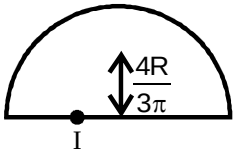
EXERCISE – III

SUBJECTIVE PROBLEMS

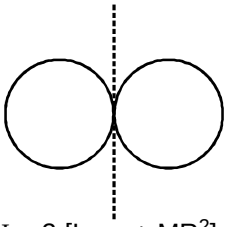
1.



$$I_{COM} = I - M\left(\frac{4R}{3\pi}\right)^2$$

$$= \frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2$$

2.

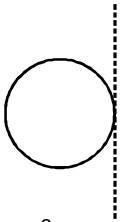


$$I = 2\,[I_{COM} + MR^2]$$

$$= 2\left[\frac{2}{5}MR^2 + MR^2\right]$$

$$= \frac{14}{5}\,MR^2$$

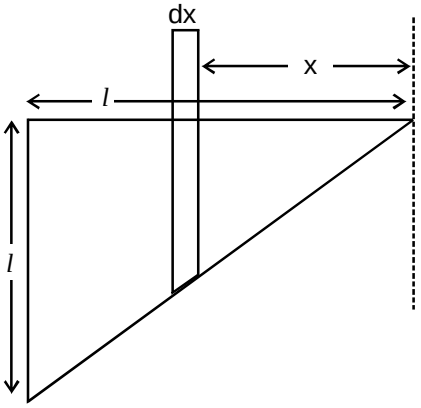
3.



$$MK^2 = MR^2 + MR^2$$

$$K = \sqrt{2}\,R$$

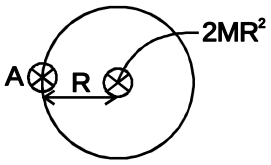
4.



$$\sigma = \frac{M}{\frac{1}{2}\ell^2}, \qquad dI = \sigma x dx\, x^2$$

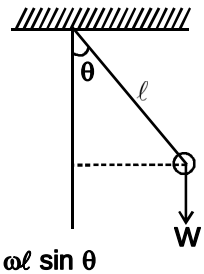
$$= \sigma\int_0^{\ell} x^3\,dx\ , = \frac{\sigma\ell^4}{4}, = \frac{2M}{\ell^2}\ .\ \frac{\ell^4}{4} = \frac{M\ell^2}{2}$$

5.



$$\frac{2MR^2 + 2MR^2}{2}$$

6.



$\tau = \omega\ell \sin\theta$

$\tau = 0$

When $\theta = 0°$

7. $\vec{r}_1 = [1-(-1)]\,\hat{i} + (1-0)\,\hat{j} + (0-1)\,\hat{k}$

$= 2\hat{i} + \hat{j} - \hat{k}$

$\vec{r}_2 = [0-(-1)]\,\hat{i} + (1-0)\,\hat{j} + (2-1)\,\hat{k}$

$= \hat{i} + \hat{j} + \hat{k}$

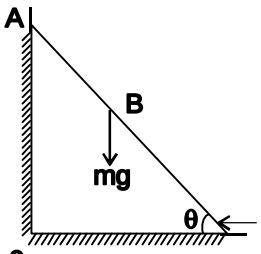
So $\vec{\tau} = \vec{r}_1 \times \vec{f}_1 + \vec{r}_2 \times \vec{f}_2$

$= (2\hat{i} + \hat{j} - \hat{k}) \times (2\hat{i} - 5\hat{j} - 6\hat{k})$

$+ (\hat{i} + \hat{j} + \hat{k}) \times (-\hat{i} + 2\hat{j} - \hat{k})$

$\vec{\tau} = -14\hat{i} + 10\hat{j} - 9\hat{k}$

8.



$\tau_A = 0$

$mg\,\frac{\ell}{2}\cos\theta - P\,\ell\sin\theta = 0$

$p = \frac{mg}{2}\cot\theta$

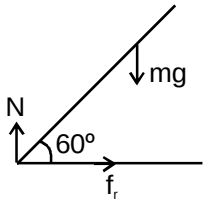
9. $\tau = F\,.\,\frac{L}{4} = I\alpha$

$\alpha = \frac{FL}{4I} = \frac{FL}{\frac{4ML^2}{12}} = \frac{3F}{ML}$

$\theta = \frac{1}{2}\ \alpha t^2$

$\theta = \frac{1}{2} \cdot \frac{3F}{ML}\ t^2$

10. (a)



$\tau = mg\ \frac{\ell}{2}\ \cos 60°$

$= \frac{m\ell^2}{3} \cdot \alpha$

$\Rightarrow\ \alpha = \frac{3g}{4\ell}$ (clock wise)

(b) $mg - N = m\left(\frac{\alpha\ell}{2}\right)\cos 60°$ [In vertical]

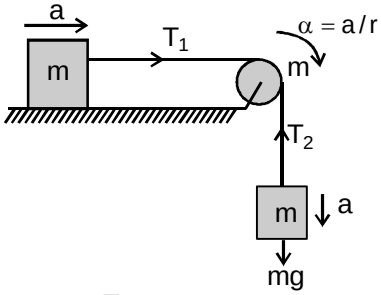
$N = mg - m\ \frac{3g}{4\ell} \cdot \frac{\ell}{2} \cdot \frac{1}{2}$

$N = \frac{13}{16}\ mg$

$f_r = m\left(\frac{\alpha\ell}{2}\right)\sin 60° = \frac{3mg}{4\ell} \cdot \frac{\ell}{2} \cdot \frac{\sqrt{3}}{2}$

$f_r = \frac{3\sqrt{3}}{16}\ mg$

11.



$mg - T_2 = ma$ ...(1)

$(T_2 - T_1)r = I\alpha = \frac{mr^2}{2} \cdot \frac{a}{r}$

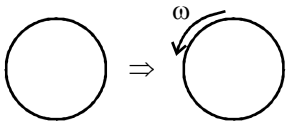
$T_2 - T_1 = \frac{ma}{2}$ ...(2)

$T_1 = ma$ ...(3)

$mg - ma - ma = \frac{ma}{2}$

$mg = 2\ ma + \frac{ma}{2}$

12.



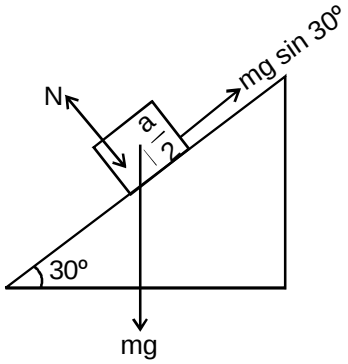
$I = \left[3mR^2 + \frac{MR^2}{2}\right] = \frac{(6m+M)R^2}{2}$

$mgR = \frac{1}{2}\ I\omega^2 \Rightarrow \quad \omega^2 = \frac{2mgR}{I} = \frac{4mg}{(6m+M)R}$

$= \frac{4 \times 0.1 \times 10}{(0.6+1)(0.5)} = 5$

$\omega^2 = 5\ \Rightarrow\ \omega = \sqrt{5}$ rad/sec.

13.

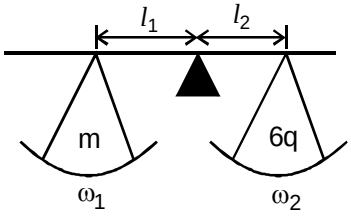


$Nx = \frac{a}{2}\ mg \sin 30°$, $N = mg \cos 30°$

Now, $mg \cos 30° \cdot x = \frac{a}{2}\ mg \sin 30°$

$x = \frac{a}{2\sqrt{3}}$

14.



$\omega_1\ell_1 = \omega_2\ell_2$

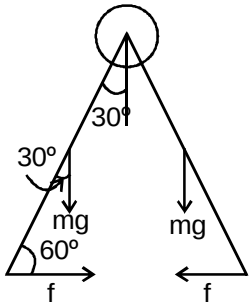
$(m + \omega_1)\ \ell_1 = (6 + \omega_2)\ \ell_2$

$m\ell_1 = 6\ell_2$ ...(1)

Similarly $24\ell_1 = m\ell_2$ ...(2)

$m^2 = 6 \times 24 \quad \Rightarrow\ m = 12$ gm

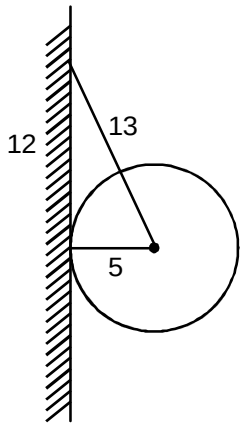
15.



$f \cdot \sin 60^\circ \cdot \ell = mg \sin 30 \times \frac{\ell}{2}$

$f = \frac{mg}{2\sqrt{3}} = \frac{100}{\sqrt{3}}$ N

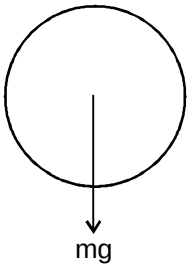
16.



$T \cos\theta = \omega$

$T \cdot \frac{12}{13} = \omega,\ T = \frac{13\omega}{12}$

17.

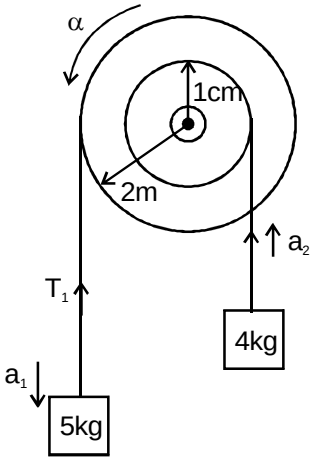


$T = mg, \quad mg \cdot R = \frac{mR^2}{2}\alpha$

$\alpha = \frac{2g}{R}, \quad \omega^2 = 2 \cdot \frac{2g\theta}{R}$

$\theta = \frac{\omega^2R^2}{4g}$, $\begin{cases} 2\pi R \to 2\pi \\ 1 \to R \\ \theta \to \frac{\omega^2R^2}{4g} \end{cases}$

18.



$\alpha \cdot 2 = a_1$ ...(1)

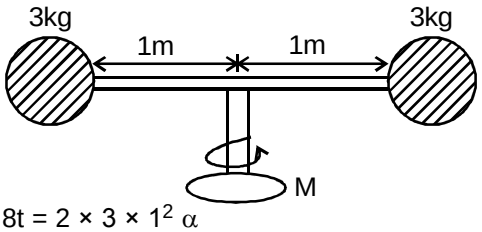
$\alpha \cdot 1 = a_2$ ...(2)

$5g - T_1 = 5a_1$ ...(3)

$T_2 - 4g = 4a_2$ ...(4)

$-T_1 \cdot 1 + T_1 \times 2 = 4\alpha$ ...(5)

19.



$8t = 2 \times 3 \times 1^2\ \alpha$

$\alpha = \frac{4t}{3}$

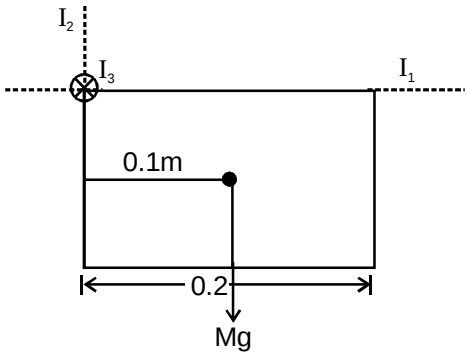
$v = R\omega \Rightarrow 3 = \omega \times 1$

$\Rightarrow \omega = 3$ rad/sec

$\int_0^3 d\omega = \int_0^t \frac{4tdt}{3}$

$3 = \frac{2t^2}{3} \Rightarrow t = 1.5\sqrt{2}$ sec

20.



$I_1 = \frac{20(0.2)^2}{3} = \frac{0.8}{3}$

$I_2 = \frac{20(0.15)^2}{3} = \frac{0.45}{3}$

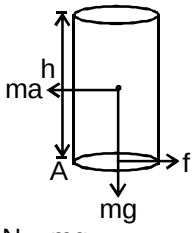
$e = I\alpha$

$I_3 = I_1 + I_2 = \frac{1.25}{3}$

$\because \quad e = I\alpha$

$200.01 = I_3\alpha \Rightarrow \alpha = 48^\circ$

21.



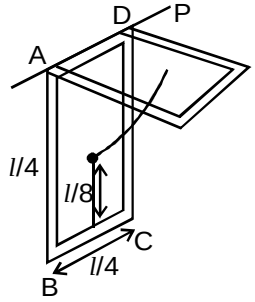
$N = mg$

$V = a + bt^2$

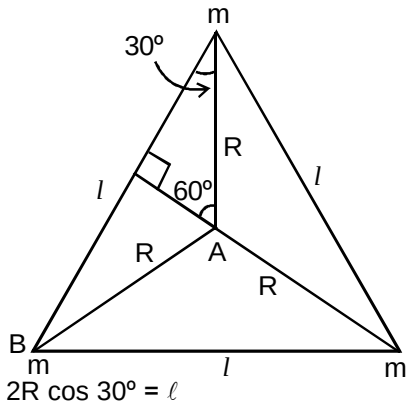
$a = \frac{dv}{dt} = 2bt$, $N \cdot 2r = ma\,\frac{h}{2} + mgr$

$mgr = mh\,bt$, $t = \frac{gr}{bh}$

22. $\omega = mg\left(\frac{\ell}{8}\right)$



23.



$2R\cos 30° = \ell$

$R = \frac{\ell}{\sqrt{3}}$

$I_A = 3m\,R^2 = 3m \cdot \frac{\ell^2}{3}$

$I_B = 2m\ell^2$

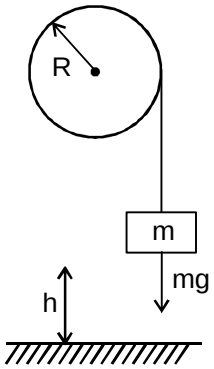
24. $4g - T_1 = 4a$ $\theta = \frac{1}{2} \propto (10)^2$

$T_2 - 2g = 2a$

$(T_1 - T_2)\,r = \frac{40r^2}{2} \cdot \frac{a}{r}$

$2g = 26a$, $a = \frac{10}{13}$ m/s^2

25.

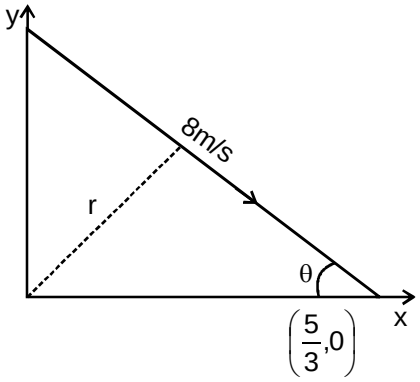


$mg - T = m \cdot a$

$\Rightarrow$ $mg - T = m \cdot a$

$T \cdot R = \frac{mR^2}{2} \cdot \alpha \Rightarrow T = \frac{ma}{2}$, $a = \frac{2m.g}{(2m+M)}$

26.



$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = 37°$

$L = mvr = 2 \times 8 \times \frac{5}{3} \sin\theta$

$= 2 \times 8 \times \frac{5}{3} \times \frac{3}{5} = 16$ kg m^2/s

27. $\vec{r} = \hat{i} + \hat{j}$

$\vec{L} = m(\vec{r} \times \vec{V})$

$= 2[(\hat{i} + \hat{j}) \times (2\hat{i} + 3\hat{j})]$

$= 2\hat{k}$ kg m^2/s

28. $I_d = \frac{ma^2}{12} = \frac{2\times100}{12} = \frac{50}{3}$ kg cm

$= \frac{50\times10^{-4}}{3}$ kg m^2

$0.10 = \frac{50\times10^{-4}}{3} \cdot \alpha$

$= \alpha = 60$ Rad/sec^2

$\omega = \alpha \times t = 60 \times 5 = 300$ Rad/sec

So $L = I\omega$

$= \frac{50\times10^{-4}}{3} \times 300$

$= 0.5$ kg $\frac{m^2}{sec}$

$E = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{0.5}{3}\times10^{-2}\right)(9 \times 10^4)$

$= 75$ J

29. $I_{new} = I_{old} + m_p R^2$

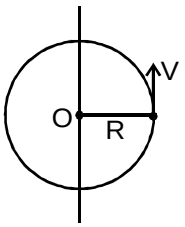
$= 0.5 + (0.2)\,(0.2)^2$

$= 0.5 + 0.008 = 0.508$

So $I\omega = I_{new}\,\omega_{new}$

$$\omega_{new} = \frac{0.5}{0.508} \times 20 = 19.7 \text{ rad/sec}$$

30.



$V \rightarrow$ velocity of man w.r.t. ground

$$t = \frac{2\pi R}{v}$$

$0 = mvR - I\omega$

$$mvR = \left(\frac{22mR^2}{2} + mR^2\right)\omega$$

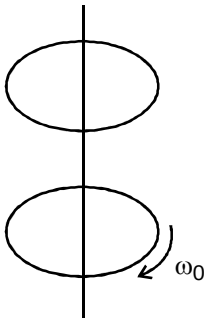
$$\omega = \frac{mvR \times 2}{24mR^2} = \frac{v}{12R}$$

$$\because \quad \omega = \frac{\theta}{t} \quad \Rightarrow \quad \theta = \omega t$$

$$\Rightarrow \quad \theta = \frac{v}{12R} \times \frac{2\pi R}{v}$$

$$\theta = \frac{\pi}{6} = 30^\circ$$

31.



$$K.E_i = \frac{1}{2}\, I\omega_0^{2}, \qquad = \frac{1}{2}\,\frac{MR^2}{2}\,\omega_0^{2}$$

$I_1\omega_0 = I_2\omega$

$$I_1 = \frac{MR^2}{2}, \qquad I_2 = \frac{2MR^2}{2}$$

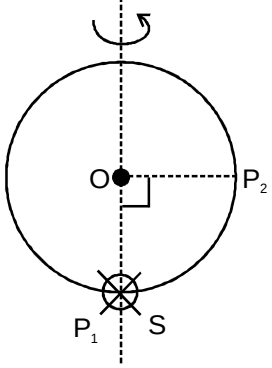
$$\frac{MR^2}{2}\,\omega_0 = \frac{2MR^2}{2}\,\omega', \qquad \omega' = \frac{\omega_0}{2}$$

$$K.E._f = \frac{1}{2}\, I'\,\omega^2 = \frac{1}{2} \cdot \frac{2MR^2}{2} \cdot \frac{\omega_0^2}{4}$$

$$= \frac{1}{2}\,\frac{MR^2\omega_0^2}{4}, \qquad K.E._f = \frac{1}{2}\, K.E._i$$

Axis of Rotation

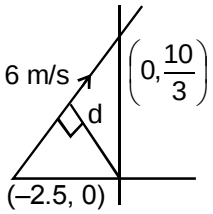
32.



$$\frac{MR^2}{2}\,\omega_0 = \left(\frac{MR^2}{2} + MR^2\right)\omega$$

$$\frac{\omega_0}{2} = \frac{3}{2}\,\omega \quad , \qquad \omega = \frac{\omega_0}{3}$$

33.



Angular momentum
= mvd

34. $\omega = v/R$

$$\text{So} \quad K.E. = \frac{1}{2} I\omega^2 + \frac{1}{2}\, mV^2$$

$$= \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{V^2}{R^2} + \frac{1}{2}mV^2$$

$$= \frac{7}{10}\, mV^2$$

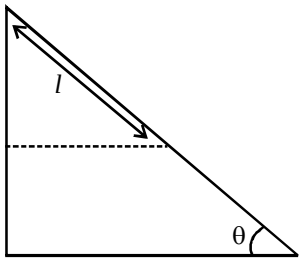
35. $2V = 50$ m/s

36. $mgh = \frac{1}{2}\, I\omega^2 + \frac{1}{2}\, mV^2, \qquad \omega = \frac{V}{R}$

$$mgh = \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot \frac{V^2}{R^2} + \frac{1}{2}\, mV^2,$$

$$= \frac{7}{10}\, mV^2, \qquad V = \sqrt{\frac{10}{7}gh}$$

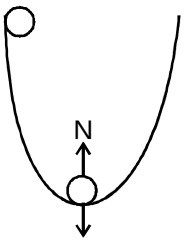
37.



$mg\ell \sin\theta = \frac{7}{10}\ mV^2$

$V = \sqrt{\frac{10}{7} g\ell \sin\theta}$

38.



$mg + \frac{mV^2}{R}$

$mgR = \frac{7}{10}\ mV^2$

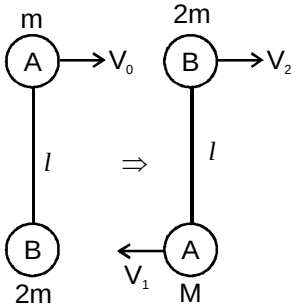
$\Rightarrow\ mV^2 = \frac{10}{7}\ mgR$

$\therefore\ N = mg + \frac{mV^2}{R}$

$N = m\ g + \frac{10}{7}\ mg$

$= \frac{17}{7} mg$

39.



$mV_0 = 2mV_2 - mV_1$

$V_0 = 2V_2 - V_1$ ...(1)

$\frac{1}{2} mV_0^2 = \frac{1}{2}\ .\ 2mV_2^2 + \frac{1}{2} mV_1^2$

$V_0^2 = 2V_2^2 + V_1^2$ ...(2)

From (1) & (2)

$V_0^2 = 2\left(\frac{V_0 + V_1}{2}\right)^2 + V_1^2$

$2V_0^2 = (V_0 + V_1)^2 + 2V_1^2$

$2V_0^2 = V_0^2 + V_1^2 + 2V_1V_0 + 2V_1^2$

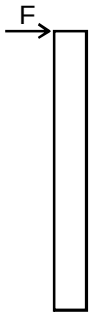
$3V_1^2 + 2V_1V_0 - V_0^2 = 0$

$V_1 = \frac{-2V_0 \pm \sqrt{4V_0^2 - 4\times3\times(-V_0^2)}}{2\times3}$

$V_1 = \frac{-2V_0 \pm 4V_0}{6} = \frac{V_0}{3}, -V_0,\ V_2 = \frac{2V_0}{3}$

40. (a) $Ft = mV_C$

$V_C = \frac{Ft}{m}$

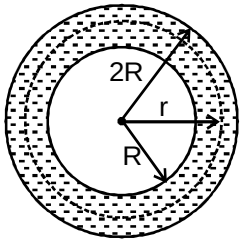


(b) $\tau_t = F.\frac{\ell}{2}t = I_C\ \omega$

$\omega = \frac{F\ell t}{2} = I_C\omega$

$\omega = \frac{F\ell t}{2\frac{m\ell^2}{12}} = \frac{6Ft}{m\ell}$

41.



$\sigma = \frac{M}{\pi(4R^2 - R^2)}$

$= \frac{M}{3\pi R^2}$

$dm = \frac{M}{3\pi R^2}\ .\ 2\pi r dr$

$dI = \frac{2Mr}{3R^2}\ .\ r^2 dr$

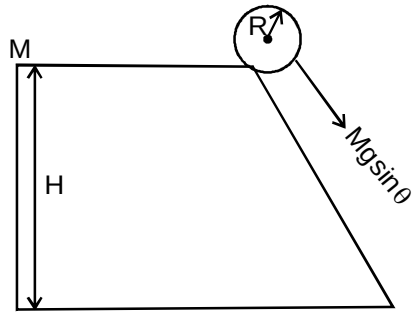
$I = \frac{2}{3}\ \frac{M}{R}\ \int_R^{2R} r^3 dr = \frac{2}{3}\ \frac{M}{R^2}\ \frac{R^4(16-1)}{4}$

$= \frac{10}{4}\ MR^2 = \frac{5}{2}\ MR^2$

$\therefore\ K.E. = \frac{1}{2}\ MV^2 + \frac{1}{2}\ .\ \frac{10}{4}\ MR^2\ \frac{V^2}{4R^2}$

$= \frac{13}{16} MV^2$

42.



$Mg \sin \theta - f = Ma$

$f \cdot R = CMR^2 \cdot \frac{a}{R}$

$a = \frac{g\sin\theta}{(c+1)}$

$\omega_f = 0$

$MgH = \frac{1}{2} CMR^2 \cdot \frac{V^2}{R^2} + \frac{1}{2} MV^2$

$MgH = \frac{1}{2} MV^2 (C + 1)$

$C + 1 = \frac{14}{8} \quad \Rightarrow \quad C = \frac{3}{4}$