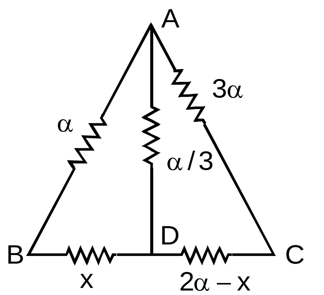
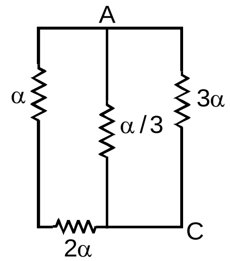
EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

1. Now

$$R_{eq} = \frac{1}{\alpha + x} + \frac{3}{\alpha} + \frac{1}{5\alpha - x}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{1}{R_{eq}}\right) = 0 \text{ for } (R_{eq})_{max}$$





$$\Rightarrow \frac{1}{(5\alpha - x)^2} - \frac{1}{(\alpha - x)^2} = 0$$

$\Rightarrow x = 2\alpha$

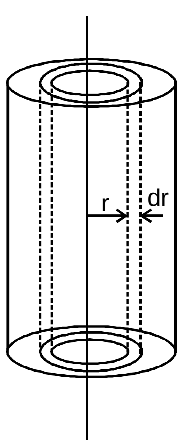
Now circuit is $R_{eq} = \left(\frac{3}{11}\right)\alpha$

2. ***A***

(a) from I = JA

$$\int_0^I dI = \int_0^R I_0\left(1 - \frac{r}{R}\right) 2\pi r dr$$

$$I = \frac{\pi J_0 R^2}{3}$$

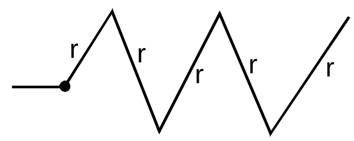


$$I = \frac{J_0 A}{3}$$

(b) $$\int_0^I dI = \int_0^R \frac{J_0 r}{R} . 2\pi r dr$$

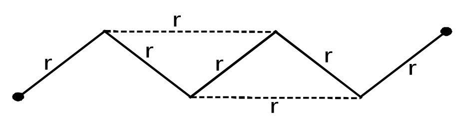
$$I = \frac{2\pi J_0}{R}\left(\frac{R^3}{3}\right) = \frac{2}{3}\pi J_0 R^2 \Rightarrow I = \frac{2J_0 A}{3}$$

3. **Case - I**



$R_1 = 5r$

Case - II



$R_2 = r + (2r \,||\, 2r) + r = 3r$ (with the help of W.S.B)

$$\therefore \frac{R_2}{R_1} = \frac{3}{5}$$

4. (a) $0.44 = 0.2\, V^{5/2}$

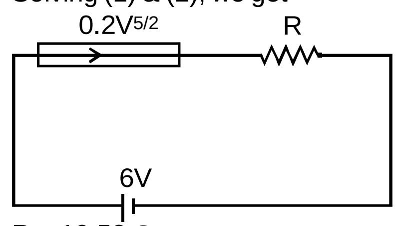
$\Rightarrow V^{5/2} = 2.2$

...(1)

$6 - V = 0.44\, R$

...(2)

Solving (1) & (2), we get



$R = 10.52\ \Omega$

(b) $0.2\, V^{5/2} \times V = 2 \times (0.2\, V^{5/2})^2 R$

$0.2\, V^{7/2} = 0.08\, V^5 R$

$1 = 0.4\, V^{3/2} R$

$6 - V = 0.2\, V^{5/2} R$

$6 = V(1.5)$

$V = 4V$

$$\Rightarrow R = \frac{1}{3.2} = 0.3125$$

5. Given circuit is

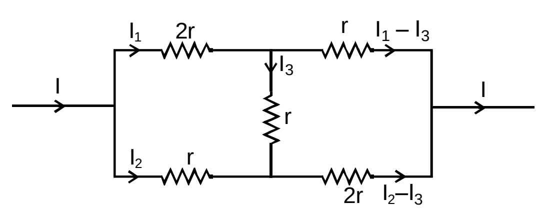
Applying KVL :

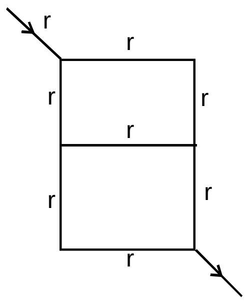
$2r\, I_1 + I_3\, r - I_2\, r = 0$

$\Rightarrow 2I_1 + I_3 - I_2 = 0$

...(1)

$(I_1 - I_3)\, r - (I_2 + I_3)\, 2r - I_3\, r = 0$





$\Rightarrow I_1 - 2 I_2 - 4 I_3 = 0$ ...(2)

Solving (1) & (2), we get

$I_2 = -3 I_3$, $4I_1 = -2I_3$

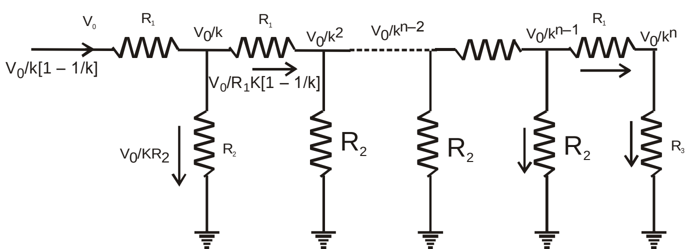
$I = I_1 + I_2 = -5I_3 \Rightarrow \frac{I_3}{I} = \frac{1}{5}$

6. **(i)**

$$\left[1-\frac{1}{K}\right]\frac{V_0}{R_1} = \frac{V_0}{KR_2} + \frac{V_0}{KR_1}\left[1-\frac{1}{K}\right]$$

$$\frac{K-1}{R_1} = \frac{1}{R_2} + \frac{K-1}{KR_1}$$

$$\frac{K-1}{R_1}\left[1-\frac{1}{K}\right] = \frac{1}{R_2}$$



$$\frac{R_1}{R_2} = \frac{(K-1)^2}{K}$$

Now from again apply KCL

$$\left[\frac{V_0}{k^{n-2}} - \frac{V_0}{K^{n-1}}\right]\frac{1}{R_1} = \frac{V_0}{k^{n-1}R_2} + \frac{V_0}{k^nR_3}$$

$$\Rightarrow \frac{k^2-k}{R_1} = \frac{k}{R_2} + \frac{1}{R_3}$$

$$\frac{R_2}{R_1}K(K-1) = K + \frac{R_2}{R_3} \quad \Rightarrow \quad \frac{R_2}{R_3} = \frac{K}{K-1}$$

(ii) $i = \frac{V_0}{KR_2} = \frac{V_0(K-1)}{K^2R_3} = \frac{V_0}{R_3}\frac{K-1}{K^2}$

7. Power given to turbines is x

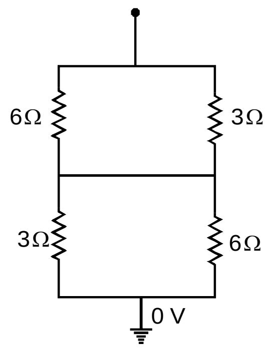
$\Rightarrow$ 90% of x = 40 W

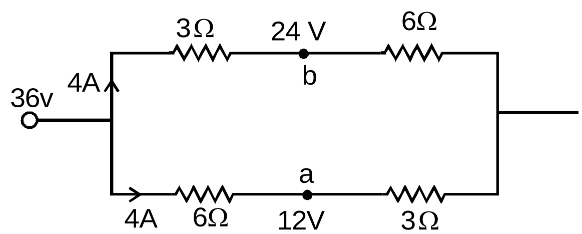
$x = \frac{400}{9}$ W , mgh = $\frac{400}{9}$ $\Rightarrow$ m = $\frac{4}{9}$ kg/sec.

$t = \frac{200}{4} \times 9$ = 450 sec.

8. (a) When switch is open

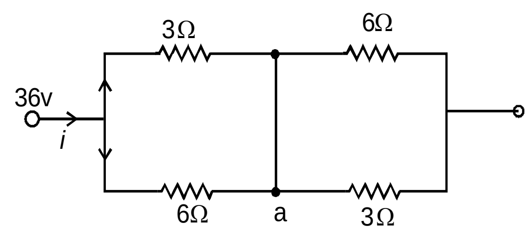
$$i = \frac{36}{9/2} = 8A$$

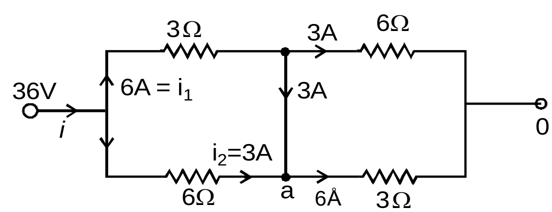




$v_b - v_a = 12$ V

8(ii)





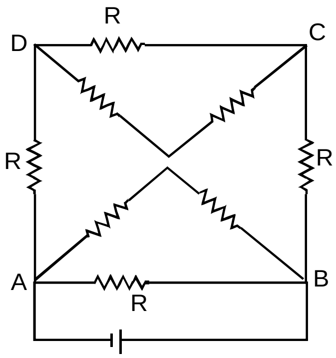
$$i = \frac{36}{4} = 9A$$

9. Let us assume each wire have cross sectional area is A and square of length ℓ then

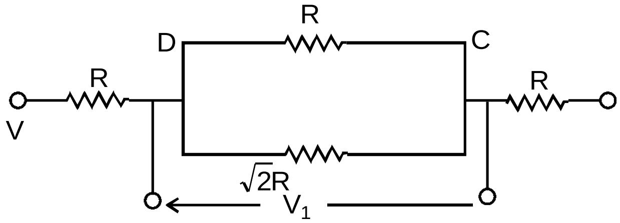
$$R = \frac{\rho\ell}{A}$$

$$P_{AB} = \frac{V^2}{R}$$

for power in DC



$$v_1 = \frac{v\left(\frac{R\sqrt{2}}{\sqrt{2}+1}\right)}{\frac{R\sqrt{2}}{\sqrt{2}+1}+2R} = \frac{V\sqrt{2}}{3\sqrt{2}+2}, \quad P_{DC} = \frac{v_1^2}{R}$$

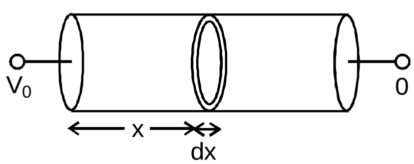


$$\frac{P_{AB}}{P_{DC}} = \frac{(3\sqrt{2}+2)^2}{2} = 11+6\sqrt{2}$$

10. Given $\rho = \rho_0 e^{-x/L}$

(a) $dR = \frac{\rho_0 e^{-x/L} dx}{A}$

$$R = \frac{1}{A}\int_0^L \rho_0 e^{-x/L} dx = \frac{-L\rho_0}{A}[e^{-x/L}]_0^L$$



$$R = \frac{\rho_0 L}{A}\left[1-\frac{1}{e}\right] \Rightarrow i = \frac{V}{R} = \frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)$$

(b) V(x) = I R (x)

$$\int_{v_0}^{v_x} dv = \frac{v_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)\frac{\rho_0}{A}\int_0^x e^{-x/L} dx$$

$$\Rightarrow v_x - v_0 = \frac{v_0}{L}\left(\frac{e}{e-1}\right)\left(\frac{-L}{1}\right)\left[e^{-x/L}\right]_0^x$$

$$v_x - v_0 = \frac{v_0 e}{(1-e)}[e^{-x/L} - 1]$$

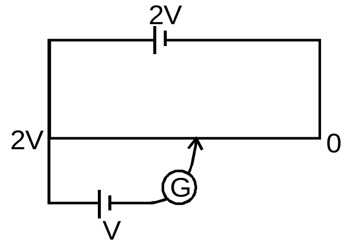
$$\Rightarrow v_x = v_0\left[\frac{(e^{-x/L}-1)}{(1-e^{-1})}+1\right] \Rightarrow v_x = v_0\left[\frac{e^{-x/L}-e^{-1}}{1-e^{-1}}\right]$$

11. Resistane of potentiometer wire = 11.5 × 10 = 115

Current in the circuit

$$i = \frac{2}{115} A$$

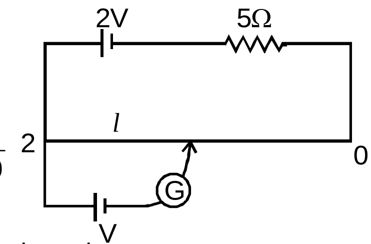
Now cell is balanced at 6.9 m



So i (6.9 × 11.5) = V, $V = 6.9 \times 11.5 \times \frac{2}{115} = 1.38\,V$

after 5 Ω is also connected then

$i = \frac{2}{120}$

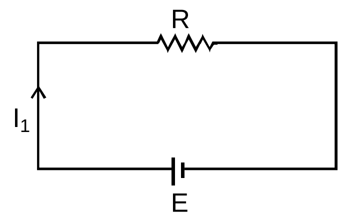


At Balanced condt.

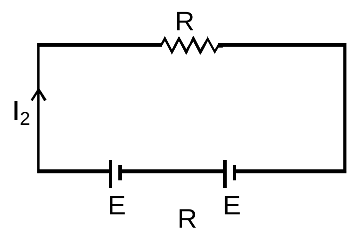
i (ℓ × 11.5) = 1.38

ℓ = 7.2 m

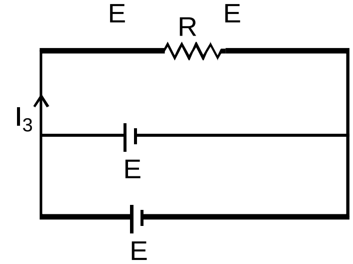
12.



$\Rightarrow E = I_1 R$



$\Rightarrow 2E = I_2 R$



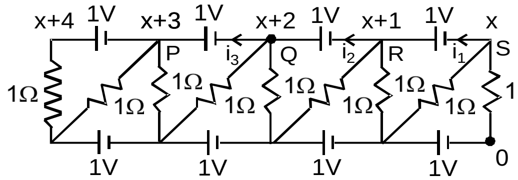
$\Rightarrow E = I_3 R$

Now $3I_3 I_2 = \frac{6E^2}{R^2}$, $2I_1(I_2 + I_3) = \frac{6E^2}{R^2}$

13. KCL at point P, Q, R, S then find out

$9x = 4 \Rightarrow x = \frac{4}{9}$

$\Rightarrow v_B - 1 - 1 - \frac{4}{9} = v_A$



$$v_A - v_B = -\frac{22}{9}\text{volt}$$

14. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\Rightarrow \frac{1}{R_{eq}(1+\alpha_{eff})} = \frac{1}{R(1+\alpha t)} + \frac{1}{3R(1+2\alpha t)} \Rightarrow R_{eq} = \frac{3R}{4}$$

$$\frac{4}{3(1+\alpha_{eff}t)} = \frac{1}{(1+\alpha t)} + \frac{1}{3(1+2\alpha t)}$$

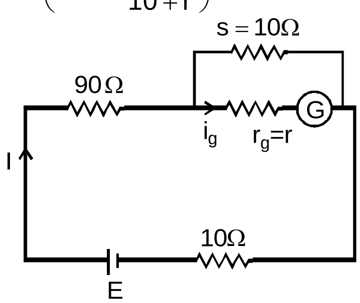
$$\Rightarrow \frac{4}{3}(1-\alpha_{eff}t) = (1-\alpha t) + \frac{1}{3}(1-2\alpha t)$$

$$\frac{4}{3}(1-\alpha_{eff}t) = \frac{3-3\alpha t+1-2\alpha t}{3}$$

$$\Rightarrow 4 - 4\,\alpha_{eff}\,t = 4 - 5\,\alpha\,t,\quad \alpha_{eff} = \frac{5}{4}\alpha$$

15. Current sensitivity = $\frac{300}{50}$ = 6 mA/div.

$$I = \left(\frac{E}{100+\frac{10r}{10+r}}\right)\text{Amp.}$$



$$i_g = \frac{10I}{(10+r)} = \frac{10}{(10+r)}\left[\frac{E}{100+\frac{10r}{10+r}}\right]$$

In this situation i_g = 9 × 6 = 54 mA

$$\Rightarrow 54 \times 10^{-3} = \frac{10E}{(10+r)\left[100+\frac{10r}{10+r}\right]} \quad ..(1)$$

Similarly $30\times 6\times 10^{-3} = \frac{50E}{(50+r)\left[100+\frac{50r}{50+r}\right]}$ ...(2)

from eq. (1) & (2)

r = 233.33 Ω

E = 144 V

16. **(a)** $i = \frac{10}{10} = 1A$ Now

$$\left(\frac{9}{12}\right)L(1) = 4.5 \quad \Rightarrow \quad L = 6\,m$$

(b) Now i = $\frac{10}{20}$ = 0.5 A

So at balancing condt.

$$V = \frac{1}{2}\times 6 = 3V \quad \Rightarrow 4.5 - \left(\frac{4.5}{2+r}\right)r = 3 \quad \Rightarrow \quad r = 1\ \Omega$$

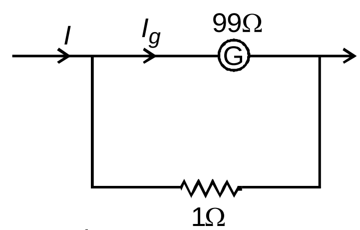
17. For ammeter 99 I_g = (I – I_g) 1

or I = 100 I_g ...(i)

I_g is the full scale deflection current of the galvanometer and I the range of ammeter.

For the circuit in figure-1, given in the question

$$\frac{12V}{2+r+\frac{99\times 1}{99+1}} = 3A \quad \Rightarrow \quad r = 1.01\ \Omega$$



For voltmeter, range

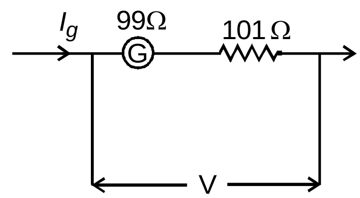
V = I_g(99 + 101)

V = 200 I_g

Also resistance of the voltmeter = 99 + 101 = 200 Ω.

In figure - 2 resistance across the terminals of the battery.

$$R_1 = r + \frac{200\times 2}{200+2} = 2.99\,\Omega$$



∴ Current drawn from the battery, $I_1 = \frac{12}{2.99} = 4.01A$

∴ Voltmeter reading $\frac{4}{5}V = 12 - I_1 r$ = 12 – 4.01 × 1.01

$$\Rightarrow \quad V = 7.96\times\frac{5}{4} = 9.95\,\text{volt}$$

Using (ii), $I_g = \frac{9.95}{200} = 0.05\,A$

Using (i) range of the ammeter I = 100 I_g = 5 A