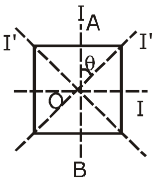
EXERCISE – V

JEE QUESTIONS

1.



$2I = 2I'$
$I' = I$

2. $\vec{\tau} = \vec{A} \times \vec{L}$

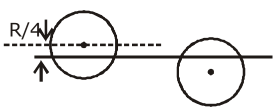
$\vec{\tau} \perp \vec{L}$ $\vec{\tau} \perp \vec{A}$

and $\vec{\tau} = \frac{dL}{dt}$

component of $\vec{\tau}$ along $\vec{A}$ is zero

there is no change of $\vec{L}$ along $\vec{A}$

3.



$2 \times mg \times R/4 + mg\ 2 \times 5 \times R/4 =$

$\frac{1}{2}\left(\frac{MR^2}{4} + M\left(\frac{R}{4}\right)^2 + M\left(\frac{5R}{4}\right)^2\right)\omega^2$

$\Rightarrow \quad \frac{12mgR}{4} = \frac{1}{2}\frac{mR^2[4+1+25}{16}.\omega^2$

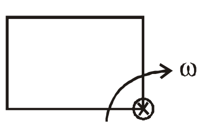
$\Rightarrow \quad 3mgR = \frac{1}{2}\frac{mR^2 \times 30.w^2}{10}$

$\Rightarrow \quad W = \sqrt{\frac{2g}{10R}}$

Speed $= W.\ 5R/4 = \sqrt{\frac{2g \times 16}{10R}}.\frac{5R}{4}$

Speed $= \sqrt{5gR}$

4. $Mv\frac{a}{2} = \frac{2 \times M \times a^2 \times w}{3}$



$\frac{2 \times Ma^2}{3}$

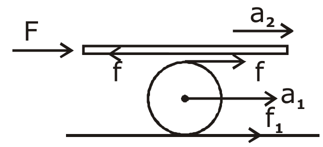
$W = \frac{3v}{4a}$

5. W remains unchanged
Because friction is absent.

6. Angular Momentum about 0
$= I_{disc}\, W + mvR$

$= \frac{1}{2}mR^2W + m(WR)R = \frac{3}{2}mR^2w$

7.

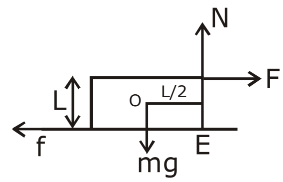


$a_2 = a_1 + \alpha R$ .....(1)
$F - f = m_2\,(a_2)$ .....(2)
$f + f_1 = m_1 a_1$ .....(3)

$f.R - f_1 R = \frac{m_1 R^2}{\alpha} \times \frac{a_1}{R}$ .....(4)

8. In a limiting case
Normal Reaction

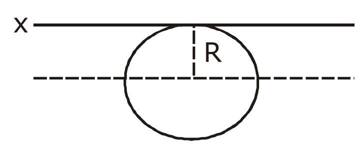
will through E.



so torque about O is

$F\frac{L}{2} + f\frac{L}{2} = \frac{NL}{2}$

9.



$I = \frac{MR^2}{2}$

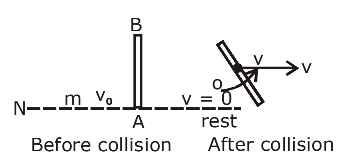
$I_{xx'} = I + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

$\because M = \rho L$ and $R = \frac{L}{2\pi}$

$I_{xx'} = \frac{3}{8}\ \frac{\rho L^3}{\pi^2}$

10. As the beads are released the distance from AO is continuoesly change and speed is continuoesly change.
so w $\neq$ const value
and L about AO is change continously (due to r change)
Net τ about AO is zero is the system so L conserved.
T Energy is conserved because increase its K.E.= change is P.E.

11.



Angular momentum consevation about N

$0 + 0 = \frac{Ml^2}{12}\ W - Mv\frac{L}{2}$

$\Rightarrow \quad \boxed{W = \frac{6v}{l}}$ ...... (1)

From Linear Momentum Conservation

$mV_0 = MV$ ...... (2)

$e = 1 = \frac{v + wL/2 - 0}{v_0}$

$\Rightarrow \quad v_0 = v + \frac{wL}{2}$ ..... (3)

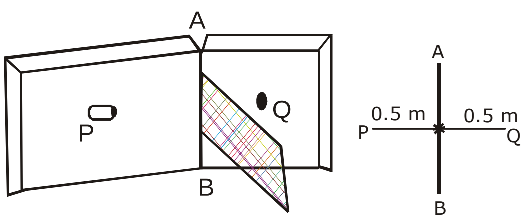
From equation (2)

$\frac{m}{M} = \frac{v}{v_0}$

From equations we get

$\frac{m}{M} = \frac{1}{4}$

12.



Let N be the $\perp$ distance of C.M. from AB

Impulse = change in linear Momentum

$6 = M(v_f + v_i) \quad \begin{Bmatrix} v_i = r(1) \\ v_f = rw \end{Bmatrix}$

$6 = 30(rw + r)$

$r(1 + w) = \frac{1}{5}$ ....(1)

linear speed after time t in combined motion (Rotational + Translational)

$v' = 2v \sin\left(\frac{wt}{2}\right)$

From equation (1), (2), (3)

$v_0 = 4v$

$w = \frac{3v_0}{2l}, \quad v' = \frac{v_0}{2\sqrt{2}}$

Angular Impulse = change in angular Momentum

$6(0.5) = I_{AW}(w + 1)$

$I_{AB} = I_{CM} + Mr^2 = 1.2 + 30r^2$

$\Rightarrow \quad 3 = (`1.2 + 30r^2)(1 + w)$ ....(2)

From equation (1) & (2)

r = 0.1 m & 0.4 m

(Reject, because w = –ve)

Put r = 0.1 m in equation (1)

w = 1 rad /sec

13. Mass of complete disc = 4M

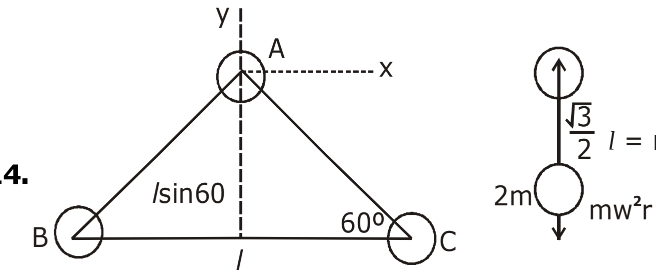
$I_{disc} = \frac{1}{2}(4M)R^2$

$I_{section} = \frac{I_{disc}}{4} = \frac{MR^2}{2}$

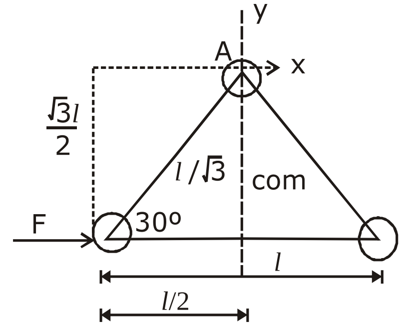
Since sheet return with same Angular velocity of 1 rad/sec.

So sheet will never comes to Rest.

14.



$F_{hnz} = mw^2r = 2mw^2 \frac{\sqrt{3}}{2} l = \sqrt{3} mw^2 l$.



Torque about A $\tau_A = F\frac{\sqrt{3}l}{2} = 2\, m l^2 \alpha$

$\alpha = \frac{\sqrt{3}F}{4ml}$

Now $a_t \left(\frac{\sqrt{3}F}{4ml}\right)\left(\frac{l}{\sqrt{3}}\right) = \frac{F}{4m}$

acceleration of C.O.M. along x axes

Let F_x be be the force applied by the hinged along x-axis then.

$F_x + F = (3m)(a_t)$

$F_x = -\frac{F}{4}$

F has no component along y direction

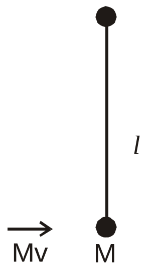
$\Rightarrow$ Contripletal force $\quad F_y = \sqrt{3}mw^2 l$

15. About the center

Beccause about center τ of $\frac{mv^2}{r}$ is zero.

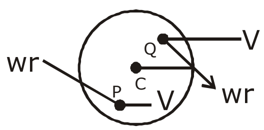
16. Given

$\int F.\, dt = mv$



Angular impulse
= change in angular
momentum
$mvl = ml^2w$

$w = \frac{v}{l}$

17. 

$V_Q > V_C > V_P$

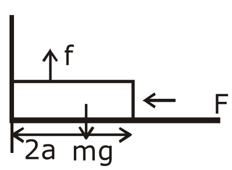
18. Let initial moment of inertia is I and angular velocity is w.

So $K = \frac{1}{2}Iw^2$

From angular momentum conservaiton
$IW = 2IW_f$

$W_f = \frac{w}{2}$

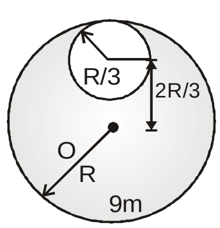
$K_f = \frac{1}{2}.2IW_f^2 = \frac{1}{2}.2I.\frac{W^2}{4} = \frac{K}{2}$

19. 

Block held fixed f = mg
normal force N = F
normal may produce torque because
it may shift
F will not produce torque due to passing
C.O.M.

20. $I_0 = \frac{9mR^2}{2}$
Moment of inertia of

R/3 disc about is



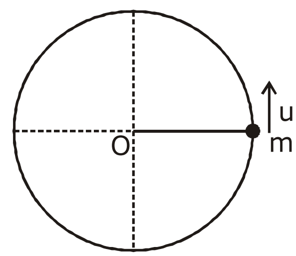
$I = I_{com} + M\left(\frac{2R}{3}\right)^2$

$= \frac{m(R/3)^2}{2} + \frac{4R^2}{9}M$

$I = \frac{MR^2}{2}$

remaining disc $= \frac{9MR^2}{2} - \frac{MR^2}{2}$

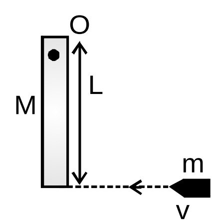
$= 4MR^2$

21. 

$\vec{L}$ is not constant due decreasing in speed, only direction of L will constant.

22. Angular momentum
about O is conserved

$L_i = L_f$

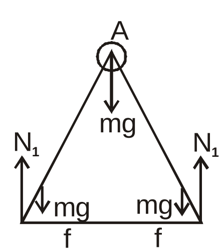


$mvL = \left[mL^2 + \frac{mL^2}{3}\right]w$

$w = \frac{3mv}{L(3m+M)}$

23. From Notes ; $a = \frac{g\sin\theta}{1+c}$

For cylinder ; $c = \frac{1}{2}$

24. 

System is in equilibrium
S0 $2N_1 = (12M + m)g$ ..(1)
Torque about A

$Mg\frac{1}{2}\cos\theta + fL\sin\theta = N_1L\cos\theta$

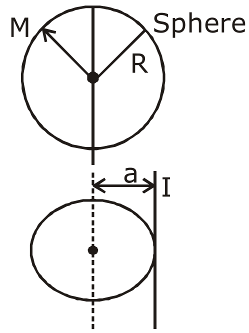
$f\sin\theta = N_1\cos\theta - \frac{Mg\cos\theta}{2}$

Put the value of N_1 from equation (1), we get

$f = \frac{(M+m)}{2}g\cot\theta$

25. $I = \frac{2}{5}MR^2$

Now, $I = \frac{Ma^2}{2} + Ma^2$



$\therefore \quad \frac{2}{5}MR^2 = \frac{Ma^2}{2} + Ma^2$

$a = \frac{2R}{\sqrt{15}}$

26. If friction is sufficient for pure ralling

then $f = \frac{mg\sin\theta}{1+\frac{1}{C}}$

For cylinder $C = \frac{1}{2}$

$f = \frac{mg\sin\theta}{3}$, $f = \frac{mg\sin\theta}{3}$

A $\theta \downarrow$ $f \uparrow$

In pure rolling friction opposes translation motion and support Rotational (Provided Net Tarque)

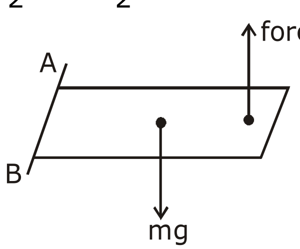
27. At B

$\frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

At C

$\frac{1}{2}mv_1^2 + \frac{1}{2}Iw^2$

28.



Now if plate is in equilibrium then

$\tau_{AB} = 0$

From

$\int \tau.dt = \text{chagnein A.M.}$

$Mg\,\frac{b}{2} = n.\,m.\,\frac{3b}{2}v(A)$

$v = \frac{3\times 10}{100\times 0.01\times 3}\left(\frac{ab}{2}\right)$

$\text{Area} = \frac{ab}{2} = 1m^2$

v = 10 m/sec

29. $\frac{1}{2}Kx_1^2 = \frac{1}{2}I(2w)^2$ ....(1)

$\frac{1}{2}Kx_2^2 = \frac{1}{2}2I(w)^2$ ....(2)

equation (1) / (2)

$\frac{x_1}{x_2} = \sqrt{2}$

30. From angular momentum conservation (Friction is internal force)

2 W(I) + w(2I) = (2I + I) w'

$w' = \frac{4wI}{3I} = \frac{4}{3}w$

$\int \tau_{friction} \times t$ = change is angular momentum

$\tau_{(friction)t} = \frac{4}{3}wI - 2Iw \Rightarrow \tau_{friction} = \frac{2Iw}{2t}$

31. Initial $K.E_0 = \frac{1}{2}I(4w^2) + \frac{1}{2}.2Iw^2$

$= 3Iw^2$

Final $K.E_f = \frac{1}{2}(3I)\frac{16w^2}{9} = \frac{8}{3}Iw^2$

Loss in K.E. $= -\frac{8}{3}Iw^2 + 3Iw^2 = \frac{Iw^2}{3}$

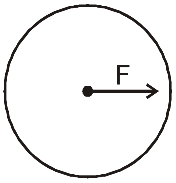
32. $\frac{1}{2}mv^2 + \frac{1}{2}Iw^2 = mg\left(\frac{3v^2}{4g}\right)$

Put $w = \frac{v}{R}$

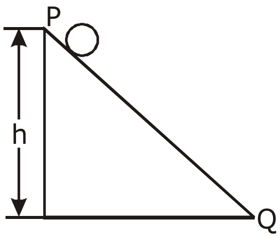
$\frac{1}{2}\frac{I}{R^2} = \frac{3}{4}m - \frac{1}{2}m$

$I = \frac{mR^2}{2}$ (disc)

33. Force at centre increase linear velocity



34.



For solid cylinder $C = \frac{1}{2}$

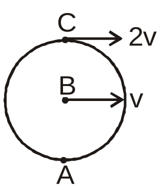
For hellow cylinder C = 1

$a = -g\frac{\sin\theta}{1+c}$

$a_{HC} < a_{sc}$ So $t_{HC} > t_{SC}$
Kinetic energy of both cylinder is same (mgh)

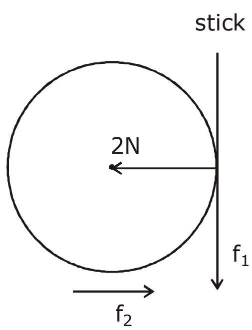
35. $F_{external} = 0$
P = consereved

36.



$V_A = 0$, $V_B = V$
and $V_C = 2V$

37. 0004



$2 - f_2 = 2a = 0.6 \Rightarrow f_2 = 1.4$

$\tau = I\alpha \Rightarrow (f_2 - f_1)\,R = MR^2\,\frac{a}{R}$

$1.4 - f_1 = Ma = 0.6$

$f_1 = 0.8 = \mu\,(2) = \frac{P}{10}\times 2$

P = 4

38. $\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega)$

$\tau = \omega\frac{dI}{dt} = \omega\,\frac{d}{dt}(I_{rod} + I_m)$

as I_{rod} = com

$\Rightarrow \tau = \frac{wd}{dt}\left(I_{insect}\right)$

$= \omega\frac{d}{dt}(mr^2)$

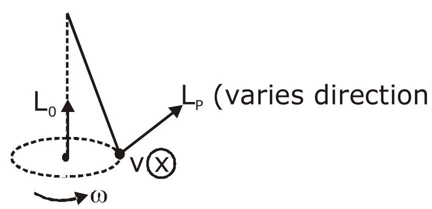
$= m\omega\left(2r\frac{dr}{dt}\right) = 2m\,r\omega v$

$= 2m(vt)\omega v$

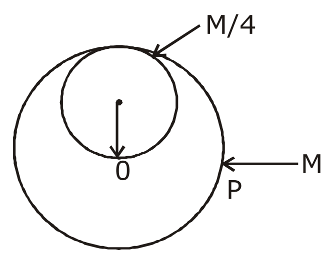
$\tau \propto t$

39.

L_0 remains cons. in magnitude and direction but L_P changes its direction continously hence L_P is variable



40.



Let σ be the density off disc.

$\therefore\ \sigma = \frac{M}{\pi(2R)^2} = \frac{M}{4\pi R^2}$

Here M $\rightarrow$ Mass of disc without cavity

$\therefore$ Mass of cavity = σ x πR^2 = $M/4\pi R^2$ x πR^2 = M/4

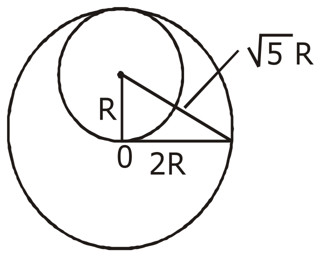
I_o = MI of disc with non cavity
- MI of cavity (About O)

$I_o = \left[\frac{1}{2}M(2R)^2\right] - \left[\frac{1}{2}\left(\frac{M}{4}\right)R^2 + \frac{M}{4}R^2\right]$

$I_o = \frac{4MR^2}{2} - \frac{1}{8}MR^2 - \frac{M}{4}R^2 = \frac{16MR^2}{8} - \frac{3MR^2}{8}$

$= \dfrac{13MR^2}{8}$

Now,



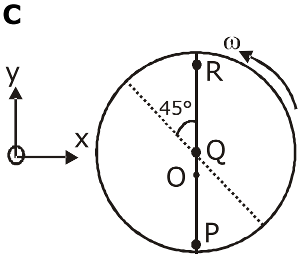
I_p = M.I. of Disc without cavity about P - M.I. of
cavity(aboutP)

$$= \left[\frac{1}{2}M(2R)^2 + M(2R)^2\right] - \left[\frac{1}{2}\left(\frac{M}{4}\right)R^2 + \frac{M}{4}\left(\sqrt{5}R\right)^2\right]$$

$$I_P = 37\frac{MR^2}{8}$$

$$\therefore \frac{I_P}{I_o} = \frac{37}{13} \approx 2.8 \approx 3$$

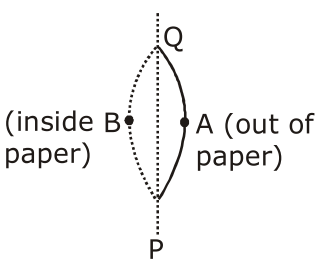
41. C



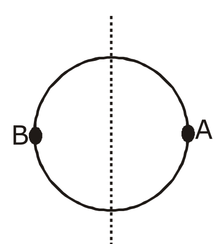
At 45° P & Q both land in unshaded region.

42. A

Consider case (a)

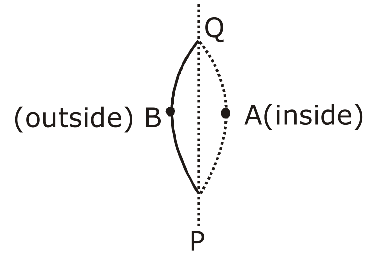


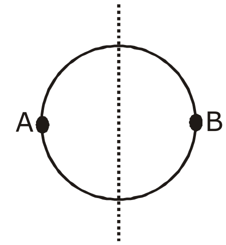
at t = 0



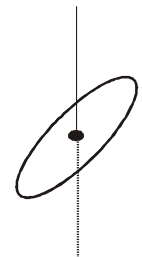
at t = T/4

at t = T/2 at t = 3T/4





Hence axis is vertical.
For case (b)

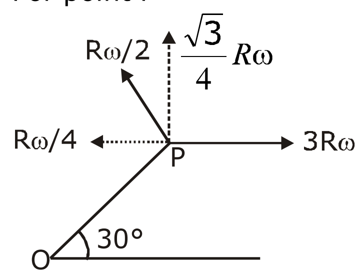


43. D

Given ω is same.

44. A,B

For point P



$$V_p = 3\,R\omega\,\hat{i} - R\omega/4\,\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

$$= 11/4\,R\omega\,\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

45. D

$I_P > I_Q$

$$a = \frac{g\sin\theta}{1 + I/MR^2}$$

Hence $a_p < a_0$
$t_p > t_Q$
$V_p < v_Q$
And as $\omega = v/R$
So $\omega_P < \omega_Q$