Exercise - V JEE-Problems

1. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillations is a, then its time period T is
(A) proportional to $1/\sqrt{a}$
(B) independent of a
(C) proportional to $\sqrt{a}$
(D) proportional to $a^{3/2}$ **[JEE' 98]**

2. A particle free to move along the x-axis has potential energy given by $U(x) = k[1 - \exp(-x^2)]$ for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then
(A) at point away from the origin, the particle is in unstable equilibrium.
(B) for any finite nonzero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic. **[JEE' 99]**

3. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then
(A) the resultant amplitude is $(1+\sqrt{2})a$
(B) the phase of the resultant motion relative to the first is 90°.
(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion.
(D) the resulting motion is not simple harmonic.
[JEE' 99]

4. The period of oscillation of simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α is given by
[JEE' 2000]

(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

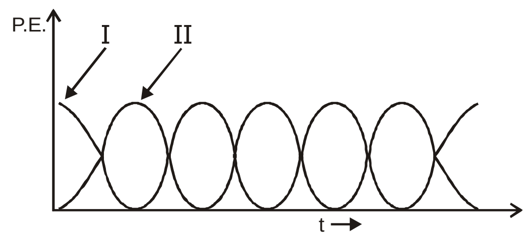
(C) $2\pi\sqrt{\frac{L}{g}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

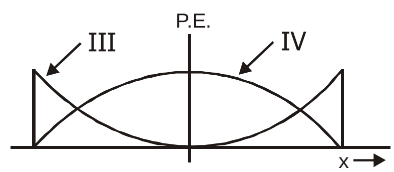
5. A bob of mass M is attached to the lower end of a vertical string of length L and cross sectional area A. The Young's modulus of the material of the string is Y. If the bob executes SHM in the vertical direcion, find the frequency of these oscillations. **[REE' 2000]**

6. A particle executes simple harmonic motion between x = –A and x = +A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then
[JEE (Scr)' 2001]
(A) $T_1 < T_2$ (B) $T_1 > T_2$
(C) $T_1 = T_2$ (D) $T_1 = 2T_2$

7. A diatomic molecule has atoms of masses m_1 and m_2. The potential energy of the molecule for the interatomic separation r is given by $V(r) = -A + B(r - r_0)^2$, where r_0 is the equilibrium separation, and A and B are positive constants. The atoms are compressed towards each other from their equilibrium positions released. What is the vibrational frquency of the molecule?
[REE' 2001]

8. A particle is executing SHM according to $y = a \cos \omega t$. Then which of the graphs represents variations of potential energy :
[JEE (Scr)' 2003]

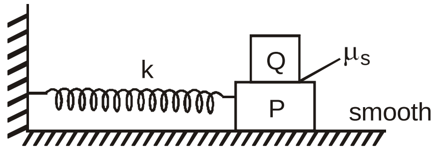




(A) (I) & (III) (B) (II) & (IV)
(C) (I) & (IV) (D) (II) & (III)

9. Two masses m_1 and m_2 connected by a light spring of natural length l_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve x-axis pass through the origin at t = 0. At this position the string snaps. Position of mass m_1 at time is given by the equation. $x_1(t) = v_0 t - A(1 - \cos \omega t)$
Calculate :
(a) Position of the particle m_2 as a function of time.
(b) l_0 is terms of A.
[JEE' 2003]

10. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is



(A) kA (B) $\frac{kA}{2}$

(C) zero (D) $\mu_s mg$ **[JEE' 2004]**

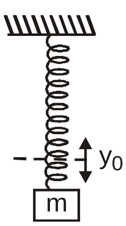
11. A simple pendulum has time period T_1. When the point of suspension moves vertically up according to the equation $y = kt^2$ where $k = 1\ m/s^2$ and 't' is time then the time period of the pendulum is T_2 then $\left(\frac{T_1}{T_2}\right)^2$ is

(A) $\frac{5}{6}$ (B) $\frac{11}{10}$

(C) $\frac{6}{5}$ (D) $\frac{5}{4}$

[JEE' 2005(Scr)]

12. A small body attached to one end of a vertically hanging spring is performing SHM about it's mean position with angular frequency ω and amplitude a. If at a height y* from the mean position the body gets detached from the spring, calculate the value of y* so that the height H attained by the mass is maximum. The body does not interact with the spring during it's subsequent motion after detachment. ($aw^2 > g$).



13. Function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM

[JEE' 2006]

(A) for any value of A, B and C (except C = 0)
(B) if A = – B ; C = 2B, amplitude = $|B\sqrt{2}|$
(C) if A = B; C = 0
(D) if A = B; C = 2B, amplitude = |B|

14. A student performs an experiment for determination of g $\left(= \frac{4\pi^2 l}{T^2}\right)$ $l \approx$ m and he commits an error of Δl.
For The takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate?

	Δl	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 sec	10	5 mm
(B)	5 mm	0.2 sec	20	5 mm
(C)	5 mm	0.1 sec	20	1 mm
(D)	1 mm	0.1 sec	50	1 mm

[JEE' 2006]

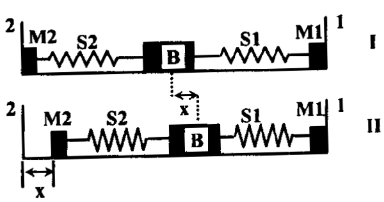
15. Column I describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(P) The object executes a SHM
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(Q) The object does not change its direction
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(R) The kinetic energy of the object keeps on decreasing

(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth. | (S) The object can change its direction only once

[JEE' 2007]

16. A block (B) is attached to two unstretched springs S1 and S2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacement x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ in Figure

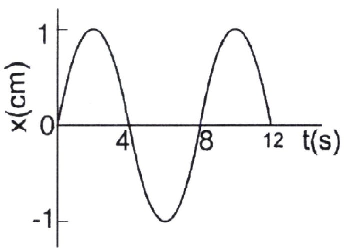


[JEE'2008]

(A) 4 (B) 2

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

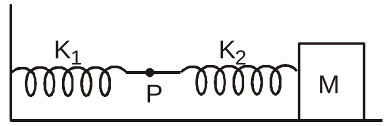
17. The *x–t* graph of particle undergoing simple harmonic motion is shown below. The acceleration of the particle at *t* = 4 / 3 s is



(A) $\frac{\sqrt{3}}{32}\pi^2 cm/s^2$ (B) $\frac{-\pi^2}{32} cm/s^2$

(C) $\frac{\pi^2}{32} cm/s^2$ (D) $-\frac{\sqrt{3}}{32}\pi^2 cm/s^2$

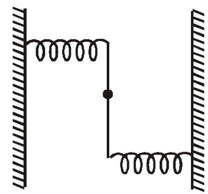
18. The mass *M* shown in the figure oscillates in simple harmonic motion with amplitude *A*. The amplitude of the point *P* is



(A) $\frac{k_2A}{k_2}$ (B) $\frac{k_2A}{k_2}$

(C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

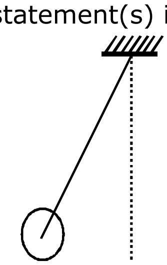
19. A uniform rod of length *L* and mass *M* is pivoted at the centre. Its two ends are attached to two springs of equal spring constants *k*. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

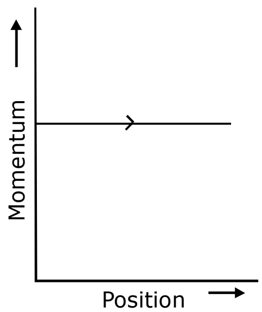
20. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached : (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. which of the following statement(s) is (are) true?



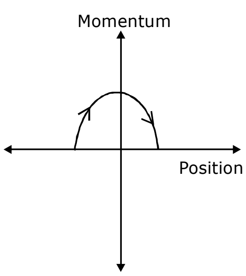
(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A < Restoring torque in case B
(C) Angular frequency for case A > Angular frequency for case B
(D) Angular frequency for case A < Angular frequency for case B

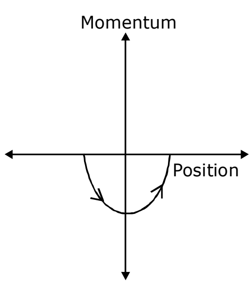
Paragraph for Question Nos. 21 to 23

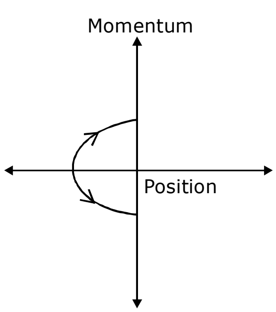
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve idicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position of momentum upwards (or to right) is positive and downwards (or to left) is negative.

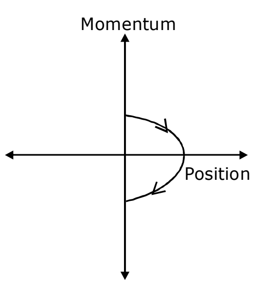


21. The phase space diagram for a ball thrown vertically up from ground is

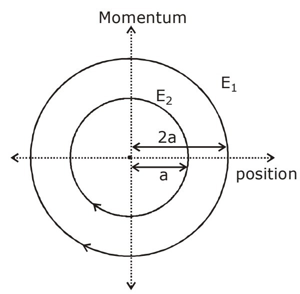
(A)


(B)


(C)


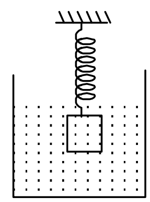
(D)


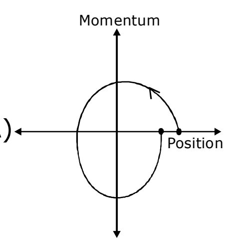
22. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

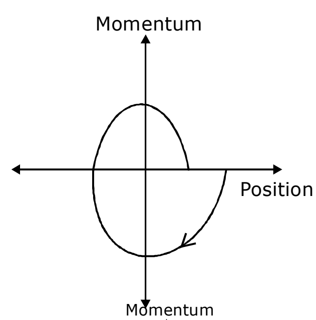


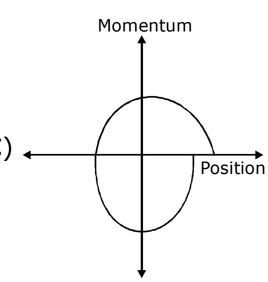
(A) $E_1 = \sqrt{2}\, E_2$ (B) $E_1 = 2E_2$
(C) $E_1 = 4E_2$ (D) $E_1 = 15E_2$

23. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


(B)


(C)


(D)