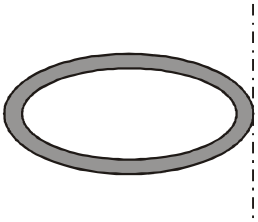
Exercise - III (Subjective Problems)

(A) MOMENT OF INERTIA

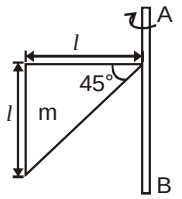
1. Find the moment of inertia of a uniform half-disc about an axis perpendicular to the plane and passing through its centre of mass. Mass of this disc is M and radius is R.

2. Find the moment of inertia of a pair of solid spheres, each having a mass m and radius r, kept in contact about the tangent passing through the point of contact.

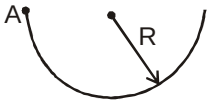
3. Find the radius of gyration of a circular ring of radius r about a line perpendicular to the plane of this ring and tangent to the ring.



4. Moment of inertial of a triangle plane of mass M shown in figure about vertical axis AB is :



5. A uniform rod of mass m is bent into the form of a semicircle of radius R. The moment of inertia of the rod about an axis passing through A and perpendicular to the plane of the paper is

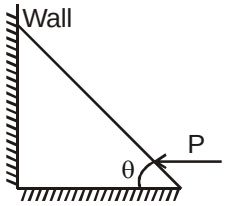


(B) TORQUE & PURE ROTATIONAL MOTION

6. A simple pendulum of length ℓ is pulled aside to made an angle θ with the vertical. Find the magnitude of the torque of the weight w of the bob about the point of suspension. When is the torque zero ?

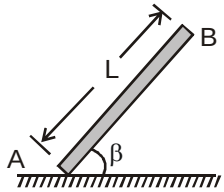
7. Two forces $\vec{F}_1 = 2\hat{i} - 5\hat{j} - 6\hat{k}$ and $\vec{F}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ are acting on a body at the points (1, 1, 0) and (0, 1, 2). Find torque acting on the body about point (–1, 0, 1).

8. Assuming frictionless contacts, determine the magnitude of external horizontal force P applied at the lower end for equilibrium of the rod. The rod is uniform and its mass is 'm'.



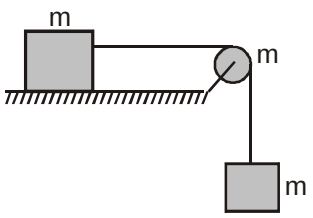
9. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis through its centre. A horizontal force of constant magnitude F acts on the rod at a distance of L/4 from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.

10. The uniform rod AB of mass m is released from rest when $\beta = 60°$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine (for the instant just after release)

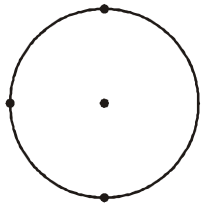


(a) The angular acceleration of the rod
(b) The normal reaction and the friction force at A.
(c) The minimum value of μ, compatible with the described motion.

11. Figure shows two blocks of mass m and m connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley (uniform disc) has mass m and it can freely rotate about this axis. Find the acceleration of the mass m assuming that the string does not slip on the pulley.



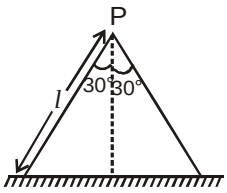
12. A solid cylinder of mass M = 1kg & radius R = 0.5m is pivoted at its centre & has three particles of mass m = 0.1 kg mounted at its perimeter as shown in the figure. The system is originally at rest. Find the angular speed of the cylinder, when it has swung through 90° in anticlockwise direction.



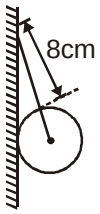
13. A cube is in limiting equilibrium on an inclined plane forming an angle of 30° with the horizontal. The line of action of the normal reaction of the plane on the cube is

14. A body weighs 6 gms when placed in one pan and 24 gms when placed on the other pan of a false balance. If the beam is horizontal when both the pans are ampty, the true weight of the body is :

15. An inverted "V" is made up of two uniform boards each weighing 200 N. Each side has the same length and makes an angle 30° with the vertical as shown in figure. The magnitude of the static frictional force that acts on each of the lower end of the V is

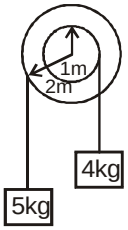


16. A uniform sphere of weight W and radius 5 cm is being held by a string as shown in the figure. The wall is smooth. The tension in the string will be

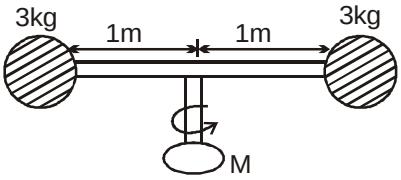


17. A light string is wrapped around a cylinder of mass 'm and radius 'R'. The string is pulled vertically upward to prevent the centre of mass from falling as the cylinder unwinds the string. Then length of the string unwound when the cylinder has reached a speed ω will be :

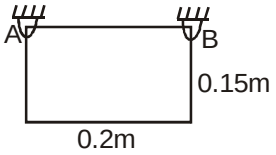
18. The moment of inertia of the pulley system as shown in the figure is 4 kgm^2. The radii of bigger and smaller pulleys 2m and 1m respectively. The angular acceleration of the pulley system is



19. The two small spheres each have a mass of 3 kg and are attached to the rod of negligible mass. A torque M = 8t Nm, where t is in seconds is applied to the rod. Find the value of time when each sphere attains a speed of 3 m/s starting from rest.

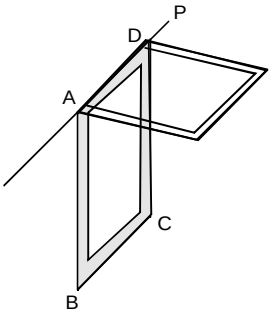


20. A rectangular plate of mass 20 kg is suspended from points A and B as shown. If pin B is removed determine the initial angular acceleration (in rad/s^2) of plate. (g = 10m/s^2)

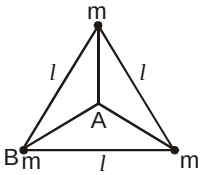


21. A solid homogeneous cylinder of height *h* and base radius *r* is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = a + bt^2$. If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

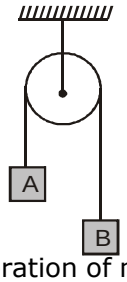
22. A square frame made up of a wire of mass m & length *l* is held in horizontal plane. It is free to rotate about AD. If the frame is released, the work done by gravity during the time frame rotates through an angle of 90° is equal to



23. Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A is equal to

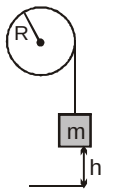


24. In the figure A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find :



(i) the linear acceleration of mass B.
(ii) the number of revolutions made by the disc at the end of 10 sec. from the start.
(iii) the tension in the string segment supporting the block A.

25. A mass m is attached to a pulley through a cord as shown in the fig. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



(C) ANGULAR MOMENTUM

26. A particle having mass 2 kg is moving along straight line 3x+ 4 y = 5 with speed 8m/s. Find angular momentum of the particle about origin, x and y are in meters.

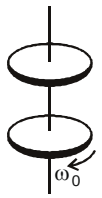
27. A particle having mass 2 kg is moving with velcoity $(2\hat{i}+3\hat{j})m/s$. Find angular momentum of the particle about origin when it is at (1, 1, 0).

28. A uniform square plate of mass 2.0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0.10 N.m. Calculate the angular momentum and the kinetic energy of the plate at the end of the fifth second after the start.

29. A wheel of moment of inertia 0.500 kg-m^2 and radius 20.0 cm is rotating about its axis at an angular speed of 20.0 rad/s. It picks up a stationary particle of mass 200 g at its edge. Find new angular speed of the wheel.

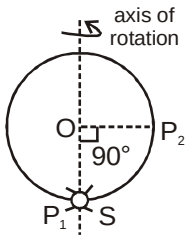
30. A uniform circular disc can rotate freely about a rigid vertical axis through its centre O. A man stands at rest at A on the edge due east of O. The mass of the disc is 22 times the mass of the man. The man starts walking anticlockwise. When he reaches the point A after completing one rotation relative to the disc he will be :

31. Two identical disks are positioned on a vertical axis. The bottom disk is rotating at angular velocity ω_0 and has rotational kinetic energy KE_0. The top disk is initially at rest. It is allowed to fall, and sticks to the bottom disk. What is the rotational kinetic energy of the system after the collision?



32. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally

reaches the point P_2 (as shown in figure). The final angular velocity of the ring will be equal to



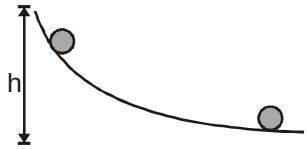
33. A particle of mass 10 kg is moving with a uniform speed of 6m/sec. in x-y plane along the line 3y = 4x+ 10 the magnitude of its angular momentum about the origin in kg –m^2/s is

(D) COMBINED TRANSLATIONAL + ROTATIONAL MOTION

34. A sphere of mass m rolls on a plane surface. Find its kinetic energy at an instant when its centre moves with speed v.

35. A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25 m/s, what is the speed of the highest point ?

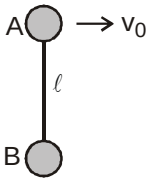
36. A small spherical ball is released from a point at a height h on a rough track shown in figure. Assuming that it does not slip anywhere, find its linear speed when it rolls on the horizontal part of the track.



37. A sphere starts rolling down an incline of inclination θ. Find the speed of its centre when it has covered a distance ℓ.

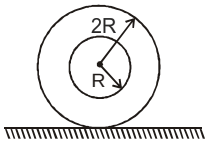
38. A solid uniform sphere of mass m is released from rest from the rim of a hemispherical cup so that it rolls without sliding along the surface. If the rim of the hemisphere is kept horizotnal, find the normal force exerted by the cup on the ball when the ball reaches the bottom of the cup.

39. Two small spheres A & B respectively of mass m & 2m are connected by a rigid rod of length ℓ & negligible mass. The two spheres are resting on a horizontal, frictionless surface. When A is suddenly given the velocity v_0 as shown. Find velocities of A & B after the rod has rotated through 180°.



40. A uniform rod of mass m and length ℓ is struck at an end by a force F perpendicular to the rod for a short time interval t. Calculate
(a) the speed of the centre of mass,
(b) the angular speed of the rod about the centre of mass,
(c) the kinetic energy of the rod and
(d) the angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that t is so small that the rod does not appreciably change its direction while the force acts.

41. A hollow cylinder with inner radius R, outer radius 2R mass M is rolling with speed of its axis v. Its kinetic energy is



42. The cylinder shown, with mass M and radius R, has a radially dependent density. The cylinder starts from rest and rolls without slipping down an inclined plane of height H. At the bottom of the plane of height H. At the bottom of the plane its translational speed is $(8gH/7)^{1/2}$. Which of the following is the rotational inertia of the cylinder?