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PHYSICS

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PINNACLE

For – JEE (Main/Advanced)

PHYSICAL WORLD AND MEASUREMENT

Physical World and Measurement

Syllabus:

Scope and excitement of physics, Technology & society. Forces in nature, Conservation laws, Examples of gravitation, electromagnetic and nuclear forces from daily life experiences (qualitative description only). Need of measurement, Units of measurement, System of units, SI units, Fundamental and derived unit; Length, mass and time measurement, Accuracy and precision of measuring instrument, errors in measurement, significant figures. Dimensions of physical quantities, dimensional analysis and its application.

Have you ever observed the nature and the various spectacular events like formation of rainbow on any rainy day?

Whenever we observe nature keenly, we can easily understand that the various events in nature like blowing of wind, flow of water, motion of planets, formation of rainbow, different forms of energies, the function of human bodies, animals, etc. are happening or taking place according to some basic laws. The systematic study of these laws of nature governing the observed events is called science. For our convenience, clear understanding and systematic study of Science is classified into various branches. Among these branches Chemistry, Mathematics, Botany, Zoology, etc. are ancient branches and Bio–technology, Bio–chemistry, Bio–Physics, Computer science, Space Science, etc. are considered to be modern branches of science and engineering. One of such ancient and reputed branches of this science is physics.

SCOPE AND EXCITEMENT OF PHYSICS

The domain of physics consists of wide variety and large number of natural phenomena. Hence, the scope of physics is very vast and obviously the excitement that one gets from the careful study of physics has got no boundaries.

Scope of Physics

For example, when we study one of the basic physical quantities called mass, we come across the values ranging from minute masses like mass of an electron (of the order of 10^{-30} kg) to heavy masses like mass of universe (10^{55} kg). Similarly, in case of other basic quantities like length and time also the range is very wide.

Hence, the scope of physics can be understood easily, only when we can classify the study of physics chiefly into three levels. They are:

(a) Macroscopic level study of physics,
(b) Mesoscopic level study of physics, and
(c) Microscopic level study of physics.

Macroscopic level study of physics: Macroscopic level study of physics mainly includes the study of basic laws of nature and several natural phenomena like gravitational force of attraction between any two bodies in the universe (in mechanics), variation of quantities like pressure, volume, temperature, etc. of gases on their thermal expansion or contraction (in thermodynamics), etc.

Microscopic level study of physics: The microscopic level study of physics deals with constitution and structure of matter at the level of atoms or nuclei. For example, interaction between elementary particles like electrons, protons and other particles , etc.

Mesoscopic level study of physics: The mesoscopic level study of physics deals with the intermediate domain of macroscopic and microscopic, where we study various physical phenomena of atoms in bulk.

So, the edifice of physics is beautiful and one can appreciate the subject as and when one pursues the same seriously.

Excitement of Physics

The study of physics is exciting in many ways as it explains us the reason behind several interesting features like (a) ***how day and nights are formed?*** (b) ***how different climatic conditions are formed in different seasons?*** (c) ***how satellite works and helps in using several devices like television, telephones, etc.?*** (d) ***how an astronaut travels to celestial space ?*** (e) ***how we can convert one form of energy to another ?*** (f) ***how different types of forces are governing different types of motion in universe ? etc***.

It is quite common and simple that every human being on the earth will be interested to know the answers for at least few of the above questions. As physics is the subject which answers them, naturally the study of physics will be exciting.

TECHNOLOGY AND SOCIETY

Physics is almost an integral part of upgradation of technology. Technology was also a branch of science where we study the application of principles of physics for practical purposes. Based on laws and principles of physics, technocrats along with scientists develop technically advanced equipment to help the society.

For example, from the principles of thermodynamics James watt invented steam engine which was responsible for a big industrial revolution in England in the 18^{th} century. Another recent example is invention of mobile phones which are creating revolution in wireless communication technology. Yet another important example is invention of micro–processors by using silicon chips which has replaced valve technology and brought the computers from the size of your study room to the size of your geometry box. These are few examples. There are many more areas where physics is involved in upgrading technology and thereby helping the society. The following table gives us a list of various branches of physics that helped the field of technology.

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Nuclear fission
Radio and Television	Propagation of electromagnetic waves
Computers	Digital logic

Lasers	Light amplification by stimulated emission of radiation (population inversion)
Production of ultra–high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electric energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Air conditioners / Refrigerators	Laws of thermodynamics
Washing machines, centrifuge, etc.	Centrifugal force
Sonar	Reflection of ultrasonic waves

The following table lists the involvement of various renowned physicists all across the world, who helped the society with their noble inventions.

Name	**Major Contribution / Discovery**	**Country of origin**
Isaac Newton	Universal law of gravitation: Laws of motion; reflecting telescope.	**U. K.**
Galileo Galilei	Law of inertia	**Italy**
Archimedes	Principle of buoyancy; principle of the lever	**Greece**
James Clerk Maxwell	Electromagnetic theory; light an electromagnetic wave	**U. K.**
W. K. Roentgen	x– rays	**Germany**
Marie Sklodowska Curie	Discovery of radium and polonium; Studies on natural radioactivity	**Poland**
Albert Einstein	Law of photo–electricity; Theory of relativity	**Germany**
S. N. Bose	Quantum statistics	**India**
James Chadwick	Neutron	**U.K.**
Niels Bohr	Quantum model of hydrogen atom	**Denmark**
Ernest Rutherford	Nuclear model of atom	**New Zealand**
C.V. Raman	Inelastic scattering of light by molecules	**India**
Christiaan Huygens	Wave theory of light	**Holland**
Michael Faraday	Laws of electromagnetic induction	**U.K.**
Edwin Hubble	Expanding universe	**U.S.A.**
Homi Jehangir Bhabha	Cascade process in cosmic radiation	**India**
Abdus Salam	Unification of weak and electromagnetic interactions	**Pakistan**
R. A. Millikan	Measurement of electronic charge	**U.S.A**
Ernest Orlando Lawrence	Cyclotron	**U.S.A.**
Wolfgang Pauli	Quantum Exclusion Principle	**Austria**
Louis victor de Broglie	Wave nature of matter	**France**

J.J. Thomson	Electron	**U.K.**
S. Chandrasekhar	Chandrasekhar limit, structure and evolution of stars	**India**
Lev Devidovich Landau	Theory of condensed matter; liquid helium	**Russia**
Heinrich Rudolf Hertz	Electromagnetic waves	**Germany**
Victor Francis Hess	Cosmic radiation	**Austria**
M. N. Saha	Thermal ionisation	**India**
G. N. Ramachandran	Triple helical structure of proteins	**India**
Thomas Alwa Edison	Electric bulb, Projector	**US**
Graham Bell	Telephone	**US**
Cavendish	Determination of 'G'	**England**
Robert Boyle	Boyle's law	**England**

So, to put it in a nut shell, science, technology and society are inseparable as they are deeply interwined.

FUNDAMENTAL FORCES IN NATURE

Force is a very common word which we normally come across in our daily life. We need force to push or pull or throw a body. Even we need it to deform or break the bodies. Sometimes, we experience force like when we are standing in a great storm, we experience the force exerted by wind. When we are sitting in a bus which is negotiating a turn, we experience an outward push. So, what is this force? Let us try to understand the concept of force in terms of physics.

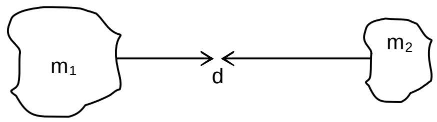
At macroscopic level study of physics, we normally encounter different kinds of forces like gravitational force, muscular force, frictional force, contact force, spring force, buoyant force, viscous force, pressure force, force due to surface tension, electrostatic force, magnetic force, etc. whereas at microscopic level of study we come across nuclear forces, interatomic forces, intermolecular forces, weak forces, etc.

After analysing these various types of forces in nature, it was concluded that all the forces can be comfortably classified into four categories, which are known as fundamental forces in nature. They are

(1) Gravitational force (2) Electromagnetic force,
(3) Nuclear force, and (4) Weak force.

That means, any force other than the above four forces can be derived from these four basic forces. For example, elastic force or spring force arises due to the net attraction or repulsion between any two neighboring atoms of the spring. When it is elongated or compressed, attractive or repulsive forces produced between the atoms can be treated as the resultant of all electromagnetic forces between charged particles of an atom. Hence, this spring force is known as derived force and electromagnetic force which is the origin of this spring force is called fundamental force. Now, we will study about fundamental forces in brief.

Gravitational Force



Newton discovered that any two bodies in universe attract each other. This force of attraction exists by virtue of their masses, and is known as gravitational force of attraction. He found that the gravitational force is directly proportional to their masses and is inversely proportional to the square of the distance between them.

i.e. $F = G\dfrac{m_1 m_2}{d^2}$ where 'G' is a Universal Gravitational Constant. This force is a universal force and is independent of any type of intervening medium between the two bodies. Though this is the weakest force in nature when compared to other types of fundamental forces, it plays vital role in governing the motion of planets around sun, natural satellites (like moon around earth), artificial satellites, etc.

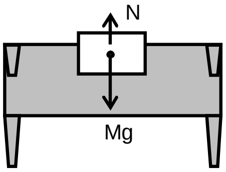
Electromagnetic Force:

The force of attraction or repulsion between any two charged particles is known as electrostatic force. If q_1 and q_2 charges are separated by a distance 'd' in air then the force of attraction or repulsion between them is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{d^2}$. This is called Coulomb's law of electric forces.

Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. In general electric and magnetic effects are inseparable and hence the name – electromagnetic force. This electromagnetic force between moving charged particles is comparatively more complicated and contains several other terms other than Coulomb's force.

In atoms electromagnetic force between electrons and protons is responsible for several molecular and atomic phenomena. Apart from this it also plays vital role in the dynamics of chemical reactions, mechanical and thermal properties of materials, tension in ropes, friction, normal force, spring force, Vander Waals force .

Example: Let us consider a block which is placed on a horizontal surface of a table as shown in the figure. The table balances the weight (Mg) and exerts a force which comes from electromagnetic force between charged constituents of atoms or molecules of surface of block and that of the table. Thus a force called normal force acts on block.



This electromagnetic force is a strong force when compared to the gravitational force. The electromagnetic force between two protons is 10^{36} times the gravitational force between them for any fixed distance.

Nuclear Force

We know that, in general, nucleus of every atom consists of two elementary particles called protons and neutrons. As neutrons are uncharged and protons are charged, the electric force of repulsion between protons will cause nucleus to break into fragments. But this is not happening, and also we know that nucleus of a non–radioactive element is a stable one.

That means there must be some other attractive force which is dominating coulombic force of repulsion between protons and keeping all the particles in nucleus together in stable condition as gravitational force can't dominate electric force. That new force existing between any two nucleons and which keeps all the particles in nucleus bound together is known as nuclear force. This force is stronger than electromagnetic force and is a charge independent force. Range of these forces is very small and will be of the order of nuclear size (10^{-21} th portion of size of an atom).

Latest developments in physics revealed that this strong nuclear force is also not a fundamental force as protons and neutrons consist of still elementary particles called quarks. And according to this latest development quark – quark force is fundamental force of nature and nuclear force is a derived force. However the study of quark – quark force is out of the scope of this book and our curriculum.

Weak Nuclear Force

This force appears only in certain nuclear processes. A neutron can change itself into a proton by emitting an electron and another elementary particle called antineutrino simultaneously. This process is called β^- decay. Similarly a proton can also change into neutron by emitting positron and a neutrino. This process is called β^+ decay. The forces which are responsible for these changes are known as weak forces. These forces are weak in nature when compared to nuclear and electromagnetic forces but stronger than gravitational forces. The range of these weak nuclear forces is exceedingly small, of the order of 10^{-15}m.

The following table gives us an overall idea about relative strengths and ranges of four fundamental forces.

Name	Relative strength	Range	Operates among
Gravitational force	10^{-38}	Infinite	**All objects in the universe**
Weak nuclear force	10^{-13}	Very short, within nuclear size ($\sim 10^{-15}$)	**Elementary particles**
Electromagnetic force	10^{-2}	Infinite	**Charged particles**
Strong nuclear force	1	Very short, within nuclear size ($\sim 10^{-15}$)	**Nucleons**

CONSERVATION LAWS

In any physical phenomena, few physical quantities associated with the phenomena may change with time and few physical quantities associated with it may not change. Those physical quantities which remain constant in time are known as conserved quantities.

For example, if a big liquid drop is sprayed into several small droplets the volume of liquid before spraying and after spraying remains same. Hence, we can say that a physical quantity called volume is conserved in this example. Similarly, we have several quantities which are conserved. Within the scope of our course, we can discuss the following conservation laws.

1. Law of conservation of linear momentum
2. Law of conservation of energy
3. Law of conservation of angular momentum
4. Law of conservation of charge.

Let us discuss them in brief.

Law of conservation of linear momentum

The linear momentum of a body is defined as the ability of a body by virtue of which it imparts its motion to other objects along a straight line. And mathematically it is equal to the product of mass of the body (m) and its velocity ($\vec{v}$) Mathematically, $\vec{P} = m.\vec{v}$.

According to this law, in absence of an external force, the total vector sum of linear momentum remains unchanged.

Example: When a bullet is fired with a gun, the total momentum vector of the system of bullet and gun is zero. After firing, bullet moves in forward direction with some momentum and gun recoils with the same amount of momentum in magnitude, but opposite in direction. Hence total vector sum of momentum after firing is also zero. Thus linear momentum of the system before and after firing is zero. Hence we can say that linear momentum is conserved.

Law of conservation of energy

According to this law the total energy of an isolated system is always constant and it never changes. But it can be transformed from one form to another. *For example* an electric cell in our daily life gives electrical energy by transforming chemical energy in it, electric motor converts electrical energy to mechanical energy, etc. However the total energy in these processes is conserved.

When an object is dropped from a certain height the total mechanical energy of the body is conserved. At its highest point all its mechanical energy will be in the form of potential energy and at its lowest point it will be in the form of kinetic energy, i.e. energy has transformed from one form into another, (i.e. potential to kinetic) but the total energy remains constant. Hence the total mechanical energy is conserved.

But this conservation of mechanical energy can't be applied in the presence of non – conservative force. For example in the above case if you consider air resistance on the freely falling body total mechanical energy does not remain constant. Here work done by air resistance gets converted into different forms of energy like heat energy. So such while applying nergy conservation principle heat energy should also be taken into consideration in such cases

Law of conservation of angular momentum

Angular momentum ($\vec{L}$) of a body about a point is defined as the cross product of its position vector about that point ($\vec{r}$) and its linear momentum at that instant ($\vec{p}$)

i.e. $\vec{L} = \vec{r} \times \vec{p}$
or $L = rp \sin\theta$ where θ is the angle between '$\vec{r}$' and '$\vec{p}$'.

According to this law the total angular momentum of the system remains conserved in absence of external torque.

Example: We know that planets revolves around sun in elliptical orbits. The angular momentum of a planet at any point during its motion in its path is conserved. We will study more clearly about this under rotatory motion concepts.

These are the few conservation laws in mechanics. Now let us discuss a conservation law in electrostatics.

Law of conservation of charge

This law states that the total electric charge of an isolated system is always conserved. Charge can neither be created nor destroyed, but it can be transferred or exchanged from one body to another.

Apart from these, there are several other physical quantities that are conserved in nature. During our further discussions in various chapters we will understand them.

MEASUREMENT AND UNITS

Physical quantity: Any meaningful term which can be measured is a physical quantity. For example length, velocity, time etc. are physical quantity. But handsomeness, beauty are not physical quantity.

Why measurement is needed?: Physics is an experimental science and experiments involve measurement of different physical quantities in which laws of physics are expressed. Without measuring results of experiments, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Units of measurement: To measure a physical quantity we need some standard unit of that quantity. For example, if a measurement of length is quoted as 5 meters, it means that the measured length is 5 times as long as the value accepted for a standard length defined to be **"one meter".**

Any set of standards of units must fulfill the following two conditions
(i) It must be accessible.
(ii) It must be invariable with the passage of time

Two more auxiliary conditions are:–
(i) It is necessary to have wide unlimited agreement about those standards.
(ii) It is inter convertible to different units of same quantity.

A measurement consists of two parts, one is numeric and the other is standard chosen. For example, 5 meter of length implies 5 times the **"standard meter"**. It is not necessary to establish a measurement standard for every physical quantity. Some quantities can be regarded as fundamental and the standard for other quantities can be derived from the fundamental ones. For example, in mechanics length, mass and time are regarded as fundamental quantities and the standard for speed (= length / time) can be derived from fundamental quantities length and time.

Quantity	SI Units	Symbols
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of Substance	mole	mol
Thermodynamic Temp.	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	Cd

And two supplementary units are

Plane Angle	Radian	rad
Solid Angle	Steradian	sr

Two other system of units compete with the international system. One is Gaussian System in terms of which much of the literature of physics is expressed. In India this system is not in use.
The other is the British system. This system is still in daily use in United states. But SI units are standard units worldwide.

C.G.S. Unit: In this system of unit, centimeter, gram and seconds are units of length, mass and time respectively.

Conversion of One System of Units to another System: The basic formula is $n_1u_1 = n_2u_2$ where n_1 and n_2 are numbers.

Illustration 1. *How many dyne–centimeter are equal to 1 N–m?*

Solution:
$$1\text{N}-\text{m} = (1\text{kg})(1\text{m})^2(1\text{s})^{-2}$$
$$1\text{dyne}-\text{centimeter} = (1\text{g})(1\text{cm})^2(1\text{ s})^{-2}$$
$$\therefore \frac{1\text{N}-\text{m}}{1\text{dyne}-\text{cm}} = \left(\frac{1000\text{g}}{1\text{g}}\right)\left(\frac{100\text{cm}}{1\text{cm}}\right)^2$$
$$= 1000 \times 10000$$
$$\therefore 1\text{N}-\text{m} = 10^7\text{ dyne}-\text{cm}$$

Exercise: ***Calculate the value of 1 erg in SI system.***

Measurement of Length

Depending upon the range of length, there are three main methods for measuring length.

(i) Direct method using measuring instruments.
(ii) Indirect method or Mathematical method
(iii) Chemical method

(i) Direct method

The simplest method measuring the length of a straight line is by means of a meter scale. But there exist some limitations in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier calliper (b) micrometer scale (screw gauge) are used .

Vernier calliper

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being slightly shorter than the divisions of the main scale.

Least count of Vernier Calliper

The least count or the Vernier constant (V.B.) is the minimum value of correct estimation of length without eye estimation. The difference between the values of one main scale division and one vernier scale division is known as vernier constant if N division of vernier scale coincides with (N−1) divisions of main scale, then vernier constant,

$$n.V.S.D. = (n-1)\ M.S.D.$$

$$1.V.S.D. = \left(\frac{n-1}{n}\right) M.S.D., \text{ and}$$

$$1.M.S.D. - 1.V.S.D. = 1.M.S.D.\ \left(\frac{n-1}{n}\right) M.S.D.$$

$$= \frac{1}{n}\ M.S.D.$$

$$= \frac{\text{Smallest Division on main scale}}{\text{No. of divisions on vernier scale}}$$

Reading a Vernier scale

Let one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions

$$\therefore\ 1.V.S.D. = \frac{9}{10}\ M.S.D. = 0.9\ mm$$

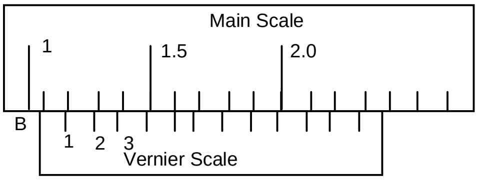
$$\therefore\ \text{Vernier constant} = 1.M.S.D - 1.V.S.D. = 1\ mm - 0.9\ mm$$
$$= 0.1\ mm = 0.01\ cm$$

The reading with vernier scale is read as given below :

(i) Firstly take the main scale reading (N) before on the left of the zero of the vernier scale.
(ii) Find the number (n) of vernier division which just coincides with any of the main scale division. Multiply this number (n) with vernier constant (V.C.)
(iii) Total reading = (N + n × V.C.)

Caution: The main scale reading with which the Vernier scale division coincides has no connection with reading
Suppose If we have to measure a length AB, the end A is coincided with the zero of the vernier scale as shown in fig. Its enlarged view is given in fig.

Length AB > 1.0 cm < 1.1. cm



Let 5th division of vernier scale coincide with 1.6 cm of main scale. From diagram it is clear that the distance between 4th division of vernier scale and 1.5 cm of main scale is equal to one V.C. and distance between zero mark of vernier scale and 1.0 cm mark on the main scale is equal to 5 times the vernier constant.

$$\therefore\quad AB = 1.0 + 5 \times v.c. = 1.0 + 5 \times 0.01 = 1.05\ cm.$$

Illustration 2. *In travelling microscope the vernier scale used has the following data.*
1 M.S.D. = 0.5 mm, 50 V.S.D. = 49 M.S.D.
and the actual reading for distance travelled by travelling microscope is 2.4 cm with 8th division coinciding with a main scale graduation . Estimate the distance travelled.

Solution : In this case vernier constant = 1.M.S.D. – 1.V.S.D.

$$= 1.M.S.D. - \frac{49}{50} M.S.D. = \frac{1}{50} M.S.D = \frac{1}{50} \times 0.5\,mm$$

$= \frac{5}{10} \times \frac{1}{50} = 0.01\,\text{mm} = 0.001\,\text{cm}$

$\therefore$ Distance travelled = 2.4 + 8 × 0.001 cm
= 2.408 cm

Illustration 3. *The Vernier scale used in Fortin's barometer has 20 divisions coinciding with the 19 main scale divisions. If the height of the mercury level measured is 5 mm and 15th division of vernier scale is coinciding with the main scale division. Then calculate the exact height of the mercury level (given that 1.M.S.D. = 1mm)*

Solution: 20 V.S.D. = 19 M.S.D. (Given)

1.V.S.D. = $\frac{19}{20}$ M.S.D.

V.C. = 1. M.S.D. – 1.V.S.D $= \left(1 - \frac{19}{20}\right)$M.S.D.

$= \frac{1}{20}$ M.S.D.

$= \frac{1}{20} \times 1\,\text{mm} = 0.05\,\text{mm}$

= 0.005 cm

Height of mercury level = 5 + 0.05 × 15
= 5.75 mm

Exercise: ***The Vernier calliper is used to measure the length of an object. The least count of such a vernier calliper is 0.2 cm and scale reads its length to be 5.6 cm. 3rd division of Vernier scale is coinciding main scale division Calculate the length of an object.***

Zero Error

If the zero marking of main scale and Vernier scale do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument. If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

Illustration 4. *Consider the following data:*
10 main scale divisions = 1cm, 10 vernier division = 9 main scale divisions,
zero of Vernier scale is to the right of the zero marking of the main scale with 6th Vernier division coinciding with a main scale division and the actual reading for length measurement is 4.3 cm with 2nd Vernier divisions coinciding with a main scale graduation. Estimate the length.

Solution: In this case, vernier constant = $\frac{1\,\text{mm}}{10} = 0.1\,\text{mm}$

Zero error = 6 × 0.1 = + 0.6 mm

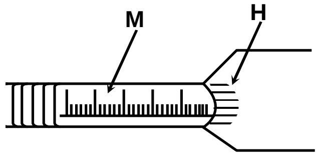
Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction
= 4.32 – 0.06 = 4.26 cm

Screw Gauge (or Micrometer Screw)

In general Vernier Callipers can measure accurately upto 0.02 mm and for greater accuracy micrometer screw devices, e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by turning it axially. The instrument is provided with two scales:

(i) The main scale or pitch scale M graduated along the axis of the screw.
(ii) The cap–scale or head scale H round the edge of the screw head.



Constants of the screw gauge:

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus if 10 rotations of cap ≡5 mm, then pitch = 0.5 mm

In general, pitch $= \frac{\text{Dis tan ce travelled by screw on main scale}}{\text{No. of rotation taken by the cap to travel that much dist an ce}}$

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration, if the total cap division is 100, then least count = 0.5 mm/100

= 0.005 mm
In general, In case of circular scale,

Least count $= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

If pitch is 1 mm and there are 100 divisions on circular scale, then

Least count = $= \frac{1\text{mm}}{100}$ = 0.01 mm = 0.001 cm

= 0.00001 m = 10^{-5} m = 10 μm

Since least count is of the order of 10 μm, So the screw is called a micrometer screw. Screw gauge and the spherometer which work on the principle of micrometer screw, consist essentially of the following two scales.

(i) Linear or Pitch scale : It is a scale running parallel to the axis of the screw.

(ii) Circular of Head scale: It is marked on the circumference of the circular disc or the cap attached to the screw.

Zero Error: In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero–error, otherwise the instrument is said to heave zero–error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and vice–versa. The corresponding corrections will be just opposite.

Illustration 5. *A screw gauge has 100 divisions on its circular scale. Circular scale travels one division on linear scale in one rotation and 10 divisions on linear scale of screw gauge is equal to 5 mm. What is the least count of a screw gauge.*

Solution: Pitch $= \frac{1\text{division on linear scale}}{1\text{rotation}} = 1\text{div.}$

10 division = 5 mm
∴ 1 division = 0.5 mm
∴ pitch = 0.5 mm

$\text{least count} = \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$= \frac{0.5\,\text{mm}}{100} = 0.005\,\text{mm}$$

Illustration 6. *The screw gauge mentioned in above illustration is used to measure thickness of a coin. The reading of the linear scale is 4^{th} div and 25^{th} division of circular scale is coinciding with it. What is the value of thickness of the coin..*

Solution: Reading = Linear scale Reading + Least count × circular scale reading
= 4^{th} division on linear scale + 0.005 mm × 2.5
= 4 × 0.5 mm + 0.125 mm
= 2 mm + 0.125 mm
= 2.125 mm

Illustration 7. *A spherometer has 250 equal divisions marked along the periphery of its disc and one full rotation of the disc advances it on the main scale by 0.0625 cm. The least count of the spherometer is*

(A) 2.5×10^{-2}cm *(B) 25×10^{-3}cm*
(C) 2.5×10^{-4} cm *(D) none of the above*

Solution: Least count = $\frac{0.0625}{250}$ cm = 2.5×10^{-4}cm

$\therefore$(C)

(ii) Indirect or Mathematical method

This method involves measurement of long distances. Main methods of this category are –

Reflection method: Suppose we want to measure the distance of a multi story building from a destination point P. If a shot be fired from P, the sound of shot travels a distance x towards the building, gets reflected from the building. The reflected sound travels the distance x to the point of P, when an echo of the shot is heard.

Let t = time interval between the firing of the shot and echo sound.

v = velocity of sound in air.

Distance = velocity x time

x + x = (v) (t)

$\Rightarrow$ x = (v) (t/2)

As v is known, x can be calculated by measuring the time t.

Illustration 8. *A rock is at the bottom of a very deep river. An ultrasonic signal is sent towards rock and received back after reflection from rock in 4 seconds. If the velocity of ultrasonic wave in water is 1.45 km/s, find the depth of river.*

Solution: Here x = ?

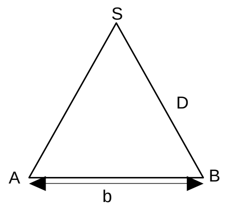
v = 1.45 km/s = 1450 m/sec.

t = 4 sec

so, x = v x t / 2 = 1450 x 4 / 2 = 2900 m.

Parallel method: This method is used for measuring distance of nearby stars.

Let we have to measure the distance D of a far away star S by this method. We observe this star from two different position A and B on the earth, separated by a distance AB = b at the same time as shown in figure. Let $\angle ASB = \theta$, the angle θ is called parallatic angle. As the star is very far away, b/D << 1 and θ is very small.



Here we can take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS=BS so that AB = b = Dθ where θ is in radians.

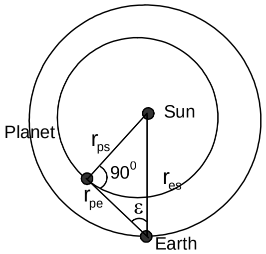
D = b/θ

Knowing b and measuring θ, we can calculate D.

Copernicus method: This method is used to measure the relative distances of the planets from the Sun.

(a) For Interior Planets: The angle formed at earth between the earth–planet direction and the earth–sun direction is called the planet's elongation. This is the angular distance of the planet from the sun as observed from earth. When the elongation attains its maximum value ε as in the figure, the planet appears farthest from Sun.

$$r_{ps} = r_{es}\sin\varepsilon$$
$$= (\sin\varepsilon)\,AU \quad (AU = \text{Astronomical Unit})$$



(b) For Exterior Planets: This method is a consequence of Kepler's 3rd law of planetary motion. For two planets P_1 and P_2 we have,

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

where a_1 and a_2 are semi–major axes, of respective orbits. Period can be ascertained by direct observation. Therefore if a_1 is measured, a_2 can be calculated.

(iii) Chemical Method

This method is used to measure distance of the order of $10^{-1\circ}$ m. Let us calculate the size of an atom.

Let m = mass of substance,

V = volume occupied by substance &

ρ = density of the substance

$\therefore v = m/\rho$ (1)

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ No. of atoms in mass m of the substance = Nm / M

If r = radius of each atom then V = volume of each atom = $\frac{4}{3}\pi r^3$

Volume of all the atoms in substance = $(\frac{4}{3}\pi r^3 \text{ x } Nm)M$.

According to Avagordo's hypothesis,

Volume of all the atoms = (2/3) x volume of substance

$$\frac{4}{3}\pi r^3 \text{ x } Nm/M = (2/3)\, m/\rho$$

$$\therefore r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$

MEASUREMENT OF MASS

Measurement of Inertial Mass

Inertial mass of a body is measured using a device which is known as inertial balance. It consists of a long metal strip. One end of the strip is clamped to a table such that its flat face is vertical, and it can easily vibrate horizontally. The other end of strip supports a pan in which the object whose inertial mass is to be found can be kept. It is found that the square of time period of vibration is directly proportional to total mass of the pan and the body placed in it.

$$t^2 \propto m$$

$$\therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \quad m_2 = m_1 \frac{t_2^2}{t_1^2}$$

Measurement of Time: The following methods are used

(a) Quartz Crystal Clock
(b) Atomic Clock
(c) Radioactive dating

Significant figures:

Each measurement involves errors. The measure results has a number that includes all reliably known digits and first unknown digit. The combination of reliable digits and first uncertain digit are significant figures.

Example: If a length is measured as 2.43 cm then 2 and 4 are reliable while 3 is uncertain. Thus the measured value has three significant figures.

Common rules for counting significant figures

(1) All non zero digits are significant.
For example: 1745 has four significant digits.
(2) All zeros present between 2 non zero digits are significant, irrespective of the position of the decimal point.

Example: 208005 has 6 significant figures.
(3) If there is no decimal point, all zeros to the right of the right–most non zero digit are considered to be significant only if they come from a measurement.

Example: 41000 has only 2 significant digits while 41000 m has 5 significant digits.
(4) All zeros to the right of a decimal point but to the left of non–zero digits are considered to be non significant, provided there should be no non zero digit to the left of the decimal point.
Example: 0.00305 has 3 significant figures.
(5) All zeros are significant if they are placed to the right of a decimal point and to the right of a non zero digit.

Example: 0.04080 has 4 significant figures
50.000 has 5 significant figures

(6)The number of significant figures does not alter in different units.
If we want to write 450 m in different units, we can write it 4.50×10^4 cm or 4.50×10^5 mm etc. in which all of them are having 3 significant figures.

Illustration 9. *State the number of significant figures in the following –*
(a) 06500310
(b) 754400
(c) 15000 kg
(d) $8.314\times10^{+2}$ J
(e) 1.6×10^{-19} C
(f) 0.0065050

Solution: (a) 7 (b) 4 (c) 5 (d) 4 (e) 2 (f) 5

Exercise: ***The number of significant figures in 0.0160 is***
(a) 2 ***(b) 3*** ***(c) 4*** ***(d) 5***

Rounding off

(1) If all the digits to be discarded are such that the first discarded digit is less than 5, the remaining digits are left unchanged.

Example:

7.499498 can be written in 4 significant figures as 7.499
(2) If the digit to be discarded is 5 followed by digits other than zero, then the preceding digit should be raised by 1.

Example:

7.45001, on being rounded off to first decimal, became 7.5
(3) If the digits to be discarded is 5 or 5 followed by zero the preceding digit remains unchanged if it is even and the preceding digit is raised by 1 if it is odd.

Example:

3.6500 will become 3.6 and 4.7500 will become 4.8 in 2 significant figures.

Arithmetic operations with significant figures:

(1) Addition and subtraction In addition and subtraction, the number of decimal places in the result is the smallest number of decimal places of terms in the operation.

Let us consider the sum of following measurements.
3.45 kg., 7.6 kg. and 10.055 kg.

$$\begin{array}{r} 3.45 \\ 7.6 \\ 10.055 \\ \hline 21.105 \\ \hline \end{array}$$

So the sum will be 21.1 kg as 7.6 kg has only 1 digit after the decimal point while the others are having more than one digit.

Multiplication and Division:

In the result of multiplication or division, the number of significant figures is same as the smallest number of significant figures among the numbers.

Illustration 9: *Multiply 1.21 and 1.1.*

Solution: $1.21 \times 1.1 = 1.331$

So the result is 1.3 as there are only 2 significant digits in 1.1

The same procedure is followed for division.

Exercise: Value of 1.2 + 1.34 + 2.342 is
(a) 4.88 (b) 4.8 (c) 4.90 (d) 5

Accuracy and Precision of measuring instrument: It is impossible to measure any physical quantity perfectly. It is due to imperfection in manufacturing and working of measuring instruments.

Accuracy: It is the degree of correctness of the measured quantity, i.e. how much close the result is to the true value of the physical quantity.

Precision: It is the degree of repeatability & refinement of a measurement.

ERRORS IN MEASUREMENT

In the experiment we may get some other value than that of the true value due to faulty equipment, carelessness or random causes. This will cause error in measurement.

There are 3 ways to express an error

(1) Absolute Error: It is the positive value of difference between the true value and measured value of the quantity. Since we don't know the correct value of quantity the best possible value can be given by mean value of all the measured value.

Arithmetic mean v, $A_m = \dfrac{A_1 + A_2 + ...A_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} A_i$

$\therefore$ The absolute error in the measurement can be given as.

$\Delta A_1 = |A_m - A_1|$ where A_m : Mean value of the measurements.

$\Delta A_2 = |A_m - A_2|$ where A_1, A_2 : Measured value of quantity.

$\Delta A_n = |A_m - A_n|$

Taking the arithmetic mean of all the absolute errors we get the mean absolute error $\Delta A_{m.}$.

$$\Delta A_m = \frac{\Delta A_1 + \Delta A_2 + ... + \Delta A_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \Delta A_i$$

So the true value of A will be such that

$$\left(A_m - \Delta A_m\right) \le A \le \left(A_m + \Delta A_m\right)$$

(2) Relative Error: It is defined as the ratio of the mean absolute error to the mean value of the quantity being measured

Relative error $= \frac{\Delta A_m}{A_m}$

(3) Percentage Error: The relative error can be expressed in percentage error as

% error = Relative error $\times 100$

Propagation of Error: Any physical quantity depends on one or more than one physical quantities. So the error in any physical quantity will lead to error in the result.

(1) Error in result involving sum or difference of quantities

Let Z is defined as

$$Z = A + B - C$$

$\therefore$ $\Delta Z = \Delta A + \Delta B - \Delta C$

$\therefore$ Maximum possible error in Z is given by

$|\Delta Z|_{max} = \Delta A + \Delta B + \Delta C$ (Since ΔC can be positive or negative)

2. Error in the result having product or division of quantities:

$$Z = \frac{A^p B^q}{C^r}$$

$\Rightarrow$ $\ln z = p\ln A + q\ln B - r\ln c$

$\Rightarrow$ $\frac{dz}{z} = \frac{pdA}{A} + \frac{qdB}{B} + \frac{rdC}{C}$

For small change $dz \approx \Delta z. \Rightarrow \frac{\Delta z}{z} = P\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

Illustration10. *Multiply 107.88 by 0.610 and express the result upto the correct number of significant figure.*

(A) 65.8068 *(B 65.807*

(C) 65.81 *(D) 65.8*

Solution: Number of significant figures in multiplication is three corresponding to the minimum number 107.88×0.610

= 65.8068 = 65.8

$\therefore$ (D)

Illustration11. *In measurement of the period of oscillation of a Helical spring, the readings comes out to be 2.15 sec, 2.25 sec, 2.36 sec, 2.45 sec and 2.54 sec, calculate the absolute errors, relative error or percentage error.*

Solution: The mean period of oscillation of the Helical spring is

$$T = \frac{2.15 + 2.25 + 2.36 + 2.45 + 2.54}{5}$$

= 2.35 sec

The absolute error in the measurements are

2.15 – 2.35 = – 0.20 sec

2.25 – 2.35 = – 0.10 sec

2.36 – 2.35 = 0.01 sec

2.45 – 2.35 = 0.10 sec

2.54 – 2.35 = 0.19 sec

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is ΔT_{mean}= [(0.20 + 0.10 + 0.01 + 0.10 + 0.19)] /5

$= \frac{0.6}{5} = 0.12\,\text{sec}$

Period of oscillation of the simple pendulum is (2.35 ± 0.12) sec.
A more correct way to write its is
(2.4 ± 0.2) sec
The relative error or the percentage error is

$= \frac{0.2}{2.4} \times 100 = 8\%$

Combination of Errors

While doing an experiment we take several measurements, we must know how the errors in all the measurements combine.
To make such estimates, we should learn how errors combine in various mathematical operations. For this we use the following procedure

(I) Error of a sum or difference: Suppose two physical quantities A and B have measured values A ± ΔA, B ± Δ B respectively where ΔA and ΔB are their absolute errors.

(a) We wish to find the error Δz in the sum
z= A + B

We have by addition, z ± Δz

= (A ± ΔA) + (B ± Δ B)

The maximum possible error in z = Δ z = ΔA + ΔB

(b) For the difference z = A – B, we have

z ± Δz = (A ± Δ A) – (B ± Δ B)

= (A – B) ± Δ A ± Δ B

or, ± Δ z = ± Δ A ± Δ B

The maximum value of the error Δz is again ΔA + ΔB.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual.

Illustration 12. *The series combination of resistances is given by $R = R_1 + R_2$*
Suppose two resistances $R_1 = (50 \pm 4)\Omega$ and $R_2 = (100 \pm 3)\ \Omega$ are connected in series. Find equivalent resistance of the series combination.

Solution: $R_{eq} = R_1 + R_2$
= (50 ± 4) Ω + (100 + 3) Ω
= (150 ± 7) Ω

(II) Error in a product or a quotient

Suppose z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then
z ± Δz = (A ± ΔA) (B ± ΔB)

= AB ± B ΔA ± A ΔB ± ΔA ΔB

Dividing L.H.S. by z and R.H.S. by AB, we have

$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right).$$

Since ΔA and ΔB are small we shall ignore their product.
Hence the maximum fractional error in Z

$$= \frac{\Delta z}{z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Similarly, we can easily verify that this is true for division also.
So, when two or more quantities multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

(III) Error due to the power of a measured quantity.

Let $Z = X^2$

Then $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta X}{X} = \frac{2\Delta X}{X}$

Hence the fractional error in X^2 is two times the error in X.

In general if $Z = \frac{X^a Y^b}{Q^c}$

then $\frac{\Delta Z}{Z} = a\frac{\Delta X}{X} + b\frac{\Delta Y}{Y} + \left[\frac{\Delta Q}{Q} \times c\right]$

Illustration 13. *Find the fractional error in Z, if* $Z = \sqrt{\frac{XY}{M}}$

Solution: $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta X}{X} + \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta M}{M}$

Illustration 14. *Find maximum possible percentage error in* $x = \frac{a^{\ell} b^m}{y^p z^k}$

Solution: $\frac{\Delta X}{X} \times 100 = \left(\ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta y}{y} + k\frac{\Delta z}{z}\right) \times 100$

Illustration 15. *In the relation $x = 3yz^2$, x, y and z represents various physical quantities, if the percentage error in measurement of y and z is 3% and 1% respectively, then final maximum possible percentage error in x.*

Solution: $\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta y}{y} + 2\frac{\Delta z}{z}\right) \times 100$

$= 3\% + 2 \times 1\% = 5\%$

PHYSICAL QUANTITIES

All the physical quantities can be expressed in terms of some combination of seven base quantities: Length [L], mass [M], time [T], electric current [I], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol]. These base quantities are considered as the seven dimensions of the physical world.

DIMENSIONS

The dimension of a physical quantity are the powers to which the fundamental (or base) quantities like mass, length and time etc. have to be raised to represent the quantity. Consider the physical quantity **"Force"**. The unit of force is Newton.

1 Newton = 1 kg m/sec^2

$kg \to M^1$ (Mass); $m \to L^1$ (Length); $s^{-2} \to T^{-2}$ (Time)

$\therefore$ Dimensions of force are $[M^1L^1T^{-2}]$

Dimensional formulae for some physical quantities

Physical quantity	**Relation with other quantity**	**Dimensional formula**
Area	Length × breadth	$L \times L = [L^2]$
Density	Mass/volume	$\frac{M}{L^3} = [ML^{-3}]$
Acceleration	$\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = [LT^{-2}]$
Force	F = ma	$[MLT^{-2}]$
Linear momentum	P = mv	$[MLT^{-1}]$
Pressure	P = F/A	$[ML^{-1}T^{-2}]$
Universal gravitational constant	$G = \frac{Fr^2}{M_1M_2}$	$[M^{-1}L^3T^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (kinetic, potential and heat)	$\frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Surface tension	$T = \frac{F}{\ell}$	$[ML^{\circ}T^{-2}]$
Strain	$e = \frac{\Delta\ell}{\ell}$	$[M^{\circ}L^{\circ}T^{\circ}]$
Modulus of elasticity	$E = \frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	$[M^{\circ}L^{\circ}T^{\circ}]$
Coefficient of viscosity	$\eta = \frac{F \times r}{A \times v}$	$[M^1L^{-1}T^{-1}]$
Planck's constant	$h = mv\lambda$	$[ML^2T^{-1}]$
Thermal resistance	$\frac{\Delta\theta t}{Q}$	$[M^{-1}L^{-2}T^3\,\theta]$
Thermal conductivity	$K = \frac{H}{At(d\theta/dx)}$	$[MLT^{-3}\theta^{-1}]$
Boltzman's constant	k = R/N	$[ML^2T^{-2}\theta^{-1}]$
Universal gas constant	$R = \frac{PV}{T}$	$[ML^2T^{-2}\theta^{-1}]$
Mechanical equivalent of heat	J = W/H	$[M^{\circ}L^{\circ}T^{\circ}]$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$	$[M^{\circ}L^{\circ}T^{-1}]$

Illustration 16. *Write the dimensions of: Impulse, Pressure, Work, Universal constant of Gravitation.*

Solution: (i) $[M^1L^1T^{-1}]$ (ii) $[M^1L^{-1}T^{-2}]$ (iii) $[M^1L^2T^{-2}]$ (iv) $[M^{-1}L^3T^{-2}]$

Four types of quantities

Dimensional constant: These are the quantities whose values are constant and they possess dimensions. For example, velocity of light in vacuum, universal gas constant etc.

Dimensional variables: These are the quantities whose values are variable, and they possess dimensions. For example, area, volume, density etc.

Dimensionless constants: These are the quantities whose values are constant, but they do not possess dimensions. For example, π, 1, 2, 3, ….. etc.

Dimensionless Variables: These are the quantities, whose values are variable, and they do not have dimensions, e.g., angle, strain, specific gravity etc.

Uses of dimensions: dimensional analysis

(1) Checking the correctness (dimensional consistency) of an equation: An equation contains several terms which are separated from each other by symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This means that we can not add velocity to force. This principle is called Principle of Homogeneity of dimensions.
Look at the equation : $v^2 = u^2 + 2as$
Dimensions of $v^2 : [L^2T^{-2}]$
Dimensions of $u^2 : [L^2T^{-2}]$
Dimensions of $2as : LT^{-2}][L] = [L^2T^{-2}]$
$\therefore$ The equation $v^2 = u^2 + 2as$ is dimensionally consistent, or dimensionally correct.

Note:
A dimensionally correct equation may not be actually correct. For example, the equation $v^2 = u^2 + 3as$ is also dimensionally correct but we know that it is not actually correct. However, all correct equations must necessarily be dimensionally correct.

Illustration 17. *Which of the following equations may be correct ?*

(i) $x = ut + \frac{1}{2}at^2$ *(ii)* $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $F = \frac{GM_1M_2}{r}$ *(iv)* $T^2 = \frac{4\pi^2R^3}{GM}$

(v) $V = \sqrt{GMR}$

Given: G = Gravitational constant, whose dimensions are $[M^{-1}L^3T^{-2}]$
M_1, M_2 *and M have dimensions of mass. L, x, r, R has dimensssions of length. And t has dimensions of Time. 'F' denotes Force and 'a' has dimensions of acceleration.*

Solution: (i) Yes (ii)Yes (iii) No (iv) Yes (v) No.

(2) Conversion of units: Dimensional methods are useful in finding the conversion factor for changing the units to a different set of base quantities. Let us consider one example, the SI unit of force is Newton. The CGS unit of force is dyne. How many dynes is equal to one newton. Now,

$$1\text{ newton} = [F] = [M^1L^1T^{-2}] = (1\,\text{kg})^1)(1\,\text{meter})^1(1\text{s})^{-2}$$

$$1\,\text{dyne} = (1\text{g})(1\text{cm})(1\text{s})^{-2}$$

$$\therefore \frac{1\,\text{newton}}{1\,\text{dyne}} = \frac{(1\,\text{kg})^1(1\,\text{meter})^1\,(1\text{s})^{-2}}{(1\text{g})(1\text{cm})(1\text{s})^{-2}} = (10^3)(10^2) = 10^5$$

$$1\text{ newton} = 10^5\text{ dynes}$$

Thus knowing the conversion factors for the base quantities, one can work out the conversion factor of any derived quantity if the dimensional formula of the derived quantity is known.

Illustration 18. *Find the conversion factor for expressing universal gravitational constant from SI units to cgs units.*

Solution: $6.67\text{x}10^{-8}\ \text{cm}^3\text{s}^{-2}\text{g}^{-1}$

(3) Deducing relation among the physical quantities:

Suppose we have to find the relationship connecting a set of physical quantities as a product type of dependence. Then dimensional analysis can be used as a tool to find the required relation. Let us consider one example. Suppose we have to find the relationship between gravitational potential energy of a body in terms of its mass ‘m’, height ‘h’ from the earth’s surface and acceleration due to gravity ‘g’, then,

Let us assume that: – Gravitational potential energy, U,

$$U = K[m]^a[g]^b[h]^c,$$

where K, a, b, and c are dimensionless constants.

Then $[ML^2T^{-2}] = [M]^a[LT^{-2}]^b[L]^c$

$$= [M^aL^{b+c}T^{-2b}]$$

$\therefore$ a =1, b + c = 2

–2b = –2

b = 1, c = 1.

$\therefore$ U = Kmgh, where K is a dimensionless constant.

Thus by dimensional analysis, we conclude that the gravitational potential energy of a body is directly proportional to its mass, acceleration due to gravity and its height from the surface of the earth.

Limitations of dimensional analysis:

This method does not give us any information about the dimensionless constants appearing in the derived formula, e.g. 1, 2, 3, .. . π etc.

We can’t derive the formula having trigonometrical functions, exponential functions etc, which have no dimensions.

The method of dimensions cannot be used to derive an exact form of relation when it consists of more that one part on any side, e.g. the formula $v^2 = u^2 + 2as$ cannot be obtained.

If a quantity depends on more than three factors having dimensions the formula cannot be derived. This is because on equating powers of M, L and T on either side of the dimensional equation, we can obtain three equations from which only three exponents can be calculated.
It gives no information whether a physical quantity is a scalar or a vector.

Illustration 19. *Using the method of dimensions, find the acceleration of a particle moving with a constant speed v in a circle of radius r.*

Solution: Assuming that the aceeleration of a particle depends on v and r
$a \propto v^x r^y \Rightarrow a = k\, v^x r^y$
Now as we know dimensions of acceleration (a) = $M^\circ LT^{-2}$
and dimensions of velocity (v) = $M^\circ LT^{-1}$
dimension of radius (r) = $M^\circ LT^\circ$
Putting all thee dimensions in (1), we get
$M^\circ LT^{-2} = k\,(M^\circ LT^{-1})^x (M^\circ LT^\circ)^y$
$M^\circ LT^{-2} = k\, M^\circ L^{x+Y} T^{-x}$
Comparing the powers, we get
$x + y = 1$
$x = 2$
$\therefore\ y = 1-2 = -1$
$\therefore\ a = k\, v^2 r^{-1}$
$a = \dfrac{kv^2}{r}$

Illustration 20. *In the expression* $\left(P + \dfrac{a}{v^2}\right)(v - b) = RT$
P is pressure and v is the volume. Calculate the dimensions of a and b.

Solution: Only physical quantities having same dimensions are added or subtracted. So $\dfrac{a}{v^2}$ has the same dimensions as that of pressure.
As pressure = $\dfrac{\text{Force}}{\text{Area}}$
Dimensions of pressure $= \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$
$\therefore$ Dimensions of $\dfrac{a}{v^2} = ML^{-1}T^{-2}$
Dimensions of a $= ML^{-1}T^{-2}(V^3)^2$
$= (ML^{-1}T^{-2})(L^3)^2$
$= ML^{-1}T^{-2}L^6 = ML^5T^{-2}$
Similarly dimensions of b is same as that of volume.
Dimensions of $b = M^0L^3T^0$.

Illustration 21. *Does* $S_{nth} = u + \dfrac{a}{2}(L_n - 1)$ *dimensionally correct?*

Solution: Yes, this expression is dimensionally correct, yet it appears to be incorrect. As we are taking it to be for n^th second. Here one second is divided through the equation.

Illustration 22. *Find the dimensions of resistivity, thermal conductivity and coefficient of viscosity.*

Solution: (i) $R = \rho \dfrac{\ell}{A}$

$\rho = \dfrac{RA}{L} = [ML^3T^{-3}A^{-2}]$

(ii) Thermal conductivity, k

$\dfrac{d\theta}{dt} = \dfrac{k\ell}{A\Delta\theta} = \dfrac{ML^2T^{-3}L}{L^2K}$

$= MLT^{-3}k^{-1}$

(iii) Coefficient of viscosity

$\therefore\ F = \eta A \dfrac{dv}{dx}$

$\eta = \dfrac{Fdx}{Adv} = \dfrac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

Illustration 23. *A displacement of a particle is given by equation y = A sin ωt, where y is in metres and A is also in metres, t is in seconds. What are the dimensions of ω.*

Solution: As the angles are always dimensionless, so

ωt = dimensionless quantity

Dimensions of $\omega t = M^\circ L^\circ T^\circ$

Dimensions of $\omega = M^\circ L^\circ T^{-1}$

Illustration 24. *If density ρ, acceleration due to gravity g and frequency f are the basic quantities, find the dimensions of force.*

Solution: We have $\rho = ML^{-3}$, $g = LT^{-2}$ $f = T^{-1}$

Solving for M, L and T in terms of ρ, g and f , we get

$M = \rho^2 g^3 f^{-6}$, $L = gf^{-2}$ & $T = f^{-1}$

Force = $[MLT^{-2}] = [\rho g^3 f^{-6}.gf^{-2}.\ f^2] = [\rho g^4 f^{-6}]$

Illustration 25. *An athlete's coach told his team that muscle times speed equals power. What dimensions does he view for "muscle"?*

(A) MLT^2 *(B) ML^2T^{-2}*
(C) MLT^{-2} *(D) L*

Solution: Power = force × velocity

= muscle times speed

$\therefore$ muscle represents force

muscle = $[MLT^{-2}]$

$\therefore$ (C)

Illustration 26. *If force, length and time would have been the fundamental units what would have been the dimensional formula for mass*

(A) $FL^{-1}T^{-2}$ *(B) $FL^{-1}T^2$*
(C) FLT^{-2} *(D) F*

Solution: Let $M = K\,F^aL^bT^c$

$= [MLT^{-2}]^a\,[L^b]\,T^c = [M^aL^{(a+b)}T^{(-2a+c)}]$

$a = 1, a + b = 0$ & $-2a + c = 0$

$\Rightarrow a = 1, b = -1, c = 2$

$\therefore$ (B)

Illustration 27. *The dimensions of the Rydberg constant are*

(A) $M^{\circ} L^{-1} T$ *(B)* MLT^{-1}

(C) $M^{\circ} L^{-1} T^{\circ}$ *(D)* $ML^{\circ} T^{2}$

Solution: From the relation $\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$R = \frac{1}{L} = L^{-1} = M^{\circ}L^{-1}T^{\circ}$

$\therefore$ (C)

Illustration 28. *The error in the measurement of the radius of a sphere is 1%. Then error in the measurement of volume is*

(A) 1% *(B) 5%*

(C) 3% *(D) 8%*

Solution: $V = \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r}\right) \times 100 = 3 \times 1 = 3\%$

$\therefore$ (C)

MISCELLANEOUS EXERCISE

1. The time period of a gas bubble formed under water oscillating with a time period depending on static pressure P, density of water ρ and ε total energy of explosion. Find the relationship between T, P, ρ and ε.

2. Name the three physical quantities having the same dimensions

3. A student measures the time period of a simple pendulum. If error in measurement of length is 2% and error in measurement of g is 2% calculate the error in measurements of Time period.

4. A physical quantity is given by x = a + bt, where x is in metres and t is in seconds. So calculate the dimensions of a and b.

5. Find the dimensional formulae of the following quantities
   (A) the universal gravitational constant (B) Surface tension
   (C) Potential energy (D) Surface energy

6. A Vernier calliper has 50 divisions on its Vernier scale. Minimum division in main scale is of 1 mm.
   (a) Find out the least count of the Vernier Calliper.
   (b) During a length measurement, zero of Vernier scale is between 12th and 13th divisions of the main scale and 26th division of the Vernier scale coincides with a main scale division. What is the length ?

7. Calculate the focal length of a spherical mirror if measured quantities u and v are as follows.
   $u = 50.1 \pm 0.5\,\text{cm}$
   $v = 20.1 \pm 0.2\ \text{cm}$

8. Young's modulus of steel is 19 x $10^{1\circ}$ N/m^2. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.

9. Convert 1 joule into ergs.

10. A goldsmith puts some gold weighting 5.42 gm in a box weighing 1.2 kg. Find the total weight of the box to correct number of significant figures.

SOLUTION TO MISCELLANEOUS EXERCISE

1. $T \propto P^{-5/6}\rho^{1/2}E^{1/3}$
2. Work, energy and torque
3. 2%
4. x = a + bt
   Dimensions of a = $M^\circ L T^\circ$
   Dimensions of b = $M^\circ L T^{-1}$
5. (a) $M^{-1}L^2T^{-2}$ (b) MLT^{-2}
   (c) ML^2T^{-2} (d) ML^2T^{-2}
6. 0.02 mm, 12.52 mm
7. 14.3 ± 0.4 cm
8. 19×10^{11} dyne / cm^2
9. 1 joule = 10^7 ergs
10. 1.2 kg

SOLVED PROBLEMS

Subjective:

Prob 1. *The Bernoulli's equation is given by,*
$P+\frac{1}{2}\rho v^2+\rho gh=constant$, *where P is pressure. Compare the unit of the quantity* $\frac{1}{2}\rho v^2$ *with the unit of pressure.*

Sol. Only same quantities can be summed up or subtracted from each other. So $\frac{1}{2}\rho v^2$ has same unit as that of pressure.

Prob 2. *The relation between velocity and time of a moving body is given as,* $v=A+\frac{B}{t}+Ct^2$. *Find the units of A, B and C.*

Sol. From the principle of homogeneity
v = A = m/sec
v = B/t ⇒ B = m
v = (ct^2)
∴ c = m/sec^3

Prob 3. *The external and internal diameters of a hollow cylinder are measured to be (4.23 ±0.01) cm and (3.89 ± 0.01) cm. What is the thickness of the wall of the cylinder?*

Sol. Thickness = (4.23 – 3.89) cm = $\frac{0.34}{2}$ = 0.17 cm.
Error = ±(0.01 + 0.01) cm = ±0.02 cm

Prob 4. *A physical quantity x is calculated from the relation* $x=\frac{a^2b^3}{c\sqrt{d}}$. *If percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively. What is the percentage error in x?*

Sol. As $x=\frac{a^2\ b^3}{c\sqrt{d}}$

$$\frac{\Delta x}{x}=\pm\left[2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right]$$

$$= \pm\ [2\times 2\% + 3\times 1\% + 3\% + \frac{1}{2}\times 4\% = \pm\ 12\%$$

Prob 5. *A physical quantity A is defined as,* $A=pkx^2y/z$. *The absolute errors in the measured of x, y, z are given as*
x = (0.26 ± 0.02)cm
y = (64 ± 2) Ω
z = (156.0 ± 0.1)cm
Find the percentage error in the quantity A.

Sol. $a = kx^2y / z$

$$\Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$= 2 \times \frac{0.02}{0.26} + \frac{2}{64} + \frac{0.1}{156}$$

$\pm\, 0.186 = 18.6\ \%$

Prob 6. *10 rotations of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 divisions. Find the least count. A reading taken for the diameter of wire with the screw gauge shows 4 complete rotations and 35 on the circular scale. Find the diameter of the wire.*

Sol. The least count = $\frac{5}{1000}$ = 0.005 mm

The diameter of the wire = $(4 \times 0.5 + 35 \times 0.005)$ mm = 2.175 mm

Prob 7. *The diameter of a sphere is 2.78 cm. Calculate its volume in proper significant figures.*

Sol. Volume = $\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi\left(\frac{2.78}{2}\right)^3$ $cm^3 = 11.2437\ cm^3$

Hence the volume in proper significant figures is $11.2\ cm^3$

Prob 8. *Calculate the number of light years in one meter.*

Sol. We know 1 light year $(\ell_y) = 9.46 \times 10^{15}$m

or 9.46×10^{15} m = $1\ell y$

$1\ m = 1.057\times10^{-16}\ell y$

Prob 9. *Find the dimensions of a and b in the relation* $P = \frac{b - x^2}{at}$

where P is power, x is distance and t is time.

Sol. The given relation is, $P = \frac{b - x^2}{at}$

As x^2 is subtracted from b therefore the dimensions of b are of x^2

i.e. $b = L^2$

We can rewrite relation as

$$P = \frac{\left[L^2\right]}{at} = \frac{L^2}{at}$$

$$a = \frac{L^2}{\left[ML^2T^{-3}\right]\ \left[T\right]} = M^{-1}L^{\circ}T^2$$

Prob 10. *It is claimed that two cesium clocks if allowed to run for 100 years free from any disturbance may differ by only about 0.02 sec. What is the accuracy of the standard cesium clock in measuring a time interval of 1 sec?*

Sol. $\because$ t = 100 years = $100\times365.25\times86400$s

$\Delta t = 0.02$s

Fractional error = $\dfrac{\Delta t}{t} = \dfrac{0.02}{100\times365.25\times86400}$

$= 0.63\times10^{-11}$

So, there is an accuracy of 10^{-11} Part in 1 or 1 sec in 10^{11} sec.

Prob11. *In screw gauge no. of division on circular scale is n and circular scale travels a distance of a units in one rotation. Calculate least count of the screw gauge.*

Sol. Pitch = a units

Least count $= \dfrac{\text{Pitch}}{\text{No. of divisions on circulat scale}}$

$= \dfrac{a}{n}$ units.

Prob 12. *The diameter of the spherical bob is measured by vernier Calipers (10 divisions of a Vernier scale coincide with a divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 12 mm and 7^{th} division of the main scale coincides with the main scale. Mass of the sphere is 4.532 g. Find the density of the sphere.*

Sol. Vernier constant = 1.M.S.D. – 1.V.S.D.

$= 1\text{ mm} - \dfrac{9}{10}\text{ mm}$

= 0.1 mm

Diameter of sphere = 12 mm + 0.1×7

= 12.7 mm

$\therefore$ Volume of sphere = $= \dfrac{4}{3}\pi\left(\dfrac{D}{2}\right)^3$

$= \dfrac{4}{3}\times3.14\times\left(\dfrac{12.7}{2}\times10^{-3}\right)^3$

Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{4.532\times3\times8\times10^{-3}}{4\times3.14\times(12.7\times10^{-3})^3}$

= 4.227 kg /m^3

= 4.23 kg/m^3 (in appropriate significant figures)

Prob 13. *A wire of length $\ell = 8\pm0.02$ cm and radius r = 0.2 ± 0.02 cm and mass m = 0.1 ± 0.001 gm. Calculate maximum percentage error in density*

Sol. $\rho = \dfrac{m}{\pi r^2\ell}$

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{2\Delta r}{r} + \dfrac{\Delta\ell}{\ell}$

$\Delta\ell = 0.02$ cm, $\ell = 8$ cm

$\Delta r = 0.02$ cm, r = 0.02 cm

m = 0.1 gm, $\Delta m = 0.001$ gm

$\dfrac{\Delta\rho}{\rho} = \dfrac{0.001}{0.1} + 2\times\dfrac{0.02}{0.2} + \dfrac{0.001}{0.1}$

$$= \left(\frac{1\times 10}{1000\times 1} + \frac{2\times 2}{100\times 2}\times 10 + \frac{1\times 10}{1000\times 1}\right)\times 100$$

$$= \frac{(1+20+1)}{100}\times 100$$

= 22%

Prob 14. *Planck's formula is given by*

$$u = \frac{\hbar\omega^3}{\pi^2 e^3}\times\frac{1}{e^{\hbar\omega/k^{-1}-1}}$$

where u is the energy radiated per unit area per unit time and h is Planck's constant. What will be the dimensions of k in the expression.

Sol. The power in exponential is always dimensionless. So,

$$\frac{\hbar\omega}{kT} = M^0L^0T^0$$

E = hv

$$\text{so, } h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^0L^0T^{-1}}$$

$= ML^2T^{-1}$

$$\therefore \quad k = \frac{\hbar\omega}{T}$$

$$= \frac{ML^2T^{-1}T^{-1}}{T} = ML^2T^{-3}$$

Prob15. *According to Stoke's law the viscous force acting on a spherical body moving fluid depends on radius r of the body, co–efficient of viscosity η of the fluid and velocity f the body. Find the relation between F, η, r, v.*

Sol. Force acting on a spherical body depends on

$F \propto \eta^a r^b v^C$

$F=k\eta^a r^b v^c$

$(MLT^{-2}) = k\,(ML^{-1}T^{-1})^a\,(L)^b\,(LT^{-1})^c$

$MLT^{-2} = k\,(M)^a\,(L)^{-a+b+c}\,(T^{-a-c})$

a = 1

–a + b + c = 1

– a –c = –2

–1 –c = – 2

c = – 2 = 1

– a + b + c = 1

– 1 + b + 1 = 1

$\Rightarrow$ b = 1

F = k η r v

Objective:

Prob 1. *An experiment measures quantities a, b, c and x is calculated from $x = ab^2/c^3$. If the maximum percentage error in a, b and c are 1%, 3% and 2% respectively, the maximum percentage error in x will be*

(A) 13% *(B) 17%*
(C) 14% *(D) 11%*

Sol. (A) Maximum percentage error in x

As $x = \frac{ab^2}{c^3}$

$\frac{\Delta x}{x} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$

$\frac{\Delta x}{x} = 1\% + 2 \times 3\% + 3 \times 2\%$

$= (1 + 6 + 6)\% = 13\%$

Prob 2. *If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second, then non–zero integers x, y and z, such that $P^xQ^yc^z$ is dimensionless, may be*

(A) x = 1, y =1, z = 1. *(B) x = 1, y =–1, z = 1.*
(C) x = –1, y =1, z = 1. *(D) x = 1, y = 1, z = 1*

Sol. As $P^xQ^{y+}C^z$ is a dimensionless

$\left(\frac{MLT^{-2}}{L^2}\right)^x\left(\frac{ML^2T^{-2}}{L^2T}\right)(LT^{-1})^2 = M^0L^0T^0$

$(M^1L^{-1}T^{-2})^x(ML^{\circ}T^{-3})^y(LT^{-1})^2 = (M^{\circ}L^{\circ}T^{\circ})$

Comparing powers, we get

$x + y = 0$ . . . (i)

$-x + z = 0$ . . . (ii)

$-2x - 5y - z = 0$ . . . (iii)

From (1) and (2),

$y = -x, z = x$

Substituting in (3), we get

If x = k

y = –k, z = k

x = 1, y = –1, z = 1

Prob 3. *The dimensional representation of Planck's constant is identical to that of*

(A) Torque *(B) Power*
(C) Linear momentum *(D) angular momentum*

Sol. (D) As Planck's constant has dimensions of $\frac{E}{\nu}$

$= \dfrac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

and Dimensions of angular momentum = r × p

$= (L \times MLT^{-1})$

$= ML^2T^{-1}$

Prob 4. *The parallel combination of two resistances is given by If the two resistances $R_1 = (2 \pm 0.2)\Omega$ and $R_2 = (1 \pm 0.1)\Omega$ are connected in parallel. Then the % error is given by*

(A) 0.1% *(B) 0.2%*

(C) 0.3% *(D) 0.4%*

Sol. (C) $R_p = \dfrac{R_1R_2}{R_1 + R_2}$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right) \times 100$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{0.2}{2} + \frac{0.1}{1} + \frac{0.2 + 0.1}{3}\right)$$

(0.1 + 0.1 + 0.1)

= 0.3%

Prob5. *If the units of M and L are quadrupled, then the units of torque becomes*

(A) 16 times *(B) 64 times*

(C) 8 times *(D) 4 times*

Sol. (B)

Dimensions of torque = $ML^2\,T^{-2}$

$= (4M)\,(4\,L\,)^2\,T^{-2}$

$= \quad 64\,M\,L^2\,T^{-2}$

Prob6. *A radar signal is beamed towards a planet from earth and its echo is received seven minutes later. If distance between the planet and earth is $6.3 \times 10^{1\,\circ}$m, then velocity of the signal will be*

(A) 3×10^8 m/s *(B) 2.97×10^6 m/s*

(C) 3.10×10^5 m/s *(D) 300 m/s*

Sol. (A).

Velocity of signal, $c = \dfrac{2x}{t} = \dfrac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ m/s

Prob7. *If speed of light c, acceleration due to gravity g and pressure P are taken as fundamental units, then the dimensions of gravitational constant is*

(A) $[c^{\circ}gP^{-3}]$ *(B) $[c^2g^3P^{-2}]$*

(C) $[c^{\circ}g^2P^{-1}]$ *(D) $[c^2g^2P^{-2}]$*

Sol. (C).

Let $G = c^x g^y P^z$

$\Rightarrow [M^{-1}L^3T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [ML^{-1}T^{-2}]^z$

$= [M^z L^{x+y} T^{-x-2y-2z}]$

Comparing powers of M, L and T on both sides, we get

$z = -1,\ x + y = 3,\ -x - 2y - 2z = -2$

On solving these equations for x, y and z, we get

$x = 0,\ y = 2,\ z = -1$

$\Rightarrow G = [c^\circ g^2 P^{-1}]$.

Prob 8. *The time dependence of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and t is time. The constant α*

(A) is dimensionless *(B) has dimensions T^{-2}*

(C) has dimensions of P *(D) has dimensions T^2*

Sol. (B).

$P = P_0 [\exp(-\alpha t^2)]$.

Since αt^2 must be dimensionless, so $\alpha = \frac{1}{T^2} = T^{-2}$

Prob 9. *The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is*

(A) hertz *(B) metre*

(C) radian *(D) second*

Sol. (A).

Here, kt is dimensionless. Hence, $k = 1/t = \sec^{-1}$ = hertz

Prob10. *The parallel of a heavenly body measured from two points diametrically opposite on the equator of earth is 1.0 minute. If the radius of earth is 6400 km, find the distance of the heavenly body from the centre of earth in AU. Take 1 AU = 1.5×10^{11} m.*

(A) 0.293 AU *(B) 0.28 AU*

(C) 2.01 AU *(D) 3.97 AU*

Sol. (A).

Here, $\theta = 1' = \frac{1^\circ}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

ℓ = diameter of earth = 2×6400 km

$= 1.28 \times 10^4$ km $= 1.28 \times 10^7$ m

Now, $\ell = r\theta$

$\Rightarrow r = \frac{1.28 \times 10^7}{(\pi/60) \times 180} = 4.4 \times 10^{10}$ m

$r = \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} = 0.293$ AU

Prob11. *Dimensions of ohm are same as (h is Planck's constant and e is charge)*

(A) $\frac{h}{e}$ (B) $\frac{h^2}{e}$

(C) $\frac{h}{e^2}$ (D) $\frac{h^2}{e^2}$

Sol. (C).

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}] = \text{resistance}$$

Prob12. *Which of the following is a derived unit?*

(A) newton *(B) joule*
(C) pascal *(D) metre*

Sol. A, B, C.
Because, they are derived from the fundamental units, i.e. kg, m and sec.

Prob13. *Which of the following equations is dimensionally correct?*

(A) Pressure = energy per unit volume
(B) Pressure = energy per unit area
(C) Pressure = force per unit volume
(D) Pressure=momentum per unit volume

Sol. $$\frac{\text{Energy}}{\text{Volume}} = \left[\frac{\frac{1}{2}mv^2}{\text{volume}}\right]$$

$$\Rightarrow \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

$\therefore$ (A)

Prob 14. *Which of the following is/are dimensional constants is*

(A) Planck's constant *(B) dielectric constant*
(C) relative density *(D) gravitational constant*

Sol. A Planck's constant and gravitational constant G have constant values and dimensions
$\therefore$ A, D

Prob 15. *Which of the following is not a unit of time*

(A) solar year *(B) tropical year*
(C) leap year *(D) light year*

Sol. Tropical year is the year in which there is total eclipse.
Light year represents distance
$\therefore$ (D)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. If force acting on a particle depends on the x–coordinates as $F = ax + bx^2$, find the dimensions of 'a' and 'b'.

2. If velocity, time and force are chosen as basic quantities, find the dimensions of mass.

3. Find the dimensional formula of
(a) Charge Q
(b) The potential V
(c) The capacitance C,
(d) The Resistance, R

4. Which of the following have same dimensions?
(A) angular momentum and linear momentum (B) work and power
(C) work and torque (D) Torque and Pressure

5. The Van der Waals interaction between two molecules separated by a distance r is given by the energy $E = -\frac{A}{r^6} + \frac{B}{r^{12}}$. Find the dimensions of A and B.

6. If error in measuring diameter of a circle is 4%, find the error in radius of circle.

7. Derive, by method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (ν) and the amplitude of vibration (r)

8. Write the dimensions of $\frac{a}{b}$ in the relation
$F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

9. Assuming that the mass of the largest stone that can be moved by a flowing river depends upon the velocity r, the density ρ and acceleration due to gravity, show that m varies with sixth power of the velocity of flow.

10. The density of a material in cgs system is 8 gcm^{-3}. In a system of units, in which unit of length is 5 cm and unit of mass is 20 g, what is the density of the material ?

11. To study the flow of a liquid through a narrow tube the following formula is used
$\eta = \frac{\pi\rho r^4}{8v\ell}$ where the letters have their usual meanings. The values of ρ, r, v and ℓ are measured to be 76 cm of Hg, 0.28 cm, 1.2 cm^3s^{-1} and 18.2 cm respectively. If these quantities are measured to the accuracy of 0.5 cm of Hg, 0.01 cm, 0.1 cm^3 s^{-1} and 0.1 cm respectively, find the percentage error in the value of η.

12. The equation of a wave is given by $y = A\sin\omega\left(\frac{x}{v} - k\right)$, where ω is angular velocity and v is linear velocity. Find the dimension of k. Given that

13. The surface tension of a liquid is 70 dyne/cm. Express it in MKS system of units?

14. Name a physical quantity which has same unit as that of Torque.

15. If all measurements in an experiment are taken upto same number of significant figures then mention two possible reasons for maximum error.

Level – I

1. The mass of a block is 87.2 g and its volume is 25 cm^3. What is its density upto correct significant figures?

2. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants, λ is wavelength. Then calculate the dimensions of A and B.

3. Suppose, the torque acting on a body, is given by $\tau = KL + \frac{MI}{\omega}$
   Where L = angular momentum, I = moment of inertia & ω = angular speed
   What is the dimensional formula for KM?

4. When a current of (2.5 ± 0.5). A flows through a wire it develops a potential difference of (20 ± 1) V. What is the resistance of wire?

5. Find out the result in proper significant figures, 291 x 0.03842 / 0.0080.

6. The radius of a sphere is (5.3 ± 0.1) cm. Find the percentage error in its volume.

7. If Planck's constant h; the velocity of light, c and Newton's gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of these quantities using dimensional notation.

8. What will be the unit of time in the system in which the unit of length is meter, unit of mass is kg and unit of force is kg. wt.?

9. Imagine a system of units in which the unit of mass is 10 kg, length is 1 km and time is 1 minute, then calculate the value of 1 J in this system.

10. A screw gauge of pitch 0.5 mm has a circular scale divided into 5 divisions. The screw gauge is used to measure the thickness of a coin. The main scale reading is 2 mm and 35^{th} circular division coincides with main scale with a positive zero error of divisions. Find the thickness of the coin

11. A Vernier Calliper is used to measure the thickness of the wall of cylinder by measuring its external and internal diameters. For external diameter, the zero if the Vernier scale coincides with the 5^{th} division of main scale and 6^{th} division of Vernier scale coincides with the main scale.
    For internal diameter, the zero of the Vernier scale coincides with the 3^{rd} division of mass scale and 2^{nd} division of Vernier scale coincides with 3^{rd} division of main scale and 2^{nd} division of Vernier scale coincides with main scale. Given that 1 main scale division is equal to 10 m
    1 V.S.D. = 0.09 cm.
    Calculate the thickness of the wall of a cylinder.

12. The time period of small oscillations of a spring mass system is given as $T = 2\pi\sqrt{\frac{m}{k}}$. What will be the accuracy in the determination of k if mass m is given as 10 kg with accuracy of 10 gm and time period is 0.5 sec measured for time of 100 oscillations with a watch of accuracy of 1 sec.

13. In a screw micrometer, main scale divisions are in mm. There are 100 cap divisions.
(a) Find out the least count of the micrometer.
(b) In fully closed condition, 4^{th} division of the cap scale coincides with the line of graduation along the screw axis. What is the zero error of the instrument ? Is it to be added or subtracted from the observed reading during a measurement ?
(c) In the above instrument, during a measurement, the cap is between 7^{th} and 8^{th} divisions of the main scale and 37^{th} division of cap scale coincides with the line of graduation of the main scale. What is the measurement corrected for zero error ?

14. The equation for energy (E) of a simple harmonic oscillator,
$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2x^2$$
is to be made "dimensionless" it with multiplying by a suitable factor, which may involve the constants, m(mass), ω(angular frequency) and h (Planck's constant). What will be the unit of momentum and Length ?

15. In in the equation $F = A\sin Bx^2 + \frac{C}{t}e^{Dt}$, F, x and t are force, position and time respectively, then give the dimensions of $\frac{A}{CB}$.

Objective:

1. Which of the following is a possible dimensionless quantity?
(A) Velocity gradient
(B) Pressure gradient
(C) Displacement gradient
(D) Force gradient

2. Dimensional formula of thermal conductivity is
(A) $ML^2T^{-3}\theta^{-1}$
(B) $ML^2T^{-2}\theta^{-4}$
(C) $ML^2T^{-2}\theta^{-1}$
(D) $MLT^{-3}\theta^{-1}$

3. The unit of power is
(A) kilowatt hour
(B) joule
(C) dyne
(D) kilo watt

4. The dimensional representation of Planck's constant is identical to that of
(A) torque.
(B) power.
(C) linear momentum.
(D) angular momentum.

5. Which of the following is a fundamental quantity?
(A) volume
(B) velocity
(C) time
(D) force

6. The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is
(A) hertz
(B) metre
(C) radian
(D) second

7. The dimensional representation of Planck's constant is identical to that of
(A) torque
(B) work
(C) stress
(D) angular momentum

8. A force F is given by $F = \frac{a}{t} + bt^2$, where t is time. The dimensions of a and b are
(A) $[MLT^{-3}]$ and $[MLT^{-4}]$
(B) $[MLT^{-4}]$ and $[MLT^{-3}]$
(C) $[MLT^{-1}]$ and $[MLT^{-4}]$
(D) $[MLT^{-2}]$ and $[MLT^{\circ}]$

9. A unit–less quantity
(A) may have non–zero dimensions
(B) always has non–zero dimensions
(C) never has a non–zero dimensions
(D) does not exist

10. Joule $\times$ sec is the unit of
(A) energy
(B) momentum
(C) angular momentum
(D) power

11. Given that v is speed, r is radius and g is gravitational acceleration, which of the following expression is dimensionless.
(A) $\frac{v^2}{gr}$
(B) $\frac{v^2 r}{g}$
(C) $\frac{v^2 g}{r}$
(D) $v^2 rg$

12. The dimensional formula for modulus of rigidity is
(A) $[ML^2T^{-2}]$ (B) $[ML^{-1}T^{-3}]$
(C) $[ML^{-2}T^{-2}]$ (D) $ML^{-1}T^{-2}]$

13. A highly rigid cubicle block A of small mass m and side L is rigidly fixed to an other similar cubical block of low modulus of rigidity η. Lower face of A completely covers the upper face of B. The lower face of B is rigidly held on horizontal surface. A small force T is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes simple harmonic motion, the time period of which is given by
(A) $2\pi\sqrt{m\eta L}$ (B) $2\pi\sqrt{m\eta / L}$
(C) $2\pi\sqrt{mL / \eta}$ (D) $2\pi\sqrt{m / \eta L}$

14. The time period of a soap bubble is $T \propto P^a d^b S^c$, where P is pressure, d is density and S is surface tension, then values of a, b and c, respectively, are
(A) –1, –2, 3 (B) –3/2, 1/2 1
(C) 1, –2, –3/2 (D) 1, 2, 1/3

15. The dimensional formula for specific resistance in term of M, L, T and Q is
(A) $[ML^3T^{-1}Q^{-2}]$ (B) $[ML^2T^{-2}Q^2]$
(C) $[MLT^{-2}Q^{-1}]$ (D) $[ML^2T^{-2}Q^{-2}]$

16. Which of the two have same dimensions?
(A) Force and strain (B) Force and stress
(C) Angular velocity and frequency (D) Energy and strain

17. The velocity of water waves depend on their wavelength λ, the density of water ρ and acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 \propto g^{-1}\lambda^{-1}$ y (B) $v^2 \propto g\lambda$ y
(C) $v^2 \propto g\lambda\rho$ y (D) $v^2 \propto g^{-1}\lambda^{-3}$ y

18. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combination which have the dimensions of angle
(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$
(C) $\frac{C}{L}$ (D) $\frac{R^2C}{L}$

19. The vernier of a circular scale is divided into 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count by the instrument is
(A) 0.1′ (B) 1′
(C) 10′ (D) 30′

20. Dimensional analysis of the equation $(\text{velocity})^x = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$ gives the value of x as
(A) 1 (B) 2
(C) 3 (D) 4

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. $[a] = M^1L^0T^{-2}, [b] = M^1L^{-1}T^{-2}$
2. FTV^{-1}
3. (a) $[Q] = IT$ (b) $[V] = ML^2I^{-1}T^{-3}$ (c) $[C] = M^{-1}L^{-2}I^2T^4$ (d) $[R] = ML^2T^{-3}I^{-2}$
4. Work and torque
5. $[A] = ML^8T^{-2}$, $[B] = ML^{14}T^{-2}$
6. 4 %
7. $E = k\, mv^2\, r^2$
8. $M^\circ L^{-1/2}\, T^2$
10. 50 units
11. 23%
12 $k = M^\circ L^\circ T$
13. $7x10^{-2}$ N/m
14. Work
15. The maximum error will be due to (i) measurement, which is least accurate.
(ii) measurement of the quantity which has maximum power in formula's.

Level – I

1. 3.5 g/cc
2. $M^\circ L^\circ T^\circ$, $M^\circ L^2 T^\circ$
3. T^{-4}
4. $(8 \pm 2)\,\Omega$
5. 1400
6. 5.7%
7. $(hc)^{1/2}\, G^{-1/2}$, $(hG)^{1/2}c^{-3/2}$, $(hG)^{1/2}\, c^{-5/2}$
8. $\frac{1}{\sqrt{9.8}}$ sec
9. 360
10. 2.25 mm
11. 1.02 cm
12. $\pm$ 5%
13. (a) 0.01mm (b) + 0.04 mm, to be subtracted (c) 7.33 mm
14. $\frac{E}{\hbar\omega} = \frac{1}{2}\frac{mv^2}{\hbar\omega} + \frac{1}{2}\frac{\omega m x^2}{\hbar}$, $\sqrt{m\omega\hbar}$, $\sqrt{\frac{\hbar}{m\omega}}$
15. L^2T^{-1}

Objective:

1.	C	2.	D	3.	D	4.	D
5.	C	6	A	7.	D	8.	C
9.	C	10.	C	11	A	12.	D
13.	D	14.	B	15.	A	16.	C
17.	B	18.	D	19.	B	20.	C

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Regards from Learnaf team

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PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

UNITS AND DIMENSION

Units and Dimension

Syllabus of IITJEE and Maharashtra Board:

Scope and excitement of physics, Technology & society. Forces in nature, Conservation laws, Examples of gravitation, electromagnetic and nuclear forces from daily life experiences (qualitative description only). Need of measurement, Units of measurement, System of units, SI units, Fundamental and derived unit; Length, mass and time measurement, Accuracy and precision of measuring instrument, errors in measurement, significant figures. Dimensions of physical quantities, dimensional analysis and its application.

Have you ever observed the nature and the various spectacular events like formation of rainbow on any rainy day?

Whenever we observe nature keenly, we can easily understand that the various events in nature like blowing of wind, flow of water, motion of planets, formation of rainbow, different forms of energies, the function of human bodies, animals, etc. are happening or taking place according to some basic laws. The systematic study of these laws of nature governing the observed events is called science. For our convenience, clear understanding and systematic study of Science is classified into various branches. Among these branches Chemistry, Mathematics, Botany, Zoology, etc. are ancient branches and Bio–technology, Bio–chemistry, Bio–Physics, Computer science, Space Science, etc. are considered to be modern branches of science and engineering. One of such ancient and reputed branches of this science is physics.

SCOPE AND EXCITEMENT OF PHYSICS

The domain of physics consists of wide variety and large number of natural phenomena. Hence, the scope of physics is very vast and obviously the excitement that one gets from the careful study of physics has got no boundaries.

Scope of Physics

For example, when we study one of the basic physical quantities called mass, we come across the values ranging from minute masses like mass of an electron (of the order of 10^{-30} kg) to heavy masses like mass of universe (10^{55} kg). Similarly, in case of other basic quantities like length and time also the range is very wide.

Hence, the scope of physics can be understood easily, only when we can classify the study of physics chiefly into three levels. They are:

(a) Macroscopic level study of physics,
(b) Mesoscopic level study of physics, and
(c) Microscopic level study of physics.

Macroscopic level study of physics: Macroscopic level study of physics mainly includes the study of basic laws of nature and several natural phenomena like gravitational force of attraction between any two bodies in the universe (in mechanics), variation of quantities like pressure, volume, temperature, etc. of gases on their thermal expansion or contraction (in thermodynamics), etc.

Microscopic level study of physics: The microscopic level study of physics deals with constitution and structure of matter at the level of atoms or nuclei. For example, interaction between elementary particles like electrons, protons and other particles , etc.

Mesoscopic level study of physics: The mesoscopic level study of physics deals with the intermediate domain of macroscopic and microscopic, where we study various physical phenomena of atoms in bulk.

So, the edifice of physics is beautiful and one can appreciate the subject as and when one pursues the same seriously.

Excitement of Physics

The study of physics is exciting in many ways as it explains us the reason behind several interesting features like (a) ***how day and nights are formed?*** (b) ***how different climatic conditions are formed in different seasons?*** (c) ***how satellite works and helps in using several devices like television, telephones, etc.?*** (d) ***how an astronaut travels to celestial space ?*** (e) ***how we can convert one form of energy to another ?*** (f) ***how different types of forces are governing different types of motion in universe ? etc***.

It is quite common and simple that every human being on the earth will be interested to know the answers for at least few of the above questions. As physics is the subject which answers them, naturally the study of physics will be exciting.

TECHNOLOGY AND SOCIETY

Physics is almost an integral part of upgradation of technology. Technology was also a branch of science where we study the application of principles of physics for practical purposes. Based on laws and principles of physics, technocrats along with scientists develop technically advanced equipment to help the society.

For example, from the principles of thermodynamics James watt invented steam engine which was responsible for a big industrial revolution in England in the 18^{th} century. Another recent example is invention of mobile phones which are creating revolution in wireless communication technology. Yet another important example is invention of micro–processors by using silicon chips which has replaced valve technology and brought the computers from the size of your study room to the size of your geometry box. These are few examples. There are many more areas where physics is involved in upgrading technology and thereby helping the society. The following table gives us a list of various branches of physics that helped the field of technology.

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Nuclear fission
Radio and Television	Propagation of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation (population inversion)

Production of ultra–high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electric energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Air conditioners / Refrigerators	Laws of thermodynamics
Washing machines, centrifuge, etc.	Centrifugal force
Sonar	Reflection of ultrasonic waves

The following table lists the involvement of various renowned physicists all across the world, who helped the society with their noble inventions.

Name	**Major Contribution / Discovery**	**Country of origin**
Isaac Newton	Universal law of gravitation: Laws of motion; reflecting telescope.	**U. K.**
Galileo Galilei	Law of inertia	**Italy**
Archimedes	Principle of buoyancy; principle of the lever	**Greece**
James Clerk Maxwell	Electromagnetic theory; light an electromagnetic wave	**U. K.**
W. K. Roentgen	x– rays	**Germany**
Marie Sklodowska Curie	Discovery of radium and polonium; Studies on natural radioactivity	**Poland**
Albert Einstein	Law of photo–electricity; Theory of relativity	**Germany**
S. N. Bose	Quantum statistics	**India**
James Chadwick	Neutron	**U.K.**
Niels Bohr	Quantum model of hydrogen atom	**Denmark**
Ernest Rutherford	Nuclear model of atom	**New Zealand**
C.V. Raman	Inelastic scattering of light by molecules	**India**
Christiaan Huygens	Wave theory of light	**Holland**
Michael Faraday	Laws of electromagnetic induction	**U.K.**
Edwin Hubble	Expanding universe	**U.S.A.**
Homi Jehangir Bhabha	Cascade process in cosmic radiation	**India**
Abdus Salam	Unification of weak and electromagnetic interactions	**Pakistan**
R. A. Millikan	Measurement of electronic charge	**U.S.A**
Ernest Orlando Lawrence	Cyclotron	**U.S.A.**
Wolfgang Pauli	Quantum Exclusion Principle	**Austria**
Louis victor de Broglie	Wave nature of matter	**France**
J.J. Thomson	Electron	**U.K.**
S. Chandrasekhar	Chandrasekhar limit, structure and	**India**

	evolution of stars	
Lev Devidovich Landau	Theory of condensed matter; liquid helium	**Russia**
Heinrich Rudolf Hertz	Electromagnetic waves	**Germany**
Victor Francis Hess	Cosmic radiation	**Austria**
M. N. Saha	Thermal ionisation	**India**
G. N. Ramachandran	Triple helical structure of proteins	**India**
Thomas Alwa Edison	Electric bulb, Projector	**US**
Graham Bell	Telephone	**US**
Cavendish	Determination of 'G'	**England**
Robert Boyle	Boyle's law	**England**

So, to put it in a nut shell, science, technology and society are inseparable as they are deeply interwined.

FUNDAMENTAL FORCES IN NATURE

Force is a very common word which we normally come across in our daily life. We need force to push or pull or throw a body. Even we need it to deform or break the bodies. Sometimes, we experience force like when we are standing in a great storm, we experience the force exerted by wind. When we are sitting in a bus which is negotiating a turn, we experience an outward push. So, what is this force? Let us try to understand the concept of force in terms of physics.

At macroscopic level study of physics, we normally encounter different kinds of forces like gravitational force, muscular force, frictional force, contact force, spring force, buoyant force, viscous force, pressure force, force due to surface tension, electrostatic force, magnetic force, etc. whereas at microscopic level of study we come across nuclear forces, interatomic forces, intermolecular forces, weak forces, etc.

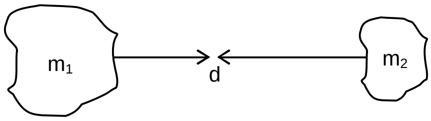
After analysing these various types of forces in nature, it was concluded that all the forces can be comfortably classified into four categories, which are known as fundamental forces in nature. They are

(1) Gravitational force (2) Electromagnetic force,
(3) Nuclear force, and (4) Weak force.

That means, any force other than the above four forces can be derived from these four basic forces. For example, elastic force or spring force arises due to the net attraction or repulsion between any two neighboring atoms of the spring. When it is elongated or compressed, attractive or repulsive forces produced between the atoms can be treated as the resultant of all electromagnetic forces between charged particles of an atom. Hence, this spring force is known as derived force and electromagnetic force which is the origin of this spring force is called fundamental force. Now, we will study about fundamental forces in brief.

Gravitational Force

Newton discovered that any two bodies in universe attract each other. This force of attraction exists by virtue of their masses, and is known as gravitational force of attraction. He found that the gravitational force is directly proportional to their masses and is inversely proportional to the square of the distance between them.



i.e. $F = G\dfrac{m_1m_2}{d^2}$ where 'G' is a Universal Gravitational Constant. This force is a universal force and is independent of any type of intervening medium between the two bodies. Though this is the weakest force in nature when compared to other types of fundamental forces, it plays vital role in governing the motion of planets around sun, natural satellites (like moon around earth), artificial satellites, etc.

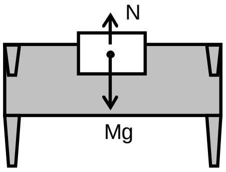
Electromagnetic Force:

The force of attraction or repulsion between any two charged particles is known as electrostatic force. If q_1 and q_2 charges are separated by a distance 'd' in air then the force of attraction or repulsion between them is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{d^2}$. This is called Coulomb's law of electric forces.

Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. In general electric and magnetic effects are inseparable and hence the name – electromagnetic force. This electromagnetic force between moving charged particles is comparatively more complicated and contains several other terms other than Coulomb's force.

In atoms electromagnetic force between electrons and protons is responsible for several molecular and atomic phenomena. Apart from this it also plays vital role in the dynamics of chemical reactions, mechanical and thermal properties of materials, tension in ropes, friction, normal force, spring force, Vander Waals force .

Example: Let us consider a block which is placed on a horizontal surface of a table as shown in the figure. The table balances the weight (Mg) and exerts a force which comes from electromagnetic force between charged constituents of atoms or molecules of surface of block and that of the table. Thus a force called normal force acts on block.



This electromagnetic force is a strong force when compared to the gravitational force. The electromagnetic force between two protons is 10^{36} times the gravitational force between them for any fixed distance.

Nuclear Force

We know that, in general, nucleus of every atom consists of two elementary particles called protons and neutrons. As neutrons are uncharged and protons are charged, the electric force of repulsion between protons will cause nucleus to break into fragments. But this is not happening, and also we know that nucleus of a non–radioactive element is a stable one.

That means there must be some other attractive force which is dominating coulombic force of repulsion between protons and keeping all the particles in nucleus together in stable condition as gravitational force can't dominate electric force. That new force existing between any two nucleons and which keeps all the particles in nucleus bound together is known as nuclear force. This force is stronger than electromagnetic force and is a charge independent force. Range of these forces is very small and will be of the order of nuclear size (10^{-21} th portion of size of an atom).

Latest developments in physics revealed that this strong nuclear force is also not a fundamental force as protons and neutrons consist of still elementary particles called quarks. And according to this latest development quark – quark force is fundamental force of nature and nuclear force is a derived force. However the study of quark – quark force is out of the scope of this book and our curriculum.

Weak Nuclear Force

This force appears only in certain nuclear processes. A neutron can change itself into a proton by emitting an electron and another elementary particle called antineutrino simultaneously. This process is called β^- decay. Similarly a proton can also change into neutron by emitting positron and a neutrino. This process is called β^+ decay. The forces which are responsible for these changes are known as weak forces. These forces are weak in nature when compared to nuclear and electromagnetic forces but stronger than gravitational forces. The range of these weak nuclear forces is exceedingly small, of the order of 10^{-15}m.

The following table gives us an overall idea about relative strengths and ranges of four fundamental forces.

Name	**Relative strength**	**Range**	**Operates among**
Gravitational force	10^{-38}	Infinite	**All objects in the universe**
Weak nuclear force	10^{-13}	Very short, within nuclear size ($\sim10^{-15}$)	**Elementary particles**
Electromagnetic force	10^{-2}	Infinite	**Charged particles**
Strong nuclear force	1	Very short, within nuclear size ($\sim10^{-15}$)	**Nucleons**

CONSERVATION LAWS

In any physical phenomena, few physical quantities associated with the phenomena may change with time and few physical quantities associated with it may not change. Those physical quantities which remain constant in time are known as conserved quantities.

For example, if a big liquid drop is sprayed into several small droplets the volume of liquid before spraying and after spraying remains same. Hence, we can say that a physical quantity called volume is conserved in this example. Similarly, we have several quantities which are conserved. Within the scope of our course, we can discuss the following conservation laws.

1. Law of conservation of linear momentum
2. Law of conservation of energy
3. Law of conservation of angular momentum
4. Law of conservation of charge.

Let us discuss them in brief.

Law of conservation of linear momentum

The linear momentum of a body is defined as the ability of a body by virtue of which it imparts its motion to other objects along a straight line. And mathematically it is equal to the product of mass of the body (m) and its velocity ($\vec{v}$) Mathematically, $\vec{P} = m.\vec{v}$.

According to this law, in absence of an external force, the total vector sum of linear momentum remains unchanged.

Example: When a bullet is fired with a gun, the total momentum vector of the system of bullet and gun is zero. After firing, bullet moves in forward direction with some momentum and gun recoils with the same amount of momentum in magnitude, but opposite in direction. Hence total vector sum of momentum after firing is also zero. Thus linear momentum of the system before and after firing is zero. Hence we can say that linear momentum is conserved.

Law of conservation of energy

According to this law the total energy of an isolated system is always constant and it never changes. But it can be transformed from one form to another. *For example* an electric cell in our daily life gives electrical energy by transforming chemical energy in it, electric motor converts electrical energy to mechanical energy, etc. However the total energy in these processes is conserved.

When an object is dropped from a certain height the total mechanical energy of the body is conserved. At its highest point all its mechanical energy will be in the form of potential energy and at its lowest point it will be in the form of kinetic energy, i.e. energy has transformed from one form into another, (i.e. potential to kinetic) but the total energy remains constant. Hence the total mechanical energy is conserved.

But this conservation of mechanical energy can't be applied in the presence of non – conservative force. For example in the above case if you consider air resistance on the freely falling body total mechanical energy does not remain constant. Here work done by air resistance gets converted into different forms of energy like heat energy. So such while applying nergy conservation principle heat energy should also be taken into consideration in such cases

Law of conservation of angular momentum

Angular momentum ($\vec{L}$) of a body about a point is defined as the cross product of its position vector about that point ($\vec{r}$) and its linear momentum at that instant ($\vec{p}$)

i.e. $\vec{L} = \vec{r} \times \vec{p}$
or $L = rp \sin\theta$ where θ is the angle between ' $\vec{r}$ ' and ' $\vec{p}$ '.

According to this law the total angular momentum of the system remains conserved in absence of external torque.

Example: We know that planets revolves around sun in elliptical orbits. The angular momentum of a planet at any point during its motion in its path is conserved. We will study more clearly about this under rotatory motion concepts.
These are the few conservation laws in mechanics. Now let us discuss a conservation law in electrostatics.

Law of conservation of charge

This law states that the total electric charge of an isolated system is always conserved. Charge can neither be created nor destroyed, but it can be transferred or exchanged from one body to another.

Apart from these, there are several other physical quantities that are conserved in nature. During our further discussions in various chapters we will understand them.

MEASUREMENT AND UNITS

Physical quantity: Any meaningful term which can be measured is a physical quantity. For example length, velocity, time etc. are physical quantity. But handsomeness, beauty are not physical quantity.

Why measurement is needed?: Physics is an experimental science and experiments involve measurement of different physical quantities in which laws of physics are expressed. Without measuring results of experiments, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Units of measurement: To measure a physical quantity we need some standard unit of that quantity. For example, if a measurement of length is quoted as 5 meters, it means that the measured length is 5 times as long as the value accepted for a standard length defined to be **"one meter".**

Any set of standards of units must fulfill the following two conditions
(i) It must be accessible.
(ii) It must be invariable with the passage of time

Two more auxiliary conditions are:–
(i) It is necessary to have wide unlimited agreement about those standards.
(ii) It is inter convertible to different units of same quantity.

A measurement consists of two parts, one is numeric and the other is standard chosen. For example, 5 meter of length implies 5 times the **"standard meter"**. It is not necessary to establish a measurement standard for every physical quantity. Some quantities can be regarded as fundamental and the standard for other quantities can be derived from the fundamental ones. For example, in mechanics length, mass and time are regarded as fundamental quantities and the standard for speed (= length / time) can be derived from fundamental quantities length and time.

Quantity	SI Units	Symbols
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of Substance	mole	mol
Thermodynamic Temp.	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	Cd

And two supplementary units are

Plane Angle	Radian	rad

Solid Angle	Steradian	sr

Two other system of units compete with the international system. One is Gaussian System in terms of which much of the literature of physics is expressed. In India this system is not in use.
The other is the British system. This system is still in daily use in United states. But SI units are standard units worldwide.

C.G.S. Unit: In this system of unit, centimeter, gram and seconds are units of length, mass and time respectively.

Conversion of One System of Units to another System: The basic formula is $n_1u_1 = n_2u_2$ where n_1 and n_2 are numbers.

Illustration 1. *How many dyne–centimeter are equal to 1 N–m?*

Solution:

$$1\text{N} - \text{m} = (1\text{kg})(1\text{m})^2(1\text{s})^{-2}$$
$$1\text{dyne} - \text{centimeter} = (1\text{g})(1\text{cm})^2(1\text{ s})^{-2}$$
$$\therefore \frac{1\text{N}-\text{m}}{1\text{dyne}-\text{cm}} = \left(\frac{1000\text{g}}{1\text{g}}\right)\left(\frac{100\text{cm}}{1\text{cm}}\right)^2$$
$$= 1000 \times 10000$$
$$\therefore 1\text{N} - \text{m} = 10^7 \text{ dyne} - \text{cm}$$

Exercise: ***Calculate the value of 1 erg in SI system.***

Measurement of Length

Depending upon the range of length, there are three main methods for measuring length.

(i) Direct method using measuring instruments.
(ii) Indirect method or Mathematical method
(iii) Chemical method

(i) Direct method

The simplest method measuring the length of a straight line is by means of a meter scale. But there exist some limitations in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier calliper (b) micrometer scale (screw gauge) are used .

Vernier calliper

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being slightly shorter than the divisions of the main scale.

Least count of Vernier Calliper

The least count or the Vernier constant (V.B.) is the minimum value of correct estimation of length without eye estimation. The difference between the values of one main scale division and one vernier scale division is known as vernier constant if N division of vernier scale coincides with (N–1) divisions of main scale, then vernier constant,

$$\text{n.V.S.D.} = (\text{n}-1)\ \text{M.S.D.}$$

$$\text{1.V.S.D.} = \left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D., and}$$

$$\text{1.M.S.D.} - \text{1.V.S.D.} = \text{1.M.S.D.}\ \left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D.}$$

$$= \frac{1}{\text{n}}\ \text{M.S.D.}$$

$$= \frac{\text{Smallest Division on main scale}}{\text{No. of divisions on vernier scale}}$$

Reading a Vernier scale

Let one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions

$$\therefore\ \text{1.V.S.D.} = \frac{9}{10}\ \text{M.S.D.} = 0.9\ \text{mm}$$

$$\therefore\ \text{Vernier constant} = \text{1.M.S.D} - \text{1.V.S.D.} = 1\ \text{mm} - 0.9\ \text{mm}$$

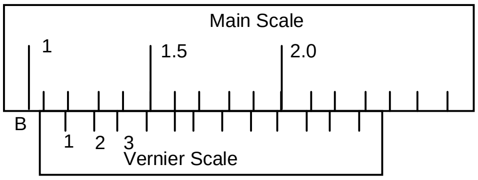
$$= 0.1\ \text{mm} = 0.01\ \text{cm}$$

The reading with vernier scale is read as given below :

(i) Firstly take the main scale reading (N) before on the left of the zero of the vernier scale.
(ii) Find the number (n) of vernier division which just coincides with any of the main scale division. Multiply this number (n) with vernier constant (V.C.)
(iii) Total reading = (N + n × V.C.)

Caution: The main scale reading with which the Vernier scale division coincides has no connection with reading
Suppose If we have to measure a length AB, the end A is coincided with the zero of the vernier scale as shown in fig. Its enlarged view is given in fig.

Length AB > 1.0 cm < 1.1. cm



Let 5th division of vernier scale coincide with 1.6 cm of main scale. From diagram it is clear that the distance between 4th division of vernier scale and 1.5 cm of main scale is equal to one V.C. and distance between zero mark of vernier scale and 1.0 cm mark on the main scale is equal to 5 times the vernier constant.

$$\therefore\quad AB = 1.0 + 5 \times \text{v.c.} = 1.0 + 5 \times 0.01 = 1.05\ \text{cm.}$$

Illustration 2. *In travelling microscope the vernier scale used has the following data.*
1 M.S.D. = 0.5 mm, 50 V.S.D. = 49 M.S.D.
and the actual reading for distance travelled by travelling microscope is 2.4 cm with 8th division coinciding with a main scale graduation . Estimate the distance travelled.

Solution : In this case vernier constant = 1.M.S.D. – 1.V.S.D.

$$= \text{1.M.S.D.} - \frac{49}{50}\text{M.S.D.} = \frac{1}{50}\text{M.S.D} = \frac{1}{50} \times 0.5\,\text{mm}$$

$= \frac{5}{10} \times \frac{1}{50} = 0.01\text{mm} = 0.001\text{cm}$

$\therefore$ Distance travelled = 2.4 + 8 × 0.001 cm
= 2.408 cm

Illustration 3. *The Vernier scale used in Fortin's barometer has 20 divisions coinciding with the 19 main scale divisions. If the height of the mercury level measured is 5 mm and 15th division of vernier scale is coinciding with the main scale division. Then calculate the exact height of the mercury level (given that 1.M.S.D. = 1mm)*

Solution: 20 V.S.D. = 19 M.S.D. (Given)

1.V.S.D. = $\frac{19}{20}$ M.S.D.

V.C. = 1. M.S.D. – 1.V.S.D $= \left(1 - \frac{19}{20}\right)$M.S.D.

$= \frac{1}{20}$ M.S.D.

$= \frac{1}{20} \times 1\text{mm} = 0.05\,\text{mm}$

= 0.005 cm

Height of mercury level = 5 + 0.05 × 15
= 5.75 mm

Exercise: ***The Vernier calliper is used to measure the length of an object. The least count of such a vernier calliper is 0.2 cm and scale reads its length to be 5.6 cm. 3rd division of Vernier scale is coinciding main scale division Calculate the length of an object.***

Zero Error

If the zero marking of main scale and Vernier scale do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument. If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

Illustration 4. *Consider the following data:*
10 main scale divisions = 1cm, 10 vernier division = 9 main scale divisions,
zero of Vernier scale is to the right of the zero marking of the main scale with 6th Vernier division coinciding with a main scale division and the actual reading for length measurement is 4.3 cm with 2nd Vernier divisions coinciding with a main scale graduation. Estimate the length.

Solution: In this case, vernier constant = $\frac{1\text{mm}}{10} = 0.1\text{mm}$

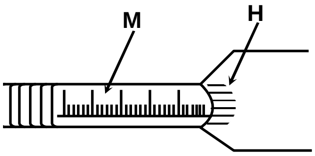
Zero error = 6 × 0.1 = + 0.6 mm

Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction
= 4.32 – 0.06 = 4.26 cm

Screw Gauge (or Micrometer Screw)

In general Vernier Callipers can measure accurately upto 0.02 mm and for greater accuracy micrometer screw devices, e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by turning it axially. The instrument is provided with two scales:



(i) The main scale or pitch scale M graduated along the axis of the screw.
(ii) The cap–scale or head scale H round the edge of the screw head.

Constants of the screw gauge:

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus if 10 rotations of cap ≡5 mm, then pitch = 0.5 mm

In general, pitch $= \dfrac{\text{Distance travelled by screw on main scale}}{\text{No. of rotation taken by the cap to travel that much distance}}$

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration, if the total cap division is 100, then least count = 0.5 mm/100

= 0.005 mm
In general, In case of circular scale,

Least count $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

If pitch is 1 mm and there are 100 divisions on circular scale, then

Least count = $= \dfrac{1\text{mm}}{100}$ = 0.01 mm = 0.001 cm

= 0.00001 m = 10^{-5} m = 10 μm.

Since least count is of the order of 10 μm, So the screw is called a micrometer screw. Screw gauge and the spherometer which work on the principle of micrometer screw, consist essentially of the following two scales.

(i) Linear or Pitch scale: It is a scale running parallel to the axis of the screw.

(ii) Circular of Head scale: It is marked on the circumference of the circular disc or the cap attached to the screw.

Zero Error: In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero–error, otherwise the instrument is said to heave zero–error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and vice–versa. The corresponding corrections will be just opposite.

Illustration 5. *A screw gauge has 100 divisions on its circular scale. Circular scale travels one division on linear scale in one rotation and 10 divisions on linear scale of screw gauge is equal to 5 mm. What is the least count of a screw gauge.*

Solution: Pitch $= \dfrac{1\,\text{division on linear scale}}{1\,\text{rotation}} = 1\,\text{div.}$

10 division = 5 mm
∴ 1 division = 0.5 mm
∴ pitch = 0.5 mm

$\text{least count} = \dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$=\frac{0.5\,\text{mm}}{100}=0.005\,\text{mm}$$

Illustration 6. *The screw gauge mentioned in above illustration is used to measure thickness of a coin. The reading of the linear scale is 4th div and 25th division of circular scale is coinciding with it. What is the value of thickness of the coin..*

Solution: Reading = Linear scale Reading + Least count × circular scale reading
= 4^{th} division on linear scale + 0.005 mm × 2.5
= 4 × 0.5 mm + 0.125 mm
= 2 mm + 0.125 mm
= 2.125 mm

Illustration 7. *A spherometer has 250 equal divisions marked along the periphery of its disc and one full rotation of the disc advances it on the main scale by 0.0625 cm. The least count of the spherometer is*
(A) 2.5×10^{-2}cm (B) 25×10^{-3}cm
(C) 2.5×10^{-4} cm (D) none of the above

Solution: Least count = $\frac{0.0625}{250}$ cm = 2.5×10^{-4}cm
$\therefore$(C)

(ii) Indirect or Mathematical method

This method involves measurement of long distances. Main methods of this category are –

Reflection method: Suppose we want to measure the distance of a multi story building from a destination point P. If a shot be fired from P, the sound of shot travels a distance x towards the building, gets reflected from the building. The reflected sound travels the distance x to the point of P, when an echo of the shot is heard.

Let t = time interval between the firing of the shot and echo sound.
v = velocity of sound in air.

Distance = velocity x time

x + x = (v) (t)

⇒ x = (v) (t/2)

As v is known, x can be calculated by measuring the time t.

Illustration 8. *A rock is at the bottom of a very deep river. An ultrasonic signal is sent towards rock and received back after reflection from rock in 4 seconds. If the velocity of ultrasonic wave in water is 1.45 km/s, find the depth of river.*

Solution: Here x = ?

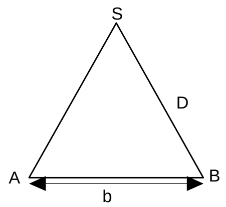
v = 1.45 km/s = 1450 m/sec.

t = 4 sec

so, x = v x t / 2 = 1450 x 4 / 2 = 2900 m.

Parallel method: This method is used for measuring distance of nearby stars.

Let we have to measure the distance D of a far away star S by this method. We observe this star from two different position A and B on the earth, separated by a distance AB = b at the same time as shown in figure. Let $\angle ASB = \theta$, the angle θ is called parallatic angle. As the star is very far away, b/D << 1 and θ is very small.



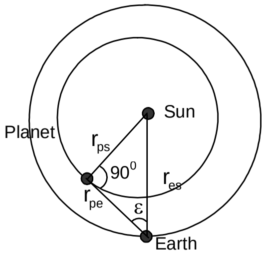
Here we can take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS=BS so that AB = b = Dθ where θ is in radians.

$D = b/\theta$

Knowing b and measuring θ, we can calculate D.

Copernicus method: This method is used to measure the relative distances of the planets from the Sun.

(a) For Interior Planets: The angle formed at earth between the earth–planet direction and the earth–sun direction is called the planet's elongation. This is the angular distance of the planet from the sun as observed from earth. When the elongation attains its maximum value ε as in the figure, the planet appears farthest from Sun.



$$r_{ps} = r_{es} \sin \varepsilon$$

$$= (\sin \varepsilon)\, AU \quad (AU = \text{Astronomical Unit})$$

(b) For Exterior Planets: This method is a consequence of Kepler's 3rd law of planetary motion. For two planets P_1 and P_2 we have,

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

where a_1 and a_2 are semi–major axes, of respective orbits. Period can be ascertained by direct observation. Therefore if a_1 is measured, a_2 can be calculated.

(iii) Chemical Method

This method is used to measure distance of the order of $10^{-1\circ}$ m. Let us calculate the size of an atom.

Let m = mass of substance,

V = volume occupied by substance &

ρ = density of the substance

$\therefore v = m/\rho \qquad (1)$

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ No. of atoms in mass m of the substance = Nm / M

If r = radius of each atom then V = volume of each atom = $\frac{4}{3}\pi r^3$

Volume of all the atoms in substance = $(\frac{4}{3}\pi r^3 \text{ x } Nm)M$.

According to Avagordo's hypothesis,

Volume of all the atoms = (2/3) x volume of substance

$$\frac{4}{3}\pi r^3 \text{ x } Nm/M = (2/3)\, m/\rho$$

$$\therefore r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$

MEASUREMENT OF MASS

Measurement of Inertial Mass

Inertial mass of a body is measured using a device which is known as inertial balance. It consists of a long metal strip. One end of the strip is clamped to a table such that its flat face is vertical, and it can easily vibrate horizontally. The other end of strip supports a pan in which the object whose inertial mass is to be found can be kept. It is found that the square of time period of vibration is directly proportional to total mass of the pan and the body placed in it.

$$t^2 \propto m$$

$$\therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \quad m_2 = m_1 \frac{t_2^2}{t_1^2}$$

Measurement of Time: The following methods are used

(a) Quartz Crystal Clock
(b) Atomic Clock
(c) Radioactive dating

Significant figures:

Each measurement involves errors. The measure results has a number that includes all reliably known digits and first unknown digit. The combination of reliable digits and first uncertain digit are significant figures.

Example: If a length is measured as 2.43 cm then 2 and 4 are reliable while 3 is uncertain. Thus the measured value has three significant figures.

Common rules for counting significant figures

(1) All non zero digits are significant.

For example: 1745 has four significant digits.

(2) All zeros present between 2 non zero digits are significant, irrespective of the position of the decimal point.

Example: 208005 has 6 significant figures.

(3) If there is no decimal point, all zeros to the right of the right–most non zero digit are considered to be significant only if they come from a measurement.

Example: 41000 has only 2 significant digits while 41000 m has 5 significant digits.

(4) All zeros to the right of a decimal point but to the left of non–zero digits are considered to be non significant, provided there should be no non zero digit to the left of the decimal point.

Example: 0.00305 has 3 significant figures.

(5) All zeros are significant if they are placed to the right of a decimal point and to the right of a non zero digit.

Example: 0.04080 has 4 significant figures
50.000 has 5 significant figures

(6)The number of significant figures does not alter in different units.
If we want to write 450 m in different units, we can write it 4.50×10^4 cm or 4.50×10^5 mm etc. in which all of them are having 3 significant figures.

Illustration 9. *State the number of significant figures in the following –*
(a) 06500310
(b) 754400
(c) 15000 kg
(d) $8.314 \times 10^{+2}$ J
(e) 1.6×10^{-19} C
(f) 0.0065050

Solution: (a) 7 (b) 4 (c) 5 (d) 4 (e) 2 (f) 5

Exercise: ***The number of significant figures in 0.0160 is***
(a) 2 ***(b) 3*** ***(c) 4*** ***(d) 5***

Rounding off

(1) If all the digits to be discarded are such that the first discarded digit is less than 5, the remaining digits are left unchanged.

Example:

7.499498 can be written in 4 significant figures as 7.499

(2) If the digit to be discarded is 5 followed by digits other than zero, then the preceding digit should be raised by 1.

Example:

7.45001, on being rounded off to first decimal, became 7.5

(3) If the digits to be discarded is 5 or 5 followed by zero the preceding digit remains unchanged if it is even and the preceding digit is raised by 1 if it is odd.

Example:

3.6500 will become 3.6 and 4.7500 will become 4.8 in 2 significant figures.

Arithmetic operations with significant figures:

(1) Addition and subtraction In addition and subtraction, the number of decimal places in the result is the smallest number of decimal places of terms in the operation.

Let us consider the sum of following measurements.
3.45 kg., 7.6 kg. and 10.055 kg.

$$\begin{array}{r} 3.45 \\ 7.6 \\ 10.055 \\ \hline 21.105 \\ \hline \end{array}$$

So the sum will be 21.1 kg as 7.6 kg has only 1 digit after the decimal point while the others are having more than one digit.

Multiplication and Division:
In the result of multiplication or division, the number of significant figures is same as the smallest number of significant figures among the numbers.

Illustration 9: *Multiply 1.21 and 1.1.*

Solution: $1.21 \times 1.1 = 1.331$

So the result is 1.3 as there are only 2 significant digits in 1.1

The same procedure is followed for division.

Exercise: Value of 1.2 + 1.34 + 2.342 is
(a) 4.88 ***(b) 4.8*** ***(c) 4.90*** ***(d) 5***

Accuracy and Precision of measuring instrument: It is impossible to measure any physical quantity perfectly. It is due to imperfection in manufacturing and working of measuring instruments.

Accuracy: It is the degree of correctness of the measured quantity, i.e. how much close the result is to the true value of the physical quantity.

Precision: It is the degree of repeatability & refinement of a measurement.

ERRORS IN MEASUREMENT
In the experiment we may get some other value than that of the true value due to faulty equipment, carelessness or random causes. This will cause error in measurement.

There are 3 ways to express an error
(1) Absolute Error: It is the positive value of difference between the true value and measured value of the quantity. Since we don't know the correct value of quantity the best possible value can be given by mean value of all the measured value.

Arithmetic mean v, $A_m = \dfrac{A_1 + A_2 + \ldots.A_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} A_i$

$\therefore$ The absolute error in the measurement can be given as.

$\Delta A_1 = |A_m - A_1|$ where A_m : Mean value of the measurements.

$\Delta A_2 = |A_m - A_2|$ where A_1, A_2 : Measured value of quantity.

$\Delta A_n = |A_m - A_n|$

Taking the arithmetic mean of all the absolute errors we get the mean absolute error ΔA_m.

$$\Delta A_m = \frac{\Delta A_1 + \Delta A_2 + \ldots\ldots\ldots\ldots\ldots\ldots + \Delta A_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \Delta A_i$$

So the true value of A will be such that

$$(A_m - \Delta A_m) \le A \le (A_m + \Delta A_m)$$

(2) Relative Error: It is defined as the ratio of the mean absolute error to the mean value of the quantity being measured

Relative error $=\dfrac{\Delta A_m}{A_m}$

(3) Percentage Error: The relative error can be expressed in percentage error as

% error = Relative error $\times 100$

Propagation of Error: Any physical quantity depends on one or more than one physical quantities. So the error in any physical quantity will lead to error in the result.

(1) Error in result involving sum or difference of quantities

Let Z is defined as

$$Z = A + B - C$$

$\therefore$ $\Delta Z = \Delta A + \Delta B - \Delta C$

$\therefore$ Maximum possible error in Z is given by

$|\Delta Z|_{max} = \Delta A + \Delta B + \Delta C$ (Since ΔC can be positive or negative)

2. Error in the result having product or division of quantities:

$$Z = \frac{A^p B^q}{C^r}$$

$\Rightarrow$ $\ln z = p\ln A + q\ln B - r\ln c$

$\Rightarrow$ $\dfrac{dz}{z} = \dfrac{pdA}{A} + \dfrac{qdB}{B} + \dfrac{rdC}{C}$

For small change $dz \approx \Delta z.$ $\Rightarrow$ $\dfrac{\Delta z}{z} = P\dfrac{\Delta A}{A} + q\dfrac{\Delta B}{B} + r\dfrac{\Delta C}{C}$

Illustration10. *Multiply 107.88 by 0.610 and express the result upto the correct number of significant figure.*

(A) 65.8068 *(B 65.807*

(C) 65.81 *(D) 65.8*

Solution: Number of significant figures in multiplication is three corresponding to the minimum number 107.88×0.610

= 65.8068 = 65.8

$\therefore$ (D)

Illustration11. *In measurement of the period of oscillation of a Helical spring, the readings comes out to be 2.15 sec, 2.25 sec, 2.36 sec, 2.45 sec and 2.54 sec, calculate the absolute errors, relative error or percentage error.*

Solution: The mean period of oscillation of the Helical spring is

$$T = \frac{2.15 + 2.25 + 2.36 + 2.45 + 2.54}{5}$$

= 2.35 sec

The absolute error in the measurements are

2.15 – 2.35 = – 0.20 sec

2.25 – 2.35 = – 0.10 sec

2.36 – 2.35 = 0.01 sec

2.45 – 2.35 = 0.10 sec

2.54 – 2.35 = 0.19 sec

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is ΔT_{mean}= [(0.20 + 0.10 + 0.01 + 0.10 + 0.19)] /5

$$= \frac{0.6}{5} = 0.12\,\text{sec}$$

Period of oscillation of the simple pendulum is (2.35 ± 0.12) sec.
A more correct way to write its is
(2.4 ± 0.2) sec
The relative error or the percentage error is

$$= \frac{0.2}{2.4} \times 100 = 8\%$$

Combination of Errors

While doing an experiment we take several measurements, we must know how the errors in all the measurements combine.
To make such estimates, we should learn how errors combine in various mathematical operations. For this we use the following procedure

(I) Error of a sum or difference: Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors.

(a) We wish to find the error Δz in the sum

$$z = A + B$$

We have by addition, $z \pm \Delta z$

$$= (A \pm \Delta A) + (B \pm \Delta B)$$

The maximum possible error in $z = \Delta z = \Delta A + \Delta B$

(b) For the difference $z = A - B$, we have

$$z \pm \Delta z = (A \pm \Delta A) - (B \pm \Delta B)$$

$$= (A - B) \pm \Delta A \pm \Delta B$$

$$\text{or,}\quad \pm \Delta z = \pm \Delta A \pm \Delta B$$

The maximum value of the error Δz is again $\Delta A + \Delta B$.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual.

Illustration 12. *The series combination of resistances is given by $R = R_1 + R_2$*
Suppose two resistances $R_1 = (50 \pm 4)\Omega$ and $R_2 = (100 \pm 3)\ \Omega$ are connected in series. Find equivalent resistance of the series combination.

Solution: $R_{eq} = R_1 + R_2$
$= (50 \pm 4)\ \Omega + (100 + 3)\ \Omega$
$= (150 \pm 7)\ \Omega$

(II) Error in a product or a quotient

Suppose $z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then
$z \pm \Delta z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B$

Dividing L.H.S. by z and R.H.S. by AB, we have

$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right).$$

Since ΔA and ΔB are small we shall ignore their product.
Hence the maximum fractional error in Z

$$= \frac{\Delta z}{z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Similarly, we can easily verify that this is true for division also.
So, when two or more quantities multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

(III) Error due to the power of a measured quantity.

Let $Z = X^2$

Then $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta X}{X} = \frac{2\Delta X}{X}$

Hence the fractional error in X^2 is two times the error in X.

In general if $Z = \frac{X^a Y^b}{Q^c}$

then $\frac{\Delta Z}{Z} = a\frac{\Delta X}{X} + b\frac{\Delta Y}{Y} + \left[\frac{\Delta Q}{Q} \times c\right]$

Illustration 13. *Find the fractional error in Z, if* $Z = \sqrt{\frac{XY}{M}}$

Solution: $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta X}{X} + \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta M}{M}$

Illustration 14. *Find maximum possible percentage error in* $x = \frac{a^\ell b^m}{y^p z^k}$

Solution: $\frac{\Delta X}{X} \times 100 = \left(\ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta y}{y} + k\frac{\Delta z}{z}\right) \times 100$

Illustration 15. *In the relation $x = 3yz^2$, x, y and z represents various physical quantities, if the percentage error in measurement of y and z is 3% and 1% respectively, then final maximum possible percentage error in x.*

Solution: $\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta y}{y} + 2\frac{\Delta z}{z}\right) \times 100$

$= 3\% + 2 \times 1\% = 5\%$

PHYSICAL QUANTITIES

All the physical quantities can be expressed in terms of some combination of seven base quantities: Length [L], mass [M], time [T], electric current [I], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol]. These base quantities are considered as the seven dimensions of the physical world.

DIMENSIONS

The dimension of a physical quantity are the powers to which the fundamental (or base) quantities like mass, length and time etc. have to be raised to represent the quantity. Consider the physical quantity **"Force"**. The unit of force is Newton.

1 Newton = 1 kg m/sec^2

$kg \rightarrow M^1$ (Mass); $m \rightarrow L^1$ (Length); $s^{-2} \rightarrow T^{-2}$ (Time)

$\therefore$ Dimensions of force are $[M^1L^1T^{-2}]$

Dimensional formulae for some physical quantities

Physical quantity	**Relation with other quantity**	**Dimensional formula**
Area	Length × breadth	$L \times L = [L^2]$
Density	Mass/volume	$\frac{M}{L^3} = [ML^{-3}]$
Acceleration	$\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = [LT^{-2}]$
Force	F = ma	$[MLT^{-2}]$
Linear momentum	P = mv	$[MLT^{-1}]$
Pressure	P = F/A	$[ML^{-1}T^{-2}]$
Universal gravitational constant	$G = \frac{Fr^2}{M_1M_2}$	$[M^{-1}L^3T^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (kinetic, potential and heat)	$\frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Surface tension	$T = \frac{F}{\ell}$	$[ML^\circ T^{-2}]$
Strain	$e = \frac{\Delta\ell}{\ell}$	$[M^\circ L^\circ T^\circ]$
Modulus of elasticity	$E = \frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	$[M^\circ L^\circ T^\circ]$
Coefficient of viscosity	$\eta = \frac{F \times r}{A \times v}$	$[M^1L^{-1}T^{-1}]$
Planck's constant	$h = mv\lambda$	$[ML^2T^{-1}]$
Thermal resistance	$\frac{\Delta\theta t}{Q}$	$[M^{-1}L^{-2}T^3\,\theta]$
Thermal conductivity	$K = \frac{H}{At(d\theta/dx)}$	$[MLT^{-3}\theta^{-1}]$
Boltzman's constant	k = R/N	$[ML^2T^{-2}\theta^{-1}]$
Universal gas constant	$R = \frac{PV}{T}$	$[ML^2T^{-2}\theta^{-1}]$
Mechanical equivalent of heat	J = W/H	$[M^\circ L^\circ T^\circ]$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$	$[M^\circ L^\circ T^{-1}]$

Illustration 16. *Write the dimensions of: Impulse, Pressure, Work, Universal constant of Gravitation.*

Solution: (i) $[M^1L^1T^{-1}]$ (ii) $[M^1L^{-1}T^{-2}]$ (iii) $[M^1L^2T^{-2}]$ (iv) $[M^{-1}L^3T^{-2}]$

Four types of quantities

Dimensional constant: These are the quantities whose values are constant and they possess dimensions. For example, velocity of light in vacuum, universal gas constant etc.

Dimensional variables: These are the quantities whose values are variable, and they possess dimensions. For example, area, volume, density etc.

Dimensionless constants: These are the quantities whose values are constant, but they do not possess dimensions. For example, π, 1, 2, 3, ….. etc.

Dimensionless Variables: These are the quantities, whose values are variable, and they do not have dimensions, e.g., angle, strain, specific gravity etc.

Uses of dimensions: dimensional analysis

(1) Checking the correctness (dimensional consistency) of an equation: An equation contains several terms which are separated from each other by symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This means that we can not add velocity to force. This principle is called Principle of Homogeneity of dimensions.

Look at the equation : $v^2 = u^2 + 2as$

Dimensions of $v^2 : [L^2T^{-2}]$

Dimensions of $u^2 : [L^2T^{-2}]$

Dimensions of $2as : LT^{-2}][L] = [L^2T^{-2}]$

$\therefore$ The equation $v^2 = u^2 + 2as$ is dimensionally consistent, or dimensionally correct.

Note:

A dimensionally correct equation may not be actually correct. For example, the equation $v^2 = u^2 + 3as$ is also dimensionally correct but we know that it is not actually correct. However, all correct equations must necessarily be dimensionally correct.

Illustration 17. *Which of the following equations may be correct ?*

(i) $x = ut + \frac{1}{2}at^2$ *(ii)* $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $F = \frac{GM_1M_2}{r}$ *(iv)* $T^2 = \frac{4\pi^2R^3}{GM}$

(v) $V = \sqrt{GMR}$

Given: G = Gravitational constant, whose dimensions are $[M^{-1}L^3T^{-2}]$

M_1, M_2 *and M have dimensions of mass. L, x, r, R has dimenssions of length. And t has dimensions of Time. 'F' denotes Force and 'a' has dimensions of acceleration.*

Solution: (i) Yes (ii)Yes (iii) No (iv) Yes (v) No.

(2) Conversion of units: Dimensional methods are useful in finding the conversion factor for changing the units to a different set of base quantities. Let us consider one example, the SI unit of force is Newton. The CGS unit of force is dyne. How many dynes is equal to one newton. Now,

$$1 \text{ newton} = [F] = [M^1L^1T^{-2}] = (1\text{kg})^1)(1\text{meter})^1(1\text{s})^{-2}$$

$$1\text{dyne} = (1\text{g})(1\text{cm})(1\text{s})^{-2}$$

$$\therefore \frac{1\text{newton}}{1\text{dyne}} = \frac{(1\text{kg})^1(1\text{meter})^1\,(1\text{s})^{-2}}{(1\text{g})(1\text{cm})(1\text{s})^{-2}} = (10^3)(10^2) = 10^5$$

$$1 \text{ newton} = 10^5 \text{ dynes}$$

Thus knowing the conversion factors for the base quantities, one can work out the conversion factor of any derived quantity if the dimensional formula of the derived quantity is known.

Illustration 18. *Find the conversion factor for expressing universal gravitational constant from SI units to cgs units.*

Solution: $6.67\text{x}10^{-8}\ \text{cm}^3\text{s}^{-2}\text{g}^{-1}$

(3) Deducing relation among the physical quantities:

Suppose we have to find the relationship connecting a set of physical quantities as a product type of dependence. Then dimensional analysis can be used as a tool to find the required relation. Let us consider one example. Suppose we have to find the relationship between gravitational potential energy of a body in terms of its mass 'm', height 'h' from the earth's surface and acceleration due to gravity 'g', then,

Let us assume that: – Gravitational potential energy, U,

$$U = K[m]^a[g]^b[h]^c,$$

where K, a, b, and c are dimensionless constants.

Then $[ML^2T^{-2}] = [M]^a[LT^{-2}]^b[L]^c$

$$= [M^aL^{b+c}T^{-2b}]$$

$\therefore$ a =1, b + c = 2

–2b = –2

b = 1, c = 1.

$\therefore$ U = Kmgh, where K is a dimensionless constant.

Thus by dimensional analysis, we conclude that the gravitational potential energy of a body is directly proportional to its mass, acceleration due to gravity and its height from the surface of the earth.

Limitations of dimensional analysis:

This method does not give us any information about the dimensionless constants appearing in the derived formula, e.g. 1, 2, 3, .. . π etc.

We can't derive the formula having trigonometrical functions, exponential functions etc, which have no dimensions.

The method of dimensions cannot be used to derive an exact form of relation when it consists of more that one part on any side, e.g. the formula $v^2 = u^2 + 2as$ cannot be obtained.

If a quantity depends on more than three factors having dimensions the formula cannot be derived. This is because on equating powers of M, L and T on either side of the dimensional equation, we can obtain three equations from which only three exponents can be calculated.
It gives no information whether a physical quantity is a scalar or a vector.

Illustration 19. *Using the method of dimensions, find the acceleration of a particle moving with a constant speed v in a circle of radius r.*

Solution: Assuming that the aceeleration of a particle depends on v and r

$a \propto v^x r^y \Rightarrow a = k\, v^x r^y$

Now as we know dimensions of acceleration (a) = $M^\circ LT^{-2}$

and dimensions of velocity (v) = $M^\circ LT^{-1}$

dimension of radius (r) = $M^\circ LT^\circ$

Putting all thee dimensions in (1), we get

$M^\circ LT^{-2} = k\,(M^\circ LT^{-1})^x (M^\circ LT^\circ)^y$

$M^\circ LT^{-2} = k\, M^\circ L^{x+Y} T^{-x}$

Comparing the powers, we get

$x + y = 1$

$x = 2$

$\therefore\ y = 1 - 2 = -1$

$\therefore\ a = k\, v^2 r^{-1}$

$a = \frac{kv^2}{r}$

Illustration 20. *In the expression* $\left(P + \frac{a}{v^2}\right)(v - b) = RT$

P is pressure and v is the volume. Calculate the dimensions of a and b.

Solution: Only physical quantities having same dimensions are added or subtracted. So $\frac{a}{v^2}$ has the same dimensions as that of pressure.

As pressure = $\frac{\text{Force}}{\text{Area}}$

Dimensions of pressure $= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$\therefore$ Dimensions of $\frac{a}{v^2} = ML^{-1}T^{-2}$

Dimensions of a $= ML^{-1}T^{-2}(V^3)^2$

$= (ML^{-1}T^{-2})(L^3)^2$

$= ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

Similarly dimensions of b is same as that of volume.

Dimensions of $b = M^0L^3T^0$.

Illustration 21. *Does* $S_{nth} = u + \frac{a}{2}(L_n - 1)$ *dimensionally correct?*

Solution: Yes, this expression is dimensionally correct, yet it appears to be incorrect. As we are taking it to be for nth second. Here one second is divided through the equation.

Illustration 22. *Find the dimensions of resistivity, thermal conductivity and coefficient of viscosity.*

Solution: (i) $R = \rho\frac{\ell}{A}$

$\rho = \frac{RA}{L} = [ML^3T^{-3}A^{-2}]$

(ii) Thermal conductivity, k

$\frac{d\theta}{dt} = \frac{k\ell}{A\Delta\theta} = \frac{ML^2T^{-3}L}{L^2K}$

$= MLT^{-3}k^{-1}$

(iii) Coefficient of viscosity

$\therefore\ F = \eta A\frac{dv}{dx}$

$\eta = \frac{Fdx}{Adv} = \frac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

Illustration 23. *A displacement of a particle is given by equation y = A sin ωt, where y is in metres and A is also in metres, t is in seconds. What are the dimensions of ω.*

Solution: As the angles are always dimensionless, so

ωt = dimensionless quantity

Dimensions of $\omega t = M^\circ L^\circ T^\circ$

Dimensions of $\omega = M^\circ L^\circ T^{-1}$

Illustration 24. *If density ρ, acceleration due to gravity g and frequency f are the basic quantities, find the dimensions of force.*

Solution: We have $\rho = ML^{-3}$, $g = LT^{-2}$ $f = T^{-1}$

Solving for M, L and T in terms of ρ, g and f , we get

$M = \rho^2g^3f^{-6}$, $L = gf^{-2}$ & $T = f^{-1}$

Force = $[MLT^{-2}] = [\rho g^3f^{-6}.gf^{-2}.\ f^2] = [\rho g^4f^{-6}]$

Illustration 25. *An athlete's coach told his team that muscle times speed equals power. What dimensions does he view for "muscle"?*

(A) $ML\,T^2$ *(B) $ML^2\,T^{-2}$*
(C) $ML\,T^{-2}$ *(D) L*

Solution: Power = force × velocity

= muscle times speed

$\therefore$ muscle represents force

muscle = $[MLT^{-2}]$

$\therefore$ (C)

Illustration 26. *If force, length and time would have been the fundamental units what would have been the dimensional formula for mass*

(A) $FL^{-1}\,T^{-2}$ *(B) $FL^{-1}T^2$*
(C) $FL\,T^{-2}$ *(D) F*

Solution: Let $M = K\,F^aL^bT^c$

$= [MLT^{-2}]^a\,[L^b]\,T^c = [M^aL^{(a+b)}T^{(-2a+c)}]$

$a = 1, a + b = 0$ & $-2a + c = 0$
$\Rightarrow a = 1, b = -1, c = 2$
$\therefore$ (B)

Illustration 27. *The dimensions of the Rydberg constant are*
(A) $M^{\circ} L^{-1} T$ *(B) MLT^{-1}*
(C) $M^{\circ}L^{-1} T^{\circ}$ *(D) $ML^{\circ}T^{2}$*

Solution: From the relation $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
$R = \frac{1}{L} = L^{-1} = M^{\circ}L^{-1}T^{\circ}$
$\therefore$ (C)

Illustration 28. *The error in the measurement of the radius of a sphere is 1%. Then error in the measurement of volume is*
(A) 1% *(B) 5%*
(C) 3% *(D) 8%*

Solution: $V = \frac{4}{3}\pi r^3$
$\frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta r}{r}\right)\times 100 = 3 \times 1 = 3\%$
$\therefore$ (C)

MISCELLANEOUS EXERCISE

1. The time period of a gas bubble formed under water oscillating with a time period depending on static pressure P, density of water ρ and ε total energy of explosion. Find the relationship between T, P, ρ and ε.

2. Name the three physical quantities having the same dimensions

3. A student measures the time period of a simple pendulum. If error in measurement of length is 2% and error in measurement of g is 2% calculate the error in measurements of Time period.

4. A physical quantity is given by x = a + bt, where x is in metres and t is in seconds. So calculate the dimensions of a and b.

5. Find the dimensional formulae of the following quantities
   (A) the universal gravitational constant (B) Surface tension
   (C) Potential energy (D) Surface energy

6. A Vernier calliper has 50 divisions on its Vernier scale. Minimum division in main scale is of 1 mm.
   (a) Find out the least count of the Vernier Calliper.
   (b) During a length measurement, zero of Vernier scale is between 12^{th} and 13^{th} divisions of the main scale and 26^{th} division of the Vernier scale coincides with a main scale division. What is the length ?

7. Calculate the focal length of a spherical mirror if measured quantities u and v are as follows.
   $u = 50.1 \pm 0.5\,\text{cm}$
   $v = 20.1 \pm 0.2$ cm

8. Young's modulus of steel is $19 \times 10^{1°}$ N/m^2. Express it in dyne/cm^2. Here dyne is the CGS unit of force.

9. Convert 1 joule into ergs.

10. A goldsmith puts some gold weighting 5.42 gm in a box weighing 1.2 kg. Find the total weight of the box to correct number of significant figures.

SOLUTION TO MISCELLANEOUS EXERCISE

1. $T \propto P^{-5/6}\rho^{1/2}E^{1/3}$
2. Work, energy and torque
3. 2%
4. x = a + bt
   Dimensions of a = $M°LT°$
   Dimensions of b = $M°LT^{-1}$
5. (a) $M^{-1}L^2T^{-2}$ (b) MLT^{-2}
   (c) ML^2T^{-2} (d) ML^2T^{-2}
6. 0.02 mm, 12.52 mm
7. 14.3 ± 0.4 cm
8. 19×10^{11} dyne / cm^2
9. 1 joule = 10^7 ergs
10. 1.2 kg

SOLVED PROBLEMS

Subjective:

Prob 1. *The Bernoulli's equation is given by,*

$P+\frac{1}{2}\rho v^2+\rho gh = constant$ *, where P is pressure. Compare the unit of the quantity* $\frac{1}{2}\rho v^2$ *with the unit of pressure.*

Sol. Only same quantities can be summed up or subtracted from each other. So $\frac{1}{2}\rho v^2$ has same unit as that of pressure.

Prob 2. *The relation between velocity and time of a moving body is given as,* $v = A+\frac{B}{t}+Ct^2$ *. Find the units of A, B and C.*

Sol. From the principle of homogeneity

v = A = m/sec

v = B/t $\Rightarrow$ B = m

v = (ct^2)

$\therefore$ c = m/sec^3

Prob 3. *The external and internal diameters of a hollow cylinder are measured to be (4.23 ±0.01) cm and (3.89 ± 0.01) cm. What is the thickness of the wall of the cylinder?*

Sol. Thickness = (4.23 – 3.89) cm = $\frac{0.34}{2}$ = 0.17 cm.

Error = ±(0.01 + 0.01) cm = ±0.02 cm

Prob 4. *A physical quantity x is calculated from the relation* $x = \frac{a^2b^3}{c\sqrt{d}}$*. If percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively. What is the percentage error in x?*

Sol. As $x = \frac{a^2 \ \ b^3}{c\sqrt{d}}$

$$\frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right]$$

$$= \pm\ [2 \times 2\% + 3\times1\% + 3\% + \frac{1}{2}\times 4\% = \pm\ 12\%$$

Prob 5. *A physical quantity A is defined as,* $A = pkx^2y/z$. *The absolute errors in the measured of x, y, z are given as*

x = (0.26 ± 0.02)cm

y = (64 ± 2) Ω

z = (156.0 ± 0.1)cm

Find the percentage error in the quantity A.

Sol. $a = kx^2y / z$

$\Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$

$= 2 \times \frac{0.02}{0.26} + \frac{2}{64} + \frac{0.1}{156}$

$\pm 0.186 = 18.6\ \%$

Prob 6. *10 rotations of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 divisions. Find the least count. A reading taken for the diameter of wire with the screw gauge shows 4 complete rotations and 35 on the circular scale. Find the diameter of the wire.*

Sol. The least count = $\frac{5}{1000}$ = 0.005 mm

The diameter of the wire = $(4 \times 0.5 + 35 \times 0.005)$ mm = 2.175 mm

Prob 7. *The diameter of a sphere is 2.78 cm. Calculate its volume in proper significant figures.*

Sol. Volume = $\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi\left(\frac{2.78}{2}\right)^3$ $cm^3 = 11.2437\ cm^3$

Hence the volume in proper significant figures is $11.2\ cm^3$

Prob 8. *Calculate the number of light years in one meter.*

Sol. We know 1 light year (ℓ_y) = 9.46×10^{15}m

or 9.46×10^{15} m = $1\ell y$

$1\ m = 1.057 \times 10^{-16}\ell y$

Prob 9. *Find the dimensions of a and b in the relation P =* $\frac{b - x^2}{at}$

where P is power, x is distance and t is time.

Sol. The given relation is, P = $\frac{b - x^2}{at}$

As x^2 is subtracted from b therefore the dimensions of b are of x^2

i.e. $b = L^2$

We can rewrite relation as

$P = \frac{[L^2]}{at} = \frac{L^2}{at}$

$a = \frac{L^2}{[ML^2T^{-3}]\ [T]} = M^{-1}L^{\circ}T^2$

Prob 10. *It is claimed that two cesium clocks if allowed to run for 100 years free from any disturbance may differ by only about 0.02 sec. What is the accuracy of the standard cesium clock in measuring a time interval of 1 sec?*

Sol. $\because$ t = 100 years = $100\times365.25\times86400$s

$\Delta t = 0.02$s

Fractional error = $\dfrac{\Delta t}{t} = \dfrac{0.02}{100\times365.25\times86400}$

$= 0.63\times10^{-11}$

So, there is an accuracy of 10^{-11} Part in 1 or 1 sec in 10^{11} sec.

Prob11. *In screw gauge no. of division on circular scale is n and circular scale travels a distance of a units in one rotation. Calculate least count of the screw gauge.*

Sol. Pitch = a units

Least count $= \dfrac{\text{Pitch}}{\text{No. of divisions on circulat scale}}$

$= \dfrac{a}{n}$ units.

Prob 12. *The diameter of the spherical bob is measured by vernier Calipers (10 divisions of a Vernier scale coincide with a divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 12 mm and 7^{th} division of the main scale coincides with the main scale. Mass of the sphere is 4.532 g. Find the density of the sphere.*

Sol. Vernier constant = 1.M.S.D. – 1.V.S.D.

$= 1\text{ mm} - \dfrac{9}{10}\text{ mm}$

$= 0.1$ mm

Diameter of sphere = 12 mm + 0.1×7

= 12.7 mm

$\therefore$ Volume of sphere = $= \dfrac{4}{3}\pi\left(\dfrac{D}{2}\right)^3$

$= \dfrac{4}{3}\times3.14\times\left(\dfrac{12.7}{2}\times10^{-3}\right)^3$

Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{4.532\times3\times8\times10^{-3}}{4\times3.14\times(12.7\times10^{-3})^3}$

$= 4.227$ kg /m^3

$= 4.23$ kg/m^3 (in appropriate significant figures)

Prob 13. *A wire of length $\ell = 8 \pm 0.02$ cm and radius $r = 0.2 \pm 0.02$ cm and mass $m = 0.1 \pm 0.001$ gm. Calculate maximum percentage error in density*

Sol. $\rho = \dfrac{m}{\pi r^2 \ell}$

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{2\Delta r}{r} + \dfrac{\Delta\ell}{\ell}$

$\Delta\ell = 0.02$ cm, $\ell = 8$ cm

$\Delta r = 0.02$ cm, r = 0.02 cm

m = 0.1 gm, $\Delta m = 0.001$ gm

$\dfrac{\Delta\rho}{\rho} = \dfrac{0.001}{0.1} + 2\times\dfrac{0.02}{0.2} + \dfrac{0.001}{0.1}$

$$= \left(\frac{1\times10}{1000\times1} + \frac{2\times2}{100\times2}\times10 + \frac{1\times10}{1000\times1}\right)\times100$$

$$= \frac{(1+20+1)}{100}\times100$$

= 22%

Prob 14. *Planck's formula is given by*

$$u = \frac{\hbar\omega^3}{\pi^2 e^3}\times\frac{1}{e^{\hbar\omega/k^{-1}-1}}$$

where u is the energy radiated per unit area per unit time and h is Planck's constant. What will be the dimensions of k in the expression.

Sol. The power in exponential is always dimensionless. So,

$$\frac{\hbar\omega}{kT} = M^0L^0T^0$$

E = hv

$$\text{so, } h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^0L^0T^{-1}}$$

$= ML^2T^{-1}$

$$\therefore \quad k = \frac{\hbar\omega}{T}$$

$$= \frac{ML^2T^{-1}T^{-1}}{T} = ML^2T^{-3}$$

Prob15. *According to Stoke's law the viscous force acting on a spherical body moving fluid depends on radius r of the body, co–efficient of viscosity η of the fluid and velocity f the body. Find the relation between F, η, r, v.*

Sol. Force acting on a spherical body depends on

$F \propto \eta^a r^b v^C$

$F = k\eta^a r^b v^c$

$(MLT^{-2}) = k\,(ML^{-1}T^{-1})^a\,(L)^b\,(LT^{-1})^c$

$MLT^{-2} = k\,(M)^a\,(L)^{-a+b+c}\,(T^{-a-c})$

a = 1

–a + b + c = 1

– a –c = –2

–1 –c = – 2

c = – 2 = 1

– a + b + c = 1

– 1 + b + 1 = 1

$\Rightarrow$ b = 1

F = k η r v

Objective:

Prob 1. *An experiment measures quantities a, b, c and x is calculated from $x = ab^2/c^3$. If the maximum percentage error in a, b and c are 1%, 3% and 2% respectively, the maximum percentage error in x will be*

(A) 13% *(B) 17%*
(C) 14% *(D) 11%*

Sol. (A) Maximum percentage error in x

As $x = \frac{ab^2}{c^3}$

$$\frac{\Delta x}{x} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

$$\frac{\Delta x}{x} = 1\% + 2 \times 3\% + 3 \times 2\%$$

$= (1 + 6 + 6)\% = 13\%$

Prob 2. *If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second, then non–zero integers x, y and z, such that $P^xQ^yc^z$ is dimensionless, may be*

(A) x = 1, y =1, z = 1. *(B) x = 1, y =–1, z = 1.*
(C) x = –1, y =1, z = 1. *(D) x = 1, y = 1, z = 1*

Sol. As $P^xQ^{y+}C^z$ is a dimensionless

$$\left(\frac{MLT^{-2}}{L^2}\right)^x \left(\frac{ML^2T^{-2}}{L^2T}\right)(LT^{-1})^2 = M^0L^0T^0$$

$(M^1L^{-1}T^{-2})^x\,(ML^\circ T^{-3})^y\,(LT^{-1})^2 = (M^\circ L^\circ T^\circ)$

Comparing powers, we get

$x + y = 0$ . . . (i)

$-x + z = 0$ . . . (ii)

$-2x - 5y - z = 0$ . . . (iii)

From (1) and (2),

$y = -x,\ z = x$

Substituting in (3), we get

If x = k

$y = -k,\ z = k$

$x = 1,\ y = -1,\ z = 1$

Prob 3. *The dimensional representation of Planck's constant is identical to that of*

(A) Torque *(B) Power*
(C) Linear momentum *(D) angular momentum*

Sol. (D) As Planck's constant has dimensions of $\frac{E}{\nu}$

$= \frac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

and Dimensions of angular momentum = r × p

= (L × MLT^{-1})

= ML^{2}T^{-1}

Prob 4. *The parallel combination of two resistances is given by If the two resistances R_1= (2 ±0.2)Ω and R_2= (1 ±0.1)Ω are connected in parallel. Then the % error is given by*

(A) 0.1% *(B) 0.2%*

(C) 0.3% *(D) 0.4%*

Sol. (C) $R_p = \frac{R_1R_2}{R_1+R_2}$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right) \times 100$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{0.2}{2} + \frac{0.1}{1} + \frac{0.2+0.1}{3}\right)$$

(0.1 + 0.1 + 0.1)

= 0.3%

Prob5. *If the units of M and L are quadrupled, then the units of torque becomes*

(A) 16 times *(B) 64 times*

(C) 8 times *(D) 4 times*

Sol. (B)

Dimensions of torque = ML^2 T^{-2}

= (4M) (4 L)^2 T^{-2}

= 64 M L^2 T^{-2}

Prob6. *A radar signal is beamed towards a planet from earth and its echo is received seven minutes later. If distance between the planet and earth is 6.3 × 10^{1} °m, then velocity of the signal will be*

(A) 3 × 10^8 m/s *(B) 2.97 × 10^6 m/s*

(C) 3.10 × 10^5 m/s *(D) 300 m/s*

Sol. (A).

Velocity of signal, $c = \frac{2x}{t} = \frac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ m/s

Prob7. *If speed of light c, acceleration due to gravity g and pressure P are taken as fundamental units, then the dimensions of gravitational constant is*

(A) [$c^{\circ}gP^{-3}$] *(B) [$c^2g^3P^{-2}$]*

(C) [$c^{\circ}g^2P^{-1}$] *(D) [$c^2g^2P^{-2}$]*

Sol. (C).

Let $G = c^x g^y P^z$

$\Rightarrow [M^{-1}L^3T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [ML^{-1}T^{-2}]^z$

$= [M^z L^{x+y} T^{-x-2y-2z}]$

Comparing powers of M, L and T on both sides, we get

$z = -1$, $x + y = 3$, $-x - 2y - 2z = -2$

On solving these equations for x, y and z, we get

$x = 0$, $y = 2$, $z = -1$

$\Rightarrow G = [c^{\circ} g^2 P^{-1}]$.

Prob 8. *The time dependence of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and t is time. The constant α*

(A) is dimensionless *(B) has dimensions T^{-2}*

(C) has dimensions of P *(D) has dimensions T^2*

Sol. (B).

$P = P_0 [\exp(-\alpha t^2)]$.

Since αt^2 must be dimensionless, so $\alpha = \frac{1}{T^2} = T^{-2}$

Prob 9. *The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is*

(A) hertz *(B) metre*

(C) radian *(D) second*

Sol. (A).

Here, kt is dimensionless. Hence, $k = 1/t = \sec^{-1}$ = hertz

Prob10. *The parallel of a heavenly body measured from two points diametrically opposite on the equator of earth is 1.0 minute. If the radius of earth is 6400 km, find the distance of the heavenly body from the centre of earth in AU. Take 1 AU = 1.5×10^{11} m.*

(A) 0.293 AU *(B) 0.28 AU*

(C) 2.01 AU *(D) 3.97 AU*

Sol. (A).

Here, $\theta = 1' = \frac{1^\circ}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

ℓ = diameter of earth = 2×6400 km

$= 1.28 \times 10^4$ km $= 1.28 \times 10^7$ m

Now, $\ell = r\theta$

$\Rightarrow r = \frac{1.28 \times 10^7}{(\pi/60) \times 180} = 4.4 \times 10^{10}$ m

$r = \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} = 0.293$ AU

Prob11. *Dimensions of ohm are same as (h is Planck's constant and e is charge)*

(A) $\frac{h}{e}$ *(B)* $\frac{h^2}{e}$

(C) $\frac{h}{e^2}$ *(D)* $\frac{h^2}{e^2}$

Sol. (C).

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}] = \text{resistance}$$

Prob12. *Which of the following is a derived unit?*

(A) newton *(B) joule*

(C) pascal *(D) metre*

Sol. A, B, C.

Because, they are derived from the fundamental units, i.e. kg, m and sec.

Prob13. *Which of the following equations is dimensionally correct?*

(A) Pressure = energy per unit volume

(B) Pressure = energy per unit area

(C) Pressure = force per unit volume

(D) Pressure=momentum per unit volume

Sol. $\frac{\text{Energy}}{\text{Volume}} = \left[\frac{\frac{1}{2}mv^2}{\text{volume}}\right]$

$$\Rightarrow \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

$\therefore$ (A)

Prob 14. *Which of the following is/are dimensional constants is*

(A) Planck's constant *(B) dielectric constant*

(C) relative density *(D) gravitational constant*

Sol. A Planck's constant and gravitational constant G have constant values and dimensions

$\therefore$ A, D

Prob 15. *Which of the following is not a unit of time*

(A) solar year *(B) tropical year*

(C) leap year *(D) light year*

Sol. Tropical year is the year in which there is total eclipse.

Light year represents distance

$\therefore$ (D)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. If force acting on a particle depends on the x–coordinates as $F = ax + bx^2$, find the dimensions of 'a' and 'b'.

2. If velocity, time and force are chosen as basic quantities, find the dimensions of mass.

3. Find the dimensional formula of
   (a) Charge Q
   (b) The potential V
   (c) The capacitance C,
   (d) The Resistance, R

4. Which of the following have same dimensions?
   (A) angular momentum and linear momentum (B) work and power
   (C) work and torque (D) Torque and Pressure

5. The Van der Waals interaction between two molecules separated by a distance r is given by the energy $E = -\frac{A}{r^6} + \frac{B}{r^{12}}$. Find the dimensions of A and B.

6. If error in measuring diameter of a circle is 4%, find the error in radius of circle.

7. Derive, by method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (ν) and the amplitude of vibration (r)

8. Write the dimensions of $\frac{a}{b}$ in the relation
   $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

9. Assuming that the mass of the largest stone that can be moved by a flowing river depends upon the velocity r, the density ρ and acceleration due to gravity, show that m varies with sixth power of the velocity of flow.

10. The density of a material in cgs system is 8 gcm^{-3}. In a system of units, in which unit of length is 5 cm and unit of mass is 20 g, what is the density of the material ?

11. To study the flow of a liquid through a narrow tube the following formula is used
    $\eta = \frac{\pi\rho r^4}{8v\ell}$ where the letters have their usual meanings. The values of ρ, r, v and ℓ are measured to be 76 cm of Hg, 0.28 cm, 1.2 cm^3s^{-1} and 18.2 cm respectively. If these quantities are measured to the accuracy of 0.5 cm of Hg, 0.01 cm, 0.1 $cm^3\ s^{-1}$ and 0.1 cm respectively, find the percentage error in the value of η.

12. The equation of a wave is given by $y = A\sin\omega\left(\frac{x}{v} - k\right)$, where ω is angular velocity and v is linear velocity. Find the dimension of k. Given that

13. The surface tension of a liquid is 70 dyne/cm. Express it in MKS system of units?

14. Name a physical quantity which has same unit as that of Torque.

15. If all measurements in an experiment are taken upto same number of significant figures then mention two possible reasons for maximum error.

Level – I

1. The mass of a block is 87.2 g and its volume is 25 cm^3. What is its density upto correct significant figures?

2. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants, λ is wavelength. Then calculate the dimensions of A and B.

3. Suppose, the torque acting on a body, is given by $\tau = KL + \frac{MI}{\omega}$
   Where L = angular momentum, I = moment of inertia & ω = angular speed
   What is the dimensional formula for KM?

4. When a current of (2.5 ± 0.5). A flows through a wire it develops a potential difference of (20 ± 1) V. What is the resistance of wire?

5. Find out the result in proper significant figures, 291 x 0.03842 / 0.0080.

6. The radius of a sphere is (5.3 ± 0.1) cm. Find the percentage error in its volume.

7. If Planck's constant h; the velocity of light, c and Newton's gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of these quantities using dimensional notation.

8. What will be the unit of time in the system in which the unit of length is meter, unit of mass is kg and unit of force is kg. wt.?

9. Imagine a system of units in which the unit of mass is 10 kg, length is 1 km and time is 1 minute, then calculate the value of 1 J in this system.

10. A screw gauge of pitch 0.5 mm has a circular scale divided into 5 divisions. The screw gauge is used to measure the thickness of a coin. The main scale reading is 2 mm and 35^{th} circular division coincides with main scale with a positive zero error of divisions. Find the thickness of the coin

11. A Vernier Calliper is used to measure the thickness of the wall of cylinder by measuring its external and internal diameters. For external diameter, the zero if the Vernier scale coincides with the 5^{th} division of main scale and 6^{th} division of Vernier scale coincides with the main scale.
    For internal diameter, the zero of the Vernier scale coincides with the 3^{rd} division of mass scale and 2^{nd} division of Vernier scale coincides with 3^{rd} division of main scale and 2^{nd} division of Vernier scale coincides with main scale. Given that 1 main scale division is equal to 10 m
    1 V.S.D. = 0.09 cm.
    Calculate the thickness of the wall of a cylinder.

12. The time period of small oscillations of a spring mass system is given as $T = 2\pi\sqrt{\frac{m}{k}}$. What will be the accuracy in the determination of k if mass m is given as 10 kg with accuracy of 10 gm and time period is 0.5 sec measured for time of 100 oscillations with a watch of accuracy of 1 sec.

13. In a screw micrometer, main scale divisions are in mm. There are 100 cap divisions.
(a) Find out the least count of the micrometer.
(b) In fully closed condition, 4^{th} division of the cap scale coincides with the line of graduation along the screw axis. What is the zero error of the instrument ? Is it to be added or subtracted from the observed reading during a measurement ?
(c) In the above instrument, during a measurement, the cap is between 7^{th} and 8^{th} divisions of the main scale and 37^{th} division of cap scale coincides with the line of graduation of the main scale. What is the measurement corrected for zero error ?

14. The equation for energy (E) of a simple harmonic oscillator,
$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2x^2$$
is to be made "dimensionless" it with multiplying by a suitable factor, which may involve the constants, m(mass), ω(angular frequency) and h (Planck's constant). What will be the unit of momentum and Length ?

15. In in the equation $F = A\sin Bx^2 + \frac{C}{t}e^{Dt}$, F, x and t are force, position and time respectively, then give the dimensions of $\frac{A}{CB}$.

Objective:

1. Which of the following is a possible dimensionless quantity?
   (A) Velocity gradient (B) Pressure gradient
   (C) Displacement gradient (D) Force gradient

2. Dimensional formula of thermal conductivity is
   (A) $ML^2T^{-3}\theta^{-1}$ (B) $ML^2T^{-2}\theta^{-4}$
   (C) $ML^2T^{-2}\theta^{-1}$ (D) $MLT^{-3}\theta^{-1}$

3. The unit of power is
   (A) kilowatt hour (B) joule
   (C) dyne (D) kilo watt

4. The dimensional representation of Planck's constant is identical to that of
   (A) torque. (B) power.
   (C) linear momentum. (D) angular momentum.

5. Which of the following is a fundamental quantity?
   (A) volume (B) velocity
   (C) time (D) force

6. The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is
   (A) hertz (B) metre
   (C) radian (D) second

7. The dimensional representation of Planck's constant is identical to that of
   (A) torque (B) work
   (C) stress (D) angular momentum

8. A force F is given by $F = \frac{a}{t} + bt^2$, where t is time. The dimensions of a and b are
   (A) $[MLT^{-3}]$ and $[MLT^{-4}]$ (B) $[MLT^{-4}]$ and $[MLT^{-3}]$
   (C) $[MLT^{-1}]$ and $[MLT^{-4}]$ (D) $[MLT^{-2}]$ and $[MLT^{\circ}]$

9. A unit–less quantity
   (A) may have non–zero dimensions (B) always has non–zero dimensions
   (C) never has a non–zero dimensions (D) does not exist

10. Joule $\times$ sec is the unit of
    (A) energy (B) momentum
    (C) angular momentum (D) power

11. Given that v is speed, r is radius and g is gravitational acceleration, which of the following expression is dimensionless.
    (A) $\frac{v^2}{gr}$ (B) $\frac{v^2 r}{g}$
    (C) $\frac{v^2 g}{r}$ (D) $v^2 rg$

12. The dimensional formula for modulus of rigidity is
(A) $[ML^2T^{-2}]$ (B) $[ML^{-1}T^{-3}]$
(C) $[ML^{-2}T^{-2}]$ (D) $ML^{-1}T^{-2}]$

13. A highly rigid cubicle block A of small mass m and side L is rigidly fixed to an other similar cubical block of low modulus of rigidity η. Lower face of A completely covers the upper face of B. The lower face of B is rigidly held on horizontal surface. A small force T is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes simple harmonic motion, the time period of which is given by
(A) $2\pi\sqrt{m\eta L}$ (B) $2\pi\sqrt{m\eta / L}$
(C) $2\pi\sqrt{mL / \eta}$ (D) $2\pi\sqrt{m / \eta L}$

14. The time period of a soap bubble is $T \propto P^a d^b S^c$, where P is pressure, d is density and S is surface tension, then values of a, b and c, respectively, are
(A) –1, –2, 3 (B) –3/2, 1/2 1
(C) 1, –2, –3/2 (D) 1, 2, 1/3

15. The dimensional formula for specific resistance in term of M, L, T and Q is
(A) $[ML^3T^{-1}Q^{-2}]$ (B) $[ML^2T^{-2}Q^2]$
(C) $[MLT^{-2}Q^{-1}]$ (D) $[ML^2T^{-2}Q^{-2}]$

16. Which of the two have same dimensions?
(A) Force and strain (B) Force and stress
(C) Angular velocity and frequency (D) Energy and strain

17. The velocity of water waves depend on their wavelength λ, the density of water ρ and acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 \propto g^{-1}\lambda^{-1}$ y (B) $v^2 \propto g\lambda$ y
(C) $v^2 \propto g\lambda\rho$ y (D) $v^2 \propto g^{-1}\lambda^{-3}$ y

18. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combination which have the dimensions of angle
(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$
(C) $\frac{C}{L}$ (D) $\frac{R^2C}{L}$

19. The vernier of a circular scale is divided into 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count by the instrument is
(A) 0.1′ (B) 1′
(C) 10′ (D) 30′

20. Dimensional analysis of the equation $(\text{velocity})^x = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$ gives the value of x as
(A) 1 (B) 2
(C) 3 (D) 4

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. $[a] = M^1L^0T^{-2}$, $[b] = M^1L^{-1}T^{-2}$
2. FTV^{-1}
3. (a) $[Q] = IT$ (b) $[V] = ML^2I^{-1}T^{-3}$ (c) $[C] = M^{-1}L^{-2}I^2T^4$ (d) $[R] = ML^2T^{-3}I^{-2}$
4. Work and torque
5. $[A] = ML^8T^{-2}$, $[B] = ML^{14}T^{-2}$
6. 4 %
7. $E = k\,mv^2\,r^2$
8. $M^\circ L^{-1/2}\,T^2$
10. 50 units
11. 23%
12 $k = M^\circ L^\circ T$
13. 7×10^{-2} N/m
14. Work
15. The maximum error will be due to (i) measurement, which is least accurate.
(ii) measurement of the quantity which has maximum power in formula's.

Level – I

1. 3.5 g/cc
2. $M^\circ L^\circ T^\circ$, $M^\circ L^2 T^\circ$
3. T^{-4}
4. $(8 \pm 2)\,\Omega$
5. 1400
6. 5.7%
7. $(hc)^{1/2}\,G^{-1/2}$, $(hG)^{1/2}c^{-3/2}$, $(hG)^{1/2}\,c^{-5/2}$
8. $\frac{1}{\sqrt{9.8}}$ sec
9. 360
10. 2.25 mm
11. 1.02 cm
12. $\pm$ 5%
13. (a) 0.01mm (b) + 0.04 mm, to be subtracted (c) 7.33 mm
14. $\frac{E}{\hbar\omega} = \frac{1}{2}\frac{mv^2}{\hbar\omega} + \frac{1}{2}\frac{\omega m x^2}{\hbar}$, $\sqrt{m\omega\hbar}$, $\sqrt{\frac{\hbar}{m\omega}}$
15. L^2T^{-1}

Objective:

1.	C	2.	D	3.	D	4.	D
5.	C	6	A	7.	D	8.	C
9.	C	10.	C	11	A	12.	D
13.	D	14.	B	15.	A	16.	C
17.	B	18.	D	19.	B	20.	C

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

VECTORS

Vectors

Syllabus for IITJEE and ISC Board:
Scalar and vector quantities; Position and displacement vectors, general vectors and notation; Equality of vectors, multiplication of vectors by a real number; Addition and subtraction of vectors; Unit vector, Resolution of a vector in a plane Rectangular components, Multiplication of vectors-scalar and vector products; vectors in three dimensions (elementary idea only).

VECTORS

Definition: The physical quantities specified completely by their magnitude as well as direction are called vector quantities. The magnitude and direction alone cannot decide whether a physical quantity is a vector. In addition to the above characteristics, a physical quantity, which is a vector, should follow law of vector addition. For example, electric current has magnitude as well as direction, but does not follow law of vector addition. Hence, it is not a vector.

A vector is represented by putting an arrow over it. The length of the line drawn in a convenient scale represents the magnitude of the vector. The direction of the vector quantity is depicted by placing an arrow at the end of the line.

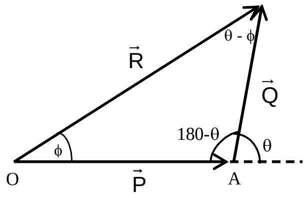
If two vectors have the same direction, they are parallel. Two vectors are said to be equal when their magnitudes and directions, both are same, e.g. if $\vec{a} = \vec{b}$ then $|\vec{a}| = |\vec{b}|$ and the directions of vectors are same. Thus, a vector is not altered by shifting it parallel to itself in the space.

The vector having same magnitude as of $\vec{a}$, but the opposite direction is defined as the negative or opposite of $\vec{a}$ and is denoted by $-\vec{a}$.

Laws of Addition of Vectors: Two or more vectors can be added to give another vector, which is called the resultant of the vectors. The resultant would produce the same effect as that of the original vectors together.

Triangle law of addition of vectors: If two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant will be represented in magnitude and direction by the third side of the triangle taken in reverse order.

To add the two vectors $\vec{P}$ and $\vec{Q}$, the vectors are drawn with the tail of $\vec{Q}$ coinciding with the terminus of $\vec{P}$. The vector sum i.e. the resultant vector $\vec{R}$ which completes the triangle drawn from the tail O of $\vec{P}$ to the terminus B of $\vec{Q}$ as shown in figure.

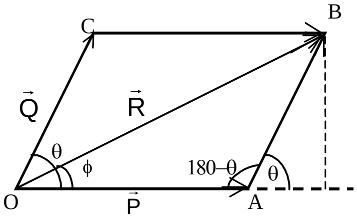


Parallelogram law of addition of vectors:

If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.

We get , $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\phi = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)$$



where ϕ is the angle that the resultant makes with $\vec{P}$

(i) $\theta = 0^o$

$\vec{P}$ and $\vec{Q}$ are in the same direction i.e. they are parallel $\cos 0^o = 1$

$\therefore |\vec{R}| = |\vec{P}| + |\vec{Q}|$ & $\phi = 0^o$

(ii) $\theta = 180^o$, $\vec{P}$ and $\vec{Q}$ are in opposite direction i.e. they are antiparallel $\cos 180^o = -1$

$\therefore |\vec{R}| = |\vec{P}| \sim |\vec{Q}|$ and $\vec{R}$ is in the direction of the larger vector.

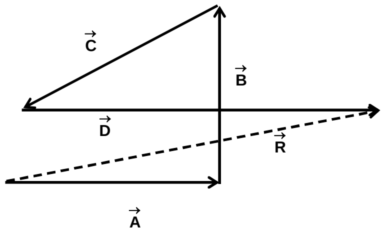
(iii) $\theta = 90^o$, $\cos 90^0 = 0$

$\vec{P}$ and $\vec{Q}$ are perpendicular to each other

$\therefore |\vec{R}| = (|\vec{P}|^2 + |\vec{Q}|^2)^{1/2}$ & $\phi = \tan^{-1}(Q/P)$

Polygon law of addition of vectors

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

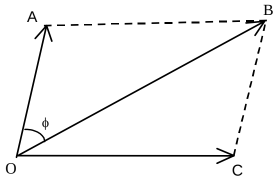


Vectors obey commutative law

i.e. $\vec{A}+\vec{B} = \vec{B}+\vec{A}$

Illustration 1. *Two forces of 60N and 80N acting at an angle of 60^0 with each other, pull an object. What single pull would replace the given forces?*

Solution: *Two forces are drawn from a common origin O, making an angle of 60^0. OA and OC represent the forces 60N and 80N respectively. The diagonal OB represents the resultant R.*



$\therefore R^2 = 60^2 + 80^2 + 2.60.80\cos 60^0$

$= 3600 + 6400 + 4800 = 14800$ $\therefore R = 121.7N$

Angle ϕ is given, $\tan\phi = \dfrac{80\sin 60^0}{60+80\cos 60^0}$

Which gives, $\phi = 34.7^0$

Exercise 1:

i) ***Is it possible that the resultant of two equal forces is equal to one of the forces?***

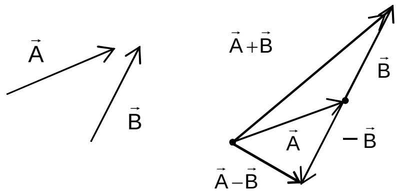
ii) ***If a vector has zero magnitude is it meaningful to call it a vector?***

iii) ***Can three vectors, not in one plane, give a zero resultant? Can four vectors do?***

Subtraction of Vectors:

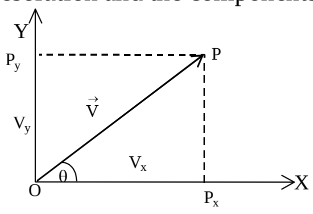
When a vector B is reversed in direction, then the reversed vector is written as - $\vec{B}$ then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Resolution of Vectors: If $\vec{P}+\vec{Q}=\vec{R}$, the resultant, then conversely $\vec{R}=\vec{P}+\vec{Q}$ i.e. the vector $\vec{R}$ can be split up so that the vector sum of the split parts equals the original vector $\vec{R}$. If the split parts are mutually perpendicular then they are known as components of $\vec{R}$ and this process is known as resolution. The orthogonal component of any vector along another direction which is at an angular separation θ is the product of the magnitude of the vector and cosine of the angle between them (θ). Therefore the component of $\vec{A}$ is A cosθ.

Note: In physics, resolution gives unique and mutually independent components only if the resolved components are mutually perpendicular to each other. Such a resolution is known as rectangular or orthogonal resolution and the components are called rectangular or orthogonal components.



O – the origin, OP – the given vector $\vec{V}$

PP_x – perpendicular to X axis.

PP_y – Perpendicular to Y axis.

$$\overrightarrow{OP_x} + \overrightarrow{P_xP} = \overrightarrow{OP} = \vec{V}$$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$V_x = V\cos\theta$ & $V_y = V\sin\theta$

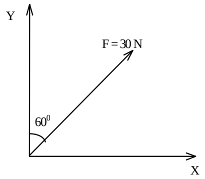
Illustration 2. *A force of 30 N is acting at an angle of 60^0 with the y-axis. Determine the components of the forces along x and y-axes.*

Solution :

$$F_x = F\sin 60^0$$

$$= \frac{30 \text{ x } \sqrt{3}}{2} = 15\sqrt{3}\text{ N}$$

$$F_y = F\cos 60^0 = 30 \text{ x} \frac{1}{2} = 15\text{ N}$$



Unit Vector: In order to make the algebraic operations with vectors simple, a given vector is expressed as a product of its magnitude and direction vector. Since the product should have the same magnitude, the direction vector having unit magnitude and is called unit vector.

A unit vector is not a physical quantity but represents only a given direction. Unit vector along the direction of $\vec{A}$ is $\hat{A}=\vec{A}$/A, Where A is magnitude of $\vec{A}$. $\hat{i}$, $\hat{j}$, $\hat{k}$, are the unit vectors along positive direction of X, Y and Z axis respectively, then the rectangular resolution of vector $\vec{A}$ can be represented.

$\vec{A}= A_x\hat{i}+A_Y\hat{j}+A_Z\hat{k}$ where A_X, A_Y, A_Z are the magnitudes of X, Y and Z components of $\vec{A}$. The magnitude of vector $\vec{A}$ is given by $|\vec{A}|=\sqrt{A_x^2+A_y^2+A_z^2}$.

Illustration 3. *Find the net displacement of a particle from its starting point if it undergoes three successive displacements given $\vec{S_1}$ = 20 m, 45^0 West of North, $\vec{S_2}$ = 15 m, 30^0 North of East; $\overrightarrow{S_3}$ =20m, due South.*

Solution: Let us set our axial system such that x-axis is along West-East and y-axis along South-North.

$\Rightarrow \ \vec{S_1} = 20\cos 45^0(-\hat{i}) + 20\sin 45^0(\hat{j})$

and $\vec{S_2} = 15\cos 30^0(\hat{i}) + 15\sin 30^0(\hat{j})$

$\vec{S_3} = 0(\hat{i}) + 20(-\hat{j})$

$\vec{S} = \vec{S_1} + \vec{S_2} + \vec{S_3}$

$= \left(-\frac{20}{\sqrt{2}} + \frac{15\sqrt{3}}{2} + 0\right)\hat{i} + \left(\frac{20}{\sqrt{2}} + \frac{15}{2} - 20\right)\hat{j}$

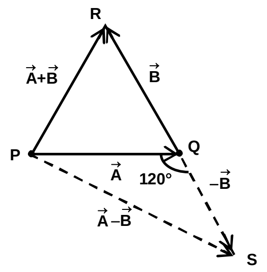
$= -1.15\hat{i} + 1.64\hat{j} = S_x\hat{i} + S_y\hat{j}$

$|\vec{S}| = \sqrt{S_x^2 + S_Y^2} = \sqrt{(-1.15)^2 + (1.64)^2} = 2\text{m}$

Direction $\theta = \tan^{-1}\frac{1.15}{1.64} = 35^0$ West of North

Illustration 4. *If the sum of two unit vectors $\vec{A}$ and $\vec{B}$ is also equal to a unit vector, find the magnitude of the vector $\vec{A} - \vec{B}$.*

Solution: Given that $|\vec{A}| = |\vec{B}| = |\vec{A} + \vec{B}| = 1$



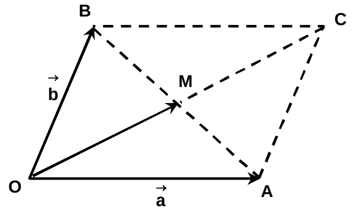
Hence the angle between $\vec{A}$ and $\vec{B}$ is 120°

Now $|\vec{PS}|^2 = |\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}||-\vec{B}|\cos 120°$

$= 1 + 1 + 2\times1\times(-1)\left(-\frac{1}{2}\right) = 3$

$\Rightarrow PS = \sqrt{3}$

Illustration 5. *If the position vector of point A and B are $\vec{a}$ and $\vec{b}$ respectively. Find the position vector of middle point of AB.*



Solution: $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

$\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OM}$

$\vec{a} + \vec{b} = 2\overrightarrow{OM}$

$\overrightarrow{OM} = \frac{1}{2}(\vec{a} + \vec{b})$

Multiplication of Vectors: Vector multiplications are of two types. One, in which we obtain a scalar quantity and the other in which we obtain a vector quantity on multiplication. The first one is called Dot Product and the other is called Cross Product.

Scalar multiplication: $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$ where θ is the angle between the two vectors, when placed tail to tail.

For $\theta = 90^0$, $\cos\theta = 0$ then $\vec{A}.\vec{B} = 0$

Now for orthogonal unit vectors, $\hat{i}\ .\ \hat{j} = \hat{j}\ .\ \hat{k} = \hat{k}\ .\ \hat{i} = 0$

Again for $\theta = 0^0$, $\cos\theta = 1$ then $\vec{A}.\vec{B} = AB$

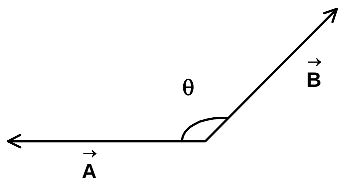
For orthogonal unit vectors, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

Let there be two vectors given by

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}.\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right).\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= A_xB_x + A_yB_y + A_zB_z$$



(in the same line)

Dot product is commutative. i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Angle between two vectors:

As we know $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$$

$$= \frac{A_xB_x + A_yB_y + A_zB_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)\left(\sqrt{B_x^2 + B_Y^2 + B_Z^2}\right)}$$

Direction cosines:

If vector $\vec{A}$ makes angle α, β and γ with x, y and z axes respectively, then

$$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{A_x}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\beta = \frac{A_y}{|\vec{A}|} = \frac{A_y}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\gamma = \frac{A_z}{|\vec{A}|} = \frac{A_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

where cos α, cos β and cos γ are called direction cosines of the vector $\vec{A}$. The unit vector in the direction of $\vec{A}$ is

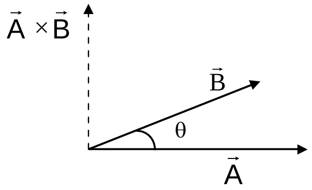
$$\hat{n} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{|\vec{A}|}$$

$\hat{n} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$. (Note: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$)

Cross product:

The cross product of two vectors $\vec{A}$ and $\vec{B}$ inclined to each other by an angle θ is given by

$\vec{A}$ x $\vec{B}$ = $\vec{C}$, a vector. where $\vec{C} = |\vec{A}|.|\vec{B}|\sin\theta\hat{n}$, where $\hat{n}$ is the unit vector along a direction which is perpendicular to plane containing $\vec{A}$ & $\vec{B}$. Its direction is given by the right hand thumb rule, or right hand screw rule.



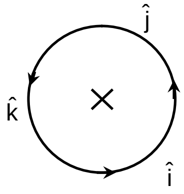
If the vectors $\vec{A}$ and $\vec{B}$ lie in the x-y plane then the product is perpendicular to the plane i.e. is parallel to z-axis.

The vector product is not commutative i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ and $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

In terms of orthogonal vectors $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$$\hat{i}\times\hat{j} = \hat{k} = -(\hat{j}\times\hat{i}),\quad \hat{j}\times\hat{k} = \hat{i} = -\left(\hat{k}\times\hat{j}\right),$$

$$\left(\hat{k}\times\hat{i}\right)=\hat{j}=-\left(\hat{i}\times\hat{k}\right)$$



If $\vec{A}=A_X\hat{i}+A_Y\hat{j}+A_Z\hat{k}$, $\vec{B}=B_X\hat{i}+B_Y\hat{j}+B_Z\hat{k}$

Then, $\vec{A}\times\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right) x \left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$

$$= \left(A_YB_Z - A_ZB_Y\right)\hat{i}+\left(A_ZB_X - A_XB_Z\right)\hat{j}+\left(A_XB_Y - A_YB_X\right)\hat{k}$$

In determinant form we have,

Then, $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_X & A_Y & A_Z\\ B_X & B_Y & B_Z\end{vmatrix}$

Cross Product of two parallel or anti parallel vectors is zero.

Note: *Division of a vector by a vector is not defined.*

Exercise 2:

(i) ***Calculate the angle between a two dyne and a three dyne force so that their sum is four dyne.***

(ii) ***Resultant of two forces which have equal magnitudes and which act at right angles to each other is 1414 dyne. Calculate the magnitude of each forces.***

(iii) ***Find the direction cosines of*** $5\hat{i}+2\hat{j}+4\hat{k}$

(iv) ***Given:*** $\vec{a}=3\hat{i}+4\hat{j}$ ***and*** $\vec{b}=3\hat{j}+4\hat{k}$ ***, Calculate the magnitude of the resultant.***

(v) ***One of the rectangular component of an acceleration of 8 m/s^2 is 4 m/s^2, calculate the other component.***

(vi) ***Find the unit vector in the direction of*** $3\hat{i}+4\hat{j}-\hat{k}$

Illustration 6. *If the magnitudes of the dot product and cross product of two vectors are equal, find the angle between the two vectors.*

Solution: $|\vec{A}\times\vec{B}|=|\vec{A}\cdot\vec{B}|$

A.B sinθ = A.B cosθ $\Rightarrow$ tanθ = 1

$\Rightarrow \theta = 45^0$.

Illustration 7. *If* $\vec{a} = 3\hat{i} + 4\hat{j}$ *and* $\vec{b} = 7\hat{i} + 24\hat{j}$, *find the vector having the same magnitude as* $\vec{b}$ *and parallel to* $\vec{a}$.

Solution: Magnitude of $\vec{a}=\left|\vec{a}\right|= \sqrt{3^2+4^2} = 5$

And magnitude of $\vec{b} = \left|\vec{b}\right| = \sqrt{7^2+24^2} = 25$

Now a unit vector parallel to $\vec{a} = \hat{a} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

$\therefore$ The vector having the same magnitude as $\vec{b}$ and parallel to $\vec{a}$

$= 25\ \hat{a} = 15\hat{i} + 20\hat{j}$

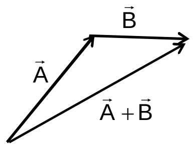
SUMMARY

1. Scalar quantities are quantities with magnitudes only and combine with the usual rules of arithmetic e.g. speed, mass and temperature.

2. Vector quantities have magnitude as well as direction and combine according to the rules of vector addition. e.g. velocity and acceleration.

3. $\vec{B} = \lambda\vec{A}$
Where λ is a real number. Magnitude of B is λ time the magnitude of A and direction is same as that of A. (If λ is positive).

4. Graphically, two vectors $\vec{A}$ and $\vec{B}$ are added by placing the tail of $\vec{B}$ at the head of $\vec{A}$. The vector sum $\vec{A}+\vec{B}$ then extends from the tail of $\vec{A}$ to the head of $\vec{B}$



5. Vector addition is
$\vec{A}+\vec{B} = \vec{B}+\vec{A}$ (Commutative)
$\left(\vec{A}+\vec{B}\right)+\vec{C} = \vec{A}+\left(\vec{B}+\vec{C}\right)$ (Associative law)

6. A vector with zero magnitude is called null vector and
$\vec{A}+\vec{0} = \vec{A}$
$\lambda\vec{0} = \vec{0}$

7. Subtraction of vectors
$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

8. Unit vectors describe directions in space. A unit vector has a magnitude of one, with no units.
The unit vectors $\hat{i},\hat{j},\hat{k}$ are vectors of unit magnitude and points in the direction of the x, y and z axes, respectively, in a right – handed coordinate system.

10. vector $\vec{A}$ can be expressed as $\vec{A} = A_X\hat{i} + A_y\,\hat{j}$ having magnitude $=\sqrt{A_x^2 + A_y{}^2}$ and angle θ with the x – axis = $\tan^{-1}\dfrac{A_y}{A_x}$.

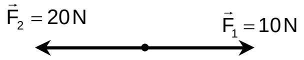
11. Scalar product of two vectors, $C = \vec{A}\cdot\vec{B} = AB\cos\phi$, where ϕ is the angle between two vectors and scalar product of two vectors is a scalar quantity. Scalar products obey the commutative and the distributive laws.

12. Cross – product of two vectors $\vec{A}$ and $\vec{B}$ is a vector quantity. $\vec{C} = \vec{A}\times\vec{B} = AB\sin\phi\ \hat{n}$ and its direction is given by right hand rule, $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ (non – commutative)

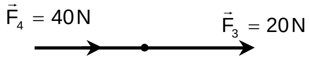
MISCELLANEOUS EXERCISE

1. Find the magnitude and direction of resultant vectors as shown in the figures below.

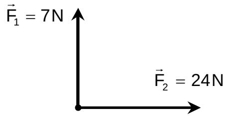
(a)



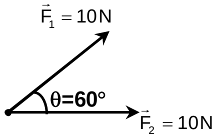
(b)



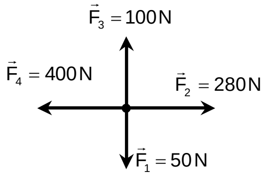
(c)



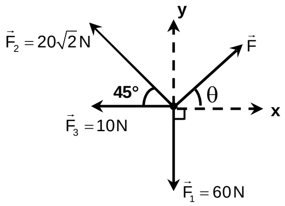
(d)



2. (a) In the adjacent figure, find the magnitude and direction of the resultant vector.



(b) In the adjacent figure, find the value of F and θ so that the sum of the vectors will be zero.



3. Show that the vectors $\vec{A} = 12\hat{i} - 10\hat{j} + 2\hat{k}$ and $\vec{B} = 4\hat{i} + 8\hat{j} + 16\hat{k}$ are perpendicular to each other.

4. Resultant of two vectors of equal magnitude makes an angle 60° with one of the vectors. Find the angle between the vectors.

5. If $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = 4\hat{k}$ find the value of $\vec{a}\cdot\vec{b}$ and $\vec{a}\times\vec{b}$. Also find the angle between $\vec{a}$ and $\vec{b}$.

6. If $\vec{A} = 3\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$, find $|\vec{A}|$, $|\vec{B}|$ and $|\vec{A} + \vec{B}|$. Also find the direction of $\vec{A} + \vec{B}$ with the x-axis. Check whether $|\vec{A}| + |\vec{B}|$ is equal to $|\vec{A} + \vec{B}|$.

7. Check whether the two vectors, $\vec{A} = -3\hat{i} - 7\hat{j} + 9\hat{k}$ and $\vec{B} = -2\hat{i} - 21\hat{j} + 6\hat{k}$ are parallel to each other.

8. Two vectors are given by $\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} + \hat{k}$. Find the magnitude and direction cosines of $\vec{A} + \vec{B}$.

9. The components of a vector $\vec{A}$ along x-axis and y-axis are 4 unit and 6 unit respectively. If the components of vector $\vec{A} + \vec{B}$ along x-axis and y-axis are 10 unit and 14 unit respectively, find the vector $\vec{B}$ and its direction with the x-axis.

10. (a) Find the unit vector which is parallel to the vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$.

(b) Find the unit vector which is perpendicular to both of the vectors $\vec{A} = 2\hat{i}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 12\hat{k}$.

SOLUTION TO MISCELLANEOUS EXERCISE

1. (a) 10 N (towards $\vec{F}_2$)
   (b) 60 N (towards $\vec{F}_3$)
   (c) 25 N, at an angle $\tan^{-1}\left(\frac{7}{24}\right)$ from $\vec{F}_2$ towards $\vec{F}_1$
   (d) $10\sqrt{3}$ at an angle 30° from $\vec{F}_2$ towards F_1

2. (a) 130 N, $\tan^{-1}\left(\frac{5}{12}\right)$ (from $\vec{F}_4$ towards $\vec{F}_3$)
   (b) 50 N, $\theta = 53°$

4. 120^0

5. 0, $4\left(2\hat{i} - 3\hat{j}\right)$, 90°

6. $\sqrt{14}$, $2\sqrt{6}$ and $\sqrt{30}$ N, $\cos^{-1}\left(\sqrt{\frac{5}{6}}\right)$; No

7. No

8. $\sqrt{90}$, $\frac{7}{\sqrt{90}}$, $\frac{4}{\sqrt{90}}$, $\frac{5}{\sqrt{90}}$

9. $\vec{B} = 6\hat{i} + 8\hat{j}$ and $\theta = 53°$

10. (a) $\frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}}$ (b) $\frac{\left(-3\hat{j} + \hat{k}\right)}{\sqrt{10}}$

SOLVED PROBLEMS

Subjective:

Prob 1. *Find the component of a vector* $\vec{A} = 3\hat{i} + 2\hat{j}$ *along the direction of* $(\hat{i} + \hat{j})$.

Sol. Unit vector along $(\hat{i}+\hat{j})$ is $\hat{n} = \dfrac{\hat{i}+\hat{j}}{(1^2+1^2)^{1/2}} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore$ The magnitude of the component of vector $\vec{A}$ along the $(\hat{i}+\hat{j})$ is

$$\vec{A}\cdot\hat{n} = (3\hat{i}+2\hat{j})\cdot\frac{\hat{i}+\hat{j}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(3+2) = \frac{5}{\sqrt{2}}$$

$\therefore$ The component vector of $\vec{A}$ along the $(\hat{i}+\hat{j})$ is

$$\vec{A}_1 = \frac{5}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j}$$

Prob 2. *At what angle do the forces* $(\vec{A}+\vec{B})$ *and* $(\vec{A}-\vec{B})$ *act so that the magnitude of their resultant is* $\sqrt{3A^2+B^2}$ *?*

Sol. $R^2 = 3A^2 + B^2 = (A + B)^2 + (A - B)^2 + 2(A^2 - B^2)\cos\theta$

where θ is the angle between (A + B) and (A–B)

or, $A^2 - B^2 = 2(A^2 - B^2)\cos\theta$

or, $\cos\theta = 1/2 \Rightarrow \theta = 60°$

Prob 3. *The resultant of two non-zero forces* $\vec{P}$ *and* $\vec{Q}$ *is of magnitude P. Prove that if* $\vec{P}$ *is doubled, the resultant force will be perpendicular to Q.*

Sol. $P^2 = P^2 + Q^2 + 2PQ\cos\theta \Rightarrow Q + 2P\cos\theta = 0$

Now, if $\vec{P}$ is doubled then

$\tan\alpha = \dfrac{2P\sin\theta}{Q+2P\cos\theta}$ [where α = angle made by the resultant with Q]

$= \dfrac{2P\sin\theta}{0} = \infty$ (undefined)

$\Rightarrow \alpha = 90°$

Prob 4. $\vec{A}$, $\vec{B}$ *and* $\vec{C}$ *satisfy the relation* $\vec{A}.\vec{B} = 0$ *and* $\vec{A}.\vec{C} = 0$. *To which vector, the vector* $\vec{A}$ *is parallel.*

Sol. $\vec{A}.\vec{B} = 0$ so $\vec{A}$ is perpendicular to $\vec{B}$

$\vec{A}.\vec{C} = 0$ so $\vec{A}$ is perpendicular to $\vec{C}$

But $(\vec{B}\times\vec{C})$ is a vector which is perpendicular to both $\vec{B}$ and $\vec{C}$.

So, $\vec{A}$ is parallel to $(\vec{B}\times\vec{C})$.

Prob 5. *Prove that* $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$.

Sol. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}$ $[\because \ \vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0$ and $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}]$

$= 2(\vec{a}\times\vec{b})$

Prob 6. *Given* $\vec{A}.\vec{B}=\vec{A}.\vec{C}$. *Is it correct to conclude* $\vec{B}=\vec{C}$ *?*

Sol. $\vec{A}.\vec{B}=\vec{A}.\vec{C}$

i.e. $AB\cos\theta_1 = AC\cos\theta_2$

[where θ_1 and θ_2 are the angles formed by $\vec{B}$ and $\vec{C}$ respectively with $\vec{A}$.]

or, $B\cos\theta_1 = C\cos\theta_2$

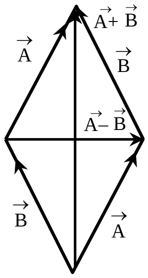
Now, if $\theta_1=\theta_2$ then $\vec{B}=\vec{C}$

But if $\theta_1\neq\theta_2$ then $\vec{B}\neq\vec{C}$

So, we can't conclusively say that $\vec{B}=\vec{C}$.

Prob 7. *Find the area of a parallelogram whose diagonals are represented by* $(3\hat{i}+\hat{j}+\hat{k})$ *and* $(\hat{i}-\hat{j}-\hat{k})$.

Sol. Let $\vec{A}$ and $\vec{B}$ be the two adjoining sides of the parallelogram drawn from a point. The area of the parallelogram = $\vec{A}\times\vec{B}$



Given that $\vec{A}+\vec{B}=3\hat{i}+\hat{j}+\hat{k}$

and $\vec{A}-\vec{B}=\hat{i}-\hat{j}-\hat{k}$

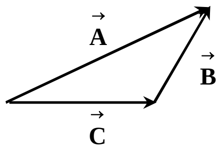
Now, $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})=-2(\vec{A}\times\vec{B})$

$\therefore$ $\vec{A}\times\vec{B}=-\frac{1}{2}[(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})]$

$$=-\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 1\\ 1 & -1 & -1\end{vmatrix}$$

$$=-\frac{1}{2}\left(0\hat{i}+4\hat{j}-4\hat{k}\right)=-2\hat{j}+2\hat{k}$$

Prob 8. *Prove that the vectors* $\vec{A}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=-\hat{i}+3\hat{j}+4\hat{k}$ *and* $\vec{C}=4\hat{i}-2\hat{j}-6\hat{k}$, *can form a triangle.*



Sol. Let us add any two of the given vectors, say $\vec{C}$ and $\vec{B}$

$\vec{C}+\vec{B}=\left(4\hat{i}-2\hat{j}-6\hat{k}\right)+\left(-\hat{i}+3\hat{j}+4\hat{k}\right)=3\hat{i}+\hat{j}-2\hat{k}=\vec{A}$

As the sum of two vectors is equal to the third vector, the three vectors can form a triangle.

Prob 9. *If* $\vec{A}=3\hat{i}+7\hat{j}$ *and* $\vec{B}=2\hat{i}+5\hat{j}$, *find the angle between* $\vec{A}$ *and* $\vec{B}$.

Sol. $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

or, $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{6+35}{\sqrt{58}\sqrt{29}} = 0.9997$ [$\because$ $|\vec{A}| = \sqrt{9+49}$ & $|\vec{B}| = \sqrt{4+25}$

$\therefore$ $\theta = 1°24'$

Prob 10. *If $\vec{A} = 3\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{B} = 4\hat{i}+2\hat{j}+\hat{k}$, find the magnitude and direction cosines of $(\vec{A}+\vec{B})$.*

Sol. $\vec{A}+\vec{B} = \left(3\hat{i}+2\hat{j}+4\hat{k}\right)+\left(4\hat{i}+2\hat{j}+\hat{k}\right) = 7\hat{i}+4\hat{j}+5\hat{k}$

$\therefore |\vec{A}+\vec{B}| = \sqrt{7^2+4^2+5^2} = \sqrt{90}$

$\therefore$ The direction cosines of $\vec{A}+\vec{B}$ are

$$\cos\alpha = \frac{\left|\vec{A}+\vec{B}\right|_x}{\left|\vec{A}+\vec{B}\right|} = \frac{7}{\sqrt{90}}$$

$$\cos\beta = \frac{\left|\vec{A}+\vec{B}\right|_y}{\left|\vec{A}+\vec{B}\right|} = \frac{4}{\sqrt{90}}$$

$$\cos\gamma = \frac{\left|\vec{A}+\vec{B}\right|_z}{\left|\vec{A}+\vec{B}\right|} = \frac{5}{\sqrt{90}}$$

Prob 11. *Prove that the vectors $\vec{A} = 2\hat{i}-3\hat{j}-\hat{k}$ and $\vec{B} = -6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.*

Sol. The given vectors will be parallel if their cross product is zero. Because if the two vectors are $\vec{A}$ and $\vec{B}$ then

$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n} = AB\sin 0°\ \hat{n}$ [if they are parallel then the angle between $\vec{A}$ and $\vec{B}$ is 0°]

$\therefore$ $\vec{A}\times\vec{B} = 0$

Now, $\left(2\hat{i}-3\hat{j}-\hat{k}\right)\times\left(-6\hat{i}+9\hat{j}+3\hat{k}\right) = (9-9)\hat{i}+(6-6)\hat{j}+(18-18)\hat{k} = 0$

Hence, the two vectors $\vec{A}$ and $\vec{B}$ are parallel to each other.

$\vec{B} = -3\vec{A} \Rightarrow \vec{B} \parallel A$.

Prob12. *For a point P (2, 4, –5) in a three dimensional co-ordinate system, find*

(a) the position vector $\vec{r}$ of point P.

(b) $|\vec{r}|$

(c) the direction cosines of vector $\vec{r}$.

Sol. (a) $\vec{r} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

(b) $|\vec{r}| = \sqrt{r_x^2+r_y^2+r_z^2}$

$= \sqrt{(2)^2+(4)^2+(-5)^2} = \sqrt{45}$

(c) $\cos\alpha = \dfrac{r_x}{|\vec{r}|} = \dfrac{2}{\sqrt{45}}$

$\cos\beta = \dfrac{r_y}{|\vec{r}|} = \dfrac{4}{\sqrt{45}}$

and $\cos\gamma = \dfrac{r_z}{|\vec{r}|} = -\dfrac{5}{\sqrt{45}}$

Prob13. *Under the action of a force $\left(10\hat{i} - 3\hat{j} + 6\hat{k}\right)$N, a body of mass 5 kg moves from position $\left(6\,\hat{i} + 5\,\hat{j} - 3\,\hat{k}\right)$m to a position $\left(10\,\hat{i} - 2\,\hat{j} + 7\,\hat{k}\right)$ m. Deduce the work done.*

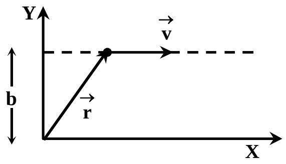
Sol. Displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= \left(10\hat{i} - 2\hat{j} + 7\hat{k}\right) - \left(6\hat{i} + 5\hat{j} - 3\hat{k}\right)$$
$$= \left(4\hat{i} - 7\hat{j} + 10\hat{k}\right) \text{m}$$

Work $W = \vec{F}\,.\,\vec{s}$

$$= \left(10\hat{i} - 3\hat{j} + 6\hat{k}\right).\left(4\hat{i} - 7\hat{j} + 10\hat{k}\right)$$
$$= 40 + 21 + 60 = 121 \text{ J}$$

Prob 14. *If a particle of mass m is moving with a constant velocity $\vec{v}$ parallel to the x-axis in x-y plane as shown in the figure, calculate its angular momentum w.r.t. origin at any time t.*



Sol. We know $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$

As the motion is in x-y plane, z = 0 and $p_z = 0$

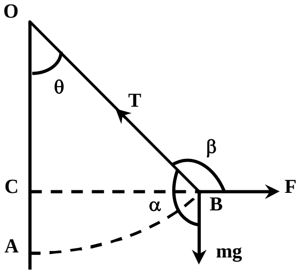
$\therefore$ $\vec{L} = \hat{k}\,(xp_y - yp_x)$

Hence $x = vt$, $y = b$, $p_x = mv$ and $p_y = 0$

$\therefore$ $\vec{L} = \hat{k}\,(vt \times 0 - bmv) = -mvb\,\hat{k}$

Prob15. *A bob weighing 50 gm hangs vertically at the end of a string 50 cm long. If 20 gm force is applied horizontally on the bob, by what distance is the bob pulled aside from its initial position when it reaches its equilibrium position?*

Sol. Let the bob be in equilibrium when it is pulled to B.



$$\frac{F}{\sin\alpha} = \frac{mg}{\sin\beta} = \frac{T}{\sin 90^\circ}$$

$$\frac{F}{\sin(\pi - \theta)} = \frac{mg}{\sin\left(\frac{\pi}{2} + \theta\right)} = T$$

or, $$\frac{F}{\sin\theta} = \frac{mg}{\cos\theta} = T$$

$\therefore$ $\frac{F}{mg} = \tan\theta = \frac{20 \times 980}{50 \times 980} = 0.4 = \tan 21^\circ 48'$

$\therefore$ $\theta = 21^\circ 48'$

$\therefore$ $CB = OB\sin\theta = OB \sin 21^\circ 48'$

$= 50 \times 0.3714 = 18.57$ cm

Prob16. *The x and y components of vector $\vec{A}$ are 4 m and 6 m, respectively. If the x and y components of vector ($\vec{A} + \vec{B}$) are 10 m and 9 m, respectively, calculate*

(a) x and y components of vector $\vec{B}$.

(b) its length and

(c) the angle made by vector $\vec{B}$ with the x-axis.

Sol. In terms of components

$\vec{A} + \vec{B} = (\hat{i}A_x + \hat{j}A_y) + (\hat{i}B_x + \hat{j}B_y)$

$\therefore\ \vec{A} + \vec{B} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y)$

According to the given Problem

$A_x + B_x = 10$ m and $A_y + B_y = 9$ m

As $A_x = 4$ m and $A_y = 6$ m (given)

(a) $B_x = 6$ m and $B_y = 3$ m

(b) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$ m

(c) $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

Objective:

Prob 1. *A force* $\vec{F} = (4\hat{i} - 5\hat{j} + 3\hat{k})N$ *is acting at a point having a position vector* $\vec{r}_1 = (\hat{i} + 2\hat{j} + 3\hat{k})$. *The torque acting about a point having a position vector* $\vec{r}_2 = (3\hat{i} - 2\hat{j} - 3\hat{k})$, *is*

(A) $42\hat{i} + 30\hat{j} - 6\hat{k}$ *(B)* $42\hat{i} + 30\hat{j} + 6\hat{k}$
(C) $42\hat{i} - 30\hat{j} + 6\hat{k}$ *(D) zero.*

Sol. Calculate torque $(\vec{\tau}) = \vec{r} \times \vec{F}$
Where $\vec{r} = \vec{r}_1 - \vec{r}_2$
Hence (A) is correct.

Prob 2. *The area of a parallelogram formed by the vectors* $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ *and* $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$ *as adjacent sides is*

(A) $8\sqrt{3}$ *units.* *(B) 64 units*
(C) 32 units. *(D)* $\sqrt{3}$ *units*

Sol. Calculate $\vec{A} \times \vec{B}$
and Area of a parallelogram $= |\vec{A} \times \vec{B}|$
Hence (A) is correct.

Prob 3. *If vectors* $\vec{A}$ *and* $\vec{B}$ *are perpendicular to each other, then which of the following statements is valid?*

(A) $\vec{A} \times \vec{B} = \vec{A} . \vec{B}$ *(B)* $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = 0$
(C) $\vec{A} . \vec{B} = 0$ *(D)* $\vec{A} . \vec{B} = |\vec{A}| |\vec{B}|$

Sol. $\vec{A} \perp \vec{B}$. Then.
$\vec{A} . \vec{B} = |A||B|\cos 90° = 0$
Hence (C) is correct.

Prob 4. *Which of the rectangular pair may be the components of a 13 N force?*

(A) 5 N, 12 N *(B) 10 N, 11 N*
(C) 6.5 N, 6.5 N *(D) 9 N, 12 N*

Sol. Rectangular components will follow

$$R = \sqrt{R_x^2 + R_y^2}$$

$\therefore \quad 13^2 = 5^2 + 12^2$

Hence (A) is correct.

Prob 5. *If* $\vec{A}$ *and* $\vec{B}$ *are two mutually perpendicular vectors, where* $\vec{A} = 5\vec{i} + 7\hat{j} + 3\hat{k}$ *and* $\vec{B} = 2\vec{i} + 2\hat{j} - a\hat{k}$, *then the value of a is*

(A) –2 *(B) 8*
(C) –7 *(D) –8*

Sol. $\vec{A} \perp \vec{B}$

$\vec{A}\,.\,\vec{B} = 0 = (5\vec{i} + 7\hat{j} + 3\hat{k})(2\vec{i} + 2\hat{j} - a\hat{k}) = 10 + 14 - 3a$

$\therefore \quad 3a = 24 \Rightarrow a = 8$

Hence (B) is correct.

Prob 6. *The unit vector perpendicular to* $\vec{i} - 2\hat{j} + \hat{k}$ *and* $3\vec{i} + \hat{j} - 2\hat{k}$ *is*

(A) $\dfrac{5\vec{i}+3\hat{j}+7\hat{k}}{\sqrt{83}}$ (B) $\dfrac{3\vec{i}+5\hat{j}+7\hat{k}}{\sqrt{83}}$

(C) $\dfrac{5\vec{i}+3\hat{j}-7\hat{k}}{\sqrt{83}}$ (D) $\dfrac{3\vec{i}-5\hat{j}+7\hat{k}}{\sqrt{83}}$

Sol. $\vec{A} \times \vec{B}$ is a vector $\perp$ to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A} \times \vec{B} = (\vec{i} - 2\hat{j} + \hat{k}) \times (3\vec{i} + \hat{j} - 2\hat{k}) = 3\vec{i} + 5\hat{j} + 7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$= \dfrac{3\vec{i}+5\hat{j}+7\hat{k}}{\sqrt{3^2+5^2+7^2}} = \dfrac{3\vec{i}+5\hat{j}+7\hat{k}}{\sqrt{83}}$

Hence (B) is correct.

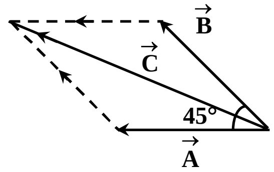
Prob 7. *For any two vectors* $\vec{A}$ *and* $\vec{B}$ *if* $\vec{A}\,.\,\vec{B} = |\vec{A} \times \vec{B}|$*, the magnitude of* $\vec{C} = \vec{A} + \vec{B}$ *is equal to*

(A) $\sqrt{A^2+B^2}$ (B) $A + B$

(C) $\left[A^2+B^2+\dfrac{AB}{\sqrt{2}}\right]^{1/2}$ (D) $(A^2 + B^2 + \sqrt{2} \times AB)^{1/2}$

Sol. $\vec{A}\,.\,\vec{B} = AB\cos\theta$ . . . (1)

$|\vec{A} \times \vec{B}| = AB\sin\theta\,\hat{n}$ . . . (2)



$\therefore \quad AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

Again given $\vec{C} = \vec{A} + \vec{B}$

$\therefore \quad |\vec{C}| = (A^2 + B^2 + 2AB\cos45°)^{1/2}$

$= (A^2 + B^2 + \sqrt{2}AB)^{1/2}$

Hence (D) is correct.

Prob 8. *What is the value of linear velocity, if* $\vec{\omega} = 3\vec{i} - 4\hat{j} + \hat{k}$ *and* $\vec{r} = 5\vec{i} - 6\hat{j} + 6\hat{k}$ *?*

(A) $6\vec{i} + 2\hat{j} - 3\hat{k}$ (B) $18\vec{i} + 13\hat{j} - \hat{k}$

(C) $4\vec{i} - 13\hat{j} + 6\hat{k}$ (D) $6\vec{i} - 2\hat{j} + 8\hat{k}$

Sol. $\vec{v}$ = tangential velocity $= \vec{r} \times \vec{\omega}$

$\vec{v} = (5\vec{i} - 6\hat{j} + 6\hat{k}) \times (3\vec{i} - 4\hat{j} + \hat{k})$

$= 18\vec{i} + 13\hat{j} - 2\hat{k}$

Hence (B) is correct.

Prob 9. *A particle moves in x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $\vec{p}$ at any time t is $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between $\vec{F}$ and $\vec{p}$ at given time t will be*

(A) $\theta = 0°$ *(B) $\theta = 30°$*
(C) $\theta = 90°$ *(D) $\theta = 180°$*

Sol. $P = \sqrt{P_x^2 + P_y^2} = 2\sqrt{\cos^2 t + \sin^2 t} = 2$, which is independent of t, which means the applied force is not changing the magnitude of velocity.

i.e. $\vec{F}$ is perpendicular to $\vec{p}$

Hence (C) is correct.

Prob10. *If $\hat{i}$ denotes a unit vector along an incident ray $\hat{r}$ the unit vector along the refracted ray in a medium of refractive index μ and $\hat{n}$ a unit vector normal to boundary of medium directed towards incident medium, law of refraction is*

(A) $\hat{i}.\hat{n} = \mu(\hat{r}.\hat{n})$ *(B) $\hat{i} \times \hat{n} = \mu(\hat{n} \times \hat{r})$*
(C) $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$ *(D) $\mu(\hat{i} \times \hat{n}) = \hat{r} \times \hat{n}$*

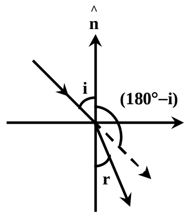
Sol. $|\hat{i} \times \hat{n}| = 1.1\sin(180° - i) = \sin i$

$|\hat{r} \times \hat{n}| = 1.1\sin(180 - r) = \sin r$

Now $\dfrac{|\hat{i} \times \hat{n}|}{|\hat{r} \times \hat{n}|} = \dfrac{\sin i}{\sin r} = \mu$

$\therefore\ \hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$, as their directions are also same.

Hence (C) is correct.



ASSIGNMENT PROBLEMS

Subjective:

1. Two forces whose magnitudes are in ratio of 3 : 5 give a resultant of 35 N. If the angle of inclination be 60°, calculate the magnitude of each force.

2. Find the unit vector of $3\hat{i}+4\hat{j}-\hat{k}$

3. Given $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. What is the vector component of $\vec{A}$ in the direction of $\vec{B}$?

4. Given $\vec{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+6\hat{k}$. Calcualte the angle made by $(\vec{A}+\vec{B})$ with x-axis.

5. Prove that the vectors $2\hat{i}-3\hat{j}-\hat{k}$ and $-6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.

6. Calculate the area of the parallelogram when adjacent sides are given by the vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-3\hat{j}+\hat{k}$.

7. What is the angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$?

8. Two vectors of magnitudes A and $\sqrt{3}$ A ar perpendicular to each other. What is the angle which their resultant makes with $\vec{A}$?

9. What should be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum ?

10. Find the direction cosines of $5\hat{i}+2\hat{j}+4\hat{k}$.

Objective:

1. Which of the following statement is correct?
   (A) A vector having zero length can have a unique direction.
   (B) If $\vec{A} \times \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
   (C) If $\vec{A} \cdot \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
   (D) The vector $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ are mutually perpendicular.

2. The magnitude of a given vector with end points (4, –4, 0) and (–2, –2, 0) must be
   (A) 6 (B) $5\sqrt{6}$
   (C) 4 (D) $2\sqrt{10}$

3. If the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units, respectively, and $\vec{A} + \vec{B} = \vec{C}$, then the angle between $\vec{A}$ and $\vec{B}$ is
   (A) zero. (B) π
   (C) $\pi/2$ (D) $\pi/4$

4. If $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j}$, then their dot product will be
   (A) 0 (B) 12
   (C) 8 (D) 16

5. A particle is acted upon by two forces of 3 N and 4 N simultaneously. Which of the following is most correct?
   (A) The resultant of these forces is 7 N.
   (B) The resultant of these forces is 1 N.
   (C) The resultant of these forces in 4 N.
   (D) The resultant of these forces lies between 1 N and 7 N

6. Find the value of c if $\vec{A} = 0.4\hat{i} + 0.3\hat{j} + c\hat{k}$ is a unit vector.
   (A) 0.5 (B) $\sqrt{0.75}$
   (C) 1 (D) none of these.

7. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. The vector $\vec{A}$ is parallel to
   (A) $\vec{B}$ (B) $\vec{C}$
   (C) $\vec{B} \cdot \vec{C}$ (D) $\vec{B} \times \vec{C}$

8. Angular momentum is
   (A) scalar. (B) an axial vector.
   (C) a polar vector. (D) a null vector.

9. Minimum number of forces having equal magnitudes, which can give a resultant zero, is
   (A) 2 (B) 4
   (C) 3 (D) 1

10. $\hat{i}\times(\hat{j}\times\hat{k})$ is
   (A) $\hat{i}+\hat{j}+\hat{k}$ (B) $\hat{i}+\hat{j}+\hat{k}$
   (C) zero vector (D) unit vector.

11. If $\vec{A} = 2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+4\hat{j}+\hat{k}$, then the unit vector along $(\vec{A}+\vec{B})$ is
   (A) $\dfrac{3\hat{i}+7\hat{j}+\hat{k}}{\sqrt{59}}$ (B) $\dfrac{2\hat{i}+3\hat{j}}{\sqrt{59}}$
   (C) $\dfrac{\hat{i}+4\hat{j}+\hat{k}}{\sqrt{18}}$ (D) $\dfrac{2\hat{i}+3\hat{j}}{\sqrt{13}}$

12. A particle moves under a force $\vec{F} = 2\hat{i}+4\hat{j}$ with a velocity $\vec{v} = 4\hat{i}-2\hat{j}$, then the power delivered by the force is
   (A) 16 W (B) zero
   (C) 8 W (D) $8\sqrt{2}$ W

13. If $\vec{A} = \vec{B}+\vec{C}$, and the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is
   (A) $\cos^{-1}(3/5)$ (B) $\cos^{-1}(4/5)$
   (C) $\pi/2$ (D) $\sin^{-1}(3/4)$

14. Given that $\vec{A}+\vec{B}+\vec{C}=0$. Out of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either vector having equal magnitudes. Then, the angle between the vectors is
   (A) 30°, 60°, 90° (B) 45°, 45°, 90°
   (C) 45°, 60°, 90° (D) 90°, 135°, 135°

15. The vector sum of N coplanar forces, each of magnitude F, when each force is making an angle of $2\pi/N$ with the preceding one is
   (A) $N\vec{F}$ (B) $N\vec{F}/2$
   (C) $\vec{F}/2$ (D) zero

16. At what angle two forces 2F and $\sqrt{2}$ F should act, so that the resultant force is F $\sqrt{10}$?
   (A) 45° (B) 60°
   (C) 120° (D) 90°

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

1. 15 N, 25 N
2. $\frac{3}{\sqrt{26}}\hat{i}+\frac{4}{\sqrt{26}}\hat{j}-\frac{1}{\sqrt{26}}\hat{k}$
3. 2.5 ($(\hat{i}+\hat{j})$
4. 45°
6. 13.96 sq. units.
7. 90°
8. 60°
9. $\theta = 0°$
10. $\frac{5}{\sqrt{45}},\frac{2}{\sqrt{45}},\frac{4}{\sqrt{45}}$

Objective:

1.	D	2.	D
3.	C	4.	A
5.	D	6.	B
7.	D	8.	B
9.	A	10.	C
11.	A	12.	B
13.	A	14.	D
15.	D	16.	A

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

KINEMATICS

Kinematics

Syllabus:
Motion in a straight line, Position–time graph, speed & velocity, Uniform and non-uniform motion, Average speed & instantaneous velocity. Uniformly accelerated motion, velocity–time, position–time graphs, Relations for uniformly accelerated motion (graphical treatment), Elementary concepts of differentiation and integration for describing motion. Motion in a plane, cases of uniform velocity and uniform acceleration, Projectile motion, uniform circular motion.

KINEMATICS

Rest and Motion
When an observer says that a particle is in motion, it means that the particle is changing its position with respect to the observer as time passes, otherwise, it is said to be at rest. For an observer standing near a lamp-post, the moving car is in motion but the building appears to be at rest. Same statement is not true for an observer in a moving car. For this observer, the co-passengers are at relative rest whereas the buildings or lamp-posts appear to be in state of motion. Thus, the states of rest or motion are relative terms, relative to the state of the observer. Thus, motion is a combined property of the object under observation and the observer as well.

In order to define motion empirically, we locate the position of the particle with respect to the origin of a coordinate system (x-y-z axis) at different times. Such a system comprising x-y-z coordinate axes with a clock (to measure time interval) is called *frame of reference*.

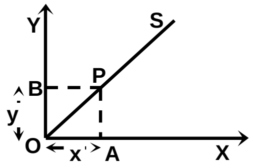
If all the three coordinates (x, y, z) of a particle P remain unchanged as time passes, we say that P is at rest relative to the frame. However, if any one or more co-ordinates changed with time, the particle is said to be in motion with respect to the frame.

Motion in a Straight Line
When a particle moves along a straight line (assuming along the x-axis of the reference frame), we need only one coordinate (here the x-coordinate) to specify its position. This is also known as motion in one dimension or one-dimensional motion or rectilinear motion of the particle.

The remaining two coordinates (y and z) remain unchanged as time passes. Motion of a particle projected vertically upward is one-dimensional motion. It is appropriate to mention here that for a particle moving in a plane along a curved path, two coordinates are required (say, x and y) to specify the position. Such motions are called motion in a plane or motion in two dimensions. Examples of two dimensional motion are: (i) circular motion, (ii) projectile motion, (iii) motion of an insect on table top along a curved path, etc.

Similarly, we require all the three coordinates (x, y and z) to locate the position of a mosquito flying in space. Such motions are called three-dimensional motion or motion in three dimensions. In this chapter, we shall describe the simplest kind of motion, i.e. the motion in a straight line only.

Illustration 1. *A particle P is moving along a straight line OS. The coordinates x = OA and y = OB are required to describe the motion of the particle. Does it indicate motion in one dimension or in two dimensions?*



Solution: Yes, it is a one-dimensional motion.

Illustration 2. *Two particles A and B start together and are moving with speed 2 m/s and 3 m/s, respectively, in the same direction. Find how far will B be from A after 10 second.*

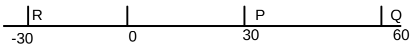
Solution: Distance travelled by A_1, $S_1 = 2 \times 10 = 20$ m,
Distance travelled by B, $S_2 = 3 \times 10 = 30$ m.
$\therefore$ Distance between them, $S = S_2 - S_1 = 30 - 20 = 10$ m.

In our discussions in the following sections, we shall treat the objects in motion as point objects or like a particle. This approximation is true in cases where the size of the object is much smaller than the distance it covers in a reasonable time interval. We consider moon as a particle during its orbital motion round the earth and even earth as a particle during its orbital motion round the sun.

POSITION, PATH LENGTH AND DISPLACEMENT

(i) When motion is along a straight line

Position: To locate the position of the particle at some time (instantaneous position) we choose an axis (*say x-axis*) with a fixed origin O in the given reference frame and find the distance from O. Thus, positions of P, Q and R are +30 cm, +60 cm and –30 cm, respectively. In vector notation, we represent the position vectors as $\overrightarrow{OP} = (+30\,\text{cm})\hat{i}$, $\overrightarrow{OQ} = +(60\,\text{cm})\hat{i}$ and $\overrightarrow{OR} = (-30\text{cm})\hat{i}$, respectively.



Path length: With reference to the above *figure*, let a particle starts moving from the origin O at time t = 0 and at subsequent times t_1, t_2 and t_3 ($t_3 > t_2 > t_1$) and it is at P, Q and R, respectively. Can you find the path length during the interval (i) t = 0 to $t = t_1$, (ii) t = 0 to $t = t_3$, (iii) $t = t_1$ to $t = t_3$?

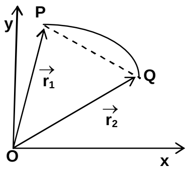
The path length is always equal to the total distance moved by the particle. Hence, the corresponding path lengths are 30 cm, 150 cm and 120 cm, respectively. Thus, path lengths add up like a scalar quantity, they have no direction but magnitude only.

Displacement: It is defined as the change in the position vectors in the given time interval. If x_i and x_f be the initial and final positions of the particle in the time interval $(t_2 - t_1)$, then the displacement $\Delta x = x_2 - x_1$. In vector notation, $\vec{x}_i = x_1\hat{i}$, $\vec{x}_f = x_2\hat{i}$, hence $\Delta\vec{x} = (x_2 - x_1)\hat{i}$, since the motion is along the x-axis only. Can you find the displacement during the same time interval as done for calculating the path length in the previous section. The displacements are (i) (+30 cm)$\hat{i}$, (ii) (–30 cm)$\hat{i}$ and (iii) (–60 cm)$\hat{i}$. Note that no sign + or – has been mentioned in expressing the path length (since it is scalar) while ± sign has been mentioned in expressing the displacements (since displacement is a vector quantity). The magnitude of the displacement may or may not be equal to the path length traversed by the particle.

In general path length ≥ magnitude of displacement. If the initial and final positions of a particle in its motion be same, the displacement is zero but path length is not zero.

(ii) When the motion is along the curved path

Position: Initial position is at P and is represented by $\overrightarrow{OP} = \vec{r}_1$. Similarly, final position is at Q and is represented by $\overrightarrow{OQ} = \vec{r}_2$. In terms of coordinates of P and Q, $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



Path length: Here, the path length is the length of the curve joining the initial and final positions (not the straight line joining P and Q as shown by dotted line) along which the particle has actually moved through.

Displacement: Magnitude of the displacement is the length of the straight line joining the initial and final positions and its direction is from the initial to the final position. We have already defined displacement as the change in position vector, hence displacement $\overrightarrow{PQ} = \vec{r}_2 - \vec{r}_1$,

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\,\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Exercise 1: ***A person moves from A to B along the semicircular path. Compare the distance moved by him and the displacement.***

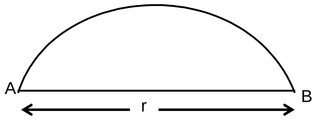
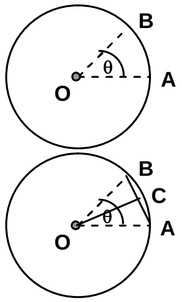


Illustration 3. *A boy travels from his house to a play ground along a straight path of length 'D' meter and return back to his house. Find the distance travelled and displacement of the boy.*

Solution: Distance = 2D meter, Displacement = Zero

Illustration 4. *A particle moves along a circle of radius R. Find the path length and magnitude of displacement from initial position A to final position B.*

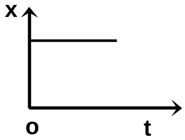


Solution: Path length = Rθ
Displacement = AB = AC + BC = 2R sin (θ/2).

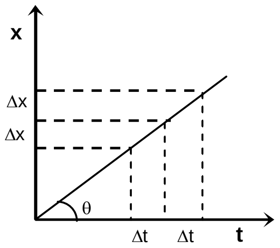
Position–Time Graph:

If we plot time t along the x-axis and the corresponding position (say x) from the origin O on the y-axis, we get a graph which is called the position–time graph. This graph is very convenient to analyse different aspects of motion of a particle. Let us consider the following case.

(i) In this case, position (x) remains constant but time changes. This indicates that the particle is stationary in the given reference frame. Hence, the straight line nature of position–time graph parallel to the time axis represents *the state of rest.* Note that its slope (tan θ) is zero.



(ii) When the x–t graph is a straight line inclined at some angle ($\theta \neq 0$) with the time axis, the particle traverses equal displacement Δx in equal interval of time Δt. The motion of the particle is said to be *uniform rectilinear motion.* The slope of the line measured by $\frac{\Delta x}{\Delta t} = \tan\theta$ represents the uniform velocity of the particle.



(iii) When the x–t graph is a curve, motion is not uniform. It either speeds up or slows down depending upon whether the slope ($\tan\theta$) successively increases or decreases with time. As shown in the figure, the motion speeds up from t = 0 to $t = t_1$ (since the slope $\tan\theta$ increases). From $t = t_1$ to $t = t_2$, AB represents a straight line indicating uniform motion. From $t = t_2$ to $t = t_3$, the motion slows down and for $t > t_3$ the particle remains at rest in the reference frame.

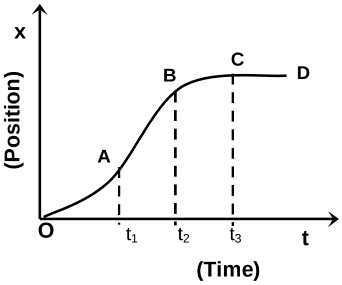
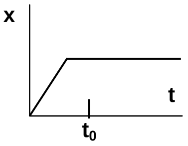


Illustration 5. *The adjacent figure shows the displacement–time graph of a particle moving on the x-axis. Choose the correct option given below.*
(A) The particle is continuously going in positive x direction.
(B) The particle is at rest.
(C) The particle moves at a constant velocity all time
(D) The particle moves at a constant velocity upto a time t_0, and then stops.



Solution: **(D).** *Upto time t_0,, particle is said to have uniform rectilinear motion and after that comes to rest as the slope is zero.*

Exercise 2:
(i) ***Distinguish between the distance covered by a body and its displacement. What are the characteristics of displacement?***
(ii) ***Under what condition will the distance and displacement of a moving object have the same magnitude.***

Speed

The term average indicates overall effect whereas instantaneous means the effect at a particular time. Hence, the average speed in a given time interval $(t_2 - t_1)$ is measured by the distance covered (path length s) divided by the time interval.

Thus, average speed $= \dfrac{s}{t_2 - t_1} = \dfrac{\text{path length}}{\text{time interval}}$

If the time interval $(t_2 - t_1)$ is divided into small segments Δt_1, Δt_2, ..., for which the corresponding path lengths be Δs_1, Δs_2, ..., then

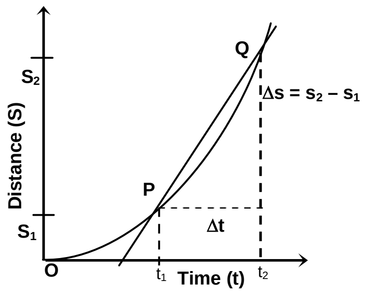
Average speed $= \dfrac{\Delta s_1 + \Delta s_2 + \Delta s_3 + ...}{\Delta t_1 + \Delta t_2 + \Delta t_3 + ...} = \dfrac{s}{t_2 - t_1}$

Hence, the average speed during one such time interval is equal to $\frac{\Delta s}{\Delta t}$. If Δt is infinitesimally small $|\Delta t \to 0|$, then we define the instantaneous speed at a time t as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Speed is a scalar quantity. It has only the magnitude and no direction. For a particle in motion in a given reference frame the instantaneous or average speed during any time interval is always positive.

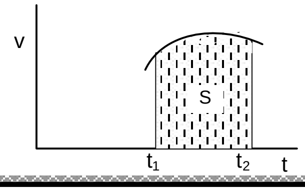
Consider the distance time graph as shown in the given figure. The average speed during the time interval Δt is $\frac{\Delta s}{\Delta t}$ which is the slope of the chord PQ. As $\Delta t \to 0$, the chord PQ becomes the tangent at P and the average speed becomes the instantaneous speed at P given by $\frac{ds}{dt} = \tan\theta$, which is the slope of the tangent at P.



Remember that the s–t graph (position–time graph) does not indicate the path of motion but represents increase in the path length as time increases, whether the particle does or does not retrace its path. Now, instantaneous speed $v = \frac{ds}{dt}$.

$\therefore$ $ds = v\,dt$ and $s = \int_{t_1}^{t_2} v\,dt$ = total distance travelled during the time interval $(t_2 - t_1)$

It is evident that the area under the speed time graph (shown by the shaded region) measures the total distance covered during the time interval $t_2 - t_1$.



Exercise 3:

i) ***Can a body moving with uniform speed have variable velocity?***

ii) ***Can a body moving with uniform velocity have variable speed?***

iii) ***Can average velocity ever become equal to instantaneous velocity ?***

Illustration 6. *A car covers the first half of the distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. What is the average speed of the car?*

Solution : Let the distance between the two places be 2x km.

$\therefore$ Time taken by the car for the first half of the journey = $\frac{x}{40}$ hr

Also, the time taken for the second half = $\frac{x}{60}$ hr

The total time of the journey = $\frac{x}{40} + \frac{x}{60} = \frac{5x}{120}$ hr

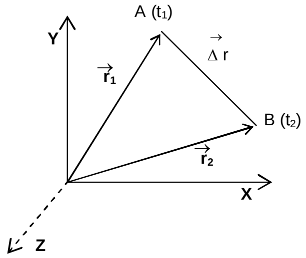
Average speed = $\frac{\text{distance}}{\text{time}} = \frac{2x}{5x/120}$ = 48 km/hr

Velocity

By definition,

Average velocity, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$

Note that the positions in between the interval of time t_1 and t_2 are to be specified in finding the average velocity. If the particle takes different paths to travel from A to B in the same time interval, the average velocity will remain same but average speed will be different and greater than the magnitude of $\vec{v}_{av}$. In the special case when the points A to B is straight, the average speed is equal to the magnitude of average velocity.



If $\Delta t \to 0$, the path length Δs during the interval Δt is equal to the Δr.

Hence, the instantaneous velocity $\vec{v} = \lim \dfrac{\Delta\vec{r}}{dt} = \dfrac{d\vec{r}}{dt}$, and

magnitude of the velocity is $v = \left|\dfrac{\Delta\vec{r}}{dt}\right| = \dfrac{|d\vec{r}|}{dt} = \dfrac{ds}{dt}$.

Hence, the instantaneous speed at any time t is the magnitude of instantaneous velocity at that time.

On a graph of position as a function of time for straight line, the instantaneous velocity at any point is equal to the slope of the tangent to the curve at that point.

The figure depicts the motion of a particle.

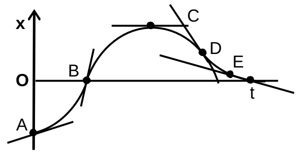
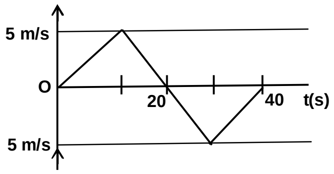
	x, t group	Motion of particle	
A	Positive slope, so $V_x > 0$	Moving in +ive x = direction	
B	Larger positive slope, so $V_x > 0$	Moving in +x direction faster than at A	
C	Zero slope, so $V_X = 0$	Instantaneously at rest	
D	Negative slope, so $V_x < 0$	Moving in –x direction	
E	Smaller negative slope, so $V_x < 0$	Moving in –ve x-direction more slowly than at D	

Illustration 7. *From the velocity– time plot shown in fig. Find*

(a) distance travelled by the particle during the first 40 seconds.

(b) displacement travelled by the particle during the first 40 seconds.

(c) Also find the average velocity during this period.



Solution: (a) Distance = area under the curve

$= \dfrac{1}{2}\times 20\times 5 + \dfrac{1}{2}\times 5\times 20$

= 50 + 50 = 100 m

For distance measurement, the curve is plotted as in Fig. (a)

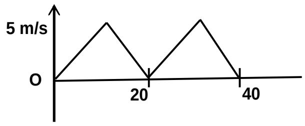


Fig. (a)

(b) Displacement = area under the curve in Fig. (b) = 0

(c) $V_{aV} = \dfrac{\text{Displacement}}{\text{time}}$

As displacement is zero,

$\therefore V_{av} = 0$

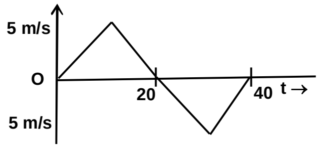


Fig. (b)

Illustration 8. *If velocity of a particle moving along a straight line changes with time as V(m/s) = 4 sin* $\left(\frac{\pi}{2}t\right)$*, its average velocity over time interval t = 0 to t = 2(2n – 1) sec, (n being any (+)ve integer) is*

(A) $\dfrac{8}{\pi(2n-1)}$ *m/s* *(B)* $\dfrac{4}{\pi(2n-1)}$ *m/s*

(C) zero (D) $\dfrac{16(2n-1)}{\pi}$ *m/s* *(D) none*

Solution: **(A).**

Displacement over the interval t = 0 to t = 2(2n –1) sec

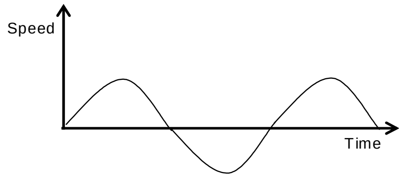
$$= 4\int_0^{2(2n-1)} \sin\left(\frac{\pi}{2}t\right)dt = -\left(\frac{8}{\pi}\right)\left|\cos\frac{\pi t}{2}\right|_0^{2(2n-1)}$$

$$= \frac{16}{\pi}\,\text{m}$$

$$\Rightarrow \text{Average velocity} = \frac{16}{2(2n-1)\pi} = \frac{8}{\pi(2n-1)}\ \text{m/s}$$

Exercise 4:

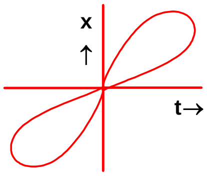
(i) ***Is the speed–time graph shown in figure is possible ?***



(ii) ***Under what circumstances, does the relationship $\Delta x = v.\ \Delta t$ hold good.***

(iii) ***Show that average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.***

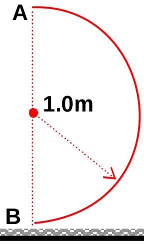
(iv) ***State with reasons why the graph in the adjacent figure cannot represent one-dimensional motion of a particle.***



(v) ***In 1.0 sec, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in the adjacent figure. The magnitude of average velocity is***

(A) 3.14 m/s ***(B) 2.0 m/s***

(C) 1.0 m/s ***(D) Zero***



Acceleration

Motion of a particle moving with constant velocity along a straight line is said to be uniform motion because neither the speed nor the direction of motion changes with the passage of time. On the other hand, the motion is said to be accelerated, if either the speed or the direction or both continuously change with time.

Definition: *Acceleration is defined as the rate of change of velocity.* It is a vector quantity and has its direction along which velocity has changed.

Average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta\vec{v}}{\Delta t}$

Note: When the velocity decreases, we say that the particle is decelerating. Deceleration is equivalent to negative acceleration.

It is also a vector quantity directed along the direction of the change $\Delta\vec{v}$ and independent of the intermediate values of velocities in between the interval $t_2 - t_1$.

Instantaneous acceleration at a time t is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{v}) = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

If the particle moves along x-axis,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i}) = \frac{dv_x}{dt}\hat{i} = \frac{d^2x}{dt^2}\hat{i}$$

If the particle moves in the xy plane,

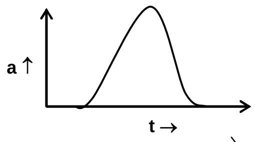
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} = a_x\hat{i} + a_y\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Exercise 5:

(i) ***Can a body have an acceleration with zero velocity?***

(ii) ***Can the direction of the velocity of a body change when its acceleration is constant ?***

(iii) ***A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time-interval. Suggest acceleration time graph for this situation.***



(iv) ***Figure shows the x–t graph of a particle moving along a straight line. What is the sign of the acceleration during the intervals OA, AB, BC and CD?***

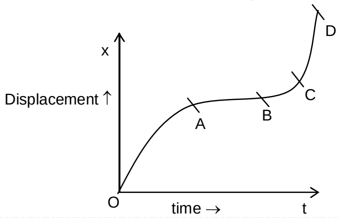


Illustration 9. *A body moving in a curved path possesses a velocity 3 m/s towards north at any instant of its motion. After 10s, the velocity of the body was found to be 4 m/s towards west. Calculate the average acceleration during this interval.*

Solution: To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity v_0 and the final velocity v are drawn from a common origin. The vector difference of them is found by the parallelogram method.

The magnitude of difference is

$|v - v_0| = OC = \sqrt{OA^2 + AC^2}$

$= \sqrt{4^2 + 3^2} = 5$ m/s

The direction is given by

$\tan\theta = \frac{3}{4} = 0.75 \quad \therefore 37^0$

$\therefore$ Average acceleration

$= \frac{v - v_0}{t} = \frac{5}{10}$

$= 0.5$ m/s^2 at 37° South of West.

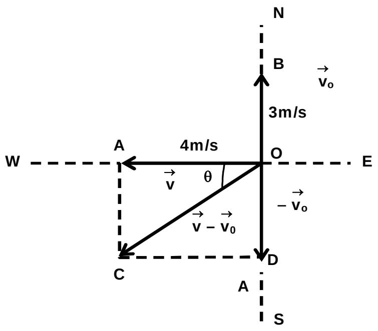


Illustration 10. *A car is moving eastwards with velocity 10 m/s. In 20 seconds, the velocity changes to 10 m/s northwards. Calculate the average acceleration in this time.*

Solution: The change in velocity

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$= 10\hat{j} - 10\hat{i}$

$\Rightarrow$ $\Delta\vec{v}$ has a magnitude of $10\sqrt{2}$ and is directed towards N-W.

So, $|\vec{a}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{1}{\sqrt{2}}$ m/s^2 towards N-W.

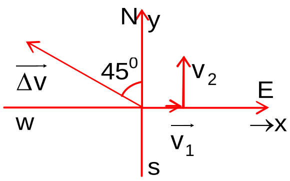


Illustration 11. *Which of the following statements is possible in a one/two-dimensional motion?*
(A) A body is having zero velocity and still accelerating.
(B) The velocity of an object reverses direction when acceleration is constant.
(C) An object increasing in speed as its acceleration decreases.
(D) None of these.

Solution: **(A), (B) and C**

(A) When a body is projected vertically upward, at the highest point its speed becomes zero while it is accelerating downwards with g = 9.8 m/s^2.

(B) When a body is projected upward, the direction of its velocity reverses during its descent while the acceleration (g) remains constant.

(C) Steel ball falling through a viscous liquid before terminal velocity is attained.

Exercise 6:

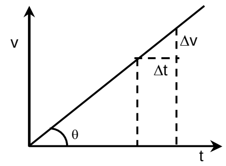
(i). ***A particle is moving along a circular path of radius 5 m with a uniform speed of 5 m/s. What will be the average acceleration when the particle completes half revolution?***
(A) Zero (B) 10 m/s^2
(C) 10π m/s^2 (D) 10/π m/s^2

(ii). ***Which of the following statement is incorrect?***
(A) Zero velocity of particle does not necessarily mean that its acceleration is zero.
(B) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
(C) If speed of a particle is constant its acceleration must be zero.
(D) None of these.

Uniformly accelerated motion

Motion of a particle is said to be uniformly accelerated if acceleration (a vector quantity) remains constant in magnitude as well as in direction. Motion of a particle falling freely under gravity is an example of uniformly accelerated motion since the acceleration $(\vec{g})$ remains constant (assuming negligible air resistance).

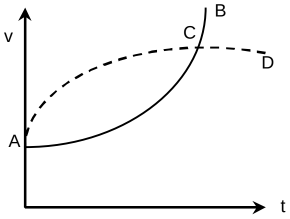
Velocity–time graph



When the v–t graph is a straight line, inclined at an angle θ with the time axis, the velocity increases equally in equal time interval. This indicates that the acceleration is uniform. Its magnitude is $a = \frac{\Delta v}{\Delta t} = \tan\theta$, the slope of v–t graph.

Let us find the area under the v–t graph:

$$A = \int dA = \int v dt = \int \frac{ds}{dt} dt = \int_i^f ds = \text{net displacement}$$



If the v–t graph is a curve, the slope continuously changes with time, which indicates that the magnitude of acceleration either increases with time (for curve AB) or it decreases with time (for curve ACD).

Note:

Features of v–t graphs:

(i) The slope of v–t graph gives the instantaneous acceleration.

(ii) The area under the v–t graph gives the net displacement (not distance) in the given time interval.

EQUATION OF MOTION IN A STRAIGHT LINE WITH UNIFORM ACCELERATION

Consider the motion of a particle moving along the x-axis with uniform acceleration.

Let u = Initial velocity (at time t = 0)
 v = Final velocity (at time t), and
 x = Net displacement in the time interval t = 0 to t = t.

The equations describing such uniformly accelerated motion are

$$v = u + at; \quad x = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2ax$$

Remember that these equations are valid or applicable if the acceleration remains constant both in magnitude and direction.

Derivation of Equations of Motion

(i) v = u + at

Calculus method:

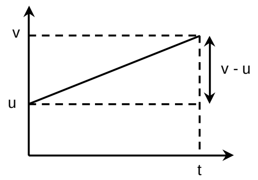
By definition $a = \frac{dv}{dt}$ or, $dv = a dt$

Integrating, $\int_u^v dv = a\int_0^t dt$ [$\because$ a = constant]

$v - u = at$, or $v = u + at$

Graphical Method:

By definition, slope of v–t graph gives the acceleration



$$\therefore \ a = \tan\theta = \frac{v-u}{t-0} = \frac{v-u}{t}$$

or $v - u = at$ $\therefore$ $v = u + at$

(ii) $\mathbf{x = ut + \frac{1}{2}at^2}$

Calculus method:

By definition, instantaneous velocity $v = \frac{dx}{dt}$

or, $dx = v\,dt$

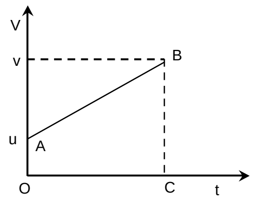
$= (u + at)dt$ $[\because v = u + at]$

$= u\,dt + at\,dt$

Integrating, $\int_0^x dx = u\int_0^t dt + a\int_0^t t\,dt$ $[\because$ u and a are constant]

Hence, $x = ut + \frac{1}{2}at^2$

Graphical method:



We know that area under the velocity– time graph gives the net displacement during the given time interval. Hence, net displacement x = area OABC.

or $$x = \frac{1}{2}(OA + BC)\times OC$$

$$= \frac{1}{2}(u+v).t = \frac{1}{2}[2u + \frac{v-u}{t}t]t$$

$$= \frac{1}{2}[2u + at]t \quad [\because a = \frac{v-u}{t}] = ut + \frac{1}{2}at^2$$

(iii) $\mathbf{v^2 = u^2 + 2ax}$

Calculus method:

By definition, $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt}$

or, $a = v.\frac{dv}{dx}$

or, $vdv = adx$

Integrating, $\int_u^v vdv = a\int_0^x dx$

or, $\frac{v^2 - u^2}{2} = ax$ $\therefore$ $v^2 = u^2 + 2ax$

Graphical method:

From the v–t graph, net displacement x = area under the v–t graph

or, $x = \frac{1}{2}(v + u)t$

Multyplying both sides by (v – u),

$$x\,(v-u) = \frac{1}{2}(v+u)(v-u)t$$

or, $x\left(\frac{v-u}{t}\right) = \frac{v^2 - u^2}{2}$

or, $x \cdot a = \frac{v^2 - u^2}{2}$ $\left[\because a = \frac{v-u}{t}\right]$

$\therefore$ $v^2 = u^2 + 2ax$

It is to be noted that u, v and a are vectors and may have positive or negative value depending on whether their directions are along the positive or negative directions of the x-axis.

Graphs Representing Motion of a Particle

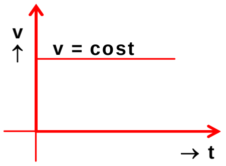
From the knowledge of calculus, we can say from Eq. (1) that:

(i) Slope of s–t graph gives velocity;
(ii) Slope of v–t graph gives acceleration;
(iii) Area under v–t graph gives displacement; and
(iv) Area under a–t graph gives change in velocity.

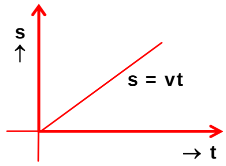
(a) **Uniform motion**

v = constant

$\Rightarrow a = \frac{dv}{dt} = 0$



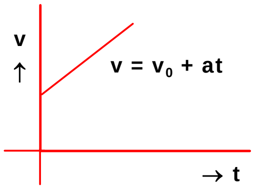
v–t graph
(a)



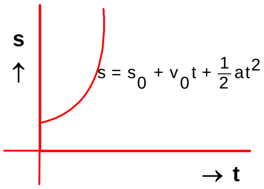
s–t graph
(b)

(b) **Uniformly accelerated motion**

a = constant

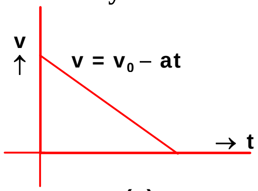


v–t graph
(a)

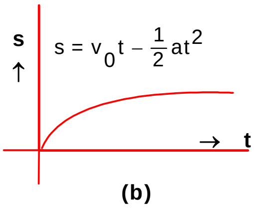


s–t graph
(b)

(c) **Uniformly retarded motion**



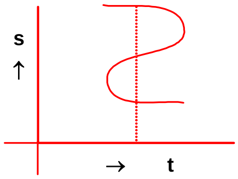
(a)



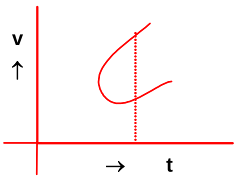
(b)

Some facts regarding graphs:

(i) Two or more values of velocity or displacement at any particular instant of time are not possible. So, the corresponding graphs should be rejected. For example, the following graphs are not possible.

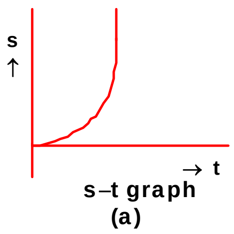


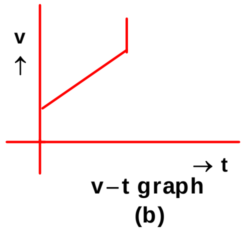
(a)



(b)

(ii) At any point, the slopes of s–t or v–t graph can never be infinite because infinite slope of s–t graph means infinite velocity and that of v–t graph means infinite acceleration, which are not possible. So, corresponding graphs are not acceptable. For example, the following graphs are all not possible.





In general, when a particle is moving with a uniform acceleration, to its motion is described by the following equations.

$$\vec{r}(t) = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\cdot\vec{s}$$

Here, $\vec{r}(t) =$ represents position vector of a particle at an instant t.

$\vec{r}_0$ = position vector of a particle at t = 0.

$\vec{u}$ = initial velocity of a particle at t = 0.

$\vec{v}$ = velocity of a particle at an instant t.

$\vec{a}$ = acceleration of the particle at an instant t.

Exercise 7:

(i) ***A boy sitting on a rail road car moving with a constant velocity tosses a coin up. Describe the path of the coin as seen by***

(a) the man on the train.

(b) the man standing on the ground near the rail.

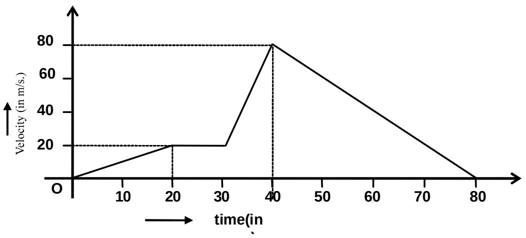
(ii) ***A particle is moving along a straight path, draw its velocity– time graph for the following cases:***

(a) When the acceleration of the particle increases.

(b) When the displacement of the particle obeys the relation $s = 4 + 5t + 2t^2$

(c) When the acceleration of the particle is given by $a = 12 \cos 6t$

Illustration 12. *The velocity–time graph of a moving object is given in the figure. Find the maximum acceleration of the body and distance travelled by the body in the interval of time in which this acceleration exists.*



Solution: Acceleration is maximum when slope is maximum.

$$a_{max} = \frac{80-20}{40-30} = 6\text{m/s}^2$$

$$S = 20\text{m/s}\times 10\text{s} + \frac{1}{2}\times 6\text{m/s}^2 \times 100\text{s}^2 = 500\text{m}.$$

Illustration 13. *A particle having initial velocity is moving with a constant acceleration 'a' for a time t.*
(a) Find the displacement of the particle in the last 1 second.
(b) Evaluate it for u = 2 m/s, a = 1 m/s^2 and t = 5 sec.

Solution: (a) The displacement of a particle at time t is given by $s = ut + \frac{1}{2}at^2$

At time (t – 1), the displacement of a particle is given by

$$S' = u(t-1) + \frac{1}{2}a(t-1)^2$$

$\therefore$ Displacement in the last 1 second is

$$S_t = S - S'$$
$$= ut + \frac{1}{2}at^2 - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$
$$= ut + \frac{1}{2}at^2 - ut + u - \frac{1}{2}a(t-1)^2$$
$$= \frac{1}{2}at^2 + u - \frac{1}{2}a(t^2 + 1 - 2t) = \frac{1}{2}at^2 + u - \frac{1}{2}at^2 - \frac{a}{2} + at$$
$$S = u + \frac{a}{2}(2t-1)$$

(b) Putting the values of u = 2m/s, a = 1m/s^2 and t = 5 sec, we get

$$S = 2 + \frac{1}{2}(2\times5-1) = 2 + \frac{1}{2}\times 9$$
$$= 2 + 4.5 = 6.5 \text{ m}$$

Illustration 14. *A car moving along a straight road with a speed of 72 km/h is brought to a stop with in a distance of 10m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?*

Solution: v = 72 km/h

$$= \frac{72\times1000}{3600} = 20\,\text{m/s}$$

using equation

$$\vec{v}^2 = \vec{u}^2 + 2a(\vec{x} - \vec{x}_0)$$
$$= (20)^2 + 2 \times a \times 10$$
$$a = -\frac{20\times20}{20} = -20\,\text{m/s}^2$$
$$\vec{v} = \vec{u} + \vec{a}t$$
$$0 = 20 - 20\,t$$
$$t = \frac{20}{20} = 1\,\text{sec}$$

So, it will take 1 sec for the car to stop.

Illustration 15. *Position of a particle moving along x-axis is given by x = 3t – 4t^2 + t^3, where x is in meters and t in seconds.*
(a) Find the position of the particle at t = 2s.
(b) Find the displacement of the particle in the time interval from t=0 to t=4s.
(c) Find the average velocity of the particle in the time interval from t=2s to t=4s.
(d) Find the velocity of the particle at t =2s.

Solution: (a) $x_{(t)} = 3t - 4t^2 + t^3$

$\Rightarrow x_{(2)} = 3 \times 2 - 4 \times (2)^2 + (2)^3 = 6 - 4 \times 4 + 8 = -2$ m.

(b) $x_{(o)} = 0$

$x_{(4)} = 3 \times 4 - 4 \times (4)^2 + (4)^3 = 12$m.

Displacement $= x_{(4)} - x_{(0)} = 12$ m.

(c) $$< v >= \frac{x_{(4)} - x_{(2)}}{(4-2)} = \frac{12-(-2)}{2} m/s = 7 m/s$$

(d) $$\frac{dx}{dt} = 3 - 8t + 3t^2$$

$$\Rightarrow v_{(2)} = \left(\frac{dx}{dt}\right)_{(2)} = 3 - 8 \times 2 + 3 \times (2)^2 = -1 m/s$$

Illustration 16. *An anti-aircraft shell is fired vertically upwards with a muzzle velocity of 294 m/s. Calculate (a) the maximum height reached by it, (b) time taken to reach this height, (c) the velocities at the ends of 20th and 40th second. (d) when will its height be 2450 m? Given g = 980 cm/s^2.*

Solution : (a) Here, the initial velocity u = 294 m/s and g = 9.8 m/s^2

$\therefore$ The maximum height reached by the shell is,

$$H = \frac{u^2}{2g} = \frac{294^2}{2 \times 9.8} = 4410 m = 4.41 km$$

(b) The time taken to reach the height is,

$$T = \frac{u}{g} = \frac{294}{9.8} = 30 s$$

(c) The velocity at the end of 20th second is given by,

v = u – gt = 294 – 9.8 x 20 = 98 m/s upward,

and the velocity at the end of 40th second is given by,

v = 294 – 9.8 x 40 = – 98 m/s

The negative sign implies that the shell is falling downward.

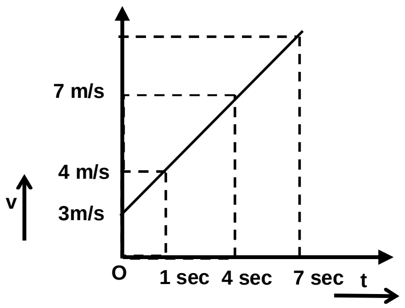
(d) From the equation

$$h = ut + \frac{1}{2}gt^2 \quad \text{or} \quad 2450 = 294\,t - \frac{1}{2} \times 9.8t^2$$

or, $t^2 - 60\,t + 500 = 0$ $\therefore$ t = 10s and 50 s.

At t = 10 s the shell is at a height of 2450 m and is ascending, and at the end of 50 s it is at the same height, but is falling.

Illustration 17. *The velocity–time graph of a particle is given as shown in the figure. Find the distance travelled by the object in 7th second.*



Solution : $$\frac{dv}{dt} = \frac{7-4}{4-1} = 1 m/s^2$$

$$\vec{s}_{t=7} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

$$= 3 + \frac{1}{2}(2 \times 7 - 1) = 9.5 m .$$

Exercise 8:

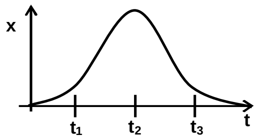
(i) ***A truck moving with constant acceleration covers the distance between two points 180 m apart in 6 seconds. Its speed as it passes the second point is 45 m/s. Find***
(a) its acceleration , and
(b) its speed when it was at the first point.

(ii) ***A body undergoing uniformly accelerated motion starts moving along +x-axis with a velocity of 5 m/s and after 5 seconds its velocity becomes 20 m/s in the same direction. What is the velocity of the body 10 seconds after the start of the motion ?***

(iii) ***What is the speed with which a stone is projected vertically upwards from the ground if it attains a maximum height of 20 m?***

(iv) ***A ball is thrown vertically upwards with a speed of 20 m/s from a hard floor. Draw a graph showing the velocity of the ball as a function of time if the ball suffers elastic collisions continuously.***

(v) ***The adjacent figure shows the x-coordinate of a particle as a function of time. Find the signs of v_x and a_x at $t = t_1$, $t = t_2$ and $t = t_3$.***



Solving problems in Kinematics using elementary concepts of differential and integral calculus

For the motion of a particle in a straight line, we always write instantaneous velocity $v = \frac{dx}{dt}$.

In case, the acceleration is non uniform and a function of displacement, we write,

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$$

Let us solve some illustrative examples:

Illustration 18. *The instantaneous velocity of a particle moving along a straight line is given by $v = \alpha t^2$ whose α is a positive constant. Find the average speed during the interval $t = 0$ to $t = T$.*

Solution: By definition, average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{\int_0^T v\,dt}{\int_0^T dt}$

$$= \frac{1}{T}\int_0^T \alpha t^2 dt = \frac{\alpha}{T}\left[\frac{t^3}{3}\right]_0^T = \frac{\alpha T^2}{3}.$$

Illustration 19. *The displacement (x) of a particle moving in one dimension under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$, x in m and t in sec. Find the displacement of the particle when its velocity is zero.*

Solution: Here, $t = \sqrt{x} + 3$
or $x = t^2 - 6t + 9$

$\therefore v = \frac{dx}{dt} = 2t - 6$

When v = 0, 2t – 6 = 0 ⇒ t = 3 sec

At t = 3 sec, $x = t^2 - 6t + 9$

= 9 –6 (3) + 9 = 0

Hence, the displacement of particle is zero when its velocity is zero.

Motion Under Gravity (Free Fall)

When a body is dropped from some height (earth's radius = 6400 km), it falls freely under gravity with constant acceleration g (=9.8 m/s^2) provided the air resistance is negligible small. The same set of three equations of kinematics (where the acceleration $\vec{a}$ remains constant) are used in solving such motion.

Here, we replace $\vec{a}$ by $\vec{g}$ and choose the direction of y-axis conveniently. When the y-axis is chosen positive along vertically downward direction, we take $\vec{g}$ as positive and use the equations as

$$v = u + gh \qquad \Rightarrow \qquad v^2 = u^2 + 2gh$$

$$h = ut + \frac{1}{2}gt^2$$

where u is initial velocity of projection in the vertically downward direction.

However, if an object is projected vertically upward with initially velocity u, we can take y – axis positive in the vertically upward direction the set of equations reduces to

$$v = u - gt \qquad \Rightarrow v^2 = u^2 - 2gh$$

$$h = ut - \frac{1}{2}gt^2$$

In order to avoid confusion in selecting $\vec{g}$ as positive or negative, it is advisable to take the y-axis as positive along vertically upward direction and point of projection as the origin. We can now write the set of three equations in the vector form:

$$\vec{v} = \vec{u} + \vec{g}t \qquad \Rightarrow \qquad \vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{g}.\vec{h}$$

$$\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

Exercise 9:

(i) ***A stone is thrown upwards with a speed v from the top of a tower. It reaches the ground with a velocity 3v, what is the height of the tower ?***

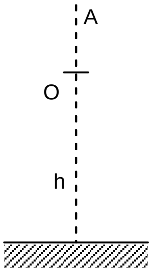
(ii) ***A stone is thrown vertically upwards with a velocity of 19.6 m/s. After 2 second, another stone is thrown upwards with a velocity of 9.8 m/s. When and where these stones will collide?***

Illustration 20. *A body is projected vertically upward, then find the velocity and acceleration of that body at it's highest point of motion?*

Solution: Velocity = 0 , acceleration = ± g

Illustration 21. *A ball is projected vertically upward with a speed of 4.0 m/s from a point 64 m above the ground. Find the time it takes to reach the ground. [g = 10 m/s^2]*

Solution: Before solving the problem, analyse the situation. As the ball will move up, it gradually slows down and attains the maximum height at A (where it comes to momentarily rest) and thereafter it retraces its path, attains the same speed at O but direction reversed down and finally it strikes the ground.



Choosing O (the point of projection) as the origin and positive y-axis as vertically upward, we collect the data in the vector notation which are given in the question.

Net displacement $\vec{y} = -h\ \hat{j} = (-64\text{ m})\,\hat{j}$

Constant acceleration $\vec{g} = -g\hat{j} = (-10\,\text{m/s}^2)\hat{j}$

Initial velocity $\vec{u} = +u\hat{j} = (+\,4.0\text{ m/s})\ \hat{j}$

Now, $\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$

Writing the values with proper sign.

$$-64\hat{j} = 4t\hat{j} + \frac{1}{2}(-10\hat{j})t^2$$

This reduces to a simple quandratic equation, $5t^2 - 4t - 64 = 0$

The solution $t = -\dfrac{16}{5}$ s is not permissible. Hence, the required time= 4.0 seconds.

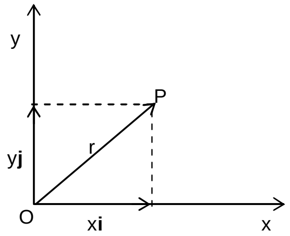
MOTION IN A PLANE

Position Vector and Displacement

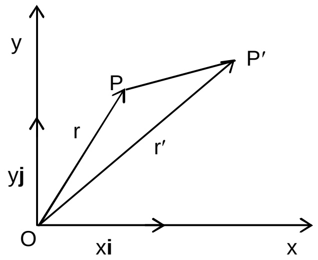
The position vector $\vec{r}$ of a particle located in a plane with reference to the origin of an x-y reference frame is

$$\vec{r} = x\hat{i} + y\hat{j}$$

where x, y are the coordinates of the object.



Let the particle be at a point P at any time t and at a point P′ at any time t′ as shown in figure.



$\therefore\ \overrightarrow{OP} = \vec{r}(t)$ and $\overrightarrow{OP}' = \vec{r}'(t')$

$\therefore$ Displacement vector

$$\vec{PP'} = \Delta\vec{r} = \vec{r}'(t') - \vec{r}(t)$$

$$= (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$$

$$= \hat{i}\Delta x + \hat{j}\Delta y$$

where $\Delta x = x' - x$ and $\Delta y = y' - y$

Velocity

The average velocity $\vec{v}$ of an object = $\dfrac{\text{displacement}}{\text{corresponding time interval}}$

$\therefore$ $$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \hat{i}\frac{\Delta x}{\Delta t} + \hat{j}\frac{\Delta y}{\Delta t}$$

or, $$\vec{v} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$$

Direction of the average velocity is same as that of displacement.
The instantaneous velocity is the average velocity as the time interval approaches to zero.

$\therefore \qquad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

The direction of the instantaneous velocity of an object at any point on the path is tangent to the path at that point and is in the direction of motion.

In component form,

$$\vec{v} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$$

Magnitude of v,

$$v = \sqrt{v_x^2 + v_y^2}$$

and the direction of v,

$$\tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

Acceleration

Average acceleration = $\frac{\text{change in velocity}}{\text{time interval}}$

$$\vec{a}_{av} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} \qquad \text{or} \qquad \vec{a}_{av} = a_x\hat{i} + a_y\hat{j}$$

Instantaneous acceleration: It is the limiting value of the average acceleration as the time interval approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta \vec{v}}{\Delta t}$$

$$\therefore \quad \vec{a} = \hat{i}\lim_{\Delta t \to 0}\frac{\Delta v_x}{\Delta t} + \hat{j}\lim_{\Delta t \to 0}\frac{\Delta v_y}{\Delta t}$$

or, $\qquad \vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$

Motion in a plane with constant acceleration:

If an object is moving in x-y plane having constant acceleration a, then by the definition of average acceleration

$$a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

where , v_0 = velocity of the object at time t = 0
and v = velocity of the object at time t.

or, $\qquad v = v_0 + at$

In terms of components,

$v_x = v_{0x} + a_x t$

$v_y = v_{0y} + a_y t$

Now, let r_0 and r be the position vectors of any particle at time zero and t and their velocities at these instant are v_0 and v, respectively. During the time interval t, $\left(\frac{v_0 + v}{2}\right)$ is the average velocity.

$\therefore$ Displacement,

$$r - r_0 = \left(\frac{v + v_0}{2}\right)t = \left(\frac{v_0 + at + v_0}{2}\right)t$$

$$r = r_0 + v_0\,t + \frac{1}{2}at^2$$

Hence, in component form,

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Therefore, a two dimensional motion can be treated as two separate simultaneous one-dimensional motions having constant acceleration along two perpendicular direction.

PROJECTILE

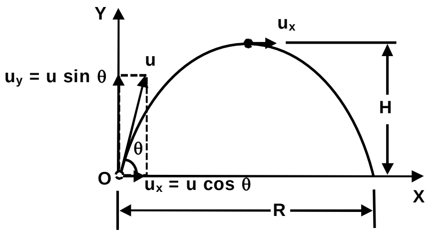
An object projected into space or air, such that it moves under the effect of gravity only, is called a projectile.

Projectile Motion

Motion in a vertical plane containing horizontal and vertical axes:

A particle when given a velocity at any arbitrary angle (other than 90°) made with the horizontal surface is known as a projectile.

If a particle is projected from point O, at any angle θ from the horizontal, with initial velocity $\vec{u}$, then the components of $\vec{u}$ in x and y directions are given as



$u_x = u\cos\theta$, $u_y = u\sin\theta$ where $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$$\Rightarrow \quad \vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

The X axis is parallel to the horizontal. Y axis is parallel to the vertical and the $\vec{u}$ lies in the X – Y plane.

The constant acceleration $\vec{a}$ is given as, $\vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = 0$ [as there is no acceleration along the X-axis].

$a_y = -g$ [the acceleration is downward and equal to g].

Now, velocity after time t is given as.

$v_{tx} = u_x + a_x t = u\cos\theta$ (as $a_x = 0$)

$v_{ty} = u_y + a_y t = u\sin\theta - gt$

$$\because \quad \vec{v} = v_x\hat{i} + v_y\hat{j} \quad \Rightarrow \quad \vec{v}_{(t)} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

The direction of $\vec{v}$ with the x axis is given by $\tan^{-1}\left(\frac{v_y}{v_x}\right)$

Co-ordinates of the projectile after time t is given by

$$x = x_o + u_x t + \frac{1}{2}a_x t^2 \quad \Rightarrow \quad x = 0 + u\cos\theta\,.t + 0$$

$$\Rightarrow \quad x = u\cos\theta\,t \qquad \text{. . . (1)}$$

and $$y = y_o + u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad y = 0 + u\sin\theta\,t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad y = u\sin\theta\,t - \frac{1}{2}gt^2 \qquad \text{. . . (2)}$$

Eliminating 't' from Eqs. (1) and (2), we get

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \ldots (3)$$

The above equation shows the relation between x and y and represents the path of the projectile known as trajectory. The inspection of eq. (3) shows that it is the equation of parabola of the form

$$y = bx + cx^2$$

where $b = \tan\theta$ = constant, and $c = -\frac{g}{2u^2\cos^2\theta}$ = constant

Time of flight: It is time interval during which the projectile remains in air.

Putting y = 0 in (2), we get

$$T = \frac{2u\sin\theta}{g},$$ where T = time of flight.

Range: The horizontal range R of the projectile is the horizontal distance between the initial point and the point where the projectile is again at same horizontal level.

If R be the horizontal range then $R = u\cos\theta \times \frac{2u\sin\theta.}{g} = \frac{u^2\sin 2\theta}{g}$

Note (i): Since $\sin 2\theta = \sin(\pi - 2\theta) = \sin 2\left(\frac{\pi}{2} - \theta\right)$

Let $(\pi/2 - \theta) = \beta \quad \Rightarrow \sin 2\theta = \sin 2\beta$

Hence, range is same for two angles of projection provided angles are complimentary.

Note (ii): For a given velocity of projection R is maximum for $\theta = 45^o$.

$$\Rightarrow \quad R_{max} = \frac{u^2}{g}$$

Maximum height: The maximum height attained by the projectile is given by

$$\because \quad v_y^2 = u_y^2 + 2a_y y \quad \text{at} \quad y = y_{max},\ v_y = 0$$

$$\Rightarrow \quad 0 = u^2\sin^2\theta - 2\,g\,y_{max} \quad \Rightarrow \quad y_{max} = \frac{u^2\sin^2\theta}{2g}$$

Exercise 10:

(i) ***A projectile is thrown horizontally from the top of a tower and strikes the ground after 3 second at an angle of 45° with the horizontal. Find the height of the tower and speed with which the body was projected. Given g = 9.8 m/s²***

(ii) ***A ball is thrown with an initial velocity of 100 m/s at an angle of 30° above the horizontal. How far from the throwing point will the ball attain its original level? Solve the problem without using formula for horizontal range.***

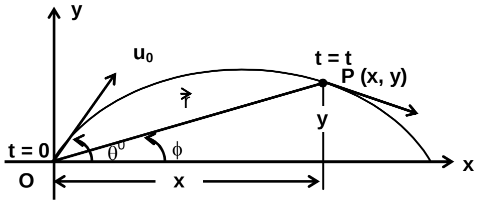
(iii) ***A bullet P is fired from a gun when the angle of elevation of the gun is 30°, another bullet Q is fired from the gun when the angle of elevation is 60° which of the two bullets would have a greater horizontal range and why ?***

(iv) ***Name the two quantities which would be reduced if air resistance is taken into account in the study of motion of oblique projectile.***

Illustration 22. *A boy throws a stone with an speed V_0 = 10 m/sec at an angle $\theta_0 = 30^o$ to the horizontal. Find the position of the stone w.r.t. the point of projection just after a time t = 1/2 sec.*

Solution: The position of the stone is given by

$\vec{r} = xi + yj$



where $x=(v_0 \cos\theta_0)t$

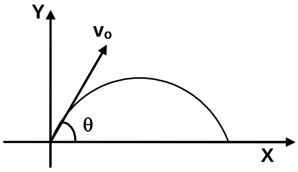
$= (10\ \cos 30)\left(\frac{1}{2}\right) = 4.33$ m.

and $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

$= (10\ \sin 30)\left(\frac{1}{2}\right) - \frac{1}{2}\times 10\times\left(\frac{1}{2}\right)^2 = 1.25$ m

$\Rightarrow \vec{r} = (4.33i\ +\ 1.25j)$ m.

Illustration 23. *A particle is projected with velocity v_o = 100 m/s at an angle $\theta = 30^o$ with the horizontal. Find*



(a) velocity of the particle after 2 s.
(b) angle between initial velocity and the velocity after 2s
(c) the maximum height reached by the projectile
(d) horizontal range of the projectile.
(e) the total time of flight

Solution: (a) $\vec{v}_{(t)} = \vec{v}_{x(t)}\hat{i} + \vec{v}_{y(t)}\hat{j}$

where $\hat{i}$ and $\hat{j}$ are the unit vectors along +ve x and +ve y axis, respectively.

$\Rightarrow \quad \vec{v}_{(t)} = (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$

Here, $u_x = v_o \cos\theta = 50\sqrt{3}$ m/s

$a_x = 0$

$u_y = v_o \sin\theta = 50$m/s

$a_y = -g$ ($\because$ g acts downward)

$\Rightarrow \quad \vec{v}_{(t)} = v_o\cos\theta\hat{i} + (v_o\sin\theta - gt)\hat{j}$

$\vec{v}_{(2)} = 50\sqrt{3}\hat{i} + (50 - 10\times 2)\hat{j} = 50\sqrt{3}\hat{i} + 30\hat{j}$ m/s

$\Rightarrow \quad |\vec{v}_2| = \sqrt{v_x^2 + v_y^2} = 91.65$ m / s

(b) $\vec{v}_o = 50\sqrt{3}\hat{i} + 50\hat{j}$, and

$\vec{v}_{(t=2\sec)} = 50\sqrt{3}\hat{i} + 30\hat{j}$

$\Rightarrow \quad \vec{v}_o.\vec{v}_{(2)} = 7500 + 1500 = 9000$

If α is the angle between $\vec{v}_o$ and $\vec{v}_{(2s)}$.

Then, $\cos\alpha = \dfrac{\vec{v}_o.\vec{v}_{(2s)}}{|\vec{v}_o|\times|\vec{v}_{(2s)}|} = \dfrac{9000}{1000\times 91.65}$

or $\alpha = \cos^{-1}(0.98) = 10.8^o$.

(c) $v_y^2 - u_y^2 = 2a_y y$

At $y = y_{max}$, $v_y = 0$

$\Rightarrow\ 0 - v_0^2 \sin^2\theta = 2(-g)y_{max}$

$\Rightarrow\ y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g} = 125\text{m}$

(d) $R = \dfrac{u^2 \sin 2\theta}{g} = 866$ m

(e) $T = \dfrac{2v_0 \sin\theta}{g} = 50$ sec.

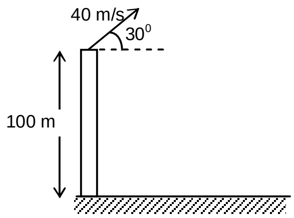
Illustration 24. *A ball is thrown at a speed of 50 m/s at an angle of 60° with the horizontal. Find*
(a) the maximum height reached, and.
(b) the range of ball. (Take g = 10 m/s^2)

Solution: (a) Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

$$= \frac{(50)^2}{2\times 10}\times\left(\frac{\sqrt{3}}{2}\right)^2 \text{ m} = 93.75 \text{ m}$$

(b) Range, $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(50)^2 \times \sin 120}{10} = 216.5$ m

Illustration 25. *A stone is projected with a speed of 40 m/s at an angle of 30° with the horizontal from a tower of height 100 m above ground. Find*
(a) the maximum height attained by the stone, and
(b) the horizontal distance from the tower where it hits the ground.



Solution: (a) Maximum height above the tower, using $v^2 = u^2 + 2as$ in vertical direction.

$(u \sin 30°)^2 = 2gh$ As $u = 40$ m/s, $\theta = 30°$

$\dfrac{40\times 40\times 1}{4} = 2 \times 10 \times h \quad \Rightarrow h = \dfrac{1600}{80} = 20\text{m}$

$\therefore$ Height above ground $= 100 + 20 = 120$m.

(b) Range, time of flight = t, $H = u \sin\theta t - \dfrac{1}{2}gt^2$, $H = -100$ m,

$-100 = (40 \times \dfrac{1}{2})t - \dfrac{1}{2}\times 10\times t^2$

$-100 = 20t - 5t^2$

$t^2 - 4t - 20 = 0$, $t = 6.9$ sec.

$R = u \cos\theta \times t$, $R \rightarrow$ distance from tower

$R = 40 \times \dfrac{\sqrt{3}}{2}\times 6.9 = 238.9$ m.

Illustration 26. *The position of a particle at time t = 0 is P = (–1, 2, –1). It starts moving with an initial velocity $\vec{u} = 3\hat{i} + 4\hat{j}$ and with uniform acceleration $-4\hat{i} + 4\hat{j}$. Find the final position and the magnitude of displacement after 4 sec.*

Solution: Initial position vector of the particle $= \left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

Final position of the particle after 4 seconds

$$S_f = S_i + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

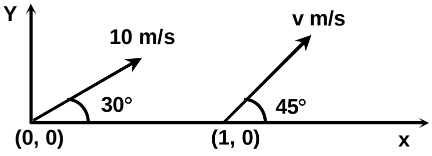
$$= \left(-\hat{i} + 2\hat{j} - \hat{k}\right) + \left(3\hat{i} + 4\hat{j}\right) \times 4 + \frac{1}{2} \times \left(-4\hat{i} + 3\hat{j}\right) \times 16$$

Final position $= -21\hat{i} + 42\hat{j} - \hat{k}$,

Displacement $= -20\,\hat{i} + 40\,\hat{j}$.

Magnitude of displacement $= \sqrt{(20)^2 + (40)^2} = 20\sqrt{5}$ m

Illustration 27. *Two particles projected vertically upward from points (0, 0) and (1, 0) with uniform velocity 10 m/s and v m/s, respectively, as shown in the figure. It is found that they collide after time t in space. Find v and t.*



Solution:

$x_1 = 10\cos 30°t$

$x_2 = v\cos 45°t$

$y_1 = 10\sin 30°t - \frac{1}{2}gt^2$

$y_2 = v\sin 45°t - \frac{1}{2}gt^2$

For collision:

$y_1 = y_2$

$10 \times \frac{1}{2} = \frac{v}{\sqrt{2}}$

$\Rightarrow v = 5\sqrt{2}$ m/s

$\Rightarrow x_1 = x_2 + 1$

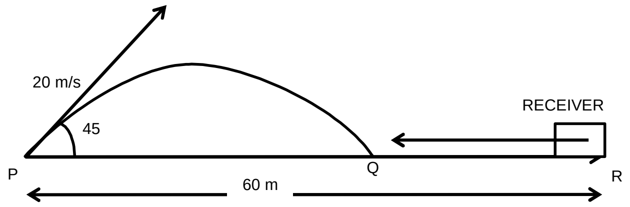
$10\cos 30°t = 5\sqrt{2}\cos 45°t + 1$

$t\left(5\sqrt{3} - 5\right) = 1$

and $t = \dfrac{1}{5\left(\sqrt{3} - 1\right)}$ sec

Illustration 28. *A football is kicked off with an initial speed of 20m/s at an angle of projection of 45°. A receiver on the goal line at a distance of 60 m away in the direction of the kick starts running to meet the ball at that instant. What must be his speed if he is to catch the ball before it hits the ground? [Take $g = 10m/s^2$]*

Solution:



Let u = 20 m/s, $\theta = 45^\circ$ and v = speed of the receiver.
The ball is projected from Pand the receiver starts running from R to receive the ball at Q.
Let t be the time after which they meet.
So t is the time taken by the ball to go from P to Qin which the receiver goes from R to Q.

$\therefore PQ = \frac{u^2}{g}\sin 2\theta$ and QR = vt

$PR = 60 \Rightarrow \frac{u^2}{g}\sin 2\theta + vt = 60 \ldots(i)$

Putting the value of t (i.e. the time of flight) = $\frac{2u\sin\theta}{g}$

in equation (I) we get,

$$\frac{u^2}{g}\sin 2\theta + v\left(\frac{2u\sin\theta}{g}\right) = 60$$

$$\Rightarrow \quad v = \frac{60g - u^2\sin 2\theta}{2u \sin\theta}$$

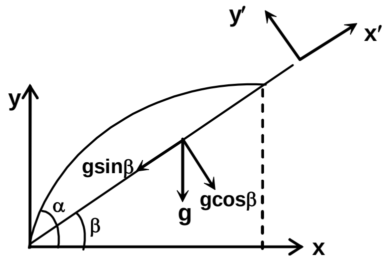
$$= \frac{600-400}{2(20)}\sqrt{2}$$

$$= 5\sqrt{2}\ \text{m/s}$$

The Projectile on an Inclined Plane

In case the projection is from an inclined plane, we consider two axes x′ and y′, along and perpendicular to the inclined plane.

Motion up the plane



In x′-y′ plane,

$u_{x'} = v_0\cos(\alpha - \beta)$, $u_{y'} = v_0\sin(\alpha - \beta)$
$a_{x'} = -g\sin\beta$, $a_{y'} = -g\cos\beta$

Since $y' = v_o\sin(\alpha - \beta)t - \frac{1}{2}g\cos\beta t^2$

at $t = T$, $y' = 0$, where T = time of flight.

$\Rightarrow \quad T = \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$ again $x = (v_0\cos\alpha).T$

$$x = v_o\cos\alpha\ \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$$

So range along inclined plane (R) = $x' = x/\cos\beta$

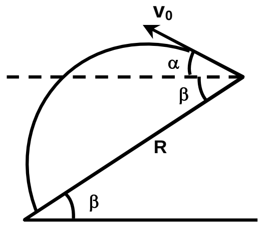
$\therefore \quad x' = \frac{2v_0^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$ [Apply formula 2cosA sin B = sin(A + B) – sin(A–B)]

$$x' = R = \frac{v_0^2\left[\sin(2\alpha-\beta)-\sin\beta\right]}{g\cos^2\beta}$$

Now, R will be maximum when $\sin(2\alpha - \beta)$ is maximum, i.e. $\sin(2\alpha - \beta) = 1$.

$\Rightarrow \quad R_{max} = \dfrac{v_0^2[1-\sin\beta]}{g(1-\sin^2\beta)} \Rightarrow \quad R_{max} = \dfrac{v_0^2}{g(1+\sin\beta)}$ up the plane

Motion down the plane



Let the particle be thrown at a velocity v_0 at an angle 'α' with the horizontal as shown in figure.

$V_0 \sin(\alpha + \beta)T - \dfrac{1}{2}\cos\beta T^2 = 0$ [for y′ = 0]

$\Rightarrow \quad T = \dfrac{2v_o\sin(\alpha+\beta)}{g\cos\beta}$

$R = v_o\cos(\alpha + \beta)T + \dfrac{1}{2}g\sin\beta T^2 = \dfrac{v_0^2}{g}\left[\dfrac{\sin(2\alpha+\beta)+\sin\beta}{1-\sin^2\beta}\right]$

For R to be maximum;

$\sin(2\alpha + \beta) = 1$

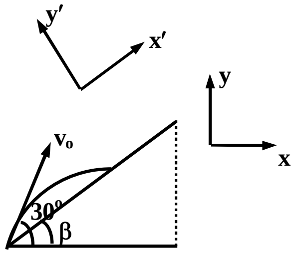
and $\quad R_{max} = \dfrac{v_0^2}{g}\left[\dfrac{1+\sin\beta}{1-\sin^2\beta}\right]$

$= \dfrac{v_0^2}{g(1-\sin\beta)}$ down the plane.

Illustration 29. *Name a quantity which remains unchanged during the flight of projectile on an inclined plane.*

Solution: Horizontal component of velocity.

Illustration 30. *From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30° with the horizontal (a) up the plane (b) down the plane. Find the range in each case.*



Solution : Let β be the inclination of the plane.

Hence, $\sin\beta = \dfrac{7}{25}$, and $\cos\beta = \dfrac{24}{25}$

(a) $v_{ox'} = v_o\cos(30^o - \beta)$

and $a_x' = -g\sin\beta$

$v_{oy'} = v_o\sin(30^o - \beta)$

and $a_y' = -g\cos\beta$

$\because \quad y' = v_{oy}'t + \dfrac{1}{2}a_y't^2$

If T = time of flight, then at t = T, y′ = 0

$\Rightarrow \quad 0 = v_o\sin(30^o - \beta)T - \dfrac{1}{2}g\cos\beta T^2$

$\Rightarrow \quad T = \dfrac{2v_o\sin(30^o-\beta)}{g\cos\beta}$

If R_1 be the range then $R_1\cos\beta = x = v_o\cos30^o.T$

$\Rightarrow \quad R_1\cos\beta = v_o\cos30.\dfrac{2v_o\sin(30°-\beta)}{g\cos\beta}$

$$\Rightarrow \quad R_1 = \frac{2v_0^2 \cos 30^o \sin(30-\beta)}{g\cos^2\beta}$$

Solving we get, $R_1 = 1749.8$ m

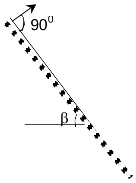
(b) For down the plane,

$$T = \frac{2v_o \sin(30^o + \beta)}{g\cos\beta}$$

Hence $\quad R_2 \cos\beta = v_o\cos 30^o .T$

$$\Rightarrow \quad R_2 = \frac{2v_0^2 \cos 30^o \sin(30^o + \beta)}{g\cos^2\beta}.$$

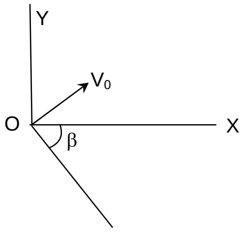
Illustration 31. *A projectile is launched from an inclined plane with an initial velocity v_0 as shown in the figure. Find the time after which the projectile hits the plane for the first time.*



Solution: Let the projectile hit the plane after time t.

The horizontal displacement $x = (v_0 \sin\beta)$ t

The vertical displacement $y = (v_0 \cos\beta)\, t - \frac{1}{2}gt^2$



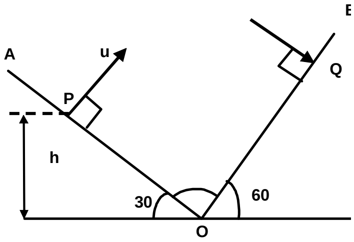
$y = -(\tan\beta)x$ for the plane

$$\therefore \quad t = \frac{2v_0}{g\cos\beta}$$

Illustration 32. *Two inclined planes of inclinations 30° and 60°, respectively, meet at 90° as shown in figure. A particle is projected from point P on the first inclined plane with a velocity u = $10\sqrt{3}$ m/s in a direction perpendicular to the inclined plane and it is observed to hit the other inclined plane at 90°.*

Find (a) the height of point P from ground,

(b) the length of $\overline{PQ}$.



Solution: (a) We observe the motion of projectile fixing y-axis with OP and x-axis with OQ. Hence, velocity at any instant t along x-axis:

$v_x = 10\sqrt{3} - (g \sin 60°)t$

$v_y = 0 - (g\cos 60°)t$

As $v_x = 0$ at the time of hitting,

Time of flight $= T = 2$ sec.

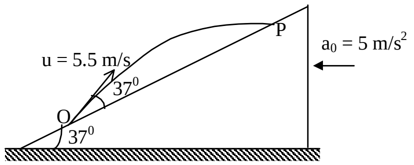
Displacement OP during this time $= \frac{1}{2}(g\cos 60°)t^2 = \frac{1}{2} \times 10 \times \frac{1}{2} \times 4 = 10$ m

Hence, $h = OP \sin 30^\circ = 10 \times \frac{1}{2} = 5m$

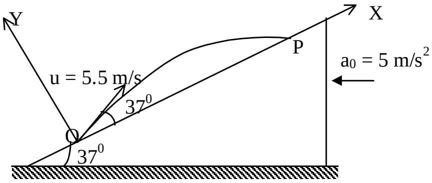
(b) Similarly, displacement $OQ = (10\sqrt{3})(2) - \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times 4 = 10\sqrt{3}$ m

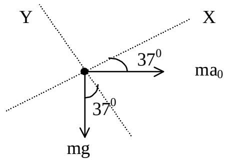
Hence, $PQ = \sqrt{OP^2 + OQ^2} = 20$ m.

Illustration 33. *A particle is projected up a large inclined plane from a point O on it as shown in the figure. The projection velocity has a magnitude of 5.5 m/s and its direction makes an angle of 37° with the inclined plane. The inclination of the plane is also 37°. The inclined plane starts moving towards left with an acceleration $a_0 = 5\ m/s^2$ at the moment the particle is projected. The particle strikes the inclined plane at a point P. Find the time taken by the particle to move from O to P. Also find the magnitude of displacement along the inclined plane as it moves from O to P. (Take $\sin 37^\circ = 3/5$)*



Solution: Let us take x and y axes as shown in the figure. The magnitude of pseudo force acting on the particle has a magnitude of ma_0 and its direction will be towards right as shown in the free body diagram.





F.B.D. of the particle

The components of the acceleration of the particle are

$$a_x = \frac{ma_o \cos 37^\circ - mg \sin 37^\circ}{m}$$

$$= 5 \times \frac{4}{5} - 10 \times \frac{3}{5} = -2 m/s^2$$

$$a_y = \frac{-(mg\cos 37^\circ + ma_o \sin 37^\circ)}{m}$$

$$= -\left(10 \times \frac{4}{5} + 5 \times \frac{3}{5}\right) = -11 m/s^2$$

$$u_x = u\cos 37^\circ = 5.5 \times \frac{4}{5} = 4.4 \text{ m/s}.$$

$u_y = u\sin 37^\circ = 5.5 \times \frac{3}{5} = 3.3$ m/s

Displacement of the particle along y – axis

$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = 3.3t - \frac{1}{2} \times 11t^2$

When the particle strikes the plane y = 0

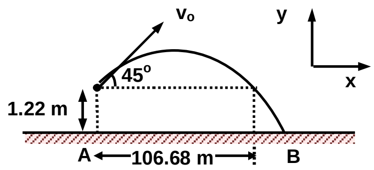
$\Rightarrow t = \frac{2u_y}{-a_y} = \frac{2 \times 3.3}{11} = 0.6$ sec.

$OP = x = u_x t + \frac{1}{2} a_x t^2$

$= 4.4 \times 0.6 - \frac{1}{2} \times 2 \times (0.6)^2 = 2.28$ m.

Illustration 34. *A batsman hits a ball at a height of 1.22 m above the ground so that ball leaves the bat at an angle of 45^o with the horizontal. A 7.31 m high wall is situated at a distance of 97.53 m from the position of the batsman. Will the ball clear the wall if its range is 106.68m? Take g = 10 m/s^2.*

Solution :



$$R(\text{range}) = \frac{v_0^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad v_0^2 = \frac{Rg}{\sin 2\theta} = Rg \quad \text{as} \quad \theta = 45^o.$$

$$\Rightarrow \quad v_o = \sqrt{Rg} \qquad \ldots (1)$$

Equation of trajectory

$$y = x \tan 45^o - \frac{gx^2}{2v_0^2 \cos^2 45^o} = x - \frac{gx^2}{2Rg\frac{1}{2}} \quad \text{[using (1)]}$$

Putting x = 97.53, we get

$$y = 97.53 - \frac{10 \times (97.53)^2}{106.68 \times 10} = 8.35$$

Hence, height of the ball from the ground level is

h= 8.35 + 1.22 = 9.577m.

As height of the wall is 7.31m, so the ball will clear the wall.

Illustration 35. *A particle is projected with velocity u and angle θ with the horizontal. Find the time after which the velocity will be perpendicular to the initial velocity.*

Solution :

$$\bar{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

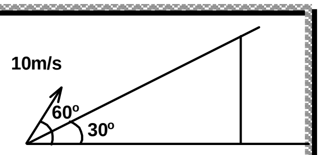
$$\bar{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

$$\bar{u}\cdot\bar{v} = 0 = u^2\cos^2\theta + u^2\sin^2\theta - (u\sin\theta)gt$$

$$\therefore \quad t = \frac{u}{g\sin\theta}$$

Exercise 11:

(i) ***A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of*** $60°$ ***with the horizontal, from a point on an incline plane, making an angle of*** $30°$ ***with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is***



(A) $\frac{2}{\sqrt{3}}$ sec　　*(B)* $\frac{1}{\sqrt{3}}$ sec

(C) $\sqrt{3}$ sec　　*(D)* $\frac{1}{2\sqrt{3}}$ sec

(ii) ***An object projected with the same speed at two different angles covers the same horizontal range R. If the two times of flight be t_1 and t_2, prove that R = $\frac{1}{2}gt_1t_2$.***

(iii). ***Is it important in the long jump that how much height you take for jumping?***

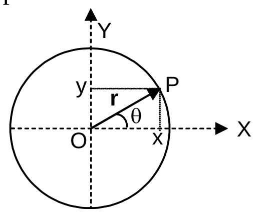
UNIFORM CIRCULAR MOTION

As another small illustration of motion of a particle in two dimensions let's analyse the uniform circular motion of a particle.

In uniform circular motion, the particle moves in a circular path with constant speed.

Let's choose the centre of the circular path as the origin of the reference frame. Point 'P' is an arbitrary point on the path whose position vector

$$\vec{r} = x\hat{i} + y\hat{j}.$$



where r, the radius of the circular path, is related to x and y by following equations

$$x = r\cos\theta,\ \ y = r\sin\theta\ \text{ and } x^2 + y^2 = r^2$$

$\Rightarrow$ $\quad \vec{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$

Now, the velocity of particle 'P' is given as

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d(r\cos\theta)}{dt}\hat{i} + \frac{d(r\sin\theta)}{dt}\hat{j}$$

$\Rightarrow$ $\quad \vec{v} = -r\sin\theta\frac{d\theta}{dt}\hat{i} + r\cos\theta\frac{d\theta}{dt}\hat{j}$

but $\quad \frac{d\theta}{dt} = \omega = \text{const.}$ $\qquad$ [for uniform circular motion]

Thus, $\quad \vec{v} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Now, $\quad \vec{v}\,.\,\vec{r} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})(r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}) = \omega r^2(-\sin\theta\cos\theta + \cos\theta\sin\theta) = 0$

$\Rightarrow$ $\quad \vec{v}$ is perpendicular to $\vec{r}$

$$|\vec{v}| = \omega r\sqrt{\sin^2\theta + \cos^2\theta} = \omega r$$

Now, acceleration $\vec{a}$ is given as

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega r\left(-\cos\theta.\frac{d\theta}{dt}\hat{i} - \sin\theta\frac{d\theta}{dt}\hat{j}\right) \quad \Rightarrow \quad \vec{a} = -\omega^2 r\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$\Rightarrow$ $\quad \vec{a} = \omega^2(-\vec{r})$

which shows that $\vec{a}$ is directed in the opposite direction of '$\vec{r}$'. Thus, $\vec{a}$ is always directed towards the centre.

Magnitude of $\vec{a}$, $|\vec{a}| = \omega^2 r\sqrt{\cos^2\theta + \sin^2\theta} = \omega^2 r$

Note: If the circular motion is non-uniform, then tangential acceleration $a_t = dv/dt$ exists apart from normal acceleration $\omega^2 r$.

Illustration 36. *Does velocity remain constant in uniform circular motion ?*

Solution: No, magnitude remains constant but direction keep on changing.

Illustration 37. *Find the magnitude of average acceleration of the tip of the second hand of length 10 cm during 10 seconds.*

Solution: Average acceleration has the magnitude

$a = \Delta v/\Delta t$, where $\Delta v = 2v\sin\theta/2$

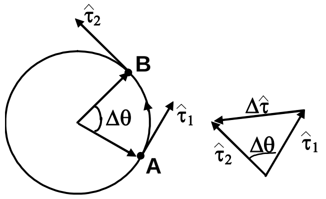
$\Rightarrow$ $\quad a = \frac{2V\sin\theta/2}{\Delta t}$

Putting v = π/300 m/sec (obtained earlier), Δt = 10 seconds and $\theta = 60°$, we obtain

$$a = \frac{2(\pi/300)\sin 30°}{10}$$

$$\Rightarrow \quad a = \frac{\pi}{3000} \text{ m/sec}^2.$$

CIRCULAR MOTION



$|\vec{v}| = r\omega$ (variable)

Let $\hat{\tau}$ = unit vector along the tangent. and

$\hat{u}$ = unit vector along radius (outwards)

Since the velocity of the particle describing circular motion is along the tangent, hence it can be given by the expression.

$\vec{v} = v\hat{\tau}$ where v = magnitude of the velocity

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt}$$

Take A and B two positions of the particle.

Change in $\hat{\tau} = \Delta\hat{\tau} = \Delta\theta(-\hat{u})$

Negative sign shows that it is towards the centre as (S = R□)

$$\Rightarrow \quad \frac{d\hat{\tau}}{dt} = \frac{d\theta}{dt}(-\hat{u}) \qquad \Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\theta}{dt}(-\hat{u})$$

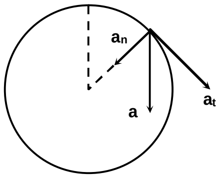
$$\Rightarrow \quad \vec{a} = \vec{a}_{tangential} + \vec{a}_{radial}, \qquad \text{where} \quad \vec{a}_{tangential} = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

If 'v' is a constant then $\vec{a}_{tangential} = 0$ and $\quad \vec{a}_{radial} = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r} = \omega^2 r$

$$|\vec{a}| = \sqrt{a_r^2 + a_T^2}$$

Illustration 38. *A point moves along a circle with velocity v = at where a = 0.5m/s². Find the total acceleration of the point at the moment when it covered (1/10)th of the circle after beginning of motion.*

Solution : We know $S = ut + \frac{1}{2}at^2$



Here, $S = \frac{2\pi r}{10} = \frac{\pi r}{5}$, $a_t = 0.5$ m/s² and u = 0

$$\therefore \quad \frac{\pi r}{5} = 0 + \frac{1}{2}0.5t^2, \quad t = \sqrt{\frac{4\pi r}{5}}$$

$$\therefore \quad v = at = 0.5\sqrt{\frac{4\pi r}{5}} = \sqrt{\frac{\pi r}{5}}$$

$$\therefore \quad a_n = \frac{v^2}{r} = \frac{\pi r}{5}\frac{1}{r} = \frac{\pi}{5}$$

$$\therefore \quad a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{\pi}{5}\right)^2 + 0.5^2} = \sqrt{\frac{\pi^2}{25} + \frac{1}{4}} = 0.8 \text{ m/s}^2$$

Exercise 12:

(i) ***By using vector method, show that direction of acceleration vector $\vec{a}$ is towards the centre of the circle in which body is revolving.***

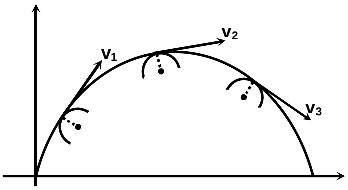
(ii) ***What is the direction of velocity vector of a particle in circular motion ?***

(iii) ***What is the acceleration associated with a body having variable speed in a circular path ?***

(iv) ***Two cars having masses m_1 and m_2 move in circles of radii r_1 and r_2, respectively. If they complete the circles in equal time, find the ratio of the their angular speeds $\frac{\omega_1}{\omega_2}$.***

RADIUS OF CURVATURE

In a curvilinear motion, every small path may be assumed to be an arc of a circular path, and here the radius of curvature will be different at different points. So if a particle moves on a curved path then radius of curvature is given by $R = \frac{v^2}{a_R}$.



where v = instantaneous velocity at any time at that point and a_R = acceleration acting normal to the path towards the centre.

Illustration 39. *Where is radius of curvature maximum at the highest point or at the point of projection?*

Solution: At the point of projection

Illustration 40. *Find the ratio of radius of curvature at the highest point of projectile to that just after its projection if the angle of projection is 30°.*

Solution : If $\vec{v}_0$ is the initial velocity

$v_p = v_0 \cos\theta$

Normal acceleration at $0 = g\cos\theta$

Normal acceleration at P = g

Hence, if r_0 and r_p be radii of curvature at O and P, respectively.

$$r_0 = \frac{v_0^2}{g\cos\theta} \quad \text{and} \quad r_p = \frac{v_0^2\cos^2\theta}{g}$$

Hence, the required ratio $= \frac{r_p}{r_0} = \cos^3\theta = \frac{3\sqrt{3}}{8}$.

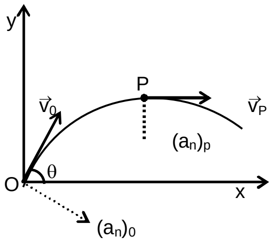
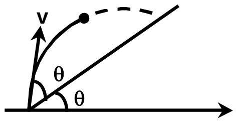


Illustration 41. *A particle is projected with a velocity u at an angle θ with an inclined plane which makes an angle θ< 45° with the horizontal. Calculate the radius of curvature of the path of projectile when velocity of projectile becomes parallel to the plane.*



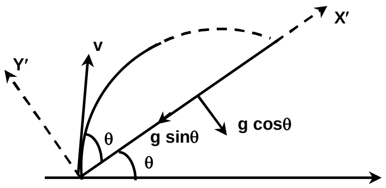
Solution : Along y′,

$$u\sin\theta - g\cos\theta t = 0$$

$$\Rightarrow \quad t = \frac{u}{g}\tan\theta$$

Along x′,

$$v'_x = u\cos\theta - g\sin\theta.t = u\cos\theta - g\sin\theta.\frac{u}{g}\tan\theta$$



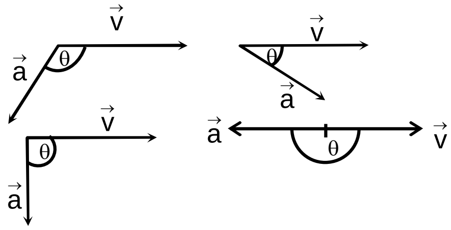
$$= \frac{u\cos^2\theta - u\sin^2\theta}{\cos\theta} = \frac{u\cos 2\theta}{\cos\theta}$$

$$r = \frac{(v'_x)^2}{a_n} = \frac{u^2\cos^2 2\theta}{g\cos^3\theta}$$

Exercise 13:

i) ***The tangential acceleration change the speed of the particle whereas the normal acceleration changes its direction. State whether the statement true or false?***

ii) ***At a certain moment, the angle between velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$ of the particle is θ. What will be the motion of the particle at this moment for different θ's: rectilinear or curvilinear, accelerated or decelerated?***

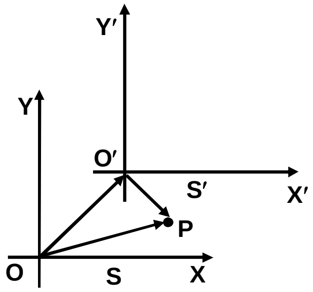


RELATIVE VELOCITY

The position, velocity and acceleration of a particle depend on the reference frame chosen.

A particle P is moving and is observed from two frames 'S' and 'S′'. The frame S is stationary and the frame S′ is in motion.

Let at any time position vector of the particle P with respect to S is



$\overrightarrow{OP} = \vec{r}_{p,s}$ and with respect to S′ is $\overrightarrow{O'P} = \vec{r}_{p,s'}$

Position vector of the origin of S′ with respect to origin of S is

$\overrightarrow{OO'} = \vec{r}_{s',s}$

From vector triangle OO′P, we get

$$\overrightarrow{O'P} = \overrightarrow{OP} - \overrightarrow{OO'} \Rightarrow \vec{r}_{p,s'} = \vec{r}_{p,s} - \vec{r}_{s',s}$$

$$\Rightarrow \quad \frac{d}{dt}\left(\vec{r}_{p,s'}\right) = \frac{d}{dt}\left(\vec{r}_{p,s}\right) - \frac{d}{dt}\left(\vec{r}_{s',s}\right)$$

$$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p,s} - \vec{v}_{s',s}$$

$$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p(\text{absolute})} - \vec{v}_{s'\,(\text{absolute})}$$

If $\vec{v}_r$ and $\vec{v}_m$ are the absolute velocities of the rain and the man, respectively, then the relative velocity of rain w.r.t. (as seen by) the man is $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$.

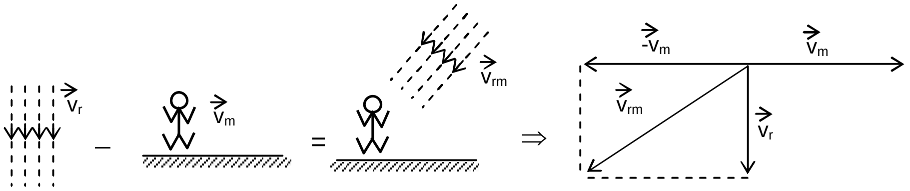


Illustration 42. *Two men are moving with same velocity in the same direction. What is there relative velocity?*

Solution: zero

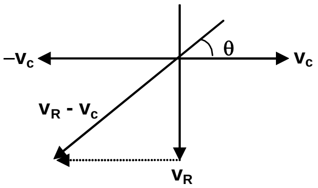
Illustration 43. *A stationary person observes that rain is falling vertically down at 30 km/hr. A cyclist is moving on the level road at 10 km/hr. In which direction the cyclist should hold his umbrella to prevent himself from rain?*

Solution: Relative to stationary frame, velocity of rain is 30 km/hr downward. Take horizontal axis as x-axis and vertical axis as y-axis and $\hat{i}, \hat{j}$ are the unit vectors along X- and Y-axes, respectively.

$\vec{v}_R = 0 - 30\hat{j}$, $\vec{v}_c = 10\hat{i}$

$\because$ $\vec{v}_{R,c} = \vec{v}_R - \vec{v}_c$

$= -30\hat{j} - 10\hat{i} = -10\hat{i} - 30\hat{j}$



If angle between horizontal and the $\vec{v}_{R,c}$ is θ, then

$$\tan\theta = \frac{-30}{-10} = 3$$

$\Rightarrow$ $\theta = \tan^{-1}3$ $\Rightarrow$ $\theta = 72^o$.

Therefore, to prevent himself from rain the cyclist should hold the umbrella at angle of 72^o from horizontal.

Illustration 44. *A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.*

Solution: Let velocity of the wind be

$\vec{v}_w = v_1\hat{i} + v_2\hat{j}$ m/s

And velocity of the man is

$\vec{v}_m = 5\hat{i}$

$\therefore$ $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (v_1 - 5)\hat{i} + v_2\hat{j}$

In first case,

$v_1 - 5 = 0$

$\Rightarrow$ $v_1 = 5$ m/s.

In the second case, $\tan 45° = \dfrac{v_2}{v_1 - 10}$

$\Rightarrow$ $v_2 = v_1 - 10 = -5$ m/s.

$\Rightarrow$ $\vec{v}_w = 5\hat{i} - 5\hat{j}$ m/s.

Illustration 45. *From a lift moving upward with a uniform acceleration 'a', a man throws a ball vertically upwards with a velocity v relative to the lift. The ball comes back to the man after a time t. Show that a + g = 2 v/t.*

Solution: Let us consider all the motion from lift frame. Then, the acceleration, displacement and velocity everything will be considered from the lift frame itself.
As the ball comes again to the man, therefore displacement from the lift frame is zero.
Again, the velocity with respect to the lift frame is v.
Similarly, the acceleration with respect the lift frame is

$$g - (-a) = a + g \quad \text{(downwards)}$$

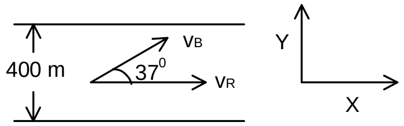
Now, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 0 = vt - \frac{1}{2}(a + g)t^2$$

or $\quad a + g = 2\frac{v}{t}$.

Illustration 46. *A river 400 m wide is flowing at a rate of 4 m/s. A boat is sailing at a velocity of 20 m/s with respect to the still water in a direction making an angle of 37° with the direction of river flow.*
(a) Find time taken by the boat to reach the opposite bank.
(b) How far from the starting point does the boat reach on the opposite bank?

Solution: (a) Resultant velocity of the boat is



$$v = (v_R + v_B \cos 37^\circ)\, i + v_B \sin 37^\circ\, j$$
$$= 4i + 20 \times \frac{4}{5} i + 20 \times \frac{3}{5} j$$
$$= 20\, i + 12\, j \text{ m/s}$$

Time taken by boat to cross the river

$$= \frac{\text{distance travelled in y-direction}}{\text{velocity in y-direction}} = \frac{400}{12} = \frac{100}{3} \text{ sec.}$$

(b) Displacement along x = v t

$$= 20 \times \frac{100}{3} = \frac{2000}{3} \text{ m}$$

Distance from starting point $= \sqrt{(400)^2 + \left(\frac{2000}{3}\right)^2} = \frac{400}{3}\sqrt{34}$ m.

Illustration 47. *A stone is projected from a balloon which is ascending with a velocity 2 m/s. The velocity of the stone w.r.t. balloon is $\sqrt{2}$ m/s at an angle of 45°. Find the velocity of the stone with respect to ground.*

Solution :

$$\vec{v}_{SB} = v\cos 45^0\,\hat{i} + v\sin 45^0\,\hat{j}$$
$$= \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{j} = (\hat{i} + \hat{j})\text{m/s}$$
$$\vec{v}_{BG} = 2\hat{j}\text{m/s}$$

Thus, $\vec{v}_{S,G} = \vec{v}_{SB} + \vec{v}_{BG}$

$= 2\hat{j} + (\hat{i} + \hat{j}) = ((\hat{i} + 3\hat{j})$

$v = \sqrt{1^2 + 3^2} = \sqrt{10}$ m/s and $\tan\theta = \dfrac{3}{1}$

$\theta = \tan^{-1}(3)$.

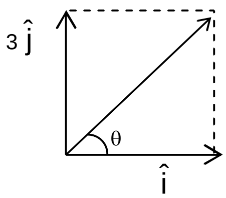
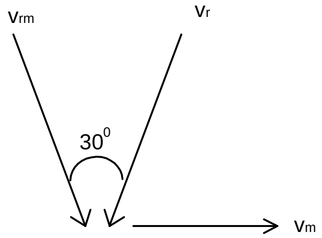


Illustration 48. *A man standing on a road has to hold his umberella at 30° with the vertical to keep the rain away. He throws the umbrella and runs at 10 kmph. He finds that rain drops are hitting his head vertically. find the speed of raindrops with respect to*
(a) the road
(b) the moving man.



Solution : Velocity ofrain w.r.t. road is $\vec{v}_r$ and velocity of rain w.r.t. moving man is $\vec{v}_{rm}$ but

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

$= -v_r \sin 30\,\hat{i} - v_r \cos 30\,\hat{j} - 10\,\hat{i}$

$= (-v_r \sin 30 - 10)\,\hat{i} - v_r \cos 30\,\hat{j}$

But $-v_r \sin 30 - 10 = 0$ $\quad\therefore\ v_r \sin 30 = -10$

$v_r = \dfrac{-10}{\sin 30} = -20$ m/s.

But v_r is not negative

$\therefore\quad \vec{v}_m = -10\hat{i}$

and $\vec{v}_{rm} = -[20\cos 30]$

$= 20\cos 30\,\hat{j} = 10\sqrt{3}\,\hat{j}$.

Illustration 49. *A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle of 30° to the path of A.. Find the velocity of B relative to A.*

Solution: $|v_B - v_A| = \sqrt{v_B^2 + v_A^2 - 2v_A v_B \cos 30^0}$

$= \sqrt{5^2 + 3^2 - 2\times5\times3\times(\sqrt{3/2})}$

$= \sqrt{8.02} = 2.832$ m/sec.

Using sine rule, $\dfrac{3}{\sin\theta} = \dfrac{2.832}{\sin 30^0}$

$\Rightarrow\ \theta = 32°$

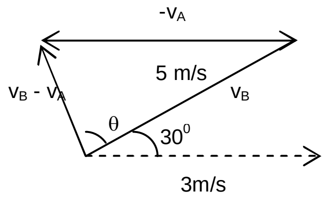
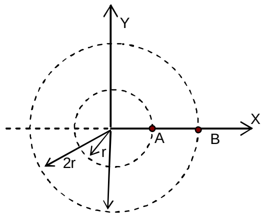


Illustration 50. *Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω, respectively.*
(a) Find the relative velocity of B w.r.t. A after time $t = \pi/\omega$.
(Initial position of particles A and B are shown in figure.)
(b) Also find the relative position vector of B w.r.t. A.



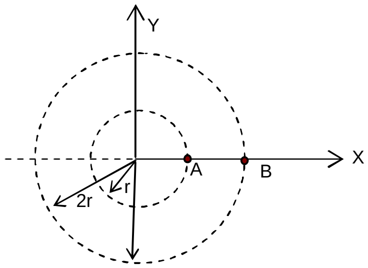
Solution:

$\theta_A = 2\omega\dfrac{\pi}{\omega} = 2\pi,\ \ v_A = 2\omega r\,\hat{j}$

$\theta_B = \omega\dfrac{\pi}{\omega} = \pi,\ \ v_B = 2\omega r(-\hat{j})$

(a) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 2\omega r(-\hat{j}) - 2\omega r(\hat{j}) = -4\omega r\,\hat{j}$

(b) $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A = 2r\,(-\hat{i}) - (r)\,\hat{i} = -3r\,\hat{i}$.



Exercise 14:

(i) ***How long will a boy sitting near the window of a train travelling at 54 km/h see a train passing by in the opposite direction with a speed of 36 km/hr? The length of the slow moving train is 100 m.***

(ii) ***Two particles A and B are moving with speeds of 2 km/hr and 3 km/hr, respectively, in the same direction. Find how far will B be from A after 1 hour ?***

(iii) ***Two cars are moving in the same direction with the same speed 30 km/hr. They are separated by a distance of 5 km. What is the speed of a car moving in the opposite direction if it meets these two cars at an internal of 4 minutes?***

(iv) ***A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr. He finds raindrops are hitting his head vertically. Find the speed of raindrops with respect to:***
(a) the road, and (b) the moving man.

(v) ***A boat travels downstream from point A to point B in two hours and upstream in four hours. Find the time taken by a log of wood to cover the distance from point A to point B.***

SUMMARY

Motion in a straight line:

1. Distance is the total length of the path traversed by an object.

2. Displacement is the change in position $\Delta x = x_2 - x_1$

3. Average speed $= \dfrac{\text{distance traversed}}{\text{time interval}}$

4. Speed: The speed of an object is equal to the distance traversed by it in a very short time interval divided by time interval.

5. Instantaneous velocity: It is defined as the limit of average velocity as the time interval Δt becomes infinitesimally small,
$v = \lim\limits_{\Delta t \to 0} \bar{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
Slope of the tangent drawn on position–time graph at any instant gives the velocity at that particular time.

6. Average acceleration = $\dfrac{\text{change in velocity}}{\text{time interval}}$

7. Instantaneous acceleration: It is defined as the limit of the average acceleration as the time interval Δt goes to zero
i.e. $a = \lim\limits_{\Delta t \to 0} \bar{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$
At any particular instant, the slope of the velocity–time graph gives the acceleration of an object at that particular instant.

8. (i) The area under the speed–time graph gives the distance traversed by the object in the corresponding time interval.
(ii) The area under a velocity– time graph gives the displacement of the object.

9. For uniformly accelerated rectilinear motion, three equations of motion are
$v = u + at$
$x = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2ax$
where
u = initial velocity
v = final velocity
t = time taken
a = acceleration

Motion in a plane:

1. The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y and z.
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

2. $$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to o} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{r}}{dt}$$

3. $$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{av} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

4. In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity component are simple functions of time, and the shape of the path is always a parabola.

5. When a particle moves in a circular path of radius R, its acceleration $\vec{a}$ is directed towards the centre of the circle and perpendicular to $\vec{v}$.

$$a_{radial} = \frac{v^2}{R}$$

where $v = \frac{2\pi R}{T}$

6. If the speed is not constant in circular motion, there is still a radial component of $\vec{a}$ but there is also a component of $\vec{a}$ parallel to the path.

$$a_{rad} = \frac{v^2}{R}$$

$$a_{tan\,gential} = \frac{dv}{dt}$$

7. When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B.

$\therefore \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

where $\vec{v}_{BA}$ is the velocity of B with respect to A.

Both observers measure the same acceleration for the particle; that is

$\vec{a}_{PA} = \vec{a}_{PB}$

MISCELLANEOUS EXERCISE

1. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement?

2. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed. Do you agree with this statement?

3. Is the vertical height taken by a long jumper important while taking the jump?

4. A woman standing on the edge of a cliff throws a ball straight up with a speed of 8 km/h and then throws another ball straight down with a speed of 8 km/h from the same position. What is the ratio of the speeds with which the balls hit the ground ?

5. Find the average velocity during the time of flight, if a particle is projected with v at an angle θ with horizontal plane.

6. Establish the relation $x(t) = v(0)t + \frac{1}{2}at^2$ by calculus method.

7. Derive the velocity–time relationship by (i) calculus method, (ii) graphical method.

8. A stone is thrown upwards from the top of a tower 85 m high. It reaches the ground in 5 second. Calculate (i) the greatest height above the ground, (ii) the velocity with which it reaches the ground and (iii) the time taken to reach the maximum height.
Given: $g = 10\ m/s^2$.

9. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m/s. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the values of acceleration, retardation and total time taken.

10. Prove that there are two times for which a projectile travels the same vertical distance. Also prove that the sum of the two times is equal to the time of flight.

SOLUTION TO MISCELLANEOUS EXERCISE

1. No, not always. In case of uniform circular motion, the particle is accelerating but its speed is neither decreasing nor increasing, only direction of velocity changes.

2. This is true as motion under gravity is independent of mass of the body and so the body comes back to the point of projection with the same speed.

3. Yes, because for the longest jump the player should throw himself at an angle of 45° wrt horizontal. The vertical height required for this purpose should be $\frac{u^2}{4g}$, where u is velocity of

throw. If the vertical height is different from $\frac{u^2}{4g}$ then the angle will be different from 45° and the horizontal distance covered also will be less.

4. 1 : 1, both the balls will hit the ground with the same speed.

5. v cos θ

8. h = 3.2 m, v = 42 m/s, t = 0.8 sec.

9. 0.8 m/s^2, 0.5 m/s^2 and 86 sec.

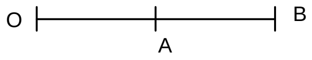
SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *A car starts from rest and moves with a constant acceleration of 2.0 m/s² for 30 seconds. The brakes are then applied and the car comes to rest in another 60 seconds. Find*
(a) total distance covered by the car.
(b) Maximum speed attained by the car
(c) Find shortest distance from initial point to the point when its speed is half of maximum speed.

Sol. Final velocity at A $v_A = 2 \times t_1 = 2 \times 30 = 60$ m/sec.



For AB, Let the retardation be 'b'

$\therefore\ 0 = v_A + bt^2$

$\therefore\ b = -\dfrac{v_A}{t} = -\dfrac{60}{60} = -1$ m/s²

(a) Total distance = OA + AB

$$OB = \frac{1}{2}at_1^2 + (v_A t_2 - \frac{1}{2}bt_2^2)$$
$$= (\frac{1}{2}\times 2\times 30\times 30 + 60\times 60\times 60 - \frac{1}{2}\times 1\times 60\times 60)$$
$$= 900 + 3600 - 1800 = 2700 \text{ m.}$$

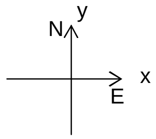
(b) Maximum speed $v_A = 60$ m/s.

(c) $v^2 = 2 \times a \times s$

$$s = \frac{(v_A/2)^2}{2\times a} = \frac{30\times 30}{2\times 2} = 225 \text{ m.}$$

Prob 2. *A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 minutes to reach the field,*
(a) what distance he has to walk to reach the field ?
(b) what is his displacement from his house to the field ?
(c) what is the average speed of farmer during the walk ?
(d) what is the average velocity of farmer during the walk ?

Sol. (a) Distance = 500 + 400 + 200 = 1100m



(b) Displacement = $500(\hat{j}) + 400(\hat{i}) + 200(-\hat{j}) = 300\vec{j} + 400\hat{i}$

Magnitude of displacement = $\sqrt{(400)^2 + (300)^2} = 500$m

(c) Average speed = $\dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{1100}{20\times 60} = \dfrac{11}{12}$ m/s

(d) Average velocity = $\dfrac{\text{Displacement}}{\text{Time}} = \dfrac{500}{20\times 60} = \dfrac{5}{12}$ m/s

$\theta = \tan^{-1}\left(\dfrac{300}{400}\right) = 37^\circ$ due North of East.

Prob 3. *A body is projected up such that its position vector varies with time as* $\vec{r} = 6t\hat{i} + (8t - 5t^2)\hat{j}$. *Find the (a) initial velocity (b) time of flight*

Sol. (a) The position of the body at any time t is given as $\vec{r} = 6t\,\hat{i} + (8t-5t^2)\,\hat{j}$. When t = 0, r = 0. That means the body is projected from the origin of the coordinate system. Differentiating both sides w.r.t. time 't', we obtain

$$\frac{d\vec{r}}{dt} = 6\,\hat{i} + (8-10t)\,j \qquad \Rightarrow \vec{v} = 6\,\hat{i} + (8-10t)\,j.$$

Putting t = 0, we obtain the initial velocity (velocity of projection) given as

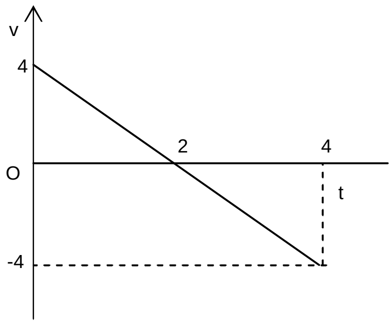
$$(\vec{v})_{t=0} = \vec{v}_0 = 6\,\hat{i} + 8\,\hat{j} \quad \Rightarrow v_0 = 10 \text{ m/sec;}$$

(b) The time of flight $T = \dfrac{2v_0 \sin\theta_0}{g}$

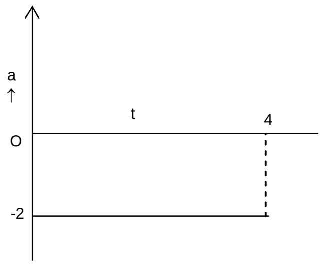
$$\Rightarrow T = \frac{2(v_y)_0}{g} \quad \text{where } (v_y)_0 = 8$$

$$\Rightarrow \quad T = \frac{2\times 8}{10} = 1.6 \text{ sec.}$$

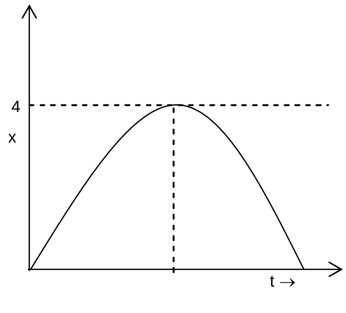
Prob 4. *A particle starts from origin at t = 0 along +ve x axis. It's velocity–time graph is shown in the figure. Draw*
(i) a, t graph
(ii) x, t graph.



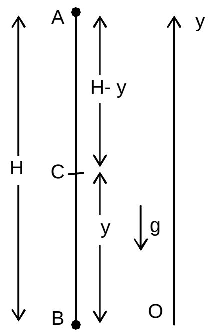
Sol. (i) Velocity is decreasing
so, a = –4/2 = –2



(ii)



Prob 5. *A stone 'A' is dropped from the top of a tower 20 m high. Simultaneously another stone 'B' is thrown up from the bottom of the tower so that it can reach just on the top of the tower. What is the distance of the stones from the ground while they pass each other?*



Sol. Let t be the time when they pass one another

For stone B, $y = v_B t + \frac{1}{2}(-g)t^2$ … (i)

Ffor stone A, $H - y = \frac{1}{2}gt^2$ … (ii)

From (i) and (ii),

$H = v_B t$ … (iii)

Stone B can reach just one the top of tower. We can calculate the velocity of stone B,

$v_f^2 = v_i^2 + 2a_y y$

$u_f = 0$, for $y_{max} = H = 20$ m

$v_i = v_B$; $a_y = -g$; $v_B = 20$ m/s

From (iii) $t = \frac{20m}{20m/s} = 1sec.$

From equation (i), the required distance (BC) from ground = $20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15$ m

IITJEE TYPE

Prob 6. *A rocket is fired vertically and ascends with constant vertical acceleration of 20m/s^2 for 1 minute. Its fuel is then all used and it continues as a free particle. Find the*
(a) maximum height reached by the rocket
(b) total time elapsed from the take off till the rocket strikes the earth.(g=10m/s^2).

Sol. (a) For the time interval from 0 to 60 seconds rocket accelerates and thereafter it moves under gravity. Distance moved by it in 60 seconds is given by

$$S_1 = \frac{1}{2} \times \frac{20m}{s^2} \times (60s)^2 = 36000m.$$

$$v_{(60s)} = \frac{20m}{s^2} \times 60s = 1200m/s$$

If H be the maximum height reached.

Then, $0 = \left(1200\frac{m}{s}\right)^2 - 2g(H - 36000)$, ($v^2 = u^2 + 2as$)

$\Rightarrow H = 36000 + \frac{1200 \times 1200}{2 \times 10}$ m

$\Rightarrow H = 108000$m

(b) Time taken to ascend is

$$t_1 = 60s + \frac{1200}{10}s = 180 \text{ s}, \quad [t = t_1 + \frac{u}{a}]$$

Let time taken to descend is t_2 then

$$108000 = \frac{1}{2}gt_2^2$$

$\Rightarrow t_2 = \sqrt{\frac{2 \times 108000}{10}} = 146.96s$

Total time $T = t_1 + t_2 = 180 + 146.96 = 326.96$ s.

Prob 7. *The position of a particle moving along the x-axis depends on the time as $x = at^2 - bt^3$ where $a = 3.0\ m/s^2$ and $b = 1.0\ m/s^3$ respectively.*
(a) At what time does the particle reach its maximum positive x-position ?
(b) What total path length does the particle cover in the first 2.0 sec?
(c) Does the particle cover equal path length in the opposite direction in the subsequent 2.0 sec. ? If not, explain why?
(d) Find the total path length covered in the first 4.0 sec.
(e) Find the displacement during the first 4.0 sec.
(f) What is the particles speed and acceleration at the end of first 3.0 sec.?

Sol. Here, the position (x) is time dependent as $x = at^2 - bt^3 = 3t^2 - t^3$

Instantaneous velocity $v = \frac{dx}{dt} = 6t - 3t^2$ . . . (i)

And acceleration $a = \frac{dv}{dt} = 6 - 6t$ . . . (ii)

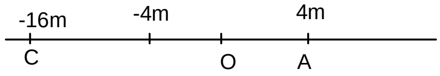
Note that the acceleration is not uniform (like gravity) but time dependent.

(a) At the maximum positive x-position the particle comes to momentary rest (v = 0) and then moves in the negative x-direction with non uniform acceleration.
From equation (i), $v = 0 = 6t - 3t^2$
$\therefore$ Required time = 2.0 seconds.

(b) The x-coordinate of the particle increases from zero to $(x)_{max}$ during the first 2 seconds.
$\therefore$ path length $= (x)_{t = 2s} = (3t^2 - t^3)_{t = 2s} = 4.0$ m

(c) For t > 2s, the particle moves in the backward direction with time dependent acceleration. Hence, subsequent motion is not repeated (as we have seen in free fall) where the acceleration $|\vec{g}|$ remains constant. Hence, the path length for the subsequent 2 seconds will be different.

(d) Position at t = 2 sec. is



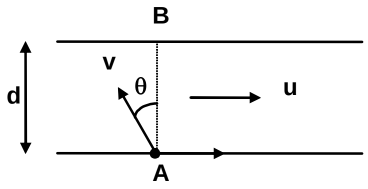
$(x)_{t = 2} = 4.0$ m and
$(x)_{t = 4} = (3t^2 - t^3)_{t = 4} = 48 - 64$
$= -16$ m.
Hence, the path length during the first 4.0 is
OA + AO + OC
= 4 + 4 + 16 = 24.0 m

(e) Displacement during the first 4.0 sec is – 16.0 m.

(f) Speed at the end of 3.0 sec is
$(v)_{t = 3} = (6t - 3t^2)_{t = 3s} = -9.0$ m/s.
Negative sign indicates that motion is along the negative x-direction.
Acceleration $(a)_{t = 3s} = (6 - 6t)_{t = 3s} = -12\ m/s^2$.

Prob 8. *A man can row a boat with a speed of 4 km/hr in still water. He is crossing a river where the speed of current is 2 km/hr.*
(a) In what direction will his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?
(b) If width of the river is 4 km how long will it take him to cross the river, with the condition in part 'a' ?
(c) In what direction should he head the boat if he wants to cross the river in shortest time?
(d) How long will it take him to row 2 kms up the stream and then back to his starting point?

Sol. B is a point directly opposite to the starting point A.
Let the man heads the boat in a direction making an angle θ with the line AB.

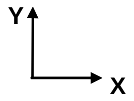


Here $\vec{v}_w = 2\hat{i}$

$\vec{v}_{bw} = -4\sin\theta\hat{i} + 4\cos\theta\hat{j}$

$\because$ $\vec{v}_{(absolute)} = \vec{v}_{bw} + \vec{v}_w$

$= (2 - 4\sin\theta)\hat{i} + 4\cos\theta\hat{j}$



$\Rightarrow v_{bx} = 2 - 4\sin\theta$ and $v_{by} = 4\cos\theta$

(a) For directly opposite point $v_{bx} = 0$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30^o \Rightarrow \theta = 30^o$

Hence, to reach the point directly opposite to starting point he should head the boat an angle $\beta = (90^o + 30^o) = 120^o$ with the river flow.

(b) $t = \frac{y}{v_{by}} = \frac{d}{4\cos\theta} = \frac{4}{4\cos 30^o} = \frac{2}{\sqrt{3}}$ hr.

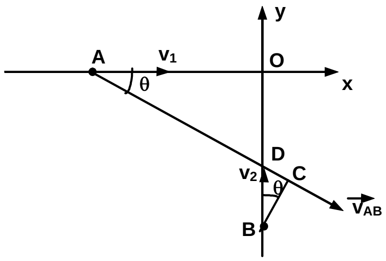
(c) For t to be minimum $\cos\theta = 1 \Rightarrow \theta = 0^o$

$\Rightarrow t_{min} = \frac{4}{4\cos 0} = 1$ hr.

(d) $T = \frac{2}{(4-2)}\text{hr} + \frac{2}{(4+2)}\text{hr} = \left(1 + \frac{1}{3}\right)\text{hr} = \frac{4}{3}\text{hr}.$

Prob 9. *Two particles A and B move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines towards the intersection point O. At moment t = 0, the particles were located at distances l_1 and l_2 from O, respectively. Find the time, when they are nearest and also this shortest distance.*

Sol. $\because$ $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v_1\hat{i} - v_2\hat{j}$



Minimum distance is the length of the perpendicular to $\vec{v}_{AB}$ from B.

If θ is the angle between the x-axis and $\vec{v}_{AB}$, then

$\tan\theta = \left|-\frac{v_2}{v_1}\right| = \frac{v_2}{v_1}$

In ΔAOD, OD = OA tan θ = $\frac{v_2}{v_1} l_1$

Therefore, BD = l_2 – OD = $\frac{v_1 l_2 - v_2 l_1}{v_1}$

In ΔBCD, $\cos\theta = \frac{BC}{BD}$

$\Rightarrow BC = BD\cos\theta = \frac{v_1 l_2 - v_2 l_1}{v_1} \times \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$

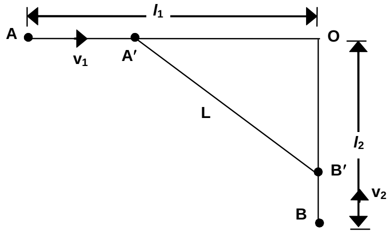
$\Rightarrow BC = \frac{|v_1 l_2 - v_2 l_1|}{\sqrt{v_1^2 + v_2^2}}$

The required time $t = \dfrac{AC}{|\vec{v}_{AB}|} = \dfrac{AD + DC}{|\vec{v}_{AB}|}$

$$\Rightarrow \quad \frac{\ell_1 \sec\theta + BC\tan\theta}{\sqrt{v_1^2 + v_2^2}} = \frac{\dfrac{\ell_1}{v_1}\sqrt{v_1^2+v_2^2} + \dfrac{v_1\ell_2 - v_2\ell_1}{\sqrt{v_1^2+v_2^2}}\dfrac{v_2}{v_1}}{\sqrt{v_1^2+v_2^2}}$$

$$= \frac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$$

Alternatively



After time 't' , the position of the point A and B are
$(\ell_1 - v_1 t)$ and $(\ell_2 - v_2 t)$, respectively.

The distance L between the points A′ and B′ are
$L^2 = (\ell_1 - v_1 t)^2 + (\ell_2 - v_2 t)^2 \; . . . \text{(i)}$

Differentiating with respect to time,

$2L\dfrac{dL}{dt} = 2(\ell_1 - v_1 t)(-v_1) + 2(\ell_2 - v_2 t)(-v_2)$ For minimum value of L, $\dfrac{dL}{dt} = 0$

$(v_1^2 + v_2^2)t = \ell_1 v_1 + \ell_2 v_2$

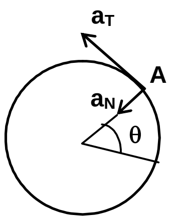
or $\quad t = \dfrac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$

Putting the value of t in equation (i)

$$L_{min} = \frac{|\ell_1 v_2 - \ell_2 v_1|}{\sqrt{v_1^2 + v_2^2}}.$$

Prob 10. *A wheel rotates around a stationary axis so that rotation angle θ varies as $\theta = Pt^2$, where $P = 0.20$ rad/s^2. Find the total acceleration a of the point A at the rim at the moment $t = 2.55$ sec, if the linear velocity of the point A at this moment is $v = 0.65$ m/s.*

Sol. Total acceleration of a body moving in a circular path



$\vec{a} = \vec{a}_R + \vec{a}_t$

$|\vec{a}| = \sqrt{|\vec{a}_R^2| + |\vec{a}_t^2|}$

The radial acceleration a_R is the centripetal acceleration

$a_R = \dfrac{v^2}{R} = \omega^2 R \qquad = \left(\dfrac{d\theta}{dt}\right)^2 R$

$$= \left\{\frac{d}{dt}(Pt^2)\right\}^2 R = 4P^2t^2R \qquad . . . \text{(i)}$$

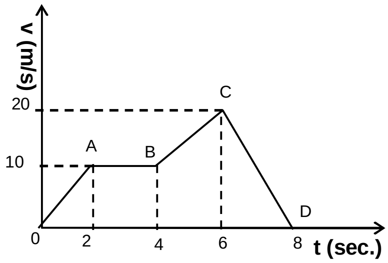
Tangential acceleration $a_T = \dfrac{d}{dt}(v)$

$$= \frac{d}{dt}(\omega R) = \frac{R\, d^2(\theta)}{dt^2} = 2PR \qquad . . . \text{(ii)}$$

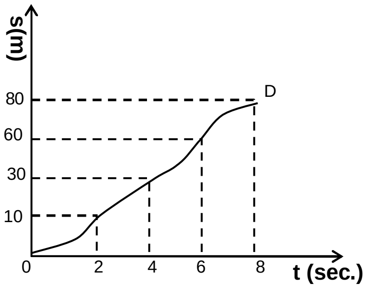
$$\therefore \qquad a = \sqrt{\left(\frac{v^2}{R}\right)^2 + (2PR)^2}$$

$$= \sqrt{(4P^2t^2R)^2 + (2PR)^2} = 2PR\sqrt{1 + 4P^2t^4} = \frac{v}{t}\sqrt{1+4P^2t^4} = 0.7 \text{ m/s}^2.$$

Prob 11. *Velocity–time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement–time graph of a particle if at t = 0 displacement s = 0.*



Sol.



Prob 12. *A particle moves in a circle of radius 20 cm at a speed given by $v = 1 + t + t^2$ m/s where t is time in s. Find (a) the initial tangential and normal acceleration. (b) the angle covered by the radius in first 2 s.*

Sol. (a) Tangent acceleration $a_t = \frac{dv}{dt} = 2t + 1$

Normal acceleration $a_n = \frac{v^2}{R}$ $\therefore (a_t)_{t=0} = 1\ m/s^2$

$(a_n)_{t=0} = \frac{v_0^{\ 2}}{R} = \frac{1}{(0.2)} = 5\ m/s^2$

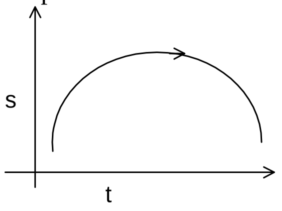
(b) $v = R\frac{d\theta}{dt}$

$R\,d\theta = (1+t+t^2)dt$ $\therefore R\int_0^{\theta'} d\theta = \int_0^2 \left(1+t+t^2\right)dt$

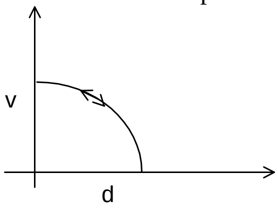
$\theta = 33.3$ rad

Prob 13. *A body of mass m is projected vertically upwards with a speed v_0. It goes up and comes back to the same point. For this motion draw displacement–time, velocity–time, acceleration–time and speed–displacement graphs.*

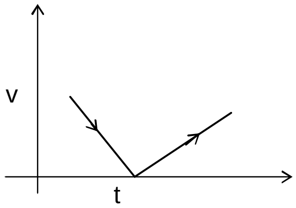
Sol. Displacement–time



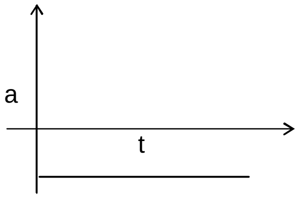
Speed–displacement



Speed time



Acceleration time



Prob 14. *A car starts moving from rest with an acceleration whose value linearly increases with time from zero to 6 m/s^2 in 6 sec after which it moves with constant velocity. Find the time taken by the car to travel first 72 m from starting point.*

Sol. Since acceleration varies linearly with time therefore

$a \propto t$

$\Rightarrow a = kt \quad \Rightarrow \int_0^6 da = k\int_0^6 dt \qquad \Rightarrow k = 1$

then, $\dfrac{dv}{dt} = t \quad \Rightarrow \; v = \dfrac{t^2}{2}$ m/s

then, $\dfrac{ds}{dt} = \dfrac{t^2}{2}$ or, $s = \dfrac{t^3}{6}$

at the end of t = 6 sec. Acceleration becomes zero.

Distance moved by car at t = 6 sec is

$S_1 = \dfrac{6\times6\times6}{6}$ = 36 m

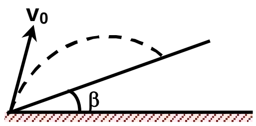
Speed of the car $= \dfrac{6\times 6}{2}$ = 18 m/s

Remaining distance = 72 – 36 = 36 m.

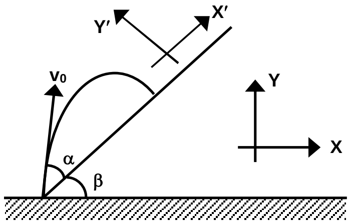
so time taken to cover this distance = $t_2 = \dfrac{36}{18}\text{sec.} = 2\,\text{sec.}$

Total time = 6+2 = 8 sec.

Prob 15. *A particle projected with velocity v_0 from an inclined plane whose angle of inclination with the horizontal is β. If afterwards the projectile strikes the inclined plane perpendicular to it. Find the height of the point struck, from horizontal plane through the point of projection.*



Sol. Let α be the angle between the velocity of projection and the inclined plane.



$v_{0x'} = v_0\cos\alpha,\; v_{0y'} = v_0 \sin\alpha$

$a_{x'} = -g\sin\beta \qquad a_{y'} = -g\cos\beta$

$\Rightarrow \quad v_{x'}(t) = v_0\cos\alpha - g\sin\beta t$

At the point of impact $v_{x'} = 0$

$$\Rightarrow \quad t = \frac{v_0\cos\alpha}{g\sin\beta} \qquad ..(1)$$

Also y′ at the point is zero.

$$\Rightarrow \quad v_0 \sin\alpha t - \frac{1}{2}g\cos\beta t^2 = 0$$

$$\Rightarrow \quad t = \frac{2v_0\sin\alpha}{g\cos\beta} \qquad ..(2)$$

From (1) and (2) $\qquad \tan\alpha = \dfrac{\cot\beta}{2} \qquad ..(3)$

$$x = v_0\cos(\alpha+\beta)t$$

$$= v_0\left[\cos\alpha\cos\beta - \sin\alpha\sin\beta\right].\frac{v_0\cos\alpha}{g\sin\beta}$$

$$= \frac{v_0^2}{g}\left[\cos^2\alpha\cot\beta - \sin\alpha\cos\alpha\right]$$

$$= \frac{v_0^2}{g}\left[\left(\frac{2}{\sqrt{4+\cot^2\beta}}\right)^2 \cot\beta - \frac{\cot\beta}{\sqrt{4+\cot^2\beta}}.\frac{2}{\sqrt{4+\cot^2\beta}}\right]$$

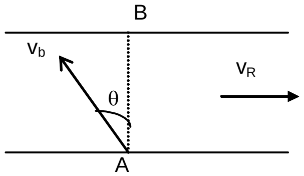
$$= \frac{v_0^2}{g}\frac{2\cot\beta}{4+\cot^2\beta}$$

$$\therefore \qquad y = x\tan\beta = \frac{v_0^2}{g}.\frac{2\cot\beta}{4+\cot^2\beta}\tan\beta$$

$$\Rightarrow \qquad y = \frac{2v_0^2}{g\left(4+\cot^2\beta\right)}$$

Prob 16. *The velocity of a boat in still water is n times less than the velocity of flow of a river. At what angle to the stream direction must the boat move so that drift is minimised ? If n = 2, show that the angle $\theta = 120°$.*

Sol. Given $v_b = \frac{v_R}{n}$



$\vec{v}_b = (-v_b \sin\theta)\hat{i} + (v_b\cos\theta)\hat{j}$

Resultant velocity of boat

$= \vec{v}_b + \vec{v}_R$

$= (v_R - v_b\sin\theta)\ \hat{i}\ + (v_b\cos\theta)\ \hat{j}$

If w = width of the river, time for crossing is

$$T = \frac{W}{v_b\cos\theta}$$

Drift during time T is $(v_R - v_b \sin\theta)$ T

$\Rightarrow$ Drift $x = v_b(n - \sin\theta)\frac{w}{v_b\cos\theta} = w(n\sec\theta - \tan\theta)$

For x to be minimum, $\frac{dx}{d\theta} = 0$ lead to $\theta = \sin^{-1}(1/n)$

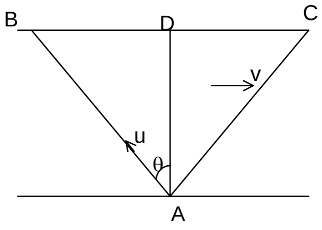
Direction of boat w.r.t. stream is

$90° + \theta = 90° + \sin^{-1}(1/n)$

For n = 1/2, the required angle $= 90° + 30° = 120°$

Prob 17. *A man can row a boat in still water at 3 km/h He can walk at a speed of 5 km/h on the shore. The water in the river flows at 2 km/h. If the man rows across the river and walks along the shore to reach the opposite point on the river bank find the direction in which he should row the boat so that he could reach the opposite shore in the least possible time. The width of the river is 500 m.*

Sol. Let the points towards B and reches at C



t_1 : the time taken by the boat to reach C

$t_1 = \frac{AD}{u\cos\theta}$ $\qquad CD = (v - u\sin\theta)t_1$

$t_1 = \frac{500\times10^{-3}}{3\cos\theta}$ hr $= \frac{1}{6\cos\theta}$

$CD = (-3\sin\theta + 2)\frac{1}{6\cos\theta}$

$= -0.5 \tan\theta + \dfrac{1}{3\cos\theta}$

t_2 : time taken by the man from C to D

$t_2 = \dfrac{CD}{v_s} = -\dfrac{0.5\tan\theta}{5} + \dfrac{1}{3\cos\theta \times 5} = \dfrac{1}{10}\tan\theta + \dfrac{1}{15\cos\theta}$

$= -\dfrac{\sin\theta}{10\cos\theta} + \dfrac{1}{15\cos\theta}$

$= \dfrac{(-3\sin\theta + 2)}{30\cos\theta}$

Total time $t = t_1 + t_2 = \dfrac{1}{6\cos\theta} + \dfrac{-3\sin\theta + 2}{30\cos\theta} = \dfrac{7 - 3\sin\theta}{30\cos\theta}$

$= \dfrac{7}{30}\sec\theta - \dfrac{1}{10}\tan\theta$

For minimum t

$\dfrac{dt}{d\theta} = 0 \quad \Rightarrow \quad \dfrac{7}{30}\sec\theta\tan\theta - \dfrac{1}{10}\sec^2\theta = 0$

$\Rightarrow \dfrac{1}{10}\sec\theta\left(\dfrac{7}{3}\tan\theta - \sec\theta\right) = 0 \Rightarrow \quad \dfrac{7}{3}\tan\theta - \sec\theta = 0$

$\dfrac{7\sin\theta - 3}{3\cos\theta} = 0 \quad \Rightarrow \quad \theta = \sin^{-1}(3/7)$

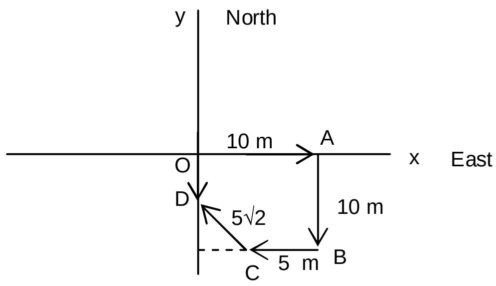
Prob 18. *A cyclist moves with constant speed 5 m/s along eastward for 2 seconds, and along southward for 2 seconds. Then, he moves along west for one second and finally along north-west for $\sqrt{2}$ seconds. Find*

(a) Distance and displacement of cyclist for whole journey.

(b) Average speed and average velocity for whole journey

(c) Average acceleration of cyclist for whole journey.

Sol. (a) In figure, shown final displacement



$\vec{OD} = -5\hat{j}$m

Distance = OA + AB + BC + CD

$= (25 + 5\sqrt{2})$ m

(b) Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$= \dfrac{25 + 5\sqrt{2}}{5 + \sqrt{2}} = 5$ m/s.

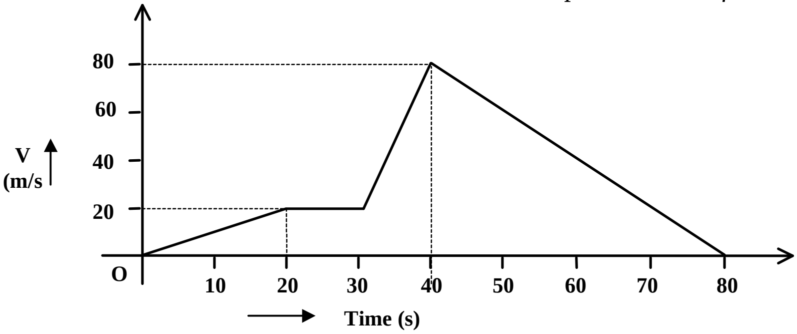
Average velocity $= \dfrac{-5\hat{j}}{(5 + \sqrt{2})}$ m/s

(c) For average acceleration $= \dfrac{\bar{v}_f - \bar{v}_i}{\Delta t}$

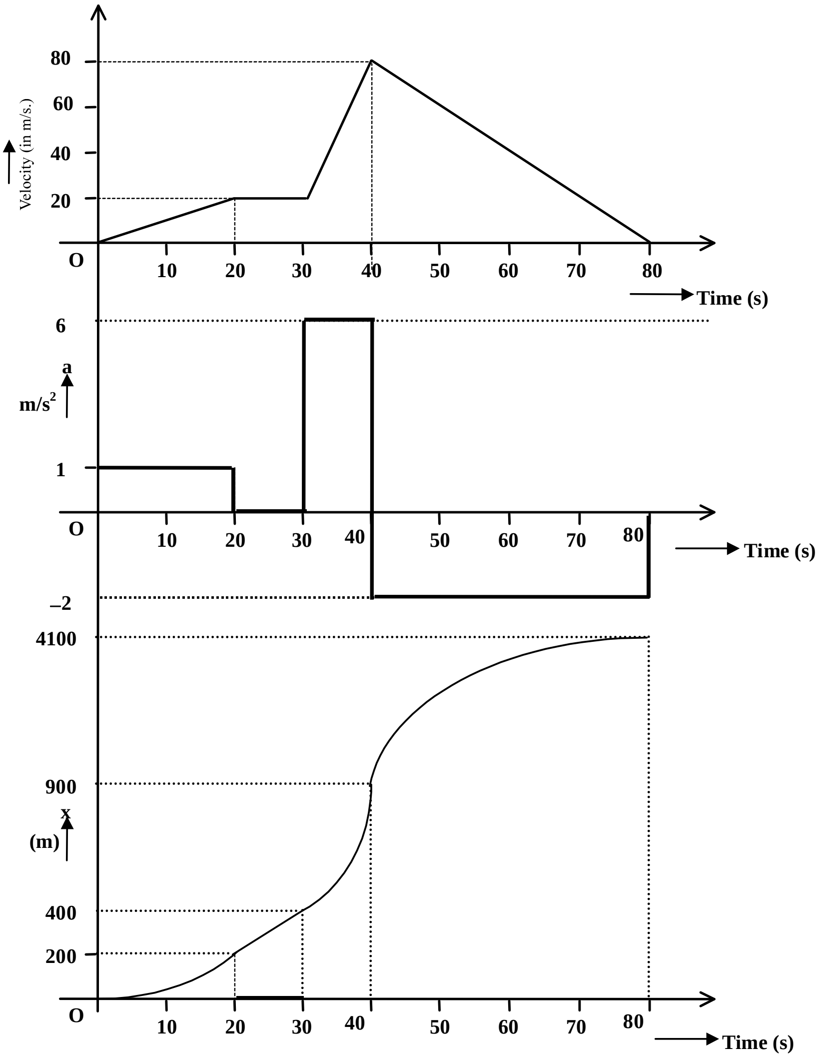
$\bar{v}_f = -5\hat{i} + 5\hat{j}$, $v_i = 5\hat{i}$

average $= \dfrac{-5\hat{i} + 5\hat{j} - 5\hat{j}}{(5 + \sqrt{2})} = \dfrac{-10\hat{i} + 5\hat{j}}{5 + \sqrt{2}}$ m/s^2

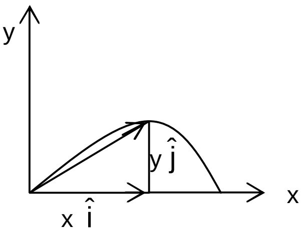
Prob 19. *The velocity-time graph of moving object is given in the figure. Draw the acceleration versus time and displacement versus time graph. Find the distance travelled during the time interval when the acceleration is maximum. Assume that the particle starts from origin.*



Sol.



Prob 20. *A projectile is fired with speed v_0 at an angle θ with the horizontal on a horizontal plane, Find*
(a) the average velocity of projectile in half of time of flight.
(b) the time in which the speed of projectile becomes perpendicular to its initial velocity.
(c) the radius of curvature of projectile at the instant when it is at its maximum height.



Sol. (a) $\vec{v}_{av} = \dfrac{\text{displacement}}{\text{time}} = \dfrac{x\hat{i} + y\hat{j}}{\dfrac{v_0 \sin\theta}{2g}}$

x = half of range $= \dfrac{v_0^2 \sin 2\theta}{2g}$

y = Max. height $= \dfrac{v_0^2 \sin^2\theta}{2g}$

$$\bar{v}_{av} = \frac{\dfrac{v_0^2 \sin 2\theta}{2g}\hat{i} + \dfrac{v_0^2 \sin^2\theta}{2g}\hat{j}}{\dfrac{v_0 \sin\theta}{2g}} = \frac{v_0 \sin 2\theta}{\sin\theta}\hat{i} + \frac{v_0 \sin^2\theta}{\sin\theta}\hat{j}$$

$= 2v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\ \hat{j}$

(b) Let the perpendicular velocity be $\vec{v}$

$\bar{v}\cdot\bar{v} = 0 \quad \therefore\ \bar{v}\cdot\bar{v} = (v_0\cos\theta\ \hat{i} + v_0\sin\theta\,\hat{j}).\ (v_0\cos\theta\ \hat{i} + (v_0\sin\theta - gt)\,\hat{j}$

$\therefore\ v_0^2\cos^2\theta + v_0\sin\theta(v_0\sin\theta - gt) = 0$

$v_0^2 - v_0\sin\theta gt = 0\ \therefore\ t = \dfrac{v_0}{g\sin\theta}$

(c) Radius of curvature $= \dfrac{v^2}{g} = \dfrac{v_0^2\cos^2\theta}{g}$

Prob 21. *An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with a constant acceleration of 1.2 m/s^2. Two seconds after it starts, a bolt begins to fall from the ceiling of the elevator. Find*

(a) the bolt's free fall time,

(b) the displacement and the distance covered by the bolt during the fall in the reference frame fixed to the ground. (Use g = 9.8 m/s^2.)

Sol. (a) Since a = 1.2 m/sec^2 is the constant acceleration of the elevator car while ascending and h = 2.7 m is the separation between the floor and the ceiling, therefore, the free fall time is given by

$\Rightarrow h = \dfrac{1}{2}(g + a)t^2 \Rightarrow t = \sqrt{\dfrac{2h}{g+a}} = 0.7$ sec

(b) Velocity of elevator at t = 2 sec is v = (1.2 m/s^2) (2 s) = 2.4 m/sec.

Thus, with respect to the reference frame fixed to the ground i.e. with respect to a stationary observer, the displacement in the course of free fall is

$y = (-2.4\text{ m/s})(0.7\text{ s}) + \dfrac{1}{2}(9.8\text{ m/s}^2)(0.7\text{ s})^2 = 0.72$ m

Total distance covered w.r.t. the ground during the free fall times is

s = y + 2h

$= 0.72 + 2 \times \dfrac{(2.4)^2}{2\times 9.8} = 1.31$ m.

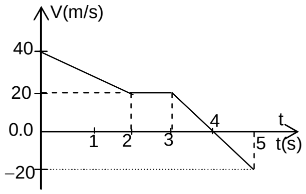
$v^2 = u^2 + 2gh$

$0 = (2.4\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)h$

$\Rightarrow \quad h = \dfrac{(2.4\,\text{m/s})^2}{2\times\left(9.8\,\text{m/s}^2\right)}$

Objective:

Prob 1. *In the given v–t graph, the distance travelled by the body in 5 sec will be*
(A) 100 m
(B) 80 m
(C) 40 m
(D) 20 m



Sol. **A.**
Distance travelled = area under the v–t curve
$$=\frac{20\times2}{2}+20\times2+20\times1+\frac{20\times1}{2}+\frac{20\times1}{2} = 100\text{ m}$$

Prob 2. *In Question 1, the displacement of the body in 5 sec will be*
(A) 100 m *(B) 80 m*
(C) 40 m *(D) 20 m*

Sol. **B.**
Displacement is a vector and is equal to algebraic sum of area under the v–t graph.
= 20 + 40 + 20 + 10 – 10 = 80 m.

Prob 3. *In Question 1, the average velocity of the body in 5 seconds is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **B.**
Average velocity $=\frac{\text{displacement}}{\text{time}}=\frac{80}{5}=16\,\text{m/s}$

Prob 4. *In above Question, the average speed of the body during 5 sec is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **A.**
Average speed $=\frac{\text{distance}}{\text{time}}=\frac{100}{5}=20\,\text{m/s}$

Prob 5. *A body when projected vertically up, covers a total distance D during its time of flight. If there were no gravity, the distance covered by it during the same time is equal to*
(A) 0 *(B) D*
(C) 2D *(D) 4D*

Sol. **C.**
The displacement of the body during the time t as it reaches the point of projection
$\Rightarrow S = 0 \Rightarrow v_0t - \frac{1}{2}gt^2 = 0 \qquad \Rightarrow t = \frac{2v_0}{g}$
During the same time t, the body moves in absence of gravity through a distance
D′ = v.t, because in absence of gravity g = 0

$\Rightarrow D' = v_0 \left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g}$ …(1)

In presence of gravity, the total distance covered is

$= D = 2H = 2\,\frac{v_0^2}{2g} = \frac{v_0^2}{g}$ …(2)

$(1) \div (2) \Rightarrow D' = 2D$.

Prob 6. *A particle is projected vertically upward with initial velocity 25 ms^{-1}. During third second of its motion, which of the following statement is correct?*

(A) displacement of the particle is 30 m

(B) distance covered by the particle is 30 m

(C) distance covered by the particle is 2.5 m

(D) none of these

Sol. **C.**

Displacement of the particle during third second of the motion (i.e. between t = 2s and t = 3s) is zero. Hence, t = 2.5 sec is the turning point of the motion.

For distance $S_{t=2} = 25 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 30$ m

and $S_{t=2.5} = 25 \times 2.5 - \frac{1}{2} \times 10 \times 2.5^2 = 31.25$

Hence, distance covered by the particle during third second of motion

$= 2\,(31.25 - 30) = 2.5$ m.

Prob 7. *A particle is projected from a point A with a velocity v at an angle θ (upward) with the horizontal. At a certain point B, it moves at right angle to its initial direction. It follows that*

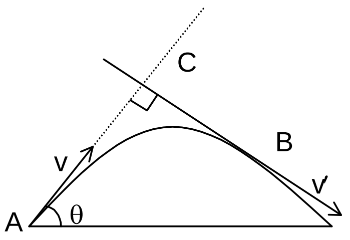
(A) velocity of the particle at B is v.

(B) velocity of the particle at B is v cos θ.

(C) velocity of the particle at B is v tan θ.

(D) the time of flight from A to B is $\frac{v}{g\sin\theta}$.

Sol. **D.**



$\vec{v} = \vec{u} + \vec{a}t$

Considering along the line AC

$$0 = v - g\sin\theta t \Rightarrow t = \frac{v}{g\sin\theta}$$

Now, consider along the line CB

$$v' = 0 + g\cos\theta\,\frac{v}{g\sin\theta} = v\cot\theta$$

Prob 8. *A particle is projected horizontally from the top of a cliff of height H with a speed $\sqrt{2gH}$. The radius of curvature of the trajectory at the instant of projection will be*

(A) H/2 *(B) H*

(C) 2H *(D) ∞*

Sol. **C.**

Since, $\vec{g} \perp \vec{v}$

Radial acceleration $a_r = g$

$\Rightarrow \dfrac{v_0^2}{r} = g$ where r is the radius of curvature.

$\Rightarrow \dfrac{2gH}{r} = g \quad (\because v = \sqrt{2gH})$

$\Rightarrow r = 2H$

Prob 9. *If a boat can have a speed of 4 km/hr in still water, for what values of speed of river flow, it can be managed to row boat right across the river, without any drift?*

(A) $\geq$ 4 km/hr *(B) greater than zero but less than 4 km/hr*
(C) only 4 km /hr *(D) none of these*

Sol. **B.**

Drift $(\Delta x) = (v_{b,x})\,\Delta t = (v_{br}\cos\theta + v_r)\Delta t$

where $v_{b,x}$ = velocity of boat w.r.t. ground

$v_{\perp, r}$ = velocity of boat w.r.t river

v_r = velocity of river w.r.t. ground

For $\Delta x = 0$, $v_r = -v_{br}\cos\theta$

$\Rightarrow (v_r)_{max} = v_{br}$

For, $v_r > v_{br}$ we can not have zero drift.

Prob 10. *A swimmer crosses a river of width d flowing at velocity v. While swimming, he keeps himself always at an angle of 120° with the river flow and on reaching the other end he finds a drift of d/2 in the direction of flow of river. The speed of the swimmer with respect to the river is*

(A) $(2-\sqrt{3})\,v$ *(B) $2\,(2-\sqrt{3})\,v$*
(C) $4\,(2-\sqrt{3})\,v$ *(D) $(2+\sqrt{3})\,v$*

Sol. **C.**

Drift = $d/2 = (V_r - V_s\sin30)d/V_s\cos30$

$\Rightarrow V_s = 4\,(2-\sqrt{3})V$

Prob 11. *A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400 m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum, are*

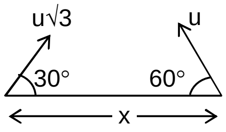
(A) (400, 100) *(B) (200, 100)*
(C) (400, 200) *(D) (200, 200)*

Sol. **B.**

When the horizontal range is maximum, the maximum height attained is R/4 = 100 m. The velocity of the projectile is minimum at the highest point.

$\therefore$ Required point is (200, 100).

Prob 12. *Two particles are separated at a horizontal distance as shown in the adjacent figure. They are projected along the same line with different initial speeds. The time after which the horizontal distance between them becomes zero is*



(A) $\dfrac{x}{u}$ *(B)* $\dfrac{x}{2u}$

(C) $\dfrac{2u}{x}$ *(D) none of these*

Sol. **B.**

Both particles will collide at the highest point of their path. At highest point, only the horizontal component exists.

$V_1 = u\sqrt{3}\cos 30° = \frac{3u}{2}$

$V_2 = u\cos 120° = -\frac{u}{2}$

Relative velocity of the particle 1 w.r.t. particle 2 in x-direction = $\frac{3u}{2}+\frac{u}{2}=2u$

$\Rightarrow$ Required time $=\frac{x}{2u}$

Prob 13. *A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s, then maximum height attained by it is: (g = 10 m/s^2)*

(A) 8 m *(B) 20 m*
(C) 10 m *(D) 16 m*

Sol. **C.**

Suppose after travelling distance s, particle has the velocity 10 m/s.

So, $v^2 = u^2 - 2as$

$\Rightarrow$ $(10)^2 = u^2 - 2 \times 10s$ …(1)

At the maximum height, i.e. 2s, v = 0

$\Rightarrow$ $0 = u^2 - 2g(2s)$

$\Rightarrow$ $u^2 = 40s$ …(2)

From Eqs. (1) and (2), s = 5 m

$\Rightarrow$ 2s = 10 m

Prob 14. *A person walks up a stationary escalator in 90 sec. If the escalator moves with the person, first standing on it, it will take 1 minute to reach the top from ground. How much time it would take him to walk up the moving escalator?*

(A) 24 sec *(B) 48 sec*
(C) 36 sec *(D) 40 sec*

Sol. **C.**

Let L be the length of escalator.

$\therefore$ Relative speed $=\frac{L}{90}+\frac{L}{60}=\frac{L}{36}$

$\therefore$ Time taken to walk up the moving escalator $=\left(\frac{L}{L/36}\right)=36\,\text{sec}$

Prob 15. *A driver applies brakes on seeing a traffic signal 400 m ahead. At the time of applying the brakes the vehicle was moving with 15 m/s and retarding with 0.3 m/s^2. The distance of vehicle after 1 min from the traffic light is*

(A) 25 m *(B) 375 m*
(C) 360 m *(D) 40 m*

Sol. **A.**

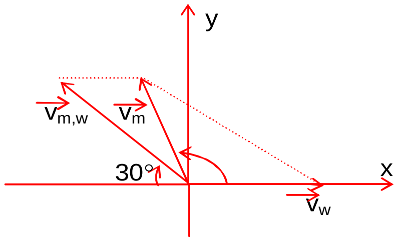
The maximum distance covered by the vehicle before coming to rest $=\frac{v^2}{2a}=\frac{(15)^2}{2(0.3)}=375$ m

The corresponding time = t = $\frac{v}{a}=\frac{15}{0.3}=50$ sec

$\therefore$ The distance of the vehicle from the traffic signal after one minute = 400 – 375 = 25 m

Prob 16. *A motorboat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. The velocity of the motorboat wrt water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle*
(A) 30° wrt the line of destination from starting point
(B) 60° wrt normal to the bank
(C) 120° wrt stream direction
(D) None of these

Sol. **C.**
The velocity of motorboat,



$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$
$$= -5\sqrt{3}\cos 30°\ \hat{i} + 5\sqrt{3}\sin 30°\,\hat{j} + 5\hat{i}$$
$$= -2.5\ \hat{i} + \frac{5\sqrt{3}}{2}\hat{j}$$
$$\phi = \tan^{-1}\left(-\frac{5\sqrt{3}}{2\times 2.5}\right) = \tan^{-1}\left(-\sqrt{3}\right) = 120^0$$

Prob 17. *The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be*

(A) $v_0 t + \frac{1}{6}bt^3$ *(B)* $v_0 t + \frac{1}{3}bt^3$
(C) $v_0 t + \frac{1}{3}bt^2$ *(D)* $v_0 t + \frac{1}{2}bt^2$

Sol. **A.**
Given, acceleration a = bt
$$\Rightarrow \frac{dv}{dt} = bt \quad \Rightarrow v = \frac{bt^2}{2} + c$$
At t = 0, v = v_0 ⇒ c = v_0
$$\text{So, } v = \frac{bt^2}{2} + v_0$$
$$\Rightarrow \frac{ds}{dt} = \frac{bt^2}{2} + v_0$$
$$\Rightarrow s = \frac{bt^3}{6} + v_0 t$$

Fill in the Blanks

Prob 1. *The position of a body w.r.t. time is given by $x = 3t^3 - 6t^2 + 12t + 6$. At time t = 0, its acceleration is ____________.*

Sol.
$$\frac{dx}{dt} = 6t^2 - 12t + 12$$
$$\frac{d^2x}{dt^2} = 12t - 12$$
$$\left.\frac{d^2x}{dt^2}\right|_{t=0} = -12$$

Prob 2. *A body thrown up from the ground vertically passes the height of 10.2 m twice in an interval of 10 sec. Its initial velocity was ________ m/s and its time of journey upwards was ________sec ($g = 10\ m/s^2$).*

Sol. It takes 5 sec from its maximum height to the height of 10.2 m, travelling from rest at acceleration of 10 m/s^2. Hence, if this distance be s, then

$$s = \frac{1}{2} \times 10 \times 5^2 = 125\text{ m.}$$

So, $u^2 = 2 \times 10 \times (125 + 10.2)$

$\Rightarrow \quad u = 52$ m/s

$\Rightarrow \quad t = \dfrac{52}{10} = 5.2$ sec.

Prob 3. *For a projectile projected at an angle ____________, the maximum height and horizontal range are equal.*

Sol. $\tan^{-1}(4)$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\Rightarrow \sin 2\theta = \frac{\sin^2\theta}{2}$$

$$\Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1} 4.$$

Prob 4. *A car covers the first half of its distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. The average speed of the car is ________ km/hr.*

Sol. 48 km/hr.
Let total distance be 2x km

$$\therefore \text{Total time taken } = \left(\frac{x}{40} + \frac{x}{60}\right)\text{hr} = \frac{x}{24}\text{hr}$$

$$\text{Therefore, average speed } = \frac{2x}{\left(\frac{x}{24}\right)} = 48 \text{ km/hr.}$$

Prob 5. *In a uniform motion, the particle travels in a ______________ and traces equal _________however ______________intervals of ________________ be taken.*

Sol. Straight line, displacements, small, time.

True or False Type Questions

Prob 1. *A particle in one-dimensional motion with positive value of acceleration must be speeding up.*

Sol. False.
If the velocity of the body is negative, then even in case of positive acceleration the body speeds down, e.g. a body projected up slows down even when acceleration is positive.

Prob 2. *A particle in one-dimensional motion with constant speed must have zero acceleration.*

Sol. True.
As the direction of motion remains unchanged, therefore, if the speed is zero the acceleration must also be zero.

Prob 3. *A particle moves with a uniform velocity in a straight line. If another particle moves such that it is always directed towards the first particle then the motion of the second particle is also along a straight line.*

Sol. False.
Because the second particle is always directed towards the first particle, the motion of the second particle can be straight line only in the special case when it follows the uniformly moving first particle along the same straight line.

Prob 4. *A particle in one-dimensional motion with zero speed at any instant may have non-zero acceleration at that instant.*

Sol. True.
When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 m/s^2

Prob 5. *If a base ball player can throw a ball to a maximum distance d over the ground, then the maximum vertical height to which he can throw it will be equal to d/2. Assume that initial speed of the ball is same in both the cases.*

Sol. True.
$R_{max} = d = \frac{u^2}{g}$ and $H_{max} = \frac{u^2}{2g} = \frac{d}{2}$

Prob 6. *A bus moving towards north takes a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.*

Sol. False.
The direction will change.

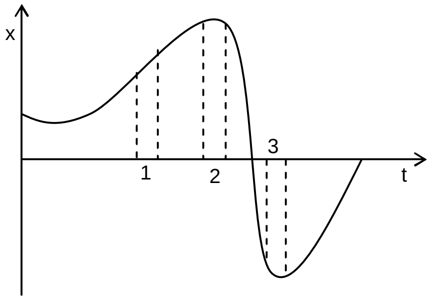
ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the cliff upward with a velocity 100 m/s. After what time the bullet and the block meet ?

2. Figure shows the x–t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



3. Two balls of different masses are thrown vertically upwards with the same speed. During their downward journey, they pass through the point of projection with the same speed. Neglect air resistance. Is this statement correct?

4. Galileo stated that "For elevations which exceed or fall short of 45° by equal amounts, the ranges are equal." Prove this statement.

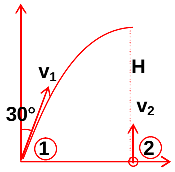
5. A block slides down a smooth inclined plane when released from the top while another falls freely from the same point. Which one of them will strike the ground earlier ?

6. A stone is thrown horizontally with a speed $\sqrt{2gh}$ from the top of a wall of height h. What is the distance from the wall when it reaches the ground?

7. What is the angle θ of projection with horizontal plane of a projectile if its range is $\frac{\sqrt{3}v^2}{2g}$, where v is velocity of projection?

8. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B desires to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past driver of A, calculate the original distance between the guard of B and the driver of A.

9. A particle 1 is projected with speed v_1 from a point A making an angle of 30° with the vertical. At the same instant, a second particle 2 is thrown vertically upwards from position B with velocity v_2. The two particles reach height H, the highest point on the parabolic path of particle 1 simultaneously. Calculate the ratio $\frac{v_1}{v_2}$.



10. A car is moving with a speed of 30 m/s on a circular path of radius 500 m. Its speed is increasing at a rate of 2 m/s^2. What is the acceleration of the car?

11. A particle is thrown vertically upward. Its velocity at half of the maximum height is 10 m/s, then calculate the maximum height attained by it. ($g=10\ m/s^2$)

12. A car moving with a speed of 40 km/hr can be stopped, by applying brakes, in 2 meters. If the same car is moving with a speed of 80 km/hr, what is the minimum stopping distance?

13. The position of a particle moving along x-axis is given by $x = a + bt^2$ where x is in meter and t in seconds. The constants a and b are 4.5 m and 3.5 m/s^2 respectively. Find
    (a) initial velocity
    (b) velocity at t = 3 seconds.
    (c) average velocity during the time interval t = 1 s to t = 3 s.

14. A body dropped from a height h, with initial velocity zero, strikes the ground with velocity 3 m/s. Another body of the same mass is dropped from the height h with an initial velocity of 4 m/s. Find the final velocity with which it strikes the ground.

15. The velocity of a train increases uniformly from 20 km/hr to 60 km/hr in 4 hours. Find the distance travelled by the train during this period.

16. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then find the maximum velocity acquired by the car.

17. An aeroplane is flying horizontally with a velocity of 216 km/hr and at a height of 1960 m. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. Find the distance AB.

18. A body is projected horizontally with a speed of 20 m/s from the top of a tower. What will be its speed after nearly 5 sec? ($g = 10\ m/s^2$)

19. A bus moves a distance of 200 m. It covers the first half of the distance at speed 40 km/hr and the second half of the distance at speed v. The average speed is 48 km/hr. Find the value of v.

20. A ball is dropped from height of 90 m on a floor. The ball loses one tenth of its speed. Put the speed–time graph of its motion between t = 0 and 12 sec. ($g = 10\ m/s^2$)

Level - I

1. The position of a particle along the x-axis is given in centimeters by $x=9.75+1.50t^3$, where t is in seconds. Consider the time interval t = 2 s to t = 3 s and calculate
   (a) the average velocity
   (b) instantaneous velocity at t = 2 s;
   (c) the instantaneous velocity when t = 2.5 s;
   (d) the instantaneous velocity when the particle is mid way between its position at t = 2 s and t = 3 s.

2. A train started from rest and moved with constant acceleration. At one time it was travelling at 33.0 m/s and 160 m farther it was travelling at 54.0 m/s. Calculate
   (a) the acceleration.
   (b) the time required to travel the 160 m.
   (c) the time required to attain the speed of 33.0 m/s.
   (d) the distance moved from rest to the time the train had a speed of 33 m/s.

3. A body travelling in a straight line travels 2 m in the first two seconds and 2.2 m in the next four seconds with constant retardation. What will be its velocity at the end of the seventh second from the start?

4. A motorcyclist moving with uniform retardation takes 10 s and 20 s to travel successive quarter kilometer. How much further he will travel before coming to rest?

5. A car is moving on a straight road with a speed 20 m/s. At t = 0, the driver of the car applies the brakes after watching an obstacle 150 m ahead. After application of brakes the car retards with 2 m/s^2. Find the position of the car from the obstacle at t =15 s.

6. A ball is thrown with a velocity of 100 ms^{-1} at an angle of 30° to the horizontal and meets the same horizontal plane later. Find
   (a) its time of flight
   (b) the horizontal distance it travels
   (c) the velocity with which it strikes the ground at the end of its flight. [g = 9.8 ms^{-2}]

7. A projectile shot at an angle of 60° above the horizontal strikes a wall 30 m away at a point 15 m above the point of projection.
   (a) Find the speed of projection.
   (b) Find the magnitude of velocity of the projectile when it strikes the wall.

8. A ball is thrown vertically up with a certain velocity from the top of a tower of height 40 m. At 4.5 m above the top of the tower its speed is exactly half of that it will have at 4.5 m below the top of the tower. Find the maximum height reached by the ball above the ground?

9. A rotating fan completes 1200 revolutions every minute. Consider a point on the tip of the blade, which has a radius of 0.15 m
   (a) Through what distance does the point move in one revolution?
   (b) What is the speed of the point? (c) What is its acceleration?

10. A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle 30° to the path of A. Find the velocity of B relative to A.

11. A ball dropped from some height covers half of its total height during the last second of its free fall. Find
(a) time of flight
(b) height of its fall
(c) speed with which it strikes the ground.

12. From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30^o with the horizontal up the plane. Find the range.

13. A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.

14. The acceleration experienced by a moving boat after its engine is cut off, is given by $\frac{d\omega}{dt} = -kv^3$, where k is a constant if v_0 is the magnitude of the velocity at cutoff find the magnitude of the velocity at time t after the cut off.

15. A boy throws a ball vertically upward with an initial speed of 15.0 m/s. The ball was released when it was at 2.00 m above ground. The boy catches it at the same point as the point of projection.
(a) What is maximum height reached by the ball ?
(b) How long is the ball in the air?

Level- II

1. The equation of motion of a particle moving along a straight line is given as $x = \frac{1}{2}vt$ when x, v t, have usual meaning, prove that the acceleration is constant.

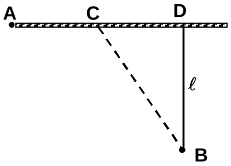
2. A point moving in a straight line traversed half the distance with a velocity v_o. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

3. At the instant the traffic light turns green, an automobile starts with a constant acceleration of 2.2 m/s^2. At the same instant a truck, travelling with a constant speed of 9.5 m/s, overtakes and passes the automobile.
   (a) How far beyond the starting point will the automobile overtake the truck?
   (b) How fast will the car be traveling at the instant?
   (It is instructive to plot a qualitative graph of 'x' versus t for each vehicle.)

4. A balloon is ascending vertically with an acceleration of 1 m/s^2. Two stones are dropped from it at an interval of 2 s. Find the distance between them 1.5 sec after the second stone is released.

5. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point in space and moved with velocities v_1 = 3.0 m/s and v_2 = 4.0 m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

6. From point A located on a highway (Fig.) one has to get by car as soon as possible to point B located in the field at a distance ℓ from the highway. It is known that the car moves in the field n times slower than the highway. At what distance from point D one must turn off the highway?



7. To a man walking at 7 km/hr due west the wind appears to blow from the north west, but when he walks at 3 km/hr due west the wind appears to blow from the north. What is the actual direction of the wind and what is its velocity?

8. A particle is projected with a velocity u at an angle θ with the horizontal. Find the radius of the curvature of the parabola traced out by the particle at the point where velocity makes an angle (θ/2) with the horizontal.

9. A ship A streams due north at 16 km/hr and a ship B due west at 12 km/hr. At a certain instant B is 10 km north-east of A. Find the velocity of A relative to B. Find also the nearest distance of approach of ships.

10. A particle moves in x-y plane with constant acceleration 'a' directed along the negative y-axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the particle at the origin.

11. The position vector of a particle varies with time t as x = kt(1– αt), where k is a constant vector and α is a positive factor. Find
(a) the velocity v and the acceleration a of the particle as functions of time.
(b) the time interval Δt taken by the particle to return to the initial points, and the distance covered during that time.

12. A balloon starts rising from the surface of the Earth. The ascent rate is constant and equal to v_o. Due to the wind the balloon gathers the horizontal velocity component v_x = ay, where a is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent.
(a) The horizontal drift of the balloon x(y);
(b) The total, tangential, and normal accelerations of the balloon.

13. Two boats A and B move away from buoy anchored at the middle of a river along mutually perpendicular straight lines, the boat A along the river and the boat B across the river. Having moved off an equal distance from the buoy the boat returned. Find the times of motion of boats t_A / t_B if the velocity of each boat with respect to water is n times greater than the stream velocity.

14. A ball starts failing with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

15. A man standing in an elevator observes a screw fall from the ceiling. The ceiling is 3m above the floor.
(a) If the elevator is moving upward with a speed of 2.2 m/s, how long does it take for the screw to hit the floor.
(b) How long is the screw in the air if the elevator starts from rest when the screw falls and moves upwards with a constant acceleration of a = 4.0 m/s^2.

Objective:

Level- I

1. A bus starts moving with an acceleration of 2 m/s^2. A cyclist 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus?
(A) 4 sec (B) 8 sec
(C) 12 sec (D) 16 sec

2. A body is projected at an angle of 30° to the horizontal with a speed of 30 m/s. What will be the angle with the horizontal after 1.5 seconds? [Take g = 10 m/s^2]
(A) 0° (B) 30°
(C) 60° (D) 90°

3. A ball rolls off the top of stairway with a horizontal velocity of magnitude 1.8 m/s. The steps are 0.20 m high and 0.2 m wide. Which step will the ball hit first?
(A) First (B) Second
(C) Third (D) Fourth

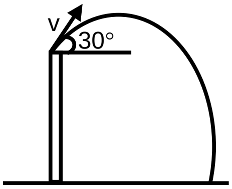
4. In a projectile motion, the velocity is
(A) never perpendicular to the acceleration
(B) always perpendicular to the acceleration
(C) perpendicular to acceleration at one instant only
(D) perpendicular to acceleration at two instants only

5. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of the boat is 10 km/hr, then velocity of the river is
(A) 12.8 km/hr (B) 6 km/hr
(C) 8 km/hr (D) 10 km/hr

6. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is
(A) parallel to position vector (B) perpendicular to position vector
(C) directed towards the origin (D) directed away from the origin

7. A particle starts from rest with constant acceleration. The ratio of space-average velocity to the time-average velocity is
(A) 1/2 (B) 3/4
(C) 4/3 (D) 3/2

8. A ball is projected with a speed of 20 m/s at an angle of 30° from a point on the top of a very high tower. The time after which its velocity becomes perpendicular to the velocity of projection (take g = 10 m/s^2) is
(A) 0.5 sec (B) 2 sec
(C) 4 sec (D) never



9. A particle is moving along a circular path of radius r with uniform speed v. Through what angle does its angular velocity change when it completes half of the circular path?
(A) 0° (B) 45°
(C) 180° (D) 360°

10. Three particles start moving simultaneously from a point on a horizontal smooth plane. First particle moves with speed v_1 towards east, second particle moves towards north with speed v_2 and third one moves towards north east. The velocity of the third particle, so that the three always lie on a straight line, is

(A) $\frac{v_1 + v_2}{2}$ (B) $\sqrt{v_1 v_2}$ s

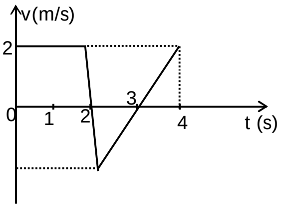
(C) $\frac{v_1 v_2}{v_1 + v_2}$ (D) $\sqrt{2}\frac{v_1 v_2}{v_1 + v_2}$

11. A particle is moving along a circular path of radius 5 m and with uniform speed 5 m/s. What will be the average acceleration when the particle completes half revolution?

(A) zero (B) 10 m/s^2

(C) 10 π m/s^2 (D) 10/π m/s^2

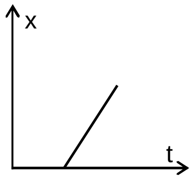
12. The velocity v versus t graph of a body in a straight line is as shown in the adjacent figure. The displacement of the body in 4 sec is

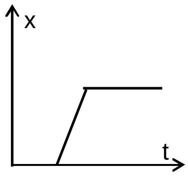


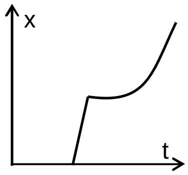
(A) 2 m (B) 4 m

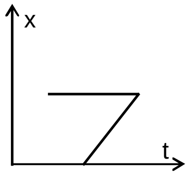
(C) 6 cm (D) 8 m

13. Which of the following displacement–time graph is not possible?

(A)


(B)


(C)


(D)


14. A train of length 100 m travelling at 50 m/s overtakes another train of length 200 m moving at 30 m/s. The time taken by the first train to overtake the second is

(A) 5 sec (B) 10 sec

(C) 15 sec (D) 20 sec

15. A balloon starts from the ground with an acceleration of 1.25 m/s^2. After 8 sec, a stone is released from the balloon. The stone will

(A) cover a distance of 40 m

(B) have a displacement of 50 m

(C) reach the ground in 4 sec

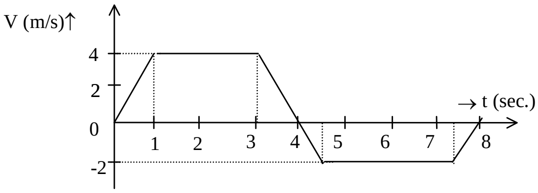
(D) begin to move down after being released.

16. A body moving with a uniform acceleration has velocities of u and v when passing through points A and B in its path. The velocity of the body midway between A and B is

(A) $\frac{u+v}{2}$ (B) $\sqrt{\frac{u^2+v^2}{2}}$

(C) $\sqrt{uv}$ (D) None of these

17. The velocity–time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is



(A) 5 m

(B) 16 m

(C) 8 m

(D) 6 m

18. A ball is thrown up vertically with speed u. At the same instant another ball B is released from rest from a height h. At time t, the speed of A relative to B is

(A) u (B) $u - 2gt$

(C) $\sqrt{u^2 - 2gh}$ (D) $u - gt$

19. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw will be:

(A) h/2 (B) h

(C) 2 h (D) 4 h

20. A motor boat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. Velocity of motor boat with respect to water is $5\sqrt{3}$ m/sec. The driver should steer the boat an angle:

(A) 30° w.r.t. the line of destination from starting point

(B) 60° w.r.t.. normal to the bank

(C) 120° w.r.t. stream direction

(D) None of these

Fill in the Blanks

1. A particle moves in a circle of radius R. In half the period of revolution its displacement is _______________________ and distance covered is _______________________.

2. A particle is projected with an initial velocity of 200 m/s in a direction which makes an angle of 30° with the vertical, the horizontal distance travelled by the particle in 3 sec is ____________ m.

3. A stone is released from an elevator going up with an acceleration a. the acceleration of the stone after the release is _________________________.

4. For angles of projection which exceed or fall short of 45° by equal amounts, the ranges are ____________ .

5. The weight of a body in projectile motion is ______________________.

True or False Type Questions

1. If the displacement y of a particle is proportional to t^2, i.e. if $y \propto t^2$, then its initial velocity will be non-zero.

2. The instantaneous velocity vector is always in the direction of motion.

3. The magnitude of the sum of two displacement vectors must be greater than the magnitude of either displacement vectors.

4. A particle can move with constant velocity and constant acceleration simultanesouly.

5. The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval (infinite acceleration are not allowed).

Level - II

1. A stone is thrown vertically upward with an initial velocity v_0 . The distance travelled in time $4v_0/3g$ is

(A) $\frac{2v_0^2}{g}$ (B) $\frac{v_0^2}{2g}$

(C) $\frac{4v_0^2}{3g}$ (D) $\frac{4v_0^2}{9g}$

2. The motion of a body depends on time according to the equation $\frac{dv}{dt}$ = 6.0 – 3v, where v is speed in m/s and t is time in second. If the body was at rest at t = 0 which of the following statements is correct?
(A) The speed of the body approaches 2 m/s after long time
(B) The speed varies linearly with time
(C) The acceleration remains constant
(D) The initial acceleration is zero

3. If the angle (θ) between velocity vector and the acceleration vector is $90° < \theta < 180°$. The body is moving on a:
(A) Straight path with retardation (B) Straight path with acceleration
(C) Curvilinear path with acceleration (D) Curvilinear path with retardation

4. The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. The retardation is: (if v is velocity of the particle)

(A) $2\alpha v^3$ (B) $2\beta v^2$
(C) $2\alpha\beta v^2$ (D) $2\beta^2 v^3$

5. Two particles start moving along the same straight line starting at the same moment from the same point. The first moves with constant velocity u and the second with constant acceleration f. During the time that elapses before second catches the first, the greatest distance between the particles is

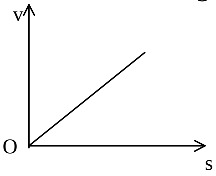
(A) $\frac{u}{f}$ (B) $\frac{u^2}{2f}$

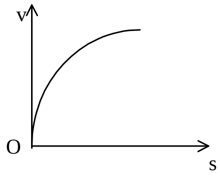
(C) $\frac{f}{2u^2}$ (D) $\frac{u^2}{f}$

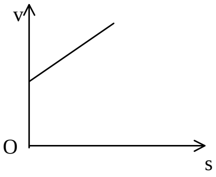
6. A particle is projected horizontally in air at a height of 25 m from the ground with a speed of 10 m/s. The speed of the particle after 2 seconds will be
(A) 10 m/s (B) 22.4 m/s
(C) 25 m/s (D) 28.4 m/s

7. A man can swim at a speed of 5 km/h w.r.t. water. He wants to cross a 1.5 km wide river flowing at 3 km/h. He keeps himself always at an angle of 60^o with the flow direction while swimming. The time taken by him to cross the river will be
(A) 0.25 hr. (B) 0.35 hr.
(C) 0.45 hr. (D) 0.55 hr.

8. A body starts from rest and moves along a straight line with constant acceleration. The variation of speed v with distance s is given by graph

(A)


(B)


(C)


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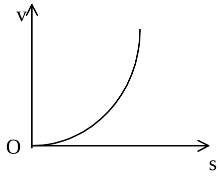
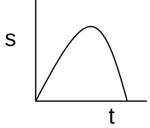
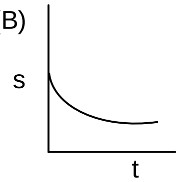
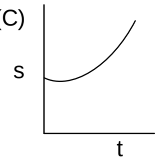
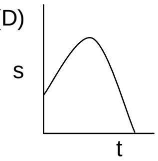


9. The displacement of a particle in a straight line motion is given by $s = 1 + 10t - 5t^2$. The correct representation of the motion is

(A)


(B)


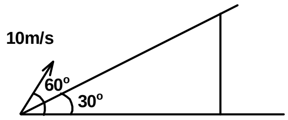
(C)


(D)


10. The position of a particle along x-axis at time t is given by $x = 1 + t - t^2$. The distance travelled by the particle in first 2 seconds is
(A) 1m (B) 2m
(C) 2.5 m (D) 3m

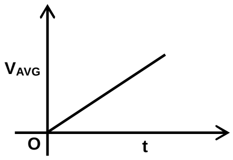
11. From the top of a tower, two particles A and B are projected simultaneously with speeds of 3 m/s and 4 m/s, respectively, in horizontally opposite directions at time t = 0. At time t = $\left(2\sqrt{3}/10\right)$ sec, the angle between their velocities is
(A) 60° (B) 45°
(C) 90° (D) 30°

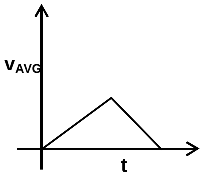
12. A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of 60° with the horizontal, from a point on an incline plane, making an angle of 30° with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is

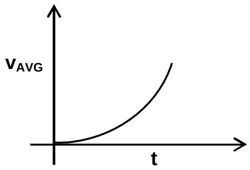


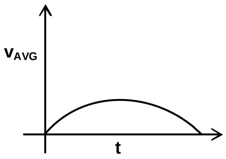
(A) $\frac{2}{\sqrt{3}}$sec (B) $\frac{1}{\sqrt{3}}$sec
(C) $\sqrt{3}$sec (D) $\frac{1}{2\sqrt{3}}$sec

13. A particle is moving along a straight line with a velocity of $\frac{1}{2}kt^2$, where k is a constant. Then, the average velocity of the particle as a function of time is best represented by

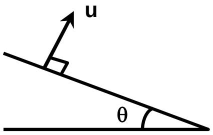
(A)


(B)


(C)


(D)


14. A particle is projected perpendicularly to an inclined plane as shown in the adjacent figure. If the initial velocity of the particle is u, calculate how far from the point of projection does it hit the plane again if the distance is measured along the plane?



(A) $\frac{2u^2}{g}$ (B) zero
(C) $\frac{2u^2}{g}\sin\theta$ (D) $\frac{2u^2}{g}\tan\theta\sec\theta$

15. A box is moving up on an inclined plane of inclination 30° with a constant acceleration of $5\,m/s^2$. A particle is projected with a velocity of $5\sqrt{3}$ m/s inside a box, at an angle of 30° with the base. Then, the time after which it again strikes the same base of the box is (assume during its flight, particle does not hit any other side of the box)
(A) 1 sec (B) 2 sec
(C) 1.5 sec (D) data insufficient

16. The height y and distance x along the horizontal for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is
(A) 8 m/s (B) 9 m/s
(C) 10 m/s (D) (10/3) m/s

More than one choice are correct:

17. Read and examine the following statements. Which of the following is /are correct/ true?
(A) $a_x \neq 0$, $a_y = 0$, $a_z = 0$ is necessarily a case of one dimensional motion.
(B) $v_x \neq 0$, $v_y = 0$, $v_z = 0$ is necessarily a case of one dimensional motion.
(C) If $v_x \neq 0$, $a_x \neq 0$; $v_y \neq 0$, $a_y \neq 0$; $v_z = 0$, $a_z = 0$ is necessarily a case of motion in one plane.
(D) If $a_x = a_y = a_z = 0$ is necessarily a case of one dimensional motion.

18. The rain is falling vertically downwards. A man walking on the road holds his umberalla tilted. Now, suddenly the rain stops and there is afternoon sun just above the head. In order to protect himself from sun-rays, he holds the umberalla vertical. The reason assigned can be
(A) The speed of light is much higher than that of speed of rain drops.
(B) The speed of light is much higher than that of speed of man.

(C) Actually, the angle of tilt of umberalla in sun-light is very small in comparison to the angle of tilt of umberalla in rain.
(D) Light is behaving as a wave, not as a particle here.

19. A swimmer swims in a flowing river.
$\vec{v}_{s,r}$ = velocity of swimmer with respect to (w.r.t.) river water.
$\vec{v}_{r,g}$ = velocity of river water w.r.t. ground,
$\vec{v}_{s,g}$ =velocity of swimmer w.r.t. ground.
The swimmer intends to reach at the opposite bank of the river. It is possible only when,
(A) $v_{r,g} > v_{s,r}$ (B) $v_{r,g} < v_{s,r}$
(C) $v_{s,g} < v_{s,r}$ (D) none of these.

True or False Type Questions

1. The instantaneous velocity of a body is equal to its average velocity when it is moving with uniform velocity.

2. The average velocity is always equal to the mean value of the initial and final velocities.

3. If the displacement y of a particle is proportional to time, i.e. if $y \propto t$, then the displacement of the particle will be non-zero.

4. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. The two bullets will hit the ground simultaneously.

5. A man while walking observes that the rain is falling vertically downward, if he suddenly stops walking then the rain drops will strike him on his back.

Fill in the Blanks

1. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, its acceleration will be ___________.

2. A boat takes 2 hours to travel 8 km and back in a still water lake with water velocity 4 km/hr. The time taken for going up-stream 8 km and coming back is _______________ minutes.

3. The velocity of a particle moving with constant acceleration at an instant t is 10 m/s. After 5 sec the velocity is 20 m/s. The velocity at 3 sec before was ______________________.

4. A food packet is released from a helicopter which is rising steadily at 2 m/s. After 2 sec the velocity of the packet is ________________ ($g = 9.8$ m/s^2).

5. A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D meter away, and the acceleration due to gravity is ‘g’, the speed of water as it leaves the tank in terms of g, h and D is ________________.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. $-x = -100t + \frac{1}{2}gt^2$; $100 - x = \frac{1}{2}gt^2$

   $-x = -100\,t + 100 - x \quad \Rightarrow t = 1$ s.

2. Greatest in 3, least in 2; v > 0 in 1 and 2, v < 0 in 3.

3. The given statement is true. Both the balls have equal acceleration due to gravity. Both the balls would attain the same height and would pass through the point of projection with the same speed.

4. The values of sin (90 + 2θ) and sin (90 – 2θ) are the same, equal to cos 2θ. Therefore, range are equal for elevation which exceed or falls short of 45° by equal amount θ.

5. The block will reach the ground earlier which falls freely.

6. $x = 2h$

7. $\theta = 30^\circ$

9. $\frac{2}{\sqrt{3}}$

10. $2.7\ m/s^2$

11. 10 m

12. 8m

13. (a) 0 m/s. (b) 21.0 m/s. (c) 14.0 m/s.

14. 5 m/s

15. 160 km

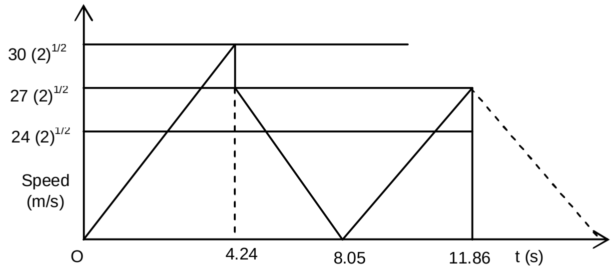
16. $\frac{\alpha\beta t}{\alpha + \beta}$

17. 1200 m

18. 54 m/s

19. 60 km/hr.

20.



Level - I

1. (a) 28.5 cm/s (b) 18.0 cm/s (c) 28.1cm/s (d) 30.4 cm/s
2. (a) 5.71m/s^2 (b) 3.68s (c) 5.78s (d) 95.4m
3. 0.1m/s
4. 10.42 m
5. 50 m
6. (a) 10.20s; (b)883.67 m; (c) 100 m/s
7. (a) 21.8 m/s, (b) 13.55 m/s
8. 47.5 m
9. (a) 94cm (b) 19m/s (c) 2400 m/s^2.
10. [2.832 m/s at an angle of 32^o with $\overrightarrow{V_B}$
11. (a) 3.41 seconds (b) 58.14 m(c) 34.10 m/s
12. 1749.8 m
13. $\left(5\hat{i} - 5\hat{j}\right)$ m/s
14. $\dfrac{v_0}{\sqrt{1+2kv_0^2 t}}$
15. (i) 13.5 m, (ii) 3.06 sec.

Level – II

2. $\dfrac{2v_o\left(v_1+v_2\right)}{2v_o+v_1+v_2}$
3. (A) 82 m; (B) 19 m/s
4. 55m
5. 2.5 m
6. $CD = \dfrac{\ell}{\sqrt{n^2-1}}$
7. 5 km/hr, 53^o North of East.
8. $\dfrac{u^2\cos^2\theta}{g\ \ \cos^3\dfrac{\theta}{2}}$
9. 20 km/hr at an angle $\tan^{-1}\dfrac{3}{4}$ or 37^o east of north, $\sqrt{2}$ km
10. $\sqrt{\dfrac{a\ \ \left(p^2+1\right)}{2q}}$
11. (a) $v = k\,(1-2\alpha t)$, $a = -\,2\alpha k$ (b) $\Delta t = 1/\alpha$, $s = k/2\alpha$
12. (a) $x = \left(\dfrac{a}{2v_o}\right)y^2$ (b) $\omega = av_o$, $\omega_t = \dfrac{a^2v_0y}{\sqrt{v_0^2+a^2y^2}}$, $\omega_n = \dfrac{av_0^2}{\sqrt{v_0^2+a^2y^2}}$
13. $\left[\dfrac{n}{\sqrt{n^2-1}}\right]$
14. $\ell = 8\,h\sin\alpha$
15. (a) 0.78 sec. (b) 0.66 sec.

Objective:

Level – I

1.	B	2.	A	3.	D
4.	C	5.	B	6.	B
7.	C	8.	C	9.	A
10.	D	11.	D	12.	B
13.	D	14.	C	15.	C
16.	B	17.	A	18.	A
19.	C	20.	A		

Fill In The Blanks

1.	2R and πR	2.	300 m
3.	downward	4.	equal
5.	zero		

True or False Type Questions

1.	False	2.	True
3.	False	4.	False
5.	False		

Level – II

1.	D	2.	A	3.	D
4.	A	5.	B	6.	B
7.	B	8.	D	9.	D
10.	C	11.	C	12.	B
13.	C	14.	D	15.	A
16.	C	17.	C, D	18.	B, C
19.	B, C				

True or False Type Questions

1.	True	2.	False
3.	True	4.	True
5.	True		

Fill In the Blanks

1.	$-8\ m/s^2$	2.	160
3.	4 m/s	4.	-17.6 m/s
5.	$D\sqrt{\frac{g}{2h}}$		

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PHYSICS

FIITJEE

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For – JEE (Main/Advanced)

LAWS OF MOTION

Laws of Motion

Syllabus:

Force and inertia, Newton's first law of motion; Momentum, Newton's second law of motion, Impulse; Newton's third law of motion; Law of conservation of linear momentum and its applications; Equilibrium of concurrent forces; Static and kinetic friction, laws of friction, rolling friction, lubrication; Examples of variable mass situation. Dynamics of uniform circular motion; Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road); Inertial and non- inertial frame (elementary idea).

Introduction:

In the introduction of the preceding chapter "Kinematics" we studied that mechanics can be broadly classified into two categories namely *"Kinematics and Dynamics"*.

In kinematics our prime concern was to define the physical quantities like position, velocity, acceleration and to establish the relations among them. But we never tried to answer the questions like "What causes the bodies to move from one place to another? What makes the body to gain or loose its speed? . . . etc.

Dynamics is that branch of mechanics which gives not only qualitative but quantitative description of the above quantities. This branch of mechanics also explains few basic laws which governs the motion of bodies.

Before you start studying this chapter try to analyse the answers for the following questions:

(i) Why to set a ball into motion in playground someone has to kick it or throw it?

(ii) Why a navigator has to row the boat in still water to move the boat ?

(iii) Why the branches of a tree swing when wind blows ?

In all these cases one can observe that some external agency (like hand or legs of player or wind) is coming into contact with the respective objects that are going into motion.

(iv) When a ball is released from the top of a building, the ball falls by itself, though no one is pushing it in downward direction. Why ?

(v) When a piece of iron is placed near a magnet, magnet attracts it from certain distance itself. How ?

In these cases we can observe that even though no external agency is physically coming into contact with the objects still the objects are moving.

Hence we can conclude that an animate or inanimate external agency is required to change the state of a body (i.e. from rest to motion or vice versa). To understand the logic behind this type of questions more clearly, we should know about two basic physical quantities namely *inertia and force*.

Inertia: It is a very common observation for all of us that any book kept on our study table will not move by itself, i.e. until and unless it is acted upon by any external force, it will not change its state of rest.

To explain the reason behind this type of questions Italian scientist Galileo has defined a new physical quantity known as *inertia*. Though it was introduced by Galileo the effective use of this term and its usage for explaining the motion of the bodies was done by another reputed physicist, Sir Isaac Newton.
"Inertia is an inherent property of a body by virtue of which it cannot change its state (i.e. rest or motion) by itself."

It says that every body in the universe does have a property which is hidden in itself and because of this property the body is unable to change its state by itself, i.e. from state of rest to state of motion or vice versa or even it's direction. This inertia is of 3 types, namely
(a) inertia of rest (b) inertia of motion and (c) inertia of direction

Inertia of rest: Inertia of rest is the inability of a body by virtue of which it can't change its state of rest to state of motion. That means any body which is at rest continues to be in the state of rest only and it can't go further into state of motion by itself.

Examples:
(i) *Passengers standing or sitting loosely in a bus experience jerk in the backward direction when the bus suddenly starts moving.* This is due to the fact that when the bus suddenly starts its motion, the lower parts of the human body shares the motion but the upper part tends to remain at rest due to inertia of rest.

(ii) *When a bullet is fired into a tightly- fitted glass pane from a reasonably close range, it makes a clear circular hole in the glass pane.* This is due to the fact that particles of glass around the hole tend to remain at rest due to inertia of rest. So they are unable to share the fast motion of the bullet.

Inertia of motion: Inertia of motion is the inability of a body by virtue of which it can not change its state of uniform motion along a straight line to state of rest. That means any body which is in uniform motion can't come to rest by itself until and unless some external force acts on it.

Examples:
(i) *A passenger standing in a moving bus falls forward when the bus stops suddenly*. This is due to fact that the lower part of the body comes to rest along with the bus but the upper part of the body remains in a state of motion on account of "inertia of motion".

(ii) *An athlete runs for some distance before taking a long jump.* In this way, the athlete gains momentum and this inertia of motion helps him in taking longer jump.

Inertia of direction: Inertia of direction is inability of a body by virtue of which it can't change its direction by itself. This means a body moving along a straight line can't change its direction by itself, until and unless it is acted upon by any external force.

Examples:
(i) *When a running car suddenly takes a turn, the passengers experience a jerk in the outward direction.* This is because the passengers tend to maintain their original direction of motion due to inertia of direction.

(ii) *A stone tied to one end of a string is whirled in a horizontal circle. When string breaks, the stone tends to fly off tangentially along a straight line.* This is due to inertia of direction.

Note: The mass of the body is the indirect measure of the inertia of that body.

Exercise 1.
(i) ***Why does the sparks coming out tangentially from the grid store when knife or any other such objects are sharpened ?***
(ii) ***A clothes line hangs between two poles. No matter how tightly the line is stretched, it always sags a little at the centre. Explain.***

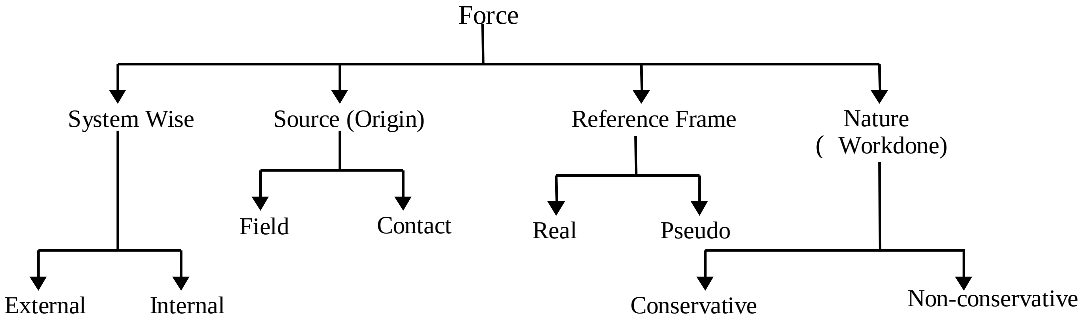
Now let us try to understand another physical quantity "*force*", with the help of which only the mechanical state of a body changes.

"Force is that which pushes or pulls the body or tends to change the state of rest or of uniform motion in a straight line."

(a) It produces or tries to produce motion in a body at rest.
(b) It stops or tries to stop a moving body.
(c) It changes or tries to change the direction of motion of body.
(d) It produces a change in the shape of the body.

CLASSIFICATION OF FORCES

There are different types of forces in our universe. Based on the nature of the interaction between two bodies, forces may be broadly classified as under



Since we are going to encounter these forces in our analysis we will briefly discuss each force and how it acts between two bodies, its nature etc and how we are going to take it into account.

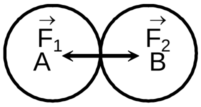
(a) Contact force: The force exerted by one surface over the surface of another body when they are physically in contact with each other is known as *contact force.*

If two surfaces that are coming into contact are perfectly smooth, then the entire contact force will act only perpendicularly (normal) to their surface of contact and it is known as "*Normal force or Normal reaction.*"

If two surfaces that are coming into contact are rough surfaces, then one component of this contact force acts perpendicular to their surface of contact and the other component of this force acts in tangential direction to their surface of contact and this component is known as "*force of friction*."

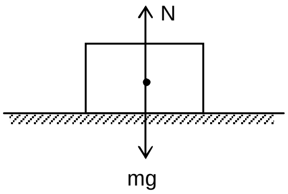
Normal Reaction, Tension, Friction, etc. are the examples of various contact forces.

Normal reaction: The forces $\vec{F}_1$, $\vec{F}_2$ shown in the diagram acting on A and B respectively act away from the surface of contact, and prevent the two bodies from " occupying the same space".

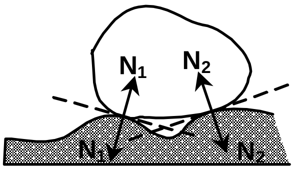


If $\vec{F}_1$ is the action, $\vec{F}_2$ is reaction: they are equal in magnitude but opposite in direction. Further, $\vec{F}_1$ and $\vec{F}_2$ are both perpendicular to the surfaces in contact and note that they act on two different bodies.

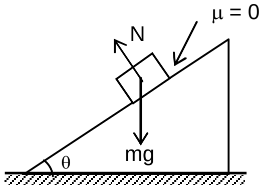
Examples:



N = mg

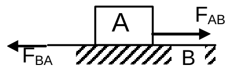


N is the normal reaction of the ground on the stone.



$N = mg\cos\theta$

Friction: It is a force that acts between bodies in contact with each other along the surface of contact and it opposes relative motion between the two bodies. The direction of frictional force on A is opposite to that of direction of frictional force on surface B and magnitude is same for both.

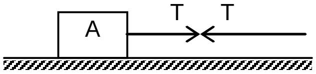


Tension (T): When a string, thread or wire is held taut, the ends of the string or thread (or wire) pull on whatever bodies are attached to them in the direction of the string. This force is known as Tension.

If the string is massless then the tension T has the same magnitude at all points throughout the string.

Examples:

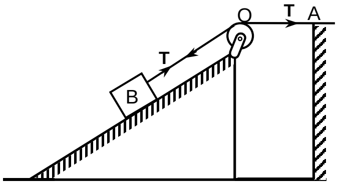
(i) Tension in a string: For a block A pulled by a string,



(ii) The direction of tension is always away from the point of attachment to the body. In the given figure two segments of tension act at O towards A and B.

For the wedge, there are two segments of thread at the point of attachment O to the body. Hence, two tensions act on the wedge; one along OB and the other along OA.

Hence a tension acts away from the point of attachment along BO.

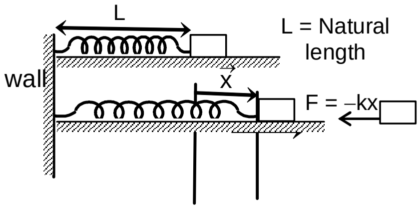


Spring forces: Whenever a spring is compressed or extended, the elastic force developed in the spring which helps the spring to restore its original position is known as spring force.

In an extended (or compressed) spring, force is proportional to the magnitude of extension (or compression).

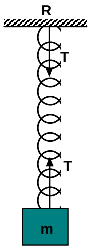
$F \propto x$, in magnitude, but opposite in direction. So, $F = -kx$, where k is a positive constant, also known as the spring constant of the spring.

x is the compression or elongation from the natural length.



Example:

In case of the spring the tensions are oppositely directed on the block 'm' and on the roof R.



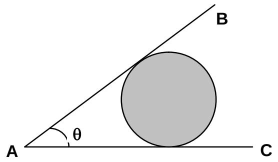
(b) Non-contact force: Bodies can exert forces on each other without actual physical contact. This is known as action at a distance. Such forces are known as *non-contact forces* (or) field forces, e.g. gravitational force, electrostatic forces, etc.

For the moment, we deal with actual forces. Suffice it to say that there exist pseudo-forces acting in a non-inertial frame of reference, which we will discuss later.

Forces may be conservative or non-conservative depending on whether work done against them by an external agent is recoverable or otherwise. This will be discussed in later chapters.

Exercise 2. ***(i). Is it necessary to have normal reaction whenever two surfaces are in contact with each other?***

(ii). Find the contact forces acting on a stationary sphere weighing 10 N is placed in a fixed frame BAC as shown in the diagram? (Assume θ is constant)



Now having understood about inertia and force, we can easily understand the three basic laws of motion which are known as Newton's laws of motion.

Newton's first law

Every body continues to be in the state of rest or of uniform motion in a straight line until and unless it is compelled to change the state of the body by an unbalanced force.

For better understanding we can divide this statement into two parts.

(i) "Every body continues to be in its state of rest until and unless some external force compels it to change the state of rest."

This part of the law is self explanatory and self evident as we come across several examples in our daily life like all inanimate objects will continue to be in the same place where they are put until they are disturbed by some external agents.

(ii) "Every body continues in its state of uniform motion in a straight line unless external force compels it to change that state."

The second part of the statement can't be readily understood as on the surface of the earth because of various types of frictional or resisting forces. For example when a ball is rolled on a horizontal surface the ball will come to halt after some time however smooth the surface may be, as we can't eliminate force of friction completely.

Momentum (Linear):

Till now we studied about inertia (translational) which is the inability of a body. Now we will study about another physical quantity called 'momentum' which is the ability of body.

Momentum is defined as the ability of a body by virtue of which it imparts or tends to impart its motion along a straight line.

Mathematically, momentum (p) is measured as the product of mass (m) and velocity (v) of the body. As velocity is a vector quantity, momentum is also a vector quantity.

$\vec{p} = m\vec{v}$ or $p = mv$

Unit : Its unit is kg-m/s in SI system and gm-cm/s in CGS system.
Dimensions : Its dimensions are MLT^{-1}

Exercise 3.
(i) If a body is at rest, can we say that no force is acting on the body?
(ii) A car and a lorry are travelling with same velocity on a straight horizontal road. Which of the two has got greater momentum?

Illustration 1. *A block of mass 2 kg is moving with a velocity of* $2\hat{i} - \hat{j} + 3\hat{k}$ *m/s. Find the magnitude and direction of momentum of the block with the x-axis.*

Solution: The magnitude of momentum is $2\sqrt{14}$ kg m/s and the direction is at an angle of $\tan^{-1}\sqrt{\frac{2}{7}}$ with x-axis.

Illustration 2. *A block rests on air inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane, down the plane or sideways? Why?*

Solution: It is easier to push it down the plane because we have to apply the force only to overcome from the rest part of friction force.

Illustration 3. *When a force of constant magnitude always act perpendicular to the motion of a particle then :*

(A) Velocity is constant *(B) Acceleration is constant*
(C) KE is constant *(D) None of the above*

Solution : Force will provide centripetal acceleration, so it will move in a circular path therefore KE is constant because speed remains unchanged.
Option (C) is correct.

Newton's second law of motion

We have already studied the Newton's 1st law which has given us a qualitative idea about force. Now, we will study about Newton's IInd law which gives us a quantitative idea about force.

Whenever a cricketer catches a ball he allows a longer time for his hands to stop the ball. Otherwise the ball will hurt the cricketer. If you observe this incident carefully you can easily understand that cricketer is applying some force on ball in order to make the momentum of the body zero. And also we can understand that the magnitude of the retarding force that cricketer applies on the ball in order to stop depends on two factors.
(1) The momentum of the ball and
(2) Time for which he is applying the force

These type of observations lead Newton to state his second law of motion.

The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of the force.

So for a body with constant mass,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \qquad \text{or,} \qquad \frac{d}{dt}(m\vec{v}) \propto \vec{F}$$

$$\text{or,} \qquad m\frac{d\vec{v}}{dt} \propto \vec{F} \qquad ; \qquad \vec{F} = km\left(\frac{d\vec{v}}{dt}\right),$$

where k is a constant. With proper choice of units, k = 1. Thus,

$$\mathbf{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}} \ .$$

Unit of Force : Its unit is newton in SI system and dyne in CGS system.
Dimensions : $[MLT^{-2}]$

Note (1): The second law of motion is a vector law. These are actually three equations, one for each component of the vectors:

$$F_x = \frac{dp_x}{dt} = ma_x \; ; \; F_y = \frac{dp_y}{dt} = ma_y \; ; \quad F_z = \frac{dp_z}{dt} = ma_z$$

Note (2): The second law of motion is strictly applicable to a single point particle. The force F here stands for net external force on the particle.

Exercise 4. ***A body is acted upon by a number of external forces. Can it remain at rest ?***

Illustration 4. *A body of mass m = 1 kg falls from a height h = 20 m from the ground level*
(a) What is the magnitude of total change in momentum of the body before it strikes the ground?
(b) What is the corresponding average force experienced by it? (g = 10m/sec^2).

Solution: (a) Since the body falls from rest (u = 0) through a distance h before striking the ground, the speed v of the body is given by kinematics equation.
$v^2 = u^2 + 2\,as$; Putting a = g and s = h
we obtain $v = \sqrt{2gh}$
$\Rightarrow$ The magnitude of total change in momentum of the body
$= \Delta p = |\,mv - 0\,| = mv$, Where $v = \sqrt{2gh}$
$\Rightarrow \Delta p = m\sqrt{2gh} \quad \Rightarrow \Delta p = (1)\sqrt{(2\times10\times20)}$ kg m/sec
$\Rightarrow \Delta p = 20$ kg m/sec.
(b) The average force experienced by the body = $F_{av} = \dfrac{\Delta p}{\Delta t}$
where Δt = time of motion of the body = t(say). We know Δp = 20 kg m/sec. Therefore we will have to find t using the given data. We know from kinematics that,

$$S = ut + \frac{1}{2}at^2 \Rightarrow h = \frac{1}{2}g\,t^2 \qquad (u = 0)$$

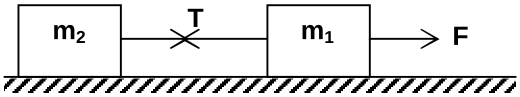
$$\Rightarrow t = \sqrt{\frac{2h}{g}} \qquad \therefore \; F_{av} = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{t}$$

Putting the general values of ΔP and t we obtain

$$F_{av} = \frac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg \qquad \Rightarrow \vec{F}_{av} = m\vec{g}\,.$$

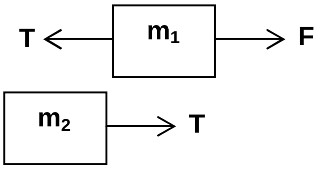
Where mg is the weight (W) of the body and $\vec{g}$ is directed vertically downward. Therefore the body experiences a constant vertically downward force of magnitude mg.

Illustration 5. *Two masses connected with a light string are placed on a horizontal frictionless surface. One mass is pulled by a constant force of F directed along the string. The acceleration of mass m_1 is*



(A) $\dfrac{F}{m_1}$ *(B)* $\dfrac{F}{m_2}$
(C) $\dfrac{F}{m_1 + m_2}$ *(D) zero*

Solution:

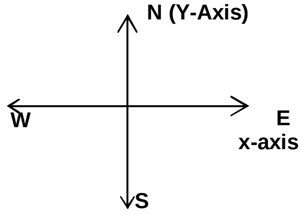


$$F - T = m_1 a$$
$$T = m_2 a$$
$$F = (m_1 + m_2) a$$
$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

So option (C) is correct.

Illustration 6. *A body of mass 2 kg is moved towards east with a uniform speed of 2 m/s. A force of 3 N is applied to it towards north. Calculate the magnitude of the displacement of the body 2S after the application.*

Solution : According to the given reference frame:



$u_x = 2\,m/s$ and $a_x = 0$

$u_y = 0$ and $a_y = \frac{F_y}{m} = \frac{3}{2}$

$= 1.5\,m/s^2$

After 2 seconds, the displacement of the body along x-axis

$S_x = v_x t = 4m$

Along y-axis

$S_y = \frac{1}{2} a_y t^2 = \frac{1}{2} x 1.5 x 4 = 3m$

So magnitude of the displacement $= \sqrt{S_x^2 - S_y^2} = 5\,m$

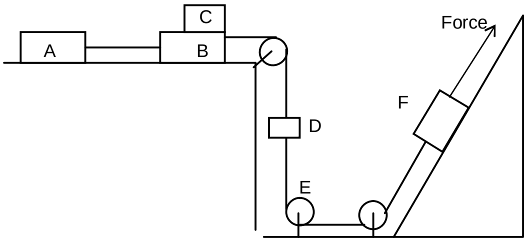
Working with Newton's first and second law

Before trying to write an equation from Newton's law, we should very clearly understand which particle we are considering. In any particle situation, we deal with extended bodies which are collection of a large number of particles. The laws stated a force ma be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing equations from Newton's law is as follow:

1st step : Decide the system

The first step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

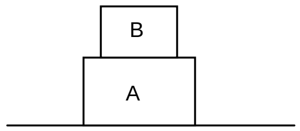
Here the distance covered by all the blocks is same but they all can not be taken as a system because even though magnitude of acceleration is same but the direction of acceleration in all the blocks is not same.



2nd step : Identify the forces

Once the system is decided, make a list of forces acting on the system due to all the objects other than system. Any force applied by the system should not be included in the list of the forces.

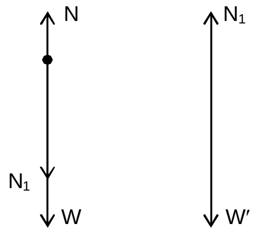
Consider the situation shown in the figure where small block of mass m is kept on bigger block of mass M. The load presses lower block, the lower pushes the upper block, the bigger block presses the floor downward, the floor pushes the block upward, the earth attracts the block.



3rd step: Make a free body diagram

Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin.

The forces may lie along a line, may be distributed in a plane (coplanar) or may be distributed in the space (non-planer). Indicate the magnitude and directions of the forces in the diagram. This is called a free body diagram.



Block B Block A

4th step: Choose axes and write equations:

Any three mutually perpendicular directions may be chosen as the x-y-z axes. Some suggestion for choosing the axes to solve problems are,

(a) If the forces are coplaner, only two axes, say x and y, taken in the plane of forces are needed.
(b) Choose the x-axis along the direction in which the system is known to have or is likely to have the acceleration.
(c) If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes.
(d) Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration. Write the components of the forces along the y-axis and equate the sum to zero.

Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

Impulse: A large force acting for a short time to produce a finite change in momentum is called *impulse* and the force acted is called *impulsive force* or force of impulse.

Mathematically it is described as the product of force and time.

$\therefore$ Impulse (J) = F . t

$\therefore$ Impulse (J) = mv – mu and since force is variable, hence $J = \int_{t_1}^{t_2} F\,dt$

The area under F – t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$.

Unit of Impulse **:** The unit in S.I. system is kgm/sec or newton -second.
Dimension **:** $M^1L^1T^{-1}$

Examples:

(i) *Automobiles are provided with spring shocker systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, there by reducing the impulse of force. This minimises the damage to the vehicle.

(ii) *A man falling from a certain height receives more injuries when he falls on a marble floor than when he falls on a heap of sand.* This is because the marble floor does not yield under the weight of the man. The

man is stopped abruptly. A large change of momentum takes place in a very short interval of time. But when he falls on a heap of sand, the sand yields under the weight of the man and this increases the time interval. So it reduces the force exerted by sand on man.
(iii) *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is because cricket ball will have more momentum than tennis ball due to its heavier mass. The change in momentum in case of cricket ball is more. Hence more force is required to stop cricket ball than tennis ball.

Exercise 5. ***When a swimmer dives into water just before piercing himself into water, he stretches himself. Why ?***

Illustration 7. *A cricket ball of mass 200 gm moving with velocity 15 m/s is brought to rest by a player in 0.05 sec. What is the impulse of the ball and average force exerted by player ?*

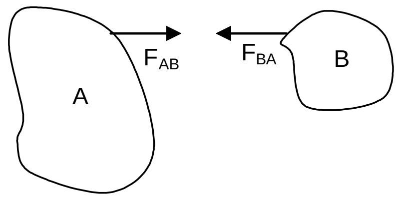
Solution: Impulse = change in momentum = m(v – u) = 0.2 (0 – 15) = – 3 Ns

Average force = Impulse / Time = $\frac{3}{0.05} = 60$ N

Newton's third law of motion

Now we have understood the qualitative and quantitative definitions of force from Newton's first and second laws. But how are the forces between two bodies related to each other if at all ? The answer is provided by the third law of motion.

Every action has an equal and opposite reaction, *which are equal in magnitude and opposite in direction.*



Consider two bodies A and B interacting with each other, by means of forces

$\vec{F}_{AB}$: the force exerted by body B on A

$\vec{F}_{BA}$: The force exerted by the body A on B.

According ot Newton's 3rd law : $\vec{F}_{AB} = -\vec{F}_{BA}$ (equal in magnitude & opposite in direction)

That may look fine, but it, apparently, raises a lot of questions. For example, if a horse pulls a cart and cart pulls the horse backward, how does the cart moves forward at all ?

If we observe we will find that the forces acting on the horse and the cart, though equal and opposite, they are acting not on the same body, rather, two bodies. It cannot produce equilibrium neither in horse nor in cart.

Exercise 6.

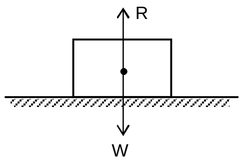
i) ***If action and reaction are equal and opposite to each other then how can a man move a box on a floor?***

ii) ***In a tug of war if the parties on both ends of the rope apply equal and opposite forces on each other then how can one party win?***

(iii) ***Suppose you are standing on a boat in the middle of a swimming pool. The boat is loaded with a bag of stones. Can you get to the shore using the stones? If yes, explain the concept behind it.***

Examples:

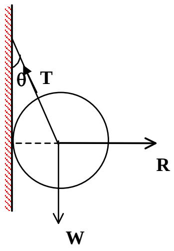
(i) Consider a body of weight W resting on a horizontal surface. The body exerts a force (action) equal to weight W on the surface. The surface exerts a reaction R on the body in the upward direction such that W = R or in vector notation, $\vec{W} = -\vec{R}$



(ii) In a lawn sprinkler, when water comes out of the curved nozzles, a backward force is experienced by the sprinkler. Consequently, the sprinkler starts rotating and sprinkles water in all directions.

(iii) In order to swim, a man pushes the water backwards with his hands. As a result of the reaction offered by water to the man, the man is pushed forward.

Illustration 8. *A metal of sphere is hung by by a string fixed to a wall. The force acting on the sphere are shown in figure. Which of the following statement is correct*



(A) $\vec{F}+\vec{T}+\vec{W} = 0$ (B) $T^2 = R^2 + W^2$

(C) $T = R + W$ (D) $R = W \tan\theta$

Solution : By the free body diagram

$T\cos\theta = w$

$T\sin\theta = R$

$T^2 = W^2 + R^2$

Option (B) is correct.

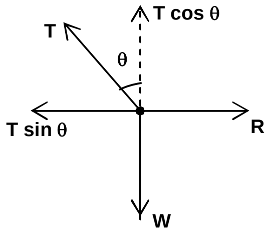
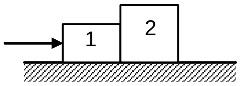


Illustration 9. *Two blocks of mass 2 kg and 4 kg are kept in contact with each other on a smooth horizontal surface. A horizontal force of 12 N is applied on the first block due to which they move with certain constant acceleration. Calculate the force between the blocks.*



Solution : Let a = common acceleration of blocks,

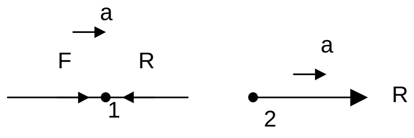
R = Force between the two blocks

From FBD of Ist block

$F - R = m_1 a$ . ... (i)

From FBD of block 2,

$R = m_2 a$ . . . .(ii)



F.B.D.

$\therefore$ From (i) & (ii), $F = (m_1 + m_2).\ a$

$$\therefore\ a = \frac{F}{m_1+m_2} = \frac{12}{6} = 2\ \text{m/sec}^2$$

$\therefore$ Force between blocks $= R = m_2 a = (4 \times 2)$ N = 8 N.

Illustration 10. *A body of mass 5 kg is supported by a light cord. Find the tension in the cord.*

Solution:
(i) The body is isolated.
(ii) The two forces acting on it are :
Tension (T) in the cord (upwards)
Weight (mg) (downwards)
(iii) Equations of equilibrium :
$\Sigma F_x = 0$
$\Sigma F_y = 0 \Rightarrow T - mg = 0$
or $T = (5)(10) = 50$ N
(iv) Tension in the cord equals the weight of the body.

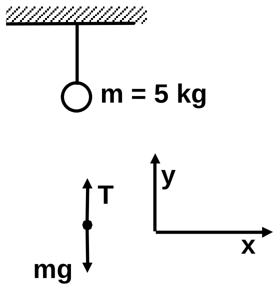
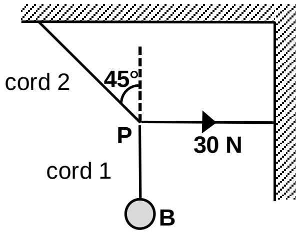


Illustration 11. *In the shown figure the tension in the horizontal cord is 30 N. Find the weight of the body B.*



Solution:

(i) Isolate the point P

(ii) The forces acting on it are :
Unknown tension T_2 in cord 2
Unknown tension T_1 in cord1.
Known tension of 30N in the horizontal cord.

(iii) T_2 is resolved along x and y axes.

(iv) Condition of equilibrium:

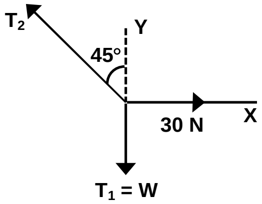
$\Sigma F_x = 0 \Rightarrow 30 - T_2\sin 45° = 0$ (1)

and $\Sigma F_y = 0 \Rightarrow T_2\cos 45° - T_1 = 0$ (2)

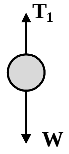
Since body B is also in equilibrium,

Hence, $T_1 = W$ (3)

(v) After solving these equations,
we get W = 30 N



For body *B*



FRAME OF REFERENCE

It is a conveniently chosen co-ordinate system, which describes the position and motion of a body in space.

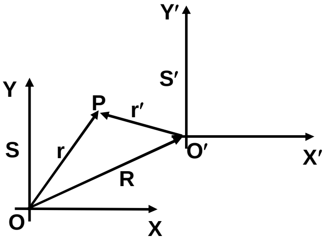
Inertial frame of reference

A reference frame which is either at rest or in uniform motion along the straight line. Newton's laws are strictly valid only for inertial frame.

Non-inertial frame of reference

A reference frame which accelerates or rotates with respect to an inertial reference frame.

Motion of a particle (P) is studied from two frames of references S and S′. S is an inertial frame of reference and S′ is a non inertial frame of reference. At any time, position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}\,'$ respectively. At the same moment position vector of the origin of S′ is $\vec{R}$ with respect to S as shown in the figure.



From the vector triangle OO′P , we get

$$\vec{r}\,' = \vec{r} - \vec{R}$$

Differentiating this equation twice with respect to time we get

$$\frac{d^2\vec{r}\,'}{dt^2} = \frac{d^2(\vec{r})}{dt^2} - \frac{d^2}{dt^2}(\vec{R})$$

$$\Rightarrow \vec{a}\,' = \vec{a} - \vec{A}$$

Here $\vec{a}\,'$ = acceleration of the particle P relative to S′

$\vec{a}$ = Acceleration of the particle relative to S

$\vec{A}$ = Acceleration of S′ relative to S.

Multiplying the above equation by m (mass of the particle) we get

$$m\vec{a}' = m\vec{a} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} + \left(-m\vec{A}\right).$$

In non-inertial frame of reference an extra force is taken into account in order to apply Newton's laws of motion. The magnitude of this force is equal to the product of the mass of the body and acceleration of the frame and it is always directed opposite to the acceleration of the frame. This force is known as ***Pseudo force***, because this force does not exist in the inertial frame of reference.

Exercise 7. *Apparent weight of a person, standing in an elevator which is going up or down with certain acceleration, changes. Why?*

Illustration 12. *A block slides down an inclined plane of slope angle θ with constant velocity. If it is then projected up the same plane with an initial speed v_0, the distance in which it will come to rest is*

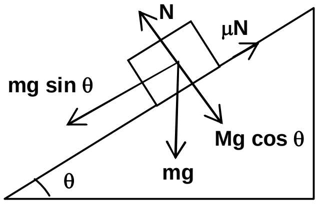
(A) $\dfrac{v_0^2}{g\tan\theta}$ *(B)* $\dfrac{v_0^2}{4g\sin\theta}$

(C) $\dfrac{v_0^2}{2g}$ *(D)* $\dfrac{v_0^2}{2g\sin\theta}$

Solution : From the free body diagram
If the body move with constant velocity $mg\sin\theta = \mu N$

$N - mg\cos\theta = 0$

So, $\mu = \tan\theta$



Now in case (ii) if it is projected with velocity v_0 upward then net retardation against it is

$$a = \frac{mg\sin\theta + \mu\, mg\cos\theta}{m} = g\sin\theta + \mu g\cos\theta$$

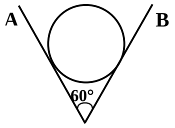
$$= g\sin\theta + g\sin\theta$$

$$= 2g\sin\theta$$

So distance travelled = $= \dfrac{v_0^{\ 2}}{2a} = \dfrac{v_0^{\ 2}}{4g\sin\theta}$

So option (B) is correct.

Illustration 13. *An iron sphere weights 10 N and rests in a V shaped trough whose sides form a angle 60°. The net force exerted by the walls on the sphere in case as shown in figure is*



(A) 0 N *(B) 10 N*

(C) $\dfrac{10}{\sqrt{3}}$ *N* *(D) 5 N*

Solution : By Symmetry $N_1 = N_2$

Therefore

$N_1 \sin 30^0 + N_2 \sin 30^0 = mg$

And $\frac{N_1 + N_2}{2} = mg$

So $N = mg = 10\,N$

So option (B) is correct.

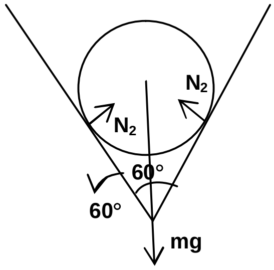


Illustration14. *A block of mass m is placed on an inclined plane. With what acceleration should the system move towards right on a horizontal surface so that m does not slide on the surface of inclined plane? Assume all surfaces are smooth.*

Solution : From ground frame of reference the forces acting on the block m are:

(i) its weight mg and

(ii) normal reaction R and its acceleration is rightward. If we analyse the motion of m relative to the inclined plane, its acceleration is zero and the forces acting are its weight, the normal reaction and a pseudo force of magnitude ma towards left.

$R\cos\theta = mg \therefore a = g\tan\theta$

$R\sin\theta = ma$

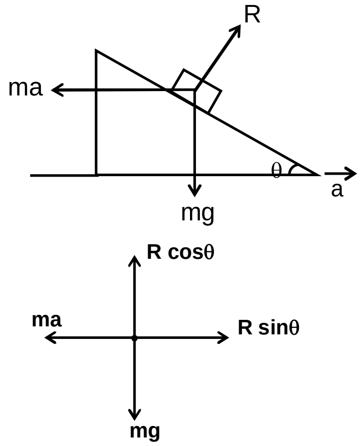
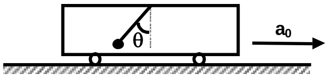


Illustration 15. *A pendulum of mass m is hanging from the ceiling of a car having an acceleration a_o with respect to the road in the direction shown. Find the angle made by the string with the vertical.*



Solution: Since bob of the pendulum is stationary relative to car

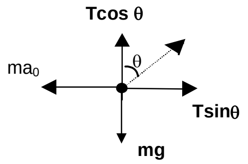
Hence

$T\sin\theta = ma_o$ (pseudo force) ...(i)

$T\cos\theta = mg$ ... (ii)

Dividing (i) by (ii), we get

$\tan\theta = \frac{a_o}{g} \Rightarrow \theta = \tan^{-1}\frac{a_o}{g}$



F.B.D. of pendulum relative to car

CONSTRAINT RELATIONS

The equations showing the relation of the motions of a system of bodies, in which motion of one body is constrained by motion of other bodies, are called the constraint relations.

Applying Newton's Laws alone is not sufficient in some cases where the number of equations is less than the number of unknowns.

In the given diagram for finding the acceleration of the masses there are three unknowns, tensions T, acceleration a_1 and a_2 of masses m_1 and m_2. However we will get only two equations. Clearly Newton's laws are not sufficient to solve the problem and constraint relations provide additional equations. When the motions of bodies in a system is constrained because of pulleys, strings, wedges or other factors, we use geometry to develop additional equations.

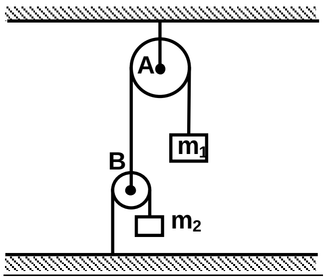
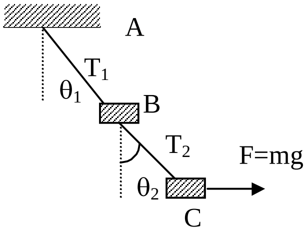


Illustration 16. *The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.*

(A) tan $\theta_1 = 1$ *(B) tan $\theta_2 = 1/2$*

(C) $T_1 = \sqrt{5}$ mg *(D) $T_2 = \sqrt{5}$ mg*



Solution : From F.B.D.

$T_2 \cos\theta_2 = mg$

$T_2 \sin\theta_2 = mg$

$\Rightarrow T_2 = \sqrt{2}\,mg$

and $\theta_2 = 45^o$

Also $T_1 \sin\theta_1 = T_2 \sin\theta_2$

and $T_2 \cos\theta_2 + mg = T_1 \cos\theta_1$

$T_1 \cos\theta_1 = mg + mg = 2mg$

and $T_1 \sin\theta_1 = mg$

Therefore $T_1 = \sqrt{5}\,mg$

So option (C) is correct.

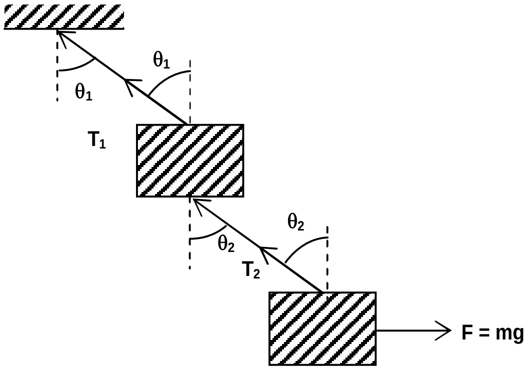


Illustration 17: *A rod is sliding along the wall as shown in figure. Find the ratio of velocity of its ends at the given instant.*

Solution : From the figure

The length of the rod should be constant, so

$v_A \cos\alpha = v_B \sin\alpha$

$\frac{v_A}{v} = \tan\alpha$

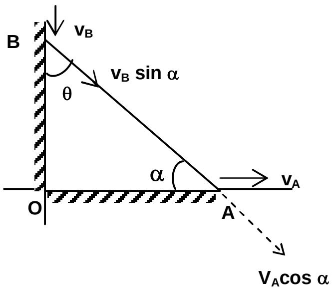
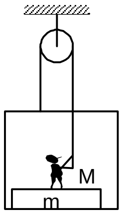
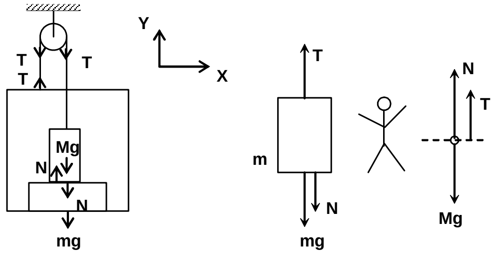


Illustration 18. *A man of mass M is standing on a plank kept in a box. The plank and box as a whole has mass m. A light string passing over a fixed smooth pulley connects the man and box. If the box remains stationary, find the tension in the string and the force exerted by the man on the plank.*



Solution: The fixed pulley is taken as frame of reference. The forces on man and box with plank are shown in figure.



The forces are as follows:

i. Weight of the man $= Mg$
ii. The tension in the string = T.
iii. The normal contact force between the man and the plank = N.
iv. The weight of the plank and box = mg

Referring to figure. The equation of motion of the man is given as

$T + N - Mg = M a$

since M > m and the box remains at rest, the man will have to be at rest

$T + N - Mg = 0$ ...(1)

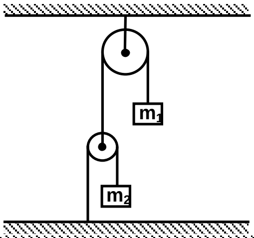
Similarly referring to figure,

$T - N - mg = ma = 0$ . . . (2)

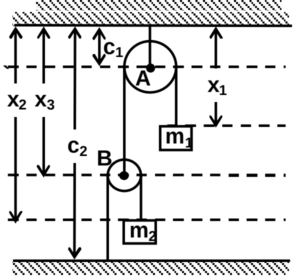
Solving (1) and (2) we obtain,

$$T = \frac{(M+m)g}{2} \text{ and } N = \frac{(M-m)g}{2}$$

Illustration 19. *Find the acceleration of the two blocks m_1 and m_2. Assume pulleys are massless and frictionless and the strings are inextensible.*



Solution: Let the acceleration of blocks m_1, m_2 and pulley B be a_1, a_2 and a_3 respectively.



Constraint relationship for the string attached to block of mass m_1:

$x_1 + x_3 - c_1 = \text{constant}$

Differentiating twice w.r.t. time we get,

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_3}{dt^2} = 0$$

$\Rightarrow a_1 = - a_3$ . . . (i)

The minus sign signifies that acceleration of pulley B is opposite to that of block of mass m_1

Constraint relationship for the string attached to block of mass m_2:

$c_2 - x_3 + x_2 - x_3 = \text{constant}$

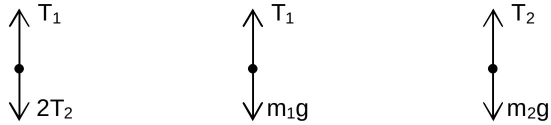
Differentiating twice w.r.t. time we get,

$$\frac{d^2x_3}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

$\Rightarrow a_2 = 2a_3$ . . . (ii)

$a_2 = 2a_1$ . . . (iii) (from equations (i) and (ii))

Taking the magnitudes only and ignoring the sign.



FBD of pulley B FBD of block m_1 FBD of block m_2

$T_1 = 2T_2$ . . . (iv)

$m_1g - T_1 = m_1a_1$ . . . (v)

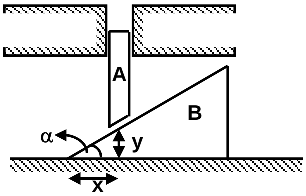
$T_2 - m_2\,g = m_2a_2$ . . . .(vi)

Solving equations from (iii) to (vi) for a_1 and a_2 we get,

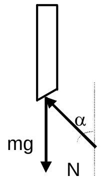
$$a_1 = \frac{m_1 - 2m_2}{m_1 + 4m_2}$$

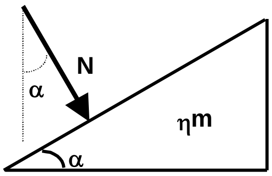
$$a_2 = 2a_1 = \frac{2(m_1 - 2m_2)}{m_1 + 4m_2}.$$

Illustration 20. *Find the accelerations of the rod A and the wedge B in the arrangement shown in the figure if the ratio of the mass of the wedge to that of the rod equals η and the friction between all surfaces is negligible.*



Solution: $y = x\tan\alpha \Rightarrow \frac{d^2y}{dt^2} = \frac{d^2x}{dt^2}\tan\alpha$





$a_{rod} = a_{wedge}.\tan\alpha$ (constraint equation)

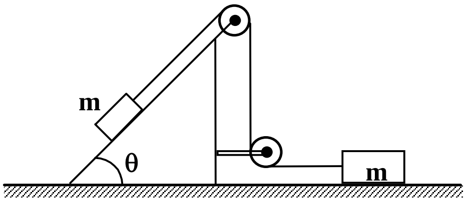
$mg - N\cos\alpha = ma_R$ & $N\sin\alpha = (\eta m).a_{wedge}$

$\Rightarrow mg - N\cos\alpha = m\,a_w\tan\alpha$

Solving we get,

$$a_{wedge} = \frac{g}{\tan\alpha + \eta\cot\alpha} \text{ and } a_{Rod} = \frac{g}{1+\eta\cot^2\alpha}$$

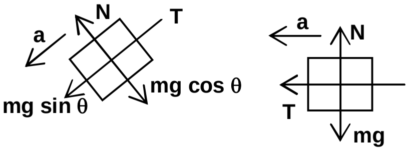
Illustration 21. *For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be*



(A) $\frac{2}{3}mg\sin\theta$ *(B)* $\frac{3}{2}mg\sin\theta$

(C) $\frac{1}{2}mg\sin\theta$ *(D)* $2mg\sin\theta$

Solution: Form F.B.D.



$T = ma$

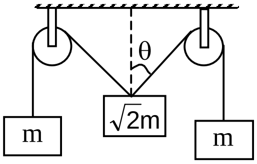
and $mg\sin\theta - T = ma$

So, $a = \frac{g\sin\theta}{2}$

Therefore, $T = \frac{mg\sin\theta}{2}$

So, option (C) is correct.

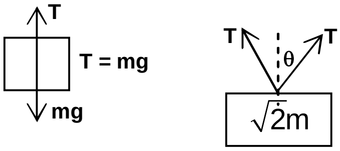
Illustration 22. *The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be*



(A) 0° *(B) 30°*
(C) 45° *(D) 60°*

Solution: $\Rightarrow 2T\cos\theta = \sqrt{2}\,mg$

$\cos\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45^{O}$



So option (C) is correct.

Application of Newton's laws of motion: techniques and approach

A separate point diagram of the body is drawn showing the different forces exerted by the bodies in the environment, this is known as free body diagram.

Application of Newton's Laws to any system (consisting of one or more objects) can be done by following a systematic method. We recommend the following steps in the order given below –

(i) Draw the complete free body diagram (FBD), showing all the forces acting on each separate body.

(ii) Select proper coordinates for analysing the motion of each body.
Include any pseudo forces within the FBD if required.

(iii) If there are any constraints, write the proper constraint equations.

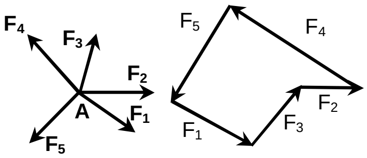
(iv) Apply Newton's 2nd law of motion : $\vec{F} = m\vec{a}$ for each body. This leads to a system of equations.

(v) Solve these equations.
(a) Identify the known and unknown quantities. Check that the number of equations equals the number of unknowns.
(b) Check the equations using dimensional analysis.
(c) After solving, check the final solution using back substitution.

(vi) If the velocity ($\vec{v}$) or position ($\vec{x}$) is required, proceed from a knowledge of acceleration ($\vec{a}$) as found from equations in step (v) and apply kinematics, e.g.
$\dfrac{d\vec{v}}{dt} = \vec{a}$ (known) and then integrate.

Equilibrium of concurrent forces

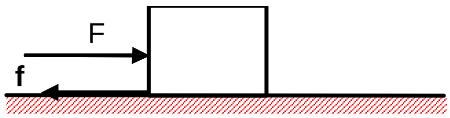
If the number of forces that are acting on a particle can be taken along the sides of any polygon both in direction as well as in magnitude, it will be in equilibrium.
Suppose that the force F_1, F_2, F_3, F_4 and F_5 are acting on the particle A and if they are in equilibrium then they will form a pentagon.

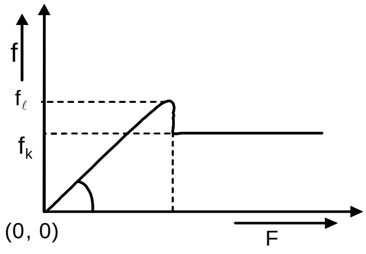


FRICTIONAL FORCE

Frictional force comes into play between two surfaces whenever there is relative motion or a tendency of relative motion between two surfaces in contact. Frictional force has the tendency to stop relative motion between the surfaces in contact.

Friction is a self-adjusting force. It changes its direction and magnitude according to the applied force or the force, which causes a tendency in the body to move. If the force increases then the opposing force also increases until the body moves beyond which it remains constant. If the applied force is plotted against the frictional force we obtain a graph as shown.





The graph shows that first frictional force increases to a certain maximum value f_ℓ with F and then suddenly decreases to a constant value f_k. For the range from 0 to f_ℓ frictional force is equal and opposite to F and hence block does not move. In this range, friction force is static.

Thus, friction can be classified as

(a) Static friction: It acts between surfaces in contact not in relative motion. It opposes the tendency of relative motion.

(b) Kinetic friction: It act acts between surfaces in contact which are in relative motion. It opposes the relative motion between the surfaces. Kinetic friction can be further classified as sliding friction and rolling friction.

Rolling friction: When a body rolls on a rough surface the frictional force developed is known as rolling friction. It is generally less than the kinetic friction or limiting friction.

Laws of static friction

Static Friction, acting between the surfaces in contact, (not in relative motion) opposes the tendency of relative motion between the surfaces.

The frictional force acts tangentially along the surfaces in contact, and the maximum value (or limiting value) of this force is proportional to the normal reaction between the two surfaces. The force of friction between two bodies is an *adjustable* force, only its maximum or limiting value is proportional to the normal reaction. Secondly, the direction of this force is determined by *all* other forces acting on the body that is by the forces that *tend* to cause relative motion. The force of static friction acts in a direction so as to *oppose* the other forces that tend to cause relative motion between the surfaces in contact.

Now, $f_{s(max)} \propto N$ where $f_\ell = f_{s(max)} \Rightarrow f_{s(max)} = \mu_s N$

Here μ_s = co-efficient of static friction.

N = normal reaction of the block from the surface.

$$0 \le f_s \le \mu N$$

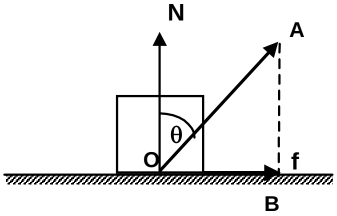
When F exceeds f_ℓ block starts moving and frictional force decreases to a constant value f_k. f_k is called kinetic friction and it has unique value which is given by

$$f_k = \mu_k N$$

Here μ_k = co-efficient of kinetic friction.

N = normal reaction.

Angle of friction: The angle made by the resultant reaction force with the vertical *(normal reaction)* is known as the angle of the friction.



Now, in the triangle OAB,

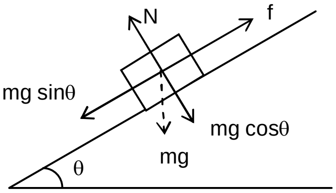
$$\frac{AB}{OB} = \cot\theta$$

$\Rightarrow OB = AB \tan\theta$

or, $\tan\theta = \frac{f}{N}$

Angle of Repose:

The angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide. Consider an inclined plane, whose inclination with horizontal is gradually increased, till the body placed on its surface just begins to slide down, then the angle made by the plane with horizontal is called angle of repose.



From the diagram:

$f = mg \sin\theta$ . . . (i)

$N = Mg \cos\theta$

Dividing (i) by (ii)

$$\frac{f}{N} = \frac{Mg\sin\theta}{Mg\cos\theta} = \tan\theta$$

Since $\frac{f}{N} = \mu$, $\tan\theta = \mu$

Therefore, coefficient of limiting friction is equal to the tangent to the angle of repose thus angle of repose is equal to the angle of friction.

Exercise 8.

(i) ***What causes the motion of a car on a road?***

(ii) ***Why the centrifugal force is never included in the free body diagram of circular motion. Explain why?***

Illustration 23. *A block weighing 2 kg rests on a horizontal surface. The coefficient of static friction between the block and surface is 0.40 and kinetic friction is 0.20.*

(a) How large is the friction force acting on the block ?

(b) How large will the friction force be if a horizontal force of 5N is applied on the block?

(c) What is the minimum force that will start the block in motion?

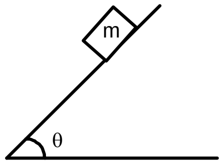
Solution: (a) As the block rests on the horizontal surface and no other force parallel to the surface is on the block, the friction force is zero.

(b) With the applied force parallel to the surfaces in contact 5 N, opposing friction becomes equal and opposite. Further the limiting friction is $\mu_s N = \mu_s Mg = 8$ N

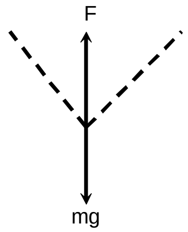
$\therefore$ Force of friction is 5N.

(c)The minimum force that can start motion is the limiting one. $\mu_s N = \mu_s mg = 8$ N

Illustration 24. *A block of mass m is at rest on a rough inclined plane of inclination θ as shown in the figure*
(a) Find the force exerted by the inclined plane on the block.
(b) What are the tangential and normal contact forces?



Solution: (a) The forces acting on the block are the field force $m\vec{g}$, vertically downward and total contact force $\vec{F}$ given by inclined plane on the block. As the block is at rest net force on the block is zero.



$\therefore \quad \vec{F} + mg = 0$ [as shown in Figure]

$\therefore \quad \vec{F} = -mg$

$\therefore$ The force exerted by the inclined plane on the block is $-\overrightarrow{mg}$ in vertically upward direction.

(b) The normal contact force N and tangential contact force f are shown in F.B.D. (Figure)

$f = mg \sin\theta$
$N = mg \cos\theta$

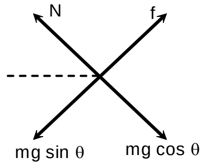
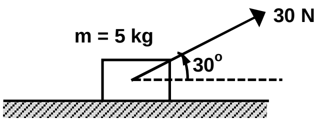
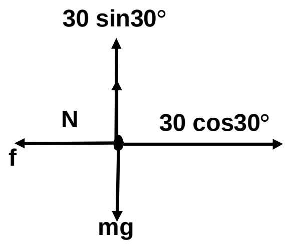


Illustration 25. *A 5 kg box is being moved across the floor at a constant velocity by a force of 30 N, as shown in the figure.*
(a) What is the force of friction acting on the box ?
(b) Find μ_k between the box and the floor.



Solution: The free body diagram of the box is shown in the figure.
Conditions of equilibrium :

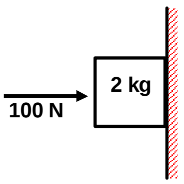
$\Sigma F_y = 0$
$\Rightarrow 30 \sin 30° + N - mg = 0$
or $N = (5)(10) - 30(1/2) = 35$ N
$\Sigma F_x = 0$
$\Rightarrow 30 \cos 30° - f = 0$
or $f = 30 \left(\frac{\sqrt{3}}{2}\right) = 15\sqrt{3}$ N



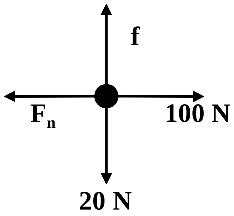
As there is a relative motion between the surfaces,
Hence $f = \mu_k N$, where μ_k = coefficient of kinetic friction.

$$\Rightarrow \mu_k = \frac{f}{N} = \frac{15\sqrt{3}}{35} = \frac{3\sqrt{3}}{7} = 0.74$$

Illustration 26. *A block of mass 2 kg is pressed against a rigid vertical wall by a horizontal force of 100 N. If co-efficient of static and kinetic friction are each equal to 0.3 then find the magnitude and direction of frictional force on the block.*



Solution: Since weight of block is 20 N which is acting downward, it has tendency to move the block downward. Hence, the frictional force will be upward. Maximum value of frictional force can be $f_{s(max)} = \mu F_n$



F.B.D. of the block

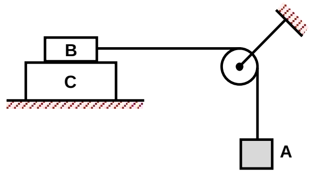
As block is in equilibrium along horizontal

$F_n = 100$ N

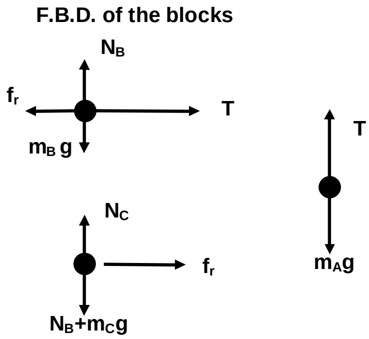
$\Rightarrow$ $f_{s(max)} = 0.3 \times 100 \text{ N} = 30 \text{ N}$

Since weight of the block is less than the limiting friction, it will not slide. Therefore, for vertical equilibrium f = 20 N.

Illustration 27. *In the figure shown co-efficient of friction between the block B and C is 0.4. There is no friction between the block C and the surface on which it is placed. The system of blocks is released from rest in the shown situation. Find the distance moved by the block C when block A descends through a distance 2 m. Given masses of the blocks are m_A = 3 kg, m_B = 5 kg and m_C = 10 kg.*



Solution: Let there is no relative motion between the blocks B and C



Hence

$T = (m_B + m_C)a$ . . . (1)

And

$m_Ag - T = m_Aa$ . . . (2)

From (1) and (2), we get

$$a = \frac{m_A g}{m_A + m_B + m_C} = \frac{30}{18} = \frac{5}{3} \text{ m/s}^2$$

$\Rightarrow$ Net force on the block C is, $F = m_Ca = 10 \times (5/3) \text{ N} = 16.6 \text{ N}$

If maximum value of frictional force acting on block C is f_{max} , then

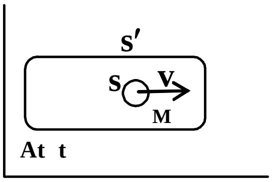
$f_{(max)} = \mu m_B g = 0.4 \times 5 \times 10 = 20 \text{ N}$ $\because F \le f_{max}$

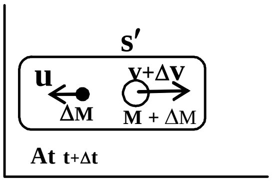
Hence there is no relative motion between the block B and C. Therefore, distance moved by C is 2 m only.

Lubrication: In some cases friction acts as a hindrance when there are moving parts in contact. A great amount of energy is lost in such type of machines like an automobile or pump or any motor. This energy converted to heat can damage the machines. So friction is reduced by suitable lubricants like oil grease, graphite etc.

Variable Mass System: Till now, we were discussing mass which remains constant with time. But if the mass changes with time continuously, then can we apply the conservation of momentum as we discussed earlier.

Let a body move continuously either by ejecting mass or absorbing mass. At any instant let the mass of the body be M and its velocity be $\vec{v}$ from a given inertial frame of reference. Suppose that the mass increases by ΔM and the velocity





increases by Δv in a time Δt. As the mass increases by ΔM, the remainder of the system has a mass $-\Delta M$ and moves with a speed u, relative to the inertial frame.

The change in momentum of the system s′ will be:

$$\Delta p = (M + \Delta M)(v + \Delta v) + (-\Delta M)u - Mv$$

To generalize, let us imagine an external force F_{ext} acting on the sub-system M.

The external force acting on the mass is

$$\lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{(M+\Delta M)\ (v+\Delta v) - \Delta M u - Mv}{\Delta t}$$

$$F_{ext} = M\frac{dv}{dt} + (v-u)\frac{dM}{dt} \quad \left[\because \frac{\Delta v \Delta M}{\Delta t} \text{ is very small}\right]$$

The relative velocity of the mass ΔM with respect to the sub-system is $(u - v) = v_{rel}$

$$\therefore F_{ext} = M\frac{Mdv}{dt} + v_{rel}\frac{dM}{dt}$$

Situations of variable mass

(i) Rocket Equation:

In distant space $F_{ext} = 0$ and $\frac{dv}{dt}$ is in positive direction, the direction of v_{rel} is negative. Now

$$M\frac{dv}{dt} = -v_{rel}\frac{dM}{dt}$$

$$\Rightarrow \quad dv = -v_{rel}\frac{dM}{M}$$

If the original mass of the rocket at t = 0 was M_0 and mass of the fuel burnt is m then

$$\int_{v_i}^{v_f} dv = -v_{rel}\int_{M_0}^{M_0-m}\frac{dM}{M} \quad \Rightarrow \quad v_f - v_i = -v_{rel}\ln\frac{M_0-m}{M_0}$$

$$M_f = M_0\, e^{-v_f/v_{rel}} \quad \text{where } M_f = M_0 - m \text{ and } v_i = 0$$

(ii) Rain drops accumulating in a moving rail road card is another example of variable mass situation.

(iii) A vertical chain falling on a fixed table also represents variable mass situation.

Exercise 9. ***A container filled with liquid has a hole on the side wall near bottom. Will momentum of the container and liquid system remain constant?***

Illustration 28. *A rocket of mass 40 kg has 360 kg of fuel. The exhaust velocity of the fuel is 2.0 km/sec. Calculate minimum rate of consumption of fuel so that the rocket may rise from the ground.*

Solution: We have relation $M\frac{dv}{dt} = F_{ext} + v_{rel}\frac{dM}{dt}$.

In the question, we have the force by the gravity as the external force. As the rocket is to lift from the ground the minimum acceleration of the rocket to do so is, o, beyond which, with a slight increase it will zoom into the sky.

Therefore, $\frac{dv}{dt} = 0$

Putting these into the equation, we obtain

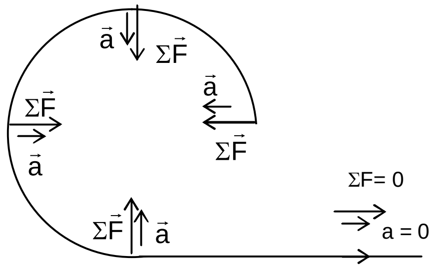
$$0 = Mg + v_{rel}\left(-\frac{dM}{dt}\right)$$

$\left(\frac{dM}{dt}\right)$ is the rate of decrease of mass hence taken negative. Mg is in the direction of the relative velocity of the fuel with respect to the rocket

Hence, $\frac{dM}{dt} = \frac{M}{v_{rel}}g = \frac{400 \times 9.8}{2 \times 1(10^3)} = 1.96\,\text{kg/sec}$

CIRCULAR MOTION

If a particle moves in a circular path at a constant speed, the velocity of the particle at any point of its path is directed along the tangent at that point. Due to continuous change in the direction of velocity, the particle has an acceleration. It is found that this acceleration is always directed radially inwards. It is known as radial or centripetal acceleration. So the particle is always acted on by a force directed radially inward, known as centripetal force.



Thus, centripetal force is defined as the force which acts towards the centre along the radius of a circular path on which a body is moving with a uniform speed.

Expression for the centripetal force:

(i) For uniform motion:

Acceleration towards centre

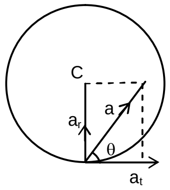
$a_r = r\omega^2 = \frac{v^2}{r}$ [$\because$ v = rω]. Hence centripetal force, F = $\frac{mv^2}{r}$

(ii) For non uniform motion:

In this case a body has two accelerations

(a) Radial acceleration : $a_r = \frac{v^2}{r}$, it occurs due to change in direction of the body and

(b) Tangential acceleration, $a_t = \frac{dv}{dt}$, it occurs due to change in speed of the body.



Hence the total acceleration, a = $\sqrt{a_r^2 + a_t^2}$

$\tan\theta = \frac{a_r}{a_t}$

Exercise 10. ***For uniform circular motion does the direction of the centripetal force depend upon the sense of revolution?***

Illustration 29. *A stone, tied to the end of a string 80 cm long, is whirled in a horizontal circle with a constant speed. If the stone makes 5 revolutions in 10 s, what is the magnitude and direction of acceleration of the stone ?*

Solution: Acceleration a = rω^2 = r $(2\pi n)^2$

$$= 0.8 \times 4 \times \pi^2 \times \left(\frac{5}{10}\right)^2 = 8\ \text{m/s}^2 \qquad \text{(Towards centre)}$$

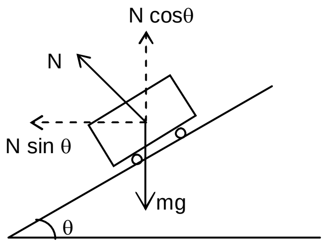
Vehicle moving round a circular path on horizontal road:
When a vehicle takes a turn on a road, it has a tendency to skid away from the centre of curvature of the road due to inertia. This tendency to skid brings into action a frictional force between the road and the tyres, directed towards the centre. This frictional force provides the necessary centripetal force.

If the maximum speed of a vehicle without skidding is v, then frictional force,

$$f = \mu mg = \frac{mv^2}{r}$$

$$\Rightarrow \quad v = \sqrt{\mu rg}$$

Banking of tracks:
Friction is not always sufficient to provide the required centripetal force. Moreover, it causes damage of the tyres. Friction can be avoided by banking the road at a suitable angle θ to the horizontal.



The horizontal component of the normal reaction N provides the centripetal force, i.e.

$$N \sin \theta = \frac{mv^2}{r} \quad \ldots \text{(i)}$$

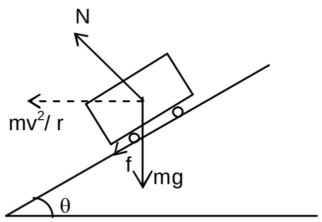
And the vertical component of normal reaction balances the weight, i.e.

$$N \cos \theta = mg \quad \ldots \text{(ii)}$$

From (i) and (ii)

$$\tan \theta = \frac{v^2}{rg}$$

If the above relation is not satisfied, then frictional force will come into play. It may act down the plane or up the plane depending on whether the vehicle has a tendency to skid outward or inward.



If the vehicle has a tendency to skid outward, then the component of frictional force and the normal reaction in the horizontal direction produces necessary centripetal force,

$$N \sin \theta + f \cos \theta = \frac{mv^2}{r} \quad \ldots \text{(i)}$$

And the component of frictional force and the normal reaction in upward direction balances the weight of the vehicle.

i.e. $N \cos \theta - f \sin \theta = mg \quad \ldots$ (ii)

From (i) and (ii),

$$\frac{v^2}{rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

We know frictional force, $f = \mu N$.

Hence, $\dfrac{v^2}{rg} = \dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$

$$\Rightarrow \frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

This is the maximum velocity of a vehicle without slipping. If $\mu = 0$, we get the optimum speed for least damage of tyres as

$$V = \sqrt{rg\tan\theta}$$

Illustration 30. *The road at a circular turn of radius 10 m is banked by an angle of 10^0. With what speed should a vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force?* *[tan 10° = 0.176]*

Solution: $\tan\theta = v^2 / rg$

or, $v = \sqrt{rg\tan\theta}$

$= \sqrt{(10)(9.8)\tan 10^0} = 4.2$ m/s

Illustration 31. *A cyclist is riding with a speed of 27 km/hr. As he approaches a circular turn on the road of radius 80 m. He applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude of the net acceleration of the cyclist?*

Solution: Tangential acceleration $= 0.5\,\text{m/s}^2$

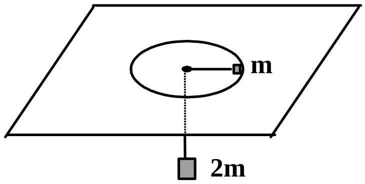
Centripetal acceleration $= \dfrac{v^2}{R} = \dfrac{\left(27\times\dfrac{5}{18}\right)^2}{80} = \dfrac{225}{320}$

$= 0.703\,\text{m/s}^2$

Net acceleration $\sqrt{a_T^2 + a_R^2}$

$= \sqrt{0.25 + 0.49} = \sqrt{0.74} = 0.86$ m/s^2

Illustration 32. *A mass m rotating freely in a horizontal circle of radius 1m on a frictionless smooth table supports a mass 2m in equilibrium attached to the other end of the string hanging vertically. If the instantaneous acceleration of the mass 2m is g m/s^2 vertically upward then angular velocity of rotation is*



(A) 5.78 rad/s *(B) 6.32 rad/s*
(C) 5.94 rad/s *(D) 6.11 rad/s*

Solution: Mass 2m is moving upward with an acceleration g

So, T – 2 mg = (2 m)g

$\Rightarrow$ T = 4 mg

Now for mass m, Tension will provide the necessary centripetal acceleration

4 mg = mRω^2

$\omega = 2\sqrt{g} = 6.32$ rad/sec.

So option (B) is correct.

SUMMARY

When a body is in equilibrium in an inertial frame of reference, the vector sum of forces acting on it must be zero. Free body diagrams are essential in identifying the forces that act on the body being considered.

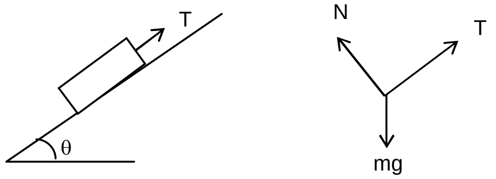
Newton's third law is also frequently needed in equilibrium problems. The forces in an action –reaction pair never act on the same body.

Vector form

$\Sigma\vec{F} = 0$

Component form

$\Sigma F_x = 0, \ \Sigma F_y = 0$

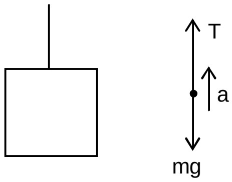


If the vector sum of forces on a body in not zero, the body accelerates. Its acceleration is given by Newton's 2nd law.

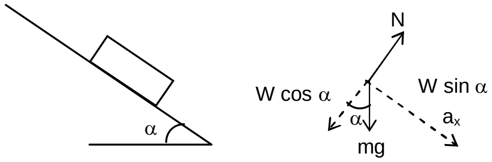
As they are for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law.

Vector form: $\Sigma\vec{F} = m\vec{a}$

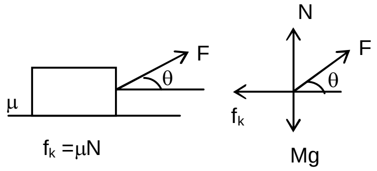
component form $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$



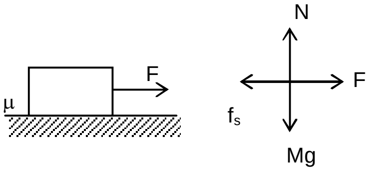
The contact force between two bodies can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force f parallel to the surface. The normal force exerted on a body by a surface is not always equal to the body's weight.



When a body is sliding over the surface, the friction force is called kinetic friction. Its magnitude f_k is approximately equal to the normal force magnitude N multiplied by the coefficient of kinetic friction μ_k.



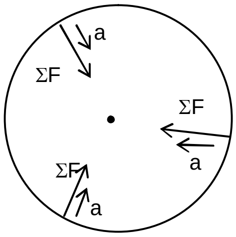
When a body is not moving relative to the surface, the friction force is called static friction. The maximum possible static friction force is approximately equal to the magnitude N of the normal force multiplied by the coefficient of static friction.



The actual static force may be anything from zero to this maximum value, depending on the situation usually μ_s is greater than μ_k for a given pair of surface in contact.

In uniform circular motion, the acceleration vector is directed towards the centre of the circle. Just as in any other dynamic problem, the motion is governed by Newton's second law $\Sigma\vec{F} = m\vec{a}$.

$a_{rad} = v^2/R$.



MISCELLANEOUS EXERCISE

1. If the net force acting on a body be zero, then will the body remain necessarily in rest position?

2. Can a body remain in rest position when external forces are acting on it?

3. The two ends of a spring-balance are pulled each by a force of 10 kg-wt. What will be the reading of the balance?

4. A force of 5 N changes the velocity of a body from 10 m/s to 20 m/s in 5 sec. How much force is required to bring out the same change in 2 sec.

5. An impulsive force of 100 N acts on a body for 1 s. What is the change in its linear momentum.

6. Two bodies of masses M and m are allowed to fall freely from the same height. If air resistance for each body is same, then will both the bodies reach the earth simultaneously?

7. When a ball is thrown upward, its momentum first decreases then increases. Is conservation of linear momentum violated in this process?

8. Four blocks of same mass m connected by cords are pulled by a force F on a smooth horizsontal surface, as shown in figure. Determine the tensions T_1, T_2 and T_3 in the cords.

SOLUTIONS TO MISCELLANEOUS EXERCISE

1. No, the body may be moving uniformly along a straight line.
2. Yes, if vector sum of all the forces acting on the body is zero.
3. The reading of balance will be 10 kg-wt.
4. From $F_1 = \frac{dp_1}{dt_1}$ and $F_2 = \frac{dp_2}{dt_2} \Rightarrow \frac{F_2}{F_1} = \frac{dt_1}{dt_2} \times \frac{dp_2}{dp_1}$

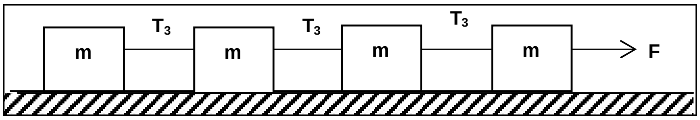
   Now, $dp_1 = dp_2$

   So, $\frac{F_2}{F_1} = \frac{dt_1}{dt_2} = \frac{5}{2} \Rightarrow F_2 = \frac{5}{2}F_1 = \frac{5\times5}{2} = 12.5\,\text{N}$
5. Change in linear momentum = Impulse = F× t = 100 N-s.
6. No, the net force on the body of mass M is (Mg — F). Therefore, its acceleration,

   $a = \frac{Mg - F}{M} = \left(g - \frac{F}{M}\right)$

   Thus, acceleration, a of a body of larger mass will be greater and it will appear lighter than before.
7. No, the momentum conservation principle is not violated. This is because vector sum of linear momentum of the ball and the earth remain constant.

8. Let a be the common acceleration of the whole system



   $\therefore$ F = 4 ma $\Rightarrow$ a = $\frac{F}{4m}$

   Applying Newton's 2^{nd} law for each blocks :

   $F - T_1 = ma \Rightarrow T_1 = \frac{3}{4}F$

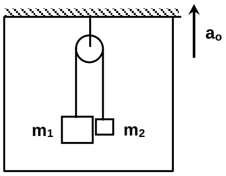
   $T_1 - T_2 = ma \Rightarrow T_2 = \frac{F}{2} \;\; \Rightarrow T_2 - T_3 = ma \Rightarrow T_3 = \frac{F}{4}$

SOLVED PROBLEMS

Subjective:

BOARD TYPE

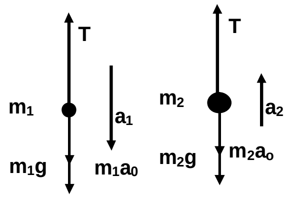
Prob 1. *A pulley fixed to the ceiling of an elevator car carries a thread whose ends are attached to the masses m_1 and m_2. The car starts going up with an acceleration a_o. Assuming the masses of the pulley and the thread as well as the friction to be negligible, find :*



(a) the acceleration of the load m_1 relative to the elevator shaft and relative to the car.
(b) the force exerted by the pulley on the ceiling of the car. Given, $m_1 > m_2$.

Sol. The elevator is an accelerated frame which is non-inertial, pseudo force $m_1a_0 < m_2a_0$ have been taken into consideration for m_1 and m_2 respectively.

F.B.D. of the blocks relative to elevator.



$m_1g + m_1a_o - T = m_1a_1$ . . . (i)
$T - m_2g - m_2a_o = m_2a_2$ . . . (ii)
$a_1 = a_2$ . . . (iii)

Solving the above equations , we get,

$$a_1 = \frac{(m_1 - m_2).(a_o + g)}{(m_1 + m_2)}$$

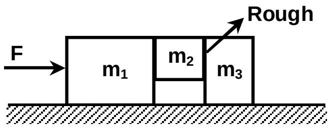
This is the acceleration of m_1 w.r.t car. Acceleration of the mass m_1 w.r.t the elevator shaft

$$= a_o - a_1 = \frac{2a_om_2 - g(m_1 - m_2)}{(m_1 + m_2)}$$

Force exerted by the pulley on the ceiling of the car

$$= 2.T = (m_1a_o + m_{\square}g - m_1a_1) \times 2 = \frac{4m_1.m_2(a_o + g)}{(m_1 + m_2)}.$$

Prob 2. *In the arrangement shown in the figure, the system of masses m_1, m_2 and m_3 is being pushed by a force F applied on m_1 horizontally, in order to prevent the downward slipping of m_2 between m_1 and m_3. If coefficient of friction between m_2 and m_3 is μ and all the other surfaces are smooth, What is the minimum value of force F?*

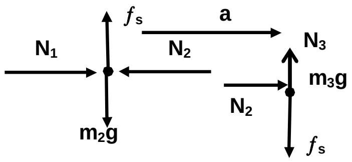


Sol. $f_s = m_2g$; $f_s \le \mu N_2$; $N_2 = m_3a$

$\therefore m_2g \le \mu m_3a$

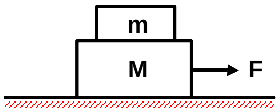
$$\Rightarrow a \ge \left(\frac{m_2g}{\mu m_3}\right)$$

$$F \ge (m_1 + m_2 + m_3)\frac{m_2g}{\mu \quad m_3}.$$



F.B.D of m_2 F.B.D of m_3

Prob 3. *A block of mass m is placed on another block of mass M lying on a smooth horizontal surface as shown in the figure. The co-efficient of friction between the blocks is μ. What maximum horizontal force F can be applied to the block M so that the blocks move together ?*



Sol. If there is no relative motion between the blocks then acceleration of the blocks is

$$a = \frac{F}{M+m}$$

For vertical equilibrium $N = mg$

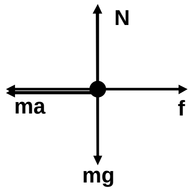
For horizontal equilibrium $f - ma = 0$

$$f \le \mu N$$

$\Rightarrow \quad \mu mg \ge m\dfrac{F}{(M+m)}$

$\Rightarrow \quad F \le (M + m)\mu g$

$\Rightarrow \quad F_{max} = (M + m)\,\mu g$



F.B.D. of m relative to M

Prob 4. *A mass of 200 kg. is placed on a rough inclined plane of angle 30°. If coefficient of limiting friction is* $\frac{1}{\sqrt{3}}$*, Find the greatest and the least forces in Newton, acting parallel to the plane to keep the mass in equilibrium .*

Sol. From the figure shown

$R = mg\cos\theta$

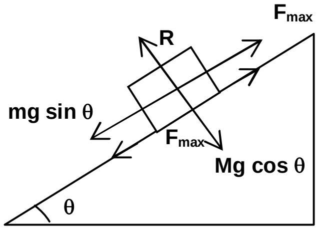
$F_{max} = \mu R = \mu\, mg\cos\theta$

So, the greatest force required to kept the body in equilibrium

$F_{max} = mg\sin\theta + f_{max}$

$= mg\sin\theta + \mu\, mg\cos\theta$

$$= 200\times 9.8\left[\frac{1}{\sqrt{3}}\cos 30^\circ\right] = 1960\text{ N}$$



From the figure (b)

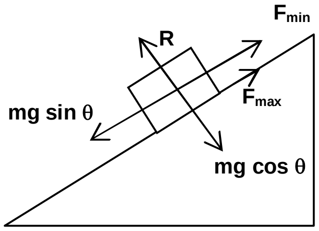
$F_{min} = mg\sin\theta - f_{max}$

$= mg\sin\theta - \mu\, mg\cos\theta$

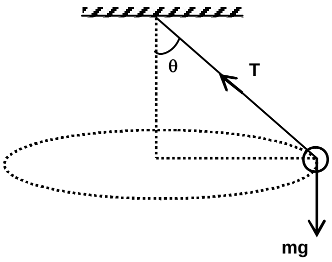
$= mg[\sin\theta - \mu\cos\theta]$

$$= 200\times 9.8\left[\sin 30^{O} - \frac{1}{\sqrt{3}}\cos 30^{O}\right]$$

= zero



Prob 5. *A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can with stand a maximum tension of 200 N?*



Sol. From figure

$T \cos \theta = mg$

$T \sin \theta = mr\,\omega^2$

So, $T = m\sqrt{g^2 + r^2\omega^4}$

Now $\omega = 40\,\text{rev/min} = 2\pi \times \frac{40}{60}\,\text{rad/sec}$

$= \frac{4\pi}{3}\,\text{rad/sec}$

So, Tension (T) $= \sqrt{g^2 + (r\omega^2)^2}) \times 0.25$

$= \left(\sqrt{100 + \left(1.5 \times \frac{16\pi^2}{9}\right)^2}\right)^{0.25}$

$= \left(\sqrt{100 + \left(\frac{64}{9}\right)\pi^4}\right) \times 0.25 = 7.0\,\text{N}$

So, maximum speed, by the given data

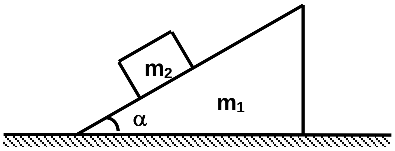
$T_{max} = 200$ N

So $200 = \sqrt{g^2 + r^2\omega_{max}^4})0.25 = 7.0\,\text{N}$

$\omega_{max} = 23.1$ rad/sec

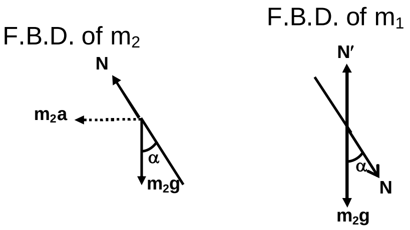
IITJEE TYPE

Prob 6. *A wedge 1 of mass m_1 and with angle α rests on a horizontal surface. Block 2 of mass m_2 is placed on the wedge. Assuming the friction to be negligible, find the acceleration of the wedge.*



Sol. Let us solve this problem by considering the motion of m_2 in non-inertial frame of wedge. In that frame the block is at rest along the normal to the inclined plane. Hence it is under equilibrium along the normal to the plane.

Due to the acceleration of the frame towards right pseudo force acts on the block towards left. As shown in the F.B.D.



$m_2 a \sin \alpha + N = m_2 g \cos \alpha$ ...(1)

and for m_1, $N \sin \alpha = m_1 a$ ...(2)

Multiplying (1) by $\sin \alpha$ and substituting (2) in it,

$m_2 \sin^2\alpha + m_1 a = m_2 g \sin \alpha \cos \alpha$

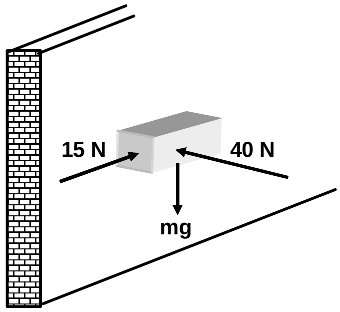
$\Rightarrow \quad a = \frac{g \sin \alpha . \cos \alpha}{\sin^2 \alpha + (m_1/m_2)}$.

Prob 7. *A block of mass 2 kg is pushed normally against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction and with what minimum acceleration ? If no, find the frictional force exerted by the wall on the block.*

Sol. The force which may cause the tendency of motion or motion in the body is its own weight and the applied horizontal force of 15 N. The resultant of the forces

$$F = \sqrt{20^2 + 15^2} = 25N$$

In a direction $\tan^{-1}\left(\frac{15}{20}\right) = 37°$ with the vertical.



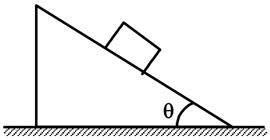
The friction will, by its very virtue of opposing the tendency of relative motion will act in a direction opposite to the resultant force. Now, for the acceleration to be minimum.

The minimum force required = F – μN (as, μN is the maximum frictional force)

$$= 25 - 0.5 \times 40 = 5 \text{ N}$$

$\therefore$ Minimum acceleration is $\frac{5}{2} = 2.5 \text{ m/s}^2$.

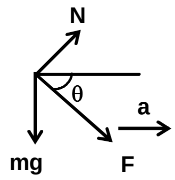
Prob 8. *A small block is resting on an inclined plane (coefficient of friction $\mu > \tan\theta$) as shown in the figure. The inclined plane is given a constant horizontal acceleration 'a' towards right.*



(a) Find the range of 'a' such that the block does not slide on the plane.

(b) Find the value of 'a' such that the friction force between the block and the plane is zero.

Sol. (a) As $\mu > \tan\theta$ the block does not slide when a = 0 which is the lower limit.



Newton's 2nd law

$$N - mg\cos\theta = ma\sin\theta \quad \ldots (1)$$

$$F + mg\sin\theta = ma\cos\theta \quad \ldots (2)$$

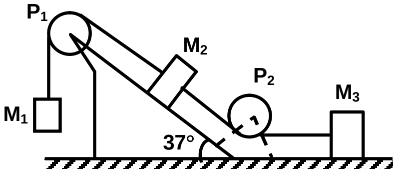
Force of friction $F \le \mu N$

$\therefore$ $a \le \frac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$ (no slide condition)

$\therefore$ the range of a is O to $\frac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$

(b) setting F = 0 we get $a = g\tan\theta$

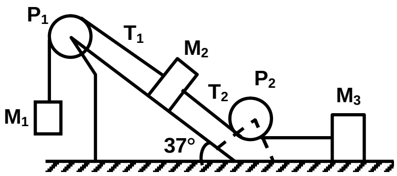
Prob 9. *Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal.*



The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity find the

(a) mass of M_1

(b) tension in the horizontal portion of the string. ($g = 9.8 \text{ m/s}^2$ and $\sin 37° \approx 3/5$)



Sol. Let T_1 be the tension in the string connecting M_1 and M_2 and T_2 be the tension in the string connecting M_2 and M_3. From the figure.

$M_1g = T_1$

$T_2 = \mu M_3 g = (0.25)4g$

or, $T_2 = g = 9.8$ N

Also, $T_1 = T_2 + (0.25) \times 4g\cos 37° + 4g\sin 37°$

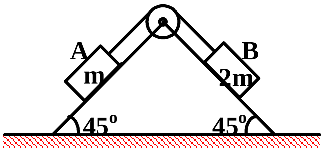
$= g\left(1+\frac{4}{5}+\frac{4\times 3}{5}\right)$

$T_1 = \frac{21}{5}g$

or, $M_1 g = \frac{21}{5}g$

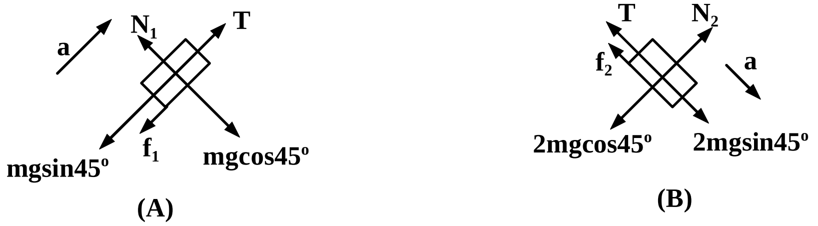
$\therefore$ $M_1 = \frac{21}{5} = 4.2$ kg

Prob10. *Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a light and inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3. If the system of A and B is released from rest, then find*
(a) the acceleration of A
(b) tension in the string
(c) the magnitude and direction of the frictional force acting on A.



Sol. (a) In the absence of friction the block B will move down the plane and the block A will move up the plane. Frictional force opposes this motion.

F.B.D. of the blocks.



$\Rightarrow$ $T - mg\sin 45° - f_1 = ma$ . . . (1)

and, $2mg\sin 45° - f_2 - T = 2ma$ . . . (2)

Adding (1) and (2), we get

$mg\sin 45° - (f_1 + f_2) = 3ma$

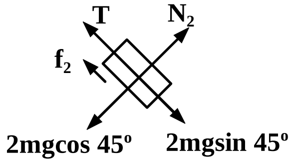
For a to be non-zero $mg\sin 45°$ must be greater than the maximum value of $(f_1 + f_2)$

$\because$ $(f_1 + f_2)_{max} = (\mu_1 m_1 + \mu_2 m_2)g\cos 45° = \frac{4}{3}mg\cos 45°$

$\Rightarrow$ $mg\sin 45° < (f_1 + f_2)_{max}$

Hence blocks will remain stationary

(b) F.B.D. of the block B



$f_{(2)max} = \frac{1}{3} 2mg\cos 45^o = \frac{2}{3\sqrt{2}} mg$ &

$2\, mg \sin 45° = \frac{2mg}{\sqrt{2}}$

$\because$ $2\, mg \sin 45° > f_{2(max)}$, therefore block B has tendency to slide down the plane.

For block B to be at rest

$T + f_{2(max)} = 2\, mg \sin 45°$

$\Rightarrow T = \frac{mg}{\sqrt{2}}\left(2 - \frac{2}{3}\right) = \frac{4mg}{3\sqrt{2}}$

$\Rightarrow T = \frac{2\sqrt{2}}{3} mg$

(c) $mg \cos 45° = \frac{mg}{\sqrt{2}}$

$\because$ T(tension) is greater than mg cos 45°.

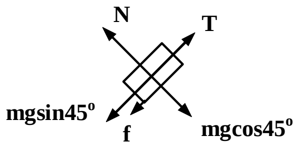
Hence block A has tendency to move up the plane, therefore frictional force on the block A will be down the plane.

For A to be at rest

$mg \sin 45^o + f = T$

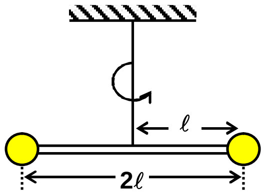
$\Rightarrow f = T - mg\sin 45^o$

$= \frac{2\sqrt{2}mg}{3} - \frac{mg}{\sqrt{2}} \Rightarrow f = \frac{mg}{3\sqrt{2}}$



F.B.D. of A

Prob 11. *Consider two small balls connected to a rigid rod of length '2ℓ' which in turn is suspended by a thread & rotated about the thread at angular velocity ω. What would be the magnitude & direction of total force exerted by rod on one of the balls?*



Sol. Consider the ball in ground's reference frame. It is observed to be doing uniform circular motion for which each ball must experience a resultant force of magnitude $m\omega^2 \ell$ directed towards centre of its circular path.

$\Rightarrow \quad \vec{R} + m\vec{g} = m\omega^2 \ell\, \hat{n}$ Where $\vec{R}$ is the force exerted by rod on one of the two balls.

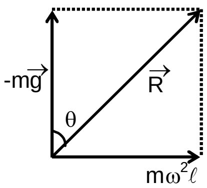
$\Rightarrow \quad \vec{R} = m\omega^2 \ell\, \hat{n} - m\vec{g}$

As $\hat{n}$ and $\vec{g}$ are mutually perpendicular

$$|\vec{R}| = \sqrt{(m\omega^2\ell)^2 + (mg)^2} = m\sqrt{(\omega^2\ell)^2 + g^2}$$

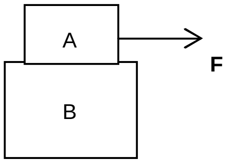
To decide direction of $\vec{R}$, consider the following vector diagram.

Thus, the angle that $\vec{R}$ makes with vertical to equals $\tan^{-1}\left(\frac{\omega^2\ell}{g}\right)$.



Prob 12. *A block A of mass 2 kg is placed on another blocks of mass 5 kg and a horizontal force F is applied on the block A . If co-efficient of friction between block A and B is 0.3 and between block B and the floor is frictionless, then what is the maximum value of F so both blocks will move together and what is the value of this acceleration?*

Sol. Suppose both blocks will move with common acceleration a, then

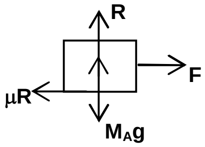


$F = (m_A + m_B)\, a$

$a = \frac{F}{2+5} = \frac{F}{7}$

Now for F.B.D. of block A.

$F - \mu R = m_A a$



$\Rightarrow \mu R = F - m_A a = F - \frac{2F}{7}$

$\Rightarrow \mu m_A g = \frac{5F}{7}$

$\Rightarrow = \frac{7 \times 0.3 \times 2 \times g}{5}$

$= 4.2 \times 2 = 8.4$ N and

$a = \frac{F}{7} = \frac{8.4}{7} = 1.2\ \text{m/s}^2$

Prob 13. *A car start from rest and accelerates uniformly with 2 m/s^2. At t = 10 s, a stone is dropped out of the window (1 m high) of the car. What are the (a) velocity and (b) acceleration of the stone at t = 10.1 sec? Neglect air resistance and take g = 9.8 m/s^2.*

Sol. At t = 10 s

Velocity of car = at = 20 m/s

(a) Horizontal component of stone V_s=20 m/s.

Now in vertical direction acceleration = g m/s^2

So at t = 10.1 sec.

$v_y = gt = 9.8 \times (10.1 - 10)$

$= 0.98$ m/s

So, resultant velocity of stone = $\sqrt{v_x^2 + v_y^2}$ = 20.02 m/s and

the angle of resultant velocity with horizontal direction is

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{0.98}{20}\right)$

$\theta = 2.8°$

(b) The moment of the stone is dropped out from the car, horizontal force on the stone =0. The only acceleration is due to gravity $a_y = g = 9.8$ m/s^2 (downwards)

Objective:

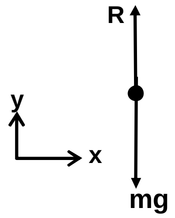
Prob 1. *An iron nail is dropped from a height h from the level of a sand bed. If it penetrates through a distance x in the sand before coming to rest, the average force exerted by the sand on the nail is,*

(A) $mg\left(\frac{h}{x}+1\right)$ (B) $mg\left(\frac{x}{h}+1\right)$

(C) $mg\left(\frac{h}{x}-1\right)$ (D) $mg\left(\frac{x}{h}-1\right)$

Sol. The nail hits the sand with a speed v_o after falling through a height h

$\Rightarrow v_0^2 = 2gh \Rightarrow v_0 = \sqrt{2gh}$ …(1)



The nail stops after sometime say t, penetrating through a distance, x into the sand. Since its velocity decreases gradually the sand exerts a retarding upward force, R (say). The net force acting on the nail is given as

$\Sigma F_y = R - mg = ma$

$\Rightarrow R = m(g + a)$ …(2)

Where a = deceleration of the nail. Since the nail penetrates a distance x

$0 - V_0^2 = -2a\,x$ …(3)

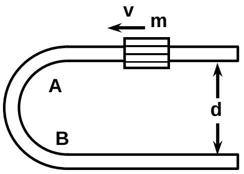
Putting V_0 from (1) and 'a' from (2) in (3), we obtain

$$2gh = 2\left(\frac{R-mg}{m}\right)x$$

$$\Rightarrow \quad R = \frac{mg(h+x)}{x}$$

$$\Rightarrow \quad R = mg\left(\frac{h}{x}+1\right)$$, Hence **(A)** is the correct choice.

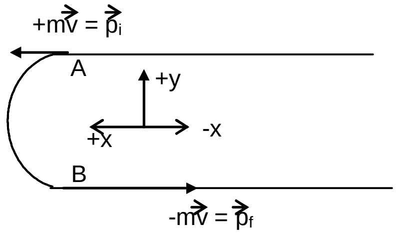
Prob 2. *A U shaped smooth wire has a semi-circular bending between A and B as shown in the figure. A bead of mass 'm' moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force exerted by the bead on the part AB of the wire is,*



(A) 0 (B) $\frac{4mv^2}{\pi d}$

(C) $\frac{2mv^2}{\pi d}$ (D) none of these.

Sol. Choosing the positive x-y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.



Therefore, the magnitude of the change in momentum between A and B is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive x-axis.

The time interval taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi . d/2}{v} = \frac{\pi d}{2v}.$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{\pi d}{2v}} = \frac{4mv^2}{\pi d}$$

Hence the correct choice is **(B)**.

Note:

1. By mistake, if the change in the magnitude of the momentum is considered, average force will be equal to zero.
2. If someone accounts carelessly d = r instead r = d/2 then he will lead to wrong choice (c).

Prob 3. *A man holds a ball of mass (1/2) kg in his hand. He throws it vertically upward. During this process his hand moves up by 40 cm and the ball leaves his hand with an upward velocity of 4 ms^{-1}. The constant force with which the man pushes the ball is*

(A) 2 N *(B) 10 N*
(C) 15 N *(D) 7 N*

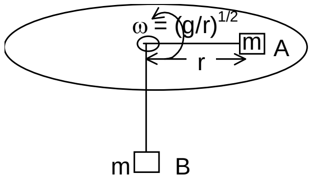
Sol. Acceleration of the ball

$$a = \frac{v^2}{2s} = \frac{4^2}{2\times 0.4} = 20\ m/s^2$$

Hence force applied by the man $= m(a+g) = \frac{1}{2}(20+10) = 15$ N

Hence the correct choice is **(C)**.

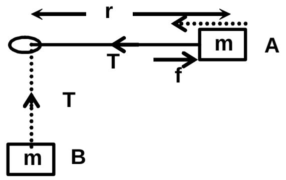
Prob 4. *Two particles A and B, each of mass m, are inter-connected by an inextensible string such that the particle B hangs below a table as shown in the figure and particle A is on a rough rotating disc at a distance r from the axis of rotation of the disc.*



If the angular speed of the disc and the block is $\omega = \sqrt{g/r}$, the frictional force developed at the interface of the particle & the disc is equal to

(A) mg/2 *(B) < mg/2*
(C) mg/√2 *(D) zero*

Sol. The particle of mass m experiences two forces (i) tension T (ii) frictional force f.



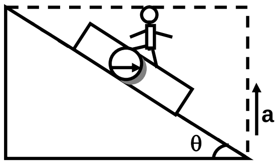
Since the particle A is rotating in a circular path of radius r, its centripetal acceleration,

$$\Rightarrow \quad r\,\omega^2 = \frac{T-f}{m}$$

Putting T = mg for equilibrium of the mass B & $\omega^2 = g/r$

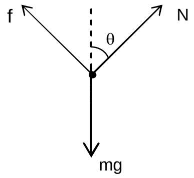
we obtain f = mg – mr g/r = 0

Prob 5. *A man of mass m = 60 kg is standing on weighing machine fixed on a triangular wedge of angle θ = 60° with horizontal as shown in the figure. The wedge is moving up with an upward acceleration a = 2 m/s^2. The weight registered by machine is*



(A) 600 N *(B) 1440 N*
(C) 360 N *(D) 240 N*

Sol. $N - mg\cos\theta = ma\cos\theta$
$N = m(g + a)\cos\theta = 60\,(10 + 2)\cos 60°$
$= 360$ N
Hence the correct choice is **(C).**



Prob 6. *A massive platform of mass M is moving with speed v=6 ms^{-1}. At t=0 a body of mass m (m << M) is gently placed on the platform. If coefficient of friction between body and platform is μ = 0.3 and g = 10 m/s^2, then*
(A) the body covers a distance 3 m on the platform in the direction of motion of the platform.
(B) the body covers a distance 3 m on the platform opposite to the direction of motion of platform before coming to rest.
(C) the body covers a distance of 6 m on the platform in the direction of motion of the platform.
(D) the body covers a distance of 6 m on the platform opposite to the direction of motion of platform before coming to rest.

Sol. Since M>>m, the velocity of M remains unchanged after m is placed on to it.

Acceleration of m, $a = \dfrac{\mu mg}{m} = \mu g$

$a_{mM} = a - 0 = a$ and initial relative velocity $v_{mM} = 0 - v = -v$

Hence $s = \dfrac{v^2}{2\mu g} = \dfrac{6^2}{2\times 0.3\times 10} = 6$ m. Hence the correct choice is **(D).**

Prob 7. *A body of mass m is kept on a rough horizontal surface of friction coefficient μ. A force is applied horizontally, but the body is not moving. The net force 'F' by the surface on the body will be*
(A) $F \le \mu\, mg$ *(B)* $F = \mu\, mg$
(C) $mg \le F \le mg\sqrt{1+\mu^2}$ *(D)* $mg \ge F \ge mg\sqrt{(1-\mu^2)}$

Sol. If the body is not moving, F = f, where f is the force of friction on the body and
$0 \le f \le \mu mg$ or $0 \le F \le \mu mg$ . . . (i)
The force by the surface on the body

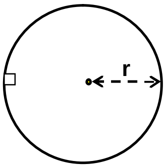
$R = \sqrt{f^2 + N^2} = \sqrt{F^2 + (mg)^2}$

or $R = mg\sqrt{\mu^2 + 1}$

$\therefore\ mg \le F \le mg\sqrt{1+\mu^2}$,

Hence the correct choice is **(C)**

Prob 8. *Consider a small cube of mass 'm' kept on a horizontal disc. If the disc is to rotate with uniform angular velocity, what could be its maximum value without causing any sliding between the cube and the disc? (Coefficient of static friction between cube & disc is μ).*



(A) $\sqrt{\frac{\mu g}{r}}$ (B) $\sqrt{\frac{2\mu g}{r}}$

(C) $\sqrt{\frac{\mu g}{2r}}$ (D) $2\sqrt{\frac{\mu g}{r}}$

Sol. In absence of any sliding, net force on the cube in the frame of reference rotating with disc will be zero. We find two forces in the plane of disc - frictional force and centrifugal force.

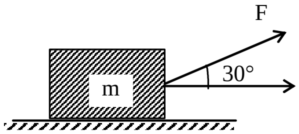
Hence, $m\omega^2 r = f$

but $f \le \mu mg$

Hence, $\omega \le \sqrt{\mu g / r} \Rightarrow \omega \le \sqrt{\mu g / r}$

$\Rightarrow \omega_{max} = \sqrt{\frac{\mu g}{r}}$, Hence **(A)** is the correct choice.

Prob 9. *A mass m rests on a horizontal surface. The coefficient of friction between the mass and the surface is μ. If the mass is pulled by a force F as shown in figure, the limiting friction between mass and the surface will be*



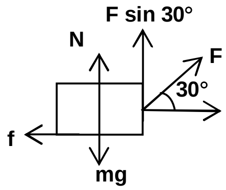
(A) μmg (B) $\mu(mg - \frac{\sqrt{3}}{2}F)$

(C) $\mu\,[mg - F/2]$ (D) $\mu\,(mg + F/2)$

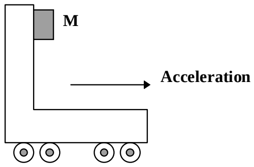
Sol. From F.B.D. $N = mg - F \sin 30°$

So Limiting friction $= \mu N = \mu\left(mg - \frac{F}{2}\right)$

So, option (C) is correct.



Prob 10. *An accelerated system with a vertical wall has co-efficient of friction μ between block and walls as shown in the figure. A block M of mass 1 kg just remains in equilibrium with the vertical wall, when the system has an acceleration of 20 m/s^2. The co-efficient of friction has a value*



(A) 0.10 (B) 0.25

(C) 0.50 (D) 1

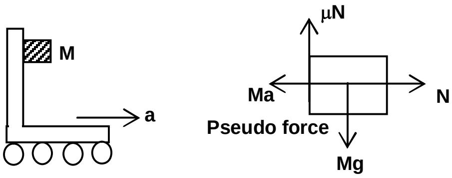
Sol. So, $N = Ma$

And if mass M is in equilibrium then

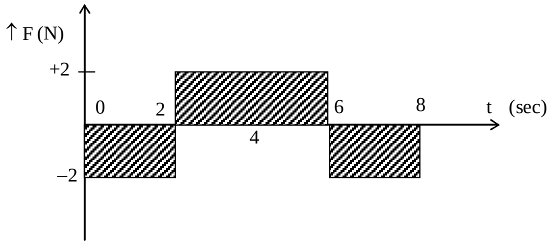
then, $\mu N - Mg = 0$

$$\mu = \frac{g}{a} = \frac{10}{20} = 0.5$$

So, option (C) is correct.

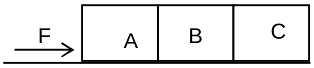


Prob 11. *Force time graph for the motion of a body of mass 2 kg is shown in figure. Change in velocity between 0 to 8 sec is*
(A) zero
(B) 4 m/s
(C) 8 m/s
(D) None of these



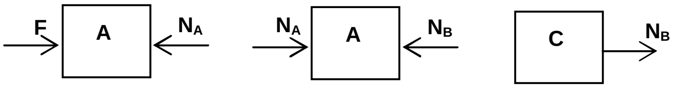
Sol. Area of graph = + [2 × (6 − 2)]− [2 × 2]−[2 × 2]
Change in momentum = $\int F.dt = 8 - 4 - 4 = 0$
Thus there is no change in velocity between 0 to 8 sec.
So option (A) is correct.

Prob 12. *A constant force F pushes three blocks A, B and C on a horizontal smooth surface. The masses of the blocks are m_A, m_B and m_C respectively. The normal reaction between the blocks B and C will be*



(A) $\dfrac{F(m_B + m_C)}{m_A + m_B + m_C}$ *(B)* $\dfrac{Fm_A}{(m_B + m_C)}$
(C) $\dfrac{Fm_C}{(m_B + m_C)}$ *(D)* $\dfrac{F.m_C}{(m_A + m_B + m_C)}$

Sol. F.B.D. of blocks



If the common acceleration of the block is a, then
F = (m_A + m_B + m_C)
So normal reaction between B and C is
$N_B = m_C a = \dfrac{F \cdot m_c}{(m_A + m_B + m_C)}$
So option (D) is correct.

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. In general the normal force is not equal to the weight. Give an example where the two forces are equal in magnitude and at least two examples where they are not.

2. A car is driven up a steep hill at constant speed. Discuss all the forces acting on the car. What pushes it up the hill?

3. If there is a net force on a particle in uniform circular motion, why does the particle's speed not change.

4. A curve in a road has the banking angle calculated for 80 km/h. However, the road is covered with ice, and you plan to creep around the highest lane at 20 km/h. What may happen to your car ? Why ?

5. A reference frame attached to the earth cannot be an inertial frame. Explain.

6. A person is sitting on a moving train and is facing the engine. He tosses up a coin which falls behind him. Find out the reason.

7. Is a 'single isolated force' possible in nature ?

8. Which of Newton's laws of motion is involved in rocket propulsion?

9. Can a body in linear motion be in equilibrium position?

10. Friction is a self – adjusting force. Is this statement correct ? If yes then justify.
    What is limiting friction ? What are the laws of limiting friction ?

11. A body moving over the surface of another body suddenly comes to rest. What happens to friction between the two surfaces ?

12. When walking on ice, one should take short steps rather than long steps. Why

13. Why does a cyclist bend inwards from his vertical position while taking a turn?

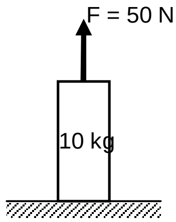
14. A stone tied to one end of a string is whirled in a circle. If the string breaks, the stone flies off tangentially. Explain.

15. Why is it easier to pull a body than to push it.

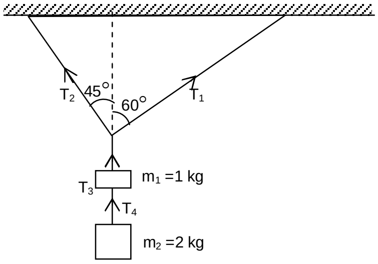
Level - I

1. A ball of mass 0.2 kg falls from a height of 45 m. On striking the ground, it rebounds in 0.1sec with two third of the velocity with which it struck the ground. Calculate
   (a) change in the momentum of the ball immediately after hitting the ground,
   (b) the average force on the ball due to the impact.

2. (a) Find the normal reaction between the block and the horizontal surface.

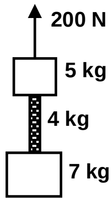


(b) Find out the tensions T_1, T_2 , T_3 and T_4
(Take g = 10 m/s^2)

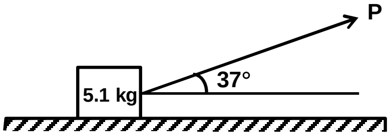


3. A body hangs from a spring balance supported from the roof of an elevator.
   (a) If the elevator has an upward acceleration of 2 m/s^2 and balance reads 240 N, what is the true weight of the body?
   (b) Under what circumstances will the balance read 160 N?
   (c) What will the balance read if the elevator cable breaks? (Take g = 10 ms^{-2})

4. Two blocks shown in figure are connected by a heavy uniform rope of mass 4 kg. An upward force of 200 N is applied as shown.
   (a) What is the acceleration of the system?
   (b) What is the tension at the top of heavy rope?
   (c) What is the tension at the mid-point of the rope?

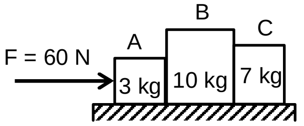


5. A 5.1 kg block is pulled along a frictionless floor by a cord that exerts a force P = 10 N at an angle θ = 37° above the horizontal, as shown in the figure.



   (a) What is the acceleration of the block?
   (b) The force P is slowly increased. What is the value of P just before the block breaks off the floor?
   (c) What is the acceleration of the block just before it is lifted off the floor?
   (d) Suppose the surfaces are rough with μ = 0.4, for what value of P the block just begins to move?

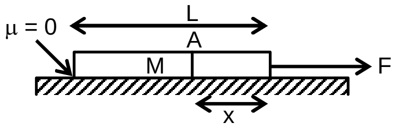
6. Find out the mutual contact forces between A and B and between blocks B and C.



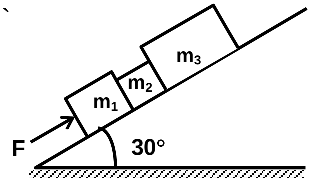
μ = 0 for all surfaces

7. A block weighing 100 kg is placed on an inclined plane of height 6 m and base 8 m. The co-efficient of friction is 0.3. ($g = 9.8\ m/s^2$)
   (a) Would the block slide down the inclined plane due to its own weight? If so, how far it will move in 1s starting from rest?
   (b) What force parallel to the inclined plane must be applied to just support the block on the plane?
   (c) What force parallel to the inclined plane is required to keep the block moving up the plane at constant velocity?
   (d) If an upward force of 940 N parallel to the inclined plane is applied to the block what will be its acceleration?
   (e) How far will the block move in 1s starting from rest?
   (f) What will happen if an upward force of 500 N parallel to the inclined plane is applied?
   (g) If an upward force of 260 N parallel to the inclined plane is applied what will happen? How far will the block move in 1s starting from rest?

8. Find the tension in rope at section A, at a distance x from the right end.

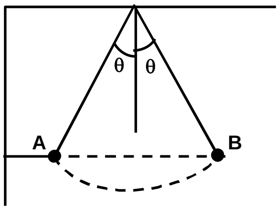


9. Three blocks $m_1 = 3$ kg, $m_2 = 2$ kg, $m_3 = 5$ kg, lie on an inclined frictionless surface as shown in the figure.
   (a) What force (F) parallel to the incline is needed to push the blocks up the plane with an acceleration $a = 2\ m/s^2$?
   (b) Find the contact force between m_1 & m_2 and m_2 and m_3.



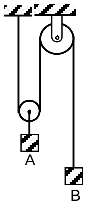
10. A child places a picnic basket on the outer rim of a merry go round that has a radius of 4.0 m and revolves once in every 24 s. What is the minimum co-efficient of static friction for the basket to stay on the merry go round?

11. A ball is held at rest in position A as shown in the figure by two light cords. The horizontal cord is cut and the ball swings as a pendulum. What is the ratio of the tensions in the supporting cord, in position A, to that in position B?

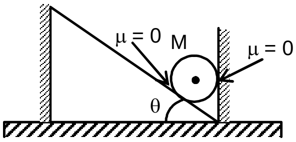


12. A balloon is descending with a constant acceleration 'a' less than the acceleration due to gravity. The mass of the balloon, with its basket and contents is M. What mass m, of ballast (Sand bags) should be released so that the balloon will begin to accelerate upward with constant acceleration 'a'? (Neglect air resistance)

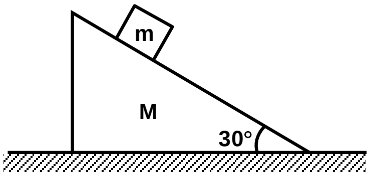
13. Two blocks 'A' and 'B' having masses m_A and m_B respectively are connected by an arrangement shown in the fig. Calculate the downward acceleration of the block B. Assume the pulleys to be massless. Under what condition the block A will have downward acceleration ?



14. Find the normal reaction forces acting on the vertical wall (say N_1) and the fixed incline surface (say N_2) respectively by the sphere of mass M.



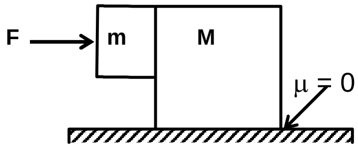
15. A block of mass m = 1 kg rests on a wedge of mass M = 9 kg, which in turn is placed on a table as shown in the figure. All the surfaces are smooth.
(a) What horizontal acceleration 'a' must M have relative to stationary table so that m remains stationary relative to the wedge?



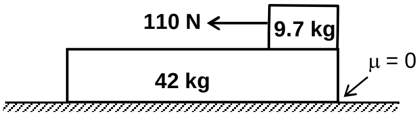
(b) Find the horizontal force required to maintain this acceleration.

Level- II

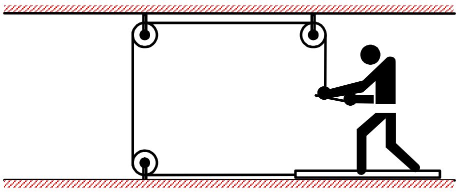
1. The two blocks m = 5 kg and M = 25 kg as shown in the figure are free to move. The coefficient of friction between the blocks is $\mu_s = 0.4$, but the coefficient of friction between ground and M is zero. What is the minimum horizontal force F required to hold m against M?



2. A 42 kg slab rests on a frictionless floor. A 9.7 kg block rests on the top of the slab as shown in the figure. The coefficient of static friction between the block and the slab is 0.53, while the coefficient of kinetic friction is 0.38. The 9.7 kg block is acted on by a horizontal force of 110 N. What are the resulting accelerations of
(a) the block? (b) the slab?

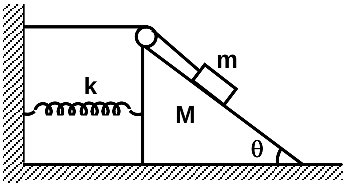


3. The friction coefficient between the board and the floor shown in figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor. The mass of man is M and the mass of plank is m.

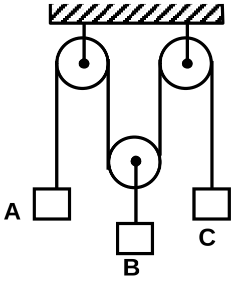


4. A car moves with constant tangential acceleration $a_T = 0.80$ m/s^2 along a horizontal surface circumscribing a circle of radius R = 40m. The coefficient of sliding friction between the wheels of the car and the surface is $\mu = 0.20$. What distance will the car ride without sliding if its initial velocity is zero ?

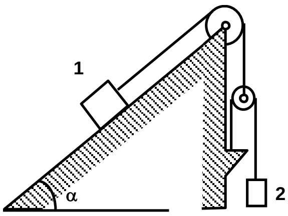
5. A wedge of mass 'M' and angle of inclination 'θ' and of mass 'm' is arranged in a manner shown in the figure. The spring of force constant 'k' attached to the wedge. Assuming the pulleys to be massless and all surfaces to be frictionless. Find the compression of the spring under equilibrium condition.



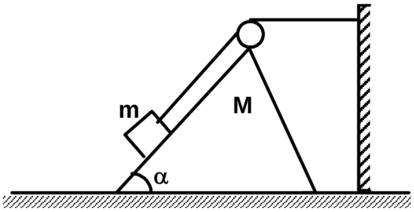
6. The pulley block system shown in the figure is released from rest. Assuming the pulleys to be light and frictionless and the string to be light and inextensible, find
(a) the acceleration of the blocks A, B and C.
(b) the tension in the string connecting the blocks. The masses of the block A, B and C are m, m and 2m respectively.



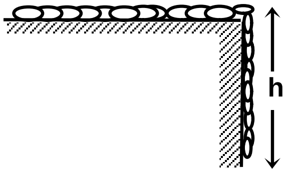
7. Find the acceleration a of body 2 in the arrangement shown in Fig. If its mass is n times as great as the mass of bar 1 and the angle that the inclined plane forms with the horizontal is equal to α. The masses of the pulleys and the threads, as well as the friction, are assumed to be negligible. Look into possible cases.



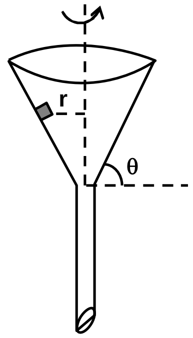
8. In the arrangement shown in the figure, the masses m of the bar and M of the wedge, as well as the wedge angle α, are known. The masses of the pulley and thread are negligible. Friction is absent. Find the acceleration of the wedge M.



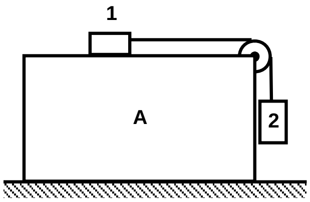
9. A uniform chain of length ℓ is released from rest on a smooth horizontal table with a portion h of the chain overhanging as shown in the figure. Find the time taken by the chain to slip off the table.



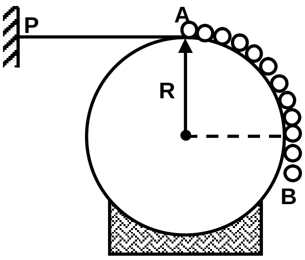
10. A small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of f revolutions per second. The wall of the funnel makes an angle θ with the horizontal. The coefficient of static friction between the cube and the funnel is μ and the centre of the cube is at a distance r from the axis of rotation find the
(a) largest and
(b) smallest value of f for which the cube will not move with respect to the funnel.



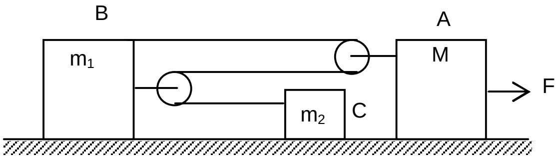
11. What is the minimum and maximum acceleration with which bar A (Fig.) should be shifted horizontally to keep bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal , and the co-efficient of friction between the bar and the bodies is equal to k. The masses of the pulley and the threads are negligible, the friction in the pulley is absent.



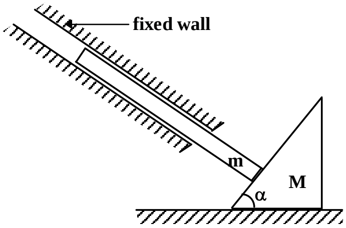
12. A uniform flexible chain of length 1.50 m rest on a fixed smooth sphere of radius $R = 2/\pi$ m such that one end A of chain is at top of the sphere while the other end B is hanging freely. Chain is held stationary by a horizontal thread PA as shown in figure.
Calculate the acceleration of the chain when the thread is burnt.



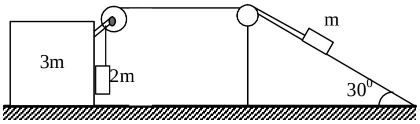
13. In the shown figure, the blocks and pulley are ideal and force of friction is absent. External horizontal force F is applied as shown in the figure. Find the acceleration of block C.



14. In the arrangement shown in the figure, the rod of mass m held by two smooth walls, always remains perpendicular to the surface of the wedge of mass M. Assuming that all the surfaces are frictionless, find the acceleration of the rod and that of the wedge.



15. Neglect friction. Find accelerations of m, 2m and 3m as shown in the figure. The wedge is fixed.



Objective:

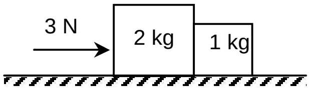
Level- I

1. When a bird of weight W sits on a stretched wire, the tension T in the wire is
   (A) > W/2 (B) = W
   (C) < W (D) None of these

2. A machine gun fires n bullets per second and the mass of each bullet is m. If v is the speed of each bullet then the force exerted on the machine gun is
   (A) mng (B) mnv
   (C) mnvg (D) mnv/g

3. A particle of mass 'm' moving with a velocity $\vec{v}$ strikes a horizontal surface at angle θ with the vertical and rebounds with the same magnitude and at the same angle with the vertical. The magnitude of change in momentum is
   (A) $2\,mv\cos\theta$ (B) $2\,m\,v\sin\theta$
   (C) 2 mv (D) 0

4. Two blocks of masses 2 kg and 1 kg are in contact with each other on a horizontal frictionless table. When a horizontal force of 3.0 N is applied to the block of mass 2 kg, the value of the force of contact between the two blocks is:



   (A) 4 N (B) 3 N
   (C) 2 N (D) 1 N

5. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force F is applied at one end of the rope, the force which the rope exerts on the block is:
   (A) $\frac{FM}{m+M}$ (B) $\frac{Fm}{m+M}$
   (C) $\frac{FM}{M-m}$ (D) $\frac{Fm}{M-m}$

6. Two masses *m* and *m′* are tied with a thread passing over a pulley, *m′* is on a frictionless horizontal surface and m is hanging freely. If acceleration due to gravity is g, the acceleration of *m′* in this arrangement will be
   (A) g (B) $\frac{mg}{(m+m')}$
   (C) $\frac{m'g}{(m+m')}$ (D) $\frac{g(m-m')}{(m+m')}$

7. Which of the following statements is true in a tug of war.
   (A) The team which applies a greater force on the rope than the other wins.
   (B) The team which applies a smaller force on the other wins.
   (C) The team which pushes harder against the ground wins.
   (D) none of these

8. Two students of equal weight try to break a rope which can break if the tension in it is equal to the sum of their weights
   (A) They should pull against each other applying a force equal to their weight on the rope.
   (B) They should hang the rope over a pulley and pull on either side of the pulley downwards.
   (C) They should climb on either side of the rope suspended over the pulley.
   (D) They should tie one-end of the rope to the ceiling and pull the other end together.

9. A massless rope passes over a frictionless pulley. A monkey holds one end of the rope and a mirror, having the same weight as the monkey, is attached to the other end of the rope at the same level. The monkey gets scared by its own image. It can get away from its image by
   (A) climbing up the rope. (B) moving down the rope.
   (C) by releasing the rope. (D) None of the above.

10. A chain of length L and mass M is hanging by fixing its upper end to a rigid support. The tension in the chain at a distance x from the rigid support is:
    (A) Zero (B) $\frac{MgL}{(L-x)}$
    (C) $Mg\frac{(L-x)}{L}$ (D) Mgx / L

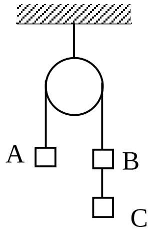
11. When a force of constant magnitude always act perpendicular to the motion of a particle then:
    (A) Velocity is constant (B) Acceleration is constant
    (C) KE is constant (D) None of these

12. A given object takes n times as much time to slide down a 45^o rough incline as it takes to slide down a perfectly smooth 45^o incline. The coefficient of kinetic friction between the object and the incline is given by
    (A) $\mu_k = \frac{1}{(1-n^2)}$ (B) $\mu_k = 1-\frac{1}{n^2}$
    (C) $\mu_k = \sqrt{\frac{1}{(1-n^2)}}$ (D) $\sqrt{1-\frac{1}{n^2}}$

13. Sand drops are falling gently at the rate of 2 kg/sec. on a conveyor belt moving horizontally with a velocity of 5 m/s. The extra force and extra power required to keep the belt moving will be respectively,
    (A) 10 N force and 20 watts (B) 10 N force and 50 watts.
    (C) 20 N force and 20 watts (D) 50 N force and 50 watts

14. A body of mass 60 kg is dragged with just enough force to start moving on a rough surface with coefficients of static and kinetic frictions 0.5 and 0.4 respectively. On applying the same force what is the acceleration ?
    (A) 0.98 m/s^2 (B) 9.8 m/s^2
    (C) 0.54 m/s^2 (D) 5.292 m/s^2

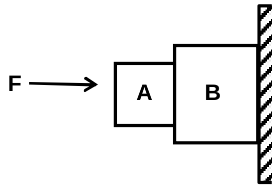
15. Three equal weights A, B, C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in figure. The tension in the string connecting weights B and C is



(A) Zero (B) 13 N
(C) 3.3 N (D) 19.6 N

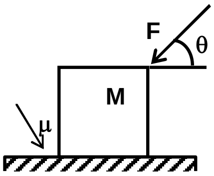
Level- II

1. Two blocks A and B are pushed against the wall with the force F. The wall is smooth but the surfaces in contact of A and B are rough. Which of the following is true for the system of the blocks?



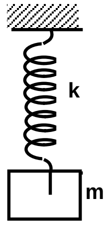
(A) F should be equal to weight of A and B.
(B) F should be less than the weight of A and B.
(C) F should be more than the weight of A and B
(D) The system cannot be in equilibrium (at rest).

2. A block of mass M lies on a rough surface of coefficient of friction μ. A force is applied on it at an angle θ to the horizontal as shown, and the block is at rest. The frictional force acting on the block will be



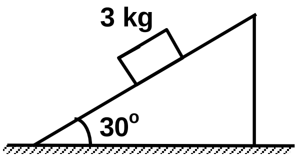
(A) $F \cos\theta$ (B) $\mu(mg + F\sin\theta)$
(C) $\mu(mg - F\sin\theta)$ (D) μmg

3. The spring mass system shown in the figure is in equilibrium. If the mass m is pulled down by a distance mg/3k and released, its instantaneous acceleration will be



(A) g/3 upward (B) 2g/3 downward
(C) g/3 downward (D) 2g/3 upward

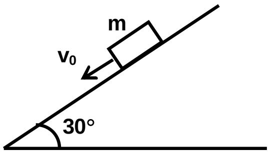
4. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be



(A) 26 N (B) 19.5 N
(C) 10 N (D) 30 N

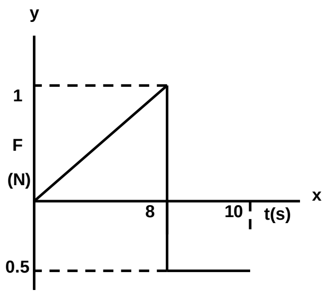
5. A body of mass 60 kg is dragged along a horizontal surface by a horizontal force which is just sufficient to start the motion of the body from rest. If the coefficients of static and kinetic friction are 0.5 and 0.4 respectively, the acceleration of the body is
(A) $0.98\ m/s^2$ (B) $9.8\ m/s^2$
(C) $0.54\ m/s^2$ (D) $5.292\ m/s^2$

6. A block of mass m is pushed down on a rough inclined plane (coefficient of friction is 0.25) with a velocity v_0 as shown in the figure. Then the block will

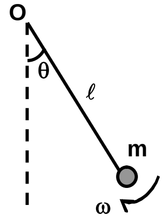


(A) decelerate and come to rest (B) accelerate downward
(C) move downward with velocity v_0 (D) first accelerate and then decelerate

7. A force time graph for the motion of a body is shown in the figure. The change in the momentum of the body between zero and 10 sec. is
(A) zero
(B) 4 kg m/s
(C) 5 kg m/s
(D) 3 kg m/s

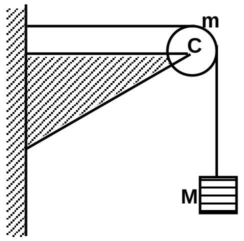


8. A simple pendulum swings in a vertical plane about the point of suspension O. In the position shown in the figure, the string has an angular velocity ω radian/second. The instantaneous tension in the string is



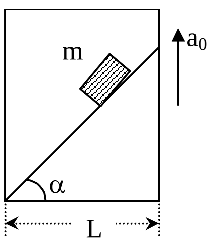
(A) mg cos θ
(B) $mg\left(\cos\theta + \frac{\ell\omega^2}{g}\right)$
(C) $mg\left(\cos\theta - \frac{\ell\omega^2}{g}\right)$
(D) $m\ell\omega^2$

9. A string of negligible mass, going over a clamped pulley of mass m, supports a block of mass M as shown in the figure. The force on the pulley by the clamp C is given by
(A) $\sqrt{2}Mg$
(B) $\sqrt{2}mg$
(C) $g\sqrt{(M+m)^2 + m^2}$
(D) $g\sqrt{(M+m)^2 + M^2}$

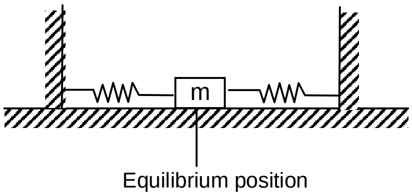


10. A long horizontal rod has a bead which can slide along its length, The bead is initially placed at a distance L from one end A of the rod. The rod is set in angular motion in the horizontal plane about the end A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping on the rod is
(A) $\sqrt{\frac{\mu}{\alpha}}$
(B) $\frac{\mu}{\sqrt{\alpha}}$
(C) $\frac{1}{\sqrt{\mu\alpha}}$
(D) infinite

11. A particle slides down a smooth inclined plane of elevation α fixed in the elevator going up with an acceleration a_0 as shown in figure. The base of the incline has a length L. The time taken by the particle to reach the bottom is
(A) $\left[\frac{2L}{(g+a_0)\sin\alpha\cos\alpha}\right]^{1/2}$
(B) $\left[\frac{2L}{g\sin\alpha\cos\alpha}\right]^{1/2}$
(C) $\left[\frac{g\sin\alpha\cos\alpha}{2L}\right]^{1/2}$
(D) $\left[\frac{2L}{a_0\sin\alpha\cos\alpha}\right]^{1/2}$



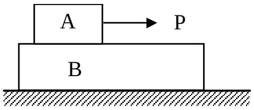
12. A body of mass 'm' is connected to two springs of spring constants K_1 and K_2 and is in equilibrium on a smooth horizontal surface as shown. If the body is displaced to the left by a small distance 'x' from the position shown, what is the velocity of the body as it passes through this position again? (springs are massless)

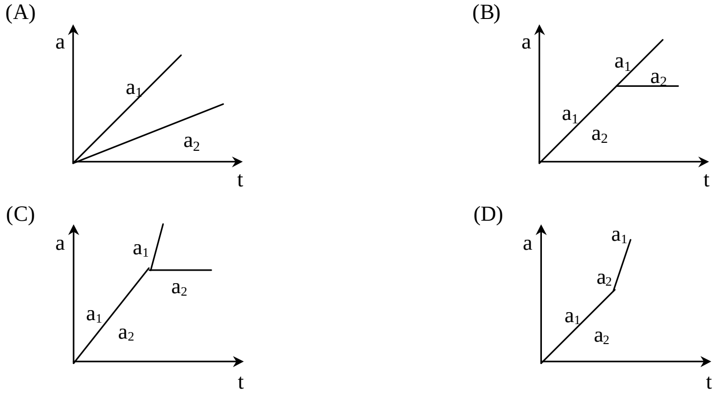


(A) 0
(B) $x\sqrt{\frac{K_1+K_2}{m}}$
(C) $(K_1 + K_2)x$
(D) can't say

13. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1m. The angle made by the rod with the track is
(A) Zero
(B) 30°
(C) 45°
(D) 60°

14. Block A is placed on block B, whose mass is greater than that of A. There is friction between the blocks, while the ground is smooth. A horizontal force P, linearly increasing with time, begins to act on A. The acceleration a_1 and a_2 of A and B respectively are plotted against time t. Choose the correct graph.





15. A block of mass M rest on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A force F = Mg acting at an angle θ with the vertical side of the block pulls it. In which of the following cases, the block can be pulled along the surface?
(A) $\tan \theta \geq \mu$
(B) $\cot \theta \geq \mu$
(C) $\tan \theta/2 \geq \mu$
(D) $\cot \theta/2 \geq \mu$

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – I

1. (a) 10 Ns (b) 100 N
2. (a) 50 N (b) $T_1 = \frac{60\,N}{1+\sqrt{3}}$, $T_2 = \frac{30\sqrt{6}}{1+\sqrt{3}}N$, $T_3 = 30$ N, $T_4 = 20$ N
3. (a) 200 N (b) When the elevator as a downward acceleration of 2 m/s^2(c) zero
4. 2.5 m/s^2, 137.5 N, 112.5 N
5. (a) 1.56 m/s^2 (b) 83.3 N (c) 13 m/s^2 (d) 19.6 N
6. $F_{AB} = 51$ N, $F_{BC} = 21$ N
7. (a) Yes, 176.4 cm; (b) 36 kg wt; (c) 84 kg wt; (d) 116.8 cm/s^2 upwards; (e) 58.4 cm; (f) remains at rest, (g) slides down with acceleration 92.8 cm/s^2, 46.4 cm
8. $F\left(\frac{L-x}{L}\right)$
9. (a) 70 N (b) 49 N, 35 N
10. 0.032
11. $\frac{T_A}{T_B} = \sec^2\theta$
12. $m = \frac{2\ Ma}{g+a}$
13. $2g\,(2m_B - m_A) / (4m_B + m_A)$; when $m_A > 2m_B$
14. $N_1 = Mg\tan\theta$, $N_2 = Mg\sec\theta$
15. (a) 5.658 m/s^2 rigtward (b) 56.58 N

Level – II

1. 150 N
2. (a) 7.6 m/s^2 leftward (b) 0.86 m/s^2 leftward
3. $\frac{\mu(M+m)g}{(1+\mu)}$
4. 45.82 m
5. $\frac{mg\sin\theta}{k}$
6. (a) $a_A = \frac{3g}{11}$ (downward), $a_B = \frac{5g}{11}$ (upward), $a_C = \frac{7g}{11}$ (downward)

   (b) $T_A = T_C = \frac{8mg}{11}$, $T_B = \frac{16mg}{11}$
7. $\frac{2g(2n-\sin\alpha)}{(4n+1)}$

8. $\dfrac{mg\ \ \sin\alpha}{M+2m(1-\cos\alpha)}$

9. $\left\{\sqrt{\dfrac{\ell}{g}}\ln\left[\dfrac{\ell}{h}+\sqrt{\left(\dfrac{\ell}{h}\right)^2-1}\right]\right\}$ sec.

10. (a) $\dfrac{1}{2\pi}\sqrt{\dfrac{g(\tan\theta+\mu\ )}{r(1-\mu\tan\theta)}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{g(\tan\theta-\mu\ )}{r(1+\mu\tan\theta)}}$

11. $a_{min} = g(1-k)/(1\ +\ k)$, $a_{max} = \left(\dfrac{1+k}{1-k}\right)g$

12. $\dfrac{4+\pi}{3\pi}g$

13. $\dfrac{2m_1F}{m(m_1+9m_2)+4m_1m_2}$

14. $\dfrac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}, \dfrac{mg\cos\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}$

15. $\dfrac{13g}{34}, \sqrt{\dfrac{397}{34}}g, \dfrac{3g}{17}$

Objective:

Level - I

1.	A	2.	B	3.	A
4.	D	5.	A	6.	B
7.	C	8.	D	9.	D
10.	C	11.	C	12.	B
13.	B	14.	A	15.	D

Level - II

1.	D	2.	A	3.	A
4.	D	5.	A	6.	B
7.	D	8.	B	9.	D
10.	A	11.	A	12.	B
13.	C	14.	C	15.	D

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Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

PHYSICAL WORLD AND MEASUREMENT

Physical World and Measurement

Syllabus:
Scope and excitement of physics, Technology & society. Forces in nature, Conservation laws, Examples of gravitation, electromagnetic and nuclear forces from daily life experiences (qualitative description only). Need of measurement, Units of measurement, System of units, SI units, Fundamental and derived unit; Length, mass and time measurement, Accuracy and precision of measuring instrument, errors in measurement, significant figures. Dimensions of physical quantities, dimensional analysis and its application.

Have you ever observed the nature and the various spectacular events like formation of rainbow on any rainy day?

Whenever we observe nature keenly, we can easily understand that the various events in nature like blowing of wind, flow of water, motion of planets, formation of rainbow, different forms of energies, the function of human bodies, animals, etc. are happening or taking place according to some basic laws. The systematic study of these laws of nature governing the observed events is called science. For our convenience, clear understanding and systematic study of Science is classified into various branches. Among these branches Chemistry, Mathematics, Botany, Zoology, etc. are ancient branches and Bio–technology, Bio–chemistry, Bio–Physics, Computer science, Space Science, etc. are considered to be modern branches of science and engineering. One of such ancient and reputed branches of this science is physics.

SCOPE AND EXCITEMENT OF PHYSICS

The domain of physics consists of wide variety and large number of natural phenomena. Hence, the scope of physics is very vast and obviously the excitement that one gets from the careful study of physics has got no boundaries.

Scope of Physics

For example, when we study one of the basic physical quantities called mass, we come across the values ranging from minute masses like mass of an electron (of the order of 10^{-30} kg) to heavy masses like mass of universe (10^{55} kg). Similarly, in case of other basic quantities like length and time also the range is very wide.

Hence, the scope of physics can be understood easily, only when we can classify the study of physics chiefly into three levels. They are:

(a) Macroscopic level study of physics,
(b) Mesoscopic level study of physics, and
(c) Microscopic level study of physics.

Macroscopic level study of physics: Macroscopic level study of physics mainly includes the study of basic laws of nature and several natural phenomena like gravitational force of attraction between any two bodies in the universe (in mechanics), variation of quantities like pressure, volume, temperature, etc. of gases on their thermal expansion or contraction (in thermodynamics), etc.

Microscopic level study of physics: The microscopic level study of physics deals with constitution and structure of matter at the level of atoms or nuclei. For example, interaction between elementary particles like electrons, protons and other particles , etc.

Mesoscopic level study of physics: The mesoscopic level study of physics deals with the intermediate domain of macroscopic and microscopic, where we study various physical phenomena of atoms in bulk.

So, the edifice of physics is beautiful and one can appreciate the subject as and when one pursues the same seriously.

Excitement of Physics

The study of physics is exciting in many ways as it explains us the reason behind several interesting features like (a) ***how day and nights are formed?*** (b) ***how different climatic conditions are formed in different seasons?*** (c) ***how satellite works and helps in using several devices like television, telephones, etc.?*** (d) ***how an astronaut travels to celestial space ?*** (e) ***how we can convert one form of energy to another ?*** (f) ***how different types of forces are governing different types of motion in universe ? etc***.

It is quite common and simple that every human being on the earth will be interested to know the answers for at least few of the above questions. As physics is the subject which answers them, naturally the study of physics will be exciting.

TECHNOLOGY AND SOCIETY

Physics is almost an integral part of upgradation of technology. Technology was also a branch of science where we study the application of principles of physics for practical purposes. Based on laws and principles of physics, technocrats along with scientists develop technically advanced equipment to help the society.

For example, from the principles of thermodynamics James watt invented steam engine which was responsible for a big industrial revolution in England in the 18^{th} century. Another recent example is invention of mobile phones which are creating revolution in wireless communication technology. Yet another important example is invention of micro–processors by using silicon chips which has replaced valve technology and brought the computers from the size of your study room to the size of your geometry box. These are few examples. There are many more areas where physics is involved in upgrading technology and thereby helping the society. The following table gives us a list of various branches of physics that helped the field of technology.

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Nuclear fission
Radio and Television	Propagation of electromagnetic waves
Computers	Digital logic

Lasers	Light amplification by stimulated emission of radiation (population inversion)
Production of ultra–high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electric energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Air conditioners / Refrigerators	Laws of thermodynamics
Washing machines, centrifuge, etc.	Centrifugal force
Sonar	Reflection of ultrasonic waves

The following table lists the involvement of various renowned physicists all across the world, who helped the society with their noble inventions.

Name	**Major Contribution / Discovery**	**Country of origin**
Isaac Newton	Universal law of gravitation: Laws of motion; reflecting telescope.	**U. K.**
Galileo Galilei	Law of inertia	**Italy**
Archimedes	Principle of buoyancy; principle of the lever	**Greece**
James Clerk Maxwell	Electromagnetic theory; light an electromagnetic wave	**U. K.**
W. K. Roentgen	x– rays	**Germany**
Marie Sklodowska Curie	Discovery of radium and polonium; Studies on natural radioactivity	**Poland**
Albert Einstein	Law of photo–electricity; Theory of relativity	**Germany**
S. N. Bose	Quantum statistics	**India**
James Chadwick	Neutron	**U.K.**
Niels Bohr	Quantum model of hydrogen atom	**Denmark**
Ernest Rutherford	Nuclear model of atom	**New Zealand**
C.V. Raman	Inelastic scattering of light by molecules	**India**
Christiaan Huygens	Wave theory of light	**Holland**
Michael Faraday	Laws of electromagnetic induction	**U.K.**
Edwin Hubble	Expanding universe	**U.S.A.**
Homi Jehangir Bhabha	Cascade process in cosmic radiation	**India**
Abdus Salam	Unification of weak and electromagnetic interactions	**Pakistan**
R. A. Millikan	Measurement of electronic charge	**U.S.A**
Ernest Orlando Lawrence	Cyclotron	**U.S.A.**
Wolfgang Pauli	Quantum Exclusion Principle	**Austria**
Louis victor de Broglie	Wave nature of matter	**France**

J.J. Thomson	Electron	**U.K.**
S. Chandrasekhar	Chandrasekhar limit, structure and evolution of stars	**India**
Lev Devidovich Landau	Theory of condensed matter; liquid helium	**Russia**
Heinrich Rudolf Hertz	Electromagnetic waves	**Germany**
Victor Francis Hess	Cosmic radiation	**Austria**
M. N. Saha	Thermal ionisation	**India**
G. N. Ramachandran	Triple helical structure of proteins	**India**
Thomas Alwa Edison	Electric bulb, Projector	**US**
Graham Bell	Telephone	**US**
Cavendish	Determination of 'G'	**England**
Robert Boyle	Boyle's law	**England**

So, to put it in a nut shell, science, technology and society are inseparable as they are deeply interwined.

FUNDAMENTAL FORCES IN NATURE

Force is a very common word which we normally come across in our daily life. We need force to push or pull or throw a body. Even we need it to deform or break the bodies. Sometimes, we experience force like when we are standing in a great storm, we experience the force exerted by wind. When we are sitting in a bus which is negotiating a turn, we experience an outward push. So, what is this force? Let us try to understand the concept of force in terms of physics.

At macroscopic level study of physics, we normally encounter different kinds of forces like gravitational force, muscular force, frictional force, contact force, spring force, buoyant force, viscous force, pressure force, force due to surface tension, electrostatic force, magnetic force, etc. whereas at microscopic level of study we come across nuclear forces, interatomic forces, intermolecular forces, weak forces, etc.

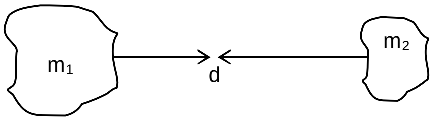
After analysing these various types of forces in nature, it was concluded that all the forces can be comfortably classified into four categories, which are known as fundamental forces in nature. They are

(1) Gravitational force (2) Electromagnetic force,
(3) Nuclear force, and (4) Weak force.

That means, any force other than the above four forces can be derived from these four basic forces. For example, elastic force or spring force arises due to the net attraction or repulsion between any two neighboring atoms of the spring. When it is elongated or compressed, attractive or repulsive forces produced between the atoms can be treated as the resultant of all electromagnetic forces between charged particles of an atom. Hence, this spring force is known as derived force and electromagnetic force which is the origin of this spring force is called fundamental force. Now, we will study about fundamental forces in brief.

Gravitational Force

Newton discovered that any two bodies in universe attract each other. This force of attraction exists by virtue of their masses, and is known as gravitational force of attraction. He found that the gravitational force is directly proportional to their masses



and is inversely proportional to the square of the distance between them.

i.e. $F = G\dfrac{m_1 m_2}{d^2}$ where 'G' is a Universal Gravitational Constant. This force is a universal force and is independent of any type of intervening medium between the two bodies. Though this is the weakest force in nature when compared to other types of fundamental forces, it plays vital role in governing the motion of planets around sun, natural satellites (like moon around earth), artificial satellites, etc.

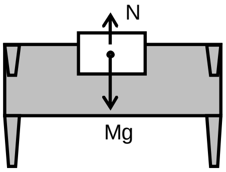
Electromagnetic Force:

The force of attraction or repulsion between any two charged particles is known as electrostatic force. If q_1 and q_2 charges are separated by a distance 'd' in air then the force of attraction or repulsion between them is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{d^2}$. This is called Coulomb's law of electric forces.

Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. In general electric and magnetic effects are inseparable and hence the name – electromagnetic force. This electromagnetic force between moving charged particles is comparatively more complicated and contains several other terms other than Coulomb's force.

In atoms electromagnetic force between electrons and protons is responsible for several molecular and atomic phenomena. Apart from this it also plays vital role in the dynamics of chemical reactions, mechanical and thermal properties of materials, tension in ropes, friction, normal force, spring force, Vander Waals force .

Example: Let us consider a block which is placed on a horizontal surface of a table as shown in the figure. The table balances the weight (Mg) and exerts a force which comes from electromagnetic force between charged constituents of atoms or molecules of surface of block and that of the table. Thus a force called normal force acts on block.



This electromagnetic force is a strong force when compared to the gravitational force. The electromagnetic force between two protons is 10^{36} times the gravitational force between them for any fixed distance.

Nuclear Force

We know that, in general, nucleus of every atom consists of two elementary particles called protons and neutrons. As neutrons are uncharged and protons are charged, the electric force of repulsion between protons will cause nucleus to break into fragments. But this is not happening, and also we know that nucleus of a non–radioactive element is a stable one.

That means there must be some other attractive force which is dominating coulombic force of repulsion between protons and keeping all the particles in nucleus together in stable condition as gravitational force can't dominate electric force. That new force existing between any two nucleons and which keeps all the particles in nucleus bound together is known as nuclear force. This force is stronger than electromagnetic force and is a charge independent force. Range of these forces is very small and will be of the order of nuclear size (10^{-21} th portion of size of an atom).

Latest developments in physics revealed that this strong nuclear force is also not a fundamental force as protons and neutrons consist of still elementary particles called quarks. And according to this latest development quark – quark force is fundamental force of nature and nuclear force is a derived force. However the study of quark – quark force is out of the scope of this book and our curriculum.

Weak Nuclear Force

This force appears only in certain nuclear processes. A neutron can change itself into a proton by emitting an electron and another elementary particle called antineutrino simultaneously. This process is called β^- decay. Similarly a proton can also change into neutron by emitting positron and a neutrino. This process is called β^+ decay. The forces which are responsible for these changes are known as weak forces. These forces are weak in nature when compared to nuclear and electromagnetic forces but stronger than gravitational forces. The range of these weak nuclear forces is exceedingly small, of the order of 10^{-15}m.

The following table gives us an overall idea about relative strengths and ranges of four fundamental forces.

Name	**Relative strength**	**Range**	**Operates among**
Gravitational force	10^{-38}	Infinite	**All objects in the universe**
Weak nuclear force	10^{-13}	Very short, within nuclear size ($\sim 10^{-15}$)	**Elementary particles**
Electromagnetic force	10^{-2}	Infinite	**Charged particles**
Strong nuclear force	1	Very short, within nuclear size ($\sim 10^{-15}$)	**Nucleons**

CONSERVATION LAWS

In any physical phenomena, few physical quantities associated with the phenomena may change with time and few physical quantities associated with it may not change. Those physical quantities which remain constant in time are known as conserved quantities.

For example, if a big liquid drop is sprayed into several small droplets the volume of liquid before spraying and after spraying remains same. Hence, we can say that a physical quantity called volume is conserved in this example. Similarly, we have several quantities which are conserved. Within the scope of our course, we can discuss the following conservation laws.

1. Law of conservation of linear momentum
2. Law of conservation of energy
3. Law of conservation of angular momentum
4. Law of conservation of charge.

Let us discuss them in brief.

Law of conservation of linear momentum

The linear momentum of a body is defined as the ability of a body by virtue of which it imparts its motion to other objects along a straight line. And mathematically it is equal to the product of mass of the body (m) and its velocity ($\vec{v}$) Mathematically, $\vec{P} = m.\vec{v}$.

According to this law, in absence of an external force, the total vector sum of linear momentum remains unchanged.

Example: When a bullet is fired with a gun, the total momentum vector of the system of bullet and gun is zero. After firing, bullet moves in forward direction with some momentum and gun recoils with the same amount of momentum in magnitude, but opposite in direction. Hence total vector sum of momentum after firing is also zero. Thus linear momentum of the system before and after firing is zero. Hence we can say that linear momentum is conserved.

Law of conservation of energy

According to this law the total energy of an isolated system is always constant and it never changes. But it can be transformed from one form to another. *For example* an electric cell in our daily life gives electrical energy by transforming chemical energy in it, electric motor converts electrical energy to mechanical energy, etc. However the total energy in these processes is conserved.

When an object is dropped from a certain height the total mechanical energy of the body is conserved. At its highest point all its mechanical energy will be in the form of potential energy and at its lowest point it will be in the form of kinetic energy, i.e. energy has transformed from one form into another, (i.e. potential to kinetic) but the total energy remains constant. Hence the total mechanical energy is conserved.

But this conservation of mechanical energy can't be applied in the presence of non – conservative force. For example in the above case if you consider air resistance on the freely falling body total mechanical energy does not remain constant. Here work done by air resistance gets converted into different forms of energy like heat energy. So such while applying nergy conservation principle heat energy should also be taken into consideration in such cases

Law of conservation of angular momentum

Angular momentum ($\vec{L}$) of a body about a point is defined as the cross product of its position vector about that point ($\vec{r}$) and its linear momentum at that instant ($\vec{p}$)

i.e. $\vec{L} = \vec{r} \times \vec{p}$
or $L = rp \sin\theta$ where θ is the angle between ' $\vec{r}$ ' and ' $\vec{p}$ '.

According to this law the total angular momentum of the system remains conserved in absence of external torque.

Example: We know that planets revolves around sun in elliptical orbits. The angular momentum of a planet at any point during its motion in its path is conserved. We will study more clearly about this under rotatory motion concepts.

These are the few conservation laws in mechanics. Now let us discuss a conservation law in electrostatics.

Law of conservation of charge

This law states that the total electric charge of an isolated system is always conserved. Charge can neither be created nor destroyed, but it can be transferred or exchanged from one body to another.

Apart from these, there are several other physical quantities that are conserved in nature. During our further discussions in various chapters we will understand them.

MEASUREMENT AND UNITS

Physical quantity: Any meaningful term which can be measured is a physical quantity. For example length, velocity, time etc. are physical quantity. But handsomeness, beauty are not physical quantity.

Why measurement is needed?: Physics is an experimental science and experiments involve measurement of different physical quantities in which laws of physics are expressed. Without measuring results of experiments, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Units of measurement: To measure a physical quantity we need some standard unit of that quantity. For example, if a measurement of length is quoted as 5 meters, it means that the measured length is 5 times as long as the value accepted for a standard length defined to be **"one meter".**

Any set of standards of units must fulfill the following two conditions
(i) It must be accessible.
(ii) It must be invariable with the passage of time

Two more auxiliary conditions are:–
(i) It is necessary to have wide unlimited agreement about those standards.
(ii) It is inter convertible to different units of same quantity.

A measurement consists of two parts, one is numeric and the other is standard chosen. For example, 5 meter of length implies 5 times the **"standard meter"**. It is not necessary to establish a measurement standard for every physical quantity. Some quantities can be regarded as fundamental and the standard for other quantities can be derived from the fundamental ones. For example, in mechanics length, mass and time are regarded as fundamental quantities and the standard for speed (= length / time) can be derived from fundamental quantities length and time.

Quantity	SI Units	Symbols
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of Substance	mole	mol
Thermodynamic Temp.	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	Cd

And two supplementary units are

Plane Angle	Radian	rad
Solid Angle	Steradian	sr

Two other system of units compete with the international system. One is Gaussian System in terms of which much of the literature of physics is expressed. In India this system is not in use.
The other is the British system. This system is still in daily use in United states. But SI units are standard units worldwide.

C.G.S. Unit: In this system of unit, centimeter, gram and seconds are units of length, mass and time respectively.

Conversion of One System of Units to another System: The basic formula is $n_1u_1 = n_2u_2$ where n_1 and n_2 are numbers.

Illustration 1. *How many dyne–centimeter are equal to 1 N–m?*

Solution:
$$1\,N - m = (1kg)(1m)^2(1s)^{-2}$$
$$1\,dyne - centimeter = (1g)(1cm)^2(1\ s)^{-2}$$
$$\therefore \frac{1\,N - m}{1\,dyne - cm} = \left(\frac{1000g}{1g}\right)\left(\frac{100cm}{1cm}\right)^2$$
$$= 1000 \times 10000$$
$$\therefore 1\,N - m = 10^7\ dyne - cm$$

Exercise: ***Calculate the value of 1 erg in SI system.***

Measurement of Length

Depending upon the range of length, there are three main methods for measuring length.
(i) Direct method using measuring instruments.
(ii) Indirect method or Mathematical method
(iii) Chemical method

(i) Direct method

The simplest method measuring the length of a straight line is by means of a meter scale. But there exist some limitations in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier calliper (b) micrometer scale (screw gauge) are used .

Vernier calliper

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being slightly shorter than the divisions of the main scale.

Least count of Vernier Calliper

The least count or the Vernier constant (V.B.) is the minimum value of correct estimation of length without eye estimation. The difference between the values of one main scale division and one vernier scale division is known as vernier constant if N division of vernier scale coincides with (N−1) divisions of main scale, then vernier constant,

$$\text{n.V.S.D.} = (\text{n}-1)\ \text{M.S.D.}$$

$$1.\text{V.S.D.} = \left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D., and}$$

$$1.\text{M.S.D.} - 1.\text{V.S.D.} = 1.\text{M.S.D.}\ \left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D.}$$

$$= \frac{1}{\text{n}}\ \text{M.S.D.}$$

$$= \frac{\text{Smallest Division on main scale}}{\text{No. of divisions on vernier scale}}$$

Reading a Vernier scale

Let one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions

$$\therefore\ 1.\text{V.S.D.} = \frac{9}{10}\ \text{M.S.D.} = 0.9\ \text{mm}$$

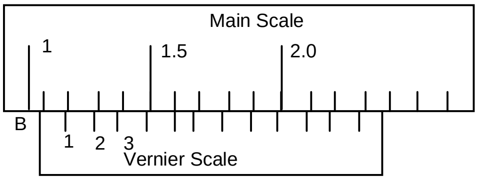
$$\therefore\ \text{Vernier constant} = 1.\text{M.S.D} - 1.\text{V.S.D.} = 1\ \text{mm} - 0.9\ \text{mm}$$
$$= 0.1\ \text{mm} = 0.01\ \text{cm}$$

The reading with vernier scale is read as given below :

(i) Firstly take the main scale reading (N) before on the left of the zero of the vernier scale.
(ii) Find the number (n) of vernier division which just coincides with any of the main scale division. Multiply this number (n) with vernier constant (V.C.)
(iii) Total reading = (N + n × V.C.)

Caution: The main scale reading with which the Vernier scale division coincides has no connection with reading
Suppose If we have to measure a length AB, the end A is coincided with the zero of the vernier scale as shown in fig. Its enlarged view is given in fig.

Length AB > 1.0 cm < 1.1. cm



Let 5th division of vernier scale coincide with 1.6 cm of main scale. From diagram it is clear that the distance between 4th division of vernier scale and 1.5 cm of main scale is equal to one V.C. and distance between zero mark of vernier scale and 1.0 cm mark on the main scale is equal to 5 times the vernier constant.

$$\therefore\quad AB = 1.0 + 5 \times \text{v.c.} = 1.0 + 5 \times 0.01 = 1.05\ \text{cm.}$$

Illustration 2. *In travelling microscope the vernier scale used has the following data.*
1 M.S.D. = 0.5 mm, 50 V.S.D. = 49 M.S.D.
and the actual reading for distance travelled by travelling microscope is 2.4 cm with 8th division coinciding with a main scale graduation . Estimate the distance travelled.

Solution : In this case vernier constant = 1.M.S.D. – 1.V.S.D.

$$= 1.\text{M.S.D.} - \frac{49}{50}\text{M.S.D.} = \frac{1}{50}\text{M.S.D} = \frac{1}{50} \times 0.5\,\text{mm}$$

$= \frac{5}{10} \times \frac{1}{50} = 0.01\text{mm} = 0.001\text{cm}$

$\therefore$ Distance travelled = 2.4 + 8 × 0.001 cm
= 2.408 cm

Illustration 3. *The Vernier scale used in Fortin's barometer has 20 divisions coinciding with the 19 main scale divisions. If the height of the mercury level measured is 5 mm and 15^{th} division of vernier scale is coinciding with the main scale division. Then calculate the exact height of the mercury level (given that 1.M.S.D. = 1mm)*

Solution: 20 V.S.D. = 19 M.S.D. (Given)

1.V.S.D. = $\frac{19}{20}$ M.S.D.

V.C. = 1. M.S.D. – 1.V.S.D $= \left(1 - \frac{19}{20}\right)$M.S.D.

$= \frac{1}{20}$ M.S.D.

$= \frac{1}{20} \times 1\text{mm} = 0.05\text{mm}$

= 0.005 cm

Height of mercury level = 5 + 0.05 × 15
= 5.75 mm

Exercise: ***The Vernier calliper is used to measure the length of an object. The least count of such a vernier calliper is 0.2 cm and scale reads its length to be 5.6 cm. 3^{rd} division of Vernier scale is coinciding main scale division Calculate the length of an object.***

Zero Error

If the zero marking of main scale and Vernier scale do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument. If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

Illustration 4. *Consider the following data:*
10 main scale divisions = 1cm, 10 vernier division = 9 main scale divisions,
zero of Vernier scale is to the right of the zero marking of the main scale with 6^{th} Vernier division coinciding with a main scale division and the actual reading for length measurement is 4.3 cm with 2^{nd} Vernier divisions coinciding with a main scale graduation. Estimate the length.

Solution: In this case, vernier constant = $\frac{1\text{mm}}{10} = 0.1\text{mm}$

Zero error = 6 × 0.1 = + 0.6 mm

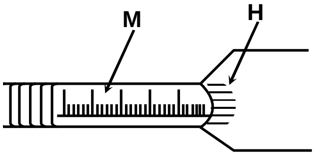
Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction
= 4.32 – 0.06 = 4.26 cm

Screw Gauge (or Micrometer Screw)

In general Vernier Callipers can measure accurately upto 0.02 mm and for greater accuracy micrometer screw devices, e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by turning it axially. The instrument is provided with two scales:

(i) The main scale or pitch scale M graduated along the axis of the screw.
(ii) The cap–scale or head scale H round the edge of the screw head.



Constants of the screw gauge:

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus if 10 rotations of cap ≡5 mm, then pitch = 0.5 mm

In general, pitch $= \dfrac{\text{Distance travelled by screw on main scale}}{\text{No. of rotation taken by the cap to travel that much distance}}$

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration, if the total cap division is 100, then least count = 0.5 mm/100

= 0.005 mm
In general, In case of circular scale,

Least count $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

If pitch is 1 mm and there are 100 divisions on circular scale, then

Least count = $= \dfrac{1\text{mm}}{100}$ = 0.01 mm = 0.001 cm

= 0.00001 m = 10^{-5} m = 10 μm

Since least count is of the order of 10 μm, So the screw is called a micrometer screw. Screw gauge and the spherometer which work on the principle of micrometer screw, consist essentially of the following two scales.

(i) Linear or Pitch scale : It is a scale running parallel to the axis of the screw.

(ii) Circular of Head scale: It is marked on the circumference of the circular disc or the cap attached to the screw.

Zero Error: In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero–error, otherwise the instrument is said to heave zero–error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and vice–versa. The corresponding corrections will be just opposite.

Illustration 5. *A screw gauge has 100 divisions on its circular scale. Circular scale travels one division on linear scale in one rotation and 10 divisions on linear scale of screw gauge is equal to 5 mm. What is the least count of a screw gauge.*

Solution: Pitch $= \dfrac{1\,\text{division on linear scale}}{1\,\text{rotation}} = 1\,\text{div.}$

10 division = 5 mm
∴ 1 division = 0.5 mm
∴ pitch = 0.5 mm

$\text{least count} = \dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$= \frac{0.5\,\text{mm}}{100} = 0.005\,\text{mm}$$

Illustration 6. *The screw gauge mentioned in above illustration is used to measure thickness of a coin. The reading of the linear scale is 4^{th} div and 25^{th} division of circular scale is coinciding with it. What is the value of thickness of the coin..*

Solution: Reading = Linear scale Reading + Least count × circular scale reading
= 4^{th} division on linear scale + 0.005 mm × 2.5
= 4 × 0.5 mm + 0.125 mm
= 2 mm + 0.125 mm
= 2.125 mm

Illustration 7. *A spherometer has 250 equal divisions marked along the periphery of its disc and one full rotation of the disc advances it on the main scale by 0.0625 cm. The least count of the spherometer is*

(A) 2.5×10^{-2}cm *(B) 25×10^{-3}cm*
(C) 2.5×10^{-4} cm *(D) none of the above*

Solution: Least count = $\frac{0.0625}{250}$ cm = 2.5×10^{-4}cm
$\therefore$(C)

(ii) Indirect or Mathematical method

This method involves measurement of long distances. Main methods of this category are –

Reflection method: Suppose we want to measure the distance of a multi story building from a destination point P. If a shot be fired from P, the sound of shot travels a distance x towards the building, gets reflected from the building. The reflected sound travels the distance x to the point of P, when an echo of the shot is heard.

Let t = time interval between the firing of the shot and echo sound.
v = velocity of sound in air.

Distance = velocity x time

x + x = (v) (t)

$\Rightarrow$ x = (v) (t/2)

As v is known, x can be calculated by measuring the time t.

Illustration 8. *A rock is at the bottom of a very deep river. An ultrasonic signal is sent towards rock and received back after reflection from rock in 4 seconds. If the velocity of ultrasonic wave in water is 1.45 km/s, find the depth of river.*

Solution: Here x = ?

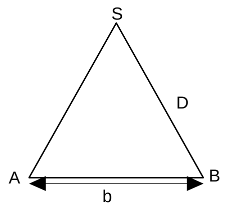
v = 1.45 km/s = 1450 m/sec.

t = 4 sec

so, x = v x t / 2 = 1450 x 4 / 2 = 2900 m.

Parallel method: This method is used for measuring distance of nearby stars.

Let we have to measure the distance D of a far away star S by this method. We observe this star from two different position A and B on the earth, separated by a distance AB = b at the same time as shown in figure. Let $\angle ASB = \theta$, the angle θ is called parallatic angle. As the star is very far away, b/D << 1 and θ is very small.



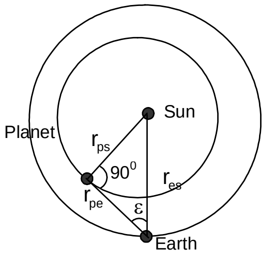
Here we can take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS=BS so that AB = b = Dθ where θ is in radians.

$D = b/\theta$

Knowing b and measuring θ, we can calculate D.

Copernicus method: This method is used to measure the relative distances of the planets from the Sun.

(a) For Interior Planets: The angle formed at earth between the earth–planet direction and the earth–sun direction is called the planet's elongation. This is the angular distance of the planet from the sun as observed from earth. When the elongation attains its maximum value ε as in the figure, the planet appears farthest from Sun.



$$r_{ps} = r_{es}\sin\varepsilon$$
$$= (\sin\varepsilon)\,\text{AU} \quad (\text{AU = Astronomical Unit})$$

(b) For Exterior Planets: This method is a consequence of Kepler's 3rd law of planetary motion. For two planets P_1 and P_2 we have,

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

where a_1 and a_2 are semi–major axes, of respective orbits. Period can be ascertained by direct observation. Therefore if a_1 is measured, a_2 can be calculated.

(iii) Chemical Method

This method is used to measure distance of the order of $10^{-1\circ}$ m. Let us calculate the size of an atom.

Let m = mass of substance,

V = volume occupied by substance &

ρ = density of the substance

$\therefore v = m/\rho$ (1)

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ No. of atoms in mass m of the substance = Nm / M

If r = radius of each atom then V = volume of each atom = $\frac{4}{3}\pi r^3$

Volume of all the atoms in substance = $(\frac{4}{3}\pi r^3 \text{ x Nm})M$.

According to Avagordo's hypothesis,

Volume of all the atoms = (2/3) x volume of substance

$$\frac{4}{3}\pi r^3 \text{ x Nm/M} = (2/3)\, m/\rho$$

$$\therefore r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$

MEASUREMENT OF MASS

Measurement of Inertial Mass

Inertial mass of a body is measured using a device which is known as inertial balance. It consists of a long metal strip. One end of the strip is clamped to a table such that its flat face is vertical, and it can easily vibrate horizontally. The other end of strip supports a pan in which the object whose inertial mass is to be found can be kept. It is found that the square of time period of vibration is directly proportional to total mass of the pan and the body placed in it.

$$t^2 \propto m$$

$$\therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \quad m_2 = m_1 \frac{t_2^2}{t_1^2}$$

Measurement of Time: The following methods are used

(a) Quartz Crystal Clock
(b) Atomic Clock
(c) Radioactive dating

Significant figures:

Each measurement involves errors. The measure results has a number that includes all reliably known digits and first unknown digit. The combination of reliable digits and first uncertain digit are significant figures.

Example: If a length is measured as 2.43 cm then 2 and 4 are reliable while 3 is uncertain. Thus the measured value has three significant figures.

Common rules for counting significant figures

(1) All non zero digits are significant.

For example: 1745 has four significant digits.

(2) All zeros present between 2 non zero digits are significant, irrespective of the position of the decimal point.

Example: 208005 has 6 significant figures.

(3) If there is no decimal point, all zeros to the right of the right–most non zero digit are considered to be significant only if they come from a measurement.

Example: 41000 has only 2 significant digits while 41000 m has 5 significant digits.

(4) All zeros to the right of a decimal point but to the left of non–zero digits are considered to be non significant, provided there should be no non zero digit to the left of the decimal point.

Example: 0.00305 has 3 significant figures.

(5) All zeros are significant if they are placed to the right of a decimal point and to the right of a non zero digit.

Example: 0.04080 has 4 significant figures

50.000 has 5 significant figures

(6)The number of significant figures does not alter in different units.
If we want to write 450 m in different units, we can write it 4.50×10^4 cm or 4.50×10^5 mm etc. in which all of them are having 3 significant figures.

Illustration 9. *State the number of significant figures in the following –*
(a) 06500310
(b) 754400
(c) 15000 kg
(d) $8.314\times10^{+2}$ J
(e) 1.6×10^{-19} C
(f) 0.0065050

Solution: (a) 7 (b) 4 (c) 5 (d) 4 (e) 2 (f) 5

Exercise: ***The number of significant figures in 0.0160 is***
(a) 2 ***(b) 3*** ***(c) 4*** ***(d) 5***

Rounding off

(1) If all the digits to be discarded are such that the first discarded digit is less than 5, the remaining digits are left unchanged.

Example:

7.499498 can be written in 4 significant figures as 7.499

(2) If the digit to be discarded is 5 followed by digits other than zero, then the preceding digit should be raised by 1.

Example:

7.45001, on being rounded off to first decimal, became 7.5

(3) If the digits to be discarded is 5 or 5 followed by zero the preceding digit remains unchanged if it is even and the preceding digit is raised by 1 if it is odd.

Example:

3.6500 will become 3.6 and 4.7500 will become 4.8 in 2 significant figures.

Arithmetic operations with significant figures:

(1) Addition and subtraction In addition and subtraction, the number of decimal places in the result is the smallest number of decimal places of terms in the operation.

Let us consider the sum of following measurements.
3.45 kg., 7.6 kg. and 10.055 kg.

$$\begin{array}{r} 3.45 \\ 7.6 \\ 10.055 \\ \hline 21.105 \\ \hline \end{array}$$

So the sum will be 21.1 kg as 7.6 kg has only 1 digit after the decimal point while the others are having more than one digit.

Multiplication and Division:

In the result of multiplication or division, the number of significant figures is same as the smallest number of significant figures among the numbers.

Illustration 9: *Multiply 1.21 and 1.1.*

Solution: $1.21 \times 1.1 = 1.331$

So the result is 1.3 as there are only 2 significant digits in 1.1

The same procedure is followed for division.

Exercise: Value of 1.2 + 1.34 + 2.342 is
(a) 4.88 (b) 4.8 (c) 4.90 (d) 5

Accuracy and Precision of measuring instrument: It is impossible to measure any physical quantity perfectly. It is due to imperfection in manufacturing and working of measuring instruments.

Accuracy: It is the degree of correctness of the measured quantity, i.e. how much close the result is to the true value of the physical quantity.

Precision: It is the degree of repeatability & refinement of a measurement.

ERRORS IN MEASUREMENT

In the experiment we may get some other value than that of the true value due to faulty equipment, carelessness or random causes. This will cause error in measurement.

There are 3 ways to express an error

(1) Absolute Error: It is the positive value of difference between the true value and measured value of the quantity. Since we don't know the correct value of quantity the best possible value can be given by mean value of all the measured value.

Arithmetic mean v, $A_m = \dfrac{A_1 + A_2 + ...A_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} A_i$

$\therefore$ The absolute error in the measurement can be given as.

$\Delta A_1 = |A_m - A_1|$ where A_m : Mean value of the measurements.

$\Delta A_2 = |A_m - A_2|$ where A_1, A_2 : Measured value of quantity.

$\Delta A_n = |A_m - A_n|$

Taking the arithmetic mean of all the absolute errors we get the mean absolute error $\Delta A_{m.}$.

$$\Delta A_m = \frac{\Delta A_1 + \Delta A_2 + ... + \Delta A_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \Delta A_i$$

So the true value of A will be such that

$$\left(A_m - \Delta A_m\right) \le A \le \left(A_m + \Delta A_m\right)$$

(2) Relative Error: It is defined as the ratio of the mean absolute error to the mean value of the quantity being measured

Relative error $= \frac{\Delta A_m}{A_m}$

(3) Percentage Error: The relative error can be expressed in percentage error as

% error = Relative error $\times 100$

Propagation of Error: Any physical quantity depends on one or more than one physical quantities. So the error in any physical quantity will lead to error in the result.

(1) Error in result involving sum or difference of quantities

Let Z is defined as

$$Z = A + B - C$$

$\therefore \quad \Delta Z = \Delta A + \Delta B - \Delta C$

$\therefore$ Maximum possible error in Z is given by

$|\Delta Z|_{max} = \Delta A + \Delta B + \Delta C$ (Since ΔC can be positive or negative)

2. Error in the result having product or division of quantities:

$$Z = \frac{A^p B^q}{C^r}$$

$\Rightarrow \quad \ln z = p\ln A + q\ln B - r\ln c$

$\Rightarrow \quad \frac{dz}{z} = \frac{pdA}{A} + \frac{qdB}{B} + \frac{rdC}{C}$

For small change $dz \approx \Delta z. \Rightarrow \frac{\Delta z}{z} = P\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

Illustration10. *Multiply 107.88 by 0.610 and express the result upto the correct number of significant figure.*

(A) 65.8068 *(B 65.807*

(C) 65.81 *(D) 65.8*

Solution: Number of significant figures in multiplication is three corresponding to the minimum number 107.88×0.610

= 65.8068 = 65.8

$\therefore$ (D)

Illustration11. *In measurement of the period of oscillation of a Helical spring, the readings comes out to be 2.15 sec, 2.25 sec, 2.36 sec, 2.45 sec and 2.54 sec, calculate the absolute errors, relative error or percentage error.*

Solution: The mean period of oscillation of the Helical spring is

$$T = \frac{2.15 + 2.25 + 2.36 + 2.45 + 2.54}{5}$$

= 2.35 sec

The absolute error in the measurements are

2.15 – 2.35 = – 0.20 sec

2.25 – 2.35 = – 0.10 sec

2.36 – 2.35 = 0.01 sec

2.45 – 2.35 = 0.10 sec

2.54 – 2.35 = 0.19 sec

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is ΔT_{mean}= [(0.20 + 0.10 + 0.01 + 0.10 + 0.19)] /5

$= \frac{0.6}{5} = 0.12\,\text{sec}$

Period of oscillation of the simple pendulum is (2.35 ± 0.12) sec.
A more correct way to write its is
(2.4 ± 0.2) sec
The relative error or the percentage error is

$= \frac{0.2}{2.4} \times 100 = 8\%$

Combination of Errors

While doing an experiment we take several measurements, we must know how the errors in all the measurements combine.
To make such estimates, we should learn how errors combine in various mathematical operations. For this we use the following procedure

(I) Error of a sum or difference: Suppose two physical quantities A and B have measured values A ± ΔA, B ± Δ B respectively where ΔA and ΔB are their absolute errors.

(a) We wish to find the error Δz in the sum
z= A + B

We have by addition, z ± Δz

= (A ± ΔA) + (B ± Δ B)

The maximum possible error in z = Δ z = ΔA + ΔB

(b) For the difference z = A – B, we have

z ± Δz = (A ± Δ A) – (B ± Δ B)

= (A – B) ± Δ A ± Δ B

or, ± Δ z = ± Δ A ± Δ B

The maximum value of the error Δz is again ΔA + ΔB.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual.

Illustration 12. *The series combination of resistances is given by $R = R_1 + R_2$*
Suppose two resistances $R_1 = (50 \pm 4)\Omega$ and $R_2 = (100 \pm 3)\ \Omega$ are connected in series. Find equivalent resistance of the series combination.

Solution: $R_{eq} = R_1 + R_2$
= (50 ± 4) Ω + (100 + 3) Ω
= (150 ± 7) Ω

(II) Error in a product or a quotient

Suppose z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then
z ± Δz = (A ± ΔA) (B ± ΔB)

= AB ± B ΔA ± A ΔB ± ΔA ΔB

Dividing L.H.S. by z and R.H.S. by AB, we have

$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right).$$

Since ΔA and ΔB are small we shall ignore their product.
Hence the maximum fractional error in Z

$$= \frac{\Delta z}{z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Similarly, we can easily verify that this is true for division also.
So, when two or more quantities multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

(III) Error due to the power of a measured quantity.

Let $Z = X^2$

Then $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta X}{X} = \frac{2\Delta X}{X}$

Hence the fractional error in X^2 is two times the error in X.

In general if $Z = \frac{X^a Y^b}{Q^c}$

then $\frac{\Delta Z}{Z} = a\frac{\Delta X}{X} + b\frac{\Delta Y}{Y} + \left[\frac{\Delta Q}{Q} \times c\right]$

Illustration 13. *Find the fractional error in Z, if* $Z = \sqrt{\frac{XY}{M}}$

Solution: $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta X}{X} + \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta M}{M}$

Illustration 14. *Find maximum possible percentage error in* $x = \frac{a^{\ell} b^m}{y^p z^k}$

Solution: $\frac{\Delta X}{X} \times 100 = \left(\ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta y}{y} + k\frac{\Delta z}{z}\right) \times 100$

Illustration 15. *In the relation $x = 3yz^2$, x, y and z represents various physical quantities, if the percentage error in measurement of y and z is 3% and 1% respectively, then final maximum possible percentage error in x.*

Solution: $\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta y}{y} + 2\frac{\Delta z}{z}\right) \times 100$

= 3% + 2 × 1% = 5%

PHYSICAL QUANTITIES

All the physical quantities can be expressed in terms of some combination of seven base quantities: Length [L], mass [M], time [T], electric current [I], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol]. These base quantities are considered as the seven dimensions of the physical world.

DIMENSIONS

The dimension of a physical quantity are the powers to which the fundamental (or base) quantities like mass, length and time etc. have to be raised to represent the quantity. Consider the physical quantity **"Force"**. The unit of force is Newton.

1 Newton = 1 kg m/sec^2

$kg \to M^1$ (Mass); $m \to L^1$ (Length); $s^{-2} \to T^{-2}$ (Time)

$\therefore$ Dimensions of force are $[M^1L^1T^{-2}]$

Dimensional formulae for some physical quantities

Physical quantity	**Relation with other quantity**	**Dimensional formula**
Area	Length × breadth	$L \times L = [L^2]$
Density	Mass/volume	$\frac{M}{L^3} = [ML^{-3}]$
Acceleration	$\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = [LT^{-2}]$
Force	F = ma	$[MLT^{-2}]$
Linear momentum	P = mv	$[MLT^{-1}]$
Pressure	P = F/A	$[ML^{-1}T^{-2}]$
Universal gravitational constant	$G = \frac{Fr^2}{M_1M_2}$	$[M^{-1}L^3T^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (kinetic, potential and heat)	$\frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Surface tension	$T = \frac{F}{\ell}$	$[ML^{\circ}T^{-2}]$
Strain	$e = \frac{\Delta\ell}{\ell}$	$[M^{\circ}L^{\circ}T^{\circ}]$
Modulus of elasticity	$E = \frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	$[M^{\circ}L^{\circ}T^{\circ}]$
Coefficient of viscosity	$\eta = \frac{F \times r}{A \times v}$	$[M^1L^{-1}T^{-1}]$
Planck's constant	$h = mv\lambda$	$[ML^2T^{-1}]$
Thermal resistance	$\frac{\Delta\theta t}{Q}$	$[M^{-1}L^{-2}T^3\,\theta]$
Thermal conductivity	$K = \frac{H}{At(d\theta/dx)}$	$[MLT^{-3}\theta^{-1}]$
Boltzman's constant	k = R/N	$[ML^2T^{-2}\theta^{-1}]$
Universal gas constant	$R = \frac{PV}{T}$	$[ML^2T^{-2}\theta^{-1}]$
Mechanical equivalent of heat	J = W/H	$[M^{\circ}L^{\circ}T^{\circ}]$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$	$[M^{\circ}L^{\circ}T^{-1}]$

Illustration 16. *Write the dimensions of: Impulse, Pressure, Work, Universal constant of Gravitation.*

Solution: (i) $[M^1L^1T^{-1}]$ (ii) $[M^1L^{-1}T^{-2}]$ (iii) $[M^1L^2T^{-2}]$ (iv) $[M^{-1}L^3T^{-2}]$

Four types of quantities

Dimensional constant: These are the quantities whose values are constant and they possess dimensions. For example, velocity of light in vacuum, universal gas constant etc.

Dimensional variables: These are the quantities whose values are variable, and they possess dimensions. For example, area, volume, density etc.

Dimensionless constants: These are the quantities whose values are constant, but they do not possess dimensions. For example, π, 1, 2, 3, ….. etc.

Dimensionless Variables: These are the quantities, whose values are variable, and they do not have dimensions, e.g., angle, strain, specific gravity etc.

Uses of dimensions: dimensional analysis

(1) Checking the correctness (dimensional consistency) of an equation: An equation contains several terms which are separated from each other by symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This means that we can not add velocity to force. This principle is called Principle of Homogeneity of dimensions.

Look at the equation : $v^2 = u^2 + 2as$

Dimensions of $v^2 : [L^2T^{-2}]$

Dimensions of $u^2 : [L^2T^{-2}]$

Dimensions of $2as : LT^{-2}][L] = [L^2T^{-2}]$

$\therefore$ The equation $v^2 = u^2 + 2as$ is dimensionally consistent, or dimensionally correct.

Note:

A dimensionally correct equation may not be actually correct. For example, the equation $v^2 = u^2 + 3as$ is also dimensionally correct but we know that it is not actually correct. However, all correct equations must necessarily be dimensionally correct.

Illustration 17. *Which of the following equations may be correct ?*

(i) $x = ut + \frac{1}{2}at^2$ *(ii)* $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $F = \frac{GM_1M_2}{r}$ *(iv)* $T^2 = \frac{4\pi^2R^3}{GM}$

(v) $V = \sqrt{GMR}$

Given: G = Gravitational constant, whose dimensions are $[M^{-1}L^3T^{-2}]$

M_1, M_2 *and M have dimensions of mass. L, x, r, R has dimensssions of length. And t has dimensions of Time. 'F' denotes Force and 'a' has dimensions of acceleration.*

Solution: (i) Yes (ii)Yes (iii) No (iv) Yes (v) No.

(2) Conversion of units: Dimensional methods are useful in finding the conversion factor for changing the units to a different set of base quantities. Let us consider one example, the SI unit of force is Newton. The CGS unit of force is dyne. How many dynes is equal to one newton. Now,

$$1 \text{ newton} = [F] = [M^1L^1T^{-2}] = (1\text{kg})^1)(1\text{meter})^1(1\text{s})^{-2}$$

$$1\text{dyne} = (1\text{g})(1\text{cm})(1\text{s})^{-2}$$

$$\therefore \frac{1\text{newton}}{1\text{dyne}} = \frac{(1\text{kg})^1(1\text{meter})^1(1\text{s})^{-2}}{(1\text{g})(1\text{cm})(1\text{s})^{-2}} = (10^3)(10^2) = 10^5$$

$$1 \text{ newton} = 10^5 \text{ dynes}$$

Thus knowing the conversion factors for the base quantities, one can work out the conversion factor of any derived quantity if the dimensional formula of the derived quantity is known.

Illustration 18. *Find the conversion factor for expressing universal gravitational constant from SI units to cgs units.*

Solution: $6.67\text{x}10^{-8}\ \text{cm}^3\text{s}^{-2}\text{g}^{-1}$

(3) Deducing relation among the physical quantities:

Suppose we have to find the relationship connecting a set of physical quantities as a product type of dependence. Then dimensional analysis can be used as a tool to find the required relation. Let us consider one example. Suppose we have to find the relationship between gravitational potential energy of a body in terms of its mass 'm', height 'h' from the earth's surface and acceleration due to gravity 'g', then,

Let us assume that: – Gravitational potential energy, U,

$$U = K[m]^a[g]^b[h]^c,$$

where K, a, b, and c are dimensionless constants.

Then $[ML^2T^{-2}] = [M]^a[LT^{-2}]^b[L]^c$

$$= [M^aL^{b+c}T^{-2b}]$$

$\therefore$ a =1, b + c = 2

–2b = –2

b = 1, c = 1.

$\therefore$ U = Kmgh, where K is a dimensionless constant.

Thus by dimensional analysis, we conclude that the gravitational potential energy of a body is directly proportional to its mass, acceleration due to gravity and its height from the surface of the earth.

Limitations of dimensional analysis:

This method does not give us any information about the dimensionless constants appearing in the derived formula, e.g. 1, 2, 3, .. . π etc.

We can't derive the formula having trigonometrical functions, exponential functions etc, which have no dimensions.

The method of dimensions cannot be used to derive an exact form of relation when it consists of more that one part on any side, e.g. the formula $v^2 = u^2 + 2as$ cannot be obtained.

If a quantity depends on more than three factors having dimensions the formula cannot be derived. This is because on equating powers of M, L and T on either side of the dimensional equation, we can obtain three equations from which only three exponents can be calculated.
It gives no information whether a physical quantity is a scalar or a vector.

Illustration 19. *Using the method of dimensions, find the acceleration of a particle moving with a constant speed v in a circle of radius r.*

Solution: Assuming that the aceeleration of a particle depends on v and r

$a \propto v^x r^y \Rightarrow a = k\, v^x r^y$

Now as we know dimensions of acceleration (a) = $M^\circ LT^{-2}$

and dimensions of velocity (v) = $M^\circ LT^{-1}$

dimension of radius (r) = $M^\circ LT^\circ$

Putting all thee dimensions in (1), we get

$M^\circ LT^{-2} = k\,(M^\circ LT^{-1})^x (M^\circ LT^\circ)^y$

$M^\circ LT^{-2} = k\,M^\circ L^{x+Y} T^{-x}$

Comparing the powers, we get

$x + y = 1$

$x = 2$

$\therefore\ y = 1 - 2 = -1$

$\therefore\ a = k\, v^2 r^{-1}$

$a = \dfrac{kv^2}{r}$

Illustration 20. *In the expression* $\left(P + \dfrac{a}{v^2}\right)(v - b) = RT$

P is pressure and v is the volume. Calculate the dimensions of a and b.

Solution: Only physical quantities having same dimensions are added or subtracted. So $\dfrac{a}{v^2}$ has the same dimensions as that of pressure.

As pressure = $\dfrac{\text{Force}}{\text{Area}}$

Dimensions of pressure $= \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$\therefore$ Dimensions of $\dfrac{a}{v^2} = ML^{-1}T^{-2}$

Dimensions of a $= ML^{-1}T^{-2}(V^3)^2$

$= (ML^{-1}T^{-2})(L^3)^2$

$= ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

Similarly dimensions of b is same as that of volume.

Dimensions of $b = M^0L^3T^0$.

Illustration 21. *Does* $S_{nth} = u + \dfrac{a}{2}(L_n - 1)$ *dimensionally correct?*

Solution: Yes, this expression is dimensionally correct, yet it appears to be incorrect. As we are taking it to be for nth second. Here one second is divided through the equation.

Illustration 22. *Find the dimensions of resistivity, thermal conductivity and coefficient of viscosity.*

Solution: (i) $R = \rho\frac{\ell}{A}$

$\rho = \frac{RA}{L} = [ML^3T^{-3}A^{-2}]$

(ii) Thermal conductivity, k

$\frac{d\theta}{dt} = \frac{k\ell}{A\Delta\theta} = \frac{ML^2T^{-3}L}{L^2K}$

$= MLT^{-3}k^{-1}$

(iii) Coefficient of viscosity

$\therefore\ F = \eta A\frac{dv}{dx}$

$\eta = \frac{Fdx}{Adv} = \frac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

Illustration 23. *A displacement of a particle is given by equation y = A sin ωt, where y is in metres and A is also in metres, t is in seconds. What are the dimensions of ω.*

Solution: As the angles are always dimensionless, so
ωt = dimensionless quantity
Dimensions of ωt = $M^\circ L^\circ T^\circ$
Dimensions of ω = $M^\circ L^\circ T^{-1}$

Illustration 24. *If density ρ, acceleration due to gravity g and frequency f are the basic quantities, find the dimensions of force.*

Solution: We have $\rho = ML^{-3}$, $g = LT^{-2}$ $f = T^{-1}$
Solving for M, L and T in terms of ρ, g and f , we get
$M = \rho^2g^3f^{-6}$, $L = gf^{-2}$ & $T = f^{-1}$
Force = $[MLT^{-2}] = [\rho g^3f^{-6}.gf^{-2}.\ f^2] = [\rho g^4f^{-6}]$

Illustration 25. *An athlete's coach told his team that muscle times speed equals power. What dimensions does he view for "muscle"?*

(A) MLT^2 *(B) ML^2T^{-2}*
(C) MLT^{-2} *(D) L*

Solution: Power = force × velocity
= muscle times speed
∴ muscle represents force
muscle = $[MLT^{-2}]$
∴ (C)

Illustration 26. *If force, length and time would have been the fundamental units what would have been the dimensional formula for mass*

(A) $FL^{-1}T^{-2}$ *(B) $FL^{-1}T^2$*
(C) FLT^{-2} *(D) F*

Solution: Let $M = K\,F^aL^bT^c$
$= [MLT^{-2}]^a\,[L^b]\,T^c = [M^aL^{(a+b)}T^{(-2a+c)}]$

$a = 1, a + b = 0 \quad \& \; -2a + c = 0$

$\Rightarrow a = 1, b = -1, \; c = 2$

$\therefore$ (B)

Illustration 27. *The dimensions of the Rydberg constant are*

(A) $M^{\circ} L^{-1} T$ *(B)* MLT^{-1}

(C) $M^{\circ} L^{-1} T^{\circ}$ *(D)* $ML^{\circ} T^{2}$

Solution: From the relation $\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$R = \frac{1}{L} = L^{-1} = M^{\circ}L^{-1}T^{\circ}$

$\therefore$ (C)

Illustration 28. *The error in the measurement of the radius of a sphere is 1%. Then error in the measurement of volume is*

(A) 1% *(B) 5%*

(C) 3% *(D) 8%*

Solution: $V = \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r}\right) \times 100 = 3 \times 1 = 3\%$

$\therefore$ (C)

MISCELLANEOUS EXERCISE

1. The time period of a gas bubble formed under water oscillating with a time period depending on static pressure P, density of water ρ and ε total energy of explosion. Find the relationship between T, P, ρ and ε.

2. Name the three physical quantities having the same dimensions

3. A student measures the time period of a simple pendulum. If error in measurement of length is 2% and error in measurement of g is 2% calculate the error in measurements of Time period.

4. A physical quantity is given by x = a + bt, where x is in metres and t is in seconds. So calculate the dimensions of a and b.

5. Find the dimensional formulae of the following quantities
   (A) the universal gravitational constant (B) Surface tension
   (C) Potential energy (D) Surface energy

6. A Vernier calliper has 50 divisions on its Vernier scale. Minimum division in main scale is of 1 mm.
   (a) Find out the least count of the Vernier Calliper.
   (b) During a length measurement, zero of Vernier scale is between 12^{th} and 13^{th} divisions of the main scale and 26^{th} division of the Vernier scale coincides with a main scale division. What is the length ?

7. Calculate the focal length of a spherical mirror if measured quantities u and v are as follows.
   $u = 50.1 \pm 0.5\,\text{cm}$
   $v = 20.1 \pm 0.2\ \text{cm}$

8. Young's modulus of steel is 19 x $10^{1°}$ N/m^2. Express it in dyne/cm^2. Here dyne is the CGS unit of force.

9. Convert 1 joule into ergs.

10. A goldsmith puts some gold weighting 5.42 gm in a box weighing 1.2 kg. Find the total weight of the box to correct number of significant figures.

SOLUTION TO MISCELLANEOUS EXERCISE

1. $T \propto P^{-5/6}\rho^{1/2}E^{1/3}$
2. Work, energy and torque
3. 2%
4. x = a + bt
   Dimensions of a = $M°LT°$
   Dimensions of b = $M°LT^{-1}$
5. (a) $M^{-1}L^2T^{-2}$ (b) MLT^{-2}
   (c) ML^2T^{-2} (d) ML^2T^{-2}
6. 0.02 mm, 12.52 mm
7. 14.3 ± 0.4 cm
8. 19×10^{11} dyne / cm^2
9. 1 joule = 10^7 ergs
10. 1.2 kg

SOLVED PROBLEMS

Subjective:

Prob 1. *The Bernoulli's equation is given by,*
$P+\frac{1}{2}\rho v^2+\rho gh = constant$ *, where P is pressure. Compare the unit of the quantity* $\frac{1}{2}\rho v^2$ *with the unit of pressure.*

Sol. Only same quantities can be summed up or subtracted from each other. So $\frac{1}{2}\rho v^2$ has same unit as that of pressure.

Prob 2. *The relation between velocity and time of a moving body is given as,* $v = A+\frac{B}{t}+Ct^2$ *. Find the units of A, B and C.*

Sol. From the principle of homogeneity
v = A = m/sec
v = B/t $\Rightarrow$ B = m
v = (ct^2)
$\therefore$ c = m/sec^3

Prob 3. *The external and internal diameters of a hollow cylinder are measured to be (4.23 ±0.01) cm and (3.89 ± 0.01) cm. What is the thickness of the wall of the cylinder?*

Sol. Thickness = (4.23 – 3.89) cm = $\frac{0.34}{2}$ = 0.17 cm.
Error = ±(0.01 + 0.01) cm = ±0.02 cm

Prob 4. *A physical quantity x is calculated from the relation* $x = \frac{a^2b^3}{c\sqrt{d}}$ *. If percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively. What is the percentage error in x?*

Sol. As $x = \frac{a^2\ b^3}{c\sqrt{d}}$

$$\frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right]$$

$$= \pm\ [2 \times 2\% + 3\times1\% + 3\% + \frac{1}{2}\times 4\% = \pm\ 12\%$$

Prob 5. *A physical quantity A is defined as,* $A = pkx^2y/z$ *. The absolute errors in the measured of x, y, z are given as*
x = (0.26 ± 0.02)cm
y = (64 ± 2) Ω
z = (156.0 ± 0.1)cm
Find the percentage error in the quantity A.

Sol. $a = kx^2y / z$

$\Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$

$= 2 \times \frac{0.02}{0.26} + \frac{2}{64} + \frac{0.1}{156}$

$\pm 0.186 = 18.6\ \%$

Prob 6. *10 rotations of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 divisions. Find the least count. A reading taken for the diameter of wire with the screw gauge shows 4 complete rotations and 35 on the circular scale. Find the diameter of the wire.*

Sol. The least count = $\frac{5}{1000}$ = 0.005 mm

The diameter of the wire = $(4 \times 0.5 + 35 \times 0.005)$ mm = 2.175 mm

Prob 7. *The diameter of a sphere is 2.78 cm. Calculate its volume in proper significant figures.*

Sol. Volume = $\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi\left(\frac{2.78}{2}\right)^3$ cm^3 = 11.2437 cm^3

Hence the volume in proper significant figures is 11.2 cm^3

Prob 8. *Calculate the number of light years in one meter.*

Sol. We know 1 light year (ℓ_y) = 9.46×10^{15}m

or 9.46×10^{15} m = $1\ell y$

1 m = $1.057 \times 10^{-16}\ell y$

Prob 9. *Find the dimensions of a and b in the relation P =* $\frac{b - x^2}{at}$

where P is power, x is distance and t is time.

Sol. The given relation is, P = $\frac{b - x^2}{at}$

As x^2 is subtracted from b therefore the dimensions of b are of x^2

i.e. $b = L^2$

We can rewrite relation as

$P = \frac{[L^2]}{at} = \frac{L^2}{at}$

$a = \frac{L^2}{[ML^2T^{-3}]\ [T]} = M^{-1}L^{\circ}T^2$

Prob 10. *It is claimed that two cesium clocks if allowed to run for 100 years free from any disturbance may differ by only about 0.02 sec. What is the accuracy of the standard cesium clock in measuring a time interval of 1 sec?*

Sol. $\because$ t = 100 years = 100×365.25×86400s

$\Delta t = 0.02s$

Fractional error = $\frac{\Delta t}{t} = \frac{0.02}{100 \times 365.25 \times 86400}$

$= 0.63 \times 10^{-11}$

So, there is an accuracy of 10^{-11} Part in 1 or 1 sec in 10^{11} sec.

Prob11. *In screw gauge no. of division on circular scale is n and circular scale travels a distance of a units in one rotation. Calculate least count of the screw gauge.*

Sol. Pitch = a units

Least count $= \frac{\text{Pitch}}{\text{No. of divisions on circulat scale}}$

$= \frac{a}{n}$ units.

Prob 12. *The diameter of the spherical bob is measured by vernier Calipers (10 divisions of a Vernier scale coincide with a divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 12 mm and 7th division of the main scale coincides with the main scale. Mass of the sphere is 4.532 g. Find the density of the sphere.*

Sol. Vernier constant = 1.M.S.D. – 1.V.S.D.

$= 1 \text{ mm} - \frac{9}{10} \text{ mm}$

= 0.1 mm

Diameter of sphere = 12 mm + 0.1 × 7

= 12.7 mm

$\therefore$ Volume of sphere = $= \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

$= \frac{4}{3} \times 3.14 \times \left(\frac{12.7}{2} \times 10^{-3}\right)^3$

Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{4.532 \times 3 \times 8 \times 10^{-3}}{4 \times 3.14 \times (12.7 \times 10^{-3})^3}$

= 4.227 kg /m^3

= 4.23 kg/m^3 (in appropriate significant figures)

Prob 13. *A wire of length ℓ = 8 ± 0.02 cm and radius r = 0.2 ± 0.02 cm and mass m = 0.1 ± 0.001 gm. Calculate maximum percentage error in density*

Sol. $\rho = \frac{m}{\pi r^2 \ell}$

$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell}$

$\Delta\ell$ = 0.02 cm, ℓ = 8 cm

Δr = 0.02 cm, r = 0.02 cm

m = 0.1 gm, Δm = 0.001 gm

$\frac{\Delta \rho}{\rho} = \frac{0.001}{0.1} + 2 \times \frac{0.02}{0.2} + \frac{0.001}{0.1}$

$$= \left(\frac{1\times10}{1000\times1} + \frac{2\times2}{100\times2}\times10 + \frac{1\times10}{1000\times1}\right)\times100$$

$$= \frac{(1+20+1)}{100}\times100$$

= 22%

Prob 14. *Planck's formula is given by*

$$u = \frac{\hbar\omega^3}{\pi^2 e^3}\times\frac{1}{e^{\hbar\omega/k^{-1}-1}}$$

where u is the energy radiated per unit area per unit time and h is Planck's constant. What will be the dimensions of k in the expression.

Sol. The power in exponential is always dimensionless. So,

$$\frac{\hbar\omega}{kT} = M^0L^0T^0$$

E = hv

$$\text{so, } h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^0L^0T^{-1}}$$

$= ML^2T^{-1}$

$$\therefore \quad k = \frac{\hbar\omega}{T}$$

$$= \frac{ML^2T^{-1}T^{-1}}{T} = ML^2T^{-3}$$

Prob15. *According to Stoke's law the viscous force acting on a spherical body moving fluid depends on radius r of the body, co–efficient of viscosity η of the fluid and velocity f the body. Find the relation between F, η, r, v.*

Sol. Force acting on a spherical body depends on

$F \propto \eta^a r^b v^C$

$F = k\eta^a r^b v^c$

$(MLT^{-2}) = k\,(ML^{-1}T^{-1})^a\,(L)^b\,(LT^{-1})^c$

$MLT^{-2} = k\,(M)^a\,(L)^{-a+b+c}\,(T^{-a-c})$

a = 1

–a + b + c = 1

– a –c = –2

–1 –c = – 2

c = – 2 = 1

– a + b + c = 1

– 1 + b + 1 = 1

$\Rightarrow$ b = 1

F = k η r v

Objective:

Prob 1. *An experiment measures quantities a, b, c and x is calculated from $x = ab^2/c^3$. If the maximum percentage error in a, b and c are 1%, 3% and 2% respectively, the maximum percentage error in x will be*
(A) 13% *(B) 17%*
(C) 14% *(D) 11%*

Sol. (A) Maximum percentage error in x

As $x = \frac{ab^2}{c^3}$

$\frac{\Delta x}{x} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$

$\frac{\Delta x}{x} = 1\% + 2 \times 3\% + 3 \times 2\%$

$= (1 + 6 + 6)\% = 13\%$

Prob 2. *If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second, then non–zero integers x, y and z, such that $P^xQ^yc^z$ is dimensionless, may be*
(A) x = 1, y =1, z = 1. *(B) x = 1, y =–1, z = 1.*
(C) x = –1, y =1, z = 1. *(D) x = 1, y = 1, z = 1*

Sol. As $P^xQ^{y+}C^z$ is a dimensionless

$\left(\frac{MLT^{-2}}{L^2}\right)^x \left(\frac{ML^2T^{-2}}{L^2T}\right)(LT^{-1})^2 = M^0L^0T^0$

$(M^1L^{-1}T^{-2})^x (ML^\circ T^{-3})^y (LT^{-1})^2 = (M^\circ L^\circ T^\circ)$

Comparing powers, we get

$x + y = 0$ . . . (i)

$-x + z = 0$ . . . (ii)

$-2x - 5y - z = 0$ . . . (iii)

From (1) and (2),

$y = -x,\ z = x$

Substituting in (3), we get

If x = k

$y = -k,\ z = k$

$x = 1,\ y = -1,\ z = 1$

Prob 3. *The dimensional representation of Planck's constant is identical to that of*
(A) Torque *(B) Power*
(C) Linear momentum *(D) angular momentum*

Sol. (D) As Planck's constant has dimensions of $\frac{E}{\nu}$

$= \frac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

and Dimensions of angular momentum = r × p

$= (L \times MLT^{-1})$

$= ML^2T^{-1}$

Prob 4. *The parallel combination of two resistances is given by If the two resistances $R_1 = (2 \pm 0.2)\Omega$ and $R_2 = (1 \pm 0.1)\Omega$ are connected in parallel. Then the % error is given by*

(A) 0.1% *(B) 0.2%*

(C) 0.3% *(D) 0.4%*

Sol. (C) $R_p = \frac{R_1R_2}{R_1 + R_2}$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right) \times 100$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{0.2}{2} + \frac{0.1}{1} + \frac{0.2 + 0.1}{3}\right)$$

(0.1 + 0.1 + 0.1)

= 0.3%

Prob5. *If the units of M and L are quadrupled, then the units of torque becomes*

(A) 16 times *(B) 64 times*

(C) 8 times *(D) 4 times*

Sol. (B)

Dimensions of torque = ML^2T^{-2}

$= (4M)(4L)^2T^{-2}$

$= 64\, ML^2T^{-2}$

Prob6. *A radar signal is beamed towards a planet from earth and its echo is received seven minutes later. If distance between the planet and earth is $6.3 \times 10^{1\,o}$m, then velocity of the signal will be*

(A) 3×10^8 m/s *(B) 2.97×10^6 m/s*

(C) 3.10×10^5 m/s *(D) 300 m/s*

Sol. (A).

Velocity of signal, $c = \frac{2x}{t} = \frac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ m/s

Prob7. *If speed of light c, acceleration due to gravity g and pressure P are taken as fundamental units, then the dimensions of gravitational constant is*

(A) $[c^o g P^{-3}]$ *(B) $[c^2 g^3 P^{-2}]$*

(C) $[c^o g^2 P^{-1}]$ *(D) $[c^2 g^2 P^{-2}]$*

Sol. (C).

Let $G = c^x g^y P^z$

$\Rightarrow [M^{-1}L^3T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [ML^{-1}T^{-2}]^z$

$= [M^z L^{x+y} T^{-x-2y-2z}]$

Comparing powers of M, L and T on both sides, we get

$z = -1$, $x + y = 3$, $-x - 2y - 2z = -2$

On solving these equations for x, y and z, we get

$x = 0$, $y = 2$, $z = -1$

$\Rightarrow G = [c^\circ g^2 P^{-1}]$.

Prob 8. *The time dependence of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and t is time. The constant α*

(A) is dimensionless *(B) has dimensions T^{-2}*

(C) has dimensions of P *(D) has dimensions T^2*

Sol. (B).

$P = P_0 [\exp(-\alpha t^2)]$.

Since αt^2 must be dimensionless, so $\alpha = \frac{1}{T^2} = T^{-2}$

Prob 9. *The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is*

(A) hertz *(B) metre*

(C) radian *(D) second*

Sol. (A).

Here, kt is dimensionless. Hence, $k = 1/t = \text{sec}^{-1}$ = hertz

Prob10. *The parallel of a heavenly body measured from two points diametrically opposite on the equator of earth is 1.0 minute. If the radius of earth is 6400 km, find the distance of the heavenly body from the centre of earth in AU. Take 1 AU = 1.5×10^{11} m.*

(A) 0.293 AU *(B) 0.28 AU*

(C) 2.01 AU *(D) 3.97 AU*

Sol. (A).

Here, $\theta = 1' = \frac{1^\circ}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

ℓ = diameter of earth = 2×6400 km

$= 1.28 \times 10^4$ km $= 1.28 \times 10^7$ m

Now, $\ell = r\theta$

$\Rightarrow r = \frac{1.28 \times 10^7}{(\pi/60) \times 180} = 4.4 \times 10^{10}$ m

$r = \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} = 0.293$ AU

Prob11. *Dimensions of ohm are same as (h is Planck's constant and e is charge)*

(A) $\frac{h}{e}$ (B) $\frac{h^2}{e}$

(C) $\frac{h}{e^2}$ (D) $\frac{h^2}{e^2}$

Sol. (C).

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}] = \text{resistance}$$

Prob12. *Which of the following is a derived unit?*

(A) newton *(B) joule*
(C) pascal *(D) metre*

Sol. A, B, C.
Because, they are derived from the fundamental units, i.e. kg, m and sec.

Prob13. *Which of the following equations is dimensionally correct?*

(A) Pressure = energy per unit volume
(B) Pressure = energy per unit area
(C) Pressure = force per unit volume
(D) Pressure=momentum per unit volume

Sol. $\frac{\text{Energy}}{\text{Volume}} = \left[\frac{\frac{1}{2}\ mv^2}{\text{volume}}\right]$

$\Rightarrow \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

$\therefore$ (A)

Prob 14. *Which of the following is/are dimensional constants is*

(A) Planck's constant *(B) dielectric constant*
(C) relative density *(D) gravitational constant*

Sol. A Planck's constant and gravitational constant G have constant values and dimensions
$\therefore$ A, D

Prob 15. *Which of the following is not a unit of time*

(A) solar year *(B) tropical year*
(C) leap year *(D) light year*

Sol. Tropical year is the year in which there is total eclipse.
Light year represents distance
$\therefore$ (D)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. If force acting on a particle depends on the x–coordinates as $F = ax + bx^2$, find the dimensions of 'a' and 'b'.

2. If velocity, time and force are chosen as basic quantities, find the dimensions of mass.

3. Find the dimensional formula of
   (a) Charge Q
   (b) The potential V
   (c) The capacitance C,
   (d) The Resistance, R

4. Which of the following have same dimensions?
   (A) angular momentum and linear momentum (B) work and power
   (C) work and torque (D) Torque and Pressure

5. The Van der Waals interaction between two molecules separated by a distance r is given by the energy $E = -\frac{A}{r^6} + \frac{B}{r^{12}}$. Find the dimensions of A and B.

6. If error in measuring diameter of a circle is 4%, find the error in radius of circle.

7. Derive, by method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (ν) and the amplitude of vibration (r)

8. Write the dimensions of $\frac{a}{b}$ in the relation
   $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

9. Assuming that the mass of the largest stone that can be moved by a flowing river depends upon the velocity r, the density ρ and acceleration due to gravity, show that m varies with sixth power of the velocity of flow.

10. The density of a material in cgs system is 8 gcm^{-3}. In a system of units, in which unit of length is 5 cm and unit of mass is 20 g, what is the density of the material ?

11. To study the flow of a liquid through a narrow tube the following formula is used
    $\eta = \frac{\pi \rho r^4}{8v\ell}$ where the letters have their usual meanings. The values of ρ, r, v and ℓ are measured to be 76 cm of Hg, 0.28 cm, 1.2 cm^3s^{-1} and 18.2 cm respectively. If these quantities are measured to the accuracy of 0.5 cm of Hg, 0.01 cm, 0.1 $cm^3\ s^{-1}$ and 0.1 cm respectively, find the percentage error in the value of η.

12. The equation of a wave is given by $y = A\sin\omega\left(\frac{x}{v} - k\right)$, where ω is angular velocity and v is linear velocity. Find the dimension of k. Given that

13. The surface tension of a liquid is 70 dyne/cm. Express it in MKS system of units?

14. Name a physical quantity which has same unit as that of Torque.

15. If all measurements in an experiment are taken upto same number of significant figures then mention two possible reasons for maximum error.

Level – I

1. The mass of a block is 87.2 g and its volume is 25 cm^3. What is its density upto correct significant figures?

2. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants, λ is wavelength. Then calculate the dimensions of A and B.

3. Suppose, the torque acting on a body, is given by $\tau = KL + \frac{MI}{\omega}$

   Where L = angular momentum, I = moment of inertia & ω = angular speed
   What is the dimensional formula for KM?

4. When a current of (2.5 ± 0.5). A flows through a wire it develops a potential difference of (20 ± 1) V. What is the resistance of wire?

5. Find out the result in proper significant figures, 291 x 0.03842 / 0.0080.

6. The radius of a sphere is (5.3 ± 0.1) cm. Find the percentage error in its volume.

7. If Planck's constant h; the velocity of light, c and Newton's gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of these quantities using dimensional notation.

8. What will be the unit of time in the system in which the unit of length is meter, unit of mass is kg and unit of force is kg. wt.?

9. Imagine a system of units in which the unit of mass is 10 kg, length is 1 km and time is 1 minute, then calculate the value of 1 J in this system.

10. A screw gauge of pitch 0.5 mm has a circular scale divided into 5 divisions. The screw gauge is used to measure the thickness of a coin. The main scale reading is 2 mm and 35th circular division coincides with main scale with a positive zero error of divisions. Find the thickness of the coin

11. A Vernier Calliper is used to measure the thickness of the wall of cylinder by measuring its external and internal diameters. For external diameter, the zero if the Vernier scale coincides with the 5th division of main scale and 6th division of Vernier scale coincides with the main scale.
    For internal diameter, the zero of the Vernier scale coincides with the 3rd division of mass scale and 2nd division of Vernier scale coincides with 3rd division of main scale and 2nd division of Vernier scale coincides with main scale. Given that 1 main scale division is equal to 10 m
    1 V.S.D. = 0.09 cm.
    Calculate the thickness of the wall of a cylinder.

12. The time period of small oscillations of a spring mass system is given as $T = 2\pi\sqrt{\frac{m}{k}}$. What will be the accuracy in the determination of k if mass m is given as 10 kg with accuracy of 10 gm and time period is 0.5 sec measured for time of 100 oscillations with a watch of accuracy of 1 sec.

13. In a screw micrometer, main scale divisions are in mm. There are 100 cap divisions.
(a) Find out the least count of the micrometer.
(b) In fully closed condition, 4^{th} division of the cap scale coincides with the line of graduation along the screw axis. What is the zero error of the instrument ? Is it to be added or subtracted from the observed reading during a measurement ?
(c) In the above instrument, during a measurement, the cap is between 7^{th} and 8^{th} divisions of the main scale and 37^{th} division of cap scale coincides with the line of graduation of the main scale. What is the measurement corrected for zero error ?

14. The equation for energy (E) of a simple harmonic oscillator,
$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2x^2$$
is to be made "dimensionless" it with multiplying by a suitable factor, which may involve the constants, m(mass), ω(angular frequency) and h (Planck's constant). What will be the unit of momentum and Length ?

15. In in the equation $F = A \sin Bx^2 + \frac{C}{t}e^{Dt}$, F, x and t are force, position and time respectively, then give the dimensions of $\frac{A}{CB}$.

Objective:

1. Which of the following is a possible dimensionless quantity?
(A) Velocity gradient (B) Pressure gradient
(C) Displacement gradient (D) Force gradient

2. Dimensional formula of thermal conductivity is
(A) $ML^2T^{-3}\theta^{-1}$ (B) $ML^2T^{-2}\theta^{-4}$
(C) $ML^2T^{-2}\theta^{-1}$ (D) $MLT^{-3}\theta^{-1}$

3. The unit of power is
(A) kilowatt hour (B) joule
(C) dyne (D) kilo watt

4. The dimensional representation of Planck's constant is identical to that of
(A) torque. (B) power.
(C) linear momentum. (D) angular momentum.

5. Which of the following is a fundamental quantity?
(A) volume (B) velocity
(C) time (D) force

6. The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is
(A) hertz (B) metre
(C) radian (D) second

7. The dimensional representation of Planck's constant is identical to that of
(A) torque (B) work
(C) stress (D) angular momentum

8. A force F is given by $F = \frac{a}{t} + bt^2$, where t is time. The dimensions of a and b are
(A) $[MLT^{-3}]$ and $[MLT^{-4}]$ (B) $[MLT^{-4}]$ and $[MLT^{-3}]$
(C) $[MLT^{-1}]$ and $[MLT^{-4}]$ (D) $[MLT^{-2}]$ and $[MLT^{\circ}]$

9. A unit–less quantity
(A) may have non–zero dimensions (B) always has non–zero dimensions
(C) never has a non–zero dimensions (D) does not exist

10. Joule $\times$ sec is the unit of
(A) energy (B) momentum
(C) angular momentum (D) power

11. Given that v is speed, r is radius and g is gravitational acceleration, which of the following expression is dimensionless.
(A) $\frac{v^2}{gr}$ (B) $\frac{v^2 r}{g}$
(C) $\frac{v^2 g}{r}$ (D) $v^2 rg$

12. The dimensional formula for modulus of rigidity is
(A) $[ML^2T^{-2}]$
(B) $[ML^{-1}T^{-3}]$
(C) $[ML^{-2}T^{-2}]$
(D) $ML^{-1}T^{-2}]$

13. A highly rigid cubicle block A of small mass m and side L is rigidly fixed to an other similar cubical block of low modulus of rigidity η. Lower face of A completely covers the upper face of B. The lower face of B is rigidly held on horizontal surface. A small force T is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes simple harmonic motion, the time period of which is given by
(A) $2\pi\sqrt{m\eta L}$
(B) $2\pi\sqrt{m\eta / L}$
(C) $2\pi\sqrt{mL / \eta}$
(D) $2\pi\sqrt{m / \eta L}$

14. The time period of a soap bubble is $T \propto P^a d^b S^c$, where P is pressure, d is density and S is surface tension, then values of a, b and c, respectively, are
(A) –1, –2, 3
(B) –3/2, 1/2 1
(C) 1, –2, –3/2
(D) 1, 2, 1/3

15. The dimensional formula for specific resistance in term of M, L, T and Q is
(A) $[ML^3T^{-1}Q^{-2}]$
(B) $[ML^2T^{-2}Q^2]$
(C) $[MLT^{-2}Q^{-1}]$
(D) $[ML^2T^{-2}Q^{-2}]$

16. Which of the two have same dimensions?
(A) Force and strain
(B) Force and stress
(C) Angular velocity and frequency
(D) Energy and strain

17. The velocity of water waves depend on their wavelength λ, the density of water ρ and acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 \propto g^{-1}\lambda^{-1}$ y
(B) $v^2 \propto g\lambda$ y
(C) $v^2 \propto g\lambda\rho$ y
(D) $v^2 \propto g^{-1}\lambda^{-3}$ y

18. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combination which have the dimensions of angle
(A) $\frac{1}{RC}$
(B) $\frac{R}{L}$
(C) $\frac{C}{L}$
(D) $\frac{R^2C}{L}$

19. The vernier of a circular scale is divided into 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count by the instrument is
(A) 0.1′
(B) 1′
(C) 10′
(D) 30′

20. Dimensional analysis of the equation $(\text{velocity})^x = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$ gives the value of x as
(A) 1
(B) 2
(C) 3
(D) 4

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. $[a] = M^1L^0T^{-2}, [b] = M^1L^{-1}T^{-2}$
2. FTV^{-1}
3. (a) $[Q] = IT$ (b) $[V] = ML^2I^{-1}T^{-3}$ (c) $[C] = M^{-1}L^{-2}I^2T^4$ (d) $[R] = ML^2T^{-3}I^{-2}$
4. Work and torque
5. $[A] = ML^8T^{-2}$, $[B] = ML^{14}T^{-2}$
6. 4 %
7. $E = k\, mv^2\, r^2$
8. $M^\circ L^{-1/2}\, T^2$
10. 50 units
11. 23%
12 $k= M^\circ L^\circ T$
13. $7x10^{-2}$ N/m
14. Work
15. The maximum error will be due to (i) measurement, which is least accurate.
(ii) measurement of the quantity which has maximum power in formula's.

Level – I

1. 3.5 g/cc
2. $M^\circ L^\circ T^\circ$, $M^\circ L^2 T^\circ$
3. T^{-4}
4. $(8 \pm 2)\ \Omega$
5. 1400
6. 5.7%
7. $(hc)^{1/2}\, G^{-1/2}$, $(hG)^{1/2}c^{-3/2}$, $(hG)^{1/2}\, c^{-5/2}$
8. $\frac{1}{\sqrt{9.8}}$ sec
9. 360
10. 2.25 mm
11. 1.02 cm
12. $\pm$ 5%
13. (a) 0.01mm (b) + 0.04 mm, to be subtracted (c) 7.33 mm
14. $\frac{E}{\hbar\omega} = \frac{1}{2}\frac{mv^2}{\hbar\omega} + \frac{1}{2}\frac{\omega m x^2}{\hbar}$, $\sqrt{m\omega\hbar}$, $\sqrt{\frac{\hbar}{m\omega}}$
15. L^2T^{-1}

Objective:

1.	C	2.	D	3.	D	4.	D
5.	C	6	A	7.	D	8.	C
9.	C	10.	C	11	A	12.	D
13.	D	14.	B	15.	A	16.	C
17.	B	18.	D	19.	B	20.	C

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

UNITS AND DIMENSION

Units and Dimension

Syllabus of IITJEE and Maharashtra Board:
Scope and excitement of physics, Technology & society. Forces in nature, Conservation laws, Examples of gravitation, electromagnetic and nuclear forces from daily life experiences (qualitative description only). Need of measurement, Units of measurement, System of units, SI units, Fundamental and derived unit; Length, mass and time measurement, Accuracy and precision of measuring instrument, errors in measurement, significant figures. Dimensions of physical quantities, dimensional analysis and its application.

Have you ever observed the nature and the various spectacular events like formation of rainbow on any rainy day?
Whenever we observe nature keenly, we can easily understand that the various events in nature like blowing of wind, flow of water, motion of planets, formation of rainbow, different forms of energies, the function of human bodies, animals, etc. are happening or taking place according to some basic laws. The systematic study of these laws of nature governing the observed events is called science. For our convenience, clear understanding and systematic study of Science is classified into various branches. Among these branches Chemistry, Mathematics, Botany, Zoology, etc. are ancient branches and Bio–technology, Bio–chemistry, Bio–Physics, Computer science, Space Science, etc. are considered to be modern branches of science and engineering. One of such ancient and reputed branches of this science is physics.

SCOPE AND EXCITEMENT OF PHYSICS

The domain of physics consists of wide variety and large number of natural phenomena. Hence, the scope of physics is very vast and obviously the excitement that one gets from the careful study of physics has got no boundaries.

Scope of Physics

For example, when we study one of the basic physical quantities called mass, we come across the values ranging from minute masses like mass of an electron (of the order of $10^{-3\circ}$ kg) to heavy masses like mass of universe (10^{55} kg). Similarly, in case of other basic quantities like length and time also the range is very wide.

Hence, the scope of physics can be understood easily, only when we can classify the study of physics chiefly into three levels. They are:

(a) Macroscopic level study of physics,
(b) Mesoscopic level study of physics, and
(c) Microscopic level study of physics.

Macroscopic level study of physics: Macroscopic level study of physics mainly includes the study of basic laws of nature and several natural phenomena like gravitational force of attraction between any two bodies in the universe (in mechanics), variation of quantities like pressure, volume, temperature, etc. of gases on their thermal expansion or contraction (in thermodynamics), etc.

Microscopic level study of physics: The microscopic level study of physics deals with constitution and structure of matter at the level of atoms or nuclei. For example, interaction between elementary particles like electrons, protons and other particles , etc.

Mesoscopic level study of physics: The mesoscopic level study of physics deals with the intermediate domain of macroscopic and microscopic, where we study various physical phenomena of atoms in bulk.

So, the edifice of physics is beautiful and one can appreciate the subject as and when one pursues the same seriously.

Excitement of Physics

The study of physics is exciting in many ways as it explains us the reason behind several interesting features like (a) ***how day and nights are formed?*** (b) ***how different climatic conditions are formed in different seasons?*** (c) ***how satellite works and helps in using several devices like television, telephones, etc.?*** (d) ***how an astronaut travels to celestial space ?*** (e) ***how we can convert one form of energy to another ?*** (f) ***how different types of forces are governing different types of motion in universe ? etc***.

It is quite common and simple that every human being on the earth will be interested to know the answers for at least few of the above questions. As physics is the subject which answers them, naturally the study of physics will be exciting.

TECHNOLOGY AND SOCIETY

Physics is almost an integral part of upgradation of technology. Technology was also a branch of science where we study the application of principles of physics for practical purposes. Based on laws and principles of physics, technocrats along with scientists develop technically advanced equipment to help the society.

For example, from the principles of thermodynamics James watt invented steam engine which was responsible for a big industrial revolution in England in the 18^{th} century. Another recent example is invention of mobile phones which are creating revolution in wireless communication technology. Yet another important example is invention of micro–processors by using silicon chips which has replaced valve technology and brought the computers from the size of your study room to the size of your geometry box. These are few examples. There are many more areas where physics is involved in upgrading technology and thereby helping the society. The following table gives us a list of various branches of physics that helped the field of technology.

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Nuclear fission
Radio and Television	Propagation of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation (population inversion)

Production of ultra–high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electric energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Air conditioners / Refrigerators	Laws of thermodynamics
Washing machines, centrifuge, etc.	Centrifugal force
Sonar	Reflection of ultrasonic waves

The following table lists the involvement of various renowned physicists all across the world, who helped the society with their noble inventions.

Name	Major Contribution / Discovery	Country of origin
Isaac Newton	Universal law of gravitation: Laws of motion; reflecting telescope.	**U. K.**
Galileo Galilei	Law of inertia	**Italy**
Archimedes	Principle of buoyancy; principle of the lever	**Greece**
James Clerk Maxwell	Electromagnetic theory; light an electromagnetic wave	**U. K.**
W. K. Roentgen	x– rays	**Germany**
Marie Sklodowska Curie	Discovery of radium and polonium; Studies on natural radioactivity	**Poland**
Albert Einstein	Law of photo–electricity; Theory of relativity	**Germany**
S. N. Bose	Quantum statistics	**India**
James Chadwick	Neutron	**U.K.**
Niels Bohr	Quantum model of hydrogen atom	**Denmark**
Ernest Rutherford	Nuclear model of atom	**New Zealand**
C.V. Raman	Inelastic scattering of light by molecules	**India**
Christiaan Huygens	Wave theory of light	**Holland**
Michael Faraday	Laws of electromagnetic induction	**U.K.**
Edwin Hubble	Expanding universe	**U.S.A.**
Homi Jehangir Bhabha	Cascade process in cosmic radiation	**India**
Abdus Salam	Unification of weak and electromagnetic interactions	**Pakistan**
R. A. Millikan	Measurement of electronic charge	**U.S.A**
Ernest Orlando Lawrence	Cyclotron	**U.S.A.**
Wolfgang Pauli	Quantum Exclusion Principle	**Austria**
Louis victor de Broglie	Wave nature of matter	**France**
J.J. Thomson	Electron	**U.K.**
S. Chandrasekhar	Chandrasekhar limit, structure and	**India**

	evolution of stars	
Lev Devidovich Landau	Theory of condensed matter; liquid helium	**Russia**
Heinrich Rudolf Hertz	Electromagnetic waves	**Germany**
Victor Francis Hess	Cosmic radiation	**Austria**
M. N. Saha	Thermal ionisation	**India**
G. N. Ramachandran	Triple helical structure of proteins	**India**
Thomas Alwa Edison	Electric bulb, Projector	**US**
Graham Bell	Telephone	**US**
Cavendish	Determination of 'G'	**England**
Robert Boyle	Boyle's law	**England**

So, to put it in a nut shell, science, technology and society are inseparable as they are deeply interwined.

FUNDAMENTAL FORCES IN NATURE

Force is a very common word which we normally come across in our daily life. We need force to push or pull or throw a body. Even we need it to deform or break the bodies. Sometimes, we experience force like when we are standing in a great storm, we experience the force exerted by wind. When we are sitting in a bus which is negotiating a turn, we experience an outward push. So, what is this force? Let us try to understand the concept of force in terms of physics.

At macroscopic level study of physics, we normally encounter different kinds of forces like gravitational force, muscular force, frictional force, contact force, spring force, buoyant force, viscous force, pressure force, force due to surface tension, electrostatic force, magnetic force, etc. whereas at microscopic level of study we come across nuclear forces, interatomic forces, intermolecular forces, weak forces, etc.

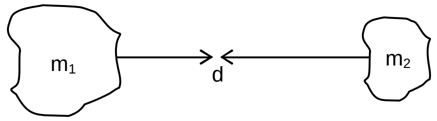
After analysing these various types of forces in nature, it was concluded that all the forces can be comfortably classified into four categories, which are known as fundamental forces in nature. They are

(1) Gravitational force (2) Electromagnetic force,
(3) Nuclear force, and (4) Weak force.

That means, any force other than the above four forces can be derived from these four basic forces. For example, elastic force or spring force arises due to the net attraction or repulsion between any two neighboring atoms of the spring. When it is elongated or compressed, attractive or repulsive forces produced between the atoms can be treated as the resultant of all electromagnetic forces between charged particles of an atom. Hence, this spring force is known as derived force and electromagnetic force which is the origin of this spring force is called fundamental force. Now, we will study about fundamental forces in brief.

Gravitational Force

Newton discovered that any two bodies in universe attract each other. This force of attraction exists by virtue of their masses, and is known as gravitational force of attraction. He found that the gravitational force is directly proportional to their masses and is inversely proportional to the square of the distance between them.



i.e. $F = G\dfrac{m_1m_2}{d^2}$ where 'G' is a Universal Gravitational Constant. This force is a universal force and is independent of any type of intervening medium between the two bodies. Though this is the weakest force in nature when compared to other types of fundamental forces, it plays vital role in governing the motion of planets around sun, natural satellites (like moon around earth), artificial satellites, etc.

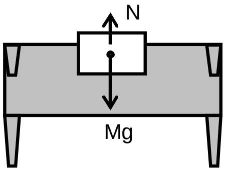
Electromagnetic Force:

The force of attraction or repulsion between any two charged particles is known as electrostatic force. If q_1 and q_2 charges are separated by a distance 'd' in air then the force of attraction or repulsion between them is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{d^2}$. This is called Coulomb's law of electric forces.

Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. In general electric and magnetic effects are inseparable and hence the name – electromagnetic force. This electromagnetic force between moving charged particles is comparatively more complicated and contains several other terms other than Coulomb's force.

In atoms electromagnetic force between electrons and protons is responsible for several molecular and atomic phenomena. Apart from this it also plays vital role in the dynamics of chemical reactions, mechanical and thermal properties of materials, tension in ropes, friction, normal force, spring force, Vander Waals force .

Example: Let us consider a block which is placed on a horizontal surface of a table as shown in the figure. The table balances the weight (Mg) and exerts a force which comes from electromagnetic force between charged constituents of atoms or molecules of surface of block and that of the table. Thus a force called normal force acts on block.



This electromagnetic force is a strong force when compared to the gravitational force. The electromagnetic force between two protons is 10^{36} times the gravitational force between them for any fixed distance.

Nuclear Force

We know that, in general, nucleus of every atom consists of two elementary particles called protons and neutrons. As neutrons are uncharged and protons are charged, the electric force of repulsion between protons will cause nucleus to break into fragments. But this is not happening, and also we know that nucleus of a non–radioactive element is a stable one.

That means there must be some other attractive force which is dominating coulombic force of repulsion between protons and keeping all the particles in nucleus together in stable condition as gravitational force can't dominate electric force. That new force existing between any two nucleons and which keeps all the particles in nucleus bound together is known as nuclear force. This force is stronger than electromagnetic force and is a charge independent force. Range of these forces is very small and will be of the order of nuclear size (10^{-21} th portion of size of an atom).

Latest developments in physics revealed that this strong nuclear force is also not a fundamental force as protons and neutrons consist of still elementary particles called quarks. And according to this latest development quark – quark force is fundamental force of nature and nuclear force is a derived force. However the study of quark – quark force is out of the scope of this book and our curriculum.

Weak Nuclear Force

This force appears only in certain nuclear processes. A neutron can change itself into a proton by emitting an electron and another elementary particle called antineutrino simultaneously. This process is called β^- decay. Similarly a proton can also change into neutron by emitting positron and a neutrino. This process is called β^+ decay. The forces which are responsible for these changes are known as weak forces. These forces are weak in nature when compared to nuclear and electromagnetic forces but stronger than gravitational forces. The range of these weak nuclear forces is exceedingly small, of the order of 10^{-15}m.

The following table gives us an overall idea about relative strengths and ranges of four fundamental forces.

Name	Relative strength	Range	Operates among
Gravitational force	10^{-38}	Infinite	**All objects in the universe**
Weak nuclear force	10^{-13}	Very short, within nuclear size ($\sim 10^{-15}$)	**Elementary particles**
Electromagnetic force	10^{-2}	Infinite	**Charged particles**
Strong nuclear force	1	Very short, within nuclear size ($\sim 10^{-15}$)	**Nucleons**

CONSERVATION LAWS

In any physical phenomena, few physical quantities associated with the phenomena may change with time and few physical quantities associated with it may not change. Those physical quantities which remain constant in time are known as conserved quantities.

For example, if a big liquid drop is sprayed into several small droplets the volume of liquid before spraying and after spraying remains same. Hence, we can say that a physical quantity called volume is conserved in this example. Similarly, we have several quantities which are conserved. Within the scope of our course, we can discuss the following conservation laws.

1. Law of conservation of linear momentum
2. Law of conservation of energy
3. Law of conservation of angular momentum
4. Law of conservation of charge.

Let us discuss them in brief.

Law of conservation of linear momentum

The linear momentum of a body is defined as the ability of a body by virtue of which it imparts its motion to other objects along a straight line. And mathematically it is equal to the product of mass of the body (m) and its velocity ($\vec{v}$) Mathematically, $\vec{P} = m.\vec{v}$.

According to this law, in absence of an external force, the total vector sum of linear momentum remains unchanged.

Example: When a bullet is fired with a gun, the total momentum vector of the system of bullet and gun is zero. After firing, bullet moves in forward direction with some momentum and gun recoils with the same amount of momentum in magnitude, but opposite in direction. Hence total vector sum of momentum after firing is also zero. Thus linear momentum of the system before and after firing is zero. Hence we can say that linear momentum is conserved.

Law of conservation of energy

According to this law the total energy of an isolated system is always constant and it never changes. But it can be transformed from one form to another. *For example* an electric cell in our daily life gives electrical energy by transforming chemical energy in it, electric motor converts electrical energy to mechanical energy, etc. However the total energy in these processes is conserved.

When an object is dropped from a certain height the total mechanical energy of the body is conserved. At its highest point all its mechanical energy will be in the form of potential energy and at its lowest point it will be in the form of kinetic energy, i.e. energy has transformed from one form into another, (i.e. potential to kinetic) but the total energy remains constant. Hence the total mechanical energy is conserved.

But this conservation of mechanical energy can't be applied in the presence of non – conservative force. For example in the above case if you consider air resistance on the freely falling body total mechanical energy does not remain constant. Here work done by air resistance gets converted into different forms of energy like heat energy. So such while applying nergy conservation principle heat energy should also be taken into consideration in such cases

Law of conservation of angular momentum

Angular momentum ($\vec{L}$) of a body about a point is defined as the cross product of its position vector about that point ($\vec{r}$) and its linear momentum at that instant ($\vec{p}$)

i.e. $\vec{L} = \vec{r} \times \vec{p}$
or $L = rp \sin\theta$ where θ is the angle between ' $\vec{r}$ ' and ' $\vec{p}$ '.

According to this law the total angular momentum of the system remains conserved in absence of external torque.

Example: We know that planets revolves around sun in elliptical orbits. The angular momentum of a planet at any point during its motion in its path is conserved. We will study more clearly about this under rotatory motion concepts.
These are the few conservation laws in mechanics. Now let us discuss a conservation law in electrostatics.

Law of conservation of charge

This law states that the total electric charge of an isolated system is always conserved. Charge can neither be created nor destroyed, but it can be transferred or exchanged from one body to another.

Apart from these, there are several other physical quantities that are conserved in nature. During our further discussions in various chapters we will understand them.

MEASUREMENT AND UNITS

Physical quantity: Any meaningful term which can be measured is a physical quantity. For example length, velocity, time etc. are physical quantity. But handsomeness, beauty are not physical quantity.

Why measurement is needed?: Physics is an experimental science and experiments involve measurement of different physical quantities in which laws of physics are expressed. Without measuring results of experiments, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Units of measurement: To measure a physical quantity we need some standard unit of that quantity. For example, if a measurement of length is quoted as 5 meters, it means that the measured length is 5 times as long as the value accepted for a standard length defined to be **"one meter".**

Any set of standards of units must fulfill the following two conditions
(i) It must be accessible.
(ii) It must be invariable with the passage of time

Two more auxiliary conditions are:–
(i) It is necessary to have wide unlimited agreement about those standards.
(ii) It is inter convertible to different units of same quantity.

A measurement consists of two parts, one is numeric and the other is standard chosen. For example, 5 meter of length implies 5 times the **"standard meter"**. It is not necessary to establish a measurement standard for every physical quantity. Some quantities can be regarded as fundamental and the standard for other quantities can be derived from the fundamental ones. For example, in mechanics length, mass and time are regarded as fundamental quantities and the standard for speed (= length / time) can be derived from fundamental quantities length and time.

Quantity	SI Units	Symbols
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of Substance	mole	mol
Thermodynamic Temp.	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	Cd

And two supplementary units are

Plane Angle	Radian	rad

Solid Angle	Steradian	sr

Two other system of units compete with the international system. One is Gaussian System in terms of which much of the literature of physics is expressed. In India this system is not in use.
The other is the British system. This system is still in daily use in United states. But SI units are standard units worldwide.

C.G.S. Unit: In this system of unit, centimeter, gram and seconds are units of length, mass and time respectively.

Conversion of One System of Units to another System: The basic formula is $n_1u_1 = n_2u_2$ where n_1 and n_2 are numbers.

Illustration 1. *How many dyne–centimeter are equal to 1 N–m?*

Solution:

$$1\,\text{N}-\text{m} = (1\text{kg})(1\text{m})^2(1\text{s})^{-2}$$
$$1\,\text{dyne}-\text{centimeter} = (1\text{g})(1\text{cm})^2(1\text{ s})^{-2}$$
$$\therefore \frac{1\,\text{N}-\text{m}}{1\,\text{dyne}-\text{cm}} = \left(\frac{1000\text{g}}{1\text{g}}\right)\left(\frac{100\text{cm}}{1\text{cm}}\right)^2$$
$$= 1000 \times 10000$$
$$\therefore 1\,\text{N}-\text{m} = 10^7\text{ dyne}-\text{cm}$$

Exercise: ***Calculate the value of 1 erg in SI system.***

Measurement of Length

Depending upon the range of length, there are three main methods for measuring length.

(i) Direct method using measuring instruments.
(ii) Indirect method or Mathematical method
(iii) Chemical method

(i) Direct method

The simplest method measuring the length of a straight line is by means of a meter scale. But there exist some limitations in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier calliper (b) micrometer scale (screw gauge) are used .

Vernier calliper

It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being slightly shorter than the divisions of the main scale.

Least count of Vernier Calliper

The least count or the Vernier constant (V.B.) is the minimum value of correct estimation of length without eye estimation. The difference between the values of one main scale division and one vernier scale division is known as vernier constant if N division of vernier scale coincides with (N–1) divisions of main scale, then vernier constant,

$$n.V.S.D. = (n-1)\ M.S.D.$$

$$1.V.S.D. = \left(\frac{n-1}{n}\right) M.S.D., \text{ and}$$

$$1.M.S.D. - 1.V.S.D. = 1.M.S.D. \left(\frac{n-1}{n}\right) M.S.D.$$

$$= \frac{1}{n}\ M.S.D.$$

$$= \frac{\text{Smallest Division on main scale}}{\text{No. of divisions on vernier scale}}$$

Reading a Vernier scale

Let one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions

$$\therefore\ 1.V.S.D. = \frac{9}{10}\ M.S.D. = 0.9\ mm$$

$$\therefore\ \text{Vernier constant} = 1.M.S.D - 1.V.S.D. = 1\ mm - 0.9\ mm$$
$$= 0.1\ mm = 0.01\ cm$$

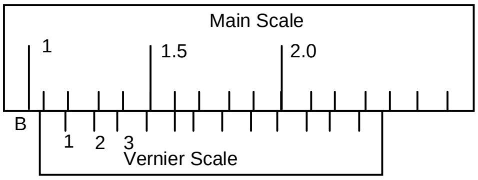
The reading with vernier scale is read as given below :

(i) Firstly take the main scale reading (N) before on the left of the zero of the vernier scale.
(ii) Find the number (n) of vernier division which just coincides with any of the main scale division. Multiply this number (n) with vernier constant (V.C.)
(iii) Total reading = (N + n × V.C.)

Caution: The main scale reading with which the Vernier scale division coincides has no connection with reading

Suppose If we have to measure a length AB, the end A is coincided with the zero of the vernier scale as shown in fig. Its enlarged view is given in fig.

Length AB > 1.0 cm < 1.1. cm



Let 5^{th} division of vernier scale coincide with 1.6 cm of main scale. From diagram it is clear that the distance between 4^{th} division of vernier scale and 1.5 cm of main scale is equal to one V.C. and distance between zero mark of vernier scale and 1.0 cm mark on the main scale is equal to 5 times the vernier constant.

$$\therefore\quad AB = 1.0 + 5 \times v.c. = 1.0 + 5 \times 0.01 = 1.05\ cm.$$

Illustration 2. *In travelling microscope the vernier scale used has the following data.*
1 M.S.D. = 0.5 mm, 50 V.S.D. = 49 M.S.D.
and the actual reading for distance travelled by travelling microscope is 2.4 cm with 8^{th} *division coinciding with a main scale graduation . Estimate the distance travelled.*

Solution : In this case vernier constant = 1.M.S.D. – 1.V.S.D.

$$= 1.M.S.D. - \frac{49}{50} M.S.D. = \frac{1}{50} M.S.D = \frac{1}{50} \times 0.5\,mm$$

$= \frac{5}{10} \times \frac{1}{50} = 0.01\,\text{mm} = 0.001\,\text{cm}$

$\therefore$ Distance travelled = 2.4 + 8 × 0.001 cm
= 2.408 cm

Illustration 3. *The Vernier scale used in Fortin's barometer has 20 divisions coinciding with the 19 main scale divisions. If the height of the mercury level measured is 5 mm and 15th division of vernier scale is coinciding with the main scale division. Then calculate the exact height of the mercury level (given that 1.M.S.D. = 1mm)*

Solution: 20 V.S.D. = 19 M.S.D. (Given)

1.V.S.D. = $\frac{19}{20}$ M.S.D.

V.C. = 1. M.S.D. – 1.V.S.D $= \left(1 - \frac{19}{20}\right)$M.S.D.

$= \frac{1}{20}$ M.S.D.

$= \frac{1}{20} \times 1\,\text{mm} = 0.05\,\text{mm}$

= 0.005 cm

Height of mercury level = 5 + 0.05 × 15
= 5.75 mm

Exercise: ***The Vernier calliper is used to measure the length of an object. The least count of such a vernier calliper is 0.2 cm and scale reads its length to be 5.6 cm. 3rd division of Vernier scale is coinciding main scale division Calculate the length of an object.***

Zero Error

If the zero marking of main scale and Vernier scale do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument. If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

Illustration 4. *Consider the following data:*
10 main scale divisions = 1cm, 10 vernier division = 9 main scale divisions,
zero of Vernier scale is to the right of the zero marking of the main scale with 6th Vernier division coinciding with a main scale division and the actual reading for length measurement is 4.3 cm with 2nd Vernier divisions coinciding with a main scale graduation. Estimate the length.

Solution: In this case, vernier constant = $\frac{1\text{mm}}{10} = 0.1\text{mm}$

Zero error = 6 × 0.1 = + 0.6 mm

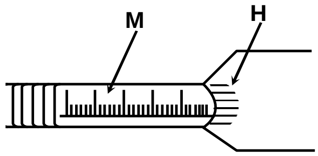
Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction
= 4.32 – 0.06 = 4.26 cm

Screw Gauge (or Micrometer Screw)

In general Vernier Callipers can measure accurately upto 0.02 mm and for greater accuracy micrometer screw devices, e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by turning it axially. The instrument is provided with two scales:

(i) The main scale or pitch scale M graduated along the axis of the screw.
(ii) The cap–scale or head scale H round the edge of the screw head.



Constants of the screw gauge:

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus if 10 rotations of cap ≡5 mm, then pitch = 0.5 mm

In general, pitch $= \dfrac{\text{Distance travelled by screw on main scale}}{\text{No. of rotation taken by the cap to travel that much distance}}$

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration, if the total cap division is 100, then least count = 0.5 mm/100

= 0.005 mm
In general, In case of circular scale,

Least count $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

If pitch is 1 mm and there are 100 divisions on circular scale, then

Least count = $= \dfrac{1\text{mm}}{100}$ = 0.01 mm = 0.001 cm

= 0.00001 m = 10^{-5} m = 10 μm.

Since least count is of the order of 10 μm, So the screw is called a micrometer screw. Screw gauge and the spherometer which work on the principle of micrometer screw, consist essentially of the following two scales.

(i) Linear or Pitch scale: It is a scale running parallel to the axis of the screw.

(ii) Circular of Head scale: It is marked on the circumference of the circular disc or the cap attached to the screw.

Zero Error: In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero–error, otherwise the instrument is said to heave zero–error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and vice–versa. The corresponding corrections will be just opposite.

Illustration 5. *A screw gauge has 100 divisions on its circular scale. Circular scale travels one division on linear scale in one rotation and 10 divisions on linear scale of screw gauge is equal to 5 mm. What is the least count of a screw gauge.*

Solution: Pitch $= \dfrac{1\,\text{division on linear scale}}{1\,\text{rotation}} = 1\,\text{div.}$

10 division = 5 mm
∴ 1 division = 0.5 mm
∴ pitch = 0.5 mm

$\text{least count} = \dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$= \frac{0.5\,\text{mm}}{100} = 0.005\,\text{mm}$

Illustration 6. *The screw gauge mentioned in above illustration is used to measure thickness of a coin. The reading of the linear scale is 4^th^ div and 25^th^ division of circular scale is coinciding with it. What is the value of thickness of the coin..*

Solution: Reading = Linear scale Reading + Least count × circular scale reading
= 4^{th} division on linear scale + 0.005 mm × 2.5
= 4 × 0.5 mm + 0.125 mm
= 2 mm + 0.125 mm
= 2.125 mm

Illustration 7. *A spherometer has 250 equal divisions marked along the periphery of its disc and one full rotation of the disc advances it on the main scale by 0.0625 cm. The least count of the spherometer is*
(A) 2.5×10^{-2}cm (B) 25×10^{-3}cm
(C) 2.5×10^{-4} cm (D) none of the above

Solution: Least count = $\frac{0.0625}{250}$ cm = 2.5×10^{-4}cm
$\therefore$(C)

(ii) Indirect or Mathematical method

This method involves measurement of long distances. Main methods of this category are –

Reflection method: Suppose we want to measure the distance of a multi story building from a destination point P. If a shot be fired from P, the sound of shot travels a distance x towards the building, gets reflected from the building. The reflected sound travels the distance x to the point of P, when an echo of the shot is heard.

Let t = time interval between the firing of the shot and echo sound.
v = velocity of sound in air.

Distance = velocity x time

x + x = (v) (t)

$\Rightarrow$ x = (v) (t/2)

As v is known, x can be calculated by measuring the time t.

Illustration 8. *A rock is at the bottom of a very deep river. An ultrasonic signal is sent towards rock and received back after reflection from rock in 4 seconds. If the velocity of ultrasonic wave in water is 1.45 km/s, find the depth of river.*

Solution: Here x = ?

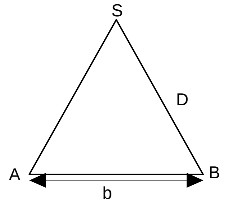
v = 1.45 km/s = 1450 m/sec.

t = 4 sec

so, x = v x t / 2 = 1450 x 4 / 2 = 2900 m.

Parallel method: This method is used for measuring distance of nearby stars.

Let we have to measure the distance D of a far away star S by this method. We observe this star from two different position A and B on the earth, separated by a distance AB = b at the same time as shown in figure. Let $\angle ASB = \theta$, the angle θ is called parallatic angle. As the star is very far away, b/D << 1 and θ is very small.



Here we can take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS=BS so that AB = b = $D\theta$ where θ is in radians.

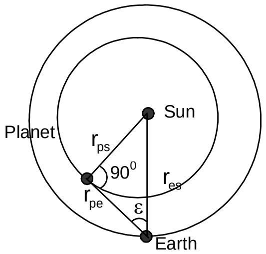
$D = b/\theta$

Knowing b and measuring θ, we can calculate D.

Copernicus method: This method is used to measure the relative distances of the planets from the Sun.

(a) For Interior Planets: The angle formed at earth between the earth–planet direction and the earth–sun direction is called the planet's elongation. This is the angular distance of the planet from the sun as observed from earth. When the elongation attains its maximum value ε as in the figure, the planet appears farthest from Sun.

$$r_{ps} = r_{es} \sin \varepsilon$$
$$= (\sin \varepsilon)\, AU \quad (AU = \text{Astronomical Unit})$$



(b) For Exterior Planets: This method is a consequence of Kepler's 3rd law of planetary motion. For two planets P_1 and P_2 we have,

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

where a_1 and a_2 are semi–major axes, of respective orbits. Period can be ascertained by direct observation. Therefore if a_1 is measured, a_2 can be calculated.

(iii) Chemical Method

This method is used to measure distance of the order of $10^{-1\circ}$ m. Let us calculate the size of an atom.

Let m = mass of substance,

V = volume occupied by substance &

ρ = density of the substance

$\therefore v = m/\rho \qquad (1)$

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ No. of atoms in mass m of the substance = Nm / M

If r = radius of each atom then V = volume of each atom = $\frac{4}{3}\pi r^3$

Volume of all the atoms in substance = $(\frac{4}{3}\pi r^3 \text{ x } Nm)M$.

According to Avagordo's hypothesis,

Volume of all the atoms = (2/3) x volume of substance

$$\frac{4}{3}\pi r^3 \text{ x } Nm/M = (2/3)\, m/\rho$$

$$\therefore r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$

MEASUREMENT OF MASS

Measurement of Inertial Mass

Inertial mass of a body is measured using a device which is known as inertial balance. It consists of a long metal strip. One end of the strip is clamped to a table such that its flat face is vertical, and it can easily vibrate horizontally. The other end of strip supports a pan in which the object whose inertial mass is to be found can be kept. It is found that the square of time period of vibration is directly proportional to total mass of the pan and the body placed in it.

$$t^2 \propto m$$

$$\therefore \quad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \quad m_2 = m_1 \frac{t_2^2}{t_1^2}$$

Measurement of Time: The following methods are used

(a) Quartz Crystal Clock
(b) Atomic Clock
(c) Radioactive dating

Significant figures:

Each measurement involves errors. The measure results has a number that includes all reliably known digits and first unknown digit. The combination of reliable digits and first uncertain digit are significant figures.

Example: If a length is measured as 2.43 cm then 2 and 4 are reliable while 3 is uncertain. Thus the measured value has three significant figures.

Common rules for counting significant figures

(1) All non zero digits are significant.
For example: 1745 has four significant digits.
(2) All zeros present between 2 non zero digits are significant, irrespective of the position of the decimal point.

Example: 208005 has 6 significant figures.
(3) If there is no decimal point, all zeros to the right of the right–most non zero digit are considered to be significant only if they come from a measurement.

Example: 41000 has only 2 significant digits while 41000 m has 5 significant digits.
(4) All zeros to the right of a decimal point but to the left of non–zero digits are considered to be non significant, provided there should be no non zero digit to the left of the decimal point.
Example: 0.00305 has 3 significant figures.
(5) All zeros are significant if they are placed to the right of a decimal point and to the right of a non zero digit.

Example: 0.04080 has 4 significant figures
50.000 has 5 significant figures

(6)The number of significant figures does not alter in different units.
If we want to write 450 m in different units, we can write it 4.50×10^4 cm or 4.50×10^5 mm etc. in which all of them are having 3 significant figures.

Illustration 9. *State the number of significant figures in the following –*
(a) 06500310
(b) 754400
(c) 15000 kg
(d) $8.314\times10^{+2}$ J
(e) 1.6×10^{-19} C
(f) 0.0065050

Solution: (a) 7 (b) 4 (c) 5 (d) 4 (e) 2 (f) 5

Exercise: ***The number of significant figures in 0.0160 is***
(a) 2 (b) 3 (c) 4 (d) 5

Rounding off

(1) If all the digits to be discarded are such that the first discarded digit is less than 5, the remaining digits are left unchanged.

Example:

7.499498 can be written in 4 significant figures as 7.499

(2) If the digit to be discarded is 5 followed by digits other than zero, then the preceding digit should be raised by 1.

Example:

7.45001, on being rounded off to first decimal, became 7.5

(3) If the digits to be discarded is 5 or 5 followed by zero the preceding digit remains unchanged if it is even and the preceding digit is raised by 1 if it is odd.

Example:

3.6500 will become 3.6 and 4.7500 will become 4.8 in 2 significant figures.

Arithmetic operations with significant figures:

(1) Addition and subtraction In addition and subtraction, the number of decimal places in the result is the smallest number of decimal places of terms in the operation.

Let us consider the sum of following measurements.
3.45 kg., 7.6 kg. and 10.055 kg.

$$\begin{array}{r} 3.45 \\ 7.6 \\ 10.055 \\ \hline 21.105 \\ \hline \end{array}$$

So the sum will be 21.1 kg as 7.6 kg has only 1 digit after the decimal point while the others are having more than one digit.

Multiplication and Division:

In the result of multiplication or division, the number of significant figures is same as the smallest number of significant figures among the numbers.

Illustration 9: *Multiply 1.21 and 1.1.*

Solution: $1.21\times 1.1 = 1.331$

So the result is 1.3 as there are only 2 significant digits in 1.1

The same procedure is followed for division.

Exercise: Value of 1.2 + 1.34 + 2.342 is

(a) 4.88 (b) 4.8 (c) 4.90 (d) 5

Accuracy and Precision of measuring instrument: It is impossible to measure any physical quantity perfectly. It is due to imperfection in manufacturing and working of measuring instruments.

Accuracy: It is the degree of correctness of the measured quantity, i.e. how much close the result is to the true value of the physical quantity.

Precision: It is the degree of repeatability & refinement of a measurement.

ERRORS IN MEASUREMENT

In the experiment we may get some other value than that of the true value due to faulty equipment, carelessness or random causes. This will cause error in measurement.

There are 3 ways to express an error

(1) Absolute Error: It is the positive value of difference between the true value and measured value of the quantity. Since we don't know the correct value of quantity the best possible value can be given by mean value of all the measured value.

Arithmetic mean v, $A_m = \dfrac{A_1 + A_2 + ...A_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} A_i$

$\therefore$ The absolute error in the measurement can be given as.

$\Delta A_1 = |A_m - A_1|$ where A_m : Mean value of the measurements.

$\Delta A_2 = |A_m - A_2|$ where A_1, A_2 : Measured value of quantity.

$\Delta A_n = |A_m - A_n|$

Taking the arithmetic mean of all the absolute errors we get the mean absolute error ΔA_m.

$$\Delta A_m = \frac{\Delta A_1 + \Delta A_2 + ... + \Delta A_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\Delta A_i$$

So the true value of A will be such that

$$\left(A_m - \Delta A_m\right) \le A \le \left(A_m + \Delta A_m\right)$$

(2) Relative Error: It is defined as the ratio of the mean absolute error to the mean value of the quantity being measured

Relative error $=\dfrac{\Delta A_m}{A_m}$

(3) Percentage Error: The relative error can be expressed in percentage error as

% error = Relative error $\times 100$

Propagation of Error: Any physical quantity depends on one or more than one physical quantities. So the error in any physical quantity will lead to error in the result.

(1) Error in result involving sum or difference of quantities

Let Z is defined as

$Z = A + B - C$

$\therefore \quad \Delta Z = \Delta A + \Delta B - \Delta C$

$\therefore$ Maximum possible error in Z is given by

$|\Delta Z|_{max} = \Delta A + \Delta B + \Delta C$ (Since ΔC can be positive or negative)

2. Error in the result having product or division of quantities:

$$Z = \frac{A^p B^q}{C^r}$$

$$\Rightarrow \quad \ln z = p\ln A + q\ln B - r\ln c$$

$$\Rightarrow \quad \frac{dz}{z} = \frac{pdA}{A} + \frac{qdB}{B} + \frac{rdC}{C}$$

For small change $dz \approx \Delta z.\ \Rightarrow \dfrac{\Delta z}{z} = P\dfrac{\Delta A}{A} + q\dfrac{\Delta B}{B} + r\dfrac{\Delta C}{C}$

Illustration10. *Multiply 107.88 by 0.610 and express the result upto the correct number of significant figure.*

(A) 65.8068 *(B 65.807*

(C) 65.81 *(D) 65.8*

Solution: Number of significant figures in multiplication is three corresponding to the minimum number 107.88×0.610

= 65.8068 = 65.8

$\therefore$ (D)

Illustration11. *In measurement of the period of oscillation of a Helical spring, the readings comes out to be 2.15 sec, 2.25 sec, 2.36 sec, 2.45 sec and 2.54 sec, calculate the absolute errors, relative error or percentage error.*

Solution: The mean period of oscillation of the Helical spring is

$$T = \frac{2.15 + 2.25 + 2.36 + 2.45 + 2.54}{5}$$

= 2.35 sec

The absolute error in the measurements are

2.15 – 2.35 = – 0.20 sec

2.25 – 2.35 = – 0.10 sec

2.36 – 2.35 = 0.01 sec

2.45 – 2.35 = 0.10 sec

2.54 – 2.35 = 0.19 sec

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is ΔT_{mean}= [(0.20 + 0.10 + 0.01 + 0.10 + 0.19)] /5

$$= \frac{0.6}{5} = 0.12 \sec$$

Period of oscillation of the simple pendulum is (2.35 ± 0.12) sec.
A more correct way to write its is
(2.4 ± 0.2) sec
The relative error or the percentage error is

$$= \frac{0.2}{2.4} \times 100 = 8\%$$

Combination of Errors

While doing an experiment we take several measurements, we must know how the errors in all the measurements combine.
To make such estimates, we should learn how errors combine in various mathematical operations. For this we use the following procedure

(I) Error of a sum or difference: Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors.

(a) We wish to find the error Δz in the sum
$z = A + B$

We have by addition, $z \pm \Delta z$

$= (A \pm \Delta A) + (B \pm \Delta B)$

The maximum possible error in $z = \Delta z = \Delta A + \Delta B$

(b) For the difference $z = A - B$, we have

$z \pm \Delta z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \pm \Delta B$

or, $\pm \Delta z = \pm \Delta A \pm \Delta B$

The maximum value of the error Δz is again $\Delta A + \Delta B$.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual.

Illustration 12. *The series combination of resistances is given by $R = R_1 + R_2$
Suppose two resistances $R_1 = (50 \pm 4)\Omega$ and $R_2 = (100 \pm 3)\ \Omega$ are connected in series. Find equivalent resistance of the series combination.*

Solution: $R_{eq} = R_1 + R_2$
$= (50 \pm 4)\ \Omega + (100 + 3)\ \Omega$
$= (150 \pm 7)\ \Omega$

(II) Error in a product or a quotient

Suppose $z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then
$z \pm \Delta z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B$

Dividing L.H.S. by z and R.H.S. by AB, we have

$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right).$$

Since ΔA and ΔB are small we shall ignore their product.
Hence the maximum fractional error in Z

$$= \frac{\Delta z}{z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Similarly, we can easily verify that this is true for division also.
So, when two or more quantities multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

(III) Error due to the power of a measured quantity.

Let $Z = X^2$

Then $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta X}{X} = \frac{2\Delta X}{X}$

Hence the fractional error in X^2 is two times the error in X.

In general if $Z = \frac{X^a Y^b}{Q^c}$

then $\frac{\Delta Z}{Z} = a\frac{\Delta X}{X} + b\frac{\Delta Y}{Y} + \left[\frac{\Delta Q}{Q} \times c\right]$

Illustration 13. *Find the fractional error in Z, if* $Z = \sqrt{\frac{XY}{M}}$

Solution: $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta X}{X} + \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta M}{M}$

Illustration 14. *Find maximum possible percentage error in* $x = \frac{a^{\ell} b^m}{y^p z^k}$

Solution: $\frac{\Delta X}{X} \times 100 = \left(\ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta y}{y} + k\frac{\Delta z}{z}\right) \times 100$

Illustration 15. *In the relation $x = 3yz^2$, x, y and z represents various physical quantities, if the percentage error in measurement of y and z is 3% and 1% respectively, then final maximum possible percentage error in x.*

Solution: $\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta y}{y} + 2\frac{\Delta z}{z}\right) \times 100$

$= 3\% + 2 \times 1\% = 5\%$

PHYSICAL QUANTITIES

All the physical quantities can be expressed in terms of some combination of seven base quantities: Length [L], mass [M], time [T], electric current [I], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol]. These base quantities are considered as the seven dimensions of the physical world.

DIMENSIONS

The dimension of a physical quantity are the powers to which the fundamental (or base) quantities like mass, length and time etc. have to be raised to represent the quantity. Consider the physical quantity **"Force"**. The unit of force is Newton.

1 Newton = 1 kg m/sec^2

kg $\to M^1$ (Mass); m $\to L^1$ (Length); $s^{-2} \to T^{-2}$ (Time)

$\therefore$ Dimensions of force are $[M^1L^1T^{-2}]$

Dimensional formulae for some physical quantities

Physical quantity	Relation with other quantity	Dimensional formula
Area	Length × breadth	$L \times L = [L^2]$
Density	Mass/volume	$\frac{M}{L^3} = [ML^{-3}]$
Acceleration	$\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = [LT^{-2}]$
Force	F = ma	$[MLT^{-2}]$
Linear momentum	P = mv	$[MLT^{-1}]$
Pressure	P = F/A	$[ML^{-1}T^{-2}]$
Universal gravitational constant	$G = \frac{Fr^2}{M_1M_2}$	$[M^{-1}L^3T^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (kinetic, potential and heat)	$\frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Surface tension	$T = \frac{F}{\ell}$	$[ML^\circ T^{-2}]$
Strain	$e = \frac{\Delta\ell}{\ell}$	$[M^\circ L^\circ T^\circ]$
Modulus of elasticity	$E = \frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	$[M^\circ L^\circ T^\circ]$
Coefficient of viscosity	$\eta = \frac{F \times r}{A \times v}$	$[M^1L^{-1}T^{-1}]$
Planck's constant	$h = mv\lambda$	$[ML^2T^{-1}]$
Thermal resistance	$\frac{\Delta\theta t}{Q}$	$[M^{-1}L^{-2}T^3\,\theta]$
Thermal conductivity	$K = \frac{H}{At(d\theta/dx)}$	$[MLT^{-3}\theta^{-1}]$
Boltzman's constant	k = R/N	$[ML^2T^{-2}\theta^{-1}]$
Universal gas constant	$R = \frac{PV}{T}$	$[ML^2T^{-2}\theta^{-1}]$
Mechanical equivalent of heat	J = W/H	$[M^\circ L^\circ T^\circ]$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$	$[M^\circ L^\circ T^{-1}]$

Illustration 16. *Write the dimensions of: Impulse, Pressure, Work, Universal constant of Gravitation.*

Solution: (i) $[M^1L^1T^{-1}]$ (ii) $[M^1L^{-1}T^{-2}]$ (iii) $[M^1L^2T^{-2}]$ (iv) $[M^{-1}L^3T^{-2}]$

Four types of quantities

Dimensional constant: These are the quantities whose values are constant and they possess dimensions. For example, velocity of light in vacuum, universal gas constant etc.

Dimensional variables: These are the quantities whose values are variable, and they possess dimensions. For example, area, volume, density etc.

Dimensionless constants: These are the quantities whose values are constant, but they do not possess dimensions. For example, π, 1, 2, 3, ….. etc.

Dimensionless Variables: These are the quantities, whose values are variable, and they do not have dimensions, e.g., angle, strain, specific gravity etc.

Uses of dimensions: dimensional analysis

(1) Checking the correctness (dimensional consistency) of an equation: An equation contains several terms which are separated from each other by symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This means that we can not add velocity to force. This principle is called Principle of Homogeneity of dimensions.

Look at the equation : $v^2 = u^2 + 2as$

Dimensions of $v^2 : [L^2T^{-2}]$

Dimensions of $u^2 : [L^2T^{-2}]$

Dimensions of $2as : LT^{-2}][L] = [L^2T^{-2}]$

$\therefore$ The equation $v^2 = u^2 + 2as$ is dimensionally consistent, or dimensionally correct.

Note:

A dimensionally correct equation may not be actually correct. For example, the equation $v^2 = u^2 + 3as$ is also dimensionally correct but we know that it is not actually correct. However, all correct equations must necessarily be dimensionally correct.

Illustration 17. *Which of the following equations may be correct ?*

(i) $x = ut + \frac{1}{2}at^2$ *(ii)* $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $F = \frac{GM_1M_2}{r}$ *(iv)* $T^2 = \frac{4\pi^2R^3}{GM}$

(v) $V = \sqrt{GMR}$

Given: G = Gravitational constant, whose dimensions are $[M^{-1}L^3T^{-2}]$

M_1, M_2 *and M have dimensions of mass. L, x, r, R has dimenssions of length. And t has dimensions of Time. 'F' denotes Force and 'a' has dimensions of acceleration.*

Solution: (i) Yes (ii)Yes (iii) No (iv) Yes (v) No.

(2) Conversion of units: Dimensional methods are useful in finding the conversion factor for changing the units to a different set of base quantities. Let us consider one example, the SI unit of force is Newton. The CGS unit of force is dyne. How many dynes is equal to one newton. Now,

$$1 \text{ newton} = [F] = [M^1L^1T^{-2}] = (1\text{kg})^1)(1\text{meter})^1(1\text{s})^{-2}$$

$$1\text{dyne} = (1\text{g})(1\text{cm})(1\text{s})^{-2}$$

$$\therefore \frac{1\text{newton}}{1\text{dyne}} = \frac{(1\text{kg})^1(1\text{meter})^1(1\text{s})^{-2}}{(1\text{g})(1\text{cm})(1\text{s})^{-2}} = (10^3)(10^2) = 10^5$$

$$1 \text{ newton} = 10^5 \text{ dynes}$$

Thus knowing the conversion factors for the base quantities, one can work out the conversion factor of any derived quantity if the dimensional formula of the derived quantity is known.

Illustration 18. *Find the conversion factor for expressing universal gravitational constant from SI units to cgs units.*

Solution: $6.67\text{x}10^{-8}\ \text{cm}^3\text{s}^{-2}\text{g}^{-1}$

(3) Deducing relation among the physical quantities:

Suppose we have to find the relationship connecting a set of physical quantities as a product type of dependence. Then dimensional analysis can be used as a tool to find the required relation. Let us consider one example. Suppose we have to find the relationship between gravitational potential energy of a body in terms of its mass 'm', height 'h' from the earth's surface and acceleration due to gravity 'g', then,

Let us assume that: – Gravitational potential energy, U,

$$U = K[m]^a[g]^b[h]^c,$$

where K, a, b, and c are dimensionless constants.

Then $[ML^2T^{-2}] = [M]^a[LT^{-2}]^b[L]^c$

$$= [M^aL^{b+c}T^{-2b}]$$

$\therefore$ a =1, b + c = 2

–2b = –2

b = 1, c = 1.

$\therefore$ U = Kmgh, where K is a dimensionless constant.

Thus by dimensional analysis, we conclude that the gravitational potential energy of a body is directly proportional to its mass, acceleration due to gravity and its height from the surface of the earth.

Limitations of dimensional analysis:

This method does not give us any information about the dimensionless constants appearing in the derived formula, e.g. 1, 2, 3, .. . π etc.

We can't derive the formula having trigonometrical functions, exponential functions etc, which have no dimensions.

The method of dimensions cannot be used to derive an exact form of relation when it consists of more that one part on any side, e.g. the formula $v^2 = u^2 + 2as$ cannot be obtained.

If a quantity depends on more than three factors having dimensions the formula cannot be derived. This is because on equating powers of M, L and T on either side of the dimensional equation, we can obtain three equations from which only three exponents can be calculated.
It gives no information whether a physical quantity is a scalar or a vector.

Illustration 19. *Using the method of dimensions, find the acceleration of a particle moving with a constant speed v in a circle of radius r.*

Solution: Assuming that the aceeleration of a particle depends on v and r

$a \propto v^x r^y \Rightarrow a = k\, v^x r^y$

Now as we know dimensions of acceleration (a) = $M°LT^{-2}$

and dimensions of velocity (v) = $M°LT^{-1}$

dimension of radius (r) = $M°LT°$

Putting all thee dimensions in (1), we get

$M°LT^{-2} = k\,(M°LT^{-1})^x\,(M°LT°)^y$

$M°LT^{-2} = k\,M°L^{x+Y}T^{-x}$

Comparing the powers, we get

$x + y = 1$

$x = 2$

$\therefore\ y = 1–2 = –1$

$\therefore\ a = k\, v^2 r^{-1}$

$a = \dfrac{kv^2}{r}$

Illustration 20. *In the expression* $\left(P + \dfrac{a}{v^2}\right)(v - b) = RT$

P is pressure and v is the volume. Calculate the dimensions of a and b.

Solution: Only physical quantities having same dimensions are added or subtracted. So $\dfrac{a}{v^2}$ has the same dimensions as that of pressure.

As pressure = $\dfrac{\text{Force}}{\text{Area}}$

Dimensions of pressure $= \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$\therefore$ Dimensions of $\dfrac{a}{v^2} = ML^{-1}T^{-2}$

Dimensions of a $= ML^{-1}T^{-2}(V^3)^2$

$= (ML^{-1}T^{-2})(L^3)^2$

$= ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

Similarly dimensions of b is same as that of volume.

Dimensions of $b = M^0L^3T^0$.

Illustration 21. *Does* $S_{nth} = u + \dfrac{a}{2}(L_n - 1)$ *dimensionally correct?*

Solution: Yes, this expression is dimensionally correct, yet it appears to be incorrect. As we are taking it to be for n^th second. Here one second is divided through the equation.

Illustration 22. *Find the dimensions of resistivity, thermal conductivity and coefficient of viscosity.*

Solution: (i) $R = \rho\dfrac{\ell}{A}$

$\rho = \dfrac{RA}{L} = [ML^3T^{-3}A^{-2}]$

(ii) Thermal conductivity, k

$\dfrac{d\theta}{dt} = \dfrac{k\ell}{A\Delta\theta} = \dfrac{ML^2T^{-3}L}{L^2K}$

$= MLT^{-3}k^{-1}$

(iii) Coefficient of viscosity

$\therefore\ F = \eta A\dfrac{dv}{dx}$

$\eta = \dfrac{Fdx}{Adv} = \dfrac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

Illustration 23. *A displacement of a particle is given by equation y = A sin ωt, where y is in metres and A is also in metres, t is in seconds. What are the dimensions of ω.*

Solution: As the angles are always dimensionless, so

ωt = dimensionless quantity

Dimensions of ωt = $M^\circ L^\circ T^\circ$

Dimensions of ω = $M^\circ L^\circ T^{-1}$

Illustration 24. *If density ρ, acceleration due to gravity g and frequency f are the basic quantities, find the dimensions of force.*

Solution: We have $\rho = ML^{-3}$, $g = LT^{-2}$ $f = T^{-1}$

Solving for M, L and T in terms of ρ, g and f , we get

$M = \rho^2 g^3 f^{-6}$, $L = gf^{-2}$ & $T = f^{-1}$

Force = $[MLT^{-2}] = [\rho g^3 f^{-6}.gf^{-2}.\ f^2] = [\rho g^4 f^{-6}]$

Illustration 25. *An athlete's coach told his team that muscle times speed equals power. What dimensions does he view for "muscle"?*

(A) MLT^2 *(B) ML^2T^{-2}*

(C) MLT^{-2} *(D) L*

Solution: Power = force × velocity

= muscle times speed

∴ muscle represents force

muscle = $[MLT^{-2}]$

∴ (C)

Illustration 26. *If force, length and time would have been the fundamental units what would have been the dimensional formula for mass*

(A) $FL^{-1}T^{-2}$ *(B) $FL^{-1}T^2$*

(C) FLT^{-2} *(D) F*

Solution: Let $M = K\,F^aL^bT^c$

$= [MLT^{-2}]^a\,[L^b]\,T^c = [M^aL^{(a+b)}T^{(-2a+c)}]$

$a = 1, a + b = 0 \quad \& \; -2a + c = 0$

$\Rightarrow a = 1, b = -1, \; c = 2$

$\therefore$ (B)

Illustration 27. *The dimensions of the Rydberg constant are*

(A) $M^\circ L^{-1} T$ (B) MLT^{-1}

(C) $M^\circ L^{-1} T^\circ$ (D) $ML^\circ T^2$

Solution: From the relation $\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$R = \frac{1}{L} = L^{-1} = M^\circ L^{-1} T^\circ$

$\therefore$ (C)

Illustration 28. *The error in the measurement of the radius of a sphere is 1%. Then error in the measurement of volume is*

(A) 1% (B) 5%

(C) 3% (D) 8%

Solution: $V = \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r}\right) \times 100 = 3 \times 1 = 3\%$

$\therefore$ (C)

MISCELLANEOUS EXERCISE

1. The time period of a gas bubble formed under water oscillating with a time period depending on static pressure P, density of water ρ and ε total energy of explosion. Find the relationship between T, P, ρ and ε.

2. Name the three physical quantities having the same dimensions

3. A student measures the time period of a simple pendulum. If error in measurement of length is 2% and error in measurement of g is 2% calculate the error in measurements of Time period.

4. A physical quantity is given by x = a + bt, where x is in metres and t is in seconds. So calculate the dimensions of a and b.

5. Find the dimensional formulae of the following quantities
   (A) the universal gravitational constant
   (B) Surface tension
   (C) Potential energy
   (D) Surface energy

6. A Vernier calliper has 50 divisions on its Vernier scale. Minimum division in main scale is of 1 mm.
   (a) Find out the least count of the Vernier Calliper.
   (b) During a length measurement, zero of Vernier scale is between 12^{th} and 13^{th} divisions of the main scale and 26^{th} division of the Vernier scale coincides with a main scale division. What is the length ?

7. Calculate the focal length of a spherical mirror if measured quantities u and v are as follows.
   $u = 50.1 \pm 0.5\,\text{cm}$
   $v = 20.1 \pm 0.2$ cm

8. Young's modulus of steel is 19×10^{10} N/m^2. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.

9. Convert 1 joule into ergs.

10. A goldsmith puts some gold weighting 5.42 gm in a box weighing 1.2 kg. Find the total weight of the box to correct number of significant figures.

SOLUTION TO MISCELLANEOUS EXERCISE

1. $T \propto P^{-5/6}\rho^{1/2}E^{1/3}$
2. Work, energy and torque
3. 2%
4. x = a + bt
   Dimensions of a = $M^{\circ}LT^{\circ}$
   Dimensions of b = $M^{\circ}LT^{-1}$
5. (a) $M^{-1}L^{2}T^{-2}$ (b) MLT^{-2}
   (c) $ML^{2}T^{-2}$ (d) $ML^{2}T^{-2}$
6. 0.02 mm, 12.52 mm
7. 14.3 ± 0.4 cm
8. 19×10^{11} dyne / cm^2
9. 1 joule = 10^7 ergs
10. 1.2 kg

SOLVED PROBLEMS

Subjective:

Prob 1. *The Bernoulli's equation is given by,*
$P+\frac{1}{2}\rho v^2+\rho gh = constant$ *, where P is pressure. Compare the unit of the quantity* $\frac{1}{2}\rho v^2$ *with the unit of pressure.*

Sol. Only same quantities can be summed up or subtracted from each other. So $\frac{1}{2}\rho v^2$ has same unit as that of pressure.

Prob 2. *The relation between velocity and time of a moving body is given as,* $v = A+\frac{B}{t}+Ct^2$. *Find the units of A, B and C.*

Sol. From the principle of homogeneity
v = A = m/sec
v = B/t $\Rightarrow$ B = m
v = (ct^2)
$\therefore$ c = m/sec^3

Prob 3. *The external and internal diameters of a hollow cylinder are measured to be (4.23 ±0.01) cm and (3.89 ± 0.01) cm. What is the thickness of the wall of the cylinder?*

Sol. Thickness = (4.23 – 3.89) cm = $\frac{0.34}{2}$ = 0.17 cm.
Error = ±(0.01 + 0.01) cm = ±0.02 cm

Prob 4. *A physical quantity x is calculated from the relation* $x = \frac{a^2b^3}{c\sqrt{d}}$. *If percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively. What is the percentage error in x?*

Sol. As $x = \frac{a^2\ \ b^3}{c\sqrt{d}}$

$$\frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right]$$

$$= \pm\ [2 \times 2\% + 3\times1\% + 3\% + \frac{1}{2}\times 4\% = \pm\ 12\%$$

Prob 5. *A physical quantity A is defined as,* $A = \mathrm{pkx^2y/z}$. *The absolute errors in the measured of x, y, z are given as*
x = (0.26 ± 0.02)cm
y = (64 ± 2) Ω
z = (156.0 ± 0.1)cm
Find the percentage error in the quantity A.

Sol. $a = kx^2y / z$

$\Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$

$= 2 \times \frac{0.02}{0.26} + \frac{2}{64} + \frac{0.1}{156}$

$\pm 0.186 = 18.6\ \%$

Prob 6. *10 rotations of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 divisions. Find the least count. A reading taken for the diameter of wire with the screw gauge shows 4 complete rotations and 35 on the circular scale. Find the diameter of the wire.*

Sol. The least count = $\frac{5}{1000}$ = 0.005 mm

The diameter of the wire = $(4 \times 0.5 + 35 \times 0.005)$ mm = 2.175 mm

Prob 7. *The diameter of a sphere is 2.78 cm. Calculate its volume in proper significant figures.*

Sol. Volume = $\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi\left(\frac{2.78}{2}\right)^3$ cm^3 = 11.2437 cm^3

Hence the volume in proper significant figures is 11.2 cm^3

Prob 8. *Calculate the number of light years in one meter.*

Sol. We know 1 light year (ℓ_y) = 9.46×10^{15}m

or 9.46×10^{15} m = $1\ell y$

1 m = $1.057 \times 10^{-16} \ell y$

Prob 9. *Find the dimensions of a and b in the relation* $P = \frac{b - x^2}{at}$

where P is power, x is distance and t is time.

Sol. The given relation is, $P = \frac{b - x^2}{at}$

As x^2 is subtracted from b therefore the dimensions of b are of x^2

i.e. $b = L^2$

We can rewrite relation as

$P = \frac{[L^2]}{at} = \frac{L^2}{at}$

$a = \frac{L^2}{[ML^2T^{-3}]\ [T]} = M^{-1}L^{\circ}T^2$

Prob 10. *It is claimed that two cesium clocks if allowed to run for 100 years free from any disturbance may differ by only about 0.02 sec. What is the accuracy of the standard cesium clock in measuring a time interval of 1 sec?*

Sol. $\because$ t = 100 years = 100×365.25×86400s

$\Delta t = 0.02s$

Fractional error = $\frac{\Delta t}{t} = \frac{0.02}{100\times365.25\times86400}$

$= 0.63 \times 10^{-11}$

So, there is an accuracy of 10^{-11} Part in 1 or 1 sec in 10^{11} sec.

Prob11. *In screw gauge no. of division on circular scale is n and circular scale travels a distance of a units in one rotation. Calculate least count of the screw gauge.*

Sol. Pitch = a units

Least count $= \frac{\text{Pitch}}{\text{No. of divisions on circulat scale}}$

$= \frac{a}{n}$ units.

Prob 12. *The diameter of the spherical bob is measured by vernier Calipers (10 divisions of a Vernier scale coincide with a divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 12 mm and 7th division of the main scale coincides with the main scale. Mass of the sphere is 4.532 g. Find the density of the sphere.*

Sol. Vernier constant = 1.M.S.D. – 1.V.S.D.

$= 1\text{ mm} - \frac{9}{10}\text{ mm}$

= 0.1 mm

Diameter of sphere = 12 mm + 0.1 × 7

= 12.7 mm

$\therefore$ Volume of sphere = $= \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

$= \frac{4}{3}\times3.14\times\left(\frac{12.7}{2}\times10^{-3}\right)^3$

Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{4.532\times3\times8\times10^{-3}}{4\times3.14\times(12.7\times10^{-3})^3}$

= 4.227 kg /m^3

= 4.23 kg/m^3 (in appropriate significant figures)

Prob 13. *A wire of length ℓ = 8 ± 0.02 cm and radius r = 0.2 ± 0.02 cm and mass m = 0.1 ± 0.001 gm. Calculate maximum percentage error in density*

Sol. $\rho = \frac{m}{\pi r^2 \ell}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta\ell}{\ell}$

$\Delta\ell$ = 0.02 cm, ℓ = 8 cm

Δr = 0.02 cm, r = 0.02 cm

m = 0.1 gm, Δm = 0.001 gm

$\frac{\Delta\rho}{\rho} = \frac{0.001}{0.1} + 2\times\frac{0.02}{0.2} + \frac{0.001}{0.1}$

$$= \left(\frac{1\times10}{1000\times1} + \frac{2\times2}{100\times2}\times10 + \frac{1\times10}{1000\times1}\right)\times100$$

$$= \frac{(1+20+1)}{100}\times100$$

= 22%

Prob 14. *Planck's formula is given by*

$$u = \frac{\hbar\omega^3}{\pi^2 e^3}\times\frac{1}{e^{\hbar\omega/k^{-1}-1}}$$

where u is the energy radiated per unit area per unit time and h is Planck's constant. What will be the dimensions of k in the expression.

Sol. The power in exponential is always dimensionless. So,

$$\frac{\hbar\omega}{kT} = M^0L^0T^0$$

E = hv

$$\text{so, } h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^0L^0T^{-1}}$$

$= ML^2T^{-1}$

$$\therefore \quad k = \frac{\hbar\omega}{T}$$

$$= \frac{ML^2T^{-1}T^{-1}}{T} = ML^2T^{-3}$$

Prob15. *According to Stoke's law the viscous force acting on a spherical body moving fluid depends on radius r of the body, co–efficient of viscosity η of the fluid and velocity f the body. Find the relation between F, η, r, v.*

Sol. Force acting on a spherical body depends on

$F \propto \eta^a r^b v^C$

$F = k\eta^a r^b v^c$

$(MLT^{-2}) = k\,(ML^{-1}T^{-1})^a\,(L)^b\,(LT^{-1})^c$

$MLT^{-2} = k\,(M)^a\,(L)^{-a+b+c}\,(T^{-a-c})$

a = 1

–a + b + c = 1

– a –c = –2

–1 –c = – 2

c = – 2 = 1

– a + b + c = 1

– 1 + b + 1 = 1

$\Rightarrow$ b = 1

F = k η r v

Objective:

Prob 1. *An experiment measures quantities a, b, c and x is calculated from $x = ab^2/c^3$. If the maximum percentage error in a, b and c are 1%, 3% and 2% respectively, the maximum percentage error in x will be*

(A) 13% *(B) 17%*
(C) 14% *(D) 11%*

Sol. (A) Maximum percentage error in x

As $x = \frac{ab^2}{c^3}$

$$\frac{\Delta x}{x} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

$$\frac{\Delta x}{x} = 1\% + 2 \times 3\% + 3 \times 2\%$$

$= (1 + 6 + 6)\% = 13\%$

Prob 2. *If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second, then non–zero integers x, y and z, such that $P^xQ^yc^z$ is dimensionless, may be*

(A) x = 1, y =1, z = 1. *(B) x = 1, y =–1, z = 1.*
(C) x = –1, y =1, z = 1. *(D) x = 1, y = 1, z = 1*

Sol. As $P^xQ^{y+}C^z$ is a dimensionless

$$\left(\frac{MLT^{-2}}{L^2}\right)^x \left(\frac{ML^2T^{-2}}{L^2T}\right)(LT^{-1})^2 = M^0L^0T^0$$

$(M^1L^{-1}T^{-2})^x (ML^{\circ}T^{-3})^y (LT^{-1})^2 = (M^{\circ}L^{\circ}T^{\circ})$

Comparing powers, we get

$x + y = 0$ . . . (i)

$-x + z = 0$ . . . (ii)

$-2x - 5y - z = 0$ . . . (iii)

From (1) and (2),

$y = -x, z = x$

Substituting in (3), we get

If x = k

y = –k, z = k

x = 1, y = –1, z = 1

Prob 3. *The dimensional representation of Planck's constant is identical to that of*

(A) Torque *(B) Power*
(C) Linear momentum *(D) angular momentum*

Sol. (D) As Planck's constant has dimensions of $\frac{E}{\nu}$

$= \dfrac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

and Dimensions of angular momentum = r × p

$= (L \times MLT^{-1})$

$= ML^2T^{-1}$

Prob 4. *The parallel combination of two resistances is given by If the two resistances $R_1 = (2 \pm 0.2)\Omega$ and $R_2 = (1 \pm 0.1)\Omega$ are connected in parallel. Then the % error is given by*

(A) 0.1% *(B) 0.2%*

(C) 0.3% *(D) 0.4%*

Sol. (C) $R_p = \dfrac{R_1R_2}{R_1 + R_2}$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right) \times 100$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{0.2}{2} + \frac{0.1}{1} + \frac{0.2 + 0.1}{3}\right)$$

(0.1 + 0.1 + 0.1)

= 0.3%

Prob5. *If the units of M and L are quadrupled, then the units of torque becomes*

(A) 16 times *(B) 64 times*

(C) 8 times *(D) 4 times*

Sol. (B)

Dimensions of torque = ML^2T^{-2}

$= (4M)(4L)^2T^{-2}$

$= 64\,ML^2T^{-2}$

Prob6. *A radar signal is beamed towards a planet from earth and its echo is received seven minutes later. If distance between the planet and earth is $6.3 \times 10^{1°}$m, then velocity of the signal will be*

(A) 3×10^8 m/s *(B) 2.97×10^6 m/s*

(C) 3.10×10^5 m/s *(D) 300 m/s*

Sol. (A).

Velocity of signal, $c = \dfrac{2x}{t} = \dfrac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ m/s

Prob7. *If speed of light c, acceleration due to gravity g and pressure P are taken as fundamental units, then the dimensions of gravitational constant is*

(A) $[c^{\circ}gP^{-3}]$ *(B) $[c^2g^3P^{-2}]$*

(C) $[c^{\circ}g^2P^{-1}]$ *(D) $[c^2g^2P^{-2}]$*

Sol. (C).

Let $G = c^x g^y P^z$

$\Rightarrow [M^{-1}L^3T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [ML^{-1}T^{-2}]^z$

$= [M^z L^{x+y} T^{-x-2y-2z}]$

Comparing powers of M, L and T on both sides, we get

$z = -1,\ x + y = 3,\ -x - 2y - 2z = -2$

On solving these equations for x, y and z, we get

$x = 0,\ y = 2,\ z = -1$

$\Rightarrow G = [c^\circ g^2 P^{-1}]$.

Prob 8. *The time dependence of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and t is time. The constant α*

(A) is dimensionless *(B) has dimensions T^{-2}*

(C) has dimensions of P *(D) has dimensions T^2*

Sol. (B).

$P = P_0 [\exp(-\alpha t^2)]$.

Since αt^2 must be dimensionless, so $\alpha = \frac{1}{T^2} = T^{-2}$

Prob 9. *The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is*

(A) hertz *(B) metre*

(C) radian *(D) second*

Sol. (A).

Here, kt is dimensionless. Hence, $k = 1/t = \sec^{-1}$ = hertz

Prob10. *The parallel of a heavenly body measured from two points diametrically opposite on the equator of earth is 1.0 minute. If the radius of earth is 6400 km, find the distance of the heavenly body from the centre of earth in AU. Take 1 AU = 1.5×10^{11} m.*

(A) 0.293 AU *(B) 0.28 AU*

(C) 2.01 AU *(D) 3.97 AU*

Sol. (A).

Here, $\theta = 1' = \frac{1^\circ}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

ℓ = diameter of earth = 2×6400 km

$= 1.28 \times 10^4$ km $= 1.28 \times 10^7$ m

Now, $\ell = r\theta$

$\Rightarrow r = \frac{1.28 \times 10^7}{(\pi/60) \times 180} = 4.4 \times 10^{10}$ m

$r = \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} = 0.293$ AU

Prob11. *Dimensions of ohm are same as (h is Planck's constant and e is charge)*

(A) $\frac{h}{e}$ *(B)* $\frac{h^2}{e}$

(C) $\frac{h}{e^2}$ *(D)* $\frac{h^2}{e^2}$

Sol. (C).

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}] = \text{resistance}$$

Prob12. *Which of the following is a derived unit?*

(A) newton *(B) joule*
(C) pascal *(D) metre*

Sol. A, B, C.
Because, they are derived from the fundamental units, i.e. kg, m and sec.

Prob13. *Which of the following equations is dimensionally correct?*

(A) Pressure = energy per unit volume
(B) Pressure = energy per unit area
(C) Pressure = force per unit volume
(D) Pressure=momentum per unit volume

Sol. $\frac{\text{Energy}}{\text{Volume}} = \left[\frac{\frac{1}{2} mv^2}{\text{volume}}\right]$

$\Rightarrow \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

$\therefore$ (A)

Prob 14. *Which of the following is/are dimensional constants is*

(A) Planck's constant *(B) dielectric constant*
(C) relative density *(D) gravitational constant*

Sol. A Planck's constant and gravitational constant G have constant values and dimensions
$\therefore$ A, D

Prob 15. *Which of the following is not a unit of time*

(A) solar year *(B) tropical year*
(C) leap year *(D) light year*

Sol. Tropical year is the year in which there is total eclipse.
Light year represents distance
$\therefore$ (D)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. If force acting on a particle depends on the x–coordinates as $F = ax + bx^2$, find the dimensions of 'a' and 'b'.

2. If velocity, time and force are chosen as basic quantities, find the dimensions of mass.

3. Find the dimensional formula of
   (a) Charge Q
   (b) The potential V
   (c) The capacitance C,
   (d) The Resistance, R

4. Which of the following have same dimensions?
   (A) angular momentum and linear momentum
   (B) work and power
   (C) work and torque
   (D) Torque and Pressure

5. The Van der Waals interaction between two molecules separated by a distance r is given by the energy $E = -\frac{A}{r^6} + \frac{B}{r^{12}}$. Find the dimensions of A and B.

6. If error in measuring diameter of a circle is 4%, find the error in radius of circle.

7. Derive, by method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (ν) and the amplitude of vibration (r)

8. Write the dimensions of $\frac{a}{b}$ in the relation
   $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

9. Assuming that the mass of the largest stone that can be moved by a flowing river depends upon the velocity r, the density ρ and acceleration due to gravity, show that m varies with sixth power of the velocity of flow.

10. The density of a material in cgs system is 8 gcm^{-3}. In a system of units, in which unit of length is 5 cm and unit of mass is 20 g, what is the density of the material ?

11. To study the flow of a liquid through a narrow tube the following formula is used
    $\eta = \frac{\pi\rho r^4}{8v\ell}$ where the letters have their usual meanings. The values of ρ, r, v and ℓ are measured to be 76 cm of Hg, 0.28 cm, 1.2 cm^3s^{-1} and 18.2 cm respectively. If these quantities are measured to the accuracy of 0.5 cm of Hg, 0.01 cm, 0.1 $cm^3\ s^{-1}$ and 0.1 cm respectively, find the percentage error in the value of η.

12. The equation of a wave is given by $y = A \sin \omega \left(\frac{x}{v} - k \right)$, where ω is angular velocity and v is linear velocity. Find the dimension of k. Given that

13. The surface tension of a liquid is 70 dyne/cm. Express it in MKS system of units?

14. Name a physical quantity which has same unit as that of Torque.

15. If all measurements in an experiment are taken upto same number of significant figures then mention two possible reasons for maximum error.

Level – I

1. The mass of a block is 87.2 g and its volume is 25 cm^3. What is its density upto correct significant figures?

2. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants, λ is wavelength. Then calculate the dimensions of A and B.

3. Suppose, the torque acting on a body, is given by $\tau = KL + \frac{MI}{\omega}$

   Where L = angular momentum, I = moment of inertia & ω = angular speed
   What is the dimensional formula for KM?

4. When a current of (2.5 ± 0.5). A flows through a wire it develops a potential difference of (20 ± 1) V. What is the resistance of wire?

5. Find out the result in proper significant figures, 291 x 0.03842 / 0.0080.

6. The radius of a sphere is (5.3 ± 0.1) cm. Find the percentage error in its volume.

7. If Planck's constant h; the velocity of light, c and Newton's gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of these quantities using dimensional notation.

8. What will be the unit of time in the system in which the unit of length is meter, unit of mass is kg and unit of force is kg. wt.?

9. Imagine a system of units in which the unit of mass is 10 kg, length is 1 km and time is 1 minute, then calculate the value of 1 J in this system.

10. A screw gauge of pitch 0.5 mm has a circular scale divided into 5 divisions. The screw gauge is used to measure the thickness of a coin. The main scale reading is 2 mm and 35^{th} circular division coincides with main scale with a positive zero error of divisions. Find the thickness of the coin

11. A Vernier Calliper is used to measure the thickness of the wall of cylinder by measuring its external and internal diameters. For external diameter, the zero if the Vernier scale coincides with the 5^{th} division of main scale and 6^{th} division of Vernier scale coincides with the main scale.
    For internal diameter, the zero of the Vernier scale coincides with the 3^{rd} division of mass scale and 2^{nd} division of Vernier scale coincides with 3^{rd} division of main scale and 2^{nd} division of Vernier scale coincides with main scale. Given that 1 main scale division is equal to 10 m
    1 V.S.D. = 0.09 cm.
    Calculate the thickness of the wall of a cylinder.

12. The time period of small oscillations of a spring mass system is given as $T = 2\pi\sqrt{\frac{m}{k}}$. What will be the accuracy in the determination of k if mass m is given as 10 kg with accuracy of 10 gm and time period is 0.5 sec measured for time of 100 oscillations with a watch of accuracy of 1 sec.

13. In a screw micrometer, main scale divisions are in mm. There are 100 cap divisions.
    (a) Find out the least count of the micrometer.
    (b) In fully closed condition, 4^{th} division of the cap scale coincides with the line of graduation along the screw axis. What is the zero error of the instrument ? Is it to be added or subtracted from the observed reading during a measurement ?
    (c) In the above instrument, during a measurement, the cap is between 7^{th} and 8^{th} divisions of the main scale and 37^{th} division of cap scale coincides with the line of graduation of the main scale. What is the measurement corrected for zero error ?

14. The equation for energy (E) of a simple harmonic oscillator,
    $E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2x^2$
    is to be made "dimensionless" it with multiplying by a suitable factor, which may involve the constants, m(mass), ω(angular frequency) and h (Planck's constant). What will be the unit of momentum and Length ?

15. In in the equation $F = A \sin Bx^2 + \frac{C}{t}e^{Dt}$, F, x and t are force, position and time respectively, then give the dimensions of $\frac{A}{CB}$.

Objective:

1. Which of the following is a possible dimensionless quantity?
   (A) Velocity gradient (B) Pressure gradient
   (C) Displacement gradient (D) Force gradient

2. Dimensional formula of thermal conductivity is
   (A) $ML^2T^{-3}\theta^{-1}$ (B) $ML^2T^{-2}\theta^{-4}$
   (C) $ML^2T^{-2}\theta^{-1}$ (D) $MLT^{-3}\theta^{-1}$

3. The unit of power is
   (A) kilowatt hour (B) joule
   (C) dyne (D) kilo watt

4. The dimensional representation of Planck's constant is identical to that of
   (A) torque. (B) power.
   (C) linear momentum. (D) angular momentum.

5. Which of the following is a fundamental quantity?
   (A) volume (B) velocity
   (C) time (D) force

6. The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is
   (A) hertz (B) metre
   (C) radian (D) second

7. The dimensional representation of Planck's constant is identical to that of
   (A) torque (B) work
   (C) stress (D) angular momentum

8. A force F is given by $F = \frac{a}{t} + bt^2$, where t is time. The dimensions of a and b are
   (A) $[MLT^{-3}]$ and $[MLT^{-4}]$ (B) $[MLT^{-4}]$ and $[MLT^{-3}]$
   (C) $[MLT^{-1}]$ and $[MLT^{-4}]$ (D) $[MLT^{-2}]$ and $[MLT^{\circ}]$

9. A unit–less quantity
   (A) may have non–zero dimensions (B) always has non–zero dimensions
   (C) never has a non–zero dimensions (D) does not exist

10. Joule $\times$ sec is the unit of
    (A) energy (B) momentum
    (C) angular momentum (D) power

11. Given that v is speed, r is radius and g is gravitational acceleration, which of the following expression is dimensionless.
    (A) $\frac{v^2}{gr}$ (B) $\frac{v^2 r}{g}$
    (C) $\frac{v^2 g}{r}$ (D) $v^2 rg$

12. The dimensional formula for modulus of rigidity is
(A) $[ML^2T^{-2}]$ (B) $[ML^{-1}T^{-3}]$
(C) $[ML^{-2}T^{-2}]$ (D) $ML^{-1}T^{-2}]$

13. A highly rigid cubicle block A of small mass m and side L is rigidly fixed to an other similar cubical block of low modulus of rigidity η. Lower face of A completely covers the upper face of B. The lower face of B is rigidly held on horizontal surface. A small force T is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes simple harmonic motion, the time period of which is given by
(A) $2\pi\sqrt{m\eta L}$ (B) $2\pi\sqrt{m\eta / L}$
(C) $2\pi\sqrt{mL/\eta}$ (D) $2\pi\sqrt{m/\eta L}$

14. The time period of a soap bubble is $T \propto P^a d^b S^c$, where P is pressure, d is density and S is surface tension, then values of a, b and c, respectively, are
(A) –1, –2, 3 (B) –3/2, 1/2 1
(C) 1, –2, –3/2 (D) 1, 2, 1/3

15. The dimensional formula for specific resistance in term of M, L, T and Q is
(A) $[ML^3T^{-1}Q^{-2}]$ (B) $[ML^2T^{-2}Q^2]$
(C) $[MLT^{-2}Q^{-1}]$ (D) $[ML^2T^{-2}Q^{-2}]$

16. Which of the two have same dimensions?
(A) Force and strain (B) Force and stress
(C) Angular velocity and frequency (D) Energy and strain

17. The velocity of water waves depend on their wavelength λ, the density of water ρ and acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 \propto g^{-1}\lambda^{-1}$ y (B) $v^2 \propto g\lambda$ y
(C) $v^2 \propto g\lambda\rho$ y (D) $v^2 \propto g^{-1}\lambda^{-3}$ y

18. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combination which have the dimensions of angle
(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$
(C) $\frac{C}{L}$ (D) $\frac{R^2C}{L}$

19. The vernier of a circular scale is divided into 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count by the instrument is
(A) 0.1′ (B) 1′
(C) 10′ (D) 30′

20. Dimensional analysis of the equation $(\text{velocity})^x = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$ gives the value of x as
(A) 1 (B) 2
(C) 3 (D) 4

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. $[a] = M^1L^0T^{-2}$, $[b] = M^1L^{-1}T^{-2}$
2. FTV^{-1}
3. (a) $[Q] = IT$ (b) $[V] = ML^2I^{-1}T^{-3}$ (c) $[C] = M^{-1}L^{-2}I^2T^4$ (d) $[R] = ML^2T^{-3}I^{-2}$
4. Work and torque
5. $[A] = ML^8T^{-2}$, $[B] = ML^{14}T^{-2}$
6. 4 %
7. $E = k\,mv^2\,r^2$
8. $M°L^{-1/2}\,T^2$
10. 50 units
11. 23%
12 k= M°L°T
13. $7x10^{-2}$ N/m
14. Work
15. The maximum error will be due to (i) measurement, which is least accurate.
(ii) measurement of the quantity which has maximum power in formula's.

Level – I

1. 3.5 g/cc
2. $M°L°T°$, $M°L^2T°$
3. T^{-4}
4. $(8 \pm 2)\,\Omega$
5. 1400
6. 5.7%
7. $(hc)^{1/2}\,G^{-1/2}$, $(hG)^{1/2}c^{-3/2}$, $(hG)^{1/2}\,c^{-5/2}$
8. $\frac{1}{\sqrt{9.8}}$ sec
9. 360
10. 2.25 mm
11. 1.02 cm
12. $\pm$ 5%
13. (a) 0.01mm (b) + 0.04 mm, to be subtracted (c) 7.33 mm
14. $\frac{E}{\hbar\omega} = \frac{1}{2}\frac{mv^2}{\hbar\omega} + \frac{1}{2}\frac{\omega m x^2}{\hbar}$, $\sqrt{m\omega\hbar}$, $\sqrt{\frac{\hbar}{m\omega}}$
15. L^2T^{-1}

Objective:

1.	C	2.	D	3.	D	4.	D
5.	C	6	A	7.	D	8.	C
9.	C	10.	C	11	A	12.	D
13.	D	14.	B	15.	A	16.	C
17.	B	18.	D	19.	B	20.	C

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Thank you for your love and support, we hope you are always being happy and get success in your life, we are happy to see you again.

Regards from Learnaf team

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PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

VECTORS

Vectors

Syllabus for IITJEE and ISC Board:
Scalar and vector quantities; Position and displacement vectors, general vectors and notation; Equality of vectors, multiplication of vectors by a real number; Addition and subtraction of vectors; Unit vector, Resolution of a vector in a plane Rectangular components, Multiplication of vectors-scalar and vector products; vectors in three dimensions (elementary idea only).

VECTORS

Definition: The physical quantities specified completely by their magnitude as well as direction are called vector quantities. The magnitude and direction alone cannot decide whether a physical quantity is a vector. In addition to the above characteristics, a physical quantity, which is a vector, should follow law of vector addition. For example, electric current has magnitude as well as direction, but does not follow law of vector addition. Hence, it is not a vector.

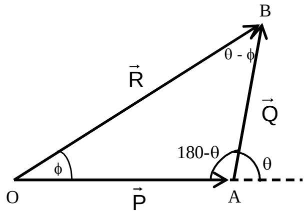
A vector is represented by putting an arrow over it. The length of the line drawn in a convenient scale represents the magnitude of the vector. The direction of the vector quantity is depicted by placing an arrow at the end of the line.

If two vectors have the same direction, they are parallel. Two vectors are said to be equal when their magnitudes and directions, both are same, e.g. if $\vec{a} = \vec{b}$ then $|\vec{a}| = |\vec{b}|$ and the directions of vectors are same. Thus, a vector is not altered by shifting it parallel to itself in the space.

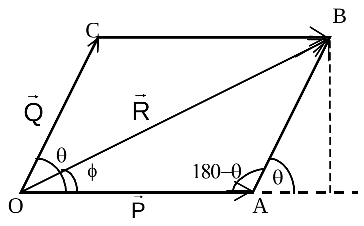
The vector having same magnitude as of $\vec{a}$, but the opposite direction is defined as the negative or opposite of $\vec{a}$ and is denoted by $-\vec{a}$.

Laws of Addition of Vectors: Two or more vectors can be added to give another vector, which is called the resultant of the vectors. The resultant would produce the same effect as that of the original vectors together.

Triangle law of addition of vectors: If two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant will be represented in magnitude and direction by the third side of the triangle taken in reverse order.
To add the two vectors $\vec{P}$ and $\vec{Q}$, the vectors are drawn with the tail of $\vec{Q}$ coinciding with the terminus of $\vec{P}$. The vector sum i.e. the resultant vector $\vec{R}$ which completes the triangle drawn from the tail O of $\vec{P}$ to the terminus B of $\vec{Q}$ as shown in figure.



Parallelogram law of addition of vectors:
If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.



We get , $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\phi = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

where ϕ is the angle that the resultant makes with $\vec{P}$

(i) $\theta = 0^o$

$\vec{P}$ and $\vec{Q}$ are in the same direction i.e. they are parallel $\cos 0^o = 1$

$\therefore |\vec{R}| = |\vec{P}| + |\vec{Q}|$ & $\phi = 0^o$

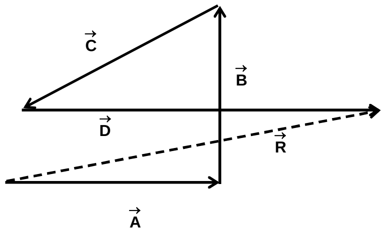
(ii) $\theta = 180^o$, $\vec{P}$ and $\vec{Q}$ are in opposite direction i.e. they are antiparallel $\cos 180^o = -1$

$\therefore |\vec{R}| = |\vec{P}| \sim |\vec{Q}|$ and $\vec{R}$ is in the direction of the larger vector.

(iii) $\theta = 90^o$, $\cos 90^0 = 0$

$\vec{P}$ and $\vec{Q}$ are perpendicular to each other

$\therefore |\vec{R}| = (|\vec{P}|^2 + |\vec{Q}|^2)^{1/2}$ & $\phi = \tan^{-1}(Q/P)$



Polygon law of addition of vectors

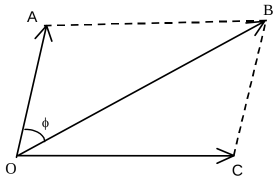
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Vectors obey commutative law

i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Illustration 1. *Two forces of 60N and 80N acting at an angle of 60^0 with each other, pull an object. What single pull would replace the given forces?*

Solution: *Two forces are drawn from a common origin O, making an angle of 60^0. OA and OC represent the forces 60N and 80N respectively. The diagonal OB represents the resultant R.*



$\therefore R^2 = 60^2 + 80^2 + 2.60.80\cos 60^0$

$= 3600 + 6400 + 4800 = 14800$ $\therefore R = 121.7N$

Angle ϕ is given, $\tan\phi = \dfrac{80\sin 60^0}{60 + 80\cos 60^0}$

Which gives, $\phi = 34.7^0$

Exercise 1:

i) ***Is it possible that the resultant of two equal forces is equal to one of the forces?***

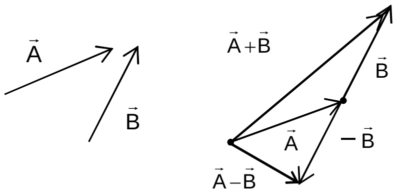
ii) ***If a vector has zero magnitude is it meaningful to call it a vector?***

iii) ***Can three vectors, not in one plane, give a zero resultant? Can four vectors do?***

Subtraction of Vectors:

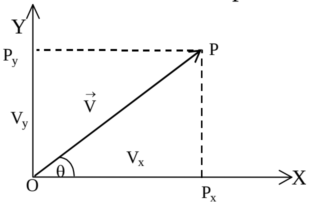
When a vector B is reversed in direction, then the reversed vector is written as $-\vec{B}$ then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Resolution of Vectors: If $\vec{P}+\vec{Q}=\vec{R}$, the resultant, then conversely $\vec{R}=\vec{P}+\vec{Q}$ i.e. the vector $\vec{R}$ can be split up so that the vector sum of the split parts equals the original vector $\vec{R}$. If the split parts are mutually perpendicular then they are known as components of $\vec{R}$ and this process is known as resolution. The orthogonal component of any vector along another direction which is at an angular separation θ is the product of the magnitude of the vector and cosine of the angle between them (θ). Therefore the component of $\vec{A}$ is A cosθ.

Note: In physics, resolution gives unique and mutually independent components only if the resolved components are mutually perpendicular to each other. Such a resolution is known as rectangular or orthogonal resolution and the components are called rectangular or orthogonal components.



O – the origin, OP – the given vector $\vec{V}$

PP_x – perpendicular to X axis.

PP_y – Perpendicular to Y axis.

$$\overrightarrow{OP_x} + \overrightarrow{P_xP} = \overrightarrow{OP} = \vec{V}$$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$V_x = V\cos\theta$ & $V_y = V\sin\theta$

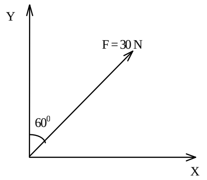
Illustration 2. *A force of 30 N is acting at an angle of 60^0 with the y-axis. Determine the components of the forces along x and y-axes.*

Solution :

$F_x = F\sin 60^0$

$$= \frac{30 \text{ x } \sqrt{3}}{2} = 15\sqrt{3}\text{ N}$$

$F_y = F\cos 60^0 = 30 \text{ x} \frac{1}{2} = 15\text{ N}$



Unit Vector: In order to make the algebraic operations with vectors simple, a given vector is expressed as a product of its magnitude and direction vector. Since the product should have the same magnitude, the direction vector having unit magnitude and is called unit vector.

A unit vector is not a physical quantity but represents only a given direction. Unit vector along the direction of $\vec{A}$ is $\hat{A}=\vec{A}/A$, Where A is magnitude of $\vec{A}$. $\hat{i}, \hat{j}, \hat{k}$, are the unit vectors along positive direction of X, Y and Z axis respectively, then the rectangular resolution of vector $\vec{A}$ can be represented.

$\vec{A} = A_x\hat{i}+A_Y\hat{j}+A_Z\hat{k}$ where A_X, A_Y, A_Z are the magnitudes of X, Y and Z components of $\vec{A}$. The magnitude of vector $\vec{A}$ is given by $|\vec{A}|=\sqrt{A_x^2+A_y^2+A_z^2}$.

Illustration 3. *Find the net displacement of a particle from its starting point if it undergoes three successive displacements given $\vec{S_1}$ = 20 m, 45^0 West of North, $\vec{S_2}$ = 15 m, 30^0 North of East; $\overrightarrow{S_3}$ =20m, due South.*

Solution: Let us set our axial system such that x-axis is along West-East and y-axis along South-North.

$\Rightarrow \ \overrightarrow{S_1} = 20\cos 45^0(-\hat{i}) + 20\sin 45^0(\hat{j})$

and $\overrightarrow{S_2} = 15\cos 30^0(\hat{i}) + 15\sin 30^0(\hat{j})$

$\overrightarrow{S_3} = 0(\hat{i}) + 20(-\hat{j})$

$\vec{S} = \overrightarrow{S_1} + \overrightarrow{S_2} + \overrightarrow{S_3}$

$= \left(-\frac{20}{\sqrt{2}} + \frac{15\sqrt{3}}{2} + 0\right)\hat{i} + \left(\frac{20}{\sqrt{2}} + \frac{15}{2} - 20\right)\hat{j}$

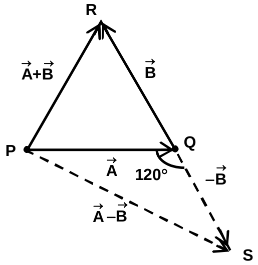
$= -1.15\hat{i} + 1.64\hat{j} = S_x\hat{i} + S_y\hat{j}$

$|\vec{S}| = \sqrt{S_x^2 + S_Y^2} = \sqrt{(-1.15)^2 + (1.64)^2} = 2\text{m}$

Direction $\theta = \tan^{-1}\frac{1.15}{1.64} = 35^0$ West of North

Illustration 4. *If the sum of two unit vectors $\vec{A}$ and $\vec{B}$ is also equal to a unit vector, find the magnitude of the vector $\vec{A} - \vec{B}$.*

Solution: Given that $|\vec{A}| = |\vec{B}| = |\vec{A} + \vec{B}| = 1$



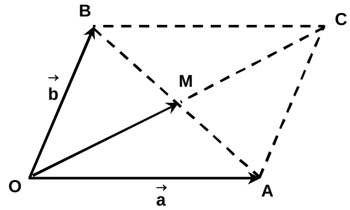
Hence the angle between $\vec{A}$ and $\vec{B}$ is 120°

Now $|\overrightarrow{PS}|^2 = |\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}||-\vec{B}|\cos 120°$

$= 1 + 1 + 2\times1\times(-1)\left(-\frac{1}{2}\right) = 3$

$\Rightarrow PS = \sqrt{3}$

Illustration 5. *If the position vector of point A and B are $\vec{a}$ and $\vec{b}$ respectively. Find the position vector of middle point of AB.*



Solution:

$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

$\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OM}$

$\vec{a} + \vec{b} = 2\overrightarrow{OM}$

$\overrightarrow{OM} = \frac{1}{2}(\vec{a} + \vec{b})$

Multiplication of Vectors: Vector multiplications are of two types. One, in which we obtain a scalar quantity and the other in which we obtain a vector quantity on multiplication. The first one is called Dot Product and the other is called Cross Product.

Scalar multiplication: $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$ where θ is the angle between the two vectors, when placed tail to tail.

For $\theta = 90^0$, $\cos\theta = 0$ then $\vec{A}.\vec{B} = 0$

Now for orthogonal unit vectors, $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Again for $\theta = 0^0$, $\cos\theta = 1$ then $\vec{A}.\vec{B} = AB$

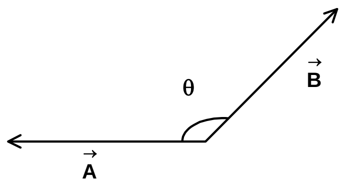
For orthogonal unit vectors, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

Let there be two vectors given by

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}.\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right).\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= A_xB_x + A_yB_y + A_zB_z$$



(in the same line)

Dot product is commutative. i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Angle between two vectors:

As we know $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$$

$$= \frac{A_xB_x + A_yB_y + A_zB_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)\left(\sqrt{B_x^2 + B_Y^2 + B_Z^2}\right)}$$

Direction cosines:

If vector $\vec{A}$ makes angle α, β and γ with x, y and z axes respectively, then

$$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{A_x}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\beta = \frac{A_y}{|\vec{A}|} = \frac{A_y}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\gamma = \frac{A_z}{|\vec{A}|} = \frac{A_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

where cos α, cos β and cos γ are called direction cosines of the vector $\vec{A}$. The unit vector in the direction of $\vec{A}$ is

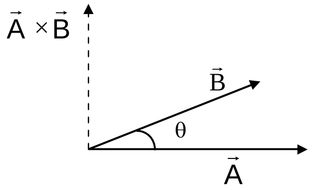
$$\hat{n} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{|\vec{A}|}$$

$\hat{n} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$. (Note: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$)

Cross product:

The cross product of two vectors $\vec{A}$ and $\vec{B}$ inclined to each other by an angle θ is given by

$\vec{A} \times \vec{B} = \vec{C}$, a vector. where $\vec{C} = |\vec{A}|.|\vec{B}|\sin\theta\hat{n}$, where $\hat{n}$ is the unit vector along a direction which is perpendicular to plane containing $\vec{A}$ & $\vec{B}$. Its direction is given by the right hand thumb rule, or right hand screw rule.



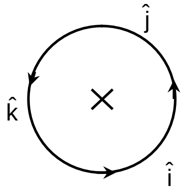
If the vectors $\vec{A}$ and $\vec{B}$ lie in the x-y plane then the product is perpendicular to the plane i.e. is parallel to z-axis.

The vector product is not commutative i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ and $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

In terms of orthogonal vectors $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$$\hat{i}\times\hat{j} = \hat{k} = -(\hat{j}\times\hat{i}),\quad \hat{j}\times\hat{k} = \hat{i} = -\left(\hat{k}\times\hat{j}\right),$$

$$\left(\hat{k}\times\hat{i}\right)=\hat{j}=-\left(\hat{i}\times\hat{k}\right)$$



If $\vec{A}=A_X\hat{i}+A_Y\hat{j}+A_Z\hat{k}$, $\vec{B}= B_X\hat{i}+B_Y\hat{j}+B_Z\hat{k}$

Then, $\vec{A}\times\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\ x\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$

$$= \left(A_YB_Z-A_ZB_Y\right)\hat{i}+\left(A_ZB_X-A_XB_Z\right)\hat{j}+\left(A_XB_Y-A_YB_X\right)\hat{k}$$

In determinant form we have,

Then, $$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_X & A_Y & A_Z\\ B_X & B_Y & B_Z\end{vmatrix}$$

Cross Product of two parallel or anti parallel vectors is zero.

Note: *Division of a vector by a vector is not defined.*

Exercise 2:

(i) ***Calculate the angle between a two dyne and a three dyne force so that their sum is four dyne.***

(ii) ***Resultant of two forces which have equal magnitudes and which act at right angles to each other is 1414 dyne. Calculate the magnitude of each forces.***

(iii) ***Find the direction cosines of*** $5\hat{i}+2\hat{j}+4\hat{k}$

(iv) ***Given:*** $\vec{a}=3\hat{i}+4\hat{j}$ ***and*** $\vec{b}=3\hat{j}+4\hat{k}$ ***, Calculate the magnitude of the resultant.***

(v) ***One of the rectangular component of an acceleration of 8 m/s^2 is 4 m/s^2, calculate the other component.***

(vi) ***Find the unit vector in the direction of*** $3\hat{i}+4\hat{j}-\hat{k}$

Illustration 6. *If the magnitudes of the dot product and cross product of two vectors are equal, find the angle between the two vectors.*

Solution: $|\vec{A}\times\vec{B}|=|\vec{A}\cdot\vec{B}|$

A.B sinθ = A.B cosθ $\Rightarrow$ tanθ = 1

$\Rightarrow \theta = 45^0$.

Illustration 7. *If* $\vec{a} = 3\hat{i} + 4\hat{j}$ *and* $\vec{b} = 7\hat{i} + 24\hat{j}$, *find the vector having the same magnitude as* $\vec{b}$ *and parallel to* $\vec{a}$.

Solution: Magnitude of $\vec{a}=\left|\vec{a}\right|= \sqrt{3^2+4^2} = 5$

And magnitude of $\vec{b} = \left|\vec{b}\right| = \sqrt{7^2+24^2} = 25$

Now a unit vector parallel to $\vec{a} = \hat{a} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

$\therefore$ The vector having the same magnitude as $\vec{b}$ and parallel to $\vec{a}$

$= 25\ \hat{a} = 15\hat{i} + 20\hat{j}$

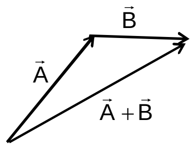
SUMMARY

1. Scalar quantities are quantities with magnitudes only and combine with the usual rules of arithmetic e.g. speed, mass and temperature.

2. Vector quantities have magnitude as well as direction and combine according to the rules of vector addition. e.g. velocity and acceleration.

3. $\vec{B} = \lambda\vec{A}$
Where λ is a real number. Magnitude of B is λ time the magnitude of A and direction is same as that of A. (If λ is positive).

4. Graphically, two vectors $\vec{A}$ and $\vec{B}$ are added by placing the tail of $\vec{B}$ at the head of $\vec{A}$. The vector sum $\vec{A}+\vec{B}$ then extends from the tail of $\vec{A}$ to the head of $\vec{B}$



5. Vector addition is
$\vec{A}+\vec{B}=\vec{B}+\vec{A}$ (Commutative)
$\left(\vec{A}+\vec{B}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$ (Associative law)

6. A vector with zero magnitude is called null vector and
$\vec{A}+\vec{0}=\vec{A}$
$\lambda\vec{0}=\vec{0}$

7. Subtraction of vectors
$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

8. Unit vectors describe directions in space. A unit vector has a magnitude of one, with no units.
The unit vectors $\hat{i},\hat{j},\hat{k}$ are vectors of unit magnitude and points in the direction of the x, y and z axes, respectively, in a right – handed coordinate system.

10. vector $\vec{A}$ can be expressed as $\vec{A} = A_X\hat{i} + A_y\hat{j}$ having magnitude $=\sqrt{A_x^2 + A_y{}^2}$ and angle θ with the x – axis = $\tan^{-1}\dfrac{A_y}{A_x}$.

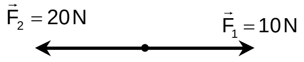
11. Scalar product of two vectors, $C = \vec{A}\cdot\vec{B} = AB\cos\phi$, where ϕ is the angle between two vectors and scalar product of two vectors is a scalar quantity. Scalar products obey the commutative and the distributive laws.

12. Cross – product of two vectors $\vec{A}$ and $\vec{B}$ is a vector quantity. $\vec{C}=\vec{A}\times\vec{B}=AB\sin\phi\ \hat{n}$ and its direction is given by right hand rule, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ (non – commutative)

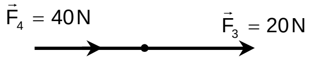
MISCELLANEOUS EXERCISE

1. Find the magnitude and direction of resultant vectors as shown in the figures below.

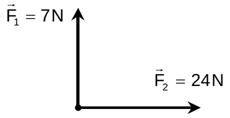
(a)



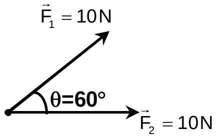
(b)



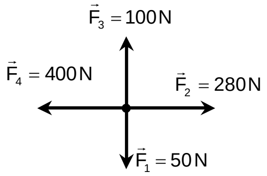
(c)



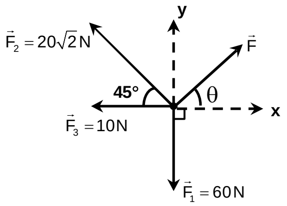
(d)



2. (a) In the adjacent figure, find the magnitude and direction of the resultant vector.



(b) In the adjacent figure, find the value of F and θ so that the sum of the vectors will be zero.



3. Show that the vectors $\vec{A} = 12\hat{i} - 10\hat{j} + 2\hat{k}$ and $\vec{B} = 4\hat{i} + 8\hat{j} + 16\hat{k}$ are perpendicular to each other.

4. Resultant of two vectors of equal magnitude makes an angle 60° with one of the vectors. Find the angle between the vectors.

5. If $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = 4\hat{k}$ find the value of $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$. Also find the angle between $\vec{a}$ and $\vec{b}$.

6. If $\vec{A} = 3\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$, find $|\vec{A}|$, $|\vec{B}|$ and $|\vec{A} + \vec{B}|$. Also find the direction of $\vec{A} + \vec{B}$ with the x-axis. Check whether $|\vec{A}| + |\vec{B}|$ is equal to $|\vec{A} + \vec{B}|$.

7. Check whether the two vectors, $\vec{A} = -3\hat{i} - 7\hat{j} + 9\hat{k}$ and $\vec{B} = -2\hat{i} - 21\hat{j} + 6\hat{k}$ are parallel to each other.

8. Two vectors are given by $\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} + \hat{k}$. Find the magnitude and direction cosines of $\vec{A} + \vec{B}$.

9. The components of a vector $\vec{A}$ along x-axis and y-axis are 4 unit and 6 unit respectively. If the components of vector $\vec{A} + \vec{B}$ along x-axis and y-axis are 10 unit and 14 unit respectively, find the vector $\vec{B}$ and its direction with the x-axis.

10. (a) Find the unit vector which is parallel to the vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$.

(b) Find the unit vector which is perpendicular to both of the vectors $\vec{A} = 2\hat{i}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 12\hat{k}$.

SOLUTION TO MISCELLANEOUS EXERCISE

1. (a) 10 N (towards $\vec{F}_2$)
   (b) 60 N (towards $\vec{F}_3$)
   (c) 25 N, at an angle $\tan^{-1}\left(\frac{7}{24}\right)$ from $\vec{F}_2$ towards $\vec{F}_1$
   (d) $10\sqrt{3}$ at an angle 30° from $\vec{F}_2$ towards F_1
2. (a) 130 N, $\tan^{-1}\left(\frac{5}{12}\right)$ (from $\vec{F}_4$ towards $\vec{F}_3$)
   (b) 50 N, $\theta = 53°$
4. 120^0
5. 0, $4\left(2\hat{i} - 3\hat{j}\right)$, 90°
6. $\sqrt{14}$, $2\sqrt{6}$ and $\sqrt{30}$ N, $\cos^{-1}\left(\sqrt{\frac{5}{6}}\right)$; No
7. No
8. $\sqrt{90}$, $\frac{7}{\sqrt{90}}$, $\frac{4}{\sqrt{90}}$, $\frac{5}{\sqrt{90}}$
9. $\vec{B} = 6\hat{i} + 8\hat{j}$ and $\theta = 53°$
10. (a) $\frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}}$ (b) $\frac{\left(-3\hat{j} + \hat{k}\right)}{\sqrt{10}}$

SOLVED PROBLEMS

Subjective:

Prob 1. *Find the component of a vector $\vec{A} = 3\hat{i} + 2\hat{j}$ along the direction of $\left(\hat{i} + \hat{j}\right)$.*

Sol. Unit vector along $\left(\hat{i}+\hat{j}\right)$ is $\hat{n} = \dfrac{\hat{i}+\hat{j}}{\left(1^2+1^2\right)^{1/2}} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore$ The magnitude of the component of vector $\vec{A}$ along the $\left(\hat{i}+\hat{j}\right)$ is

$$\vec{A}\cdot\hat{n} = \left(3\hat{i}+2\hat{j}\right)\cdot\frac{\hat{i}+\hat{j}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(3+2) = \frac{5}{\sqrt{2}}$$

$\therefore$ The component vector of $\vec{A}$ along the $\left(\hat{i}+\hat{j}\right)$ is

$$\vec{A}_1 = \frac{5}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j}$$

Prob 2. *At what angle do the forces $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ act so that the magnitude of their resultant is $\sqrt{3A^2+B^2}$?*

Sol. $R^2 = 3A^2 + B^2 = (A + B)^2 + (A - B)^2 + 2(A^2 - B^2)\cos\theta$
where θ is the angle between (A + B) and (A–B)
or, $A^2 - B^2 = 2(A^2 - B^2)\cos\theta$
or, $\cos\theta = 1/2 \quad \Rightarrow \quad \theta = 60°$

Prob 3. *The resultant of two non-zero forces $\vec{P}$ and $\vec{Q}$ is of magnitude P. Prove that if $\vec{P}$ is doubled, the resultant force will be perpendicular to Q.*

Sol. $P^2 = P^2 + Q^2 + 2PQ\cos\theta \Rightarrow Q + 2P\cos\theta = 0$
Now, if $\vec{P}$ is doubled then
$\tan\alpha = \dfrac{2P\sin\theta}{Q+2P\cos\theta}$ [where α = angle made by the resultant with Q]
$= \dfrac{2P\sin\theta}{0} = \infty$ (undefined)
$\Rightarrow \alpha = 90°$

Prob 4. *$\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$. To which vector, the vector $\vec{A}$ is parallel.*

Sol. $\vec{A}.\vec{B} = 0$ so $\vec{A}$ is perpendicular to $\vec{B}$
$\vec{A}.\vec{C} = 0$ so $\vec{A}$ is perpendicular to $\vec{C}$
But $(\vec{B}\times\vec{C})$ is a vector which is perpendicular to both $\vec{B}$ and $\vec{C}$.
So, $\vec{A}$ is parallel to $(\vec{B}\times\vec{C})$.

Prob 5. *Prove that* $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$.

Sol. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}$ [$\because$ $\vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0$ and $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$]

$= 2(\vec{a}\times\vec{b})$

Prob 6. *Given* $\vec{A}.\vec{B}=\vec{A}.\vec{C}$. *Is it correct to conclude* $\vec{B}=\vec{C}$ *?*

Sol. $\vec{A}.\vec{B}=\vec{A}.\vec{C}$

i.e. $AB\cos\theta_1 = AC\cos\theta_2$

[where θ_1 and θ_2 are the angles formed by $\vec{B}$ and $\vec{C}$ respectively with $\vec{A}$.]

or, $B\cos\theta_1 = C\cos\theta_2$

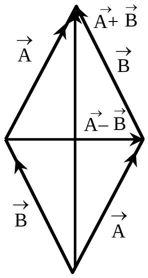
Now, if $\theta_1=\theta_2$ then $\vec{B}=\vec{C}$

But if $\theta_1\neq\theta_2$ then $\vec{B}\neq\vec{C}$

So, we can't conclusively say that $\vec{B}=\vec{C}$.

Prob 7. *Find the area of a parallelogram whose diagonals are represented by* $(3\hat{i}+\hat{j}+\hat{k})$ *and* $(\hat{i}-\hat{j}-\hat{k})$.

Sol. Let $\vec{A}$ and $\vec{B}$ be the two adjoining sides of the parallelogram drawn from a point. The area of the parallelogram = $\vec{A}\times\vec{B}$



Given that $\vec{A}+\vec{B}=3\hat{i}+\hat{j}+\hat{k}$

and $\vec{A}-\vec{B}=\hat{i}-\hat{j}-\hat{k}$

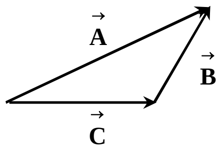
Now, $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})=-2(\vec{A}\times\vec{B})$

$\therefore$ $\vec{A}\times\vec{B}=-\frac{1}{2}[(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})]$

$$=-\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 1\\ 1 & -1 & -1\end{vmatrix}$$

$$=-\frac{1}{2}\left(0\hat{i}+4\hat{j}-4\hat{k}\right)=-2\hat{j}+2\hat{k}$$

Prob 8. *Prove that the vectors* $\vec{A}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=-\hat{i}+3\hat{j}+4\hat{k}$ *and* $\vec{C}=4\hat{i}-2\hat{j}-6\hat{k}$, *can form a triangle.*



Sol. Let us add any two of the given vectors, say $\vec{C}$ and $\vec{B}$

$\vec{C}+\vec{B}=\left(4\hat{i}-2\hat{j}-6\hat{k}\right)+\left(-\hat{i}+3\hat{j}+4\hat{k}\right)=3\hat{i}+\hat{j}-2\hat{k}=\vec{A}$

As the sum of two vectors is equal to the third vector, the three vectors can form a triangle.

Prob 9. *If* $\vec{A}=3\hat{i}+7\hat{j}$ *and* $\vec{B}=2\hat{i}+5\hat{j}$, *find the angle between* $\vec{A}$ *and* $\vec{B}$.

Sol. $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

or, $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{6+35}{\sqrt{58}\sqrt{29}} = 0.9997$ [$\because$ $|\vec{A}| = \sqrt{9+49}$ & $|\vec{B}| = \sqrt{4+25}$

$\therefore$ $\theta = 1°24'$

Prob 10. *If $\vec{A} = 3\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{B} = 4\hat{i}+2\hat{j}+\hat{k}$, find the magnitude and direction cosines of $(\vec{A}+\vec{B})$.*

Sol. $\vec{A}+\vec{B} = \left(3\hat{i}+2\hat{j}+4\hat{k}\right)+\left(4\hat{i}+2\hat{j}+\hat{k}\right) = 7\hat{i}+4\hat{j}+5\hat{k}$

$\therefore |\vec{A}+\vec{B}| = \sqrt{7^2+4^2+5^2} = \sqrt{90}$

$\therefore$ The direction cosines of $\vec{A}+\vec{B}$ are

$$\cos\alpha = \frac{\left|\vec{A}+\vec{B}\right|_x}{\left|\vec{A}+\vec{B}\right|} = \frac{7}{\sqrt{90}}$$

$$\cos\beta = \frac{\left|\vec{A}+\vec{B}\right|_y}{\left|\vec{A}+\vec{B}\right|} = \frac{4}{\sqrt{90}}$$

$$\cos\gamma = \frac{\left|\vec{A}+\vec{B}\right|_z}{\left|\vec{A}+\vec{B}\right|} = \frac{5}{\sqrt{90}}$$

Prob 11. *Prove that the vectors $\vec{A} = 2\hat{i}-3\hat{j}-\hat{k}$ and $\vec{B} = -6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.*

Sol. The given vectors will be parallel if their cross product is zero. Because if the two vectors are $\vec{A}$ and $\vec{B}$ then

$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n} = AB\sin 0°\ \hat{n}$ [if they are parallel then the angle between $\vec{A}$ and $\vec{B}$ is 0°]

$\therefore$ $\vec{A}\times\vec{B} = 0$

Now, $\left(2\hat{i}-3\hat{j}-\hat{k}\right)\times\left(-6\hat{i}+9\hat{j}+3\hat{k}\right) = (9-9)\hat{i}+(6-6)\hat{j}+(18-18)\hat{k} = 0$

Hence, the two vectors $\vec{A}$ and $\vec{B}$ are parallel to each other.

$\vec{B} = -3\vec{A} \Rightarrow \vec{B} \parallel A$.

Prob12. *For a point P (2, 4, –5) in a three dimensional co-ordinate system, find*

(a) the position vector $\vec{r}$ of point P.

(b) $|\vec{r}|$

(c) the direction cosines of vector $\vec{r}$.

Sol. (a) $\vec{r} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

(b) $|\vec{r}| = \sqrt{r_x^2+r_y^2+r_z^2}$

$= \sqrt{(2)^2+(4)^2+(-5)^2} = \sqrt{45}$

(c) $\cos\alpha = \dfrac{r_x}{|\vec{r}|} = \dfrac{2}{\sqrt{45}}$

$\cos\beta = \dfrac{r_y}{|\vec{r}|} = \dfrac{4}{\sqrt{45}}$

and $\cos\gamma = \dfrac{r_z}{|\vec{r}|} = -\dfrac{5}{\sqrt{45}}$

Prob13. *Under the action of a force $\left(10\hat{i}-3\hat{j}+6\hat{k}\right)$N, a body of mass 5 kg moves from position $\left(6\hat{i}+5\hat{j}-3\hat{k}\right)$m to a position $\left(10\hat{i}-2\hat{j}+7\hat{k}\right)$ m. Deduce the work done.*

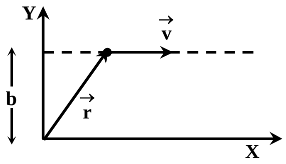
Sol. Displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= \left(10\hat{i}-2\hat{j}+7\hat{k}\right) - \left(6\hat{i}+5\hat{j}-3\hat{k}\right)$$
$$= \left(4\hat{i}-7\hat{j}+10\hat{k}\right)\text{ m}$$

Work $W = \vec{F}\cdot\vec{s}$

$$= \left(10\hat{i}-3\hat{j}+6\hat{k}\right)\cdot\left(4\hat{i}-7\hat{j}+10\hat{k}\right)$$
$$= 40 + 21 + 60 = 121\text{ J}$$

Prob 14. *If a particle of mass m is moving with a constant velocity $\vec{v}$ parallel to the x-axis in x-y plane as shown in the figure, calculate its angular momentum w.r.t. origin at any time t.*



Sol. We know $\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$

As the motion is in x-y plane, z = 0 and $p_z = 0$

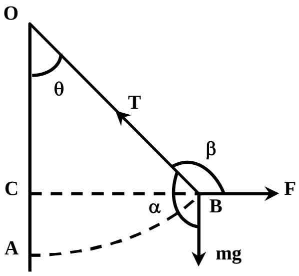
$\therefore$ $\vec{L} = \hat{k}\,(xp_y - yp_x)$

Hence $x = vt$, $y = b$, $p_x = mv$ and $p_y = 0$

$\therefore$ $\vec{L} = \hat{k}\,(vt\times 0 - bmv) = -mvb\,\hat{k}$

Prob15. *A bob weighing 50 gm hangs vertically at the end of a string 50 cm long. If 20 gm force is applied horizontally on the bob, by what distance is the bob pulled aside from its initial position when it reaches its equilibrium position?*

Sol. Let the bob be in equilibrium when it is pulled to B.



$$\frac{F}{\sin\alpha} = \frac{mg}{\sin\beta} = \frac{T}{\sin 90°}$$

$$\frac{F}{\sin(\pi-\theta)} = \frac{mg}{\sin\left(\frac{\pi}{2}+\theta\right)} = T$$

or, $$\frac{F}{\sin\theta} = \frac{mg}{\cos\theta} = T$$

$\therefore$ $$\frac{F}{mg} = \tan\theta = \frac{20\times 980}{50\times 980} = 0.4 = \tan 21°48'$$

$\therefore$ $\theta = 21°48'$

$\therefore$ $CB = OB\sin\theta = OB\sin 21°48'$

$= 50\times 0.3714 = 18.57$ cm

Prob16. *The x and y components of vector $\vec{A}$ are 4 m and 6 m, respectively. If the x and y components of vector $(\vec{A}+\vec{B})$ are 10 m and 9 m, respectively, calculate*

(a) x and y components of vector $\vec{B}$.

(b) its length and

(c) the angle made by vector $\vec{B}$ with the x-axis.

Sol. In terms of components

$\vec{A} + \vec{B} = (\hat{i}A_x + \hat{j}A_y) + (\hat{i}B_x + \hat{j}B_y)$

$\therefore \vec{A} + \vec{B} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y)$

According to the given Problem

$A_x + B_x = 10$ m and $A_y + B_y = 9$ m

As $A_x = 4$ m and $A_y = 6$ m (given)

(a) $B_x = 6$ m and $B_y = 3$ m

(b) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$ m

(c) $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

Objective:

Prob 1. *A force $\vec{F} = (4\hat{i} - 5\hat{j} + 3\hat{k})$N is acting at a point having a position vector $\vec{r}_1 = (\hat{i} + 2\hat{j} + 3\hat{k})$. The torque acting about a point having a position vector $\vec{r}_2 = (3\hat{i} - 2\hat{j} - 3\hat{k})$, is*

(A) $42\hat{i} + 30\hat{j} - 6\hat{k}$ *(B)* $42\hat{i} + 30\hat{j} + 6\hat{k}$

(C) $42\hat{i} - 30\hat{j} + 6\hat{k}$ *(D) zero.*

Sol. Calculate torque $(\vec{\tau}) = \vec{r} \times \vec{F}$

Where $\vec{r} = \vec{r}_1 - \vec{r}_2$

Hence (A) is correct.

Prob 2. *The area of a parallelogram formed by the vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$ as adjacent sides is*

(A) $8\sqrt{3}$ *units.* *(B) 64 units*

(C) 32 units. *(D)* $\sqrt{3}$ *units*

Sol. Calculate $\vec{A} \times \vec{B}$

and Area of a parallelogram $= |\vec{A} \times \vec{B}|$

Hence (A) is correct.

Prob 3. *If vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then which of the following statements is valid?*

(A) $\vec{A} \times \vec{B} = \vec{A} . \vec{B}$ *(B)* $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = 0$

(C) $\vec{A} . \vec{B} = 0$ *(D)* $\vec{A} . \vec{B} = |\vec{A}|\,|\vec{B}|$

Sol. $\vec{A} \perp \vec{B}$. Then.

$\vec{A} . \vec{B} = |A||B|\cos 90° = 0$

Hence (C) is correct.

Prob 4. *Which of the rectangular pair may be the components of a 13 N force?*

(A) 5 N, 12 N *(B) 10 N, 11 N*

(C) 6.5 N, 6.5 N *(D) 9 N, 12 N*

Sol. Rectangular components will follow

$$R = \sqrt{R_x^2 + R_y^2}$$

$\therefore$ $13^2 = 5^2 + 12^2$

Hence (A) is correct.

Prob 5. *If $\vec{A}$ and $\vec{B}$ are two mutually perpendicular vectors, where $\vec{A} = 5\vec{i} + 7\hat{j} + 3\hat{k}$ and $\vec{B} = 2\vec{i} + 2\hat{j} - a\hat{k}$, then the value of a is*

(A) –2 *(B) 8*

(C) –7 *(D) –8*

Sol. $\vec{A} \perp \vec{B}$

$\vec{A} \cdot \vec{B} = 0 = (5\hat{i} + 7\hat{j} + 3\hat{k})(2\hat{i} + 2\hat{j} - a\hat{k}) = 10 + 14 - 3a$

$\therefore \quad 3a = 24 \Rightarrow a = 8$

Hence (B) is correct.

Prob 6. *The unit vector perpendicular to* $\hat{i} - 2\hat{j} + \hat{k}$ *and* $3\hat{i} + \hat{j} - 2\hat{k}$ *is*

(A) $\dfrac{5\hat{i} + 3\hat{j} + 7\hat{k}}{\sqrt{83}}$ *(B)* $\dfrac{3\hat{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

(C) $\dfrac{5\hat{i} + 3\hat{j} - 7\hat{k}}{\sqrt{83}}$ *(D)* $\dfrac{3\hat{i} - 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

Sol. $\vec{A} \times \vec{B}$ is a vector $\perp$ to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A} \times \vec{B} = (\hat{i} - 2\hat{j} + \hat{k}) \times (3\hat{i} + \hat{j} - 2\hat{k}) = 3\hat{i} + 5\hat{j} + 7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$$= \frac{3\hat{i} + 5\hat{j} + 7\hat{k}}{\sqrt{3^2 + 5^2 + 7^2}} = \frac{3\hat{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$$

Hence (B) is correct.

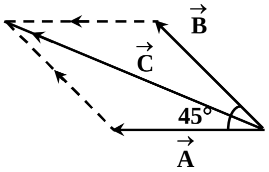
Prob 7. *For any two vectors* $\vec{A}$ *and* $\vec{B}$ *if* $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$, *the magnitude of* $\vec{C} = \vec{A} + \vec{B}$ *is equal to*

(A) $\sqrt{A^2 + B^2}$ *(B)* $A + B$

(C) $\left[A^2 + B^2 + \dfrac{AB}{\sqrt{2}}\right]^{1/2}$ *(D)* $(A^2 + B^2 + \sqrt{2} \times AB)^{1/2}$

Sol. $\vec{A} \cdot \vec{B} = AB\cos\theta$ . . . (1)

$|\vec{A} \times \vec{B}| = AB\sin\theta\,\hat{n}$ . . . (2)



$\therefore \quad AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

Again given $\vec{C} = \vec{A} + \vec{B}$

$\therefore \quad |\vec{C}| = (A^2 + B^2 + 2AB\cos 45°)^{1/2}$

$= (A^2 + B^2 + \sqrt{2}AB)^{1/2}$

Hence (D) is correct.

Prob 8. *What is the value of linear velocity, if* $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ *and* $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ *?*

(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ *(B)* $18\hat{i} + 13\hat{j} - \hat{k}$

(C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ *(D)* $6\hat{i} - 2\hat{j} + 8\hat{k}$

Sol. $\vec{v}$ = tangential velocity $= \vec{r} \times \vec{\omega}$

$\vec{v} = (5\hat{i} - 6\hat{j} + 6\hat{k}) \times (3\hat{i} - 4\hat{j} + \hat{k})$

$= 18\hat{i} + 13\hat{j} - 2\hat{k}$

Hence (B) is correct.

Prob 9. *A particle moves in x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $\vec{p}$ at any time t is $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between $\vec{F}$ and $\vec{p}$ at given time t will be*

(A) $\theta = 0°$ *(B) $\theta = 30°$*
(C) $\theta = 90°$ *(D) $\theta = 180°$*

Sol. $P = \sqrt{P_x^2 + P_y^2} = 2\sqrt{\cos^2 t + \sin^2 t} = 2$, which is independent of t, which means the applied force is not changing the magnitude of velocity.

i.e. $\vec{F}$ is perpendicular to $\vec{p}$

Hence (C) is correct.

Prob10. *If $\hat{i}$ denotes a unit vector along an incident ray $\hat{r}$ the unit vector along the refracted ray in a medium of refractive index μ and $\hat{n}$ a unit vector normal to boundary of medium directed towards incident medium, law of refraction is*

(A) $\hat{i}.\hat{n} = \mu(\hat{r}.\hat{n})$ *(B) $\hat{i}\times\hat{n} = \mu(\hat{n}\times\hat{r})$*
(C) $\hat{i}\times\hat{n} = \mu(\hat{r}\times\hat{n})$ *(D) $\mu(\hat{i}\times\hat{n}) = \hat{r}\times\hat{n}$*

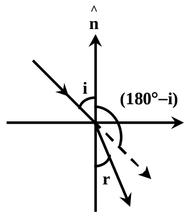
Sol. $|\hat{i}\times\hat{n}| = 1.1\sin(180° - i) = \sin i$

$|\hat{r}\times\hat{n}| = 1.1\sin(180 - r) = \sin r$

Now $\dfrac{|\hat{i}\times\hat{n}|}{|\hat{r}\times\hat{n}|} = \dfrac{\sin i}{\sin r} = \mu$

$\therefore\ \hat{i}\times\hat{n} = \mu(\hat{r}\times\hat{n})$, as their directions are also same.

Hence (C) is correct.



ASSIGNMENT PROBLEMS

Subjective:

1. Two forces whose magnitudes are in ratio of 3 : 5 give a resultant of 35 N. If the angle of inclination be 60°, calculate the magnitude of each force.

2. Find the unit vector of $3\hat{i}+4\hat{j}-\hat{k}$

3. Given $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. What is the vector component of $\vec{A}$ in the direction of $\vec{B}$?

4. Given $\vec{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+6\hat{k}$. Calcualte the angle made by $(\vec{A}+\vec{B})$ with x-axis.

5. Prove that the vectors $2\hat{i}-3\hat{j}-\hat{k}$ and $-6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.

6. Calculate the area of the parallelogram when adjacent sides are given by the vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-3\hat{j}+\hat{k}$.

7. What is the angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$?

8. Two vectors of magnitudes A and $\sqrt{3}$ A ar perpendicular to each other. What is the angle which their resultant makes with $\vec{A}$?

9. What should be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum ?

10. Find the direction cosines of $5\hat{i}+2\hat{j}+4\hat{k}$.

Objective:

1. Which of the following statement is correct?
   (A) A vector having zero length can have a unique direction.
   (B) If $\vec{A} \times \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
   (C) If $\vec{A} . \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
   (D) The vector $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ are mutually perpendicular.

2. The magnitude of a given vector with end points (4, –4, 0) and (–2, –2, 0) must be
   (A) 6 (B) $5\sqrt{6}$
   (C) 4 (D) $2\sqrt{10}$

3. If the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units, respectively, and $\vec{A} + \vec{B} = \vec{C}$, then the angle between $\vec{A}$ and $\vec{B}$ is
   (A) zero. (B) π
   (C) $\pi/2$ (D) $\pi/4$

4. If $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j}$, then their dot product will be
   (A) 0 (B) 12
   (C) 8 (D) 16

5. A particle is acted upon by two forces of 3 N and 4 N simultaneously. Which of the following is most correct?
   (A) The resultant of these forces is 7 N.
   (B) The resultant of these forces is 1 N.
   (C) The resultant of these forces in 4 N.
   (D) The resultant of these forces lies between 1 N and 7 N

6. Find the value of c if $\vec{A} = 0.4\hat{i} + 0.3\hat{j} + c\hat{k}$ is a unit vector.
   (A) 0.5 (B) $\sqrt{0.75}$
   (C) 1 (D) none of these.

7. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A} . \vec{B} = 0$ and $\vec{A} . \vec{C} = 0$. The vector $\vec{A}$ is parallel to
   (A) $\vec{B}$ (B) $\vec{C}$
   (C) $\vec{B} . \vec{C}$ (D) $\vec{B} \times \vec{C}$

8. Angular momentum is
   (A) scalar. (B) an axial vector.
   (C) a polar vector. (D) a null vector.

9. Minimum number of forces having equal magnitudes, which can give a resultant zero, is
   (A) 2 (B) 4
   (C) 3 (D) 1

10. $\hat{i} \times (\hat{j} \times \hat{k})$ is
(A) $\hat{i} + \hat{j} + \hat{k}$
(B) $\hat{i} + \hat{j} + \hat{k}$
(C) zero vector
(D) unit vector.

11. If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j} + \hat{k}$, then the unit vector along $(\vec{A} + \vec{B})$ is
(A) $\frac{3\hat{i} + 7\hat{j} + \hat{k}}{\sqrt{59}}$
(B) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{59}}$
(C) $\frac{\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{18}}$
(D) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$

12. A particle moves under a force $\vec{F} = 2\hat{i} + 4\hat{j}$ with a velocity $\vec{v} = 4\hat{i} - 2\hat{j}$, then the power delivered by the force is
(A) 16 W
(B) zero
(C) 8 W
(D) $8\sqrt{2}$ W

13. If $\vec{A} = \vec{B} + \vec{C}$, and the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is
(A) $\cos^{-1}(3/5)$
(B) $\cos^{-1}(4/5)$
(C) $\pi/2$
(D) $\sin^{-1}(3/4)$

14. Given that $\vec{A} + \vec{B} + \vec{C} = 0$. Out of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either vector having equal magnitudes. Then, the angle between the vectors is
(A) 30°, 60°, 90°
(B) 45°, 45°, 90°
(C) 45°, 60°, 90°
(D) 90°, 135°, 135°

15. The vector sum of N coplanar forces, each of magnitude F, when each force is making an angle of $2\pi/N$ with the preceding one is
(A) $N\vec{F}$ (B)
$N\vec{F}/2$
(C) $\vec{F}/2$
(D) zero

16. At what angle two forces 2F and $\sqrt{2}$ F should act, so that the resultant force is F $\sqrt{10}$?
(A) 45°
(B) 60°
(C) 120°
(D) 90°

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

1. 15 N, 25 N
2. $\frac{3}{\sqrt{26}}\hat{i}+\frac{4}{\sqrt{26}}\hat{j}-\frac{1}{\sqrt{26}}\hat{k}$
3. 2.5 ($(\hat{i}+\hat{j})$
4. 45°
6. 13.96 sq. units.
7. 90°
8. 60°
9. $\theta = 0°$
10. $\frac{5}{\sqrt{45}},\frac{2}{\sqrt{45}},\frac{4}{\sqrt{45}}$

Objective:

1.	D	2.	D
3.	C	4.	A
5.	D	6.	B
7.	D	8.	B
9.	A	10.	C
11.	A	12.	B
13.	A	14.	D
15.	D	16.	A

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

KINEMATICS

Kinematics

Syllabus:
Motion in a straight line, Position–time graph, speed & velocity, Uniform and non-uniform motion, Average speed & instantaneous velocity. Uniformly accelerated motion, velocity–time, position–time graphs, Relations for uniformly accelerated motion (graphical treatment), Elementary concepts of differentiation and integration for describing motion. Motion in a plane, cases of uniform velocity and uniform acceleration, Projectile motion, uniform circular motion.

KINEMATICS

Rest and Motion

When an observer says that a particle is in motion, it means that the particle is changing its position with respect to the observer as time passes, otherwise, it is said to be at rest. For an observer standing near a lamp-post, the moving car is in motion but the building appears to be at rest. Same statement is not true for an observer in a moving car. For this observer, the co-passengers are at relative rest whereas the buildings or lamp-posts appear to be in state of motion. Thus, the states of rest or motion are relative terms, relative to the state of the observer. Thus, motion is a combined property of the object under observation and the observer as well.

In order to define motion empirically, we locate the position of the particle with respect to the origin of a coordinate system (x-y-z axis) at different times. Such a system comprising x-y-z coordinate axes with a clock (to measure time interval) is called *frame of reference*.

If all the three coordinates (x, y, z) of a particle P remain unchanged as time passes, we say that P is at rest relative to the frame. However, if any one or more co-ordinates changed with time, the particle is said to be in motion with respect to the frame.

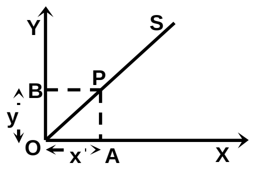
Motion in a Straight Line

When a particle moves along a straight line (assuming along the x-axis of the reference frame), we need only one coordinate (here the x-coordinate) to specify its position. This is also known as motion in one dimension or one-dimensional motion or rectilinear motion of the particle.

The remaining two coordinates (y and z) remain unchanged as time passes. Motion of a particle projected vertically upward is one-dimensional motion. It is appropriate to mention here that for a particle moving in a plane along a curved path, two coordinates are required (say, x and y) to specify the position. Such motions are called motion in a plane or motion in two dimensions. Examples of two dimensional motion are: (i) circular motion, (ii) projectile motion, (iii) motion of an insect on table top along a curved path, etc.

Similarly, we require all the three coordinates (x, y and z) to locate the position of a mosquito flying in space. Such motions are called three-dimensional motion or motion in three dimensions. In this chapter, we shall describe the simplest kind of motion, i.e. the motion in a straight line only.

Illustration 1. *A particle P is moving along a straight line OS. The coordinates x = OA and y = OB are required to describe the motion of the particle. Does it indicate motion in one dimension or in two dimensions?*



Solution: Yes, it is a one-dimensional motion.

Illustration 2. *Two particles A and B start together and are moving with speed 2 m/s and 3 m/s, respectively, in the same direction. Find how far will B be from A after 10 second.*

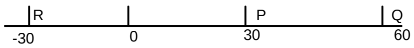
Solution: Distance travelled by A_1, $S_1 = 2 \times 10 = 20$ m,
Distance travelled by B, $S_2 = 3 \times 10 = 30$ m.
$\therefore$ Distance between them, $S = S_2 - S_1 = 30 - 20 = 10$ m.

In our discussions in the following sections, we shall treat the objects in motion as point objects or like a particle. This approximation is true in cases where the size of the object is much smaller than the distance it covers in a reasonable time interval. We consider moon as a particle during its orbital motion round the earth and even earth as a particle during its orbital motion round the sun.

POSITION, PATH LENGTH AND DISPLACEMENT

(i) When motion is along a straight line

Position: To locate the position of the particle at some time (instantaneous position) we choose an axis (*say x-axis*) with a fixed origin O in the given reference frame and find the distance from O. Thus, positions of P, Q and R are +30 cm, +60 cm and –30 cm, respectively. In vector notation, we represent the position vectors as $\overrightarrow{OP} = (+30\,\text{cm})\hat{i}$, $\overrightarrow{OQ} = +(60\,\text{cm})\hat{i}$ and $\overrightarrow{OR} = (-30\text{cm})\hat{i}$, respectively.



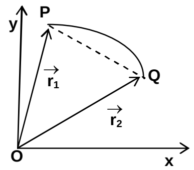
Path length: With reference to the above *figure*, let a particle starts moving from the origin O at time t = 0 and at subsequent times t_1, t_2 and t_3 ($t_3 > t_2 > t_1$) and it is at P, Q and R, respectively. Can you find the path length during the interval (i) $t = 0$ to $t = t_1$, (ii) $t = 0$ to $t = t_3$, (iii) $t = t_1$ to $t = t_3$?

The path length is always equal to the total distance moved by the particle. Hence, the corresponding path lengths are 30 cm, 150 cm and 120 cm, respectively. Thus, path lengths add up like a scalar quantity, they have no direction but magnitude only.

Displacement: It is defined as the change in the position vectors in the given time interval. If x_i and x_f be the initial and final positions of the particle in the time interval $(t_2 - t_1)$, then the displacement $\Delta x = x_2 - x_1$. In vector notation, $\vec{x}_i = x_1\hat{i}$, $\vec{x}_f = x_2\hat{i}$, hence $\Delta\vec{x} = (x_2 - x_1)\hat{i}$, since the motion is along the x-axis only. Can you find the displacement during the same time interval as done for calculating the path length in the previous section. The displacements are (i) (+30 cm)$\hat{i}$, (ii) (–30 cm)$\hat{i}$ and (iii) (–60 cm)$\hat{i}$. Note that no sign + or – has been mentioned in expressing the path length (since it is scalar) while ± sign has been mentioned in expressing the displacements (since displacement is a vector quantity). The magnitude of the displacement may or may not be equal to the path length traversed by the particle.

In general path length ≥ magnitude of displacement. If the initial and final positions of a particle in its motion be same, the displacement is zero but path length is not zero.

(ii) When the motion is along the curved path



Position: Initial position is at P and is represented by $\overrightarrow{OP} = \vec{r}_1$. Similarly, final position is at Q and is represented by $\overrightarrow{OQ} = \vec{r}_2$. In terms of coordinates of P and Q, $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Path length: Here, the path length is the length of the curve joining the initial and final positions (not the straight line joining P and Q as shown by dotted line) along which the particle has actually moved through.
Displacement: Magnitude of the displacement is the length of the straight line joining the initial and final positions and its direction is from the initial to the final position. We have already defined displacement as the change in position vector, hence displacement $\overrightarrow{PQ} = \vec{r}_2 - \vec{r}_1$,

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\,\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Exercise 1: ***A person moves from A to B along the semicircular path. Compare the distance moved by him and the displacement.***

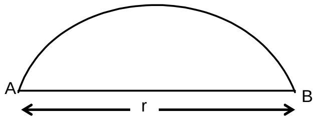
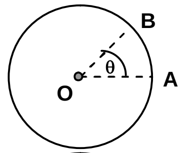


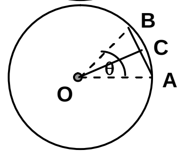
Illustration 3. *A boy travels from his house to a play ground along a straight path of length 'D' meter and return back to his house. Find the distance travelled and displacement of the boy.*

Solution: Distance = 2D meter, Displacement = Zero

Illustration 4. *A particle moves along a circle of radius R. Find the path length and magnitude of displacement from initial position A to final position B.*



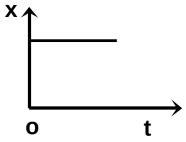
Solution: Path length = Rθ
Displacement = AB = AC + BC = 2R sin (θ/2).



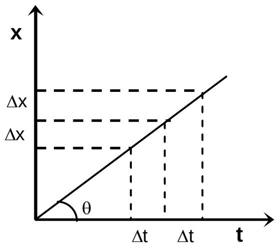
Position–Time Graph:

If we plot time t along the x-axis and the corresponding position (say x) from the origin O on the y-axis, we get a graph which is called the position–time graph. This graph is very convenient to analyse different aspects of motion of a particle. Let us consider the following case.

(i) In this case, position (x) remains constant but time changes. This indicates that the particle is stationary in the given reference frame. Hence, the straight line nature of position–time graph parallel to the time axis represents *the state of rest*. Note that its slope (tan θ) is zero.



(ii) When the x–t graph is a straight line inclined at some angle ($\theta \neq 0$) with the time axis, the particle traverses equal displacement Δx in equal interval of time Δt. The motion of the particle is said to be *uniform rectilinear motion*. The slope of the line measured by $\frac{\Delta x}{\Delta t} = \tan\theta$ represents the uniform velocity of the particle.



(iii) When the x–t graph is a curve, motion is not uniform. It either speeds up or slows down depending upon whether the slope ($\tan\theta$) successively increases or decreases with time. As shown in the figure, the motion speeds up from t = 0 to $t = t_1$ (since the slope $\tan\theta$ increases). From $t = t_1$ to $t = t_2$, AB represents a straight line indicating uniform motion. From $t = t_2$ to $t = t_3$, the motion slows down and for $t > t_3$ the particle remains at rest in the reference frame.

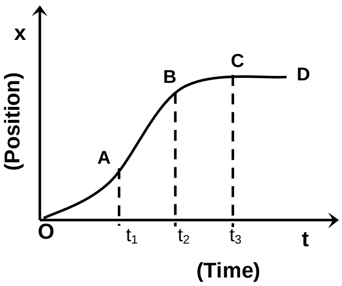
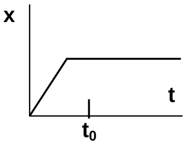


Illustration 5. *The adjacent figure shows the displacement–time graph of a particle moving on the x-axis. Choose the correct option given below.*
(A) The particle is continuously going in positive x direction.
(B) The particle is at rest.
(C) The particle moves at a constant velocity all time
(D) The particle moves at a constant velocity upto a time t_0, and then stops.



Solution: **(D).** *Upto time t_0,, particle is said to have uniform rectilinear motion and after that comes to rest as the slope is zero.*

Exercise 2:
(i) ***Distinguish between the distance covered by a body and its displacement. What are the characteristics of displacement?***
(ii) ***Under what condition will the distance and displacement of a moving object have the same magnitude.***

Speed

The term average indicates overall effect whereas instantaneous means the effect at a particular time. Hence, the average speed in a given time interval ($t_2 - t_1$) is measured by the distance covered (path length s) divided by the time interval.

Thus, average speed $= \frac{s}{t_2 - t_1} = \frac{\text{path length}}{\text{time interval}}$

If the time interval ($t_2 - t_1$) is divided into small segments Δt_1, Δt_2, ..., for which the corresponding path lengths be Δs_1, Δs_2, ..., then

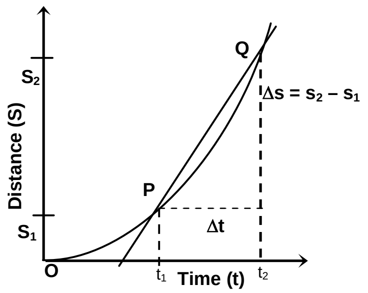
Average speed $= \frac{\Delta s_1 + \Delta s_2 + \Delta s_3 + ...}{\Delta t_1 + \Delta t_2 + \Delta t_3 + ...} = \frac{s}{t_2 - t_1}$

Hence, the average speed during one such time interval is equal to $\frac{\Delta s}{\Delta t}$. If Δt is infinitesimally small $|\Delta t \to 0|$, then we define the instantaneous speed at a time t as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Speed is a scalar quantity. It has only the magnitude and no direction. For a particle in motion in a given reference frame the instantaneous or average speed during any time interval is always positive.

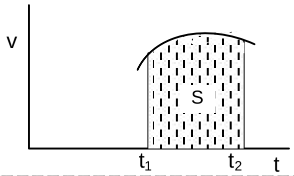
Consider the distance time graph as shown in the given figure. The average speed during the time interval Δt is $\frac{\Delta s}{\Delta t}$ which is the slope of the chord PQ. As $\Delta t \to 0$, the chord PQ becomes the tangent at P and the average speed becomes the instantaneous speed at P given by $\frac{ds}{dt} = \tan\theta$, which is the slope of the tangent at P.



Remember that the s–t graph (position–time graph) does not indicate the path of motion but represents increase in the path length as time increases, whether the particle does or does not retrace its path. Now, instantaneous speed $v = \frac{ds}{dt}$.

$\therefore$ ds = v dt and $s = \int_{t_1}^{t_2} v dt$ = total distance travelled during the time interval $(t_2 - t_1)$

It is evident that the area under the speed time graph (shown by the shaded region) measures the total distance covered during the time interval $t_2 - t_1$.



Exercise 3:

i) ***Can a body moving with uniform speed have variable velocity?***

ii) ***Can a body moving with uniform velocity have variable speed?***

iii) ***Can average velocity ever become equal to instantaneous velocity ?***

Illustration 6. *A car covers the first half of the distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. What is the average speed of the car?*

Solution : Let the distance between the two places be 2x km.

$\therefore$ Time taken by the car for the first half of the journey = $\frac{x}{40}$ hr

Also, the time taken for the second half = $\frac{x}{60}$ hr

The total time of the journey = $\frac{x}{40} + \frac{x}{60} = \frac{5x}{120}$ hr

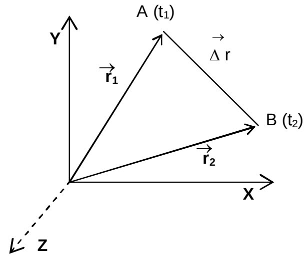
Average speed = $\frac{\text{distance}}{\text{time}} = \frac{2x}{5x/120}$ = 48 km/hr

Velocity

By definition,

Average velocity, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$

Note that the positions in between the interval of time t_1 and t_2 are to be specified in finding the average velocity. If the particle takes different paths to travel from A to B in the same time interval, the average velocity will remain same but average speed will be different and greater than the magnitude of $\vec{v}_{av}$. In the special case when the points A to B is straight, the average speed is equal to the magnitude of average velocity.



If $\Delta t \to 0$, the path length Δs during the interval Δt is equal to the Δr.

Hence, the instantaneous velocity $\vec{v} = \lim \dfrac{\Delta\vec{r}}{dt} = \dfrac{d\vec{r}}{dt}$, and

magnitude of the velocity is $v = \left|\dfrac{\Delta\vec{r}}{dt}\right| = \dfrac{|d\vec{r}|}{dt} = \dfrac{ds}{dt}$.

Hence, the instantaneous speed at any time t is the magnitude of instantaneous velocity at that time.

On a graph of position as a function of time for straight line, the instantaneous velocity at any point is equal to the slope of the tangent to the curve at that point.

The figure depicts the motion of a particle.

	x, t group	Motion of particle
A	Positive slope, so $V_x > 0$	Moving in +ive x = direction
B	Larger positive slope, so $V_x > 0$	Moving in +x direction faster than at A
C	Zero slope, so $V_X = 0$	Instantaneously at rest
D	Negative slope, so $V_x < 0$	Moving in –x direction
E	Smaller negative slope, so $V_x < 0$	Moving in –ve x-direction more slowly than at D

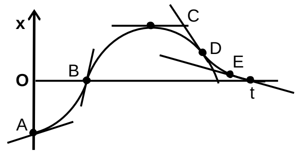
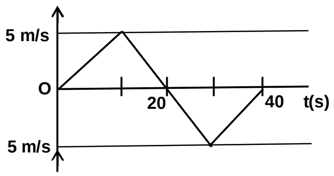


Illustration 7. *From the velocity– time plot shown in fig. Find*

(a) distance travelled by the particle during the first 40 seconds.

(b) displacement travelled by the particle during the first 40 seconds.

(c) Also find the average velocity during this period.



Solution: (a) Distance = area under the curve

$= \dfrac{1}{2} \times 20 \times 5 + \dfrac{1}{2} \times 5 \times 20$

= 50 + 50 = 100 m

For distance measurement, the curve is plotted as in Fig. (a)

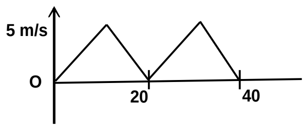


Fig. (a)

(b) Displacement = area under the curve in Fig. (b) = 0

(c) $V_{aV} = \dfrac{\text{Displacement}}{\text{time}}$

As displacement is zero,

$\therefore V_{av} = 0$

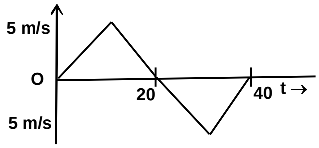


Fig. (b)

Illustration 8. *If velocity of a particle moving along a straight line changes with time as V(m/s) = 4 sin $\left(\frac{\pi}{2}t\right)$, its average velocity over time interval t = 0 to t = 2(2n – 1) sec, (n being any (+)ve integer) is*

(A) $\dfrac{8}{\pi(2n-1)}$ *m/s* *(B)* $\dfrac{4}{\pi(2n-1)}$ *m/s*

(C) zero (D) $\dfrac{16(2n-1)}{\pi}$ *m/s* *(D) none*

Solution: **(A).**

Displacement over the interval t = 0 to t = 2(2n –1) sec

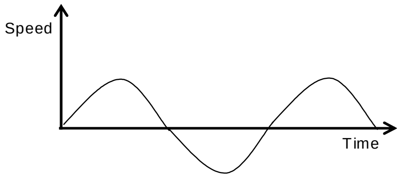
$$= 4\int_0^{2(2n-1)} \sin\left(\frac{\pi}{2}t\right)dt = -\left(\frac{8}{\pi}\right)\left|\cos\frac{\pi t}{2}\right|_0^{2(2n-1)}$$

$$= \frac{16}{\pi}\text{m}$$

$$\Rightarrow \text{Average velocity } = \frac{16}{2(2n-1)\pi} = \frac{8}{\pi(2n-1)}\text{ m/s}$$

Exercise 4:

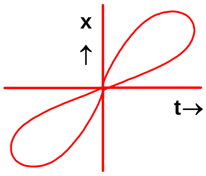
(i) ***Is the speed–time graph shown in figure is possible ?***



(ii) ***Under what circumstances, does the relationship Δx = v. Δt hold good.***

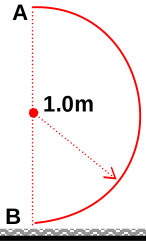
(iii) ***Show that average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.***

(iv) ***State with reasons why the graph in the adjacent figure cannot represent one-dimensional motion of a particle.***



(v) ***In 1.0 sec, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in the adjacent figure. The magnitude of average velocity is***

(A) 3.14 m/s ***(B) 2.0 m/s***
(C) 1.0 m/s ***(D) Zero***



Acceleration

Motion of a particle moving with constant velocity along a straight line is said to be uniform motion because neither the speed nor the direction of motion changes with the passage of time. On the other hand, the motion is said to be accelerated, if either the speed or the direction or both continuously change with time.

Definition: *Acceleration is defined as the rate of change of velocity.* It is a vector quantity and has its direction along which velocity has changed.

Average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta\vec{v}}{\Delta t}$

Note: When the velocity decreases, we say that the particle is decelerating. Deceleration is equivalent to negative acceleration.

It is also a vector quantity directed along the direction of the change $\Delta\vec{v}$ and independent of the intermediate values of velocities in between the interval $t_2 - t_1$.

Instantaneous acceleration at a time t is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{v}) = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

If the particle moves along x-axis,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i}) = \frac{dv_x}{dt}\hat{i} = \frac{d^2x}{dt^2}\hat{i}$$

If the particle moves in the xy plane,

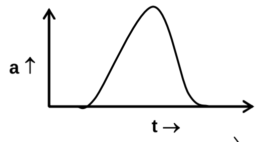
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} = a_x\hat{i} + a_y\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Exercise 5:

(i) ***Can a body have an acceleration with zero velocity?***

(ii) ***Can the direction of the velocity of a body change when its acceleration is constant ?***

(iii) ***A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time-interval. Suggest acceleration time graph for this situation.***



(iv) ***Figure shows the x–t graph of a particle moving along a straight line. What is the sign of the acceleration during the intervals OA, AB, BC and CD?***

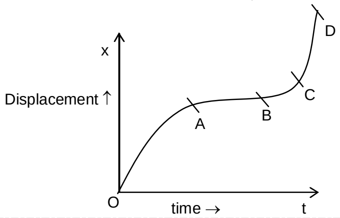


Illustration 9. *A body moving in a curved path possesses a velocity 3 m/s towards north at any instant of its motion. After 10s, the velocity of the body was found to be 4 m/s towards west. Calculate the average acceleration during this interval.*

Solution: To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity v_0 and the final velocity v are drawn from a common origin. The vector difference of them is found by the parallelogram method.

The magnitude of difference is

$|v - v_0| = OC = \sqrt{OA^2 + AC^2}$

$= \sqrt{4^2 + 3^2} = 5 \text{ m/s}$

The direction is given by

$\tan\theta = \frac{3}{4} = 0.75 \quad \therefore\ 37^0$

$\therefore$ Average acceleration

$= \frac{v - v_0}{t} = \frac{5}{10}$

$= 0.5\ m/s^2$ at 37° South of West.

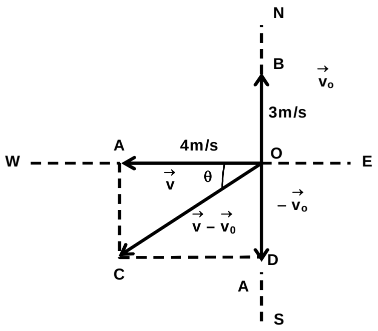


Illustration 10. *A car is moving eastwards with velocity 10 m/s. In 20 seconds, the velocity changes to 10 m/s northwards. Calculate the average acceleration in this time.*

Solution: The change in velocity

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$= 10\hat{j} - 10\hat{i}$

$\Rightarrow\ \Delta\vec{v}$ has a magnitude of $10\sqrt{2}$ and is directed towards N-W.

So, $|\vec{a}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{1}{\sqrt{2}}\ m/s^2$ towards N-W.

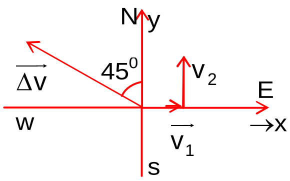


Illustration 11. *Which of the following statements is possible in a one/two-dimensional motion?*
(A) A body is having zero velocity and still accelerating.
(B) The velocity of an object reverses direction when acceleration is constant.
(C) An object increasing in speed as its acceleration decreases.
(D) None of these.

Solution: **(A), (B) and C**

(A) When a body is projected vertically upward, at the highest point its speed becomes zero while it is accelerating downwards with $g = 9.8\ m/s^2$.

(B) When a body is projected upward, the direction of its velocity reverses during its descent while the acceleration (g) remains constant.

(C) Steel ball falling through a viscous liquid before terminal velocity is attained.

Exercise 6:

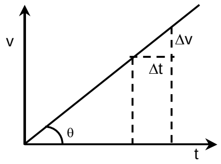
(i). ***A particle is moving along a circular path of radius 5 m with a uniform speed of 5 m/s. What will be the average acceleration when the particle completes half revolution?***
(A) Zero(B) 10 m/s^2
(C) $10\pi\ m/s^2$ (D) $10/\pi\ m/s^2$

(ii). ***Which of the following statement is incorrect?***
(A) Zero velocity of particle does not necessarily mean that its acceleration is zero.
(B) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
(C) If speed of a particle is constant its acceleration must be zero.
(D) None of these.

Uniformly accelerated motion

Motion of a particle is said to be uniformly accelerated if acceleration (a vector quantity) remains constant in magnitude as well as in direction. Motion of a particle falling freely under gravity is an example of uniformly accelerated motion since the acceleration ($\vec{g}$) remains constant (assuming negligible air resistance).

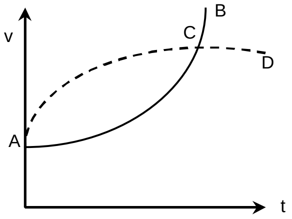
Velocity–time graph



When the v–t graph is a straight line, inclined at an angle θ with the time axis, the velocity increases equally in equal time interval. This indicates that the acceleration is uniform. Its magnitude is $a = \frac{\Delta v}{\Delta t} = \tan\theta$, the slope of v–t graph.

Let us find the area under the v–t graph:

$$A = \int dA = \int v dt = \int \frac{ds}{dt} dt = \int_i^f ds \ = \text{net displacement}$$



If the v–t graph is a curve, the slope continuously changes with time, which indicates that the magnitude of acceleration either increases with time (for curve AB) or it decreases with time (for curve ACD).

Note:

Features of v–t graphs:

(i) The slope of v–t graph gives the instantaneous acceleration.

(ii) The area under the v–t graph gives the net displacement (not distance) in the given time interval.

EQUATION OF MOTION IN A STRAIGHT LINE WITH UNIFORM ACCELERATION

Consider the motion of a particle moving along the x-axis with uniform acceleration.

Let u = Initial velocity (at time t = 0)

v = Final velocity (at time t), and

x = Net displacement in the time interval t = 0 to t = t.

The equations describing such uniformly accelerated motion are

$$v = u + at; \quad x = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2ax$$

Remember that these equations are valid or applicable if the acceleration remains constant both in magnitude and direction.

Derivation of Equations of Motion

(i) v = u + at

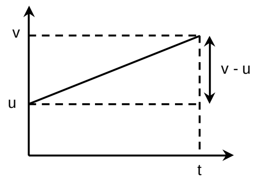
Calculus method:

By definition $a = \frac{dv}{dt}$ or, $dv = a dt$

Integrating, $\int_u^v dv = a\int_0^t dt$ $[\because a = \text{constant}]$

$v - u = at$, or $v = u + at$

Graphical Method:



By definition, slope of v–t graph gives the acceleration

$$\therefore \ a = \tan\theta = \frac{v-u}{t-0} = \frac{v-u}{t}$$

or $v - u = at$ $\therefore$ $v = u + at$

(ii) $\mathbf{x = ut + \frac{1}{2}at^2}$

Calculus method:

By definition, instantaneous velocity $v = \dfrac{dx}{dt}$

or, $dx = v\,dt$

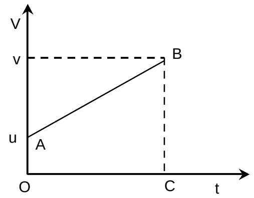
$= (u + at)dt$ $[\because v = u + at]$

$= u\,dt + at\,dt$

Integrating, $\int_0^x dx = u\int_0^t dt + a\int_0^t t dt$ $[\because$ u and a are constant$]$

Hence, $x = ut + \dfrac{1}{2}at^2$

Graphical method:



We know that area under the velocity– time graph gives the net displacement during the given time interval. Hence, net displacement x = area OABC.

or $x = \dfrac{1}{2}(OA + BC) \times OC$

$= \dfrac{1}{2}(u+v).t = \dfrac{1}{2}[2u + \dfrac{v-u}{t}t]t$

$= \dfrac{1}{2}[2u + at]t$ $[\because a = \dfrac{v-u}{t}] = ut + \dfrac{1}{2}at^2$

(iii) $\mathbf{v^2 = u^2 + 2ax}$

Calculus method:

By definition, $a = \dfrac{dv}{dt} = \dfrac{dv}{dx}.\dfrac{dx}{dt}$

or, $a = v.\dfrac{dv}{dx}$

or, $vdv = adx$

Integrating, $\int_u^v vdv = a\int_0^x dx$

or, $\dfrac{v^2 - u^2}{2} = ax$ $\therefore$ $v^2 = u^2 + 2ax$

Graphical method:

From the v–t graph, net displacement x = area under the v–t graph

or, $x = \dfrac{1}{2}(v + u)t$

Multyplying both sides by (v – u),

$$x\,(v - u) = \frac{1}{2}(v+u)(v-u)t$$

or, $$x\left(\frac{v-u}{t}\right)=\frac{v^2-u^2}{2}$$

or, $$x \cdot a=\frac{v^2-u^2}{2} \qquad \left[\because a=\frac{v-u}{t}\right]$$

$\therefore$ $$v^2 = u^2 + 2ax$$

It is to be noted that u, v and a are vectors and may have positive or negative value depending on whether their directions are along the positive or negative directions of the x-axis.

Graphs Representing Motion of a Particle

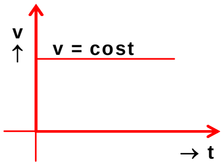
From the knowledge of calculus, we can say from Eq. (1) that:

(i) Slope of s–t graph gives velocity;
(ii) Slope of v–t graph gives acceleration;
(iii) Area under v–t graph gives displacement; and
(iv) Area under a–t graph gives change in velocity.

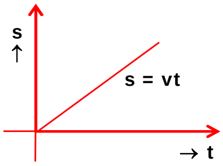
(a) Uniform motion

v = constant

$\Rightarrow a = \frac{dv}{dt} = 0$



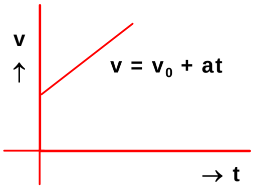
v–t graph
(a)



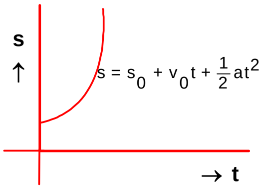
s–t graph
(b)

(b) Uniformly accelerated motion

a = constant

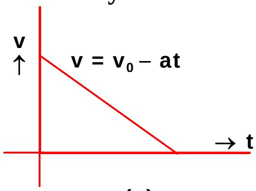


v–t graph
(a)

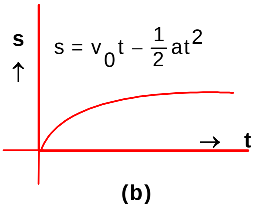


s–t graph
(b)

(c) Uniformly retarded motion



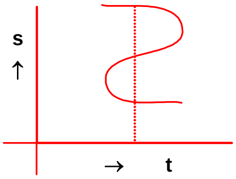
(a)



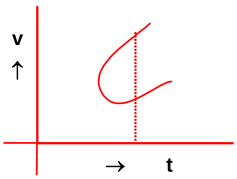
(b)

Some facts regarding graphs:

(i) Two or more values of velocity or displacement at any particular instant of time are not possible. So, the corresponding graphs should be rejected. For example, the following graphs are not possible.

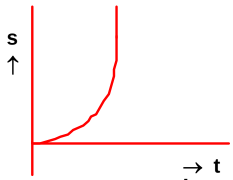


(a)

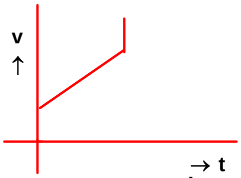


(b)

(ii) At any point, the slopes of s–t or v–t graph can never be infinite because infinite slope of s–t graph means infinite velocity and that of v–t graph means infinite acceleration, which are not possible. So, corresponding graphs are not acceptable. For example, the following graphs are all not possible.



s–t graph
(a)



v–t graph
(b)

In general, when a particle is moving with a uniform acceleration, to its motion is described by the following equations.

$$\vec{r}(t) = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\cdot\vec{s}$$

Here, $\vec{r}(t) =$ represents position vector of a particle at an instant t.

$\vec{r}_0$ = position vector of a particle at t = 0.

$\vec{u}$ = initial velocity of a particle at t = 0.

$\vec{v}$ = velocity of a particle at an instant t.

$\vec{a}$ = acceleration of the particle at an instant t.

Exercise 7:

(i) ***A boy sitting on a rail road car moving with a constant velocity tosses a coin up. Describe the path of the coin as seen by***

(a) the man on the train.

(b) the man standing on the ground near the rail.

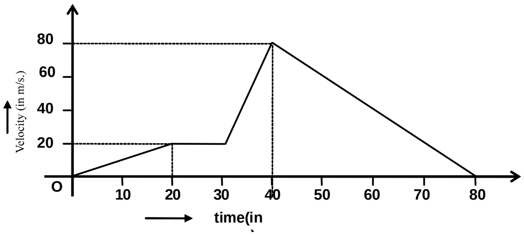
(ii) ***A particle is moving along a straight path, draw its velocity– time graph for the following cases:***

(a) When the acceleration of the particle increases.

(b) When the displacement of the particle obeys the relation $s = 4 + 5t + 2t^2$

(c) When the acceleration of the particle is given by $a = 12\cos 6t$

Illustration 12. *The velocity–time graph of a moving object is given in the figure. Find the maximum acceleration of the body and distance travelled by the body in the interval of time in which this acceleration exists.*



Solution: Acceleration is maximum when slope is maximum.

$$a_{max} = \frac{80-20}{40-30} = 6\,\text{m/s}^2$$

$$S = 20\,\text{m/s} \times 10\,\text{s} + \frac{1}{2} \times 6\,\text{m/s}^2 \times 100\,\text{s}^2 = 500\,\text{m}.$$

Illustration 13. *A particle having initial velocity is moving with a constant acceleration 'a' for a time t.*
(a) Find the displacement of the particle in the last 1 second.
(b) Evaluate it for u = 2 m/s, a = 1 m/s² and t = 5 sec.

Solution: (a) The displacement of a particle at time t is given by $s = ut + \frac{1}{2}at^2$

At time (t – 1), the displacement of a particle is given by

$$S' = u(t-1) + \frac{1}{2}a(t-1)^2$$

$\therefore$ Displacement in the last 1 second is

$$S_t = S - S'$$

$$= ut + \frac{1}{2}at^2 - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$

$$= ut + \frac{1}{2}at^2 - ut + u - \frac{1}{2}a(t-1)^2$$

$$= \frac{1}{2}at^2 + u - \frac{1}{2}a(t^2 + 1 - 2t) = \frac{1}{2}at^2 + u - \frac{1}{2}at^2 - \frac{a}{2} + at$$

$$S = u + \frac{a}{2}(2t-1)$$

(b) Putting the values of u = 2m/s, a = 1m/s² and t = 5 sec, we get

$$S = 2 + \frac{1}{2}(2\times5-1) = 2 + \frac{1}{2}\times9$$

$$= 2 + 4.5 = 6.5 \text{ m}$$

Illustration 14. *A car moving along a straight road with a speed of 72 km/h is brought to a stop with in a distance of 10m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?*

Solution: v = 72 km/h

$$= \frac{72\times1000}{3600} = 20\,\text{m/s}$$

using equation

$$\vec{v}^2 = \vec{u}^2 + 2a(\vec{x} - \vec{x}_0)$$

$$= (20)^2 + 2 \times a \times 10$$

$$a = -\frac{20\times20}{20} = -20\,\text{m/s}^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$0 = 20 - 20\,t$$

$$t = \frac{20}{20} = 1\text{sec}$$

So, it will take 1 sec for the car to stop.

Illustration 15. *Position of a particle moving along x-axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds.*
(a) Find the position of the particle at t = 2s.
(b) Find the displacement of the particle in the time interval from t=0 to t=4s.
(c) Find the average velocity of the particle in the time interval from t=2s to t=4s.
(d) Find the velocity of the particle at t =2s.

Solution: (a) $x_{(t)} = 3t - 4t^2 + t^3$

$\Rightarrow x_{(2)} = 3 \times 2 - 4 \times (2)^2 + (2)^3 = 6 - 4 \times 4 + 8 = -2$ m.

(b) $x_{(o)} = 0$

$x_{(4)} = 3 \times 4 - 4 \times (4)^2 + (4)^3 = 12$m.

Displacement $= x_{(4)} - x_{(0)} = 12$ m.

(c) $$< v >= \frac{x_{(4)} - x_{(2)}}{(4-2)} = \frac{12-(-2)}{2} \text{m/s} = 7\text{m/s}$$

(d) $$\frac{dx}{dt} = 3 - 8t + 3t^2$$

$$\Rightarrow v_{(2)} = \left(\frac{dx}{dt}\right)_{(2)} = 3 - 8\times 2 + 3\times(2)^2 = -1\text{m/s}$$

Illustration 16. *An anti-aircraft shell is fired vertically upwards with a muzzle velocity of 294 m/s. Calculate (a) the maximum height reached by it, (b) time taken to reach this height, (c) the velocities at the ends of 20th and 40th second. (d) when will its height be 2450 m? Given g = 980 cm/s^2.*

Solution : (a) Here, the initial velocity u = 294 m/s and g = 9.8 m/s^2

$\therefore$ The maximum height reached by the shell is,

$$H = \frac{u^2}{2g} = \frac{294^2}{2 \text{ x } 9.8} = 4410\text{m} = 4.41\text{km}$$

(b) The time taken to reach the height is,

$$T = \frac{u}{g} = \frac{294}{9.8} = 30\text{s}$$

(c) The velocity at the end of 20th second is given by,

v = u – gt = 294 – 9.8 x 20 = 98 m/s upward,

and the velocity at the end of 40th second is given by,

v = 294 – 9.8 x 40 = – 98 m/s

The negative sign implies that the shell is falling downward.

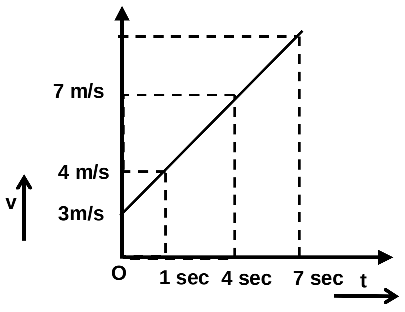
(d) From the equation

$h = ut + \frac{1}{2}gt^2$ or $2450 = 294\,t - \frac{1}{2}\times 9.8t^2$

or, $t^2 - 60\,t + 500 = 0$ $\therefore$ t = 10s and 50 s.

At t = 10 s the shell is at a height of 2450 m and is ascending, and at the end of 50 s it is at the same height, but is falling.

Illustration 17. *The velocity–time graph of a particle is given as shown in the figure. Find the distance travelled by the object in 7th second.*



Solution : $$\frac{dv}{dt} = \frac{7-4}{4-1} = 1\text{m/s}^2$$

$$\vec{s}_{t=7} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

$$= 3 + \frac{1}{2}(2\times 7 - 1) = 9.5\text{m}\,.$$

Exercise 8:

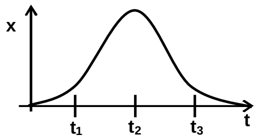
(i) ***A truck moving with constant acceleration covers the distance between two points 180 m apart in 6 seconds. Its speed as it passes the second point is 45 m/s. Find***
(a) its acceleration , and
(b) its speed when it was at the first point.

(ii) ***A body undergoing uniformly accelerated motion starts moving along +x-axis with a velocity of 5 m/s and after 5 seconds its velocity becomes 20 m/s in the same direction. What is the velocity of the body 10 seconds after the start of the motion ?***

(iii) ***What is the speed with which a stone is projected vertically upwards from the ground if it attains a maximum height of 20 m?***

(iv) ***A ball is thrown vertically upwards with a speed of 20 m/s from a hard floor. Draw a graph showing the velocity of the ball as a function of time if the ball suffers elastic collisions continuously.***

(v) ***The adjacent figure shows the x-coordinate of a particle as a function of time. Find the signs of v_x and a_x at $t = t_1$, $t = t_2$ and $t = t_3$.***



Solving problems in Kinematics using elementary concepts of differential and integral calculus

For the motion of a particle in a straight line, we always write instantaneous velocity $v = \frac{dx}{dt}$.

In case, the acceleration is non uniform and a function of displacement, we write,

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$$

Let us solve some illustrative examples:

Illustration 18. *The instantaneous velocity of a particle moving along a straight line is given by $v = \alpha t^2$ whose α is a positive constant. Find the average speed during the interval $t = 0$ to $t = T$.*

Solution: By definition, average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{\int_0^T v\,dt}{\int_0^T dt}$

$$= \frac{1}{T}\int_0^T \alpha t^2 dt = \frac{\alpha}{T}\left[\frac{t^3}{3}\right]_0^T = \frac{\alpha T^2}{3}.$$

Illustration 19. *The displacement (x) of a particle moving in one dimension under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$, x in m and t in sec. Find the displacement of the particle when its velocity is zero.*

Solution: Here, $t = \sqrt{x} + 3$
or $x = t^2 - 6t + 9$

$\therefore v = \frac{dx}{dt} = 2t - 6$

When v = 0, 2t – 6 = 0 $\Rightarrow$ t = 3 sec

At t = 3 sec, $x = t^2 - 6t + 9$

= 9 –6 (3) + 9 = 0

Hence, the displacement of particle is zero when its velocity is zero.

Motion Under Gravity (Free Fall)

When a body is dropped from some height (earth's radius = 6400 km), it falls freely under gravity with constant acceleration g (=9.8 m/s^2) provided the air resistance is negligible small. The same set of three equations of kinematics (where the acceleration $\vec{a}$ remains constant) are used in solving such motion.

Here, we replace $\vec{a}$ by $\vec{g}$ and choose the direction of y-axis conveniently. When the y-axis is chosen positive along vertically downward direction, we take $\vec{g}$ as positive and use the equations as

$$v = u + gh \qquad \Rightarrow \qquad v^2 = u^2 + 2gh$$

$$h = ut + \frac{1}{2}gt^2$$

where u is initial velocity of projection in the vertically downward direction.

However, if an object is projected vertically upward with initially velocity u, we can take y – axis positive in the vertically upward direction the set of equations reduces to

$$v = u - gt \qquad \Rightarrow v^2 = u^2 - 2gh$$

$$h = ut - \frac{1}{2}gt^2$$

In order to avoid confusion in selecting $\vec{g}$ as positive or negative, it is advisable to take the y-axis as positive along vertically upward direction and point of projection as the origin. We can now write the set of three equations in the vector form:

$$\vec{v} = \vec{u} + \vec{g}t \qquad \Rightarrow \qquad \vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{g}.\vec{h}$$

$$\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

Exercise 9:

(i) ***A stone is thrown upwards with a speed v from the top of a tower. It reaches the ground with a velocity 3v, what is the height of the tower ?***

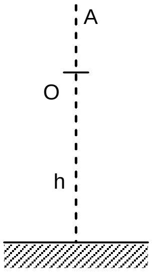
(ii) ***A stone is thrown vertically upwards with a velocity of 19.6 m/s. After 2 second, another stone is thrown upwards with a velocity of 9.8 m/s. When and where these stones will collide?***

Illustration 20. *A body is projected vertically upward, then find the velocity and acceleration of that body at it's highest point of motion?*

Solution: Velocity = 0 , acceleration = ± g

Illustration 21. *A ball is projected vertically upward with a speed of 4.0 m/s from a point 64 m above the ground. Find the time it takes to reach the ground. [g = 10 m/s^2]*

Solution: Before solving the problem, analyse the situation. As the ball will move up, it gradually slows down and attains the maximum height at A (where it comes to momentarily rest) and thereafter it retraces its path, attains the same speed at O but direction reversed down and finally it strikes the ground.



Choosing O (the point of projection) as the origin and positive y-axis as vertically upward, we collect the data in the vector notation which are given in the question.

Net displacement $\vec{y} = -h\ \hat{j} = (-64\text{ m})\,\hat{j}$

Constant acceleration $\vec{g} = -g\hat{j} = (-10\,\text{m/s}^2)\hat{j}$

Initial velocity $\vec{u} = +u\hat{j} = (+\,4.0\text{ m/s})\ \hat{j}$

Now, $\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$

Writing the values with proper sign.

$$-64\hat{j} = 4t\hat{j} + \frac{1}{2}(-10\hat{j})t^2$$

This reduces to a simple quandratic equation, $5t^2 - 4t - 64 = 0$

The solution $t = -\dfrac{16}{5}$ s is not permissible. Hence, the required time= 4.0 seconds.

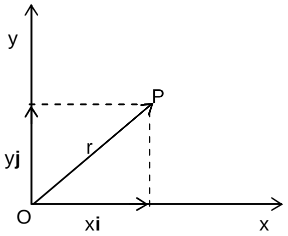
MOTION IN A PLANE

Position Vector and Displacement

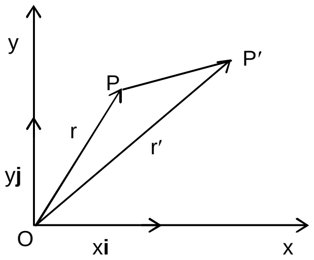
The position vector $\vec{r}$ of a particle located in a plane with reference to the origin of an x-y reference frame is

$$\vec{r} = x\hat{i} + y\hat{j}$$

where x, y are the coordinates of the object.



Let the particle be at a point P at any time t and at a point P′ at any time t′ as shown in figure.



$\therefore\ \overrightarrow{OP} = \vec{r}(t)$ and $\overrightarrow{OP}' = \vec{r}\,'(t')$

$\therefore$ Displacement vector

$$\vec{PP'} = \Delta\vec{r} = \vec{r}\,'(t') - \vec{r}(t)$$

$$= (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$$

$$= \hat{i}\Delta x + \hat{j}\Delta y$$

where $\Delta x = x' - x$ and $\Delta y = y' - y$

Velocity

The average velocity $\vec{v}$ of an object $= \dfrac{\text{displacement}}{\text{corresponding time interval}}$

$\therefore$ $$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \hat{i}\frac{\Delta x}{\Delta t} + \hat{j}\frac{\Delta y}{\Delta t}$$

or, $$\vec{v} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$$

Direction of the average velocity is same as that of displacement.
The instantaneous velocity is the average velocity as the time interval approaches to zero.

$\therefore \qquad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

The direction of the instantaneous velocity of an object at any point on the path is tangent to the path at that point and is in the direction of motion.

In component form,

$$\vec{v} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$$

Magnitude of v,

$$v = \sqrt{v_x^2 + v_y^2}$$

and the direction of v,

$$\tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

Acceleration

Average acceleration = $\frac{\text{change in velocity}}{\text{time interval}}$

$$\vec{a}_{av} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} \qquad \text{or} \qquad \vec{a}_{av} = a_x\hat{i} + a_y\hat{j}$$

Instantaneous acceleration: It is the limiting value of the average acceleration as the time interval approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta \vec{v}}{\Delta t}$$

$$\therefore \quad \vec{a} = \hat{i}\lim_{\Delta t \to 0}\frac{\Delta v_x}{\Delta t} + \hat{j}\lim_{\Delta t \to 0}\frac{\Delta v_y}{\Delta t}$$

or, $\qquad \vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$

Motion in a plane with constant acceleration:

If an object is moving in x-y plane having constant acceleration a, then by the definition of average acceleration

$$a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

where , v_0 = velocity of the object at time t = 0
and v = velocity of the object at time t.

or, $\qquad v = v_0 + at$

In terms of components,

$v_x = v_{0x} + a_xt$

$v_y = v_{0y} + a_yt$

Now, let r_0 and r be the position vectors of any particle at time zero and t and their velocities at these instant are v_0 and v, respectively. During the time interval t, $\left(\frac{v_0 + v}{2}\right)$ is the average velocity.

$\therefore$ Displacement,

$$r - r_0 = \left(\frac{v + v_0}{2}\right)t = \left(\frac{v_0 + at + v_0}{2}\right)t$$

$$r = r_0 + v_0\, t + \frac{1}{2}at^2$$

Hence, in component form,

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Therefore, a two dimensional motion can be treated as two separate simultaneous one-dimensional motions having constant acceleration along two perpendicular direction.

PROJECTILE

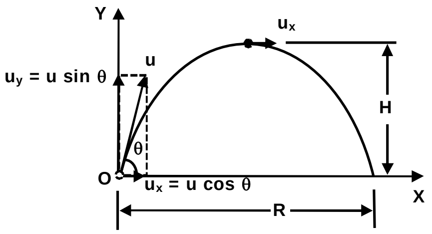
An object projected into space or air, such that it moves under the effect of gravity only, is called a projectile.

Projectile Motion

Motion in a vertical plane containing horizontal and vertical axes:

A particle when given a velocity at any arbitrary angle (other than 90°) made with the horizontal surface is known as a projectile.

If a particle is projected from point O, at any angle θ from the horizontal, with initial velocity $\vec{u}$, then the components of $\vec{u}$ in x and y directions are given as



$u_x = u \cos\theta$, $u_y = u \sin\theta$ where $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\Rightarrow \quad \vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

The X axis is parallel to the horizontal. Y axis is parallel to the vertical and the $\vec{u}$ lies in the X – Y plane.

The constant acceleration $\vec{a}$ is given as, $\quad \vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = 0$ [as there is no acceleration along the X-axis].

$a_y = -g$ [the acceleration is downward and equal to g].

Now, velocity after time t is given as.

$v_{tx} = u_x + a_x t = u\cos\theta \quad (\text{as} \quad a_x = 0)$

$v_{ty} = u_y + a_y t = u\sin\theta - gt$

$\because \quad \vec{v} = v_x\hat{i} + v_y\hat{j} \quad \Rightarrow \quad \vec{v}_{(t)} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

The direction of $\vec{v}$ with the x axis is given by $\tan^{-1}\left(\frac{v_y}{v_x}\right)$

Co-ordinates of the projectile after time t is given by

$$x = x_o + u_x t + \frac{1}{2}a_x t^2 \quad \Rightarrow \quad x = 0 + u\cos\theta\,.t + 0$$

$$\Rightarrow \quad x = u\cos\theta\, t \qquad \ldots (1)$$

and $\quad y = y_o + u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad y = 0 + u\sin\theta\, t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad y = u\sin\theta\, t - \frac{1}{2}gt^2 \qquad \ldots (2)$$

Eliminating 't' from Eqs. (1) and (2), we get

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$\Rightarrow \quad y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta} \qquad \ldots (3)$

The above equation shows the relation between x and y and represents the path of the projectile known as trajectory. The inspection of eq. (3) shows that it is the equation of parabola of the form

$$y = bx + cx^2$$

where $b = \tan\theta$ = constant, and $c = -\dfrac{g}{2u^2\cos^2\theta}$ = constant

Time of flight: It is time interval during which the projectile remains in air.
Putting y = 0 in (2), we get

$$T = \frac{2u\sin\theta}{g}, \text{ where T = time of flight.}$$

Range: The horizontal range R of the projectile is the horizontal distance between the initial point and the point where the projectile is again at same horizontal level.

If R be the horizontal range then $R = u\cos\theta \times \dfrac{2u\sin\theta.}{g} = \dfrac{u^2\sin 2\theta}{g}$

Note (i): Since $\sin 2\theta = \sin(\pi - 2\theta) = \sin 2\left(\dfrac{\pi}{2} - \theta\right)$

Let $(\pi/2 - \theta) = \beta \quad \Rightarrow \sin 2\theta = \sin 2\beta$

Hence, range is same for two angles of projection provided angles are complimentary.

Note (ii): For a given velocity of projection R is maximum for $\theta = 45^o$.

$$\Rightarrow \quad R_{max} = \frac{u^2}{g}$$

Maximum height: The maximum height attained by the projectile is given by

$\because \quad v_y^2 = u_y^2 + 2a_y y \quad$ at $\quad y = y_{max},\ v_y = 0$

$\Rightarrow \quad 0 = u^2\sin^2\theta - 2\,g\,y_{max} \quad \Rightarrow \quad y_{max} = \dfrac{u^2\sin^2\theta}{2g}$

Exercise 10:

(i) ***A projectile is thrown horizontally from the top of a tower and strikes the ground after 3 second at an angle of 45° with the horizontal. Find the height of the tower and speed with which the body was projected. Given g = 9.8 m/s²***

(ii) ***A ball is thrown with an initial velocity of 100 m/s at an angle of 30° above the horizontal. How far from the throwing point will the ball attain its original level? Solve the problem without using formula for horizontal range.***

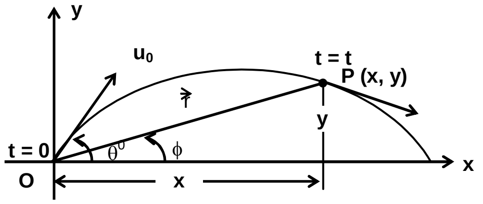
(iii) ***A bullet P is fired from a gun when the angle of elevation of the gun is 30°, another bullet Q is fired from the gun when the angle of elevation is 60° which of the two bullets would have a greater horizontal range and why ?***

(iv) ***Name the two quantities which would be reduced if air resistance is taken into account in the study of motion of oblique projectile.***

Illustration 22. *A boy throws a stone with an speed V_0 = 10 m/sec at an angle $\theta_0 = 30^o$ to the horizontal. Find the position of the stone w.r.t. the point of projection just after a time t = 1/2 sec.*

Solution: The position of the stone is given by

$$\vec{r} = x\text{i} + y\text{j}$$



where $x=(v_0 \cos\theta_0)t$

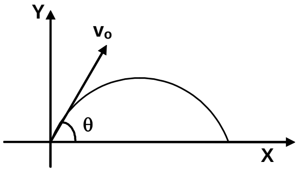
$= (10 \ \cos 30)\left(\frac{1}{2}\right) = 4.33$ m.

and $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

$= (10 \ \sin 30)\left(\frac{1}{2}\right) - \frac{1}{2}\times 10 \times \left(\frac{1}{2}\right)^2 = 1.25$ m

$\Rightarrow \vec{r} = (4.33\text{i} \ + \ 1.25\text{j}) \ \text{m}$.

Illustration 23. *A particle is projected with velocity v_o = 100 m/s at an angle $\theta = 30^o$ with the horizontal. Find*



(a) velocity of the particle after 2 s.

(b) angle between initial velocity and the velocity after 2s

(c) the maximum height reached by the projectile

(d) horizontal range of the projectile.

(e) the total time of flight

Solution: (a) $\vec{v}_{(t)} = \vec{v}_{x(t)}\hat{i} + \vec{v}_{y(t)}\hat{j}$

where $\hat{i}$ and $\hat{j}$ are the unit vectors along +ve x and +ve y axis, respectively.

$\Rightarrow \quad \vec{v}_{(t)} = (u_x + a_x t)\hat{i} + (u_y + a_{y^t})\hat{j}$

Here, $u_x = v_o \cos\theta = 50\sqrt{3}$ m/s

$a_x = 0$

$u_y = v_o \sin\theta = 50$m/s

$a_y = -g$ ($\because$ g acts downward)

$\Rightarrow \quad \vec{v}_{(t)} = v_o \cos\theta\hat{i} + (v_o \sin\theta - gt)\hat{j}$

$\vec{v}_{(2)} = 50\sqrt{3}\hat{i} + (50 - 10\times 2)\hat{j} = 50\sqrt{3}\hat{i} + 30\hat{j}$ m/s

$\Rightarrow \quad |\vec{v}_2| = \sqrt{v_x^2 + v_y^2} = 91.65 \text{ m/s}$

(b) $\vec{v}_o = 50\sqrt{3}\hat{i} + 50\hat{j}$, and

$\vec{v}_{(t=2\sec)} = 50\sqrt{3}\hat{i} + 30\hat{j}$

$\Rightarrow \quad \vec{v}_o.\vec{v}_{(2)} = 7500 + 1500 = 9000$

If α is the angle between $\vec{v}_o$ and $\vec{v}_{(2s)}$.

Then, $\cos\alpha = \frac{\vec{v}_o.\vec{v}_{(2s)}}{|\vec{v}_o|\times|\vec{v}_{(2s)}|} = \frac{9000}{1000\times 91.65}$

or $\alpha = \cos^{-1}(0.98) = 10.8^o$.

(c) $v_y^2 - u_y^2 = 2a_y y$

At $y = y_{max}$, $v_y = 0$

$\Rightarrow\ 0 - v_0^2 \sin^2\theta = 2(-g)\,y_{max}$

$\Rightarrow\ y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g} = 125\text{m}$

(d) $R = \dfrac{u^2 \sin 2\theta}{g} = 866$ m

(e) $T = \dfrac{2v_0 \sin\theta}{g} = 50$ sec.

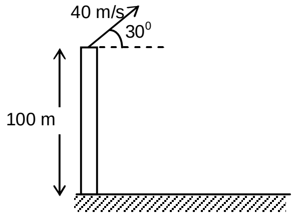
Illustration 24. *A ball is thrown at a speed of 50 m/s at an angle of 60° with the horizontal. Find*
(a) the maximum height reached, and.
(b) the range of ball. (Take g = 10 m/s²)

Solution: (a) Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

$$= \frac{(50)^2}{2\times 10}\times\left(\frac{\sqrt{3}}{2}\right)^2 \text{ m } = 93.75 \text{ m}$$

(b) Range, $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(50)^2 \times \sin 120}{10} = 216.5$ m

Illustration 25. *A stone is projected with a speed of 40 m/s at an angle of 30° with the horizontal from a tower of height 100 m above ground. Find*
(a) the maximum height attained by the stone, and
(b) the horizontal distance from the tower where it hits the ground.



Solution: (a) Maximum height above the tower, using $v^2 = u^2 + 2as$ in vertical direction.

$(u \sin 30°)^2 = 2gh$ As $u = 40$ m/s, $\theta = 30°$

$\dfrac{40\times 40\times 1}{4} = 2 \times 10 \times h \quad \Rightarrow h = \dfrac{1600}{80} = 20\text{m}$

$\therefore$ Height above ground $= 100 + 20 = 120$m.

(b) Range, time of flight = t, $H = u \sin\theta t - \frac{1}{2}gt^2$, $H = -100$ m,

$-100 = (40 \times \frac{1}{2})t - \frac{1}{2}\times 10\times t^2$

$-100 = 20t - 5t^2$

$t^2 - 4t - 20 = 0$, $t = 6.9$ sec.

$R = u\cos\theta \times t$, $R \to$ distance from tower

$R = 40 \times \dfrac{\sqrt{3}}{2}\times 6.9 = 238.9$ m.

Illustration 26. *The position of a particle at time t = 0 is P = (–1, 2, –1). It starts moving with an initial velocity* $\vec{u} = 3\hat{i} + 4\hat{j}$ *and with uniform acceleration* $-4\hat{i} + 4\hat{j}$. *Find the final position and the magnitude of displacement after 4 sec.*

Solution: Initial position vector of the particle $= \left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

Final position of the particle after 4 seconds

$$S_f = S_i + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

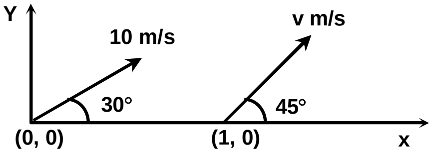
$$= \left(-\hat{i} + 2\hat{j} - \hat{k}\right) + \left(3\hat{i} + 4\hat{j}\right) \times 4 + \frac{1}{2} \times \left(-4\hat{i} + 3\hat{j}\right) \times 16$$

Final position $= -21\hat{i} + 42\hat{j} - \hat{k}$,

Displacement $= -20\,\hat{i} + 40\,\hat{j}$.

Magnitude of displacement $= \sqrt{(20)^2 + (40)^2} = 20\sqrt{5}$ m

Illustration 27. *Two particles projected vertically upward from points (0, 0) and (1, 0) with uniform velocity 10 m/s and v m/s, respectively, as shown in the figure. It is found that they collide after time t in space. Find v and t.*



Solution:

$x_1 = 10\cos 30°t$

$x_2 = v\cos 45°t$

$y_1 = 10\sin 30°t - \frac{1}{2}gt^2$

$y_2 = v\sin 45°t - \frac{1}{2}gt^2$

For collision:

$$y_1 = y_2$$

$$10 \times \frac{1}{2} = \frac{v}{\sqrt{2}}$$

$\Rightarrow v = 5\sqrt{2}$ m/s

$\Rightarrow x_1 = x_2 + 1$

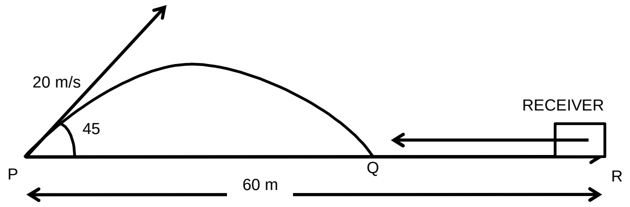
$$10\cos 30°t = 5\sqrt{2}\cos 45°t + 1$$

$$t\left(5\sqrt{3} - 5\right) = 1$$

and $t = \dfrac{1}{5\left(\sqrt{3} - 1\right)}$ sec

Illustration 28. *A football is kicked off with an initial speed of 20m/s at an angle of projection of 45°. A receiver on the goal line at a distance of 60 m away in the direction of the kick starts running to meet the ball at that instant. What must be his speed if he is to catch the ball before it hits the ground? [Take g = 10m/s²]*

Solution:



Let u = 20 m/s, $\theta = 45^\circ$ and v = speed of the receiver.
The ball is projected from Pand the receiver starts running from R to receive the ball at Q.
Let t be the time after which they meet.
So t is the time taken by the ball to go from P to Qin which the receiver goes from R to Q.

$\therefore$ PQ = $\frac{u^2}{g}\sin 2\theta$ and QR = vt

PR = 60 $\Rightarrow \frac{u^2}{g}\sin 2\theta + vt = 60$ …(i)

Putting the value of t (i.e. the time of flight) = $\frac{2u\sin\theta}{g}$

in equation (I) we get,

$$\frac{u^2}{g}\sin 2\theta + v\left(\frac{2u\sin\theta}{g}\right) = 60$$

$$\Rightarrow \quad v = \frac{60g - u^2\sin 2\theta}{2u \sin\theta}$$

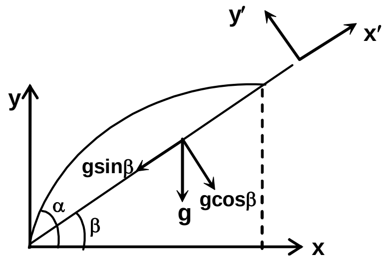
$$= \frac{600-400}{2(20)}\sqrt{2}$$

$$= 5\sqrt{2}\ \text{m/s}$$

The Projectile on an Inclined Plane

In case the projection is from an inclined plane, we consider two axes x′ and y′, along and perpendicular to the inclined plane.

Motion up the plane



In x′-y′ plane,

$u_{x'} = v_0\cos(\alpha-\beta)$, $u_{y'} = v_0\sin(\alpha-\beta)$
$a_{x'} = -g\sin\beta$, $a_{y'} = -g\cos\beta$

Since $y' = v_o\sin(\alpha-\beta)t - \frac{1}{2}g\cos\beta t^2$

at t = T, y′ = 0, where T = time of flight.

$\Rightarrow$ $T = \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$ again $x = (v_0\cos\alpha).T$

$$x = v_o\cos\alpha\ \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$$

So range along inclined plane (R) = x′ = $x/\cos\beta$

$\therefore$ $x' = \frac{2v_0^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$ [Apply formula 2cosA sin B = sin(A + B) – sin(A–B)]

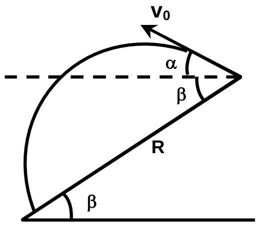
$$x' = R = \frac{v_0^2\left[\sin(2\alpha-\beta)-\sin\beta\right]}{g\cos^2\beta}$$

Now, R will be maximum when $\sin(2\alpha-\beta)$ is maximum, i.e. $\sin(2\alpha-\beta) = 1$.

$\Rightarrow \quad R_{max} = \dfrac{v_0^2[1-\sin\beta]}{g(1-\sin^2\beta)} \Rightarrow \quad R_{max} = \dfrac{v_0^2}{g(1+\sin\beta)}$ up the plane

Motion down the plane

Let the particle be thrown at a velocity v_0 at an angle 'α' with the horizontal as shown in figure.



$V_0 \sin(\alpha + \beta)T - \dfrac{1}{2}\cos\beta T^2 = 0 \qquad$ [for $y' = 0$]

$\Rightarrow \quad T = \dfrac{2v_o\sin(\alpha+\beta)}{g\cos\beta}$

$R = v_o\cos(\alpha + \beta)T + \dfrac{1}{2}g\sin\beta T^2 = \dfrac{v_0^2}{g}\left[\dfrac{\sin(2\alpha+\beta)+\sin\beta}{1-\sin^2\beta}\right]$

For R to be maximum;

$\sin(2\alpha + \beta) = 1$

and $\quad R_{max} = \dfrac{v_0^2}{g}\left[\dfrac{1+\sin\beta}{1-\sin^2\beta}\right]$

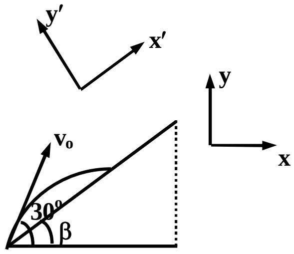
$= \dfrac{v_0^2}{g(1-\sin\beta)}$ down the plane.

Illustration 29. *Name a quantity which remains unchanged during the flight of projectile on an inclined plane.*

Solution: Horizontal component of velocity.

Illustration 30. *From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30^o with the horizontal (a) up the plane (b) down the plane. Find the range in each case.*

Solution : Let β be the inclination of the plane.



Hence, $\sin\beta = \dfrac{7}{25}$, and $\cos\beta = \dfrac{24}{25}$

(a) $v_{ox'} = v_o\cos(30^o - \beta)$

and $a_x' = -g\sin\beta$

$v_{oy'} = v_o\sin(30^o - \beta)$

and $a_y' = -g\cos\beta$

$\because \quad y' = v_{oy}'t + \dfrac{1}{2}a_y't^2$

If T = time of flight, then at $t = T$, $y' = 0$

$\Rightarrow \quad 0 = v_o\sin(30^o - \beta)T - \dfrac{1}{2}g\cos\beta T^2$

$\Rightarrow \quad T = \dfrac{2v_o\sin(30^o-\beta)}{g\cos\beta}$

If R_1 be the range then $R_1\cos\beta = x = v_o\cos30^o . T$

$\Rightarrow \quad R_1\cos\beta = v_o\cos30 . \dfrac{2v_o\sin(30°-\beta)}{g\cos\beta}$

$$\Rightarrow \quad R_1 = \frac{2v_0^2 \cos 30^o \sin(30-\beta)}{g\cos^2\beta}$$

Solving we get, R_1 = 1749.8 m

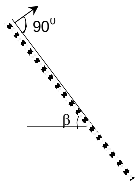
(b) For down the plane,

$$T = \frac{2v_o \sin(30^o+\beta)}{g\cos\beta}$$

Hence $\qquad R_2 \cos\beta = v_o \cos 30^o .T$

$$\Rightarrow \quad R_2 = \frac{2v_0^2 \cos 30^o \sin(30^o+\beta)}{g\cos^2\beta}.$$

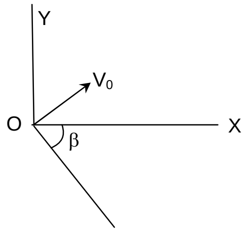
Illustration 31. *A projectile is launched from an inclined plane with an initial velocity v_0 as shown in the figure. Find the time after which the projectile hits the plane for the first time.*



Solution: Let the projectile hit the plane after time t.

The horizontal displacement x = (v_0 sin β) t

The vertical displacement y = (v_0 cos β) t $-\frac{1}{2}gt^2$



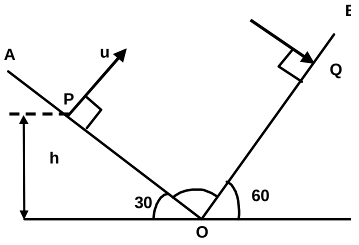
y = –(tan β)x for the plane

$$\therefore \quad t = \frac{2v_0}{g\cos\beta}$$

Illustration 32. *Two inclined planes of inclinations 30° and 60°, respectively, meet at 90° as shown in figure. A particle is projected from point P on the first inclined plane with a velocity u = 10$\sqrt{3}$ m/s in a direction perpendicular to the inclined plane and it is observed to hit the other inclined plane at 90°.*

Find (a) the height of point P from ground,

(b) the length of $\overline{PQ}$.



Solution: (a) We observe the motion of projectile fixing y-axis with OP and x-axis with OQ. Hence, velocity at any instant t along x-axis:

$v_x = 10\sqrt{3} - (g \sin 60°)t$

$v_y = 0 - (g \cos 60°)t$

As $v_x = 0$ at the time of hitting,

Time of flight = T = 2 sec.

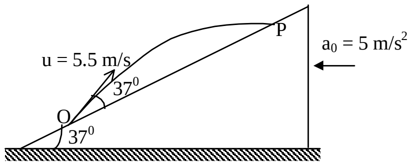
Displacement OP during this time = $\frac{1}{2}(g \cos 60°)t^2 = \frac{1}{2} \times 10 \times \frac{1}{2} \times 4 = 10$ m

Hence, h = OP sin 30° = 10 × $\frac{1}{2}$ = 5m

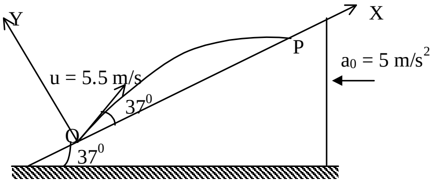
(b) Similarly, displacement OQ = $(10\sqrt{3})(2) - \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times 4 = 10\sqrt{3}$ m

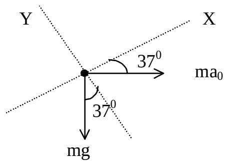
Hence, PQ = $\sqrt{OP^2 + OQ^2}$ = 20 m.

Illustration 33. *A particle is projected up a large inclined plane from a point O on it as shown in the figure. The projection velocity has a magnitude of 5.5 m/s and its direction makes an angle of 37° with the inclined plane. The inclination of the plane is also 37°. The inclined plane starts moving towards left with an acceleration $a_0 = 5\ m/s^2$ at the moment the particle is projected. The particle strikes the inclined plane at a point P. Find the time taken by the particle to move from O to P. Also find the magnitude of displacement along the inclined plane as it moves from O to P. (Take sin 37° = 3/5)*



Solution: Let us take x and y axes as shown in the figure. The magnitude of pseudo force acting on the particle has a magnitude of ma_0 and its direction will be towards right as shown in the free body diagram.





F.B.D. of the particle

The components of the acceleration of the particle are

$$a_x = \frac{ma_o \cos 37^\circ - mg \sin 37^\circ}{m}$$

$$= 5 \times \frac{4}{5} - 10 \times \frac{3}{5} = -2 m/s^2$$

$$a_y = \frac{-(mg\cos 37^o + ma_o \sin 37^o)}{m}$$

$$= -\left(10 \times \frac{4}{5} + 5 \times \frac{3}{5}\right) = -11 m/s^2$$

$u_x = u\cos37° = 5.5 \times \frac{4}{5} = 4.4$ m/s.

$u_y = u\sin37° = 5.5 \times \frac{3}{5} = 3.3$ m/s

Displacement of the particle along y – axis

$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = 3.3t - \frac{1}{2} \times 11t^2$

When the particle strikes the plane y = 0

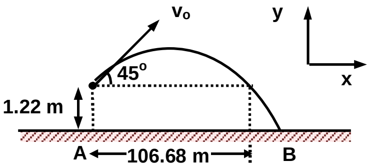
$\Rightarrow t = \frac{2u_y}{-a_y} = \frac{2 \times 3.3}{11} = 0.6$ sec.

$OP = x = u_x t + \frac{1}{2} a_x t^2$

$= 4.4 \times 0.6 - \frac{1}{2} \times 2 \times (0.6)^2 = 2.28$ m.

Illustration 34. *A batsman hits a ball at a height of 1.22 m above the ground so that ball leaves the bat at an angle of 45^o with the horizontal. A 7.31 m high wall is situated at a distance of 97.53 m from the position of the batsman. Will the ball clear the wall if its range is 106.68m? Take g = 10 m/s^2.*

Solution :



$$R(\text{range}) = \frac{v_0^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad v_0^2 = \frac{Rg}{\sin 2\theta} = Rg \quad \text{as } \theta = 45^o.$$

$$\Rightarrow \quad v_o = \sqrt{Rg} \qquad \ldots (1)$$

Equation of trajectory

$$y = x \tan 45^o - \frac{gx^2}{2v_0^2 \cos^2 45^o} = x - \frac{gx^2}{2Rg\frac{1}{2}} \quad [\text{using (1)}]$$

Putting x = 97.53, we get

$$y = 97.53 - \frac{10 \times (97.53)^2}{106.68 \times 10} = 8.35$$

Hence, height of the ball from the ground level is

h= 8.35 + 1.22 = 9.577m.

As height of the wall is 7.31m, so the ball will clear the wall.

Illustration 35. *A particle is projected with velocity u and angle θ with the horizontal. Find the time after which the velocity will be perpendicular to the initial velocity.*

Solution :

$$\bar{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

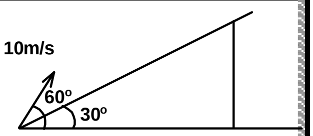
$$\bar{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

$$\bar{u} \cdot \bar{v} = 0 = u^2 \cos^2\theta + u^2\sin^2\theta - (u\sin\theta)gt$$

$$\therefore \; t = \frac{u}{g\sin\theta}$$

Exercise 11:

(i) ***A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of 60° with the horizontal, from a point on an incline plane, making an angle of 30° with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is***



(A) $\frac{2}{\sqrt{3}}$ sec (B) $\frac{1}{\sqrt{3}}$ sec

(C) $\sqrt{3}$ sec (D) $\frac{1}{2\sqrt{3}}$ sec

(ii) ***An object projected with the same speed at two different angles covers the same horizontal range R. If the two times of flight be t_1 and t_2, prove that R = $\frac{1}{2}gt_1t_2$.***

(iii). ***Is it important in the long jump that how much height you take for jumping?***

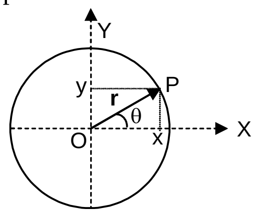
UNIFORM CIRCULAR MOTION

As another small illustration of motion of a particle in two dimensions let's analyse the uniform circular motion of a particle.

In uniform circular motion, the particle moves in a circular path with constant speed.

Let's choose the centre of the circular path as the origin of the reference frame. Point 'P' is an arbitrary point on the path whose position vector

$\vec{r} = x\hat{i} + y\hat{j}$.



where r, the radius of the circular path, is related to x and y by following equations

$x = r\cos\theta,\ \ y = r\sin\theta$ and $x^2 + y^2 = r^2$

$\Rightarrow \quad \vec{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$

Now, the velocity of particle 'P' is given as

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d(r\cos\theta)}{dt}\hat{i} + \frac{d(r\sin\theta)}{dt}\hat{j}$$

$$\Rightarrow \quad \vec{v} = -r\sin\theta\frac{d\theta}{dt}\hat{i} + r\cos\theta\frac{d\theta}{dt}\hat{j}$$

but $\frac{d\theta}{dt} = \omega = \text{const.}$ [for uniform circular motion]

Thus, $\vec{v} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Now, $\vec{v}\,.\,\vec{r} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})(r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}) = \omega r^2(-\sin\theta\cos\theta + \cos\theta\sin\theta) = 0$

$\Rightarrow \quad \vec{v}$ is perpendicular to $\vec{r}$

$|\vec{v}| = \omega r\sqrt{\sin^2\theta + \cos^2\theta} = \omega r$

Now, acceleration $\vec{a}$ is given as

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega r\left(-\cos\theta.\frac{d\theta}{dt}\hat{i} - \sin\theta\frac{d\theta}{dt}\hat{j}\right) \quad \Rightarrow \quad \vec{a} = -\omega^2 r\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$\Rightarrow \quad \vec{a} = \omega^2(-\vec{r})$$

which shows that $\vec{a}$ is directed in the opposite direction of '$\vec{r}$'. Thus, $\vec{a}$ is always directed towards the centre.

Magnitude of $\vec{a}$, $|\vec{a}| = \omega^2 r\sqrt{\cos^2\theta + \sin^2\theta} = \omega^2 r$

Note: If the circular motion is non-uniform, then tangential acceleration $a_t = dv/dt$ exists apart from normal acceleration $\omega^2 r$.

Illustration 36. *Does velocity remain constant in uniform circular motion ?*

Solution: No, magnitude remains constant but direction keep on changing.

Illustration 37. *Find the magnitude of average acceleration of the tip of the second hand of length 10 cm during 10 seconds.*

Solution: Average acceleration has the magnitude

$a = \Delta v/\Delta t$, where $\Delta v = 2v\sin\theta/2$

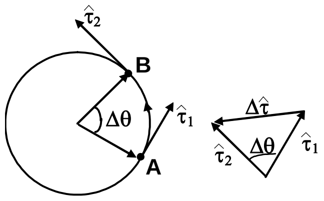
$$\Rightarrow \quad a = \frac{2V\sin\theta/2}{\Delta t}$$

Putting $v = \pi/300$ m/sec (obtained earlier), $\Delta t = 10$ seconds and $\theta = 60°$, we obtain

$$a = \frac{2(\pi/300)\sin 30°}{10}$$

$$\Rightarrow \quad a = \frac{\pi}{3000} \text{ m/sec}^2.$$

CIRCULAR MOTION



$|\vec{v}| = r\omega$ (variable)

Let $\hat{\tau}$ = unit vector along the tangent. and

$\hat{u}$ = unit vector along radius (outwards)

Since the velocity of the particle describing circular motion is along the tangent, hence it can be given by the expression.

$\vec{v} = v\hat{\tau}$ where v = magnitude of the velocity

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt}$$

Take A and B two positions of the particle.

Change in $\hat{\tau} = \Delta\hat{\tau} = \Delta\theta(-\hat{u})$

Negative sign shows that it is towards the centre as (S = R□)

$$\Rightarrow \quad \frac{d\hat{\tau}}{dt} = \frac{d\theta}{dt}(-\hat{u}) \qquad \Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\theta}{dt}(-\hat{u})$$

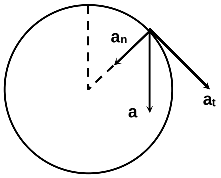
$$\Rightarrow \quad \vec{a} = \vec{a}_{\text{tangential}} + \vec{a}_{\text{radial}}, \qquad \text{where} \quad \vec{a}_{\text{tangential}} = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

If 'v' is a constant then $\vec{a}_{\text{tangential}} = 0$ and $\quad \vec{a}_{\text{radial}} = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r} = \omega^2 r$

$$|\vec{a}| = \sqrt{a_r^2 + a_T^2}$$

Illustration 38. *A point moves along a circle with velocity v = at where a = 0.5m/s². Find the total acceleration of the point at the moment when it covered (1/10)th of the circle after beginning of motion.*

Solution : We know $S = ut + \frac{1}{2}at^2$



Here, $S = \frac{2\pi r}{10} = \frac{\pi r}{5}$, $a_t = 0.5$ m/s² and u = 0

$$\therefore \quad \frac{\pi r}{5} = 0 + \frac{1}{2}0.5t^2, \ t = \sqrt{\frac{4\pi r}{5}}$$

$$\therefore \quad v = at = 0.5\sqrt{\frac{4\pi r}{5}} = \sqrt{\frac{\pi r}{5}}$$

$$\therefore \quad a_n = \frac{v^2}{r} = \frac{\pi r}{5}\frac{1}{r} = \frac{\pi}{5}$$

$$\therefore \quad a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{\pi}{5}\right)^2 + 0.5^2} = \sqrt{\frac{\pi^2}{25} + \frac{1}{4}} = 0.8 \text{ m/s}^2$$

Exercise 12:

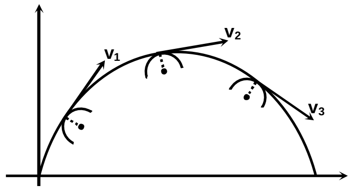
(i) ***By using vector method, show that direction of acceleration vector $\vec{a}$ is towards the centre of the circle in which body is revolving.***

(ii) ***What is the direction of velocity vector of a particle in circular motion ?***

(iii) ***What is the acceleration associated with a body having variable speed in a circular path ?***

(iv) ***Two cars having masses m_1 and m_2 move in circles of radii r_1 and r_2, respectively. If they complete the circles in equal time, find the ratio of the their angular speeds $\frac{\omega_1}{\omega_2}$.***

RADIUS OF CURVATURE



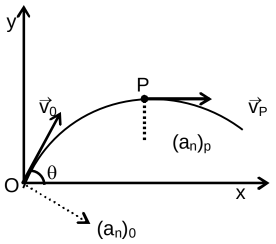
In a curvilinear motion, every small path may be assumed to be an arc of a circular path, and here the radius of curvature will be different at different points. So if a particle moves on a curved path then radius of curvature is given by $R = \frac{v^2}{a_R}$.

where v = instantaneous velocity at any time at that point and a_R = acceleration acting normal to the path towards the centre.

Illustration 39. *Where is radius of curvature maximum at the highest point or at the point of projection?*

Solution: At the point of projection

Illustration 40. *Find the ratio of radius of curvature at the highest point of projectile to that just after its projection if the angle of projection is 30°.*



Solution : If $\vec{v}_0$ is the initial velocity

$v_p = v_0 \cos\theta$

Normal acceleration at 0 = g cos θ

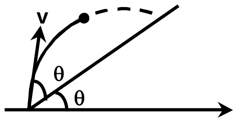
Normal acceleration at P = g

Hence, if r_0 and r_p be radii of curvature at O and P, respectively.

$$r_0 = \frac{v_0^2}{g\cos\theta} \quad \text{and} \quad r_p = \frac{v_0^2\cos^2\theta}{g}$$

Hence, the required ratio = $\frac{r_p}{r_0} = \cos^3\theta = \frac{3\sqrt{3}}{8}$.

Illustration 41. *A particle is projected with a velocity u at an angle θ with an inclined plane which makes an angle θ< 45° with the horizontal. Calculate the radius of curvature of the path of projectile when velocity of projectile becomes parallel to the plane.*

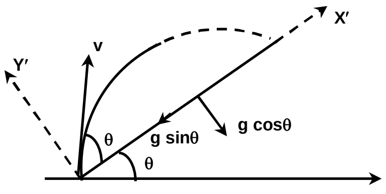


Solution : Along y′,

$$u\sin\theta - g\cos\theta t = 0$$

$$\Rightarrow \quad t = \frac{u}{g}\tan\theta$$

Along x′,



$$v_x' = u\cos\theta - g\sin\theta.t = u\cos\theta - g\sin\theta.\frac{u}{g}\tan\theta$$

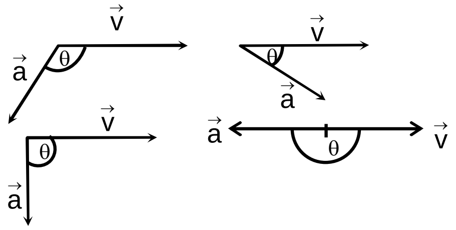
$$= \frac{u\cos^2\theta - u\sin^2\theta}{\cos\theta} = \frac{u\cos 2\theta}{\cos\theta}$$

$$r = \frac{(v'_x)^2}{a_n} = \frac{u^2\cos^2 2\theta}{g\cos^3\theta}$$

Exercise 13:

i) ***The tangential acceleration change the speed of the particle whereas the normal acceleration changes its direction. State whether the statement true or false?***

ii) ***At a certain moment, the angle between velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$ of the particle is θ. What will be the motion of the particle at this moment for different θ's: rectilinear or curvilinear, accelerated or decelerated?***

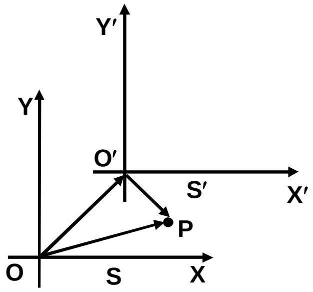


RELATIVE VELOCITY

The position, velocity and acceleration of a particle depend on the reference frame chosen.

A particle P is moving and is observed from two frames 'S' and 'S′'. The frame S is stationary and the frame S′ is in motion.

Let at any time position vector of the particle P with respect to S is



$\overrightarrow{OP} = \vec{r}_{p,s}$ and with respect to S′ is $\overrightarrow{O'P} = \vec{r}_{p,s'}$

Position vector of the origin of S′ with respect to origin of S is

$\overrightarrow{OO'} = \vec{r}_{s',s}$

From vector triangle OO′P, we get

$$\overrightarrow{O'P} = \overrightarrow{OP} - \overrightarrow{OO'} \Rightarrow \vec{r}_{p,s'} = \vec{r}_{p,s} - \vec{r}_{s',s}$$

$$\Rightarrow \quad \frac{d}{dt}\left(\vec{r}_{p,s'}\right) = \frac{d}{dt}\left(\vec{r}_{p,s}\right) - \frac{d}{dt}\left(\vec{r}_{s',s}\right)$$

$$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p,s} - \vec{v}_{s',s}$$

$$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p(\text{absolute})} - \vec{v}_{s'\,(\text{absolute})}$$

If $\vec{v}_r$ and $\vec{v}_m$ are the absolute velocities of the rain and the man, respectively, then the relative velocity of rain w.r.t. (as seen by) the man is $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$.

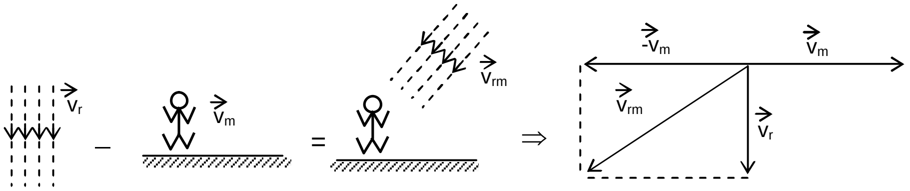


Illustration 42. *Two men are moving with same velocity in the same direction. What is there relative velocity?*

Solution: zero

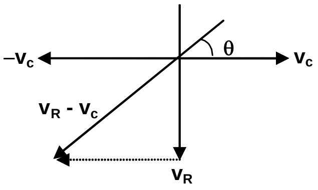
Illustration 43. *A stationary person observes that rain is falling vertically down at 30 km/hr. A cyclist is moving on the level road at 10 km/hr. In which direction the cyclist should hold his umbrella to prevent himself from rain?*

Solution: Relative to stationary frame, velocity of rain is 30 km/hr downward. Take horizontal axis as x-axis and vertical axis as y-axis and $\hat{i}, \hat{j}$ are the unit vectors along X- and Y-axes, respectively.

$$\vec{v}_R = 0 - 30\hat{j}, \quad \vec{v}_c = 10\hat{i}$$

$$\because \quad \vec{v}_{R,c} = \vec{v}_R - \vec{v}_c$$

$$= -30\hat{j} - 10\hat{i} = -10\hat{i} - 30\hat{j}$$



If angle between horizontal and the $\vec{v}_{R,c}$ is θ, then

$$\tan\theta = \frac{-30}{-10} = 3$$

$\Rightarrow \theta = \tan^{-1}3 \quad \Rightarrow \theta = 72^o$.

Therefore, to prevent himself from rain the cyclist should hold the umbrella at angle of 72^o from horizontal.

Illustration 44. *A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.*

Solution: Let velocity of the wind be

$$\vec{v}_w = v_1\hat{i} + v_2\hat{j} \text{ m/s}$$

And velocity of the man is

$$\vec{v}_m = 5\hat{i}$$

$$\therefore \quad \vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (v_1 - 5)\hat{i} + v_2\hat{j}$$

In first case,

$v_1 - 5 = 0$

$\Rightarrow v_1 = 5$ m/s.

In the second case, $\tan 45° = \dfrac{v_2}{v_1 - 10}$

$\Rightarrow v_2 = v_1 - 10 = -5$ m/s.

$\Rightarrow \vec{v}_w = 5\hat{i} - 5\hat{j}$ m/s.

Illustration 45. *From a lift moving upward with a uniform acceleration 'a', a man throws a ball vertically upwards with a velocity v relative to the lift. The ball comes back to the man after a time t. Show that a + g = 2 v/t.*

Solution: Let us consider all the motion from lift frame. Then, the acceleration, displacement and velocity everything will be considered from the lift frame itself.

As the ball comes again to the man, therefore displacement from the lift frame is zero.

Again, the velocity with respect to the lift frame is v.

Similarly, the acceleration with respect the lift frame is

$g - (-a) = a + g$ (downwards)

Now, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 0 = vt - \frac{1}{2}(a + g)t^2$

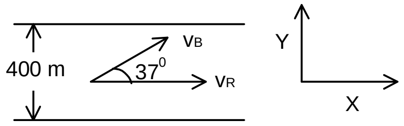
or $\quad a + g = 2\frac{v}{t}$.

Illustration 46. *A river 400 m wide is flowing at a rate of 4 m/s. A boat is sailing at a velocity of 20 m/s with respect to the still water in a direction making an angle of 37° with the direction of river flow.*

(a) Find time taken by the boat to reach the opposite bank.

(b) How far from the starting point does the boat reach on the opposite bank?

Solution: (a) Resultant velocity of the boat is



$v = (v_R + v_B \cos 37^\circ)\, i + v_B \sin 37^\circ\, j$

$= 4i + 20 \times \frac{4}{5} i + 20 \times \frac{3}{5} j$

$= 20\, i + 12\, j$ m/s

Time taken by boat to cross the river

$$= \frac{\text{distance travelled in y-direction}}{\text{velocity in y-direction}} = \frac{400}{12} = \frac{100}{3} \text{ sec.}$$

(b) Displacement along x $= v\,t$

$$= 20 \times \frac{100}{3} = \frac{2000}{3} \text{ m}$$

Distance from starting point $= \sqrt{(400)^2 + \left(\frac{2000}{3}\right)^2} = \frac{400}{3}\sqrt{34}$ m.

Illustration 47. *A stone is projected from a balloon which is ascending with a velocity 2 m/s. The velocity of the stone w.r.t. balloon is $\sqrt{2}$ m/s at an angle of 45°. Find the velocity of the stone with respect to ground.*

Solution : $\vec{v}_{SB} = v\cos 45^0\,\hat{i} + v\sin 45^0\,\hat{j}$

$$= \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{j} = (\hat{i} + \hat{j})\text{m/s}$$

$\vec{v}_{BG} = 2\hat{j}\text{m/s}$

Thus, $\vec{v}_{S,G} = \vec{v}_{SB} + \vec{v}_{BG}$

$= 2\hat{j} + (\hat{i} + \hat{j}) = ((\hat{i} + 3\hat{j})$

$v = \sqrt{1^2 + 3^2} = \sqrt{10}$ m/s and $\tan\theta = \frac{3}{1}$

$\theta = \tan^{-1}(3)$.

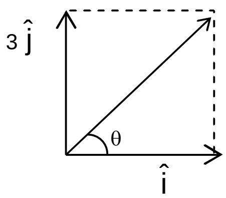
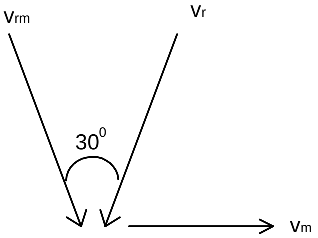


Illustration 48. *A man standing on a road has to hold his umberella at 30° with the vertical to keep the rain away. He throws the umbrella and runs at 10 kmph. He finds that rain drops are hitting his head vertically. find the speed of raindrops with respect to*
(a) the road
(b) the moving man.



Solution : Velocity ofrain w.r.t. road is $\vec{v}_r$ and velocity of rain w.r.t. moving man is $\vec{v}_{rm}$ but

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

$= -v_r \sin 30\ \hat{i} - v_r \cos 30\ \hat{j} - 10\ \hat{i}$

$= (-v_r \sin 30 - 10)\ \hat{i} - v_r \cos 30\ \hat{j}$

But $-v_r \sin 30 - 10 = 0$ $\therefore$ $v_r \sin 30 = -10$

$v_r = \frac{-10}{\sin 30} = -20$ m/s.

But v_r is not negative

$\therefore$ $\vec{v}_m = -10\hat{i}$

and $\vec{v}_{rm} = -[20\cos 30]$

$= 20 \cos 30\ \hat{j} = 10\sqrt{3}\,\hat{j}$.

Illustration 49. *A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle of 30° to the path of A.. Find the velocity of B relative to A.*

Solution: $|v_B - v_A| = \sqrt{v_B^2 + v_A^2 - 2v_A v_B \cos 30^0}$

$= \sqrt{5^2 + 3^2 - 2\times5\times3\times(\sqrt{3/2})}$

$= \sqrt{8.02} = 2.832$ m/sec.

Using sine rule, $\frac{3}{\sin\theta} = \frac{2.832}{\sin 30^0}$

$\Rightarrow \theta = 32°$

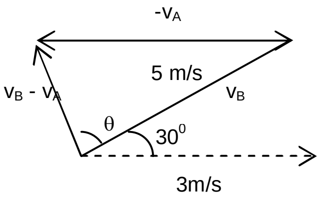
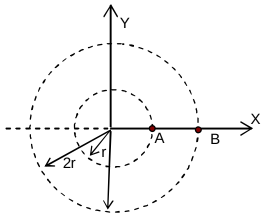


Illustration 50. *Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω, respectively.*
(a) Find the relative velocity of B w.r.t. A after time $t = \pi/\omega$.
(Initial position of particles A and B are shown in figure.)
(b) Also find the relative position vector of B w.r.t. A.

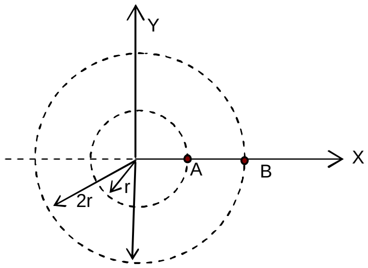


Solution: $\theta_A = 2\omega\dfrac{\pi}{\omega} = 2\pi$, $v_A = 2\omega r\,\hat{j}$

$\theta_B = \omega\dfrac{\pi}{\omega} = \pi$, $v_B = 2\omega r(-\hat{j})$

(a) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 2\omega r(-\hat{j}) - 2\omega r(\hat{j}) = -4\omega r\,\hat{j}$

(b) $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A = 2r\,(-\hat{i}) - (r)\,\hat{i} = -3r\,\hat{i}$.



Exercise 14:

(i) ***How long will a boy sitting near the window of a train travelling at 54 km/h see a train passing by in the opposite direction with a speed of 36 km/hr? The length of the slow moving train is 100 m.***

(ii) ***Two particles A and B are moving with speeds of 2 km/hr and 3 km/hr, respectively, in the same direction. Find how far will B be from A after 1 hour ?***

(iii) ***Two cars are moving in the same direction with the same speed 30 km/hr. They are separated by a distance of 5 km. What is the speed of a car moving in the opposite direction if it meets these two cars at an internal of 4 minutes?***

(iv) ***A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr. He finds raindrops are hitting his head vertically. Find the speed of raindrops with respect to:***
(a) the road, and (b) the moving man.

(v) ***A boat travels downstream from point A to point B in two hours and upstream in four hours. Find the time taken by a log of wood to cover the distance from point A to point B.***

SUMMARY

Motion in a straight line:

1. Distance is the total length of the path traversed by an object.

2. Displacement is the change in position $\Delta x = x_2 - x_1$

3. Average speed $= \dfrac{\text{distance traversed}}{\text{time interval}}$

4. Speed: The speed of an object is equal to the distance traversed by it in a very short time interval divided by time interval.

5. Instantaneous velocity: It is defined as the limit of average velocity as the time interval Δt becomes infinitesimally small,
$v = \lim\limits_{\Delta t \to 0} \bar{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
Slope of the tangent drawn on position–time graph at any instant gives the velocity at that particular time.

6. Average acceleration = $\dfrac{\text{change in velocity}}{\text{time interval}}$

7. Instantaneous acceleration: It is defined as the limit of the average acceleration as the time interval Δt goes to zero
i.e. $a = \lim\limits_{\Delta t \to 0} \bar{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$
At any particular instant, the slope of the velocity–time graph gives the acceleration of an object at that particular instant.

8. (i) The area under the speed–time graph gives the distance traversed by the object in the corresponding time interval.
(ii) The area under a velocity– time graph gives the displacement of the object.

9. For uniformly accelerated rectilinear motion, three equations of motion are
$v = u + at$
$x = ut + \dfrac{1}{2}at^2$
$v^2 = u^2 + 2ax$
where
u = initial velocity
v = final velocity
t = time taken
a = acceleration

Motion in a plane:

1. The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y and z.
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

2. $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$

$\vec{v} = \lim_{\Delta t \to o} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{r}}{dt}$

3. $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$

$\vec{a}_{av} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

4. In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity component are simple functions of time, and the shape of the path is always a parabola.

5. When a particle moves in a circular path of radius R, its acceleration $\vec{a}$ is directed towards the centre of the circle and perpendicular to $\vec{v}$.

$a_{radial} = \frac{v^2}{R}$

where $v = \frac{2\pi R}{T}$

6. If the speed is not constant in circular motion, there is still a radial component of $\vec{a}$ but there is also a component of $\vec{a}$ parallel to the path.

$a_{rad} = \frac{v^2}{R}$

$a_{tangential} = \frac{dv}{dt}$

7. When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B.

$\therefore \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

where $\vec{v}_{BA}$ is the velocity of B with respect to A.

Both observers measure the same acceleration for the particle; that is

$\vec{a}_{PA} = \vec{a}_{PB}$

MISCELLANEOUS EXERCISE

1. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement?

2. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed. Do you agree with this statement?

3. Is the vertical height taken by a long jumper important while taking the jump?

4. A woman standing on the edge of a cliff throws a ball straight up with a speed of 8 km/h and then throws another ball straight down with a speed of 8 km/h from the same position. What is the ratio of the speeds with which the balls hit the ground ?

5. Find the average velocity during the time of flight, if a particle is projected with v at an angle θ with horizontal plane.

6. Establish the relation $x(t) = v(0)t + \frac{1}{2}at^2$ by calculus method.

7. Derive the velocity–time relationship by (i) calculus method, (ii) graphical method.

8. A stone is thrown upwards from the top of a tower 85 m high. It reaches the ground in 5 second. Calculate (i) the greatest height above the ground, (ii) the velocity with which it reaches the ground and (iii) the time taken to reach the maximum height.
Given: $g = 10\ m/s^2$.

9. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m/s. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the values of acceleration, retardation and total time taken.

10. Prove that there are two times for which a projectile travels the same vertical distance. Also prove that the sum of the two times is equal to the time of flight.

SOLUTION TO MISCELLANEOUS EXERCISE

1. No, not always. In case of uniform circular motion, the particle is accelerating but its speed is neither decreasing nor increasing, only direction of velocity changes.

2. This is true as motion under gravity is independent of mass of the body and so the body comes back to the point of projection with the same speed.

3. Yes, because for the longest jump the player should throw himself at an angle of 45° wrt horizontal. The vertical height required for this purpose should be $\frac{u^2}{4g}$, where u is velocity of

throw. If the vertical height is different from $\frac{u^2}{4g}$ then the angle will be different from 45° and the horizontal distance covered also will be less.

4. 1 : 1, both the balls will hit the ground with the same speed.

5. $v \cos \theta$

8. h = 3.2 m, v = 42 m/s, t = 0.8 sec.

9. 0.8 m/s^2, 0.5 m/s^2 and 86 sec.

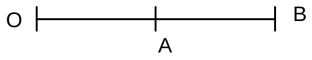
SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *A car starts from rest and moves with a constant acceleration of 2.0 m/s² for 30 seconds. The brakes are then applied and the car comes to rest in another 60 seconds. Find*
(a) total distance covered by the car.
(b) Maximum speed attained by the car
(c) Find shortest distance from initial point to the point when its speed is half of maximum speed.

Sol. Final velocity at A $v_A = 2 \times t_1 = 2 \times 30 = 60$ m/sec.



For AB, Let the retardation be 'b'

$\therefore\ 0 = v_A + bt^2$

$\therefore\ b = -\frac{v_A}{t} = -\frac{60}{60} = -1\ m/s^2$

(a) Total distance = OA + AB

$$OB = \frac{1}{2}at_1^2 + (v_A t_2 - \frac{1}{2}bt_2^2)$$

$$= (\frac{1}{2}\times 2\times 30\times 30 + 60\times 60\times 60 - \frac{1}{2}\times 1\times 60\times 60)$$

$$= 900 + 3600 - 1800 = 2700 \text{ m.}$$

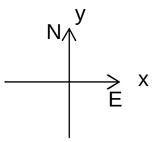
(b) Maximum speed $v_A = 60$ m/s.

(c) $v^2 = 2 \times a \times s$

$$s = \frac{(v_A/2)^2}{2\times a} = \frac{30\times 30}{2\times 2} = 225 \text{ m.}$$

Prob 2. *A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 minutes to reach the field,*
(a) what distance he has to walk to reach the field ?
(b) what is his displacement from his house to the field ?
(c) what is the average speed of farmer during the walk ?
(d) what is the average velocity of farmer during the walk ?

Sol. (a) Distance = 500 + 400 + 200 = 1100m



(b) Displacement = $500(\vec{j}) + 400(\hat{i}) + 200(-\hat{j}) = 300\vec{j} + 400\hat{i}$

Magnitude of displacement = $\sqrt{(400)^2 + (300)^2} = 500$m

(c) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{1100}{20\times 60} = \frac{11}{12}$ m/s

(d) Average velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{500}{20\times 60} = \frac{5}{12}$ m/s

$\theta = \tan^{-1}\left(\frac{300}{400}\right) = 37^\circ$ due North of East.

Prob 3. *A body is projected up such that its position vector varies with time as* $\vec{r} = 6t\hat{i} + (8t - 5t^2)\hat{j}$. *Find the (a) initial velocity (b) time of flight*

Sol. (a) The position of the body at any time t is given as $\vec{r} = 6t\,\hat{i} + (8t - 5t^2)\,\hat{j}$. When t = 0, r = 0. That means the body is projected from the origin of the coordinate system. Differentiating both sides w.r.t. time 't', we obtain

$$\frac{d\vec{r}}{dt} = 6\,\hat{i} + (8-10t)\,j \qquad \Rightarrow \vec{v} = 6\,\hat{i} + (8-10t)\,j.$$

Putting t = 0, we obtain the initial velocity (velocity of projection) given as

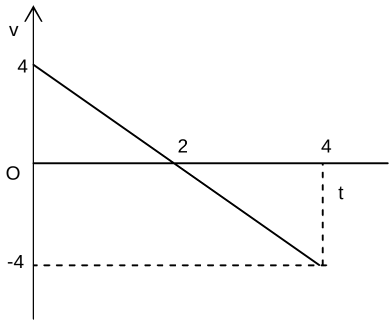
$$(\vec{v})_{t=0} = \vec{v}_0 = 6\,\hat{i} + 8\,\hat{j} \qquad \Rightarrow v_0 = 10 \text{ m/sec;}$$

(b) The time of flight $T = \dfrac{2v_0 \sin\theta_0}{g}$

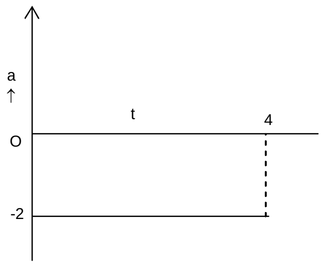
$$\Rightarrow T = \frac{2(v_y)_0}{g} \quad \text{where } (v_y)_0 = 8$$

$$\Rightarrow \quad T = \frac{2\times 8}{10} = 1.6 \text{ sec.}$$

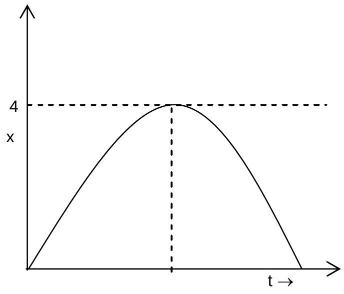
Prob 4. *A particle starts from origin at t = 0 along +ve x axis. It's velocity–time graph is shown in the figure. Draw*
(i) a, t graph
(ii) x, t graph.



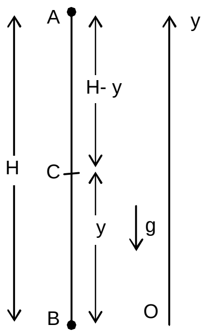
Sol. (i) Velocity is decreasing
so, a = –4/2 = –2



(ii)



Prob 5. *A stone 'A' is dropped from the top of a tower 20 m high. Simultaneously another stone 'B' is thrown up from the bottom of the tower so that it can reach just on the top of the tower. What is the distance of the stones from the ground while they pass each other?*



Sol. Let t be the time when they pass one another

For stone B, $y = v_B t + \frac{1}{2}(-g)t^2$ … (i)

Ffor stone A, $H - y = \frac{1}{2}gt^2$ … (ii)

From (i) and (ii),

$H = v_B t$ … (iii)

Stone B can reach just one the top of tower. We can calculate the velocity of stone B,

$v_f^2 = v_i^2 + 2a_y y$

$u_f = 0$, for $y_{max} = H = 20$ m

$v_i = v_B$; $a_y = -g$; $v_B = 20$ m/s

From (iii) $t = \frac{20m}{20m/s} = 1sec.$

From equation (i), the required distance (BC) from ground $= 20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15$ m

IITJEE TYPE

Prob 6. *A rocket is fired vertically and ascends with constant vertical acceleration of $20m/s^2$ for 1 minute. Its fuel is then all used and it continues as a free particle. Find the*
(a) maximum height reached by the rocket
(b) total time elapsed from the take off till the rocket strikes the earth.($g=10m/s^2$).

Sol. (a) For the time interval from 0 to 60 seconds rocket accelerates and thereafter it moves under gravity. Distance moved by it in 60 seconds is given by

$$S_1 = \frac{1}{2} \times \frac{20m}{s^2} \times (60s)^2 = 36000m.$$

$$v_{(60s)} = \frac{20m}{s^2} \times 60s = 1200m/s$$

If H be the maximum height reached.

Then, $0 = \left(1200\frac{m}{s}\right)^2 - 2g(H - 36000)$, $(v^2 = u^2 + 2as)$

$\Rightarrow H = 36000 + \frac{1200 \times 1200}{2 \times 10}m$

$\Rightarrow H = 108000m$

(b) Time taken to ascend is

$$t_1 = 60s + \frac{1200}{10}s = 180 \text{ s}, \quad [t = t_1 + \frac{u}{a}]$$

Let time taken to descend is t_2 then

$$108000 = \frac{1}{2}gt_2^2$$

$\Rightarrow t_2 = \sqrt{\frac{2 \times 108000}{10}} = 146.96s$

Total time $T = t_1 + t_2 = 180 + 146.96 = 326.96$ s.

Prob 7. *The position of a particle moving along the x-axis depends on the time as $x = at^2 - bt^3$ where $a = 3.0\ m/s^2$ and $b = 1.0\ m/s^3$ respectively.*
(a) At what time does the particle reach its maximum positive x-position ?
(b) What total path length does the particle cover in the first 2.0 sec?
(c) Does the particle cover equal path length in the opposite direction in the subsequent 2.0 sec. ? If not, explain why?
(d) Find the total path length covered in the first 4.0 sec.
(e) Find the displacement during the first 4.0 sec.
(f) What is the particles speed and acceleration at the end of first 3.0 sec.?

Sol. Here, the position (x) is time dependent as $x = at^2 - bt^3 = 3t^2 - t^3$

Instantaneous velocity $v = \dfrac{dx}{dt} = 6t - 3t^2$ . . . (i)

And acceleration $a = \dfrac{dv}{dt} = 6 - 6t$ . . . (ii)

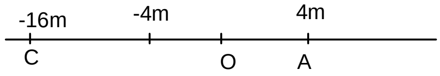
Note that the acceleration is not uniform (like gravity) but time dependent.

(a) At the maximum positive x-position the particle comes to momentary rest (v = 0) and then moves in the negative x-direction with non uniform acceleration.
From equation (i), $v = 0 = 6t - 3t^2$
$\therefore$ Required time = 2.0 seconds.

(b) The x-coordinate of the particle increases from zero to $(x)_{max}$ during the first 2 seconds.
$\therefore$ path length $= (x)_{t = 2s} = (3t^2 - t^3)_{t = 2s} = 4.0$ m

(c) For t > 2s, the particle moves in the backward direction with time dependent acceleration. Hence, subsequent motion is not repeated (as we have seen in free fall) where the acceleration $|\vec{g}|$ remains constant. Hence, the path length for the subsequent 2 seconds will be different.

(d) Position at t = 2 sec. is
$(x)_{t=2} = 4.0$ m and
$(x)_{t=4} = (3t^2 - t^3)_{t=4} = 48 - 64$
$= -16$ m.

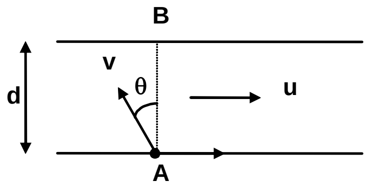


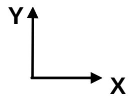
Hence, the path length during the first 4.0 is
OA + AO + OC
= 4 + 4 + 16 = 24.0 m

(e) Displacement during the first 4.0 sec is – 16.0 m.

(f) Speed at the end of 3.0 sec is
$(v)_{t=3} = (6t - 3t^2)_{t=3s} = -9.0$ m/s.
Negative sign indicates that motion is along the negative x-direction.
Acceleration $(a)_{t=3s} = (6 - 6t)_{t=3s} = -12\ m/s^2$.

Prob 8. *A man can row a boat with a speed of 4 km/hr in still water. He is crossing a river where the speed of current is 2 km/hr.*
(a) In what direction will his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?
(b) If width of the river is 4 km how long will it take him to cross the river, with the condition in part 'a' ?
(c) In what direction should he head the boat if he wants to cross the river in shortest time?
(d) How long will it take him to row 2 kms up the stream and then back to his starting point?





Sol. B is a point directly opposite to the starting point A.
Let the man heads the boat in a direction making an angle θ with the line AB.

Here $\vec{v}_w = 2\hat{i}$

$\vec{v}_{bw} = -4\sin\theta\hat{i} + 4\cos\theta\hat{j}$

$\because \quad \vec{v}_{(absolute)} = \vec{v}_{bw} + \vec{v}_w$

$= (2 - 4\sin\theta)\hat{i} + 4\cos\theta\hat{j}$

$\Rightarrow v_{bx} = 2 - 4\sin\theta$ and $v_{by} = 4\cos\theta$

(a) For directly opposite point $v_{bx} = 0$

$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin 30^o \Rightarrow \theta = 30^o$

Hence, to reach the point directly opposite to starting point he should head the boat an angle $\beta = (90^o + 30^o) = 120^o$ with the river flow.

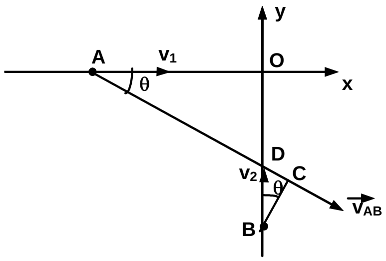
(b) $t = \frac{y}{v_{by}} = \frac{d}{4\cos\theta} = \frac{4}{4\cos 30^o} = \frac{2}{\sqrt{3}}$ hr.

(c) For t to be minimum $\cos\theta = 1 \Rightarrow \theta = 0^o$

$\Rightarrow \quad t_{min} = \frac{4}{4\cos 0} = 1$ hr.

(d) $T = \frac{2}{(4-2)}\text{hr} + \frac{2}{(4+2)}\text{hr} = \left(1 + \frac{1}{3}\right)\text{hr} = \frac{4}{3}\text{hr}.$

Prob 9. *Two particles A and B move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines towards the intersection point O. At moment t = 0, the particles were located at distances l_1 and l_2 from O, respectively. Find the time, when they are nearest and also this shortest distance.*



Sol. $\because \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v_1\hat{i} - v_2\hat{j}$

Minimum distance is the length of the perpendicular to $\vec{v}_{AB}$ from B.

If θ is the angle between the x-axis and $\vec{v}_{AB}$, then

$\tan\theta = \left|-\frac{v_2}{v_1}\right| = \frac{v_2}{v_1}$

In ΔAOD, $OD = OA\tan\theta = \frac{v_2}{v_1} l_1$

Therefore, $BD = l_2 - OD = \frac{v_1 l_2 - v_2 l_1}{v_1}$

In ΔBCD, $\cos\theta = \frac{BC}{BD}$

$\Rightarrow BC = BD\cos\theta = \frac{v_1 l_2 - v_2 l_1}{v_1} \times \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$

$\Rightarrow BC = \frac{|v_1 l_2 - v_2 l_1|}{\sqrt{v_1^2 + v_2^2}}$

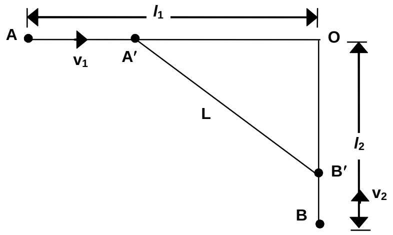
The required time $t = \dfrac{AC}{|\vec{v}_{AB}|} = \dfrac{AD + DC}{|\vec{v}_{AB}|}$

$$\Rightarrow \quad \frac{\ell_1 \sec\theta + BC\tan\theta}{\sqrt{v_1^2+v_2^2}} = \frac{\dfrac{\ell_1}{v_1}\sqrt{v_1^2+v_2^2} + \dfrac{v_1\ell_2 - v_2\ell_1}{\sqrt{v_1^2+v_2^2}}\dfrac{v_2}{v_1}}{\sqrt{v_1^2+v_2^2}}$$

$$= \frac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$$

Alternatively

After time 't' , the position of the point A and B are $(\ell_1 - v_1 t)$ and $(\ell_2 - v_2 t)$, respectively.



The distance L between the points A′ and B′ are

$$L^2 = (\ell_1 - v_1 t)^2 + (\ell_2 - v_2 t)^2 \; . \; . \; . \text{ (i)}$$

Differentiating with respect to time,

$2L\dfrac{dL}{dt} = 2(\ell_1 - v_1 t)(-v_1) + 2(\ell_2 - v_2 t)(-v_2)$ For minimum value of L, $\dfrac{dL}{dt} = 0$

$$(v_1^2 + v_2^2)t = \ell_1 v_1 + \ell_2 v_2$$

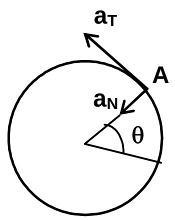
or $\quad t = \dfrac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$

Putting the value of t in equation (i)

$$L_{min} = \frac{|\ell_1 v_2 - \ell_2 v_1|}{\sqrt{v_1^2 + v_2^2}}.$$

Prob 10. *A wheel rotates around a stationary axis so that rotation angle θ varies as $\theta = Pt^2$, where $P = 0.20$ rad/s^2. Find the total acceleration a of the point A at the rim at the moment $t = 2.55$ sec, if the linear velocity of the point A at this moment is $v = 0.65$ m/s.*

Sol. Total acceleration of a body moving in a circular path



$$\vec{a} = \vec{a}_R + \vec{a}_t$$

$$|\vec{a}| = \sqrt{|\vec{a}_R^2| + |\vec{a}_t^2|}$$

The radial acceleration a_R is the centripetal acceleration

$$a_R = \frac{v^2}{R} = \omega^2 R = \left(\frac{d\theta}{dt}\right)^2 R$$

$$= \left\{\frac{d}{dt}(Pt^2)\right\}^2 R = 4P^2t^2R \qquad . \; . \; . \text{ (i)}$$

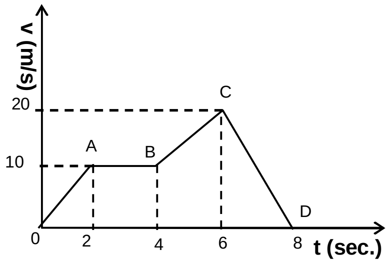
Tangential acceleration $a_T = \dfrac{d}{dt}(v)$

$$= \frac{d}{dt}(\omega R) = \frac{R\; d^2(\theta)}{dt^2} = 2PR \qquad . \; . \; . \text{ (ii)}$$

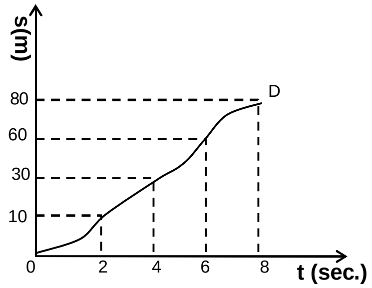
$$\therefore \quad a = \sqrt{\left(\frac{v^2}{R}\right)^2 + (2PR)^2}$$

$$= \sqrt{(4P^2t^2R)^2 + (2PR)^2} = 2PR\sqrt{1 + 4P^2t^4} = \frac{v}{t}\sqrt{1+4P^2t^4} = 0.7 \text{ m/s}^2.$$

Prob 11. *Velocity–time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement–time graph of a particle if at t = 0 displacement s = 0.*



Sol.



Prob 12. *A particle moves in a circle of radius 20 cm at a speed given by $v = 1 + t + t^2$ m/s where t is time in s. Find (a) the initial tangential and normal acceleration. (b) the angle covered by the radius in first 2 s.*

Sol. (a) Tangent acceleration $a_t = \dfrac{dv}{dt} = 2t + 1$

Normal acceleration $a_n = \dfrac{v^2}{R}$ $\therefore (a_t)_{t=0} = 1\ m/s^2$

$(a_n)_{t=0} = \dfrac{v_0^{\ 2}}{R} = \dfrac{1}{(0.2)} = 5\ m/s^2$

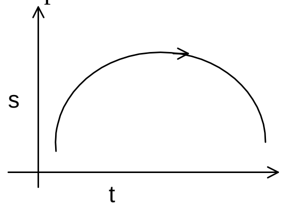
(b) $v = R\dfrac{d\theta}{dt}$

$R\, d\theta = (1+t+t^2)dt$ $\therefore R\int_0^{\theta'} d\theta = \int_0^2 \left(1+t+t^2\right)dt$

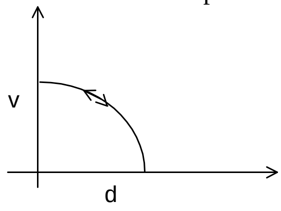
$\theta = 33.3$ rad

Prob 13. *A body of mass m is projected vertically upwards with a speed v_0. It goes up and comes back to the same point. For this motion draw displacement–time, velocity–time, acceleration–time and speed–displacement graphs.*

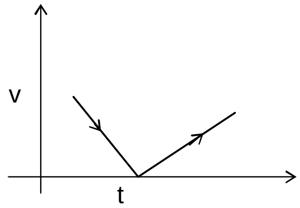
Sol. Displacement–time



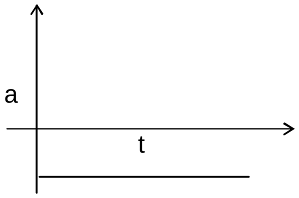
Speed–displacement



Speed time



Acceleration time



Prob 14. *A car starts moving from rest with an acceleration whose value linearly increases with time from zero to 6 m/s² in 6 sec after which it moves with constant velocity. Find the time taken by the car to travel first 72 m from starting point.*

Sol. Since acceleration varies linearly with time therefore

$a \propto t$

$\Rightarrow a = kt \quad \Rightarrow \int_0^6 da = k\int_0^6 dt \qquad \Rightarrow k = 1$

then, $\frac{dv}{dt} = t \quad \Rightarrow v = \frac{t^2}{2}$ m/s

then, $\frac{ds}{dt} = \frac{t^2}{2}$ or, $s = \frac{t^3}{6}$

at the end of t = 6 sec. Acceleration becomes zero.

Distance moved by car at t = 6 sec is

$S_1 = \frac{6\times6\times6}{6} = 36$ m

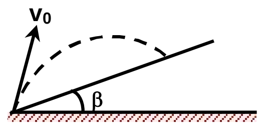
Speed of the car $= \frac{6\times6}{2} = 18$ m/s

Remaining distance = 72 – 36 = 36 m.

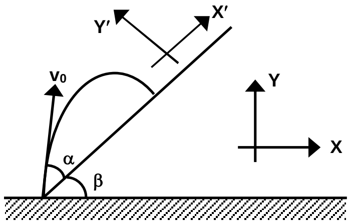
so time taken to cover this distance = $t_2 = \frac{36}{18}\text{sec.} = 2\,\text{sec.}$

Total time = 6+2 = 8 sec.

Prob 15. *A particle projected with velocity v_0 from an inclined plane whose angle of inclination with the horizontal is β. If afterwards the projectile strikes the inclined plane perpendicular to it. Find the height of the point struck, from horizontal plane through the point of projection.*



Sol. Let α be the angle between the velocity of projection and the inclined plane.



$v_{0x'} = v_0\cos\alpha,\ v_{0y'} = v_0\sin\alpha$

$a_{x'} = -g\sin\beta \qquad a_{y'} = -g\cos\beta$

$\Rightarrow \quad v_{x'}(t) = v_0\cos\alpha - g\sin\beta t$

At the point of impact $v_{x'} = 0$

$$\Rightarrow \quad t = \frac{v_0\cos\alpha}{g\sin\beta} \qquad ..(1)$$

Also y′ at the point is zero.

$$\Rightarrow \quad v_0\sin\alpha t - \frac{1}{2}g\cos\beta t^2 = 0$$

$$\Rightarrow \quad t = \frac{2v_0\sin\alpha}{g\cos\beta} \qquad ..(2)$$

From (1) and (2) $\qquad \tan\alpha = \frac{\cot\beta}{2} \qquad ..(3)$

$$x = v_0\cos(\alpha+\beta)t$$

$$= v_0\left[\cos\alpha\cos\beta - \sin\alpha\sin\beta\right].\frac{v_0\cos\alpha}{g\sin\beta}$$

$$= \frac{v_0^2}{g}\left[\cos^2\alpha\cot\beta - \sin\alpha\cos\alpha\right]$$

$$=\frac{v_0^2}{g}\left[\left(\frac{2}{\sqrt{4+\cot^2\beta}}\right)^2\cot\beta-\frac{\cot\beta}{\sqrt{4+\cot^2\beta}}.\frac{2}{\sqrt{4+\cot^2\beta}}\right]$$

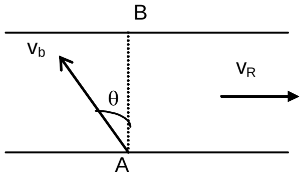
$$=\frac{v_0^2}{g}\frac{2\cot\beta}{4+\cot^2\beta}$$

$$\therefore \quad y=x\tan\beta=\frac{v_0^2}{g}.\frac{2\cot\beta}{4+\cot^2\beta}\tan\beta$$

$$\Rightarrow \quad y=\frac{2v_0^2}{g\left(4+\cot^2\beta\right)}$$

Prob 16. *The velocity of a boat in still water is n times less than the velocity of flow of a river. At what angle to the stream direction must the boat move so that drift is minimised ? If n = 2, show that the angle $\theta = 120°$.*

Sol. Given $v_b = \frac{v_R}{n}$



$\vec{v}_b = (-v_b \sin\theta)\hat{i}+(v_b\cos\theta)\hat{j}$

Resultant velocity of boat

$= \vec{v}_b+\vec{v}_R$

$= (v_R - v_b \sin\theta)\ \hat{i}\ + (v_b \cos\theta)\ \hat{j}$

If w = width of the river, time for crossing is

$$T=\frac{W}{v_b\cos\theta}$$

Drift during time T is $(v_R - v_b \sin\theta)$ T

$\Rightarrow$ Drift $x = v_b(n - \sin\theta)\frac{w}{v_b\cos\theta}$ $= w(n \sec\theta - \tan\theta)$

For x to be minimum, $\frac{dx}{d\theta} = 0$ lead to $\theta = \sin^{-1}(1/n)$

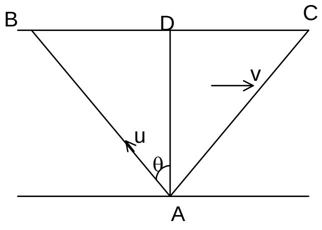
Direction of boat w.r.t. stream is

$90° + \theta = 90° + \sin^{-1}(1/n)$

For n = 1/2, the required angle $= 90° + 30° = 120°$

Prob 17. *A man can row a boat in still water at 3 km/h He can walk at a speed of 5 km/h on the shore. The water in the river flows at 2 km/h. If the man rows across the river and walks along the shore to reach the opposite point on the river bank find the direction in which he should row the boat so that he could reach the opposite shore in the least possible time. The width of the river is 500 m.*

Sol. Let the points towards B and reches at C



t_1 : the time taken by the boat to reach C

$t_1 = \frac{AD}{u\cos\theta}$ $\quad CD = (v - u\sin\theta)t_1$

$t_1 = \frac{500\times10^{-3}}{3\cos\theta}\ \text{hr} = \frac{1}{6\cos\theta}$

$CD = (-3\sin\theta + 2)\frac{1}{6\cos\theta}$

$= -0.5 \tan\theta + \dfrac{1}{3\cos\theta}$

t_2 : time taken by the man from C to D

$t_2 = \dfrac{CD}{v_s} = -\dfrac{0.5\tan\theta}{5} + \dfrac{1}{3\cos\theta \times 5} = \dfrac{1}{10}\tan\theta + \dfrac{1}{15\cos\theta}$

$= -\dfrac{\sin\theta}{10\cos\theta} + \dfrac{1}{15\cos\theta}$

$= \dfrac{(-3\sin\theta + 2)}{30\cos\theta}$

Total time $t = t_1 + t_2 = \dfrac{1}{6\cos\theta} + \dfrac{-3\sin\theta + 2}{30\cos\theta} = \dfrac{7 - 3\sin\theta}{30\cos\theta}$

$= \dfrac{7}{30}\sec\theta - \dfrac{1}{10}\tan\theta$

For minimum t

$\dfrac{dt}{d\theta} = 0 \quad \Rightarrow \quad \dfrac{7}{30}\sec\theta\tan\theta - \dfrac{1}{10}\sec^2\theta = 0$

$\Rightarrow \dfrac{1}{10}\sec\theta\left(\dfrac{7}{3}\tan\theta - \sec\theta\right) = 0 \Rightarrow \quad \dfrac{7}{3}\tan\theta - \sec\theta = 0$

$\dfrac{7\sin\theta - 3}{3\cos\theta} = 0 \quad \Rightarrow \quad \theta = \sin^{-1}(3/7)$

Prob 18. *A cyclist moves with constant speed 5 m/s along eastward for 2 seconds, and along southward for 2 seconds. Then, he moves along west for one second and finally along north-west for $\sqrt{2}$ seconds. Find*

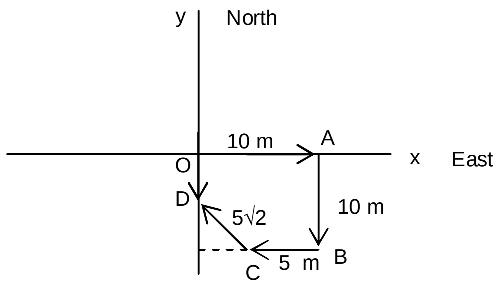
(a) Distance and displacement of cyclist for whole journey.

(b) Average speed and average velocity for whole journey

(c) Average acceleration of cyclist for whole journey.

Sol. (a) In figure, shown final displacement

$\vec{OD} = -5\hat{j}\,m$



Distance = OA + AB + BC + CD

= (25 + 5$\sqrt{2}$) m

(b) Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$= \dfrac{25 + 5\sqrt{2}}{5 + \sqrt{2}} = 5$ m/s.

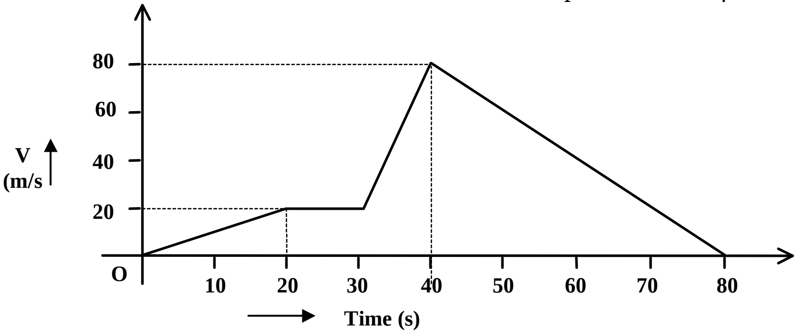
Average velocity $= \dfrac{-5\hat{j}}{(5 + \sqrt{2})}$ m/s

(c) For average acceleration $= \dfrac{\bar{v}_f - \bar{v}_i}{\Delta t}$

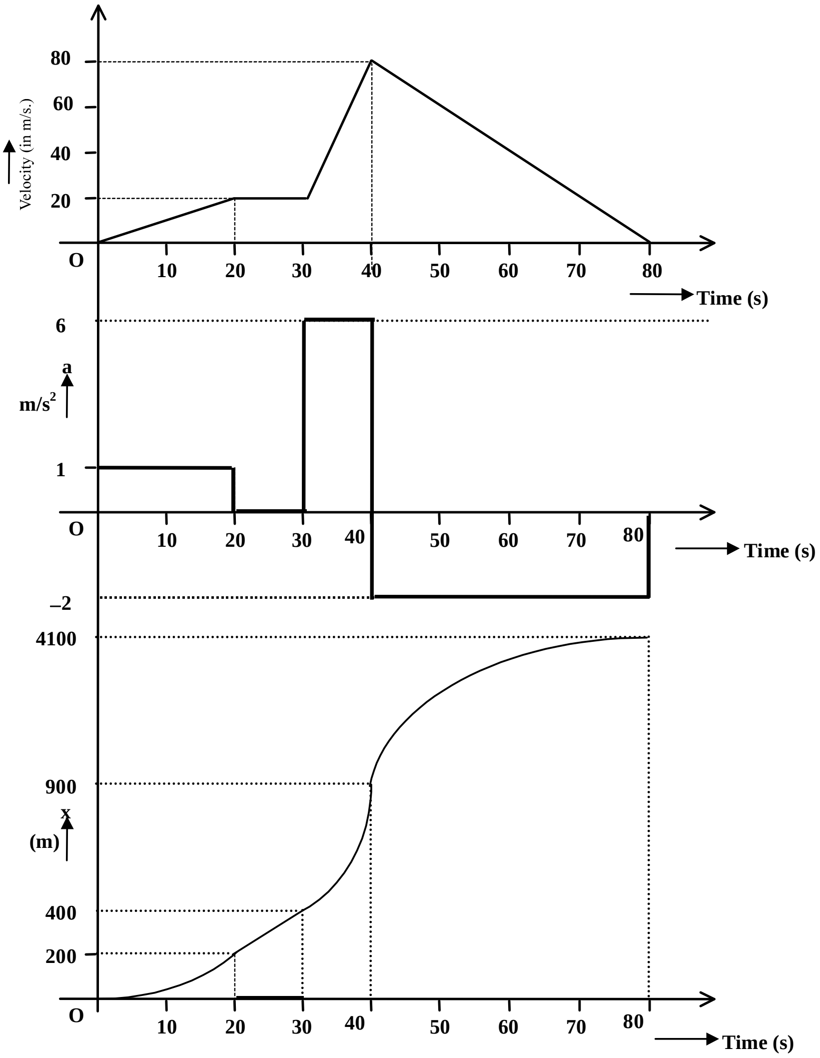
$\bar{v}_f = -5\hat{i} + 5\hat{j}$, $v_i = 5\hat{i}$

average $= \dfrac{-5\hat{i} + 5\hat{j} - 5\hat{j}}{(5 + \sqrt{2})} = \dfrac{-10\hat{i} + 5\hat{j}}{5 + \sqrt{2}}$ m/s^2

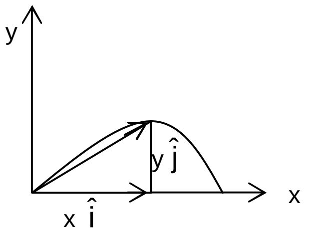
Prob 19. *The velocity-time graph of moving object is given in the figure. Draw the acceleration versus time and displacement versus time graph. Find the distance travelled during the time interval when the acceleration is maximum. Assume that the particle starts from origin.*



Sol.



Prob 20. *A projectile is fired with speed v_0 at an angle θ with the horizontal on a horizontal plane, Find*
(a) the average velocity of projectile in half of time of flight.
(b) the time in which the speed of projectile becomes perpendicular to its initial velocity.
(c) the radius of curvature of projectile at the instant when it is at its maximum height.



Sol. (a) $\vec{v}_{av} = \dfrac{\text{displacement}}{\text{time}} = \dfrac{x\hat{i} + y\hat{j}}{\dfrac{v_0 \sin\theta}{2g}}$

x = half of range $= \dfrac{v_0^2 \sin 2\theta}{2g}$

y = Max. height $= \dfrac{v_0^2 \sin^2\theta}{2g}$

$$\overline{v}_{av} = \dfrac{\dfrac{v_0^2 \sin 2\theta}{2g}\hat{i} + \dfrac{v_0^2 \sin^2\theta}{2g}\hat{j}}{\dfrac{v_0 \sin\theta}{2g}} = \dfrac{v_0 \sin 2\theta}{\sin\theta}\hat{i} + \dfrac{v_0 \sin^2\theta}{\sin\theta}\hat{j}$$

$= 2v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\ \hat{j}$

(b) Let the perpendicular velocity be $\vec{v}$

$\overline{v}\cdot\overline{v} = 0 \quad \therefore\ \overline{v}\cdot\overline{v} = (v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\,\hat{j}).\ (v_0 \cos\theta\ \hat{i} + (v_0 \sin\theta - gt)\,\hat{j}$

$\therefore\ v_0^2 \cos^2\theta + v_0 \sin\theta(v_0 \sin\theta - gt) = 0$

$v_0^2 - v_0 \sin\theta gt = 0 \ \therefore\ t = \dfrac{v_0}{g\sin\theta}$

(c) Radius of curvature $= \dfrac{v^2}{g} = \dfrac{v_0^2 \cos^2\theta}{g}$

Prob 21. *An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with a constant acceleration of 1.2 m/s^2. Two seconds after it starts, a bolt begins to fall from the ceiling of the elevator. Find*

(a) the bolt's free fall time,

(b) the displacement and the distance covered by the bolt during the fall in the reference frame fixed to the ground. (Use g = 9.8 m/s^2.)

Sol. (a) Since a = 1.2 m/sec^2 is the constant acceleration of the elevator car while ascending and h = 2.7 m is the separation between the floor and the ceiling, therefore, the free fall time is given by

$\Rightarrow h = \dfrac{1}{2}(g + a)t^2 \ \Rightarrow t = \sqrt{\dfrac{2h}{g+a}}$ = 0.7 sec

(b) Velocity of elevator at t = 2 sec is v = (1.2 m/s^2) (2 s) = 2.4 m/sec.

Thus, with respect to the reference frame fixed to the ground i.e. with respect to a stationary observer, the displacement in the course of free fall is

$y = (-2.4\text{ m/s})\ (0.7\text{ s}) + \dfrac{1}{2}\ (9.8\text{ m/s}^2)\ (0.7\text{ s})^2 = 0.72\text{ m}$

Total distance covered w.r.t. the ground during the free fall times is

s = y + 2h

$= 0.72 + 2 \times \dfrac{(2.4)^2}{2\times 9.8} = 1.31\text{ m.}$

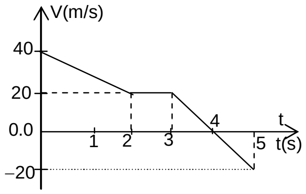
$v^2 = u^2 + 2gh$

$0 = (2.4\text{ m/s})^2 + 2\ (-\ 9.8\text{ m/s}^2)h$

$\Rightarrow \quad h = \dfrac{(2.4\,\text{m/s})^2}{2\times\left(9.8\,\text{m/s}^2\right)}$

Objective:

Prob 1. *In the given v–t graph, the distance travelled by the body in 5 sec will be*
(A) 100 m
(B) 80 m
(C) 40 m
(D) 20 m



Sol. **A.**
Distance travelled = area under the v–t curve

$$= \frac{20\times 2}{2} + 20\times 2 + 20\times 1 + \frac{20\times 1}{2} + \frac{20\times 1}{2} = 100 \text{ m}$$

Prob 2. *In Question 1, the displacement of the body in 5 sec will be*
(A) 100 m *(B) 80 m*
(C) 40 m *(D) 20 m*

Sol. **B.**
Displacement is a vector and is equal to algebraic sum of area under the v–t graph.
= 20 + 40 + 20 + 10 – 10 = 80 m.

Prob 3. *In Question 1, the average velocity of the body in 5 seconds is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **B.**
Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{80}{5} = 16\,\text{m/s}$

Prob 4. *In above Question, the average speed of the body during 5 sec is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **A.**
Average speed $= \frac{\text{distance}}{\text{time}} = \frac{100}{5} = 20\,\text{m/s}$

Prob 5. *A body when projected vertically up, covers a total distance D during its time of flight. If there were no gravity, the distance covered by it during the same time is equal to*
(A) 0 *(B) D*
(C) 2D *(D) 4D*

Sol. **C**.
The displacement of the body during the time t as it reaches the point of projection
$\Rightarrow S = 0 \Rightarrow v_0t - \frac{1}{2}gt^2 = 0 \qquad \Rightarrow t = \frac{2v_0}{g}$
During the same time t, the body moves in absence of gravity through a distance
D′ = v.t, because in absence of gravity g = 0

$\Rightarrow D' = v_0 \left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g}$ …(1)

In presence of gravity, the total distance covered is

$= D = 2H = 2\ \frac{v_0^2}{2g} = \frac{v_0^2}{g}$ …(2)

(1) ÷ (2) $\Rightarrow D' = 2D$.

Prob 6. *A particle is projected vertically upward with initial velocity 25 ms^{-1}. During third second of its motion, which of the following statement is correct?*
(A) displacement of the particle is 30 m
(B) distance covered by the particle is 30 m
(C) distance covered by the particle is 2.5 m
(D) none of these

Sol. **C.**
Displacement of the particle during third second of the motion (i.e. between t = 2s and t = 3s) is zero. Hence, t = 2.5 sec is the turning point of the motion.

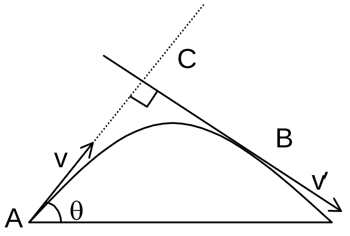
For distance $S_{t=2} = 25 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 30$ m

and $S_{t=2.5} = 25 \times 2.5 - \frac{1}{2} \times 10 \times 2.5^2 = 31.25$

Hence, distance covered by the particle during third second of motion
$= 2\,(31.25 - 30) = 2.5$ m.

Prob 7. *A particle is projected from a point A with a velocity v at an angle θ (upward) with the horizontal. At a certain point B, it moves at right angle to its initial direction. It follows that*
(A) velocity of the particle at B is v.
(B) velocity of the particle at B is v cos θ.
(C) velocity of the particle at B is v tan θ.
(D) the time of flight from A to B is $\frac{v}{g\sin\theta}$.

Sol. **D.**



$\vec{v} = \vec{u} + \vec{a}t$

Considering along the line AC

$$0 = v - g\sin\theta t \Rightarrow t = \frac{v}{g\sin\theta}$$

Now, consider along the line CB

$$v' = 0 + g\cos\theta\ \frac{v}{g\sin\theta} = v\cot\theta$$

Prob 8. *A particle is projected horizontally from the top of a cliff of height H with a speed $\sqrt{2gH}$. The radius of curvature of the trajectory at the instant of projection will be*
(A) H/2 *(B) H*
(C) 2H *(D) ∞*

Sol. **C.**

Since, $\vec{g} \perp \vec{v}$

Radial acceleration $a_r = g$

$\Rightarrow \dfrac{v_0^2}{r} = g$ where r is the radius of curvature.

$\Rightarrow \dfrac{2gH}{r} = g \quad (\because v = \sqrt{2gH})$

$\Rightarrow r = 2H$

Prob 9. *If a boat can have a speed of 4 km/hr in still water, for what values of speed of river flow, it can be managed to row boat right across the river, without any drift?*

(A) $\geq$ 4 km/hr *(B) greater than zero but less than 4 km/hr*
(C) only 4 km /hr *(D) none of these*

Sol. **B.**

Drift $(\Delta x) = (v_{b,x})\,\Delta t = (v_{br}\cos\theta + v_r)\Delta t$

where $v_{b,x}$ = velocity of boat w.r.t. ground

$v_{\perp, r}$ = velocity of boat w.r.t river

v_r = velocity of river w.r.t. ground

For $\Delta x = 0$, $v_r = -v_{br}\cos\theta$

$\Rightarrow (v_r)_{max} = v_{br}$

For, $v_r > v_{br}$ we can not have zero drift.

Prob 10. *A swimmer crosses a river of width d flowing at velocity v. While swimming, he keeps himself always at an angle of 120° with the river flow and on reaching the other end he finds a drift of d/2 in the direction of flow of river. The speed of the swimmer with respect to the river is*

(A) $(2-\sqrt{3})\,v$ *(B) $2\,(2-\sqrt{3})\,v$*
(C) $4\,(2-\sqrt{3})\,v$ *(D) $(2+\sqrt{3})\,v$*

Sol. **C.**

Drift = $d/2 = (V_r - V_s\sin30)d/V_s\cos30$

$\Rightarrow V_s = 4\,(2-\sqrt{3})V$

Prob 11. *A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400 m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum, are*

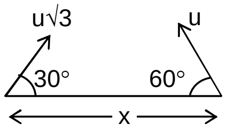
(A) (400, 100) *(B) (200, 100)*
(C) (400, 200) *(D) (200, 200)*

Sol. **B.**

When the horizontal range is maximum, the maximum height attained is R/4 = 100 m. The velocity of the projectile is minimum at the highest point.

$\therefore$ Required point is (200, 100).

Prob 12. *Two particles are separated at a horizontal distance as shown in the adjacent figure. They are projected along the same line with different initial speeds. The time after which the horizontal distance between them becomes zero is*



(A) $\dfrac{x}{u}$ *(B)* $\dfrac{x}{2u}$

(C) $\dfrac{2u}{x}$ *(D) none of these*

Sol. **B.**

Both particles will collide at the highest point of their path. At highest point, only the horizontal component exists.

$$V_1 = u\sqrt{3}\cos 30° = \frac{3u}{2}$$

$$V_2 = u\cos 120° = -\frac{u}{2}$$

Relative velocity of the particle 1 w.r.t. particle 2 in x-direction = $\frac{3u}{2}+\frac{u}{2}=2u$

$\Rightarrow$ Required time $=\frac{x}{2u}$

Prob 13. *A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s, then maximum height attained by it is: (g = 10 m/s^2)*

(A) 8 m *(B) 20 m*
(C) 10 m *(D) 16 m*

Sol. **C.**

Suppose after travelling distance s, particle has the velocity 10 m/s.

So, $v^2 = u^2 - 2as$

$\Rightarrow$ $(10)^2 = u^2 - 2 \times 10s$ …(1)

At the maximum height, i.e. 2s, v = 0

$\Rightarrow$ $0 = u^2 - 2g(2s)$

$\Rightarrow$ $u^2 = 40s$ …(2)

From Eqs. (1) and (2), s = 5 m

$\Rightarrow$ 2s = 10 m

Prob 14. *A person walks up a stationary escalator in 90 sec. If the escalator moves with the person, first standing on it, it will take 1 minute to reach the top from ground. How much time it would take him to walk up the moving escalator?*

(A) 24 sec *(B) 48 sec*
(C) 36 sec *(D) 40 sec*

Sol. **C.**

Let L be the length of escalator.

$\therefore$ Relative speed $=\frac{L}{90}+\frac{L}{60}=\frac{L}{36}$

$\therefore$ Time taken to walk up the moving escalator $=\left(\frac{L}{L/36}\right)=36\,\text{sec}$

Prob 15. *A driver applies brakes on seeing a traffic signal 400 m ahead. At the time of applying the brakes the vehicle was moving with 15 m/s and retarding with 0.3 m/s^2. The distance of vehicle after 1 min from the traffic light is*

(A) 25 m *(B) 375 m*
(C) 360 m *(D) 40 m*

Sol. **A.**

The maximum distance covered by the vehicle before coming to rest $=\frac{v^2}{2a}=\frac{(15)^2}{2(0.3)}=375$ m

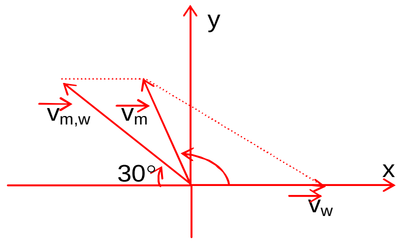
The corresponding time = t = $\frac{v}{a}=\frac{15}{0.3}=50$ sec

$\therefore$ The distance of the vehicle from the traffic signal after one minute = 400 – 375 = 25 m

Prob 16. *A motorboat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. The velocity of the motorboat wrt water is 5 √3 m/s. The driver should steer the boat at an angle*
(A) 30° wrt the line of destination from starting point
(B) 60° wrt normal to the bank
(C) 120° wrt stream direction
(D) None of these

Sol. **C.**
The velocity of motorboat,

$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$
$$= -5\sqrt{3}\cos 30°\ \hat{i} + 5\sqrt{3}\sin 30°\ \hat{j} + 5\hat{i}$$
$$= -2.5\ \hat{i} + \frac{5\sqrt{3}}{2}\hat{j}$$
$$\phi = \tan^{-1}\left(-\frac{5\sqrt{3}}{2\times 2.5}\right) = \tan^{-1}\left(-\sqrt{3}\right) = 120^0$$



Prob 17. *The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be*

(A) $v_0 t + \frac{1}{6}bt^3$ *(B)* $v_0 t + \frac{1}{3}bt^3$
(C) $v_0 t + \frac{1}{3}bt^2$ *(D)* $v_0 t + \frac{1}{2}bt^2$

Sol. **A.**
Given, acceleration a = bt

$$\Rightarrow \frac{dv}{dt} = bt \quad \Rightarrow \quad v = \frac{bt^2}{2} + c$$

At t = 0, v = v_0 ⇒ c = v_0

$$\text{So,}\ v = \frac{bt^2}{2} + v_0$$
$$\Rightarrow \frac{ds}{dt} = \frac{bt^2}{2} + v_0$$
$$\Rightarrow s = \frac{bt^3}{6} + v_0 t$$

Fill in the Blanks

Prob 1. *The position of a body w.r.t. time is given by $x = 3t^3 - 6t^2 + 12t + 6$. At time t = 0, its acceleration is ____________.*

Sol.
$$\frac{dx}{dt} = 6t^2 - 12t + 12$$
$$\frac{d^2x}{dt^2} = 12t - 12$$
$$\left.\frac{d^2x}{dt^2}\right|_{t=0} = -12$$

Prob 2. *A body thrown up from the ground vertically passes the height of 10.2 m twice in an interval of 10 sec. Its initial velocity was ________ m/s and its time of journey upwards was ________sec ($g = 10\ m/s^2$).*

Sol. It takes 5 sec from its maximum height to the height of 10.2 m, travelling from rest at acceleration of 10 m/s^2. Hence, if this distance be s, then

$$s = \frac{1}{2} \times 10 \times 5^2 = 125 \text{ m.}$$

So, $u^2 = 2 \times 10 \times (125 + 10.2)$

$\Rightarrow$ u = 52 m/s

$\Rightarrow$ $t = \frac{52}{10} = 5.2$ sec.

Prob 3. *For a projectile projected at an angle ____________, the maximum height and horizontal range are equal.*

Sol. $\tan^{-1}(4)$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$

$\Rightarrow \sin 2\theta = \frac{\sin^2 \theta}{2}$

$\Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1} 4.$

Prob 4. *A car covers the first half of its distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. The average speed of the car is ________ km/hr.*

Sol. 48 km/hr.

Let total distance be 2x km

$\therefore$ Total time taken $= \left(\frac{x}{40} + \frac{x}{60}\right) \text{hr} = \frac{x}{24} \text{hr}$

Therefore, average speed = $\frac{2x}{\left(\frac{x}{24}\right)} = 48$ km/hr.

Prob 5. *In a uniform motion, the particle travels in a ______________ and traces equal _________however ______________intervals of ________________ be taken.*

Sol. Straight line, displacements, small, time.

True or False Type Questions

Prob 1. *A particle in one-dimensional motion with positive value of acceleration must be speeding up.*

Sol. False.

If the velocity of the body is negative, then even in case of positive acceleration the body speeds down, e.g. a body projected up slows down even when acceleration is positive.

Prob 2. *A particle in one-dimensional motion with constant speed must have zero acceleration.*

Sol. True.

As the direction of motion remains unchanged, therefore, if the speed is zero the acceleration must also be zero.

Prob 3. *A particle moves with a uniform velocity in a straight line. If another particle moves such that it is always directed towards the first particle then the motion of the second particle is also along a straight line.*

Sol. False.
Because the second particle is always directed towards the first particle, the motion of the second particle can be straight line only in the special case when it follows the uniformly moving first particle along the same straight line.

Prob 4. *A particle in one-dimensional motion with zero speed at any instant may have non-zero acceleration at that instant.*

Sol. True.
When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 m/s^2

Prob 5. *If a base ball player can throw a ball to a maximum distance d over the ground, then the maximum vertical height to which he can throw it will be equal to d/2. Assume that initial speed of the ball is same in both the cases.*

Sol. True.
$R_{max} = d = \frac{u^2}{g}$ and $H_{max} = \frac{u^2}{2g} = \frac{d}{2}$

Prob 6. *A bus moving towards north takes a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.*

Sol. False.
The direction will change.

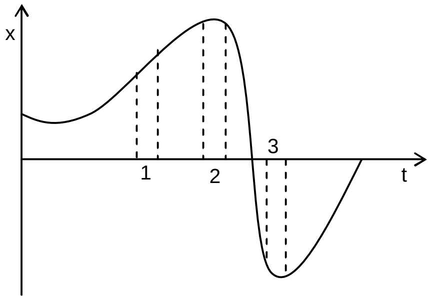
ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the cliff upward with a velocity 100 m/s. After what time the bullet and the block meet ?

2. Figure shows the x–t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



3. Two balls of different masses are thrown vertically upwards with the same speed. During their downward journey, they pass through the point of projection with the same speed. Neglect air resistance. Is this statement correct?

4. Galileo stated that "For elevations which exceed or fall short of 45° by equal amounts, the ranges are equal." Prove this statement.

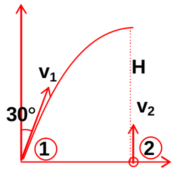
5. A block slides down a smooth inclined plane when released from the top while another falls freely from the same point. Which one of them will strike the ground earlier ?

6. A stone is thrown horizontally with a speed $\sqrt{2gh}$ from the top of a wall of height h. What is the distance from the wall when it reaches the ground?

7. What is the angle θ of projection with horizontal plane of a projectile if its range is $\frac{\sqrt{3}v^2}{2g}$, where v is velocity of projection?

8. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B desires to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past driver of A, calculate the original distance between the guard of B and the driver of A.

9. A particle 1 is projected with speed v_1 from a point A making an angle of 30° with the vertical. At the same instant, a second particle 2 is thrown vertically upwards from position B with velocity v_2. The two particles reach height H, the highest point on the parabolic path of particle 1 simultaneously. Calculate the ratio $\frac{v_1}{v_2}$.



10. A car is moving with a speed of 30 m/s on a circular path of radius 500 m. Its speed is increasing at a rate of 2 m/s^2. What is the acceleration of the car?

11. A particle is thrown vertically upward. Its velocity at half of the maximum height is 10 m/s, then calculate the maximum height attained by it. ($g=10\ m/s^2$)

12. A car moving with a speed of 40 km/hr can be stopped, by applying brakes, in 2 meters. If the same car is moving with a speed of 80 km/hr, what is the minimum stopping distance?

13. The position of a particle moving along x-axis is given by $x = a + bt^2$ where x is in meter and t in seconds. The constants a and b are 4.5 m and 3.5 m/s^2 respectively. Find
(a) initial velocity
(b) velocity at t = 3 seconds.
(c) average velocity during the time interval t = 1 s to t = 3 s.

14. A body dropped from a height h, with initial velocity zero, strikes the ground with velocity 3 m/s. Another body of the same mass is dropped from the height h with an initial velocity of 4 m/s. Find the final velocity with which it strikes the ground.

15. The velocity of a train increases uniformly from 20 km/hr to 60 km/hr in 4 hours. Find the distance travelled by the train during this period.

16. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then find the maximum velocity acquired by the car.

17. An aeroplane is flying horizontally with a velocity of 216 km/hr and at a height of 1960 m. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. Find the distance AB.

18. A body is projected horizontally with a speed of 20 m/s from the top of a tower. What will be its speed after nearly 5 sec? ($g = 10\ m/s^2$)

19. A bus moves a distance of 200 m. It covers the first half of the distance at speed 40 km/hr and the second half of the distance at speed v. The average speed is 48 km/hr. Find the value of v.

20. A ball is dropped from height of 90 m on a floor. The ball loses one tenth of its speed. Put the speed–time graph of its motion between t = 0 and 12 sec. ($g = 10\ m/s^2$)

Level - I

1. The position of a particle along the x-axis is given in centimeters by $x=9.75+1.50t^3$, where t is in seconds. Consider the time interval t = 2 s to t = 3 s and calculate
   (a) the average velocity
   (b) instantaneous velocity at t = 2 s;
   (c) the instantaneous velocity when t = 2.5 s;
   (d) the instantaneous velocity when the particle is mid way between its position at t = 2 s and t = 3 s.

2. A train started from rest and moved with constant acceleration. At one time it was travelling at 33.0 m/s and 160 m farther it was travelling at 54.0 m/s. Calculate
   (a) the acceleration.
   (b) the time required to travel the 160 m.
   (c) the time required to attain the speed of 33.0 m/s.
   (d) the distance moved from rest to the time the train had a speed of 33 m/s.

3. A body travelling in a straight line travels 2 m in the first two seconds and 2.2 m in the next four seconds with constant retardation. What will be its velocity at the end of the seventh second from the start?

4. A motorcyclist moving with uniform retardation takes 10 s and 20 s to travel successive quarter kilometer. How much further he will travel before coming to rest?

5. A car is moving on a straight road with a speed 20 m/s. At t = 0, the driver of the car applies the brakes after watching an obstacle 150 m ahead. After application of brakes the car retards with 2 m/s^2. Find the position of the car from the obstacle at t =15 s.

6. A ball is thrown with a velocity of 100 ms^{-1} at an angle of 30° to the horizontal and meets the same horizontal plane later. Find
   (a) its time of flight
   (b) the horizontal distance it travels
   (c) the velocity with which it strikes the ground at the end of its flight. [g = 9.8 ms^{-2}]

7. A projectile shot at an angle of 60° above the horizontal strikes a wall 30 m away at a point 15 m above the point of projection.
   (a) Find the speed of projection.
   (b) Find the magnitude of velocity of the projectile when it strikes the wall.

8. A ball is thrown vertically up with a certain velocity from the top of a tower of height 40 m. At 4.5 m above the top of the tower its speed is exactly half of that it will have at 4.5 m below the top of the tower. Find the maximum height reached by the ball above the ground?

9. A rotating fan completes 1200 revolutions every minute. Consider a point on the tip of the blade, which has a radius of 0.15 m
   (a) Through what distance does the point move in one revolution?
   (b) What is the speed of the point? (c) What is its acceleration?

10. A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle 30^o to the path of A. Find the velocity of B relative to A.

11. A ball dropped from some height covers half of its total height during the last second of its free fall. Find
(a) time of flight
(b) height of its fall
(c) speed with which it strikes the ground.

12. From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30^o with the horizontal up the plane. Find the range.

13. A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.

14. The acceleration experienced by a moving boat after its engine is cut off, is given by $\frac{d\omega}{dt} = -kv^3$, where k is a constant if v_0 is the magnitude of the velocity at cutoff find the magnitude of the velocity at time t after the cut off.

15. A boy throws a ball vertically upward with an initial speed of 15.0 m/s. The ball was released when it was at 2.00 m above ground. The boy catches it at the same point as the point of projection.
(a) What is maximum height reached by the ball ?
(b) How long is the ball in the air?

Level- II

1. The equation of motion of a particle moving along a straight line is given as $x = \frac{1}{2}vt$ when x, v t, have usual meaning, prove that the acceleration is constant.

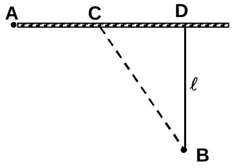
2. A point moving in a straight line traversed half the distance with a velocity v_o. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

3. At the instant the traffic light turns green, an automobile starts with a constant acceleration of 2.2 m/s^2. At the same instant a truck, travelling with a constant speed of 9.5 m/s, overtakes and passes the automobile.
 (a) How far beyond the starting point will the automobile overtake the truck?
 (b) How fast will the car be traveling at the instant?
 (It is instructive to plot a qualitative graph of 'x' versus t for each vehicle.)

4. A balloon is ascending vertically with an acceleration of 1 m/s^2. Two stones are dropped from it at an interval of 2 s. Find the distance between them 1.5 sec after the second stone is released.

5. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point in space and moved with velocities v_1 = 3.0 m/s and v_2 = 4.0 m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

6. From point A located on a highway (Fig.) one has to get by car as soon as possible to point B located in the field at a distance ℓ from the highway. It is known that the car moves in the field n times slower than the highway. At what distance from point D one must turn off the highway?



7. To a man walking at 7 km/hr due west the wind appears to blow from the north west, but when he walks at 3 km/hr due west the wind appears to blow from the north. What is the actual direction of the wind and what is its velocity?

8. A particle is projected with a velocity u at an angle θ with the horizontal. Find the radius of the curvature of the parabola traced out by the particle at the point where velocity makes an angle (θ/2) with the horizontal.

9. A ship A streams due north at 16 km/hr and a ship B due west at 12 km/hr. At a certain instant B is 10 km north-east of A. Find the velocity of A relative to B. Find also the nearest distance of approach of ships.

10. A particle moves in x-y plane with constant acceleration 'a' directed along the negative y-axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the particle at the origin.

11. The position vector of a particle varies with time t as x = kt(1– αt), where k is a constant vector and α is a positive factor. Find
    (a) the velocity v and the acceleration a of the particle as functions of time.
    (b) the time interval Δt taken by the particle to return to the initial points, and the distance covered during that time.

12. A balloon starts rising from the surface of the Earth. The ascent rate is constant and equal to v_o. Due to the wind the balloon gathers the horizontal velocity component v_x = ay, where a is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent.
    (a) The horizontal drift of the balloon x(y);
    (b) The total, tangential, and normal accelerations of the balloon.

13. Two boats A and B move away from buoy anchored at the middle of a river along mutually perpendicular straight lines, the boat A along the river and the boat B across the river. Having moved off an equal distance from the buoy the boat returned. Find the times of motion of boats t_A / t_B if the velocity of each boat with respect to water is n times greater than the stream velocity.

14. A ball starts failing with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

15. A man standing in an elevator observes a screw fall from the ceiling. The ceiling is 3m above the floor.
    (a) If the elevator is moving upward with a speed of 2.2 m/s, how long does it take for the screw to hit the floor.
    (b) How long is the screw in the air if the elevator starts from rest when the screw falls and moves upwards with a constant acceleration of a = 4.0 m/s^2.

Objective:

Level- I

1. A bus starts moving with an acceleration of 2 m/s^2. A cyclist 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus?
(A) 4 sec (B) 8 sec
(C) 12 sec (D) 16 sec

2. A body is projected at an angle of 30° to the horizontal with a speed of 30 m/s. What will be the angle with the horizontal after 1.5 seconds? [Take g = 10 m/s^2]
(A) 0° (B) 30°
(C) 60° (D) 90°

3. A ball rolls off the top of stairway with a horizontal velocity of magnitude 1.8 m/s. The steps are 0.20 m high and 0.2 m wide. Which step will the ball hit first?
(A) First (B) Second
(C) Third (D) Fourth

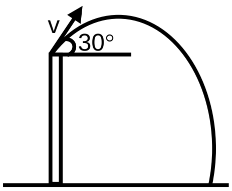
4. In a projectile motion, the velocity is
(A) never perpendicular to the acceleration
(B) always perpendicular to the acceleration
(C) perpendicular to acceleration at one instant only
(D) perpendicular to acceleration at two instants only

5. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of the boat is 10 km/hr, then velocity of the river is
(A) 12.8 km/hr (B) 6 km/hr
(C) 8 km/hr (D) 10 km/hr

6. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is
(A) parallel to position vector (B) perpendicular to position vector
(C) directed towards the origin (D) directed away from the origin

7. A particle starts from rest with constant acceleration. The ratio of space-average velocity to the time-average velocity is
(A) 1/2 (B) 3/4
(C) 4/3 (D) 3/2

8. A ball is projected with a speed of 20 m/s at an angle of 30° from a point on the top of a very high tower. The time after which its velocity becomes perpendicular to the velocity of projection (take g = 10 m/s^2) is
(A) 0.5 sec (B) 2 sec
(C) 4 sec (D) never



9. A particle is moving along a circular path of radius r with uniform speed v. Through what angle does its angular velocity change when it completes half of the circular path?
(A) 0° (B) 45°
(C) 180° (D) 360°

10. Three particles start moving simultaneously from a point on a horizontal smooth plane. First particle moves with speed v_1 towards east, second particle moves towards north with speed v_2 and third one moves towards north east. The velocity of the third particle, so that the three always lie on a straight line, is

(A) $\frac{v_1 + v_2}{2}$ (B) $\sqrt{v_1 v_2}$ s

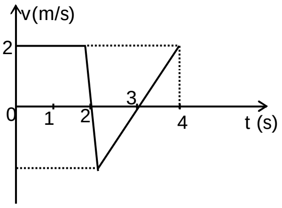
(C) $\frac{v_1 v_2}{v_1 + v_2}$ (D) $\sqrt{2}\frac{v_1 v_2}{v_1 + v_2}$

11. A particle is moving along a circular path of radius 5 m and with uniform speed 5 m/s. What will be the average acceleration when the particle completes half revolution?

(A) zero (B) 10 m/s^2

(C) 10 π m/s^2 (D) $10/\pi$ m/s^2

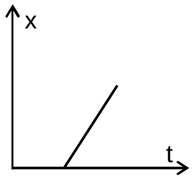
12. The velocity v versus t graph of a body in a straight line is as shown in the adjacent figure. The displacement of the body in 4 sec is

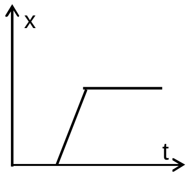


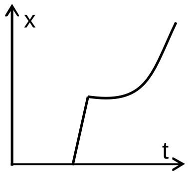
(A) 2 m (B) 4 m

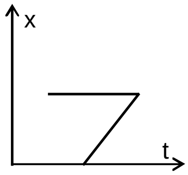
(C) 6 cm (D) 8 m

13. Which of the following displacement–time graph is not possible?

(A)


(B)


(C)


(D)


14. A train of length 100 m travelling at 50 m/s overtakes another train of length 200 m moving at 30 m/s. The time taken by the first train to overtake the second is

(A) 5 sec (B) 10 sec

(C) 15 sec (D) 20 sec

15. A balloon starts from the ground with an acceleration of 1.25 m/s^2. After 8 sec, a stone is released from the balloon. The stone will

(A) cover a distance of 40 m

(B) have a displacement of 50 m

(C) reach the ground in 4 sec

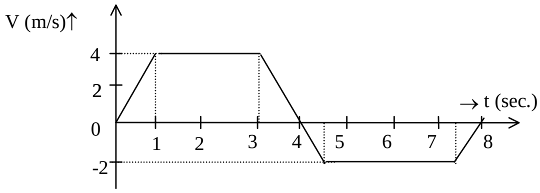
(D) begin to move down after being released.

16. A body moving with a uniform acceleration has velocities of u and v when passing through points A and B in its path. The velocity of the body midway between A and B is

(A) $\frac{u+v}{2}$ (B) $\sqrt{\frac{u^2+v^2}{2}}$

(C) $\sqrt{uv}$ (D) None of these

17. The velocity–time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is



(A) 5 m

(B) 16 m

(C) 8 m

(D) 6 m

18. A ball is thrown up vertically with speed u. At the same instant another ball B is released from rest from a height h. At time t, the speed of A relative to B is

(A) u (B) u – 2gt

(C) $\sqrt{u^2-2gh}$ (D) u – gt

19. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw will be:

(A) h/2 (B) h

(C) 2 h (D) 4 h

20. A motor boat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. Velocity of motor boat with respect to water is $5\sqrt{3}$ m/sec. The driver should steer the boat an angle:

(A) 30° w.r.t. the line of destination from starting point

(B) 60° w.r.t.. normal to the bank

(C) 120° w.r.t. stream direction

(D) None of these

Fill in the Blanks

1. A particle moves in a circle of radius R. In half the period of revolution its displacement is ________________________ and distance covered is ________________________.

2. A particle is projected with an initial velocity of 200 m/s in a direction which makes an angle of 30° with the vertical, the horizontal distance travelled by the particle in 3 sec is _____________ m.

3. A stone is released from an elevator going up with an acceleration a. the acceleration of the stone after the release is __________________________.

4. For angles of projection which exceed or fall short of 45° by equal amounts, the ranges are _____________ .

5. The weight of a body in projectile motion is ________________________.

True or False Type Questions

1. If the displacement y of a particle is proportional to t^2, i.e. if $y \propto t^2$, then its initial velocity will be non-zero.

2. The instantaneous velocity vector is always in the direction of motion.

3. The magnitude of the sum of two displacement vectors must be greater than the magnitude of either displacement vectors.

4. A particle can move with constant velocity and constant acceleration simultanesouly.

5. The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval (infinite acceleration are not allowed).

Level - II

1. A stone is thrown vertically upward with an initial velocity v_0 . The distance travelled in time $4v_0/3g$ is

(A) $\frac{2v_0^2}{g}$ (B) $\frac{v_0^2}{2g}$

(C) $\frac{4v_0^2}{3g}$ (D) $\frac{4v_0^2}{9g}$

2. The motion of a body depends on time according to the equation $\frac{dv}{dt}$ = 6.0 – 3v, where v is speed in m/s and t is time in second. If the body was at rest at t = 0 which of the following statements is correct?
(A) The speed of the body approaches 2 m/s after long time
(B) The speed varies linearly with time
(C) The acceleration remains constant
(D) The initial acceleration is zero

3. If the angle (θ) between velocity vector and the acceleration vector is $90° < \theta < 180°$. The body is moving on a:
(A) Straight path with retardation (B) Straight path with acceleration
(C) Curvilinear path with acceleration (D) Curvilinear path with retardation

4. The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. The retardation is: (if v is velocity of the particle)

(A) $2\ \alpha v^3$ (B) $2\ \beta v^2$
(C) $2\ \alpha\beta v^2$ (D) $2\ \beta^2 v^3$

5. Two particles start moving along the same straight line starting at the same moment from the same point. The first moves with constant velocity u and the second with constant acceleration f. During the time that elapses before second catches the first, the greatest distance between the particles is

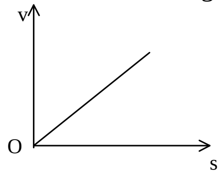
(A) $\frac{u}{f}$ (B) $\frac{u^2}{2f}$

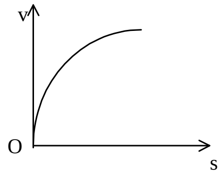
(C) $\frac{f}{2u^2}$ (D) $\frac{u^2}{f}$

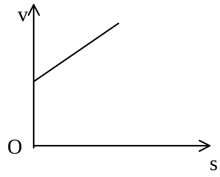
6. A particle is projected horizontally in air at a height of 25 m from the ground with a speed of 10 m/s. The speed of the particle after 2 seconds will be
(A) 10 m/s (B) 22.4 m/s
(C) 25 m/s (D) 28.4 m/s

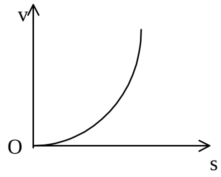
7. A man can swim at a speed of 5 km/h w.r.t. water. He wants to cross a 1.5 km wide river flowing at 3 km/h. He keeps himself always at an angle of 60^o with the flow direction while swimming. The time taken by him to cross the river will be
(A) 0.25 hr. (B) 0.35 hr.
(C) 0.45 hr. (D) 0.55 hr.

8. A body starts from rest and moves along a straight line with constant acceleration. The variation of speed v with distance s is given by graph

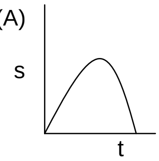
(A)


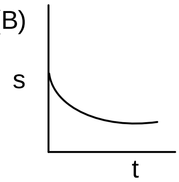
(B)


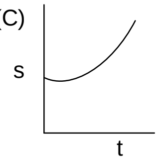
(C)


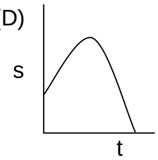
(D)


9. The displacement of a particle in a straight line motion is given by $s = 1 + 10t - 5t^2$. The correct representation of the motion is

(A)


(B)


(C)


(D)


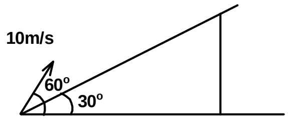
10. The position of a particle along x-axis at time t is given by $x = 1 + t - t^2$. The distance travelled by the particle in first 2 seconds is

(A) 1m (B) 2m
(C) 2.5 m (D) 3m

11. From the top of a tower, two particles A and B are projected simultaneously with speeds of 3 m/s and 4 m/s, respectively, in horizontally opposite directions at time t = 0. At time t = $\left(2\sqrt{3}/10\right)$ sec, the angle between their velocities is

(A) 60° (B) 45°
(C) 90° (D) 30°

12. A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of 60° with the horizontal, from a point on an incline plane, making an angle of 30° with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is

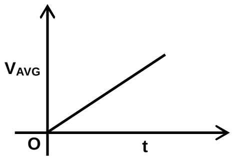


(A) $\frac{2}{\sqrt{3}}$sec (B) $\frac{1}{\sqrt{3}}$sec

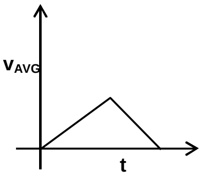
(C) $\sqrt{3}$sec (D) $\frac{1}{2\sqrt{3}}$sec

13. A particle is moving along a straight line with a velocity of $\frac{1}{2}kt^2$, where k is a constant. Then, the average velocity of the particle as a function of time is best represented by

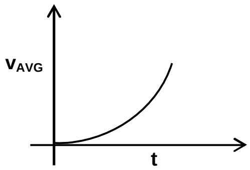
(A)



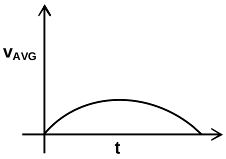
(B)



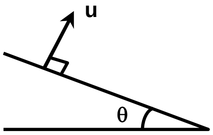
(C)



(D)



14. A particle is projected perpendicularly to an inclined plane as shown in the adjacent figure. If the initial velocity of the particle is u, calculate how far from the point of projection does it hit the plane again if the distance is measured along the plane?



(A) $\frac{2u^2}{g}$ (B) zero
(C) $\frac{2u^2}{g}\sin\theta$ (D) $\frac{2u^2}{g}\tan\theta\sec\theta$

15. A box is moving up on an inclined plane of inclination 30° with a constant acceleration of $5\,m/s^2$. A particle is projected with a velocity of $5\sqrt{3}$ m/s inside a box, at an angle of 30° with the base. Then, the time after which it again strikes the same base of the box is (assume during its flight, particle does not hit any other side of the box)
(A) 1 sec (B) 2 sec
(C) 1.5 sec (D) data insufficient

16. The height y and distance x along the horizontal for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is
(A) 8 m/s (B) 9 m/s
(C) 10 m/s (D) (10/3) m/s

More than one choice are correct:

17. Read and examine the following statements. Which of the following is /are correct/ true?
(A) $a_x \neq 0$, $a_y = 0$, $a_z = 0$ is necessarily a case of one dimensional motion.
(B) $v_x \neq 0$, $v_y = 0$, $v_z = 0$ is necessarily a case of one dimensional motion.
(C) If $v_x \neq 0$, $a_x \neq 0$; $v_y \neq 0$, $a_y \neq 0$; $v_z = 0$, $a_z = 0$ is necessarily a case of motion in one plane.
(D) If $a_x = a_y = a_z = 0$ is necessarily a case of one dimensional motion.

18. The rain is falling vertically downwards. A man walking on the road holds his umberalla tilted. Now, suddenly the rain stops and there is afternoon sun just above the head. In order to protect himself from sun-rays, he holds the umberalla vertical. The reason assigned can be
(A) The speed of light is much higher than that of speed of rain drops.
(B) The speed of light is much higher than that of speed of man.

(C) Actually, the angle of tilt of umberalla in sun-light is very small in comparison to the angle of tilt of umberalla in rain.
(D) Light is behaving as a wave, not as a particle here.

19. A swimmer swims in a flowing river.
$\vec{v}_{s,r}$ = velocity of swimmer with respect to (w.r.t.) river water.
$\vec{v}_{r,g}$ = velocity of river water w.r.t. ground,
$\vec{v}_{s,g}$ =velocity of swimmer w.r.t. ground.
The swimmer intends to reach at the opposite bank of the river. It is possible only when,
(A) $v_{r,g} > v_{s,r}$ (B) $v_{r,g} < v_{s,r}$
(C) $v_{s,g} < v_{s,r}$ (D) none of these.

True or False Type Questions

1. The instantaneous velocity of a body is equal to its average velocity when it is moving with uniform velocity.

2. The average velocity is always equal to the mean value of the initial and final velocities.

3. If the displacement y of a particle is proportional to time, i.e. if $y \propto t$, then the displacement of the particle will be non-zero.

4. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. The two bullets will hit the ground simultaneously.

5. A man while walking observes that the rain is falling vertically downward, if he suddenly stops walking then the rain drops will strike him on his back.

Fill in the Blanks

1. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, its acceleration will be ___________.

2. A boat takes 2 hours to travel 8 km and back in a still water lake with water velocity 4 km/hr. The time taken for going up-stream 8 km and coming back is _______________ minutes.

3. The velocity of a particle moving with constant acceleration at an instant t is 10 m/s. After 5 sec the velocity is 20 m/s. The velocity at 3 sec before was ______________________.

4. A food packet is released from a helicopter which is rising steadily at 2 m/s. After 2 sec the velocity of the packet is ________________ ($g = 9.8\ m/s^2$).

5. A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D meter away, and the acceleration due to gravity is ‘g’, the speed of water as it leaves the tank in terms of g, h and D is ________________.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. $-x = -100t + \frac{1}{2}gt^2$; $100 - x = \frac{1}{2}gt^2$

   $-x = -100\,t + 100 - x \quad \Rightarrow t = 1$ s.

2. Greatest in 3, least in 2; $v > 0$ in 1 and 2, $v < 0$ in 3.

3. The given statement is true. Both the balls have equal acceleration due to gravity. Both the balls would attain the same height and would pass through the point of projection with the same speed.

4. The values of $\sin(90 + 2\theta)$ and $\sin(90 - 2\theta)$ are the same, equal to $\cos 2\theta$. Therefore, range are equal for elevation which exceed or falls short of 45° by equal amount θ.

5. The block will reach the ground earlier which falls freely.

6. $x = 2h$

7. $\theta = 30^\circ$

9. $\frac{2}{\sqrt{3}}$

10. $2.7\ m/s^2$

11. 10 m

12. 8m

13. (a) 0 m/s. (b) 21.0 m/s. (c) 14.0 m/s.

14. 5 m/s

15. 160 km

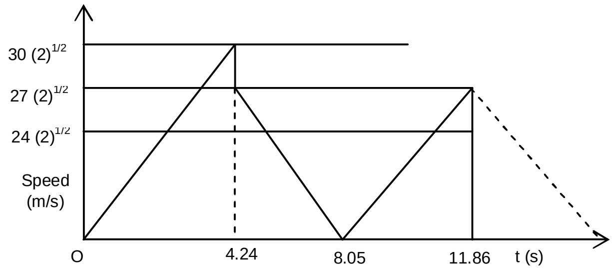
16. $\frac{\alpha\beta t}{\alpha+\beta}$

17. 1200 m

18. 54 m/s

19. 60 km/hr.

20.



Level - I

1. (a) 28.5 cm/s (b) 18.0 cm/s (c) 28.1cm/s (d) 30.4 cm/s
2. (a) $5.71 m/s^2$ (b) 3.68s (c) 5.78s (d) 95.4m
3. 0.1m/s
4. 10.42 m
5. 50 m
6. (a) 10.20s; (b)883.67 m; (c) 100 m/s
7. (a) 21.8 m/s, (b) 13.55 m/s
8. 47.5 m
9. (a) 94cm (b) 19m/s (c) 2400 m/s^2.
10. [2.832 m/s at an angle of 32^o with $\overrightarrow{V_B}$
11. (a) 3.41 seconds (b) 58.14 m(c) 34.10 m/s
12. 1749.8 m
13. $\left(5\hat{i}-5\hat{j}\right)$ m/s
14. $\dfrac{v_0}{\sqrt{1+2kv_0^2 t}}$
15. (i) 13.5 m, (ii) 3.06 sec.

Level – II

2. $\dfrac{2v_o\left(v_1+v_2\right)}{2v_o+v_1+v_2}$
3. (A) 82 m; (B) 19 m/s
4. 55m
5. 2.5 m
6. $CD = \dfrac{\ell}{\sqrt{n^2-1}}$
7. 5 km/hr, 53^o North of East.
8. $\dfrac{u^2\cos^2\theta}{g\ \cos^3\dfrac{\theta}{2}}$
9. 20 km/hr at an angle $\tan^{-1}\dfrac{3}{4}$ or 37^o east of north, $\sqrt{2}$ km
10. $\sqrt{\dfrac{a\ \left(p^2+1\right)}{2q}}$
11. (a) $v = k(1-2\alpha t)$, $a = -2\alpha k$ (b) $\Delta t = 1/\alpha$, $s = k/2\alpha$
12. (a) $x = \left(\dfrac{a}{2v_o}\right)y^2$ (b) $\omega = av_o$, $\omega_t = \dfrac{a^2v_0y}{\sqrt{v_0^2+a^2y^2}}$, $\omega_n = \dfrac{av_0^2}{\sqrt{v_0^2+a^2y^2}}$
13. $\left[\dfrac{n}{\sqrt{n^2-1}}\right]$
14. $\ell = 8\ h \sin\alpha$
15. (a) 0.78 sec. (b) 0.66 sec.

Objective:

Level – I

1.	B	2.	A	3.	D
4.	C	5.	B	6.	B
7.	C	8.	C	9.	A
10.	D	11.	D	12.	B
13.	D	14.	C	15.	C
16.	B	17.	A	18.	A
19.	C	20.	A		

Fill In The Blanks

1.	2R and πR	2.	300 m
3.	downward	4.	equal
5.	zero		

True or False Type Questions

1.	False	2.	True
3.	False	4.	False
5.	False		

Level – II

1.	D	2.	A	3.	D
4.	A	5.	B	6.	B
7.	B	8.	D	9.	D
10.	C	11.	C	12.	B
13.	C	14.	D	15.	A
16.	C	17.	C, D	18.	B, C
19.	B, C				

True or False Type Questions

1.	True	2.	False
3.	True	4.	True
5.	True		

Fill In the Blanks

1.	-8 m/s^2	2.	160
3.	4 m/s	4.	-17.6 m/s
5.	$D\sqrt{\frac{g}{2h}}$		

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

LAWS OF MOTION

Laws of Motion

Syllabus:

Force and inertia, Newton's first law of motion; Momentum, Newton's second law of motion, Impulse; Newton's third law of motion; Law of conservation of linear momentum and its applications; Equilibrium of concurrent forces; Static and kinetic friction, laws of friction, rolling friction, lubrication; Examples of variable mass situation. Dynamics of uniform circular motion; Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road); Inertial and non- inertial frame (elementary idea).

Introduction:

In the introduction of the preceding chapter "Kinematics" we studied that mechanics can be broadly classified into two categories namely *"Kinematics and Dynamics"*.

In kinematics our prime concern was to define the physical quantities like position, velocity, acceleration and to establish the relations among them. But we never tried to answer the questions like "What causes the bodies to move from one place to another? What makes the body to gain or loose its speed? . . . etc.

Dynamics is that branch of mechanics which gives not only qualitative but quantitative description of the above quantities. This branch of mechanics also explains few basic laws which governs the motion of bodies.

Before you start studying this chapter try to analyse the answers for the following questions:

(i) Why to set a ball into motion in playground someone has to kick it or throw it?

(ii) Why a navigator has to row the boat in still water to move the boat ?

(iii) Why the branches of a tree swing when wind blows ?

In all these cases one can observe that some external agency (like hand or legs of player or wind) is coming into contact with the respective objects that are going into motion.

(iv) When a ball is released from the top of a building, the ball falls by itself, though no one is pushing it in downward direction. Why ?

(v) When a piece of iron is placed near a magnet, magnet attracts it from certain distance itself. How ?

In these cases we can observe that even though no external agency is physically coming into contact with the objects still the objects are moving.

Hence we can conclude that an animate or inanimate external agency is required to change the state of a body (i.e. from rest to motion or vice versa). To understand the logic behind this type of questions more clearly, we should know about two basic physical quantities namely *inertia and force*.

Inertia: It is a very common observation for all of us that any book kept on our study table will not move by itself, i.e. until and unless it is acted upon by any external force, it will not change its state of rest.

To explain the reason behind this type of questions Italian scientist Galileo has defined a new physical quantity known as *inertia*. Though it was introduced by Galileo the effective use of this term and its usage for explaining the motion of the bodies was done by another reputed physicist, Sir Isaac Newton.
"Inertia is an inherent property of a body by virtue of which it cannot change its state (i.e. rest or motion) by itself."

It says that every body in the universe does have a property which is hidden in itself and because of this property the body is unable to change its state by itself, i.e. from state of rest to state of motion or vice versa or even it's direction. This inertia is of 3 types, namely
(a) inertia of rest (b) inertia of motion and (c) inertia of direction

Inertia of rest: Inertia of rest is the inability of a body by virtue of which it can't change its state of rest to state of motion. That means any body which is at rest continues to be in the state of rest only and it can't go further into state of motion by itself.

Examples:
(i) *Passengers standing or sitting loosely in a bus experience jerk in the backward direction when the bus suddenly starts moving.* This is due to the fact that when the bus suddenly starts its motion, the lower parts of the human body shares the motion but the upper part tends to remain at rest due to inertia of rest.

(ii) *When a bullet is fired into a tightly- fitted glass pane from a reasonably close range, it makes a clear circular hole in the glass pane.* This is due to the fact that particles of glass around the hole tend to remain at rest due to inertia of rest. So they are unable to share the fast motion of the bullet.

Inertia of motion: Inertia of motion is the inability of a body by virtue of which it can not change its state of uniform motion along a straight line to state of rest. That means any body which is in uniform motion can't come to rest by itself until and unless some external force acts on it.

Examples:
(i) *A passenger standing in a moving bus falls forward when the bus stops suddenly*. This is due to fact that the lower part of the body comes to rest along with the bus but the upper part of the body remains in a state of motion on account of "inertia of motion".

(ii) *An athlete runs for some distance before taking a long jump.* In this way, the athlete gains momentum and this inertia of motion helps him in taking longer jump.

Inertia of direction: Inertia of direction is inability of a body by virtue of which it can't change its direction by itself. This means a body moving along a straight line can't change its direction by itself, until and unless it is acted upon by any external force.

Examples:
(i) *When a running car suddenly takes a turn, the passengers experience a jerk in the outward direction.* This is because the passengers tend to maintain their original direction of motion due to inertia of direction.

(ii) *A stone tied to one end of a string is whirled in a horizontal circle. When string breaks, the stone tends to fly off tangentially along a straight line.* This is due to inertia of direction.

Note: The mass of the body is the indirect measure of the inertia of that body.

Exercise 1.
(i) ***Why does the sparks coming out tangentially from the grid store when knife or any other such objects are sharpened ?***
(ii) ***A clothes line hangs between two poles. No matter how tightly the line is stretched, it always sags a little at the centre. Explain.***

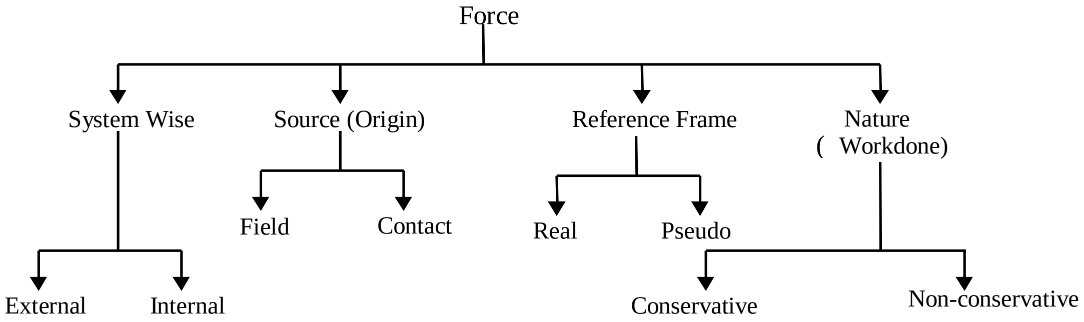
Now let us try to understand another physical quantity "*force*", with the help of which only the mechanical state of a body changes.

"Force is that which pushes or pulls the body or tends to change the state of rest or of uniform motion in a straight line."

(a) It produces or tries to produce motion in a body at rest.
(b) It stops or tries to stop a moving body.
(c) It changes or tries to change the direction of motion of body.
(d) It produces a change in the shape of the body.

CLASSIFICATION OF FORCES

There are different types of forces in our universe. Based on the nature of the interaction between two bodies, forces may be broadly classified as under



Since we are going to encounter these forces in our analysis we will briefly discuss each force and how it acts between two bodies, its nature etc and how we are going to take it into account.

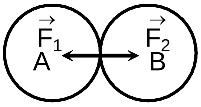
(a) Contact force: The force exerted by one surface over the surface of another body when they are physically in contact with each other is known as *contact force.*

If two surfaces that are coming into contact are perfectly smooth, then the entire contact force will act only perpendicularly (normal) to their surface of contact and it is known as "*Normal force or Normal reaction.*"

If two surfaces that are coming into contact are rough surfaces, then one component of this contact force acts perpendicular to their surface of contact and the other component of this force acts in tangential direction to their surface of contact and this component is known as "*force of friction*."

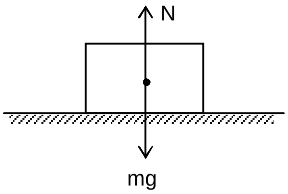
Normal Reaction, Tension, Friction, etc. are the examples of various contact forces.

Normal reaction: The forces $\vec{F}_1$, $\vec{F}_2$ shown in the diagram acting on A and B respectively act away from the surface of contact, and prevent the two bodies from " occupying the same space".

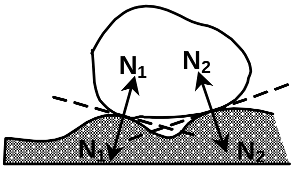


If $\vec{F}_1$ is the action, $\vec{F}_2$ is reaction: they are equal in magnitude but opposite in direction. Further, $\vec{F}_1$ and $\vec{F}_2$ are both perpendicular to the surfaces in contact and note that they act on two different bodies.

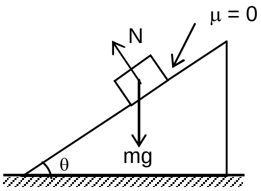
Examples:



N = mg

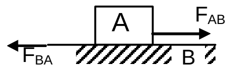


N is the normal reaction of the ground on the stone.



$N = mg \cos\theta$

Friction: It is a force that acts between bodies in contact with each other along the surface of contact and it opposes relative motion between the two bodies. The direction of frictional force on A is opposite to that of direction of frictional force on surface B and magnitude is same for both.

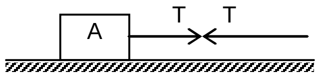


Tension (T): When a string, thread or wire is held taut, the ends of the string or thread (or wire) pull on whatever bodies are attached to them in the direction of the string. This force is known as Tension.

If the string is massless then the tension T has the same magnitude at all points throughout the string.

Examples:

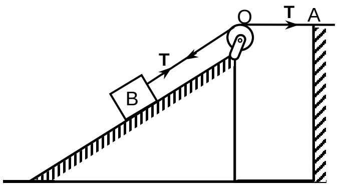
(i) Tension in a string: For a block A pulled by a string,



(ii) The direction of tension is always away from the point of attachment to the body. In the given figure two segments of tension act at O towards A and B.

For the wedge, there are two segments of thread at the point of attachment O to the body. Hence, two tensions act on the wedge; one along OB and the other along OA.

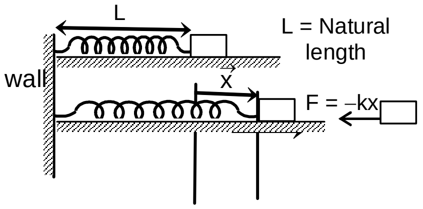
Hence a tension acts away from the point of attachment along BO.



Spring forces: Whenever a spring is compressed or extended, the elastic force developed in the spring which helps the spring to restore its original position is known as spring force.

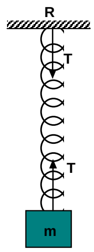
In an extended (or compressed) spring, force is proportional to the magnitude of extension (or compression).

$F \propto x$, in magnitude, but opposite in direction. So, $F = -kx$, where k is a positive constant, also known as the spring constant of the spring.
x is the compression or elongation from the natural length.



Example:

In case of the spring the tensions are oppositely directed on the block 'm' and on the roof R.



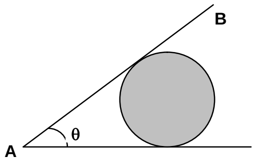
(b) Non-contact force: Bodies can exert forces on each other without actual physical contact. This is known as action at a distance. Such forces are known as *non-contact forces* (or) field forces, e.g. gravitational force, electrostatic forces, etc.

For the moment, we deal with actual forces. Suffice it to say that there exist pseudo-forces acting in a non-inertial frame of reference, which we will discuss later.

Forces may be conservative or non-conservative depending on whether work done against them by an external agent is recoverable or otherwise. This will be discussed in later chapters.

Exercise 2. ***(i). Is it necessary to have normal reaction whenever two surfaces are in contact with each other?***

(ii). Find the contact forces acting on a stationary sphere weighing 10 N is placed in a fixed frame BAC as shown in the diagram? (Assume θ is constant)



Now having understood about inertia and force, we can easily understand the three basic laws of motion which are known as Newton's laws of motion.

Newton's first law

Every body continues to be in the state of rest or of uniform motion in a straight line until and unless it is compelled to change the state of the body by an unbalanced force.

For better understanding we can divide this statement into two parts.

(i) "Every body continues to be in its state of rest until and unless some external force compels it to change the state of rest."

This part of the law is self explanatory and self evident as we come across several examples in our daily life like all inanimate objects will continue to be in the same place where they are put until they are disturbed by some external agents.

(ii) "Every body continues in its state of uniform motion in a straight line unless external force compels it to change that state."

The second part of the statement can't be readily understood as on the surface of the earth because of various types of frictional or resisting forces. For example when a ball is rolled on a horizontal surface the ball will come to halt after some time however smooth the surface may be, as we can't eliminate force of friction completely.

Momentum (Linear):

Till now we studied about inertia (translational) which is the inability of a body. Now we will study about another physical quantity called 'momentum' which is the ability of body.

Momentum is defined as the ability of a body by virtue of which it imparts or tends to impart its motion along a straight line.

Mathematically, momentum (p) is measured as the product of mass (m) and velocity (v) of the body. As velocity is a vector quantity, momentum is also a vector quantity.

$\vec{p} = m\vec{v}$ or $p = mv$

Unit : Its unit is kg-m/s in SI system and gm-cm/s in CGS system.
Dimensions : Its dimensions are MLT^{-1}

Exercise 3.
(i) If a body is at rest, can we say that no force is acting on the body?
(ii) A car and a lorry are travelling with same velocity on a straight horizontal road. Which of the two has got greater momentum?

Illustration 1. *A block of mass 2 kg is moving with a velocity of* $2\hat{i} - \hat{j} + 3\hat{k}$ *m/s. Find the magnitude and direction of momentum of the block with the x-axis.*

Solution: The magnitude of momentum is $2\sqrt{14}$ kg m/s and the direction is at an angle of $\tan^{-1}\sqrt{\frac{2}{7}}$ with x-axis.

Illustration 2. *A block rests on air inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane, down the plane or sideways? Why?*

Solution: It is easier to push it down the plane because we have to apply the force only to overcome from the rest part of friction force.

Illustration 3. *When a force of constant magnitude always act perpendicular to the motion of a particle then :*

(A) Velocity is constant *(B) Acceleration is constant*
(C) KE is constant *(D) None of the above*

Solution : Force will provide centripetal acceleration, so it will move in a circular path therefore KE is constant because speed remains unchanged.
Option (C) is correct.

Newton's second law of motion

We have already studied the Newton's 1st law which has given us a qualitative idea about force. Now, we will study about Newton's IInd law which gives us a quantitative idea about force.

Whenever a cricketer catches a ball he allows a longer time for his hands to stop the ball. Otherwise the ball will hurt the cricketer. If you observe this incident carefully you can easily understand that cricketer is applying some force on ball in order to make the momentum of the body zero. And also we can understand that the magnitude of the retarding force that cricketer applies on the ball in order to stop depends on two factors.
(1) The momentum of the ball and
(2) Time for which he is applying the force

These type of observations lead Newton to state his second law of motion.

The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of the force.

So for a body with constant mass,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \qquad \text{or,} \qquad \frac{d}{dt}(m\vec{v}) \propto \vec{F}$$

$$\text{or,} \qquad m\frac{d\vec{v}}{dt} \propto \vec{F} \qquad ; \qquad \vec{F} = km\left(\frac{d\vec{v}}{dt}\right),$$

where k is a constant. With proper choice of units, k = 1. Thus,

$$\mathbf{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}} \ .$$

Unit of Force : Its unit is newton in SI system and dyne in CGS system.
Dimensions : $[MLT^{-2}]$

Note (1): The second law of motion is a vector law. These are actually three equations, one for each component of the vectors:

$$F_x = \frac{dp_x}{dt} = ma_x \; ; \; F_y = \frac{dp_y}{dt} = ma_y \; ; \quad F_z = \frac{dp_z}{dt} = ma_z$$

Note (2): The second law of motion is strictly applicable to a single point particle. The force F here stands for net external force on the particle.

Exercise 4. ***A body is acted upon by a number of external forces. Can it remain at rest ?***

Illustration 4. *A body of mass m = 1 kg falls from a height h = 20 m from the ground level*
(a) What is the magnitude of total change in momentum of the body before it strikes the ground?
(b) What is the corresponding average force experienced by it? (g = 10m/sec^2).

Solution: (a) Since the body falls from rest (u = 0) through a distance h before striking the ground, the speed v of the body is given by kinematics equation.
$v^2 = u^2 + 2\,as$; Putting a = g and s = h
we obtain $v = \sqrt{2gh}$
$\Rightarrow$ The magnitude of total change in momentum of the body
$= \Delta p = |\, mv - 0\,| = mv$, Where $v = \sqrt{2gh}$
$\Rightarrow \Delta p = m\sqrt{2gh} \;\; \Rightarrow \Delta p = (1)\sqrt{(2 \times 10 \times 20)}$ kg m/sec
$\Rightarrow \Delta p = 20$ kg m/sec.
(b) The average force experienced by the body $= F_{av} = \dfrac{\Delta p}{\Delta t}$
where Δt = time of motion of the body = t(say). We know Δp = 20 kg m/sec. Therefore we will have to find t using the given data. We know from kinematics that,

$$S = ut + \frac{1}{2}at^2 \Rightarrow h = \frac{1}{2}g\,t^2 \qquad (u = 0)$$

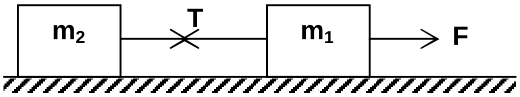
$$\Rightarrow t = \sqrt{\frac{2h}{g}} \qquad \therefore \; F_{av} = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{t}$$

Putting the general values of ΔP and t we obtain

$$F_{av} = \frac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg \qquad \Rightarrow \vec{F}_{av} = m\vec{g}\,.$$

Where mg is the weight (W) of the body and $\vec{g}$ is directed vertically downward. Therefore the body experiences a constant vertically downward force of magnitude mg.

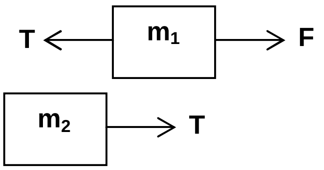
Illustration 5. *Two masses connected with a light string are placed on a horizontal frictionless surface. One mass is pulled by a constant force of F directed along the string. The acceleration of mass* m_1 *is*



(A) $\dfrac{F}{m_1}$ *(B)* $\dfrac{F}{m_2}$

(C) $\dfrac{F}{m_1 + m_2}$ *(D) zero*

Solution:



$$F - T = m_1 a$$
$$T = m_2 a$$
$$F = (m_1 + m_2)a$$
$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

So option (C) is correct.

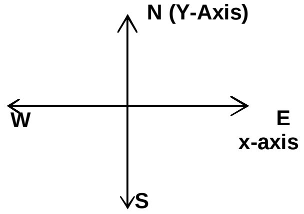
Illustration 6. *A body of mass 2 kg is moved towards east with a uniform speed of 2 m/s. A force of 3 N is applied to it towards north. Calculate the magnitude of the displacement of the body 2S after the application.*

Solution : According to the given reference frame:

$u_x = 2\,m/s$ and $a_x = 0$

$u_y = 0$ and $a_y = \frac{F_y}{m} = \frac{3}{2}$

$= 1.5\,m/s^2$



After 2 seconds, the displacement of the body along x-axis

$$S_x = v_x t = 4m$$

Along y-axis

$$S_y = \frac{1}{2} a_y t^2 = \frac{1}{2} x 1.5 x 4 = 3m$$

So magnitude of the displacement $= \sqrt{S_x^2 - S_y^2} = 5\,m$

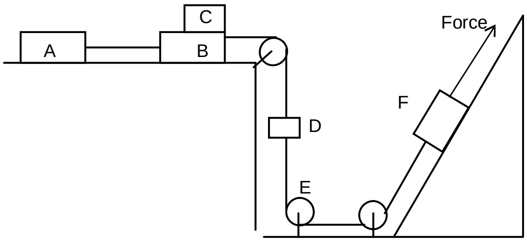
Working with Newton's first and second law

Before trying to write an equation from Newton's law, we should very clearly understand which particle we are considering. In any particle situation, we deal with extended bodies which are collection of a large number of particles. The laws stated a force ma be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing equations from Newton's law is as follow:

1st step : Decide the system

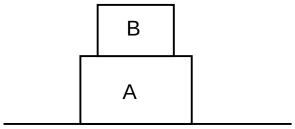
The first step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

Here the distance covered by all the blocks is same but they all can not be taken as a system because even though magnitude of acceleration is same but the direction of acceleration in all the blocks is not same.



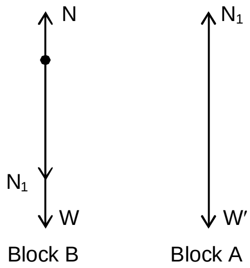
2nd step : Identify the forces
Once the system is decided, make a list of forces acting on the system due to all the objects other than system. Any force applied by the system should not be included in the list of the forces.

Consider the situation shown in the figure where small block of mass m is kept on bigger block of mass M. The load presses lower block, the lower pushes the upper block, the bigger block presses the floor downward, the floor pushes the block upward, the earth attracts the block.



3rd step: Make a free body diagram
Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin.

The forces may lie along a line, may be distributed in a plane (coplanar) or may be distributed in the space (non-planer). Indicate the magnitude and directions of the forces in the diagram. This is called a free body diagram.



4th step: Choose axes and write equations:
Any three mutually perpendicular directions may be chosen as the x-y-z axes. Some suggestion for choosing the axes to solve problems are,

(a) If the forces are coplaner, only two axes, say x and y, taken in the plane of forces are needed.
(b) Choose the x-axis along the direction in which the system is known to have or is likely to have the acceleration.
(c) If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes.
(d) Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration. Write the components of the forces along the y-axis and equate the sum to zero.

Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

Impulse: A large force acting for a short time to produce a finite change in momentum is called *impulse* and the force acted is called *impulsive force* or force of impulse.

Mathematically it is described as the product of force and time.

$\therefore$ Impulse (J) = F . t

$\therefore$ Impulse (J) = mv – mu and since force is variable, hence $J = \int_{t_1}^{t_2} F dt$

The area under F – t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$.

Unit of Impulse **:** The unit in S.I. system is kgm/sec or newton -second.
Dimension **:** $M^1L^1T^{-1}$

Examples:
(i) *Automobiles are provided with spring shocker systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, there by reducing the impulse of force. This minimises the damage to the vehicle.

(ii) *A man falling from a certain height receives more injuries when he falls on a marble floor than when he falls on a heap of sand.* This is because the marble floor does not yield under the weight of the man. The

man is stopped abruptly. A large change of momentum takes place in a very short interval of time. But when he falls on a heap of sand, the sand yields under the weight of the man and this increases the time interval. So it reduces the force exerted by sand on man.
(iii) *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is because cricket ball will have more momentum than tennis ball due to its heavier mass. The change in momentum in case of cricket ball is more. Hence more force is required to stop cricket ball than tennis ball.

Exercise 5. ***When a swimmer dives into water just before piercing himself into water, he stretches himself. Why ?***

Illustration 7. *A cricket ball of mass 200 gm moving with velocity 15 m/s is brought to rest by a player in 0.05 sec. What is the impulse of the ball and average force exerted by player ?*

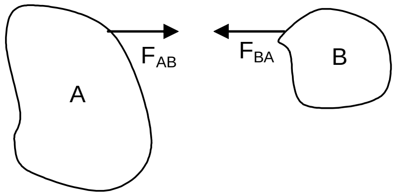
Solution: Impulse = change in momentum = m(v – u) = 0.2 (0 – 15) = – 3 Ns

Average force = Impulse / Time $= \dfrac{3}{0.05} = 60$ N

Newton's third law of motion

Now we have understood the qualitative and quantitative definitions of force from Newton's first and second laws. But how are the forces between two bodies related to each other if at all ? The answer is provided by the third law of motion.

Every action has an equal and opposite reaction, *which are equal in magnitude and opposite in direction.*



Consider two bodies A and B interacting with each other, by means of forces

$\vec{F}_{AB}$: the force exerted by body B on A

$\vec{F}_{BA}$: The force exerted by the body A on B.

According ot Newton's 3rd law : $\vec{F}_{AB} = -\vec{F}_{BA}$ (equal in magnitude & opposite in direction)

That may look fine, but it, apparently, raises a lot of questions. For example, if a horse pulls a cart and cart pulls the horse backward, how does the cart moves forward at all ?

If we observe we will find that the forces acting on the horse and the cart, though equal and opposite, they are acting not on the same body, rather, two bodies. It cannot produce equilibrium neither in horse nor in cart.

Exercise 6.

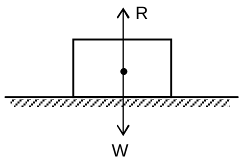
i) ***If action and reaction are equal and opposite to each other then how can a man move a box on a floor?***

ii) ***In a tug of war if the parties on both ends of the rope apply equal and opposite forces on each other then how can one party win?***

(iii) ***Suppose you are standing on a boat in the middle of a swimming pool. The boat is loaded with a bag of stones. Can you get to the shore using the stones? If yes, explain the concept behind it.***

Examples:

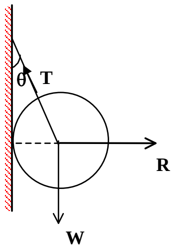
(i) Consider a body of weight W resting on a horizontal surface. The body exerts a force (action) equal to weight W on the surface. The surface exerts a reaction R on the body in the upward direction such that W = R or in vector notation, $\vec{W} = -\vec{R}$



(ii) In a lawn sprinkler, when water comes out of the curved nozzles, a backward force is experienced by the sprinkler. Consequently, the sprinkler starts rotating and sprinkles water in all directions.

(iii) In order to swim, a man pushes the water backwards with his hands. As a result of the reaction offered by water to the man, the man is pushed forward.

Illustration 8. *A metal of sphere is hung by by a string fixed to a wall. The force acting on the sphere are shown in figure. Which of the following statement is correct*



(A) $\vec{F}+\vec{T}+\vec{W} = 0$ *(B)* $T^2 = R^2 + W^2$

(C) $T = R + W$ *(D)* $R = W \tan\theta$

Solution : By the free body diagram

$T\cos\theta = w$

$T\sin\theta = R$

$T^2 = W^2 + R^2$

Option (B) is correct.

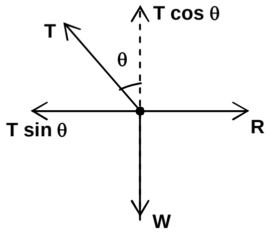
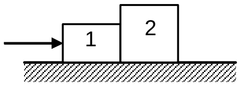


Illustration 9. *Two blocks of mass 2 kg and 4 kg are kept in contact with each other on a smooth horizontal surface. A horizontal force of 12 N is applied on the first block due to which they move with certain constant acceleration. Calculate the force between the blocks.*



Solution : Let a = common acceleration of blocks,

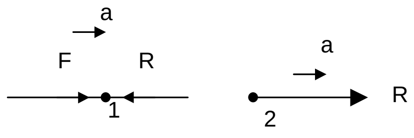
R = Force between the two blocks

From FBD of Ist block

$F - R = m_1 a$ . .. . (i)

From FBD of block 2,

$R = m_2 a$ . . . .(ii)



F.B.D.

$\therefore$ From (i) & (ii), $F = (m_1 + m_2).\, a$

$$\therefore\ a = \frac{F}{m_1 + m_2} = \frac{12}{6} = 2 \text{ m/sec}^2$$

$\therefore$ Force between blocks $= R = m_2 a = (4 \times 2)$ N = 8 N.

Illustration 10. *A body of mass 5 kg is supported by a light cord. Find the tension in the cord.*

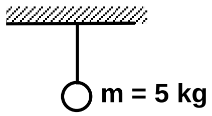
Solution:

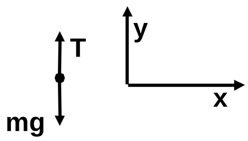
(i) The body is isolated.

(ii) The two forces acting on it are :
Tension (T) in the cord (upwards)
Weight (mg) (downwards)

(iii) Equations of equilibrium :
$\Sigma F_x = 0$
$\Sigma F_y = 0 \Rightarrow T - mg = 0$
or $T = (5)(10) = 50$ N

(iv) Tension in the cord equals the weight of the body.





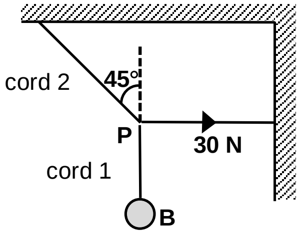
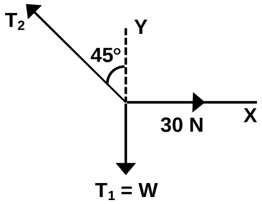


Illustration 11. *In the shown figure the tension in the horizontal cord is 30 N. Find the weight of the body B.*



Solution:
(i) Isolate the point P
(ii) The forces acting on it are :
Unknown tension T_2 in cord 2
Unknown tension T_1 in cord1.
Known tension of 30N in the horizontal cord.
(iii) T_2 is resolved along x and y axes.
(iv) Condition of equilibrium:

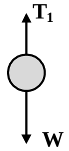
$$\Sigma F_x = 0 \Rightarrow 30 - T_2 \sin 45° = 0 \qquad (1)$$

and $\Sigma F_y = 0 \Rightarrow T_2 \cos 45° - T_1 = 0 \qquad (2)$

Since body B is also in equilibrium,
Hence, $T_1 = W \qquad (3)$

(v) After solving these equations,
we get W = 30 N

For body *B*



FRAME OF REFERENCE

It is a conveniently chosen co-ordinate system, which describes the position and motion of a body in space.

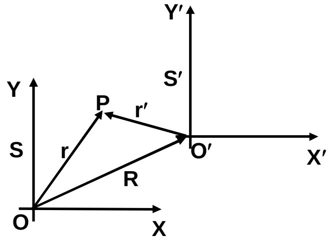
Inertial frame of reference

A reference frame which is either at rest or in uniform motion along the straight line. Newton's laws are strictly valid only for inertial frame.

Non-inertial frame of reference

A reference frame which accelerates or rotates with respect to an inertial reference frame.

Motion of a particle (P) is studied from two frames of references S and S′. S is an inertial frame of reference and S′ is a non inertial frame of reference. At any time, position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}\,'$ respectively. At the same moment position vector of the origin of S′ is $\vec{R}$ with respect to S as shown in the figure.



From the vector triangle OO′P , we get

$$\vec{r}\,' = \vec{r} - \vec{R}$$

Differentiating this equation twice with respect to time we get

$$\frac{d^2\vec{r}\,'}{dt^2} = \frac{d^2(\vec{r})}{dt^2} - \frac{d^2}{dt^2}(\vec{R})$$

$$\Rightarrow \vec{a}\,' = \vec{a} - \vec{A}$$

Here $\vec{a}\,'$ = acceleration of the particle P relative to S′
$\vec{a}$ = Acceleration of the particle relative to S
$\vec{A}$ = Acceleration of S′ relative to S.

Multiplying the above equation by m (mass of the particle) we get

$$m\vec{a}' = m\vec{a} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} + \left(-m\vec{A}\right).$$

In non-inertial frame of reference an extra force is taken into account in order to apply Newton's laws of motion. The magnitude of this force is equal to the product of the mass of the body and acceleration of the frame and it is always directed opposite to the acceleration of the frame. This force is known as ***Pseudo force***, because this force does not exist in the inertial frame of reference.

Exercise 7. *Apparent weight of a person, standing in an elevator which is going up or down with certain acceleration, changes. Why?*

Illustration 12. *A block slides down an inclined plane of slope angle θ with constant velocity. If it is then projected up the same plane with an initial speed v_0, the distance in which it will come to rest is*

(A) $\dfrac{v_0^2}{g\tan\theta}$ *(B)* $\dfrac{v_0^2}{4g\sin\theta}$

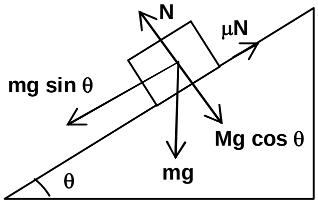
(C) $\dfrac{v_0^2}{2g}$ *(D)* $\dfrac{v_0^2}{2g\sin\theta}$

Solution : From the free body diagram

If the body move with constant velocity $mg\sin\theta = \mu N$

$N - mg\cos\theta = 0$

So, $\mu = \tan\theta$



Now in case (ii) if it is projected with velocity v_0 upward then net retardation against it is

$$a = \frac{mg\sin\theta + \mu mg\cos\theta}{m} = g\sin\theta + \mu g\cos\theta$$

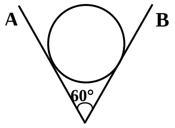
$$= g\sin\theta + g\sin\theta$$

$$= 2g\sin\theta$$

So distance travelled = $= \dfrac{{v_0}^2}{2a} = \dfrac{{v_0}^2}{4g\sin\theta}$

So option (B) is correct.

Illustration 13. *An iron sphere weights 10 N and rests in a V shaped trough whose sides form a angle 60°. The net force exerted by the walls on the sphere in case as shown in figure is*



(A) 0 N *(B) 10 N*

(C) $\dfrac{10}{\sqrt{3}}$ *N* *(D) 5 N*

Solution : By Symmetry $N_1 = N_2$

Therefore

$N_1 \sin 30^0 + N_2 \sin 30^0 = mg$

And $\frac{N_1 + N_2}{2} = mg$

So $N = mg = 10\,N$

So option (B) is correct.

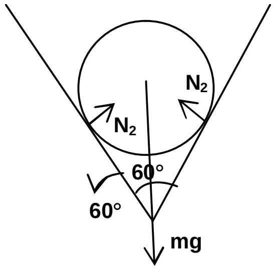


Illustration14. *A block of mass m is placed on an inclined plane. With what acceleration should the system move towards right on a horizontal surface so that m does not slide on the surface of inclined plane? Assume all surfaces are smooth.*

Solution : From ground frame of reference the forces acting on the block m are:

(i) its weight mg and

(ii) normal reaction R and its acceleration is rightward. If we analyse the motion of m relative to the inclined plane, its acceleration is zero and the forces acting are its weight, the normal reaction and a pseudo force of magnitude ma towards left.

$R\cos\theta = mg \;\therefore\; a = g\tan\theta$

$R\sin\theta = ma$

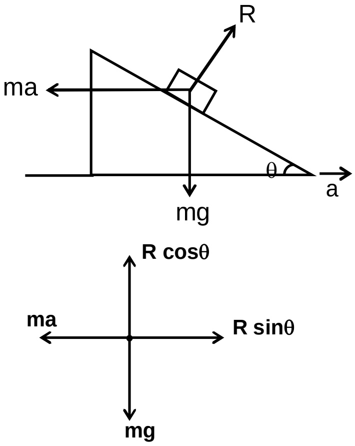
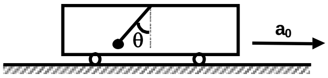


Illustration 15. *A pendulum of mass m is hanging from the ceiling of a car having an acceleration a_o with respect to the road in the direction shown. Find the angle made by the string with the vertical.*



Solution: Since bob of the pendulum is stationary relative to car

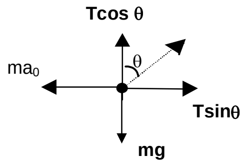
Hence

$T \sin\theta = ma_o$ (pseudo force) . . .(i)

$T \cos\theta = mg$ . . . (ii)

Dividing (i) by (ii), we get

$\tan\theta = \frac{a_o}{g} \Rightarrow \theta = \tan^{-1}\frac{a_o}{g}$



F.B.D. of pendulum relative to car

CONSTRAINT RELATIONS

The equations showing the relation of the motions of a system of bodies, in which motion of one body is constrained by motion of other bodies, are called the constraint relations.

Applying Newton's Laws alone is not sufficient in some cases where the number of equations is less than the number of unknowns.

In the given diagram for finding the acceleration of the masses there are three unknowns, tensions T, acceleration a_1 and a_2 of masses m_1 and m_2. However we will get only two equations. Clearly Newton's laws are not sufficient to solve the problem and constraint relations provide additional equations. When the motions of bodies in a system is constrained because of pulleys, strings, wedges or other factors, we use geometry to develop additional equations.

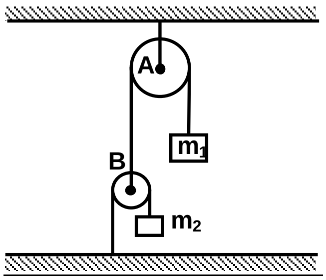
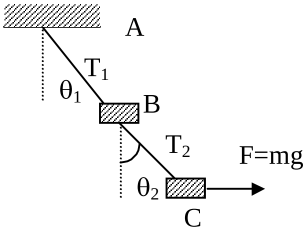


Illustration 16. *The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.*

(A) tan $\theta_1 = 1$ *(B) tan $\theta_2 = 1/2$*
(C) $T_1 = \sqrt{5}$ mg *(D) $T_2 = \sqrt{5}$ mg*



Solution : From F.B.D.

$T_2 \cos\theta_2 = mg$

$T_2 \sin\theta_2 = mg$

$\Rightarrow T_2 = \sqrt{2}\,mg$

and $\theta_2 = 45^\circ$

Also $T_1 \sin\theta_1 = T_2 \sin\theta_2$

and $T_2 \cos\theta_2 + mg = T_1 \cos\theta_1$

$T_1 \cos\theta_1 = mg + mg = 2mg$

and $T_1 \sin\theta_1 = mg$

Therefore $T_1 = \sqrt{5}\,mg$

So option (C) is correct.

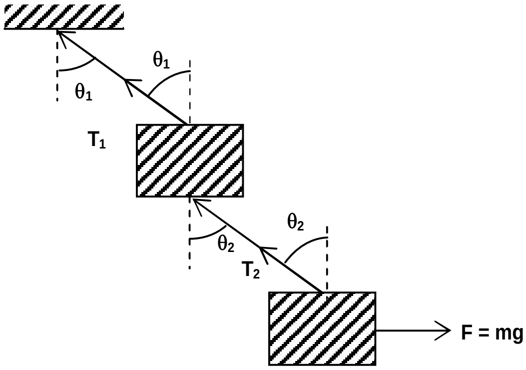


Illustration 17: *A rod is sliding along the wall as shown in figure. Find the ratio of velocity of its ends at the given instant.*

Solution : From the figure
The length of the rod should be constant, so

$v_A \cos\alpha = v_B \sin\alpha$

$\frac{v_A}{v} = \tan\alpha$

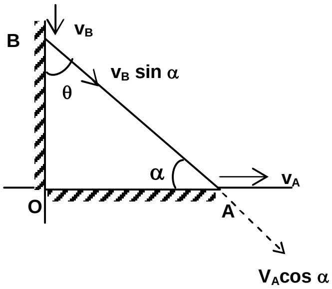
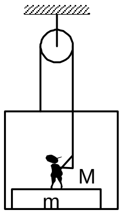
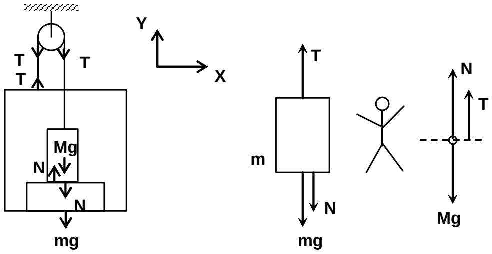


Illustration 18. *A man of mass M is standing on a plank kept in a box. The plank and box as a whole has mass m. A light string passing over a fixed smooth pulley connects the man and box. If the box remains stationary, find the tension in the string and the force exerted by the man on the plank.*



Solution: The fixed pulley is taken as frame of reference. The forces on man and box with plank are shown in figure.



The forces are as follows:

i. Weight of the man = Mg
ii. The tension in the string = T.
iii. The normal contact force between the man and the plank = N.
iv. The weight of the plank and box = mg

Referring to figure. The equation of motion of the man is given as

$T + N - Mg = M a$

since M > m and the box remains at rest, the man will have to be at rest

$T + N - Mg = 0$ …(1)

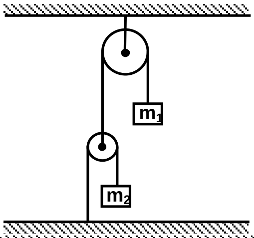
Similarly referring to figure,

$T - N - mg = ma = 0$ . . . (2)

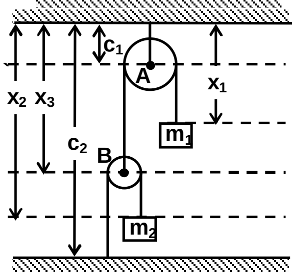
Solving (1) and (2) we obtain,

$$T = \frac{(M+m)g}{2} \text{ and } N = \frac{(M-m)g}{2}$$

Illustration 19. *Find the acceleration of the two blocks m_1 and m_2. Assume pulleys are massless and frictionless and the strings are inextensible.*



Solution: Let the acceleration of blocks m_1, m_2 and pulley B be a_1, a_2 and a_3 respectively.



Constraint relationship for the string attached to block of mass m_1:

$x_1 + x_3 - c_1 = \text{constant}$

Differentiating twice w.r.t. time we get,

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_3}{dt^2} = 0$$

$\Rightarrow a_1 = - a_3$ . . . (i)

The minus sign signifies that acceleration of pulley B is opposite to that of block of mass m_1

Constraint relationship for the string attached to block of mass m_2:

$c_2 - x_3 + x_2 - x_3 = \text{constant}$

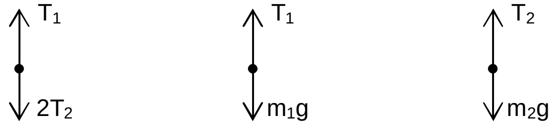
Differentiating twice w.r.t. time we get,

$$\frac{d^2x_3}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

$\Rightarrow a_2 = 2a_3$ . . . (ii)

$a_2 = 2a_1$ . . . (iii) (from equations (i) and (ii))

Taking the magnitudes only and ignoring the sign.



FBD of pulley B FBD of block m_1 FBD of block m_2

$T_1 = 2T_2$ . . . (iv)

$m_1g - T_1 = m_1a_1$ . . . (v)

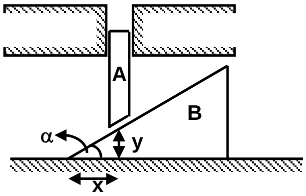
$T_2 - m_2 g = m_2a_2$ . . . .(vi)

Solving equations from (iii) to (vi) for a_1 and a_2 we get,

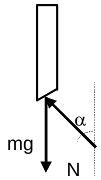
$$a_1 = \frac{m_1 - 2m_2}{m_1 + 4m_2}$$

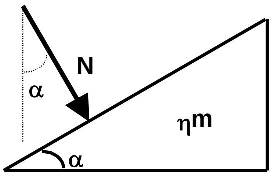
$$a_2 = 2a_1 = \frac{2(m_1 - 2m_2)}{m_1 + 4m_2}.$$

Illustration 20. *Find the accelerations of the rod A and the wedge B in the arrangement shown in the figure if the ratio of the mass of the wedge to that of the rod equals η and the friction between all surfaces is negligible.*



Solution: $y = x \tan\alpha \Rightarrow \frac{d^2y}{dt^2} = \frac{d^2x}{dt^2} \tan\alpha$





$a_{rod} = a_{wedge}.\tan\alpha$ (constraint equation)

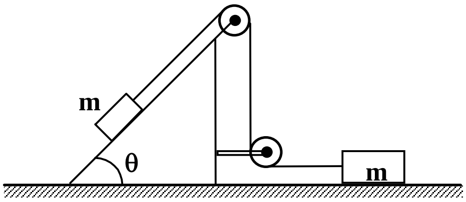
$mg - N\cos\alpha = ma_R$ & $N\sin\alpha = (\eta m).a_{wedge}$

$\Rightarrow mg - N\cos\alpha = m\,a_w \tan\alpha$

Solving we get,

$$a_{wedge} = \frac{g}{\tan\alpha + \eta\cot\alpha} \quad \text{and} \quad a_{Rod} = \frac{g}{1+\eta\cot^2\alpha}$$

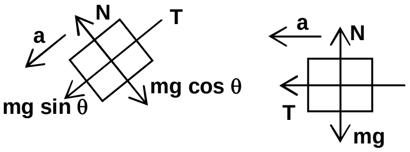
Illustration 21. *For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be*



(A) $\frac{2}{3}mg\sin\theta$ *(B)* $\frac{3}{2}mg\sin\theta$

(C) $\frac{1}{2}mg\sin\theta$ *(D)* $2mg\sin\theta$

Solution: Form F.B.D.



$T = ma$

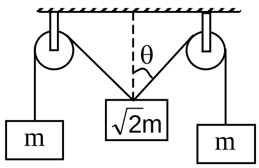
and $mg\sin\theta - T = ma$

So, $a = \frac{g\sin\theta}{2}$

Therefore, $T = \frac{mg\sin\theta}{2}$

So, option (C) is correct.

Illustration 22. *The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be*

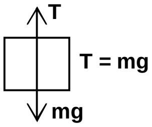


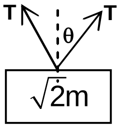
(A) 0° *(B) 30°*
(C) 45° *(D) 60°*

Solution:

$$\Rightarrow 2T\cos\theta = \sqrt{2}\,mg$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^{O}$$





So option (C) is correct.

Application of Newton's laws of motion: techniques and approach

A separate point diagram of the body is drawn showing the different forces exerted by the bodies in the environment, this is known as free body diagram.

Application of Newton's Laws to any system (consisting of one or more objects) can be done by following a systematic method. We recommend the following steps in the order given below –

(i) Draw the complete free body diagram (FBD), showing all the forces acting on each separate body.

(ii) Select proper coordinates for analysing the motion of each body.
Include any pseudo forces within the FBD if required.

(iii) If there are any constraints, write the proper constraint equations.

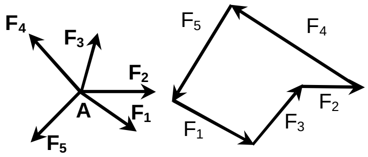
(iv) Apply Newton's 2nd law of motion : $\vec{F} = m\vec{a}$ for each body. This leads to a system of equations.

(v) Solve these equations.
(a) Identify the known and unknown quantities. Check that the number of equations equals the number of unknowns.
(b) Check the equations using dimensional analysis.
(c) After solving, check the final solution using back substitution.

(vi) If the velocity ($\vec{v}$) or position ($\vec{x}$) is required, proceed from a knowledge of acceleration ($\vec{a}$) as found from equations in step (v) and apply kinematics, e.g.
$\frac{d\vec{v}}{dt} = \vec{a}$ (known) and then integrate.

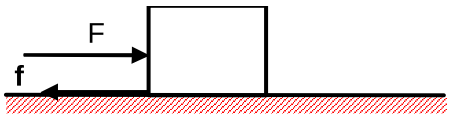
Equilibrium of concurrent forces

If the number of forces that are acting on a particle can be taken along the sides of any polygon both in direction as well as in magnitude, it will be in equilibrium.
Suppose that the force F_1, F_2, F_3, F_4 and F_5 are acting on the particle A and if they are in equilibrium then they will form a pentagon.

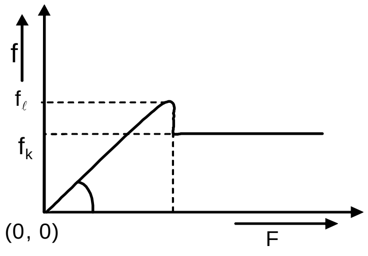


FRICTIONAL FORCE

Frictional force comes into play between two surfaces whenever there is relative motion or a tendency of relative motion between two surfaces in contact. Frictional force has the tendency to stop relative motion between the surfaces in contact.



Friction is a self-adjusting force. It changes its direction and magnitude according to the applied force or the force, which causes a tendency in the body to move. If the force increases then the opposing force also increases until the body moves beyond which it remains constant. If the applied force is plotted against the frictional force we obtain a graph as shown.



The graph shows that first frictional force increases to a certain maximum value f_ℓ with F and then suddenly decreases to a constant value f_k. For the range from 0 to f_ℓ frictional force is equal and opposite to F and hence block does not move. In this range, friction force is static.
Thus, friction can be classified as

(a) Static friction: It acts between surfaces in contact not in relative motion. It opposes the tendency of relative motion.

(b) Kinetic friction: It act acts between surfaces in contact which are in relative motion. It opposes the relative motion between the surfaces. Kinetic friction can be further classified as sliding friction and rolling friction.

Rolling friction: When a body rolls on a rough surface the frictional force developed is known as rolling friction. It is generally less than the kinetic friction or limiting friction.

Laws of static friction

Static Friction, acting between the surfaces in contact, (not in relative motion) opposes the tendency of relative motion between the surfaces.

The frictional force acts tangentially along the surfaces in contact, and the maximum value (or limiting value) of this force is proportional to the normal reaction between the two surfaces. The force of friction between two bodies is an *adjustable* force, only its maximum or limiting value is proportional to the normal reaction. Secondly, the direction of this force is determined by *all* other forces acting on the body that is by the forces that *tend* to cause relative motion. The force of static friction acts in a direction so as to *oppose* the other forces that tend to cause relative motion between the surfaces in contact.

Now, $f_{s(max)} \propto N$ where $f_\ell = f_{s(max)} \Rightarrow f_{s(max)} = \mu_s N$

Here μ_s = co-efficient of static friction.

N = normal reaction of the block from the surface.

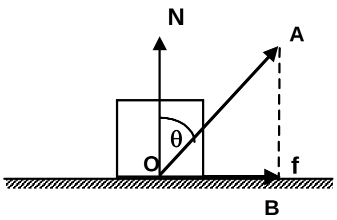
$$0 \le f_s \le \mu N$$

When F exceeds f_ℓ block starts moving and frictional force decreases to a constant value f_k. f_k is called kinetic friction and it has unique value which is given by

$$f_k = \mu_k N$$

Here μ_k = co-efficient of kinetic friction.

N = normal reaction.



Angle of friction: The angle made by the resultant reaction force with the vertical *(normal reaction)* is known as the angle of the friction.

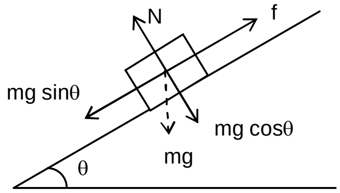
Now, in the triangle OAB,

$$\frac{AB}{OB} = \cot\theta$$

$\Rightarrow OB = AB \tan\theta$

or, $\tan\theta = \frac{f}{N}$

Angle of Repose:



The angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide. Consider an inclined plane, whose inclination with horizontal is gradually increased, till the body placed on its surface just begins to slide down, then the angle made by the plane with horizontal is called angle of repose.

From the diagram:

$f = mg \sin\theta$ . . . (i)

$N = Mg \cos\theta$

Dividing (i) by (ii)

$$\frac{f}{N} = \frac{Mg\sin\theta}{Mg\cos\theta} = \tan\theta$$

Since $\frac{f}{N} = \mu$, $\tan\theta = \mu$

Therefore, coefficient of limiting friction is equal to the tangent to the angle of repose thus angle of repose is equal to the angle of friction.

Exercise 8.

(i) ***What causes the motion of a car on a road?***

(ii) ***Why the centrifugal force is never included in the free body diagram of circular motion. Explain why?***

Illustration 23. *A block weighing 2 kg rests on a horizontal surface. The coefficient of static friction between the block and surface is 0.40 and kinetic friction is 0.20.*

(a) How large is the friction force acting on the block ?

(b) How large will the friction force be if a horizontal force of 5N is applied on the block?

(c) What is the minimum force that will start the block in motion?

Solution: (a) As the block rests on the horizontal surface and no other force parallel to the surface is on the block, the friction force is zero.

(b) With the applied force parallel to the surfaces in contact 5 N, opposing friction becomes equal and opposite. Further the limiting friction is $\mu_s N = \mu_s Mg = 8$ N

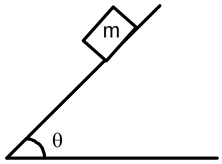
$\therefore$ Force of friction is 5N.

(c)The minimum force that can start motion is the limiting one. $\mu_s N = \mu_s mg = 8$ N

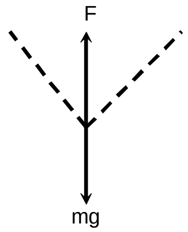
Illustration 24. *A block of mass m is at rest on a rough inclined plane of inclination θ as shown in the figure*

(a) Find the force exerted by the inclined plane on the block.

(b) What are the tangential and normal contact forces?



Solution: (a) The forces acting on the block are the field force $m\vec{g}$, vertically downward and total contact force $\vec{F}$ given by inclined plane on the block. As the block is at rest net force on the block is zero.



$\therefore \quad \vec{F} + mg = 0$ [as shown in Figure]

$\therefore \quad \vec{F} = -mg$

$\therefore$ The force exerted by the inclined plane on the block is $-\overrightarrow{mg}$ in vertically upward direction.

(b) The normal contact force N and tangential contact force f are shown in F.B.D. (Figure)

$f = mg \sin\theta$

$N = mg \cos\theta$

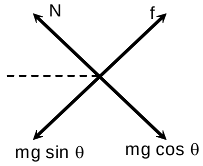
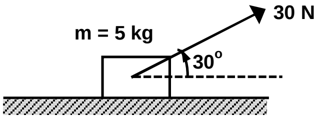


Illustration 25. *A 5 kg box is being moved across the floor at a constant velocity by a force of 30 N, as shown in the figure.*

(a) What is the force of friction acting on the box ?

(b) Find μ_k between the box and the floor.



Solution: The free body diagram of the box is shown in the figure.

Conditions of equilibrium :

$\Sigma F_y = 0$

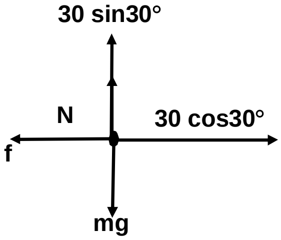
$\Rightarrow 30 \sin 30° + N - mg = 0$

or $N = (5)(10) - 30(1/2) = 35$ N

$\Sigma F_x = 0$

$\Rightarrow 30 \cos 30° - f = 0$

or $f = 30 \left(\frac{\sqrt{3}}{2}\right) = 15\sqrt{3}$ N

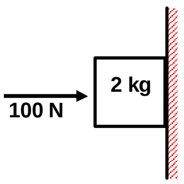


As there is a relative motion between the surfaces,

Hence $f = \mu_k N$, where μ_k = coefficient of kinetic friction.

$\Rightarrow \mu_k = \frac{f}{N} = \frac{15\sqrt{3}}{35} = \frac{3\sqrt{3}}{7} = 0.74$

Illustration 26. *A block of mass 2 kg is pressed against a rigid vertical wall by a horizontal force of 100 N. If co-efficient of static and kinetic friction are each equal to 0.3 then find the magnitude and direction of frictional force on the block.*

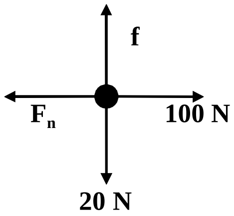


Solution: Since weight of block is 20 N which is acting downward, it has tendency to move the block downward. Hence, the frictional force will be upward. Maximum value of frictional force can be $f_{s(max)} = \mu F_n$

As block is in equilibrium along horizontal

$F_n = 100$ N

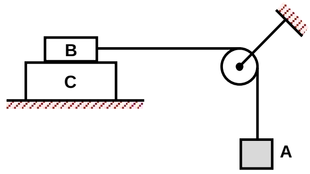
$\Rightarrow f_{s(max)} = 0.3 \times 100$ N = 30 N



F.B.D. of the block

Since weight of the block is less than the limiting friction, it will not slide. Therefore, for vertical equilibrium f = 20 N.

Illustration 27. *In the figure shown co-efficient of friction between the block B and C is 0.4. There is no friction between the block C and the surface on which it is placed. The system of blocks is released from rest in the shown situation. Find the distance moved by the block C when block A descends through a distance 2 m. Given masses of the blocks are m_A = 3 kg, m_B = 5 kg and m_C = 10 kg.*



Solution: Let there is no relative motion between the blocks B and C

Hence

$T = (m_B + m_C)a$ . . . (1)

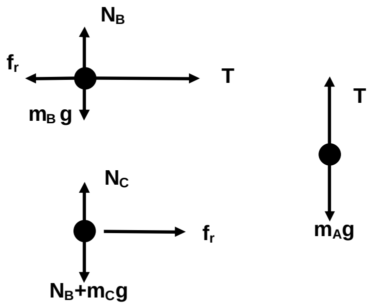
And

$m_Ag - T = m_Aa$ . . . (2)

From (1) and (2), we get

$$a = \frac{m_A g}{m_A + m_B + m_C} = \frac{30}{18} = \frac{5}{3}\ m/s^2$$

F.B.D. of the blocks



$\Rightarrow$ Net force on the block C is, $F = m_Ca = 10 \times (5/3)$ N = 16.6 N

If maximum value of frictional force acting on block C is f_{max}, then

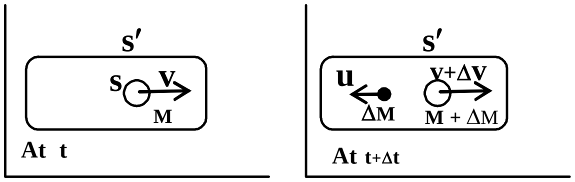
$f_{(max)} = \mu m_Bg = 0.4 \times 5 \times 10 = 20$ N $\because F \le f_{max}$

Hence there is no relative motion between the block B and C. Therefore, distance moved by C is 2 m only.

Lubrication: In some cases friction acts as a hindrance when there are moving parts in contact. A great amount of energy is lost in such type of machines like an automobile or pump or any motor. This energy converted to heat can damage the machines. So friction is reduced by suitable lubricants like oil grease, graphite etc.

Variable Mass System: Till now, we were discussing mass which remains constant with time. But if the mass changes with time continuously, then can we apply the conservation of momentum as we discussed earlier.

Let a body move continuously either by ejecting mass or absorbing mass. At any instant let the mass of the body be M and its velocity be $\vec{v}$ from a given inertial frame of reference. Suppose that the mass increases by ΔM and the velocity



increases by Δv in a time Δt. As the mass increases by ΔM, the remainder of the system has a mass $-\Delta M$ and moves with a speed u, relative to the inertial frame.

The change in momentum of the system s′ will be:

$$\Delta p = (M + \Delta M)(v + \Delta v) + (-\Delta M)u - Mv$$

To generalize, let us imagine an external force F_{ext} acting on the sub-system M.

The external force acting on the mass is

$$\lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{(M+\Delta M)\ (v+\Delta v) - \Delta M u - Mv}{\Delta t}$$

$$F_{ext} = M\frac{dv}{dt} + (v-u)\frac{dM}{dt} \quad \left[\because \frac{\Delta v \Delta M}{\Delta t} \text{ is very small}\right]$$

The relative velocity of the mass ΔM with respect to the sub-system is $(u - v) = v_{rel}$

$$\therefore F_{ext} = M\frac{Mdv}{dt} + v_{rel}\frac{dM}{dt}$$

Situations of variable mass

(i) Rocket Equation:

In distant space $F_{ext} = 0$ and $\frac{dv}{dt}$ is in positive direction, the direction of v_{rel} is negative. Now

$$M\frac{dv}{dt} = -v_{rel}\frac{dM}{dt}$$

$$\Rightarrow \quad dv = -v_{rel}\frac{dM}{M}$$

If the original mass of the rocket at t = 0 was M_0 and mass of the fuel burnt is m then

$$\int_{v_i}^{v_f} dv = -v_{rel}\int_{M_0}^{M_0-m}\frac{dM}{M} \quad \Rightarrow \quad v_f - v_i = -v_{rel}\ln\frac{M_0-m}{M_0}$$

$$M_f = M_0\, e^{-v_f/v_{rel}} \quad \text{where } M_f = M_0 - m \text{ and } v_i = 0$$

(ii) Rain drops accumulating in a moving rail road card is another example of variable mass situation.

(iii) A vertical chain falling on a fixed table also represents variable mass situation.

Exercise 9. ***A container filled with liquid has a hole on the side wall near bottom. Will momentum of the container and liquid system remain constant?***

Illustration 28. *A rocket of mass 40 kg has 360 kg of fuel. The exhaust velocity of the fuel is 2.0 km/sec. Calculate minimum rate of consumption of fuel so that the rocket may rise from the ground.*

Solution: We have relation $M\frac{dv}{dt} = F_{ext} + v_{rel}\frac{dM}{dt}$.

In the question, we have the force by the gravity as the external force. As the rocket is to lift from the ground the minimum acceleration of the rocket to do so is, o, beyond which, with a slight increase it will zoom into the sky.

Therefore, $\frac{dv}{dt} = 0$

Putting these into the equation, we obtain

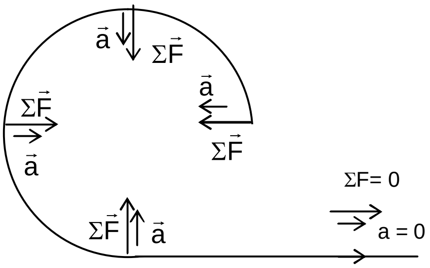
$$0 = Mg + v_{rel}\left(-\frac{dM}{dt}\right)$$

$\left(\frac{dM}{dt}\right)$ is the rate of decrease of mass hence taken negative. Mg is in the direction of the relative velocity of the fuel with respect to the rocket

Hence, $\frac{dM}{dt} = \frac{M}{v_{rel}}g = \frac{400\times 9.8}{2\times 1\left(10^3\right)} = 1.96\,\text{kg/sec}$

CIRCULAR MOTION

If a particle moves in a circular path at a constant speed, the velocity of the particle at any point of its path is directed along the tangent at that point. Due to continuous change in the direction of velocity, the particle has an acceleration. It is found that this acceleration is always directed radially inwards. It is known as radial or centripetal acceleration. So the particle is always acted on by a force directed radially inward, known as centripetal force.



Thus, centripetal force is defined as the force which acts towards the centre along the radius of a circular path on which a body is moving with a uniform speed.

Expression for the centripetal force:

(i) For uniform motion:

Acceleration towards centre

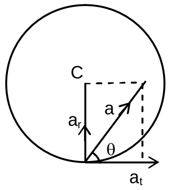
$a_r = r\omega^2 = \frac{v^2}{r}$ $\quad [\because v = r\omega]$. Hence centripetal force, $F = \frac{mv^2}{r}$

(ii) For non uniform motion:

In this case a body has two accelerations

(a) Radial acceleration : $a_r = \frac{v^2}{r}$, it occurs due to change in direction of the body and

(b) Tangential acceleration, $a_t = \frac{dv}{dt}$, it occurs due to change in speed of the body.



Hence the total acceleration, $a = \sqrt{a_r^2 + a_t^2}$

$\tan\theta = \frac{a_r}{a_t}$

Exercise 10.	***For uniform circular motion does the direction of the centripetal force depend upon the sense of revolution?***

Illustration 29. *A stone, tied to the end of a string 80 cm long, is whirled in a horizontal circle with a constant speed. If the stone makes 5 revolutions in 10 s, what is the magnitude and direction of acceleration of the stone ?*

Solution: Acceleration $a = r\omega^2 = r\,(2\pi n)^2$

$$= 0.8 \times 4 \times \pi^2 \times \left(\frac{5}{10}\right)^2 = 8\ \text{m/s}^2 \qquad \text{(Towards centre)}$$

Vehicle moving round a circular path on horizontal road:

When a vehicle takes a turn on a road, it has a tendency to skid away from the centre of curvature of the road due to inertia. This tendency to skid brings into action a frictional force between the road and the tyres, directed towards the centre. This frictional force provides the necessary centripetal force.

If the maximum speed of a vehicle without skidding is v, then frictional force,

$$f = \mu mg = \frac{mv^2}{r}$$

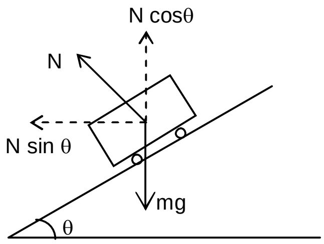
$$\Rightarrow \quad v = \sqrt{\mu rg}$$

Banking of tracks:

Friction is not always sufficient to provide the required centripetal force. Moreover, it causes damage of the tyres. Friction can be avoided by banking the road at a suitable angle θ to the horizontal.

The horizontal component of the normal reaction N provides the centripetal force, i.e.

$$N \sin\theta = \frac{mv^2}{r} \quad \ldots \text{(i)}$$



And the vertical component of normal reaction balances the weight, i.e.

$$N \cos\theta = mg \quad \ldots \text{(ii)}$$

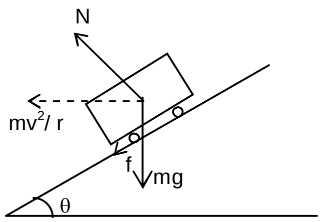
From (i) and (ii)

$$\tan\theta = \frac{v^2}{rg}$$

If the above relation is not satisfied, then frictional force will come into play. It may act down the plane or up the plane depending on whether the vehicle has a tendency to skid outward or inward.

If the vehicle has a tendency to skid outward, then the component of frictional force and the normal reaction in the horizontal direction produces necessary centripetal force,

$$N \sin\theta + f \cos\theta = \frac{mv^2}{r} \quad \ldots \text{(i)}$$



And the component of frictional force and the normal reaction in upward direction balances the weight of the vehicle.

i.e. $N\cos\theta - f\sin\theta = mg \quad \ldots$ (ii)

From (i) and (ii),

$$\frac{v^2}{rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

We know frictional force, $f = \mu N$.

Hence, $$\frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

This is the maximum velocity of a vehicle without slipping. If $\mu = 0$, we get the optimum speed for least damage of tyres as

$$V = \sqrt{rg\tan\theta}$$

Illustration 30. *The road at a circular turn of radius 10 m is banked by an angle of 10^0. With what speed should a vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force? [tan 10° = 0.176]*

Solution: $\tan\theta = v^2 / rg$

or, $v = \sqrt{rg\tan\theta}$

$= \sqrt{(10)(9.8)\tan 10^0} = 4.2$ m/s

Illustration 31. *A cyclist is riding with a speed of 27 km/hr. As he approaches a circular turn on the road of radius 80 m. He applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude of the net acceleration of the cyclist?*

Solution: Tangential acceleration $= 0.5\,m/s^2$

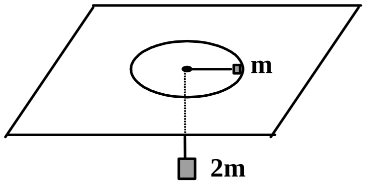
Centripetal acceleration $= \frac{v^2}{R} = \frac{\left(27\times\frac{5}{18}\right)^2}{80} = \frac{225}{320}$

$= 0.703\,m/s^2$

Net acceleration $\sqrt{a_T^2 + a_R^2}$

$= \sqrt{0.25+0.49} = \sqrt{0.74} = 0.86$ m/s^2

Illustration 32. *A mass m rotating freely in a horizontal circle of radius 1m on a frictionless smooth table supports a mass 2m in equilibrium attached to the other end of the string hanging vertically. If the instantaneous acceleration of the mass 2m is g m/s^2 vertically upward then angular velocity of rotation is*



(A) 5.78 rad/s *(B) 6.32 rad/s*
(C) 5.94 rad/s *(D) 6.11 rad/s*

Solution: Mass 2m is moving upward with an acceleration g

So, T – 2 mg = (2 m)g

$\Rightarrow$ T = 4 mg

Now for mass m, Tension will provide the necessary centripetal acceleration

4 mg = mRω^2

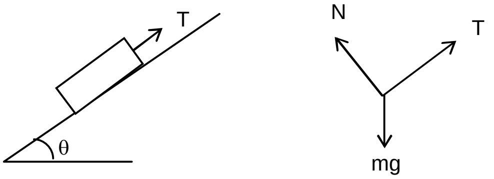
$\omega = 2\sqrt{g} = 6.32$ rad/sec.

So option (B) is correct.

SUMMARY

When a body is in equilibrium in an inertial frame of reference, the vector sum of forces acting on it must be zero. Free body diagrams are essential in identifying the forces that act on the body being considered.

Newton's third law is also frequently needed in equilibrium problems. The forces in an action –reaction pair never act on the same body.



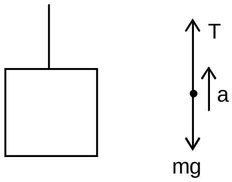
Vector form

$\Sigma\vec{F} = 0$

Component form

$\Sigma F_x = 0,\ \Sigma F_y = 0$

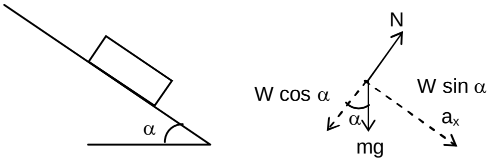
If the vector sum of forces on a body in not zero, the body accelerates. Its acceleration is given by Newton's 2nd law.



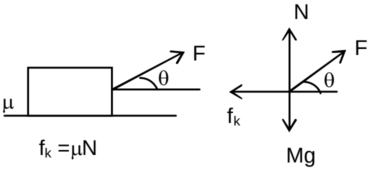
As they are for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law.

Vector form: $\Sigma\vec{F} = m\vec{a}$

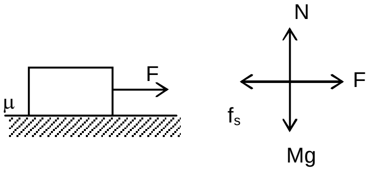
component form $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$



The contact force between two bodies can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force f parallel to the surface. The normal force exerted on a body by a surface is not always equal to the body's weight.

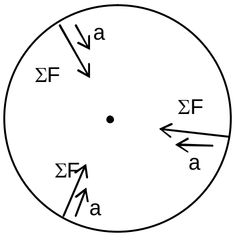


When a body is sliding over the surface, the friction force is called kinetic friction. Its magnitude f_k is approximately equal to the normal force magnitude N multiplied by the coefficient of kinetic friction μ_k.



When a body is not moving relative to the surface, the friction force is called static friction. The maximum possible static friction force is approximately equal to the magnitude N of the normal force multiplied by the coefficient of static friction.

The actual static force may be anything from zero to this maximum value, depending on the situation usually μ_s is greater than μ_k for a given pair of surface in contact.



In uniform circular motion, the acceleration vector is directed towards the centre of the circle. Just as in any other dynamic problem, the motion is governed by Newton's second law $\Sigma\vec{F} = m\vec{a}$.

$a_{rad} = v^2/R$.

MISCELLANEOUS EXERCISE

1. If the net force acting on a body be zero, then will the body remain necessarily in rest position?

2. Can a body remain in rest position when external forces are acting on it?

3. The two ends of a spring-balance are pulled each by a force of 10 kg-wt. What will be the reading of the balance?

4. A force of 5 N changes the velocity of a body from 10 m/s to 20 m/s in 5 sec. How much force is required to bring out the same change in 2 sec.

5. An impulsive force of 100 N acts on a body for 1 s. What is the change in its linear momentum.

6. Two bodies of masses M and m are allowed to fall freely from the same height. If air resistance for each body is same, then will both the bodies reach the earth simultaneously?

7. When a ball is thrown upward, its momentum first decreases then increases. Is conservation of linear momentum violated in this process?

8. Four blocks of same mass m connected by cords are pulled by a force F on a smooth horizsontal surface, as shown in figure. Determine the tensions T_1, T_2 and T_3 in the cords.

SOLUTIONS TO MISCELLANEOUS EXERCISE

1. No, the body may be moving uniformly along a straight line.
2. Yes, if vector sum of all the forces acting on the body is zero.
3. The reading of balance will be 10 kg-wt.
4. From $F_1 = \frac{dp_1}{dt_1}$ and $F_2 = \frac{dp_2}{dt_2}$ $\Rightarrow \frac{F_2}{F_1} = \frac{dt_1}{dt_2} \times \frac{dp_2}{dp_1}$

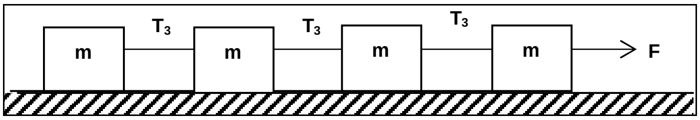
   Now, $dp_1 = dp_2$

   So, $\frac{F_2}{F_1} = \frac{dt_1}{dt_2} = \frac{5}{2} \Rightarrow F_2 = \frac{5}{2}F_1 = \frac{5\times 5}{2} = 12.5\,\text{N}$
5. Change in linear momentum = Impulse = F× t = 100 N-s.
6. No, the net force on the body of mass M is (Mg — F). Therefore, its acceleration,

   $a = \frac{Mg - F}{M} = \left(g - \frac{F}{M}\right)$

   Thus, acceleration, a of a body of larger mass will be greater and it will appear lighter than before.
7. No, the momentum conservation principle is not violated. This is because vector sum of linear momentum of the ball and the earth remain constant.

8. Let a be the common acceleration of the whole system



   $\therefore$ F = 4 ma $\Rightarrow$ a = $\frac{F}{4m}$

   Applying Newton's 2nd law for each blocks :

   $F - T_1 = ma \Rightarrow T_1 = \frac{3}{4}F$

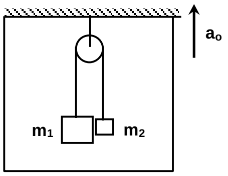
   $T_1 - T_2 = ma \Rightarrow T_2 = \frac{F}{2}$ $\Rightarrow T_2 - T_3 = ma \Rightarrow T_3 = \frac{F}{4}$

SOLVED PROBLEMS

Subjective:

BOARD TYPE

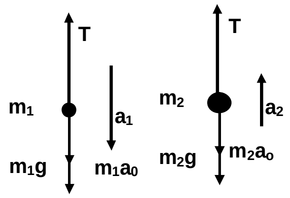
Prob 1. *A pulley fixed to the ceiling of an elevator car carries a thread whose ends are attached to the masses m_1 and m_2. The car starts going up with an acceleration a_o. Assuming the masses of the pulley and the thread as well as the friction to be negligible, find :*



(a) the acceleration of the load m_1 relative to the elevator shaft and relative to the car.
(b) the force exerted by the pulley on the ceiling of the car. Given, $m_1 > m_2$.

Sol. The elevator is an accelerated frame which is non-inertial, pseudo force $m_1a_0 < m_2a_0$ have been taken into consideration for m_1 and m_2 respectively.

F.B.D. of the blocks relative to elevator.



$m_1g + m_1a_o - T = m_1a_1$ . . . (i)

$T - m_2g - m_2a_o = m_2a_2$ . . . (ii)

$a_1 = a_2$ . . . (iii)

Solving the above equations , we get,

$$a_1 = \frac{(m_1 - m_2).(a_o + g)}{(m_1 + m_2)}$$

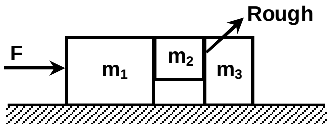
This is the acceleration of m_1 w.r.t car. Acceleration of the mass m_1 w.r.t the elevator shaft

$$= a_o - a_1 = \frac{2a_o m_2 - g(m_1 - m_2)}{(m_1 + m_2)}$$

Force exerted by the pulley on the ceiling of the car

$$= 2.T = (m_1a_o + m_{\square}g - m_1a_1) \text{ x } 2 = \frac{4m_1.m_2(a_o + g)}{(m_1 + m_2)}.$$

Prob 2. *In the arrangement shown in the figure, the system of masses m_1, m_2 and m_3 is being pushed by a force F applied on m_1 horizontally, in order to prevent the downward slipping of m_2 between m_1 and m_3. If coefficient of friction between m_2 and m_3 is μ and all the other surfaces are smooth, What is the minimum value of force F?*

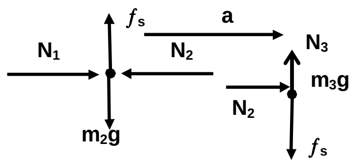


Sol. $f_s = m_2g$; $f_s \le \mu N_2$; $N_2 = m_3a$

$\therefore m_2g \le \mu m_3a$

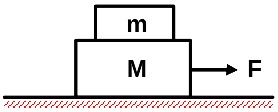
$$\Rightarrow a \ge \left(\frac{m_2g}{\mu m_3}\right)$$

$$F \ge (m_1 + m_2 + m_3)\frac{m_2g}{\mu \quad m_3}.$$



F.B.D of m_2 F.B.D of m_3

Prob 3. *A block of mass m is placed on another block of mass M lying on a smooth horizontal surface as shown in the figure. The co-efficient of friction between the blocks is μ. What maximum horizontal force F can be applied to the block M so that the blocks move together ?*



Sol. If there is no relative motion between the blocks then acceleration of the blocks is

$$a = \frac{F}{M+m}$$

For vertical equilibrium $N = mg$

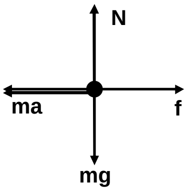
For horizontal equilibrium $f - ma = 0$

$$f \le \mu N$$

$\Rightarrow \quad \mu mg \ge m \dfrac{F}{(M+m)}$

$\Rightarrow \quad F \le (M + m)\mu g$

$\Rightarrow \quad F_{max} = (M + m)\,\mu g$



F.B.D. of m relative to M

Prob 4. *A mass of 200 kg. is placed on a rough inclined plane of angle 30°. If coefficient of limiting friction is $\frac{1}{\sqrt{3}}$, Find the greatest and the least forces in Newton, acting parallel to the plane to keep the mass in equilibrium .*

Sol. From the figure shown

$R = mg\cos\theta$

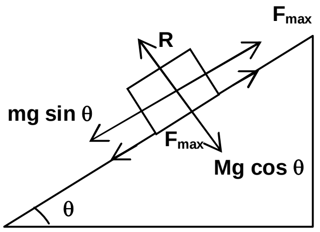
$F_{max} = \mu R = \mu\, mg\cos\theta$

So, the greatest force required to kept the body in equilibrium

$F_{max} = mg\sin\theta + f_{max}$

$= mg\sin\theta + \mu\, mg\cos\theta$

$$= 200\times 9.8\left[\frac{1}{\sqrt{3}}\cos 30^\circ\right] = 1960\text{ N}$$



From the figure (b)

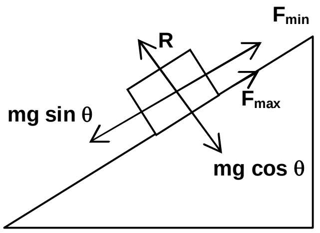
$F_{min} = mg\sin\theta - f_{max}$

$= mg\sin\theta - \mu\, mg\cos\theta$

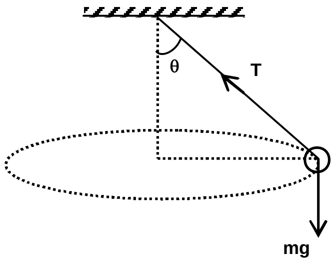
$= mg[\sin\theta - \mu\cos\theta]$

$$= 200\times 9.8\left[\sin 30^{O} - \frac{1}{\sqrt{3}}\cos 30^{O}\right]$$

= zero



Prob 5. *A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can with stand a maximum tension of 200 N?*



Sol. From figure

$T \cos\theta = mg$

$T \sin\theta = mr\,\omega^2$

So, $T = m\sqrt{g^2 + r^2\omega^4}$

Now $\omega = 40\,\text{rev/min} = 2\pi \times \frac{40}{60}\,\text{rad/sec}$

$= \frac{4\pi}{3}\,\text{rad/sec}$

So, Tension (T) $= \sqrt{g^2 + (r\omega^2)^2}) \times 0.25$

$$= \left(\sqrt{100 + \left(1.5 \times \frac{16\pi^2}{9}\right)^2}\right)^{0.25}$$

$$= \left(\sqrt{100 + \left(\frac{64}{9}\right)\pi^4}\right) \times 0.25 = 7.0\,\text{N}$$

So, maximum speed, by the given data

$T_{max} = 200$ N

So $200 = \sqrt{g^2 + r^2\omega^4_{max}})0.25 = 7.0\,\text{N}$

$\omega_{max} = 23.1$ rad/sec

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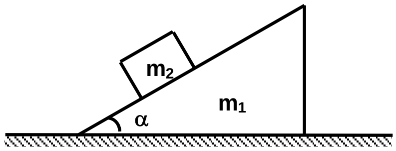
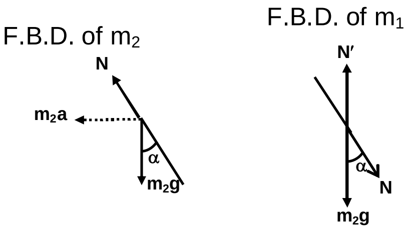
Prob 6. *A wedge 1 of mass m_1 and with angle α rests on a horizontal surface. Block 2 of mass m_2 is placed on the wedge. Assuming the friction to be negligible, find the acceleration of the wedge.*



Sol. Let us solve this problem by considering the motion of m_2 in non-inertial frame of wedge. In that frame the block is at rest along the normal to the inclined plane. Hence it is under equilibrium along the normal to the plane.

Due to the acceleration of the frame towards right pseudo force acts on the block towards left. As shown in the F.B.D.



$m_2 a \sin\alpha + N = m_2 g \cos\alpha$ ...(1)

and for m_1, $N\sin\alpha = m_1 a$ ...(2)

Multiplying (1) by $\sin\alpha$ and substituting (2) in it,

$m_2 \sin^2\alpha + m_1 a = m_2 g \sin\alpha \cos\alpha$

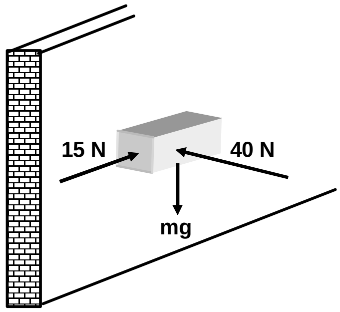
$$\Rightarrow \quad a = \frac{g\sin\alpha.\cos\alpha}{\sin^2\alpha + (m_1/m_2)}.$$

Prob 7. *A block of mass 2 kg is pushed normally against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction and with what minimum acceleration ? If no, find the frictional force exerted by the wall on the block.*

Sol. The force which may cause the tendency of motion or motion in the body is its own weight and the applied horizontal force of 15 N. The resultant of the forces

$$F = \sqrt{20^2 + 15^2} = 25N$$

In a direction $\tan^{-1}\left(\frac{15}{20}\right) = 37°$ with the vertical.



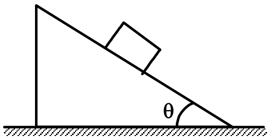
The friction will, by its very virtue of opposing the tendency of relative motion will act in a direction opposite to the resultant force. Now, for the acceleration to be minimum.

The minimum force required = F – μN (as, μN is the maximum frictional force)

$$= 25 - 0.5 \times 40 = 5 \text{ N}$$

$\therefore$ Minimum acceleration is $\frac{5}{2} = 2.5 \text{ m/s}^2$.

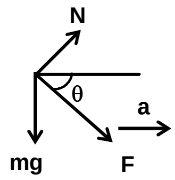
Prob 8. *A small block is resting on an inclined plane (coefficient of friction $\mu > \tan\theta$) as shown in the figure. The inclined plane is given a constant horizontal acceleration 'a' towards right.*



(a) Find the range of 'a' such that the block does not slide on the plane.

(b) Find the value of 'a' such that the friction force between the block and the plane is zero.

Sol. (a) As $\mu > \tan\theta$ the block does not slide when a = 0 which is the lower limit.



Newton's 2nd law

$$N - mg\cos\theta = ma\sin\theta \quad \ldots (1)$$

$$F + mg\sin\theta = ma\cos\theta \quad \ldots (2)$$

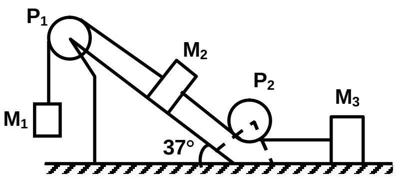
Force of friction $F \le \mu N$

$\therefore \quad a \le \dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$ (no slide condition)

$\therefore \quad$ the range of a is O to $\dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$

(b) setting F = 0 we get $\quad a = g\tan\theta$

Prob 9. *Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal.*

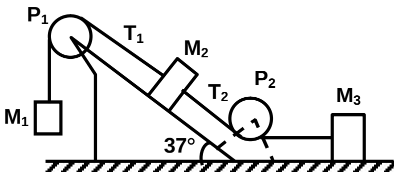


The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity find the

(a) mass of M_1

(b) tension in the horizontal portion of the string. ($g = 9.8 \text{ m/s}^2$ and $\sin 37° \approx 3/5$)

Sol. Let T_1 be the tension in the string connecting M_1 and M_2 and T_2 be the tension in the string connecting M_2 and M_3. From the figure.



$M_1g = T_1$

$T_2 = \mu M_3 g = (0.25)4g$

or, $T_2 = g = 9.8$ N

Also, $T_1 = T_2 + (0.25) \times 4g\cos 37° + 4g\sin 37°$

$= g\left(1 + \frac{4}{5} + \frac{4\times 3}{5}\right)$

$T_1 = \frac{21}{5}g$

or, $M_1 g = \frac{21}{5}g$

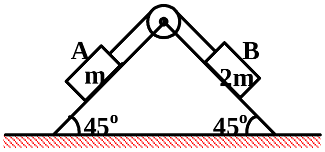
$\therefore$ $M_1 = \frac{21}{5} = 4.2$ kg

Prob10. *Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a light and inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3. If the system of A and B is released from rest, then find*

(a) the acceleration of A

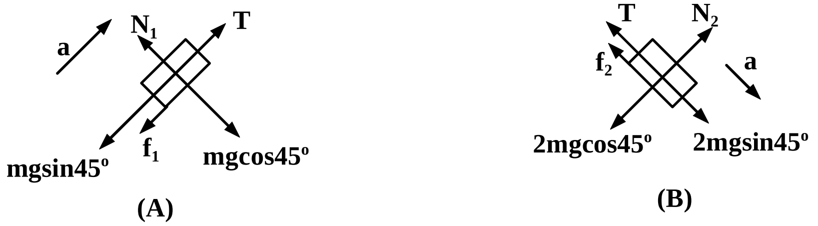
(b) tension in the string

(c) the magnitude and direction of the frictional force acting on A.



Sol. (a) In the absence of friction the block B will move down the plane and the block A will move up the plane. Frictional force opposes this motion.

F.B.D. of the blocks.



$\Rightarrow$ $T - mg\sin 45° - f_1 = ma$ . . . (1)

and, $2mg\sin 45° - f_2 - T = 2ma$ . . . (2)

Adding (1) and (2), we get

$mg\sin 45° - (f_1 + f_2) = 3ma$

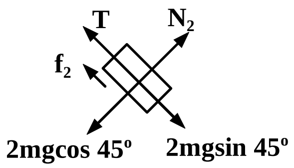
For a to be non-zero mg sin 45° must be greater than the maximum value of $(f_1 + f_2)$

$\because$ $(f_1 + f_2)_{max} = (\mu_1 m_1 + \mu_2 m_2)g\cos 45° = \frac{4}{3}mg\cos 45°$

$\Rightarrow$ $mg\sin 45° < (f_1 + f_2)_{max}$

Hence blocks will remain stationary

(b) F.B.D. of the block B



$f_{(2)max} = \frac{1}{3} 2mg \cos 45^o = \frac{2}{3\sqrt{2}} mg$ &

$2 \text{ mg} \sin 45° = \frac{2mg}{\sqrt{2}}$

$\because$ 2 mg sin 45° > $f_{2(max)}$, therefore block B has tendency to slide down the plane.

For block B to be at rest

$T + f_{2(max)} = 2 \text{ mg} \sin 45°$

$\Rightarrow T = \frac{mg}{\sqrt{2}}\left(2 - \frac{2}{3}\right) = \frac{4mg}{3\sqrt{2}}$

$\Rightarrow T = \frac{2\sqrt{2}}{3} mg$

(c) $mg \cos 45° = \frac{mg}{\sqrt{2}}$

$\because$ T(tension) is greater than mg cos 45°.

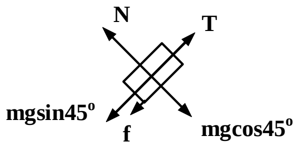
Hence block A has tendency to move up the plane, therefore frictional force on the block A will be down the plane.

For A to be at rest

$mg \sin 45^o + f = T$

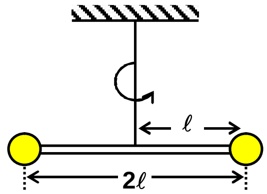
$\Rightarrow f = T - mg \sin 45^o$

$= \frac{2\sqrt{2}mg}{3} - \frac{mg}{\sqrt{2}} \Rightarrow f = \frac{mg}{3\sqrt{2}}$



F.B.D. of A

Prob 11. *Consider two small balls connected to a rigid rod of length '2ℓ' which in turn is suspended by a thread & rotated about the thread at angular velocity ω. What would be the magnitude & direction of total force exerted by rod on one of the balls?*



Sol. Consider the ball in ground's reference frame. It is observed to be doing uniform circular motion for which each ball must experience a resultant force of magnitude $m\omega^2 \ell$ directed towards centre of its circular path.

$\Rightarrow \quad \vec{R} + m\vec{g} = m\omega^2 \ell \hat{n}$ Where $\vec{R}$ is the force exerted by rod on one of the two balls.

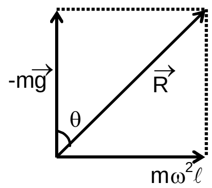
$\Rightarrow \quad \vec{R} = m\omega^2 \ell \hat{n} - m\vec{g}$

As $\hat{n}$ and $\vec{g}$ are mutually perpendicular

$$|\vec{R}| = \sqrt{(m\omega^2 \ell)^2 + (mg)^2} = m\sqrt{(\omega^2 \ell)^2 + g^2}$$

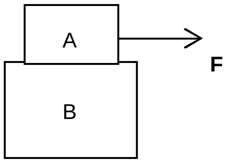
To decide direction of $\vec{R}$, consider the following vector diagram.

Thus, the angle that $\vec{R}$ makes with vertical to equals $\tan^{-1}\left(\frac{\omega^2\ell}{g}\right)$.



Prob 12. *A block A of mass 2 kg is placed on another blocks of mass 5 kg and a horizontal force F is applied on the block A . If co-efficient of friction between block A and B is 0.3 and between block B and the floor is frictionless, then what is the maximum value of F so both blocks will move together and what is the value of this acceleration?*

Sol. Suppose both blocks will move with common acceleration a, then

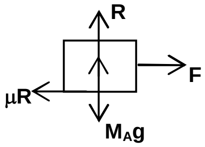


$F = (m_A + m_B)\,a$

$a = \frac{F}{2+5} = \frac{F}{7}$

Now for F.B.D. of block A.

$F - \mu R = m_A a$



$\Rightarrow \mu R = F - m_A a = F - \frac{2F}{7}$

$\Rightarrow \mu m_A g = \frac{5F}{7}$

$\Rightarrow = \frac{7 \times 0.3 \times 2 \times g}{5}$

$= 4.2 \times 2 = 8.4$ N and

$a = \frac{F}{7} = \frac{8.4}{7} = 1.2 \text{ m/s}^2$

Prob 13. *A car start from rest and accelerates uniformly with 2 m/s^2. At t = 10 s, a stone is dropped out of the window (1 m high) of the car. What are the (a) velocity and (b) acceleration of the stone at t = 10.1 sec? Neglect air resistance and take g = 9.8 m/s^2.*

Sol. At t = 10 s

Velocity of car = at = 20 m/s

(a) Horizontal component of stone V_s=20 m/s.

Now in vertical direction acceleration = g m/s^2

So at t = 10.1 sec.

$v_y = gt = 9.8 \times (10.1 - 10)$

$= 0.98$ m/s

So, resultant velocity of stone = $\sqrt{v_x^2 + v_y^2}$ = 20.02 m/s and

the angle of resultant velocity with horizontal direction is

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{0.98}{20}\right)$

$\theta = 2.8°$

(b) The moment of the stone is dropped out from the car, horizontal force on the stone =0. The only acceleration is due to gravity $a_y = g = 9.8$ m/s^2 (downwards)

Objective:

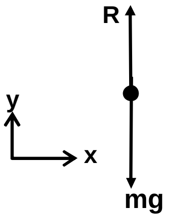
Prob 1. *An iron nail is dropped from a height h from the level of a sand bed. If it penetrates through a distance x in the sand before coming to rest, the average force exerted by the sand on the nail is,*

(A) $mg\left(\frac{h}{x}+1\right)$ (B) $mg\left(\frac{x}{h}+1\right)$

(C) $mg\left(\frac{h}{x}-1\right)$ (D) $mg\left(\frac{x}{h}-1\right)$

Sol. The nail hits the sand with a speed v_o after falling through a height h

$\Rightarrow v_0^2 = 2gh \Rightarrow v_0 = \sqrt{2gh}$ …(1)



The nail stops after sometime say t, penetrating through a distance, x into the sand. Since its velocity decreases gradually the sand exerts a retarding upward force, R (say). The net force acting on the nail is given as

$\Sigma F_y = R - mg = ma$

$\Rightarrow R = m(g + a)$ …(2)

Where a = deceleration of the nail. Since the nail penetrates a distance x

$0 - V_0^2 = -2a\,x$ …(3)

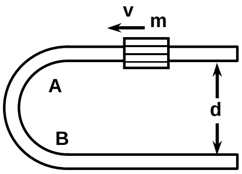
Putting V_0 from (1) and 'a' from (2) in (3), we obtain

$$2gh = 2\left(\frac{R - mg}{m}\right) x$$

$$\Rightarrow \quad R = \frac{mg(h+x)}{x}$$

$\Rightarrow \quad R = mg\left(\frac{h}{x}+1\right)$, Hence **(A)** is the correct choice.

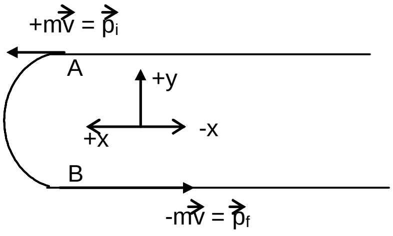
Prob 2. *A U shaped smooth wire has a semi-circular bending between A and B as shown in the figure. A bead of mass 'm' moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force exerted by the bead on the part AB of the wire is,*



(A) 0 (B) $\frac{4mv^2}{\pi d}$

(C) $\frac{2mv^2}{\pi d}$ (D) none of these.

Sol. Choosing the positive x-y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.



Therefore, the magnitude of the change in momentum between A and B is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive x-axis.

The time interval taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi . d/2}{v} = \frac{\pi d}{2v}.$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{\pi d}{2v}} = \frac{4mv^2}{\pi d}$$

Hence the correct choice is **(B)**.

Note:

1. By mistake, if the change in the magnitude of the momentum is considered, average force will be equal to zero.
2. If someone accounts carelessly d = r instead r = d/2 then he will lead to wrong choice (c).

Prob 3. *A man holds a ball of mass (1/2) kg in his hand. He throws it vertically upward. During this process his hand moves up by 40 cm and the ball leaves his hand with an upward velocity of 4 ms^{-1}. The constant force with which the man pushes the ball is*

(A) 2 N *(B) 10 N*
(C) 15 N *(D) 7 N*

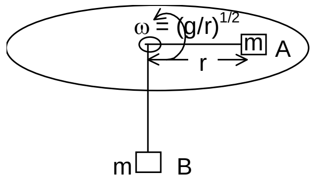
Sol. Acceleration of the ball

$$a = \frac{v^2}{2s} = \frac{4^2}{2 \times 0.4} = 20 \text{ m/s}^2$$

Hence force applied by the man $= m(a+g) = \frac{1}{2}(20+10) = 15$ N

Hence the correct choice is **(C)**.

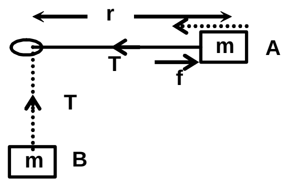
Prob 4. *Two particles A and B, each of mass m, are inter-connected by an inextensible string such that the particle B hangs below a table as shown in the figure and particle A is on a rough rotating disc at a distance r from the axis of rotation of the disc.*



If the angular speed of the disc and the block is $\omega = \sqrt{g/r}$, the frictional force developed at the interface of the particle & the disc is equal to

(A) mg/2 *(B) < mg/2*
(C) mg/√2 *(D) zero*

Sol. The particle of mass m experiences two forces (i) tension T (ii) frictional force f.

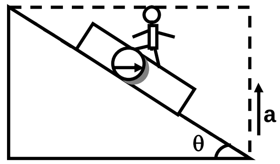


Since the particle A is rotating in a circular path of radius r, its centripetal acceleration,

$$\Rightarrow \quad r\,\omega^2 = \frac{T - f}{m}$$

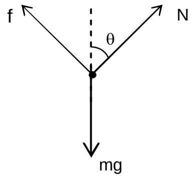
Putting T = mg for equilibrium of the mass B & ω^2 = g/r
we obtain f = mg – mr g/r = 0

Prob 5. *A man of mass m = 60 kg is standing on weighing machine fixed on a triangular wedge of angle θ = 60° with horizontal as shown in the figure. The wedge is moving up with an upward acceleration a = 2 m/s^2. The weight registered by machine is*



(A) 600 N *(B) 1440 N*
(C) 360 N *(D) 240 N*

Sol. N – mg cos θ = ma cos θ
N = m(g + a) cos θ = 60 (10 + 2) cos 60°
= 360 N
Hence the correct choice is **(C).**



Prob 6. *A massive platform of mass M is moving with speed v=6 ms^{-1}. At t=0 a body of mass m (m << M) is gently placed on the platform. If coefficient of friction between body and platform is μ = 0.3 and g = 10 m/s^2, then*
(A) the body covers a distance 3 m on the platform in the direction of motion of the platform.
(B) the body covers a distance 3 m on the platform opposite to the direction of motion of platform before coming to rest.
(C) the body covers a distance of 6 m on the platform in the direction of motion of the platform.
(D) the body covers a distance of 6 m on the platform opposite to the direction of motion of platform before coming to rest.

Sol. Since M>>m, the velocity of M remains unchanged after m is placed on to it.

Acceleration of m, $a = \dfrac{\mu mg}{m} = \mu g$

$a_{mM} = a - 0 = a$ and initial relative velocity $v_{mM} = 0 - v = -v$

Hence $s = \dfrac{v^2}{2\mu g} = \dfrac{6^2}{2\times 0.3\times 10} = 6$ m. Hence the correct choice is **(D).**

Prob 7. *A body of mass m is kept on a rough horizontal surface of friction coefficient μ. A force is applied horizontally, but the body is not moving. The net force 'F' by the surface on the body will be*
(A) $F \le \mu mg$ *(B) $F = \mu mg$*
(C) $mg \le F \le mg\sqrt{1+\mu^2}$ *(D) $mg \ge F \ge mg\sqrt{(1-\mu^2)}$*

Sol. If the body is not moving, F = f, where f is the force of friction on the body and
$0 \le f \le \mu mg$ or $0 \le F \le \mu mg$ . . . (i)
The force by the surface on the body

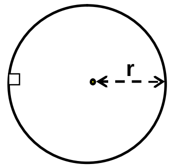
$R = \sqrt{f^2 + N^2} = \sqrt{F^2 + (mg)^2}$

or $R = mg\sqrt{\mu^2 + 1}$

$\therefore mg \le F \le mg\sqrt{1+\mu^2}$,

Hence the correct choice is **(C)**

Prob 8. *Consider a small cube of mass 'm' kept on a horizontal disc. If the disc is to rotate with uniform angular velocity, what could be its maximum value without causing any sliding between the cube and the disc? (Coefficient of static friction between cube & disc is μ).*



(A) $\sqrt{\frac{\mu g}{r}}$ (B) $\sqrt{\frac{2\mu g}{r}}$

(C) $\sqrt{\frac{\mu g}{2r}}$ (D) $2\sqrt{\frac{\mu g}{r}}$

Sol. In absence of any sliding, net force on the cube in the frame of reference rotating with disc will be zero. We find two forces in the plane of disc - frictional force and centrifugal force.

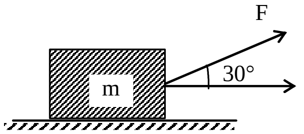
Hence, $m\omega^2 r = f$

but $f \le \mu mg$

Hence, $\omega \le \sqrt{\mu g / r} \Rightarrow \omega \le \sqrt{\mu g / r}$

$\Rightarrow \omega_{max} = \sqrt{\frac{\mu g}{r}}$, Hence **(A)** is the correct choice.

Prob 9. *A mass m rests on a horizontal surface. The coefficient of friction between the mass and the surface is μ. If the mass is pulled by a force F as shown in figure, the limiting friction between mass and the surface will be*



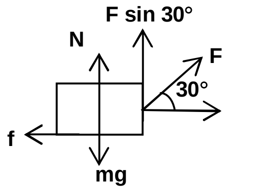
(A) μmg (B) $\mu(mg - \frac{\sqrt{3}}{2}F)$

(C) $\mu\, [mg - F/2]$ (D) $\mu\,(mg + F/2)$

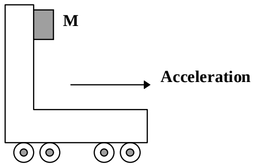
Sol. From F.B.D. N = mg — F sin 30°

So Limiting friction $= \mu N = \mu\left(mg - \frac{F}{2}\right)$

So, option (C) is correct.



Prob 10. *An accelerated system with a vertical wall has co-efficient of friction μ between block and walls as shown in the figure. A block M of mass 1 kg just remains in equilibrium with the vertical wall, when the system has an acceleration of 20 m/s^2. The co-efficient of friction has a value*



(A) 0.10 (B) 0.25

(C) 0.50 (D) 1

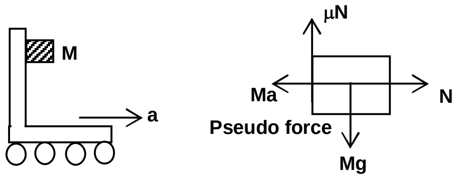
Sol. So, N = Ma

And if mass M is in equilibrium then

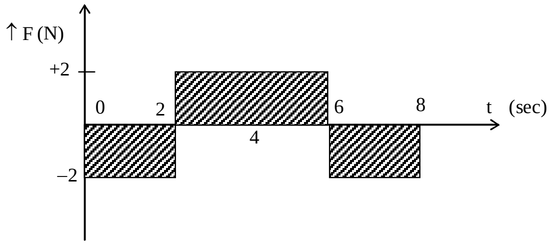
then, $\mu N - Mg = 0$

$$\mu = \frac{g}{a} = \frac{10}{20} = 0.5$$

So, option (C) is correct.



Prob 11. *Force time graph for the motion of a body of mass 2 kg is shown in figure. Change in velocity between 0 to 8 sec is*



(A) zero
(B) 4 m/s
(C) 8 m/s
(D) None of these

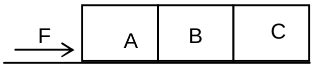
Sol. Area of graph = + $[2 \times (6-2)] - [2 \times 2] - [2 \times 2]$

Change in momentum = $\int F.dt = 8 - 4 - 4 = 0$

Thus there is no change in velocity between 0 to 8 sec.

So option (A) is correct.

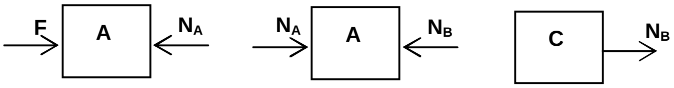
Prob 12. *A constant force F pushes three blocks A, B and C on a horizontal smooth surface. The masses of the blocks are m_A, m_B and m_C respectively. The normal reaction between the blocks B and C will be*



(A) $\dfrac{F(m_B + m_C)}{m_A + m_B + m_C}$ *(B)* $\dfrac{Fm_A}{(m_B + m_C)}$

(C) $\dfrac{Fm_C}{(m_B + m_C)}$ *(D)* $\dfrac{F.m_C}{(m_A + m_B + m_C)}$

Sol. F.B.D. of blocks



If the common acceleration of the block is a, then

$F = (m_A + m_B + m_C)$

So normal reaction between B and C is

$N_B = m_C a = \dfrac{F \cdot m_c}{(m_A + m_B + m_C)}$

So option (D) is correct.

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. In general the normal force is not equal to the weight. Give an example where the two forces are equal in magnitude and at least two examples where they are not.

2. A car is driven up a steep hill at constant speed. Discuss all the forces acting on the car. What pushes it up the hill?

3. If there is a net force on a particle in uniform circular motion, why does the particle's speed not change.

4. A curve in a road has the banking angle calculated for 80 km/h. However, the road is covered with ice, and you plan to creep around the highest lane at 20 km/h. What may happen to your car ? Why ?

5. A reference frame attached to the earth cannot be an inertial frame. Explain.

6. A person is sitting on a moving train and is facing the engine. He tosses up a coin which falls behind him. Find out the reason.

7. Is a 'single isolated force' possible in nature ?

8. Which of Newton's laws of motion is involved in rocket propulsion?

9. Can a body in linear motion be in equilibrium position?

10. Friction is a self – adjusting force. Is this statement correct ? If yes then justify.
    What is limiting friction ? What are the laws of limiting friction ?

11. A body moving over the surface of another body suddenly comes to rest. What happens to friction between the two surfaces ?

12. When walking on ice, one should take short steps rather than long steps. Why

13. Why does a cyclist bend inwards from his vertical position while taking a turn?

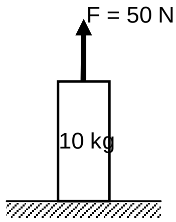
14. A stone tied to one end of a string is whirled in a circle. If the string breaks, the stone flies off tangentially. Explain.

15. Why is it easier to pull a body than to push it.

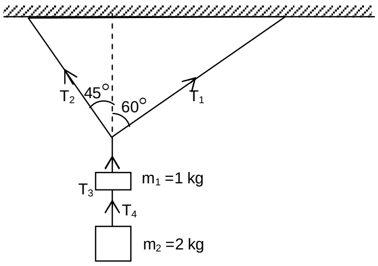
Level - I

1. A ball of mass 0.2 kg falls from a height of 45 m. On striking the ground, it rebounds in 0.1sec with two third of the velocity with which it struck the ground. Calculate
   (a) change in the momentum of the ball immediately after hitting the ground,
   (b) the average force on the ball due to the impact.

2. (a) Find the normal reaction between the block and the horizontal surface.

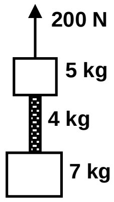


   (b) Find out the tensions T_1, T_2 , T_3 and T_4
   (Take g = 10 m/s^2)

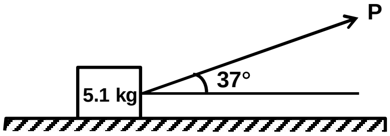


3. A body hangs from a spring balance supported from the roof of an elevator.
   (a) If the elevator has an upward acceleration of 2 m/s^2 and balance reads 240 N, what is the true weight of the body?
   (b) Under what circumstances will the balance read 160 N?
   (c) What will the balance read if the elevator cable breaks? (Take g = 10 ms^{-2})

4. Two blocks shown in figure are connected by a heavy uniform rope of mass 4 kg. An upward force of 200 N is applied as shown.
   (a) What is the acceleration of the system?
   (b) What is the tension at the top of heavy rope?
   (c) What is the tension at the mid-point of the rope?

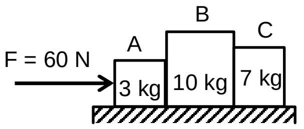


5. A 5.1 kg block is pulled along a frictionless floor by a cord that exerts a force P = 10 N at an angle θ = 37° above the horizontal, as shown in the figure.



   (a) What is the acceleration of the block?
   (b) The force P is slowly increased. What is the value of P just before the block breaks off the floor?
   (c) What is the acceleration of the block just before it is lifted off the floor?
   (d) Suppose the surfaces are rough with μ = 0.4, for what value of P the block just begins to move?

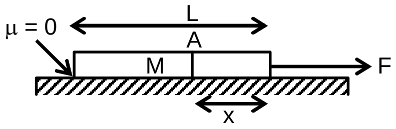
6. Find out the mutual contact forces between A and B and between blocks B and C.



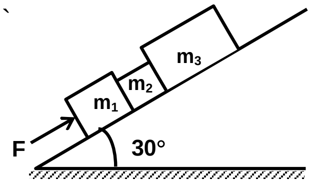
μ = 0 for all surfaces

7. A block weighing 100 kg is placed on an inclined plane of height 6 m and base 8 m. The co-efficient of friction is 0.3. ($g = 9.8\ m/s^2$)
   (a) Would the block slide down the inclined plane due to its own weight? If so, how far it will move in 1s starting from rest?
   (b) What force parallel to the inclined plane must be applied to just support the block on the plane?
   (c) What force parallel to the inclined plane is required to keep the block moving up the plane at constant velocity?
   (d) If an upward force of 940 N parallel to the inclined plane is applied to the block what will be its acceleration?
   (e) How far will the block move in 1s starting from rest?
   (f) What will happen if an upward force of 500 N parallel to the inclined plane is applied?
   (g) If an upward force of 260 N parallel to the inclined plane is applied what will happen? How far will the block move in 1s starting from rest?

8. Find the tension in rope at section A, at a distance x from the right end.

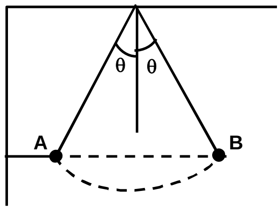


9. Three blocks $m_1 = 3$ kg, $m_2 = 2$ kg, $m_3 = 5$ kg, lie on an inclined frictionless surface as shown in the figure.
   (a) What force (F) parallel to the incline is needed to push the blocks up the plane with an acceleration $a = 2\ m/s^2$?
   (b) Find the contact force between m_1 & m_2 and m_2 and m_3.



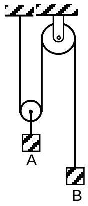
10. A child places a picnic basket on the outer rim of a merry go round that has a radius of 4.0 m and revolves once in every 24 s. What is the minimum co-efficient of static friction for the basket to stay on the merry go round?

11. A ball is held at rest in position A as shown in the figure by two light cords. The horizontal cord is cut and the ball swings as a pendulum. What is the ratio of the tensions in the supporting cord, in position A, to that in position B?

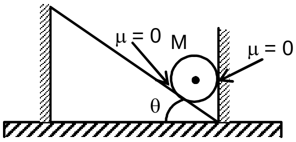


12. A balloon is descending with a constant acceleration ‘a’ less than the acceleration due to gravity. The mass of the balloon, with its basket and contents is M. What mass m, of ballast (Sand bags) should be released so that the balloon will begin to accelerate upward with constant acceleration ‘a’? (Neglect air resistance)

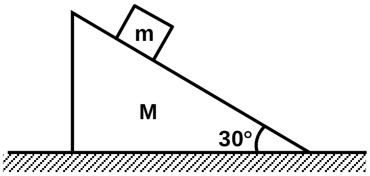
13. Two blocks ‘A’ and ‘B’ having masses m_A and m_B respectively are connected by an arrangement shown in the fig. Calculate the downward acceleration of the block B. Assume the pulleys to be massless. Under what condition the block A will have downward acceleration ?



14. Find the normal reaction forces acting on the vertical wall (say N_1) and the fixed incline surface (say N_2) respectively by the sphere of mass M.



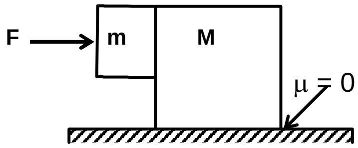
15. A block of mass m = 1 kg rests on a wedge of mass M = 9 kg, which in turn is placed on a table as shown in the figure. All the surfaces are smooth.
(a) What horizontal acceleration 'a' must M have relative to stationary table so that m remains stationary relative to the wedge?



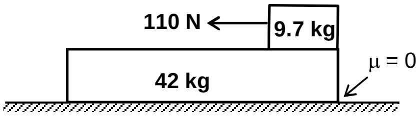
(b) Find the horizontal force required to maintain this acceleration.

Level- II

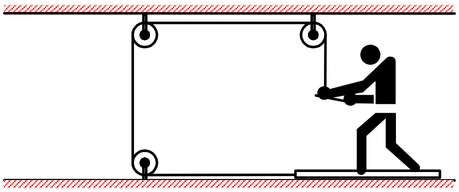
1. The two blocks m = 5 kg and M = 25 kg as shown in the figure are free to move. The coefficient of friction between the blocks is $\mu_s = 0.4$, but the coefficient of friction between ground and M is zero. What is the minimum horizontal force F required to hold m against M?



2. A 42 kg slab rests on a frictionless floor. A 9.7 kg block rests on the top of the slab as shown in the figure. The coefficient of static friction between the block and the slab is 0.53, while the coefficient of kinetic friction is 0.38. The 9.7 kg block is acted on by a horizontal force of 110 N. What are the resulting accelerations of
(a) the block? (b) the slab?

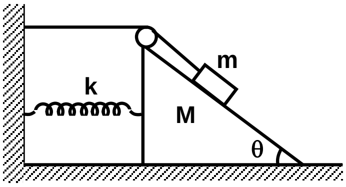


3. The friction coefficient between the board and the floor shown in figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor. The mass of man is M and the mass of plank is m.

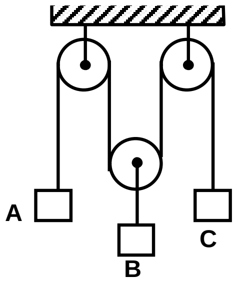


4. A car moves with constant tangential acceleration $a_T = 0.80\ m/s^2$ along a horizontal surface circumscribing a circle of radius R = 40m. The coefficient of sliding friction between the wheels of the car and the surface is μ = 0.20. What distance will the car ride without sliding if its initial velocity is zero ?

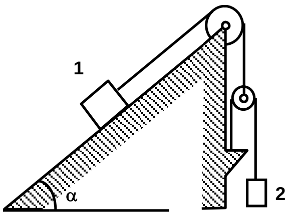
5. A wedge of mass 'M' and angle of inclination 'θ' and of mass 'm' is arranged in a manner shown in the figure. The spring of force constant 'k' attached to the wedge. Assuming the pulleys to be massless and all surfaces to be frictionless. Find the compression of the spring under equilibrium condition.



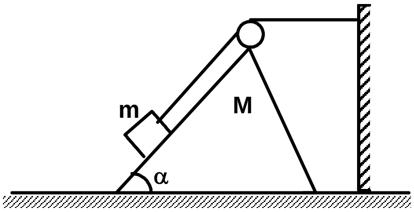
6. The pulley block system shown in the figure is released from rest. Assuming the pulleys to be light and frictionless and the string to be light and inextensible, find
(a) the acceleration of the blocks A, B and C.
(b) the tension in the string connecting the blocks. The masses of the block A, B and C are m, m and 2m respectively.



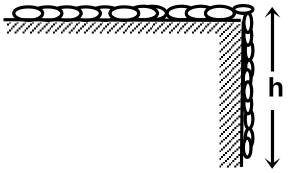
7. Find the acceleration a of body 2 in the arrangement shown in Fig. If its mass is n times as great as the mass of bar 1 and the angle that the inclined plane forms with the horizontal is equal to α. The masses of the pulleys and the threads, as well as the friction, are assumed to be negligible. Look into possible cases.



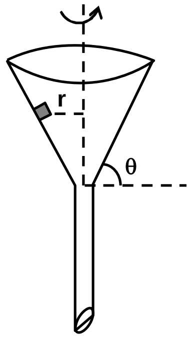
8. In the arrangement shown in the figure, the masses m of the bar and M of the wedge, as well as the wedge angle α, are known. The masses of the pulley and thread are negligible. Friction is absent. Find the acceleration of the wedge M.



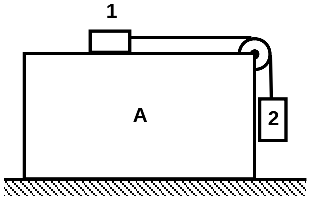
9. A uniform chain of length ℓ is released from rest on a smooth horizontal table with a portion h of the chain overhanging as shown in the figure. Find the time taken by the chain to slip off the table.



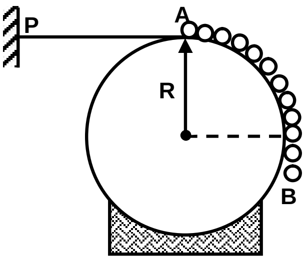
10. A small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of f revolutions per second. The wall of the funnel makes an angle θ with the horizontal. The coefficient of static friction between the cube and the funnel is μ and the centre of the cube is at a distance r from the axis of rotation find the
(a) largest and
(b) smallest value of f for which the cube will not move with respect to the funnel.



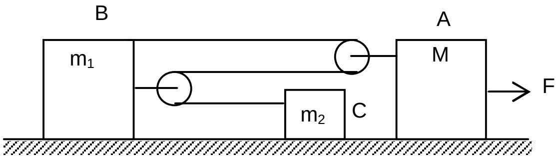
11. What is the minimum and maximum acceleration with which bar A (Fig.) should be shifted horizontally to keep bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal , and the co-efficient of friction between the bar and the bodies is equal to k. The masses of the pulley and the threads are negligible, the friction in the pulley is absent.



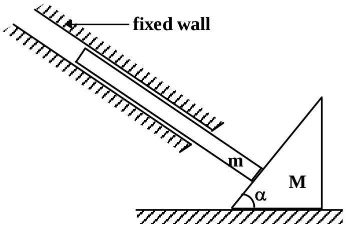
12. A uniform flexible chain of length 1.50 m rest on a fixed smooth sphere of radius $R = 2/\pi$ m such that one end A of chain is at top of the sphere while the other end B is hanging freely. Chain is held stationary by a horizontal thread PA as shown in figure.
Calculate the acceleration of the chain when the thread is burnt.



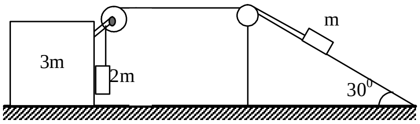
13. In the shown figure, the blocks and pulley are ideal and force of friction is absent. External horizontal force F is applied as shown in the figure. Find the acceleration of block C.



14. In the arrangement shown in the figure, the rod of mass m held by two smooth walls, always remains perpendicular to the surface of the wedge of mass M. Assuming that all the surfaces are frictionless, find the acceleration of the rod and that of the wedge.



15. Neglect friction. Find accelerations of m, 2m and 3m as shown in the figure. The wedge is fixed.



Objective:

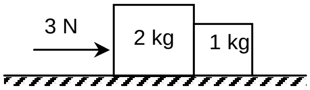
Level- I

1. When a bird of weight W sits on a stretched wire, the tension T in the wire is
(A) > W/2 (B) = W
(C) < W (D) None of these

2. A machine gun fires n bullets per second and the mass of each bullet is m. If v is the speed of each bullet then the force exerted on the machine gun is
(A) mng (B) mnv
(C) mnvg (D) mnv/g

3. A particle of mass 'm' moving with a velocity $\vec{v}$ strikes a horizontal surface at angle θ with the vertical and rebounds with the same magnitude and at the same angle with the vertical. The magnitude of change in momentum is
(A) 2 mv cos θ (B) 2 m v sin θ
(C) 2 mv (D) 0

4. Two blocks of masses 2 kg and 1 kg are in contact with each other on a horizontal frictionless table. When a horizontal force of 3.0 N is applied to the block of mass 2 kg, the value of the force of contact between the two blocks is:



(A) 4 N (B) 3 N
(C) 2 N (D) 1 N

5. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force F is applied at one end of the rope, the force which the rope exerts on the block is:
(A) $\frac{FM}{m+M}$ (B) $\frac{Fm}{m+M}$
(C) $\frac{FM}{M-m}$ (D) $\frac{Fm}{M-m}$

6. Two masses *m* and *m′* are tied with a thread passing over a pulley, *m′* is on a frictionless horizontal surface and m is hanging freely. If acceleration due to gravity is g, the acceleration of *m′* in this arrangement will be
(A) g (B) $\frac{mg}{(m+m')}$
(C) $\frac{m'g}{(m+m')}$ (D) $\frac{g(m-m')}{(m+m')}$

7. Which of the following statements is true in a tug of war.
(A) The team which applies a greater force on the rope than the other wins.
(B) The team which applies a smaller force on the other wins.
(C) The team which pushes harder against the ground wins.
(D) none of these

8. Two students of equal weight try to break a rope which can break if the tension in it is equal to the sum of their weights
(A) They should pull against each other applying a force equal to their weight on the rope.
(B) They should hang the rope over a pulley and pull on either side of the pulley downwards.
(C) They should climb on either side of the rope suspended over the pulley.
(D) They should tie one-end of the rope to the ceiling and pull the other end together.

9. A massless rope passes over a frictionless pulley. A monkey holds one end of the rope and a mirror, having the same weight as the monkey, is attached to the other end of the rope at the same level. The monkey gets scared by its own image. It can get away from its image by
(A) climbing up the rope.
(B) moving down the rope.
(C) by releasing the rope.
(D) None of the above.

10. A chain of length L and mass M is hanging by fixing its upper end to a rigid support. The tension in the chain at a distance *x* from the rigid support is:
(A) Zero
(B) $\frac{MgL}{(L-x)}$
(C) $Mg\frac{(L-x)}{L}$
(D) Mgx / L

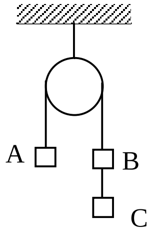
11. When a force of constant magnitude always act perpendicular to the motion of a particle then:
(A) Velocity is constant
(B) Acceleration is constant
(C) KE is constant
(D) None of these

12. A given object takes n times as much time to slide down a 45^o rough incline as it takes to slide down a perfectly smooth 45^o incline. The coefficient of kinetic friction between the object and the incline is given by
(A) $\mu_k = \frac{1}{(1-n^2)}$
(B) $\mu_k = 1-\frac{1}{n^2}$
(C) $\mu_k = \sqrt{\frac{1}{(1-n^2)}}$
(D) $\sqrt{1-\frac{1}{n^2}}$

13. Sand drops are falling gently at the rate of 2 kg/sec. on a conveyor belt moving horizontally with a velocity of 5 m/s. The extra force and extra power required to keep the belt moving will be respectively,
(A) 10 N force and 20 watts
(B) 10 N force and 50 watts.
(C) 20 N force and 20 watts
(D) 50 N force and 50 watts

14. A body of mass 60 kg is dragged with just enough force to start moving on a rough surface with coefficients of static and kinetic frictions 0.5 and 0.4 respectively. On applying the same force what is the acceleration ?
(A) 0.98 m/s^2
(B) 9.8 m/s^2
(C) 0.54 m/s^2
(D) 5.292 m/s^2

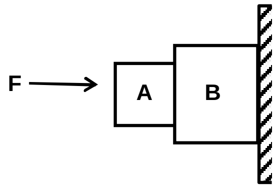
15. Three equal weights A, B, C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in figure. The tension in the string connecting weights B and C is



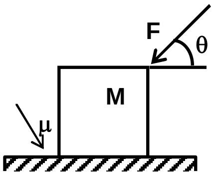
(A) Zero (B) 13 N
(C) 3.3 N (D) 19.6 N

Level- II

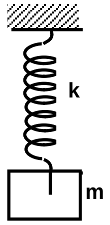
1. Two blocks A and B are pushed against the wall with the force F. The wall is smooth but the surfaces in contact of A and B are rough. Which of the following is true for the system of the blocks?
(A) F should be equal to weight of A and B.
(B) F should be less than the weight of A and B.
(C) F should be more than the weight of A and B
(D) The system cannot be in equilibrium (at rest).



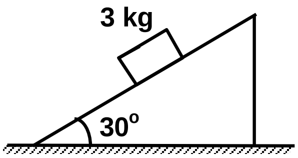
2. A block of mass M lies on a rough surface of coefficient of friction μ. A force is applied on it at an angle θ to the horizontal as shown, and the block is at rest. The frictional force acting on the block will be
(A) $F\cos\theta$ (B) $\mu(mg + F\sin\theta)$
(C) $\mu(mg - F\sin\theta)$ (D) μmg



3. The spring mass system shown in the figure is in equilibrium. If the mass m is pulled down by a distance mg/3k and released, its instantaneous acceleration will be
(A) g/3 upward (B) 2g/3 downward
(C) g/3 downward (D) 2g/3 upward

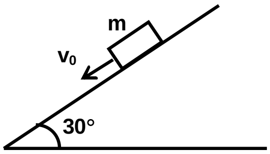


4. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be
(A) 26 N (B) 19.5 N
(C) 10 N (D) 30 N



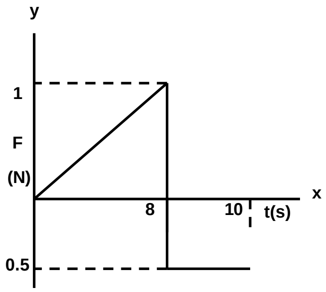
5. A body of mass 60 kg is dragged along a horizontal surface by a horizontal force which is just sufficient to start the motion of the body from rest. If the coefficients of static and kinetic friction are 0.5 and 0.4 respectively, the acceleration of the body is
(A) $0.98\ m/s^2$ (B) $9.8\ m/s^2$
(C) $0.54\ m/s^2$ (D) $5.292\ m/s^2$

6. A block of mass m is pushed down on a rough inclined plane (coefficient of friction is 0.25) with a velocity v_0 as shown in the figure. Then the block will

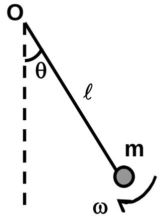


(A) decelerate and come to rest (B) accelerate downward
(C) move downward with velocity v_0 (D) first accelerate and then decelerate

7. A force time graph for the motion of a body is shown in the figure. The change in the momentum of the body between zero and 10 sec. is
(A) zero
(B) 4 kg m/s
(C) 5 kg m/s
(D) 3 kg m/s

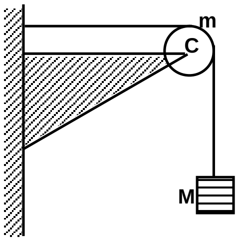


8. A simple pendulum swings in a vertical plane about the point of suspension O. In the position shown in the figure, the string has an angular velocity ω radian/second. The instantaneous tension in the string is



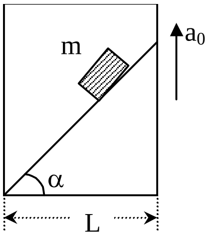
(A) mg cos θ
(B) $mg\left(\cos\theta + \frac{\ell\omega^2}{g}\right)$
(C) $mg\left(\cos\theta - \frac{\ell\omega^2}{g}\right)$
(D) $m\ell\omega^2$

9. A string of negligible mass, going over a clamped pulley of mass m, supports a block of mass M as shown in the figure. The force on the pulley by the clamp C is given by
(A) $\sqrt{2}Mg$
(B) $\sqrt{2}mg$
(C) $g\sqrt{(M+m)^2 + m^2}$
(D) $g\sqrt{(M+m)^2 + M^2}$

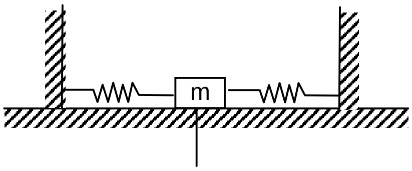


10. A long horizontal rod has a bead which can slide along its length, The bead is initially placed at a distance L from one end A of the rod. The rod is set in angular motion in the horizontal plane about the end A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping on the rod is
(A) $\sqrt{\frac{\mu}{\alpha}}$
(B) $\frac{\mu}{\sqrt{\alpha}}$
(C) $\frac{1}{\sqrt{\mu\alpha}}$
(D) infinite

11. A particle slides down a smooth inclined plane of elevation α fixed in the elevator going up with an acceleration a_0 as shown in figure. The base of the incline has a length L. The time taken by the particle to reach the bottom is
(A) $\left[\frac{2L}{(g+a_0)\sin\alpha\cos\alpha}\right]^{1/2}$
(B) $\left[\frac{2L}{g\sin\alpha\cos\alpha}\right]^{1/2}$
(C) $\left[\frac{g\sin\alpha\cos\alpha}{2L}\right]^{1/2}$
(D) $\left[\frac{2L}{a_0\sin\alpha\cos\alpha}\right]^{1/2}$



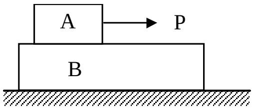
12. A body of mass 'm' is connected to two springs of spring constants K_1 and K_2 and is in equilibrium on a smooth horizontal surface as shown. If the body is displaced to the left by a small distance 'x' from the position shown, what is the velocity of the body as it passes through this position again? (springs are massless)

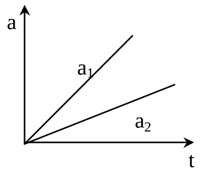


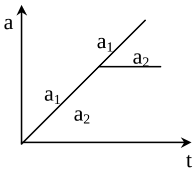
(A) 0
(B) $x\sqrt{\frac{K_1 + K_2}{m}}$
(C) $(K_1 + K_2)x$
(D) can't say

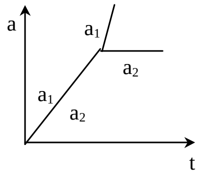
13. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1m. The angle made by the rod with the track is
(A) Zero
(B) 30°
(C) 45°
(D) 60°

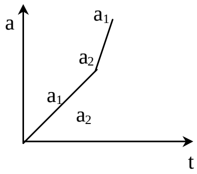
14. Block A is placed on block B, whose mass is greater than that of A. There is friction between the blocks, while the ground is smooth. A horizontal force P, linearly increasing with time, begins to act on A. The acceleration a_1 and a_2 of A and B respectively are plotted against time t. Choose the correct graph.



(A)


(B)


(C)


(D)


15. A block of mass M rest on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A force F = Mg acting at an angle θ with the vertical side of the block pulls it. In which of the following cases, the block can be pulled along the surface?
(A) $\tan\theta \geq \mu$
(B) $\cot\theta \geq \mu$
(C) $\tan\theta/2 \geq \mu$
(D) $\cot\theta/2 \geq \mu$

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – I

1. (a) 10 Ns (b) 100 N
2. (a) 50 N (b) $T_1 = \frac{60\,N}{1+\sqrt{3}}$, $T_2 = \frac{30\sqrt{6}}{1+\sqrt{3}}N$, $T_3 = 30$ N, $T_4 = 20$ N
3. (a) 200 N (b) When the elevator as a downward acceleration of 2 m/s^2(c) zero
4. 2.5 m/s^2, 137.5 N, 112.5 N
5. (a) 1.56 m/s^2 (b) 83.3 N (c) 13 m/s^2 (d) 19.6 N
6. $F_{AB} = 51$ N, $F_{BC} = 21$ N
7. (a) Yes, 176.4 cm; (b) 36 kg wt; (c) 84 kg wt; (d) 116.8 cm/s^2 upwards; (e) 58.4 cm; (f) remains at rest, (g) slides down with acceleration 92.8 cm/s^2, 46.4 cm
8. $F\left(\frac{L-x}{L}\right)$
9. (a) 70 N (b) 49 N, 35 N
10. 0.032
11. $\frac{T_A}{T_B} = \sec^2\theta$
12. $m = \frac{2\ \ Ma}{g+a}$
13. $2g\,(2m_B - m_A) / (4m_B + m_A)$; when $m_A > 2m_B$
14. $N_1 = Mg\tan\theta$, $N_2 = Mg\sec\theta$
15. (a) 5.658 m/s^2 rigtward (b) 56.58 N

Level – II

1. 150 N
2. (a) 7.6 m/s^2 leftward (b) 0.86 m/s^2 leftward
3. $\frac{\mu(M+m)g}{(1+\mu)}$
4. 45.82 m
5. $\frac{mg\sin\theta}{k}$
6. (a) $a_A = \frac{3g}{11}$ (downward), $a_B = \frac{5g}{11}$ (upward), $a_C = \frac{7g}{11}$ (downward)

   (b) $T_A = T_C = \frac{8mg}{11}$, $T_B = \frac{16mg}{11}$
7. $\frac{2g(2n-\sin\alpha)}{(4n+1)}$

8. $\dfrac{mg \sin\alpha}{M+2m(1-\cos\alpha)}$

9. $\left\{\sqrt{\dfrac{\ell}{g}}\ln\left[\dfrac{\ell}{h}+\sqrt{\left(\dfrac{\ell}{h}\right)^2-1}\right]\right\}$ sec.

10. (a) $\dfrac{1}{2\pi}\sqrt{\dfrac{g(\tan\theta+\mu)}{r(1-\mu\tan\theta)}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{g(\tan\theta-\mu)}{r(1+\mu\tan\theta)}}$

11. $a_{min} = g(1-k)/(1+k)$, $a_{max} = \left(\dfrac{1+k}{1-k}\right)g$

12. $\dfrac{4+\pi}{3\pi}g$

13. $\dfrac{2m_1F}{m(m_1+9m_2)+4m_1m_2}$

14. $\dfrac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}, \dfrac{mg\cos\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}$

15. $\dfrac{13g}{34}, \sqrt{\dfrac{397}{34}}g, \dfrac{3g}{17}$

Objective:

Level - I

1.	A	2.	B	3.	A
4.	D	5.	A	6.	B
7.	C	8.	D	9.	D
10.	C	11.	C	12.	B
13.	B	14.	A	15.	D

Level - II

1.	D	2.	A	3.	A
4.	D	5.	A	6.	B
7.	D	8.	B	9.	D
10.	A	11.	A	12.	B
13.	C	14.	C	15.	D