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Exercise - I (ON	ILY ONE OPTION IS CORRECT)	
Q.1 At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R) ? (A) R/4 (B) R (C) 3R/8 (D) R/2 Sol.		
Q.2 Two masses $m_1 \& m_2$ are initially at rest and are separated by a very large distance. If the masses	Q.4 If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same ? (A) $1/25$ (B) $1/5$ (C) $1/\sqrt{5}$ (D) 5 Sol.	
approach each other subsequently, due to gravitational attraction between them, their relative velocity of approach at a separation distance of d is (A) $\frac{2Gd}{(m_1+m_2)}$ (B) $\frac{(m_1+m_2)G}{2d}$ (C) $\left[(m_1+m_2)\frac{2G}{d} \right]^{1/2}$ (D) $(m_1+m_2)^{1/2} 2Gd$	Q.5 A man of mass m starts falling towards a planet	
Sol.	of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass 2M/3 and a point mass M/3 at the centre. Change in the force of gravity experienced by the man is	
	(A) $\frac{2}{3} \frac{\text{GMm}}{\text{B}^2}$ (B) 0 (C) $\frac{1}{3} \frac{\text{GMm}}{\text{B}^2}$ (D) $\frac{4}{3} \frac{\text{GMm}}{\text{B}^2}$	
Q.3 Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole (d << R). The value of d is (A) $\frac{\omega^2 R^2}{g}$ (B) $\frac{\omega^2 R^2}{2g}$ (C) $\frac{2\omega^2 R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$ Sol.	Sol.	
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Q.6 The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth ?

(A) $\sqrt{2}$ second

(B)
$$2\sqrt{2}$$
 seconds

(C) $\frac{1}{\sqrt{2}}$ second

Sol.

(D)
$$\frac{1}{2\sqrt{2}}$$
 second

Q.8 A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.

(A) $V_e = 2V$ (B) $V_e = V$ (C) $V_e = V/2$ (D) $V_e = \sqrt{3} V$ Sol.

Q.7 A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also

(C) Inside the shell, gravitational field alone is zero

(D) Neither gravitational field nor gravitational potential is zero inside the shell

Sol.

Q.9 Two planets A and B have the same material density. If the radius of A is twice that of B, then the

ratio of the escape velocity $\frac{v_A}{v_B}$ is

(A) 2 (B) $\sqrt{2}$ (C) $1/\sqrt{2}$ (D) 1/2Sol.



Sol.

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Q.10 The escape velocity for a planet is v_e . A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(A) v_{e} (B) $\frac{v_{e}}{\sqrt{2}}$ (C) $\frac{v_{e}}{2}$ (D) zero **Sol.**

Q.11 A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre
(A) increases
(B) decreases
(C) remains same
(D) during the compression increases then returns at the previous value

Q.12 If a tunnel is cut at any orientation through

earth, then a ball released from one end will reach the

(B) 42.3 minutes

(D) depends on orientation

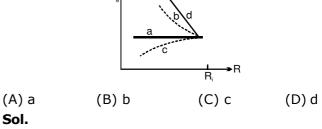
other end in time (neglect earth rotation)

(A) 84.6 minutes

(C) 8 minutes

Sol.

Q.13 A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse ?



Q.14 & 15

Two uniform spherical stars made of same material have radii R and 2R. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.

Q.14 Kinetic energy of the system just after the collision is

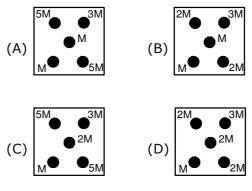
(A)
$$\frac{8Gm^2}{3R}$$
 (B) $\frac{2Gm^2}{3R}$ (C) $\frac{4Gm^2}{3R}$
(D) cannot be determined

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Motionscience

Sol.

Q.16 A mass is at the center of a square, with four masses at the corners as shown.



Rank the choices according to the magnitude of the gravitational force on the center mass.

```
(A) F_A = F_B < F_C = F_D (B) F_A > F_B < F_D < F_C
(C) F_A = F_B > F_C = F_D (D) none
Sol.
```

Q.15 The maximum separation between their centres after their first collision
(A) 4R
(B) 6R
(C) 8R
(D) 12R
Sol.



 Q.17 A satellite of the earth is revolving in circular orbit with a uniform velocity V. If the gravitational force suddenly disappears, the statellite will (A) continue to move with the same velocity in the same orbit (B) move tangentially to the original orbit with velocity V (C) fall down with increasing velocity (D) come to a stop somewhere in its original orbit Sol. Q.18 A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is (A) 48 hrs (B) 24 hrs (C) 12 hrs (D) never Sol.	 Q.20 Select the correct choice(s) : (A) The gravitational field inside a spherical cavity, within a spherical planet must be non zero and uniform. (B) When a body is projected horizontally at an appreciable large height above the earth, with a velocity less than for a circular orbit, it will fall to the earth along a parabolic path (C) A body of zero total mechanical energy placed in a gravitational field will escape the field (D) Earth's satellite must be in equatorial plane. Sol.
Q.19 Two point masses of mass 4m and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be	Q.21 A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is
(A) 1:4 (B) 1:5 (C) 1:1 (D) 1:2 Sol.	(A) $\frac{\sqrt{3}}{4}$ mgR (B) $\frac{1}{2}$ mgR (C) $\frac{1}{4}$ mgR (D) $\frac{3}{4}$ mgR Sol.



Q.22 The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct state ment about the

curves A, B and C

(A) A shows the kinetic energy, B the total energy and C the potential energy of the system

Energy

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system
(D) A and B are kinetic and potential energies and C is the total energy of the system

Sol.

Q.23 When a satellite moves around the earth in a		
certain orbit, the quantity which remains constant is		
(A) angular velocity	(B) kinetic energy	
(C) aerial velocity	(D) potential energy	
Sol.		

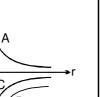
Q.25 A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the center is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$ Sol.

Q.24 A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass 4M is in (A) bound orbit (B) unbound orbit (C) partially bound orbit (D) data is insufficient to determine the nature of the orbit

Q.26 A planet of mass m is in an elliptical orbit about the sun (m << M_{sun}) with an orbital period T. If A be the area of orbit, then its angular momentum would be

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(C) partially bound orbit (D) data is insufficient to determine the nature of the orbit	(A) $\frac{2mA}{T}$	(B) mAT	(C) <u>mA</u> 2T	(D) 2mAT



Sol.

Sol.

Q. 27 & 28 Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.	Q.30 Figure shows the variation of energy with the orbit radius r of a satellite in a circular motion. Select the correct statement.
Q.27 If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then (A) $t_1 = t_2$ (B) $t_1 > t_2$ (C) $t_1 < t_2$ (D) nothing can be concluded Sol.	 (A) Z is total energy, Y is kinetic energy and X is potential energy (B) X is kinetic energy, Y is total energy and Z is potential energy (C) X is kinetic energy, Y is potential energy and Z is total energy (D) Z is kinetic energy, X is potential energy and Y is total energy Sol.
Q.28 If U is the potential energy and K kinetic energy then $ U > K $ at (A) Only D (B) Only C (C) both D & C (D) neither D nor C Sol.	Q.31 Statement - I : Assuming zero potential at
Q.29 Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is (A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16 Sol.	 infinity, gravitational potential at a point cannot be positive. Statement - 2 : Magnitude of gravitational force between two particle has inverse square dependence on distance between two particles. (A) Statement - 1 istrue, statement-2 is true and statement-2 is correct explanation for statement-1 (B) Statement -1 is true, statement-2 is true and statement - 2 is NOT the correct explanation for statement-1 (C) Statement - 1 is true, statement - 2 is false. (D) Statement - 1 is false, statement - 2 is true.



Exercise - II (ONE OR MOR	RE THAN ONE OPTION IS CORRECT)
 1. Assuming the earth to be a sphere of uniform density the acceleration due to gravity (A) at a point outside the earth is inversely proportional to the square of its distance from the center (B) at a point outside the earth is inversely proportional to its distance from the centre (C) at a point inside is zero (D) at a point inside is proportional to its distance from the centre Sol. 	 3. In side a hollow spherical shell (A) everywhere gravitational potential is zero (B) everywhere gravitational field is zero (C) everywhere gravitational potential is same (D) everywhere gravitational field is same Sol.
 2. Two masses m₁ and m₂ (m₁ < m₂) are released from rest from a finite distance. They start under their mutual gravitational attraction (A) acceleration of m₁ is more than that of m₂ (B) acceleration of m₂ is more than that of m₁ (C) centre of mass of system will remain at rest in all the references frame (D) total energy of system remains constant Sol. 	4. A geostationary satellite is at a height h above the surface of earth. If earth radius is R (A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1} (R/R + h)$ (B) The maximum colatitudes on earth upto which the satellite can be used for communication is $\sin^{-1} (R/R + h)$ (C) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \sin \theta)$ (D) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \cos \theta)$ Sol.





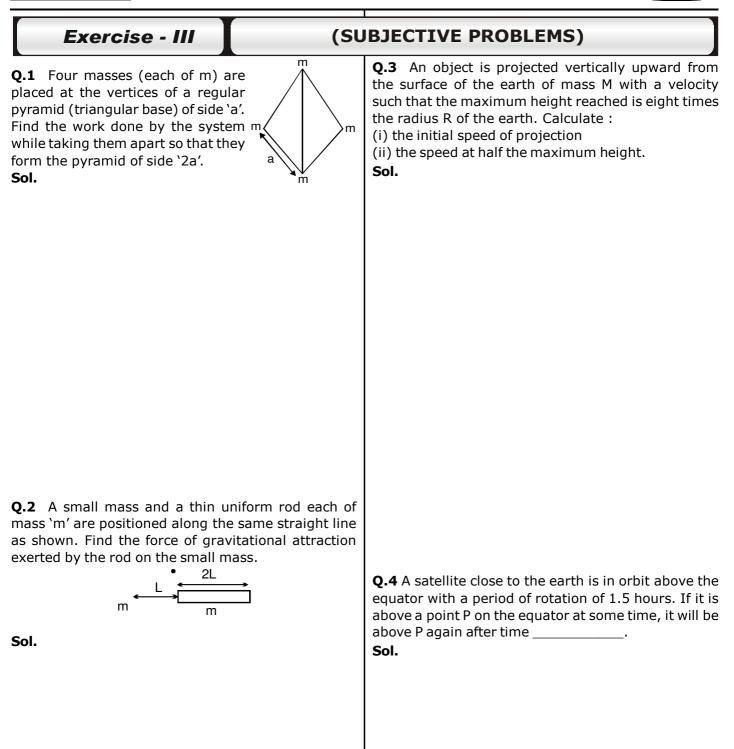
	 7. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change (A) gravitational potential energy (B) angular velocity (C) linear orbital velocity (D) centripetal acceleration
 5. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then (A) its kinetic energy increases (B) its kinetic energy decreases (C) its angular momentum about the earth decreases (D) its period of revolution around the earth increases Sol. 	
6. A communications Earth satellite (A) goes round the earth from east to west (B) can be in the equatorial plane only (C) can be vertically above any place on the earth (D) goes round the earth from west to east Sol.	 8. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth (A) the acceleration of S is always directed towards the centre of the earth (B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant (C) the total mechanical energy of S varies periodically with time (D) the linear momentum of S remains constant in magnitude Sol.



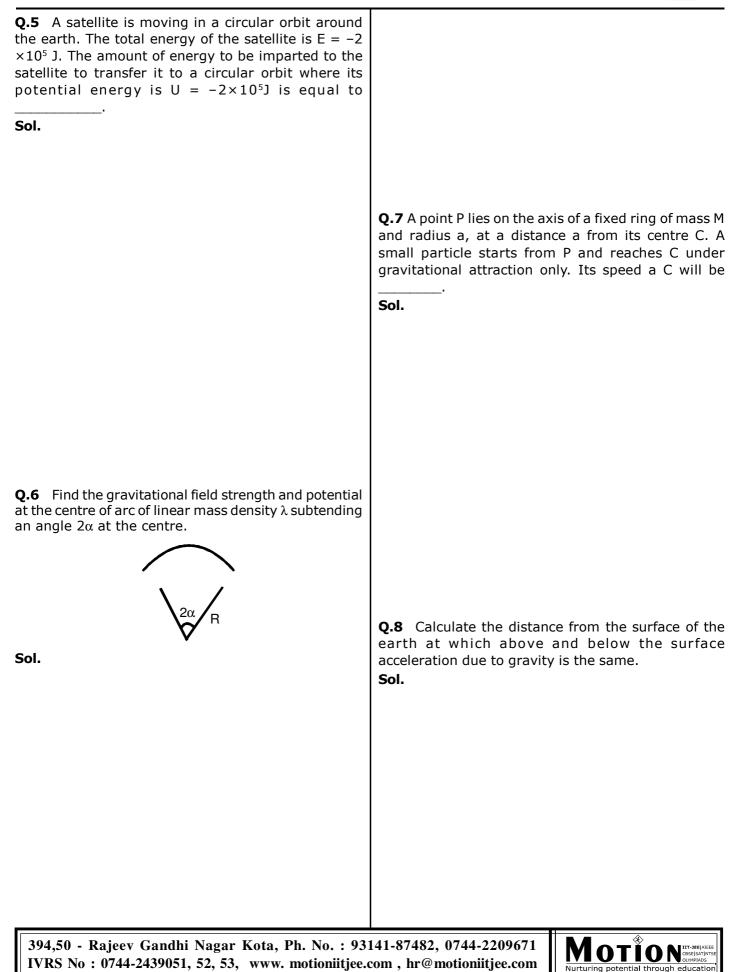
 9. If a satellite orbits as close to the earth's surface as possible, (A) its speed is maximum (B) time period of its rotation is minimum (C) the total energy of the 'earth plus satellite' system is minimum (D) the total energy of the 'earth plus satellite' system is maximum Sol. 	10. For a satellite to orbit around the earth, which of the following must be true ? (A) It must be above the equator at some time (B) It cannot pass over the poles at any time (C) Its height above the surface cannot exceed 36,000 km (D) Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth Sol.



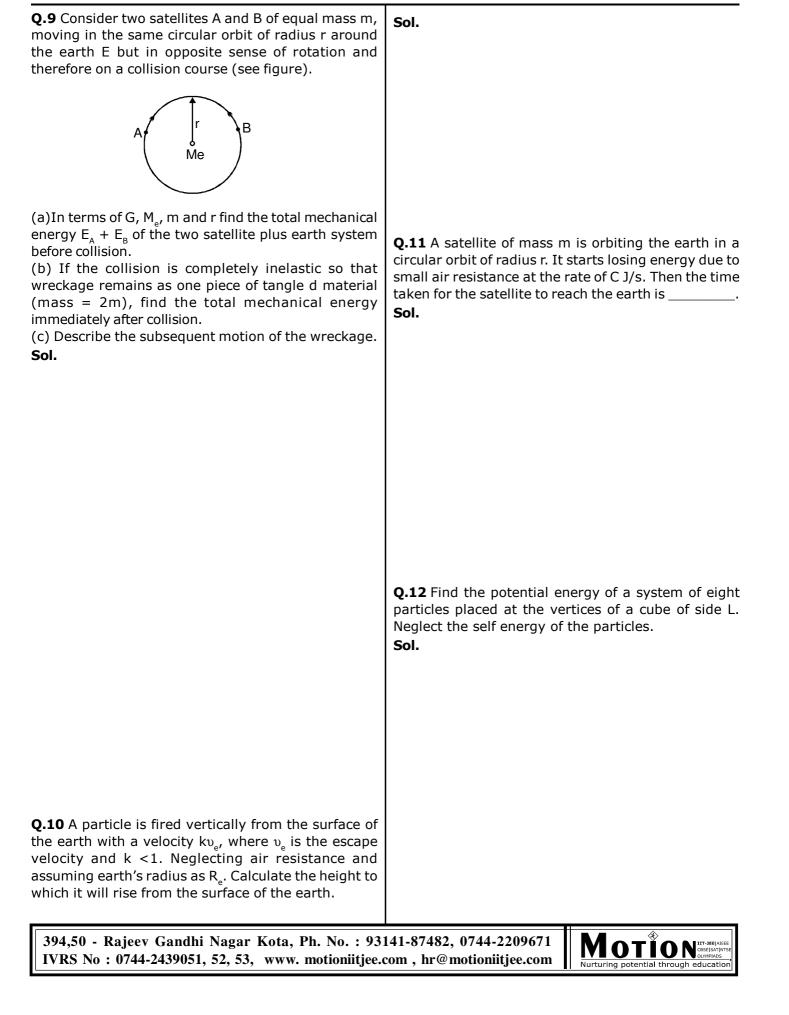
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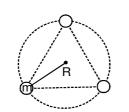




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Q.13 A hypothetical planet of mass M has three moons each of equal mass 'm' each revolving in the same circular orbit of radius R. The masses are equally spaced and thus form an equilateral triangle. Find (i) the total RE of the system



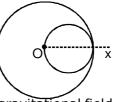
(i) the total P.E. of the system

(ii) the orbital speed of each moon such that they maintain this configuration.

Sol.

Q.14 Two small dense stars rotate about their common centre of mass as a binary system with the period 1year for each. One star is of double the mass of the other and the mass of the lighter one is 1/3 of the mass of the sun. Find the distance between the stars if distance between the earth & the sun is R. **Sol.**

Q.15 A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius r = R/2 whose centre is at x = R/2 as in fig. (a) Find gravitational field at points on the axis for x > R



(ii) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.

Sol.



Q.17 A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the paint diametrically expectite.
the hole to the point diametrically opposite.
Sol.

Q.16 A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be

(a) then velocity in circular orbits.

(b) maximum distance between the two fragments before collision and

(c) magnitude of their relative velocity just before they collide.

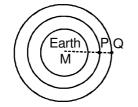
Sol.

Q.18 A remote sensing satellite is revolving in an orbit of radius x the equator of earth. Find the area on earth surface in which satellite can not send message.

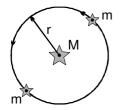


Exercise - IV

Q.1 A satellite P is revolving around the earth at a height h = radius of earth (R) above equator. Another satellite Q is at a height 2h revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this is situation.



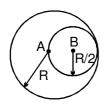
Q.2 A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit, see figure. Derive an expression for the period of revolution of the stars; the radius of the orbit is r.



Q.3 Find the gravitational force of interaction between the mass m and an infinite rod of varying mass density λ such that $\lambda(x) = \lambda/x$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in figure.

$$\bigcap_{m} \frac{d}{\lambda(x)} = \frac{\lambda}{x}$$

Q.4 Inside an isolated fixed sphere of radius R and uniform density r, there is a spherical cavity of radius R/2 such that the surface of the cavity passes through the centre of the sphere as in figure. A particle of mass m is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere.



(TOUGH SUBJECTIVE PROBLEMS)

Q.5 A ring of radius R is made from a thin wire of radius r. If ρ is the density of the material of wire then what will be the gravitational force exerted by the ring on the material particle of mass m placed on the axis of ring at a distance x from its centre. Show that the force will be maximum when $x=R/\sqrt{2}$ and the maximum value of force will be given as $4\pi^2 Gr^2 om$

$$\mathsf{F}_{\max} = \frac{4\pi^2 \mathrm{Gr}^2 \rho \mathrm{m}}{(3)^{3/2} \mathrm{R}}$$

Q.6 A man can jump over b = 4m wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth = 6400 km.

Q.7 A launching pad with a spaceship is moving along a circular orbit of the moon, whose radius R is triple that of moon Rm. The ship leaves the launching pad with a relative velocity equal to the launching pad's

initial orbital velocity \vec{v}_0 and the launching pad then falls to the moon. Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.

Q.8 A satellite of mass m is in an elliptical orbit around the earth of mass M(M >>m). The speed of the satellite

at its nearest point to the earth (perigee) is $\sqrt{\frac{6GM}{5R}}$

where R = its closest distance to the earth. It is desired to transfer this satellite into a circular orbit around the earth of radius equal its largest distance from the earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

Q.9 A body is launched from the earth's surface a an angle $\alpha = 30^{\circ}$ to the horizontal at a speed $\sqrt{15 \text{ GM}}$

 $v_0 = \sqrt{\frac{1.5 \text{ GM}}{\text{R}}}$. Neglecting air resistance and earth's rotation, find (a) the height to which the body will

rise. (ii) The radius of curvature of trajectory at its top point.

Q.10 Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to reach centre of earth if it is projected into the tunnel from surface of earth with speed needed for it to escape the gravitational field of earth.



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Exercise - V	(JEE PROBLEMS)
Q.1 If the distance between the earth and the sum were half its present value, the number of days in a year would have been [JEE' 96] (A) 64.5 (B) 129 (C) 182.5 (C) 730 Sol.	
Q.2 Distance between the centres of two stars is 10a. The masses of these starts are M and 16M and their radii a and 2a respectively. A body of a mass m is fired at night from the surface of the larger star towards the maller star. What should be its minimum initial speed to reach the surface of the smaller start? Obtain the expression in terms of G, M and a. [JEE'96] Sol.	Q.3 An artificial satellite moving in a circular orbit around the earth has a total (K.E. + P.E.)E ₀ . Its potential energy is [JEE'97] (A) $-E_0$ (B) 1.5 E ₀ (C) 2E ₀ (D) E ₀ Sol.

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GRAVITATION

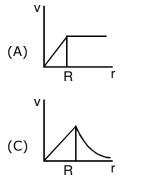


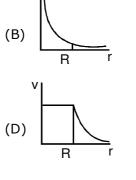
Q.4 A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near earth surafce. Assume that the spaceship and the astronaut fall on a straight line from the earth centre. The radius of the earth is 6400 km. [REE 98] Sol.	Sol.
	Q.6 A body is projected vertically upwards from the bottom of a crater of moon of depth R/100 where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. [JEE' 2003] Sol.
In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1 , W_2 , W_3 respectively then $B \xrightarrow{(3)} C$	
(1) (A) $W_1 = W_2 = W_3$ (B) $W_1 > W_2 > W_3$ (C) $W_1 = W_2 > W_3$ (D) $W_1 < W_2 < W_3$ (394,50 - Rajeev Gandhi Nagar Kota, Ph. No. : 931 IVRS No : 0744-2439051, 52, 53, www. motioniitjee.c	141-87482, 0744-2209671 com , hr@motioniitjee.com

Q.7 A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then **[JEE 2006]** (A) $T_A > T_B$ (if $r_A > r_B$) (B) $T_A > T_B$ (if $m_A > m_B$) (C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$ (D) $T_A = T_B$ **Sol. Q.8** A spherically symmetric gravitational system of particles has a mass density **[JEE 2008]**

$$\rho = \begin{cases} \rho_0 \text{ for } r \leq R \\ 0 \text{ for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r (0 < r < ∞) from the centre of the system is represented by





Sol.

Q.9 STATEMENT-1

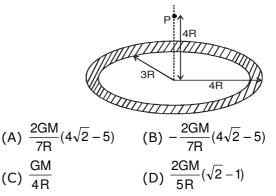
An astronaut in an orbiting space station above the Earth experiences weightlessness. [JEE 2008] and

STATEMENT-2

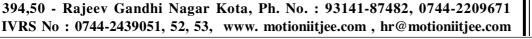
An object moving around the Earth under the influence of Earth's gravitaitonal force is in a state of 'free-fall'. (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1 (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False (D) STATEMENT-1 is False, STATEMENT-2 is True **Sol.**

Q.10 A thin uniform annular disc (see figure) of mass M has outer radius 4 R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is [JEE 2010]



Sol.





GRAVITATION

Q.11 A binary star consists of two stars A (mass 2.2 M) and B (mass 1 M), where M, is the mass of the sun. They are separated by distance d and arrotation pit about the earth. An object of mass im 'is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of a pinare star to the angular momentum of that is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of is ejector, the kinetic energy of the object is (A)
$$\frac{1}{2}$$
 mV² (B) mV² (C) $\frac{3}{2}$ mV² (D) 2mV² (S).

