ROTATIONAL DYNAMICS



Exercise - I

(A) MOMENT OF INERTIA

1. The moment of inertia of a uniform semicircular wire of mass M and radius r about a line perpendicular to the plane of the wire through the centre is

(A) Mr^2

(B)
$$\frac{1}{2}$$
Mr²

(C)
$$\frac{1}{4}$$
Mr² (D) $\frac{2}{5}$ M

2. Let I_A and I_B be moments of inertia of a body about two axes A and B respectively. The axis A passes through the centre of mass of the body but B does not.

(A) $I_A < I_B$ (B) If $I_A < I_B$, the axes are parallel.

(C) If the axes are parallel, $I_A < I_B$

(D) If the axes are not parallel, $I_{A} \ge I_{B}$

3. Three bodies have equal masses m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina) (1)

4. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

(A) a disc of radius a

(B) a ring of radius a

(C) a square lamina of side 2a

(D) four rods forming a square of side 2a

5. Two rods of equal mass m and length *l* lie along the x axis and y axis with their centres origin. What is the moment of inertia of both about the line x = y:

(A)
$$\frac{ml^2}{3}$$
 (B) $\frac{ml^2}{4}$
(C) $\frac{ml^2}{12}$ (D) $\frac{ml^2}{6}$

6. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate :





(Objective Problems)

(A) about P = I/2(B) about R = I/2(C) about P > I/2

(D) about R > I/2

7. A thin uniform rod of mass M and length L has its moment of inertia I, about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2 . The ratio of $I_1 : I_2$ will be

$$\begin{array}{ll} (A) < 1 & (B) > 1 \\ (C) = 1 & (D) \ can't \ be \ said \end{array}$$

8. Moment of inertia of a thin semicircular disc (mass = M & radius = R) about an axis through point O and perpendicular to plane of disc, is given by :



9. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by

 $I = 2x^2 - 12x + 27$ The x-coordinate of centre of mass is

(A) x = 2	(B) x = 0
(C) x = 1	(D) x = 3

10. A square plate of mass M and edge L is shown in figure. The moment of inertia of the plate about the axis in the plane of plate passing through one of its vertex making an angle 15° from horizontal is.



(C)
$$\frac{7 \text{ ML}^2}{12}$$
 (D) none

11. Consider the following statements

Assertion (A): The moment of inertia of a rigid body reduces to its minimum value as compared to any other parallel axis when the axis of rotation

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Reason (R) : The weight of a rigid body always acts through its centre of mass in uniform gravitational field. Of these statements :

(A) both A and R are true and R is the correct explanation of A $% \left(A_{n}^{A}\right) =0$

(B) both A and R are true but R is not a correct explanation of A $% \left(A_{n}^{A}\right) =0$

- (C) A is true but R is false
- (D) A is false but R is true

Question No. 12 to 14 (3 questions)

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90° . The apex lies at the origin and the base is parallel to X - axis.



12. The moment of inertia of the plate about the z-axis is

(A)
$$\frac{ML^2}{12}$$
 (B) $\frac{ML^2}{24}$
(C) $\frac{ML^2}{6}$ (D) none of these

13. The moment of inertia of the plate about the x-axis is

(A)
$$\frac{ML^2}{8}$$
 (B) $\frac{ML^2}{32}$

(C) $\frac{}{24}$ (D) -

14. The moment of inertia of the plate about its base parallel to the x-axis is



15. The moment of inertia of the plate about the y-axis is

(A) $\frac{ML^2}{6}$	(B) $\frac{ML^2}{8}$
(C) $\frac{ML^2}{24}$	(D) none of these

SECTION (D); FIXED AXIS

16. A body is rotating uniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is (A) vertical

(B) horizontal and skew with the axis

(C) horizontal and intersecting the axis

(D) none of these

17. One end of a uniform rod of mass m and length I is clamped. The rod lies on a smooth horizontal surface and rotates on it about the clamped end at a uniform angular velocity ω . The force exerted by the clamp on the rod has a horizontal component

(A) $m\omega^2 l$ (B) zero

18. A rod of length 'L' is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

 $\frac{1}{2}m\omega^2\ell$

(A)
$$\sqrt{\frac{2g}{L}}$$
 (B) $\sqrt{\frac{3g}{L}}$
(C) $\sqrt{\frac{g}{2L}}$ (D) $\sqrt{\frac{g}{L}}$

(B) TORQUE AND PURE ROTATIONAL MOTION

19. A horizontal force F = mg/3 is applied on the upper surface of a uniform cube of mass `m' and side `a' which is resting on a rough horizontal surface having $\mu_s = 1/2$. The distance between lines of action of `mg' and normal reaction `N' is : (A) a/2 (B) a/3 (C) a/4 (D) None

20. A man can move on a horizontal plank supported symmetrically as shown. The variation of normal reaction on support A with distance x of the man from the end of the plank is best represented by :



21. A weightless rod is acted on by upward parallel forces of 2N and 4N ends A and B respectively. The total length of the rod AB = 3m. To keep the rod in equilibrium a force of 6N should act in the following manner :





(B) right half

- (A) Downwards at any point between A and B.
- (B) Downwards at mid point of AB.
- (C) Downwards at a point C such that AC = 1m.
- (D) Downwards at a point D such that BD = 1m.

22. A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A is :



23. In an experiment with a beam balance on unknown mass m is balanced by two known mass m is balanced by two known masses of 16 kg and 4 kg as shown in figure.



The value of the unknown mass m is(A) 10 kg(B) 6 kg(C) 8 kg(D) 12 kg

24. A uniform cube of side 'b' and mass M rest on a rough horizontal table. A horizontal force F is applied normal to one of the face at a point, at a height 3b/4 above the base. What should be the coefficient of friction (μ) between cube and table so that is will tip about an edge before it starts slipping?



(C) $\mu > \frac{3}{2}$ (D) none **25.** A homogeneous cubical brick lies motionless

on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



- (A) left half (B) (C) both applies equal pressure
- (D) the answer depend upon coefficient of friction

26. Consider the following statements

Assertion (A) : A cyclist always bends inwards while negotiating a curve

Reason (R) : By bending he lowers his centre of gravity Of these statements,

(A) both A and R are true and R is the correct explanation of A

(B) both A and R are true but R is not the correct explanation of A $% \left(A_{1}^{A}\right) =0$

- (C) A is true but R is false
- (D) A is false but R is true

27. A solid cone hangs from a frictionless pivot

at the origin O, as shown. If \hat{i} , \hat{j} and \hat{k} are unit vectors, and a, b, and c are positive constants, which of the following forces F applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_7 ?



(D) None

28. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :



29. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,



(A) its angular velocity increases, but force on hinge remains constant

(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

(D) its angular velocity remains same and force on hinge is constant.

30. The angular momentum of a flywheel having a moment of inertia of 0.4 kg m² decreases from 30 to 20 kg m²/s in a period of 2 second. The average torque acting on the flywheel during this period is :

(A) 10 N.m (B) 2.5 N.m (C) 5 N.m (D) 1.5 N.m

31. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then the maximum angle ' θ ' made by the hinged upper half with the vertical is :



32. A block of mass m is attached to a pulley disc of equal mass m, radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut will be





moves with uniform velocity of $3\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :



(C) ANGULAR MOMENTUM

34. A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin.

(A) is zero (B) remains constant

(C) goes on increasing

(D) goes on decreasing.

35. A thin circular ring of mass 'M' and radius 'R' is rotating about its axis with a constant angular velocity ... Two objects each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velcoity.

(A)
$$\frac{\omega M}{(M+m)}$$
 (B) $\frac{\omega M}{(M+2m)}$

(C)
$$\frac{\omega(M+3m)}{(M-2m)}$$
 (D) $\frac{\omega(M+3m)}{M}$

36. A person sitting firmly over a rotating stool has his arms streatched. If he folds his arms, his angular momentum about the axis of rotation (A) increases (B) decreases (C) remains unchanged (D) doubles.

37. A small bead of mass m moving with velocity v gets threaded on a stationary semicircular ring of mass m and radius R kept on a horizontal table. The ring can freely rotate about its centre. The bead comes to rest relative to the ring. What will be the final angular velocity of the system?



38. A man, sitting firmly over a rotating stool has his arms streched. If he folds his arms, the work done by the man is

- (A) zero
- (B) positive
- (C) negative

(D) may be positive or negative.

39. A particle of mass 2 kg located at the position

 $(\hat{i} + \hat{j})$ m has a velocity $2(+\hat{i} - \hat{j} + \hat{k})$ m/s. Its angular momentum about z-axis in kg-m²/s is : (A) zero (B) +8 (C) 12

(D) - 8

40. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10



Ns at its lowest point, normal to its length as shown in figure. The kinetic energy of rod just after impact is

> 10 NS (B) 100 J

(A) 75 J (B) 100 J (C) 200 J (D) none

41. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is :



(A) 2 mvd(B) 2 mvd(C) $2 mvd \sin\theta$ (D) zero

42. A uniform rod of mass M is hinged at its upper end. A particle of mass m moving horizontally strikes the rod at its mid point elastically. If the particle comes to rest after collision find the value of M/m = ?



(A) 3/4 (C) 2/3

43. A child with mass *m* is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω . See figure given below. The child jumps off the edge of the disc with tangential velocity v with respect to the ground. The new angular velocity of the disc is



Question No. 44 & 45 (2 questions)

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



44. What is the direction of the rod's angular momentum vector (calculated about its lower end) (A) vertically downwards

(B) down at 20° to the horizontal

(C) up at 20° to the horizontal

(D) vertically upwards

45. Is there a torque acting on it, and if so in what direction?

(A) yes, vertically (B) yes, horizontally

(C) yes at 20° to the horizontal

(D) no

46. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2 ?



(B) $x = vt + 0.5b \sin (3vt/b), y = 0.5b \cos(3vt/b)$ (C) $x = 0.5vt + 0.5b \sin (3vt/b), y = 0.5b \cos(3vt/b)$ (D) $x = 0.5vt + 0.5b \sin (6vt/b), y = 0.5b \cos(6vt/b)$

47. Two equal masses each of mass M are joined by a massless rod of length L. Now an impulse MV is given to the mass M making an angle of 30°



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with the length of the rod. The angular velocity of the rod just after imparting the impulse is



48. A uniform rod AB of length L and mass M is lying on a smooth table. A small particle of mass m strike the rod with a velocity v_0 at point C at a distance x from the centre O. The particle comes to rest after collision. The value of x, so that point A of the rod remains ststionary just after collision is :



49. Two particles of equal mass m at A and B are connected by a rigid light rod AB lying on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular at AB. Then the velocity of particle at A is :

(A)
$$\frac{J}{2m}$$
 (B) $\frac{J}{m}$
(C) $\frac{2J}{m}$ (D) zero

50. A uniform rod of mass M has an impulse applied at right angles to one end. If the other end begins to move with speed V, the magnitude of the impulse is

(A) MV (B) $\frac{MV}{2}$

(C) 2MV (D)
$$\frac{2N}{3}$$

51. A uniform rod AB of mass m and length *l* is at rest on a smooth horizontal surface. An impulse J is applied to the end B, perpendicular to the rod in the horizontal direction. Speed of particle P at

	a	distance	$\frac{l}{6}$	from	the	centre	towards A of the	2
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rod after time
$$t = \frac{\pi m l}{12 J}$$
 is

(A)
$$2\frac{J}{m}$$
 (B) $\frac{J}{\sqrt{2}m}$
(C) $\frac{J}{m}$ (D) $\sqrt{2}\frac{J}{m}$

(D) COMBINED TRANSLATIONAL + ROTATIONAL MOTION

52. The centre of a wheel rolling without slipping in a plane surface moves with speed v_0 . A particle on the rim of the wheel at the same level as the centre will be moving at speed.

(A) zero (B)
$$v_0$$

(C) $\sqrt{2}v_0$ (D) $2v_0$

53. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. The smallest kinetic energy at the bottom of the incline will be achieved by

(A) the solid sphere (B) the hollow sphere

(C) the disc

(D) all will achieve same kinetic energy.

54. A solid sphere, a hollow sphere and a disc, all having smooth incline and released. Least time will be taken in reaching the bottom by

(A) the solid sphere (B) the hollow sphere

(C) the disc (D) all will take same time.

55. Fig. shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is related to the acceleration a of the car as a = g tan θ . If the sphere is set in pure rotation on the incline.



(A) it will continue pure rolling

(B) it will slip down the plane

(C) its linear velocity will increase

(D) its linear velocity will decrease.

56. A straight rod of length L is released on a frictionless horizontal floor in a vertical position. As it falls + slips, the distance of a point on the rod from the lower end, which follows a quarter circular locus is

(A) L/2 (B) L/4 (C) L/8 (D) None

57. A wheel of radius r rolling on a straight line, the velocity of its centre being v. At a certain instant the point of contact of the wheel with





the grounds is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is :

(A) The velocity of any point P of the wheel is proportional to MP.

(B) Points of the wheel moving with velocity greater than v form a larger area of the wheel than points moving with velocity less than v.

(C) The point of contact M is instantaneously at rest.

(D) The velocities of any two parts of the wheel which are equidistant from centre are equal.

58. There is rod of length *l*. The velocities of its two ends are v_1 and v_2 in opposite directions normal to the rod. The distance of the instantaneous axis of rotation from v_1 is :

(A) zero
(B)
$$\frac{V_2}{V_1 + V_2} l$$

(C) $\frac{V_1 l}{V_1 + V_2}$
(D) $l/2$

59. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle $\alpha = 30^{\circ}$ with the horizontal. Then the speed of the ladder's center must be

(A) $_{2v}/\sqrt{3}$ (B) v/2 (C) v (D) none

60. In the previous question, if dv/dt = 0, then the angular acceleration of the ladder when $\alpha = 45^{\circ}$ is

) v²/2L²

(C) $\sqrt{2}[v^2/L^2]$ (D) None

61. A ring of radius R rolls without sliding with a constant velocity. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be

(A) R (B) 2R (C) 4R (D) none

62. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards

(B) Thread winds, spool rotates clockwise and friction acts leftwards

(C) Thread winds, spool moves to the right anf friction act rightwards

(D) Thread winds, spool moves to the right and friction does not come into existence.

63. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. The ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is :





64. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is :



(A) up the plane (E (C) horizontal (E

(B) down the plane (D) zero

65. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is incorrect.



(A) Acceleration of the centre of sphere is less than that of the plank

(B) Work done by friction acting on the sphere is equal to its total kinetic energy.

(C) Total kinetic energy of the system is equal to work done by the force F

(D) None of the above

66. A time varying force F = 2t is applied on a spool rolling as shown in figure. The angular momentum of the spool at time t about bottom most point is :



(D) data is insufficient

 ${\bf 67.}$ A ring of mass m and radius R has three particles attached to the ring as shown in the



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figure. The centre of the ring has speed v_0 . The kinetic energy of the system is (Slipping is absent)



(A) 6mv₀² (C) 4 mv₀²

68. The linear speed of a uniform spherical shell after rolling down an inclined plane of vertical height h from rest, is :

(A)
$$\sqrt{\frac{10\text{gh}}{7}}$$
 (B) $\sqrt{\frac{4\text{gh}}{5}}$ (C) $\sqrt{\frac{6\text{gh}}{5}}$ (D) $\sqrt{2\text{gh}}$

69. A uniform sphere of radius R is placed on a rough horizontal surface and given a linear velocity v_0 angular velocity ω_0 as shown. The sphere comes to rest after moving some distance to the right. It follows that :



70. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :

(A)	thin pipe		(B)	uniforn	n cy	lin	de	r
-	-	-	-					

(C) uniform sphere (D) thin spherical shell

71. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere :

(A) must be forward (in direction of v)

(B) must be backward (opposite to v)

(C) cannot be zero

(D) none of the above

72. A cylinder is pure rolling up an incline plane. It stops momentarily and then rolls back. The force of friction.

(A) on the cylinder is zero throughout the journey(B) is directed opposite to the velocity of the centre of mass throughout the journey

(C) is directed up the plane throughout the journey (D) is directed down the plane throughout the journey

73. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in



74. A ball rolls down an inclined plane, figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/ are correct?



(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.

(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.

(iii) The ball has twice as much K.E. at O when rolling from Q as it does when rolling from P.

A) i,	ii only	(B) ii, iii only
~		

(C) i only (D) iii only

75. Starting from the rest, at the same time, a ring, a coin and a solid ball of same mass roll down an incline without slipping. The ratio of their translational kinetic energies at the bottom will be

(A) 1 : 1 : 1	(B) 10 : 5 : 4
(C) 21 : 28 : 30	(D) none

76. In the figure shown a ring A is initially rolling without sliding with a velocity v on the horizontal surface of the body B (of same mass as A). All surfaces are smooth. B has no initial velocity. What will be the maximum height reached by A on B.





77. A Cubical bloc of mass M and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude.

(A) zero (B) Mga (C) Mga sin θ (D) $\frac{1}{2}$ Mga sin θ

