Exercise - V JEE-Problems

1. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

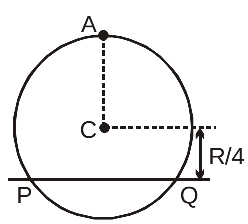
(A) I (B) I $\sin^2\theta$
(C) I $\cos^2\theta$ (D) I $\cos^2(\theta/2)$

[JEE' 98]

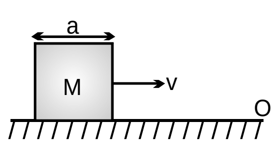
2. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that **[JEE' 98]**

(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time
(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(C) the magnitude of $\vec{L}$ does not change with time
(D) $\vec{L}$ does not change with time

3. A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the wedge of the disc as in fig. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position. **[JEE' 98]**



4. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :

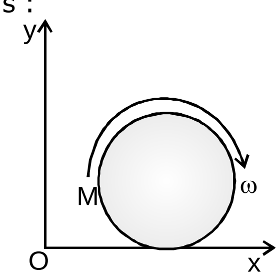


(A) 3v/4a (B) 3v/2a
(C) $\sqrt{3v}/\sqrt{2a}$ (D) zero **[JEE' 99]**

5. A smooth sphere A is moving on a frictionless horizontal p lane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angularspeeds are ω_A and ω_B, respectively. Then **[JEE' 99]**

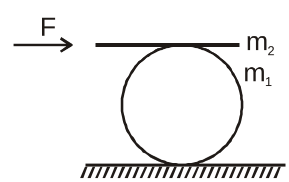
(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$
(C) $\omega_A = \omega$ (D) $\omega_B = \omega$

6. A disc of mass M and radius R is rolling with angular speed w on a horizontal as shown. The magnitude of angular momentum of the disc about the origin O is : **[JEE' 99]**



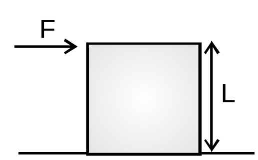
(A) (1/2) $MR^2\,\omega$ (B) $MR^2\,\omega$
(C) (3/2) $MR^2\,\omega$ (D) $2MR^2\,\omega$

7. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F. Find



(a) the accelerations of the plank and the center of mass of the cylinder, and
(b) the magnitudes and directions of frictional forces at contact points. **[JEE' 99]**

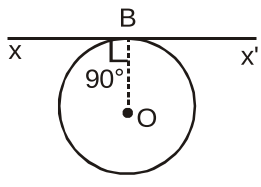
8. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is :



(A) infinitesimal (B) mg/4
(C) mg/2 (D) mg$(1-\mu)$

[JEE' 2000]

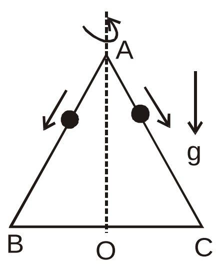
9. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX′ is :



(A) $\rho L^3/8\pi^2$ (B) $\rho L^3/16\pi^2$
(C) $5\rho L^3/16\pi^2$ (D) $3\rho L^3/8\pi^2$

[JEE' 2000]

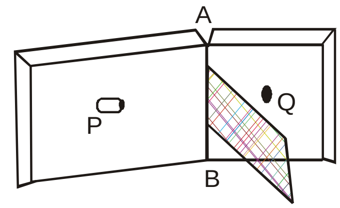
10. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at AO. Then, the beads are released from rest simultaneously and allowed to slide down, one along AB and the other AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are :



(A) angular velocity and total energy (kinetic and potential)
(B) total angular momentum and total energy
(C) angular velocity and moment of inertia about the axis of rotation
(D) total angular momentum and moment of inertia about the axis of rotation. **[JEE' 2000]**

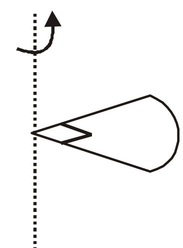
11. A rod AB of Mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest. **[JEE' 2000]**
(a) Find the ratio m/M
(b) A point P on the rod is at rest immediately after the collision. Find the distance AP
(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision

12. Two heavy metallic plates are joined together at 90º to each other. A laminar sheet of mass 30 Kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 Kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6N-s



(a) Find the location of the centre of mass of the laminar sheet from AB
(b) At what angular velocity does the laminar sheet come back after the first impact ?
(c) After how many impacts, does the laminar sheet come to rest ? **[JEE' 2001]**

13. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

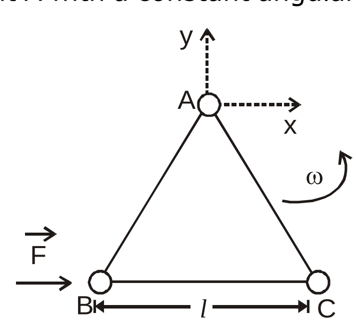


(A) $\frac{1}{2}MR^2$ (B) $\frac{1}{4}MR^2$

(C) $\frac{1}{8}MR^2$ (D) $\sqrt{2}MR^2$

[JEE'(Scr) 2001]

14. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.



(a) Find the magnitude of the horizontal force exerted by the hinge on the body

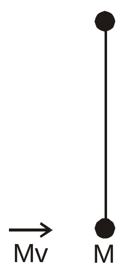
(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exterted by the hinge on the body, immediately after time T.

15. A particle is moving in a horizontal uniform circular motion. The angular momentum of the particle is conserved about the point :

[JEE'(Scr) 2003]

(A) Centre of the circle (B) Outside the circle
(C) Inside the circle
(D) Point on circumference

16. Two particles each of mass M are connected by a massless rod of length l. The rod is lying on the smooth surface. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be

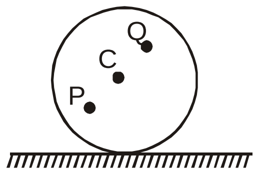


(A) v/l (B) $2v/l$

(C) $v/2l$ (D) none

[JEE'(Scr) 2003]

17. A disc is rolling (without slipping) on a horizontal surface. C is its center and Q and P are two points equidistant from C. Let V_p, V_Q and V_C be the magnitude of velocities of points P, Q and C respectively, then



(A) $V_Q > V_C > V_P$ (B) $V_Q < V_C < V_P$

(C) $V_Q = V_p$, $V_C = \frac{1}{2}V_P$ (D) $V_Q < V_C > V_P$

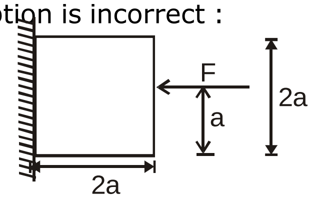
[JEE'(Scr) 2004]

18. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

[JEE'(Scr) 2004]

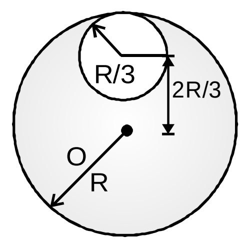
(A) 2K (B) K/2
(C) K/4 (D) 4K

19. A block of mass m is held fixed against a wall by a applying a hor izontal force F. Which of the following option is incorrect :



(A) friction force = mg
(B) F will not produce torque
(C) normal will not produce torque
(D) normal reaction = F

20. A disc has mass 9m. A hole of radius R/3 is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is :



(A) $8\ mR^2$ (B) $4\ mR^2$

(C) $\frac{40}{9}mR^2$ (D) $\frac{37}{9}mR^2$

[JEE'(Scr) 2005]

21. A particle moves in circular path with decreasing speed. Which of the following is correct

(A) $\vec{L}$ is constant

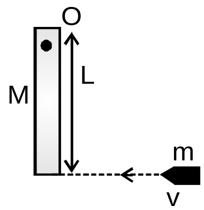
(B) only direction of $\vec{L}$ is constant

(C) acceleration $\vec{a}$ is towards the centre

(D) it will move in a spiral and finally reach the centre

[JEE'(Scr) 2005]

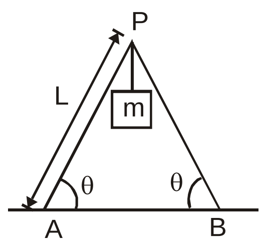
22. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O.



[JEE' 2005]

23. A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. **[JEE' 2005]**

24. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B. **[JEE' 2005]**



25. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be **[JEE' 2006]**

(A) $2R/\sqrt{15}$ (B) $R\sqrt{2/15}$

(C) $4R/\sqrt{15}$ (D) R/4

26. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and incline is μ. Then **[JEE' 2006]**

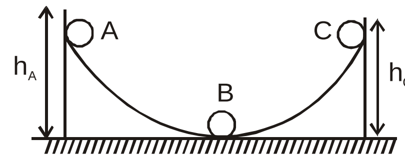
(A) frictional force is always $\mu mg \cos\theta$

(B) friction is a dissipative force

(C) by decreasing θ, frictional force decreases

(D) friction opposes translation and supports rotation

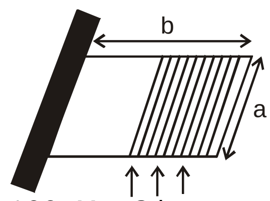
27. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respe0ctively. Then **[JEE' 2006]**



(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$

(C) $h_A = h_C$; $K_B = K_C$ (D) $h_A < h_C$; $K_B > K_C$

28. There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v ? **[JEE' 2006]**



It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m, a = 1m; g = 10 m/s^2.

Paragraph for Question Nos. 29 to 31 (3 questions)

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

Q.29 The ratio x_1/x_2 is **[JEE' 2007]**

(A) 2 (B) 1/2

(C) $\sqrt{2}$ (D) $1/\sqrt{2}$

30. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is **[JEE' 2007]**

(A) $2I\omega/(3t)$ (B) $9I\omega/(2t)$

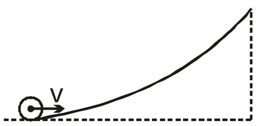
(C) $9I\omega/(4t)$ (D) $3I\omega/(2t)$

31. The loss of kinetic energy during the above process is **[JEE' 2007]**

(A) $I\omega^2/2$ (B) $I\omega^2/3$

(C) $I\omega^2/4$ (D) $I\omega^2/6$

32. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2/(4g)$ with respect to the initial position. The object is **[JEE' 2007]**



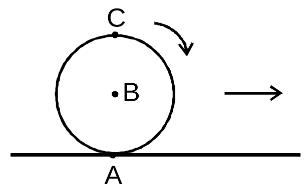
(A) ring (B) solid sphere
(C) hollow sphere (D) disc

33. STATEMENT-1 If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant
because
STATEMENT-2
The linear momentum of an isolated system remains constant.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
[JEE 2007]

34. STATEMENT-1
Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.
STATEMENT-2
By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True
[JEE-2008]

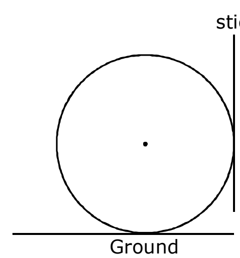
35. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009]**
(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

36. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure *A* is the point of contact, *B* is the centre of the sphere and *C* is its topmost point Then, **[JEE 2009]**

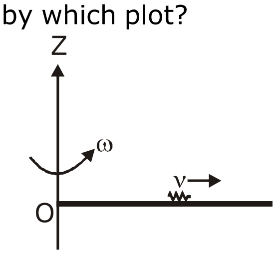


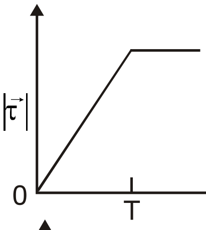
(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$
(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$
(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

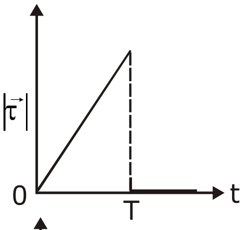
37. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is? **[JEE 2011]**

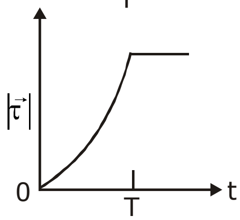


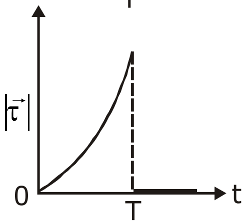
38. A thin uniform rod, pivoted at O is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, small insect starts from O and moves with constant speed v with respect to the rod towards the other end. it reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?



(A) 

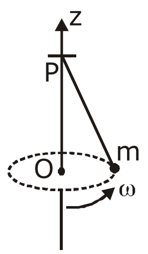
(B) 

(C) 

(D) 

[JEE 2012]

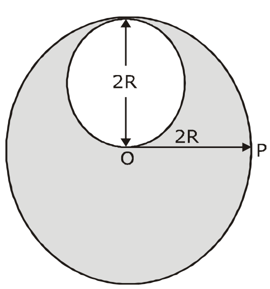
39. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_0$ and $\vec{L}_P$ respectively, then.



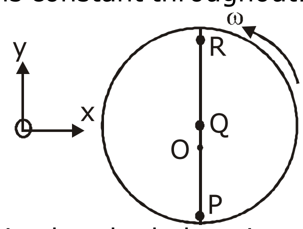
(A) $\vec{L}_0$ and $\vec{L}_P$ do not vary with time
(B) $\vec{L}_0$ varies with time while $\vec{L}_P$ remains constant
(C) $\vec{L}_0$ remains constant while $\vec{L}_P$ varies with time
(D) $\vec{L}_0$ and $\vec{L}_P$ both vary with time.

[JEE 2012]

40. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_o and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_o}$ to the nearest integer is **[JEE 2012]**



41. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then

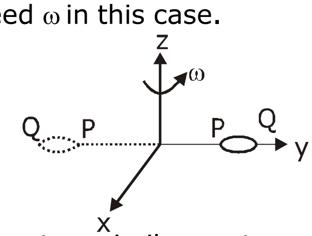


(A) P lands in the shaded region and Q in the unshaded region
(B) P lands in the unshaded region and Q in the shaded region
(C) Both P and Q land in the unshaded region
(D) Both P and Q land in the shaded region

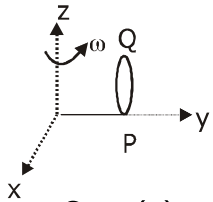
[JEE 2012]

Paragraph for Question Nos. 42 to 43

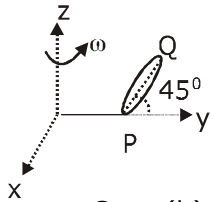
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

42. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct ?

(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a); and is at 45^o to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a); and is at 45^o to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a); and is at 45^o to the x-z plane and is normal to the plane of the disc for case (b). **[JEE 2012]**

43. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct

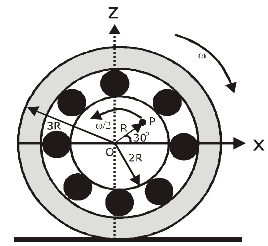
(A) It is $\sqrt{2}\omega$ for both the cases.

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).

(D) It is ω for both the cases. **[JEE 2012]**

44. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30^o with the horizontal. Then with respect to the horizontal surface.



(A) the point O has a linear velocity $3R\omega\hat{i}$.

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

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45. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time.

(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

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