EXERCISE – V

JEE QUESTIONS

1. $V(x) = k[x]^3$

Dimension of K = $\dfrac{ML^2T^{-2}}{L^3} = ML^{-1}\,T^{-2}$

$T \propto (\text{mass}) \times (\text{Amplitude})^y\,(k)^2$

$[M^0L^0T] = [M]^x\,[L]^y\,[ML^{-1}T^{-2}]^2$

$[M^0L^0T] = [M]^x\,[L]^y\,[ML^{-1}T^{-2}]^2$

$1 = -2z \Rightarrow z = -\dfrac{1}{2}$

$\Rightarrow 0 = y - z \qquad \Rightarrow y = z - \dfrac{1}{2}$

$T \propto (\text{Amplitude})^{-1/2}$

$T \propto \dfrac{1}{\sqrt{a}}$

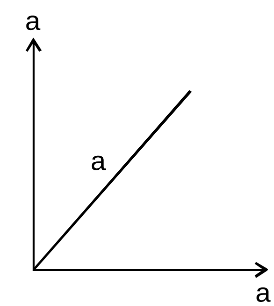
2. $V(x) = k\,[1 - e^{-x^2}]$

$F = -\dfrac{dU}{dx} = -2x\,k\,e^{-x^2}$

$F = -2kx\,(1 - x^2 + ...)$

If x is very small

F = – 2kx S.H.M.

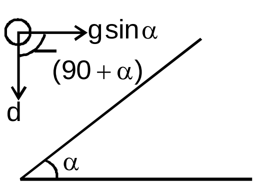
3. 

$a_{net} = a + \sqrt{2}\,a$

$= a(1 + \sqrt{2})$

$\dfrac{1}{2}\,ka_{net}^2 = \dfrac{1}{2}\,k\,(a + a\sqrt{2})^2$

$= \dfrac{1}{2}\,ka^2\,[3 + 2\sqrt{2}\,]$

4. 

$g_{eff} = [g^2 + (g\sin\alpha)^2 + 2g\,(g\sin\alpha)\cos(90 + \alpha)]^{1/2}$

$= \sqrt{1 + \sin^2\alpha - 2\sin^2\alpha}$

$= g\cos\alpha$

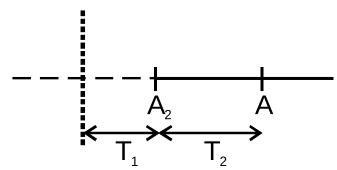
$\Rightarrow T = 2p\sqrt{\dfrac{L}{g\cos\alpha}}$

5. $Y = \dfrac{F/A}{\dfrac{\Delta\ell}{\ell}}$

$F = \left(\dfrac{YA}{\ell}\right)\Delta\ell$

$F \propto \Delta\ell \;\Rightarrow\;$ S.H.M.

$T = 2\pi\sqrt{\dfrac{m\ell}{YA}}$

6. 

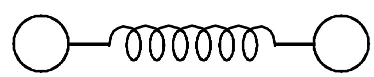
$T_1 < T_1$

7. $V(r) = -A + B\,(r - r_0)^2$

$F = -\dfrac{dv(r)}{dt} = -82\,\dfrac{(r - r_0)}{x}$

It is S.H.M.

It can be considered as two block system

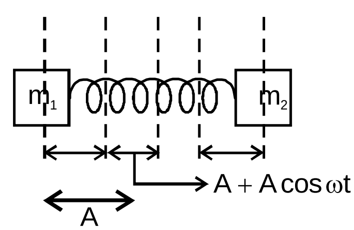


$T = 2p\sqrt{\dfrac{m_1m_2}{(m_1 + m_2)2B}}$

8. Potential Energy = $\dfrac{1}{2}\,ky^2$

$= \dfrac{1}{2}\,ka^2\cos^2\omega t$

$\Rightarrow$ III and I

9. 

w.r.t. = C.O.M. it perform S.H.M.

$x_1(t) = v_0t - A + A\cos\omega t$

$m_1 \,.\, A = m_2 \,.\, A_2$

$$A_2 = \frac{m_1}{m_2} A$$

$A + A_2 = \ell_0$

$A_2 = (\ell_0 - A)$

$$= V_0 t + \frac{m_1}{m_2} A\,(1 - \cos \omega t)$$

10. At E.P.

$$a = Aw^2 = \frac{Ak}{2m}$$

$$\text{Force} = f = \frac{Ak}{2m} \cdot m = \frac{Ak}{2}$$

11. $y = kt^2 \quad \Rightarrow \frac{d^2y}{dt^2} = 2\text{m/s}^2$

$$\therefore\ T_1 = 2\pi\sqrt{\frac{\ell}{g}}\ ;\ \ T_2 = 2\pi\sqrt{\frac{\ell}{(g+2)}}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{g+2}{g}\right) = \frac{12}{10} = \frac{6}{5}$$

12. $V^2 = w^2\,(a^2 - y^2)$

$$V^2 = \frac{k}{m}\ (a^2 - y^2),\ \ \text{Now } H = \frac{V^2}{2g} + y$$

$$H = \frac{k}{m}\ \frac{(a^2 - y^2)}{2g} + y$$

$$\frac{dx}{dy} = -\ \frac{-2yk}{2gm} + 1 = 0\ ,\ y = \frac{m}{2k} \times 2g = \frac{mg}{k}$$

13. B

14. $dg = \frac{4\pi^2}{T^2}\ d\ell - \frac{4\pi^2 \ell \times 2.dT}{T^2}$

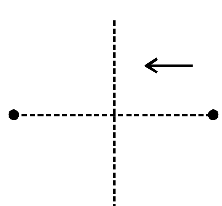
$$\frac{dg}{g} = \frac{d\ell}{\ell} - 2\ \frac{dT}{T}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{nT}$$

15. $V = C_1\sqrt{C_2 - x^2}$

$V^2 = C_1^{\,2}\,(C_2 - x^2)$

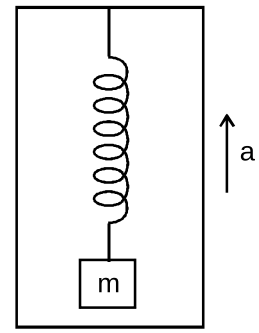
$a = V\ \frac{dv}{dx} = -\,C_1^{\,2}x \qquad$ (S.H.M.)



$V = -\,kx$

It will move towards s

Origin and will stop at origin

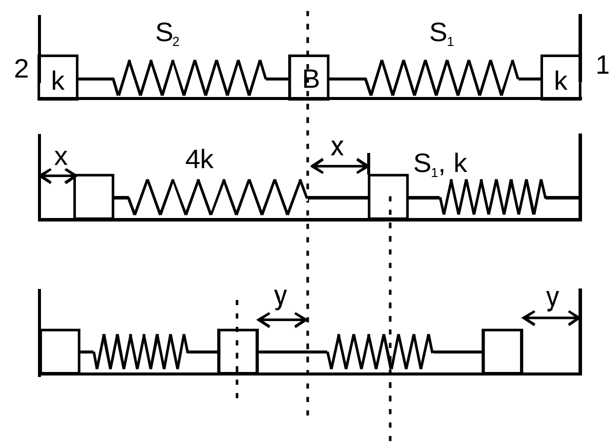


$v = \sqrt{4gRe}$

$v > \sqrt{2gRe}$

16. $\frac{1}{2}kx^2 = \frac{1}{2} \times 4(y)^2$

$$\frac{y}{x} = \frac{1}{2}$$



$$V_{avg} = \frac{4\sqrt{2}}{T}\ A$$

Maximum displacement will be close to

M.P. = $\sqrt{2}$ A

$$v_{avg} = \frac{4\sqrt{2}}{T} A$$

17. D

$y = A \sin \omega t$

$$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$y = \sin \frac{\pi}{4} t$$

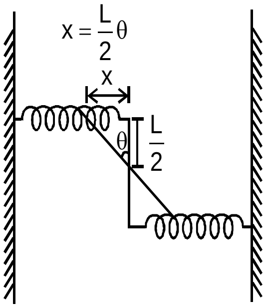
$$\text{at } t = \frac{4}{3}\ \text{sec} \Rightarrow \quad y < \frac{\sqrt{3}}{2}$$

$$a = -\omega^2 y = -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi^2}{32}$$

18. $\frac{4d^2y}{dt^2} + 9y = 0$

$$\omega^2 = \frac{9}{4} \Rightarrow \quad \omega = \frac{3}{2}$$

19.



$$\tau = I\alpha$$

$$k \times \left(\frac{L}{2}\right) + kx\left(\frac{L}{2}\right) = \frac{ML^2}{12}\ \alpha$$

$$kxL = \frac{ML^2}{12}\ \omega^2\theta$$

$$x = \frac{L}{2\theta}$$

$$\frac{12 \times L\theta}{ML \times 2} = \omega^2\theta$$

$$\omega = \sqrt{\frac{6k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

20. A,D

$$\frac{\omega_A}{\omega_B} = \sqrt{\frac{I_B}{I_A}} \Rightarrow \omega_A < \omega_B$$

21. D

$V^2 = u^2 - 2gs$

$p = mv = \pm m\sqrt{u^2 - 2gs}$

so its parabola

Initially position = 0

Momentum is +ve

22. C

$\omega_1 = \omega_2$

$$E_1 = \frac{1}{2}\ m\omega^2\ (2a)^2$$

$$= 4\left(\frac{1}{2}m\omega^2a^2\right) \qquad \text{...... (1)}$$

$$E_2 = \frac{1}{2}m\omega^2\ a^2 \qquad \text{...... (2)}$$

from equation (1) and (2)

$E_1 = 4E_2$

23. B