PHYSICS

STUDY MATERIAL

ELECTROMAGNETIC INDUCTION

IIT-JEE

श्रमेव जयते

THE NARAYANA GROUP

NARAYANA INSTITUTE OF CORRESPONDENCE COURSES

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PREFACE

Dear Student,

Heartiest congratulations on making up your mind and deciding to be an engineer to serve the society.

As you are planning to take various Engineering Entrance Examinations, we are sure that this **STUDY PACKAGE** is going to be of immense help to you.

At NARAYANA we have taken special care to design this package according to the **Latest Pattern of IIT-JEE,** which will not only help but also guide you to compete for IIT-JEE, AIEEE & other State Level Engineering Entrance Examinations.

The salient features of this package include :

- Power packed division of units and chapters in a scientific way, with a correlation being there.
- Sufficient number of solved examples in Physics, Chemistry & Mathematics in all the chapters to motivate the students attempt all the questions.
- All the chapters are followed by various types of exercises, including Objective - Single Choice Questions, Objective - Multiple Choice Questions, Passage Based Questions, Matching Type Questions, Assertion-Reason & Subjective Type Questions.

These exercises are followed by answers in the last section of the chapter including Hints & Solutions wherever required. *This package will help you to know* what to study, how to study, time management, your weaknesses and improve your performance.

We, at NARAYANA, strongly believe that quality of our package is such that the students who are not fortunate enough to attend to our Regular Classroom Programs, can still get the best of our quality through these packages.

We feel that there is always a scope for improvement. We would welcome your suggestions & feedback.

Wish you success in your future endeavours.

THE NARAYANA TEAM

ACKNOWLEDGEMENT

While preparing the study package, it has become a wonderful feeling for the NARAYANA TEAM to get the wholehearted support of our Staff Members including our Designers. They have made our job really easy through their untiring efforts and constant help at every stage.

We are thankful to all of them.

THE NARAYANA TEAM

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ELECTROMAGNETIC INDUCTION

ELECTROMAGNETIC INDUCTION

IIT-JEE-Syllabus

Faraday's law, Lenz's law; Self and mutual inductance

INTRODUCTION

Whenever magnetic flux linked with a circuit changes, an e.m.f. is induced in the circuit. If the circuit is closed, a current is also induced in it. The e.m.f. and current produced lasts as long as the flux linked with the circuit changes. The phenomenon is called **electromagnetic induction (EMI)**.

FARADAY'S LAWS OF ELECTRO-MAGNETIC INDUCTION

On the basis of experimental observations Faraday summarized the phenomenon of electromagnetic induction by giving following laws.

a) Whenever magnetic flux linked with a closed coil changes, an induced e.m.f. (or induced current) is set up in the coil

b) This induced e.m.f. (or induced current) lasts as long as the change in magnetic flux continues.

c) The magnitude of induced e.m.f. is proportional to the rate of change of magnetic flux linked with the circuit. If ϕ is magnetic flux linked with the circuit at any instant t, then induced e.m.f.

$$\xi \propto \frac{d\phi}{dt} \qquad \ldots \text{(i)}$$

d) The direction of induced e.m.f. (or induced current, if circuit is closed) is such that it opposes the change in flux that produced it. This law is also called **Lenz's law.**

In view of Lenz's law, equation (i) takes the form

$$\xi \propto -\frac{d\phi}{dt}$$

Conventionally, the change in flux is given with one turn and if the coil contains N turns, then

$$\xi \propto -N\frac{d\phi}{dt}$$

If R is resistance of the circuit, then current induced

$$I = \frac{\xi}{R} = -\frac{N}{R}\frac{d\phi}{dt}$$

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The charge induced in time dt is given by

$$dq = Idt = -\frac{N}{R}\frac{d\phi}{dt}dt = -\frac{N}{R}d\phi$$

$$\Rightarrow \quad q = \int_0^q dq = -\frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi$$

$$\Rightarrow \quad |q| = \left(\frac{N}{R}\right)\Delta\phi$$

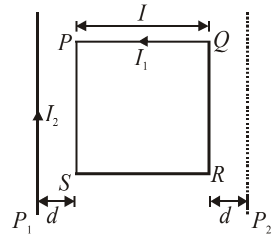
$$\Rightarrow \quad q = \frac{\text{Net Change in flux}}{\text{Resistance}}$$

So, we observe that the charge induced is independent of time.

Illustration 1. *A square wire frame PQRS of each side ℓ carrying a steady current I_1 is placed near a long straight conductor carrying a current I_2. The frame and the conductor are in one plane, with the length of the conductor parallel to the side PS and QR of the square frame as shown in the figure.*

Prove that the work done in moving the conductor from position P_1 to position P_2 is given by

$$W = \frac{-\mu_0}{\pi}(I_1 I_2 \ell)\log_e\left(\frac{d+\ell}{d}\right)$$



Solution : On changing the position of the long conductor from P_1 to P_2, there is a change of magnetic flux linked with the square frame *PQRS* carrying a current I_1.

The magnetic flux Φ_1 linked with loop due to the current I_2 in the conductor lying at position P_1 is

$$\Phi_1 = \int_d^{d+\ell} \frac{\mu_0 I_2}{2\pi r}\ell dr$$

$$\Rightarrow \quad \Phi_1 = \frac{\mu_0 I_2 \ell}{2\pi}\log_e\left(\frac{d+\ell}{d}\right) \cos 180^\circ$$

$$= -\frac{\mu_0 I_2 \ell}{2\pi}\log_e\left(\frac{d+\ell}{d}\right)$$

Similarly, the magnetic flux Φ_2 linked with the wire frame due to current I_2 in the conductor at position P_2 is given by

$$\Phi_2 = \frac{\mu_0 I_2 \ell}{2\pi}\log_e\left(\frac{d+\ell}{d}\right) \cos 0^\circ$$

$$= \frac{\mu_0 I_2 \ell}{2\pi}\log_e\left(\frac{d+\ell}{d}\right)$$

So, change in the value of the flux linked with the loop when the wire moves form first position to the second is given by

$$\Delta\Phi = \Phi_2 - \Phi_1 = \frac{\mu_0 I_2 \ell}{\pi}\log_e\left(\frac{d+\ell}{d}\right)$$

Further by definition, work done is given by

$$\Delta W = I_1\Delta\Phi = -\frac{\mu_0 I_1 I_2 \ell}{\pi}\log_e\left(\frac{d+\ell}{d}\right)$$

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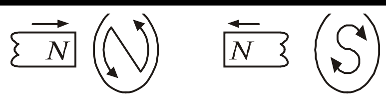
LENZ'S LAW

Lenz's law is based on Law of Conservation of Energy and it gives the nature of induced e.m.f. or direction of induced current in the coil. When north pole of a magnet is moved towards the coil, the induced current flows in a direction so as to oppose the motion of the magnet towards the coil. This is only possible when nearer face of the coil acts as a magnetic north pole which makes an anticlockwise current to flow in the coil. Then the repulsion between two similar poles opposes the motion of the magnet towards the coil.

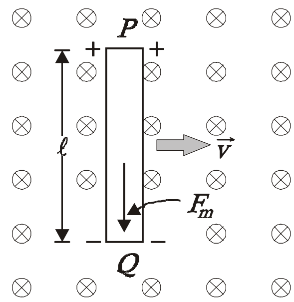


(a) (b)

Similarly, when the magnet is moved away from the coil, the direction of induced current is such as to make the nearer face of the coil as a south pole which makes a clockwise induced current to flow in the coil. Then the attraction between two opposite poles opposes the motion of the magnet away from the coil. In either case, therefore work has to be done in moving the magnet. It is this mechanical work, which appears as electrical energy in the coil. Hence the production of induced e.m.f. or induced current in the coil is in accordance with the Law of Conservation of Energy.

INDUCED E.M.F. IN A CONDUCTING ROD MOVING THROUGH A UNIFORM MAGNETIC FIELD

Let a thin conducting rod *PQ* of length ℓ move in a uniform magnetic field *b* directed perpendicular to plane of paper, inwards. Let the velocity v of rod be in the plane of paper towards right.



By Fleming's left hand rule a positive charge (q) in the rod suffers magnetic force qvB directed from Q to P along the rod while an electron will experience a evB directed from P to Q along the length of rod. Due to this force, the free electrons of rod move from P to Q, thus making end Q negative and end P positive. This causes a potential difference along the ends of rod. The potential difference developed is called induced emf ξ. If E is electric field developed in the rod, then

$$E = \frac{\xi}{\ell}$$

ξ being e.m.f. induced across the rod.

For equilibrium of charges;

Electrical force = Magnetic force

$$\Rightarrow \quad eE = evB$$

$$\Rightarrow \quad E = vB$$

So, induced e.m.f. $\xi = E\,\ell = B\,\ell v$

If the rod moves across the magnetic field moving at an angle θ with it, then induced e.m.f. $\xi = B_n v\,\ell$,

where B_n is component of magnetic field normal to *v*. Here $B_n = B\sin\theta$

$$\Rightarrow \quad \text{induced e.m.f. } \xi = Bv\,\ell\sin\theta$$

IMPORTANT

If $\vec{\ell}$ is a vector directed along the direction of induced current, then a general notation for induced e.m.f. ξ is $\xi = \vec{B}.\left(\vec{\ell}\times\vec{v}\right)$

So, for e.m.f. or induced current to exist we must make sure that $\vec{B}$, $\vec{\ell}$ and $\vec{v}$ must never be coplanar.

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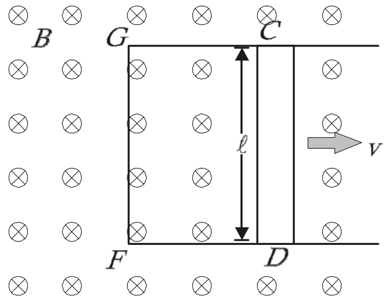
FLEMING'S RIGHT HAND RULE

The direction of induced current is given by Fleming's Right Hand Rule which states that stretch the First Finger, Middle Finger and the Thumb of Right Hand in such a way that all three are mutually perpendicular to each other. First finger points in the direction of field. Thumb points in the direction of motion of conductor, then Middle figure point along the direction of Induced Conventional Current.

INDUCED E.M.F. IN A LOOP BY CHANGING ITS AREA



(i) Consider a straight conductor *CD* moving with velocity v towards right on a *U*-shaped conducting guide placed in a uniform magnetic field *B* directed into the page. As the conductor moves, the area of the loop *CDFG* increases, causing a change in flux and hence an e.m.f. is induced in the coil. Let the loop move through *dx* in a time *dt*.

$$\xi = \frac{d\phi}{dt} = \frac{d}{dt}(BA) \qquad \text{(in magnitude)}$$

$$\Rightarrow \quad \xi = B\frac{dA}{dt}$$

$$\Rightarrow \quad \xi = B\ell\left(\frac{dx}{dt}\right) \qquad \{\because A = \ell dx\}$$

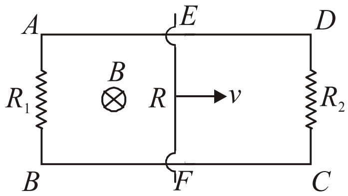
$$\Rightarrow \quad \xi = B\ell v$$

If R is the resistance of the loop then the induced current is

$$I = \frac{B\ell v}{R}$$

Direction of induced current is given by Fleming's Right Hand rule and it will flow anticlockwise.

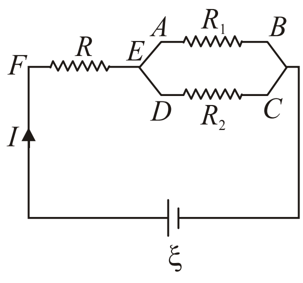
Illustration 2. *A rectangular loop with a sliding connector of length ℓ is located in a uniform magnetic field perpendicular to the plane of the loop. The magnetic induction is equal to B.*



The connector has an electric resistance R, the sides AB and CD have resistances R_1 and R_2 respectively. Neglecting the self inductance of the loop, find the current flowing in the connector during its motion with a constant velocity.

Solution : The magnitude of induced e.m.f. is

$$|\xi| = B\frac{dA}{dt} = B\ell v$$



The induced e.m.f. is distributed over the entire circuit in such a way that the current I flowing through R is divided at E into two resistances R_1 and R_2 and finally returns to F. According to Lenz's law, the current in loop *AEFB* must be in anticlockwise direction (so that field produced by this current is normally upwards, opposite to *B*) and in *EDCF,* current flows in clockwise direction. The effective resistance R_{eff} of the circuit is given by

$$R_{eff} = R + \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

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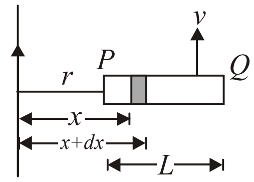
$$\Rightarrow \quad I = \frac{\xi}{R_{eff}} = \frac{B\ell v}{R + \left(\frac{R_1 R_2}{R_1 + R_2}\right)}$$

Illustration 3. *A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current I. The end P remains at a fixed perpendicular distance r from the wire. Calculate the e.m.f. induced across the rod.*

Solution : The rod moves in the magnetic field produced by the current carrying wire as a result of which an e.m.f. is induced across the rod. Let us consider a small element of length dx of the rod at a distance x and $x + dx$ from the wire. The e.m.f. induced across the element, $d\xi = Bvdx$ ...(1)



The magnetic field B at a distance x from a wire carrying a current I is given by

$$B = \frac{\mu_0 I}{2\pi x} \qquad \text{...(2)}$$

Using (2) in (1), we get

$$d\xi = \frac{\mu_0 I}{2\pi x} v dx \qquad \text{...(3)}$$

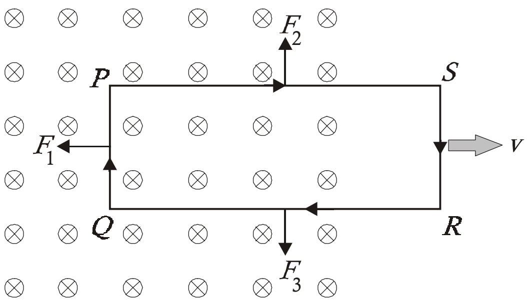
Induced emf across the entire length of the rod PQ is given by

$$\xi = \int d\xi = \int_P^Q \frac{\mu_0 I}{2\pi x} v\, dx$$

$$\Rightarrow \quad \xi = \int_r^{r+L} \frac{\mu_0 I}{2\pi x} v\, dx \qquad \Rightarrow \quad \xi = \frac{\mu_0 I v}{2\pi} \int_r^{r+L} \frac{dx}{x} = \frac{\mu_0 I v}{2\pi} [\log_e x]_r^{r+L}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I v}{2\pi} [\log_e (r+L) - \log_e r] \qquad \Rightarrow \quad \xi = \frac{\mu_0 I v}{2\pi} \log_e \left(\frac{r+L}{r}\right)$$

(ii) Consider the situation shown in the figure in which let the current flows clockwise.



Further, the current in the loop will cause force F_1, F_2 and F_3 to act on the three arms PQ, PS and QR respectively. F_2 and F_3, being equal and opposite, will cancel out, F_1 is given by

$$F_1 = BI\ell = \frac{B^2 \ell^2 v}{R}$$

$$\left\{\because I = \frac{B\ell v}{R}\right\}$$

The power required to pull the loop is

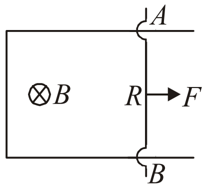
$$P = F_1 v = \frac{B^2 \ell^2 v^2}{R}$$

The rate at which Joule's heat is produced in the loop is

$$H = I^2 R = \frac{B^2 \ell^2 v^2}{R}$$

Which is same as P, as expected, according to the law of conservation of Energy.

Illustration 4. *A connector AB can slide without friction along π-shaped conductor located in a horizontal plane. The connector has a length ℓ, mass m and resistance R. The whole system is located in a uniform magnetic field of induction B directed inwards perpendicular to the plane of the paper. Initially, a constant horizontal force F starts acting on the connector translating to the right. Find how the velocity of the connector varies with time t. The inductance of the loop and the resistance of the π-shaped conductor are assumed to be negligible.*



Solution : Magnetic force on connector AB due to field is

$$F_m = BI\ell$$

$$F_m = B\left(\frac{\xi}{R}\right)\ell = B\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{R} \qquad \left\{I = \frac{\xi}{R} = \frac{B\ell v}{R}\right\}$$

Net force on the connector along the direction of motion is

$$F - F_m = ma$$

$$\Rightarrow F - \frac{B^2\ell^2 v}{R} = ma = m\frac{dv}{dt} \qquad \Rightarrow \frac{mdv}{\left(F - B^2\ell^2 v/R\right)} = dt$$

Integrating we get

$$\Rightarrow \int_0^v \frac{mdv}{\left(F - B^2\ell^2 v/R\right)} = \int_0^t dt \qquad \Rightarrow -\frac{m}{\left(B^2\ell^2/R\right)}\log_e\left(F - \frac{B^2\ell^2 v}{R}\right)\Bigg|_0^v = t$$

$$\Rightarrow \log_e\left(\frac{F - B^2\ell^2 v/R}{F}\right) = -\frac{B^2\ell^2}{mR}t \qquad \Rightarrow \left(\frac{F - B^2\ell^2 v/R}{F}\right) = e^{-\frac{B^2\ell^2}{mR}t}$$

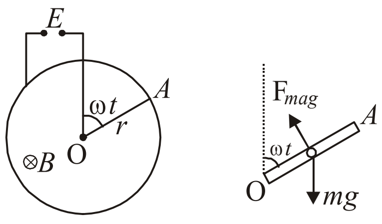
$$\Rightarrow v = \frac{RF}{B^2\ell^2}\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right) \qquad \Rightarrow v = \frac{F}{m\alpha}\left(1 - e^{-\alpha t}\right), \text{ where } \alpha = \frac{B^2\ell^2}{mR}$$

(iii) If a rod is rotated in the field with angular velocity ω, then the induced e.m.f. is

$$\xi = \frac{B\,(\text{Area swept by the Rod})}{\text{Time to complete one Revolution}}$$

$$\Rightarrow \xi = \frac{B\left(\pi\ell^2\right)}{\frac{2\pi}{\omega}} \qquad \Rightarrow \xi = \frac{1}{2}B\omega\ell^2$$

Illustration 5. *A metal rod OA of mass m can rotate about a horizontal axis O, sliding along a circular conducting ring of radius r as shown in figure. This arrangement is located in a uniform magnetic field of induction B, directed perpendicular to the plane of the ring. The axis O and the rim of ring are connected to a source of e.m.f. to form a non-inductive closed circuit of resistance R. Neglecting friction and ring resistance, find the expression for the e.m.f. in terms of time t, so that the rod rotates with constant angular velocity ω.*



(a) (b)

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Solution : Following forces are acting on the rod as shown in figure. The induced e.m.f. across the ends of the rod is

$$\xi = \frac{1}{2}B\omega r^2$$

Net e.m.f. in rod – ring circuit is

$$E = E_0 - \xi$$

If I is the current in the circuit then

$$I = \frac{E}{R} = \frac{E_0 - B\omega r^2/2}{R} \qquad \ldots(1)$$

The magnetic force on the rod opposing the motion is

$$F_m = BIr \qquad \{\text{perpendicular to the rod at mid-point}\}$$

Since the rod rotates with constant angular velocity so any point on the rod once taken will have a constant velocity throughout the entire tenure of rotation. So for equilibrium we have

$$F_m = mg\sin\omega t$$

$$\Rightarrow \quad BIr = mg\sin\omega t$$

$$\Rightarrow \quad I = \frac{mg\sin\omega t}{Br} \qquad \ldots(2)$$

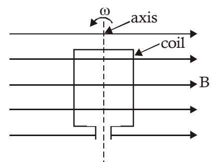
From equation (1) and (2), equating the value of I we get

$$\left\{\frac{\left(E_0 - B\omega r^2/2\right)}{R}\right\} rB = mg\sin\omega t$$

$$\Rightarrow \quad 2E_0 rB - r^3B^2\omega = 2mgR\sin\omega t$$

$$\Rightarrow \quad E_0 = \frac{1}{2rB}\left[r^3B^2\omega + 2mgR\sin\omega t\right]$$

(iv) Production of induced e.m.f. in a coil by rotating it in a magnetic field (The A.C. Generator) Suppose a coil of N turns, and area A is rotated in a uniform magnetic field B with angular velocity ω. As the coil rotates, the flux through it changes.



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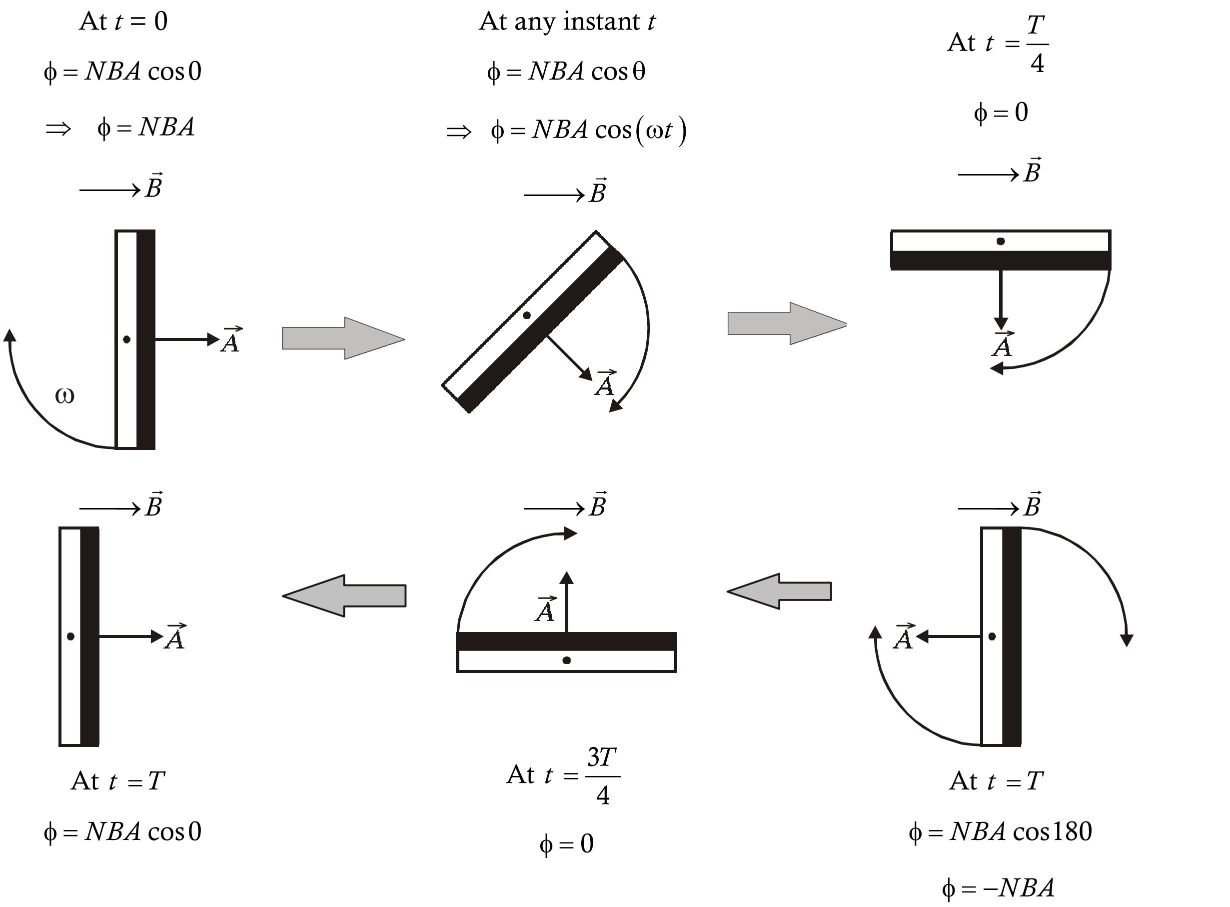


Figure showing cross-sectional view of coil rotating clockwise in a uniform magnetic field with uniform angular velocity ω

Due to this change in flux an induced e.m.f. is set up in the coil.

Since , $\quad \xi = \frac{d\phi}{dt}$

At any instant.

$$\phi = N\vec{B}.\vec{A} - NBA\cos\theta = BA\cos(\omega t)$$

So, $\quad \xi = -\frac{d}{dt}\left[NBA\cos(\omega t)\right]$

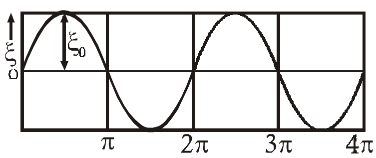
$\Rightarrow \quad \xi = NBA\omega\sin(\omega t)$

$\Rightarrow \quad \xi = \xi_0\sin(\omega t)$

where $\xi_0 = NBA\omega =$ Peak value of AC voltage developed.

We note that the induced e.m.f. ξ has a sinusoidal variation, having the peak value $NBA\omega$.

This forms the basic principle of the Alternating Current generator, which is a device converting mechanical energy into electric energy.



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BACK E.M.F. IN D.C. MOTOR

A motor is the reverse of generator—it converts electrical energy to mechanical energy. When current is passed through a coil placed in a magnetic field, it rotates. As the coil rotates, the magnetic flux linked with it changes, giving rise to an induced e.m.f. This e.m.f. opposes the applied e.m.f. (ξ) and is, therefore, called back e.m.f. (e). If R is the resistance of the coil, the current through it is given by

$$I = \frac{\xi - e}{R}$$

The efficiency of the motor is

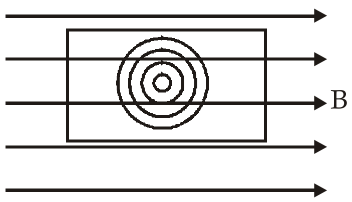
$$\eta = \frac{e}{\xi}$$

Maximum efficiency of a motor is 50%

i.e. $$\eta_{max} = \frac{e}{\xi} = \frac{1}{2}$$

EDDY CURRENTS OR FOUCAULT'S CURRENTS

When a metallic body is moved in a magnetic field in such a way that the flux through it changes or is placed in a changing magnetic field, induced currents circulate throughout the volume of the body. These are called eddy currents. If the resistance of the said conductor is small, then the magnitude of eddy currents are large and the metal gets heated up.



This heating effect is a source of power loss in iron-cored devices such as dynamos, motors and transformers. The effect can be reduced by laminating the core. Application of eddy currents are given below :

INDUCTION FURNACE

Joule's heat causes the melting of a metal piece placed in a rapidly changing magnetic field.

DEAD-BEAT GALVANOMETER

If a moving coil galvanometer is intended to attain steady value quickly, damping is necessary to prevent oscillation. This is achieved by winding the coil on a metallic frame – the large eddy currents induced in the frame provide electromagnetic damping.

MAGNETIC BRAKES

A drum is attached to the axle of the wheels of a train. It rotates when the train is moving. To stop the train, a strong magnetic field is applied to the rotating drum. The eddy currents set up in the drum oppose the rotation of the drum and the train stops.

SELF INDUCTANCE

Whenever the electric current flowing through a circuit changes, the magnetic flux linked with the circuit also changes. As a result of this, an induced e.m.f. is set up in the circuit. This phenomenon is called **self induction** and the induced e.m.f. is called the **back e.m.f.**

So, the phenomenon of the production of an induced e.m.f. in the coil when the flux linked with the coil changes is called phenomenon of self induction.

If I is the current flowing in the circuit, then flux linked with the circuit is proportional to I.

$\Rightarrow \quad \phi \propto I$

$\Rightarrow \quad \phi = LI$

where L is called coefficient of self-induction or self-inductance or simply inductance of the coil and its *SI* unit is henry (*H*)

$\Rightarrow \qquad I = 1 \text{ WbA}^{-1}$

if $\qquad I = 1$ A,

then $\qquad L = \phi$ (numerically)

So, inductance of a coil is numerically equal to the flux linked with the coil when the current in the coil is 1 A.

since , $\qquad \xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \qquad \xi = -\dfrac{d}{dt}(LI)$

$\Rightarrow \qquad \xi = L\dfrac{dI}{dt}$ (in magnitude)

if $\qquad \dfrac{dI}{dt} = 1\text{As}^{-1}$,

then $\qquad L = \xi$ (numerically)

So, inductance of a coil is numerically equal to the e.m.f. induced in the coil when the current in the coil changes at the rate of 1 As^{-1}.

IMPORTANT

Here we must note that if we are asked to calculate the induced e.m.f. in an inductor, then we have

$$\xi = -L\frac{dI}{dt}$$

But when we are asked to calculate the voltage (V) across the inductor then

$$V = |\xi| = L\frac{dI}{dt}$$

ENERGY STORED IN AN INDUCTOR

If dW is the infinitesimal work done in time *dt*, then

$$dW = VIdt$$

where V is voltage across an inductor.

Since, $\qquad V = L\dfrac{dI}{dt}$

$\Rightarrow \qquad dW = \left(\dfrac{LdI}{dt}\right)Idt = LIdI$

$\Rightarrow \qquad W = \displaystyle\int_0^I LIdI = \frac{1}{2}LI^2$

This work done is stored in the form of energy of the magnetic field in an inductor.

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Also, $L = \frac{2W}{I^2}$

If $I = 1A$

$L = 2W$

Thus the self-inductance of a circuit is numerically equal to twice the work done against the induced e.m.f. in establishing a current of 1 A in the coil.

IMPORTANT

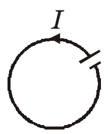
The role of self-inductance in an electrical circuit is the same as that of the inertia in mechanical motion. Thus the self-inductance of a coil is a measure of its ability to oppose the change in current through it and hence is also called electrical inertia.

CALCULATION OF SELF-INDUCTANCE

FOR A CIRCULAR COIL

Consider a circular coil of radius r and number of turns N. If current I passes in the coil, then magnetic field at centre of coil

$$B = \frac{\mu_0 NI}{2r}$$



The effective magnetic flux linked with this coil

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A$$

Since, by definition

$$L = \frac{\phi}{I}$$

$$\Rightarrow \quad L = \frac{\mu_0 N^2 A}{2r} = \frac{\mu_0 N^2 \pi r^2}{2r} \qquad \Rightarrow \quad L = \frac{\mu_0 N^2 \pi r}{2}$$

FOR A SOLENOID

Consider a solenoid with n number of turns per metre. Let current I flow in the windings of solenoid, then magnetic field inside solenoid is given by

$$B = \mu_0 nI$$

The magnetic flux linked with its length ℓ is $\phi = NBA$, where N is total number of turns in length ℓ of solenoid.

$$\Rightarrow \quad \phi = (n\ell)BA = n\ell(\mu_0 nI)A$$

Since, $L = \frac{\phi}{I}$

$$L = \mu_0 n^2 A\ell$$

Since, $n = \frac{N}{\ell}$

$$\Rightarrow \quad \text{Self-inductance, } L = \frac{\mu_0 N^2 A}{\ell}$$

MAGNETIC ENERGY DENSITY (U_m)

Since, $W = \frac{1}{2}LI^2$

For a solenoid

$$L = \mu_0 n^2 A\ell \text{ and } B = \mu_0 nI$$

$$\Rightarrow \quad W = \frac{1}{2}\left(\mu_0 n^2 A\ell\right)\left(\frac{B}{\mu_0 n}\right)$$

$$\Rightarrow \quad W = \left(\frac{B^2}{2\mu_0}\right)\left(A\ell\right)$$

Since, $\frac{W}{A\ell} = \frac{\text{Energy}}{\text{Volume}} = U_m =$ Magnetic Energy Density

$$\Rightarrow \quad U_m = \frac{B^2}{2\mu_0}$$

Thus making us conclude that the energy is stored in the form of magnetic field in a solenoid.

Illustration 6. *A long coaxial cable consists of two concentric cylinders of radii a and b. The central conductor of the cable carries a steady current I and the outer conductor provides the return path of the current. Calculate*

(i) the energy stored in the magnetic field of length ℓ of such a cable,

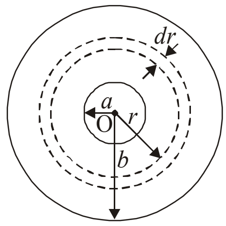
(ii) the self inductance of this length ℓ of the cable.

Solution : **(i)** The magnetic field *B* in the space between the two conductors is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where *I* = current in either of two conductors

r = distance of the point from the axis .



According to Ampere's Law

$\oint B.d\ell = \mu_0 \times$ current enclosed by the path

$$\Rightarrow \quad B(2\pi r) = \mu_0 I$$

The energy density in the space between the conductors

$$u = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left\{\frac{\mu_0 I}{2\pi r}\right\}^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}\frac{\text{joule}}{\text{metre}^3}$$

Consider a volume element *dV* in the form of a cylindrical shell of radii *r* and (*r* + *dr)*

$$\text{Energy } dW = udV = \frac{\mu_0 I^2}{8\pi^2 r^2} \times 2\pi r\ell dr$$

$$\Rightarrow \quad dW = \frac{\mu_0 I^2 \ell}{4\pi}\left(\frac{dr}{r}\right)$$

Total magnetic energy is obtained by integrating this expression from $r = a$ to $r = b$.

So, $W = \int dW = \frac{\mu_0 I^2 \ell}{4\pi} \int_a^b \left(\frac{dr}{r} \right)$

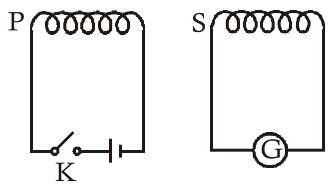
$\Rightarrow W = \frac{\mu_0 I^2 \ell}{4\pi} \log_e \left(\frac{b}{a} \right)$

(ii) If L be the self inductance of length ℓ of the cable, then the energy in magnetic field will be $\left(\frac{1}{2} \right) LI^2$.

Hence, $\frac{1}{2} LI^2 = \frac{\mu_0 \ell^2}{4\pi} \log_e \left(\frac{b}{a} \right)$

$\Rightarrow L = \frac{\mu_0 \ell}{2\pi} \log_e \left(\frac{b}{a} \right)$

MUTUAL INDUCTANCE



Consider two coils P (Primary) and S (Secondary) placed close to each other such that if a current passes in coil P, the coil S is in the magnetic field of coil P and vice-versa.

When the key K is closed then the current flowing through a coil (P) changes, the magnetic flux liked with the neighbouring coil (Q) also changes. As a result of this an induced e.m.f. and hence an induced current is set up in coil Q.

The circuit, in which an emf is induced due to change in current in the neighbouring coil, is called the secondary circuit. This phenomenon of production of e.m.f. in a coil when the current in neighbouring coil changes is called mutual induction. The neighbouring circuit it which e.m.f. is induced is called the secondary circuit.

If I_1 is current flowing through primary coil then at any instant, the flux linked with secondary coil is given by

$$\phi_2 \propto I_1$$

$$\Rightarrow \quad \phi_2 = MI_1 \qquad \ldots\text{(i)}$$

where M is called the coefficient of mutual induction or mutual inductance of the coils.

If $I_1 = 1A$, then $\phi_2 = M$ (numerically)

So, M is numerically equal to the flux in the secondary coil when the current in the primary is 1 A.

Also induced e.m.f. in secondary coil,

$$\xi_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(MI_1) = -M\frac{dI_1}{dt} \qquad \ldots\text{(ii)}$$

If $\frac{dI_1}{dt} = 1\text{As}^{-1}$, then $M = \xi_2$ (numerically)

So, M is numerically equal to the e.m.f. induced in the secondary coil when the rate of change of current in the primary coil is 1 As^{-1}.

Like self-inductance, the unit of mutual inductance is henry (H). The direction of induced e.m.f. or induced current arising due to a change in magnetic flux in all cases is given by **Lenz's law.**

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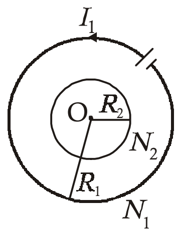
CALCULATION OF MUTUAL INDUCTANCE

FOR A TWO COIL SYSTEM

Consider two co-axial coils having number of turns N_1 and N_2 and radii R_1 and R_2 respectively. If I_1 is current in outer coil, the magnetic field at its centre,

$$B_1 = \frac{\mu_0 N_1 I_1}{2R_1}$$



The flux linked with the inner-coil,

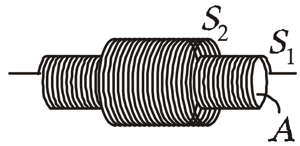
$$\phi_1 = N_2 B_1 A_2$$

$$\Rightarrow \qquad \phi_2 = N_2\left(\frac{\mu_0 N_1 I_1}{2R_1}\right)A_2$$

$$\Rightarrow \qquad M = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A_2}{2R_1} = \frac{\mu_0 N_1 N_2 \left(\pi R_2^2\right)}{2R_1}$$

FOR A SOLENOID-COIL SYSTEM

Let n_1 be number of turns per metre length of a long solenoid S_1 having area of cross-section A. Let a coil S_2 of n_2 turns per unit length surround the solenoid completely. If I_1 is current in solenoid, then magnetic field within it,



$$B_1 = \mu_0 n_1 I_1$$

The magnetic field does not exist inside the annular region (region between S_1 and S_2).

The magnetic flux linked with coil,

$$\phi_2 = N_2 B_1 A$$

$$= \left(n_2 \ell\right)\left(\mu_0 n_1 I_1\right) A$$

hence, $$M = \frac{\phi_2}{I_1} = \mu_0 n_1 n_2 A \ell$$

MUTUAL INDUCTANCE OF TWO CLOSE COILS AND COEFFICIENT OF COUPLING

If two coils of self-inductances L_1 and L_2 are placed near each other, then mutual inductance

$$M = K\sqrt{L_1 L_2}$$

where K is constant, called **coefficient of coupling**

If flux linkage between coils is 100% then

$K = 1$ and so

$$M = \sqrt{L_1 L_2}$$

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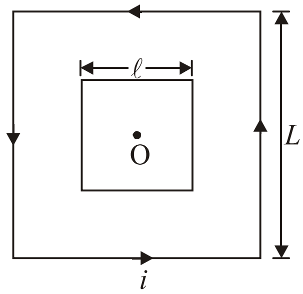
Illustration 7. *A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L $(\gg \ell)$. The loops are coplanar and their centers coincide. What is the inductance of the system ?*

Solution : The larger loop consists of four wires. The field at the center *O*, at a distance $\frac{L}{2}$ from each wire is given by

$$B = 4\left[\frac{\mu_0}{4\pi}\cdot\frac{I}{d}\cdot 2\sin 45^\circ\right] = \frac{\mu_0}{4\pi}\times\frac{8I}{(L/2)}\times\frac{1}{\sqrt{2}}$$



$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\times\frac{8\sqrt{2}}{L}I$$

So, flux linked with smaller loop is

$$\Phi = BA = \left(\frac{\mu_0}{4\pi}\times\frac{8\sqrt{2}}{L}\times I\right)\times\ell^2$$

$$\Rightarrow \quad M = \frac{\Phi}{I} = \frac{\mu_0}{4\pi}\times\frac{8\sqrt{2}}{L}\times\ell^2$$

$$\Rightarrow \quad M = 2\sqrt{2}\times\frac{\mu_0}{\pi}\times\frac{\ell^2}{L}$$

INDUCTANCES IN SERIES AND PARALLEL

SERIES

If two coils of self-inductances L_1 and L_2 having mutual inductance are connected in series and are far from each other, so that the mutual induction between them is negligible, then net self inductance

$$L_S = L_1 + L_2$$

when they are situated close to each other, then net inductance

$$L_S = L_1 + L_2 \pm 2M$$

PARALLEL

If two coils of self-inductance L_1 and L_2 having mutual inductance are connected parallel and are far from each other, then net inductance *L* is

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\Rightarrow \qquad L_P = \frac{L_1 L_2}{L_1 + L_2}$$

When they are situated close to each other, then

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$$

INDUCTION COIL

Induction coil is based on mutual induction and is used to produce a larger e.m.f. from a source of low e.m.f.

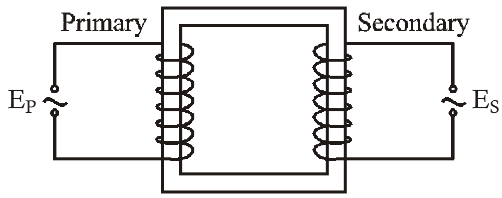
It has two coils, Primary and Secondary. When current in primary coil changes, the magnetic flux linked with the secondary also changes thus setting up an induced e.m.f. in the secondary. During self-induction of primary the rate of growth of current is slow but when circuit breaks rate of change of current $\left|\frac{dI}{dt}\right|$ in the secondary is very large and so induced e.m.f. in the secondary is also very large.

Induction coil can be used to produce an e.m.f. of the order of 50,000 V from a 12 V battery. Induction coil is often used to operate a discharge tube.

TRANSFORMER

A transformer is a device for converting high voltage at low current to low voltage at high current and vice-versa.

It consists of two coils would on a soft iron core. The primary coils is connected to an a.c. source. The secondary coil is connected to the load which may be resistor or any other electrical device.



There are two types of transformers.

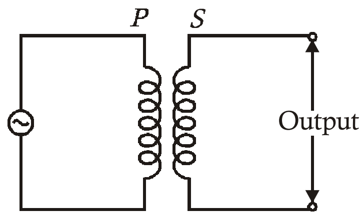
STEP UP TRANSFORMER

It converts low voltage at high current into high voltage at low current.

STEP DOWN TRANSFORMER

It converts high voltage at low current into low voltage at high current.

The principle of transformer is based on mutual induction and a transformer works on AC only. The input is applied across primary terminals and output across secondary terminals. If the resistance of primary coils is zero then voltage across the primary is equal to the applied voltage.



$P \rightarrow$ Primary Coil
$S \rightarrow$ Secondary Coil

The ratio of number of turns in secondary and primary is called **turn ratio**. So,

$$\text{Turn Ratio } N = \frac{N_S}{N_P}$$

If E_P and E_S are alternating voltages, I_P and I_S the alternating currents, ϕ_P and ϕ_S be the magnetic flux across primary and secondary terminal respectively, then

For an Ideal Transformer (Efficiency = 1) we have

$$\eta = 1 = \frac{\text{Output Power}}{\text{Input Power}} = \frac{\xi_S I_S}{\xi_P I_P}$$

$$\Rightarrow \quad \frac{\xi_S}{\xi_P} = \frac{I_P}{I_S}$$

Also we observe that

$$\frac{\xi_S}{\xi_P} = \frac{\phi_S}{\phi_P} = \frac{N_S}{N_P}$$

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So for an Ideal Transformer

$$\frac{\xi_S}{\xi_P} = \frac{\phi_S}{\phi_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

For a non-ideal transformer (with efficiency $\eta < 1$)

We have $\eta = \dfrac{\xi_S I_S}{\xi_P I_P}$

$$\frac{\xi_S}{\xi_P} = \eta \frac{I_P}{I_S}$$

So, we get

$$\frac{\xi_S}{\xi_P} = \frac{N_S}{N_P} = \frac{\phi_S}{\phi_P} = \frac{\eta I_P}{I_S}$$

In actual transformer, there is some power loss. The main sources of power loss are :

a) I^2R loss due to Joule heat in copper windings.

b) Heating produced due to Eddy currents in the iron core. This reduced by using laminated core.

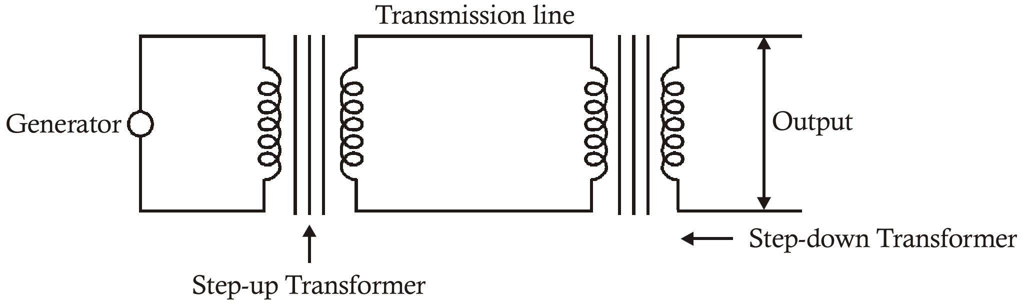
c) Hysteresis loss due to repeated magnetization of the iron core.

d) Loss due to flux leakage.

When all the losses are minimized, the efficiency of the transformer becomes very high (90—99%).

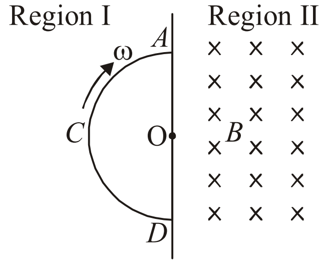
LONG DISTANCE TRANSMISSION OF ELECTRIC POWER

Two minimize the I^2R loss during long distance transmission of electric power, the voltage is first stepped up to a high value using a transformer.



Since $P = VI$ is constant, as V is increased I decreases and hence I^2R loss becomes small. By increasing V to a very high value, this power loss can be made negligible.

Illustration 8. *Space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the plane of paper. ACD is semicircular conducting loop of radius r with center at O (show in figure), the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The effective resistance of the loop is R.*



(i) Obtain an expression for the magnitude of the induced current in the loop.

(ii) Show the direction of the current when the loop is entering into the region II.

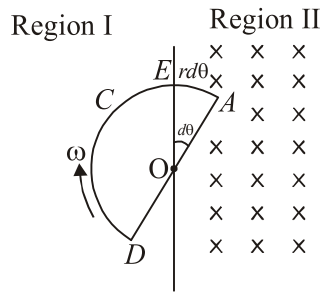
Solution : (i) When the loop is in region I, the magnetic field linked with the loop is zero. As soon as the loop enters magnetic field in region II (shown in figure), the magnetic flux linked with it, is given by

$$\Phi = BA$$

So, e.m.f. induced $\xi = -\frac{d\Phi}{dt} = -\frac{d(BA)}{dt}$

$$\Rightarrow \quad \xi = -B\frac{dA}{dt} = B\frac{dA}{dt}$$

Let $d\theta$ be the angle rotated by the loop in time dt, then



dA = Area of the triangle

$$OEA = \left(\frac{1}{2}\right)r\cdot(rd\theta) = \frac{1}{2}r^2 d\theta$$

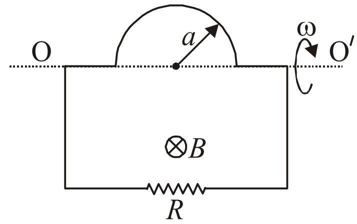
$$\therefore \quad \xi = \frac{B\left(\frac{1}{2}r^2 d\theta\right)}{dt} = \frac{1}{2}Br^2\frac{d\theta}{dt} = \frac{1}{2}Br^2\omega$$

If I is the induced current then

$$I = \frac{\xi}{R} = \frac{1}{2}\left(\frac{Br^2\omega}{R}\right)$$

(ii) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. Hence, the direction of current must be anticlockwise.

Illustration 9. *A wire shaped as a semi – circle of radius a rotates about an axis OO' with an angular velocity ω in a uniform magnetic field of induction B (shown in figure). The rotation axis is perpendicular to the field direction. The total resistance of the circuit is equal to R. Neglecting the magnetic field of induced current, calculate the mean amount of thermal power being generated in the loop during one rotation period.*



Solution : $\Phi = \vec{B}\cdot\vec{A} = B\left(\frac{\pi a^2}{2}\right)\cos(\omega t)$

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Since, $\xi = -\frac{d\Phi}{dt} = B\left(\frac{\pi a^2}{2}\right)\omega \sin(\omega t)$

Induced current, $I = \frac{\xi}{R} = \frac{B\pi a^2}{2R}\omega \sin(\omega t)$

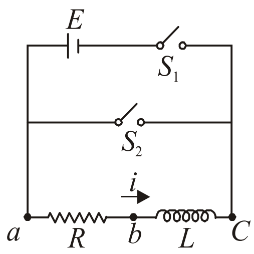
At any moment *t,* the thermal power generated in the circuit is

$$P_t = \xi \times I = \left(\frac{B\pi a^2\omega}{2}\right)^2 \frac{1}{2}\sin^2(\omega t)$$

Mean power

$$\langle P \rangle = \frac{\left(\frac{B\pi a^2\omega}{2}\right)^2 \frac{1}{R}\int_0^T \sin^2 \omega t}{\int_0^T dt} = \frac{1}{2R}\left(\frac{\pi\omega a^2 B}{2}\right)^2$$

GROWTH AND DECAY OF CURRENT IN AN L- R CIRCUIT



Let us consider a circuit consisting of a battery of emf *E*, a coil of self inductance *L* and a resistor *R*. The resistor *R* may be a separate circuit element or it may be the resistance of the inductor windings. By closing switch S_1, we connect *R* and *L* in series with constant emf *E*. Let *I* be the current at some time *t* after switch S_1 is closed and *di/dt* be its rate of change at that time.

$$E - V_{ab} - V_{bc} = 0$$

$$\Rightarrow \quad E = IR + L\frac{dI}{dt}$$

$$\Rightarrow \quad E - IR = L\frac{dI}{dt}$$

$$\Rightarrow \quad \frac{dI}{E - IR} = \frac{dt}{L}$$

$$\Rightarrow \quad \int_0^I \frac{dI}{E - IR} = \int_0^t \frac{dt}{L}$$

$$\therefore \quad I = \frac{E}{R}\left(1 - e^{\frac{Rt}{L}}\right)$$

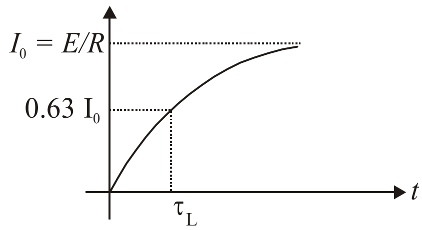
Since, $\frac{E}{R} = I_0$ and $\frac{L}{R} = \tau_L$, the above expression reduces to

$$I = I_0\left(1 - e^{-t/\tau_L}\right)$$

Here, $\tau_0 = \frac{E}{R}$ is the current as $t \to \infty$. It is also called the steady state current or maximum current in the circuit.

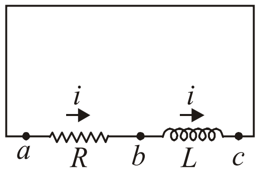
And $\tau_L = \frac{L}{R}$ is called time constant of the *LR* circuit. At a time equal to one time constant the current rises to $\left(1-\frac{1}{e}\right)$ or about 63% of its final value I_0.

The $I - t$ graph is as shown in figure.



DECAY OF CURRENT

Now suppose switch s_1 in the circuit shown in figure has been closed for a long time and that the current has reached its steady state value I_0. Resetting our stopwatch to redefine the initial time we close switch S_2 to by pass the battery. The current through *L* and *R* does not instantaneously go to zero but decays exponentially.



$$IR + L\left(\frac{dI}{dt}\right) = 0$$

$$\Rightarrow \quad \frac{dI}{I} = -\frac{R}{L}dt$$

$$\therefore \quad \int_{I_0}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow I = I_0 e^{-\frac{t}{\tau_L}}$$

where $\tau_L = \frac{L}{R}$, is the time for current to decrease to $\frac{1}{e}$ or about 37% of its original value. The $i - t$ graph shown in figure.

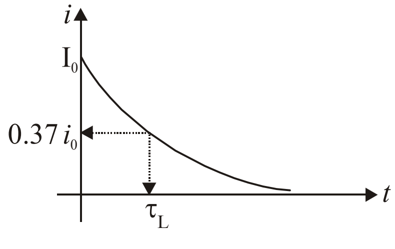


Illustration 10. *A circuit containing a two position switch S is shown in figure.*

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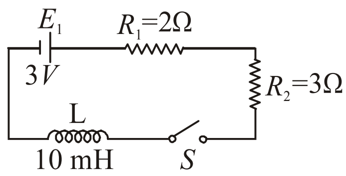
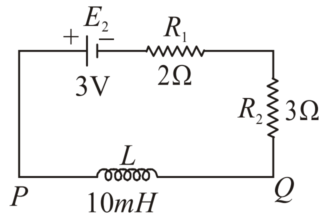


If the switch S is connected at t = 0, find

(i) steady current in R_2 and

(ii) the time when the current in R_2 is half the steady value. Also calculate the energy stored in the inductor L at the time.

Solution : (i)



Total resistance of the circuit

$$R = R_2 + R_4 = 5\Omega$$

$$L = 10\text{mH} = 10\times 10^{-3}\,\text{H} = 10^{-2}\,\text{H}$$

Steady current $i_0 = \dfrac{E}{R} = \dfrac{3}{5} = 0.6$ amp {$\because$ In steady state, there is no role of inductor}

(ii) The growth of the current in $L-C$ circuit is given by

$$i = i_0\left[1-\exp\left(-\frac{R}{L}\cdot t\right)\right]$$

$$\Rightarrow \quad \frac{i}{i_0} = \left[1-\exp\left(-\frac{R}{L}\cdot t\right)\right]$$

According to given problem, we have

$$i = \frac{i_0}{2}$$

Thus after time *t*,

$$\frac{i}{i_0} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{2} = \left[1-\exp\left(-\frac{R}{L}\cdot t\right)\right]$$

$$\Rightarrow \quad \exp\left(-\frac{R}{L}t\right) = \frac{1}{2}$$

Taking log on both sides, we get

$\Rightarrow \quad \frac{R}{L}t = \log_e 2$

$\Rightarrow \quad t = \frac{L}{R}(\log_e 2)$

$t = \frac{10^{-2}}{5} \times 0.693$

$\Rightarrow \quad t = 1.386 \times 10^{-3}$ sec.

The energy stored in the inductor is given by

$U = \frac{1}{2}Li^2$

When $i = \frac{i_0}{2} = \left(\frac{0.6}{2}\right)\text{A} = 0.3\text{A}$, we have

$\Rightarrow \quad U = \frac{1}{2} \times (10 \times 10^{-3})(0.3)^2$ joule

$\Rightarrow \quad U = 4.5 \times 10^{-4}$ J

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SOLVED SUBJECTIVE PROBLEMS

Problem 1. *The two rails of a railway track. insulated from each other and the ground, are connected to a millivoltmeter. What is the reading of the millivoltmeter when a train travels at a speed of 18 km/h along the track given that the vertical components of earth's magnetic field is 0.2 ×10^{-4} weber/m^2 and the rails are separated by 1 m ? Track is south to north.*

Solution : Potential difference between the two rails :

$V = Bv\ell$ (when $\vec{B}, \vec{v}$ and $\vec{l}$ all are mutually perpendicular)

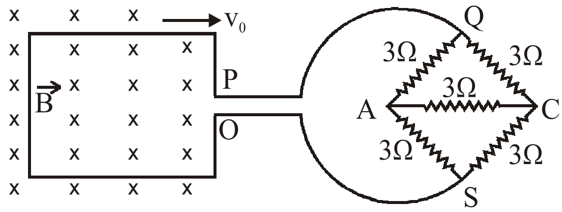
$= \left(0.2 \times 10^{-4}\right)\left(180 \times \frac{5}{18}\right)(1)$

$= 10^{-3}\,\text{V}$

$= 1\ \text{mV}$

Problem 2. *A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 weber/m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohm.*

The resistance of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in the loop ? Give the direction of current in the loop.



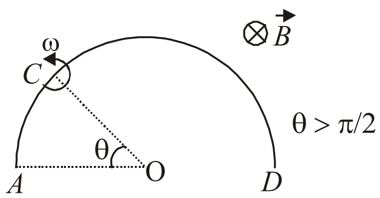
Solution : Given network forms a balanced Wheatstone's bridge. The net resistance of the circuit is therefore $3\Omega + 1\Omega = 4\Omega$ emf of the circuit is $Bv_0\ell$. Therefore, current in the circuit would be

$$I = \frac{BV_0\ell}{R}$$

or $$v_0 = \frac{iR}{B\ell} = \frac{\left(1 \times 10^{-3}\right)(4)}{2 \times 0.1} = 0.02\ \text{m/s}$$

Cross magnetic field passing through the loop is decreasing. Therefore, direction of induced current is clockwise.

Problem 3. *A uniform wire of resistance per unit length λ is bent into a semicircle of radius a. The wire rotates with angular velocity ω in a vertical plane about an axis passing through C. A uniform magnetic field B exists in space in a direction perpendicular to paper inwards.*

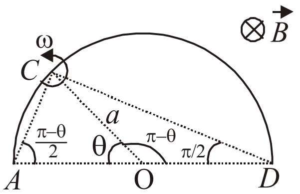


(i) Calculate potential difference between points A and D. Which point is at a higher potential ?

(ii) If point A and D are connected by a conducting wire

of zero resistance find the potential difference between A and C .

Solution : (i) Length of straight wire AC, is $\ell_1 = 2a\sin\left(\frac{\theta}{2}\right)$



Therefore, the motional *emf* (or potential difference) between point C and A is,

$$V_{CA} = V_C - V_A = \frac{1}{2}B\omega\ell_1^2 = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right) \qquad \ldots(1)$$

From Right Hand Rule, we have $V_C > V_A$

Similarly, length of straight wire CD is

$$\ell_2 = 2a\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = 2a\cos\left(\frac{\theta}{2}\right) \qquad \ldots(2)$$

Therefore, the P.D between points C and D is

$$V_{CP} = V_C - V_D = \frac{1}{2}B\omega\ell_2^2 = 2aB\omega\cos^2\left(\frac{\theta}{2}\right)$$

With $V_C > V_D$

Equation (2) – (1) gives,

$$V_A - V_D = 2a^2B\omega\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) = 2a^2B\omega\cos\theta$$

A is at higher potential.

(b) On connecting A and D from a wire, current starts flowing in the circuit and the potential difference between C and A has now a value different from $2a^2B\omega\sin^2\left(\frac{\theta}{2}\right)$.

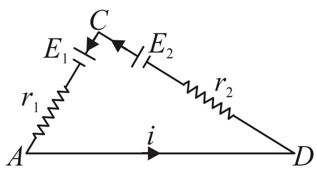
Resistance between A and C is,

$$r_1 = (\text{length of arc } AC)\lambda = a\theta\lambda$$

and between C and D, is

$$r_2 = (\text{length of arc} CD)\lambda = (\pi - \theta)a\lambda$$

Now, the equivalent circuit can be drawn as shown in figure.



where, $E_1 = 2a^2B\omega\sin^2\left(\frac{\theta}{2}\right)$

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and $\quad E_2 = 2a^2 B\omega\cos^2\left(\frac{\theta}{2}\right)$

with $\quad E_2 > E_1$

If i is the current in the circuit, then

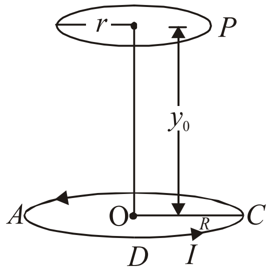
$$i = \frac{E_2 - E_1}{r_1 + r_2} = \frac{2a^2 B\omega\cos\theta}{\pi a\lambda} = \frac{2aB\omega\cos\theta}{\pi\lambda}$$

and potential difference between points C and A is,

$$V'_{CA} = E_1 + ir_1 = 2a^2 B\omega\sin^2\left(\frac{\theta}{2}\right) + \left(\frac{2aB\omega\cos\theta}{\pi\lambda}\right)(a\theta\lambda)$$

$$\Rightarrow \quad V'_{CA} = 2a^2 B\omega\left\{\sin^2\left(\frac{\theta}{2}\right) + \frac{\theta\cos\theta}{\pi}\right\}$$

Problem 4. *A coil ACD of radius R and number of turns n carries a current I amp. and is placed in the plane of paper. A small conducting ring P of radius r is placed at a distance y_0 from the center and above the coil ACD. Calculate the induced e.m.f. produced in the ring when the ring is allowed to fall freely. Express induced e.m.f. in terms of speed of the ring.*



Solution : Magnetic induction at a point on the axis of the current carrying coil at a distance y from its center is given by

$$B = \frac{\mu_0}{2}\frac{nIR^2}{\left(R^2 + y^2\right)^{3/2}}$$

The magnetic flux linked with ring P is

$$\Phi = BA = \frac{\mu_0}{2}\frac{nIR^2}{\left(R^2 + y^2\right)^{3/2}}\pi r^2$$

where $A = \pi r^2$ is the area of the loop P.

So, $\Phi = \frac{\mu_0}{2}\frac{\pi nIR^2 r^2}{\left(R^2 + y^2\right)^{3/2}}$

Let the ring P fall with velocity v. At any instant the y varies as

$$y = y_0 - vt$$

The induced e.m.f. in the ring is

$$\xi = -\frac{d\Phi}{dt} = -\frac{\mu_0\pi nIR^2 r^2}{2}\frac{d}{dt}\left(R^2 + y^2\right)^{-3/2}$$

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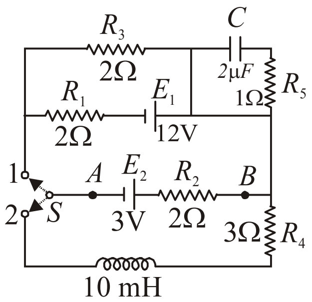
$$\Rightarrow \quad \xi = \frac{3}{2}\mu_0 \pi n I R^2 r^2 \left(R^2 + y^2\right)^{-5/2} 2y \left(\frac{dy}{dt}\right)$$

$$\Rightarrow \quad \xi = \frac{3}{2}\frac{\mu_0 \pi I R^2 r^2}{\left(R^2 + y^2\right)^{5/2}} y\,(-v) \qquad \left\{\because \frac{dy}{dt} = -v\right\}$$

$$\therefore \quad |\xi| = \frac{3}{2}\frac{\mu_0 n I R^2 r^2 y v}{\left(R^2 + y^2\right)^{5/2}}$$

Problem 5. *A circuit containing a two positions switch S is shown in figure.*



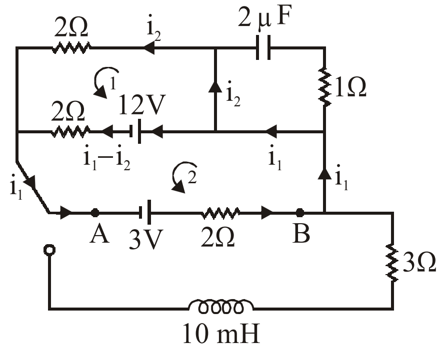
(a) *The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.*

(b) *If now switch S is put in position 2 at t = 0, find*

   (i) *steady, current in R_4 and*

   (ii) *the time when the current in R_4 is half the steady value . Also calculate the energy stored in the inductor L at that time.*

Solution : In steady state no current will flow through capacitor. Applying Kirchhoff's second law in loop 1 :



$$-2i_2 + 2\left(i_1 - i_2\right) + 12 = 0$$

$$\therefore \; 2i_1 - 4i_2 = -12$$

or $i_1 - 2i_2 = -6$ ...(i)

Applying Kirchhoff's second low in loop 2 :

$$-12 - 2\left(i_1 - i_2\right) + 3 - 2i_1 = 0$$

$$\Rightarrow \quad 4i_1 - 2i_2 = -9 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

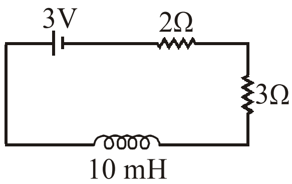
$i_2 = 2.5 \text{ A}$ and $i_1 = -1 \text{ A}$

Now $V_A + 3 - 2i_1 = V_B$

or $V_A - V_B = 2i_1 - 3 = 2(-1) - 3 = -5$ volt

$P_{R_1} = (i_1 - i_2)^2 R_1 = (-1 - 2.5)^2 (2) = 24.5$ watt

(b) **In position 2** : Circuit is as under



Steady current in R_4 :

$$i_0 = \frac{3}{3+2} = 0.6 \text{ A}$$

Time when current in R_4 is half the steady value :

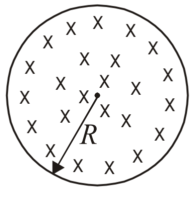
$$t_{1/2} = \frac{\ln(2)}{1/\tau_L} = \tau_L(\ln 2) = \frac{L}{R}\ln(2)$$

$$= \frac{(10 \times 10^{-3})}{5}\ln(2) = \mathbf{1.386 \times 10^{-3}\,s}$$

Energy stored in inductor at that time $U = \frac{1}{2}LI^2 = \frac{1}{2}(10 \times 10^{-3}) \times (0.3)^2$

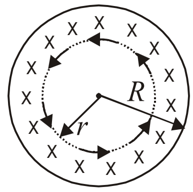
$= \mathbf{4.5 \times 10^{-4}\,J}$

Problem 6. *The magnetic field at all points within the cylindrical region whose cross – section is indicated in the accompanying figure starts increasing at a constant rate* $\beta \text{WbM}^{-2}\text{s}^{-1}$ *. Find the magnitude of electric field as a function of r, the distance from the geometric center of the region.*



Solution : Here, we shall discuss three situations i.e. when *r* lies inside, at the surface and outside the region

CASE I : For r ≤ R (inside) :



Using , $E(2\pi r) = A\left|\frac{dB}{dt}\right|$

$\Rightarrow$ $E(2\pi r) = (\pi r^2)\beta$

$\Rightarrow$ $E = \frac{1}{2}r\beta$

$\therefore$ $E \propto r$,

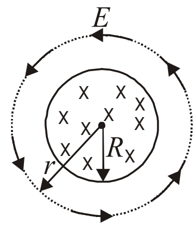
So, $E - r$ graph is a straight line passing through origin.

CASE II : At $r = R$ (*at the surface*)

$$E\left(2\pi R\right)=\left(\pi R^2\right)\beta$$

$\therefore \qquad E=\dfrac{R\beta}{2}$

CASE III : For $r \geq R$ (outside)



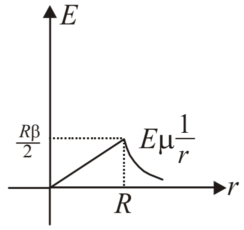
Using, $\qquad E\left(2\pi r\right)=A\left|\dfrac{dB}{dt}\right|$

$\Rightarrow \qquad E\left(2\pi r\right)=\left(\pi R^2\right)(\beta)$

$\Rightarrow \qquad E=\dfrac{\alpha R^2}{2r}$

$\therefore \qquad E\propto\dfrac{1}{r}$

So, *E-r* graph is a rectangular hyperbola.
The *E-r* graph is as shown in figure. Direction of electric field is shown in figure.



Problem 7. *A square frame with side a and a straight conductor carrying a constant current I are located in the same plane. The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current carrying conductor by a distance b. Find the electric charge having flown through the frame.*

Solution : Total charge flowing through the wire is

$$q=\int Idt=-\frac{1}{R}\int\left(\frac{d\Phi}{dt}+L\frac{dI}{dt}\right)dt$$

$$\Rightarrow \quad q=-\left(\frac{1}{R}\Delta\Phi+L\Delta I\right)$$

Since the current in the coil before and after the rotation remains the same so,

$$\Delta I=0$$

$$\Rightarrow \quad q=-\frac{1}{R}\Delta\Phi$$

Further

$$\Delta\Phi=\int d\Phi=\int Badr=\frac{\mu_0 2Ia}{2\pi}\int_{a-b}^{a+b}\frac{dr}{r}$$

$$\Rightarrow \quad \Delta\Phi=\frac{\mu_0}{4\pi}2Ia\log_e\left(\frac{a+b}{b-a}\right)$$

$$\Rightarrow \quad q=\frac{|\Delta\Phi|}{R}=\frac{\mu_0 aI}{2\pi R}\log_e\left(\frac{a+b}{b-a}\right)$$

Problem 8. *Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (shown in figure). There is a uniform magnetic field of*

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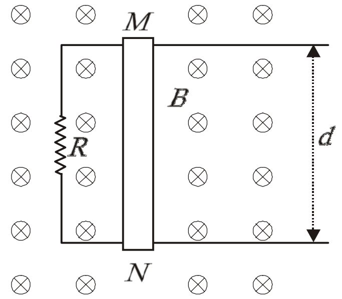
induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current I flows through R. Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.



Solution : Let F be the instantaneous force acting on the rod MN at any instant t when the rod is at a distance x. The instantaneous flux Φ is given by

$$\Phi = BA = B(xd)$$

The instantaneous induced e.m.f. is given by

$$\xi = -\frac{d\Phi}{dt} = -Bd\left(\frac{dx}{dt}\right)$$

The total resistance of the circuit at the instant = $R + 2\lambda x$

By Ohm's law, the current I in the circuit is

$$I = \frac{\xi}{\text{Resistance}} = \frac{Bd}{(R+2\lambda x)}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \text{ Velocity} = \frac{dx}{dt} = \frac{I(R+2\lambda x)}{Bd}$$

The instantaneous acceleration a is given by

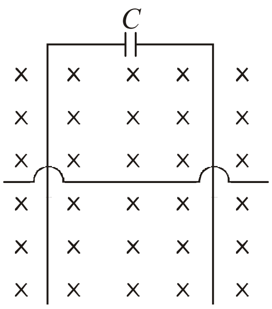
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{2I\lambda}{Bd}\left(\frac{dx}{dt}\right) = \frac{2I\lambda}{Bd}\left\{\frac{I(R+2\lambda x)}{Bd}\right\} = \frac{2I^2\lambda(R+2\lambda x)}{B^2d^2}$$

$$\left\{\because \frac{dv}{dt} = \frac{2I\lambda}{Bd}\left(\frac{dx}{dt}\right)\right\}$$

So, Instantaneous applied force F is given by $F - IBd = ma$

$$\Rightarrow F = m\frac{2I^2\lambda(R+2\lambda x)}{B^2d^2} + IBd$$

Problem 9. *Two metal bars are fixed vertically and are connected on the top by a capacitor C. A sliding conductor of length ℓ and mass m slides with its ends in contact with the bars. The arrangement is placed in a uniform horizontal magnetic field directed normal to the plane of the figure. The conductor is released from rest. Find the displacement x(t) of the conductor as a function of time t.*



Solution: Due to the motion of the conductor in magnetic field, an e.m.f. is induced in it. As a result, a current flows through the conductor. According to Lenz's law, a force $Bi\ell$ (due to induced current) opposes the motion of the conductor. Let at some instant *t*, velocity of the conductor be *v*.

The net accelerating force on conductor is $F = mg - Bi\ell$ …(1)

Here, induced e.m.f. = $B\ell v$

Charge on the capacitor,

$$q = Ce = C(B\ell v)$$

Since, v is increasing, the charge and hence the current through the capacitor is also increasing. The current through capacitor is given by

$$i_C = \frac{dq}{dt} = CB\ell\frac{dv}{dt} \quad \text{…(2)}$$

From equations (1) and (2), we get

$$m\frac{dv}{dt} = mg - B\left(C\ B\ \ell\frac{dv}{dt}\right)\ell$$

$$\Rightarrow\ m\frac{dv}{dt} = mg - B^2\ell^2C\frac{dv}{dt}$$

$$\Rightarrow\ \frac{dv}{dt}\left[m + B^2\ell^2C\right] = mg$$

$$\Rightarrow\ a = \frac{dv}{dt} = \frac{mg}{\left(m + B^2\ell^2C\right)}$$

$$\Rightarrow\ x(t) = \frac{1}{2}at^2 = \frac{mgt^2}{2\left(m + B^2\ell^2C\right)}$$

Problem 10. *A thermocole vessel contains 0.5 kg of distilled water at* 30^o C. *A metal coil of area* 5×10^{-3} m^2, *number of turns 100, mass 0.06 kg and resistance* $1.6\,\Omega$ *is lying horizontally at the bottom of the vessel. A uniform, time varying magnetic field is set up to pass vertically through the coil at time t = 0. The field is first increased from zero to* 0.8 T *at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. This cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as functions of time for the first two cycles. Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of the metal = 500* J kg^{-1} K^{-1} *and the specific heat of water = 4200* J kg^{-1} K^{-1}. *Neglect the inductance of the coil.*

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Solution : Since, Induced e.m.f.

$$\xi = -N\frac{d\Phi}{dt} = -NA\frac{dB}{dt}$$

So, Induced current is given by

$$i = \frac{\xi}{R} = -\frac{NA\,dB}{dt}$$

Given that $\left(\frac{dB}{dt}\right) = 0.4\text{Ts}^{-1}$ for time interval 0 to 0.2 sec.

Hence current for this interval will be

$$(i)_{\min} = -\frac{100\times(5\times10^{-3})}{1.6}\times\left(\frac{0.8}{2}\right)$$

$\Rightarrow \quad (i)_{\min} = +1.25$ amp

Further, $\left(\frac{dB}{dt}\right) = -0.4$ Tesla sec^{-1} for time interval 0.2 to 0.4 sec.

Hence current in this interval will be

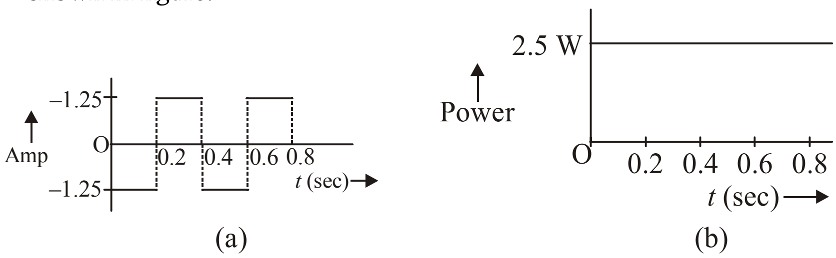
$$(i)_{\max} = \frac{100\times(5\times10^{-3})}{1.6}\times\left(\frac{0.8}{2}\right)$$

$\Rightarrow \quad (i)_{\max} = -1.25$ amp

Now, Power $P = \frac{dW}{dt} = I^2R$

$\Rightarrow \quad P = (1.25)^2\times(1.6) = 2.5$ W (for all time)

The sketches of the current through the coil as functions of time for the first two cycles are shown in figure.



Total time for which the magnetic field varies $= 0.4\times12000 = 4800$ s

Amount of heat generated in the coil = $I^2Rt = (1.25)^2\times1.6\times4800 = 12000$ joule

Let T be the final temperature, then

$$m_w s_w \Delta T + m_m s_m \Delta T = 12000$$

$\Rightarrow \quad (0.5)(4200)(T-30)+(0.06)(500)(T-30) = 1200$

Solving, we get

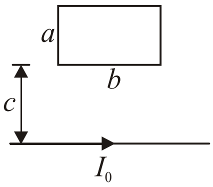
$$T = 35.6°\text{C}$$

SOLVED OBJECTIVE PROBLEMS

Problem 1. *A rectangular loop of sides a and b, has a resistance R and lies at a distance c from an infinite straight wire carrying current I_0. The current decreases to zero in time* τ

$$I(t) = I_0 \frac{(\tau - t)}{\tau}, 0 < t < \tau$$

The charge flowing through the rectangular loop is



(a) $\mu_0 I_0 \tau$ (b) $\mu_0 I_0 \frac{ab}{c^2}\tau$

(c) $\frac{1}{2}\frac{\mu_0 b I_0}{\pi R}\ln\left(\frac{c+a}{c}\right)$ (d) $\frac{\mu_0 I \tau}{R}\frac{ba}{c^2}$

Solution : Consider an infinitesimal element of length b, thickness dx at a distance x from the wire.

Since $d\phi = BdA$

$$\Rightarrow d\phi = \frac{\mu_0 I}{2\pi x}(bdx)$$

$$\Rightarrow \phi = \int d\phi = \frac{\mu_0 I b}{2\pi}\int_c^{c+a}\frac{dx}{x}$$

$$\Rightarrow \phi = \frac{\mu_0 b I}{2\pi}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow \phi = \frac{\mu_0 b}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left[I_0\left(1-\frac{t}{\tau}\right)\right] \qquad \left\{\because I = I_0\left(1-\frac{t}{\tau}\right)\right\}$$

Since, $\varepsilon = -\frac{d\phi}{dt}$

$$\Rightarrow \varepsilon = -\frac{\mu_0 b I_0}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left(-\frac{1}{\tau}\right)$$

$$\Rightarrow \varepsilon = \frac{\mu_0 b I_0}{2\pi\tau}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow I = \frac{\varepsilon}{R} = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow \frac{dq}{dt} = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow dq = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)dt$$

$$\Rightarrow q = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)\int_0^{\tau}dt$$

$$\Rightarrow q = \frac{\mu_0 b I_0}{2\pi R}\log_e\left(\frac{c+a}{c}\right)$$

$\therefore$ (C) Ans.

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Problem 2. *The time constant of an inductance coil is 2×10^{-3} s. When a 90Ω resistance is joined in series, the same constant becomes $0.5\times10^{-3} s$. The inductance and resistance of the coil are*

(a) $30mH\,;30\,\Omega$ *(b)* $30mH\,;60\,\Omega$

(c) $60mH\,;30\,\Omega$ *(d)* $60mH\,;60\,\Omega$

Solution : $\frac{L}{R}=2\times10^{-3}$ ...(1)

$\frac{L}{R+90}=0.5\times10^{-3}$ ...(2)

From (1) and (2), on solving we get

L = 60 mH and R = 30 Ω

$\therefore$ (C) Ans.

Problem 3. *A coil of inductance 8.4* mH *and resistance* 6Ω *is connected to a* 12V *battery. The current in the coil is 1 A at approximately the time*

(a) *500 s* *(b)* *20 ms*
(c) *35 ms* *(d)* *1 ms*

Solution : $I=I_0\left(1-e^{-\frac{Rt}{L}}\right)$, where

$$I_0=\frac{12}{6}A=2A$$

$$\Rightarrow\quad 1=2\left(1-e^{-\frac{6t}{8.4\times10^{-3}}}\right)$$

$$\Rightarrow\quad e^{-\frac{6t}{8.4\times10^{-3}}}=\frac{1}{2}$$

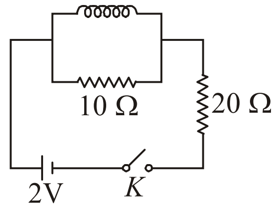
$$\Rightarrow\quad \frac{6t}{8.4\times10^{-3}}=\ln 2$$

$$\Rightarrow\quad t=(1.4\times0.693)ms$$

$$\Rightarrow\quad t=0.97ms$$

$\therefore$ (D) Ans.

Problem 4. *Two resistors of* 10Ω *and* 20Ω *and an ideal inductor of 10 H are connected to a 2 V battery as shown. The key K is inserted at time t = 0. The initial (t = 0) and final* $(t\to\infty)$ *currents through battery are*



(a) $\frac{1}{15}$A, $\frac{1}{10}$A *(b)* $\frac{1}{10}$A, $\frac{1}{15}$A

(c) $\frac{2}{15}$A, $\frac{1}{10}$A *(d)* $\frac{1}{15}$A, $\frac{2}{25}$A

Solution : At $t = 0$ i.e. when the key is just pressed, no current exists inside the inductor. So 10Ω and 20Ω resistors are in series and a net resistance of (10 + 20) = 30Ω exists across the circuit.

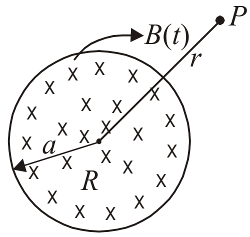
Hence $I_1 = \frac{2}{30} = \frac{1}{15} A$

As $t \to \infty$, the current in the inductor grows to attain a maximum value i.e. the entire current passes through the inductor and no current passes through 10Ω resistor

Hence $I_2 = \frac{2}{20} = \frac{1}{10} A$

$\therefore$ (A) Ans.

Problem 5. *A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the center of the circular region*



(a) *is ZERO*

(b) *decreases as* $\frac{1}{r}$

(c) *increases as r*

(d) *decreases as* $\frac{1}{r^2}$

Solution : $\oint \vec{E} \cdot d\vec{r} = (2\pi r)E = \xi = -\frac{d\phi}{dt}$

$\Rightarrow \quad 2\pi r E = \pi a^2 - \frac{dB}{dt}$

$\Rightarrow \quad E = -\frac{1}{2}\frac{a^2}{r}\frac{dB}{dt}$

$\therefore$ (B) Ans.

Problem 6. *A circuit contains an inductance L, a resistance R and a battery of emf E. The circuit is switched on at t = 0. The charge flown through the battery in one time constant* (τ) *is*

(a) $\frac{2E\tau}{\mathrm{Re}}$

(b) $\frac{E\tau}{2\,\mathrm{Re}}$

(c) $\frac{E\tau}{\mathrm{Re}}$

(d) *ZERO*

Solution : At any time t, the current in the circuit grows in accordance with formula

$$I = I_0\left(1 - e^{-Rt/L}\right) = I_0\left(1 - e^{-t/\tau}\right)$$

$\Rightarrow \quad I = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$

$\Rightarrow \quad \frac{dq}{dt} = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$

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$$\Rightarrow \quad dq = \frac{E}{R}dt - \frac{E}{R}e^{-t/\tau}dt$$

$$\Rightarrow \quad q = \frac{E}{R}\int_0^{\tau} dt - \frac{E}{R}\int_0^{\tau} e^{-t/\tau}dt$$

$$\Rightarrow \quad q = \frac{E\tau}{R} - \left[-\frac{E\tau}{\mathrm{Re}} + \frac{E\tau}{R}\right] = \frac{E\tau}{\mathrm{Re}}$$

$\therefore$ (C) Ans.

Problem 7. *A small square loop of wire of side 1 is placed inside a large square loop of wire of side $L(L >> 1)$. The loops are co-planar and their centers coincide. The mutual inductance of the system is proportional to*

(a) $\frac{1}{L}$ *(b)* $\frac{1^2}{L}$

(c) $\frac{L}{1}$ *(d)* $\frac{L^2}{1}$

Solution : $B = \frac{2\sqrt{2}\mu_0 I}{\pi L}$

The magnetic flux ϕ_{12} linking big loop with the small square loop of side $\ell(\ll L)$ is $\phi_{12} = B\ell^2 = \frac{2\sqrt{2}\mu_0 I}{\pi}\left(\frac{\ell^2}{L}\right)$

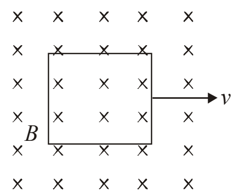
Mutual Inductance is $M_{12} = \frac{\phi_{12}}{I}$

$$\Rightarrow \quad M_{12} = \frac{2\sqrt{2}\mu_0}{\pi}\left(\frac{\ell^2}{L}\right)$$

$$\Rightarrow \quad M \propto \frac{\ell^2}{L}$$

$\therefore$ (B) Ans.

Problem 8. *A conducting square loop of side L is moving with uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into plane of the loop exists everywhere. Current induced in the loop is*



(a) $\frac{BLv}{R}$ *clockwise* *(b)* $\frac{BLv}{R}$ *anticlockwise*

(c) $\frac{2BLv}{R}$ *anticlockwise* *(d) ZERO*

Solution : Since the flux linked with the loop does not change. Hence the induced current is zero.

$\therefore$ (D) Ans.

Problem 9. *A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Ignoring the field variation across the cross-section of the toroid, the self-inductance of the toroid is*

(a) 4.6 mH *(b) 6.9 mH*

(c) 2.3 mH *(d) 9.2 mH*

Solution : Magnetic field due to toroid is $B = \mu_0 n I$, where $n = \dfrac{N}{2\pi r}$ (n is number of turns per unit length)

$\Rightarrow \quad B = \dfrac{\mu_0 N I}{2\pi r}$

Further, flux ϕ is given by NBA

$\Rightarrow \quad \phi = NBA = \dfrac{\mu_0 N^2 A I}{2\pi r}$

Now, by definition of L we have

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

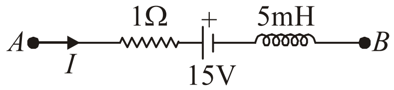
$$\Rightarrow \quad L = \frac{4\pi\times10^{-7}\times(1200)^2\times\left(\frac{12}{1000}\right)}{2\pi\left(\frac{15}{100}\right)}$$

$\Rightarrow \quad L = 2.3\times10^{-3} H$

$\Rightarrow \quad L = 2.3 mH$

$\therefore$ (C) Ans.

Problem 10. *The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5 A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals*



(a) 20 V *(b) 15 V*

(c) 10 V *(d) 5 V*

Solution : $V_A - (5)(1) + 15 - (5\times10^{-3})(10^3) - V_B = 0$

$\Rightarrow \quad V_A - V_B = 15V$

$\therefore$ (B) Ans.

Problem 11. *A planar loop of wire of area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{B}$ varies in time according to the expression $B = B_0 e^{-at}$, where a is a constant and B_0 is the value of field at t = 0. The induced e.m.f.*

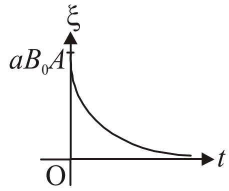
(a) increases exponentially with time with minimum value aB_0A at t = 0

(b) decreases exponentially with time with maximum value aB_0A at t = 0

(c) increases exponentially with time to a value $\dfrac{aB_0A}{e}$ at $t = \dfrac{1}{a}$.

(d) decreases exponentially with time to a value $\dfrac{aB_0A}{e}$ at $t = \dfrac{1}{a}$

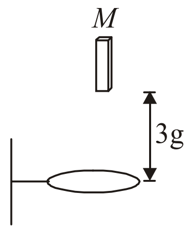
Solution : $\xi = -A\dfrac{dB}{dt}$



$\Rightarrow \quad \xi = AB_0 a e^{-at}$

$\therefore$ (B) and (D) Ans.

Problem 12. *A bar magnet M is allowed to fall towards a fixed conducting ring C. If g is the acceleration due to gravity, v is the velocity of the magnet at t = 2 s and s is the distance travelled by it in the same time then,*



(a) $v > 2g$ (b) $v < 2g$
(c) $s > 2g$ (d) $s < 2g$

Solution : v < g(t)

$\Rightarrow$ v < 2g {due to Lenz's Law}

Also, $s < \dfrac{1}{2}gt^2$

$\Rightarrow$ s < 2g {due to Lenz's Law}

$\therefore$ (B) and (D) Ans.

Problem 13. *A circular ring is rotated about is own axis in a magnetic field*

(a) No emf is induced in it if magnetic field is uniform

(b) An emf is induced in it if magnetic field is non-uniform

(c) No emf is induced in it if magnetic field is non-uniform

(d) None of these

Solution : When a circular ring is rotated about its own axis in a magnetic field then magnetic flux linked with the ring remains constant. It does not vary with time whether the magnetic field then magnetic flux linked with the ring remains constant. It does not vary with time whether the magnetic field is uniform or non-uniform. Hence no emf is induced in the ring.

Hence (A) and (c) are correct.

Problem 14. *Two different coils have self-inductances* $L_1 = 8\text{mH}$, $L_2 = 2\text{mH}$. *The current in one coil is increased at a constant rate. the current in the second coil is also increased at the same constant rate. AT a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are* i_1, V_1 *and* w_1 *respectively. Corresponding values for the second coil at that instant are* i_2, V_2 *and* w_2 *respectively. Then:*

(a) $\dfrac{i_1}{i_2} = \dfrac{1}{4}$ (b) $\dfrac{i_1}{i_2} = 4$

(c) $\dfrac{w_2}{w_1} = 4$ (d) $\dfrac{V_2}{V_1} = \dfrac{1}{4}$

Solution : Since rate of increase of current in both the coils is same, therefore $\frac{di}{dt}$ is same for both the coils or $\frac{di_1}{dt} = \frac{di_2}{dt}$

Induced emf in a solenoid is $|V| = L\frac{di}{dt}$

Therefore, $\frac{V_2}{V_1} = \frac{L_2}{L_1} = 1/4$

Hence (d) is correct.

Power supplied to the solenoid is $p = Vi$ since power supplied to both the coils is same, therefore $V_1 i_1 = V_2 i_2$

or $\frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$

Hence (a) is correct.

Energy stored in a solenoid is $W = \frac{1}{2}Li^2$

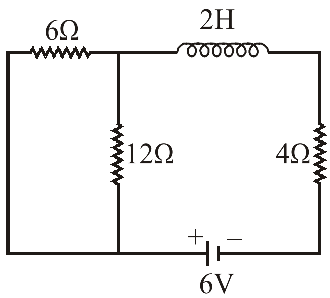
Hence, $\frac{W_2}{W_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{I_2}{I_1}\right)^2$

$= \frac{1}{4}.(4)^2 = 4$

Hence (c) is correct.

Ans. (a,c,d)

Problem 15. *For the circuit shown in fig.*



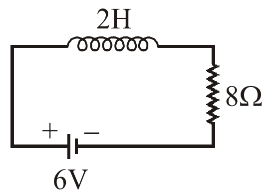
(a) Its time constant is 0.25 second.

(b) In steady state, current through inductance will be equal to zero

(c) In steady state, current through the battery will be equal to 0.75 amp.

(d) None of these

Solution : In the circuit shown in fig, 6Ω and 12Ω resistances are parallel with each other and this parallel combination is in series with 4Ω and the inductance of 2H. Hence, equivalent resistance of these three resistance is equal to 8Ω. Therefore, this circuit may be reduced to the circuit as shown in fig. The time constant for the circuit is



$\lambda = \frac{L}{R} = 2/8 = 0.25\,\text{sec}$

Hence (a) is correct.

In steady state, no emf will be induced in the inductance. Hence current through the circuit will be equal to E/R where R is equivalent resistance. Hence, the steady state current will be equal to 6/8 – 0.75 amp.

Hence (c) is correct and (b) is wrong.

Ans. (a,c,d)

PASSAGE BASED PROBLEMS

✍ *Write-up I [Questions 1 to 3]*

A coil of radius R carries a current I. Another concentric coil of radius $r\left(r << R\right)$ carries a current *i*. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Both the coils are now released. The mass of the coils are M and m respectively. Assume all contact surfaces to be frictionless.

Problem 1. *The mutual inductance between the two coils initially is*

(a) $\frac{\mu_0 \pi r^2}{2R}$ *(b)* $\frac{\mu_0 \pi r^2}{R}$

(c) zero *(d)* $\frac{\mu_0 \pi R^2}{2r}$

Problem 2. *Which statement is correct regarding the angular momentum of the system?*

(a) The total angular momentum of the system has a finite non-zero value.
(b) The angular momentum is not conserved.
(c) The angular momentum of both coils are equal at any instant.
(d) The total angular momentum of the system is zero.

Problem 3. *The maximum kinetic energy of the smaller coil is*

(a) $\frac{\mu_0 \pi IiMRr^2}{2\left(MR^2 + mr^2\right)}$ *(b)* $\frac{\mu_0 \pi IiMRr^2}{MR^2 + mr^2}$

(c) $\frac{\mu_0 \pi IiMmr^2}{m + M}$ *(d) none of these*

Solution :

1. Zero. since no flux cross from bigger coil to smaller coil when they are mutually perpendicular

   Ans. **(a)**

2. No external torque acting on the system. Hence angular momentum is conserved. The initial angular momentum of the system is zero. Hence angular momentum is zero.

***Ans.* (d)**

3. Kinetic energy is maximum when coils are coplanar potential energy of river coil $= \text{MB}\left(1-\cos\theta\right)$

Initially $= \theta = 90^0;\ v_1 = MB = \pi \quad r^2 i \dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 i I \pi \quad r^2}{2R}$

Finally when coplanar, $\theta = O^0,\ v_2 = 0$

Momentum conservation : $I_1 w_2$ (i)

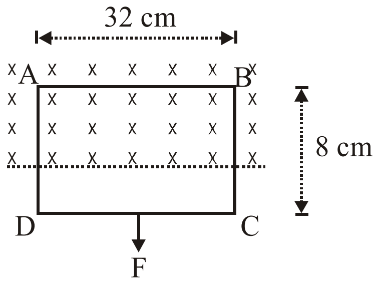
From energy conservation $\dfrac{1}{2}J_1 w^2 + \dfrac{1}{2}J_2 w_2^2 = v_1 - v$

Solving (i) and (ii), KE of smaller coil = $\dfrac{\mu_0 \pi \quad I \quad i \quad MR \quad r^2}{2\left(MR^2 + mr^2\right)}$

***Ans*. (a)**

✍ *Write-up II [Questions 4 to 6]*

A rectangular metallic frame ABCD of length 32cm and breadth 8cm has total resistance of 2 ohm. A magnetic field B=0.02 Tesla acts normal to and into the plane of the frame. The frame is pulled out of the magnetic field by a constant force F. It is observed that a current of 0.02 A flow through the frame and the frame moves with constant speed. Now answer the following questions.



Problem 4. *During the motion, with the part CD out of magnetic region, emf is induced in the portion*

(a) only along AB   *(b) only along CD*
(c) only along BC and AD   *(d) we can never predict*

Problem 5. *The value of constant force is*

(a) $6.4 \times 10^{-4}\,N$ (b) $1.28 \times 10^{-4}\,N$
(c) $1.6 \times 10^{-4}\,N$ *(d) None of these*

Problem 6. *The potential difference between A and B is*

(a) $1.6 \times 10^{-2}\,V$ (b) $1.8 \times 10^{-2}\,V$
(c) $3.6 \times 10^{-2}\,V$ (d) $36 \times 10^{-2}\,V$

***Solution* :** 4. Emf is induced across the wire AB

Ans. (a)

5. The magnetic force = $F_m = I\ell B$

$= 0.02 \times 0.32 \times 0.02$

$= 1.28 \times 10^{-4}$ N

Ans. (b)

6. The potential difference between AB,

$V_{AB} = IR$

$= 0.02 \times \left(\frac{2}{80} \times 32\right)$

$= 1.6 \times 10^{-2}$ volts

Ans. (a)

MATCHING TYPE PROBLEMS

Problem 7.

	Column-I		*Column-II*
(A)	A plane metallic loop rotating about an axis parallel to the magnetic field.	(P)	Conservation of energy.
(B)	Lenz's law	(Q)	Direction of magnetic field.
(C)	If a current in the loop decreases.	(R)	No emf is induced in the loop.
(D)	Fleming's rule	(S)	emf is induced in the loop.

Solution : Magnetic flux does not change. Hence no emf is induced in the loop.

Lenz's law is explained by conservation of energy.

Fleming's rule explains direction of magnetic field.

ASSERTION-REASON TYPE PROBLEMS

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion
(b) Both Assertion and Reason are true and Reason is not the correct explanation of assertion
(c) Assertion is true but reason is false
(d) Assertion is false but Reason is true

Problem 8. **Assertion :** *It is more difficult to push a magnet into a coil with more loops.*

Reason : *This is because emf. induced in each current loop resists the motion if the magnet.*

(a) A *(b) B*

(c) C *(d) D*

Solution : Whenever there is a change in magnetic flux linked with the coil a current is induced in the coil, which produces a magnetic field and hence it opposes the movement of the magnet.

Ans. (a)

Problem 9. **Assertion :** *When number of turns in a coil is doubled, coefficient of self inductance of the coil doubled, coefficient of self inductance of the coil becomes 4 times.*

Reason : *This is because* $L \propto n^2$

(a) A (b) B

(c) C (d) D

Solution : Self inductance of the coil, $L = \mu_0 n^2 A\ell_0$

Ans. (a)

Problem 10. **Assertion :** *The energy stored in a coil of 50 mH on passing 2A current is 0.2 J.*

Reason : *The magnetic energy stored,* $U = \frac{1}{2}LI^2$

(a) A (b) B

(c) C (d) D

Solution : The magnetic energy stored in the coil $U = \frac{1}{2}LI^2$

$$U = \frac{1}{2}\times\left(50\times10^{-3}\right)\times(2)^2$$

$= 0.1$ J

Ans. (d)

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SECTION - I

SUBJECTIVE TYPE QUESTIONS

LEVEL - I

1. A solenoid of self inductance 4 H and resistance 5 ohm is connected to a battery of 10 V and negligible resistance. After how long will the current in it rise to 1 A ?

2. A condenser is charged from a d.c. source through a resistance of 2×10^6 ohm. After half a second this charge reaches three quarters of its maximum value. Find the capacity of the condenser ?

3. A $2\mu F$ condenser is connected to a 220 V supply through a 0.5 megaohm resistance. Calculate the potential difference between the terminals of the condenser 0.1 second after the application of the voltage. Calculate also the initial charging current.

4. How many time constants one should wait for the current in an L – R circuit to grow within 0.1% of its steady state value.

5. A 3 H inductor is placed in series with 10 ohm resistor and an e.m.f. of 3V is applied to the combination. Find

   (a) the current at 0.3 second,
   (b) the rate of increase of current at 0.3 second,
   (c) the rate at which energy is dissipated as heat at $t = 0.3$ second,
   (d) the rate at which energy is stored in the magnetic field at $t = 0.3$ second,
   (e) the rate at which energy is delivered by the battery,
   (f) the energy stored when the current has attained a steady value

6. A 100 V potential difference is suddenly applied to a coil of inductance 100 mH and resistance 50 ohm. At what is the current increasing at 1 millisecond ?

7. An inductor of self inductance 500 mH and resistance 5 ohm is connected to battery of negligible internal resistance. Calculate the time in which the current will attain half of its final steady value ?

8. A coil of inductance 0.5 H and resistance 20 ohm is switched to direct current 200 V supply. Calculate the rate of increase of current
   (a) at the instant of closing the switch,
   (b) at $t = \frac{L}{R}$ second after the switch is closed.

9. The time constant of an inductive coil is 20×10^{-3} sec. When 90 ohm resistances is added in series, the time constant reduces to 0.5×10^{-3} sec. Find the inductance and resistance of the coil ?

10. An inductor of 3 H and resistance 6 ohm is connected to the terminals of a battery of e.m.f. 12 V and of negligible internal resistance. Calculate

    (a) the initial rate of increase of current in the circuit,
    (b) the rate of increase of current at the instant when the current in the circuit is 1 A,
    (c) the instantaneous value of current 0.2 second after the circuit is closed,
    (d) the final steady state current,
    (e) the power input to the inductor at the instant when the current is 0.5 A,
    (f) the rate of development of heat at this instant,
    (g) the rate at which the energy of the magnetic field is increasing at this instant ,
    (h) the energy stored in the magnetic field when the circuit has attained its steady value.

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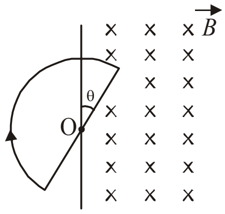
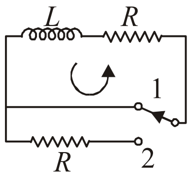
LEVEL - II

1. A wire loop enclosing a semicircle of radius R is located on the boundary of uniform magnetic field B. At the moment t = 0, the loop is set into rotation with a constant angular acceleration α about an axis O coinciding with a line of vector $\vec{B}$ on the boundary. Find the emf induced in the loop as a function of time. Draw the approximate plot of this function. The arrow in the figure shows the emf direction taken to be positive.



2. A long straight wire carries a current $I = I_0 \sin(\omega t + \phi)$ and lies in the plane of a rectangular loop of N turns of wire of length ℓ, width b with length parallel to the long straight wire and the nearer end is at a distance a from long wire. The quantities ℓ_0, ω and ϕ are all constants. Determine the e.m.f. induced in the loop by the magnetic field due to the current in the straight wire.

3. In the circuit shown in figure, the initial current through the inductor at t = 0 is I_0. After a time $t = \frac{L}{R}$, the switch is quickly shifted to position 2.

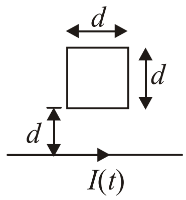


(a) Plot a graph showing the variation of current with time.

(b) Calculate the value of current in the inductor at $t = \frac{3L}{2R}$.

4. A square loop of side d, resistance R, lies at a distance d from the wire which carries current $I(t)$ which goes down gradually as :

$$I(t) = \begin{cases} (1-\alpha t)I_0 & for\, 0 \le t \le \frac{1}{\alpha} \\ 0 & for\, t > \frac{1}{\alpha} \end{cases}$$



5. A thin non-conducting ring of mass m radius a carrying a charge q rotates freely about its own axis which is vertical. At the initial moment, the ring was at rest and no magnetic field was present. At instant t = 0, a uniform magnetic field is switched on which is vertically downwards and increases with time according to the law $B = B_0 t$. Neglecting magnetism induced due to rotational motion of the ring, calculate

(a) angular acceleration of the ring and its direction of rotation as seen from above and
(b) power developed by the force acting on the ring as function of time.

6. A very small circular loop of radius a is initially coplanar and concentric with a much larger circular loop of radius b (>>a). A constant current I is passed in the large loop which is kept fixed in space and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is R and the inductance is negligible.

(a) Find the current in the small loop as a function of time.

(b) Calculate how much torque must be exerted on the small loop to rotate it.

(c) Calculate the induced EMF in the large loop due to current [found in part (a)] in smaller loop as a function of time.

7. A metal disc of radius R = 25 cm rotates with a constant angular velocity ω = 130 rad/s about its axis. Find the potential difference between the centre and rim of the disc if :

(a) the external magnetic field is absent.

(b) the external uniform magnetic field B = 5.0 mT is directed perpendicular to the disc.

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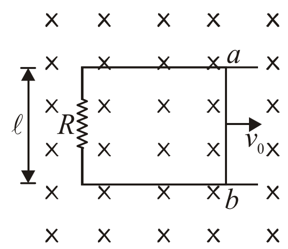
8. A conducting rod shown in figure of mass m and length ℓ moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the page. The rod is given an initial velocity v_0 to the right and is released at $t = 0$. Find as a function of time,
   (a) the velocity of the rod
   (b) the induced current and
   (c) the magnitude of the induced emf.

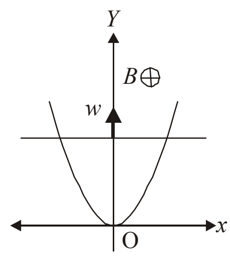


9. An inductance L and a resistance R are connected in series with a battery of e.m.f. E. are connected in series with a battery of e.m.f. E. What is the maximum rate at which the energy is stored in magnetic field ?

10. An L-C circuit consists of an inductor with $L = 0.0900$ H and a capacitor $C = 4.00\times10^{-4}$ F . The initial charge on the capacitor is $5.00\mu C$, and the initial current in the inductor is zero.
    (a) What is the maximum voltage across the capacitor ?
    (b) What is the maximum current in the inductor ?
    (c) What is the maximum energy stored in the inductor ?
    (d) When the current in the inductor has half its maximum value, what is the charge on the capacitor and what is the energy stored in the inductor ?

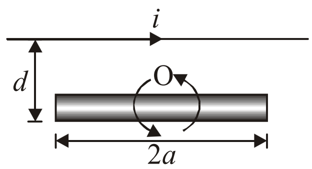
LEVEL - III

1. A wire bent as a parabola $y = ax^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the plane x, y. At the moment $t = 0$ a connector starts sliding translation wise from the parabola apex with a constant acceleration w as shown in figure. Find the emf of electromagnetic induction in the loop thus formed as a function of y.

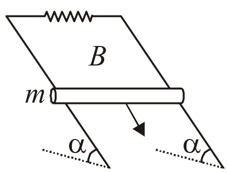


2. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\Phi = at(\tau - t)$. Find the amount of heat generated in the loop during that time. The inductance of the loop is to be neglected.

3. A rod of length $2a$ is free to rotate in a vertical plane, about a horizontal axis – O passing through its midpoint. A long straight, horizontal wire is in the same plane and is carrying a constant current I as shown in figure. Initially, the rod is horizontal and starts to rotate with constant angular velocity ω, calculate emf induced in the rod as a function of time.



4. A copper connector of mass m slides down on two smooth copper bars, set at an angle α to the horizontal, due to gravity as shown in figure. At the top the bars are interconnected through a resistance R. The separation between the bars is equal to ℓ. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.



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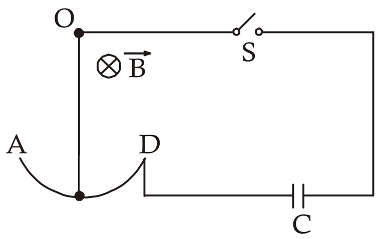
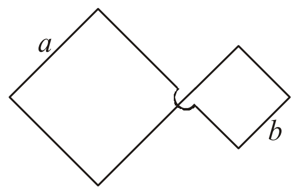
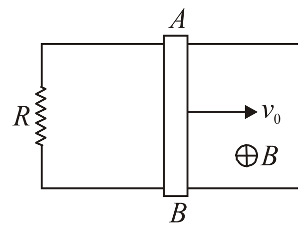
5. A simple pendulum consists of a small conducting ball of mass m and a light conducting rod of length ℓ. The pendulum oscillates with angular amplitude θ_0 in a vertical plane about a horizontal axis passing through O such that the ball is always just in contact with a metallic strip *AD* bent into a circular arc of radius ℓ as shown in the figure. In the space, a uniform magnetic field of induction B normal to plane of oscillation exists in the space. At time $t = 0$ when the ball is at its lowest position and moving towards right, the switch *S* is closed neglecting self inductance of the circuit calculate external torque τ required to keep the pendulum oscillating as before. Assume that θ_0 is small.



6. A plane loop shown in figure is shaped as two squares with sides a = 20 cm and b = 10 cm and is introduced into a uniform magnetic field at right angles to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin\omega t$, where $B_0 = 10\,\text{mT}$ and $\omega = 100\text{s}^{-1}$. Find the amplitude of the current induced in the loop if its resistance per unit length is equal to $\rho = 50\,\text{m}\Omega\text{m}^{-1}$. The inductance of the loop is to be neglected.

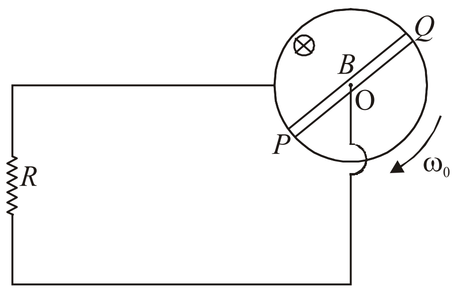


7. A conducting rod AB of mass *m* slides without friction over two long conducting rails separated by a distance ℓ. At the left end the rails are interconnected by a resistance *R*. The system is located in a uniform magnetic field perpendicular to the plane of the loop. At the moment $t = 0$ the rod *AB* starts moving to the right with an initial velocity v_0. Neglecting the resistances of the rails and the rod *AB*, as well as the self-inductance, find :



(a) the distance covered by the rod until it comes to a standstill;
(b) the amount of heat generated in the resistance *R* during this process.

8. A conducting rod *PQ* of mass *M* rotates without friction on a horizontal plane about O on circular rails on diameter ℓ. The centre O and the periphery are connected by resistance *R*. The system is located in a uniform magnetic field perpendicular to the plane of the loop. At t = 0, *PQ* starts rotating clockwise with angular velocity ω_0. Neglecting the resistance of the rails and rod, as well as self inductance, find
(a) magnitude of current as a function time.
(b) total charge flown through the resistance.
(c) the heat generated in the circuit by $t \to \infty$.



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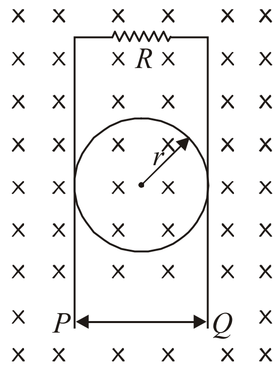
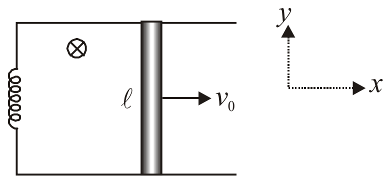
9. P and Q are two infinite conducting planes kept parallel to each other and separated by a distance 2r. A conducting ring of radius r falls vertically between the planes such that planes are always tangent to the ring. Both the planes are connected by a resistance R. There exists a uniform magnetic field of strength B perpendicular to the plane of ring as shown in figure. Plane Q is smooth and friction between the plane P and the ring is enough to prevent slipping . At t = 0, the ring was at rest and neglect the resistance of the planes and the ring. Find



(a) the current trough R as a function of time
(b) terminal velocity of the ring (assume g to be constant)

10. A loop is formed by two parallel conductors connected by a solenoid with inductance L and a conducting rod of mass m which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane in a uniform vertical magnetic field B. The distance between the conductor is ℓ. At the moment $t = 0$, the rod is imparted an initial velocity v_0 directed [to the right. Find the law of its motion $n(t)$ if the electric] resistance of the loop is negligible.



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SECTION - II

SINGLE CHOICE QUESTIONS

1. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(a) $\frac{BR}{A}$ (b) $\frac{AB}{R}$

(c) ABR (d) $\frac{B^2A}{R^2}$

2. A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns Ignoring the field variation across the cross-section of the toroid, the self-inductance of the toroid is

(a) 4.6 mH (b) 6.9 mH

(c) 2.3 mH (d) 9.2 mH

3. An emf of 15 V is applied in a circuit containing 5 H inductance and 10Ω resistance. The ratio of the currents at time $t = \infty$ and at $t = 1$ s is

(a) $\frac{e^{1/2}}{e^{1/2}-1}$ (b) $\frac{e^2}{e^2-1}$

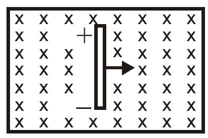
(c) $1-e^{-1}$ (d) e^{-1}

4. A rectangular loop of sides 8 cm and 2 cm is lying in a uniform magnetic field of magnitude 0.5 T with its plane normal to the field. The field is now gradually reduced at the rate of 0.02 Ts^{-1}. If the resistance of the loop is 1.6Ω, then the power dissipated by the loop as heat is

(a) 6.4×10^{-10} W (b) 3.2×10^{-10} W

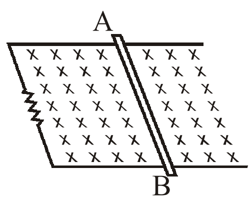
(c) 6.4×10^{-5} W (d) 3.2×10^{-5} W

5. A conducting wire is moving towards right in a magnetic field. The direction of induced emf in the wire is as shown in the figure. The direction of magnetic field is



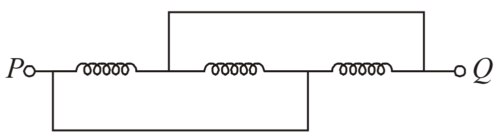
(a) in the plane of the paper pointing towards right.
(b) in the plane of the paper pointing towards left..
(c) perpendicular to the plane of the paper and downwards.
(d) perpendicular to the plane of the paper and upwards.

6. Consider the situation shown. The wire AB is sliding on fixed rails with a constant velocity. If the wire AB is replaced by semi-circular wire, the magnitude of induced e.m.f. will



(a) increases
(b) decreases.
(c) remain the same
(d) increase or decrease depending on whether the semi – circle bulges towards the resistance or away from it.

7. Pure inductors each of inductance 3 H are connected as shown. The equivalent induction of the circuit is



(a) 1 H (b) 2 H

(c) 3 H (d) 9 H

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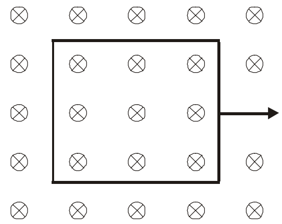
8. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. The current induced in the loop is



(a) $\frac{BLv}{R}$ clockwise (b) $\frac{BLv}{R}$ anticlockwise

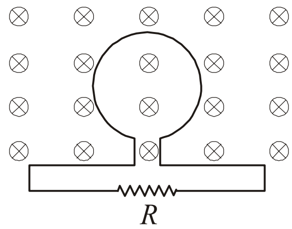
(c) $\frac{2BLv}{R}$ anticlockwise (d) ZERO

9. A copper rod of length 1 is rotated about one end perpendicular to a magnetic field B with constant angular velocity ω. The induced emf between the two ends is

(a) $\frac{1}{2}B\omega l^2$ (b) $\frac{3}{4}B\omega l^2$

(c) $B\omega l^2$ (d) $2B\omega l^2$

10. In the figure the flux through the loop perpendicular to the plane of the coil and directed into the paper is varying according to the relation $\Phi = 6t^2 + 7t + 1$ where Φ is in milliweber and t is in second. The magnitude of the emf induced in the loop at t = 2 s and the direction of induced current through R are



(a) 39 mV; right to left (b) 39 mV; left to right

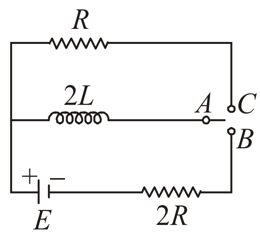
(c) 31 mV; right to left (d) 31 mV; left to right

11. A uniformly wound solenoidal coil of self inductance 2.4×10^{-4} H and resistance $8\,\Omega$ is broken up into two identical coils. These identical coils are then connected in parallel across a 12 V battery of negligible resistance. The time constant for the current in the circuit is

(a) 3×10^{-5}s (b) 3×10^{-4}s

(c) 3×10^{-3}s (d) 3×10^{4}s

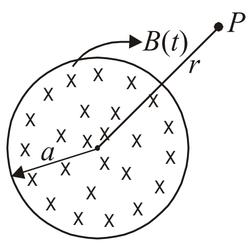
12. In the circuit shown, A is joined to B for a long time, and then A is joined to C. The total heat produced in R, after A is connected to C, is



(a) $\frac{LE^2}{R^2}$ (b) $\frac{LE^2}{2R^2}$

(c) $\frac{LE^2}{4R^2}$ *(d)* $\frac{LE^2}{8R^2}$

13. A uniform but time varying magnetic field B (t) exists in a circular region of radius a and is directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$

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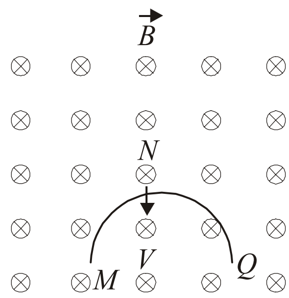
14. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v, and the potential difference developed across the ring is



(a) ZERO
(b) $\frac{BV\pi R^2}{2}$ and M is at a higher potential
(c) πRBv and Q is at a higher potential
(d) $2RBv$ and Q is at a higher potential

15. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge – length d is placed with its edges along X and Y – axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is

(a) ZERO
(b) $v_0 B_0 d$
(c) $\frac{v_0 B_0 d^3}{a^2}$
(d) $\frac{v_0 B_0 d^2}{a}$

16. A capacitor of $1\mu F$ initially charged to 10 V is connected across an ideal inductor of 0.1 mH. The maximum current in the circuit is

(a) 0.5 A
(b) 1A
(c) 1.5 A
(d) 2 A

17. Initially an inductor of zero resistance is joined to a cell of emf E through a resistance. The current increases with a time constant τ. The emf across the coil after time t is

(a) $E\left(1-e^{-\frac{t}{\tau}}\right)$
(b) $\frac{Et}{\tau}$
(c) $Ee^{-\frac{2t}{\tau}}$
(d) $Ee^{-\frac{t}{\tau}}$

18. Two inductors, each of inductance L, are connected in parallel but are well separated from each other. The effective inductance is

(a) $\frac{L}{4}$
(b) $\frac{L}{2}$
(c) L
(d) $2L$

19. A current of 2A flowing through a coil of 100 turns gives rise to a magnetic flux of 5×10^{-5} Wb per turn. The magnetic energy associated with the coil is

(a) 5 J
(b) 0.5 J
(c) 0.05 J
(d) 0.005 J

20. A capacitor of capacitance $2\mu F$ is first charged and then discharged through a resistance of $1\ M\Omega$. The time in which the charge on the capacitor will fall to 50% of its initial value is

(a) 1.38 second
(b) 1.04 second
(c) 0.35 second
(d) 0.693 second

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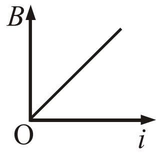
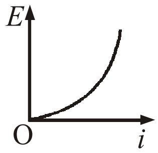
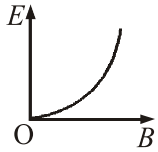
SECTION - III

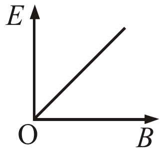
MULTIPLE CHOICE QUESTIONS

1. If B and E denote induction of magnetic field and energy density at mid-point of a long solenoid, carrying a current I, then which of the following graphs are correct ?

(a) 

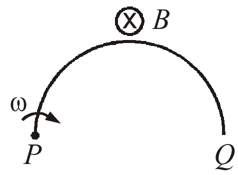
(b) 

(c) 

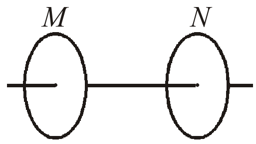
(d) 

2. If a current is flowing in a solenoid :
   (a) It has tendency to increase its radius if no external magnetic field exists in the space.
   (b) It may have tendency to increase its radius if an external magnetic field exists is the space.
   (c) It may have tendency to decrease its radius if an external magnetic field exists in the space.
   (d) None of these

3. A wire is bent to form a semi-circle of radius a. The wire rotates about its one end with angular velocity ω. Axis of rotation being perpendicular to plane of the semicircle. In the space, a uniform a magnetic field of induction B exists along the axis of rotation as shown in figure. Then
   (a) potential difference between P and Q is equal to $2B\,\omega\,a^2$.
   (b) potential difference between P and Q is equal to $\frac{1}{2}B\,\omega\,a^2$
   (c) P is at higher potential than Q.
   (d) P is at lower potential than Q.

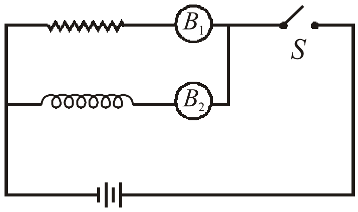


4. Two identical circular coils M and N are arranged coaxilly as shown in figure Separation between the coils is large as compared to their radii. The arrangement is viewed from left along the common axis. The sign convention adopted is that currents are taken to be positive when they appear to flow in clockwise direction. Then



   (a) If M carries a constant positive current and is moved towards N, a positive current is induced in N
   (b) If M carries a constant positive current and N is move towards M, an negative current is induced in N.
   (c) If a positive current in M is switched off, a positive current is momentarily induced in N
   (d) If both coils carry positive currents, they will attract each other.

5. The fig shows two bulbs B_1 and B_2, resistor R and inductor L. When the switch S is turned off, then :



The current (s) flowing through

(a) both B_1 and B_2 reach their steady values promptly

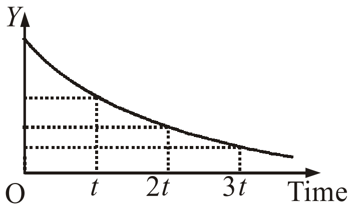
(b) both B_1 and B_2 reach their steady value with some delay.

(c) B_1 reaches its steady value promptly

(d) B_2 reaches its steady value with some delay

6. A solenoid is connected to a source of constant emf for a long time. A soft iron piece is inserted into it. Then:
(a) self-inductance of the solenoid gets increased.
(b) flux linked with the solenoid increase, hence steady state current gets decreased.
(c) energy stored in the solenoid gets increased
(d) magnetic moment of the solenoid gets increased

7. The Y-axis of the following graph may represent :



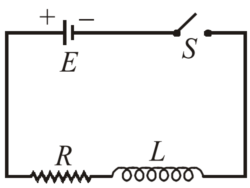
(a) a current in a circuit containing a source of constant emf a pure resistance and a pure inductor, when the source is shorted at a time $t = 0$

(b) the number of disintegrated nuclei in a large population of identical radioactive nuclei.

(c) the p.d. between the plates of charged capacitor, which is shorted through a pure resistance at time = t =0.

(d) the temperature difference between a body and comparatively slightly cooler enclosure of constant temperature, in which the body is suspended.

8. Switch S of the circuit shown in the figure is closed at $t = 0$. If e denotes the induced emf in L



and I, the current flowing through the circuit at time t, which of the following graphs are correct ?

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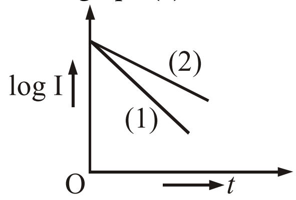
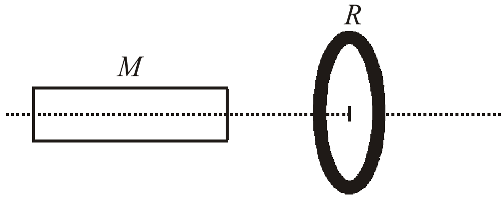
9. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor R. At time $t = 0$, the key is closed. If I = current at time t, a plot of log I against t is as shown as (1) in the graph. Later, one of the parameters i.e. V , R or C is changed keeping the other two constant, and the graph (2) is recorded. Then :



(a) C is reduced.
(b) C is increased.
(c) R is reduced
(d) R is increased.

10. A conducting ring R is placed on the axis of a bar magnet M. The plane of R is perpendicular to this axis. M can move along this axis :



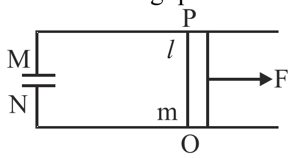
(a) M will repel R when it is moving towards R
(b) M will attract R when it is moving towards R.
(c) M will repel R when it is moving away from R
(d) M will attract R when it is moving away from R.

SECTION - IV

PASSAGE BASED QUESTIONS

Write-up I [Questions 1 to 3]

A parallel plate capacitor of capacitance C is connected between two horizontal metallic rails where a uniform vertical magnetic field B is applied, so that it acts into the plane. A metallic rod of length l and mass m lies and can slides freely on them. The distance between the rails is also l. A constant horizontal force F acts on the rod. The resistance of the system is negligible. Answer the following questions.



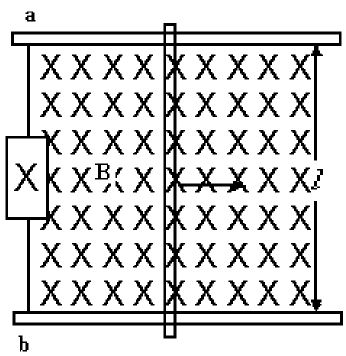
1. During the motion of the rod
(a) The current increases with time
(b) The current keeps on decreasing
(c) The current remains constant
(d) No current flows through the circuit

2. During the motion of the rod
(a) The charge on the capacitor increases with time
(b) The charge on the capacitor decreases with time
(c) The charge on the capacitor remains constant
(d) The charge on the capacitor is always zero

3. During the motion of the rod
(a) The plate M becomes +ve
(b) The plate N becomes +ve
(c) The plate M becomes -ve
(d) The plate N becomes -ve

Write-up II [Questions 4 to 6]

A wire of mass m and length l can freely slide on a pair of parallel smooth thick conducting rails placed in a horizontal plane. The rails are connected by an unknown element X as shown in figure A uniform magnetic field B exists perpendicular to the plane of the rails Neglect the resistance of the rails and wire.



4. The wire is given a initial velocity v_0 to the right and released.

a) If X is a capacitor, then velocity decreases exponentially.
b) If X is a resistor, then its velocity remains constant.
c) If X is a inductor, the wire executes SHM.
d) If X is a capacitor the wire executes oscillatory motion

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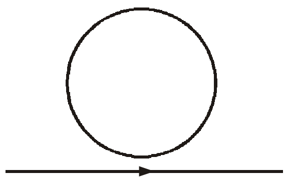
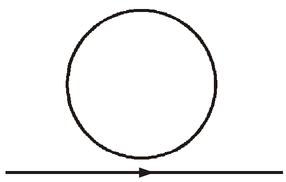
5. The element X is a battery of emf E with its positive polarity at *a* and internal resistance *r*. If the wire is released from rest, then
(a) The wire begins to move towards X
(b) The wire remains stationary
(c) The velocity of the wire increases before it reaches a terminal velocity.
(d) The wire moves with an acceleration of constant magnitude.

6. The element X is a constant current source which drives a current directed from *b* to *a*. The wire is released very close to *a b*. Then its distance from *a b* as a function of time is proportional to
(a) t (b) t^2
(c) $e^{-\alpha t}, \alpha = const$ (d) none of these

MATCHING TYPE QUESTIONS

7.	Column-I		Column II
(A)	North pole of magnet approaching the coil	(P)	The current is induced in the loop clockwise.
(B)	South pole of magnet approaching the coil	(Q)	The current is induced in the loop anti clockwise.
(C)	Current in straight wire is increasing as shown in figure. 	(R)	The face of the coil behaves as north pole.
(D)	Current in straight wire is decreasing as shown in figure. 	(S)	The face of the coil behaves as south pole.

8.	Column-I		Column II
(A)	Mutual inductance exists	(P)	Two coils completely overlap.
(B)	Mutual inductance is maximum	(Q)	Self inductance
(C)	Mutual inductance depends on	(R)	Spacing between two coils
(D)	Mutual inductance is related to	(S)	Between two coils

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ASSERTION-REASON TYPE QUESTIONS

(A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion
(B) Both Assertion and Reason are true and Reason is not the correct explanation of assertion
(C) Assertion is true but reason is false
(D) Assertion is false but Reason is true

9. ***Assertion :*** Lenz's law violates the principle of conservation of energy.

Reason : Induced emf always opposes the change in magnetic flux responsible for its production.

(a) A (b) B
(c) C (d) D

10. ***Assertion :*** Electricity is generated by rotating a copper coil in a magnetic field.

Reason : On rotating the coil, θ changes. Therefore, magnetic flux ϕ linked with the coil changes. Hence an emf is induced.

(a) A (b) B
(c) C (d) D

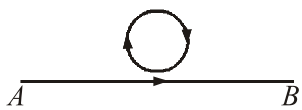
11. ***Assertion :*** In the phenomenon of mutual induction between two current carrying coils, self inductance of each of the coils does not contribute to the Faraday emf.

Reason : Self induction arises when strength of current in one coil changes, and there is back-emf in that coil.

(a) A (b) B
(c) C (d) D

12. ***Assertion :*** When current flowing through a straight conductor is increased, an induced electric field is also produced around it.

Reason : Whenever a current flows through a conductor, a magnetic field is produced around it.



(a) A (b) B
(c) C (d) D

13. ***Assertion :*** When an induction coil is working, self induced emf at break is much greater than self induced emf at make.

Reason : $e = LdI/dt$. Symbols have standard meaning.

(a) A (b) B
(c) C (d) D

14. ***Assertion :*** A 25 kW D.C. generator produces a potential difference of 250 V. If the resistance of transmission line is 1 Ω, the percentage loss of power in transmission is 40%.

Reason : Loss of power = V^2/R, where V is the voltage of the generator, and R is resistance of the transmission line

(a) A (b) B
(c) C (d) D

15. ***Assertion :*** When plane of a rotating coil is perpendicular to the magnetic field, magnetic flux linked with the coil is maximum but induced emf is zero.

Reason : $\phi = nAB\cos\theta$ and $e = \dfrac{d\phi}{dt}$

(a) A (b) B
(c) C (d) D

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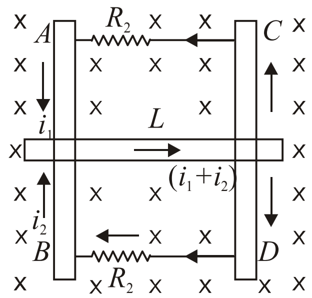
SECTION - V

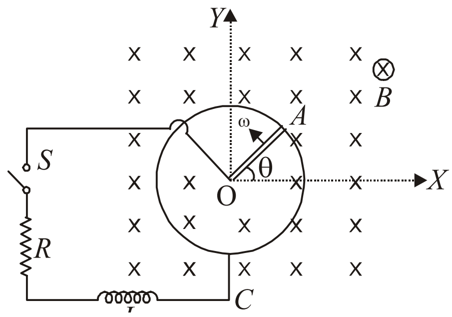
PROBLEMS ASKED IN IIT-JEE

SUBJECTIVE

1. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.760 W and 1.2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **[IIT – JEE 1994]**



2. A metal rod OA of mass m and length l is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic field induction B is applied perpendicular and into the plane of rotation as shown in figure. An inductor and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(a) What is the induced e.m.f. across the terminals of the switch ?
(b) The switch S is closed at time t = 0.
   (i) Obtain an expression for the current as a function of time.
   (ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X – axis at $t = 0$.

[IIT – JEE 1995]

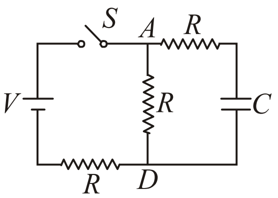
3. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC.

(a) When Q = 100 μC, what is the value of $\left|\dfrac{dI}{dt}\right|$?
(b) When Q = 200 μC, what is the value of I?
(c) Find the maximum value of I.
(d) When I is equal to half its maximum value, what is the value of |Q|? **[IIT-JEE 1998]**

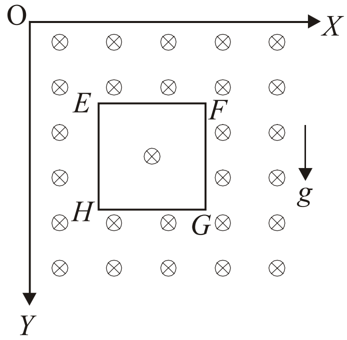
4. In the circuit shown in figure, the battery is an ideal one with e.m.f. V. The capacitor is initially uncharged . The switch S is closed at time t = 0.

   (a) Find the charge q on the capacitor at time

   (b) Find the current in AD at time t. What is the limiting value as $t \to \infty$? **[IIT-JEE 1998]**



5. A magnetic field $\vec{B} = \left(\dfrac{B_0 y}{a}\right)\hat{k}$ is into the paper in the +Z direction. B_0 and a are positive constants. A square loop EFGH of side a and mass m and resistance R, in X – Y plane , start falling under the influence of gravity. Note the directions of X and Y axes in figure, find.



   (a) the induced current in the loop and indicate its direction.

   (b) the total Lorentz force acting on the loop and indicate its direction, and

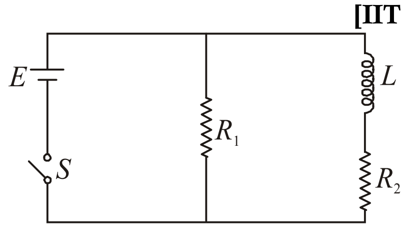
   (c) an expression for the speed of the loop, v (t) and its terminal velocity.

[IIT – JEE 1999]

6. A thermocole vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area $5 \times 10^{-3} m^2$, number of turns 100, mass 0.06 kg and resistance 1.6Ω is lying horizontally at the bottom of the vessel. A uniform , time varying magnetic field is set up to pass vertically through the coil at time t = 0. The field is first increased from zero to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. This cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as functions of time for the first two cycles, Clearly indicate the magnitudes of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of the water under thermal equilibrium. Specific heat of the metal = 500 $Jkg^{-1}K^{-1}$ and the specific heat of water =4200J $kg^{-1}K$. Neglect the inductance of the coil.

[IIT – JEE 2000]

7. An inductor of inductance L = 400 mH and resistor or resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of e.m.f. E of 12 V as shown in figure. The internal resistance of the battery is negligible. The switch is closed at time t = 0.



   What is the potential drop across L as a function of time ? After the steady state is reached, the switch is opened. What is the direction and magnitude of current through R_1 as a function of time ?

[IIT – JEE 2001]

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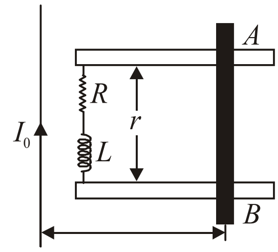
8. A meter bar AB can slide on two parallel thick metallic rails separated by a distance L. A resistance R and an inductance L are connected to the rails as shown in figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At t = 0, it is made to slide on the rails away from the wire. Answer the following questions:

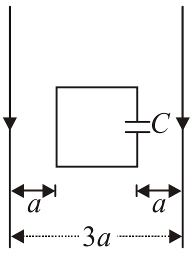


(a) Find a relation among, $i, \frac{di}{dt}$ and $\frac{d\Phi}{dt}$, where i is the current in the circuit and Φ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to t = T.

(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. Find the value of $\frac{L}{R}$ in terms of the other given quantities. **[IIT – JEE 2002]**

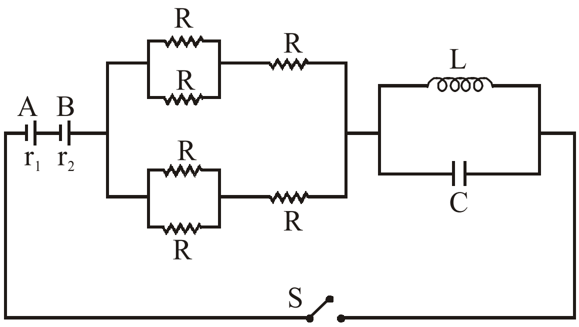
9. Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite directions are placed at a distance 3a apart as shown in figure. A square loop of side a of negligible resistance with a capacitor of capacitance C is



placed in the plane of wires. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive. **[IIT – JEE 2003]**

10. In the circuit shown, A and B are two cells of same emf E but different internal resistances r_1 and $r_2(r_1>r_2)$ respectively. Find the value of R such that the potential difference across the terminals of cell A is zero a long time after the key K is closed

[IIT-JEE 2004]



OBJECTIVE

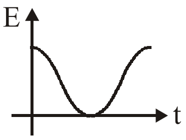
1. A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns was to be quadrupled and the wire radius halved, the electric power dissipated would be

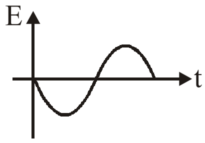
[IIT-JEE 2002]

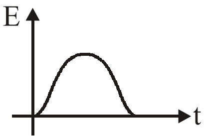
(a) halved
(b) the same
(c) doubled
(d) quadrupled

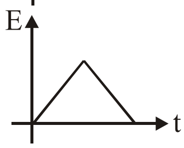
2. The variation of induced emf (E) with time (t) in a coil if a short bar magnetic is moved along its axis with a constant velocity is best represented as :

[IIT-JEE 2004]

(a)


(b)


(c)


(d)


3. Two difference coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_1, V_2 and W_2 respectively. Then **:**

(a) $\frac{i_1}{i_2} = \frac{1}{4}$
(b) $\frac{i_1}{i_2} = 4$
(c) $\frac{W_1}{W_2} = \frac{1}{4}$
(d) $\frac{V_1}{V_2} = 4$

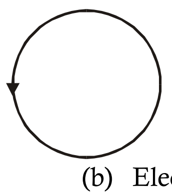
[IIT-JEE 1994]

4. An infinitely long cylinder is kept parallel to an uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be :

(a) clockwise of the +ve z-axis
(b) anticlockwise of the +ve z-axis
(c) zero
(d) along the magnetic field

[IIT-JEE 2005]

5. A field line is shown in the figure. This field cannot represent.



(a) Magnetic field
(b) Electrostatic field
(c) Induced electric field
(d) Gravitational field

[IIT-JEE 2006]

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ANSWERS

SECTION - I

SUBJECTIVE TYPE QUESTIONS

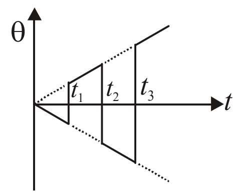
LEVEL I

1. 0.5544 s
2. $0.1803\mu F$
3. 20.944 V, 440×10^{-6} A
4. 6.9 time constants
5. (a) 0.19 A, (b) 0.368 As^{-1}, (c) 0.36 W
   (d) 0.21 W, (e) 0.57 W, (f) 0.135 J
6. 606 As^{-1}
7. 0.0693 second
8. (a) 400 As^{-1}, (b) 147.04 As^{-1}
9. R = 30 ohm, L = 60 mH
10. (a) 4 As^{-1} (b) 2 As^{-1}
    (c) 0.65 A (d) 2 A
    (e) 6 W (f) 0.357 cal s^{-1}
    (g) 4.5 Js^{-1} (h) 6 J

LEVEL II

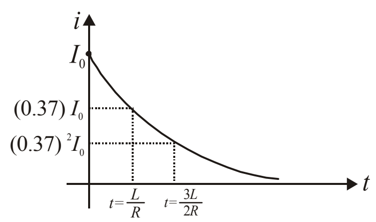
1. $e = (-1)^n \left(\frac{1}{2} BR^2 \alpha t\right)$

   Here n = 1, 2, 3 .... is the number of half revolutions that the loop performs at the given moment t. The e-t graph is as shown in figure.



2. $\xi = \frac{\mu_0 N I_0 \ell \omega}{2\pi} \log_e \left(\frac{a+b}{a}\right) \cos(\omega t + \phi)$

3.



4. Anticlockwise : $\Delta q = \frac{\mu_0 I_0 d}{2\pi} \ln(2)$

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5. (a) $\alpha = \frac{B_0 q}{2m}$

(b) $P = \frac{q^2 B_0^2 a^2 t}{4m}$

6. (a) $\frac{\mu_0 \omega \pi a^2 I}{2bR} \sin(\omega t)$

(b) $\frac{\mu_0 \omega \pi^2 a^4 I^2}{4b^2 R} \sin(\omega t)$

(c) $\frac{\mu_0^2 \pi^2 a^4 \omega^2 I^2}{4b^2} \cos(\omega t)$

7. (a) 3.0 nV

(b) 20 mV

8. (a) $v = v_0 e^{-t/\tau}$; where, $\tau = \frac{mR}{B^2 I^2}$

(b) $i = \frac{B\ell v_0}{R} e^{-t/\tau}$

(c) $e = iR = B\ell v_0 e^{-t/\tau}$

9. $P_{\max} = \frac{E^2}{4R}$

10. (a) 1.25×10^{-2} V

(b) 8.33×10^{-4} A

(c) 3.125×10^{-8} J

(d) 7.8×10^{-7} J

LEVEL III

1. By $\sqrt{\frac{8w}{a}}$

2. Heat Generated $= \frac{a^2 \tau^3}{3R}$

3. $\frac{\mu_0 i \omega}{2\pi \sin(\omega t)} \left[2a + \frac{d}{\sin(\omega t)} \ln \left\{ \frac{d - a\sin(\omega t)}{d + a\sin(\omega t)} \right\} \right]$

4. $v = \frac{mgR \sin\alpha}{B^2 l^2}$

5. $\tau_{\text{clockwise}} = \frac{\pi^2 \theta_0 B^2 \ell^2 C}{T^2} \sin\left(\frac{2\pi t}{T}\right)$

6. $I = \frac{\omega B_0 (a - b)}{4\rho} = 0.5$ A

7. (a) $s = \frac{v_0 m R}{\ell^2 B^2}$

(b) $Q = \frac{1}{2} m v_0^2$

8. (a) $i(t)=\frac{B\omega_0\ell^2}{8R}e^{-\alpha t}$; where, $\alpha=\frac{3B^2l^2}{8RM}$

(b) Total charge $=\frac{\omega_0 M}{3B}$

(c) Heat Generated = $\frac{Ml^2\omega_0^2}{24}$

9. (a) $i(t)=\frac{mg}{2Br}\left\{1-e^{-\left(\frac{2B^2r^2}{mR}\right)t}\right\}$

(b) Terminal Velocity $=\frac{mgR}{4B^2r^2}$

10. $x(t)=\frac{v_0}{\omega}\sin(\omega t)$, where $\omega=\frac{Bl}{\sqrt{mL}}$

SECTION - II

SINGE CHOICE QUESTIONS

1.	(b)	2.	(c)	3.	(b)	4.	(a)
5.	(c)	6.	(c)	7.	(a)	8.	(d)
9.	(a)	10.	(d)	11.	(a)	12.	(c)
13.	(b)	14.	(d)	15.	(d)	16.	(b)
17.	(d)	18.	(b)	19.	(d)	20.	(a)

SECTION - III

MULTIPLE CHOICE QUESTIONS

1.	(a,b,c)	2.	(a,b,c)	3.	(a,d)	4.	(b,c,d)	5.	(c,d)
6.	(a,c,d)	7.	(a,b,c,d)	8.	(c,d)	9.	(b,d)	10.	(a,d)

SECTION - IV

PASSAGE BASED QUESTIONS

1.	(c)	2.	(a)	(a)	3.	(c)	4.	(c)	5.	(b)

MATCHING TYPE QUESTONS

7. (A)→(R)
   (B)→(S)
   (C)→(P & S)
   (D)→(Q & R)

8. (A)→(S)
   (B)→(P)
   (C)→(R & Q)
   (D)→(R & Q)

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ASSERTION-REASON TYPE QUESTIONS

9.	(c)	10.	(a)	11.	(a)	12.	(b)
13.	(a)	14.	(c)	15.	(a)		

SECTION - V

PROBLEMS ASKED IN IIT – JEE

SUBJECTIVE

1. $v_T = 1\text{ ms}^{-1};\ R_1 = 0.47\Omega;\ R_2 = 0.3\Omega$

2. (a) $e = -\frac{1}{2}Br^2\omega$

   (b) (i) $I = \frac{Br^2\omega}{2R}\left\{1 - e^{-\left(\frac{Rt}{L}\right)}\right\}$

   (ii) $\tau = \frac{B^2r^4\omega}{4R}\left\{1 - e^{-\left(\frac{Rt}{L}\right)}\right\}$

3. (a) 10^4 As^{-1}
   2. (b) Zero (c) 2.0 A (d) 173 μC

4. (a) $q = \frac{VC}{2}\left\{1 - e^{-\left(\frac{2t}{3CR}\right)}\right\}$

   (b) $\frac{V}{2R} - \frac{V}{6R}e^{-\left(\frac{2t}{3CR}\right)}; \frac{V}{2R}$

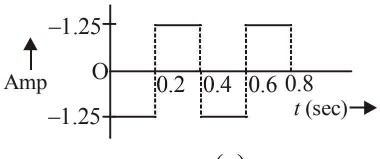
5. (a) $\frac{B_0av}{R}$; Anticlockwise

   (b) $\vec{F} = \frac{B_0^2a^2v}{R}\left(-\hat{j}\right)$

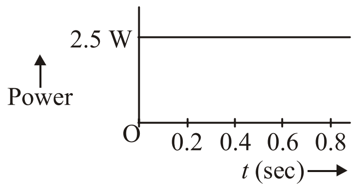
   (c) $v = \frac{mgR}{B_0^2a^2}\left\{1 - e^{-\left(\frac{B_0^2a^2t}{mr}\right)}\right\}$;

   $v_{\text{terminal}} = \frac{mgR}{B_0^2a^2}$

6.



(a)



(b)

$T = 35.6°\text{C}$

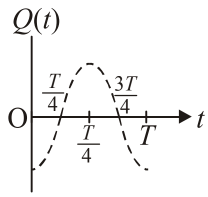
7. $12e^{-5t}; i = 6e^{-10t}$; The direction of current in R_1 is from E to B.

8. (a) $\left|\frac{d\Phi}{dt}\right| = L\frac{di}{dt} + R$

   (b) $\Delta q = \frac{\left(\frac{\mu_0 I_0 I}{2\pi}\right)[\log_e 2] - Li_1}{R}$

   (c) $\frac{L}{R} = \frac{2\log_e 2}{T}$

9. $I_{\max} = \left\{\frac{C\mu_0 I_0 a\omega^2}{\pi}\right\}\log_e(2)$



10 $\frac{4}{3}\left(r_1 - r_2\right)$

OBJECTIVE

1.	(b)	2.	(b)	3.	(a,c, b)	4.	(c)	5.	(a,d)

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