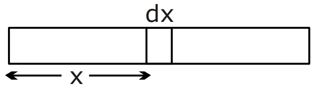
EXERCISE – III

SUBJECTIVE PROBLEMS

1.



$\lambda = Ax + B$

when $x = 0$, $\lambda = \lambda_0$

$\Rightarrow B = \lambda_0$

and when $x = \ell$, $\lambda = 2\lambda_0$

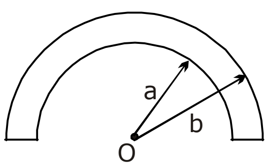
$\Rightarrow 2\lambda_0 = A\ell + \lambda_0$

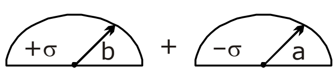
$$A = \frac{\lambda_0}{\ell}$$

$$\therefore \lambda = \frac{\lambda_0}{\ell} x + \lambda_0$$

$$x_{COM} = \frac{\int_0^{\ell} \lambda \,.\, dx \,.\, x}{\int_0^{\ell} \lambda \,.\, dx} = \frac{5}{9}\ell$$

2. So C.M. from 0





$$y = \frac{\sigma\left(\frac{\pi b^2}{2}\right)\left(\frac{4b}{3\pi}\right) - \sigma\left(\frac{\pi a^2}{2}\right)\frac{4a}{3\pi}}{\sigma\frac{\pi b^2}{2} - \sigma\left(\frac{\pi a^2}{2}\right)}$$

$$y = \frac{4}{3\pi}\left[\frac{b^3 - a^3}{b^2 - a^2}\right]$$

3. $y = \pm kx^2$

$$x_{COM} = \frac{\int_0^a x \; dm}{\int_0^a dm}$$

$\because dm = (2y\, dx)\sigma = +2kx^2dx$

$$\text{So } x_{COM} = \frac{\sigma 2k\int_0^a x \,.\, x^2 dx}{\sigma 2k\int_0^a x^2 dx}$$

$$= \frac{a^4 \times 3}{4 \times a^3} = \frac{3a}{4}$$

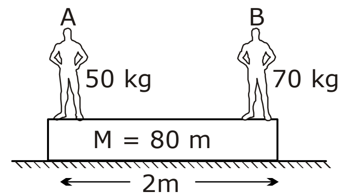
Now, $Y_{COM} = 0$ [$\because$ Symmetric for x-axis]

4. $V_{COM} = \dfrac{m \times 50 + m \times 30}{2m} = 40$ m/s

$V_{COM} = -g$

$H_{COM} = \dfrac{(40)^2}{2 \times 10} = 80$ m

5.



(i) zero

(ii) Right

(iii) $50(2 + x) + 80(x) = 70(2 - x)$

$100 + 50x + 80x = 140 - 70x$

$x(50 + 80 + 70) = 140 - 100$

$x = 0.2$ m or $x = 20$ cm

(iv) Distance moved by A with respect to ground is $= 2 + x = 2.2$ m

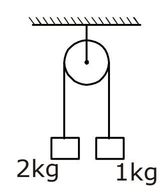
(v) Distance moved by B with respect to ground $= (2 - x)$ $= 2 - 0.2 = 1.8$ m

6. $2g - T = 2a$

$T - g = a$

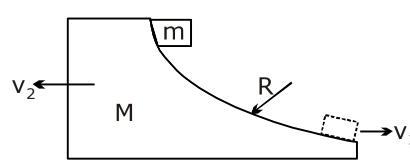
$g = 3a$

$a = g/3$



$$a_{COM} = \frac{2 \times \frac{g}{3} - \frac{g}{3}}{3} = \frac{g}{9}\downarrow \text{ (downwards)}$$

7. By momentum conservation



$mv_1 = Mv_2$

By energy conservation

$$mgR = \frac{1}{2}mv_1^2 - \frac{1}{2}Mv_2^2$$

$$mgR = \frac{1}{2}mv_1 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$$

$$v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$$

8. Given : Trolley + child = 200 kg ;
mass of child = 20 kg
u = 36 km/hr
Let the new velocity of trolley = V
New velocity of boy = (V + 10) m/s
By momentum conservation

$$200 \times 36 \times \frac{1000}{3600}$$

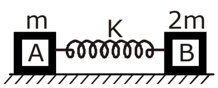
$= 20\,(v + 10) + (200 - 20)v$
$2000 = 20v + 200 + 200v - 20v$

$$v = \frac{1800}{200} = 9 \text{ m/s}$$

$$\text{Time taken} = \frac{\text{Length of Trolley}}{\text{Relative speed of boy}}$$

$$= \frac{10}{10} = 1 \text{ sec.}$$

Required distance = 9 m

9. 

(a) By momentum conservation
$mv = 2mv'$

$$v' = \left(\frac{v}{2}\right)$$

By energy conservation

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\frac{x_0^2}{4}$$

$$kx_0^2 = mv^2 + \frac{mv^2}{2} + \frac{kx_0^2}{4}$$

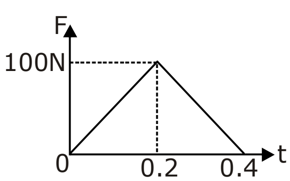
$$v = \sqrt{\frac{kx_0^2}{2m}}$$

(b) Work done = ΔK

$$\text{W.D.} = \frac{1}{2}m\left(\frac{kx_0^2}{2m}\right) - 0$$

$$= \frac{kx_0^2}{4}$$

10. (i) By graph



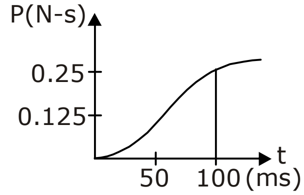
Area = ΔP

$$= \frac{1}{2} \times 0.2 \times 100 + \frac{1}{2} \times 0.2 \times 100$$

= 20 Ns.

(ii) $F_{avg.}\ \Delta t = \Delta P$
$F_{avg}\ \Delta t = 20$

$$F_{avg} = \frac{200}{4} = 50 \text{ N}$$

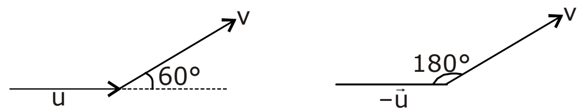
11. 

12. $F_{Th} = V_r \frac{dm}{dt}$ $\quad [\because m = \rho Ax \ \ \frac{dm}{dt} = \rho Av]$

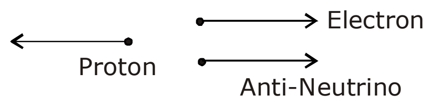
$= \rho Av^2$
$= 1000 \times 300 \times 10^{-6} \times (25)^2$
$= 187.5$ N

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{187.5}{300 \times (10^{-3})^2}$$

= 625 kPa

13. 

$$|\Delta P| = m\sqrt{m^2 + v^2 - uv}$$

14. 

(a) By momentum conservation
$m_p v_p = m_e v_e + m_A v_A$
$m_P v_P = P_1 + P_2$

$$v_p = \frac{P_1 + P_2}{m_P} = \frac{1.4 \times 10^{-28} + 65 \times 10^{-27}}{1.67 \times 10^{-27}}$$

= 12.3 m/s

(b) $\frac{\sqrt{P_1^2 + P_2^2}}{m_P} = 9.4$ m/s

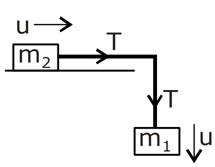
15. Speed at time of collision

$v = \sqrt{2gh} = \sqrt{80}$

(a) So I = ΔP = mv – m(–v)
= 2mv
= $4\sqrt{5}$ m/s

(b) I = $A_{avg}\ \Delta t = 4\sqrt{5}$

$$F_{avg} = \frac{4\sqrt{5}}{0.002} = 2000\sqrt{5} \text{ N}$$

16. 

(0.7 – 0.25)m = 0.45 m
So speed of A after height = 0.45 m
Let then,
$v^2 = 0^2 + 2g\,(0.45)$

$v = \sqrt{9} = 3\text{m/sec}$

Now, tension will give impulsive force and both block move with same velocity.

So $I = m_1(-u) - m_1(-v)$ .... (1)

$I = m_2 u$ .... (2)

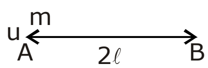
from (1) and (2)

$m_2 u = -m_1 u + m_1 v$

$$u = \frac{m_1 v}{m_1 + m_2} = \frac{2\times 3}{2+3} = \frac{6}{5}$$

and $I = m_2 u = 3 \times \frac{6}{5} = 3.6$

17. (a)



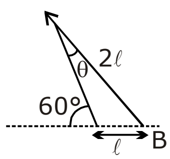
mu = 2mv'

$v' = \frac{u}{2}$

$I = \Delta P$

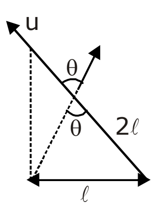
$I = mu/2$

(b) $mu\cos\theta = 2mv'$



$$v' = \frac{u\cos\theta}{2}$$

(c) $mu\cos\theta = 2mv'$



$$v' = u\frac{\sqrt{3}}{4}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

18. t = 0

t = T

m 5u = 2mv' – mu

v' = 3u

$$\text{W.D.} = \frac{1}{2}mu^2 + \frac{1}{2}.2m\,(3u)^2 - \frac{1}{2}m(5u)^2$$

$= -3mu^2$

19. This is a case of purely inelastic collision for x-direction.

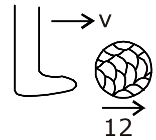
$p_i = p_f$

$mv = (m + \rho Ax)$

$$v = \frac{mv}{m + \rho Ax}$$

20. $e = \frac{v-0}{12-v}$

v = 6 m/sec.



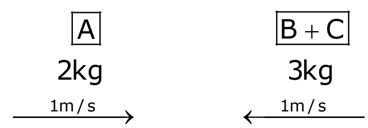
21. $m_2v_2 + m_3v_3 = (m_2 + m_3)v'$

$1 - 4 = 3v'$

$v' = -1$ m/s

$$\Delta K = \frac{1}{2}\times 1\times 1^2 + \frac{1}{2}\times 2\times 2^2 - \frac{1}{2}\times 3\times 1^2$$

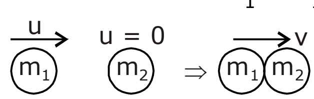
$\Delta K = 3$



$\Delta P_A = (2\times -1/5) - (2\times 1) = 12/5\text{Ns}$

$2\times 1 + 3\times -1 = 5V_1 \Rightarrow V_1 = -1/5$ m/s

22.



$P_i = P_f$

Given $m_1u = (m_1 + m_2)v$ ....(i)

$K.E_f = 2/3 K.E_i$

$$\frac{1}{2}(m_1 + m_2)v = \frac{2}{3}\times\frac{1}{2}m_1u^2 \quad ....(ii)$$

Solving $\frac{m_1}{m_2} = \frac{2}{1}$

23. (a) Total distance

$$= \frac{10^2}{g} + \frac{e^2 10^2}{g} + \frac{e^4 10^2}{g} + ...$$

$$= \frac{10^2}{g}\{1 + e^2 + e^4 + ...\}$$

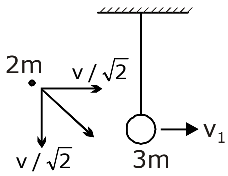
$$= 5\left\{\frac{1}{1-\frac{1}{4}}\right\} = \frac{40}{3}\text{m}$$

(b) Time elapsed = $\frac{2u}{g} + \frac{2(eu)}{g} + \frac{1}{2} \times \frac{2(e^2u)}{g}$

$= \frac{2 \times 10}{10} + \frac{2 \times 5}{10} + \frac{1}{2} \times 2 \times \frac{\left(\frac{1}{4} \times 10\right)}{10}$

$= \frac{13}{4} = 3.25$ s

24. By energy conservation



$\frac{1}{2} \cdot 3mv_1^2 = (3m)g\ell$

$v_1 = \sqrt{2g\ell}$

In horizontal direction, momentum conserved.

$2m\frac{v}{\sqrt{2}} = 3mv_1$

$v_1 = \frac{\sqrt{2}}{3}v$..... (2)

From (1) & (2)

$\sqrt{2g\ell} = \frac{\sqrt{2}}{3}v$

$v = \sqrt{9g\ell} = 3\sqrt{g\ell}$

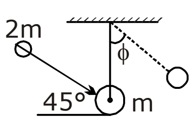
(b) By energy conservation

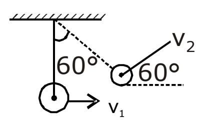
$\frac{1}{2} \cdot 3mv_1^2 = \frac{1}{2} \cdot 3mv_2^2 + 3mg\ell\,[1 - \cos 60°]$

$v_1^2 = v_2^2 + g\ell$

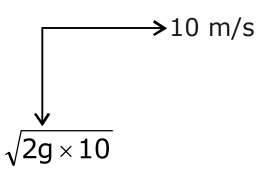
$\frac{2}{9} \times 9g\ell = v_2^2 + g\ell$

$v_2^2 = g\ell \Rightarrow v_2 = \sqrt{g\ell}$

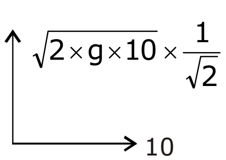




25.



after impact



$e = \frac{1}{\sqrt{2}}$

(a) Before first collision

$u_x = 10$ m/s $u_y = \sqrt{2gh}$

$v = \sqrt{u_x^2 + u_y^2} = \sqrt{(10)^2 + \left(10\sqrt{2}\right)^2} = 10\sqrt{3}$

(b) $\tan\theta = \frac{u_y}{u_x} = \frac{10\sqrt{2}}{10}$

$\theta = \tan^{-1}(\sqrt{2})$

(c) After striking

$V_x = u_x$; $v_y = eu_y$

$\tan\phi = \frac{v_y}{v_x} = \frac{eu_y}{u_x} = \frac{1}{\sqrt{2}} \times \frac{10\sqrt{2}}{10}$

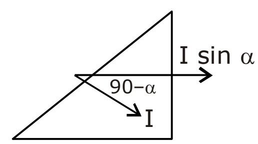
$\phi = 45°$

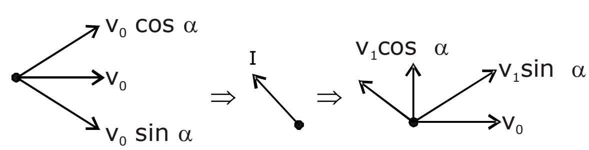
(d) $R' = \frac{2u_x(ev_y)}{g} = \frac{2 \times 10 \times \frac{1}{\sqrt{2}} \times 10\sqrt{2}}{10}$

R' = 20 m

26. $I \sin\alpha = 2mv_2$

$I = \frac{mv_0}{\sin\alpha}$





Now, $v_1 \sin\alpha = v_0 \cos\alpha$

$v_1 = v_0 \cot\alpha$

$e = \frac{v_1 \cos\alpha + v_2 \sin\alpha}{v_0 \sin\alpha}$

$e = \frac{v_0 \sin\alpha\,[\cot^2\alpha + 1/2]}{v_0 \sin\alpha}$

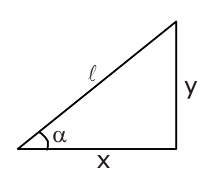
$= \frac{2\cot^2\alpha}{2} = \frac{3}{4}$

$v_{pw} = v_p - v_w$

$v_{pw} = v_0 \cot\alpha\,\hat{j} - \frac{v_0}{2}\,\hat{i}$

Let particle hit after length ℓ and time t on wedge, then

$\tan\alpha = \frac{y}{x}$ .... (i)



$-y = v_0 \cot\alpha\, t - \frac{1}{2}gt^2$ .....(2)

and $x = \frac{v_0}{2} \cdot t$ .....(3)

on solving $-2 = \frac{10\cot\alpha - 5t}{5}$

t = 3 sec.

27. (a) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.5}$

$v = \sqrt{30}$

$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$

$= \frac{2 \times \sqrt{30} + 4 \times \frac{3}{4}\left(-\sqrt{30}\right)}{2 + 4}$

$v_1 = -\frac{\sqrt{30}}{6} = -\sqrt{\frac{30}{36}}$

$v_1 = -\sqrt{\frac{g}{12}}$

(b) $v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$

$= \frac{2 \times \sqrt{30} + 2 \times \frac{3}{4}\left(\sqrt{30}\right)}{6}$

$= \frac{\sqrt{30}}{6}\left(2 + \frac{3}{2}\right) = \frac{7\sqrt{30}}{12}$

$S_{max} = \frac{u^2}{2a}$ $a = \frac{F_x}{m_2} = 5$

$= \frac{49 \times 30}{12 \times 12 \times 2 \times 5} = \frac{49}{48}$

28. (a) $mv_0 = 2mv'$

$v = \frac{v_0}{3} \geq \sqrt{5g\ell}$

(b) $mv_0 = 3mv'$

$v' = \frac{v_0}{3}$

For complete motion

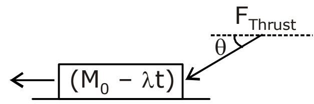
$\frac{v_0}{3} \geq \sqrt{5gR}$

or $v_0 = v_3\sqrt{5gR}$

29. $F_{thrust} = v_{rel}\frac{dm}{dt} = u\lambda$

At time t

$f_r = \mu[(M_0 - \lambda t)g + F_{Thrust}\sin\theta]$



So $F_{thrust}\cos\theta - f_r = (M_0 - \lambda t)a$

$\int_0^t \frac{\mu\lambda\left[\cos\theta - \mu\sin\theta\right]}{(M_0 - \lambda t)} - \mu g = \int_0^v dv$

$v = u[\cos\theta - \mu\sin\theta]\ln\left(\frac{M_0}{M - \lambda t}\right) - \mu gt$