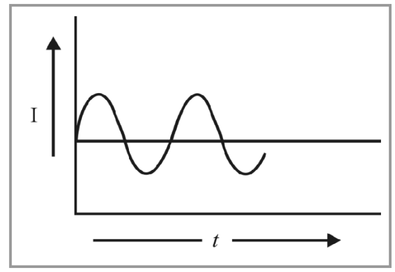
1. PHASORS

Alternating Current



- A time varying periodic current with constant amplitude which reverses its direction at the end of every half cycle is referred to as alternating current.
- In its sinusoidal form, it can be represented as follows:
- The corresponding voltage is called alternating voltage.
- Most of the electric power is generated and used in the a.c form.

Advantages of AC

- Alternating voltage can be easily increased or decreased by means of transformers.
- Alternating current energy can be transmitted and distributed over long distances without much loss of energy.

Mathematical Representation

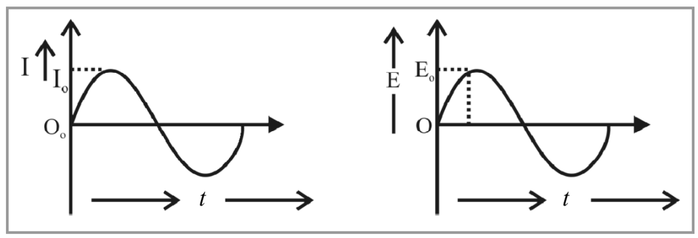
a.c. is represented as

$I = I_o \sin \omega t$

Or $I = I_o \cos \omega t$

Where, I is the instantaneous value of current at any instant t, I_o is the peak value or maximum value of ac (called amplitude) and ω is the angular frequency of a.c.

Also $\omega = \frac{2\pi}{T} = 2\pi f$



Where, T is the time period of a.c.

Similarly, a.c. voltage can be represented as,

$E = E_o \sin \omega t$ *Or* $E = E_o \cos \omega t$

- The effective value of ac is given in terms of its rms value. The rms value of ac is the value of dc that would produce the same amount of heat across a conductor over a given time. For ac in sinusoidal form, the rms value is $\frac{1}{\sqrt{2}}$ times the peak value.

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \quad , \quad E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

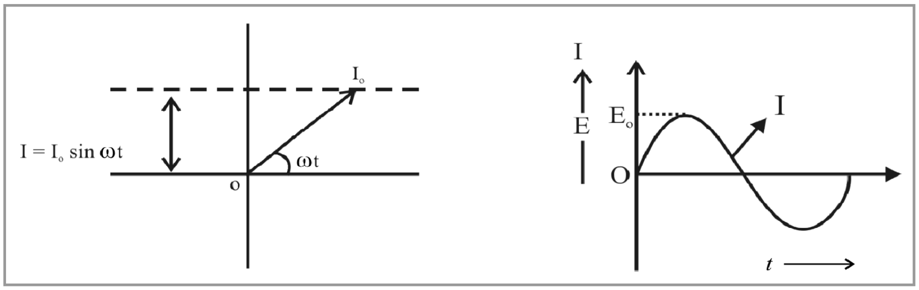
Note: For triangular waves, $I_{rms} = \frac{I_0}{\sqrt{3}}$ $\Rightarrow$ For square waves, $I_{rms} = I_0$

- The mean value of ac over a full cycle is zero. As a practical measure, the average value is calculated as the mathematical mean of all the points' absolute values. In sinusoidal form, the average value of ac, so calculated, is approximately 0.637 of its peak value.

  $I_{av} = 0.637\, I_0$

Phasor Diagrams

- A phasor is a vector which rotates about the origin with an angular speed ω.
- The phasor can be current phasor or voltage phasor.
- The length of the phasor gives the magnitude of the quantity (*i.e.* I or E).
- Phasors are inclined to horizontal axis at an angle equal to 'ωt' and rotates in the anticlockwise direction.
- Projection of the phasor on any axis represents the instantaneous value of the quantity.
- In sine form, projection is taken on the vertical axis.
- In cosine form, projection is taken on the horizontal axis.
- Phase difference between two alternating quantities is represented by the angle between the two vectors $\vec{E_o}$ and $\vec{I_o}$
- The diagram or plot of phasors for analysing an a.c circuit is called phasor diagram.



2. MEAN OR AVERAGE VALUE OF A.C. & EM.F

The mean or average value of a.c. over any half cycle is defined as that value of steady current which would send the same amount of charge thorugh a circuit in the time of half cycle (i.e. T/2) as is sent by the a.c. through the same circuit, in the same time.

To calculate its value let an alternating current be represented by

If the strength of current is assumed to remain constant for a small time, dt, then small amount of charge sent in a small time dt is

$$dq = I\ dt$$

Let q be the total charge sent by a.c. in the first half cycles $(i.e\ 0 \rightarrow T/2)$

$$\therefore \qquad q = \int_0^{T/2} I dt$$

$$q = \int_0^{T/2} I_0 \sin\omega t.dt = I_0\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{I_0}{\omega}\left[\cos\omega\frac{T}{2} - \cos 0^o\right]$$

$$= -\frac{I_0}{\omega}\left[\cos\pi - \cos 0^o\right]$$

$$= -\frac{I_0}{\omega}\left[-1-1\right] = \frac{2I_0}{\omega} \qquad \text{... (i)}$$

If Im represents the mean or average value of a.c. over the 1st half cycle, then

$$q = I_m \times \frac{T}{2} \qquad \text{.. (ii)}$$

From (i) and (ii), we get

$$I_m \times \frac{T}{2} = 2\frac{I_0}{\omega} = \frac{2I_0.T}{2\pi}$$

or $$I_m = \frac{2}{\pi} I_0 = 0.637\, I_0$$

(b) Mean or Average value of Alternating E.M.F

The mean or average value of alternating e.m.f. over a half cycle is that value of constant e.m.f which would send the same amount of charge through a circuit in the time of half cylce (T/2), as is sent by alternating e.m.f. through the same circuit in the same time.

To calculate its value, let an alternating e.m.f. be represented by $E = E_0 \sin\omega t$

If I is the value of current at instant t, then $I = \frac{E}{R} = \frac{E_0}{R}\sin\omega t$

where R is resistance of the circuit.
If this current remains constant for a small time dt, then small amount of charge sent by alternating e.m.f in the small time dt is

$$dq = Idt = \frac{E_0}{R}\sin\omega t\, dt$$

$\therefore$ Time charge sent by alternating e.m.f in the first half cycle $(0 \to T/2)$ would be

$$q = \int_0^{T/2} \frac{E_0}{R}\sin\omega t\, dt = \frac{E_0}{R}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{E_0}{\omega R}\left[\cos\omega\frac{T}{2} - \cos 0^o\right]$$

$$= -\frac{E_0}{\omega R}\left[\cos\pi - \cos 0^o\right]$$

$$q = -\frac{E_0}{\omega R}(-1-1) = \frac{2E_0}{\omega R} \qquad \text{... (iii)}$$

If E_m is mean or average value of alternating e.m.f. over the first half cycle, then $q = \frac{E_m}{R} \times \frac{T}{2}$ ... (iv)

from (iii) and (iv),

$$\frac{E_m}{R}\frac{T}{2} = \frac{2E_0}{\omega R} = \frac{2E_0}{\left(\frac{2\pi}{T}\right)R} = \frac{2\ E_0\ T}{2\ \pi\ R}$$

3. ROOT MEAN SQUARE VLAUE OF ALTERNATING CURRENT

The root mean square (r.m.s) value of a.c. is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time,a s is done by the a.c. when passed thorugh the same resistance for the same time.
The r.m.s value is also called effective value of a.c. or virtual vlaue of a.c. It is represented by I_{rms} or I_{eff} or I_v. to calculate it suppose, an alternating current is represented by

$$I = I_0 \sin\omega t$$

Let this current flow thorugh a resistance R. In a small time dt, the amount of heat produced in resistance R is

$$dH = I^2 R\, dt$$

In one complete cycle $(time\ 0 \to T)$, the total amount of heat produced in the resistance R would be

$$H = \int_0^T I^2 R\ dt$$

Using, we get

$$H = \int_0^T \left(I_0^2 \sin^2 \omega t\right) R\, dt$$

$$= I_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T 1\ dt - \int_0^T \cos 2\omega t.dt\right] = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_0^2 R}{2}\left[(T-0) - \left(\frac{\sin 2\omega t}{2\omega} - \sin\ 0^o\right)\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{\sin 2 \times 2\pi}{2\omega}\right]$$

$$H = \frac{I_0^2 RT}{2} \qquad \text{... (v)}$$

If r.m.s value or virtual vlaue of a.c. is represented by I_v, then the amount of heat produced in the same resistance R, in the same time t would be

$$H = I_v^2 RT \qquad \text{... (vi)}$$

From (v) and (vi), we get

$$I_v^2 RT = \frac{I_0^2 RT}{2}$$

$$I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

(b) Root mean square value of alternating E.M.F

The root mean square (r.m.s) value of alternating e.mf. is defined as the value of steady voltage; which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating e.m.f, when applied to the same resistance for the same time.

The r.m.s. value is also called effective value or virtual value of alternating e.m.f. it is represented by E_{rms} or E_{eff} or E_v to calculate it, suppose the alternating e.m.f is represented by $E = E_0 \sin \omega t$

small amount of heat produced when this alternating e.m.f is applied to a resistance R for a small time dt is

$$dH = \frac{E^2}{R}dt = \frac{\left(E^2 \sin \omega t\right)}{R}dt$$

$$dH = \frac{E_0^2}{R}\sin^2 \omega t\ dt$$

In one complete cycle $(0 \to T)$, total amount of heat produced in resistance R is

$$\text{H} = \int_0^T \frac{E_0^2}{R}\sin^2 \omega t\ dt$$

$$= \frac{E_0^2}{R}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right)dt = \frac{E_0^2}{2R}\int_0^T 1\ dt - \frac{E_0^2}{2R}\int_0^T \cos 2\ \omega\ t\ dt$$

$$\text{H} = \frac{E_0^2}{2R}[t]_0^T - zero$$

$$\left[\because \int_0^T \cos 2\omega t\ dt = 0\right]$$

$$H = \frac{E_0^2}{2R}(T-0) = \frac{E_0^2 T}{2R} \qquad \text{......(vii)}$$

If E_v is the rms value fo alternating e.m.f., then amount of heat produced in the same resistance R in the same time T is

$$H = \frac{E_0^2}{R}T \qquad \text{.....(viii)}$$

from (vii) and (viii) we get $\dfrac{E_v^2}{R}T = \dfrac{E_0^2 T}{2R}$

$$E_v^2 = \frac{E_0^2}{2} \qquad \text{or} \qquad E_v = \frac{E_0}{\sqrt{2}}$$

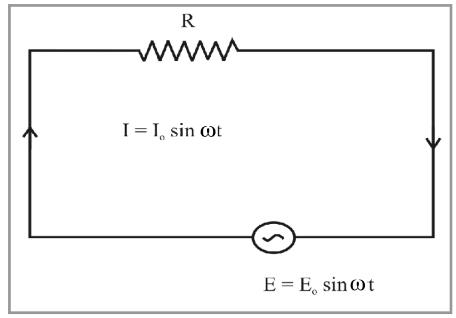
i.e. $E_v = \frac{E_0}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{E_0 \times 1.414}{2} = 0.707\, E_0$

Hence r.m.s vaue of alternating e.m.f is 0.707 times the peak value of alternating e.m.f.

4. ALTERNATING CURRENT THROUGH A PURE RESISTIVE CIRCUIT

An a.c resistive circuit is a circuit having only resistance component in it (*i.e.* a resistor R).

In the a.c circuit shown an a.c source of alternating e.m.f (E) is connected to a pure resistance 'R'



The alternating emf is given by $E = E_o \sin \omega t$ ... (*i*)

As soon as the circuit is closed, a.c. current I starts flowing such that,

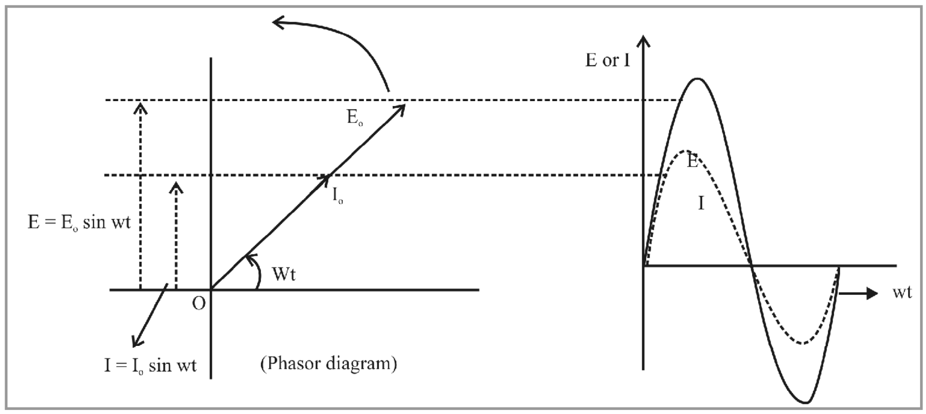
$$IR = E \quad \Rightarrow \quad I = \frac{E}{R}$$

Using (*i*) we have,

$$I = \frac{E_o}{R} \sin \omega t$$

$$I = I_o \sin \omega t \qquad \ldots (ii)$$

I_o is the maximum value of current.



From $I_0 = \frac{E_o}{R}$, it is clear that an a.c circuit too satisfies ohm's law and the behaviour of R in a.c. circuit is same as it is in *d.c* circuit, *i.e.* R can reduce a.c as well as *d.c* equally effectively.

Comparing equations (*i*) and (*ii*) it is clear that E and I are in phase.

Phasor diagram for a resistance circuit

The diagram shows that the current and voltage phasors are at the same phase.

5. AC THROUGH AN INDUCTOR

An inductive circuit is a circuit having only inductor as a component in it.

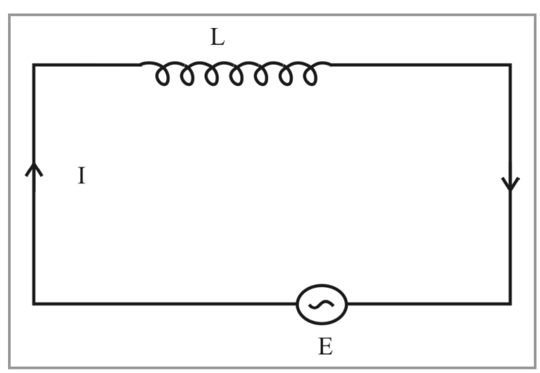
The circuit can be as shown

If $\frac{dI}{dt}$ is the rate of change of current through L at any instant, then the induced emf in the inductor = $\frac{-L\,dI}{dt}$.

The – ve sign indicates that induced emf opposes the change of current.

In order to maintain the flow of current in the circuit, applied emf must be equal and opposite of the induced voltage.

i.e., $$E = -\left(-L\frac{dI}{dt}\right) = E_o \sin \omega t$$



$$dI = \frac{E_o}{L} \sin \omega t dt \qquad \ldots (1)$$

Integrating both sides of (1), we get

$$I = \frac{E_o}{L} \int \sin \omega t dt$$

$$I = \frac{E_o}{L}\left[\frac{-\cos \omega t}{\omega}\right] \Rightarrow I = \frac{-E_o}{\omega L} \cos \omega t$$

$$I = \frac{-E_o}{\omega L}\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$I = \frac{-E_o}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$$

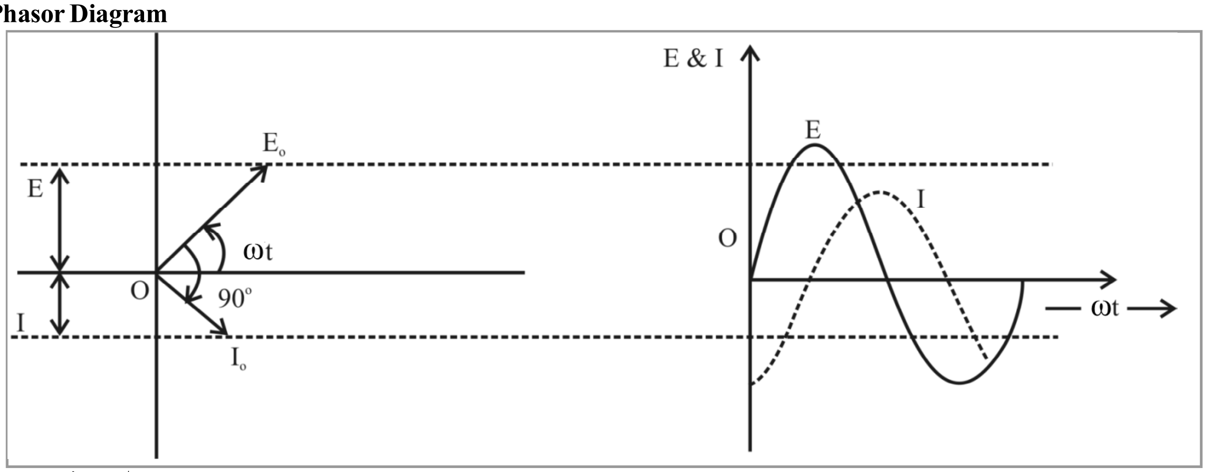
where $I_o = \frac{E_o}{\omega L}$ or $I_o = \frac{E_o}{X_L}$

In inductive circuit, ac current lags behind the ac voltage by a phase angle of 90^o by $\frac{1}{4}$th of a period.

Also $X_L = \omega L = 2\pi f L$ = Inductive reactance

In dc circuits, $f = 0$ therefore, $X_L = 0$

Phasor Diagram



$E = E_o \sin \omega t$

and $I = I_o \sin(\omega t - 90^o)$

Thus, it is clear that in an inductive circuit, current I lags behind the alternating voltage E by a phase angle of 90^o.

6. AC THROUGH A CAPACITOR

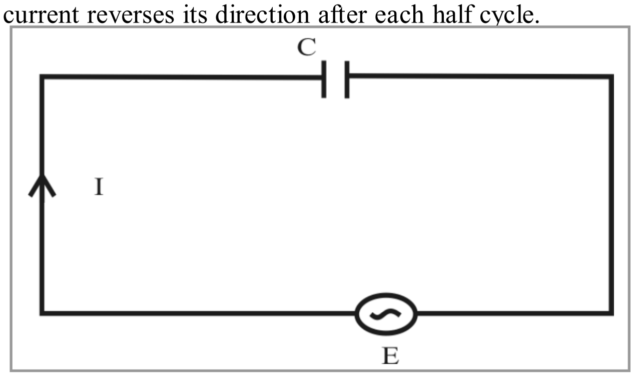
A capacitive circuit is a circuit having only capacitor as a component in it.

The circuit can be drawn as: Here, $E = E_o \sin \omega t$

The current flows as soon as the circuit is closed and the charge in transferred from the emf source 'E' to the plates of the capacitor.

This produces a potential difference between the plates.

The capacitor is charged and discharged alternatively as the current reverses its direction after each half cycle.



$\therefore \quad V = \frac{q}{C}$

At every instant, $V = E$

$\therefore \quad V = \frac{q}{C} = E = E_o \sin \omega t$ *Or*

$q = C E_o \sin \omega t$

If I = current in the circuit at instant 't', then

$$I = \frac{dq}{dt} = \frac{d}{dt}(C E_o \sin \omega t)$$

$$I = C E_o (\cos \omega t)\, \omega$$

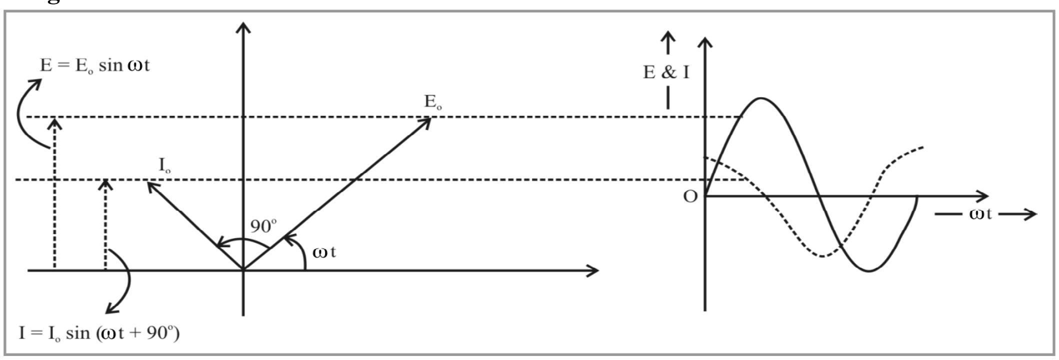
$$I = \frac{E_o}{\frac{1}{\omega c}} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \Rightarrow \quad I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where $I_o = \dfrac{E_o}{\frac{1}{\omega C}} = \dfrac{E_o}{X_c}$

$X_c = \dfrac{1}{\omega C}$ and X_c is called capacitive reactance

Thus in a capacitive circuit, alternating current leads the alternating emf by a phase angle of 90°.

Phasor Diagram



From phasor diagram it is clear that current phasor leads the e.m.f phasor by a phase angle of 90°.

7. ALTERNATING CURRENT THROUGH A SERIES LCR CIRCUIT

A circuit containing L, C and R connected in series is called series LCR circuit.

Let the voltage applied across the combination be $E = E_0 \sin \omega t$

Let I be the current in the circuit and V_L, V_C and V_R be the potential difference across inductance, capacitor and resistor, respectively.

The phasor diagram is given below.

Here,

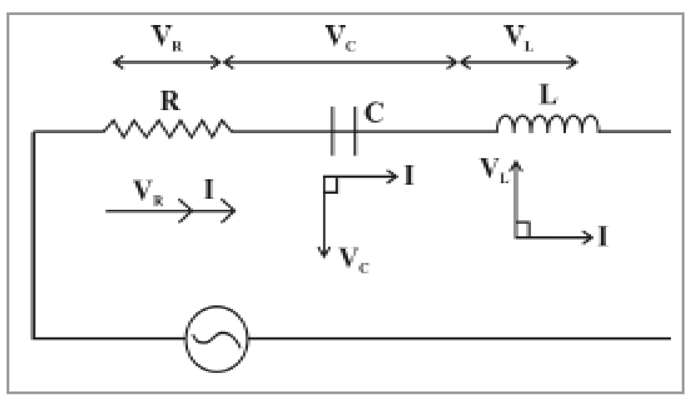
$$OE^2 = OA^2 + OD^2$$

Or $$E^2 = V_R^2 + (V_L - V_C)^2$$

Or $$E^2 = (IR)^2 + (IX_L - IX_C)^2$$

Or $$E = \left[IR^2 + \left(I\omega L - \frac{I}{\omega C}\right)^2\right]^{\frac{1}{2}}$$

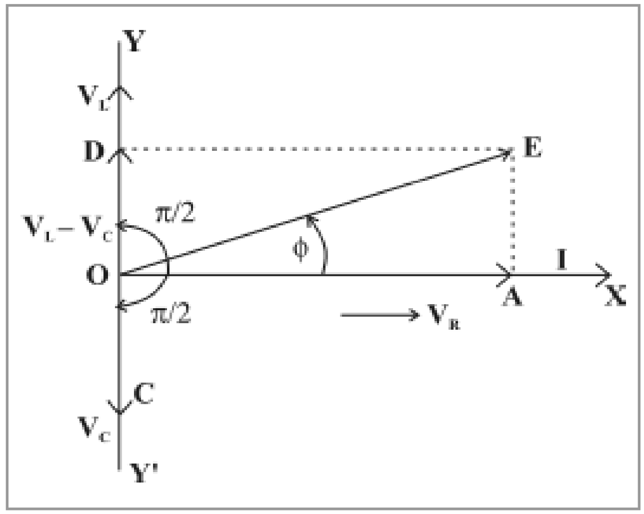
Or $$\frac{E}{I} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}}$$



Comparing the above equation with Ohm's law, we get the expression for impedance as

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}}$$

It is also clear from phasor diagram that voltage V makes phase angle ϕ with current I.



Now, $\tan\phi = \frac{AE}{OA} = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

The expression for current in this regard is:

$I = I_0 \sin(\omega t - \phi)$

Special cases:

(*i*) When $\omega L > \frac{1}{\omega C}$

Then it follows from the phase equation that $\tan\phi$ is positive, *i.e.* ϕ is positive and voltage leads the current.

(*ii*) When $\omega L < \frac{1}{\omega C}$, then phase angle ϕ is negative, *i.e.* voltage lags the current.

(*iii*) When $\omega L = \frac{1}{\omega C}$, then it follows from phase equation that $\tan\phi = 0$ or $\phi = 0$

That is voltage and current are in same phase. Here, impedance Z is minimum (=R) and the current would reach a maximum value and this condition is known as condition of resonance.

The angular frequency (ω_0) at which this condition is achieved is termed as natural or resonant angular frequency.

Thus, at $\omega = \omega_0$,

Impedance $Z = \left[R^2 + \left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2\right]^{\frac{1}{2}} = R$

Also, $\omega_0 L - \frac{1}{\omega_0 L} = 0$ *Or* $\omega_0 = \frac{1}{\sqrt{LC}}$ *Or* $f_0 = \frac{1}{2\pi\sqrt{LC}}$

f_0 is the frequency of applied source and ω_0 is the angular resonant frequency.

The graphical representation of variation of circuit current with the changing frequency of applied voltage is shown as below.

Here, Q is the quality factor.

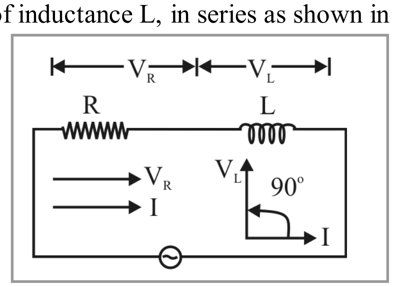
It is clear from the graph that for the frequencies greater than or less than f_0, the values of current is less than the maximum value. Further, at $f = f_0$; current I is maximum (I_{max}).

Also $I_{max} \propto \frac{1}{R}$.

A series resonance circuit admits maximum current through it and is therefore called an acceptor circuit.

8. A.C. CIRCUIT CONTAINING RESISTANCE AND INDUCTANCE

Let a source of alternating e.m.f be connected to an ohmic resistance R and a coil of inductance L, in series as shown in fig.

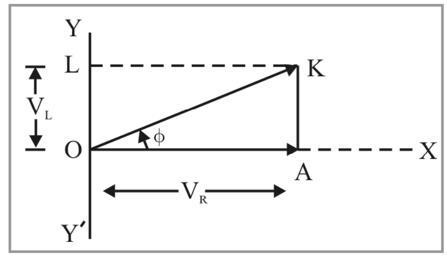


Proceeding as in (1) and ignoring $\overrightarrow{V_C}$ as there is no capacitor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_L^2}$$

Fig. represents phasor diagram of RL circuit.

We find that in RL circuit, voltage leads the current by a phase angle ϕ, where

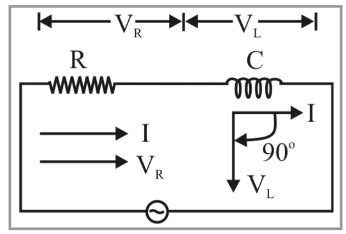


$$\tan\phi = \frac{AK}{OA} = \frac{OL}{OA} = \frac{V_L}{V_R} = \frac{I_0X_L}{I_0R}$$

$$\tan\phi = \frac{X_L}{R}$$

9. A.C. CIRCUIT CONTAINING RESISTANCE AND CAPACITANCE

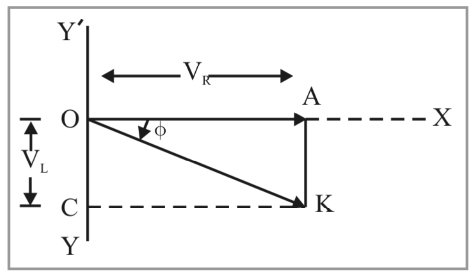
Let a source of alternating e.m.f. be connected to an ohmic resistance R and a condenser of capacity C, in series as shown in fig.



Proceeding as in (1) and ignoring $\vec{V}_L$ as there is no inductor in the circuit, we shall obtain

$$Z = \sqrt{R^2 + X_C^2}$$

Fig . represents phasor diagram of RC circuit. We find that in RC circuit, voltage lags behind the current by a phase angle ϕ, where



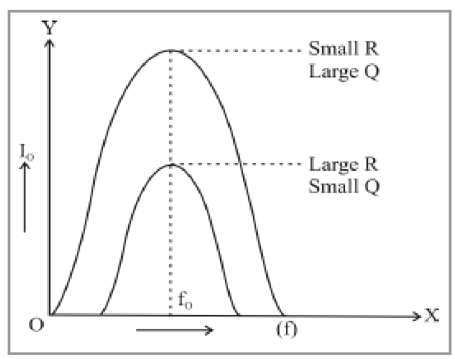
$$\tan\phi = \frac{AK}{OA} = \frac{OC}{OA} = \frac{V_C}{V_R} = \frac{I_0X_C}{I_0R}$$

$$\tan\phi = \frac{X_C}{R}$$

Q-Factor

Q-factor or quality factor determines the characteristic of a series resonant circuit. It defines the sharpness of turning at resonance.

Mathematically, $Q = \dfrac{\text{Voltage across L or C}}{\text{Applied voltage}}$



For inductor, $Q = \dfrac{(\omega_0 L)}{RI} I = \dfrac{\omega_0 L}{R}$ … (*i*)

For capacitor, $Q = \dfrac{\left(\dfrac{1}{\omega_0 C}\right) I}{RI} = \dfrac{\dfrac{1}{\omega_0 C}}{R}$ … (*ii*)

Using $\omega_0 = \sqrt{\dfrac{1}{LC}}$

We obtain $Q = \dfrac{L}{R}\dfrac{1}{\sqrt{LC}}$ from equation (*i*)

And $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ from equation (*ii*)

Quality factor, Q is just a number. It is clear from the above expression that $Q \propto \dfrac{1}{R}$

Electronic circuits with high Q values would respond to a very narrow range of frequencies and vice versa. Value of quality factor Q varies from 10 to 100. However, Q is 200 for circuits dealing with very high frequencies.

Q factor could also be defined as the ratio of resonant angular frequency to the bandwidth of power curve at half the maximum power.

Mathematically, $Q = \dfrac{\omega_0}{\Delta\omega}$

10. AVERAGE POWER ASSOCIATED WITH RESISTANCE OR NON INDUCTIVE CIRCUIT

Power, we know, is defined as the rate of doing work, In a.d.c circuit, power is given by the product of voltage and current. However, in an a.c circuit, values of voltage and current change every instant. Therefore, power in an a.c. circuit at any instant is the product of instantaneous voltage (E) and instantaneous current (I)

In a pure resistance, the alternating current developed is in phase with the alternating voltage applied i.e.

when $E = E_0 \sin\omega t$ then

$$I = I_0 \sin\omega t$$

Instantaneous power = E I

$$= (E_0 \sin\omega t)(I_0 \sin \omega t)$$

$$= E_0 I_0 \sin^2 \omega t$$

If the instantaneous powr reamins constant for a small time dt, then small amount of work done in maintaining the current for a small time dt it

$$dW = E_0 I_0 \sin^2 \omega t \ dt$$

Total work done or energy spent in maintaining current over one full cycle

$$W = \int_0^T E_0 I_0 \sin^2 \omega t \ dt$$

$$= E_0 I_0 \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{E_0 I_0}{2}\int_0^T dt - \frac{E_0 I_0}{2}\int_0^T \cos 2\omega t \, dt$$

$$W = \frac{E_0 I_0}{2} T - 0$$

$\therefore$ Average power supplied to R over a complete cycle

$$P = \frac{W}{T} = \frac{E_0 I_0}{2} . \frac{T}{T} = \frac{E_0}{\sqrt{2}} . \frac{I_0}{\sqrt{2}}$$

$$P = E_v . I_v$$

(b) Average Power Associated with an Inductor

Thus net power supplied by the source in a complete cycle is zero.

Mathematically, work done in one complete cycle is

$$W = \int_0^T EI \ dt$$

If the alternating voltage is $E = E_0 \sin\omega t$ then thrugh L, as current lags behind E by a phase angle of $\frac{\pi}{2}$, therefore

$$I = I_0 \sin(\omega t - \pi/2) = -I_0 \cos\omega t$$

Put in (52), $W = -\int_0^T E_0 \sin \omega t . I_0 \cos \omega t \ dt$

$$W = -E_0 I_0 \int_0^T \sin \omega t \cos \omega t \ dt = \frac{-E_0 I_0}{2}\int_0^T 2\sin \omega t \cos \omega t \ dt = \frac{-E_0 I_0}{2}\int_0^T \sin 2\omega t \, dt = \frac{-E_0 I_0}{2}\left[\left(-\frac{\cos 2\omega t}{2\omega}\right)\right]_0^T$$

W = zero

Therefore, average power over a complete cycle of a.c. through an ideal inductor is zero. Infact, whatever energy is needed in building up current in L is returned back during the decay of current.

Energy Stored in an Inductor

When a.c. is applied to an inductor of inductance L, the current in it grows from zero to maximum steady value I_0. if I is the current at any instant t, then the induced e.m.f. developed in the inductor at that instant is

$$E = -L\frac{dI}{dt}$$

This tends to prevent the growth of current. To maintain the grrowth, power has to be supplied from the external source. Power supplied at instant t,

$$\frac{dW}{dt} = EI = L\frac{dI}{dt}.I$$

Small amount of work done in a small time dt

$$dW = \left(L\frac{dI}{dt}\,I\right)dt = LI\ dI$$

$\therefore$ Total work done by the external source in building up current from zero to I_0 is

$$W = \int_0^{I_0} LI\ dI = L\left[\frac{I^2}{2}\right]_0^{I_0} = \frac{1}{2}LI_0^2$$

i.e., $E = W = \frac{1}{2}LI_0^2$

This is the energy stoed in inductor.

(c) Average Power Associated with a Capacitor

Mathematically if alternating voltage applied is $E = E_0 \sin\omega t$

then through C, as currrent leads the e.m.f. by a phase angle of $\pi/2$

$\therefore$ $I = I_0\sin(\omega t + \pi/2)$

$I = I_0 \cos\omega t$

Work done over a complete cycle is

$$W = \int_0^T EI\ dt$$

$$= \int_0^T (E_0\sin\omega t)(I_0\cos\omega t)dt = \frac{E_0I_0}{2}\int_0^T 2\sin\omega t\ \cos\omega t\ dt$$

$$= \frac{E_0I_0}{2}\int_0^T \sin 2\ \omega\ t\ dt$$

$$W = \frac{E_0I_0}{2}\left[-\frac{\cos 2\omega\ t}{2\omega}\right]_0^T$$

$= \text{Zero}$

(d) Average Power in RLC circuit of inductive Circuit

Let the alternating e.m.f applied to an RLC circuit be

$E = E_0\ \sin\omega t$

If alternating current developed lags behind the applied e.m.f. by a phase angle ϕ, then

$I = I_0\sin(\omega t - \phi)$

Power at instant t,

$$\frac{dW}{dt} = E\ I$$

$$\frac{dW}{dt} = E_0 \sin \omega t \times I_0 \sin(\omega t - \phi)$$

$$= E_0 I_0 \sin \omega t\ (\sin \omega t \cos\phi - \cos \omega\ t \sin\ \phi)$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - E_0 I_0 \sin \omega t \cos\ \omega\ t \sin\ \phi$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\ \phi$$

If this instantaneous power is assumed to remain constant for a small time dt, then small amount of work done in this time is

$$dW = \left(E_0 I_0 \sin^2 \omega\ t\ \cos\ \phi - \frac{E_0 I_0}{2} \sin 2\omega\ t\ \sin\theta \right) dt$$

total work done over a complete cycle is

$$W = \int_0^T E_0 I_0 \sin^2 \omega\ t\ \cos\ \phi\ dt - \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi\, dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2 \omega\ t\ dt - \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t\ dt$$

As $\int_0^T \sin^2 \omega\ t\ dt = \frac{T}{2}$ and $\int_0^T \sin 2\omega t\ dt = 0$

$$\therefore \quad W = E_0 I_0 \cos\phi \times \frac{T}{2}$$

$$P = \frac{W}{T} = \frac{E_0 I_0 \cos\phi}{\mathrm{T}} . \frac{T}{2} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$P = E_v I_v \cos\phi$$

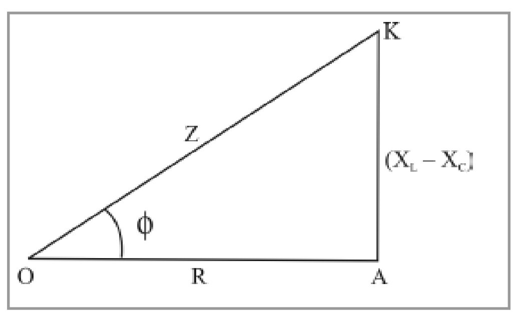
11. POWER FACTOR (PF) OF AN AC CIRCUIT

The power dissipated across a pure resistor is called true power.

The power dissipated across the impedence of an ac circuit is called apparent power.

Power factor (PF) is the ratio of true power to apparent power.

$$\therefore \quad \mathrm{PF} = \cos\varphi = \frac{\text{True Power}}{\text{Apparent Power}}$$



As voltage across inductor leads the current by $\frac{\pi}{2}$ and voltage across capacitor lags current by $\frac{\pi}{2}$. Therefore, X_L and X_C have opposite signs and total reactance $= \pm(X_L - X_C)$.

Z represents the impedance of LCR circuit.

$$PF = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

So, PF, $\cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$

In a non- inductive circuit,

$X_L = X_C$

Therefore, $\cos\phi = \frac{R}{\sqrt{R^2}} = 1 \Rightarrow \phi = 90°$

This is the maximum value of power factor.

In a pure inductor or a capacitor

$\phi = 90°$, *i.e.* PF = 0

Therefore, average power consumed in such circuits is

$P = V_{rms} \cdot I_{rms} \cos 90° = 0$

Current through pure inductor L or capacitor C consumes no power for its maintenance in the circuit and that is why this is called the idle current or wattless current.

For LCR circuit it can be shown that the average power consumed by the circuit

$P_{av} = E_{rms} I_{rms} \cos\phi$

At resonance (as cos ϕ = 0),

$$P_{av} = E_{rms} I_{rms} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}$$

Or, $P_{av} = \frac{1}{2} E_0 I_0$

12. LC OSCILLATIONS

Oscillation can be generated by using an LC circuit as shown,

I_C = Charging current

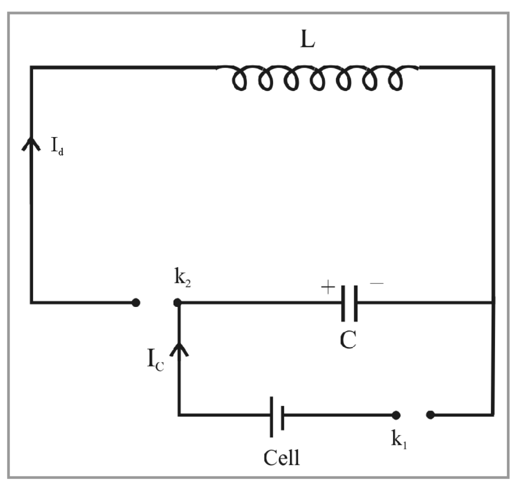
I_d = Discharging current

When k_1 is on and k_2 is off then current from cell charges the capacitor C

Such that V = $\frac{q}{c}$

And energy stored by the capacitor

$$U_E = \frac{q^2}{2C}$$



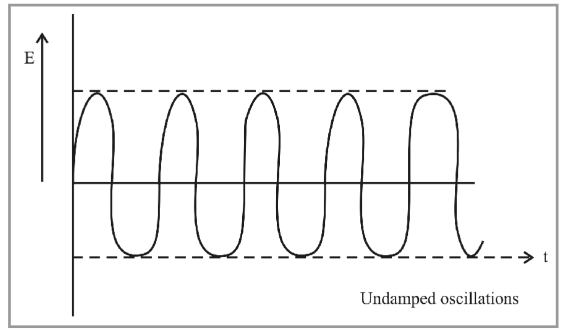
Switch k_1 off and make k_2 on. The charged capacitor gets discharged through L. The complete energy of capacitor gets transferred to inductor such that

$$U_B = \frac{1}{2} L I^2$$

As soon as the capacitor is completely discharged, current stops and magnetic field in 'L' starts collapsing. Thus, an induced emf develops and starts recharging the capacitor but in opposite direction.

When condenser is charged completely, the magnetic field in 'L' starts building up due to discharging of capacitor and the cycle repeats itself.

Thus, transfer of energy takes place between 'C' and 'L'. In the absence of any resistance, no loss will occur in energy transfer and thus oscillations produced will be of constant amplitude called undamped oscillations.



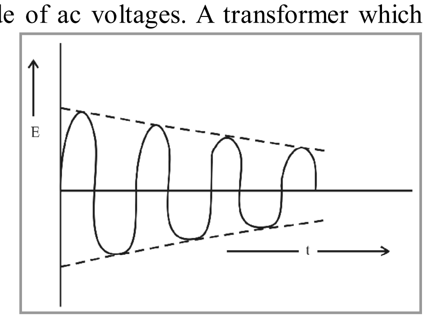
The frequency of oscillations in this case is given by $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Or, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

In case if $R \neq 0$, there will be loss of energy. Then the oscillations don't have constant amplitude, resulting in oscillations called damped oscillations.

13. TRANSFORMER

A transformer is an electrical device which is used for changing the magnitude of ac voltages. A transformer which increases the ac voltage is called step up transformer. Step down transformers reduce the ac voltage.

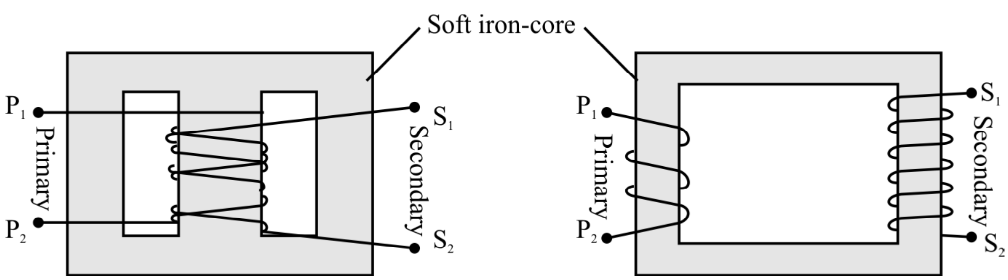


Principle

Works on the principle of mutual induction, *i.e.* when even the amount of magnetic flux linked with a coil changes an emf is induced in the neighbuoring coil.

Construction

It consists of a rectangular soft iron core made of laminated sheets, insulated from one another.



(*a*) (*b*)

Two arrangements for winding of primary and secondary coil in a transformer:
(*a*) two coils on top of each other, (*b*) two coils on separate limbs of the core.

Two coils P_1P_2 (primary coil) and $S_1\,S_2$ (secondary coil) are wound on the same core but well insulated from each other. Source of ac voltage is connected to $P_1\,P_2$ and load resistance (R_L) is connected to S_1S_2.

Theory and working: The applied input AC voltage is

$E = E_o \sin \omega t$

It is applied to $P_1\,P_2$ and because of this voltage, an ac starts flowing in $P_1\,P_2$. This ac current in turn induces an alternating magnetic flux ϕ_B that gets linked with S_1S_2.

As per Faraday's law of electromagnetic induction, emf induced per turn (E_{turn}) is same for S_1S_2 and $P_1\,P_2$.

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_p}{n_p} = \frac{E_s}{n_s}$$

n_p and n_s are the total number of turns in primary and secondary respectively.
E_p and E_s are the voltages across primary and secondary respectively.

$$\therefore \quad E_s = E_p \frac{n_s}{n_p}$$

$$E_s = k\ E_P \text{ Where } k = \frac{n_s}{n_p} = \text{Transformation ratio.}$$

If $n_s > n_p$; $E_s > E_p$ $\Rightarrow$ step up transformer

$n_s < n_p$; $E_s < E_p$ $\Rightarrow$ step down transformer

As per conservation of energy no energy is lost,

i.e. $I_p E_p = I_s E_s$

$$I_s = I_P \frac{E_P}{E_S}$$

For stepup transformer
$E_s > E_p$
$k > 1$
$I_s < I_p$
For step down transformer,
$E_s < E_p$
$k < 1$
$I_s > I_p$

Also, Efficiency $= \dfrac{\text{Output power}}{\text{Input power}}$

$$q = \frac{E_s I_s}{E_p I_p}$$

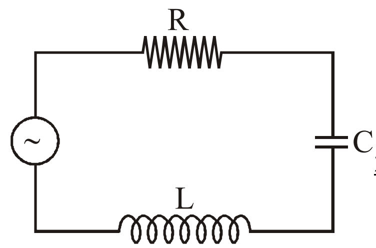
Uses of Transformer

Used in almost all ac applications like,

(*i*) In voltage regulators for T.V., Refrigerators etc.
(*ii*) In induction furnaces.
(*iii*) Step down transformer is used in welding purposes.
(*iv*) In power transmission of ac. over long distances.

2012

1. The figure shows a series LCR circuit with L = 10.0 H, C = 40 μF, R = 60 Ω connected to a variable frequency 240 V source, calculate (i) the angular frequency of the source which drives the circuit at resonance, (ii) the current at the resonating frequency, (iii) the rms potential drop across the inductor at resonance. **(3 marks)**



2011

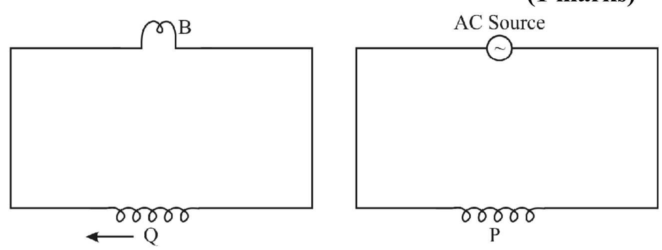
2011

1. Define the term 'wattless current'. **(1 marks)**

2. (i) With the help of a labelled diagram, describe briefly the underlying principle and working of a step up transformer.

(ii) Write any two sources of energy loss in a transformer.

(iii) A step up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain. **(5 marks)**



3. Derive an expression for the impedance of a series LCR circuit connected to an AC supply of variable frequency.

Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set. **(5 marks)**

2010

1. A coil Q is connected to low voltage bulb B and placed near another coil P as shown in the figure. Give reasons to explain the following observations: **(2 Marks)**

(*a*) The bulb 'B' lights

(*b*) Bulb gets dimmer if the coil Q is moved towards left.

2. Describe briefly, with the help of a labelled diagram, the basic elements of an A.C. generator. State its underlying principle. Show diagrammatically how an alternating emf is generated by a loop of wire rotating in a magnetic field. Write the expression for the instantaneous value of the emf induced in the rotating loop.

(5 Marks)

3. A series LCR circuit is connected to a source having voltage $v = v_m \sin \omega t$. Derive the expression for the instantaneous current I and its phase relationship to the applied voltage.

Obtain the condition for resonance to occur. Define 'Power factor'. State the conditions under which it is (*i*) maximum and (*ii*) minimum. **(5 Marks)**

2009

1. (*a*) Derive an expression for the average power consumed in a series LCR circuit connected to a.c. source in which the phase difference between the voltage and the current i the circuit is ϕ.

(*b*) Define the quality factor in an a.c. circuit. Why should the quality factor have high value in receiving circuits. Name the factors on which it depends. **(5 Marks)**

2. (*a*) Derive the relationship between the peak and the rms value of current in an a.c. circuit.

(*b*) Describe briefly, with the help of a labelled diagram, working of a step-up transformer.

A step-up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy. Explain **(5 Marks)**

2008

1. An inductor 200mH, capacitor 500 μ F, resistor 10 Ω are connected in series with a 100 V, variable frequency ac source. Calculate the

(*i*) frequency at which the power factor of the circuit is unity

(*ii*) current amplitude at this frequency

(*iii*) Q-factor **(3 Marks)**

2007

1. In a series LCR circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between the applied voltage and the current in the circuit. **(1 Mark)**

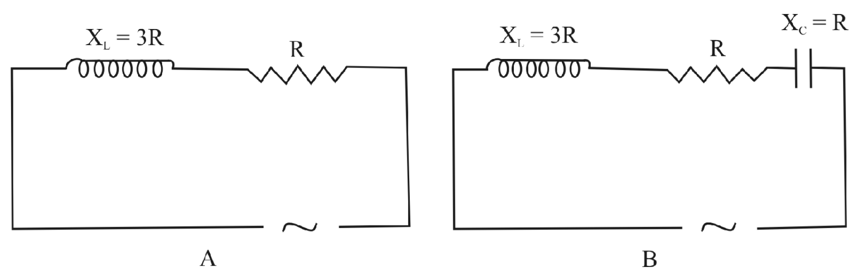
2. Distinguish between the terms 'average value' and 'rms value' of an alternating current. The instantaneous current from an ac source is I = 5 sin (314 t) ampere. What are the average and rms values of the current. **(2 Marks)**

3. Calculate the current drawn by the primary coil of a transformer which steps down 200 V to 20 V to operate a device of resistance. Assume the efficiency of the transformer to be 80%. **(2 Marks)**

4. An ac voltage of 100 V, 50 Hz is connected across a 20 ohm resistor and mH inductor in series. Calculate (*i*) impedance of the circuit, (*ii*) rms current in the circuit. **(2 Marks)**

5. Explain with the help of a labelled diagram the underlying principle and working of a step-up transformer. Why cannot such a device be used to step-up d.c. voltage. **(3 Marks)**

6. Given below are two electric circuits A and B. Calculate the ratio of power factor of the circuit B to the power factor of circuit A. **(3 Marks)**



7. In a series LCR circuit, define the quality factor (Q) at resonance. Illustrate its significance by giving one example. Show that power dissipated at resonance in LCR circuit is maximum. **(3 Marks)**

8. A resistor of 200 Ω and a capacitor of 40 μF are connected in series to 220 V ac source with angular frequency $\omega = 300\,\text{Hz}$. Calculate the voltages (rms) across the resistor and the capacitor. Why is the algebraic sum of these voltages more than the source voltage. How do you resolve this paradox. **(3 Marks)**

9. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage. An ac voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance L. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\frac{\pi}{2}$. **(5 Marks)**

10. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage. An ac voltage $E = E_0 \sin \omega t$ is applied across a pure capacitor of capacitance C. Show mathematically that the current flowing through it leads the applied voltage by a phase angle of $\frac{\pi}{2}$. **(5 Marks)**

2006

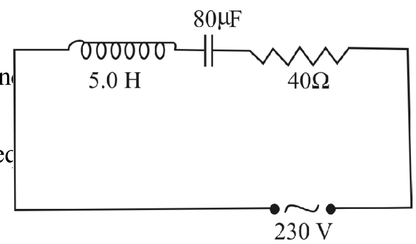
1. A capacitor and a resistor are connected in series with an ac source. If the potential difference across C, R are 120 V, 90 V rspectively and if the r.m.s. current of the circuit is 3 A, calculate the (*i*) impedance, (*ii*) power factor of the circuit. **(2 Marks)**

2. An inductor 200 mH, a capacitor C and a resistor 10 ohm are connected in series with a 100 V, 50 s^{-1} ac source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor. **(2 Marks)**

3. When an inductor L and a resistor R in series are connected across a 12 V, 50Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\frac{\pi}{3}$ radians. Calculate the value of R. **(3 Marks)**

4. In the following circuit, calculate,

(*i*) the capacitance 'C' of the capacitor, if the power factor of the circuit is unity, an
circuit



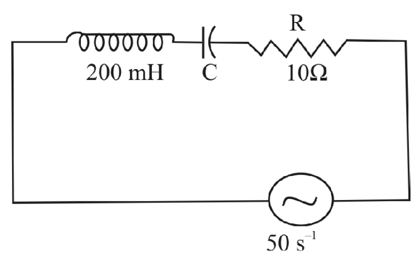
5. The given circuit diagram shows a series LCR circuit connected to a variable frec

(*a*) Determine the source frequency which drives the circuit in resonance.

(*b*) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(*c*) Determine the rms potential drops across the three elements of the circuit.

(*d*) How do you explain the observation that the algebraic sum of the voltages across the three elements obtained in (*c*) is greater than the supplied voltage.



6. The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and power are 220 V and 1100 W respectively. Calculate:

(*i*) number of turns in the secondary

(*ii*) the current in the primary

(*iii*) voltage across the secondary

(*iv*) the current in the secondary

(*v*) power in the secondary **(5 Marks)**

2005

1. The power factor of an ac circuit is 0.5. What will be the phase difference between voltage and current in this circuit. **(1 Mark)**

2. A bulb and a capacitor are connected in series to an ac source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the ac source. Give reason. **(1 Mark)**

3. In a series LCR circuit, the voltage across an inductor, capacitor and resistor are 20 V, 20 V and 40 V respectively. What is the phase difference between the applied voltage and the current in the circuit. **(1 Mark)**

4. State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of ac source in a series LCR circuit. **(2 Marks)**

5. (*a*) Define the term 'impedance of series LCR circuit'. Derive a mathematical expression for it using phasor diagram.

(*b*) Obtain the reasonant frequency of a series LCR circuit with L = 2.0 H, C = 32 μF and R= 10 Ω **(5 Marks)**