ELASTICITY & THERMAL EXPANSION

EXERCISE – I

SINGLE CORRECT

1. **C.**

$\Delta L = L\alpha\,\Delta T$

6×10^{-5} mm = (1mm) $(12 \times 10^{-6})\,\Delta t$

$\Delta t = 5°C$

2. **A.**

$$\frac{\Delta L}{L} = \alpha\Delta T = \frac{F}{A\gamma}$$

$F = \alpha A\,\gamma\Delta t$

$= (10^{-5})(0\cdot8 \times 10^{-4}) \times (2 \times 10^{10}) \times 10$

$= 160$ N

3. **C.**

$L_1 = L + L\alpha_1\,\Delta t$

$L_2 = L + L\alpha_2\,\Delta t$

$$\frac{\text{Stress}_1}{\text{Stress}_2} = \frac{\gamma_1 L\alpha_1\Delta t}{L}\cdot\frac{L}{\gamma_2 L\alpha_2\Delta t}$$

$$1 = \frac{2}{3}\frac{\gamma_1}{\gamma_2}$$

4. **C.**

$I = CMR^2$

$dI = 2CMRdR = 2CMR\,[R\alpha\Delta T]$

$= 2\alpha I\Delta T$

5. **B.**

$$F = A\gamma\frac{\Delta L}{L} = A\gamma\alpha\Delta T$$

$$f = k\sqrt{\frac{F}{\mu}} = K\sqrt{\frac{A\gamma\alpha\Delta T}{\rho A}}$$

$$\Rightarrow f\ \alpha\sqrt{\frac{\gamma\alpha}{\rho}}$$

6. **B.**

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \gamma\,(\text{strain})^2\ \text{volume}$$

$$U = \frac{1}{2}\gamma\,(\alpha\Delta T)^2\,AL$$

$U \propto \Delta T^2$

$U \propto (t - 20)^2$

7. **B.**

$$\frac{\Delta L}{L} = \alpha\Delta t = -\alpha 20$$

means read more so actual is less

8. **B.**

$0\cdot075 = 20\,\alpha_1\,(100)$

$0\cdot045 = 20\,\alpha_2\,(100)$

Let for third rod L_1 and $L_2 = 20 - L_1$

So $\Delta L_3 = \Delta L_1 + \Delta L_2$

$\Rightarrow 0\cdot06 = L_1\alpha_1\,100 + (20 - L_1)\,\alpha_2\,100$

$L_1 = 10$ cm.

9. **A.**

$\rho_{Spere} = \rho_l'$

$$\Rightarrow \frac{266.5}{\frac{4}{3}\pi\left(\frac{7}{2}\right)^3} = \frac{1.527}{1+35\gamma}$$

$\Rightarrow \gamma = 8.3 \times 10^{-4}$/.c

10. **B.**

$$h = \frac{V_m' - V_b'}{A_0'} = \frac{V_0\left(1+\gamma\Delta T\right) - V_0(1+3a_g\Delta T)}{A_0\left(1+2a_g\,\Delta T\right)}$$

$$= \frac{V_0 T\left(\gamma - 3a_g\right)}{A_0\left(1+2a_g T\right)}$$

11. **B.**

$$F = A\gamma\frac{\Delta L}{L(1+\alpha\Delta t)}$$

$$F = \frac{A\varepsilon\alpha t}{(1+\alpha t)}$$

12. **A.**

At 0°C

then $\rho_l\,v_s\,g = W_0$ ...(1)

At t°C $\rho_l'\,v_s'g = W$ ...(2)

$\rho_s v_s = \rho_s'\,v_s' = m$ ...(3)

$(2) - (1) \Rightarrow (\rho_l'\,V_s' - \rho_l\,V_s)\,g = W - W_0$

$W = W_0 + (\rho_l\,\{1 - \gamma_l t\}\,v_s\,(1 + \gamma_s t) - \rho_l v_s)g$

$= W_0\,[1 - (\gamma_l - \gamma_s)t]$

13. **C.**

$$\text{Initially } P = \frac{V_b\rho_b}{A_c},\ P' = \frac{V_b'\,\rho_b'}{A_c'}$$

$$P' = \frac{V_b\left(1+10^{-3}\times10\right)}{A_c\left(1+2\times10^{-3}\times10\right)} \times \frac{\rho_b}{\left(1+10^{-3}\times10\right)}$$

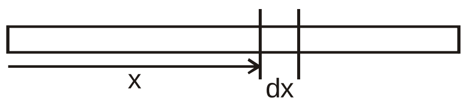
$$P' = \frac{P}{1+2\times10^{-2}}$$

$$\left(\frac{P'}{P}-1\right)\times 100 = \frac{1-(1+2\times 10^{-2})}{1}\times 100$$

$= -2\%$

14 C.

$dx = \Delta dx$



$$\int_0^{\Delta L} \Delta dx = \int_0^L dx(3x+2)\times 10^{-6}(20-0)$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3x^2}{2}+2x\right)_0^L$$

$$\Delta L = (20\times 10^{-6})\left(\frac{3L^2}{2}+2L\right) = 1\cdot 2\text{ cm}$$

$L_{new} = L + \Delta L$

15. C.

Let eq^n. temp = t then

$m_R s_R t = m_s s_s (100 - t)$ ...(1)

$d_R' = d_R(1 + \alpha_R t)$ ...(2)

$d_s' = d_s[1 - \alpha_s(100 - t)]$ ...(3)

Now $d_R' = d_s'$ ...(4)

So. $d_R(1 + \alpha_R t) = d_s[1 - \alpha_s(100 - t)]$

$$t = \frac{d_s(1-\alpha_s 100) - d_R}{[d_R\alpha_R - d_s\alpha_s]}$$

Put the above value of t in eq. 1.

$$\left(\frac{m_R s_R}{m_S s_s}+1\right)t = 100$$

$$\frac{m_s}{m_r} = \frac{23}{54}$$

16. C.

$\alpha_x + \alpha_y$ for x – y plane

$\beta_{CDEH} = 3\times 10^{-5}$ per °C

17. D.

$\gamma_{oil} = \gamma_{vessel} \Rightarrow$ D.

Volume increases but mass remains same.

18. C.

$\because \gamma_m < \gamma_{Al}$ $\rho_m >> \rho_{ac}$

$\Delta V_m < \Delta V_{al}$ So completely Immersed

$\Delta\rho_m < \Delta\rho_{Al}$

So $W_2 > W_1$ [$\because$ Displaced mass of alchol) is less]

19. D.

Iuitially ρ_s ρ_l and V

$$\frac{B_T' - B_T}{B_T}\times 100 = \frac{V_s'\rho_l'g - V_s\rho_l g}{V_s\rho_l g}\times 100$$

$\Rightarrow [(1 + \gamma_s \Delta t)(1 - \gamma_l \Delta t) - 1]\times 100$

$= -0.05$ (decreases)

20. B.

$$\frac{\Delta L}{L}\times 100 = 1 = 100\,\alpha\Delta t = 100\,\alpha(T_2 - T_1)$$

$$\frac{\Delta A}{A}\times 100 = 200\,\alpha\Delta t = 2\%$$

21. C.

$\Delta L = \Delta L_1 + \Delta L_2$

$(3L)\alpha_{net}\Delta t = L\alpha\Delta t = (2L)(2\alpha)\Delta t$

$$\alpha_{net} = \frac{\alpha+4\alpha}{3} = \frac{5\alpha}{3}$$

22. B.

$$\beta = -\frac{\Delta P}{\Delta V/V} \Rightarrow \Delta P = -\beta\frac{\Delta V}{V}$$

$\Delta P = \beta(3\alpha\Delta T)$

$= 7.14\times 10^7$ Pa.

23. A.

At 40°C

1 Unit will be $= 1(1 + \alpha_s \Delta t)$ units

$= 1(1 + 12\times 10^{-6}\times 40)$ Units

So 100 Unit will be $= 100(1 + 12\times 10^{-6}\times 40)$ = Actual

$100(1 + 40\times 12\times 10^{-6}) = l_0(1 + (2\times 10^{-6})\,40)$

$l_0 = 100[1 + 400\times 10^{-6}] > 100$mm.

24. C.

$V_{air} = V_{flask} - V_m = V'_{flask} - V_m'$

$V_{flask} - 300 = V_{flask}[1 + 3\times(9\times 10^{-6})\Delta t] - 300[1 + 8\times 10^{-4}\Delta t]$

$$V_{flask} = \frac{(300\times 1.8\times 10^{-4})}{27\times 10^{-6}\,\Delta t}\Delta t = 2000\text{ cm}^3$$

25. B.

Because floating

$$\rho_s Vg = \rho_\ell\left(\frac{V}{2}\right)g$$

$2\rho_s = \rho_\ell$

26. A.

if $\gamma_L > \gamma_S$ then submerged more else come out of liquid respectively

and $\gamma_L > \gamma_s$ (always)

27. A.

$V' = V[1 + \gamma_s \Delta t]$

$\rho_l' = \rho_l[1 - \gamma_l \Delta t]$

$$\rho_l\left(\frac{V}{2}\right)g = \rho_l'\left(\frac{V'}{2}\right)g$$

$$\rho_l\left(\frac{V}{2}\right)g = \rho_l(1-\gamma_l\,\Delta t)\left(\frac{V}{2}\right)(1+\gamma_s\,\Delta t)g$$

$(1-\gamma_l\,\Delta t)\,(1+\gamma_s\,\Delta t) = 1$

$(1-\gamma_l\,\Delta t)\,(1+3\alpha_s\,\Delta t) = 1$

$3\alpha_s - \gamma_l = 0$

28. A.

initially $\rho_l(A_s h)g = (\rho_s A_s h_o)g$ ...(1)

Now $\rho_l'(A_s' h)\,g = (\rho_s' A_s' h_o')g$ ...(2)

$\rho_l\,(1-\gamma_l\,\Delta t)h = \rho_s(1-3\alpha_s\,\Delta t)h_o\,(1+\alpha_s\,\Delta t)$

$\gamma_L = 2\alpha_s$

29. A.

$\rho_l' < r_s$ or $\frac{\rho_l}{2}$

$$\frac{\rho_l}{1+\gamma_l\,\Delta t} < \frac{\rho_l}{2}$$

$1+\gamma_l\,\Delta t > 2$

$$\Delta t > \frac{1}{\gamma_l}$$

$$T_F - T > \frac{1}{\gamma_l} \quad \Rightarrow T_F > T + \frac{1}{\gamma_l}$$

30. C.

$$\left.\begin{aligned}\text{Given}\quad \gamma_l - \gamma_c &= c\\ \text{and}\quad \gamma_l - \gamma_c &= s\end{aligned}\right] \Rightarrow \gamma_s = c + \gamma_c - s = 3\alpha_s$$

$$\alpha_s = \frac{c+\gamma_c - s}{3}$$

31. B.

$\Delta Q_0 = 100 \times 4 \times 60 = 24000$ cal.

for 0°C = water

$\Delta Q_1 = (100 \times 0.2 \times 20) + (200 \times 0.5 \times 20)$

$+ (200 \times 80)$

$= 18400$ cal.

So let temp is t then.

$24000 - 18400 = (200 \times 1 + 100 \times 0.2)t$

$t = 25.5°C$

32. C.

$\rho_{0°C}\,h_1\,g = \rho_{30°C}\,h_2\,g$

$\rho_0\,(120) = \rho_0\,(1-\gamma 30)\,(124)$

$$\gamma = \left(1-\frac{120}{124}\right)\frac{1}{30} = 11\times10^{-4}\,/°C$$

33. A.

$$\frac{(212-37)°F}{(100-0)°C}\times 25°C = 45°F$$

34. D.

at 0°C

$V_{0x} = 20A$

$V_{0y} = 30A$

Now at time T y read 120°C

So. $V'_{0y} = A(120) = 30A\,(1+\gamma_m\,T)$

and $V'_{0x} = Ah = 20A\,(1+\gamma_m\,T)$

Dividing $\frac{120}{h} = \frac{30}{20} \Rightarrow h = 80.$

Multiple Choice Question

35. C, D

for Adiabatic

PV^{γ} = const.

$$P \propto \frac{1}{V^{\gamma}}$$

PV = nRT

36. B, C

Strain → Same

Stress = $\frac{F}{A}$ = constant

$F \propto A$

$\Rightarrow \quad F \propto r^2$

Energy = $\frac{1}{2}$ stress × strain × volume

$\propto$ Area

$\propto r^2$

37. A, C, D

Gravitational Potential Energy $U_G = Mgl$

Elastic Potential Energy U_e =

$= \frac{1}{2}$ stress × strain × volume

$$= \frac{1}{2}\frac{F}{A}\frac{\ell}{\ell_0}\times V \;\left|\; \begin{aligned}F &= mg\\ V &= A\,\ell_0\end{aligned}\right.$$

$$= \frac{1}{2}mgl$$

Heat Produced = $U_e = \frac{1}{2}Mgl$

38. A, C, D

(A) % rise in area = $\beta\Delta T$

$= 2(\alpha\Delta T)$

$= 2 \times 0.2 = 0.4\%$

(C) % rise is volume = $3\,\alpha\Delta T$

$= 3 \times 0.2 = 0.6\%$

(D) $\alpha = \frac{0.2}{80\times100} = 0.25 \times 10^{-4}/°C$