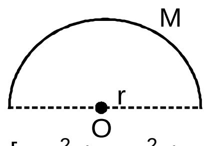
ROTATIONAL DYNAMICS

EXERCISE – I   SINGLE CORRECT

1. **A**



$I = [m_1r^2 + m_2r^2 + ... m_nr^2]$
$I = r^2 (m_1 + m_2 + ... m_n)$
$I = Mr^2$

2. **C**

$I_B = I_A + Md^2$

3. **B**

Cylinder $= \frac{MR^2}{2}$, Square lamina $= \frac{MR^2}{6}$,

Solid sphere $= \frac{2}{5} MR^2$

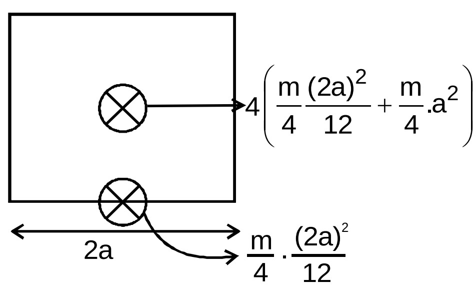
4. **D**

(a) $\frac{Ma^2}{2}$

(b) Ma^2

(c) $\frac{2}{3} Ma^2$

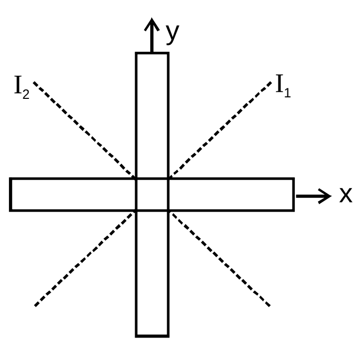
(d)



$\therefore\ 4\left(\frac{m}{4}\frac{(2a)^2}{12} + \frac{m}{4}.a^2\right)$

$= \frac{ma^2}{3} + ma^2 = \frac{4ma^2}{3}$

5. **C**

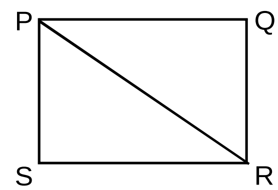


$I_x = \frac{ML^2}{12}$; $I_y = \frac{ML^2}{12}$

$I_x + I_y = I_z = I_1 + I_2$

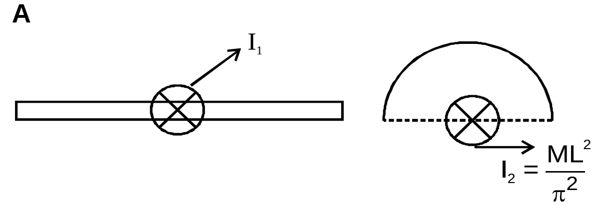
$\frac{2.ML^2}{12} = 2I_1 \Rightarrow I_1 = \frac{ML^2}{12}$

6. **C**



In case PQR
r is larger.

7. **A**

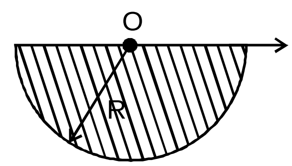


$I_1 = \frac{ML^2}{12}$

$l = \pi R \Rightarrow R = \frac{l}{\pi}$

$I_1 < I_2$

8. **B**



$I = \frac{MR^2}{2}$

(pasing through 0)

9. **D**

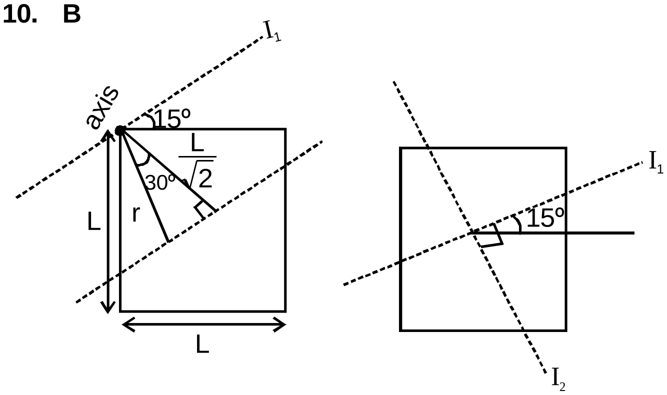
M.O.I. about C.O.M. is Minimum

$I = I_{C.M.} + Mx_0^2$
$I = 2x^2 - 12x + 27$

$\therefore\ \frac{dI}{dx} = 4x - 12 = 0$

$\Rightarrow x = 3$

10. **B**



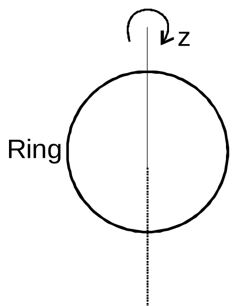
$r = \frac{L}{\sqrt{2}}\cos 30^\circ \qquad I_1 + I_2 = \frac{ML^2}{6}$

$= \frac{L\sqrt{3}}{2\sqrt{2}} \qquad \Rightarrow \quad I_1 = \frac{ML^2}{12}$

$\therefore \quad I' = \frac{ML^2}{12} + \frac{ML^2.3}{8} = \frac{2ML^2 + 9ML^2}{24}$

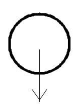
$I' = \frac{11ML^2}{24}$

11. B

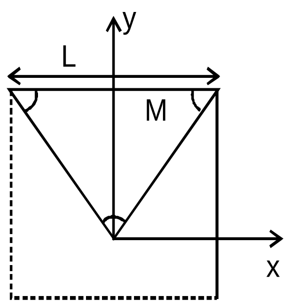


$I_z = MR^2$
$I_x = I_y = \frac{MR^2}{2}$

From parallel axis theorm
$I = I_{cm} + md^2$

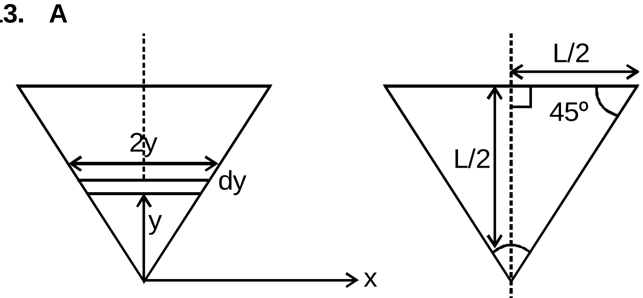
and 

12. C



$I_z = \frac{4ML^2}{4\times 6} = \frac{ML^2}{6}$

13. A

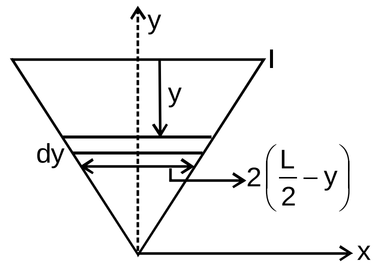


$dI = \int \sigma.2y.dy.y^2 \qquad \sigma = \frac{M}{\text{area}}$

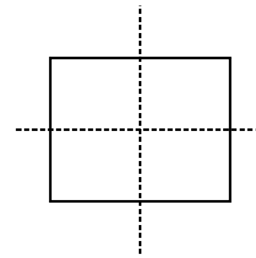
$I = 2\sigma\left[\frac{y^4}{4}\right]_0^{L/2} = \frac{\sigma}{2} \times \frac{L^4}{16} = \frac{4\times M}{L^2\times 2} \times \frac{L^4}{16}$

$I = \frac{ML^2}{8}$

14. C



$\sigma = 4M/L^2$



$I = \int_0^{L/2} \sigma.2\left(\frac{L}{2} - y\right).dy.y^2$

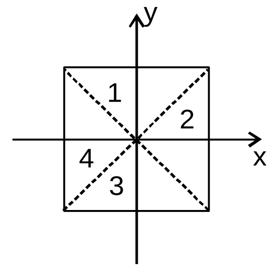
$= \sigma\,.\,2\left[\int_0^{L/2}\frac{L}{2}.y^2dy - \int_0^{L/2}y^3dy\right]$

$= \sigma.\,2\left[\frac{L}{2}.\frac{1}{3}\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2\times 4}\right)^4\right]$

$= \frac{4M}{L^2}.2\left[\frac{L^4}{3\times 16} - \frac{L^4}{16\times 4}\right]$

$= \frac{ML^2}{2}.\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{ML^2}{24}$

15. C



$I_x = I_{1x} + I_{2x} + I_{3x} + I_{4x}$

$= 2[I_{1x} + I_{2x}] \quad \left\{\begin{matrix} I_{1x} = I_{3x} \\ I_{2x} = I_{4x}\end{matrix}\right\}$

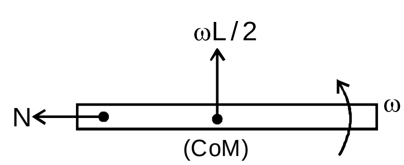
$\frac{4mL^2}{24} = \frac{2ML^2}{24} + 2I_4$

$I_4 = \frac{ML^2}{24}$

16. C

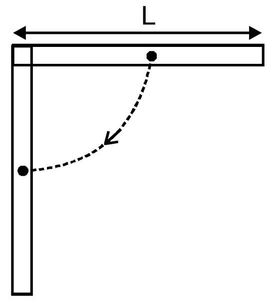
ω = constant, $\alpha = 0$, $\tau = 0$
Torque along horizontal axis is zero.

17. D



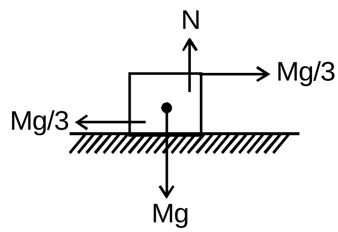
$N = \frac{M}{L/2}\left(\frac{\omega L}{2}\right)^2, \quad N = \frac{ML\omega^2}{2}$

18. B



$Mg\frac{L}{2} = \frac{1}{2}I\omega^2, \ \omega = \sqrt{\frac{3Mg\ell}{ML^2}} = \sqrt{\frac{3g}{L}}$

19. B



$f_{max} = \frac{1}{2}Mg$

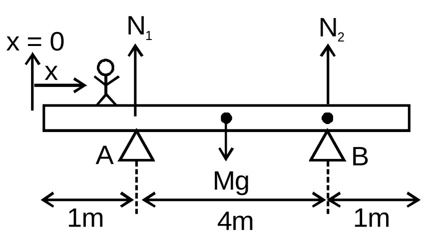
$f = Mg/3$

Torque Blance $\frac{Mg}{3}\cdot\frac{a}{2} + \frac{Mg}{3}\cdot\frac{a}{2} = N.\,x$

$\frac{Mga}{3} = mg\,x$

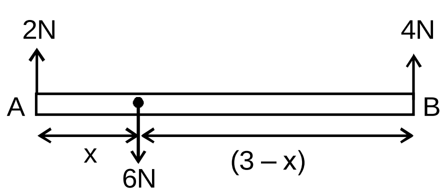
$x = \frac{a}{3}$

20. B



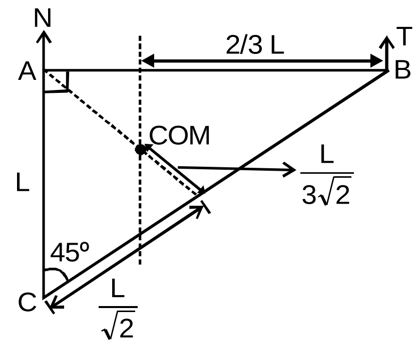
$N_1 + N_2 = (M + m)\,.\,g$
$\tau_B = 0$
$N_1\,.\,4 = Mg \times 2 + m\,(5 - x)$

21. D



$2 \times 3 = 6\,(3 - x)$
$6 = 18 - 6x$
$6x = 12$
$x = 2m$

22. B



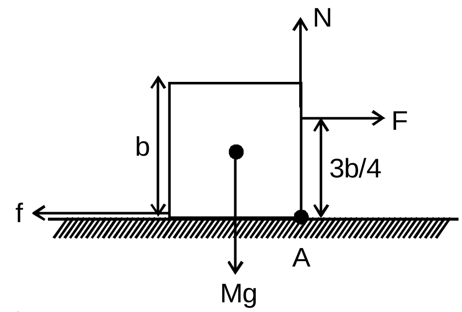
$T + N = Mg$ ....(1)
$\tau_B = 0$

$N.L = Mg\,.\,\frac{2}{3}L, \quad \boxed{N = \frac{2}{3}Mg}$

23. C

By torque balance
$16\,L_1 = ML_2$ ....(1)
$ML_1 = 4L_2$ ....(2)
$16 \times 4 = M^2$
$M = 8kg$

24. A



$f_{max} = \mu N$ ....(1)
$f = F$ ....(2)
$\tau_A = 0$ ....(3)

$F.\,\frac{3b}{4} = Mg.b/2$

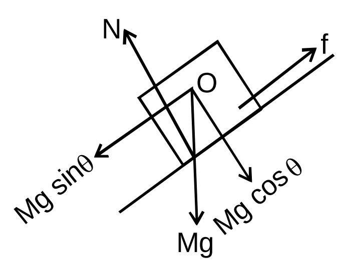
$f = F = 2\,Mg/3$

$\therefore \quad f > \mu N \quad \Rightarrow \quad 2Mg/3 > \mu.\,Mg$

$\mu > \frac{2}{3}$

25. A

To Balance torque N shifts Downwards



26. B

Centripetal force is necessary for a circular path.

27. C

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (-b\hat{i} - c\hat{k}) \times a\hat{j}$

$= (-b\hat{k} - c(-\hat{i}))$

$= -b\hat{k} + c\hat{i}$

28. B

$P = \vec{\tau} \cdot \vec{\omega}$

$P = \tau\,\omega$

$P = \tau\ \alpha\ t$

Constant

29. A

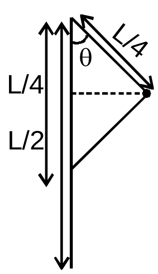
Its angular velocity increases

But force on hinge is constant

30. C

$\tau_{avg} = \frac{\Delta L}{\Delta T} = \frac{10}{2} = 5\text{N-m}$

31. C



$\omega = \sqrt{\frac{3g}{L}}$

By Energy Conservation

$\frac{1}{2}\ \frac{M}{2 \times 3} \times \left(\frac{L}{2}\right)^2 \times \frac{3g}{L} = \frac{Mg}{2}\ \frac{L}{4}\ (1 - \cos\theta)$

$\frac{ML^2}{4L} \times g = \frac{MgL}{2}\ (1 - \cos\theta)$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

32. D

$-\int T.dt = m.v - m \times 5$ ...(1)

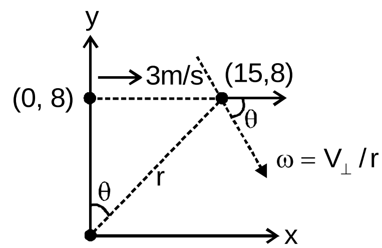
$\int T.dt\ .\ r = \frac{mr^2}{2}\ .\ \omega$ ...(2)

$\omega = \frac{v}{r}$ ...(3)

$\int T.dt = \frac{mv}{2}$

$5m - mv = \frac{mv}{2}$, $5 = \frac{3v}{2}$, $v = \frac{10}{3}\ \frac{m}{sec}$

33. C



$\omega = V_{\perp}/r$

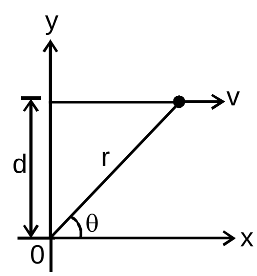
$\omega = 3\cos\theta/r$

$\omega = \frac{3}{r} \times \frac{8}{r} = \frac{24}{r^2}$

$r^2 = (15)^2 + (8)^2 = 289$

$\omega = \frac{24}{289}$ rad/s

34. B



Angular momentum

$= mvr \sin\theta$

$= mvd$ (constnat)

35. B

$I_{new} = I_{old} + 2mR^2$

$= MR^2 + 2mR^2$

By angular momentum conservation

$I_{old}\omega = I_{new}\ \omega_{new}$

$\Rightarrow MR^2\ \omega = (M + 2m)\ R^2\ \omega_{new}$

$\boxed{\omega_{new} = \frac{M\omega}{M + 2m}}$

36. C

$I\omega$ = Constant

37. C

Angular momentum conservation

$MVR = (MR^2 + MR^2)\ .\ \omega$

$\frac{V}{2R} = \omega$

38. B

Angular momentum

$I_1\omega_1 = I_2\omega_2$

$\therefore$ K.E. $= \frac{L^2}{2I}$ $I\downarrow$ K.E.$\uparrow$

$\therefore$ W.D. = + ve

39. D

$\vec{L} = m(\vec{r} \times \vec{V})$

$= 2 \times 2\ [(\hat{i} + \hat{j}) \times (\hat{i} - \hat{j} + \hat{k})]$

$= 4(-\hat{k} - \hat{j} - \hat{k} + \hat{i}) = 4(\hat{i} - \hat{j} - 2\hat{k})$

$\vec{L}$ = Angular Momentum along z-axis is the compoent of angular momentum along z-axis.

i.e. = – 8 kg-m^2/sec

40. A

$\int \tau . dt = I\omega - 0$

$10 \times 1 = \frac{2 \times (1)^2}{3} \times \omega \Rightarrow 15$ rad/sec

$\omega = 15$ rad/sec

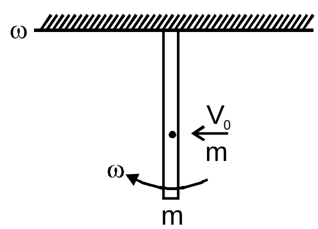
K.E. $= \frac{1}{2} \times \frac{2 \times (1)^2}{3} \times (15)^2 = 75$ Joule

41. B

Change in momentum = $2mV\cos\theta = \int F.dt$

$\therefore$ Change in impulse = $\int Fd.dt = 2m\,Vd\cos\theta$

42. A



$\left[e = -\frac{(V_1 - V_2)}{u_1 - u_2}\right]$, $I = \frac{\frac{\omega L}{2} - 0}{V}$

$\Rightarrow \frac{\omega L}{2} = V$

$\frac{mVL}{2} = \frac{ML^2}{3} . \omega \Rightarrow \frac{mVL}{2} = \frac{ML^2}{3} \times \frac{2V}{L}$

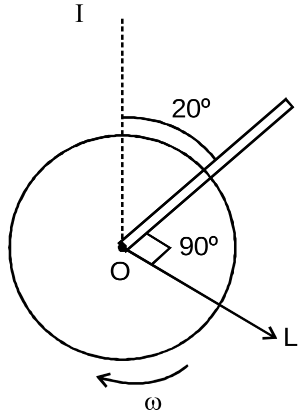
$\Rightarrow \frac{M}{m} = \frac{3}{4}$

43. D

$(I + mR^2) . \omega = I\omega' + mvR$

$\omega' = \frac{(I + mR^2).\omega - mVR}{I}$

44. B



$L \perp$ to r and V

Direction of L vector is $\perp$ to r and V.

45. B

Direction of $\vec{L}$ is continuously changing but not Magnitude

$\therefore$ Torque is present

$\therefore$ If there is a torque along the axis then it will increase ω

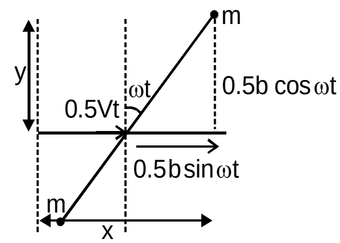
$\therefore$ ω is constant $\Rightarrow$ In H-direction

46. C

Angular Momentum conservation about C.O.M.

$2m.v.\frac{b}{2} + mv\frac{b}{2} = \left(2m.\frac{b^2}{4}.\omega\right) + 0$

$\Rightarrow \frac{3mvb}{2} = \frac{mb^2}{2}.\omega$ $\boxed{\omega = \frac{3V}{b}}$



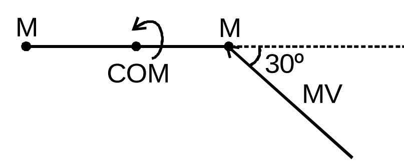
L.M.C. $2mV - mV = 2mV'$

$V' = 0.5\ V$

$x = 0.5Vt + 0.5b \sin \omega t$

$y = 0.5 \cos\omega t$ where $\omega = \frac{3V}{b}$

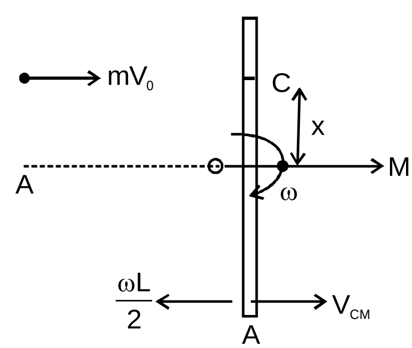
47. C



$\int \vec{L}dt$ = change in angular momentum

$MV \sin 30^\circ \frac{L}{2} = \frac{2ML^2\omega}{4}$ $\boxed{\frac{V}{2L} = \omega}$

48. B

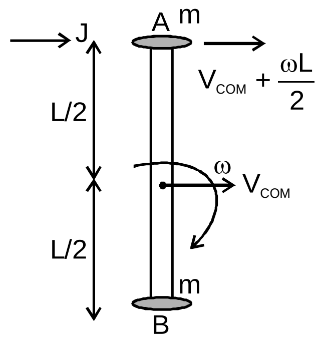


L.M.C. $mV_0 = MV_{CM}$
A.M.C. (A)

$$mv_0x = \frac{ML^2}{12}\omega$$

$$\frac{\omega L}{2} = V_{CM} \Rightarrow \boxed{x = \frac{L}{6}}$$

49. B



$$J.\frac{L}{2} = I\omega$$

$$J.\frac{L}{2} = 2\frac{ML^2}{4}.\omega$$

$$\Rightarrow \omega = \frac{J}{ML}$$

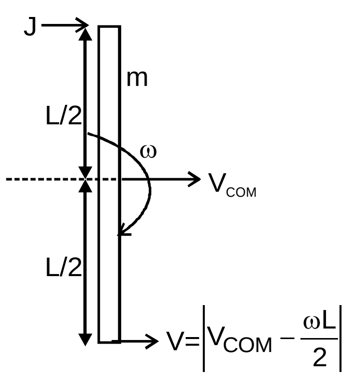
$$J = 2M\,V_{COM}$$

$$V_{COM} = \frac{J}{2M}$$

Now, $V_A = \frac{J}{2M} + \frac{J}{ML}\cdot\frac{L}{2}$

$$= \frac{J}{M}$$

50. B



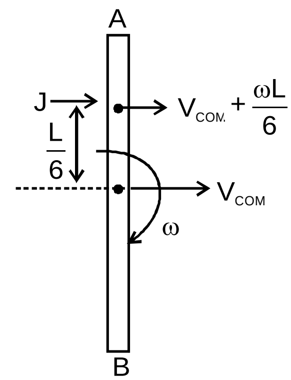
$$J = MV_{COM} \Rightarrow V_{COM} = \frac{J}{M}$$

$$J\frac{L}{2} = \frac{ML^2}{12}\omega$$

$$V = \left|\frac{J}{M} - \frac{6J}{ML}\frac{L}{2}\right|$$

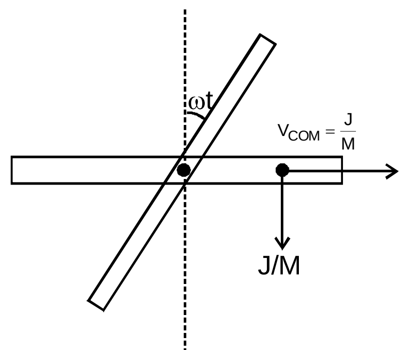
$$J = \frac{MV}{2}$$

51. D



$$J = M.V_{COM} \Rightarrow V_{COM} = \frac{J}{M}$$

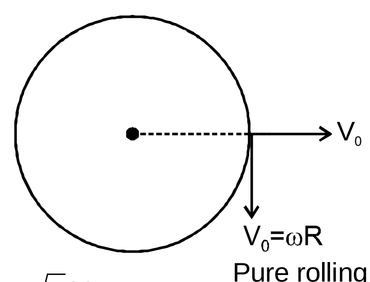
$$\frac{J.L}{2} = \frac{ML^2\omega}{12} \Rightarrow \omega = \frac{6J}{ML}$$



$$\theta = \omega t = \frac{6J}{ML}\cdot\frac{\pi ML}{12J} = \frac{\pi}{2}$$

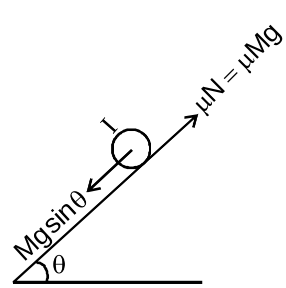
$$\boxed{V = \frac{\sqrt{2}J}{M}}$$

52. C



$$\Rightarrow V_{net} = \sqrt{2}\,V$$

53. B



$Mg \sin\theta - \mu Mg \cos\theta = Ma$

$a = g [\sin\theta - \mu \cos \theta]$ ...(1)

$\alpha = \frac{(\mu N)R}{I} = \frac{\mu Mg \cos\theta R}{I}$ ...(2)

a is same for all

α is maximum for hollow sphere.

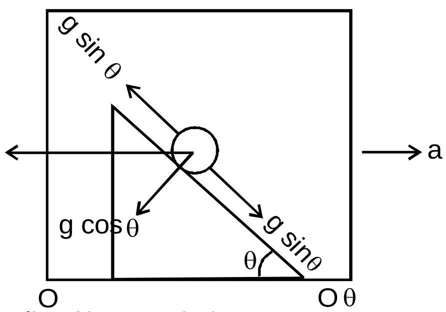
So kinetic energy is more for hollow sphere.

54. D

$\because \mu = 0$

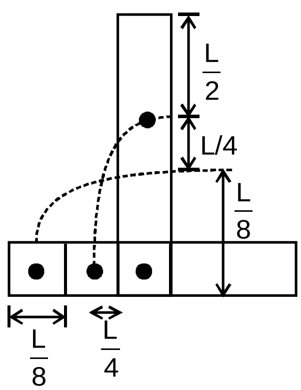
$a = g\sin\theta,\ t = \sqrt{\frac{2h}{g\sin\theta}}$

55. A

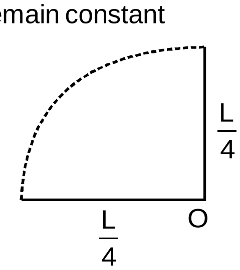


$\because$ along inclined in Pseudo frame acceleration = 0.It is a case of pure rolling.

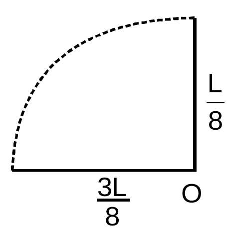
56. B



There is no force in Horizontal direction C.O.M. will remain constant



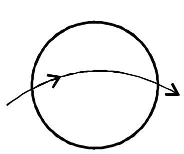
Quarter circle with Radius $\frac{L}{4}$



It is not circle

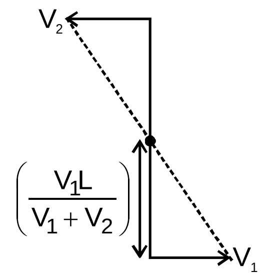
57. D

(a) M is instantaneous axis of Rotation (I.A.R.)(b)

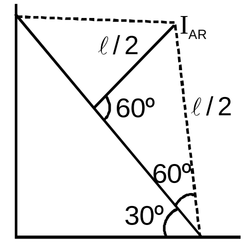


Magnitude is same but direction is different

58. C



59. C

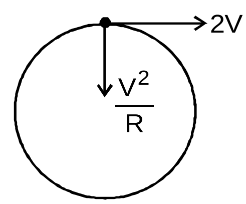


$\omega = \frac{V}{\ell \sin\theta} = \frac{2v}{\ell}$

$\frac{v_c}{\frac{\ell}{2}} = \frac{2v}{\ell}, \quad v_C = v$

60. A

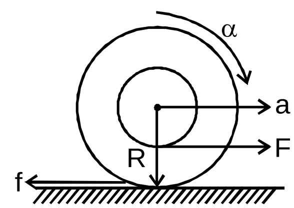
61. C



$\frac{V^2}{R} = \frac{(2V)^2}{R'}$

$R' = 4R$

62. B



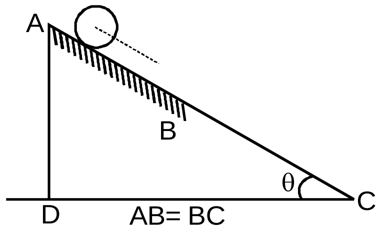
$\alpha = \frac{a}{R}$

$F - f = Ma$

$f \cdot R - F.r = I. \alpha$

assumed direction of friction is same so spool rotates clockwise and thread winds.

63. B



For pure rolling

Mg sin θ – f = Ma

$$f \cdot R = \frac{MR^2}{2} \cdot \frac{a}{R}$$

$$Mg \sin\theta = \frac{3Ma}{2} \Rightarrow a = \frac{2g\sin\theta}{3}$$

Now $\frac{L}{2} = \frac{1}{2} \cdot \frac{2g\sin\theta . t^2}{3}$

$$V_B^2 = \frac{2\times 2g\sin\theta}{3} \times \frac{L}{2} = \frac{2g\sin\theta L}{3}$$

$$\omega_B = \frac{V_B}{R}$$

$$\therefore \quad V_C^2 - V_B^2 = 2g\sin\theta \cdot \frac{L}{2} = gL\sin\theta$$

$$V_C^2 = g\sin\theta\, L + \frac{2gL\sin\theta}{3}$$

$$= \frac{5gL\sin\theta}{3}$$

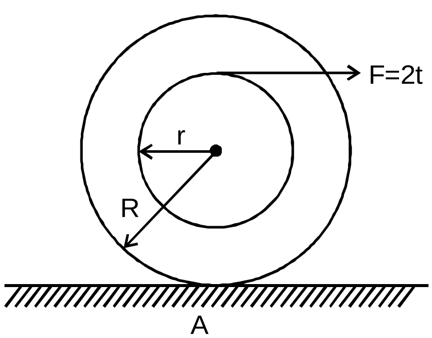
$$\therefore \text{ Rotational kinetic energy} = \frac{1}{2} \cdot \frac{MR^2}{2} \cdot \frac{V_B^2}{R^2}$$

$$= \frac{1}{2} \cdot \frac{M.2g\sin\theta L}{3}$$

64. D

65. D

66. C



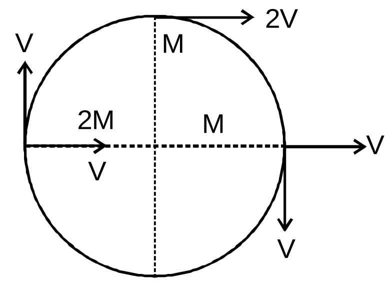
2t – f = 0

τ_A = wt (R + r)

$$\int_0^L dL = \int_0^t 2t(R+r)dt$$

L = (R + r) t^2

67. A

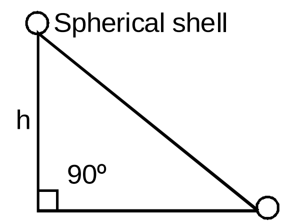


$$= \frac{1}{2}MV^2 + \frac{1}{2}\frac{MR^2 \times V^2}{R^2} + \frac{1}{2}M(2V^2)$$

$$+ \frac{1}{2}M(2V)^2 + \frac{1}{2} \cdot 2M(2V)^2$$

$$= 6MV^2$$

68. C



$$mgh = \frac{1}{2}Iw^2 + \frac{1}{2}mV^2, \; I_{shell} = \frac{2}{3}MR^2$$

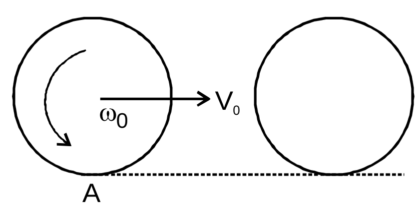
$$mgh = \frac{1}{2} \times \frac{2}{3}MR^2 \cdot \frac{V^2}{R^2} + \frac{1}{2}MV^2$$

$$mgh = \frac{1}{3}MV^2 + \frac{1}{2}mV^2$$

$$mgh = \frac{5}{6}mV^2 \Rightarrow V^2 = \frac{6gh}{5}$$

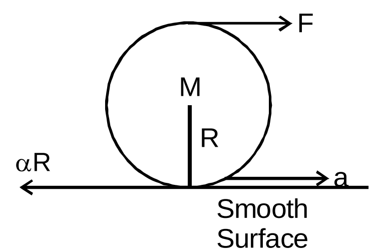
69. C

Angular momentum conservation (about A)



$$\frac{2}{5}MR^2\omega_0 = MV_0R \qquad \boxed{5V_0 = 2\omega_0 R}$$

70. A



$\vec{F} = Ma$

$\Rightarrow \quad a = \frac{F}{M}$

For pure rolling

$a = \alpha R$

$F \times R = I\alpha$

$\alpha = \frac{FR}{I}$

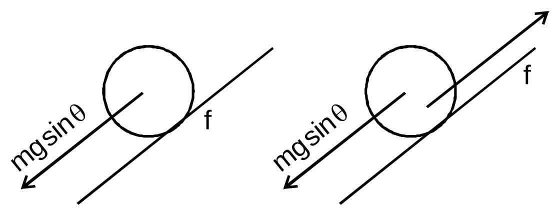
$\frac{F}{m} = \frac{FR.R}{I}$

$I = MR^2$

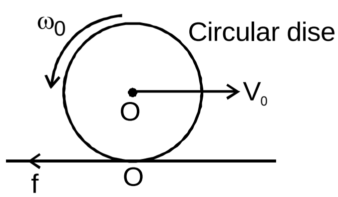
MR^2 is the moment of inertia of chin pipe.

71. **D**

72 C



73. **A**



α is conserved about 0

$I\omega_0 - mVR = 0$

$I\omega_0 = mVR$

$\frac{MR^2}{2}\omega_0 = mV_0R$

$\omega_0 = \frac{2V_0}{R} \quad \Rightarrow \quad \frac{V_0}{\omega_0 R} = \frac{1}{2}$

74. **D**

$a = \frac{5g\sin\theta}{7}$

(i) $25 = \frac{1}{2}\, a\, t^2_{Q\ to\ 0}$ ...(1)

$5 = \frac{1}{2}\, a\, t^2_{P\ to\ 0}$ ...(2)

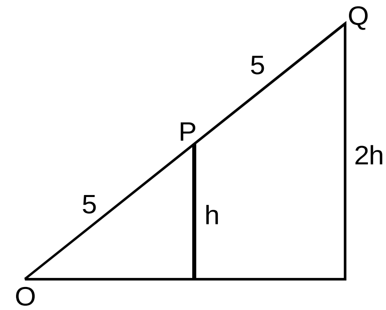
$t_{Q\ to\ 0} = \sqrt{\frac{45}{a}}$

$t_{P\ to\ 0} = \sqrt{\frac{25}{a}}$

(ii) $Mg\sin\theta - f = ma$

$fR = I\alpha$

$mg\sin\theta - \frac{I\alpha}{R} = ma$



$mg\sin\theta = I\frac{\alpha}{R} + ma$ $\quad \boxed{a = \alpha R}$

$mg\sin\theta = \frac{Ia}{R^2} + ma$

$mg\sin\theta = a\left(\frac{I}{R^2} + m\right)$

$a = \frac{mg\sin\theta}{m\left(1 + \frac{I}{mR^2}\right)} \qquad I = \frac{2}{5}mR^2$

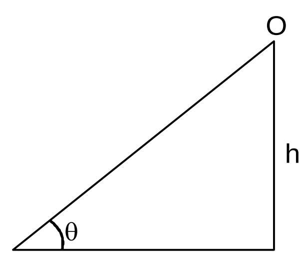
$a = \frac{g\sin\theta}{1 + \frac{2}{5}\frac{MR^2}{MR^2}}$

$a = \frac{g\sin\theta}{1 + \frac{2}{5}} = \frac{5g\sin\theta}{7}$

(iii) $K.E_{at\ O\ from\ P} = mgh$

$K.E_{at\ O\ from\ P} = 2\ mgh$

75. **C**



$mgh = \frac{1}{2}\ I\omega^2 + \frac{1}{2}\ mV^2$

$mgh = \frac{1}{2}\left(mR^2 \times \frac{V^2}{R^2}\right) + \frac{1}{2}\ mV^2$

$2mgh = mV^2\,(1 + C)$

$V^2 = \frac{2gh}{1+C}$

$K.E. = \frac{1}{2}\ mV^2 = \frac{1}{2}\ m\ .\ \frac{2gh}{1+C} = \frac{mgh}{1+C}$

$K.E_{ring} = \frac{mgh}{1+1} = \frac{mgh}{2}$

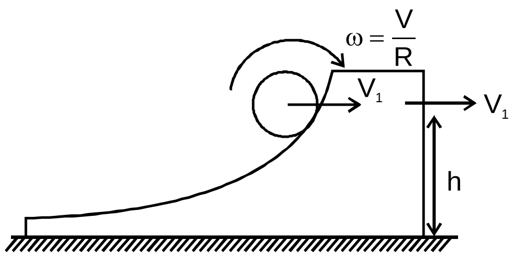
$K.E_{coin} = \frac{mgh}{1+\frac{1}{2}} = \frac{2}{3}\ mgh$

$K.E_{solid\ sphere} = \frac{mgh}{1+\frac{2}{5}} = \frac{5}{7}\ mgh$

Ratio = $\frac{1}{2} : \frac{2}{3} : \frac{5}{7}$

= 21 : 28 : 30

76. B



$MV_0 = 2MV_1$

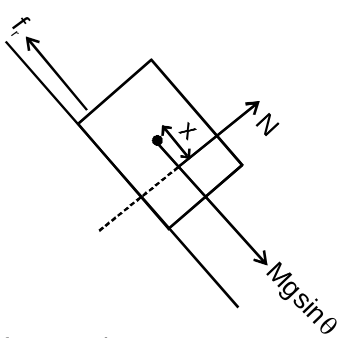
$V_1 = \frac{V_0}{2}$

$\frac{1}{2} mV_0^2 = \frac{1}{2}\ .\ 2m\ .\ V_1^2 + mgh$

$\frac{mV_0^2}{4} = mgh$

$h = \frac{v_0^2}{4g}$

77. D



$f_r = Mg\sin\theta = \mu Mg\cos\theta$

$f_r\ .\ \frac{a}{2} = N.x = \tau_N$

$\boxed{Mg\frac{a}{2}\sin\theta = \tau_N}$