1. MAGNETIC FIELD AND LORENTZ FORCE

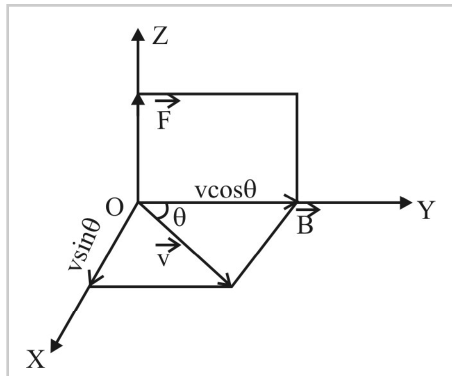
Magnetic Field

Magnetic field is the space around a magnet or the space around a conductor carrying current in which magnetic influence can be experienced.

Force on a charge moving in a uniform magnetic field $\vec{B}$.

Consider a positive charge q moving in a uniform magnnetic field $\vec{B}$, with a velocity $\vec{v}$. Let the angle between $\vec{v} \times \vec{B}$ be θ.

Change experiences a force. Which depends on a factors given below.



(i) $F \propto q$

(ii) $F \propto \text{v}\sin\theta$

(iii) $F \propto B$

$F \propto qVB\sin\theta$

$\text{F} = KqVB\sin\theta$

$\text{k} = 1$

$F = qvB\sin\theta$

$\left|\vec{F}\right| = q\left|\vec{v} \times \vec{B}\right|$

$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$

The direction of $\vec{F}$ is perpendicular to the plane containing $\vec{v} \times \vec{B}$. It is directed as given by right hand rule.

If a charged particle moves through a region in which both electric and magnetic fields are present, the resultant force will be the sum of forces due to the individual fields.

(*i*) **Force due to electric field:** When a charged particle carrying charge $+q$ is moving in a region of electric field $\vec{E}$, it experiences a force given by:

$$\vec{F}_e = +q\vec{E}$$

The direction of force is the same as that of $\vec{E}$

(*ii*) **Force due to magnetic field:** If the charged particle is moving in a magnetic field $\vec{B}$ with a velocity $\vec{v}$, it experiences a force given by,

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

The direction of the force is determined by right hand screw rule, which is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

The total force experienced by the charged particle due to both the electric and magnetic fields will be given by Lorentz relation,

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \text{... (}i\text{)}$$

This is also known as Lorentz force.

In the absence of the electric field, (*i*) reduces to:

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \qquad \text{... }(ii)$$

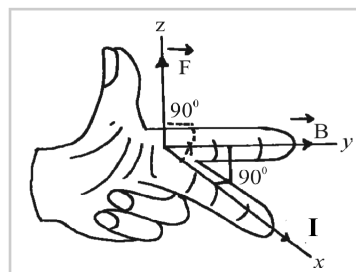
Then:

- F depends on q, $\vec{v}$ and $\vec{B}$
- F = 0, if the charge is stationary ($\vec{v}$ = 0)
- F vanishes if $\vec{v}$ and $\vec{B}$ are parallel or anti-parallel to each other
- F is maximum when $\vec{v}$ and $\vec{B}$ are mutually perpendicular

Here, F = qvB ... (*iii*)

The Direction of This Force is Given by Fleming's Left Hand Rule

According to this rule, if the first finger, the central finger and the thumb of left hand are stretched mutually perpendicular to one another in such a way that the first finger indicates the direction of magnetic field and the central finger, the direction of electric current (*i.e.*, the direction of moving positive charge), then the thumb represents the direction of force experienced by the charged particle.



Unit of Magnetic Field

From (*iii*), $B = \dfrac{F}{qv}$

$$Unit\ of\ B = \frac{N}{Cms^{-1}} = \frac{Ns}{Cm}$$

This unit is called tesla (T).

A smaller unit called Gauss (G) is also used to express the strength of the magnetic field.

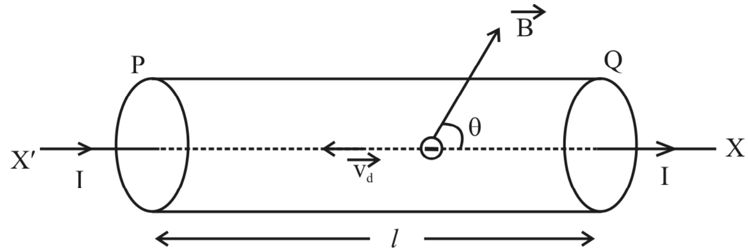
1 T = 10^4 G

Note: *When the strength of magnetic field is expressed as the magnetic flux over unit area, then its unit is weber per square metre (Wbm^{-2}). This unit is equivalent to tesla (T).*

(1 T = 1 Wb m^{-2})

2. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD

Consider a striaght conductor PQ of length l area of cross section A carrying current I placed in a uniform magnetic field of induction $\vec{B}$. Let the conductor be placed in a uniform magnetic field of induction B. Let the conductor be placed along x-axis & magnetic field be acting in xy plane making on angle θ with the x-axis. Suppose the current I flows through the conductor from end P to Q.



Let $\vec{V}_d$ = drift velocity of electron

– e = charge on each electron

Magnetic Lorentz force on an electron

$$\vec{f} = -e\left(\vec{v}_d \times \vec{B}\right)$$

If n is the number density of free electron

N = n(Al)

Total force on the conductor

$$\vec{F} = N\vec{f} = -n\text{Al e}\left(\vec{v}_d \times \vec{B}\right) \text{ .....(i)}$$

$$I = AneV_d$$

$$Il = AneV_d.l$$

$$I\vec{l} = -nAle\vec{V}_d \text{ ....(ii)} \qquad [\because I\vec{l} \;\&\; \vec{V}_d \text{ have opposite direction}]$$

From (i) & (ii)

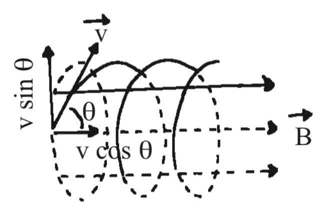
$$\vec{F} = I\vec{l} \times \vec{B}$$

$$\left|\vec{F}\right| = I\left|\vec{l} \times \vec{B}\right|$$

$$F = IlB\sin\theta$$

3. MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Let a particle of mass m and charge q enter a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ making an angle θ with the direction of the field acting in the plane of the paper.



The rectangular components of $\vec{v}$, are $\vec{v}_1 = v\cos\theta\ \hat{n}$ along the direction of magnetic field and $\vec{v}_2 = v\sin\theta\ \hat{n}$ perpendicular to the direction of magnetic field.

For velocity $\vec{v}_2$, the force acting on the charged particle is

$$\vec{F} = q\left(\vec{v}_2 \times \vec{B}\right)$$

As v_2 is perpendicular to $\vec{B}$,

F = qv_2B

The existence of $\vec{v}_1$ and the effect of $\vec{F}$ make the charged particle describe a helical path.

If the charged particle of charge q enters the magnetic field $\vec{B}$ with velocity $\vec{v}$ perpendicular to the field, then it will move in a circle.

Magnetic Force

F = qvB

Centripetal force required (if mass of the particle is m and r is the radius of the circle)

$$F_c = \frac{mv^2}{r}$$

As the magnetic force will supply the necessary centripetal force,

$F = F_c$

$\Rightarrow \quad qvB = \dfrac{mv^2}{r} \qquad \Rightarrow \quad \boxed{r = \dfrac{mv}{qB}}$

Radius of the circle described by the charged particle is given by the above expression.

Angular velocity (ω) of the particle: $v = r\omega \Rightarrow \omega = \dfrac{v}{r} = \dfrac{qB}{m}$

Time period for one complete revolution: $\boxed{T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB}}$

Frequency of the charged particle, $\boxed{v = \dfrac{1}{T} = \dfrac{qB}{2\pi m}}$

4. MOTION IN COMBINED ELECTRIC AND MAGNETIC FIELD

Velocity Selector

The force experienced by a particle of charge q moving with velocity v in combined electric and magnetic fields of strength E and B respectively is: $\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$

Specialiases

(*i*) $\vec{v}, \vec{E}$ and $\vec{B}$ are all collinear, $q\vec{v} \times \vec{B} = 0$

Force is purely electrical, $\vec{F} = q\vec{E}$

$\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{q\vec{E}}{m}$

- Particle passes in a straight line
- Parallel to the electric and magnetic fields
- With changes in velocity, momentum and kinetic energy
- Without change in the direction.

(*ii*) $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular. If $\vec{E}$ and $\vec{B}$ are such that $q\vec{v} \times \vec{B} = q\vec{E}$ and are in opposite direction, then the particle goes undeviated.

Here, $qvB = qE \Rightarrow v = \dfrac{E}{B}$

This condition can be used to select certain charged particles of specific velocities.

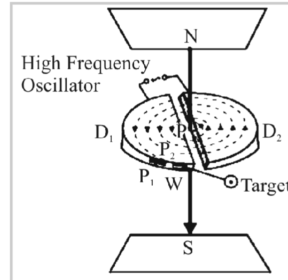
Note: *Particles with speed equivalent to E/B only can pass through crossed fields undeviated.*

5. CYCLOTRON

Cyclotron is used for accelerating positively charged particles like proton, deuteron, etc. such that they acquire sufficient energy to carry out nuclear reactions. It is also known as magnetic resonance accelerator.

(*i*) Principle: A positively charged particle can acquire sufficiently high energy (*i.e.* can be accelerated) with a comparatively small alternating potential difference by making them cross the same electric field frequently with the use of a strong magnetic field.

(*ii*) **Construction:** It primarily consists of two D-shaped hollow evacuated semicircular metal chambers (D_1 and D_2) called dees placed horizontally with a small gap separating them. These are connected to a high frequency oscillator, which can produce a potential difference of the order of 10^4 V at a frequency of 10^7 Hz. The dees are enclosed in a metal steel box, which is evacuated and well insulated. The whole apparatus is placed in a strong magnetic field produced by two poles of strong electromagnets NS.



(*iii*) **Working and Theory:** The particle which requires to be accelerated is produced at P. It is supposed at any time, D_2 is at a negative potential while D_1 at a positive potential. The particle will be accelerated towards D_2. When it reaches inside D_2, it will be in a field free space moving with constant speed v. Due to perpendicular magnetic field B, the particle describes the circular path of radius r in D_2 given by:

$$qvB = \frac{mv^2}{r} \qquad Or, \qquad r = \frac{mv}{qB}$$

m and *q* being the mass and charge of the particle.

Period of revolution, $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Frequency, $\nu_c = \frac{1}{T} = \frac{qB}{2\pi m}$

This frequency is termed as cyclotron frequency.

The frequency ν_a of the applied voltage is adjusted so that the polarity of the dees is reversed in the same time that it takes the particle to complete one half of the revolution.

The condition $\nu_a = \nu_c$ is called resonance condition and hence the name resonance accelerator.

The particle will again arrive in a gap between the two dees at the same time, when the polarity of the two dees is reversed. It goes on accelerating every time it enters the gap with increasing radius of the path and acquiring more and more energy. The accelerated particle can be removed out of the dees from window W by application of electric field across the deflecting plates P_1 and P_2.

If R is the radius of the trajectory at exit, the speed of the particle: $v = \frac{qBR}{m}$

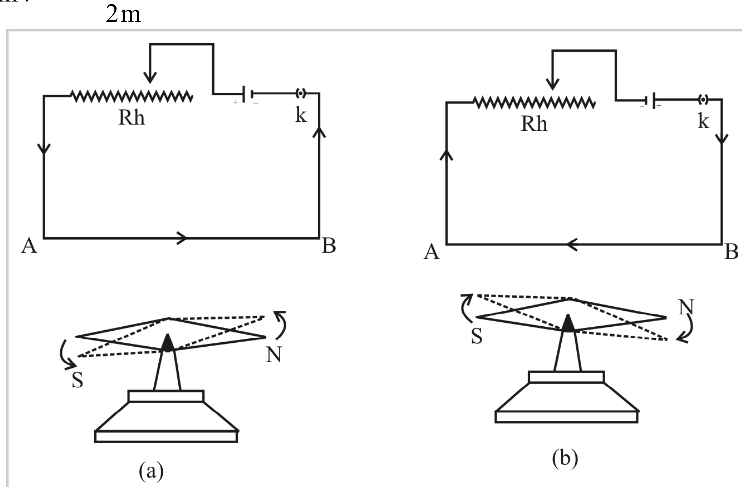
And kinetic energy of the particle: $K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$

6. OERSTED'S EXPERIMENT

Earlier it was thought that there is no connection between electricity & magnetism. However in the year 1820 Oersted showed that the electric current through the wire deflects the magnetic needle held below wire. The direction of deflection of the magnetic needle is reversed. This shows that the magnetic field is associated with a current carrying wire.

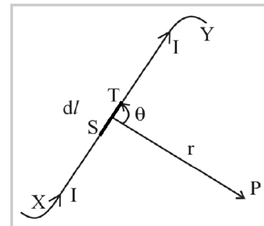
Snow Rule:

If a current flowing in a wire from south to north direction then north of the needle deflected toward west.



7. BIOT - SAVART'S LAW

Biot-Savart law is an experimental law that deals with the magnetic field induction at a point due to a small current element. By Biot - Savart's law, a small current element TS (in the figure) induces a magnetic field dB at the point P, that is at a distance r from the current element, such that



$$dB \propto \frac{I\, d\ell\, \sin\theta}{r^2}$$

where, I is the current, dl is the length of the current element, θ is the angle between the current element and the position vector of P from the current element.

Or, $$dB = k\frac{I\, d\ell\, \sin\theta}{r^2}$$

In vacuum, taking SI units, $k = \frac{\mu_0}{4\pi}$, μ_0 being the absolute permeability of free space and has value $4\pi\times10^{-7}$ T m A^{-1}.

Then, $dB = \frac{\mu_0}{4\pi}\frac{I\ d\ell\ \sin\theta}{r^2}$. In vector form, $\boxed{d\vec{B} = \frac{_0}{4\pi}\frac{I(d\vec{\ell}\times\vec{r})}{r^3}}$

The direction of $d\vec{B}$ is the direction of the vector product $\left(d\vec{\ell}\times\vec{r}\right)$..

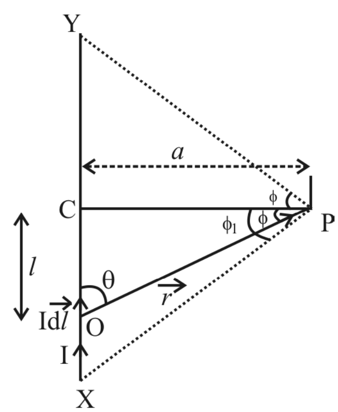
8. MAGNETIC FILED DUE TO A STRAIGHT CONDUCTOR CARRYING CURRENT

Consider a straight conductor XY lying in the plane of paper carrying current I in the direction X to Y, figure. Let P be a point at a perpendicular distance a from the straight conductor. Clearly, PC = a. Consider a small current element $I\vec{dl}$ of the straight conductor at O. Let $\vec{r}$ be the position vector of P w.r.t. current element and θ be the angle between $I\vec{dl}$ and $\vec{r}$ and CO = l.

According to Biot-Savart's law, the magnetic field induction (i.e. magnetic flux density) at point P due to current element $I\vec{dl}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}.\frac{I\vec{dl}\times\vec{r}}{r^3}$$

or $$dB = \frac{\mu_0}{4\pi}.\frac{Idl\sin\theta}{r^2} \quad ....(i)$$



In right angled ΔPOC, $\theta + \phi = 90°$

or $\theta = 90° - \phi$

and $\sin\theta = \sin(90° - \phi) = \cos\phi \quad ....(ii)$

Also $\cos\phi = \frac{a}{r}$ or $r = \frac{a}{\cos\phi}$ ....(iii)

And $\tan\phi = \frac{l}{a}$ or $l = a\tan\phi$

Differentiating it, we get $dl = a\sec^2\phi d\phi \quad ......(iv)$

Putting the values in (i) from (ii), (iii) and (iv), we get

$$dB = \frac{\mu_0}{4\pi}\frac{I\left(a\sec^2\phi d\phi\right)\cos\phi}{\left(\frac{a^2}{\cos^2\phi}\right)} = \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi d\phi \quad ....(v)$$

The direction of $d\vec{B}$, according to right hand thumb rule, will be perpendicular to the plane of paper and directed inwards. As all the current elements of the conductor will also produced magnetic field in the same direction, therefore, the total magnetic field at point P due to current through the whole straight conductor XY can be obtained by integrating equ. (v) within the limits $-\phi_1$ and ϕ_2. Thus

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0 I}{4\pi a}\int_{-\phi_1}^{\phi_2} \cos\phi d\phi = \frac{\mu_0 I}{4\pi a}\left[\sin\phi\right]_{-\phi_1}^{\phi_2} = \frac{\mu_0 I}{4\pi a}\left[\sin\phi_2 - \sin(-\phi_1)\right] = \frac{\mu_0 I}{4\pi a}\left(\sin\phi_1 + \sin\phi_2\right) \quad \text{....(vi)}$$

Special Cases. (i) When the conductor XY is of infinite length and the point P lies near the centre of the conductor then $\phi_1 = \phi_2 = 90°$

So, $B = \dfrac{\mu_0 I}{4\pi a}\left[\sin 90° + \sin 90°\right] = \dfrac{\mu_0 2I}{4\pi a}$ .....(vii)

(ii) When the conductor XY is of infinite length but the point P lies near the end Y (or X).

then $\phi_1 = 90°$ and $\phi_2 = 0°$.

So, $B = \dfrac{\mu_0 I}{4\pi a}\left[\sin 90° + \sin 0°\right] = \dfrac{\mu_0 I}{4\pi a}$ ......(viii)

Thus magnetic field due to an infinite long linear conductor carrying current near its centre is twice than that near its one of the ends.

(iii) If length of conductor is finite say L and point P lies on right bisector of conductor, then $\phi_1 = \phi_2 = \phi$ and

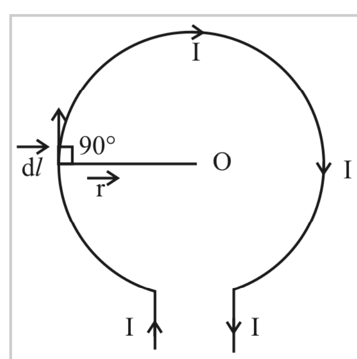
$$\sin\phi = \frac{L/2}{\sqrt{a^2 + (L/2)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

Then $B = \dfrac{\mu_0 I}{4\pi a}\left[\sin\phi + \sin\phi\right] = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a}\sin\phi = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a}\dfrac{L}{\sqrt{4a^2 + L^2}}$

9. MAGNETIC FIELD AT THE CENTRE OF THE CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius r with centre O, lying with its plane in the plane of paper, Let I be the current flowing in the circular coil in the direction shown, figure. Supppose the circular coil is made of a large number of current elements each of length dl.

According to Biot-Savart's law, the magnetic field at the centre of the circular coil due to the current element $I\vec{dl}$ is given by



$$d\vec{B} = \frac{\mu_0}{4\pi} I\left(\frac{d\vec{l} \times \vec{r}}{r^3}\right)$$

where $\vec{r}$ is the position vector of point O from the current element.

Since the angle between $d\vec{l}$ and $\vec{r}$ is 90° (*i.e.*, θ = 90°).

$\therefore \quad d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Idl\, r\sin\theta}{r^3} = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin 90°}{r^2}$

or $\quad dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl}{r^2}$ ....(i)

In this case, the direction of $d\vec{B}$ is perpendicular to the plane of the current loop and is directed inwards. Since the current through all the elements of the circular coil will contribute to the magnetic field in the same direction, therefore, the total magnetic field at point O due to current in the whole circular coil can be obtained by integrating equation (v). Thus

$$B = \int dB = \int \frac{{}_0}{4\pi}\frac{Idl}{r^2} = \frac{{}_0}{4\pi}\frac{I}{r^2}\int dl$$

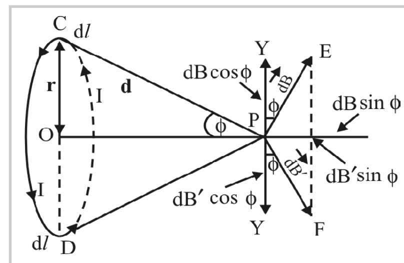
But $\int dl$ = total lengthof the circular coil = circumference of the current loop = $2\pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}.2\pi r = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$$

If the circular coil consists of *n* turnes, then $B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = \frac{\mu_0}{4\pi}\frac{I}{r}\times 2\pi n$.

10. MAGNETIC FIELD AT A POINT ON THE AXIS OF A CIRCULAR CURRENT LOOP

A circular loop of radius *r* and centre O with its plane perpendicular to the plane of the paper carries a current I as shown in the figure.



Point P is on the axis of the loop at a distance *x* from the centre O.

Taking two small elements each of length d*l* at the diametrically opposite edges C and D, we have,

$$PC = PD = d = \sqrt{r^2 + x^2}$$

and $\angle CPO = \angle DPO = \phi$

As r is very small, the angle between $d\vec{\ell}$ and $\vec{d}$ can be taken as 90°. The magnitude of field at P due to each element at C and D is $dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{d^2}\sin 90° = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{\left(r^2 + x^2\right)}$

The direction of $d\vec{B}$ by the element at C is along PE perpendicular to CP and that due to the element at D is along PF perpendicular to PD.

Both can be resolved into rectangular components as shown in the figure. Vertical components (cos) cancel each other as they are equal and opposite. The horizontal components (sine) add up as they are in the same direction.

This is true for all the pairs of diametrically opposite current elements and the net field at P is given by,

$$B = \int dB\ \sin\phi = \int \frac{\mu_0 I\ d\ell\ \sin\phi}{4\pi\left(r^2 + x^2\right)} = \frac{\mu_0 I\ \sin\phi}{4\pi\left(r^2 + x^2\right)}\int d\ell$$

But, $\sin\phi = \frac{d}{\sqrt{r^2 + x^2}}$ and $\int d\ell = 2\pi r$

Then, $B = \frac{\mu_0 I}{4\pi\left(r^2 + x^2\right)}\frac{r}{\sqrt{r^2 + x^2}}\times 2\pi r$

Or, $B = \frac{\mu_0 I r^2}{2\left(r^2 + x^2\right)^{\frac{3}{2}}}$, directed along PX

If the field is along the *x*-axis, it can be given as $\vec{B} = B_x\hat{i} = \frac{\mu_0 I r^2}{2\left(r^2 + x^2\right)^{\frac{3}{2}}}\hat{i}$

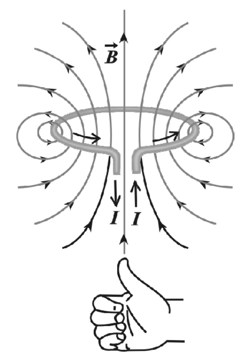
At the centre, *x* = 0 and so

$$\vec{B}_0 = \frac{{}_0 I}{2r}\hat{i}$$

Magnitude of the field at the centre, $B_0 = \frac{\mu_0 I}{2r}$

Note: For a coil of N turns

$B = \frac{{}_0 I N r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}$ at a point on the axis and $B = \frac{{}_0 NI}{2r}$ at the centre of the coil



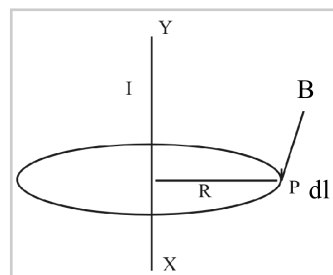
The direction of magnetic field due to a circular loop carrying current

Right-hand thumb rule gives the direction of the magnetic field due to a circular current loop.

Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current. The right-hand thumb gives the direction of the magnetic field.

11. AMPERE'S CIRCUITAL LAW

According to Ampere's circuital law, the line integral of magnetic field around any closed path (or circuit) in vacuum is equal to μ_0 (absolute permeability of space) times the total current (I) threading the closed path.



Mathematically, $\oint \vec{B}.d\vec{\ell} = {}_0 I$

It can be proved for a closed path of any size and shape carrying a current.

Magnetic field due to an infinitely long straight conductor carrying current

Let XY be an infinitely long wire carrying a current I. Let P be a point situated at a distance R from the wire. Draw a circle of radius R around the wire as shown in the figure above.

By symmetry, magnetic field $\vec{B}$ is directed tangentially to the circle at all points.

$$\therefore \quad \oint \vec{B}.\vec{dl} = \oint B\,dl = B\oint dl = B(2\pi R)$$

But by Ampere's law, $\oint \vec{B}.\vec{dl} = {}_0 I$

Comparing the two results, we have $B 2\pi R = \mu_0 I$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R}$$

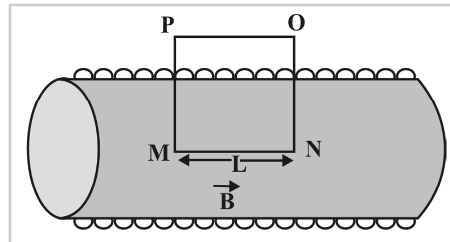
12. MAGNETIC FIELD DUE TO SOLENOID AND TOROID

A solenoid is a tightly and closely wound helical coil made of a long insulated wire. Its length is very large as compared to its diameter.

Let *n* be the number of turns per unit length in a very long straight solenoid with current I passing through it. A magnetic field is set up in the solenoid as shown in the diagram.

Let us consider a rectangle MNOP as shown in the above figure where MN = L. The line integral of magnetic field induction over the closed path MNOP is

$$\oint \vec{B}.d\vec{\ell} = \int_M^N \vec{B}.d\vec{\ell} + \int_N^O \vec{B}.d\vec{\ell} + \int_O^P \vec{B}.d\vec{\ell} + \int_P^M \vec{B}.d\vec{\ell}$$



We have $\int_M^N \vec{B}.d\vec{\ell} = \int_M^N B\,d\ell \cos 0 = BL$ and

and $\int_N^O \vec{B}.d\vec{\ell} = \int_N^O B\,d\ell \cos 90 = 0 \quad and \quad \int_O^P \vec{B}.d\vec{\ell} = \int_P^M \vec{B}.d\vec{\ell} = 0$

since outside the solenoid, B = 0

$\Rightarrow \oint \vec{B}.d\vec{\ell} = BL \qquad (i)$

By Ampere's Circital law,

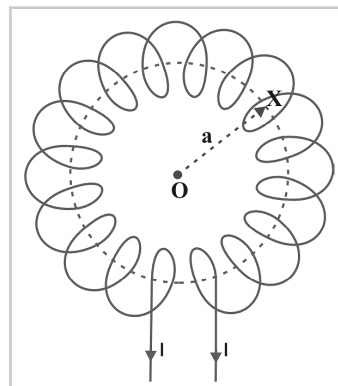
$\oint \vec{B}.d\vec{\ell} = {}_0 \times total \;\; current$ in $the \;\; rec\tan gle \;\; MNOP$

$= {}_0 nLI \quad \ldots (ii)$

(where n is the number of turns of the coil per unit length)

From (i) and (ii), $B = {}_0 nI$

At any end point of a solenoid, $B = \dfrac{{}_0 nI}{2}$



Magnetic Field Due to a Current Carrying Toroid

Let a toroid with n number of turns per unit length carry a current I. X is a point inside the turns of the toroid which is at a distance 'a' from its centre O. The magnetic field at X is to be found out.

By Ampere's circuital law,

$\oint \vec{B}.d\vec{\ell} = {}_0 \times$ total current passing through the circle of radius 'a'

$= \mu_0 2\pi a \; nI \qquad \ldots (iii)$ But $\oint \vec{B}.d\vec{\ell} = B \times 2\pi a \qquad \ldots (iv)$

From (iii) and (iv) $B = {}_0 nI$

Inside the toroid, the field has a constant magnitude and it is always tangential to the circular closed path.

13. FORCE BETWEEN TWO PARALLEL LIVE CONDUCTORS

Figure shows two infinite long straight conductors in the plane of the paper at a distance d apart carrying currents I_1 and I_2 in the same direction.

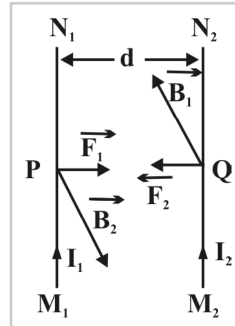
Each conductor is in the magnetic field produced by the other and hence experiences a force. Magnetic field induction at a point P on conductor M_2N_2 due to current I_1 passing through M_1N_1 is

$$B_1 = \frac{{}_0 I_1}{2\pi d} \quad \ldots (i)$$

Using the right hand rule, the direction of magnetic field B is perpendicular to the plane of paper, directed inwards.

Unit length of M_2N_2 will experience a force given by $F_2 = B_1 I_2 \times 1 = B_1 I_2 \quad \ldots (ii)$

From (i) and (ii), $F_2 = \frac{{}_0 I_1 I_2}{2\pi d}$



By Fleming's left hand rule, this force acts in the plane of the paper perpendicular to M_2N_2 towards M_1N_1.

Similarly, M_1N_1 also experiences a force given by $F_1 = \frac{{}_0 I_1 I_2}{2\pi d}$

It acts in the plane of paper, perpendicular to M_1N_1 and directed towards M_2N_2.

Two linear parallel conductors carrying current in the same direction attract each other and those carrying current in opposite directions repel each other with a force given by

$$F = \frac{{}_0 I_1 I_2}{2\pi d} \qquad \ldots(iii)$$

Unit of the Intensity of Electric current - Ampere

In expression (iii), if $I_1 = I_2 = 1$ A and $d = 1$ m, then $F = \frac{{}_0 \times 1 \times 1}{2\pi \times 1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}\ N$

One ampere is the value of that steady current which, when maintained in each of the two very long straight conductors of negligible cross section placed 1 m apart in vacuum, would produce on each of these conductors a force of 2×10^{-7} N per unit length.

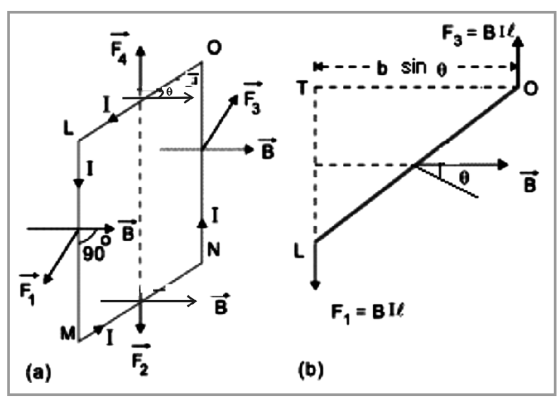
14. TORQUE ON A CURRENT LOOP, MAGNETIC DIPOLE

Torque on a Rectangular Current Loop in a Uniform Magnetic Field

Consider a rectangular coil LMNO carrying current I in the direction LMNO, suspended in a uniform magnetic field as in the figure. Let LM = NO = ℓ and MN = OL = b. Let θ be the angle between the plane of the coil and the magnetic field. There will be forces acting on the four arms of the coil.

Let $\vec{F}_1, \vec{F}_2, \vec{F}_3$ and $\vec{F}_4$ denote the forces acting on the four current carrying arms LM, MN, NO and OL respectively.

The force on arm OL is given by



$$\vec{F}_4 = I\left(\overrightarrow{OL} \times \vec{B}\right)$$

and $F_4 = I\,(OL)\,B\sin(180° - \theta) = I b B \sin\theta$

The direction of this force is in the plane of the coil, directed upwards.

The force on MN is $\vec{F}_2 = I\left(\overrightarrow{MN} \times \vec{B}\right)$ and $F_2 = I b B \sin\theta$

The direction of this force is in the plane of the coil, directed downwards.

Since $\vec{F}_2$ and $\vec{F}_4$ are equal and opposite, they cancel each other.

Force on arm LM is $\vec{F}_1 = I(\overrightarrow{LM} \times \vec{B})$

and $F_1 = I(LM)B\sin 90° = I\ell B$

From Fleming's left-hand rule, the direction of force is perpendicular to the plane of the coil and is directed outwards.

Force on arm NO is

$\vec{F}_3 = I(\overrightarrow{NO} \times \vec{B})$ *Or* $F_3 = I(NO)B\sin 90^\circ = I\ell B$

Again, Fleming's left-hand rule suggests that the direction of this force is perpendicular to the plane of the coil and directed away from the paper.

Forces acting on LM and NO are equal, opposite and parallel, constituting a couple. It produces rotation in the coil in anticlockwise direction.

The torque on the coil is $\tau = \textit{Either force} \times \textit{arm of the couple}$

Arm of the couple, $OT = LO\ \sin\theta = b\sin\theta$

Therefore, $\tau = I\ell B \times b\sin\theta = I\,B\,A\sin\theta$

Also, $\tau = mB\sin\theta$

where *m* is the magnetic moment of the current loop, which is given by:

$m = I\,A$

For a rectangular coil of n turns,

$\tau = nI\,A\,B\sin\theta$ and $m = n\,I\,A$

Torque in vector form, $\vec{\tau} = \vec{m} \times \vec{B}$

15. CIRCULAR CURRENT LOOP AS A MAGNETIC DIPOLE

The magnetic field B on the axis of a circular loop of radius R, carrying a steady current *I* is given by:

$$B = \frac{{}_0IR^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Its direction is given by the right hand thumb rule. When $x >> R$, the above expression becomes

$$B = \frac{{}_0IR^2}{2x^3} \qquad \textit{Or}, \qquad B = \frac{{}_0IR^2}{2x^3} \qquad (\because\ A = \pi R^2\text{, the area of the loop})$$

The magnetic moment of a circular current loop is $M = IA$

Therefore, $B = \dfrac{{}_0m}{2\pi x^3} = \dfrac{{}_0}{4\pi}\dfrac{2m}{x^3}$

The above expression resembles the expression for electric field due to an electric dipole at a point on the axial line that is *x* distance away from the dipole, which is:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2P}{x^3}$$

An expression similar to that for the electric field along the equatorial line of an electric dipole can be derived in the case of the magnetic field due to a planar current loop also.

Thus it can be shown that a planar current loop is equivalent to a magnetic dipole of dipole moment m that is given by:

$m = IA$

16. THE MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

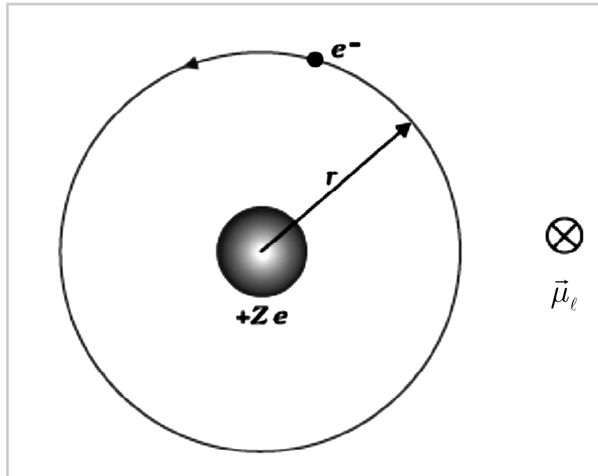
Let an electron of charge *–e* perform uniform circular motion around a stationary heavy nucleus of charge *+Ze*. This constitutes a current I, such that

$I = \frac{e}{T}$ ...(*i*)

where T is the period of revolution of the electron.

If the radius of the orbital and speed of electron are r and v respectively, then

$T = \frac{2\pi r}{v}$ ...(*ii*)



From (*i*) and (*ii*), $I = \frac{ev}{2\pi r}$ ...(*iii*)

Magnetic moment is given by:

$\mu_\ell = IA = I\pi r^2$ ...(*iv*)

From (*iii*) and (*iv*), ${}_\ell = \frac{evr}{2}$ ...(*v*)

This magnetic moment is directed into the plane of paper/ screen.

If m_e is the mass of the electron, equation (v) can be rewritten as:

${}_\ell = \frac{e}{2m_e}(m_e vr)$ *Or* ${}_\ell = \frac{e}{2m_e}\ell$ ...(*vi*)

l is the magnitude of the angular momentum of electron about the central nucleus.

In vector form, $\vec{}_\ell = -\frac{e}{2m_e}\vec{\ell}$ (–ve sign indicates that the direction of angular momentum of the electron is opposite to that of its magnetic moment).

From (vi), $\frac{{}_\ell}{\ell} = \frac{e}{2m_e}$ ...(*vii*)

Equation (*vii*) gives the gyromagnetic ratio. For electron, its value is 8.8×10^{10} C kg^{-1}.

According to postulates of Bohr model, the angular momentum of electron has discrete sets of values given by:

$\ell = \frac{nh}{2\pi}$; (n = 1, 2, 3, …)

With this, equation (vi) becomes

${}_\ell = \frac{nhe}{4\pi m_e}$; (n = 1, 2, 3, …)

Putting n = 1, the elementary dipole moment of electron is:

${}_{\ell(\min)} = \frac{he}{4\pi m_e}$

This works out to be 9.27×10^{-24} Am^2. This value is referred to as Bohr magneton.

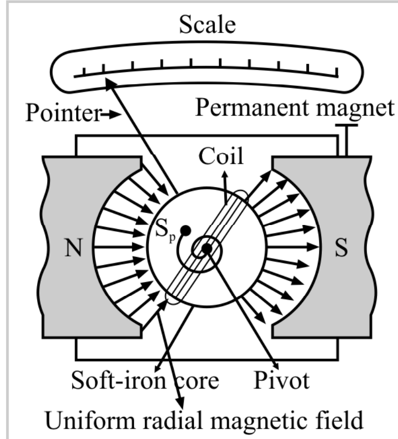
Electron possesses orbital and spin magnetic moments, which have the same numerical values.

Not only electron, but also any charged particle in revolution has magnetic dipole moment that can be calculated using expression similar to equation (*vi*)

17. MOVING COIL GALVANOMETER (MCG)

Principle and Working of a Moving Coil Galvanometer

Principle: When a coil carrying current is freely suspended in a magnetic field, it experiences a torque and tends to be twisted.



Construction and Working:

A labeled diagram of moving coil galvanometer has been shown in adjoining figure

As the pivoted coil is placed in a radial magnetic field, hence on passing current I through it a deflecting torque acts on the coil which is given be

$\tau = NA\,IB$

Where N = total number of turns in the coil, A = area of coil, B = magnetic field.

The spring S_p attached to the coil provides the counter torque and in equilibrium state balances the deflecting torque. If θ be the steady angular deflecting then counter torque is $C\theta$, where C = torsional constant (restoring torque per unit twist) of the spring.

In equilibrium state

$$NIAB = C\theta$$

$$I = \frac{C}{NAB}\theta$$

Thus $I = k\theta$ where $k = \frac{C}{NAB}$ and is called the galvanometer constant.

The deflection per unit current, called the current sensitivity [C.S.] of the galvanometer is given by:

$$C.S = \frac{\theta}{I} = \frac{d\theta}{dI} = \frac{NAB}{C} = \frac{1}{k}$$

It is also known as sensitiveness of the galvanometer

Also, the deflection per unit pd across the coil is called voltage sensitivity (V.S.)

$$V.S = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{C.S}{R}$$

The sensitivity of galvanometer is high if a small current causes a large deflection.

18. CONVERSION OF GALVANOMETER INTO AN AMMETER

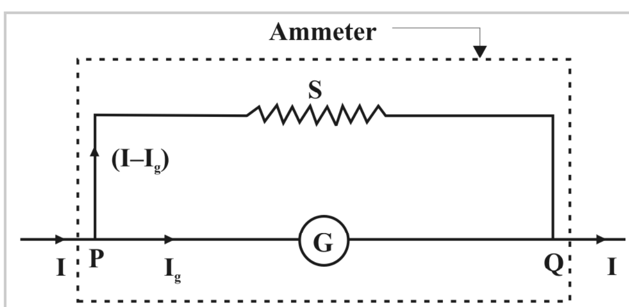
An ammeter is a galvanometer with a low resistance used to measure currents in electrical circuits. A galvanometer can be converted into an ammeter by connecting a low resistance (shunt) in parallel.

If G be the resistance, K, the fgure of merit or current for one scale deflection in the galvanometer and n, the number of scale divisions in the galvanometer, then the current causing full-scale deflection in the galvanometer is given by

$$I_g = nK$$

If S is the shunt resistance to be determined,

From the figure,



$$I_g G = (I - I_g)S$$

Or $$S = \frac{I_g G}{(I - I_g)}$$

Effective resistance R_p of ammeter is:

$$\frac{1}{R_p} = \frac{1}{G} + \frac{1}{S} = \frac{S+G}{GS} \Rightarrow R_p = \frac{GS}{G+S}$$

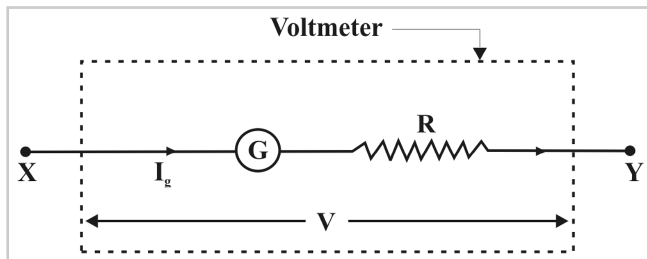
An ideal ammeter has zero resistance.

Conversion of Galvanometer into a Voltmeter

For this, a high resistance is connected in series with the galvanometer. If potential difference V is to be measured by the galvanometer, a resistance R is connected in series with the galvanometer such that when a potential difference V is applied across the terminals A and B, a current I_g flows through it.

Total resistance of voltmeter = G + R

Using Ohm's law,



$$I_g = \frac{V}{G+R} \qquad \textit{Or,} \quad G + R = \frac{V}{I_g}$$

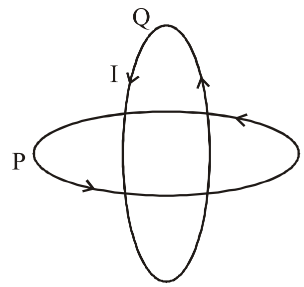
i.e. $$R = \frac{V}{I_g} - G$$

Thus, when resistance R is connected in series with the galvanometer, the range of the voltmeter becomes 0 to V volt.

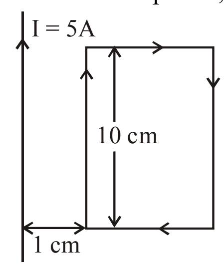
An ideal voltmeter has infinite resistance.

2012

1. Two identical circular wires P and Q each of radius R and carrying current 'I' are kept in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils. **(2 Marks)**



2. A rectangular loop of wire of size 4 cm × 10 cm carries a steady current of 2 A. A straight long wire carrying 5 A current is kept near the loop as shown. If the loop and the wire are coplanar, find **(3 Marks)**



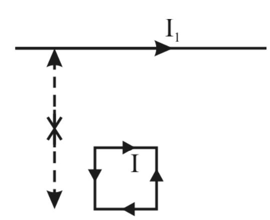
2011

1. State Biot-Savart law, giving the mathematical expression for it.
Use this law to derive the expression for the magnetic field due to a circular coil carrying curent at a point along its axis.
How does a circular loop carrying current behave as a magnet? **(5 Marks)**

2. With the help of a labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it workd to accelerate the charged particles.
Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason. **(5 Marks)**

2010

1.(a) Write the expression for the magnetic moment ($\vec{m}$) due to a planar square loop of side 'l' carrying a steady current I in a vector form.
In the given figure this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reason to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop. **(2 Marks)**



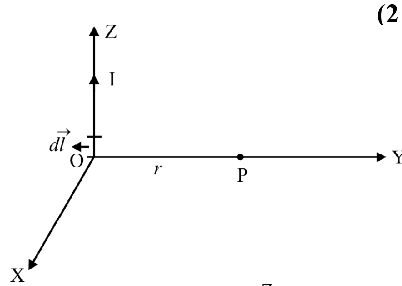
(b) A long straight wire of a circular cross-section of radius 'a' carries a steady current 'I'. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point 'r' in the region for (i) r < a and (ii) r > a. **(3 Marks)**

2. State the underlying principle of working of a moving coil galvanometer. Write two reasons why a galvanometer can not be used as such to measure current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends. **(5 Marks)**

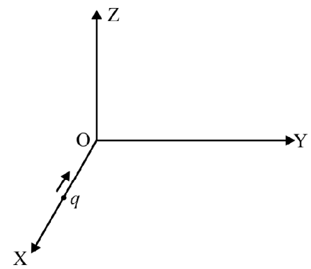
2009

1. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why. **(1 Marks)**

2. State Biot-Savart law. **(2 Marks)**
A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element $\vec{dl}$ at point P situated at a distance $\vec{r}$ from the element as shown in the figure.



3. A charge 'q' moving along the X-axis with a velocity $\vec{v}$ is subjected to a uniform magnetic field B acting along the Z-axis as it crosses the origin O. **(2 Marks)**



(*i*) Trace its trajectory.
(*ii*) Does the charge gain kinetic energy as it enters the magnetic field. Justity your answer.

4. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere. **(3 Marks)**

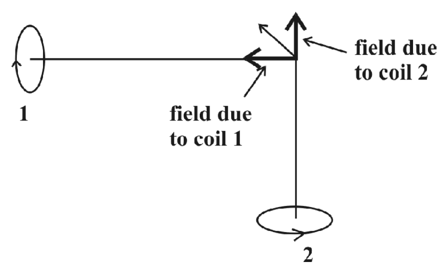
5. (*a*) Define self inductance. Write its S.I. units.
(*b*) Derive an expression for self inductance of a long solenoid of length l, cross sectional area A having N number of turns. **(3 Marks)**

2008

1. What is the direction of the force acting on a charged particle q, moving with a velocity $\vec{V}$ in a uniform magnetic field $\vec{B}$. **(2 Marks)**

2. Define the term: magnetic dipole moment of a current loop. Write the expression for the magnetic moment when an electron revolves at a speed 'v' around an orbit of radius 'r' in hydrogen atom. **(2 Marks)**

3. (*a*) Using Biot-Savart's law, derive an expression for the magnetic field at the centre of a circular coil of radius R, number of turns N, carrying current i. **(2½ Marks)**
(*b*) Calculate the resultant field produced by the two current carrying coils 1 and 2 at P as shown in the figure. **(2½ Marks)**



4. Draw a schematic diagram of a cyclotron. Explain its underlying principle and working, stating clearly the function of the electric and magnetic fields applied on a charged particle. Deduce an expression for the period of revolution of the charged particle and show that it does not depend on the speed of the charged particle. **(5 Marks)**

2007

1. An electron is moving a along +ve x-axis in the presence of uniform magnetic field along +ve y-axis. What is the direction of the force acting on it. **(1 Mark)**

2. Write the relation for the force acting on a charge carrier q moving with a velocity through a magnetic field in vector notation. Using this relation, deduce the conditions under which this force will be:
(*i*) Maximum (*ii*) Minimum. **(2 Marks)**

3. A galvanometer has a resistance of 5Ω. It gives full scale deflection with a current of 2 mA. Calculate the value of the resistance needed to convert it into an ammeter of range 0 – 0.3 A. **(3-marks)**

4. State Ampere's circuital law. Write the expression for the magnetic field at the centre of a circular coil of radius R carrying a current I. Draw the magnetic field lines due to this coil. **(3 Marks)**

5. Write the expression for the force acting on a charged particle of charge q moving with velocity in the presence of magnetic field. Show that in the presence of this force: **(3 Marks)**
(*i*) The kinetic energy of the particle does not change. (*ii*) Its instantaneous power is zero.

6. Draw a labelled diagram of a moving coil galvanometer. State the principle on which it works. Deduce an expression for the torque acting on a rectangular current carrying loop kept in a uniform magnetic field. Write two factors on which the current sensitivity of a moving coil galvanometer depends. **(5 Marks)**

7. State Biot-Savart law. Use it to derive an expression for the magnetic field at the centre of a circular loop of radius R carrying a steady current I. Sketch the magnetic field lines for such a current carrying loop. **(5 Marks)**

2006

1. State the principle of working of a cyclotron. Write two uses of this machine. Derive an expression for the maximum force experienced by a straight conductor of length *l,* carrying current I and kept in a uniform magnetic field, B
(2 Marks)

2. Using Biot — Savart law derive the expression for the magnetic field at a distance x along the axis from the centre of a current carrying circular loop. **(3 Marks)**

3. With the help of a neat and labelled diagram, explain the underlying principle and working of a moving coil galvanometer. In such a device, what is the function of: **(5 Marks)**
(*i*) Uniform radial field.
(*ii*) Soft iron core.

4. Derive a mathematical expression for the force per unit length experienced by each of the two long current carrying conductors placed parallel to each other in air. Hence define one ampere of current.
(5 Marks)

5. Explain why two parallel straight conductors carrying current in the opposite direction kept near each other in air repel.
(5 Marks)

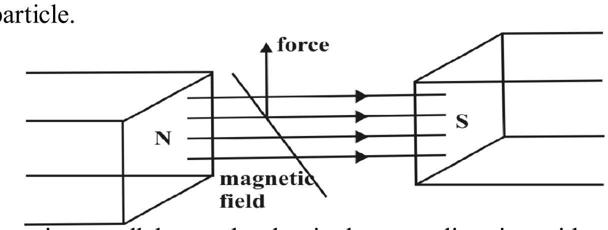
Draw a neat and labelled diagram of a cyclotron. State the underlying principle and explain how a positively charged particle gets accelerated in this ma chine. Show mathematically that the cyclotron frequency does not depend upon the speed of the particle.

6. State the Biot - Savart law for the magnetic field due to a current carrying element. Use this law to obtain a formula for magnetic field at the centre of a circular loop of radius R carrying a steady current I. Sketch the magnetic field lines for a current loop clearly indicating the direction of the field.

2005

1. Under what conditions is the force acting on a charge moving through a uniform magnetic field minimum.
(1 Mark)

2. A charged particle enters into a uniform magnetic field and experiences an upward force as indicated in the figure. What is the charge sign on the particle. **(1 Mark)**



3. An electron and a proton, moving parallel to each other in the same direction with equal momenta, enter into a uniform magnetic field which is at right angles to their velocities. Trace their trajectories in the magnetic field.
(1 Mark)

4. Two wires of equal lengths are bent in the form of two loops. One of the loops is square shaped whereas the other loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque. Give reasons. **(1 Mark)**

5. A galvanometer with a coil of resistance 120 ohm shows full scale deflection for a current of 2.5 mA. How will you convert the galvanometer into an ammeter of range 0 to 7.5 A. Determine the net resistance of the ammeter. When an ammeter is put in a circuit, does it read slightly less or more than the actual current in the original circuit. Justify your answer. **(3 Marks)**

6. (*a*) With the help of a labelled diagram, explain the principle and working of a moving coil galvanometer.
(2½ × 2 = 5 Marks)
(*b*) Two parallel coaxial circular coils of equal radius 'R' and equal number of turns 'N', carry equal currents ' I ' in the same direction and are separated by a distance '2R'. Find the magnitude and direction of the net magnetic field produced at the mid-point of the line joining their centres.

