Exercise - III (Subjective Problems)

LINEAR S.H.M

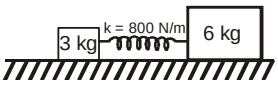
1. The equation of a particle executing SHM is $x = (5m)\sin\left[(\pi s^{-1})t + \frac{\pi}{6}\right]$. Write down the amplitude, phase constant, time period and maximum speed.

2. A particle having mass 10 g oscillates according to the equation x = (2.0 cm) sin [100 s^{-1}] t + $\frac{\pi}{6}$]. Find (a) the amplitude, the time period and the force constant (b) the position, the velocity and the acceleration at t = 0.

3. The equation of motion of a particle started at t = 0 is given by x = 5 sin (20 t + π/3) where x is in centimetre and t in second. When does the particle.
(a) first come to rest ?
(b) first have zero acceleration ?
(c) first have maximum speed ?

4. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

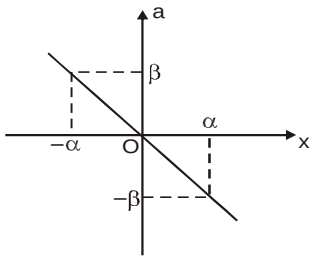
5. The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6 cm and then released. Find



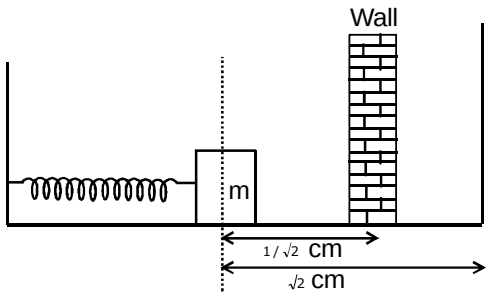
(a) Time period (b) Amplitude of 3 kg block
(c) Maximum momentum of 6 kg block

6. A body undergoing SHM about the origin has its equation is given by x = 0.2 cos 5πt. Find its average speed from t = 0 to t = 0.7 sec.

7. The acceleration-displacement (a – x) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



8. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it make elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.



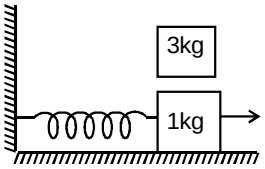
9. A force f = −10x + 2 acts on a particle of mass 0.1 kg, where 'k' is in m and F in newton. If it is released from rest at x = −2 m, find :
(A) Amplitude (b) Time period
(c) Equation of motion

10. Potential energy (U) of a body of unit mass moving in a one-dimension conservative force fileld is given by, U = (x^2 – 4x + 3). All units are in S.I.
(i) Find the equilibrium position of the body.
(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its timeperiod.
(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

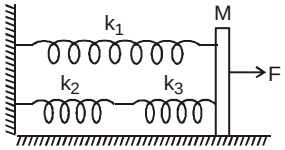
11. The resulting amplitude A′ and the phase of the vibrations δ S = A cos (ωt) + $\frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right)$ + $\frac{A}{4}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right)$ = A′ cos (ωt + δ) are ..... and ..... respectively.

12. A body is executing SHM under the action of force whose maximum magnitude is 50N. Find the magnitude of force acting on the particle at the time when its energy is half kinetic and half potential.

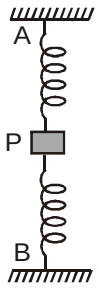
13. A 1kg block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and the amplitude of the motion.



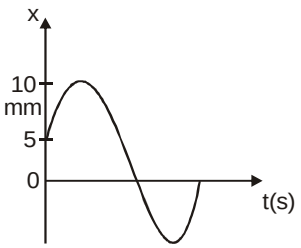
14. The springs shown in the figure are all unstretched in the beginning when a man starts pulling the block. The man exerts a constant force F on the block. Find the amplitude and the frequency of the motion of the block.



15. Two identical springs are attached to a small block P. The other ends of the springs are fixed at A and B. When P is in equilibrium the extension of top spring is 20 cm and extension of bottom spring is 10 cm. Find the period of small vertical oscillations of P about its equilibrium position. (use $g = 9.8\ m/s^2$)

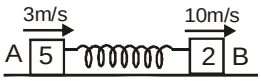


16. The figure shows the displacement - time graph of a particle executing SHM. If the time period of oscillation is 2s, then the equation of motion is given by x = ...



17. Two particles A and B execute SHM along the same line with the same amplitude a, same frequency and same equilibrium position O. If the phase difference between them is $\phi = 2\ \sin^{-1}(0.9)$, then find the maximum distance between the two.

18. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120 N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(a) Find the maximum extension of the spring.
(b) When does the first maximum compression occurs after start.

19. The motion of a particle is described by $x = 30 \sin(\pi t + \pi/6)$, where x is in cm and t in sec. Potential energy of the particle is twice of kinetic energy for the first time after t = 0 when the particle is at position ..... after ..... time.

20. A particle is performing SHM with accleration $a = 8\pi^2 - 4\pi^2 x$ where x is coordinate of the particle w.r.t. the origin. The parameters are in S.I. units. The particle is at rest at x = –2 at t = 0. Find coordinate of the particle w.r.t. origin at any time.

21. (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$ J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.
(b) A simple pendulum of length l = 0.5 m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

22. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point x = 0.04m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is _____.

23. A body of mass 1 kg is suspended from a weightless spring having force constant 600 N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0 m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

24. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.

25. A particle of mass m moves in a one-dimensional potential energy $U(x) = -ax^2 + bx^4$, where 'a' and 'b' are positive constants. Then what is the angular frequency of small oscillations about the minima of the potential energy.

26. A pendulum having time period equal to two seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a seconds pendulum at a place where $g = \pi^2$ m/s^2

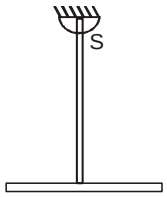
27. The angle made by the string of a simple pendulum with the vertical depends on time as $\theta = \frac{\pi}{90}\sin[(\pi s^{-1})t]$. Find the length of the pendulum if $g = \pi^2$ m/s^2.

28. A pendulum is suspended in a lift and its period of oscillation is T_0 when the lift is stationary.
(i) What will the period T of oscillation of pendulum be, if the lift begins to accelerate downwards with an acceleration equal to $\frac{3g}{4}$?
(ii) What must be the acceleration of the lift for the period of oscillation of the pendulum to be $\frac{T_0}{2}$?

29. A simple pendulum of length ℓ is suspended through the ceiling of an elevator. Find the time period of small oscillations if the elevator (a) is going up with an acceleration a_0 (b) is going down with an acceleration a_0 and (c) is moving with a uniform velocity.

30. A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3.99 seconds. Making an approximate analysis, find the acceleration of the car.

31. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of supension). Find the time period of such small oscillations.



32. A simple pendulum has a time period T = 2 sec when it swings freely. The pendulum is hung as shown in figure. so that only one-fourth of its total length is free to swing to the left of obstacle. It is displaced to position A and released. How long does it take to swing to extreme displacement B and return to A? Assume that displacement angle is always small.