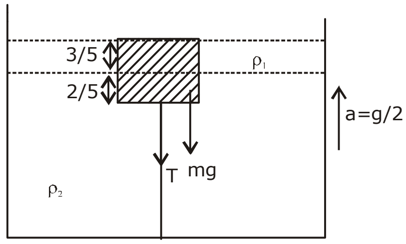
EXERCISE – IV

TOUGH SUBJECTIVE PROBLEMS

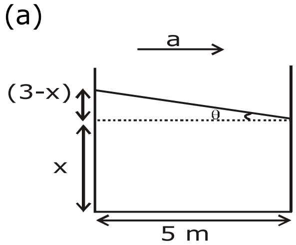
1.



$$T+mg = \left[\left(\frac{2}{5}x10^{-3}\right)x\left(1500x\frac{3g}{2}\right)\right]$$

$$+\left[\left(\frac{3}{5}x10^{-3}\right)x\left(1000x\frac{3g}{2}\right)\right] \qquad T = 6\text{ N}$$

2. (a)

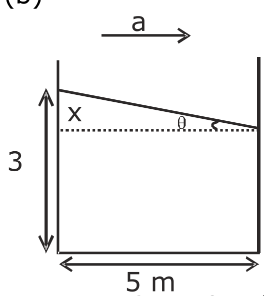


$$2\times4\times5=(5\times x\times4)+\frac{1}{2}\times5(3-x)\times4$$

$$\Rightarrow x=1\,m \qquad \tan\theta=\frac{a}{g}=\frac{3-x}{5}$$

$$\frac{a}{10}=\frac{2}{5}\Rightarrow a=4\,m/sec^2$$

(b)



a = 4 x 1.2= 4.8 m/sec^2

$$\tan\theta=\frac{a}{g}=\frac{x}{5} \qquad x=2.4\,m$$

Water split out

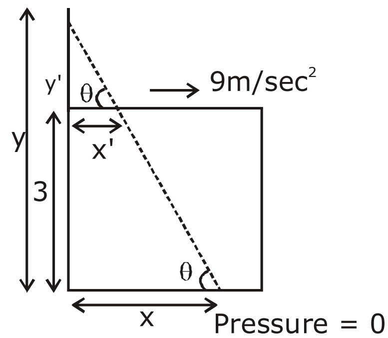
$$=(2\times4\times5)-(0.6\times5.4)-\frac{1}{2}\times2.4\times4\times5$$

= 40 - 12 - 24 = 40 - 36 = 4m^3

Inital = 2 x 4 x 5 = 40m^3

$$\% = \frac{4}{40}x100=10\,\%$$

(c)



$$\tan\theta = \frac{9}{10}=\frac{y'}{x'} = 0.9=y'/x'$$

$$\tan\theta=\frac{9}{10}=\frac{y}{x}=\frac{y'+3}{x}$$

y' = 0.9x - 3

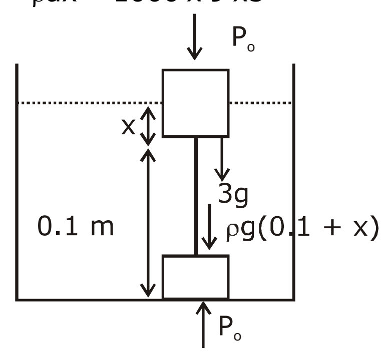
$$40 = \frac{1}{2}(y'+3)xXx4-\frac{1}{2}y'x4x\frac{y'}{0.9}$$

x = 5

Pressure at rear wall

= ρax = 1000 x 9 x5 = 45 KPa

3.



$$\rho g/(0.1+X)\pi(0.05)^2+3g=\rho g\pi(0.1)^2x$$

x = 0.16

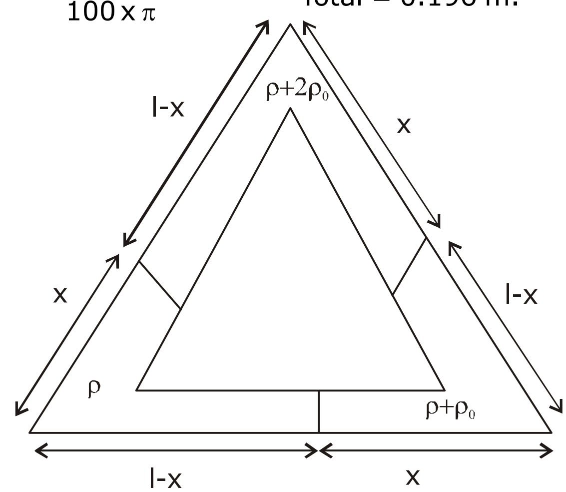
h= 0.1 + 0.16 = 0.26 m

(b)

30 = $\pi(0.1)^2$ x $\times$ 1000 $\times$ 10

$$x = \frac{30}{100x\pi}=0.096\,m$$ Total = 0.196 m.

4.



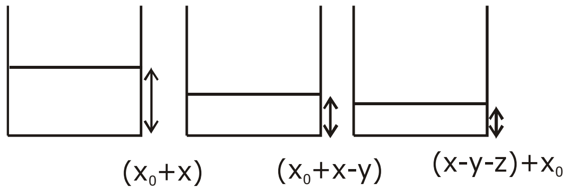
$\rho x \sin 60^0 g + (\rho+2\rho_0)(l-x)$

$\sin 60^0 = (\rho+2\rho_0) x \sin 60^0 g + (\rho+\rho_0)(l-x)\sin 60^0 g$

$$\rho x + (\rho + 2\rho_o)(l - x) = (\rho + 2\rho_o)x + (\rho + \rho_o)(l - x)$$

$x = l/3$

5. Initial depth in sea water = x_0



Water

$W_S + W_C = x_0 A\rho_{sw} g$ ....(1)

$W_S + W_C = (x_o + x)A\rho_w g$ ....(2)

$W_S = (x_o + x - y)A\rho_w g$ ....(3)

$W_S = (x_o + x - y - z)A\rho_{sw} g$ ....(4)

(1)/(2)

$$1 = \frac{\rho_{SW}}{\rho_W}\cdot\frac{x_o}{x_o + x},$$

$$x_o\left[\frac{\rho_{SW} - \rho_W}{\rho_W}\right] = x \Rightarrow x_o = \frac{\rho_W x}{\rho_{SW} + \rho_W}$$

(3)/(4) $$1 = \frac{x_o + x - y}{x_o + x - y - z}\frac{\rho_W}{\rho_{SW}}$$

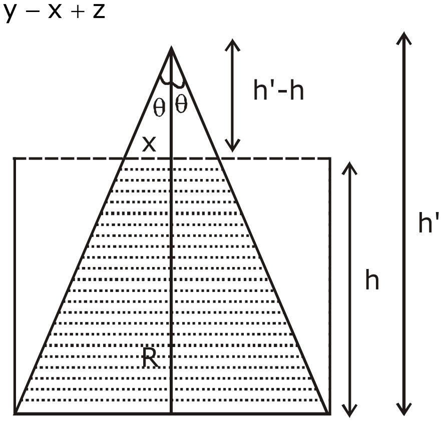
$$x_0 + x - y - z = \frac{\rho_W}{\rho_{SW}}x_o + (x - y)\frac{\rho_W}{\rho_{SW}}$$

$$x_0\left[\frac{\rho_{SW} - \rho_W}{\rho_{SW}}\right] = (x - y)\frac{\rho_W}{\rho_{SW}} - x + y + z$$

$$\frac{\rho_W}{\rho_{SW}}x = (x - y)\frac{\rho_W}{\rho_{SW}} - x + y + z$$

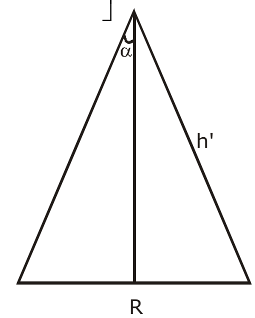
$$\frac{\rho_{SW}}{\rho_W} = \frac{y}{y - x + z}$$

6.



Upward force excuted by the liquid on the flask

$$\rho g\left[\pi R^2 h - \{\frac{1}{3}\pi R^2 h' - \frac{1}{3}\pi x^2(h' - h)\}\right] = W$$



$$\tan\alpha = \frac{R}{h'} \Rightarrow h' = R\cot\alpha$$

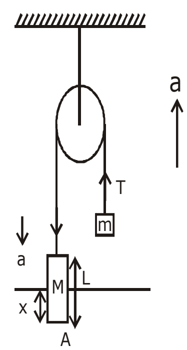
$$\frac{R}{x} = \frac{h'}{h' - h} \Rightarrow R(R\cot\alpha - h) = xR\cot\alpha$$

$$x = \frac{R\cot\alpha - h}{\cot\alpha}$$

$$w = \rho g\left[\pi R^2 h - \{\frac{1}{3}\pi R^2 R\cot\alpha - \frac{1}{3}\pi\left(\frac{R\cot\alpha - h}{\cot\alpha}\right)^2 (R\cot\alpha - h)\}\right]$$

$$\rho = \frac{W}{\pi g h^2 \tan\alpha(R - \frac{1}{3}h\tan\alpha)}$$

7.



$$\rho = \frac{M}{LA} \Rightarrow A = \frac{M}{\rho L}$$

$Mg - T - Ax\,\rho_w g = Ma$ ...(1)

$T - mg = ma$ ...(2)

$Mg - mg - \frac{M}{\rho L}\cdot\rho_w x g$

$= (M+m)\,a$

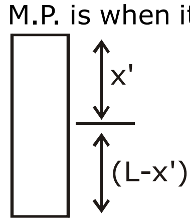
$$(M - m) - \frac{M}{L}\frac{\rho_w}{\rho}xg = (M + m)a$$

$$a = \left(\frac{M - m}{M + m}\right)g - \left(\frac{M - m}{M + m}\right)\frac{gx}{L}$$

(b)

a = 0 initial x = L

M.P. is when it is just submerged.



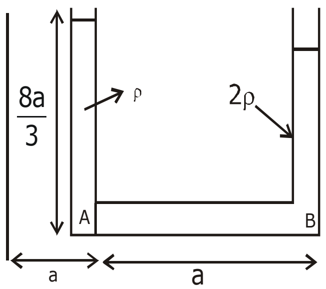
$$a = \left(\frac{M-m}{M+m}\right)g\left[1 - \frac{L-x'}{L}\right]$$

$$= \left(\frac{M-m}{M+m}\right)\frac{gx'}{L}$$

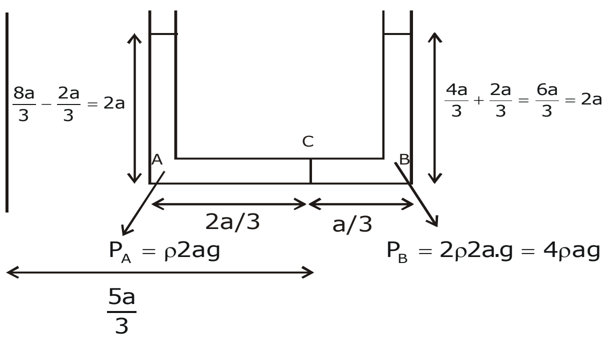
$$\omega^2 = \left(\frac{M-m}{M+m}\right)\frac{g}{L} \qquad \omega = \sqrt{\left(\frac{M-m}{M+m}\right)\frac{g}{L}}$$

$$T = 2\pi\sqrt{\left(\frac{M+m}{M-m}\right)\frac{L}{g}} \qquad \frac{T}{4} = \frac{\pi}{2}\sqrt{\left(\frac{M+m}{M-m}\right)\frac{L}{g}}$$

8.



$$\rho g x\frac{8a}{3} = 2\rho.xg \qquad x = \frac{4a}{3}$$



$$(P_C - P_A)S = \rho s\omega^2\int_a^{5a/3} x\,dx = \frac{\rho\omega^2}{2}\left[\frac{25a^2}{9} - a^2\right]$$

$$= \frac{8}{9}\rho s\omega^2 a^2 ....(1)$$

$$(P_B - P_C)s = 2\rho s\omega^2\int_{5a/3}^{2a} x\,dx$$

$$= \frac{2\rho s\omega^2}{2}\left[4a^2 - \frac{25a^2}{9}\right] = \rho s\omega^2 a^2 x\frac{11}{9}..(2)$$

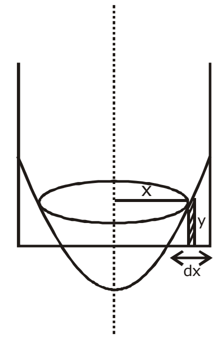
1 + 2

$$(P_B - P_A)S = \rho s\omega^2 a^2\left[\frac{8}{9} + \frac{11}{9}\right]$$

$$4\rho ag - 2\rho ag = \rho\omega^2 a^2 x\frac{19}{9}$$

$$2g = \omega^2 a\frac{19}{9} \Rightarrow \omega = \sqrt{\frac{18g}{19a}}$$

9.



$$y = \frac{\omega^2x^2}{2g} = \frac{(20)^2x^2}{2\times10} = 20x^2$$

Volume = $y dx\, 2\pi x$

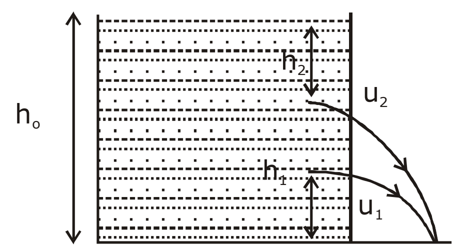
$1.5 = 20x^2.2\pi x dx \qquad 1.5 = 40\pi x^3 dx$

$$1.5 = 40\pi\int_x^{0.5} x^3 dx \qquad 1.5 = \frac{40\pi}{4}\left[(0.5)^4 - x^4\right]$$

$$x = (0.0155)^{\frac{1}{4}}$$

Area = $\pi x^2 = 0.39 m^2$.

10.



$$u_1 = \sqrt{2g(h_0 - h_1)} \qquad u_2 = \sqrt{2gh_2}$$

$$u_1\sqrt{\frac{2h_1}{g}} = u_2\sqrt{\frac{2(h_o - h_2)}{g}}$$

$$u_1^2 h_1 = u_2^2(h_0 - h_2)$$

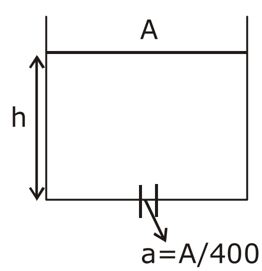
$$2g(h_o - h_1)h_1 = 2gh_2(h_o - h_2)$$

$$h_o(h_1 - h_2) = h_1^2 - h_2^2$$

$$h_o = h_1 + h_2 \qquad h_1 - h_2 = 0$$

$$h_1 = h_2$$

11.



$\rho g h = \frac{1}{2}\rho x^2 \qquad v = \sqrt{2gh}$

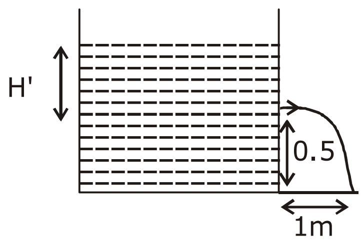
(a) $-Adh = \frac{A}{400} vdt \qquad \int_1^0 \frac{-dh}{\sqrt{2gh}} = \int_0^t \frac{dt}{400}$

$t = 80\sqrt{5}$ sec.

(b) $v = \sqrt{2gh} = \sqrt{2x10x1} = \sqrt{20}\ m/sec$

$1 x A = \sqrt{20} tx \frac{A}{400} \qquad t = 40\sqrt{5}$ sec.

12.



Range $V\sqrt{\frac{2x0.5}{g}} = 1$

$V = \sqrt{10} = \sqrt{2gH'} \qquad H' = \frac{1}{2} = 0.5m$

$-Adh = avdt \qquad \int_{0.81}^{0.5} \frac{-dh}{\sqrt{2gh}} = \int_0^t \frac{a}{A}.dt$

$\frac{2}{\sqrt{20}}\left[\sqrt{0.81} - \sqrt{0.5}\right] = \frac{1x10^{-4}}{0.5} t$

t = 431 sec.

13. (i) $\frac{1}{2}\rho v^2 = (600 \times 10 \times 0.6) + (900 \times 10 \times 0.4)$

$\frac{1}{2} 900 v^2 = 3600 + 3600 = 7200$

$v \approx 4m/sec.$

(ii) $F = (\rho av)v = \rho av^2$

$= 900 \times 5 \times 10^{-4} \times (4)^2 = 7.2 N$

(iii)

$F_{min} = 0$

$\mu mg = 0.01\left[900 \times 0.6 \times 0.5 + 600 \times 0.6 \times 0.5\right] \times 10$

= 45 N

F_{max} = 45 + 7.2 = 52.2 N.

(iv)

$A_1 v_1 = A_2 v_2 \qquad 0.5 v_1 = 5 \times 10^{-4} \times 4$

$v_1 = 4 \times 10^{-3}$ m/sec for both

15. $5 = (0.8) \times 10^3 \times (0.1)^2 x$

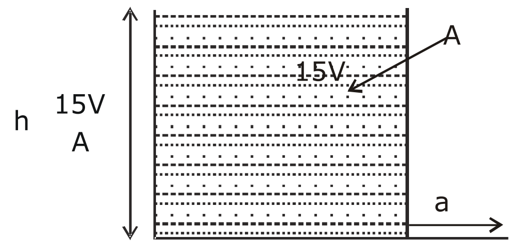
$x = 6.25 cm$

$v_i = v_f$

$15 \times 15 \times 8 = 15 \times 15 \times H - 10 \times 10 \times 6.25$

H = 10.7 cm = $\frac{388}{36}$ cm

16. One glass of juice = Vm^3



-adh = $a\sqrt{2gh}$ dt

14V/A

$-\int \frac{dh}{\sqrt{2gh}} = \int \frac{a}{A} dt\ cm$

$\frac{15V}{A}$

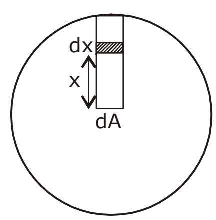
$\frac{2}{\sqrt{2g}}\left[\sqrt{\frac{15V}{A}} - \sqrt{\frac{14V}{A}}\right] = \frac{a}{A} x 12....(1)$

$\frac{2}{\sqrt{2g}}\left[\sqrt{\frac{14V}{A}} - 0\right] = \frac{a}{A} t....(2)$

(2)/(1)

$\frac{t}{12} = \frac{\sqrt{14}}{\sqrt{15} - \sqrt{14}} \Rightarrow \frac{12\sqrt{14}}{\sqrt{15} - \sqrt{14}}$

17.



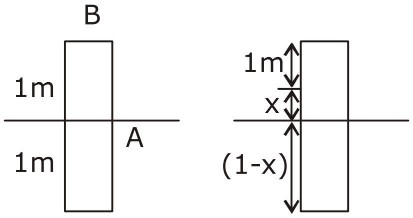
$g_{eff} = \frac{GMr}{R^3} \qquad r = x$

$dF = (\rho dA.dx)\frac{gx}{R} \qquad F = \rho dA \frac{g}{R}\int_0^R x dx$

$F = \frac{\rho g}{R}.\frac{R^2}{2} dA = \frac{\rho g R}{2} dA$

Pressure = $\frac{\rho g R}{2}$

18. (a)



$$F_{net} = (1-x)\rho Ag - \frac{2A\rho}{2}g$$

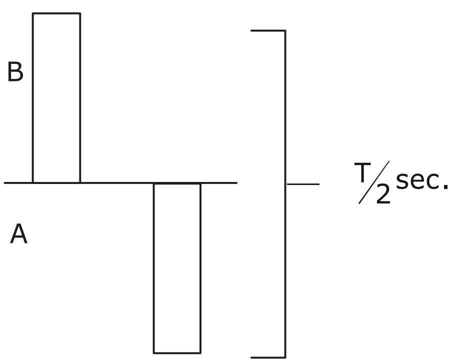
$= -\rho Agx$

$K_{eff} = \rho Ag$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2\times A\times\rho/2}{A\rho g}} = 2\pi\sqrt{\frac{1}{g}}$$

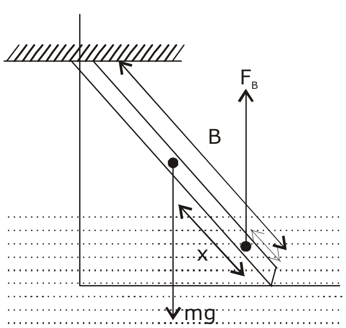
$$= T = 2\pi\sqrt{\frac{1}{\pi^2}} \approx 2\,\text{sec.}$$

(b)



$\Rightarrow 1\,\text{sec.}$

19.



$$(\rho_R Abg)\frac{b}{2}\sin\theta = \rho_W Axg.(b - \frac{x}{2})\sin\theta$$

$$\frac{5}{9}.\frac{b^2}{2} = x\left(b - \frac{x}{2}\right) \Rightarrow \frac{5b^2}{18} = bx - \frac{x^2}{2}$$

x= b/3

20. (a)
cylinder floats so
mg = Buoyancy force

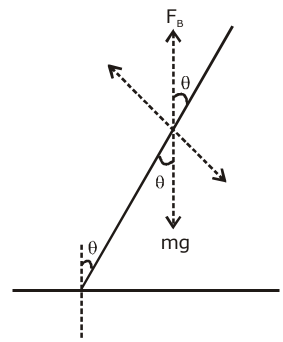
$$\left(\frac{A}{5}L\rho_c\right)g = \left(\frac{3L}{4}.\frac{A}{5}\right)dg + \left(\frac{L}{4}.\frac{A}{5}\right)2dg$$

$$\rho_c = \frac{3d}{4} + \frac{d}{2} = \frac{5}{4}d$$

(ii) B
x = v (time of flight)

$$= \sqrt{(3H-4h)\frac{g}{2}}\sqrt{\frac{2h}{g}} = \sqrt{h(3H-4h)}$$

21.



$$\tau_{net} = (F_B - mg)\sin\theta.\frac{L}{2}$$

if θ is small.

$$\tau_{net} = (F_B - mg)\theta.\frac{L}{2}$$

$F_B = SLd_2g$

$mg = SLd_1g \qquad \Rightarrow \tau_{net} = (d_2 - d_1)SLg\theta.\frac{1}{2}$

$$\alpha = \frac{(d_2-d_1)SLg(3)}{mL^2}.\frac{\theta L}{2}$$

$$\alpha = \frac{3g(d_2-d_1)}{d_1L^2}.\frac{\theta L}{2} \qquad \omega = \sqrt{\frac{3g(d_2-d_1)}{2d_1L}}$$

23. from $A_1V_1 = A_2V_2$

$(4\text{x}10^{-3})(1) = (8\text{x}10^{-3})V_2 \qquad V_2 = \frac{1}{2}\,m/s$

from Bernoulli's

$$P_p + \rho gh_p + \frac{1}{2}\rho V_P^2 = P_Q + \rho gh_Q + \frac{1}{2}\rho V_Q^2$$

$$= 10^3\text{x}10\,(2-5) + \frac{1}{2}\text{x}10^3(1 - \frac{1}{4})$$

= - 30000+375

$\therefore P_Q - P_P = -29625$