EXERCISE – III

SUBJECTIVE PROBLEMS

1. $t = \frac{d}{v_{avg}} = \frac{d}{\sqrt{\frac{8}{\pi}\frac{RT}{m_0}}}$

$= \frac{12800 \times 10^3}{\frac{8}{\pi} \times \frac{8.314 \times 300}{32 \times 10^{-3}}}$

$= 28.7236 \times 10^3$ sec

2. $mv_{avg} = m\sqrt{\frac{8}{\pi}\frac{RT}{M_0}}$

$= 664 \times 10^{-27}\sqrt{\frac{8 \times 8.314 \times 273}{\pi \times (4 \times 10^{-3})}}$

$= 8 \times 10^{-24}$ kgm/sec

3. $\sqrt{\frac{8RT_H}{\pi M_H}} = \sqrt{\frac{8RT_{He}}{\pi M_{He}}}$

$\frac{T_H}{T_{He}} = \frac{M_H}{M_{He}} = \frac{2}{4} = \frac{1}{2}$

4. $\frac{V_H}{V_N} = \sqrt{\frac{\frac{1}{M_H}}{\frac{1}{M_{N_2}}}} = \sqrt{\frac{\frac{1}{2}}{\frac{1}{28}}} = \sqrt{14}$

5. $\Delta u = n_0\frac{f_0}{2}RT + n_{Ar}\frac{f_{Ar}}{2}RT$

$= [2 \times 5 + 4 \times 3]\frac{RT}{2}$

$\Delta u = 11$ RT

6. V = constant $\Rightarrow \Delta W = 0$

$\Rightarrow \Delta Q = \Delta U = 12 = \frac{nf}{2}R(\Delta T)$

$\Rightarrow 24 = \left(\frac{0.04}{4}\right)3R(T - 100)$

$\Rightarrow \frac{800}{R} + 100 = T$

$T = 196.22^0$ C

7. $u = n\frac{f}{2}RT = \frac{f}{2}PV$

$\therefore$ PV = constant

so u=constant

8. $U_T = n\frac{f}{2}RT, f = 3, n = 1$

$U_T = 1 \times \frac{3}{2}R(0 + 273)$

$U_T = \frac{3 \times 273}{2}R = 3.40 \times 10^3 J$

9. VP^2 = constant

$\frac{VT^2}{V^2} = \text{constant}$

$\frac{T^2}{V} = \text{constant}$

$\frac{T_1^2}{V_1} = \frac{T_2^2}{V_2} \Rightarrow \frac{T^2}{V} = \frac{T_2^2}{2V}$

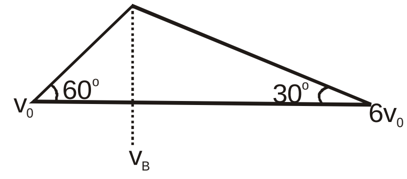
$T_2 = \sqrt{2}T$

10. $\frac{V_{rms1}}{V_{rms2}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{P_1 v_1}{P_2 v_2}}$

$\therefore$ n = constant

$= \sqrt{\frac{10^5 \times 2}{(2 \times 10^5) \times 2}} = \frac{1}{\sqrt{2}}$

11. $\frac{T_B}{T_A} = \frac{P_B V_B / nR}{P_A V_A / nR} = \frac{3P_0 V_B}{P_0 V_0}$



$\Rightarrow (V_B - V_0) \tan 60^0 = (6 V_0 - V_B) \tan 30°$

$\Rightarrow V_B = \frac{9}{4}V_0$

So $\frac{T_B}{T_A} = \frac{3\times 9}{4} = \frac{27}{4}$

12. $\rho_0 = \frac{m}{V_A} = \frac{m}{\left(\frac{nRT_0}{P}\right)}$ ....(1)

Now $\rho_B = \frac{m}{V_B} = \frac{m}{\left(\frac{nR(2T_0)}{3P}\right)}$ ......(2)

From (1) & (2) $\frac{\rho_B}{\rho_0} = \frac{1}{2/3} \Rightarrow \rho_B = \frac{3}{2}\rho_0$

13. $\frac{V}{T} = \frac{nR}{P}$ tan 53° = 4/3

$P = \frac{3}{4}nR = \frac{3}{4}\times 2\times 0.0821$ atm

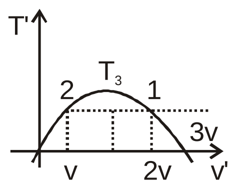
= 1.23×10^4 Pa

14. (i) $P_1 < P_2$ $T_2 > T_1$ for same v

(ii) $P' = \left(\frac{2P-P}{v-2v}\right)v' + 3P$

$\Rightarrow \frac{nRT'}{v'} = \frac{-P}{v}v' + 3P$

$T' = V'(3P - \frac{P}{V}V')$



15. Then $P_{air} + 75 = 76$

$P_{air} = 1$ cm Hg

Now $P_{air} \times [10\,A] = P'_{air}[(10+1)A]$

$\because$ (T = Constant)

$\Rightarrow P'_{air} = \frac{10}{11}P_{air}$

So Actual reading

$P_T = P'_{air} + 74 = \frac{10}{11}[1\text{ cm of Hg}] + 74\text{ Hg}$

$P_T = (74 + 0.9)$ cm Hg

16. $W = \int_{v_1}^{v_2} \frac{nR}{\sqrt{Vk}}dv = \frac{2nR}{\sqrt{K}}\frac{\left(\sqrt{v_2}-\sqrt{v_1}\right)}{1}$

$= 2nR(T_2 - T_1)$

$W = \int_{v_1}^{v_2} P dv$

$\because v = KT^2, v = K\left(\frac{Pv}{nR}\right)^2$

$P = \frac{nR}{\sqrt{vk}}$

= 2nR(60) = 120(1) R = 120 R

17. $\Delta Wg = P\Delta V$

$= 10^5\left(\frac{v}{2} - v\right)$

$= -10^5\frac{v}{2} = -\frac{10^5}{2}\times\left(\frac{nRT}{P}\right)$

$= -\frac{10^5}{2}\times\frac{1\times 8.314\times 360}{105}$

= – 180R J

= – 1496.52 J

So work done on gas $= -\Delta Wg$

= 1496.52 J

18. W $= P\Delta V$

$= (1\times 10^5)(1.091-1)\times(10^{-2})^3$

$= 9100\times 10^{-6}$ J = 0.0091 J

19. ΔW = Area

= (2P – P) (2v – v)

= Pv

20. ΔW = Area $= -\pi R^2$

$= -\pi\left(\frac{30-10^2}{2}\right)^2 (KP_a)(u_r)$

$= -100\,\pi J$

21. Area under the curve $= \Delta W$

$\Delta W = \frac{1}{2}[4P_1 - P_1]\times[3v_1 - v_1]$

$= 3P_1v_1$

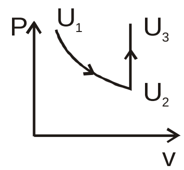
22. ΔW = Area of semi circle

$\frac{1}{2}\pi R^2 = \frac{1}{2}\pi(2-1)\left(\frac{3-1}{2}\right)$

$= \frac{\pi}{2}$ atm lt.

23. $U_2 - U_1 = -W$ $\therefore \Delta Q = 0$

W = 0 $\therefore$ V = constant



$\Delta Q = \Delta U = U_3 - U_2$

$\Rightarrow \Delta U = U_3 - U_2 = Q$

Now $(U_3 - U_2) + (U_2 - U_1) = U_3 - U_1$

$\Rightarrow Q + (-W) = U_3 - U_1$

$\Rightarrow U_3 = Q + U_1 - W$

$U_3 - U_1 = Q - W$

24. $\Delta U_{acb} = \Delta U_{adb}$

Now $\Delta U_{acb} = \Delta Q_{acb} - \Delta W_{acb}$

$\Delta U_{acb} = 200 - 80 = 120$ J

Now $\Delta U_{adb} + \Delta W_{adb} = \Delta Q_{adb}$

$\Delta W_{adb} = \Delta Q_{adb} - \Delta U_{adb}$

$= \Delta Q_{adb} - \Delta U_{acb}$

$\Delta W_{adb} = (144 - 120) = 24$ J

25. $\Delta Q = ms\Delta T = 2 \times 4200 \times 4 = 33600$ J

$\Delta W = Pa\Delta V = 10^5 \left(\frac{2}{1000} - \frac{2}{999.9}\right)$

$= -\frac{200}{9999} \cong -0.02$

$\Delta U = \Delta Q - \Delta W$

$= (133600 + 0.02)$ J

26. $\Delta W = \frac{1}{2} \times 100 \times 10^3 \times 200 \times 10^{-6}$

$= 10$ J

2.4 Cal $\times$ J $= 10$ J

$J = \frac{10}{2.4} \frac{J}{Cal} = \frac{25}{6} J/Cal$

27. $\Delta W = (0.05 - 0.02)\ m^3 \times (200\ K - 0)$ Pa

$= 6K\ Pa\ m^3 = 6000$ J

Now $\Delta Q = \Delta U + \Delta W$

2.625 J $= (5000 + 6000)$ J

$J = \frac{11000}{2625} = \frac{88}{21} J/Cal$

28. $\Delta U = \Delta Q - \Delta W$

$= \Delta Q - 0 \ \therefore (V \Rightarrow \text{Constant})$

$\Delta U = \Delta Q$

$= 100$ J

29. $\Delta W = 0 \quad \therefore V = \text{Const.}$

$\Delta Q = \Delta U = n_{He} \frac{f_{He}}{2} R\Delta T + n_{N_2} \frac{f_{N_2}}{2} R\Delta T$

$\Delta Q = \frac{R\Delta T}{2}\left(n_{He} f_{He} + n_{N_2} f_{N_2}\right)$ ...... (i)

Now $v = k\sqrt{T}$

$v \to 2v$ is $T \to 4T$

$\Rightarrow \Delta T = 4T - T = 3T$

So $\Delta Q = \frac{3RT}{2}[1 \times 3 + 1 \times 5]$

$= 12 \times 300$ R

$\Rightarrow \Delta Q = 3600$ R

30. (i) $\Delta Q_T = \Delta W_T$ in cyclic process

$[5960 + (-5585) + (-2980) + 3645]$

$= [2200 - 825 - 1100 + W_4]$

$\Rightarrow W_4 = 765$ J

31. (ii) $\eta = \frac{\Delta Wr}{\Delta Q(\text{only the})} \times 100$

$\left[\frac{2200 - 825 - 1100 + 765}{5960 + 3645}\right] \times 100$

$\eta = \frac{104000}{9605} = \frac{208}{1921} \times 100$

31. $Q = \frac{Q}{2} + \Delta U$

$\Rightarrow \Delta U = \frac{Q}{2} = \frac{3}{2} \eta R\Delta T$

$\Rightarrow Q = 3\eta R\Delta T$

$\Rightarrow nC_P\Delta T = 3\eta R\Delta T$

$C_P = 3R$

32. $\Delta W = \int Pdv$

$= \int_{v_1}^{v_2} kvdv$

$= \frac{kv_2^2 - kv_1^2}{2}$

$= \frac{p_2 v_2 - p_1 v_1}{2}$

$= \frac{nR\Delta T}{2}$

$nc_P\Delta T = nc_v\Delta T + nR\frac{\Delta T}{2}$

$\Rightarrow C_p = C_v + \frac{R}{2}$

33. dW = Pdv
dQ = dW + du
molar specific heat capacity zero
$\Rightarrow \Delta Q = 0$
So adiabatic
PV^r = Constant
Now $PV^{-b} = a$
$= -b = \gamma$
$\Rightarrow b = -\gamma$

34. $\frac{(n_1+n_2)C'_p\,\Delta T}{(n_1+n_2)C'_V\,\Delta T} = \frac{n_1C_{P_1}\Delta T + n_2C_{P_2}\Delta T}{n_1C_{V_1}\Delta T + n_2C_{V_2}\Delta T}$

$\frac{C'_P}{C'_V} = \frac{n_1\frac{\gamma}{\gamma-1}R + 2n_1\frac{\gamma}{\gamma-1}R}{n_1\frac{\gamma}{\gamma-1}R + 2n_1\frac{\gamma}{\gamma-1}R}$

$\frac{C'_P}{C'_V} = \gamma$

35. $(n_1 + n_2)\,C_V\Delta T = n_1C_{V_1}\Delta T + n_2C_{V_2}\Delta T$

$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2} = 2R$

Similerly $C_P = 3R$

$Y = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$

36. $\frac{(n_1+n_2)C_P\Delta T}{(n_1+n_2)C_V\Delta T} = \frac{n_1C_{P_1}\Delta T + n_2C_{P_2}\Delta T}{n_1C_{V_1}\Delta T + n_2C_{V_2}\Delta T}$

$\frac{C_P}{C_V} = \frac{\left(\frac{16}{4}\right)\left(\frac{3}{2}+1\right) + \frac{16}{32}\left(\frac{5}{2}+1\right)}{\frac{16}{4}\left(\frac{3}{2}\right) + \frac{16}{32}\left(\frac{5}{2}\right)}$

$\frac{C_P}{C_V} = \frac{\frac{5}{2} + \frac{7}{2 \times 8}}{\frac{3}{2} + \frac{1}{8} \times \frac{5}{2}} = \frac{47}{29}$

$\frac{C_P}{C_V} = \frac{47}{29}$

37. $\Delta Q = \Delta u + \Delta W = \frac{f}{2}nR\Delta T + nR\Delta T$

$\Delta Q = \left(1+\frac{f}{2}\right)Rn\Delta T \Rightarrow \frac{2500}{100} = \left(\frac{1+f}{2}\right)R$

$\Rightarrow f = 4$

$r = 1 + \frac{2}{f} = \frac{3}{2}$

38. $\Delta Q = \Delta U + \Delta W$
$CdT = C_V\,dT + PdV$ ....(1)
Now $T = T_0\,e^{\alpha v}$
$dT = T_0\,e^{\alpha v}\,\alpha dv$ ....(2)
(1) & (2)

$C = C_v + \frac{P}{T_0\alpha e^{\alpha v}}$

$= C_V + \frac{P}{\alpha T} \Rightarrow C = C_V + \frac{1}{\alpha}\frac{R}{V}$

39. $V = m\left(\frac{1}{T}\right)$

$\Rightarrow VT$ = Const. $\Rightarrow$ TdV + VdT = 0 ...(1)
$\Delta Q = \Delta V + \Delta W$
$CdT = C_V dT + PdV$ ....(2)
(1) & (2)

$CdT = C_V\,dT - \frac{P}{T}VdT$

$C = C_v - R = \frac{3}{2}R - R$

$\Rightarrow C = \frac{R}{2}$

40. $6300 = nC_v\Delta T$
$6300 = nC_v\,150$..........(1)
So $nC_v\,300 = \Delta U_f = 2 \times 6300$
$\Rightarrow \Delta U_f = 12600J$

41. Given

$70 = n\left(\frac{f}{2}+1\right)R\Delta T$

$70 = n\frac{f}{2}R\Delta T + nR\Delta T$

$\Delta Q_v = n\frac{f}{2}R\Delta T = 70 - 2R(45-40)$

$= 70 - 10\left(\frac{8.314}{4.2}\right)$

$\approx 50\,\text{cal}$

42. $\Delta Q = \Delta U + \Delta W$ and $nRT^3 = aP$

$$\frac{nR}{\gamma-1}dT + Pdv \qquad ...(1)$$

$Pv = nRT \Rightarrow PdV + VdP = nRdT$.....(2)

$$v = \frac{a}{T^2} \Rightarrow dv = \frac{-2a}{T^3}dT \qquad .....(3)$$

(1), (2) and (3)

$$dQ = \left(\frac{nR}{\gamma-1} - \frac{2aP}{T^3}\right)dT$$

$$= R\Delta T\,\frac{(3-2\gamma)}{\gamma-1}$$

43. $C_P = C_V + R$

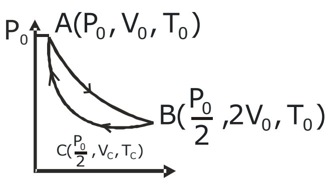
$(0.2M) = (0.15M) + R$ {for per mole}

$$m = \frac{R}{0.05} = \frac{2}{0.05} = 40gm$$

44. Adiabatic Process

$$P_{atm}V^r = P_{now}\left(\frac{V}{4}\right)^r$$

$$P_{new} = P_{atm}4^{3/2} = 8P_{atm}$$

45. 

For AC

$$P_0V_0^{\gamma} = \frac{P_0}{2}V_C^{\gamma} \Rightarrow V_C = 2^{1/\gamma}V_0$$

Now $T_0V_0^{\gamma-1} = T_C(2^{1/\gamma}V_0)^{\gamma-1}$

$$\Rightarrow T_C = T_0 2^{\frac{1-\gamma}{\gamma}}$$

$$\Delta W = \left(nRT_0 \ln\frac{2V_0}{V_0}\right) + \frac{P_0}{2}\left(2^{1/\gamma}V_0 - 2V_0\right)_{BC}$$

$$-\frac{nR}{\gamma-1}(T_0 - T_C)$$

$$\Rightarrow \Delta W = nRT_0 \ln 2 + \frac{nRT_0}{2}\left(2^{1/\gamma} - 2\right)$$

$$-\frac{nRT_0}{\gamma-1}\left(1 - 2^{-(\gamma-1)/\gamma}\right)$$

$$= nRT_0\left[\ln 2 + (2^{\frac{1}{\gamma}-1} - 1)\frac{-1-2^{\frac{-(\gamma-1)}{\gamma}}}{\gamma-1}\right]$$

Now $\Delta Q_{AB} = \Delta W_{AB}$ {$\because \Delta U_{AB} = 0$

$= nRT_0 \ln 2$

So $n = \dfrac{\Delta W}{\Delta Q_{AB}} = 1 - \dfrac{3(1 - \frac{1}{2^{1/3}})}{\ln 2}$

46. PV^m = const.

$$\Rightarrow V^m dP + mv^{m-1} Pd\,v = 0$$

$$\Rightarrow \frac{dP}{dV} = \frac{-mP}{V} = \tan(180 - 37^0)$$

$$\Rightarrow \frac{3}{4} = m\frac{2 \times 10^5}{4 \times 10^5} \Rightarrow m = 3/2$$

47. $C\Delta T = \Delta Q \le 2\Delta U = \dfrac{2f}{2}R\Delta T$

$C = 5R$

48. $\beta = -\dfrac{\Delta P}{\Delta V/V} = -v\dfrac{dP}{dV} \qquad .....(1)$

$VP^n = K$

$\Rightarrow dV.\,P^n + nP^{n-1}VdP = 0$

$\Rightarrow \dfrac{dP}{dV} = \dfrac{-P}{nV} \qquad .....(2)$

From 1 & 2

$$\beta = -V\left(\frac{-P}{nV}\right)$$

$$\beta = \frac{P}{V}$$

49. Free expansion

So $\Delta W = 0$ and $\Delta Q = 0$

So $\Delta u = 0$

$\Rightarrow$ T = 300 K