## EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

**1.** $K.E. = 8\times10^{-3}$ J

$\frac{1}{2}m\omega^2A^2 = 8\times10^{-3}$ J

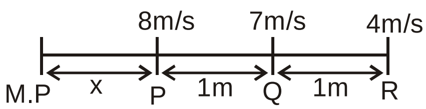
$\frac{0.1}{2}\omega^2(.1)^2 = 8\times10^{-3}$

$\omega = 4$

$\phi = 45^\circ = \pi/4$

$x = (.1)\sin(4t+\pi/4)$

**2.**




As $V^2 = \omega^2(A^2 - x^2)$

For P, $64 = \omega^2(A^2 - x^2)$ ...(1)

For Q, $49 = \omega^2[A^2 - (x+1)^2]$ ...(2)

For, R, $16 = \omega^2[A^2 - (x+2)^2]$ ...(3)

(1) – (2)

$15 = \omega^2(2x+1)$

(2) – (3)

$33 = \omega^2(2x+3)$

$\frac{15}{33} = \frac{(2x+1)}{(2x+3)}$

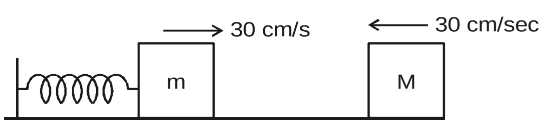
$x = \frac{1}{3}$

Putting the value in equation above

$\omega = 3$

$A = \frac{\sqrt{65}}{3}$, Max. Speed $= A\omega = \sqrt{65}$

**3.**




we know that $\omega^2 = \frac{k}{m}$

$k = m\omega^2 = (1)(10)^2 = 100$ N/m

At t = 0 block of mass m is at mean position

x = 10 cm.

velocity of block $m = v_m = \frac{dx}{dt} = 30\cos 10t$

at t = 0 $v_m$ = 30 cm/sec.

from momentum conservation

(M + m) v = M(30) – m(30)

v = 15 cm/sec

Now $\frac{1}{2}(M+m)v^2 = \frac{1}{2}kA^2$

on solving A = 3 cm

(b) New $\omega$ of the system having mass (M + m)

$\omega' = \sqrt{\frac{K}{M+m}} = \sqrt{\frac{100}{4}} = 5$ rad/s

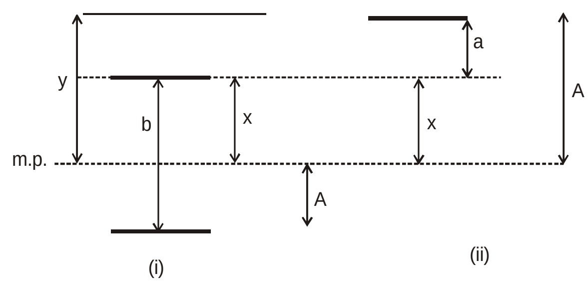
$x' = 10 - 3\sin 5t$

(c) Losse of energy during collision = Energy before collision – Energy after collision

$= \frac{1}{2}m(0.3)^2 + \frac{1}{2}M(0.3)^2 - \frac{1}{2}(M+m)(0.15)^2$

$= 0.135$ Jule

**4.**




(A) from figure (i) b = A + x ...(1)

from figure (ii) A = a + x ...(2)

from eq. (1) & (2)

b = a + 2x ⇒ 2x = b – a

and x = mg/k

⇒ $K = \frac{2mg}{b-a}$

(B) Oscillation frequency $= \frac{1}{2\pi}\sqrt{\frac{K}{m_{total}}}$

$= \frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$

(C) By energy conservation.

***5.*** (a) Both the spring have same force so.

It is parallel equivalent of spring

$k_{eq} = k_1 + k_2 = 0.2$ N/m

Now the problem change in two block system in which reduced mass is

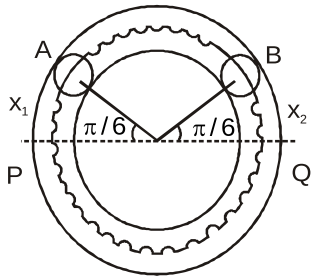
$m = \frac{m_1m_2}{m_1+m_2} = \frac{0.1\times0.1}{0.1+0.1} = 0.05$ kg

$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.20}{0.05}} = \frac{1}{\pi}H_2$

(b) Balls are at rest in position A & B so

Total energy is in potential energy for

$$E = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$




$$= \frac{1}{2}kx^2 + \frac{1}{2}kx^2 \qquad \begin{cases} x_1 = x_2 \\ R_1 = R_2 \end{cases}$$

$$E = kx^2$$

$$x = x_1 + x_2 \qquad = R\,\pi/6 + R\,\pi/6$$

$$= 0.02\pi\,m$$

Now $E = kx^2$

$$= (0.1)\,(0.02\pi)^2 = 4\pi^2 \times 10^{-5}\,J$$

(c) At P & Q no stretch in spring so complete energy is in the kinetic form

$$\Rightarrow \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = E$$

$$m_1 = m_2 = 0.1\,kg$$

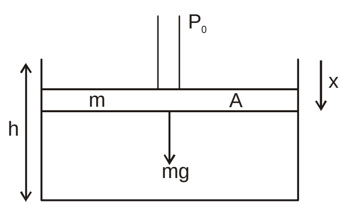
$$v_1 = v_2 = v \Rightarrow 0.1v^2 = 4\pi^2 \times 10^{-5}$$

$$v = 2\pi \times 10^{-2}\,m/sec$$

**6.** $P = P_0 + \frac{mg}{A}$

Bulk modulus B = $\gamma p$

$$\frac{-\Delta P}{\frac{\Delta v}{v_0}} = \gamma p$$




$$\Delta P = \frac{-\gamma p\,\Delta V}{V_0}$$

$$F_{net} = A\,\Delta P = \frac{-A\gamma p\,\Delta v}{v_0}$$

$$= -A\gamma P\frac{x\,A}{h\,A} \qquad \begin{Bmatrix} \because \quad \Delta v = xA \\ v_0 = hA \end{Bmatrix}$$

$$= -A\gamma P\frac{x}{h}$$

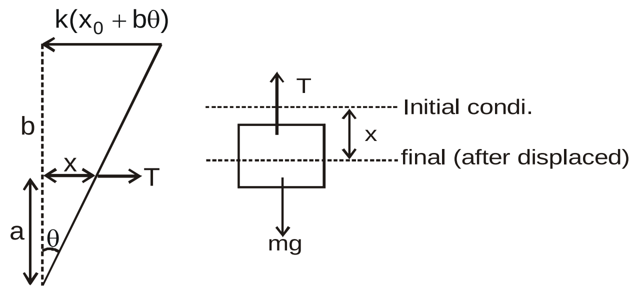
$$F_{net} = -A\gamma\left[P_0 + \frac{mg}{A}\right]\frac{x}{h} \Rightarrow k = mw^2 = A\gamma\left(P_0 + \frac{mg}{A}\right)\frac{1}{h}$$

$$\omega^2 = \frac{A\gamma}{mh}\left(P_0 + \frac{mg}{A}\right) \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{A\gamma}{mh}\left(P_0 + \frac{mg}{A}\right)}$$

**7.** At equilibrium condition we assume elongation is spring is $x_0$ then

$mg(a) = Kx_0 b$ ...(1)

Now rod is moved small angle $\theta$ then




$$\Rightarrow \quad T.a = K(x_0 + b\theta).b$$

$$T = \frac{k(x_0 + b\theta)b}{a}$$

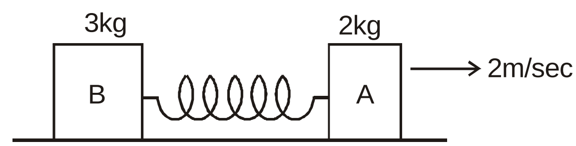
On block of mass m $F_{net} = mg - T$

$$F_{net} = mg - \frac{k(x_0 + b\theta)b}{a}$$

$$= mg - \frac{k\,x_0 b}{a} - \frac{kb^2\theta}{a} = \frac{-kb^2\theta}{a} \qquad \left\{\text{and} \quad \theta = \frac{x}{a}\right.$$

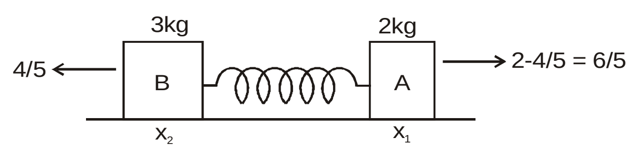
$$= \frac{-kb^2 x}{a^2} \Rightarrow \qquad T = 2\pi\sqrt{\frac{ma^2}{kb^2}}$$

**8.**




$V_{com}$ = 4/5

In frame of chita :-




Let us assume that elongation in spring is x then

$x_1 + x_2 = x$ ....(1)

$2x_1 = 3x_2$ ....(2)

(Centre of mass is at rest)

from (1) & (2)

$$x_1 + \frac{2x_1}{3} = x \qquad ...(3)$$

from energy conservation

$$\frac{1}{2} \times 2 \times \left(\frac{6}{5}\right)^2 + \frac{1}{2} \times 3 \times \left(\frac{4}{5}\right)^2 = \frac{1}{2}kx^2$$

x = 0.2 ...(3)

from (2) & (3)

$x_1 = 0.12$ m

Maximum velocity = $A\omega = 6/5$

$x_1\omega = 6/5$

$(0.12)\ \omega = 6/5$

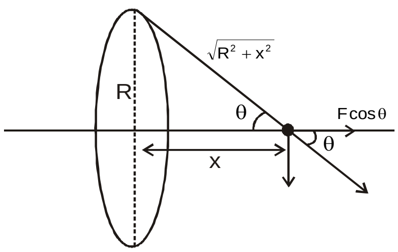
$\omega = 10$

then equation of block A

$$x = \left(\frac{4}{5}\right)t + 0.12\sin 10t$$

**9.** $F_{net} = f\cos\theta$

$$\cos\theta = \frac{x}{\sqrt{R^2 + x^2}}$$




$$\cos\theta = \frac{x}{R} \qquad \because \quad x^2 \cong 0 \quad \text{(for small distance)}$$

$$F = -\frac{GM_1M_2}{R^3}x$$

$$T = 2\pi\sqrt{\frac{M_2}{K}} \qquad \therefore \quad M_1 = \rho \times 2\pi R$$

$$= 2\pi\sqrt{\frac{R^3}{G\rho 2\pi R}} \Rightarrow T = \frac{2\pi R}{\sqrt{G\rho 2\pi}} = \sqrt{\frac{2\pi R^2}{G\rho}}$$