SHM

EXERCISE – I SINGLE CORRECT

1. A

$F = ma \Rightarrow F \alpha - x$ (for SHM) $\Rightarrow a \alpha - x$

2 C

In one time period travel distance is
4(distance in T/4 time) = 4A

3. B

4. C

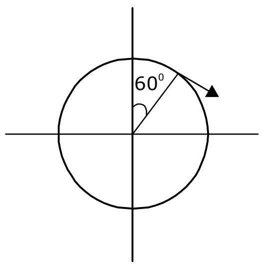
$y = A(1 + \cos 2\omega t)$ $y = 2A\sin\left(\omega t + \pi/3\right)$

$y = A + A\cos 2\omega t$ $V_{max} = 2A\omega$

$V_{max} = A \times 2\omega$ Ratio = 1:1

5. A

$t = \dfrac{2\pi}{3.2\pi} = \dfrac{1}{3}$ sec.



6. A

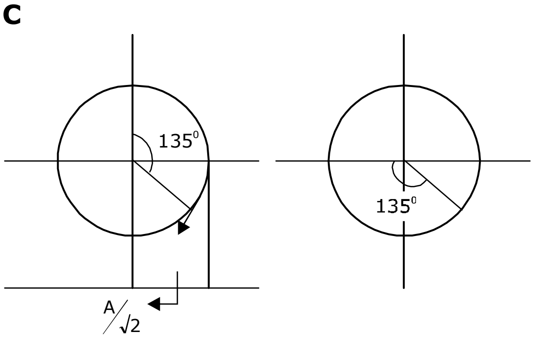
$y = 5\sin\pi(t+4) \Rightarrow y = 5\sin(\pi t + 4\pi)$ ---(1)

standard equation $y = A\sin(\omega t + \phi)$ ----(2)

compare equation (1) & (2)

A = 5m, $\omega = \pi$ $T = \dfrac{2\pi}{\omega} = 2$ sec.

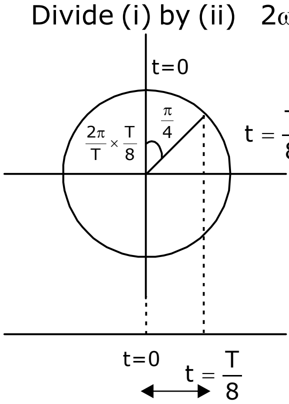
7. C



8. C

$2A_1\omega_2^2 = 2A_1\omega_2^2$ ---(i), $2A_1\omega_1 = 2A_2\omega_2$ ---(ii)

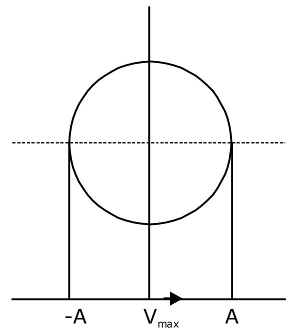
Divide (i) by (ii) $2\omega_1 = \omega_2$ $A_2 = A_1/2$



$A\sin\pi/4 = A/\sqrt{2}$

displacement $= A/\sqrt{2}$

10. A



11. D

Total Acc^n = 0

$a_{av.} = \dfrac{\text{Total change in Velocity in one T.P.}}{\text{Time Period}}$

$a_{av} = \dfrac{0}{T} = 0$

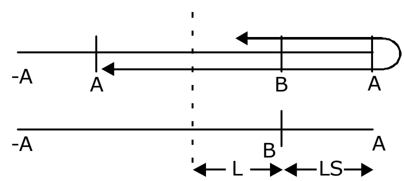
12. B

$v^2 = \omega^2(A^2 - x^2) = -\omega^2x^2 + \omega^2A^2$

$a^2 = \omega^2x^2$

$\Rightarrow v^2 = -a^2 + \omega^2 a^2$ straight line with a -ve slope.

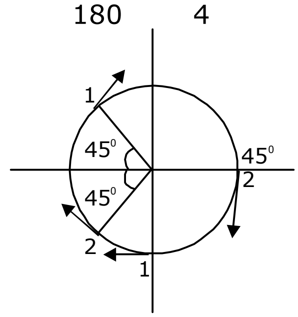
13. B



$\dfrac{T}{4} = 2$ sec. T = 8 sec.

14. B

$\dfrac{135 \times \pi}{180} = \dfrac{3\pi}{4}$



$\therefore t = \dfrac{3\pi}{4 \times 2\pi}T = \dfrac{3T}{8}$

15. A

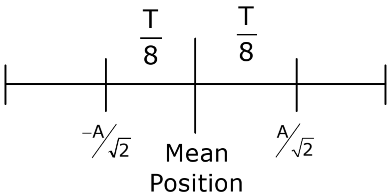
$x = A\sin\omega t$ if t = 1 is t=0

$v = A\omega\,\text{co}\,\omega t$ $\Rightarrow 0.25 = A \times \dfrac{2\pi}{6}\cos\left(\dfrac{\pi}{3} - 1\right)$

$$0.25 = A \times \frac{2\pi}{6} \times \frac{1}{2} \qquad \Rightarrow A = \frac{3}{2\pi}$$

16. D

$$\text{Max. Average velocity} = \frac{\text{Total max displacement}}{\text{Total time}}$$



Max^m displacement will be close to $M.P. = \sqrt{2}A$

$$V_{avg} = \frac{4\sqrt{2}}{T}A$$

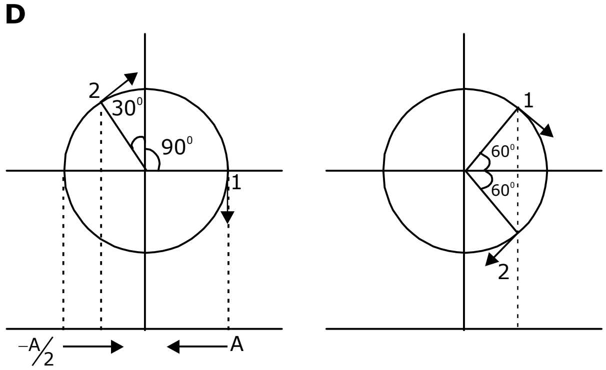
17. Dt=1

$$\frac{A.\sqrt{2}}{A\sqrt{2}\left(\sqrt{2}-1\right)} = \left(\sqrt{2}+1\right)$$

18. C

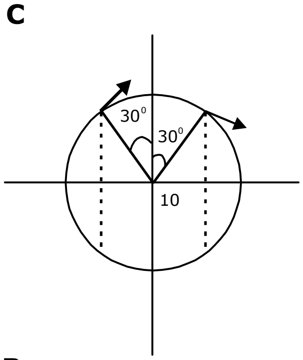
$$\frac{4d^2y}{dt^2} + 9y = 0 \Rightarrow \omega^2 = \frac{9}{4} \Rightarrow \omega = \frac{3}{2}$$

19. D



$$t = \frac{2\pi}{3} \times \frac{T}{2\pi} \qquad\qquad t = T/6$$

20. C



21. B

Slope of F-x curve gives K

$$slope = \frac{13.5}{1.5} \qquad F = -Kx \Rightarrow K = 9$$

$$\omega^2 = \frac{K}{m} = 9 \qquad \omega = 3\,, \quad T = 2\pi/3$$

22. B

$$\frac{1}{2}KA^2 = 8 \times 10^{-3}J \Rightarrow \text{A = 0.1 m,} \quad \text{M=0.1 Kg}$$

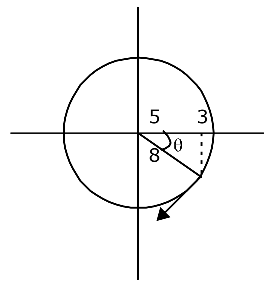
$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.6}{0.1}} = 4$$

$$\phi = 45^\circ = \frac{\pi}{4} \Rightarrow \quad x = 0.1\sin(4t + \frac{\pi}{4})$$

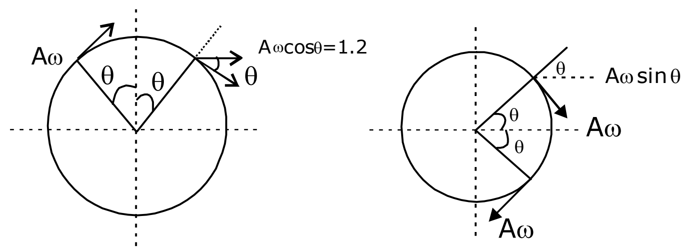
23. C

$$\cos\theta = \frac{5}{8} \Rightarrow \qquad \theta = \cos^{-1}(5/8) = \omega t = 51^\circ$$

$t = 0.17\,\text{sec}.$



24. D



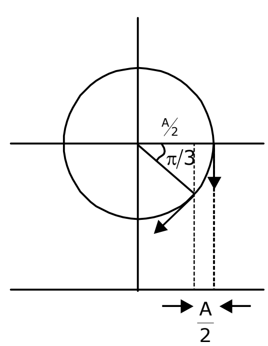
$A\omega\cos\theta = 1.2$ -- (1) $\quad A\omega\sin\theta = 1.6$ -- (2)

$$\tan\theta = 4/3 \Rightarrow \quad \theta = 53^\circ$$

$$\therefore A\omega.\frac{3}{5} = 1.2 \Rightarrow A\omega = 2m/\text{sec}.$$

25. C

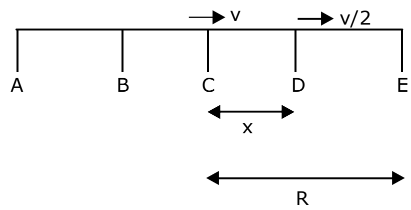
$$\theta = \pi/3 = \omega t$$



$$t = \frac{T}{6}$$

$$\therefore V_{avg} = \frac{A/2}{T/6} = \frac{3A}{T}$$

26. C



$v/2 = \omega\sqrt{R^2 - x^2}$ $\quad$ $R\omega/2 = \omega\sqrt{R^2 - x^2}$

$R^2/4 = R^2 - x^2$ $\quad$ $x = \frac{\sqrt{3}}{2}R$

Distance $= 2x = \sqrt{3}R$

27. C

$T = 2\pi\sqrt{\frac{M}{K}}$ When the rubber ribbon slacks it will not exert any force.

28. C

K will remain unchanged

$n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} =$ constant

29. B

$T_1 = 2\pi\sqrt{\frac{m}{K_1}}$, $\quad T_2 = 2\pi\sqrt{\frac{m}{K_2}}$ $\quad T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$K_{eq}\frac{K_1K_2}{K_1+K_2}$ (for series) $t_1^2 = \frac{2\pi m}{K_1}$,

$t_2^2 = \frac{2\pi m}{K_2}$ and $T^2 = 2\pi m\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$

$\Rightarrow T^2 = t_1^2 + t_2^2$

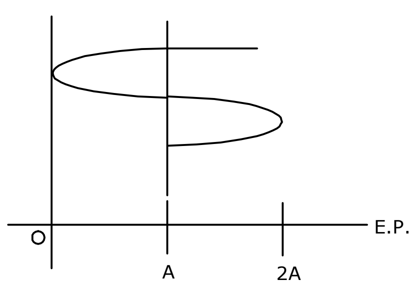
30. C

$x = A + A\sin\omega t$

$t = \frac{2.5\pi}{\omega}$

$t = \frac{2\pi}{\omega} + \frac{0.5\pi}{\omega}$

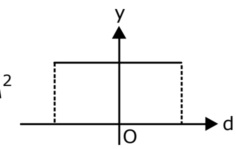
= 5A



31. A

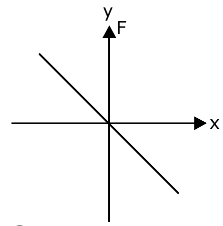
Total energy is constant during S.H.M.

$\Rightarrow E = \frac{1}{2}KA^2$



32. D

33. D



34. C

A = 2cm. $\Rightarrow x = 1cm = \frac{A}{2}$

$\Rightarrow \; v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{3} \Rightarrow \; a = w^2.1$

$\omega = \sqrt{3} = 2\pi n \;\Rightarrow\; n = \frac{\sqrt{3}}{2\pi}$

35. D

Total M.E.= T.E. at M.P.+Total oscillation energy

9 = 5 + 4

Total oscillation energy = $\frac{1}{2}Ka^2$ $\quad$ = 4

$\Rightarrow K = 8\times10^4 \Rightarrow \quad T = 2\pi\sqrt{M/K} \quad \Rightarrow T = \pi/100$

36. B

$T_1 = 2\pi\sqrt{\frac{2m}{K}}$, $\quad T_2 = 2\pi\sqrt{\frac{m}{2K}}$

series $K_{eq}^n = \frac{K^2}{2K} = \frac{K}{2}$, parallel $\quad K_q^n = 2K$

37. B

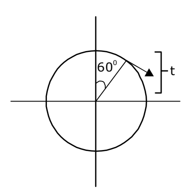
$V_{max} = \omega A$ $\qquad \omega_p A_p = \omega_Q A_Q$

$\frac{A_p}{A_Q} = \frac{\omega_Q}{\omega_p} = \sqrt{\frac{K_2}{K_1}}$

38. A

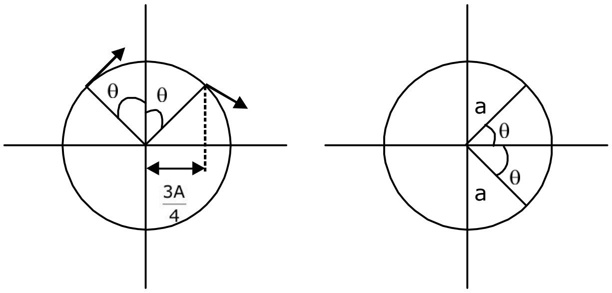
$y = \sin\omega t + \sqrt{3}\cos\omega t$

$y = 2\sin\left(\omega t + \pi/3\right)$ $\qquad A\omega^2 = g$



$\Rightarrow \omega = \sqrt{g/2} \Rightarrow t = \frac{\pi}{6}\sqrt{\frac{2}{g}}$

39. B



$\Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$

$\Rightarrow a\cos\theta = \frac{a\sqrt{7}}{4}$

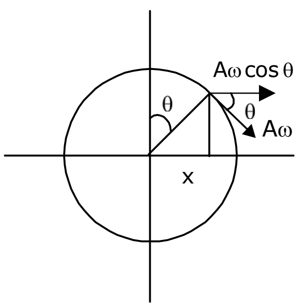
40. B

$\theta = \omega\frac{T}{12} = \frac{\pi}{6}$, $x = A\sin\frac{\pi}{6} = \frac{A}{2}$

$P.E. = \frac{1}{2}Kx^2 = \frac{1}{2}\frac{KA^2}{4}$

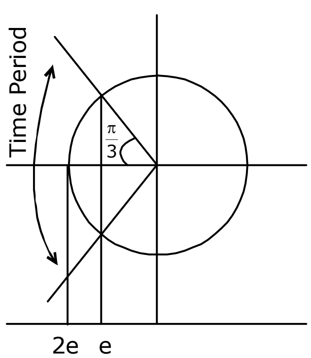
$v = A\omega\cos\theta = \frac{A\omega\sqrt{3}}{2}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}K\left(\frac{3}{4}A^2\right) \Rightarrow \frac{KE}{PE} = 3$



41. A

$\omega t = \frac{2\pi}{3} \Rightarrow \frac{2\pi}{T}t = \frac{2\pi}{3} \Rightarrow t = \frac{T}{3} \Rightarrow t = \frac{2\pi}{3}\sqrt{\frac{m}{K}}$



42. C

$E = \frac{1}{2}KA^2$ remains unchanged

43. A

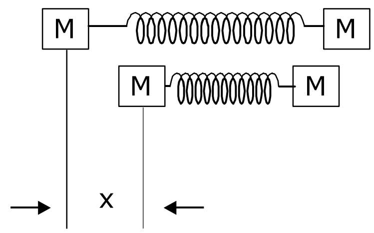
$v_1 = \omega\sqrt{A^2 - x^2}$ $v_1 = \omega A = v$

44. C

$\mu = \frac{m_2}{2m} = \frac{m}{2}$

It can be considered as a two block system for $\frac{T}{2}$ time

$t = \frac{T}{2} = \pi\sqrt{\frac{m}{2K}} \Rightarrow t = \pi\sqrt{\frac{2}{2.\pi^2}} = 1s.$



45. A Elastic Collision

46. A

x = 2 sin ωt

$y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$

from Lissajous figures if $\phi = \frac{\pi}{4}$ then the path of particle is an ellipse.

47. C

$a_1 = 1$, $a_2 = 1$ $\theta = \frac{\pi}{3}$

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta} \Rightarrow a = \sqrt{3}$

48. B

$E = \frac{1}{2}KA_{eq}^2$ $A_{eq} = \sqrt{A^2 + A^2}$

$E = \frac{1}{2}m\omega^2\left(\sqrt{2}A\right)^2$ $= \sqrt{2}A$

$E = m\omega^2A^2$

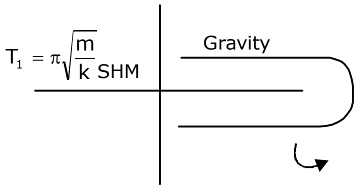
49. C

$K_{A_{eq}} = K/3$, $K_B = 3K \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{K_B}{K_A}} = \frac{3}{1}$

50. C

for x < 0 perform SHM

$\frac{1}{2}mU^2 = E$



$U = \sqrt{\frac{2E}{m}}$

$$\Rightarrow T_2 = \frac{2}{g}\sqrt{\frac{2E}{m}}$$

$$\therefore T = T_1 + T_2 \Rightarrow T = \pi\sqrt{\frac{m}{K}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$$

51. B

$F_{net} = 2\left(L/2 - x\right) - \left(L/2 + x\right) \Rightarrow F_{net} = -4x$

$T = 2\pi$, $\omega = 1$, $T/6 \Rightarrow \therefore \pi/3$

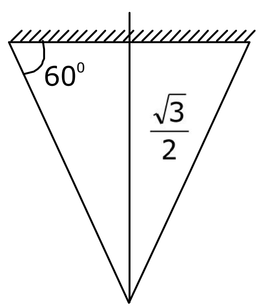
52. C

$T_p = 2\pi\sqrt{\frac{m}{K}}$ $\qquad T_s = 2\pi\sqrt{\frac{\ell}{g+a}}$

T_p = Remain same $\qquad T_s$ decreases

53. B

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

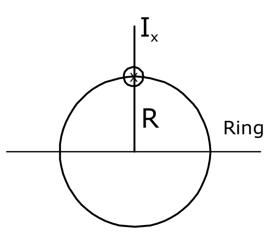


$$= 2\pi\sqrt{\frac{m\left(\frac{\sqrt{3}}{2}\ell\right)^2}{mg\frac{\sqrt{3}}{2}\ell}}$$

$$= 2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$$

54. C

$$I_x = \frac{mR^2}{2} + mR^2$$



$$= \frac{3}{2}mR^2$$

$$T = 2\pi\sqrt{\frac{3}{2}\frac{mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} \Rightarrow \ell = \frac{3}{2}$$

55. A

$n_1T_1 = n_2T_2 \qquad n_1T = \frac{5T}{4} \times n_2$

$\frac{n_1}{n_2} = \frac{5}{4} \Rightarrow n_1 = 5, \qquad n_2 = 4$

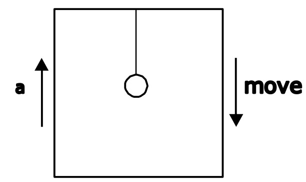
56. $T_1 = 2\pi\sqrt{\ell/g} = T \Rightarrow T_2 = 2\pi\sqrt{\ell/4g} = \frac{T}{2}$

$$T_{eq} = \frac{T_1}{2} + \frac{T_2}{2} \Rightarrow = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

57. A

V = const. $\qquad \Rightarrow a = 0 \qquad$ No effect

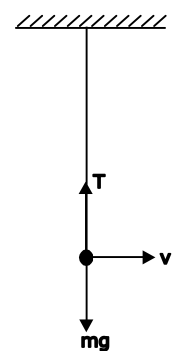
$T_1 = T = 2\pi\sqrt{\frac{\ell}{g}} \qquad g_{eff} = (g + a)$



$T_2 = 2\pi\sqrt{\frac{\ell}{g+a}}$

$T_1 > T_2$

58. C



$$T_{max} = mg + \frac{mv^2}{\ell}$$

$v = A\omega$

$\omega = \sqrt{g/\ell}$

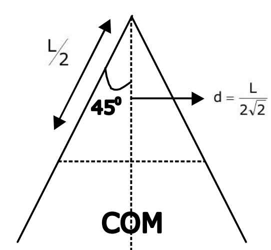
$v^2 = \frac{g}{\ell}A^2$

$T_{max} = mg[1 + (A/\ell)^2]$

59. B

$T = 2\pi\sqrt{\frac{I}{mgd}} \qquad I = \frac{2md^2}{3}$

$$T = 2\pi\sqrt{\frac{2md^2}{3}\frac{2\sqrt{2}}{(2m)gd}}$$

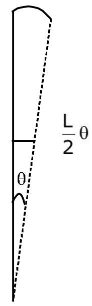


$$T = 2\pi\sqrt{\frac{2\sqrt{2}d}{3g}}$$

60. C

$$KL^2\theta + \frac{KL^2\theta}{4} > mg.\frac{L}{2}.\theta$$

K> 2mg/5L



61. A

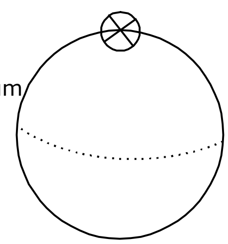
$$\tau_{net} = KL^2\theta + \frac{KL^2\theta}{4} - mg.\frac{L\theta}{2} \quad \because K = mg/L$$

$$I = mL^2/3 \Rightarrow \quad \tau_{net} = \frac{3}{4}KL^2\theta \;\Rightarrow\; K_{eq} = \frac{3}{4}KL^2$$

$$\omega = \sqrt{\frac{K_{eq}}{I}} = \frac{3}{2}\left(\frac{K}{m}\right)^{1/2}$$

62. A
T = 2 sec.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



$$2 = 2\pi\sqrt{\frac{2mR^2}{mgR}} \qquad R = .5m$$

63. D

$$T_A = 2\pi\sqrt{\frac{I_A}{mgx}} \quad \text{-----}\ (1)$$

$$T_B = 2\pi\sqrt{\frac{I_B}{mg(0.25-x)}} \quad \text{-----}(2)$$

Eq. (1) & (2)

$$\frac{3}{4} = \sqrt{\frac{I_A(0.25-x)}{I_B x}} \Rightarrow \frac{3}{4} = \sqrt{\frac{9}{4}\left(\frac{0.25-x}{x}\right)}$$

$$\Rightarrow \frac{9}{16} = \frac{9}{4}\left(\frac{0.25-x}{x}\right) \Rightarrow x = 0.20m$$