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4. FORCES AND LAWS OF MOTION

1. INTRODUCTION

In this chapter, we study in detail the actual consequences due to motion, i.e., the concept of force, which we specifically define as a push or pull experienced by a particular body or system. As we are of aware of the fact that the equation(s) of motion is/are governed by the choice of reference frame made, we, therefore, also concentrate on the same by involving different types of reference frames.

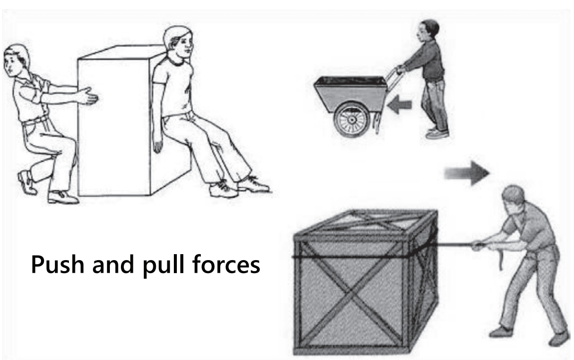


Figure: 4.1

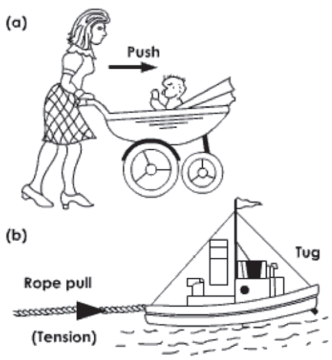


Figure: 4.2

2. FORCE

Force, by its nature, is better understood as any influence that causes an object to undergo a certain change, which maybe with respect to its movement, direction, or even geometrical construction. To be succinct, suffice it to say that a force can facilitate an object with mass to change its velocity, either to accelerate or deform a flexible object, or both. However, we can also define force using intuitive concepts such as a push or a pull. As mandated for a vector quantity, force has both magnitude and direction. We generally measure force based on the SI unit (of Newton) and represent the same using the symbol 'F.' It is imperative to understand, therefore, that in case if a body is subjected to more than one force, then the actual net force acting on that particular body is invariably a vector addition of all the forces in operation.

3. FREE BODY DIAGRAM

Suppose that we indicate all the operative external forces on an object, then the representation of the same is what we call as a free body diagram (FBD) of that particular object.

(a) Weight of a body/object: Weight of a body or an object is generally regarded as the force with which earth attracts that particular body/object toward its center. For example, if we consider 'M' as the mass of a body/

object and 'g' as its acceleration due to gravity, then we can conveniently express the weight of that particular body/object as Mg. However, we always consider that the weight of a body/object is in a direction that is vertically downward.

(b) Normal force: To understand the concept of normal force, let us consider a book resting on a table, as an example. The book has a specific weight, specifically in vertically downward direction and is at rest to begin with. Therefore, we understand that there is definitely one more force that is operative on the block but in an opposite direction, which helps to balance its weight. The source of this force is none other than the table and we hence call the same as a normal force. This signifies the fact that if in case two bodies are in contact with each other, then a contact force arises; further, if the contact surface is smooth, then the direction of the force is usually normal to the plane of contact. As stressed earlier, we always mean that its direction is towards the body under consideration.

Figure: 4.3

(c) Tension in a string: Let us assume that there is a block hanging from a fixed surface by a string. The weight of this block is acting vertically downward although it is not under motion; hence, its weight is adequately balanced by a force originating from the string. We call this force as 'tension in string.' Thus, we define 'tension' as a resisting force that is operative in a stretched string. Further we understand that its direction is along the string but away from the body/object under consideration.

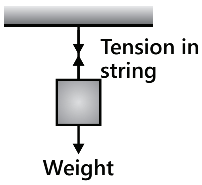


Figure: 4.4

Illustration 1: A cylinder of weight w is resting on groove V as shown in the Figure 4.5. Draw the FBD of the same. **(JEE MAIN)**

Figure: 4.5

Sol: Weight acts vertically downwards and contact force from the surface is normal to the surface at the point of contact.
The FBD of the cylinder is as shown in the Figure 4.6.

Here, w = weight of the cylinder and represent the normal reactions between the cylinder and the two inclined walls.

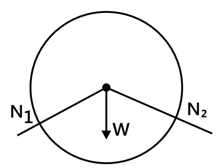


Figure: 4.6

Illustration 2: A block of mass m is attached with two strings as shown in the Figure 4.7. Draw the FBD of the same. **(JEE ADVANCED)**

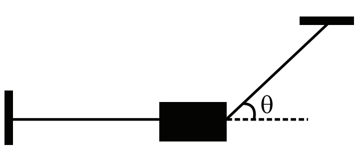


Figure: 4.7

Sol: Weight acts vertically downwards and tension forces along the length of the strings. The FBD of the block is as shown in the Figure 4.8.

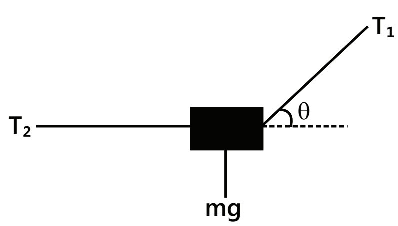


Figure: 4.8

4. NEWTON'S FIRST LAW OF MOTION

Suppose that if a body is observed from an inertial frame i.e., a frame which is at rest or moving with uniform velocity, then it will remain at rest or continue to move with uniform velocity unless an external force is applied on it. This property due to which a body remains at rest or continues its motion with uniform velocity is called as inertia. Force is push or pull, which disturbs or tends to disturb inertia of rest or inertia of uniform motion of a body. Thus, Newton's first law of motion defines inertia, force and intertial frame of reference. One example in this regard is the straight line motion of a body in the absence of the constraining force.

Illustration 3: A heavy particle of mass 0.50 kg is hanging from a string fixed with a roof. Find the force exerted by the string on the particle. (Take $g = 9.8\ m/s^2$) **(JEE MAIN)**

Sol: The weight of the particle is balanced by the force of tension in the string.

The forces acting on the heavy particle are

(a) Pull of the earth $0.50\ kg \times 9.8\ m/s^2 = 4.9\ N$, vertically downward

(b) Pull of the string, T, vertically upward.

The heavy particle is at rest with reference to position of the earth (which we assume to be an inertial frame). Hence, the sum of forces should be zero. Therefore, T is 4.9 N when acting vertically upward.

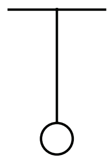


Figure: 4.9

Illustration 4: The given diagram shows the forces in operation on a block. Determine whether the block is under acceleration or not. **(JEE MAIN)**

Sol: If the net force on the block is non-zero then the block accelerates. If the net force on the block is zero, then acceleration is zero.

To check whether the particle will have any acceleration or not, let us confirm if net force is zero or not by resolving the forces in both horizontal and vertical directions.

Net force in horizontal direction $= 4\cos 30° - 4\cos 30° = 0$

Net force in vertically downward direction $= 8 - 4\sin 30° - 4\sin 30° \neq 0$

The net force is not zero. Therefore, the particle will have downward acceleration.

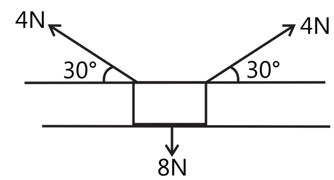


Figure: 4.10

5. INERTIA

Inertia is the resistance of any physical object to any change in its state of motion (including a change in direction). However, we need to understand that inertia is actually a passive property. Further, it does not permit a body to do anything but resists active agents such as torques and forces. In other words, it is tendency of objects to keep moving in a straight line at constant linear velocity.

5.1 Types of Inertia

There are basically three types of inertia.

5.1.1 Inertia of Rest

The inability of a body to change its state of rest by itself is known as inertia of rest. For example.

When we happen to shake the branch of a tree, we observe that the leaves or the fruits fall down. This is because the branches comes in motion, whereas the leaves or the fruits tend to remain at rest and hence fall down.

5.1.2 Inertia of Motion

The inability of a body to change its state of uniform motion by itself its state of uniform motion is known as inertia of motion.

Example: (i) When a moving car suddenly stops, we know that the person sitting in the car falls in the forward direction. This is because the lower portion of the person's body in contact with the car comes to rest, whereas the upper portion tends to remain in motion due to inertia of motion.

(ii) A person runs a certain distance before taking a long jump. This is mainly because the velocity acquired by running prior to attempting a long jump is added at the time of jump, so that he or she can cover a long distance.

5.1.3 Inertia of Direction

The inability of a body to change by itself its direction of motion is referred to as inertia of direction.

Example: (i) When a car moves around a curve, a person sitting inside it is thrown outward. This is to ensure his or her direction of motion.

5.2 Linear Momentum

The principle of linear momentum helps us to have a measure of an object's translational motion. The linear momentum p of a single particle is defined as the product of the mass m and velocity v of a particle in motion.

i.e., p = mv.

Linear momentum is a vector quantity. Its direction is in accordance with the direction of the velocity. The net momentum of a system of particles is the sum of momenta. In a system of two particles with masses m_1 and m_2 and, having velocities v_1 and v_2, respectively, the total momentum, $p = p_1 + p_2 = m_1v_1 + m_2v_2$

In the same manner, the momenta of more than two particles can be added.

6. NEWTON'S SECOND LAW OF MOTION

Newton's second law states that the net force on an object is equal to the rate of change of its linear momentum, p (i.e., the derivative) in an inertial reference frame: $F = \frac{dp}{dt} = \frac{d(mv)}{dt}$.

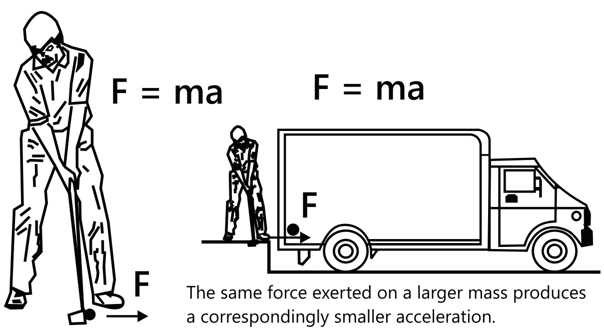


Figure: 4.11

However, the second law can also be stated in terms of an object's acceleration. As the law is valid only for constant-mass systems, the mass can be considered outside of the differentiation operator by the constant factor rule in differentiation. Thus,

$F = m\frac{dv}{dt} = ma$, Where F is the net force applied, m is the mass of the body, and a is the body's acceleration.

Thus, we now know that the net force applied to a body results in a proportional acceleration. In other words, if a body is in an accelerating mode, then there is force acting on it. Both force and acceleration are vector quantities (as denoted by the bold type in the Figure 4.11). This shows that they have both a magnitude (size) and a direction relative to some reference frame.

Illustration 5: Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 2 kg mass. If = 10 N and = 5 N, find the acceleration. **(JEE MAIN)**

Sol: Apply Newton's second law of motion.

Acceleration, as we already know, will be in the direction of the net force and hence will have magnitude as given by

$\sum\vec{F} = m\vec{a}$; $\vec{F} = \vec{F}_1 + \vec{F}_2 \Rightarrow |\vec{F}| = \sqrt{10^2 + 5^2 + 2.10.5\cos 120^o} + 5\sqrt{3}N$

$\Rightarrow |\vec{a}| = 2.5\sqrt{3}m / sec^2$

Further, if the resultant force is at angle α with $\vec{F}_1$

$\tan\alpha = \dfrac{5\sin120°}{10+5\cos120°} \Rightarrow \alpha = 30°$

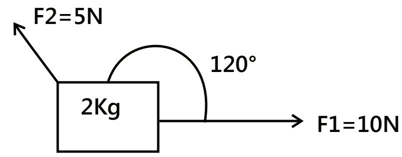


Figure: 4.12

Therefore acceleration is $2.5\sqrt{3}m / sec^2$ at an angle 30° with the direction of $\vec{F}_1$

Illustration 6: A block of mass M is pulled on a smooth horizontal table by a string making an angle with the horizontal as shown in the Figure 4.13. If the acceleration of the block is a, find the force applied by both the string and the table on the block. **(JEE ADVANCED)**

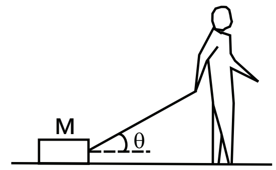


Figure: 4.13

Sol: List all the forces acting on the block. Take components of forces along the horizontal and the vertical. Apply Newton's second law along the horizontal and along the vertical.

Let us consider the block as the whole system. Therefore, the forces acting on the block are

(a) Pull of the earth, Mg, vertically downward,

(b) Contact force by the table, N, vertically upward, and

(c) Pull of the string, T, along the string.

Please observe the provided free body diagram for the block.

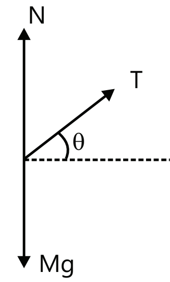


Figure: 4.14

The acceleration of the block is horizontal and toward the right. Now, take this direction as the x-axis and vertically upward direction as the y-axis. Therefore, we have

Component of Mg along the x-axis = 0; component of N along the x-axis = 0

Component of T along the Xx-axis = T cos

Hence, the total force along the x-axis = T cos.

Now, applying Newton's second law, T cos = Ma. ... (i)

Component of Mg along the y-axis =-Mg

Component of N along the y-axis = N

Component of T along the y-axis = T sin

The total force along the y-axis = N + T sin − Mg.

Again applying Newton's second law, N + T sin − Mg = 0; ... (ii)

From equation (i), $T = \dfrac{Ma}{\cos\theta}$. Substituting this in equation (ii) $N = Mg - Ma\tan\theta$.

7. BASIC FORCES IN NATURE

The various types of forces in nature can be grouped into four categories as listed hereunder:

(a) Gravitational, (b) Electromagnetic, (c) Nuclear, and (d) Weak.

7.1 Gravitational Force

Any two bodies attract each other by virtue of their masses. Now, the force of attraction between two masses is $F = G\dfrac{m_1m_2}{r^2}$. where, m_1 and m_2 are the masses of the particles and r is the distance between the particles, and G is universal constant having the value $6.67\times10^{-11}\ N - m^2 / kg^2$

Similarly, the force of attraction exerted by the earth on any object is called gravity. The force exerted by the earth on a small body of mass m, kept near the earth's surface is mg in the vertically downward direction.

7.2 Electromagnetic Force

Consider two particles having charges at rest with respect to the observer. Now, the force between them has magnitude $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ where $\varepsilon = 8.85419 \times 10^{-12}\ C^2/N\text{–}m^2$ is a constant

The quantity $\frac{1}{4\pi\varepsilon_0}$ is $9.0 \times 10^9 \frac{N-m^2}{C^2}$

This is called Coulomb force and it acts along the line joining the particles. Electromagnetic force is realized in many forms in our day-to-day life. Some examples having practical importance in this regard are listed hereunder:

(1) Forces between two surfaces in contact, (2) Tension in a string or a rope, and (3) Force due to spring.

7.3 Nuclear Force

The nuclear force (or nucleon–nucleon interaction or residual strong force) is the actual force between two or more nucleons. However, its fundamental laws are unknown as of now unlike the laws of Coulomb and Newton. This force is responsible for binding protons and neutrons in an atomic nucleus.

7.4 Weak Force

Weak force is a fundamental force of nature that underlies some forms of radioactivity. This force controls the decay of unstable subatomic particles such as mesons, and initiates the nuclear fusion reaction that fuels the Sun. We should know that the weak force acts upon all known fermions—i.e., elementary particles with half-integer values of intrinsic angular momentum, or spin. Particles are known to interact through the weak force by exchanging force-carrier particles known as the W- and Z particles. These particles are generally heavy, with masses of about 100 times the mass of a proton. It is precisely their heavier nature that defines the extremely short-range nature of the weak force. Understandably, therefore, this makes the weak force appear weak at the low energies associated with radioactivity.

8. NEWTON'S THIRD LAW OF MOTION

According to this law, when two bodies interact, they apply forces to one another that are equal in magnitude but opposite in direction.

However, for simplicity we state this law as, "To every action there is an equal and opposite reaction". Therefore, the third law is known also as the law of action and reaction.

But what is the meaning of action and reaction? Further, which force is "action" and which force is "reaction"? We know that every force that acts on a body is due to the presence other bodies in environment. Suppose that a body A experiences a force due to other body B. Then, the body B will also experience a force due to A. As per Newton's third law, two forces are equal in magnitude and opposite in direction. Therefore, mathematically we represent it as $\vec{F}_{AB} = -\vec{F}_{BA}$ Here, in this case, we can take either $\vec{F}_{AB}$ or $\vec{F}_{BA}$ as action force and the other will be the reaction force. Another important thing is that these two forces always act on different bodies.

Practical examples of law of motion

(a) First Law: "Every object persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."

Before release: Object is in state of rest, air speed is zero, and there is weight but no drag. When the object is released: Object accelerates – airspeed increases.

As drag depends on airspeed – drag increases.

When drag is equal to weight: Object no longer accelerates but holds a constant velocity – terminal velocity.

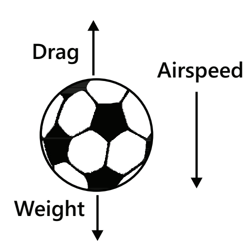


Figure: 4.15

(b) Second Law: Differential form: Force = change of momentum with change of time, i.e., $F = \frac{d(mv)}{dt}$ with mass constant: Force = mass × acceleration F=ma

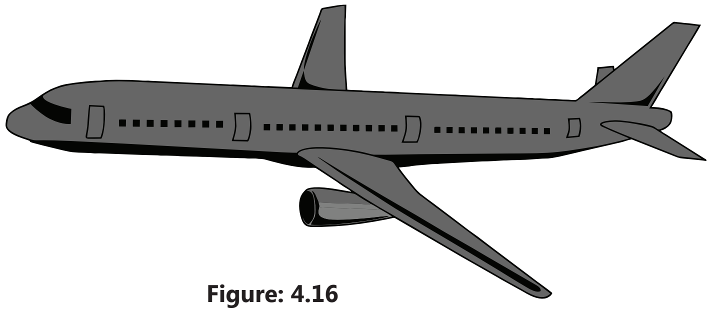


Figure: 4.16

Force = mass × change in velocity with time

$$F = \frac{m(V_1 - V_0)}{(t_1 - t_0)}$$

Hence, each has both magnitude and direction.

(c) Third Law: For every action, there is an equal and opposite reaction.

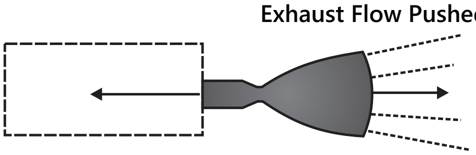


Figure 4.17: Rocket Engine Thrust

PLANCESS CONCEPTS

Working with laws of motion

Step 1: Decide the system: The first step is to decide the system on which the laws of motion to be applied .The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string, etc. The only restriction is that all parts of the system should have identical acceleration.

Step 2: Identify the forces: Once the system is decided, make a list of the forces acting on the system due to all the objects other than the system. Any force applied by the system should not be included in the list of forces.

Step 3: Make a Free Body Diagram (FBD): Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin.

Step 4: Choose the axes and Write Equations: Any three mutually perpendicular directions may be chosen as X-Y-Z axes.

Some suggestions are given below for choosing the axes to solve the problems

PLANCESS CONCEPTS

If the forces are coplanar, only two axes say X and Y, taken in the plane of forces are needed. Choose the X-axis along the direction in which the system is known to have or is likely to have acceleration. A direction perpendicular to it may be chosen as the Y-axis. If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes. Write the components of all the forces along the X-axis and equate their sum to the product of the mass of the system and its acceleration. This gives you an equation. Write the components of the forces along the Y-axis and equate the sum to zero. This gives you another equation. If the forces are collinear, this second equation is not needed.

If necessary you can go to step 1, choose another object as the system, repeat steps 2, 3 and 4 to get more equations. These are called equations of motion. Use mathematical techniques to get unknown quantities out of these equations. This completes the algorithm.

Note: (i) If the system is in equilibrium we will write the two equations as: $\sum F_x = 0$ and $\sum F_y = 0$ (ii) If the system is in collinear, the second equation, i.e. $\sum F_y = 0$ is not needed.

Nivvedan (JEE 2009, AIR 113)

9. IMPULSE

Definition: The impulse of a force is defined as the product of the average force $\vec{F}$ and the time interval Δt during which the force acts: $\text{Impulse} = \vec{F}\Delta t$.

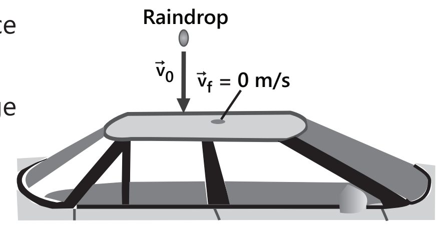


Figure: 4.18

Impulse, hence, is a vector quantity and has the same direction as the average force. The SI unit of impulse is Newton-second (Ns).

However, we can also define impulse as the change in the linear momentum of a body. Forces acting for a very short duration are called impulsive forces.

9.1 Impulse Linear Momentum Theorem

When a net force is in operation on an object, then the impulse of the net force is equal to the change in momentum of the object: $\text{Impulse} = \text{Change in momentum } \bar{F}\Delta t = m\overrightarrow{v_f} - m\overrightarrow{v_0}$

Illustration 7: A truck of mass travelling at 4 m/s is brought to rest in 2 s when it strikes a wall. What force (assume constant) is exerted by the wall? **(JEE MAIN)**

Sol: Force on the truck is the change in momentum per unit time.

Using the relation, impulse = change in linear momentum

We have, $F.t = mv_f - mv_0 = m(v_f - v_0)$ or $F(2) = 2\times10^3\left[0-(-4)\right]$ or $2F = 8x10^3$ or $F = 4x10^3$

Illustration 8: Assume that on a certain day rain comes down at a velocity of –15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof. **(JEE MAIN)**

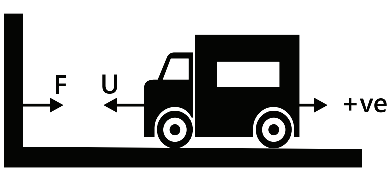


Figure: 4.19

Sol: Force on the roof of the car is equal to the momentum imparted to it per second by rain drops.

$\left(\sum \vec{F}\right)\Delta t = m\vec{v_f} - m\vec{v_0}$; $\quad \vec{F}\Delta t = m\vec{v_f} - m\vec{v_0} \Rightarrow \vec{F} = -\left(\frac{m}{\Delta t}\right)\vec{v_0}$

$\vec{F} = -(0.060\,kg/s)(-15\,m/s) = +0.90\ N$ [Hint: Third law of motion]

Illustration 9: A bullet of mass strikes an obstacle and moves at to its original direction. If its speed also changes from 20 m/s to 10 m/s, then find the magnitude of impulse acting on the bullet. **(JEE ADVANCED)**

Sol: Find the impulse along the initial line of motion and along the perpendicular to the initial line of motion.

Mass of the bullet, m = 10^{-3}kg

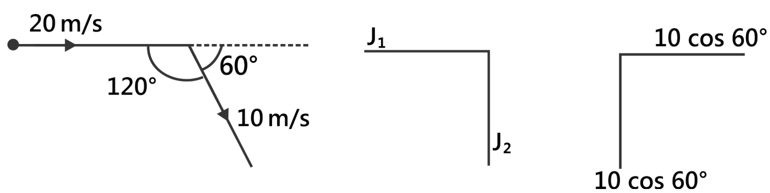


Figure: 4.20

Consider components parallel to J_1 $J_1 = 10^{-3}\left[-10\cos 60° - (-20)\right]$ $J_1 = 15\times10^{-3}$ N.s

Put 8 & 8.1 alter a and adjust the numbering

Similarly, parallel to J_2, we have $J_2 = 10^{-3}\left[10\sin 60° - 0\right] = 5\sqrt{3}\times10^{-3}$ N.s

The magnitude of the resultant impulse is given by

$J = \sqrt{J_1^2 + J_2^2} = 10^{-3}\sqrt{(15)^2 + (5\sqrt{3})^2}$ or $J = \sqrt{3}\times10^{-2}$N.s

10. APPLICATION OF LAWS OF MOTION

10.1 Two Blocks in Contact

If two blocks of masses m_1 and m_2 are in contact on a horizontal frictionless surface, so that a force F applied horizontally imparts an acceleration a and F_c is the contact force, which is equal and opposite for and. , then Newton's second law, when applied to free body diagram, gives the following equations:

$F - F_c = m_1 a$; $\quad F_c = m_2 a$

Adding $a = \frac{F}{m_1 + m_2}$

$\therefore \quad Fc = \frac{m_2 F}{m_1 + m_2}$

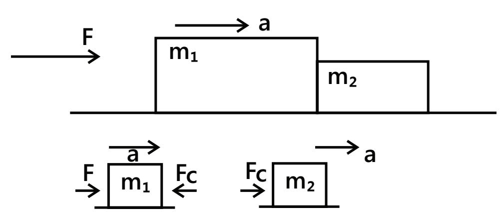


Figure: 4.21

10.2 Blocks Connected by Strings

If two blocks of masses and are connected by an inextensible string so that if force F is applied to and there is an equal and opposite tension T in the string and if a is acceleration of the masses, then Newton's law gives

$F - T = m_1 a$; $T = m_2 a$ Adding $F = (m_1 + m_2)a$ or $a = \frac{F}{m_1 + m_2}$; $\quad T = \frac{m_2 F}{m_1 + m_2}$

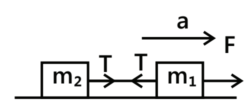


Figure: 4.22

If three blocks of masses, , and are connected by two strings with tension and when a force F applied to imparts an acceleration a to all the blocks, then Newton's law gives the following relations for these three blocks: $F - T_1 = m_1 a$; $T_1 - T_2 = m_2 a$ $T_2 = m_3 a$

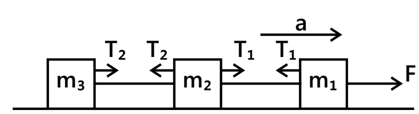


Figure: 4.23

Adding $F = (m_1 + m_2 + m_3)a$ or $a = \frac{F}{m_1 + m_2 + m_3}$

$$T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3};\ T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$$

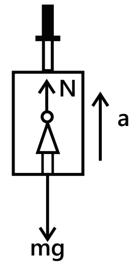


Figure: 4.24

10.3 Apparent Weight in a Lift

If a person is standing in a stationary lift, his or her weight mg acts downward and his or her normal reaction on the floor of the lift acts upward so that N = mg as per Newton's third law of motion. However, if the lift is moving with a constant velocity, then N is equal and opposite to mg as now the net force is zero.

$$N - mg = 0 \quad \text{or} \quad N = mg$$

Thus, the apparent weight is equal to true weight.

Further, if the lift is moving upward with acceleration a, $N - mg = ma$ or $N = m(g + a)$

Thus, the apparent weight is greater than the actual weight.

However, if the lift is accelerating downward with a′, $mg - N = ma'$ or $N = m(g - a')$

Therefore, in this case, the apparent weight is lesser than the actual weight.

10.4 Horse Cart Problem

To analyze this properly, it is probably best to individually consider the cart and the horse. The cardinal rules while dealing with introductory physics courses are first identify and isolate the body that you intend to apply Newton's second law to, and then identify all forces acting on that body and add them (as vectors) to get the net force, and finally, use the relation $F_{net} = ma$.

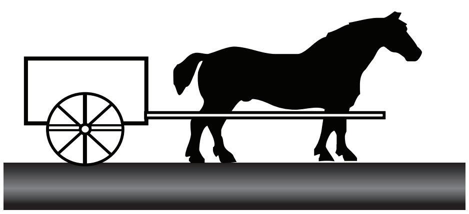
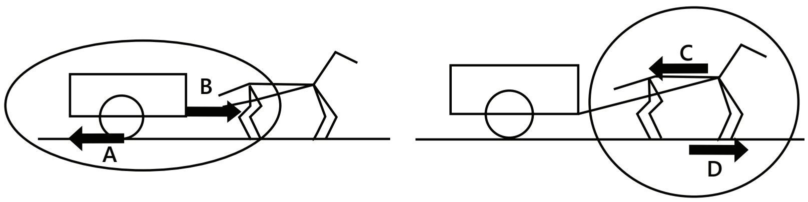


Figure: 4.25

In the diagrams provided, we have used an oval or a circle to enclose the subsystem being analyzed. The forces acting on the cart include the forward force that the horse exerts on the cart and the backward force due to friction at the ground, acting on the wheels. However, at rest, or at constant velocity, these two are equal in magnitude, because the acceleration of the cart is zero.

On the contrary, the forces that are acting on the horse include the backward force the cart exerts on the horse and the forward force of the ground on its hooves. However, at rest, or at constant velocity, these two are equal in magnitude, because the acceleration of the horse is zero. Therefore, $\vec{C} = -\vec{D}$. Similarly, for the cart, $\vec{A} = -\vec{B}$.

By Newton's third law, the force the horse exerts on the cart is of equal size and opposite in direction to the force the cart exerts on the horse. (These two forces are an action–reaction pair.) Therefore, $\vec{B} = -\vec{C}$, and this is true whether or not anything is accelerating.



Forces on the cart, no acceleration. Forces on the horse, no acceleration.

Figure: 4.26

Since the horse is not accelerating, $\vec{C} = -\vec{D}$, by Newton's second law, and, finally we see that all the forces shown in the diagram are of the same size.

Illustration 10: Two blocks of masses and are placed in contact with each other on a frictionless horizontal surface as shown in the Figure 4.27. Constant forces and are applied on and as shown in the Figure 4.27. Find the magnitude of acceleration of the system. Also, calculate the contact force between the blocks. **(JEE MAIN)**

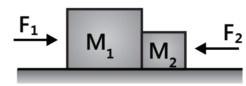


Figure: 4.27

Sol: Draw the FBD of each block. Apply Newton's first law along the vertical and Newton's second law along the horizontal.

In this problem, acceleration of both blocks will be the same as they are rigid and in contact with each other. As the surfaces are frictionless, contact force on any surface will be normal force only. Let us assume that the acceleration of blocks be a and contact forces, and N as shown in free body diagrams of blocks.

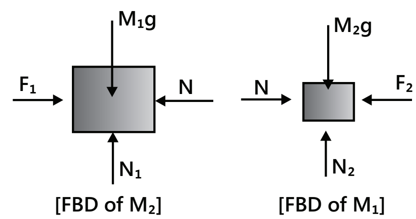


Figure: 4.28

Therefore, by applying, Newton's second law for

$F_1 - N = M_1 a$ ...... (i) and $M_1 g - N_1 = 0$ ....(ii)

Applying, Newton's second law for

$N - F_1 = M_2 a$ ...... (iii) and $M_2 g - N_2 = 0$ ....(iv)

By solving (i) and (iii) $a = \dfrac{F_1 - F_2}{M_1 + M_2}$ and $N = \dfrac{M_2F_1 + M_1F_2}{M_1 + M_2}$

Illustration 11: A rope of length L is pulled by a constant force F. What is the tension in the rope at a distance x from one end where the force is applied? **(JEEMAIN)**

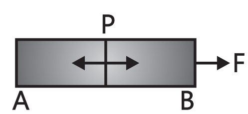


Figure: 4.29

Sol: Acceleration of all parts of the rope will be same. Net force on a part of rope is equal to acceleration multiplied by the mass of that part.

Let AB be a string of length L and F the constant force pulling the rope as shown in the Figure 4.29 provided.

Mass per unit length of rope $= \dfrac{M}{L}$

where M is the total mass. Let P be a point at a distance x from B. If T is the tension in the rope at P then for the part AP, the tension is toward right while for the part PB it is toward left. If a is the acceleration produced in the rope, then for part PB

$F - T = \text{mass of PB} \times a = \dfrac{Mx}{L} a \cdot$ Also for rope, $F = Ma$ $\quad \therefore\ T = \dfrac{F(L-x)}{L}$

Illustration 12: Two blocks each having mass of 20 kg rest on frictionless surfaces as are shown in the Figure 4.30. Assume that the pulleys to be light and frictionless. Now, find:

(a) The time required for the block A to move 1 m down the plane, starting from rest;

(b) The tension in the cord connecting the blocks. $\sin\theta = 3/5$ **(JEE ADVANCED)**

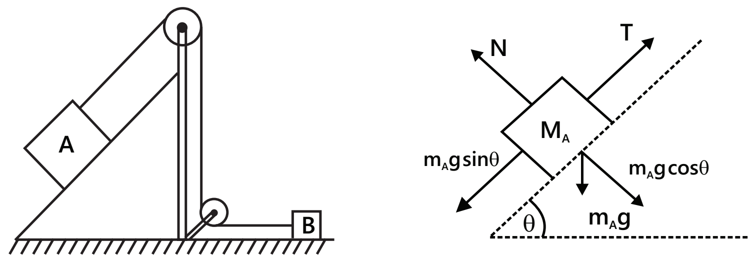


Figure: 4.30

Sol: Draw the FBD of each block. Apply Newton's second law along the direction of motion for each block. Solve the equations obtained to get the values for two variables T and a.

Blocks A and B are considered as two systems. The free body diagrams for the blocks A and B are shown in the Figure 4.31 where T is tension in the string.

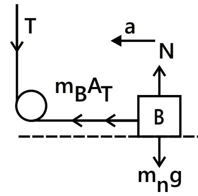


Figure: 4.31

$m_A g\sin\theta - T = m_A a$ ....(i)

$N = m_A g\cos\theta$ ....(ii)

$T = m_B a$ ....(iii)

Adding equation (i) and (iii), $m_A g\sin\theta = (m_A + m_B)a$

$$\Rightarrow a = \left(\frac{m_A}{m_A + m_B}\right)g\sin\theta = \left(\frac{20}{20+20}\right)(10)\left(\frac{3}{5}\right) = 3m/s^2$$

$$\text{(a) } s = \frac{1}{2}at^2; t = \left(\frac{2s}{a}\right)^{\frac{1}{2}} = \left(2\times\frac{1}{3}\right)^{\frac{1}{2}} = 0.82$$

(b) $T = m_B a = 20\times 3 = 60N.$

11. LAW OF CONSERVATION OF LINEAR MOMENTUM

The law of conservation of linear momentum states that if no external forces act on the system of objects, then the vector sum of the linear momentum of each body remains constant and is not affected by their mutual interaction.

By applying the principle of conservation of linear momentum

(a) Decide which objects are included in the system.

(b) Identify the internal and external forces relative to the system.

(c) Verify that the system is isolated.

(d) Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

Illustration 13: From a gun (mass = M) a bullet (mass = m) is fired with speed relative to barrel of the gun which is inclined at an angle of with horizontal. The gun is placed over a smooth horizontal surface. Find the recoil speed of the gun. **(JEE ADVANCED)**

Sol: Apply the law of conservation of linear momentum along the horizontal direction.

Let the recoil speed of the gun is v. By taking gun + bullet as the system, the net external force on the system in horizontal direction is zero. Initially, the system was at rest. Therefore, applying the principle of the conservation of linear momentum in horizontal direction,

we get $Mv - m(v_r\cos 60^\circ - v) = 0$

$$v = \frac{mv_r\cos 60^\circ}{M+m} \text{ or } v = \frac{mv_r}{2(M+m)}$$

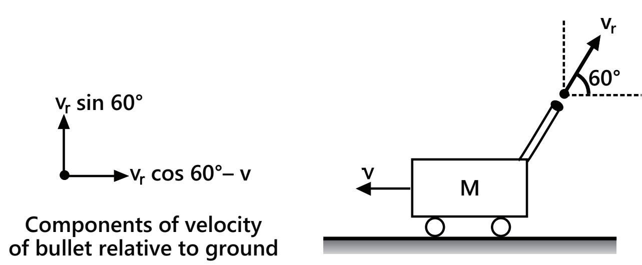


Figure: 4.32

Illustration 14: A man of mass is standing on a platform of mass kept on a smooth horizontal surface. Now, the man starts moving on the platform with velocity relative to the platform. Based on the above, find the recoil velocity of the platform. **(JEE MAIN)**

Sol: Apply the law of conservation of linear momentum along the horizontal direction.

Absolute velocity of the man = – v where v = recoil velocity of the platform. By considering together the platform and the man as a system, the net external force acting on the system in horizontal direction is zero. However, the linear momentum of the system remains

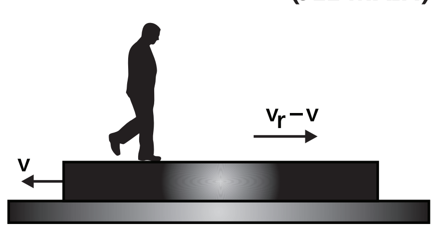


Figure: 4.33

constant. Initially, both the man and the platform were at rest.

Hence, $0 = m_1(v_r - v) - m_2 v$ $\quad \therefore v = \dfrac{m_1 v_r}{m_1 + m_2}$

12. VARIABLE MASS

Problems related to variable mass can be solved in the following three steps

(a) Make a list of all the forces acting on the main mass and then apply them on it.

(b) Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\dfrac{dm}{dt}\right)\right|$ and direction is given by $\vec{v}_r$, in case the mass is increasing otherwise the direction of $-\vec{v}_r$, if it is decreasing.

(c) Find the net force on the mass and then apply $\vec{F}_{net} = m\dfrac{d\vec{v}}{dt}$ (m = mass at that particular instant)

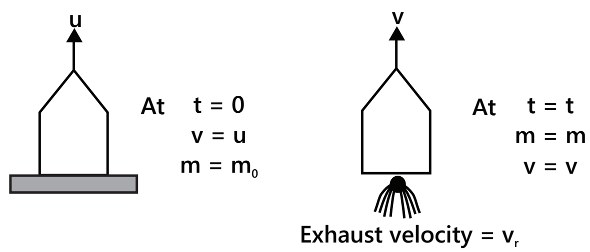


Figure: 4.34

Let m_0 be the mass of the rocket at time t = 0. Let m be its mass at any time t and v its velocity at that moment. Initially, let us suppose that velocity of the rocket is u.

Further, let $\left(-\dfrac{dm}{dt}\right)$ be the mass of gas ejected per unit time and, the exhaust velocity of the gases. Usually $\left(-\dfrac{dm}{dt}\right)$ and, are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time t = t,

(a) Thrust force on the rocket $\quad F_t = v_r\left(-\dfrac{dm}{dt}\right)$ (upwards)

(b) Weight of the rocket $\quad W = mg$ (downwards)

(c) Net force on the rocket $\quad F_{net} = F_t - W$ (upwards)

or $\quad F_{net} = v_r\left(-\dfrac{dm}{dt}\right) - mg$

(d) Net acceleration of the rocket $\quad a = \dfrac{F}{m}$ or $\dfrac{dv}{dt} = \dfrac{v_r}{m}\left(\dfrac{-dm}{dt}\right) - g$ or $dv = v_r\left(\dfrac{-dm}{m}\right) - gdt$

or $\int_u^v dv = v_r \int_{m_0}^{m} \dfrac{-dm}{m} - g\int_0^t dt$ or $v - u = v_r \ln\left(\dfrac{m_0}{m}\right) - gt$

Thus, $$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right) \quad \text{... (i)}$$

Note: $F_t = V_r\left(-\dfrac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\dfrac{dm}{dt}$ is negative.

If gravity is ignored and initial velocity of the rocket u = 0, eq(i) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$

Illustration 15: (a) A rocket set for vertical firing weighs 50 kg and contains 450 kg of fuel. It can have a maximum exhaust velocity of 2 km/s. What should be its minimum rate of fuel consumption?

(a) (i) To just lifting it off from the launching pad?

(ii) To give it an acceleration of 20 m/?

(b) What will be the speed of the rocket when the rate of consumption of the fuel is 10 kg/s after whole of the fuel is consumed? (take g = 9.8 m/). **(JEE ADVANCED)**

Sol: Use the equation of motion for variable mass.

(a) (i) to just lift it off from the launching pad

Weight = thrust force or $mg = v_r\left(\frac{-dm}{dt}\right)$ or $\left(\frac{-dm}{dt}\right) = \frac{mg}{v_r}$

Substituting the value, we get $\left(\frac{-dm}{dt}\right) = \frac{(450+50)(9.8)}{2\times10^3} = 2.45\ kg/s$

(ii) net acceleration $a = 20 m/s^2$

$ma = F_t - mg$ or $a = \frac{F_t}{m} - g$ or $a = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$

This gives $\left(\frac{-dm}{dt}\right) = \frac{m(g+a)}{v_r}$

Substituting the values, we get $\left(\frac{-dm}{dt}\right) = \frac{(450+50)(9.8+20)}{2\times10^3} = 7.45 kg/s.$

(b) The rate of fuel consumption is 10 kg/s. So, the time for the consumption of entire fuel is t = 450/10 = 45 s

Using Eq. (i), i.e., $v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$

Here $u = 0, v_r = 2\times10^3 m/s, m_0 = 500 kg$ and $m = 50 kg$

Substituting the values, we get $v = 0 - (9.8)(45) + (2\times10^3)\ln\left(\frac{500}{50}\right)$

Other Example of Variable Mass System is Falling raindrop

Illustration 16: Suppose that a raindrop falls through a cloud and accumulates mass at a rate of kmv where k > 0 is a constant, m is the mass of the raindrop, and v its velocity. What is the speed of the raindrop at a given time if it starts from rest, and what is its mass? **(JEE ADVANCED)**

Sol: Use the equation of motion for variable mass.

Then, the external force is its weight mg and so we have

$$mg = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} = m\frac{dv}{dt} + kmv^2$$

Since we know that dm/dt = kmv. Cancelling the mass and rearranging $\frac{dv}{dt} = g - kv^2$,

So that, $\int_0^v \frac{dv}{g-kv^2} = \int_0^t dt = t$

Now set $V^2 = g/k$ and use partial fractions to get

$$t = \int_0^v \frac{dv}{g-kv^2} = \frac{1}{2kV}\int_0^v \frac{1}{V+v} + \frac{1}{V-v}dv = \frac{1}{2kV}\log\left(\frac{V+v}{V-v}\right)$$

so, $V + v = (V - v)e^{2kVt}$, i.e. $v = V\left(\frac{e^{2kVt}-1}{e^{2kVt}+1}\right) = V\tanh(Vkt)$, so that $v = \sqrt{\frac{g}{k}}\tanh(\sqrt{kg}t)$

Now we may find the mass : we have $\frac{dm}{dt} = kmv = km\sqrt{\frac{g}{k}}\tanh(\sqrt{kg}t) = m\sqrt{kg}\tanh(\sqrt{kg}t)$.

Thus, $\int_0^t \frac{1}{m}\frac{dm}{dt}dt = \int_0^t \sqrt{kg}\tanh(\sqrt{kg}t)dt$

$$\int_{m_0}^{m} \frac{dm}{m} = \int_0^t \sqrt{kg}\tanh(\sqrt{kg}t)dt$$

$$\log m - \log m_0 = \log\cosh(\sqrt{kg}t)$$

which gives $m = m_0\cosh(\sqrt{kg}t)$

13. EQUILIBRIUM

Equilibrium is the condition of a system, when net external farce is zero.

13.1 Equilibrium of Concurrent Forces

A simple mechanical body is said to be in equilibrium, if it does not experience any linear acceleration; however, unless it is disturbed by an outside force, it will continue in that condition indefinitely. For a body facing concurrent forces, equilibrium arises if the vector sum of all forces acting upon the body is zero. There are two types of equilibrium as listed hereunder:

(a) Static equilibrium: When a body is at rest under the influence of external forces acting on the it.

(b) Dynamic equilibrium: If net external force is zero but the velocity of a body is not zero, i.e., body moves with a constant velocity.

13.2 Constrained Motion

When a motion of a body can be controlled, then the motion is said to be a constrained motion. For example, when a body tied with a string is lowered under the effect of gravity, then its motion is a constrained motion. Also, motion of masses and can be controlled by choosing an appropriate value for and.

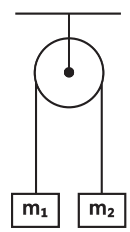


Figure: 4.35

13.2.1 Masses Connected by Pulley and Constraint Relation

Let us consider blocks of masses and connected by a string and passing over the pulley as shown in the Figure 4.36. Let be the acceleration of downward and be the acceleration of upward. Let T is the tension in the string, so that the pulley moves clockwise. For block,

As there are three unknown parameters, we take the following steps for writing the constraint relation and hence find the parameters:

(a) Assume direction of acceleration of each body.

(b) Locate position of each block from any fixed point like, for example, center of the pulley.

(c) Identify the constraint and write the equation of constraint in terms of distance.

(d) Write the equation of constraint and hence differentiate twice to find one of the parameters.

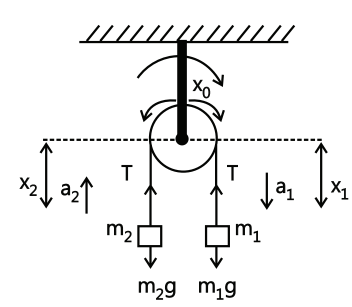


Figure: 4.36

In this case, the string is inextensible; therefore, the constraint the length of string remains constant. If is the length of the string passing over the pulley, and lengths of string from the pulley to and respectively, then the total length L of the string remains constant.

$\therefore x_1 + x_2 + x_0 = L = \text{constant}$

Differentiate, $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$; Differentiate, $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

As and have opposite directions,

$a_1 - a_2 = 0$; $\quad a_1 = a_2 = a \quad \therefore m_1g - T = m_1a$; $\quad T - m_2g = m_2a$

Adding $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$; $\quad T = m_2a + m_2g = \frac{m_2(m_1 - m_2)g}{m_1 + m_2} + m_2g$; $\quad T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$

If the pulley is pulled in upward direction with an acceleration a, then $T = \left(\frac{2m_1m_2}{m_1 + m2}\right)(g + a)$

Illustration 17: Find the relationship between accelerations of blocks A and B based on the Figure 4.37. **(JEE MAIN)**

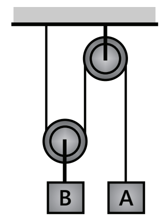


Figure: 4.37

Sol: Measure all distances of pulley and block from a fixed point (stationary point).

The physical property that we can use here is the inextensibility of string,

i.e., ab + bc + cd + de + ef = constant.

Let at any moment A and B are distances and from the support as shown in the Fig. 4.37.

Let us take gh= and ik= and hence express the length of string in equation (i) in terms of, l_1 and l_2.

We hence obtain $X_B - l_1 + bc + (X_B - l_1 - l_2) + de + (X_A - l_2) = \text{constant}$ Here,, bc and de are constants.

$\therefore 2X_B + X_A = \text{constant}$ ... (i)

Let at time Δt, changes to + Δ and changes to - Δ

[therefore, B is assumed to move downward]

Then, $2(X_B + \Delta X_B) + (X_A - \Delta X_B) = \text{constant}$ ... (ii)

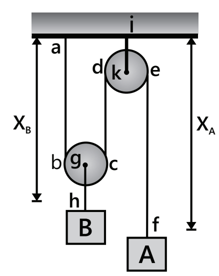


Figure: 4.38

From (i) and (ii) $2\Delta X_B - \Delta X_A = 0$

Also, $\left(\frac{2\Delta X_B}{\Delta t}\right) - \left(\frac{\Delta X_A}{\Delta t}\right) = 0$; $\quad 2V_B - V_A = 0$ also, $2\Delta V_B - \Delta V_A = 0$

$\frac{2\Delta V_B}{\Delta t} - \frac{\Delta V_A}{\Delta t} = 0 \quad \therefore 2a_B = a_A$

Hence, we prove that magnitude of acceleration of A is twice the magnitude of acceleration of B.

Let us assume that B moves by a distance x during an interval of time, and this will cause movement of pulley g by x. Now, an extra length of 2x of string will come to the left of pulley k. This must be coming from the right side of the pulleys. Hence, displacement of A will be 2x. On the basis of this discussion, we can say that if acceleration of block B is a, then the acceleration of A will be 2a.

14 PSEUDO FORCE

4.1 Inertial and Non-inertial Frames of References

Non-accelerated frames of reference are called inertial frames, whereas accelerated frames are called non-inertial frames. If one is travelling in a train which is accelerating forward, the body in the train is pushed backward and he or she is pushed forward when the brakes are applied. This is due to inertia of the body. Such an accelerated frame is called a non-inertial frame.

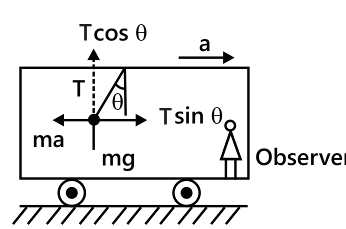


Figure: 4.39

In order to make Newton's laws applicable to such a frame, a fictitious force or pseudo force is applied on the body. Based on the above discussion, we now understand that the magnitude of this pseudo force is equal to the product of the mass m of the body and acceleration a of the reference frame and its direction is opposite to the acceleration of the frame.

$\therefore$ pseudo force, $F = -m \times a$. Thus in a non-inertial frame trolley moving with an acceleration a hanging bob of mass m will be deflected through an angle due to a pseudo force acting in backward direction. In the non-inertial frame of reference, this bob is in equilibrium under the action of force due to tension T, weight mg and the pseudo force ma in a direction making an angle with the vertical.

$T\sin\theta = ma$; $T\cos\theta = mg$; $\tan\theta = \frac{a}{g}$ or $\theta = \tan^{-1}\left(\frac{a}{g}\right)$

Example: Motion of a block on an inclined plane is an example of accelerated frame of motion.

Motion of a Block on a Smooth Inclined Plane: Let us consider a block of mass m placed on a frictionless inclined plane, inclined at an angle to the horizontal. We observe that the normal reaction N acts perpendicular to the plane and its weight is resolved into component mg sin along the plane which slides the block downward with acceleration a and component mg cos perpendicular to the plane downward, which is equal and opposite to the normal reaction.

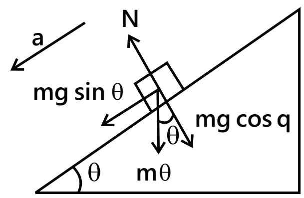


Figure: 4.40

$\therefore mg\sin\theta = ma$ $\quad mg\cos\theta = N$ $\quad$ or $a = g\sin\theta$ However, if the plane is provided with a horizontal acceleration a′ in the horizontal direction as shown in the Figure 4.44, then the body lies in an accelerating frame of reference and a pseudo force ma′ acts horizontally in a direction opposite to that of a′ because an inertial force ma′ acts on it in the direction of a′. Thus ma′ can be resolved into a component ma′cos up the plane and ma′sinperpendicular to the plane in the downward direction as shown in the Figure 4.44. From Newton's second law of motion, we know that: $mg\sin\theta - ma'\cos\theta = ma$ or $a = g\sin\theta - a'\cos\theta$

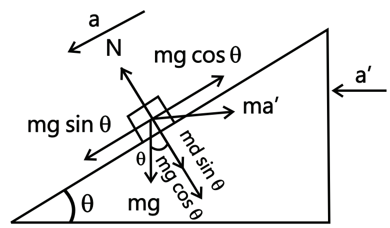


Figure: 4.41

$N = m(g\cos\theta + a'\sin\theta)$. If the body is at rest relative to the inclined plane, then a=0 or $g\sin\theta = a'\cos\theta$ or $a' = g\tan\theta$

Illustration 18: A frictionless block 3 carries two other frictionless blocks 1 and 2 connected by a light string passing over a weightless and frictionless pulley as shown in the Figure 4.42. What horizontal force must continuously be applied to block 3 so that 1 and 2 do not move relative to 3? **(JEE ADVANCED)**

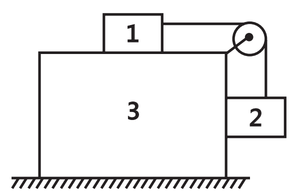


Figure: 4.42

Sol: Analyze the motion of blocks 1 and 2 in the reference frame of block 3. As block 3 is accelerated, blocks 1 and 2 experience pseudo forces in the frame of block 3.

Let a_1, a_2 and a_3 be the accelerations of 1, 2, and 3, respectively. Let a_{1x}, a_{2x} and a_{3x} be the absolute horizontal acceleration of 1, 2 and 3 to the right and a_{1y}, a_{2y} and a_{3y} be their downward accelerations. According to the constraints of the problem $a_{1y} = 0$, $a_{3y} = 0$

Let a_{13x} = relative acceleration of 1 w.r.t. 3. $= a_{1x} - a_{3x} = 0 \Rightarrow a_{3x} = a_{1x}$

a_{23y} = relative acceleration of 2 w.r.t 3. $= a_{2y} - a_{3y} = a_{2y} - 0 \Rightarrow a_{2y} = 0$

a_{23x} = relative acceleration of 2 w.r.t. 3. $= a_{2x} - a_{3x} = 0 \Rightarrow a_{2x} = a_{3x}$

$\therefore a_{1x} = a_{2x} = a_{3x}$ and $a_{1y} = a_{2y} = a_{3y} = 0$

Consider the free-body diagrams of blocks 1 to 3.

From the FBD, we obtain the following equations.

$T = m_1 a_{1x}$ ....(i)

$N - m_1 g = 0$ ....(ii)

} For Block I

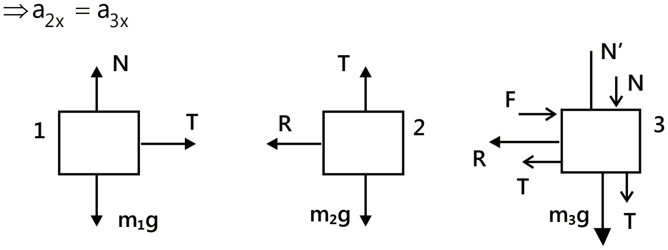


Figure: 4.43

$$\left.\begin{array}{ll} R = m_2 a_{2x} & \text{........(iii)} \\ T - m_2 g = 0 & \text{.......(iv)} \end{array}\right\} \text{For Block II}$$

$$\left.\begin{array}{ll} F - R - T = m_3 a_{3x} & \text{.......(v)} \\ N' - N - T - m_3 g = 0 & \text{........(vi)} \end{array}\right\} \text{For Block III}$$

Adding (i), (iii) and (v)

$$F = m_1 a_{1x} + m_2 a_{2x} + m_3 a_{3x} = a_{1x}(m_1 + m_2 + m_3) \quad (\therefore a_{1x} = a_{2x} = a_{3x})$$

From (i) and (iv), $a_{1x} = \frac{m_2}{m_1}.g$; $F = (m_1 + m_2 + m_3).\frac{m_2}{m_1}.g$

15. FRICTION

If there are two bodies in contact with each other, then the force which opposes the relative motion between two bodies in contact is called force of friction. Further, the magnitude of the frictional force depends upon the nature of two surfaces in contact. This is primarily due to surface irregularities at molecular levels, with the result that even a highly polished surface has irregularities. This results in producing interlocking of uneven surfaces. Once there is smooth motion of the body, the friction is less than the maximum force of static friction or limiting friction.The variation of force of friction with the applied force is shown in the graph when any block is moving over another surface. However, when any block is at rest, the resultant force of static friction is equal to the force applied. Then, it reaches to a maximum value at A, the limiting friction. Once the motion resumes, a lesser force is required for maintaining uniform motion.

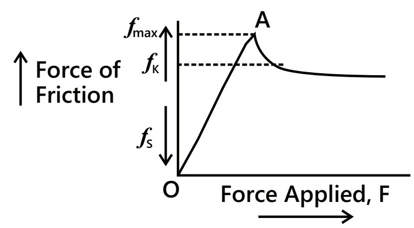


Figure: 4.44

5.1 Static Friction

Suppose that if the force applied in the horizontal direction on a surface is less, then the body does not have any motion because an equal and opposite frictional force is present. Hence, it is clear that static friction is in operation only between surfaces that are at rest with respect to each other. As F is increased the frictional force too increases continuously until a stage is reached when the body is just at the point of sliding. The force of friction at this stage is called a limiting friction or maximum static friction.

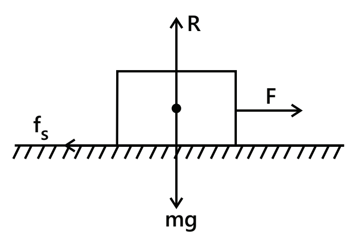


Figure: 4.45

15.2 Coefficient of Friction

The coefficient of friction, generally specified by the Greek letter mu (μ), is the ratio of limiting or maximum value of force of friction   to reaction R between surfaces when the body is just about to move. $\mu = \frac{f_{max}}{R}$; $f_{max} = f_{limiting} = \mu_s R$

Thus if the body is not in motion, the static frictional force   and external applied force parallel to the surface are equal in magnitude but opposite in direction and hence F is directly proportional to . However, if external force F exceeds, then the body slides on the surface and magnitude of frictional force decreases than, the frictional force for sliding = = R where is the coefficient of kinetic friction. As, is less than, the coefficient of kinetic friction is less than the coefficient of static friction.

15.3 Angle of Friction

The angle between the normal reaction R and the resultant of limiting friction with normal reaction is called the angle of friction and is denoted by λ.

$\tan\lambda = \frac{f_{max}}{R} = \mu$ or $\lambda = \tan^{-1}(\mu)$ Suppose that if a body of mass m is placed on an inclined

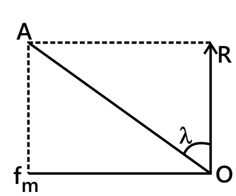


Figure: 4.46

plane whose inclination is gradually increasing. Then the body just starts sliding down at a certain angle of inclination θ. Now, the weight mg can be resolved into a component mg sin due to which the body is about to slide down against maximum or limiting value of friction and therefore the second component mg cos balances the normal reaction R perpendicular to the inclined plane.

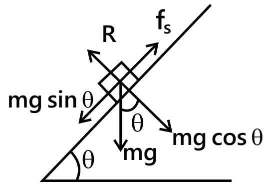


Figure: 4.47

$\therefore f_s = mg\sin\theta$; $R = mg\cos\theta$; $\therefore \frac{f_s}{R} = \tan\theta = \mu$ as $\mu = \tan\lambda$; $\theta = \lambda = \tan^{-1}(\mu)$

Here, the angle θ is called angle of inclination. We now know that the angle of friction λ is that minimum angle of inclination of the inclined plane at which a body placed at rest on the inclined plane just starts sliding down.

However, when $\theta < \lambda$, then the body is in equilibrium and does not slide. On the contrary, when $\theta > \lambda$, then the body starts sliding down with an acceleration.

15.4 Motion of a Block on a Rough Inclined Plane

Let us assume that a block of mass m is moving down an inclined plane with an acceleration a. Now, the coefficient of friction between the block and inclined plane equal to μ, the force of friction μN will be acting along the plane upward as shown in the Figure 4.48. Thus, the weight mg of the block is resolved into component mg cosθ opposite of normal reaction and component mg sin downward opposite to μN.

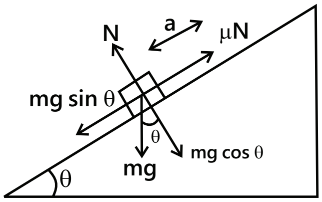


Figure: 4.48

Thus, from Newton's second law of motion, $mg\sin\theta - \mu N = ma$

$N = mg\cos\theta$ or $mg\sin\theta - \mu mg\cos\theta = ma$ $\therefore a = g(\sin\theta - \mu\cos\theta)$

However, if the block is moving upward and its retardation is a, where the frictional force acts downward, then

$ma = mg\sin\theta + \mu mg\cos\theta$ $\therefore a = g\sin\theta + \mu g\cos\theta$.

PLANCESS CONCEPTS

Value of friction is not always equal to μN. Further, μN is the maximum value of friction. Friction does not oppose motion; rather, it opposes relative motion between two surfaces.

Anand K (JEE 2011, AIR 47)

Illustration 19: A heavy box of mass 20 kg is pulled on a horizontal surface by applying a horizontal force. If the coefficient of kinetic friction between the box and the horizontal surface is 0.25, then find the force of friction exerted by the horizontal surface on the box. **(JEE MAIN)**

Sol: Force of friction on a body sliding on a surface is equal to the normal reaction multiplied by the coefficient of kinetic friction between the pair of surfaces.

The situation is shown in the Figure 4.49. In the vertical direction, there is no acceleration; therefore, N=mg.

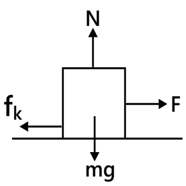


Figure: 4.49

As the box slides on the horizontal surface, the surface exerts kinetic friction on the box. Therefore, the magnitude of the kinetic friction is $f_k = \mu_k N = \mu_k Mg$

$= 0.25 \times (20\text{kg}) \times (9.8\text{m/s}^2) = 49\text{N}$. This force thus acts in the direction opposite to that of the pull.

Illustration 20: Two blocks, M_1 and M_2, connected by a massless string slide down an inclined plane, having an angle of inclination of. The masses of the two blocks are = 4 kg and = 2 kg, respectively and the coefficients of friction of and with inclined plane are 0.75 and 0.25, respectively. Assuming the string to be taut, find

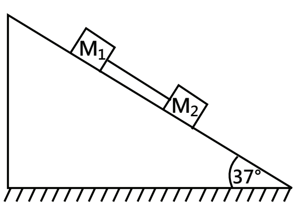


Figure: 4.50

(a) The common acceleration of the two masses and

(b) The tension in the string. (note: sin =0.6, cos =0.8) **(JEE ADVANCED)**

Sol: Let each block is having acceleration a down the incline plane. Draw the FBD of each block and apply the Newton's second law of motion along the direction of motion. Solve the equations obtained to get the value of two variables a and T.

Let a be the common acceleration of the system and T be the tension in the string $\mu_1 = 3/4,\ \mu_2 = 1/4$

Equation of motion for M_1 and M_2 are

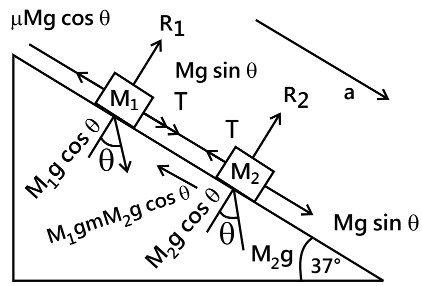


Figure: 4.51

$M_1a = M_1 g\sin 37° + T - \mu_1 M_1 g\cos 37°$ ......(i)

$M_2a = M_2 g\sin 37° - T - \mu_2 M_2 g\cos 37°$ .....(ii)

Now, by adding, (i) and (ii)

$(M_1 + M_2)a = (M_1 + M_2)g\sin 37° - (\mu_1 M_1 + \mu_2 M_2)g\cos 37°$

$\therefore (4+2)a = (4+2)g\times(0.6) - \left(4\times\frac{3}{4} + \frac{1}{4}\times 2\right)g\times 0.8$

$6a = g[3.6 - 2.8]$ or $a = \frac{9.8\times 0.8}{6} = \frac{7.64}{6} = 1.27\text{m/sec}^2$

From (ii) $T = M_2 g\sin 37° - \mu_2 M_2 g\cos 37° - M_2 a = 2\times 10\times\frac{6}{10} - \frac{1}{4}\times 2\times 10\times\frac{8}{10} - 2\times 1.27 = 12 - 4 - 2.54 = 5.46$ Newtons

16. CIRCULAR DYNAMICS

The motion of a particle particularly along a circular path is called its circular motion and it can be uniform, with constant angular rate (and constant speed), or non-uniform with a changing rate. In uniform circular motion, a resultant non-zero force is in operation on the particle. This is because a particle moving in a circular path is accelerated even if speed of the particle remains constant. This acceleration is due to change in direction of the velocity vector. As we have already seen that in uniform circular motion tangential acceleration (a_t) is zero, the acceleration of the particle is toward the center and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r is the radius of the circle. The direction of the resultant force F is, therefore, toward the center and its magnitude is

$F = ma$ or $F = \frac{mv^2}{r}$ Or $F = mr\omega^2$ (as $v = r\omega$)

Here, ω denotes the angular speed of the particle. The force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is required to keep the particle moving in a circular path with constant speed. This force is generally provided by some external sources such as friction, magnetic force, Coulomb force, gravitation, tension, etc.

Illustration 21: Assume that a small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius 25 cm. If the block takes 2 s to complete one round, then find the normal contact force by the side wall of the groove. **(JEE MAIN)**

Sol: The normal contact force provides the necessary centripetal force to the block to move in a circle.

The speed of the block is $v = \frac{2\pi\times(25\text{ cm})}{2.0\text{s}} = 0.785\text{m/s}$.

The acceleration of the block is $a = \frac{v^2}{r} = \frac{(0.785\text{m/s})^2}{0.25\text{m}} = 2.5\text{m/s}^2$

However, toward the center, the only force in this direction is the normal contact force due to the side walls. Thus, from Newton's second law, this force is $N = ma = (0.100\text{kg})(2.5\text{m/s}^2) = 0.25\text{N}$

PLANCESS CONCEPTS

I have found students often confused over the concept of centripetal force. They think that this force acts on a particle moving in a circle. This force does not act but required for moving in a circle which is being provided by the other forces acting on the particle. Let us take an example, Suppose a particle of mass 'm' is moving in a vertical circle with the help of a string of length l fixed at point O.

Let v be the speed of the particle at its lowest position. When I ask the students what forces are acting on the particle in this position, they immediately say, three forces are acting on the particle: (i) tension, T (ii) weight, mg and (iii) centripetal force, $\frac{mv^2}{l}$ (r=l). However, they are wrong. Only the first two forces T and mg are acting on the particle.

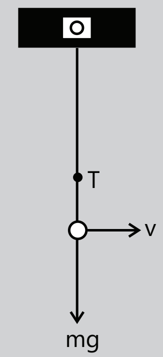


Figure: 4.52

The third force $\frac{mv^2}{l}$ is required for circular motion which is being provided by T and mg. Thus, the resultant of these two forces is $v^2 = \mu rg$ or $v = \sqrt{\mu rg}$ toward O. Or we can write $T - mg = \frac{mv^2}{l}$

Ankit Rathore (JEE Advanced 2013, AIR 158)

17. UNIFORM CIRCULAR MOTION

If a particle moves on a circular path with constant speed, its motion is called as a uniform circular motion. In this type of motion, angular speed of the particle is also constant. Further, linear acceleration in such motion will not have any tangential component; therefore, the particle possesses only radial or centripetal acceleration. Therefore in case of uniform circular motion the particle will have acceleration toward the center only and is called as centripetal acceleration having magnitude $\frac{v^2}{R}$ or $\omega^2 R$. However, the magnitude of acceleration remains constant but its direction changes with time. If a particle moving on circular path is observed from an inertial frame, then we know that it has an acceleration $\omega^2 R$ or $\frac{v^2}{R}$ acting toward the center. Therefore, from Newton's second law of motion, there must be a force acting on the particle toward the center of magnitude $m\omega^2 R$ or $\frac{mv^2}{R}$. This required force

for a particle to move on a circular path is called as centripetal force. $\therefore$ Centripetal force $= \frac{mv^2}{R}$

The term "centripetal force" merely signifies a force toward the center; however, it tells nothing about its nature or origin. Further, the centripetal force may be a single force due to a rope, a string, the force of gravity, friction, and so forth or it may be resultant of several forces.

Illustration 22: A ball of mass 0.5 kg is attached to the end of a cord whose length is 1.50 m. The ball is whirled in a horizontal circular path. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can have before the cord breaks? **(JEE MAIN)**

Sol: The tension force in the cord provides the necessary centripetal force to the ball to move in a circular path.

Because the centripetal force in this case is the force T exerted by the cord on the ball, we have $T = m\frac{v^2}{r}$; therefore, solving for v, we have $v = \sqrt{\frac{Tr}{m}}$

The maximum speed that the ball can have corresponds to the maximum tension. Hence, we find

$$v_{max} = \sqrt{\frac{T_{max} r}{m}} = \sqrt{\frac{(50.0N)(1.50m)}{0.500kg}} = 12.2m/s$$

18. NON-UNIFORM CIRCULAR MOTION

If the speed of a particle moving in a circle is not constant, then the acceleration has both radial and tangential components. These radial and tangential accelerations are given as: $a_r = \omega^2 r = \frac{v^2}{r}$; $a_t = \frac{dv}{dt}$

Then, the magnitude of the resultant acceleration will be: $a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

However, if the direction of the resultant acceleration makes an angle β with the radius, where $\tan\beta = \frac{dv/dt}{v^2/r}$

then, in the direction $\tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$ with radius of circle, acceleration needs a resultant force of $m\sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

Illustration 23: A car moves on a horizontal circular road of radius R. The speed of the car is increasing at a rate $\frac{dv}{dt} = a$. The frictional coefficient between the road and tire is μ. Find the speed at which the car will skid.

(JEE ADVANCED)

Sol: The net acceleration of the car is the vector sum of the centripetal acceleration and the tangential acceleration. By Newton's second law the friction force on the car is (mass)$\times$(net acceleration).

Here, at any time t, the speed of the car becomes V; therefore, the net acceleration in the plane of the road is $\sqrt{\left(\frac{v^2}{R}\right)^2 + (a^2)}$. This acceleration is provided by the frictional force. At the moment, the car will slide if it reaches the speed as given by $M\sqrt{\left(\frac{v^2}{R}\right)^2 + (a^2)} = \mu Mg \Rightarrow v = \left[R^2(\mu^2 g^2 - a^2)\right]^{1/4}$

Illustration 24: A large mass M and a small mass m hang at the two ends of the string that passes through a smooth tube as shown in the Figure 4.53. The small mass m, which lies in the horizontal plane, moves around in a circular path. The length of the string from the mass m to the top of the tube is l and θ is the angle this length makes with the vertical. What should be the frequency of rotation of the small mass m so that the large mass M remains stationary? **(JEE MAIN)**

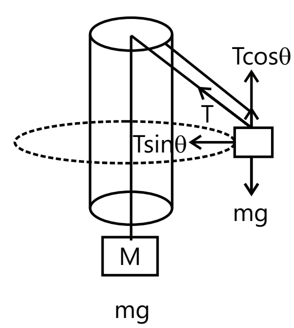


Figure: 4.53

Sol: For the mass M to be stationary the tension in the string should balance the weight of M. For mass m the horizontal component of tension in the string provides the centripetal force. The vertical component of tension balances the weight of m.

The forces acting on the small mass m and the large mass M are shown in the Figure 4.56. When mass M is stationary, then T = Mg ... (i)

where T is tension in the string.

For the smaller mass, the vertical component of tension T cos θ balances mg and the horizontal component T sin θ supplies the necessary centripetal force.

$T\cos\theta = mg$ ... (ii)

$T\sin\theta = mr\omega^2$ ... (iii)

ω being the angular velocity and r is the radius of horizontal circular path.

From (i) and (iii), Mg sinθ = $mr\omega^2 \Rightarrow \omega = \sqrt{\frac{Mg\sin\theta}{mr}} = \sqrt{\frac{Mg\sin\theta}{ml\sin\theta}} = \sqrt{\frac{Mg}{ml}}$

Frequency of rotation $= \frac{1}{T'} = \frac{1}{2\pi/\omega} = \frac{\omega}{2\pi}$ $\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$

19. CENTRIPETAL FORCE

An observer in a rotating system is another example of a non-inertial observer category. Suppose that a block of mass m lying on a horizontal frictionless turntable is connected to a string as shown in the Figure 4.54. Then, according to an inertial observer, if the block rotates uniformly it hence undergoes an acceleration of magnitude $\frac{v^2}{r}$ where v is the tangential speed. The inertial observer hence concludes that this centripetal acceleration is provided by the force exerted by the string T and writes as per Newton's second law $T = \frac{mv^2}{r}$.

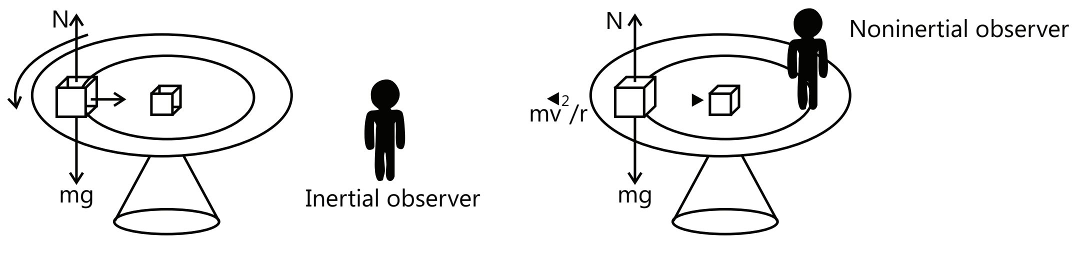


Figure: 4.54

However, according to a non-inertial observer attached to the turntable, the block is at rest. Therefore, by applying Newton's second law, this observer introduces a fictitious outward force of magnitude $\frac{mv^2}{r}$. According to the non-inertial observer, this outward force balances the force exerted by the string and therefore $T - \frac{mv^2}{r} = 0$. In fact, the centrifugal force is sufficient pseudo force only if we were analyzing the particles at rest in a uniformly rotating frame. In contrast, if we analyze the motion of a particle that moves in the rotating frame then we may have to assume other pseudo forces together with the centrifugal force, such forces are called Coriolis forces. The Coriolis force, named after the 19th century French engineer-mathematician, is perpendicular not only to the velocity of the particle but also to the axis of rotation of the frame. Once again we should be remembering that all these pseudo forces, centrifugal or Coriolis, are needed only if the reference frame is rotating. We must know that if we work from an inertial frame, then there is no need to apply any pseudo force. However, we should be aware of the fact that there should not be a misconception that centrifugal force acts on a particle because the particle describes a circle. Therefore, when we are working from a frame of reference that is rotating at a constant angular velocity ω w.r.t. an inertial frame, then we have to obviously assume that a force $m\omega^2 r$ acts radially outward on a particle of mass m kept at the distance r from the axis of rotation. Then only we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called centrifugal force. One should be careful when using fictitious forces to describe such physical phenomena. Remember that fictitious forces are used only in non-inertial frames of references. Therefore, when solving problems of this nature, it is often best to use an inertial frame.

Illustration 25: A table with smooth horizontal surface is fixed in a cabin that rotates with angular speed ω in a circular path of radius R. A smooth groove AB of length L(<<R) is made on the surface of table as shown in the Figure 4.55. A small particle is kept at the point A in the groove and is released to move. Find the time taken by the particle to reach the point B. **(JEE MAIN)**

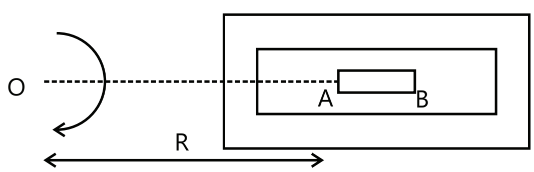


Figure: 4.55

Sol: In the reference frame of cabin the particle experiences centrifugal force in the radial direction. This force can be assumed constant for L<<R.

Let us analyze the motion of particle with respect to the table which is moving with cabin at an angular speed of ω. Now, along the smooth groove AB centrifugal force of magnitude $m\omega^2 R$ will act at A on the particle which can be treated as constant from A to B as L<<R.

∴ Acceleration of the particle along AB with respect to the cabin a = $\omega^2 R$ (constant)

Therefore, required time "t" is given by $s = ut + \frac{1}{2}at^2 \Rightarrow L = 0 + \frac{1}{2} \times \omega^2 R t^2 \Rightarrow t = \sqrt{\frac{2L}{\omega^2 R}}$

19.1 Applications of Centripetal Force

19.1.1 Circular Turning of Roads

When vehicles go through turnings, we observe that they travel along a nearly circular arc. Naturally, there must be some force which will produce the required centripetal acceleration. However, if the vehicles travel in a horizontal circular path, then this resultant force is also horizontal. The necessary centripetal force, which we are discussing about, is being provided to the vehicles by any one or combination of the following three ways:

(a) By friction only, **(b)** By banking of roads only, or **(c)** By both friction and banking of roads

In real life, the necessary centripetal force is provided by both friction and banking of roads. Now, let us write equations of motion in each of these three cases separately and find out what the constraints in each case are.

(a) By Friction only: Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, therefore, the necessary centripetal force to the car will be provided by force of friction, f, acting toward the center. Thus, $f = \frac{mv^2}{r}$ Further, limiting value of f is μN Or $f_L = \mu$; $N = \mu mg$ (N=mg)

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$ or $\frac{mv^2}{r} \le \mu mg$ or $\mu \ge \frac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

Here, however, two situations may arise. If μ and r are known to us, then the speed of the vehicle should not exceed and if v and r are known to us, then the coefficient of friction should be greater than $\frac{v^2}{rg}$.

PLANCESS CONCEPTS

You might have seen that if the speed of a car is too high, the car starts skidding outward with the radius of the circle increased of the necessary centripetal force is reduced.

centripetal force $\propto \frac{1}{r}$

Anurag Saraf (JEE 2011, AIR 71)

(b) By Banking of Roads only: It is a common fact that friction is not always reliable particularly at circular turns when in high speeds and where sharp turns are also involved. To avoid dependence on friction, the roads are banked at the turn in such a way that the outer part of the road is somewhat lifted compared to the inner part.

Now, by applying Newton's second law along the radius and the first law in the vertical direction, we obtain $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg$

Thus, from the above two equations, we obtain

$\tan\theta = \frac{v^2}{rg}$ ... (i) or $v = \sqrt{rg\tan\theta}$ ... (ii)

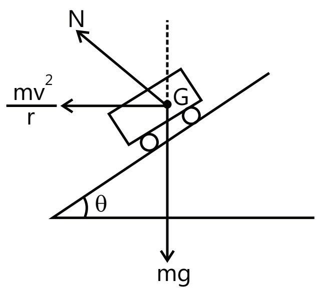


Figure: 4.56

PLANCESS CONCEPTS

This is the speed at which a car does not slide down even if a track is smooth. If the track is smooth and speed is less than $\sqrt{rg\tan\theta}$, then the vehicle will move down so that r gets decreased and if speed is more than this, then the vehicle will move up.

Vijay Senapathi (JEE 2011, AIR 71)

(c) By Both Friction and Banking of Road: If a vehicle is moving on a circular road which is both rough and banked, then three forces may act on the vehicle, and out of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of the second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction f can be either inward or outward while its magnitude can be varied up to a maximum limit ($f_L = \mu N$). Therefore, the magnitude of normal reaction, N and direction plus magnitude of friction, f, are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center.

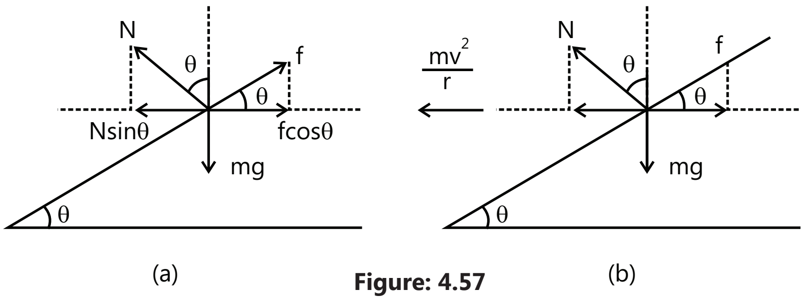


Figure: 4.57

Therefore, magnitude of N and direction plus magnitude of friction mainly depends on the speed of vehicle, v. Thus, the situation varies on a case-to-case basis even though we can observe the following scenarios:

(i) Friction f is outward if the vehicle is at rest or v = 0. Because, in this case, the component of weight mg sin θ is balanced by f.

(ii) Friction f is inward if $v > \sqrt{rg\tan\theta}$

(iii) Friction f is outward if $v < \sqrt{rg\tan\theta}$

(iv) Friction f is zero if $v = \sqrt{rg\tan\theta}$

Let us now observe how the force of friction and normal reaction changes as speed is gradually increased.

In Figure 4.57 (a): When the car is at rest, then the force of friction is upward. However, we can resolve the forces in any two mutually perpendicular directions. Let us resolve them in both horizontal and vertical directions.

$$\sum F_H = 0 \;\therefore\; N\sin\theta - f\cos\theta = 0 \quad \text{... (i)}$$

$$\sum F_v = 0 \;\therefore\; N\cos\theta + f\sin\theta = mg \quad \text{... (ii)}$$

In Figure 4.57 (b): Now the car is given a small speed v, so that a centripetal force $\frac{mv^2}{r}$ is now required in horizontal
direction toward the center. Therefore, Eq. (i) will now become $N\sin\theta - f\cos\theta = \frac{mv^2}{r}$

Or we can say, in case (a) $N\sin\theta$ and $f\cos\theta$ were equal while in case (b) their difference is $\frac{mv^2}{r}$. This can occur in any of the following three ways:

(i) N increases while f remains same

(ii) N remains same while f decreases

(iii) N increases and f decreases

But only the third case is possible, i.e., N will increase but f will decrease. This is because Eq. (ii), $N\cos\theta + f\sin\theta = mg$ is still has to be valid.

Therefore, to keep $N\cos\theta + f\sin\theta$ to be constant (=mg), N should increase but f should decrease (as θ =constant). Now, as speed goes on increasing, force of friction first decreases But becomes zero at v= $\sqrt{rg\tan\theta}$ and then reverses it direction. Let us show an example which illustrates this theory.

Illustration 26: A turn of radius 20 m is banked for the vehicle of mass 200 kg moving at a speed of 10 m/s. Find the direction and magnitude of frictional force acting on a vehicle if it moves with a speed (a) 5 m/s and (b) 15 m/s. Assume that friction is sufficient to prevent slipping. (g = 10 m/s^2) **(JEE ADVANCED)**

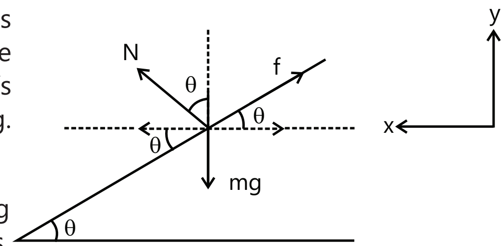


Figure: 4.58

Sol: At the correct speed for which the road is banked, the self-adjusting static friction acting on the vehicle is zero. When the speed decreases, the vehicle has a tendency to slip downwards, thus the static friction acts upwards. When the speed increases, the vehicle has a tendency to slip upwards, thus static friction acts downwards.

(a) the turn is banked for speed v = 10 m/s

Therefore, $\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{(20)(10)} = \frac{1}{2}$ Now, as the speed is decreased, force of friction f acts upward.

Using the equations $\sum F_X = \frac{mv^2}{r}$ and $\sum F_y = 0$ we obtain

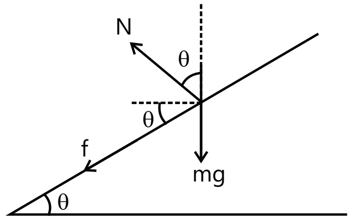


Figure: 4.59

$N\sin\theta - f\cos\theta = \frac{mv^2}{r}$ ... (i)

and $N\cos\theta + f\sin\theta = mg$ ... (ii)

Now, by substituting, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$, v = 5 m/s, m = 200 kg and r = 20 m, in the above equations,

we obtain $f = 300\sqrt{5}N$ (outward)

(b) In the second case, force of friction f will act downward.

Using $\sum F_X = \frac{mv^2}{r}$ and $\sum F_y = 0$ we obtain

$N\sin\theta + f\cos\theta = \frac{mv^2}{r}$ ... (iii)

$N\cos\theta - f\sin\theta = mg$ ... (iv)

Substituting, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$, v = 15 m/s, m = 200 kg and r = 20 m, in the above equations, we obtain $f = 500\sqrt{5}$ N (downward).

19.1.2 Conical Pendulum

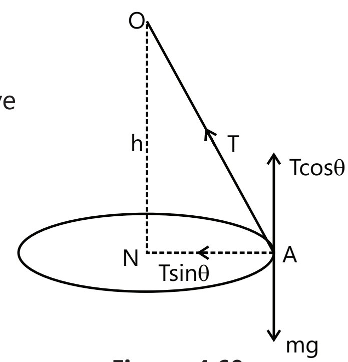


Figure: 4.60

A conical pendulum consists of a string OA whose upper end O is fixed and a bob is tied at the free end. When a horizontal push is given to the bob by drawing aside and let it describe a horizontal circle with uniform angular velocity ω in such way that the string makes an angle θ with the vertical, then the string traces the surface of a cone of semi-vertical angle θ. It is called a conical pendulum. Let us assume that T be the tension in string, l be the length and r be the radius of the horizontal circle described. Now, the vertical component of tension balances the weight, whereas the horizontal component supplies the centripetal force.

$T\cos\theta = mg$; $T\sin\theta = mr\omega^2$ $\therefore \tan\theta = \frac{r\omega^2}{g}$

$\omega = \sqrt{\frac{g\tan\theta}{r}}$; r= $l\sin\theta$ $\therefore \omega = \frac{2\pi}{T}$

T being the period, i.e., time for one revolution $\therefore \frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{l\sin\theta}}$; $T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$ where $h = l\cos\theta$

PLANCESS CONCEPTS

This is similar to the case, when necessary centripetal force to vehicles is provided by the property of banking. The only difference here is that the normal reaction is being replaced by the tension.

Yashwanth Sandupatla (JEE 2012, AIR 821)

19.1.3 Death Well or Rotor

In the case of a 'death well', a person drives a bicycle on a vertical surface of a large wooden well, while in the case of rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In the death well, all walls are at rest and the person revolves while in case of the rotor the person is at rest but the walls rotate.

In both the cases, friction balances the weight of the person while reaction provides centripetal force for circular motion, i.e., f = mg and $N = \frac{mv^2}{r} = mr\omega^2$

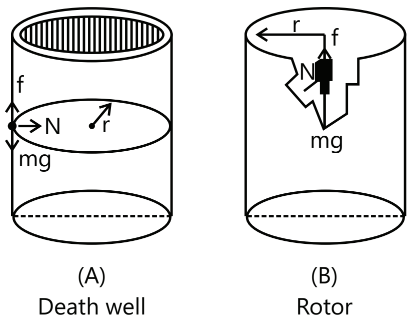


Figure: 4.61

A cyclist on the bend of a road: In the Figure 4.62, $F = \sqrt{N^2 + f^2}$

When the cyclist is inclined to the center of the rounding along its path, the resultant N, f and mg are directed horizontally to the center of the circular path of the cycle. This resultant force naturally imparts a centripetal acceleration to the cyclist. The resultants of N and f, i.e., F should pass through G, the center of gravity of the cyclist (for complete equilibrium, rotational as well as translational). Hence,

$\tan\theta = \frac{f}{N}$ where $f = \frac{mv^2}{r}$ and $N = mg$

$\therefore \tan\theta = \frac{v^2}{rg}$

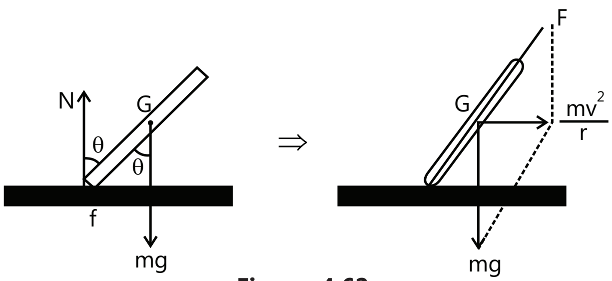


Figure: 4.62

20. CENTRIFUGAL FORCE

We know that Newton's laws are valid only in inertial frames. This is because in non-inertial frames a pseudoforce $-m\vec{a}$ has to be applied on a particle of mass m ($\vec{a}$ =acceleration of frame of reference). After applying the pseudo force, then one can apply Newton's laws in their usual form. Now, suppose that a frame of reference is rotating with constant angular velocity ω in a circle of radius 'r'. Then, it will become a non-inertial frame of acceleration θ toward the center. Now, if we observe an object of mass 'm' from this frame then obviously a pseudo force of magnitude $mr\omega^2$ will have to be applied to this object in a direction away from the center. This pseudo force is what we call as centrifugal force. After applying this force, we can apply Newton's laws in their usual form. The following example will illustrate this concept clearly.

Illustration 27: A particle of mass m is placed over a horizontal circular table rotating with an angular velocity ω about a vertical axis passing through its center. The distance of the object from the axis is r. Based on the above, find the force of friction f between the particle and the table. **(JEE ADVANCED)**

Sol: The particle is stationary in the rotating reference frame rigidly fixed to the rotating table. In the list of all the forces acting on the particle, include the centrifugal force (pseudo force) acting on the particle radially outwards.

Let us solve this problem from both the frames. The one is a frame fixed on the ground while the other is a frame fixed on the table itself.

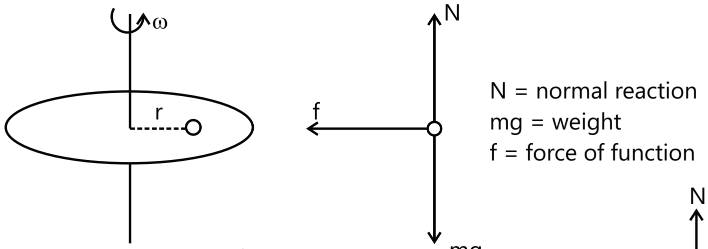


Figure: 4.63

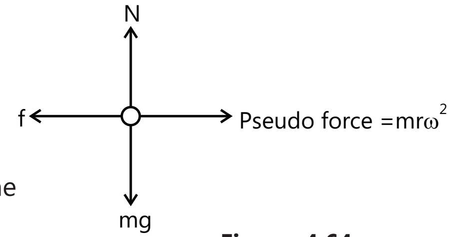


Figure: 4.64

From frame of reference fixed on ground (inertial)

Here, N will balance its weight and force of friction, whereas f will provide the necessary centripetal force.

Thus, $f = mr\omega^2$

From frame of reference fixed on table itself (non-inertial)

In the FBD of particle with respect to table, in addition to the above three forces (N, mg and f) a pseudo force of magnitude $mr\omega^2$ will have to be applied in a direction away from the center. But one significant point here is that in this frame the particle is in equilibrium, i.e., N will balance its weight in vertical direction, whereas f will balance the pseudo force in horizontal direction. $f = mr\omega^2$

Thus, we observe that 'f' equals $mr\omega^2$ from both the frames. Now, let us work up few more examples of circular motion.

Illustration 28: A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at the upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle α with the vertical. Based on the above, find the tension in the string and the magnitude of net force on the bob at that instant.

(JEE MAIN)

Sol: Apply Newton's second law on the bob in two perpendicular directions. One along the string and the other along the tangent to its circular path, i.e. along the perpendicular to the string.

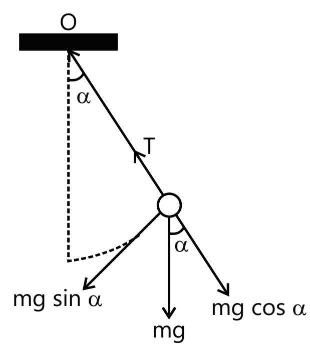


Figure: 4.65

The forces acting on the bob are:

(a) Tension, T and

(b) Weight, mg

As observe that the bob moves in a circle of radius L with center at O, it is imperative that a centripetal force of magnitude $\frac{mv^2}{L}$ is required toward O.

However, this force will be provided by the resultant of T and mg cos α. Thus,

(i) $T - mg\cos\alpha = \frac{mv^2}{L}$ or $T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$

(ii) $\left|\vec{F}_{net}\right| = \sqrt{(mg\sin\alpha)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$

Illustration 29: Suppose that a hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. Further, a small ball kept in the bowl rotates with the bowl but without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is $\sqrt{5gR}$, then find the angular speed at which the bowl is rotating. **(JEE MAIN)**

Sol: The horizontal component of normal contact force acting on the ball will provide the necessary centripetal acceleration to move in a circular path. The vertical component of normal contact force acting on the ball will balance its weight.

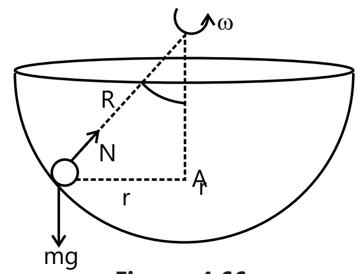


Figure: 4.66

Let us assume that ω be the angular speed of rotation of the bowl. Now, the two forces acting on the ball are

(a) Normal reaction, N and

(b) Weight, mg

We know that the ball is rotating in a circle of radius r (=$R\sin\alpha$) with center A at an angular speed ω.

Thus, $N\sin\alpha = mr\omega^2 = mR\omega^2\sin\alpha$ ... (i)

and $N\cos\alpha = mg$ ... (ii)

Thus, by dividing Eq. (i) by (ii), we obtain $\frac{1}{\cos\alpha} = \frac{\omega^2 R}{g}$ $\quad \therefore\ \omega = \sqrt{\frac{g}{R\cos\alpha}}$

21. EFFECT OF EARTH'S ROTATION ON CIRCULAR WEIGHT

We are very familiar with the fact that the earth rotates about its own axis at an angular speed of one revolution for every 24 hours. We also know that the line joining the north and the south poles is the axis of its rotation, and every point on the earth moves in a circular path. Further, a point at the equator moves in a circle of radius equal to that of the radius of the earth and the center of the circle is the same as the center of the earth. However, for any other point on the earth, the circle of rotation is smaller than this. Now, consider a place P on the earth.

If we drop a perpendicular PC from P to the axis SN, then the place P rotates in a circle with the center at C, and the radius of this circle is CP. Now, the angle between the axis SN and the radius OP through P is called the colatitude of the place P. We have $CP = OP\sin\theta$ or, $r = R\sin\theta$ where R is the radius of the earth.

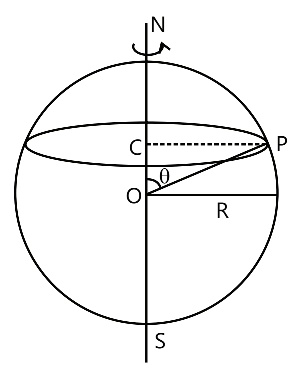


Figure: 4.67

However, if we work from the frame of reference of the earth, then we shall have to assume the existence of the pseudoforces. In particular, a centrifugal force, $m\omega^2 r$ has to be assumed on any particle of mass m placed at P. Here ω is the angular speed of the earth. In contrast, if we discuss the equilibrium of bodies at rest in the earth's frame, then no other pseudo force is needed. Let us now consider a heavy particle of mass m that is suspended through a string from the ceiling of a laboratory at colatitude θ. Observing from the earth's frame it appears that the particle is in equilibrium and hence the forces acting on it are

(a) Gravitational attraction, mg, toward the center of the earth, i.e., vertically downward,

(b) Centrifugal force, $m\omega^2 r$, toward CP, and

(c) The tension in the string, T, along the string.

As the particle is in equilibrium (in the frame of earth), the three forces on the particle therefore should add up to zero.

The resultant of mg and

$$m\omega^2 r \quad = \sqrt{(mg)^2 + (m\omega^2 r)^2 + 2(mg)(m\omega^2 r)\cos(90° + \theta)}$$

$$= m\sqrt{g^2 + \omega^4 R^2 \sin^2\theta - 2g\omega^2 R\sin^2\theta} = mg'$$

where $g' = \sqrt{g^2 + \omega^4 R^2 \sin^2\theta - 2g\omega^2 R\sin^2\theta}$

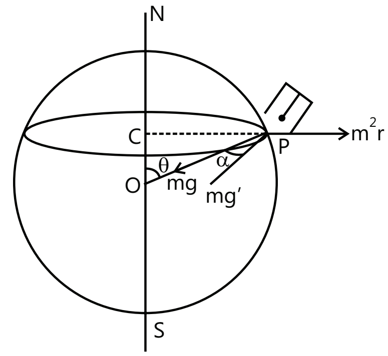


Figure: 4.68

Also, it is obvious that the direction of the resultant makes an angle α with the vertical OP, where

$$\tan\alpha = \frac{m\omega^2 r\sin(90^\circ + \theta)}{mg + m\omega^2 r\cos(90^\circ + \theta)} = \frac{\omega^2 R\sin\theta\cos\theta}{g - \omega^2 R\sin^2\theta}$$

From the above, as the three forces acting on the particle must add up to zero, we understand that the force of tension must be both equal and opposite to the resultant of the rest two. Therefore, the magnitude of the tension in the string must be mg′ and the direction of the string hence should make an angle α with the true vertical.

The direction of g′ is in the apparent vertical direction. This is because a plumb line stays in this direction only. However, the walls of the buildings are constructed by making them parallel to g′ and not to g. Further, the water surface placed at rest is perpendicular to g′.

We observe further that the magnitude of g′ is also different from g. As 2g > $\omega^2 R$, it is clear from the above equation that g′ < g. Here, one way of measuring the weight of the body is to suspend it by a string and hence find the tension in the string. This is because the tension itself is taken as a representative measure of the weight. As T = mg′, the weight so observed is clearly less than the true weight, mg. We call this quantity as the apparent weight. Similarly, if a person stands on the platform of a weighing machine, the platform then exerts a normal force N which is equal to mg′. The reading of the weighing machine responds to the force exerted on it and hence the weight recorded is the apparent weight, mg′ of the person. At the equator, we know that $\theta = 90^\circ$ and hence Eq. (7.14) gives

$$g' = \sqrt{g^2 - 2g\omega^2 R + \omega^4 R^2} = g - \omega^2 R \quad \text{Or,}\ mg' = mg - m\omega^2 R.$$

This value, however, can be obtained in a more straightforward way. At the equator, we know that $m\omega^2 R$ is directly opposite to mg and the resultant is simply mg − $m\omega^2 R$. Also, it is clear that this resultant is towards the center of the earth so that at the equator the plumb line is along the true vertical.

At poles, invariably θ =0 and the first equation gives g′=g and by the next equation we know that α = 0. Thus, at the poles there is no apparent change in g. This is basically because of the fact that the poles do not rotate and hence the effect of earth's rotation is negligible there.

Illustration 30: A body weighs 98 N on a spring balance at the North Pole. What will be its weight recorded on the same scale if it is shifted to the equator? Use $g = GM/R^2 = 9.8\ m/s^2$ and the radius of the earth R=6400 km.

(JEE MAIN)

Sol: At the equator due to the rotation of earth the body experiences centrifugal force, directed away from the center of earth. So the apparent weight at equator is less than the actual weight at the poles.

At poles, we know that the apparent weight is the same as the true weight. Thus, 98 N = mg = m (9.8 m/s^2)

or, m = 10 kg. At the equator, however, the apparent weight is mg′ = mg − $m\omega^2 R$.

It is given that the radius of the earth is 6400 km and hence the angular speed is $\omega = \frac{2\pi\ \text{rad}}{24 \times 60 \times 60\ \text{s}} = 7.27 \times 10^{-5}\ \text{rad/s}$

Thus, mg′= 98N – (10 kg) $(7.27 \times 10^{-5} s^{-1})^2$ (6400 km) = 97.66 N.

PROBLEM SOLVING TACTICS

Step 1: First, construct a big schematic diagram of the physical situation. Then, while reading and rereading the problem statement construct your diagram accordingly including every available information from the statement on the diagram. Thereafter, if applicable, attach appropriate symbols to each important parameter in the problem irrespective of the fact that whether the value of the parameter is known or not. Eventually, make straight lines straight, parallel lines parallel, perpendicular lines perpendicular, etc., to the best of your ability in order to avoid confusion later on.

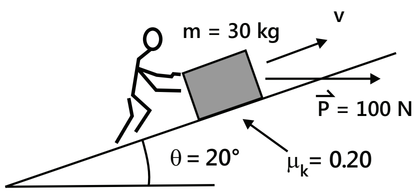


Figure: 4.69

Example at right. (Note how the following items recorded from the statement of the problem into specific items on the drawing: "... 30 kg suitcase ... moves upward ... 20 degree incline ... applied horizontal force of 100 N ... kinetic coefficient of friction ... is 0.20. ")

Step 2: Select a «system» to which you intend to apply Newton›s second law. In some problems, however, there may be more than one candidate for the "system." You may not choose the best one always the first time. That should not be a case for worry; just choose another one and do it again.

Example: We will select the "suitcase" as our system because it is the thing to which many obvious forces are being applied and it is the thing whose acceleration we want to find.

Step 3: Identify all the forces acting on «the system.» You can do this by drawing a dotted line around the system chosen in step 2 and identifying all physical objects that come in contact with the system. Each of these will exert a definite force on the system. Then, look for «field» forces—those forces that act without touching through the intermediary of a field of some sort. We know that in introductory mechanics the only "field" force is the force of gravity. It is a force exerted by the earth (or some other very massive body) on the system through the intermediary of the gravitational field. **Important!** It should be understood that every force on a system is exerted by some physical object exterior to the system. If you cannot identify that object and the method of interaction (contact or field), then the force DOES NOT EXIST! Listed here are the some commonly encountered forces and some tips on dealing with them:

(a) Ropes or strings: These exert «tension» forces on the system in question. They are always directed away from the system and along the direction of extension of the rope or string used.

(b) Contacts with surfaces: We generally split up the force due to contact with a surface into two components called the "normal"—meaning "perpendicular"—force and the "frictional" force. The normal force is generally a "push" type of force directed toward the system, unless the surface is sticky enabling it to exert a «pull» type of force. In contrast, the frictional force is parallel to the surface, opposes motion or potential motion (i.e., a system on the verge of «slipping») and is often assumed to be related to the normal force through a «coefficient of friction.» Readers may kindly refer to the discussion on the topic, "coefficient of friction" in this regard.

(c) Hinges or Pins: These exert forces of arbitrary magnitude and direction as required so as to ensure that the point of attachment remains stationary.

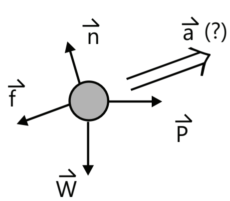


Figure: 4.70

(d) General pushes or pulls: If a working problem specifies that some object is being pushed or pulled in some direction, then you may have to assume that the force specified is being exerted by some physical object. Therefore, it is very important that you do not forget to include the same.

(e) Air resistance: Air may is not visible, but it is very likely that it does establish a physical contact with your system. Quite often we neglect air resistance because its effects are deemed negligible. However, if a problem specifies a certain amount of air resistance is involved or tells you that the air resistance depends in some way on velocity or other parameters, then do not forget to include it.

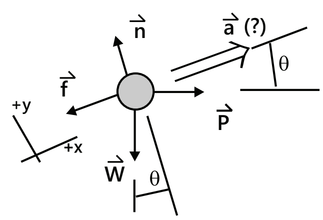


Figure: 4.71

(f) Gravitational force: We are aware of the fact that the gravitational force—commonly called the "weight" of the system—is the only force that acts without being in physical contact with the system (at least until you learn about electric and magnetic forces later on.) Generally, it acts in the downward direction (by definition!) and is equal to the mass of the system times the local gravitational field strength g—commonly, but misleadingly called "the acceleration due to gravity."

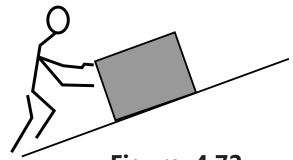


Figure: 4.72

The example at right. Here, we find two objects in contact with the system—one being the "surface" and other one "pusher." Thus, we find a total of four forces—the normal force, the frictional force (from the surface), the push (from the pusher), and the weight (due to the only force—so far—that acts without needing to touch—gravity.)

Step 4: Draw an "FBD." Now, the system may be represented by a simple circle or square; however, we want to focus our attention on the forces on and the resulting acceleration of the system. Now, draw each force with its tail at the surface of the system extending in the proper direction. Further, include the acceleration vector as well, but distinguish it from the force vectors by drawing it in a different way.

Example at the right. In this example, note that the normal force is directed perpendicular to the surface (not shown in the FBD), the frictional force is directed opposite to the direction that the system slips with respect to the surface, the push is in its given direction, and the weight is directed "down." We also show the acceleration as a different-looking vector that is directed upward along the plane, but we do not know this for certain; it may be directed downward along the incline. To keep us remind ourselves of this fact, we put a "(?)" next to the acceleration vector.

Step 5: Now, pick a coordinate system and hence determine the angles that the forces and accelerations make with the coordinate axes. It is usually "clever" and preferable to pick a coordinate system that minimizes the number of unknown vectors that will have to be broken down into components. The answers you obtain thereafter must and will be independent of your choice of coordinate system, but clever choices will help us to arrive at equations that are more easy to solve. However, you may need to do some geometrical work on another sheet of paper to figure out how the angles are related to those given in the problem statement.

Example at right. In this example, we have chosen a coordinate system that requires us to determine the components of only the weight and the push—the two forces about which we know a lot. These two forces lie at the angle theta (given in the problem statement as 20 degrees) from one of the axis directions.

Step 6: Now, write Newton's second law. This law is the basic physical principle you are applying; i.e., the "starting point" for your calculations. Just proceed to do it! Example: $\sum \vec{F} = m\vec{a}$

Step 7: Thence, apply the basic equation to this problem. Now, simply write what the "sum of forces" is in this case. If the acceleration is zero, then use that fact to simplify the equation too. Example: $\vec{n} + \vec{f} + \vec{P} + \vec{W} = m\vec{a}$

Step 8: Now, continue by writing the component equations. This is simply a matter of recognizing that every vector equation is shorthand for two (or, more generally, three) scalar equations. Then, simply rewrite the vector equation for each component direction with each vector quantity rewritten as the corresponding component. Examples: $x : n_x + f_x + P_x + W_x = ma_x$ and $n_y + f_y + P_y + W_y = ma_y$

Step 9: Now, determine what each component is in terms of the vector magnitude and trigonometric functions of the associated angles. In this step, it is imperative that we explicitly indicate the signs of the vector components. This is also a good time to explicitly substitute "mg" for "W" if you really happen to know the mass of the system.

Example: Notice that the normal force is purely in the +y direction, the frictional force is purely in the –x direction, the push has a positive x-component and a negative y-component, the weight has negative x- and y-components, and the assumed acceleration is purely in the +x direction. Thus, we have: $x :\ 0 + (-f) + (+P\cos\theta) + (-mg\sin\theta) = m(+a)$
$y :\ (+n) + 0 + (-P\sin\theta) + (-mg\cos\theta) = m(0)$

Step 10: To conclude this procedure, as a final step, simplify the resulting equations and figure out where to go from here. This is the end of "the method." Example: $P\cos\theta - f - mg\sin\theta = ma \quad n - P\sin\theta - mg\cos\theta = 0$

FORMULAE SHEET

(a) $F_1 = F\cos\theta$ = component of $\vec{F}$ along AC $\quad F_2 = F\sin\theta$ = component of $\vec{F}$ perpendicular to AC

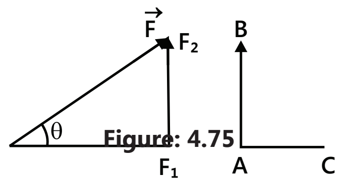


Figure: 4.73

(b) $F = \dfrac{dp}{dt} = \dfrac{d(mv)}{dt}$

(c) $\sum \vec{F} = \vec{F}_{net} = m\vec{a}$ or $\vec{a} = \dfrac{\vec{F}_{net}}{m}$

$\sum F_x = 0 \Rightarrow T\sin\theta = ma$

$\sum F_y = 0 \Rightarrow T\cos\theta = mg$

$\theta = \tan^{-1}\left(\dfrac{a}{g}\right)$

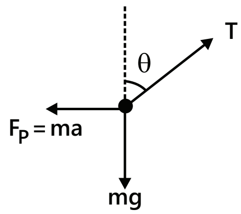
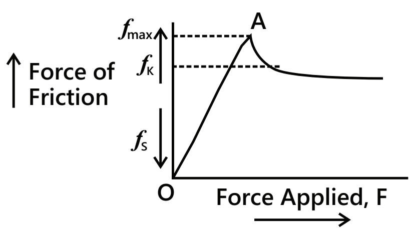


Figure: 4.74

(d) **Impulse = change in momentum** $\quad \vec{F}\Delta t = m\vec{v}_f - m\vec{V}_o$

(e) $\mu = \dfrac{f_{max}}{R}$



(f) $f_{max} = f_{limiting} = \mu_s R$

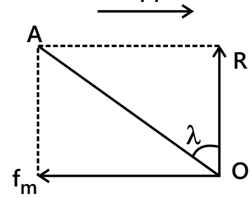


Figure: 4.76

(g) **Angle of Friction:** $\tan\lambda = \dfrac{f_{max}}{R} = \mu \quad$ or $\lambda = \tan^{-1}(\mu)$

(h) **Pseudo force:** F = –ma; where m = mass of the object, a = acceleration of the reference frame

(i) A particle in circular motion may have two types of velocities as listed hereunder.

(i) Linear velocity v and

(ii) Angular velocity ω.These two are related by the equation $v = R\omega$ (R = radius of circular path)

(j) Acceleration of a particle in a circular motion may have two components as listed hereunder.

(i) Tangential component (a_t) and

(ii) Normal or radial component (a_n).

As the name suggests, the tangential component is tangential to the circular path, given by a_t = rate of change of speed

(k) $\frac{dv}{dt} = \frac{d|\vec{v}|}{dt} = R\alpha$ where α = angular acceleration = rate of change of angular velocity $= \frac{d\omega}{dt}$

The normal or radial component, also known as centripetal acceleration is toward the center and is given by

$a_n = R\omega^2 = \frac{v^2}{R}$

(l) Net acceleration of a particle is the resultant of two perpendicular components, a_n and a_t. Hence, $a = \sqrt{a_n^2 + a_t^2}$

(m) Tangential component a_t is responsible for change of speed of a particle. This can be positive, negative or zero, depending upon the situation whether the speed of the particle is increasing, decreasing or remains constant.

(n) In general, in any curvilinear motion, direction of instantaneous velocity is tangential to the path, while acceleration may assume any direction. If we resolve the acceleration in two normal directions, one parallel to velocity and another perpendicular to velocity, then the first component is a_t while the other is a_n.

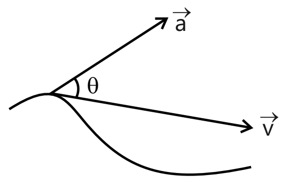


Figure: 4.77

Thus, a_t = component of $\vec{a}$ along $\vec{v}$ = $a\cos\theta = \frac{\vec{a}.\vec{v}}{v} = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt}$ = rate of change of speed.

Further, a_n = component of $\therefore a_{1x} = a_{2x} = a_{3x}$ perpendicular to $\vec{v}$ $= \sqrt{a^2 - a_t^2} = \frac{v^2}{R}$

Here, v is the speed of the particle at that instant and R is called the radius of curvature to the curvilinear path at that point.

(o) In $a_t = a\cos\theta$, if θ is acute, a_t will be positive and speed increases. However, if θ is obtuse a_t will be negative and speed will decrease. If θ is $90°$, a_t is zero and speed will remain constant.

(p) Now, depending upon the value of a_t, circular motion may be of three types as listed hereunder.

(i) Uniform circular motion in which speed remains constant or $a_t = 0$.

(ii) Circular motion of increasing speed, in which a_t is positive.

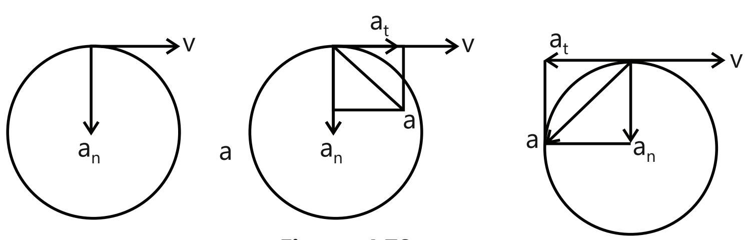


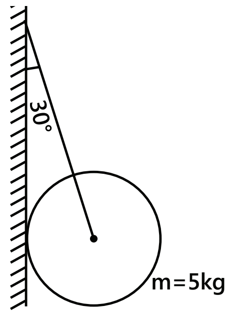
Figure: 4.78

(iii) Circular motion of decreasing speed, in which a_t is negative.

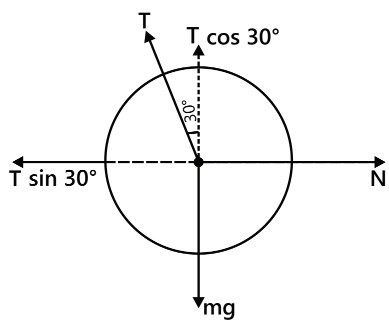
Solved Examples

JEE Main/Boards

Example 1: A spherical shell is resting against the vertical wall which makes an angle 30° with the vertical as shown in the Figure. Determine the normal reaction at the wall and tension in the string.



Sol: Draw the FBD of the sphere. Resolve the forces in horizontal and vertical directions. Apply Newton's first law along the horizontal and vertical directions.



FBD of the sphere is provided. Since sphere is in equilibrium, hence,

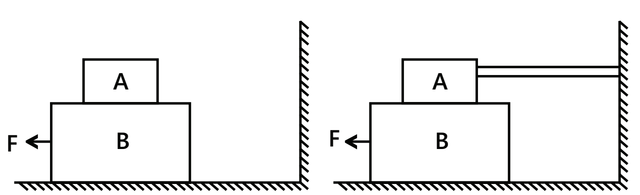
$N - T\sin 30 = 0$ ... (i)

and $\Rightarrow T\cos 30 = mg$ ... (ii)

$$\Rightarrow T = \frac{mg}{\cos 30} = \frac{5\times 10\times 2}{\sqrt{3}} = \left(\frac{100}{\sqrt{3}}\right)\text{N}.$$

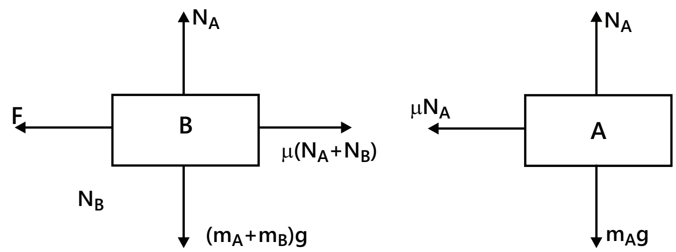
from (i) $N = T\sin 30 = \frac{100}{\sqrt{3}}\times\frac{1}{2} = \frac{50}{\sqrt{3}}\text{N}.$

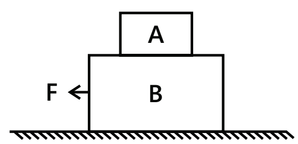
Example 2: The Figure below shows blocks A and B weighing 4N and 8N, respectively and the coefficient of sliding friction between any two surfaces is 0.25. Find the force necessary to drag the block B to the left with constant velocity in all the cases when (a) A is kept over B and (b) A is held firmly over B.



Sol: In the first case, the blocks A and B move together. The friction force will be exerted on the bottom surface of B. In the second case only block B moves. The friction force will be exerted both on bottom and top surface of block B.

Let us consider free diagrams of A and B as two separate systems shown as follows:





(a) $W_A = m_A g = 4\text{ N};\quad W_B = m_B g = 8\text{ N}$

$N_B = N_A + m_B g = (m_A + m_B)g = 4 + 8 = 12\text{N}$

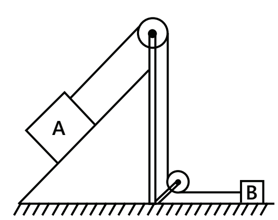
$F = \mu\times 12 = 0.25\times 12 = 3\text{N}$

(b) $F = \mu(N_A + N_B)$ because B is sliding over horizontal rough surface under it and B will be sliding under A also.

$= \mu(m_A + m_B) + \mu m_A g = \mu(2m_A + m_B)g = \mu(2W_A + W_B)$

$= 0.25(8+8) = 4\text{N}$

Example 3: Two blocks each having mass of 20 kg rest on frictionless surfaces as shown in the Figure. Assume that the pulleys to be light and frictionless. Now, find (a) the time required for the block A to move 1 m down the plane, starting from rest and (b) the tension in the cord connecting the blocks.



Sol: For block A apply Newton's second law of motion along the inclined and for block B apply Newton's second law along the horizontal.

Both the blocks A and B are considered as two independent systems. The FBDs for the blocks A and B are shown in the Figure and

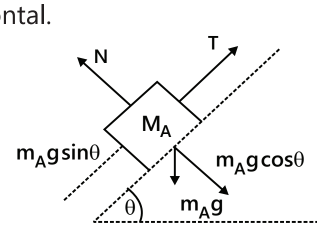


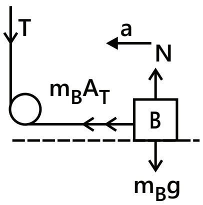
Figure where T is tension in the string.

$m_A g\sin\theta - T = m_A a$ ... (i)

$N = m_A g\cos\theta$ ... (ii)

$T = m_B a$ ... (iii)

Now, by adding equations (i) and (iii), we obtain



$m_A g\sin\theta = (m_A + m_B) g\sin\theta$

$$\Rightarrow a = \left(\frac{m_A}{m_A + m_B}\right) g\sin\theta = \left(\frac{20}{20+20}\right)(10)\left(\frac{3}{5}\right) = 3\ m/s^2$$

$$\text{(a)}\ s = \frac{1}{2}at^2;\quad t = \left(\frac{2s}{a}\right)^{\frac{1}{2}} = \left(2\times\frac{1}{3}\right) = 0.82$$

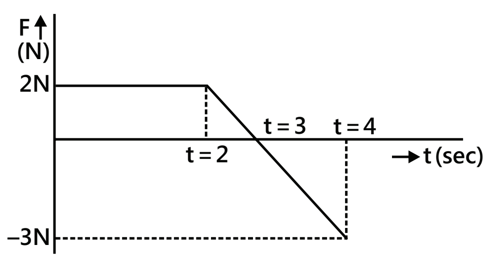
$$\text{(b)}\ T = m_B a = 20\times 3 = 60\text{N}.$$

Example 4: A body of mass = 2 kg starts from rest whose force time graph is shown in the following graph.

(a) What is momentum of the body at t = 4 seconds?

(b) What is velocity of the body at t = 3 seconds?

Sol: Area under the force-time graph and the time axis is equal to the change in momentum.



(a) Area under the curve from t = 0→2 sec.

$A_1 = 2\times 2 = 4\text{N.sec}$. Area from t = 2→3 sec.

$$A_2 = \frac{1}{2}\times 1\times 2 = 1\text{N.sec}.$$

Area from t = 3→4 sec.

$$A_3 = -\frac{1}{2}\times 1\times 3 = -1.5\text{N.sec}.$$

Therefore, the net impulse = 4 + 1 – 1.5 = 3.5 N sec

$P_f = \text{impulse} + P_i = 3.5 + 0 = 3.5$ N.s or kg.m / s

(b) Impulse from t = 0→3 sec

$= A_1 + A_2 = 4 + 1 = 5\text{N.sec}$

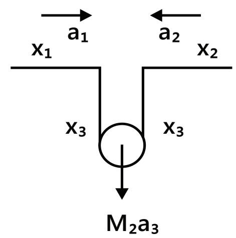
Momentum at t = 3 sec = 5N sec (at t = 0, P = 0)

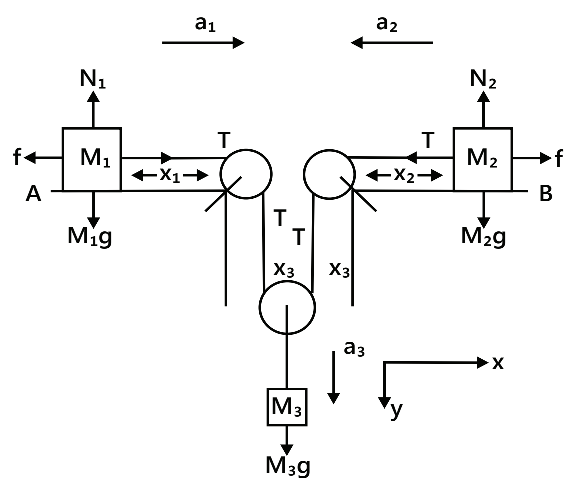
$mv = 5;\ v = \frac{5}{2} = 2.5\text{m/sec}.$

Example 5: Three blocks of masses, and are connected by inextensible strings passing over three massless pulleys as shown in the Figure. The coefficient of friction between the masses and horizontal surfaces is μ. Assume that M_1 and M_2 are sliding. Now, find

(a) Relation between accelerations a_1, a_2 and a_3

(b) Tension T in the strings





Sol: To find the constraint relation between the acceleration of the blocks, measure the distances of blocks from the stationary pulleys. Draw the FBD for each block. For M_1 and M_2 apply Newton's second law in horizontal direction. For M_3 apply Newton's second law in vertical direction.

(a) Forces of friction f, tension T and reaction are marked for the blocks M_1, M_2 and M_3.

Now, take the horizontal line AB as the reference line, i.e., x-axis and vertically downward as y-axis.

If x_1, x_2 and x_3 are the lengths of the strings, then

$x_1 + x_2 + 2x_3 = L$ where L is the constant length of the string.

Now, differentiating twice, $a_1 + a_2 + 2a_3 = 0$

As a_3 is increasing, a_1 and a_2 are decreasing.

Thus, the constraint relation shows that

$a_1 + a_2 = 2a_3$

(b) The equations of motion are given as follows

For M_3, $M_3g - T - T = M_3a_3$ ... (i)

For M_1, $T - \mu M_1 g = M_1 a_1$ ... (ii)

For M_2, $T - \mu M_2 g = M_2 a_2$ ... (iii)

$a_1 + a_2 = 2a_3$ ... (iv)

Dividing (i) by M_3, $g - \frac{2T}{M_3} = a_3$

Dividing (ii) by M_1, $\frac{T}{M_1} - \mu g = a_1$

Dividing (iii) by M_2, $\frac{T}{M_2} - \mu g = a_2$

Using (iv), $a_1 + a_2 = 2a_3$

$$\frac{T}{M_1} - \mu g + \frac{T}{M2} - \mu g = 2g - \frac{4T}{M_3}$$

$$T\left[\frac{1}{M_1} + \frac{1}{M_2} + \frac{4}{M_3}\right] = 2\mu g + 2g = 2g\left[\mu + 1\right]$$

$$T = \frac{\left[2g(\mu+1)\right]}{\frac{1}{M_1} + \frac{1}{M_2} + \frac{4}{M_3}} = \frac{(\mu+1)g}{\frac{1}{2M_1} + \frac{1}{2M_2} + \frac{2}{M_3}}$$

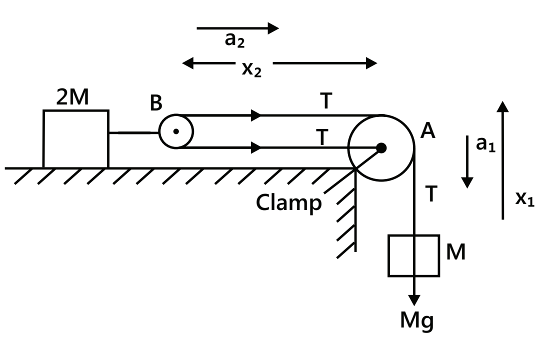
Example 6: Masses M and 2M are connected through pulleys A and B with strings as shown in the Figure. Assume that both the pulleys and the strings are light and all the surfaces are frictionless.

(a) Find the acceleration of the block of mass M.

(b) Find the tension in the string.

(c) Calculate the force exerted on the clamp.

Sol: To find the constraint relation between accelerations of blocks M and 2M, measure all distances from the fixed pulley A. Apply Newton's second law in horizontal direction for block 2M and Newton's second law in vertical direction for block M.



(a) Let L be the length of the string. Let x_1 be the length of the vertical string and x_2 be the length of each string in the horizontal direction. The constraint relation for the string of length L is $x_1 + 2x_2 = L$ Now, by differentiating twice, $a_1 + 2a_2 = 0$

If a_1 is +ve, then a_2 is –ve,

$a_1 - 2a_2 = 0$ or $a_2 = \frac{a_1}{2} = \frac{a}{2}$

Let $a_1 =$ a be the acceleration of M and $\frac{a}{2}$ be the acceleration of 2M.

$\therefore$ $Mg - T = Ma$ ... (i)

$2M \times \frac{a}{2} = 2T$ ... (ii)

$2T = Ma$ or $T = \frac{Ma}{2}$

Now, by substituting for T in (i)

$$Mg - \frac{Ma}{2} = Ma; \frac{3Ma}{2} = Mg; \therefore a = \frac{2g}{3}$$

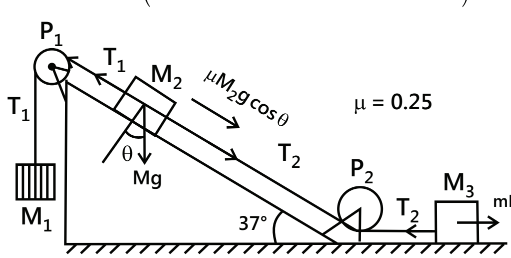
(b) $T = \frac{Ma}{2} = \frac{2gM}{2 \times 3} = \frac{Mg}{3}$

(c) Force on clamp $C = \sqrt{(2T)^2 + (T)^2} = \sqrt{5}T = \frac{\sqrt{5}Mg}{3}$.

Example 7: Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the Figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the incline and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 0.4 kg each and the coefficient of kinetic friction between masses and surfaces is 0.25. The inclined plane makes an angle of $37°$ with the horizontal, however, the mass M_1 moves with uniform velocity downwards. Now, find

(a) The tension in the horizontal portion of the string

(b) The mass M_1 $\left(g = 9.8\ ms^{-2}, \sin 37° = 3/5\right)$.



Sol: Apply Newton's first law for each of the blocks as the velocity of each block is constant.

Let T_1 be the tension between M_1 and M_2 and T_2 be the tension between M_2 and M_3.

Let μ be the coefficient of kinetic friction. Then

(a) $T_2 = \mu M_3 g = 0.25 \times 4.0 \times 9.8 = 9.8N$

(b) $T_1 = M_1 g$

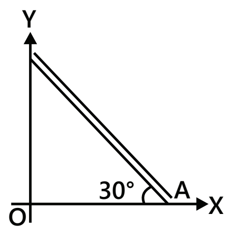
$$T_1 - T_2 - \mu M_2 g\cos\theta - M_2 g\sin\theta = 0$$

$$M_1 g - 9.8 - 0.25 \times 4 \times 9.8 \times \frac{4}{5} - 4 \times 9.8 \times \frac{3}{5} = 0$$

$$M_1 \times 9.8 = 9.8 + 9.8 \times \frac{4}{5} + 9.8 \times \frac{12}{5} = 0$$

$$\therefore M_1 = 1 + \frac{4}{5} + \frac{12}{5} = \frac{21}{5} = 4.2 \text{ kg}$$

Example 8: A rod AB rests with the end A on rough horizontal ground and the end B against smooth vertical wall. The rod is of uniform length and of weight W. If the rod is in equilibrium in the position shown in the Figure, then find:

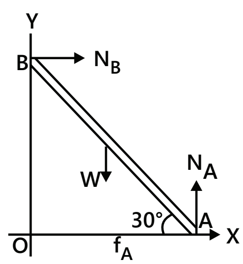


(a) Frictional force at A

(b) Normal reaction at A

(c) Normal reaction at B.

Sol: For translational equilibrium, the vector sum of all the forces acting on the rod is zero. Take component of forces along horizontal (x-axis) and vertical (y-axis) direction. Sum of components of forces along the x and y axes will be zero. For rotational equilibrium, the net torque of all the forces acting on the rod relative to a fixed point (say O) is zero.



Let the length of the rod be 2l. Using the three conditions of equilibrium, the anticlockwise moment is taken as positive.

(i) $\sum F_x = 0 \quad \therefore N_B - f_A = 0$ or $N_B = f_A$ ... (i)

(ii) $\sum F_y = 0 \therefore N_A - W = 0$ or $N_A = W$ ... (ii)

(iii) $\sum \tau_0 = 0$

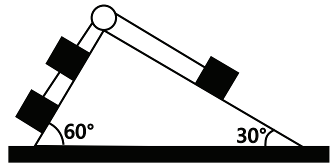
$$\therefore N_A(2l\cos 30°) - N_B(2l\sin 30°) - W(l\cos 30°) = 0$$

or $\sqrt{3}N_A - N_B - \frac{\sqrt{3}}{2}W = 0$ ... (iii)

Solving the above three equations, we obtain

(a) $f_A = \frac{\sqrt{3}}{2}W$ (b) $N_A = W$ (c) $N_B = \frac{\sqrt{3}}{2}W$

Example 9: In the adjacent Figure, masses of A, B and C are 1 kg, 3 kg and 2 kg, respectively. Find (a) the acceleration of the system and (b) tension in the string. Neglect friction ($g = 10 m/s^2$)



Sol: Draw the FBD of each block and apply Newton's second law along the incline plane for each block.

(a) In this case, the net pulling force

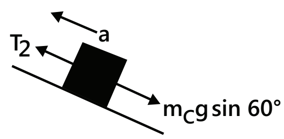
$= m_A g\sin 60° + m_B g\sin 60° - m_C g\sin 60°$

$$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\frac{\sqrt{3}}{2} - (2)(10)\frac{1}{2} = 24.64N$$

Therefore, the total mass being pulled = 1+3+2 = 6 kg

$\therefore$ Acceleration of the system $a = \frac{21.17}{6} = 4.1 m/s^2$

(b) For the tension in the string between A and B.



FBD of A is $m_A g\sin 60° - T_1 = (m_A)(a) = m_A(g\sin 60° - a)$

$$\therefore T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56N$$

(b) For the tension in the string between B and C. FBD of C, $T_2 - m_c g\sin 30° = m_c a$

$$\therefore T_2 = m_c(a + g\sin 30°) \therefore T_2 = 2\left[3.53 + 10\left(\frac{1}{2}\right)\right] = 18.2N$$

Example 10: A small smooth ring of mass m is threaded on a light inextensible string of length 8L which has its ends fixed at points in the same vertical line at distance 4L apart. The ring describes horizontal circles at constant speed with both parts of the string taut and with the lower portion of the string horizontal. Find the speed of the ring and tension in the string. The ring is then tied at the midpoint of the string and made to perform horizontal circles at constant speed of $3\sqrt{gL}$. Find the tension in each part of the string.

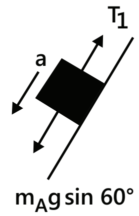
Sol: Apply Newton's second law in the radial direction in each case.

When the string passes through the ring, the tension in the string is the same in both the parts. Also from geometry, BP=3L and AP=5L

$T\cos\theta = \frac{4}{5}T = mg$

$T + T\sin\theta = T\left(1+\frac{3}{5}\right) = \frac{8}{5}T$ ... (i)

$= \frac{mv^2}{BP} = \frac{mv^2}{3L}$ ... (ii)



Dividing (ii) by (i) $\frac{v2}{3Lg} = 2$

$v = \sqrt{6Lg}$ From (i) $T = \frac{mg}{4/5} = \frac{5}{4}mg$. In the second case, ABP is an equilateral triangle

$T_1\cos 60^o = mg + T_2\cos 60^o$

$T_1 - T_2 = \frac{mg}{\cos 60°} = 2mg$ ... (iii)

$T_1\sin 60° + T_2\sin 60° = \frac{mv^2}{r} = \frac{9mgL}{4L\sin 60°}$

$T_1 + T_2 = \frac{9mg}{4\sin^2 60°} = 3mg$ ... (iv)

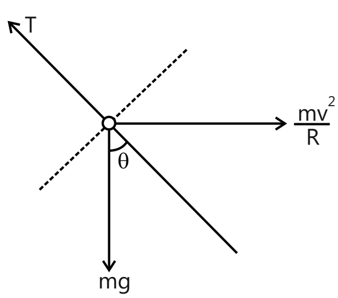
By solving equations (iii) and (iv), we have

$T_1 = \frac{5}{2}mg;\ T_2 = \frac{1}{2}mg$

Example 11: A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from roof by a light rigid rod of length 1 m. What is the angle made by the rod with the track?

Sol: In the reference frame of the car the bob will experience a centrifugal force radially outwards. The vector sum of the three forces acting on the bob (the weight, the tension and the centrifugal force) will be equal to zero.

The different forces acting on the bob are shown in the Figure. Resolving the force along the length and perpendicular to the rod, we have



$mg\cos\theta + \frac{mv^2}{R}\sin\theta = T;\ mg\sin\theta = \frac{mv^2}{R}\cos\theta$

$\tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)} = 1;\ \theta = \tan^{-1}(1) = 45°$

Example 12: A car moves on a horizontal circular road of radius R, the speed increases at a rate $\frac{dv}{dt} = a$. The frictional coefficient between the road and the tire is μ. Now, find the speed at which the car will skid.

Sol: The net acceleration of the car is the vector sum of centripetal acceleration and tangential acceleration. By Newton's second law the net force of friction acting on the car is equal to mass multiplied by net acceleration.

Here, at any time t, the speed of the car becomes V,

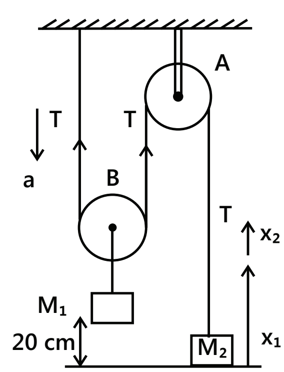
the net acceleration in the plane road is $\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2}$.

This acceleration is provided by the frictional force. At the moment car will slide

$m\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2} = \mu Mg \Rightarrow v = \left[R^2\left(\mu^2 g^2 - a^2\right)\right]^{1/4}$

JEE Advanced/Boards

Example 1: In the system of two pulleys connected as shown in the figure, $M_1 = 4M_2$ and mass M_1 is 20 cm above the ground, whereas mass M_2 is lying on the ground. Find the distance covered by when the system is released. ($g = 10\,m/s^2$).



Sol: To find the constraint relation between accelerations of M_1 and M_2, measure their distances from fixed pulley A. Apply Newton's second law in vertical direction for each block.

$M_1 = 4M_2$

As M_1 is heavier, itwill move down with acceleration a and M_2 will move upward with acceleration 2a because the strings around the pulley B will move through half the distance as compared to that of A.

$\therefore M_1g - 2T = M_1a \quad T - M_2g = M_2 \times 2a$ or $a = \frac{g}{4}$.

Therefore, the time taken for M_1 to reach the ground at 20 cm distance $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$

or $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 20 \times 4}{10 \times 100}} \quad \left(\because s = \frac{20}{100}\right)$

or $= \sqrt{\frac{16}{100}} = \frac{4}{10} = 0.45$

Distance travelled by M_2

$x_1 = \frac{1}{2} \times (2a) \times t^2 = \frac{1}{2} \times 2 \times \frac{10}{4} \times (0.4)^2 = 0.4\text{m}$

Velocity of M_2 after 0.4 seconds $= v = u + 2at$

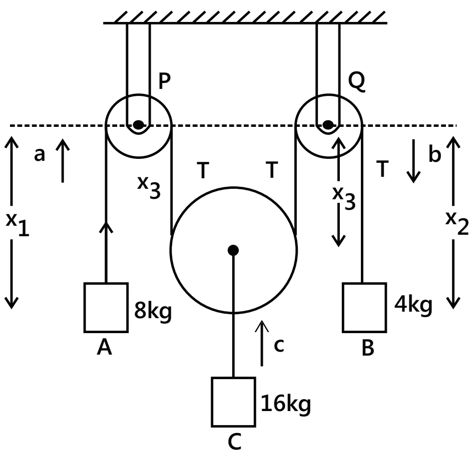
$= 0 + 2 \times \frac{10}{4} \times 0.4; \qquad v = 2\text{ms}^{-1}$

Distance covered by M_2 with velocity 2ms^{-1}upwards before coming to rest

$x_2 = \frac{v^2}{2g} = \frac{(2)^2}{2 \times 10} = 0.2\text{ m}$

Distance covered by M_2 before coming to rest $= x = x_1 + x_2 = 0.4 + 0.2 = 0.6\text{ m}$

Example 2: Masses 4 kg and 8 kg are attached to the free end of an inextensible light string passing over two fixed pulleys as shown in the Figure. The movable pulley carries a mass of 16 kg. Assuming frictionless motion, calculate the acceleration of the three masses.



Sol: To find the constraint relation between accelerations of blocks measure their distances from the fixed pulleys. Apply Newton's second law in vertical direction for each block.

Let a, b and c be the respective accelerations of masses A (8 kg), B (4 kg), and C (16 kg) such that a and b are downward and c is upward. Let x_1 and x_2 be the distances of strings from axial line passing through P and Q to the blocks A and B, respectively. Let x_3 be the length of the string from the axial line PQ to the center of the movable pulley. If L is the length of the string, then the constraint relation gives

$x_1 + x_2 + x_3 + x_3 = L = \text{constant}$

Differentiating $\frac{dx_1}{dt} + \frac{dx_2}{dt} + 2\frac{dx_3}{dt} = 0$

Differentiating again $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$

or $a + b - 2c = 0$ or $a + b = 2c$.

As tension T is equal in all the strings as it passes over smooth pulleys, equations for the strings are as follows:

$8g - T = 8a$ ... (i)

$2T - 16g = 16c$ ... (ii)

$4g - T = 4b$ ... (iii)

$a + b = 2c$ ... (iv)

From Eqs. (ii) and (iv), we obtain

$2T - 16g = 8 \times 2c = 8a + 8b$

By substituting a and b from Eqs (i) and (iii)

$2T - 16g = 8g - T + 8g - 2T = 16g - 3T$

$5T = 32g$ or $T = \frac{32}{5}g$.

Now, from Eq (i)

$8a = 8g - T = 8g - \frac{32}{5} = \frac{32}{5}; a = \frac{g}{5}$

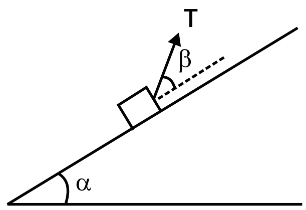
From Eq (iii),

$4b = 4g - T = 4g - \frac{32g}{5} = \frac{-12g}{5}$

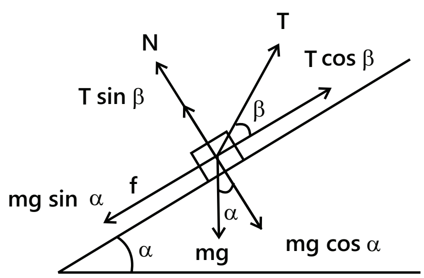
$c = \frac{a+b}{2} = \frac{\left(\frac{g}{5} - \frac{3g}{5}\right)}{2} \quad \therefore c = -\frac{g}{5}$

$\therefore$ 16 kg and 8 kg go downward and 4 kg go upward.

Example 3: A block of mass m is pulled up by means of a thread up and inclined plane forming an angle α with the horizontal. The coefficient of friction is equal to μ. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. Also, find the value of minimum tension.



Sol: Draw the FBD of the block. Apply Newton's first law along the perpendicular to the inclined plane and Newton's second law along the inclined plane for the block.



When the body is just about to move up, the force of friction f is acting downward. If N is the normal reaction, the force of friction f is equal to μN. Further, T and mg can be resolved into rectangular components parallel and perpendicular to the inclined plane as shown in the Figure.

$\therefore T\cos\beta = mg\sin\alpha + \mu N$ ....(i)

$N + T\sin\beta = mg\cos\alpha$ or $N = mg\cos\alpha - T\sin\beta$

Now, by substituting in Eq.(i), we obtain

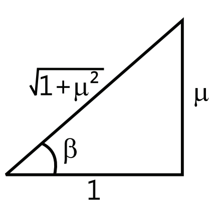
$T\cos\beta = mg\sin\alpha + \mu mg\cos\alpha - \mu T\sin\beta$

or $T\cos\beta + \mu T\sin\beta = mg(\sin\alpha + \mu\cos\alpha)$

$$T = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\cos\beta + \mu\sin\beta}$$

For T to be minimum, $\cos\beta + \mu\sin\beta$ should be maximum.

$$\therefore \frac{d}{d\beta}(\cos\beta + \mu\sin\beta) = 0$$



and $\frac{d^2}{d\beta^2}(\cos\beta + \mu\sin\beta)$ is negative.

$$\therefore -\sin\beta + \mu\cos\beta = 0 \text{ or } \mu = \frac{\sin\beta}{\cos\beta} = \tan\beta$$

Also, $\frac{d}{d\beta}(-\sin\beta + \mu\cos\beta) = -\cos\beta + \mu\sin\beta$

which is negative.

$\therefore$ For minimum T, $\beta = \tan^{-1}\mu$

The value of T_{min} can be found by writing β in terms of μ.

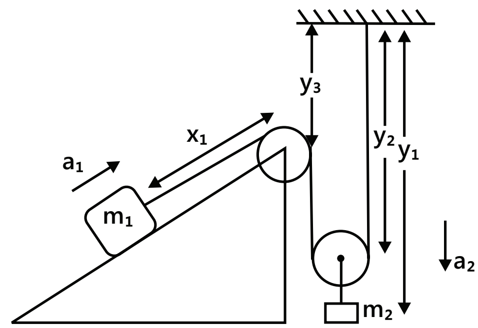
$$\cos\beta = \frac{1}{\sqrt{1+\mu^2}}, \sin\beta = \frac{1}{\sqrt{1+\mu^2}}$$

$$\therefore T_{min} = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\cos\beta + \mu\sin\beta}$$

$$= \frac{mg(\sin\alpha + \mu\cos\alpha)}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\frac{1+\mu^2}{\sqrt{1+\mu^2}}}$$

$$= \frac{mg(\sin\alpha + \mu\cos\alpha)}{\sqrt{1+\mu^2}}$$

Example 4: Find the constraint relation in the Figure.



Sol: To find the constraint relation between accelerations of blocks measure their distances from stationary points. For block m_1 measure the distance from fixed pulley on the wedge. For block m_2 measure the distance from the fixed roof.

Since length of each string is constant

$x_1 + (y_2 - y_3) + y_2 = c_1$ ... (i)

$y_1 - y_2 = c_2 \Rightarrow 2y_1 - 2y_2 = 2c_2$ ... (ii)

By adding (i) and (ii), we obtain

$(x_1 - y_3 + 2y_1) = c_1 + 2c_2$

$x_1 + 2y_1 = y_3 + c_1 + 2c_2 = c$ ... (iii)

(since y_3 = constant)

Differentiating (iii) w.r.t. t $\frac{d^2x_1}{dt^2} + 2\frac{d^2y_1}{dt^2} = 0$

$-a_1 + 2a_2 = 0 \Rightarrow a_1 = 2a_2$ ... (iv)

Example 5: A pendulum is hanging from the ceiling of a car having acceleration a_0 with respect to the ground. Find the angle made by the string with the vertical.

Sol: In the reference frame of the car the pendulum bob will experience a pseudo force. For the bob to be in equilibrium, the vector sum of all the forces acting on it in the frame of the car should be zero.

The situation is shown in the Figure. Suppose that the mass of the bob is m and string makes an angle θ with the vertical. We shall now proceed based on the car frame. This frame is non-inertial as it has acceleration $\vec{a_0}$ with respect to an inertial frame (the road). Hence, if we use Newton's second law we shall have to include a pseudo force.

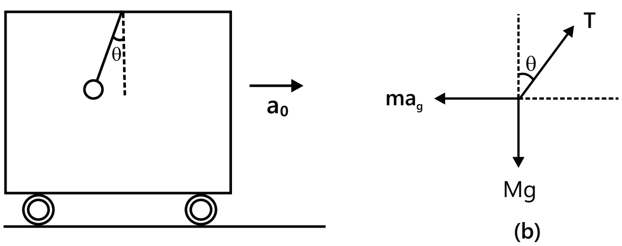
Now, consider the bob as the complete system.

Then, the forces acting on it are:

(a) T along the string, by the string

(b) mg downward, by the earth

(c) m a_0 towards left (pseudo force).



(b)

(a)

The FBD is shown in the Figure. As the bob is at rest (remember we are discussing the motion with respect to car) the force in (a), (b) and (c) should add to zero. Take the X-axis along the forward horizontal direction and the Y-axis along the upward vertical direction.

The components of the forces along the X-axis give

$T\sin\theta - ma_0 = 0$ or, $T\sin\theta = ma_0$ ... (i)

And the components along the Y-axis give

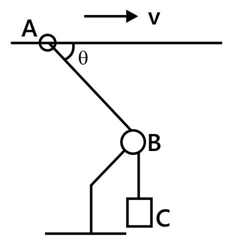
$T\cos\theta - mg = 0$ or, $T\cos\theta = mg$ ... (ii)

Dividing (i) by (ii) $\tan\theta = a_0 / g$

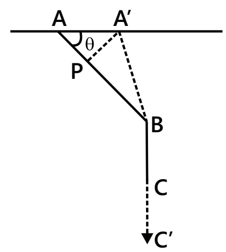
Thus, the string makes an angle

$\tan^{-1}(a_0 / g)$ with the vertical

Example 6: A smooth ring A of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley B and carries a block C of mass M(= 2m) as shown in the Figure. At an instant the string between the ring and pulley makes an angle θ with the rod. (a) Show that, if the ring slides with a speed v, the block descends with speed $v\cos\theta$. (b) With what acceleration will the ring start moving if the system is released from rest with $\theta = 30°$?



Sol: Find the constraint relation between the acceleration of the ring and the block. Measure the distances of ring and the block from the fixed pulley B.



(a) Suppose in a small time interval Δt the ring is displaced from A to A' and the block from C to C'. Drop a perpendicular A'P from A' to AB. For small displacement A'B = PB. Since the length of the string is constant,

we have $AB + BC = A'B + BC'$

or, $AP + PB + BC = A'B + BC$

or, $AP = B'C - BC = CC'$ (as $A'B = PB$)

or, $AA'\cos\theta = CC'$

or, $\dfrac{AA'\cos\theta}{\Delta t} = \dfrac{CC'}{\Delta t}$

or, (velocity of the ring) $\cos\theta$ = (velocity of the block).

(b) If the initial acceleration of the ring is a, that of the block will be $a\cos\theta$. Let T be the tension in the string at this instant. Consider the block as the system. The forces acting on the block are

(i) Mg downward due to earth, and

(ii) T upward due to string.

equation of motion of the block is

$Mg - T = Ma\cos\theta$ ... (ii)

Now, consider the ring as the system. The forces acting on the ring are

(i) Mg downward due to gravity,

(ii) N upward due to the rod,

(iii) T along the string due to string.

Taking components along the rod, the equation of motion of the ring is

$T\cos\theta = ma$ ... (ii)

From (i) and (ii)

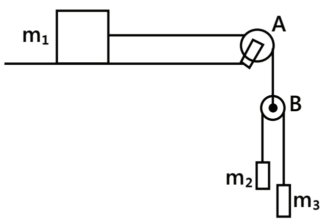
$$Mg - \frac{ma}{\cos\theta} = Ma\cos\theta$$

or, $$a = \frac{Mg\cos\theta}{m + M\cos^2\theta}$$

Putting $\theta = 30°$, M = 2 m and $g = 9.8\ m/s^2$;

therefore, $A = 6.78\ m/s^2$

Example 7: Three blocks of masses m_1, m_2 and m_3 are connected as shown in the Figure. All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of m_1.



Sol: Draw the FBD of all the blocks and the pulley B. The acceleration of pulley B is same in magnitude as the acceleration of m_1. In the frame of pulley B blocks m_2 and m_3 will experience pseudo forces.

Suppose the acceleration of m_1 is a_0 toward the right. That will also be the downward acceleration of the pulley B because the string connecting m_1 and B is constant in length. This implies that the decrease in the separation between m_2 and B equals the increase in the separation between m_3 and B. Therefore, the upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be a.

The acceleration of with respect to the ground = $a_0 - a$ (downward) and the acceleration of with respect to the ground = $a_0 + a$ (downward).

These accelerations will be used in Newton's laws. Let the tension be T in the upper string and T′ in the lower string. Consider the motion of the pulley B.

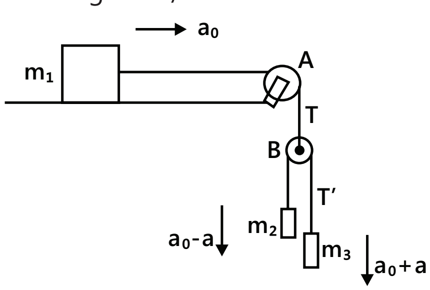
The forces on this light pulley are

(a) T upward by the upper string and

(b) 2T′ downward by the lower string.

As the mass of the pulley is negligible,

2T′ – T = 0 Giving T′= T/2. ... (i)



Motion of: The acceleration is a_0, in the horizontal direction. The forces on m_1 are

(a) T by the string (horizontal).

(b) m_1 g by the earth (vertically downward) and

(c) N by the table (vertically upward).

In the horizontal direction, the equation is

$T = m_1 a_0$ ... (ii)

Motion of: Acceleration is $a_0 - a$ in the downward direction. The forces on m_2 are

(a) m_2 g downward by the earth and

(b) T′ = T/2 upward by the string.

Thus $m_2g - T/2 = m_2(a_0 - a)$ ... (iii)

Motion of m_3: Acceleration is $(a_0 + a)$ in the downward direction. The forces on are

(a) $m_3 g$ downward by the earth and

(b) T′= T/2 upward by the string.

Thus $m_3g - T/2 = m_3(a_0 + a)$ ... (iv)

We want to calculate a_0, so we shall eliminate T and a from (ii), (iii), and (iv).

Putting T from (ii) in (iii) and (iv),

$$a_0 - a = \frac{m_2g - m_1a_0/2}{m_2} = g - \frac{m_1a_0}{2m_2}$$

and $$a_0 + a = \frac{m_3g - m_1a_0/2}{m_3} = g - \frac{m_1a_0}{2m_3}$$

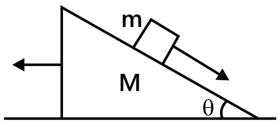
Adding, $$2a_0 = 2g - \frac{m_1a_0}{2}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0 = g - \frac{m_1a_0}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right] = g$$

or, $$a_0 = \frac{g}{\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right]}$$

Example 8: All the surfaces shown in the figure. are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.



Sol: Draw the FBD of both the blocks. In the reference frame of block M, the block m will experience pseudo force. Apply Newton's second law on the block m along the inclined plane and Newton's first law along the perpendicular to the inclined plane. For block M apply Newton's second law along the horizontal.

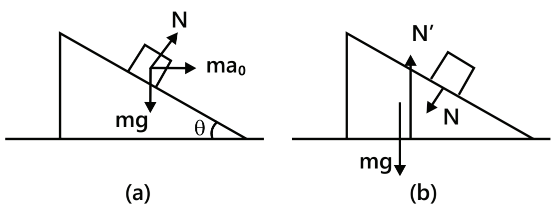
Let the acceleration of the prism be in the backward direction. Consider the motion of the smaller block from the frame of the prism.

The forces on the block are

(i) N normal force,

(ii) mg downward (gravity), and

(iii) m a_0 forward(pseudo).



The block slides down the plane. Components of the forces parallel to the incline give

$ma_0\cos\theta + mg\sin\theta = ma$

or $a = a_0\cos\theta + g\sin\theta$ ... (i)

Components of the force perpendicular to the incline give $N + ma_0\sin\theta = mg\cos\theta$. ... (ii)

Now, consider the motion of the prism from the lab frame. No pseudo force is needed as the frame used is inertial. The forces acting now are

(i) Mg downward,

(ii) N normal to the incline (by the block), and

(iii) N' upward (by the horizontal surface).

Horizontal components give,

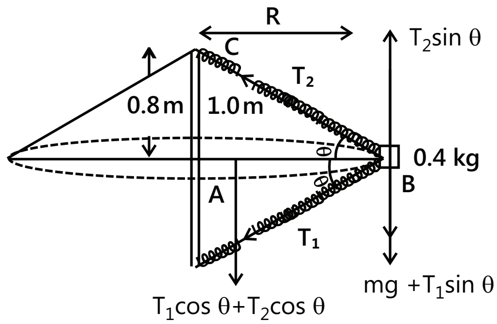
$N\sin\theta = Ma_0$ or, $N = Ma_0/\sin\theta$. ... (iii)

Replacing in (ii) $\dfrac{Ma_0}{\sin\theta} + Ma_0\sin\theta = mg\cos\theta$

or, $a_0 = \dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$

From (i), $a_0 = \dfrac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} + g\sin\theta = \dfrac{(M+m)g\sin\theta}{M + m\sin^2\theta}$

Example 9: A block of mass 0.4 kg is attached to a vertical rotating spindle of length 1.6 m by two springs each of length 1 m of equal lengths as shown in the Figure. The period of rotation is 1.2 seconds. Find the tension in the springs.



Sol: The sum of the horizontal components of tensions in the two springs will provide the necessary centripetal acceleration to the block. The vector sum of the vertical components of tensions in the two springs will balance the weight of the block.

Let T_1 and T_2 be the tension in the springs when these springs subtend an angle θ each with the horizontal direction. Let AB = R be the radius of circular path traversed by mass B in the horizontal plane.

$R = AB = \sqrt{1-(0.8)^2} = \sqrt{1-0.64} = \sqrt{0.36} = 0.6$

$\sin\theta = \dfrac{AC}{BC} = \dfrac{0.8}{1} = 0.8$ m

$\cos\theta = \dfrac{AB}{BC} = \dfrac{0.6}{1} = 0.6$ m

Angular velocity $= \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{1.2} = \dfrac{\pi}{0.6}$

Resolving T_1 and T_2 into rectangular component

$T_2\sin\theta = T_1\sin\theta + mg$

$(T_1 + T_2)\cos\theta = m\omega^2 R$ .....(i)

$T_2\cos\theta = m\omega^2 R - T_1\cos\theta$ .....(ii)

Multiply (i) by $\cos\theta$,

$T_2\sin\theta\cos\theta = T_1\sin\theta\cos\theta + mg\cos\theta$

Multiply (ii) by $\sin\theta$,

$T_2\sin\theta\cos\theta = m\omega^2 R\sin\theta - T_1\sin\theta\cos\theta$

Adding, $2T_2\sin\theta\cos\theta = m\omega^2 R\sin\theta + mg\cos\theta$

$2T_2 = \dfrac{m\omega^2 R\sin\theta}{\sin\theta\cos\theta} + \dfrac{mg\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{m\omega^2 R}{\cos\theta} + \dfrac{mg}{\sin\theta} = \dfrac{0.4\times\pi^2\times 0.6}{(0.6)^2\times 0.6} + \dfrac{0.4\times 10}{0.8} = \dfrac{0.4\times\pi^2}{(0.6)^2} + 5$

$2T_2 = 10.97 + 5 = 15.97 \quad \therefore T_2 = 7.99\ N \approx 8\ N$

Subtracting the above mentioned terms,

$$2T_1 = \frac{m\omega^2 R}{\cos\theta} - \frac{mg}{\sin\theta} = 10.97 - 5 = 5.97$$

$$T_1 = 2.99 \approx 3\ N$$

Example 10: A block of mass m is pulled by means of a thread up an inclined plane forming an angle θ with the horizontal. The coefficient of friction is μ. Find the inclination of the thread with the horizontal so that tension in the thread is minimum. What is the value of the minimum tension?

Sol: Draw the FBD of the block. Apply Newton's second law along the direction of the incline and Newton's first law along the direction perpendicular to the incline.

Let the mass moves up the plane with acceleration a.

Writing the equation of motion, we obtain

$$R + T\sin\alpha = mg\cos\theta$$

$$R = mg\cos\theta - T\sin\alpha \quad \text{... (i)}$$

$$T\cos\alpha - mg\sin\theta - f = ma \quad \text{... (ii)}$$

where t is the force of friction

$$f = \mu(mg\cos\theta - T\sin\alpha) \quad \text{... (iii)}$$

Substituting the value of f from Eq (iii)

in Eq (ii) $T\cos\alpha - mg\sin\theta - \mu mg\cos\theta + \mu T\sin\alpha = ma$

$$T(\cos\alpha + m\sin\alpha) = ma + mg\sin\theta + \mu mg\cos\theta$$

$$T = \frac{ma + mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha} \quad \text{... (iv)}$$

For T to be minimum $(\cos\alpha + \mu\sin\alpha)$ should be

maximum $\frac{d}{d\alpha}(\cos\alpha + \mu\sin\alpha) = 0$

$$\frac{d^2}{d\alpha^2}(\cos\alpha + \mu\sin\alpha) = -ve$$

$$\frac{d}{d\alpha}(\cos\alpha + \mu\sin\alpha) = -\sin\alpha + m\cos\alpha = 0$$

$$\mu = \tan\alpha \quad \alpha = \tan^{-1}(\mu)$$

It can be shown that $\frac{d^2}{d\alpha^2}$ is negative.

T will have minimum value when a = 0 and

$\alpha = \tan^{-1}(\mu)$. From Eq. (iv)

$$T_{min} = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha}$$

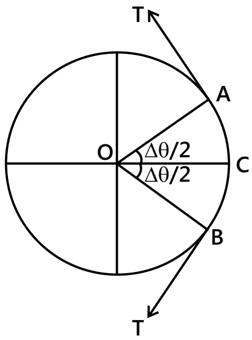
$$\cos\alpha + \mu\sin\alpha = \cos\alpha + \mu(\mu\cos\alpha)$$

$$= \cos\alpha + \mu^2\cos\alpha = \cos\alpha(1+\mu^2)$$

$$= \frac{1+\mu^2}{\sec\alpha} = \frac{1+\mu^2}{\sqrt{1+\tan^2\alpha}} = \frac{1+\mu^2}{\sqrt{1+\mu^2}} = \sqrt{1+\mu^2}$$

$$\therefore T_{min} = \frac{mg\sin\theta + \mu mg\cos\theta}{\sqrt{1+\mu^2}}$$

Example 11: A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of the ring moves with velocity v. Based on the above facts, find the tension in the ring.



Sol: Each small part of the ring will experience a centrifugal force radially outwards. So the ring will tend to expand, i.e. the radius and circumference will tend to increase. By virtue of its elasticity the ring will oppose its expansion. So each part of the ring will experience a force of pull or tension from the other part.

Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the center as shown in the Figure. Let the tension in the ring be T.

The forces on this elementary portion ACB are:

(i) Tension T by the part of the ring left to A

(ii) Tension T by the part of the ring right to B

(iii) Weight $(\Delta m)g$

(iv) Normal force N by the table

As the elementary portion ACB moves in a circle of radius R at constant speed v, its acceleration toward the centre is $\frac{(\Delta m)v^2}{R}$ Resolving the forces along the radius CO

$$T\cos\left(90° - \frac{\Delta\theta}{2}\right) + T\cos\left(90° - \frac{\Delta\theta}{2}\right) = \Delta m\frac{v^2}{R} \quad \text{... (i)}$$

$$2T\sin\frac{\Delta\theta}{2} = \Delta m\frac{v^2}{R} \quad \text{... (ii)}$$

Thus the length of the part ACB = $R\Delta\theta$. The mass per unit length of the ring is $\frac{m}{2\pi R}$

$\therefore$ Mass of this portion ACB, $\Delta m = \frac{R\Delta\theta m}{2\pi R} = \frac{m\Delta\theta}{2\pi}$

Putting the value of Δm in (ii)

$$2T\sin\frac{\Delta\theta}{2}=\frac{m\Delta\theta}{2\pi}\frac{v^2}{R}$$

$$\therefore T=\frac{mv^2}{2\pi R}\left(\frac{\frac{\Delta\theta}{2}}{\left(\sin\left(\frac{\Delta\theta}{2}\right)\right)}\right)$$

Since $\left(\frac{\frac{\Delta\theta}{2}}{\left(\sin\left(\frac{\Delta\theta}{2}\right)\right)}\right)$ is equal to 1; $T=\frac{mv^2}{2\pi R}$

JEE Main/Boards

Exercise 1

Forces and Laws of Motion

Q.1 What is meant by law of inertia?

Q.2 State the laws of motion.

Q.3 A cricket player lowers his hands while catching a ball. Why?

Q.4 An impulsive force of 100N acts on a body for 1 s. What is the change in its linear momentum?

Q.5 A force of 5N changes the velocity of a body from 10 ms^{-1} to 20 ms^{-1} in 5 sec. How much force is required to bring about the same change in 2 sec?

Q.6 State and explain Newton's first law of motion.

Q.7 What are the three types of inertia? Give at least two examples of each type.

Q.8 State and explain Newton's first law of motion. Hence deduce the relation F = ma, where the symbols have their usual meaning.

Q.9 Define absolute and gravitational units of force. What are the dimensions of force?

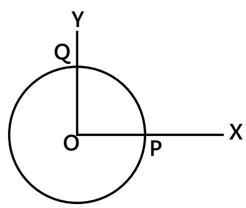
Q.10 Mention some of the consequences of the Newton's second law of motion.

Q.11 Explain the term 'impulse'. Discuss some of the applications of this concept.

Q.12 State and explain Newton's third law of motion. Give at least two Illustrations.

Q.13 Discuss the apparent weight of a man in a lift/ elevator.

Q.14 Two bodies of masses 11 kg and 11.5 kg are connected by a long light string passing over a smooth pulley. Calculate velocity and height ascended/ descended by each body at the end of 4s.



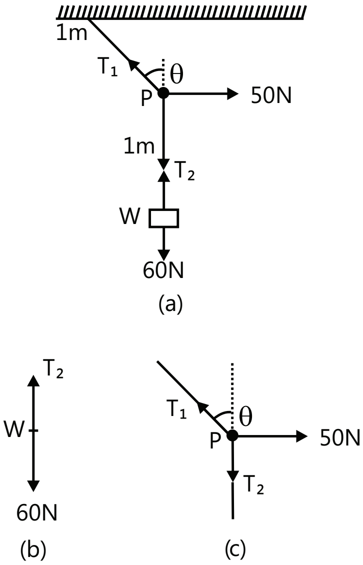
Q.15 A rope of mass 0.5 kg is pulling a block of mass 10 kg under the action of force of 31.5 N. If the block is resting on a smooth horizontal surface, calculate the force of reaction exerted by the block on the rope.

Q.16 Two bodies of masses 4 kg and 3 kg respectively are connected by a light string passing over a smooth frictionless pulley. Calculate the acceleration of the masses and tension in the string.

Q.17 Two bodies whose masses are m_1=50 kg and m_2=50 kg are tied by a light string and are placed on a frictionless horizontal surface. When m_1 is pulled by a force F, an acceleration of 5 ms^{-2} is produced in both the bodies. Calculate the value of F. What is the tension in the string 1?

Q.18 See Figure wherein a mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50N in the horizontal direction is applied at midpoint P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium?

(take g = 10 ms^{-2}) Neglect mass of the rope.



Q.19 A body builder exerts a force of 150N against a bull worker and compresses it by 20 cm. Calculate the spring constant of the spring in the bull worker.

Q.20 A lift of mass 2000 kg is supported by thick steel ropes. If maximum upward acceleration of the lift be $1.2\,m/s^2$, and the breaking stress for the ropes be $2.8\times10^8\ Nm^{-2}$, what would be the minimum diameter of the rope?

Q.21 A car of mass one metric ton travelling at 32 m/s dashes into rear of a truck of mass 8000 kg moving in the same direction with the velocity of 4 m/s. After the collision, the car bounces backward with the velocity 8 m/s. What is the velocity of the truck after the impact?

Q.22 The force on a particle of mass 10 g is (10i+5j)N. If it starts from rest, what would be its position at time t = 5s?

Q.23 A projectile is fired vertically from the earth's surface with an initial velocity of 10 km/s. Neglecting atmospheric retardation, how far above the surface of the earth would it go? Take the earth's radius as 6400 km.

Q.24 Two balls of mass m each are hung side by side two long threads, each of length l. If the distance between the upper end is r then find the distance r' between the centres of the ball in terms of g, r, l and m.

Circular Motion

Q.25 Calculate the centripetal acceleration of a point on the equator of earth due to the rotation of earth about its own axis.

Radius of earth = 6400 km.

Q.26 A cyclist is riding with a speed of 27 kmh^{-1}. As he approaches a circular turn on the road of radius 80.0 m, he applies brakes and reduces his speed at a constant rate of 0.5 ms^{-1} per second. Find the magnitude of the net acceleration of the cyclist.

Q.27 A particle moves in a circle of radius 4.0 cm clockwise at constant seed of 2 cms^{-1}. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X-axis and Y-axis, respectively, find the acceleration of the particle at the instant half-way between P and Q in the Figure.

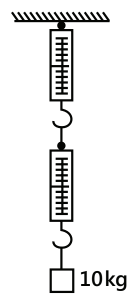
Q.28 A cyclist is riding with a speed of 36 kmh^{-1}. As he approaches a circular turn on the road of radius 140 m, he applies brakes and reduces his speed at the constant rate of 1 ms^{-2}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Exercise 2

Forces and Laws of Motion

Single Correct Question

Q.1 A block of mass 10kg is suspended through two light spring balances as shown in given Figure.



(A) Both the scales will read 10 kg

(B) Both the scales will read 5 kg.

(C) The upper scales will read 10 kg and the lower zero.

(D) The readings may be anything but their sum will be 10 kg

Q.2 A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of $12\,m/s^2$. Find the displacement of the block during the first 0.2 s after the start. Take g=10$\,m/s^2$.

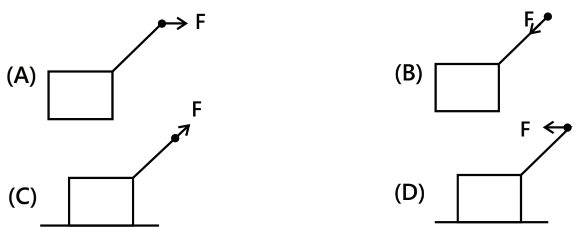
(A) 10 cm (B) 20 cm (C) 30 cm (D) 40 cm

Q.3 A body of mass m is kept on a rough horizontal surface (friction coefficient=μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on the body is F where

(A) F=mg (B) F=μmg

(C) $mg \le F \le mg\sqrt{1+\mu^2}$ (D) $mg \ge F \ge mg\sqrt{1-\mu^2}$

Q.4 Which of the following case correctly represents the applied force on a string under tension. End of string is represented with dot.



Q.5 A balloon is descending at a constant acceleration a. The mass of the balloon is M.

When a mass m is released from the balloon it starts rising with acceleration a. Assuming that volume does not change when the mass is released, what is the value of m? [Assume same upward buoyant force]

(A) $\frac{2a}{(a+g)}M$ (B) $\left(\frac{a+g}{2a}\right)M$

(C) $\frac{2a}{(a+g)M}$ (D) $\frac{Ma}{a+g}$

Q.6 A small cart with a sphere suspended from ceiling by a string is moving up an inclined plane at a speed V. The direction of string supporting the sphere is

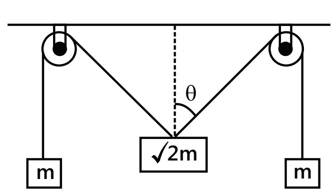
(A) Vertical

(B) Horizontal

(C) Perpendicular to the inclined plane

(D) None of these

Q.7 The pulleys and strings shown in the figure are smooth and of negligible mass. For the system of remain in equilibrium, the angle θ should be



(A) 0° (B) 30° (C) 45° (D) 60°

Q.8 While walking on ice, one should take small steps to avoid slipping. This is because smaller steps ensure

(A) Larger friction (B) Smaller friction

(C) Larger normal force (D) Smaller normal force

Q.9 Two masses m and m′ are tied with a thread passing over a pulley. M′ is on a frictionless horizontal surface and m is hanging freely. If acceleration due to gravity is g, the acceleration of m′ in this arrangement will be

(A) g (B) mg/(m + m′)

(C) mg/m′ (D) mg/(m-m′)

Q.10 A body of mass 60 kg is dragged with just enough force to start moving on a rough surface with coefficients of static and kinetic frictions 0.5 and 0.4 respectively. On continuing (g=9.8 m/s^2) the same force what is the acceleration:

(A) 0.98 m/s^2 (B) 9.8 m/s^2

(C) 0.54 m/s^2 (D) 5.292 m/s^2

Q.11 Which of the following represents 2nd law of motion most correctly.

(A) $\vec{F} = m\vec{a}$ (B) $\vec{F} = m\frac{d\vec{v}}{dt}$

(C) $\vec{F} = \frac{d\vec{p}}{dt}$ (D) $\vec{F} = m\vec{v}$

Q.12 Two objects A and B are thrown upward simultaneously with the same speed. The mass of A is greater than the mass of B. Suppose the air exerts a constant and equal force of resistance on the two bodies.

(A) The two bodies will reach the same height.

(B) A will go higher than B

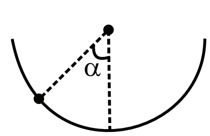
(C) B will go higher than A

(D) Any of the above three may happen depending on the speed with which the objects are thrown.

Q.13 A heavy uniform chain party lies on a horizontal table. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over edge of the table is

(A) 20% (B) 25% (C) 33% (D) 15%.

Q.14 An insect crawls up hemispherical surface very slowly as shown in Figure. The coefficient of friction between insect and surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the max. Possible value of α is given by

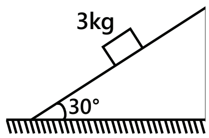


(A) cot α=3 (B) sec α=3 (C) cosec α=3 (D) None

Q.15 When a bird of weight W alights on a stretched wire, the tension T in the wire may be:

(A)>W/2 (B)=W (C)<W (D) None of these.

Q.16 A block of mass 3kg is at rest on a rough inclined plane as shown in the Figure. The magnitude of net force exerted by the surface on the block will be

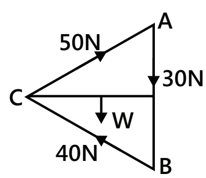


(A) 26N (B) 19.5N (C) 10N (D) 30N

Q.17 With what minimum acceleration can a fireman slides down a rope whose breaking strength is two third of his weight?

(A) g/2 (B) 2g/3 (C) g/3 (D) 3g/4

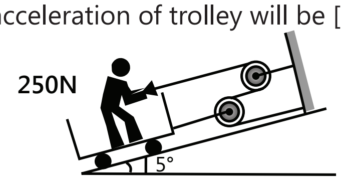
Q.18 Forces of 30N, 40N and 50N act along the sides $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{CA}$ of an equilateral triangle ABC. The triangle is of mass 0.5 kg and kept in a vertical plane as shown in the Figure. With the side AB vertical. The net vertical force acting on the triangle will be ($g=10\ m/s^2$)



(A) 125 N upwards (B) 5 N downwards

(C) 10 N upwards (D) 25 N downwards

Q.19 A trolley is being pulled up an incline plane by a man sitting on it (as shown in Figure). He applies a force of 250N. If the combined mass of the man and trolley is 100 kg, the acceleration of trolley will be $[\sin 15° = 0.26]$



(A) 2.4 m/s² (B) 9.4 m/s²

(C) 6.9 m/s² (D) 4.9 m/s²

Q.20 A body is placed on a rough inclined plane of inclination. As the angle θ is increased from $0°$ to $90°$, the contact force between the block and plane

(A) Remains constant

(B) First remains constant then decreases

(C) First decreases then increases

(D) First increases then decreases

Q.21 A uniform chain of length ℓ is placed on a rough table with length $n\ell$ hanging over the edge ($n<\ell$). If the chain just begins to slide off the table by itself from this position, the coefficient of friction between chain and table is

(A) $\frac{1}{n}$ (B) $\frac{n}{1-n}$ (C) $\frac{1}{n+1}$ (D) $\frac{1-n}{1+n}$

Circular Dynamics

Single Correct Question

Q.22 A particle moves in a circle of radius R with a constant speed under a centripetal force F. The work done F in completing a full circle is:

(A) $\left(Mv^2/R\right)2\pi R$ (B) $\pi R^2 F$

(C) $2\pi RF$ (D) zero

Q.23 When a particle is rotated in a vertical plane with constant angular velocity magnitude of centripetal force is:

(A) Maximum at highest point

(B) Maximum at lowest point

(C) Same at all points

(D) Zero

Q.24 In uniform circular motion, the quantity that remains constant is:

(A) Linear velocity (B) Centripetal force

(C) Acceleration (D) Speed

Q.25 Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is:

(A) $\frac{r_2}{r_1}$ (B) $\sqrt{\frac{r_2}{r_1}}$ (C) $\left(\frac{r_1}{r_2}\right)^2$ (D) $\left(\frac{r_2}{r_1}\right)^2$

Q.26 A 500kg car takes a round turn of radius 50m with a velocity of 36km/hr. The centripetal force is:

(A) 250 N (B) 750 N (C) 1000 N (D) 1200 N

Q.27 A body of mass 5kg is moving in a circle of radius 1m with an angular velocity of 2 radian/sec. The centripetal force is:

(A) 10 N (B) 20 N (C) 30 N (D) 40 N

Q.28 A motorcycle is going on an overbridge of radius R. The driver maintains a constant speed. As the motorcycles is ascending on the overbridge, the normal force on it:

(A) Increase

(B) Decreases

(C) Remains constant

(D) First increases then decreases.

Q.29 If a particle of mass m is moving in a horizontal circle of radius r with a centripetal force $\left(-k/r^2\right)$, the total energy of the particle is:

(A) $-\frac{k}{2r}$ (B) $-\frac{k}{r}$ (C) $-\frac{2k}{r}$ (D) $-\frac{4k}{r}$

Q.30 A person with his hands in his pocket is skating on ice at the rate of 10m/s and describes a circle of radius 50m. What is his inclination to the vertical:

(A) $\tan^{-1}\left(1/2\right)$ (B) $\tan^{-1}\left(1/5\right)$

(C) $\tan^{-1}\left(3/5\right)$ (D) $\tan^{-1}\left(1/10\right)$

Q.31 A ball tied to a string (in vertical plane) is swinging in a vertical circle. Which of the following remains constant during the motion?

(A) Tension in the string

(B) Speed of the ball

(C) Centripetal force

(D) Gravitational force on the ball

Q.32 A heavy particle hanging vertically from a point by a light inextensible string of length l is started so as to make a complete revolution in a vertical plane. The sum of the tension at the ends of any diameter:

(A) First increase then decreases

(B) Is constant

(C) First decrease then increases

(D) Decreases continuously

Q.33 In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be:

(A) $\sqrt{2gR}$ (B) $2gR$ (C) $\sqrt{3gR}$ (D) $\sqrt{gR}$

Previous Years' Questions

Forces and Laws of Motion

Q.1 A ship of mass 3×10^7 kg initially at rest, is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming that the resistance due to water is negligible, the speed of the ship is ***(1980)***

(A) 1.5 m/s (B) 60 m/s

(C) 0.1 m/s (D) 5 m/s

Q.2 A block of mass 2 kg rests on a rough inclined plane making an angle of with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is ***(1980)***

(A) $9.8N$ (B) $0.7\times9.8\times\sqrt{3}N$

(C) $9.8\times\sqrt{3}N$ (D) $0.7\times9.8N$

Q.3 A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. The angle made by the rod with vertical is (Take $g=10m/s^2$) ***(1992)***

(A) Zero (B) 30^o (C) 45^o (D) 60^o

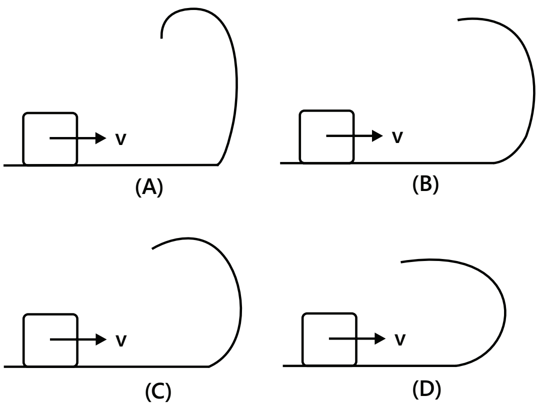
Q.4 A block of mass 0.1 kg is held against a wall applying a horizontal force of 5N on the block. If the coefficient of friction between the block and wall is 0.5, the magnitude of the frictional force acting on the block is ***(1994)***

(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

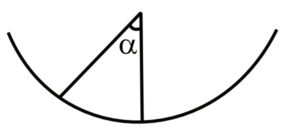
Q.5 A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is ***(2000)***

(A) $\sqrt{\frac{\mu}{a}}$ (B) $\frac{\mu}{\sqrt{a}}$ (C) $\frac{1}{\sqrt{\mu a}}$ (D) infinitesimal

Q.6 A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all the cases. At the highest point of track, the normal reaction is maximum in ***(2001)***



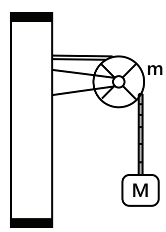
Q.7 An insect crawls up a hemispherical surface very slowly (see the Figure). The coefficient of friction between the surface and insect is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given ***(2001)***



(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$

(C) $\sec \alpha = 3$ (D) $\operatorname{cosec} \alpha = 3$

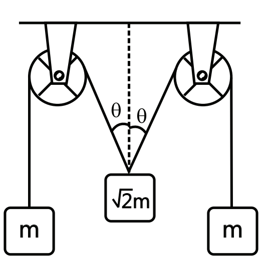
Q.8 A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the Figure. The force on the pulley by the clamp is given by ***(2001)***



(A) $\sqrt{2}Mg$ (B) $\sqrt{2}mg$

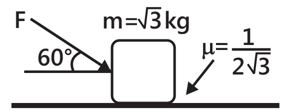
(C) $\sqrt{(M+m)^2 + m^2}g$ (D) $\left(\sqrt{(M+m)^2 + M^2}\right)g$

Q.9 The pulleys and strings shown in the Figure. are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be ***(2001)***



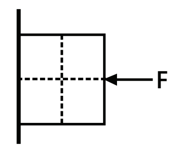
(A) Zero (B) 30° (C) 45° (D) 60°

Q.10 What is the maximum value of force F such that the block shown in the arrangement. does not move? ***(2003)***



(A) 20 N (B) 10 N (C) 12 N (D) 15 N

Q.11 A block of mass m is at rest under the action of force F against a wall as shown in Figure. Which of the following statement is incorrect? ***(2005)***



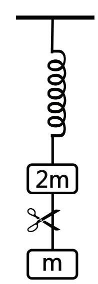
(A) f=mg (where f is the frictional force)

(B) F=N (where N is the normal force)

(C) F will not produce torque

(D) N will not produce torque

Q.12 System shown in Figure is in equilibrium and at rest. The spring and string are massless. Now the string is cut. The acceleration of mass 2m and m just after the string is cut will be ***(2006)***



(A) g/2 upwards, g downwards

(B) g upwards, g/2 downwards

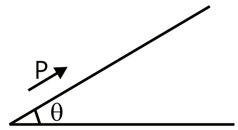
(C) g upwards, 2g downwards

(D) 2g upwards, g downwards

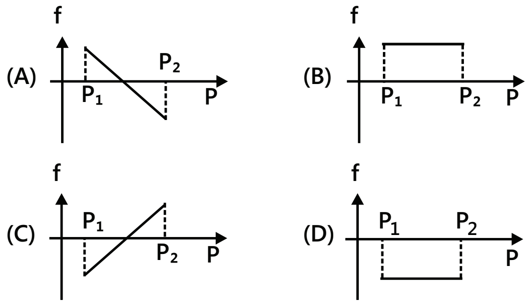
Q.13 A piece of wire is bent in the shape of a parabola (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is ***(2009)***

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

Q.14 A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and plane is μ and $\tan \theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to the positive. As P is varied from



$P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$ the frictional force f versus P graph look like ***(2010)***



Q.15 A reference frame attached to the earth ***(1986)***

(A) Is an inertial frame by definition.

(B) Cannot be an inertial frame because the earth is revolving round the sun.

(C) Is an inertial frame because Newton's law are applicable in this frame.

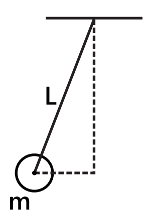
(D) Cannot be an inertial frame because the earth is rotating about its own axis

Circular Dynamics

Q.16 A car is moving in a circular horizontal track of radius 10m with a constant speed of 10m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. The angle made by the rod with the vertical is (Take $g=10/s^2$) ***(1992)***

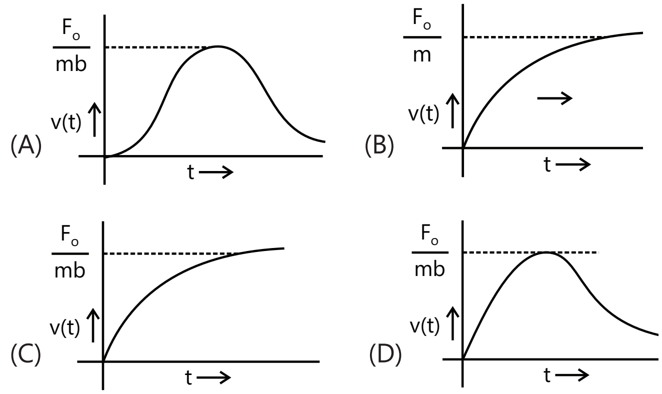
(A) Zero (B) 30° (C) 45° (D) 60°

Q.17 A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) $3kg - ms^{-1}$ ***(2011)***



(A) 9 (B) 18 (C) 27 (D) 36

Q.18 A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves? ***(2012)***



Q.19 A block of mass m is placed on a surface with a vertical cross section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: ***(2014)***

(A) $\frac{1}{3}$ m (B) $\frac{1}{2}$ m (C) $\frac{1}{6}$ m (D) $\frac{2}{3}$ m

Q.20 Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is ***(2015)***

(A) 80 N (B) 120 N (C) 150 N (D) 100 N

Q.21 A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. ***(2016)***

The values of the coefficient of friction μ and the distance x(=QR), are, respectively close to :

(A) 0.2 and 3.5 m (B) 0.29 and 3.5 m

(C) 0.29 and 6.5 m (D) 0

JEE Advanced/Boards

Exercise 1

Forces and Laws of Motion

Q.1 A man of mass 70 kg stands on weighting scale in a lift which is moving

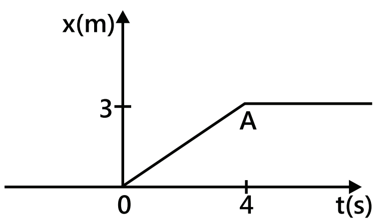
(a) Upwards with a uniform speed of $10\ ms^{-1}$.

(b) Downwards with a uniform acceleration of $5\ ms^{-2}$.

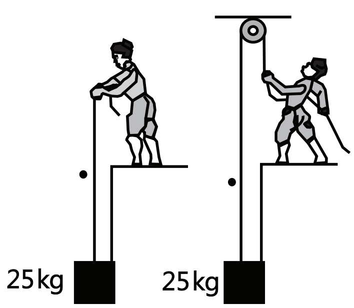
(c) Upwards with a uniform acceleration of $5\ ms^{-2}$.

(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity?

What would be the readings on the scale in each case?

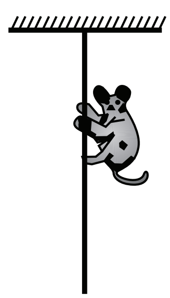


Q.2 A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should man adopt to lift the block without the floor yielding?



(a) (b)

Q.3 A monkey of mass 40 kg climbs on a rope which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey



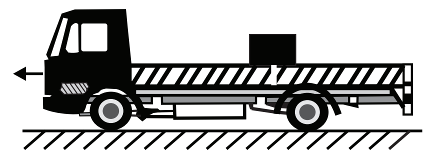
(a) Climbs up with an acceleration of $6ms^{-2}$

(b) Climbs down with an acceleration of $4ms^{-2}$

(c) Climbs up with a uniform speed of $5ms^{-2}$

(d) Falls down the rope nearly freely under gravity.

Q.4 The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Figure. The co-efficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with $2\ ms^{-2}$ At what distance from the starting point does the box fall of the truck? (Ignore the size of the box).



Q.5 A helicopter of mass 1000 kg rises with a vertical acceleration of $15\ ms^{-2}$. The crew and the passenger weigh 300 kg. Give the magnitude and direction of the

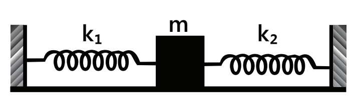
(a) Force on the floor by the crew and passengers.

(b) Action of the rotor of the helicopter on the surrounding air.

(c) Force on the helicopter due to the surrounding air.

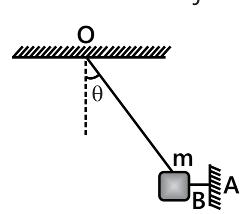
Q.6 A block of mass 15 kg is placed on a long trolley. The co-efficient of static friction between the block and trolley is 0.18. The trolley accelerates from rest with $0.5\ ms^{-2}$ for 20 s and then moves with uniform velocity. Discuss the motion of the block viewed by (a) a stationary observer on the ground. (b) an observer moving with the trolley.

Q.7 Both the springs shown in the Figure are un-stretched. If the block is displaced by a distance x and released, what will be the initial acceleration?

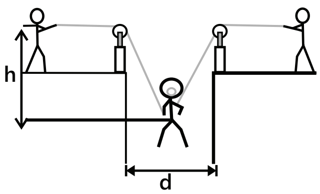


Q.8 Three equal weights of 2 kg each are hanging over the frictionless pulley. Find the acceleration of the system and tension of the string connecting weights A and B. ($g=10\ m/s^2$)

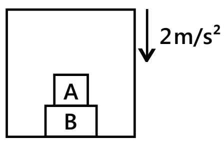
Q.9 Find the tension in OB and AB in the given Figure. Also, calculate the tension in OB when just after the string AB is burnt.



Q.10 A man of mass m has fallen into a ditch of width d and two of his friends are slowly pulling him out using a light rope and two fixed pulleys as shown in Figure. Show that the force (assumed equal for both the friends) exerted by each friend on the rope increases as the man move up. Find the force when the man is at a depth h.



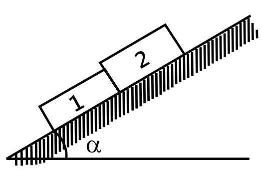
Q.11 The elevator shown in the Figure is descending with an acceleration of 2m/s². The mass of block A is 0.5 kg. What force es exerted by the block A on the block B?



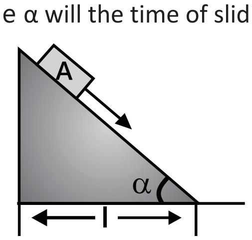
Q.12 The force of buoyancy exerted by the atmosphere on a balloon is B in the upward direction and remains constant. The force of air resistance on the balloon acts opposite to the direction of velocity and is proportional to it. The balloon carries a mass M and is found to fall down near the earth's surface with a constant velocity v. How much mass should be removed from the balloon so that it may rise with a constant velocity v?

Q.13 Two touching bars 1 and 2 are placed on an inclined plane forming an angle α with the horizontal shown in Figure. The masses of the bars are equal to m_1 and m_2 and the coefficients of friction between the inclined plane and the bars are equal to k_1 and k_2 respectively, with $k_1 > k_2$.

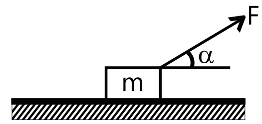
Find (a) The force of interaction of the bars in the process of motion; (b) The minimum value of the angle α at which the bars start sliding down.



Q.14 A small body A starts sliding down from the top of fixed wedge (as shown in the Figure) whose base is equal to l=2.10 m. The coefficient of friction between the body and the wedge surface is k=0.140. At what value of the angle α will the time of sliding be the least?

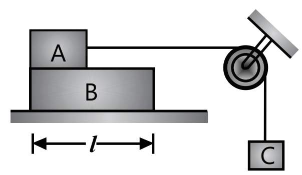


Q.15 At the moment t=0 the force F=at is applied to a small body of mass m resting on a smooth horizontal plane (a) is constant). The permanent direction of this force forms as angle α with the horizontal. Find (a) The velocity of the body at the moment of its breaking off the plane; (b) The distance traversed by the body up to this moment.

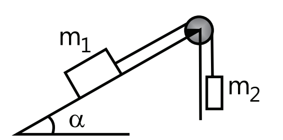


Q.16 Two identical block A and B each of mass M are connected through a light inextensible string. Coefficient of friction between blocks and surfaces are μ as shown. Initially string is relaxed in its normal length. Force F is applied on block A as shown. Find the force of friction on blocks and tension in the string.

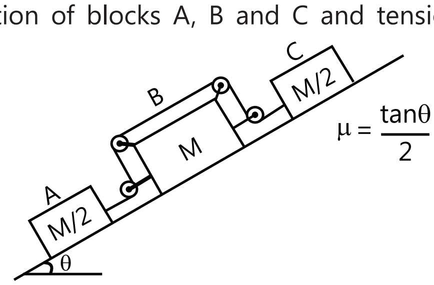
Q.17 In the Figure block A is one fourth the length of the block B and there is no friction between block B and surface on which it is placed. The coefficient of sliding friction between A and B. Block C and block A have the same mass and mass of B is four times mass of A. when the system is released, calculate the distance the block B moves when only three fourth of block A is still on the block B.



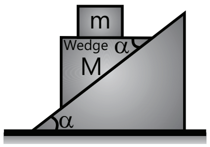
Q.18 The inclined plane of forms an angle α=30° with the horizontal. The mass ratio. The coefficient of friction between the body and inclined plane is equal to k-0.10. The masses of the pulley and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the system of masses starts moving.



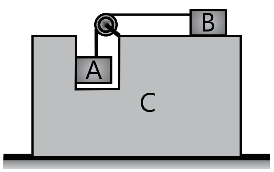
Q.19 As shown in the Figure blocks of masses M/2, M and M/2 are connected through a light string as shown, pulleys are light and smooth. Friction is only between block C and floor. System is released from rest. Find the acceleration of blocks A, B and C and tension in the string.



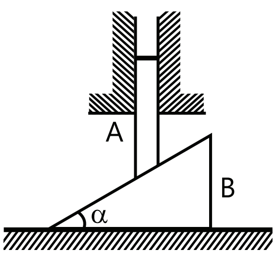
Q.20 On a smooth inclined plane of angle α is placed on in such a way that the upper wedge face is horizontal. On this horizontal face is placed a block of mass m. Find the resultant acceleration of the block in subsequent motion.



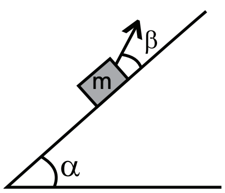
Q.21 In the system shown in Figure. $m_A = 4m, m_B = 3$ m and $m_C = 8m$. Friction is absent everywhere. String is light and inextensible. If the system is released from rest find the acceleration of each block.



Q.22 Find the accelerations of rod A and wedge B in the arrangement shown in the Figure. If the ratio of the mass of the wedge of that of the rod equals, and the friction between all contact surfaces is negligible.



Q.23 A particle of mass m is pulled by means of a thread up an inclined plane forming an angle α with the horizontal as shown in the Figure. The coefficient of friction is equal to μ. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be maximum. What is it equal to?



Q.24 A plank of mass with a block of mass m_2 placed on it lies on a smooth horizontal plane. A horizontal force growing with time t as F=at(a is constant) is applied to the plank. Find how the accelerations of the plank and of the bar w_2 depend on t, if the coefficient of friction between the plank and block is equal to k. Draw approximate plots of these dependences.

Q.25 A horizontal plane with the coefficient of friction k supports two bodies: a bar and an electric motor with a battery on a block. A thread attached to the bar is wound on the shaft of the electric motor. The distance between the bar and electric motor is equal to l. When the motor is switched on, the bar, whose mass is twice as great as that of the other body, starts moving with a constant acceleration w. How soon will the bodies collide?

Q.26 Two particle of equal masses m and m are connected up a light string of length 2l. A constant force F is applied continuously at the middle of the string, always along the perpendicular bisector of the line joining the two particles. Show that when the distance between the particles is 2x, the acceleration of approach of particles is $\dfrac{fx}{m(\ell^2 - x^2)^{\frac{1}{2}}}$.

Q.27 Determine the acceleration of bodies A and B and the tension in the cable due to application of the 300 N force. Neglect all friction and the masses of pulleys.

Q.28 Two blocks A and B having masses $m_1 = 1$ kg and $m_2 = 4$ kg are arranged as shown in Figure. The pulleys P and Q are light and frictionless. All blocks are resting on the horizontal floor and pulleys are held such that strings remain just taut. At moment t=0, a force F=30N starts acting on the pulley P along vertically upward direction as shown in the Figure: Determine

(a) The time when blocks A and B lose contact with ground,

(b) The velocity of A when B loses contact with ground,

(c) The height raised by A till this instant.

Circular Dynamics

Q.29 An astronaut is rotating in a rotor having vertical axis and radius 4m. If he can withstand upto acceleration of 10 g. Then what is the maximum number of permissible revolutions per second? 60°

Q.30 A racing-car of 1000kg moves round a banked track at a constant speed of 108 km ms^{-2}. Assuming the total reaction at the wheels is normal to the track and the horizontal radius of inclination of the track to the horizontal and the reaction at the wheels.

Q.31 A man whirls a stone around his lead on the end of a string 4metre long. If the stone has a mass of 0.4 kg and the string will break if the tension in it exceeds 8 N, what is the smallest angle the string can make with the horizontal? What is the speed of the stone? 40°

Q.32 A boy whirls a stone in a horizontal circle of radius 1.5m and 2m above the ground by means of a string. The string breaks and the stone files off horizontally, striking the ground 10m away. What is the centripetal acceleration during circular motion?

Q.33 A stone is fastened to one end of a string and is whirled in a vertical circle of radius R. Find the minimum speed the stone can have at the highest point of the circle.

Q.34 A stone of mass 1kg is attached to one end of a string of length 1m and breaking strength 500N, and is whirled in a horizontal circle on a frictionless table top. The other end of the string is kept fixed. Find the maximum speed the stone can attain without breaking the string.

Q.35 A circular automobile test track has a radius of 200m. The track is so designed that when a car travels at a speed of 100 kilometer per hour, the force between the automobile and the track is normal to the surface of track. Find the angle of the bank.

Q.36 A block of mass M is kept on a horizontal ruler. The friction coefficient between the ruler and block is $v = \frac{mg^2 \cos\alpha}{2a \sin^2 \alpha}$. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end. Find the maximum angular speed for which block will slip.

Q.37 A motorcycle has to move with a constant speed on an over bridge which is in the form of a circular are of radius R and has a total length L. Suppose the motorcycle starts from the highest point. (a) What can its maximum velocity be for which contact with road is not broken at the highest point? (b) If the motorcycle goes at speed $gr\phi^2(r - r\phi) = 2lgm$ times the maximum found in part (a). Where will it lose the contact with the road? (c) What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

Q.38 A simple pendulum is suspended from the ceiling of a car taking a turn of radius 10m at a speed of 36km/h. Find the angle made by the string of the pendulum with the vertical if this angle does not change during the turn. Take kmh^{-2}.

Q.39 A heavy particle hanging from a fixed point by a light inextensible string of length ms^{-2} is projected horizontally with speed $-\left(\hat{x}+\hat{y}\right)/\sqrt{2}cm/s^2$, find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Q.40 A hemispherical bowl of radius R is rotated about its axis of symmetry which is kept vertical. A small block is kept in the bowl at a position where the radius makes angle ms^{-2} with the vertical. The block rotates with the bowl without any shipping. The frictional coefficient between the block and the bowl is β=54°28′. Find the range of angular speed for which the block will not slip.

Q.41 A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity m/s^2 in a circular path of radius R=700m. A smooth groove AB of length L=7m is made on the surface of the table. The groove makes an angle $\frac{(k_1+k_2)x}{m}$ with the radius OA of the circle in which the cabin rotates. A small particle if kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.

Q.42 A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R. A smooth pulley of small radius is fastened to the table. Two masses of m and 2m are placed on the table connected through a string going over the pulley. Initially the masses were at rest. Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.

Q.43 A particle of mass m moves along a horizontal circle of radius R such that normal acceleration of particle varies with time as T_{ab}=mgtanθ, T_{ab}=mg/cosθ, T′=mgcosθ. where K is a constant. Calculate

(i) Tangential force on particle at time t

(ii) Total force on particle at time t

(iii) Power developed by total force at time t and

(iv) Average power developed by total force over first t second

Q.44 A smooth sphere of radius R is made to translate in a straight line with a constant acceleration a. A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle $\frac{Mg}{4h}\sqrt{a^2+4h^2}$ it slides.

Q.45 A uniform circular ring of mass per unit length $\frac{2(Mg-B)}{g}$ and radius R is rotating with angular velocity

$f=(k_1-k_2)\frac{mg^2\cos\alpha}{m_1+m_2}$ about its own axis in a gravity free space. Find the tension in the ring.

Q.46 If a particle is rotating in a circle of radius R with velocity at an instant v and the tangential acceleration is a. Find the net acceleration of the particle.

Q.47 A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of the ring moves with speed v. Find the tension in the ring.

Q.48 A car goes on a horizontal circular road of radius R the speed is increasing at a constant rate $\cos\alpha_{min}=\frac{k_1 m_1+k_2 m_2}{m_1+m_2}$ a. The friction coefficient is $\alpha=\frac{1}{2}\tan^{-1}\left(-\frac{1}{\mu}\right)$. Find the speed at which the car will just skid.

Exercise 2

Forces and Laws of Motion

Single Correct Choice Type

Q.1 A chain of length L and mass M is hanging by fixing its upper end to rigid support. The tension in the chain at a distance x from the rigid support is

$$s=\frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$$

Q.2 A block A kept on an inclined surface just begins to slide if the inclination is 30°. The block is replaced by another block B and it is found that it just begins to slide if the inclination is 40°.

(A) Mass of A > mass of B.

(B) Mass of A < mass of B

(C) Mass of A = mass of B

(D) All the three are possible.

Q.3 The arrangement shown in the Figure, the system of masses m_1, m_2 and m_3 is being pushed by a force F applied on m_1 horizontally. In order to prevent the downwards slipping of m_2 between m_1 and m_3. If coefficient of friction between m_2 and m_3 is μ and all the other surfaces are smooth, the minimum value of F

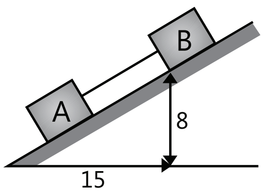
(A) $f_A=\frac{3}{4}\mu Mg,\ f_B=0,\ T=0$

(B) $f_A=\mu Mg,\ f_B=\frac{\mu Mg}{2},\ T=\frac{\mu Mg}{2}$

(C) $F\geq(m_1+m_2+m_3)\mu g$

(D) $F\leq(m_1+m_2+m_3)\mu g$

Q.4 Blocks A and B in Figure are connected by a bar of negligible weight. If mass of A and M are 170 kg each and $\mu_A=0.2$ and $\mu_B=0.4$, where μ_A and μ_B are the coefficients of limiting friction between blocks and plane, calculate the force in the bar. $\left(g=10\ m/s^2\right)$



(A) 150 N (B) 75 N (C) 200 N (D) 250 N

Q.5 A person, standing on the floor of an elevator, drops a coin. The coin reaches the floor of the elevator in a time t_1. If the elevator is stationary and in time t_2 if it is moving uniformly. Then

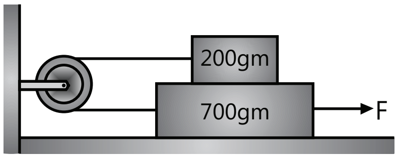
(A) $t_1=t_2$

(B) $t_1<t_2$

(C) $t_1>t_2$

(D) $t_1>t_2$ or $t_1<t_2$ depending on whether the lift is going up or down.

Q.6 How large must F be in the Figure shown to given the 700 gm block an acceleration of $30\ cm/s^2$? The coefficient of friction between all surfaces is 0.15.



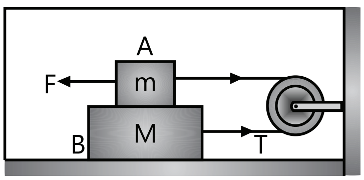
(A) 4 N (B) 2.18 N (C) 3.18 N (D) 6N

Q.7 If the force which acting parallel to an inclined plane of angle α just sufficient to draw the weight up in n times the force which will just let it be on the point of sliding down, the coefficient of friction will be

(A) $\mu=\frac{(n-1)}{n+1}\tan\alpha$ (B) $\mu=\frac{(n+1)}{n-1}\tan\alpha$

(C) $\mu=n\tan\alpha$ (D) $\mu=(n+1)\tan\alpha$

Q.8 Two blocks A and B of masses m and M are placed in a platform as shown in the Figure. The friction coefficient between A and B is μ but there is no friction between B and the platform. The whole arrangement is placed inside an elevator which is coming down with an acceleration a(a < g). What maximum horizontal force F can be applied to A without disturbing the equilibrium of the system?



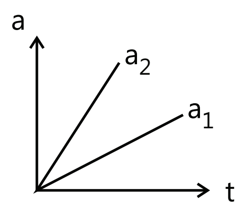
(A) 2μmg (B) 2μm(g – a)

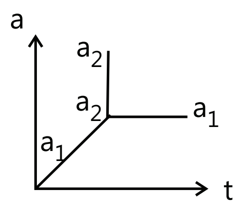
(C) 2μm(g+a) (D) 2μma

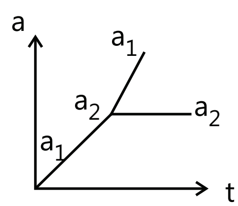
Q.9 A body of mass m_1 is placed on a horizontal plank of mass m_2 which rests on a smooth horizontal table. The coefficient of friction between the mass m_1 and plank is μ. A gradually increasing force F depending on time t as F=at where a is constant is applied to the plank. The time t_0 at which the plank starts sliding under the mass is

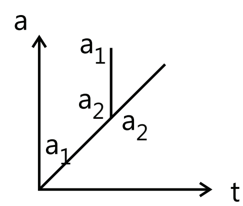
(A) $\frac{m_1\mu g}{a}$ (B) $\frac{(m_1+m_2)\mu g}{a}$ (C) $\frac{m_2\mu g}{a}$ (D) $\frac{m_1 m_2\mu g}{a}$

Q.10 Block A is placed on block B whose mass is greater than that of A. There is friction between blocks while the ground is smooth. A horizontal force P increasing linearly with time begins to act on A. The accelerations a_1 and a_2 of A and B respectively are plotted in a graph against time. Which of the following graphs represents the real situation?

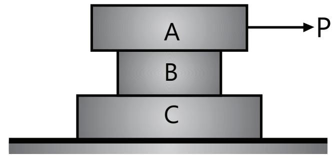
(A)


(B)


(C)


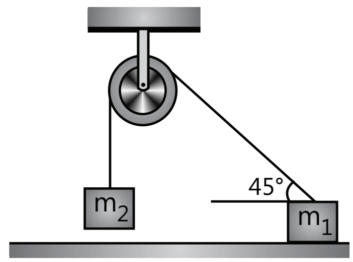
(D)


Q.11 Find the least horizontal force P to start motion of any part of the system of the three blocks resting upon one another as shown in Figure. The weight of blocks are A=300 N, B= 100 N, C=200 N. Between A and B, μ=0.3. Between B and C, μ=0.2. Between C and the ground μ=0.1.



(A) 90 N (B) 60 N (C) 80 N (D) 100 N

Q.12 A block of mass rests on a rough horizontal plane relative to which the coefficient of friction is μ. A light string attached to the body passes over a light pulley and carries at its other end a mass m_2. When the system just begins to move, the value of μ is

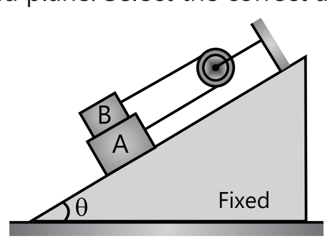


(A) $\frac{m_2}{\sqrt{2}m_1 - m_2}$ (B) $\frac{m_2}{\sqrt{2}m_1 + m_2}$

(C) $\frac{m_2}{\sqrt{2}m_2 + m_1}$ (D) $\frac{m2}{\sqrt{2}m_2 - m_1}$

Multiple Correct Choice Type

Q.14 In the arrangement shown in the Figure pulley is smooth and massless and string is light. Friction coefficient between A and B is μ. Friction is absent between A and plane. Select the correct alternative(s)



(A) acceleration of the system is zero if

$\mu \geq \frac{m_B - m_A}{2m_B}\tan\theta$ and $m_B > m_A$

(B) Force of friction between A and B is zero if

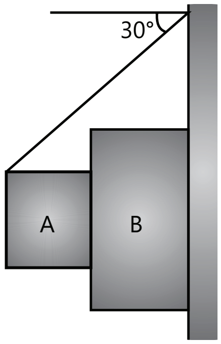
$m_A = m_B$

(C) B moves upwards if $m_B < m_A$

(D) Tension in the string is $mg(\sin\theta - \mu\cos\theta)$ if

$m_A = m_B = m$

Q.15 (A, D) Two blocks A and B of mass 10 kg and 20 kg respectively are placed as shown in Figure. Coefficient of friction between all the surfaces is 0.2 ($g=10\ m/s^2$)



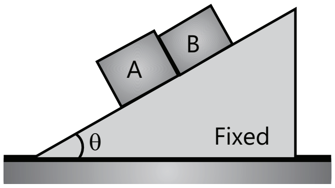
(A) Tension in the string is 306 N

(B) Tension in the string is 132 N

(C) Acceleration of block B is $2.6 m/s^2$

(D) Acceleration of block B is $2.6 m/s^2$

Q.16 In the arrangement shown in the Figure. all surface are smooth. Select the correct alternative(s)



(A) For any value of θ acceleration of A and B are equal

(B) Contact force between the two blocks is zero if $m_A / m_B = \tan\theta$

(C) Contact force between the two is zero for any value of m_A or m_B

(D) Normal reactions exerted by the wedge on the block are equal.

Q.17 Two blocks A and B of equal mass m are connected through a massless string and arranged as shown in Figure. Friction is absent everywhere. When the system is released from rest.

(A) Tension in string is mg/2

(B) Tension in string is mg/4

(C) Acceleration in string is g/2

(D) Acceleration in string is 3g/2

Assertion Reasoning Type

Each of the question given below consists of two statements, an assertion and reason. Select the number corresponding to the appropriate alternative as follows

(A) If both assertion and reason are true and reason is the correct explanation of assertion

(B) If both assertion and reason are true but reason is not the correct explanation of assertion

(C) If assertion is true but reason is false

(D) If assertion is false but reason is true.

Q.18 Assertion: The law of conservation of linear momentum, as applied to a single particle, is equivalent to Newton's first law of motion.

Reason: As Newton's first law states in the absence of external force state of motion of a body does not change.

Q.19 Assertion: The impulse of a force can be zero even if force is not zero.

Reason: The impulse of a force is equal to change in momentum of a body.

Q.20 Assertion: If a book is placed on table at rest then force exerted by table on the book and weight of the book formation reaction pairs according to Newton 3rd law of motion

Reason: Since both are equal in magnitude and opposite in directions.

Q.21 Assertion: The mass of a body can be regarded as a quantitative measure of the resistance of a body to acceleration by a given force.

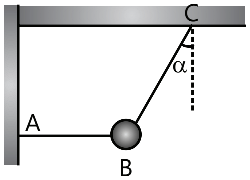
Reason: The acceleration produced by a given force is inversely proportional to mass being accelerated.

Q.22 Assertion: While conserving the linear momentum of the system we must specify the reference frame.

Reason: Like velocity, momentum also depends on the reference from of observer.

Comprehension Type

Paragraph 1: A ball of mass m is connected with the string AB and BC respectively as shown in the figure. Now string AB is cut. Answer the following questions



Q.23 Tension in the string AB and BC respectively the string AB is cut

(A) mg cotB, mg cosB (B) mg tanB, mg cosB

(C) mg tanB, mg secB (D) mg cotB, mg secB

Q.24 Tension in the string BC just after the string AB is cut

(A) mg sinB (B) mg cosB

(C) mg tanB (D)mg secB

Q.25 If string BC is cut instead of AB, what is the tension in the string AB just after

(A) mg cosB (B) mg tanB

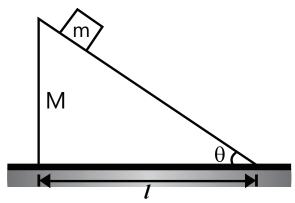
(C) mg sinB (D) zero

Q.26 If the whole system is placed in an automobile, what is the acceleration required to be given to it so that even after cutting the string AB, it remains in the same position

(A) g tanB, right ward (B) g cotB, right ward

(C) g tanB, left ward (D) g cotB, left ward

Passage 2: A block of mass m slides down a smooth incline of mass M and length l, solely as a result of the force of gravity. The incline is placed on a smooth horizontal table as shown in Figure. Let us denote the coordinate system relative to the table as S_1 and the coordinate system relative to the incline as S_ϕ



Q.27 The acceleration of m in the S' frame is

(A) $\frac{(M+m)g\sin\theta}{M+m\sin^2\theta}$ (B) $\frac{(M+m)g\sin\theta}{m+M\sin^2\theta}$

(C) $\frac{(M-m)g\sin\theta}{M+m\sin^2\theta}$ (D) $\frac{(M+m)g\sin\theta}{M+m\sin\theta}$

Q.28 The acceleration of the incline in the S frame

(A) $\left(\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$ (B) $-\left(\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$

(C) $\left(\frac{Mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$ (D) $-\left(\frac{Mg\sin\theta\cos\theta}{M+m\sin^2\theta}\right)$

Q.29 The force exerted by the small m on the wedge of mass M

(A) $mg\cos\theta$ (B) $\frac{Mmg}{M+m\sin^2\theta}$

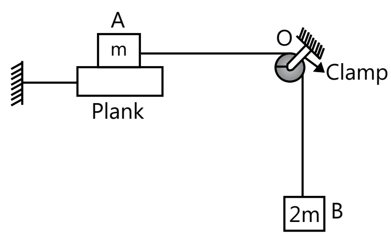
(C) $\frac{mg}{\cos\theta}$ (D) None

Q.30 At what acceleration a_x (in the S frame) must the incline be accelerated to prevent m from sliding

(A) $-g\tan\theta$ (B) $+g\tan\theta$

(C) $-\frac{g\tan\theta}{2}$ (D) $+\frac{g\tan\theta}{2}$

Passage 3: An arrangement designed to measure the acceleration due to gravity at a place consist of two blocks A and B, of mass m and 2m respectively connected to each other by means of a light inextensible string passing over a light frictionless pulley as shown in the Figure. A light and very rough plank, rigidly held in position, supports the block A. The system, it is observed does not move at all. The portion of the string OA, is initially horizontal. Assume that the acceleration due to gravity, $\left(g = 10\ m/s^2\right)$



Q.31 The net force due to plank, acting on the block A, has magnitude

(A) 2mg (B) mg (C) $\sqrt{3}$mg (D) $\sqrt{5}$mg

Q.32 The magnitude of the force exerted on the pulley by the clamp is, when the system is in equilibrium

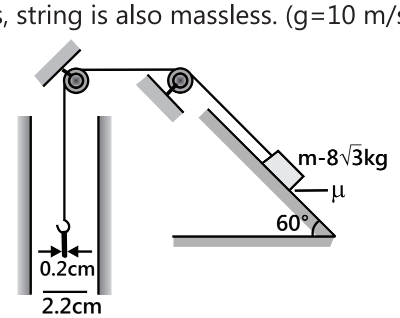
(A) 4mg (B) 4mg / 3 (C) $\frac{2\sqrt{2}\,mg}{3}$ (D) $2\sqrt{2}$mg

Q.33 The plank is suddenly broken by an impulsive force, acting downwards. The instantaneous accelerations of A and B, just after the plank is removed, are respectively,

(A) 10 m/s^2 and 10 m/s^2 (B) 20 m/s^2 and 3.33 m/s^2

(C) 12 m/s^2 and 6.66 m/s^2 (D) None of the above

Passage 4: A vertical gap 2.2 cm wide of infinite extent contains a fluid of viscosity $2.0\ NS/m^2$ and specific gravity 0.9. A metallic plate 1m X 1m X 0.2cm, which is in the middle of the gap, is to be lifted up with a constant speed 0.15 m/sec through the gap. The weight of the plate is 48N. Assuming pulley is massless and frictionless, string is also massless. (g=10 m/s^2)



Q.34 Buoyant force acting on the plate

(A) 1800 N (B) 900 N (C) 180 N (D) 18 N

Q.35 Net frictional force exerted by the liquid on the plate

(A) 30 N (B) 60 N (C) 15 N (D) 120 N

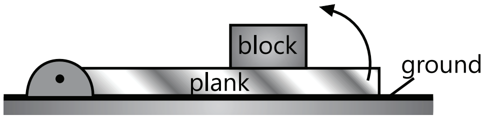
Q.36 Tension in the string

(A) 90 N (B) 108 N (C) 240 N (D) 120 N

Q.37 For doing so the kinetic friction between the inclined plane and the block should be equals to

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{8}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

Q.38 A block of mass m is placed on a plank, which is pivoted at one end. The plank is slowly turned as shown in Figure. The friction coefficient between block and plank is 0.8. Angle between ground and plank friction force between block and plank at which the block starts sliding is



(A) 20° (B) 45° (C) 30° (D) 35°

Circular Dynamics

Single Correct Choice Type

Q.39 Two bodies of mass 10k and 5kg moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is:

(A) R/r (B) r/R (C) R^2/r^2 (D) r^2/R^2

Q.40 A string breaks if its tension exceeds 10 newton. A stone of mass 250 mg tied to this string of length 10cm is rotated in a horizontal circle. The maximum angular velocity of rotation can be:

(A) 20 rad/s (B) 40 rad/s (C) 100 rad/s (D) 200 rad/s

Q.41 A stone of mass of 16kg is attached to a string 144m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16 newton. The maximum speed of revolution of the stone without breaking it, will be:

(A) $20ms^{-1}$ (B) $16ms^{-1}$ (C) $14ms^{-1}$ (D) $12ms^{-1}$

Q.42 Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is

(A) 3:5:7 (B) 3:4:5 (C) 7:11:6 (D) 3:5:6

Q.43 The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as k= as^2 where a is a constant. The total force acting on the particle is:

(A) $2a\frac{s^2}{R}$ (B) $2as\left(\frac{s^2}{R^2}\right)^{1/2}$ (C) 2as (D) $2as\frac{R^2}{s^2}$

Multiple Correct Choice Type

Q.44 A car of mass M is moving on a horizontal circular path of radius r. At an instant its speed is v and is increasing at a rate a-

(A) The acceleration of the car is towards the centre of the path

(B) The magnitude of the frictional force on the car is greater than mv^2/R

(C) The friction coefficient between the ground and the car is not less than a/g

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1} v^2/Rg$

Q.45 A circular road of radius r is banked for a speed of v=40km/h. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible. Then-

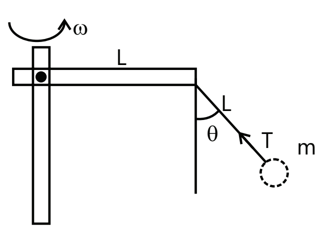
(A) The car cannot make a turn without skidding

(B) If the car turn at a speed less than 40km/h, it will slip down.

(C) If the car turns at the correct speed of 40km/h the force by the road on the car is equal to mv^2/r

(D) If the car turns at the correct speed of 40 km/h, the force by the road on the car is greater that mg as well as greater than mv^2/r

Q.46 Figure shows a rod of length L pivoted near an end and which is made to rotate in a horizontal plane with a constant angular speed. A ball of mass m is suspended by a string also of length L from the other end of the rod. If θ is the angle made by sting with the vertical, then-



(A) $T\sin\theta = m\omega^2 L(1+\sin\theta)$ (B) $T\cos\theta = mg$

(C) $\tan\theta = \frac{\omega^2 L(1+\sin\theta)}{g}$ (D) None of above

Q.47 A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle move in a circle of radius r with a uniform speed v.
(A) This is not possible
(B) There are other forces also on the particle
(C) The resultant of other forces is mv^2/r towards centre
(D) The resultant of the other forces varies in magnitude as well as direction

Assertion Reasoning Type

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it/of the statements, mark the correct answer as
(A) If both assertion and reason are true and reason is the correct explanation of assertion.
(B) If both assertion and reason are true but reason is not the correct explanation of assertion.
(C) If assertion is true but reason is false.
(D) If assertion is false but reason is true.
(E) If both assertion and reason are false.

Q.48 **Assertion:** Centripetal and centrifugal forces cancel each other
Reason: This is because they are always equal and opposite.

Q.49 **Assertion:** A cyclist bends inwards from his vertical position, while turning to secure the necessary centripetal force.
Reason: Friction between the tyres and road provides him the necessary centripetal force.

Q.50 **Assertion:** The tendency of skidding or overturning is quadrupled, when a cyclist double his speed of turning.

Reason: Angle of bending increases as velocity of vehicle increases.

Q.51 **Assertion:** On banked curved track, vertical component of normal reaction provide the necessary centripetal force.
Reason: Centripetal force is always required for motion in curved path.

Q.52 **Assertion:** A cyclist always bends inwards while negotiating a curve
Reason: By bending he lowers his centre of gravity

Q.53 **Assertion:** The tendency of skidding/overturning is quadrupled, when a cyclist doubles his speed of turning.
Reason: $\tan\theta = \frac{v^2}{rg}$

Q.54 **Assertion:** On a banked curved track, vertical component of normal reaction provides the necessary centripetal force.
Reason: Centripetal force is always required for turning.

Comprehension Type Questions

Passage 1: A block of mass m moves on a horizontal circle against the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the block and the side wall is μ. The block is given initial velocity v_o. Then answer the following questions.

Q.55 What is the tangential acceleration of the block?
(A) μg (B) $-\mu g$ (C) $\mu v^2/R$ (D) $-\mu v^2/R$

Q.56 What is the value of velocity v as the function of time t?

(A) $\frac{1}{v} = \frac{1}{v_0} + \frac{\mu t}{2R}$ (B) $\frac{1}{v} = \frac{1}{v_0} - \frac{\mu t}{2R}$

(C) $\frac{1}{v} = \frac{1}{v_0} + \frac{\mu t}{R}$ (D) $\frac{1}{v} = \frac{1}{v_0} - \frac{\mu t}{R}$

Q.57 What is the value of velocity v as the function of distance x travelled on the circumference?

(A) $v = v_o e^{-\frac{2\mu}{R}}$ (B) $v = v_o e^{-\frac{\mu}{R}x}$

(C) $v = v_o\left(1 - e^{-\frac{2\mu}{R}x}\right)$ (D) $v = v_o$

Passage 2: In a rotor, a hollow vertical cylindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If the radius of the rotor is 2m and the coefficient of static friction between the wall and the person is 0.2. Find the following parameters and relations.

Q.58 If v is the velocity of rotation of rotor and N be the reaction of wall, then-

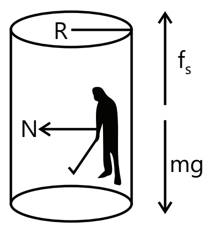
(A) N=mg

(B) $\vec{F}=\vec{F}_1+\vec{F}_2 \Rightarrow |\vec{F}|=\sqrt{10^2+5^2+2.10.5\cos 120^o}=5\sqrt{3}N$

(C) $N=\sqrt{(mg)^2+\left(\frac{mv^2}{r}\right)^2}$

(D) None of these

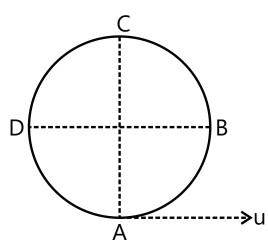
Q.59 In order to man remain in equilibrium we must have



(A) $\mu mg = N$ (B) $f_2 = \mu mg$

(C) $\mu N = mg$ (D) None of these

Q.60 The value of velocity will be given by –



(A) $v=\sqrt{\mu rg}$ (B) $v=\sqrt{\frac{rg}{\mu}}$

(C) $v=\sqrt{\frac{g}{\mu r}}$ (D) $v=\sqrt{\frac{\mu g}{r}}$

Match the Columns

Q.61 A particle is suspended from a string of length 'R'. It is given a velocity $u=3\sqrt{Rg}$. Match the following

Column I	Column II
(A) Velocity at B	(p) 7 mg
(B) Velocity at C	(q) $\sqrt{5gR}$
(C) Tension at B	(r) $\sqrt{7gR}$
(D) Tension at C	(s) 4mg

Q.62 The bob of a simple pendulum is given a velocity 10m/s at its lowest point. Mass of the bob is 1kg and string length is 1 m.

Column I	Column II
(A) Minimum tension in string (in Newton)	(p) 50
(B) Magnitude of acceleration of bob when the string is horizontal (in m/s^2)	(q) 60
(C) Minimum magnitude of acceleration of bob (in m/s^2)	(r) zero
(D) Tangential acceleration at the highest point (in m/s^2)	(s) $10\sqrt{65}$

Q.63 A car of mass 500kg is moving in a circular road of radius $35/\sqrt{3}$. Angle of the banking of road is 30. Coefficient of friction between road and tires is $\mu=\frac{1}{2\sqrt{3}}$.

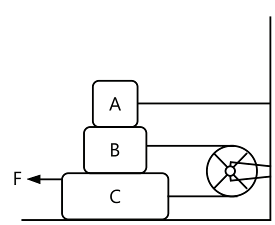
Match the following:

Column I	Column II
(A) Maximum speed (in m/s) of car for safe turning	(p) $5\sqrt{2}$
(B) Minimum speed (in m/s) of car for safe turning	(q) 12.50
(C) Speed (in m/s) at which friction force between tires and road is zero	(r) $\sqrt{210}$
(D) Friction force (in 10^2 Newton) between tires and road if speed is $\sqrt{\frac{350}{6}}m/s$	(s) $\sqrt{\frac{350}{3}}$

Previous Years' Questions

Forces and Laws of Motion

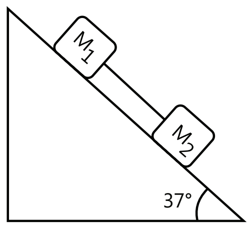
Q.1 In the Figure, the blocks A, B and C have masses 3 kg, 4 kg, and 8 kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall, while B and C are connected by a light flexible cord passing around a fixed frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at a constant speed. Assume that the arrangement shown in the Figure. i.e. B on C and A on B, is maintained throughout. (Take $g=10ms^2$) ***(1978)***



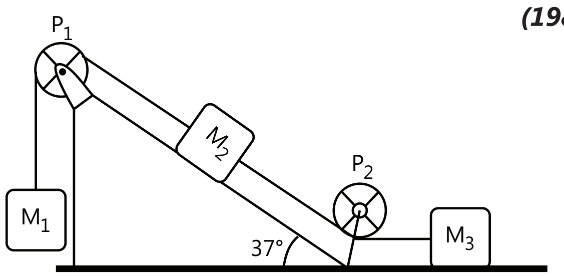
Q.2 A uniform rope of length L and mass M lying on a smooth table is pulled by a constant force F. What is the tension in the rope at a distance l from the end where the force is applied? ***(1978)***

Q.3 A block of mass 2 kg slides on an inclined plane which makes an angle of with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{3}/\sqrt{2}$. What force along the plane should be applied to the block so that it moves (a) down and (b) up without any acceleration (Take $g = 10\ m/s^2$) ***(1978)***

Q.4 Two blocks connected by a massless string slides down an inclined plane having an angle of inclination 37°. The masses of the two blocks are M_1=4 kg and M_2=2 kg respectively and coefficients of friction of M_1 and M_2 with the inclined plane are 0.75 and 0.25 respectively. Assuming the string to be taut, find (a) the common acceleration of two masses and (b) the tension in the string. (sin37°=0.6, cos37°=0.8) (Take g=9.8 m/s²) ***(1979)***

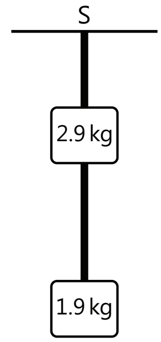


Q.5 Masses M_1, M_2 and M_3 are connected by strings of negligible mass which passes over massless and frictionless pulleys P_1 and P_2 as shown in Figure. The masses move such that portion of the string between P_1 and P_2 is parallel to the inclined plane and portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and coefficient of kinetic friction between the masses and surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. ***(1981)***



Q.6 A block of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and direction in which it has to be applied. ***(1987)***

Q.7 Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 m. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks, wires and support have an upward acceleration of $0.2\ m/s^2$. The acceleration due to gravity is g=9.8 m/s². ***(1989)***



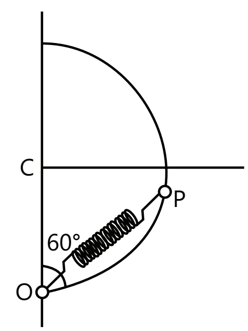
Q.8 A hemispherical bowl of radius R=0.1 m is rotating about its own axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2} kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at height h from the bottom of the bowl.

(a) Obtain the relation between h and ω. What is the minimum value of ω needed, in order to have a non-zero value of h?

(b) It is desired to measure (acceleration due to gravity) using the setup by measuring h accurately. Assuming that R and ω are known precisely and that the least count in the measurement of h is 10^{-4} m, what is minimum possible error Δg in the measured value of g?

(1993)

Q.9 A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length 3R/4 is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle with the vertical. The spring constant k=mg/R. Consider the instant when the ring is making an angle 60° with the vertical. The spring is released (a) Draw the free body diagram of the ring. (b) Determine the tangential acceleration of the ring and the normal reaction? ***(1996)***



Q.10 Two blocks of mass m_1 = 10 kg and m_2 = 5 kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about the vertical axis passing through centre O. The masses are placed along the diameter of the table on either side of center O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn of table. ***(1997)***

(A) Calculate the frictional force on m_1.

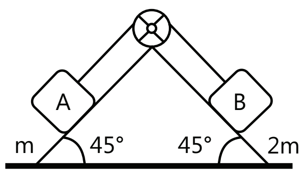
(B) What should be the minimum angular speed of the turn table, so that the masses will slip from this position?

(C) How should the masses be placed with the string remaining taut so that there is no frictional force acting on the mass m_2?

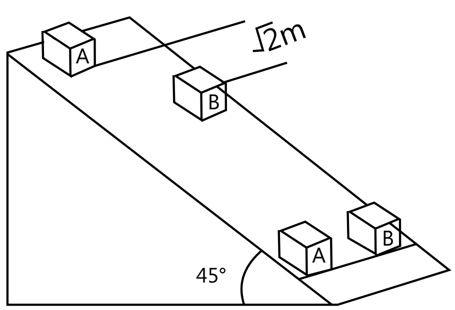
Q.11 Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in Figure. The wedge is inclined at 45° to the horizontal on both the sides. The coefficient of friction between block A and wedge is 2/3 and that between block B and the wedge is 1/3. If the blocks A and B released from rest, find (A) the acceleration A,

(B) Tension in the string and

(C) The magnitude and direction of the force of friction acting on A ***(1997)***



Q.12 Two blocks A and B of equal masses are released from an inclined plane of inclination 45° at t=0. Both the blocks are initially at rest. The coefficient of kinetic friction between the block A and inclined plane is 0.2 while it is 0.3 for block B. Initially the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line? ***(2004)***

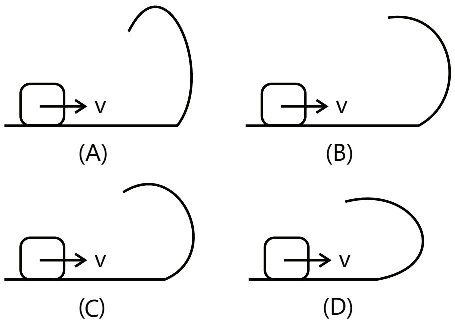


Circular Dynamics

Q 13. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is ***(2000)***

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) Infinitesimal

Q 14. A small block is shot into each of the four track as shown below. Each of the track rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in ***(2001)***



Q.15 A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and v respectively. The following relations hold good under the above conditions. ***(1986)***

(A) $T\cos\theta = Mg$

(B) $T - Mg\cos\theta = \frac{Mv^2}{L}$

(C) The magnitude of the tangential acceleration of the bob $|\alpha_T| = g\sin\theta$

(D) $T = Mg\cos\theta$

Q.16 A reference frame attached to the earth ***(1986)***

(A) Is an inertial frame by definition

(B) Cannot be an inertial frame because the earth is revolving round the sun

(C) Is an inertial frame because Newton's laws are applicable in this frame

(D) Cannot be an inertial frame because the earth is rotating about its own axis

Q.17 A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of $2ms^{-1}$. Which of the following statement(s) is/are correct for the system of these two masses. ***(2010)***

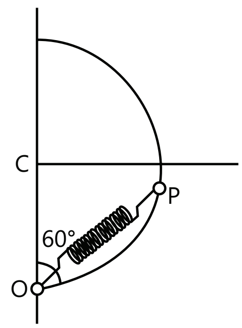
(A) Total momentum of the system is $3kg\text{–}ms^{-1}$

(B) Momentum of 5 kg mass after collision is 4kg–ms $^{-1}$

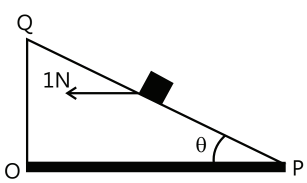
(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4J

Q.18 A smooth semicircular wire track of radius R is fixed in a vertical plane (Figure). One end of a massless spring of natural length 3R/4 is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationery at point P such that the spring makes an angle 60° with the vertical. The spring constant k=mg/R. Consider the instant when the ring is making an angle 60° with the vertical. The spring is released (a) Draw the free body diagram of the ring. (b) Determine the tangential acceleration of the ring and the normal reaction. ***(1996)***



Q. 19 A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle q with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2) ***(2012)***



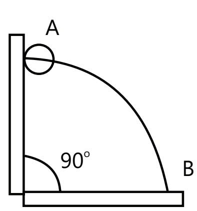
(A) θ = 45°

(B) θ > 45° and a frictional force acts on the block towards P.

(C) θ > 45° and a frictional force acts on the block towards Q.

(D) θ < 45° and a frictional force acts on the block towards Q.

Q.20 A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is ***(2014)***



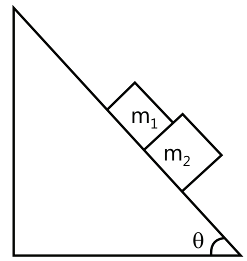
(A) Always radially outwards.

(B) Always radially inwards.

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

Q.21 A block of mass m_1 = 1 kg another mass m_2 = 2kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List II expressions for the friction on the block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option.



The acceleration due to gravity is denoted by g.

[Useful information: tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3] ***(2014)***

	List I		List II
(P)	$\theta = 5°$	(1)	$m_2 g \sin\theta$
(Q	$\theta = 10°$	(2)	$(m_1+m_2)\, g \sin\theta$
(R)	$\theta = 15°$	(3)	$\mu m_2 g \cos\theta$
(S)	$\theta = 20°$	(4)	$\mu(m_1 + m_2) g \cos\theta$

Code:

(A) (P)→(1), (Q)→(1), (R)→(1), (S)→(1)

(B) (P)→(2), (Q)→(2), (R)→(2), (S)→(3)

(C) (P)→(2), (Q)→(2), (R)→(2), (S)→(4)

(D) (P)→(2), (Q)→(2), (R)→(3), (S)→(3)

Q.22 The net reaction of the disc on the block is ***(2016)***

(A) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$

(B) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} - mg\hat{k}$

(C) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-\omega t})\hat{j} - mg\hat{k}$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 18 Q.21 Q.26
Q.27 Q.28

Exercise 2

Q.5 Q.14 Q.18

Previous Years' Questions

Q.51 Q.59

JEE Advanced/Boards

Exercise 1

Q.1 Q.4 Q.13 Q.17
Q.20 Q.29 Q.38 Q.43

Exercise 2

Q.3 Q.6 Q.8 Q.11
Q.14 Q.15 Q.16 Q.17
Q.44 Q.45

Previous Years' Questions

Q.12 Q.14 Q.17

Answer Key

JEE Main/Boards

Exercise 1

Forces and Laws of Motion

Q.4 100 Ns

Q.5 12.5 N

Q.14 0.872 m/s, 1.744 m

Q.15 30N

Q.16 1.4 ms^{-2}, 33.6 N

Q.17 1000 N, 750 N

Q.18 40°

Q.19 750 N/m

Q.20 1 cm

Q.21 9 m/s.

Q.22 $\vec{r} = (\hat{i}12500 + \hat{j}6250)\text{m}$

Q.23 $2.5 \times 10^4 \text{km}$

Q.24 $gr\phi^2(r - r\phi) = 2lgm$

Circular Dynamics

Q.25 0.03 m/s^2

Q.26 $0.86\, ms^{-2}$

Q.27 $-\left(\hat{x} + \hat{y}\right) / \sqrt{2} cm / s^2$

Q.28 1.22 m/s^2; $\beta = \tan^{-1}\left(\frac{10}{7}\right)$

Exercise 2

Forces and Laws of Motion

Single Correct Choice Type

Q.1 A	**Q.2** B	**Q.3** C	**Q.4** C	**Q.5** A	**Q.6** A	**Q.7** C
Q.8 B	**Q.9** B	**Q.10** A	**Q.11** C	**Q.12** B	**Q.13** A	**Q.14** A
Q.15 A	**Q.16** D	**Q.17** C	**Q.18** C	**Q.19** D	**Q.20** B	**Q.21** B

Circular Dynamics

Single Correct Choice Type

Q.22 D	**Q.23** C	**Q.24** D	**Q.25** A	**Q.26** C	**Q.27** B	**Q.28** A
Q.29 A	**Q.30** B	**Q.31** D	**Q.32** B	**Q.33** D		

Previous Years' Questions

Forces and Laws of Motion

Q.1 C	**Q.2** A	**Q.3** C	**Q.4** A	**Q.5** A	**Q.6** A	**Q.7** A
Q.8 D	**Q.9** C	**Q.10** A	**Q.11** D	**Q.12** A	**Q.13** B	**Q.14** A
Q.15 B, D						

Circular Dynamics

Q.16 C	**Q.17** D	**Q.18** C	**Q.19** C	**Q.20** B	**Q.21** B

JEE Advanced/Boards

Exercise 1

Forces and Laws of Motion

Q.1 (a) 70 kg, (b) 35 kg, (c) 105 kg, (d) zero

Q.2 (a) 750 N, (b) 250 N, Mode (b) should not be adopted

Q.3 (a) T=640 N, (b) T=240 N, (c) T=400 N, (d) T=0, Rope will break in case (a).

Q.4 15 m

Q.5 (a) 7500 N downwards, (b) 32500 N downwards, (c) 32500 N upward

Q.6 (a) accelerated with acceleration 0.5 m/s^2, (b) at rest.

Q.7 $\frac{(k_1+k_2)x}{m}$

Q.8 g/3, 2g/3

Q.9 $T_{ab} = mg\tan\theta, T_{ob} = mg/\cos\theta, T' = mg\cos\theta$

Q.10 $\frac{mg}{4h}\sqrt{a^2+4h^2}$

Q.11 4N

Q.12 $\frac{2(Mg-B)}{g}$

Q.13 (a) $f=(k_1-k_2)\frac{mg^2\cos\alpha}{m_1+m_2}$,

(b) $\cos\alpha_{min}=\frac{k_1 m_1+k_2 m_2}{m_1+m_2}$

Q.14 $\alpha=\frac{1}{2}\tan^{-1}\left(-\frac{1}{\mu}\right)$

Q.15 (a) $v=\frac{mg^2\cos\alpha}{2a\sin^2\alpha}$, (b) $s=\frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$

Q.16 (a) $f_A=\frac{3}{4}\mu Mg,\ f_B=0,\ T=0$

(b) $f_A=\mu Mg,\ f_B=\frac{\mu Mg}{2},\ T=\frac{\mu Mg}{2}$

Q.17 $\frac{13\mu\ell}{16(2-3\mu)}$

Q.18 $w_2=\frac{g(\eta-\sin\alpha-\cos\alpha)}{\eta+1}=0.5g$

Q.19 $a_A=a_C=\frac{3}{4}g\sin\theta, a_B=g\sin\theta, T=\frac{Mg\sin\theta}{8}$

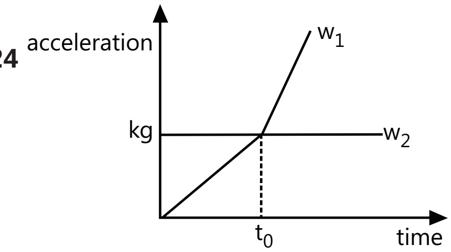
Q.20 $f=\frac{(M+m)g\sin^2\alpha}{M+m\sin^2\alpha}$

Q.21 Acceleration of block A is g/8 in horizontal direction and 5g/8 in vertical direction. Acceleration of block B is g/2 leftwards. Acceleration of block C is g/8 rightwards

Q.22 $a_A=\frac{g}{1+\eta\cot^2\alpha}, a_B=\frac{g}{\tan\alpha+\eta\cot\alpha}$

Q.23 $\tan B=\mu,\ T_{min}=\frac{mg(\sin\alpha+\mu\cos\alpha)}{\sqrt{1+\mu^2}}$

Q.24



When $t\le t_0$ (where $t_0=\frac{\mu(m_1+m_2)g}{a}$) $w_1=w_2=kg$

$t>t_0$ $w_1=at/m_1-km_2g/m_1, w_2=kg$

Q.25 $t=\sqrt{\frac{2l}{(3w+kg)}}$

Q.27 $a_A=2.34\ m/s^2,\ a_B=1.558 m/s^2,\ T=81.8N$

Q.28 (a) $t_A=1 sec, tB=2 sec$, (b) $v_A=5m/s$ (c) $\frac{5}{3}m$

Circular Dynamics

Q.29 $f_{max}=\frac{5}{2\pi}rev/sec$

Q.30 $45°, \sqrt{2}\times10^4 N$

Q.31 $\theta=30°, v=7.7m/s$

Q.32 $163.3m/s^2$

Q.33 $\sqrt{Rg}$

Q.34 $22.36m/s$

Q.35 21°29'

Q.36 $\sqrt{\frac{\mu g}{L}}$

Q.37 (a) $\sqrt{Rg}$

(b) a distance $\frac{\pi R}{3}$ along the bridge from the highest point

(c) $\sqrt{gR\cos(L/2R)}$

Q.38 45°

Q.39 $v=\sqrt{\frac{\ell g}{3}}$

Q.40 $\left(\dfrac{g(\sin\theta-\mu\cos\theta)}{R\sin\theta(\cos\theta+\mu\sin\theta)}\right)^{1/2}$ to $\left(\dfrac{g(\sin\theta+\mu\cos\theta)}{R\sin\theta(\cos\theta-\mu\sin\theta)}\right)^{1/2}$

Q.41 $\sqrt{\dfrac{2L}{\omega^2 R\cos\theta}}$

Q.42 $\dfrac{\omega^2 R}{3}, \dfrac{4}{3}m\omega^2 R$

Q.43 (i) $m\sqrt{KR}$ (ii) $m\sqrt{K\left(R+Kt^4\right)}$

(iii) $mKRt$ (iv) $\dfrac{1}{2}mKRt$

Q.44 $\left[2R\left(a\sin\theta+g-g\cos\theta\right)\right]^{1/2}$

Q.45 $\lambda R^2\omega^2$

Q.46 $\sqrt{a^2+\left(\dfrac{v^2}{R}\right)^2}$

Q.47 $\dfrac{mv^2}{2\pi R}$

Q.48 $\left[\left(\mu^2 g^2-a^2\right)R^2\right]^{1/4}$

Exercise 2

Forces and Laws of Motion

Single Correct Choice Type

Q.1 C **Q.2** A **Q.3** A **Q.4** A **Q.5** A **Q.6** B **Q.7** A
Q.8 B **Q.9** B **Q.10** C **Q.11** B **Q.12** A

Multiple Correct Choice Type

Q.14 A, B **Q.15** A, D **Q.16** A, C **Q.17** B, D

Assertion Reasoning Type

Q.18 A **Q.19** D **Q.20** D **Q.21** A **Q.22** A

Comprehension Type

Q.23 C **Q.24** B **Q.25** D **Q.26** A **Q.27** A **Q.28** B **Q.29** B
Q.30 B **Q.31** D **Q.32** D **Q.33** C **Q.34** D **Q.35** B **Q.36** A
Q.37 A

Match the Columns

Q38 A $\rightarrow$ q; B $\rightarrow$ r; C $\rightarrow$ r; D $\rightarrow$ q

Circular Dynamics

Single Correct Choice Type

Q.39 A **Q.40** A **Q.41** D **Q.42** D **Q.43** B

Multiple Correct Choice Type

Q.44 B, C **Q.45** B, D **Q.46** A, B, C **Q.47** B, D

Assertion Reasoning Type

Q.48 D **Q.49** C **Q.50** B **Q.51** E **Q.52** B **Q.53** A **Q.54** D

Comprehension Type

Q.55 D **Q.56** C **Q.57** B **Q.58** B **Q.59** C **Q.60** B

Match the Columns

Q.61 $A \rightarrow r; B \rightarrow q; C \rightarrow p; D \rightarrow s$

Q.62 $A \rightarrow p; B \rightarrow s; C \rightarrow q; D \rightarrow r$

Q.63 $A \rightarrow r; B \rightarrow p; C \rightarrow s; D \rightarrow q$

Previous Years Questions

Forces and Laws of Motion

Q.1 37.5 N **Q.2** F(1-l/L) **Q.3** a=11.21 N, b=31.21N

Q.4 $a = 1.5 m/s^2$, T=5.2 N **Q.5** (a) 4.2 kg, (b) 9.8 N **Q.6** $mg\sin\theta$

Q.7 (a) 20N, (b) 50N **Q.8** (a) 9.89 rad/sec, (b) $9.8 \times 10^{-3} m/s^2$

Q.9 (a) mg/4, (b) $a_{tan} = 5\sqrt{3}g/8, N = 3mg/8$

Q.10 (a) f=36N inwards, (b) 11.67 rad/sec, (c) m2 at 0.2m and m1 at 0.1 m from O

Q.11 (a) 0, (b) T= $2\sqrt{2}mg/3$ (c) f= $mg/3\sqrt{2}$ (down the plane)

Q.12 After A travel a distance of $8\sqrt{2}$ m down the plane

Circular Dynamics

Q.13 A **Q.14** A **Q.15** B, C **Q.16** B, D **Q.17** A, C **Q.18** B $\frac{5\sqrt{3}}{8}g, \frac{3mg}{8}$

Q.19 A, C **Q.20** D **Q.21** D **Q.22** C

Solutions

JEE Main/Boards

Exercise 1

Forces and Laws of Motion

Sol 1: A body will preserve its velocity and direction as long as no force acts on it in its motion. Inertia is in fact the resistance of any physical object to any change in its motion.

Sol 2:

Forces are balanced

↓

$a = 0\ m/s^2$

↙ ↘

Objects at rest ($v = 0$ m/s) → Stay at rest

Objects in motion ($v \neq 0$ m/s) → Stay in motion (same speed and direction)

Now

Forces are Unbalanced

↓

There is an acceleration

↙ ↘

Acceleration depends on the 'net force'

Acceleration depends Inversely upon the mass of object

Sol 3: While taking a catch, a cricket player moves his hands backwards. He has to apply retarding force to stop the moving ball in his hands. If he catches the ball abruptly, then he has to apply a large retarding force for a short time. So he gets hurt. On the other hand if he moves his hands backwards then the player applies force for longer time to bring the ball at rest. In this case he has to apply less retarding force.

Sol 4: $\Delta p = F\Delta t$

$\Delta p = 100 . 1$ Ns

$\Delta p = 100$ Ns.

Sol 5: $F = ma$ and $a = \dfrac{v_f - v_i}{t} \Rightarrow \dfrac{\Delta v}{\Delta t}$

In case (i) $\dfrac{\Delta v}{\Delta t} = \dfrac{20-10}{5} = \dfrac{10}{5} = 2$ m/s

$\therefore F = ma \Rightarrow 5 = m(2)$ ... (i)

Now further, we want this ΔV in in 2 s.

$a_{new} = \dfrac{\Delta V}{\Delta t_{new}} = \dfrac{20-10}{2} = \dfrac{10}{2} = 5\ m/s^2$

$\therefore F_{new} = m(5)$ ... (ii)

Dividing equation (i) by (ii)

$\Rightarrow \dfrac{5}{F_{new}} = \dfrac{m(2)}{m(5)}$

$F_{new} = \dfrac{25}{2}$ N

$\Rightarrow F_{new} \equiv 12.5$ N.

Sol 6: Conceptual. Refer to the reading manual.

Sol 7: 1. Linear inertia: In an isolated system, a body at rest will remain at rest and a body moving with constant velocity will continue to do so, unless disturbed by an external force.

2. Gyroscopic Inertia: A body that is set spinning has a tendency to keep spinning in its original orientation if no external force is applied.

3. Rotational Inertia: An object resists any change in its state of rotation. If no external force is applied.

Sol 8: Conceptual, Refer to reading manual.

Sol 9: Absolute unit of weight is Newton (N)

Gravitational unit is kg-weight.

1 N = 9. 8 kg. wt

Sol 10: $\vec{F} \propto \dfrac{d\vec{p}}{dt}$; $\quad \vec{F} \propto m\dfrac{d\vec{v}}{dt}$

$\vec{F} = Kma$, K= 1

$\therefore F = m\vec{a}$

Consequences

1. No force is required to move a body uniformly in a straight line.

2. Accelerated motion is always due to an external force.

Sol 11: Impulse is defined as the product of the average force and change in time.

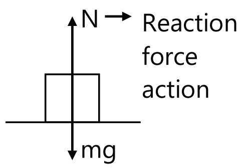
$J = F_{avg}(t_2 - t_1)$; $J = \int_{t_1}^{t_2} F\,dt$

$F = \dfrac{dp}{dt}$; $J = \int_{t_1}^{t_2} \dfrac{dp}{dt}\,dt$

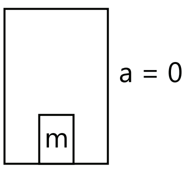
$J = \int_{p_1}^{p_2} dp$; $J = P_2 - P_1 = \Delta P.$

Sol 12: Every action has an equal and opposite reaction.

Example (1)



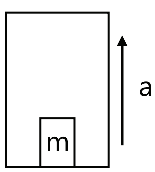
Sol 13: Lift moving uniformly

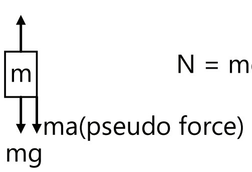


Then $N - mg = 0$

$\therefore\ N \equiv w = mg.$

Lift acceleration upward



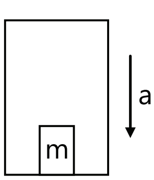


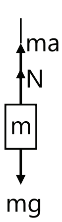
$N = mg + ma$

$N \equiv w = m(g + a)$

$\therefore$ weight Increases

Lift accelerating downwards:





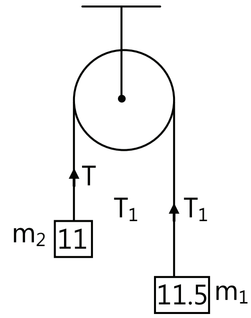
$N + ma = mg$

$N = mg - ma$

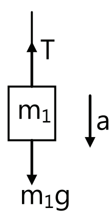
$\therefore\ N \equiv w = m(g - a)$

Hence weight decreases.

Sol 14: writing down the equations of motion

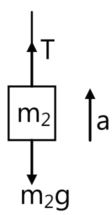


$m_1g - T = m_1a$ ... (i)



And for second body

$T - m_2g = m_2a$ ... (ii)



Adding (i) and (ii)

$(m_1 - m_2)g = (m_1 + m_2)a$

$a = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)g$

Here m_1 = 11.5 kg, m_2 = 11 kg, g = 10 m/s^2

Now m_1 will descend down by height 'h' and m_2 moves up by the same height h;

$H = ut + \dfrac{1}{2}at^2$

$\Rightarrow\quad h = 0.\,t + \dfrac{1}{2} \times (0.\,2)\,(4)^2 = 1.\,6$ m.

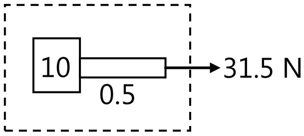
And for velocity

$v = u + at$

$v = 0 + (0.\,2)\,(4)$

$v = 0.\,8$ m/s.

Sol 15:

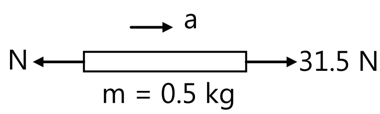


Let us say the whole system moves forward with an acceleration 'a'.

Then $a = \left(\frac{31.5}{10+0.5}\right)$ m/s

$a = 3$ m/s^2

Now let us consider the string.



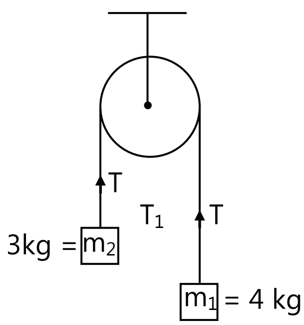
Now, $31.5 - N = ma$

$\Rightarrow 31.5 - ma = N$

$N = 31.5 - (0.5)(3)$

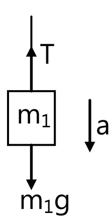
$N = 30$ Newton.

Sol 16: Constraint Equation:

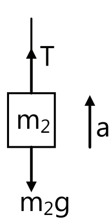


$a_{m_1} + a_{m_2} = 0$. [∵ length of string is constant]

Let us say m_1 moves down with an acceleration 'a', then m_2 will move up by an acceleration 'a'.



$m_1g - T = m_1a$ ... (i)



$T - m_2g = m_2a$ ... (ii)

(i) + (ii) $\Rightarrow (m_1 - m_2)\,g = (m_1 + m_2)a$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g = \left(\frac{4-3}{4+3}\right)10 = \frac{10}{7} = 1.4\ \text{ms}^{-2}$$

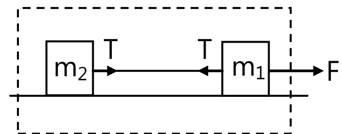
And using this value, find the value of T in equation (i) or equation (ii)

$m_1g - m_1a = T \Rightarrow \boxed{T = m_1(g-a)}$

now put $m_1 = 4$ kg $m_2 = 3$kg

to get the numerical, after putting values of m_1, m_2 and $a \Rightarrow T = m_1(g - a) = 33.6$ N

Sol 17:



The total external Horizontal force applied on the system is F.

∴ Acceleration 'a' of the system $= \frac{F}{m_1 + m_2}$ m/s^2

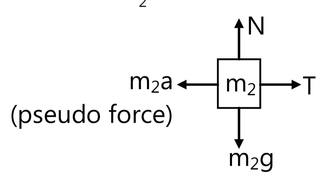
Given $a = 5$ m/s^2

$\therefore 5 = \frac{F}{50+150}$ m/s^2 $\quad \therefore F = 200 \times 5$ N

$\boxed{F = 1000\ \text{N}}$

Now for finding the tension;

Consider m_2



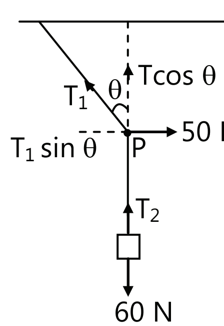
$T - m_2a = 0$

$\because\ T = m_2a$.

$T = 150 \times 5$

$T = 750$ N.

Sol 18:



At point P

For equilibrium;

$T_1 \sin\theta = 50$ ... (i)

$T_1 \cos\theta = T_2$ ... (ii)

And for the mass;

$T_2 = 60$ N ... (iii)

From (i) and (ii) $\tan\theta = \frac{50}{T_2}$

$\tan\theta = \frac{50}{60}$

$\theta = \tan^{-1}(5/6) = 40°$

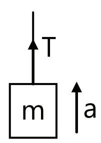
Sol 19: F = kx.

x = 20 cm = 0. 2 m

150 = k (0. 2)

$k = \frac{150}{0.2} = \frac{15}{2} \times 10^2$ N/m $= 7.5 \times 10^2$ N/m.

Sol 20:



T – mg = ma

Now for a_{max}, we have T_{max}

$T_{max} - mg = ma$

$T_{max} = m(g + a)$ N $= m(9.8 + 1.2)$ N $= 2000 (11)$

$\boxed{T_{max} = 22\times10^3\,N}$

Now T_{max} = (Breaking stress) Area

$\therefore 22 \times 10^3 = (2.8 \times 10^8)(pR^2)$

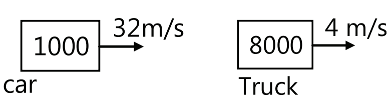
$R = \sqrt{\frac{22\times10^3}{28\times10^7\pi}}$

$R = \sqrt{25\times10^{-6}}$ m

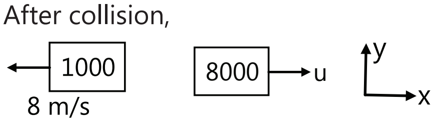
$R = 5 \times 10^{-3}$ m

Diameter $= 2R \equiv 10 \times 10^{-3}$ m $\equiv 10^{-2}$ m.

Sol 21: Before collision



After collision,



In the whole process, linear momentum along the x-direction is conserved.

$\therefore$Initial momentum $= 10^3 \times 32 + 8 \times 10^3 \times 4$

$P_i = 64 \times 10^3$ kg m/s $(\hat{i})$

Now in the final state

Momentum of car$=10^3 \times (-8) = -8 \times 10^3 (\hat{i})$

Momentum of truck $= 8 \times 10^3 (v\hat{i})$

$= 8v \times 10^3 \hat{i}$

$P_{final} = (-8 + 8v) \times 10^3 (\hat{i})$

$P_{initial} = p_{final}$

$\Rightarrow 64 \times 10^3 = (-8 + 8v) \times 10^3$

$\therefore v = \frac{64+8}{8}$ m/s; v = 9 m/sec

Sol 22: $\vec{F} = m\vec{a}$

$\vec{a} = \frac{\vec{F}}{m}$; $m = 10g = 10 \times 10^{-3}$ kg $= 10^{-2}$ kg

$\therefore \vec{a} = \frac{(10\hat{i}+5\hat{j})}{10^{-2}}$

$\boxed{\vec{a} = 10^3\hat{i} + 5\times10^2\hat{j}}$

$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Since $\vec{u} = 0$, $\vec{r} = \frac{1}{2}\vec{a}t^2$

$\vec{u} = \frac{10^3\times25}{2}\hat{i} + \frac{500\times25}{2}\hat{j}$

Sol 23: This is just an energy conservation problem on surface of earth;

$E_i = \frac{1}{2}mv_0^2 + U_i$; $U_i = -\frac{Gm}{R}$

$\therefore E_i = \frac{1}{2}mv_0^2 - \frac{Gm}{R}$

Now finally;

V = 0

$E_f = 0 + \left(-\frac{Gm}{R+h}\right)$ ... (i)

And $E_f = E_i$

$\therefore \frac{1}{2}mv_0^2 - \frac{Gm}{R} = -\frac{Gm}{R+h}$

$\Rightarrow \frac{v_0^2}{2} - \frac{G}{R} = \frac{-G}{R+h}$

$$\Rightarrow \frac{1}{R+h} = \frac{1}{R} - \frac{v_0^2}{2G}$$

$$\Rightarrow R + h = \frac{1}{\left(\frac{1}{R} - \frac{v_0^2}{2G}\right)}$$

$$\boxed{h = \frac{1}{\left(\frac{1}{R} - \frac{v_0^2}{2G}\right)} - R}$$

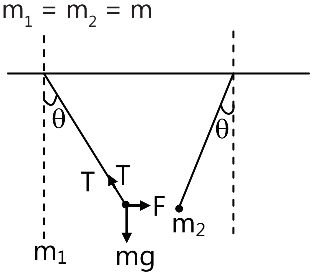
$G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$

$R = 64 \times 10^5$ m.

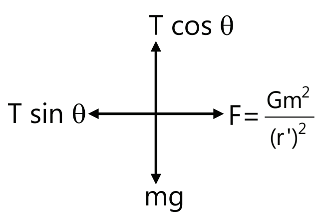
$v_0 = 10^4$ m/s

After putting above values we get, $h = 2.5 \times 10^4$ km

Sol 24: $m_1 = m_2 = m$



FBD of m_1;



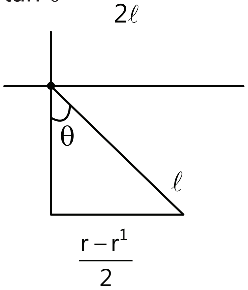
$T \sin\theta = F$ ... (i)

$T \cos\theta = mg$ ... (ii)

$$\text{(i)/(ii)} \Rightarrow \tan\theta = \frac{Gm^2}{(r')^2 mg}$$

$$\tan\theta = \frac{Gm}{(r')^2 g} \quad \text{... (iii)}$$

$$\tan\theta = \frac{r - r'}{2\ell}$$

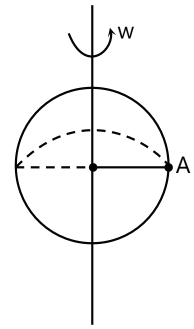


$$\therefore \frac{r - r'}{2\ell} = \frac{Gm}{(r')^2 g}$$

Solving for r′, We get the value of r′.

Circular Motion

Sol 25:



Earth completes 1 rotation in 1 day

$$\text{i. e., } \omega = 1. \frac{\text{rotation}}{\text{day}}$$

$$\omega = 1. \frac{2\pi}{24 \times 60 \times 60} \text{ rad/s}$$

$$\omega = \frac{\pi}{432} \times 10^{-2} \text{ rad/s}$$

and now acceleration at point A;

$a = r\omega^2$

$r = 6400$ km $= 6400 \times 10^3$ m; $\quad r = 64 \times 10^5$ m

$$\therefore a = 64 \times 10^5 \times \frac{\pi^2}{(432)^2} \times 10^{-4} \text{ m/s}^2$$

$a = 0.03$ m/s^2

Sol 26: $v = 27 \text{ km/h} = 27 \times \frac{5}{18}$ m/s

$$v = \frac{15}{2} \text{ m/s}$$

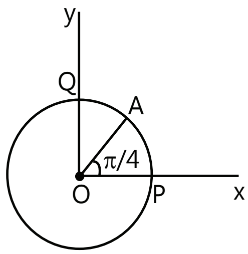
$$\vec{a}_r = \frac{v^2}{R} = \frac{(15)^2}{4 \times 80} = 0.7$$

$$\vec{a}_t = 0.5 \text{ m/s}^2 = \frac{1}{2} \text{ m/s}^2$$

$$\vec{a}_{net} = \vec{a}_r + \vec{a}_t = \sqrt{(0.7)^2 + (0.5)^2}$$

$$\vec{a}_{net} = 0.86 \text{ m/s}^2$$

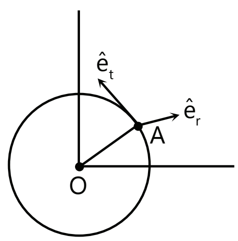
Sol 27:



At point the acceleration will be centripetal acceleration which is radially directed towards point O. i.e.

Physically: $\vec{a} = \frac{v^2}{r}(-\hat{e}_r)$

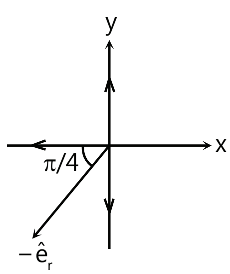
Remember $\hat{e}_r$ and $\hat{e}_t$ are the unit vectors along radial and tangential directions respectively.



Refer to the figure.

So in this case also $\vec{a}_A = \frac{v^2}{r}(-\hat{e}_r)$

Now, since the point is in between the points P and Q,



angle between $\overrightarrow{OA}$ and $\overrightarrow{OP}$ will be $\frac{\pi}{4}$

Now let us resolve $(-\hat{e}_r)$ into $\hat{i}$ and $\hat{j}$.

$(-\hat{e}_r) = |-\hat{e}_r|.\cos\frac{\pi}{4}(-\hat{i}) + |-\hat{e}_r|\sin\frac{\pi}{4}(-\hat{j})$

But since $\hat{e}_r$ and $\hat{e}_t$ are unit vectors

$|\hat{e}_r| = |\hat{e}_t| = 1$

$\because (-\hat{e}_r) = -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

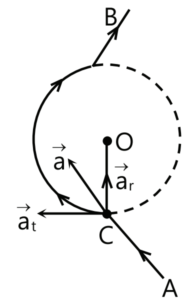
Now $\vec{a}_A = \frac{v^2}{r}\left(-\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})\right)$

$\vec{a}_A = -\frac{v^2}{r\sqrt{2}}(\hat{i}+\hat{j})$

Put v = 2 cm/s and r = 4 cm, to find $\vec{a}_A$.

After putting above values we get, $\vec{a}_A = -(\hat{x}+\hat{y})/\sqrt{2}\ cm/s^2$

Sol 28:



Let us say the circular turn is of the shape AB.

Now at the starting point of the track i. e. C;

$\vec{a} = \vec{a}_r + \vec{a}_t$

$\vec{a}_r$ = centripetal acceleration = $\frac{v^2}{R}(-\hat{e}_r)$

v = 36 km/h = 36 × $\frac{5}{18}$ m/s = 10 m/s

R = 140 m

$\vec{a}_r = \frac{(10)^2}{140} = \frac{5}{7}$ m/s² $(-\hat{e}_r)$

and given that $\frac{dv}{dt}$ = 1 m/s

$\therefore \vec{a}_t = \frac{dv}{dt}(\hat{e}_t)$; $\quad \vec{a}_t$ = 1 m/s²$(\hat{e}_t)$

Now $\vec{a} = \vec{a}_r + \vec{a}_t$

$\vec{a}$ = (0. 7 $(-\hat{e}_r)$) + 1 $\hat{e}_t$) m/s²

$|a| = \sqrt{(0.7)^2+1} = \sqrt{0.49+1} = \sqrt{1.49}$ m/s² = 1. 22 m/s²

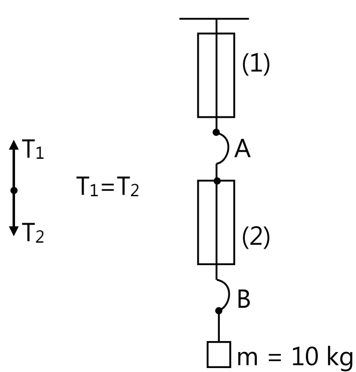
and $\tan\beta = \left(\frac{1}{0.7}\right) \Rightarrow \beta = \tan^{-1}\left(\frac{10}{7}\right)$

Exercise 2

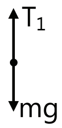
Forces and Laws of Motion

Single Correct Choice Type

Sol 1: (A) At point A;



At point B;

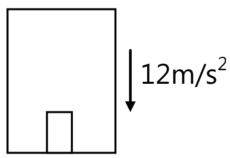


$mg = T_2$

$T_2 = 10g \equiv T_1$

$\therefore$ Both the spring show a reading of 10 kg

Sol 2: (B) Here acceleration of the lift is 12 m/s² which is greater than 'g'.



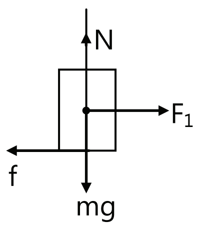
at t = 0

The body will undergo a free fall condition. Actually the body loses the contact with the floor of the lift.

$\therefore \quad s = \frac{1}{2} g\, t^2 \quad = \frac{1}{2} \times 10\,(0.2)^2\, m$

S = 20 cm.

Sol 3: (C) Here we need to understand the concept of friction



We are given that the body is not moving. Hence balancing the forces in both the directions;

$N - mg = 0$ ... (i)

$F_1 - f = 0$ ... (ii)

$\Rightarrow N = mg$ and $f = F_1$.

Now we don't know anything about F_1.

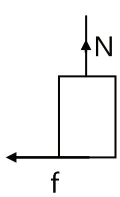
But we know that the force F_1 must be less than maximum static friction i.e. μmg for the body to be at rest.

$\therefore f = F_1 \le \mu mg$. And minimum F_1 can be zero.

$\therefore \; 0 \le f \le \mu\, mg$ ... (iii)

Now we know that contact force on the body is

$F = \sqrt{N^2 + f^2}$



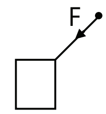
Using (i) and (iii) here,

$$\sqrt{0 + (mg)^2} \le F \le \sqrt{(mg)^2 + (\mu mg)^2}$$

$$mg \le F \le mg\sqrt{1 + \mu^2}$$

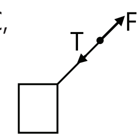
Sol 4: (C) Tension will always act along the length of the string and opposing the applied force.

In option B,



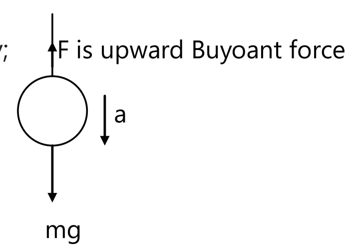
Tension has to act opposite to the applied force, but there is no string after the end point. Hence the string collapses.

In option C,



The tension in the string acts towards the body, thus making the string tough. Hence this is the correct representation.

Sol 5: (A) Initially;

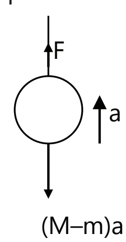


$Mg - F = Ma$

$Mg - Ma = F$ ... (i)

Now when the mass 'm' is released,

Balloon starts rising upwards with an acceleration 'a'.

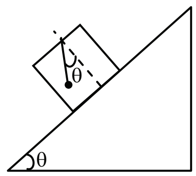


$F - (M - m)g = (M - m)a$ ... (ii)

Solving (i) and (ii); we get

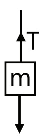
$$m = \left(\frac{2a}{a+g}\right)M$$

Sol 6: (A) Let us assume that the string makes an angle of 'θ_1' with the normal of the plane.



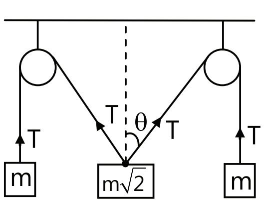
The only external force acting on the sphere is 'mg' which is vertically downward. Hence the string also becomes vertical so as to balance the force mg.

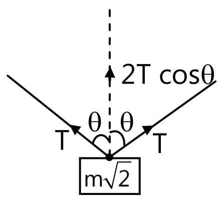
Sol 7: (C) F. B. D of (1)



$T - mg = 0$ ... (i)

$\Rightarrow T = mg$





(1) (2) (3)

$2T\cos\theta - m\sqrt{2}g = 0$ ... (ii)

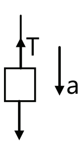
From (i) and (ii): $2(mg)\cos\theta = m\sqrt{2}g$

$\text{Cos}\,\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

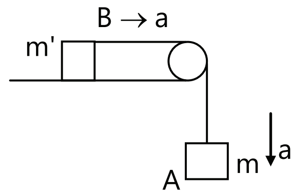
Sol 8: (B) The reason for small steps is that the lateral forces are decreased. Imagine taking a large step on concrete. When you put your foot down well in front of you, it will be pushing forwards on the concrete. And at the end of that step, when that foot is well behind you, it will be pushing backward on the concrete. The larger the step, the larger there forward and backward forces.

Our shoes on Ice can only provide or sustain small forward/backward forces, before they slip. Hence we try to reduce the friction.

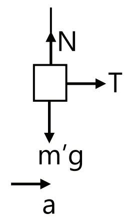
Sol 9: (B) FBD of A;



$mg - T = ma$ ... (i)



FBD of B;



$\therefore$ $N - m'g = 0$ ...(ii)

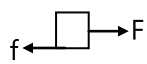
$T - m'a = 0$ ...(iii)

Using (i) and (iii)

$mg = ma + m'a$ [(i) + (iii)]

$a = \left(\frac{m}{m+m'}\right)g$

Sol 10: (A) Now, the force required to just start the motion would be the static friction (f_s)

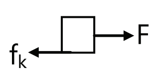


$\therefore F = f_s = \mu_\sigma mg$

i.e. after this point the body starts moving.

When the body is moving, kinetic friction acts on the body (i. e μ_k mg)

FBD of the body;



$\to$ a

$F - f_k = ma$

$\mu_s mg - \mu_k mg = ma$

$\Rightarrow (\mu_s - \mu_k)mg = ma \quad \Rightarrow a = (\mu_s - \mu_k)g$

$a = 0.98\ m/s^2$

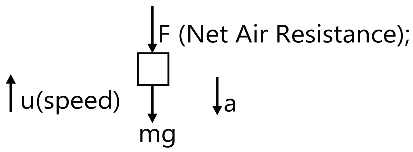
Sol 11: (C) Newton's second law states that the net force on an object is equal to the rate of change of its linear momentum.

$\Rightarrow \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$

if m is constant, then

$\equiv m.\frac{d\vec{v}}{dt} \equiv m\vec{a}$

Sol 12: (B) FBD of the body;



mg + F = ma

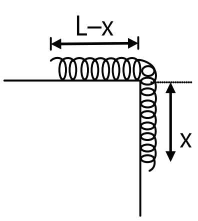
$a = g + \frac{F}{m}$; which is downwards. (i. e opposite) to the direction of displacement (till it reaches maximum height)

Since $m_A > m_B$; $a_A < a_B$

i.e. Body 'A' has less downward acceleration when composed to Body 'B'. Hence A will go higher than B.

Sol 13: (A) Let 'x' be the maximum length that can hang hand from the table.

Now say f_s be the static friction



$f_s = \left(\frac{M}{L}\right).\ x.\ g$ ... (i)

[∵ Condition for Equilibrium]

And also we know that $f_s = \mu N$.

$N = \frac{M}{L}(L-x)g$

$f_s = \frac{\mu M}{L}\ (L-x)\ g$ ... (ii)

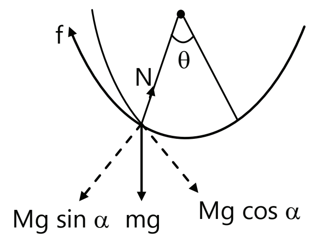
from (i) and (ii)

$\frac{Mx}{L}g = \mu\frac{M}{L}\ (L-x)\ g$

$\Rightarrow \frac{x}{L} = \left(\frac{\mu}{1+\mu}\right)$

$\left(\frac{x}{L}\times 100\right) = \left(\frac{\mu}{1+\mu}\times 100\right) = \frac{1/4}{5/4}\times 100 = 20\%$

Sol 14: (A)



Given that insect moves very lowly;

∴ V = 0; Acceleration of the body is also zero.

$f = Mg\cos\alpha$

$N = Mg\sin\alpha$

Now for the maximum case;

$f = f_s = mN$.

∴ $mN = Mg\cos\alpha$

$\mu\ (Mg\sin\alpha) = Mg\cos\alpha$

$\tan\alpha = \frac{1}{\mu} \Rightarrow \tan\alpha = 3$

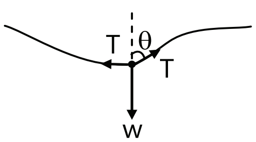
But we want to express in terms of θ;

$\alpha + \theta = 90^o, \rightarrow \alpha = 90^o - \theta$

$\tan\alpha = \tan(90^o - \theta)$

$3 = \cot\theta$

Sol 15: (A)



When the bird alights on the wire; the wire makes a curve of small angle.

$2\ T\sin\theta = w$

$\sin\theta = \left(\frac{W}{2T}\right)$

we know that $\sin\theta \le 1$

$\Rightarrow \frac{W}{2T} < 1 \Rightarrow \left(T > \frac{W}{2}\right)$

Sol 16: (D) Now Balancing the forces parallel and perpendicular to the incline surface;

$f = mg\sin\theta$

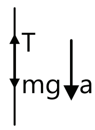
$N = mg\cos\theta$

And Net force by surface $= \sqrt{f^2 + N^2}$

$= \sqrt{(mg\sin\theta)^2 + (mg\cos\theta)^2} = mg = 30\ N.$

Sol 17: (C) While descending down;

The fireman tries to pull the rope down and so there will be a tension 'T' upwards.

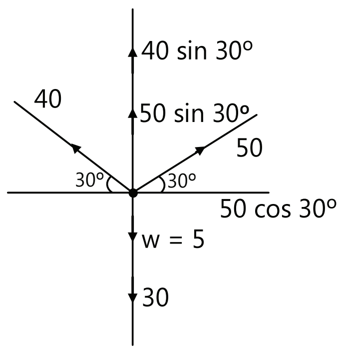


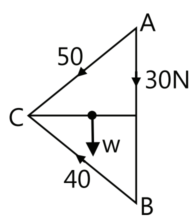
mg – T = ma; mg – ma = T

Now given $T_{max} = \frac{2mg}{3}$

$\therefore\ a_{min} = mg - \frac{2mg}{3}\Big/m$ $\therefore\ a_{min} = g/3$

Sol 18: (C)

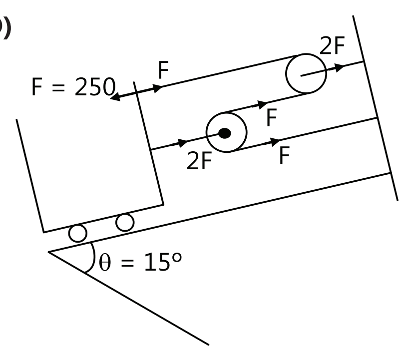




F_{net} = 90 sin 30° – (30 + 5) = 45 – 35

F_{net} = 10 N upwards

Sol 19: (D)



3F – mg sin α = ma

$a = \left(\frac{3F}{m} - g\sin\alpha\right)$

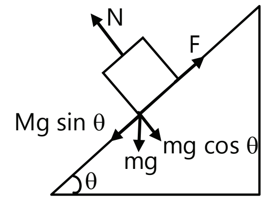
$a = \frac{250\times3}{100} - 10(0.26) = 7.5 - 2.6\ m/s^2 = 4.9 m/s^2$

Sol 20: (B) Here in the problem, two cases arises;

(i) when the body is at rest

(ii) when the body just starts sliding and slides down

For case I;



As long as body doesn't slide;

F = mg sin θ;

N = mg cos θ

$\therefore\ F = \sqrt{f^2 + N^2} = mg$

$\therefore$ It remains constant till a particular 'θ'.

For case II;

When the body is sliding down,

f = mN

N = mg cos θ

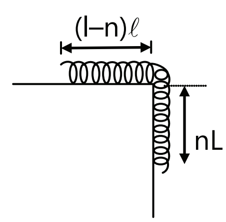
$\therefore F = \sqrt{(\mu N)^2 + N^2} = N\left(\sqrt{\mu^2+1}\right)$

$= mg\cos\theta\left(\sqrt{\mu^2+1}\right)$

As θ increases; cos θ decreases.

Hence F decreases.

Sol 21: (B)



λ (mass per unit length) = $\left(\frac{m}{L}\right)$

Now mass of the part which is hanging= $(nL)\left(\frac{m}{L}\right)$ = nm

And mass of the part which is on the table = (1 – n)m

Now total downward force = (nm) g ≡ nmg.

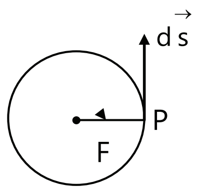
This force has to be balanced by the frictional force which is μN ≡ μ [(1 – n) mg]

$\therefore\ \mu(1-n)\,mg = n\,mg$

$\mu = \left(\frac{n}{1-n}\right)$

Circular Dynamics

Sol 22: (D) Force acting on the particle at any instant is $mR\omega^2$ towards the center.



i.e. $\vec{F} = mR\omega^2\ \hat{e}_r$ [Radial dire ction]

And the displacement of the particle will be 'ds' along tangential direction.

i.e. $\vec{ds} = ds\,\hat{e}_t$

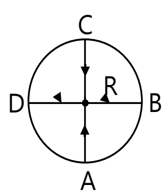
Now work = $\vec{F}\,.\,\vec{ds}$

$W = mr\omega^2\ ds\left(\hat{e}_r.\hat{e}_t\right)$

W = Zero (As $\hat{e}_r, \hat{e}_t$ are perpendicular to each other)

Hence the work done by the Centripetal force is zero.

Sol 23: (C) Centripetal force = $mR\omega^2$



Now at any point in the circle this value remains the same. Its only that the direction keeps changing.

Sol 24: (D) In uniform circular motion, ω is constant Now in the options, A, B, C the quantities are constant in magnitude but keep changing in direction.

And since they are vector Quantities, we can't say they are constant. For speed, its only magnitude that matters. Since it's a Scalar Quantity.

And Speed = $R\omega$ $\because$ Constant

Hence option D.

Sol 25: (A) $m_1 = m_2 = m$; $v_1 = v_2 = v$

Now $F_1 = \dfrac{m_1 v_1^2}{r_1} = \dfrac{mv^2}{r_1}$

$F_2 = \dfrac{m_2 v_2^2}{r_2} = \dfrac{mv^2}{r_2}$

$\dfrac{F_1}{F_2} = \left(\dfrac{r_2}{r_1}\right)$

Sol 26: (C) Centripetal force = $\dfrac{mv^2}{R}$

$v = 36$ km/hr $= 36\left[\dfrac{1000}{3600} m/s\right] = 36\ \dfrac{5}{18}$ m/s

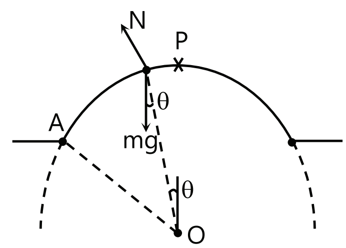
v = 10 m/s

$F = \dfrac{(500)(10)^2}{50}$

F = 1000 N

Sol 27: (B) Use $F = mr\omega^2$

Sol 28: (A)



$N = mg\cos\theta - \dfrac{mv^2}{R}$

As one goes from A to P; θ decreases, so cos θ increase.

$\therefore$ N increases

Sol 29: (A) Centripetal force = $-\dfrac{k}{r^2}$

$\Rightarrow \dfrac{mv^2}{r} = -\dfrac{k}{r^2}$

$\Rightarrow mv^2 = -\dfrac{k}{r}$

$\Rightarrow \dfrac{1}{2}mv^2 = -\dfrac{k}{2r}$

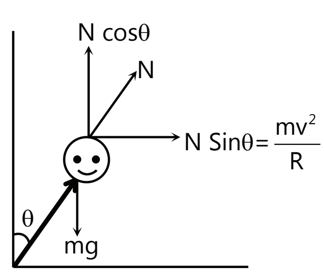
$\Rightarrow$ kinetic energy $K = -\dfrac{k}{2r}$

And since the motion is horizontal motion; let us assume the potential energy same as that of ground i.e. zero

$\therefore$ total energy $= K + U = -\dfrac{k}{2r} + 0$

$E = -\dfrac{k}{2r}$

Sol 30: (B)



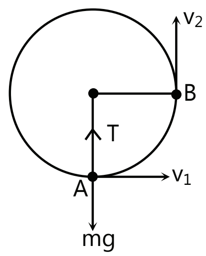
$N \sin\theta = \frac{mv^2}{R}$

$N \cos\theta = mg$

$\tan\theta = \frac{v^2}{Rg}$

$\theta = \tan^{-1} = \tan^{-1} = \tan^{-1}\left(\frac{1}{5}\right)$

Sol 31: (D)



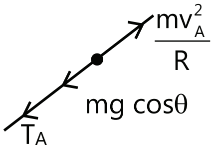
In a vertical motion, the speed of ball doesn't remain constant and as we discussed earlier, centripetal force can't be constant in direction itself, so its ruled out.

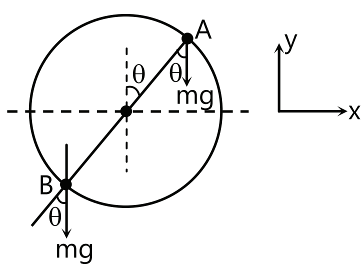
And for tension, consider two points A, B

$\vec{T}_A = \left(mg + \frac{mv_A^2}{R}\right)(-\hat{j})$ and $\vec{T}_B = \frac{mv_B^2}{R}(\hat{i})$

Hence tension is also not constant. Now gravitational force on the ball is (mg) at any point on the circle.

Sol 32: (B) At point A

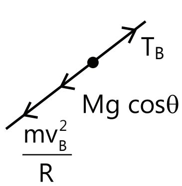




$T_A + mg\cos\theta = \frac{mv_A^2}{R}$

$T_A = \frac{mv_A^2}{R} - mg\cos\theta$

and for point B,



$T_B = mg\cos\theta + \frac{mv_B^2}{R}$

Now $T_A - T_B = \frac{mv_A^2}{R} - \frac{mv_B^2}{R}$

$T_A - T_B = \frac{m}{R}(v_A^2 - v_B^2)$ (i)

Now using conservation of energy theorem;

At point A; $E_A = \frac{1}{2}mv_A^2 + U_A$

At point B; $E_B = \frac{1}{2}mv_B^2 + U_B$

$E_A = E_B$

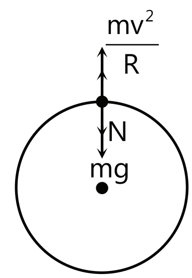
$\frac{1}{2}m(v_A^2 - v_B^2) = U_B - U_A$

But we can observe that both points A and B are at same heights from the center.

$\therefore U_A = -U_B \quad \because T_A - T_B = \frac{m}{R}\cdot\frac{2}{m}(U_B - U_A) = \frac{2}{R}(U_B - U_A)$

$\therefore$ is constant

Sol 33: (D)



$mg + N = \frac{mv^2}{R}$

$v = \sqrt{\frac{R}{m}(mg + N)}$

Now for minimum case; let us say he just loses contact i.e. N = 0

$\therefore v = \sqrt{gR}$. This is the minimum speed.

Previous Years' Questions

Forces and Laws of Motion

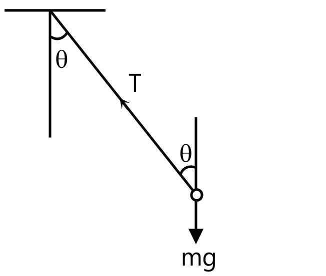
Sol 1: (C) $a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7} = \frac{5}{3} \times 10^{-3}$ m/s^2

$v = \sqrt{2as} = \sqrt{2\times\frac{5}{3}\times10^{-3}\times3} = 0.01$ m/s

Sol 2: (A) Since, mg cos θ > mg sinθ

$\therefore$ force of friction is f = mg sinθ

Sol 3: (C)



FBD of bob is $T\sin\theta = \frac{mv^2}{R}$

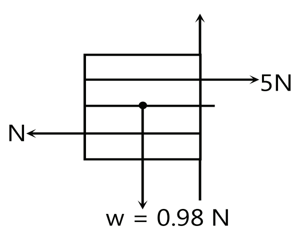
and $T\cos\theta = mg$

$\therefore \tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)}$

$\tan\theta = 1$ or $\theta = 45°$

Sol 4: (A) N = 5N

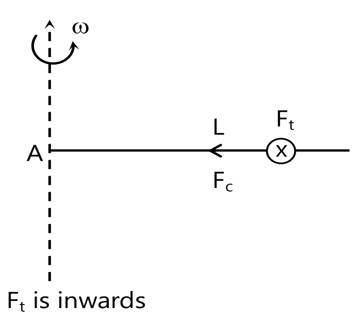
$(f)_{max} = \mu N = (0.5)(5) = 2.5$ N



For vertical equilibrium of the block

$F = mg = 0.98\text{ N} < (f)_{max}$

Sol 5: (A) Tangential force (F_t) of the bead will be given by the normal reaction (N), while centripetal force (F_c) is provided by friction (f_r). The bead starts sliding when the centripetal force is just equal to the limiting friction.



Therefore,

$F_t = ma = m\alpha L = N$

$\therefore$ Limiting value of friction

$(f_r)_{max} = \mu N = \mu m\alpha L$ ... (i)

Angular velocity at time t is $\omega = \alpha t$

$\therefore$ Centripetal force at time t will be

$F_c = mL\omega^2 = mL\alpha^2t^2$ ... (ii)

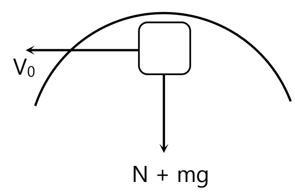
Equating equation (i) and (ii), we get

$t = \sqrt{\frac{\mu}{\alpha}}$

For $t > \sqrt{\frac{\mu}{\alpha}}$, $F_c > (f_r)_{max}$ i.e. , the bead starts sliding.

In the figure F_t is perpendicular to the paper inwards.

Sol 6: (A) Since, the block rises to the same heights in all the four cases, from conservation of energy, speed of the block at highest point will be same in all four cases. Say it is v_0.



Equation of motion will be

$N + mg = \frac{mv_0^2}{R}$ or $N = \frac{mv_0^2}{R} - mg$

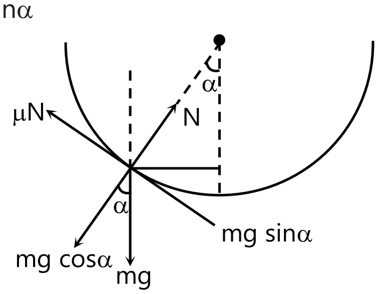
R (The radius of curvature) in first case is minimum. Therefore, normal reaction N will be maximum in first case.

Note in the question it should be mentioned that all the four tracks are frictionless. Otherwise, v_0 will be different in different tracks.

Sol 7: (A) Equilibrium of insect give

$N = mg\cos\alpha$

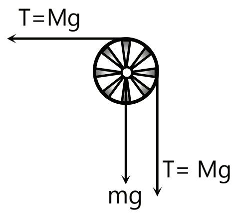
$\mu N = mg\sin\alpha$



From Equation (i) and (ii). We get

$\cot\alpha = 1/\mu = 3$

Sol 8: (D)



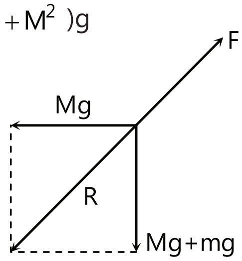
Free body diagram of pulley is shown in figure. Pulley is in equilibrium under four forces. Three forces as shown in figure and the fourth, which is equal and opposite to the resultant of these three forces, is the force applied by the clamp on the pulley (say F).

Resultant R of these three forces is

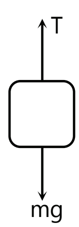
$R = (\sqrt{(M+m)^2 + M^2})g$

Therefore, the force F is equal and opposite to R as shown in figure.

$\therefore\ F = (\sqrt{(M+m)^2 + M^2})g$

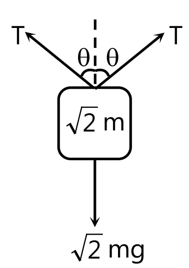


Sol 9: (C) Free body diagram of m is



$T = mg$ ... (i)

Free body diagram of mass $\sqrt{2}$ m is

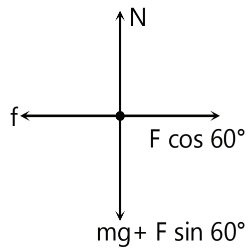


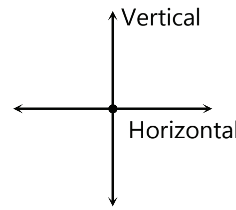
$2T\cos\theta = \sqrt{2}\,mg$ ... (ii)

Dividing Eq. (ii) by Eq. (i) we get

$\cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = 45°$

Sol 10: (A) Free body diagram (FBD) of the block (shown by a dot) is shown in figure.





For vertical equilibrium of the block

$N = \mu g + F\sin 60° = \sqrt{3}\,g + \sqrt{3}\,\frac{F}{2}$ ... (i)

For no motion, force of friction

$f \geq F\cos 60°$

or $\mu N \geq F\cos 60°$

or $\frac{1}{2\sqrt{3}}(\sqrt{3}g + \frac{\sqrt{3}F}{2}) \geq \frac{F}{2}$

or $g \geq \frac{F}{2}$ or $F \leq 2g$ or 20 N

Therefore, maximum value of F is 20 N.

Sol 11: (D) This is the equilibrium of coplanar forces. Hence,

$\Sigma F_x = 0$

$\therefore\quad F = N$

$\therefore\quad \Sigma F_y = 0,\ f = mg$

$St_c = 0\ \therefore\ \vec{\tau_N} + \vec{\tau_f} = 0$

$\therefore$ Since, $\vec{\tau_f} \neq 0$

$\therefore\quad \vec{\tau_N} \neq 0$

Sol 12: (A) Initially under equilibrium of mass m

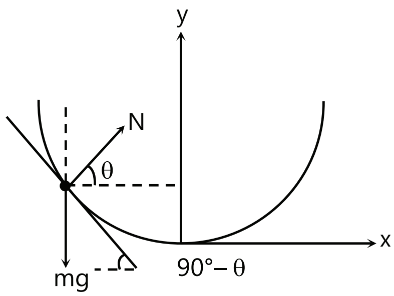
$T = mg$

Now, the string is cut. Therefore, T = mg force is decreased on mass m upwards and downwards on mass 2m.

$\therefore\quad a_m = \frac{mg}{m} = g$ (downwards) and

$a_{2m} = \frac{mg}{2m} = \frac{g}{2}$ (upwards)

Sol 13: (B)



N sin θ = mg

N cos θ = ma

$\tan\theta = \dfrac{g}{a}$

$\cot\theta = \dfrac{a}{g} = \tan(90° - \theta) = \dfrac{dy}{dx} = 2kx$

$\therefore\ x = \dfrac{a}{2kg}$

Sol 14: (A) When

P = mg (sin θ – μ cos θ)

F = μ mg cos θ (upwards)

when P = mg sin θ

f = 0

and when P = mg(sin θ + μ cos θ)

f = μmg cos θ (downwards)

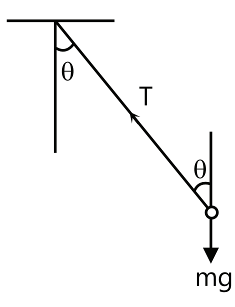
Hence friction is first positive, then zero and then negative.

∴ Correct option is (A).

Sol 15: (B,D) A rotating/revolving frame is acceleration and hence non-inertial. Therefore, correct options are (B) and (D).

Circular Dynamics

Sol 16: (C)



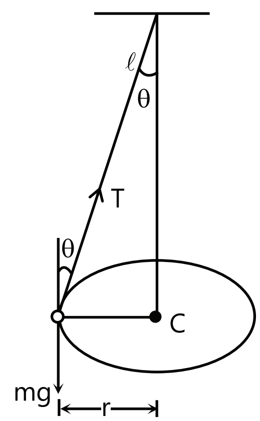
FBD of bob is $T\sin\theta = \dfrac{mv^2}{R}$

and T cos θ = mg

$\therefore\ \tan\theta = \dfrac{v^2}{Rg} = \dfrac{(10)^2}{(10)(10)}$

tan θ = 1 or θ = 45°

Sol 17: (D)



$R = \ell\sin\theta$

T cosθ component will cancel mg.

T sinθ component will provide necessary centripetal force to the ball towards centre C.

$\therefore\ T\sin\theta = mr\omega^2 = m(\ell\sin\theta)\,\omega^2$

or $T = ml\omega^2$

$\therefore\quad \omega = \sqrt{\dfrac{T}{m\ell}}$

or $\omega_{max} = \sqrt{\dfrac{T_{max}}{m\ell}} = \sqrt{\dfrac{324}{0.5\times0.5}} = 36\text{rad/s}$

Sol 18: (C) $F = F_0e^{-bt}$

$\Rightarrow\quad a = \dfrac{F}{m} = \dfrac{F_0}{m}e^{-bt}$

$\Rightarrow\quad \dfrac{dv}{dt} = \dfrac{F_0}{m}e^{-bt}$

$\Rightarrow\quad \int dv = \int_0^t \dfrac{F}{m}e^{-bt}dt$

$\Rightarrow\quad v = \dfrac{F}{m}\left[\dfrac{-1}{b}\right]\left[e^{-bt}\right]_0^1$

$\Rightarrow\quad v = \dfrac{F}{mb}\left[e^{-bt}\right]$

$v = 0$ at $t = 0$

and $\quad v \rightarrow \frac{F}{mb}$ as $t \rightarrow \infty$

So, velocity increases continuously and attains a maximum value of $v = \frac{F}{mb}$ as $t \rightarrow \infty$

Sol 19: (C) $mg \sin\theta = \mu mg \cos\theta$

$\tan\theta = \mu$

$\Rightarrow \frac{dy}{dx} = \tan\theta = \mu = \frac{1}{2}$

$\Rightarrow \frac{x^2}{2} = \frac{1}{2}, x = \pm 1 \qquad \Rightarrow y = \frac{1}{6}m$

Sol 20: (B) Normal force on block A due to B and between B and wall will be F.

Friction on A due to B = 20 N

$\therefore$ Friction on B due to wall = 100 + 20 = 120 N

Sol 21: (B) Since work done by friction on parts PQ and QR are equal

$-\mu mg \times \frac{\sqrt{3}}{2} \times 4 = -\mu mgx \qquad$ (QR=x)

$\Rightarrow x = 2\sqrt{3}$ m ≈ 3.5m

Applying work energy theorem from P to R

$mg \sin 30^0 \times 4 - \mu mg\frac{\sqrt{3}}{2} \times 4 - \mu mgx = 0$

$\Rightarrow \mu = \frac{1}{2\sqrt{3}} \approx 0.29$

JEE Advanced/Boards

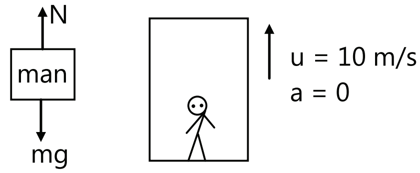
Exercise 1

Forces and Laws of Motion

Sol 1: The reading shown by the weighting scale is the normal reaction between the man and the weighing scale.

Now, in **Case (I)**

In this case,

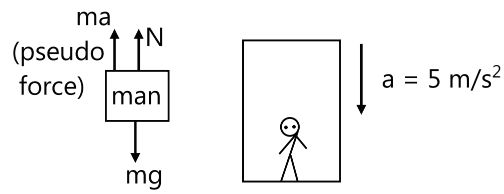


$N - mg = 0 \Rightarrow N = mg = 70 \times 10 = 700$ Newton.

$\Rightarrow$ reading by the scale = 70 kg

Case (II)

In the frame of the lift;



$\Rightarrow N + ma = mg$

$\Rightarrow N = m(g - a)$

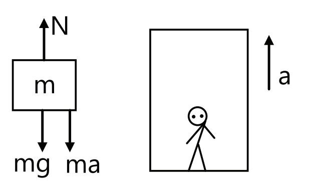
$\Rightarrow N = 70\ (10 - 5)$

$\Rightarrow N = 70 \times 5$ N

N = 350 Newton

$\Rightarrow$ Reading by the scale = 35 kg

Case (III)



$N = mg + ma$

$\Rightarrow N = m(a + g)$

$\Rightarrow N = 70\ (10 + 5)$

$\Rightarrow N = 70\ (15)$

N = 1050 Newton

$\Rightarrow$ reading by the scale = 105 kg

Now In this case a = g downward,

$\therefore$ from case (b);

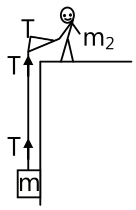
$\Rightarrow N = m\ (g - a)$

$\Rightarrow N = m(g - g)$

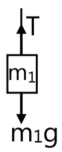
N = 0

i. e the man is in free fall.

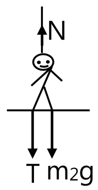
Sol 2:



Here rope tries to pull the man down.



$T = m_1g$

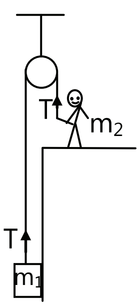


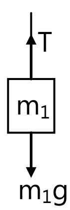
$T + m_2g = N$

$\Rightarrow N = (m_1 + m_2)g$ ...(i)

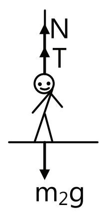
In case II;

Now rope pulls the man up





$m_1g = T$



$T + N = m_2g$

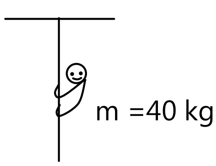
$\Rightarrow N = m_2g - T$

$\Rightarrow N = m_2g - m_1g$

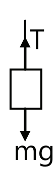
$\Rightarrow N = (m_2 - m_1)g$

Hence normal force is less in second case.

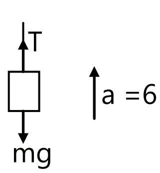
Sol 3: In climbing the rope, monkey tries to pull down the rope, and the rope pulls the monkey upwards.



∴ On monkey;



Now in **Case (a)**



$T - mg = ma$

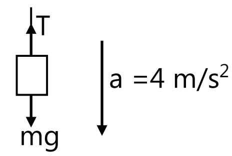
$T = m(g + a)$

$T = 40(10 + 6)$

$T = 640$ N

But $T_{max} = 600$ N, hence the string breaks.

Case b:



$mg - T = ma$

$T = m(g - a)$ ... (i)

$T = 40(10 - 4)$

$T = 40 \times 6$

$T = 240N$

$T < T_{max}$

Case c:

u = 5 m/s uniformly i.e. a = 0

$T = mg = 40(10)$

$T = 400$ N

$T \le T_{max}$

Case d:

In this case;

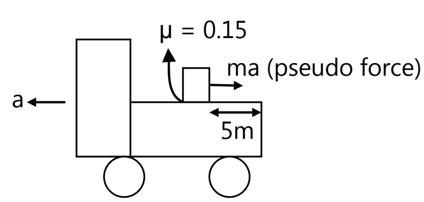
Put a = g in case (b)

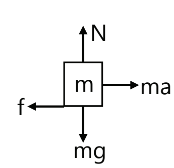
We get $t = m(g - a)$

$T = m(g - g)$

$T = 0$

Sol 4: Now with respect to the truck; forces on the mass 'm' are





ma – f = ma′

N = mg

And f = μN = μmg

ma – μmg = ma′

a′=a–μg

a′ = 2 – (0.15) (10)

A′ = 2 – 1.5

A′ = 0.5 m/s^2

Now this fairly a relative motion problem

Box has to cover a distance of 5 m to fall off from the truck;

$s = 0t + \frac{1}{2}at^2 \quad \Rightarrow 5 = \frac{1}{2}(0.5)t^2$

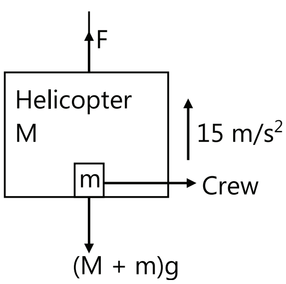
$T = \sqrt{20}$ s

Now in the meantime, distance traveled by the truck in

$s = \frac{1}{2}(2)(20) = 20m$

∴Distance from the starting point where the box lands is 15m.

Sol 5:



F is the force on helicopter due to the surrounding air

$\therefore F - (M + m)g = (M + m)a$

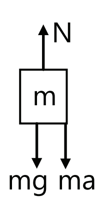
$\Rightarrow F = (M + m)g + (M + m)a$

$\Rightarrow F = (M + m)(g + a)$

$\Rightarrow F = (1300)(25)$ N = 32500 N upwards

Now using newton's third law, force by helicopter on surrounding air is F downward, i.e. 32500 downwards.

Now if we consider the crew,

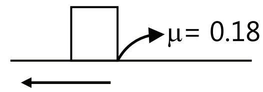


$\Rightarrow N = m(a + g) = 300(25)$

N = 7500 N upwards.

∴ Force on the floor by the crew is 7500 N downwards.

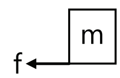
Sol 6:



a = 0.5 m/s $0 < t \le 20$

a=0   t > 20

For an observer on ground, this is how he depicts the FBD of mass,



f = ma

Now let us check for any sliding.

$f \le f_s$ ... (i) [Condition for no sliding]

$f_s = \mu mg = (0.18)(15 \times 10) = 27$ N.

and f = ma ≡ 15(0. 5) = 7. 5 N.

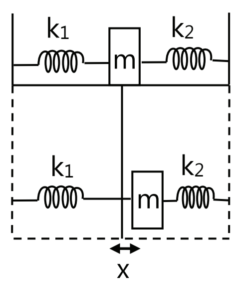
hence no sliding.

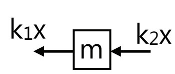
The observer will find the body to move with acceleration of 0. 5 m/s.

Now since there is no sliding, there is no relative motion w. r. t. the trolley.

Hence observer on trolley will find the mass to be at rest.

Sol 7:

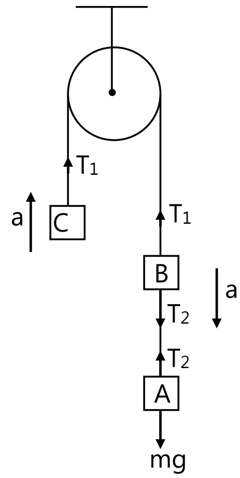


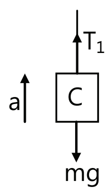


$k_1x + k_2x = ma$

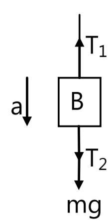
$a = \frac{(k_1 + k_2)x}{m}$

Sol 8:

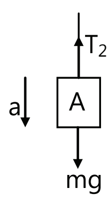




$T_1 - mg = ma$ ... (i)



$T_2 + mg - T_1 = ma$ ... (ii)



$mg - T_2 = ma$ ... (iii)

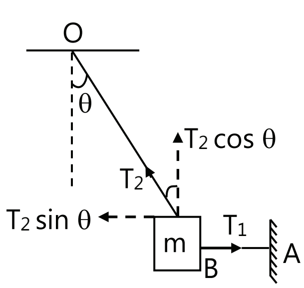
now, (i) + (ii)

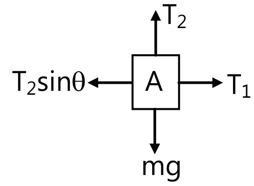
gives $T_2 = 2ma$

Now using this is equation (iii)

$\boxed{a = \frac{g}{3}}$ and $\boxed{T_2 = \frac{2g}{3}}$

Sol 9:





$T_2 \cos\theta = mg$ ... (i)

$T_2 \sin\theta = T_1$ ... (ii)

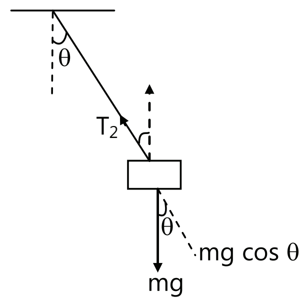
$T_2 = mg \sec\theta$

From (ii) & (i) $\Rightarrow \tan\theta = \frac{T_1}{mg}$

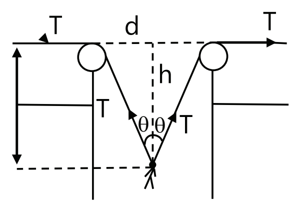
$\Rightarrow T = mg\tan\theta$

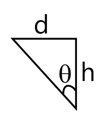
Now just after the string AB is burnt,

$T_2 = mg\cos\theta$



Sol 10:





$2T\cos\theta = mg$

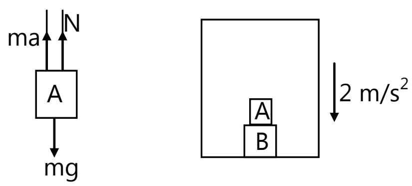
$T = \frac{mg}{2}\sec\theta$

$\sec\theta = \frac{\sqrt{h^2 + \left(\frac{d}{2}\right)^2}}{h}$

$\therefore T = \frac{mg}{2} \cdot \frac{\sqrt{h^2 + (d/2)^2}}{h}$

$\therefore$ We can see that when h decreases, T increases.

Sol 11: FBD of A;



mg = N + ma

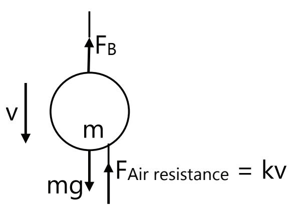
$\Rightarrow$ N = m (g – a)

$\Rightarrow$ N = 0.5 (10 – 2)

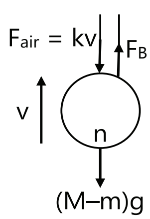
$\Rightarrow$ N = $\frac{1}{2}$(8) = 4 newton.

Sol 12: Initially,

$F_B + F_A = mg$ ... (i)



Let us say mass 'm' is removed to achieve case b; finally;

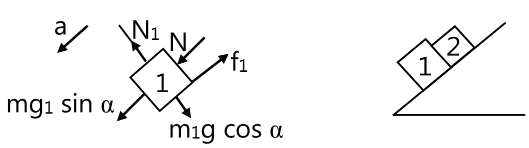


$(M - m)g + F_A = F_B$ ... (ii)

From equation (i) and (ii), eliminating F_A;

We get m = $\frac{2(Mg - B)}{g}$

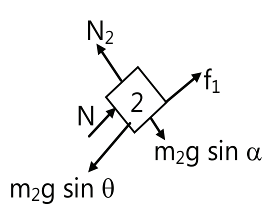
Sol 13:



$m_1 g \sin\alpha + N - f_1 = m_1 a$ ... (i)

$N_1 = mg\cos\alpha$ ... (ii)

$f_1 = \mu_1 N_1 = k_1 N_1$ ... (iii)



$N_2 = mg\cos a$ ... (iv)

$m_2 g\sin\alpha - N - f_2 = m_2 a$ ... (v)

$f_2 = k_2 N_2$ ... (vi)

Now (i)/(v)

$$g\sin\alpha + \frac{N}{m_1} - \frac{f_1}{m_1} = g\sin\alpha - \frac{N}{m_2}, - \frac{f_2}{m_2}$$

Solving; $f_1 = k_1 m_1 g\cos a$

$f_2 = k_2\, m_2 g\cos\alpha$

$$\therefore \qquad N = \frac{g\cos\alpha(k_1 - k_2)m_1 m_2}{(m_1 + m_2)}$$

Adding (i) + (v)

$(m_1 + m_2)\, g\sin\alpha - (f_1 + f_2) = (m_1 + m_2)\, a$

But for just sliding case, a = 0

$(m_1 + m_2)\, g\sin\alpha = f_1 + f_2$

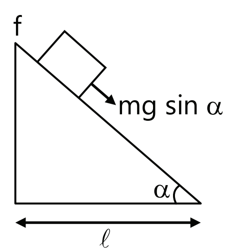
$f_1 = k_1 N_1$; $\qquad f_2 = k_2 N_2$

$\therefore (m_1 + m_2\, g\sin\alpha = k_1 m_1\, g\cos\alpha + k_2\, m_2\, g\cos\alpha$

$$\tan\alpha = \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2}$$

Sol 14: mg sin α – f = ma

f = μ mg cos α [∵ μ N]



mg sin a – μ mg cos α = ma

$\therefore$ a = g sin α – mg cos α

a = g(sin α – μ cos α)

Now time taken by the block to reach point O;

s= ot + $\frac{1}{2}at^2$ $\quad \therefore s = \ell\cos\alpha$

$\ell\cos\alpha = \frac{1}{2}g\,(\sin\alpha - \mu\cos\alpha)\,t^2$

$$t = \sqrt{\frac{2\ell\cos\alpha}{g(\sin\alpha - \mu\cos\alpha)}}$$

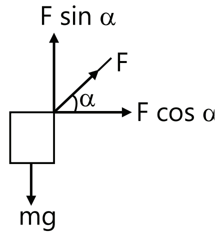
for minimum t;

$\frac{dt}{d\alpha}$ = 0.

We get $\alpha = \frac{1}{2}\tan^{-1}\left(\frac{-1}{\mu}\right)$

Sol 15: f cos α = ma ... (i)

f sin α + N = mg ... (ii)



Now at the moment, contact is lost;

N = 0

F sin α = mg

$at_0 \sin\alpha = mg$

$t_0 = \left(\frac{mg}{a\sin\alpha}\right)$

now F cos α $= ma \equiv m\frac{dv}{dt}$

$\therefore$ at cos α = $m\frac{dv}{dt}$

Integrating on both sides

$$\int_0^{t_0}(a\cos\alpha)t\,dt = m\int_0^{v}dv$$

$$\frac{a\cos\alpha}{2}\cdot\frac{t_0^2}{2} = vm \quad \text{... (i)}$$

$$\Rightarrow v = \frac{a\cos\alpha}{2m}\cdot\frac{m^2g^2}{a^2\sin^2\alpha} \Rightarrow v = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$$

We see that in equation (i)

$$\Rightarrow v = \frac{a\cos\alpha}{2m}t^2$$

$$v = \frac{dx}{dt} = \frac{a\cos\alpha}{2m}t^2$$

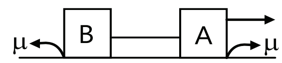
$$dx = \frac{a\cos\alpha}{2m}t^2\,dt$$

Integrating on both sides;

$$\Rightarrow \int_0^x dx = \int_0^{t_0}\frac{a\cos\alpha}{2m}t^2\,dt \Rightarrow x = \frac{a\cos\alpha}{2m}\left.\frac{t^3}{3}\right|_0^{t_0}$$

$$x = \left(\frac{a\cos\alpha}{6m}\right)\left(\frac{mg}{a\sin\alpha}\right)^3$$

Sol 16: First let us calculate the limiting friction on blocks 'A' and 'B'.



$f_{sA} = \mu mg$

$f_{sB} = \mu mg$

Now when a force of $\frac{3}{4}$ μ mg acts on the block A; it doesn't cause any motion in A.

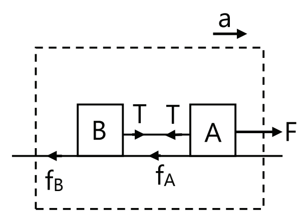
Hence; $F = f_A = \frac{3}{4}\mu mg$

And string is left unaltered. Hence tension is zero. And hence f_B = T = zero

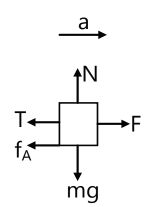
(b) Now when force of $\frac{3}{2}$ μ mg is applied,

Body A will tend to move forward. ($F \geq f_s$)

Let us assume that the whole system moves with on acceleration 'a'.



On body A;

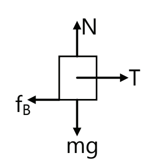


$F - T - f_A = ma$ ... (i)

mg = N ... (ii)

$f_A = \mu mg$ ... (iii)

On body B;



$T - f_B = ma$ ... (iv)

$f_B = \mu mg$ ... (v)

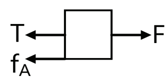
Adding (i) and (iv);

$F - (f_A + f_B) = 2ma$

$\frac{3}{2}\mu mg - (2\mu mg) = 2ma$

a is negative

It means that our assumption that both the bodies move is false.

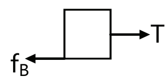


$F - T - f_A = 0$

$T = F - f_A = \frac{3}{2}\mu mg - \mu mg$

$T = \mu\frac{mg}{2}$

Now we can see that



$T = f_B = \frac{\mu mg}{2} \le f_s \equiv \mu mg$

$\therefore$ Block B cannot move. Since they both are connected to each other, even A can't move.

Sol 17: Length of block A = $\frac{\ell}{4}$

$\Rightarrow$ Distance travelled by A relative to B

$= \frac{3\ell}{4} + \frac{1}{4}\left(\frac{\ell}{4}\right)$

$l_0 = \frac{13\ell}{16}$

Let mass of A be $m_A = m$

$m_c = m;$ $m_B = 4m$

friction force = $\mu m_A g$

Acceleration of B $a_B = \frac{f}{m_b} = \frac{\mu m_A g}{m_B} = \frac{\mu g}{4}$

Acceleration of A $a_A = \frac{m_c g - \mu m_A g}{m_c + m_A}$

$= \frac{mg(1-\mu)}{2m} = g\frac{(1-\mu)}{2}$

Relative acceleration $a = a_A - a_B$

$= \frac{g(1-\mu)}{2} - \frac{\mu g}{4} = \frac{g}{4}(2-3\mu)$

$\frac{1}{2}at^2 = l$

$\therefore \frac{1}{2}\frac{g}{4}(2-3\mu)t^2 = \frac{13\ell}{16}$

$\Rightarrow t^2 = \frac{13\ell}{2(2-3\mu)g}$

Distance travelled by B = $\frac{1}{2}a_B t^2$

$= \frac{1}{2}\times\frac{\mu g}{4}.\frac{13\ell}{2(2-3\mu)g} = \frac{13\mu\ell}{16(2-3\mu)}$

Sol 18: mass $m_2 = hm_1$

Friction force on m_1 $f = km_1 g \cos a$

Gravitational force on m_1, $f_1 = m_1 g \sin\alpha$

acceleration $a = \frac{m_2 g - f - f_1}{m_1 + m_2}$

$= \frac{\eta m_1 g - km_1 g\cos\alpha - m_1 g\sin\alpha}{m_1 + \eta m_1}$

$a = \frac{g(\eta - k\cos\alpha - \sin\alpha)}{(n+1)}$

Sol 19: By constrains of string,

Acceleration of A equals to B

$\Rightarrow a_A = a_c = a$

$\Rightarrow (m_A + m_C)a = (m_A + m_C) g\sin\theta - \mu(m_C) g\cos\theta$

$\Rightarrow 2ma = 2mg\sin\theta - \mu mg\cos\theta$

$a = \frac{1}{2}(2g\sin\theta - \tan\theta\, g\cos\theta) = \frac{3}{4} g\sin\theta$

$\therefore a_A = a_C = \frac{3}{4}g\sin\theta$

Now for B, tensions of string cancel each other and no friction exists.

Hence the only acceleration is due to gravity

$\therefore a_B = g\sin\theta$

Sol 20: m will have acceleration vertically downword. Let call it a_m.

M will have acceleration along inclined plane lets call is a_M

$\because$ m, M have no relative acceleration vertically downward,

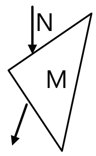
$a_M \sin\alpha = a_m$ ... (i)

Let normal force on block be N,

$mg - N = m a_m$

$N = m(g - a_m)$

From free body diagram of wedge



$Mg \sin\ \alpha + N \sin \alpha = M a_m$

$\therefore\ mg \sin \alpha + m(g - a_m) \sin \alpha = Ma_m$

$mg \sin \alpha + (m\ (g - a_M \sin \alpha\ ) \sin \alpha = M\ a_m$

$$\Rightarrow a_m = \frac{(m+M)g\sin\alpha}{M+m\sin^2\alpha}$$

$$\Rightarrow\ a_m = \frac{(m+M)g\sin^2\alpha}{M+m\sin^2\alpha} \qquad (a_m = a_M \sin \alpha)$$

Sol 22: Let mass of A = m

Mass of B = ηm

Let normal reaction between surfaces be N

$$a_B = \frac{N\sin\alpha}{m_B} = \frac{N\sin\alpha}{\eta m}$$

$$a_A = \frac{mg - N\cos\alpha}{m}$$

$a_A = a_B \tan \alpha$

$$\Rightarrow g - \frac{N}{m}\cos\alpha = \frac{N\sin\alpha\tan\alpha}{\eta m}$$

$$\Rightarrow g = \frac{N}{m}\left(\cos\alpha + \frac{\sin\alpha\tan\alpha}{\eta}\right)$$

$$\Rightarrow \frac{N}{\eta m} = \frac{g}{\eta\cos\alpha + \sin\alpha\tan\alpha}$$

$$a_B = \frac{N}{\eta m}\sin\alpha = \frac{g}{\eta\cot\alpha + \tan\alpha}$$

$$a_A = a_B \tan\alpha = \frac{g}{\eta\cot^2\alpha + 1}$$

Sol 23: Let tension in string be T

Net force perpendicular to plane

$N = mg \cos \alpha - T \sin B$

For minimum tension acceleration is zero

$\therefore\ mg \sin \alpha = T \cos \beta - \mu N$

$mg \sin \alpha = T \cos \beta - \mu\ mg \cos \alpha + \mu T \sin \beta$

$$T = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\cos\beta + \mu\sin\beta}$$

$$\frac{dT}{d\beta} = \frac{mg(\sin\alpha + \mu\cos\alpha)}{-(\cos\beta + \mu\sin\beta)^2}\frac{d}{d\beta}(\cos\beta + \mu\sin\beta)$$

For minimum T, $\frac{dT}{d\beta} = 0$

$$\Rightarrow \frac{d}{d\beta}(\cos\beta + \mu\sin\beta) = 0$$

$\Rightarrow -\sin \beta + \mu \cos \beta = 0$

$\Rightarrow \mu = \tan \beta$

$$\Rightarrow \beta = \tan^{-1}\mu \Rightarrow \cos\beta = \frac{1}{\sqrt{\mu^2+1}} \text{ and } \sin\beta = \frac{\mu}{\sqrt{\mu^2+1}}$$

$$T = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu^2}{\sqrt{\mu^2+1}}} = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\sqrt{\mu^2+1}}$$

Sol 24: frictional force $f = km_2g$

$$a_1 = \frac{F - f}{m_1} \qquad (F > f)$$

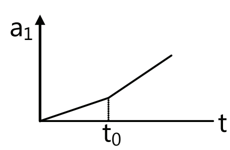
$$a_1 = \frac{at - km_2g}{m_1}$$

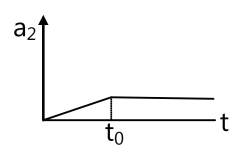
$$a_1 = \frac{at - km_2g}{m_1} \quad (t > t_0)$$

$$a_2 = \frac{km_2g}{m_2} = kg \qquad (t > t_0)$$

for $t < t_o$, f acts as internal force as there is no sliding

$$\therefore\ a_1 = a_2 = \frac{at}{m_1 + m_2} \qquad (t < t_0)$$





Till time t_o , the bodies move together.

at $t = t_0$, $f = k\ m_2g$

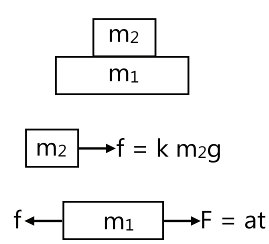
$km_2g = m_2a_2$

$kg = a_2 = a_1$

$at_o - km_2g = m_1a_1$

$at_o = km_2g + m_1a_1$

$$t_o = \frac{k(m_1+m_2)g}{a}$$



Sol 25: Let mass of motor = m

mass of bar = 2m

$2m\ w = T - 2mg\ k$

$\Rightarrow\ T = 2\ m\ (w + kg)$

Let acceleration or motor be a_m

$m\ a_m = T - mg\ K$

$\Rightarrow a_m = \frac{1}{m}[2mw + 2m\,kg - mkg]$

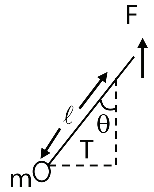
$\Rightarrow a_m = 2w + kg$

Relative accelerator $a = a_m + w = 3w + k\,g$

$\frac{1}{2}at^2 = l$

$t = \sqrt{\frac{2\ell}{a}} = \sqrt{\frac{2\ell}{3w + kg}}$

Sol 26:



$2T\cos\theta = F$

$\Rightarrow T = \frac{F}{2\cos\theta}$

Horizontal acceleration $a_x = \frac{T\sin\theta}{m}$

$= \frac{F}{2\cos\theta}\frac{\sin\theta}{m} = \frac{F\tan\theta}{2m} = \frac{F}{2m}\cdot\frac{x}{\sqrt{(\ell)^2 - (x)^2}}$

$a_x = \frac{Fx}{2m(\ell^2 - x^2)^{\frac{1}{2}}}$

Acceleration of approach $= 2a_x = \frac{fx}{m(\ell^2 - x^2)^{\frac{1}{2}}}$

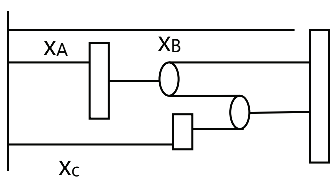
Sol 27: Let tension in thread = T

$F - 3T = 35\,a_B$

$2T = 70\,a_A$

$\Rightarrow F = 35\,(a_B + 3a_A)$

Constrain equation x_B



$2(x_B - x_A) + (x_B - x_C) = \text{Constant}$

$\Rightarrow 3a_B - 2a_A = 0 \quad \Rightarrow a_A = \frac{3}{2}a_B$

$\Rightarrow F = 35\left(\frac{11}{2}\right)a_B$

$\Rightarrow a_B = \frac{300\times 2}{385} = 1.558\ ms^{-2}$

$a_A = \frac{3}{2}a_B = 2.338\ ms^2$

T = 81. 8 N

Sol 28: F = 30t N

$\Rightarrow T = 10\,t$

wt. of A = 10 m_1 = 10 N

(a) Block A loses contact when T = weight

10t = 10

t = 1 s

Similarly 2T = 10m_2 when B loses contact

20t = 10(4)

t = 2s

(b) Net force on A $F_A = 10\,t - 10\ (t > 1)$

$a_A = \frac{1}{m_1}(10t - 10)$

$a_A = (10t - 10)$

$\frac{dv_A}{dt} = 10t - 10$

$v_A = \int_1^2 (10t - 10).dt = 5t^2 - 10t\Big|_1^2$

$v = 5\ ms^{-1}$

(c) $v_A = \int_1^t (10t - 10)dt = 5t^2 - 10t\Big|_1^t$

$v_A = 5t^2 - 10t + 5$

$\frac{dh}{dt} = 5t^2 - 10t + 5$

$H = \int dh = \int_1^2 (5t^2 - 10t + 5)dt = \frac{5}{3}t^3 - 5t^2 + 5t\Big|_1^2 = \frac{5}{3}$ m.

Circular Dynamics

Sol 29: Acceleration inside a rotor = $R\omega^2$

$\vec{a} = R\omega^2$

Now for $\vec{a}_{max}$

$a_{max} = R\,w_{max}^2$

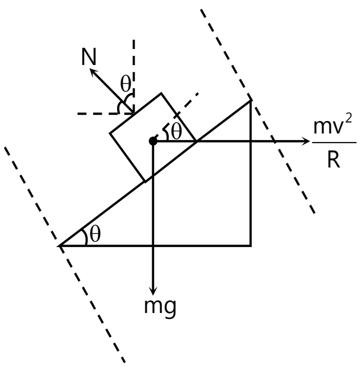
Given $a_{max} = 10g = 100\ m/s^2$

$\omega_{max} = \sqrt{\frac{100}{4}} = \frac{10}{2}$ rad/s = 5 rad/s

we know that 1 rad = $\frac{1}{2\pi}$ rev

$\therefore\ \omega_m = \frac{5}{2\pi}$ Rev/s

Sol 30:



$N \sin\theta = \frac{mv^2}{R}$ ... (i)

$N \cos\theta = mg$ ... (ii)

Dividing (i) and (ii)

$\Rightarrow \tan\theta = \frac{v^2}{Rg}$

$\Rightarrow v = 108$ km/h $= 108 \times \frac{5}{18}$ m/s

v = 30 m/s

R = 90 m

$\therefore\ \tan\theta = \frac{30.30}{90.10} = 1$

$\Rightarrow\ \theta = \frac{\pi}{4}$

Squaring (i) and (ii) and adding them

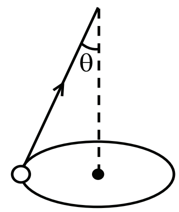
$\Rightarrow N^2 (\sin^2\theta + \cos^2\theta) = \left(\frac{mv^2}{R}\right)^2 + (mg)^2$

$\Rightarrow N = \sqrt{(mg)^2 + \left(\frac{mv^2}{R}\right)^2}$

$\Rightarrow N = m\sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$ m Newton

$\Rightarrow N = 10^4.\ \sqrt{2}$ N.

Sol 31:



$T \cos\theta = mg$

$T \sin\theta = \frac{mv^2}{R}$

Now the component $T \cos\theta$ has to balance the weight of the body

$\therefore\ T_{max} \cos\theta = mg \Rightarrow 8 \cos\theta = 0.4 \times 10$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

$\therefore$ Angle with the horizontal is $(90° - \theta) = 30°$

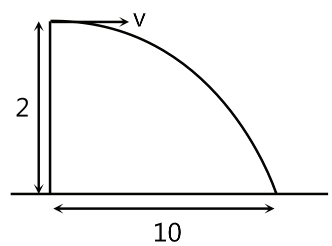
and $T \sin\theta = \frac{mv^2}{R}$

$8.\ \frac{\sqrt{3}}{2} = \frac{0.4 \times v^2}{4}$

$v = \sqrt{40(\sqrt{3})}$ m/s

v = 8. 3 m/s

Sol 32: Speed of the particle just before the string breaks is v. Now after the string is broken; the path of the stone will be;



Writing the equations of motion;

along y : $2 = \frac{1}{2}gt^2$ ... (i)

along x : 10=vt ... (ii)

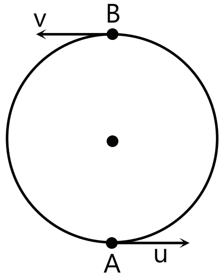
Solving for v; we get

v = 15.8 m/s

and centripetal acceleration = $\frac{v^2}{R}$

$a = \frac{(15.8)^2}{1.5} = 168.3$ m/s²

Sol 33:



Writing down the equation of motions at point A and B;

At B:

$T_B = \frac{mv^2}{R} - mg$

At A:- $T_A = \frac{mu^2}{R} + mg$

Now for completing the circle;

Tension at the highest point has to be non-zero; or else the particle will fall down.

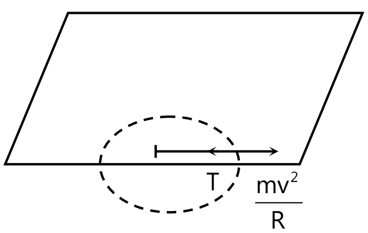
So for the minimum case, $T \approx 0$

$\therefore T_B = 0$

$\Rightarrow \frac{mv^2}{R} = mg$

$v = \sqrt{Rg}$

Sol 34:



$T = \frac{mv^2}{R}$

Now for v_{max}

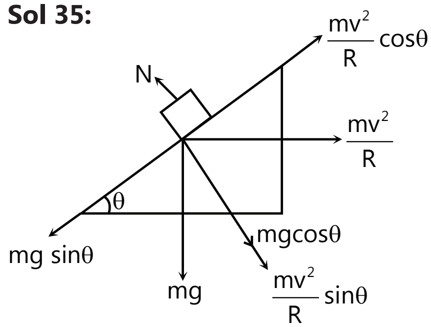
we have $T_{max} = 500$ N

$\Rightarrow 500 = \frac{1v^2}{1}$

$v = \sqrt{500} = 10\sqrt{5}$ m/s

v = 22.36 m/s

Sol 35:



$N = \frac{mv^2}{R} \sin\theta + mg\cos\theta$

$f = \frac{mv^2}{R} \cos\theta - mg\sin\theta$

Contact force is N + f

And the angle with which the force and the surface of the contact lie is

$\tan^{-1}\left(\frac{f}{N}\right)$

But here given that the force is normal to the surface

$\Rightarrow$ Friction force = 0

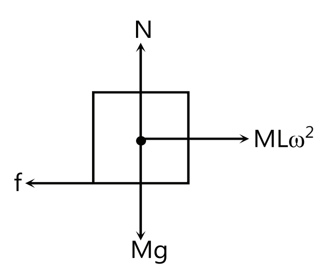
$\therefore \frac{mv^2}{R}\cos\theta - mg\sin\theta = 0$

$\Rightarrow \tan\theta = \frac{v^2}{Rg} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$

$v = 100 \text{ km/h} = 100.\frac{5}{18} = \frac{250}{9}$ m/s

Find θ now !

Sol 36: FBD of M;



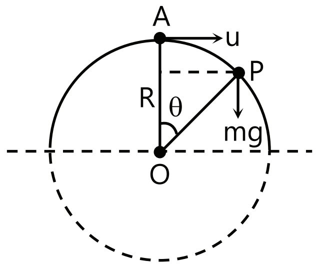
$f = ML\omega^2$; $N = Mg$

and for static conditions;

$f = \mu N = \mu Mg \qquad \Rightarrow \mu Mg = ML\omega^2$

$\omega = \sqrt{\frac{\mu g}{L}}$

Sol 37: Let u be the speed at the highest point of the bridge



$$\frac{mu^2}{R} + N = mg$$

$$N = mg - \frac{mu^2}{R}$$

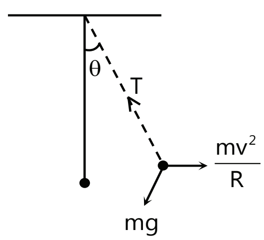
Now for maximum speed where contact is broken;

$N = 0$

$$\therefore mg = \frac{mu^2}{R}$$

$$u = \sqrt{Rg}$$

Sol 38:



$$T \sin\theta = \frac{mv^2}{R} \quad \text{... (i)}$$

$$T \cos\theta = mg \quad \text{... (ii)}$$

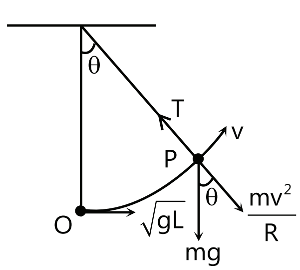
$$\frac{(i)}{(ii)} = \tan\theta = \frac{v^2}{Rg}$$

$$v = 36 \text{ km/h} = 36\ \frac{5}{18} = 10 \text{ m/s}$$

$$\tan\theta = \frac{10\times 10}{10\times 10}; \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Sol 39:



Let us call the point where tension is equal to the weight of the particle as 'P'.

Now at point P,

$$T = \frac{mv^2}{R} + mg\cos\theta \quad \text{... (i)}$$

Given that T = mg

$$mg = \frac{mv^2}{R} + mg\cos\theta$$

$$mg(1 - \cos\theta) = \frac{mv^2}{R} \quad \text{... (ii)}$$

Now Total energy at point O

$$0 = \frac{1}{2} m\left(\sqrt{gL}\right)^2 + 0$$

$$E_0 = \frac{mgL}{2}$$

Total energy at point P = $\frac{1}{2} m(v^2) + mgL(1 - \cos\theta)$

$E_0 = E_P$

$$\therefore \frac{mgL}{2} = \frac{mv^2}{2} + mgL(1 - \cos\theta)$$

$$-\frac{mgL}{2} + mg\,L\cos\theta = \frac{mv^2}{2}$$

$$\frac{mv^2}{2} = 2\,mg\cos\theta - mg \quad \text{... (iii)}$$

Now using this value of $\frac{mv^2}{L}$ in eq^n (ii)

$2\,mg\cos\theta - mg = mg(1 - \cos\theta)$

$3\,mg\cos\theta = 2\,mg$

$$\cos\theta = \frac{2}{3} \qquad \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

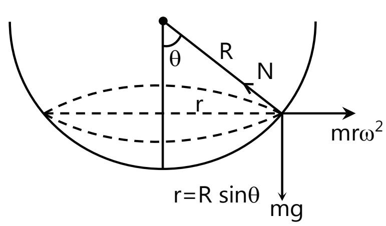
Considering eq^n – 3

$$\frac{mv^2}{L} = 2mg\left(\frac{2}{3}\right) - mg$$

$$\frac{mv^2}{L} = \frac{mg}{3}$$

$$v = \sqrt{\frac{g\ell}{3}}$$

Sol 40:

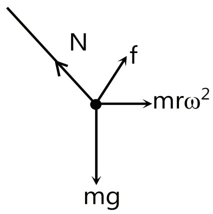


FBD of body:

(a) For minimum ω;

Body tends to slip down

$\therefore$ friction acts upwards



$N = mg\cos\theta + mr\,\omega^2\sin\theta$

$F = mg\sin\theta - mr\,\omega^2\cos\theta$

We know that $f = \mu N$

$\Rightarrow mg\sin\theta - mr\omega^2\cos\theta = \mu[mg\cos\theta + mr\omega^2\sin\theta]$

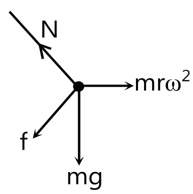
Separating all ω^2 terms to one side;

$(\mu r\sin\theta + r\cos\theta)\omega^2 = g\sin\theta - \mu g\cos\theta$

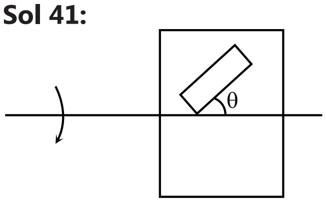
$$\omega = \sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\mu\sin\theta + \cos\theta)}}$$

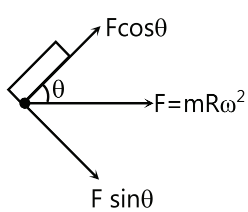
Now for maximum limit case;

Solve exactly as above



Sol 41:





Now $mR\omega^2\cos\theta = ma$

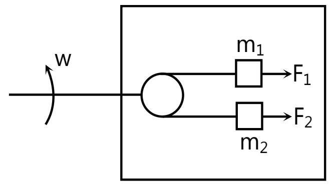
$\therefore a = R\omega^2\cos\theta$

Now $s = ut + \frac{1}{2}at^2$

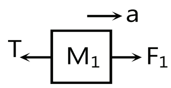
$L = 0 + \frac{1}{2}R\omega^2\cos\theta\ t^2$

$$t = \sqrt{\frac{2L}{R\omega^2\cos\theta}}$$

Sol 42:

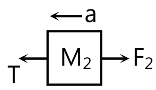


FBD of m_1;



$F_1 - T = m_1 a$ ... (i)

FBD of m_2 :



$T - F_2 = m_2 a$ ... (ii)

Adding equation (i) and (ii)

$F_1 - F_2 = (m_1 + m_2)a$

$F_1 = mR\omega^2 \qquad F_2 = 2\,mR\omega^2$

$\therefore -mR\omega^2 = 3m\,a$

$$a = -\frac{R\omega^2}{3}$$

Using equation (i)

$$mR\omega^2 - T = m\left(-\frac{R\omega^2}{3}\right)$$

$$T = mR\omega^2 + \frac{mR\omega^2}{3}$$

$$T = \frac{4}{3}mR\omega^2$$

Sol 43: Given Normal acceleration $a_n = Kt^2$

But we know that $ma_n = \frac{mv^2}{R}$

$\therefore \frac{v^2}{R} = Kt^2$

$v = \sqrt{KR}\,t$ ... (i)

$$\frac{dv}{dt} = \sqrt{KR}$$

Tangential force $= m.\frac{dv}{dt} = (\sqrt{KR})m = m\sqrt{KR}$

Total force $= m|\vec{a}|$

$\vec{a} = \vec{a}_n + \vec{a}_t$

$$|\vec{a}| = \sqrt{a_n^2 + a_t^2} = \sqrt{(Kt^2)^2 + (\sqrt{KR})^2}$$

Total force $= m.|\vec{a}| = m\sqrt{K(R + Kt^4)}$

Now we know that work done by normal force in a circular motion is zero

$\therefore \omega_N = 0$

Now only work is done by tangential force

$(m.\sqrt{KR})ds$ ... (i)

We know that

$$\text{Power} = \frac{d\omega}{dt} \equiv \frac{d\omega}{ds} \cdot \frac{ds}{dt}$$

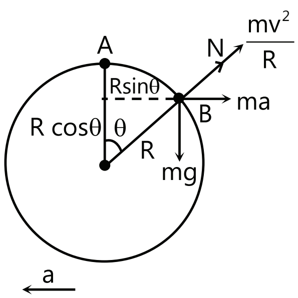
$$P = v.\frac{d\omega}{ds} = \sqrt{KR} \times t \times m\sqrt{KR}$$

$$P = mKRt$$

$$\text{Avg power} = \frac{\int_0^t mKRt\,dt}{\int_0^t dt}$$

$$P_{avg} = \frac{1}{2} mKRt$$

Sol 44:



In this case, there will be a pseudo force acting on the body. Now we use Work-Energy theorem, i.e. work done by all the forces is equal to change in kinetic energy. We know that, work done by normal force and centripetal force is zero

Work done by pseudo force = ma. (R sin θ)

$W_{PF} = maR\sin\theta$

Work done by gravitational force = mg(R – Rcos θ)

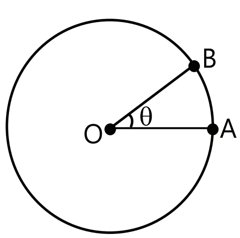
$W_{mg} = mgR(1 - \cos\theta)$

Net work done = maR sinθ + mgR (1–cos θ)

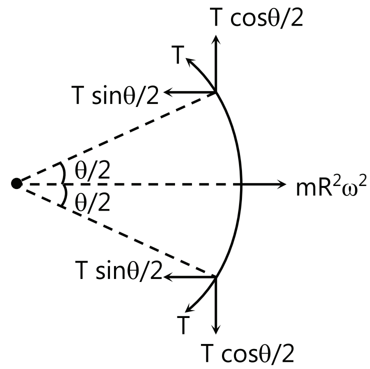
$$\equiv \frac{1}{2}mv^2 = Rm(a\sin\theta + g(1 - \cos\theta))$$

$$v = \sqrt{2R(a\sin\theta + g(1-\cos\theta))}$$

Sol 45:



Let us consider the part OAB;



m is the mass of the part OAB.

$$\Rightarrow 2T\sin\frac{\theta}{2} = mR^2\omega^2$$

now for small values of θ; sinθ = θ;

$$2T.\left(\frac{\theta}{2}\right) = mR\omega^2$$

$T\theta = mRw^2$ ... (i)

Now m = (λ)(Length) = (λ). Rθ

$\therefore T\theta = (\lambda R\theta)R\omega^2$

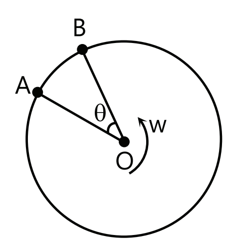
$T = \lambda R^2\omega^2$

Sol 46: $\vec{a}_{net} = \vec{a}_{radial} + \vec{a}_{tangential}$

$\vec{a}_r = \frac{v^2}{R}.(-\hat{e}_r)$; $\vec{a}_t = a(\hat{e}_t)$

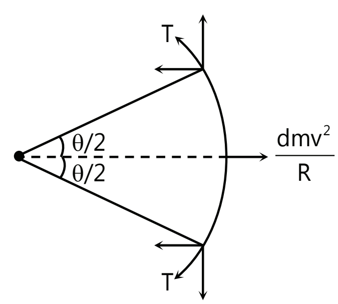
$$|\vec{a}_{net}| = \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2} \text{ m/s}^2$$

Sol 47:



Consider the part OAB;

Let the mass of this strip be 'dm'



$$2T\sin\left(\frac{\theta}{2}\right) = \frac{dm.v^2}{R}$$

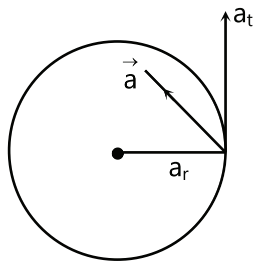
For very small values of θ; $\sin\theta \approx \theta$

$$\therefore 2T\left(\frac{\theta}{2}\right) = \frac{dm.v^2}{R}; \quad T.\theta = \frac{dm.v^2}{R}$$

$$\text{Now } dm = \frac{m}{2\pi R}.R.\theta = \left(\frac{m\theta}{2\pi}\right) \Rightarrow T\theta = \frac{m\theta}{2\pi}.\frac{v^2}{R}$$

$$T = \frac{mv^2}{2\pi R}$$

Sol 49:



$$\vec{a}_{net} = \vec{a}_r + \vec{a}_t$$

$$\vec{a}_r = \frac{v^2}{R}; \qquad \vec{a}_t = \frac{dv}{dt} = a$$

$$\vec{a}_{net} = \frac{v^2}{R}(-\hat{e}_r) + a(\hat{e}_t); \qquad |\vec{a}_{net}| = \sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

$$f = m|\vec{a}_{net}|$$

$\therefore$ Under static conditions

$$\mu mg = m\sqrt{a^2 + \left(\frac{v^2}{R}\right)^2}$$

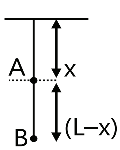
$$v = [(\mu^2 g^2 - a^2)R^2]^{1/4}$$

Exercise 2

Forces and Laws of Motion

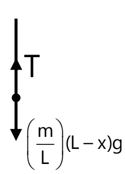
Single Correct Choice Type

Sol 1: (C)



λ (linear density) of chain $= \left(\frac{m}{L}\right)$

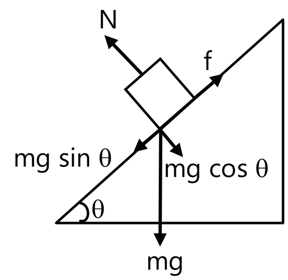
Now at point A;



The mass of the part AB of chain has to be supported by the rest of the chain.

$$T = \frac{m}{L}(L-x)g$$

Sol 2: (A) $mg\sin\theta = f$ and $N = mg\cos\theta$



For the condition of just sliding;

$f = \mu N$

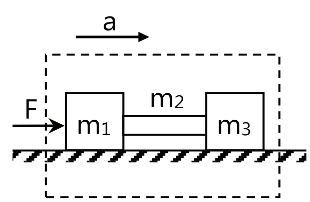
$\Rightarrow$ $mg\sin\theta = \mu\, mg\cos\theta$

$\Rightarrow$ $\tan\theta = \mu \Rightarrow \theta = \tan^{-1}(\mu)$.

Hence the angle of inclination has nothing to do with the mass of the body.

Here the angles are different because of the change in 'μ' from one block to another.

Sol 3: (A)

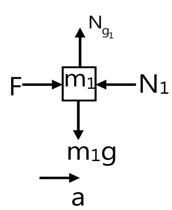


Now let us say the whole system moves with an acceleration 'a'.

$\therefore F = (m_1 + m_2 + m_3)\, a$ ... (i)

Let as consider Individual masses;

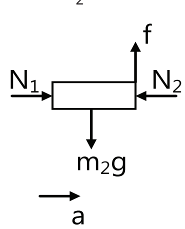
For m_1;



$F - N_1 = m_1 a$; ...(ii)

$N_{g_1} - m_1 g = 0$; ...(iii)

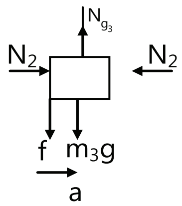
For m_2:



$N_1 - N_2 = m_2 a$; ...(iv)

$m_2 g - f = 0$; ...(v)

For m_3;



$N_2 = m_3a$; ...(vi)

$N_{g_3} = f + m_3g$; ...(vii)

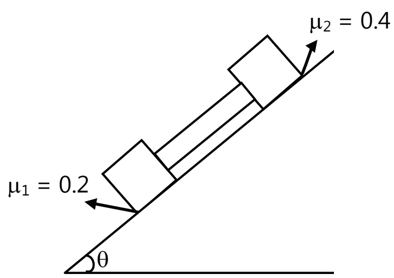
We know that $f_{max} = \mu N_2 = \mu m_3 a$

$f = m_2 g \le f_{max} \equiv \mu m_3 a$

$\Rightarrow a \ge \left(\dfrac{m_2 g}{\mu m_3}\right)$ ... (viii)

$\Rightarrow F \ge (m_1 + m_2 + m_3)\left(\dfrac{m_2 g}{\mu m_3}\right)$ (from (viii) & (i))

Sol 4: (A)



Here both the particles are constrained to move together. Hence $a_A = a_B$

Now let us first find the net force down the incline;

i. e $(m_1 + m_2)g \sin\theta$

$F_{net} = 340 \times 10 \times \dfrac{8}{17}$

$F_{net} = 1600$ N.

Now let us calculate the $f_{s_1} + f_{s_2}$

$f_{s_1} = \mu_1 . (m_1 g \cos\theta) = (0.2)(170 \times 10 \times \dfrac{15}{17}) = 300$ N

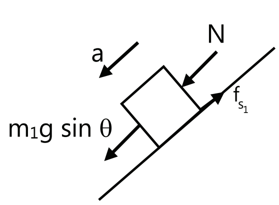
$f_{s_2} = \mu_2 . (m_2 g \cos\theta) = 0.4 (170 \times 10 \times \dfrac{15}{17}) = 600$ N

$\therefore f_{s_1} + f_{s_2} = 900$ N.

$\therefore$ Net Acceleration of the system

$= \left(\dfrac{1600 - 900}{170 + 170}\right) = \dfrac{700}{340}$ m/s^2 $\therefore a = \dfrac{35}{17}$ m/s^2

Now on A;



$m_1 g \sin\theta + N - f_{s_1} = m_1 a$

$N = m_1 a - m_1 g \sin\theta + f_{s_1}$

$N = 170\left(\dfrac{35}{17}\right) - 800 + 300$

$N = -150$ N

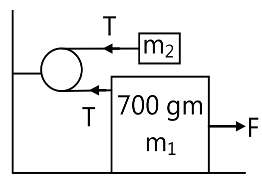
i.e. force in the bar is 150 N.

Sol 5: (A) Lift moving uniformly means lift is moving without any acceleration.

Hence in both the cases; acceleration of the coin is 'g'.

$\therefore t_1 = t_2$.

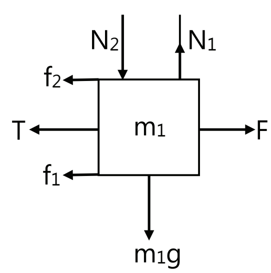
Sol 6: (B)



Now if m_1 moves with an acceleration 'a' towards right; m_2 will have an acceleration of 'a' towards left.

[∵ string constraint]

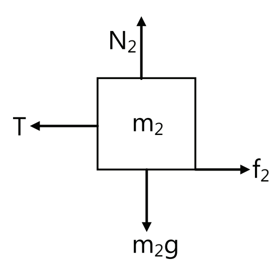
FBD of m_1;



$F - f_1 - f_2 - T = m_1 a$ ... (i)

$m_1 g + N_2 = N_1$ ... (ii)

FDB of m_2;



$N_2 - m_2 g = 0$ ... (iii)

$T - f_2 = m_2 a$ ... (iv)

$f_2 = \mu N_2 = \mu m_2 g$ ... (v)

$\therefore T = m_2 a + \mu m_2 g$

$T = (a + \mu g) m_2$ ... (vi)

$f_1 = \mu N_1 = \mu(m_1 g + m_2 g) = \mu g (m_1 + m_2)$

∴In equation (i)

$F - \mu g\,(m_1 + m_2) - \mu\, m_2 g - (a + \mu g)m_2 = m_1 a$

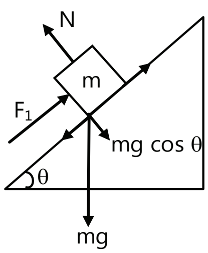
$\therefore\ F = (m_1 + m_2)\,a + 3\mu m_2 g + \mu m_1 g$

$\Rightarrow F = (m_1 + m_2)a + \mu g\,(m_1 + 3m_2)$

Put $a = 0.3\ m/s^2$ and $m_1 = 0.7$ kg, $m_2 = 0.2$ kg to get the value of force.

Hence, we get F = 2.18 N

Sol 7: (A)



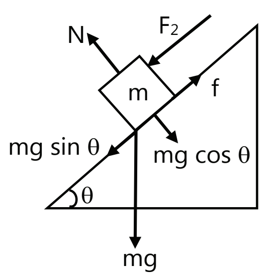
As force F tends to push the mass upwards, friction will tend to oppose it. So, it will act downwards.

$\therefore\ F = f + mg\sin\alpha$

$f = \mu N = \mu\ mg\cos\alpha$

$\Rightarrow F_1 = \mu\, mg\cos\alpha + mg\sin\alpha$ ... (i)

Now when pushing downwards, friction will be acting upwards,



$\therefore\ F_2 + f = mg\sin\theta$

$F_2 = mg\sin\theta - f$

$f = \mu\, mg\cos\theta$

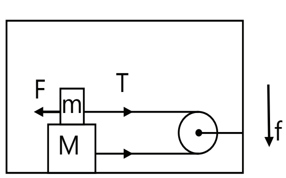
$\Rightarrow F_2 = mg\sin\theta - \mu\, mg\cos\theta$ ... (ii)

Given that $F_1 = nF_2$

$\therefore\ \mu mg\cos\theta + mg\sin\theta = n(mg\sin\theta - \mu mg\cos\theta)$

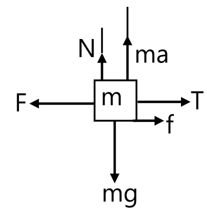
$$\Rightarrow\ \mu = \frac{n-1}{n+1}\tan\theta$$

Sol 8: (B)



For maximum force, F; the friction on 'M' will be towards Right.

∴ FBD of m;



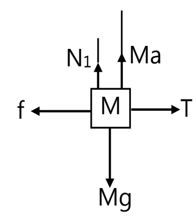
$F - T - f = 0$

$N + ma = mg$

$\Rightarrow F = f + T$ ... (i)

$N = mg - ma$ ... (ii)

∴ FBD of M;



$T - f = 0$ ... (iii)

$N_1 + Ma = Mg$ ... (iv)

From (i) and (iii);

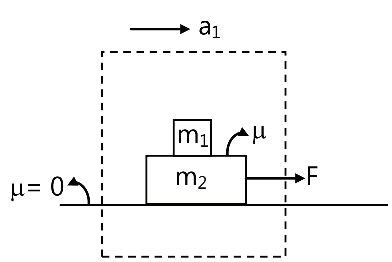
$\Rightarrow F = f + f$

$\Rightarrow F = 2f\ ;\ f = \mu N = \mu(mg - ma)$

$F = \mu m\,(g - a)$

$\Rightarrow F = 2\,\mu m\,(g - a).$

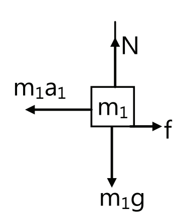
Sol 9: (B)



Let us say the whole system moves with an acceleration a_1.

$$\therefore\ a_1 = \left(\frac{F}{m_1 + m_2}\right) \quad \text{... (i)}$$

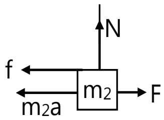
Now FBD of m_1;



$m_1a_1 - f = 0$

$\Rightarrow m_1a_1 = f$ ... (ii)

FBD of m_2



$F - f - m_2a = 0$

$\Rightarrow F = m_2a + f$ ... (iii)

Now when the mass m_1 just tends to slide;

$f = \mu N = \mu m_1g$

$\therefore m_1a_1 = \mu m_1g$ (from (ii))

$\therefore a_1 = \mu g$.

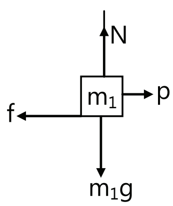
Now from (i)

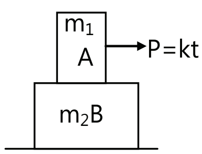
$F = (m_1 + m_2)\mu g$

$at = (m_1 + m_2)\mu g$

$$t = \frac{(m_1 + m_2)\mu g}{a}$$

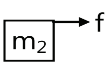
Sol 10: (C)





$p - f = m_1a$ ... (i)

$N = m_1g$ ... (ii)



$f = m_2a$ ... (iii)

Now for $f \le \mu m_1g$;

Both the block will move together;

$\therefore$ Adding (i) and (iii);

$P = (m_1 + m_2)a$.

$$a = \left(\frac{P}{m_1 + m_2}\right) = \left(\frac{k}{m_1 + m_2}\right)t$$

Now for $f = \mu m_1g$; this is the maximum frictional force;

$\therefore f = m_2a_2$

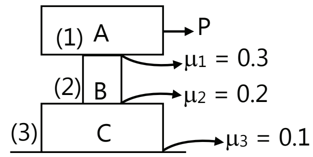
$\mu m_1g = m_2a_2$

$$\Rightarrow a_2 = \frac{\mu m_1 g}{m_2} \text{ which is constant}$$

And $P - f = m_1a \quad \Rightarrow P - \mu mg = m_1a$

$$a_1 = \frac{kt - \mu m_1 g}{m_1}$$

Sol 11: (B) Now let us check the limiting frictions between the three surfaces



$f_{s_1} = \mu_1(m_A\, g) = 90$ N

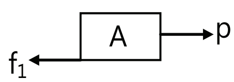
$f_{s_2} = \mu_2(m_A + m_B)g = 80$ N

$f_{s_3} = \mu_3(m_A + m_B + m_C)\, g = 60$ N.

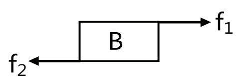
$\therefore$ Now let us assume P would be greater than 60 N and less than 80 N.

For this P;

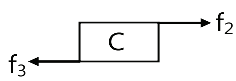
$f_1 = P \qquad [\because f_1 < f_{s_1} \equiv 90]$



$f_1 = f_2 = P \qquad [\because f_2 < f_{s_2} \equiv 80]$



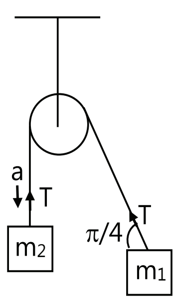
Now $f_2 - f_3 = m_3a$



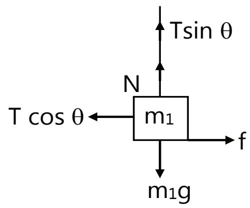
$[\because f_2 = P > 60 \equiv f_{s_3}]$

$\therefore$ Here f_2 is greater than the maximum static friction between C and ground. Hence the block C will slide on the ground. There by all the three blocks will slide for a minimum force of 60 N.

Sol 12: (A) $m_2g - T = m_2a_1$ ... (i)



On m_1;



$N + T \sin\theta = m_1g$

$T \cos\theta - f = m_2a_2$

Now for just initiating the motion;

$a_1 = a_2 = 0$

$\therefore\ m_2g - T = 0$ ... (i)

$T \cos\theta - f = 0$ ... (ii)

$m_2g \cos\theta = f$ ... (iii)

$f = \mu N = \mu(m_1g - T \sin\theta) = \mu(m_1g - m_2g \sin\theta)$

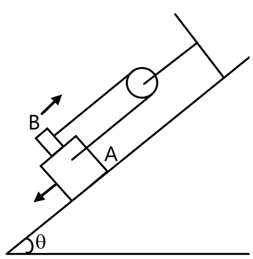
$\Rightarrow\ m_2g\cos\theta = \mu(m_1g - m_2g \sin\theta)$

$$m = \left(\frac{m_2 \cos\theta}{m_1 - m_2 \sin\theta}\right)$$

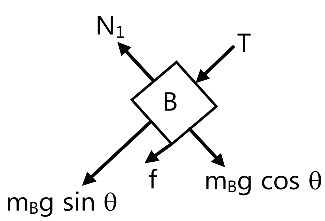
put $\theta = \pi/4$.

Multiple Correct Choice Type

Sol 14: (A, B)



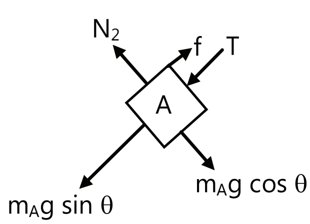
FBD of B;



$T - f - m_Bg \sin\theta = m_B a_B$ ... (i)

$N_1 = m_Bg \cos\theta$ ... (ii)

FBD of A;



$m_Ag \sin\theta - f - T = m_A a_A$ ... (iii)

$N_2 = m_Ag \cos\theta$ ... (iv)

$a_A = a_B$ (constraint equation)

By adding (i) and (iii)

$(m_A - m_B) g \sin\theta - 2f = (m_A + m_B) a$ ... (v)

For limiting condition, a= 0.

$\Rightarrow (m_A - m_B) g \sin\theta = 2f$

Here $f \le f_s = mN_1 = \mu\, m_B\, g \cos\theta$

$(m_A - m_B) g \sin\theta \le 2\, \mu\, m_B\, g \cos\theta$

$$\mu \ge \frac{m_A - m_B}{2m_B} \tan\theta$$

Now in equation (v)

If $m_1 = m_2$;

Tension itself balances both the masses.

So, no necessity for any friction.

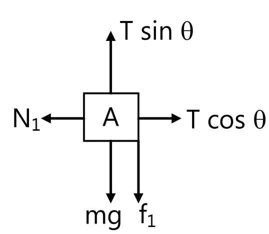
C: we cannot explicitly say that. We need more information on μ.

D: when $m_A = m_B$;

Put friction f = 0 in (i) and (iii)

And subtract them to get Tension 'T'.

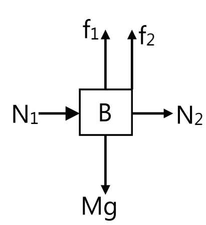
Sol 15: (A, D)



Acceleration of block will be zero. Since its constrained.

$\therefore\ mg + f_1 = T \sin\theta$ ... (i)

$N_1 = T \cos\theta$ ... (ii)



$Mg - f_1 - f_2 = Ma$ ... (iii)

$N_1 = N_2 = T \cos\theta$

$f_2 = \mu(N_2) = \mu\, T \cos\theta$

$f_2 = \mu N_1 = \mu T \cos\theta$

$\therefore$ From equation (i)

$mg + \mu T\cos\theta = T\sin\theta$

$mg = T(\sin\theta - \mu\cos\theta)$

$$T = \left(\frac{mg}{\sin\theta - \mu\cos\theta}\right) = \frac{100}{0.5 - 0.2\left(\frac{\sqrt{3}}{2}\right)}$$

T = 306 N.

Now using equation (iii)

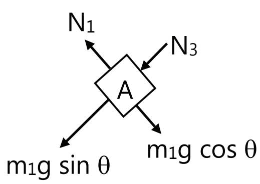
$Mg - f_1 - f_2 = Ma$

$Mg - 2\mu T\cos\theta = Ma$

$$a = g - \frac{2\mu T\cos\theta}{M}$$

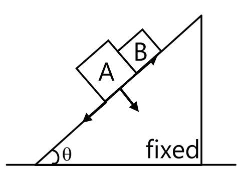
$a = 4.7\ m/s^2$

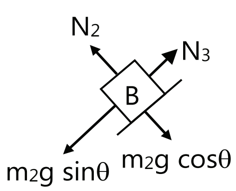
Sol 16: (A, C)



$m_1 g\sin\theta + N_3 = m_1 a_1$ ... (i)

$N_1 = m_1 g\cos\theta$ ... (ii)





$m_2 g\sin\theta - N_3 = m_2 a_2$ ... (iii)

$N_2 = m_2 g\cos\theta$ ... (iv)

now let us assume; $a_1 \neq a_2$, then;

Both of them will lose contact

$\therefore$ $N_3 = 0$.

But we then find $a_1 = a_2 = g\sin\theta$.

Hence both of them will have same acceleration.

Now putting $a_1 = a_2 = a$

And adding equation (i) and (iii);

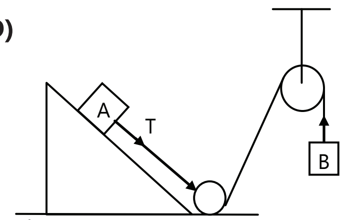
$$a = \frac{(m_1 + m_2)g\sin\theta}{(m_1 + m_2)}$$

$a = g\sin\theta$ ... (v)

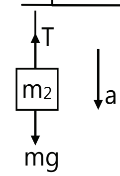
now using this we can find, N_3;

N_3 = zero, for all m_1 and m_2.

Sol 17: (B, D)

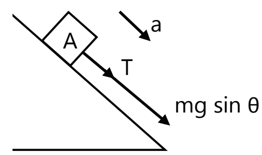


FBD of B;



$mg - T = ma$ ... (i)

FBD of A;



$mg\sin\theta + T = ma$ ... (ii)

By adding (i) and (ii)

$mg + mg\sin\theta = 2\,ma$

$$mg + \frac{mg}{2} = 2ma$$

$$\Rightarrow \boxed{a = \frac{3g}{4}}$$

Now using equation (i)

$mg - T = ma.$

$$T = mg - ma = mg - \frac{3mg}{4}$$

$$\boxed{T = \frac{mg}{4}}$$

Assertion Reasoning Type

Sol 18: (A) Conceptual. Conservation of linear momentum for a single particle do mean that the state of the body is conserved or constant unless an external force acts on the body.

Sol 19: (D) Assertion: If the force is non-constant and reverses itself over time, it can give a zero impulse.

For example: spring force would give a zero impulse over one period of oscillation.

Sol 20: (D) Here; weight of the book is because of the Gravitational Attraction Between earth and book. There will also be a gravitational force between book and table, which is very small, hence always neglected.

That Gravitational force between table and book form an Action-Reaction pair.

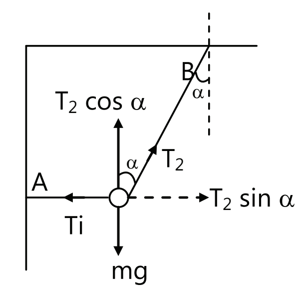
Sol 21: (A) Both assertion and reason are statements of Newton's laws.

Sol 22: (A) Momentum $= m\vec{u}$.

We have to specify reference frame, because velocities will vary in different frames. So, momentum which implicitly depends on velocity might also very.

Comprehension Type

Paragraph 1:

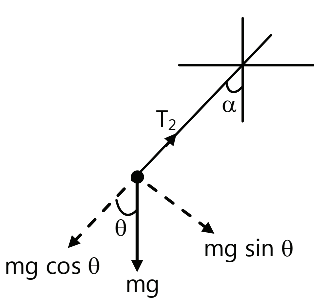


$T_2 \cos\alpha = mg$; $T_2 \sin\alpha = T_1$

$\Rightarrow T_2 = mg \sec\alpha$; $T_1 = mg \tan\alpha$

Now just after the string AB is cut;

$T_2 = mg \cos\theta$



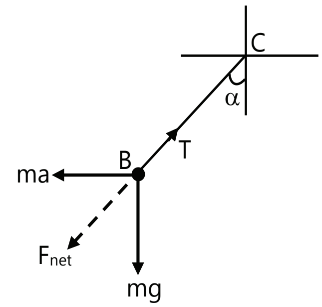
Now when string BC is cut;

Mass 'm' will just have force fall. Hence tension in string AB is zero.

Now suppose it is keep in a moving automobile;

In automobile's frame of reference, there is a pseudo force acting on the mass.

The resultant force should be along BC.



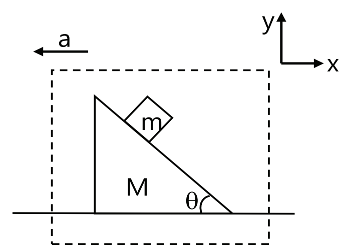
$\therefore \frac{ma}{mg} = \tan\alpha \Rightarrow a = g \tan a$

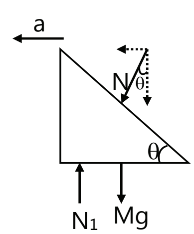
Since its acting leftwards, the vehicle should move rightwards.

Paragraph 2:

In s' frame;

FBD of M.

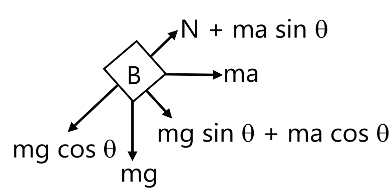




$N \sin\theta = Ma$ ... (i)

$N \cos\theta + Mg = N_1$ ... (ii)

FBD of m;



$N + ma \sin\theta = mg \cos\theta$ ... (iii)

$mg \sin\theta + ma \cos\theta = ma'$ ... (iv)

from (i) and (iii);

$N = mg \cos\theta - mg \sin\theta$

Now in equation (i)

$N \sin\theta = Ma$

$(mg \cos\theta - ma \sin\theta) \sin\theta = Ma$

$a = \left(\dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}\right)$

Since its in –ve x direction; we add a '–' sign.

$\therefore\ a = -\left(\dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}\right)$

Now using this value of a, solving equation (iv);

We get

$mg\sin\theta + m\left(\dfrac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}\right)\cos\theta = ma'$

$\Rightarrow\ g\sin\theta + \dfrac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} = a'$

$\Rightarrow\ \dfrac{Mg\sin\theta + mg\sin^3\theta + mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} = a'$

$\Rightarrow\ a' = \dfrac{Mg\sin\theta + mg\sin\theta(\sin^2\theta + \cos^2\theta)}{M + m\sin^2\theta}$

$a' = \dfrac{Mg\sin\theta + mg\sin\theta}{M + m\sin^2\theta}$ $\qquad \therefore\ a' = \dfrac{(M+m)g\sin\theta}{M + m\sin^2\theta}$

This is the acceleration of the block 'm' with respect to the incline.

Force exerted by the mass 'm' on wedge is 'N'.

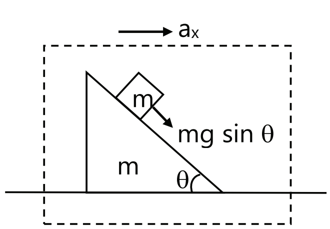
We can find this by; equation (iii)

$\therefore\ N + ma\sin\theta = mg\cos\theta$

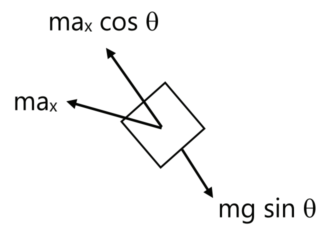
$\Rightarrow N = mg\cos\theta - ma\sin\theta$

$= mg\cos\theta - m\left(\dfrac{mg\sin^2\theta\cos\theta}{M + m\sin^2\theta}\right)$

$N = \left(\dfrac{Mmg}{M + m\sin^2\theta}\right)$



Now in this question; the downward component of mg sin θ has to be balanced.



$\therefore\ ma_x\cos\theta = mg\sin\theta$

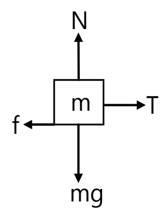
$a_x = g\tan\theta$ in positive x direction,

Paragraph 3:

Given that the plank has very rough surface.

$\mu >> 0$

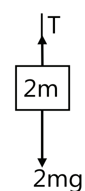
$\therefore$FBD of A;



$T - f = 0$ ... (i) $\Leftrightarrow T = f$

$mg - N = 0$ ... (ii) $\Leftrightarrow N = mg$

FBD of B;



$T - 2mg = 0$ ... (iii) $\Leftrightarrow T = 2mg$

$\therefore\ f = 2mg$

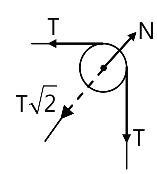
Net Contact force acting between block A and plank;

is $\sqrt{N^2 + f^2} = \sqrt{(mg)^2 + (2mg)^2}$

$F = mg\sqrt{5}$

On the pulley;

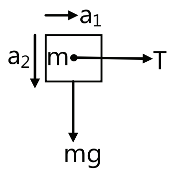
$N = T\sqrt{2} = 2\sqrt{2}$ mg.



Now just after this instant;

Normal reaction becomes zero.

On body A;

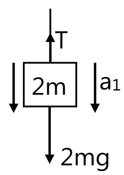


$T = ma_1$ ... (i)

$mg = ma_2$.

$\Rightarrow \boxed{a_2 = g}$

on body B



$2mg - T = 2ma_1$ ... (ii)

From (i) and (ii)

$a_1 = \frac{2g}{3} = 6.66 \text{ m/s}^2$

Now $a_A = a_1(\hat{i}) + a_2(-\hat{j})$

$= \frac{-2g}{3}\hat{i} - g\hat{j}$

$|a_A| = \sqrt{\frac{4g^2}{9} + g^2}$

$|a_A| = \frac{\sqrt{13}}{3} g$

$|a_A| = 12 \text{ m/s}^2$

Paragraph 4:

Buoyant force $= \rho g V_{imm} = \rho g v$

$F = (0.9)(10^3).(10)(0.2 \times 1 \times 1 \times \times 10^{-2})$

$F_B = 18$ N.

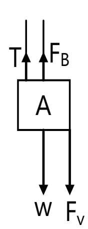
Now $T + F_B = W + F_v$.

$T = W + F_v - F_B$

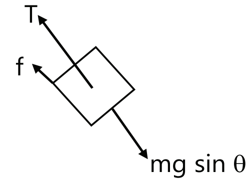
$F_v = 60$ N.

$T = 48 + 60 - 18$

$T = 90$ N.



Now for this the acceleration of the block should be zero.



$mg \sin\theta = f + T$

$120 - 90 = f$

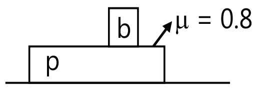
$f = 30$ N

$\mu(mg\cos\theta) = 30$

$\mu(8\sqrt{3}.10.\frac{1}{2}) = 30$

$\mu = \frac{\sqrt{3}}{4}$.

Sol 38: (B)

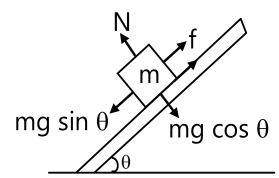


At an angle θ;

$mg\sin\theta - f = 0$

$N - mg\cos\theta = 0$

Now $f_{max} = f_s = \mu N$.



At this point the block starts sliding.

$\therefore\ f_{max} = \mu\, mg\cos\theta$

$\therefore\ mg\sin\theta - \mu\, mg\cos\theta = 0$

$\Rightarrow\ \tan\theta = \mu$

$\Rightarrow\ \theta = \tan^{-1}(0.8)$

$\theta = 40^o$

Now till this angle; $f = mg\sin\theta$

$\therefore$ for $\theta = 30^o$,

$f = mg/2$

Now for $\theta = 45^o$, let us say body is not sliding $mg\sin\theta - f = 0$

$N = mg\cos\theta$

$f_s = \mu mg\cos\theta = \mu mg/\sqrt{2} = 0.8\,(mg/\sqrt{2})$

$f = mg\sin\theta = \left(\frac{mg}{\sqrt{2}}\right)$

But for our assumption;

$f \le f_s$

$\Rightarrow \left(\frac{mg}{\sqrt{2}}\right) \le (0.8)\left(\frac{mg}{\sqrt{2}}\right)$

which is not true.

Hence the body would have started sliding

$f = f_s = \mu N = \mu mg \cos\theta = \mu mg/\sqrt{2}$

Circular Dynamics

Sol 39: (A) Centripetal acceleration = $r\omega^2\left(\text{or}\,\frac{v^2}{r}\right)$

Given that both have same periods.

So $\omega_1 = \omega_2$

$a_1 = R\omega^2 \qquad a^2 = r\omega^2$

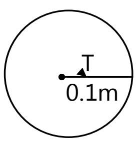
$\frac{a_1}{a_2} = \frac{R}{r}$

Sol 40: (A) Max Tension the string can sustain

$T_{max} = 10$ N.

Mass of the sto
ne = 250 gm = $\frac{1}{4}$ kg

Length of string = 10 cm = 0.1 m



$T = mr\omega^2$

$T_{max} = mr\,\omega_{max}^2$

$\omega_{max} = \sqrt{\frac{T_{max}}{mr}}$

$\omega_{max} = \sqrt{\frac{10}{\frac{1}{4}\times 0.1}} = \sqrt{400}$ rad/s

$w_{max} = 20$ rad/s.

Sol 41: (D) Already discussed in Q. 40 So try this yourself

Sol 42: (D) Let the angular speed of the thread is ω.

For particle C $\qquad T_3 = m\omega^2 3l$

For particle B $\qquad T_2 \to T_3 = m\omega^2 2l \to T_2 = m\omega^2 5l$

For particle A $\qquad T_1 \to T_2 = m\omega^2 l \to T_1 = m\omega^2 6l$

Sol 43: (B) Kinetic energy $k = \frac{1}{2}mv^2$

But given that $k = as^2$

$\therefore \frac{1}{2}mv^2 = as^2$

$v = \sqrt{\frac{2a}{m}}\,s$

Now $\vec{a} = \vec{a}_r + \vec{a}_t$

$\vec{a}_r = \frac{v^2}{R} = \frac{2as^2}{mR}$

$\vec{a}_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt}$

$\vec{a}_t = v.\frac{dv}{ds}$

$\frac{dv}{ds} = \sqrt{\frac{2a}{m}}$

$\Rightarrow \vec{a}_t = \frac{2as}{m}$

$\vec{a} = \vec{a}_r + \vec{a}_t$

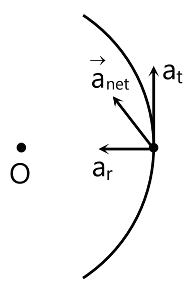
$|a| = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2}$

$|a| = \frac{2as}{m}\sqrt{1+\frac{s^2}{R^2}}$

$|\vec{F}| = m\,|a| = 2as\sqrt{1+\frac{s^2}{R^2}}$

Multiple Correct Choice Type

Sol 44: (B, C)



Given speed = v; and $\frac{dv}{dt} = a$

$\vec{a}_r = \frac{v^2}{r}\hat{e}_r$

$\vec{a}_t = a\,\hat{e}_t$

$\vec{a}_{net} = \vec{a}_r + \vec{a}_t$

$\vec{a}_{net} = \frac{v^2}{r}\hat{e}_r + a\,\hat{e}_t$

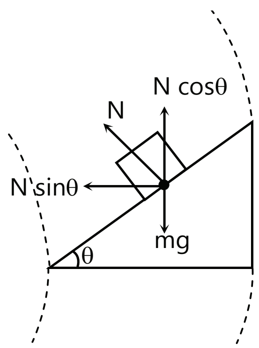
$|\vec{a}_{net}| = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$

Now friction force $f = m\,\vec{a}_{net}$

$$f = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$$

$$f = \sqrt{\left(\frac{mv^2}{r}\right)^2 + (ma)^2} \quad \text{and} \quad f = \mu mg$$

Sol 45: (B, D)



Since $\mu \approx 0$, there would be no frictional force.

$N\cos\theta = mg$

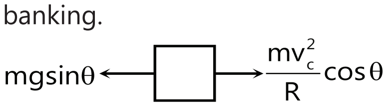
$N\sin\theta = \frac{mv^2}{R}$

$$\Rightarrow N = \sqrt{\left(\frac{mv^2}{R}\right)^2 + (mg)^2}$$

$\therefore$ N > mg as well as $N > \frac{mv^2}{R}$

Now when speed of the car is less than v_c = 40 km/hr and if we consider the frame of car;

Both these forces are made equal through proper banking.

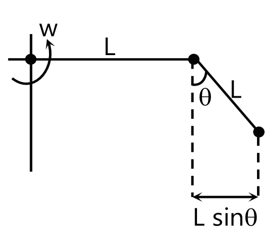


Now if $v < v_{critical}$,

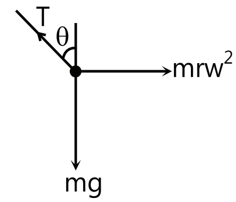
Then $mg\sin\theta > \frac{mv^2}{R}\cos\theta$

$\Rightarrow$ It slips downwards.

Sol 46: (A, B, C)



Free body diagram of mass m;



Resolving into components

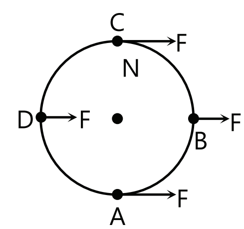
$T\cos\theta = mg$

$T\sin\theta = mr\omega^2$; $r = L + L\sin\theta$

$\Rightarrow T\sin\theta = m\omega^2 L(1 + \sin\theta)$

$$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{\omega^2 L(1+\sin\theta)}{g}$$

Sol 47: (B, D)



Consider the figure, with force F on the particle at different instants of time.

So it is evident that there should be some other forces such that particle will have uniform circular motion

$\therefore \vec{F} + \vec{F}_2 = m\vec{a}$

Since it's a uniform circular motion

$\vec{a}_t = 0$

$\therefore \vec{a} = \vec{a}_r = \frac{v^2}{R}$

$\therefore \vec{F} + \vec{F}_2 = \left(\frac{mv^2}{R}\right)$

Now resultant of both the forces $\vec{F}$ and $\vec{F}_2$ is $\frac{mv^2}{r}$ which in turn keeps changing both in direction as well as magnitude.

$\therefore \vec{F}_2 = \frac{mv^2}{R}\hat{e}_r - \vec{F}$

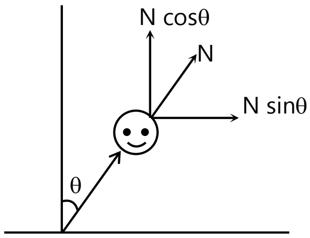
Angle between $\hat{e}_r$ and $\vec{F}$ keeps varying.

Assertion Reasoning Type

So 48: (D) Concept of centrifugal force comes into picture only in a non-inertial frame. So, both of them cannot co-exist in a same frame.

Although it is true that they are equal and opposite they can't cancel each other because of this.

Sol 49: (C)



$N \sin \theta = \frac{mv^2}{R}$

$N \cos \theta = mg$

It is not the friction between the tyres that provide him centripetal force, but it is component of Normal force.

Sol 50: (B) From the above solution;

We can write $N \sin\theta = \frac{mv^2}{R}$

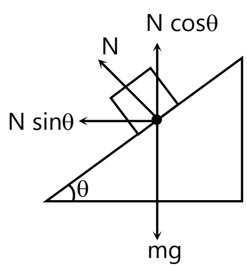
$N \cos \theta = mg$

Now when v is doubled, $\frac{m(2v)^2}{R} = 4. \frac{mv^2}{R}$

$\therefore$ Tendency is quadrupled

And also $\tan \theta = \left(\frac{v^2}{rg}\right)$ as $v \uparrow \theta \uparrow$

Sol 51: (E)



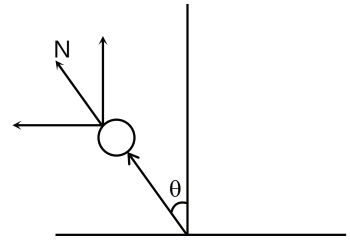
$N \sin \theta = \frac{mv^2}{R}$

Horizontal component of normal force provides the centripetal force. Hence false.

Reason:-

A curved path need not always be circular path. In case of elliptical paths, the force is not necessarily centripetal.

Sol 52: (B)

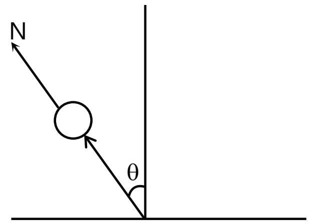


$N \cos\theta = mg$

$N \sin\theta = \frac{mv^2}{R}$

So bending inwards is always essential. He does it so as to get horizontal component of normal force as centripetal force. Although bending lowers his center of gravity, it's not the reason.

Sol 53: (A)



$N \cos\theta = mg$

$N \sin\theta = \frac{mv^2}{R}$

$\tan \theta = \frac{v^2}{Rg}$

when velocity is doubled

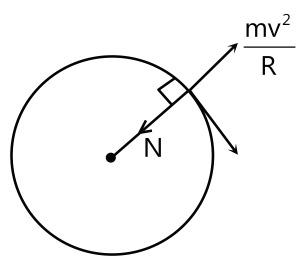
$\tan \theta_f = \frac{(2v)^2}{Rg} = 4. \frac{V^2}{Rg}$

Hence skidding tendency is quadrupled.

Sol 54: (D) Assertion is explained in Q. 46 and Reason is true (It is conceptual)

Comprehension Type

Paragraph 1:



At any instant, say speed is v. Normal force against wall,

$N = \frac{mv^2}{R}$

Now frictional force, f = μN

$F = \frac{\mu mv^2}{R}(-\hat{e}_t)$ [tangential]

And tangential acceleration say $\vec{a}_t$

Now $m\vec{a}_t = \frac{\mu mv^2}{R}(-\hat{e}_t)$

$\vec{a}_t = \frac{\mu v^2}{R}(-\hat{e}_t)$

and also $\vec{a}_t = \frac{dv}{dt}$

$\frac{dv}{dt} = -\frac{\mu v^2}{R}$

$\frac{dv}{v^2} = -\frac{\mu}{R}.dt$

Integrating both sides

$\int_{v_0}^{v}\frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt \Rightarrow -\left[\frac{1}{v}\right]_{v_0}^{v} = -\frac{\mu t}{R}$

$\frac{1}{v} = \frac{1}{v_0} + \frac{\mu t}{R}$

Sol 57: (B) $a_t = -\frac{\mu v^2}{R}$

$a_t = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$

$\Rightarrow v\frac{dv}{dx} = -\frac{\mu v^2}{R}$

$\frac{dv}{v} = -\frac{\mu}{R}dx$

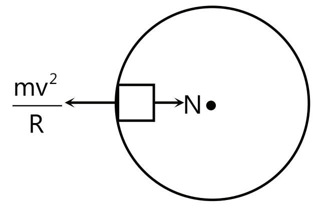
Integrating both sides;

$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\mu}{R}\int_0^x dx$

$\ln\left(\frac{v}{v_0}\right) = -\frac{\mu x}{R}$

$v = v_0 e^{-\frac{\mu x}{R}}$

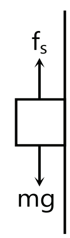
Paragraph 2:



Top view of the rotor

$N = \frac{mv^2}{R}$; $f = \mu N = \frac{\mu.mv^2}{R}$

For equilibrium;



$f_s = mg$

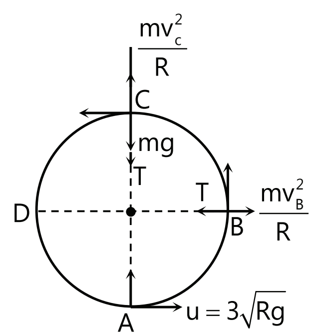
$\mu N = mg$

And this is $\frac{\mu.mv^2}{R} = mg$

$v = \sqrt{\frac{Rg}{\mu}}$

Match the Columns

Sol 61:



At point B;

$T_B = \frac{mv_B^2}{R}$

And also total energy at point A;

$E = \frac{1}{2}m(u)^2 + U_A$

Now assume ground at the point A itself

$\therefore U_A = 0$

$E_A = \frac{1}{2}m(9Rg) = \frac{9mRg}{2}$

And total energy at point B;

$E_B = \frac{1}{2}m(v_B^2) + mg(R)$

According to conversation of energy

$E_A = E_B$

$\therefore \frac{1}{2}mv_B^2 + mgR = \frac{9mgR}{2}$

$\frac{1}{2}mv_B^2 = \frac{7}{2}mgR$

$v_B = \sqrt{7gR}$

and $T_B = \frac{mv_B^2}{R} = 7mg$

for point C;

$T_c + mg = \frac{mv_c^2}{R}$

$T_c = \frac{mv_c^2}{R} - mg$

Total energy at point C is

$E_c = \frac{1}{2}mv_c^2 + mg(2R)$

$E_c = \frac{1}{2}mv_c^2 + 2mgR$

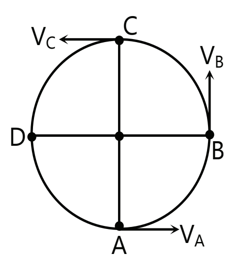
$E_c = E_A$

$\Rightarrow \frac{1}{2}mv_c^2 + 2mgR = \frac{9mgR}{2}$

$\frac{mv_c^2}{2} = \frac{5mgR}{2} \Rightarrow v_c = \sqrt{5gR}$

$\therefore T_c = \frac{mv_c^2}{R} - mg = 5mg - mg = 4\ mg$

Sol 62:



At point A; $T_A = mg + \frac{mv_A^2}{R}$ ... (i)

At point B; $T_B = \frac{mv_B^2}{R}$ ... (ii)

At point C; $T_C = \frac{mv_C^2}{R} - mg$ ... (iii)

Energy at point A = $\frac{1}{2}mv_A^2$ (point A is assumed to be ground)

$E_B = \frac{1}{2}mv_B^2 + mgR$

$E_C = \frac{1}{2}mv_C^2 + 2mgR$

Now given that $v_A = 10$

So; $E_A = E_B$ (using conservation of energy)

$\frac{1}{2}m(10)^2 = \frac{1}{2}m(v_B^2) + mg(1)$. [R = 1, v_A = 10, m = 1]

$v_B = \sqrt{80}$ m/s

and similarly

$E_A = E_C \Rightarrow \frac{1}{2}(10)^2 = \frac{1}{2}v_C^2 + g(2)$

$v_c = \sqrt{60}$ m/s

$\therefore$ from (i), (ii), (iii)

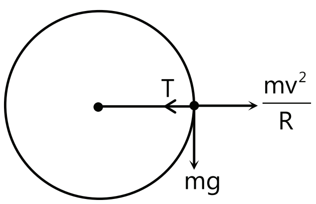
$T_A = 10 + 100 = 110$ N

$T_B = 80$ N

$T_C = 50$ N

$\therefore$ minimum tension is 50 N

When string is horizontal i.e. at point B;



$\vec{a}_r = \frac{v_B^2}{R} = 80\ m/s^2(-\hat{i})$

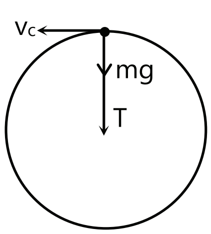
$\vec{a}_t = g = 10\ m/s^2(-\hat{j})$

$\vec{a}_{net} = \vec{a}_r + \vec{a}_t$

$|\vec{a}_{net}| = \sqrt{(80)^2 + (10)^2}$

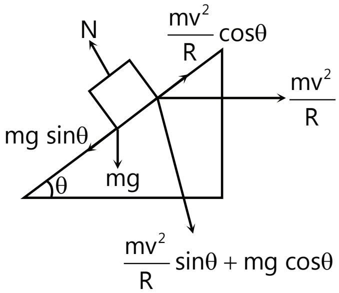
$|\vec{a}| = 10\sqrt{65}\ m/s^2$

At point C; tangential acceleration is zero



$\therefore \vec{a}_{net} = \vec{a}_r = \frac{v_c^2}{R} = 60\ m/s^2$

Sol 63:



$N = \frac{mv^2}{R}\sin\theta + mg\cos\theta$ ... (i)

Now depending on condition, friction can be upwards or downwards.

For maximum speed, friction is downwards.

$\therefore\ f = \frac{mv^2}{R}\cos\theta - mg\sin\theta$ ... (ii)

And also $f = \mu N$

$$\Rightarrow \mu\left(\frac{mv^2}{R}\sin\theta + mg\cos\theta\right) = \frac{mv^2}{R}\cos\theta - mg\sin\theta$$

$$v^2\left(\frac{\mu m\sin\theta}{R} - \frac{m\cos\theta}{R}\right) = -(mg\sin\theta + \mu mg\cos\theta)$$

$$v = \sqrt{\frac{mg(\sin\theta + \mu\cos\theta)}{\frac{m}{R}(-\mu\sin\theta + \cos\theta)}}$$

$$v_{max.} = \sqrt{\frac{Rg(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$$

And for the minimum speed;

friction will be acting upwards

$\therefore\ f = mg\sin\theta - \frac{mv^2}{R}\cos\theta$ ... (iii)

And following the same argument

for $f = 0$

$$\tan\theta = \left(\frac{v^2}{Rg}\right)$$

$v = \sqrt{Rg\tan\theta}$ ... (iv)

Previous Years' Questions

Forces and Laws of Motion

Sol 1: Maximum friction between A and B= $\mu m_A g$

or $f_1 = 0.25(3)(10) = 7.5$ N

Maximum friction between B and C = $\mu\,(m_A + m_B)g$

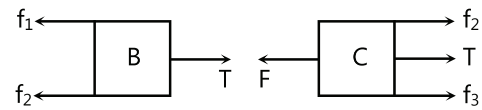
or $f_2 = 0.25\,(3 + 4)(10) = 17\ 5$ N

Maximum friction between C and ground

$f_3 = \mu\,(m_A + m_B + m_C)g$

$= 0.25(3 + 4 + 8)(10) = 37.5$ N

Block C and hence block B are moving in opposite directions with constant velocities and block A is at rest. Hence, net force no all three blocks should be zero. Free body diagrams have been shown below (Only horizontal forces are shown)



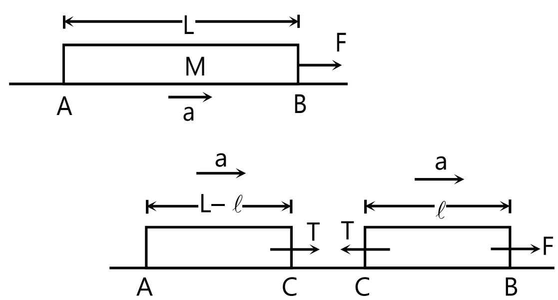
For equilibrium of B

$T = f_1 + f_2 = 25$ N

For equilibrium of C

$F = T + f_2 + f_3 = 80$ N

Sol 2: Acceleration of rope $a = \frac{F}{M}$



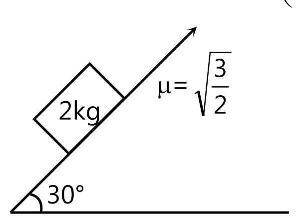
Now to find tension at point C, a distance ℓ from point B, we can write equation of motion of any one part (AC or CB), both moving with acceleration a.

Equation of motion of part AC is

T = (mass of AC) × (acceleration) = $\frac{M}{L}(L - \ell)\left(\frac{F}{M}\right)$

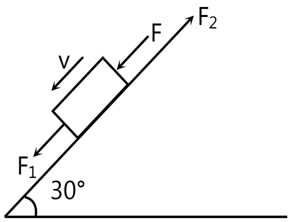
$= F\left(1 - \frac{\ell}{L}\right)$

So 3: $mg\sin\theta = (2)(10)\left(\frac{1}{2}\right) = 10\text{ N} = F_1$ (say)



$\mu mg\cos\theta = \left(\sqrt{\frac{3}{2}}\right)(2)(10)\left(\frac{\sqrt{3}}{2}\right) = 21.21\text{ N} = F_2$ (say)

(a) Force required to move the block down the plane with constant velocity.

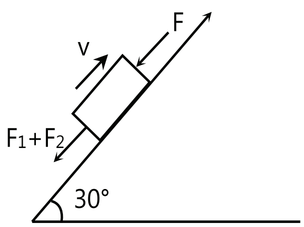


F_1 will be acting downwards, while F_2 upwards.

Since $F_2 > F_1$, force required

$F = F_2 - F_1 = 11.21$ N

(b) Force required to move the block up the plane with constant velocity.



F_1 and F_2 both will be acing downwards.

$F = F_1 + F_2 = 31.21$ N

Sol 4: Maximum force of friction between M_1 and inclined plane

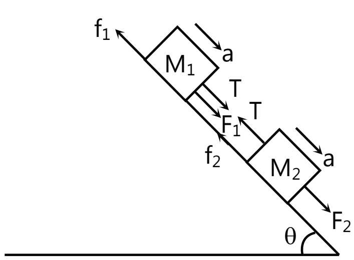
$f_1 = \mu_1 M_1 g\cos\theta = (0.75)(4)(9.8)(0.8) = 23.52$ N

$M_1 g\sin\theta = (4)(9.8)(0.6) = 23.52\text{ N} = F_1$ (say)

Maximum force of friction between M_2 and inclined plane

$f_2 = \mu_2 M_2 g\cos\theta = (0.25)(2)(9.8)(0.8) = 3.92$ N

$M_2 g\sin\theta = (2)(9.8)(0.6) = 11.76\text{ N} = F_2$ (say)



Both the blocks will be moving downwards with same acceleration a. Different forces acting on two blocks are as shown in figures.

Equation of motion of M_1

$T + F_1 - f_1 = M_1 a$ ... (i)

or $T = 4a$

Equation of motion M_2

$F_2 - T - f_2 = M_2 a$

or $7.84 - T = 2a$ ... (ii)

Solving eqs. (i) and (ii), we get

$a = 1.3\text{ m/s}^2$ and $T = 5.2$ N

Sol 5: Constant velocity means net acceleration of the system is zero. Or net pulling force on the system is zero. While calculating the pulling force, tension forces are not taken into consideration. Therefore,

(a) $M_1 g = M_2 g\sin 37° + \mu M_2 g\cos 37° + \mu M_3 g$

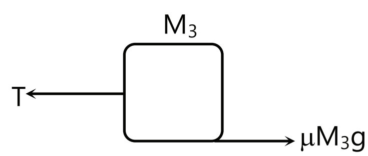
or $M_1 = M_2\sin 37° + \mu M_2\cos 37° + \mu M_3$

Substituting the values

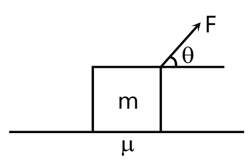
$M_1 = (4)\left(\frac{3}{5}\right) + (0.25)(4)\left(\frac{4}{5}\right) + (0.25)(4) = 4.2$ kg

(b) Since, M_3 is moving with uniform velocity

$T = \mu M_3 g = (0.25)(4)(9.8) = 9.8$ N



Sol 6:



Let F be applied at angle θ as shown in figure. Normal reaction in this case will be

$N = mg - F\sin\theta$

The limiting friction is therefore

$f_L = \mu N = \mu(mg - F\sin\theta)$

For the block to move,

$F\cos\theta = f_L = \mu(mg - F\sin\theta)$

or $F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$ ... (i)

For F to be minimum, denominator should be maximum.

or $\dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$

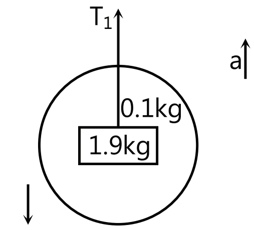
or $-\sin\theta + \mu\cos\theta = 0$

or $\tan\theta = \mu$ or $\theta = \tan^{-1}(\mu)$

Substituting this value of θ in Eq. (i), we get

$F_{min} = mg\sin\theta$

Sol 7: (a) To find tension at mid-point of the lower wire we cut the string at this point. Draw the free body diagram of lower portion.

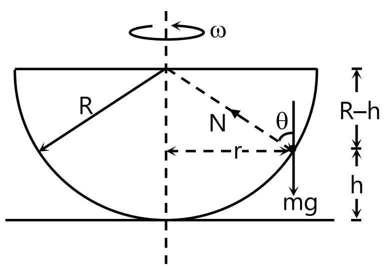


The equation of motion gives

$T_1 - 2.0g = (2.0)a$

or $T_1 = (2.0)(g + a) = (2.0)(9.8 + 0.2) = 20$ N

Sol 8: Given : R = 0 1m, m = 10^{-2} kg



(a) FBD of particle in ground frame of reference is shown in figure. Hence,

$\tan\theta = \dfrac{r}{R-h}$

$N\cos\theta = mg$ ... (i)

and $N\sin\theta = mr\omega^2$ ... (ii)

Dividing Eq. (ii) by Eq. (i), we obtain

$\tan\theta = \dfrac{r\omega^2}{g}$ or $\dfrac{r}{R-h} = \dfrac{r\omega^2}{g}$

or $\omega^2 = \dfrac{g}{R-h}$ ... (iii)

This is the desired relation between ω and h.

From Eq. (iii)

$h = R - \dfrac{g}{\omega^2}$

Form non-zero value of h

$R > \dfrac{g}{\omega^2}$ or $\omega > \sqrt{g/R}$

Therefore, minimum value of ω should be

$\omega_{min} = \sqrt{\dfrac{g}{R}} = \sqrt{\dfrac{9.8}{0.1}}$ rad/s

or $\omega_{min} = 9.89$ rad/s

(b) Eq. (iii) can be written as $h = R - \dfrac{g}{\omega^2}$

If R and ω are known precisely, then

$\Delta h = -\dfrac{\Delta g}{\omega^2}$ or $\Delta g = \omega^2\,\Delta h$ (neglecting the negative sign)

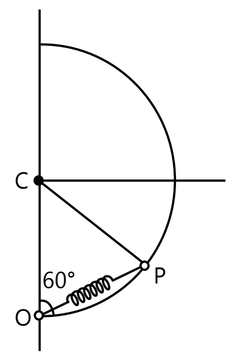
$(\Delta g)_{min} = (\omega_{min})^2\,\Delta h$, $(\Delta g)_{min} = 9.8\times10^{-3}$ m/s^2

Sol 9: (a) CP = CO = Radius of circle (R)

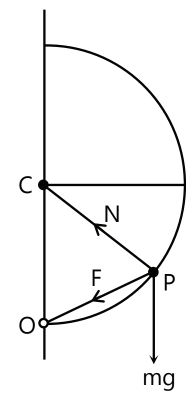
$\therefore \angle CPO = \angle POC = 60°$

$\therefore \angle OCP$ is also 60°

Therefore, ΔOCP is an equilateral triangle.



Hence, OP = R



Natural length of spring is 3R/4.

$\therefore$ Extension in the spring

$x = R - \dfrac{3R}{4} = \dfrac{R}{4}$

$\Rightarrow$ Spring force,

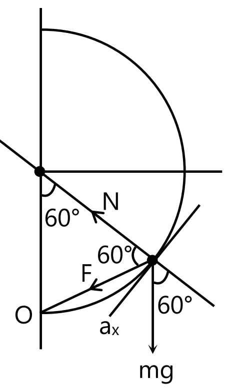
$F = kx = \left(\frac{m\,g}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$

The free body diagram of the ring will be as shown.

Here, $F = kx = \frac{mg}{4}$

and N = Normal reaction.

(b) **Tangential acceleration a_T** : The ring will move forwards the x-axis just after the release. So net force along x-axis



$F_x = F \sin 60° + mg \sin 60°$

$= \left(\frac{m\,g}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$

$F_x = \frac{5\sqrt{3}}{8}mg$

Therefore, tangential acceleration of the ring,

$a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$

Normal reaction N : Net force along y-axis on the ring just after the release will be zero.

$F_y = 0$

$\therefore N + F\cos 60° = mg \cos 60°$

$\therefore N = mg\cos 60° - F\cos 60°$

$= \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$

$N = \frac{3mg}{8}$

Sol 10: Given,

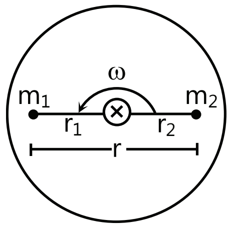
$m_1 = 10$ kg, $m_2 = 5$ kg, $\omega = 10$ rad/s

$r = 0.3$ m, $r_1 = 0.124$ m

$\therefore r_2 = r - r_1 = 0.176$ m

(a) Masses m_1 and m_2 are at rest with respect to rotating table.

Let f be the friction between mass m_1 and table.



Free body diagram of m_1 and m_2 with respect to ground

$m_1 \bullet\!\longrightarrow T + f \quad T \longleftarrow\!\bullet\, m_2$

$T = m_2 r_2 \omega^2$ ... (i)

Since, $m_2 r_2 \omega^2 < m_1 r_1 \omega^2$

Therefore, $m_1 r_1 \omega^2 > T$

and friction on m_1 will be inward (toward centre)

$f + T = m_1 r_1 \omega^2$ ... (ii)

from equations (i) and (ii), we get

$f = m_1 r_1 \omega^2 - m_2 r_2 \omega^2$ ... (iii)

$= (m_1 r_1 - m_2 r_2)\omega^2$

$= (10 \times 0.124 - 5\times 0.176)(10)^2$ N = 36 N

Therefore, frictional force on m_1 is 36 N (inwards)

(b) From eq. (iii)

$f = (m_1 r_1 - m_2 r_2)\,\omega^2$

Masses will start slipping when this force is greater than f_{max} or

$(m_1 r_1 - m_2 r_2)\omega^2 > f_{max} > \mu m_1 g$

$\therefore$ Minimum value of ω is

$\omega_{min} = \sqrt{\frac{\mu m_1 g}{m_1 r_1 - m_2 r_2}} = \sqrt{\frac{0.5\times 10\times 9.8}{10\times 0.124 - 5\times 0.176}}$

$\omega_{min} = 11.67$ rad/s

(c) From Eq. (iii), frictional force f = 0

where $m_1 r_1 = m_2 r_2$ or $\frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{5}{10} = \frac{1}{2}$

and $r = r_1 + r_2 = 0.3$ m

$\therefore r_1 = 0.1$ m and $r_2 = 0.2$ m

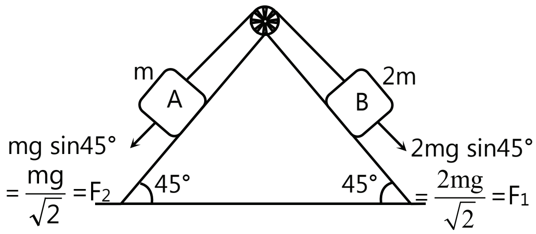
i. e. , mass m_2 should be placed at 0.2 m and m_1 at 0.1 m from the centre O.

Sol 11: Acceleration of block A

Maximum friction force that can be obtained at A is

$(f_{max})_A = \mu_A(mg \cos 45°)$

$= \frac{2}{3}(mg/\sqrt{2}) = \frac{\sqrt{2}mg}{3}$



Similarly,

$(f_{max})_B = \mu_B(2mg\cos 45°)$

$= \frac{1}{3}(2mg/\sqrt{2}) = \frac{\sqrt{2}mg}{3}$

Therefore, maximum value of friction that can be obtained on the system is

$$(f_{max}) = (f_{max})_A + (f_{max})_B = \frac{2\sqrt{2}mg}{3} \quad \text{... (i)}$$

Net pulling force on the ststem is

$$F = F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we can see that

Net pulling force < f_{max}

Therefore, the system will not move or the acceleration of block A will be zero.

(b) and (c) Tension in the string and friction at A

Net pulling force on the system (block A and B)

$F = F_1 - F_2 = mg/\sqrt{2}$

Therefore, total friction force on the blocks should also be equal to $\frac{mg}{\sqrt{2}}$

or $f_A + f_B = F = mg/\sqrt{2}$

Now since the blocks will start moving from block B first (if they move), therefore, f_B will reach its limiting value first and if still some force is needed, it will be provided by f_A

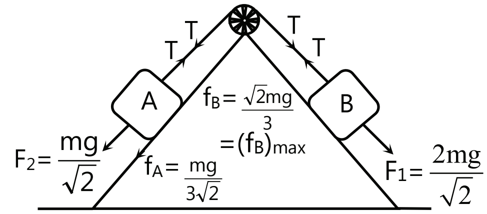
Here, $(f_{max})_B < F$

Therefore, f_B will be in its limiting value and rest will be provided by f_A.

Hence $f_B = (f_{max})_B = \frac{\sqrt{2}mg}{3}$

and $f_A = F - f_B = \frac{mg}{\sqrt{2}} - \frac{\sqrt{2}mg}{3} = \frac{mg}{3\sqrt{2}}$

The FBD of the whole system will be as shown in the figure



Therefore, friction on A is

$f_A = mg/3\sqrt{2}$ (down the plane)

Now for tension T in the string, we may consider either equilibrium of A or B

Equilibrium of A gives

$$T = F_2 + f_A = \frac{mg}{\sqrt{2}} + \frac{mg}{3\sqrt{2}} = \frac{4mg}{3\sqrt{2}} \text{ or } \frac{2\sqrt{2}mg}{3}$$

Similarly, equilibrium of B gives $T + f_B = F_1$

or $T = F_1 - f_B = \frac{2mg}{\sqrt{2}} - \frac{\sqrt{2}mg}{3} = \frac{4mg}{3\sqrt{2}}$

or $\frac{2\sqrt{2}mg}{3}$

Therefore, tension in the string is $\frac{2\sqrt{2}mg}{3}$

Sol 12: Acceleration of A down the plane,

$a_A = g\sin 45° - \mu_A g\cos 45°$

$= (10)\left(\frac{1}{\sqrt{2}}\right) - (0.2)(10)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ m/s}^2$

Similarly acceleration of B down the plane,

$a_B = g\sin 45° - \mu_B g\cos 45°$

$(10)\left(\frac{1}{\sqrt{2}}\right) - (0.3)(10)\left(\frac{1}{\sqrt{2}}\right) = 3.5\sqrt{2}\text{ m/s}^2$

The front face of A and B will come in a line when,

$s_A = s_B + \sqrt{2}$

or $\frac{1}{2}a_A t^2 = \frac{1}{2}a_B t^2 + \sqrt{2}$

$\frac{1}{2} \times 4\sqrt{2} \times t^2 = \frac{1}{2} \times 3.5\sqrt{2} \times t^2 + \sqrt{2}$

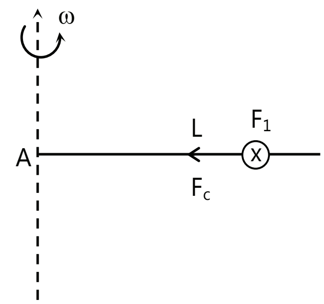
Solving this equation, we get $t = 2s$

Further, $s_A = \frac{1}{2}a_A t^2 = \frac{1}{2} \times 4\sqrt{2} \times (2)^2 = 8\sqrt{2}\text{ m}$

Hence, both the blocks will come in a line after A has travelled a distance $8\sqrt{2}$ m down the plane.

Circular Dynamics

Sol 13: (A) Tangential force (F_1) of the bead will be given by the normal reaction (N), while centripetal force (F_c) is provided by friction (f_r). The bead starts sliding when the centripetal force is just equal to the limiting friction.



F_t is inwards

Therefore, $F_t = ma = m\alpha L = N$

$\therefore$ Limiting value of friction

$(f_r)_{max} = \mu N = \mu m\alpha L$ ... (i)

Angular velocity at time t is $\omega = \alpha t$

$\therefore$ Centripetal force at time t will be

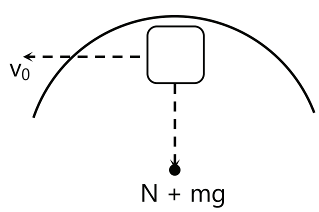
$F_c = mL\omega^2 = mL\alpha^2t^2$ ... (ii)

Equating equation (i) and (ii), we get $t = \sqrt{\dfrac{\mu}{\alpha}}$

For $t > \sqrt{\dfrac{\mu}{\alpha}}$, $F_c > (f_r)_{max}$ i.e. , the bead starts sliding.

In the figure F_t is perpendicular to the paper inwards.

Sol 14: (A) Since, the block rises to the same heights in all the four cases, from conservation of energy, speed of the block at highest point will be same in all four cases. Say it is v_0.



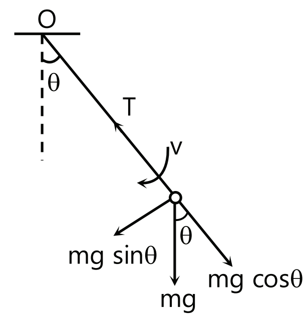
Equation of motion will be

$N + mg = \dfrac{mv_0^2}{R}$ or $N = \dfrac{mv_0^2}{R} - mg$

R (the radius of curvature) in first case is minimum. Therefore, normal reaction N will be maximum in first case.

Note In the question it should be mentioned that all the four tracks are frictionless. Otherwise, v_0 will be different in different tracks.

Sol 15: (B, C) Motion of pendulum is part of a circular motion. In circular motion it is better to resolve the forces in two perpendicular directions. First along radius (towards centre) and second along tangential. Along radius net force should be equal to $\dfrac{mv^2}{R}$ and along tangentitshouldbeequalto ma_T, where a_T is the tangential acceleration in the figure.



$T - Mg\cos\theta = \dfrac{mv^2}{L}$ and

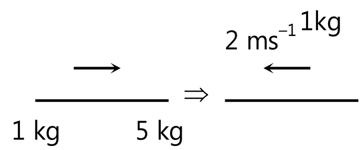
$Mg\sin\theta = Ma_T$

or $a_T = g\sin\theta$

$\therefore$ Correct options are (b) and (c).

Sol 16: (B, D) A rotating/revolving frame is accelerating and hence non-inertial. Therefore, correct options are (b) and (d).

Sol 17: (A, C)



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$-2 = \left(\frac{1-5}{1+5}\right)v_1 + 0 \qquad (\text{as } v_2 = 0)$$

$\therefore v_1 = 3\ ms^{-1}$

$$v_2' = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

$$= 0 + \left(\frac{2\times 1}{6}\right)(3) = 1\ ms^{-1}$$

$P_{CM} = P_i = (1)(3) = 3$ kg-m/s

$P_5' = (5)(1) = 5$ kg-m/s

$$K_{CM} = \frac{P_{CM}^2}{2M_{CM}} = \frac{9}{2\times 6} = 0.75\ J$$

$$K_{total} = \frac{1}{2} \times 1 \times (3)^2 = 4.5\ J$$

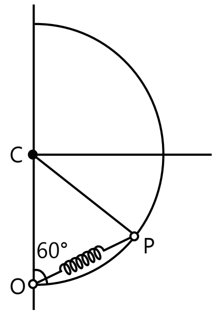
$\therefore$ Correct options are (a) and (c).

Sol 18: (B) (a) CP = CO = Radius of circle (R)

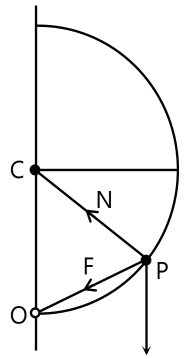
$\therefore \angle CPO = ?POC = 60°$

$\therefore$ $\angle OCP$ is also 60°

Therefore, ΔOCP is an equilateral triangle.



Hence, OP = R



Natural length of spring is 3R/4.

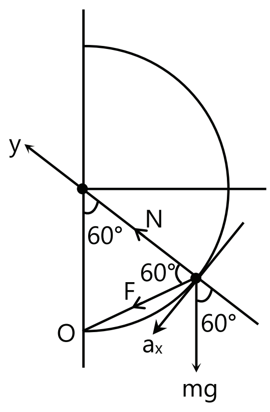
$\therefore$ Extension in the spring $x = R - \frac{3R}{4} = \frac{R}{4}$

$\Rightarrow$ Spring force, $F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$

The free body diagram of the ring will be as shown.

Here, $F = kx = \frac{mg}{4}$ and N = Normal reaction.

(b)Tangential acceleration a_T the ring will move forwards the x-axis just after the release. So, net force along x-axis



$F_x = F\sin 60° + mg\sin 60° = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$

$F_x = \frac{5\sqrt{3}}{8}$ mg

Therefore, tangential acceleration of the ring,

$a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$

Normal reaction N Net force along y-axis on the ring just after the release will be zero

$F_y = 0$

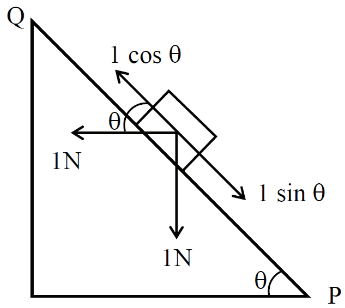
$\therefore N + F\cos 60° = mg\cos 60°$

$\therefore N = mg\cos 60° - F\cos 60°$

$= \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$

$N = \frac{3mg}{8}$

Sol 19: (A, C)



If $\theta = 45°$ then $\cos\theta = \sin\theta$ hence block will be at rest.

If plane is rough & $\theta > 45°$ then $\sin\theta > \cos\theta$ so friction will act up the plane

If plane is rough & $\theta < 45$ then $\cos\theta > \sin$ so friction will act down the plane so (A, C) are correct

Sol 20: (D) Initially bead is applying radially inward normal force.

During motion at an instant, N = 0, after that N will act radially outward.

Sol 21: (D) Condition for not sliding,

$f_{max} > (m_1 + m_2)\, g\sin\theta$

$\mu N > (m_1 + m_2)\, g\sin\theta$

$0.3\, m_2\, g\cos\theta \geq 30\sin\theta$

$6 \geq 30\tan\theta$

$1/5 \geq \tan\theta$

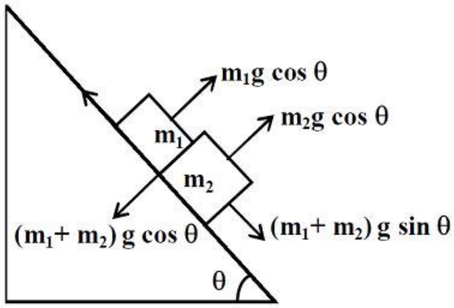
$0.2 \geq \tan\theta$

$\therefore$ for P, Q

$f = (m_1 + m_2)\, g\sin\theta$

For R and S

$F = f_{max} = \mu m_2 g\sin\theta$



Sol 22: (C) $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{v}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega}$

$$= -m\omega^2 r\hat{i} + 2mv_{rot}\omega\left(-\hat{j}\right) + m\omega^2 r\hat{i} = -2mv_{rot}\omega\hat{j}$$

$$v_{rot} = \frac{dr}{dt} = \frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)$$

$$\vec{F}_{rot} = -\frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

$$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k} = \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

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