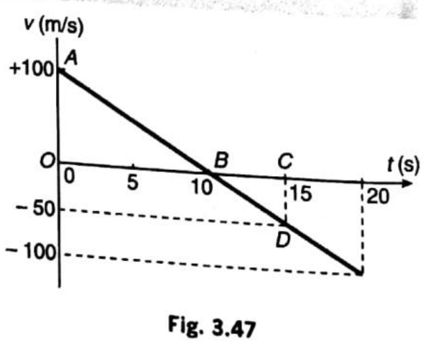


Fig. 3.47

(i) Maximum height attained by the ball = Area of ΔAOB

$$= \frac{1}{2} \times 10 \times 100 = 500 \text{ m}$$

(ii) Height attained after 15 s = Area of ΔAOB + Area of ΔBCD

$$= 500 + \frac{1}{2}(15-10) \times (-50) = 500 - 125 = 375 \text{ m}$$

Example 11 *A rocket is fired vertically up from the ground with a resultant vertical acceleration of* $10 \, m/s^2$. *The fuel is finished in 1 min and it continues to move up.*

(a) What is the maximum height reached? (b) After how much time from then will the maximum height be reached? (Take $g = 10 \, m/s^2$*)*

Solution (a) The distance travelled by the rocket in 1 min (= 60 s) in which resultant acceleration is vertically upwards and 10 m/s^2 will be

$$h_1 = (1/2) \times 10 \times 60^2 = 18000 \text{ m} = 18 \text{ km} \qquad \ldots\text{(i)}$$

and velocity acquired by it will be $\quad v = 10 \times 60 = 600 \text{ m/s} \qquad \ldots$(ii)

Now, after 1 min the rocket moves vertically up with velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height h_2 till its velocity becomes zero such that

$$0 = (600)^2 - 2gh_2$$

or

$$h_2 = 18000 \text{ m} \qquad [\text{as } \; g = 10 \text{ m/s}^2] \qquad \ldots\text{(iii)}$$

$$= 18 \text{ km}$$

So, from Eqs. (i) and (iii) the maximum height reached by the rocket from the ground

$$h = h_1 + h_2 = 18 + 18 = 36 \text{ km}$$ **Ans.**

(b) As after burning of fuel the initial velocity from Eq. (ii) is 600 m/s and gravity opposes the motion of rocket, so the time taken by it to reach the maximum height (for which $v = 0$),

$$0 = 600 - gt \quad \text{or} \quad t = 60 \text{ s}$$

i.e., after finishing fuel the rocket further goes up for 60 s, or 1 min. **Ans.**

Level 2

Note 1. *In one dimensional motion, if a = constant, we use $v = u + at$, etc., with proper signs and if $a \neq$ constant, we go for integration and differentiation.*

2. *Sometimes the standard results are written in different manners and the students unnecessarily go on integrating or differentiating. The standard results which are usually altered are :*

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

} *These are the equations of motion in one dimension with constant acceleration*

(iv) $v = \omega\sqrt{A^2 - x^2}$

(v) $a = -\omega^2 x$

} *These are the equations of simple harmonic motion.*

The above point will be more clear after going through following two examples 1 and 2.

Example 1 *Velocity of a particle moving in a straight line varies with its displacement as* $v = (\sqrt{4+4s})$ *m/s. Displacement of particle at time* $t = 0$ *is* $s = 0$. *Find displacement of particle at time* $t = 2$ *s.*

Solution Squaring the given equation, we get

$$sv^2 = 4 + 4s$$

Now, comparing it with

$$v^2 = u^2 + 2as$$

we get,

$$u = 2 \text{ m/s} \quad \text{and} \quad a = 2 \text{ m/s}^2$$

∴ Displacement at $t = 2\text{s}$ is

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad s = (2)(2) + \frac{1}{2}(2)(2)^2$$

or

$$s = 8 \text{ m}$$

Example 2 *Velocity of a particle varies with its displacement as*

$$v = (\sqrt{9 - x^2}) \text{ m/s}$$

Find the magnitude of maximum acceleration of the particle.

Solution Comparing the given equation with standard velocity-displacement equation of simple harmonic motion, *i.e.*, $v = \omega\sqrt{A^2 - x^2}$, we get

$$\omega = 1 \text{ rad/s} \quad \text{and} \quad A = 3 \text{ m}$$

The magnitude of maximum acceleration of the particle in SHM is $= \omega^2 A$

$$= (1)^2(3) \text{ m/s}^2 = 3 \text{ m/s}^2$$

Example 3 *Fig. 3.48 shows a rod of length l resting on a wall and the floor. Its lower end A is pulled towards left with a constant velocity v. Find the velocity of the other end B downward when the rod makes an angle θ with the horizontal.*

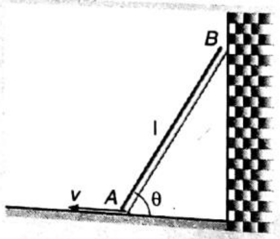


Fig. 3.48

Solution In such type of problems, when velocity of one part of a body is given and that of other is required, we first find the relation between the two displacements, then differentiate them with respect to time. Here, if the distance from the corner to the point A is x and that up to B is y. Then,

$$v = \frac{dx}{dt}$$

and

$$v_B = -\frac{dy}{dt} \qquad \text{(– sign denotes that } y \text{ is decreasing)}$$

Further,

$$x^2 + y^2 = l^2$$

Differentiating with respect to time t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$xv = yv_B$$

$$v_B = \frac{x}{y} v = v \cot \theta$$

Example 4 *A particle is moving in a straight line with constant acceleration. If x, y and z be the distances described by a particle during the pth, qth and rth second respectively, prove that*

$$(q-r)x + (r-p)y + (p-q)z = 0$$

Solution As $\quad s_{nth} = u + an - \frac{1}{2}a = u + \frac{a}{2}(2n-1)$

$\therefore$
$$x = u + \frac{a}{2}(2p-1) \qquad \text{...(i)}$$

$$y = u + \frac{a}{2}(2q-1) \qquad \text{...(ii)}$$

$$z = u + \frac{a}{2}(2r-1) \qquad \text{...(iii)}$$

Subtracting Eq. (iii) from Eq. (ii),

$$y - z = \frac{a}{2}(2q-2r) \quad \text{or} \quad q - r = \frac{y-z}{a}$$

or
$$(q-r)x = \frac{1}{a}(yx - zx) \qquad \text{...(iv)}$$

Similarly, we can show that

$$(r-p)y = \frac{1}{a}(zy - xy) \qquad \text{...(v)}$$

and
$$(p-q)z = \frac{1}{a}(xz - yz) \qquad \text{...(vi)}$$

Adding Eqs. (iv), (v) and (vi), we get

$$(q-r)x + (r-p)y + (p-q)z = 0$$

Example 5 *A balloon starts rising from the earth's surface. The ascension rate is constant and equal to* v_0. *Due to the wind, the balloon gathers the horizontal velocity component* $v_x = k\,y$, *where k is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent:*
(a) the horizontal drift of the balloon $x(y)$.
(b) the total, tangential and normal accelerations of the balloon.

Solution (a)
$$\frac{dy}{dt} = v_0 \qquad \text{...(i)}$$

$$\frac{dx}{dt} = ky \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{dy}{dx} = \frac{v_0}{ky} \quad \text{or} \quad dx = \frac{ky}{v_0}\,dy$$

Intergrating, we get $$x = \frac{k}{v_0}\left(\frac{y^2}{2}\right)$$

This is the desired trajectory of the particle, which is an equation of a parabola.

(b) For finding the tangential and normal accelerations, we require an expression for the speed as a function of height y

$$v_y = v_0 \quad \text{and} \quad v_x = ky$$

$\therefore$ $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + k^2 y^2}$$

Therefore, tangential acceleration,

$$a_t = \frac{dv}{dt} = \frac{k^2 y}{\sqrt{v_0^2 + k^2 y^2}} \frac{dy}{dt}$$

$$= \frac{k^2 y v_0}{\sqrt{v_0^2 + k^2 y^2}}$$

or $$a_t = \frac{k^2 y}{\sqrt{1 + k^2 y^2 / v_0^2}}$$

Now, the total acceleration is,

$$a = \sqrt{\left(\frac{dv_y}{dt}\right)^2 + \left(\frac{dv_x}{dt}\right)^2}$$

$$= \frac{dv_x}{dt} = k\frac{dy}{dt} = kv_0$$

$\therefore$ Normal acceleration, $$a_n = \sqrt{a^2 - a_t^2} = \frac{kv_0}{\sqrt{1 + \left(\frac{ky}{v_0}\right)^2}}$$

Example 6 *Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side d at time t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other?*

Solution Velocity of A is v along AB. The velocity of B is along BC. Its component along BA is $v \cos 60° = v/2$. Thus, the separation AB decreases at the rate

$$v + \frac{v}{2} = \frac{3v}{2}$$

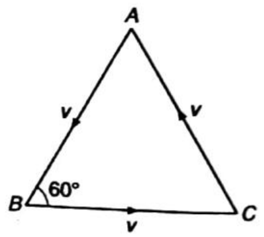


Fig. 3.49

Since, this rate is constant, the time taken in reducing the separation AB from d to zero is

$$t = \frac{d}{(3v/2)} = \frac{2d}{3v}$$

Example 7 *An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 m/s^2. 2 s after the start a bolt begins falling from the ceiling of the car. Find:*

(a) the time after which bolt hits the floor of the elevator.

(b) the net displacement and distance travelled by the bolt, with respect to earth.

(Take $g = 9.8\ m/s^2$)

Solution (a) If we consider elevator at rest, then relative acceleration of the bolt is $a_r = 9.8 + 1.2$

$$= 11\ \text{m/s}^2 \text{ (downwards)}$$

After 2 s velocity of lift is $v = at = (1.2)(2) = 2.4$ m/s. Therefore initial velocity of the bolt is also 2.4 m/s and it gets accelerated with relative acceleration 11 m/s^2. With respect to elevator initial velocity of bolt is zero and it has to travel 2.7 m with 11 m/s^2. Thus, time taken can be directly given as

$$\sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2.7}{11}} = 0.7\ \text{s}.$$

(b) Displacement of bolt relative to ground in 0.7 s.

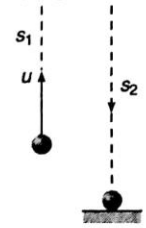


Fig. 3.50

$$s = ut + \frac{1}{2}at^2$$

or

$$s = (2.4)(0.7) + \frac{1}{2}(-9.8)(0.7)^2$$

$$s = -0.72\ \text{m}$$

Velocity of bolt will become zero after a time

$$t_0 = \frac{u}{g} \qquad (v = u - gt)$$

$$= \frac{2.4}{9.8} = 0.245\ \text{s}$$

Therefore, distance travelled by the bolt $= s_1 + s_2$

$$= \frac{u^2}{2g} + \frac{1}{2}g(t - t_0)^2$$

$$= \frac{(2.4)^2}{2 \times 9.8} + \frac{1}{2} \times 9.8(0.7 - 0.245)^2 = 1.3\ \text{m}$$

Example 8 *A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in figure. What minimum speed relative to water should the man have so that he can reach point B? In which direction should he swim?*

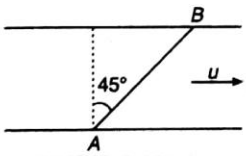


Fig. 3.51

Solution Let v be the speed of boatman in still water.

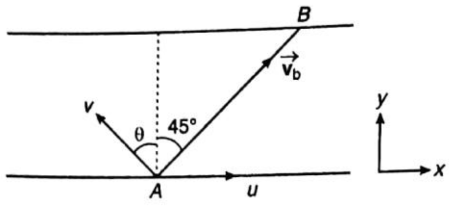


Fig. 3.52

Resultant of v and u should be along AB. Components of $\vec{v}_b$ (absolute velocity of boatman) along x and y-direction are,

$$v_x = u - v\sin\theta \quad \text{and} \quad v_y = v\cos\theta$$

Further,
$$\tan 45° = \frac{v_y}{v_x}$$

or
$$1 = \frac{v\cos\theta}{u - v\sin\theta}$$

$\therefore$
$$v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

v is minimum at,
$$\theta + 45° = 90° \quad \text{or} \quad \theta = 45°$$

and
$$v_{\min} = \frac{u}{\sqrt{2}}$$

EXERCISES

AIEEE Corner

Subjective Questions (Level 1)

Basic Definitions

1. A car moves with 60 km/h in first one hour and with 80 km/h in next half an hour. Find :
   (a) total distance travelled by the car, (b) average speed of car in total 1.5 hours.
2. A particle moves in a straight line with initial velocity 4 m/s and a constant acceleration of $6\ \text{m/s}^2$. Find the average velocity of the particle in a time interval from
   (a) $t = 0$ to $t = 2\ \text{s}$ (b) $t = 2\ \text{s}$ to $t = 4\ \text{s}$.
3. A particle is projected upwards from the roof of a tower 60 m high with velocity 20 m/s. Find :
   (a) the average speed and
   (b) average velocity of the particle upto an instant when it strikes the ground. Take $g = 10\ \text{m/s}^2$.
4. A block moves in a straight line with velocity v for time t_0. Then, its velocity becomes $2v$ for next t_0 time. Finally its velocity becomes $3v$ for time T. If average velocity during the complete journey was 2.5 v, then find T in terms of t_0.
5. A particle starting from rest has a constant acceleration of $4\ \text{m/s}^2$ for 4 s. It then retards uniformly for next 8 s and comes to rest. Find during the motion of particle :
   (a) average acceleration, (b) average speed,
   (c) average velocity.
6. A particle moves in a circle of radius $R = \frac{21}{22}$ m with constant speed 1m/s. Find :
   (a) magnitude of average velocity and (b) magnitude of average acceleration in 2 s.
7. A particle is moving in x-y plane. At time $t = 0$, particle is at (1m, 2m) and has velocity $(4\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$ m/s. At $t = 4$ s, particle reaches at (6m, 4m) and has velocity $(2\hat{\mathbf{i}} + 10\hat{\mathbf{j}})$ m/s. In the given time interval, find :
   (a) average velocity,
   (b) average acceleration and
   (c) from the given data, can you find average speed?

Uniform Acceleration

(a) One dimensional motion

8. Two diamonds begin a free fall from rest from the same height, 1.0 s apart. How long after the first diamond begins to fall will the two diamonds be 10 m apart? Take $g = 10\ \text{m/s}^2$.

9. Two bodies are projected vertically upwards from one point with the same initial velocity v_0. The second body is projected t_0 s after the first. How long after will the bodies meet?

10. A stone is dropped from the top of a tower. When it crosses a point 5 m below the top, another stone is let fall from a point 25 m below the top. Both stones reach the bottom of the tower simultaneously. Find the height of the tower. Take $g = 10\ \text{m/s}^2$.

11. A point mass starts moving in a straight line with constant acceleration. After time t_0 the acceleration changes its sign, remaining the same in magnitude. Determine the time t from the beginning of motion in which the point mass returns to the initial position.

12. A football is kicked vertically upward from the ground and a student gazing out of the window sees it moving upwards past her at 5.00 m/s. The window is 15.0 m above the ground. Air resistance may be ignored. Take $g = 10\ \text{m/s}^2$.
   (a) How high does the football go above ground?
   (b) How much time does it take to go from the ground to its highest point?

13. A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passes the second point was 15.0 m/s.
   (a) What is the speed at the first point?
   (b) What is the acceleration?
   (c) At what prior distance from the first was the car at rest?
   (d) Graph s *versus* t and v *versus* t for the car, from rest ($t = 0$).

14. A train stopping at two stations 4 km apart takes 4 min on the journey from one of the station to the other. Assuming that it first accelerates with a uniform acceleration x and then that of uniform retardation y, prove that $\frac{1}{x} + \frac{1}{y} = 2$.

15. A particle moves along the x-direction with constant acceleration. The displacement, measured from a convenient position, is 2m at time $t = 0$ and is zero when $t = 10$ s. If the velocity of the particle is momentary zero when $t = 6$ s, determine the acceleration a and the velocity v when $t = 10$ s.

(b) Two or three dimensional motion

16. Net force acting on a particle of mass 2 kg is 10 N in north direction. At $t = 0$, particle was moving eastwards with 10 m/s. Find displacement and velocity of particle after 2 s.

17. At time $t = 0$, a particle is at (2m, 4m). It starts moving towards positive x-axis with constant acceleration $2\ \text{m/s}^2$ (initial velocity = 0). After 2 s an acceleration of $4\ \text{m/s}^2$ starts acting on the particle in negative y-direction also. Find after next 2 s :
   (a) velocity and
   (b) coordinates of particle.

18. A particle moving in x-y plane is at origin at time $t = 0$. Velocity of the particle is $(2\hat{i} - 4\hat{j})$ m/s and acceleration is $(4\hat{i} + \hat{j})\ \text{m/s}^2$. Find at $t = 2$ s :
   (a) velocity of particle and
   (b) coordinates of particle.

19. A particle starts from the origin at $t = 0$ with a velocity of $8.0\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(4.0\hat{i} + 2.0\hat{j})\ \text{m/s}^2$. At the instant the particle's x-coordinate is 29 m, what are :
   (a) its y-coordinate and
   (b) its speed ?

20. At time $t = 0$, the position vector of a particle moving in the x-y plane is $5\hat{\mathbf{i}}$ m. By time $t = 0.02$ s, its position vector has become $(5.1\hat{\mathbf{i}} + 0.4\hat{\mathbf{j}})$ m determine the magnitude v_{av} of the average velocity during this interval and the angle θ made by the average velocity with the positive x-axis.

Non-uniform Acceleration

21. x-coordinate of a particle moving along this axis is : $x = (2 + t^2 + 2t^3)$. Here, x is in metre and t in seconds. Find :

(a) position of particle from where it started its journey,
(b) initial velocity of particle and
(c) acceleration of particle at $t = 2$ s.

22. The velocity of a particle moving in a straight line is decreasing at the rate of 3 m/s per metre of displacement at an instant when the velocity is 10 m/s. Determine the acceleration of the particle at this instant.

23. The position of a particle along a straight line is given by $s = (t^3 - 9t^2 - 15t)$ m, here t is in seconds. Determine its maximum acceleration during the time interval $0 \le t \le 10$ s.

24. The acceleration of a particle is given by $a(t) = (3.00\text{ m/s}^2) - (2.00\text{ m/s}^3)t$.

(a) Find the initial speed v_0 such that the particle will have the same x-coordinate at $t = 5.00$ s as it had at $t = 0$.
(b) What will be the velocity at $t = 5.00$ s?

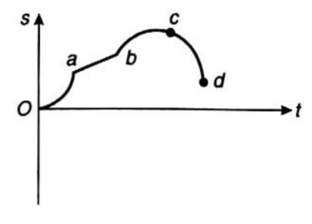
25. A particle moves along a horizontal path, such that its velocity is given by $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance travelled by the particle in time interval from $t = 0$ to $t = 3.5$ s and the particle's average velocity and average speed druing the same time interval.

26. A particle travels in a straight line, such that for a short time $2\text{ s} \le t \le 6\text{ s}$, its motion is described by $v = (4/a)$ m/s, where a is in m/s^2. If $v = 6$ m/s when $t = 2$ s, determine the particle's acceleration when $t = 3$ s.

27. If the velocity v of a particle moving along a straight line decreases linearly with its displacement from 20 m/s to a value approaching zero at $s = 30$ m, determine the acceleration of the particle when $s = 15$ m.

Graphs

28. Displacement-time graph of a particle moving in a straight line is as shown in figure.



(a) Find the sign of velocity in regions oa, ab, bc and cd.
(b) Find the sign of acceleration in the above region.

29. Let us call a motion as :

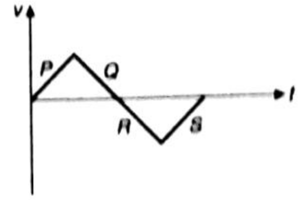
$M_1 \rightarrow$ if velocity and acceleration both are positive.

$M_2 \rightarrow$ if velocity is positive but acceleration is negative.

$M_3 \rightarrow$ if velocity and acceleration both are negative.

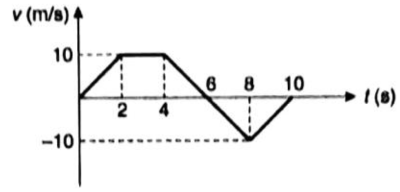
$M_4 \rightarrow$ if velocity is negative but acceleration is positive.

(a) State, in which of the above four motions, magnitude of velocity is increasing and in which it is decreasing.

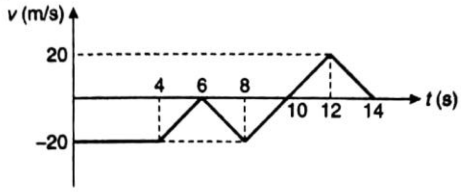


(b) v-t graph of a particle moving in a straight line is as shown in figure. The whole graph is made up of four straight lines P, Q, R and S. These four straight lines indicate four type of motions ($M_1, \ldots M_4$) discussed above. State, which straight line corresponds to which type of motion.

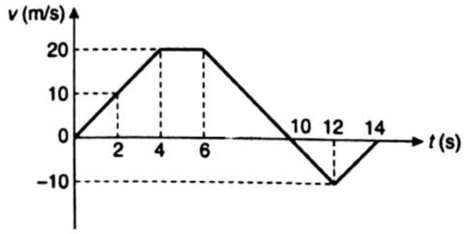
30. Velocity-time graph of a particle moving in a straight line is shown in figure. At time $t = 0$, $s = -10$ m. Plot corresponding a-t and s-t graphs.



31. Velocity-time graph of a particle moving in a straight line is shown in figure. At time $t = 0$, $s = 20$ m. Plot a-t and s-t graphs of the particle.



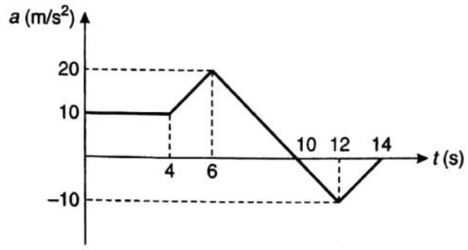
32. Velocity-time graph of a particle moving in a straight line is shown in figure. In the time interval from $t = 0$ to $t = 14$ s, find :



(a) average velocity and

(b) average speed of the particle

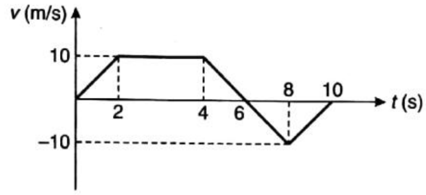
33. Acceleration-time graph of a particle moving in a straight line is as shown in figure. At time $t = 0$, velocity of the particle is zero. Find :



(a) average acceleration in a time interval from $t = 6\text{ s}$ to $t = 12\text{ s}$,

(b) velocity of the particle at $t = 14\text{ s}$.

34. Velocity-time graph of a particle moving in a straight line is shown in figure. At time $t = 0$, displacement of the particle from mean position is 10 m. Find :



(a) acceleration of particle at $t = 1\text{ s}, 3\text{ s}$ and 9 s.

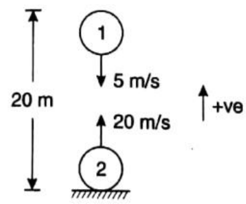
(b) position of particle from mean position at $t = 10\text{ s}$.

(c) write down s-t equation for time interval :

(i) $0 \le t \le 2\text{ s}$, (ii) $4\text{ s} \le t \le 8\text{ s}$

Relative Motion

35. Two particles 1 and 2 are thrown in the directions shown in figure simultaneously with velocities 5 m/s and 20 m/s. Initially particle 1 is at height 20 m from the ground. Taking upwards as the positive direction, find :



(a) acceleration of 1 with respect to 2

(b) initial velocity of 2 with respect to 1

(c) velocity of 1 with respect to 2 after time $t = 1/2\text{ s}$

(d) time when the particles will collide.

36. A person walks up a stalled 15 m long escalator in 90 s. When standing on the same escalator, now moving, the person is carried up in 60 s. How much time would it take that person to walk up the moving escalator? Does the answer depend on the length of the escalator?

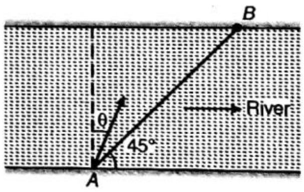
37. A ball is thrown vertically upward from the 12 m level with an initial velocity of 18 m/s. At the same instant an open platform elevator passes the 5 m level, moving upward with a constant velocity of 2 m/s. Determine :
   (a) when and where the ball will meet the elevator,
   (b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.

38. An automobile and a truck start from rest at the same instant, with the automobile initially at some distance behind the truck. The truck has a constant acceleration of $2.2\ \text{m/s}^2$ and the automobile has an acceleration of $3.5\ \text{m/s}^2$. The automobile overtakes the truck when it (truck) has moved 60 m.
   (a) How much time does it take the automobile to overtake the truck ?
   (b) How far was the automobile behind the truck initially ?
   (c) What is the speed of each during overtaking ?

39. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is t. Show that the lift is moving up with an acceleration $\frac{2u - tg}{t}$.

40. A river is 20 m wide. River speed is 3 m/s. A boat starts with velocity $2\sqrt{2}$ m/s at angle 45° from the river current (relative to river)
   (a) Find the time taken by the boat to reach the opposite bank.
   (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank?

41. Given $|\vec{v}_{br}| = 4$ m/s = magnitude of velocity of boatman with respect to river, $\vec{v}_r = 2$ m/s in the direction shown. Boatman wants to reach from point A to point B. At what angle θ should he row his boat.



42. An aeroplane has to go from a point P to another point Q, 1000 km away due north. Wind is blowing due east at a speed of 200 km/h. The air speed of plane is 500 km/h.
   (a) Find the direction in which the pilot should head the plane to reach the point Q.
   (b) Find the time taken by the plane to go from P to Q.

Objective Questions (Level 1)

Single Correct Option

1. A packet is released from a rising balloon accelerating upward with acceleration a. The acceleration of the stone just after the release is
   (a) a upward (b) g downward (c) $(g - a)$ downward (d) $(g + a)$ downward

2. A ball is thrown vertically upwards from the ground. If T_1 and T_2 are the respective time taken in going up and coming down, and the air resistance is not ignored, then
   (a) $T_1 > T_2$ (b) $T_1 = T_2$ (c) $T_1 < T_2$ (d) nothing can be said

3. The length of a seconds hand in watch is 1 cm. The change in velocity of its tip in 15 s is

(a) zero (b) $\frac{\pi}{30\sqrt{2}}$ cms^{-1} (c) $\frac{\pi}{30}$ cms^{-1} (d) $\frac{\pi\sqrt{2}}{30}$ cms^{-1}

4. A particle moving along a straight line travels half of the distance with uniform speed 30 ms^{-1} and the remaining half of the distance with speed 60 ms^{-1}. The average speed of the particle is

(a) 45 ms^{-1} (b) 42 ms^{-1} (c) 40 ms^{-1} (d) 50 ms^{-1}

5. A boat is moving with a velocity $(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ with respect to ground. The water in the river is moving with a velocity $-3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ with respect to ground. The relative velocity of the boat with respect to water is

(a) $8\hat{\mathbf{j}}$ (b) $-6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$ (c) $6\hat{\mathbf{i}} + 9\hat{\mathbf{j}}$ (d) zero

6. During the first 18 min of a 60 min trip, a car has an average speed of 11 ms^{-1}. What should be the average speed for remaining 42 min so that car is having an average speed of 21 ms^{-1} for the entire trip ?

(a) 25.3 ms^{-1} (b) 29.2 ms^{-1} (c) 31 ms^{-1} (d) 35.6 ms^{-1}

7. A particle moves along a straight line. Its position at any instant is given by $x = 32t - \frac{8t^3}{3}$ where x is in metre and t in second. Find the acceleration of the particle at the instant when particle is at rest.

(a) $-16\,\text{ms}^{-2}$ (b) $-32\,\text{ms}^{-2}$ (c) 32 ms^{-2} (d) 16 ms^{-2}

8. A car starts from rest and accelerates at constant rate in a straight line. In the first second the car covers a distance of 2 m. The velocity of the car at the end of second (sec) will be

(a) 4.0 ms^{-1} (b) 8.0 ms^{-1} (c) 16 ms^{-1} (d) None of these

9. A particle is moving along x-axis whose position is varying with time according to the relation $x = -3t + t^3$ where x is in metre and t is in second. The displacement of particle for $t = 1$ s to $t = 3$ s is

(a) +16 m (b) −16 m (c) +20 m (d) −20 m

10. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(a) $v_0t + \frac{1}{6}bt^3$ (b) $v_0t + \frac{1}{3}bt^3$ (c) $v_0t + \frac{1}{3}bt^2$ (d) $v_0t + \frac{1}{2}bt^2$

11. Water drops fall at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap, the instant the first drop touches the ground. How far above the ground is the second drop at that instant. $(g = 10\,\text{ms}^{-2})$

(a) 1.25 m (b) 2.50 m (c) 3.75 m (d) 4.00 m

12. A stone is dropped from the top of a tower and one second later, a second stone is thrown vertically downward with a velocity 20 ms^{-1}. The second stone will overtake the first after travelling a distance of $(g = 10\,\text{ms}^{-2})$

(a) 13 m (b) 15 m (c) 11.25 m (d) 19.5 m

13. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height then the ball should be thrown with velocity

(a) $\sqrt{3}\,v_0$ (b) $3\,v_0$ (c) $9\,v_0$ (d) $3/2\,v_0$

14. A particle moves in the x-y plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$ s, the equation of the path is

(a) $x = y^2 - y + 2$ (b) $x = y^2 - 2$ (c) $x = y^2 + y - 6$ (d) None of these

15. The horizontal and vertical displacements of a particle moving along a curved line are given by $x = 5t$ and $y = 2t^2 + t$. Time after which its velocity vector makes an angle of 45° with the horizontal is

(a) 0.5 s (b) 1 s (c) 2 s (d) 1.5 s

16. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = (8t - 5t^2)$ metre and $x = 6t$ metre where t is in second. The velocity of projection is

(a) $8\ ms^{-1}$ (b) $6\ ms^{-1}$ (c) $10\ ms^{-1}$ (d) data insufficient

17. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second ?

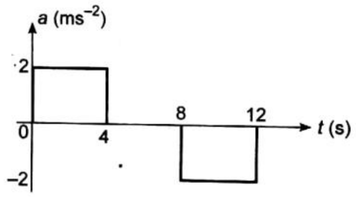
(a) $\frac{h}{9}$ metre from the ground (b) $(7h/9)$ metre from the ground

(c) $(8h/9)$ metre from the ground (d) $(17h/18)$ metre form the ground

18. An ant is at a corner of a cubical room of side a. The ant can move with a constant speed u. The minimum time taken to reach the farthest corner of the cube is

(a) $\frac{3a}{u}$ (b) $\frac{\sqrt{3}a}{u}$ (c) $\frac{\sqrt{5}a}{u}$ (d) $\frac{(\sqrt{2}+1)a}{u}$

19. A lift starts from rest. Its acceleration is plotted against time. When it comes to rest its height above its starting point is



(a) 20 m (b) 64 m (c) 32 m (d) 36 m

20. A lift performs the first part of its ascent with uniform acceleration a and the remaining with uniform retardation $2a$. If t is the time of ascent, find the depth of the shaft.

(a) $\frac{at^2}{4}$ (b) $\frac{at^2}{3}$ (c) $\frac{at^2}{2}$ (d) $\frac{at^2}{8}$

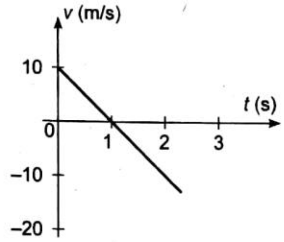
21. Two objects are moving along the same straight line. They cross a point A with an acceleration a, $2a$ and velocity $2u$, u at time $t = 0$. The distance moved by the object when one overtakes the other is

(a) $\frac{6u^2}{a}$ (b) $\frac{2u^2}{a}$ (c) $\frac{4u^2}{a}$ (d) $\frac{8u^2}{a}$

22. A cart is moving horizontally along a straight line with constant speed $30\ ms^{-1}$. A particle is to be fired vertically upwards from the moving cart in such a way that it returns to the cart at the same point from where it was projected after the cart has moved 80 m. At what speed (relative to the cart) must the projectile be fired? (Take $g = 10\ ms^{-2}$)

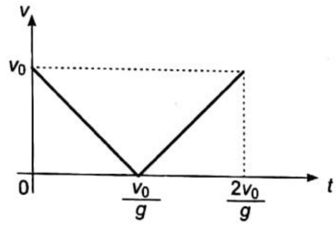
(a) $10\ ms^{-1}$ (b) $10\sqrt{8}\ ms^{-1}$ (c) $\frac{40}{3}\ ms^{-1}$ (d) None of these

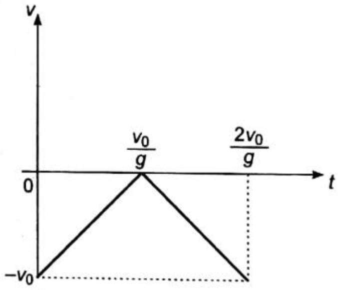
23. The figure shows velocity–time graph of a particle moving along a straight line. Identify the correct statement.

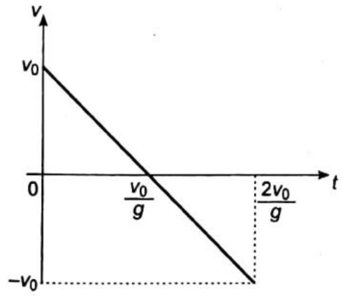


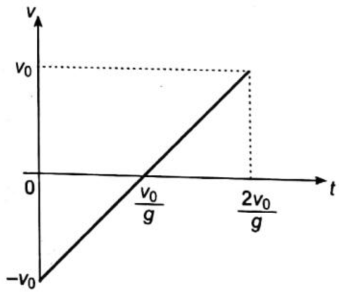
(a) The particle starts from the origin
(b) The particle crosses it initial position at $t = 2$s
(c) The average speed of the particle in the time interval, $0 \le t \le 2$s is zero
(d) All of the above

24. A ball is thrown vertically upwards with a velocity v_0. If the vertical downward direction is considered to be positive, then the correct variation of velocity with time t is represented by

(a)


(b)


(c)


(d)


25. A boy standing in an elevator accelerating upward throws a ball upward with a velocity v_0. The ball returns in his hands after a time t. The acceleration of the lift is
(a) $\frac{2v_0 - gt}{t}$ (b) $\frac{2v_0 + gt}{t}$ (c) $\frac{v_0 - gt}{t}$ (d) $\frac{v_0 + gt}{t}$

26. A body starts moving with a velocity $v_0 = 10\,\text{ms}^{-1}$. It experiences a retardation equal to $0.2v^2$. Its velocity after 2s is given by
(a) $+2\,\text{ms}^{-1}$ (b) $+4\,\text{ms}^{-1}$ (c) $-2\,\text{ms}^{-1}$ (d) $+6\,\text{ms}^{-1}$

27. Two trains are moving with velocities $v_1 = 10\,\text{ms}^{-1}$ and $v_2 = 20\,\text{ms}^{-1}$ on the same track in opposite directions. After the application of brakes if their retarding rates are $a_1 = 2\,\text{ms}^{-2}$ and $a_2 = 1\,\text{ms}^{-2}$ respectively, then the minimum distance of separation between the trains to avoid collision is
(a) 150 m (b) 225 m (c) 450 m (d) 300 m

28. Two identical balls are shot upward one after another at an interval of 2s along the same vertical line with same initial velocity of 40 ms^{-1}. The height at which the balls collide is

(a) 50 m (b) 75 m (c) 100 m (d) 125 m

29. A particle is projected vertically upwards and reaches the maximum height H in time T. The height of the particle at any time t will be

(a) $g(t-T)^2$ (b) $H-g(t-T)^2$ (c) $\frac{1}{2}g(t-T)^2$ (d) $H-\frac{1}{2}g(t-T)^2$

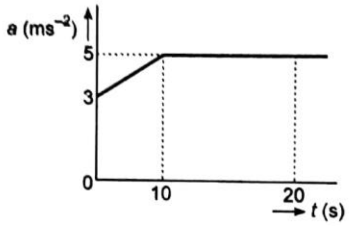
30. A particle moves along the curve $y=\frac{x^2}{2}$. Here x varies with time as $x=\frac{t^2}{2}$. Where x and y are measured in metre and t in second. At $t=2$ s, the velocity of the particle (in ms^{-1}) is

(a) $2\hat{i}-4\hat{j}$ (b) $2\hat{i}+4\hat{j}$ (c) $4\hat{i}+2\hat{j}$ (d) $4\hat{i}-2\hat{j}$

31. If the displacement of a particle varies with time as $\sqrt{x}=t+3$

(a) velocity of the particle is inversely proportional to t
(b) velocity of particle varies linearly with t
(c) velocity of particle is proportional to $\sqrt{t}$
(d) initial velocity of the particle is zero

32. The graph describes an airplane's acceleration during its take-off run. The airplane's velocity when it lifts off at $t=20$ s is



(a) 40 ms^{-1} (b) 50 ms^{-1} (c) 90 ms^{-1} (d) 180 ms^{-1}

33. A particle moving in a straight line has velocity-displacement equation as $v=5\sqrt{1+s}$. Here v is in ms^{-1} and s in metre. Select the correct alternative.

(a) Particle is initially at rest
(b) Initially velocity of the particle is 5 m/s and the particle has a constant acceleration of 12.5 ms^{-2}
(c) Particle moves with a uniform velocity
(d) None of these

34. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce a retardation of 2 ms^{-2}. The ratio of time of ascent to time of descent is ($g=10\ ms^{-2}$)

(a) 1 : 1 (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{3}{2}}$

35. A river is flowing from west to east with a speed of 5 m min^{-1}. A man can swim in still water with a velocity 10 m min^{-1}. In which direction should the man swim so as to take the shortest possible path to go to the south?

(a) 30° east of south (b) 30° west of south (c) South (d) 30° west of north

36. A body of mass 10 kg is being acted upon by a force $3t^2$ and an opposing constant force of 32 N. The initial speed is 10 ms^{-1}. The velocity of body after 5 s is

(a) 14.5 ms^{-1} (b) 6.5 ms^{-1} (c) 3.5 ms^{-1} (d) 4.5 ms^{-1}

37. A stone is thrown vertically upwards. When stone is at a height half of its maximum height, its speed is $10\ \text{ms}^{-1}$; then the maximum height attained by the stone is $(g = 10\ \text{ms}^{-2})$

(a) 25 m (b) 10 m (c) 15 m (d) 20 m

38. A ball is thrown vertically upwards from the ground and a student gazing out of the window sees it moving upward past him at $10\ \text{ms}^{-1}$. The window is at 15 m above the ground level. The velocity of ball 3 s after it was projected from the ground is [Take $g = 10\ \text{ms}^{-2}$]

(a) 10 m/s, up (b) $20\ \text{ms}^{-1}$, up (c) $20\ \text{ms}^{-1}$, down (d) $10\ \text{ms}^{-1}$, down

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*
(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*
(*c*) *If **Assertion** is true, but the **Reason** is false.*
(*d*) *If **Assertion** is false but the **Reason** is true.*

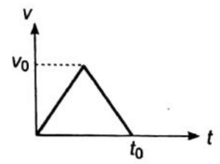
1. **Assertion :** Velocity and acceleration of a particle are given as,

$$\vec{\mathbf{v}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} \text{ and } \vec{\mathbf{a}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

This is a two dimensional motion with constant acceleration.
Reason : Velocity and acceleration are two constant vectors.

2. **Assertion :** Displacement-time graph is a parabola corresponding to straight line velocity-time graph.
Reason : If $v = u + at$, then $s = ut + \frac{1}{2}at^2$

3. **Assertion :** In v-t graph shown is figure, average velocity in time interval from 0 t_0 to depends only on v_0. It is independent of t_0.
Reason : In the given time interval average velocity is $\frac{v_0}{2}$.



4. **Assertion :** We know the relation $a = v.\frac{dv}{ds}$. Therefore, if velocity of a particle is zero, then acceleration is also zero.
Reason : In the above equation, a is the instantaneous acceleration.

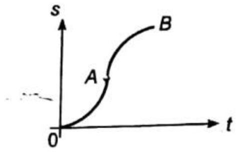
5. **Assertion :** Speed of a particle may decrease, even if acceleration is increasing.
Reason : This will happen if acceleration is positive.

6. **Assertion :** Starting from rest with zero acceleration if acceleration of particle increases at a constant rate of $2\ \text{ms}^{-3}$ then velocity should increase at constant rate of $1\ \text{ms}^{-2}$.
Reason : For the given condition.

$$\frac{da}{dt} = 2\ \text{ms}^{-3} \qquad \therefore \qquad a = 2t.$$

7. **Assertion :** Average velocity can't be zero in case of uniform acceleration.
Reason : For average velocity to be zero, velocity should not remain constant.

8. **Assertion :** In displacement-time graph of a particle as shown in figure. Velocity of particle changes its direction at point A.



Reason : Sign of slope of s-t graph decides the direction of velocity.

9. **Assertion :** Displacement-time equation of two particles moving in a straight line are, $s_1 = 2t - 4t^2$ and $s_2 = -2t + 4t^2$. Relative velocity between the two will go on increasing.

Reason : If velocity and acceleration are of same sign then speed will increase.

10. **Assertion :** Acceleration of a moving particle can change its direction without any change in direction of velocity.

Reason : If the direction of change in velocity vector changes, the direction of acceleration vector also changes.

11. **Assertion :** A body is dropped from height h and another body is thrown vertically upwards with a speed $\sqrt{gh}$. They meet at height $h/2$.

Reason : The time taken by both the blocks in reaching the height $h/2$ is same.

12. **Assertion :** Two bodies of unequal masses m_1 and m_2 are dropped from the same height. If the resistance offered by air to the motion of both bodies is the same, the bodies will reach the earth at the same time.

Reason : For equal air resistance, acceleration of fall of masses m_1 and m_2 will be different.

Objective Questions (Level 2)

Single Correct Option

1. When a man moves down the inclined plane with a constant speed 5 ms^{-1} which makes an angle of 37° with the horizontal, he finds that the rain is falling vertically downward. When he moves up the same inclined plane with the same speed, he finds that the rain makes an angle $\theta = \tan^{-1}\left(\frac{7}{8}\right)$ with the horizontal. The speed of the rain is

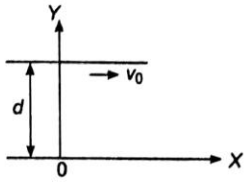
(a) $\sqrt{116}$ ms^{-1} (b) $\sqrt{32}$ ms^{-1} (c) 5 ms^{-1} (d) $\sqrt{73}$ ms^{-1}

2. Equation of motion of a body is $\frac{dv}{dt} = -4v + 8$, where v is the velocity in ms^{-1} and t is the time in second. Initial velocity of the particle was zero. Then

(a) the initial rate of change of acceleration of the particle is 8 ms^{-2}
(b) the terminal speed is 2 ms^{-1}
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

3. Two particles A and B are placed in gravity free space at (0, 0, 0) m and (30, 0, 0) m respectively. Particle A is projected with a velocity $(5\hat{i} + 10\hat{j} + 5\hat{k})$ ms^{-1}, while particle B is projected with a velocity $(10\hat{i} + 5\hat{j} + 5\hat{k})$ ms^{-1} simultaneously. Then

(a) they will collide at (10, 20, 10) m
(b) they will collide at (10, 10, 10) m
(c) they will never collide
(d) None of the above



4. Velocity of the river with respect to ground is given by v_0. Width of the river is d. A swimmer swims (with respect to water) perpendicular to the current with acceleration $a = 2t$ (where t is time) starting from rest from the origin O at $t = 0$. The equation of trajectory of the path followed by the swimmer is

(a) $y = \dfrac{x^3}{3v_0^3}$ (b) $y = \dfrac{x^2}{2v_0^2}$

(c) $y = \dfrac{x}{v_0}$ (d) $y = \sqrt{\dfrac{x}{v_0}}$

5. The relation between time t and displacement x is $t = \alpha x^2 + \beta x$ where α and β are constants. The retardation is

(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$

6. A street car moves rectilinearly from station A to the next station B with an acceleration varying according to the law $f = a - bx$, where a and b are constants and x is the distance from station A. The distance between the two stations and the maximum velocity are

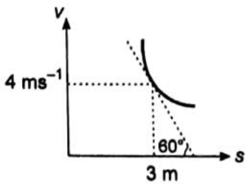
(a) $x = \dfrac{2a}{b}, v_{max} = \dfrac{a}{\sqrt{b}}$ (b) $x = \dfrac{b}{2a}, v_{max} = \dfrac{a}{b}$ (c) $x = \dfrac{a}{2b}, v_{max} = \dfrac{b}{\sqrt{a}}$ (d) $x = \dfrac{a}{b}, v_{max} = \dfrac{\sqrt{a}}{b}$

7. A particle of mass m moves on positive x-axis under the influence of force acting towards the origin given by $-kx^2\hat{\mathbf{i}}$. If the particle starts from rest at $x = a$, the speed it will attain when it crosses the origin is

(a) $\sqrt{\dfrac{k}{ma}}$ (b) $\sqrt{\dfrac{2k}{ma}}$ (c) $\sqrt{\dfrac{ma}{2k}}$ (d) $\sqrt{\dfrac{2ka^3}{m}}$

8. A particle is moving in X-Y plane such that $v_x = 4 + 4t$ and $v_y = 4t$. If the initial position of the particle is (1, 2). Then the equation of trajectory will be

(a) $y^2 = 4x$ (b) $y = 2x$ (c) $x^2 = \dfrac{y}{2}$ (d) None of these



9. A particle is moving along a straight line whose velocity-displacement graph is as shown in the figure. What is the magnitude of acceleration when displacement is 3 m ?

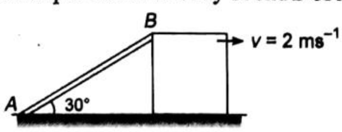
(a) $4\sqrt{3}\ ms^{-2}$ (b) $3\sqrt{3}\ ms^{-2}$

(c) $\sqrt{3}\ ms^{-2}$ (d) $\dfrac{4}{\sqrt{3}}\ ms^{-2}$

10. A particle is falling freely under gravity. In first t second it covers distance x_1 and in the next t second it covers distance x_2, then t is given by

(a) $\sqrt{\dfrac{x_2 - x_1}{g}}$ (b) $\sqrt{\dfrac{x_2 + x_1}{g}}$ (c) $\sqrt{\dfrac{2(x_2 - x_1)}{g}}$ (d) $\sqrt{\dfrac{2(x_2 + x_1)}{g}}$

11. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity 2 ms^{-1} towards right. The vertical component of velocity of end B of rod at the instant shown in figure is



(a) $\sqrt{3}\ ms^{-1}$ (b) $2\ ms^{-1}$ (c) $2\sqrt{3}\ ms^{-1}$ (d) $4\ ms^{-1}$

12. A thief in a stolen car passes through a police check post at his top speed of 90 kmh^{-1}. A motorcycle cop, reacting after 2 s, accelerates from rest at 5 ms^{-2}. His top speed being 108 kmh^{-1}. Find the maximum separation between policemen and thief.
(a) 112.5 m (b) 115 m (c) 116.5 m (d) None of these

13. Sachin (S) hits a ball along the ground with a speed u in a direction which makes an angle 30° with the line joining him and the fielder Prem (P). Prem runs to intercept the ball with a speed $\frac{2u}{3}$. At what angle θ should he run to intercept the ball ?

(a) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]$ (b) $\sin^{-1}\left[\frac{2}{3}\right]$ (c) $\sin^{-1}\left[\frac{3}{4}\right]$ (d) $\sin^{-1}\left[\frac{4}{5}\right]$

14. A car is travelling on a straight road. The maximum velocity the car can attain is 24 ms^{-1}. The maximum acceleration and deceleration it can attain are 1 ms^{-2} and 4 ms^{-2} respectively. The shortest time the car takes from rest to rest in a distance of 200 m is,
(a) 22.4 s (b) 30 s (c) 11.2 s (d) 5.6 s

15. A car is travelling on a road. The maximum velocity the car can attain is 24 ms^{-1} and the maximum deceleration is 4 ms^{-2}. If car starts from rest and comes to rest after travelling 1032 m in the shortest time of 56 s, the maximum acceleration that the car can attain is
(a) 6 ms^{-2} (b) 1.2 ms^{-2} (c) 12 ms^{-2} (d) 3.6 ms^{-2}

16. Two particles are moving along two long straight lines, in the same plane with same speed equal to 20 cms^{-1}. The angle between the two lines is 60° and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O and are moving towards O. Subsequently, the shortest distance between them will be
(a) 50 cm (b) $40\sqrt{2}$ cm (c) $50\sqrt{2}$ cm (d) $50\sqrt{3}$ cm

Passage : Q 17 to 20

An elevator without a ceiling is ascending up with an acceleration of 5 ms^{-2}. A boy on the elevator shoots a ball in vertical upward direction from a height of 2 m above the floor of elevator. At this instant the elevator is moving up with a velocity of 10 ms^{-1}, and floor of the elevator is at a height of 50 m from the ground. The initial speed of the ball is 15 ms^{-1} with respect to the elevator. Consider the duration for which the ball strikes the floor of elevator in answering following questions. ($g = 10\ ms^{-2}$)

17. The time in which the ball strikes the floor of elevator is given by
(a) 2.13 s (b) 2.0 s (c) 1.0 s (d) 3.12 s

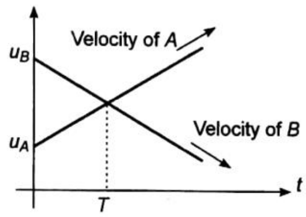
18. The maximum height reached by ball, as measured from the ground would be
(a) 73.65 m (b) 116.25 m (c) 82.56 m (d) 63.25 m

19. Displacement of ball with respect to ground during its flight would be
(a) 16.25 m (b) 8.76 m (c) 20.24 m (d) 30.56 m

20. The maximum separation between the floor of elevator and the ball during its flight would be
(a) 12 m (b) 15 m (c) 9.5 m (d) 7.5 m

Passage : Q 21 to Q 23

A situation is shown in which two objects A and B start their motion from same point in same direction. The graph of their velocities against time is drawn. u_A and u_B are the initial velocities of A and B respectively. T is the time at which their velocities become equal after start of motion. You cannot use the data of one question while solving another question of the same set. So all the questions are independent of each other.



21. If the value of T is 4 s, then the times after which A will meet B is
(a) 12 s (b) 6 s (c) 8 s (d) data insufficient

22. Let v_A and v_B be the velocities of the particles A and B respectively at the moment A and B meet after start of the motion. If $u_A = 5\,\text{ms}^{-1}$ and $u_B = 15\,\text{ms}^{-1}$, then the magnitude of the difference of velocities v_A and v_B is
(a) $5\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$ (c) $15\ \text{ms}^{-1}$ (d) data insufficient

23. After 10 s of the start of motion of both objects A and B, find the value of velocity of A if $u_A = 6\,\text{ms}^{-1}$, $u_B = 12\,\text{ms}^{-1}$ and at T velocity of A is $8\ \text{ms}^{-1}$ and $T = 4$ s
(a) $12\ \text{ms}^{-1}$ (b) $10\ \text{ms}^{-1}$ (c) $15\ \text{ms}^{-1}$ (d) None of these

More than One Correct Options

1. A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.
(a) The particle comes to rest at $t = \dfrac{2\sqrt{v_0}}{\alpha}$
(b) The particle will come to rest at infinity.
(c) The distance travelled by the particle before coming to rest is $\dfrac{2v_0^{3/2}}{\alpha}$
(d) The distance travelled by the particle before coming to rest is $\dfrac{2v_0^{3/2}}{3\alpha}$

2. At time $t = 0$, a car moving along a straight line has a velocity of $16\ \text{ms}^{-1}$. It slows down with an acceleration of $-0.5t\ \text{ms}^{-2}$, where t is in second. Mark the correct statement (s).
(a) The direction of velocity changes at $t = 8$ s
(b) The distance travelled in 4 s is approximately 58.67 m
(c) The distance travelled by the particle in 10 s is 94 m
(d) The speed of particle at $t = 10$ s is $9\ \text{ms}^{-1}$

3. An object moves with constant acceleration $\vec{\mathbf{a}}$. Which of the following expressions are also constant ?

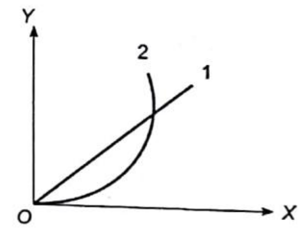
(a) $\dfrac{d|\vec{\mathbf{v}}|}{dt}$ (b) $\left|\dfrac{d\vec{\mathbf{v}}}{dt}\right|$ (c) $\dfrac{d(v^2)}{dt}$ (d) $\dfrac{d\left(\dfrac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}\right)}{dt}$

4. Ship A is located 4 km north and 3 km east of ship B. Ship A has a velocity of 20 kmh^{-1} towards the south and ship B is moving at 40 kmh^{-1} in a direction 37° north of east. X and Y-axes are along east and north directions, respectively
(a) Velocity of A relative to B is $-32\hat{i} - 44\hat{j}$
(b) Position of A relative to B as a function of time is given by
$$\vec{r}_{AB} = (3-32t)\hat{i} + (4-44t)\hat{j}$$
(c) Velocity of A relative to B is $32\hat{i} - 44\hat{j}$
(d) Position of A relative to B as a function of time is given by $(32t\hat{i} - 44t\hat{j})$

5. Starting from rest a particle is first accelerated for time t_1 with constant acceleration a_1 and then stops in time t_2 with constant retardation a_2. Let v_1 be the average velocity in this case and s_1 the total displacement. In the second case it is accelerating for the same time t_1 with constant acceleration $2a_1$ and come to rest with constant retardation a_2 in time t_3. If v_2 is the average velocity in this case and s_2 the total displacement, then
(a) $v_2 = 2v_1$ (b) $2v_1 < v_2 < 4v_1$ (c) $s_2 = 2s_1$ (d) $2s_1 < s_2 < 4s_1$

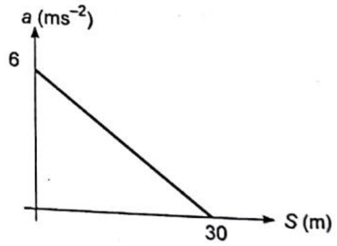
6. A particle is moving along a straight line. The displacement of the particle becomes zero in a certain time ($t > 0$). The particle does not undergo any collision.
(a) The acceleration of the particle may be zero always
(b) The acceleration of the particle may be uniform
(c) The velocity of the particle must be zero at some instant
(d) The acceleration of the particle must change its direction

7. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time according to law $F = \alpha t$, where α is a positive constant. From figure, which of the following statements are correct ?



(a) Curve 1 can be the plot of acceleration against time
(b) Curve 2 can be the plot of velocity against time
(c) Curve 2 can be the plot of velocity against acceleration
(d) Curve 1 can be the plot of displacement against time

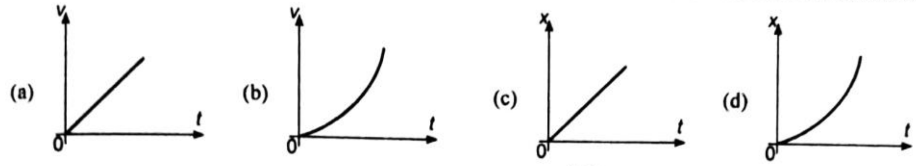
8. A train starts from rest at $S = 0$ and is subjected to acceleration as shown



(a) velocity at the end of 10 m displacement is 20 ms^{-1}
(b) velocity of the train at $S = 10$ m is 10 ms^{-1}
(c) The maximum velocity attained by train is $\sqrt{180}$ ms^{-1}
(d) The maximum velocity attained by the train is 15 ms^{-1}

9. For a moving particle which of the following options may be correct?
(a) $|\vec{v}_{av}| < v_{av}$ (b) $|\vec{v}_{av}| > v_{av}$ (c) $\vec{v}_{av} = 0$ but $v_{av} \neq 0$ (d) $\vec{v}_{av} \neq 0$ but $\vec{v}_{av} = 0$

10. Identify the correct graph representing the motion of a particle along a straight line with constant acceleration.



11. A man who can swim at a velocity v relative to water wants to cross a river of width b, flowing with a speed u.

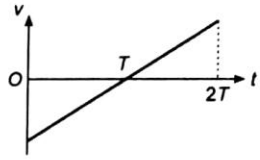
(a) The minimum time in which he can cross the river is $\frac{b}{v}$

(b) He can reach a point exactly opposite on the bank in time $t = \frac{d}{\sqrt{v^2 - u^2}}$ if $v > u$

(c) He cannot reach the point exactly opposite on the bank if $u > v$

(d) He cannot reach the point exactly opposite on the bank if $v > u$

12. The figure shows the velocity (v) of a particle plotted against time (t).



(a) The particle changes its direction of motion at some points

(b) The acceleration of the particle remains constant

(c) The displacement of the particle is zero

(d) The initial and final speeds of the particle are the same

13. The speed of a train increases at a constant rate α from zero to v and then remains constant for an interval and finally decreases to zero at a constant rate β. The total distance travelled by the train is l. The time taken to complete the journey is t. Then

(a) $t = \frac{l(\alpha + \beta)}{\alpha\beta}$

(b) $t = \frac{l}{v} + \frac{v}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

(c) t is minimum when $v = \sqrt{\frac{2l\alpha\beta}{(\alpha - \beta)}}$

(d) t is minimum when $v = \sqrt{\frac{2l\alpha\beta}{(\alpha + \beta)}}$

14. A particle moves in x-y plane and at time t is at the point $(t^2, t^3 - 2t)$, then which of the following is/are correct?

(a) At $t = 0$, particle is moving parallel to y-axis

(b) At $t = 0$, direction of velocity and acceleration are perpendicular.

(c) At $t = \sqrt{\frac{2}{3}}$, particle is moving parallel to x-axis.

(d) At $t = 0$, particle is at rest.

15. A car is moving with uniform acceleration along a straight line between two stops X and Y. Its speed at X and Y are $2\ \text{ms}^{-1}$ and $14\ \text{ms}^{-1}$, Then

(a) its speed at mid-point of XY is $10\ \text{ms}^{-1}$

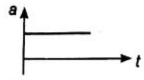
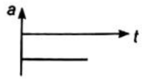
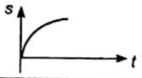
(b) its speed at a point A such that $XA : AY = 1:3$ is $5\ \text{ms}^{-1}$

(c) the time to go from X to the mid-point of XY is double of that to go from mid-point to Y

(d) the distance travelled in first half of the total time is half of the distance travelled in the second half of the time

Match the Columns

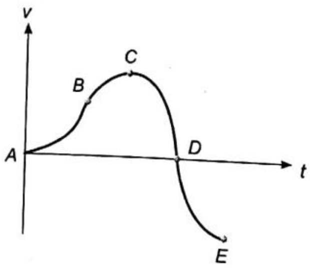
1. Match the following two columns.

Column I	Column II
(a) 	(p) speed must be increasing
(b) 	(q) speed must be decreasing
(c) 	(r) speed may be increasing
(d) 	(s) speed may be decreasing

2. Match the following two columns.

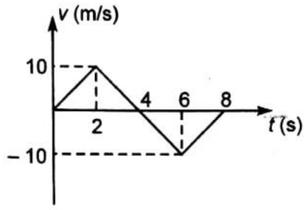
Column I	Column II
(a) $\vec{\mathbf{v}} = -2\hat{\mathbf{i}}, \vec{\mathbf{a}} = -4\hat{\mathbf{j}}$	(p) speed increasing
(b) $\vec{\mathbf{v}} = 2\hat{\mathbf{i}}, \vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$	(q) speed decreasing
(c) $\vec{\mathbf{v}} = -2\hat{\mathbf{i}}, \vec{\mathbf{a}} = +2\hat{\mathbf{i}}$	(r) speed constant
(d) $\vec{\mathbf{v}} = 2\hat{\mathbf{i}}, \vec{\mathbf{a}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$	(s) Nothing can be said

3. The velocity-time graph of a particle moving along X-axis is shown in figure. Match the entries of Column I with entries of Column II.



Column I	Column II
(a) For AB, particle is	(p) Moving in +ve X-direction with increasing speed
(b) For BC, particle is	(q) Moving in +ve X-direction with decreasing speed
(c) For CD, particle is	(r) Moving in −ve X-direction with increasing speed
(d) For DE, particle is	(s) Moving in +ve X-direction with decreasing speed

4. Corresponding to velocity-time graph in one dimensional motion of a particle is shown in figure, match the following two columns.



Column I	Column II
(a) Average velocity between zero sec and 4 s	(p) 10 SI units
(b) Average acceleration between 1 s and 4 s	(q) 2.5 SI units
(c) Average speed between zero sec and 6 s	(r) 5 SI units
(d) Rate of change of speed at 4 s	(s) None

5. A particle is moving along x-axis. Its x-coordinate varies with time as :
$x = -20 + 5t^2$

For the given equation match the following two columns

Column I	Column II
(a) Particle will cross the origin at	(p) zero sec
(b) At what time velocity and acceleration are equal	(q) 1 s
(c) At what time particle changes its direction of motion	(r) 2 s
(d) At what time velocity is zero	(s) None

6. x and y-coordinates of particle moving in x - y plane are,

$$x = 1 - 2t + t^2 \text{ and } y = 4 - 4t + t^2$$

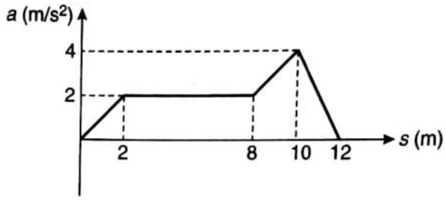
For the given situation match the following two columns.

Column I	Column II
(a) y-component of velocity when it crosses the y-axis	(p) + 2 SI unit
(b) x-component of velocity when it crosses the x-axis	(q) – 2 SI units
(c) Initial velocity of particle	(r) + 4 SI units
(d) Initial acceleration of particle	(s) None

Subjective Questions (Level 2)

1. To test the quality of a tennis ball, you drop it onto the floor from a height of 4.00 m. It rebounds to a height of 2.00 m. If the ball is in contact with the floor for 12.0 ms, what is its average acceleration during that contact? Take $g = 9.8 \text{ m/s}^2$.

2. The acceleration-displacement graph of a particle moving in a straight line is as shown in figure, initial velocity of particle is zero. Find the velocity of the particle when displacement of the particle is $s = 12$ m.



3. At the initial moment three points A, B and C are on a horizontal straight line at equal distances from one another. Point A begins to move vertically upward with a constant velocity v and point C vertically downward without any initial velocity but with a constant acceleration a. How should point B move vertically for all the three points to be constantly on one straight line. The points begin to move simultaneously.

4. A particle moves in a straight line with constant acceleration a. The displacements of particle from origin in times t_1, t_2 and t_3 are s_1, s_2 and s_3 respectively. If times are in AP with common difference d and displacements are in GP, then prove that $a = \dfrac{(\sqrt{s_1} - \sqrt{s_3})^2}{d^2}$.

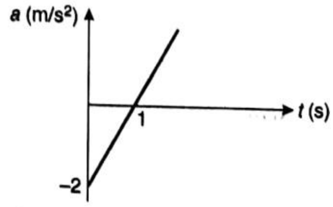
5. A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 14 m above the ground. If the elevator can have maximum acceleration of 0.2 m/s^2 and maximum deceleration of 0.1 m/s^2 and can reach a maximum speed of 2.5 m/s, determine the shortest time to make the lift, starting from rest and ending at rest.

6. To stop a car, first you require a certain reaction time to begin braking; then the car slows under the constant braking deceleration. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h and 24.4 m when its initial speed is 48.3 km/h. What are :
   (a) your reaction time and
   (b) the magnitude of the deceleration?

7. An elevator without a ceiling is ascending with a constant speed of 10 m/s. A boy on the elevator shoots a ball directly upward, from a height of 2.0 m above the elevator floor. At this time the elevator floor is 28 m above the ground. The initial speed of the ball with respect to the elevator is 20 m/s. (Take $g = 9.8 \text{ m/s}^2$)
   (a) What maximum height above the ground does the ball reach?
   (b) How long does the ball take to return to the elevator floor?

8. A particle moves along a straight line and its velocity depends on time as $v = 3t - t^2$. Here, v is in m/s and t in second. Find :
   (a) average velocity and
   (b) average speed for first five seconds.

9. The acceleration of particle varies with time as shown.



(a) Find an expression for velocity in terms of t.

(b) Calculate the displacement of the particle in the interval from $t = 2$ s to $t = 4$ s. Assume that $v = 0$ at $t = 0$.

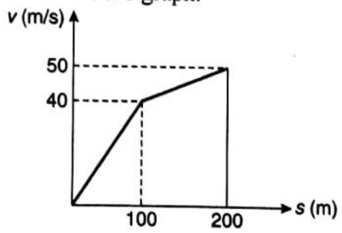
10. A man wishes to cross a river of width 120 m by a motorboat. His rowing speed in still water is 3 m/s and his maximum walking speed is 1 m/s. The river flows with velocity of 4 m/s.

(a) Find the path which he should take to get to the point directly opposite to his starting point in the shortest time.

(b) Also, find the time which he takes to reach his destination.

11. The current velocity of river grows in proportion to the distance from its bank and reaches the maximum value v_0 in the middle. Near the banks the velocity is zero. A boat is moving along the river in such a manner that the boatman rows his boat always perpendicular to the current. The speed of the boat in still water is u. Find the distance through which the boat crossing the river will be carried away by the current, if the width of the river is c. Also determine the trajectory of the boat.

12. The v-s graph for an airplane travelling on a straight runway is shown. Determine the acceleration of the plane at $s = 50$ m and $s = 150$ m. Draw the a-s graph.



13. A river of width a with straight parallel banks flows due north with speed u. The points O and A are on opposite banks and A is due east of O. Coordinate axes Ox and Oy are taken in the east and north directions respectively. A boat, whose speed is v relative to water, starts from O and crosses the river. If the boat is steered due east and u varies with x as : $u = x(a - x)\dfrac{v}{a^2}$. Find :

(a) equation of trajectory of the boat,

(b) time taken to cross the river,

(c) absolute velocity of boatman when he reaches the opposite bank,

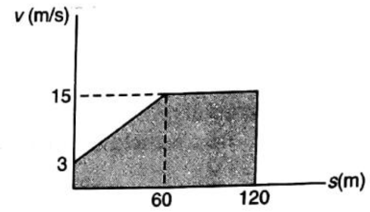
(d) the displacement of boatman when he reaches the opposite bank from the initial position.

14. A river of width ω is flowing with a uniform velocity v. A boat starts moving from point P also with velocity v relative to the river. The direction of resultant velocity is always perpendicular to the line joining boat and the fixed point R. Point Q is on the opposite side of the river. P, Q and R are in a straight line. If $PQ = QR = \omega$, find :

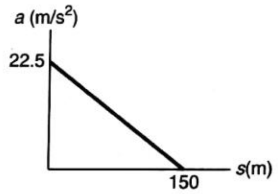
(a) the trajectory of the boat,

(b) the drifting of the boat and
(c) the time taken by the boat to cross the river.

15. The v-s graph describing the motion of a motorcycle is shown in figure. Construct the a-s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 120$ m. Given $\ln 5 = 1.6$



16. The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has travelled 60 m.



17. A particle leaves the origin with an initial velocity $\vec{\mathbf{v}} = (3.00\hat{\mathbf{i}})$ m/s and a constant acceleration $\vec{\mathbf{a}} = (-1.00\hat{\mathbf{i}} - 0.500\hat{\mathbf{j}})$ m/s^2. When the particle reaches its maximum x coordinate, what are
(a) its velocity and
(b) its position vector?

18. The speed of a particle moving in a plane is equal to the magnitude of its instantaneous velocity, $v = |\vec{\mathbf{v}}| = \sqrt{v_x^2 + v_y^2}$.

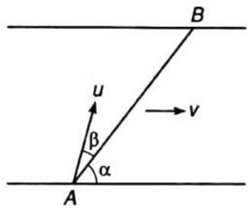
(a) Show that the rate of change of the speed is $dv/dt = (v_x a_x + v_y a_y)/\sqrt{v_x^2 + v_y^2}$.

(b) Show that the rate of change of speed can be expressed as $dv/dt = \vec{\mathbf{v}} \cdot \vec{\mathbf{a}}/v$, and use this result to explain why dv/dt is equal to a_t the component of $\vec{\mathbf{a}}$ that is parallel to $\vec{\mathbf{v}}$.

19. A man with some passengers in his boat, starts perpendicular to flow of river 200 m wide and flowing with 2 m/s. Speed of boat in still water is 4 m/s. When he reaches half the width of river the passengers asked him that they want to reach the just opposite end from where they have started.
(a) Find the direction due which he must row to reach the required end.
(b) How many times more time, it would take to that if he would have denied the passengers?

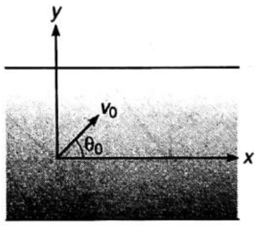
20. A child in danger of drowning in a river is being carried downstream by a current that flows uniformly at a speed of 2.5 km/h. The child is 0.6 km from shore and 0.8 km upstream of a boat landing when a rescue boat sets out. If the boat proceeds at its maximum speed of 20 km/h with respect to the water, what angle does the boat velocity v make with the shore? How long will it take boat to reach the child?

21. A launch plies between two points A and B on the opposite banks of a river always following the line AB. The distance S between points A and B is 1200 m. The velocity of the river current $v = 1.9$ m/s is constant over the entire width of the river. The line AB makes an angle $\alpha = 60°$ with the direction of the current. With what velocity u and at what angle β to the line AB should the launch move to cover the distance AB and back in a time $t = 5$ min? The angle β remains the same during the passage from A to B and from B to A.

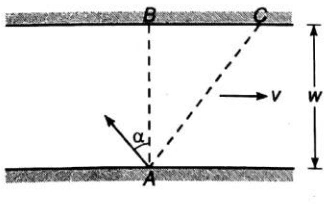


22. The slopes of wind screen of two cars are $\alpha_1 = 30°$ and $\alpha_2 = 15°$ respectively. At what ratio $\frac{v_1}{v_2}$ of the velocities of the cars will their drivers see the hail stones bounced back by the wind screen on their cars in vertical direction? Assume hail stones fall vertically downwards and collisions to be elastic.

23. A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity v_0 as shown. If the liquid develops a frictional or drag resistance on the projectile which is proportional to its velocity, *i.e.*, $F = -kv$ where k is a positive constant, determine the x and y components of its velocity at any instant. Also find is the maximum distance x_{max} that it travels?



24. A man in a boat crosses a river from point A. If he rows perpendicular to the banks he reaches point C $(BC = 120\,\text{m})$ in 10 min. If the man heads at a certain angle α to the straight line AB (AB is perpendicular to the banks) against the current he reaches point B in 12.5 min. Find the width of the river w, the rowing velocity u, the speed of the river current v and the angle α. Assume the velocity of the boat relative to water to be constant and the same magnitude in both cases.



ANSWERS

Introductory Exercise 3.1

1. True **2.** g (downwards) **3.** $\frac{\pi}{15}$ cm/s, $\frac{2\sqrt{2}}{15}$ cm/s

4. (a) Yes, in uniform circular motion (b) No, yes (projectile motion), yes

5. (a) 25.13 s (b) 1 cm/s, 0.9 cm/s, 0.23 cm/s^2 **6.** 5.2 m/s

Introductory Exercise 3.2

1. acceleration **2.** $s_t = \left\{ut + \frac{1}{2}at^2\right\} - \left\{u(t-1) + \frac{1}{2}a(t-1)^2\right\} = (u \cdot 1) + (at)(1) - \frac{1}{2}a(1)^2$

3. Yes, in simple harmonic motion **4.** $t^{7/4}, t^{-1/4}$ **5.** 60 m, 100 m **6.** 2 s, 6 s, $2(2+\sqrt{7})$ s

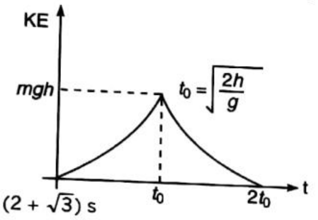
7. $u + \frac{1}{2}at$ **8.** True **9.** 25 m/s (downwards) **10.** (a) 1 m/s^2 (b) 43.5 m **11.** $2\sqrt{7}$ m/s, $4\sqrt{3}$ m

12. $(2\hat{j})$ m/s^2, $(2\hat{i} + \hat{j})$ m, yes **13.** (a) $x^2 = 4y$ (b) $(2\hat{i} + 2\hat{j})$ units (c) $(2\hat{j})$ units

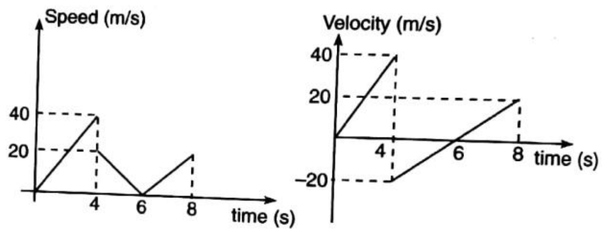
Introductory Exercise 3.3

1. v_{t_1}, a_{t_1} and a_{t_2} are positive while v_{t_2} is negative

2.



3.



4. $(2+\sqrt{3})$ s

Introductory Exercise 3.4

1. zero **2.** straight line passing through origin **3.** (a) 40 s (b) 80 m

4. (a) $\sin^{-1}\left(\frac{1}{15}\right)$ east of the line AB (b) 50 min **5.** 8 m

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Subjective Questions (Level 1)

1. (a) 100 km (b) 66.67 kmh^{-1} **2.** (a) 10 ms^{-1} (b) 22 ms^{-1} **3.** (a) 16.67 ms^{-1} (b) 10 ms^{-1}(downwards)

4. $T = 4t_0$ **5.** (a) zero (b) 8 ms^{-1} (c) 8 ms^{-1} **6.** (a) $\frac{21\sqrt{3}}{44}$ ms^{-1} (b) $(\sqrt{3}/2)$ ms^{-2}

7. (a) $(1.25\hat{i} + 0.5\hat{j})$ ms^{-1}(b) $(-0.5\hat{i} + \hat{j})$ ms^{-2} (c) No **8.** 1.5 s **9.** $\frac{v_0}{g} + \frac{t_0}{2}$ **10.** 45 m

11. $(3.414)\,t_0$ **12.** (a) 16.25 m (b) 1.8 s **13.** (a) 5 ms^{-1} (b)1.67 ms^{-2} (c) 7.5 m

15. 0.2 ms^{-2}, 0.8 ms^{-1}

16. $10\sqrt{5}$ m at $\cos^{-1}(2)$ from east towards north, $10\sqrt{2}$ ms^{-1} at 45° from east towards north.

17. (a) $(8\hat{i} - 8\hat{j})$ ms^{-1} (b) (18 m , – 4m) **18.** (a) $(10\hat{i} - 2\hat{j})$ ms^{-1} (b) (12 m, – 6 m)

19. (a) 45 m (b) 22 ms^{-1} **20.** 20.6 ms^{-1}, $\tan^{-1}(4)$ **21.** (a) $x = 2.0$ m (b) zero (c) 26 ms^{-2}

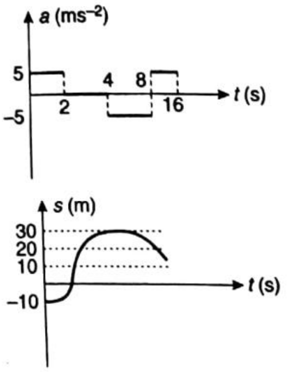
22. – 30 ms^{-2} **23.** 42 ms^{-2} **24.** (a) 0.833 ms^{-1} (b) – 9.17 ms^{-1} **25.** 14.125 m, 1.75 ms^{-1}, 4.03 ms^{-1}

26. $0.603\ ms^{-2}$ **27.** $(-20/3)\ ms^{-2}$

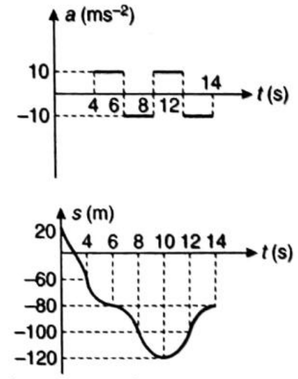
28. (a) positive, positive, positive, negative (b) positive, zero, negative, negative

29. (a) In M_1 and M_3 magnitude is increasing, in M_2 and M_4 magnitude is decreasing
(b) $P \to M_1$; $Q \to M_2$; $R \to M_3$; $S \to M_4$

30.



31.



32. (a) $(50/7)\ ms^{-1}$ (b) $10\ ms^{-1}$ **33.** (a) $5\ ms^{-2}$ (b) $90\ ms^{-1}$

34. (a) $5\ ms^{-2}$, zero, $5\ ms^{-2}$ (b) $s = 30$ m (c) (i) $s = 10 + 2.5t^2$ (ii) $s = 40 + 10(t-4) - 2.5(t-4)^2$

35. (a) zero (b) $25\ ms^{-1}$ (c) $-25\ ms^{-1}$ (d) 0.8 s **36.** 36 s, No

37. (a) 3.65 s, at 12.30 m level (b) $19.8\ ms^{-1}$ (downwards)

38. (a) 7.39 s (b) 35.5 m (c) automobile $25.9\ ms^{-1}$, truck $16.2\ ms^{-1}$ **40.** (a) 10 s (b) 50 m

41. $45° - \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right) \approx 24.3°$ **42.** (a) at an angle $\theta = \sin^{-1}(0.4)$ west of north (b) $\frac{10}{\sqrt{21}}\ h$

Objective Questions (Level 1)

1.(b)	**2.**(c)	**3.**(d)	**4.**(c)	**5.**(c)	**6.**(a)	**7.**(b)	**8.**(b)	**9.**(c)	**10.**(a)
11.(c)	**12.**(c)	**13.**(a)	**14.**(a)	**15.**(b)	**16.**(c)	**17.**(c)	**18.**(c)	**19.**(b)	**20.**(b)
21.(a)	**22.**(c)	**23.**(b)	**24.**(d)	**25.**(a)	**26.**(a)	**27.**(b)	**28.**(b)	**29.**(d)	**30.**(b)
31.(b)	**32.**(c)	**33.**(b)	**34.**(b)	**35.**(b)	**36.**(b)	**37.**(b)	**38.**(d)		

JEE Corner

Assertion and Reason

1.(d)	**2.**(d)	**3.**(a)	**4.**(d)	**5.**(c)	**6.**(d)	**7.**(d)	**8.**(d)	**9.**(d)	**10.**(a,b)
11.(a)	**12.**(d)								

Objective Questions (Level 2)

1.(b)	**2.**(b)	**3.**(c)	**4.**(a)	**5.**(a)	**6.**(a)	**7.**(d)	**8.**(d)	**9.**(a)	**10.**(a)
11.(c)	**12.**(a)	**13.**(c)	**14.**(a)	**15.**(b)	**16.**(d)	**17.**(a)	**18.**(c)	**19.**(d)	**20.**(c)
21.(c)	**22.**(b)	**23.**(d)							

More than One Correct Options

1.(a,d)	**2.**(all)	**3.**(b)	**4.**(a,b)	**5.**(a,d)	**6.**(b,c)	**7.**(a,b)	**8.**(b,c)	**9.**(a,c)	**10.**(a,d)
11.(a,b,c)	**12.**(all)	**13.**(b,d)	**14.**(a,b,c)	**15.**(a,c)					

Match the Columns

1. (a) → r,s (b) → r,s (c) → p (d) → q
2. (a) → p (b) → p (c) → q (d) → q
3. (a) → p (b) → p (c) → q (d) → r
4. (a) → (r) (b) → (s) (c) → (r) (d) → (r)
5. (a) → (r) (b) → (q) (c) → (s) (d) → (p)
6. (a) → (q) (b) → (p) (c) → (s) (d) → (s)

Subjective Questions (Level 2)

1. 1.26×10^3 ms^{-2} (upward) **2.** $4\sqrt{3}$ ms^{-1}

3. B moves up with initial velocity $\frac{v}{2}$ and downward acceleration $-\frac{a}{2}$ **5.** 20.5 s

6. (a) 0.74 s (b) 6.2 ms^{-2} **7.** (a) 76 m (b) 4.2 s **8.** (a) – 0.833 ms^{-1} (b) 2.63 ms^{-1}

9. (a) $v = t^2 - 2t$ (b) 6.67 m **10.** (a) 90° + $\sin^{-1}(3/5)$ from river current (b) 2 min 40 s

11. $\frac{cv_0}{2u}$, $y^2 = \frac{ucx}{v_0}$ **12.** 8 ms^{-2}, 4.5 ms^{-2} **13.** (a) $y = \frac{x^2}{2a} - \frac{x^3}{3a^2}$ (b) $\frac{a}{v}$ (c) v (due east) (d) $a\hat{i} + \frac{a\hat{j}}{6}$

14. (a) circle (b) $\sqrt{3}\,\omega$ (c) $\frac{1.317\,\omega}{v}$ **15.** 12.0 s **16.** 46.47 ms^{-1}

17. (a) $(-1.5\hat{j})$ ms^{-1} (b) $(4.5\hat{i} - 2.25\hat{j})$ m

19. (a) At an angle (90° + 2 θ) from river current (upstream). Here : $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ (b) $\frac{4}{3}$

20. 37°, 3 min **21.** $u = 8$ ms^{-1}, $\beta = 12°$ **22.** $\frac{v_1}{v_2} = 3$

23. $v_x = v_0 \cos\theta_0\, e^{-kt/m}$, $v_y = \frac{m}{k}\left[\left(\frac{k}{m} v_0 \sin\theta_0 + g\right) e^{-\frac{kt}{m}} - g\right]$ $X_m = \frac{mv\cos\theta}{k}$

24. 200 m, 20 m min^{-1}, 12 m min^{-1}, 36°50′.

Chapter – 4 Projectile Motion

4

Projectile Motion

Chapter Contents

4.1 Projectile Motion

If a constant force (and hence constant acceleration) acts on a particle at an angle θ ($\neq 0°$ or $180°$) with the direction of its initial velocity ($\neq$ zero), the path followed by the particle is a parabola and the motion of the particle is called projectile motion. Projectile motion is a two dimensional motion, *i.e.*, motion of the particle is constrained in a plane.

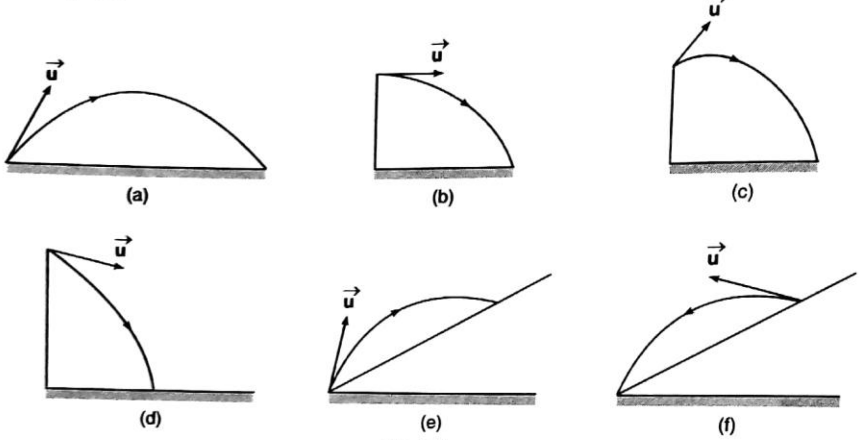


Fig. 4.1

When a particle is thrown obliquely near the earth's surface it moves in a parabolic path, provided the particle remains close to the surface of earth and the air resistance is negligible. This is an example of projectile motion. The different types of projectile motion we come across are shown in Fig. 4.1.

In all the above cases acceleration of the particle is g downwards.

Let us first make ourselves familiar with certain terms used in projectile motion.

Fig. 4.2 shows a particle projected from the point O with an initial velocity u at an angle α with the horizontal. It goes through the highest point A and falls at B on the horizontal surface through O. The point O is called the **point of projection**, the angle α is called the **angle of projection**, the distance OB is called the **horizontal range (R)** or simply range and the vertical height AC is called the **maximum height (H)**. The total time taken by the particle in describing the path OAB is called the **time of flight (T)**.

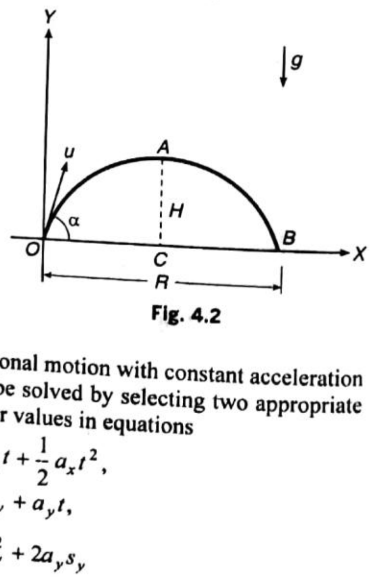


Fig. 4.2

As we have already discussed, projectile motion is a two dimensional motion with constant acceleration (normally g). Problems related to projectile motion of any type can be solved by selecting two appropriate mutually perpendicular directions (x and y) and substituting the proper values in equations

$$v_x = u_x + a_x t, \qquad s_x = u_x t + \frac{1}{2} a_x t^2,$$

$$v_x^2 = u_x^2 + 2a_x s_x, \qquad v_y = u_y + a_y t,$$

$$s_y = u_y t + \frac{1}{2} a_y t^2 \quad \text{and} \quad v_y^2 = u_y^2 + 2a_y s_y$$

In any problem of projectile motion we usually follow the three steps given below :

Step 1. Select two mutually perpendicular directions x and y.

Step 2. Write down the proper values of u_x, a_x, u_y and a_y with sign.

Step 3. Apply those equations from the six listed above which are required in the problem.

What should be the directions x and y or which equations are to be used, this you will learn after solving some problems of projectile motion. Using the above methodology let us first prove the three standard results of time of flight (T), horizontal range (R) and the maximum height (H).

Time of Flight (T)

Refer Fig. 4.2. Here, x and y-axes are in the directions shown in figure. Axis x is along horizontal direction and axis y is vertically upwards. Thus,

$$u_x = u\cos\alpha, \quad u_y = u\sin\alpha, a_x = 0 \quad \text{and} \quad a_y = -g$$

At point B, $s_y = 0$. So, applying

$$s_y = u_y t + \frac{1}{2}a_y t^2, \text{ we have}$$

$$0 = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$\therefore \qquad t = 0, \ \frac{2u\sin\alpha}{g}$$

Both $t = 0$ and $t = \dfrac{2u\sin\alpha}{g}$ correspond to the situation where $s_y = 0$. The time $t = 0$ corresponds to point O and time $t = \dfrac{2u\sin\alpha}{g}$ corresponds to point B. Thus, time of flight of the projectile is :

$$T = t_{OAB} \quad \text{or} \quad T = \frac{2u\sin\alpha}{g}$$

Horizontal Range (R)

Distance OB is the range R. This is also equal to the displacement of particle along x-axis in time $t = T$. Thus, applying $s_x = u_x t + \frac{1}{2}a_x t^2$, we get

$$R = (u\cos\alpha)\left(\frac{2u\sin\alpha}{g}\right) + 0$$

as $$a_x = 0 \quad \text{and} \quad t = T = \frac{2u\sin\alpha}{g}$$

$$\therefore \qquad R = \frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{u^2\sin 2\alpha}{g}$$

or $$R = \frac{u^2\sin 2\alpha}{g}$$

Here, two points are important regarding the range of a projectile.

(i) Range is maximum where $\sin 2\alpha = 1$ or $\alpha = 45°$ and this maximum range is:

$$R_{max} = \frac{u^2}{g} \qquad (\text{at } \alpha = 45°)$$

(ii) For given value of u range at α and range at $90° - \alpha$ are equal although times of flight and maximum heights may be different. Because

$$R_{90°-\alpha} = \frac{u^2 \sin 2(90° - \alpha)}{g} = \frac{u^2 \sin (180° - 2\alpha)}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g} = R_\alpha$$

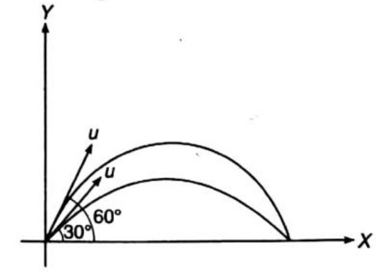


Fig. 4.3

So, $R_{30°} = R_{60°}$ or $R_{20°} = R_{70°}$

This is shown in Fig. 4.3.

Maximum Height (H)

At point A vertical component of velocity becomes zero, *i.e.*, $v_y = 0$. Substituting the proper values in

$$v_y^2 = u_y^2 + 2a_y s_y$$

we have, $$0 = (u \sin \alpha)^2 + 2(-g)(H)$$

$\therefore$ $$H = \frac{u^2 \sin^2 \alpha}{2g}$$

Check Points

- As we have seen in the above derivations that $a_x = 0$, *i.e.*, motion of the projectile in horizontal direction is uniform. Hence, horizontal component of velocity $u \cos \alpha$ does not change during its motion.
- Motion in vertical direction is first retarded then accelerated in opposite direction. Because u_y is upwards and a_y is downwards. Hence, vertical component of its velocity first decreases from O to A and then increases from A to B. This can be shown as in Fig. 4.4.

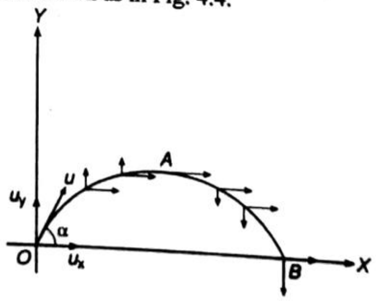


Fig. 4.4

$R = \frac{u^2 \sin 2\theta}{g}$; $\frac{2u^2 \sin\theta \cos\theta}{g}$, $\frac{2u^2 \cos^2\theta}{g}$

- The coordinates and velocity components of the projectile at time t are

$$x = s_x = u_x t = (u \cos \alpha) t$$

$$y = s_y = u_y t + \frac{1}{2} a_y t^2 = (u \sin \alpha) t - \frac{1}{2} g t^2$$

$$v_x = u_x = u \cos \alpha$$

and $$v_y = u_y + a_y t = u \sin \alpha - g t$$

Therefore, speed of projectile at time t is $v = \sqrt{v_x^2 + v_y^2}$ and the angle made by its velocity vector with positive x-axis is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

- Equation of trajectory of projectile

$$x = (u \cos \alpha) t \qquad \therefore \quad t = \frac{x}{u \cos \alpha}$$

Substituting this value of t in, $y = (u \sin \alpha) t - \frac{1}{2} g t^2$, we get

$$y = x \tan \alpha - \frac{g x^2}{2u^2 \cos^2 \alpha}$$

$y = x \tan\theta - \frac{g x^2}{2u^2 \cos^2\theta}$

$$= x \tan \alpha - \frac{g x^2}{2u^2} \sec^2 \alpha$$

$$= x \tan \alpha - \frac{g x^2}{2u^2} (1 + \tan^2 \alpha)$$

These are the standard equations of trajectory of a projectile. The equation is quadratic in x. This is why the path of a projectile is a parabola. The above equation can also be written in terms of range (R) of projectile as

$$y = x\left(1 - \frac{x}{R}\right) \tan \alpha$$

Now, let us take few examples based on the above theory.

- Projectile motion is a two dimensional motion with constant acceleration (g). So, we can use $\vec{v} = \vec{u} + \vec{a} t$, $\vec{s} = \vec{u} t + \frac{1}{2} \vec{a} t^2$, etc., in projectile motion as well. Here,

$$\vec{u} = u \cos \alpha \hat{i} + u \sin \alpha \hat{j} \quad \text{and} \quad \vec{a} = -g \hat{j}$$

Now, suppose we want to find velocity at time t.

$$\vec{v} = \vec{u} + \vec{a} t$$

$$= (u \cos \alpha \hat{i} + u \sin \alpha \hat{j}) - g t \hat{j}$$

or $$\vec{v} = u \cos \alpha \hat{i} + (u \sin \alpha - g t) \hat{j}$$

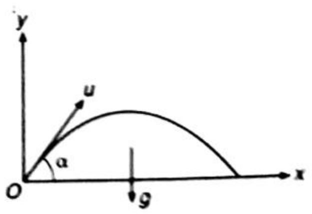


Fig. 4.5

Similarly, displacement at time t will be

$$\vec{\mathbf{s}} = \vec{\mathbf{u}}\,t + \frac{1}{2}\,\vec{\mathbf{a}}\,t^2$$

$$= (u\cos\alpha\hat{\mathbf{i}} + u\sin\alpha\hat{\mathbf{j}})\,t - \frac{1}{2}gt^2\,\hat{\mathbf{j}}$$

$$= ut\cos\alpha\hat{\mathbf{i}} + \left(ut\sin\alpha - \frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

Sample Example 4.1 *Find the angle of projection of a projectile for which the horizontal range and maximum height are equal.*

Solution Given, $R = H$

$\therefore$
$$\frac{u^2\sin 2\alpha}{g} = \frac{u^2\sin^2\alpha}{2g}$$

or
$$2\sin\alpha\cos\alpha = \frac{\sin^2\alpha}{2}$$

or
$$\frac{\sin\alpha}{\cos\alpha} = 4 \quad \text{or} \quad \tan\alpha = 4$$

$\therefore$
$$\alpha = \tan^{-1}(4)$$

Sample Example 4.2 *Prove that the maximum horizontal range is four times the maximum height attained by the projectile; when fired at an inclination so as to have maximum horizontal range.*

Solution For $\theta = 45°$, the horizontal range is maximum and is given by

$$R_{max} = \frac{u^2}{g}$$

Maximum height attained

$$H_{max} = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

or
$$R_{max} = 4\,H_{max}$$

$\sin 45 = \frac{1}{\sqrt{2}}$, $\sin^2 45 = \frac{1}{2}$

Proved.

Sample Example 4.3 *There are two angles of projection for which the horizontal range is the same. Show that the sum of the maximum heights for these two angles is independent of the angle of projection.*

Solution There are two angles of projection α and $90° - \alpha$ for which the horizontal range R is same.

Now,
$$H_1 = \frac{u^2\sin^2\alpha}{2g}$$

and
$$H_2 = \frac{u^2\sin^2(90° - \alpha)}{2g} = \frac{u^2\cos^2\alpha}{2g}$$

Therefore,
$$H_1 + H_2 = \frac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha) = \frac{u^2}{2g}$$

Clearly the sum of the heights for the two angles of projection is independent of the angles of projection.

Sample Example 4.4 *Show that there are two values of time for which a projectile is at the same height. Also show mathematically that the sum of these two times is equal to the time of flight.*

Solution For vertically upward motion of a projectile,

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

or $$\frac{1}{2}gt^2 - (u\sin\alpha)t + y = 0$$

This is a quadratic equation in t. Its roots are

$$t_1 = \frac{u\sin\alpha - \sqrt{u^2\sin^2\alpha - 2gy}}{g}$$

and $$t_2 = \frac{u\sin\alpha + \sqrt{u^2\sin^2\alpha - 2gy}}{g}$$

$\therefore$ $$t_1 + t_2 = \frac{2u\sin\alpha}{g} = T \quad \text{(time of flight of the projectile)}$$

Sample Example 4.5 *A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find:*

(a) the time taken by the projectile to reach the ground,

(b) the distance of the point where the particle hits the ground from foot of the hill and

(c) the velocity with which the projectile hits the ground.

$(g = 9.8\ m/s^2)$

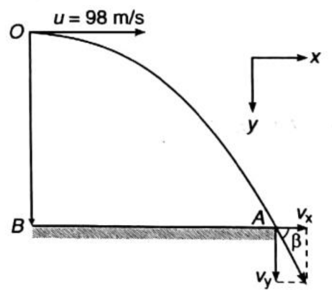


Fig. 4.6

Solution In this problem we cannot apply the formulae of R, H and T directly. We will have to follow the three steps discussed in the theory. Here, it will be more convenient to choose x and y directions as shown in figure.

Here, $u_x = 98$ m/s, $a_x = 0$, $u_y = 0$ and $a_y = g$

(a) At A, $s_y = 490$ m. So, applying

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$\therefore$ $$490 = 0 + \frac{1}{2}(9.8)t^2$$

$\therefore$ $$t = 10\,\text{s}$$

(b) $$BA = s_x = u_x t + \frac{1}{2}a_x t^2$$

or $$BA = (98)(10) + 0$$

or $$BA = 980\,\text{m}$$

(c) $$v_x = u_x = 98 \text{ m/s}$$

$$v_y = u_y + a_y t = 0 + (9.8)(10) = 98 \text{ m/s}$$

$\therefore$ $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(98)^2 + (98)^2} = 98\sqrt{2} \text{ m/s}$$

and $$\tan \beta = \frac{v_y}{v_x} = \frac{98}{98} = 1$$

$\therefore$ $$\beta = 45°$$

Thus, the projectile hits the ground with a velocity $98\sqrt{2}$ m/s at an angle of $\beta = 45°$ with horizontal as shown in Fig. 4.6.

Sample Example 4.6 *A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. Take $g = 9.8\ m/s^2$.*

Solution As shown in the figure of Sample Example 4.5.

$$u_y = 0 \quad \text{and} \quad a_y = g = 9.8 \text{ m/s}^2 \quad s_y = u_y t + \frac{1}{2} a_y t^2$$

$$= 0 \times 3 + \frac{1}{2} \times 9.8 \times (3)^2$$

$$= 44.1 \text{ m}$$

Further, $$v_y = u_y + a_y t = 0 + (9.8)(3)$$

$$= 29.4 \text{ m/s}$$

As the resultant velocity v makes an angle of 45° with the horizontal, so

$$\tan 45° = \frac{v_y}{v_x} \quad \text{or} \quad 1 = \frac{29.4}{v_x}$$

$$v_x = 29.4 \text{ m/s}$$

Therefore, the speed with which the body was projected (horizontally) is 29.4 m/s.

Introductory Exercise 4.1

1. Projectile motion is a 3-dimensional motion. Is this statement true or false.
2. Projectile motion (at low speeds) is uniformly accelerated motion. Is this statement true or false.
3. A particle is projected with speed u at angle θ with vertical. Find :
   (a) time of flight (b) maximum height
   (c) range (d) maximum range and corresponding value of θ.
4. A particle is projected from ground with velocity $40\sqrt{2}$ m/s at 45°. Find :
   (a) velocity and
   (b) displacement of the particle after 2 s. ($g = 10\ m/s^2$)
5. A particle is projected from ground with velocity $20\sqrt{2}$ m/s at 45°. At what time particle is at height 15 m from ground? ($g = 10\ m/s^2$)

6. A particle is projected from ground with velocity 40 m/s at 60° with horizontal. Find speed of particle when its velocity is making 45° with horizontal. Also find the times (s) when it happens. ($g = 10\ m/s^2$)

7. What is the average velocity of a particle projected from the ground with speed u at an angle α with the horizontal over a time interval from beginning till it strikes the ground again?

8. What is the change in velocity in the above question?

9. Under what conditions the formulae of range, time of flight and maximum height can be applied directly in case of a projectile motion?

10. A body is projected up such that its position vector varies with time as $\vec{r} = \{3t\hat{i} + (4t - 5t^2)\hat{j}\}$ m. Here, t is in seconds.
Find the time and x-coordinate of particle when its y-coordinate is zero.

11. A particle is projected at an angle 60° with horizontal with a speed $v = 20$ m/s. Taking $g = 10\ m/s^2$. Find the time after which the speed of the particle remains half of its initial speed.

4.2 Projectile Motion in Inclined Plane

Here, two cases arise. One is up the plane and the other is down the plane. Let us discuss both the cases separately.

(i) Up the Plane : In this case direction x is chosen up the plane and direction y is chosen perpendicular to the plane. Hence,

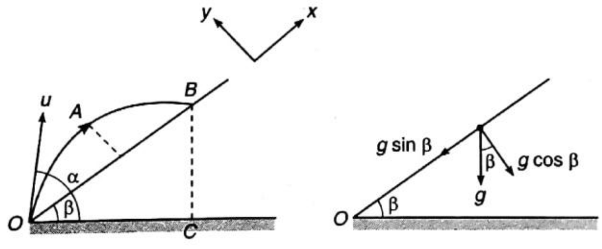


Fig. 4.7

$$u_x = u\cos(\alpha - \beta), \qquad a_x = -g\sin\beta$$
$$u_y = u\sin(\alpha - \beta) \text{ and } a_y = -g\cos\beta$$

Now, let us derive the expressions for time of flight (T) and range (R) along the plane.

Time of Flight

At point B displacement along y-direction is zero. So, substituting the proper values in $s_y = u_y t + \frac{1}{2}a_y t^2$, we get

$$0 = ut\sin(\alpha - \beta) + \frac{1}{2}(-g\cos\beta)t^2$$

$$\therefore \qquad t = 0 \text{ and } \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

$t = 0$, corresponds to point O and $t = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$ corresponds to point B. Thus,

$$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

Note *Substituting* $\beta = 0$, *in the above expression, we get* $T = \frac{2u \sin \alpha}{g}$ *which is quite obvious because* $\beta = 0$ *is the situation shown in Fig. 4.8.*

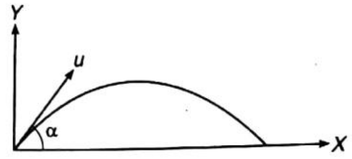


Fig. 4.8

Range

Range (R) or the distance OB can be found by following two methods:

Method 1. Horizontal component of initial velocity is:

$$u_H = u \cos \alpha$$

$$\therefore \quad OC = u_H T \qquad \text{(as } a_H = 0\text{)}$$

$$= \frac{(u \cos \alpha)\, 2u \sin (\alpha - \beta)}{g \cos \beta}$$

$$= \frac{2u^2 \sin (\alpha - \beta) \cos \alpha}{g \cos \beta}$$

$$\therefore \quad R = OB = \frac{OC}{\cos \beta}$$

$$= \frac{2u^2 \sin (\alpha - \beta) \cos \alpha}{g \cos^2 \beta}$$

Using, $\sin C - \sin D = 2 \sin \left(\frac{C - D}{2} \right) \cos \left(\frac{C + D}{2} \right)$,

Range can also be written as,

$$R = \frac{u^2}{g \cos^2 \beta} [\sin (2\alpha - \beta) - \sin \beta]$$

This range will be maximum when

$$2\alpha - \beta = \frac{\pi}{2} \quad \text{or} \quad \alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

and $$R_{\max} = \frac{u^2}{g \cos^2 \beta} [1 - \sin \beta]$$

Here, also we can see that for $\beta = 0$, range is maximum at $\alpha = \frac{\pi}{4}$ or $\alpha = 45°$

and $$R_{\max} = \frac{u^2}{g \cos^2 0°} (1 - \sin 0°) = \frac{u^2}{g}$$

Method 2. Range (R) or the distance OB is also equal to the displacement of projectile along x-direction in time $t = T$. Therefore,

$$R = s_x = u_x T + \frac{1}{2} a_x T^2$$

Substituting the values of u_x, a_x and T, we get the same result.

(ii) Down the Plane : Here, x and y-directions are down the plane and perpendicular to plane respectively as shown in Fig. 4.9. Hence,

$$u_x = u \cos(\alpha + \beta), \quad a_x = g \sin \beta$$
$$u_y = u \sin(\alpha + \beta), \quad a_y = -g \cos \beta$$

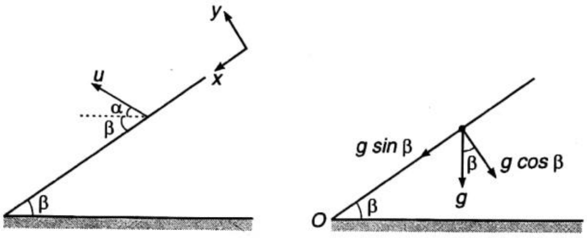


Fig. 4.9

Proceeding in the similar manner, we get the following results:

$$T = \frac{2u \sin(\alpha + \beta)}{g \cos \beta}, \quad R = \frac{u^2}{g \cos^2 \beta}[\sin(2\alpha + \beta) + \sin \beta]$$

From the above expressions, we can see that if we replace β by $-\beta$, the equations of T and R for up the plane and down the plane are interchanged provided α (angle of projection) in both the cases is measured from the horizontal not from the plane.

Sample Example 4.7 *A man standing on a hill top projects a stone horizontally with speed v_0 as shown in figure. Taking the co-ordinate system as given in the figure. Find the co-ordinates of the point where the stone will hit the hill surface.*

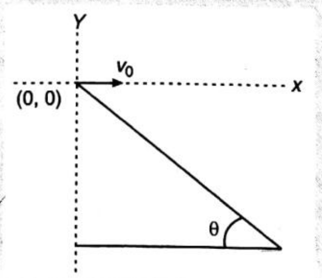


Fig. 4.10

Solution Range of the projectile on an inclined plane (down the plane) is,

$$R = \frac{u^2}{g \cos^2 \beta}[\sin(2\alpha + \beta) + \sin \beta]$$

Here, $u = v_0$, $\alpha = 0$ and $\beta = \theta$

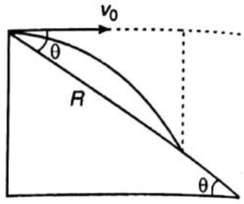


Fig. 4.11

$$\therefore \qquad R = \frac{2v_0^2 \sin\theta}{g\cos^2\theta}$$

Now,
$$x = R\cos\theta = \frac{2v_0^2 \tan\theta}{g}$$

and
$$y = -R\sin\theta = -\frac{2v_0^2 \tan^2\theta}{g}$$

4.3 Relative Motion between Two Projectiles

Let us now discuss the relative motion between two projectiles or the path observed by one projectile of the other. Suppose that two particles are projected from the ground with speeds u_1 and u_2 at angles α_1 and α_2 as shown in Fig. 4.13 and 4.14. Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

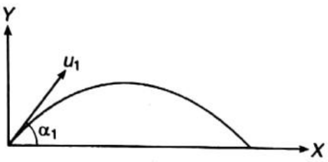


Fig. 4.12

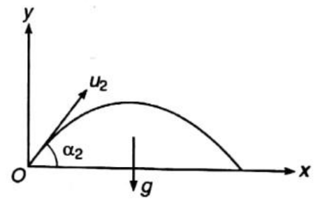


Fig. 4.13

$$a_{12} = a_1 - a_2 = g - g = 0$$

i.e., the relative motion between the two particles is uniform. Now,

$$u_{1x} = u_1\cos\alpha_1, \qquad u_{2x} = u_2\cos\alpha_2$$
$$u_{1y} = u_1\sin\alpha_1 \text{ and } u_{2y} = u_2\sin\alpha_2$$

Therefore,
$$u_{12x} = u_{1x} - u_{2x} = u_1\cos\alpha_1 - u_2\cos\alpha_2$$

and
$$u_{12y} = u_{1y} - u_{2y} = u_1\sin\alpha_1 - u_2\sin\alpha_2$$

u_{12x} and u_{12y} are the x and y components of relative velocity of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight line at an angle $\theta = \tan^{-1}\left(\frac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.

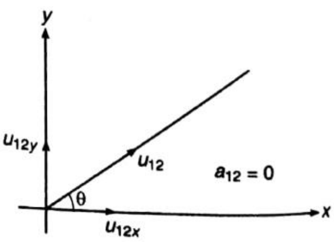


Fig. 4.14

Now, if $u_{12x} = 0$ or $u_1\cos\alpha_1 = u_2\cos\alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90°$). Similarly, if $u_{12y} = 0$ or $u_1\sin\alpha_1 = u_2\sin\alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

Note *Relative acceleration between two projectiles is zero. Relative motion between them is uniform. Therefore, condition of collision of two particles in air is that relative velocity of one with respect to the other should be along line joining them, i.e., if two projecticles A and B collide in mid air, then* $\vec{V}_{AB}$ *should be along AB or* $\vec{V}_{BA}$ *along BA.*

Sample Example 4.8 *A particle A is projected with an initial velocity of 60 m/s at an angle 30° to the horizontal. At the same time a second particle B is projected in opposite direction with initial speed of 50 m/s from a point at a distance of 100 m from A. If the particles collide in air, find (a) the angle of projection α of particle B, (b) time when the collision takes place and (c) the distance of P from A, where collision occurs.* ($g = 10\ m/s^2$)

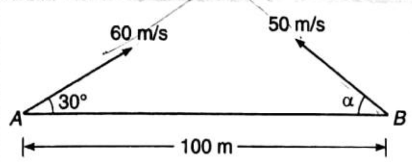


Fig. 4.15

Solution (a) Taking x and y-directions as shown in figure.

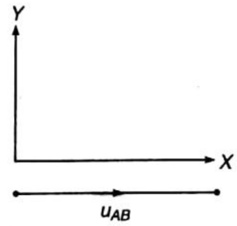


Fig. 4.16

Here,
$$\vec{\mathbf{a}}_A = -g\hat{\mathbf{j}}$$
$$\vec{\mathbf{a}}_B = -g\hat{\mathbf{j}}$$
$$u_{Ax} = 60\cos 30° = 30\sqrt{3}\ \text{m/s}$$
$$u_{Ay} = 60\sin 30° = 30\ \text{m/s}$$
$$u_{Bx} = -50\cos\alpha$$
and
$$u_{By} = 50\sin\alpha$$

Relative acceleration between the two is zero as $\vec{\mathbf{a}}_A = \vec{\mathbf{a}}_B$. Hence, the relative motion between the two is uniform. It can be assumed that B is at rest and A is moving with $\vec{\mathbf{u}}_{AB}$. Hence, the two particles will collide, if $\vec{\mathbf{u}}_{AB}$ is along AB. This is possible only when

$$u_{Ay} = u_{By}$$

i.e., component of relative velocity along y-axis should be zero.

or
$$30 = 50\sin\alpha$$
∴
$$\alpha = \sin^{-1}(3/5)$$

(b) Now,
$$|\vec{\mathbf{u}}_{AB}| = u_{Ax} - u_{Bx}$$
$$= (30\sqrt{3} + 50\cos\alpha)\ \text{m/s}$$
$$= \left(30\sqrt{3} + 50\times\frac{4}{5}\right)\text{m/s}$$
$$= (30\sqrt{3} + 40)\ \text{m/s}$$

Therefore, time of collision is

$$t = \frac{AB}{|\vec{\mathbf{u}}_{AB}|} = \frac{100}{30\sqrt{3}+40}$$

or
$$t = 1.09\,\text{s}$$

(c) Distance of point P from A where collision takes place is

$$s = \sqrt{(u_{Ax}t)^2 + \left(u_{Ay}t - \frac{1}{2}gt^2\right)^2}$$

$$= \sqrt{(30\sqrt{3} \times 1.09)^2 + \left(30 \times 1.09 - \frac{1}{2} \times 10 \times 1.09 \times 1.09\right)^2}$$

or $$s = 62.64 \text{ m}$$

Introductory Exercise 4.2

1. A particle is projected along an inclined plane as shown in figure. What is the speed of the particle when it collides at point A? $(g = 10 \text{ m/s}^2)$

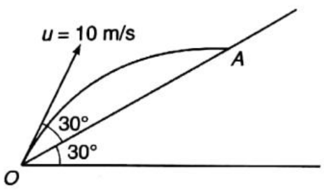


Fig. 4.17

2. In the above problem what is the component of its velocity perpendicular to the plane when it strikes at A?

3. Two particles A and B are projected simultaneously from the two towers of height 10 m and 20 m respectively. Particle A is projected with an initial speed of $10\sqrt{2}$ m/s at an angle of 45° with horizontal, while particle B is projected horizontally with speed 10 m/s. If they collide in air, what is the distance d between the towers?

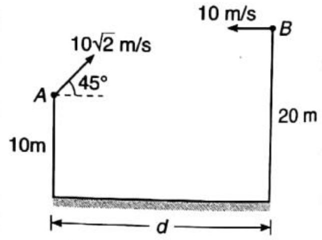


Fig. 4.18

4. Two particles A and B are projected from ground towards each other with speeds 10 m/s and $5\sqrt{2}$ m/s at angles 30° and 45° with horizontal from two points separated by a distance of 15 m. Will they collide or not?

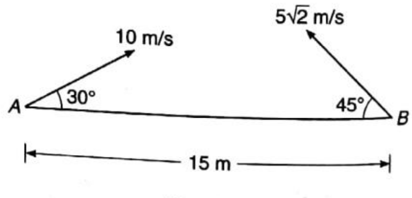


Fig. 4.19

5. A particle is projected from the bottom of an inclined plane of inclination 30°. At what angle α (from the horizontal) should the particle be projected to get the maximum range on the inclined plane.

6. A particle is projected from the bottom of an inclined plane of inclination 30° with velocity of 40 m/s at an angle of 60° with horizontal. Find the speed of the particle when its velocity vector is parallel to the plane. Take $g = 10 \text{ m/s}^2$.

7. Two particles A and B are projected simultaneously in the directions shown in figure with velocities $v_A = 20$ m/s and $v_B = 10$ m/s respectively. They collide in air after $\frac{1}{2}$ s. Find :

(a) the angle θ (b) the distance x.

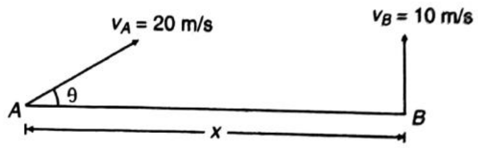


Fig. 4.20

Extra Points

- In projectile motion speed (and hence kinetic energy) is minimum at highest point.

  Speed = $(\cos\theta)$ times the speed of projection

  and kinetic energy = $(\cos^2\theta)$ times the initial kinetic energy

  Here, θ = angle of projection

- Path of a particle depends on nature of acceleration and the angle between initial velocity $\vec{\mathbf{u}}$ and acceleration $\vec{\mathbf{a}}$. Following are few paths which are observed frequently.

  (a) If $\vec{\mathbf{a}}$ = constant and θ is either 0° or 180°, then path of the particle is straight line.

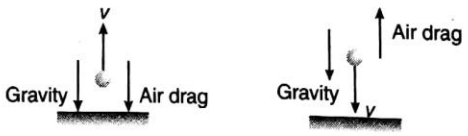
  (b) If $\vec{\mathbf{a}}$ = constant but θ is other then 0° or 180°, then path of the particle is parabola (as in projectile motion).

  (c) If $|\vec{\mathbf{a}}|$ = constant and $\vec{\mathbf{a}}$ is always perpendicular to velocity vector $\vec{\mathbf{v}}$, then path of the particle is a circle.

- In projectile motion it is sometimes better to write the equations of H, R and T in terms of u_x and u_y as under.

$$T = \frac{2u_y}{g}, \quad H = \frac{u_y^2}{2g} \quad \text{and} \quad R = \frac{2u_x u_y}{g}$$

- In projectile motion $H = R$, when $u_y = 4u_x$ or $\tan\theta = 4$.

- 

  If a particle is projected vertically upwards, then during upward journey gravity forces (weight) and air drag both are acting downwards. Hence, |retardation| > $|g|$. During its downward journey air drag is upwards while gravity is downwards. Hence, acceleration < g. Therefore we may conclude that,

  time of ascent < time of descent

  Exercise : In projectile motion, if air drag is taken into consideration than state whether the H, R and T will increase, decrease or remain same.

- In JEE problems from the present chapter kinematics are often based on collision of two particles. Following are two tips to solve such problems.

  (a) At the time of collision coordinates of particles should be same, *i.e.*,

  $$x_1 = x_2, \quad \text{and} \quad y_1 = y_2 \quad \text{(for a 2-D motion)}$$

  Similarly $$x_1 = x_2, \quad y_1 = y_2 \quad \text{and} \quad z_1 = z_2 \quad \text{(for a 3-D motion)}$$

  (b) Two particles collide at the same moment. Of course their time of journies may be different, *i.e.*, they may start at different times (t_1 and t_2 may be different). If they start together then $t_1 = t_2$.

Solved Examples

Level 1

Example 1 *A particle is projected from horizontal making an angle 60° with initial velocity 40 m/s. Find the time taken to the particle to make angle 45° from horizontal.*

Solution At 45°, $v_x = v_y$

or $$u_x = u_y - gt$$

$\therefore$ $$t = \frac{u_y - u_x}{g} = \frac{40(\sin 60° - \sin 30°)}{9.8}$$

$$= 1.5\,\text{s}$$

Example 2 *A ball rolls off the edge of a horizontal table top 4 m high. If it strikes the floor at a point 5 m horizontally away from the edge of the table, what was its speed at the instant it left the table.*

Solution Using $h = \frac{1}{2}gt^2$, we have

$$h_{AB} = \frac{1}{2}gt_{AC}^2$$

or $$t_{AC} = \sqrt{\frac{2h_{AB}}{g}} = \sqrt{\frac{2\times 4}{9.8}} = 0.9\,\text{s}$$

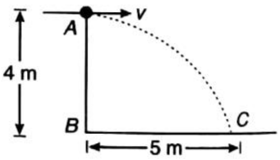


Fig. 4.21

Further, $$BC = vt_{AC}$$

or $$v = \frac{BC}{t_{AC}} = \frac{5.0}{0.9} = 5.55\,\text{m/s}$$

Example 3 *An aeroplane is flying in a horizontal direction with a velocity 600 km/h at a height of 1960 m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.*

Solution From $h = \frac{1}{2}gt^2$

we have, $$t_{OB} = \sqrt{\frac{2h_{OA}}{g}} = \sqrt{\frac{2\times 1960}{9.8}} = 20\,\text{s}$$

Horizontal distance $$AB = vt_{OB}$$

$$= \left(600 \times \frac{5}{18}\,\text{m/s}\right)(20\,\text{s})$$

$$= 3333.33\,\text{m} = 3.33\,\text{km}$$

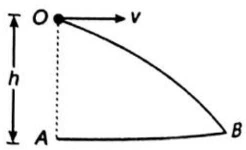


Fig. 4.22

Example 4 *A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the particle at the origin of co-ordinates.*

Solution Comparing the given equation with the equation of a projectile motion,

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

we find that $\quad g = a, \quad \tan\theta = p \quad$ and $\quad \frac{a}{2u^2}(1+\tan^2\theta) = q$

$\therefore \quad u =$ velocity of particle at origin

$$= \sqrt{\frac{a(1+\tan^2\theta)}{2q}} = \sqrt{\frac{a(1+p^2)}{2q}}$$

Level 2

Example 1 *A car accelerating at the rate of $2\ m/s^2$ from rest from origin is carrying a man at the rear end who has a gun in his hand. The car is always moving along positive x-axis. At $t = 4$ s, the man fires from the gun and the bullet hits a bird at $t = 8$ s. The bird has a position vector $40\hat{\mathbf{i}} + 80\hat{\mathbf{j}} + 40\hat{\mathbf{k}}$. Find velocity of projection of the bullet. Take the y-axis in the horizontal plane. ($g = 10\ m/s^2$)*

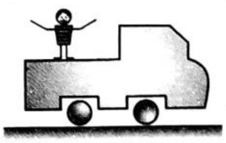


Fig. 4.23

Solution Let velocity of bullet be,

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

At $t = 4$ s, x-coordinate of car is

$$x_c = \frac{1}{2}at^2 = \frac{1}{2}\times 2\times 16 = 16\ \text{m}$$

x-coordinate of bird is $x_b = 40$ m

$\therefore \quad x_b = x_c + v_x(8-4)$

or $\quad 40 = 16 + 4v_x$

$\therefore \quad v_x = 6$ m/s

Similarly, $\quad y_b = y_c + v_y(8-4)$

or $\quad 80 = 0 + 4v_y$

or $\quad v_y = 20$ m/s

and $\quad z_b = z_c + v_z(8-4) - \frac{1}{2}g(8-4)^2$

or $\quad 40 = 0 + 4v_z - \frac{1}{2}\times 5\times 16$

or

$$v_z = 20 \text{ m/s}^2$$

∴ Velocity of projection of bullet

$$\vec{v} = (6\hat{i} + 20\hat{j} + 20\hat{k}) \text{ m/s}$$

Example 2 *The velocity of a projectile when it is at the greatest height is $\sqrt{2/5}$ times its velocity when it is at half of its greatest height. Determine its angle of projection.*

Solution Suppose the particle is projected with velocity u at an angle θ with the horizontal. Horizontal component of its velocity at all height will be $u \cos \theta$.

At the greatest height, the vertical component of velocity is zero, so the resultant velocity is

$$v_1 = u \cos \theta$$

At half the greatest height during upward motion,

$$y = h/2, \quad a_y = -g, \quad u_y = u \sin \theta$$

Using

$$v_y^2 - u_y^2 = 2a_y y$$

we get,

$$v_y^2 - u^2 \sin^2 \theta = 2(-g)\frac{h}{2}$$

or

$$v_y^2 = u^2 \sin^2 \theta - g \times \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 \theta}{2} \qquad \left[\because h = \frac{u^2 \sin^2 \theta}{2g}\right]$$

or

$$v_y = \frac{u \sin \theta}{\sqrt{2}}$$

Hence, resultant velocity at half of the greatest height is

$$v_2 = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 \cos^2 \theta + \frac{u^2 \sin^2 \theta}{2}}$$

Given,

$$\frac{v_1}{v_2} = \sqrt{\frac{2}{5}}$$

∴

$$\frac{v_1^2}{v_2^2} = \frac{u^2 \cos^2 \theta}{u^2 \cos^2 \theta + \frac{u^2 \sin^2 \theta}{2}} = \frac{2}{5} \quad \text{or} \quad \frac{1}{1 + \frac{1}{2}\tan^2 \theta} = \frac{2}{5}$$

or

$$2 + \tan^2 \theta = 5 \quad \text{or} \quad \tan^2 \theta = 3$$

or

$$\tan \theta = \sqrt{3}$$

∴

$$\theta = 60°$$

Example 3 *A particle is thrown over a triangle from one end of a horizontal base and after grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that* $\tan \theta = \tan \alpha + \tan \beta$.

Solution The situation is shown in Fig. 4.24.

From figure, we have

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$$

$$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)} \quad \text{...(i)}$$

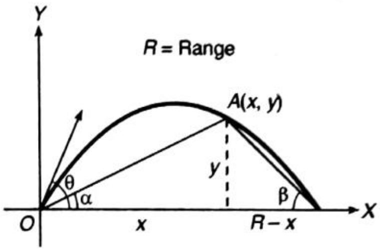


Fig. 4.24

Equation of trajectory is $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

or, $$\tan\theta = \frac{yR}{x(R-x)} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\tan\theta = \tan\alpha + \tan\beta$$

Example 4 *Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m/s along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q. Calculate*

(a) time of flight,

(b) velocity with which the particle strikes the plane OB,

(c) height h of point P from point O,

(d) distance PQ. (Take g = 10 m/s²)

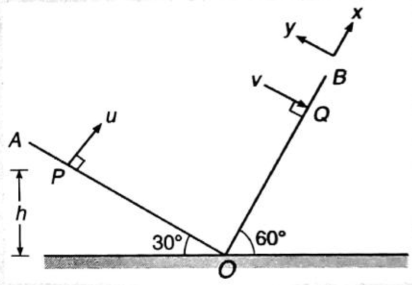


Fig. 4.25

Solution Let us choose the x and y directions along OB and OA respectively. Then,

$$u_x = u = 10\sqrt{3}\text{ m/s}, \; u_y = 0$$

$$a_x = -g\sin 60° = -5\sqrt{3}\text{ m/s}^2 \quad \text{and} \quad a_y = -g\cos 60° = -5\text{ m/s}^2$$

(a) At point Q, x-component of velocity is zero. Hence, substituting in

$$v_x = u_x + a_x t$$

$$0 = 10\sqrt{3} - 5\sqrt{3}t$$

or $$t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2\text{s}$$

(b) At point Q, $$v = v_y = u_y + a_y t$$

∴ $$v = 0 - (5)(2) = -10\text{ m/s}$$

Here, negative sign implies that velocity of particle at Q is along negative y direction.

(c) Distance $PO = |$ displacement of particle along y-direction $| = |s_y|$

Here, $$s_y = u_y t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}(5)(2)^2 = -10\text{m}$$

∴ $$PO = 10\text{ m}$$

Therefore, $$h = PO \sin 30° = (10)\left(\frac{1}{2}\right)$$

or $$h = 5 \text{ m}$$

(d) Distance $OQ =$ displacement of particle along x-direction $= s_x$

Here, $$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3} \text{ m}$$

or $$OQ = 10\sqrt{3} \text{ m}$$

∴ $$PQ = \sqrt{(PO)^2 + (OQ)^2}$$

$$= \sqrt{(10)^2 + (10\sqrt{3})^2}$$

$$= \sqrt{100 + 300} = \sqrt{400}$$

∴ $$PQ = 20 \text{ m}$$

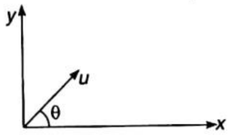
EXERCISES

AIEEE Corner

Subjective Questions (Level 1)

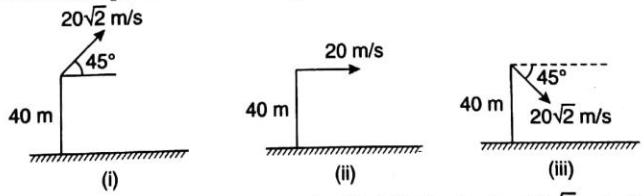
Projectile Motion from Ground to Ground

1. A particle is projected from ground with initial velocity $u = 20\sqrt{2}$ m/s at $\theta = 45°$. Find :



   (a) R, H and T,
   (b) velocity of particle after 1 s
   (c) velocity of particle at the time of collision with the ground (x-axis).

2. In the figures shown, three particles are thrown from a tower of height 40 m as shown in figure. In each case find the time when the particles strike the ground and the distance of this point from foot of tower.



3. A particle is projected from ground at angle 45° with initial velocity $20\sqrt{2}$ m/s. Find :
   (a) change in velocity,
   (b) magnitude of average velocity in a time interval from $t = 0$ to $t = 3$ s.

4. The coach throws a baseball to a player with an initial speed of 20 m/s at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m from the coach. At what speed and in what direction must the player run to catch the ball at the same height at which it was released? $(g = 10\ \text{m/s}^2)$

5. At time $t = 0$ a smail ball is projected from point A with a velocity of 60 m/s at 60° angle with horizontal. Neglect atmospheric resistance and determine the two times t_1 and t_2 when the velocity of the ball makes an angle of 45° with the horizontal x-axis.

6. A particle moves in the xy-plane with constant acceleration a directed along the negative y-axis. The equation of path of the particle has the form $y = bx - cx^2$, where b and c are positive constants. Find the velocity of the particle at the origin of coordinates.

7. A ball is shot from the ground into the air. At a height of 9.1 m, its velocity is observed to be $\vec{v} = 7.6\hat{i} + 6.1\hat{j}$ in metre per second ($\hat{i}$ is horizontal, $\hat{j}$ is upward). Give the approximate answers.
   (a) To what maximum height does the ball rise?
   (b) What total horizontal distance does the ball travel?
   What are :
   (c) the magnitude and
   (d) the direction of the ball's velocity just before it hits the ground?

8. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located over a tower at one point and moved with velocities $v_1 = 3$ m/s and $v_2 = 4$ m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

9. A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall. Find the angle of projection of ball.

10. A particle is projected with velocity $2\sqrt{gh}$, so that it just clears two walls of equal height h which are at a distance of $2h$ from each other. Show that the time of passing between the walls is $2\sqrt{\frac{h}{g}}$.

   [Hint : First find velocity at height h. Treat it as initial velocity and $2h$ as the range.]

11. A particle is projected at an angle of elevation α and after t second it appears to have an elevation of β as seen from the point of projection. Find the initial velocity of projection.

12. A projectile aimed at a mark, which is in the horizontal plane through the point of projection, falls a cm short of it when the elevation is α and goes b cm far when the elevation is β. Show that, if the speed of projection is same in all the cases the proper elevation is :

$$\frac{1}{2}\sin^{-1}\left[\frac{b\sin 2\alpha + a\sin 2\beta}{a+b}\right]$$

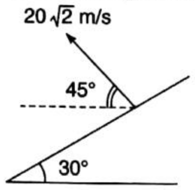
13. Two particles are simultaneously thrown in horizontal direction from two points on a riverbank, which are at certain height above the water surface. The initial velocities of the particles are $v_1 = 5$ m/s and $v_2 = 7.5$ m/s respectively. Both particles fall into the water at the same time. First particle enters the water at a point $s = 10$ m from the bank. Determine :
   (a) the time of flight of the two particles,
   (b) the height from which they are thrown,
   (c) the point where the second particle falls in water.

14. A balloon is ascending at the rate $v = 12$ km/h and is being carried horizontally by the wind at $v_w = 20$ km/h. If a ballast bag is dropped from the balloon at the instant $h = 50$ m, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, find the speed with which bag the strikes the ground?

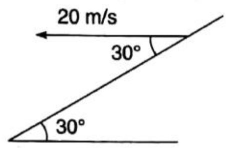
Projectile Motion in Inclined Plane

15. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s^2)

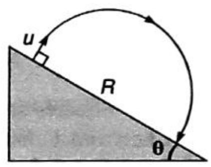
16. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10\ m/s^2$)



17. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10\ m/s^2$)



18. A projectile is fired with a velocity u at right angles to the slope, which is inclined at an angle θ with the horizontal. Derive an expression for the distance R to the point of impact.



Relative Motion in Projectiles

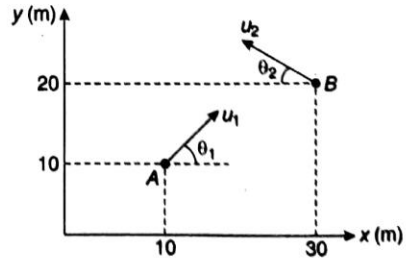
Note *The problems can also be solved without using the concept of relative motion.*

19. A particle is projected upwards with velocity 20 m/s. Simultaneously another particle is projected with velocity $20\sqrt{2}$ m/s at 45°. ($g = 10\ m/s^2$)
   (a) What is acceleration of first particle relative to the second?
   (b) What is initial velocity of first particle relative to the other?
   (c) What is distance between two particles after 2 s?

20. Passenger of a train just drops a stone from it. The train was moving with constant velocity. What is path of the stone as observed by
   (a) the passenger itself,
   (b) a man standing on ground?

21. An elevator is going up with an upward acceleration of $1\ m/s^2$. At the instant when its velocity is 2 m/s, a stone is projected upward from its floor with a speed of 2 m/s relative to the elevator, at an elevation of 30°.
   (a) Calculate the time taken by the stone to return to the floor.
   (b) Sketch the path of the projectile as observed by an observer outdside the elevator.
   (c) If the elevator was moving with a downward acceleration equal to g, how would the motion be altered?

22. Two particles A and B are projected simultaneously in a vertical plane as shown in figure. They collide at time t in air. Write down two necessary equations for collision to take place.



Objective Questions (Level 1)

Single Correct Option

1. A particle has initial velocity, $\vec{v} = 3\hat{i} + 4\hat{j}$ and a constant force $\vec{F} = 4\hat{i} - 3\hat{j}$ acts on it. The path of the particle is
(a) straight line (b) parabolic (c) circular (d) elliptical

2. Identify the correct statement related to the projectile motion.
(a) It is uniformly accelerated everywhere
(b) It is uniformly accelerated everywhere except at the highest position where it is moving with constant velocity
(c) Acceleration is never perpendicular to velocity
(d) None of the above

3. A ball is projected with a velocity 20 ms^{-1} at an angle to the horizontal. In order to have the maximum range. Its velocity at the highest position must be
(a) 10 ms^{-1} (b) 14 ms^{-1} (c) 18 ms^{-1} (d) 16 ms^{-1}

4. Two bodies are thrown with the same initial velocity at angles θ and $(90° - \theta)$ respectively with the horizontal, then their maximum heights are in the ratio
(a) 1 : 1 (b) $\sin\theta : \cos\theta$ (c) $\sin^2\theta : \cos^2\theta$ (d) $\cos\theta : \sin\theta$

5. A gun is firing bullets with velocity v_0 by rotating through 360° in the horizontal plane. The maximum area covered by the bullets is
(a) $\frac{\pi v_0^2}{g}$ (b) $\frac{\pi^2 v_0^2}{g}$ (c) $\frac{\pi v_0^4}{g^2}$ (d) $\frac{\pi^2 v_0^4}{g}$

6. A body is projected at an angle 60° with the horizontal with kinetic energy K. When the velocity makes an angle 30° with the horizontal, the kinetic energy of the body will be
(a) $K/2$ (b) $K/3$ (c) $2K/3$ (d) $3K/4$

7. The range of a projectile at an angle θ is equal to half of the maximum range if thrown at the same speed. The angle of projection θ is given by
(a) 15° (b) 30° (c) 60° (d) data insufficient

8. If T_1 and T_2 are the times of flight for two complementary angles, then the range of projectile R is given by
(a) $R = 4gT_1T_2$ (b) $R = 2gT_1T_2$ (c) $R = \frac{1}{4}gT_1T_2$ (d) $R = \frac{1}{2}gT_1T_2$

9. A grass hopper can jump maximum distance 1.6 m. It spends negligible time on ground. How far can it go in $10\sqrt{2}$ s.
(a) 45 m (b) 30 m (c) 20 m (d) 40 m

10. Average velocity of a particle in projectile motion between its starting point and the highest point of its trajectory is (projection speed = u, angle of projection from horizontal = θ)
(a) $u\cos\theta$ (b) $\frac{u}{2}\sqrt{1 + 3\cos^2\theta}$ (c) $\frac{u}{2}\sqrt{2 + \cos^2\theta}$ (d) $\frac{u}{2}\sqrt{1 + \cos^2\theta}$

11. A train is moving on a track at 30 ms^{-1}. A ball is thrown from it perpendicular to the direction of motion at 30 ms^{-1} at 45° from horizontal. Find the distance of ball from the point of projection on train to the point where it strikes the ground.
(a) 90 m (b) $90\sqrt{3}$ m (c) 60 m (d) $60\sqrt{3}$ m

12. A body is projected at time $t = 0$ from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacements x and y (in metre) respectively vary with time t in second as, $x = (10\sqrt{3})\,t$ and $y = 10t - t^2$. The maximum height attained by the body is
(a) 200 m (b) 100 m (c) 50 m (d) 25 m

13. A particle is fired horizontally from an inclined plane of inclination 30° with horizontal with speed $50\,\text{ms}^{-1}$. If $g = 10\,\text{ms}^{-2}$, the range measured along the incline is
(a) 500 m (b) $\frac{1000}{3}$ m (c) $200\sqrt{2}$ m (d) $100\sqrt{3}$ m

14. Two stones are projected with the same speed but making different angles with the horizontal. Their horizontal ranges are equal. The angle of projection of one is $\frac{\pi}{3}$ and the maximum height reached by it is 102 m. Then the maximum height reached by the other in metres is
(a) 336 (b) 224 (c) 56 (d) 34

15. A ball is projected upwards from the top of a tower with a velocity $50\,\text{ms}^{-1}$ making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds from the instant of throwing, will the ball reach the ground. ($g = 10\,\text{ms}^{-2}$)
(a) 2 s (b) 5 s (c) 7 s (d) 9 s

16. A fixed mortar fires a bomb at an angle of 53° above the horizontal with a muzzle velocity of $80\,\text{ms}^{-1}$. A tank is advancing directly towards the mortar on level ground at a constant speed of 5m/s. The initial separation (*at the instant mortar is fired*) between the mortar and tank, so that the tank would be hit is [Take $g = 10\,\text{ms}^{-2}$]
(a) 662.4 m (b) 526.3 m (c) 486.6 m (d) None of these

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

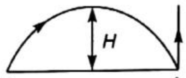
(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***
(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***
(*c*) *If **Assertion** is true, but the **Reason** is false.*
(*d*) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : A particle can follow a parabolic path only if acceration is constant.
Reason : In projectile motion path is parabolic, as acceleration is assumed to be constant at low heights.

2. Assertion : Projectile motion is called a two dimensional motion, although it takes place in space.
Reason : In space it takes place in a plane.

3. Assertion : If time of flight in a projectile motion is made two times, its maximum height will become four times.
Reason : In projectile motion $H \propto T^2$, where H is maximum height and T the time of flight.

4. **Assertion :** A particle is projected with velocity $\vec{u}$ at angle 45° with ground. Let $\vec{v}$ be the velocity of particle at time t ($\neq 0$), then value of $\vec{u}.\vec{v}$ can be zero.
Reason : Value of dot product is zero when angle between two vectors is 90°.
5. **Assertion :** A particle has constant acceleration is x-y plane. But neither of its acceleration components (a_x and a_y) is zero. Under this condition particle can not have parabolic path.
Reason : In projectile motion, horizontal component of acceleration is zero.
6. **Assertion :** In projectile motion at any two positions $\frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ always remains constant.
Reason : The given quantity is average acceleration, which should remain constant as acceleration is constant.
7. **Assertion :** Particle A is projected upwards. Simultaneouly particle B is projected as projectile as shown. Particle A returns to ground is 4 s. At the same time particle B collides with A. Maximum height H attained by B would be 20 m. ($g = 10\ \text{ms}^{-2}$)



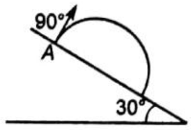
Reason : Speed of projection of both the particles should be same under the given condition.
8. **Assertion :** Two projectiles have maximum heights $4H$ and H respectively. The ratio of their horizontal components of velocities should be $1:2$ for their horizontal ranges to be same.
Reason : Horizontal range = horizontal component of velocity X time of flight.
9. **Assertion :** If $g = 10\ \text{m/s}^2$ then in projectile motion speed of particle in every second will change by $10\ \text{ms}^{-1}$.
Reason : Acceleration is nothing but rate of change of velocity.
10. **Assertion:** In projectile motion if particle is projected with speed u, then speed of particle at height h would be $\sqrt{u^2 - 2gh}$.
Reason : If particle is projected with vertical component of velocity u_y. Then vertical component at the height h would be $\pm\sqrt{u_y^2 - 2gh}$.

Objective Questions (Level 2)

Single Correct Option

1. Two bodies were thrown simultaneously from the same point, one straight up, and the other, at an angle of $\theta = 30°$ to the horizontal. The initial velocity of each body is $20\ \text{ms}^{-1}$. Neglecting air resistance, the distance between the bodies at $t = 1.2$ later is
(a) 20 m (b) 30 m (c) 24 m (d) 50 m
2. A particle is dropped from a height h. Another particle which is initially at a horizontal distance d from the first is simultaneously projected with a horizontal velocity u and the two particles just collide on the ground. Then :
(a) $d^2 = \frac{u^2 h}{2h}$ (b) $d^2 = \frac{2u^2 h}{g}$
(c) $d = h$ (d) $gd^2 = u^2 h$

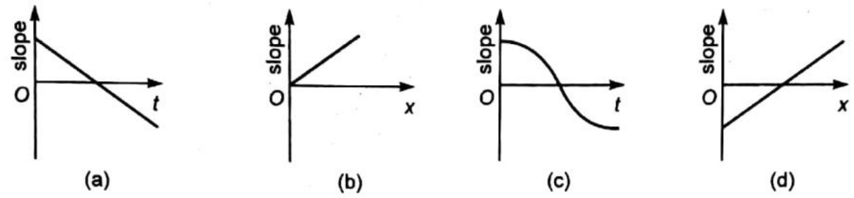
3. A ball is projected from point A with velocity $10\ \text{ms}^{-1}$ perpendicular to the inclined plane as shown in figure. Range of the ball on the inclined plane is



(a) $\frac{40}{3}$ m (b) $\frac{20}{3}$ m

(c) $\frac{12}{3}$ m (d) $\frac{60}{3}$ m

4. A heavy particle is projected with a velocity at an angle with the horizontal into the uniform gravitational field. The slope of the trajectory of the particle varies as



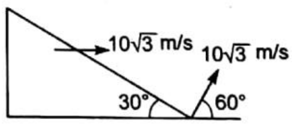
5. A particle starts from the origin of coordinates at time $t = 0$ and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction is

(a) variable (b) $\sqrt{\frac{2\alpha}{\beta}}$ (c) $\frac{\alpha}{2\beta}$ (d) $\sqrt{\frac{\alpha}{2\beta}}$

6. A projectile is projected with speed u at an angle of $60°$ with horizontal from the foot of an inclined plane. If the projectile hits the inclined plane horizontally, the range on inclined plane will be

(a) $\frac{u^2\sqrt{21}}{2g}$ (b) $\frac{3u^2}{4g}$ (c) $\frac{u^2}{2\beta}$ (d) $\frac{\sqrt{21}\,u^2}{8g}$

7. A particle is projected at an angle $60°$ with speed $10\sqrt{3}$ m/s, from the point A, as shown in the figure. At the same time the wedge is made to move with speed $10\sqrt{3}$ m/s towards right as shown in the figure. Then the time after which particle will strike with wedge is



(a) 2 s (b) $2\sqrt{3}$ s (c) $\frac{4}{\sqrt{3}}$ s (d) None of these

8. A particle moves along the parabolic path $x = y^2 + 2y + 2$ in such a way that Y-component of velocity vector remains $5\ \text{ms}^{-1}$ during the motion. The magnitude of the acceleration of the particle is

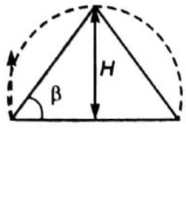
(a) $50\ \text{ms}^{-2}$ (b) $100\ \text{ms}^{-2}$

(c) $10\sqrt{2}\ \text{ms}^{-2}$ (d) $0.1\ \text{ms}^{-2}$

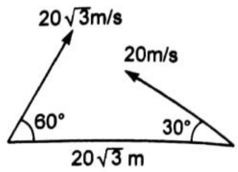
9. A shell fired from the base of a mountain just clears it. If α is the angle of projection, then the angular elevation of the summit β is

(a) $\frac{\alpha}{2}$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{\tan\alpha}{2}\right)$ (d) $\tan^{-1}(2\tan\alpha)$



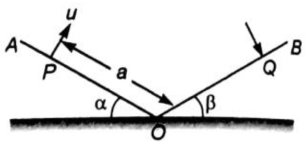
10. In the figure shown, the two projectiles are fired simultaneously, The minimum distance between them during their flight is



(a) 20 m
(b) $10\sqrt{3}$ m
(c) 10 m
(d) None of these

Passage (Q. No. 11-12)

Two inclined planes OA and OB intersect in a horizontal plane having their inclinations α and β with the horizontal as shown in figure. A particle is projected from point P with velocity u along a direction perpendicular to plane OA. The particle strikes plane OB perpendicularly at Q.



11. If $\alpha = 30°, \beta = 30°$, the time of flight from P to Q is
(a) $\frac{u}{g}$
(b) $\frac{\sqrt{3}\,u}{g}$
(c) $\frac{\sqrt{2}\,u}{g}$
(d) $\frac{2u}{g}$

12. If $\alpha = 30°, \beta = 30°$ and $a = 4.9$ m, the initial velocity of projection is
(a) 9.8 ms^{-1}
(b) 4.9 ms^{-1}
(c) $4.9\sqrt{2}$ ms^{-1}
(d) 19.6 ms^{-1}

More than One Correct Options

1. Two particles projected from the same point with same speed u at angles of projection α and β strike the horizontal ground at the same point. If h_1 and h_2 are the maximum heights attained by the projectile, R is the range for both and t_1 and t_2 are their times of flights, respectively , then
(a) $\alpha + \beta = \frac{\pi}{2}$
(b) $R = 4\sqrt{h_1 h_2}$
(c) $\frac{t_1}{t_2} = \tan\alpha$
(d) $\tan\alpha = \sqrt{\frac{h_1}{h_2}}$

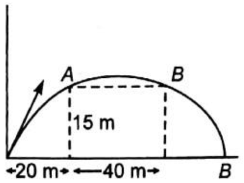
2. A ball is dropped from a height of 49 m. The wind is blowing horizontally. Due to wind a constant horizontal acceleration is provided to the ball. Choose the correct statement (s). [Take $g = 9.8\ ms^{-2}$]
(a) Path of the ball is a straight line
(b) Path of the ball is a curved one
(c) The time taken by the ball to reach the ground is 3.16 s
(d) Actual distance travelled by the ball is more then 49 m

3. A particle is projected from a point P with a velocity v at an angle θ with horizontal. At a certain point Q it moves at right angles to its initial direction. Then
(a) velocity of particle at Q is $v\sin\theta$
(b) velocity of particle at Q is $v\cot\theta$
(c) time of flight from P to Q is $(v/g)\operatorname{cosec}\theta$
(d) time of flight from P to Q is $(v/g)\sec\theta$

4. At a height of 15 m from ground velocity of a projectile is $\vec{v} = (10\hat{i} + 10\hat{j})$. Here, $\hat{j}$ is vertically upwards and $\hat{i}$ is along horizontal direction then ($g = 10\ ms^{-2}$)
(a) particle was projected at an angle of 45° with horizontal
(b) time of flight of projectile is 4 s
(c) horizontal range of projectile is 100 m
(d) maximum height of projectile from ground is 20 m

5. Which of the following quantities remain constant during projectile motion?
(a) Average velocity between two points
(b) Average speed between two points
(c) $\frac{d\vec{v}}{dt}$
(d) $\frac{d^2\vec{v}}{dt^2}$

6. In the projectile motion shown is figure, given $t_{AB} = 2\,\text{s}$ then $(g = 10\,\text{ms}^{-2})$



(a) particle is at point B at 3 s

(b) maximum height of projectile is 20 m

(c) initial vertical component of velocity is $20\,\text{ms}^{-1}$

(d) horizontal component of velocity is $20\,\text{ms}^{-1}$

Match the Columns

1. Particle-1 is just dropped from a tower. 1 s later particle-2 is thrown from the same tower horizontally with velocity $10\,\text{ms}^{-1}$. Taking $g = 10\,\text{ms}^{-2}$, match the following two columns at $t = 2\,\text{s}$.

Column I	Column II
(a) Horizontal displacement between two	(p) 10 SI units
(b) Vertical displacement between two	(q) 20 SI units
(c) Magnitude of relative horizontal component of velocity	(r) $10\sqrt{2}$ SI units
(d) Magnitude of relative vertical component of velocity	(s) None

2. In a projectile motion, given $H = \frac{R}{2} = 20\,\text{m}$. Here, H is maximum height and R the horizontal range. For the given condition match the following two columns.

Column I	Column II
(a) Time of flight	(p) 1
(b) Ratio of vertical component of velocity and horizontal component of velocity	(q) 2
(c) Horizontal component of velocity	(r) 10
(d) Vertical component of velocity	(s) None

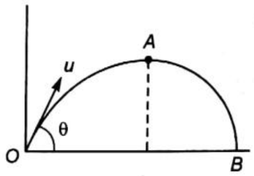
3. A particle can be thrown at a constant speed at different angles. When it is thrown at 15° with horizontal, it falls at a distance of 10 m from point of projection. For this speed of particle match following two columns.

Column I	Column II
(a) Maximum horizontal range which can be taken with this speed	(p) 10 m
(b) Maximum height	(q) 20 m
(c) Range at 75°	(r) 15 m
(d) Height at 30°	(s) None

4. In projectile motion if vertical component of velocity is increased to two times, keeping horizontal component unchanged, then

Column I	Column II
(a) Time of flight	(p) will remain same
(b) Maximum height	(q) will become two times
(c) Horizontal range	(r) will become four times
(d) Angle of projection with horizontal	(s) None

5. In projectile motion shown in figure.

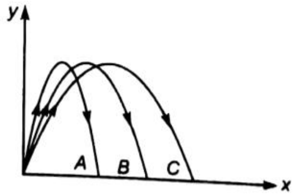


Column I	Column II
(a) Change in velocity between O and A	(p) $u\cos\theta$
(b) Average velocity between O and A	(q) $u\sin\theta$
(c) Change in velocity between O and B	(r) $2u\cos\theta$
(d) Average velocity between O and B	(s) None

6. Particle-1 is projected from ground (take it origin) at time $t=0$, with velocity $(30\hat{\mathbf{i}}+30\hat{\mathbf{j}})\,\text{ms}^{-1}$. Particle-2 is projected from (130 m, 75 m) at time $t=1$ second with velocity $(-20\hat{\mathbf{i}}+20\hat{\mathbf{j}})\,\text{ms}^{-1}$. Assuming $\hat{\mathbf{j}}$ to be vertically upward and $\hat{\mathbf{i}}$ to be in horizontal direction, match the following two columns at $t=2\,\text{s}$.

Column I	Column II
(a) horizontal distance between two	(p) 30 SI units
(b) vertical distance between two	(q) 40 SI units
(c) relative horizontal component of velocity between two	(r) 50 SI units
(d) relative vertical component of velocity between two	(s) None

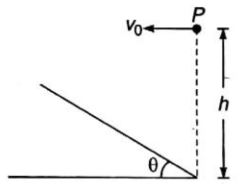
7. The trajectories of the motion of 3 particles are shown in the figure. Match the entries of column I with the entries of column II.



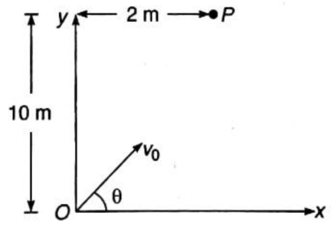
Column I	Column II
(a) Time of flight is least for	(p) A
(b) Vertical component of velocity is greatest for	(q) B
(c) Horizontal component of velocity is greatest for	(r) C
(d) Launch speed is least for	(s) can't say

Subjective Questions (Level 2)

1. Determine the horizontal velocity v_0 with which a stone must be projected horizontally from a point P, so that it may hit the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and point P is at a height h above the foot of the incline, as shown in the figure.



2. A particle is dropped from point P at time $t = 0$. At the same time another particle is thrown from point O as shown in the figure and it collides with the particle P. Acceleration due to gravity is along the negative y-axis. If the two particles collide 2 s after they start, find the initial velocity v_0 of the particle which was projected from O. Point O is not necessarily on ground.



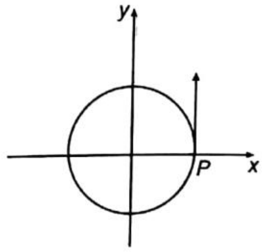
3. Two particles are simultaneously projected in the same vertical plane from the same point with velocities u and v at angles α and β with horizontal. Find the time that elapses when their velocities are parallel.

4. A projectile takes off with an initial velocity of 10 m/s at an angle of elevation of 45°. It is just able to clear two hurdles of height 2 m each, separated from each other by a distance d. Calculate d. At what distance from the point of projection is the first hurdle placed? Take $g = 10 \text{ m/s}^2$.

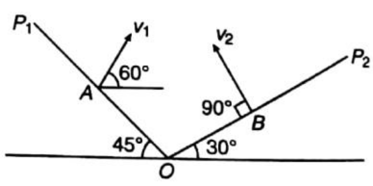
5. A stone is projected from the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and attains the maximum height of $2h$ above the ground. If at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocity of bird and the horizontal component of velocity of stone, if the stone hits the bird while descending.

6. A particle is released from a certain height $H = 400\text{m}$. Due to the wind the particle gathers the horizontal velocity component $v_x = ay$ where $a = \sqrt{5}\ \text{s}^{-1}$ and y is the vertical displacement of the partricle from the point of release, then find :
(a) the horizontal drift of the particle when it stikes the ground,
(b) the speed with which particle strikes the ground.
(Take $g = 10\ \text{m/s}^2$)

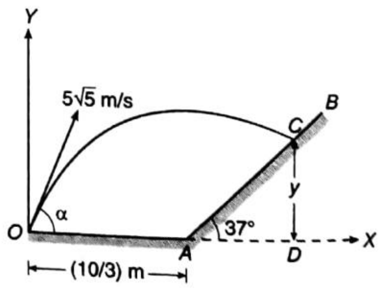
7. A train is moving with a constant speed of 10 m/s in a circle of radius $\frac{16}{\pi}$ m. The plane of the circle lies in horizontal x-y plane. At time $t = 0$ train is at point P and moving in counter-clockwise direction. At this instant a stone is thrown from the train with speed 10 m/s relative to train towards negative x-axis at an angle of 37° with vertical z-axis. Find
(a) the velocity of particle relative to train at the highest point of its trajectory.
(b) the co-ordinates of points on the ground where it finally falls and that of the highest point of its trajectory.
Take $g = 10\ \text{m/s}^2$, $\sin 37° = \frac{3}{5}$



8. A particle is projected from an inclined plane OP_1 from A with velocity $v_1 = 8\ \text{ms}^{-1}$ at an angle 60° with horizontal. An another particle is projected at the same instant from B with velocity $v_2 = 16\ \text{ms}^{-1}$ and perpendicular to the plane OP_2 as shown in figure. After time $10\sqrt{3}$ sec there separation was minimum and found to be 70 m. Then find distance AB.



9. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}$ m/s at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of 37° with horizontal as shown in figure. If the particle does not rebound, calculate
(a) coordinates of point C in reference to coordinate system as shown in the figure.
(b) maximum height from the ground to which the particle rises. ($g = 10\ \text{m/s}^2$).



10. A plank fitted with a gun is moving on a horizontal surface with speed of 4 m/s along the positive x-axis. The z-axis is in vertically upward direction. The mass of the plank including the mass of the gun is 50 kg. When the plank reaches the origin, a shell of mass 10 kg is fired at an angle of 60° with the positive x-axis with a speed of $v = 20$ m/s with respect to the gun in x-z plane. Find the position vector of the shell at $t = 2\text{s}$ after firing it. Take $g = 9.8\ \text{m/s}^2$.

ANSWERS

Introductory Exercise 4.1

1. False **2.** True **3.** (a) $T = \dfrac{2u\cos\theta}{g}$ (b) $H = \dfrac{u^2\cos^2\theta}{2g}$ (c) $R = \dfrac{u^2\sin 2\theta}{g}$ (d) $R_{max} = \dfrac{u^2}{g}$ at $\theta = 45°$

4. $20\sqrt{5}$ m/s at angle $\tan^{-1}\left(\dfrac{1}{2}\right)$ with horizontal (b) 100 m **5.** 1 s and 3 s

6. $20\sqrt{2}$ m/s, $2(\sqrt{3} \pm 1)$ s **7.** $u\cos\alpha$ **8.** $2u\sin\alpha$ (downwards)

9. Between two points lying on the same horizontal line. **10.** time = 0, 0.8 s, x-coordinate = 0, 2.4 m

11. $\sqrt{3}$ s

Introductory Exercise 4.2

1. $\dfrac{10}{\sqrt{3}}$ m/s **2.** 5 m/s **3.** 20 m **4.** No **5.** 60° **6.** $\dfrac{40}{\sqrt{3}}$ m/s **7.** (a) 30° (b) $5\sqrt{3}$ m

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Subjective Questions (Level 1)

1. (a) 80 m, 20 m, 4 s (b) $(20\hat{i} + 10\hat{j})$ ms^{-1} (c) $(20\hat{i} - 20\hat{j})$ ms^{-1}

2. 5.46 s, 2.83 s, 1.46 s, 109.2 m, 56.6 m, 29.2 m

3. (a) 30 ms^{-1} (vertically downwards) (b) 20.62 ms^{-1} **4.** $\dfrac{5}{\sqrt{2}}$ ms^{-1} **5.** $t_1 = 2.19$ s, $t_2 = 8.20$ s

6. $v = \sqrt{\dfrac{a}{2c}(1 + b^2)}$ **7.** (a) 11 m (b) 23 m (c) 16.6 ms^{-1} (d) $\tan^{-1}(2)$, below horizontal

8. 2.5 m **9.** $\tan^{-1}\left(\dfrac{2}{3}\right)$ **11.** $u = \dfrac{gt\cos\beta}{2\sin(\alpha - \beta)}$ **13.** (a) 2 s (b) 19.6 m (c) 15 m

14. 3.55 s, 32.0 m/s **15.** $T = \dfrac{4(\sqrt{3}+1)}{\sqrt{3}}$ s = 1.69 s, $R = \dfrac{160}{3}(\sqrt{3} - 1)$ m = 39 m

16. $T = \dfrac{4(\sqrt{3}+1)}{\sqrt{3}}$ s = 6.31 s, $\dfrac{160}{3}(\sqrt{3} + 1)$ m = 145.71 m **17.** $T = \dfrac{4}{\sqrt{3}}$ s = 2.31 s, $R = \dfrac{160}{3}$ m = 53.33 m

18. $R = \dfrac{2u^2}{g}\tan\theta\sec\theta$ **19.** (a) zero (b) 20 ms^{-1} in horizontal direction (c) 40 m

20. (a) A vertical straight line (b) A parabola **21.** (a) 0.18 s

22. $(u_1\cos\theta_1 + u_2\cos\theta_2)\,t = 20$ ...(i) $(u_1\sin\theta_1 - u_2\sin\theta_2)\,t = 10$ ...(ii)

Objective Questions (Level 1)

1.(b)	**2.**(a)	**3.**(b)	**4.**(c)	**5.**(c)	**6.**(b)	**7.**(a)	**8.**(d)	**9.**(d)	**10.**(b)
11.(a)	**12.**(d)	**13.**(b)	**14.**(d)	**15.**(c)	**16.**(d)				

JEE Corner

Assertion and Reason

1.(d)	**2.**(a)	**3.**(a)	**4.**(b)	**5.**(d)	**6.**(a)	**7.**(c)	**8.**(a or b)	**9.**(d)	**10.**(b)

Objective Questions (Level 2)

1.(c) 2.(b) 3.(a) 4.(a) 5.(d) 6.(d) 7.(a) 8.(a) 9.(c) 10.(b)
11.(b) 12.(a)

More than One Correct Options

1. (all) 2. (a,c,d) 3. (b,c) 4. (a,b,d) 5. (c,d) 6. (all)

Match the Columns

1. (a) → (p) (b) → (s) (c) → (p) (d) → (p)
2. (a) → (s) (b) → (q) (c) → (r) (d) → (s)
3. (a) → (q) (b) → (p) (c) → (p) (d) → (s)
4. (a) → (q) (b) → (r) (c) → (q) (d) → (s)
5. (a) → (q) (b) → (s) (c) → (s) (d) → (p)
6. (a) → (r) (b) → (r) (c) → (r) (d) → (s)
7. (a) → (s) (b) → (s) (c) → (r) (d) → (p)

Subjective Questions (Level 2)

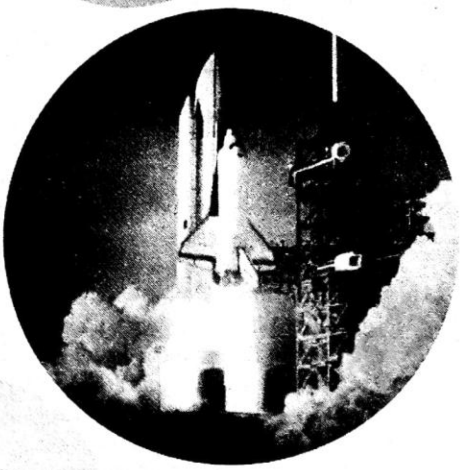
1. $V_0 = \sqrt{\dfrac{2gh}{2 + \cot^2\theta}}$ 2. $\sqrt{26}\ \text{ms}^{-1}$ at angle $\theta = \tan^{-1}(5)$ with x-axis 3. $t = \dfrac{uv\sin(\alpha - \beta)}{g(v\cos\beta - u\cos\alpha)}$

4. 4.47 m, 2.75 m 5. $\dfrac{2}{\sqrt{2}+1}$ 6. (a) 2.67 km (b) 0.9 kms^{-1}

7. (a) $(-6\hat{i} + 10\hat{j})\ \text{ms}^{-1}$ (b) (–4.5 m, 16 m, 0), (0.3 m, 8.0 m, 3.2 m) 8. (a) 250 m

9. (a) (5 m, 1.25 m) (b) 4.45 m 10. $[24\hat{i} + 15\hat{k}]$ m.

Chapter – 5 Laws of Motion



5

Laws of Motion

Chapter Contents

5.1 Types of Forces

There are basically three forces which are commonly encountered in mechanics.

(a) Field Forces

These are the forces in which contact between two objects is not necessary. Gravitational force between two bodies and electrostatic force between two charges are two examples of field forces. Weight $(W = mg)$ of a body comes in this category.

(b) Contact Forces

Two bodies in contact exert equal and opposite forces on each other. If the contact is frictionless the contact force is perpendicular to the common surface and known as **normal reaction.**

If, however, the objects are in rough contact and move (or have a tendency to move) relative to each other without losing contact then **frictional force** arise which oppose such motion. Again each object exerts a frictional force on the other and the two forces are equal and opposite. This force is perpendicular to normal reaction. Thus, the contact force (F) between two objects is made up of two forces.

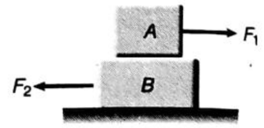


Fig. 5.1

(i) Normal reaction (N) (ii) Force of friction (f)

and since these two forces are mutually perpendicular.

$$F = \sqrt{N^2 + f^2}$$

Consider two wooden blocks A and B being rubbed against each other.

In Fig. 5.1, A is being moved to the right while B is being moved leftward. In order to see more clearly which forces act on A and which on B, a second diagram is drawn showing a space between the blocks but they are still supposed to be in contact.

In Fig. 5.2 the two normal reactions each of magnitude N are perpendicular to the surface of contact between the blocks and the two frictional forces each of magnitude f act along that surface, each in a direction opposing the motion of the block upon which it acts.

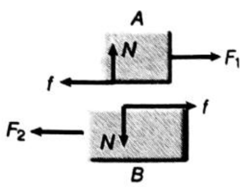


Fig. 5.2

Note *Forces on block B from the ground are not shown in the figure.*

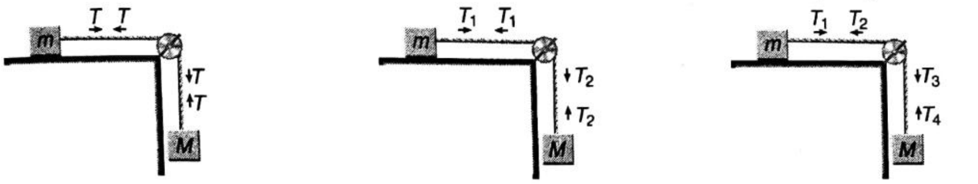
(c) Attachment to Another Body

Tension (T) in a string and **spring force** $(F = kx)$ come in this group. Regarding the tension and string, the following three points are important to remember.

1. If a string is inextensible the magnitude of acceleration of any number of masses connected through string is always same.

Fig. 5.3

2. If a string is massless, the tension in it is same everywhere. However, if a string has a mass, tension at different points will be different.
3. If there is friction between string and pulley, tension is different on two sides of the pulley, but if there is no friction between pulley and string, tension will be same on both sides of the pulley.



String is massless and there is no friction between pulley and string

String is massless and there is friction between string and pulley

String is not massless and there is friction between pulley and string

Fig. 5.4

Last two points can be understood in diagram as follows:

Spring force ($F = kx$) has been discussed in detail in the chapter of work, energy and power.

5.2 Free Body Diagram

No system, natural or man made, consists of a single body alone or is complete in itself. A single body or a part of the system can, however be isolated from the rest by appropriately accounting for its effect on the remaining system.

A free body diagram (FBD) consists of a diagrammatic representation of a single body or a sub-system of bodies isolated from its surroundings showing all the forces acting on it.

Consider, for example, a book lying on a horizontal surface.

A free body diagram of the book alone would consist of its weight ($W = mg$), acting through the centre of gravity and the reaction (N) exerted on the book by the surface.

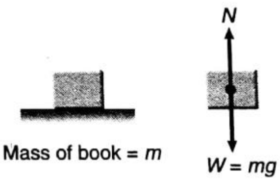


Fig. 5.5

Sample Example 5.1 *A cylinder of weight w is resting on a V-groove as shown in figure. Draw its free body diagram.*

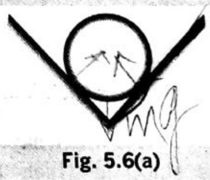


Fig. 5.6(a)

Solution The free body diagram of the cylinder is as shown in Fig. 5.6(b)

Here, w = weight of cylinder and N_1 and N_2 are the normal reactions between the cylinder and the two inclined walls.

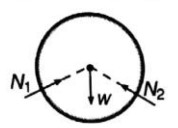


Fig. 5.6(b)

Sample Example 5.2 *Three blocks A, B and C are placed one over the other as shown in figure. Draw free body diagrams of all the three blocks.*

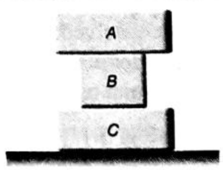


Fig. 5.7

Solution Free body diagrams of A, B and C are shown below:

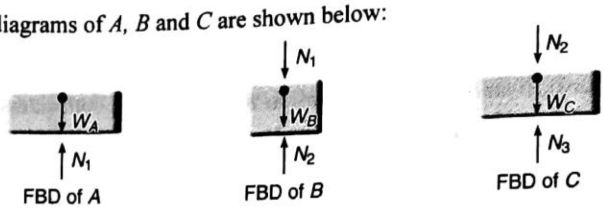


Fig. 5.8

Here, N_1 = normal reaction between A and B.

N_2 = normal reaction between B and C.

N_3 = normal reaction between C and ground.

Sample Example 5.3 *A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.*

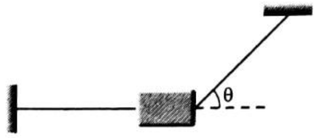


Fig. 5.9

Solution The free body diagram of the block is as shown in Fig. 5.10.

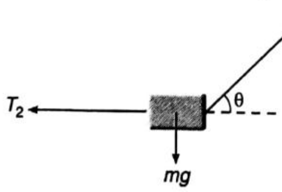


Fig. 5.10

5.3 Equilibrium

Forces which have zero linear resultant and zero turning effect will not cause any change in the motion of the object to which they are applied. Such forces (and the object) are said to be in equilibrium. For understanding the equilibrium of an object under two or more concurrent or coplanar forces let us first discuss the resolution of force and moment of a force about some point.

(a) Resolution of a Force

When a force is replaced by an equivalent set of components, it is said to be resolved. One of the most useful ways in which to resolve a force is to choose only two components (although a force may be resolved in three or more components also) which are at right angles also. The magnitude of these components can be very easily found using trigonometry.

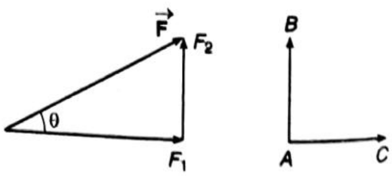


Fig. 5.11

In Fig. 5.11,

$$F_1 = F\cos\theta = \text{component of } \vec{\mathbf{F}} \text{ along } AC$$

$$F_2 = F\sin\theta = \text{component of } \vec{\mathbf{F}} \text{ perpendicular to } AC \text{ or along } AB$$

Finding such components is referred to as resolving a force in a pair of perpendicular directions. Note that the component of a force in a direction perpendicular to itself is zero. For example, if a force of 10 N is applied on an object in horizontal direction then its component along vertical is zero. Similarly, the component of a force in a direction parallel to the force is equal to the magnitude of the force. For example component of the above force in the direction of force (horizontal) will be 10 N.

Sample Example 5.4 *Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.*

Solution Component perpendicular to the plane

$$W_\perp = W\cos 30° = (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

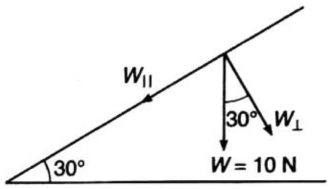


Fig. 5.12

and component parallel to the plane

$$W_{||} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

Sample Example 5.5 *Resolve horizontally and vertically a force F = 8 N which makes an angle of 45° with the horizontal.*

Solution Horizontal component of $\vec{\mathbf{F}}$ is

$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

and vertical component of $\vec{\mathbf{F}}$ is $F_V = F\sin 45°$

$$= (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

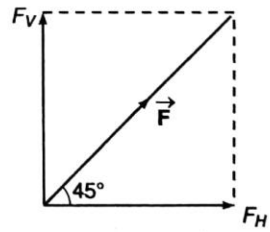


Fig. 5.13

Sample Example 5.6 *A body is supported on a rough plane inclined at 30° to the horizontal by a string attached to the body and held at an angle of 30° to the plane. Draw a diagram showing the forces acting on the body and resolve each of these forces:*

(a) horizontally and vertically,

(b) parallel and perpendicular to the plane.

Solution The forces are :

The tension in the string T.

The normal reaction with the plane N.

The weight of the body W

and the friction f.

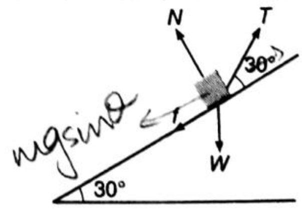


Fig. 5.14

(a) Resolving horizontally and vertically:

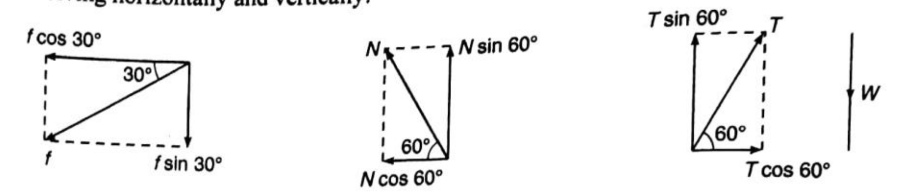


Fig. 5.15

(b) Resolving parallel and perpendicular to the plane:

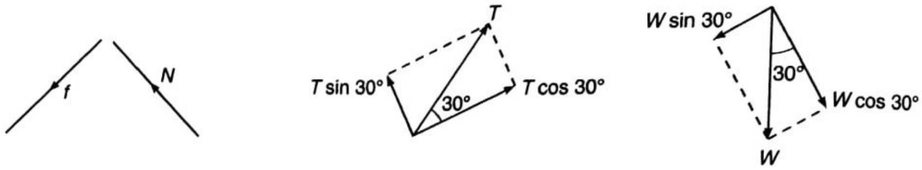


Fig. 5.16

Resolving horizontally and vertically in the senses OX and OY as shown, the components are:

Force	Components	
	Parallel to OX (horizontal)	**Parallel to OY (vertical)**
f	$-f \cos 30°$	$-f \sin 30°$
N	$-N \cos 60°$	$N \sin 60°$
T	$T \cos 60°$	$T \sin 60°$
W	0	$-W$

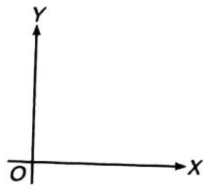


Fig. 5.17

Resolving parallel and perpendicular to the plane in the senses OX' and OY' as shown, the components are :

Force	Components	
	Parallel to OX′ (parallel to plane)	**Parallel to OY′ (perpendicular to plane)**
f	$-f$	0
N	0	N
T	$T \cos 30°$	$T \sin 30°$
W	$-W \sin 30°$	$-W \cos 30°$

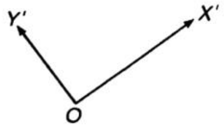


Fig. 5.18

(b) Moment of a Force

The general name given to any turning effect is **torque.** The magnitude of torque, also known as the moment of a force F is calculated by multiplying together the magnitude of the force and its perpendicular distance $r_\perp$ from the axis of rotation. This is denoted by C or τ (tau).

i.e.,

$$C = Fr_\perp \quad \text{or} \quad \tau = Fr_\perp$$

Direction of Torque

The angular direction of a torque is the sense of the rotation it would cause.

Consider a lamina that is free to rotate in its own plane about an axis perpendicular to the lamina and passing through a point A on the lamina. In the diagram the moment about the axis of rotation of the force F_1 is F_1r_1 anticlock-wise and the moment of the force F_2 is F_2r_2 clockwise. A convenient way to differentiate between clockwise and anticlock-wise torques is to allocate a positive sign to one sense (usually, but not invariably, this is anticlockwise) and negative sign to the other.

With this convention, the moments of F_1 and F_2 are $+F_1r_1$ and $-F_2r_2$ (when using a sign convention in any problem it is advisable to specify the chosen positive sense).

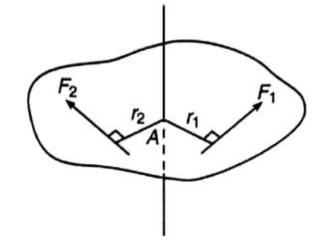


Fig. 5.19

Zero moment → doubt

If the line of action of a force passes through the axis of rotation, its perpendicular distance from the axis is zero. Therefore, its moment about that axis is also zero.

Note *Later in the chapter of rotation we will see that torque is a vector quantity.*

Sample Examle 5.7 *ABCD is a square of side 2 m and O is its centre. Forces act along the sides as shown in the diagram. Calculate the moment of each force about:*

(a) an axis through A and perpendicular to the plane of square.
(b) an axis through O and perpendicular to the plane of square.

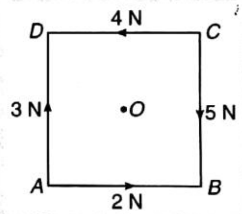


Fig. 5.20

Solution Taking anticlockwise moments as positive we have:

(a)	Magnitude of force	2 N	5 N	4 N	3 N
	Perpendicular distance from A	0	2 m	2 m	0
	Moment about A	0	−10 N-m	+8 N-m	0

(b)	Magnitude of force	2 N	5 N	4 N	3 N
	Perpendicular distance from O	1 m	1 m	1 m	1 m
	Moment about O	+2 N-m	−5 N-m	+4 N-m	−3 N-m

Sample Example 5.8 *Forces act as indicated on a rod AB which is pivoted at A. Find the anticlockwise moment of each force about the pivot.*

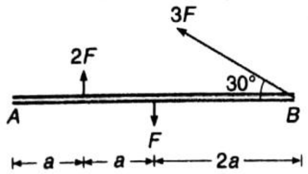


Fig. 5.21

Solution

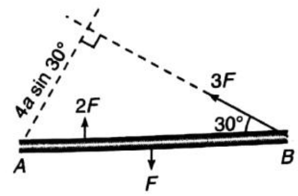


Fig. 5.22

Magnitude of force	2F	F	3F
Perpendicular distance from A	a	$2a$	$4a \sin 30° = 2a$
Anticlockwise moment about A	$+2\,Fa$	$-2\,Fa$	$+6\,Fa$

Coplanar Forces in Equilibrium

When an object is in equilibrium under the action of a set of two or more coplanar forces, each of three factors which comprise the possible movement of the object must be zero, *i.e.*, the object has

(i) no linear movement along any two mutually perpendicular directions ox and oy.

(ii) no rotation about any axis.

The set of forces must, therefore, be such that

(a) the algebraic sum of the components parallel to ox is zero or $\Sigma F_x = 0$

(b) the algebraic sum of the components parallel to oy is zero or $\Sigma F_y = 0$

(c) the resultant moment about any specified axis is zero or $\Sigma \tau_{\text{any axis}} = 0$

Thus, for the equilibrium of a set of two or more coplanar forces:

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma \tau_{\text{any axis}} = 0$$

Using the above three conditions, we get only three set of equations. So, in a problem number of unknowns should not be more than three.

Sample Example 5.9 *A rod AB rests with the end A on rough horizontal ground and the end B against a smooth vertical wall. The rod is uniform and of weight W. If the rod is in equilibrium in the position shown in figure. Find :*

(a) frictional force at A

(b) normal reaction at A

(c) normal reaction at B.

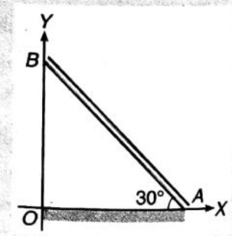


Fig. 5.23

Solution Let length of the rod be $2l$. Using the three conditions of equilibrium. Anticlockwise moment is taken as positive.

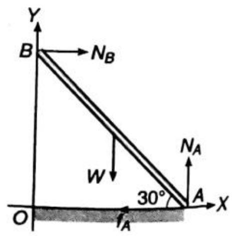


Fig. 5.24

(i) $\Sigma F_x = 0$

$\therefore \qquad N_B - f_A = 0$

or $\qquad N_B = f_A \qquad \ldots\text{(i)}$

(ii) $\Sigma F_y = 0$

$\therefore$ $$N_A - W = 0$$

or $$N_A = W \quad \text{...(ii)}$$

(iii) $\Sigma \tau_o = 0$

$\therefore$ $$N_A\,(2l \cos 30°) - N_B\,(2l \sin 30°) - W(l \cos 30°) = 0$$

or $$\sqrt{3}N_A - N_B - \frac{\sqrt{3}}{2}W = 0 \quad \text{...(iii)}$$

Solving these three equations, we get

(a) $f_A = \frac{\sqrt{3}}{2}W$ (b) $N_A = W$ (c) $N_B = \frac{\sqrt{3}}{2}W$

Equilibrium of Concurrent Coplanar Forces

If an object is in equilibrium under two or more concurrent coplanar forces the algebraic sum of the components of forces in any two mutually perpendicular directions *ox* and *oy* should be zero, *i.e.*, the set of forces must be such that:

(a) the algebraic sum of the components parallel to *ox* is zero, *i.e.*, $\Sigma F_x = 0$.

(b) the algebraic sum of the components parallel to *oy* is zero, *i.e.*, $\Sigma F_y = 0$.

Thus, for the equilibrium of two or more concurrent coplanar forces:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

The third condition of zero moment about any specified axis is automatically satisfied if the moment is taken about the point of intersection of the forces. So, here we get only two equations. Thus, number of unknown in any problem should not be more than two.

Sample Example 5.10 *An object is in equilibrium under four concurrent forces in the directions shown in figure. Find the magnitude of $\vec{F}_1$ and $\vec{F}_2$.*

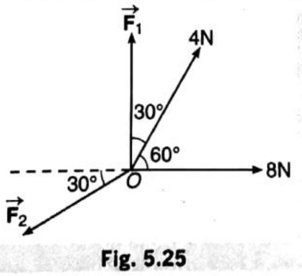


Fig. 5.25

Solution The object is in equilibrium. Hence,

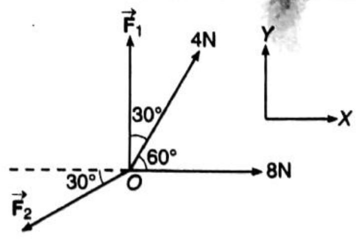


Fig. 5.26

(i) $\Sigma F_x = 0$

$\therefore$ $$8 + 4\cos 60° - F_2 \cos 30° = 0$$

or $$8 + 2 - F_2\frac{\sqrt{3}}{2} = 0$$

or $$F_2 = \frac{20}{\sqrt{3}}\text{ N}$$

(ii) $\Sigma F_y = 0$

$\therefore$ $$F_1 + 4\sin 60° - F_2 \sin 30° = 0$$

or $$F_1 + \frac{4\sqrt{3}}{2} - \frac{F_2}{2} = 0$$

or $$F_1 = \frac{F_2}{2} - 2\sqrt{3} = \frac{10}{\sqrt{3}} - 2\sqrt{3}$$

or $$F_1 = \frac{4}{\sqrt{3}} \text{ N}$$

Lami's Theorem : If an object O is in equilibrium under three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ as shown in figure. Then,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

This property of three concurrent forces in equilibrium is known as Lami's theorem and is very useful method of solving problems related to three concurrent forces in equilibrium.

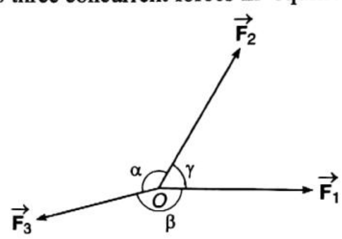


Fig. 5.27

Sample Example 5.11 *One end of a string 0.5 m long is fixed to a point A and the other end is fastened to a small object of weight 8 N. The object is pulled aside by a horizontal force F, until it is 0.3 m from the vertical through A. Find the magnitudes of the tension T in the string and the force F.*

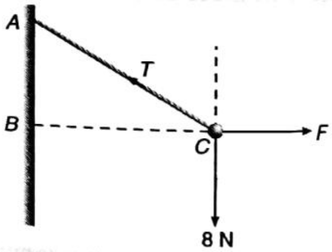


Fig. 5.28

Solution $AC = 0.5\text{ m}$, $BC = 0.3\text{ m}$

$\therefore$ $$AB = 0.4 \text{ m}$$

and if $$\angle BAC = \theta.$$

Then $$\cos\theta = \frac{AB}{AC} = \frac{0.4}{0.5} = \frac{4}{5}$$

and $$\sin\theta = \frac{BC}{AC} = \frac{0.3}{0.5} = \frac{3}{5}$$

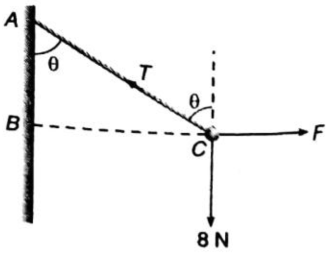


Fig. 5.29

Here, the object is in equilibrium under three concurrent forces. So, we can apply Lami's theorem.

or $$\frac{F}{\sin(180^\circ - \theta)} = \frac{8}{\sin(90^\circ + \theta)} = \frac{T}{\sin 90^\circ}$$

or $$\frac{F}{\sin\theta} = \frac{8}{\cos\theta} = T$$

$\therefore$ $$T = \frac{8}{\cos\theta} = \frac{8}{4/5} = 10\,\text{N}$$

and $$F = \frac{8\sin\theta}{\cos\theta} = \frac{(8)(3/5)}{(4/5)} = 6\,\text{N}$$ **Ans.**

Introductory Exercise 5.1

1. The diagram shows a rough plank resting on a cylinder with one end of the plank on rough ground. Neglect friction between plank and cylinder. Draw diagrams to show:
   (a) the forces acting on the plank,
   (b) the forces acting on the cylinder.

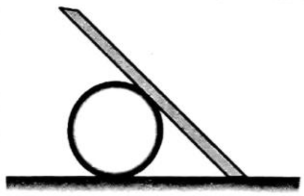


Fig. 5.30

2. Two spheres A and B are placed between two vertical walls as shown in figure. Friction is absent everywhere. Draw the free body diagrams of both the spheres.

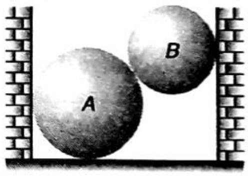


Fig. 5.31

3. A point A on a sphere of weight W rests in contact with a smooth vertical wall and is supported by a string joining a point B on the sphere to a point C on the wall. Draw free body diagram of the sphere.

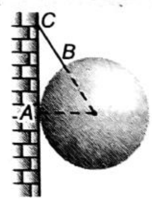


Fig. 5.32

4. Write down the components of four forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ along ox and oy directions as shown in Fig. 5.33.

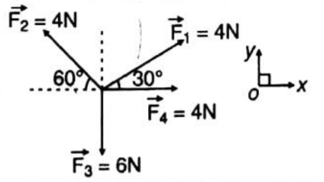


Fig. 5.33

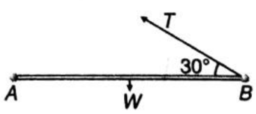


Fig. 5.34

5. A uniform rod AB of weight W is hinged to a fixed point at A. It is held in the horizontal position by a string, one end of which is attached to B as shown in Fig. 5.34. Find in terms of W, the tension in the string.
6. In Question 3 of the same exercise the radius of the sphere is a. The length of the string is also a. Find tension in the string.
7. Find the values of the unknown forces if the given set of forces shown in Fig. 5.35 are in equilibrium.

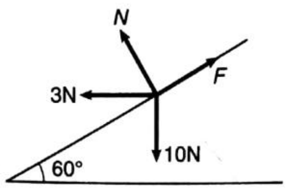


Fig. 5.35

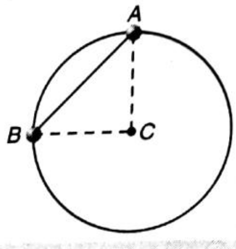


Fig. 5.36

8. Two beads of equal masses m are attached by a string of length $\sqrt{2}a$ and are free to move in a smooth circular ring lying in a vertical plane as shown in Fig. 5.36. Here, a is the radius of the ring. Find the tension and acceleration of B just after the beads are released to move.

5.4 Newton's Laws of Motion

It is interesting to read Newton's original version of the laws of motion.

Law I. Every body continues in its state of rest or in uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.

Law II. The change of motion is proportional to the magnitude of force impressed; and is made in the direction of the straight line in which that force is impressed.

Law III. To every action there is always an equal and opposite reaction; or, the mutual actions of two bodies upon each other are always directed to contrary parts.

The modern version of these laws is:

1. A body continues in its initial state of rest or motion with uniform velocity unless acted on by an unbalanced external force.
2. The acceleration of a body is inversely proportional of its mass and directly proportional to the resultant external force acting on it, *i.e.*,

$$\Sigma \vec{F} = \vec{F}_{net} = m\vec{a} \quad \text{or} \quad \vec{a} = \frac{\vec{F}_{net}}{m}$$

3. Forces always occur in pairs. If body A exerts a force on body B, an equal but opposite force is exerted by body B on body A.

Working with Newton's First and Second Laws

Normally any problem relating to Newton's laws is solved in following four steps:

1. First of all we decide the system on which the laws of motion are to be applied. The system may be a single particle, a block or a combination of two or more blocks, two blocks connected by a string, etc. The only restriction is that all parts of the system should have the same acceleration.
2. Once the system is decided, we make the list of all the forces acting on the system. Any force applied by the system on other bodies is not included in the list of the forces.
3. Then we make a free body diagram of the system and indicate the magnitude and directions of all the forces listed in step 2 in this diagram.

4. In the last step we choose any two mutually perpendicular axes say x and y in the plane of the forces in case of coplanar forces. Choose the x-axis along the direction in which the system is known to have or is likely to have the acceleration. A direction perpendicular to it may be chosen as the y-axis. If the system is in equilibrium, any mutually perpendicular directions may be chosen. Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration, *i.e.*,

$$\Sigma F_x = ma \qquad \ldots\text{(i)}$$

This gives us one equation. Now, we write the components of the forces along the y-axis and equate the sum to zero. This gives us another equation, *i.e.*,

$$\Sigma F_y = 0 \qquad \ldots\text{(ii)}$$

Note (i) *If the system is in equilibrium we will write the two equations as:*

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

(ii) *If the forces are collinear, the second equation, i.e.,* $\Sigma F_y = 0$ *is not needed.*

Sample Example 5.12 *Two blocks of mass 4 kg and 2 kg are placed side by side on a smooth horizontal surface as shown in the figure. A horizontal force of 20 N is applied on 4 kg block. Find :*

(a) the acceleration of each block.

(b) the normal reaction between two blocks.

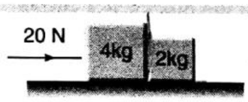


Fig. 5.37

Solution (a) Since, both the blocks will move with same acceleration (say a) in horizontal direction.

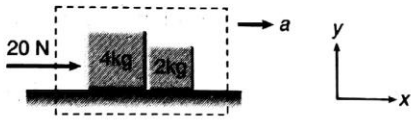


Fig. 5.38

Let us take both the blocks as a system. Net external force on the system is 20 N in horizontal direction.

Using $\Sigma F_x = ma_x$

$$20 = (4+2)a = 6a$$

or $a = \dfrac{10}{3}\ \text{m/s}^2$

(b) The free body diagram of both the blocks are as shown in Fig. 5.39.

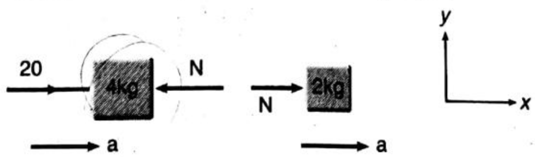


Fig. 5.39

Using $\Sigma F_x = ma_x$

For 4 kg block $\quad 20 - N = 4a = 4 \times \dfrac{10}{3}$

$$N = 20 - \frac{40}{3} = \frac{20}{3}\ \text{N}$$

This can also be solved as under

For 2 kg block $N = 2a = 2 \times \frac{10}{3} = \frac{20}{3}$ N

Here, N is the normal reaction between the two blocks.

Note *In free body diagram of the blocks we have not shown the forces acting on the blocks in vertical direction, because normal reaction between the blocks and acceleration of the system can be obtained without using* $\Sigma F_y = 0$.

Sample Example 5.13 *Three blocks of mass 3 kg, 2 kg and 1 kg are placed side by side on a smooth surface as shown in figure. A horizontal force of 12 N is applied on 3 kg block. Find the net force on 2 kg block.*

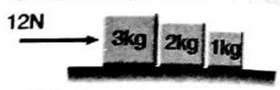


Fig. 5.40

Solution Since, all the blocks will move with same acceleration (say a) in horizontal direction. Let us take all the blocks as a system.

Net external force on the system is 12 N in horizontal direction.

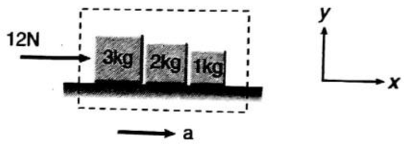


Fig. 5.41

Using $\Sigma F_x = ma_x$, we get

$$12 = (3 + 2 + 1)a = 6a$$

or $a = \frac{12}{6} = 2 \text{ m/s}^2$

Now, let F be the net force on 2 kg block in x-direction, then using $\Sigma F_x = ma_x$ for 2 kg block, we get

$$F = (2)(2) = 4 \text{ N}$$

Note *Here, net force F on 2 kg block is the resultant of* N_1 *and* N_2 $(N_1 > N_2)$

where N_1 = *normal reaction between 3 kg and 2 kg block,*

and N_2 = *normal reaction between 2 kg and 1 kg block.*

Thus, $F = N_1 - N_2$

Sample Example 5.14 *In the arrangement shown in figure. The strings are light and inextensible. The surface over which blocks are placed is smooth. Find :*

(a) the acceleration of each block,

(b) the tension in each string.

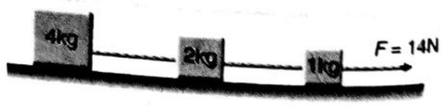


Fig. 5.42

Solution (a) Let 'a' be the acceleration of each block and T_1 and T_2 be the tensions, in the two strings as shown in figure.

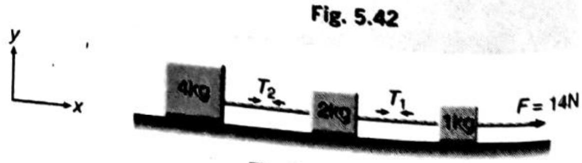


Fig. 5.43

Taking the three blocks and the two strings as the system.

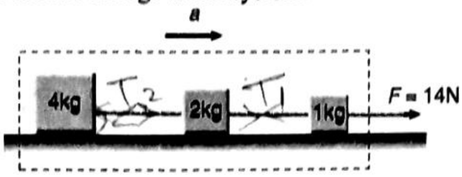


Fig. 5.44

Using $\quad \Sigma F_x = ma_x$

or $\quad 14 = (4 + 2 + 1)a$

or $\quad a = \dfrac{14}{7} = 2 \text{ m/s}^2$

(b) Free body diagram (showing the forces in x-direction only) of 4 kg block and 1 kg block are shown in figure.

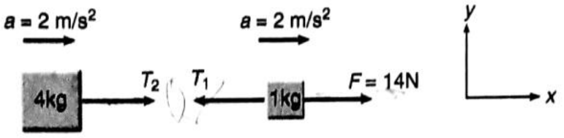


Fig. 5.45

Using $\quad \Sigma F_x = ma_x$

For 1 kg block, $\quad F - T_1 = (1)(a)$

or $\quad 14 - T_1 = (1)(2) = 2$

$\therefore \quad T_1 = 14 - 2 = 12 \text{ N}$

For 4 kg block, $\quad T_2 = (4)(a)$

$\therefore \quad T_2 = (4)(2) = 8 \text{ N}$

Sample Example 5.15 *Two blocks of mass 4 kg and 2 kg are attached by an inextensible light string as shown in figure. Both the blocks are pulled vertically upwards by a force $F = 120$ N. Find :*

(a) the acceleration of the blocks,

(b) tension in the string. (Take $g = 10$ m/s^2).

Fig. 5.46

Solution (a) Let a be the acceleration of the blocks and T the tension in the string as shown in figure.

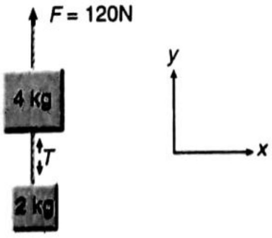


Fig. 5.47

Taking the two blocks and the string as the system.
Using $\Sigma F_y = ma_y$, we get

$$F - 4g - 2g = (4 + 2)a$$

or $$120 - 40 - 20 = 6a$$

or $$60 = 6a$$

$\therefore$ $$a = 10 \text{ m/s}^2$$

(b) Free body diagram of 2 kg block is as shown in Fig. 5.49.

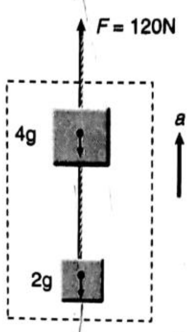


Fig. 5.48

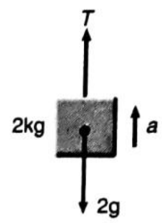


Fig. 5.49

Using $$\Sigma F_y = ma_y \text{ we get}$$

$$T - 2g = 2a$$

or $$T - 20 = (2)(10)$$

$\therefore$ $$T = 40 \text{ N}$$

Note *If the string is having some mass, tension in it is different at different points. Under such condition tension on the string at some point is calculated as under :*

In the adjoining figure the length of the string connecting the two blocks is 2 m and mass is 2 kg. Tension at A, B and C (centre point) can be calculated by considering the motion of system below A, B and C. For example :

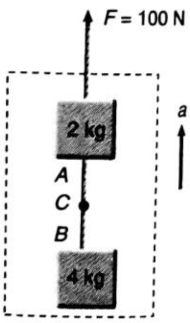


Fig. 5.50

$(g = 10 \text{ m/s}^2)$

$$a = \frac{F - \text{weight of 2 kg} - \text{weight of 4 kg} - \text{weight of string}}{\text{mass of 2 kg} + \text{mass of 4 kg} + \text{mass of string}}$$

$$= \frac{100 - 20 - 40 - 20}{2 + 4 + 2}$$

$$= \frac{20}{8} = 2.5 \text{ m/s}^2$$

Refer Fig. (a)

$$T_A - m_{AB}g - 40 = (m_{AB} + 4)a$$

or $$T_A - 20 - 40 = (2 + 4)(2.5)$$

or $$T_A = 75 \text{ N}$$

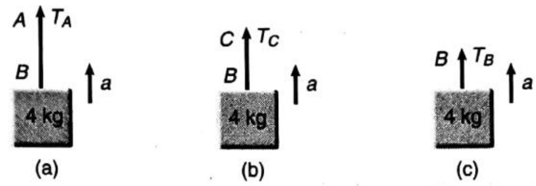


Fig. 5.51

Refer Fig. (b)

$$T_C - m_{BC}g - 40 = (m_{BC} + 4)a$$

or $$T_C - 10 - 40 = (1 + 4)(2.5)$$

or $$T_C = 62.5\text{ N}$$

Refer Fig. (c) $$T_B - 40 = 4a$$

or $$T_B = 40 + 4 \times 2.5$$

or $$T_B = 50\text{ N}$$

Introductory Exercise 5.2

1. Three blocks of mass 1 kg, 4 kg and 2 kg are placed on a smooth horizontal plane as shown in figure. Find:
(a) the acceleration of the system,
(b) the normal force between 1 kg block and 4 kg block,
(c) the net force on 2 kg block.

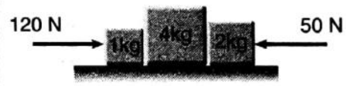


Fig. 5.52

2. Two blocks of mass 2 kg and 4 kg are released from rest over a smooth inclined plane of inclination 30° as shown in figure. What is the normal force between the two blocks?

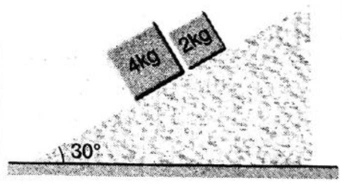


Fig. 5.53

3. What should be the acceleration 'a' of the box shown in figure so that the block of mass m exerts a force $\frac{mg}{4}$ on the floor of the box?

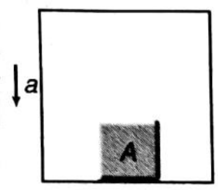


Fig. 5.54

4. A plumb bob of mass 1 kg is hung from the ceiling of a train compartment. The train moves on an inclined plane with constant velocity. If the angle of incline is 30°. Find the angle made by the string with the normal to the ceiling. Also, find the tension in the string. (g = 10 m/s^2)

5. Repeat both parts of the above question, if the train moves with an acceleration $a = g/2$ up the plane.

6. Two blocks of mass 1 kg and 2 kg are connected by a string AB of mass 1 kg. The blocks are placed on a smooth horizontal surface. Block of mass 1 kg is pulled by a horizontal force F of magnitude 8 N. Find the tension in the string at points A and B.

Fig. 5.55

5.5 Pulleys

As an author I personally feel that problems based on pulleys become very simple using **pulling force** method. Now, let us see what is this pulling force method with the help of an example.

Suppose two unequal masses m and $2m$ are attached to the ends of a light inextensible string which passes over a smooth massless pulley. We have to find the acceleration of the system. We can assume that the mass $2m$ is pulled downwards by a force equal to its weight, *i.e.*, $2mg$. Similarly, the mass m is being pulled by a force of mg downwards. Therefore, net pulling force on the system is $2mg - mg = mg$ and total mass being pulled is $2m + m = 3m$.

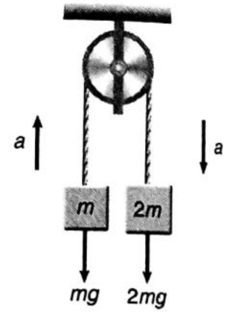


Fig. 5.56

$\therefore$ Acceleration of the system is

$$a = \frac{\text{Net pulling force}}{\text{Total mass to be pulled}}$$

$$= \frac{mg}{3m} = \frac{g}{3}$$

Let us now take few examples based on pulling force method.

Note *While considering net pulling force, take the forces (or their components) which are in the direction of motion (or opposite to it) and are single (i.e., they are not forming pairs of equal and oppsite forces). For example weight (mg) or some applied force F. Tension makes an equal and opposite pair. So, they will not be included, unless the system in broken at some place and only one tension is considered on the remaining system.*

Sample Example 5.16 *In the system shown in figure pulley is smooth. String is massless and inextensible. Find acceleration of the system a, tensions T_1 and T_2. ($g = 10\ m/s^2$)*

Fig. 5.57

Solution Here, net pulling force will be:

Weight of 4 kg and 6 kg blocks on one side – weight of 2 kg block on the other side. Therefore,

$$a = \frac{\text{Net pulling force}}{\text{Total mass}}$$

$$= \frac{(6 \times 10) + (4 \times 10) - (2)(10)}{6 + 4 + 2}$$

$$= \frac{20}{3} \text{ m/s}^2$$

For T_1, let us consider FBD of 2 kg block. Writing equation of motion, we get

$$T_1 - 20 = 2a$$

or $$T_1 = 20 + 2 \times \frac{20}{3} = \frac{100}{3} \text{ N}$$

For T_2, we may consider FBD of 6 kg block. Writing equation of motion, we get

$$60 - T_2 = 6a$$

$\therefore$ $$T_2 = 60 - 6a$$

$$= 60 - 6\left(\frac{20}{3}\right)$$

$$= \frac{60}{3} \text{ N}$$

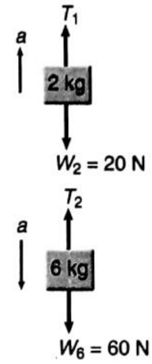


Fig. 5.58

Exercise : Draw FBD of 4 kg block. Write down the equation of motion for it and check whether the values calculated above are correct or not.

Sample Example 5.17 *In the system shown in figure all surfaces are smooth. String is massless and inextensible. Find acceleration a of the system and tension T in the string.* ($g = 10\, m/s^2$)

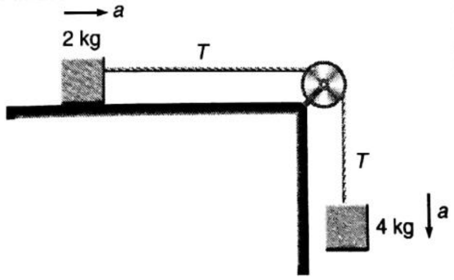


Fig. 5.59

Solution Here, weight of 2 kg is perpendicular to motion (or a). Hence, it will not contribute in net pulling force. Only weight of 4 kg block will be included.

$\therefore$ $$a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{(4)(10)}{(4+2)}$$

$$= \frac{20}{3} \text{ m/s}^2$$

For T, consider FBD of 4 kg block. Writing equation of motion.

$$40 - T = 4a$$

$$\therefore \quad T = 40 - 4a$$

$$= 40 - 4\left(\frac{20}{3}\right)$$

$$= \frac{40}{3}\text{ N}$$

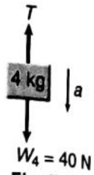


Fig. 5.60

Exercise : Draw FBD of 2 kg block and write down equation of motion for it. Check whether the values calculated above are correct or not.

Sample Example 5.18 *In the adjacent figure, mass of A, B and C are 1 kg, 3 kg and 2 kg respectively. Find :*

(a) the acceleration of the system and

(b) tension in the string.

Neglect friction. $(g = 10\ m/s^2)$

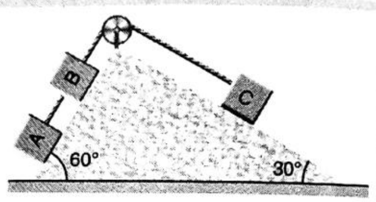


Fig. 5.61

Solution (a) In this case net pulling force

$$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$$

$$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$

$$= 24.64\text{ N}$$

Total mass being pulled $= 1 + 3 + 2 = 6$ kg

$\therefore$ Acceleration of the system $a = \dfrac{21.17}{6} = 4.1\ \text{m/s}^2$

(b) For the tension in the string between A and B.

FBD of A

$$m_A\ g \sin 60° - T_1 = (m_A)(a)$$

$$\therefore \quad T_1 = m_A\ g \sin 60° - m_A\ a$$

$$= m_A\ (g \sin 60° - a)$$

$$\therefore \quad T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right)$$

$$= 4.56\text{ N}$$

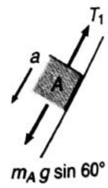


Fig. 5.62

For the tension in the string between B and C.

FBD of C

$$T_2 - m_C g \sin 30° = m_C\, a$$

$$\therefore \quad T_2 = m_C\,(a + g \sin 30°)$$

$$\therefore \quad T_2 = 2\left[3.53 + 10\left(\frac{1}{2}\right)\right]$$

$$= 18.2\text{ N}$$

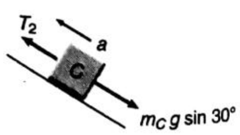


Fig. 5.63

Introductory Exercise 5.3

1. In the arrangement shown in figure what should be the mass of block A, so that the system remains at rest? Neglect friction and mass of strings.

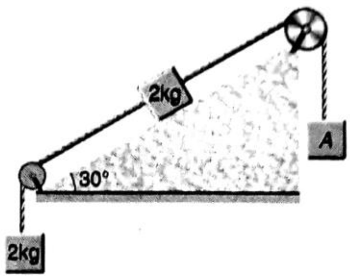


Fig. 5.64

2. In the arrangement shown in figure, find the ratio of tensions in the strings attached with 4 kg block and that with 1 kg block.

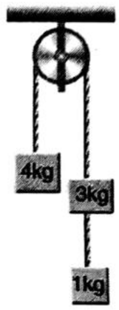


Fig. 5.65

3. Two unequal masses of 1 kg and 2 kg are connected by a string going over a clamped light smooth pulley as shown in figure. The system is released from rest. The larger mass is stopped for a moment 1.0 s after the system is set in motion. Find the time elapsed before the string is tight again.

Fig. 5.66

4. Two unequal masses of 1 kg and 2 kg are connected by an inextensible light string passing over a smooth pulley as shown in figure. A force $F = 20$ N is applied on 1 kg block. Find the acceleration of either block. ($g = 10\ m/s^2$).

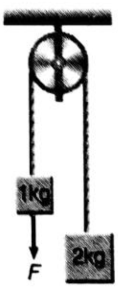


Fig. 5.67

5.6 Constraint Equations

These equations basically establish the relation between accelerations (or velocities) of different masses attached by string(s). Usually it is observed that the number of constraint equations are as many as the number of strings in the system under consideration. From the following few examples we can better understand the method.

Sample Example 5.19 *Using constraint method find the relation between accelerations of* 1 *and* 2.

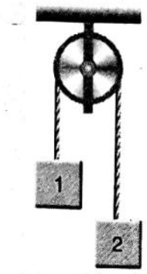


Fig. 5.68

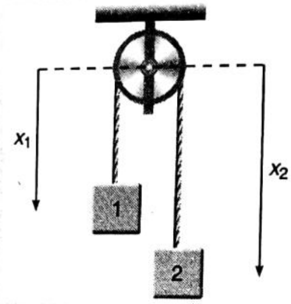


Fig. 5.69

Solution At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line (shown dotted).

Then,

$$x_1 + x_2 = \text{constant}$$

or

$$x_1 + x_2 = l \qquad \text{(length of string)}$$

Differentiating with respect to time, we have

$$v_1 + v_2 = 0 \quad \text{or} \quad v_1 = -v_2$$

Again differentiating with respect to time, we get

$$a_1 + a_2 = 0 \quad \text{or} \quad a_1 = -a_2$$

This is the required relation between a_1 and a_2, *i.e.*, accelerations of 1 and 2 are equal but in opposite directions.

Sample Example 5.20 *Find the constraint relation between* a_1, a_2 *and* a_3.

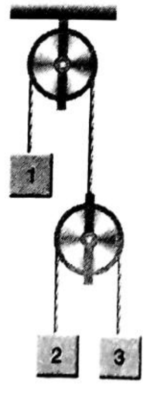


Fig. 5.70

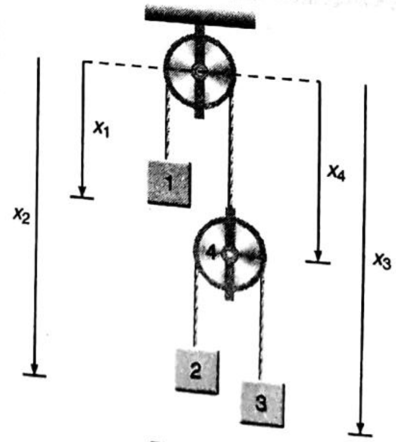


Fig. 5.71

Solution Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4.

We have,

$$x_1 + x_4 = l_1 \quad \text{(length of first string)} \quad ...(i)$$

and $$(x_2 - x_4) + (x_3 - x_4) = l_2 \quad \text{(length of second string)}$$

or $$x_2 + x_3 - 2x_4 = l_2 \quad ...(ii)$$

On double, differentiating with respect to time, we get

$$a_1 + a_4 = 0 \quad ...(iii)$$

and $$a_2 + a_3 - 2a_4 = 0 \quad ...(iv)$$

But since $$a_4 = -a_1 \quad \text{[From Eq. (iii)]}$$

We have, $$a_2 + a_3 + 2a_1 = 0$$

This is the required constraint relation between a_1, a_2 and a_3.

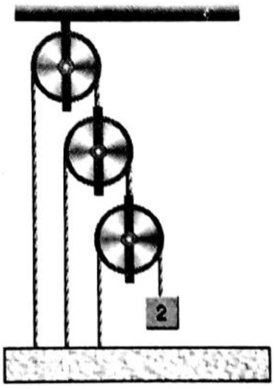


Fig. 5.72

Sample Example 5.21 *Using constraint equations find the relation between a_1 and a_2.*

Solution Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4

$$x_1 + x_3 = l_1$$

$$(x_1 - x_3) + (x_4 - x_3) = l_2$$

$$(x_1 - x_4) + (x_2 - x_4) = l_3$$

On double differentiating with respect to time, we will get following three constraint relations

$$a_1 + a_3 = 0 \quad ...(i)$$

$$a_1 + a_4 - 2a_3 = 0 \quad ...(ii)$$

$$a_1 + a_2 - 2a_4 = 0 \quad ...(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$a_2 = -7a_1$$

Which is the desired relation between a_1 and a_2.

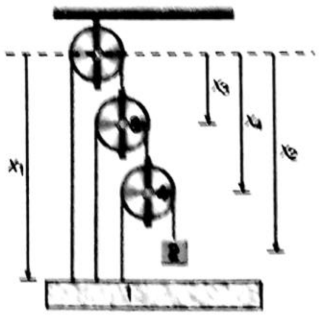


Fig. 5.73

Sample Example 5.22 *At certain moment of time, velocities of* 1 *and* 2 *both are* 1 m/s *upwards. Find the velocity of* 3 *at that moment.*

Solution In Instance 20, we have found

$$a_2 + a_3 + 2a_1 = 0$$

Similarly, we can find

$$v_2 + v_3 + 2v_1 = 0$$

Taking, upward direction as positive we are given:

$$v_1 = v_2 = 1 \text{ m/s}$$

$$\therefore \quad v_3 = -3 \text{ m/s}$$

i.e., velocity of block 3 is 3 m/s (downwards).

Fig. 5.74

Introductory Exercise 5.4

1. Consider the situation shown in figure. Both the pulleys and the string are light and all the surfaces are smooth.
   (a) Find the acceleration of 1 kg block.
   (b) Find the tension in the string.
   $(g = 10 \text{ m/s}^2)$.

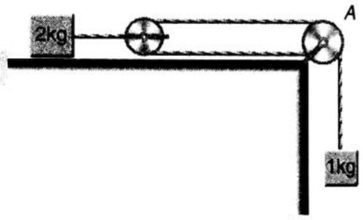


Fig. 5.75

2. Calculate the acceleration of either blocks and tension in the string shown in figure. The pulley and the string are light and all surfaces are smooth.

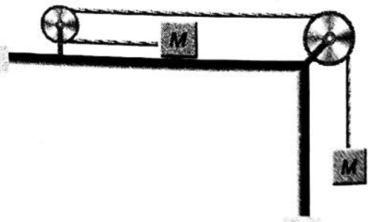


Fig. 5.76

3. Find the mass M so that it remains at rest in the adjoining figure. Both the pulley and string are light and friction is absent everywhere. $(g = 10 \text{ m/s}^2)$.

Fig. 5.77

4. In figure assume that there is negligible friction between the blocks and table. Compute the tension in the cord connecting m_2 and the pulley and acceleration of m_2 if $m_1 = 300g$, $m_2 = 200g$ and $F = 0.40N$.

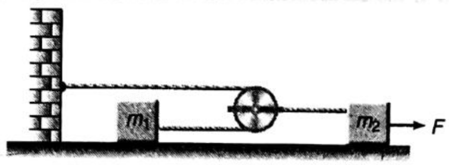


Fig. 5.78

5.7 Pseudo Force

Before studying pseudo force let us first discuss frame of reference. A system of coordinate axes which defines the position of a particle or an event in two or three dimensional space is called a frame of reference. The simplest frame of reference is, of course, the familiar cartesian system of coordinates, in which the position of the particle is specified by its three coordinates x, y and z. Frame of references are of two types :

(a) Inertial frame of reference

A non-accelerating frame of reference is called an inertial frame of reference. A frame of reference moving with a constant velocity is an inertial frame of reference.

(b) Non-inertial frame of reference

An accelerating frame of reference is called a non-inertial frame of reference.

Note *A rotating frame of reference is a non-inertial frame of reference, because it is also an accelerating one.*

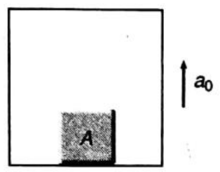


Fig. 5.79

Now, let us come to the pseudo force. Newton's first two laws hold good in an intertial frame only. However, we people spend most of our time on the earth which is an (approximate) inertial frame. We are so familiar with the Newton's laws that we will still like to use 'total force equals mass times acceleration' even when we use a non-inertial frame. This can be done if we agree to call $(-m\vec{\mathbf{a}}_0)$ a force acting on the particle. Then while preparing the list of the forces acting on a particle P, we include all the (real) forces acting on P by all other objects and also include an imaginary force $-m\vec{\mathbf{a}}_0$. Here, $\vec{\mathbf{a}}_0$ is the acceleration of the non-inertial frame under consideration. After applying this additional imaginary force (called pseudo force) $-m\vec{\mathbf{a}}_0$ we can now use 'total force equals mass time acceleration' even in non-inertial frames also. Now, with the help of a simple example let us see what problem arises if we don't apply the pseudo force $-m\vec{\mathbf{a}}_0$ while using $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ (second law) in non-inertial frame. Suppose a block A of mass m is placed on a lift ascending with an acceleration a_0. Let N be the normal reaction between the block and the floor of the lift. Free body diagram of A in ground frame of reference (inertial) is shown in Fig. 5.80.

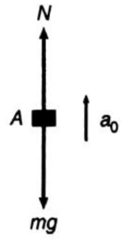


Fig. 5.80

$\therefore$ $$N - mg = ma_0$$

or $$N = m(g + a_0) \quad \text{...(i)}$$

But if we draw the free body diagram of A with respect to the elevator (a non-inertial frame of reference) without applying the pseudo force, as shown in Fig. 5.81, we get

$$N' - mg = 0 \quad \text{or} \quad N' = mg \qquad \ldots\text{(ii)}$$

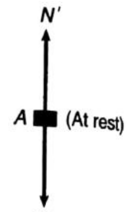


Fig. 5.81

Since, $N' \neq N$, either of the equations is wrong. But if we apply a pseudo force in non-inertial frame of reference, N' becomes equal to N as shown in Fig. 5.82. Acceleration of block with respect to elevator is zero.

$\therefore \quad N' - mg - ma_0 = 0$

or $\quad N' = m(g + a_0) \qquad \ldots\text{(iii)}$

$\therefore \quad N' = N$

Pseudo force is given by $\vec{\mathbf{F}}_P = -m\vec{\mathbf{a}}_0$. Here, $\vec{\mathbf{a}}_0$ is the acceleration of the non-inertial frame of reference and m the mass of the body under consideration. In the whole chapter, we will show the pseudo force by $\vec{\mathbf{F}}_P$.

Thus, we may conclude that pseudo force is not a real force. When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass), but when the free body diagram is drawn from a non-inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, valid in this frame also.

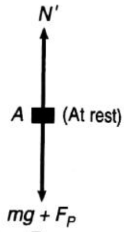


Fig. 5.82

Note *In case of rotating frame of reference this pseudo force is called the centrifugal force when applied for centripetal acceleration. Let us take few examples of pseudo forces.*

Sample Example 5.23 *In the adjoining figure, the coefficient of friction between wedge (of mass M) and block (of mass m) is μ.*

Find the minimum horizontal force F required to keep the block stationary with respect to wedge.

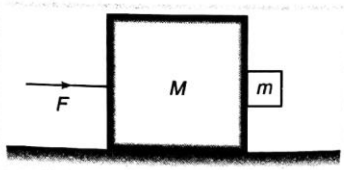


Fig. 5.83

Solution Such problems can be solved with or without using the concept of pseudo force. Let us, solve the problem by both the methods.

$$a = \text{acceleration of (wedge + block) in horizontal direction} = \frac{F}{M + m}.$$

Inertial frame of reference (Ground)

FBD of block with respect to ground (only real forces have to applied) with respect to ground block is moving with an acceleration 'a'. Therefore,

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma F_x = ma$$

$$mg = \mu N \quad \text{and} \quad N = ma$$

$$\therefore \quad a = \frac{g}{\mu}$$

$$\therefore \quad F = (M + m)a = (M + m)\frac{g}{\mu}$$

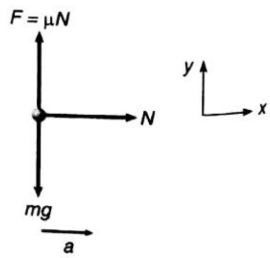


Fig. 5.84

Non-inertial frame of reference (Wedge)

FBD of 'm' with respect to wedge (real + one pseudo force) with respect to wedge block is stationary.

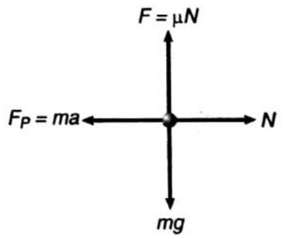


Fig. 5.85

$\therefore \quad \Sigma F_x = 0 = \Sigma F_y$

$\therefore \quad mg = \mu N \quad \text{and} \quad N = ma$

$\therefore \quad a = \dfrac{g}{\mu}$

and $\quad F = (M + m)a$

$= (M + m)\dfrac{g}{\mu}$

From the above discussion, we can see that from both the methods results are same.

Sample Example 5.24 *All surfaces are smooth in following figure. Find F, such that block remains stationary with respect to wedge.*

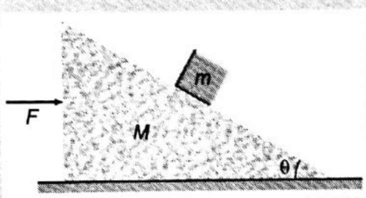


Fig. 5.86

Solution Acceleration of (block + wedge) $a = \dfrac{F}{(M + m)}$

Let us solve the problem by both the methods.

From inertial frame of reference (Ground)

FBD of block w.r.t. ground (Apply real forces):

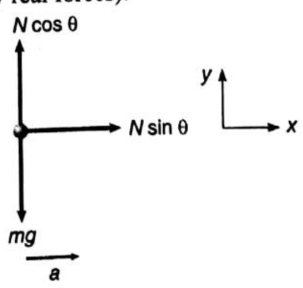


Fig. 5.87

With respect to ground block is moving with an acceleration a.

$\therefore \quad \Sigma F_y = 0 \Rightarrow N\cos\theta = mg \quad$ ...(i)

and $\quad \Sigma F_x = ma \Rightarrow N\sin\theta = ma \quad$ ...(ii)

From Eqs. (i) and (ii)

$$a = g \tan\theta$$

$$\therefore \quad F = (M+m)a$$

$$= (M+m)g\tan\theta$$

From non-inertial frame of reference (Wedge)

FBD of block w.r.t. wedge (real forces + pseudo force)

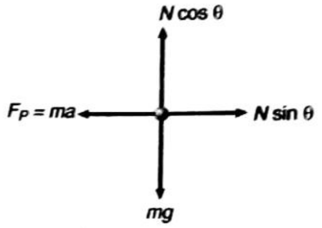


Fig. 5.88

w.r.t. wedge, block is stationary

$\therefore$

$$\Sigma F_y = 0 \Rightarrow N\cos\theta = mg \quad \text{...(iii)}$$

$$\Sigma F_x = 0 \Rightarrow N\sin\theta = ma \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we will get the same result

i.e., $F = (M+m)g\tan\theta.$

Sample Example 5.25 *A bob of mass m is suspended from the ceiling of a train moving with an acceleration a as shown in figure. Find the angle θ in equilibrium position.*

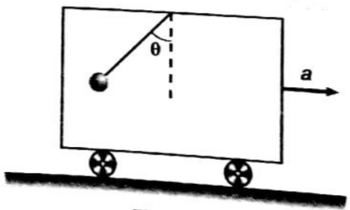


Fig. 5.89

Solution This problem can also be solved by both the methods.

Inertial frame of reference (Ground)

FBD of bob w.r.t. ground (only real forces) :

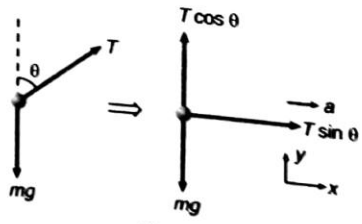


Fig. 5.90

With respect to ground, bob is also moving with an acceleration 'a'.

$\therefore$ $\Sigma F_x = 0 \Rightarrow T \sin\theta = ma$ ...(i)

and $\Sigma F_y = 0 \Rightarrow T \cos\theta = mg$ ...(ii)

From Eqs. (i) and (ii), we get

$$\tan\theta = \frac{a}{g} \text{ or } \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Non-inertial frame of reference (Train)

FBD of bob w.r.t train. (real forces + pseudo force):

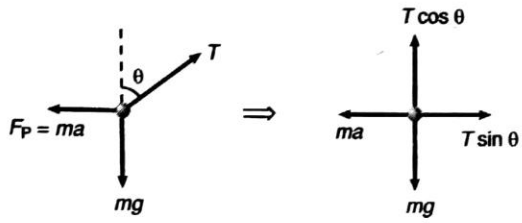


Fig. 5.91

with respect to train, bob is in equilibrium

$\therefore$ $\Sigma F_x = 0 \Rightarrow T \sin\theta = ma$ ...(iii)

$\therefore$ $\Sigma F_y = 0 \Rightarrow T \cos\theta = mg$ ...(iv)

From Eqs. (iii) and (iv), we get the same result, *i.e.*,

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

Sample Example 5.26 *In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration a. A block of mass m is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.*

Fig. 5.92

Solution Since, acceleration of block w.r.t. wedge (an accelerating or non-inertial frame of reference) is to be find out.

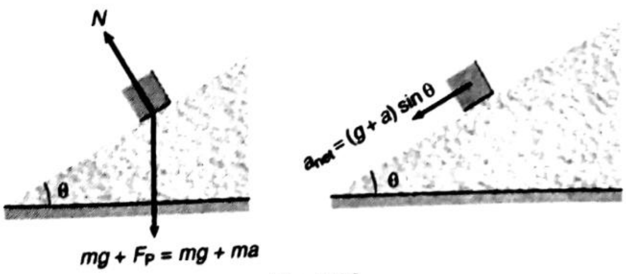


Fig. 5.93

FBD of 'block' w.r.t. 'wedge' is shown in Fig. 5.93.

The acceleration would had been $g \sin\theta$ (down the plane) if the lift were stationary or when only weight (*i.e.*, mg) acts downwards.

Here, downward force is $m(g+a)$.

$\therefore$ Acceleration of the block (of course w.r.t. wedge) will be $(g+a)\sin\theta$ down the plane.

5.8 Friction

As we have discussed in Article 5.1 friction is the parallel component of contact force between two bodies in contact. These forces are basically electromagnetic in nature. Friction can operate between a given pair of solids between a solid and a fluid or between a pair of fluids. Frictional force exerted by fluids is called viscous force. When two bodies slip over each other the force of friction is called kinetic friction, but when they do not slip but have a tendency to do so the force of friction is called static friction.

Regarding friction it is worth noting that:

1. If a body is at rest and no pulling force is acting on it, force of friction on it is zero.
2. If a force is applied to pull the body and it does not move, the friction acts which is equal in magnitude and opposite in direction to the applied force, *i.e.*, friction is self adjusting force. Further, as the body is at rest the friction is called static friction.
3. If the applied force is increased the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum force of static friction upto which body does not move is called limiting friction. Thus, static friction is a self adjusting force with an upper limit called limiting friction.
4. This limiting force of friction (f_L) is found experimentally to depend on normal reaction (N). Hence,

$$f_L \propto N$$

or

$$f_L = \mu_s N$$

Here, μ_s is a dimensionless constant and called coefficient of static friction, which depends on nature of surfaces in contact.

5. If the applied force is further increased, the friction opposing the motion is called kinetic or sliding friction. Experimentally, it is well established that kinetic friction is lesser than limiting friction and is given by

$$f_k = \mu_k N$$

where μ_k is coefficient of kinetic friction and less than μ_s.

Note (i) *In problems if μ_s and μ_k are separately not given but only μ is given. Then use*

$$f_L = f_k = \mu N$$

(ii) *If more than two blocks are placed one over the other on a horizontal ground then normal reaction between two blocks will be equal to the weight of the blocks over the common surface.*

For example N_1 = *normal reaction between A and B*

$= m_A g$

N_2 = *normal reaction between B and C*

$= (m_A + m_B) g$ *and so on.*

The theory of static and kinetic friction can he better understood by the following simple example.

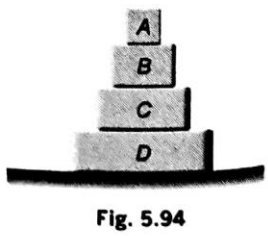


Fig. 5.94

Sample Example 5.27 *Suppose a block of mass 1 kg is placed over a rough surface and a horizontal force F is applied on the block as shown in figure. Now, let us see what are the values of force of friction f and acceleration of the block a if the force F is gradually increased. Given that* $\mu_s = 0.5$, $\mu_k = 0.4$ *and* $g = 10\,m/s^2$.

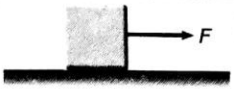


Fig. 5.95

Solution Free body diagram of block is

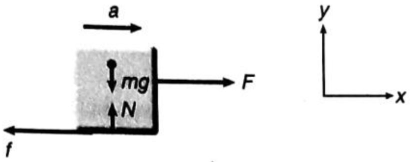


Fig. 5.96

$$\Sigma F_y = 0$$

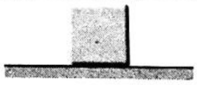
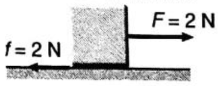
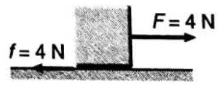
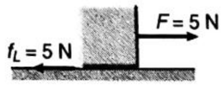
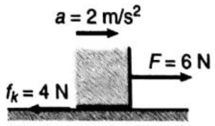
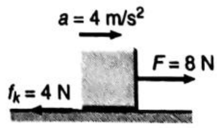
∴ $$N - mg = 0$$

or $$N = mg = (1)(10) = 10\,\text{N}$$

$$f_L = \mu_s N = (0.5)(10) = 5\,\text{N}$$

and $$f_k = \mu_k N = (0.4)(10) = 4\,\text{N}$$

Below is explained in tabular form, how the force of friction f depends on the applied force F.

F	f	$F_{net} = F - f$	Acceleration of block $a = \frac{F_{net}}{m}$	Diagram
0	0	0	0	
2 N	2 N	0	0	
4 N	4 N	0	0	
5 N	5 N	0	0	
6 N	4 N	2 N	2 m/s^2	
8 N	4 N	4 N	4 m/s^2	

Graphically this can be understood as under :

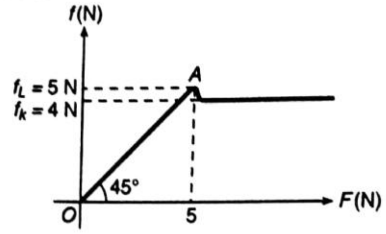


Fig. 5.97

Note that $f = F$ till $F \le f_L$. Therefore, slope of line OA will be 1 ($y = mx$) or angle of line OA with F-axis is 45°.

Here, $a = 0$ for $F \le 5$ N

and $$a = \frac{F - f_K}{m} = \frac{F - 4}{1} = F - 4 \text{ for } F > 5 \text{ N}$$

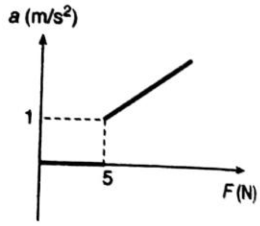


Fig. 5.98

a-F graph is shown in figure. When F is slightly increased from 5 N, acceleration of block increases from 0 to 1 m/s^2. Think why?

Note *Henceforth, we will take coefficient of friction as* μ *unless and until specially mentioned in the question* μ_s *and* μ_k *separately.*

Angle of friction (λ)

At a point of rough contact, where slipping is about to occur the two forces acting on each object are the normal reaction N and frictional force μN.

The resultant of these two forces is F and it makes an angle λ with the normal where

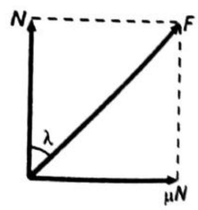


Fig. 5.99

$$\tan \lambda = \frac{\mu N}{N} = \mu$$

or $$\lambda = \tan^{-1}(\mu) \quad \text{...(i)}$$

This angle λ is called the angle of friction.

Angle of repose (α)

Suppose a block of mass m is placed on an inclined plane whose inclination θ can be increased or decreased. Let, μ be, the coefficient of friction between the block and the plane. At a general angle θ,

Normal reaction $N = mg \cos \theta$

Limiting friction $f_L = \mu N = \mu mg \cos \theta$

and the driving force (or pulling force)

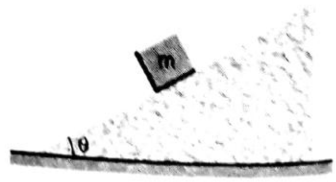


Fig. 5.100

$$F = mg \sin \theta \quad \text{(Down the plane)}$$

From these three equations we see that when θ is increased from 0° to 90°, normal reaction N and hence, the limiting friction f_L is decreased while the driving force F is increased. There is a critical angle called angle of repose (α) at which these two forces are equal. Now, if θ is further increased, then the driving force F becomes more than the limiting friction f_L and the block starts sliding.

Thus, $f_L = F \quad \text{at} \quad \theta = \alpha$

or $\mu\, mg \cos \alpha = mg \sin \alpha$

or $\tan \alpha = \mu$

or $\alpha = \tan^{-1} (\mu)$ ...(ii)

From Eqs. (i) and (ii), we see that angle of friction (λ) is numerically equal to the angle of repose.

or $\lambda = \alpha$

From the above discussion we can conclude that

If $\theta < \alpha$, $F < f_L$ the block is stationary

If $\theta = \alpha$, $F = f_L$ the block is on the verge of sliding

and if $\theta > \alpha, F > f_L$ the block slides down with acceleration

$$a = \frac{F - f_L}{m} = g\,(\sin\theta - \mu\cos\theta)$$

How, N, f_L and F varies with θ, this can be shown graphically as shown in Fig. 5.101.

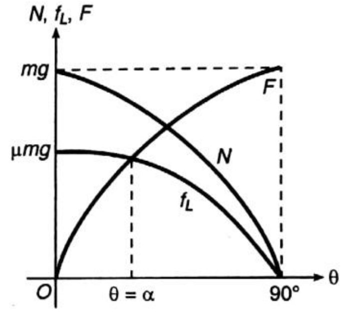


Fig. 5.101

$N = mg \cos \theta$

or $N \propto \cos \theta$

$f_L = \mu mg \cos \theta$

or $f_L \propto \cos \theta$

$F = mg \sin \theta$

or $F \propto \sin \theta$

Normally $\mu < 1$,

So, $f_L < N$.

Sample Example 5.28 *A particle of mass 1 kg rests on rough contact with a plane inclined at 30° to the horizontal and is just about to slip. Find the coefficient of friction between the plane and the particle.*

Solution The given angle 30° is really the angle of repose α. Hence,

$$\mu = \tan 30° = \frac{1}{\sqrt{3}}$$

Sample Example 5.29 *A block of weight W rests on a horizontal plane with which the angle of friction is λ. A force P inclined at an angle θ to the plane is applied to the plane until it is on the point of moving. Find the value of θ for which the value of P will be least.*

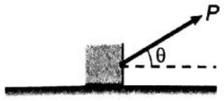


Fig. 5.102

Solution In the limiting case contact force F is inclined at λ to the normal. Only three forces act on the block. Applying Lami's theorem, we get

$$\frac{P}{\sin(180° - \lambda)} = \frac{W}{\sin(90° - \theta + \lambda)}$$

$$= \frac{F}{\sin(90° + \theta)}$$

or $$\frac{P}{\sin\lambda} = \frac{W}{\cos(\theta - \lambda)}$$

or $$P = \frac{W \sin\lambda}{\cos(\theta - \lambda)}$$

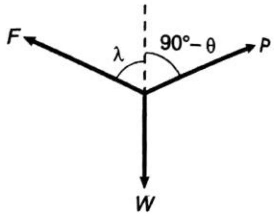


Fig. 5.103

P will be least when $\cos(\theta - \lambda)$ is greatest because W and λ are constant.

i.e., when $\cos(\theta - \lambda) = 1$ and $\theta - \lambda = 0°$

or $\theta = \lambda$ **Ans.**

Sample Example 5.30 *Figure shows two blocks in contact sliding down an inclined surface of inclination* $30°$. *The friction coefficient between the block of mass* 2.0 *kg and the incline is* $\mu_1 = 0.20$ *and that between the block of mass* 4.0 *kg and the incline is* $\mu_2 = 0.30$. *Find the acceleration of* 2.0 *kg block.* ($g = 10\ \text{m/s}^2$).

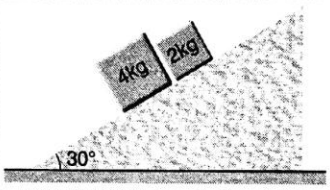


Fig. 5.104

Solution Since, $\mu_1 < \mu_2$, acceleration of 2 kg block down the plane will be more than the acceleration of 4 kg block, if allowed to move separately. But as the 2.0 kg block is behind the 4.0 kg block both of them will move with same acceleration say 'a'. Taking both the blocks as a system.

Force down the plane on the system $= (4 + 2) g \sin 30°$

$$= (6)(10)\left(\frac{1}{2}\right) = 30\ \text{N}$$

Force up the plane on the system

$$= \mu_1 (2)(g) \cos 30° + \mu_2 (4)(g) \cos 30°$$
$$= (2\mu_1 + 4\mu_2) g \cos 30°$$
$$= (2 \times 0.2 + 4 \times 0.3)(10)(0.86)$$
$$\approx 13.76\ \text{N}$$

$\therefore$ Net force down the plane is $F = 30 - 13.76 = 16.24$ N

$\therefore$ Acceleration of both the blocks down the plane will be a.

$$a = \frac{F}{4 + 2} = \frac{16.24}{6} = 2.7\ \text{m/s}^2$$

Introductory Exercise 5.5

1. In figure $m_1 = 1$ kg and $m_2 = 4$ kg. Find the mass M of the hanging block which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light

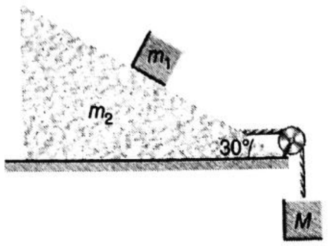


Fig. 5.105

Note *In exercises 2 to 4 the situations described take place in a box car which has initial velocity v = 0 but acceleration* $\vec{a} = (5\ m/s^2)\hat{i}$. *(Take* $g = 10\ m/s^2$*)*

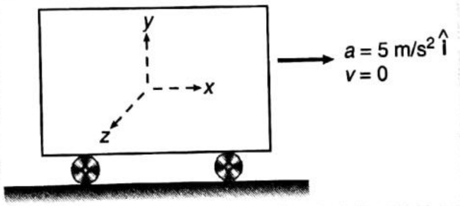


Fig. 5.106

2. A 2 kg object is slid along the frictionless floor with initial velocity $(10\ m/s)\hat{i}$ (a) Describe the motion of the object relative to car (b) when does the object reach its original position relative to the box car.

3. A 2 kg object is slid along the frictionless floor with initial transverse velocity $(10\ m/s)\hat{k}$. Describe the motion (a) in car's frame (b) in ground frame.

4. A 2 kg object is slid along a rough floor (coefficient of sliding friction = 0.3) with initial velocity $(10\ m/s)\hat{i}$. Describe the motion of the object relative to car assuming that the coefficient of static friction is greater than 0.5.

5. A block is placed on an inclined plane as shown in figure. What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at $3\ m/s^2 \left(\sin 37° = \frac{3}{5}\right)$? (Take $g = 10\ m/s^2$)

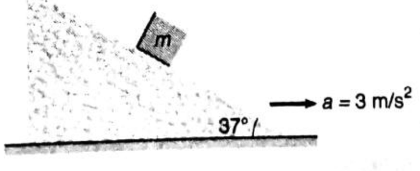


Fig. 5.107

Extra Points

- The common problem which I feel, students often face is the resolution of forces. Following two rules can be made in this regard.
  Rule 1 : If the body is in equilibrium, you can resolve the forces in any direction. Net force should be zero in all directions. A body moving with constant velocity is also in equilibrium.
  Rule 2 : If the body is accelerated, resolve the forces along acceleration and perpendicular to it. Net force along acceleration $= m\vec{a}$ and net force perpendicular to acceleration is zero.
- To find net force on a body find the acceleration of the body. Net force = mass × acceleration.

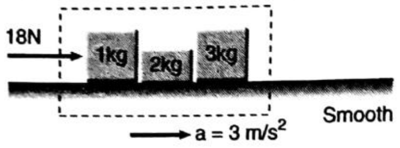


Fig. 5.108

For example, in the figure shown,

$$a=\frac{18}{1+2+3}=3\ \text{m/s}^2$$

Therefore,net force on 1 kg block $= 1 \times 3 = 3$ N.
This 3 N is the resultant of the applied force 18 N and normal reaction between 1 kg block and 2 kg block.
Similarly, net force on 2 kg block $= 2 \times 3 = 6$ N.
This is the resultant of normal reactions (1 kg and 2 kg) and (2 kg and 3 kg) blocks.

- Force of friction does not oppose the motion of a body but it opposes the relative motion between two bodies in contact.
  As far as motion of individual body is concerned it is sometimes friction which is responsible for its motion.
  For example, in the figure shown the 1 kg block moves with 2 kg block only due to friction.

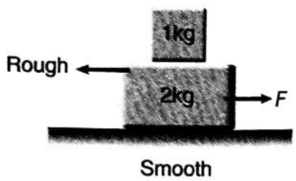


Fig. 5.109

- Mathematically a body is said to be in equilibrium, if
  (a) Net force acting on it is zero, *i.e.*, $\vec{F}_{net} = 0$.
  (b) Net moments of all the forces acting on it about any axis is zero. Physically the body at rest is said to be in equilibrium, if it is permanently at rest (unless some other force is applied on it, which may disturb its equilibrium). If a body is at rest just for a moment, it does not mean it is in equilibrium.

For example, when a ball is thrown upwards, at highest point of its journey it momentarily comes at rest, but there it is not in equilibrium. A net force (equal to its weight) is acting downward. Due to that force it moves downwards.

If a problem is asked on equilibrium, check whether the body is in equilibrium (permanent rest) or it is at rest just for a moment.

Now, if the body is in equilibrium, you may resolve the forces in any direction (x, y, z whatsoever). Net force on the body should be zero in all directions.

But if the body is **momentarily at rest but not in equilibrium** (I call it Mr. Ne. momentary rest not equilibrium); you will have to be very careful. First see in what direction will the body move just after few seconds. Obviously the net force on the body should point in that particular direction. Therefore components of all the forces in a direction perpendicular to the net force or perpendicular to the direction in which motion is likely to occur after few seconds should be zero.

- If a pulley is massless, net force on it is zero even, if it is accelerated. For example in the adjoining figure: $T_1 = 2T_2$ whether the pulley is accelerated on not provided the pulley is massless. This is because $\vec{F}_{net}$ = mass x acceleration and $\vec{F}_{net}$ will be zero if pulley is massless.

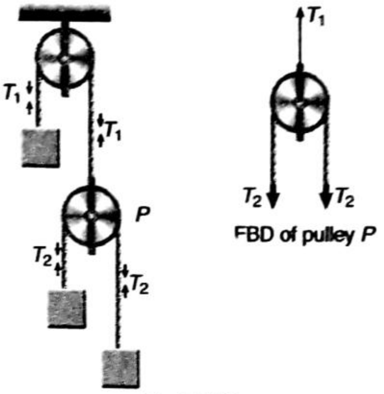


Fig. 5.110

- The direction of friction force on each of them is such as it either stops the relative motion or attempts to do so. For example, if a force F is applied on block A of a two block system, the direction of frictional forces at different contacts on different bodies will be as shown:

Here, f_1 = force of friction between A and B

and f_2 = force of friction between B and ground

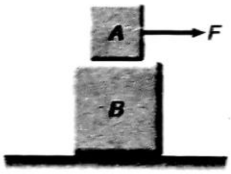


Fig. 5.111

- Force of friction $f = 0$, if no driving force is applied.

$$f \le f_L \quad (= \mu_s N),$$

If driving force is applied but no relative motion is there.

and $$f = \mu_k N$$

If relative motion is there.

So, apply kinetic friction whenever you see the relative motion between two bodies in contact. But don't apply $f = f_L$ in case there is no relative motion. Because being a self adjusting force only that much amount will act which is required for stopping the relative motion. So, it may be less than f_L also.

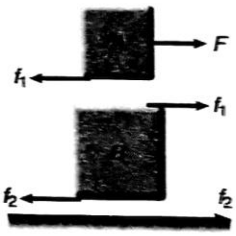


Fig. 5.112

- A common mistake which the students err in hurry is that they always write $f_L = \mu mg$ (in case of horizontal ground) or $f_L = \mu mg \cos\theta$ (in inclined surface). The actual formula is $f_L = \mu N$. Here, N is equal to mg or $mg \cos\theta$ upto when no force is acting at some angle ($\neq 0°$) with the plane.

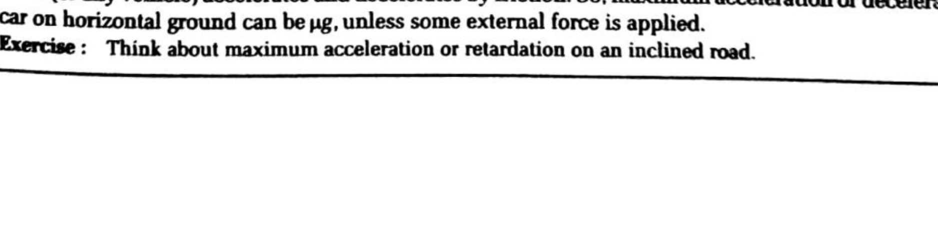


Fig. 5.113

- A car (or any vehicle) accelerates and decelerates by friction. So, maximum acceleration or deceleration of a car on horizontal ground can be μg, unless some external force is applied.

Exercise : Think about maximum acceleration or retardation on an inclined road.

Solved Examples

Level 1

Example 1 *Determine the tensions T_1 and T_2 in the strings as shown in figure.*

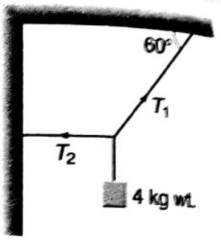


Fig. 5.114

Solution Resolving the tension T_1 along horizontal and vertical directions. As the body is in equilibrium,

$$T_1 \sin 60° = 4 \times 9.8 \text{ N} \qquad \text{...(i)}$$

$$T_1 \cos 60° = T_2 \qquad \text{...(ii)}$$

$$T_1 = \frac{4 \times 9.8}{\sin 60°}$$

$$= \frac{4 \times 9.8 \times 2}{\sqrt{3}} = 45.26 \text{ N}$$

$$T_2 = T_1 \cos 60° = 45.26 \times 0.5 = 22.63 \text{ N}$$

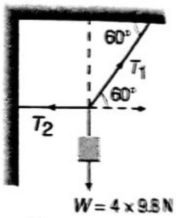
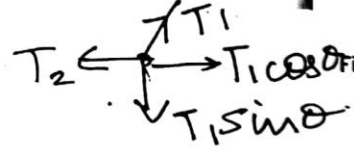


Fig. 5.115



Example 2 *A ball of mass 1 kg hangs in equilibrium from two strings OA and OB as shown in figure. What are the tensions in strings OA and OB? (Take $g = 10\ m/s^2$).*

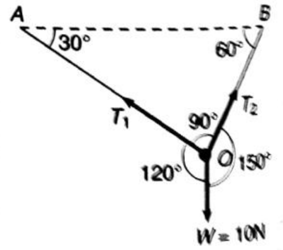


Fig. 5.116

Solution Various forces acting on the ball are as shown in Fig 5.116. The three concurrent forces are in equilibrium. Using Lami's theorem,

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{10}{\sin 90°}$$

or

$$\frac{T_1}{\sin 30°} = \frac{T_2}{\sin 60°} = \frac{10}{1}$$

$\therefore$

$$T_1 = 10 \sin 30° = 10 \times 0.5 = 5 \text{ N}$$

and $$T_2 = 10 \sin 60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{N}$$

Example 3 *A 4 m long ladder weighing 25 kg rests with its upper end against a smooth wall and lower end on rough ground. What should be the minimum coefficient of friction between the ground and the ladder for it to be inclined at 60° with the horizontal without slipping? (Take $g = 10\ m/s^2$).*

Solution In Fig 5.117 AB is a ladder of weight W which acts at its centre of gravity G.

$$\angle ABC = 60°$$

$$\therefore \quad \angle BAC = 30°$$

Let N_1 be the reaction of the wall, and N_2 the reaction of the ground. Force of friction f between the ladder and the ground acts along BC.

For horizontal equilibrium,

$$f = N_1 \quad \ldots\text{(i)}$$

For vertical equilibrium,

$$N_2 = W \quad \ldots\text{(ii)}$$

Taking moments about B, we get for equilibrium,

$$N_1(4\cos 30°) - W(2\cos 60°) = 0 \quad \ldots\text{(iii)}$$

Here, $W = 250\text{N}$

Solving these three equations, we get

$$f = 72.17\text{N} \quad \text{and} \quad N_2 = 250\ \text{N}$$

$$\therefore \quad \mu = \frac{f}{N_2} = \frac{72.17}{250} = 0.288$$

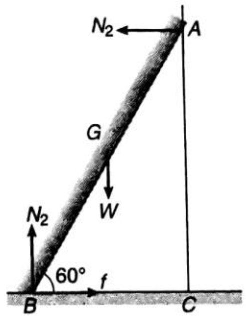


Fig. 5.117

Example 4 *A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding? (Take $g = 9.8\ m/s^2$).*

Solution In mode (a), the man applies a force equal to 25 kg weight in upward direction. According to Newton's third law of motion, there will be a downward force of reaction on the floor.

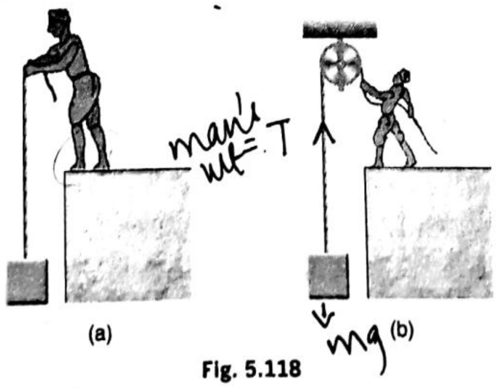


Fig. 5.118

$\therefore$ Total action on the floor by the man

$$= 50\ \text{kg-wt} + 25\ \text{kg-wt} = 75\ \text{kg-wt}$$

$$= 75 \times 9.8\ \text{N} = 735\ \text{N}$$

In mode (b), the man applies a downward force equal to 25 kg-wt. According to Newton's third law, the reaction will be in the upward direction.

$\therefore$ Total action on the floor by the man

$$= 50\text{ kg-wt} - 25\text{ kg-wt} = 25\text{ kg-wt}$$
$$= 25 \times 9.8\text{ N} = 245\text{ N}$$

As the floor yields to a downward force of 700 N, so the man should adopt mode (b).

Example 5 *A block of mass 200 kg is set into motion on a frictionless horizontal surface with the help of frictionless pulley and a rope system as shown in figure (a). What horizontal force F should be applied to produce in the block an acceleration of* $1\ m/s^2$*?*

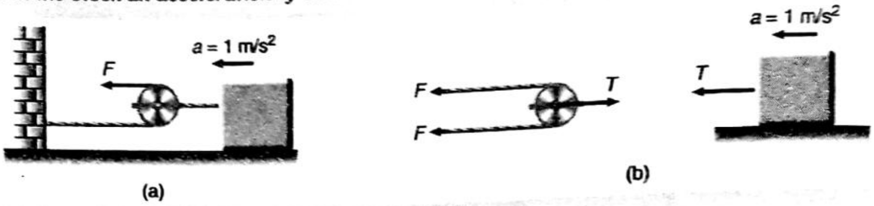


Fig. 5.119

Solution As shown in Fig. (b), when force F is applied at the end of the string, the tension in the lower part of the string is also F. If T is the tension in string connecting the pulley and the block, then,

$$T = 2F$$

But $\quad T = ma = (200)(1) = 200\text{ N}$

$\therefore \quad 2F = 200\text{ N}$

or $\quad F = 100\text{ N}$

Example 6 *A block of mass 1 kg is pushed against a rough vertical wall with a force of 20 N, coefficient of static friction being* $\frac{1}{4}$. *Another horizontal force of 10 N is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction? If no, find the frictional force exerted by the wall on the block.* $(g = 10\ m/s^2)$

Solution Normal reaction on the block from the wall will be

$$N = F = 20\text{ N}$$

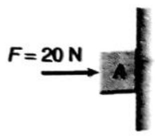


Fig. 5.120

Therefore, limiting friction

$$f_L = \mu N = \left(\frac{1}{4}\right)(20) = 5\text{ N}$$

Weight of the block is

$$W = mg = (1)(10) = 10\text{ N}$$

A horizontal force of 10 N is applied to the block. The resultant of these two forces will be $10\sqrt{2}$ N in the direction shown in figure. Since, this resultant is greater than the limiting friction. The block will move in the direction of $\vec{F}_{net}$ with acceleration

$$a = \frac{F_{net} - f_L}{m} = \frac{10\sqrt{2} - 5}{1} = 9.14\text{ m/s}^2$$

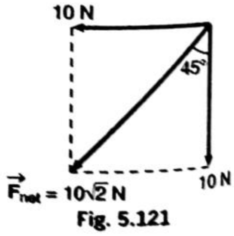


Fig. 5.121

Example 7 *Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with* $1\,ms^{-2}$. *What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration of the belt can the man continue to be stationary relative to the belt? Mass of the man = 65 kg.* $(g = 9.8\,m/s^2)$

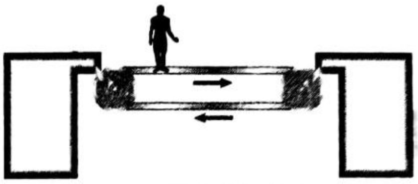


Fig. 5.122

Solution As the man is standing stationary w.r.t. the belt,

$\therefore$ Acceleration of the man = Acceleration of the belt

$$= a = 1\,ms^{-2}$$

Mass of the man, $m = 65$ kg

Net force on the man $= ma = 65 \times 1 = 65$ N

Given coefficient of friction, $\mu = 0.2$

$\therefore$ Limiting friction, $f_L = \mu mg$

If the man remains stationary with respect to the maximum acceleration a_0 of the belt, then

$$ma_0 = f_L = \mu mg$$

$$\therefore \quad a_0 = \mu g = 0.2 \times 9.8 = 1.96\,ms^{-2}$$

Example 8 *A block of mass m is at rest on a rough wedge as shown in figure. What is the force exerted by the wedge on the block?*

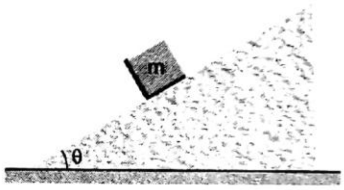


Fig. 5.123

Solution Since, the block is permanently at rest, it is in equilibrium. Net force on it should be zero. In this case only two forces are acting on the block.

(1) Weight = *mg* (downwards).

(2) Contact force (resultant of normal reaction and friction force) applied by the wedge on the block.

For the block to be in equilibrium these two forces should be equal and opposite.

Therefore, force exerted by the wedge on the block is *mg* (upwards).

Note *(i) From Newton's third law of motion-force exerted by the block on the wedge is also mg but downwards.*

(ii) The result can also be obtained in a different manner. The normal force on the block is $N = mg\cos\theta$ *and the friction force on the block is* $f = mg\sin\theta$ *(not* $\mu\, mg\cos\theta$*)*

These two forces are mutually perpendicular.

$\therefore$ *Net contact force would be* $\sqrt{N^2 + f^2}$ *or* $\sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2}$ *which is equal to mg.*

Problem 9 *A ball of mass 1 kg is at rest in position P by means of two light strings OP and RP. The string RP is now cut and the ball swings to position Q. If $\theta = 45°$. Find the tensions in the strings in positions OP (when RP was not cut) and OQ (when RP was cut). (Take $g = 10\ m/s^2$).*

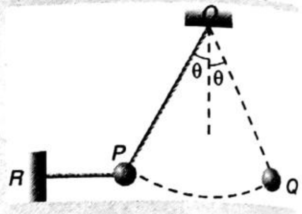


Fig. 5.124

Solution In the first case, ball is in equilibrium (permanent rest). Therefore, net force on the ball in any direction should be zero.

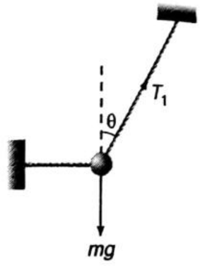


Fig. 5.125

$\therefore (\Sigma \vec{\mathbf{F}})$ in vertical direction = 0

or $$T_1 \cos\theta = mg$$

or $$T_1 = \frac{mg}{\cos\theta}$$

Substituting $m_1 = 1\ \text{kg}, g = 10\ \text{m/s}^2$ and $\theta = 45°$.

we get, $$T_1 = 10\sqrt{2}\ \text{N}$$

Note *Here, we deliberately resolved all the forces in vertical direction because component of the tension in RP in vertical direction is zero. Although, since, the ball is in equilibrium, net force on it in any direction is zero. But in a direction other than vertical we will have to consider component of tension in RP also, which will unnecessarily increase the calculation.*

In the second case ball is not in equilibrium (temporary rest). After few seconds it will move in a direction perpendicular to OQ. Therefore, net force on the ball at Q is perpendicular to OQ, or net force along $OQ = 0$.

$\therefore$ $$T_2 = mg\cos\theta$$

Substituting the values, we get $$T_2 = 5\sqrt{2}\ \text{N}$$

Here, we can see that $$T_1 \neq T_2$$

Level 2

Example 1 *Two blocks of mass $m = 5$ kg and $M = 10$ kg are connected by a string passing over a pulley B as shown. Another string connects the centre of pulley B to the floor and passes over another pulley A as shown. An upward force F is applied at the centre of pulley A. Both the pulleys are massless. Find the acceleration of blocks m and M, if F is:*

(a) 100 *N*
(b) 300 *N*
(c) 500 *N. (Take $g = 10\ m/s^2$)*

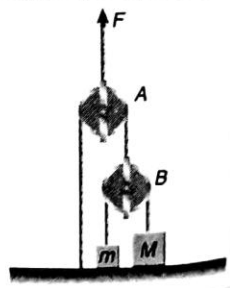


Fig. 5.126

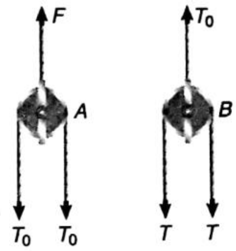


Fig. 5.127

Solution Let T_0 = tension in the string passing over A

T = tension in the string passing over B

$$2T_0 = F \quad \text{and} \quad 2T = T_0$$

$\Rightarrow$ $$T = F/4$$

(a) $$T = F/4 = 25\text{ N}$$

weights of blocks are $mg = 50\text{ N}$

$Mg = 100\text{ N}$

As $T < mg$ and Mg both, the blocks will remain stationary on the floor.

(b) $$T = F/4 = 75\text{ N}$$

As $T < Mg$ and $T > mg$, M will remain stationary on the floor, whereas m will move.

acceleration of m,

$$a = \frac{T - mg}{m} = \frac{75 - 50}{5} = 5\text{ m/s}^2$$

(c) $$T = F/4 = 125\text{ N}$$

As $T > mg$ and Mg, both the blocks will accelerate upwards.

Acceleration of m,

$$a_1 = \frac{T - mg}{m} = \frac{125 - 50}{5} = 15\text{ m/s}^2$$

Acceleration of M,

$$a_2 = \frac{T - Mg}{M} = \frac{125 - 100}{10} = 2.5\text{ m/s}^2$$

Example 2 *Consider the situation shown in figure the block B moves on a frictionless surface, while the coefficient of friction between A and the surface on which it moves is 0.2. Find the acceleration with which the masses move and also the tension in the strings. (Take $g = 10\ m/s^2$).*

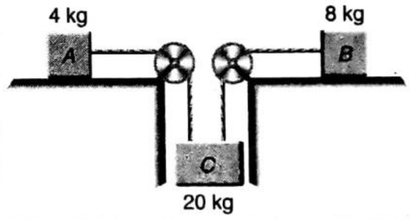


Fig. 5.128

Solution Let a be the acceleration with which the masses move and T_1 and T_2 be the tensions in left and right strings. Friction on mass A is $\mu mg = 8$ N. Then equations of motion of masses A, B and C are

For mass A $$T_1 - 8 = 4a \quad \text{...(i)}$$

For mass B $$T_2 = 8a \quad \text{...(ii)}$$

For mass C $$200 - T_1 - T_2 = 20a \quad \text{...(iii)}$$

Adding the above three equations, we get $32a = 192$

or $$a = 6\text{ m/s}^2$$

From Eqs. (i) and (ii), we have $$T_2 = 48\text{ N}$$

and $$T_1 = 32\text{ N}$$

Example 3 *Three blocks of mass m_1, m_2 and m_3 are connected as shown in the figure. All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of m_1.*

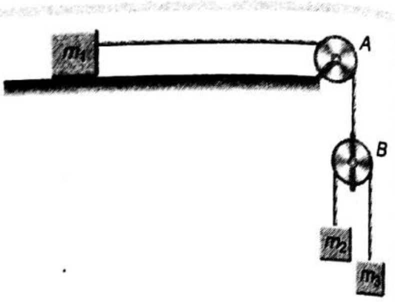


Fig. 5.129

Solution Suppose the acceleration of m_1 is a_0 towards right. The acceleration of pulley B will also be a_0 downwards because the string connecting m_1 and B is constant in length. Also the string connecting m_2 and m_3 has a constant length. This implies that the decrease in the separation between m_2 and B equals the increase in the separation between m_3 and B. So, the upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be a_r.

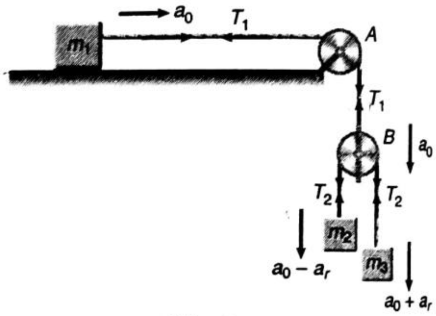


Fig. 5.130

The acceleration of m_2 with respect to the ground $= a_0 - a_r$ (downward) and the acceleration of m_3 with respect to the ground $= a_0 + a_r$ (downward).

Let the tension be T_1 in the upper string and T_2 in the lower string. Consider the motion of the pulley B.

The forces on this light pulley are

(a) T_1 upwards by the upper string and

(b) $2T_2$ downwards by the lower string.

As the mass of the pulley is negligible,

$$2T_2 - T_1 = 0 \quad \text{giving} \quad T_2 = \frac{T_1}{2} \qquad \text{...(i)}$$

Motion of m_1 : In the horizontal direction, the equation is

$$T_1 = m_1 a_0 \qquad \text{...(ii)}$$

Motion of m_2 : $$m_2 g - T_1/2 = m_2(a_0 - a_r) \qquad \text{...(iii)}$$

Motion of m_3 : $$m_3 g - T_1/2 = m_3(a_0 + a_r) \qquad \text{...(iv)}$$

Solving these four equations, we get

$$a_0 = \frac{g}{1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)}$$

Example 4 *Two blocks A and B of mass* 1 kg *and* 2 kg *respectively are connected by a string, passing over a light frictionless pulley. Both the blocks are resting on a horizontal floor and the pulley is held such that string remains just taut.*

At moment $t = 0$, a force $F = 20t$ newton starts acting on the pulley along vertically upward direction as shown in figure. Calculate:

(a) velocity of A when B loses contact with the floor.

(b) height raised by the pulley upto that instant. (Take, $g = 10\ m/s^2$)

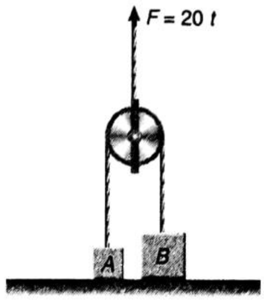


Fig. 5.131

Solution (a) Let T be the tension in the string. Then,

$$2T = 20t \quad \text{or} \quad T = 10t \text{ newton}$$

Let the block A loses its contact with the floor at time $t = t_1$. This happens when the tension in string becomes equal to the weight of A. Thus,

$$T = mg \quad \text{or} \quad 10t_1 = 1 \times 10 \quad \text{or} \quad t_1 = 1\text{s} \qquad \text{...(i)}$$

Similarly, for block B, we have

$$10t_2 = 2 \times 10 \quad \text{or} \quad t_2 = 2\text{s} \qquad \text{...(ii)}$$

i.e., the block B loses contact after 2 s. For block A, at time t such that $t \geq t_1$ let a be its acceleration in upward direction. Then,

$$10t - 1 \times 10 = 1 \times a = (dv/dt)$$

or

$$dv = 10(t - 1)\,dt \qquad \text{...(iii)}$$

Integrating this expression, we get

$$\int_0^v dv = 10\int_1^t (t-1)\,dt$$

or

$$v = 5t^2 - 10t + 5 \qquad \text{...(iv)}$$

Substituting $t = t_2 = 2\text{s}$

$$v = 20 - 20 + 5 = 5 \text{ m/s} \qquad \text{...(v)}$$

(b) From Eq. (iv),

$$dy = (5t^2 - 10t + 5)\,dt \qquad \text{...(vi)}$$

Where y is the vertical displacement of block A at time t $(\geq t_1)$.

Integrating, we have

$$\int_{y=0}^{y=h} dy = \int_{t=1}^{t=2} (5t^2 - 10t + 5)\,dt$$

$$h = 5\left[\frac{t^3}{3}\right]_1^2 - 10\left[\frac{t^2}{2}\right]_1^2 + 5[t]_1^2 = \frac{5}{3} \text{ m}$$

$\therefore$ Height raised by pulley upto that instant $= \dfrac{h}{2} = \dfrac{5}{6}$ m

Example 5 *Find the acceleration of the body of mass m_2 in the arrangement shown in Fig. 5.132. If the mass m_2 is η times great as the mass m_1, and the angle that the inclined plane forms with the horizontal is equal to θ. The masses of the pulleys and threads, as well as the friction, are assumed to be negligible.*

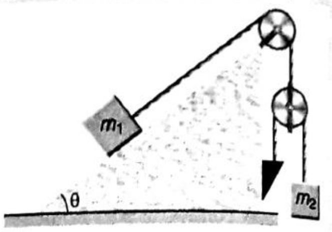


Fig. 5.132

Solution Here, by constraint relation we can see that the acceleration of m_2 is two times that of m_1. So, we assume if m_1 is moving up the inclined plane with an acceleration a, the acceleration of mass m_2 going down is $2a$. The tensions in different strings are shown in figure.

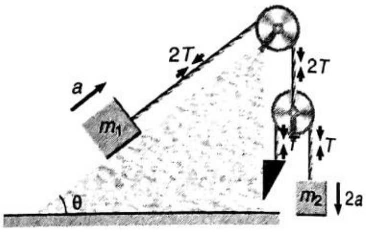


Fig. 5.133

The dynamic equations can be written as

For mass m_1 $\quad 2T - m_1 g \sin\theta = m_1 a$ ...(i)

For mass m_2 $\quad m_2 g - T = m_2(2a)$ ...(ii)

Substituting $m_2 = \eta m_1$ and solving Eqs. (i) and (ii), we get

$$\text{Acceleration of } m_2 = 2a = \frac{2g(2\eta - \sin\theta)}{4\eta + 1}$$

Example 6 *In the arrangement shown in Fig. 5.134 the mass of the ball is η times as great as that of the rod. The length of the rod is l, the masses of the pulleys and the threads, as well as the friction, are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite the upper end of the rod?*

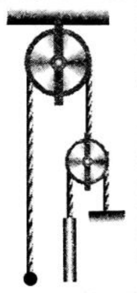


Fig. 5.134

Solution From constraint relation we can see that the acceleration of the rod is double than that of the acceleration of the ball. If ball is going up with an acceleration a, rod will be coming down with the acceleration $2a$, thus, the relative acceleration of the ball with respect to rod is $3a$ in upward direction. If it takes time t seconds to reach the upper end of the rod, we have

$$t = \sqrt{\frac{2l}{3a}} \qquad \text{...(i)}$$

Let mass of ball be m and that of rod is M, the dynamic equations of these are

For rod $\quad Mg - T = M(2a)$ ...(ii)

For ball $\quad 2T - mg = ma$ ...(iii)

Substituting $m = \eta M$ and solving Eqs. (ii) and (iii), we get

$$a = \left(\frac{2-\eta}{\eta+4}\right) g$$

From Eq. (i), we have

$$t = \sqrt{\frac{2l(\eta+4)}{3g(2-\eta)}}$$

Example 7 *Figure shows a small block A of mass m kept at the left end of a plank B of mass $M = 2m$ and length l. The system can slide on a horizontal road. The system is started towards right with the initial velocity v. The friction coefficients between the road and the plank is 1/2 and that between the plank and the block is 1/4. Find:*

(a) the time elapsed before the block separates from the plank.

(b) displacement of block and plank relative to ground till that moment.

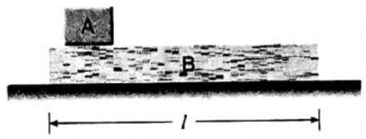


Fig. 5.135

Solution There will be relative motion between block and plank and plank and road. So at each surface limiting friction will act. The direction of friction forces at different surfaces are as shown in figure.

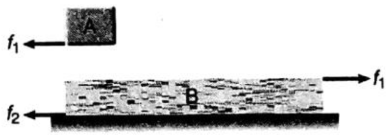


Fig. 5.136

Here, $$f_1 = \left(\frac{1}{4}\right)(mg)$$

and $$f_2 = \left(\frac{1}{2}\right)(m+2m)g = \left(\frac{3}{2}\right)mg$$

Retardation of A is $$a_1 = \frac{f_1}{m} = \frac{g}{4}$$

and retardation of B is $$a_2 = \frac{f_2 - f_1}{2m} = \frac{5}{8}g$$

Since, $$a_2 > a_1$$

Relative acceleration of A with respect to B is

$$a_r = a_2 - a_1 = \frac{3}{8}g$$

Initial velocity of both A and B is v. So, there is no relative initial velocity. Hence,

(a) Applying $$s = \frac{1}{2}at^2 \quad \text{or} \quad l = \frac{1}{2}a_r t^2 = \frac{3}{16}gt^2$$

$$\therefore \quad t = 4\sqrt{\frac{l}{3g}}$$

(b) Displacement of block $$s_A = u_A t - \frac{1}{2}a_A t^2$$

or $$s_A = 4v\sqrt{\frac{l}{3g}} - \frac{1}{2}\cdot\frac{g}{4}\cdot\left(\frac{16l}{3g}\right) \qquad \left(a_A = a_1 = \frac{g}{4}\right)$$

or $$s_A = 4v\sqrt{\frac{l}{3g}} - \frac{2}{3}l$$

Displacement of plank $$s_B = u_B t - \frac{1}{2}a_B t^2$$

or $$s_B = 4v\sqrt{\frac{l}{3g}} - \frac{1}{2}\left(\frac{5}{8}g\right)\left(\frac{16l}{3g}\right) \qquad \left(a_B = a_2 = \frac{5}{8}g\right)$$

or $$s_B = 4v\sqrt{\frac{l}{3g}} - \frac{5}{3}l$$ **Ans.**

Note *We can see that $s_A - s_B = l$. Which is quite obvious because block A has moved a distance l relative to plank.*

Example 8 *Two blocks A and B of mass 2 kg and 4 kg are placed one over the other as shown in figure. A time varying horizontal force $F = 2t$ is applied on the upper block as shown in figure. Here t is in second and F is in newton. Draw a graph showing accelerations of A and B on y-axis and time on x-axis. Coefficient of friction between A and B is $\mu = \frac{1}{2}$ and the horizontal surface over which B is placed is smooth. ($g = 10\ m/s^2$)*

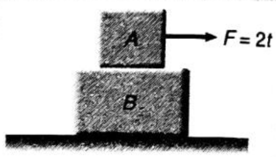


Fig. 5.137

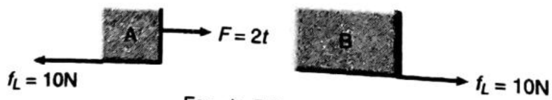
Solution Limiting friction between A and B is

$$f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)\ (10) = 10\ \text{N}$$

Block B moves due to friction only. Therefore, maximum acceleration of B can be

$$a_{\max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5\ \text{m/s}^2$$

Thus, both the blocks move together with same acceleration till the common acceleration becomes $2.5\ \text{m/s}^2$, after that acceleration of B will become constant while that of A will go on increasing. To find the time when the acceleration of both the blocks becomes $2.5\ \text{m/s}^2$ (or when slipping will start between A and B) we will write



For $t \geq 7.5$ sec
Fig. 5.138

$$2.5 = \frac{F}{(m_A + m_B)} = \frac{2t}{6}$$

$\therefore$ $$t = 7.5\text{s}$$

Hence, for $$t \leq 7.5\text{s}$$

$$a_A = a_B = \frac{F}{m_A + m_B} = \frac{2t}{6} = \frac{t}{3}$$

Thus, a_A *versus* t or a_B *versus* t graph is a straight line passing through origin of slope $\frac{1}{3}$.

For, $$t \geq 7.5\,\text{s}$$

$$a_B = 2.5\ \text{m/s}^2 = \text{constant}$$

and $$a_A = \frac{F - f_L}{m_A}$$

or $$a_A = \frac{2t - 10}{2} \quad \text{or} \quad a_A = t - 5$$

Thus, a_A *versus* t graph is a straight line of slope 1 and intercept −5. While a_B *versus* t graph is a straight line parallel to t axis. The corresponding graph is as shown in Fig. 5.139.

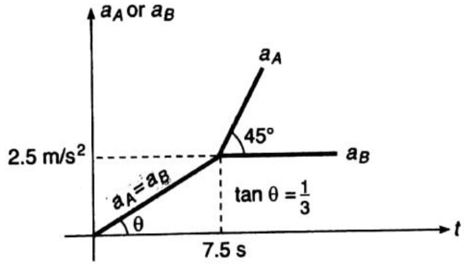


Fig. 5.139

EXERCISES

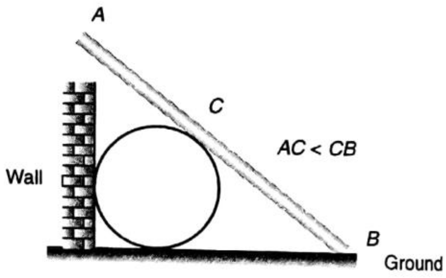
AIEEE Corner

Subjective Questions (Level 1)

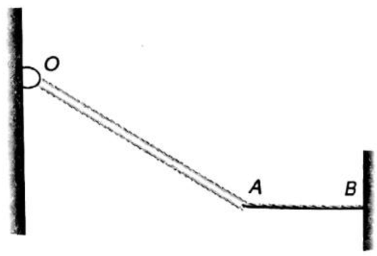
Free Body Diagram

1. A rod AB of weight W_1 is placed over a sphere of weight W_2 as shown in figure. Ground is rough and there is no friction between rod and sphere and sphere and wall. Draw free body diagrams of sphere and rod separately.

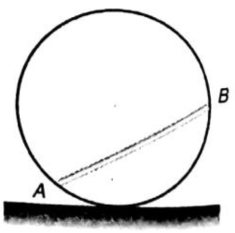
Note *No friction will act between sphere and ground, think why ?*



2. A rod OA is suspended with the help of a massless string AB as shown in figure. Rod is hinged at point O. Draw free body diagram of the rod.



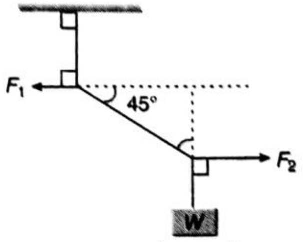
3. A rod AB is placed inside a rough spherical shell as shown in figure. Draw the free body diagram of the rod.



Equilibrium of Forces

(a) Concurrent forces

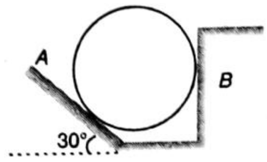
4. In figure the tension in the diagonal string is 60 N.



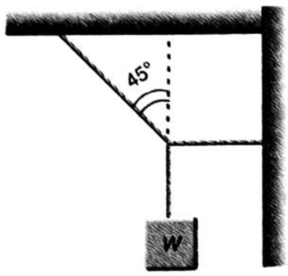
(a) Find the magnitude of the horizontal forces $\vec{F}_1$ and $\vec{F}_2$ that must be applied to hold the system in the position shown.

(b) What is the weight of the suspended block ?

5. The 50 kg homogeneous smooth sphere rests on the 30° incline A and against the smooth vertical wall B. Calculate the contact forces at A and B.

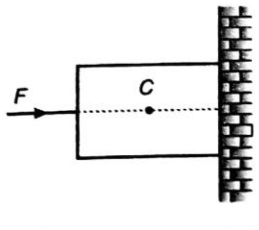


6. All the strings shown in figure are massless. Tension in the horizontal string is 30 N. Find W.



(b) Coplanar forces

7. A cube of mass 2 kg is held stationary against a rough wall by a force $F = 40\,\text{N}$ passing through centre C. Find perpendicular distance of normal reaction between wall and cube from point C. Side of the cube is 20 cm. Take $g = 10\,\text{m/s}^2$.

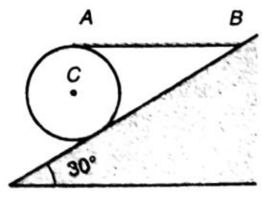


8. A sphere of weight $W = 100\,\text{N}$ is kept stationary on a rough inclined plane by a horizontal string AB as shown in figure. Find :

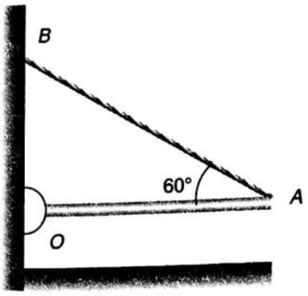
(a) tension in the string,

(b) force of friction on the sphere and

(c) normal reaction on the sphere by the plane.



9. A rod OA of mass 4 kg is held in horizontal position by a massless string AB as shown in figure. Length of the rod is 2 m. Find :

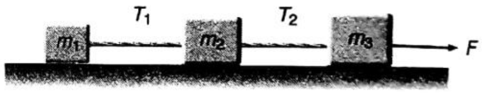


(a) tension in the string,
(b) net force exerted by hinge on the rod. ($g = 10\ \text{m/s}^2$)

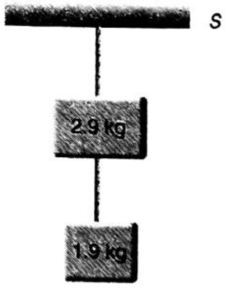
Newton's Laws

10. In the figure shown all surfaces are smooth. Find :
(a) acceleration of all the three blocks,
(b) net force on 6 kg, 4 kg and 10 kg blocks and
(c) force acting between 4 kg and 10 kg blocks.

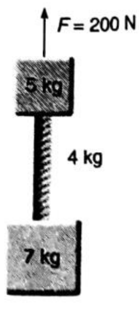
11. Three blocks $m_1 = 10\,\text{kg}$, $m_2 = 20\,\text{kg}$ and $m_3 = 30\,\text{kg}$ are on a smooth horizontal table, connected to each other by light horizontal strings. A horizontal force $F = 60$ N is applied to m_3, towards right. Find :
(a) tensions T_1 and T_2 and
(b) tension T_2 if all of a sudden the string between m_1 and m_2 snaps.



12. Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextenible wires each of length 1 m, as shown in the figure. The upper wire has negligible mass and the lower wire has a uniformly distributed mass of 0.2 kg. The whole system of blocks, wires and support have an upward acceleration of $0.2\ \text{m/s}^2$. Acceleration due to gravity is $9.8\ \text{m/s}^2$.
(a) Find the tension at the mid-point of the lower wire.
(b) Find the tension at the mid-point of the upper wire.



13. Two blocks shown in figure are connected by a heavy uniform rope of mass 4 kg. An upward force of 200 N is applied as shown.
(a) What is the acceleration of the system ?
(b) What is the tension at the top of the rope ?
(c) What is the tension at the mid-point of the rope ?
(Take $g = 9.8\ \text{m/s}^2$)



14. A 20 kg monkey has a firm hold on a light rope that passes over a frictionless pulley and is attached to a 20 kg bunch of bananas. The monkey looks upward, sees the bananas, and starts to climb the rope to get them.

(a) As the monkey climbs, do the bananas move up, move down, or remain at rest ?

(b) As the monkey climbs, does the distance between the monkey and the bananas decrease, increase, or remain constant ?

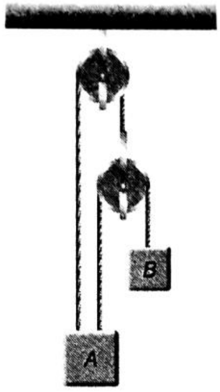
(c) The monkey releases her hold on the rope. What happens to the distance between the monkey and the bananas while she is falling ?

(d) Before reaching the ground, the monkey grabs the rope to stop her fall. What do the bananas do ?

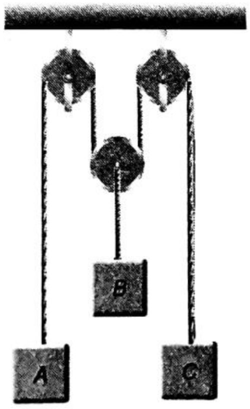
Constraint Equations

Note (Q. 15 *to* Q. 23) *Assume massless strings, massless and smooth pulleys.*

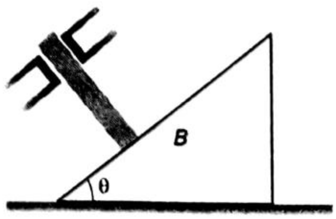
15. In the pulley-block arrangement shown in figure. Find the relation between acceleration of blocks A and B.



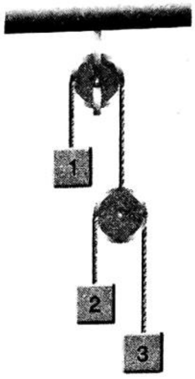
16. In the pulley-block arrangement shown in figure. Find relation between a_A, a_B and a_C.



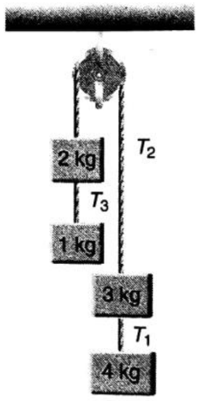
17. In the figure shown find relation between magnitudes of $\vec{a}_A$ and $\vec{a}_B$.



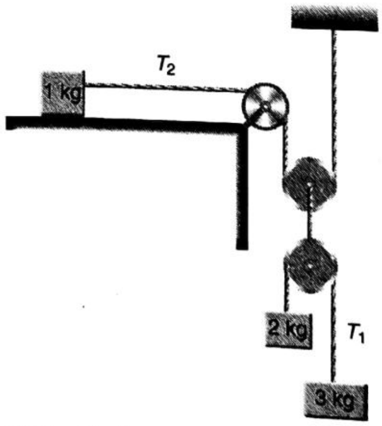
18. In the figure shown, $a_3 = 6\,\text{m/s}^2$ (downwards) and $a_2 = 4\,\text{m/s}^2$ (upwards). Find acceleration of 1.



19. In the figure shown, find acceleration of the system and tensions T_1, T_2 and T_3. (Take $g = 10\,\text{m/s}^2$)

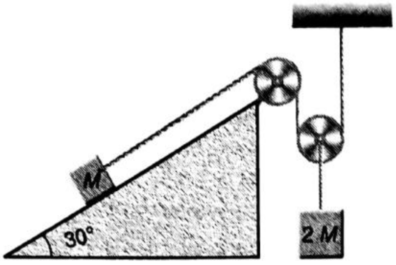


20. In the figure shown. Find : ($g = 10\,\text{m/s}^2$)

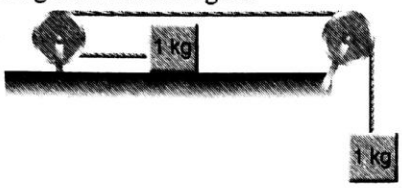


(a) acceleration of 1 kg, 2 kg and 3 kg blocks and
(b) tensions T_1 and T_2.

21. Find the acceleration of the block of mass M in the situation shown in the figure. All the surfaces are frictionless.

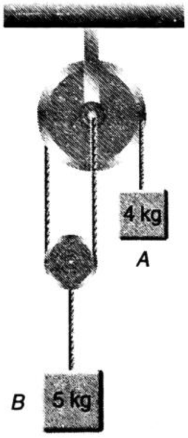


22. Calculate the tension in the string shown in the figure.



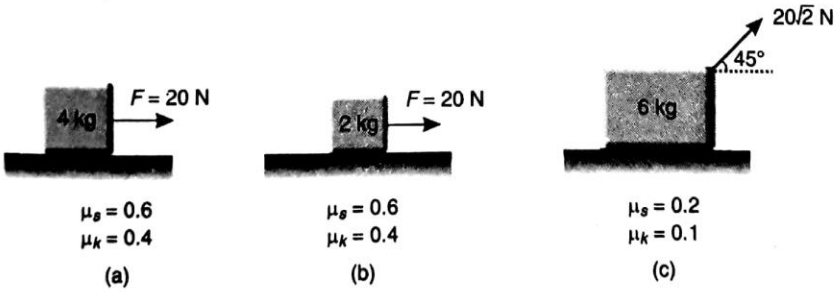
All the surfaces are frictionless. (Take $g = 10\,\text{m/s}^2$)

23. Find the acceleration of the blocks A and B in the situation shown in the figure.

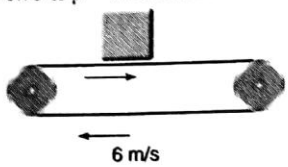


Friction

24. In the three figures shown, find acceleration of block and force of friction on it in each case.



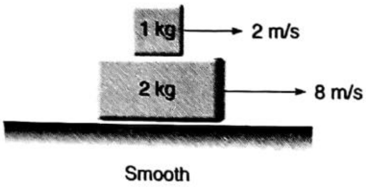
25. A conveyor belt is moving with constant speed of 6 m/s. A small block is just dropped on it. Coefficient of friction between the two is $\mu = 0.3$. Find :



(a) The time when relative motion between them will stop.
(b) Displacement of block upto that instant. ($g = 10 \text{m/s}^2$).

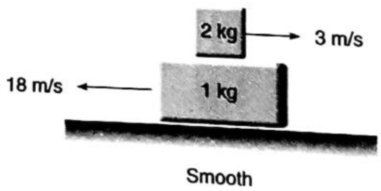
Note (*Q.* 26 *and Q.* 27) : *Assume that lower block is very long.*

26. Coefficient of friction between two blocks shown in figure is $\mu = 0.4$. The blocks are given velocities of 2 m/s and 8 m/s in the directions shown in figure. Find :



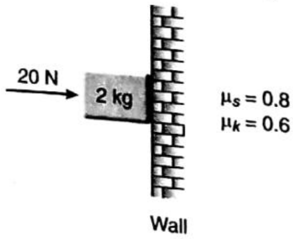
(a) The time when relative motion between them will stop.
(b) The common velocities of blocks upto that instant.
(c) Displacements of 1 kg and 2 kg blocks upto that instant. ($g = 10 \text{m/s}^2$)

27. Coefficient of friction between two blocks shown in figure is $\mu = 0.6$. The blocks are given velocities in the directions shown in figure. Find :

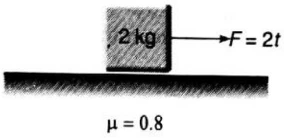


(a) Time when relative motion between them is stopped.
(b) The common velocity of the two blocks.
(c) The displacements of 1 kg and 2 kg blocks upto that instant. (Take $g = 10 \text{m/s}^2$)

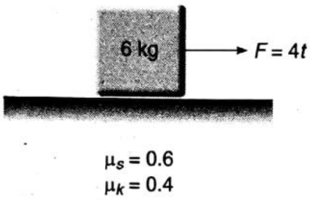
28. A 2 kg block is pressed against a rough wall by a force $F = 20 \text{N}$ as shown in figure. Find acceleration of the block and force of friction acting on it. (Take $g = 10 \text{m/s}^2$)



29. A 2 kg block is kept over a rough ground with coefficient of friction $\mu = 0.8$ as shown in figure. A time varying force $F = 2t$ (F in newton and t in second) is applied on the block. Plot a graph between acceleration of block *versus* time. ($g = 10 \text{m/s}^2$)



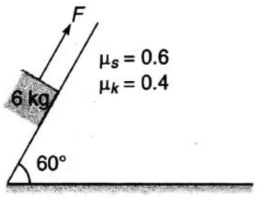
30. A 6 kg block is kept over a rough surface with coefficients of friction $\mu_s = 0.6$ and $\mu_k = 0.4$ as shown in figure. A time varying force $F = 4t$ (F in newton and t in second) is applied on the block as shown. Plot a graph between acceleration of block and time. (Take $g = 10 \text{m/s}^2$)



31. A 6 kg block is kept on an inclined rough surface as shown in figure. Find the force F required to :

(a) keep the block stationary,
(b) move the block downwards with constant velocity and
(c) move the block upwards with an acceleration of 4 m/s^2.

(Take $g = 10 \text{m/s}^2$)



Objective Questions (Level 1)

Single Correct Option

1. Two balls A and B of same size are dropped from the same point under gravity. The mass of A is greater than that of B. If the air resistance acting on each ball is same, then

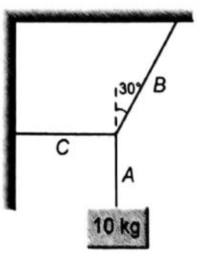
(a) both the balls reach the ground simultaneously
(b) the ball A reaches earlier
(c) the ball B reaches earlier
(d) nothing can be said

2. Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure. The tension in the string connecting weights B and C is

(a) zero (b) 13 N
(c) 3.3 N (d) 19.6 N

3. In a figure a block of mass 10 kg is in equilibrium. Identify the string in which the tension is zero.

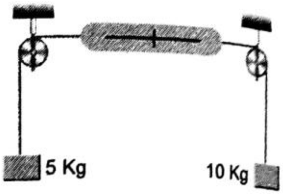
(a) B
(b) C
(c) A
(d) None of the above



4. At what minimum acceleration should a monkey should slide a rope whose breaking strength $\frac{2}{3}$ rd of its weight ?

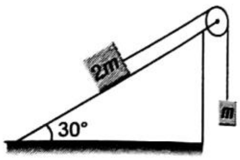
(a) $\frac{2g}{3}$ (b) g (c) $\frac{g}{3}$ (d) zero

5. For the arrangement shown in the figure the reading of spring balance is



(a) 50 N
(b) 100 N
(c) 150 N
(d) None of the above

6. For the arrangement shown in figure, the tension in the string is given by



(a) $\frac{mg}{2}$ (b) $\frac{3}{2}mg$
(c) mg (d) $2\,mg$

7. The time taken by a body to slide down a rough 45° inclined plane is twice that required to slide down a smooth 45° inclined plane. The coefficient of kinetic friction between the object and rough plane is given by

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\sqrt{\frac{3}{4}}$ (d) $\sqrt{\frac{2}{3}}$

8. The force required to just move a body up the inclined plane is double the force required to just prevent the body from sliding down the plane. The coefficient of friction is μ. If θ is the angle of inclination of the plane than $\tan\theta$ is equal to

(a) μ (b) 3μ (c) 2μ (d) $0.5\,\mu$

9. A block of mass m is placed at rest on an inclined plane of inclination θ to the horizontal. If the coefficient of friction between the block and the plane is μ, then the total force the inclined plane exerts on the block is

(a) mg (b) $\mu\, mg\cos\theta$ (c) $mg\sin\theta$ (d) $\mu\, mg\tan\theta$

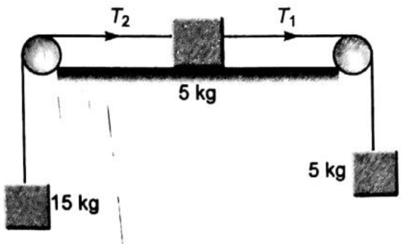
10. A force F_1 accelerates a particle from rest to a velocity v. Another force F_2 decelerates the same particle from v to rest, then

(a) F_1 is always equal to F_2
(b) F_2 is greater than F_1
(c) F_2 may be smaller than, greater than or equal to F_1
(d) F_2 cannot be equal to F_1

11. A particle is placed at rest inside a hollow hemisphere of radius R. The coefficient of friction between the particle and the hemisphere is $\mu = \frac{1}{\sqrt{3}}$. The maximum height up to which the particle can remain stationary is

(a) $\frac{R}{2}$ (b) $\left(1-\frac{\sqrt{3}}{2}\right)R$ (c) $\frac{\sqrt{3}}{2}R$ (d) $\frac{3R}{8}$

12. In the figure shown, the frictional coefficient between table and block is 0.2. Find the ratio of tensions in the right and left strings.



(a) 17 : 24 (b) 34 : 12 (c) 2 : 3 (d) 3 : 2

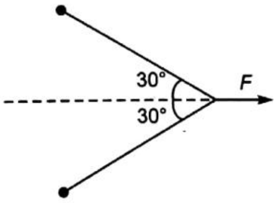
13. A smooth inclined plane of length L having inclination θ with the horizontal is inside a lift which is moving down with a retardation a. The time taken by a body to slide down the inclined plane from rest will be.

(a) $\sqrt{\dfrac{2L}{(g+a)\sin\theta}}$ (b) $\sqrt{\dfrac{2L}{(g-a)\sin\theta}}$ (c) $\sqrt{\dfrac{2L}{a\sin\theta}}$ (d) $\sqrt{\dfrac{2L}{g\sin\theta}}$

14. A block rests on a rough inclined plane making an angle of 30° with horizontal. The coefficient of static friction between the block and inclined plane is 0.8. If the frictional force on the block is 10N, the mass of the block in kg is ($g = 10\,\text{m/s}^2$)

(a) 2.0 (b) 4.0 (c) 1.6 (d) 2.5

15. In figure two identical particles each of mass m are tied together with an inextensible string. This is pulled at its centre with a constant force F. If the whole system lies on a smooth horizontal plane, then the acceleration of each particle towards each other is



(a) $\dfrac{\sqrt{3}}{2}\dfrac{F}{m}$ (b) $\dfrac{1}{2\sqrt{3}}\dfrac{F}{m}$ (c) $\dfrac{2}{\sqrt{3}}\dfrac{F}{m}$ (d) $\sqrt{3}\dfrac{F}{m}$

16. A block of mass m is placed at rest on a horizontal rough surface with angle of friction ϕ. The block is pulled with a force F at an angle θ with the horizontal. The minimum value of F required to move the block is

(a) $\dfrac{mg\sin\phi}{\cos(\theta-\phi)}$ (b) $\dfrac{mg\cos\phi}{\cos(\theta-\phi)}$ (c) $mg\tan\phi$ (d) $mg\sin\phi$

17. A block of mass 4 kg is placed on a rough horizontal plane. A time dependent horizontal force $F = kt$ acts on the block, $k = 2\,\text{Ns}^{-1}$. The frictional force between the block and plane at time $t = 2$ s is ($\mu = 0.2$)

(a) 4 N (b) 8 N (c) 12 N (d) 10 N

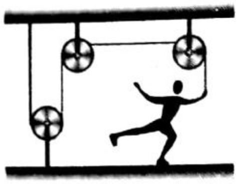
18. A body takes times t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is $2t$. The coefficient of friction of the rough surface is

(a) $\dfrac{3}{4}\tan\theta$ (b) $\dfrac{2}{3}\tan\theta$ (c) $\dfrac{1}{4}\tan\theta$ (d) $\dfrac{1}{2}\tan\theta$

19. A man of mass m slides down along a rope which is connected to the ceiling of an elevator with deceleration a relative to the rope. If the elevator is going upward with an acceleration a relative to the ground, then tension in the rope is

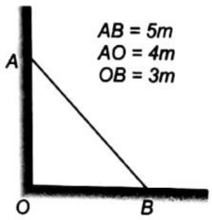
(a) mg (b) $m(g+2a)$ (c) $m(g+a)$ (d) zero

20. A 50 kg person stands on a 25kg platform. He pulls on the rope which is attached to the platform *via* the frictionless pulleys as shown in the figure. The platform moves upwards at a steady rate if the force with which the person pulls the rope is



(a) 500 N (b) 250 N (c) 25 N (d) None of these

21. A ladder of length 5 m is placed against a smooth wall as shown in figure. The coefficient of friction is μ between ladder and ground. What is the minimum value of μ, if the ladder is not to slip?



(a) $\mu = \frac{1}{2}$ (b) $\mu = \frac{1}{4}$ (c) $\mu = \frac{3}{8}$ (d) $\mu = \frac{5}{8}$

22. If a ladder weighing 250 N is placed against a smooth vertical wall having coefficient of friction between it and floor 0.3, then what is the maximum force of friction available at the point of contact between the ladder and the floor?

(a) 75 N (b) 50 N (c) 35 N (d) 25 N

23. A rope of length L and mass M is being pulled on a rough horizontal floor by a constant horizontal force $F = Mg$. The force is acting at one end of the rope in the same direction as the length of the rope. The coefficient of kinetic friction between rope and floor is 1/2. Then, the tension at the midpoint of the rope is

(a) $\frac{Mg}{4}$ (b) $\frac{2Mg}{5}$ (c) $\frac{Mg}{8}$ (d) $\frac{Mg}{2}$

24. A heavy body of mass 25 kg is to be dragged along a horizontal plane $\left(\mu = \frac{1}{\sqrt{3}}\right)$. The least force required is

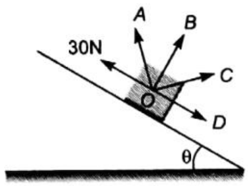
(a) 25 kgf (b) 2.5 kgf (c) 12.5 kgf (d) 6.25 kgf

25. A block A of mass 4 kg is kept on ground. The coefficient of friction between the block and the ground is 0.8. The external force of magnitude 30 N is applied parallel to the ground. The resultant force exerted by the ground on the block is

(a) 40 N (b) 30 N (c) zero (d) 50 N

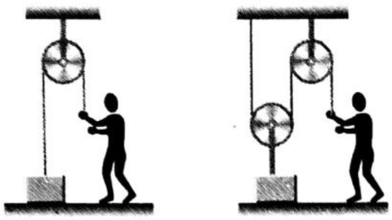
26. A block A of mass 2 kg rests on another block B of mass 8 kg which rests on a horizontal floor. The coefficient of friction between A and B is 0.2 while that between B and floor is 0.5. When a horizontal force F of 25 N is applied on the block B, the force of friction between A and B is
(a) 3 N (b) 4 N (c) 2 N (d) zero

27. A body of mass 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ with the horizontal. When the force of 30 N is applied on the block parallel to and upward the plane, the total force by the plane on the block is nearly along



(a) OA (b) OB
(c) OC (d) OD

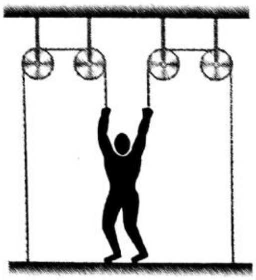
28. In the figure shown, a person wants to raise a block lying on the ground to a height h. In which case he has to exert more force. Assume pulleys and strings are light



(i) (ii)

(a) Fig. (i) (b) Fig. (ii)
(c) Same in both (d) Cannot be determined

29. A man of mass m stands on a platform of equal mass m and pulls himself by two ropes passing over pulleys as shown in figure. If he pulls each rope with a force equal to half his weight, his upward acceleration would be



(a) $\frac{g}{2}$
(b) $\frac{g}{4}$
(c) g
(d) zero

30. A varying horizontal force $F =$ at acts on a block of mass m kept on a smooth horizontal surface. An identical block is kept on the first block. The coefficient of friction between the blocks is μ. The time after which the relative sliding between the blocks prevails is
(a) $\frac{2mg}{a}$ (b) $\frac{2\mu mg}{a}$ (c) $\frac{\mu mg}{a}$ (d) $2\mu mga$

31. Two particles start together from a point O and slide down along straight smooth wires inclined at 30° and 60° to the vertical plane and on the same side of vertical through O. The relative acceleration of second with respect to first will be of magnitude
(a) $\frac{g}{2}$ (b) $\frac{\sqrt{3}g}{2}$ (c) $\frac{g}{\sqrt{3}}$ (d) g

JEE Corner

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*

(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*

(*c*) *If **Assertion** is true, but the **Reason** is false.*

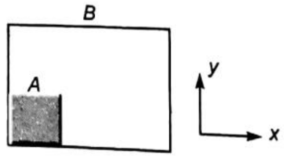
(*d*) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** If net force on a rigid body in zero, it is either at rest or moving with a constant linear velocity. Nothing else can happen.
Reason : Constant velocity means linear acceleration is zero.

2. **Assertion :** Three concurrent forces are $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Angle between $\vec{F}_1$ and $\vec{F}_2$ is 30° and between $\vec{F}_1$ and $\vec{F}_3$ is 120°. Under these conditions, forces cannot remain in equilibrium.
Reason : At least one angle should be greater than 180°.

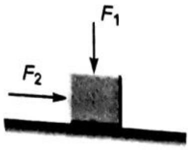
3. **Assertion :** Two identical blocks are placed over a rough inclined plane. One block is given an upward velocity to the block and the other in downward direction. If $\mu = \frac{1}{3}$ and $\theta = 45°$ the ratio of magnitudes of accelerations of two is 2 : 1.
Reason : The desired ratio is $\frac{1+\mu}{1-\mu}$.

4. **Assertion :** A block A is just placed inside a smooth box B as shown in figure. Now, the box is given an acceleration $\vec{a} = (3\hat{j} - 2\hat{i})\,ms^{-2}$. Under this acceleration block A can not remain in the position shown.

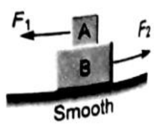


Reason : Block will require $m\vec{a}$ force for moving with acceleration $\vec{a}$.

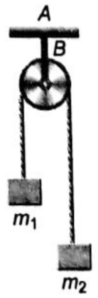
5. **Assertion :** A block is kept at rest on a rough ground as shown. Two forces F_1 and F_2 are acting on it. If we increase either of the two forces F_1 or F_2, force of friction will increase.
Reason : By increasing F_1, normal reaction from ground will increase.



6. **Assertion :** In the figure shown force of friction on A from B will be right wards.
Reason : Friction always opposes the relative motion between two bodies in contact.



7. **Assertion :** In the figure shown tension in string AB always lies between $2m_1g$ and $2m_2g$. ($m_1 \neq m_2$)

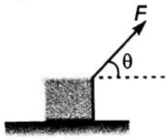


Reason : Tension in massless string is uniform throughout.

8. **Assertion :** Two frames S_1 and S_2 are noninertial. Then frame S_2 when observed from S_1 is also noninertial.
Reason : A frame in motion is not necessarily a non-inertial frame.

9. **Assertion :** Moment of concurrent forces about any point is constant.
Reason : If vector sum of all the concurrent forces is zero, then moment of all the forces about any point is also zero.

10. **Assertion :** Minimum force is needed to move a block on rough surface, if θ = angle of friction.
Reason : Angle of friction and angle of repose are numerically same.

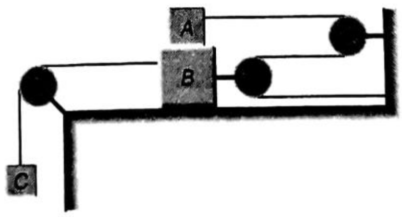


11. **Assertion :** When a person walks on a rough surface, the frictional force exerted by surface on the person is opposite to the direction of his motion.
Reason : It is the force exerted by the road on the person that causes the motion.

Objective Questions (Level 2)

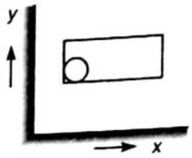
Single Correct Option

1. What is the largest mass of C in kg that can be suspended without moving blocks A and B ? The static coefficient of friction for all plane surface of contact is 0.3. Mass of block A is 50kg and block B is 70kg. Neglect friction in the pulleys



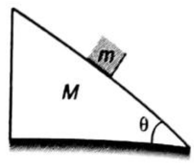
(a) 120 kg (b) 92 kg (c) 81 kg (d) None of these

2. A sphere of mass 1 kg rests at one corner of a cube. The cube is moved with a velocity $\vec{v} = 8t\,\hat{i} - 2t^2\,\hat{j}$. Where t is time in second. The force by sphere on the cube at $t = 1$ s is ($g = 10\,\text{ms}^{-2}$) [Figure shows vertical plane of the cube]



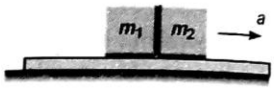
(a) 8 N (b) 10 N (c) 20 N (d) 6 N

3. A smooth block of mass m is held stationary on a smooth wedge of mass M and inclination θ as shown in figure. If the system is released from rest, then the normal reaction between the block and the wedge is



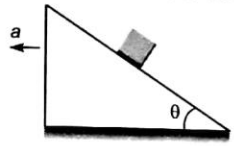
(a) $mg \cos \theta$
(b) less than $mg \cos \theta$
(c) greater than $mg \cos \theta$
(d) may be less or greater than $mg \cos \theta$ depending upon whether m is less or greater than m

4. Two blocks of masses m_1 and m_2 are placed in contact with each other on a horizontal platform as shown in figure. The coefficient of friction between m_1 and platform is 2μ and that between block m_2 and platform is μ. The platform moves with an acceleration a. The normal reaction between the blocks is



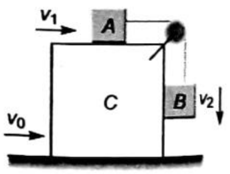
(a) zero in all cases
(b) zero only if $m_1 = m_2$
(c) non zero only if $a > 2\mu g$
(d) non zero only if $a > \mu g$

5. A block of mass m is resting on a wedge of angle θ as shown in the figure. With what minimum acceleration a should the wedge move so that the mass m falls freely?



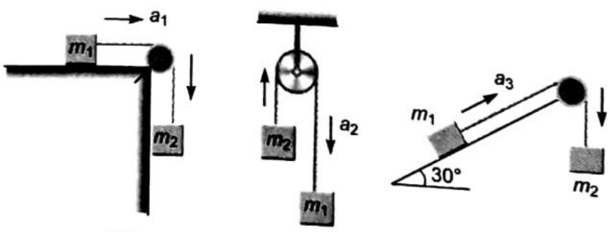
(a) g (b) $g \cos\theta$ (c) $g \cot\theta$ (d) $g \tan\theta$

6. To a ground observer the block C is moving with v_0 and the blocks A and B are moving with v_1 and v_2 relative to C as shown in the figure. Identify the correct statement.



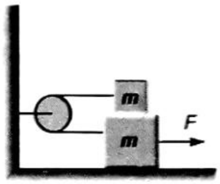
(a) $v_1 - v_2 = v_0$
(b) $v_1 = v_2$
(c) $v_1 + v_0 = v_2$
(d) None of these

7. In each case $m_1 = 4\,\text{kg}$ and $m_2 = 3\,\text{kg}$. If a_1, a_2 and a_3 are the respective accelerations of the block m_1 in given situations, then



(a) $a_1 > a_2 > a_3$ (b) $a_1 > a_2 = a_3$ (c) $a_1 = a_2 = a_3$ (d) $a_1 > a_3 > a_2$

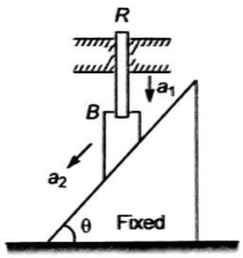
8. For the arrangement shown in figure the coefficient of friction between the two blocks is μ. If both the blocks are identical, then the acceleration of each block is



(a) $\frac{F}{2m}-2\mu g$ (b) $\frac{F}{2m}$

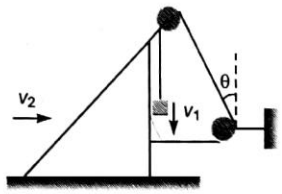
(c) $\frac{F}{2m}-\mu g$ (d) zero

9. In the arrangement shown in the figure the rod R is restricted to move in the vertical direction with acceleration a_1, and the block B can slide down the fixed wedge with acceleration a_2. The correct relation between a_1 and a_2 is given by



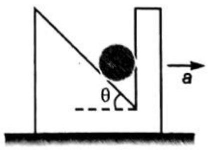
(a) $a_2 = a_1 \sin\theta$
(b) $a_2 \sin\theta = a_1$
(c) $a_2 \cos\theta = a_1$
(d) $a_2 = a_1 \cos\theta$

10. In figure the block moves downwards with velocity v_1, the wedge moves rightwards with velocity v_2. The correct relation between v_1 and v_2 is



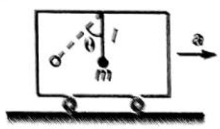
(a) $v_2 = v_1$ (b) $v_2 = v_1 \sin\theta$ (c) $2v_2 \sin\theta = v_1$ (d) $v_2(1+\sin\theta) = v_1$

11. In the figure, the minimum value of a at which the cylinder starts rising up the inclined surface is



(a) $g\tan\theta$ (b) $g\cot\theta$ (c) $g\sin\theta$ (d) $g\cos\theta$

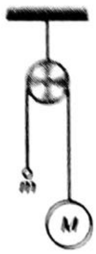
12. When the trolley shown in figure is given a horizontal acceleration a, the pendulum bob of mass m gets deflected to a maximum angle θ with the vertical. At the position of maximum deflection, the net acceleration of the bob with respect to trolley is



(a) $\sqrt{g^2+a^2}$ (b) $a\cos\theta$

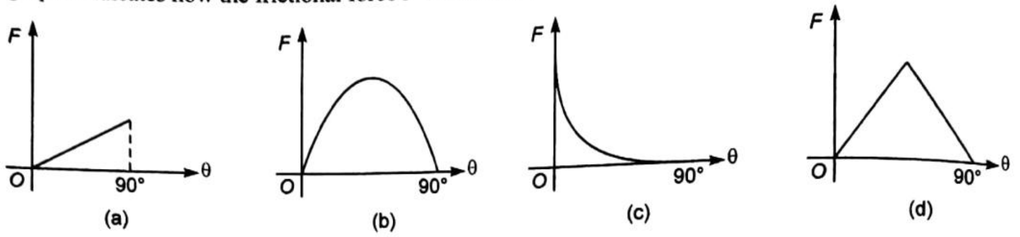
(c) $g\sin\theta - a\cos\theta$ (d) $a\sin\theta$

13. In the arrangement shown in figure the mass M is very heavy compared to m ($M >> m$). The tension T in the string suspended from the ceiling is

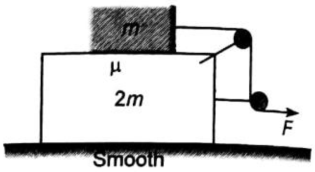


(a) 4 mg
(b) 2 mg
(c) zero
(d) None of the above

14. A block rests on a rough plane whose inclination θ to the horizontal can be varied. Which of the following graphs indicates how the frictional force F between the block and the plane varies as θ is increased?



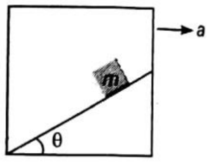
15. The minimum value of μ between the two blocks for no slipping is



(a) $\frac{F}{mg}$ (b) $\frac{F}{3mg}$

(c) $\frac{2F}{3mg}$ (d) $\frac{4F}{3mg}$

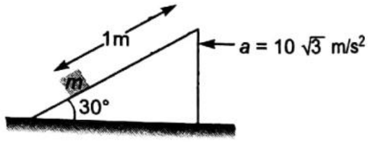
16. A block is sliding along incline as shown in figure. If the acceleration of chamber is a as shown in the figure. The time required to cover a distance L along incline is



(a) $\sqrt{\frac{2L}{g\sin\theta - a\cos\theta}}$ (b) $\sqrt{\frac{2L}{g\sin\theta + a\sin\theta}}$

(c) $\sqrt{\frac{2L}{g\sin\theta + a\cos\theta}}$ (d) $\sqrt{\frac{2L}{g\sin\theta}}$

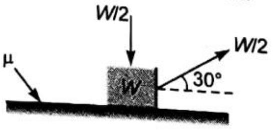
17. In the figure, the wedge is pushed with an acceleration of $10\sqrt{3}\ \text{m/s}^2$. It is seen that the block starts climbing up on the smooth inclined face of wedge. What will be the time taken by the block to reach the top?



(a) $\frac{2}{\sqrt{5}}$ s (b) $\frac{1}{\sqrt{5}}$ s

(c) $\sqrt{5}$ s (d) $\frac{\sqrt{5}}{2}$ s

18. A block of weight W is kept on a rough horizontal surface (friction coefficient μ). Two forces $\frac{W}{2}$ each are applied as shown in the figure. Choose the correct statement.



(a) For $\mu > \frac{\sqrt{3}}{5}$ block will move

(b) For $\mu < \frac{\sqrt{3}}{5}$, work done by friction force is zero (in ground frame)

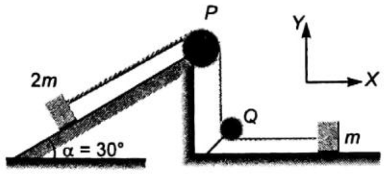
(c) For $\mu > \frac{\sqrt{3}}{5}$, friction force will do positive work (in ground frame)

(d) For $\mu < \frac{\sqrt{3}}{5}$ block will move

19. Two blocks A and B are separated by some distance and tied by a string as shown in the figure. The force of friction in both the blocks at $t = 2$ s is

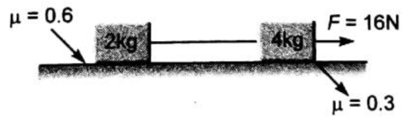
(a) 4 N (→), 5 N(←) (b) 2 N(→), 5 N(←) (c) 0 N(→), 10 N(←) (d) 1 N(←), 10 N(←)

20. All the surfaces and pulleys are frictionless in the shown arrangement. Pulleys P and Q are massless. The force applied by clamp on pulley P is



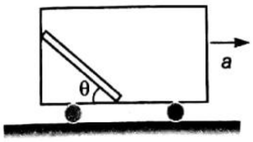
(a) $\frac{mg}{6}(-\sqrt{3}\,\hat{i} - 3\,\hat{j})$ (b) $\frac{mg}{6}(\sqrt{3}\,\hat{i} + 3\,\hat{j})$ (c) $\frac{mg}{6}\sqrt{2}$ (d) None of these

21. Two blocks of masses 2 kg and 4 kg are connected by a light string and kept on horizontal surface. A force of 16 N is acted on 4kg block horizontally as shown in figure. Besides it is given that coefficient of friction between 4 kg and ground is 0.3 and between 2kg block and ground is 0.6. Then frictional force between 2 kg block and ground is



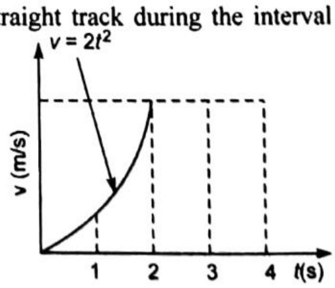
(a) 12 N (b) 6 N (c) 4 N (d) zero

22. A smooth rod of length l is kept inside a trolley at an angle θ as shown in the figure. What should be the acceleration a of the trolley so that the rod remains in equilibrium with respect to it?



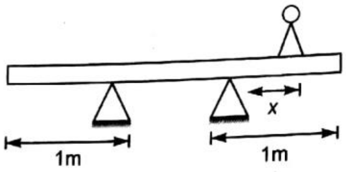
(a) $g\tan\theta$ (b) $g\cos\theta$ (c) $g\sin\theta$ (d) $g\cot\theta$

23. A car begins from rest at time $t = 0$, and then accelerates along a straight track during the interval $0 < t \le 2$s and thereafter with constant velocity as shown in the graph. A coin is initially at rest on the floor of the car. At $t = 1$ s, the coin begins to slip and its stops slipping at $t = 3$ s. The coefficient of static friction between the floor and the coin is ($g = 10\ \text{m/s}^2$)



(a) 0.2 (b) 0.3
(c) 0.4 (d) 0.5

24. A horizontal plank is 10.0 m long with uniform density and mass 10kg. It rests on two supports which are placed 1.0 m from each end as shown in the figure. A man of mass 80kg can stand upto distance x on the plank without causing it to tip. The value of x is



(a) $\frac{1}{2}$ m (b) $\frac{1}{4}$ m (c) $\frac{3}{4}$ m (d) $\frac{1}{8}$ m

25. A block is kept on a smooth inclined plane of angle of inclination θ that moves with a constant acceleration so that the block does not slide relative to the inclined plane. If the inclined plane stops, the normal contact force offered by the plane on the block changes by a factor

(a) $\tan\theta$ (b) $\tan^2\theta$ (c) $\cos^2\theta$ (d) $\cot\theta$

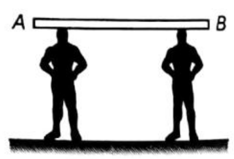
26. A uniform cube of mass m and side a is resting in equilibrium on a rough 45° inclined surface. The distance of the point of application of normal reaction measured from the lower edge of the cube is

(a) zero (b) $\frac{a}{3}$ (c) $\frac{a}{\sqrt{2}}$ (d) $\frac{a}{4}$

27. A horizontal force $F = \frac{mg}{3}$ is applied on the upper surface of a uniform cube of mass m and side a which is resting on a rough horizontal surface having $\mu = \frac{1}{2}$. The distance between lines of action of mg and normal reaction is

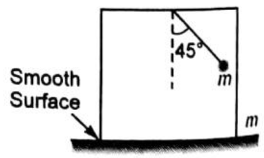
(a) $\frac{a}{2}$ (b) $\frac{a}{3}$ (c) $\frac{a}{4}$ (d) None of these

28. Two persons of equal height are carrying a long uniform wooden plank of length l. They are at distance $\frac{l}{4}$ and $\frac{l}{6}$ from nearest end of the rod. The ratio of normal reaction at their heads is



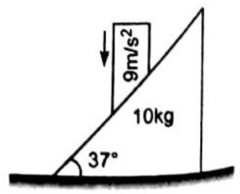
(a) 2 : 3 (b) 1 : 3
(c) 4 : 3 (d) 1 : 2

29. A ball connected with string is released at an angle 45° with the vertical as shown in the figure. Then the acceleration of the box at this instant will be (mass of the box is equal to mass of ball)



(a) $\frac{g}{4}$ (b) $\frac{g}{3}$
(c) $\frac{g}{2}$ (d) g

30. In the system shown in figure all surfaces are smooth. Rod is moved by external agent with acceleration 9 ms^{-2} vertically downwards. Force exerted on the rod by the wedge will be



(a) 120 N (b) 200 N
(c) 160 N (d) 180 N

31. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration $4\ ms^{-2}$. A bead can slide on the rod and friction coefficient between them is 0.5. If the bead is released from rest at the top of the rod, it will reach the bottom in

(a) $\sqrt{2}$ s (b) 1 s (c) 2 s (d) 0.5 s

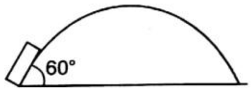
32. Three solid hemispheres of radii R each, are placed in contact with each other with their flat faces on a rough horizontal surface. A sphere of mass m and radius R is placed symmetrically on top of them. The normal reaction between the top sphere and any hemisphere assuming the system to be in static equilibrium is

(a) $\frac{mg}{3}$ (b) $\frac{mg}{\sqrt{6}}$ (c) $\frac{mg}{\sqrt{3}}$ (d) None of these

33. Mr. X of mass 80 kg enters a lift and selects the floor he wants. The lift now accelerates upwards at $2\ ms^{-2}$ for 2 s and then moves with constant velocity. As the lift approaches his floor, it decelerates at the same rate as it previously accelerates. If the lift cables can safely withstand a tension of 2×10^4 N and the lift itself has a mass of 500 kg, how many Mr.X's could it safely carry at one time?

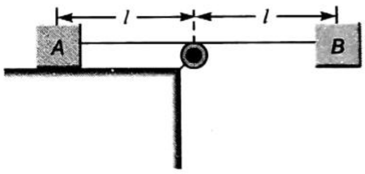
(a) 22 (b) 14 (c) 18 (d) 12

34. A particle when projected in vertical plane moves along smooth surface with initial velocity $20\ ms^{-1}$ at an angle of 60°, so that its normal reaction on the surface remains zero throughout the motion. Then the slope of the tangent to the surface at height 5m from the point of projection will be



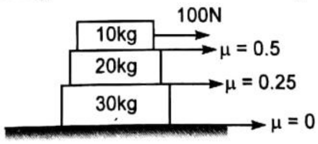
(a) 30° (b) 45° (c) $\tan^{-1} 2$ (d) None of these

35. Two blocks A and B, each of same mass are attached by a thin inextensible string through an ideal pulley. Initially block B is held in position as shown in figure. Now, the block B is released. Block A will slide to right and hit the pulley in time t_A. Block B will swing and hit the surface in time t_B. Assume the surface as frictionless, then



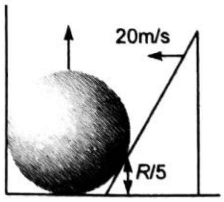
(a) $t_A > t_B$ (b) $t_A < t_B$
(c) $t_A = t_B$ (d) data insufficient

36. Three blocks are kept as shown in figure. Acceleration of 20 kg block with respect to ground is



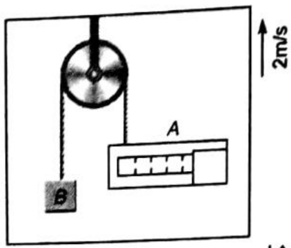
(a) $5\ ms^{-2}$ (b) $2\ ms^{-2}$ (c) $1\ ms^{-2}$ (d) None of these

37. A sphere of radius R is in contact with a wedge. The point of contact is $\frac{R}{5}$ from the ground as shown in the figure. Wedge is moving with velocity $20\ ms^{-1}$ towards left then the velocity of the sphere at this instant will be



(a) $20\ ms^{-1}$ (b) $15\ ms^{-1}$
(c) $16\ ms^{-1}$ (d) $12\ ms^{-1}$

38. In the figure it is shown that the velocity of lift is 2 ms^{-1} while string is winding on the motor shaft with velocity 2 ms^{-1} and shaft A is moving downward with velocity 2 ms^{-1} with respect to lift, then find out the velocity of block B

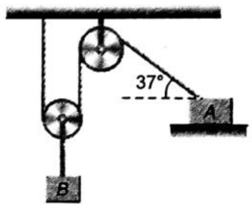


(a) 2 $ms^{-1}\uparrow$ (b) 2 $ms^{-1}\downarrow$ (c) 4 $ms^{-1}\uparrow$ (d) None of these

39. A monkey pulls the midpoint of a 10 cm long light inextensible string connecting two identical objects A and B lying on smooth table of masses 0.3 kg continuously along the perpendicular bisector of line joining the masses. The masses are found to approach each other at a relative acceleration of 5 ms^{-2} when they are 6 cm apart. The constant force applied by monkey is

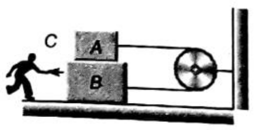
(a) 4 N (b) 2 N (c) 3 N (d) None of these

40. In the figure shown the block B moves with the velocity 10 ms^{-1}. The velocity of A in the position shown is



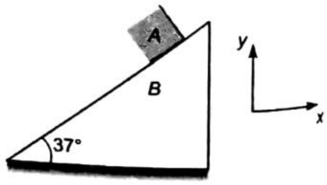
(a) 12.5 ms^{-1} (b) 25 ms^{-1} (c) 8 ms^{-1} (d) 16 ms^{-1}

41. In the figure $m_A = m_B = m_c = 60$ kg. The coefficient of friction between C and ground is 0.5, B and ground is 0.3, A and B is 0.4. C is pulling the string with the maximum possible force without moving. Then the tension in the string connected to A will be



(a) 120 N (b) 60 N (c) 100 N (d) zero

42. In the figure shown the acceleration of A is $\vec{a}_A = 15\hat{i} + 15\hat{j}$. Then the acceleration of B is (A remains in contact with B)



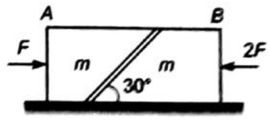
(a) $6\hat{i}$

(b) $-15\hat{i}$

(c) $-10\hat{i}$

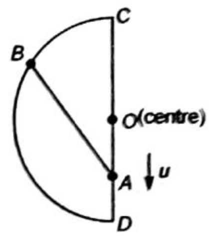
(d) $-5\hat{i}$

43. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal forces F and $2F$ are applied on the blocks A and B respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is



(a) F (b) $\frac{F}{2}$ (c) $\frac{F}{\sqrt{3}}$ (d) $3F$

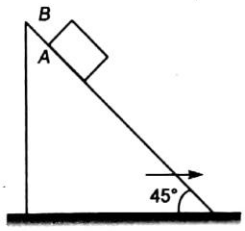
44. Two beads A and B move along a semicircular wire frame as shown in figure. The beads are connected by an inelastic string which always remains tight. At an instant the speed of A is u, $\angle BAC = 45°$ and $BOC = 75°$, where O is the centre of the semicircular arc. The speed of bead B at that instant is



(a) $\sqrt{2}u$ (b) u

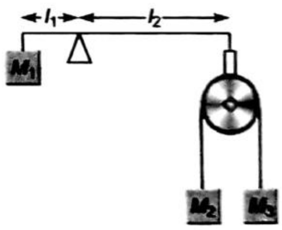
(c) $\frac{u}{2\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}u$

45. If the coefficient of friction between A and B is μ, the maximum acceleration of the wedge A for which B will remain at rest with respect to the wedge is



(a) μg (b) $g\left(\frac{1+\mu}{1-\mu}\right)$ (c) $g\left(\frac{1-\mu}{1+\mu}\right)$ (d) $\frac{g}{\mu}$

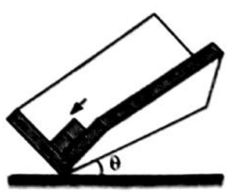
46. A pivoted beam of negligible mass has a mass suspended from one end and an Atwood's machine suspended from the other. The frictionless pulley has negligible mass and dimension. Gravity is directed downward and $M_2 = 3M_3,\ l_2 = 3l_1$. Find the ratio $\frac{M_1}{M_2}$ which will ensure that the beam has no tendency to rotate just after the masses are released



(a) $\frac{M_1}{M_2} = 2$ (b) $\frac{M_1}{M_2} = 3$

(c) $\frac{M_1}{M_2} = 4$ (d) None of these

47. A block of mass m slides down an inclined right angled trough. If the coefficient of friction between block and the trough is μ_k, acceleration of the block down the plane is



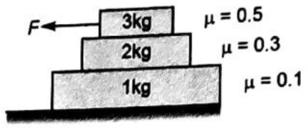
(a) $g(\sin\theta + \sqrt{2}\mu_k \cos\theta)$

(b) $g(\sin\theta + \mu_k \cos\theta)$

(c) $g(\sin\theta - \sqrt{2}\mu_k \cos\theta)$

(d) $g(\sin\theta - \mu_k \cos\theta)$

48. If force F is increasing with time and at $t = 0$, $F = 0$, where will slipping first start?



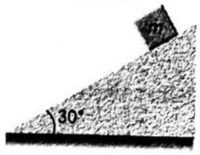
(a) between 3 kg and 2 kg
(b) between 2 kg and 1 kg
(c) between 1 kg and ground
(d) Both (a) and (b)

49. A plank of mass 2kg and length 1 m is placed on horizontal floor. A small block of mass 1kg is placed on top of the plank, at its right extreme end. The coefficient of friction between plank and floor is 0.5 and that between plank and block is 0.2. If a horizontal force = 30N starts acting on the plank to the right, the time after which the block will fall off the plank is ($g = 10\,ms^{-2}$)

(a) $\left(\frac{2}{3}\right)$ s (b) 1.5 s (c) 0.75 s (d) $\left(\frac{4}{3}\right)$ s

Passage 1 (Q. No. 50 to 54)

A man wants to slide down a block of mass m which is kept on a fixed inclined plane of inclination 30° as shown in the figure. Initially the block is not sliding.



To just start sliding the man pushes the block down the incline with a force F. Now, the block starts accelerating. To move it downwards with constant speed the man starts pulling the block with same force. Surfaces are such that ratio of maximum static friction to kinetic friction is 2. Now, answer the following questions.

50. What is the value of F?

(a) $\frac{mg}{4}$ (b) $\frac{mg}{6}$ (c) $\frac{mg\sqrt{3}}{4}$ (d) $\frac{mg}{2\sqrt{3}}$

51. What is the value of μ_s, the coefficient of static friction?

(a) $\frac{4}{3\sqrt{3}}$ (b) $\frac{2}{3\sqrt{3}}$ (c) $\frac{3}{3\sqrt{3}}$ (d) $\frac{1}{2\sqrt{3}}$

52. If the man continues pushing the block by force F, its acceleration would be

(a) $\frac{g}{6}$ (b) $\frac{g}{4}$ (c) $\frac{g}{2}$ (d) $\frac{g}{3}$

53. If the man wants to move the block up the incline, what minimum force is required to start the motion?

(a) $\frac{2}{3}mg$ (b) $\frac{mg}{2}$ (c) $\frac{7mg}{6}$ (d) $\frac{5mg}{6}$

54. What minimum force is required to move it up the incline with constant speed?

(a) $\frac{2}{3}mg$ (b) $\frac{mg}{2}$ (c) $\frac{7mg}{6}$ (d) $\frac{5mg}{6}$

Passage 2 (Q. No. 55 to 56)

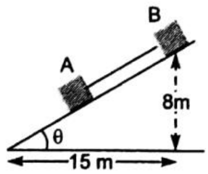
A lift with a mass 1200 kg is raised from rest by a cable with a tension 1350 g newton. After some time the tension drops to 1000 g Newton and the lift comes to rest at a height of 25 m above its initial point. (1 g newton = 9.8 N)

55. What is the height at which the tension changes?
(a) 10.8 m (b) 12.5 m (c) 14.3 m (d) 16 m

56. What is greatest speed of lift?
(a) 9.8 ms^{-1} (b) 7.5 ms^{-1} (c) 5.92 ms^{-1} (d) None of these

Passage 3 (Q. No. 57 to 58)

Blocks A and B shown in the figure are connected with a bar of negligible weight. A and B each has mass 170 kg, the coefficient of friction between A and the plane is 0.2 and that between B and the plane is 0.4 ($g = 10\ ms^{-2}$)

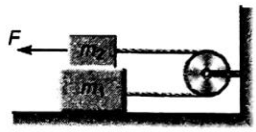


57. What is the total force of friction between the blocks and the plane?
(a) 900 N (b) 700 N (c) 600 N (d) 300 N

58. What is the force acting on the connecting bar?
(a) 150 N (b) 100 N (c) 75 N (d) 125 N

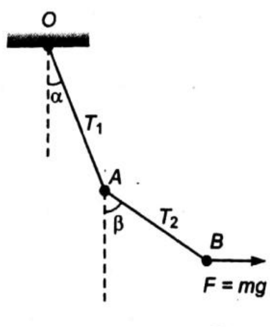
More than One Correct Options

1. Two blocks each of mass 1 kg are placed as shown. They are connected by a string which passes over a smooth (massless) pulley.



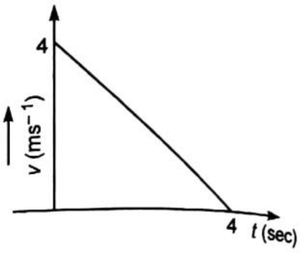
There is no friction between m_1 and the ground. The coefficient of friction between m_1 and m_2 is 0.2. A force F is applied to m_2. Which of the following statements is/are correct
(a) The system will be in equilibrium if $F \leq 4$ N
(b) If $F > 4$ N tension in the string will be 4 N
(c) If $F > 4$ N the frictional force between the blocks will be 2 N
(d) If $F = 6$ N tension in the string will be 3 N

2. Two particles A and B, each of mass m are kept stationary by applying a horizontal force $F = mg$ on particle B as shown in figure. Then

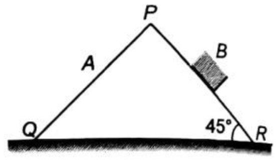


(a) $\tan\beta = 2\tan\alpha$ (b) $2T_1 = 5T_2$ (c) $\sqrt{2}\,T_1 = \sqrt{5}T_2$ (d) None of these

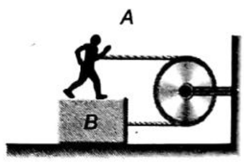
3. The velocity-time graph of the figure shows the motion of a wooden block of mass 1 kg which is given an initial push at $t = 0$ along a horizontal table.
(a) The coefficient of friction between the block and the table is 0.1
(b) The coefficient of friction between the block and the table is 0.2
(c) If the table was half of its present roughness, the time taken by the block to complete the journey is 4 s
(d) If the table was half of its present roughness, the time taken by the block to complete the journey is 8 s



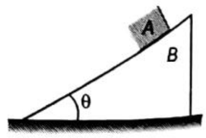
4. A block B of mass 0.6 kg slides down the smooth face PR of a wedge A of mass 1.7 kg which can move freely on a smooth horizontal surface. The inclination of the face PR to the horizontal is 45°. Then
(a) the acceleration of A is $\frac{3g}{40}$
(b) the vertical component of the acceleration of B is $23\frac{g}{40}$
(c) the horizontal component of the acceleration of B is $17\frac{g}{40}$
(d) the acceleration of A is $\frac{g}{3}$



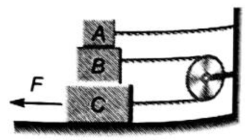
5. As shown in the figure, A is a man of mass 60kg standing on a block of mass 40 kg kept on ground. The coefficient of friction between the feet of the man and the block is 0.3 and that between B and the ground is 0.2. If the person pulls the string with 125 N force, then
(a) B will slide on ground
(b) A and B will move with acceleration 0.5 ms^{-2}
(c) the force of friction acting between A and B will be 40 N
(d) the force of friction acting between A and B will be 180 N



6. In the figure shown A and B are free to move. All the surfaces are smooth. Mass of A is m. Then
(a) the acceleration of A will be more than g sin θ
(b) the acceleration of A will be less than g sin θ
(c) normal reaction on A due to B will be more than mg cos θ
(d) normal reaction on A due to B will be less than mg cos θ



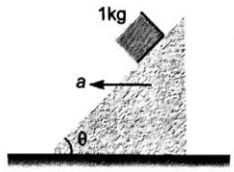
7. $M_A = 3$ kg, $M_B = 4$ kg, and $M_C = 8$ kg. μ between any two surfaces is 0.25. Pulley is frictionless and string is massless. A is connected to wall through a massless rigid rod.
(a) the value of F to keep C moving with constant speed is 80 N.
(b) the value of F to keep C moving with constant speed is 120 N.
(c) if F is 200 N then acceleration of B is 10 ms^{-2}
(d) to slide C towards left, F should be at least 50 N. (take $g = 10\ ms^{-2}$)



8. A man pulls a block of mass equal to himself with a light string. The coefficient of friction between the man and the floor is greater than that between the block and the floor
(a) if the block does not move, then the man also does not move
(b) the block can move even when the man is stationary

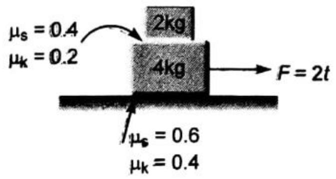
(c) if both move then the acceleration of the block is greater than the accèleration of man
(d) if both move then the acceleration of man is greater than the acceleration of block

9. A block of mass 1 kg is at rest relative to a smooth wedge moving leftwards with constant acceleration $a = 5\ \text{ms}^{-2}$. Let N be the normal reaction between the block and the wedge. Then ($g = 10\,\text{ms}^{-2}$)



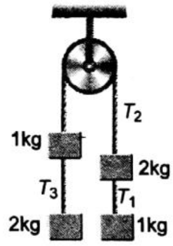
(a) $N = 5\sqrt{5}$ N (b) $N = 15$ N
(c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = 2$

10. For the given situation shown in figure, choose the correct options, ($g = 10\,\text{ms}^{-2}$)



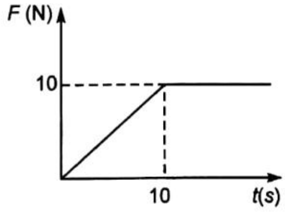
(a) at $t = 1$ s, force of friction between 2 kg and 4 kg is 2N
(b) At $t = 1$ s, force of friction between 2 kg and 4 kg is zero
(c) at $t = 4$ s force of friction between 4 kg and ground is 8 N
(d) At $t = 15$ s acceleration of 2kg is 1 ms^{-2}

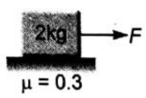
11. In the figure shown, all the strings are massless and friction is absent everywhere. Choose the correct options.



(a) $T_1 > T_3$
(b) $T_3 > T_1$
(c) $T_2 > T_1$
(d) $T_2 > T_3$

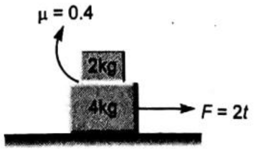
12. Force acting on a block *versus* time graph is as shown in figure. Choose the correct options. ($g = 10\,\text{ms}^{-2}$)





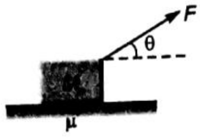
(a) At $t = 2$ s, force of friction is 2N (b) At $t = 8$ s, force of friction is 6 N
(c) At $t = 10$ s, acceleration of block is 2 ms^{-2} (d) At $t = 12$ s, velocity of block is 8 ms^{-1}

13. For the situation shown in figure, mark the correct options.



(a) At $t = 3$ s, pseudo force on 4 kg when applied from 2 kg is 4 N in forward direction
(b) At $t = 3$ s, pseudo force on 2 kg when applied from 4 kg is 2 N in backward direction
(c) Pseudo force does not make an equal and opposite pairs
(d) Pseudo force also makes a pair of equal and opposite forces.

14. For the situation shown in figure, mark the correct options.



(a) Angle of friction is $\tan^{-1}(\mu)$

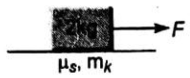
(b) Angle of repose is $\tan^{-1}(\mu)$

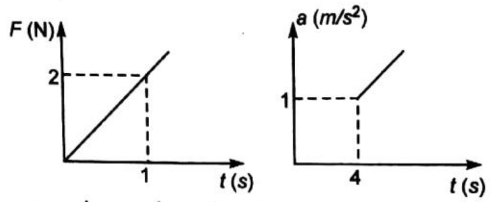
(c) At $\theta = \tan^{-1}(\mu)$, minimum force will be required to move the block

(d) Minimum froce required to move the block is $\frac{\mu Mg}{\sqrt{1+\mu^2}}$.

Match the Columns

1.

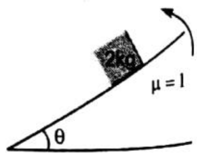




Force acting on a block *versus* time and acceleration *versus* time graph are as shown in figure. Taking value of $g = 10\,ms^{-2}$, match the following two columns.

Column I	Column II
(a) Coefficient of static friction	(p) 0.2
(b) Coefficient of kinetic friction	(q) 0.3
(c) Force of friction at $t = 0.1$ s	(r) 0.4
(d) Value of $\frac{a}{10}$, where a is acceleration of block at $t = 8$ s	(s) 0.5

2. Angle θ is gradually increased as shown in figure. For the given situation match the following two columns. ($g = 10\,ms^{-2}$)

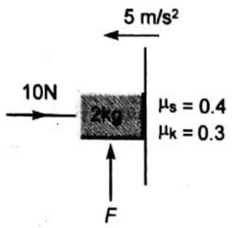


Column I	Column II
(a) Force of friction when θ = 0°	(p) 10 N
(b) Force of friction when θ = 90°	(q) $10\sqrt{3}$ N
(c) Force of friction when θ = 30°	(r) $\frac{10}{\sqrt{3}}$ N
(d) Force of friction when θ = 60°	(s) None

3. Match the following two columns regarding fundamental forces of nature.

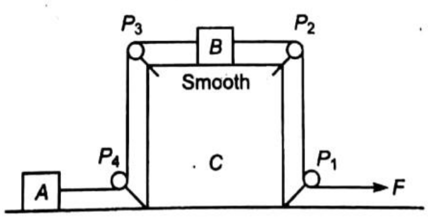
Column I	Column II
(a) Force of friction	(p) field force
(b) Normal reaction	(q) contact force
(c) Force between two neutrons	(r) electromagnetic force
(d) Force between two protons	(s) nuclear force

4. In the figure shown, match the following two columns. ($g = 10\,\text{ms}^{-2}$)



Column I	Column II
(a) Normal reactoin	(p) 5 N
(b) Force of friction when $F = 15\,\text{N}$	(q) 10 N
(c) Minimum value of F for stopping the block moving down	(r) 15 N
(d) Minimum value of F for stopping the block moving up	(s) None

5. There is no friction between blocks B and C. But ground in rough. Pulleys are smooth and massless and strings are light. For $F = 10\,\text{N}$, whole system remains stationary. Match the following two columns. ($m_B = m_C = 1\,\text{kg}$ and $g = 10\,\text{ms}^{-2}$)



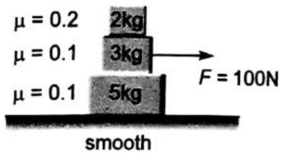
Column I	Column II
(a) Force of friction between A and ground	(p) 10 N
(b) Force of friction between C and ground	(q) 20 N
(c) Normal reaction on C from ground	(r) 5 N
(d) Tension in string between P_3 and P_4	(s) None

6. Match Column I with Column II.

Note *Applied force is parallel to plane.*

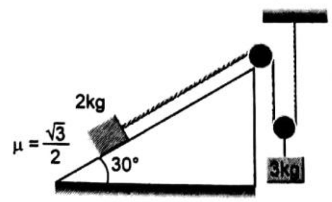
Column I	Column II
(a) If friction force is less than applied force then friction may be	(p) Static
(b) If friction force is equal to the force applied, then friction may be	(q) Kinetic
(c) If object is moving, then friction is	(r) Limiting
(d) If object is at rest, then friction may be	(s) No conclusion can be drawn

7. For the situation shown in figure, in Column I, the statements regarding friction forces are mentioned, while in Column II some information related to friction forces are given. Match the entries of Column I with the entries of Column II (Take $g = 10\,ms^{-2}$)



Column I	Column II
(a) Total friction force on 3 kg block is	(p) Towards right
(b) Total friction force on 5 kg block is	(q) Towards left
(c) Friction force on 2 kg block due to 3 kg block is	(r) Zero
(d) Friction force on 3 kg block due to 5 kg block is	(s) Non-zero

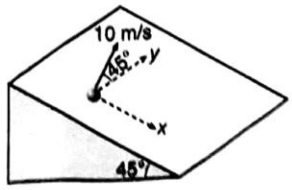
8. If the system is released from rest, then match the following columns.



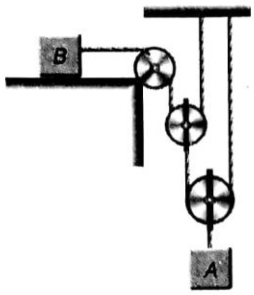
Column I	Column II
(a) Acceleration of 2 kg mass	(p) 2 SI unit
(b) Acceleration of 3 kg mass	(q) 5 SI unit
(c) Tension in the string connecting 2 kg mass	(r) Zero
(d) Frictional forces on 2 kg mass	(s) None of these

Subjective Questions (Level 2)

1. A small marble is projected with a velocity of 10 m/s in a direction 45° from the y-direction on the smooth inclined plane. Calculate the magnitude v of its velocity after 2s. (Take $g = 10\,m/s^2$)



2. Determine the acceleration of the 5 kg block A. Neglect the mass of the pulley and cords. The block B has a mass of 10 kg. The coefficient of kinetic friction between block B and the surface is $\mu_k = 0.1$. (Take $g = 10 \text{m/s}^2$)

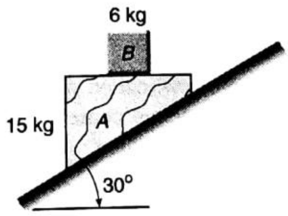


3. A 30 kg mass is initially at rest on the floor of a truck. The coefficient of static friction between the mass and the floor of truck in 0.3 and coefficient of kinetic friction is 0.2. Initially the truck is travelling due east at constant speed. Find the magnitude and direction of the friction force acting on the mass, if : (Take $g = 10 \text{ m/s}^2$)

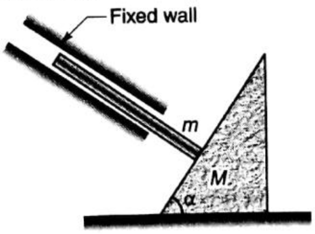
(a) The truck accelerates at 1.8 m/s^2 eastward,

(b) The truck accelerates at 3.8 m/s^2 westward.

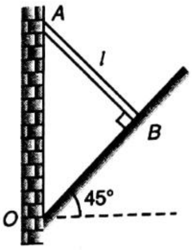
4. A 6 kg block B rests as shown on the upper surface of a 15 kg wedge A. Neglecting friction, determine immediately after the system is released from rest (a) the acceleration of A, (b) the acceleration of B relative to A. (Take $g = 10 \text{m/s}^2$)



5. In the arrangement shown in the figure, the rod of mass m held by two smooth walls, remains always perpendicular to the surface of the wedge of mass M. Assuming all the surfaces are frictionless, find the acceleration of the rod and that of the wedge.



6. At the bottom edge of a smooth vertical wall, an inclined plane is kept at an angle of 45°. A uniform ladder of length l and mass M rests on the inclined plane against the wall such that it is perpendicular to the incline.

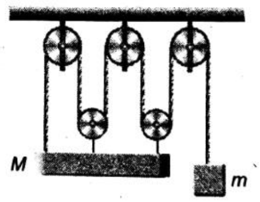


(a) If the plane is also smooth, which way will the ladder slide.

(b) What is the minimum coefficient of friction necessary so that the ladder does not slip on the incline?

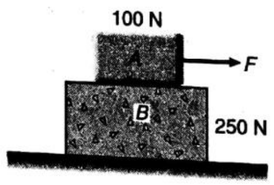
7. A plank of mass M is placed on a rough horizontal surface and a constant horizontal force F is applied on it. A man of mass m runs on the plank. Find the accelerations of the man so that the plank does not move on the surface. Coefficient of friction between the plank and the surface is μ. Assume that the man does not slip on the plank.

8. Find the acceleration of the two masses as shown in figure. The pulleys are light and frictionless and strings are light and inextensible.



9. The upper portion of an inclined plane of inclination α is smooth and the lower portion is rough. A particle slides down from rest from the top and just comes to rest at the foot. If the ratio of smooth length to rough length is $m:n$, find the coefficient of friction.

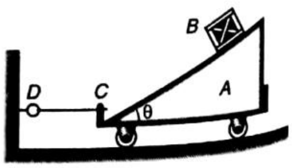
10. Block B rests on a smooth surface. If the coefficient of static friction between A and B is $\mu = 0.4$. Determine the acceleration of each, if :



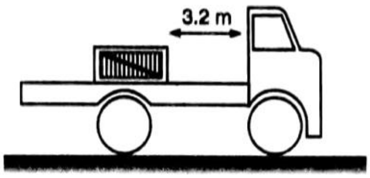
(a) $F = 30$ N and

(b) $F = 250$ N. ($g = 10\,\text{m/s}^2$)

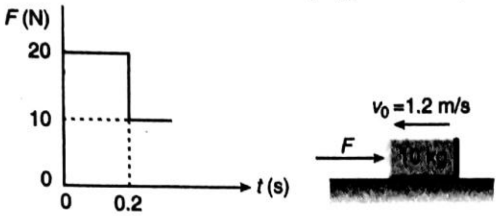
11. Block B has a mass m and is released from rest when it is on top of wedge A, which has a mass $3m$. Determine the tension in cord CD while B is sliding down A. Neglect friction.



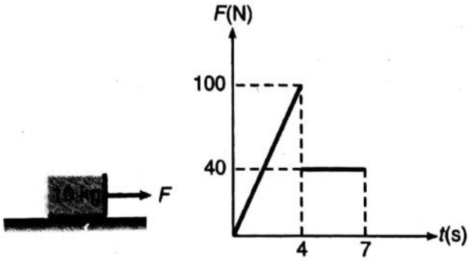
12. Coefficients of friction between the flat bed of the truck and crate are $\mu_s = 0.8$ and $\mu_k = 0.7$. The coefficient of kinetic friction between the truck tires and the road surface is 0.9. If the truck stops from an initial speed of 15 m/s with maximum braking (wheels skidding). Determine where on the bed the crate finally comes to rest. (Take $g = 10\,\text{m/s}^2$)



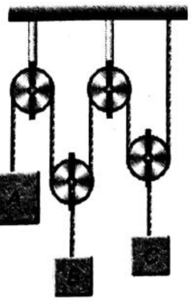
13. The 10 kg block is moving to the left with a speed of 1.2 m/s at time $t = 0$. A force F is applied as shown in the graph. After 0.2 s the force continues at the 10 N level. If the coefficient of kinetic friction is $\mu_k = 0.2$. Determine the time t at which the block comes to a stop. ($g = 10\,\text{m/s}^2$)



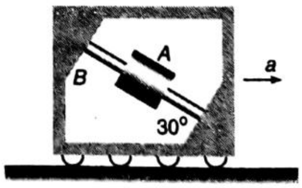
14. The 10 kg block is resting on the horizontal surface when the force F is applied to it for 7 s. The variation of F with time is shown. Calculate the maximum velocity reached by the block and the total time t during which the block is in motion. The coefficients of static and kinetic friction are both 0.50. ($g = 9.8\,\text{m/s}^2$)



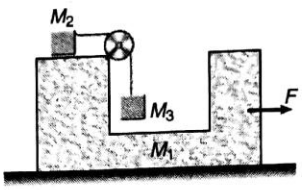
15. If block A of the pulley system is moving downward with a speed of 1 m/s while block C is moving up at 0.5 m/s, determine the speed of block B.



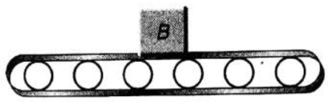
16. The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the horizontal acceleration a of the frame necessary to maintain the collar in a fixed position on the shaft. ($g = 9.8\,\text{m/s}^2$)



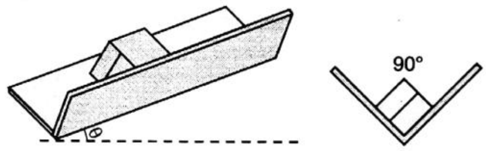
17. In the adjoining figure all surfaces are frictionless. What force F must by applied to M_1 to keep M_3 free from rising or falling?



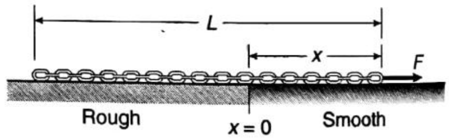
18. The conveyor belt is designed to transport packages of various weights. Each 10 kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor belt is 5 m/s, and then it suddenly stops, determine the distance the package will slide before coming to rest. ($g = 9.8 \text{ m/s}^2$)



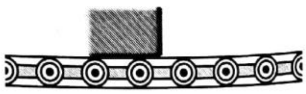
19. In figure, a crate slides down an inclined right-angled trough. The coefficient of kinetic friction between the crate and the trough is μ_k. What is the acceleration of the crate in terms of μ_k, θ and g ?



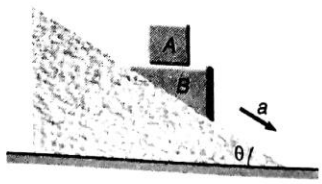
20. A heavy chain with a mass per unit length ρ is pulled by the constant force F along a horizontal surface consisting of a smooth section and a rough section. The chain is initially at rest on the rough surface with $x = 0$. If the coefficient of kinetic friction between the chain and the rough surface is μ_k, determine the velocity v of the chain when $x = L$. The force F is greater than $\mu_k \rho g L$ in order to initiate the motion.



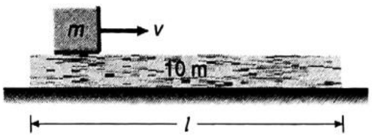
21. A package is at rest on a conveyor belt which is initially at rest. The belt is started and moves to the right for 1.3 s with a constant acceleration of 2 m/s^2. The belt then moves with a constant deceleration a_2 and comes to a stop after a total displacement of 2.2 m. Knowing that the coefficients of friction between the package and the belt are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine (a) the deceleration a_2 of the belt, (b) the displacement of the package relative to the belt as the belt comes to a stop. ($g = 9.8 \text{ m/s}^2$)



22. Determine the normal force the 10 kg crate A exerts on the smooth cart B, if the cart is given an acceleration of $a = 2 \text{ m/s}^2$ down the plane. Also, find the acceleration of the crate. Set $\theta = 30°$. ($g = 10 \text{ m/s}^2$).

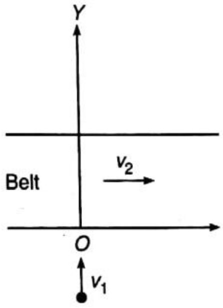


23. A small block of mass m is projected on a larger block of mass $10\,m$ and length l with a velocity v as shown in the figure. The coefficient of friction between the two blocks is μ_2 while that between the lower block and the ground is μ_1. Given that $\mu_2 > 11\mu_1$.

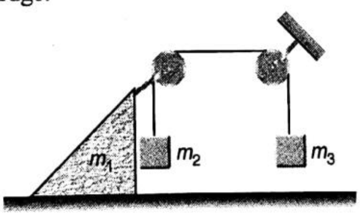


(a) Find the minimum value of v, such that the mass m falls off the block of mass $10\,m$.
(b) If v has this minimum value, find the time taken by block m to do so.

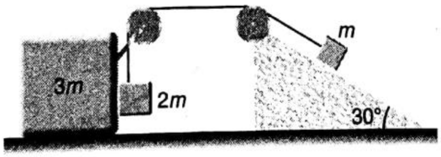
24. A particle of mass m and velocity v_1 in positive y direction is projected on to a belt that is moving with uniform velocity v_2 in x-direction as shown in figure. Coefficient of friction between particle and belt is μ. Assuming that the particle first touches the belt at the origin of fixed x-y co-ordinate system and remains on the belt, find the co-ordinates (x, y) of the point where sliding stops.



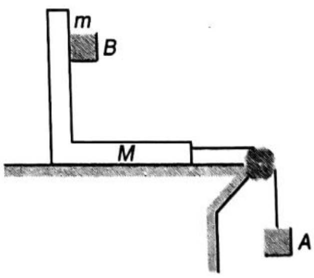
25. In the shown arrangement, both pulleys and the string are massless and all the surfaces are frictionless. Find the acceleration of the wedge.



26. Neglect friction. Find accelerations of m, $2m$ and $3m$ as shown in the figure. The wedge is fixed.



27. The figure shows an L shaped body of mass M placed on smooth horizontal surface. The block A is connected to the body by means of an inextensible string, which is passing over a smooth pulley of negligible mass. Another block B of mass m is placed against a vertical wall of the body. Find the minimum value of the mass of block A so that block B remains stationary relative to the wall. Coefficient of friction between the block B and the vertical wall is μ.



ANSWERS

Introductory Exercise 5.1

4. $F_{1x} = 2\sqrt{3}$ N, $F_2x = -2$ N, $F_3x = 0$, $F_4x = 4$ N, $F_1y = 2$N, $F_{2y} = 2\sqrt{3}$ N, $F_{3y} = -6$ N, $F_{4y} = 0$

5. W **6.** $\frac{2}{\sqrt{3}}W$ **7.** $F = 10.16$ newton, $N = 2.4$ newton **8.** $\frac{mg}{\sqrt{2}}, \frac{g}{2}$

Introductory Exercise 5.2

1. (a) 10 ms^{-2} (b) 110 N (c) 20 N **2.** zero **3.** $\frac{3g}{4}$ **4.** 30°, 10N

5. $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) 5\sqrt{7}$ N **6.** 4 N, 6 N

Introductory Exercise 5.3

1. 3 kg **2.** 4 **3.** $\frac{1}{3}$ s **4.** $\frac{10}{3}$ ms^{-2}

Introductory Exercise 5.4

1. (a) $\frac{2g}{3}$ (b) $\frac{10}{3}$ N **2.** (a) $\frac{g}{2}, \frac{Mg}{2}$ **3.** 4.8 kg **4.** $\frac{12}{35}$ N, $\frac{2}{7}$ ms^{-2}

Introductory Exercise 5.5

1. 6.83 kg **2.** (a) $x = x_0 + 10t - 2.5t^2$, $v = 10 - 5t$ (b) $t = 4$ s

3. (a) $x = x_0 - 2.5t^2$, $z = z_0 + 10t$, $v_x = -5t$, $v_z = 10$ ms^{-1} (b) $x = x_0$, $z = z_0 + 10t$, $v_x = 0$, $v_z = 10$ ms^{-1}

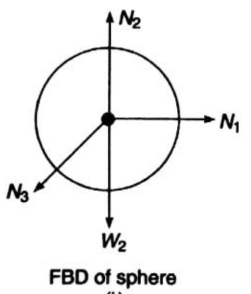
4. $x = x_0 + 10t - 4t^2$, $V = 10 - 8t$ for $0 < t < 1.25$ s object stops at $t = 1.25$ s and remains at rest relative to car.

5. $\frac{9}{25}mg$

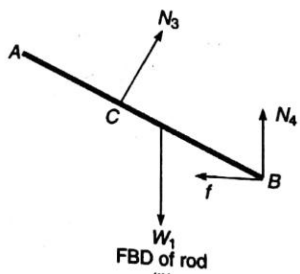
AIEEE Corner

Subjective Questions Level 1

1.



FBD of sphere
(i)



FBD of rod
(ii)

In the figure :

N_1 = normal reaction between sphere and wall,

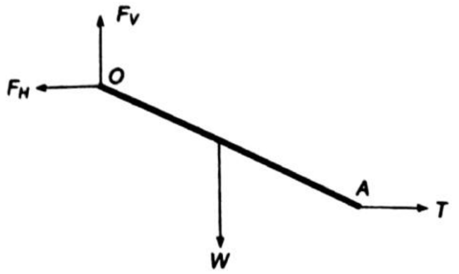
N_2 = normal reaction between sphere and ground

N_3 = normal reaction between sphere and rod and

N_4 = normal reaction between rod and ground

f = force of friction between rod and ground

2.



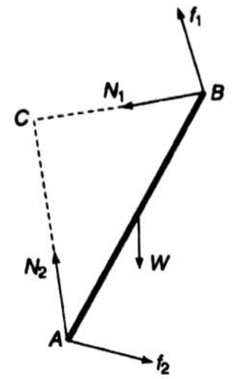
FBD of rod

In the figure :

T = tension in the string, W = weight of the rod, F_v = vertical force exerted by hinge on the rod

F_H = horizontal force exerted by hinge on the rod

3.



FBD of the rod

In the figure :

N_1 = Normal reaction at B, f_1 = force of friction at B, N_2= normal reaction at A, f_2 = force of friction at A

W = weight of the rod.

4. (a) $F_1 = F_2 = 30\sqrt{2}$ N (b) $w = 30\sqrt{2}$ N **5.** $N_A = \frac{1000}{\sqrt{3}}$ N, $N_B = \frac{500}{\sqrt{3}}$ N **6.** 30 N **7.** 5 cm

8. (a) 26.8 N (b) 26.8 N (c) 100 N **9.** (a) $\frac{40}{\sqrt{3}}$ N (b) $\frac{40}{\sqrt{3}}$ N

10. (a) 3 ms^{-2} (b) 18 N, 12 N, 30 N (c) 70 N **11.** (a) 10 N, 30 N (b) 24 N **12.** (a) 20 N (b) 50 N

13. (a) 2.69 ms^{-2} (b) 137.5 N (c) 112.5 N **14.** (a) move up (b) constant (c) constant (d) stop

15. $a_B = -3a_A$ **16.** $a_A + 2a_B + a_C = 0$ **17.** $a_A = a_B \sin\theta$ **18.** 1 ms^{-2} (upwards)

19. 4 ms^{-2}, 24 N, 42 N, 14 N

20. (a) $a_1 = \frac{120}{11}$ ms^{-2}, $a_2 = \frac{50}{11}$ ms^{-2} (downwards), $a_3 = \frac{70}{11}$ ms^{-2} (downwards) (b) $T_1 = T_2 = \frac{120}{11}$ N

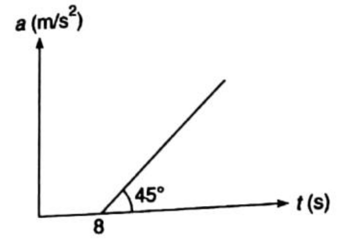
21. $\frac{g}{3}$ (up the plane) **22.** 5 N **23.** $\frac{2}{7}g$ (downwards), $\frac{g}{7}$ (upwards)

24. (a) zero, 20 N (b) 6 ms^{-2}, 8 N (c) $\frac{8}{3}$ ms^{-2}, 4 N **25.** (a) 2s (b) 6 m

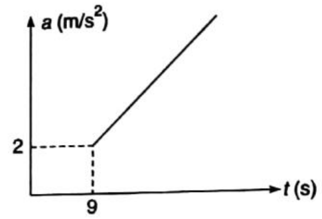
26. (a) 1 s (b) 6 ms^{-1} (c) 4 m, 7 m (both towards right)

27. (a) $\frac{7}{6}$ s (b) 4 ms^{-1} (c) 12.83 m (towards left), 0.58 m (towards left)

28. 4 ms^{-2} (downwards), 12 N (upwards) **29.** $a = 0$ for $t \le 8$s, $a = t - 8$ for $t \ge 8$s



30. $a = 0$ for $t \le 9$s, $a = \left(\frac{2}{3}t - 4\right)$ for $t \ge 9$ s



31. (a) 34 N (b) 40 N (c) 88 N

Objective Questions (Level 1)

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (b)
21. (c)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (d)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (b)
31. (a)									

JEE Corner

Assertion and Reason

1. (d)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (b)
11. (d)									

Objective Questions (Level 2)

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (b)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (b)
31. (d)	**32.** (d)	**33.** (b)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (d)
41. (d)	**42.** (d)	**43.** (d)	**44.** (a)	**45.** (b)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (b)
51. (a)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (c)	**57.** (a)	**58.** (a)		

More than One Correct Options

1. (a,c,d)	**2.** (a,c)	**3.** (a,d)	**4.** (b,c)	**5.** (a,b)	**6.** (a,c)	**7.** (a,c)
8. (a,b,c)	**9.** (a,c)	**10.** (b,c,d)	**11.** (b,c,d)	**12.** (all)	**13.** (b,c)	**14.** (all)

Match the Columns

1. (a) → (r) (b)→ (q) (c)→ (p) (d)→ (s)

2. (a) → (s) (b) → (s) (c) → (p) (d) → (p)

3. (a) → (q,r) (b) → (q,r) (c) → (p,s) (d) → (s)

4. (a) → (s) (b)→ (p) (c)→ (s) (d)→ (s)

5. (a) → (p) (b) → (s) (c) → (q) (d) → (p)

6. (a) → (q) (b) → (p,r) (c) → (q) (d) → (p,r)

7. (a) → (q, s) (b)→ (p,s) (c)→ (p,s) (d)→ (q,s)

8. (a) → (r) (b)→ (r) (c)→ (s) (d)→ (q)

Subjective Questions (Level 2)

1. 10 ms^{-1} **2.** $\frac{2}{33}$ ms^{-2} **3.** (a) 54 N (due east) (b) 60 N (due west) **4.** (a) 6.36 ms^{-2} (b) 5.5 ms^{-2}

5. $\frac{mg\cos\alpha\sin\alpha}{m\sin\alpha + \frac{M}{\sin\alpha}}, \frac{mg\cos\alpha}{m\sin\alpha + \frac{M}{\sin\alpha}}$ **6.** (a) Clockwise (b) $\frac{1}{3}$ **7.** $\frac{F}{m} - \frac{\mu(M+m)g}{m} \le a \le \frac{F}{m} + \frac{\mu(M+m)g}{m}$

8. $a_M = \left(\frac{5m - M}{25m + M}\right)g$. (upwards) $a_m = 5a_M$ **9.** $\mu = \left(\frac{m+n}{m}\right)\tan\alpha$

10. (a) $a_A = a_B = 0.857$ m/s^2 (b) $a_A = 21$ m/s^2, $a_B = 1.6$ m/s^2 **11.** $\frac{mg}{2}\sin 2\theta$ **12.** 2.77 m

13. $t = 0.33$ s **14.** 5.2 m/s , 5.55 s **15.** zero **16.** 5.66 m/s^2 **17.** $\frac{M_3}{M_2}(M_1 + M_2 + M_3)g$

18. 8.5 m **19.** $g(\sin\theta - \sqrt{2}\mu_k\cos\theta)$ **20.** $\sqrt{\frac{2F}{\rho} - \mu_k gL}$

21. (a) 6.63 m/s^2 (b) 0.33 m, 22.90 N, 1 m/s^2 **23.** (a) $v_{min} = \sqrt{\frac{22(\mu_2 - \mu_1)gl}{10}}$ (b) $t = \sqrt{\frac{20l}{11g(\mu_2 - \mu_1)}}$

24. $x = v_2\frac{\sqrt{v_1^2 + v_2^2}}{2\mu g}$, $y = \frac{v_1\sqrt{v_1^2 + v_2^2}}{2\mu g}$ **25.** $\frac{2m_1m_3g}{(m_2 + m_3)(m_1 + m_2) + m_2m_3}$

26. $a_m = \frac{13}{34}g$, $a_{2m} = \frac{\sqrt{397}}{34}g$, $a_{3m} = \frac{3}{17}g$ **27.** $m_A = \frac{M+m}{\mu - 1}$ but $\mu > 1$