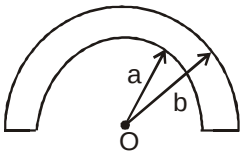
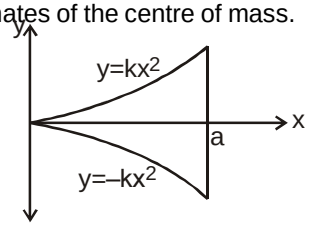
Exercise - III — Subjective Problem

1. The mass of an uniform ladder of length *l* increases uniformly from one end A to the other end B,
(a) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end.
(b) find the position of the centre of mass from end A.

2. Find the distance of centre of mass of a uniform plate having semicircular inner and other boundaries of radii a and b from the centre O.

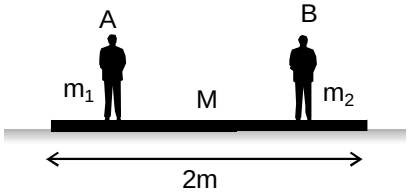


3. A thin sheet of metal of uniform thickness is cut into the shape bounded by the line x = a and y = ± k x^2, as shown. Find the coordinates of the centre of mass.



4. Two balls of equal masses are projected upwards simultaneously, one from the ground with speed 50 m/s and other from a 40m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.

5. In the figure shown, when the persons A and B exchange their positions, then

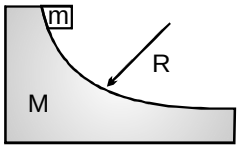


m_1=50kg, m_2 = 70kg, M = 80 kg

(i) the distance moved by the centre of mass of the system is_____.
(ii) the plank moves towards_____
(iii) the distance moved by the plank is _____.
(iv) the distance moved by A with respect to ground is _____
(v) the distance moved by B with respect to ground is _____.

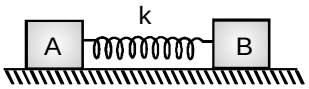
6. In the arrangement, m_A = 2 kg and m_B = 1 kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.

7. A small cube of mass m slides down a circular path of radius R cut into a large block of mass M. M rests on a table and both blocks move without friction. The blocks initially are at rest and m starts from the top of the path. Find the velocity v of the cube as it leaves the block.



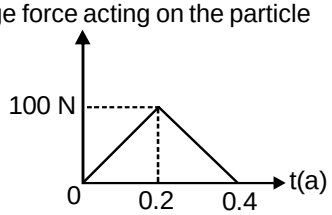
8. A (trolley + child) of total mass 200 kg is moving with a uniform speed of 36 km/h on a frictionless track. The child of mass 20 kg starts running on the trolley from one end to the other (10 m away) with a speed of 10 ms^{-1} relative to the trolley in the direction of the trolley's motion and jumps out of the trolley with the same relative velocity. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

9. In the figure shown the spring is compressed by 'x_0' and released. Two blocks 'A' and 'B' of masses 'm' and '2m' respectively are attached at the ends of the spring. Blocks are kept on a smooth horizontal surface and released.

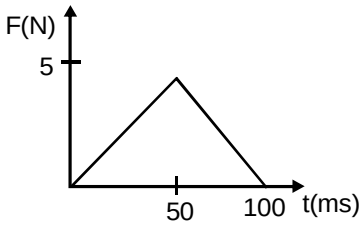


(a) Find the speed of block A by the time compression of the spring is reduced to $x_0/2$.
(b) Find the work done by the spring on 'A' by the time compression of the spring reduced to $x_0/2$.

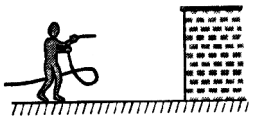
10. The figure showns the force versus time graph for a particle.
(i) Find the change in momentum Δp of the particle (ii) Find the average force acting on the particle



11. A force F acts on an object (mass = 1kg) which is initially at rest as shown in the figure. Draw the graph showing the momentum of the object varying during the time for which the force acts.



12. A man hosing down his driveway hits the wall by mistake. Knowing that the velocity of the stream is 25 m/s and the cross-sectional area of the stream is 300 mm^2, determine the force exerted on the wall. Assume that streamstrikes wall horizontally and after striking the wall, stream comes to rest. Also find the pressure exerted on the wall by stream.



13. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet.

14. A neutro intially at rest, decays into a proton, an electron and an antineutrio. The ejected electron has a momentum of $p_1 = 1.4 \times 10^{-28}$ kg-m/s and the antineutrino $p_2 = 6.5 \times 10^{-27}$ kg-m/s. Find the recoil speed of the proton (a) if the electron and the antineutrino are ejected along the the same direection and (b) if they are ejected along perpendicular direction. Mass of the proton $m_p = 1.67 \times 10^{-27}$ kg.

15. A steel ball of mass 0.5 kg is dropped from a height of 4 m on to a horizontal heavy steel slab. The collision is elastic and the ball rebounds to its original height.
(a) Calculate the impulse delivered to the ball during impact.
(b) If the ball is in contact with the slab for 0.002 s, find the average reaction force on the ball during impact.

16. A particle A of mass 2 kg lies on the edge of a table of height 1m. It is connected by a light inelastic string of length 0.7 m to a second particle B of mass 3 kg which is lying on the table 0.25 m from the edge (line joining A & B is perpendicular to the edge). If A is pushed gently so that it start falling from table then, find the speed of B when it starts to move. Also find the imulsive tension in the string at that moment.

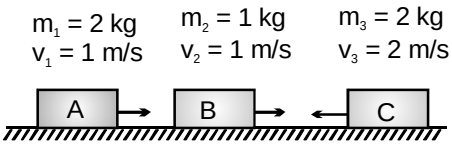
17. Two particles, each of mass *m*, are connected by a light inextensible string of length 2*l*. Initially they lie on a smooth horizontal table at points A and B distant *l* apart. The particle at A is projected across the table with velocity *u*. Find the speed with which the second particle begins to move if the direction of *u* is, (a) along BA, (b) at an angle of 120° with AB, (c) perpendicular to AB. In each case calculate (in terms of *m* and *u*) the impulsive tension in the string.

18. Two particle P of mass 2m and Q of mass m are subjected to mutual force of attraction and no other act on them. At time t = 0, P is at rest at a fixed O and Q is directly moving away from O with a speed 5 u. At a later instant when t = T before any collision has taken place Q is moving towards O with speed u.
(a) Find in terms of m and u the total work done by the forces of attraction during the time interval $0 \le t \le T$.
(b) At the instantt = T, impulses of magnitude J and K and applied to P and Q bringing them to rest. Find the values of J and K

19. A block of mass m moving with a velocity v, enters a region where it starts colliding with the stationary dust particles. If the desnsity of dust particles is ρ, & all colliding particle stick to its front surface of cross-sectional area A. The velocity of block after it has covered a distance x in this region is _____.

20. A football approaches a player at v = 12 m/s. At what speed u and in which direction should the player's foot move in order to stop the ball upon contact? Assume that the mass of the foot is much greater than that of the ball and that the collision is elastic.

21. Three carts move on a frictioless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.

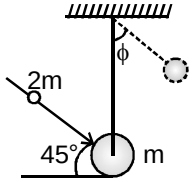


(a) Find loss of mechanical energy when B & C stick together.
(b) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).

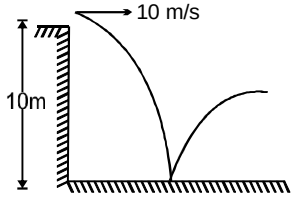
22. A sphere of mass m_1 in motion hits directly another sphere of mass m_2 at rest and sticks to it, the total kinetic energy after collision is 2/3 of their total K.E. before collision. Find the ratio of $m_1 : m_2$.

23. A body is thrown vertically upwards from ground with a speed of 10 m/s. If coefficient of restitution of ground, e = 1/2. Find
(a) the total distance travelled by the time it almost stops.
(b) time elapsed (after the ball has been thrown) when it is at its subsequent maximum height for the third time.

24. A ball of mass 'm' is suspended by a massless string of length 'ℓ' from a fixed point. A ball of mass 2m strikes in the direction of $\theta = 45°$ from horizontal and sticks to it.
(a) What should be the initial velocity of 2m so that system deflects by $\phi = \frac{\pi}{2}$
(b) If at $\phi = 60°$ the stirng is cut then what will be the velocity at highest point of trajectory.

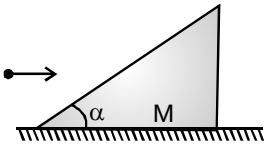


25. A ball is thrown horizontally from a cliff 10 m high with a velocity of 10 m/s. It strikes the smooth ground and rebounds as shown. The coefficient of restitution e for collision with the ground e = $1/\sqrt{2}$. Find



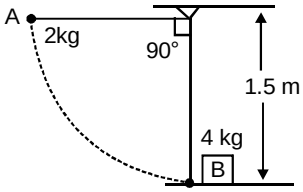
(a) velocity of ball just before striking ground.
(b) angle of velocity vector with horizontal before striking.
(c) angle of velocity vector with horizotal after striking.
(d) range of ball after first collision.

26. A wedge free to move of mass 'M' has one face making an angle α with horizotnal and is resting on a smooth rigid floor. A particle of mass 'm' hits the inclined face of the wedge with a horizontal velocity v_0. It is observed that the particle rebounds in vertical direction after impact. Neglect friction between particle and the wedge & take M = 2m, v_0 = 10 m/s, tan α = 2, g = 10 m/s^2



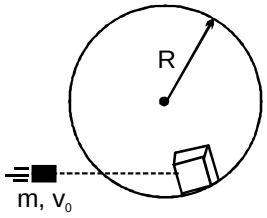
(a) Determine the coefficient of restitution for the impact.
(b) Assume that the inclined face of the wedge is sufficiently long so that the particle hits the same face once more during its downward motion. Calculate the time elapsed between the two impacts.

27. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and μ_k = 0.5 between B and the support, determine
(a) the velocity of A just after the impact



(b) the maximum displacement of B after the impact.

28. A small block of mass 2m initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms m, v_0, R and g.



(a) The speed of the masses immediately after the impact.

(b) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop.

29. A Cart of total mass M_0 is at rest on a rough horizontal road. It ejects bullets at rate of λ kg/s at an angle θ with the horizontal and at velocity 'u' (constant) relative to the cart. The coefficient of friction between the cart and the ground is μ. Find the velocity of the cart in terms of time 't'. The cart moves with sliding.