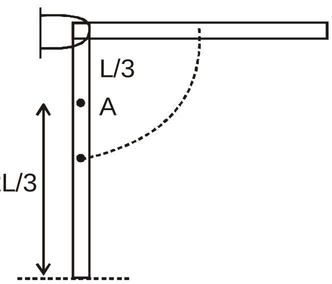
EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

1. From energy conservation

$$Mg\frac{L}{2} = \frac{1}{2}\frac{m\ell^2}{3} \times \omega^2$$

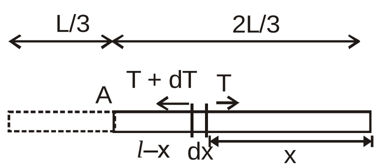


$$\omega^2 = \frac{3g}{\ell}$$

Now $\quad dT = \frac{m}{\ell}.dx\,\omega^2(\ell - x)$

for Tension at A we integrate about experssion with time 0 to $2\ell/3$

$$\Rightarrow \int_0^T dT = \int_0^{2\ell/3} \frac{m}{\ell} dx\omega^2(\ell - x)$$



after solving

$$T = \frac{m}{\ell}.\omega^2 \times \frac{4\ell^2}{9}$$

Put the value of ω

$$\Rightarrow T = \frac{m}{\ell}.\frac{3g}{\ell}.\frac{4\ell^2}{9} = \frac{4}{3}mg \text{ (due to circular)}$$

Total jension at point A is

$$T_{total} = T + \frac{2M}{3}g \text{ (due to weight)}$$

$$= \frac{4}{3}mg + \frac{2m}{3}g = 2mg$$

2. Given $M_{rod} = 0.75$ kg $\qquad M_{ring} = 1$ kg

L = 40 cm

from angular momentum conservation

$$\left(\frac{Mr^2}{12} + 2 \times mr^2\right)(30) = \left(\frac{Mr^2}{12} + 2x^2\right)\omega \Rightarrow 0.9 =$$

$(0.01 + 2x^2)\ \omega$

$$\omega = \frac{0.9}{0.01 + 2x^2}$$

Now $\qquad \omega^2 x = a$

$$\Rightarrow \frac{vdv_0}{dx} = \left(\frac{0.9}{0.01 + 2x^2}\right)x$$

$$\int vdv = \int_{0.2}^{0.1}\left(\frac{0.9}{0.01 + 2x^2}\right)^2 xdx$$

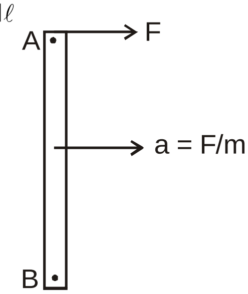
$$\Rightarrow \frac{v^2}{2} = \frac{18}{4}$$

$\Rightarrow$ v = 3 m/sec

3. From $\qquad \tau = I\,\alpha$

$$\frac{F\ell}{2} = \frac{M\ell^2}{12}\alpha \Rightarrow \qquad \alpha = \frac{6F}{M\ell}$$

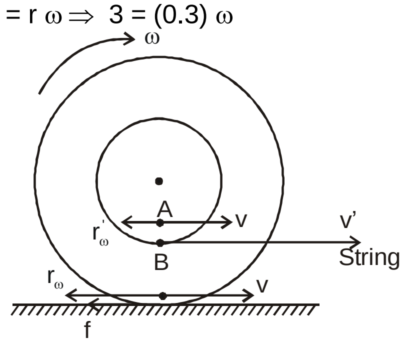
at point B. $a_B = a_t - a$



$$= \alpha.\frac{\ell}{2} - \frac{F}{m}$$

$$= \frac{3F}{m} - \frac{F}{m} \Rightarrow a_B = \frac{2F}{m}$$

4. from v = r $\omega \Rightarrow$ 3 = (0.3) ω



ω = 10 rad/sec

Both point A & B are together when

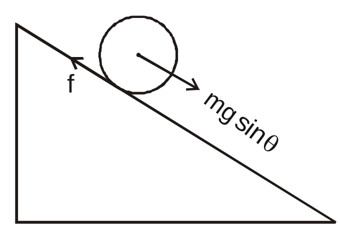
$v - r'\omega = v'$

$\Rightarrow$ 3 – (0.1) 10 = v′

v′ = 2 m/sec

5. from $\tau = I\,\alpha$

$$f.R = \frac{mR^2}{2}.\alpha \qquad \Rightarrow \alpha = \frac{2f}{mR}$$



Now $\quad a = \alpha R \Rightarrow a = \frac{2f}{mR}.R$

$f = \frac{ma}{2}$

6. Let ρ is the mass density of the material then
$M = (\pi R^2 l)\,\rho$
when radius R/2 then

$M_f = \pi(R/2)^2 \ell\rho = \frac{M}{4}$

Initial P.E. = MgR

final P.E. = $\frac{Mg}{4}\frac{R}{2} = \frac{MgR}{8}$

Change in potartial energy = increase in rotational K.E.

$$\Rightarrow mgR - \frac{mgR}{8} = K_T + K_R$$

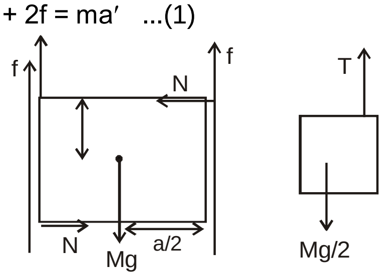
$$\Rightarrow \frac{7}{8}mgR = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}I\omega^2$$

$$\frac{7}{8}mgR = \frac{1}{2}\frac{m}{a}v^2 + \frac{1}{2}\frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 . \frac{R}{2}\omega$$

$$\Rightarrow \frac{7}{8}mgR = \frac{3}{16}mv^2$$

$$v = \sqrt{14\frac{gR}{3}}$$

7. Mg – T + 2f = ma′ ...(1)



$\frac{Nb}{2} + \frac{Nb}{2} = \frac{Ta}{2}$ ...(2)

$f = \mu N$ ...(3)

$T - \frac{mg}{2} = ma'$ ...(4)

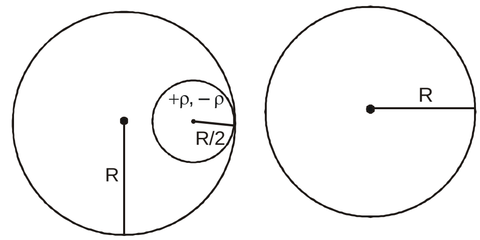
8. $F_{1y} + F_{2y} - mg = 0 \Rightarrow F_{1y} + F_{2y} = F_{ynet} = mg$

$\vec{F}_x = \vec{N}_1 + \vec{N}_2 = ma_x = 0$

$\Rightarrow N_1 = -N_2$ (equal & opp.)

$\tau_B = \frac{\omega}{2} \times mg - \left(N \times \frac{h}{3}\right) = 0 \Rightarrow N = \frac{3\omega mg}{2h}$

9. $\rho = \frac{M}{\frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)} = \frac{24M}{4 \times 7\pi R^3}$



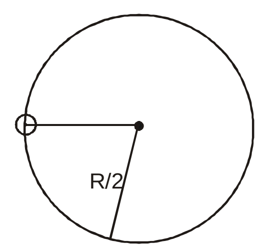
$$I' = \frac{2}{5}\left(M + \rho\frac{4}{3}\pi\frac{R^3}{8}\right)R^2$$

$$I' = \frac{2}{5}MR^2 + \frac{2}{5} \times \frac{24M}{28\pi R^3}.\frac{4}{3}\frac{\pi R}{8}$$

$$I' = \frac{2}{5}MR^2 + \frac{2}{35}MR^2$$

$$\Rightarrow I' = \frac{14MR^2 + 2MR^2}{35} = \frac{16MR^2}{35}$$

$$I'' = \frac{2}{5}\rho\frac{4}{3}\pi\frac{R^3}{8}.\frac{R^2}{4} + \rho\frac{4}{3}\pi\frac{R^3}{8}.\frac{R^2}{4}$$



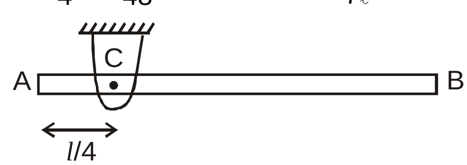
$$\Rightarrow \frac{MR^2}{70} + \frac{MR^2}{28}$$

$$\frac{2MR^2 + 5MR^2}{140} = \frac{7MR^2}{140}$$

$$\Rightarrow 1 = \frac{16MR^2}{35} - \frac{7MR^2}{140} = \frac{(64-7)MR^2}{140} = \frac{57}{140}MR^2$$

10. $I_C = I_{CM} + Ma^2 \quad \Rightarrow I_C = \frac{M\ell^2}{12} + M\left(\frac{\ell}{4}\right)^2 = \frac{7m\ell^2}{40}$

$Mg\frac{\ell}{4} = \frac{7M\ell^2}{48}\alpha \quad \Rightarrow \alpha = \frac{12g}{7\ell}$



$a_t = \alpha.\,R = \frac{12g}{7\ell}.\frac{3\ell}{4} = \frac{9g}{7}$

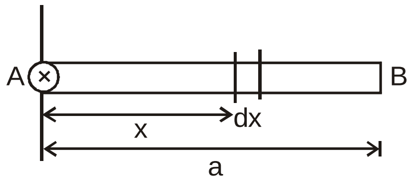
Now mg – N = ma = m (α.R′)

$mg - N = \frac{m.12g}{7\ell}.\frac{\ell}{4} = \frac{2mg}{7}$

$$N = \frac{4mg}{7}$$

11. Given $\rho = \rho_0\left(1+\frac{x}{a}\right)$

(a) $dm = \rho d\omega$



$$\int dm = \int_0^a \rho_0\left(1+\frac{x}{a}\right)dx$$

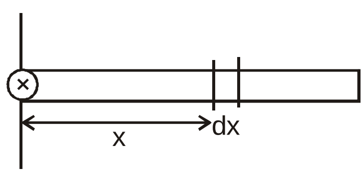
$$= \rho_0\left[x+\frac{x^2}{2a}\right]_0^a = \rho_0\left(a+\frac{a}{2}\right) = \frac{3}{2}a\rho_0$$

(b) Centre of mass

$$\int_0^x dx = \frac{\int dm.x}{\int dm} = \frac{\int x\rho dx}{\frac{3}{2}a\rho_0} \Rightarrow x = \frac{\int \rho_0 x\left(1+\frac{x}{a}\right)dx}{\frac{3}{2}a\rho_0}$$

$$\Rightarrow x = \frac{\left[\frac{x^2}{2}+\frac{x^3}{3a}\right]_0^a}{\frac{3}{2}a\rho_0} = \frac{\frac{a^2}{2}+\frac{a^2}{3}}{\frac{3}{2}a} = \frac{5a^2}{6\times 3a}\times 2 = \frac{5a}{9}$$

(c)



$$\int dI = \int \rho.dx.x^2 = \int_0^a \rho_0\left(1+\frac{x}{a}\right)x^2.dx$$

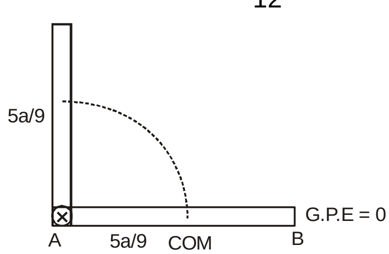
$$= \rho_0\int_0^a\left(x^2+\frac{x^3}{a}\right)dx$$

$$= \rho_0\left[\frac{x^3}{3}+\frac{x^4}{49}\right]_0^a = \rho_0\left[\frac{a^3}{3}+\frac{a^3}{4}\right] = \frac{7a^3\rho_0}{12}$$

(d) Angular Impulse = change in angular momentum

$$P.a = I\,\omega \Rightarrow Pa = \frac{7a^3\rho_0}{12}\omega \Rightarrow \omega = \frac{12\rho}{7a^2\rho_0}$$

(e)



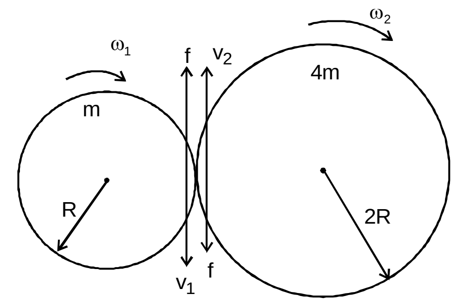
from energy conservation

$$\Rightarrow \frac{1}{2}I\omega^2 = Mg.\frac{5a}{9}$$

$$\frac{1}{2}\times\frac{7}{124}a^3\rho_0\omega^2 = \frac{3}{2}a\rho_0 g.\frac{5a_0}{9} \Rightarrow \omega^2 = \frac{20g}{7a}$$

Now Impulse $= \frac{I\omega}{a}$

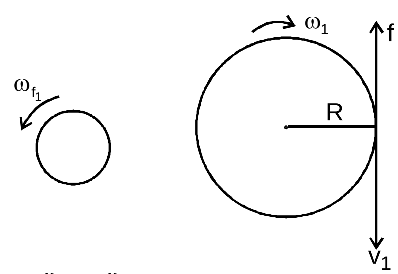
12.



after slipping between the cylinders stops then

$$v_1 = v_2$$

$$\omega_{f_1}R_1 = w_{f_2}R_2$$



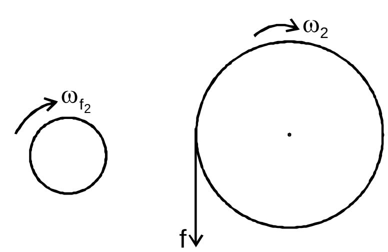
$$\Rightarrow \omega_{f_1} = \omega_{f_2}$$

for I[st] cylinder ⊙ +ve ⊗ – ve

$$\Rightarrow \int f.Rdt = I_1\omega_{f_1} + I_1\omega_1 \quad ...(1)$$

for II[nd] cylinder

$$\int f.2R\,dt = -I_2\omega_{f_2} + I_2\omega_2 \quad ...(1)$$



$$\int f.R\,dt = \frac{I_2\omega_2 - I_2\omega_{f_2}}{2} \quad ...(2)$$

from eq. (1) & (2)

$$I_1\omega_{f_1} + I_1\omega_1 = \frac{I_2\omega_2 - I_2\omega_{f_2}}{2}$$

$$2I_1\omega_{f_2} + I_1\omega_1 = \frac{I_2\omega_2 - I_2\omega_{f_2}}{2}$$

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$$\frac{2MR^2}{2}.\omega_{f_2} + \frac{MR^2}{2}\times 100 = \frac{4M4R^2}{2\times 2}\left[200-\omega_{f_2}\right]$$

$$\omega_{f_2} + 50 = 800 - 4\omega_{f_2}$$

$$5\ \omega_{f_2} = 750$$

$$\omega_{f_2} = 150 \text{ rad/sec}, \quad \omega_{f_1} = 300 \text{ rad/sec}$$

13. $I_D\omega_D = I_i\omega_i \Rightarrow I_D = m_D\frac{R^2}{2}$

or $I_D = 3m_i\frac{R^2}{2}$, or, $I_D = I_D = \frac{3}{2}I_i \Rightarrow \frac{I_D}{I_i} = \frac{3}{2}$

$\therefore \frac{\omega_i}{\omega_{Di}} = \frac{3}{2}$

given $\omega_D = 2\pi$

$\Rightarrow \omega_i = 3\pi$, $\omega_r = \omega_D + \omega_i = 5\pi$ rad/s

$T = \frac{2\pi}{\omega_i} = \frac{2\pi}{5\pi} = \frac{2}{5}s$

$Q_D = \omega_D T = 2\pi.\frac{2}{5} = \frac{4\pi}{5}rad$

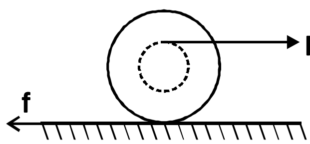
14. $Mg - f = ma$ ...(1)

$MgR + 3fR = MR^2\alpha$ ...(2)

$a_C = 2(3R)$

by solving (1) & (2)

$\alpha = \frac{2}{5}\frac{g}{R}$



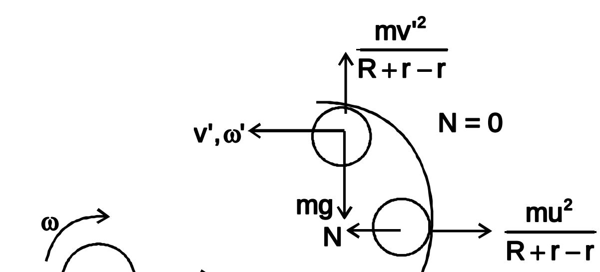
$f = \frac{m\alpha R}{2}$

$a_t = 4R\alpha = 4R\ \frac{2}{5}\frac{g}{R} = 16$

15. Just completing the circle N = 0 at top most point.

$\Rightarrow \frac{mv'^2}{R+r-\gamma} = mg$ ...(1)

Now E.C.



$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 2mg(R+r-1) + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

$\omega = \frac{v}{r}$ $\quad\quad \omega' = \frac{v'}{r}$

$$\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mr^2\frac{v^2}{r^2} = 2mg(R+r-r) + \frac{1}{2}mv'^2 + \frac{1}{2}\frac{2}{5}mr^2\frac{v'^2}{r^2}$$

$$\frac{7}{10}v^2 = 2g(R+r-r) + \frac{7}{10}v'^2 \quad ...(2)$$

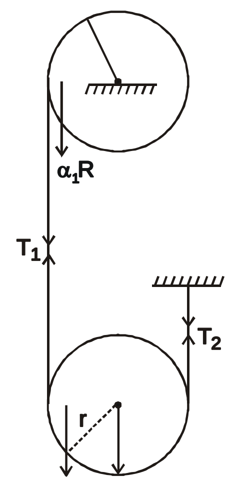
from (3) & (1) we get

$v = \sqrt{\frac{27}{7}g(R+r-r)} \Rightarrow v = \sqrt{\frac{27}{7}g(R)}$

16. from figure

$\alpha_1 R = a + \alpha_2 R$

$T_1 R = \frac{mr^2}{2}.\alpha_1$



$-T_1R + T_2R = \frac{mr^2}{2}.\alpha_2$

$mg - (T_1 + T_2) = ma$

$\alpha_2 = \frac{a}{R}$

from the above equation

$a = 2g/7$

then $\quad v^2 = 2as$

$v^2 = \frac{2.2g}{7}\times 1.2 = 4\sqrt{\frac{3}{7}}m/sec$

17. From Notes

18. $M v_0 l_0 = Mv(l_0 + l_0/10)$

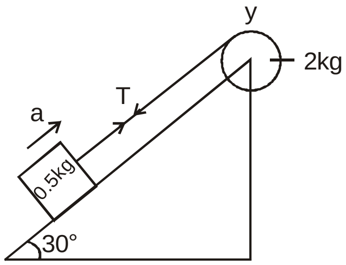
$v = 10$ m/s

$1/2\ mv_0^2 = 1/2\ mv^2 + 1/2\ kh^2$

19. **(a)** when block x moves upward then

$mg\sin\theta - T = ma$

$T = \frac{1}{2}(g/2 - a)$ ...(1)



Due to motion of Y

$T \times r = I\,\alpha$

$\alpha = a/r$ and $I = \frac{1}{2}(2)r^2$

$\Rightarrow$ $T.r = \frac{1}{2}(2)r^2.\frac{a}{r}$

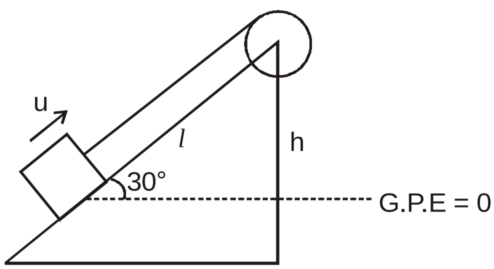
$T = a$ ...(2)

from (1) & (2) T = 1.63 N

(b) from energy conservation

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$h = l \sin 30° = l/2$



from $v = r\omega$

$\Rightarrow$ $(0.5)\, g\, \frac{\ell}{2} = \frac{1}{2} \times (0.5)\,(r\omega)^2 + \frac{1}{2}.\left(\frac{1}{2}2r^2\right)\omega^2$

$\ell = 1.22$ m