disha
Nurturing Ambitions

JEE MAIN

MOCK TEST - 02

Student Name : ______________________________ **MM : 360**

Time : 3 Hours **Date : ....**

SYLLABUS

PHYSICS : COMPLETE SYLLABUS

CHEMISTRY : COMPLETE SYLLABUS

MATHEMATICS : COMPLETE SYLLABUS

INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper.

Subject	No. of Questions	Mark per Question	Negative Marking	Total Marks
Physics	30	4	–1	120
Chemistry	30	4	–1	120
Maths	30	4	–1	120
Total	**90**			**360**

6. Each question consists of four marks.

   ¼ (one fourth) mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet.

   Use of pencil is strictly prohibited.
8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.
9. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
11. Do not fold or make any stray marks on the Answer Sheet.

SPACE FOR ROUGH WORK

PART A – PHYSICS

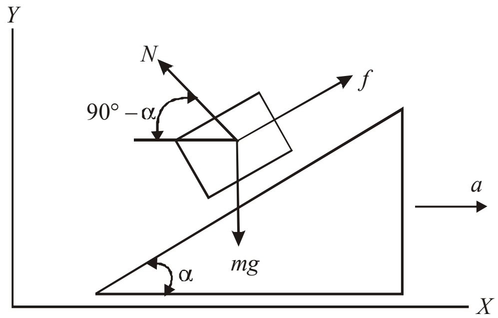
DIRECTIONS : There are 30 multiple choice questions numbered 1 to 30. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. A long straight wire along the Z-axis carries a current I in the negative X-direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $Z = 0$ plane is

(a) $\dfrac{\mu_0 I\left(y\hat{i} - x\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}$ (b) $\dfrac{\mu_0 I\left(x\hat{i} + y\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}$

(c) $\dfrac{\mu_0 I\left(x\hat{j} - y\hat{i}\right)}{2\pi\left(x^2 + y^2\right)}$ (d) $\dfrac{\mu_0 I\left(x\hat{i} - y\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}$

2. The inclined plane shown in figure has an acceleration 'a' to the right. The block will side on the plane if ($\mu_s \equiv \tan\theta$ is the coefficient of static friction for the contacting surfaces)



(a) $a < g \tan(\theta - \alpha)$ (b) $a > g \tan(\theta + \alpha)$
(c) $a > g \tan(\theta - \alpha)$ (d) $a > g \cot(\theta - \alpha)$

3. If E is the electric field intensity and μ_0 is the permeability of free space, then the quantity E^2/μ_0 has the dimensions of

(a) $[M^0LT^{-2}]$ (b) $[MLT^{-4}]$
(c) $[ML^0T^{-4}]$ (d) $[M^2L^2T^0]$

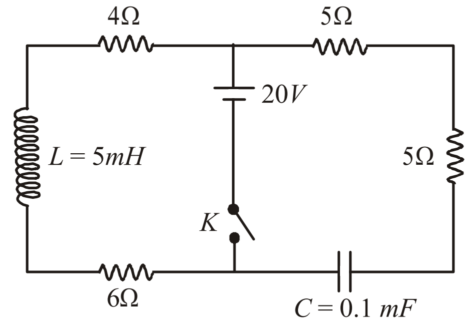
4. A uniform coil of self-inductance 1.8×10^{-4} H and resistance 6W is broken up into two identical coils. These two coils are then connected in parallel across a 12 V battery. The circuit time constant and steady state current through the battery respectively are :

(a) 30 μs, 8 A (b) 30 ms, 8 mA
(c) 30 s, 8 A (d) 300 s, 800 A

5. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J

6. In the circuit shown, the key (K) is closed at $t = 0$, the current through the key at the instant $t = 10^{-3} \ln 2$, is



(a) 2A (b) 3.5A
(c) 2.5A (d) zero

7. The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at

(a) t = 1s (b) t = 0.5 s
(c) t = 0.25 s (d) t = 2s

8. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = \left(40\hat{i} + 50\hat{j}\right)$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is (take g = 10 ms^{-2})

(a) $\tan^{-1}\dfrac{2}{3}$ (b) $\tan^{-1}\dfrac{3}{2}$

(c) $\tan^{-1}\dfrac{7}{4}$ (d) $\tan^{-1}\dfrac{4}{5}$

9. A point charge 50 μC is located in the x-y plane at a point whose position vector is $\vec{r} = (2\hat{i} + 3\hat{j})\, m$. Then electric field at the point whose position vector is $\vec{r} = (8\hat{i} - 5\hat{j})\, m$ (in vector form) will be

(a) $90\,(-3\hat{i} + 4\hat{j})$ V/m (b) $900\,(3\hat{i} - 4\hat{j})$ V/m
(c) $90\,(3\hat{i} - 4\hat{j})$ V/m (d) $900\,(-3\hat{i} + 4\hat{j})$ V/m

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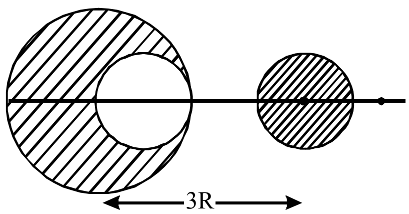
10. From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two sphere is

(a) $\frac{41\,GM^2}{3600\,R^2}$

(b) $\frac{41\,GM^2}{450\,R^2}$

(c) $\frac{59\,GM^2}{450\,R^2}$

(d) $\frac{GM^2}{225\,R^2}$



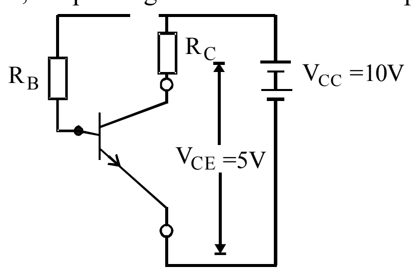
11. Light of wavelength 200 Å fall on aluminium surface. Work function of aluminium is 4.2 eV. What is the kinetic energy of the fastest emitted photoelectrons?

(a) 2 eV (b) 1 eV (c) 4 eV (d) 0.2 eV

12. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is :

(a) 1.75 mm (b) 2.0 mm (c) 1.50 mm (d) 1.25 mm

13. A transistor connected in common emitter configuration has input resistance $R_B = 2$ kΩ and load resistance of 5 kΩ. If $\beta = 60$ and an input signal 12 mV is applied, calculate the voltage gain, the power gain and the value of output voltage



(a) $A_v = 150$, $V_{out} = 1.8$V, and power gain = 9000

(b) $A_v = 20$, $V_{out} = 1$V, and power gain = 2000

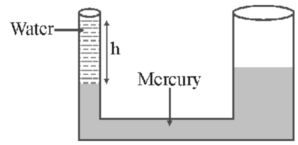
(c) $A_v = 150$, $V_{out} = 1.5$V, and power gain = 8500

(d) $A_v = 20$, $V_{out} = 1.5$V, and power gain = 2000

14. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(a) 802 nm (b) 823 nm (c) 1882 nm (d) 1648 nm

15. Two communicating vessels contain mercury. The diameter of one vessel is n times larger than the diameter of the other. A column of water of height h is poured into the left vessel. The mercury level will rise in the right-hand vessel (s = relative density of mercury and ρ = density of water) by



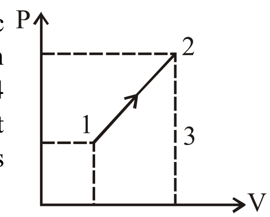
(a) $\frac{n^2 h}{(n+1)^2 s}$

(b) $\frac{h}{(n^2+1)s}$

(c) $\frac{h}{(n+1)^2 s}$

(d) $\frac{h}{n^2 s}$

16. A process 1 → 2 using diatomic gas is shown on the P-V diagram below. $P_2 = 2P_1 = 10^6$ N/m^2, $V_2 = 4V_1 = 0.4$ m^3. The molar heat capacity of the gas in this process will be :



(a) 35R/12 (b) 25R/13

(c) 35R/11 (d) 22R/7

17. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it's suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:

(a) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$

(b) $\frac{\gamma M^2 v}{2R}K$

(c) $\frac{(\gamma-1)}{2R}Mv^2K$

(d) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

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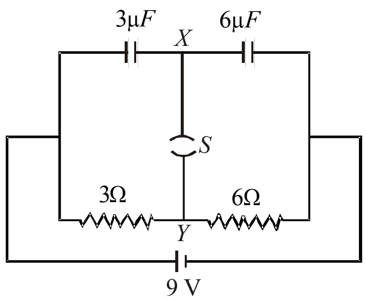
18. An electromagnetic wave, going through vacuum is described by $E = E_0 \sin(kx - \omega t)$. Which of the following is independent of wavelength ?

(a) k (b) ω
(c) k/ω (d) $k\omega$

19. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t$ $(\omega \neq \omega_0)$ is applied to the oscillator. The displacement of the oscillator will be proportional to

(a) $\frac{1}{m(\omega_0^2+\omega^2)}$ (b) $\frac{1}{m(\omega_0^2-\omega^2)}$
(c) $\frac{m}{\omega_0^2-\omega^2}$ (d) $\frac{m}{(\omega_0^2+\omega^2)}$

20. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is



(a) 0 (b) 54 μC
(c) 27μC (d) 81 μC

21. A radioactive source in the form of metal sphere of diameter 10^{-3} m emits beta particle at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0 volt, assuming that 80% of the emitted beta particles escape from the source?

(a) 6.95 μ sec (b) 0.95 μ sec
(c) 1.95 μ sec (d) 2.15 μ sec

22. A short bar magnet with its north pole facing north forms a neutral point at P in the horizonatal plane. If the magnet is rotated by 90° in the horizonatal plane, the net magnetic induction at P is (Horizontal component of earth's magnetic field = B_H)

(a) 0 (b) $2B_H$ (c) $\frac{\sqrt{5}}{2}B_H$ (d) $\sqrt{5}B_H$

23. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is

(a) 286 cps (b) 292 cps (c) 294 cps (d) 288 cps

24. The maximum peak to peak voltage of an AM wire is 24 mV and the minimum peak to peak voltage is 8 mV. The modulation factor is

(a) 10% (b) 20% (c) 25% (d) 50%

25. A mixture of n_1 moles of monoatomic gas and n_2 moles of diatomic gas has $\frac{C_p}{C_v} = \gamma = 1.5$ then

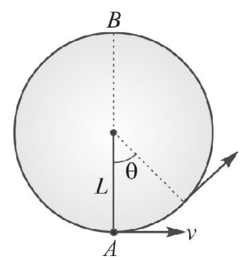
(a) $n_1 = n_2$ (b) $2n_1 = n_2$
(c) $n_1 = 2n_2$ (d) $2n_1 = 3n_2$

26. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

27. A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

(a) $\theta = \frac{\pi}{4}$
(b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$
(d) $\frac{3\pi}{4} < \theta < \pi$.



28. A thin glass (refractive index 1.5) lens has optical power of $-5\,D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be

(a) $-1D$ (b) $1D$ (c) $-25\,D$ (d) $25\,D$

SPACE FOR ROUGH WORK

29. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) zero (b) $\frac{1}{2}(K-1)\,CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

30. An electrical cable of copper has just one wire of radius 9 mm. Its resistance is 5 ohm. This single copper wire of the cable is replaced by 6 different well insulated copper wires each of radius 3 mm. The total resistance of the cable will now be equal to

(a) 7.5 ohm (b) 45 ohm (c) 90 ohm (d) 270 ohm

PART B – CHEMISTRY

DIRECTIONS : There are 30 multiple choice questions numbered 31 to 60. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

31. For a spontaneous reaction the ΔG, equilibrium constant (K) and E°_{cell} will be respectively

(a) – ve, > 1, – ve (b) – ve, < 1, – ve

(c) + ve, > 1, – ve (d) – ve, > 1, + ve

32. Pick out the correct statements from the following and choose the correct answer from the codes given below:

1. Hexa-1, 5-diene is a conjugated diene
2. Prop-1, 2-diene is conjugated diene
3. Hexa-1, 3-diene is a conjugated diene
4. Buta-1,3-diene is an isolated diene
5. Prop-1, 2-diene is a cumulative diene

(a) 1, 2 (b) 2, 3

(c) 4, 5 (d) 3, 5

33. Two fast moving particles X and Y are associated with de Broglie wavelengths 1 nm and 4 nm respectively. If mass of X is nine times the mass of Y, the ratio of kinetic energies of X and Y would be

(a) 3 : 1 (b) 9 : 1

(c) 5 : 12 (d) 16 : 9

34. Which of the following orders regarding the bond order is correct ?

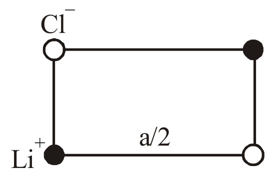
(a) $O_2^- > O_2 > O_2^+$ (b) $O_2^- < O_2 < O_2^+$

(c) $O_2^- > O_2 < O_2^+$ (d) $O_2^- < O_2 > O_2^+$

35. In order to convert R – X to R – R′ which reaction is most suitable

(a) Corey House reaction
(b) Kolbe's reaction
(c) Williamson's synthesis
(d) Wurtz reaction

36. Lithium chloride has a cubic structure as shown below. If the edge length is 400 pm., then the radii of Cl^- ions is



(a) 100 pm (b) 200 pm

(c) 141.4pm (d) 282.8pm

37. Which of the following order of root mean square speed of different gases at same temperature is true?

(a) $(u_{rms})_{H_2} > (u_{rms})_{CH_4} > (u_{rms})_{NH_3} > (u_{rms})_{CO_2}$

(b) $(u_{rms})_{H_2} < (u_{rms})_{CH_4} < (u_{rms})_{NH_3} < (u_{rms})_{CO_2}$

(c) $(u_{rms})_{H_2} < (u_{rms})_{CH_4} > (u_{rms})_{NH_3} > (u_{rms})_{CO_2}$

(d) $(u_{rms})_{H_2} > (u_{rms})_{CH_4} < (u_{rms})_{NH_3} < (u_{rms})_{CO_2}$

38. When Br_2 is treated with aqueous solutions of NaF, NaCl and NaI separately

(a) F_2, Cl_2 and I_2 are liberated
(b) only F_2 and Cl_2 are liberated
(c) only I_2 is liberated
(d) only Cl_2 is liberated

39. Identify the feasible reaction among the following:

(a) $K_2CO_3 \xrightarrow{\Delta} K_2O + CO_2$

(b) $Na_2CO_3 \xrightarrow{\Delta} Na_2O + CO_2$

(c) $Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

(d) $Rb_2CO_3 \longrightarrow Rb_2O + CO_2$

40. Though the five d-orbitals are degenerate, the first four d-orbitals are similar to each other in shape whereas the fifth d-orbital is different from others. What is the designation of the fifth orbital?

(a) $d_{x^2-y^2}$ (b) d_{z^2} (c) d_{xz} (d) d_{xy}

41. Oxyacids of phosphorous and the starting materials for their preparation are given below.

	Oxyacid		**Materials for preparation**
(A)	H_3PO_2	(i)	Red P + alkali
(B)	H_3PO_3	(ii)	$P_4O_{10} + H_2O$
(C)	H_3PO_4	(iii)	$P_2O_3 + H_2O$
(D)	$H_4P_2O_6$	(iv)	White P + alkali

Choose the correct answer from the codes given below

	(A)	(B)	(C)	(D)
(a)	(iv)	(iii)	(ii)	(i)
(b)	(i)	(iii)	(ii)	(iv)
(c)	(iv)	(iii)	(i)	(ii)
(d)	(ii)	(iii)	(i)	(iv)

42. Five gaseous homogenous equilibrium reactions are given below. Choose the reaction in which both increase in pressure and increase in temperature favour the formation of products.

(a) $2A + B \rightleftharpoons C + D; \Delta H = -78\ kJ\ mol^{-1}$
(b) $2M + 3N \rightleftharpoons P + 2Q; \Delta H = +105\ kJ\ mol^{-1}$
(c) $W + X \rightleftharpoons 2Y + 3Z; \Delta H = +92\ kJ\ mol^{-1}$
(d) $S + T \rightleftharpoons E + F; \Delta H = -80\ kJ\ mol^{-1}$

43. To prepare 3-ethylpentane-3-ol the reagents needed are

(a) $CH_3CH_2\ MgBr + CH_3COCH_2CH_3$
(b) $CH_3MgBr + CH_3CH_2CH_2COCH_2CH_3$
(c) $CH_3CH_2MgBr + CH_3CH_2COCH_2CH_3$
(d) $CH_3CH_2CH_2MgBr + CH_3COCH_2CH_3$

44. How many P–H and O–H bonds respectively, are present in $H_4P_2O_7$ molecule?

(a) 1 and 3 (b) 0 and 4
(c) 4 and 0 (d) 2 and 3

45. The correct order of boiling points of 2, 2-dimethylpropane, 2-methylbutane and *n*-pentane is

(a) *n*-pentane > 2, 2-dimethylpropane > 2-methylbutane
(b) *n*-pentane > 2-methylbutane > 2, 2-dimethylpropane
(c) 2, 2-dimethylpropane > 2-methylbutane > *n*-pentane
(d) 2-methylbutane > *n*-pentane > 2, 2-dimethylpropane

46. The half life of a first order reaction is 10 min. If initial amount is 0.08 mol L^{-1} and concentration at some instant is 0.01 mol L^{-1}. What is the time elapsed ?

(a) 10 min (b) 20 min (c) 30 min (d) 40 min

47. Which of the following statements is not correct for chemisorption and physisorption?

(a) Physical adsorption occurs at a low temperature and chemisorption occurs at all temperatures.
(b) Magnitude of chemisorption decreases with rise in temperature while physisorption increases with rise in temperature.
(c) Chemisorption is irreversible and physisorption is reversible.
(d) In physisorption activation energy is low while in chemisorption it is high.

48. Which one of the following on treatment with 50% aqueous sodium hydroxide yields the corresponding alcohol and acid?

(a) C_6H_5CHO (b) $CH_3CH_2CH_2CHO$
(c) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ (d) $C_6H_5CH_2CHO$

49. The complexes of Nickel (II) can be

(a) square planar, tetrahedral and octahedral
(b) square planar and octahedral
(c) tetrahedral and octahedral
(d) square planar only

50. Which of the following represents the correct order of stability?

(a) $H-C\equiv C:^{-} > H_2C=CH:^{-} > H_3C-CH_2:^{-}$
(b) $H_3C-CH_2:^{-} > H_2C=CH:^{-} > H-C\equiv C:^{-}$
(c) $H_3C-CH_2:^{-} > H_2C=CH:^{-} > H-C\equiv C:^{-}$
(d) None of these

51. Identify *x, y, z* for the following metallurgical process.

Metal sulphide $\xrightarrow{x}$ Metal oxide $\xrightarrow{y}$ Impure metal $\xrightarrow{z}$ Pure metal.

x, y and *z* are respectively

(a) roasting, smelting, electrolysis
(b) roasting, calcination, smelting
(c) calcination, auto-reduction, bassemerisation
(d) None of the above is correct

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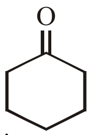
52.  $\xrightarrow{NH_2OH}$ A $\xrightarrow{H_2SO_4}$ B. The product B is

(a) Caprolactam (b) Lactone
(c) Perlone (d) Nylon-6

53. On heating chromite ($FeCr_2O_4$) with Na_2CO_3 in air, which of the following product is obtained?

(a) $Na_2Cr_2O_7$ (b) FeO
(c) Fe_3O_4 (d) Na_2CrO_4

54. During steam distillation of a mixture of *o*-nitrophenol and *p*-nitrophenol

(a) vapours of *o*-nitrophenol are carried by steam because of its lower boiling point due to chelation.
(b) vapours of *o*-nitrophenols are carried by steam because of its lower boiling point and solubility in steam.
(c) vapours of *o*-nitrophenol are carried by steam because its boiling point is reduced by steam.
(d) vapours of *p*-nitrophenol are carried by steam because of its lower boiling point.

55. The hybridization states of the central atoms in the complexes $[Fe(CN)_6]^{3-}$, $[Fe(CN)_6]^{4-}$ and $[Co(NO_2)_6]^{3-}$ are

(a) d^2sp^3, sp^3 and d^4s^2 respectively
(b) d^2sp^3, sp^3d and sp^3d^2 respectively
(c) d^2sp^3, sp^3d^2 and dsp^2 respectively
(d) for all d^2sp^3

56. In nucleophilic addition reactions, the reactivity of carbonyl compounds follows the order

(a) $HCHO > RCHO > ArCHO > R_2CO > Ar_2CO$
(b) $HCHO > R_2CO > Ar_2CO > RCHO > ArCHO$
(c) $Ar_2CO > R_2CO > ArCHO > RCHO > HCHO$
(d) $ArCHO > Ar_2CO > RCHO > R_2CO > HCHO$

57. Which of the following is most basic in nature-?

(a) $Lu(OH)_3$ (b) $Nd(OH)_3$
(c) $Ce(OH)_3$ (d) All are equally basic

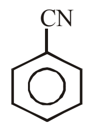
58. At constant external pressure of one atmosphere, 4 moles of a metallic oxide MO_2 undergoes complete decomposition at 227°C in an open vessel according to the equation

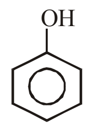
$$2MO_2(s) \rightarrow 2MO(s) + O_2(g)$$

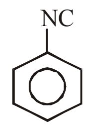
The work done by the system in kJ is (R = 8.3 JK^{-1} mol^{-1})

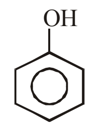
(a) –16.6 (b) –24.9
(c) –8.3 (d) –4.15

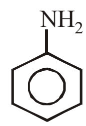
59. A mixture of two aromatic compounds A and B is separated by dissolving in chloroform followed by extraction with aq. KOH solution. The alkaline aqueous layer gives a mixture of two isomeric compounds on treatment with carbon tetrachloride. The organic layer containing compound A gives an unpleasant odour on treatment with alcoholic solution of KOH. Compounds A and B respectively are

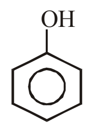
(a) 

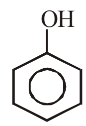
and 

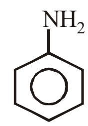
(b) 

and 

(c) 

and 

(d) 

and 

60. The resistance of 0.01 N solution of an electrolyte was found to be 220 ohm at 298 K using a conductivity cell with a cell constant of 0.88cm^{-1}. The value of equivalent conductance of solution is –

(a) 400 mho cm^2 g eq^{-1} (b) 295 mho cm^2 g eq^{-1}
(c) 419 mho cm^2 g eq^{-1} (d) 425 mho cm^2 g eq^{-1}

PART C – MATHEMATICS

DIRECTIONS : There are 30 multiple choice questions numbered 61 to 90. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

61. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4 y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneo

(a) 1/2 (b) 1
(c) 3/2 (d) 2

62. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

63. The distance between two points *P* and *Q* is *d* and the length of their projections of *PQ* on the co-ordinate planes are d_1, d_2, d_3. Then $d_1^2 + d_2^2 + d_3^2 = kd^2$ where '*k*' is

(a) 1 (b) 5

(c) 3 (d) 2

64. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8

(c) 4/5 (d) 5/4

65. If $\alpha \le 2\sin^{-1} x + \cos^{-1} x \le \beta$, then

(a) $\alpha = -\frac{\pi}{2}, \beta = \frac{\pi}{2}$ (b) $\alpha = 0, \beta = \pi$

(c) $\alpha = -\frac{\pi}{2}, \beta = \frac{3\pi}{2}$ (d) $\alpha = 0, \beta = 2\pi$

66. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 (b) 27

(c) 81 (d) 512

67. If $p + q + r = 0 = a + b + c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is :

(a) 0 (b) $pa + qb + rc$

(c) 1 (d) None of these

68. A is a 2 × 2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The sum of the elements of A is

(a) –1 (b) 0

(c) 2 (d) 5

69. If $0 < x < 1$, then

$$\sqrt{1+x^2}\left[(x\cos[\cot^{-1}x] + \sin[\cot^{-1}x])^2 - 1\right]^{1/2} =$$

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) x

(c) $x\sqrt{1+x^2}$ (d) $\sqrt{1+x^2}$

70. The function $f(x) = \cot x$ is discontinuous on the set

(a) $\{x = n\pi; n \in Z\}$ (b) $\{x = 2n\pi; n \in Z\}$

(c) $\left\{x = (2n+1)\frac{\pi}{2}; n \in Z\right\}$ (d) $\left\{x = \frac{n\pi}{2}; n \in Z\right\}$

71. A stone is dropped into a quiet lake and waves move in a circle at a speed of 3.5 cm/sec. At the instant when the radius of the circular wave is 7.5 cm. How fast is the enclosed area increasing?

(a) $32.5\pi\ cm^2/sec$ (b) $31.5\ \pi\ cm^2/sec$

(c) $52.5\pi\ cm^2/sec$ (d) None of these

72. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(a) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(c) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(d) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

73. The interval in which the function $2x^3 + 15$ increases less rapidly than the function $9x^2 - 12x$, is

(a) $(-\infty, 1)$ (b) $(1, 2)$

(c) $(2, \infty)$ (d) None of these

74. $\int_{-\pi/2}^{\pi/2} \frac{\ln(\cos x)}{1+e^x . e^{\sin x}} dx =$

(a) $-2\pi \ln 2$ (b) $-\frac{\pi}{4} \ln 2$

(c) $-\pi \ln 2$ (d) $-\frac{\pi}{2} \ln 2$

75. The value of the integral

$\int_1^3 \left(\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right) dx$ is equal to

(a) π (b) 2π

(c) 4π (d) None of these

76. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$, is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all $b > 1$, then f(x) is

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$

(c) $\sqrt{x^2+1}$ (d) $\frac{x}{\sqrt{1+x^2}}$

77. The solution of the differential equation

$\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$ is

(a) $\frac{y}{4} + \frac{1}{x^2+y^2} = c$ (b) $\frac{y}{x} - \frac{1}{x^2+y^2} = c$

(c) $\frac{x}{y} - \frac{1}{x^2+y^2} = c$ (d) None of these

78. If $\vec{a} + \vec{b} + \vec{c} = \alpha \vec{d}$, $\vec{b} + \vec{c} + \vec{d} = \beta \vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ is equal to : [$\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar]

(a) $\beta \vec{b}$ (b) $\alpha \vec{a}$

(c) $\vec{0}$ (d) None of these

79. An equation of the plane passing through the points $(3, 2, -1)$, $(3, 4, 2)$ and $(7, 0, 6)$ is $5x + 3y - 2z = \lambda$, where λ is

(a) 23 (b) 21

(c) 19 (d) 27

80. A drunken man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps, he is one step away from the starting point.

(a) $24 \times (0.36)^6$

(b) $462 \times (0.24)^6$

(c) $24 \times (0.36)^5$

(d) $462 \times (0.24)^5$

81. If $A_1, A_3, ..., A_{2n-1}$ are n skew-symmetric matrices of same order, then $B = \sum_{r=1}^{n} (2r-1)(A_{2r-1})^{2r-1}$ will be

(a) symmetric

(b) skew-symmetric

(c) neither symmetric nor skew-symmetric

(d) data is adequate

82 Let S be any set and P(S) be its power set. We define a relation R on P(S) by ARB to mean $A \subseteq B$; $\forall A, B \in P(S)$. Then, R is

(a) equivalence relation

(b) not an equivalence but partial order relation

(c) both equivalence and partial order relation

(d) None of these

83. Let S be a finite set containing n elements. Then the total number of binary operations on S is:

(a) n^{n^2} (b) n^n

(c) 2^{n^2} (d) n^2

84. A binary operation o is defined on the set of integers I by $p \, o \, q = 3p^2 + 2q^2 - 5pq$

If $a \, o \, 1 = 1$, then a is equal to :

(a) -1 (b) 1

(c) -2 (d) None of these

85. Value of $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$ is

(a) $\frac{-1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(b) $\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(c) $\frac{1}{22}\tan^{-1}\left(\frac{\tan x+2}{2}\right)+C$

(d) None of these

86. The probability of the simultaneous occurrence of two events A and B is p. If the probability that exactly one of the events occurs is q, then which of the following is not correct?

(a) $P(A')+P(B')=2+2q-p$

(b) $P(A')+P(B')=2-2p-q$

(c) $P(A\cap B\,|\,A\cup B)=\frac{p}{p+q}$

(d) $P(A'\cap B')=1-p-q$

87. Derivative of $\sec^{-1}\left\{\frac{1}{2x^2-1}\right\}$ w.r.t $\sqrt{1+3x}$ at $x=-\frac{1}{3}$ is

(a) 0 (b) 1/2

(c) 1/3 (d) None of these

88 If $y=f\left(\frac{5x+1}{10x^2-3}\right)$ and $f'(x)=\cos x$, then $\frac{dy}{dx}=$

(a) $\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$

(b) $\frac{5x+1}{10x^2-3}\cos\left(\frac{5x+1}{10x^2-3}\right)$

(c) $\cos\left(\frac{5x+1}{10x^2-3}\right)$

(d) None of these

89. The area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is

(a) $3(\pi-2)$ (b) $\frac{3}{2}\pi$

(c) $\frac{3}{2}(\pi-2)$ (d) $\frac{2}{3}(\pi-2)$

90. The interval in which the function $f(x)=\frac{4x^2+1}{x}$ is decreasing is :

(a) $\left(-\frac{1}{2},\frac{1}{2}\right)$ (b) $\left[-\frac{1}{2},\frac{1}{2}\right]$

(c) $(-1,1)$ (d) $[-1,1]$

JEE MAIN

SOLUTIONS MOCK TEST-2

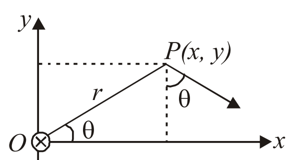
PART A – PHYSICS

1. (a) The wire carries a current I in the negative z-direction. We have to consider the magnetic vector field $\vec{B}$ at (x, y) in the $z = 0$ plane.

Magnetic field $\vec{B}$ is perpendicular to OP.

$\therefore\ \vec{B} = B\sin\theta\hat{i} - B\cos\theta\hat{j}$

$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}\ B = \frac{\mu_0 I}{2\pi r}$



$\therefore\ \vec{B} = \frac{\mu_0 I}{2\pi r^2}\left(y\hat{i} - x\hat{j}\right)$

or $\vec{B} = \frac{\mu_0 I\left(y\hat{i} - x\hat{j}\right)}{2\pi\left(x^2 + y^2\right)}.$

2. (c) If the block is not to slide, it must have the same acceleration as the plane.

Hence, $f\cos\alpha - N\sin\alpha = ma$

$f\sin\alpha + N\cos\alpha - mg = 0$

From these,

$f = m\,(a\cos\alpha + g\sin\alpha)$

$N = m\,(g\cos\alpha - a\sin\alpha)$

and $\frac{f}{N} = \frac{a\cos\alpha + g\sin\alpha}{g\cos\alpha - a\sin\alpha} = \frac{a + g\tan\alpha}{g - a\tan\alpha}$

Now the maximum value of f/N in the absence of slipping is $\mu_s = \tan\theta$. Thus the acceleration a must satisfy

$$\frac{a + g\tan\alpha}{g - a\tan\alpha} \le \tan\theta$$

or $a \le g\,\frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha} = g\tan(\theta - \alpha)$

If $a > g\tan(\theta - \alpha)$ the block will slide.

3. (b) Write E^2/μ_0 as $(E^2\varepsilon_0)/(\mu_0\varepsilon_0)$ and note that the numerator has the dimensions of energy per unit volume whereas the denominator has the dimensions of square of reciprocal of speed.

$$\left[\frac{E^2}{\mu_0}\right] = \frac{[E^2]}{\mu_0}$$

We have,

$[E] = MLT^{-3}I^{-1}$ $\left(\text{using } E = \frac{V}{d}\right)$

$[\mu_0] = MLT^{-2}I^{-2}$ $\left(\text{using } dB = \frac{\mu_0}{4\pi}.\frac{Id\sin\theta}{r^2}\right)$

$$\therefore\ \left[\frac{E^2}{\mu_0}\right] = \frac{M^2L^2T^{-6}I^{-2}}{MLT^{-2}I^{-2}} = MLT^{-4}$$

4. (a) When coil is broken into two identical parts, then resistance of each part

$$R' = \frac{R}{2} = \frac{6}{2} = 3\Omega$$

Inductance of each part

$$L' = \frac{L}{2} = \frac{1.8}{2}\times 10^{-4} = 0.9\times 10^{-4}\,\text{H}$$

Now, time constant $\tau = \frac{L'}{R'}$

$$\tau = \frac{L}{2\times\frac{R}{2}} = \frac{L}{R} = \frac{1.8\times 10^{-4}}{6} = 30\mu\text{s}$$

Now, effective resistance when both coils are connected in parallel

$$R'' = \frac{R'\times R'}{R' + R'}$$

So, maximum current drawn from battery

$$i = \frac{E}{R''} = \frac{12}{6/9} = 8\text{A}$$

5. (c) Initial kinetic energy of the system

$$\text{K.E}_i = \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2$$

$$= \frac{1}{2}\times 0.5\times 2\times 2 + 0 = 1\text{J}$$

For collision, applying conservation of linear momentum

$m\times u = (m + M)\times v$

$\therefore\ 0.5\times 2 = (0.5 + 1)\times v \ \Rightarrow\ v = \frac{2}{3}\text{m/s}$

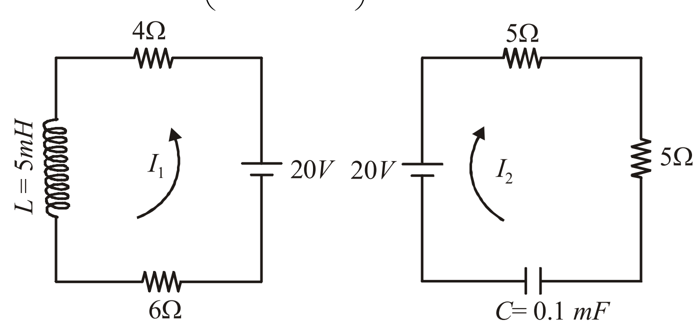
Final kinetic energy of the system is

$$\text{K.E}_f = \frac{1}{2}(m + M)v^2$$

$$= \frac{1}{2}(0.5 + 1)\times\frac{2}{3}\times\frac{2}{3} = \frac{1}{3}\text{J}$$

$\therefore$ Energy loss during collision $= \left(1 - \frac{1}{3}\right)\text{J} = 0.67\text{J}$

6. (c) $I_1 = \frac{20}{10}\left(1 - e^{-\frac{t}{5\times 10^4}}\right) = \frac{3}{2} = 1.5A$



$$I_2 = \frac{20}{10} e^{-\frac{t}{10^{-3}}} = 1.0\,\text{A}$$

From superposition, $I = I_1 + I_2 = 2.5\text{A}$

7. (a) When angular acceleration (α) is zero then torque on the wheel becomes zero.

$\theta(t) = 2t^3 - 6t^2$

$$\Rightarrow \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\Rightarrow \alpha = \frac{d^2\theta}{dt^2} = 12t - 12 = 0$$

$\therefore$ t = 1 sec.

8. (c) From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\,\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2} g t^2$

$$\Rightarrow u_y \times 2 + \frac{1}{2}(-10)\times 4$$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35\,\text{m/s}$

$$\therefore\ \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

$$\Rightarrow \text{Angle } \theta = \tan^{-1}\frac{7}{4}$$

9. (b) $\vec{E} = \frac{kQ}{r^3}\vec{r} = \frac{9\times 10^9 \times 50 \times 10^{-6}}{|\vec{r} - \vec{r}_0|^3} \times (\vec{r} - \vec{r}_0)$

where $\vec{r} - \vec{r}_0 = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j}) = 6\hat{i} - 8\hat{j}$

$$\vec{E} = 900\left(3\hat{i} - 4\hat{j}\right)\ \text{v/m}$$

10. (a) Volume of removed sphere

$$V_{\text{remo}} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi R^3\left(\frac{1}{8}\right)$$

Volume of the sphere (remaining)

$$V_{\text{remain}} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3\left(\frac{1}{8}\right) = \frac{4}{3}\pi R^3\left(\frac{7}{8}\right)$$

Therefore mass of sphere carved and remaining sphere are at respectively $\frac{1}{8}$M and $\frac{7}{8}$M.

Therefore, gravitational force between these two spheres,

$$F = \frac{GMm}{r^2}$$

$$= \frac{G\frac{7M}{8}\times\frac{1}{8}M}{(3R)^2} = \frac{7}{64\times 9}\frac{GM^2}{R^2} \simeq \frac{41}{3600}\frac{GM^2}{R^2}$$

11. (a) By Einstein's equation of photo-electric effect, the maximum kinetic energy of emiited photo-electrons is given by

$E_k = h\nu - W$ or $E_k = \frac{hc}{\lambda} - W$

Where, h = Planck's constant

v = frequency of incident light

W = work function of metal

λ = wavelength of incident light

$$E_k = \frac{6.6\times 10^{-34}\times 3\times 10^8}{2000\times 10^{-10}\times 1.6\times 10^{-19}}\text{eV} - 4.2\text{eV}$$

So, $E_k = 2\text{eV}$

12. (d) $Y_c \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow 1\times 10^{11}\times\left(\frac{1\times 10^{-3}}{1}\right) = 2\times 10^{11}\times\left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore\ \Delta L_s = \frac{0.5\times 10^{-3}}{2} = 0.25\ \text{mm}$$

Therefore, total extension of the composite wire

$= \Delta L_c + \Delta L_s$

$= 1\,\text{mm} + 0.25\,\text{m} = 1.25\,\text{m}$

13. (a) Voltage gain

$$A_v = \beta\frac{R_C}{R_B} = 60\times\frac{5\times 10^3}{2\times 10^3} = 150$$

$$\text{Power gain } = \beta^2\frac{R_C}{R_B} = 60\times 60\times 2.5 = 9000$$

$$\text{Base current} = I_B = \frac{12\times 10^{-3}}{2\times 10^3} = 6\times 10^{-6}\,\text{A}$$

$I_C = \beta I_B = 60\times 6\times 10^{-6} = 3.6\times 10^{-4}\text{A}$

Output $= I_C R_L = 3.6\times 10^{-4}\times 5\times 10^3\Omega = 1.8\text{V}$

14. (b) The smallest frequency and largest wavelength in ultraviolet region will be for transition of electron from orbit 2 to orbit 1.

$$\therefore\ \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{122\times 10^{-9}m} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow R = \frac{4}{3\times 122\times 10^{-9}}m^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

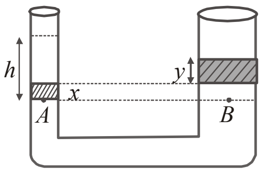
$$\therefore\ \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{4}{3\times 122\times 10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore\ \lambda = \frac{3\times 122\times 9\times 10^{-9}}{4} = 823.5\text{nm}$$

15. (b) $x \times \frac{\pi d^2}{4} = y\frac{\pi(nd)^2}{4}$

$\therefore \quad x = n^2 y$



From Pascal's law

$\rho_w gh = \rho_{Hg} g(x+y)$

or $\quad 1 \times gh = sg(n^2y + y)$

$\therefore \quad y = \frac{h}{s(1+n^2)}$

16. (d) $P_2 = 2P_1, V_2 = 4V_1, n = 1$

$C = C_V + \frac{PdV}{dT}$

$dW = PdV = \text{Area} = \frac{1}{2}[(P_1 + P_2)(V_2 - V_1)]$

$= \frac{1}{2}(3P_1 \times 3V_1) = \frac{9}{2}P_1V_1$

$dT = T_2 - T_1 = \frac{P_2V_2}{R} - \frac{P_1V_1}{R}$

$= \frac{2P_1 \times 4V_1}{R} - \frac{P_1V_1}{R} = \frac{7P_1V_1}{R}$

$C = \frac{5}{2}R + \frac{9}{2}\frac{P_1V_1R}{7P_1V_1} = \frac{22R}{7}$

17. (c) Here, work done is zero.

So, loss in kinetic energy = change in internal energy of gas

$\frac{1}{2}mv^2 = nC_v\Delta T = n\frac{R}{\gamma - 1}\Delta T$

$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\gamma - 1}\Delta T$

$\therefore \Delta T = \frac{Mv^2(\gamma - 1)}{2R}K$

18. (c) The angular wave number $k = \frac{2\pi}{\lambda}$; where λ is the wave length. The angular frequency is $\omega = 2\pi\nu$.

The ratio $\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi\nu} = \frac{1}{\nu\lambda} = \frac{1}{c}$ = constant

19. (b) Equation of displacement is given by

$x = A\sin(\omega t + \phi)$

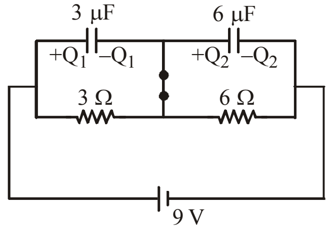
where $A = \frac{F_0}{m\sqrt{({\omega_0}^2 - \omega^2)^2}} = \frac{F_0}{m({\omega_0}^2 - \omega^2)}$

Here damping effect is considered to be zero

$\therefore x \propto \frac{1}{m({\omega_0}^2 - \omega^2)}$

20. (c) When steady state is reached, the current I coming from the battery is

$9 = I(3+6) \quad \Rightarrow \quad I = 1A$



$\therefore$ Potential difference across 3Ω resistance = $3V$ and potential difference across 6Ω resistance = 6V

$\therefore$ p.d. across 3 μF capacitor = $3V$

and p.d. across 6 μF capacitor = $6V$

$\therefore$ Charge on 3 μF capacitor $Q_1 = 3 \times 3 = 9\ \mu C$

Charge on 6 μF capacitor $Q_2 = 6 \times 6 = 36\ \mu C$

$\therefore$ Charge $(-Q_1)$ is shifted from the positive plate of 6 μF capacitor. The remaining charge on the positive plate of 6 μF capacitor is shifted through the switch.

$\therefore$ Charge passing the switch = $36 - 9 = 27\ \mu C$

21. (a) Let t = time for the potential of metal sphere to rise by one volt.

Now β – particles emitted in this time

$= (6.25 \times 10^{11}) \times t$

Number of β-particles escaped in this time

$= (80/100) \times (6.25 \times 10^{10})t = 5 \times 10^{10}\,t$

$\therefore$ Charge acquired by the sphere in t sec.

$Q = (5 \times 10^{10}\,t) \times (1.6 \times 10^{-19})$

$= 8 \times 10^{-19}\,t$ coulomb ...(i)

($\because$ emission of β-particle lends to a charge e on metal sphere)

The capacitance C of a metal sphere is given by

$C = 4\pi\varepsilon_0 \times r$

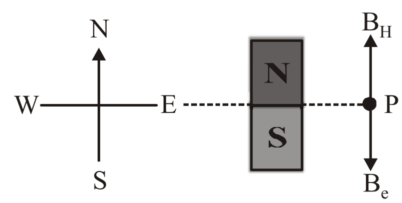
$= \left(\frac{1}{9 \times 10^9}\right) \times \left(\frac{10^{-3}}{2}\right) = \frac{10^{-12}}{18}$ farad ...(ii)

we know that $Q = C \times V$ {Here V = 1 volt}

$\therefore \quad (8 \times 10^{-9})t = \left(\frac{10^{-12}}{18}\right) \times 1$

Solving it for t, we get $t = 6.95\ \mu$ sec.

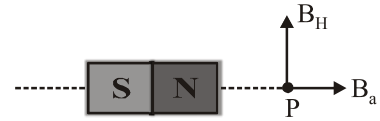
22. (d) Initially



Neutral point obtained on equatorial line and at neutral point $|B_H| = |B_e|$

where B_H = Horizontal component of earth's magnetic field, B_e = Magnetic field due to bar magnet on it's equatorial line

Finally



Point P comes on axial line of the magnet and at P, net magnetic field $B = \sqrt{B_a^2 + B_H^2}$

$= \sqrt{(2B_e)^2 + (B_H)^2} = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}B_H$

23. (b) A tuning fork produces 4 beats/sec with another tuning fork of frequency 288 cps. From this information we can conclude that the frequency of unknown fork is 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

24. (d) Here $V_{max} = \frac{24}{2} = 12$ mV an

$V_{min} = \frac{8}{2} = 4$ mV

Now, $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-4}{12+4} = \frac{8}{16} = \frac{1}{2} = 0.5 = 50\%$

25. (a) The average number of degrees of freedom per molecule,

$$f = \frac{\text{total number of degree of freedom}}{\text{total number of molecules}}$$

$$= \frac{n_1 N_A f_1 + n_2 N_A f_2}{n_1 N_A + n_2 N_A}$$

where N_A = Avogadro constant.
Here, $f_1 = 3, f_2 = 5$

$$\therefore \quad f = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_1} = \frac{3n_1 + 5n_2}{n_1 + n_2}$$

Also, $\gamma = 1 + \frac{2}{f} = 1.5$ or $f = 4$

$$\therefore \quad \frac{3n_1 + 5n_2}{n_1 + n_2} = 4 \Rightarrow n_1 = n_2$$

26. (d) Let $a_1 = a, I_1 = a_1^2 = a^2$

$a_2 = 2a, I_2 = a_2^2 = 4a^2$

$I_2 = 4I_1$

$I_r = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$

$= \mathbf{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$

$I_r = \mathbf{I_1 + 4I_1 + 2\sqrt{4I_1^2}\cos\phi}$

$\Rightarrow \quad \mathbf{I_r = 5I_1 + 4I_1\cos\phi} \quad \ldots(1)$

Now, $\mathbf{I_{max} = (a_1 + a_2)^2 = (a + 2a)^2 = 9a^2}$

$\mathbf{I_{max} = 9I_1 \Rightarrow I_1 = \frac{I_{max}}{9}}$

Substituting in equation (1)

$$\mathbf{I_r = \frac{5I_{max}}{9} + \frac{4I_{max}}{9}\cos\phi}$$

$$\mathbf{I_r = \frac{I_{max}}{9}[5 + 4\cos\phi]}$$

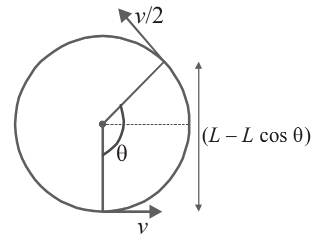
$$\mathbf{I_r = \frac{I_{max}}{9}\left[5 + 8\cos^2\frac{\phi}{2} - 4\right]}$$

$$\mathbf{I_r = \frac{I_{max}}{9}\left[1 + 8\cos^2\frac{\phi}{2}\right]}$$

27. (c) If v is the required speed, then

$0 = v^2 - 2g(2L)$,

$\therefore \quad v = 2\sqrt{gL}$.



By conservation of mechanical energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mg(L - L\cos\theta)$$

$$\frac{1}{2}m[(2\sqrt{gL})^2] = \frac{1}{2}m(\sqrt{gL})^2 + mgL(1 - \cos\theta)$$

$\therefore \quad \cos\theta = -\frac{1}{2}$ or $\theta = 120°$

28. (b) $\frac{1}{f_a} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots.(i)$

$$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_m} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots.(ii)$$

Dividing (i) by (ii), $\frac{f_m}{f_a} = \left(\frac{1.5 - 1}{\frac{1.5}{1.6} - 1}\right) = -8$

$$P_a = -5 = \frac{1}{f_a} \Rightarrow f_a = -\frac{1}{5}$$

$$\Rightarrow f_m = -8 \times f_a = -8 \times -\frac{1}{5} = \frac{8}{5}$$

$$P_m = \frac{\mu}{f_m} = \frac{1.6}{8} \times 5 = 1\text{D}$$

29. (a) The potential energy of a charged capacitor is given by $U = \frac{Q^2}{2C}$.

If a dielectric slab is inserted between the plates, the energy is given by $\frac{Q^2}{2KC}$, where K is the dielectric constant. Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

30. (a) Let the resistance of single copper wire be R_1. If ρ is the specific resistance of copper wire, then

$$R_1 = \frac{\rho \times \ell}{A_1} = \frac{\rho \times \ell}{\pi r_1^2} \quad \ldots(1)$$

When the wire is replaced by six wires, let the resistance of each wire be R_2. Then

$$R_2 = \frac{\rho \times \ell}{A_2} = \frac{\rho \times \ell}{\pi r_2^2} \quad \ldots(2)$$

From eqs. (1) and (2), we get

$$\frac{R_1}{R_2} = \frac{r_2^2}{r_1^2} \text{ or } \frac{5}{R_2} = \frac{(3 \times 10^3)^2}{(9 \times 10^{-3})^2}; \quad R_2 = 45\Omega$$

These six wires are in parallel. Hence the resistance of the combination would be $R_2 = 7.5\Omega$

PART B – CHEMISTRY

31. (d) $\Delta G^\circ = -2.303\ RT\ \log K_{eq}$

$\Delta G^\circ = -nFE^\circ_{cell}$

If a cell reaction is spontaneous (proceeding in forward side), it means

$K_{eq} > 1$ and $E^\circ_{cell} = +ve$

Thus, $\Delta G^\circ = -ve$

32. (d) Structures of given alkenes are following.

$\overset{6}{H_2C} = \overset{5}{CH} - \overset{4}{CH_2} - \overset{3}{CH_2} - \overset{2}{CH} = \overset{1}{CH_2}$

Hexa-1,5-diene is an isolated diene not conjugated diene.

$H_2\overset{3}{C}=\overset{2}{C}=\overset{1}{C}H_2$

Prop-1,2-diene is a cumulative diene.

$\overset{6}{H_3C} - \overset{5}{CH_2} - \overset{4}{CH} = \overset{3}{CH} - \overset{2}{CH} = \overset{1}{CH_2}$

Hexa-1,3-diene is a conjugated diene with an alternate single and double bond.

$\overset{4}{CH_2} = \overset{3}{CH} - \overset{2}{CH} = \overset{1}{CH_2}$

Buta-1,3-diene is a conjugated diene and not an isolated diene.

33. (d) de Broglie wavelength $\lambda = \dfrac{h}{mv}$

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1};\ \frac{1}{4} = \frac{1}{9} \times \frac{v_2}{v_1}$$

$$\frac{v_2}{v_1} = \frac{9}{4}$$

$$\frac{v_1}{v_2} = \frac{4}{9}$$

$$KE = \frac{1}{2}mv^2$$

$$\frac{KE_1}{KE_2} = \frac{m_1}{m_2} \times \frac{v_1^2}{v_2^2} = \frac{9}{1} \times \left(\frac{4}{9}\right)^2 = \frac{16}{9}$$

34. (b) Bond Order $= \dfrac{N_b - N_a}{2}$

N_a = No. of electrons in antibonding MO's

N_b = No. of electrons in bonding MOs

M.O. configurations :

$$O_2 : (\sigma 2s)^2\ (\sigma^* 2s)^2\ (\sigma 2p_x)^2\ (\pi 2p_y)^2$$
$$= (\pi 2p_z)^2\ (\pi^* 2p_y)^1 (\pi^* 2p_z)^1$$

$$O_2^+ : (\sigma 2s)^2\ (\sigma^* 2s)^2\ (\sigma 2p_x)^2\ (\pi 2p_y)^2$$
$$= (\pi 2p_z)^2\ (\pi^* 2p_y)^1 (\pi^* 2p_z)^0$$

$$O_2^- : (\sigma 2s)^2\ (\sigma^* 2s)^2\ (\sigma 2p_x)^2\ (\pi 2p_y)^2$$
$$= (\pi 2p_z)^2\ (\pi^* 2p_y)^2 (\pi^* 2p_z)^1$$

B.O. of $O_2 = \dfrac{8-4}{2} = 2$; B.O. of $O_2^- = \dfrac{8-5}{2} = 1.5$

B.O. of $O_2^+ = \dfrac{8-3}{2} = 2.5$

35. (a) To prepare unsymmetrical alkanes,

Corey – House reaction is most important.

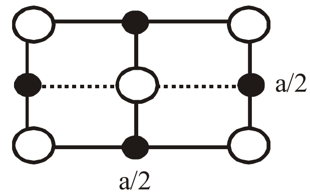
$$R - X + R'_2CuLi \xrightarrow[\text{ether}]{\text{dry}} R - R' + R'Cu + LiX$$

36. (c) For a cube as given, the Cl^- ions are at the corners and one each in the face centre i.e. it is a *ccp* structure.

For a *ccp* structure $4r^- = \sqrt{2}$ a, The face diagonal $= \sqrt{2}$a

On the face diagonal there are only Cl^- ions

$\therefore 4r^- = \sqrt{2}$ a or $r^- = 1.414 \times 400/4 = 141.4$pm



37. (a) $u_{rms} = \sqrt{3RT/M}$, i.e. $u_{rms} \propto \sqrt{1/M}$

$M_{H_2} < M_{CH_4} < M_{NH_3} < M_{CO_2}$

38. (c) Br_2 reacts with NaI only to get I_2.

$2NaI + Br_2 \rightarrow 2NaBr + I_2$

39. (c) The thermal stability of carbonates is in the following order.

$Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3 < Cs_2CO_3$

Li_2CO_3 is considerably less stable and decompose readily.

$Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

40. (b) The five *d*-orbitals are degenerate and have equal energy. The shape of first four orbitals is similar which are d_{xy}, d_{yz}, d_{zx}, $d_{x^2-y^2}$ but the fifth one, d_{z^2} has a different shape.

41. (a) Correct option is (a) as :

H_3PO_2 – white P + H_2O

$H_3PO_3 - P_2O_3 + H_2O$

$H_3PO_4 - P_4O_{10} + H_2O$

$H_4P_2O_6$ – red + alkali

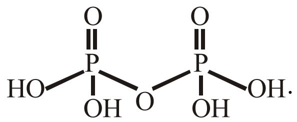
42. (a) The reaction (b) is an endothermic reaction hence increase in temperature will favour the forward reaction i.e. formation of products. Further reaction is occuring by decrease in volume, hence increase in pressure will favour the formation of product.

43. (c) $CH_3CH_2-\overset{\overset{C_2H_5}{|}}{C}=O + C_2H_5MgBr \longrightarrow$

$CH_3CH_2-\underset{\underset{C_2H_5}{|}}{\overset{\overset{C_2H_5}{|}}{C}}-OMgBr \xrightarrow[H^+]{H_2O\,(\ell)}$

$CH_3CH_2-\underset{\underset{C_2H_5}{|}}{\overset{\overset{C_2H_5}{|}}{C}}-OH + MgOH$

3-ethylpentan-3-ol

44. **(b)** 

$[H_4P_2O_7]$ tetrabasic

45. **(b)** Boiling point among isomeric alkanes decreases with branching. Branching decreases surface area of molecule on which van der Waal's forces of attraction depends thereby decreasing intermolecular attraction

46. **(c)** $a = \frac{a_0}{2^n}$, $2^n = \frac{a_0}{a} = \frac{0.08}{0.01}$ Thus $2^n = 8$ or $n = 3$; $T = 3 \times t_{1/2} = 3 \times 10 = 30$ min

47. **(b)** Chemisorption increases and physisorption decreases with rise in temperature.

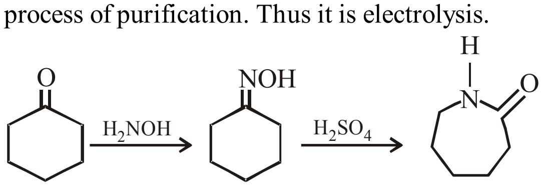
48. **(a)** Aldehydes containing no α-hydrogen atom on warming with 50% NaOH or KOH undergo disproportionation i.e. self oxidation - reduction known as Cannizzaro's reaction.

$$2C_6H_5CHO + NaOH \xrightarrow{50\%\ NaOH} C_6H_5COONa + C_6H_5CH_2OH$$

49. **(a)** Octahedral complexes result from d^2sp^3 (inner orbital) and sp^3d^2 (outer orbital) hybridisation eg. $[Ni(NH_3)_6]^{2+}$. Tetrahedral complexes are formed by sp^3 hybridisation. e.g. $Ni(CO)_4$
and square planar complexes are formed by dsp^2 hybridisation. e.g. $[Ni(CN)_4]^{2-}$.

50. **(a)** *sp*-hybridized carbon atom is more electronegative than sp^2 hybridized carbon which in turn is more electronegative than sp^3 hybridized carbon. More electronegative atom accommodates the negative charge more easily.

51. **(a)** The conversion of metal sulphide to metal oxide involves the process of **roasting** (i.e., *x* is roasting).
The metal oxides can then be converted to impure metal by reduction. Of the given choices in (a) and (b) the reduction process is that of **smelting**. (i.e., '*y*' is smelting)
The conversion of impure metal to pure metal involves a process of purification. Thus it is electrolysis.

52. **(a)** 

Caprolactum

53. **(d)** By heating chromite with Na_2CO_3, Na_2CrO_4 is obtained

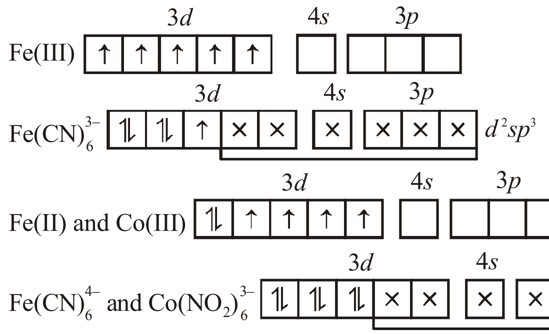
$$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \longrightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$$

54. **(b)** Only *o*-nitrophenol is capable of forming intramolecular H-bonding (chelation) which leads to its lower b.p. (less than 100°C) and also lower solubility in water (steam).

55. **(d)** In $Fe(CN)_6^{3-}$ the oxidation number of Fe = +3 and so it has a $3d^5$ configuration

In $Fe(CN)_6^{4-}$ the oxidation number of Fe = +2 and so it has a $3d^6$ configuration.

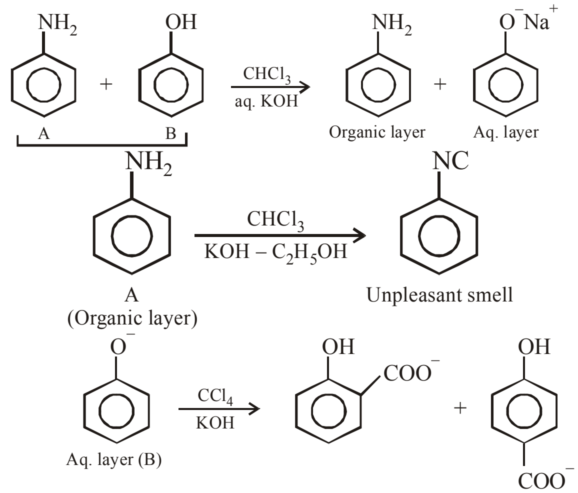
In $Co(NO_2)_6^{3-}$ the oxidation number of Co = +3 and so it has a $3d^6$ configuration.



56. **(a)** Aldehydes are more reactive than ketones towards nucleophilic addition reactions. Aromatic aldehydes and ketones are less reactive than corresponding aliphatic aldehydes and ketones.

57. **(c)** As the size of lanthanide ions decreases from Ce^{3+} to Lu^{3+}, the covalent character of M – OH bond increases and hence the basic strength decreases. Thus $Ce(OH)_3$ is most basic while $Lu(OH)_3$ is least basic.

58. **(c)** $2MO_2(s) \longrightarrow 2MO(s) + O_2(g)$
Work done = – PΔV
We know, PV = nRT
$\therefore\ -P\Delta V = -nRT = -2 \times 8.31 \times 500\ J = -8.3$ kJ

59. **(c)** 

60. **(a).** $\Lambda_{eq} = \kappa \times \frac{1000}{N} = \frac{1}{R} \times \frac{l}{a} \times \frac{1000}{N}$

$= \frac{1}{R} \times \text{cell constant} \times \frac{1000}{N} = \frac{1}{220} \times 0.88 \times \frac{1000}{0.01}$

$= 400$ mho cm^2 g eq^{-1}

PART C – MATHEMATICS

61. **(c)** $\because y = u^n$

$\therefore \frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$

On substituting the values of y and $\frac{dy}{dx}$ in the given equation, then

$$2x^4.u^n.nu^{n-1}\frac{du}{dx} + u^{4n} = 4x^6$$

$\Rightarrow \quad \dfrac{du}{dx} = \dfrac{4x^6 - u^{4n}}{2nx^4u^{2n-1}}$

Since, it is homogeneous. Then, the degree of $4x^6 - u^{4n}$ and $2nx^4\,u^{2n-1}$ must be same.

$\therefore \quad 4n = 6$ and $4 + 2n - 1 = 6$

Then, we get $n = \dfrac{3}{2}$

62. (d) Clearly $\vec{a} = -\dfrac{8}{7}\vec{c}$

$\Rightarrow \vec{a} \,||\, \vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

63. (d) Here, $d_1 = d\cos(90° - \alpha)$

$d_2 = d\cos(90° - \beta)$

$d_3 = d\cos(90° - \gamma)$

$\Rightarrow \quad d_1^2 + d_2^2 + d_3^2 = d^2\left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma\right)$

$\Rightarrow \quad d_1^2 + d_2^2 + d_3^2 = 2d^2;\ \therefore k = 2$

64. (d) The event follows binomial distribution with $n = 5, p = 3/6 = 1/2.$

$q = 1 - p = 1/2.;$

$\therefore$ Variance $= npq = 5/4.$

65. (b) We have, $-\dfrac{\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}$

$-\dfrac{\pi}{2} + \dfrac{\pi}{2} \le \sin^{-1}x + \dfrac{\pi}{2} \le \dfrac{\pi}{2} + \dfrac{\pi}{2}$

$0 \le \sin^{-1}x + \sin^{-1}x + \cos^{-1}x \le \pi$

$0 \le 2\sin^{-1}x + \cos^{-1}x \le \pi$

66. (d) The number of places in a 3×3 matrix is 9.

Also, each place has two choices i.e., either 2 or 0.

$\therefore$ The number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 i.e., 512.

67. (a) Let $A = \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$

$= pa\,(a^2qr - p^2bc) - qb\,(q^2ac - b^2pr) + rc\,(c^2pq - r^2ab)$

$= a^3pqr - p^3abc - q^3abc + b^3pqr + c^3pqr - r^3abc$

$= -abc\,[p^3 + q^3 + r^3] + pqr\,[a^3 + b^3 + c^3]$

$= 0 \qquad (\because\ p + q + r = a + b + c = 0)$

68. (d) $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \ldots(1)$

$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \ldots(2)$

Let A be given by $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The first equation gives

$a - b = -1 \qquad \ldots(3)$

$c - d = 2 \qquad \ldots(4)$

For second equation gives

$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = A\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = A\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

This gives

$-a + 2b = 1 \qquad \ldots(5)$

$-c + 2d = 0 \qquad \ldots(6)$

Eqs. (3) + (5) $\Rightarrow b = 0$

and $a = -1$

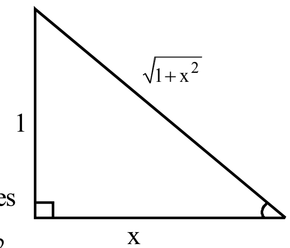
Eqs (4) + (6) $\Rightarrow d = 2$

and $c = 4$

so the sum

$a + b + c + d = 5$

69. (c) $\cos(\cot^{-1}x) = \dfrac{x}{\sqrt{1 + x^2}}$



$\sin(\cot^{-1}x) = \dfrac{1}{\sqrt{1 + x^2}}$

The given expression becomes

$\sqrt{1 + x^2}\left[\left(\dfrac{x^2 + 1}{\sqrt{1 + x^2}}\right)^2 - 1\right]^{1/2} = x\sqrt{1 + x^2}.$

70. (a) $f(x) = \cot x$ is discontinuous if $\cot x \to \infty$

$\Rightarrow \quad \cot x = \cot 0$

$\Rightarrow \quad x = n\pi\ \forall\ n \in Z.$

71. (c) The waves move in a circle at a speed of

$v = 3.5 \text{ cm/sec} = \dfrac{dr}{dt}$

where r is the instantaneous radius of circle.

Let A be the area of circular wave at time t

Then $\qquad A = \pi r^2$

On differentiating both sides w.r.t. 't', we get

$\left[\dfrac{dA}{dt}\right]_r = 2\pi r\dfrac{dr}{dt}$

$\left[\dfrac{dA}{dt}\right]_{7.5} = 2\pi \times 7.5 \times 3.5 = 52.5\,\pi$

$\therefore$ Rate of increasing area $= 52.5\pi$ cm^2/sec.

72. (b) Let $y = x^3 - px + q \Rightarrow \dfrac{dy}{dx} = 3x^2 - p$

For $\dfrac{dy}{dx} = 0 \Rightarrow 3x^2 - p = 0 \Rightarrow x = \pm\sqrt{\dfrac{p}{3}}$

$\dfrac{d^2y}{dx^2} = 6x,\ \left.\dfrac{d^2y}{dx^2}\right|_{x = \sqrt{\frac{p}{3}}} = +ve$ and $\left.\dfrac{d^2y}{dx^2}\right|_{x = -\sqrt{\frac{p}{3}}} = -ve$

$\therefore$ y has minima at $x = \sqrt{\dfrac{p}{3}}$ and maxima at $x = -\sqrt{\dfrac{p}{3}}$

73. (b) Let $f(x) = 2x^3 + 15$ and $g(x) = 9x^2 - 12x$, then

$f'(x) = 6x^2\ \forall\ x \in R$

$\therefore$ f(x) is increasing function $\forall\ x \in R$

Also, $g'(x) > 0 \Rightarrow 18x - 12 > 0 \Rightarrow x > \dfrac{2}{3}$

Thus, f(x) and g(x) both increases for $x > \dfrac{2}{3}$

Let $F(x) = f(x) - g(x),\ F'(x) < 0$

($\because$ f(x) increases less rapidly than the function g(x))

$\Rightarrow \quad 6x^2 - 18x + 12 < 0$

$\Rightarrow \quad 1 < x < 2$

74. (d) $I=\int_{-\pi/2}^{\pi/2}\frac{\ln(\cos x)}{1+e^{x}.e^{\sin x}}dx$

Replace x by (–x)

$I=\int_{-\pi/2}^{\pi/2}\frac{\ln(\cos x)}{1+e^{-(x+\sin x)}}dx$

$2I=\int_{-\pi/2}^{\pi/2}\frac{\ln(\cos x)}{1+e^{x+\sin x}}(1+e^{(x+\sin x)})\,dx$

$=\int_{-\pi/2}^{\pi/2}\ln(\cos x)\,dx$

$2I=2\int_{0}^{\pi/2}\ln(\cos x)\,dx \Rightarrow I=-\frac{\pi}{2}\ln 2$

75. (a) $\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx=$

$\int_1^3\left(\tan^{-1}\frac{x}{x^2+1}+\cot^{-1}\frac{x}{x^2+1}\right)dx=\int_1^3\frac{\pi}{2}dx=\pi$

[As $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$]

76. (d) Given $\int_1^b f(x)dx=\sqrt{b^2+1}-\sqrt{2}$

Differentiate with respect to b

$f(b)=\frac{b}{\sqrt{b^2+1}}\Rightarrow f(x)=\frac{x}{\sqrt{x^2+1}}$

77. (b) Given $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=x^2+2y^2+\frac{y^4}{x^2}$

$\Rightarrow \frac{d(x^2+y^2)}{(x^2+y^2)^2}=2\frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}\right)^2}$

Integrating, we get

$-\frac{1}{x^2+y^2}=\frac{-1}{x/y}+c \Rightarrow c=\frac{y}{x}-\frac{1}{x^2+y^2}$

78. (c) Given,

$\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$; $\vec{a}+\vec{b}+\vec{c}+\vec{d}=(\alpha+1)\vec{d}$

Also, $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a} \Rightarrow \vec{a}+\vec{b}+\vec{c}+\vec{d}=(\beta+1)\vec{a}$

If $\alpha\neq-1$

$\vec{d}=\left(\frac{\beta+1}{\alpha+1}\right)\vec{a}\Rightarrow\vec{a}+\vec{b}+\vec{c}=\alpha\left(\frac{\beta+1}{\alpha+1}\right)\vec{a}$

$\Rightarrow\left(1-\frac{\alpha(\beta+1)}{\alpha+1}\right)\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar, which is contrary to the given condition. Hence, $\alpha=-1$, $\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$

79. (a) Equation of plane through (3, 2, –1) is

$a(x-3)+b(y-2)+c(z+1)=0$ …(1)

Also, (3, 4, 2) and (7, 0, 6) lie on equation (1), then

$0\cdot a+2b+3c=0$ …(2)

and $4a-2b+7c=0$ …(3)

On eliminating a, b, c from equations (1), (2) and (3), we get

$$\begin{vmatrix}x-3 & y-2 & z+1\\ 0 & 2 & 3\\ 4 & -2 & 7\end{vmatrix}=0$$

i.e., $5x+3y-2z=23$

$\therefore \lambda=23$

80. (d) The man is one step away from starting point after 11 steps. This can happen in following two ways:

(i) he takes 5 steps forward and 6 steps backward

(ii) he takes 6 steps forward and 5 steps backward

Probability in first case $={}^{11}C_5(0.4)^5(0.6)^6$

Probability in second case $={}^{11}C_6(0.4)^6(0.6)^5$

Hence, required probability

$={}^{11}C_5(0.4)^5(0.6)^6+{}^{11}C_6(0.4)^6(0.6)^5$

$={}^{11}C_5(0.4)^5(0.6)^5(0.6+0.4)$

$={}^{11}C_5(0.24)^5\,.\,1$

$=462\times(0.24)^5$

81 (b) $B=A_1+3A_3^3+\ldots.(2n-1)(A_{2n-1})^{2n-1}$

$B^T=-[A_1+3A_3^3+\ldots.(2n-1)(A_{2r-1})^{2r-1}]$

$=-B$, so skew-symmetric.

82. (b) (i) $A\subseteq A$ i.e., ARA, $\forall A\in P(S)$

$\therefore$ R is reflexive.

(ii) $A\subseteq B \not\Rightarrow B\subseteq A$

$\therefore$ ARB $\not\Rightarrow$ BRA. So, R is not symmetric.

(iii) ARB and BRA $\Rightarrow A\subseteq B$ and $B\subseteq A\Rightarrow A=B$.

Thus, R is anti-symmetric.

(iv) ARB and BRC $\Rightarrow A\subseteq B$ and $B\subseteq C$

$\Rightarrow A\subseteq C$

$\Rightarrow$ ARC

$\therefore$ R is transitive relation.

Thus, R is partially ordered relation but not an equivalence relation.

83. (a) We know that binary operation is function from $S\times S$. Let S be a finite set containing n elements. Then total number of binary operation on S is n^{n^2}.

84. (d) Given : $p\,o\,q=3p^2+2q^2-5pq$ and

$a\,o\,1=1$

Consider $a\,o\,1=3a^2+2(1)^2-5(a)(1)$

$=3a^2+2-5a=3a^2-5a+2$

But $a\,o\,1=1$

$\Rightarrow 3a^2-5a+2=1\Rightarrow 3a^2-5a+1=0$

$\Rightarrow a=\frac{5\pm\sqrt{25-12}}{6}=\frac{5\pm\sqrt{13}}{6}$

85. (b) After dividing by $\cos^2 x$ to numerator and denominator of integration

$$I=\int\frac{\sec^2 x\,dx}{4\tan^2 x+4\tan x+5}$$
$$=\int\frac{\sec^2 x\,dx}{(2\tan x+1)^2+4}$$
$$=\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$$

86. (a) It is given that

$P(A\cap B)=p$ and $P(A'\cap B)+P(A\cap B')=q$.

since $P(A'\cap B)=P(B)-P(A\cap B)$, we get

$$=P(B)-P(A\cap B)+P(A)-P(A\cap B)$$
$$q=P(A)+P(B)=q+2p$$
$$P(A')+P(B')=1-P(A)+1-P(B)$$
$$=2-q-2p,$$

showing that (b) is correct. The answer (c) is also correct because

$$P(A\cap B\mid A\cup B)=\frac{P[(A\cap B)\cap(A\cup B)]}{P(A\cup B)}$$
$$=\frac{P(A\cap B)}{P(A\cup B)}$$
$$=\frac{P(A\cap B)}{P(A)+P(B)-P(A\cap B)}$$
$$=\frac{p}{q+2p-p}=\frac{p}{p+q}$$

Finally, (d) is correct because

$$P(A'\cap B')=1-P(A\cup B)$$
$$=1-[P(A)+P(B)-P(A'-P(A\cap B)]$$
$$=1-(q+2p-p)$$
$$=1-p-q.$$

87. (a) $\sec^{-1}\dfrac{1}{(2x^2-1)}=2\cos^{-1}x$

$\therefore\ y=2\cos^{-1}x, z=\sqrt{1+3x}$

$$\frac{dy}{dz}=\frac{dy}{dx}\div\frac{dz}{dx}=-\frac{2}{\sqrt{1-x^2}}.\frac{2\sqrt{1+3x}}{3}=0\left(at\ x=-\frac{1}{3}\right).$$

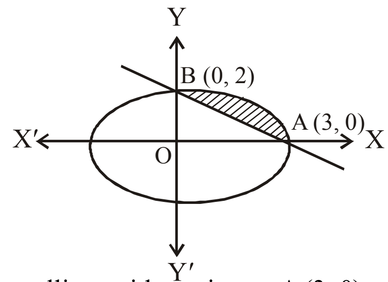
88. (a) Suppose that $t=\dfrac{5x+1}{10x^2-3}$, so $y=f(t)$

$\therefore\ \dfrac{dy}{dx}=f'(t).\dfrac{dt}{dx}$ [Since $f'(x)=\cos x$]

$$\frac{dy}{dx}=\cos\left(\frac{5x+1}{10x^2-3}\right)\frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right).$$

89. (c) The given ellipse is

$$\frac{x^2}{9}+\frac{y^2}{4}=1\ \Rightarrow\ y^2=\frac{4}{9}(9-x^2)$$
$$\Rightarrow y=\frac{2}{3}\sqrt{9-x^2}$$



It is an ellipse with vertices at A (3, 0) and B (0, 2) and length of the major axis $=2(3)=6$ and length of the minor axis $=2(2)=4$

Line $\dfrac{x}{3}+\dfrac{y}{2}=1\ \Rightarrow\ y=\left(\dfrac{6-2x}{3}\right)$

It is a straight line passing thorugh A (3, 0) and B (0, 2).

Smaller area common to both is shaded.

Shaded Area

$$=\frac{2}{3}\int_0^3\sqrt{9-x^2}\,dx-\int_0^3\left(\frac{6-2x}{3}\right)dx=\frac{2}{3}I_1-\frac{1}{3}I_2$$

where, $I_1=\int_0^3\sqrt{9-x^2}\,dx$ and $I_2=\int_0^3(6-2x)\,dx$

For I_1, put $x=3\sin\theta$ so that $dx=3\cos\theta\,d\theta$

When, $x=0, \theta=0$ and when $x=3$, $\theta=\dfrac{\pi}{2}$

$$\therefore\ I_1=\int_0^{\frac{\pi}{2}}\sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\,d\theta$$
$$=\frac{9}{2}\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)\,d\theta$$
$$=\frac{9}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}=\frac{9}{2}\left(\frac{\pi}{2}-0\right)=\frac{9\pi}{4}\quad\text{and}$$
$$I_2=\int_0^3(6-2x)\,dx=\left[6x-x^2\right]_0^3=18-9=9$$

$$\text{Required area}=\frac{2}{3}\times\frac{9\pi}{4}-\frac{1}{3}\times 9=\frac{3\pi}{2}-3$$
$$=\frac{3}{2}(\pi-2)\text{ sq. units.}$$

90. (a) Given $f(x)=\dfrac{4x^2+1}{x}$

Thus $f'(x)=4-\dfrac{1}{x^2}$

$f(x)$ will be decreasing if $f'(x)<0$

Thus $4-\dfrac{1}{x^2}<0$

$$\Rightarrow\ \frac{1}{x^2}>4\Rightarrow\ \frac{-1}{2}<x<\frac{1}{2}$$

Thus interval in which $f(x)$ is decreasing, is $\left(\dfrac{-1}{2},\dfrac{1}{2}\right)$.