F.R.M.

[Final Revision Module]

JEE Main - MATHEMATICS

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JEE - MAIN

MATHEMATICS

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CONTENTS

Answer Key & Solutions

IIT-JEE SYLLABUS

CALCULUS

functions;. one-one, into and onto functions, composition of functions. Real - valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic and exponential functions, inverse functions. Graphs of simple functions. Limits, continuity and differentiability. Differentiation of the sum, difference, product and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite and implicit functions; derivatives of order upto two. Rolle's and Lagrange's Mean Value Theorems. Applications of derivatives: Rate of change of quantities, monotonic - increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals. Integral as an anti - derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities. Evaluation of simple integrals of the type Integral as limit of a sum. Fundamental Theorem of Calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form. Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equations of the type: $\frac{dy}{dx} + p(x)\,y = q(x)\,dx$

TRIGONOMETRY

Trigonometrical identities and equations. Trigonometrical functions. Inverse trigonometrical functions and their properties. Heights and Distances.

ALGEBRA

Sets and their representation; Union, intersection and complement of sets and their algebraic properties; Power set; Relation, Types of relations, equivalence relations Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a+ib and their representation in a plane, Argand diagram, algebra of complex numbers, modulus and argument (or amplitude) of a complex number, square root of a complex number, triangle inequality, Quadratic equations in real and complex number system and their solutions. Relation between roots and co-efficients, nature of roots, formation of quadratic equations with given roots. Matrices, algebra of matrices, types of matrices, determinants and matrices of order two and three. Properties of determinants, evaluation of determinants, area of triangles using determinants. Adjoint and evaluation of inverse of a square matrix using determinants and elementary transformations, Test of consistency and solution of simultaneous linear equations in two or three variables using determinants and matrices. Fundamental principle of counting, permutation as an arrangement and combination as selection, Meaning of P (n,r) and C (n,r), simple applications. Principle of Mathematical Induction and its simple applications. Binomial theorem for a positive integral index, general term and middle term, properties of Binomial coefficients and simple applications. Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers. Relation between A.M. and G.M. Sum upto n terms of special series: S n, S n2, Sn3. Arithmetico – Geometric progression. Measures of Dispersion: Calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance and mean deviation for grouped and ungrouped data. Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and Binomial distribution. Statements, logical operations and, or, implies, implied by, if and only if. Understanding of tautology, contradiction, converse and contrapositive.

CO-ORDINATE GEOMETRY

Cartesian system of rectangular co-ordinates 10 in a plane, distance formula, section formula, locus and its equation, translation of axes, slope of a line, parallel and perpendicular lines, intercepts of a line on the coordinate axes. Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocentre and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines. Standard form of equation of a circle, general form of the equation of a circle, its radius and centre, equation of a circle when the end points of a diameter are given, points of intersection of a line and a circle with the centre at the origin and condition for a line to be tangent to a circle, equation of the tangent. Sections of cones, equations of conic sections (parabola, ellipse and hyperbola) in standard forms, condition for $y = mx + c$ to be a tangent and point (s) of tangency. Coordinates of a point in space, distance between two points, section formula, direction ratios and direction cosines, angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms, intersection of a line and a plane, coplanar lines. Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.

1 CALCULUS

EXERCISE

1. Range of the function f defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ (where [*] and {*} respectively denotes the greatest integer and the fractional part function) is

(A) I, the set of integers
(B) N, the set of natural numbers
(C) W, the set of whole numbers
(D) Q, the set of rational numbers

2. If f(x) is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, where g(x) is an odd function, then the value of f(5) is

(A) 0 (B) $\frac{37}{75}$ (C) 4 (D) $\frac{51}{77}$

3. $\lim\limits_{x\to\pi/2} \frac{\sin(x\cos x)}{\cos(x\sin x)}$ is equal to

(A) 1 (B) $\pi/2$ (C) $2/\pi$ (D) does not exist

4. $\lim\limits_{x\to 0}\left(\frac{\ln(a+x)-\ln a}{x}\right) + k\lim\limits_{x\to e}\frac{\ln x - 1}{x-e} = 1$ then

(A) $k = e\left(1-\frac{1}{a}\right)$
(B) k = e (1 + a)
(C) k = e (2 – a)
(D) The equality is not possible

5. The set of all points, where $f(x) = \sqrt[3]{x^2|x|} - |x| - 1$ is not differentiable is

(A) {0} (B) {–1, 0, 1} (C) {0, 1} (D) none of these

6. If $f(x) = \{x^2\} - (\{x\})^2$, where {x} denotes the fractional part of x, then

(A) f(x) is continuous at x = 2 but not at x = –2
(B) f(x) is continuous at x = –2 but not at x = 2
(C) f(x) is continuous at x = –2 but not at x = –2
(D) f(x) is discontinuous at x = 2 and x = –2

7. The set of values of a for which the function $f(x) = (4a-3)(x+\ln 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$ does not possess critical points is

(A) $(1, \infty)$ (B) $[1, \infty)$ (C) $(-\infty, 2)$ (D) $(-\infty, -4/3) \cup (2, \infty)$

8. If f(x) = x + tan x and f is inverse of g, then g′(x) is equal to

(A) $\frac{1}{1+(g(x)-x)^2}$ (B) $\frac{1}{1-(g(x)-x)^2}$ (C) $\frac{1}{2+(g(x)-x)^2}$ (D) $\frac{1}{2-(g(x)-x)^2}$

9. Tangents are drawn from the origin to the curve y = sin x, then their point of contact lie on the curve

(A) $x^2 + y^2 = 1$ (B) $x^2 - y^2 = 1$ (C) $\frac{1}{x^2} + \frac{1}{y^2} = 1$ (D) $\frac{1}{y^2} - \frac{1}{x^2} = 1$

10. The slope of the normal at the point with abscissa x = –2 of the graph of the function $f(x) = |x^2 - |x||$ is

(A) –1/6 (B) –1/3 (C) 1/6 (D) 1/3

11. Let $y = x^2 e^{-x}$, then the interval in which y increases with respect to x is

(A) (–∞, ∞) (B) (–2, 0) (C) (2, ∞) (D) (0, 2)

12. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing ?

(A) (0, π/2) (B) (0, 1) (C) $\left(\frac{\pi}{2}, \pi\right)$ (D) None of these

13. The point (0, 5) is closest to the curve $x^2 = 2y$ at

(A) $(2\sqrt{2}, 0)$ (B) (2, 2) (C) $(-2\sqrt{2}, 0)$ (D) $(2\sqrt{2}, 4)$

14. The global maxima of f(x) = [2{–x² + x + 1}] is (where {*} denotes fractional part of x and [*] denotes greatest integer function)

(A) 2 (B) 1 (C) 0 (D) none of these

15. |ln x| dx equals (0 < x < 1)

(A) x + x |ln x| + c (B) x |ln x| – x + c (C) x + |ln x| + c (D) x – |ln x | + c

16. $\int \sqrt{(x-3)}\ \{\sin^{-1}(\ln x) + \cos^{-1}(\ln x)\}\,dx$ equals

(A) $\frac{\pi}{3}(x-3)^{3/2} + c$ (B) 0 (C) does not exist (D) none of these

17. $\int \frac{3+2\cos x}{(2+3\cos x)^2}\,dx$ is equal to

(A) $\left(\frac{\sin x}{2+3\cos x}\right)+c$ (B) $\left(\frac{2\cos x}{2+3\sin x}\right)+c$ (C) $\left(\frac{2\cos x}{2+3\cos x}\right)+c$ (D) $\left(\frac{2\sin x}{2+3\sin x}\right)+c$

18. The value of $\int_0^{11} [x]^3 .\,dx$, where [*] denotes the greatest integer function, is

(A) 0 (B) 14400 (C) 2200 (D) 3025

19. If $I_1 = \int_0^{\pi/2} \cos(\sin x)\,dx$; $I_2 = \int_0^{\pi/2} \sin(\cos x)\,dx$ and $I_3 = \int_0^{\pi/2} \cos x\,dx$, then

(A) $I_1 > I_2 > I_3$ (B) $I_2 > I_3 > I_1$ (C) $I_3 > I_1 > I_2$ (D) $I_1 > I_3 > I_2$

20. The value of the integral $\int_{-10}^{0} \frac{\left|\frac{2[x]}{3x-[x]}\right|}{\left(\frac{2[x]}{3x-[x]}\right)} dx$, where [*] denotes the greatest integer function, is

(A) 0 (B) –10 (C) 10 (D) none of these

21. The area bounded by the curves $y = \sin^{-1}|\sin x|$ and $y = (\sin^{-1}|\sin x|)^2$, $0 \le x \le 2\pi$ is

(A) $\left(\frac{\pi^3}{3} + \frac{4}{3}\right)$ sq unit (B) $\left(\frac{\pi^3}{6} - \frac{\pi^2}{2} + \frac{4}{3}\right)$ sq unit

(C) $\left(\frac{\pi^2}{2} - \frac{4}{3}\right)$ sq unit (D) $\left(\frac{\pi^2}{6} - \frac{\pi}{4} + \frac{4}{3}\right)$ sq unit

22. The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is

(A) $\frac{7}{120}$ sq unit (B) $\frac{9}{120}$ sq unit (C) $\frac{11}{120}$ sq unit (D) $\frac{13}{120}$ sq unit

23. Solution of the differential equation $\frac{dy}{dx} = \frac{y(x - y\ln y)}{x(x\ln x - y)}$ is

(A) $\frac{x\ln x + y\ln y}{xy} = c$ (B) $\frac{x\ln x - y\ln y}{xy} = c$ (C) $\frac{\ln x}{x} + \frac{\ln y}{y} = c$ (D) $\frac{\ln x}{x} - \frac{\ln y}{y} = c$

24. Solution of differential equations $(x\cos x - \sin x)\,dx = \frac{x}{y}\sin x\,dy$ is

(A) $\sin x = \ln|xy| + c$ (B) $\ln\left|\frac{\sin x}{x}\right| = y + c$

(C) $\left|\frac{\sin x}{xy}\right| = c$ (D) none of these

25. The solution of the equation $\frac{dy}{dx} + x(x + y) = x^3(x + y)^3 - 1$ is

(A) $\frac{1}{(x+y)^2} = x^2 + 1 + ce^x$ (B) $\frac{1}{(x+y)} = x^2 + 1 + ce^x$

(C) $\frac{1}{(x+y)^2} = x^2 + 1 + ce^{x^2}$ (D) $\frac{1}{(x+y)} = x^2 + 1 + ce^{x^2}$

26. The graph of $f(x) = \left|\left|\left(\frac{1}{|x|} - n\right)\right| - n\right|$ is lie in the (n > 0)

(A) I and II quadrant (B) I and III quadrant (C) I and IV quadrant (D) II and III quadrant

27. If $f(x) = -\frac{x|x|}{1+x^2}$, then $f^{-1}(x)$ equals

(A) $\sqrt{\frac{|x|}{1-|x|}}$ (B) $(\text{sgn } x)\sqrt{\frac{|x|}{1-|x|}}$ (C) $-\sqrt{\frac{x}{1-x}}$ (D) none of these

28. The function $f(x) = \sin\left(\frac{\pi x}{n!}\right) - \cos\left(\frac{\pi x}{(n+1)!}\right)$ is

(A) non periodic (B) periodic, with period 2(n !)
(C) periodic, with period (n + 1) (D) none of the above

29. $\lim_{x \to a^-}\left(\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right)$ (a > 0), where [*] denotes the greatest integer less than or equal to x, is

(A) $a^2 - 2$ (B) $a^2 - 1$ (C) a^2 (D) $a^2 + 1$

30. $\lim_{n \to \infty} \frac{n^{\alpha} \sin^2 n!}{n+1}$, $0 < \alpha < 1$, is equal to

(A) 0 (B) 1 (C) ∞ (D) none of these

31. $\lim_{x \to 0} \frac{1}{x}\left(\int_y^c e^{\sin^2 t}\, dt - \int_{x+y}^c e^{\sin^2 t}\, dt\right)$ is equal to (where c is a constant)

(A) $e^{\sin^2 y}$ (B) $\sin 2y\, e^{\sin^2 y}$ (C) 0 (D) none of these

32. If $f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x - 2|, & 1 \le x < 2 \end{cases}$ ([*] denotes the greatest integer function), then f(x) is

(A) continuous and non-differentiable at x = −1 and x = 1
(B) continuous and differentiable at x = 0
(C) discontinuous at x = 1/2
(D) continuous but not differentiable at x = 2

33. The function defined by $f(x) = (-1)^{[x^3]}$ ([*] denotes greatest integer function) satisfies

(A) discontinuous for $x = n^{1/3}$, where n is any integer
(B) f(3/2) = 1
(C) $f'(x) = 0$ for $-1 < x < 1$
(D) none of the above

34. If $g(x) = \begin{cases} [f(x)], & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3, & x = \pi/2 \end{cases}$ where [*] denotes the greatest integer function and

$$f(x) = \frac{2(\sin x - \sin^n x) + |\sin x - \sin^n x|}{2(\sin x - \sin^n x) - |\sin x - \sin^n x|},\ n \in R, \text{ then}$$

(A) g(x) is continuous and differentiable at $x = \pi/2$, when $0 < n < 1$
(B) g(x) is continuous and differentiable at $x = \pi/2$, when $n > 1$
(C) g(x) is continuous but not differentiable at $x = \pi/2$, when $0 < n < 1$
(D) g(x) is continuous but not differentiable, at $x = \pi/2$, when $n > 1$

35. If $\sqrt{(1 - x^{2n})} + \sqrt{(1 - y^{2n})} = a\,(x^n - y^n)$, then $\sqrt{\left(\frac{1 - x^{2n}}{1 - y^{2n}}\right)}\ \frac{dy}{dx}$ is equal to

(A) $\frac{x^{n-1}}{y^{n-1}}$ (B) $\frac{y^{n-1}}{x^{n-1}}$ (C) $\frac{x}{y}$ (D) 1

36. If $y = \left(\frac{ax + b}{cx + d}\right)$, then $2\,\frac{dy}{dx} \cdot \frac{d^3y}{dx^3}$ is equal to

(A) $\left(\frac{d^2y}{dx^2}\right)^2$ (B) $3\frac{d^2y}{dx^2}$ (C) $3\left(\frac{d^2y}{dx^2}\right)^2$ (D) $3\frac{d^2x}{dy^2}$

37. If $x^y \cdot y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is

(A) –1   (B) 0   (C) 1   (D) none of these

38. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{(1 + 9u^2)}}\,du$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{\sqrt{(1 + 9y^2)}}$ (B) $\sqrt{(1 + 9y^2)}$ (C) $(1 + 9y^2)$ (D) $\frac{1}{(1 + 9y^2)}$

39. The differential coefficient of $f(\log_e x)$ w.r.t. x, where $f(x) = \log_e x$ is
(A) $x/\log_e x$ (B) $\log_e x/x$ (C) $(x \log_e x)^{-1}$ (D) none of these

40. If $\sqrt{(x^2 + y^2)} = a.\,e^{\tan^{-1}(y/x)}$ a > 0, then y'' (0) is equal to

(A) $\frac{a}{2}\ e^{-\pi/2}$ (B) $ae^{\pi/2}$ (C) $-\frac{2}{a}\ e^{-\pi/2}$ (D) not exist

41. The equation of the tangent to the curve $y = 1 - e^{x/2}$ at the point of intersection with the y = axis is

(A) $x + 2y = 0$ (B) $2x + y = 0$ (C) $x - y = 2$ (D) none of these

42. The slope of the tangent to the curve $y = \int_0^x \frac{dx}{1+x^3}$ at the point where x = 1 is

(A) 1/4 (B) 1/2 (C) 1 (D) none of these

43. The acute angles between the curves $y=|x^2 - 1|$ and $y=|x^2 - 3|$ at their points of intersection is

(A) $\pi/4$ (B) $\tan^{-1}(4\sqrt{2}/7)$ (C) $\tan^{-1}(4\sqrt{7})$ (D) none of these

44. The slope of the normal at the point with abscissa x = –2 of the graph of the function $f(x) = |x^2 - x|$ is

(A) –1/6 (B) –1/3 (C) 1/6 (D) 1/3

45. The value of a in order that $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real value of x, is given by

(A) $a < 1$ (B) $a \geq 1$ (C) $a \geq \sqrt{2}$ (D) $a < \sqrt{2}$

46. The difference between the greatest and the least values of the function $f(x) = \int_0^x (at^2 + 1 + \cos t)\,dt$, $a > 0$ for $x \in [2, 3]$ is

(A) $\frac{19}{3}a + 1 + (\sin 3 - \sin 2)$ (B) $\frac{18}{3}a + 1 + 2\sin 3$

(C) $\frac{18}{3}a - 1 + 2\sin 3$ (D) none of these

47. $\int x^x(1 + \ln|x|)\,dx$ is equal to

(A) $x^x \ln|x| + c$ (B) $e^{x^x} + c$ (C) $x^x + c$ (D) none of these

48. If $f(x) = \lim_{x\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}$, x > 1, then $\int \frac{xf(x)\ln(x + \sqrt{(1+x^2)})}{\sqrt{(1+x^2)}}\,dx$ is

(A) $\ln(x + \sqrt{(1+x^2)}) - x + c$ (B) $\frac{1}{2}\{(x^2 \ln(x + \sqrt{(1+x^2)}) - x^2\} + c$

(C) $x\ln(x + \sqrt{(1+x^2)}) - \ln(x + \sqrt{(1+x^2)}) + c$ (D) none of the above

49. If $\int f(x)\ \sin x \cos x\ dx = \frac{1}{2(b^2 - a^2)}$ ln f(x) + c, then f(x) is equal to

(A) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$

(B) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$

(C) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$

(D) none of these

50. $\int \frac{(x^4 - x)^{1/4}}{x^5}$ dx is equal to

(A) $\frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$

(B) $\frac{4}{5}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$

(C) $\frac{4}{15}\left(1 + \frac{1}{x^3}\right)^{5/4} + c$

(D) none of these

51. The value of $\int_{-1}^{15} \text{sgn}(\{x\})$ dx, where {*} denotes the fractional part function, is

(A) 8 (B) 16 (C) 24 (D) 0

52. The value of the integral $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin x / 2}$ dx ($n \in N$) is

(A) π (B) 2π (C) 3π (D) none of these

53. $\int_0^{\pi/4} \sin x\ d\,(x - [x])$ is equal to (where [*] denotes the greatest integer function)

(A) 1/2 (B) $1 - \frac{1}{\sqrt{2}}$ (C) 1 (D) none of these

54. Let f(x) = minimum $\left(|x|, 1 - |x|, \frac{1}{4}\right)$, $\forall\ x \in R$, then the value of $\int_{-1}^{1} f(x)$ dx is equal to

(A) $\frac{1}{32}$ (B) $\frac{3}{8}$ (C) $\frac{3}{32}$ (D) none of these

55. The value of the definite integral $\int_0^1 \frac{x\ dx}{(x^3 + 16)}$ lies in the interval [a,b]. Then smallest such interval is

(A) $\left[0, \frac{1}{17}\right]$ (B) [0, 1] (C) $\left[0, \frac{1}{27}\right]$ (D) none of these

56. If $f(x) = \cos x - \int_0^x (x-t)\, f(t)\, dt$, then $f''(x) + f(x)$ equals

(A) $-\cos x$ (B) 0 (C) $\int_0^x (x-t)\, f(t)\, dt$ (D) $-\int_0^{-x} (x-t)\, f(t)\, dt$

57. The area bounded by the graph $y = |[x - 3]|$, the x-axis and the lines $x = -2$ and $x = 3$ is ([*] denotes the greatest integer function)

(A) 7 sq unit (B) 15 sq unit (C) 21 sq unit (D) 28 sq unit

58. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 sq unit (B) 2 sq unit (C) $2\sqrt{2}$ sq unit (D) $4\sqrt{2}$ sq unit

59. The area bounded by $y = 2 - |2 - x|$, $y = \frac{3}{|x|}$ is

(A) $\left(\frac{5-4\ln 2}{3}\right)$ sq unit (B) $\left(\frac{2-\ln 3}{2}\right)$ sq unit

(C) $\left(\frac{4-3\ln 3}{2}\right)$ sq unit (D) none of these

60. If $y = f(x)$ passing through (1, 2) satisfies the differential equation $y(1 + xy)\,dx - x\,dy = 0$, then

(A) $f(x) = \frac{2x}{2-x^2}$ (B) $f(x) = \frac{x+1}{x^2+1}$ (C) $f(x) = \frac{x-1}{4-x^2}$ (D) $f(x) = \frac{4x}{1-2x^2}$

61. Solution of $2y \sin x \frac{dy}{dx} = 2 \sin x \cos x - y^2 \cos x$, $x = \frac{\pi}{2}$, $y = 1$ is given by

(A) $y^2 = \sin x$ (B) $y = \sin^2 x$ (C) $y^2 = 1 + \cos x$ (D) none of these

62. Solution of differential equation $(2x \cos y + y^2 \cos x)\,dx + (2y \sin x - x^2 \sin y)\,dy = 0$ is

(A) $x^2 \cos y + y^2 \sin x = c$ (B) $x \cos y - y \sin x = c$

(C) $x^2 \cos^2 y + y^2 \sin^2 x = c$ (D) none of these

63. Given $f(x) = \frac{1}{(1-x)}$, $g(x) = f\{f(x)\}$ and $h(x) = f[f\{f(x)\}]$. Then the value of $f(x).\,g(x).\,h(x)$ is

(A) 0 (B) -1 (C) 1 (D) 2

64. The interval into which the function $y = \frac{(x-1)}{(x^2-3x+3)}$ transforms the entire real line is

(A) $\left[\frac{1}{3}, 2\right]$ (B) $\left[-\frac{1}{3}, 1\right]$ (C) $\left[-\frac{1}{3}, 2\right]$ (D) none of these

65. Consider the function f(x) given by double limit as $f(x) = \lim_{n\to\infty}\lim_{t\to 0} \frac{\sin^2 n!\pi x}{\sin^2(n!\pi x + t^2)}$; x is irrational

(A) f(x) = 0 (B) f(x) = 1 (C) f(x) not defined (D) None

66. Let $f(x) = \begin{cases} (1+|\cos x|)^{ab/|\cos x|}, & n\pi < x < (2n+1)\pi/2 \\ e^a.e^b, & x = (2n+1)\,\pi/2 \\ e^{\cot 2x/\cot 8x}, & (2n+1)\,\pi/2 < x < (n+1)\pi \end{cases}$ If f(x) is continuous in (nπ, (n + 1)π), then

(A) a = 1, b = 2 (B) a = 2, b = 2 (C) a = 2, b = 3 (D) a = 3, b = 4

67. Let $f(x) = \begin{cases} \dfrac{x(1+a\cos x) - b\sin x}{x^3} & ; \ x \neq 0 \\ 1 & ; \ x = 0 \end{cases}$; If f(x) is continuous at x = 0 then a & b are given by

(A) $\frac{5}{2}, \frac{3}{2}$ (B) −5, −3 (C) $-\frac{5}{2}, -\frac{3}{2}$ (D) none

68. If $\sqrt{(x+y)} + \sqrt{(y-x)} = a$, then $\frac{d^2y}{dx^2}$ equals

(A) 2/a (B) $-2/a^2$ (C) $2/a^2$ (D) none of these

69. If P(x) be a polynomial of degree 4, with P(2) = −1, P′(2) = 0, P″ (2) = 2, P‴(2) = −12 and P^{iv} (2) = 24, then P″ (1) is equal to

(A) 22 (B) 24 (C) 26 (D) 28

70. If x = a cos θ, y = b sin θ, then $\frac{d^3y}{dx^3}$ is equal to

(A) $\left(\frac{-3b}{a^3}\right)$ $\text{cosec}^4\theta \cot^4\theta$ (B) $\left(\frac{3b}{a^3}\right)$ $\text{cosec}^4\theta \cot^4\theta$

(C) $\left(\frac{-3b}{a^3}\right)$ $\text{cosec}^4\theta \cot\theta$ (D) none of the above

71. If $F(x) = \frac{1}{x^2}\int_4^x \{4t^2 - 2F'(t)\}\,dt$, then F′ (4) equals

(A) 32/9 (B) 64/3 (C) 64/9 (D) none of these

72. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point

(A) (1, 1) (B) at no point (C) (0, 1) (D) (1, 0)

73. The slope of the tangent to the curve y = $\int_{x}^{x^2} \cos^{-1} t^2\, dt$ at x = $\frac{1}{\sqrt[4]{2}}$ is

(A) $\left(\frac{\sqrt[4]{8}}{2} - \frac{3}{4}\right)\pi$ (B) $\left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4}\right)\pi$ (C) $\left(\frac{\sqrt[5]{8}}{4} - \frac{1}{3}\right)\pi$ (D) none of these

74. For the curve x = t^2 −1, y = t^2 − t, the tangent line is perpendicular to x-axis when

(A) t = 0 (B) t = ∞ (C) t = $\frac{1}{\sqrt{3}}$ (D) t = − $\frac{1}{\sqrt{3}}$

75. Point of contact of tangents to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2) is/are :

(A) (2, 2 ± $\sqrt{3}$) (B) (2, 1 ± $\sqrt{3}$) (C) (1, 1 ± $\sqrt{3}$) (D) None of these

76. The length of the sub-tangent to the curve $x^2 + xy + y^2 = 7$ at (1, −3) is

(A) 3 (B) 5 (C) 15 (D) 3/5

77. Abscissae of points on the curve xy = $(c + x)^2$, the normal at which cuts of numerically equal intercepts from the axes of co-ordinates is/are

(A) c$\sqrt{2}$/2 (B) ± c/2 (C) ± c/$\sqrt{2}$ (D) ± c$\sqrt{2}$

78. If f(x) = $\int_{2}^{x^2} \frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}}$ dt, then the value of $(1 - x^2)\,\{f''(x)\}^2 - 2f'(x)$ at x = $\frac{1}{\sqrt{2}}$ is

(A) 2 − π (B) 3 + π (C) 4 − π (D) none of these

79. The value of $\int_{0}^{16\pi/3} |\sin x|\, dx$ is

(A) 17/2 (B) 19/2 (C) 21/2 (D) none of these

80. The value of $\int_{0}^{n^2} [\sqrt{x}]\, dx$, ([*] denotes the greatest integer function), n ∈ N, is

(A) $\left\{\frac{n(n+1)}{2}\right\}^2$ (B) $\frac{1}{6}$ n (n − 1) (4n + 1)

(C) Σn^2 (D) $\frac{n(n+1)(n+2)}{6}$

81. The value of the integral $\int_{0}^{1} \frac{x^{\alpha} - 1}{\ln x}$ dx, is

(A) ln α (B) 2ln (α + 1) (C) 3 ln α (D) none of these

82. If $\int_{-1}^{4} f(x)\, dx = 4$ and $\int_{2}^{4} (3 - f(x))\, dx = 7$, the value of $\int_{2}^{-1} f(x)\, dx$ is

(A) 2 (B) −3 (C) −5 (D) none of these

83. If $I_1 = \int_0^x e^{zx}e^{-z^2}\,dz$ and $I_2 = \int_0^x e^{-z^2/4}\,dz$, then

(A) $I_1 = e^x I_2$ (B) $I_1 = e^{x^2} I_2$ (C) $I_1 = e^{x^2/2} I_2$ (D) none of these

84. If f(x) is differentibale and $\int_0^{t^2} x f(x)\,dx = \frac{2}{5}t^5$ then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) −5/2 (C) 1 (D) 5/2

85. $\int e^{\tan^{-1}x}(1+x+x^2)\,d(\cot^{-1}x) =$

(A) $e^{\tan^{-1}x}+c$ (B) $-e^{\tan^{-1}x}+c$ (C) $-x\,e^{\tan^{-1}x}+c$ (D) $x e^{\tan^{-1}x}+c$

86. $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}} = a(\tan^2 x + b)\sqrt{\tan x}+c$, then

(A) $a = \sqrt{2}/5$, $b = 1/\sqrt{5}$

(B) $a = \sqrt{2}/5$, $b = 5$

(C) $a = \sqrt{2}/5$, $b = \sqrt{5}$

(D) $a = \sqrt{2}/5$, $b = -1/\sqrt{5}$

87. Area of the region bounded by the curves, $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is given by

(A) $(e - e^{-1} + 2)$ sq unit

(B) $(e - e^{-1} - 2)$ sq unit

(C) $(e + e^{-1} - 2)$ sq unit

(D) none of these

88. Area bounded by $y = 5x^2$ & $y - 9 = 2x^2$ is

(A) $6\sqrt{2}$ (B) $12\sqrt{2}$ (C) $4\sqrt{3}$ (D) $12\sqrt{3}$

89. Area bounded between $y = x.e^{-x^2}$, $y = 0$ and max. ordinate is

(A) $\frac{1}{2}$ (B) $\frac{1}{2\sqrt{e}}$ (C) $\frac{1}{2}\left(1-\frac{1}{\sqrt{e}}\right)$ (D) $\frac{1}{2}\left(1+\frac{1}{e}\right)$

90. Solution of the differential equation, $\sin y \frac{dy}{dx} = \cos y\,(1 - x\cos y)$ is

(A) $\sec y = x - 1 - ce^x$

(B) $\sec y = x + 1 + ce^x$

(C) $\sec y = x + e^x + c$

(D) none of these

91. The differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is constant)

(A) $[1 + (y')^2]^3 = a^3\, y''$

(B) $[1 + (y')^2]^3 = a^2\, (y'')^2$

(C) $[1 + (y')^3] = a^2\, (y'')^2$

(D) none of these

92. Differential equation of all circles with centres on y-axis is

(A) $1 + y_1^2 + yy_2 = 0$

(B) $y_1 + y_1^3 - xy_2 = 0$

(C) $y_1 + y_1^3 + xy_2 = 0$

(D) $1 + y_1^2 - xy_2 = 0$

93. Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real number x and y. If f(30) = 20 then the value f(40) is

(A) 15 (B) 20 (C) 40 (D) 60

94. If $|x| - x + y = 10$ and $x + |y| + y = 12$ then x + y is equal to

(A) $\frac{26}{5}$ (B) 0 (C) $-\frac{26}{5}$ (D) $\frac{42}{5}$

95. For what value of α the limit $\lim_{x\to\infty} \sqrt{2\alpha^2x^2 + \alpha x + 7} - \sqrt{2\alpha^2x^2 + 7}$ will be $\frac{1}{2\sqrt{2}}$

(A) Any value of α (B) $\alpha \neq 0$ (C) $\alpha = 1$ (D) $\alpha = -1$

96. If $\lim_{n\to\infty} n \cos\frac{\pi}{4n} \sin\frac{\pi}{4n} = k$ then k is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) π (D) none

97. Let $f(x) \begin{cases} \frac{x}{2} - 1 & ; \ 0 \le x \le 1 \\ \frac{1}{2} & ; \ 1 < x \le 2 \end{cases}$ & $g(x) = (2x + 1)(x - k) + 3$; $0 \le x < \infty$ then g(f(x)) is continuous at x = 1, if k equals

(A) $\frac{1}{2}$ (B) $\frac{11}{6}$ (C) $\frac{1}{6}$ (D) $\frac{13}{6}$

98. For the function $f(x) = \begin{cases} \frac{1-x}{|x-1|} & ; \ x < 1 \\ 1 & ; \ x = 1 \\ x^2 & ; \ x > 1 \end{cases}$ which of the following is true

(A) it is continuous at all points
(B) it is continuous at all points except at x = 1
(C) it is differentiable at all points
(D) none

99. The derivative of $\cos^{-1}\left(\frac{x^{-1} - x}{x^{-1} + x}\right)$ at x = −1 is

(A) −2 (B) −1 (C) 0 (D) 1

100. If $\int_{\pi/2}^{x} \sqrt{(3 - 2\sin^2 t)} + \int_0^y \cos t \, dt = 0$, then $\left(\frac{dy}{dx}\right)_{\pi,\pi}$ is equal to

(A) −3 (B) 0 (C) $\sqrt{3}$ (D) none of these

101. If $\tan^{-1} y - y + x = 0$ then $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{-2(1+y^2)}{y^5}$ (B) $\frac{1+y^2}{y^5}$ (C) $\frac{2(1+y^2)}{y^4}$ (D) $\frac{2(1+y^2)}{y^5}$

102. If $f(x) = \cos(x^2 - 4[x])$ for $0 < x < 1$, where $[x]$ = G.I.F., then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is equal to

(A) $-\sqrt{\frac{\pi}{2}}$ (B) $\sqrt{\frac{\pi}{2}}$ (C) 0 (D) $\sqrt{\frac{\pi}{4}}$

103. $y = \sin^{-1}(\cos x)$ and $y'(x)$ is identicaly equal to $g(x)$ on $R - \{k\pi, k \in I\}$ then $g(x)$ is equal to
(A) $|\sin x|$ (B) $-\text{sgn}(\sin x)$ (C) $\text{sgn}(\sin x)$ (D) None

104. Let f and g be increasing and decreasing function respectively from $[0, \infty)$ to $[0, \infty)$ $h(x) = f\{g(x)\}$. If $h(0) = 0$, then $h(x) - h(1)$ is :
(A) always 0 (B) always positive (C) always negative (D) none of these

105. Let $f''(x) > 0\ \forall\ x \in R$ and $g(x) = f(2 - x) + f(4 + x)$. Then $g(x)$ is increasing in :
(A) $(-\infty, -1)$ (B) $(-\infty, 0)$ (C) $(-1, \infty)$ (D) None

106. If the radius of a spherical balloon is measured within 1% the error (in percent) in the volume is :
(A) $4\pi r^2\%$ (B) 3% (C) (88/7)% (D) None

107. Let $y_1 = P(x_1)$ and $y_2 = P(x_2)$ be maximum and minimum values of a cubic $P(x)$. $P(-1)=10$, $P(1)=-6$ and $P(x)$ has maximum at $x = -1$ and $P'(x)$ has minimum at $x = 1$. Then distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is :

(A) $\sqrt{56}$ (B) $\sqrt{65}$ (C) $2\sqrt{65}$ (D) $4\sqrt{65}$

108. $f(x) = \begin{cases} \cos\frac{\pi x}{2} : & x > 0 \\ x + a : & x \le 0 \end{cases}$. Then $x = 0$ will be a point of local maximum for $f(x)$ if :

(A) $a \in (-1, 1)$ (B) $a \in (0, 1)$ (C) $a \le 0$ (D) $a \ge 1$

109. The function $f(x) = x^x$ decreases on the interval
(A) $(0, e)$ (B) $(0, 1)$ (C) $(0, 1/e)$ (D) None of these

110. Let $f(x) = \int e^x (x - 1)(x - 2)\,dx$, then f decreases in the interval

(A) $(-\infty, -2)$ (B) $(-2, -1)$ (C) $(1, 2)$ (D) $(2, \infty)$

111. A function f such that $f'(2) = f''(2) = 0$ and f has a local maximum of -17 at 2 is
(A) $(x - 2)^4$ (B) $3 - (x - 2)^4$ (C) $-17 - (x - 2)^4$ (D) none of these

112. If $f(x) = a \log_e |x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$, then

(A) $a = -3/4, b = -1/8$ (B) $a = 3/4, b = -1/8$
(C) $a = -3/4, b = 1/8$ (D) none of these

113. If $\frac{1}{\sqrt{a}} \int_1^a \left(\frac{3}{2}\sqrt{x} + 1 - \frac{1}{\sqrt{x}}\right) dx < 4$, then 'a' may take values :

(A) 0 (B) 4 (C) 9 (D) $\frac{13 \pm \sqrt{313}}{2}$

114. If $n \in N$, then $\int_{-n}^{n} (-1)^{[x]} dx$ equals

(A) 2n (B) n (C) n^2 (D) None of these

115. The point of extremum of $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)\,dt$ are

(A) $x = 1, -1$ (B) $x = -1, 2$ (C) $x = 2, 1$ (D) $x = -2, 1$

116. If $f(n) = \frac{1}{n}$ $[(n + 1)(n + 2)(n + 3) \ldots\ldots (n + n)]^{1/n}$ then $\lim_{n\to\infty} f(n)$ equals

(A) e (B) 1/e (C) 2/e (D) 4/e

117. The value of $\int_1^{7\sqrt{2}} \frac{1}{x(2x^7+1)} dx$ is

(A) log (6/5) (B) 6 log (6/5) (C) (1/7) log(6/5) (D) (1/12) log(6/5)

118. If $a < \int_0^{2\pi} \frac{1}{10+3\cos x} dx$ dx < b, then the ordered pair (a,b) is

(A) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$ (C) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ (D) None of these

119. The value of the integral $\int_0^{3\alpha} \operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)\,dx$ is

(A) $2 \sec \alpha \log \left(\frac{1}{2}\operatorname{cosec} \alpha\right)$ (B) $2 \sec \alpha \log \left(\frac{1}{2}\sec \alpha\right)$

(C) $2 \operatorname{cosec} \alpha \log(\sec \alpha)$ (D) $2 \operatorname{cosec} \alpha \log \left(\frac{1}{2}\sec \alpha\right)$

120. For which of following values of m area bounded between $y = x - x^2$ & $y = mx$ equals $\frac{9}{2}$

(A) −4 (B) −2 (C) 2 (D) 0

121. Area bounded between $4x^2 + y^2 - 8x + 4y - 4 = 0$ is

(A) 3π (B) 4π (C) 6π (D) 5π

122. Area between max. $\{|x|, |y|\} = 1$ & $\pi(x^2 + y^2) = 3$ is

(A) 1 (B) 3 (C) 4 (D) 2

123. Let the function x(t) & y(t) satisfy the differential equation $\frac{dx}{dt} + ax = 0$, $\frac{dy}{dt} + by = 0$.

If $x(0) = 2$, $y(0) = 1$ and $\frac{x(1)}{y(1)} = \frac{3}{2}$ then $x(t) = y(t)$ for t =

(A) $\log_{2/3} 2$ (B) $\log_{4/3} 2$ (C) $\log_2 2$ (D) $\log_3 4$

124. If $\frac{dy}{dx} = \cos(x + y)$, $y\left(\frac{\pi}{2}\right) = 0$, then y(0) =

(A) $\tan^{-1}\left(\frac{\pi}{2} - 1\right)$ (B) $\tan^{-1}\left(\frac{\pi}{2} + 1\right)$ (C) $2\tan^{-1}\left(\frac{\pi}{2} - 1\right)$ (D) $-2\tan^{-1}\left(\frac{\pi}{2} - 1\right)$

125. If $xdy = (2y + 2x^4 + x^2)\,dx$, $y(1) = 0$ then y(e) =

(A) 1 (B) e (C) e^2 (D) e^4

126. The complete set of values of x in the domain of function $f(x) = \sqrt{\log_{x+2\{x\}}([x]^2 - 5[x] + 7)}$ (where [x] and {x} denotes greatest integer and fractional part function respectively)

(A) $\left(-\frac{1}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (2, \infty)$ (B) $(0, 1), \cup (1, \infty)$

(C) $\left(-\frac{2}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (1, \infty)$ (D) $\left(-\frac{1}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (1, \infty)$

127. The function $f(x) = |x| + \frac{|x|}{x}$ is

(A) discontinuous at the origin because |x| is discontinuous there

(B) continous at the origin

(C) discontinuous at the origin because both |x| and $\frac{|x|}{x}$ are discontinous there

(D) discontinous at the origin because $\frac{|x|}{x}$ is discontinuous there

128. If n is even and $g(x, y) = x^n + y^n - nxy + n - 2$ then the number of real solutions of $g(x, y) = 0$ is

(A) 2 (B) 4 (C) 6 (D) 8

129. If [∗] represents greatest integer function, then the solution set of the equation $[x] = [3x]$ is

(A) ϕ (B) $\left[-\frac{1}{3}, 0\right]$ (C) $\left[0, \frac{1}{3}\right]$ (D) $\left[-\frac{1}{3}, \frac{1}{3}\right)$

130. The range of $f(x) = [\sin x + [\cos x + [\tan x + [\sec x]]]]$, $x \in (0, \pi/3)$ is (where [∗] denotes the greatest integer function)

(A) {0, 1} (B) {–1, 0, 1} (C) {1} (D) {1,2}

Questions based on statements (Q. 131 - 150)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

131. **Statement-I :** Let f : $[0, \infty) \to [0, \infty)$, be a function defined by y= f(x) = x^2, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = 1$.

Statement-II : $\left(\frac{dy}{dx}\right)\left(\frac{dx}{dy}\right) = 1$.

132. **Statement-I :** $\int \frac{dx}{\sqrt{4-(x^2)^2}} = \sin^{-1}\frac{x^2}{2} + c$

Statement-II : $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

133. **Statement-I :** Function $f(x) = x^2 + \tan^{-1} x$ is a non-periodic function.
Statement-II : The sum of two non-periodic function is always non-periodic.

134. **Statement-I :** $\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+.....+\frac{x^2}{x^3}\right) = \lim_{x\to\infty}\frac{1^2}{x^3} + \lim_{x\to\infty}\frac{2^2}{x^3} +....+ \lim_{x\to\infty}\frac{x^2}{x^3} = 0$

Statement-II : $\lim_{x\to a}(f_1(x)+f_2(x)+...+f_n(x)) = \lim_{x\to a} f_1(x) + \lim_{x\to a} f_2(x) +...+ \lim_{x\to a} f_n(x)$, where $n \in N$.

135. **Statement-I :** $\lim_{x\to 0} [x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ (where [*] represents greatest integer function) does not exist

Statement-II : $\lim_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist.

136. **Statement-I :** $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is non-differentiable at x = ± 1.

Statement-II : Principal value of $\tan^{-1} x$ are $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

137. **Statement-I :** Let f : R → R is a real-valued function ∀ x, y ∈ R such that $|f(x) - f(y)| \le |x - y|^3$, then f(x) is a constant function.
Statement-II : If derivative of the function w.r.t. x is zero, then function is constant.

138. Consider function f(x) satisfies the relation, $f(x + y^3) = f(x) + f(y^3)$, ∀ x, y ∈ R and differentiable for all x.
Statement-I : If f′(2) = a, then f′(−2) = a.
Statement-II : f9x) is an odd function.

139. Statement-I : Conditions of Lagrange's mean value theorem fail in $f(x) = |x - 1| (x - 1)$.
Statement-II : $|x - 1|$ is not differentiable at $x = 1$.

140. Statement-I : If $27a + 9b + 3c + d = 0$, then the equation $f(x) = 4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one real root lying between $(0, 3)$.
Statement-II : If $f(x)$ is continuous in $[a, b]$, derivable in (a, b) such that $f(a) = f(b)$, then at least one point $c \in (a, b)$ such that $f'(c) = 0$.

141. Statement-I : If $f(a) = f(b)$, then Rolle's theorem is applicable for $x \in (a, b)$.
Statement-II : The tangent at $x = 1$ to the curve $y=x^3-x^2 - x + 2$ again meets the curve at $x=-1$.

142. Statement-I : $f(x) = |x - 1| + |x - 2| + |x - 3|$ has point of minima at $x = 3$.
Statement-II : $f(x)$ is non-differentiable at $x = 3$.

143. Statement-I : $f(x) = x + \cos x$ is strictly increasing for $\forall\ x \in R$.
Statement-II : If $f(x)$ is strictly increasing, then $f'(x)$ many vanish at some finite number of points.

144. Statement-I : $\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + c$.

Statement-II : $\int e^x (f(x) + f'(x))\, dx = e^x f(x) + c$.

145. Let $f(x)$ is continuous and positive for $x \in [a, b]$, $g(x)$ is continuous for $x \in [a, b]$ and

$\int_a^b |g(x)|dx > \left|\int_a^b g(x)dx\right|$, then

Statement-I : The value of $\int_a^b f(x)g(x)dx$ can be zero.

Statement-II : Equation $g(x) = 0$ has at least oen root in $x \in (a, b)$.

146. Consider $I_1 = \int_0^{\pi/4} e^{x^2} dx$, $I_2 = \int_0^{\pi/4} e^x dx$, $I_3 = \int_0^{\pi/4} e^{x^2} \cos x dx$, $I_4 = \int_0^{\pi/4} e^{x^2} \sin x dx$.

Statement-I : $I_2 > I_1 > I_3 > I_4$
Statement-II : for $x \in (0, 1)$, $x > x^2$ and $\sin x > \cos x$.

147. Statement-I : Area bounded by $2 \geq \max. \{|x - y|, |x + y|\}$ is 8 sq. units.
Statement-II : Area of the square of side length 4 is 16 sq. units.

148. $f(x)$ is a polynomial of degree 3 passing through origin having local extrema at $x = \pm 2$.
Statement-I : Ratio of areas in which $f(x)$ cuts the circle $x^2 + y^2 = 36$ is $1 : 1$.
Statement-II : Both $y = f(x)$ and the circle as symmetric about origin.

149. Statement-I : The differential equation of all circles in a plane must be of order 3.
Statement-II : There is only one circle passing through three non-collinear points.

150. Statement-I : The order of the differential equation whose general solutionis

$$y = c_1 \cos 2x + c_2\sin^2 x + c_3\cos^2 x + c_4e^{2x} + c_5 e^{2x} + c_6 \text{ is } 3.$$

Statement-II : Total number of arbitrary parameters in the given general solution in the statement (I) is 3.

2 TRIGONOMETRY

EXERCISE

151. If k_1 = tan 27θ – tan θ and $k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

(A) $k_1 = 2k_2$ (B) $k_1 = k_2 + 4$ (C) $k_1 = k_2$ (D) none of these

152. If x = sin θ|sin θ|, y = cos θ | cos θ|, where $\frac{99\pi}{2} \le \theta \le 50\pi$, then

(A) x – y = 1 (B) x + y = –1 (C) x + y = 1 (D) x – x = 1

153. If $\frac{\sec^8\theta}{a} + \frac{\tan^8\theta}{b} = \frac{1}{a+b}$, then for every real value of $\sin^2\theta$

(A) ab ≤ 0 (B) ab ≥ 0 (C) a + b = 0 (D) none of these

154. If $\sin x + \cos x = \sqrt{\left(y + \frac{1}{y}\right)}$, $x \in [0, \pi]$, then

(A) $x = \frac{\pi}{4}$, y = 1 (B) y = 0 (C) y = 2 (D) $x = \frac{3\pi}{4}$

155. The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{(8\cos^2 x)}$ = 1 are in AP with common difference

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{8}$ (D) $\frac{5\pi}{8}$

156. If 0 < θ < 2π and $2\sin^2\theta - 5\sin\theta + 2 > 0$, then the range of θ is

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(0, \frac{5\pi}{6}\right) \cup (\pi, 2\pi)$ (C) $\left(0, \frac{\pi}{6}\right) \cup (\pi, 2\pi)$ (D) none of these

157. The sum of the infinite ters of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{4}{33}\right) + ...$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

158. The greatest of tan 1, $\tan^{-1}$ 1, $\sin^{-1}$ 1, sin 1, cos 1, is

(A) sin 1 (B) tan 1 (C) $\tan^{-1}$ 1 (D) noen of these

159. If [$\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x$] = 1, where [*] denotes the greatest integer function, then x belongs to the interval

(A) [tan sin cos 1, tan sin cos sin 1]
(B) (tan sin cos 1, tan sin cos sin 1)
(C) [–1, 1]
(D) [sin cos tan 1, sin cos sin tan 1]

160. In a triangle ABC, $r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$ is equal to (where r is inradius and r_1, r_2, r_3 are exradii a, b, c are the sides of ΔABC)

(A) $2R^2$ (B) $4R^2$ (C) $8R^2$ (D) $16R^2$

161. In a Δ ABC, angles A, B, C are in AP. Then $\lim\limits_{x \to c} \dfrac{\sqrt{(3 - 4\sin A \sin C)}}{|A - C|}$ is

(A) 1 (B) 2 (C) 3 (D) 4

162. If r and R are respectively the radii of the inscribed and circumscribed circles of a regular polygon of n sides such that $\dfrac{R}{r} = \sqrt{5} - 1$, then n is equal to

(A) 5 (B) 10 (C) 6 (D) 18

163. If A = cos (cos x) + sin (cos x) the least and greatest value of A are

(A) 0 and 2 (B) –1 and 1 (C) $-\sqrt{2}$ and $\sqrt{2}$ (D) 0 and $\sqrt{2}$

164. Let n be a fixed positive integer such that $\sin\dfrac{\pi}{2n} + \cos\dfrac{\pi}{2n} = \dfrac{\sqrt{n}}{2}$, then

(A) n = 4 (B) n = 5 (C) n = 6 (D) none of these

165. If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation $x^3 - px^2 - r = 0$, then the value of $(1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$ is equal to

(A) $(p - r)^2$ (B) $1 + (p - r)^2$ (C) $1 - (P - r)^2$ (D) none of these

166. The number of solutions of the equation tan x + sec x = 2 cos x lying in the interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

167. The solution set of (2 cos x – 1) (3 + 2 cos x) = 0 in the interval $0 \le x \le 2\pi$ is

(A) $\left\{\dfrac{\pi}{3}\right\}$ (B) $\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}$ (C) $\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}, \cos^{-1}\left(-\dfrac{3}{2}\right)\right\}$ (D) none of these

168. The number of solutions of the equation $\sin^5 x - \cos^5 x = \dfrac{1}{\cos x} - \dfrac{1}{\sin x}$ ($\sin x \ne \cos x$) is

(A) 0 (B) 1 (C) infinite (D) none of these

169. The number of solutions of the equation $\cos^{-1}(1 - x) + m\cos^{-1} x = \dfrac{n\pi}{2}$, where m > 0, $n \le 0$, is

(A) 0 (B) 1 (C) 2 (D) infinite

170. $\tan\left\{2\tan^{-1}\left(\dfrac{1}{5}\right) - \dfrac{\pi}{4}\right\}$ is equal to

(A) $\dfrac{5}{4}$ (B) $\dfrac{5}{16}$ (C) $-\dfrac{7}{17}$ (D) $\dfrac{7}{17}$

171. The value of $\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$ is equal to

(A) 0 (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

172. In a triangle ABC, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then the numerical value of cos B is equal to

(A) 0 (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

173. If G is the centroid of a Δ ABC, then $GA^2 + GB^2 + GC^2$ is equal to

(A) $(a^2 + b^2 + c^2)$ (B) $\frac{1}{3}(a^2 + b^2 + c^2)$ (C) $\frac{1}{2}(a^2 + b^2 + c^2)$ (D) $\frac{1}{3}(a + b + c)^2$

174. In an isosceles triangle ABC, AB = AC. If vertical angle a is 20º, then $a^3 + b^3$ is equal to

(A) $3a^2b$ (B) $3b^2c$ (C) $3c^2a$ (D) abc

175. The least value of $\operatorname{cosec}^2 x + 25\sec^2 x$ is

(A) 0 (B) 26 (C) 28 (D) 36

176. If θ is an acute angle and $\tan\theta = \frac{1}{\sqrt{7}}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is

(A) 3/4 (B) 1/2 (C) 2 (D) 5/4

177. If A + C = B, then tan A tan B tan C is equal to

(A) tan A + tan B + tan C (B) tan B – tan C – tan A

(C) tan A + tan C – tan B (D) – (tan A tan B + tan C)

178. If $1 + \sin\theta + \sin^2\theta + \ldots\ldots \infty = 4 + 2\sqrt{3}$, $0 < \theta < \pi$, $\theta \neq \frac{\pi}{2}$, then

(A) $\theta = \frac{\pi}{6}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$ (D) $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

179. The number of roots of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(A) 1 (B) 2 (C) 3 (D) infinite

180. The general value of θ such that $\sin 2\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ is given by

(A) $n\pi + \frac{7\pi}{6}$, $n \in I$ (B) $n\pi \pm \frac{7\pi}{6}$, $n \in I$ (C) $2n\pi + \frac{7\pi}{6}$, $n \in I$ (D) none of these

181. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{12}$ (C) $\frac{3\pi}{4}$ (D) $\frac{13\pi}{12}$

182. If $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is independent of x, then

(A) $x \in [1, \infty)$ (B) $x \in [-1, 1]$ (C) $x \in (-\infty, -1]$ (D) none of these

183. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{4\pi}{3}$

184. In a triangle ABC, $(a + b + c)(b + c - a) = kbc$ if

(A) $k < 0$ (B) $k > 6$ (C) $0 < k < 4$ (D) $k > 4$

185. If the area of a triangle ABC is given by $\Delta = a^2 - (b - c)^2$, then $\tan\left(\frac{A}{2}\right)$ is equal to

(A) -1 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

186. In a Δ ABC, if $r = r_2 + r_3 - r_1$ and $\angle A > \frac{\pi}{3}$, then the range of $\frac{s}{a}$ is equal to

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(\frac{1}{2}, \infty\right)$ (C) $\left(\frac{1}{2}, 3\right)$ (D) $(3, \infty)$

187. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x$ is equal to

(A) -1 (B) 0 (C) 1 (D) 2

188. If $0 < \alpha < \pi/6$ and $\sin\alpha + \cos\alpha = \sqrt{7}/2$, then $\tan\alpha/2$ is equal to

(A) $\frac{\sqrt{7}-2}{3}$ (B) $\frac{\sqrt{7}+2}{3}$ (C) $\frac{\sqrt{7}}{3}$ (D) none of these

189. If $\alpha,\beta,\gamma \in \left(0, \frac{\pi}{2}\right)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha + \sin\beta + \sin\gamma}$ is

(A) < 1 (B) > 1 (C) $= 1$ (D) none of these

190. Solutions of the equation $|\cos x| = 2[x]$ are (where [*] denotes the greatest integer function)

(A) nil (B) $x = \pm 1$ (C) $x = \frac{\pi}{3}$ (D) none of these

191. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by

(A) $x \in \left(-\frac{\pi}{10}, \pi\right)$ (B) $x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (C) $x \in \left(-\pi, \frac{3\pi}{10}\right)$ (D) $x \in (-\pi, \pi)$

192. The number of solutions of the equation $1 + \sin x \sin^2\left(\frac{x}{2}\right) = 0$ in $[-\pi, \pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

193. The solution of the inequality $(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$ is

(A) $(\cot 3, \cot 2)$ (B) $(-\infty, \cot 3) \cup (\cot 2, \infty)$
(C) $(\cot 2, \infty)$ (D) None of these

194. If in a Δ ABC, $a^2 + b^2 + c^2 = 8R^2$, where R = circumradius, the the triangle is

(A) equilateral (B) isosceles (C) right angled (D) none of these

195. In any triangle ABC, $\sum \frac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than

(A) 9 (B) 3 (C) 27 (D) none of these

196. If in a triangle ABC, $2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then the value of the angle A is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

197. If in a triangle, R and r are the circumradius and inradius respectively, then the Harmonic mean of the exradii of the triangle is

(A) 3r (B) 2R (C) R + r (D) none of these

198. For what and only what values of α lying between 0 and π is the inequality $\sin \alpha \cos^3 \alpha > \sin^3 \alpha \cos \alpha$ valid ?

(A) $a \in (0, \pi/4)$ (B) $a \in (0, \pi/2)$ (C) $a \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) none of these

199. The minimum and maximum values of $ab \sin x + b\sqrt{(1 - a^2)} \cos x + c$ $(|a| < 1, b > 0)$ respectively are

(A) $\{b - c, b + c\}$ (B) $\{b + c, b - c\}$ (C) $\{c - b, b + c\}$ (D) none of these

200. If $2 \cos \theta + \sin \theta = 1$, then $7 \cos \theta + 6 \sin \theta$ equals

(A) 1 or 2 (B) 2 or 3 (C) 2 or 4 (D) 2 or 6

201. If $\sin \alpha = \frac{336}{625}$ and $450° < \alpha < 540°$, then $\sin (\alpha/4)$ is equal to

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{7}{25}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$

202. Minimum value of $4x^2 - 4x |\sin \theta| - \cos^2 \theta$ is equal to

(A) -2 (B) -1 (C) $-1/2$ (D) 0

203. If $\cos^4\theta\sec^2\alpha$, $\frac{1}{2}$ and $\sin^4\theta\operatorname{cosec}^2\alpha$ are in AP, then $\cos^8\theta\sec^6\alpha$, $\frac{1}{2}$ and $\sin^8\theta\operatorname{cosec}^6\alpha$ are in

(A) AP (B) GP (C) HP (D) none of these

204. The maximum value of the expression $\left|\sqrt{(\sin^2 x + 2a^2)} - \sqrt{(2a^2 - 1 - \cos^2 x)}\right|$, where a and x are real numbers is

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) $\sqrt{5}$

205. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are

(A) $-\frac{\pi}{2}, 0$ (B) $-\frac{\pi}{2}, 0, \frac{\pi}{2}$ (C) $\frac{\pi}{2}, 0$ (D) $0, \frac{\pi}{4}, \frac{\pi}{2}$

Questions based on statements (Q. 206 - 210)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

206. Statement–I : If $\sin^2\theta_1 + \sin^2\theta_2 + \ldots + \sin^2\theta_n = 0$, then the different sets of values of $(\theta_1, \theta_2, \ldots, \theta_n)$ for which $\cos\theta_1 + \cos\theta_2 + \ldots + \cos\theta_n = n - 4$ is $n(n - 1)$.

Statement–II : If $\sin^2\theta_1 + \sin^2\theta_2 + \ldots + \sin^2\theta_n = 0$, then $\cos\theta_1 \cdot \cos\theta_2 \cdot \cos\theta_n = \pm 1$.

207. Let α, β and γ satisfy $0 < \alpha < \beta < \gamma < 2\pi$ and $\cos(x + \alpha) + \cos(x + \beta) + \cos(x + \gamma) = 0\ \forall\ x \in R$.

Statement–I : $\gamma - \alpha = \frac{2\pi}{3}$.

Statement–II : $\cos\alpha + \cos\alpha + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$.

208. **Statement–I :** The equation $\sin(\cos x) = \cos(\sin x)$ has no real solution.

Statement–II : $\sin x \pm \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$.

209. **Statement–I :** $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$.

Statement–II : $\sin^{-1} x > \tan^{-1} y$ for $x > y$, $\forall\ x, y \in (0, 1)$.

210. **Statement–I :** In any ΔABC, the maximum value of $r_1 + r_2 + r_3 = 9R/2$.

Statement–II : In any ΔABC, $R \geq 2r$.

3 ALGEBRA

EXERCISE

211. If $\log_2 x + \log_2 y \geq 6$, then the least value of x + y is
(A) 4 (B) 8 (C) 16 (D) 32

212. Suppose $f(x, n) = \sum_{k=1}^{n} \log_x\left(\frac{k}{x}\right)$, then the value of x satisfying the equation f(x, 10) = f(x, 11) is
(A) 9 (B) 10 (C) 11 (D) none

213. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is
(A) 8 (B) 16 (C) 24 (D) 32

214. If the roots of the equation $ax^2 + bx + c = 0$, are of the form $\alpha/(\alpha - 1)$ and $(\alpha + 1)/\alpha$, then the value of $(a + b + c)^2$ is
(A) $2b^2 - ac$ (B) $b^2 - 2ac$ (C) $b^2 - 4ac$ (D) $4b^2 - 2ac$

215. The maximum value of the sum of the AP 50, 48, 46, 44, ... is
(A) 648 (B) 450 (C) 558 (D) 650

216. If $x^a = y^b = z^c$, where a, b, c are unequal positive numbers and x,y,z are in GP, then $a^3 + c^3$ is
(A) $\geq 2b^3$ (B) $> 2b^3$ (C) $\leq 2b^3$ (D) $< 2b^3$

217. If $0 \leq [x] < 2$; $-1 \leq [y] < 1$ and $1 \leq [z] < 3$. where [*] denotes the greatest integer function, then the maximum value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is
(A) 2 (B) 4 (C) 6 (D) 8

218. The equations $\lambda x - y = 2$, $2x - 3y = -\lambda$, $3x - 2y = -1$ are consistent for
(A) $\lambda = -4$ (B) $\lambda = -1, 4$ (C) $\lambda = -1$ (D) $\lambda = 1, -4$

219. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $p^T (Q^{2005})p$ is equal to
(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$

220. If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ is equal to

(A) A^4 (B) A^3 (C) $I + A$ (D) none of these

221. The solution of $x - 1 = (x - [x])(x - \{x\})$ (wher $[x]$ and $\{x\}$ are the integral and fractional part of x) is

(A) $x \in R$ (B) $x \in R \sim [1, 2)$ (C) $x \in [1, 2)$ (D) $x \in R \sim [1, 2]$

222. The value of p for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$, are less than 2, lies in

(A) $(4/5, 2)$ (B) $(2, \infty)$ (C) $(-1, -4/5)$ (D) $(-\infty, -1)$

223. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $m = (a + b)(c + d)$ satisfies the relation

(A) $0 < m \leq 1$ (B) $1 \leq m \leq 2$ (C) $2 \leq m \leq 3$ (D) $3 < m \leq 4$

224. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a > b > c$ and $a + b + c = 3/2$, then the value of a is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2} + \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} + \frac{1}{\sqrt{2}}$

225. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & 1 & 2 \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2} f(x)\, dx$ is equal to

(A) 5 (B) 3 (C) 1 (D) 0

226. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then

(A) $abc > 1$ (B) $abc > -8$ (C) $abc < -8$ (D) $abc > -2$

227. If $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$, then

(A) $x + y = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & -2 \end{bmatrix}$ (B) $x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ (C) $x - y = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ (D) $y = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$

228. If $A = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{pmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then f(A) equals

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (C) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $(2 + i\sqrt{3})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

229. If the matrices A, B, A + B are non singular, then $[A(A + B)^{-1}B]^{-1}$, is equal to

(A) $A^{-1} + B^{-1}$ (B) $A + B$ (C) $A(A + B)^{-1}$ (D) None of these

230. The number of solution of $|[x] - 2x| = 4$, where $[*]$ denotes the greatest integer $\le x$, is
(A) infinite (B) 4 (C) 3 (D) 2

231. If $\sum_{i=1}^{21} a_i = 693$, where $a_1, a_2, ..., a_{21}$ are in AP, then the value of $\sum_{i=0}^{10} a_{2r+1}$ is
(A) 361 (B) 363 (C) 365 (D) 398

232. If $\log_2 (a + b) + \log_2 (c + d) \ge 4$. Then the minimum value of the expression $a + b + c + d$ is
(A) 2 (B) 4 (C) 8 (D) none of these

233. If $xyz = -2007$ and $\Delta = \begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = 0$, then value of $ayz + bzx + cxy$ is
(A) -2007 (B) 2007 (C) 0 (D) $(2007)^2$

234. If $\Delta_r = \begin{vmatrix} 1 & r & 2^r \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$, then value of $\sum_{r=1}^{n} \Delta_r$ is
(A) n (B) 2n (C) n^2 (D) $-2n$

235. If A is an orthogonal matrix, then A^{-1}, equals
(A) A (B) A′ (C) A^2 (D) none of these

236. If A is a square matrix, then adj A^T – (adj $A)^T$ is equal to
(A) $2|A|$ (B) $2|A|$ I (C) null matrix (D) unit matrix

237. The sum of all values of x, so that $16^{(x^2+3x-1)} = 8^{(x^2+3x+2)}$, is
(A) 0 (B) 3 (C) -3 (D) -5

238. If α, β, γ are the roots of $ax^3 + bx + c = 1$ such that $\alpha + \beta = 0$, then
(A) c = 0 (B) c = 1 (C) b = 0 (D) b = 1

239. If p,q,r are three positive real numbers are in AP, then the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for
(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$ (C) all p and r (D) no p and r

240. If the arithmetic progression whose common difference is none zero, the sum of first 3n terms is equal to the sum of the next n terms. Then the ratio of the sum of the first 2n terms to the next 2n terms is
(A) 1/5 (B) 2/3 (C) 3/4 (D) none of these

241. If A is square matrix of order n, a = maximum number of distinct entries. If A is triangular matrix, b = maximum number of distinct entries. If A is a diagonal matrix. c = minimum number of zeros. If A is a triangular matrix if a + 5 = c + 2b
(A) 12 (B) 4 (C) 8 (D) None of these

242. Matrix M_r is defiend as $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r \in N$ value of det (M_1) + det (M_2) + det (M_3) +....+ det (M_{2007}) is

(A) 2007 (B) 2008 (C) 2008^2 (D) 2007^2

243. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the matrix reflection in the line

(A) x = 1 (B) x + y = 1 (C) y = 1 (D) x = y

244. If the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular, then λ is equal to

(A) 3 (B) 4 (C) 2 (D) 5

245. If the roots of the quadratic equation $ax^2 - 5x + 6 = 0$ are in the ratio 2 : 3, then 'a' is equal to

(A) 3 (B) 1 (C) 2 (D) –1

246. If the sides of a right angled triangle form an AP, then the sines of the acute angles are

(A) $\frac{3}{5}, \frac{4}{5}$ (B) $\sqrt{3}, \frac{1}{3}$ (C) $\sqrt{\left(\frac{\sqrt{5}-1}{2}\right)}, \sqrt{\left(\frac{\sqrt{5}+1}{2}\right)}$ (D) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

247. If $x \in \{1,2,3 ....9\}$ and $f_n(x)$ = xxx.x (n digits), then $f_n^2(3) + f_n(2)$ is equal to

(A) $2f_{2n}(1)$ (B) $f_n^2(1)$ (C) $f_{2n}(1)$ (D) $-f_{2n}(4)$

248. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(A) 128B (B) –128B (C) 4B (D) –64B

249. If $x_1, x_2, ..., x_{20}$ are in H.P. and $x_1, 2, x_{20}$ are in G.P., then $\sum_{r=1}^{19} x_r x_{r+1} =$

(A) 76 (B) 80 (C) 84 (D) none of these

250. If m and x are two real numbers, then $e^{2mi\cot^{-1}x}\left(\frac{xi+1}{xi-1}\right)^m$ (where $i = \sqrt{-1}$) is equal to

(A) cos x + i sin x (B) m/2 (C) 1 (D) (m + 1)/2

251. If $|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$, (where $i = \sqrt{-1}$), then z lies on

(A) Re (z) = 2 (B) Im (z) = 2 (C) Re (z) + Im (z) = 2 (D) none of these

252. If z be complex number such that equation $|z - a^2| + |z - 2a| = 3$ always represents an ellipse, then range of a ($\in R^+$) is

(A) $(1, \sqrt{2})$ (B) $[1, \sqrt{3}]$ (C) (–1, 3) (D) (0, 3)

253. The total number of integral solution for x, y, z such that xyz = 24, is

(A) 3 (B) 60 (C) 90 (D) 120

254. If a, b, c are odd positive integers, then number of integral solutions of a + b + c = 13, is
(A) 14 (B) 21 (C) 28 (D) 56

255. Number of point having position vector
$a\hat{i}+b\hat{j}+c\hat{k}$ where a, b, c $\in$ {1, 2, 3, 4, 5} such that $2^a + 3^b + 5^c$ is divisible by 4 is-
(A) 140 (B) 70 (C) 100 (D) None of these

256. Number of terms in the expansion of $\left(\frac{x^3+1+x^6}{x^3}\right)^{\Sigma n}$ (where n $\in$ N) is
(A) $\Sigma n + 1$ (B) ${}^{\Sigma n+2}C_2$ (C) 2n + 1 (D) $n^2 + n + 1$

257. If 5^{40} is divided by 11, then remainder is α and when 2^{2003} is divided by 17, then remainder is β, then the value of $\beta - \alpha$ is
(A) 3 (B) 5 (C) 7 (D) 8

258. The term independent of x in the expansion of $\left(\sqrt{\left(\frac{x}{3}\right)}+\sqrt{\left(\frac{3}{2x^2}\right)}\right)^{10}$ is
(A) 5/12 (B) 1 (C) ${}^{10}C_1$ (D) none of these

259. The probability that the length of a randomly chosen chord of a circle lies between $\frac{2}{3}$ and $\frac{5}{6}$ of its diameter is
(A) 1/4 (B) 5/12 (C) 1/16 (D) 5/16

260. A die is rolled three times, the probability of getting a large number than the previous number is
(A) $\frac{1}{54}$ (B) $\frac{5}{54}$ (C) $\frac{5}{108}$ (D) $\frac{13}{108}$

261. A dice is thrown (2n + 1) times. The probability that faces with even numbers appear odd number is times is
(A) $\frac{2n+1}{2n+3}$ (B) $\frac{n+1}{2n+1}$ (C) $\frac{n}{2n+1}$ (D) none of these

262. If $x_r = \cos(\pi/3^r) - i\sin(\pi/3^r)$, (where i = $\sqrt{-1}$). then value of $x_1 \cdot x_2 \cdots \infty$, is
(A) 1 (B) –1 (C) –i (D) i

263. Let $z_1 = 6 + i$ and $z_2 = 4 - 3i$ (where i = $\sqrt{-1}$). Let z be a complex number such that
$\arg\left(\frac{z-z_1}{z_2-z}\right) = \frac{\pi}{2}$, then z satisfies
(A) $|z - (5 - i)| = 5$ (B) $|z - (5 - i)| = \sqrt{5}$ (C) $|z - (5 + i)| = 5$ (D) $|z - (5 + i)| = \sqrt{5}$

264. If $z \neq 0$, then $\int_{x=0}^{100} [\arg|z|]\, dx$ is (where [*] denotes the greatest integer function)
(A) 0 (B) 10 (C) 100 (D) not defined

265. The maximum number of points of intersection of 8 circles, is

(A) 16 (B) 24 (C) 28 (D) 56

266. Every one of the 10 available lamps can be switched on to illuminate certain Hall. The total number of ways in which the hall can be illuminated, is

(A) 55 (B) 1023 (C) 2^{10} (D) 10 !

267. The total number of 3 digit even numbers that can be composed from the digitis 1, 2, 3, ..., 9 when the repetition of digits is not allowed, is

(A) 224 (B) 280 (C) 324 (D) 405

268. The number of ways in which a score of 11 can be made from a through by three persons, each throwing a single die once, is

(A) 45 (B) 18 (C) 27 (D) 68

269. The greatest coefficient in the expansion of $(1 + x)^{2n + 2}$ is

(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n+2)!}{\{(n+1)!\}^2}$ (C) $\frac{(2n+2)!}{n!(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

270. The first integral term in the expansion of $(\sqrt{3} + \sqrt[3]{2})^9$, is its

(A) 2nd term (B) 3rd term (C) 4th term (D) 5th term

271. The number of irrational terms in the expansion of $(2^{1/5} + 3^{1/10})^{55}$ is

(A) 47 (B) 56 (C) 50 (D) 48

272. 10 bulbs out of a sample of 100 bulbs manufactured by a company are defective. The probability that 3 out of 4 bulbs, bought by a customer will not be defective, is

(A) ${}^4C_3/{}^{100}C_4$ (B) ${}^{90}C_3/{}^{96}C_4$ (C) ${}^{90}C_3/{}^{100}C_4$ (D) $({}^{90}C_3 \times {}^{10}C_1)/{}^{100}C_4$

273. Two persons each makes a single throw with a pair of dice. The probability that the throws are unequal is given by

(A) $\frac{1}{6^3}$ (B) $\frac{73}{6^3}$ (C) $\frac{51}{6^3}$ (D) none of these

274. Let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8}. An element (a, b) of their cartesian product A × B is chosen at random. The probability that a + b = 9, is

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

275. The point of intersection of the curves $\arg(z - 3i) = 3\pi/4$ and $\arg(2z + 1 - 2i) = \pi/4$ (where $i = \sqrt{-1}$) is

(A) 1/4 (3 + 9i) (B) 1/4 (3 – 9i) (C) 1/2 (3 + 2i) (D) no point

276. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

(A) 0 (B) 2 (C) 7 (D) 17

277. If $|z - i| \le 2$ and $z_1 = 5 + 3i$, (where $i = \sqrt{-1}$) then the maximum value of $|iz + z_1|$ is

(A) $2 + \sqrt{31}$ (B) 7 (C) $\sqrt{31} - 2$ (D) $\sqrt{31} + 2$

278. If $|z - 1| + |z + 3| \le 8$, then the range of values of $|z - 4|$, (where $i = \sqrt{-1}$) is
(A) (0, 7) (B) (1, 8) (C) [1, 9] (D) [2, 5]

279. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no lines passes through both points A and B, and no two are parallel, then the number of intersection points the lines have is equal to
(A) 535 (B) 601 (C) 728 (D) 963

280. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7, so that digits do not repeat and the terminal digits are even is
(A) 144 (B) 72 (C) 288 (D) 720

281. In a polygon, no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70, then number of diagonals of polygon is :
(A) 8 (B) 20 (C) 28 (D) None

282. The coefficient of $a^{10} b^7 c^3$ in the expansion of $(bc + ca + ab)^{10}$ is
(A) 30 (B) 60 (C) 120 (D) 240

283. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 ...... + a_{2n}x^{2n}$. Then value of $a_0 + a_3 + a_6 ....$ is equal to
(A) 3^n (B) 3^{n+1} (C) 3^{n-1} (D) None of these

284. The value of $\sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to
(A) 5(2n – 9) (B) 10n (C) 9(n – 4) (D) None

285. If two events A and B are such that P(A) > 0 and P(B) ≠ 1, then $p(\overline{A} / \overline{B})$ is equal to
(A) 1 – P (A/B) (B) $1 - P(\overline{A}/B)$ (C) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (D) $\frac{P(A)}{P(\overline{B})}$

286. Two numbers x and y are chosen at random from the set {1, 2, 3,....,30}. The probability that $x^2 - y^2$ is divisible by 3 is
(A) 3/29 (B) 4/29 (C) 5/29 (D) none of these

287. Consider $f(x) = x^3 + ax^2 + bx + c$. parameters a, b, c, are chosen, respectively, by throwing a die three times. Then the probability that f(x) is an increasing function is
(A) 5/36 (B) 8/36 (C) 4/9 (D) 1/3

288. Number of solutions of the equation $|z|^2 + 7\overline{z} = 0$ is/are
(A) 1 (B) 2 (C) 4 (D) 6

289. $(1 + i)^6 + (1 - i)^6 =$
(A) $15i$ (B) $-15i$ (C) 15 (D) 0

290. If 1, ω, ω^2,.... ω^{n-1} are n, nth roots of unity, then the value of $(9 - \omega)(9 - \omega^2).....(9 - \omega^{n-1})$ will be
(A) n (B) 0 (C) $\frac{9^n - 1}{8}$ (D) $\frac{9^n + 1}{8}$

291. The number of non-negative integral solutions to the system of equations $x + y + z + u + t = 20$ and $x + y + z = 5$ is-
(A) 336 (B) 346 (C) 246 (D) None of these

292. If 'n' objects are arranged in a row, then number of ways of selecting three of these objects, so that no two of them are next to each other is :
(A) ${}^{n-3}C_3$ (B) ${}^{n-3}C_2$ (C) ${}^{n-2}C_3$ (D) None

293. The sum of 20 terms of a series of which every even term is 2 times the term before it, and every odd term is 3 times the term before it, the first term being unity is
(A) $\left(\frac{2}{7}\right)(6^{10} - 1)$ (B) $\left(\frac{3}{7}\right)(6^{10} - 1)$ (C) $\left(\frac{3}{5}\right)(6^{10} - 1)$ (D) none of these

294. A fair coin is tossed 100 times. The probability of getting tails 1, 3, ... 49 times is
(A) 1/2 (B) 1/4 (C) 1/8 (D) 1/16

295. A six-faced fair dice is thrown until 1 comes. Then the probability that 1 comes in even number of trials is
(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

296. Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is
(A) reflexive and transitive (B) reflexive and symmetric
(C) reflexive and antisymmetric (D) None of the above

297. If the heights of 5 persons are 144 cm, 153 cm, 150 cm, 158 cm and 155 cm respectively, then mean height is-
(A) 150 cm (B) 151 cm (C) 152 cm (D) None of these

298. Arithmetic mean of the following frequency distribution :

x :	4	7	10	13	16	19
f :	7	10	15	20	25	30

is -
(A) 13.6 (B) 13.8 (C) 14.0 (D) None of these

299. If p and q are two statements. Then negation of compound statement $(\sim p \vee q)$ is
(A) $\sim p \wedge q$ (B) $p \vee q$ (C) $p \wedge \sim q$ (D) None

300. Negation of statement : if we control population growth we prosper, is
(A) if we do not control population growth, we prosper
(B) if we control population, we do not prosper
(C) we control population and we do not prosper
(D) we do not control population, but we prosper

301. If a set A = {a, b, c} then the number of subsets of the set A is
(A) 3 (B) 6 (C) 8 (D) 9

302. The weighted mean of first n natural number if their weight are the same as the number is-
(A) $\frac{n(n+1)}{2}$ (B) $\frac{n+1}{2}$ (C) $\frac{2n+1}{3}$ (D) None of these

303. The mean income of a group of persons is Rs.400. Another group of persons has mean income Rs.480. If the mean income of all the persons in the two groups together is Rs.430, then ratio of the number of persons in the groups:

(A) $\frac{4}{3}$ (B) $\frac{5}{4}$ (C) $\frac{5}{3}$ (D) None of these

304. If $p \Rightarrow q$ can also be written as

(A) $p \Rightarrow \sim q$ (B) $\sim p \vee q$ (C) $\sim p \Rightarrow \sim q$ (D) none

305. If $p \Rightarrow (\sim p \vee q)$ is false, the truth values of p and q are respectively.

(A) T, F (B) T, T (C) F, T (D) F, F

306. The value of n $\{P[P(\phi)]\}$ is equal to

(A) 0 (B) 2 (C) 3 (D) 4

307. The mean of a set of number is $\bar{x}$ if each number is increased by λ, then mean of the new set is-

(A) $\bar{x}$ (B) $\bar{x} + \lambda$ (C) $\lambda \bar{x}$ (D) None of these

308. Mean of 25 observations was found to be 78.4. But later on it was found that 96 was misread 69. The correct mean is

(A) 79.24 (B) 79.48 (C) 80.10 (D) None of these

309. If p, q, r are simple statement. Then $(p \wedge q) \wedge (q \wedge r)$ is true. Then

(A) p, q, r are all false
(B) p, q are true and r is false
(C) p, q, r are all true
(D) p is true and q and r are false

310. If p, q, r are simple statement with truth values T, F, T then truth values of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is

(A) True (B) False (C) True if r is false (D) None

311. If A = $\{x : x = 2n + 1, n \in Z$ and B = $\{x : x = 2n, n \in Z\}$, then $A \cup B$ is

(A) set of natural numbers
(B) set of irrational numbers
(C) set of integers
(D) none of these

312. If $\bar{x}$ is the mean of $x_1, x_2, ..., x_n$ then mean of $x_1 + a, x_2 + a, ..., x_n + a$ where a is any number positive or negative is-

(A) $\bar{x} + a$ (B) $\bar{x}$ (C) $a\bar{x}$ (D) None of these

313. Mean wage from the following data

Wage (In Rs.)	800	820	860	900	920	980	1000	
No. of workers	7	14	19	25	20	10	5	is

(A) Rs.889 (B) Rs. 890.4 (C) Rs.891.2 (D) None of these

314. Negation of '' 3 is an odd number and 7 is a rational number is -

(A) 3 is not an odd number and 7 is not a rational number
(B) 3 is an odd number or 7 is a rational number
(C) 3 is an odd number or 7 is not a rational number
(D) 3 is not an odd number or 7 is not a rational number.

315. The negation of statement $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ is -

(A) $(p \vee \sim q) \wedge (p \vee q)$` (B) $(p \wedge \sim q) \vee (p \vee q)$

(C) $(\sim p \vee q) \vee (\sim p \wedge \sim q)$ (D) $(p \wedge \sim q) \wedge (p \vee q)$

316. If $A = \{x : x = 3n, n \in Z\}$ and $B = \{x : x = 4n, n \in Z\}$ then $A \cap B$ is

(A) $\{x : x = n, n \in Z\}$ (B) $\{x : x = n/2, n \in Z\}$

(C) $\{x : x = n-1, n \in Z\}$(D) $\{x : x = 12n, n \in Z\}$

317. The geometric mean of numbers $7, 7^2, 7^3, ...,7^n$ is-

(A) 7^n (B) $7^{n/2}$ (C) $7^{\frac{n+1}{2}}$ (D) None of these

318. Harmonic mean of 2, 4, 5 is.....

(A) 4.21 (B) 3.16 (C) 2.98 (D) None of these

319. The negation of statement $(p \wedge q) \vee (q \vee \sim r)$

(A) $(p \wedge q) \vee (\sim q \vee \sim r)$ (B) $(\sim p \wedge \sim q) \wedge (\sim q \wedge r)$

(C) $(\sim p \vee \sim q) \wedge (\sim q \wedge r)$ (D) None of these

320. The statement $(p \wedge \sim q) \vee p$ is logically equivalent to -

(A) p (B) $\sim p$ (C) q (D) $\sim q$

321. If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$?

(A) 3 (B) 6 (C) 9 (D) 10

322. The number of runs scored by 11 players of a cricket team of school are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27. The median is-

(A) 21 (B) 27 (C) 30 (D) None of these

323. The median for the following frequency distribution :

x :	1	2	3	4	5	6	7	8	9
f :	8	10	11	16	20	25	15	9	6 is :

(A) 4 (B) 5 (C) 6 (D) None of these

324. If $(p \wedge \sim q) \vee (q \wedge r)$ is true and. q and r both true then p is -

(A) True (B) False (C) may be true or false (D) none

325. Negation of the statement If a number is prime then it is odd' is.

(A) A number is not prime but odd. (B) A number is prime and it is not odd.

(C) A number is neither primes nor odd. (D) None of these

326. If the number of elements in A is m and number of element in B is n then the number of elements in the power set of $A \times B$ is

(A) 2^n (B) 2^m (C) 2^{mn} (D) none of these

327. Median from the following distribution

Class	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	
frequency	5	6	15	10	5	4	2	2	is

(A) 19.0 (B) 19.2 (C) 19.3 `(D) 19.5

328. Mode of the data 3, 2, 5, 2, 3, 5, 6, 6, 5, 3, 5, 2, 5 is-

(A) 6 (B) 4 (C) 5 (D) 3

329. If p, q, r are substatements with truth values. T, T, F then the Statement $r \rightarrow (p \wedge \sim q) \vee (\sim q \wedge \sim r)$ will be

(A) True (B) False

(C) may be true or false (D) None of these

330. The Negation of the statement $(p \wedge q) \rightarrow r$ is -

(A) $(\sim p \vee \sim q) \rightarrow r$ (B) $(\sim p \wedge \sim q) \wedge \sim r$ (C) $(p \wedge q) \wedge \sim r$ (D) $(\sim p \vee \sim q) \wedge r$

331. Let A and B be two non-empty sets having elements in common, then $A \times B$ and $B \times A$ have elements in common, is

(A) n (B) n – 1 (C) n^2 (D) none of these

332. If the value of mode and mean is 60 and 66 respectively, then the value of median is-

(A) 60 (B) 64 (C) 68 (D) None of these

333. The mean deviation about median from the following data :

340, 150, 210, 240, 300, 310, 320, is

(A) 52.4 (B) 52.5 (C) 52.8 (D) None of these

334. If p is any statement, t is tautology & c is a contradiction, then which of following is not correct-

(A) $p \vee (\sim p) \equiv c$ (B) $p \vee t = t$ (C) $p \wedge t = p$ (D) $p \wedge c = c$

335. $(\sim p \vee q)$ is logically equal to -

(A) $p \rightarrow q$ (B) $q \rightarrow p$ (C) $\sim (p \rightarrow q)$ (D) $\sim (q \rightarrow p)$

336. Let R be the relation on the set N of natural numbers defined by

$R : \{(x, y)\} : x + 3y = 12 \ x \in N, y \in N\}$ then domain of R

(A) {1, 2, 3} (B) {2, 3, 5} (C) {9, 6, 3} (D) none of these

337. Mean deviation about mean from the following data :

x_i :	3	9	17	23	27
f_i :	8	10	12	9	5

is -

(A) 7.15 (B) 7.09 (C) 8.05 (D) None of these

338. Marks of 5 students of a tutorial group are 8, 12, 13, 15, 22 then variance is:

(A) 21 (B) 21.2 (C) 21.4 (D) None of these

339. The statement $p \Leftrightarrow q$ is equal to -

(A) $(\sim p \vee q) \vee (p \vee q)$ (B) $(p \wedge q) \vee (\sim p \wedge \sim q)$

(C) $(\sim p \vee q) \wedge (p \vee \sim q)$ (D) $(p \wedge q) \vee (p \vee q)$

340. The statement $(p \wedge q) \Leftrightarrow \sim p$ is a

(A) Tautology (B) contradiction

(C) Neither tautology nor contradiction (D) None of these

341. Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$ i.e., $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y=10\}$. Then domains of R^{-1}

(A) {2, 4, 6, 8} (B) {4, 3, 2, 1} (C) {1, 2, 4} (D) none of these

342. Variance of the data given below

size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5	
frequency	3	7	22	60	85	32	8	is-

(A) 1.29 (B) 2.19 (C) 1.32 (D) None of these

343. If the mean and variance of a variate X having a binomial distribution are 6 and 4 respectively. then the number of values of the variate in the distribution is

(A) 10 (B) 12 (C) 16 (C) 18

344. The statement $p \to p \vee q$ is a -

(A) Tautology (B) Contradiction

(C) Neither tautology nor contradiction (D) None of these

345. The statement $(p \to \sim q) \Leftrightarrow (p \wedge q)$ is a -

(A) Tautology (B) Contradiction

(C) Neither tautology nor contradiction (D) None of these

Questions based on statements (Q. 346 - 360)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

346. **Statement-I :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4ac = 1$

Statement-II : If a,b,c are odd integer then the roots of the equation $4abc\, x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

347. **Statement-I :** If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$, then x,y,z are in H.P.

Statement-II : If $a_1^2 + a_2^2 + ... + a_n^2 = 0$, then $a_1 = a_2 = a_3 = ... = a_n = 0$

348. **Statement-I :** The number of zeroes at the end of 100! is 24.

Statement-II : The exponent of prime 'p' in n! is $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] ..... + \left[\frac{n}{p^r}\right]$

where r is a natural number such that $p^r \le n < p^{r+1}$

349. **Statement-I :** $\sum_{r=0}^{n} \frac{1}{r+1} {}^{n}C_r x^r = \frac{1}{(n+1)x}[(1+x)^{n+1} - 1]$

Statement-II : $\sum_{r=0}^{n} \frac{{}^{n}C_r}{r+1} = \frac{2^{n+1}}{n+1}$

350. **Statement–I :** If all real values of x obtained from the equation $4^x - (a-3)2^x + (a-4) = 0$ are non-positive, then $a \in (4, 5]$.
Statement–II : If $ax^2 + bx + c$ is non-positive for all real values of x, then $b^2 - 4ac$ must be negative or zero and 'a' mst be negative.

351. **Statement–I :** If $\arg(z_1 z_2) = 2\pi$, then both z_1 and z_2 are purely real (z_1 and z_2 have principal arguments).
Statement–II : Principal argument of complex number and between $-\pi$ and π.

352. If $z_1 \neq -z_2$ and $|z_1 + z_2| = |(1/z_1) + (1/z_2)|$ then
Statement–I : $z_1 z_2$ is unimodular.
Statement–II : z_1 and z_2 both are unimodular.

353. **Statement–I :** If an infinite G.P. has 2nd term x and its sum is 4, then x belongs to (–8, 1).
Statement–II : Sum of an infinite G.P. is finite if for its common ratio r, $0 < |r| < 1$.

354. **Statement–I :** $1^{99} + 2^{99} + \ldots\ldots + 100^{99}$ is divisible by 10100.
Statement–II : $a^n + b^n$ is divisible by $a + b$ if n is odd.

355. **Statement–I :** When number of ways of arranging 21 objects of which r objects are identical of one type and remaining are identical of second type is maximum, then maximum value of ${}^{13}C_r$ is 78.
Statement–II : ${}^{2n+1}C_r$ is maximum when $r = n$.

356. **Statement–I :** Number of ways of selecting 10 objects from 42 objects of which, 21 objects are identical and remaiing objects are distinct is 2^{20}.
Statement–II : ${}^{42}C_0 + {}^{42}C_1 + {}^{42}C_2 + \ldots\ldots + {}^{42}C_{21} = 2^{41}$.

357. **Statement–I :** Greatest term in the expansion of $(1 + x)^{12}$, when $x = 11/10$ is 7th.
Statement–II : 7th term in the expansion of $(1 + x)^{12}$ has the factor ${}^{12}C_6$ which is greatest value of ${}^{12}C_r$.

358. **Statement–I :** If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$, then triangle may not be equilateral.

Statement–II : If any two rows of a determinant are the same, then the value of that determinant is zero.

359. **Statement–I :** $A = \begin{bmatrix} 4 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Then $(AB)^{-1}$ does not exist.

Statement–II : Since $|A| = 0$, $(AB)^{-1} = B^{-1}A^{-1}$ is meaningless.

360. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ….., 20}.
Statement – 1 :
The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.
Statement – 2 :
If the four chosen numbers form an AP, then the set of all possible values of common difference is
$\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

4 CO-ORDINATE GEOMETRY

EXERCISE

361. The image of P(a, b) on y = –x is Q and the image of Q on the line y = x is R. Then the mid point of R is

(A) (a + b, b + a) (B) $\left(\frac{a+b}{2}, \frac{b+a}{2}\right)$ (C) (a – b, b – a) (D) (0, 0)

362. The line 3x – 4y + 7 = 0 is rotated through an angle $\frac{\pi}{4}$ in the clockwise direction about the point (–1, 1). The equation of the line in its new position is

(A) 7y + x – 6 = 0 (B) 7y – x – 6 = 0 (C) 7y + x + 6 = 0 (D) 7y – x + 6 = 0

363. Area of the parallelogram formed by the lines y = mx, y = mx + 1, y = nx and y = nx + 1 equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

364. If 5a + 4b + 20c = t, then the value of t for which the line ax + by + c - 1 = 0 always passes through a fixed point is

(A) 0 (B) 20 (C) 30 (D) none of these

365. The point A (2, 1) is translated parallel to the line x – y = 3 by a distance 4 units. If the new position A' is in third quadrant, then the coordinates of A' are

(A) $(2 + 2\sqrt{2}, 1 + 2\sqrt{2})$ (B) $(-2 + \sqrt{2}, -1 -2\sqrt{2})$

(C) $(2 - 2\sqrt{2}, 1 - 2\sqrt{2})$ (D) none of these

366. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line 5x – 2y + 6 = 0 at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

367. The lines 2x – 3y = 5 and 3x – 4y = 7 are the diameters of a circle of area 154 sq unit. The equation of this circle is (π = 22/7)

(A) $x^2 + y^2 + 2x - 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 47$

(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

368. The range of values of 'a' such that the angle θ between the pair of tangents drawn from (a, 0) to the circle $x^2 + y^2 = 1$ satisfies $\frac{\pi}{2} < \theta < \pi$, is

(A) (1, 2) (B) $(1, \sqrt{2})$ (C) $(-\sqrt{2}, -1)$ (D) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$

369. The locus of a point such that the tangents drawn from it to the circle $x^2 + y^2 - 6x - 8y = 0$ are perpendicular to each other is

(A) $x^2 + y^2 - 6x - 8y - 25 = 0$ (B) $x^2 + y^2 + 6x - 8y - 5 = 0$

(C) $x^2 + y^2 - 6x + 8y - 5 = 0$ (D) $x^2 + y^2 - 6x - 8y + 25 = 0$

370. The locus of the point ($\sqrt{(3h+2)}$, $\sqrt{3k}$). If (h, k) lies on x + y = 1 is

(A) a pair of straight lines (B) a circle

(C) a parabola (D) an ellipse

371. The shortest distance between the parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is

(A) 2 (B) $\sqrt{5}$ (C) 3 (D) none of these

372. A parabola is drawn with focus at (3, 4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is

(A) $x^2 - 6x - 8y + 25 = 0$ (B) $y^2 - 8x - 6y + 25 = 0$

(C) $x^2 - 6x + 8y - 25 = 0$ (D) $x^2 + 6x - 8y - 25 = 0$

373. Two perpendicular tangents PA and PB are drawn to $y^2 = 4ax$, minimum length of AB is equal to

(A) a (B) 4a (C) 8a (D) 2a

374. The locus of the points of trisection of the double ordinates of the parabola $y^2 = 4ax$ is

(A) $y^2 = ax$ (B) $9y^2 = 4ax$ (C) $9y^2 = ax$ (D) $y^2 = 9ax$

375. If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then PA. PB is equal to

[where P $\equiv$ ($\sqrt{3}$, 0)]

(A) $\frac{4(\sqrt{3}+2)}{3}$ (B) $\frac{4(2-\sqrt{3})}{3}$ (C) $\frac{4\sqrt{3}}{2}$ (D) $\frac{2(\sqrt{3}+2)}{3}$

376. The point, at shortest distance from the line x + y = 7 and lying on an ellipse $x^2 + 2y^2 = 6$, has coordinates

(A) ($\sqrt{2}$, $\sqrt{2}$) (B) (0, $\sqrt{3}$) (C) (2, 1) (D) $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

377. The length of the common chord of the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ and the circle $(x - 1)^2 + (y - 2)^2 = 1$ is

(A) zero (B) one (C) three (D) eight

378. The eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latusrectum is half of its minor axis is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) none of these

379. The locus of midpoints of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(A) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$ (B) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{ex}{a}$ (C) $x^2 + y^2 = a^2 + b^2$ (D) none of these

380. An ellipse slides between two perpendicular straight lines. Then the locus of its centre is a/an
(A) parabola (B) ellipse (C) hyperbola (D) circle

381. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of b^2 is
(A) 3 (B) 16 (C) 9 (D) 12

382. If $(a - 2)x^2 + ay^2 = 4$ represents rectangular hyperbola, then a equals
(A) 0 (B) 2 (C) 1 (D) 3

383. The eccentricity of the hyperbola whose asymptotes are $3x + 4y = 2$ and $4x - 3y + 5 = 0$ is
(A) 1 (B) 2 (C) $\sqrt{2}$ (D) none of these

384. The equation of the hyperbola whose foci are (6, 5), (–4, 5) and eccentricity 5/4 is

(A) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=1$ (B) $\frac{x^2}{16}-\frac{y^2}{9}=1$

(C) $\frac{(x-1)^2}{16}+\frac{(y-5)^2}{9}=1$ (D) none of these

385. Area of the triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is

(A) $|a|$ (B) $\frac{1}{2}|a|$ (C) a^2 (D) $\frac{1}{2}a^2$

386. The coordinates of a point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) nearer the origin are
(A) (9, –13, 4) (B) $(8\sqrt{14}, -12, -1)$ (C) $(-8\sqrt{14}, 12, 1)$ (D) (–7, 11, –4)

387. The distance of the point A(–2, 3, 1) from the line PQ through P(–3, 5, 2) which make equal angles with the axes is

(A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{\frac{14}{3}}$ (C) $\frac{16}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{3}}$

388. The line joining the points (1, 1, 2) and (3, –2, 1) meets the plane $3x + 2y + z = 6$ at the point
(A) (1, 1, 2) (B) (3, –2, 1) (C) (2, –3, 1) (D) (3, 2, 1)

389. The point equidistant from the points (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0) is

(A) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$ (B) (a, b, c) (C) $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$ (D) none of these

390. The intercepts made on the axes by the plane which bisects the line joining the points (1, 2, 3) and (–3, 4, 5) at right angles are

(A) $\left(-\frac{9}{2},9,9\right)$ (B) $\left(\frac{9}{2},9,9\right)$ (C) $\left(9,-\frac{9}{2},9\right)$ (D) $\left(9,\frac{9}{2},9\right)$

391. A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The coordinates of the point A are

(A) $\left(\frac{13}{5},0\right)$ (B) $\left(\frac{5}{13},0\right)$ (C) (–7, 0) (D) none of these

392. The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in
(A) I quadrant (B) II Quadrant (C) III Quadrant (D) IV Quadrant

393. In Δ ABC if orthocentre be (1, 2) and circumcentre be (0, 0), then centroid of ΔABC is
(A) (1/2, 2/3) (B) (1/3, 2/3) (C) (2/3, 1) (D) none of these

394. If the point (a, a) fall between the lines |x + y| = 2, then

(A) |a| = 2 (B) |a| = 1 (C) |a| < 1 (D) $|a| < \frac{1}{2}$

395. If f (x + y) = f(x) f(y), ∀ x, y ∈ R and f(1) = 2,then area enclosed by 3 |x| + 2 |y| ≤ 8 is

(A) f(4) sq unit (B) $\left(\frac{1}{2}\right)$ f(6) sq unit (C) $\frac{1}{3}$ f(6) sq unit (D) $\frac{1}{3}$ f(5) sq unit

396. The four points of intersection of the lines (2x – y + 1) (x – 2y + 3) = 0 with the axes lie on a circle whose centre is at the point
(A) (–7/4, 5/4) (B) (3/4, 5/4) (C) (9/4, 5/4) (D) (0, 5/4)

397. Origin is a limiting point of a coaxial system of which $x^2 + y^2 - 6x - 8y + 1 = 0$ is a member. The other limiting point is

(A) (–2, –4) (B) $\left(\frac{3}{25},\frac{4}{25}\right)$ (C) $\left(-\frac{3}{25},-\frac{4}{25}\right)$ (D) $\left(\frac{4}{25},\frac{3}{25}\right)$

398. The shortest distance from the point (2, –7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is
(A) 1 (B) 2 (C) 3 (D) 4

399. The equation of the image of the circle $(x - 3)^2 + (y - 2)^2 = 1$ by the mirror x + y = 19 is
(A) $(x - 14)^2 + (y - 13)^2 = 1$ (B) $(x - 15)^2 + (y - 14)^2 = 1$
(C) $(x - 16)^2 + (y - 15)^2 = 1$ (D) $(x - 17)^2 + (y - 16)^2 = 1$

400. The locus of centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touch the y-axis is given by the equation
(A) $x^2 - 6x - 10y + 14 = 0$ (B) $x^2 - 10x - 6y + 14 = 0$
(C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

401. If tangents at A and B on the parabola $y^2 = 4ax$ intersect at point C, then ordinates of A, C and B are
(A) always in AP (B) always in GP (C) always in HP (D) none of these

402. Let P be any point on the parabola $y^2 = 4ax$ whose focus is S. If normal at P meet x-axis at Q. Then Δ PSQ is always
(A) isosceles (B) equilateral (C) right angled (D) None of these

403. The vertex of the parabola whose focus is (–1, 1) and directrix is $4x + 3y - 24 = 0$ is
(A) (0, 3/2) (B) (0, 5/2) (C) (1, 3/2) (D) (1/ 5/2)

404. The equation of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is
(A) $x + 2y + 4 = 0$ (B) $2x + y - 4 = 0$ (C) $x - 2y - 4 = 0$ (D) $x - 2y + 4 = 0$

405. The locus of point of intersection of tangents to the parabolas $y^2 = 4(x + 1)$ and $y^2 = 8(x + 2)$ which are perpendicular to each other is
(A) $x + 7 = 0$ (B) $x - y = 4$ (C) $x + 3 = 0$ (D) $y - x = 12$

406. AB is a diameter of $x^2 + 9y^2 = 25$. The eccentric angle of A is $\pi/6$, then the eccentric angle of B is
(A) $5\pi/6$ (B) $-5\pi/6$ (C) $-2\pi/3$ (D) none of these

407. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and parabola $y^2 = 4x$ above the x-axis is
(A) $\sqrt{3}y = 3x + 1$ (B) $\sqrt{3}y = -(x + 3)$ (C) $\sqrt{3}y = x + 3$ (D) $\sqrt{3}y = -(3x + 1)$

408. The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K, then the angle which KP subtends at its focus is
(A) 30º (B) 45º (C) 60º (D) 90º

409. Sides of an equilateral Δ ABC touch the parabola $y^2 = 4ax$ then the points A, B and C lie on
(A) $y^2 = (x + a)^2 + 4ax$ (B) $y^2 = 3(x + a)^2 + ax$
(C) $y^2 = 3(x + a)^2 + 4ax$ (D) $y^2 = (x + a)^2 + ax$

410. The common tangent of the parabolas $y^2 = 4x$ and $x^2 = -8y$ is
(A) $y = x + 2$ (B) $y = x - 2$ (C) $y = 2x + 3$ (D) None of these

411. If a triangle is inscribed in a rectagular hyperbola, its orthocentre lies
(A) inside the curve (B) outside the curve (C) on the curve (D) none of these

412. Tangents drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$, then tangents are at angle
(A) $\pi/4$ (B) $\pi/2$ (C) $\pi/3$ (D) $2\pi/3$

413. A ray emanating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8, then the equation of the reflected ray after first reflection is (P lies in first quadrant)
(A) $\sqrt{3}x - y + 7 = 0$ (B) $3\sqrt{3}x - 13y + 15\sqrt{3} = 0$
(C) $3\sqrt{3}x + 13y - 15\sqrt{3} = 0$ z (D) $\sqrt{3}x + y - 14 = 0$

414. The symmetric form of the equations of the line $x + y - z = 1$, $2x - 3y + z = 2$ is

(A) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ (B) $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{5}$ (C) $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$ (D) $\frac{x}{3} = \frac{y}{2} = \frac{z}{5}$

415. The equation of the line passing through the point (1, 1, –1) and perpendicular to the plane $x - 2y - 3z = 7$ is

(A) $\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z+1}{3}$ (B) $\frac{x-1}{-1} = \frac{y-1}{-2} = \frac{z+1}{3}$

(C) $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$ (D) none of these

416. The projections of a line on the axes are, 9, 12 and 8. The length of the line is
(A) 7 (B) 17 (C) 21 (D) 25

417. The three planes $4y + 6z = 5$; $2x + 3y + 5z = 5$; $6x + 5y + 9z = 10$.
(A) meet in a point (B) have a line in common
(C) form a triangular prism (D) none of these

418. The foot of the perpendicular from P(1, 0, 2) to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point

(A) (1, 2, –3) (B) $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$ (C) (2, 4, –6) (D) (2, 3, 6)

419. The plane containing the two lines $\frac{x-3}{1} = \frac{y-2}{4} = \frac{z-1}{5}$ and $\frac{x-2}{1} = \frac{y+3}{-4} = \frac{z+1}{5}$ is

$11x + my + nz = 28$, where
(A) m = –1, n = 3 (B) m = 1, n = –3 (C) m = –1, n = –3 (D) m = 1, n = 3

420. The projection of the line $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-1}{3}$ on the plane $x - 2y + z = 6$ is the line of intersection of this plane with the plane
(A) $2x + y + 2 = 0$ (B) $3x + y - z = 2$ (C) $2x - 3y + 8z = 3$ (D) none of these

421. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 0$, then $((\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))$ is equal to

(A) $48\vec{b}$ (B) $-48\vec{b}$ (C) $48\vec{a}$ (D) $-48\vec{a}$

422. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following Statements is true ?

(A) $\vec{a}$ is parallel to $\vec{b}$ (B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$ (D) none of these

423. Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}$, $\vec{b} = 2\hat{i} - 6\hat{j}$ and $\vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$.

Then the resolutionof the vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by

(A) $3\vec{a} - 2\vec{b}$ (B) $2\vec{a} - 3\vec{b}$ (C) $3\vec{b} - 2\vec{a}$ (D) none of these

424. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed

(A) 4 (B) 9 (C) 8 (D) 6

425. If unit vector $\vec{c}$ makes an angle $\frac{\pi}{3}$ with $\hat{i} + \hat{j}$, then minimum and maximum values of $(\hat{i} \times \hat{j}).\vec{c}$ respectively are

(A) $0, \frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$ (C) $-1, \frac{\sqrt{3}}{2}$ (D) none of these

426. If the lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ (a,b,c being distinct $\neq 1$) are concurrent, then the value of $\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$ is

(A) -1 (B) 0 (C) 1 (D) none of these

427. The equation of a line through the point (1, 2) whose distance from the point (3, 1) has the greatest possible value is

(A) $y = x$ (B) $y = 2x$ (C) $y = -2x$ (D) $y = -x$

428. If $(-6, -4)$, $(3, 5)$, $(-2, 1)$ are the vertices of a parallelogram, then remaining vertex cannot be

(A) $(0, -1)$ (B) $(-1, 0)$ (C) $(-11, -8)$ (D) $(7, 10)$

429. Length of the median from B on AC where, A $(-1, 3)$ B $(1, -1)$, C $(5, 1)$ is

(A) $\sqrt{18}$ (B) $\sqrt{10}$ (C) $2\sqrt{3}$ (D) 4

430. The incentre of the triangle formed by the lines $x = 0$, $y = 0$ and $3x + 4y = 12$ is at

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $(1, 1)$ (C) $\left(1, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, 1\right)$

431. Centre of the circle whose radius is 3 and which touches internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point $(-1, -1)$ is

(A) $\left(\frac{7}{5}, \frac{-4}{5}\right)$ (B) $\left(\frac{4}{5}, \frac{7}{5}\right)$ (C) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (D) $\left(\frac{7}{5}, \frac{3}{5}\right)$

432. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at

(A) $(6, 7)$ (B) $(-6, 7)$ (C) $(6, -7)$ (D) $(-6, -7)$

433. The locus of a point which moves so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is 2 : 3 is

(A) $5x^2 + 5y^2 + 60x - 7 = 0$ (B) $5x^2 + 5y^2 - 60x - 7 = 0$

(C) $5x^2 + 5y^2 + 60x + 7 = 0$ (D) $5x^2 + 5y^2 + 60x + 12 = 0$

434. The set of values of 'c' so that the equations $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ have no solution, is

(A) $(-\infty, -3) \cup (3, \infty)$ (B) $(-3, 3)$

(C) $(-\infty, 5\sqrt{2}) \cup (5\sqrt{2}, \infty)$ (D) $(5\sqrt{2} - 4, \infty)$

435. PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than
(A) 40 (B) 45 (C) 32 (D) 48

436. If $(\alpha^2, \alpha - 2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points (2, 2) and (8, –4), then α belongs to the interval
(A) $(-2 + 2\sqrt{2}, 2)$ (B) $(-2 + 2\sqrt{2}, \infty)$ (C) $(-2 - 2\sqrt{2}, \infty)$ (D) none of these

437. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then
(A) $-2 \le T \le 2$ (B) $T \in (-\infty, -8) \cup (8, \infty)$
(C) $T^2 < 8$ (D) $T^2 \ge 8$

438. If tangents at extremities of a focal chord AB of the parabola $y^2 = 4ax$ intersect at a point C, then $\angle$ ACB is equal to
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

439. The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is
(A) 27/4 sq unit (B) 9 sq unit (C) 27/2 sq unit (D) 27 sq unit

440. If the tangent at the point $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then the value of ϕ is
(A) $\pm \pi/2$ (B) $\pm \pi/4$ (C) $\pm \pi/3$ (D) $\pm \pi/6$

441. If roots of quadratic equation $ax^2 + 2bx + c = 0$ are not real, then $ax^2 + 2bxy + cy^2 + dx + ey + f = 0$ represents a/an
(A) ellipse (B) circle (C) parabola (D) hyperbola

442. The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is
(A) 4 (B) 6 (C) 10 (D) 8

443. Equation of the rectangular hyperbola whose focus is (1, –1) and the corresponding directrix $x - y + 1 = 0$ is
(A) $x^2 - y^2 = 1$ (B) $xy = 1$
(C) $2xy - 4x + 4y + 1 = 0$ (D) $2xy + 4x - 4y - 1 = 0$

444. If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then PA. PB is equal to [where $P \equiv (\sqrt{3}, 0)$]
(A) $\frac{4(\sqrt{3}+2)}{3}$ (B) $\frac{4(2-\sqrt{3})}{3}$ (C) $\frac{4\sqrt{3}}{2}$ (D) $\frac{2(\sqrt{3}+2)}{3}$

445. The point $(-2m, m + 1)$ is an interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$. Then m belongs to the interval
(A) $-5 - 2\sqrt{6} < m < 1$ (B) $0 < m < 4$
(C) $-1 < m < \frac{3}{5}$ (D) $-1 < m < -5 + 2\sqrt{6}$

446. Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 120^\circ$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, then

$\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 275 (C) 325 (D) 300

447. If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}|$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

448. for non-zero vectros $\vec{a}, \vec{b}, \vec{c}$ $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds iff

(A) $\vec{a}\cdot\vec{b} = 0, \vec{b}\cdot\vec{c} = 0$ (B) $\vec{b}\cdot\vec{c} = 0, \vec{c}\cdot\vec{a} = 0$

(C) $\vec{c}\cdot\vec{a} = 0, \vec{a}\cdot\vec{b} = 0$ (D) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

449. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$, and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linear dependent vectros and $|\vec{c}| = \sqrt{3}$, then

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

450. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$ is equal to

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

451. $(\vec{r}\cdot\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}\cdot\hat{j})(\vec{r} \times \hat{j}) + (\vec{r}\cdot\hat{k})(\vec{r} \times \hat{k})$ is equal to

(A) $3\vec{r}$ (B) $\vec{r}$ (C) $\vec{0}$ (D) none of these

452. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30º, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 3

453. Let $\vec{a}$ and $\vec{b}$ are two vectors making angles θ with each other, then unit vectors along bisector of $\vec{a}$ and $\vec{b}$ is

(A) $\pm\frac{\hat{a}+\hat{b}}{2}$ (B) $\pm\frac{\hat{a}+\hat{b}}{2\cos\theta}$ (C) $\pm\frac{\hat{a}+\hat{b}}{2\cos\theta/2}$ (D) $\pm\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$

454. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a} \neq 0$ and $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}$, $|\vec{a}| = |\vec{c}| = 1$, $|\vec{b}|=4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$, if $\vec{b} - 2\vec{c} = \lambda\vec{a}$. Then λ equals

(A) 1 (B) −1 (C) 2 (D) −4

455. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero and non-coplanar vectros and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$, $\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$. If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is V_1 and that of the parallelopiped determined by $\vec{p}, \vec{q}$ and $\vec{r}$ is V_2 then $V_2 : V_1$ is equal to

(A) 3 : 1 (B) 7 : 1 (C) 11 : 1 (D) 15 : 1

456. The line joining the points $6\vec{a} - 4\vec{b} - 5\vec{c}$, $-4\vec{c}$ and the line joining the points $-\vec{a} - 2\vec{b} - 3\vec{c}$, $\vec{a} + 2\vec{b} - 5\vec{c}$ intersect at

(A) $2\vec{c}$ (B) $-4\vec{c}$ (C) $8\vec{c}$ (D) none of these

457. A vector $\vec{a} = (x, y, z)$ makes an obtuse angle with y-axis, equal angles with $\vec{b} = (y, -2z, 3x)$

and $\vec{c}$ =(2z, 3x, – y) and $\vec{a}$ is perpendicular to $\vec{d}$ = (1, –1, 2) if $|\vec{a}|$ = 2, then vector $\vec{a}$ is
(A) (1, 2, 3) (B) (2, –2, –2) (C) (–1, 2, 4) (D) none of these

458. The position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ of four points A, B, C and D on a plane are such that $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$, then the point D is
(A) centroid of Δ ABC (B) orthocentre of Δ ABC
(C) circumcentre of Δ ABC (D) none of these

459. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is $\vec{a} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector
(A) $-9\hat{i} + 5\hat{j} + 2\hat{k}$ (B) $9\hat{i} + 5\hat{j} - 2\hat{k}$ (C) $9\hat{i} - 5\hat{j} - 2\hat{k}$ (D) $9\hat{i} + 5\hat{j} + 2\hat{k}$

460. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a}$ and $\vec{b}$ is -
(A) $\frac{19}{5\sqrt{43}}$ (B) $\frac{19}{3\sqrt{43}}$ (C) $\frac{19}{2\sqrt{45}}$ (D) $\frac{19}{6\sqrt{43}}$

461. If 3a + 2b + 6c = 0, then family of straight lines ax + by + c = 0 passes through a fixed point whose coordinates are given by
(A) (1/2, 1/3) (B) (2, 3) (C) (3, 2) (D) (1/3, 1/2)

462. Equation of a straight line passing through the point of intersection of x – y + 1 = 0 and 3x + y – 5 = 0 are perpendicular to one of them is
(A) x + y + 3 = 0 (B) x + y – 3 = 0 (C) x – 3y – 5 = 0 (D) x + 3y + 5 = 0

463. The points (p + 1,1), (2p + 1,3) and (2p + 2, 2p) are collinear, if
(A) p = –1 (B) p = 1/2 (C) p = 2 (D) $p = -\frac{1}{3}$

464. If m_1 and m_2 are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$ and if area of the triangle formed by the lines $y = m_1 x$, $y = m_2 x$, and y = c is $(a + b)c^2$, then the value of 2008 $(a^2 + b^2)$ must be
(A) 5050 (B) 2255 (C) 5522 (D) none of these

465. If the lines x = a + m, y = –2 and y = mx are concurrent, the least value of |a| is :
(A) 0 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) none of these

466. A focal chord of $y^2 = 4ax$ meets in P and Q. If S is the focus, then $\frac{1}{SP} + \frac{1}{SQ}$ is equal to
(A) $\frac{1}{a}$ (B) $\frac{2}{a}$ (C) $\frac{4}{a}$ (D) none

467. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)}$ represents an ellipse with major axis as y-axis and f is a decreasing function, then
(A) $a \in (-\infty, 1)$ (B) $a \in (5, \infty)$ (C) $a \in (1, 4)$ (D) $a \in (-1, 5)$

468. The set of positive value of m for which a line with slope m is a comon tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and parabola $y^2 = 4ax$ is given by

(A) (2, 0) (B) (3, 5) (C) (0, 1) (D) none of these

469. From a point on the line y = x + c, c (parameter), tangents are drawn to the hyperbola $\frac{x^2}{2}-\frac{y^2}{1}=1$ such that chords of contact pass through a fixed point (x_1, y_1). Then $\frac{x_1}{y_1}$ is equal to

(A) 2 (B) 3 (C) 4 (D) none

470. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of b^2 is

(A) 3 (B) 16 (C) 9 (D) 12

471. Column 2 contains curves satisfying the condition in column I.

Column – I	**Column – II**
(a) Subtangent is constant	(p) Parabola
(b) Subnormal is constant	(q) Ellipse
(c) Subtangent is equal to twice the abscissa	(r) Hyperbola
(d) Subnormal is equal to twice the abscissa	(s) Exponential curve

(A) a → s, b → p, c → p, d → r (B) a → p, b → p, c → r, d → r
(C) a → s, b → r, c → p, d → r (D) None of these

472. The distance of the point (2, 1, –2) from the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$ measured parallel to the plane x + 2y + z = 4 is

(A) $\sqrt{10}$ (B) $\sqrt{20}$ (C) $\sqrt{5}$ (D) $\sqrt{30}$

473. The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (C) $3\sqrt{6}$ (D) $5\sqrt{6}$

474. The equation of the line passing through (1, 2, 3) and parallel to the planes x – y + 2z = 5 and 3x + y + z = 6 is

(A) $\frac{x-1}{-3}=\frac{y-2}{5}=\frac{z-3}{4}$ (B) $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-1}{4}$

(C) $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-1}{-4}$ (D) None

475. The line $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ and the plane 4x + 5y + 3z – 5 = 0 intersect at a point

(A) (3, –1, 1) (B) (3, –2, 1) (C) (2, –1, 3) (D) (–1, –2, –3)

476. A line makes angles of 45º and 60º with the positive axes of X and Y respectively. The angle made by the same line with the positive Z axis is

(A) 30º or 60º (B) 60º or 90º (C) 90º or 120º (D) 60º or 120º

477. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ meets the co-ordinate axes in A, B, C. The centroid of the triangle ABC is

(A) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (B) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ (C) $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ (D) (a, b, c)

478. The line $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, if

(A) k = 0 or –1 (B) k = 0 or 1 (C) k = 0 or –3 (D) k = 3 or –3

479. The position vectors of the points P and Q with respet to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of POQ, then $\overrightarrow{OM}$ is

(A) $2\left(\hat{i} - \hat{j} + \hat{k}\right)$ (B) $2\hat{i} + \hat{j} - 2\hat{k}$ (C) $2(-\hat{i} + \hat{j} - \hat{k})$ (D) $2(\hat{i} + \hat{j} + \hat{k})$

480. If $\vec{b}$ is vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio k : 1 and terminal point is origin and $|\vec{b}| \le \sqrt{37}$, then k lies in the interval

(A) [–6, –1/6] (B) $(-\infty, -6] \cup [-1/6, \infty)$ (C) [0, 6] (D) None of these

481. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then one equatin of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

482. If the vertices of a triangle have integral co-ordinates, the triangle can not be

(A) an equilateral triangle (B) a right angled triangle

(C) an isosceles triangle (D) none of the above

483. If $P(a_1, b_1)$ and $Q(a_2, b_2)$ are two points, then OP, OQ cos ($\angle$ POQ) is (O is origin)

(A) $a_1a_2 + b_1b_2$ (B) $a_1^2 + a_2^2 + b_1^2 + b_2^2$

(C) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (D) none of tehse

484. If points A (x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are such that x_1, x_2, x_3, and y_1, y_2, y_3 are in G.P., with same common ratio then

(A) A,B and C are concyclic points (B) A,B and C are collinear points

(C) A,B and C are vertices of an equilateral triangle (D) None of the above

485. The equation of the line parallel to lines $L_1 \equiv x + 2y - 5 = 0$ and $L_2 \equiv x + 2y + 9 = 0$ and dividing the distance between L_1 and L_2 in the ratio 1 : 6 (internally), is

(A) x + 2y – 3 = 0 (B) x + 2y + 2 = 0 (C) x + 2y + 7 = 0 (D) None of these

486. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three vectors having magnitudes 1, 5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c}$, then tan θ is equal to

(A) 0 (B) 2/3 (C) 3/5 (D) 3/4

487. $\vec{a}$ and $\vec{c}$ are unit vectors and $|\vec{b}| = 4$. If angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$ and $\vec{a} \times \vec{b} = 2\vec{a}\times\vec{c}$, then $\vec{b} = \lambda\vec{a} + 2\vec{c}$, where λ is equal to

(A) $\pm \frac{1}{4}$ (B) $\pm \frac{1}{2}$ (C) ± 1 (D) ± 4

488. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors, then the triple product $[\vec{a} + \vec{b} + \vec{c} \quad \vec{a} + \vec{b} \quad \vec{b} + \vec{c}]$ equals

(A) 0 (B) 1 or –1 (C) 1 (D) 3

489. If $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = a\vec{\delta}$ and $\vec{\beta} + \vec{\gamma} + \vec{\delta} = b\vec{\alpha}$ and $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are non-coplanar and $\vec{\alpha}$ is not parallel to $\vec{\delta}$, then $\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta}$ equals

(A) $a\vec{\alpha}$ (B) $b\vec{\delta}$ (C) 0 (D) $(a + b)\vec{\gamma}$

490. The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to

(A) ± 1 (B) $\pm \frac{1}{3}$ (C) $\pm \sqrt{5}$ (D) None

Questions based on statements (Q. 491 - 500)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

491. **Statement–I :** The lines $(a + b)x + (a - b)y - 2ab = 0$, $(a - b)x + (a + b)y - 2ab = 0$ and $x + y = 0$ form an isosceles triangle.
Statement–II : If internal bisector of any angle of triangle is perpendiuclar to the oppoiste side, then the given triangle is isosceles.

492. **Statement–I :** The chord of contact of tangent from three points A, B, C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B, C will be collinear.
Statement–II : A, B, C always lies on the normal to the circle $x^2 + y^2 = a^2$

493. Let C_1 be the circle with centre O_1 (0, 0) and radius 1 and C_2 be the circle with centre O_2 $(t, t^2 + 1)$ $(t \in R)$ and radius 2.
Statement–I : Circles C_1 and C_2 always have at least one common tangent for any value of t.
Statement–II: For the two circles, $O_1 O_2 \geq |r_1 - r_2|$, where r_1 and r_2 are their radii for any value of t.

494. **Statement–I :** If end points of two normal chords AB and CD (normal at A and C) of a parabola $y^2=4ax$ are concyclic, then the tangents at A and C will intersect on the axis of the parabola.

Statement–II : If four point on the parabola $y^2 = 4ax$ are concyclic, then sum of their ordinates is zero.

495. **Statement–I :** Locus o fthe centre of a variable circle touching two cicles $(x-1)^2+(y-2)^2=25$ and $(x - 2)^2 + (y - 1)^2 = 16$ is an ellipse.

Statement–II : If a circle $S_2 = 0$ lies completely inside the circle $S_1 = 0$, then locus of the centre of a variable circle $S = 0$ that touches both the circles is an ellipse.

496. **Statement–I :** Given the base BC of the triagle and the ratio radius of the ex-circles opposite to the angles B and C. Then locus of the vertex A is hyperbola.

Statement–II : $|S'P - SP| = 2a$, where S and S′ are the two foci, 2a = length of the transverse axis and P be any point on the hyperbola.

497. Let $\vec{r}$ be a non-zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non-zero vectors $\vec{a}.\vec{b}$ and $\vec{c}$.

Statement–I : $[\vec{a}-\vec{b} \;\; \vec{b}-\vec{c} \;\; \vec{c}-\vec{a}] = 0$

Statement–II : $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

498. **Statement–I :** $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are non-coplanar. If $[\vec{d}\,\vec{b}\,\vec{c}] = [\vec{d}\,\vec{a}\,\vec{b}] = [\vec{d}\,\vec{c}\,\vec{a}] = 1$, then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$.

Statement–II : $[\vec{d}\,\vec{b}\,\vec{c}] = [\vec{d}\,\vec{a}\,\vec{b}] = [\vec{d}\,\vec{c}\,\vec{a}] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

499. The equation of two straight lines are $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{-3}$ and $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z+3}{2}$.

Statement–I : The given lines are coplanar.

Statement–II : The equations $2x_1 - y_1 = 1$, $x_1 + 3y_1 = 4$ and $3x_1 + 2y_1 = 5$ are consistent.

500. **Statement–I :** There exist two points on the line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+2}{2}$ which are at a distance of 2 units from point (1, 2, –4).

Statement–II : Perpendicular distance of point (1, 2, –4) from the line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+2}{2}$ is 1 unit.

ANSWER KEY

CALCULUS

1.	B	**2.**	A	**3.**	B	**4.**	A	**5.**	D	**6.**	A
7.	D	**8.**	C	**9.**	D	**10.**	D	**11.**	D	**12.**	D
13.	D	**14.**	B	**15.**	A	**16.**	C	**17.**	A	**18.**	D
19.	D	**20.**	D	**21.**	B	**22.**	A	**23.**	A	**24.**	C
25.	C	**26.**	A	**27.**	D	**28.**	D	**29.**	C	**30.**	A
31.	D	**32.**	C	**33.**	A	**34.**	B	**35.**	A	**36.**	C
37.	A	**38.**	B	**39.**	C	**40.**	C	**41.**	A	**42.**	B
43.	B	**44.**	D	**45.**	B	**46.**	A	**47.**	C	**48.**	D
49.	A	**50.**	A	**51.**	B	**52.**	A	**53.**	B	**54.**	B
55.	A	**56.**	A	**57.**	B	**58.**	B	**59.**	C	**60.**	A
61.	A	**62.**	A	**63.**	B	**64.**	B	**65.**	B	**66.**	B
67.	C	**68.**	C	**69.**	C	**70.**	C	**71.**	A	**72.**	D
73.	B	**74.**	A	**75.**	A	**76.**	C	**77.**	C	**78.**	D
79.	C	**80.**	B	**81.**	D	**82.**	C	**83.**	D	**84.**	A
85.	C	**86.**	B	**87.**	C	**88.**	D	**89.**	C	**90.**	B
91.	B	**92.**	B	**93.**	A	**94.**	A	**95.**	B	**96.**	A
97.	A	**98.**	A	**99.**	B	**100.**	C	**101.**	A	**102.**	A
103.	B	**104.**	A	**105.**	C	**106.**	B	**107.**	D	**108.**	D
109.	C	**110.**	C	**111.**	C	**112.**	A	**113.**	A	**114.**	D
115.	A	**116.**	D	**117.**	C	**118.**	B	**119.**	D	**120.**	B
121.	C	**122.**	A	**123.**	B	**124.**	D	**125.**	D	**126.**	D

127. D	**128.** A	**129.** D	**130.** D	**131.** D	**132.** A
133. C	**134.** D	**135.** B	**136.** B	**137.** A	**138.** A
139. D	**140.** A	**141.** D	**142.** D	**143.** B	**144.** A
145. A	**146.** C	**147.** B	**148.** A	**149.** A	**150.** A

TRIGONOMETRY

151. A	**152.** D	**153.** A	**154.** A	**155.** B	**156.** A
157. B	**158.** D	**159.** A	**160.** D	**161.** A	**162.** A
163. C	**164.** C	**165.** B	**166.** C	**167.** B	**168.** A
169. A	**170.** C	**171.** A	**172.** D	**173.** B	**174.** C
175. D	**176.** A	**177.** B	**178.** D	**179.** **C**	**180.** D
181. C	**182.** A	**183.** A	**184.** C	**185.** C	**186.** A
187. C	**188.** A	**189.** A	**190.** A	**191.** B	**192.** A
193. B	**194.** C	**195.** A	**196.** C	**197.** A	**198.** A
199. C	**200.** D	**201.** A	**202.** B	**203.** A	**204.** B
205. B	**206.** D	**207.** D	**208.** A	**209.** A	**210.** A

ALGEBRA

211. C	**212.** C	**213.** B	**214.** C	**215.** D	**216.** B
217. B	**218.** B	**219.** A	**220.** D	**221.** C	**222.** D
223. A	**224.** D	**225.** D	**226.** B	**227.** B	**228.** D
229. A	**230.** B	**231.** B	**232.** C	**233.** B	**234.** D

235. B	**236.** C	**237.** C	**238.** B	**239.** A	**240.** A
241. B	**242.** D	**243.** D	**244.** A	**245.** B	**246.** A
247. C	**248.** A	**249.** A	**250.** C	**251.** D	**252.** D
253. D	**254.** B	**255.** B	**256.** D	**257.** C	**258.** D
259. A	**260.** B	**261.** D	**262.** C	**263.** B	**264.** A
265. D	**266.** B	**267.** A	**268.** C	**269.** B	**270.** C
271. C	**272.** D	**273.** D	**274.** A	**275.** D	**276.** B
277. B	**278.** C	**279.** A	**280.** D	**281.** B	**282.** C
283. C	**284.** A	**285.** C	**286.** D	**287.** C	**288.** B
289. D	**290.** C	**291.** A	**292.** C	**293.** C	**294.** B
295. A	**296.** B	**297.** C	**298.** B	**299.** C	**300.** C
301. C	**302.** C	**303.** C	**304.** B	**305.** A	**306.** D
307. B	**308.** B	**309.** C	**310.** A	**311.** C	**312.** A
313. C	**314.** A	**315.** B	**316.** D	**317.** C	**318.** B
319. C	**320.** A	**321.** B	**322.** B	**323.** B	**324.** C
325. B	**326.** C	**327.** D	**328.** C	**329.** A	**330.** C
331. C	**332.** B	**333.** C	**334.** A	**335.** A	**336.** C
337. B	**338.** B	**339.** C	**340.** C	**341.** B	**342.** C
343. D	**344.** A	**345.** B	**346.** D	**347.** A	**348.** A
349. C	**350.** B	**351.** A	**352.** C	**353.** D	**354.** A
355. D	**356.** C	**357.** B	**358.** A	**359.** A	**360.** C

CO–ORDINATE GEOMETRY

361. D	**362.** A	**363.** D	**364.** B	**365.** C	**366.** C
367. C	**368.** D	**369.** A	**370.** B	**371.** B	**372.** A
373. B	**374.** B	**375.** A	**376.** C	**377.** A	**378.** C
379. A	**380.** D	**381.** B	**382.** C	**383.** C	**384.** A
385. A	**386.** D	**387.** B	**388.** B	**389.** C	**390.** A
391. A	**392.** A	**393.** B	**394.** C	**395.** C	**396.** A
397. B	**398.** B	**399.** D	**400.** D	**401.** A	**402.** A
403. D	**404.** D	**405.** C	**406.** B	**407.** C	**408.** D
409. C	**410.** D	**411.** C	**412.** B	**413.** B	**414.** C
415. C	**416.** B	**417.** B	**418.** B	**419.** C	**420.** A
421. A	**422.** D	**423.** B	**424.** B	**425.** B	**426.** C
427. B	**428.** A	**429.** B	**430.** B	**431.** B	**432.** D
433. C	**434.** D	**435.** C	**436.** A	**437.** D	**438.** C
439. D	**440.** C	**441.** A	**442.** D	**443.** C	**444.** A
445. D	**446.** D	**447.** C	**448.** D	**449.** D	**450.** A
451. C	**452.** B	**453.** C	**454.** D	**455.** D	**456.** B
457. B	**458.** B	**459.** B	**460.** A	**461.** A	**462.** B
463. C	**464.** C	**465.** C	**466.** A	**467.** D	**468.** C
469. A	**470.** B	**471.** A	**472.** D	**473.** B	**474.** A
475. A	**476.** D	**477.** D	**478.** C	**479.** B	**480.** B
481. C	**482.** A	**483.** A	**484.** B	**485.** A	**486.** D
487. D	**488.** B	**489.** C	**490.** C	**491.** A	**492.** C
493. A	**494.** A	**495.** D	**496.** D	**497.** B	**498.** B
499. A	**500.** C				

1. **B**

$\because 0 < \{x\} < 1 \Rightarrow 0 < \sin\{x\} < \sin 1$

Also, $\dfrac{1}{\sin\{x\}} > 0$ Then, $\left[\dfrac{1}{\sin\{x\}}\right] = 1, 2, 3, \ldots$

$\therefore R_f = N$

2. **A**

$\because x^2 f(x) - 2f\left(\dfrac{1}{x}\right) = g(x)$ ....(i)

$\Rightarrow$ Replacing x by $\dfrac{1}{x}$, then

$\dfrac{1}{x^2} f\left(\dfrac{1}{x}\right) - 2f(x) = g\left(\dfrac{1}{x}\right)$

or $2f\left(\dfrac{1}{x}\right) - 4x^2 f(x) = 2x^2 g\left(\dfrac{1}{x}\right)$ ....(ii)

Adding Eqs. (i) and (ii), then

$-3x^2 f(x) = g(x) + 2x^2 g\left(\dfrac{1}{x}\right)$

or $f(x) = -\left(\dfrac{g(x) + 2x^2 g\left(\frac{1}{x}\right)}{3x^2}\right)$

$\Rightarrow f(-x) = -\left(\dfrac{g(-x) + 2x^2 g\left(-\frac{1}{x}\right)}{3x^2}\right)$

$f(x) = -f(x) \quad \therefore f(x) = 0$

($\because$ g(x) is an odd and f(x) is an even functions)

Then, $f(5) = 0$

3. **B**

$$\lim_{x\to\pi/2} \frac{\sin(x\cos x)}{\sin\left(\frac{\pi}{2} - x\sin x\right)}$$

$$= \lim_{x\to\pi/2} \frac{\sin(x\cos x)}{(x\cos x)} \cdot \frac{x\cos x}{\sin\left(\frac{\pi}{2} - x\sin x\right)} \cdot \frac{\left(\frac{\pi}{2} - x\sin x\right)}{\left(\frac{\pi}{2} - x\sin x\right)}$$

$$= 1.1. \lim_{x\to\pi/2} \frac{x\cos x}{\left(\frac{\pi}{2} - x\sin x\right)}$$

Put $x = \pi/2 + h$

then $= \lim_{h\to 0} \dfrac{\left(\frac{\pi}{2} + h\right)\cos\left(\frac{\pi}{2} + h\right)}{\frac{\pi}{2} - \left(\frac{\pi}{2} + h\right)\sin\left(\frac{\pi}{2} + h\right)}$

$= \lim_{h\to 0} \dfrac{-\left(\frac{\pi}{2} + h\right)\sin h}{\frac{\pi}{2}(1 - \cos h) - h\cos h}$

$= \lim_{h\to 0} \dfrac{-\left(\frac{\pi}{2} + h\right)\left(\frac{\sin h}{h}\right)}{\frac{\pi}{2}\frac{(1 - \cos h)}{h} - \cos h}$

(divide above and below by h)

$= \dfrac{-\left(\frac{\pi}{2} + 0\right).1}{0 - 1} = \dfrac{\pi}{2}$

4. **A**

$\because f'(a) = \lim_{x\to a} \dfrac{f(x+a) - f(a)}{x}$

and $\lim_{x\to 0} \dfrac{\ln(a+x) - \ln a}{x} + k \lim_{x\to e} \dfrac{\ln x - \ln e}{x - e} = 1$

$\therefore f'(a) + k\, f'(e) = 1$ (where $f(x) = \ln x$)

$\Rightarrow \dfrac{1}{a} + \dfrac{k}{e} = 1 \Rightarrow k = e\left(1 - \dfrac{1}{a}\right)$

5. **D**

$f(x) = -x + x - 1, x < 0$

$= x - x - 1, x \geq 0$

$f(x) = -1$ = a constant function,

Hence, differentiable for all x.

6. **A**

$f(2)=0, f(2 - 0)=1-1 = 0, f(2 + 0) = 0-0=0,$

$f(-2) = 0, f(-2 - 0) = 0 - 1 = -1$

$\Rightarrow$ f(x) is continuous at 2 but not at -2.

7. **D**

$\because f'(x) = (4a - 3)(1) + (a - 7)\cos x$

$= \left(\dfrac{(4a-3)}{(a-7)} + \cos x\right)(a - 7)$

$f'(x) \neq 0$ (for non existence of critical points)

$\dfrac{(4a-3)}{(a-7)} > 1$ or $\dfrac{(4a-3)}{(a-7)} < -1$ ($\because -1 \leq \cos x \leq 1$)

$\Rightarrow \frac{3a+4}{a-7} > 0$ or $\frac{5a-10}{a-7} < 0$

$\therefore a \in \left(-\infty, -\frac{4}{3}\right) \cup (7, \infty)$ or $a \in (2, 7)$

Hence, $a \in (-\infty, -4/3) \cup (2, \infty)$
($\because$ at $a = 7$, $f'(x) \neq 0$)

8. C

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ then $f(x) = x + \tan x$

$\Rightarrow y = f^{-1}(y) + \tan(f^{-1}(y))$

$\Rightarrow y = g(y) + \tan(g(y))$ or $x = g(x) + \tan(g(x))$ ...(i)

Differentiating both sides, then we get

$1 = g'(x) + \sec^2 g(x).\ g'(x)$

$\therefore g'(x) = \frac{1}{1+\sec^2(g(x))} = \frac{1}{1+1+\tan^2(g(x))}$

$= \frac{1}{2+(x-g(x))^2} = \frac{1}{2+(g(x)-x)^2}$ [from Eq.(i)]

9. D

The equation of tangent at (x, y) is

$Y - y = \cos x\,(X - x)$ Its pass through (0, 0)

Then, $0 - y = \cos x\,(0 - x)$

or $\cos x = \frac{y}{x}$ ....(i) Given, $\sin x = y$ ....(ii)

Squaring and adding Eqs. (i) and (ii), then

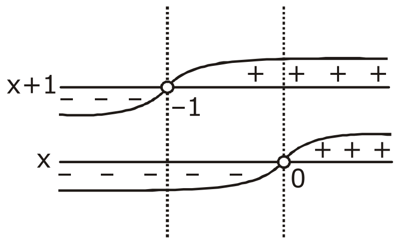
$1 = \frac{y^2}{x^2} + y^2 \Rightarrow \frac{1}{y^2} - \frac{1}{x^2} = 1$

10. D

For $x < 0$

$f(x) = |x^2 + x| = |x(x+1)| = x(x+1)\ (x < -1)$

$f'(x) = 2x + 1$



$f'(-2) = -4 + 1 = -3 \therefore$ Slope of normal $= \frac{1}{3}$

11. D

$\frac{dy}{dx} = x^2(-e^{-x}) + e^{-x}(2x) > 0$

$\Rightarrow e^{-x}\,x\,(2 - x) > 0$

$\because e^{-x} > 0 \qquad \therefore x(2 - x) > 0$

$\Rightarrow x(x - 2) < 0 \qquad \Rightarrow 0 < x < 2$

12. D

$f'(x) = 100\,x^{99} + \cos x$

$f'(x) > 0$ in $x \in (0, \pi/2)$

$f'(x) > 0$ in $x \in (0, 1)$

$f'(x) > 0$ in $x \in \left(\frac{\pi}{2}, \pi\right)$

13. D

$\therefore$ Let $P\left(t, \frac{t^2}{2}\right)$ be a point on $x^2=2y$ and a be (0,5)

If AP = d $\Rightarrow z = d^2 = t^2 + \left(\frac{t^2}{2} - 5\right)^2$

$\therefore \frac{dz}{dt} = 2t + 2\left(\frac{t^2}{2} - 5\right).t = t^3 - 8t = t(t^2 - 8)$

$\Rightarrow \frac{d^2z}{dt^2} = 3t^2 - 8$

$\therefore \frac{dz}{dt} = 0 \Rightarrow t = 0$ or $t = \pm 2\sqrt{2}$

At t = 0, $\frac{d^2z}{dt^2}$ is –ive

At $t = \pm 2\sqrt{2}$, $\frac{d^2z}{dt^2}$ is + ive

Hence, the cloest point is $(2\sqrt{2}, 4)$

14. B

$\because 0 \leq \{-x^2 + x + 1) < 1$

$\Rightarrow 0 \leq 2\{-x^2 + x + 1\} < 2$

$\therefore [2\{-x^2 + x + 1\} = 0, 1$

$\therefore$ Global maximum of f(x) is 1.

15. A

$\because 0 < x < 1 \quad \therefore |\ln x| = -\ln x$

Then, $\int |\ln x|\,dx = -\int \ln x \cdot 1dx$

$= -(x \ln x - x) + c = x + x\,|\ln x| + c$

16. C

$\because \sqrt{(x-3)}$ is defined only when $x \geq 3$

and $\sin^{-1}(\ln x) + \cos^{-1}(\ln x)$ is defined

only when $-1 \leq \ln x \leq 1 \Rightarrow \frac{1}{e} \leq x \leq e$

Then, $[3, \infty] \cap \left[\frac{1}{e}, e\right] = \phi$

Hence, the given integral does not exist.

17. A

Divide above and below by $\sin^2 x$, then

$$\int \frac{(3\,\text{cosec}^2 x + 2\,\text{cosec}\,x\cot x)}{(2\,\text{cosec}\,x + 3\cot x)^2}\,dx$$

Put 2 cosec x + 3 cot x = t

$\therefore$ (–2 cosec x cot x – 3 cosec^2 x) dx = dt

$$= \int -\frac{dt}{t^2} = \frac{1}{t} + c = \frac{1}{2\,\text{cosec}\,x + 3\cot x} + c$$

$$= \frac{\sin x}{(2+3\cos x)} + c$$

18. D

$$\int_0^{11} [x]^3 dx = \sum_{r=0}^{10} \int_r^{r+1} [x]^3 dx = \sum_{r=0}^{10} \int_r^{r+1} r^3 dx$$

$$= \sum_{r=0}^{10} (r^3) = 0^3 + 1^3 + 2^3 + ...+10^3$$

$$= \left(\frac{10.11}{2}\right)^2 = (55)^2 = 3025$$

19. D

$\because$ x > 0

$\therefore$ sin x < x $\Rightarrow$ cos (sin x) > cos x ....(i)

Also $\because$ $0 < x < \frac{\pi}{2}$ $\quad\therefore$ 1 > cos x > 0

Now, sin x < x for $x \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ sin (cos x) < cos x ....(ii)

$\therefore$ From Eqs. (i) and (ii), we get

cos (sin x) > cos x > sin (cos x)

or $\int_0^{\pi/2} \cos(\sin x)\,dx > \int_0^{\pi/2} \cos x\,dx$

$$> \int_0^{\pi/2} \sin(\cos x)\,dx$$

$\Rightarrow I_1 > I_3 > I_2$

20. D

Let $f(x) = \frac{2[x]}{3x-[x]}$

It is clear that f(x) is not defined if x = 0 and if 3x = [x]

So, in (–10, 0), f is not defined at $x = -\frac{1}{3}$

Case I : $x \in \left(-10, -\frac{1}{3}\right)$

[x] < 0 and 3x – [x] < 0 so $\frac{[x]}{3x-[x]} > 0$

$\therefore \left|\frac{2[x]}{3x-[x]}\right| = \frac{2[x]}{3x-[x]}$

$\Rightarrow \int_{-10}^{-1/3} 1dx = -\frac{1}{3} + 10 = \frac{29}{3}$

Case II : $x \in \left(-\frac{1}{3}, 0\right)$

[x] < 0 and 3x – [x] > 0 so $\frac{[x]}{3x-[x]} < 0$

$\therefore \left|\frac{2[x]}{3x-[x]}\right| = -\left(\frac{2[x]}{3x-[x]}\right)$

$\Rightarrow \int_{-\frac{1}{3}}^{0} (-1)\,dx = -\left(0+\frac{1}{3}\right) = -\frac{1}{3}$

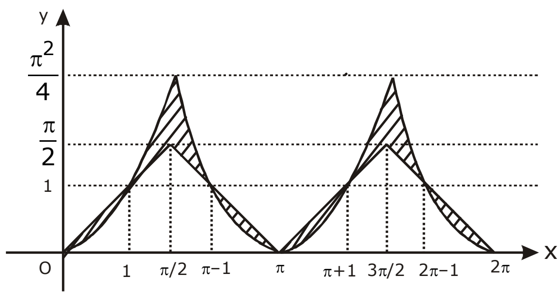
Hence $\int_{-10}^{0} \frac{|f(x)|}{f(x)} dx = \int_{-10}^{-1/3} \frac{|f(x)|}{f(x)} dx + \int_{-1/3}^{0} \frac{|f(x)|}{f(x)} dx$

$$= \frac{29}{3} - \frac{1}{3} = \frac{28}{3}$$

21. B

$$\because \sin^{-1}|\sin x| = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \\ -\pi + x, & \pi \le x \le \frac{3\pi}{2} \\ 2\pi - x, & \frac{3\pi}{2} \le x \le 2\pi \end{cases}$$

$$\therefore (\sin^{-1}|\sin x|)^2 = \begin{cases} x^2, & 0 \le x \le \frac{\pi}{2} \\ (\pi - x)^2, & \frac{\pi}{2} \le x \le \pi \\ (x-\pi)^2, & \pi \le x \le \frac{3\pi}{2} \\ (2\pi - x)^2, & \frac{3\pi}{2} \le x \le 2\pi \end{cases}$$



$\therefore$ Required area

$$=4\left\{\int_0^1 (x-x^2)\,dx+\int_1^{\pi/2}(x^2-x)\,dx\right\}$$

$$=4\left\{\left[\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\right]_0^1+\left[\left(\frac{x^3}{3}-\frac{x^2}{2}\right)\right]_1^{\pi/2}\right\}$$

$$=4\left\{\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{\pi^3}{24}-\frac{\pi^2}{8}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)\right\}$$

$$=4\left\{1-\frac{2}{3}+\frac{\pi^3}{24}-\frac{\pi^2}{8}\right\}=\left\{\frac{\pi^3}{6}-\frac{\pi^2}{2}+\frac{4}{3}\right\}\text{ Sq unit}$$

22. A

$y = 2x^4 - x^2 \;\therefore\; \frac{dy}{dx} = 8x^3 - 2x,$

for max. or min. $\frac{dy}{dx} = 0 \Rightarrow x = -\frac{1}{2}, 0, \frac{1}{2}$

Then, $\left(\frac{d^2y}{dx^2}\right)_{x=-\frac{1}{2}} > 0,\ \left(\frac{d^2y}{dx^2}\right)_{x=0} < 0$

and $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0$

$\therefore$ Required area $= \left|\int_{-1/2}^{1/2}(2x^4-x^2)\,dx\right| = \frac{7}{120}$

23. A

$$\frac{dy}{dx}=\frac{y(x-y\ln y)}{x(x\ln x-y)}$$

$\Rightarrow x^2 \ln x\,dy - x\,y dy = xy\,dx - y^2 \ln y\,dx$

On dividing by x^2y^2, then

$$\Rightarrow \frac{\ln x}{y^2}dy - \frac{1}{xy}dy = \frac{1}{xy}dx - \frac{\ln y}{x^2}dx$$

$$\Rightarrow\left(\ln x\left(-\frac{1}{y^2}dy\right)+\frac{1}{xy}dx\right)$$

$$+\left(\ln y\left(-\frac{1}{x^2}dx\right)+\frac{1}{xy}dy\right)=0$$

$$\Rightarrow d\left(\frac{\ln x}{y}\right)+d\left(\frac{\ln y}{x}\right)=0$$

On intergrating, we get

$$\frac{\ln x}{y}+\frac{\ln y}{x}=c \text{ or } \frac{x\ln x+y\ln y}{xy}=c$$

24. C

$\because (x\cos x - \sin x)\,dx = \frac{x}{y}\sin x\,dy$

$\Rightarrow (xy\cos x - y\sin x)\,dx = x\sin x\,dy$

$\Rightarrow xy\cos x\,dx = (y\,dx + x\,dy)\sin x$

$\Rightarrow \cot x\,dx = \frac{d(xy)}{xy}$

On integrating, we get

$\ln|\sin x| = \ln|xy| + \ln c$ or $\frac{|\sin x|}{|xy|} = c$

25. C

$$\Rightarrow\left(\frac{dy}{dx}+1\right)+x(x+y)=x^3(x+y)^3$$

or $\frac{(dx+dy)}{dx} + x(x+y) = x^3(x+y)^3$

Put $x + y = v$ or $dx + dy = dv$

Then, $\frac{dv}{dx} + vx = v^3x^3$

or $v^{-3}\frac{dv}{dx} + v^{-2}x = x^3$ .....(i)

Now, put $v^{-2} = t \quad \therefore\ -2v^{-3}\frac{dv}{dx} = \frac{dt}{dx}$

Then, from Eq. (i) $\frac{dt}{dx} - 2tx = -2x^3$

IF $= e^{\int -2x dx} = e^{-x^2}$

$\therefore$ Solution is $t(e^{-x^2}) = \int(-2x^3)(e^{-x^2})\,dx$

$t(e^{-x^2}) = x^2 e^{-x^2} + e^{-x^2} + c$

or $t = x^2 + 1 + ce^{x^2}$

or $\frac{1}{(x+y)^2} = x^2 + 1 + ce^{x^2}$

26. A

$\because f(x) > 0\ \forall\ x \in R \sim \{0\}$

ie, I and II quadrant ($\therefore x > 0$ and $x < 0$)

27. D

$$\text{(say) } y = f(x) = \begin{cases}\frac{-x^2}{1+x^2}, & \text{if } x \geq 0\\ \frac{x^2}{1+x^2}, & \text{if } x < 0\end{cases}$$

$$f^{-1}(y) = \begin{cases} \sqrt{\dfrac{-y}{1+y}}, & y < 0 \\ -\sqrt{\dfrac{y}{1-y}}, & y > 0 \end{cases}$$

or $$f^{-1}(x) = \begin{cases} \sqrt{\dfrac{-x}{1+x}}, & x < 0 \\ -\sqrt{\dfrac{x}{1-x}}, & x > 0 \end{cases}$$

$$\therefore \quad f^{-1}(x) = -\begin{cases} \sqrt{\left(\dfrac{-x}{1+x}\right)}, & x < 0 \\ \sqrt{\left(\dfrac{-x}{1-x}\right)}, & x > 0 \end{cases}$$

$$= (\operatorname{sgn} x)\sqrt{\frac{|x|}{1-|x|}}$$

28. D

Period of $\sin\left(\dfrac{\pi x}{n!}\right) = \dfrac{2\pi}{\pi / n!} = 2n!$ and period of

$\cos\left(\dfrac{\pi x}{(n+1)!}\right) = \dfrac{2\pi}{\pi / (n+1)!} = 2(n+1)!$

$\therefore$ Period of $f(x)$ = LCM of $\{2n!, 2(n+1)!\}$
$= 2(n+1)!$

29. C

For $a - 1 < x < a$, $\left[\dfrac{x}{a}\right] = 0$

$$\therefore \lim_{x\to a^-}\left(\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right) = \lim_{x\to a^-}\left(\frac{|x|^3}{a} - 0\right)$$

$$= \lim_{h\to 0}\left(\frac{|a-h|^3}{a}\right) = \frac{a^3}{a} = a^2$$

30. A

$$\lim_{n\to\infty}\frac{n^{\alpha}\sin^2 n!}{n\left(1+\dfrac{1}{n}\right)} = \lim_{n\to\infty}\frac{\sin^2 n!}{n^{1-\alpha}\left(1+\dfrac{1}{n}\right)} = 0$$

$(\because 0 < \sin^2 n! < 1)$

31. D

$$\lim_{x\to 0}\frac{1}{x}\left(\int_y^c e^{\sin^2 t}\,dt - \int_{x+y}^c e^{\sin^2 t}\,dt\right)$$

$$= \lim_{x\to 0}\frac{1}{x}\left(\int_y^c e^{\sin^2 t}\,dt + \int_c^{x+y} e^{\sin^2 t}\,dt\right)$$

$$= \lim_{x\to 0}\frac{\int_y^{x+y} e^{\sin^2 t}\,dt}{x} = \lim_{x\to 0}\frac{e^{\sin^2(x+y)} - 0}{1}$$

$$= e^{\sin^2 y}$$

32. C

We have, $f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x-2|, & 1 \le x < 2 \end{cases}$

$$= 2 - x,\ 1 \le x < 2 \quad \begin{cases} -1, & \frac{1}{2} < x < 1 \\ 0, & 0 < x \le \frac{1}{2} \\ 1, & x = 0 \\ 0, & -\frac{1}{2} \le x < 0 \\ -1, & -\frac{3}{2} < x < -\frac{1}{2} \end{cases}$$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$.

33. A

Let $x^3 = n \in I \quad \therefore x = n^{1/3}$
then, $f(x) = (-1)^n = \pm 1$

34. B

For $0 < n < 1$, $\sin x < \sin^n x$
and for $n > 1$, $\sin x > \sin^n x$
Now, for $0 < n < 1$,

$$f(x) = \frac{2(\sin x - \sin^n x) - (\sin x - \sin^n x)}{2(\sin x - \sin^n x) + (\sin x - \sin^n x)} = \frac{1}{3}$$

and for $n > 1$,

$$f(x) = \frac{2(\sin x - \sin^n x) + (\sin x - \sin^n x)}{2(\sin x - \sin^n x) - (\sin x - \sin^n x)} = 3$$

For $n > 1$, $g(x) = 3$, $x \in (0, \pi)$
$\therefore g(x)$ is continuous and differentiable at

$x = \dfrac{\pi}{2}$, and for $0 < n < 1$,

$$g(x) = \begin{cases} \left[\dfrac{1}{3}\right] = 0, & x \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right) \\ 3 & x = \dfrac{\pi}{2} \end{cases}$$

$\therefore$ g(x) is not continuous at $x = \frac{\pi}{2}$.

Hence, g(x) is also not differentiable at $x=\frac{\pi}{2}$

35. **A**

Put $x^n = \sin\theta$ and $y^n = \sin\phi$

then, $(\cos\theta + \cos\phi) = a(\sin\theta - \sin\phi)$

$$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$= 2a\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$$

$$\Rightarrow \left(\frac{\theta-\phi}{2}\right) = \cot^{-1} a \Rightarrow \theta - \phi = 2\cot^{-1} a$$

or $\sin^{-1} x^n - \sin^{-1} y^n = 2\cot^{-1} a$

Differentiating both sides, we have

$$\frac{nx^{n-1}}{\sqrt{(1-x^{2n})}} - \frac{ny^{n-1}}{\sqrt{(1-y^{2n})}}\frac{dy}{dx} = 0$$

$$\therefore \sqrt{\left(\frac{1-x^{2n}}{1-y^{2n}}\right)}\frac{dy}{dx} = \frac{x^{n-1}}{y^{n-1}}$$

36. **C**

$$\because y = \left(\frac{ax+b}{cx+d}\right) \text{ or } c\,xy + dy = ax + b$$

Differentiating both sides w.r.t. x, then

$$c\left\{x\frac{dy}{dx} + y.1\right\} + d\frac{dy}{dx} = a$$

$$\text{or} \quad x\frac{dy}{dx} + y + \left(\frac{d}{c}\right)\frac{dy}{dx} = \left(\frac{a}{c}\right)$$

Again differentiating both sides w.r.t. x, then

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} + \left(\frac{d}{c}\right)\frac{d^2y}{dx^2} = 0$$

$$\text{or} \quad x + \frac{2\frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)} + \frac{d}{c} = 0$$

Again differentiating both sides w.r.t. x, then

$$1 + \frac{\left(\frac{d^2y}{dx^2}.2\frac{d^2y}{dx^2} - 2\frac{dy}{dx}.\frac{d^3y}{dx^3}\right)}{\left(\frac{d^2y}{dx^2}\right)^2} + 0 = 0$$

$$\therefore 2\frac{dy}{dx}.\frac{d^3y}{dx^3} = 3\left(\frac{d^2y}{dx^2}\right)^2$$

37. **A**

$x^y.\, y^x = 16 \;\therefore \log_e x^y + \log_e y^x = \log_e 16$

$\Rightarrow y\log_e x + x\log_e y = 4\log_e 2$

Now, differentiating both sides w.r.t. x

$$\frac{y}{x} + \log_e x\frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log_e y.1 = 0$$

$$\therefore \frac{dy}{dx} = -\frac{\left(\log_e y + \frac{y}{x}\right)}{\left(\log_e x + \frac{x}{y}\right)}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{(\log_e 2 + 1)}{(\log_e 2 + 1)} = -1$$

38. **B**

$$\frac{dx}{dy} = \frac{1}{\sqrt{(1+9y^2)}} \quad \therefore \frac{dy}{dx} = \sqrt{(1+9y^2)}$$

39. **C**

$$\text{Let } u = f(\log_e x) \;\therefore \frac{du}{dx} = f'(\log_e x).\frac{1}{x} \quad ....(i)$$

$$\text{Given } f(x) = \log_e x \qquad \therefore f'(x) = \frac{1}{x}$$

$$\therefore f'(\log_e x) = \frac{1}{\log_e x} \quad ....(ii)$$

From Eqs. (i) and (ii),

$$\frac{du}{dx} = \frac{1}{x\log_e x} = (x\log_e x)^{-1}$$

40. **C**

$$\sqrt{(x^2+y^2)} = ae^{\tan^{-1}(y/x)} \qquad ....(i)$$

$$\frac{1}{2\sqrt{x^2+y^2}}(2x+2yy') = a.e^{\tan^{-1}(y/x)} \times \frac{1}{\left(1+\frac{y^2}{x^2}\right)} \times \frac{xy'-y}{x^2}$$

$$\Rightarrow \frac{x+yy'}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \times \frac{x^2}{(x^2+y^2)} \times \frac{xy'-y}{x^2}$$

[from Eq. (i)]

$\therefore x + yy' = xy' - y \Rightarrow y' = \frac{x+y}{x-y}$

$\therefore y'' = \frac{2(xy'-y)}{(x-y)^2}$

$$y''(0) = \frac{2(0-y(0))}{\{0-y(0)\}^2} = \frac{-2}{y(0)} = \frac{-2}{ae^{\pi/2}} = \frac{-2}{a}e^{-\pi/2}$$

41. A

For y - axis, x = 0 $\therefore$ y = 1 − e^0 = 1 − 1 = 0

$\therefore \frac{dy}{dx} = 0 - \frac{1}{2}e^{x/2}, \left(\frac{dy}{dx}\right)_{(0,0)} = -\frac{1}{2}$

$\therefore$ Equation of tangent

$y - 0 = -\frac{1}{2}(x-0) \Rightarrow x + 2y = 0$

42. B

$\frac{dy}{dx} = \frac{1}{1+x^3}, \left(\frac{dy}{dx}\right)_{x=1} = -\frac{1}{2}$

43. B

The point of intersection is $x^2 = 2$, y = 1
the given equations represent four parabolas $y = \pm(x^2-1)$ and $y = \pm(x^2-3)$
The curves intersect when $1 < x^2 < 3$

or $1 < x < \sqrt{3}$ or $-\sqrt{3} < x < -1$

$\therefore$ $y = x^2 - 1$ and $y = -(x^2-3)$
The point of intersection are

$(\pm\sqrt{2}, 1)$ at $(\sqrt{2}, t)$

$m_1 = 2x = 2\sqrt{2}$, $m_2 = -2x = -2\sqrt{2}$

$$\therefore \tan\theta = \left|\frac{4\sqrt{2}}{1-8}\right| = \frac{4\sqrt{2}}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

44. D

For x < 0, $f(x) = |x^2 + x| = |x||x+1|$
For x < −1, $f(x) = (-x)(-x-1) = x^2 + x$
$\therefore f'(x) = 2x + 1$
Slope of tangent = 2(−2) + 1 = −3
$\therefore$ Slope of normal = 1/3

45. B

Given, $f'(x) < 0 \ \forall \ x \in R$

$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0 \ \forall \ x \in R$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a \ \forall \ x \in R$

$\Rightarrow \sin(x + \pi/3) < a \ \forall \ x \in R$
$\sin(x + \pi/3) < a$
$\Rightarrow a \ge 1$ [$\because \sin(x + \pi/3) \le 1$]

46. A

$f'(x) = ax^2 + 1 + \cos x > 0$, for a > 0 and $\forall \ x \in R$
$\Rightarrow$ f(x) is an increasing function
$\Rightarrow$ max f(x) = f(3) and min f(x) = f(2)

$\Rightarrow$ Difference=f(3)−f(2)=$\int_2^3 (at^2 + 1 + \cos t)\,dt$

$\frac{19}{3}a + 1 + (\sin 3 - \sin 2)$

47. C

Put $x^x = t \Rightarrow x^x(1 + \log_e|x|)\,dx = dt$

$\therefore \int x^x(1+\log_e|x|)dx = t + c = x^x + c$

48. D

$\because f(x) = \lim_{n\to\infty}\frac{x^n - x^{-n}}{x^n + x^{-n}}, x > 1 = \lim_{n\to\infty}\frac{1-x^{-2n}}{1+x^{-2n}}$

$= \frac{1-0}{1+0} = 1$ $(\because x > 1, \therefore x^{-\infty} - 0)$

Then, $\int \frac{xf(x)\ln(x+\sqrt{1+x^2})}{\sqrt{(1+x^2)}}dx$

$= \int \frac{x\ln(x+\sqrt{1+x^2})}{\sqrt{(1+x^2)}}dx$

Put $\ln(x + \sqrt{(1+x^2)}) = t \Rightarrow x + \sqrt{(1+x^2)} = e^t$

$$\therefore \frac{1+\frac{x}{\sqrt{(1+x^2)}}}{(x+\sqrt{(1+x^2)})}dx = dt \Rightarrow \frac{dx}{\sqrt{(1+x^2)}} = dt$$

and $(e^t - x)^2 = 1 + x^2 \Rightarrow e^{2t} - 2x\,e^t = 1$

or $x = \frac{1}{2}(e^t - e^{-t})$

$$\therefore \int \frac{x\ln(x+\sqrt{1+x^2})}{\sqrt{(1+x^2)}}dx = \int \frac{1}{2}(e^t - e^{-t}).t\,dt$$

$$= \frac{1}{2}\{t(e^t + e^{-t}) - (e^t - e^{-t})\} + c$$

$$= \sqrt{(1+x^2)} \,.\, \ln(x + \sqrt{(1+x^2)}) - x + c$$

49. A

$$\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \ln f(x) + c$$

Differentiating both sides w.r.t. x, then

$$f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} \cdot f'(x)$$

$$\Rightarrow \frac{f'(x)}{\{f(x)\}^2} = 2(b^2 - a^2) \sin x \cos x$$

$$\Rightarrow -\frac{f'(x)}{\{f(x)\}^2} = -2b^2 \sin x \cos x + 2a^2 \sin x \cos x$$

Integrating both sides w.r.t. x, then

$$\frac{1}{f(x)} = b^2 \cos^2 x + a^2 \sin^2 x + c$$

$$\text{for } c = 0,\ f(x) = \frac{1}{(a^2 \sin^2 x + b^2 \cos^2 x)}$$

50. A

$$\text{Let } I = \int \frac{(x^4 - x)^{1/4}}{x^5} dx$$

$$= \int \frac{x\left(1 - \frac{1}{x^3}\right)^{1/4}}{x^5} dx = \int \frac{\left(1 - \frac{1}{x^3}\right)^{1/4}}{x^4} dx$$

$$\text{Put } 1 - \frac{1}{x^3} = t^4 \Rightarrow \frac{3}{x^4} dx = 4t^3 \, dt$$

$$\therefore I = \frac{4}{3} \int_0^x t.t^3 \, dt = \frac{4}{3} \cdot \left(\frac{t^5}{5}\right) + c$$

$$= \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$$

51. B

$$\int_{-1}^{15} \text{sgn}(\{x\}) \, dx = \int_0^{16} \text{sgn}(\{x - 1\}) \, dx$$

(by property)

$$= \int_0^{16} \text{sgn}(\{x\}) \, dx = 16 \int_0^1 \text{sgn}(\{x\}) \, dx$$

$$= 16 \int_0^1 \text{sgn}(x) \, dx = 16 \int_0^1 1. \, dx = 16$$

52. A

$$\text{Let } I_n = \int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)}{\sin x / 2} dx$$

$$\therefore I_n - I_{n-1} = \int_0^{\pi} \frac{\left\{\sin\left(n + \frac{1}{2}\right)x - \sin\left(n - \frac{1}{2}\right)x\right\}}{\sin(x/2)} dx$$

$$= \int_0^{\pi} \frac{2\cos nx.\sin(x/2)}{\sin(x/2)} dx = \int_0^{\pi} 2\cos nx \, dx$$

$$= 2\left\{\frac{\sin nx}{n}\right\}_0^{\pi} = 0 - 0 = 0 \Rightarrow I_n = I_{n-1}$$

Replacing n by n – 1, n – 2, ..., then we get

$$I_n = I_{n-1} = I_{n-2} = ... = I_1$$

$$\Rightarrow I_n = I_1 = \int_0^{\pi} \frac{\sin(3x/2)}{\sin(x/2)} dx = \int_0^{\pi}\left(\frac{\sin 2x + \sin x)}{\sin x}\right) dx$$

$$= \int_0^{\pi} (2\cos x + 1) \, dx = \{2 \sin x + x\}_0^{\pi} = \pi$$

Hence, $I_n = \pi$

53. B

$$\because 0 \le x < \pi/4 \quad \therefore [x] = 0$$

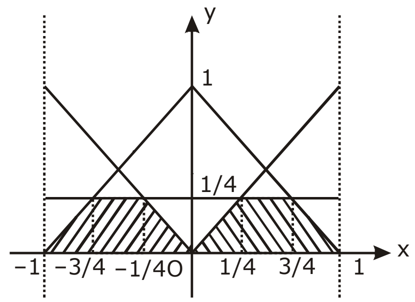
$$\text{Then, } \int_0^{\pi/4} \sin x \, d(x - [x]) = \int_0^{\pi/4} \sin x \, dx$$

$$= -\{\cos x\}_0^{\pi/4} = -\left(\frac{1}{\sqrt{2}} - 1\right) = 1 - \frac{1}{\sqrt{2}}$$

54. B

$$\int_{-1}^{1} f(x) \, dx = \text{Area of shaded region}$$

$$= 2 \times \frac{1}{2} \times \left(1 + \frac{1}{2}\right) \times \frac{1}{4} = \frac{3}{8}$$



55. A

We have

$\because\ (1-0)\left(\frac{0}{0^3+16}\right) \le \int_0^1 \frac{x\,dx}{(x^3+16)}$

$\le (1-0)\left(\frac{1}{1^3+16}\right)$ (by property)

$0 \le \int_0^1 \frac{x\,dx}{x^3+16} \le \frac{1}{17} \therefore \int_0^1 \frac{x\,dx}{x^3+16} \in \left[0, \frac{1}{17}\right]$

56. A

$f(x) = \cos x - \int_0^x (x-t)\, f(t)\, dt$

$f(x) = \cos x - x\int_0^x f(t)\, dt + \int_0^x t\, f(t)\, dt$

$\therefore f'(x) = -\sin x - \left\{x f(x) + \int_0^x f(t)\, dt\right\} + x f(x)$

$= -\sin x - \int_0^x f(t)\, dt$

$\therefore f''(x) = -\cos x - f(x) \Rightarrow f''(x) + f(x) = -\cos x$

57. B

$\therefore$ Required area $= \int_{-2}^{3} [\![x-3]\!]\, dx$

$= \int_{-2}^{-1} [\![x-3]\!]\, dx + \int_{-1}^{0} [\![x-3]\!]\, dx$

$+ \int_{0}^{1} [\![x-3]\!]\, dx + \int_{1}^{2} [\![x-3]\!]\, dx$

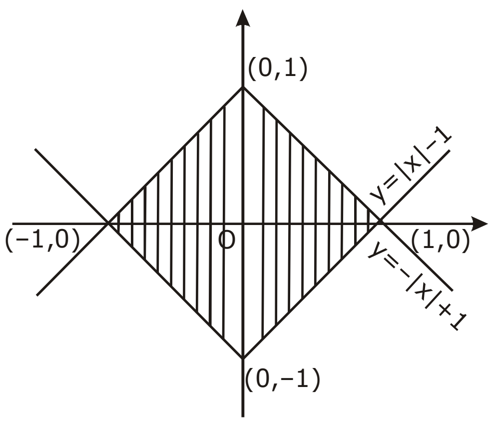
$+ \int_{2}^{3} [\![x-3]\!]\, dx$

$= \int_{-2}^{-1} 5.dx + \int_{-1}^{0} 4.dx + \int_{0}^{-1} 3.dx + \int_{1}^{2} 2.dx + \int_{2}^{3} 1.dx$

$= 5(1) + 4(1) + 3(1) + 2(1) + 1(1) = 15$ sq unit

58. B

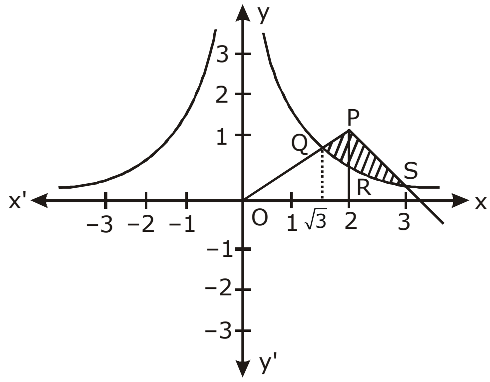
Required area = shaded area = $(\sqrt{2})^2 = 2$ sq unit.



59. C

$y = 2 - |2-x|,\ y = \frac{3}{|x|}$

$y = \begin{cases} x, & x \le 2 \\ 4-x, & x \ge 2 \end{cases};\ y = \begin{cases} \frac{3}{x}, & x > 0 \\ -\frac{3}{x}, & x < 0 \end{cases}$



Hence, required area

PQRSP = area PQRP + area PRSP

$= \left|\int_{\sqrt{3}}^{2}\left(x - \frac{3}{x}\right) dx\right| + \left|\int_{2}^{3}\left((4-x) - \frac{3}{x}\right) dx\right|$

$= \frac{4 - 3\ln 3}{2}$ sq unit.

60. A

Since, y = f(x) and given differential equation

$y(1 + xy)\, dx - x\, dy = 0$

$\Rightarrow (y\,dx - x\,dy) + xy^2\, dx = 0$

or $\frac{ydx - xdy}{y^2} + x\,dx = 0 \Rightarrow d\left(\frac{x}{y}\right) + x\,dx = 0$

On integrating, then $\frac{x}{y} + \frac{x^2}{2} = c$

which passes through (1 ,2) $\therefore c = 1$

Then, curve is $\frac{x}{y} + \frac{x^2}{2} = 1$ or $y = \frac{2x}{2-x^2}$

$\therefore f(x) = \frac{2x}{2-x^2}$

61. A

$\because\ 2y \sin x \frac{dy}{dx} = 2 \sin x \cos x - y^2 \cos x$

$\Rightarrow 2y \sin x \frac{dy}{dx} + y^2 \cos x = 2 \sin x \cos x$

$\Rightarrow \frac{d}{dx}(y^2 \sin x) = 2 \sin x \cos x$

On integrating, we get $y^2 \sin x = \sin^2 x + c$
When, $x = \pi/2$, $y = 1$ $\therefore 1 = 1 + c \Rightarrow c = 0$
$\therefore y^2 \sin x = \sin^2 x \Rightarrow y^2 = \sin x$

62. A

$(2x \cos y + y^2 \cos x)\, dx$
$+ (2y \sin x - x^2 \sin y)\, dy = 0$
$\Rightarrow d(x^2 \cos y) + d(y^2 \sin x) = 0$
On integrating, $x^2 \cos y + y^2 \sin x = c$

63. B

$$g(x) = f\{f(x)\} = f\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = -\frac{1-x}{x}$$

and $h(x) = f(f\{f(x)\}) = f\{g(x)\}$

$$= \frac{1}{1-g(x)} \Rightarrow \frac{1}{1+\frac{1-x}{x}} = x$$

$$\therefore f(x).g(x).h(x) = \frac{1}{1-x}\left\{-\frac{1-x}{x}\right\}.x = -1$$

64. B

$$y = \frac{x-1}{x^2-3x+3} \Rightarrow x^2y - 3xy + 3y = x - 1$$

$\Rightarrow x^2y - x(3y + 1) + 3y + 1 = 0$
$\therefore D \ge 0 \Rightarrow (3y + 1)^2 - 4y(3y + 1) \ge 0$
$\Rightarrow -3y^2 + 2y + 1 \ge 0 \Rightarrow 3y^2 - 2y - 1 \le 0$

$$\Rightarrow y^2 - \frac{2y}{3} - \frac{1}{3} \le 0 \Rightarrow \left(y - \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{1}{3} \le 0$$

$$\Rightarrow \left(y - \frac{1}{3}\right)^2 \le \frac{4}{9} \Rightarrow -\frac{2}{3} \le y - \frac{1}{3} \le \frac{2}{3}$$

$$\therefore -\frac{1}{3} \le y \le 1 \qquad \therefore y \in \left[-\frac{1}{3}, 1\right]$$

65. B

$$f(x) = \lim_{n\to\infty} \lim_{t\to 0} \frac{\sin^2 \lfloor n \pi x}{(\lfloor n \pi x + t^2)}$$

where x is irrational $f(x) = 1$

66. B

$$\text{LHL} = \lim_{x\to(n\pi+\pi/2)-} f(x) = \lim_{h\to 0} f(n\pi + \pi/2 - h)$$

$$= \lim_{h\to 0} (1 + |\cos(n\pi + \pi/2 - h)|)^{ab/|\cos[n\pi+\pi/2-h]|}$$

$$= \lim_{h\to 0} (1 + |\sin(n\pi - h)|)^{ab/|\sin(n\pi - h)|}$$

$$= \lim_{h\to 0} (1 + |(-1)^n \sin h|)^{ab|1(-1)^n \sin h|}$$

$$= \lim_{h\to 0} (1 + \sin h)^{ab/\sin h} = e^{ab}$$

$$\left[\because \lim_{h\to 0}(1+h)^{h/\eta} = e^{\lambda}\right]$$

V.F. $= f(n\pi + \pi/2) = e^a.\, e^b$

$$\text{RHL} = \lim_{x\to(n\pi+\pi/2)+} f(x) = \lim_{h\to 0} f\left(n\pi + \frac{\pi}{2} + h\right)$$

$$= \lim_{h\to 0} e^{\cot(2n\pi + \pi + 2h)/\cot(8n\pi + 4\pi + 8h)}$$

$$= e^{\lim_{h\to 0}(\cot 2h/\cot 8h)} = e^{\lim_{h\to 0} \tan 8h/\tan 2h}$$

$$= e^{4 \lim_{h\to 0} \tan 8h/8h.\lim_{x\to 0} 2h/\tan 2h} = e^{4.1.1} = e^4$$

$\because f(x)$ is continuous in $(n\pi, (n + 1)\pi)$
$\therefore$ LHL = RHL = V.F. $\quad e^{ab} = e^4 = e^{a+b}$
$\Rightarrow ab = 4 = a + b \Rightarrow a = b = 2$

67. C

$f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$

$$\lim_{x\to 0} \frac{x\left\{1 + a\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)\right\} - b\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)}{x^3} = 1$$

For existing of above $1 + a - b = 0$ .....(1)

$$\frac{-a}{2} + \frac{b}{6} = 1 \text{ ....(2)}$$

on solving (1) & (2) we get $a = -5/2$, $b = -3/2$

68. C

Given $\sqrt{(x+y)} + \sqrt{(y-x)} = a$ ....(i)

$$\therefore \sqrt{x+y} - \sqrt{y-x} = \frac{2x}{a} \qquad \text{....(ii)}$$

Adding Eqs. (i) and (ii), then $2\sqrt{x+y} = a + \dfrac{2x}{a}$

$$\text{Squaring, } 4x + 4y = a^2 + \frac{4x^2}{a^2} + 4x$$

$$\therefore 4 + 4\frac{dy}{dx} = 0 + \frac{8x}{a^2} + 4$$

$$\therefore 0 + 4\frac{d^2y}{dx^2} = \frac{8}{a^2} \quad \therefore \frac{d^2y}{dx^2} = \frac{2}{a^2}$$

69. C

Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$P(2) = 16a + 8b + 4c + 2d + e = -1$ ...(i)

$P'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$P'(2) = 32a + 12b + 4c + d = 0$ ....(ii)

$P''(x) = 12ax^2 + 6bx + 2c$

$P''(2) = 48a + 12b + 2c = 2$ ....(iii)

$\therefore P'''(x) = 24ax + 6b$

$\therefore P'''(2) = 48a + 6b = -12$ ...(iv)

and $P^{iv}(x) = 24a \Rightarrow P^{iv}(2) = 24a \Rightarrow 24 = 24a$

$\therefore a = 1,$

From Eq. (iv), $b = -10$

From Eq. (iii), $c = 37$

$\therefore P''(x) = 12x^2 - 60x + 74$

$\therefore P''(1) = 12 - 60 + 74 = -48 + 74 = 26$

70. C

$\because x = a\cos\theta \Rightarrow \dfrac{dx}{d\theta} = -a\sin\theta$

and $y = b\sin\theta \Rightarrow \dfrac{dy}{d\theta} = b\cos\theta$

$\therefore \dfrac{dy}{dx} = -\dfrac{b}{a}\cot\theta$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}\text{cosec}^2\theta\,\frac{d\theta}{dx} = -\frac{b}{a^2}\text{cosec}^3\theta$$

$$\therefore \frac{d^3y}{dx^3} = -\frac{3b}{a^2}\text{cosec}^2\theta\,(-\text{cosec}\,\theta\cot\theta).\frac{d\theta}{dx}$$

$$= \frac{3b}{a^2}\text{cosec}^3\theta\cot\theta\left(-\frac{1}{a\sin\theta}\right)$$

$$= -\frac{3b}{a^3}\text{cosec}^4\theta\cot\theta$$

71. A

$$\because F(x) = \frac{1}{x^2}\int_4^x \{4t^2 - 2F'(t)\}\,dt$$

$$\text{or } x^2F(x) = \int_4^x \{4t^2 - 2F'(t)\}\,dt$$

Differentiating both sides w.r.t. x, then

$x^2F'(x) + F(x).2x = 4x^2 - 2F'(x)$

Put $x = 4 \Rightarrow 16F'(4) + 8F(4) = 64 - 2F'(4)$

$\therefore 18F'(4) + 0 = 64$ [$\because F(4) = 0$, from Eq. (i)]

$\therefore F'(4) = \dfrac{32}{9}$

72. D

$y - e^{xy} + x = 0$

$$\therefore \frac{dy}{dx} - e^{xy}\left\{x\frac{dy}{dx} + y.1\right\} + 1 = 0$$

$$\frac{dy}{dx} = \left(\frac{e^{xy}.y - 1}{(1 - xe^{xy})}\right) = \infty$$

$\therefore 1 - xe^{xy} = 0 \Rightarrow xe^{xy} = 1 \quad (x, y) \equiv (1, 0)$

73. B

$$\because y = \int_x^{x^2}\cos^{-1}t^2\,dt$$

$$\therefore \frac{dy}{dx} = \cos^{-1}(x^4).\,2x - \cos^{-1}(x^2)\,.\,1$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{1}{\sqrt[4]{2}}} = \cos^{-1}\left(\frac{1}{2}\right)\frac{2}{\sqrt[4]{2}} - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{3}.\,2^{3/4} - \frac{\pi}{4} = \left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4}\right)\pi$$

74. A

$$\because \frac{dy}{dx} = \tan 90° = \infty \;\therefore\; \frac{dx}{dy} = 0$$

$$\Rightarrow \frac{dx}{dt} = 0 \text{ and } \frac{dy}{dt} \neq 0$$

$$\Rightarrow 2t = 0 \text{ and } 2t - 1 \neq 0 \;\therefore\; t = 0 \text{ and } t \neq \frac{1}{2}$$

75. A

Let point be (x_1, y_1)

differentiate the curve,

$$2y\frac{dy}{dx} - 6x^2 - 4\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6x^2}{2y - 4}$$

equation of tanget $y - y_1 = \dfrac{3x_1^2}{y_1 - 2}(x - x_1)$.

it passes through (1, 2). so

$2 - y_1 = \dfrac{3x_1^2}{y_1 - 2}(1 - x_1)$ ....(1)

and (x_1, y_1) lies on the curve

$y_1^2 - 2x_1^3 - 4y_1 + 8 = 0$ .....(2)

solving (1) and (2) we get $x = 2$; $y = 2 \pm \sqrt{3}$

76. C

direct use the formula $L = \left|\dfrac{y_1}{(dy/dx)_{(x_1,y_1)}}\right|$

77. C

$xy = (x+c)^2 \Rightarrow x.\dfrac{dy}{dx} + y = 2(x+c)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2(x+c)-y}{x} = -1 \Rightarrow 2(x+c) - y = -x$

Now solving this equation with the curve we get points

78. D

$\because\ f(x) = \displaystyle\int_2^{x^2} \frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}}\,dt$

$\therefore\ f'(x) = \dfrac{(\sin^{-1}x)^2}{x}.(2x-0)$

$f'(x) = 2(\sin^{-1}x)^2$ ....(i)

$f''(x) = \dfrac{4\sin^{-1}x}{\sqrt{(1-x^2)}}$

$\therefore\ [f''(x)]^2 = \dfrac{16(\sin^{-1}x)^2}{(1-x^2)}$ .....(ii)

From Eqs. (i) and (ii), we have

$(1-x^2)[f''(x)]^2 - 2f'(x) = 12(\sin^{-1}x)^2$

$= 12\left(\sin^{-1}\dfrac{1}{\sqrt{2}}\right)^2 = 12\left(\dfrac{\pi}{4}\right)^2 = \dfrac{3}{4}\pi^2$

79. C

Let $I = \displaystyle\int_0^{16\pi/3}|\sin x|\,dx = \int_0^{5\pi+\frac{\pi}{3}}|\sin x|\,dx$

$= \displaystyle\int_0^{5\pi}|\sin x|\,dx + \int_{5\pi}^{5\pi+\frac{\pi}{3}}|\sin x|\,dx$

$= 5\displaystyle\int_0^{\pi}|\sin x|\,dx + \int_0^{\pi/3}|\sin x|\,dx$ (by property)

$= 5\displaystyle\int_0^{\pi}\sin x\,dx + \int_0^{\pi/3}\sin x\,dx$

$= 5[-\cos x]_0^{\pi} + [-\cos x]_0^{\pi/3}$

$= 5(-1-1) - \left(\dfrac{1}{2}-1\right) = 10 + 1 - \dfrac{1}{2} = \dfrac{21}{2}$

80. B

$\displaystyle\int_0^{n^2}[\sqrt{x}]\,dx = \int_0^{1^2}[\sqrt{x}]\,dx + \int_{1^2}^{2^2}[\sqrt{x}]\,dx$

$+ \displaystyle\int_{2^2}^{3^2}[\sqrt{x}]\,dx + ... + \int_{(n-1)^2}^{n^2}[\sqrt{x}]\,dx$

$= 0 + 1.(2^2 - 1^2) + 2.(3^2 - 2^2) + ...$

$+ .... + (n-1)[n^2 - (n-1)^2]$

$-1^2 - 2^2 - 3^3 - ..... - (n-1)^2 + (n-1)n^2$

$= -(1^2 + 2^2 + 3^2 + ... + (n-1)^2 + (n-1)n^2$

$= -\dfrac{(n-1)n(2n-1)}{6} + (n-1)n^2$

$= \dfrac{1}{6}n(n-1)(4n+1)$

81. D

Let $I(\alpha) = \displaystyle\int_0^1 \frac{x^{\alpha}-1}{\ln x}\,dx$ .....(i)

$\therefore\ I'(\alpha) = \displaystyle\int_0^1 \frac{x^{\alpha}.\ln x}{\ln x}\,dx = \int_0^1 x^{\alpha}\,dx$

$= \left[\dfrac{x^{\alpha+1}}{\alpha+1}\right]_0^1 = \dfrac{1}{(\alpha+1)}$

Now, $I(\alpha) = \displaystyle\int\frac{d\alpha}{(\alpha+1)} = \ln|\alpha+1| + c$

Put $\alpha = 0$, then $I(0) = \ln 1 + c = 0$ [from Eq. (i)]

$\Rightarrow 0 + c = 0\ \therefore\ c = 0$ Hence, $I(\alpha) = \ln|\alpha+1|$

82. C

$\because\ \displaystyle\int_{-1}^{4}f(x)\,dx = 4$ and $\displaystyle\int_2^4((3-f(x))\,dx = 7$

or $\displaystyle\int_4^{-1}f(x)\,dx = -4$ and $6 - \displaystyle\int_2^4 f(x)\,dx = 7$

$\therefore\ \displaystyle\int_4^{-1}f(x)\,dx = -4,\ \int_2^4 f(x)\,dx = -1$

$\displaystyle\int_2^{-1}f(x)\,dx = \int_2^4 f(x)\,dx + \int_4^{-1}f(x)\,dx = -1-4 = -5$

83. D

$I_1 = \displaystyle\int_0^x e^{zx}.e^{-z^2}\,dz = \int_0^x e^{-(z^2-zx)}\,dz$

$= \displaystyle\int_0^x e^{-\left\{\left(z-\frac{x}{2}\right)^2-\frac{x^2}{4}\right\}}\,dz = e^{x^2/4}\int_0^x e^{-\left(z-\frac{x}{2}\right)^2}\,dz$

$$= e^{x^2/4}\int_{-x/2}^{x/2} e^{-z^2}\,dz = \frac{1}{2}\,e^{x^2/4}\int_{-x}^{x} e^{-z^2/4}\,dz$$

$= e^{x^2/4}\int_0^x e^{-z^2/4}\,dz = e^{x^2/4} I_2$ Hence, $I_1 = e^{x^2/4} I_2$

84. **A**

$$\int_0^{t^2} x f(x)\,dx = \frac{2}{5}t^5$$

differentiate both side w.r.t. t

$t^2 f(t^2)\,2t = \frac{10t^4}{5} \Rightarrow f(t^2) = t$

put t = 2/5, f(4/ 25) = 2/5

85. **C**

$$I = -\int e^{\tan^{-1}x}\,(1 + x + x^2).\frac{1}{1+x^2}dx$$

$$= -\int e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)dx$$

$$= -\int e^{\tan^{-1}x}dx - \int \frac{e^{\tan^{-1}x}}{1+x^2}.x\,dx$$

$$= -\int e^{\tan^{-1}x}dx - \{e^{\tan^{-1}x}.x - \int e^{\tan^{-1}x}.1dx\} + c$$

$$= -xe^{\tan^{-1}x} + c$$

86. **B**

$$I = \frac{1}{\sqrt{2}}\int \frac{dx}{\cos^{7/2}x\sin^{1/2}x} = \frac{1}{\sqrt{2}}\int \frac{dx}{\cos^4 x\sqrt{\tan x}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{(1+\tan^2)\sec^2 x dx}{\sqrt{\tan x}}$$

put tan x = t. Then

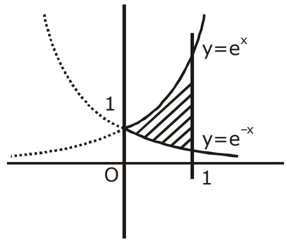
$$I = \frac{1}{\sqrt{2}}\int \frac{(1+t^2)dt}{\sqrt{t}} = \frac{1}{\sqrt{2}}(t^{-1/2} + t^{3/2})\,dx$$

$$= \sqrt{2}\sqrt{\tan x} + \frac{\sqrt{2}}{5}(\tan x)^{5/2} + c$$

$\Rightarrow a = \sqrt{2}/5,\ ab = \sqrt{2} \Rightarrow b = 5.$

87. **C**

$\therefore$ Required area $= \int_0^1 (e^x - e^{-x})\,dx$



$= [e^x + e^{-x}]_0^1 = (e + e^{-1}) - (e^0 + e^{-0})$

$= (e + e^{-1} - 2)$ sq unit.

88. **D**

$5x^2 = 9 - 2x^2 \Rightarrow x = \pm\sqrt{3}$

$$A = \int_{-\sqrt{3}}^{\sqrt{3}} (2+2x^2-x^2)\,dx = 2\int_0^{\sqrt{3}} 9-3x^2dx = 12\sqrt{3}$$

89. **C**

$y = x.e^{-x^2} \Rightarrow y'(x) = (1-2x^2)e^{-x^2} = 0 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Area between curve & max. ordinate is

$$A = \int_0^{1/\sqrt{2}} x.e^{-x^2}\,dx$$

90. **B**

$\because \sin y\frac{dy}{dx} = \cos y\,(1 - x\cos y)$

$\Rightarrow \tan y\frac{dy}{dx} = 1 - x\cos y$

$\Rightarrow \sec y\tan y\frac{dy}{dx} - \sec y = -x$ ....(i)

Put $\sec y = v$, $\therefore \sec y\tan y\frac{dy}{dx} = \frac{dv}{dx}$

Then, from Eq.(i), $\frac{dv}{dx} - v = -x$

$\therefore$ IF $= e^{\int -1dx} = e^{-x}$

Then, $v.(e^{-x}) = \int(-x)e^{-x}dx$

$\Rightarrow v.e^{-x} = (-x)(-e^{-x}) + e^{-x} + c$

or $v = x + 1 + ce^x$ or $\sec y = x + 1 + ce^x$

91. **B**

$(x-h)^2 + (y-k)^2 = a^2$ .....(i)

Differentiating both sides w.r.t. x, then

$2(x-h) + (y-k)y' = 0$

or $x - h + (y-k)y' = 0$ .....(ii)

Again differentiating both sides w.r.t. x, then

$1 + (y-k)y'' + y'.y' = 0$

$\therefore (y-k) = -\left(\frac{1+(y')^2}{y''}\right)$ .....(iii)

From Eqs. (ii) and (iii),

$(x-h) = \frac{y'(1+(y')^2)}{y''}$ ......(iv)

Substituting the values of (x – h) and (y–k) from Eqs. (iii) and (iv) in (i), then

$$\frac{(y')^2(1+(y')^2)}{(y'')^2} + \frac{(1+(y')^2)^2}{(y'')^2} = a^2$$

$\therefore (1+(y')^2)^3 = a^2(y'')^2$

92. **B**

$x^2 + (y-k)^2 = r^2$

diff. $x + (y-k)y_1 = 0$

diff. $1 + (y-k)y_2 + y_1^2 = 0$

Eliminate (y – k)

93. **A**

$f(xy) = \frac{f(x)}{y}$ .....(1)

put x = 1, y = 30 $\Rightarrow f(30) = \frac{f(1)}{30}$

$\Rightarrow f(1) = 30 \times 20$

Now put x = 1, y = 40 in (1)

$f(40) = \frac{f(1)}{40} \Rightarrow f(40) = \frac{30\times 20}{40} = 15$

94. **A**

$|x| - x + y = 10$

case (i): $x \ge 0 \Rightarrow x - x + y = 10$

$\Rightarrow y = 10$...(1) $x + |y| + y = 12$

from (1)$x + 10 + 10 = 12$

$\Rightarrow x = -8$ (contradict the case(i))

case (ii): Let $y < 0$ $\therefore x + |y| + y = 12$

$\Rightarrow x - y + y = 12 \Rightarrow x = 12$

Now from $|x| - x + y = 10$

$12 - 12 + y = 10 \Rightarrow y = 10$ (contradict case (ii))

case(iii): $x < 0$ & $y > 0$

$\therefore -x - x + y = 10 \Rightarrow -2x + y = 10$

$x + y + y = 12 \Rightarrow x + 2y = 12$

on solving y = 34/5

x = –8/5 $\therefore$ x + y = 26/5

95. **B**

$$\lim_{x\to\infty} \sqrt{2\alpha^2x^2+\alpha x+7} - \sqrt{2\alpha^2x^2+7} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{(2\alpha^2x^2+\alpha x+7)-(2\alpha^2x^2+7)}{\sqrt{2\alpha^2x^2+\alpha x+7}+\sqrt{2\alpha^2x^2+7}} = \frac{1}{2\sqrt{2}}$$

$$\lim_{x\to\infty} \frac{\alpha x}{x\left(\sqrt{2\alpha^2+\frac{\alpha}{x}+\frac{7}{x^2}}+\sqrt{2\alpha^2+\frac{7}{x^2}}\right)} = \frac{1}{2\sqrt{2}}$$

$\Rightarrow \frac{\alpha}{2\sqrt{2}\alpha} = \frac{1}{2\sqrt{2}}$ this is possible when $\alpha \ne 0$ (most correct)

96. **A**

$$\lim_{n\to\infty} n\cos\frac{\pi}{4n}\sin\frac{\pi}{4n} = k$$

$$\Rightarrow \lim_{n\to\infty} \frac{\sin\frac{\pi}{2n}}{\frac{2}{n}} = k \Rightarrow k = \frac{\pi}{4}$$

97. **A**

$g(f(x)) = (2f(x)+1)(f(x)-k)+3$

$$\begin{cases} (2(\frac{x}{2}-1)+1)(\frac{x}{2}-1-k)+3; & 0 \le x \le 1 \\ (2\times\frac{1}{2}+1)(\frac{1}{2}-k)+3; & 1 < x \le 2 \end{cases}$$

$\therefore$ g(f(x)) is continuous at x = 1 $\therefore$ f(1) = R.H.L.

$0 + 3 = 2\left(\frac{1}{2}-k\right) + 3 \Rightarrow k = \frac{1}{2}$

98. **A**

$$f(x) = \begin{cases} \frac{1-x}{|x-1|} & ; x < 1 \\ 1 & ; x = 1 \\ x^2 & ; x > 1 \end{cases} \Rightarrow f(x) = \begin{cases} 1; & x < 1 \\ 1; & x = 1 \\ x^2; & x > 1 \end{cases}$$

function is continuous every where

99. **B**

Let $y = \cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right)$

$$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{x^{-1}-x}{x^{-1}+x}\right)^2}}$$

$$\times \frac{(x^{-1}+x)(-x^{-2}-1)-(x^{-1}-x)(-x^{-2}+1)}{(x^{-1}+x)^2}$$

$$= \frac{-1(x^{-1}+x)(-x^{-2}-1)-(x^{-1}-x)(-x^{-2}+1)}{2\left|x^{-1}+x\right|}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=-1} = \frac{-\{(-1-1)(-1-1)-0\}}{2\left|-1-1\right|} = -1$$

100. C

Differentiating both sides w.r.t. x, then

$$\sqrt{(3-2\sin^2 x)}.1 + \cos y\ \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{\sqrt{(3-2\sin^2 x)}}{\cos y}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(\pi,\pi)} = -\frac{\sqrt{3}}{(-1)} = \sqrt{3}$$

101. A

$\tan^{-1} y - y + x = 0$

differentiating w.r.t. x both side

$$\frac{1}{1+y^2}\frac{dy}{dx} - \frac{dy}{dx} + 1 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+y^2}{y^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2y}{y^4}\frac{dy}{dx}$$

$$= \frac{-2}{y^3} \times \frac{(1+y^2)}{y^2}$$

102. A

$f(x) = \cos(x^2 - 4[x])$ at $x = \frac{\sqrt{\pi}}{2}$; $[x] = 0$

$\therefore f(x) = \cos x^2 \Rightarrow f'(x) = -2x \sin x^2$

$$f'\left(\frac{\sqrt{\pi}}{2}\right) = -2.\frac{\sqrt{\pi}}{2}.\sin\frac{\pi}{4}$$

$$= -\sqrt{\pi}.\frac{1}{\sqrt{2}} = -\sqrt{\frac{\pi}{2}}$$

103. B

$y = \sin^{-1}(\cos x)$

$$\frac{dy}{dx} = \frac{-\sin x}{|\sin x|};\ x \in R-\{n\pi : n \in I)$$

$$\therefore \frac{dy}{dx} = -\text{sgn}(\sin x)$$

104. A

$f: [0, \infty) \to [0, \infty)$, $g:[0, \infty) \to [0, \infty)$

g(x) is decreasing function and f(x) is increasing function $\therefore x > 0 \Rightarrow g(x) \le g(0)$.

$\Rightarrow f(g(x)) \le f(g(0))$ [f is increasing]

$\Rightarrow h(x) \le h(0) = 0$

But range of h(x) is $[0, \infty)$, $\therefore$ (x) < 0

hence h(x) = (0) $\forall\ x \ge 0$

$\therefore h(x) - h(1) = 0 - 0 = 0\ \forall\ x$

105. C

$f''(x) > 0\ \forall\ x \in R$

$\Rightarrow f'(x)$ is increasing $\forall\ x \in R$

$g(x) = f(2 - x) + f(4 + x)$

$\Rightarrow g'(x) = -f'(2 - x) + f'(4 + x)$

$g'(x) > 0 \Rightarrow f'(4 + x) > f'(2 - x)$

$\Rightarrow 4 + x > 2 - x \Rightarrow x > -1$

106. B

It is given that $(\delta r/r) \times 100 = 1$.

$v = (4/3)\pi r^3$; $\log v = \log(4\pi/3) + 3 \log r$

$$\frac{1}{v}\delta v = \frac{3}{r}\delta r \Rightarrow \frac{1}{v}.\delta v \times 100 = 3\left(\frac{1}{r}\delta r \times 100\right) = 3$$

Hence, error in volume is with in 3%

107. D

Let $P(x) = ax^3 + bx^2 + cx + d$

$P(-1) = -a + b - c + d = 10$

$P(1) = a + b + c + d = -6$

$P'(-1) = 3a - 2b + c = 0$

$P''(1) = 6x + 2b = 0$

Solving above equations,

$P(x) = x^3 - 3x^2 - 9x + 5$

$P'(x) = 3x^2 - 6x - 9 = 3(x + 1)(x - 3)$

x = –1 is point of maximum and x = 3 is point of minimum of P(x).

Also $P''(x) = 6x - 6 = 6(x - 1)$

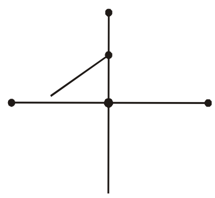
$\therefore$ x = 1 is point of minimum of P′(x)

$\therefore A(x_1, y_1) = (-1, 10)$ and $B(x_2, y_2) = (3, -22)$

$\therefore AB = \sqrt{16+32^2} = 4\sqrt{65}$

108. D

$$f(x) = \begin{cases} \cos\left(\frac{\pi}{2}x\right)\downarrow;\ x > 0 \\ x + a\uparrow:\ x \le 0 \end{cases}$$



$$f'(x) = \begin{cases} -\frac{\pi}{2}.\sin\left(\frac{\pi}{2}x\right) < 0;\ x > 0 \\ 1 > 0;\ x < 0 \end{cases}$$

so form max. value > RHL

109. C

$\because\ f(x) = x^x$

$\therefore\ f'(x) = x^x(1 + \ln x) < 0$ (given)

$\because\ x^x > 0\ \therefore\ 1 + \ln x < 0 \Rightarrow \ln x < -1$

$\Rightarrow\ x < e^{-1}\ \therefore\ x \in (0, 1/e)\quad (\because x > 0)$

110. C

$f'(x) = e^x(x - 1)(x - 2) < 0$

$\Rightarrow (x - 1)(x - 2) < 0\quad \therefore 1 < x < 2$

ie, $x \in (1, 2)$

111. C

Since, $f'(2) = 0 = f''(2)$

Let $f(x) = A + (x - 2)^n$, $n \geq 3$

but f has local maximum at $x = 2$

So, n may be taken as 4 and since maximum value is -17.Therefore, $f(x) = -17 - (x - 2)^4$

112. A

For $x > 0$ or $x < 0\qquad f'(x) = \frac{a}{x} + 2bx + 1$

$\because\ f'(1) = 0 \Rightarrow a + 2b + 1 = 0\quad$ ....(i)

and $f'(3) = 0 \Rightarrow \frac{a}{3} + 6b + 1 = 0\quad$ ....(ii)

Solving Eqs. (i) and (ii), we get

$a = -3/4$, $b = -1/8$

113. A

$$\frac{1}{\sqrt{a}}\int_1^a\left(\frac{3}{2}\sqrt{x} + 1 - \frac{1}{\sqrt{x}}\right)dx < 4$$

$$\Rightarrow \frac{1}{\sqrt{a}}(a\sqrt{a} - 1 + a - 1 - 2\sqrt{a} + 2) < 4$$

$$\Rightarrow a + \sqrt{a} - 6 < 0 \Rightarrow (\sqrt{a})^2 + \sqrt{a} - 6 < 0$$

$\sqrt{a} \in (-3, 2)$

114. D

$(-1)^{[x]}$ = odd function

115. A

$\phi'(x) = e^{-x/2}(1 - x^2)$

$\phi'(x) = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

116. D

$$A = \lim_{n\to\infty}\frac{1}{n}[(n + 1)(n + 2)(n + 3)...(n + n)]^{1/n}$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{h}\right)+\left(1+\frac{2}{h}\right)\left(1+\frac{3}{h}\right)+...\left(1+\frac{n}{n}\right)\right]^{1/n}$$

$$\log A = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right)$$

$$= \int_0^1 \log(1+x)\,dt \Rightarrow A = \frac{4}{e}$$

117. C

Put $x^7 = t$ & solve

118. B

$$7 \leq 10 + 3\cos x \leq 13 \Rightarrow \frac{1}{13} \leq \frac{1}{10+3\cos x} \leq \frac{1}{7}$$

$$\Rightarrow \frac{2\pi}{13} \leq \int_0^{2\pi}\frac{dx}{10+3\cos x} \leq \frac{2\pi}{7}$$

119. D

$$I = \frac{1}{\sin\alpha}\int_0^{3\alpha}\frac{\sin\alpha\,dt}{\sin(x-\alpha)\sin(x-2\alpha)}$$

$$= \frac{1}{\sin\alpha}\int_0^{3\alpha}\frac{\sin\{(x-\alpha)-(x-2\alpha)\}dx}{\sin(x-\alpha)\sin(x-2\alpha)}$$

$$= \frac{1}{\sin\alpha}\int_0^{3\alpha}\cot(x-2\alpha)-\cot(x-\alpha)\,dx$$

$$= 2\operatorname{cosec}\alpha.\ \log\left(\frac{1}{2}\sec\alpha\right)$$

120. B

They meet at $x = 0$ & $x = 1 - m$

$$\text{Area} = \int_0^{1-m} x - x^2 - mx\,dx = \pm\frac{9}{2}$$

$$\Rightarrow \int_0^{1-m}(x(1-x)-x^2)\,dx = \pm\frac{9}{2}$$

$$\Rightarrow \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \pm\frac{9}{2}$$

$$\Rightarrow (1 - m)^3 = \pm 27 \Rightarrow 1 - m = \pm 3 \Rightarrow m = -2, 4$$

121. C

$4(x^2 - 2x) + y^2 + 4y = 4$

$4(x - 1)^2 + (y + 2) = 12$

$$\frac{(x-1)^2}{3} + \frac{(y+2)^2}{12} = 1$$

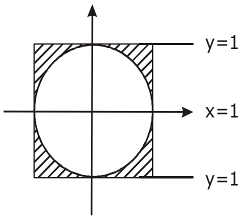
It is ellipse $\therefore A = \sqrt{3}\ \sqrt{12}\ \pi = 6\pi$

122. A

Area

= sq. circle

$= 2 \times 2 - \pi \times \frac{3}{\pi} = 1.$



123. B

$\frac{dx}{dt} = -ax \Rightarrow \frac{dx}{x} = -adt \Rightarrow \ln x = -at$

$\Rightarrow x = ce^{-at} \Rightarrow x(t) = ce^{-at}$

Similarly $y(t) = e^{-bt} \Rightarrow \frac{x(1)}{y(1)} = \frac{3}{2} \Rightarrow e^{a-b} = \frac{4}{3}$

$x(t) = y(t) \Rightarrow 2e^{-at} = e^{-bt} \Rightarrow e^{(a-b)t} = 2$

$\Rightarrow \left(\frac{4}{3}\right)^{t} = 2 \Rightarrow t = \log_{4/3}2$

124. D

Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} = 1 + \cos t \Rightarrow x = \tan\frac{t}{2} + C$

125. D

$\frac{dy}{dx} - \frac{2}{x}.\,y = 2x^3 + x$ IF $= e^{-\int\frac{2}{x}dx} = \frac{1}{x^2}$

$\Rightarrow \frac{y}{x^2} = \int 2x + \frac{1}{x}.\,dx = x^2 + \ln x + c$ & solve

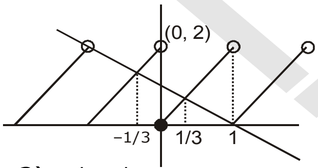
126. D

$x + 2\{x\} > 1$

$2\{x\} > 1 - x$

$[x]^2 - 5[x] + 7$

$= ([x] - 2)([x] - 3) + 1 > 1$



127. D

128. A

If n is even x^n, y^n are positive

$\therefore \frac{x^n + y^n + 1 + 1 + \ldots\ldots 1}{n} \ge \sqrt[n]{x^n y^n}$

$x^n + y^n + (n-2) = nxy$

But given $x^n + y^n + (n-2) = nxy$

$\therefore x = y = 1$ or $x = y = -1$.

129. D

130. D

Given f(x)=[sinx+[cosx+[tanx + [secx]]]]

= [sin + p],where p=[cosx+[tanx + [secx]]]

= [sin x] + p, (as p is an integer)

= [sin x] + [cos x + [tan x + [sec x]]]

= [sin x] + [cos x] + [tan x] + [sec x]

Now, for $x \in (0, \pi/3)$, $\sin x \in \left(0, \frac{\sqrt{3}}{2}\right)$,

$\cos x \in \left(\frac{1}{2}, 1\right)$, $\tan x \in (0, \sqrt{3})$, $\sec x \in (1, 2)$

$\Rightarrow$ [sin x]=0, [cos x]=0, [tan x]=0 or 1,[sec x]=1

$\Rightarrow$ The range of f(x) is {1, 2}

131. D

$\frac{dy}{dx} = 2x, \frac{d^2y}{dx^2} = 2$

$\frac{dx}{dy} = \frac{1}{2\sqrt{y}} = \frac{1}{2x}, \frac{d^2x}{dy^2} = -\frac{1}{4y\sqrt{y}} = -\frac{1}{4x^3}$

132. A

Do your self

133. C

Obviously, $f(x) = x^2 + \tan^{-1}x$ is non-periodic, but sum of two non-periodic functionis not always non-periodic, as $f(x) = x$ and $g(x) = -[x]$, where [∗] represents the greatest integer function. $f(x) + g(x) = x - [x] = \{x\}$ is a periodic function ({ ∗ } represents the fractional part function)

134. D

$\lim_{x\to\infty}\left(\frac{1^2}{x^3} + \frac{2^2}{x^3} + \frac{3^2}{x^3} + \ldots + \frac{x^2}{x^3}\right)$

$= \lim_{x\to\infty}\frac{x(x+1)(2x+1)}{6x^3} = \frac{1}{3}$

135. B

$\lim_{x\to 0^+}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$

$= \lim_{h\to 0}[h]\left(\frac{1-e^{-1/h}}{1+e^{-1/h}}\right) = 0 \times 1 = 0$

$\lim_{x\to 0^-}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$

$$= \lim_{h \to 0} [-h] \left(\frac{e^{-1/h} - 1}{e^{-1/h} + 1} \right) = -1 \times (-1) = 1$$

Thus, given limit does not exists. Also

$\lim_{x \to 0} \left(\frac{e^{1/x} - 1}{e^{1/x} + 1} \right)$ does not exist, but this canot be taken as only reason for non-existence of $\lim_{x \to 0} [x] \left(\frac{e^{1/x} - 1}{e^{1/x} + 1} \right)$.

136. B

Statement II is obviously true.

But $f(x) = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$ is non-differentiable at $x = \pm 1$

as $\frac{2x}{1 - x^2}$ is not defined at $x = \pm 1$.

Hence statement I is true but statement II is not correct explanation of statement I.

137. A

Since $|f(x) - f(y)| \le |x - y|^3$, where $x \neq y$

$\therefore \left| \frac{f(x) - f(y)}{x - y} \right| < |x - y|^2$

Taking lim as $y \to x$, we get

$$\lim_{y \to x} \left| \frac{f(x) - f(y)}{x - y} \right| \le \lim_{y \to x} |x - y|^2$$

$$\Rightarrow \left| \lim_{y \to x} \frac{f(x) - f(y)}{x - y} \right| \le \lim_{y \to x} |(x - y)|^2$$

$\Rightarrow |f'(x)| \le 0 \quad (\because |f'(x)| \ge 0)$

$\Rightarrow |f'(x)| = 0 \quad \therefore f'(x) = 0$

$\Rightarrow f(x) = c$ (constant)

138. A

Given $f(x + y^3) = f(x) + f(y^3) \ \forall \ x, y \in R$

Put $x = y = 0$, we get $f(0 + 0) = f(0) + f(0)$

$\Rightarrow f(0) = 0$.

Now, put $y = -x^{1/3}$, we get $f(0) = f(x) + f(-x)$

$\Rightarrow f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function

$\Rightarrow f'(x)$ is an even function $\Rightarrow f'(-2) = a$

139. D

Though $|x - 1|$ is non-differentiable at $x = 1$, $(x - 1)|x - 1|$ is differentiable at $x = 1$, for which Lagrange's mean value theorem is applicable.

140. A

Consider $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$\Rightarrow f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f(0) = e$ and $f(3) = 81a + 27b + 9c + 3d + e$

$= 3(27a + 9b + 3c + d) + e = 0$

Hence, Rolle's theorem is applicable for $f(x)$,

$\Rightarrow$ there exists at least one c in (a, b) such that $f'(c) = 0$.

141. D

when $x = 1, y = 1$

$$\frac{dy}{dx} = 3x^2 - 2x - 1 \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 0$$

$\Rightarrow$ Equation of the tangent is $y = 1$.

Solving with the curve, $x^3 - x^2 - x + 2 = 1$

$\Rightarrow x^3 - x^2 - x + 1 = 0 \Rightarrow x = -1, 1$

(1 is repeated root)

$\therefore$ the tangent meets the curve agai at $x = -1$

$\therefore$ statement 1 is false and statement 2 is true.

142. D

Statement 2 is true as f(x) is non-differentiable at $x = 1, 2, 3$.

But f(x) has a point of minima at $x = 2$ and not at $x = 3$.

143. B

$f(x) = x + \cos x$

$\therefore f'(x) = 1 - \sin x > 0 \ \forall \ x \in R$, except

at $x = 2n\pi + \frac{\pi}{2}$ and $f'(x) = 0$ at $x = 2n\pi + \frac{\pi}{2}$

$\therefore$ f(x) is strictly increasing.

Statement II is true but does not explain statement I. $\because$ according to Statement II, $f'(x)$ may vanish at finite number of point but in statement I $f'(x)$ vanishes at infinite number of points.

144. A

$$\int e^x \sin x dx$$

$$= \frac{1}{2} \int e^x (\sin x + \cos x + \sin x - \cos x) dx$$

$$= \frac{1}{2} \left(\int e^x (\sin x + \cos x) dx - \int e^x (\cos x - \sin x) dx \right)$$

$$= \frac{1}{2}\,(e^x \sin x - e^x \cos x) + c$$

$$= \frac{1}{2}\, e^x (\sin x - \cos x) + c$$

145. A

Given that $\int_a^b |g(x)|\,dx > \left|\int_a^b g(x)\,dx\right|$

$\Rightarrow$ y = g(x) cuts the graph at least once, then y = f(x) g(x) changes sign at least once in (a, b), hence $\int_a^b f(x)g(x)\,dx$ can be zero.

146. C

$$x > x^2 \ \forall\ x \in \left(0, \frac{\pi}{4}\right) \Rightarrow e^x > e^{x^2} \ \forall\ x \in \left(0, \frac{\pi}{4}\right)$$

$$\cos x > \sin x \ \forall\ x \in \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow e^{x^2} \cos x > e^{x^2} \sin x$$

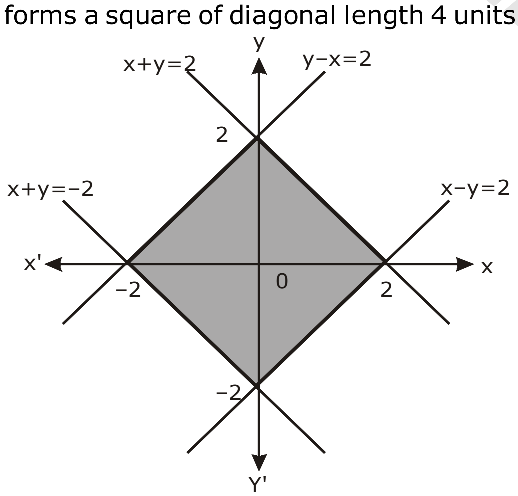
$$\Rightarrow e^x > e^{x^2} > e^{x^2}\cos x > e^{x^2} \sin x \ \forall\ x \in \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow I_2 > I_1 > I_3 > I_4$$

147. B

$2 \geq \max.\{|x - y|, |x + y|\}$

$\Rightarrow$ $|x - y| \leq 2$ and $|x + y| \leq 2$, which forms a square of diagonal length 4 units.



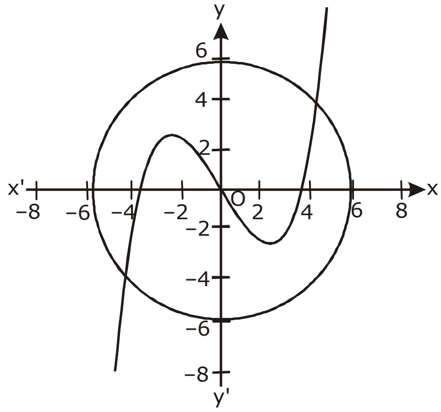
$\Rightarrow$ The area of the region is

$\frac{1}{2} \times 4 \times 4 = 8$ sq. units.

This is equal to the area of the square of side length $2\sqrt{2}$.

148. A

Statement 2 is correct as y = f(x) is odd and hence statement 1 is correct.



149. A

The equation of circle contains. Three independent constants if it passes through three non-collinear points, therefore a is true and follows from R.

150. A

$$y = c_1\cos 2x + c_2\sin^2 x + c_3 \cos^2 x + c_4 e^{2x} + c_5 e^{2x} + c_6$$

$$= c_1\cos 2x + c_2\left(\frac{1 - \cos 2x}{2}\right) + c_3\left(\frac{\cos 2x - 1}{2}\right)$$

$$+ c_4 e^{2x} + c_5 e^{2x} + c_6$$

$$= \left(c_1 + \frac{c_2}{2} + \frac{c_3}{2}\right)\cos 2x + \left(\frac{c_2}{2} - \frac{c_3}{2}\right) + (c_4 + c_5)e^{2x} + c_6$$

$$= l_1 \cos 2x + l_2 e^{2x} + l_3$$

$\Rightarrow$ Total number of independent parameters in the given general solution is 3. Hence statement I is true, also statement II is true which explains statement I.

151. A

we have $k_1 = \tan 27\theta - \tan\theta$
$= \tan(27\theta - \tan 9\theta) + (\tan 9\theta - \tan 3\theta) + (\tan 3\theta - \tan\theta)$

Now, $\tan 3\theta - \tan\theta = \dfrac{\sin 2\theta}{\cos 3\theta \cos\theta} = \dfrac{2\sin\theta}{\cos 3\theta}$

Similarly, $\tan 9\theta - \tan 3\theta = \dfrac{2\sin 3\theta}{\cos 9\theta}$

and $\tan 27\theta - \tan 9\theta = \dfrac{2\sin 9\theta}{\cos 27\theta}$

$\therefore k_1 = 2\left[\dfrac{\sin 9\theta}{\cos 27\theta} + \dfrac{\sin 3\theta}{\cos 9\theta} + \dfrac{\sin\theta}{\cos 3\theta}\right] = 2k_2$

152. D

$\dfrac{99\pi}{2} \le \theta \le 50\pi$ is equivalent to $\dfrac{3\pi}{2} \le \theta \le 2\pi$

$\therefore x = -\sin^2\theta$ and $y = \cos^2\theta$
$\therefore y - x = 1$

153. A

$\dfrac{\sec^8\theta}{a} + \dfrac{\tan^8\theta}{b} = \dfrac{1}{a+b}$

or $a^2\sin^8\theta + ab + b^2 = ab(\cos^8\theta - \sin^8\theta)$

$= ab(1 - 2\sin^2\theta\cos^2\theta)\cos 2\theta$

$= ab(1 - 2\sin^2\theta)\left(1 - \dfrac{1}{2}\sin^2 2\theta\right)$

$\Rightarrow a^2\sin^8\theta - 2ab\sin^4\theta + b^2 = -2ab\sin^2\theta$

$+ ab\sin^2\theta\sin^2 2\theta - \dfrac{ab}{2}\sin^2 2\theta - 2ab\sin^4\theta$

$(a\sin^4\theta - b)^2 = 2ab\sin^2\theta(-\sin^2\theta - 1 + 2\sin^2\theta\cos^2\theta - \cos^2\theta)$
$\Rightarrow 2ab\sin^2\theta(-2 + 2\sin^2\theta\cos^2\theta) \ge 0$
$\Rightarrow 4ab\sin^2\theta(\sin^4\theta - \sin^2\theta + 1) \ge 0$
$\Rightarrow ab \le 0$

154. A

$\because \dfrac{y + \frac{1}{y}}{2} \ge \sqrt{y.\frac{1}{y}} \Rightarrow \sqrt{\left(y + \frac{1}{y}\right)} \ge \sqrt{2}$

But $|\sin x + \cos x| < \sqrt{2}$
which is possible only when

$y + \dfrac{1}{y} = 2 \;\therefore y = 1$ and $x = \dfrac{\pi}{4}$

155. B

we have, $\sin x\sqrt{(8\cos^2 x)} = 1$

$\Rightarrow \sin x.\, 2\sqrt{2}\,|\cos x| = 1$

or $\sin x\,|\cos x| = \dfrac{1}{2\sqrt{2}}$

Case I : when $\cos x > 0$

$\Rightarrow \sin 2x = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$

$\Rightarrow 2x = n\pi + (-1)^n\dfrac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi}{2} + (-1)^n\dfrac{\pi}{8}$

$\therefore x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}$ $(\because 0 < x < 2\pi)$

But $\cos x > 0$, $\therefore x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}$

Case II : when $\cos x < 0$

$\Rightarrow \sin 2x = -\dfrac{1}{\sqrt{2}} = -\sin\dfrac{\pi}{4}$

$\therefore 2x = n\pi - (-1)^n\dfrac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi}{2} - (-1)^n\dfrac{\pi}{8}$

$\therefore x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

But $\cos x < 0$ $\therefore x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$

Hence, the values of x satisfying the given equation which lies between 0 and 2π are

$\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$ these are in AP with

common difference $\dfrac{\pi}{4}$.

156. A

$2\sin^2\theta - 5\sin\theta + 2 > 0$
$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) > 0$
$\because \sin\theta - 2 < 0$ (always)

$\therefore 2\sin\theta - 1 < 0 \Rightarrow \sin\theta < \dfrac{1}{2}$

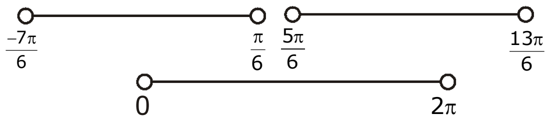
$\Rightarrow -\cos\theta > -\frac{1}{2} \Rightarrow \cos(\pi+\theta) > \cos\frac{2\pi}{3}$

or $2n\pi - \frac{2\pi}{3} < \frac{\pi}{2} + \theta < 2n\pi + \frac{2\pi}{3}$

$\Rightarrow 2n\pi - \frac{7\pi}{6} < \theta < 2n\pi + \frac{\pi}{6}$

For, n = 0, $\frac{7\pi}{6} < \theta < \frac{\pi}{6}$

For n = 1, $\frac{5\pi}{6} < \theta < \frac{13\pi}{6}$



$\therefore \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

Alternative Method :

$\sin\theta < \frac{1}{2} = \sin\frac{\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{5\pi}{6}$

$\therefore \theta \in \left(0, \frac{\pi}{6}\right)$ or $\theta \in \left(\frac{5\pi}{6}, 2\pi\right)$

$\therefore \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

157. B

$\because T_r = \tan^{-1}\left(\frac{2^r - 1}{1 + 2^{2r-1}}\right) = \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1 + 2^r . 2^{r-1}}\right)$

$= \tan^{-1}(2^r) - \tan^{-1}(2^{r-1})$

$\therefore S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \tan^{-1}(2^r) - \tan^{-1}(2^{r-1})$

$= \tan^{-1}(2^n) - \tan^{-1}(2^0)$

$= \tan^{-1}(2^n) - \tan^{-1}(1)$

$= \tan^{-1}(2^n) - \frac{\pi}{4}$

Hence, $S_\infty = \tan^{-1}(2^\infty) - \frac{\pi}{4}$

$= \tan^{-1}(\infty) - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

158. D

$\because$ 1 Radian = 57°17′ 44.8″

$\therefore$ $\sin^{-1} 1 = \frac{\pi}{2}$ is the greatest.

159. A

We have,

$1 < \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \sin 1 \le \cos^{-1}\sin^{-1}\tan^{-1} x \le 1$

$\Rightarrow \cos\sin 1 \ge \sin^{-1}\tan^{-1} x \ge \cos 1$

$\Rightarrow \sin\cos\sin 1 \ge \tan^{-1} x \ge \sin\cos 1$

$\Rightarrow \tan\sin\cos\sin 1 \ge x \ge \tan\sin\cos 1$

$\therefore x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$

160. D

$\because (r_1 + r_2 + r_3 - r)^2$

$= r_1^2 + r_2^2 + r_3^2 + r^2$

$+ 2(r_1r_2 + r_2r_3 + r_3r_1) - 2r(r_1 + r_2 + r_3)$

$\Rightarrow (4R)^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 2s^2$

$-2\{(s-a)(s-b)+(s-b)(s-c)+(s-c)(s-a)\}$

$\Rightarrow 16R^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 2s^2$

$-2\{(3s^2 - 2s(a+b+c) + ab + bc + ca\}$

$\Rightarrow 16R^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 4s^2$

$-2(ab + bc + ca)$

$= r^2 + r_1^2 + r_2^2 + r_3^2 + (a+b+c)^2$

$- 2(ab + bc + ca)$

$= r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$

Hence, $r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$

$= 16 R^2$

161. A

$\because$ A, B, C are in AP. $\therefore$ 2B = A + C

and A + B + C = 180° $\Rightarrow$ 3B = 180° $\therefore$ B = 60°

$\therefore \cos B = \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow a^2 + c^2 = b^2 + ac \Rightarrow (a-c)^2 = b^2 - ac$

$\Rightarrow |a - c| = \sqrt{(b^2 - ac)}$

or $|\sin A - \sin C| = \sqrt{(\sin^2 B - \sin A \sin C)}$

$\Rightarrow 2\cos\left(\frac{A+C}{2}\right)\left|\sin\left(\frac{A-C}{2}\right)\right|$

$= \sqrt{\left(\frac{3}{4} - \sin A \sin C\right)}$

$\Rightarrow 2\left|\sin\left(\frac{A-C}{2}\right)\right| = \sqrt{\left(\frac{3}{4} - \sin A \sin C\right)}$

$\therefore \lim_{x\to c} \frac{\sqrt{(3 - 4\sin A \sin C}}{|A - C|} = \lim_{x\to c} \frac{2\left|\sin\left(\frac{A-C}{2}\right)\right|}{|A - C|}$

$$= \lim_{x\to c}\left|\frac{\sin\left(\frac{A-C}{2}\right)}{\left(\frac{A-C}{2}\right)}\right| = |1| = 1$$

162. A

Let a be length of side of regular polygon,

then $R = \frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$ and $r = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

$$\therefore \frac{R}{r} = \frac{\operatorname{cosec}\left(\frac{\pi}{n}\right)}{\cot\left(\frac{\pi}{n}\right)} = \frac{1}{\cos\left(\frac{\pi}{n}\right)} = \sqrt{5} - 1$$

$$\therefore \cos\left(\frac{\pi}{n}\right) = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4} = \cos\left(\frac{\pi}{5}\right)$$

$\therefore n = 5$

163. C

$A = \cos(\cos x) + \sin(\cos x)$

$$= \sqrt{2}\left\{\cos(\cos x)\cos\frac{\pi}{4} + \sin(\cos x)\sin\frac{\pi}{4}\right\}$$

$$= \sqrt{2}\left\{\cos\left(\cos x - \frac{\pi}{4}\right)\right\}$$

$$\because -1 \le \cos\left(\cos x - \frac{\pi}{4}\right) \le 1$$

$$\therefore -\sqrt{2} \le A \le \sqrt{2}$$

164. C

$$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$$

$$\Rightarrow \frac{\sqrt{n}}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$$

So for $n > 1$, $\frac{\sqrt{n}}{2\sqrt{2}} = \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) > \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ or $n > 4$

Since, $\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$ for all $n > 2$,

we get $\frac{\sqrt{n}}{2\sqrt{2}} < 1$ or $n < 8$

So that $4<n<8$. By actual verification we find that only $n = 6$ satisfies the given relation.

165. B

From the given equations we have $\Sigma \tan\alpha = p$,

$\Sigma \tan\alpha\tan\beta = 0$ and $\tan\alpha\tan\beta\tan\gamma = r$

So that $(1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$

$= 1 + \Sigma\tan^2\alpha + \Sigma\tan^2\alpha\tan^2\beta + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + (\Sigma\tan\alpha)^2 - 2\Sigma\tan\alpha\tan\beta + (\Sigma\tan\alpha\tan\beta)^2$

$-2\tan\alpha\tan\beta\tan\gamma\Sigma\tan\alpha + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + p^2 - 2pr + r^2 = 1 + (p - r)^2$

166. C

Given, $\tan x + \sec x = 2\cos x$

Multiplying by $\cos x \ne 0$

$\therefore \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin x)(1 + \sin x)$

$\Rightarrow (\sin x + 1)(2\sin x - 1)$

$\therefore \sin x = -1$ and $\sin x = \frac{1}{2}$

$\sin x \ne -1$ $\quad (\because \cos x \ne 0)$

$\therefore \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

167. B

Since, $(2\cos x - 1)(3 + 2\cos x) = 0$

$\because \cos x \ne -\frac{3}{2}$

$\therefore \cos x = \frac{1}{2} \Rightarrow x = 2n\pi \pm \frac{\pi}{3}, n \in I$

and given $0 \le x \le 2\pi$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ (for n = 0, 1)

168. A

Given that

$$\sin^5 x - \cos^5 x = \frac{\sin x - \cos x}{\sin x\cos x}$$

$$\sin x\cos x\left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$$\sin x \cos x \{\sin^4 x+\sin^3 x \cos x+\sin^2 x \cos^2 x + \sin x \cos^3 x + \cos^4 x \} = 1$$

$$\sin x \cos x [(\sin^4 x+\cos^4 x)+\sin^2 x \cos^2 x + \sin x \cos x (\sin^2 x + \cos^2 x)] = 1$$

$$\sin x \cos x \{(\sin^2 x+\cos^2 x)^2-\sin^2 x \cos^2 x + \sin x \cos x\} = 1$$

$$\Rightarrow \frac{1}{2} \sin 2x \left[1-\frac{1}{4}\sin^2 2x+\frac{1}{2}\sin 2x\right] = 1$$

$\sin 2x (\sin^2 2x - 2 \sin 2x - 4) = -8$
$\sin^3 2x - 2 \sin^2 x - 4 \sin 2x + 8 = 0$
$(\sin 2x - 2)^2 (\sin 2x + 2) = 0$
$\Rightarrow \sin 2x = \pm 2$ which is impossible

169. A

For $\cos^{-1} (1 - x)$
$\Rightarrow -1 < 1 - x < 1 \Rightarrow 1 \geq -1 + x \geq -1$
or $2 \geq x \geq 0$ or $x \in [0, 2]$ .....(i)
For $\cos^{-1} x \Rightarrow -1 \leq x \leq 1$ .....(ii)
From Eqs. (i) and (ii), we get $0 \leq x \leq 1$
$\therefore$ L.H.S. ≥ 0, but R.H.S. ≤ 0 ($\because x \leq 0$)
Equality holds if L.H.S. = 0 and R.H.S.=0
$\therefore \cos^{-1} (1 - x) + m \cos^{-1} x = 0$
$\Rightarrow \cos^{-1} (1 - x) = 0$ and $\cos^{-1} x = 0$
which is impossible i.e. no solution
Hence, number of solution = 0

170. C

$$\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right)-\tan^{-1}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{10}{24}\right)-\tan^{-1}1\right\}$$

$$= \tan\tan^{-1}\left\{\frac{\frac{10}{24}-1}{1+\frac{10}{24}}\right\} = -\frac{14}{34} = -\frac{7}{17}$$

171. A

$$\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$$

$$= \sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$$

$= \sin^{-1} [\cot \{15° + 30° + 45°\}]$
$= \sin^{-1} [\cot \pi/2] = \sin^{-1} (0) = 0$

172. D

$\because 2a^2 + 4b^2 + c^2 = 4ab + 2ac$
$\Rightarrow (a - 2b)^2 + (a - c)^2 = 0$
Which is possible only when
$a - 2b = 0$ and $a - c = 0$

$$\text{or } \frac{a}{1} = \frac{b}{\frac{1}{2}} = \frac{c}{1} = \lambda \quad \text{(say)}$$

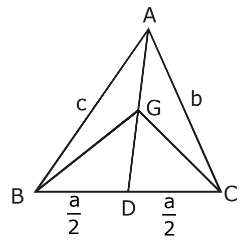
$$\therefore a = \lambda,\ b = \frac{\lambda}{2},\ c = \lambda$$

$$\therefore \cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{\lambda^2+\lambda^2-\frac{\lambda^2}{4}}{2\lambda^2}$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

173. B

By appolloneous theorem



$(GB)^2 + (GC)^2$
$= 2 \{(GD)^2 + (DC)^2\}$
$\Rightarrow (GB)^2 + (GC)^2$

$$= 2\left\{\left(\frac{1}{2}GA\right)^2+\left(\frac{a}{2}\right)^2\right\}$$

$$\Rightarrow (GB)^2 + (GC)^2 = \frac{(GA)^2}{2} + \frac{a^2}{2} \text{....(i)}$$

Similarly,

$$(GC)^2 + (GA)^2 = \frac{(GB)^2}{2} + \frac{b^2}{2} \qquad \text{...(ii)}$$

$$\text{and } (GA)^2 + (GB)^2 = \frac{(GC)^2}{2} + \frac{c^2}{2} \text{ ....(iii)}$$

Adding Eqs. (i), (ii) and (iii), we get
$2 \{(GA)^2 + (GB)^2 + (GC)^2\}$

$$= \frac{1}{2}\{(GA)^2+(GB)^2+(GC)^2\}+\frac{(a^2+b^2+c^2)}{2}$$

$$\text{or } \frac{3}{2}\{(GA)^2+(GB)^2+(GC)^2\}=\frac{(a^2+b^2+c^2)}{2}$$

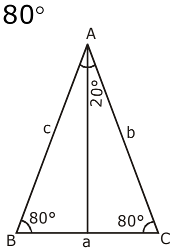
$$\Rightarrow (GA)^2 + (GB)^2 + (GC)^2 = \left(\frac{a^2+b^2+c^2}{3}\right)$$

174. C

$\because \angle A = 20° \quad \therefore \angle B = \angle C = 80°$

Then, $b = c$

$\therefore \dfrac{a}{\sin 20°} = \dfrac{b}{\sin 80°} = \dfrac{c}{\sin 80°}$



or $\dfrac{a}{\sin 20°} = \dfrac{b}{\cos 10°}$

$\Rightarrow a = 2b \sin 10°$ ....(i)

$\therefore a^3 + b^3$

$= 8b^3 \sin^3 10° + b^3$

$= b^3 \{2(4 \sin^3 10°) + 1\}$

$= b^3 \{2 (3 \sin 10° - \sin 30°) + 1\}$

$= b^3 \{6 \sin 10°\} = 3b^2 (2b \sin 10°)$

$= 3b^2 a = 3 ac^2$ [from Eq. (i)] ($\because b = c$)

175. D

$\text{cosec}^2 x + 25 \sec^2 x = 26 + \cot^2 x + 25 \tan^2 x$

$= 26 + 10 + (\cot x - 5 \tan x)^2 \geq 36$

176. A

$\because \tan\theta = \dfrac{1}{\sqrt{7}}$

$\therefore \dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \dfrac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$= \dfrac{(\cot^2\theta - \tan^2\theta)}{2 + \tan^2\theta + \cot^2\theta}$

$= \dfrac{7 - \frac{1}{7}}{2 + \frac{1}{7} + 7} = \dfrac{48}{14+1+49} = \dfrac{48}{64} = \dfrac{3}{4}$

177. B

$A + C = B \Rightarrow \tan (A + C) = \tan B$

$\Rightarrow \dfrac{\tan A + \tan C}{1 - \tan A \tan C} = \tan B$

$\Rightarrow \tan A \tan B \tan C = \tan B - \tan A - \tan C$

178. D

$1 + \sin\theta + \sin^2\theta + \ldots\ldots = 4 + 2\sqrt{3}$

$\dfrac{1}{1-\sin\theta} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin\theta = \dfrac{1}{4+2\sqrt{3}} = \dfrac{4-2\sqrt{3}}{4}$

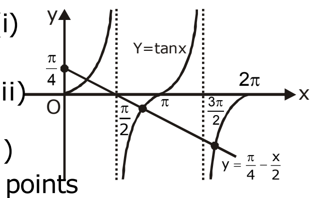
$\therefore \sin\theta = \dfrac{\sqrt{3}}{2} \quad \therefore \theta = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

179. C

Since, $x + 2\tan x = \dfrac{\pi}{2}$ or $\tan x = \dfrac{\pi}{4} - \dfrac{x}{2} = y$ (say)

$\therefore y = \tan x$ ...(i)

and $y = \dfrac{\pi}{4} - \dfrac{x}{2}$ ...(ii)



Graphs (i) and (ii) intersect at three points

$\therefore$ No. of solutions is 3.

180. D

$\sin 2\theta = \dfrac{\sqrt{3}}{2} = \sin\left(\dfrac{\pi}{3}\right), \sin\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow 2\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ or $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{3}$ .....(i)

and $\tan\theta = \dfrac{1}{\sqrt{3}} \quad \therefore \theta = \dfrac{\pi}{6}, \pi + \dfrac{\pi}{6}$

$\theta = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$ ......(ii)

From Eqs. (i) and (ii) common value of θ is $\dfrac{\pi}{6}$

Hence, general value of θ is $2n\pi + \dfrac{\pi}{6}$, $n \in I$

181. C

$\tan^{-1} 1 + \cos^{-1}\left(-\dfrac{1}{2}\right) + \sin^{-1}\left(-\dfrac{1}{2}\right)$

$= \dfrac{\pi}{4} + \dfrac{2\pi}{3} - \dfrac{\pi}{6} = \dfrac{3\pi}{4}$

182. A

Let $x = \tan\theta$

Then, $2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$= 2\theta + \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = 2\theta + \sin^{-1}(\sin 2\theta)$

If $-\dfrac{\pi}{2} \leq 2\theta \leq \dfrac{\pi}{2}$ Then,

$2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = 2\theta + 2\theta = 4\theta$

$= 4\tan^{-1} x$

Which is not independent of x

and if $-\frac{\pi}{2} \le \pi - 2\theta \le \frac{\pi}{2}$

Then, $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= 2\theta + \sin^{-1}\sin(\pi - 2\theta) = 2\theta + \pi - 2\theta$
$= \pi$ = independent of x

$\therefore \theta \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ Principal value of

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \therefore \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$ Hence, $x \in [1, \infty)$

183. A

$\because \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right) = \pi$

184. C

$(a + b + c)(b + c - a) = kbc$

$\Rightarrow 2s(2s - 2a) = kbc \Rightarrow \frac{s(s-a)}{bc} = \frac{k}{4}$

$\cos^2\left(\frac{A}{2}\right) = \frac{k}{4} \quad \because 0 < \cos^2\left(\frac{A}{2}\right) < 1$

$\therefore 0 < \frac{k}{4} < 4 \Rightarrow 0 < k < 4$

185. C

$\because \Delta = a^2 - (b - c)^2 = (a + b - c)(a - b + c)$
$= 2(s - c) \cdot 2(s - b)$

$\sqrt{s(s-a)(s-b)(s-c)} = 4(s - b)(s - c)$

$\Rightarrow \frac{1}{4} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan\frac{A}{2}$

$\therefore \tan\frac{A}{2} = \frac{1}{4}$

186. A

$r = r_2 + r_3 - r_1$

$\Rightarrow \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s-a}$

$\Rightarrow \frac{1}{s} + \frac{1}{s-a} = \frac{1}{s-b} + \frac{1}{s-c}$

$\Rightarrow \frac{2s-a}{2s-b-c} = \frac{s(s-a)}{(s-b)(s-c)}$

$\Rightarrow \frac{2s-a}{a} = \cot^2\frac{A}{2}$

$\Rightarrow \frac{s}{a} = \frac{1}{2}\left(\cot^2\frac{A}{2} + 1\right) \Rightarrow \frac{s}{a} \in \left(\frac{1}{2}, 2\right)$

187. C

$\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x$
$\Rightarrow \sin^2 x = \cos^4 x \Rightarrow 1 - \cos^2 x = \cos^4 x$
$\Rightarrow \cos^4 x + \cos^2 x = 1$
Squaring both sides,
then $\cos^8 x + \cos^4 x + 2\cos^6 x = 1$
Hence, $\cos^8 x + 2\cos^6 x + \cos^4 x = 1$

188. A

$\because 0 < \alpha < \frac{\pi}{6}$

$\Rightarrow 0 < \frac{\alpha}{2} < \frac{\pi}{12}$ (in first quadrant),

then $\tan \alpha/2$ is always positive

$\because \sin\alpha + \cos\alpha = \frac{\sqrt{7}}{2}$

$\Rightarrow \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{\sqrt{7}}{2}$

$\Rightarrow$

$2\left(2\tan\frac{\alpha}{2} + 1 - \tan^2\frac{\alpha}{2}\right) = \sqrt{7}\left(1 + \tan^2\frac{\alpha}{2}\right)$

$\Rightarrow (\sqrt{7} + 2)\tan^2\frac{\alpha}{2} - 4\tan\frac{\alpha}{2} + (\sqrt{7} - 2) = 0$

$\therefore \tan\frac{\alpha}{2} = \frac{4 \pm \sqrt{16 - 4(\sqrt{7}+2)(\sqrt{7}-2)}}{2(\sqrt{7}+2)}$

$= \frac{4 \pm \sqrt{16-12}}{2(\sqrt{7}+2)} = \frac{4 \pm 2}{2(\sqrt{7}+2)} = \frac{2 \pm 1}{(\sqrt{7}+2)}$

$= \frac{1}{\sqrt{7}+2}$ or $\frac{3}{\sqrt{7}+2} = \frac{\sqrt{7}-2}{3}$ or $\sqrt{7} - 2$

Hence, $\tan(\alpha/2) = \frac{\sqrt{7}-2}{3}\left(\because 0 < \left(\frac{\alpha}{2}\right) < \frac{\pi}{12}\right)$

189. A

$\because \sin(\alpha + \beta) < \sin\alpha + \sin\beta$ in $(0, \pi/2)$

$\therefore \sin(\alpha + \beta + \gamma) < \sin\alpha + \sin\beta + \sin\gamma$

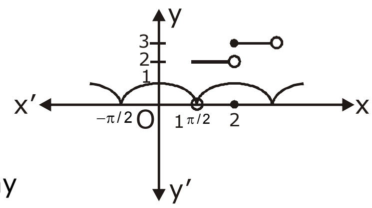
$$\Rightarrow \frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma} < 1$$

190. A

We have, $|\cos x| = 2[x] = y$ (say)

$\therefore$ $y = |\cos x|$ and

$y = 2[x]$



From graph $|\cos x|$ and $2[x]$ don't cut each other for any real value of x.

Hence, number of solution is nil.

191. B

we have, $|4\sin x - 1| < \sqrt{5}$

$\Rightarrow -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$

$$\Rightarrow -\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \left(\frac{\sqrt{5}+1}{4}\right)$$

$$\Rightarrow -\sin\frac{\pi}{10} < \sin x < \cos\frac{\pi}{5}$$

$$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{\pi}{2}-\frac{\pi}{5}\right)$$

$$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{3\pi}{10}\right)$$

$$x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right) \quad \{\because\ x \in (-\pi, \pi)\}$$

192. A

$$1 + \sin x \sin^2\frac{x}{2} = 0$$

$$1 + \sin x\left(\frac{1-\cos x}{2}\right) = 0$$

$\Rightarrow 2 + \sin x - \sin x\cos x = 0$

$\Rightarrow 4 + 2\sin x = \sin 2x$

LHS $\in [2, 6]$ but RHS $\in [-1, 1]$

Hence, no solution

i.e., Number of solutions = zero

193. B

$(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$

$\Rightarrow (\cot^{-1} x - 3)(\cot^{-1} x - 2) > 0$

Then, $\cot^{-1} x < 2$ and $\cot^{-1} x > 3$

$\Rightarrow x > \cot 2$ and $x < \cot 3$

hence, $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

194. C

Since, $a^2 + b^2 + c^2 = 8R^2$

$\Rightarrow (2R\sin A)^2+(2R\sin B)^2+(2R\sin C)^2=8R^2$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow \cos^2 A - \sin^2 B + \cos^2 C = 0$

$\Rightarrow \cos(A + B)\cos(A - B) + \cos^2 C = 0$

$\Rightarrow \cos(\pi - C)\cos(A - B) + \cos^2 C = 0$

$\Rightarrow -\cos C\{\cos(A - B) - \cos C\} = 0$

$\Rightarrow -\cos C\{\cos(A - B) + \cos(A + B)\}=0$

$\Rightarrow -2\cos A\cos B\cos C = 0$

$\therefore \cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

$$\Rightarrow A = \frac{\pi}{2} \text{ or } B = \frac{\pi}{2} \text{ or } C = \frac{\pi}{2}$$

195. A

$$\frac{\sin^2 A+\sin A+1}{\sin A} = \sin A + \frac{1}{\sin A} + 1$$

$$= \left(\sqrt{\sin A}-\frac{1}{\sqrt{\sin A}}\right)^2 + 3 \geq 3$$

Minimum value of $\dfrac{\sin^2 A+\sin A+1}{\sin A} = 3$

$\therefore$ Minimum value of $\Sigma\dfrac{\sin^2 A+\sin A+1}{\sin A}$

$= 3 + 3 + 3 = 9$

196. C

We have,

$$2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

$$\Rightarrow 2\left(\frac{b^2+c^2-a^2}{2abc}\right) + \left(\frac{c^2+a^2-b^2}{2abc}\right)$$

$$+ 2\left(\frac{a^2+b^2-c^2}{2abc}\right) = \frac{a^2+b^2}{abc}$$

$$\Rightarrow b^2 + c^2 = a^2 \Rightarrow \angle A = \frac{\pi}{2}$$

197. A

HM of exradii = $\dfrac{1}{\dfrac{\Sigma \dfrac{1}{r^1}}{3}} = \dfrac{3}{\dfrac{1}{r}} = 3r$

198. A

We have, $\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$

$\Rightarrow \sin\alpha\cos\alpha\,(\cos^2\alpha - \sin^2\alpha) > 0$

$\Rightarrow \cos^3\alpha\sin\alpha\,(1 - \tan^2\alpha) > 0$

$(\because \sin\alpha > 0 \text{ for } 0 < \alpha < \pi)$

$\Rightarrow \cos\alpha\,(1 - \tan^2\alpha) > 0$

$\Rightarrow \cos\alpha > 0$ and $1 - \tan^2\alpha > 0$

or $\cos\alpha < 0$ and $1 - \tan^2\alpha < 0$

$\Rightarrow \alpha \in (0, \pi/4)$ or $\alpha \in \left(\dfrac{3\pi}{4}, \pi\right)$

199. C

$ab\sin x + b\sqrt{(1-a^2)}\cos x$

Now, $\sqrt{(ab)^2 + (b\sqrt{(1-a^2)})^2}$

$= \sqrt{a^2b^2 + b^2(1-a^2)} = b\sqrt{(a^2+1-a^2)} = b$

$\Rightarrow b\,\{(a\sin x + \sqrt{(1-a^2)}\cos x)\}$

Let $a = \cos\alpha$,

$\therefore \sqrt{(1-a^2)} = \sin\alpha \Rightarrow b\sin(x + a)$

$\because -1 \le \sin(x+\alpha) \le 1$

$\therefore c - b \le b\sin(x+\alpha) + c \le b + c$

$\therefore b\sin(x+\alpha) + c \in [c - b, c + b]$

200. D

$\because 2\cos\theta + \sin\theta = 1$

$\therefore \dfrac{2(1-\tan^2\theta/2)}{(1+\tan^2\theta/2)} + \dfrac{(2\tan\theta/2)}{(1+\tan^2\theta/2)} = 1$

$\Rightarrow 2 - 2\tan^2\theta/2 + 2\tan\theta/2 = 1 + \tan^2\theta/2$

$\Rightarrow 3\tan^2(\theta/2) - 2\tan(\theta/2) - 1 = 0$

$\therefore \tan\theta/2 = 1,\ \tan\theta/2 = -\dfrac{1}{3}$

$\Rightarrow \theta = 90°,\ \tan\theta/2 = -\dfrac{1}{3}$

$\therefore 7\cos\theta + 6\sin\theta$ at $\theta = 90°$ is $0+6 = 6$

and $7\cos\theta + 6\sin\theta$

$= \dfrac{7(1-\tan^2\theta/2)}{(1+\tan^2\theta/2)} + \dfrac{12\tan\theta/2}{(1+\tan^2\theta/2)}$

$= \dfrac{7\left(1-\dfrac{1}{9}\right) + 12\left(-\dfrac{1}{3}\right)}{1+\dfrac{1}{9}} = 2 \quad \left(\because \tan\theta/2 = -\dfrac{1}{3}\right)$

201. A

$\because \sin\alpha = \dfrac{336}{625}$ and $450° < \alpha < 540°$

$\therefore \sin(\alpha/4) = \sqrt{\dfrac{1-\cos(\alpha/2)}{2}}$

$= \sqrt{\dfrac{1}{2}\left\{1 - \sqrt{\left(\dfrac{1+\cos\alpha}{2}\right)}\right\}} = \sqrt{\dfrac{1}{2}\left\{1 - \sqrt{\dfrac{1+\dfrac{527}{625}}{2}}\right\}}$

$= \sqrt{\dfrac{1}{2}\left\{1 - \dfrac{24}{25}\right\}} = \sqrt{\dfrac{1}{50}} = \dfrac{1}{5\sqrt{2}}$

202. B

$4x^2 - 4x|\sin\theta| - (1 - \sin^2\theta)$

$= -1 + (2x - |\sin\theta|)^2$ $\therefore$ Minimum value $= -1$

203. A

$\because \cos^4\theta\sec^2\alpha,\ \dfrac{1}{2}$ & $\sin^4\theta\,\text{cosec}^2\alpha$ are in AP

$1 = \cos^4\theta\sec^2\alpha + \sin^4\theta\,\text{cosec}^2\alpha$

$\Rightarrow 1 = \dfrac{\cos^4\theta}{\cos^2\alpha} + \dfrac{\sin^4\theta}{\sin^2\alpha}$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^2 = \dfrac{\cos^4\theta}{\cos^2\alpha} + \dfrac{\sin^4\theta}{\sin^2\alpha}$

$\Rightarrow \cos^4\theta\left(\dfrac{1}{\cos^2\alpha} - 1\right) + \sin^4\theta\left(\dfrac{1}{\sin^2\alpha} - 1\right)$

$-2\sin^2\theta\cos^2\theta = 0$

$\Rightarrow \sin^4\alpha\cos^4\theta + \sin^4\theta\cos^4\alpha$

$-2\sin^2\theta\cos^2\theta\sin^2\alpha\cos^2\alpha = 0$

$\Rightarrow (\sin^2\alpha\cos^2\theta - \cos^2\alpha\sin^2\theta)^2 = 0$

$\Rightarrow \tan^2\theta = \tan^2\alpha \quad \therefore \theta = n\pi \pm \alpha,\ n \in I$

Now, $\cos^8\theta\sec^6\alpha = \cos^8\alpha\sec^6\alpha = \cos^2\alpha$

and $\sin^8\theta\,\text{cosec}^6\alpha = \sin^8\alpha.\ \text{cosec}^6\alpha = \sin^2\alpha$

Hence, $\cos^8\theta\sec^6\alpha,\ \dfrac{1}{2},\ \sin^8\theta,\ \text{cosec}^6\alpha$

ie, $\cos^2\alpha,\ \dfrac{1}{2},\ \sin^2\alpha$ are in AP.

204. B

For maximum value $2a^2 - 1 - \cos^2 x = 0$

$\therefore \cos^2 x = 2a^2 - 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = (2 - 2a^2)$

$\Rightarrow 2a^2 + \sin^2 x = 2$

$\therefore$ Maximum value of

$$\left| \sqrt{(\sin^2 x + 2a^2)} - \sqrt{(2a^2 - 1 - \cos^2 x)} \right|$$

$= |\sqrt{2} - 0| = \sqrt{2}$

205. B

$\because \quad \cos^7 x \le \cos^2 x \quad .....(i)$

and $\quad \sin^4 x \le \sin^2 x \quad .....(ii)$

Adding Eqs. (i) and (ii), then

$\cos^7 x + \sin^4 x \le 1$

But given $\cos^7 x + \sin^4 x = 1$

Equality holds only, if

$\cos^7 x = \cos^2 x$ and $\sin^4 x = \sin^2 x$

Both are satisfied by $x = \pm \frac{\pi}{2}, 0$.

206. D

$\sin^2 \theta_1 + \sin^2 \theta_2 + ... + \sin^2 \theta_n = 0$

$\Rightarrow \sin \theta_1 = \sin \theta_2 = ... = \sin \theta_n = 0$

$\Rightarrow \cos^2 \theta_1, \cos^2 \theta_2, ..., \cos^2 \theta_n = 1$

$\Rightarrow \cos \theta_1, \cos \theta_2 ....., \cos \theta_n = \pm 1$

Now $\cos \theta_1 + \cos \theta_2 + ... + \cos \theta_n = n-4$ means two of $\cos \theta_1, \cos \theta_2, ..., \cos \theta_n$ must be -1 and the other are 1. Now two values from $\cos \theta_1, \cos \theta_2 ....., \cos \theta_n$ can be selected in ${}^nC_2$ ways. Hence,

the number of solutions is ${}^nC_2 = \frac{n(n-1)}{2}$.

Hence, statement 1 is false, but statement 2 is correct.

207. D

Given $\cos x \sum \cos \alpha - \sin x \sum \sin \alpha = 0, \forall x \in R$

Hence, $\cos \alpha + \cos \beta + \cos \gamma = 0$

and $\sin \alpha + \sin \beta + \sin \gamma = 0$

Hence, statement 2 is true.

Now $(\cos \alpha + \cos \beta)^2 = (-\cos \gamma)^2$

and $(\sin \alpha + \sin \beta)^2 = (-\sin \gamma)^2$

Adding, we get

$2 + 2\cos(\alpha-\beta) = 1 \Rightarrow \cos(\alpha-\beta) = -1/2$

Similarly, $\cos(\beta - \gamma) = -1/2$ and $\cos(\gamma - \alpha) = -1/2$

Now $0 < \alpha < \beta < \gamma < 2\pi \Rightarrow \beta - \alpha < \gamma - \alpha$

Hence, $\beta - \alpha = \frac{2\pi}{3}$ and $\gamma - \alpha = \frac{4\pi}{3}$

Statement 1 is false.

208. A

$\cos(\sin x) = \sin(\cos x)$

$\Rightarrow \cos(\sin x) = \cos[(\pi/2) - \cos x]$

$\Rightarrow \sin x = 2n\pi \pm (\pi/2 - \cos x), n \in Z$

Taking + ve sign, we get

$\sin x = 2n\pi + \pi/2 - \cos x$

or $(\cos x + \sin x) = \frac{1}{2}(4n + 1)\pi$

Now L.H.S. $\in [-\sqrt{2}, \sqrt{2}]$,

hence $-\sqrt{2} \le \frac{1}{2}(4n + 1)\pi \le \sqrt{2}$.

$\Rightarrow \frac{-2\sqrt{2} - \pi}{4\pi} \le n \le \frac{2\sqrt{2} - \pi}{4\pi}$,

which is not satisfied by any integer n. Similary, taking –ve sign, we have $\sin x - \cos x = (4n - 1)\pi/2$, which is also not satisfied for any integer n. Hence, there is no solution.

209. A

$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} > \tan^{-1} x > \tan^{-1} y$

$\left[\because x > y, \frac{x}{\sqrt{1-x^2}} > x\right]$

Therefore, statement 2 is true.

Now, $e < \pi \Rightarrow \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{\pi}}$

By statement 2, we have

$\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Therefore, statement 1 is true.

210. A

In any ΔABC, we have $r_1 + r_2 + r_3 = 4R + r \le 9(R/2)$

211. C

$\because \log_2 x + \log_2 y \ge 6 \Rightarrow \log_2 (xy) \ge 6$

$\therefore xy \ge 2^6$ or $\sqrt{xy} \ge 2^3$

$\because \frac{x+y}{2} \ge \sqrt{xy}$ or $x + y \ge 2\sqrt{xy} \ge 16$

$(\because \text{AM} \ge \text{GM}) \quad \therefore x + y \ge 16.$

212. C

$f(x, 10) = f(x, 11)$

$$\Rightarrow \sum_{k=1}^{10} \log_x \left(\frac{k}{x}\right) = \sum_{k=1}^{11} \log_x \left(\frac{k}{x}\right)$$

$$\Rightarrow 0 = \log_x \left(\frac{11}{x}\right) \Rightarrow \frac{11}{x} = 1 \Rightarrow x = 11$$

213. B

We know that only even prime is 2,
then $(2)^2 - \lambda(2) + 12 = 0 \Rightarrow \lambda = 8$ ....(i)
and $x^2 + \lambda x + \mu = 0$ has equal roots
$\therefore \lambda^2 - 4\mu = 0$ or $(8)2 - 4\mu = 0 \Rightarrow \mu = 16$
[from Eq. (i)]

214. C

By hypothesis $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$

and $\frac{\alpha}{\alpha-1} . \frac{\alpha+1}{\alpha} = \frac{c}{a}$

$\Rightarrow \frac{2\alpha^2-1}{\alpha^2-\alpha} = -\frac{b}{a}$ and $\alpha = \frac{c+a}{c-a}$

$\Rightarrow (c + a)^2 + 4ac = -2b (c + a)$
$\Rightarrow (c + a)^2 + 2b (c + a) + b^2 = b^2 - 4ac$
$\Rightarrow (a + b + c)^2 = b^2 - 4ac.$

215. D

For maximum value of the given sequence to n terms, when the nth term is either zero or the smallest positive number of the sequence i.e., $50 + (n - 1) (-2) = 0$

$\Rightarrow n = 26 \therefore S_{26} = \frac{26}{2} (50 + 0) = 26 \times 25 = 650$

216. B

$\because x^a = y^b = z^c = \lambda$ (say)
$\therefore x = \lambda^{1/a}, y = \lambda^{1/b}, z = \lambda^{1/c}$
Now, $\because$ x, y, z are in GP
$\therefore y^2 = zx$
$\Rightarrow \lambda^{2/b} = \lambda^{1/c}. \lambda^{1/a} \Rightarrow \lambda^{2/b} = \lambda^{(1/c + 1/a)}$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \therefore$ a,b,c are in HP

Now, GM > HM $\Rightarrow \sqrt{ac} > b$ .....(i)
Now, for three numbers a^3, b^3, c^3
AM > GM

$\Rightarrow \frac{a^3+c^3}{2} > (\sqrt{ac})^3 > b^3$ [from Eq. (i)]

$\therefore a^3 + c^3 > 2b^3$

217. B

$\because 0 \le [x] < 2 \Rightarrow [x] = 0,1$
$-1 \le [y] < 1 \Rightarrow [y] = -1, 0$
and $1 \le [z] < 3 \Rightarrow [z] = 1, 2$
Now, aplying in the given determinant
$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, then

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= ([x] + 1)(1 - 0)-[y] (-1 -0) + [z] (0+1)$
$= [x] + [y] + [z] + 1 = 1 + 0 + 2 + 1 = 4$
$(\therefore$ for maximum value $[x] = 1, [y]=0,[z]= 2)$

218. B

$$\begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda (-3 + 2\lambda) + 1 (2 - 3\lambda) - 2(-4 + 9) + 0$
$\Rightarrow 2\lambda^2 - 6\lambda - 8 = 0 \Rightarrow \lambda^2 - 3\lambda - 4 = 0$
$\Rightarrow (\lambda - 4) (\lambda + 1) = 0 \quad \therefore \lambda = -1, 4$

219. A

$\because Q = PAP^T \Rightarrow P^T Q = AP^T \quad (\because PP^T = I)$
$\therefore P^T Q^{2005} P = AP^T Q^{2004} P$
$= A^2 P^T Q^{2003} P = A^3 P^T Q^{2002} P = ...$
$= A^{2004} P^T (QP) = A^{2004} P^T (PA)$
$(\because Q = PAP^T \Rightarrow QP = PA)$

$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

220. D

$\because A^4 (I - A) = A^4 I - A^5 = A^4 - O = A^4 \ne I$
$A^3 (I - A) = A^3 I - A^4 = A^3 - A^4 \ne I$
and $(I + A) (I - A) = I^2 - A^2 = I - A^2 \ne I.$

221. C

$\because (x - 1) = (x - [x]) (x - \{x\})$
$\Rightarrow x = 1 + \{x\}[x] \Rightarrow [x]+\{x\} = 1 + \{x\} [x]$
$\Rightarrow (\{x\} - 1) ([x] - 1) = 0$

$\Rightarrow \{x\} - 1 \neq 0, \therefore [x] - 1 = 0 \Rightarrow [x] = 1$
$\Rightarrow x \in [1, 2)$

222. D

Discriminant ≥ 0 $\therefore \pi < 22/5$
Roots less than 2 $\therefore f(2) > 0$
$\therefore p^2 - p - 2 > 0 \Rightarrow p > 2$ or $p < -1$
combine both casses, we get

$p \in (-\infty, -1)\ e\left(2, \frac{22}{5}\right]$

223. A

$\because$ a, b, c, d are positive real numbers.
$\therefore m > 0$ ....(i) Now, AM $\geq$ GM

$\Rightarrow \frac{(a+b)+(c+d)}{2} > \sqrt{(a+b)(c+d)}$

$\Rightarrow \frac{2}{2} \geq \sqrt{m}$ or $m \leq 1$ ....(ii)

From Eqs. (1) and (ii), we get, $0 < m \leq 1$

224. D

Let $b = a + d$, $c = a + 2d$ ...(i)
$\because a^2, b^2, c^2$ are in GP $\therefore (b^2)^2 = a^2c^2$
or $\pm b^2 = ac$ ....(ii)
$\because$ a, b, c are in AP $\therefore 2b = a + c$
Given, $a + b + c = 3/2 \Rightarrow 3b = 3/2 \Rightarrow b = 1/2$

From Eq. (i), $a = \frac{1}{2} - d$, $c = \frac{1}{2} + d$

$\therefore$ From Eq. (ii),

$\pm \frac{1}{4} = \left(\frac{1}{2} - d\right)\left(\frac{1}{2} + d\right) \Rightarrow \pm \frac{1}{4} = \frac{1}{4} - d^2$

Taking (–ve) sign, $\therefore d = \pm \frac{1}{\sqrt{2}}$

$\therefore a = \frac{1}{2} - d = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$

$\Rightarrow a = \frac{1}{2} + \frac{1}{\sqrt{2}}$ $(\because a > b)$

225. D

$$\therefore f(-x) = \begin{vmatrix} -x & \cos x & e^{x^2} \\ -\sin x & x^2 & \sec x \\ -\tan x & 1 & 2 \end{vmatrix} = -f(x)$$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)\,dx = 0$ [$\because$ f(x) is an odd function]

226. B

$\Delta > 0 \Rightarrow abc + 2 > 3(abc)^{1/3}$
Let $(abc)^{1/3} = x$
$x^3 + 2 > 3x \Rightarrow (x-1)^2(x+2) > 0$
$\therefore x + 2 > 0 \Rightarrow x > -2$
$(abc)^{1/3} > -2 \Rightarrow abc > -8$

227. B

$\because 2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ ....(i)

$\Rightarrow 4x - 2y = \begin{bmatrix} 6 & -6 & 0 \\ 6 & 6 & 4 \end{bmatrix}$ ....(ii)

and $x + 2y = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$ .....(iii)

Adding Eqs. (ii) and (iii), then

$5x = \begin{bmatrix} 10 & -5 & 5 \\ 5 & 10 & 0 \end{bmatrix}$ $\therefore x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

From Eq. (iii), $2x+4y = \begin{bmatrix} 8 & 2 & 10 \\ -2 & 8 & -8 \end{bmatrix}$ ....(iv)

Substracting Eq. (i) from (iv), then

$5y = \begin{bmatrix} 5 & 5 & 10 \\ -5 & 5 & -10 \end{bmatrix}$ $\therefore y = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$

228. D

$\because \omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Also, $\omega^3 = 1$ and $\omega + \omega^2 = -1$

Then, $A = \begin{pmatrix} \frac{\omega}{i} & \frac{\omega^2}{i} \\ -\frac{\omega^2}{i} & -\frac{\omega}{i} \end{pmatrix} = \frac{\omega}{i}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}$

$\therefore A^2 = \frac{\omega^2}{i^2}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}$

$= -\omega^2\begin{pmatrix} 1-\omega^2 & 0 \\ 0 & 1-\omega^2 \end{pmatrix} = \begin{pmatrix} -\omega^2+\omega^4 & 0 \\ 0 & -\omega^2+\omega^4 \end{pmatrix}$

$= \begin{pmatrix} -\omega^2+\omega & 0 \\ 0 & -\omega^2+\omega \end{pmatrix}$

$\because f(x) = x^2 + 2$ $\therefore f(A) = A^2 + 2I$

$= \begin{pmatrix} -\omega^2+\omega & 0 \\ 0 & -\omega^2+\omega \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} -\omega^2+\omega+2 & 0 \\ 0 & -\omega^2+\omega+2 \end{pmatrix}$

$= (-\omega^2 + \omega + 2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= (3 + 2\omega) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \left(3 + 2\left(\frac{-1 + i\sqrt{3}}{2}\right)\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= (2 + i\sqrt{3}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

229. A

$[A(A + B)^{-1} B]^{-1}$
$= B^{-1}(A + B) A^{-1}$
$= (B^{-1}A + I) A^{-1} = B^{-1} + A^{-1}$

230. B

$|[x] - 2x| = 4 \Rightarrow |[x] - 2([x] + \{x\})| = 4$
$\Rightarrow |[x] + 2\{x\}| = 4$
Which is possible only when $2\{x\} = 0, 1$.
If $\{x\} = 0$, then $[x] = \pm 4$ and then $x = -4, 4$

and if $\{x\} = \frac{1}{2}$, then $[x] + 1 = \pm 4$

$\Rightarrow [x] = 3, -5 \therefore x = 3 + \frac{1}{2}$ and $-5 + \frac{1}{2}$

$\Rightarrow x = 7/2, -9/2$ Hence, $x = -4, -9/2, 7/2, 4$

231. B

$\because a_1, a_2, \ldots a_{21}$, are in AP

$\therefore a_1 + a_2 + \ldots + a_{21} = \frac{21}{2}(a_1 + a_{21})$

$\Rightarrow 693 = \frac{21}{2}(a_1 + a_{21})$ (given)

$\therefore a_1 + a_{21} = 66$ ....(i)

$\therefore \sum_{r=0}^{10} a_{2r+1} = a_1 + a_3 + a_5 + a_7 + a_9 + \ldots + a_{21}$
$= (a_1 + a_{21}) + (a_3 + a_{19}) + (a_5 + a_{17}) + (a_7 + a_{15}) + (a_9 + a_{13}) + a_{11}$
$= 5 \times (a_1 + a_{21}) + a_{11}$ $(\because T_n + T_n' = a + \ell)$
$= 5 \times 66 + a_{11} = 330 + a_{11}$

$= 330 + \left(\frac{a_1 + a_{21}}{2}\right)$ ($\because a_{11}$ is middle term)

$= 330 + 33 = 363$

232. C

$\log_2(a + b) + \log_2(c + d) \geq 4$
$\Rightarrow \log_2\{(a + b)(c + d)\} \geq 4$
$\Rightarrow (a + b)(c + d) \geq 2^4$
But AM $\geq$ GM

$\therefore \frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)} = 2^2$

$\therefore a + b + c + d \geq 8$

233. B

$R_2 \to R_2 - R_3$, $R_1 \to R_1 - R_2$

$$\begin{vmatrix} x & -y & 0 \\ 0 & y & -z \\ a & b & c+z \end{vmatrix} = 0$$

$\Rightarrow x(cy + yz + bz) + y(az) = 0$
$cxy + xyz + bzx + ayz = 0$
$cxy + bzx + ayz = 2007$

234. D

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \Sigma(1) & \Sigma r & \Sigma 2^r \\ 2 & n & n^2 \\ n & \frac{(n)(n+1)}{2} & 2^{n+1} \end{vmatrix}$$

$$= \begin{vmatrix} n & \frac{(n)(n+1)}{2} & 2^{n+1} - 2 \\ 2 & n & n^2 \\ n & \frac{(n)(n+1)}{2} & 2^{n+1} \end{vmatrix} = \begin{vmatrix} 0 & 0 & -2 \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix} = -2n$$

235. B

$\because$ A is orthogonal, $\therefore AA' = I \Rightarrow A^{-1} = A'$

236. C

By property, adj $A^T - (\text{adj } A)^T = O$ (null matrix)

237. C

Given that $16^{x^2+3x-1} = 8^{x^2+3x+2}$

$\Rightarrow 2^{4(x^2+3x-1)} = 2^{3(x^2+3x+2)}$
$\Rightarrow 4(x^2 + 3x - 1) = 3(x^2 + 3x + 2)$
$\Rightarrow x^2 + 3x - 10 = 0$
$\Rightarrow (x + 5)(x - 2) = 0 \Rightarrow x = -5, 2$
Sum of all values $= -5 + 2 = -3$

238. B

Sum of the roots $\alpha + \beta + \gamma = 0 \Rightarrow \gamma = 0$
$\therefore$ 0 is a root of the equation $\Rightarrow c - 1 = 0$
$\Rightarrow c = 1$

239. A

$\because$ p, q, r are in AP $\therefore 2q = p + r$ ...(i)

$\because$ Roots of $px^2 + qx + r = 0$ are all real, then $q^2 - 4pr \geq 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$ [from Eq. (i)]

$\Rightarrow (p + r)^2 - 16\ pr \geq 0 \Rightarrow p^2 + r^2 - 14pr \geq 0$

$\Rightarrow \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0$

$\Rightarrow \left(\frac{r}{p} - 7\right)^2 \geq 48 \Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

240. A

Let $S_n = Pn^2 + Qn$ = Sum of first n terms according to question, Sum of first 3n terms = sum of the next n terms

$\Rightarrow S_{3n} = S_{4n} - S_{3n}$ or $2S_{3n} = S_{4n}$

or $2[P(3n)^2 + Q(3n)] = P(4n)^2 + Q(4n)$

$\Rightarrow 2Pn^2 + 2Qn = 0$ or $Q = -nP$ ....(i)

Then $\frac{\text{Sum of first 2n terms}}{\text{Sum of next 2n terms}} = \frac{S_{2n}}{S_{4n} - S_{2n}}$

$= \frac{P(2n)^2 + Q(2n)}{[P(4n)^2 + Q(4n)] - [P(2n)^2 + Q(2n)]}$

$= \frac{2nP + Q}{6Pn + Q} = \frac{nP}{5nP} = \frac{1}{5}$ [from Eq. (i)]

241. B

$a + 5 = c + 2b$

$\frac{n^2 - n}{2} + n + 1 + 5 = \frac{n^2 - n}{2} + 2n + 2 \Rightarrow n = 4$

242. D

$\text{Det}(M_r) = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix} = 2r - 1$

$\sum_{r=1}^{2007} \det(M_r) = 2\sum_{r=1}^{2007} r - 2007$

$= 2\ \frac{(2007)(2008)}{2} - 2007 = (2007)^2$

243. D

$\because \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$

Then, X = y and Y = x ie, y = x

244. A

$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$

$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow \lambda = 3$

245. B

Let the roots of $ax^2 - 5x + 6 = 0$ be $2\alpha, 3\alpha$,

$5\alpha = \frac{5}{a}$ and $6\alpha^2 = \frac{6}{a} \Rightarrow \frac{6}{a^2} = \frac{6}{a} \Rightarrow a = 1$

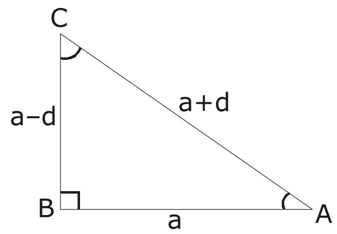
246. A

Let sides be a − d, a, a + d

$\therefore (a + d)^2 = (a - d)^2 + a^2$

$\Rightarrow 4ad = a^2 \ \therefore a = 4d$

Then sides are 3d, 4d and 5d



$\sin A = \frac{a-d}{a+d} = \frac{3d}{5d} = \frac{3}{5}$

and $\sin C = \frac{a}{a+d} = \frac{4d}{5d} = \frac{4}{5}$

247. C

$f_n(x) = x\,x\,x \ldots x(\text{n digits}) = x\left\{\frac{(10^n - 1)}{9}\right\}$

$\therefore f_n^2(3) = 3^2\left\{\frac{(10^n - 1)}{9}\right\}^2 = \frac{(10^n - 1)^2}{9}$

and $f_n(2) = \frac{2(10^n - 1)}{9}$

$\therefore f_n^2(3) + f_n(2) = \left(\frac{10^n - 1}{9}\right)(10^n - 1 + 2)$

$= \frac{(10^n - 1)(10^n + 1)}{9} = \frac{10^{2n} - 1}{9} = f_{2n}(1).$

248. A

$A^2 = -B^2 \Rightarrow A^2 = \begin{vmatrix} 2 & -2 \\ -2 & 2 \end{vmatrix} = -2B$

$A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$A^8 = 64B^2 = 128B$

249. A

$x_1, x_2, x_3 \ldots\ldots\ldots x_{20}$ are in HP

then $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3} \ldots\ldots\ldots \frac{1}{x_{20}}$ are in A.P.

Let $\left(\frac{1}{x_i} = a_i\right)$ then $a_1, a_2, a_3 \ldots a_{20}$ are in A.P.

$x_1x_2 + x_2x_3 + x_3x_4 + \ldots\ldots\ldots\ldots + x_{19}x_{20}$

$$= \frac{1}{a_1.a_2} + \frac{1}{a_2.a_3} + \ldots\ldots + \frac{1}{a_{19}.a_{20}}$$

$$= \frac{1}{d}\left[\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \ldots\ldots + \left(\frac{1}{a_{19}} - \frac{1}{a_{20}}\right)\right]$$

$$= \frac{1}{d}\left[\frac{a_{20} - a_1}{a_1a_{20}}\right] = \frac{1}{d}\left[\frac{a_1 + 19d - a_1}{a_1a_{20}}\right]$$

$$= \frac{19}{a_1a_{20}} = 19.\ x_1x_{20} = 19 \times 4 = 76$$

250. C

Let $\cot^{-1} x = \theta$ $\quad\therefore \cot\theta = x$

$$\Rightarrow \left(\frac{xi+1}{xi-1}\right) = \frac{i\cot\theta + 1}{i\cot\theta - 1} = \frac{\cot\theta - i}{\cot\theta + i}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos\theta + i\sin\theta} = \frac{e^{-i\theta}}{e^{i\theta}} = \frac{1}{e^{2i\theta}}.$$

$$\Rightarrow e^{2i\theta}\left(\frac{xi+1}{xi-1}\right) = 1 \Rightarrow e^{2\,mi\theta}\left(\frac{xi+1}{xi-1}\right)^m = 1$$

$$\therefore e^{2mi\cot^{-1}x}\left(\frac{xi+1}{xi-1}\right)^m = 1.$$

251. D

$|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$ If $z = x + iy$ then $|x + iy - ix| = |x + iy - y|$

$$\Rightarrow \sqrt{x^2 + (y-x)^2} = \sqrt{(x-y)^2 + y^2}$$

or $x^2 = y^2$ $\therefore x = \pm y \Rightarrow \text{Re}(z) = \pm \text{Im}(z)$

$\Rightarrow \text{Re}(z) + \text{Im}(z) = 0$ and $\text{Re}(z) - \text{Im}(z) = 0$

252. D

$|a^2 - 2a| < 3 \Rightarrow -3 < a^2 - 2a < 3$

$\Rightarrow -3 + 1 < a^2 - 2a + 1 < 3 + 1$

$\Rightarrow -2 < (a-1)^2 < 4$

$\therefore 0 \le (a-1)^2 < 4 \Rightarrow -2 < a - 1 < 2$

or $-1 < a < 3$ But $a \in R^+$

$\therefore 0 < a < 3 \Rightarrow a \in (0, 3)$.

253. D

$\because$ $xyz = 2^3 \times 3^1$

Let $\alpha + \beta + \gamma = 3,\ \alpha + \beta + \gamma = 1$

Number of integral positive solutions

$= {}^{3+3-1}C_{3-1} \times {}^{1+3-1}C_{3-1}$

$= {}^5C_2 \times {}^3C_2 = 30$

Since, negative values of x, y, z is also allowed but since product is positive and hence any two of them may be negative.

$\therefore$ Number of negative integral solutions $= {}^3C_2 \times 30 = 90$

Hence, total number of integral solutions of $xyz = 24$ is $30 + 90 = 120$

254. B

Let $a = 2x + 1,\ b = 2y + 1,\ c = 2z + 1$

where $x, y, z \in$ whole number

$\therefore$ $a + b + c = 13$

$\Rightarrow$ $2x + 1 + 2y + 1 + 2z + 1 = 13$

or $x + y + z = 5$

The number of integrals solutions

$$= {}^{5+3-1}C_{3-1} = {}^7C_2 = \frac{7.6}{1.2} = 21$$

255. B

$4m = 2^a + 3^b + 5^c = 2^a + (4-1)^b + (4+1)^c$

$= 4k + 2^a + (-1)^b + 1^c$

$\because$ $a = 1$, b = even, c = any number

OR

$a \ne 1$, b = odd, c any number.

Required number $1 \times 2 \times 5 + 4 \times 3 \times 5 = 70$

256. D

Let $\Sigma n = \lambda$ $\quad\ldots\ldots(i)$

$$\therefore \left(\frac{x^3 + 1 + x^6}{x^3}\right)^{\Sigma n} = \left[1 + \left(x^3 + \frac{1}{x^3}\right)\right]^{\lambda}$$

$$= 1 + {}^{\lambda}C_1\left(x^3 + \frac{1}{x^3}\right) + {}^{\lambda}C_2\left(x^3 + \frac{1}{x^3}\right)^2 + \ldots.$$

$$\ldots\ldots + {}^{\lambda}C_{\lambda}\left(x^3 + \frac{1}{x^3}\right)^{\lambda}$$

On expanding each term, two dissimilar terms are added in the expansion

$$\because \left(x^3 + \frac{1}{x^3}\right)^3 = x^9 + \frac{1}{x^9} + 3\left(x^3 + \frac{1}{x^3}\right)$$

Only x^9 and $\frac{1}{x^9}$ are new terms. Coefficient of x^3 and $\frac{1}{x^3}$

have occured earlier in ${}^{\lambda}C_1\left(x^3+\frac{1}{x^3}\right)$

Hence, number of terms
$= 1 + 2 + 2 + 2 + \ldots$ upto λ

$= 1 + 2\lambda = 1 + 2\,\Sigma n = 1 + 2.\,\frac{n(n+1)}{2}$

$= 1 + n + n^2$

257. C

$\because\ 5^{40} = (5^2)^{20} = (22+3)^{20} = 22\lambda + 3^{20},\ \lambda \in N$

Also, $3^{20} = (3^2)^{10} = (11-2)^{10} = 11\mu + 2^{10},\ \mu \in N$

Now, $2^{10} = 1024 = 11 \times 93 + 1$

$\therefore$ Remainder $= 1$ ie, $\alpha = 1$

Also, $2^{2003} = 2^3.\ 2^{2000} = 8\ (2^4)^{500} = 8\ (16)^{500}$
$= 8(17-1)^{500} = 8(17v + 1),\ v \in N$
$= 8 \times 17v + 8$

$\therefore$ Remainder $= 8$ ie, $\beta = 8$

$\Rightarrow \beta - \alpha = 8 - 1 = 7$

258. D

$$(r+1)^{th}\ \text{term} = T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^{r}$$

$$= {}^{10}C_r\left(\frac{x}{3}\right)^{5-\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{r}{2}} = {}^{10}C_r\,\frac{x^{5-\frac{r}{2}-r}}{3^{5-\frac{r}{2}-\frac{r}{2}}.2^{\frac{r}{2}}}$$

For independent of x, Put $5 - \frac{r}{2} - r = 0$

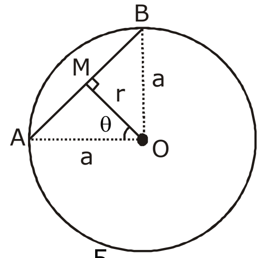
$\therefore 5 = \frac{3r}{2} \Rightarrow r = \frac{10}{2}$ impossible $\because r \neq$ whole number

259. A

Let a be the radius of the circle, ℓ be the length of the chord and r be the distance of the mid point of the chord from the centre of the circle.

Let $\angle$ AOM $= \theta$

$\therefore \sin\theta = \frac{AM}{a}$



and $\cos\theta = \frac{r}{a}$

$\Rightarrow r = a\cos\theta$,

$\ell = 2$ AM $= 2a\sin\theta$

Given, $(2a)\frac{2}{3} <$ AB $< (2a)\ \frac{5}{6}$

$\Rightarrow \frac{4a}{3} < \ell < \frac{10a}{6} \Rightarrow \frac{4a}{3} < 2a\sin\theta < \frac{10a}{6}$

$\Rightarrow \frac{2}{3} < \sin\theta < \frac{5}{6} \Rightarrow \frac{4}{9} < 1 - \cos^2\theta < \frac{25}{36}$

or $\frac{4}{9} - 1 < -\cos^2\theta < \frac{25}{36} - 1$

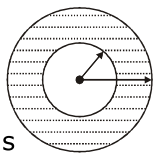
$\Rightarrow \frac{5}{9} > \cos^2\theta > \frac{11}{36}$

$\Rightarrow \frac{\sqrt{11}}{6}\ a < a\cos\theta < \frac{\sqrt{5}}{3}\ a$

or $\frac{\sqrt{11}a}{6} < r < \frac{\sqrt{5}a}{6}$

$\therefore$ The given condition is satisfied, if the mid point of the chord lies within the region between the concentric circles of radius

$\frac{\sqrt{11}a}{6}$ and $\frac{\sqrt{5}}{3}\ a$



Hence, the required probability

$$= \frac{\text{the area of the circular annulus}}{\text{area of the given circle}}$$

$$= \frac{\pi\left(\frac{5}{9}a^2 - \frac{11}{36}a^2\right)}{\pi a^2} = \frac{5}{9} - \frac{11}{36} = \frac{20-11}{36} = \frac{9}{36} = \frac{1}{4}$$

260. B

Let S be the sample space and E be the event of getting a large number than the previous number. $\therefore$ n(S) $= 6 \times 6 \times 6 = 216$

Now, we count the number of favourable ways. Obviously, the second number has to be greater than 1. If the second number is i (i > 1), then the number of favourable ways, $= (i-1) \times (6-i)$

$\therefore$ n(E) = Total number of favourable ways

$$= \sum_{i=1}^{6}(i-1)\times(6-i)$$

$= 0 + 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1 + 0 = 20$

Therefore, the required probability = $\dfrac{n(E)}{n(S)}$

$= \dfrac{20}{216} = \dfrac{5}{54}$

261. D

Even numbers are 2,4,6

$\therefore$ The probability that an even number appear = $\dfrac{3}{6} = \dfrac{1}{2}$ $\therefore$ The required probability

= P(that an even number occurs once or thrice or five times....or (2n + 1) times)

$$= {}^{2n+1}C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{2n} + {}^{2n+1}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{2n-2}$$

$$+ {}^{2n+1}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{2n-4} + \ldots +$$

$${}^{2n+1}C_{2n+1} \left(\frac{1}{2}\right)^{2n+1} \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^{2n+1}} \{{}^{2n+1}C_1 + {}^{2n+1}C_3 + {}^{2n+1}C_5 + \ldots + {}^{2n+1}C_{2n+1}\}$$

$$= \frac{2^{2n}}{2^{2n+1}} = \frac{1}{2}$$

262. C

Since $x_r = \cos\left(\dfrac{\pi}{3^r}\right) - i\sin\left(\dfrac{\pi}{3^r}\right)$

$$\therefore x_1 \cdot x_2 \cdot x_3 \ldots \infty = \cos\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots \infty\right)$$

$$- i\sin\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots \infty\right)$$

$$= \cos\left(\frac{\pi/3}{1-1/3}\right) - i\sin\left(\frac{\pi/3}{1-1/3}\right)$$

$= \cos(\pi/2) - i\sin(\pi/2) = -i$

263. B

$\arg\left(\dfrac{z-z_1}{z_2-z}\right) = \dfrac{\pi}{2}$ $\therefore z_1, z_2, z_3$ lie on a circle

$\Rightarrow z_1$ and z_2 are the end points of diameter

$\therefore$ center $(z_0) = \dfrac{z_1+z_2}{2}$

$\therefore z_0 = 5 - i$ and radius $= |z_1 - z_0| = |1 + 2i|$ $r = \sqrt{5}$

$\therefore$ Equation circle is $|z - z_0| = r$ or $|z - (5 - i)| = \sqrt{5}$

264. A

$\because$ $|z|$ = Real and positive, imaginary part is zero

$\therefore \arg|z| = 0 \Rightarrow [\arg|z|] = 0$

$$\therefore \int_{x=0}^{100} [\arg|z|]\,dx = \int_{x=0}^{100} 0\,dx = 0$$

265. D

Maximum number of points = ${}^8P_2 = 56$

266. B

$2^{10} - 1 = 1023$

267. A

8 type 7 type 2 or 4 or 6 or 8

Total number = ${}^8P_2 \times 4 = 224$

268. C

Coefficient of x^{11} in

$(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^3$

Coefficient of x^{11} in

$x^3 (1 + x + x^2 + x^3 + x^4 + x^5)^3$

or Coefficient of x^{11} in

$x^3 (1 - x^6)^3 (1 - x)^{-3}$

or Coefficient of x^8 in

$(1 - 3x^6)(1 + {}^3C_1 x + {}^4C_2 x^2 + \ldots)$

$= {}^{10}C_8 - 3.{}^4C_2$

$= {}^{10}C_2 - 3.{}^4C_2$

$= 45 - 18 = 27$

269. B

Here, 2n + 2 is even $\therefore$ Greatest coefficient

$${}^{2n+2}C_{\frac{2n+2}{2}} = {}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!} = \frac{(2n+2)!}{\{(n+1)!\}^2}$$

270. C

$(\sqrt{3} + \sqrt[3]{2})^9 = (3^{1/2} + 2^{1/3})^9$

$\therefore T_{r+1} = {}^9C_r (3^{1/2})^{9-r} (2^{1/3})^r = {}^9C_r\, 3^{(9-r)/2}\, 2^{r/3}$

For first integral term for r = 3

$T_{3+1} = {}^9C_3\, 3^2\, 2^1$ ie, $T_{3+1} = T_4$ (4th term)

271. C

$(2^{1/5} + 3^{1/10})^{55}$

Total terms = 55 + 1 = 56

$T_{r+1} = {}^{55}C_r 2^{(55-r)/5} 3^{r/10}$

Here, r = 0 , 10, 20, 30, 40, 50

Number of rational terms = 6

$\therefore$ Number of irrational terms = 56 – 6 = 50

272. D

100 bulbs = 10 defective + 90 non defective

Probability that 3 out of 4 bulbs, bought by a customer will not to be defective.

$$= \frac{{}^{90}C_3 \times {}^{10}C_1}{{}^{100}C_4}$$

273. D

Let A denotes the event that the throws of the two persons are unequal. Then A′ denote the event that the throws of the two persons are equal. The total number of cases for A′ is $(36)^2$. we now proceed to find out the number of favourable cases for A′. suppose

$(x + x^2 + x^3 + ...x^6)^2 = a_2x^2 + a_3x^3 + ...+a_{12}x^{12}$

The number of favourable ways for

$A' = a_2^2 + a_3^2 + ... + a_{12}^2$

= coefficient of constant term in

$$(a_2x^2+a_3x^3 + ...+a_{12}x^{12}) \times \left(\frac{a_2}{x^2} + \frac{a_3}{x^3} + ... + \frac{a_{12}}{x^{12}}\right)$$

= coefficient of constant term in

$$\frac{(1-x^6)^2(1-1/x^6)^2}{(1-x)^2(1-1/x)^2}$$

= coefficient of x^{10} in $(1 - x^6)^4 (1 - x)^{-4}$

= coefficient of x^{10} in

$(1 - 4x^6 + 6x^{12}) (1 + {}^4C_1x + {}^5C_2x^2 + {}^6C_3x^3+...)$

$= {}^{13}C_{10} - 4.\ {}^7C_4 = 146$

$$\therefore P(A') = \frac{146}{36^2} = \frac{73}{648} \Rightarrow P(A) = 1 - P(A') = \frac{575}{648}$$

274. A

A × B = (1, 3, 5, 7, 9) × (2, 4, 6, 8)

= (1, 2), (1, 4), (1, 6), (1, 8), (3, 2), (3, 4), (3, 6), (3, 8), (5, 2), (5, 4), (5, 6), (5, 8), (7, 2), (7, 4), (7, 6), (7, 8), (9, 2), (9, 4), (9, 6), (9, 8) Total ways = 5 × 4 = 20

Favourable case:(1, 8), (3, 6), (5, 4), (7, 2) ($\because$ a + b = 9)

$\therefore$ Number of favourable cases = 4

$$\therefore \text{Required probability} = \frac{4}{20} = \frac{1}{5}$$

275. D

$\because$ arg (z − 3i) = arg (x + iy − 3i) = $3\pi/4$

$$\Rightarrow x < 0, y - 3 > 0 \left(\because \frac{3\pi}{4} \text{is in II quadrant}\right)$$

$$\frac{y-3}{x} = \tan\frac{3\pi}{4} = -1 \Rightarrow y = -x + 3 \text{ .....(i)}$$

$\forall$ x < 0 and y > 3

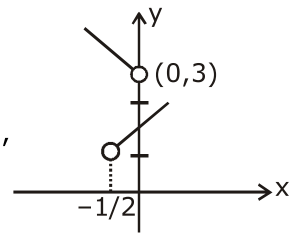
& arg (2z + 1 − 2i)=arg [(2x+1) + i (2y−2)=$\pi/4$

$\Rightarrow$ 2x + 1 > 0, 2y − 2 > 0 ($\because$ $\pi/4$ is in I quadrant)

$$\therefore \frac{2y-2}{2x+1} = \tan \pi/4 = 1 \Rightarrow 2y - 2 = 2x + 1$$

$\Rightarrow$ y = x + 3/2 $\forall$ x > −1/2, y > 1 ....(ii)

From Eqs. (i) and (ii),



it is clear from the graph No point of intersection

276. B

$\because |z_1 - z_2| = |z_1 - (z_2 - 3 - 4i) - (3 + 4i)|$
$\geq |z_1| - |z_2 - 3 - 4i| - |3 + 4i|$
$= 12 - 5 - 5 = 2 \Rightarrow |z_1 - z_2| \geq 2.$

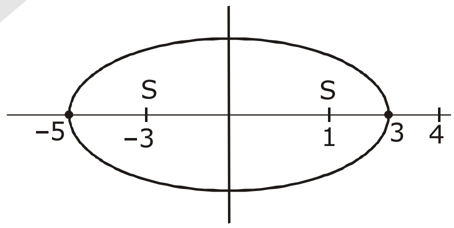
277. B

$|iz + z_1| = |i (z - i) + z_1 - 1|$
$\leq |i (z - i)| + |z_1 - 1|$
$= |z - i| + |z_1 - 1|$
$\leq 2 + |4 + 3i| = 2 + 5 \leq 7.$

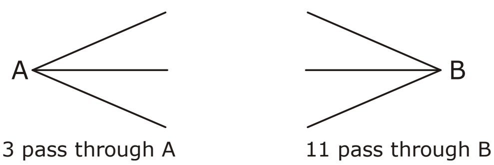
278. C

$\because |z - 1| + |z + 3| \leq 8$

$\because$ z lies inside or on the ellipse whose foci are (1, 0) and (−3, 0) and vertices are (−5, 0) and (3, 0) Now minimum and maximum value of |z − 4| are 1 and 9 respectively $\therefore |z - 4| \in [1, 9]$



279. A



$\therefore$ Number of intersection points

$= {}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2$

($\because$ two points A and B) = 535

280. **D**

Terminal digits are the first and last digits.

$\because$ Terminal digits are even

$\therefore$ Ist place can be filled in 3 ways and last place can be filled in 2 ways and remaining places can be filled in ${}^5P_4 = 120$ ways. Hence, the number of six digit numbers, the terminal digits are even, is $= 3 \times 120 \times 2 = 720$

281. **B**

${}^nC_4 = 70$

$n(n-1)(n-2)(n-3) = 1680 \Rightarrow n = 8$

Diagonals $= {}^nC_2 - n$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2} = \frac{8 \times 5}{2} = 20$$

282. **C**

$(bc + ca + ab)^{10}$

General term $= \dfrac{10!}{p!q!r!}(bc)^p(ca)^q(ab)^r$

$$= \frac{10!}{p!q!r!} a^{q+r}. b^{r+p}. c^{p+q}$$

Let $q + r = 10$, $r + p = 7$, $p + q = 3$

$\therefore p + q + r = 10 \quad \therefore p = 0, q = 3, r = 7$

$\therefore$ Coefficient of $a^{10} b^7 c^3$ is

$$\frac{10!}{0!3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{1 \cdot 1 \cdot 2 \cdot 3 \cdot 7!} = 120$$

283. **C**

Put $x = 1, \omega, \omega^2$ & add them

$3(a_0 + a_3 + \ldots) = 3^n \Rightarrow a_0 + a_3 + a_6 \ldots = 3^{n-1}$

284. **A**

$$\Sigma r. \frac{n-r+1}{r} = \Sigma (n+1) - r$$

$$= (n+1)\Sigma 1 - \Sigma r = (n+1). 10 - \frac{10.11}{2}$$

$= 10n + 10 - 55 = 10n - 45 = 5(2n - 9)$

285. **C**

$$P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})}$$

286. **D**

The total number of ways of choosing two numbers out of 1, 2, 3,...,30 is ${}^{30}C_2 = 435$

Since, $x^2 - y^2$ is divisible by 3 iff either a and b are divisible by 3 or none of a and b divisible by 3. Thus, the favourable number of cases ${}^{10}C_2 + {}^{20}C_2 = 235$

$\therefore$ Required probability $= \dfrac{235}{435} = \dfrac{47}{87}$

287. **C**

Total ways $= 6 \times 6 \times 6 = 216$

for increasing function $f'(x) \geq 0$

$\Rightarrow 3x^2 + 2ax + b \geq 0$

$\Rightarrow D \leq 0 \Rightarrow 4(a^2 - 3b) \leq 0 \Rightarrow a^2 \leq 3b$

favourable

a	b
1	1, 2, 3, 4, 5, 6
2	2, 3, 4, 5, 6
3	3, 4, 5, 6
4	6
5	–
6	–

Total favourable for (a, b) = 16

c can be any out of 1, 2, 3, 4, 5, 6

$$\therefore P = \frac{16 \times 6}{6 \times 6 \times 6} = \frac{4}{9}$$

288. **B**

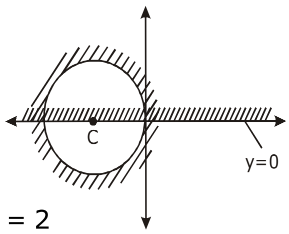
$|z|^2 + 7(\bar{z}) = 0 \Rightarrow x^2 + y^2 + 7(x - iy) = 0 + i0$

$\Rightarrow (x^2 + y^2 + 7x) - i(7y) = 0 + i0$

$\Rightarrow x^2 + y^2 + 7x = 0$ and $-7y = 0$

centre $= \left(\dfrac{-7}{2}, 0\right)$

radius $= \dfrac{7}{2}$



Circle and x-axis cut at 2 points

$\therefore$ no. of solution = 2

289. **D**

$(1 + i)^6 + (1 - i)^6$

$= 2\,[{}^6C_0 (i)^0 + {}^6C_2 (i)^2 + {}^6C_4 (i)^4\, {}^6C_4 (i)^6]$

$= 2\,[1 + (-15) + 15 - 1] = 0$

290. **C**

Let $x = (1)^{1/n} \Rightarrow x^n - 1 = 0$

or $x^n - 1 = (x - 1)(x - \omega)(x - \omega^2)\ldots(x - \omega^{n-1})$

$$\Rightarrow \frac{x^n - 1}{x - 1} = (x - \omega)(x - \omega^2)\ldots\ldots(x - \omega^{n-1})$$

Putting x = 9 in both sides, we have

$$(9 - \omega)(9 - \omega^2)(9 - \omega^3)\ldots(9 - \omega^{n-1}) = \frac{9^n - 1}{8}$$

291. A

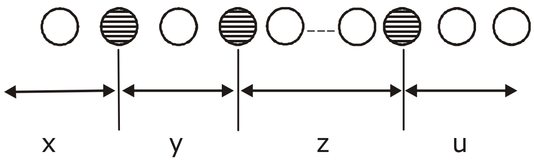
Given x + y + z + u + t = 20

x + y + z = 5

x + y + z = 5 and u + t = 15

Required number = ${}^{7}C_5 \times {}^{16}C_{15} = 336$

292. C



$\therefore$ x + y + z + u = n – 3; $y \geq 1$

$z \geq 1$

$x \geq 0$

$u \geq 0$

$\therefore$ x + y + z + u = n – 1 $\quad \therefore n = {}^{n-2}C_3$

293. C

Series is 1 + 2 + 6 + 12 + 36 + 72 +...20 term

(1+6+36+...10 terms) + (2 + 12 + 72...10 terms)

$$= 3\left(\frac{6^{10}-1}{6-1}\right) = \frac{3}{5}\left(6^{10}-1\right)$$

294. B

$$P = \frac{{}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + ...+ {}^{100}C_{49}}{{}^{100}C_0 + {}^{100}C_1 + ...+ {}^{100}C_{100}}$$

$$P = \frac{s}{(2)^{100}}$$

where $s = {}^{100}C_1 + {}^{100}C_3 + ...+ {}^{100}C_{49}$

But $({}^{100}C_1 + {}^{100}C_3 + ...+ {}^{100}C_{49})$

$$+({}^{100}C_{51} + {}^{100}C_{53} + ...+ {}^{100}C_{99}) = \frac{2^{100}}{2} = 2^{99}$$

$s + s = 2^{99}$

$$s = 2^{98} \Rightarrow P = \frac{2^{98}}{2^{100}} = \frac{1}{4}$$

295. A

1st	2nd	3ed	4th	5th	6th	.......∞
$\bar{1}$	1					
$\bar{1}$	$\bar{1}$	$\bar{1}$	1			
$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$	1	

....

....

....

∞

$$P = \left(\frac{5}{6}\cdot\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + ...\infty$$

$$= \frac{\frac{5}{36}}{1-\frac{25}{36}} = \frac{5}{11}$$

296. B

reflexive and symmetric

297. C

$$\text{Mean Height} = \frac{144+153+150+158+155}{5}$$

$$= \frac{760}{5} = 152 \text{ cm.}$$

298. B

The given frequency distribution is-

x_i	f_i	$\Sigma f_i x_i$
4	7	28
7	10	70
10	15	150
13	20	260
16	25	400
19	30	570
Σf_i = 107		$\Sigma f_i x_i$ = 1478

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1478}{107} = 13.81$$

299. C

$\sim(\sim p \vee q) = \sim(\sim p) \wedge \sim q = p \wedge \sim q$

300. C

p : we control population growth

q : we prosper

So, negative of $(p \to q)$ is $\sim(p \to q) \equiv p \wedge \sim q$

301. C

Since n(A) = 3

$\therefore$ number of subsets of A is $2^3 = 8$

302. C

Here the numbers are 1, 2, 3,....., n and their weights also are respectively 1, 2, 3.....n so

$$\text{weighted mean} = \frac{\sum wx}{\sum w}$$

$$= \frac{1.1+2.2+3.3+......+n.n}{1+2+3+.....+n}$$

$$= \frac{1^2+2^2+3^2+.....n^2}{1+2+3+.....+n}$$

$$= \frac{n(n+1)(2n+1)}{6} \times \frac{2}{n(n+1)} = \frac{2n+1}{3}$$

303. C

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} \because \bar{x}_1 = 400, \bar{x}_2 = 480, \bar{x} = 430$$

$$\therefore \quad 430 = \frac{n_1(400)+n_2(480)}{n_1+n_2}$$

$$\Rightarrow 30n_1 = 50n_2 \Rightarrow \frac{n_1}{n_2} = \frac{5}{3}$$

304. B

Because $(p \Rightarrow q) \equiv (\sim p \vee q)$

305. A

p	~p	q	$(\sim p \vee q)$	$p \Rightarrow (\sim p \vee q)$
T	F	F	F	F

306. D

We have $P(\phi) = \{\phi\} \therefore P(P(\phi)) = \{\phi, \{\phi\}\}$

$\Rightarrow P[P(P(\phi))] = \{ \phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$.

Hence, $n\{P[P(\phi)]\} = 4$

307. B

$$\bar{x} = \frac{x_1+x_2+.......+x_n}{n} = \frac{\sum x_i}{n} \quad \therefore \sum x_i = n\bar{x}$$

$$\text{New mean} = \frac{\sum(x_i+\lambda)}{n} = \frac{\sum x_i + n\lambda}{n} = \bar{x} + \lambda$$

308. B

Mean $\bar{x} = \frac{\sum x}{n}$ or $\sum x = n\bar{x}$

$\sum x = 25 \times 78.4 = 1960$ But this $\sum x$ is incorrect as 96 was misread as 69.

$\therefore$ correct $\sum x = 1960 + (96 - 69) = 1987$

$\therefore$ correct mean $= \frac{1987}{25} = 79.48$

309. C

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge (q \wedge r)$
T	T	T	T	T	T

310. A

p	q	r	~p	~r	$(\sim p \vee q)$	$(\sim p \vee q) \wedge \sim r$	$(\sim p \vee q) \wedge \sim r \Rightarrow p$
T	F	T	F	F	F	F	T

311. C

$A \cup B = \{x : x \text{ is an odd integer}\}$
$\cup \{x : x \text{ is an even integer}\}$
$= \{x : x \text{ is an integer}\} = Z$

312. A

We have $\bar{x} = \frac{x_1+x_2+....+x_n}{n}$ Let $\bar{x}'$ be the mean of x_1 + a, x_2 + a,...., x_n + a then

$$\bar{x}' = \frac{(x_1+a)+(x_2+a)+....+(x_n+c)}{n}$$

$$= \frac{(x_1+x_2+....+x_n)....+na}{n}$$

$$= \frac{x_1+x_2+....+x_n}{n} + a = \bar{x} + a$$

313. C

Let the assumed mean be, A = 900.The given data can be written as under :

Wage (in Rs.) x_i	No. of workers f_i	$d_i = x_i - A = x_i - 900$	$u_i = \frac{x_i - 900}{20}$	f_iu_i
800	7	– 100	– 5	– 35
820	14	– 80	– 4	– 56
860	19	– 40	– 2	– 38
900	25	0	0	0
920	20	20	1	20
980	10	80	4	40
1000	5	100	5	25

$\mathbf{N} = \sum f_i = 100$ $\qquad \sum f_i u_i = -44$

Here A = 900, h = 20

$$\therefore \text{Mean} = \bar{X} = A + h\left(\frac{1}{N}\sum f_i u_i\right)$$

$= 900 + 20\left(-\frac{44}{100}\right) = 891.2$

Hence, mean wage = Rs. 891.2.

314. A

use the property~(p and q)=~p or ~q

3 is not an odd number or 7 is not a rational number.

315. B

Use the property $\sim(a \wedge b) = \sim a \vee \sim b$

$(p \wedge \sim q) \vee (p \vee q)$

316. D

We have, $x \in A \cap B \Leftrightarrow x = 3n, n \in Z$ and $x = 4n, n \in Z$

$\Leftrightarrow$ x is a multiple of 3 and x is a multiple of 4

$\Leftrightarrow$ x is a multiple of 3 and 4 both

$\Leftrightarrow$ x is a multiple of 12 $\Leftrightarrow x = 12n, n \in Z$

Hence $A \cap B = \{x : x = 12n, n \in Z\}$

317. C

Geometric mean of number

$7, 7^2, 7^3, \ldots, 7^n = (7.7^2.7^3 \ldots\ldots 7^n)^{1/n}$

$= (7^{1+2+3+\ldots+n})^{1/n}$

$$= \left[7^{\frac{n(n+1)}{2}}\right]^{1/n} = 7^{\frac{(n+1)}{2}}$$

318. B

The harmonic mean of 2, 4 and 5 is

$$\frac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{5}} = \frac{60}{19} = 3.16$$

319. C

use the property $\sim(a \vee b) = \sim a \wedge \sim b$

$(\sim p \vee \sim q) \wedge (\sim q \wedge r)$

320. A

p	q	~q	$p \wedge \sim q$	$(p \wedge \sim q) \vee p$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	F	F

Hence statement $(p \wedge \sim q) \vee p$ is logically equal to statement $p \Rightarrow p$

321. B

We have, $n(A \cup B)=n(A)+n(B) - n(A \cap B)$

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case-I: When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$ This is possible only when $A \cap B = \phi$. In this case, $n(A \cup B) = n(A)+n(B) - 0 = n(A) + n(B) = 3 + 6=9$.

So, maximum number of elements in $A \cup B$ is 9

Case-II: When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$.

In this case, $n(A \cap B) = 3$

$\therefore n(A \cup B)=n(A)+n(B)-n(A \cap B)=(3 + 6 - 3)=6$

So, minimum number of elements in $A \cup B$ is 6

322. B

Let us arrange the value in ascending order 0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$\therefore$ Median M $= \left(\frac{n+1}{2}\right)^{th}$ value

$= \left(\frac{11+1}{2}\right)^{th}$ value $= 6^{th}$ value

Now 6^{th} value in data is 27

$\therefore$ Median = 27 runs.

323. B

x	f	c.f.
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120

$N = 120 = \sum f_i$

$\therefore \frac{N}{2} = 60$ We find that the c.f. just greater than $\frac{N}{2}$ is 65 and the value of x corresponding to 65 is 5, therefore median is 5.

324. C

Let p be true then statement $(p \wedge \sim q) \vee (q \wedge r) = (T \wedge F) \vee (T \wedge T) = F \vee T = T$. also let p be false then statement $(p \wedge \sim q) \vee (q \wedge r) = (F \wedge F) \vee (T \wedge T) = F \vee T = T$. $\therefore$ p may be true or false.

325. B

use the property $\sim (p \to q) = p \wedge \sim q$ Hence (B) is correct option.

326. C

Since $n(A)=m$; $n(B) = n$ then $n(A \times B) = mn$

So number of subsets of $A \times B = 2^{mn}$

$\Rightarrow n(P(A \times B)) = 2^{mn}$

327. D

Class	Frequency	Cumulative frequency
5 - 10	5	5
10 - 15	6	11
15 - 20	15	26
20 - 25	10	36
25 - 30	5	41
30 - 35	4	45
35 - 40	2	47
40 - 45	2	49
	N = 49	

We have N = 49. $\therefore \frac{N}{2} = \frac{49}{2} = 24.5$

The cumulative frequency just greater than N/2, is 26 and the corresponding class is 15-20. Thus 15-20 is the median class such that $\ell=15$, $f=15$, $F=11$, $h=5$.

$\therefore$ Median $= \ell + \frac{N/2 - F}{f} \times h$

$= 15 + \frac{24.5 - 11}{15} \times 5 = 15 + \frac{13.5}{3} = 19.5$

328. C

Since 5 is repeated maximum number of times, therefore mode of the given data is 5.

329. A

$F \to (T \wedge F) \vee (F \wedge T)$ $F \to (F \vee F)$

$F \to F = T$ $\therefore$ True

330. C

use the property $\sim (a \to b) = a \wedge \sim b$. Hence (C) is correct option

331. C

We have $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

On replacing C by B and D by A, we get

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

It is given that AB has n elements so

$(A \cap B) \times (B \cap A)$ has n^2 elements

But $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

$\therefore$ $(A \times B) \cap (B \times A)$ has n^2 elements

Hence $A \times B$ and $B \times A$ have n^2 elements in common.

332. B

Mode = 3 Median – 2 mean

$\therefore$ Median $= \frac{1}{3}$(mode + 2 mean)

$= \frac{1}{3}(60 + 2 \times 66) = 64$

333. C

Arranging the observations in ascending order of magnitude, we have 150, 210, 240, 300, 310, 320, 340. Clearly, the middle observation is 300. So, median = 300 Calculation of Mean deviation

x_i	$\lvert d_i \rvert = \lvert x_i - 300 \rvert$
340	40
150	150
210	90
240	60
300	0
310	10
320	20
Total	$\sum \lvert d_i \rvert = \sum \lvert x_i - 300 \rvert$ = **370**

Mean deviation $= \frac{1}{n} \sum |d_i| = \frac{1}{7} \sum |x_i - 300|$

$= \frac{370}{7} = 52.8$

334. A

$p \vee (\sim p) \equiv t$

So (1) is incorrect.

335. A

$\sim(p \to q) = p \wedge \sim q$, $p \to q = \sim p \vee q$. Hence (A) is correct option

336. C

We have, $x + 3y = 12 \Rightarrow x = 12 - 3y$

Putting y = 1, 2, 3, we get x = 9, 6, 3 respectively For y = 4, we get $x = 0 \notin N$.

Also for $y > 4$, $x \notin N$

$\therefore R = \{(9, 1), (6, 2), (3, 3)\}$

Domain of R = {9, 6, 3}

337. B

Calculation of mean deviation about mean.

x_i	f_i	f_ix_i	$\|x_i - 15\|$	$f_i\|x_i-15\|$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60

$\mathbf{N = \sum f_i = 44}$ $\mathbf{\sum f_i x_i = 660}$ $\mathbf{\sum f_i |x_i - 15| = 312}$

$$\text{Mean} = \bar{X} = \frac{1}{N}\left(\sum f_i x_i\right) = \frac{660}{44} = 15$$

$$\text{Mean deviation} = \text{M.D.} = \frac{1}{N}\sum f_i |x_i - 15|$$

$$= \frac{312}{44} = 7.09.$$

338. B

$$\bar{x} = \frac{8+12+13+15+22}{5} = 14$$

CALCULATION OF VARIANCE

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
8	– 6	36
12	– 2	4
13	– 1	1
15	1	1
22	8	64
		$\mathbf{\sum(x_i - \bar{x})^2 = 106}$

$\because$ n = 5, $\sum(x_i - \bar{x})^2 = 106$

$$\therefore \text{var}(x) = \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{106}{5} = 21.2$$

339. C

We know that $\sim(p \Leftrightarrow q)$

$= (p \wedge \sim q) \vee (\sim p \wedge q)$ $(p \Leftrightarrow q)$

$= (\sim p \vee q) \wedge (p \vee \sim q)$

Hence (C) is correct option

340. C

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \Leftrightarrow \sim p$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	F
F	F	F	T	F

Hence the statement neither tautology nor contradiction.

341. B

We have $(x, y) \in R \Leftrightarrow x + 2y = 10$

$\Leftrightarrow y =$, $x, y \in A$ where A

= {1,2,3,4,5,6,7,8,9,10}

Now, $x = 1 \Rightarrow y =$ $\notin A$. This shows that 1 is not related to any element in A. Similarly we can observe that 3, 5, 7, 9 and 10 are not related to any element of a under the defined relation. Further we find that

for x = 2, y = = 4 $\in A$ $\therefore (2, 4) \in R$

for x = 4, y = = 3 $\in A$ $\therefore (4, 3) \in R$

for x = 6, y = = 2 $\in A$ $\therefore (6, 2) \in R$

for x = 8, y = = 1 $\in A$ $\therefore (8, 1) \in R$

Thus R = {(2, 4), (4, 3), (6, 2), (8, 1)}

$\Rightarrow R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$

Clearly, Dom (R)={2, 4, 6, 8}=Range (R^{-1}) and Range (R) = {4, 3, 2, 1} = Dom (R^{-1})

342. C

Let the assumed mean be A = 6.5

Calculation of variance

size of item x_i	f_i	$d_i = x_i - 6.5$	d_i^2	f_id_i	$f_id_i^2$
3.5	3	–3	9	–9	27
4.5	7	–2	4	–14	28
5.5	22	–1	1	–22	22
6.5	60	0	0	0	0
7.5	85	1	1	85	85
8.5	32	2	4	64	128
9.5	8	3	9	24	72

$\mathbf{N = \sum f_i = 217}$ $\mathbf{\sum f_i d_i = 128}$ $\mathbf{\sum f_i d_i^2 = 362}$

Here, N =217, $\sum f_i d_i = 128$ and $\sum f_i d_i^2 = 362$

$$\therefore \text{Var }(X) = \left(\frac{1}{N}\sum f_i d_i^2\right) - \left(\frac{1}{N}\sum f_i d_i\right)^2$$

$$= \frac{362}{217} - \left(\frac{128}{217}\right)^2 = 1.668 - 0.347 = 1.321$$

343. D

Here np = 6, npq = 4

$\Rightarrow q = \frac{2}{3}, \; p = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ np = 6 $\Rightarrow$ n = 18

344. A

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Hence the statement is a tautology

.

345. B

p	q	$\sim q$	$p \to \sim q$	$p \wedge q$	$(p \to \sim q) \Leftrightarrow p \wedge q$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

Hence the statement is a contradiction

346. D

Given equation $x^2 - bx + c = 0$

Let α, β, be two root's such that $|\alpha - \beta| = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \Rightarrow b^2 - 4c = 1$

Statement-II given equation abc

$x^2 + (b^2 - 4ac)\, x - b = 0$

$D = (b^2 - 4ac)^2 + 16b^2ac$

$D = (b^2 + 4ac)^2 > 0$

Hence root's are real and unequal.

347. A

$x^2 + 9y^2 + 25z^2 - 15yz - 5xz - 3xy = 0$

$\Rightarrow 2x^2 + 18y^2 + 50z^2 - 30yz - 10xz - 6xy = 0$

$\Rightarrow (x - 3y)^2 + (3y - 5z)^2 + (5z - x)^2 = 0$

$\therefore x = 3y = 5z = k$ (say)

$\Rightarrow x = k,\; y = \frac{k}{3},\; z = \frac{k}{5} \Rightarrow k, \frac{k}{3}, \frac{k}{5}$ are in H.P.

$\Rightarrow$ x,y,z are in H.P.

348. A

Exponent of 5

$$= \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] = 20 + 4 = 24$$

Both correct & correct explanation

349. C

$$\Sigma\, {}^nC_r\, x^r = (1 + x)^n \Rightarrow \Sigma\, {}^nC_r \frac{x^{r+1}}{r+1} = \frac{(1+x)^{n+1} - 1}{(n+1)}$$

(I) is true & (II) is false

350. B

The equation can be written as

$(2^x)^2 - (a - 3)2^x + (a - 4) = 0$

$\Rightarrow$ $2^x = 1$ and $2^x = a - 4$

We have,

$x \le 0$ and $2^x = a - 4$ [$\because$ x is non-positive]

$\therefore$ $0 < a - 4 \le 1 \Rightarrow 4 < a \le 5 \;\therefore\; a \in (4, 5]$

351. A

$\arg(z_1 z_2) = 2\pi \Rightarrow \arg(z_1) + \arg(z_2) = 2\pi$

$\Rightarrow \arg(z_1) = \arg(z_2) = \pi$, as principal arguments are from $-\pi$ to π.

Hence both the complex numbers are purely real. Hence both the statements are true and statement 2 is correct explanation of statement 1.

352. C

$$|z_1 + z_2| = \left|\frac{z_1 + z_2}{z_1 z_2}\right|$$

$$\Rightarrow |z_1 + z_2|\left(1 - \frac{1}{|z_1 z_2|}\right) = 0 \Rightarrow |z_1 z_2| = 1$$

Hence, statement 1 is unimodular.

However, it is not necessary that $|z_1| = |z_2| = 1$. Hence, statement 2 is false.

353. D

Sum $= \frac{x/r}{1-r} = 4$ (where r is common ratio)

$x = 4r(1 - r) = 4(r - r^2)$

For $r \in (-1, 1) - \{0\}$

$r - r^2 \in \left(-2, \frac{1}{4}\right) - \{0\} \Rightarrow x$ d $(-8, 1) - \{0\}$

354. A

Statement 2 is ture as

$$\frac{a^n + b^n}{a+b} = \frac{a^n - (-b)^n}{a-(-b)}$$

$= a^{n-1} - a^{n-2} b + a^{n-3}b^2 - .....(-1)^{n-1} b^{n-1}$

Now,

$1^{99} + 2^{99} + ... + 100^{99} = (1^{99} + 100^{99})$
$+ (2^{99} + 99^{99}) + ... + (50^{99} + 51^{99})$

Each bracket is divisible by 101 ; hence the sum is divisible by 101. Also,

$1^{99} + 2^{99} + L ...... 100^{99} = (1^{99} + 100^{99})$
$+ (2^{99} + 98^{99}) + ... + (49^{99} + 51^{99})$
$+ 50^{99} + 100^{99}$

Here, each bracket and 50^{99} and 100^{99} are divisible by $101 \times 100 = 10100$.

355. D

Number of ways of arranging 21 identical objects when r is identical of one type and remaining are identical of second type is

$\frac{21!}{r!(21-r)!} = {}^{21}C_r$ which is maximum when

r=10 or 11. Therefore, ${}^{13}C_r = {}^{13}C_{10}$ or ${}^{13}C_{11}$, hence maximum value of ${}^{13}C_r$ is ${}^{13}C_{10} = 286$. Hence, statement 1 is false, Obviously statement 2 is true.

356. C

Number of objects from 21 different objects	Number of objects from 21 identical objects	Number of ways of selections
10	0	${}^{21}C_{10} \times 1$
9	1	${}^{21}C_{9} \times 1$
⋮	⋮	⋮
0	10	${}^{21}C_{0} \times 1$

Thus, total number of ways of selection is ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} = 2^{20}$. Statement 2 is false, as gien series is not exact half series. (for details, see the theory in binomial theorem.)

357. B

$$\frac{T_{r+1}}{T_r} = \frac{12 - r + 1}{r} \ \frac{11}{10}$$

Let, $T_{r+1} \geq T_r \Rightarrow 13 - r \geq 1.1x$

$\Rightarrow 13 \geq 2.1r \quad \Rightarrow r \leq 6.19$

358. A

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0 \quad \text{(i)}$$

Then A = B or B = C or C = A, for which any two rows are same. For (1) to hold it is not necessary that all the three rows are same or A = B = C.

359. A

Statement 1 is true as $|A| = 0$.

Since $|B| \neq 0$, statement 2 is also true and correct explanation of statement 1.

360. C

S : 1 required no of groups

(1,2,3,4) ....(17,18,19,20) = 17 ways

(1,3,5,7)...........(14,16,18,20) = 14 ways

(1,4,7,10)...........(11,14,17,20) = 11 ways

(1,5,9,13)...........(8,12,16,20) = 8 ways

(1,6,11,16)...........(15,10,15,20) = 5 ways

(1,7,13,19)...........(2,8,14,20) = 2 ways

required arability

$$= \frac{(17+14+11+8+5+2)4!}{{}^{20}C_4 4!}$$

$$= \frac{57\ 4!}{20.19.18.17} = \frac{3.4.3.2.1}{20.18.17.}$$

$$= \frac{1}{85}$$

S : 1 is true.

S : 2

possible cases of common difference are $[\pm 1, +2, \pm 3, \pm 4, \pm 5, \pm 6]$

S : 2 is false

361. D

The image of P(a, b) on y=−x is Q(−b, −a) (interchange and change signs) and the image of Q(−b, −a) on y = x is R(−a, −b) (merely interchange)

$\therefore$ The mid point of PR is (0, 0).

362. A

As (−1, 1) is a point on 3x − 4y + 7 = 0, the rotation is possible.

Slope of the given line = $\frac{3}{4}$.

Slope of the line in its new position$=\dfrac{\frac{3}{4}-1}{1+\frac{3}{4}}=-\dfrac{1}{7}$

The required equation is $y-1=-\frac{1}{7}(x+1)$

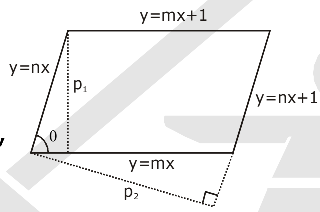
or 7y + x − 6 = 0

363. D

If p_1 and p_2 be the distance between parallel sides and θ be the angle between adjacent sides, than Required area

$= p_1 p_2 \operatorname{cosec}\theta$

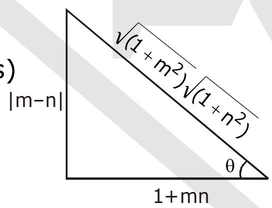
Where,



$p_1 = \dfrac{1}{\sqrt{(1+m^2)}}$,

$p_2 = \dfrac{1}{\sqrt{(1+n^2)}}$

(distance between || lines)



and $\tan\theta = \dfrac{|m-n|}{|1+mn|}$

$\therefore$ Required area

$$=\frac{1}{\sqrt{(1+m^2)}\sqrt{(1+n^2)}}\cdot\frac{\sqrt{(1+m^2)}\sqrt{(1+n^2)}}{|m=n|}=\frac{1}{|m-n|}$$

364. B

Equation of line $\frac{ax}{c-1}$ and $\frac{by}{c-1}+1=0$ has two independent parameters. It can pass through a fixed point if it contains only one independent parameter. Now there must be one relation between $\frac{a}{c-1}$ and $\frac{b}{c-1}$ independent of a,b and c so that $\frac{a}{c-1}$ can be expressed in terms of $\frac{b}{c-1}$ and straight line contains only one independent parameter. Now that given relation can be expressed as

$$\frac{5a}{c-1}+\frac{4b}{c-1}=\frac{t-20c}{c-1}$$

RHS is independent of c if t = 20.

365. C

$$\frac{x-2}{\cos 45^\circ}=\frac{y-1}{\sin 45^\circ}=-4$$

$\Rightarrow x-2=-2\sqrt{2},\ y-1=-2\sqrt{2}$

$\therefore\ x=2-2\sqrt{2},\ y=1-2\sqrt{2}$

$\Rightarrow\ A'=(2-2\sqrt{2},\ 1-2\sqrt{2})$

366. C

$\because$ Q lies on y-axis,

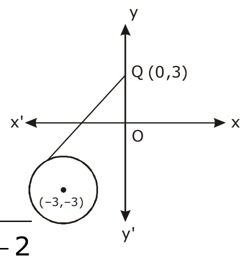
Put x = 0

in 5x − 2y + 6 = 0

$\therefore$ y = 3

$\Rightarrow$ Q(0, 3)

$\therefore$ PQ



$=\sqrt{0^2+3^2+0+6\times 3-2}$

$=\sqrt{25}=5$

367. C

The centre of the circle is the point of intersection of the given diameters 2x−3y=5 and 3x − 4y = 7. Which is (1, −1) and the radius is r, where $\pi r^2 = 154 \Rightarrow r^2$

$=154\times\frac{7}{22}\Rightarrow r=7$ and hence the required equation of the circle is

$(x-1)^2+(y+1)^2=7^2$

$\Rightarrow\ x^2+y^2-2x+2y=47$

368. D

Equation of pair of tangents by

$SS_1 = T^2$ is $(a^2-1)y^2-x^2+2ax-a^2=0$

If θ be the angle between the tangents, then

$$\tan\theta=\frac{2\sqrt{(h^2-ab)}}{a+b}=\frac{2\sqrt{-(a^2-1)(-1)}}{a^2-2}=\frac{2\sqrt{a^2-1}}{a^2-2}$$

$\because$ θ lies in II quadrant, then $\tan\theta<0$

$\therefore\ \dfrac{2\sqrt{a^2-1}}{a^2-2}<0$

$\Rightarrow a^2 - 1 > 0$ and $a^2 - 2 < 0 \Rightarrow 1 < a^2 < 2$

$\Rightarrow a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$

369. A

Given circle is $(x-3)^2 + (y-4)^2 = 25$

Since, locus of point of intersection of two perpendicular tangents is director circle, then its equation is

$(x-3)^2 + (y-4)^2 = 50$

$\Rightarrow x^2 + y^2 - 6x - 8y - 25 = 0$

370. B

Let $x_1 = \sqrt{3h+2}$ and $y_1 = \sqrt{3k}$

$\therefore \quad x_1^2 + y_1^2 = 3(h+k) + 2 = 3(1) + 2$

$(\because h + k = 1)$

$\Rightarrow x_1^2 + y_1^2 = 5$ Locus of (x_1, y_1) is $x^2 + y^2 = 5$

371. B

Shortest distance between two curves occured along the common normal.

Normal to $y^2 = 4x$ at $(m^2, 2m)$ is

$y + mx - 2m - m^3 = 0$

Normal to $y^2 = 2(x-3)$ at $\left(\frac{m^2}{2} + 3, m\right)$ is

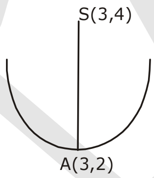
$y + m(x-3) - m - \frac{m^3}{2} = 0$

Both are same if $-2m - m^3 = -4m - \frac{1}{2}m^3$

$\Rightarrow m = 0, \pm 2$ So, points will be (4, 4) and (5, 2) or (4, –4) and (5, –2)

Hence, shortest distance will be $\sqrt{(1+4)} = \sqrt{5}$

372. A



$y^2 - 12x - 4y + 4 = 0$

$\Rightarrow (y-2)^2 = 12x$

It vertex is (0, 2) and a = 3,

its focus = (3, 2)

Hence, for the required parabola ;

focus is (3, 4) vertex = (3, 2) and a = 2

Hence, the equation of the parabola is

$(x-3)^2 = 4(2)(y-2)$ or $x^2 - 6x - 8y + 25 = 0$

373. B

Chord of contact of mutually perpendicular tangents is always a focal chord.

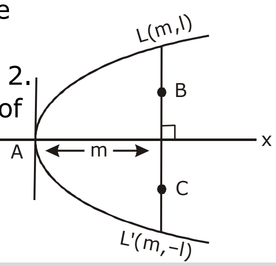
Therefore minimum length of AB is 4a

374. B

Let B and C be the trisected points.

B divides LL' in 1 : 2.

Then, coordinate of



B $\left(\frac{1.m + 2.m}{1+2}, \frac{-\ell + 2\ell}{3}\right)$

or B $(m, \ell/3)$

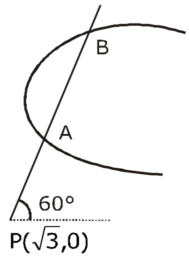
Let $x_1 = m$, $y_1 = \ell/3$

$\therefore \; (m, \ell) = (x_1, 3y_1)$

but (m, ℓ) lie on parabola, $(3y_1)^2 = 4ax_1$

$\Rightarrow 9y_1^2 = 4ax_1 \quad \therefore$ Locus $9y^2 = 4ax$

375. A



$\because P \equiv (\sqrt{3}, 0)$

$\frac{x - \sqrt{3}}{\cos 60^\circ} = \frac{y - 0}{\sin 60^\circ} = r$

or $x = \sqrt{3} + \frac{r}{2}$, $y = \frac{r\sqrt{3}}{2}$

or $\left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ lie on $y^2 = x + 2$, then

$\frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2 \Rightarrow \frac{3r^2}{4} - \frac{r}{2} - (2 + \sqrt{3}) = 0$

$\therefore PA.\,PB = r_1 r_2 = \left|\frac{-(2+\sqrt{3})}{\frac{3}{4}}\right| = \frac{4}{3}(2 + \sqrt{3})$

376. C

The tangent at the point of shortest distance from the line x + y = 7 parallel to the given line. Any point on the given ellipse is $(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta)$.

Equation of the tangent is

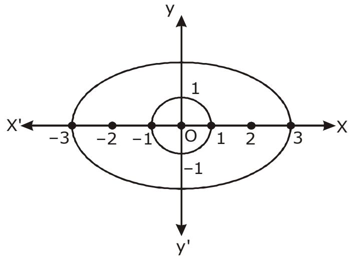
$\frac{x\cos\theta}{\sqrt{6}} + \frac{y\sin\theta}{\sqrt{3}} = 1$. it is parallel to x+y=7

$\Rightarrow \frac{\cos\theta}{\sqrt{6}} = \frac{\sin\theta}{\sqrt{3}} \Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{\sin\theta}{1} = \frac{1}{\sqrt{3}}$

The required point is (2, 1).

377. A

Centre of the ellipse is (1, 2) and length of major axis and minor axis are 6 and 4 respectively and centre and radius of the circle are (1, 2) and 1 respectively.



Hence, ellipse and circle do not touch or cut. $\therefore$ Common chord impossible.

$\therefore$ Hence, length of common chord = 0

378. C

$\because$ Latusrectum $= \dfrac{1}{2}$ (minor axis)

$\Rightarrow \dfrac{2b^2}{a} = \dfrac{1}{2}(2b) \Rightarrow 2b = a \Rightarrow 4b^2 = a^2$

$\Rightarrow 4a^2(1 - e^2) = a^2 \Rightarrow 4 - 4e^2 = 1$

$\therefore e^2 = \dfrac{3}{4} \Rightarrow e = \dfrac{\sqrt{3}}{2}$

379. A

Let mid point of focal chord is (x_1, y_1) then equation of a chord whose mid point (x_1, y_1) is $T = S_1$

$\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} - 1 = \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} - 1$

$\Rightarrow \dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2}$

Since, it is a focal chord, then its passes through focus $(\pm ae, 0)$, then

$\pm \dfrac{ex_1}{a} + 0 = \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2}$

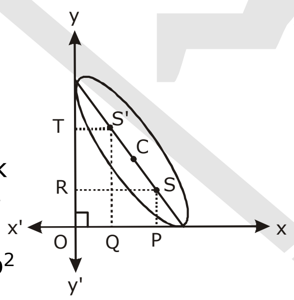
$\therefore$ Locus of mid point of focal chord is

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \pm \dfrac{ex}{a}$

380. D

Let $S \equiv (x_1, y_1)$, $S' \equiv (x_2, y_2)$

Let $C \equiv (h, k)$



$\therefore \dfrac{x_1 + x_2}{2} = h$

$\Rightarrow x_1 + x_2 = 2h$

and $y_1 + y_2 = 2k$

$\because$ SP. S′ Q $= b^2$

$\Rightarrow y_1y_2 = b^2$

and SR. S′ T $= b^2$

$\Rightarrow x_1x_2 = b^2$

Distance between foci SS′ = 2ae

$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = (2ae)$

$\Rightarrow (x_1 - x_2)^2 + (y_1 - y_2)^2 = 4a^2e^2$

$\Rightarrow (x_1+x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2 = 4a^2e^2$

$\Rightarrow 4h^2 - 4b^2 + 4k^2 - 4b^2 = 4(a^2 - b^2)$

$\Rightarrow h^2 + k^2 - 2b^2 = a^2 - b^2$

$\therefore h^2 + k^2 = a^2 + b^2$

Locus of centre is $x^2 + y^2 = a^2 + b^2$ which is a circle

381. B

If eccentricities of ellipse and hyperbola are e and e_1

$\therefore$ Foci $(\pm ae, 0)$ and $(\pm a_1e_1, 0)$

Here, $ae = a_1e_1$

$a^2e^2 = a_1^2 e_1^2$

$a^2\left(1 - \dfrac{b^2}{a^2}\right) = a_1^2\left(1 + \dfrac{b_1^2}{a_1^2}\right)$

$\Rightarrow a^2 - b^2 = a_1^2 + b_1^2$

$\Rightarrow 25 - b^2 = \dfrac{144}{25} + \dfrac{81}{25} = 9 \therefore b^2 = 16$

382. C

For rectangular hyperbola

$a - 2 = -a \qquad \therefore a = 1$

383. C

Since, asymptotes $3x + 4y = 2$ and $4x - 3y + 5 = 0$ are perpendicular to each other. Hence, hyperbola is rectangular hyperbola but we know that the eccentricity of rectangular hyperbola is $\sqrt{2}$.

384. A

Centre of hyperbola is $\left(\dfrac{6-4}{2}, \dfrac{5+5}{2}\right)$

ie, (1, 5)

Distance between foci = 2ae

$10 = 2ae \Rightarrow 5 = a \times \dfrac{5}{4} \therefore a = 4$

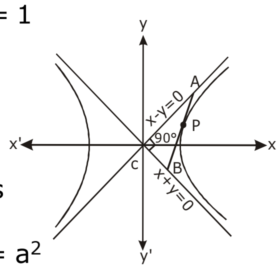
$b^2 = a^2(e^2 - 1) = 16\left(\dfrac{25}{16} - 1\right)$

$= 25 - 16 = 9$

$\therefore$ Equation of hyperbola is

$\dfrac{(x-1)^2}{16} - \dfrac{(y-5)^2}{9} = 1$

385. A



$\because x - y = 0$

and $x + y = 0$

are the asymptotes of the rectangular hyperbola $x^2 - y^2 = a^2$

Equation of tangent at $P(a \sec \phi, a \tan \phi)$ of $x^2 - y^2 = a^2$ is

$ax \sec \phi - ay \tan \phi = a^2$

or $x \sec \phi - y \tan \phi = a$ .....(i)

Solving $y = x$ and $y = -x$ with Eq. (i), then we get and

$\left\{\begin{array}{l} A(a(\sec\varphi + \tan\varphi), a(\sec\varphi + \tan\varphi)) \\ \text{and } B(a(\sec\varphi - \tan\varphi), a(\tan\varphi - \sec\varphi)) \end{array}\right\}$

$\therefore$ Area of

$\Delta CAB = \dfrac{1}{2}\left|a(\tan^2\phi - \sec^2\phi) - a(\sec^2\phi - \tan^2\phi)\right|$

$= \dfrac{1}{2}\,|-a - a| = |-a| = |a|$

386. D

Any point on given line is $(2r + 1, -3r - 1, r)$, its distance from $(1, -1, 0)$.

$\Rightarrow (2r)^2 + (-3r)^2 + r^2 = (4\sqrt{14})^2$

$\Rightarrow r = \pm 4$

$\Rightarrow$ Coordinates are $(9, -13, 4)$ and $(-7, 11, -4)$ and nearer to the origin is $(-7, 11, -4)$.

387. B

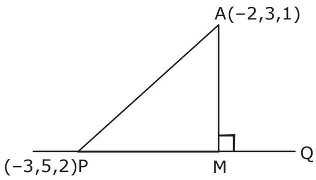
Here, $\alpha = \beta = \gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \cos\alpha = \dfrac{1}{\sqrt{3}}$

DC's of PQ are

$\left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$



PM = Projection of AP on PQ

$$= \left|(-2+3)\frac{1}{\sqrt{3}} + (3-5).\frac{1}{\sqrt{3}} + (1-2).\frac{1}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

and $AP = \sqrt{(-2+3)^2 + (3-5)^2 + (1-2)^2} = \sqrt{6}$

$$AM = \sqrt{(AP)^2 - (PM)^2} = \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$$

388. B

The straight line joining the points $(1,1,2)$ and $(3, -2, 1)$ is

$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-2}{-1} = r \text{ (say)}$$

$\therefore$ Point is $(2r + 1, 1 - 3r, 2 - r)$

which lies on $3x + 2y + z = 6$

$\therefore 3(2r + 1) + 2(1 - 3r) + 2 - r = 6$

$\therefore r = 1$ Required point is $(3, -2, 1)$

389. C

Let point is (α,β,γ)

$\therefore (\alpha - a)^2 + \beta^2 + \gamma^2 = \alpha^2 + (\beta - b)^2 + \gamma^2$

$= \alpha^2 + \beta^2 + (\gamma - c)^2 = \alpha^2 + \beta^2 + \gamma^2$

we get, $\alpha = \dfrac{a}{2}$, $\beta = \dfrac{b}{2}$ and $\gamma = \dfrac{c}{2}$

$\therefore$ Required point is $\left(\dfrac{a}{2}, \dfrac{b}{2}, \dfrac{c}{2}\right)$

390. A

Let plane is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ ....(i)

mid point of $P(1, 2, 3)$ and $Q(-3, 4, 5)$ ie, $(-1, 3, 4)$ lie on Eq. (i)

$\therefore -\dfrac{1}{a} + \dfrac{3}{b} + \dfrac{4}{c} = 1$ ....(ii)

Also, PQ is parallel to normal of the plane (i)

$$\frac{1/a}{-4} = \frac{1/b}{2} = \frac{1/c}{2}$$

$$\Rightarrow \frac{1}{-2a} = \frac{1}{b} = \frac{1}{c} = \lambda \text{ (say)}$$

$\therefore \dfrac{1}{a} = -2\lambda$, $\dfrac{1}{b} = \lambda$, $= \dfrac{1}{c} = \lambda$

$\therefore$ From Eq. (ii), $2\lambda + 3\lambda + 4\lambda = 1 \therefore \lambda = \dfrac{1}{9}$

$a = -\dfrac{1}{2\lambda}$, $b = \dfrac{1}{\lambda}$, $c = \dfrac{1}{\lambda}$

$\therefore a = -\dfrac{9}{2}$, $b = 9$, $c = 9$

Intercepts are $\left(-\dfrac{9}{2}, 9, 9\right)$

391. A

Let the coordinates of A be $(a, 0)$.

Then the slope of the reflected ray is

$\dfrac{3-0}{5-a} = \tan\theta$ (say) ....(i)

Then the slope of the incident ray

$= \dfrac{2-0}{1-a} = \tan(\pi - \theta)$ ....(ii)

From Eqs. (i) and (ii), $\tan\theta + \tan(\pi - \theta) = 0$

$$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0$$

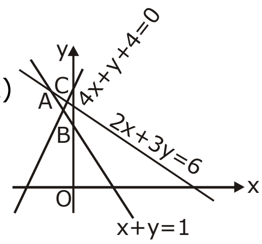
$\Rightarrow 3 - 3a + 10 - 2a = 0$; $a = \dfrac{13}{5}$

Thus, the coordinate of A is $\left(\dfrac{13}{5}, 0\right)$

392. A

Coordinates of A and B are

$(-3, 4)$ and $\left(-\dfrac{3}{5}, \dfrac{8}{5}\right)$



If orthocentre $P(h, k)$

Then, slope of PA × slope of BC $= -1$

$\Rightarrow \dfrac{k-4}{h+3} \times 4 = -1$

$\Rightarrow 4k - 16 = -h - 3$

$\Rightarrow h + 4k = 13$ .....(i)

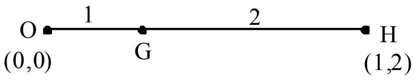
and slope of PB × slope of AC $= -1$

$$\Rightarrow \frac{k - \frac{8}{5}}{h + \frac{3}{5}} \times -\frac{2}{3} = -1 \Rightarrow \frac{5k-8}{5h+3} \times \frac{2}{3} = 1$$

$\Rightarrow 10k - 16 = 15h + 9 \Rightarrow 15h - 10k + 25 = 0$

$\Rightarrow 3h - 2k + 5 = 0$ ....(ii)

393. [B]



$G \equiv \left(\frac{1\times1+2\times0}{3}, \frac{2\times1+2\times0}{3}\right) = \left(\frac{1}{3}, \frac{2}{3}\right)$

394. C

$\because$ (a, a) fall between the lines $|x + y| = 2$,

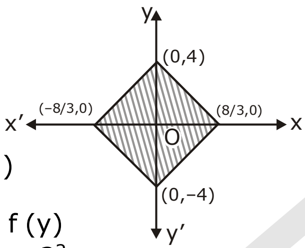
then $\frac{a+a-2}{a+a+2} < 0$

$\Rightarrow \frac{a-1}{a+1} < 0$ or $-1 < a < 1$ $\therefore |a| < 1$

395. C

Required area

$= 4 \times \frac{1}{2}\left(\frac{8}{3}\times 4\right)$



$= \frac{64}{3} = \frac{2^6}{3}$ ...(i)

$\because f(x + y) = f(x)\, f(y)$

$\therefore f(2) = f(1)\, f(1) = 2^2$

$f(3) = f(1 + 2) = f(1)\, f(2) = 2^3$

....

$\therefore f(n) = 2^n$

$\therefore$ Area $= \frac{2^6}{3} = \frac{f(6)}{3}$ sq unit.

396. A

Equation of conic is

$(2x - y + 1)(x - 2y + 3) + \lambda xy = 0$

for circle coefficient of xy = 0

ie, $-5 + \lambda = 0$, $\therefore \lambda = 5$

$\therefore$ Circle is $2x^2 + 2y^2 + 7x - 5y + 3 = 0$

$\therefore x^2 + y^2 + \frac{7}{2}x - \frac{5}{2}y + \frac{3}{2} = 0$

$\therefore$ Center is $\left(-\frac{7}{4}, \frac{5}{4}\right)$

397. B

We know, that, limiting points of the co-axial system of circles

$x^2 + y^2 + 2gx + 2fy + c = 0$

(other than origin) are given by

$\left(-\frac{gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$

Here given circle $x^2 + y^2 - 6x - 8y + 1 = 0$,

then $g = -3$, $f = -4$, $c = 1$.

Hence other limiting point is $\left(\frac{3}{25}, \frac{4}{25}\right)$

398. B

Centre $C \equiv (7, 5)$ and radius

$r = \sqrt{(49+25+151)} = 15$

If P(2, −7) The shortest distance

$= |CP - r| = \left|\sqrt{25+144} - 15\right| = |13 - 15| = 2$

399. D

The image of the circle has same radius butcentre different. If centre is (α, β),

then $\frac{\alpha-3}{1} = \frac{\beta-2}{1} = \frac{-2(3+2-19)}{1^2+1^2}$

$\Rightarrow \alpha - 3 = \beta - 2 = 14 \therefore \alpha = 17, \beta = 16$

$\therefore$ Required circle is $(x - 17)^2 + (y - 16)^2 = 1$

400. D

Let centre of the circle be (h, k)

$\because$ circle touch y-axis $\therefore$ radius = |h|

Equation of Circle is $(x - h)^2 + (y - k)^2 = h^2$ ....(i)

Given circle (i) touches externally the circle

$x^2 + y^2 - 6x - 6y + 14 = 0$

$\therefore$ Distance between centres = sum of radii

$\Rightarrow \sqrt{(h-3)^2+(k-3)^2} = |h| + 2$

$\Rightarrow (h - 3)^2 + (k - 3)^2 = h^2 + 4 + 4|h|$

$\Rightarrow k^2 - 6h - 4|h| - 6k + 14 = 0$

Here h always +ve

$\therefore$ $k^2 - 10h - 6k + 14 = 0$

$\therefore$ Locus of centre of circle is

$y^2 - 10x - 6y + 14 = 0$

401. A

Let $A \equiv (at_1^2, 2at_1)$, $B\,(at_2^2, 2at_2)$

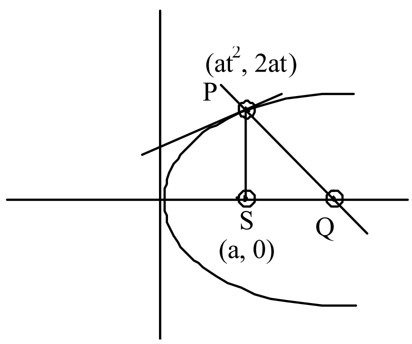
then, $C \equiv (at_1t_2, a(t_1 + t_2))$

$\because$ ordinates of A,B,C are $2at_1$, $2at_2$ and $a(t_1 + t_2)$ also,

$\frac{\text{ordinate of A} + \text{ordinate of B}}{2}$ = ordinate of C

Hence, ordinates of A, C and B are in AP.

402. [A]



$y^2 = 4ax$

$2y \frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Equation of normal at P

$y - 2at = -t(x - at^2)$

$Q(2a + at^2, 0)$

$PS = x + a = at^2 + a$

$SQ = at^2 + a$

$PS = SQ$

Isosceles

403. D

Let (α,β) be the feet of perpendicular from $(-1, 1)$ on directrix $4x + 3y - 24 = 0$, then

$$\frac{\alpha+1}{4} = \frac{\beta-1}{3} = -\left(\frac{-4+3-24}{4^2+3^2}\right) = 1$$

or $\alpha = 3, \beta = 4$ $\therefore (\alpha, \beta) \equiv (3, 4)$

Hence, vertex is the mid point of (3, 4) and (–1, 1) ie, (1, 5/2)

404. D

Equation of tangent in terms of slope of parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$ ....(i)

$\because$ Eq. (i), is also tangent of $x^2 = -32y$

then $x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow x^2 + 32mx + \frac{32}{m} = 0$

$\therefore B^2 = 4AC$ (Condition of tangency)

$\Rightarrow (32m)^2 = 4.1.\frac{32}{m} \Rightarrow m^3 = \frac{1}{8}$ or $m = \frac{1}{2}$

From Eq. (i), $y = \frac{x}{2} + 2 \Rightarrow x - 2y + 4 = 0$

405. C

$y = m(x + 1) + (1/m)$

or $y = mx + \left(m + \frac{1}{m}\right)$ ....(i)

is a tangent to the first parabola

and $y = m'(x + 2) + \frac{2}{m'}$

$= m'x + 2\left(m' + \frac{1}{m'}\right)$ .....(ii)

is a tangent to the second parabola given

$m.m' = -1$ or $m' = -\frac{1}{m}$ Then, from Eq. (ii)

$y = -\frac{x}{m} + 2\left(-\frac{1}{m} - m\right)$

$\Rightarrow y = -\frac{x}{m} - 2\left(m + \frac{1}{m}\right)$ ....(iii)

Substracting Eq. (iii) from Eq. (i), then

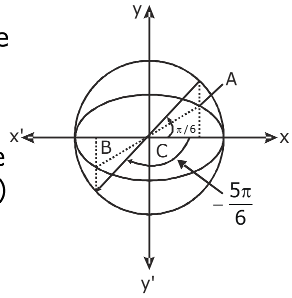
$x\left(m + \frac{1}{m}\right) + 3\left(m + \frac{1}{m}\right) = 0$ or $x + 3 = 0$

406. B

$\because$ Eccentric angle of A is $\frac{\pi}{6}$

$\therefore$ Eccentric angle of B is $-(\pi - \pi/6)$

ie, $-\frac{5\pi}{6}$



407. C

$y^2 = 4x$ ....(1)

$(x - 3)^2 + y^2 = 9$ ....(2)

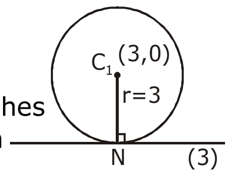
$\left\{\begin{array}{l} y^2 = 4ax \\ 4a = 4 \\ a = 1 \end{array}\right.$

Equation of tangent of parabola (1) is

$y = mx + \frac{1}{m}$ ...(3)

$\Rightarrow$ line (3) is also touches the circle (2), then

$\Rightarrow C_1N = r \Rightarrow m = ?$



408. D

Do yourself

409. C

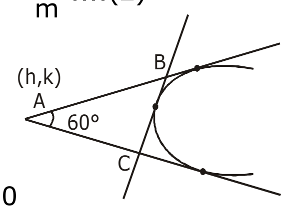
$y^2 = 4ax$ ...(1)

Equation of any tangent to the parabola is $y = mx + \frac{a}{m}$ ....(2)

If passes through A(h, k)

$\Rightarrow k = mh + \frac{a}{m}$

$\Rightarrow m^2h - mk + a = 0$



Let m_1 & m_2 be the roots, then

$\Rightarrow m_1 + m_2 = \frac{k}{h}$ & $m_1m_2 = \frac{a}{h}$

$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \Rightarrow 3(x + a)^2 = y^2 - 4ax$

410. D

$y^2 = 4x$....(1), $x^2 = -8y$ ....(2)

Equation of tangent of the parabola (1) is

$y = mx + \frac{1}{m}$ .....(3) this is also the tangent to the parabola (2), then $x^2 = -8\left(mx + \frac{1}{m}\right)$

$\Rightarrow mx^2 + 8m^2x + 8 = 0$ has equal roots

$\therefore 64m^4 = 32m \Rightarrow m = \left(\frac{1}{2}\right)^{1/3}$

$\therefore$ By (3) $\Rightarrow y = \left(\frac{1}{2}\right)^{1/3} + (2)^{1/3}$

411. C

Let rectangular hyperbola

$xy = c^2$ ....(i)

Let three points on Eq. (i) are

$A\left(ct_1, \frac{c}{t_1}\right)$, $B\left(ct_2, \frac{c}{t_2}\right)$, $C\left(ct_3, \frac{c}{t_3}\right)$

Let orthocentre is P(h, k) then slope of AP × slope of BC = −1

$$\Rightarrow \frac{k - \frac{c}{t_1}}{h - ct_1} \times \frac{\frac{c}{t_3} - \frac{c}{t_2}}{ct_3 - ct_2} = -1$$

$$\Rightarrow \frac{k - \frac{c}{t_1}}{h - ct_1} \times -\frac{1}{t_2\, t_3} = -1$$

$$\Rightarrow k - \frac{c}{t_1} = ht_2\, t_3 - ct_1t_2t_3 \quad .....(ii)$$

Similarly, $BP \perp AC$

$$\text{then } k - \frac{c}{t_2} = ht_3t_1 - ct_1t_2t_3 \quad ......(iii)$$

Substracting Eq. (iii) from Eq. (ii), then we get $h = -\frac{c}{t_1t_2t_3}$

Substituting the value h in Eq. (ii) then $k = -ct_1t_2t_3$

$\therefore$ Orthocentre is $\left(-\frac{c}{t_1t_2t_3}, -ct_1t_2t_3\right)$

which lies on $xy = c^2$

412. B

$\because x^2 + y^2 = 9 = 25 - 16$

which is director circle of the hyperbola

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

Hence, angle between tangents must be $\pi/2$

413. B

Let $P \equiv (8, y_1)$

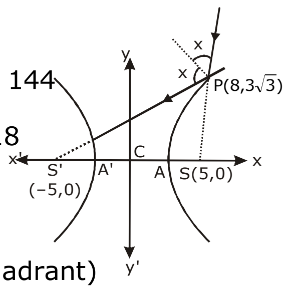
$\therefore\ 9(8)^2 - 16y_1^2 = 144$

$\Rightarrow 9 \times 8 - 2y_1^2 = 18$

$y_1 = \pm 3\sqrt{3}$

$\therefore P \equiv (8, 3\sqrt{3})$

($\because$ P lies in first quadrant)



Also, $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

$\therefore 3^2 = 4^2(e^2 - 1) \Rightarrow e = \frac{5}{4}$

then foci $\equiv (\pm 5, 0)$

$\therefore$ Equation of the reflected ray after first reflection passes through P, S′ is

$$y - 0 = \frac{3\sqrt{3} - 0}{8 + 5}(x + 5)$$

$\Rightarrow 3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

414. C

y = z = 0, then x = 1

suppose direction cosines of line of intersection are l,m,n. Then, l + m − n = 0

$2l - 3m + n = 0$ then $\frac{l}{2} = \frac{m}{3} = \frac{n}{5}$

$\therefore$ Equation of line in symmetric form is

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$$

415. C

Line is parallel to the normal of the plane $x - 2y - 3z = 7$

$\therefore$ Equation of line through (1, 1, − 1) is

$$\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$$

416. B

$r\cos\alpha = 9$, $r\cos\beta = 12$ and $r\cos\gamma = 8$

$\therefore r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 81 + 144 + 64$

$r^2 \cdot 1 = 289 \Rightarrow r = 17$

417. B

If DC's of line of intersection of planes $4y + 6z = 5$ and $2x + 3y + 5z = 5$ are l, m, n

$\therefore 0 + 4m + 6n = 0$

$2l + 3m + 5n = 0$

$$\Rightarrow \frac{l}{1} = \frac{m}{6} = \frac{n}{-4} = \frac{1}{\sqrt{53}}$$

$$\Rightarrow l = \frac{1}{\sqrt{53}}, m = \frac{6}{\sqrt{53}}, n = -\frac{4}{\sqrt{53}}$$

Also, $6\left(\frac{1}{\sqrt{53}}\right) + 5\left(\frac{6}{\sqrt{53}}\right) + 9\left(-\frac{4}{\sqrt{53}}\right) = 0$

ie, line of intersection is perpendicular to normal of third plane. Hence, three planes have a line in common.

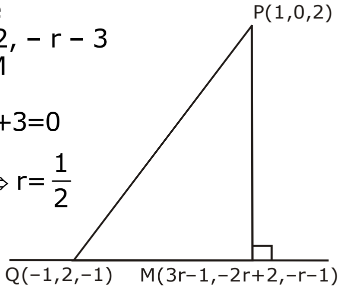
418. B

$$\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1} = r$$

$\therefore$ M(3r – 1, –2r + 2, – r – 1)
DR's of PM are
3r – 2, –2r + 2, – r – 3
DR's of line QM
are 3, –2, –1
$\therefore 9r-6+4r-4+r+3=0$

$\Rightarrow 14r-7=0 \Rightarrow r=\frac{1}{2}$

$\therefore M\left(\frac{1}{2},1,-\frac{3}{2}\right)$

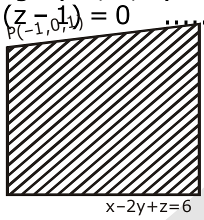


419. C

Point (3, 2, 1) and (2, –3, –1)
lies on 11x + my + nz = 28
ie, 33 + 2m + n = 28 $\Rightarrow$ 2m + n = –5 ...(i)
and 22 – 3m – n = 28 $\Rightarrow$ –3m – n = 6.....(ii)
From Eqs. (i) and (ii), m = –1 and n = –3

420. A

Equation of plane through (–1, 0, 1) is
a(x + 1) + b(y – 0) + c (z – 1) = 0 .....(i)
which is parallel to given line and perpendicular to given plane



–a + 2b + 3c = 0...(ii)
& a – 2b + c = 0...(iii)
From Eqs. (ii) and (iii), c = 0 , a = 2b
From Eqs. (i), 2b (x + 1) + by = 0
$\Rightarrow$ 2x + y + 2 = 0

421. A

$\vec{a}.\vec{b}=0$

$\therefore$ $\vec{a}$ and $\vec{b}$ are mutually perpendicular

Also $|\vec{a}|=2$ and $|\vec{b}|=3$

Let $\vec{a}=2\hat{i}$ then $\vec{b}=2\hat{i}$

such that $\vec{a}=\hat{i}$, $\vec{b}=\hat{j}$

$\therefore\ (\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))) = (2\hat{i}\times(2\hat{i}\times(2\hat{i}\times(2\hat{i}\times 3\hat{j})))$

$= 48\hat{j} = 48\vec{b}$

422. D

$3\vec{a}+4\vec{b}+5\vec{c}=0$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar
No other conclusion can be drived from it.

423. B

$\vec{\alpha}=\vec{a}+\vec{b}+\vec{c} \qquad = 6\hat{i}+12\hat{j}$

Let $\vec{\alpha}=x\vec{a}+y\vec{b} \qquad \Rightarrow 6x+2y=6$

$-3x-6y=12 \qquad \therefore x=2,\ y=-3$

$\therefore \vec{\alpha}=2\vec{a}-3\vec{b}$

424. B

Given expression = 2(1 + 1 + 1) – 2Σ $(\vec{a}\cdot\vec{b})$

$= 6-2\Sigma(\vec{a}\cdot\vec{b})$

But $(\vec{a}+\vec{b}+\vec{c})^2 \ge 0$

$\therefore (1+1+1)+2\Sigma\,\vec{a}\cdot\vec{b} \ge 0$

$\therefore 3 \ge -2\Sigma\,\vec{a}\cdot\vec{b}$

From Eqs. (i) and (ii),
Given expression $\le 6 + 3 = 9$

425. B

$(\hat{i}\times\hat{j})\cdot\vec{c} \le |\hat{i}\times\hat{j}|\,|\vec{c}|\cos\frac{\pi}{6}$

$\Rightarrow -\frac{\sqrt{3}}{2} \le (\hat{i}\times\hat{j})\cdot\vec{c} \le \frac{\sqrt{3}}{2}$

426. C

$\because \begin{vmatrix}1 & a & a\\ b & 1 & b\\ c & c & 1\end{vmatrix}=0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then, $\begin{vmatrix}1 & a-1 & a-1\\ b & 1-b & 0\\ c & 0 & 1-c\end{vmatrix}=0$

Let a – 1 = A, b – 1 = B and c – 1 = C, then

$\begin{vmatrix}1 & A & A\\ 1+B & -B & 0\\ 1+C & 0 & -C\end{vmatrix}=0$

$\Rightarrow 1(BC-0)-A(-C-BC)+A(B+BC)=0$

or $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=-2$ ....(i)

$\therefore \frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}=\frac{1+A}{A}+\frac{1+B}{B}+\frac{1+C}{C}$

$=\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+3=-2+3=1$ [from Eq. (i)]

427. B

Equation of line through (1, 2) is,
y – 2 = m (x – 1)...(i) $\Rightarrow$ mx – y – m + 2 = 0

distance from (3, 1) = $\frac{3m-1-m+2}{\sqrt{(m^2+1)}}$

Let D = $\frac{(2m+1)}{\sqrt{(m^2+1)}}$

For maximum or minimum $\frac{dD}{dm}=\frac{(2-m)}{(m^2+1)^{3/2}}=0$

$\therefore$ m = 2, $\frac{d^2D}{dm^2}$ = –ve From Eq. (i), y = 2x

428. A

If remaining vertex is (α, β), then

$\frac{\alpha+3}{2}=-4,\ \frac{\beta+5}{2}=-3/2$

$\therefore \alpha=-11,\ \beta=-8$

and $\frac{\alpha-6}{2}=\frac{3-2}{2},\ \frac{\beta-4}{2}=\frac{5+1}{2}$

$\therefore \alpha=7,\ \beta=10$

and also $\frac{\alpha-2}{2}=\frac{3-6}{2},\ \frac{\beta+1}{2}=\frac{5-4}{2}$

$\therefore \alpha=-1,\ \beta=0$

$\therefore$ Possibilities of remaining vertex are
(–11, –8) or (7, 10) or (–1, 0)

429. B

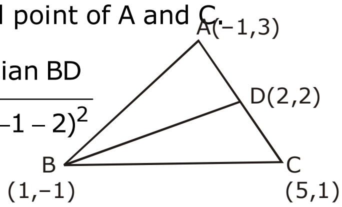
If D is the mid point of A and C

$\therefore$ D is (2, 2)

Length of median BD

$= \sqrt{(1-2)^2 + (-1-2)^2}$

$= \sqrt{10}$



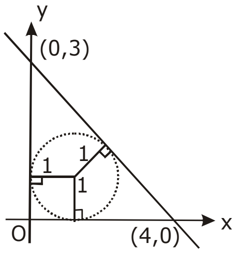
430. B

$\because 3x + 4y = 12$

$\Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

Let coordinate of incentre is (α, α)

$\therefore$ Radius is also α



Length of perpendicular from (α, α) on

$(3x + 4y - 12 = 0) = \alpha \Rightarrow \frac{|3\alpha + 4\alpha - 12|}{5} = \alpha$

$\Rightarrow 7\alpha - 12 = \pm 5\alpha \quad \therefore \alpha = 1$ and $\alpha = 6$

$\alpha \neq 6 \quad \therefore \alpha = 1$ Then, incentre (1, 1).

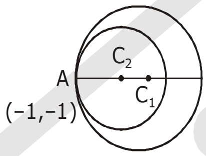
431. B

$x^2 + y^2 - 4x - 6y - 12 = 0 \quad .....(1)$

Centre C_1 (2, 3)

Radius $r_1 = 5 = C_1 A$

If C_2 (h, k) is the centre of the circle of radius 3



which touches the circle (1) internally at the point A(–1, –1), then $r_2 = C_2A = 3$ and $C_1C_2 = C_1A - C_2A = 5 - 3 = 2$ Thus, C_2 (h, k) divide C_1A in the ratio 2 : 3

internally $\therefore h = \frac{2(-1)+3(2)}{2+3} = \frac{4}{5}$

& $k = \frac{2(-1)+3(3)}{2+3} = \frac{7}{5} \quad \therefore C_2\left(\frac{4}{5}, \frac{7}{5}\right)$

432. D

$x^2 = y - 6 \quad ....(1)$

$x^2 + y^2 + 16x + 12y + c = 0 \quad .....(2)$

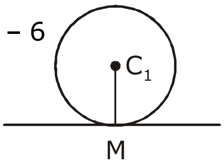
The tangent at P(1, 7) to the parabola (1)

is $\Rightarrow x(1) = \frac{1}{2}(y + 7) - 6$

$\Rightarrow y = 2x + 5 \quad ....(3)$

which is also touches the circle (2)

By (2) & (3)



$\Rightarrow x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$ must have equal roots. Let, α & β are the roots of the equal then, $\alpha + \beta = -12 \Rightarrow \alpha = -6$ ($\because \alpha = \beta$)

$\therefore x = -6$ & $y = 2x + 5 = -7$

$\therefore$ Point of contat is (–6, –7)

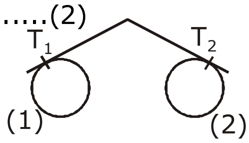
433. C

$x^2 + y^2 + 4x + 3 = 0 \quad .....(1)$

p (h, k) $\Rightarrow$ locus = ?

$x^2 + y^2 - 6x + 5 = 0 \quad .....(2)$

$\Rightarrow$ Given $\frac{PT_1}{PT_2} = \frac{2}{3}$



434. D

$y = |x| + c ...(1)$, $x^2 + y^2 - 8|x| - 9 = 0 ...(2)$

both are symmetrical about y-axis for $x > 0$,

$y = x + c \quad ....(3)$

equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$

which is parallel to line (3) is $y = x + (5\sqrt{2} - 4)$

for no solution $c > (5\sqrt{2} - 4) \therefore c \in (5\sqrt{2} - 4, \infty)$

435. C

$y^2 = 32x$

Length of focal chord

$a\left(t + \frac{1}{t}\right)^2$

$8\left(t + \frac{1}{t}\right)^2$

A.M. $\geq$ G.M

$\frac{t + \frac{1}{t}}{2} \geq \sqrt{t.\frac{1}{t}}$

$t + \frac{1}{t} \geq 2$

$\left(t + \frac{1}{t}\right)^2 \geq 4$

$8 \times 4 = 32$

436. A

Equation of chord joining (2, 2) and (8, -4) is

$y - 2 = \frac{-4-2}{8-2}(x - 2) \Rightarrow y - 2 = -x + 2$

$\therefore x + y = 4 \quad ....(i)$

Let $R(\alpha^2, \alpha - 2)$

$\because$ R is interior to the parabola, O and R are on the same side of PQ

$\therefore (\alpha - 2)^2 - 2\alpha^2 < 0$

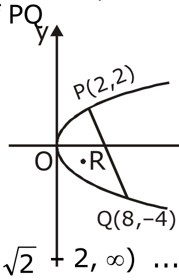
$\Rightarrow -\alpha^2 - 4\alpha + 4 < 0$

$\Rightarrow \alpha^2 + 4\alpha - 4 > 0$

$\Rightarrow (\alpha + 2)^2 > 8$

or $\alpha + 2 < -2\sqrt{2}$

and $\alpha + 2 > 2\sqrt{2}$



$\therefore \alpha \in (-\infty, -2 - 2\sqrt{2}) \cup (2\sqrt{2} + 2, \infty) \quad ...(ii)$

Also from equation (i) $\frac{\alpha - 2 + \alpha^2 - 4}{0 + 0 - 4} > 0$

$\Rightarrow \alpha^2 + \alpha - 6 < 0 \Rightarrow (\alpha + 3)(\alpha - 2) < 0$

$\therefore \quad -3 < \alpha < 2 \quad ...(iii)$

From equations (ii) and (iii), we get

$\alpha \in (2\sqrt{2} - 2, 2)$ or $\alpha \in (-2 + 2\sqrt{2}, 2)$

437. D

$\because$ Normal at $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then,

$T = -t - \dfrac{2}{t}$ or $tT = -t^2 - 2$

$\Rightarrow t^2 + tT + 2 = 0$ $\quad\because$ t is real

$\therefore T^2 - 4.1.2 \geq 0$ or $T^2 \geq 8$

438. C

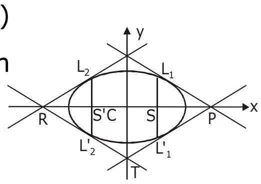
Do yourself

439. D

$\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ ....(1)

Area of parallelogram PQRT

= 4 (Area of Δ PQC)



$= 4\left\{\dfrac{1}{2}(CP)(CQ)\right\}$

440. C

The equation of the tangent at the point

$P(4\cos\phi, \dfrac{16}{\sqrt{11}}\sin\phi)$ to the ellipse

$16x^2 + 11y^2 = 256$ .....(1)

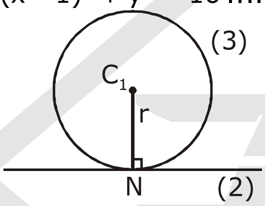
$\Rightarrow 4x\cos\phi + \sqrt{11}\,y\sin\phi = 16$ .....(2)

This touches the circle $(x - 1)^2 + y^2 = 16$ .....(3)

$\Rightarrow C_1N = r$

$\Rightarrow \cos\phi = \dfrac{1}{2}$

$\Rightarrow \phi = \pm\dfrac{\pi}{3}$



441. A

$ax^2 + 2bx + c = 0$ ....(1)

Roots of equation (1) are not real, then

$\Rightarrow D = (2b)^2 - 4ac < 0 \Rightarrow b^2 < ac$

then the equation

$ax^2 + 2bxy + cy^2 + dx + ey + f = 0$

can represents an ellipse

442. D

$x^2 - 3y^2 - 4x - 6y - 11 = 0$

$\Rightarrow (x^2 - 4x + 4 - 4) - 3(y^2 + 2y + 1 - 1) = 11$

$\Rightarrow (x - 2)^2 - 4 - 3(y + 1)^2 + 3 = 11$

$\Rightarrow (x - 2)^2 - 3(y + 1)^2 = 12$

$\Rightarrow \dfrac{(x-2)^2}{12} - \dfrac{(y+1)^2}{4} = 1 \Rightarrow b^2 = a^2(e^2 - 1)$

$\Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 = 1 + \dfrac{1}{3} \Rightarrow e = 2/\sqrt{3}$

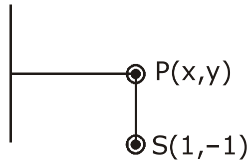
Distance between focii $2ae = 2 \times 2\sqrt{3} \times \dfrac{2}{\sqrt{3}} = 8$

443. C

Rectangular Hyperbola

$e = \sqrt{2}$

PS = ePM



$x - y + 1 = 0$

$(PS)^2 = e^2\,(PM)^2$

$(x - 1)^2 + (y + 2)^2 = 2\left(\dfrac{x - y + 1}{\sqrt{2}}\right)^2$

$x^2 - 2x + 1 + y^2 + 2y + 1$

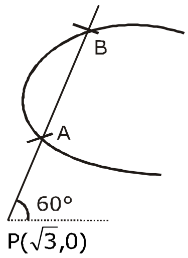
$= x^2 + y^2 + 1 - 2xy + 2x - 2y$

$2xy - 4x + 4y + 1 = 0$

444. A

$\therefore P \equiv (\sqrt{3}, 0)$

$\dfrac{x - \sqrt{3}}{\cos 60^\circ} = \dfrac{y - 0}{\sin 60^\circ} = r$



or $x = \sqrt{3} + \dfrac{r}{2}$, $y = \dfrac{r\sqrt{3}}{2}$

or $\left(\sqrt{3} + \dfrac{r}{2}, \dfrac{r\sqrt{3}}{2}\right)$ lie on $y^2 = x + 2$, then

$\dfrac{3r^2}{4} = \sqrt{3} + \dfrac{r}{2} + 2 \Rightarrow \dfrac{3r^2}{4} - \dfrac{r}{2} - (2 + \sqrt{3}) = 0$

$\therefore$ PA. PB $= r_1r_2 = \left|-\dfrac{(2+\sqrt{3})}{\frac{3}{4}}\right| = \dfrac{4}{3}(2 + \sqrt{3})$

445. D

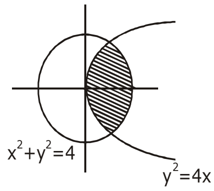
It is clear that point $(-2m, m + 1)$ lie inside the circle and parabola, then

$(-2m)^2 + (m + 1)^2 - 4 < 0$

and $(m + 1)^2 - 4(-2m) < 0$

$\therefore 5m^2 + 2m - 3 < 0$

and $m^2 + 10m + 1 < 0$



$\Rightarrow (m + 1)(5m - 3) < 0$

and $(m + 5)^2 - 24 < 0$

$\Rightarrow -1 < m < \dfrac{3}{5}$ and $-5 - 2\sqrt{6} < m < -5 + 2\sqrt{6}$

Hence $-1 < m < -5 + 2\sqrt{6}$

446. D

Since, $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos 120°$

$= 1 . 2\left(-\frac{1}{2}\right) = -1$

$\because \{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$

$= \{3\vec{a} \times \vec{a} - \vec{a} \times \vec{b} + 9\vec{b} \times \vec{a} - 3\vec{b} \times \vec{b}\}^2$

$= [0 - \vec{a} \times \vec{b} - 9\vec{a} \times \vec{b} - 0]^2$

$= [-10(\vec{a} \times \vec{b})]^2 = 100(\vec{a} \times \vec{b})^2$

$= 100\{a^2 b^2 - (\vec{a}.\vec{b})^2\}$

$= 100\{4 - 1\} = 300$

447. C

$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}-\vec{b}|^2}$

$= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}} = \sqrt{9 + 16 - 2\vec{a}.\vec{b}}$

$= \sqrt{(25 - 2\vec{a}.\vec{b})}$ ...(i)

But $|\vec{a} + \vec{b}| + 5 \quad \therefore |\vec{a} + \vec{b}|^2 = 25$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 25$

$9 + 16 + 2\vec{a}.\vec{b} = 25 \therefore \vec{a}.\vec{b} = 0$

From Eq. (i), $|\vec{a} - \vec{b}| = \sqrt{25 - 0} = 5$

448. D

$|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$

$|(\vec{a} \times \vec{b})|\,|\vec{c}|\cos\alpha = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$

{α is the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$}

$\Rightarrow |\vec{a} \times \vec{b}|\cos\alpha = |\vec{a}|\,|\vec{b}|$

$\Rightarrow |\vec{a}|\,|\vec{b}|\sin\beta\cos\alpha = |\vec{a}|\,|\vec{b}|$

(β is the angle between $\vec{a}$ and $\vec{b}$)

$\therefore \cos\alpha\cos\beta = 1$

It is possible when $\cos\alpha = 1$, $\sin\beta = 1$

$\therefore \alpha = 0$ and $\beta = \pi/2$

$\therefore \vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0,\ \vec{c}.\vec{a} = 0$

449. D

Linear combination

$1.(\hat{i} + \hat{j} + \hat{k}) + \lambda(4\hat{i} + 3\hat{j} + 4\hat{k})$

$+ \mu(\hat{i} + \alpha\hat{j} + \beta\hat{k}) = 0.\hat{i} + 0.\hat{j} + 0.\hat{k}$

$1 + 4\lambda + \mu = 0$

and $|\vec{c}| = \sqrt{1 + \alpha^2 + \beta} = \sqrt{3}$

$1 + 3\lambda + a\mu = 0 \qquad \therefore \alpha^2 + \beta^2 = 2$

and $1 + 4\lambda + \beta\mu = 0$

Solve any two then putting the value in remaining third equation.

450. A

Given, $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ and $|\vec{a}| = 1, |\vec{b}| = 1$

and $|\vec{c}| = 1 \quad \because [2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$

$(2\vec{a} - \vec{b}).\{(2\vec{b}-\vec{c}) \times (2\vec{c}-\vec{a})\}$

$=(2\vec{a} - \vec{b}).\{4\vec{b} \times \vec{c}) - 2\vec{b} \times \vec{a} - 2\vec{c} \times \vec{c} + \vec{a})\}$

$= (2\vec{a} - \vec{b}).\{4\vec{b} \times \vec{c}) + 2(\vec{a} \times \vec{b}) - 0 + (\vec{c} \times \vec{a})\}$

$= 8\vec{a}.(\vec{a} \times \vec{c}) + 4\vec{a}.(\vec{a} \times \vec{b}) + 2\vec{a}.(\vec{c} \times \vec{a})$

$-4\vec{b}.(\vec{b} \times \vec{c}) - 2\vec{b}.(\vec{a} \times \vec{b}) - \vec{b}.(\vec{c} \times \vec{a})$

$= 8[\vec{a}\ \vec{b}\ \vec{c}] + 0 + 0 - 0 - 0 - [\vec{a}\ \vec{b}\ \vec{c}]$

$= 7[\vec{a}\ \vec{b}\ \vec{c}] = 0 \qquad (\because [\vec{a}\ \vec{b}\ \vec{c}] = 0)$

451. C

Let $\vec{r} = r_1\hat{i} + r_2\hat{j} + r_3\hat{k}$

$\therefore r_1.\hat{i} = r_1,\ \vec{r}.\hat{j} = r_2,\ \vec{r}.\hat{k} = r_3$

and $\vec{r} \times \hat{i} = 0 + r_2(\hat{j} \times \hat{i}) + r_3(\hat{k} \times \hat{i})$

$= -r_2\hat{k} + r_3\hat{j}$

$\therefore (\vec{r}.\hat{i})(\vec{r} \times \hat{i}) = -r_1r_2\hat{k} + r_3r_2\hat{k}$

similarly $(\vec{r}.\hat{j})(\vec{r} \times \hat{j}) = -r_2r_3\hat{i} + r_2r_1\hat{k}$

and $(\vec{r}.\hat{k})(\vec{r} \times \hat{i}) = -r_3r_1\hat{j} + r_2r_3\hat{i}$

$\therefore (\vec{r}.\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}.\hat{j})(\vec{r} \times \hat{j})$

$+ (\vec{r}.\hat{k})(\vec{r} \times \hat{k}) = 0$

452. B

$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\therefore |(\vec{a} \times \vec{b}) \times \vec{c}| = |(\vec{a} \times \vec{b})|\,|\vec{c}|\sin 30°$

$= \sqrt{2^2 + (-2)^2 + 1^2}\ |\vec{c}|\ \frac{1}{2} = \frac{3}{2}|\vec{c}|$

Given, $|\vec{c} - \vec{a}| = 2\sqrt{2} \quad (\vec{c} - \vec{a})^2 = 8$

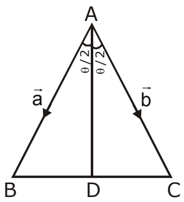
$\Rightarrow (\vec{c})^2 + (\vec{a})^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\Rightarrow (|\vec{c}|)^2 - 1)^2 = 0$

$\therefore |\vec{c}| = 1 \quad \therefore |\vec{a} \times \vec{b}) \times \vec{c}| = \frac{3}{2} \times 1 = \frac{3}{2}$

453. C



Now, in Δ ABC

$\frac{BD}{DC} = \frac{a}{b}$

$\therefore BD = ak, DC = bk$

$BC = (a + b)k$

$(BC)^2 = (AB)^2 + (AC)^2 - 2AB{\cdot}AC\cos\theta$

$\Rightarrow (a + b)^2 k^2 = a^2 + b^2 - 2ab\cos\theta$

$\therefore k^2 = \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}$

In Δ ADC and Δ ABD

$\cos\left(\frac{\theta}{2}\right) = \frac{b^2 + (AD)^2 - b^2k^2}{2bAD}$

$= \frac{a^2 + (AD)^2 - a^2k^2}{2aAD}$

$\Rightarrow (AD)^2 = ab(1 - k^2)$

$\therefore = ab\left\{1 - \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}\right\}$

[from Eq. (i)]

$= \frac{4a^2b^2\cos^2\theta/2}{(a+b)^2} \therefore AD = \frac{2ab\cos\theta/2}{(a+b)}$

$\therefore \ \overrightarrow{AD} = \pm \frac{(\vec{a}b + \vec{b}a)}{(a+b)} = \pm \frac{ab}{(a+b)} \left(\frac{\vec{a}}{a} + \frac{\vec{b}}{b}\right)$

$= \pm \frac{ab}{(a+b)} (\hat{a} + \hat{b})$

$\therefore \ \frac{\overrightarrow{AD}}{AD} = \pm \frac{(\hat{a}+\hat{b})}{2\cos\theta/2}$

454. D

If angle between $\vec{b}$ and $\vec{c}$ is α

then $|\vec{b} \times \vec{c}| = \sqrt{15}$

$\Rightarrow |\vec{b}|\,|\vec{c}| \sin\alpha = \sqrt{15} \Rightarrow \sin\alpha = \frac{\sqrt{15}}{4}$

$\therefore \cos\alpha = 1/4 \qquad \vec{b} - 2\vec{c} = \lambda\vec{a}$

$\Rightarrow (\vec{b} - 2\vec{c})^2 = \lambda^2 (\vec{a})^2$

$\Rightarrow (\vec{b})^2 + 4(\vec{c})^2 - 4\vec{b}.\vec{c} = \lambda^2(\vec{a})^2$

$\Rightarrow \ 16 + 4 - 4\{|\vec{b}|\,|\vec{c}| \cos\alpha\} = \lambda^2$

$\therefore \ \lambda^2 = 16 \Rightarrow \lambda = \pm 4$

455. D

Given, $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$

$\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$

and $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$

Given, $V_1 = [\vec{a}\ \vec{b}\ \vec{c}]$ .....(i)

$\therefore \ V_2 = [\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & 1 \\ 1 & -4 & 2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore \ V_2 = 12\ [\vec{a}\ \vec{b}\ \vec{c}]$

from Eqs. (i) and (ii), $V_2 : V_1 = 15 : 1$

456. B

Equation of line joining

$6\vec{a} - 4\vec{b} - 5\vec{c}$, and $-4\vec{c}$ is

$\vec{r} = (6\vec{a} - 4\vec{b} - 5\vec{c}) + \lambda(-6\vec{a} + 4\vec{b} + \vec{c})$

$= \vec{a}\,(6 - 6\lambda) + \vec{b}\,(-4 + 4\lambda) + \vec{c}\,(-5 + \lambda)$ ...(i)

and equation of line joining

$-\vec{a} - 2\vec{b} - 3\vec{c}$ and $\vec{a} + 2\vec{b} - 5\vec{c}$ is

$\vec{r}\ (-\vec{a} - 2\vec{b} - 3\vec{c}) + \mu(2\vec{a} + 4\vec{b} - 2\vec{c})$

$= \vec{a}\,(-1+2\mu) + \vec{b}\,(-2+4\mu) + \vec{c}\,(-3-2\mu)$ ...(ii)

Comparing Eqs. (i) and (ii), then

$6 - 6\lambda = -1 + 2\mu$

$-4 + 4\lambda = -2 + 4\mu$

and $-5 + \lambda = -3 - 2\mu$

After solving, we get

$\lambda = 1$ and $\mu = \frac{1}{2}$

Substituting the value of λ in Eq. (i)

then point of intersection is $\vec{r} = -4\vec{a}$

457. B

$\therefore \ \vec{a} = (x, y, z)$

$\vec{a}$ makes an obtuse angle with y-axis

$\therefore$ y < 0 and given $\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} = \frac{\vec{a}.\vec{c}}{|\vec{a}|.|\vec{c}|}$

$\Rightarrow \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{\vec{a}.\vec{c}}{|\vec{c}|}$

$\Rightarrow \frac{xy - 2yz + 3zx}{\sqrt{(y^2 + 4z^2 + 9x^2)}} = \frac{2zx + 3xy - yz}{\sqrt{(4z^2 + 9x^2 + y^2)}}$

$\Rightarrow xy - 2yz + 3zx = 2zx + 3xy - yz$

$\Rightarrow 2xy + yz - zx = 0$ .....(i)

and given $\vec{a}\cdot\vec{d} = 0$

$\therefore x - y + 2z = 0 \Rightarrow z = \frac{y - x}{2}$ .....(ii)

from Eqs. (i) and (ii), $2xy + \frac{(y-x)^2}{2} = 0$

$\therefore \ (y + x)^2 = 0$

$\therefore \ y = -x$ .....(iii)

$\therefore \ z = -x$ [from Eq. (ii)]

$\therefore \ \vec{a} = (x, -x, -x)$

$\therefore \ |\vec{a}| = \sqrt{x^2 + x^2 + x^2} = x\sqrt{3} = 2\sqrt{3}$

$\therefore \ x = 2, y = -2, z = -2$

$\therefore \ \vec{a} = (2, -2, -2)$

458. B

$\therefore$ AD $\perp$ BC $\quad \therefore (\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = 0$

and BD $\perp$ AC $\quad \therefore (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$

$\therefore$ D is orthocentre.

459. B

Let image of P w.r.t the given line be Q(α, β, γ). Then mid point of PQie,

$\left(\frac{\alpha+7}{2}, \frac{\beta-1}{2}, \frac{\gamma+2}{2}\right)$

lies on the line

$\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k}) = 6$

$\therefore \frac{\alpha+7}{2}\hat{i} + \frac{\beta-1}{2}\hat{j} + \frac{\gamma+2}{2}\hat{k}$

$= 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$

On comparing

$$\left.\begin{array}{l} \frac{\alpha+7}{2} = 9 + \lambda \Rightarrow \alpha = 11 + 2\lambda \\ \frac{\beta-1}{2} = 5 + 3\lambda \Rightarrow \beta = 11 + 6\lambda \\ \frac{\gamma+2}{2} = 5 + 5\lambda \Rightarrow \gamma = 8 + 10\lambda \end{array}\right\} \text{....(i)}$$

and

Also, PQ and given line are perpendicular ie, $(\alpha - 7).1 + (\beta + 1).3 + (\gamma - 2).5 = 0$

$\Rightarrow \ \alpha - 7 + 3\beta + 3 + 5\gamma - 10 = 0$

$\therefore \ \alpha + 3\beta + 5\gamma = 14$ ....(ii)

From Eqs. (i) and (ii),

$11 + 2\lambda + 3(11 + 6\lambda) + 5(8 + 10\lambda) = 14$

$\therefore \ \lambda = -1$

From Eq. (i), $(\alpha, \beta, \gamma) = (9, 5, -2)$.

460. $(3\vec{a} - 5\vec{b}).(2\vec{a} + \vec{b}) = 0$

$(\vec{a} + 4\vec{b}).(\vec{b} - \vec{a}) = 0$

461. A

$\because 3a + 2b + 6c = 0 \quad \therefore \ c = -\frac{(3a+2b)}{6}$

$\therefore ax + by + c = 0 \Rightarrow ax + by - \frac{(3a+2b)}{6} = 0$

$\Rightarrow 6ax + 6by - 3a - 2b = 0$
$\Rightarrow 3a(2x - 1) + 2b(3y - 1) = 0$

$\Rightarrow (2x - 1) + \frac{2b}{3a}(3y - 1) = 0$

$P + \lambda Q = 0, \therefore P = 0, Q = 0$
Then, $2x - 1 = 0, 3y - 1 = 0; x = 1/2, y = 1/3$

Hence, fixed points is $\left(\frac{1}{2}, \frac{1}{3}\right)$

462. B
Equation of any line through the point of intersection of the given lines is $(3x + y - 5) + \lambda(x - y + 1) = 0$ since this line is perpendicular to one of the given

lines $\frac{3+\lambda}{\lambda - 1} = -1$ or $\frac{1}{3}$

$\Rightarrow \lambda = -1$ or -5, therefore the required straight line is $x + y - 3 = 0$ or $x - 3y + 5 = 0$

463. C
Let $A \equiv (p + 1, 1)$, $B \equiv (2p + 1, 3)$, and $C \equiv (2p + 2, 2p)$
$\therefore$ Slope of AB = Slope of AC

$\Rightarrow \frac{3-1}{2p+1-p-1} = \frac{2p-1}{2p+2-p-1}$ $\therefore p = 2, -1/2$

464. C
Since m_1 and m_2 are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$

then, $m_1 + m_2 = -(\sqrt{3} + 2)$, $m_1m_2 = (\sqrt{3} - 1)$

$\therefore m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4m_1m_2}$

$= \sqrt{(3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4)} = \sqrt{11}$

and coordinates of the vertices of the given triangle are (0, 0), $(c/m_1, c)$ and $(c/m_2, c)$.
Hence, the required area of triangle

$= \frac{1}{2}\left|\frac{c}{m_1} \times c - \frac{c}{m_2} \times c\right| = \frac{1}{2} c^2 \left|\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\right|$

$= \frac{1}{2} c^2 \left|\frac{m_2 - m_1}{m_1m_2}\right| = \frac{1}{2} c^2 \frac{\sqrt{11}}{(\sqrt{3} - 1)}$

$= \frac{1}{2} c^2 \frac{\sqrt{11}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \left(\frac{\sqrt{33} + \sqrt{11}}{4}\right)$
c^2

On comparing, $a = \frac{\sqrt{33}}{4}$, $b = \frac{\sqrt{11}}{4}$

or $a = \frac{\sqrt{11}}{4}$, $b = \frac{\sqrt{33}}{4}$

$\therefore a^2 + b^2 = \frac{33}{16} + \frac{11}{16} = \frac{44}{16} = \frac{11}{4}$

$\Rightarrow 2008(a^2 + b^2) = 2008 \times \frac{11}{4}$

$= 502 \times 11 = 5522$

465. C
$x = a + m ...(1)$, $y = -2 ...(2)$, $y = mx ...(3)$
Point of intersection of (1) & (2)
$(a + m, -2)$ is lies on (3)

$\Rightarrow -2 = m(a + m) \Rightarrow -2 = am + m^2$
$\Rightarrow m^2 + am + 2 = 0 \quad \because m$ is real
$\Rightarrow D \geq 0 \Rightarrow a^2 - 8 \geq 0 \Rightarrow a^2 \geq 8 \Rightarrow |a| \geq 2\sqrt{2}$

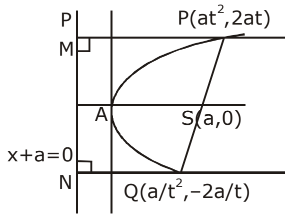
466. A

$\therefore SP = PM = a + at^2$, $SQ = QN = a + \frac{a}{t^2}$

$\frac{1}{SP} = \frac{1}{a(1+t^2)}$

$\therefore \frac{1}{SQ} = \frac{t}{a(1+t^2)}$

$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$



467. D
Since y-axis is major axis $\Rightarrow f(4a) < f(a^2 - 5)$
$\Rightarrow 4a > a^2 - 5$ ($\because$ f is decreasing)
$\Rightarrow a^2 - 4a - 5 < 0 \Rightarrow a \in (-1, 5)$

468. C
$\because$ Equation of tangent of $y^2 = 4ax$ in terms of slope (m) is $y = mx + \frac{a}{m}$

Which is also tangent of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then $\left(\frac{a}{m}\right)^2 = a^2m^2 + b^2$

$\Rightarrow a^2\left(\frac{1}{m^2} - m^2\right) = b^2 \Rightarrow \left(\frac{1}{m^2} - m^2\right) = \frac{b^2}{a^2}$

$\Rightarrow \frac{(1+m^2)(1-m^2)}{m^2} = \frac{b^2}{a^2}$

$\Rightarrow \left(\frac{1-m^2}{m^2}\right) = \frac{b^2}{a^2(1+m^2)} > 0 \Rightarrow \left(\frac{1-m^2}{m^2}\right) > 0$

$\Rightarrow \frac{m^2 - 1}{m^2} < 0 \Rightarrow 0 < m^2 < 1$
$\therefore m \in (-1, 0) \cup (0, 1)$
for positive values of m set is $m \in (0, 1)$

469. A
Let the point be $(\alpha, \beta) \Rightarrow \beta = \alpha + c$
Chord of contact of hyperbola T = 0

$\therefore \frac{x\alpha}{2} - \frac{y\beta}{1} = 1 \Rightarrow \frac{x\alpha}{2} - y(\alpha + c) = 1$

$\Rightarrow \left(\frac{x}{2} - y\right)\alpha - (yc + 1) = 0$

Since, this passes through point (x_1, y_1)
$\therefore x_1 = 2y_1$ and $y_1c + 1 = 0$

$\therefore y_1 = \frac{x_1}{2}$ hence, $\frac{x_1}{y_1} = 2$

470. B
If eccentricities of ellipse and hyperbola are e and e_1 $\therefore$ Foci $(\pm ae, 0)$ and $(\pm a_1e_1, 0)$
Here, $ae = a_1e_1 \Rightarrow a^2e^2 = a_1^2 e_1^2$

$\Rightarrow a^2\left(1-\frac{b^2}{a^2}\right) = a_1^2\left(1+\frac{b_1^2}{a_1^2}\right)$

$\Rightarrow a^2 - b^2 = a_1^2 + b_1^2$

$\Rightarrow 25 - b^2 = \frac{144}{25} + \frac{81}{25} = 9 \quad \therefore b^2 = 16$

$\therefore \cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{\lambda^2+\lambda^2-\frac{\lambda^2}{4}}{2\lambda^2} = 1 - \frac{1}{8} = \frac{7}{8}$

471. A

$a \to s$, $b \to p$, $c \to p$, $d \to r$

(a) $\frac{y}{y'} = c \Rightarrow \frac{y'}{y} = \frac{1}{c} \Rightarrow \log y = \frac{x}{c} + d$

$y = Ae^{x/c}$ Exponential curve

(b) $yy' = c \Rightarrow y^2 = 2cx + d$ [Parabola]

(c) $\frac{y}{y'} = 2x \Rightarrow \frac{2y'}{y} = \frac{1}{x}$

$\Rightarrow \ell n y^2 = \ell n x + \ell n c \Rightarrow y^2 = cx$ [Parabola]

(d) $yy' = 2x \Rightarrow \frac{y^2}{2} = x^2 + c$ [Hyperbola]

472. D

Let direction ratios of PQ are a,b,c

$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-3}$

Then, $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z+2}{c} = r$

(Let PQ = r)

$Q \equiv (ar + 2, br + 1, cr - 2)$

Which lie on $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-3}{-3}$, then

$\frac{ar+2-1}{2} = \frac{br+1+1}{2} = \frac{cr-2-3}{-3}$

$\Rightarrow \frac{ar+1}{2} = \frac{br+2}{2} = \frac{cr-5}{-3} = \lambda$ (say)

$\therefore a = \frac{2\lambda-1}{r}$, $b = \frac{2\lambda-2}{r}$, $c = \frac{-3\lambda+5}{r}$

Given, PQ is parallel to $x + 2y + z = 4$, then

$a + 2b + c = 0$

or $\frac{2\lambda-1}{r} + \frac{4\lambda-4}{r} + \frac{-3\lambda+5}{r} = 0$

$\Rightarrow 3\lambda = 0 \Rightarrow \lambda = 0$

then, $a = -\frac{1}{r}$, $b = -\frac{2}{r}$, $c = \frac{5}{r}$

$\therefore Q = (1, -1, 3)$

$\therefore PQ \equiv \sqrt{(2-1)^2+(1+1)^2+(3+2)^2} = \sqrt{30}$

473. B

Let DC's of shortest distance line are l, m, n which is perpendicular to both the given lines

$\therefore 2l - 7m + 5n = 0$ .....(i)

and $2l + m - 3n = 0$ ....(ii)

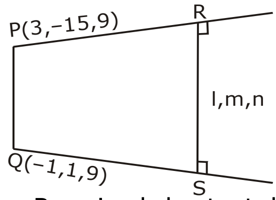
From Eqs. (i) and (ii).

l	m	n	l	m
2	−7	5	2	−7
2	1	−3	2	1

$\therefore \frac{l}{16} = \frac{m}{16} = \frac{n}{16}$

$\Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \frac{\sqrt{(l^2+m^2+n^2)}}{\sqrt{(1^2+1^2+1^2)}} = \frac{1}{\sqrt{3}}$

$\Rightarrow l = \frac{1}{\sqrt{3}}$, $m = \frac{1}{\sqrt{3}}$, $n = \frac{1}{\sqrt{3}}$



$\therefore$ Required shortest distance = Projection of PQ on RS

$= |(3-(-1))\,l + (-15-1)m + (9-9)n|$

$= |4l - 16m| = \left|\frac{-12}{\sqrt{3}}\right| = 4\sqrt{3}$

474. A

$x - y + 2z = 5$, $3x + y + z = 6$

Let the direction ratio of lies are a,b,c

$a - b + 2c = 0$, $3a + b + c = 0$

$\frac{a}{\begin{vmatrix}-1 & 2\\ 1 & 1\end{vmatrix}} = \frac{b}{\begin{vmatrix}2 & 1\\ 1 & 3\end{vmatrix}} = \frac{c}{\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}} \Rightarrow \frac{a}{-3} = \frac{b}{5} = \frac{c}{4}$

Equation of line $\frac{x-1}{-3} = \frac{y-2}{-2} = \frac{z-3}{4}$

475. A

$\frac{x+3}{3} = \frac{y-2}{-2} = \frac{z+1}{1} = r$

$M(3r - 3, -2r + 2, r - 1)$

$\Rightarrow 4x + 5y + 3z - 5 = 0$

$\Rightarrow 4'(3r - 3) + 5(-2r + 2) + 3(r - 1) - 5 = 0$

$\Rightarrow 12r - 12 - 10r + 10 + 3r - 3 - 5 = 0$

$\Rightarrow 5r = 10 \Rightarrow r = 2 \Rightarrow M(3, -1, 1)$

476. D

$\alpha = 45^o$, $\beta = 60^o \Rightarrow \ell = \cos\alpha = \cos 45^o = \frac{1}{\sqrt{2}}$

& $m = \cos\beta = \cos 60^o = \frac{1}{2}$

$\Rightarrow \ell^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{2} \Rightarrow \cos^2\gamma = \frac{1}{4}$

$\Rightarrow \cos\gamma = \pm\frac{1}{2} \Rightarrow \cos\gamma = \frac{1}{2}$ & $\cos\gamma = -\frac{1}{2}$

$\therefore \gamma = 60^o, 120^o$

477. D

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$

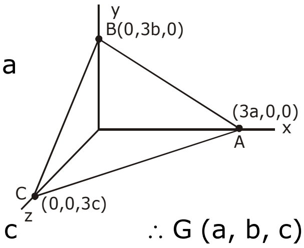
Let centroids of ΔABC is

$G(x, y, z)$

$$x = \frac{3a+0+0}{3} = a$$

$$y = \frac{0+3b+0}{3} = b$$

$$z = \frac{0+0+3c}{3} = c \qquad \therefore G\,(a, b, c)$$



478. C

$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ & $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar

$$\Rightarrow \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 + 2k) + 1\,(1 + k^2) - 1(2 - k) = 0$

$\Rightarrow 1 + 2k + 1 + k^2 - 2 + k = 0$

$\Rightarrow k^2 + 3k = 0 \Rightarrow k = 0\,,\,-3$

479. B

$\overrightarrow{OP} = \vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}\,,\ \overrightarrow{OQ} = \vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$

$$\frac{PM}{MQ} = \frac{OP}{OQ} = \frac{\sqrt{14}}{\sqrt{14}} = \frac{1}{1}$$

$$M\left(\frac{3+1}{2}, \frac{3-1}{2}, \frac{-2-2}{2}\right) \Rightarrow M(2, 1, -2)$$

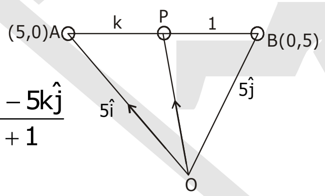
$\overrightarrow{OM} = 2i + j - 2\hat{k}$

480. B

$$\overrightarrow{OP} = \frac{5k\,\hat{j} + 5\hat{i}}{k+1}$$

$$\overrightarrow{PO} = \vec{b} = \frac{-5\hat{i} - 5k\hat{j}}{k+1}$$

$\Rightarrow |b| \le \sqrt{37}$



$$\Rightarrow \frac{25 + 25k^2}{(k+1)^2} \le 37 \Rightarrow 25 + 25k^2 \le 37k^2 + 74k + 37$$

$\Rightarrow 12k^2 + 74k + 12 \ge 0 \Rightarrow 6k^2 + 37\,k + k + 6 \ge 0$

$\Rightarrow 6k\,(k + 6) + 1\,(k + 6) \ge 0 \Rightarrow (k + 6)(6k+1) \ge 0$

$\therefore k \in (-\infty, -6] \cup [-\frac{1}{6}, \infty)$

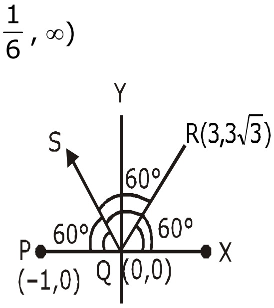
481. C

Slope of bisector

$= \tan 120°$

$= -\sqrt{3}$

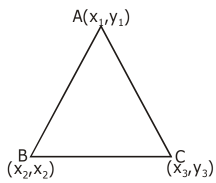
Equation $y = -\sqrt{3}\,x$



482. A

$$\text{Area } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

= A rational number



$\because$ (x_1, x_2, x_3, y_1, y_2, y_3, are integers)

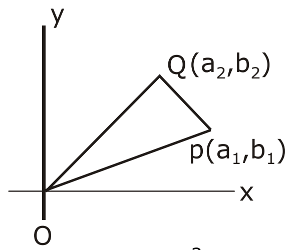
But Area of equilateral triangle

$= \frac{\sqrt{3}}{4}\,(BC)^2$ = irrational

483. A

$\cos\angle POQ$

$$= \frac{OP^2 + OQ^2 - PQ^2}{2OP.OQ}$$



484. B

$x_2 = x_1 r\,,\ y_2 = y_1 r \Rightarrow x_3 = x_1\, r^2,\ y_3 = y_1 r^2$

$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3}$ i.e. point lies on $\frac{y}{x} = k \Rightarrow y = kx$

485. A

$$x + 2y + \left(\frac{1(9) + 6(-5)}{1+6}\right) = 0 \Rightarrow x + 2y - 3 = 0$$

486. D

$|\vec{a}| = 1,\ |\vec{b}| = 5,\ |\vec{c}| = 3$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{c} \Rightarrow |\vec{a} \times (\vec{a} \times \vec{b})| = |\vec{c}|$

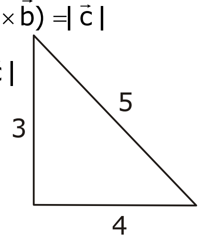
$\Rightarrow |\vec{a}| \times |\vec{a} \times \vec{b}|\, \sin 90° = |\vec{c}|$

$\Rightarrow |\vec{a}||\vec{a}||\vec{b}|\, \sin\theta = |\vec{c}|$

$\Rightarrow 5 \sin\theta = 3$

$\Rightarrow \sin\theta = 3/5$

$\therefore \tan\theta = 3/4$



487. D

$|\vec{a}| = |\vec{c}| = 1,\quad |\vec{b}| = 4$

$\Rightarrow \vec{a} \times \vec{b} = 2\vec{a} \times \vec{c} \Rightarrow \vec{b} = \lambda\vec{a} + 2\vec{c}$

$\Rightarrow \frac{\vec{b} - 2\vec{c}}{\lambda} = \vec{a} \Rightarrow \vec{a} \times \vec{b} - 2\vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times 2\vec{c} = 0 \Rightarrow \vec{a} \times (\vec{b} - 2\vec{c}) = 0$

$\therefore \vec{a}$ is collinear to $\vec{b} - 2\vec{c}$

$\Rightarrow \vec{b} - 2\vec{c} = \lambda\vec{a} \Rightarrow (\vec{b} - 2\vec{c}) = \lambda^2\, |\vec{a}|^2$

$\Rightarrow |\vec{b}|^2 + 4|\vec{c}|^2 - 4\,|\vec{b}|^2\,|\vec{c}|^2 \cos\theta = \lambda^2\, |\vec{a}|^2$

$\Rightarrow 16 + 4\,(1)^2 - 4 \times 4 \times 1 \times \frac{1}{4} = \lambda^2$

$\Rightarrow 16 + 4 - 4 = \lambda^2 \Rightarrow \lambda = \pm 4$

488. B

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\Rightarrow [\vec{a} + \vec{b} + \vec{c} \quad \vec{a} + \vec{b} \quad \vec{b} + \vec{c}]$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})\,.\ \{(\vec{a} + \vec{b}) \times (\vec{b} \times \vec{c})\}$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})\,.\ \{(\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c})\}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{a}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}]$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

$$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = 1 \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = \pm 1$

489. C

Given, $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}$ ...(i), $\vec{\beta}+\vec{\gamma}+\vec{\delta}=b\vec{\alpha}$ ...(ii)

From Eqs.(i)$\Rightarrow \vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=(a+1)\vec{\delta}$ ...(iii)

From Eqs.(ii)$\Rightarrow \vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=(b+1)\vec{\alpha}$ ...(iv)

From Eqs.(iii)&(iv)$\Rightarrow (a+1)\vec{\delta}=(b+1)\vec{\alpha}$ ...(v)

Since $\vec{\alpha}$ is not parallel to $\vec{\delta}$

$\therefore$ From Eq. (v) $\Rightarrow$ a + 1 = 0 and b + 1 = 0

From Eq. (iii) $\Rightarrow \vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=0$

490. C

We have, z = 0 for the point where the line intersects the curve $\therefore \frac{x-2}{3}=\frac{y+1}{2}=\frac{0-1}{-1}$

$\Rightarrow \frac{x-2}{3}=1$ and $\frac{y+1}{2}=1 \Rightarrow x=5$ & $y=1$

Put these values in $xy=c^2$

we get, $5=c^2 \Rightarrow c=\pm\sqrt{5}$

491. A

The given lines are

$(a+b)x+(a-b)y-2ab=0$ ...(i)

$(a-b)x+(a+b)y-2ab=0$ ...(ii)

$x+y=0$ ...(iii)

The triangle formed by the lines (i), (ii) and (iii) is an isosceles triangle if the internal bisector of the vertical angle is perpendicular to the third side. Now equations of bisectors of the angle between lines (i) and (ii) are

$$\frac{(a+b)x+(a-b)y-2ab}{\sqrt{[(a+b)^2+(a-b)^2]}}=\pm\frac{(a-b)x+(a+b)y-2ab}{\sqrt{[(a-b)^2+(a+b)^2]}}$$

or $x-y=0$ (iv)

and $x+y=2b$ (v)

Obviously the bisector (iv) is perpendicular to the third side of the triangle. Hence, the given lines form an isosceles triangle.

492. C

Equation of chord of contact from $A(x_1, y_1)$ is

$xx_1+yy_1-a^2=0$

$xx_2+yy_2-a^2=0$

$xx_3+yy_3-a^2=0$

i.e., $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0 \Rightarrow$ A, B, C are collinear.

493. A

Here $(O_1O_2)^2=t^2+(t^2+1)^2=t^4+3t^2+1\geq 0$

$\Rightarrow O_1O_2\geq 1$ and $|r_1-r_2|=1$

$\Rightarrow O_1O_2\geq|r_1-r_2|$ hence the two circles have at least one common tangent.

494. A

Let normals at points $A(at_1^2, 2at_1)$ and $C(at_3^2, 2at_3)$ meets the parabola again at points $B(at_2^2, 2at_2)$ and $D(at_4^2, 2at_4)$,`

then $t_2=-t_1-\frac{2}{t_1}$ and $t_4=-t_3-\frac{2}{t_3}$

Adding $t_2+t_4=-t_1-t_3-\frac{2}{t_1}-\frac{2}{t_3}$

$\Rightarrow t_1+t_2+t_3+t_4=-\frac{2}{t_1}-\frac{2}{t_3}$

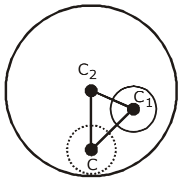
$\Rightarrow \frac{1}{t_1}+\frac{1}{t_3}=0 \qquad \Rightarrow t_1+t_3=0$

Now, point of intersection of tangent at A and C will be $(at_1t_3, a(t_1+t_3))$

Since $t_1+t_3=0$, so this point will lie on x-axis, which is axis of parabola.

495. D

Let C_1, C_2 the centres and r_1, r_2 be the radii of the two circles. Let $S_1=0$ lies completely inside the circle. $S_2=0$. Let C and r be centre and radius of the variable circle, respectively.



then $CC_2=r_2-r$ and

$C_1C=r_1+r$

$\Rightarrow C_1C+C_2C=r_1+r_2$ (constant)

$\Rightarrow$ Locus of C is an ellipse

$\Rightarrow S_2$ is true

Statement 1 is false

(two circles are intersecting).

496. D

We have $\sqrt{(\lambda-3)^2+16}-4=1$

$\Rightarrow \lambda=0$ or 6

497. B

$\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot\vec{c}=0$ only if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar. $\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=0$

Hence, Statement 2 is true.

Also, $[\vec{a}-\vec{b}\;\; \vec{b}-\vec{c}\;\; \vec{c}-\vec{a}]=0$

even if $[\vec{a}\,\vec{b}\,\vec{c}]\neq 0$.

Hence, statement 2 is not the correct expanation for statement 1

498. B

Let $\vec{d}=\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3\vec{c}$

$\Rightarrow [\vec{d}\,\vec{a}\,\vec{b}]=\lambda_3[\vec{c}\,\vec{a}\,\vec{b}]\Rightarrow\lambda_3=1$

$[\vec{c}\,\vec{a}\,\vec{b}]=1$ (because $\vec{a}$, $\vec{b}$, and $\vec{c}$ are three mutually perpendicular unit vectors)

Similarly, $\lambda_1=\lambda_2=1 \Rightarrow \vec{d}=\vec{a}+\vec{b}+\vec{c}$

Hence Statement 1 and Statement 2 are correct, but Statement 2 does not explain Statement 1 as it does not give the value of dot products.

499. A

Any point on the first line is

$(2x_1+1, x_1-3, -3x_1+2)$

Any point on the second line is

$(y_1+2, -3y_1+1, 2y_1-3)$.

If two lines are coplanar, then

$2x_1-y_1=1$, $x_1+3y_1=4$ and $3x_1+2y_1=5$ are consistent.

500. C

Any point on the line $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+2}{2}$ is $B(t+1, -t, 2t-2)$, $t\in R$.

Also, AB is perpendicular to the line, where A is (1, 2, −4).

$\Rightarrow 1(t)-(-t-2)+2(2t+2)=0$

$\Rightarrow 6t+6=0 \qquad \Rightarrow t=-1$

Point B is (0, 1, −4)

Hence, $AB=\sqrt{1+1+0}=\sqrt{2}$