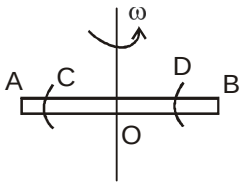
Exercise - IV

TOUGH SUBJECTIVE PROBLEMS

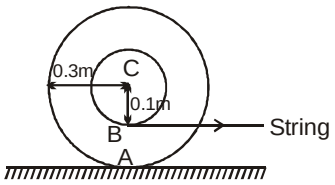
1. A thin uniform rod of mass M and length L is hinged at its upper end, and released from rest in a horizontal position. The tension at a point located at a distance L/3 from the hinge point, when the rod becomes vertical, will be

2. A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid point O. Two rings each of mass 1 kg are initially at rest a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radians per second. The velocity of each ring along the length of the rod in m/s then they reach the ends of the rod is



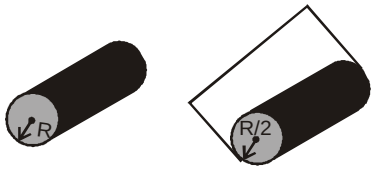
3. A straight rod AB of mass M and length L is placed on a frictionless horizontal surface. A horizontal force having constant magnitude F and a fixed direction starts acting at the end A. The rod is initially perpendicular to the force. The initial acceleration of end B is

4. A wheel is made to roll without slipping, towards right, by pulling a string wrapped around a coaxial spool as shown in figure. With what velocity the string should be pulled so that the centre of the wheel moves with a velocity of 3 m/s?

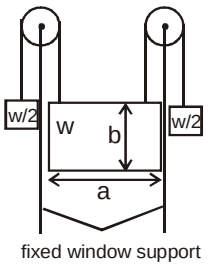


5. A solid uniform disk of mass m rolls without slipping down a fixed inclined plane with an acceleration a. The frictional force on the disk due to surface of the plane is :

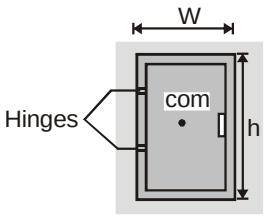
6. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be :



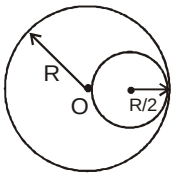
7. A slightly loosely fit window is balanced by two strings which are connected to weights w/2 each. The strings pass over the frictionless pulleys as shown in the figure. The strings are tied almost at the corner of the window. The string on the right is cut and then the window accelerates downwards. If the coefficients of friction between the window and the side supports is μ then calculate the acceleration of the window in terms of μ, a, b and g, where a is width and b is the length of the window.



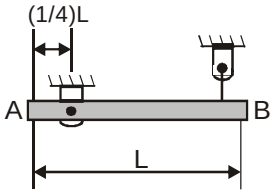
8. A uniform wood door has mass m, height h, and width w. It is hanging from two hinges attached to one side; the hinges are located h/3 and 2h/3 from the bottom of the door. Suppose that m = 20.0 kg, h = 2.20 m, and w = 1.00 m and the bottom smooth hinge is not screwed into the door frame. Find the forces acting on the door.



9. A hole of radius R/2 is cut from a solid sphere of radius R. If the mass of the remaining plate is M, then moment of inertia of the body about an axis through O perpendicular to plane is _____.



10. A uniform beam of length L and mass m is supported as shown. If the cable suddenly breaks, determine ;



(a) the acceleration of end B.
(b) the reaction at the pin support.

11. A thin rod AB of length a has variable mass per unit length $\rho_0\left(1+\frac{x}{a}\right)$ where x is the distance measured from A and ρ_0 is a constant.

(a) Find the mass M of the rod.

(b) Find the position of centre of mass of the rod.

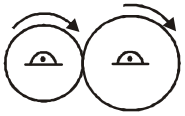
(c) Find moment of inertia of the rod about an axis passing through A and perpendicular to AB.

Rod is freely pivoted at A and is hanging in equilibrium when it is struck by a horizontal impulse of magnitude P at the point B.

(d) Find the angular velocity with which the rod begins to rotate.

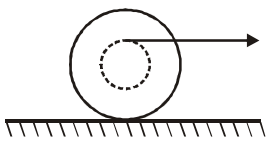
(e) Find minimum value of impulse P if B passes through a point vertically above A.

12. Two separate cylinders of masses m (= 1kg) and 4m and radii R(=10cm) and 2R rotating in clockwise direction with ω_1 = 100 rad/sec and ω_2 = 200 rad/sec. Now they are held in contact with each other as in fig. Determine their angular velocities after the slipping between the cylinders stops.



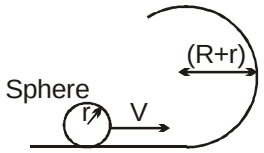
13. A circular disc of mass 300 gm and radius 20 cm can rotate freely about a vertical axis passing through its centre of O. A small insect of mass 100 gm is initially at a point A on the disc (which is initially stationary) the insect starts walking from rest along the rim of the disc with such a time varying relative velocity that the disc rotates in the opposite direction with a constant angular acceleration = 2π rad/s^2. After some time T, the insect is back at the point A. By what angle has the disc rotated till now ; as seen by a stationary earth observer ? Also find the time T.

14. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread woudn on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.

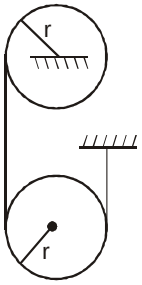


15. A sphere of mass m and radius r is pushed onto the fixed horizontal surface such that it rolls without slipping from the beginning. Determine

the minimum speed v of its mass centre at the bottom so that it rolls completely around the loop of radius (R + r) without leaving the track in between.



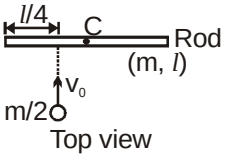
16. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine



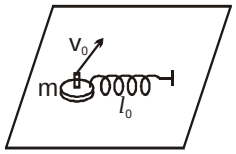
(a) the velocity of the centre of cylinder A after it has moved through 1.2 m &

(b) the tension in the portion of the belt connecting the two cylinders.

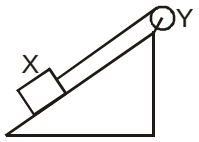
17. A uniform rod of mass m and length l is resting on a smooth horizontal surface. A particle of mass m/2 travelling with a speed v_0 hits the rod normally and elastically. Find final velocity of particle and the angular velocity of the rod.



18. One side of a spring of initial, unstretched length l_0 = 1m, lying on a frictionless table, is fixed, the other one is fastened to a small puck of mass m = 0.1kg. The puck is given velocity in a direction perpendicular to the spring, at an initial speed v_0 = 11 m/s. In the course of the motion, the maximum elongation of the spring is l = l_0/10. What is the force constant of the spring (in SI units) ?



19. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30º to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2kg and of radius 0.2 m as shown in the figure. The drum is given an initial angular velocity such that the block X starts moving up the plane.



(i) Find the tension in the string during the motion

(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad/sec. Calculate the distance travelled by X from that instant of time until it comes to rest.