FLUID

EXERCISE – I　　SINGLE CORRECT

1. B

Pressure = $h \rho g_{eff.}$

$a = g/3$

$g_{eff} = g - g/3 = 2g/3$

$\Rightarrow P = \dfrac{0.15 \times 1000 \times 2 \times 10}{3}$

P = 1KPa

2. D

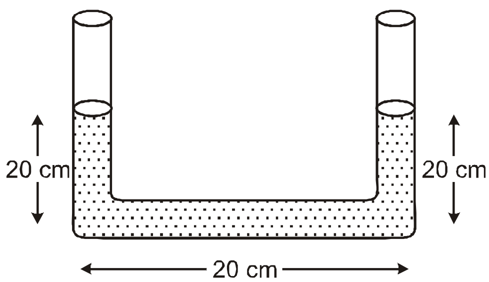
Force is same pressure is different

3. A

F = mg

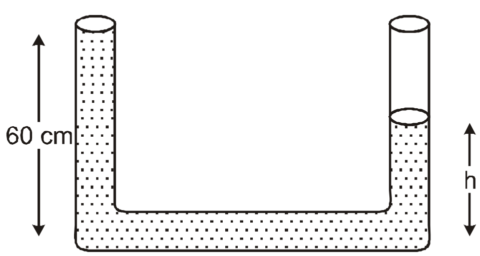
F = 10 N

4. D



$60\ \rho_w g = h\ \rho_\ell g$

$\Rightarrow 60 \times 1 \times g = h \times 4 \Rightarrow h = 15$ cm



So, volume = Ah

$= 1 \times 35 = 35\ cm^3$

5. B

Take area of projection from left

$\dfrac{2\rho g \ell h^2}{2} = \dfrac{3\rho g \ell R^2}{2}$

$h = \sqrt{\dfrac{3}{2}}.R$

6. D

O(zero) all the forces passes through O

$\therefore$ no torque.

7. B

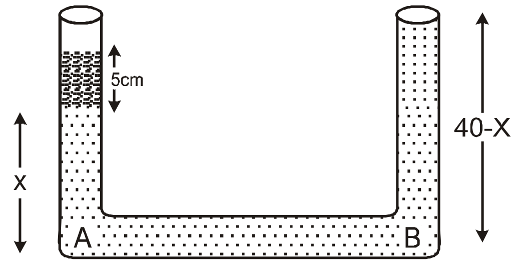
$h \rho g = 2P$

$\dfrac{4h}{5} \rho g = \dfrac{8P}{5}$ $\left\{\begin{matrix}\text{After lowering P due} \\ \text{to liquid.}\end{matrix}\right\}$

$\therefore P_T = \dfrac{8P}{5}$ + P (Atmospheric pressure)

$= \dfrac{13P}{5}$

8. C



$P_A = P_B$

$\Rightarrow 5 \times 4 \times g + x \times 1 \times g$

$= (40 - x) \times 1 \times g$

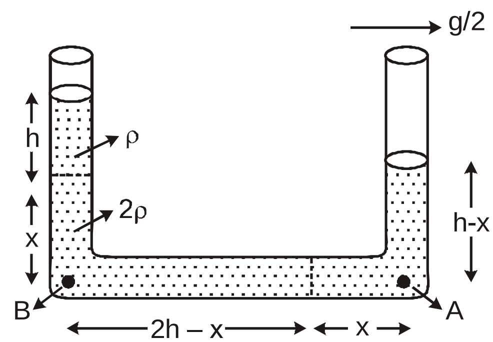
$\Rightarrow x = 10$

Now, $h_1 = x + 5 = 15$ cm

$h_2 = 40 - x = 30$ cm

$h_2/h_1 = 2$

9. B



$\rho x g/2 + 2(2h-x)g/2 = P_B - P_A$

$P_A = (h - x)\rho g$

$P_B = h\rho g + 2\rho x g$

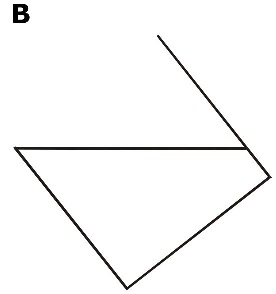
$\rho x g/2 + 2\rho(2h-x)g/2 = h\rho g + 2\rho x g - h\rho g + x\rho g$

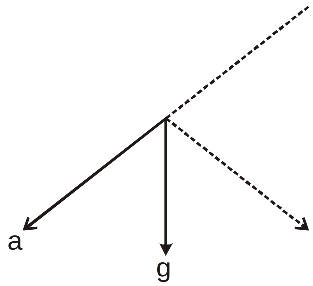
$\dfrac{4\rho h g}{2} - \dfrac{2\rho x g}{2} = 2\rho x g + \rho x g/2$

$2\rho h g - \rho x g = 5Pxg/2$

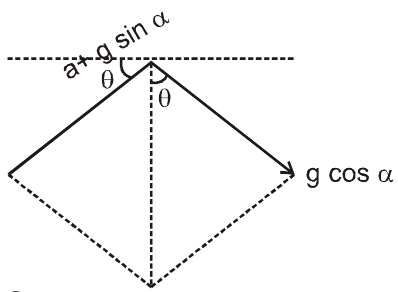
$\Rightarrow x = \dfrac{4h}{7}$

11. B

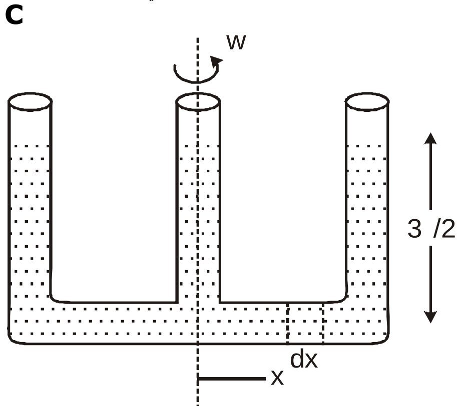




$$\tan\theta = \frac{a + g\sin\alpha}{g\cos\alpha}$$



12. C



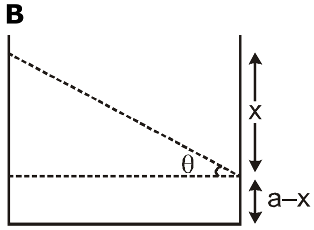
$dP \,.\, A = \rho A\, dx \,.\, \omega^2 x$

$\int dp = \int \rho\omega^2 x dx$

$$\Delta P = \frac{\rho\omega^2\ell^2}{2} = \frac{3\ell\rho g}{2}$$

$$\omega = \sqrt{\frac{3g}{\ell}}$$

13. B



$$(a\text{-}x)a^2 + \frac{1}{2}\, x\, a^2 = \frac{2}{3}a^3$$

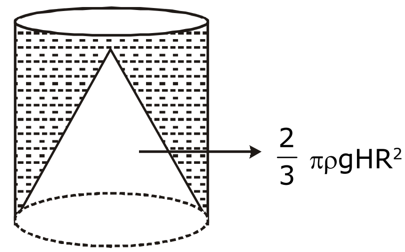
$$(a\text{-}x) + \frac{x}{2} = \frac{2}{3}a$$

$$a - \frac{x}{2} = \frac{2}{3}a$$

$$x = \frac{2a}{3}$$

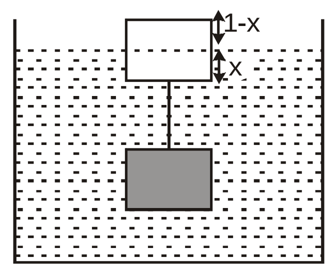
$$\therefore \tan\theta = \frac{x}{a} = \frac{acc.}{g} \qquad a = \frac{2}{3}g$$

14. D



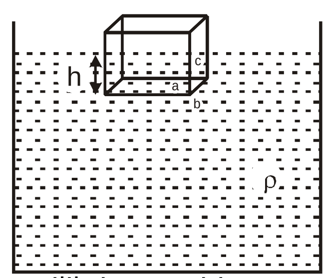
15. A

16. B



Total buoyancy
= Total Gravitation
$\Rightarrow 1^3 \times 1 \times g + 1^2 x \times 1\rho$
$= 1^3 \times 0.6 \times g + 1^3\ 1.15 \times g$
$1 + x = 0.6 + 1.15$
$x = 0.75$ m
$\therefore 1 - x = 25$ cm.

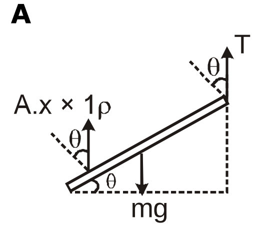
17. D



At equilibrium position
$(abc)\,(d\rho)g = (bc)\, h\rho g$
After displacing slightly x,
extra buoyancy force.
$\rho_{net} = ((bc)x)\rho g$

$$a = \frac{xbc\rho g}{abcd\rho} = \frac{xg}{ad} \qquad \omega = \sqrt{\frac{g}{ad}}$$

18. A

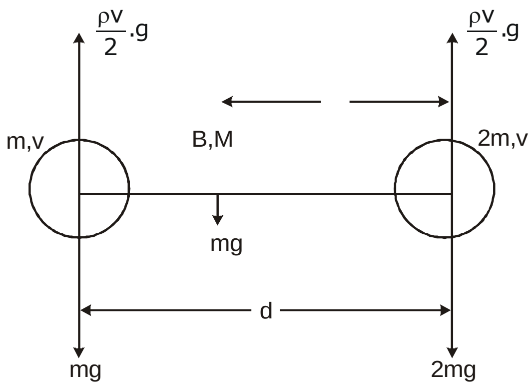


$mg = A.2L \times 0.75 \times g$
$T + Axg = A.2L \times 0.75\ g$
$T = Ag\ [1.5\ L - x]$

$A\ xg\ \cos\theta \left(\ell - \frac{x}{2}\right) = T\ \cos\theta\ \ell$

$x\left(\ell - \frac{x}{2}\right) = \ell\ [1.5\ L - x] \qquad x = \ell$

19. B



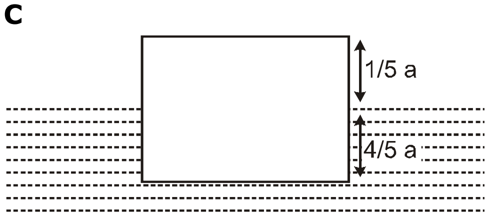
$(3M + m)g = \rho vg$
Torque balance about B

$mg\ (d - \ell) + \rho vg \frac{\ell}{2}$

$= 2\ mg\ell + \frac{\rho vg}{2}(d - \ell) \qquad \ell = \frac{d(v\rho - 2M)}{2(\rho v - 3M)}$

20. B
$\rho_1 V = \rho_2 2V$
$m_1 = m_2$
$m_1 g = 0.92\ Vg = m_2 g - xVg$
$x = 1.8\ gm/cm^3$

21. C



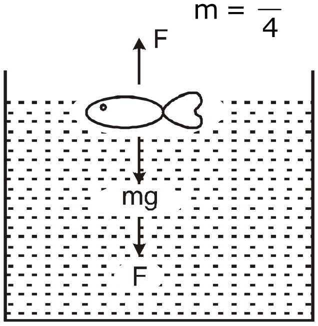
$\rho . \frac{4}{5} a^3 = M$

$(m + M) = \rho a^3$

$m = \frac{5}{4} M \qquad m = \frac{M}{4}$

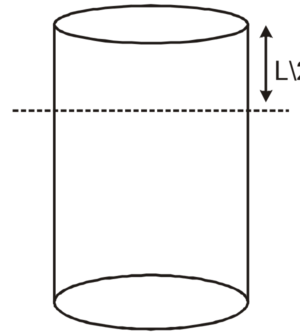
$M = 4\ m$

22. C



23. D

24. A



$d_1 AL + d_2\ AL = \frac{3}{2}\ LAd$

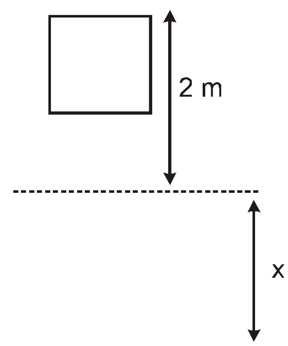
$d_1 + d_2 = \frac{3d}{2}$

$\therefore d_1 > \frac{3d}{4}$

25. B
$W - v \times 1 \times g = W_1$
$W - v \times x \times g = W_2$
$\Rightarrow W - (W - W_1) \times x = W_2$

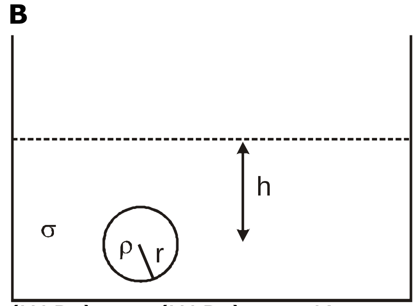
$x = \frac{W - W_2}{W - W_1}$

26. A



$mg\ (x + 2) = v \times 1 \times g \times x$
$v\ 0.8\ g\ (x + 2) = v \times 1 \times g \times x$
$0.8\ x + 1.6 = x$
$0.2x = 1.6$
$x = 8$

27. B



$(W.D.)_{mg} + (W.D.)_{F_B} = \Delta K$
$-\ mg\ (H + h) + (F_B)h = 1/2\ mv_f^2$

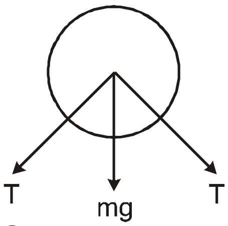
$\Rightarrow \; -\frac{4}{3}\pi r^3 \rho (H + h) g + \left(\frac{4}{3}\pi r^3\right)\sigma g h = 0$

$-\rho g H - \rho g H + \sigma g h = 0$

$gh(\sigma - \rho) = \rho g H$

$\boxed{H = (\frac{\sigma}{\rho} - 1)h}$

28. A



$\frac{2T}{\sqrt{2}} + mg = v\rho_w g$

29. C

$\therefore$ Volume where metal is present

$= \frac{9.8}{7800} = 1.256 \times 10^{-3}$

Buoyancy = $v\rho g$ = 1.5 g

$\Rightarrow v \times 1000 = 1.5$

$v = 1.5 \times 10^{-3}$

fraction of volume =

$\frac{1.5\times10^{-3} - 1.256\times10^{-3}}{1.5\times10^{-3}} \times 100$

= 16%

30. A

$\sigma \times \frac{4}{3}\pi (R^3 - r^3)g = 1 \times \frac{4}{3}\pi R^3 g$

$\frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$

31. B

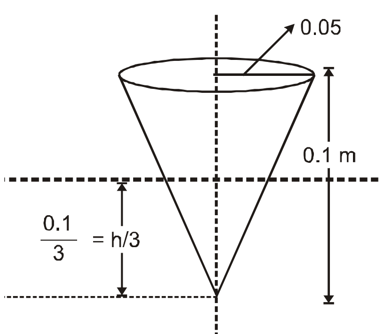
$g_{eff} = g + a$

$\Rightarrow t + mg_{eff} = F_B$

$T = Vd(g + a) - v\rho\,(g+a)$

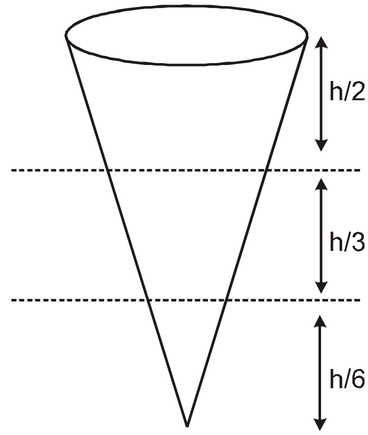
$= v[(g + a)(d - \rho)]$

32. D



$\frac{1}{3}\pi r^2 h \rho_c g = \frac{1}{3}\pi (r/3)^2 \frac{h}{3}(0.8)g$

$\rho_c = \frac{0.8}{27}$

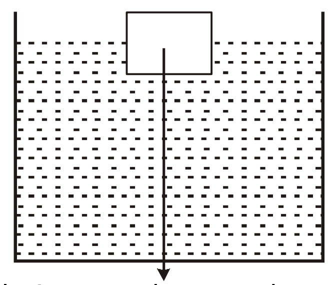


$\frac{1}{3}\pi r^2 h \rho_c g = \frac{1}{3}\pi(\frac{r}{6})^2\frac{h}{6} \times 0.8 \times g +$

$\frac{1}{3}\pi\left[\left(\frac{r}{2}\right)^2\frac{h}{2} - \left(\frac{r}{6}\right)^2\frac{h}{6}\right]\rho g$

$\Rightarrow \frac{0.8}{27} = \frac{0.8}{36\times6} + \left[\frac{1}{8} - \frac{1}{36\times6}\right]\rho \quad \rho = 1.9$

33. B



Completely inmersed means buoyancy is 1 kg upwards on stone & 1 kg downwards on water.

34. B

Initially

$W_{metal} = W_{ice}$ = Buoyancy

$V_{metal}\rho_m g + V_{ice}\rho_{ice} g = V_d \rho_\ell g$

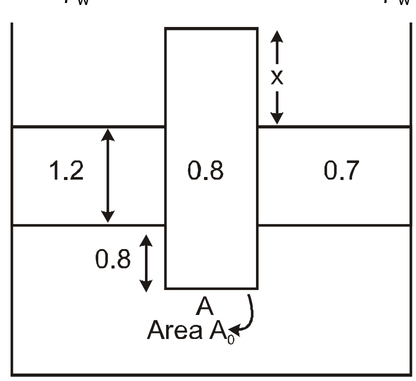
$\therefore V_d = \frac{V_{metal}\rho_m}{\rho_\ell} + \frac{V_{ice}\rho_{ice}}{\rho_\ell}$

finally volume displaced

$V = V_m + V_w$ (From ice)

$= V_m + \frac{m}{\rho_w} \qquad = V_m + \frac{V_i \ell_i}{\rho_w}$ < previous

35. B



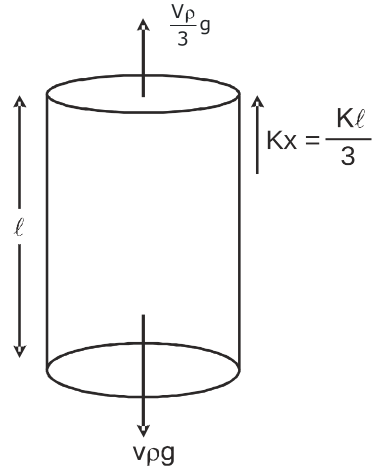
$P_A(1.2 \times 0.7 \times g + 0.8 \times 1.2\ g)$

$0.8 \times A_0\ (x + 1.2 + 0.8)\ g = \quad P_A.P_0$

$x + 1.2 + 0.8 = 0.84 + 0.96$

$x = 0.25$ cm

36. B

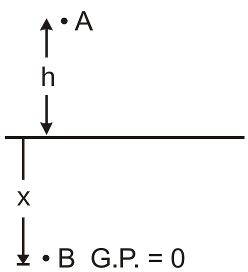


$V = A.\ell.$

Now $\quad \frac{A\ell\rho g}{3} + \frac{K\ell}{3} = A\ell\rho$

$K = 2\rho Ag$

37. C



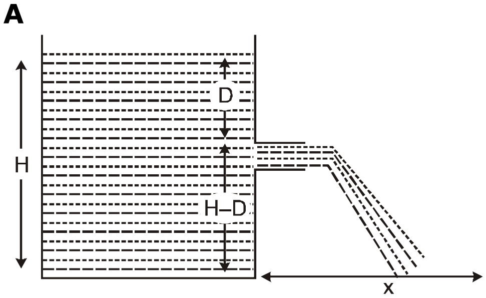
$W_{FB} = \{K_B + U_B\} - \{K_A + U_A\}$

$-V\rho x = 0 + 0 - 0 - V\rho' g(x + h)$

$\rho gx = \rho' gx + \rho' gh$

$x = \frac{\rho' h}{\rho - \rho'}$

38. A

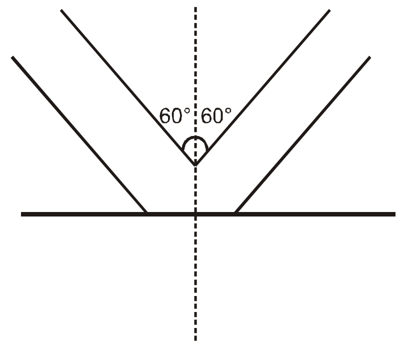


$\Delta\rho g = \frac{1}{2}\rho v^2$

$v = \sqrt{2Dg}$

$x = V\sqrt{\frac{2(H-D)}{g}} = 2\sqrt{D(H-D)}$

39. B



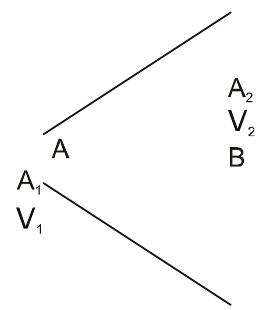
$\frac{dm}{dt} = \rho Av$

$\Delta P = F_{avg}.\ 1sec. \qquad \Rightarrow 2\rho Av^2\cos 60°$

$\Rightarrow 1000 \times 6 \times 10^{-4} \times (12) \times \frac{1}{2} \times 2$

$= 86.4$ Nt.

40. A



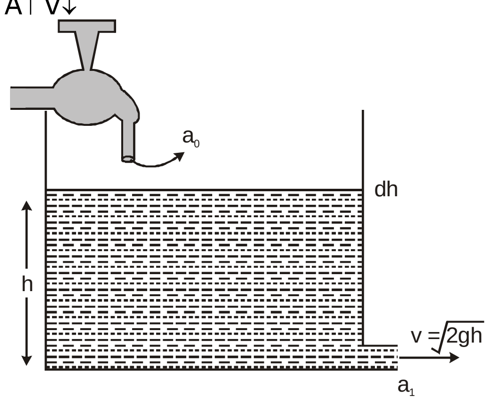
Now

$\rho gh + \frac{1}{2}\rho V_1^2 + P_A \qquad = \rho gh + \frac{1}{2}V_2^2\rho + P_B$

$A_1V_1 = A_2V_2$

as $A\uparrow V\downarrow$

41. C



$\frac{dm}{dt} = \rho A\frac{dh}{dt} = \rho a_0 v - \rho a_1\sqrt{2gh}$

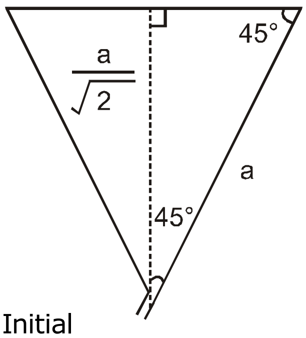
$\Rightarrow 4000\ \frac{dh}{dt} = 1 \times 2 - 0.5\sqrt{2gh}$

for $t = \infty\ \frac{dh}{dt} = 0$

$\Rightarrow 2 = 0.5\sqrt{2gh}$

$\Rightarrow h = 0.8$

42. D



Initial

$a\rho g = \frac{1}{2}\rho V^2$

$V_0 = \sqrt{2ga}$

Now $V = \sqrt{2\frac{a}{\sqrt{2}}} = \frac{V_0}{\sqrt[4]{2}}$

43. B

$A_1V_1 = A_2V_2$

$0.02 \times 2 = 0.01 \times V_2$

$V_2 = 4$ m/sec.

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$4 \times 10^4 + \frac{1}{2} \times 1000 \times 2^2$

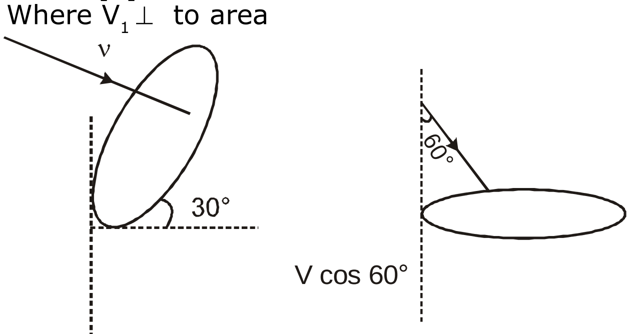
$= P_2 + \frac{1}{2} \times 1000 \times 4^2$

$P_2 = 3.4 \times 10^4$ N/m^2

44. C

From A_1V_1

Where $V_1 \perp$ to area



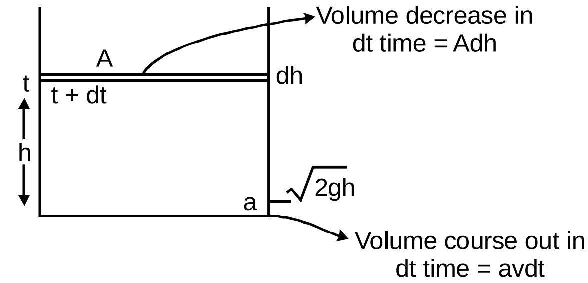
ratio $= \frac{V}{V\cos 60^\circ} = 2$

45. B

$\sqrt{2 \times 20 \times 10^{-2} \times 10}$

= 2 m/sec.

46. C



Volume decrease = Volume outlet

$Adh = a\sqrt{2gh}\ dt$

$\frac{-dh}{dt} = \frac{a\sqrt{2gh}}{A} \Rightarrow \int_H^{H/\eta} \frac{dh}{\sqrt{2gh}} = -\int_0^t \frac{a}{A} dt$

$t_1 = \left[-\sqrt{\frac{H}{\eta}} + \sqrt{H}\right]$

Similarly $t_2 = \sqrt{\frac{H}{\eta}}$

$\Rightarrow t_1 = t_2 \Rightarrow 2\sqrt{\frac{H}{\eta}} = \sqrt{H}$

$\eta = 4$

47. B

48. D

$A_1V_1 = A_2V_2$

$\pi(1 \times 10^{-2})^2 \times 3$

$= 100 \times \pi \frac{(0.05 \times 10^{-2})^2}{4} \times V_2$

$\Rightarrow V_2 = 48$ m/sec.

49. A

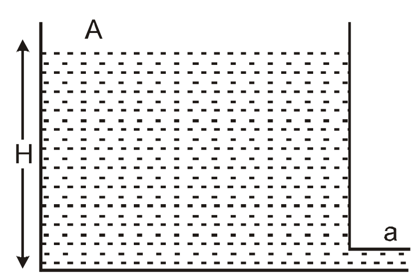
From Bernoulli's eq.

$P + \frac{1}{2}\rho V^2 + \rho gh = \text{constant}$

& $A_1V_1 = A_2V_2$

$A\downarrow\ V\uparrow\ P\downarrow$

50. D



$\frac{-dh}{dt} A = a\sqrt{2gh}$

$\int \frac{-dh}{\sqrt{2gh}} = \frac{a}{A}\int dt$

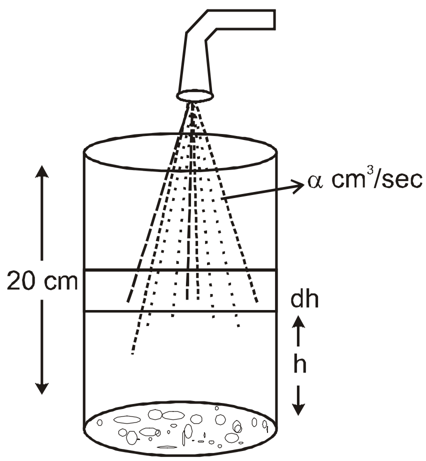
$$= \frac{-1}{\sqrt{2g}} \int_H^0 \frac{dh}{h} = \frac{a}{A} \int dt$$

$$= -\frac{1}{\sqrt{2g}} \frac{1}{2} [0 - \sqrt{H}] = \frac{a}{A} t$$

$t \propto \sqrt{H}$

51. D

Inlet = outlet

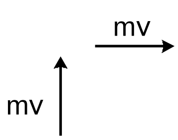


$\alpha \, dt = a\sqrt{2gh} \, dt$

$$h = \frac{\alpha^2}{2ga^2}$$

$$h = \frac{(100)^2}{2 \times (1000)(1)}$$

52. D

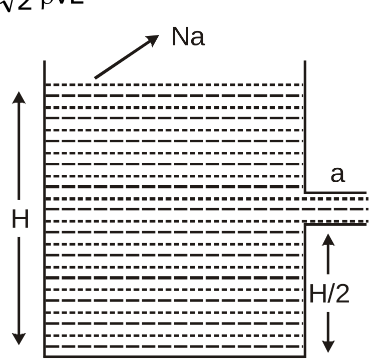


Force exerted by the water on the corner
= change in momentum in 1 sec

$= \sqrt{2}$ mv

$= \sqrt{2}\, \rho v L$

53. C



Force $= \rho a \left(\sqrt{2gh/2}\right)^2$

acceleration. $= \dfrac{\rho agh}{\rho Na.H}$

$= g/N$

54. A

From $A_P V_P = A_Q V_Q$

$$\frac{V_P}{V_Q} = \frac{A_Q}{A_P} = \frac{\pi(2 \times 10^{-2})^2}{\pi(1 \times 10^{-2})^2}$$

$V_P = 4V_Q$

55. B

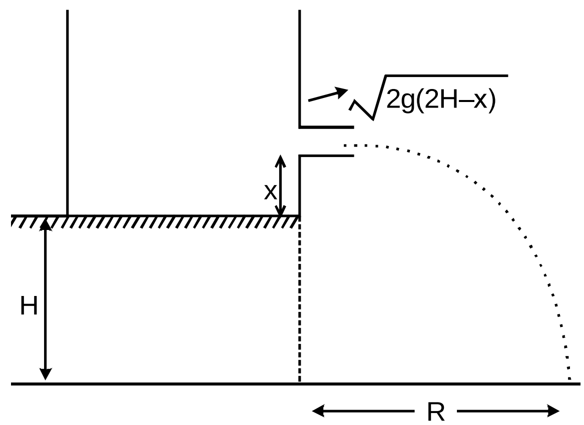
$\alpha dt = Av\, dt$

$\Rightarrow 10^{-4} = 10^{-4}\sqrt{2gh}$

$\Rightarrow h = \dfrac{1}{2g}$

h = 0.051 m

56. C



$$= \sqrt{2g(2H - x)} \sqrt{\frac{2(H + x)}{g}}$$

for max. $\dfrac{dR}{dx} = 0$

57. D

$$R = \sqrt{2g(H - h_1)}\sqrt{\frac{2h_1}{g}}$$

$$= \sqrt{2g(H - h_2)}\sqrt{\frac{2h_2}{g}}$$

$(H - h_1)\, h_1 = (H - h_2)\, h_2$

$H = h_1 + h_2$

For max. range $= \dfrac{H}{2}$

58. D

$$R = \sqrt{2g \times 10}\sqrt{\frac{2H}{g}} \quad .... (1)$$

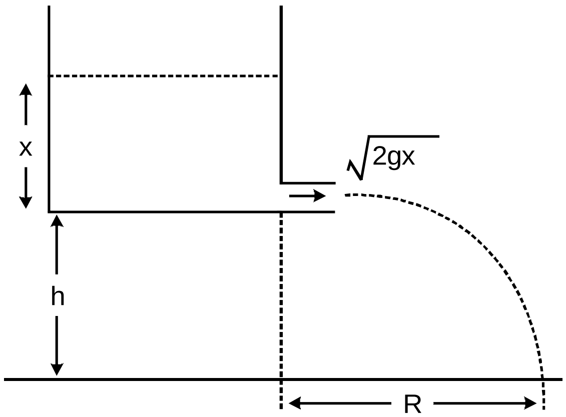
Now $\rho gh + P_O + P_E = P_O + \dfrac{1}{2}\rho V^2$

$\Rightarrow V^2 = 2gh + \frac{2P_E}{\rho}$

$\Rightarrow R' = \sqrt{(2g10) + (\frac{2P_E}{\rho})}\sqrt{\frac{2H}{g}}$ .....(2)

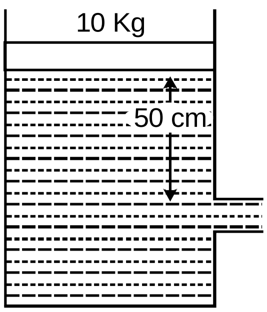
From (1) & (2) P_E = 3 atm.

59. B



$R = \sqrt{2gx}\sqrt{\frac{2h}{g}}$ $\qquad x = \frac{R^2}{4h}$

60. B



$P_0 + P_{10\,kg} + h\rho g \qquad = P_0 + \frac{1}{2}\rho V^2$

61. B

From $A_1V_1 = A_2V_2$

$(1)\ (V_1)\ (\frac{1}{2})V_2$

$\Rightarrow \frac{V_1}{V_2} = \frac{1}{2}$

$V_2 = 2V_1$

Now,

$V_2^2 = V_1^2 + 2gh$

$4V_1^2 = V_1^2 + 2(10)\left(\frac{10}{100}\right)$

$V_1 = \sqrt{\frac{2}{3}}$

Now volumetric rate of flow

$= A_1V_1$

$= \frac{1 \times 10^{-4}}{10^{-3}} \times \frac{60\sqrt{2}}{\sqrt{3}}$

= 4.9 lit/min.

62. A

Change in momentum is/sec.

$\sqrt{2}\,\rho A v^2 = 565.7$ N.

63. B

$\rho AV^2 = 1000 \times 2 \times 10^{-4} \times (10)^2$

= 20 N

64. D

$v_1 = \sqrt{2gh/2} = \sqrt{gh}$

for v_2

$\rho gh + 2\rho g\frac{h}{2} = \frac{1}{2}\,2\rho.v_2^2$

$2gh = v_2^2$

$v_2 = \sqrt{2gh}$

65. C

$A_1V_1 = A_2V_2$

$10^{-2} \times 2 = 0.5 \times 10^{-2} \times V^2$

$V_2 = 4$ m/sec.

$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$8000 + \frac{1}{2}\,1000 \times 2^2$

$= P_B + \frac{1}{2}\,1000 \times 4^2$

$P_B = 2000$ Pa

66. C

Energy required in one second is the power

$10^{-1} = A.V.$

$\Rightarrow 10^{-1} = 10^{-2} \times V$

$\Rightarrow V = 10$ m/sec.

$mgh + \frac{1}{2}mV^2 = P$

Here m = mass in one second

$P = \rho\,AVgh + \frac{1}{2}\rho AV^3$

$P = \rho AV[10 \times 10 + 50]$

= 15 Kwatt

67. A

68. A

$\mu\,mg = \frac{2}{2}\rho\pi\left(\frac{d}{2}\right)^2.2gH.$

$d = \sqrt{\frac{2\mu M}{\pi\rho H}}$

69. C

70. B

71. D

72. D

73. D

74. A

75. A