FIITJEE

CONCEPTS OF COORDINATE GEOMETRY

STRAIGHT LINE

1. Distance between two points (x_1, y_1) & (x_2, y_2)= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. If points (x, y) divides $Q(x_1, y_1)$ and $R(x_2, y_2)$ in the ratio of m : n then

$x = \frac{mx_2 + nx_1}{m+n}$: $y = \frac{my_2 + ny_1}{m+n}$ internal division

$x = \frac{mx_2 - nx_1}{m-n}$: $y = \frac{my_2 - ny_1}{m-n}$ external division

3. Area of triangle ABC with $A(x_1, y_1)$,$B(x_2, y_2)$and $C(x_3, y_3)$ as its vertices is :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$$

Note: A, B, C are collinear if $\Delta = 0$.

4. Area of polygon with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ …………….$N(x_n, y_n)$ is given by

$$A = \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + ... + (x_ny_1 - x_1y_n)]$$

5. Centroid of Δ with vertices (x_1, y_1), (x_2, y_2) & (x_3, y_3) is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Centroid divides median in the ratio 2:1

6. The In-centre of a Δ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$ where a, b, c are sides BC, CA & AB. Sides makes angle at the In-centre $\frac{\pi}{2} + \frac{A}{2}, \frac{\pi}{2} + \frac{B}{2}, \frac{\pi}{2} + \frac{C}{2}$
7. Ex-centre of Δ opposite vertex A is $\left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$
8. Orthocentre of ΔABC is: $\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}\right)$

sides makes angle at orthocentre $\pi - A, \pi - B, \pi - C$.

9. Circumcentre of the ΔABC is

$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$,sides makes angle at circumcircle 2A, 2B, 2C

10. The line passing through orthocenter, centroid and circumcentre is known as euler line and centroid divides orthocenter and circumcentre in the ratio 2:1.
11. Equation of line in various form:

[i] General form $ax + by + c = 0$

[ii] intercept form $\frac{x}{a} + \frac{y}{b} = 1$

[iii] Slope intercept form y=mx+c

[iv] Slope point form $y - y_1 = m(x - x_1)$

[v] Two points form $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

[vi] Parametric form: $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$

[vii] Normal form : $x\cos\alpha + y\sin\alpha = p$
$p \rightarrow$ length of perpendicular from origin to the line
$\alpha \rightarrow$ angle b/w perpendicular & positive x-axis

12. Length of the perpendicular from a point (x_1,y_1) to the line ax+by+c=0 is $\frac{|ax_1 + by_1 - c|}{\sqrt{a^2+b^2}}$. Length of the perpendicular from origin is $\frac{|c|}{\sqrt{a^2+b^2}}$.Perpendicular distance between two parallel line $ax+by+c_1=0$ and $ax+by+c_2=0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$

13. The mirror image of a point $P(\alpha_1,\beta_1)$ with respect to a given line ax+by+c=0 is $Q(\alpha_2,\beta_2)$ then $\frac{\alpha_2-\alpha_1}{a} = \frac{\beta_2-\beta_1}{b} = \frac{-2(a\alpha_1+b\beta_1+c)}{a^2+b^2}$

14. The ratio in which the line joining the points (x_1, y_1) and (x_2, y_2) divides the line ax+by+c=0 is $-\frac{ax_1+by_1+c}{ax_2+by_2+c}$, if point (x_1, y_1) and (x_2, y_2) are on the same side of line $ax+by+c=0$ if $\frac{ax_1+by_1+c}{ax_2+by_2+c} > 0$ and opposite side of line if $\frac{ax_1+by_1+c}{ax_2+by_2+c} < 0$

15. The side in which the origin lies is said to be the negative side of the line and other side is called positive side

16. If (x_1, y_1) lies on the negative side of the line, the length of perpendicular is + ve and vice versa.

17. Angle b/w two lines whose slopes are m_1, m_2 is $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1m_2}\right|$ $\theta = 90^0$ if $m_1m_2 = -1$; $\theta = 0^0$ or 180^0 if $m_1 = m_2$

18. Two lines $a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$

(a) parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, (b) perpendicular if $a_1a_2 + b_1b_2 = 0$ (c) identical if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

19. A line parallel to ax + by + c = 0 is ax + by + k = 0, k is a constant. A line perpendicular to ax + by + c = 0 is bx – ay + k = 0

20. Length of perpendicular from (x_1, y_1) to ax + by + c = 0 is $p = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

21. Family of lines passing through the intersection of lines $L_1 = 0$ & $L_2 = 0$ is $L_1 + \lambda L_2 = 0$

22. Three straight lines given by $\begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \\ a_3x+b_3y+c_3=0 \end{cases}$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

23. Equation of the bisectors of lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by

$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{(a_2x+b_2y+c_2)}{\sqrt{a_2^2+b_2^2}}$, provided $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ & $c_1c_2 > 0$

(a) + represents acute angle bisector if $a_1a_2 + b_1b_2$ is < 0

(b) + represents obtuse angle bisector if $a_1a_2 + b_1b_2$ is > 0

(c) – represents acute angle bisector if $a_1a_2 + b_1b_2$ is > 0

(d) – represents obtuse angle bisector if $a_1a_2 + b_1b_2$ is < 0

(e) If $a_1a_2 + b_1b_2$ is < 0 then origin lies in acute angle

(f) If $a_1a_2 + b_1b_2$ is > 0 then origin lies in obtuse angle

24. Distance between two points $P(r_1, \theta_1)$ & $Q(r_2, \theta_2)$ is $\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$.

25. The area of ΔPQR with vertices $P(r_1, \theta_1), Q(r_2, \theta_2)$ and $R(r_3, \theta_3)$ is $= \frac{1}{2}\left\{\sum r_1 r_2 \sin(\theta_1 - \theta_2)\right\}$

26. If p is the length of perpendicular from the pole to the line & α is the angle which the perpendicular makes with the initial line, then the equation is $r\cos(\theta - \alpha) = p$.

27. General equation : $\frac{k}{r} = A\cos\theta + B\sin\theta$

28. Any line parallel to the above is $\frac{k_1}{r} = A\cos\theta + B\sin\theta$.

29. Any line perpendicular to above line is $\frac{k_2}{r} = A\cos\left(\frac{\pi}{2} + \theta\right) + B\sin\left(\frac{\pi}{2} + \theta\right)$.

30. Equation of line passing through the origin is $\theta =$ constant.

PAIR OF STRAIGHT LINES

1. General equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents . any of the conic section depends on the different conditions on a, b, c, f, g, h those are as following

(a) a pair of straight lines if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ or $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

two lines are real / distinct if $h^2 > ab$
two lines are imaginary if $h^2 < ab$
two lines are coincident if $h^2 = ab$, $af^2 = bg^2 = ch^2 = abc = fgh$
two lines are parallel if $h^2 = ab$
both the lines are equally inclined on x-axis if h=0
both lines are intersecting on y-axis if $2fgh=bg^2+ch^2$, $bg^2=ch^2$

(b) Circle if $\Delta \neq 0$ and a=b & h=0

(c) Parabola if $\Delta \neq 0$ and $h^2 = ab$

(d) Ellipse if $\Delta \neq 0$ and $h^2 < ab$

(e) Hyperbola if $\Delta \neq 0$ and $h^2 > ab$

(f) Rectangular Hyperbola if $\Delta \neq 0$, $h^2 > ab$, a+b=0

2. $ax^2 + 2hxy + by^2 = 0$ is called a homogenous equation of second, it will represent a pair of straight line and both the lines will pass through origin only.

3. $ax^2 + 2hxy + by^2 + c = 0$ is a central conic whose centre is origin.

4. To find the equation of both the lines separately $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ then we will form quadratic equation either in x or in y and then solve.

5. To find out the point of intersection of both the lines we will partially differentiate the equation once with respect to x taking y as constant and then with respect to y taking x as constant and then we can solve both the equations simultaneously that is point of intersection of lines $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ can be obtained by solving $\frac{\vartheta s}{\vartheta x} = 0$ & $\frac{\vartheta s}{\vartheta y} = 0$

6. If $ax^2 + 2hxy + by^2 = 0 = b(y - m_1x)(y - m_2x)$ then, $m_1 + m_2 = -\frac{2h}{b}$ & $m_1m_2 = \frac{a}{b}$

7. Angle b/w a pair of straight line is $\theta = \tan^{-1}\frac{2\sqrt{h^2 - ab}}{a+b}$

(i) $\theta = 90^o$ if $a+b=0$ (ii) $\theta = 0^o$ if $h^2 = ab$

8. Equation of pair of straight line perpendicular to $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.

9. Equation of bisector of $ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$

If a = b, then bisectors : $y = \pm x$

If b= 0, then bisectors : $x = 0,\ y = 0$

10. If (x_1, y_1) is the pt. of intersection of such lines then the equation of bisectors of S = 0 is

$$\frac{(x-x_1)^2 - (y-y_1)^2}{a-b} = \frac{(x-x_1)(y-y_1)}{h} \qquad \left.\begin{matrix} ax_1 + hy_1 + g = 0 \\ hx_1 + by_1 + f = 0 \end{matrix}\right\} (x_1, y_1): \text{ pt. of intersection}$$

11. Equation of lines joining the origin to the points of intersection of a given line & a given curve can be obtained by making the equation of curve, homogenous equation let the equation of curve be $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0,$ & equation of line be $y = mx + c$ then equation of lines joining the origin to the point of intersection of line & the curve is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{y-mx}{c}\right) + 2fy\left(\frac{y-mx}{c}\right) + c\left(\frac{y-mx}{c}\right)^2 = 0$$

12. **Translation & rotation of axes:**

(a) If origin is shifted to (h, k) & axes are rotated through angle θ in anticlockwise dirrection then

$x = h + X\cos\theta - Y\sin\theta$

$y = k + X\sin\theta + Y\cos\theta$

(b) If the axes are rotated through an angle θ without changing the origin & in the transformed equation term xy is absent then $\tan 2\theta = \frac{2h}{a-b}$

(c) In translation of the axes, the coefficient of the term of 2^{nd} degree remain unaltered.

(d) The rotation of the axes leaves the constant term unaltered.

CIRCLE

1. In equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0,$ there are 3 independent constants and hence 3 geometrical conditions are necessary to obtain the equation of a circle, centre is (-g, -f) & radius = $\sqrt{g^2 + f^2 - c}$

2. If (x_1, y_1) & (x_2, y_2) are the extremities of diameter, then equation of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

3. Length of tangent from a point $P(x_1,y_1)$ to circle is = $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{Sp}$ where S_p is known as power of a point P.

4. The point (x_1, y_1) lie out side, on or in side the circle S = 0, according as

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c >=< 0$

5. The equation of tangent at (x_1, y_1) to circle S = 0 is $xx_1 + yy_1 = a^2$

6. $y = mx + c$ will intersect, touch or do not intersect the circle according as $c^2 <=> a^2(1+m^2)$

7. Equation of tangent is $y = mx \pm a\sqrt{1+m^2}$ to the circle $x^2 + y^2 = a^2$ & point of contact is

$$\left(\mp\frac{am}{\sqrt{1+m^2}}, \pm\frac{a}{\sqrt{1+m^2}}\right)$$

8. Length of chord of constant = $\frac{2lr}{\sqrt{r^2+l^2}}$ l= length of tangent and r = radius
9. Equation of chord of contact of tangents drawn from $P(x_1, y_1)$ to the circle is $T_1=0$ where $T_1= xx_1+yy_1-a_2$
10. The equation of chord with $P(x_1, y_1)$ as the middle point of it is $T_1 = S_1$, where T_1 stands for equation of tangent and S_1 is S (equation of circle) after (x, y) are replaced by (x_1, y_1) .
11. The equation of tangents drawn from (x_1, y_1) to the circle S = 0 is $SS_1 = T_1^2$ where S_1 is the power of point (x_1,y_1)
12. The equation of normal at (x_1, y_1) to circle $x^2+y^2+2gx+2fy+c=0$ is $y-y_1=\frac{y_1+f}{x_1+g}(x-x_1)$
13. Parametric co-ordinate of any point of the circle $(x-a)^2+(y-b)^2=R^2$ is $(a+R\cos\theta, b+R\sin\theta)$ and equation tangent to this point is $(x-a)\cos\theta+(y-b)\sin\theta=R$
14. If equation of circle is $x^2+y^2=a^2$, then any point on this circle has co-ordinate $(a\cos\theta, a\sin\theta)$ and the equation of tangent is $x\cos\theta+y\sin\theta=a$.
15. The equation of chord joining θ & φ in the circle $x^2+y^2=a^2$ is $x\cos\frac{\theta+\varphi}{2}+y\sin\frac{\theta+\varphi}{2}=a\cos\frac{\theta-\varphi}{2}$ is a
16. Equation of circle of radius r & touching both the axes is $(x-r)^2+(y-r)^2=r^2$.
17. The general equation of a tangent with slope m to the circle $x^2+y^2+2gx+2fy+c=0$ is $y+f=m(x+g)\pm\sqrt{g^2+f^2-e}\ \sqrt{1+m^2}$
18. Director circle is the locus of point of intersection of two perpendicular tangents to any circle. If the equation of circle is $x^2+y^2=a^2$, then director circle is $x^2+y^2=2a^2$.
19. The number of common tangents if the 2 circle's are such that one lies inside the other, touch internally.
20. Two circle with radii r_1 & r_2 touch one another externally, internally, intersect, do not intersect and one lies within the other if $d=r_1+r_2; d=r_1-r_2; r_1-r_2<d<r_1+r_2; d>r_1+r_2$ and $d<r_1\sim r_2$.
21. Two circles are orthogonal if $2g_1g_2+2f_1f_2=c_1+c_2$.
22. If the chord of a circle subtends a right angle at the origin, then the locus of foot of $\perp r$ from origin to these chords is a circle.
23. $S+\lambda P=0$ represents the family of circle passing through the intersection of circle $S_1=0$ & line P = 0 $\lambda\rightarrow$ a parameter.

24. If S_1 & S_2 are the intersecting circles, then $S_1+\lambda S_2=0$ represents family of circles passing through the inter section of S_1 & S_2.(S_1-S_2) represents the common chord $(\lambda=-1)$ of $S_1=0$ & $S_2=0$.
25. The equation of a family of circles passing through two given points (x_1, y_1) and (x_2, y_2) can be written in the form of $(x-x_1)(x_1-x_2)+(y-y_1)(y-y_2)+\lambda\begin{vmatrix}x & y & 1\\ x_1 & y_1 & 1\\ x_2 & y_2 & 1\end{vmatrix}=0$ where λ is a parameter.
25. $(x-x_1)^2+(y-y_1)^2+\lambda[(y-y_1)-m(x-x_1)]=0$ is the family of circles which touch $y-y_1$ $m(x-x_1)$ at (x_1, y_1) for any finite m. If m is infinite, the family is $(x-x_1)^2+(y-y_1)^2+\lambda(x-x_1)=0$
26. Radical axis of two circle $S_1=0$ & $S_2=0$ is $S_1=S_2$ provided coeff. of x^2, y^2 in S_1 & S_2 are same.

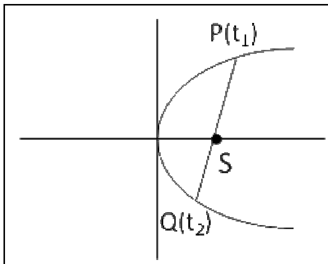
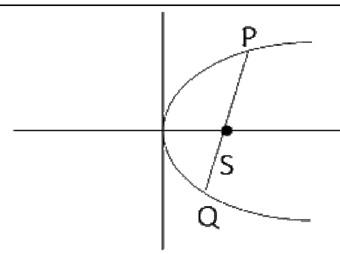
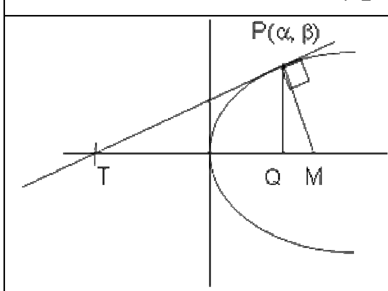
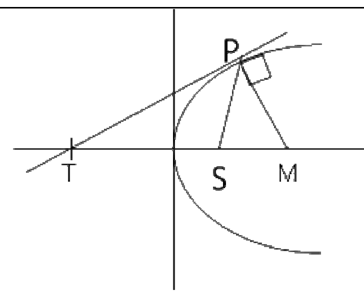
27. Point of intersection of radical axes of 3 circles is the radical centre.
28. The circle having its centre at the radical centre of 3 circle & its radius equal to the length of tangent from radical centre to any one of the circles, intersects orthogonally the 3 circle.
29. If S_1 & S_2 touch each other, then $S_1 - S_2 = 0$ is common tangent.
30. If P = 0 is a tangent to the circle S = 0 at Q, S + $\lambda P = 0$ represents a family of circle touching S = 0 at Q having P = 0 as the common tangent at Q.
31. A system of circles is said to be co-axial if every pair of circles of this family has the same radical axis eg $x^2 + y^2 + 2gx + c = 0$ & $x^2 + y^2 + 2fy + c = 0$ where g, f are parameters & c = constant
32. Centres of circles of a coaxial system lie on a straight line $\perp r$ to the radical axis.
33. Limiting points of a system of co-axial circles are the centers of the point circle belonging to the family.
34. Limiting points lie on the opposite sides of the radical axis & are equidistant from the radical axis.
35. Any point passing through the limiting points cut orthogonally every circle of the co-axial system.
36. The limiting points are conjugate w.r.t. every circle of the co-axial system.
37. For the co-axial system of circles $x^2 + y^2 + 2fy - c = 0,$ lines of centers is the x-axis. The common radical axis y – axis & $(\pm\sqrt{c},\ \ 0)$ are limiting pts.
38. If $x^2 + y^2 + 2gx + c = 0$ & $x^2 + y^2 + 2fy - c = 0$ represents the two system of co-axial circles, then
(a) Each circle of one system cuts orthogonally every circle of the other system.
(b) Limiting point of one system are the point of intersection of the other system.

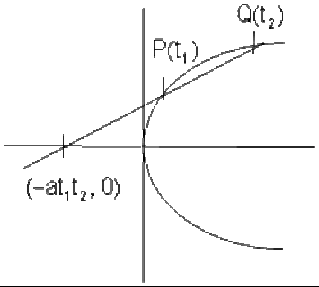
PARABOLA

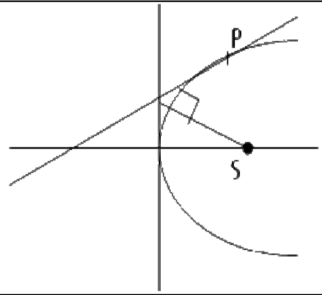
1. Conic: It is locus of a point which moves such that the ratio (ecentricity) of its distance from a fixed point(focus) to the distance from a fixed line (directrix) is a constant. fixed point must not lie on the fixed line.
If e = 1, the locus is parabola
e <1, the locus is ellipse
e > 1, the locus is hyperbola
2. If the fixed point is S(a, b) & fixed line is L: $y = mx + c,$ note that the point S must not lie on the line L then the locus of the moving point P(x, y) is given by $(x-a)^2 + (y-b)^2 = \left(\frac{-y+mx+c}{\sqrt{1+m^2}}\right)^2$ is equation of parabola
3. For parabola $y^2 = 4ax,$ the focus is (a, 0), directrix is x + a = 0; latus rectum = 4a.
4. The point (x_1, y_1) lies outside on or inside the parabola $y^2 = 4ax$ according as $y_1^2 - 4ax_1 >=< 0$
5. Point of intersection of $y = mx + c$ & $y^2 = 4ax$ are real, coincident & imaginary according as $c <=> \frac{a}{m}$
6. The equation of the tangent is $y = mx + \frac{a}{m}$ & the point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
7. Equation of normal at point $(am^2, 2am)$ to parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$
8. Foot of perpendicular from the focus to any tangent lies on the tangent to the vertex of parabola.
9. x + y + a =0 is the common tangent to the parabola $y^2 = 4ax$ & $x^2 = 4ay$.
10. The equation of tangent at (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.
11. Chord of contact of parabola with respect to point (x_1,y_1) is $S_1=T_1$
12. The equation of pair of tangents from (x_1, y_1) is $SS_1 = T_1^2$.
13. The equation of chord having (x_1, y_1) as its mid-point is $T_1 = S_1$.
14. $(at^2, 2at)$ is the co-ordinate of any point of parabola $y^2 = ax$.

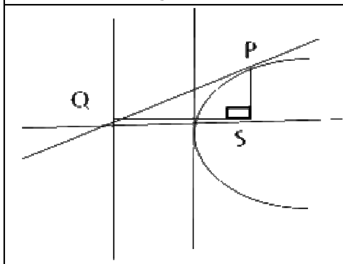
15. The equation of chord joining t_1 & t_2 is $y(t_1+t_2) = 2x + 2at_1t_2$ and it will intersect the x- axis at $(-at_1t_2, 0)$
16. Tangent at point t is $ty = x + at^2$
17. The equation of normal at point 't' is $y + tx = 2at + at^3$
18. (a) The subtangent NT is bisected by the vertex.
    (b) SP = ST
    (c) $SY \perp PT \Rightarrow SY^2 = AS \cdot SP$
    (d) PB subtends right angle at S
    (e) Perpendicular tangents intersect at x = - a
    (f) Tangent at the ends of focal chords intersect at right angle at directrix.
19. Subnormal is constant & if the normal meet the axes at G, then SG = SP.
20. By equation of normal at t. $y + tx = 2at + at^3$, we see that above equation is cubic in 't' therefore three normals can be drawn from any point to the parabola $\sum t_1 = 0; \sum t_1t_2 = \frac{2a-h}{a}, t_1t_2t_2 = k/a$, where t_1, t_2, t_3 are feet of normals from (h, k)
21. If the 3pts are feet of normal (concurrent) , then circle through these points, passes through the vertex of the parabola.
22. Sum of ordinates of the feet of normal from any point is zero.
23. The condition for normal at t_1, t_2 to intersect on the parabola is $t_1t_2 = 2$.
24. A circle cuts the parabola at 4 points P, Q, R, S
25. Algebraic sum of ordinates of P, Q, R, S is zero.
26. Chord PQ & PS are equally inclined to x-axis.
28. The locus of the middle points of a system of parallel chords of the parabola is $y = \frac{2a}{m}$, this line is parallel to x-axis.
29. The chords parallel to x-axis of parabola is called the diameter.
30. Each diameter bisects a system of parallel chords & the axis bisects all the chords perpendicular to it.

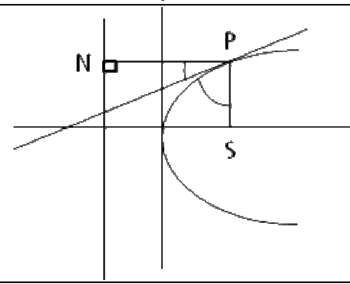
31. IMPORTANT PROPERTIES OF PARABOLA :

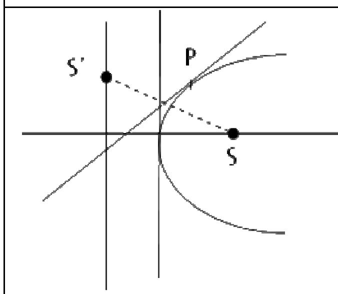
 $t_1t_2 = -1$	 $2a = \frac{2PS.SQ}{PS+SQ}$
 $TQ = 2\alpha$ (Length of sub-tangent) QM = 2a (length of sub-normal) $\beta^2 = 2\alpha .2a$ (ordinate is the G.M. of sub-tangent & sub-normal)	 PS = ST = SM

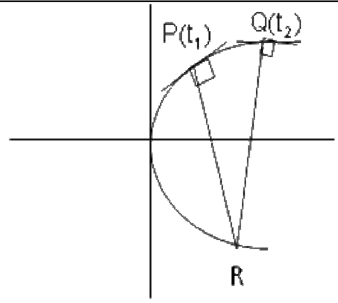




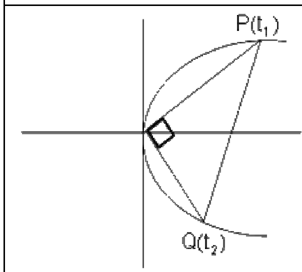




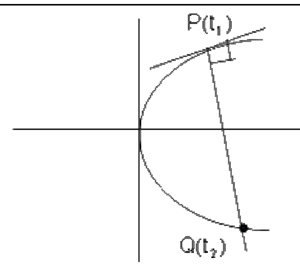




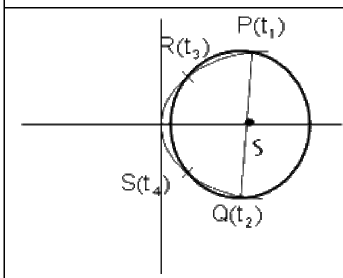
$t_1t_2 = 2$



$t_1t_2 = -4$



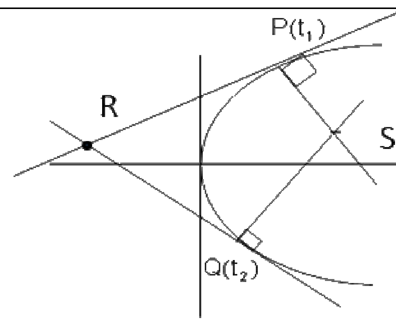
$t_2 = -t_1 - \frac{2}{t_1}$



$t_3 \cdot t_4 = 3$

(t_3 & t_4 are imaginary)

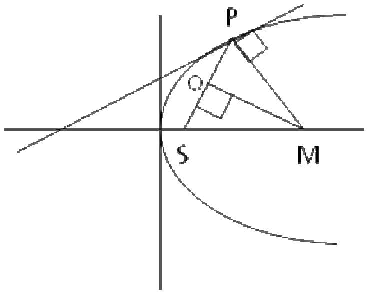
(t_1, t_2 are end points of diameter)



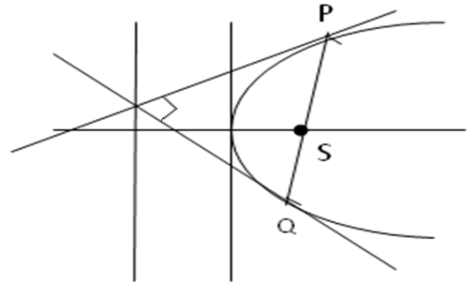
$R(at_1t_2, a(t_1 + t_2))$

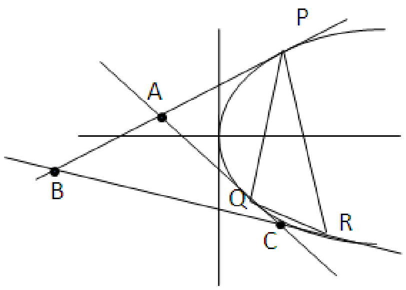
$S(2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2))$

If PQ is focal chord then slope of RS will be equal to slope of axis of parabola

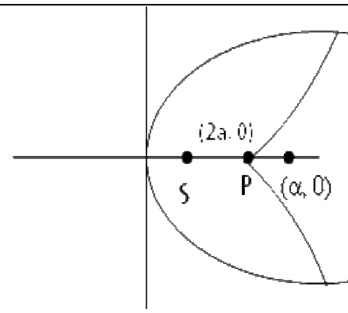


PQ = 2a





1. $Ar(\Delta\ PQR) = 2Ar(\Delta ABC)$
2. Circum circle of Δ ABC will pass through the focus.
3. Orthocentre of Δ ABC will lie on directric



If a point lies on axis of parabola and $\alpha > 2a$ Then we can draw 3 real normals to the parabola.

ELLIPSE

1. For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1 - e^2)$, foci are $(\pm ae, 0)$, directrix : $x = \pm\frac{a}{e}$, latus rectum $= \frac{2b^2}{a}$

2. If perpendicular distances P_1, P_2 of a moving point P from two perpendicular lines L_1 & L_2 are connected by the relation $\frac{P_1^2}{a^2} + \frac{P_2^2}{b^2} = 1$, the point P describes an ellipse.

3. Any point (x_1, y_1) lies outside, on or inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ according as $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 >=< 0$

4. $y = mx + c$ will intersect in real, coincident or imaginary points according as ($c^2 <=> a^2m^2 + b^2$)

   Hence $y = mx \pm \sqrt{a^2m^2 + b^2}$ is tangent & point of contact is $\left(\mp\frac{a^2m}{c}, \pm\frac{b^2}{c}\right)$

5. If $lx + my + n = 0$ is a tangent line to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $a^2l^2 + b^2m^2 = n^2$

6. Two perpendicular tangents of the ellipse intersect on director circle $x^2 + y^2 = a^2 + b^2$

7. Equation of tangent at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

8. Equation of normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

9. Chord of contact of tangents drawn from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

10. Circle described on the major axis of the ellipse as diameters is called the auxiliary circle of the ellipse & equation is $x^2 + y^2 = a^2$.
11. SY & S_1Y_1 be two perpendicular on any tangent, the feet of perpendiculars Y and Y_1 lie on auxiliary circle & $SY . S_1Y_1 = b^2$.
12. the circle on any focal distance of the point on an ellipse as diameter touches the auxiliary circle.
13. combined equation of tangents drawn from (x_1,y_1) to the ellipse is $SS_1=T_1^2$

    Where $S = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$; $\quad S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$ & $T_1 = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$
14. Locus of foot of perpendiculars to the tangent $x\cos\alpha + y\sin\alpha = p$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

    $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$ or $r^2 = a^2\cos^2\theta + b^2\sin^2\theta$ in polar co-ordinate.
15. Co-ordinates of any point on the ellipse is $(a\cos\theta, b\sin\theta)$. where θ is eccentric angle.
16. Equation of chord joining the points θ & φ is $\frac{x}{a}\cos\frac{\theta+\varphi}{2} + \frac{y}{b}\sin\frac{\theta+\varphi}{2} = \cos\frac{\theta-\varphi}{2}$

    Equation of tangent at any point θ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$.

    Point of intersection of the tangent at θ & φ are $\left(\frac{a\cos\frac{\theta+\varphi}{2}}{\cos\frac{\theta-\varphi}{2}}, \frac{b\sin\frac{\theta+\varphi}{2}}{\cos\frac{\theta-\varphi}{2}}\right)$
17. Equation of normal at θ is $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$
18. Tangent & normal at any point of an ellipse bisects the angle between the focal radii at that point.
19. If one light ray is emerging from one focus then after refection from the surface of ellipse it will pass through the second focus of the ellipse.
20. Four normals can be drawn from a given point to ellipse.
21. If the normals at the four points (x_1, y_1) on the ellipse are concurrent then

    $(x_1 + x_2 + x_3 + x_4)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right) = 4$
22. Equation of chord of an ellipse with (x_1, y_1) as its middle point is $T_1 = S_1$.
23. The locus of middle points of the parallel chords of slope m of the ellipse is $y = -\frac{b^2}{a^2m}x$
24. $y = m_1x$ bisects all chords parallel to $y = mx$ if $mm_1 = \frac{b^2}{a^2}$. Similarly $y = mx$ bisects all the chords parallel to $y = m_1x$. Such diameters are called conjugate diameters.
25. The circle on any focal distance as diameter touches the auxiliary circle.
26. Foot of perpendiculars from foci to any tangents lies on the auxiliary circle.
27. The tangent at any point on the ellipse meets the tangents at the ends of the major axis at T_1 and T_2. The circle on $T_1 T_2$ as diameter passes through focus.

PROPERTIES OF CONJUGATE DIAMETERS

28. Tangents at the extremities of a diameter are parallel to the conjugate diameter.
29. Tangents at the extremities of a chord intersect on the diameters which bisects the chord.
30. Ecentric angles of the ends of a pair of conjugate diameter differ by a right angle.
31. If P & D are extremities of two conjugate diameters of the ellipse then $CP_2 + CD_2 = a_2 + b_2$
32. Tangents at the ends of a pair of conjugate diameters of an ellipse form a parallelogram of constant area 4ab.
33. When two conjugate diameters are equal, they are called Equi-conjugate diameter.

34. $y = \pm \frac{b}{a} x \rightarrow$ combined equation of equi-conjugate diameter.
35. They are equally inclined to the major axis.
36. The length of each equi-conjugate diameter is $2(a^2 - b^2)$
37. The eccentric angle of an extremity of a equi-conjugate diameter is $\frac{\pi}{4}$
38. Tangents at extremities of the major & minor axis intersect the equi-conjugate diameters.

HYPERBOLA

1. For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2(e^2 - 1)$, transverse axis is of length 2a along the x-axis & conjugate axis of 2b along y – axis, foci are $(\pm ae,\ 0)$,latus rectum $= \frac{2b^2}{a}$, directrix x $= \pm \frac{a}{c}$
2. point (x_1, y_1) lies outside, on or inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, according as $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 <=> 0$
3. Points of intersection of $y = mx + c$ with hyperbola is real, coincident or imaginary according as $c^2 <= \text{or} > a^2m^2 - b^2$ & tangent is $y = mx \pm \sqrt{a^2m^2 - b^2}$
4. condition for $lx + my + n = 0$ to touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $a^2l^2 - b^2m^2 = n^2$
5. $x^2 + y^2 = a^2 - b^2 \rightarrow$ director circle
   $x^2 + y^2 = a^2 \rightarrow$ auxiliary circle
6. Equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
7. $y = mx$ & $y = m_1x$ are conjugate diameter if $mm_1 = \frac{b^2}{a^2}$
8. Normals at (x_1, y_1) is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 + b^2$
9. Equation of chord with middle point (x_1, y_1) is $T_1 = S_1$.
10. Pair of tangents is $SS_1 = T_1^2$.
11. $(a\sec\theta, b\tan\theta)$, and $\frac{1}{2}a\left(t + \frac{1}{t}\right)$, $\frac{1}{2}b\left(t - \frac{1}{t}\right)$ are any points on the hyperbola.
12. Equation of chord joining θ & φ is $\frac{x}{a}\cos\frac{\theta - \varphi}{2} - \frac{y}{b}\sin\frac{\theta + \varphi}{2} = \cos\frac{\theta + \varphi}{2}$
13. Equation of the tangent at θ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$
14. Equation of normal at θ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$
15. If the tangent & normal at any point of the hyperbola meets the y-axis at P & Q then circle on PQ as diameter meets the x-axis at foci of the hyperbola.
16. $y = \pm \frac{b}{a} x$ are asymptotes to hyperbola.
17. Angle between two asymptotes to the hyperbola is $2\sec^{-1}e$.
18. The intercept of any tangent to the hyperbola intercepted between the asymptotes, is bisected at the point of contact.

19. Any tangent to the hyperbola makes with the asymptotes a Δ of constant area = ab.
20. The product of the perpendiculars drawn from any point on a hyperbola to its asymptote Is constant.
21. The foot of perpendiculars from a focus to an asymptote is a point of intersection of the auxillary circle & corresponding directrix.
22. The asymptotes of a hyperbola meet the directrix on the auxillary circle.
23. If the equation of hyperbola is $(ax+by+c)(a_1x+b_1y+c_1)=k$ then equation of asymptote are $ax+by+c=0$ & $a_1x+b_1y+c_1=0$
24. Equation of conjugate hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
25. If H = 0, A = 0, C = 0 be the equation of a hyperbola, asymptotes & conjugate hyperbola then $H+\lambda=A$ and $C+H=2A$, λ can be calculated by making $\Delta=0$
26. e_1 (eccentricity) of conjugate hyperbola is $e_1^2=\frac{a^2+b^2}{b^2}$
27. Directrix of conjugate hyperbola are $y=\pm\frac{b}{e_1}$ & their foci are $(0, \pm b/e_1)$
28. Parametric representation of conjugate hyperbola is $x=a\tan\theta, y=b\sec\theta$.
29. A hyperbola & its conjugate hyperbola cannot intersect in real points.
30. The locus of middle points of the portion (intercepted between two given perpendicular lines) of a straight line which passes through a fixed point is a hyperbola with its asymptotes parallel to given lines.
31. Gradient of chord whose mid point is (x_1, y_1) is $m=\frac{b^2}{a^2}\frac{x_1}{y_1}$, so locus of midpoint (x_1, y_1) is $y=\frac{b^2}{a^2m}x$, is called diameter of a hyperbola, since it passes through centre of hyperbolas, hence mid points of all the chords of slope m lies on the diameter $y=m_1x$ where $mm_1=\frac{b^2}{a^2}$
    By symmetry mid points of of all chords of slope m_1 lies on the diameter y = mx there fore y = m_1x &m_1x are called conjugate diameters.
32. If one light ray is coming along one focus then after refection from the surface of hyperbola it will go in the direction of the second focus of the hyperbola.
33. **Rectangular hyperbola:** A hyperbola is said to be rectangular, if its asymptotes are perpendicular this$\Rightarrow a=b\Rightarrow$ equation is thus $x^2-y^2=a^2$ with $e=\sqrt{2}$
34. Asymptotes are $y=\pm x$ & director circle of rectangular hyperbola becomes $x^2+y^2=0$
35. $xy=c^2$ where $c^2=\frac{a^2}{2}$, asymptotes are coordinate axes, co-ordinate of vertices are $\left(\frac{c}{2}, \frac{c}{2}\right)$ & $\left(-\frac{c}{2}, -\frac{c}{2}\right)$
36. x = ct & $y=\frac{c}{t}$ is parametric representation of $xy=c^2$
37. Chord joining t_1 and t_2 is $x+t_1t_2y=c(t_1+t_2)$
38. Tangent at t is $x+t^2y=2ct$, normal $\rightarrow$ $tx-\frac{y}{t}=c\left(t^2-\frac{1}{t^2}\right)$
39. Intersecting points of tangents at t_1 and t_2 are $\left(\frac{2ct_1t_2}{t_1+t_2}, \frac{2c}{t_1+t_2}\right)$

40. Orthocenter of a Δ inscribed in a rectangular hyperbola lies on the rectangular hyperbola.

Concyclic Point: $x^2 + y^2 + 2gx + 2fy + k = 0$ meets the rectangular hyperbola $xy = c^2$ at four points t_1, t_2, t_3, t_4 such that $\sum t_1 = -\frac{2g}{c}$, $\sum t_1 t_2 = \frac{k}{c^2}, \sum t_1 t_2 t_3 t_4 = -\frac{2f}{c}$ & $t_1 t_2 t_3 t_4 = 1$

41. Orthocenter of Δ form by any 3 points (t_1, t_2, t_3, t_4) is a point diametrically opposite to the 4th point.

Conormal points: - From a given point (h, k), 4 normals can be drawn to a rectangular hyperbola whose feet of normal is t_1, t_2, t_3, t_4, then $\sum t_1 = \frac{h}{c}$:

$\sum t_1 t_2 = 0$ $\sum t_1 t_2 t_3 = -\frac{k}{c}$, $t_1 t_2 t_3 t_4 = -1$

A circle be drawn through any 3 points, will meet the hyperbola at a pt. diametrically opposite to the 4th point.

Extra Points

Parabola

13. The equation of chord of contact of tangents from (x_1, y_1) or the equation of polar of point (x_1, y_1) w.r.t. Parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$
14. The polar becomes the chord of contact if point (x_1, y_1) lies outside the parabola & it becomes tangent when (x_1, y_1) lies on the parabola.
15. Director is the polar of the focus.
16. Pole of line $lx + my + n = 0$ is $\left(\frac{n}{l}, \frac{2am}{l}\right)$
8. The slope of the tangent from (x_1, y_1) to $y^2 = 4ax$ is $m^2x - my + a = 0$: From this we conclude that tangents intersect at its directrix.
9. The tangent which makes complementary angles with the axis of the parabola intersect on the latus rectum of the parabola.
22. The chord joining, t_1 & t_2 is a focal chord if t_1t_2= -1.
24. The point of intersection of the tangents at t_1 & t_2 are $(at_1t_2, a(t_1 + t_2))$
26. If a normal at t_1 bisects the parabola at t_2, then $t_2 = -t_1 - \frac{2}{t_1}$.
27. The normals at t_1 & t_2 intersect on $\{2a + a(t_1^2 + t_1t_2 + t_2^2),\ -at_1t_2(t_1 + t_2)\}$
29. The circle circumscribing the Δ formed by any three tangents passes through the focus.
30. Orthocentre of this Δ lies on the directrix.

Ellipse

10. Directrix is the polar of the corresponding focus.
11. Pole of line $lx + my + n = 0$ w.r.t. ellipse is $\left(-\frac{1a^2}{n}, -\frac{mb^2}{n}\right)$