EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

1. – ve direction :

(a) $A = 4 \times 10^{-3}$

at t = 0, x = 0 , y = $2\sqrt{2} \times 10^{-3}$

$\therefore \sin\phi = \frac{1}{\sqrt{2}} \qquad \phi = \frac{\pi}{4}$

$\therefore \quad y = 4 \times 10^{-3} \qquad [\sin(\omega t + kx + \pi/4)]$

$\frac{\partial y}{\partial t} = v_w \frac{\partial y}{\partial x} \qquad \Rightarrow \quad v_\omega = \frac{20\pi \times 10^{-2}}{\tan\theta^\circ} \approx 6\,m/sec$

$\therefore \quad v_\omega = f_0\lambda$

$\Rightarrow 6 = f.(4 \times 10^{-2})$

$\Rightarrow 1.5 \times 10^2 = f$

$\therefore y = 4 \times 10^{-3} \sin\left[2\pi + 1.5\times10^{12}t + \frac{2\pi + 1.5\times10^2}{6}x + \frac{\pi}{4}\right]$

$= 4\times10^{-3}\sin 100\pi\left[3t + 0.5x + \frac{1}{400}\right]$

Power : $2\pi^2 A f^2 \mu v$

Energy : $2\pi^2 A^2 f^2 \mu v.T = 2\pi^2 A^2.\, f\mu v$

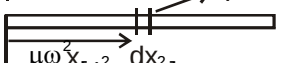
$= 2 \times (3.14)^2 \times (4 \times 10^{-3})^2 \times (1.5 \times 10^2) \times (50 + 10^{-3}) \times 6$

$= 144\ \pi^2 \quad \times 10^{-5}$ Joule.

2. $dT = -dm\ \omega^2 x$

$dT = -\mu\, dx\ \omega^2 x$

$\int_0^T dT = -\mu\omega^2 \int_x^{\ell} x\,dx \ \Rightarrow\ 0 - T_x = -\mu\omega^2 \ \frac{[\ell^2 - x^2]}{2}$



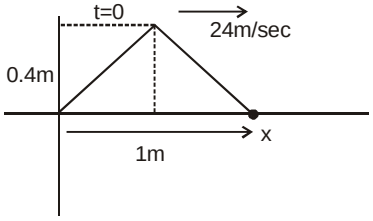
$T_x = \frac{\mu\omega^2}{2}[\ell^2 - x^2] \qquad \therefore$

$v_w = \sqrt{\frac{T}{\mu}} = \frac{\omega}{\sqrt{2}}\sqrt{(\ell^2 - x^2)}$

$\int_0^{\ell} \frac{dx}{\sqrt{\ell^2 - x^2}} = \int_0^t \frac{\omega}{\sqrt{2}}dt \ \Rightarrow\ \left[\sin^{-1}\left(\frac{x}{\ell}\right)\right]_0^{\ell}$

$\Rightarrow \frac{\pi}{2} = \frac{\omega_0}{\sqrt{2}}t \qquad \Rightarrow \qquad t = \frac{\pi}{\sqrt{2}\omega}$

3. $\frac{\partial y}{\partial t} = -v_w \frac{\partial y}{\partial x}$



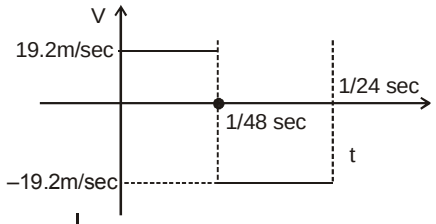
$\frac{\partial y}{\partial t} = -(24)\left(\frac{0.4}{0.5}\right) = -19.2\ m/s$

$\lambda = 1m$

$k = \frac{2\pi}{\lambda} = 2\pi$

$\omega = v_w k = 24 \times 2\,\pi = 48\,\pi$

$T = \frac{2\pi}{48\pi} = \frac{1}{24}sec.$



4. $\frac{A_1 + A_2}{A_1 - A_2} = 1.5$ $\qquad P_i = P_r + P_t$

$\frac{A_i + A_r}{A_i - A_r} = 1.5$ $\qquad P_t = P_i - P_r$

$5\,A_r = A_i$ $\qquad \frac{P_t}{P_i} = \left(1 - \frac{P_r}{P_i}\right) = 1 - \left(\frac{A_r}{A_i}\right)^2$

$\frac{A_r}{A_i} = \frac{1}{5}$ $\qquad = 1 - \frac{1}{25} = \frac{24}{25}$

$\frac{P_t}{P_i}\% = 96\%$

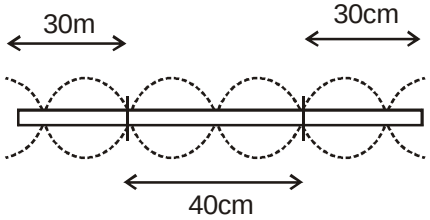
5. $v = \sqrt{\frac{T}{2.5\times10^{-2}}}, \qquad f = \frac{1}{0.25}\sqrt{\frac{T}{2.5\times10^{-2}}}$

On $T\downarrow \quad f\downarrow$

$\therefore$ f of string is 208

$208 = 4\sqrt{\frac{T}{(2.5\times10^{-2})}} \Rightarrow 52 = \frac{10}{5}\sqrt{10\,T} \quad \Rightarrow \ 26 = \sqrt{10\,T} \quad \Rightarrow \ T = 67.6\ N$

6. $\lambda = 0.4$ m



$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{144\times10^9}{9000}}$

$V = 4 \times 10^3$ m/sec

$f = \frac{4\times10^3}{0.4} = 10^4\ Hz = 10\ KHz$