1. $N_1 = N_0 e^{-\lambda t}$, $N_2 = N_0 e^{-\lambda_2 t}$

$$\frac{N_1}{N_2} = \frac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}} = e^{-(\lambda_1-\lambda_2)t} = e^{-(10\lambda-\lambda)t} = e^{-9\lambda t}$$

Given $\frac{N_1}{N_2} = e^{-1}$

Hence – $9\lambda t$ = – 1 or t = $\frac{1}{9\lambda}$

2. Beta particle in an electron which is emitted from a nucleus when a neutron decays into a proton & an electron within a nucleus. Hence the correct choice is (c).

3. Total number of undecayed atoms will continuously decrease with time.

4. The reactor produces 1000 MW power or 10^9 W power of 10^9 J/S of power. The reactor is to function for 10 years. Therefore, total energy which the reactor will supply in 10 years is
E = (power) (time)
= (10^9 J/s) (10 × 365 × 24 × 3600 s) = 3.1536 × 10^{17} J
But since the efficiency of the reactor is only 10%, therefore actual energy needed is 10 times of it or 3.1536 × 10^{18} J. One Uranium atom liberates atom liberates 200 MeV of energy of 200 × 1.6 × 10^{-13} J or 3.2 × 10^{-11} J or energy. So number of Kg-moles of uranium atoms needed are

$$= \frac{3.1536\times10^{18}}{3.2\times10^{-11}} = 0.9855 \times 10^{29}$$

or number of Kg-moles of uranium needed are

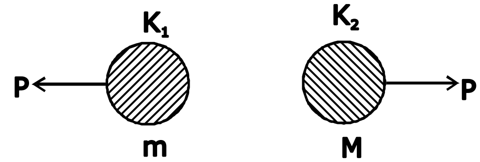
$$n = \frac{0.9855\times10^{29}}{6.02\times10^{26}} = 163.7$$

Hence total mass of uranium required is
m = (n) M = (163.7) (235) Kg
or m ≈ 38470 Kg
or m = 3.847 × 10^4 kg

5. (i) Give mass of α - particle, m = 4.002 a.m.u and mass of daughter nucleus.
M = 223.610 a.m.u. de - Broglie wavelength of α - particle.
λ = 5.76 × 10^{-15} m
So momentum of α - particle would be

$$P = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{5.76\times10^{-15}} \text{Kg}-\text{m/s}$$ or P = 1.151 × 10^{-19} Kg-m/s ...(i)

From law of conservation of linear momentum, this should also be equal to the linear momentum of the daughter nucleus (in opposite direction).



Let K_1 and K_2 be the kinetic energies of a – particle and daughter nucleus.
Then total kinetic energy in the final state is

$$K = K_1 + K_2 = \frac{P^2}{2m} + \frac{P^2}{2M}$$

$$= \frac{P^2}{2}\left(\frac{1}{m}+\frac{1}{M}\right) \Rightarrow K = \frac{P^2}{2}\left(\frac{M+m}{Mm}\right)$$

1 a.m.u. = 1.67 × 10^{-27} Kg
Substituting the values, we get

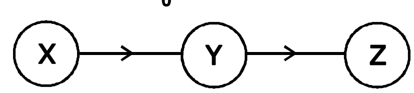
$$K = \frac{(1.151\times10^{-19})^2}{2} + \frac{(4.002+223.6)(1.67\times10^{-27})}{(4.002\times1.67\times10^{-27})(223.61\times1.67\times10^{-27})}$$

or K = 10^{-12} J.

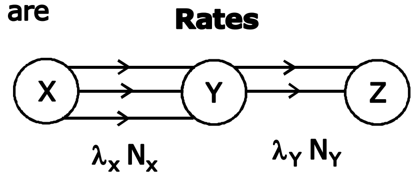
$$K = \frac{10^{-12}}{1.6\times10^{-13}} \text{ MeV} = 6.25$$

(ii) Mass defect, $\Delta m = \frac{6.25}{931.470}$ a.m.u. = 0.0067 a.m.u
Therefore, mass of parent nucleus = mass of α - particle + mass of daughter nucleus + mass defect (Δm)
= (4.002 + 223.610 + 0.0067) a.m.u
= 227.62 a.m.u.
Hence mass of parent nucleus is 227.62 a.m.u.

6. Let at time t = t, number of nuclei of Y and Z and N_Y and N_Z.
Then -
t = 0 $N_0 = 10^{20}$ Zero Zero



t = t $N_X = N_0 e^{-\lambda_X t}$ N_Y N_Z
Rate equation of the populations of X, Y and Z are



$$\left(\frac{dN_Y}{dt}\right) = -\lambda_x N_x - \lambda_y N_y$$

$$\left(\frac{dN_x}{dt}\right) = -\lambda_x N_x \qquad ...(1)$$

...(2)

and $\left(\frac{dN_Z}{dt}\right) = \lambda_y N_y$ ...(3)

(ii) Given $N_Y(t) = \frac{N_0\lambda_x}{\lambda_x - \lambda_y}\left[e^{-\lambda_y t} - e^{-\lambda_x t}\right]$

For N_Y to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

i.e., $\lambda_X N_X = \lambda_Y N_Y$ ...(4)

(From equation 2)

or $\lambda_x(N_0 e^{-\lambda_x t}) = \lambda_x \frac{N_0\lambda_x}{\lambda_x - \lambda_y}\left[e^{-\lambda_y t} - e^{-\lambda_x t}\right]$

or $\frac{\lambda_x - \lambda_y}{\lambda_y} = \frac{e^{-\lambda_y t}}{e^{-\lambda_x t}} - 1$

or $\frac{\lambda_x}{\lambda_y} = e^{(\lambda_x - \lambda_y)t}$

or $(\lambda_x - \lambda_y)t\ln(e) = \ln\left(\frac{\lambda_x}{\lambda_y}\right)$

Substituting the values of λ_x and λ_y, we have

$$t = \frac{1}{(0.1 - 1/30)}\ln\left(\frac{0.1}{1/30}\right) = 15\ln(3)$$

or t = 16.48 s

(iii) The population of X at this moment

$$N_X = N_0 e^{-\lambda_x t}$$

$= (10^{20})e^{-(0.1)(16.48)}$

$N_X = 1.92 \times 10^{19}$

$N_Y = \frac{N_X\lambda_X}{\lambda_Y}$ (From equation 4)

$= (1.92 \times 10^{19})\frac{(0.1)}{(1/30)} = 5.76 \times 10^{19}$

$\therefore$ $N_Z = N_0 - N_X - N_Y$ or $N_Z = 2.32 \times 10^{19}$

7. Since $\frac{1}{16} = \frac{1}{2^4}$, it follows that the time taken for the radioactivity to decay to $\frac{1}{16}$ of its initial value.

= four times the half – life of the sample

$= 4\ t_{1/2} = 4 \times 100 = 400\ \mu s$

8. During the emission of a gamma radiation, both the mass no. & atomic no. remain the same. Hence the answer is C

9. If m_p = mass of proton

& A = atomic no. of uranium

then the mass of uranium nucleus is

$m = m_p A$

& the volume of uranium nucleus is

$$v = 4/3\pi r^3 = 4/3\pi(r_0 A^{1/3})^3 = 4/3\pi r_0^3 A$$

$$\frac{m}{v} = \frac{m_p A}{\frac{4}{3}\pi r_0^3 A} = \frac{3m_p}{4\pi r_0^3}$$ Thus $m \propto V$

10. $K.E = \frac{(\text{momentum})^2}{2 \times \text{mass}}$

mass no. of $\propto$ particle = 4 units

mass no. of daughter nucleus = 220 – 4 = 216

If P & p $\rightarrow$ denote the momenta of daughter nucleus, then

$$Q = \frac{P^2}{2M} + \frac{p^2}{2m}$$

Since momentum is conserved

$$Q = \frac{P^2}{2}\left(\frac{1}{M} + \frac{1}{m}\right) = \frac{P^2}{2m}\left(\frac{m}{M} + 1\right)$$

Now $\frac{P^2}{2m}$ = K.E. of particle – α = E_α

$Q = E_\alpha\left(\frac{m+M}{m}\right)$ or $E_\alpha = \frac{QM}{(m+M)}$

11. Let n_0 be the number of radioactive nuclei at time t = 0. Number of nuclei decayed in time t are given $n_0(1 - e^{-2\lambda})$, which is also equal to the number of betal particles emitted the same interval of time. For the given condition,

$$n = n_0(1 - e^{-2\lambda}) \qquad ..(i)$$

$$(n + 0.75n) = n_0(1 - e^{-4\lambda}) \qquad ..(ii)$$

Dividing (ii) by (i) we get

$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$ or $1.75 - 1.75\ e^{-2\lambda} = 1 - e^{-4\lambda}$

$$\therefore\ 1.75 e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \qquad ...(iii)$$

Let us take $e^{-2\lambda} = x$

Then the above equation is,

$$x^2 - 1.75x + 0.75 = 0$$

or $x = \frac{1/75\ \sqrt{(1.75)^2 - (4)(0.75)}}{2}$ or x = 1

and $\frac{3}{4}$

$\therefore$ From equation (iii) either

$e^{-2\lambda} = 1$ or $e^{-2\lambda} = \frac{3}{4}$

but $e^{-2\lambda} = 1$ is not accepted because which means λ = 0. Hence

$$e^{-2\lambda} = \frac{3}{4}$$

or $-2\lambda \ln(e) = \ln(3) - \ln(4) = \ln(3) - 2\ln(2)$

$\therefore \lambda = \ln(2) - \frac{1}{2}\ln(3)$

Substituting the given values,

$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395\ s^{-1}$

$\therefore$ Mean life $t_{means} = \frac{1}{\lambda} = 6.947$ sec

12. $\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$, where A = Activity, n = number of half lives.

13. $\frac{0.3010}{T}$ **=** $\frac{1}{t}\log\frac{a}{a_0}$ a = Number of atoms

= 0.259

14. $4(_2He^4) = {}_8O^{16}$

Mass defect

Δm = {4(4.0026) – 15.834} = 0.011 amu.

Energy released per oxygen nuclei = (0.011) (931.48) MeV = 10.24 MeV

15. B

16. After two half lives $1/4^{th}$ fraction of nuclei will remain undecayed. or $3/4^{th}$ will decay.

Hence the propability that a nucleus decays in two half lives is 3/4.

17. (A) → P, Q ; (B) → P, R ; (C) → S, P ; (D) → P, Q, R

18. A

Rest mass of parent nuclus should be greater than the rest mass of daughter nuclei

thus (A)

19. The series in U- V region is lymen series. Longest wavelength corresponds to minimum energy which occurs in transition from n = 2 to n = 1.

$$122 = \frac{1/R}{(1/1^2 - 1/2^2)} \quad ..(i)$$

The smallest wavelength in the infrared region corresponds to max. energy of Paschen series.

$$\lambda = \frac{1/R}{(1/32 - 1/\infty)} \quad ..(ii)$$

from (i) & (ii)

λ = 823 nm

20. (A) → P,R ; (B) → Q,S ; (C) → P ; (D) → Q

21. B,D

In fusion two or more lighter nuclei combine to make a comparability heavier nucleus.

In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei further, energy will be released in a nuclear process if total binding energy increases.

22. A

$$5\mu Ci = \frac{\ln 2}{T_1}(2N_0) \Rightarrow 10\mu Ci = \frac{\ln 2}{T_2}(N_0)$$

Dividing we get $T_1 = 4T_2$

23. D

The high temperature maintained inside the reactor core

24. A

$$2 \times 15KT = \frac{Ke^2}{r} \Rightarrow T \approx 1 \times 10^9$$

25. B

deuteron density = $8.0 \times 10^{14}\ cm^{-3}$, confinement time = 9.0×10^{-1} s

26.

$$\ell n\left(\frac{dN}{dt}\right) = \ell n\lambda N_0 - \lambda t$$

By Graph $\lambda = \frac{1}{2}$ $\therefore$ T = $nt_{1/2}$

$4.16 = n \times \frac{0.693}{\lambda}$

n = 3

$N = \frac{N_0}{P} = \frac{N_0}{2^n} \Rightarrow P = 2^3 \Rightarrow P = 8$

27. 0001

$\frac{dN}{dt} = \lambda N \Rightarrow 10^{10} = \frac{1}{10^9} N$

$N = 10^{19}$

Total mass = $10^{19} \times 10^{-25} = 10^{-6}$ kg

$\Rightarrow M = 10^{-6} \times 1000 \times 10^3 = 1$ mg

28. C

29. D