EXERCISE – III

SUBJECTIVE PROBLEMS

1. $x = (5m)\sin\left[\left(\pi s^{-1}\right)t + \frac{\pi}{6}\right]$

$\because x = A\sin(\omega t + \phi)$

$\Rightarrow A = 5m, \phi = \frac{\pi}{6}, T = \frac{2\pi}{\omega} = 2\,\text{sec.}$ and

$V_{max} = A\omega = 5\pi m/s$

2. $x = 2.0\sin\left[\left(100 s^{-1}\right)t + \frac{\pi}{6}\right]$

A=0.2 cm, w=100, M=10gm

$v = (2.0\times 100)\ \text{cm/s}\cos\left[\left(100 s^{-1}\right)t + \frac{\pi}{6}\right]$

$a = 2\times 10^4 cm/s^2\left[-\sin\left[\left(100 s^{-1}\right)t + \frac{\pi}{6}\right]\right]$

$\Rightarrow a = -10^4 x \ \because F = -Kx$ and F=100 N/m

$\Rightarrow K = 10^4 \times M = 10^4 \times \frac{1}{100} = 100 N/m$

$\Rightarrow T = \frac{2\pi}{w} = \frac{2\pi}{100} = \frac{\pi}{50}\,\text{sec.}$

a at t=0 $\Rightarrow a = -200\ m/s^2 \sin\frac{\pi}{6}$

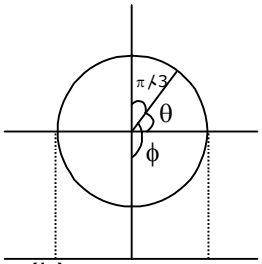
$= -100\ m/s^2$

3. ω= 20

(a) $\theta = \frac{\pi}{6} = \omega t$

$t = \frac{\pi}{120}$

(b) $\phi = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}\quad t = \frac{2\pi}{3\times 20} = \frac{\pi}{30}$



(c) Same as (b)

4. $K_1\ell_1 = K_2\ell_2 = K\ell$

$\Rightarrow \frac{K_1}{K_2} = \frac{\ell_2}{\ell_1} = \frac{3}{1} \quad \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}} = \frac{1}{\sqrt{3}}$

5. $\mu = \frac{3\times 6}{3+6} = 2$ Kg. $\omega = \sqrt{\frac{k}{\mu}} = 20$

(a) $T = 2\pi\sqrt{\frac{\mu}{K}} = \frac{\pi}{10}$

(b) $A_1 + A_2 = 6$

$\Rightarrow m_1A_1 = m_2A_2 \Rightarrow A_1 = \frac{m_2}{m_1+m_2}A$

A_1 = 4 cm, A_2 = 2 cm.

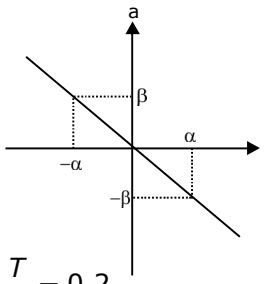
(c) $m_2V_{max} = 6.\omega A_2 = 2.4$

6. $T = \frac{2\pi}{5\pi} = 0.4\,\text{sec.}$

Av. Speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{0.2\times 7}{0.7} = 2\text{m/s}$

7. F=-Kx

$a = -\frac{K}{m}x \qquad \frac{K}{m} = \tan\theta = \frac{\beta}{\alpha}$



$\omega = 2\pi f\ ,\ f = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

8. $\frac{T}{3} = 0.2$

$T = 0.6 = 2\pi\sqrt{\frac{0.9}{K}} \Rightarrow$ K = 100 N/m

9. F=-10x + 2 = -10(x – 0.2)

x = 0.2 is M.P.

(a) Amplitude = 2 + 0.2 = 2.2 m

(b) $T = 2\pi\sqrt{\frac{0.1}{10}} = \frac{\pi}{5}s$

(c) x = 0.2 - 2.2 cos ωt

10. $U=(x^2 - 4x + 3)$

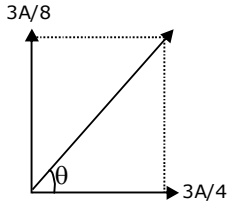
$F = -\frac{du}{dx} = -2x + 4 \quad \Rightarrow$ F = -2(x-2)

(a) For equillibrium F = 0 $\Rightarrow x = 2m$

(b) $\omega = \sqrt{\frac{2}{1}} = \sqrt{2} \ \Rightarrow T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

(c) $\frac{1}{2}KA^2 = \frac{1}{2}mv^2 \ \Rightarrow A = 2\sqrt{3}$

11. $\tan\theta = \frac{1}{2}$



$$A_{net} = \sqrt{\left(\frac{3A}{4}\right)^2 + \left(\frac{3A}{8}\right)^2} = \frac{3A}{4}\sqrt{1+\left(\frac{1}{2}\right)^2}$$

$$= \frac{3\sqrt{5}}{8}A$$

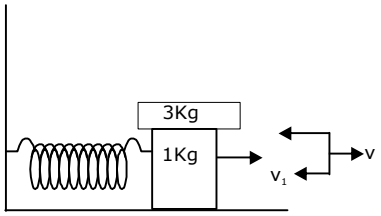
12. $F_{max} = KA = 50$

$$x = \frac{A}{\sqrt{2}} \quad \therefore F = Kx = \frac{KA}{\sqrt{2}} \quad F = \frac{F_{max}}{\sqrt{2}} = 50\sqrt{2}$$

$F = 25\sqrt{2}$ N

13. Given A=0.1m, K=100 N/m
m_1 = 1Kg, m_2 = 3Kg
Energy of system is not conserved.

$$\frac{1}{2}KA^2 = \frac{1}{2}m_1v_1^2$$



$$\Rightarrow v_1 = \frac{KA^2}{m_1}$$

$$\Rightarrow v_1 = 1 \text{ m/s}$$

$$m_1v_1 = (m_1+m_2)v \qquad \Rightarrow v = \frac{1}{(m_1+m_2)}$$

$$\Rightarrow v = 0.25 \text{ m/s}$$

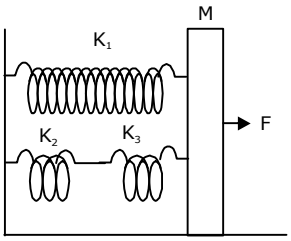
$$\because \text{freq}^n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{100}{4}} = \frac{5}{2\pi}H_2$$

Now amplitude

$$\frac{1}{2}KA'^2 = \frac{1}{2}(m_1+m_2)v^2 \Rightarrow A'^2 = \frac{(0.25)^2\times 4}{100}$$

$$\Rightarrow A' = 5cm.$$

14. $K_{eq} = K_1 + \frac{K_2K_3}{K_2+K_3}$



$$\because F = K_{eq}A$$

$$\Rightarrow A = \frac{F}{K_{eq}} = \frac{(K_2+K_3)F}{K_1K_2+K_2K_3+K_3K_1}$$

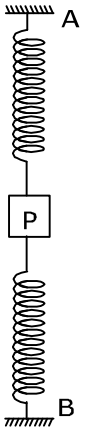
$$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M}} = \frac{1}{2\pi}\left[\frac{K_1K_2+K_2K_3+K_3K_1}{M(K_2+K_3)}\right]^{1/2}$$

15. Mg = K x 20 - K x 10

$$\Rightarrow K = \frac{mg}{10/100} \text{ N/m}$$

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$$T = 2\pi\sqrt{\frac{1}{49\times 4}} = \frac{\pi}{7}\text{sec}$$



16. General equation

$x = A\sin(\omega t+\phi)$ A=10, $\omega = \frac{2\pi}{T} = \pi$

At t = 0, x = 5 $\Rightarrow$ 5=10 sin ϕ

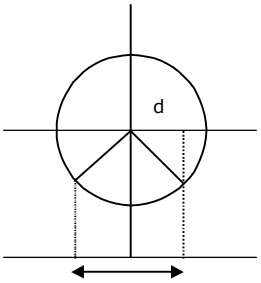
$$\phi = \frac{\pi}{6} \quad \Rightarrow x = 10\sin\left(\pi t + \frac{\pi}{6}\right)$$

17. $d = a\sin\frac{\phi}{2}$

Max distance = 2d

$$2d = 2a\sin\frac{\phi}{2}$$

= 2a x 0.9 = 1.8 a



18. By momentum conservation
5 x 3 + 10 x 2 = 7v
v = 5 m/s
By Energy Conservation

$$\frac{1}{2}.5.3^2 + \frac{1}{2}\times 2\times(10)^2 = \frac{1}{2}\times 7\times 5^2 + \frac{1}{2}Kx^2$$

x = 0.25 m, $T = 2\pi\sqrt{\frac{\mu}{K}} \Rightarrow \mu = \frac{10}{7}$

$$\Rightarrow T = \pi/14$$

first compression occurs $= \frac{3T}{4} = \frac{3\pi}{56}s$

19. $x = 30\sin(\pi t + \pi/6)$

$$\frac{1}{2}Kx^2 = 2\left(\frac{1}{2}m\omega^2(A^2-x^2)\right)$$

$\Rightarrow \frac{x^2}{2} = \left(0.09 - x^2\right)$

$\Rightarrow x^2 = 0.06 \Rightarrow x = \frac{\sqrt{6}}{10}$ m.

20. $a = 8\pi^2 - 4\pi^2 x$

$F = -4\pi^2 m[x-2]$ $\Rightarrow$ x = 2 M.P.

$T = 2\pi\sqrt{\frac{m}{4\pi^2 m}} = 1s$

$\omega = 2\pi$ rad/s

$x = 2 - 4\cos 2\pi t$

21. (a) $T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{10}{10\pi^2}} = 2s$

(b) $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} \Rightarrow g_{eff} = \sqrt{g^2 + a^2} = 5\sqrt{5}$

$T = 2\pi\sqrt{\frac{0.5}{5\sqrt{5}}} = \frac{2}{(5)^{1/4}}$

22. $\frac{1}{2}Kx^2 = 0.4$ $\omega = 2\pi \times \frac{25}{\pi}$

$x^2 = \frac{0.8}{K}$, ω =50, K = 500

$\frac{1}{2}m\omega^2\left(A^2 - x^2\right) = 0.5 \Rightarrow A^2 = x^2 + \frac{1}{500}$

A =0.06 m

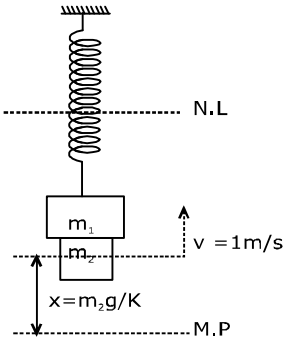
23. Momentum Conservation

0.5(3) = 1.5 v

v = 1m/s

$\omega = \sqrt{\frac{K}{m_1 + m_2}} = 20$

$v = \omega\sqrt{A^2 - x^2} \Rightarrow A^2 = \frac{v^2}{\omega^2} - x^2$



24. f = 3Hz μ = 0.72

When block does not slip

$ma_{max} < f \Rightarrow ma_{max} < \mu mg$

$\Rightarrow a_{max} < \mu g$ $\Rightarrow \omega^2 A_{max} < \mu g$

$A_{max} = \frac{\mu g}{4\pi^2 f^2} = 0.02$

25. $U(x) = -ax^2 + bx^4$

$F = \frac{-dU}{dx} \Rightarrow F = 2ax - 43bx^3$

for MP F = 0 $\Rightarrow$ x = 0, $x = \sqrt{\frac{a}{2b}}$

$F = 2a(x + x_0) - 4b(x + x_0)^3$

$= 2ax\left(1 + \frac{x_0}{x}\right) - 4bx^2\left(1 + \frac{3x_0}{x}\right)$

$= 2a\sqrt{\frac{a}{2b}}\left[1 + \frac{x_0}{x} - 1 - \frac{3x_0}{x}\right]$

$= 2a\sqrt{\frac{a}{2b}}\left[\frac{-2x_0}{x}\right] \Rightarrow 2a\sqrt{\frac{a}{2b}}\sqrt{\frac{2b}{a}}(-2x_0)$

$= -4ax_0$

$k_{eq} = 4a \Rightarrow \omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{4a}{m}}$

26. $T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \Rightarrow \ell = 1m$

27. $\theta = \frac{\pi}{90}\sin\left(\pi s^{-1}\right)t \Rightarrow \omega = \pi$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\sec.$

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \Rightarrow \ell = 1m$

28. $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}} \Rightarrow g_{eff} = g - \frac{3g}{4} = \frac{g}{4}$

(i) $T = 2\pi\sqrt{\frac{4\ell}{g}} = 2T_0$ if $T' = \frac{T_0}{2}$

(ii) $2\pi\sqrt{\frac{\ell}{g_{eff}}} = \pi\sqrt{\frac{\ell}{g}} \Rightarrow g_{eff} = 4g$

$g + a = 4g \Rightarrow a = 3g$ $\uparrow$

29. (a) $g_{eff} = g + a_0 \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g + a_0}}$

(b) $g_{eff} = g - a_0 \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g - a_0}}$

(c) $g_{eff} = g \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$

30. $T_1 = 2\pi\sqrt{\frac{\ell}{g}} \qquad T_2 = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+a^2}}}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\left(g^2+a^2\right)^{1/4}}{\sqrt{g}} \quad \Rightarrow \left(\frac{T_1}{T_2}\right)^4 g^2 = g^2 + a^2$$

$$\Rightarrow g^2 = \left(\frac{4-0.01}{4}\right)^4 \left(g^2+a^2\right)$$

$$\Rightarrow g^2 = (1-0.01)\left(g^2+a^2\right) \Rightarrow a^2 = 0.01g^2$$

$$\Rightarrow a = 0.1g \qquad \Rightarrow a = g/10$$

31. $I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\ell^2 = \frac{17}{12}m\ell^2$

$$d = \frac{3\ell}{4}$$

$$T = 2\pi\sqrt{\frac{17m\ell^2}{12.2mg.\frac{3}{4}\ell}} \Rightarrow T = 2\pi\sqrt{\frac{17\ell}{18g}}$$

32. $T = 2s$

$$T_1 = \frac{T}{2}$$

$$T_2 = 2\pi\sqrt{\frac{\ell}{4g}} = \frac{T}{2}$$

$$T' = T_1 + \frac{T_2}{2} \qquad \Rightarrow T' = \frac{T}{2} + \frac{T}{4} = \frac{3}{2}s.$$