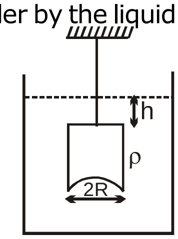
Exercise - V (JEE PROBLEMS)

1. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to : **[JEE 2000 (Scr.)]**

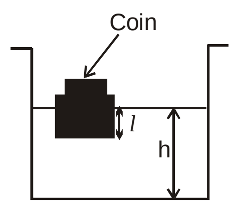
(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$ (C) L (D) $\frac{L}{2\pi}$

2. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is **[JEE 2001 (Scr.)]**



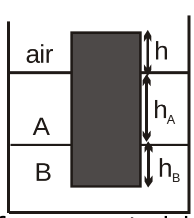
(A) Mg (B) Mg – v ρ g
(C) Mg + π R^2 h ρ g (D) ρg (V + πR^{2}h)

3. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distances l and h are shown there. After some time the coin falls into the water. Then



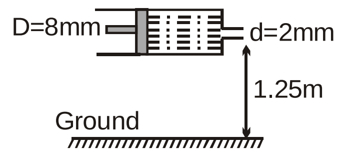
(A) l decreases and h increases
(B) l increases and h decreases
(C) both l and h increase
(D) both l and h decrease **[JEE 2002 (Scr.)]**

4. A uniform solid cylinder of density 0.8 gm/cm^3 floats in equilibrium in a combination of two non mixing liquids A and b with its axis vertical. The densities of the liquids A and B are 0.7gm/cm^3, and 1.2 g/cm^3 respectively. The height of liquid A is h_A = 1.2 cm. The length of the part of the cylinder immersed in liquid B is h_B = 0.8 cm.



(a) Find the total force exerted by liquid A on the cylinder.
(b) Find h, the length of the part of the cylinder in air.
(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. **[JEE 2002]**

5. Consider a horizontally oriented syringe containing water located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of water stream on the ground.
Take g = 10 m/s^2. **[JEE 2004]**

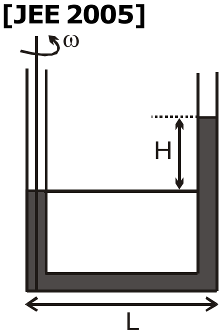


6. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[JEE 2004]**

7. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A} = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) **[JEE 2005 (Scr.)]**

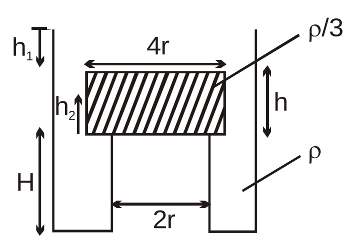
(A) 48 (B) 51 (C) 50 (D) 51.5

8. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L.. **[JEE 2005]**



Comprehension (9 - 11)

A wooden cylinder of diameter 4r, height h and density ρ/3 is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



9. If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is **[JEE 2006]**

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$ (C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$

10. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is **[JEE 2006]**

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$ (C) $\frac{2h}{3}$ (D) h

11. If height h_2 of water level is further decreased, then **[JEE 2006]**

(A) cylinder will not move up and remains at its original position

(B) for $h_2 = h/3$, cylinder again starts moving up

(C) for $h_2 = h/4$, cylinder again starts moving up

(D) for $h_2 = h/5$, cylinder again starts moving up

12. STATEMENT - 1

The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT - 2

In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct expalantion for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT,** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True **[JEE 2008]**

13. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End I has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve.

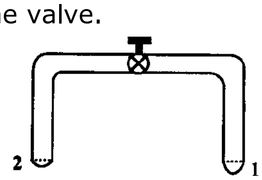


Figure.

(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

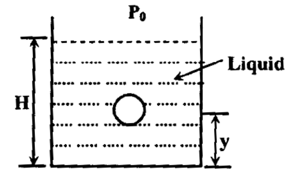
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no changes occurs

(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases **[JEE 2008]**

Paragraph for Qeustion No. 14 to 16

A small spherical monoatomic ideal gas double $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ, (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension). Figure : **[JEE 2008]**



14. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(A) Only the force of gravity

(B) The force due to gravity and the force due to the pressure of the liquid

(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) The force due to gravity and the force due to viscosity of the liquid

15. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$ (B) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$ (D) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

16. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \frac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

(B) $\frac{\rho\ nRgT_0}{(P_0+\rho\ gH)^{2/5}[P_0+\rho\ g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0 \frac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(D) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

17. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmospheric pressure = 1.0 × 10 5 Nm^{-2} , density of water = 1000 kg m^{-3} and *g* = 10 ms^{-2} . Neglect any effect of surface tension.] (Take temperature to be constant) **[JEE 2009]**

18. **Column II** shows five systems in which two objects are labbled as *X* and *Y*. Also in each case a point *P* is shown. **Column I** gives some statements about *X* and **/** or *Y* Match these statements to the appropriate system(s) from **Column II**.

[JEE 2009]

Column I

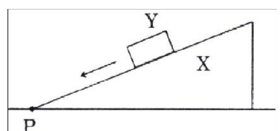
Column II

(A) The force exerted by X on Y

(P) Block Y of mass M left on a fixed inlcined plane X,

has a magnitude Mg.

slides on it with a constant velocity



(B) The gravitational potential energy

(Q) Two ring magnets Y and Z, each of mass M, are kept in

of X is continuously increasing.

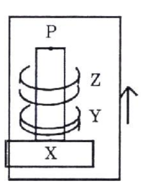
frictionless vertical plastic stand so that they repel each

other. y rests on the base X and Z hangs in air in

equilibrium. P is the topmost point of the stand on the

common axis of the two rings. The whole system is in a

lift that is going up with a constant velocity.



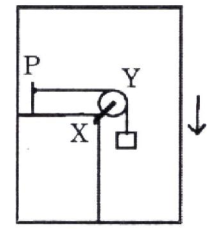
(C) Mechanical energy of the system

(R) A pulley Y of mass m_0 is fixed to a table through a clamp

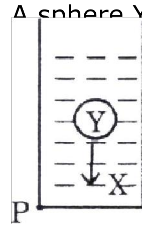
X + Y is continuously decreasing

X. A block of mass M hangs from a string that

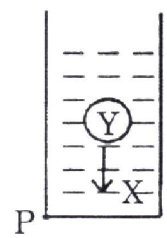
goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.



(S) A sphere Y of mass M is put is a
liquid X kept in a container
e sphere is released and it
ves down in the liquid.

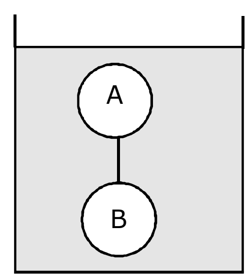


(T) A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.



[JEE 2009]

19. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **[JEE 2011]**



(A) $d_A < d_F$ (B) $d_B > d_F$
(C) $d_A > d_F$ (D) $d_A + d_B = 2\ d_F$