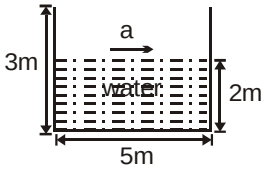
Exercise - IV (TOUGH SUBJECTIVE PROBLEMS)

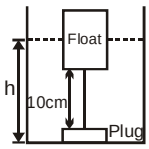
1. A solid block of volume V = 10^{-3} m^3 and density d = 800 kg/m^3 is tied to one end of a string, the other end of which is tied to the bottom of the vessel. The vessel contains 2 immiscible liquids of density ρ_1 = 1000 kg/m^3 and ρ_2 = 1500 kg/m^3. The solid block is immersed with 2/5 th of its volume in the liquid higher density & 3/5th in the liquid of lower density. The vessel is placed in an elevator which is moving up with an acceleration of a = g/2. Find the tension in the string. [g = 10 m/s^2]

2. An open rectangular tank 5m × 4m × 3m high containing water upto a height of 2m is accelerated horizontally along the longer side.

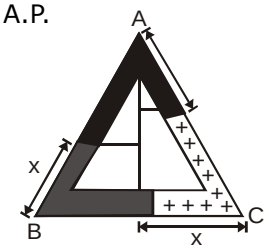


(a) Determine the maximum acceleration that can be given without spilling the water.
(b) Calculate the percentage of water split over, if this acceleration is increased by 20%
(c) If initially, the tank is closed at the top and is accelerated horizontally by 9m/s^2, find the gauge pressure at the bottom of the front and rear walls of the tank.

3. A level controller is shown in the figure. It consists of a thin circular plug of diameter 10cm and a cylindrical float of diameter 20 cm tied together with a light rigid rod of length 10 cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3kg and the plug may be assumed as massless.

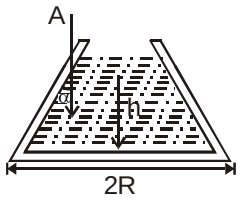


4. A closed tube in the form of an equilateral triangle of side *l* contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find x in the figure if the densities of the liquids are in A.P.

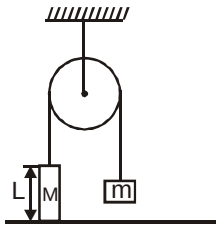


5. A ship sailing from sea into a river sinks X mm and on discharging the cargo rises Y mm. On proceeding again into sea the ship rises by Z mm. Assuming ship sides to be vertical at water line, find the specific gravity of sea water.

6. A conical vessel without a bottom stands on a table. A liquid is poured with the vessel & as soon as level reaches h, the pressure of the liquid raises the vessel. The radius of the base of vessel is R and half angle of the cone is α and the weight of the vessel is W. What is the density of the liquid?



7. As the arrangement shown in the fig is released the rod of mass M moves down into the water. Friction is negligible and the string is inextensible

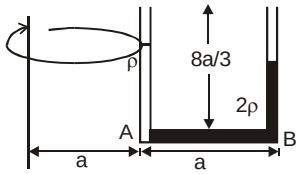


(a) Find the acceleration of the system w.r.t the distance moved by each mass.
(b) Find the time required to completely immerse the rod into water

if $\frac{m}{M} = \frac{\rho - \rho_{water}}{\rho}$.

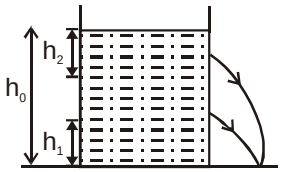
ρ = density of rod ; ρ_{water} = desity of water

8. The interface of two liquids of densities ρ and 2ρ respectively lies at the point A in a U tube at rest. The height of liquid column above A is 8a/3 where AB = a. The cross sectional area of the tube is S. With what angular velocity the tube must be whirled about a vertical axis at a distance 'a' such that the interface of the liquids shifts towards B by 2a/3.



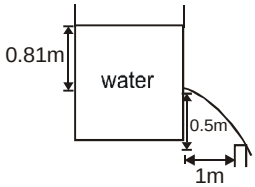
9. A closed cylindrical tank 2m high & 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20.0 rad/s, how much of the bottom of the tank is uncovered ? (The cylinder is rotated about vertical axis of symmetry passing through its length.)

10. A cylinder of height H is filled with water to a height h_0 (h_0<H), & is placed on a horizontal floor. Two small holes are punched at time t = 0 on the vertical line along the length of the cylinder, one at a height h_1 from the bottom & the other a depth h_2 below the level of water in the cylinder. Find the relation between h_1 & h_2 such that the instantaneous water jets emerging



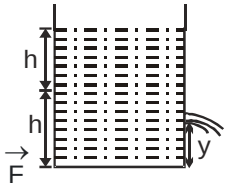
11. A cylindrical tank with a height of h = 1 m is filled with water upto its rim. What time is required to empty the tank through an orifice in its bottom? The cross sectional area of the orifice is (1/400)th of the tank. Find the time required for the same amount of water to flow out of the tank if the water level in the tank is maintained constant at a height of h = 1 m from the orifice.

12. For the arrangement shown in the figure. Find the time interval after which the water jet ceases to cross the wall.



Area of the tank = 0.5 m^2
Area of the orifice = 1 cm^2

13. A cylindrical tank having cross-sectional area A = 0.5 m^2 is filled with two liquids of densities ρ_1 = 900 kgm^{-3} & ρ_2 = 600 kgm^{-3}, to a height h = 60 cm as shown in the figure. A small hole having area a = 5 cm^2 is made in right vertical wall at a height y = 20 cm from the bottom. Calculate



(i) velocity of efflux.
(ii) horizontal force F to keep the cylinder in static equilibrium, it it is placed on a smooth horizontal plane.
(iii) minimum and maximum value of F to keep the cylinder at rest. The coefficient of friction between cylinder and the plane is μ = 0.01.
(iv) velocity of the top most layer of the liquid column and also the velocity of the boundary separating the two liquids.

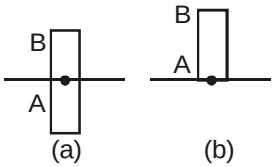
14. A cylindrical wooden float whose base area s = 4000 cm^2 & the altitude H = 50 cm drifts on the water surface. Specific weight of wood d = 0.8 gf/cm3.
(a) What work must be performed to take the float out of the water ?
(b) Compute the work to be performed to submerge completely the float into the water.

15. A 10 cm side cube weighing 5N is immersed in a liquid of relative density 0.8 contained in a rectangular tank of cross sectional area 15cm × 15cm. If the tank contained liquid to a height of 8 cm before the immersion, determine the levels of the bottom of the cube and the liquid surface.

16. A jug contains 15 glasses of orange juice. When you open the tap at the bottom it takes 12 sec to fill a glass with juice. If you leave the tap open. how long will it take to fill the remaining 14 glasses and thus empty the jug?

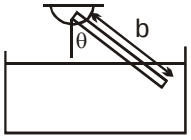
17. An interstellar explorer discovers a remarkable planet made entirely of a uniform incompressible fluid on density ρ. The radius of the planet is R and the acceleration of gravity at its surface is g. What is the pressure at the center of the planet.

18. A cylindrical rod of length l = 2m & density $\rho/2$ floats vertically in a liquid of density ρ as shown in fig.(a)

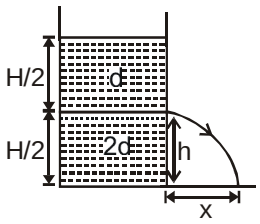


(a) Show that it performs SHM when pulled slightly up & released & find its time period. Neglect change in liquid level.
(b) Find the time taken by the rod to completely immerse when released from position shown in (b). Assume that it remains vertical throughout its motion. (take g = π^2 m/s^2)

19. A uniform rod of length b capable of tuning about its end which is out of water, rests inclined to the vertical. If its specific gravity is 5/9, find the length immersed in water.

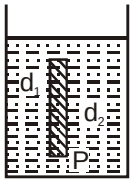


20. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous & incompressible liquids of densities d & 2d, each of height H/2 as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0.



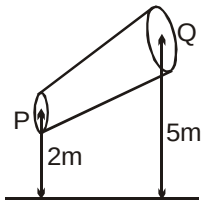
(a) A homogeneous solid cylinder of length $L\left(L<\frac{H}{2}\right)$ cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with the length L/4 in the denser liquid. Determine :
(i) The density D of the solid &
(ii) The total pressure at the bottom of the container.
(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s(s<<A) is punched on the vertical side of the container at a height $h\left(h<\frac{H}{2}\right)$. Determine :
(i) The initial speed of efflux of the liquid at the hole ;
(ii) The horizontal distance x travelled by the liquid initially &
(iii) The height h_m at which the hold should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m. [Neglect the air resistance in these calculations]. **[JEE 95, 10]**

21. A thin rod of length L & area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **[JEE' 95,5]**



22. A large open top container of negligible mass & uniform cross-sectional area A has a small hole of cross-sectional area A/100 in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at t = 0, calculate
(i) the acceleration of the container and
(ii) its velocity when 75% of the liquid has drained out. **[JEE '97,5]**

23. A nonviscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3} m^2 and 8×10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **[JEE ,97]**



24. A wooden stick of length l, and radius R and density ρ has a small metal piece of mass *m* (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters that would make the stick float vertically in equilibrium in a liquid of density σ (> ρ). **[JEE '99,10]**