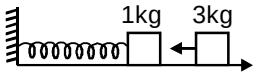
Exercise - IV (TOUGH SUBJECTIVE PROBLEMS)

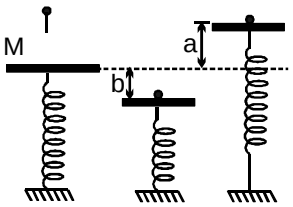
1. A point particle of mass 0.1 kg is executing SHM with amplitude of 0.1 m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

2. The particle executing SHM in a straight line has velocities 8 m/s, 7 m/s, 4 m/s at three points distant one metre from each other. What will be the maximum velocity of the particle?

3. One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to a x-axis. A block of mass m = 1 kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, t is in second and x in cm. Another block of mass M = 3kg, moving towards the origin with velocity 30 cm/s collides with the block performing SHM at t = 0 and gets struck to it, calculate :
(i) new amplitude of oscillations.
(ii) new equation for position of the combined body.
(iii) loss of energy during collision. Neglect friction.

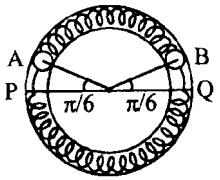


4. A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales & depth 'b' below it.
(a) Find the force constant of the spring.;



(b) Find the oscillation frequency.
(c) What is the height above the initial level from which the mass m was droped ?

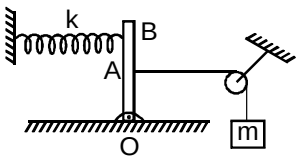
5. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in fig. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06 m. Each spring has a natural length 0.06π m and force constant 0.1 N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest



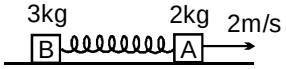
(a) Calculate the frequency of oscillation of the ball B.
(b) What is the total energy of the system.
(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.

6. An ideal gas is enclosed in a vertical cylinderical container and supports a freely moving piston of mass m. The piston and the cylinder have equal cross-sectional area A, atmospheric pressure is P_0 and when the piston is in equilibrium position. Show that the piston executes SHM and find the frequency of oscillation (system is completely isolated from the surrounding). $\gamma = C_p/C_v$. Height of the gas in equilibrium position is h.

7. A massless rod is hinged at O. A string carrying a mass m at one end is attached to point A on the rod so that OA = a. At another point B (OB = b) of the rod, a horizontal spring of force constant k is attached as shown. Find the period of small vertical oscilla tions of mass m around its equilibrium position.



8. Two blocks A (2kg) and B(3kg) rest up on a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially the spring is underformed. A is imparted a velocity of 2m/s along the line of the spring away from B. Find the displacement of A t second later.



9. Consider a fixed ring shaped uniform body of linear mass density ρ and radius R. A particle at the centre of ring is displaced along the axis by a small distance, show that the particle will execute SHM under gravitation of ring & find its time period neglecting other forces.