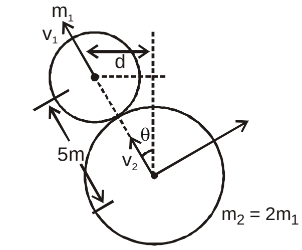
EXERCISE – IV

TOUGH SUBJECTIVE PROBLEMS

1. here $\sin\theta = d/5$



$$\cos\theta = \frac{\sqrt{5^2 - d^2}}{5}$$

After collision

Momentum conservation in line of impact

$$m_1 u\cos\theta = m_1 v_1 + (2m_1)v_2$$

$$u\cos\theta = v_1 + 2v_2 \qquad ...(1)$$

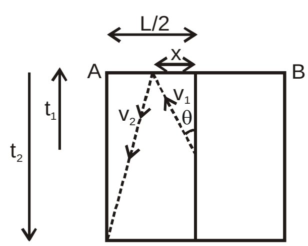
again given

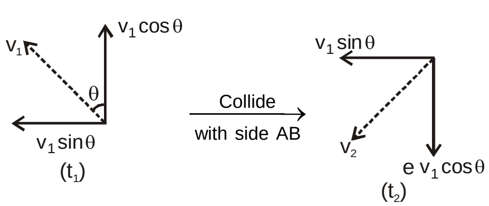
$$e = \frac{2}{3} = \frac{v_1 - v_2}{u\cos\theta} \quad \Rightarrow \quad \frac{2}{3}u\cos\theta = v_1 - v_2 \qquad ...(2)$$

eq. (1) + 2 × (2)

$$\left(\frac{2}{3}\times 2 + 1\right)u\cos\theta = 3v_1$$

or $+v_1 = \frac{7}{9}u\cos\theta \qquad ...(3)$





Now $\frac{L}{2} = v_1\sin\theta(t_1 + t_2) \qquad ...(4)$

(in x-direction)

in y direction $\frac{L}{2} = v_1\cos\theta\; t_1 \qquad ...(5)$

and $L = \frac{2}{3}v_1\cos\theta\; t_2 \qquad ....(6)$

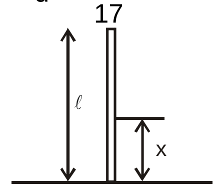
from (5) and (6) in (4) value of t_1 & t_2

$$\frac{L}{2} = v_1\frac{d}{5}\left[\frac{L}{2v_1}\frac{5}{\sqrt{25-d^2}} + \frac{3L}{2v_1}\frac{5}{\sqrt{25-d^2}}\right]$$

$$\Rightarrow \quad 1 = \frac{4d}{\sqrt{25-d^2}} \Rightarrow \quad 25 - d^2 = 16d^2$$

$$\Rightarrow \quad d^2 = \frac{25}{17} \qquad \Rightarrow \quad d = \frac{5}{\sqrt{17}} \qquad \textbf{Ans.}$$

2.



At time t_1 x length dropped and next dt time dx part dropped, then force on table

f_T = [due to weight of the chain + $\frac{dp}{dt}$]

$f_T = x\lambda g + \frac{dp}{dt}$, here $x = v_t$, $f_T = vt\lambda g + \frac{dp}{dt}$ ...(1)

$(\Delta p)_c = 0 - \lambda dx(v_2)$, $(\Delta p)_c = 0 - \lambda V^2 dt$

$\frac{(\Delta p)_c}{\Delta t} = -\lambda v^2$ on chain

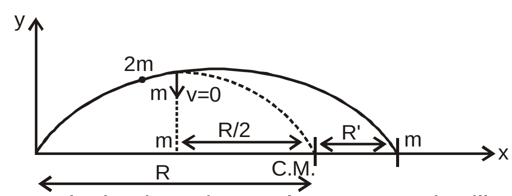
so on table $= -\frac{(dp)_c}{dt} = \lambda v^2 \qquad ...(2)$

so form eq. (1) and (2)

$$f_T = Vt\lambda g + \lambda v^2, \qquad \lambda = \frac{m}{\ell}$$

$$f_T = \frac{m}{\ell}v\,(gt + v)$$

3.



∵ Explosion is an internal process so it will not effect the position of C.M. and because no force in horizontal direction so

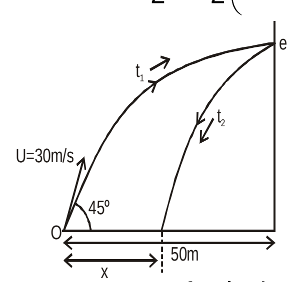
$$(x_{C.M})_{explosion} = (x_{C.M})_{without\ explosion}$$

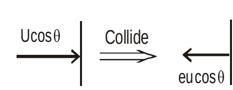
$$\frac{m(R/2) + (R + R')}{2m} = R \qquad \Rightarrow \quad R + R' = \frac{3R}{2}$$

so striking point for second particle

$$= R + R' = \frac{3R}{2} = \frac{3}{2}\left(\frac{U^2\sin 2\theta}{g}\right) = 368$$

4.





y component of velocity does not change

y direction total time if t

$t = t_1 + t_2$ ...(1)

then $0 = U\sin\theta\, t - 1/2\, gt^2$

at the time of collision if ball have y component of velocity v_y then

$v_y = v_1 - gt_1$ (here t_1 = time for collide)

$v_y = 10\sin 37^o - 10t_1$ ...(2)

for time $t_1 = \dfrac{\text{relative distance}}{[\text{relative velocity}]_{\text{in x-direction}}}$

$= \dfrac{10}{U + v_2}$

$t_1 = \dfrac{10}{U+8}$ ...(3)

from (2) and (3)

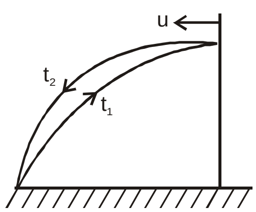
$v_y = 10\sin 37^o - \dfrac{10\times 10}{U+8}$ ...(4)

$\because$ Ball again come into his hands so total time t then

$0 = 10\sin 37^o t - 1/2\, gt^2$ in y-direction

$t = \dfrac{2(10\sin 37^o)}{g} = 1.2\,\text{sec}$

here $t_1 + t_2 = t$ ...(6)



If returning time is t_2 then

$t_2 = \dfrac{10 - ut_1}{v_2'}$

$= \dfrac{10 - ut_1}{v_2 + 2u}$ from eq. (1)

so $t_2 = \dfrac{10 - ut_1}{8 + 2u}$ ...(7)

so from eq (3), (5), (6) and (7)

putting values of t_1, t_2 and t

$\Rightarrow \dfrac{10}{U+8} + \dfrac{10 - U\left(\dfrac{10}{v+8}\right)}{8+2u} = \dfrac{12}{10}$

$\Rightarrow 3u^2 + 114 - 104 = 0$

so $u = \dfrac{-11 \pm \sqrt{(11)^2 + 4(3)(104)}}{2\times 3}$

$u = \dfrac{26}{6}$ (from + sign) $\Rightarrow u = \dfrac{13}{3}$ m/sec

or $t = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 30\times 1/\sqrt{2}}{9.8}$ sec

t = 4.33 sec ...(2)

again for time t_1

$t_1 = \dfrac{50}{u\cos\theta} = \dfrac{50}{30\times 1/\sqrt{2}}$ sec

$t_1 = 2.36$ sec ...(3)

so from eq. (1), (2) and (3)

$t_2 = t - t_1 = 4.33 - 2.36$

$t_2 = 1.97$ sec

so $x = 50 - (eu\cos\theta)t_2$

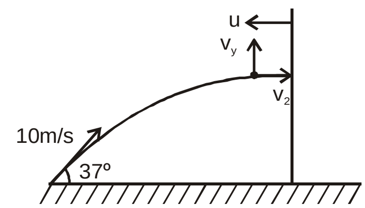
$x = 50 - e(41.8)$

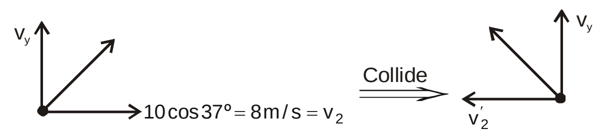
(a) e = 0 then x = 50 m

(b) e = 1 then x = 8.2 m

(c) e = 1/2 then x = 2.91 m

5.





then $e = 1 = \dfrac{v_2' - u}{v_2 + u} \Rightarrow v_2' = v_2 + 2u$ ...(1)

6. (a) Given $m_1 = m_2 = m_3 = m$

when m_1 and m_2 stickes then

Momentum Conservation

$m_1 v = (m_1 + m_2)v'$ ...(A)

$v' = \left(\dfrac{m_1 v}{m_1 - m_2}\right)$ velocity after collision of $(m_1 + m_2)$

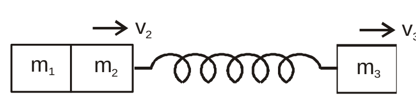
$v' = \dfrac{v}{2}$ ...(B)

and velocity of $m_3 = 0$ just after impact.

(b) Now just after impact the total energy

$E = \dfrac{1}{2}(m_1 + m_2)(v')^2 = \dfrac{1}{4}mv^2$ ...(1)

The K.E. of m_3 will max. (K.E. of m_2 will minimum) when spring is in normal position and after some time of impact at this time if velocity are v_2 and v_3 then momentum conservation



$(m_1+m_2)v' = (m_1+m_2)v_2 + m_3 v_3$

or $m_1 v = (m_1+m_2)\sqrt{2} + m_3 v_3$

from (A) $v = 2v_2 + v_3$ ...(2)

$\because$ Total energy after impact remain same for all time so

$E_f = E_i$

$\Rightarrow \frac{1}{2}2mv_2^2 + \frac{1}{2}mv_3^2 + \frac{1}{2}k(o)^2 = \frac{1}{4}mv^2$

$v^2 = 4v_2^2 + 2v_3^2$ ...(3)

from (2) and (3) element v_2

$v^2 = 4\left[\frac{v-v_3}{2}\right]^2 + 2v_3^2$

$v^2 = v^2 + v_3^2 - 2vv_3 + 2v_3^2$

$v_3 = \frac{2v}{3}$ ...(4)

$K.E_{max} = \frac{1}{2}mv_3^2 = \frac{2}{9}mv^2$

(c) from (4) and (2)

$v = 2v_2 + \frac{2v}{3} \Rightarrow v_2 = \frac{v}{6}$

so $K.E_{min} = \frac{1}{2}m\left(\frac{v}{6}\right)^2 = \frac{1}{12}mv^2$

(d) When their velocity will same from mome. con.

$m_1 v = (m_1+m_2+m_3)v'$

$v' = v/3$

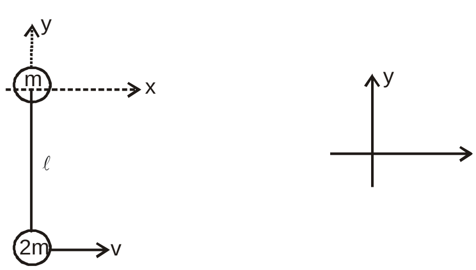
Ene. constant so

$\frac{1}{4}mv^2 = \frac{1}{2}3m(v')^2 + \frac{1}{2}k(x_m)^2$

$mv^2 = \frac{6mv^2}{9} + 2K(x_m)^2$

$\Rightarrow x_{cm} = \sqrt{\frac{m}{6k}}.v$ **Ans.**

7.



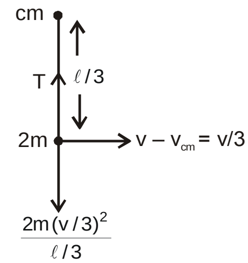
$y_{cm} = \frac{2m(-\ell)+m(o)}{(2m+m)} = -\frac{2\ell}{3}$ ...(1)

$v_{cm} = \frac{2mv+m(o)}{2m+m} = \frac{2v}{3}$ ...(2)

the velocity of 2m w.r.t. C.M.

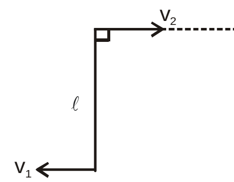
$v - v_{cm} = v/3$

Then $T = \frac{2m(v/3)^2}{\ell/3}$



$T = \frac{2}{3}m\frac{v^2}{\ell}$

8.



$(v_{cm})_x = 0 = m_A v_1 - m_B v_2$

$v_1 = \frac{m_B}{m_A}v_2$ ...(1)

E. conservation

$m_A g\ell = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_2^2$ ...(2)

(1) and (2)

$m_A g\ell = \frac{1}{2}m_A\left(\frac{m_B}{m_A}v_2\right)^2 + \frac{1}{2}m_B(v_2)^2$

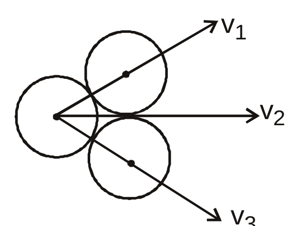
$2n_A g\ell = \left(\frac{m_B^2}{m_A} + m_B\right)v_2^2$

$v_2^2 = 2\left(\frac{m_A}{m_B}\right)^2 \frac{g\ell}{\left(1+\frac{m_A}{m_B}\right)}$ ...(3)

$T = \frac{m_A v^2}{\ell} \Rightarrow T = m_A\left(\frac{v_1+v_2}{\ell}\right)^2$

$T = 2g\left(\frac{m_A}{m_B}\right)(m_A+m_B)$

9.



After collision

then m.c.

$m10 = mv_2 + 2mv_1 \frac{\sqrt{3}}{2}$

$10 = v_2 + \sqrt{3}v_1$ ...(1)

and $e = 1 = \dfrac{v_1 - v_2\sqrt{3}/2}{10\sqrt{3}/2}$

$10\sqrt{3} = 2v_1 - \sqrt{3}v_2$ ...(2)

from (1) and (2)

$10 = v_2 + 3\left[\dfrac{10+v_2}{2}\right]$

$20 = 2v_2 + 30 + 3v_2$

$5v_2 = 10 \Rightarrow v_2 = -2$m/s

10. (a) v_{cm} remain same

$$v_{cm} = \frac{m_0 v_0 \hat{i} + m_2(0)}{(m_1+m_2)} \Rightarrow v_{cm} = \frac{v_0}{3}\hat{i}$$

(b) In x direction

$m_1 v_0 = m_1(0) + m_2(v_2 \cos\theta)$

$v_0 = 2v_2 \cos\theta$ ...(1)

In y direction $0 = m_1 v_0/2 - m_2(v_2 \sin\theta)$

$v_0 = 4v_2 \sin\theta$ ....(2)

(c) eq. (2) ÷ 1

$= \dfrac{4v_2}{2v_2}\tan\theta \Rightarrow \tan\theta = \dfrac{1}{2}$

so $\sin\theta = \dfrac{1}{\sqrt{5}}$, so from (2) $v_2 = \dfrac{\sqrt{5}}{4}v_0$

(d) $\Delta k = k_f - k_i$

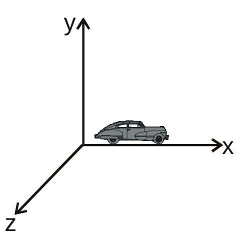
$$\Delta k = \frac{1}{2}m_1\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 v_0^2$$

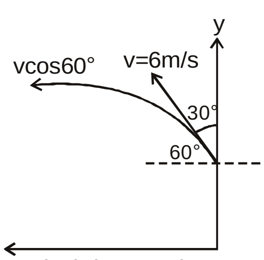
$$= \frac{1}{2}m_1\left[\frac{v_0^2}{4} + \frac{2\times 5v_0^2}{16} - v_0^2\right]$$

$$\Delta k = -\frac{1}{2}m_1 v_0^2\left(\frac{3}{8}\right) = -\frac{1}{16}m_1 v_0^2$$

So no elastic

11.





The ball will have velocity at heighest point

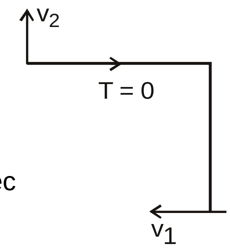
$= 4\hat{i} + v\cos 60°(\hat{k})$, $= 4\hat{i} + 6/2(\hat{k})$

when they collide then momentum conservation

$m(4\hat{i} + 3\hat{k}) = 2m\vec{v}_1$

$\vec{v}_1 = 2\hat{i} + \dfrac{3}{2}\hat{k}$,

so $|\vec{v}_1| = \sqrt{2^2 + \dfrac{9}{4}}$



$= \dfrac{\sqrt{25}}{2} = 2.5$ m/sec

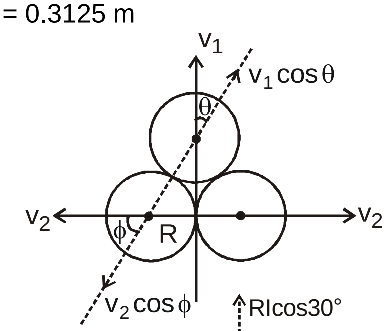
T = 0 when $v_2 = 0$ $\therefore$ $T = \dfrac{mv_2^2}{\ell}$

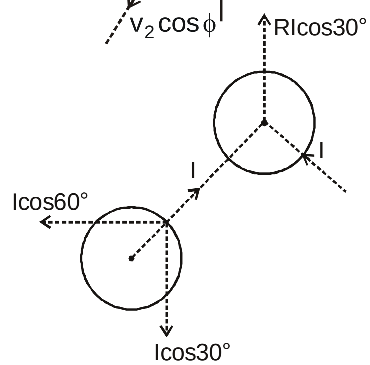
$\dfrac{1}{2}(2m)v_1^2 = 2mg\ell \Rightarrow v_1^2 = 2g\ell$

$\ell = \dfrac{v_1^2}{2g} \Rightarrow \ell = \dfrac{(2.5)^2}{8\times 10}$m

$\ell = 0.3125$ m

12.





$\sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30°$

$\cos\phi = \dfrac{1}{2} \Rightarrow \phi = 60°$

$$e = \frac{v_1\cos\theta + v_2\cos\phi}{v\sqrt{3}/2} = \frac{v_1\sqrt{3}/2 + v_2/2}{v\sqrt{3}/2}$$

$e = 1 \Rightarrow u\sqrt{3} = v_1\sqrt{3} + v_2$ ...(1)

$2I\cos 30° = mv_1 + mu$ ...(2)

$I\cos 60° = mv_2$ ...(3)

$2\sqrt{3} = \dfrac{v_1 + u}{v_2}$ ...(5)

$v_1 + u = 2\sqrt{3}[u\sqrt{3} - v_1\sqrt{3}]$

$7v_1 = 5u$

$\Rightarrow v_1 = \dfrac{5u}{7}$