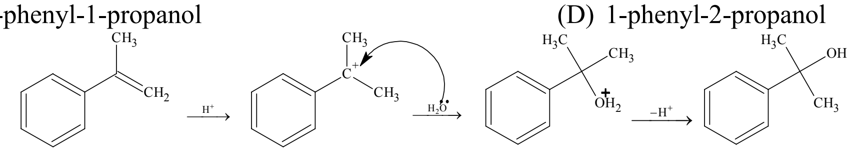
Note: *For the benefit of the students, specially the aspiring ones, the question of IIT-JEE, 2004 Screening are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Chemistry, Physics and Mathematics are **30 minutes, 25 minutes and 28 minutes** respectively.*

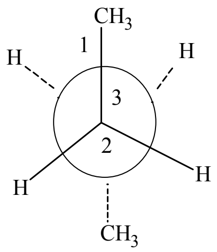
FIITJEE *solutions to IIT–JEE, 2004 Screening*

1*. 2- phenyl propene on acidic hydration gives
(A) 2-phenyl-2-propanol (B) 2-phenyl-1-propanol
(C) 3-phenyl-1-propanol (D) 1-phenyl-2-propanol

Ans. (A)

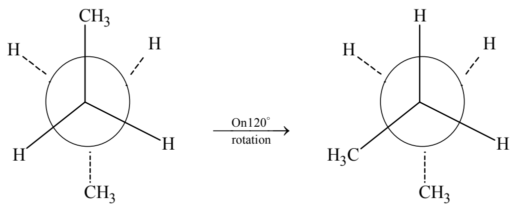


2*.



C_2 is rotated anticlockwise 120°about C_2- C_3 bond. The resulting conformer is
(A) Partially eclipsed (B) Eclipsed
(C) Gauche (D) Staggered

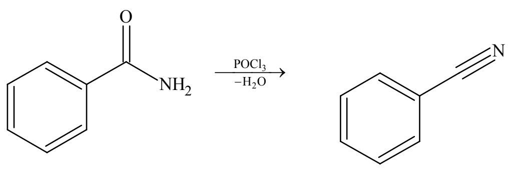
Ans. (C)



The resulting conformer is a Gauche conformer

3. Benzamide on treatment with $POCl_3$ gives
(A) Aniline (B) Benzonitrile
(C) Chlorobenzene (D) Benzyl amine

Ans. (B)

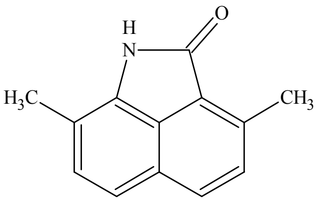


$POCl_3$ is a dehydrating agent

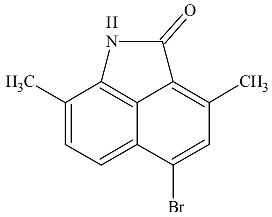
4. The methods chiefly used for the extraction of Lead & Tin from their ores are respectively
(A) Self reduction & carbon reduction (B) Self reduction & electrolytic reduction
(C) Carbon reduction & self reduction (D) Cyanide process & carbon reduction

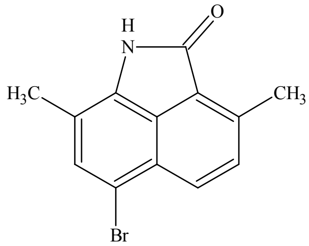
Ans (A) Factual

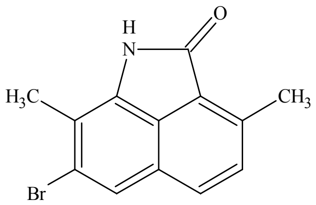
5*.

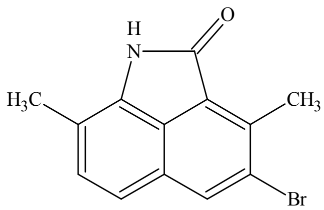


$\xrightarrow{Fe/Br_2}$ Product on monobromination of this compound is

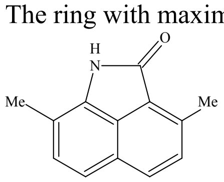
(A) 

(B) 

(C) 

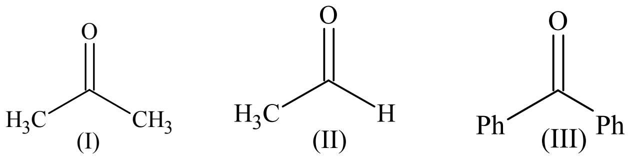
(D) 

Ans. (B) The ring with maximum electron density will be substituted by an electrophile



The ring attached with – NH- will have rich electron density due to resonance. As ortho position is blocked, the electrophile attacks the para position

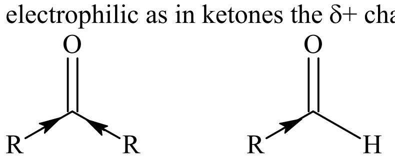
6. The order of reactivity of Phenyl Magnesium Bromide with the following compounds is



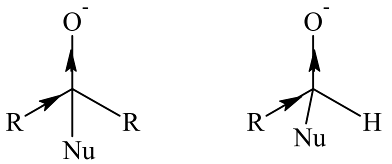
(I) (II) (III)

(A) (II) > (III) > (I)
(B) (I) > (III) > (II)
(C) (II) > (I) > (III)
(D) All react with the same rate

Ans. (C) Nucleophile attacks the most electrophilic site first. Among aldehyde and a ketone, aldehydes are more electrophilic as in ketones the δ+ charge an carbonyl carbon is decreased by +I effect of both alkyl groups



More over in the tetrahedral intermediate aldehydes have less steric repulsion than ketone and aldehydes increases the –ve charge on oxygen less in comparison to ketones



Based on the above the order of reactivity is (II) > (I) > (III)

7. $(NH_4)_2Cr_2O_7$ on heating gives a gas which is also given by
(A) Heating NH_4NO_2
(B) Heating NH_4NO_3
(C) $Mg_3N_2 + H_2O$
(D) Na(comp.) + H_2O_2

Ans. (A) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2\uparrow + Cr_2O_3 + 4H_2O$

$NH_4NO_2 \xrightarrow{\Delta} N_2\uparrow + 2H_2O$

8. Zn | Zn^{2+} (a = 0.1M) || Fe^{2+} (a = 0.01M)|Fe. The emf of the above cell is 0.2905 V. Equilibrium constant for the cell reaction is
(A) $10^{0.32/0.0591}$
(B) $10^{0.32/0.0295}$
(C) $10^{0.26/0.0295}$
(D) $e^{0.32/0.295}$

Ans. (B) $E = E° - \frac{0.0591}{n}\log Kc$

$0.2905 = E° - \frac{0.0591}{2}\log\frac{0.1}{0.01}$

$E° = 0.2905 + 0.0295 = 0.32$

$E° = \frac{0.0591}{n}\log K$

$0.32 = \frac{0.0591}{2}\log K$

$K = 10^{0.32/0.0295}$

9*. HX is a weak acid ($K_a = 10^{-5}$). It forms a salt NaX (0.1M) on reacting with caustic soda. The degree of hydrolysis of NaX is
(A) 0.01%
(B) 0.0001%
(C) 0.1%
(D) 0.5%

Ans. (A) $X^- + H_2O \longrightarrow HX + OH^-$

$K_h = \frac{K_w}{K_a} = 10^{-9}$

$K_h = C\,\alpha^2 = 10^{-9}$

$\alpha^2 = 10^{-8} \Rightarrow \alpha = 10^{-4}$

% degree of dissociation = $10^{-4} \times 100 = 0.01\%$

10. Spontaneous adsorption of a gas on solid surface is an exothermic process because
(A) ΔH increases for system
(B) ΔS increases for gas
(C) ΔS decreases for gas
(D) ΔG increases for gas

Ans. (C)For spontaneous absorption ΔG is negative as well as the degree of randomness of gas molecules decreases thereby ΔS also negative. That's why $T.\Delta S$ is –ve

$\therefore\ \Delta H = \Delta G + T.\Delta S$

Thereby ΔH is –ve

11*. For a monoatomic gas kinetic energy = E. The relation with rms velocity is
(A) $u = \left(\frac{2E}{m}\right)^{1/2}$
(B) $u = \left(\frac{3E}{2m}\right)^{1/2}$
(C) $u = \left(\frac{E}{2m}\right)^{1/2}$
(D) $u = \left(\frac{E}{3m}\right)^{1/2}$

Ans. (A) Since,

$P = \frac{1}{3}\times\frac{m}{v}\times c^2 \quad (c = \text{rms velocity}) \Rightarrow \sqrt{\frac{3PV}{m}} = c$

For 1molecule $PV = kT \Rightarrow c = \sqrt{\frac{3kT}{m}}$

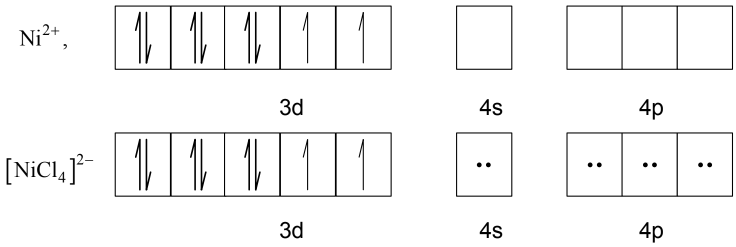
$\therefore KE = \frac{3}{2}kT \Rightarrow 2KE = 3kT \Rightarrow c = \sqrt{\frac{2KE}{m}}$

12*. The pair of compounds having metals in their highest oxidation state is
(A) MnO_2, $FeCl_3$ (B) $[MnO_4]^-$, CrO_2Cl_2
(C) $[Fe(CN)_6]^{3-}$, $[Co(CN)_3]$ (D) $[NiCl_4]^{2-}$, $[CoCl_4]^-$

Ans. (B)$[MnO_4]^-$, Mn = +7
CrO_2Cl_2, Cr = +6

13. The compound having tetrahedral geometry is
(A) $[Ni(CN)_4]^{2-}$ (B) $[Pd(CN)_4]^{2-}$
(C) $[PdCl_4]^{2-}$ (D) $[NiCl_4]^{2-}$

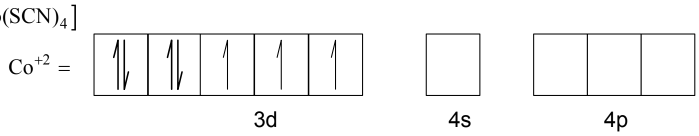
Ans. (D)



Hybridisation of $[NiCl_4]^{2-} = sp^3$
Shape of $[NiCl_4]^{2-}$ = Tetrahedral

14. Spin only magnetic moment of the compound $Hg[Co(SCN)_4]$ is
(A) $\sqrt{3}$ (B) $\sqrt{15}$
(C) $\sqrt{24}$ (D) $\sqrt{8}$

Ans. (B) $Hg[Co(SCN)_4]$



n=3
$\mu_s = \sqrt{n(n+2)} = \sqrt{3\times5} = \sqrt{15}$

15. A sodium salt of an unknown anion when treated with $MgCl_2$ gives white precipitate only on boiling. The anion is
(A) SO_4^{2-} (B) HCO_3^-
(C) CO_3^{2-} (D) NO_3^-

Ans. (B)$MgCl_2 + 2NaHCO_3 \longrightarrow \underset{\text{soluble}}{Mg(HCO_3)_2} + 2NaCl$

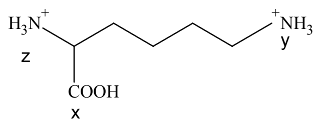
$Mg(HCO_3)_2 \xrightarrow{\Delta} \underset{\text{white}}{MgCO_3}\downarrow + H_2O + CO_2$

16*. Which Hydrogen like species will have same radius as that of Bohr orbit of Hydrogen atom?
(A) n=2, Li^{+2} (B) n=2, Be^{3+}
(C) n=2, He^+ (D) n=3, Li^{2+}

Ans. (B) $r = \frac{n^2 r_{1(H^+)}}{z}$

$\because r = r_{(H^+)} \quad \therefore n^2 = z \quad \therefore n = 2, z = 4$

so $r_{2(Be^{+3})} = r_{1(H^+)}$

17.


Arrange in order of increasing acidic strength

(A) X > Z > Y
(B) Z < X > Y
(C) X > Y > Z
(D) Z > X > Y

Ans. (A) pKa value of carboxylic group is less than pKa of $-\overset{+}{N}H_3(y)$ in amino acid and $-\overset{+}{N}H_3(z)$ will have comparatively less pKa than $-\overset{+}{N}H_3(z)$ due to –I effect of carboxylate group.

18. 0.004 M Na_2SO_4 is isotonic with 0.01 M Glucose. Degree of dissociation of Na_2SO_4 is

(A) 75%
(B) 50%
(C) 25%
(D) 85%

Ans. (A) $\pi_{Na_2SO_4} = \pi_{Glucose} = 0.01\times RT$

or $0.01\ RT = i\times 0.004\ RT$

$i = 2.5$

$$Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{-2}$$

$1-\alpha \qquad 2\alpha \qquad \alpha$

$i = 1+2\alpha = 2.5$

$\alpha = 0.75$

or 75% dissociation

19. $\Delta H_{vap} = 30$ KJ/mole and $\Delta S_{vap.} = 75\ Jmol^{-1}K^{-1}$. Find temperature of vapour, at one atmosphere

(A) 400K
(B) 350 K
(C) 298 K
(D) 250 K

Ans. (A) $\Delta G = \Delta H - T\Delta S$

At equilibrium $\Delta G = 0$

$$T = \frac{\Delta H}{\Delta S} = \frac{30\times 10^3}{75} = 400\ K$$

20. 2 mol of an ideal gas expanded isothermally & reversibly from 1 litre to 10 litres at 300 K. What is the enthalpy change?

(A) 4.98 KJ
(B) 11.47 KJ
(C) -11.47 KJ
(D) 0 KJ

Ans. (D) $H = E + PV$

and $\Delta H = \Delta E + \Delta(PV)$

or $\Delta H = \Delta E + nR\Delta T$

$\Delta T = 0$

$\Delta E = 0$

$\therefore \Delta H = 0$

21*. (A) follows first order reaction. (A) $\longrightarrow$ product

Concentration of A, changes from 0.1 M to 0.025 M in 40 minutes. Find the rate of reaction of A when concentration of A is 0.01 M

(A) 3.47×10^{-4} M min^{-1}
(B) 3.47×10^{-5} M min^{-1}
(C) 1.73×10^{-4} M min^{-1}
(D) 1.73×10^{-5} M min^{-1}

Nitin M Sir (physics-iitjee.blogspot.com)

Ans. (A)Concentration changes from 0.01 to 0.025 M in 40 minutes $\Rightarrow 2t_{1/2} = 40\,\text{min}$

$t_{1/2} = 20\,\text{min}$

$$r = K[A] = \frac{0.693}{20} \times 0.01 = 3.47 \times 10^{-4}$$

22*. 2-hexyne gives trans-2-hexene on treatment with

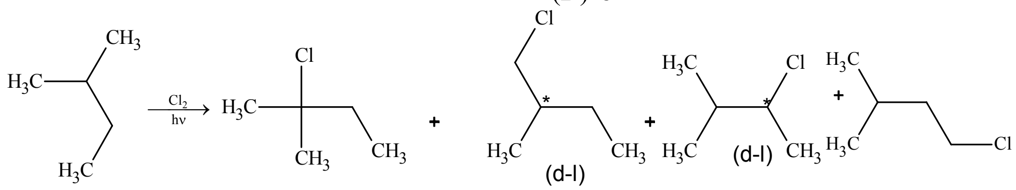
(A) Li / NH_3 (B) $Pd/ BaSO_4$
(C) $LiAlH_4$ (D) Pt/H_2

Ans. (A) Li/NH_3 brings about trans addition of H_2

23*. How many chiral compounds are possible on mono chlorination of 2-methyl butane?

(A) 2 (B) 4
(C) 6 (D) 8

Ans. (B)



24. Which of the following pairs give positive Tollen's test?

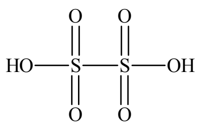
(A) Glucose, sucrose (B) Glucose, fructose
(C) Hexanal, Acetophenone (D) Fructose, sucrose

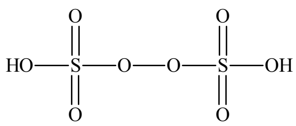
Ans. (B) Aldehydes and α- hydroxy ketones give positive Tollen's test. Glucose has an aldehydic group and fructose is an α hydroxy ketone

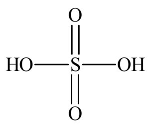
25. Which of the following has – O – O – linkage

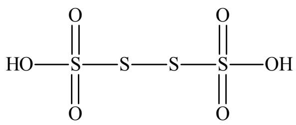
(A) $H_2S_2O_6$ (B) $H_2S_2O_8$
(C) $H_2S_2O_3$ (D) $H_2S_4O_6$

Ans. (B)

$H_2S_2O_6$ –


$H_2S_2O_8$ –

(per oxy linkage)

$H_2S_2O_3$ –


$H_2S_4O_6$ –


26. When I^- is oxidised by MnO_4^- in alkaline medium, I^- converts into

(A) IO_3^- (B) I_2
(C) IO_4^- (D) IO^-

Ans. (A) $2KMnO_4 + 2KOH \longrightarrow 2K_2MnO_4 + H_2O + O$

$$2K_2MnO_4 + 2H_2O \longrightarrow 2MnO_2 + 4KOH + 2O$$

$$2KMnO_4 + H_2O \xrightarrow{\text{alkaline}} 2MnO_2 + 2KOH + 3[O]$$

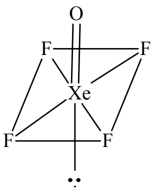
$$KI + [O] \longrightarrow KIO_3$$

$$2KMnO_4 + KI + H_2O \longrightarrow 2KOH + 2MnO_2 + KIO_3$$

27*. Number of lone pair(s) in $XeOF_4$ is/are

(A) 0 (B) 1
(C) 2 (D) 3

Ans. (B) Structure of $XeOF_4$



Number of lone pairs on the central atom is 1.

28*. According to MO Theory,

(A) O_2^+ is paramagnetic and bond order greater than O_2
(B) O_2^+ is paramagnetic and bond order less than O_2
(C) O_2^+ is diamagnetic and bond order is less than O_2
(D) O_2^+ is diamagnetic and bond order is more than O_2

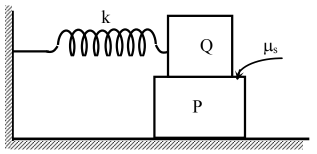
Ans. (A) O_2^+ B.O. = $\frac{1}{2}[\text{no. of bonding} - \text{no. of antibonding electrons}] = \frac{1}{2}[10-5] = 2.5$

O_2 B.O. = $\frac{1}{2}[10-6]$

$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2pz^2 \begin{bmatrix} \pi 2px^2 \\ \pi 2py^2 \end{bmatrix} \begin{bmatrix} \pi^* 2px^1 \\ \pi^* 2py^1 \end{bmatrix}$$

One unpaired e^- hence paramagnetic

29*. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks.



(A) kA/2 (B) kA
(C) $\mu_s$mg (D) zero

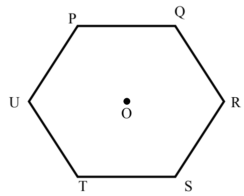
Ans. (A) $a_{max} = \frac{k}{2m}A$, hence $f_{max} = m\,a_{max} = \frac{kA}{2}$

30. A beam of white light is incident on glass air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are

(A) Violet, Indigo, Blue (B) All coolers except green
(C) Yellow, Orange, Red (D) White light

Ans. (C) Condition for light to transmit is $\sin C < 1/\mu$, $\mu_v > \mu_g > \mu_r$

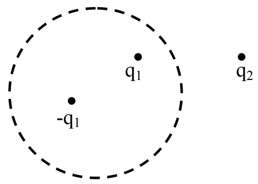
31. Six charges of equal magnitude, 3 positive and 3 negative are to be placed on PQRSTU corners of a regular hexagon, such that field at the centre is double that of what it would have been if only one +ve charge is placed at R



(A) +, +, +, -, -, - (B) -, +, +, +, -, -
(C) -, +, +, -, +, - (D) +, -, +, -, +, -

Ans. (C)

32. A Gaussian surface in the figure is shown by dotted line. The electric field on the surface will be



(A) due to q_1 and q_2 only (B) due to q_2 only
(C) zero (D) due to all

Ans. (D)

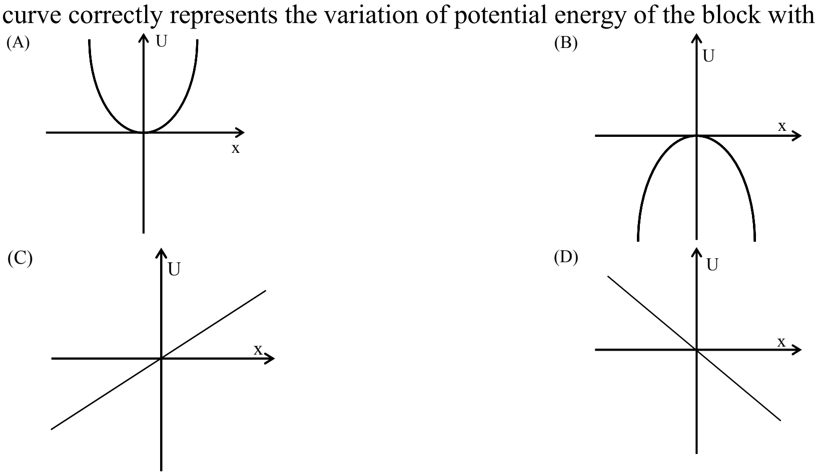
33*. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be
(A) 2K (B) K/2
(C) K (D) K/4

Ans. (B) $I\omega_0 = 2I\omega' \Rightarrow \omega' = \omega_0 / 2$

$K = \frac{1}{2} I\omega_0^2$

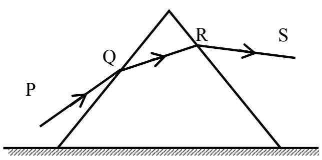
$K' = \frac{1}{2} 2I\left(\frac{\omega_0}{2}\right)^2 = \frac{K}{2}$

34*. A particle is acted by a force F = kx, where k is a +ve constant. Its potential energy at x = 0 is zero. Which curve correctly represents the variation of potential energy of the block with respect to x



Ans. (B) $U = -\int_0^x kx\,dx = -\frac{kx^2}{2}$

35. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation



(A) PQ is horizontal
(B) QR is horizontal
(C) RS is horizontal
(D) Any one will be horizontal.

Ans. (B) For minimum deviation, i = e

36*. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal.

(A) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (B) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

(C) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (D) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

Ans. (B) $\ell_1 = \frac{3}{4}\frac{v_1}{f_1}$, $\ell_2 = \frac{v_2}{f_2}$

$$\frac{3v_1}{4\ell_1} = \frac{v_2}{\ell_2}$$

$$\ell_2 = \frac{4\ell_1 v_2}{3v_1} = \frac{4\ell_1}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

37*. Three discs A, B and C having radii 2, 4, and 6 cm respectively are coated with carbon black. Wavelength for maximum intensity for the three discs are 300, 400 and 500 nm respectively. If Q_A, Q_B and Q_C are power emitted by A, B and C respectively, then

(A) Q_A will be maximum
(B) Q_B will be maximum
(C) Q_C will be maximum
(D) $Q_A = Q_B = Q_C$

Ans. (B) $\lambda_m T$ = constant

$$T_1 : T_2 : T_3 :: \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$$

$$Q = \sigma\varepsilon A T^4$$

$$Q_A : Q_B : Q_C :: \frac{2^2}{3^4} : \frac{4^2}{4^4} : \frac{6^2}{5^4}$$

Q_B will be maximum.

38. Monochromatic light of wavelength 400 nm and 560 nm are incident simultaneously and normally on double slits apparatus whose slits separation is 0.1 mm and screen distance is 1m. Distance between areas of total darkness will be

(A) 4 mm
(B) 5.6 mm
(C) 14 mm
(D) 28 mm

Ans. (D) $(2n + 1)\lambda_1 = (2m + 1)\lambda_2$

$$\frac{2n+1}{2m+1} = \frac{560}{400} = \frac{7}{5}$$

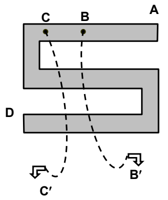
$10\,n = 14m + 2$

By inspection, for m = 2 ; n = 3

m = 7; n = 10

$$\therefore\ \Delta s = \frac{\lambda_1 D}{2d}[(2n_2+1)-(2n_1+1)] = 28\,\text{mm}.$$

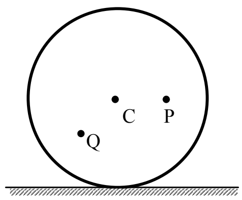
39. Shown in figure is a Post Office box. In order to calculate the value of external resistance, it should be connected between



(A) B′ and C′
(B) A and D
(C) C and D
(D) B and D

Ans. (B)

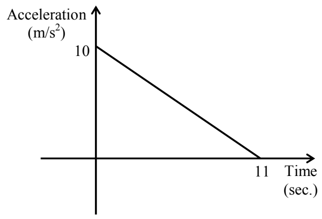
40*. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is



(A) $v_Q > v_C > v_P$
(B) $v_P > v_C > v_Q$
(C) $v_P = v_C$, $v_Q = v_C/2$
(D) $v_P < v_C > v_Q$

Ans. (B) About instantaneous axis of rotation i.e. point of contact $v = r\omega$, r_P is maximum $\therefore$ v_P is maximum.

41*. A body starts from rest at time t = 0, the acceleration time graph is shown in the figure. The maximum velocity attained by the body will be
(A) 110 m/s (B) 55 m/s
(C) 650 m/s (D) 550 m/s

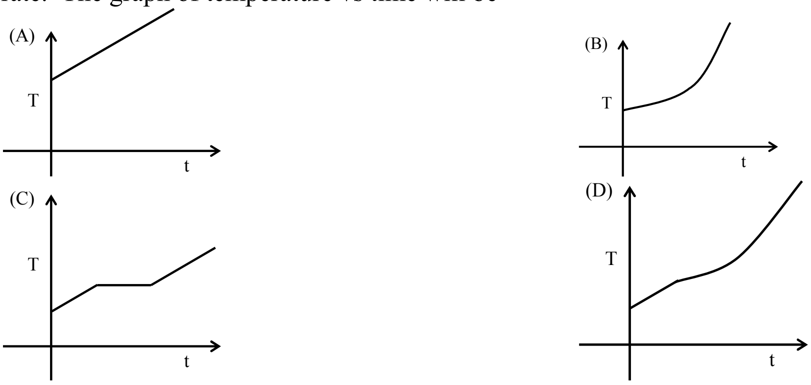


Ans. (B) Maximum velocity = area under the graph = 55 m/s.

42*. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m/s, velocity of sound in air = 300 m/s)
(A) 3000 Hz (B) 120 Hz
(C) 600Hz (D) 6000 Hz

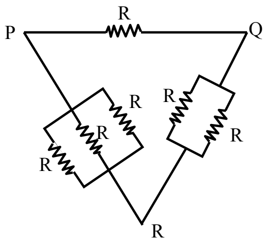
Ans. (C)

43*. If liquefied oxygen at 1 atmospheric pressure is heated from 50 k to 300 k by supplying heat at constant rate. The graph of temperature vs time will be



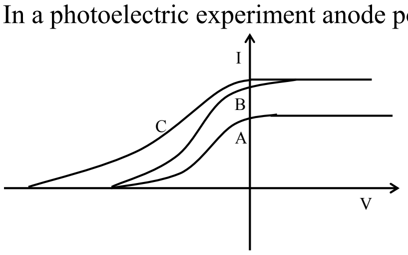
Ans. (C)

44. Six identical resistors are connected as shown in the figure. The equivalent resistance will be
(A) Maximum between P and R.
(B) Maximum between Q and R.
(C) Maximum between P and Q.
(D) all are equal.



Ans. (C)

45. In a photoelectric experiment anode potential is plotted against plate current



(A) A and B will have different intensities while B and C will have different frequencies.
(B) B and C will have different intensities while A and C will have different frequencies.
(C) A and B will have different intensities while A and C will have equal frequencies.
(D) A and B will have equal intensities while B and C will have different frequencies.

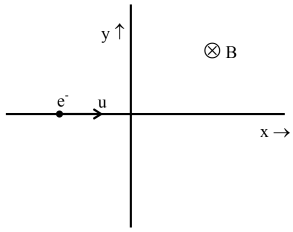
Ans. (A)

46*. Pressure depends on distance as, $P = \frac{\alpha}{\beta}\exp\left(-\frac{\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzman's constant and θ is temperature. The dimension of β are

(A) $M^0L^0T^0$ (B) $M^{-1}L^{-1}T^{-1}$
(C) $M^0L^2T^0$ (D) $M^{-1}L^1T^2$

Ans. (C) $\frac{\alpha z}{k\theta}$ should be dimensionless, hence $\alpha = MLT^{-2}$
$\alpha / \beta = ML^{-1}T^{-2} = P$, hence $\beta = M^0L^2T^0$

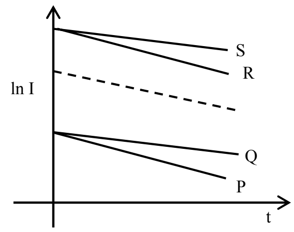
47. An electron traveling with a speed u along the positive x-axis enters into a region of magnetic field where $B = -B_0\,\hat{k}$ (x > 0). It comes out of the region with speed v then



(A) v = u at y > 0 (B) v = u at y < 0
(C) v > u at y > 0 (D) v > u at y < 0

Ans. (B) Charged particle will move in a circular path with constant speed inside the magnetic field.

48. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of ln I with respect to time. If the resistance is changed to 2x, the new graph will be



(A) P (B) Q
(C) R (D) S

Ans. (B) $I = I_0\, e^{-t/xC} \Rightarrow \ln I = \ln I_0 - t/xC$
I_0 is inversely proportional to x.

49. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later it's activity becomes 3000dps. What was its initial activity

(A) 20000 dps (B) 24000 dps
(C) 12000 dps (D) 6000 dps

Ans. (B) $A_1 = \lambda N_1 = \lambda N_0\, e^{-\lambda t_1}$, $A_2 = \lambda N_2 = \lambda N_0\, e^{-\lambda t_2}$ and $A_0 = \lambda N_0 = 24000$ dps.

50*. Two identical rods are connected between two containers one of them is at 100^0C and another is at 0^0C. If rods are connected in parallel then the rate of melting of ice q_1 gm/sec. If they are connected in series then rate is q_2. Then the ratio q_2 / q_1 is

(A) 2 (B) 4
(C) ½ (D) ¼

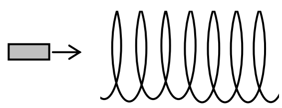
Ans. (D) $q_1 = \frac{CT}{R_T/2}$, $q_2 = \frac{CT}{2R_T}$ and $q_2 / q_1 = 1/4$

51*. A particle starts sliding down a frictionless inclined plane. If S_n is the distance traveled by it from time t = n- 1 sec to t = n sec, the ratio S_n / S_{n+1} is

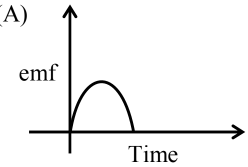
(A) $\frac{2n-1}{2n+1}$ (B) $\frac{2n+1}{2n}$
(C) $\frac{2n}{2n+1}$ (D) $\frac{2n+1}{2n-1}$

Ans. (A) $S_n = \frac{a}{2}(2n-1)$, $S_{n+1} = \frac{a}{2}(2n+2-1)$

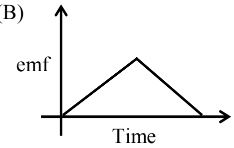
52. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in figure. Which graph best represents the relationship between emf induced with time



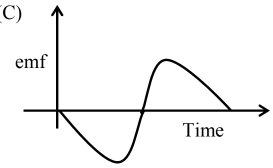
(A)



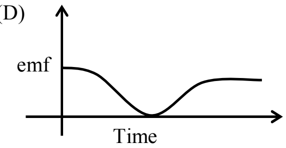
(B)



(C)



(D)



Ans. (C)

53. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is

(A) 6 cm (B) 4 cm
(C) 12 cm (D) 9 cm

Ans. (A) Object is at centre of curvature, hence image will also be at centre of curvature.

54. A proton has kinetic energy E = 100 keV which is equal to that of a photon. The wavelength of photon is λ_2 and that of proton is λ_1. The ratio of λ_1 / λ_2 is proportional to

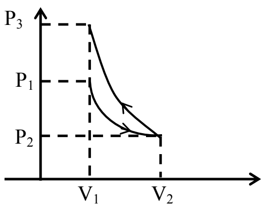
(A) E^2 (B) $E^{1/2}$
(C) E^{-1} (D) $E^{-1/2}$

Ans. (B) $\lambda_{proton} = \dfrac{h}{\sqrt{2mE}}$, $\lambda_{photon} = \dfrac{hc}{E}$.

55*. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in complete process which of the following is true

(A) $W > 0$; $P_3 > P_1$ (B) $W < 0$; $P_3 > P_1$
(C) $W > 0$; $P_3 < P_1$ (D) $W < 0$; $P_3 < P_1$

Ans. (B)



56*. A wire of length ℓ = 6 ± 0.06 cm and radius r = 0.5 ± 0.005 cm and mass m = 0.3 ± 0.003 gm. Maximum percentage error in density is

(A) 4 (B) 2
(C) 1 (D) 6.8

Ans. (A) $\rho = \dfrac{m}{\ell \pi r^2}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell} = \frac{0.003}{0.3} + \frac{2\times 0.005}{0.5} + \frac{0.06}{6} = \frac{4}{100} = 4\ \%$$

57*. The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the angles of the triangle are in the ratio

(A) 1 : 3 : 5 (B) 2 : 3 : 4
(C) 3 : 2 : 1 (D) 1 : 2 : 3

Ans. (D) Let $a = x$, $b = \sqrt{3}\,x$, $c = 2x$

$c^2 = a^2 + b^2 \Rightarrow \angle C = 90^0$

$\tan A = \frac{1}{\sqrt{3}} \Rightarrow \angle A = 30^0$.

$\Rightarrow A : B : C = 1 : 2 : 3$.

58*. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

(A) 2 sq. units (B) 4 sq. units
(C) 6 sq. units (D) 8 sq. units

Ans. (A) $x^2 - y^2 + 2y - 1 = 0$

$\Rightarrow$ Equations of lines are $y = x + 1$, $y = -x + 1$

$\Rightarrow$ angle bisectors are $y = 1$ and $x = 0$

$\Rightarrow$ area of triangle $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units

59. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is

(A) 4/25 (B) 4/35
(C) 4/33 (D) 4/1155

Ans. (D) Numbers between 1 and 100 which are divisible by both 2 and 3 are 16.

Hence the probability is $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$.

60. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is

(A) $1/\sqrt{3}$ (B) 1/2
(C) 1 (D) 1/3

Ans. (A) Points of intersection of $y = ax^2$ and $x = ay^2$ are (0, 0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$.

Hence $\int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx = 1 \Rightarrow a = \frac{1}{\sqrt{3}}$ (as $a > 0$).

61*. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to

(A) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$
(C) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left(\frac{5\pi}{6}, \pi\right]$

Ans. (B) $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$\cos\phi = \frac{1}{3} \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2} \Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$.

62*. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is

(A) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (B) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(C) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (D) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Ans. (A) Equation of tangent at any point 'θ' is $\frac{x}{\sqrt{2}}\cos\theta + y\sin\theta = 1$ and the midpoint of its intercept between the axes is $\left(\frac{\sqrt{2}}{2}\sec\theta,\ \frac{1}{2}\operatorname{cosec}\theta\right) \Rightarrow$ locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$.

63. If $f(x)$ is differentiable and $\int_0^{t^2} x\, f(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) −5/2
(C) 1 (D) 5/2

Ans. (A) Differentiating both sides, we get
$t^2\, f(t^2).2t = \frac{5t^4 \cdot 2}{5} \Rightarrow f(t^2) = t \Rightarrow f\left(\frac{4}{25}\right) = \pm\frac{2}{5}$.

64*. The value of x for which $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1} x)$ is
(A) 1/2 (B) 1
(C) 0 (D) −1/2

Ans. (D) $\frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}} \Rightarrow x^2 + 2x + 2 = x^2 + 1 \Rightarrow x = -\frac{1}{2}$.

65. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$
(A) f (x) is a strictly increasing function (B) f(x) has a local maxima
(C) f(x) is a strictly decreasing function (D) f(x) is bounded

Ans. (A) $f'(x) = 3x^2 + 2bx + c$
$D = 4b^2 - 12c = 4(b^2 - c) - 8c \Rightarrow D < 0 \Rightarrow f'(x) > 0\ \ \forall\, x \in (-\infty, \infty) \Rightarrow f(x)$ is an increasing function.

66*. If $\omega\ (\neq 1)$ be a cube root of unity and $(1 + \omega^2)^n = (1 + \omega^4)^n$, then the least positive value of n is
(A) 2 (B) 3
(C) 5 (D) 6

Ans. (B) $(1 + \omega^2)^n = (1 + \omega^4)^n \Rightarrow (-\omega)^n = (-\omega^2)^n \Rightarrow (\omega)^n = 1 \Rightarrow n = 3$.

67. If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1] is
(A) -2 (B) -1
(C) 0 (D) 1/2

Ans. (D) For function to satisfy the condition of Rolle's theorem, it should be continuous in [0, 1]
$\Rightarrow \lim_{x\to0^+} f(x) = f(0) \Rightarrow \lim_{x\to0^+}\frac{\log x}{x^{-\alpha}} = 0 \Rightarrow \lim_{x\to0^+}\frac{1/x}{-\alpha x^{-\alpha-1}} = 0 \Rightarrow \alpha > 0$.
Also $\forall\ \alpha > 0$, f(x) is differentiable in (0, 1) and $f(1) = 0 = f(0)$.

68*. For all ‘x’, $x^2 + 2ax + 10 - 3a > 0$, then the interval in which ‘a’ lies is

(A) $a < -5$
(B) $-5 < a < 2$
(C) $a > 5$
(D) $2 < a < 5$

Ans. (B) $D < 0 \Rightarrow 4a^2 - 4(10 - 3a) < 0 \Rightarrow 4a^2 + 12a - 40 < 0 \Rightarrow -5 < a < 2$.

69*. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(A) $\pi/6$
(B) $\pi/4$
(C) $\pi/3$
(D) $\pi/2$

Ans. (C) Equation of tangent is $y = mx + \frac{1}{m}$.

Since it passes through (1, 4)

$\therefore m^2 - 4m + 1 = 0 \Rightarrow m_1 + m_2 = 4, m_1m_2 = 1 \Rightarrow |m_1 - m_2| = 2\sqrt{3}$

$\therefore \tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$.

70*. If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

(A) $p^3 - q(3p - 1) + q^2 = 0$
(B) $p^3 - q(3p + 1) + q^2 = 0$
(C) $p^3 + q(3p - 1) + q^2 = 0$
(D) $p^3 + q(3p + 1) + q^2 = 0$

Ans. (A) Let the roots be α, α^2

$\Rightarrow \alpha^2 + \alpha = -p, \alpha^3 = q \Rightarrow \alpha(\alpha + 1) = -p \Rightarrow \alpha^3(\alpha^3 + 1 + 3(\alpha^2 + \alpha)) = -p^3 \Rightarrow p^3 - q(3p - 1) + q^2 = 0$.

71. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is

(A) $\frac{\pi}{2}+1$
(B) $\frac{\pi}{2}-1$
(C) -1
(D) 1

Ans. (B) Let $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}dx$. Put $x = \cos\theta \Rightarrow dx = -\sin\theta\, d\theta$ then $I = \int_0^{\pi/2} (1-\cos\theta)d\theta = \frac{\pi}{2} - 1$.

72. If $\vec{a} = (\hat{i}+\hat{j}+\hat{k})$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j}-\hat{k}$, then $\vec{b}$ is

(A) $\hat{i}-\hat{j}+\hat{k}$
(B) $2\hat{j}-\hat{k}$
(C) $\hat{i}$
(D) $2\hat{i}$

Ans. (C) $\vec{a}\times(\vec{a}\times\vec{b}) = (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b}$

$(\hat{i}+\hat{j}+\hat{k})\times(\hat{j}-\hat{k}) = (\hat{i}+\hat{j}+\hat{k}) - 3\vec{b} \Rightarrow \vec{b} = \hat{i}$.

73*. If ${}^{n-1}C_r = (k^2 - 3)\,{}^{n}C_{r+1}$, then $k \in$

(A) $(-\infty, -2]$
(B) $[2, \infty)$
(C) $\left[-\sqrt{3}, \sqrt{3}\right]$
(D) $\left(\sqrt{3}, 2\right]$

Ans. (D) ${}^{n-1}C_r = (k^2 - 3)\frac{n}{r+1}\,{}^{n-1}C_r \Rightarrow k^2 - 3 = \frac{r+1}{n}$

$\therefore 0 < k^2 - 3 \le 1$ or $3 < k^2 \le 4$.

74. If f (x) = sin x + cos x, g (x) = x^2 – 1, then g(f(x)) is invertible in the domain
(A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $[0, \pi]$

Ans. (B) g (f (x)) = $(\sin x + \cos x)^2 - 1 = \sin 2x$ which is invertible in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$.

75. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$, y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ equals
(A) 1/3 (B) 2/3
(C) -1/3 (D) 1

Ans. (A) $\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

$\ln (y + 1) = -\ln (2 + \sin x) + \ln c \Rightarrow y + 1 = \frac{c}{2+\sin x}$. Putting x = 0 and y = 1, we get c = 4

$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{1}{3}$.

76. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is
(A) 3/2 (B) 9/2
(C) –2/9 (D) –3/2

Ans. (B) General points on the lines are (2λ + 1, 3λ – 1, 4λ + 1) and (μ + 3, 2μ + k, μ)
Equating the corresponding coordinates, we get k = 9/2.

77. Given 2x – y – 2z = 2, x – 2y + z = – 4, x + y + λz = 4 then the value of λ such that the given system of equation has NO solution, is
(A) 3 (B) 1
(C) 0 (D) – 3

Ans. (D) As $\Delta_z \neq 0$, for no solution $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -3$.

78*. If the line $2x + \sqrt{6}\,y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is
(A) $(-2, \sqrt{6})$ (B) $\left(-5, 2\sqrt{6}\right)$
(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (D) $\left(4, -\sqrt{6}\right)$

Ans. (D) Equation of tangent is $xx_1 - 2yy_1 = 4$
on comparing with $2x + \sqrt{6}\,y = 2$, we get $x_1 = 4$ and $y_1 = -\sqrt{6}$.

79. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then the value of α is
(A) ± 1 (B) ± 2
(C) ± 3 (D) ± 5

Ans. (C) $|A|^3 = 125 \Rightarrow |A| = 5 \Rightarrow \alpha = \pm 3$.

80. The unit vector which is orthogonal to the vector $5\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

(A) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\frac{2\hat{i}-5\hat{j}}{\sqrt{29}}$

(C) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (D) $\frac{2\hat{i}-8\hat{j}+\hat{k}}{\sqrt{69}}$

Ans. (C) Let $\vec{a} = 5\hat{i}+2\hat{j}+6\hat{k}$, $\vec{b} = 2\hat{i}+\hat{j}+\hat{k}$, $\vec{c} = \hat{i}-\hat{j}+\hat{k}$, then required unit vector will be along $\vec{a}\times(\vec{b}\times\vec{c})$.

$\vec{a}\times(\vec{b}\times\vec{c}) = 27\hat{j}-9\hat{k} \Rightarrow$ unit vector is $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$.

81. If f(x) is differentiable and strictly increasing function , then the value of $\lim\limits_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is

(A) 1 (B) 0

(C) –1 (D) 2

Ans. (C) Using L'Hospital's rule

$$\lim_{x\to 0}\frac{2xf'(x^2)-f'(x)}{f'(x)} = -1 \quad (\because f'(x) > 0 \ \forall x)$$

82*. An infinite G.P. has first term 'x' and sum '5', then x belongs to

(A) $x < -10$ (B) $-10 < x < 0$

(C) $0 < x < 10$ (D) $x > 10$

Ans. (C) The sum of an infinite G.P. = $\frac{x}{1-r} = 5$ (given)

$|r| < 1 \Rightarrow \left|1-\frac{x}{5}\right| < 1 \Rightarrow 0 < x < 10$.

83*. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

(A) $\sqrt{3}$ (B) $\sqrt{2}$

(C) 3 (D) 2

Ans. (C) Centre is (1, 3) and radius = 2

If r = radius of second circle then $r^2 = 2^2 + (3-1)^2 + (2-1)^2 \Rightarrow r = 3$.

84. If y is a function of x and log (x + y) – 2xy = 0, then the value of $y'(0)$ is equal to

(A) 1 (B) –1

(C) 2 (D) 0

Ans. (A) At x = 0, y = 1

log(x + y) – 2xy = 0

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2y(x+y)-1}{1-2(x+y)x} \Rightarrow \left.\frac{dy}{dx}\right|_{(0,\,1)} = 1.$$

Note: ***FIITJEE solutions to IIT–JEE, 2004 Screening Test is based on Screening Test paper created using memory retention of select FIITJEE students appeared in this test and hence may not exactly be the same as the original paper. However, every effort has been made to reproduce the original paper in the interest of the aspiring students.***

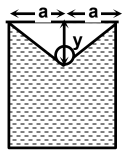
FIITJEE Solutions to IITJEE–2004 Mains Paper

Physics

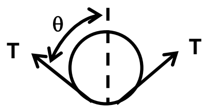
Time: 2 hours

Note: Question number 1 to 10 carries ***2 marks*** each and 11 to 20 carries ***4 marks*** each.

1. A long wire of negligible thickness and mass per unit length λ is floating in a liquid such that the top surface of liquid dips by a distance 'y'. If the length of base of vessel is 2a , find surface tension of the liquid. (y < < a)



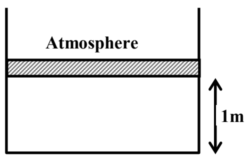
Sol. $\ell\,(2T\cos\theta) = \lambda\ell g$



$$T = \frac{\lambda g}{2\cos\theta}$$

$$\Rightarrow T = \frac{\lambda g(a^2+y^2)^{1/2}}{2y} \approx \frac{\lambda g a}{2y}.$$

2. An ideal diatomic gas is enclosed in an insulated chamber at temperature 300K. The chamber is closed by a freely movable massless piston, whose initial height from the base is 1m. Now the gas is heated such that its temperature becomes 400 K at constant pressure. Find the new height of the piston from the base.
If the gas is compressed to initial position such that no exchange of heat takes place, find the final temperature of the gas.



Sol. Process 1 is isobaric

$T_1 = 300$ K, $T_2 = 400$ K

$$\frac{V}{T} = \text{constant}$$

$$\frac{A\times 1}{300} = \frac{A\times h}{400} \Rightarrow h = \frac{4}{3}\,\text{m}$$

Process 2 is adiabatic

$$TV^{\gamma-1} = \text{constant},\ 400\left(\frac{A\times 4}{3}\right)^{\frac{7}{5}-1} = T_3\,(A\times 1)^{\frac{7}{5}-1} \Rightarrow T_3 = 400\left(\frac{4}{3}\right)^{\frac{2}{5}}\text{K}.$$

3. In Searle's apparatus diameter of the wire was measured 0.05 cm by screw gauge of least count 0.001 cm. The length of wire was measured 110 cm by meter scale of least count 0.1 cm. An external load of 50 N was applied. The extension in length of wire was measured 0.125 cm by micrometer of least count 0.001 cm. Find the maximum possible error in measurement of young's modulus.

Sol. $$Y = \frac{4F/\pi D^2}{(\Delta L)/L} = \frac{4FL}{\pi D^2(\Delta L)}$$

Maximum possible relative error

Nitin M Sir (physics-iitjee.blogspot.com)

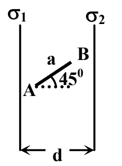
$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta(\Delta L)}{\Delta L} = \left(\frac{0.1}{110} + \frac{2\times 0.001}{0.050} + \frac{0.001}{0.125}\right)$$

Percentage error

$$100\times\frac{\Delta Y}{Y} = \frac{1}{11} + 4 + \frac{4}{5}$$

= 0.8 + 4 + 0.09 = 4.89 %.

4. Two infinitely large sheets having charge densities σ_1 and σ_2 respectively ($\sigma_1 > \sigma_2$) are placed near each other separated by distance ‘d’. A charge ‘Q’ is placed in between two plates such that there is no effect on charge distribution on plates. Now this charge is moved at an angle of 45^0 with the horizontal towards plate having charge density σ_2 by distance ‘a’ (a < d). Find the work done by electric field in the process.



Sol. $E = \frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}$

work done by electric field, $W = q\,\vec{E}\,.\,\vec{d} = E\frac{a}{\sqrt{2}}q = \frac{q(\sigma_1-\sigma_2)a}{2\sqrt{2}\varepsilon_0}$

5. An α-particle and a proton are accelerated from rest through same potential difference and both enter into a uniform perpendicular magnetic field. Find the ratio of their radii of curvature.

Sol. $r = \frac{\sqrt{2qVm}}{qB}$

$$\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha}{m_p}\times\frac{q_p}{q_\alpha}}$$

$$= \sqrt{\frac{4}{1}\times\frac{e}{2e}} = \sqrt{2}:1$$

6. A small ball of radius ‘r’ is falling in a viscous liquid under gravity. Find the dependency of rate of heat produced in terms of radius ‘r’ after the drop attains terminal velocity.

Sol. Rate of heat produced = F.v

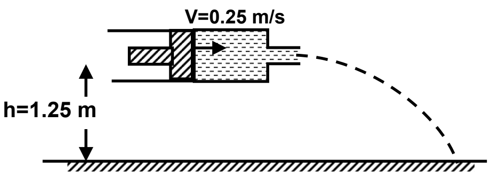
$= 6\pi\eta r v_T . v_T$

$$\frac{dQ}{dt} = 6\pi\eta r . v_T^2$$

$$v_T = \frac{2}{9}(\sigma-\rho)r^2 g/\eta$$

$$\frac{dQ}{dt} \propto r^5$$

7. A syringe of diameter D = 8 mm and having a nozzle of diameter d = 2 mm is placed horizontally at a height of 1.25 m as shown in the figure. An incompressible and non-viscous liquid is filled in syringe and the piston is moved at speed of 0.25 m/s. Find the range of liquid jet on the ground.



Sol. AV = Constant

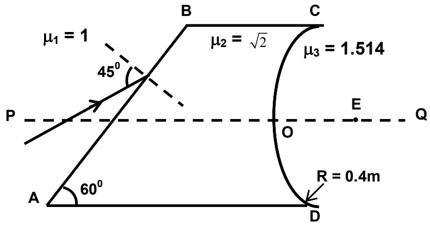
$D^2V = d^2v$

$$v = \frac{D^2}{d^2}V = \left(\frac{8}{2}\right)^2 \times 0.25$$

$$= 16 \times 0.25 = 4\,m/s$$

$$x = v\sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2\times1.25}{10}} = 4\times\frac{1}{2} = 2m$$

8. A light ray is incident on an irregular shaped slab of refractive index $\sqrt{2}$ at an angle of 45° with the normal on the incline face as shown in the figure. The ray finally emerges from the curved surface in the medium of the refractive index μ = 1.514 and passes through point E. If the radius of curved surface is equal to 0.4 m, find the distance OE correct upto two decimal places.



Sol. Using Snell's law

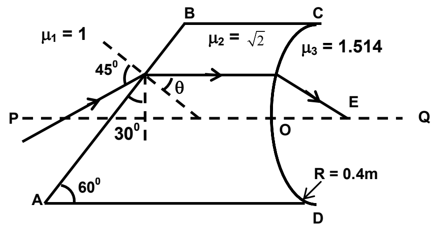
$\mu_1 \sin 45° = \mu_2 \sin\theta$

$\theta = 30°$.

i.e. ray moves parallel to axis

$$\frac{\mu_3}{OE} - \frac{\mu_2}{\infty} = \frac{(\mu_3 - \mu_2)}{R}$$

OE = 6.056 m ≈ 6.06 m



9. A screw gauge of pitch 1mm has a circular scale divided into 100 divisions. The diameter of a wire is to be measured by above said screw gauge. The main scale reading is 1mm and 47th circular division coincides with main scale. Find the curved surface area of wire in true significant figures. (Given the length of wire is equal to 5.6 cm and there is no zero-error in the screw gauge.)

Sol. Least count $= \frac{1mm}{100} = 0.01mm.$

Diameter = M. S. + No. of division coinciding with main scale × Least count.

= 1mm + 47 × 0.01 mm

= 1.47 mm = 0.147 cm.

Curved surface area $= \pi d\ell = \frac{22}{7}\times 0.147 \times 5.6 = 2.6\ cm^2$

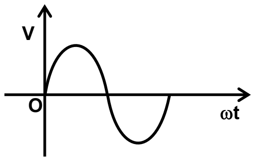
10. The age of a rock containing lead and uranium is equal to 1.5×10^9 yrs. The uranium is decaying into lead with half life equal to 4.5×10^9 yrs. Find the ratio of lead to uranium present in the rock, assuming initially no lead was present in the rock. (Given $2^{1/3} = 1.259$)

Sol.
$$\frac{N_U}{N_O} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{1.259}$$

$$\frac{N_U}{N_{Pb} + N_U} = \frac{1}{1.259}$$

$$\frac{N_{Pb}}{N_U} = 0.259.$$

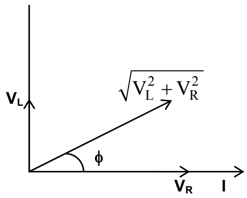
11. An inductor of inductance (L) equal to 35 mH and resistance (R) equal to 11 Ω are connected in series to an AC source. The rms voltage of a.c. source is 220 volts and frequency is 50 Hz.



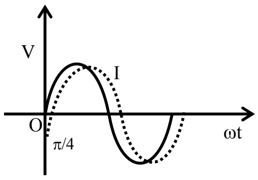
(a) Find the peak value of current in the circuit.

(b) Plot the current (I) vs (ωt) curve on the given voltage vs (ωt) curve. (Given $\pi = \frac{22}{7}$)

Sol. $Z = \sqrt{\left[(\omega L)^2 + R^2\right]}$



$I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{\sqrt{\left[(100\pi \times 35 \times 10^{-3})^2 + (11)^2\right]}} = 20\text{Amp}$



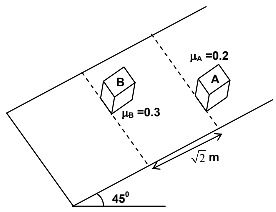
$\tan\phi = \frac{\omega L}{R} = \frac{100\pi \times 35 \times 10^{-3}}{11} = 1$

$\Rightarrow \phi = 45^0$

$I = I_0 \sin(\omega t - \frac{\pi}{4})$

$= 20 \sin(100\,\pi t - \frac{\pi}{4})$

12. Two identical blocks A and B are placed on a rough inclined plane of inclination 45^0. The coefficient of friction between block A and incline is 0.2 and that of between B and incline is 0.3. The initial separation between the two blocks is $\sqrt{2}$ m. The two blocks are released from rest, then find
(a) the time after which front faces of both blocks come in same line and
(b) the distance moved by each block for attaining above position.



Sol. $a_A = g\sin 45° - 0.2g\cos 45° = 4\sqrt{2}\ m/s^2$

$a_B = g\sin 45° - 0.3\,g\cos 45° = \frac{7}{2}\sqrt{2}\ m/s^2$

$a_{AB} = 0.5\sqrt{2}\ m/s^2$

$s_{AB} = \frac{1}{2}a_{AB}t^2$

$t^2 = \frac{2\sqrt{2}}{0.5\sqrt{2}} = 4$

t = 2 sec.

$s_B = \frac{1}{2}a_B t^2 = 7\sqrt{2}\ m$

$s_A = \frac{1}{2}a_A t^2 = 8\sqrt{2}\ m$

13. In a photoelectric setup, the radiations from the Balmer series of hydrogen atom are incident on a metal surface of work function 2eV. The wavelength of incident radiations lies between 450 nm to 700 nm. Find the maximum kinetic energy of photoelectron emitted.
(Given hc/e = 1242 eV-nm).

Sol. $\Delta E = 13.6\left[\frac{1}{4}-\frac{1}{n^2}\right] = \frac{hc}{e\lambda} = \frac{1242}{\lambda}$

$\Rightarrow \quad \lambda = \frac{1242\times 4n^2}{13.6(n^2-4)}$

λ_{min} which lies between 450 nm and 700 nm is for transition from n = 4 to n = 2 and is equal to 487.05 nm

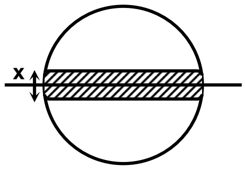
For maximum K.E. of photoelectron

$\frac{hc}{\lambda_{min}} - \phi = K.E._{max}$

$K.E_{max} = \frac{13.6\times 12}{4\times 16} - 2 = 0.55\,eV.$

14. A spherical ball of radius R, is floating in a liquid with half of its volume submerged in the liquid. Now the ball is displaced vertically by small distance inside the liquid. Find the frequency of oscillation of ball.

Sol. Restoring force $= \pi R^2 x\rho g$ (for small x)



$\Rightarrow -m\frac{d^2x}{dt^2} = \pi R^2 x\rho g$

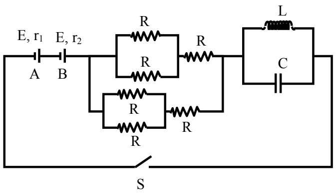
$\frac{d^2x}{dt^2} = -\frac{3}{2}\frac{g}{R}x$, (as $\frac{4}{3}\frac{\pi R^3}{2}\rho g = mg$)

$\therefore$ Motion is SHM

$\Rightarrow \omega^2 = \frac{3}{2}\frac{g}{R}$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$.

15. The two batteries A and B, connected in given circuit, have equal e.m.f. E and internal resistance r_1 and r_2 respectively ($r_1 > r_2$). The switch S is closed at t = 0. After long time it was found that terminal potential difference across the battery A is zero. Find the value of R.



Sol. Since average voltage across capacitor and inductor for D.C. sources will be zero at steady state.

$I = \frac{2E}{(R_{eq}+r_1+r_2)} = \frac{2E}{(r_1+r_2+\frac{3R}{4})}$ . . . (i)

P.D. across the battery A $= E - Ir_1 = 0$

$\Rightarrow$ $I = E/r_1$ . . . (ii)

From (i) and (ii),

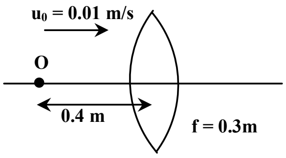
$R = \frac{4(r_1-r_2)}{3}$

16. A point object is moving with velocity 0.01 m/s on principal axis towards a convex lens of focal length 0.3 m. When object is at a distance of 0.4 m from the lens, find

(a) rate of change of position of the image, and

(b) rate of change of lateral magnification of image.

Sol. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$



$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$

$\Rightarrow \frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$ . . . . (i)

$\frac{1}{30}=\frac{1}{v}-\frac{1}{-40}$

$\Rightarrow$ v = 120 cm.

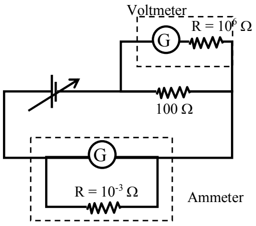
$\Rightarrow m=\frac{dv}{du}=\frac{v^2}{u^2}=\left(1-\frac{v}{f}\right)^2$

$\frac{dm}{dt}=-\frac{2}{f}\left(1-\frac{v}{f}\right)\frac{dv}{dt}$

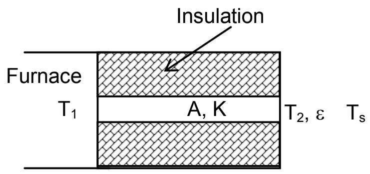
$=\frac{-2}{0.3}\left(1-\frac{120}{30}\right)\times 0.09=1.8\ s^{-1}$

17. An experiment is performed to verify Ohm's law using a resister of resistance R = 100Ω, a battery of variable potential difference, two galvanometers and two resistances of $10^6\,\Omega$ and $10^{-3}\Omega$ are given. Draw the circuit diagram and indicate clearly position of ammeter and voltmeter.

Sol.



18. A uniform rod of length L, conductivity K is connected from one end to a furnace at temperature T_1. The other end of rod is at temperature T_2 and is exposed to atmosphere. The temperature of atmosphere is T_s. The lateral part of rod is insulated. If $T_2 - T_s \ll T_s$, $T_2 = T_s + \Delta T$ & $\Delta T \propto (T_1 - T_s)$, find proportionality constant of given equation. The heat loss to atmosphere is through radiation only and the emissivity of the rod is ε.



Sol. $\frac{KA(T_1-T_2)}{L}=\varepsilon\sigma A\left(T_2^4-T_s^4\right)$

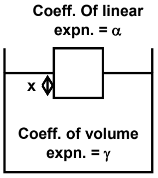
$=\varepsilon\sigma A\left[(T_s+\Delta T)^4-T_s^4\right]=4\varepsilon\sigma A T_s^3\Delta T$

$\Rightarrow \frac{K(T_1-T_S)}{L}=\Delta T\left[4\sigma\varepsilon T_S^3+\frac{K}{L}\right]$

$\Rightarrow \Delta T=\frac{K(T_1-T_S)}{L\left[4\sigma\varepsilon T_S^3+\frac{K}{L}\right]}$

$\therefore$ Proportionality constant $=\frac{K}{L\left[4\sigma\varepsilon T_S^3+\frac{K}{L}\right]}$.

19. A cubical block is floating inside a bath. The temperature of system is increased by small temperature ΔT. It was found that the depth of submerged portion of cube does not change. Find the relation between coefficient of linear expansion (α) of the cube and volume expansion of liquid (γ).



Sol. At initial temperature for the equilibrium of the block

$AL\rho_b g = Ax\rho_\ell g$

$L\rho_b = x\rho_\ell$ . . . (i)

At final temperature

$A' = A(1 + 2\alpha\Delta T)$

$\rho'_\ell = \rho_\ell(1 - \gamma\,\Delta T)$

For the equilibrium of the block

$A(1 + 2\alpha\Delta T)(x)\,\rho_\ell\,(1 - \gamma\Delta T) = AL\rho_b = Ax\rho_\ell$

$\Rightarrow 1 + 2\alpha\Delta T - \gamma\Delta\,T = 1$

$\Rightarrow \gamma = 2\alpha$

20. In a Young's double slit experiment light consisting of two wavelengths $\lambda_1 = 500$ nm and $\lambda_2 = 700$ nm is incident normally on the slits. Find the distance from the central maxima where the maximas due to two wavelengths coincide for the first time after central maxima. (Given $\frac{D}{d} = 1000$) where D is the distance between the slits and the screen and d is the separation between the slits.

Sol. $y_1 = \frac{nD\lambda_1}{d}$

$y_2 = \frac{mD\lambda_2}{d}$

$y_1 = y_2 \quad \Rightarrow n = \frac{7}{5}m$

For the first location, $m = 5, n = 7$

$\therefore\ y = 7 \times 1000 \times 5 \times 10^{-7} = 35 \times 10^{-4} = 3.5$ mm.

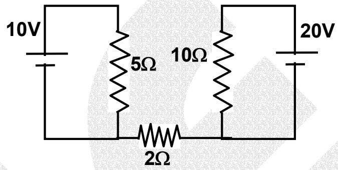
*Note: **FIITJEE** solutions to IIT–JEE, 2004 Main Papers created using memory retention of select **FIITJEE** students appeared in this test and hence may not exactly be the same as the original paper. However, every effort has been made to reproduce the original paper in the interest of the aspiring students.*

Note: ***FIITJEE** solutions to IIT–JEE, 2005 Screening Test is based on Screening Test paper created using memory retention of select **FIITJEE** students appeared in this test and hence may not exactly be the same as the original paper. However, every effort has been made to reproduce the original paper in the interest of the aspiring students.*

FIITJEE *solutions to IIT–JEE, 2005 Screening*

PHYSICS

57. Find current in 2Ω resistor
(A) 0 (B) 2 A
(C) 4 A (D) 1 A



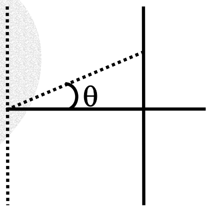
Ans. A

Sol. According to Kirchoff's junction rule no current passes through 2Ω resistor.
$\therefore i = 0$

58. In Young's double slit experiment the angular position of a point on the central maxima whose intensity is one fourth of maximum intensity
(A) $\sin^{-1}(\lambda/d)$ (B) $\sin^{-1}(\lambda/2d)$
(C) $\sin^{-1}(\lambda/3d)$ (D) $\sin^{-1}(\lambda/4d)$

Ans. C

Sol. $I = I_{max}\cos^2\phi/2$



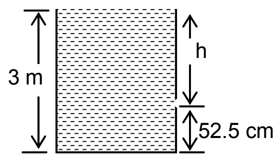
$\Rightarrow \phi = 2\pi/3$ and $\frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{3}$

$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$

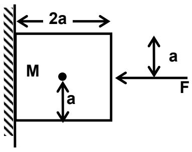
59. Ratio of area of hole to beaker is 0.1. Height of liquid in beaker is 3m, and hole is at the height of 52.5 cm from the bottom of beaker, find the square of the velocity of liquid coming out from the hole
(A) 50 $(m/s)^2$ (B) 50.5 $(m/s)^2$
(C) 51 $(m/s)^2$ (D) 42 $(m/s)^2$

Ans. A

Sol. $u^2 = \frac{2gh}{\left[1-\left(\frac{A_0}{A}\right)^2\right]} = 50\ (m/s)^2$

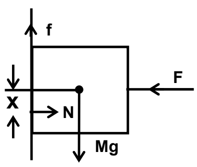


60. In the figure shown, a cubical block is held stationary against a rough wall by applying force 'F' then ***incorrect*** statement among the following is
(A) frictional force, f = Mg (B) F = N, N is normal reaction
(C) F does not apply any torque (D) N does not apply any torque

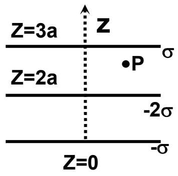


Ans. D

Sol. For equilibrium, f = Mg
F = N
For rotational equilibrium normal will shift downward.
Hence torque due to friction about centre of mass = Torque due to Normal reaction about centre of mass.



61. Three infinitely charged sheets are kept parallel to x – y plane having charge densities as shown. Then the value of electric field at 'P' is



(A) $\frac{-4\sigma}{\epsilon_0}\hat{k}$ (B) $\frac{4\sigma}{\epsilon_0}\hat{k}$

(C) $\frac{-2\sigma}{\epsilon_0}\hat{k}$ (D) $\frac{2\sigma}{\epsilon_0}\hat{k}$

Ans. **C**

Sol. $\vec{E}_P = \frac{\sigma}{2\epsilon_0}(-\hat{k}) + \frac{(-2\sigma)}{2\epsilon_0}(\hat{k}) + \frac{(-\sigma)}{2\epsilon_0}(\hat{k})$

$= \frac{-2\sigma}{\epsilon_0}\hat{k}$

62. A cylindrical conducting rod is kept with its axis along positive z-axis, where a uniform magnetic field exists parallel to z-axis. The current induced in the cylinder is

(A) zero (B) clockwise as seen from +z axis

(C) anti-clockwise as seen from +z axis (D) opposite to the direction of magnetic field.

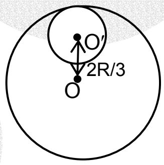
Ans. **A**

Sol. Since B is constant

$\therefore \frac{d\phi}{dt} = 0$

$\therefore i = 0$

63. A circular disc of radius R/3 is cut from a circular disc of radius R and mass 9 M as shown. Then moment of inertia of remaining disc about 'O' perpendicular to the plane of the disc is

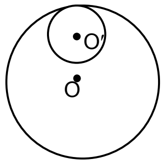


(A) 4 MR^2 (B) $9MR^2$

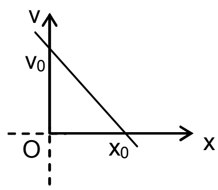
(C) $\frac{37}{9}MR^2$ (D) $\frac{40}{9}MR^2$

Ans. **A**

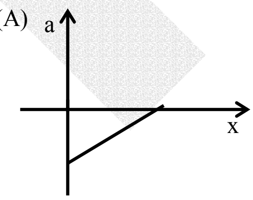
Sol. $I_0 = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] = 4MR^2$



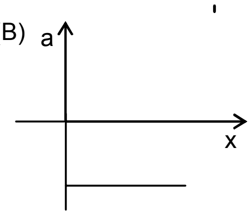
64. Depict the shown v – x graph in a– x graph.



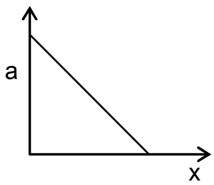
(A)



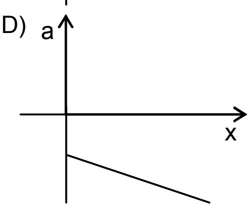
(B)



(C)

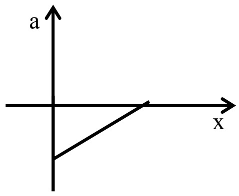


(D)



Ans. A

Sol. Equation of curve is



$$\frac{v}{v_0}+\frac{x}{x_0}=1$$

$$\therefore\ v=\left(1-\frac{x}{x_0}\right)v_0$$

$$\therefore\ a=\frac{dv}{dt}=-\frac{v_0}{x_0}(v)=-\frac{v_0^2}{x_0}\left(1-\frac{x}{x_0}\right)$$

Alternative: $a=-v\left(\frac{dv}{dx}\right)$; but dv/dx is negative and v is decreasing with the increase in x.

Hence 'a' should increase with increase of 'x'.

65. A particle is confined to rotate in a circular path with decreasing linear speed, then which of the following is correct?

(A) $\vec{L}$ (angular momentum) is conserved about the centre.
(B) only direction of angular momentum $\vec{L}$ is conserved.
(C) It spirals towards the centre.
(D) its acceleration is towards the centre.

Ans. B

66. The atomic number (Z) of an element whose k_α wavelength is λ is 11. The atomic number of an element whose k_α wavelength is 4λ is equal to

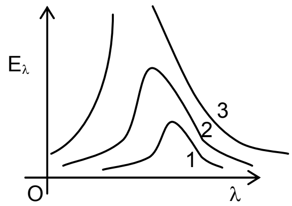
(A) 6 (B) 11
(C) 44 (D) 4

Ans. A

Sol. $(Z-1)^2\lambda$ = constant

$\therefore\ (10^2)\lambda = 4\lambda(Z-1)^2 \Rightarrow Z = 6$

67. The graph shown in the figure represents energy density E_λ versus λ for three sources sun, welding arc, tungsten filament. For λ_{max}, correct combination will be



(A) 1 – Tungsten, 2 – Welding arc, 3 – Sun
(B) 1 – Sun, 2 – Tungsten, 3 – Welding arc.
(C) 1 – Sun, 2 – Welding arc, 3 - Tungsten
(D) 1 – Welding arc, 2 – Sun, 3 - Tungsten

Ans. A

Sol. Temperature of sun would be maximum out of the given three

as $\lambda_m T$ = constant

λ_m for Sun is minimum

68. T_1 is the time period of simple pendulum. The point of suspension moves vertically upwards according to $y = kt^2$, where $k = 1\ m/s^2$. New time period is T_2, then $\frac{T_1^2}{T_2^2}$ = ? ($g = 10\ m/s^2$)

(A) 4/5 (B) 6/5
(C) 5/6 (D) 1

Ans. B

Sol. Acceleration of the point of suspension

$$a=\frac{d^2y}{dt^2}=2k=2\,m/s^2$$

$$T=2\pi\sqrt{\frac{L}{g_{eff}}}\Rightarrow T_1=2\pi\sqrt{\frac{L}{10}}\ \text{and}\ T_2=2\pi\sqrt{\frac{L}{12}}$$

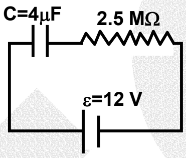
$$\therefore\ \frac{T_1^2}{T_2^2}=\frac{6}{5}$$

69. Which of the following does not have the same dimension?
(A) Electric flux, Electric field, Electric dipole moment
(B) Pressure, stress, Young's modulus
(C) Electromotive force, Potential difference, Electric voltage.
(D) Heat, Potential energy, Work done

Ans. **A**

70. A capacitor (C = 4.0 μF) is connected through a resistor (R = 2.5 MΩ) across a battery of negligible internal resistance of voltage 12 volts. The time after which the potential difference across the capacitor becomes three times to that of resistor is (ln 2 = 0.693)
(A) 13.86 sec. (B) 6.48 sec.
(C) 3.24 sec. (D) 20.52 sec.

Ans.. **A**



Sol. $q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow i = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$

$3V_R = V_C$

$\Rightarrow \varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = 3\varepsilon e^{-\frac{t}{RC}} \Rightarrow e^{-t/RC} = 1/4$

$t/RC = 2\ln 2 \therefore t = 20 \times (0.693) = 13.86$ sec

71. A photon of energy 10.2 eV collides inelastically with a Hydrogen atom in ground state. After a certain time interval of few micro seconds another photon of energy 15.0 eV collides inelastically with the same hydrogen atom, then the observation made by a suitable detector is
(A) 1 photon with energy 10.2 eV and an electron with energy 1.4 eV
(B) 2 photon with energy 10.2 eV
(C) 2 photon with energy 1.4 eV
(D) one photon with energy 3.4 eV and 1 electron with energy 1.4 eV

Ans. **A**
Sol. 10.2 eV photon on collision will excite H-atom to first excited state but Hydrogen atom will return to ground state before next collision. Second photon will provide ionization energy to Hydrogen atom, i.e., electron will be ejected with energy = 1.4 eV

72. In a resonance tube with tuning fork of frequency 512Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is
(A) 51.2 cm/s (B) 102.4 cm/s
(C) 204.8 cm/s (D) 153.6 cm/s

Ans. **C**

Sol. $\ell_1 + e = \frac{v}{4f}$ and $\ell_2 + e = \frac{3v}{4f}$

$\ell_2 - \ell_1 = \frac{2v}{4f}$

$\frac{\Delta(\ell_2 - \ell_1)}{(\ell_2 - \ell_1)} = \frac{\Delta v}{v}$

$\Delta v = 2f\Delta(\ell_2 - \ell_1) = 2f(\Delta\ell_1 + \Delta\ell_2)$ (For maximum error)

$= 2 \times 512 \times 0.2 = 204.8$ cm/s.

73. A thin concave and a thin convex lens are in contact. The ratio of the magnitude of power of two lenses is 2/3 and focal length of combination is 30cm, then the focal length of individual lenses are
(A) – 15 cm, 10 cm (B) – 75 cm, 50 cm
(C) 75 cm, - 50 cm (D) 75 cm, 50 cm

Ans. **A**

Sol. $\left|\frac{P_{concave}}{P_{convex}}\right| = \frac{2}{3}$

Nitin M Sir (physics-iitjee.blogspot.com)

$$\frac{1}{F} = \frac{1}{f_{concave}} + \frac{1}{f_{convex}}$$

$\frac{1}{30} = \frac{-2}{3f} + \frac{1}{f} = \frac{1}{3f} \Rightarrow f = 10$ cm, where f is focal length of convex lens

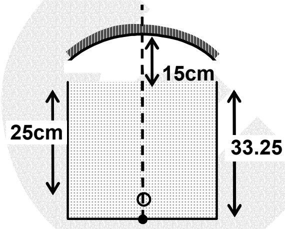
74. Which of the following process does not occur through convection

(A) Boiling of water
(B) Land breeze and Sea breeze
(C) Circulation of air around furnace
(D) Heating of glass bulb through filament

Ans. **D**

Sol. Heating of glass bulb is by radiation.

75. A tank of height 33.25 cm is completely filled with liquid (μ = 1.33). An object is placed at the bottom of tank on the axis of concave mirror as shown in the figure. Image of the object is formed 25 cm below the surface of the liquid, then focal length of the mirror is



(A) 10 cm
(B) 15 cm
(C) 20 cm
(D) 25 cm

Ans. **C**

Sol. After first refraction, position of the image = $\frac{33.25}{1.33} = 25$cm

From reflection, $\frac{1}{f} = \frac{1}{v} - \frac{1}{25+15} = \frac{1}{v} - \frac{1}{40}$

From second refraction position of the object = $\frac{25}{1.33}$

$$\frac{1}{f} = -\frac{1}{15+\frac{25}{1.33}} - \frac{1}{40} \Rightarrow f = -18.31 \text{ cm}$$

Hence magnitude of focal length of convex lens is 18.31 cm.
The nearest possible matching answer is 20 cm.

76. In YDSE, an electron beam is used to obtain interference pattern. If speed of electron is increased then

(A) no interference pattern will be observed.
(B) distance between two consecutive fringes will increase.
(C) distance between two consecutive fringes will decrease.
(D) distance between two consecutive fringes remains same.

Ans. **C**

Sol. $\lambda = \frac{h}{mv}$

v is increased, λ is decreased.
$\beta = \lambda D/d \Rightarrow \beta$ decreases.

77. A spherical body of area A and emissivity e = 0.6 is kept inside a perfectly black body. Total heat radiated by the body at temperature T

(A) $0.4\ e\ AT^4$
(B) $0.8eAT^4$
(C) $0.6\ eAT^4$
(D) $1.0eAT^4$

Ans. **D**

Sol. When a non black body is placed inside a hollow enclosure the total radiation from the body is the sum of what it would emit in the open (with e<1) and the part (1-a) of the incident radiation from the walls reflected by it. The two add up to a black body radiation . Hence the total radiation emitted by the body is $1.0\sigma AT^4$.
Probably in the examination paper 'σ' is misprinted as 'e'

78. An open organ pipe resonated with frequency 'f_1' and 2nd harmonic. Now one end is closed and the frequency is slowly increased then it resonates with frequency f_2 and nth harmonic then

(A) $n = 3,\ f_2 = \frac{3}{4} f_1$
(B) $n = 5,\ f_2 = \frac{3}{4} f_1$
(C) $n = 3,\ f_2 = \frac{5}{4} f_1$
(D) $n = 5,\ f_2 = \frac{5}{4} f_1$

Ans. **D**

Sol. $f_1 = \dfrac{1}{\ell}\sqrt{\dfrac{B}{\rho}}$

$$f_2 = \frac{n}{4\ell}\sqrt{\frac{B}{\rho}}$$

$$\frac{f_1}{f_2} = \frac{4}{n} \Rightarrow f_2 = \frac{n}{4}f_1$$

For the first resonance n = 5, $f_2 = \dfrac{5}{4}f_1$ (as frequency increases)

79. Temperature of a gas is 20^0C and pressure is changed from 1.01×10^5 Pa to 1.165×10^5 Pa . If volume is decreased isothermally by 10%. Bulk modulus of gas is

(A) 1.55×10^5 (B) 0.155×10^5
(C) 1.4×10^5 (D) 1.01×10^5

Ans. **A**

Sol. $B = -\Delta P/(\Delta V/V) = -\dfrac{(1.165-1.01)\times10^5}{0.1} = 1.55\times10^5$

80. A galvanometer with resistance 100Ω is converted to ammeter with a resistance of 0.1Ω. The galvanometer shows full scale deflection with a current of 100 μA. Then the minimum current in the circuit for full scale deflection of galvanometer will be

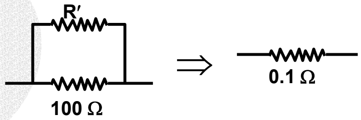
(A) 100.1mA (B) 10.01mA
(C) 1.001mA (D) 0.1001mA

Ans. **A**

Sol. $0.1 = \dfrac{100R'}{100+R'} \Rightarrow R' = \dfrac{100}{1001}$

$(100)\ (100\times10^{-6}) = R'(I - 100\times10^{-6})$

$\therefore$ I = 100.1 mA



81. One calorie is defined as the heat required to raise the temperature of 1 gm of water by 1^0C in a certain interval of temperature and at certain pressure. The temperature interval and pressure is

(A) 13.5^0C to 14.5^0C & 76 mm of Hg (B) 6.5^0C to 7.5^0C & 76 mm of Hg
(C) 14.5^0C to 15.5^0C & 760 mm of Hg (D) 98.5^0C to 99.5^0C & 760 mm of Hg

Ans. **C**

Sol. By definition.

82. If a star converts all of its Helium into oxygen nucleus, find the amount of energy released per nucleus of oxygen. He = 4.0026 amu, O = 15.9994 amu

(A) 7.26 MeV (B) 7 MeV
(C) 10.24 MeV (D) 5.12 MeV

Ans. **C**

Sol. $E = \Delta mc^2 = [4 \times 4.0026 - 15.9994] \times 931.5 = 10.24$ MeV

83. Two litre of water at initial temperature of 27^0C is heated by a heater of power 1 kW. If the lid of kettle is opened, then heat is lost at the constant rate of 160 J/s. Find the time required to raise the temperature of water to 77^0C with the lid open (Specific heat of water 4.2 kJ/kg)

(A) 5 min 40 sec (B) 14 min 20 sec
(C) 8 min 20 sec (D) 16 min 10 sec

Ans. **C**

Sol. Rate of heat gain = 1000 – 160 = 840 J/s

$\therefore$ Required time $= \dfrac{2\times4.2\times10^3\times(77-27)}{840}$ = 500 sec = 8 min 20 sec

Nitin M Sir (physics-iitjee.blogspot.com)

84. Ideal gas is contained in a thermally insulated and rigid container and it is heated through a resistance 100Ω by passing a current of 1A for five minutes, then change in internal energy of the gas is

(A) 0 kJ (B) 30 kJ
(C) 10 kJ (D) 20 kJ

Ans. **B**

Sol. $\Delta W = 0 \therefore \Delta Q = \Delta U$

$\Delta Q = \Delta U = I^2R\Delta t = (1)^2(100)(5 \times 60) = 30$ kJ

*Analyse your performance in Screening Test for evaluation of your preparation for Mains. A comprehensive analysis of your preparation on different topics would be couriered to you. Fill this sheet as per answers you have made in the IIT-JEE Screening Examination as per the sequencing provided in the solution booklet and send to nearest **FIITJEE**'s office immediately.*

FIITJEE Solutions to IITJEE–2005 Mains Paper

Physics

Time: 2 hours

Note: Question number 1 to 8 carries ***2 marks*** each, 9 to 16 carries ***4 marks*** each and 17 to 18 carries ***6 marks*** each.

Q1. A whistling train approaches a junction. An observer standing at junction observes the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed of sound = 300 m/s)

Sol. While approaching

$$f' = f_0\left(\frac{v}{v - v_s}\right)$$

$$2200 = f_0\left(\frac{300}{300 - v_s}\right)$$

While receding

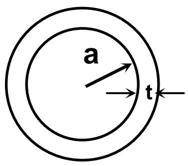
$$f'' = f_0\left(\frac{v}{v + v_s}\right)$$

$$1800 = f_0\left(\frac{300}{300 + v_s}\right)$$

On solving velocity of source (train) v_s = 30 m/s

Q2. A conducting liquid bubble of radius **a** and thickness **t** (t <<a) is charged to potential **V**. If the bubble collapses to a droplet, find the potential on the droplet.

Sol. Potential of the bubble $(V) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a}$



by conservation of volume

$$4\pi a^2 t = \frac{4}{3}\pi R^3$$

$$R = \left(3a^2 t\right)^{1/3}$$

Hence, potential on the droplet

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \text{ (as charge is conserved)}$$

$$\Rightarrow V' = \left(\frac{a}{3t}\right)^{1/3}.V$$

Q3. The potential energy of a particle of mass **m** is given by

$$V(x) = \begin{cases} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

λ_1 and λ_2 are the de–Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find λ_1/λ_2.

Sol. K.E. = $2E_0 - E_0 = E_0$ (for $0 \le x \le 1$)

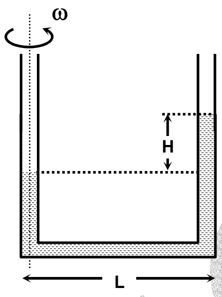
$$\lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

KE = $2E_0$ (for $x > 1$)

$$\lambda_2 = \frac{h}{\sqrt{4mE_0}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

Q4. A **U** tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L.

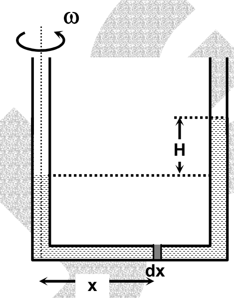


Sol.

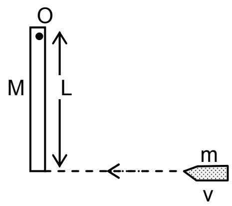
$$\Delta PA = \int_0^L dmx\omega^2$$

$$\rho gHA = \frac{\omega^2 L^2 \rho A}{2}$$

$$H = \frac{L^2\omega^2}{2g}$$



Q5. A wooden log of mass **M** and length **L** is hinged by a frictionless nail at O. A bullet of mass **m** strikes with velocity **v** and sticks to it. Find angular velocity of the system immediately after the collision about O.

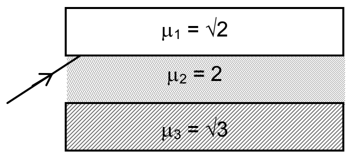


Sol. Apply conservation of angular momentum about O

$$(mv)L = (mL^2 + \frac{ML^2}{3})\omega$$

$$\omega = \frac{3mv}{(3m + M)L}$$

Q6. What will be the minimum angle of incidence such that the total internal reflection occurs on both the surfaces?



Sol. For first surface

$2 \sin c_1 = \sqrt{2} \sin 90°$

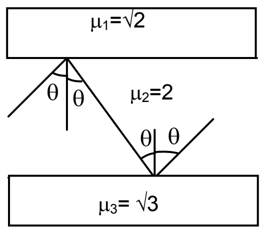
$\Rightarrow$ $c_1 = 45^0$

For second surface

$2 \sin c_2 = \sqrt{3} \sin 90°$

$\Rightarrow$ $c_2 = 60°$

$\therefore$ Minimum angle of incidence = Max $\{c_1, c_2\}$ = 60°



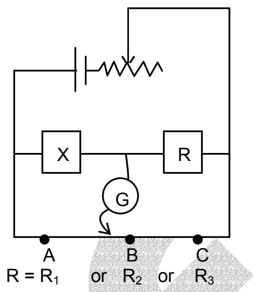
Q7. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures.

Sol. Least count of vernier callipers = $(1 - \frac{9}{10})$mm = 0.1mm

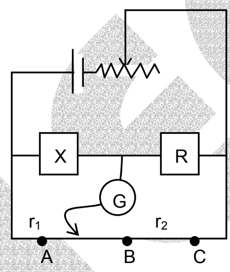
Side of the cube = 10 mm + 1 $\times$ 0.1 mm = 10.1 mm = 1.01 cm

Density = $\frac{2.736}{(1.01)^3} = 2.66 g/cm^3$

Q8. An unknown resistance **X** is to be determined using resistances **R_1, R_2** or **R_3**. Their corresponding null points are **A, B** and **C**. Find which of the above will give the most accurate reading and why?



Sol. $X = \frac{r_1 R}{r_2}$



$\left|\frac{\delta X}{X}\right| = \left|\frac{\delta r_1}{r_1}\right| + \left|\frac{\delta r_2}{r_2}\right|$

$|\delta r_1| = |\delta r_2| = \Delta$

$\left|\frac{\delta X}{X}\right| = \left(\frac{r_1 + r_2}{r_1 r_2}\right)\Delta$

For $\left|\frac{\delta X}{X}\right|$ to be minimum, $r_1 r_2$ should be maximum and as $r_1 + r_2$ is constant.

This is true for $r_1 = r_2$.

So R_2 gives most accurate value.

Q9. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform.

Sol. If amplitude of wave is A and angular frequency is ω,

$\frac{\omega A}{\omega^2 A} = \frac{3}{90} \Rightarrow \omega = 30$ rad/s

$v = \frac{\omega}{k} \Rightarrow k = \frac{3}{2} m^{-1}$

A = 10 cm

Considering sinusoidal harmonic function

$\therefore$ y = (10 cm) sin$(30t \pm \frac{3}{2}x + \phi)$

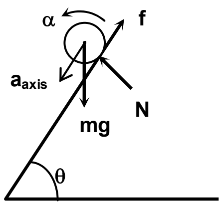
Q10. A cylinder of mass **m** and radius **R** rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder.

Sol. $mg\sin\theta - f = ma_{axis}$ (1)

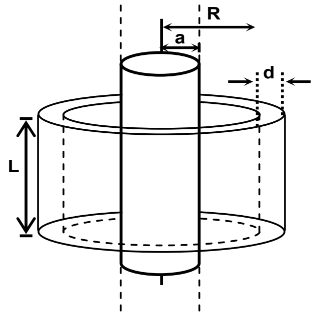
$fR = I_{axis}\,\alpha$ (2)

$a_{axis} = R\alpha$ (3)

$a_{axis} = \frac{2g\sin\theta}{3}$



Q11. A long solenoid of radius **a** and number of turns per unit length **n** is enclosed by cylindrical shell of radius **R**, thickness **d** (d<<R) and length **L**. A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.

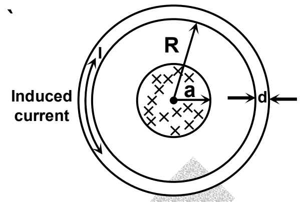


Sol. $\phi = (\mu_0 n i_0 \sin\omega t)\pi a^2$

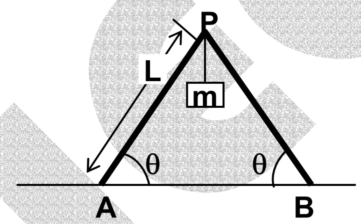
$$\varepsilon = \left|\frac{d\phi}{dt}\right| = (\mu_0 n i_0 \omega \cos\omega t)\pi a^2$$

Resistance = $\dfrac{\rho 2\pi R}{Ld}$

$$I = \frac{(\mu_0 n i_0 \omega \cos\omega t)\pi a^2 (Ld)}{\rho 2\pi R}$$



Q12. Two identical ladders, each of mass **M** and length **L** are resting on the rough horizontal surface as shown in the figure. A block of mass **m** hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.



Sol. For equilibrium of whole system,

$\Sigma F_y = 0$

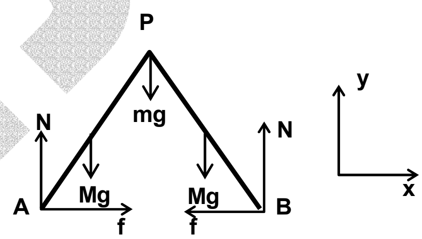
$$\Rightarrow \quad N = \left(\frac{2M+m}{2}\right)g$$

For rotational equilibrium of either ladder

Calculating torque about **P**

$$NL\cos\theta - Mg\frac{L}{2}\cos\theta - fL\sin\theta = 0$$

$$\Rightarrow \quad f = (M+m)g\frac{\cot\theta}{2}$$



Q13. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find

(a) atomic number of the nucleus.

(b) the frequency of K_α line of the X–ray produced. ($R = 1.1 \times 10^7\ m^{-1}$ and $c = 3 \times 10^8$ m/s)

Sol. $r = r_0 A^{1/3}$

$$\frac{r}{r_{He}} = \left(\frac{A}{4}\right)^{1/3} = 14^{1/3}$$

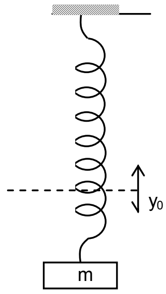
$\Rightarrow$ A = 56 and Z = (56 – 30) = 26

for K_α–line,

$$\sqrt{\nu} = \sqrt{\frac{3Rc}{4}}(Z-1)$$

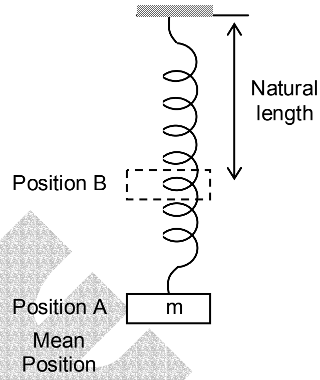
$\Rightarrow$ $\nu = 1.546 \times 10^{18}$ Hz

Q14. A small body attached to one end of a vertically hanging spring is performing SHM about it's mean position with angular frequency ω and amplitude **a**. If at a height **y*** from the mean position the body gets detached from the spring, calculate the value of y* so that the height **H** attained by the mass is maximum. The body does not interact with the spring during it's subsequent motion after detachment. ($a\omega^2 > g$)

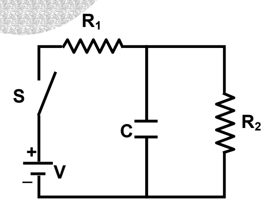


Sol. At position B as the potential energy of the spring will be zero, the total energy (Gravitational potential energy + Kinetic energy) of the block at this point will be maximum and therefore if the block gets detached at this point, it will rise to maximum height,

$$\therefore \quad y^* = \frac{mg}{k} = \frac{g}{\omega^2} < a$$



Q15. In the given circuit, the switch S is closed at time t = 0. The charge **Q** on the capacitor at any instant **t** is given by $Q(t) = Q_0\,(1-e^{-\alpha t})$. Find the value of $\mathbf{Q_0}$ and α in terms of given parameters shown in the circuit.

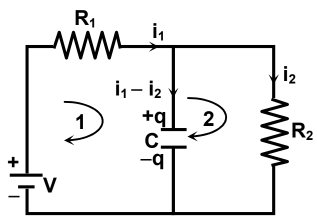


Sol. Applying KVL in loop 1 and 2,

$$V - i_1R_1 - \frac{q}{C} = 0 \qquad \ldots (1)$$

$$\frac{q}{C} - i_2R_2 = 0 \qquad \ldots (2)$$

$$i_1 - i_2 = \frac{dq}{dt} \qquad \ldots (3)$$



On solving we get,

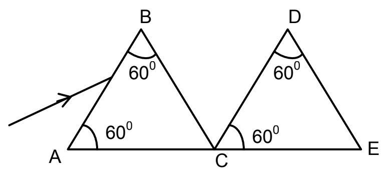
$$q = \frac{CVR_2}{R_1+R_2}\left(1-e^{-\frac{t(R_1+R_2)}{CR_1R_2}}\right)$$

$$\Rightarrow \quad Q_0 = \frac{CVR_2}{R_1+R_2} \quad \text{and} \quad \alpha = \frac{R_1+R_2}{CR_1R_2}$$

Q16. Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in the figure. A light ray strikes the first prism at face AB. Find,

(a) the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

(b) through what angle the prism DCE should be rotated about C so that the final emergent ray also has minimum deviation.

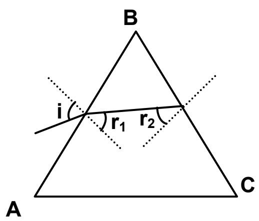


Sol. (a) For minimum deviation

$$r_1 = r_2 = \frac{\angle B}{2}$$

$$\frac{\sin i}{\sin 30^0} = \sqrt{3}$$

$$\Rightarrow \quad i = 60^0$$



(b) Prism DCE should be rotated about C in anticlockwise direction through 60° so that the final emergent ray is parallel to the incident ray and angle of deviation is zero (minimum)

Q17. A cylinder of mass 1 kg is given heat of 20000J at atmospheric pressure. If initially temperature of cylinder is 20^0C, find
(a) final temperature of the cylinder.
(b) work done by the cylinder.
(c) change in internal energy of the cylinder.
(Given that Specific heat of cylinder= 400 J kg^{-1} $^0C^{-1}$, Coefficient of volume expansion = 9×10^{-5} °C^{-1}, Atmospheric pressure = 10^5 N/m^2 and Density of cylinder = 9000 kg/m^3)

Sol. (a) $\Delta Q = ms\Delta T$

$$\Rightarrow \Delta T = \frac{20000J}{1kg \times (400J/kg\ ^0C)} = 50°C$$

$T_{final} = 70°C$

(b) $W = P_{atm}\,\Delta V = P_{atm}\,V_0\gamma\Delta T$

$$= (10^5\ N/m^2)\left(\frac{1}{9\times10^3}m^3\right)(9 \times 10^{-5}/°C)\ (50°C) = 0.05\ J$$

(c) $\Delta U = \Delta Q - W = 20000\ J - 0.05\ J = 19999.95\ J$

Q18. In a moving coil galvanometer, torque on the coil can be expressed as τ = ki, where **i** is current through the wire and **k** is constant. The rectangular coil of the galvanometer having numbers of turns **N**, area **A** and moment of inertia **I** is placed in magnetic field **B**. Find
(a) k in terms of given parameters N, I, A and B.
(b) the torsional constant of the spring, if a current $\mathbf{i_0}$ produces a deflection of $\pi/2$ in the coil.
(c) the maximum angle through which coil is deflected, if charge **Q** is passed through the coil almost instantaneously. (Ignore the damping in mechanical oscillations)

Sol. (a) $\tau = iNAB\sin\alpha$

For a moving coil galvanometer $\alpha = 90°$

$ki = iNAB \quad \Rightarrow k = NAB$

(b) $\tau = C\theta$

$$i_0NAB = C\pi/2 \quad \Rightarrow C = \frac{2i_0NAB}{\pi}$$

(c) Angular impulse = $\int\tau.\,dt = \int NABidt = NABQ$

$\Rightarrow NABQ = I\omega_0$

$$\Rightarrow \omega_0 = \frac{NABQ}{I}$$

Using energy of conservation

$$\frac{1}{2}I\omega_0^2 = \frac{1}{2}C\theta_{max}^2$$

$$\Rightarrow \theta_{max} = \omega_0\sqrt{\frac{I}{C}} = Q\sqrt{\frac{NAB\pi}{2Ii_0}}$$

FIITJEE Solutions to IITJEE–2006

Physics

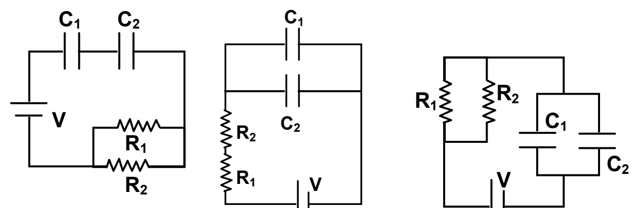
Time: 2 hours

Note: *The marking Scheme is (+3, –1) for question numbers 1 to 12 , (+5, –1) for question numbers 13 to 20, (+5, –2) for question numbers 21 to 32 and (+6, 0) for question numbers 33 to 40.*

1 Given,

$R_1 = 1\Omega$ $\quad$ $C_1 = 2\mu F$

$R_2 = 2\Omega$ $\quad$ $C_2 = 4\mu F$



(I) (II) (III)

The time constants (in μS) for the circuits I, II, III are respectively

(A) 18, 8/9, 4 (B) 18, 4, 8/9

(C) 4, 8/9, 18 (D) 8/9, 18, 4

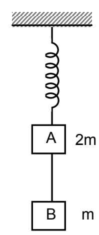
Sol. ***(D)***

$\tau_1 = 8/9\ \mu S$

$\tau_2 = 18\ \mu S$

$\tau_3 = 4\ \mu S$

2. Two blocks A and B of masses 2m and m, respectively, are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively



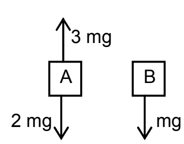
(A) g, g/2 (B) g/2, g

(C) g , g (D) g/2, g/2

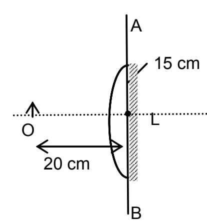
Sol. ***(B)***

$a_A = g/2$

$a_B = g$



3. A point object is placed at a distance of 20 cm from a thin plano-convex lens of focal length 15 cm, if the plane surface is silvered. The image will form at



(A) 60 cm left of AB (B) 30 cm left of AB

(C) 12 cm left of AB (D) 60 cm right of AB

Sol. ***(C)***

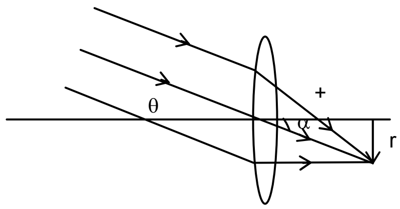
$$-\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{\infty} \Rightarrow F = -\frac{15}{2}$$

$$-\frac{2}{15} = \frac{1}{v} - \frac{1}{20}$$

$\Rightarrow v = -12$ cm i.e.12 cm left of AB

4. A biconvex lens of focal length f forms a circular image of sun of radius r in focal plane. Then
(A) $\pi r^2 \propto f$ (B) $\pi r^2 \propto f^2$
(C) if lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
(D) if f is doubled, intensity will increase

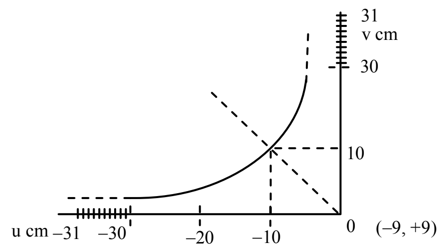
Sol. ***(B)***
$r = f\ \tan\alpha$
Hence, $\pi r^2 \propto f^2$



5. Given a sample of Radium-226 having half-life of 4 days. Find the probability, a nucleus disintegrates after 2 half lives.
(A) 1 (B) 1/2
(C) 1.5 (D) 3/4

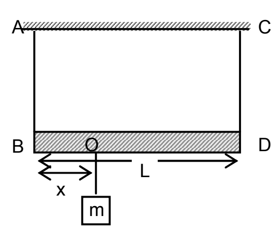
Sol. ***(B)***
Disintegration of each nuclei is independent of any factor. Hence, each nuclei has same chance of disintegration.

6. Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is
(A) 0.50 ± 0.05 cm
(B) 0.50 ± 0.10 cm
(C) 5.00 ± 0.05 cm
(D) 5.00 ± 0.10 cm



Sol. ***(D)***
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow f = 5 \text{ cm}$$
$$f = \frac{uv}{u+v}$$
$$\frac{\Delta f}{f} = \left|\frac{\Delta u}{u}\right| + \left|\frac{\Delta v}{v}\right| + \frac{|\Delta u| + |\Delta v|}{|u| + |v|}$$
$\Delta f = 0.15$ (for $f = 5$ cm)
The most appropriate answer is 5.00 ± 0.10 cm

7. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length of BO is
(A) L/5 (B) 4L/5
(C) 3L/4 (D) L/4

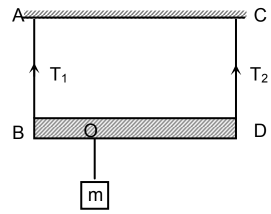


Sol. ***(A)***

$$\frac{1}{2\ell}\sqrt{\frac{T_1}{\mu}} = \frac{1}{\ell}\sqrt{\frac{T_2}{\mu}}$$

$T_2 = T_1/4$

For rotational equilibrium, $T_1 x = T_2(L - x) \Rightarrow x = L/5$



8. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then

(A) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$ (B) $T_A > T_B$ (if $r_A > r_B$)

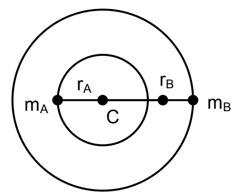
(C) $T_A > T_B$ (if $m_A > m_B$) (D) $T_A = T_B$ [+3, -1]

Sol. ***(D)***

$$\frac{Gm_A m_B}{(r_A + r_B)^2} = \frac{m_A r_A 4\pi^2}{T_A^2} = \frac{m_B r_B 4\pi^2}{T_B^2}$$

$\Rightarrow m_A r_A = m_B r_B$

$\therefore T_A = T_B$



9. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(A) $\frac{2R}{\sqrt{15}}$ (B) $R\sqrt{\frac{2}{15}}$

(C) $\frac{4R}{\sqrt{15}}$ (D) $\frac{R}{4}$ [+3, –1]

Sol. ***(A)***

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$

$$r = \frac{2R}{\sqrt{15}}$$

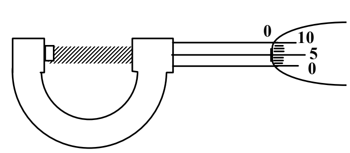
10. A student performs an experiment for determination of $g\left(= \frac{4\pi^2\ell}{T^2}\right)$, $\ell \approx 1$m, and he commits an error of $\Delta\ell$.

For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1sec. For which of the following data, the measurement of g will be most accurate?

	$\Delta\ell$	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 sec	10	5 mm
(B)	5 mm	0.2 sec	20	5 mm
(C)	5 mm	0.1 sec	20	1 mm
(D)	1 mm	0.1 sec	50	1 mm

Sol. ***(D)***

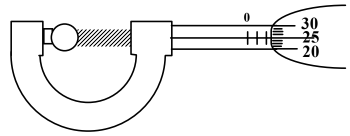
11. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is



(A) 2.25 mm (B) 2.20 mm

(C) 1.20 mm (D) 1.25 mm

Sol. ***(C)***



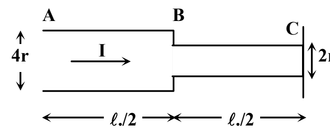
Zero error = $5 \times \frac{0.5}{50} = 0.05$ mm

Actual measurement

$= 2 \times 0.5$ mm $+ 25 \times \frac{0.5}{50} - 0.05$ mm

= 1 mm + 0.25 mm – 0.05 mm = 1.20 mm

12 Consider a cylindrical element as shown in the figure. Current flowing the through element is I and resistivity of material of the cylinder is ρ. Choose the correct option out the following.



(A) Power loss in first half is four times the power loss in second half.

(B) Voltage drop in first half is twice of voltage drop in second half.

(C) Current density in both halves are equal.

(D) Electric field in both halves is equal.

Sol. ***(B)***

$$\frac{R_1}{R_2} = \frac{A_1}{A_2} = \frac{4}{1}$$

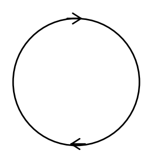
$$\frac{P_1}{P_2} = \frac{I^2R_1}{I^2R_2} = \frac{4}{1}$$

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{4}{1}$$

$$\frac{J_1}{J_2} = \frac{1}{4}$$

More than One Choice may be correct .(+5, –1)

13. In the given diagram, a line of force of a particular force field is shown. Out of the following options, it can never represent

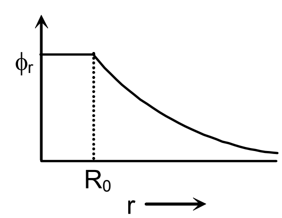


(A) an electrostatic field (B) a magnetostatic field

(C) a gravitational field of a mass at rest (D) an induced electric field

Sol. ***(A), (C)***

14. The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in the adjacent figure, and given as



$$\phi_r = \frac{q}{4\pi \in_0 r} \qquad (r \geq R_0)$$

$$\phi_r = \frac{q}{4\pi \in_0 R_0} \qquad (r \leq R_0)$$

Which of the following option(s) is/are correct?

(A) For spherical region $r \leq R_0$, total electrostatic energy stored is zero.

(B) Within $r = 2R_0$, total charge is q.

(C) There will be no charge anywhere except at $r = R_0$.

(D) Electric field is discontinuous at $r = R_0$.

Sol. ***(A), (B), (C), (D)***

The potential shown is for charged spherical conductor.

15. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and incline is μ. Then

(A) frictional force is always $\mu mg \cos\theta$
(B) friction is a dissipative force
(C) by decreasing θ, frictional force decreases
(D) friction opposes translation and supports rotation.

Sol. ***(C), (D)***

16. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM

(A) for any value of A, B and C (except C = 0)
(B) if A = –B; C = 2B, amplitude = $|B\sqrt{2}|$
(C) if A = B; C = 0
(D) if A = B; C = 2B, amplitude = $|B|$

Sol. ***(A), (B), (D)***

$$x = \frac{A}{2}(1-\cos 2\omega t) + \frac{B}{2}(1+\cos 2\omega t) + \frac{C}{2}\sin 2\omega t$$

For A = 0, B = 0

$$x = \frac{C}{2}\sin 2\omega t$$

For A = –B and C = 2B

$x = B\cos 2\omega t + B\sin 2\omega t$

Amplitude = $|B\sqrt{2}|$

For A = B; C = 0

x = A,

Hence this is not correct option

For A = B, C = 2B

$x = B + B\sin 2\omega t$

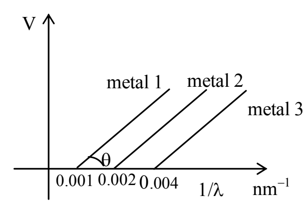
It is also represents SHM.

17. In a dark room with ambient temperature T_0, a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room. Assuming that there is no change in the ambient temperature of the room, which of the following statement(s) is/are correct?

(A) The quantity of radiation absorbed by the black body in unit time will increase.
(B) Since emissivity = absorptivity, hence the quantity of radiation emitted by black body in unit time will increase.
(C) Black body radiates more energy in unit time in the visible spectrum.
(D) The reflected energy in unit time by the black body remains same.

Sol. ***(A), (B), (C), (D)***

18. The graph between $1/\lambda$ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photo-electric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct? [Here λ is the wavelength of the incident ray].



(A) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(B) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(C) $\tan\theta$ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light.
(D) The violet colour light can eject photoelectrons from metals 2 and 3.

Sol. ***(A), (C)***

$$\frac{hc}{\lambda} - \phi = eV$$

$$V = \frac{hc}{e\lambda} - \frac{\phi}{c}$$

For plate 1: $\frac{\phi_1}{hc} = 0.001$ plate 2: $\frac{\phi_2}{hc} = 0.002$ plate 3: $\frac{\phi_3}{hc} = 0.004$

$\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$

For plate 2, threshold wavelength

$$\lambda = \frac{hc}{\phi_2} = \frac{hc}{0.002\,hc} = \frac{1000}{2} = 500 \text{ nm}$$

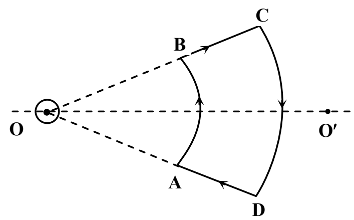
For plate 3, threshold wavelength

$$\lambda = \frac{hc}{\phi_3} = \frac{hc}{0.004hc} = \frac{1000}{4} = 250\text{nm}$$

Since violet colour light λ is 400 nm, so $\lambda_{violet} < \lambda_{threshold}$ for plate 2

So, violet colour light will eject photo-electrons from plate 2 and not from plate 3.

19. An infinite current carrying wire passes through point O and in perpendicular to the plane containing a current carrying loop ABCD as shown in the figure. Choose the correct option (s).



(A) Net force on the loop is zero.

(B) Net torque on the loop is zero.

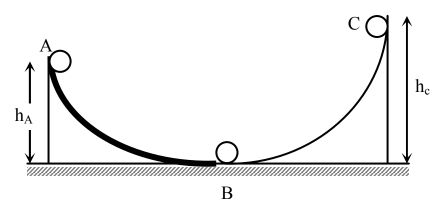
(C) As seen from O, the loop rotates clockwise.

(D) As seen from O, the loop rotates anticlockwise

Sol. ***(A), (C)***

Magnetic force on wire BC would be perpendicular to the plane of the loop along the outward direction and on wire DA the magnetic force would be along the inward normal, so net force on the wire loop is zero and torque on the loop would be along the clockwise sense as seen from O.

20. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then



(A) $h_A > h_C$; $K_B > K_C$

(B) $h_A > h_C$; $K_C > K_A$

(C) $h_A = h_C$; $K_B = K_C$

(D) $h_A < h_C$; $K_B > K_C$

Sol. ***(A), (B), (D)***

$E_A = mgh_A + K_A$

$E_B = K_B$

$E_c = mgh_C + K_C$

Using conservation of energy

$E_A = E_B = E_C$

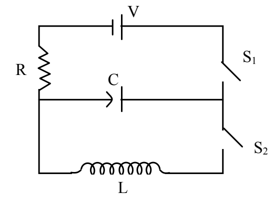
$K_B > K_C$

$K_B > K_A$

$Mg(h_A - h_C) + (K_A - K_C) = 0$

$$\Rightarrow h_A - h_C = \frac{K_C - K_A}{Mg}$$

***Comprehension –I**



The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor 'L' by closing switch S_2 and opening S_1.

21. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then

(A) after time interval τ, charge on the capacitor is CV/2

(B) after time interval 2τ, charge on the capacitor is $CV(1-e^{-2})$

(C) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged.

(D) after time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$

Sol. ***(B)***

$Q = Q_0(1 - e^{-t/\tau})$

$Q = CV(1 - e^{-t/\tau})$ after time interval 2τ.

22. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then,

(A) at t = 0, energy stored in the circuit is purely in the form of magnetic energy

(B) at any time t > 0, current in the circuit is in the same direction

(C) at t > 0, there is no exchange of energy between the inductor and capacitor

(D) at any time t > 0, instantaneous current in the circuit may $V\sqrt{\frac{C}{L}}$

Sol. ***(D)***

$q = Q_0 \cos \omega t$

$i = -\frac{dq}{dt} = Q_0\omega \sin \omega t$

$\Rightarrow i_{kax} = C\omega V = V\sqrt{\frac{C}{L}}$

23. If the total charge stored in the LC circuit is Q_0, then for $t \geq 0$

(A) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(B) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

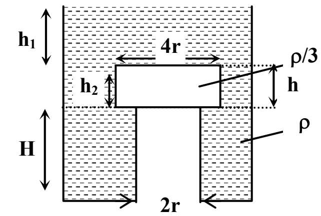
(C) the charge on the capacitor is $Q = -LC\frac{d^2Q}{dt^2}$

(D) the charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

Sol. ***(C)***

Comprehension-II

A wooden cylinder of diameter 4r, height h and density ρ/3 is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



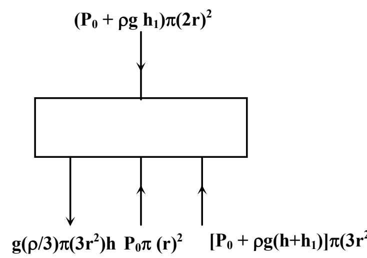
24. If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is

(A) 2h/3 (B) 5h/4
(C) 5h/3 (D) 5h/2

Sol. ***(C)***

$$[P_0 + \rho g h_1]\pi(4r^2) + \frac{\rho}{3}\pi 4r^2 hg = [P_0 + \rho g(h_1 + h_2)]\pi(3r^2) + P_0\pi r^2$$

$h_1 = 5h/3$



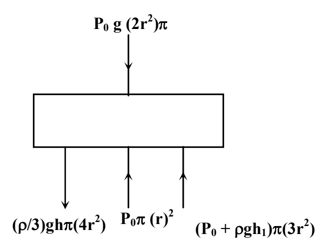
25. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is

(A) h/3 (B) 4h/9
(C) 2h/3 (D) h

Sol. ***(B)***

$$P_0\pi(4r^2) + \frac{\rho}{3}\pi 4r^2 hg = (P_0 + \rho g h_2)\pi(3r^2) + P_0\pi r^2$$

$h_1 = 4h/9$



26. If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position.
(B) for h_2 = h/3, cylinder again starts moving up
(C) for h_2 = h/4, cylinder again starts moving up
(D) for h_2 = h/5 cylinder again starts moving up

Sol. ***(A)***

For $h_2 < 4h/9$ cylinder does not moves up

27. Two waves $y_1 = A\cos(0.5\,\pi x - 100\,\pi t)$ and $y_2 = A\cos(0.46\,\pi x - 92\,\pi t)$ are travelling in a pipe placed along x-axis. Find the number of times intensity is maximum in time interval of 1 sec.

(A) 4 (B) 6
(C) 8 (D) 10

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. ***(A)***

$|f_1 - f_2| = 4\ s^{-1}$

28. Find wave velocity of louder sound

(A) 100 m/s (B) 192 m/s

(C) 200 m/s (D) 96 m/s

Sol. ***(C)***

$v_1 = v_2 = 200$ m/s

29. Find the number of times $y_1 + y_2 = 0$ at x = 0 in 1 sec

(A) 100 (B) 46

(C) 192 (D) 96

Sol. ***(D)***

$y_1 + y_2 = A\cos 100\pi t + A\cos 92\pi\, t = 0$

$\cos 100\pi t = -\cos 92\pi t$

$100\pi t = (2n + 1)\pi - 92\pi t$

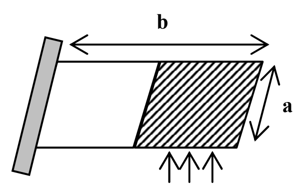
$t = \dfrac{(2n+1)}{192}$

$\Delta t = t_{n+1} - t_n = \dfrac{2}{192}$

Questions 30-32 could not be retrieved due to large length of comprehension.

33. There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?

It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1m; g = 10 m/s^2.



Sol. Torque about hinge side

$a \times \frac{b}{2} n(2\ mv) \times \frac{3b}{4} = Mg\frac{b}{2}$

$v = \frac{2}{3}\frac{Mg}{abnm} = \frac{2}{3} \times \frac{M \times 10}{2 \times 100 \times 0.01} = 10$ m/s

34. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Then, find the final temperature of the mixture.

Given, L_{fusion} = 80 cal/g = 336 J/g, $L_{vaporization}$ = 540 cal/g = 2268 J/g,

S_{ice} = 2100 J/kg K = 0.5 cal/gK and S_{water} = 4200 J/kg K = 1 cal /gK

Sol. $\sum \Delta Q = 0$

Heat lost by steam to convert into 0°C water

$H_L = 0.05 \times 540 + 0.05 \times 10 \times 1$

$= 27 + 5 = 32$ kcal

Heat required by ice to change into 0°C water

$H_g = 0.45 \times \frac{1}{2} \times 20 + 0.45 \times 80 = 4.5 + 36.00 = 40.5$ kcal

Thus, final temperature of mixture is 0°C.

35. In hydrogen-like atom (z = 11), n^{th} line of Lyman series has wavelength λ equal to the de-Broglie's wavelength of electron in the level from which it originated. What is the value of n?

Sol.

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R(11)^2\left(\frac{1}{1} - \frac{1}{n^2}\right)$$

$$\lambda = \frac{h}{\rho} = \frac{h}{mv}$$

$$\lambda = \frac{hr}{mvr} = \frac{rh2\pi}{nh} = \frac{2\pi r}{n}.$$

$$\lambda = \frac{2\pi r}{n} = \frac{\pi(0.529\times10^{-10})n^2}{(n)(11)}$$

$$\therefore \frac{1}{\lambda} = \frac{11}{2\pi(0.529\times10^{-10})n} = \frac{11}{(2\pi)(0.529\times10^{-10})n} = 1.1\times10^7(11)^2\left(1 - \frac{1}{n^2}\right)$$

$$= \frac{1}{(2\pi)(0.529\times10^{-10})(1.1\times10^2)(11)} = n - \frac{1}{n}$$

$$n - \frac{1}{n} = 25$$

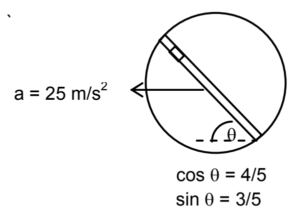
$$n^2 - 1 = 25\,n$$

$$n^2 - 25\,n - 1 = 0$$

$$n = 25$$

Hence answer = 24

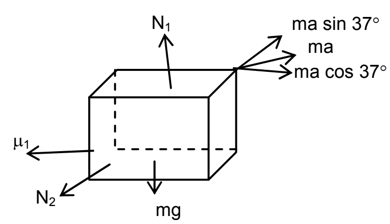
36. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of 25 m/s^2. Find the acceleration of the block with respect to disc.



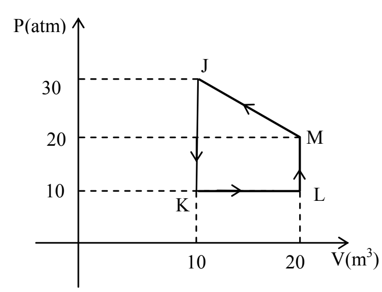
Sol. $N_1 = mg$

$N_2 = m\,a\,\sin 37°$

$$a_{bd} = \frac{ma\cos37° - \mu N_2 - \mu N_1}{m} = 10\,m/s^2.$$



37. Heat given to process is positive, match the following option of column I with the corresponding option of column II



Column I	**Column II**
(A) JK	(P) $\Delta W > 0$
(B) KL	(Q) $\Delta Q < 0$
(C) LM	(R) $\Delta W < 0$
(D) MJ	(S) $\Delta Q > 0$

Sol. ***(A)→(Q), (B)→(P), (S), (C)→(S), (D)→(Q), (R)***

38. **Match the following Columns**

Column I	Column II
(A) Nuclear fusion	(P) Converts some matter into energy
(B) Nuclear fission	(Q) Generally possible for nuclei with low atomic number
(C) β-decay	(R) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(S) Essentially proceeds by weak nuclear forces

Sol. ***(A)→(P), (Q), (B)→(P), (R), (C)→(S), (P), (D)→(P), (Q), (R)***

39 **Match the following Columns**

Column I	Column II
(A) Dielectric ring uniformly charged	(P) Time independent electrostatic field out of system
(B) Dielectric ring uniformly charged rotating with angular velocity ω	(Q) Magnetic field
(C) Constant current in ring i_0	(R) Induced electric field
(D) $i = i_0 \cos \omega t$	(S) Magnetic moment

Sol. ***(A)→(P), (B)→(Q), (S), (C)→(Q), (S), (D)→(Q), (R), (S)***

40. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_0, respectively. Then

Column I	Column II
(A) Intensity of light received by lens	(P) Radius of aperture (R)
(B) Angular magnification	(Q) Dispersion of lens
(C) Length of telescope	(R) focal length f_0, f_e
(D) Sharpness of image	(S) spherical aberration

Sol. ***(A)→(P), (B)→(R), (C)→(R), (D)→ (P), (Q), (S)***

FIITJEE Solutions to

IITJEE-2007

(Paper-I, Code-7)

Time: 3 hours ***M. Marks: 243***

Note: ***(i)*** The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.

(ii) **Section I** contains 9 multiple choice questions which have only one correct answer. Each question carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iii) **Section II** contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iv) **Section III** contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries ***+4 marks*** for correct answer and ***– 1 mark*** for wrong answer.

(v) **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries ***+6 marks*** and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is no negative marking.

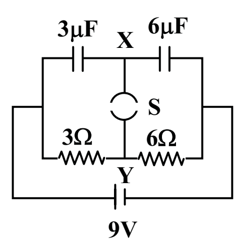
PART- I (PHYSICS)

SECTION – I

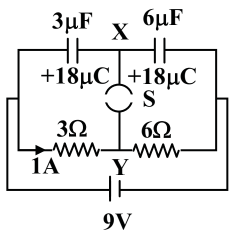
Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

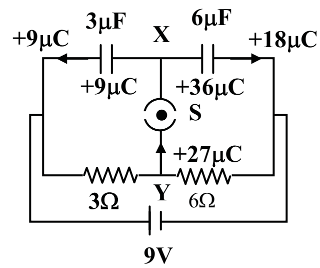
1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed the total amount of charge that flows from Y to X is
(A) 0 (B) 54μC
(C) 27 μC (D) 81 μC



Sol. **(C)**
27 μC



Initial charge distribution (when switch S is open)



Final charge distribution (when switch S is closed)

2. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.
(A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

Sol. **(A)**
$dV = -\vec{E}\cdot d\vec{r}$
and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$
where r is distance from the axis of cylindrical charge distribution (r is equal to or greater than radius of cylindrical charge distribution).

3. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is
(A) $E\left({}^{236}_{92}U\right) > E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(B) $E\left({}^{236}_{92}U\right) < E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(C) $E\left({}^{236}_{92}U\right) < E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$
(D) $E\left({}^{236}_{92}U\right) = E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$

3. **(A)**
Rest mass energy of U will be greater than the rest mass energy of the nucleus in which it breaks (as conservation of momentum is always followed)

4. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then,
(A) $x < f$
(B) $f < x < 2f$
(C) $x = 2f$
(D) $x > 2f$

Sol. **(B)**
Due to parallax

5. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is
(A) 802 nm
(B) 823 nm
(C) 1882 nm
(D) 1648 nm

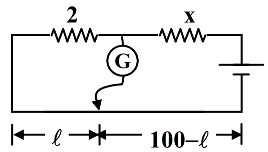
Sol. **(B)**
Transition from ∞ to n = 3 will produce smallest wavelength in infrared region.

6. A resistance of 2 Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2Ω, is connected across the other gap. When these resistance are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is
(A) 3 Ω
(B) 4 Ω
(C) 5 Ω
(D) 6 Ω

Sol. **(A)**

$$\frac{2}{x} = \frac{\ell}{100-\ell} \quad \ldots(i)$$

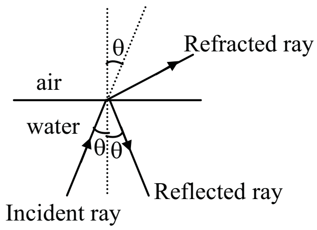
$$\frac{x}{2} = \frac{\ell+20}{80-\ell} \quad \ldots(ii)$$



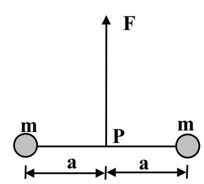
Solving (i) and (ii) x = 3Ω

7. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be
(A) only a reflected ray and no refracted ray
(B) only a refracted ray and no reflected ray
(C) a reflected ray and a refracted ray and the angle between them would be less than 180° – 2θ
(D) a reflected ray and a refracted ray and the angle between them would be greater than 180° – 2θ

Sol. **(C)**



8. Two particle of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the center P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes 2x is



(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

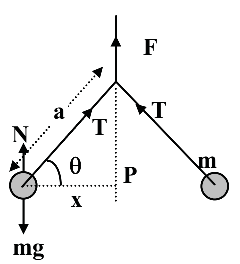
Sol. **(B)**

2T sin θ = F

T cos θ = mA

$$2\tan\theta = \frac{F}{mA}$$

$$A = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$



9. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,
(A) negative and distributed uniformly over the surface of the sphere
(B) negative and appears only at the point on the sphere closest to the point charge
(C) negative and distributed non-uniformly over the entire surface of the sphere
(D) zero

Sol. **(D)**

SECTION – II

Assertion - Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

10. **STATEMENT-1**
The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

because

STATEMENT-2
Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
(A) Statement-1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(C)**

11. **STATEMENT-1**
If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

because

STATEMENT -2
When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.
(A) Statement-1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

12. **STATEMENT-1**
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30^0 with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

because

STATEMENT-2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.
(A) Statement -1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(C)**

13. **STATEMENT-1**
In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

because

STATEMENT-2
In an elastic collision, the linear momentum of the system is conserved.
(A) Statement -1 is True, Statement-2 is True; Statement -2 is a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

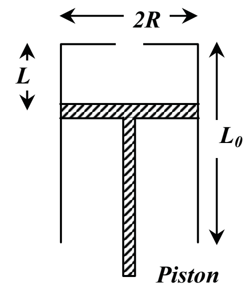
SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs $P_{14\text{-}16}$ and $P_{17\text{-}19}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

P_{14-16} : Paragraph for Question Nos. 14 to 16

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



14. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

(A) P_0 (B) $\frac{P_0}{2}$

(C) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

Sol. **(A)**

15. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0+Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2-Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2+Mg}{\pi R^2 P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0-Mg}\right)(2L)$

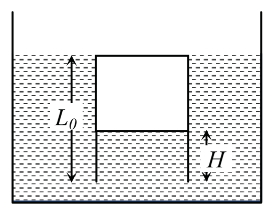
Sol. **(D)**

$Mg + P(\pi R^2) = P_0\pi R^2$

$P_0(2L\pi R^2) = P(x\pi R^2)$ ($P_1 \cdot V_1 = P_2V_2$ for isothermal process)

$x = \left(\frac{P_0\pi R^2}{\pi R^2 P_0-Mg}\right)(2L)$

16. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g(L_0-H)^2 + P_0(L_0 - H) + L_0P_0 = 0$
(B) $\rho g(L_0-H)^2 - P_0(L_0 - H) - L_0P_0 = 0$
(C) $\rho g(L_0-H)^2 + P_0(L_0 - H) - L_0P_0 = 0$
(D) $\rho g(L_0-H)^2 - P_0(L_0 - H) + L_0P_0 = 0$

Sol. **(C)**

$\pi R^2P_0L_0 = P(L_0 - H)\pi R^2$ . . . (i)

$P = P_0 + \rho g(L_0 - H)$ . . . (ii)

Solving (i) & (ii), we get the answer.

P_{17-19} : Paragraph for Question Nos. 17 to 19

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

17. The ratio of x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Sol. **(C)**

$\frac{1}{2}kx_1^2=\frac{1}{2}I(2\omega)^2$

$$\frac{1}{2}kx_2^2 = \frac{1}{2}(2I)(\omega)^2$$

$$\frac{x_1}{x_2} = \sqrt{2}$$

18. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$

(C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

Sol. **(A)**

Applying conservation of angular momentum

$$\omega' = \frac{I(2\omega) + 2I(\omega)}{3I} = \frac{4\omega}{3} \qquad \text{. . . (i)}$$

$$\omega' = \omega + \frac{\tau}{2I}t \qquad \text{. . . (ii)}$$

From (1) & (ii), $\tau = \frac{2I\omega}{3t}$

19. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$

(C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

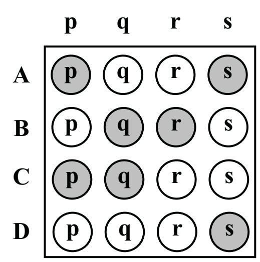
Sol. **(B)**

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct match are A-p, A-s, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



20. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II.** Match the physical quantities in **Column I** with the units in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) GM_eM_s $G \rightarrow$ universal gravitational constant, $M_e \rightarrow$ mass of the earth, $M_s \rightarrow$ mass of the Sun	(p) (volt) (coulomb) (metre)

(B) $\frac{3RT}{M}$; R → universal gas constant, T → absolute temperature, M → molar mass	(q) (kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$; F → force, q → charge, B → magnetic field	(r) $(\text{meter})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$, G → universal gravitational constant, M_e → mass of the earth, R_e → radius of the earth	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

Sol. A →(p) & (q), B →(r) & (s), C →(r) & (s), D →(r) & (s)

21. Some laws/processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

Sol. A →(p) & (r), B →(q) & (s), C →(p), D →(q)

22. **(C)olumn I** gives certain situations in which a straight metallic wire of resistance R is used and **Column II** gives some resulting effects. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) A charged capacitor is connected to the ends of the wire	(p) A constant current flows through the wire
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q) Thermal energy is generated in the wire
(C) The wire is placed in a constant electric field that has a direction along the length of the wire.	(r) A constant potential difference develops between the ends of the wire
(D) A battery of constant emf is connected to the ends of the wire	(s) Charges of constant magnitude appear at the ends of the wire

Sol. A →(q), B →(r) & (s), C →(r) & (s), D →(p), (q) & (r)

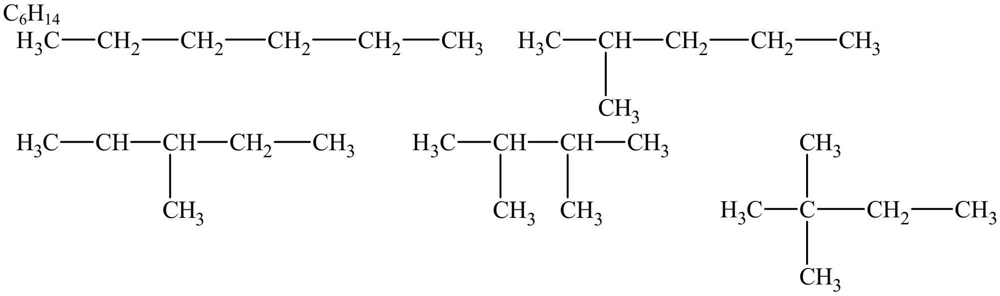
PART- II (CHEMISTRY)

SECTION – I

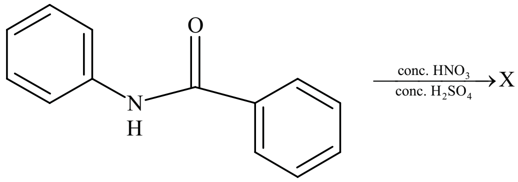
Straight Objective Type

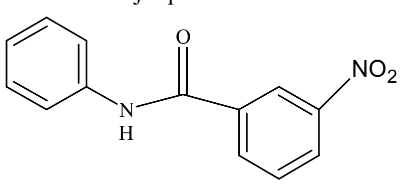
*This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

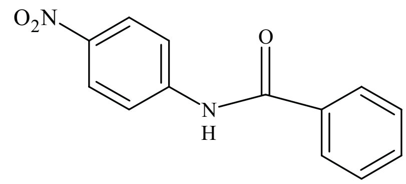
23. The number of structural isomers for C_6H_{14} is
(A) 3 (B) 4
(C) 5 (D) 6

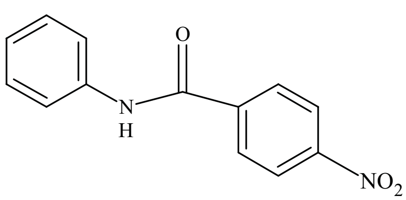
Sol. **(C)**
C_6H_{14}


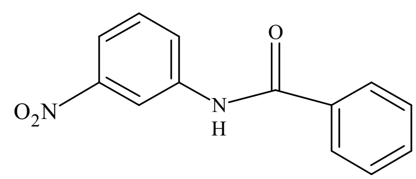
Hence (C) is correct.

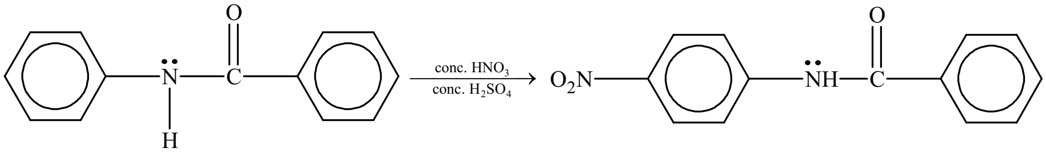
24. In the following reaction,


the structure of the major product **'X'** is
(A)


(B)


(C)


(D)


Sol. **(B)**


Due to presence of lone pair of electron on nitrogen atom, it will activate the ring and it will stabilize intermediate cation at o and p positions.

Hence (B) is correct.

25. When 20 g of naphthoic acid ($C_{11}H_8O_2$) is dissolved in 50 g of benzene (K_f = 1.72 K kg mol^{-1}), a freezing point depression of 2 K is observed. The van't Hoff factor (i) is

(A) 0.5 (B) 1
(C) 2 (D) 3

Sol. **(A)**

$\Delta T_f = K_f \times \text{molality} \times i$

$2 = 1.72 \times \frac{20}{172} \times \frac{1000}{50} \times i$

$i = 0.5$

Hence (A) is correct.

26. Among the following, the paramagnetic compound is

(A) Na_2O_2 (B) O_3
(C) N_2O (D) KO_2

Sol. **(D)**

$O_2^{2-} = \sigma 1s^2\, \sigma^* 1s^2, \sigma 2s^2\, \sigma^* 2s^2,\, \sigma 2p_z^2,\, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$

Number of unpaired electrons = 0.

$N = N \longrightarrow O$ Number of unpaired electrons = 0

$O{=}\overset{\oplus}{O}{-}\overset{\ominus}{O} \longleftrightarrow O{=}O \longrightarrow O$ Number of unpaired electrons = 0

$O_2^- = \sigma 1s^2,\, \sigma^* 1s^2 \sigma 2s^2,\, \sigma^* 2s^2,\, \sigma 2p_z^2,\, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^1$

Number of unpaired electrons = 1

Thus O_2^- is paramagnetic.

Hence (D) is correct.

27. The value of $\log_{10}K$ for a reaction $A \rightleftharpoons B$ is

(Given : $\Delta_r H^\circ_{298K} = -54.07\text{kJ mol}^{-1}$, $\Delta_r S^\circ_{298K} = 10\text{JK}^{-1}\text{mol}^{-1}$ and R= 8.314 JK^{-1}mol^{-1}; 2.303×8.314×298 = 5705)

(A) 5 (B) 10
(C) 95 (D) 100

Sol. **(B)**

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -54.07 \times 1000 - 298 \times 10 = -57050 \text{ J mol}^{-1}$

$-57050 = -5705 \log_{10}K$

$\log_{10}K = 10$

Hence (B) is correct

28. The species having bond order different from that in CO is

(A) NO^- (B) NO^+
(C) CN^- (D) N_2

Sol. **(A)**

NO^- (16 electron system)

Bond order = 2.

$NO^{\oplus}$, CN^- and N_2 are isoelectronic with CO therefore all have same bond order (= 3)

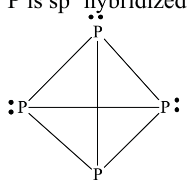
Hence (A) is correct.

29. The percentage of p-character in the orbitals forming P–P bonds in P_4 is

(A) 25 (B) 33
(C) 50 (D) 75

Sol. **(D)**

P is sp^3 hybridized in P_4.



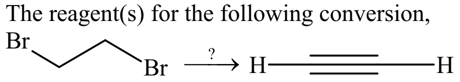
Hence (D) is correct.

Nitin M Sir (physics-iitjee.blogspot.com)

30. Extraction of zinc from zinc blende is achieved by
(A) electrolytic reduction
(B) roasting followed by reduction with carbon
(C) roasting followed by reduction with another metal
(D) roasting followed by self-reduction

Sol. **(B)**
Option (B) is correct.

31. The reagent(s) for the following conversion,



is/are
(A) alcoholic KOH
(B) alcoholic KOH followed by $NaNH_2$
(C) aqueous KOH followed by $NaNH_2$
(D) Zn/CH_3OH

Sol. **(B)**

$$\underset{\underset{Br}{|}}{H_2C}-\underset{\underset{Br}{|}}{CH_2} \xrightarrow{\text{Alc. KOH}} H_2C=\underset{\underset{Br}{|}}{CH} \xrightarrow{NaNH_2} HC\equiv CH$$

Because $CH_2 = CH - Br$ has partial $C - Br$ double bond character, it requires more stronger base to remove HBr.
Hence(B) is correct.

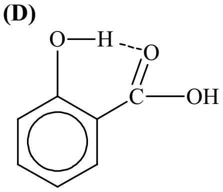
SECTION – II

Assertion-Reason Type

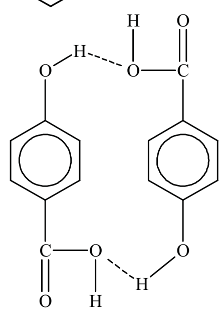
This section contains 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. **STATEMENT-1**: p-Hydroxybenzoic acid has a lower boiling point than o-hydroxybenzoic acid.
because
STATEMENT-2: o-Hydroxybenzoic acid has intramolecular hydrogen bonding.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement -2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(D)**



More stabilized by intramolecular hydrogen bonding



More stronger intermolecular forces increases the boiling point.

Hence (D) is correct.

33. STATEMENT-1: Micelles are formed by surfactant molecules above the critical micellar concentration (CMC).
because
STATEMENT-2: The conductivity of a solution having surfactant molecules decreases sharply at the CMC.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(B)**
The formation of micelles takes places only above a particular temperature called Kraft temperature (T_k) and above a particular concentration called critical micelle concentration (CMC).
Each micelle contains at least 100 molecules. Therefore conductivity of the solution decreases sharply at the CMC.
Hence (B) is correct.

34. **STATEMENT-1**: Boron always forms covalent bond.
because
STATEMENT-2: The small size of B^{3+} favours formation of covalent bond.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is True.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(A)**
According to Fajan's rule small cations having high charge density always have tendency to form covalent bond.
Hence (A) is correct

35. **STATEMENT-1**: In water, orthoboric acid behaves as a weak monobasic acid.
because
STATEMENT-2: In water, orthoboric acid acts as a proton donor.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(C)**
H_3BO_3 (orthoboric acid) is a weak lewis acid.

$$H_3BO_3 + H_2O \rightleftharpoons B(OH)_4^- + H^{\oplus}$$

It does not donate proton rather it acceptors OH^- form water.
Hence (C) is correct

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs C_{36-38} and C_{39-41}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

C_{36-38} : Paragraph for question Nos 36 to 38

Chemical reactions involve interaction of atoms and molecules. A large number of atoms/molecules (approximately 6.023×10^{23}) are present in a few grams of any chemical compound varying with their atomic/molecular masses. To handle such large numbers conveniently, the mole concept was introduced. This concept has implications in diverse areas such as analytical chemistry, biochemistry, electrochemistry and radiochemistry. The following example illustrates a typical case, involving chemical/electrochemical reaction, which requires a clear understanding of the mole concept.

A 4.0 molar aqueous solution of NaCl is prepared and 500 mL of this solution is electrolysed. This leads to the evolution of chlorine gas at one of the electrodes (atomic mass: Na = 23, Hg = 200; 1 Faraday=96500 coulombs)

36. The total number of moles of chlorine gas evolved is
(A) 0.5 (B) 1.0
(C) 2.0 (D) 3.0

Sol. **(B)**

$NaCl \longrightarrow Na^+ + Cl^-$

At anode:

$2Cl^- \longrightarrow Cl_2$

Moles of Cl^- = 2 in 500 ml.
Therefore 1 mole of Cl_2 evolves.
Hence (B) is correct.

37. If the cathode is a Hg electrode, the maximum weight (g) of amalgam formed from this solution is
(A) 200 (B) 225
(C) 400 (D) 446

Sol. **(D)**
Na – Hg (amalgam) formed = 2 moles at cathode.
Hence (D) is correct.

38. The total charge (coulombs) required for complete electrolysis is
(A) 24125 (B) 48250
(C) 96500 (D) 193000

Sol. **(D)**
2 moles of electrons (2 Faraday) are required.
1F = 96500
2F = 193000
Hence (D) is correct.

$C_{39\text{-}41}$: Paragraph for Question Nos. 39 to 41

The noble gases have closed-shell electronic configuration and are monoatomic gases under normal conditions. The low boiling points of the lighter noble gases are due to weak dispersion forces between the atoms and the absence of other interatomic interactions.

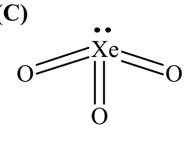
The direct reaction of xenon with fluorine leads to a series of compounds with oxidation numbers +2, +4 and +6. XeF_4 reacts violently with water to give XeO_3. The compounds of xenon exhibit rich stereochemistry and their geometries can be deduced considering the total number of electron pairs in the valence shell.

39. Argon is used in arc welding because of its
(A) low reactivity with metal (B) ability to lower the melting point of metal
(C) flammability (D) high calorific value

Sol. **(A)**
Argon is used mainly to provide an inert atmosphere in high temperature metallurgical (arc welding of metals/alloys) extraction.
Hence (A) is correct.

40. The structure of XeO_3 is
(A) linear (B) planar
(C) pyramidal (D) T-shaped

Sol. **(C)**



sp^3 hybridized pyramidal structure.

Hence (C) is correct.

41. XeF_4 and XeF_6 are expected to be
(A) oxidizing (B) reducing
(C) unreactive (D) strongly basic

Sol. **(A)**

$6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

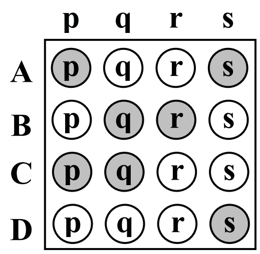
Hence (A) is correct.

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:

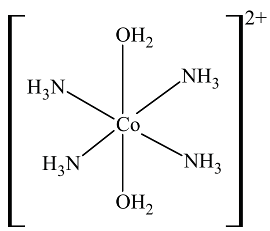


42. Match the complexes in Column-I with their properties listed in Column-II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

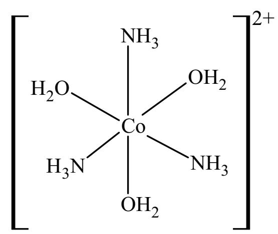
	Column-I		Column-II
(A)	$[Co(NH_3)_4(H_2O)_2]Cl_2$	(p)	geometrical isomers
(B)	$[Pt(NH_3)_2Cl_2]$	(q)	paramagnetic
(C)	$[Co(H_2O)_5Cl]Cl$	(r)	diamagnetic
(D)	$[Ni(H_2O)_6]Cl_2$	(s)	metal ion with +2 oxidation state

Sol. **A → p, q, s B → p, r, s C → q, s D → q, s**

(A)



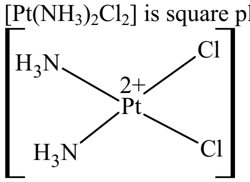
(trans)



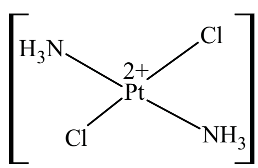
(cis)

$Co^{2+} = 3d^7$ (Paramagnetic)

(B) $[Pt(NH_3)_2Cl_2]$ is square planar.



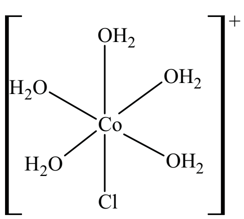
(cis)



(trans)

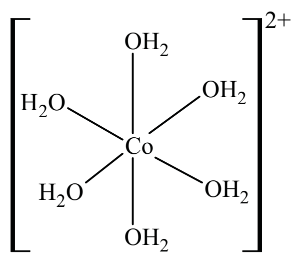
$Pt^{2+} = 5d^8 4s^0$ (diamagnetic)

(C)



$Co^{2+} = 3d^7$ (paramagnetic)

(D)



$Ni^{2+} = 3d^8$ (weak field ligand, paramagnetic)

43. Match gases under specified conditions listed in **Column-I** with their properties/laws in **Column-II.** Indicate your answer by darkening the appropriate bubsbles of the 4 × 4 matrix given in the ORS.

Column-I		**Column-II**	
(A)	hydrogen gas (P = 200 atm, T = 273K)	(p)	compressibility factor ≠ 1
(B)	hydrogen gas (P ~ 0, T = 273K)	(q)	attractive forces are dominant
(C)	CO_2 (P = 1 atm, T = 273K)	(r)	PV = nRT
(D)	real gas with very large molar volume	(s)	P(V – nb) = nRT

Sol. **A → p, s   B → r   C → p, q   D → p, s**

(A) $Z = \frac{PV_m}{RT}$ at high pressure and low temperature.

Equation $\left(P + \frac{an^2}{V^2}\right)(V - nb)$ = nRT reduces to P(V – nb) = nRT.

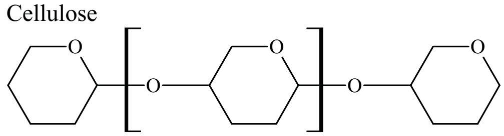
(B) For hydrogen gas value of Z = 1 at P = 0 and it increase continuously on increasing pressure.
(C) CO_2 molecules have larger attractive forces, under normal conditions.
(D) $Z = \frac{PV_m}{RT}$, at very large molar volume Z ≠ 1.

44. Match the chemical substances in Column-I with type of polymers/type of bonds in Column-II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column-I		**Column-II**	
(A)	cellulose	(p)	natural polymer
(B)	nylon-6, 6	(q)	synthetic polymer
(C)	protein	(r)	amide linkage
(D)	sucrose	(s)	glycoside linkage

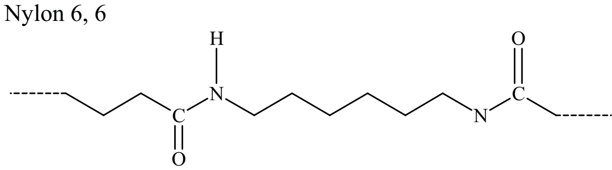
Sol. **A → p, s, B → q, r;   C → p, r   D → s**

(A) Cellulose



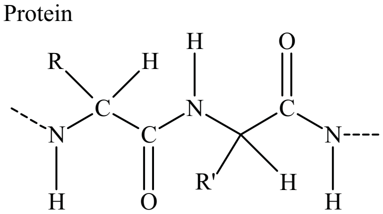
(Glycoside linkage)

(B) Nylon 6, 6



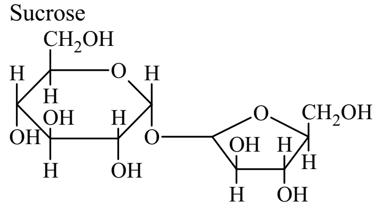
(Amide linakge)

(C) Protein



(Amide linkage)

(D) Sucrose



(Glycoside linkage)

Nitin M Sir (physics-iitjee.blogspot.com)

PART III (MATHEMATICS)

SECTION –I

Straight Objective Type

*This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

45. A hyperbola, having the transverse axis of length 2 sinθ, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(A) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ (B) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

(C) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ (D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Sol. **(A)**

The given ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow a = 2, b = \sqrt{3} \Rightarrow 3 = 4(1 - e^2) \Rightarrow e = \frac{1}{2}$

so that ae = 1

Hence the eccentricity e_1, of the hyperbola is given by

$1 = e_1 \sin\theta \Rightarrow e_1 = \operatorname{cosec}\theta$

$\Rightarrow b^2 = \sin^2\theta(\operatorname{cosec}^2\theta - 1) = \cos^2\theta$

Hence the hyperbola is $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$ or $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

46. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

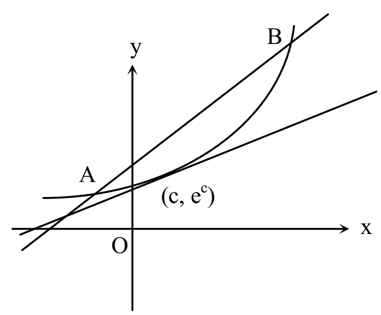
(A) on the left of x = c (B) on the right of x = c

(C) at no point (D) at all points

Sol. **(A)**

Slope of the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ is equal to $\frac{e^{c+1} - e^{c-1}}{2} > e^c$

$\Rightarrow$ tangent to the curve $y = e^x$ will intersect the given line to the left of the line x = c.



Alternative

The equation of the tangent to the curve $y = e^x$ at (c, e^c) is

$y - e^c = e^c(x - c)$ …(1)

Equation of the line joining the given points is

$y - e^{c-1} = \frac{e^c(e - e^{-1})}{2}[x - (c - 1)]$ …(2)

Eliminating y from (1) and (2), we get

$[x - (c - 1)][2 - (e - e^{-1})] = 2e^{-1}$

or $x - c = \frac{e + e^{-1} - 2}{2 - (e - e^{-1})} < 0 \Rightarrow x < c.$

$\Rightarrow$ the line (1) and (2) meet on the left of the line x = c.

47. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)e^{i\pi/4}$

(C) $(4 + 3i)e^{i\pi/4}$ (D) $(3 + 4i)e^{i\pi/4}$

Sol. **(D)**

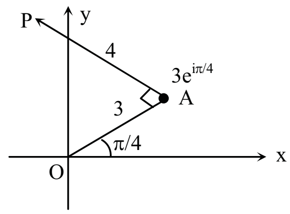
Let OA = 3, so that the complex number associated with A is $3e^{i\pi/4}$.

If z is the complex number associated with P,

then $\frac{z-3e^{i\pi/4}}{0-3e^{i\pi/4}} = \frac{4}{3}e^{-i\pi/2} = -\frac{4i}{3}$

$\Rightarrow 3z - 9e^{i\pi/4} = 12ie^{i\pi/4}$

$\Rightarrow z = (3 + 4i)e^{i\pi/4}$.



48. Let f(x) be differentiable on the interval $(0, \infty)$ such that f(1) = 1, and

$$\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x} = 1$$

for each x > 0. Then f(x) is

(A) $\frac{1}{3x}+\frac{2x^2}{3}$ (B) $-\frac{1}{3x}+\frac{4x^2}{3}$

(C) $-\frac{1}{x}+\frac{2}{x^2}$ (D) $\frac{1}{x}$

Sol. **(A)**

$\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x} = 1$

$\Rightarrow x^2f'(x) - 2xf(x) + 1 = 0$

$\Rightarrow f(x) = cx^2 + \frac{1}{3x}$ also f(1) = 1

$\Rightarrow c = \frac{2}{3}$.

Hence $f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$.

49. The number of solutions of the pair of equations

$2\sin^2\theta - \cos2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0, 2\pi]$ is

(A) zero (B) one

(C) two (D) four

Sol. **(C)**

$2\sin^2\theta - \cos2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$

also $2\cos^2\theta = 3\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow$ two solutions in $[0, 2\pi]$.

50. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

(A) $\frac{2}{9}(p-q)(2q-p)$ (B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$ (D) $\frac{2}{9}(2p-q)(2q-p)$

Sol. **(D)**

The equation $x^2 - px + r = 0$ has roots (α, β) and the equation $x^2 - qx + r = 0$ has roots $\left(\frac{\alpha}{2}, 2\beta\right)$.

$\Rightarrow r = \alpha\beta$ and $\alpha + \beta = p$ and $\frac{\alpha}{2} + 2\beta = q$

$\Rightarrow \beta = \frac{2q-p}{3}$ and $\alpha = \frac{2(2p-q)}{3}$

$\Rightarrow \alpha\beta = r = \frac{2}{9}(2q-p)(2p-q)$.

51. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero
(B) one
(C) two
(D) three

Sol. **(C)**

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda^6 - 3\lambda^2 - 2 = 0$$

$\Rightarrow (1 + \lambda^2)^2 (\lambda^2 - 2) = 0 \Rightarrow \lambda = \pm\sqrt{2}$.

52. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) 1/2
(B) 1/3
(C) 2/5
(D) 1/5

Sol. **(C)**

Let E = event when each American man is seated adjacent to his wife

A = event when Indian man is seated adjacent to his wife

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife

Again $n(E) = (5!) \times (2!)^4$

$$\Rightarrow P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}.$$

Alternative

Fixing four American couples and one Indian man in between any two couples; we have 5 different ways in which his wife can be seated, of which 2 cases are favorable.

$\therefore$ required probability $= \frac{2}{5}$.

53. $$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\, dt}{x^2 - \frac{\pi^2}{16}}$$ equals

(A) $\frac{8}{\pi} f(2)$
(B) $\frac{2}{\pi} f(2)$
(C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$
(D) $4f(2)$

Sol. **(A)**

$$\lim_{x\to\frac{\pi}{4}} \frac{\int_{2}^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}} \qquad \left(\frac{0}{0} \text{ form}\right)$$

Let $L = \lim_{x\to\frac{\pi}{4}} \frac{f(\sec^2 x) 2\sec x \sec x \tan x}{2x}$

$\therefore\ L = \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$.

SECTION –II

Assertion – Reason Type

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT – 1 (Assertion) and STATEMENT -2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ***ONLY ONE*** *is correct.*

54. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

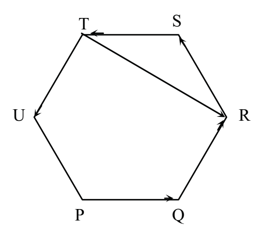
STATEMENT -1 : $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \vec{0}$.

because

STATEMENT -2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(C)**



Since $\overrightarrow{PQ} \nparallel \overrightarrow{TR}$ $\quad\because\ \overrightarrow{TR}$ is resultant of $\overrightarrow{SR}$ and $\overrightarrow{ST}$ vector.

$\Rightarrow\ \overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq 0$.

But for statement 2, we have $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$

which is not possible as $\overrightarrow{PQ} \nparallel \overrightarrow{RS}$.

Hence, statement 1 is true and statement 2 is false.

55. Let F(x) be an indefinite integral of $\sin^2 x$.

STATEMENT -1 : The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x.

because

STATEMENT -2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

$F(x) = \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx$

$\Rightarrow F(x) = \frac{1}{4}(2x - \sin 2x) + c$.

Since, $F(x + \pi) \neq F(x)$.
Hence statement 1 is false.
But statement 2 is true as $\sin^2 x$ is periodic with period π.

56. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT -1 : $P(H_i \mid E) > P(E \mid H_i) \,.\, P(H_i)$ for $i = 1, 2, \ldots, n$

because

STATEMENT -2 : $\sum_{i=1}^{n} P(H_i) = 1$

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

Statement : 1

If $P(H_i \cap E) = 0$ for some i, then $P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall$ i = 1, 2 … n, then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right) \cdot P(H_i) \qquad [\text{as } 0 < P(E) < 1]$$

Hence statement 1 may not always be true.
Statement : 2
Clearly $H_1 \cup H_2 \ldots \cup H_n = S$ (sample space)
$\Rightarrow P(H_1) + P(H_2) + \ldots + P(H_n) = 1$.

57. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

STATEMENT -1 : The tangents are mutually perpendicular.

because

STATEMENT -2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is true; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**

Since the tangents are perpendicular $\Rightarrow$ locus of perpendicular tangents to circle $x^2 + y^2 = 169$ is a director circle having equation $x^2 + y^2 = 338$.

SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs M_{58-60} and M_{61-63}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choice (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

M_{58-60} : Paragraph for question Nos. 58 to 60

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

58. The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$ (B) 1 : 2
(C) 1 : 4 (D) 1 : 8

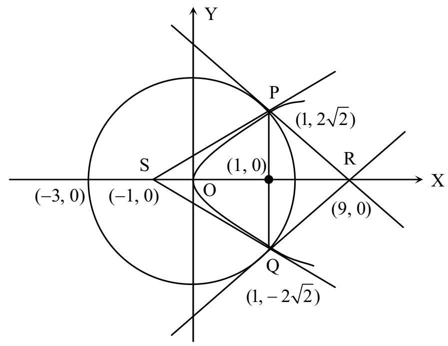
Sol. **(C)**

Coordinates of P and Q are $\left(1, +2\sqrt{2}\right)$ and $\left(1, -2\sqrt{2}\right)$.

Area of $\Delta PQR = \frac{1}{2}\cdot 4\sqrt{2}\cdot 8 = 16\sqrt{2}$

Area of $\Delta PQS = \frac{1}{2}\cdot 4\sqrt{2}\cdot 2 = 4\sqrt{2}$

Ratio of area of triangle PQS and PQR is 1 : 4.



59. The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$

(C) $3\sqrt{2}$ (D) $2\sqrt{3}$

Sol. **(B)**

Equation of circumcircle of ΔPRS is $(x + 1)(x - 9) + y^2 + \lambda y = 0$

It will pass through $\left(1, 2\sqrt{2}\right)$, then $-16 + 8 + \lambda.\, 2\sqrt{2} = 0$

$$\lambda = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$$

Equation of circumcircle is $x^2 + y^2 - 8x + 2\sqrt{2}y - 9 = 0$.

Hence its radius is $3\sqrt{3}$.

Alternative

Let $\angle PSR = \theta$

$$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow PR = 6\sqrt{2} = 2R\cdot\sin\theta \Rightarrow R = 3\sqrt{3}.$$

60. The radius of the incircle of the triangle PQR is

(A) 4 (B) 3

(C) 8/3 (D) 2

Sol. **(D)**

Radius of incircle is $r = \frac{\Delta}{s}$

as $\Delta = 16\sqrt{2}$

$$s = \frac{6\sqrt{2} + 6\sqrt{2} + 4\sqrt{2}}{2} = 8\sqrt{2}$$

$$r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2.$$

M_{61-63} : Paragraph for Question Nos. 61 to 63

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1). Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, …

61. The sum $V_1 + V_2 + \ldots + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

Sol. **(B)**

$V_r = \frac{r}{2}[2r + (r-1)(2r-1)] = \frac{1}{2}(2r^3 - r^2 + r)$

$\sum V_r = \frac{1}{12} n(n+1)(3n^2 + n + 2)$.

62. T_r is always

(A) an odd number
(B) an even number
(C) a prime number
(D) a composite number

Sol. **(D)**

$V_{r+1} - V_r = (r+1)^3 - r^3 - \frac{1}{2}[(r+1)^2 - r^2] + \frac{1}{2}(1)$

$= 3r^2 + 2r + 1$

$T_r = 3r^2 + 2r - 1 = (r+1)(3r-1)$

which is a composite number.

63. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5
(B) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6
(C) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11
(D) $Q_1 = Q_2 = Q_3 = \ldots$

Sol. **(B)**

$T_r = 3r^2 + 2r - 1$

$T_{r+1} = 3(r+1)^2 + 2(r+1) - 1$

$Q_r = T_{r+1} - T_r = 3[2r+1] + 2[1]$

$Q_r = 6r + 5$

$Q_{r+1} = 6(r+1) + 5$

Common difference $= Q_{r+1} - Q_r = 6$.

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows:

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

64. Consider the following linear equations

$ax + by + cz = 0$
$bx + cy + az = 0$
$cx + ay + bz = 0$

Match the conditions / expressions in Column I with statements in Column II and indicate your answers by darkening the appropriate bubbles in 4×4 matrix given in the ORS.

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at a single point.
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

Sol. **A – r** **B – q** **C – p** **D – s**

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A). If $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow \Delta = 0$ and $a = b = c \neq 0$
$\Rightarrow$ the equations represent identical planes.

(B). $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
$\Rightarrow \Delta = 0 \Rightarrow$ the equations have infinitely many solutions.
$ax + by = (a + b)z$
$bx + cy = (b + c)z$
$\Rightarrow (b^2 - ac)y = (b^2 - ac)z \Rightarrow y = z$
$\Rightarrow ax + by + cy = 0 \Rightarrow ax = ay \Rightarrow x = y = z.$

(C). $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
$\Rightarrow \Delta \neq 0 \Rightarrow$ the equation represent planes meeting at only one point.

(D). $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca \Rightarrow a = b = c = 0$
$\Rightarrow$ the equation represent whole of the three dimensional space.

65. Match the integrals in Column I with the values in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$	(p) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_{2}^{3} \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(D) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

Sol. **A – s** **B – s** **C – p** **D – r**

(A). $\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{\pi}{2}$

(B). $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2}$

(C). $\int_{2}^{3} \frac{dx}{1-x^2} = \frac{1}{2}\ln\frac{2}{3}$

(D). $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{3}$

66. In the following [x] denotes the greatest integer less than or equal to x.
Match the functions in Column I with the properties Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	**Column II**
(A) $x\,|x|$	(p) continuous in $(-1, 1)$
(B) $\sqrt{|x|}$	(q) differentiable in $(-1, 1)$
(C) $x + [x]$	(r) strictly increasing in $(-1, 1)$
(D) $|x - 1| + |x + 1|$	(s) not differentiable at least at one point in $(-1, 1)$

Sol. **A – p, q, r** **B – p, s** **C – r, s** **D – p, q**

(A). $x|x|$ is continuous, differentiable and strictly increasing in $(-1, 1)$.

(B). $\sqrt{|x|}$ is continuous in $(-1, 1)$ and not differentiable at $x = 0$.

(C). $x + [x]$ is strictly increasing in $(-1, 1)$ and discontinuous at $x = 0 \Rightarrow$ not differentiable at $x = 0$.

(D). $|x - 1| + |x + 1| = 2$ in $(-1, 1)$
$\Rightarrow$ the function is continuous and differentiable in $(-1, 1)$.

FIITJEE Solutions to

IITJEE-2007

(Paper-II, Code-8)

Time: 3 hours ***M. Marks: 243***

Note: ***(i)*** The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.

(ii) **Section I** contains 9 multiple choice questions which have only one correct answer. Each question carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iii) **Section II** contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iv) **Section III** contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries ***+4 marks*** for correct answer and ***– 1 mark*** for wrong answer.

(v) **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries ***+6 marks*** and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is no negative marking.

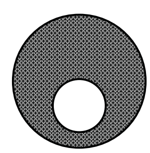
PART- I

SECTION – I

Straight Objective Type

*This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

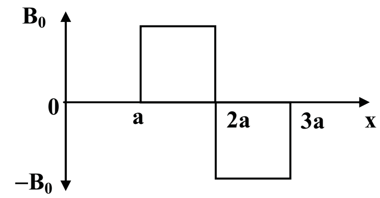
1. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is



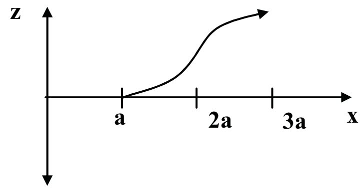
(A) zero everywhere
(B) non-zero and uniform
(C) non-uniform
(D) zero only at its center

Sol. **(B)**

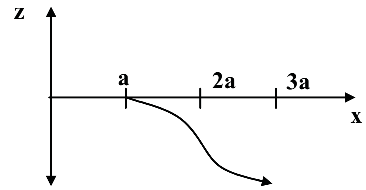
2. A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region $a < x < 2a$ and $\vec{B} = -B_0\hat{j}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x = a. The trajectory of the charge in this region can be like,



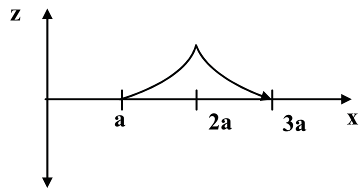
(A)



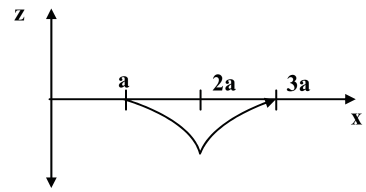
(B)



(C)



(D)

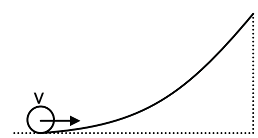


Sol. **(A)**

for $a < x < 2a$ path will be concave upward

$2a < x < 3a$ path will be concave downward

3. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(A) ring (B) solid sphere

(C) hollow sphere (D) disc

Sol. **(D)**

$$\frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

Hence $I_{cm} = \frac{1}{2}mR^2$

4. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \frac{2h}{mc}$

(C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

Sol. **(A)**

$$\lambda = \frac{h}{\sqrt{2m(eV)}} \Rightarrow eV = \frac{h^2}{2m\lambda^2}$$

$$\lambda_0 = \frac{hC}{eV}$$

5. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ±0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ±0.01 mm. Take g = 9.8 m/s^2 (exact). The Young's modulus obtained from the reading is
(A) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(B) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(C) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(D) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

Sol. **(B)**

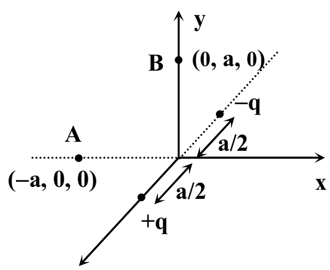
$$Y = \frac{4F\ell}{\pi d^2 \Delta\ell}$$

$$\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta(L)}{\Delta L} = 0.1125$$

$\Delta Y = 2 \times 10^{11} \times 0.1125$

6. Positive and negative point charges of equal magnitude are kept at $\left(0,\ 0,\ \frac{a}{2}\right)$ and $\left(0,\ 0,\ \frac{-a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved from (–a, 0, 0) to (0, a, 0) is
(A) positive
(B) negative
(C) zero
(D) depends on the path connecting the initial and final positions

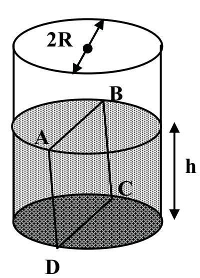
Sol. **(C)**



7. In the experiment to determine the speed of sound using a resonance column,
(A) prongs of the tuning fork are kept in a vertical plane
(B) prongs of the tuning fork are kept in a horizontal plane
(C) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(D) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

Sol. **(A)**

8. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude

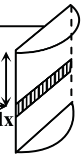


(A) $\left|2P_0Rh + \pi R^2\rho gh - 2RT\right|$
(B) $\left|2P_0Rh + R\rho gh^2 - 2RT\right|$
(C) $\left|P_0\pi R^2 + R\rho gh^2 - 2RT\right|$
(D) $\left|P_0\pi R^2 + R\rho gh^2 + 2RT\right|$

Sol. **(B)**

$$\int_0^h (P_0 + \rho g x)2R\,dx - 2RT = F$$

$$2P_0Rh + R\rho gh^2 - 2RT = F$$



9. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$, where A and k are constants. The angle between the force and the momentum is

(A) 0° (B) 30°
(C) 45° (D) 90°

Sol. **(D)**

$$\vec{F} = \frac{d\vec{P}}{dt} = AK\left[-\hat{i}\sin(kt) - \hat{j}\cos(kt)\right]$$

$$\vec{F}\cdot\vec{P} = 0$$

SECTION – II

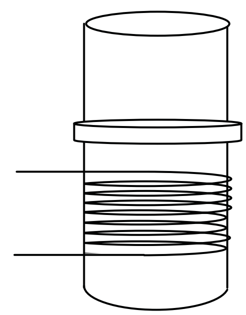
Assertion - Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

---

10. **STATEMENT-1**

A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.



Because

STATEMENT-2

In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(A)**

11. **STATEMENT-1**
If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.
because

STATEMENT-2

The linear momentum of an isolated system remains constant.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.

(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(D)**

12. **STATEMENT-1**
The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

because

STATEMENT-2

The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

13. **STATEMENT-1**
A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.
because

STATEMENT-2
For every action there is an equal and opposite reaction.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

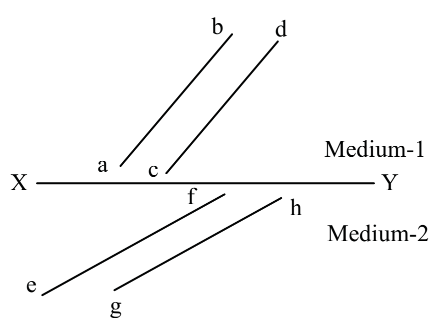
SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs P_{14-16} and P_{17-19}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ***ONLY ONE*** *is correct.*

P_{14-16} : Paragraph for Question Nos. 14 to 16

The figure shows a surface XY separating two transparent media, medium – 1 and medium -2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



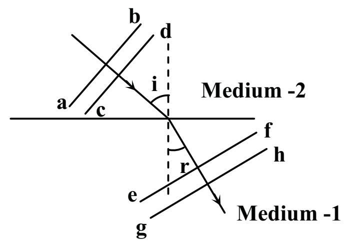
14. Light travels as a
(A) parallel beam in each medium
(B) convergent beam in each medium

Nitin M Sir (physics-iitjee.blogspot.com)

(C) divergent beam in each medium

(D) divergent beam in one medium and convergent beam in the other medium.

Sol. **(A)**



15. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$

(A) ϕ_c cannot be equal to ϕ_d

(B) ϕ_d can be equal to ϕ_e

(C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$

(D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

Sol. **(C)**

16. Speed of the light is

(A) the same in medium-1 and medium-2

(B) larger in medium-1 than in medium-2

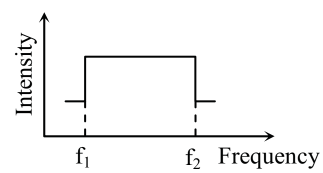
(C) larger in medium-2 than in medium-1

(D) different at b and d

Sol. **(B)**

P_{17-19} : Paragraph for Question Nos. 17 to 19

Two trains A and B are moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.



Assume that the sound of the whistle is composed of components varying in frequency from f_1 = 800 Hz to f_2 = 1120 Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

17. The speed of sound of the whistle is

(A) 340 m/s for passengers in A and 310 m/s for passengers in B

(B) 360 m/s for passengers in A and 310 m/s for passengers in B

(C) 310 m/s for passengers in A and 360 m/s for passengers in B

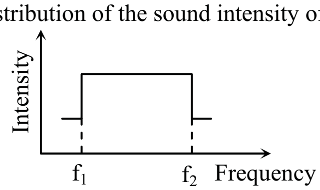
(D) 340 m/s for passengers in both the trains

Sol. **(B)**

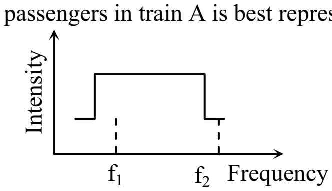
Speed of sound is frame dependent.

18. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

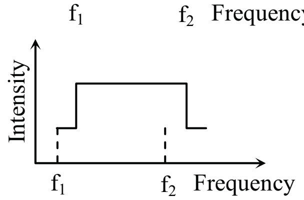
(A)



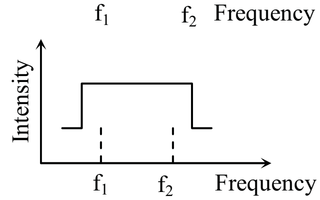
(B)



(C)



(D)



Sol. **(A)**

19. The spread of frequency as observed by the passengers in train B is
(A) 310 Hz (B) 330 Hz
(C) 350 Hz (D) 290 Hz

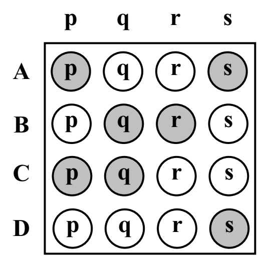
Sol. **(A)**

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



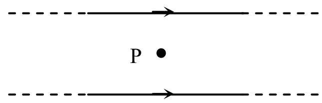
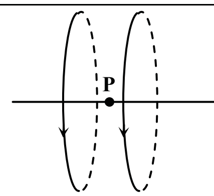
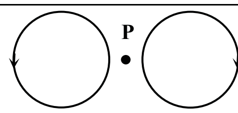
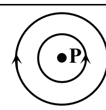
20. **Column I** describe some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situation in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where, M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Sol. A→ (p), B → (q) & (r), C → (p), D → (r) & (q)

21. Two wires each carrying a steady current I are shown in four configurations in **Column I**. Some of the resulting effects are described in **Column II**. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II

(A) Point P is situated midway between the wires.		(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(q) The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(r) There is no magnetic field at P.
(D) Point P is situated at the common center of the wires.		(s) The wires repel each other.

Sol. A→ (q) & (r), B → (p), C → (q) & (r), D → (q)

22. **Column I** gives some devices and **Column II** gives some process on which the functioning of these devices depend. Match the devices in **Column I** with the processes in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) Bimetallic strip	(p) Radiation from a hot body
(B) Steam engine	(q) Energy conversion
(C) Incandescent lamp	(r) Melting
(D) Electric fuse	(s) Thermal expansion of solids

Sol. A→ (s), B → (q), C → (p), D → (r)

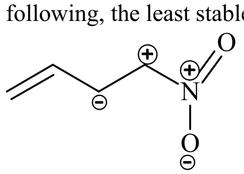
PART- II (CHEMISTRY)

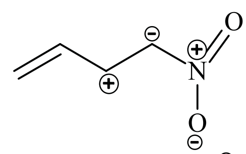
SECTION – I

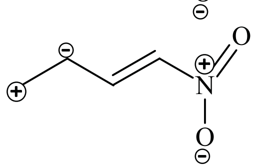
Straight Objective Type

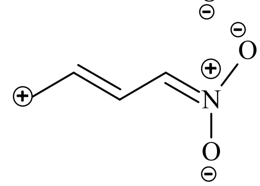
*This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

23. Among the following, the least stable resonance structure is

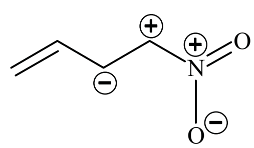
(A)


(B)


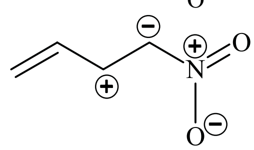
(C)


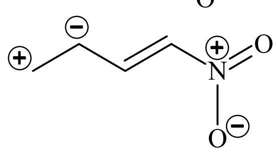
(D)


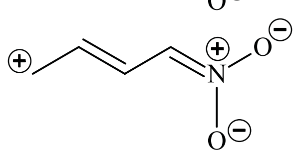
Sol. **(A)**

(A)


Same charges are present at nearest position (Less stable)

(B)


(C)


(D)


Hence (A) is correct

24. For the process $H_2O(l)$ (1 bar, 373 K) $\rightarrow$ $H_2O(g)$ (1 bar, 373 K), the correct set of thermodynamic parameters is

(A) $\Delta G = 0, \Delta S = +ve$ (B) $\Delta G = 0, \Delta S = -ve$

(C) $\Delta G = +ve, \Delta S = 0$ (D) $\Delta G = -ve, \Delta S = +ve$

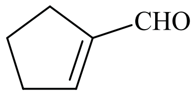
Sol. **(A)**

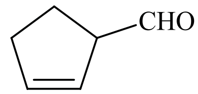
$$\underset{(1\ \text{bar}, 373\ \text{K})}{H_2O(\ell)} \rightleftharpoons \underset{(1\ \text{bar}, 373\text{K})}{H_2O(g)}$$

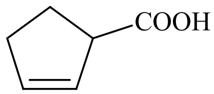
At 100°C $H_2O(\ell)$ has equilibrium with $H_2O(g)$ therefore $\Delta G = 0$.

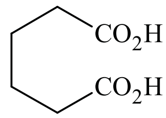
Because liquid molecules are converting into gases molecules therefore ΔS = +ve
Hence (A) is correct.

25. Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives compound E. Compound E on further treatment with aqueous KOH yields compound F. Compound F is

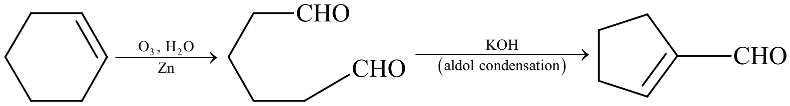
(A) 

(B) 

(C) 

(D) 

Sol. **(A)**



(E) (F)

Hence (A) is correct.

26. Consider a reaction aG + bH $\rightarrow$ Products. When concentration of both the reactants G and H is doubled, the rate increases by eight times. However, when concentration of G is doubled keeping the concentration of H fixed, the rate is doubled. The overall order of the reaction is

(A) 0 (B) 1
(C) 2 (D) 3

Sol. **(D)**

aG + bH $\longrightarrow$ Product

rate $\propto [G]^a\,[H]^b$

a = 1, b = 2

Hence (D) is correct

27. Among the following metal carbonyls, the C – O bond order is lowest in

(A) $[Mn(CO)_6]^+$ (B) $[Fe(CO)_5]$
(C) $[Cr(CO)_6]$ (D) $[V(CO)_6]^-$

Sol. **(B)**

(A) $Mn^+ = 3d^5 4s^1$ in presence of CO effective configuration = $3d^6 4s^0$.
Three lone pair for back bonding with vacant orbital of C in CO.

(B) $Fe^0 = 3d^6 4s^2$ in presence of CO effective configuration = $3d^8$ four lone pair for back bonding with CO.

(C) $Cr^0 = 3d^5 4s^1$
Effective configuration = $3d^6$.
Three lone pair for back bonding with CO.

(D) $V^- = 3d^4 4s^2$ effective configuration = $3d^6$ three lone pair for back bonding with CO.
Maximum back bonding is present in $Fe(CO)_5$ there for CO bond order is lowest here.

28. A positron is emitted from ${}^{23}Na_{11}$. The ratio of the atomic mass and atomic number of the resulting nuclide is

(A) 22/10 (B) 22/11
(C) 23/10 (D) 23/12

Sol. **(C)**

On position emission from nucleus, proton converts into neutron therefore atomic number decreases by one but atomic mass remains constant.

$$\frac{\text{Atomic mass}}{\text{atomic number}} = \frac{23}{10}$$

Hence (C) is correct.

29. Consider a titration of potassium dichromate solution with acidified Mohr's salt solution using diphenylamine as indicator. The number of moles of Mohr's salt required per mole of dichromate is

(A) 3 (B) 4
(C) 5 (D) 6

Sol. **(D)**

$Cr_2O_7^{2-} + Fe^{2+} \longrightarrow Fe^{3+} + Cr^{3+}$

n factor of $Cr_2O_7^{2-}$ = 6

n factor of Fe^{2+} = 1

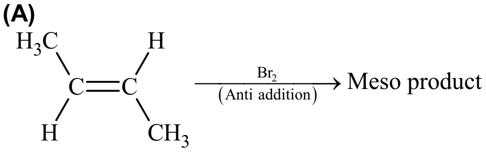
So to reduce one mole of dichromate 6 moles of Fe^{2+} are required.

Hence (D) is correct.

30. The number of stereoisomers obtained by bromination of *trans*-2-butene is

(A) 1 (B) 2
(C) 3 (D) 4

Sol. **(A)**



Hence (A) is correct.

31. A solution of metal ion when treated with KI gives a red precipitate which dissolves in excess KI to give a colourless solution. Moreover, the solution of metal ion on treatment with a solution of cobalt(II) thiocyanate gives rise to a deep blue crystalline precipitate. The metal ion is

(A) Pb^{2+} (B) Hg^{2+}
(C) Cu^{2+} (D) Co^{2+}

Sol. **(B)**

$Hg^{2+} + KI \longrightarrow HgI_2\,(\text{Red ppt.})$

$HgI_2 + \underset{(\text{excess})}{KI} \longrightarrow \underset{\text{Soluble}}{K_2HgI_4}$

$HgI^{2+} + Co(SCN)_2 \longrightarrow \underset{\text{blue crystalline precipitates}}{Hg(SCN)_2}$

SECTION – II

Assertion – Reason Type

This section 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. STATEMENT-1: Molecules that are not superimposable on their mirror images are chiral

because

STATEMENT-2: All chiral molecules have chiral centres.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. **(C)**

33. STATEMENT-1: Alkali metals dissolves in liquid ammonia to give blue solution

because

STATEMENT-2: Alkali metals in liquid ammonia give solvated species of the type $[M(NH_3)_n]^+$ (M = alkali metals).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. **(B)**

Blue colour is due to solvated electrons.

34. STATEMENT-1: Band gap in germanium is small.

because

STATEMENT-2: The energy spread of each germanium atomic energy level is infinitesimally small.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. **(C)**

35. STATEMENT-1: Glucose gives a reddish-brown precipitate with Fehling's solution.
because
STATEMENT-2: Reaction of glucose with Fehling's solution gives CuO and gluconic acid.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. **(C)**

$$C_6H_{12}O_6 + \text{Fehling solution} \longrightarrow (C_6H_{11}O_7)^- + \underset{\text{(Red ppt.)}}{Cu_2O\downarrow}$$

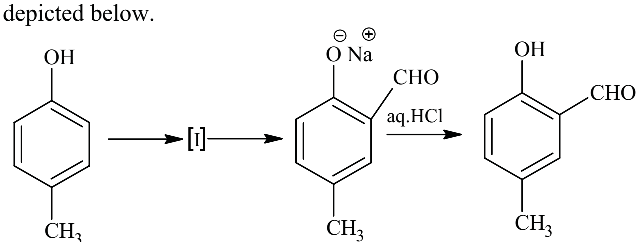
SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs C_{36-38} and C_{39-41}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

C_{36-38} : Paragraph for question Nos 36 to 38

Reimer-Tiemann reaction introduces an aldehyde group, on to the aromatic ring of phenol, *ortho* to the hydroxyl group. This reaction involves electrophilic aromatic substitution. This is a general method for the synthesis of substituted salicyladehydes as depicted below.



(I) (II) (III)

36. Which one of the following reagents is used in the above reaction?
(A) aq. NaOH + CH_3Cl
(B) aq. NaOH + CH_2Cl_2
(C) aq. NaOH + $CHCl_3$
(D) aq. NaOH + CCl_4

Sol. **(C)**

$$\overset{\ominus}{OH}_{aq} + CHCl_3 \rightleftharpoons \overset{\ominus}{C}Cl_3 + H_2O$$

$$C\overset{\ominus}{Cl_3} \longrightarrow \overset{\ominus}{Cl} + :CCl_2$$

dichlorocarbene intermediate

37. The electrophile in this reaction is
(A) : CHCl
(B) $^+CHCl_2$
(C) :CCl_2
(D) .CCl_3

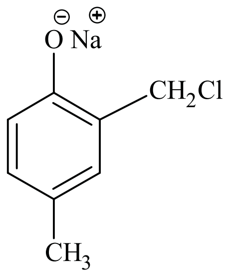
Sol. **(C)**

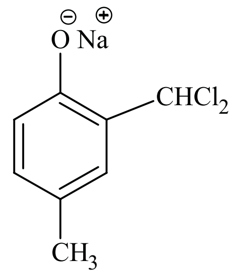
$$\overset{\ominus}{OH}_{aq} + CHCl_3 \rightleftharpoons \overset{\ominus}{C}Cl_3 + H_2O$$

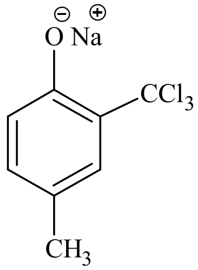
$$C\overset{\ominus}{Cl_3} \longrightarrow \overset{\ominus}{Cl} + :CCl_2$$

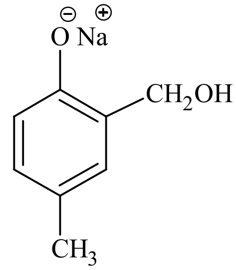
dichlorocarbene intermediate

38. The structure of the intermediate I is

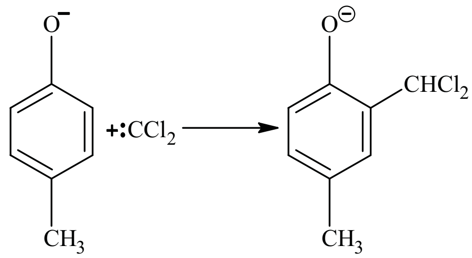
(A)


(B)


(C)


(D)


Sol. **(B)**



C_{39-41} : Paragraph for question Nos 39 to 41

Redox reactions play a pivotal role in chemistry and biology. The values of standard redox potential (E^o) of two half-cell reactions decide which way the reaction is expected to proceed. A simple example is a Daniel cell in which zinc goes into solution and copper gets deposited. Given below are a set of half-cell reactions (acidic medium) along with their E^o (V with respect to normal hydrogen electrode) values. Using this data obtain the correct explanations to Questions 39-41.

$I_2 + 2e^- \rightarrow 2I^-$ $\quad E^o = 0.54$
$Cl_2 + 2e^- \rightarrow 2Cl^-$ $\quad E^o = 1.36$
$Mn^{3+} + e^- \rightarrow Mn^{2+}$ $\quad E^o = 1.50$
$Fe^{3+} + e^- \rightarrow Fe^{2+}$ $\quad E^o = 0.77$
$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$ $\quad E^o = 1.23$

39. Among the following, identify the correct statement.
(A) Chloride ion is oxidized by O_2
(B) Fe^{2+} is oxidized by iodine
(C) Iodide ion is oxidized by chlorine
(D) Mn^{2+} is oxidized by chlorine

Sol. **(C)**
Reduction potential of I_2 is less than Cl_2.

40. While Fe^{3+} is stable, Mn^{3+} is not stable in acid solution because
(A) O_2 oxidises Mn^{2+} to Mn^{3+}
(B) O_2 oxidises both Mn^{2+} and Fe^{2+} to Fe^{3+}
(C) Fe^{3+} oxidizes H_2O to O_2
(D) Mn^{3+} oxidises H_2O to O_2

Sol. **(D)**
Reaction of Mn^{3+} with H_2O is spontaneous.

41. Sodium fusion extract, obtained from aniline, on treatment with iron (II) sulphate and H_2SO_4 in presence of air gives a Prussian blue precipitate. The blue colour is due to the formation of
(A) $Fe_4[Fe(CN)_6]_3$
(B) $Fe_3[Fe(CN)_6]_2$
(C) $Fe_4[Fe(CN)_6]_2$
(D) $Fe_3[Fe(CN)_6]_3$

Sol. **(A)**

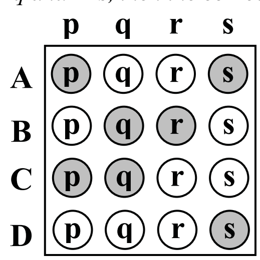
Nitin M Sir (physics-iitjee.blogspot.com)

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



42. Match the reactions in Column I with nature of the reactions/type of the products in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) $O_2^- \to O_2 + O_2^{2-}$	(p) redox reaction
(B) $CrO_4^{2-} + H^+ \to$	(q) one of the products has trigonal planar structure
(C) $MnO_4^- + NO_2^- + H^+ \to$	(r) dimeric bridged tetrahedral metal ion
(D) $NO_3^- + H_2SO_4 + Fe^{2+} \to$	(s) disproportionation

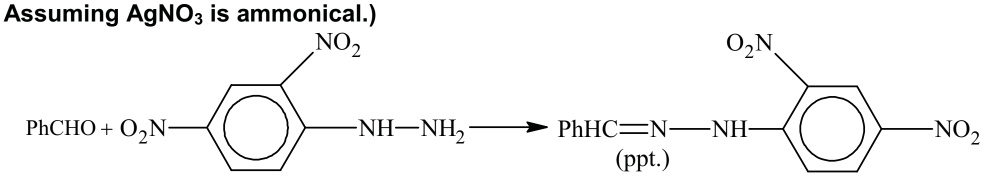
Sol. **A – p, s**
B –r
C – p, q
D – p

43. Match the compounds/ions in Column I with their properties/reactions in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

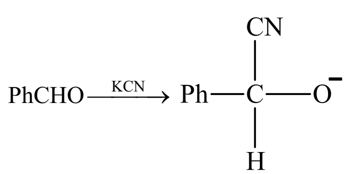
Column I	Column II
(A) C_6H_5CHO	(p) gives precipitate with 2, 4-dinitrophenylhydrazine
(B) $CH_3C \equiv CH$	(q) gives precipitate with $AgNO_3$
(C) CN^-	(r) is a nucleophile
(D) I^-	(s) is involved in cyanohydrin formation

Sol. **A – p, q, s**
B –q
C – q, r, s
D – q, r
(Note: Assuming $AgNO_3$ is ammonical.)

(A)

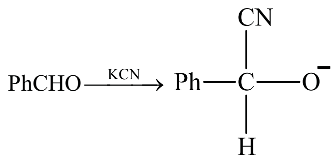


$$PhCHO + Ag_2O \xrightarrow{NH_3} PhCOO^- + \underset{(\text{white ppt.})}{Ag\downarrow}$$



(B) $$CH_3C \equiv CH \xrightarrow{\text{ammonical } AgNO_3} \underset{(\text{White ppt.})}{CH_3 - C \equiv C^-Ag^+ \downarrow}$$

(C)



$AgNO_3 + CN^- \longrightarrow AgCN\downarrow$

(D) $AgNO_3 + I^- \longrightarrow AgI\downarrow$

44. Match the crystal system/unit cells mentioned in Column I with their characteristic features mentioned in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) Simple cubic and face-centred cubic	(p) have these cell parameters $a = b = c$ and $\alpha = \beta = \gamma$
(B) cubic and rhombohedral	(q) are two crystal systems
(C) cubic and tetragonal	(r) have only two crystallography angles of 90^o
(D) hexagonal and monoclinic	(s) belong to same crystal system

Sol. **A – p, s**
B –p, q
C – q
D – q, r

Crystals class	**Axial distances**	**Angles**
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°$ $\gamma = 120°$
Trigonal and rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$
Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90°$ $\gamma \neq 90°$
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$

PART- III (MATHEMATICS)

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

---

45. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct?

(A) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

(B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$

(D) $\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ are mutually perpendicular

Sol. **(B)**

Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}=0$,

$\vec{a}, \vec{b}, \vec{c}$ represent an equilateral triangle.

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq 0$.

46. Let $f(x)=\dfrac{x}{(1+x^n)^{1/n}}$ for $n\geq 2$ and $g(x)=\underbrace{(fofo\ldots of)}_{f\text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals

(A) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

(B) $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(C) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

(D) $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

Sol. **(A)**

Here $ff(x)=\dfrac{f(x)}{[1+f(x)^n]^{1/n}}=\dfrac{x}{(1+2x^n)^{1/x}}$

$fff(x)=\dfrac{x}{(1+3x^n)^{1/n}}$

$\Rightarrow g(x)=\underbrace{(fofo\ldots of)}_{n\text{ terms}}(x)=\dfrac{x}{(1+nx^n)^{1/n}}$

Hence $I=\int x^{n-2}g(x)dx=\int\dfrac{x^{n-1}dx}{(1+nx^n)^{1/n}}$

$=\dfrac{1}{n^2}\int\dfrac{n^2x^{n-1}dx}{(1+nx^n)^{1/n}}=\dfrac{1}{n^2}\int\dfrac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}dx$

$\therefore\ I=\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+k$.

47. $\dfrac{d^2x}{dy^2}$ equals

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

(B) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$

(C) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

(D) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

Sol. **(D)**

Nitin M Sir (physics-iitjee.blogspot.com)

Since, $\frac{dx}{dy} = \frac{1}{dy/dx} = \left(\frac{dy}{dx}\right)^{-1}$

$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dx}{dy}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$.

*48. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is

(A) 360 (B) 192
(C) 96 (D) 48

Sol. **(C)**
COCHIN
The second place can be filled in ${}^4C_1$ ways and the remaining four alphabets can be arranged in 4! ways in four different places. The next 97th word will be COCHIN.
Hence, there are 96 words before COCHIN.

*49. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(A) a line not passing through the origin (B) $|z| = \sqrt{2}$
(C) the x-axis (D) the y-axis

Sol. **(D)**
Let z = cosθ + sinθ, so that

$\frac{z}{1-z^2} = \frac{\cos\theta + \sin\theta}{1-(\cos 2\theta + i\sin 2\theta)}$

$= \frac{\cos\theta + i\sin\theta}{2\sin^2\theta - 2i\sin\theta\cos\theta} = \frac{\cos\theta + i\sin\theta}{-2i\sin\theta(\cos\theta + i\sin\theta)}$

$= \frac{i}{2\sin\theta}$

Hence $\frac{z}{1-z^2}$ lies on the imaginary axis i.e., x = 0.

Alternative

Let $E = \frac{z}{1-z^2} = \frac{z}{z\bar{z}-z^2} = \frac{1}{\bar{z}-z}$

which is imaginary.

*50. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(A) 3 (B) 2
(C) 3/2 (D) 1

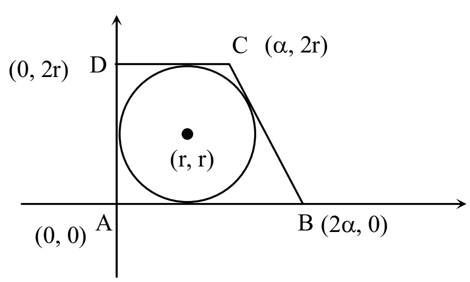
Sol. **(B)**

$18 = \frac{1}{2}(3\alpha)(2r) \Rightarrow \alpha r = 6$

Line $y = -\frac{2r}{\alpha}(x - 2\alpha)$ is tangent to $(x - r)^2 + (y - r)^2 = r^2$

$2\alpha = 3r$ and $\alpha r = 6$

r = 2.



Alternate

$\frac{1}{2}(x + 2x) \times 2r = 18$

$xr = 6$ …(1)

$\tan\theta = \frac{x - r}{r}$ $\quad\tan(90 - \theta) = \frac{2x - r}{r}$

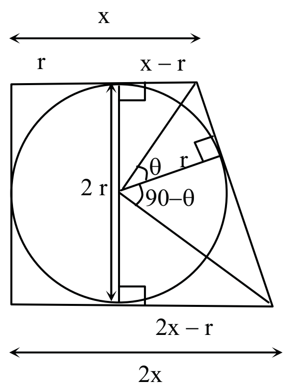
$\frac{x - r}{r} = \frac{r}{2x - r}$

$x(2x - 3r) = 0$

$x = \frac{3r}{2}$ …(2)

From (1) and (2)

$r = 2$.



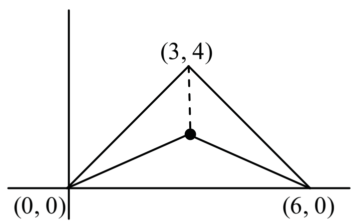
*51. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

Sol. **(C)**

Since, Δ is isosceles, hence centroid is the desired point.



52. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at (0, 1)
(B) variable radii and a fixed centre at (0, –1)
(C) fixed radius 1 and variable centres along the x-axis
(D) fixed radius 1 and variable centres along the y-axis

Sol. **(C)**

$$\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$$

$$\Rightarrow \int \frac{y}{\sqrt{1 - y^2}} dy = \int dx$$

$$\Rightarrow -\sqrt{1 - y^2} = x + c$$

$$\Rightarrow (x + c)^2 + y^2 = 1$$

centre (–c, 0); radius $\sqrt{c^2 - c^2 + 1} = 1$.

53. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^C \cap F^C \mid G)$ equals

(A) $P(E^C) + P(F^C)$ (B) $P(E^C) - P(F^C)$

(C) $P(E^C) - P(F)$ (D) $P(E) - P(F^C)$

Sol. **(C)**

$$P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)} = \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$$

$$= \frac{P(G)(1 - P(E) - P(F))}{P(G)} \quad [\because P(G) \neq 0]$$

$= 1 - P(E) - P(F)$

$= P(E^c) - P(F)$.

SECTION –II

Assertion – Reason Type

*This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT – 1 (Assertion) and STATEMENT -2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.*

54. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT -1 : For each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.

because

STATEMENT -2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

$f(x) = 2 + \cos x \ \forall \ x \in R$

Statement : 1

There exists a point $c \in [t, t + \pi]$ where $f'(c) = 0$

Hence, statement 1 is true.

Statement 2:

$f(t) = f(t + 2\pi)$ is true.

But statement 2 is not a correct explanation for statement 1.

55. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

STATEMENT -1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$

because

STATEMENT -2 : The vectors $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of the given planes.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

$3x - 6y - 2z = 15$

$2x + y - 2z = 5$

for $z = 0$, we get $x = 3$, $y = -1$

Direction vectors of plane are

$< 3 \ -6 \ -2 >$ and $<2 \ 1 \ -2>$

then the dr's of line of intersection of planes is $<14 \ 2 \ 15>$

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = \lambda$$

$\Rightarrow x = 14\lambda + 3 \quad y = 2\lambda - 1 \quad z = 15\lambda$

Hence, statement 1 is false.

But statement 2 is true.

*56. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT -1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

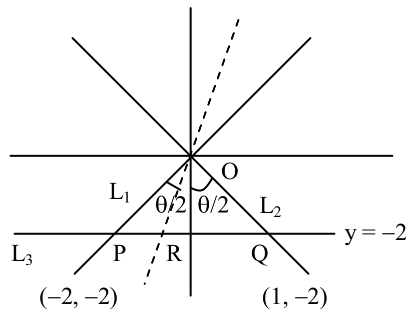
because

STATEMENT -2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(C)**

In ΔOPQ

Clearly $\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$



*57. STATEMENT -1 : The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line x = 1.

because

STATEMENT -2 : A parabola is symmetric about its axis.
(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is true; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**

$y = -\frac{x^2}{2} + x + 1$

$\Rightarrow\ y - \frac{3}{2} = -\frac{1}{2}(x-1)^2$

$\Rightarrow$ it is symmetric about x = 1.

SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs M_{58-60} and M_{61-63}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choice (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

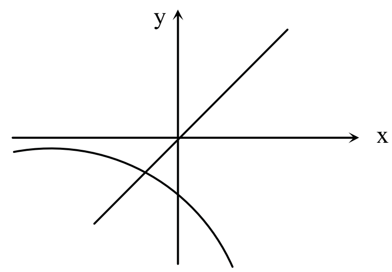
M_{58-60} : Paragraph for question Nos. 58 to 60

If a continuous f defined on the real line R, assumes positive and negative values in R then the equation f(x) = 0 has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative then the equation f(x) = 0 has a root in R.
Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

58. The line y = x meets $y = ke^x$ for $k \le 0$ at

(A) no point (B) one point
(C) two points (D) more than two points

Sol. **(B)**
Line y = x intersect the curve $y = ke^x$ at exactly one point when $k \le 0$.



59. The positive value of k for which $ke^x - x = 0$ has only one root is
(A) 1/e (B) 1
(C) e (D) $\log_e 2$

Sol. **(A)**
Let $f(x) = ke^x - x$
$f'(x) = ke^x - 1 = 0 \Rightarrow x = -\ln k$
$f''(x) = ke^x$
$f''(x)|_{x=-\ln k} = 1 > 0$
Hence $f(-\ln k) = 1 + \ln k$
For one root of given equation
$1 + \ln k = 0$
hence $k = \frac{1}{e}$.

60. For $k > 0$, the set of all values of k for which $k e^x - x = 0$ has two distinct roots is
(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$
(C) $\left(\frac{1}{e}, \infty\right)$ (D) (0, 1)

Sol. **(A)**

For two distinct roots $1 + \ln k < 0 \quad (k > 0)$
$\ln k < -1$
$k < \frac{1}{e}$
hence $k \in \left(0, \frac{1}{e}\right)$.

M_{61-63} : Paragraph for Question Nos. 61 to 63

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \ge 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

*61. Which one of the following statements is correct?
(A) $G_1 > G_2 > G_3 > \ldots$ (B) $G_1 < G_2 < G_3 < \ldots$
(C) $G_1 = G_2 = G_3 = \ldots$ (D) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

Sol. **(C)**

$$A_1 = \frac{a+b}{2}; G_1 = \sqrt{ab}; H_1 = \frac{2ab}{a+b}$$

$$A_n = \frac{A_{n-1} + H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}}, H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$$

Nitin M Sir (physics-iitjee.blogspot.com)

Clearly, $G_1 = G_2 = G_3 = \ldots = \sqrt{ab}$.

*62. Which of the following statements is correct?
(A) $A_1 > A_2 > A_3 > \ldots$ (B) $A_1 < A_2 < A_3 < \ldots$
(C) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$ (D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

Sol. **(A)**
A_2 is A.M. of A_1 and H_1 and $A_1 > H_1 \Rightarrow A_1 > A_2 > H_1$
A_3 is A.M. of A_2 and H_2 and $A_2 > H_2 \Rightarrow A_2 > A_3 > H_2$
$\therefore A_1 > A_2 > A_3 > \ldots$

*63. Which of the following statements is correct?
(A) $H_1 > H_2 > H_3 > \ldots$ (B) $H_1 < H_2 < H_3 < \ldots$
(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$ (D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

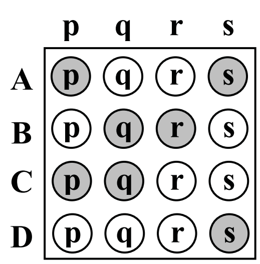
Sol. **(B)**
As above $A_1 > H_2 > H_1$, $A_2 > H_3 > H_2$
$\therefore H_1 < H_2 < H_3 < \ldots$

SECTION –IV

Matrix – Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows:



---

64. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$.
Match the conditions / expressions in **Column I** with statements in **Column II** and indicate your answers by darkening the appropriate bubbles in 4 × 4 matrix given in the ORS.

Column I	Column II
(A) If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

Sol. **A → p, r, s ; B → q, s ; C → q, s ; D → p, r, s**

$f(x) = \frac{(x-1)(x-5)}{(x-2)(x-3)}$

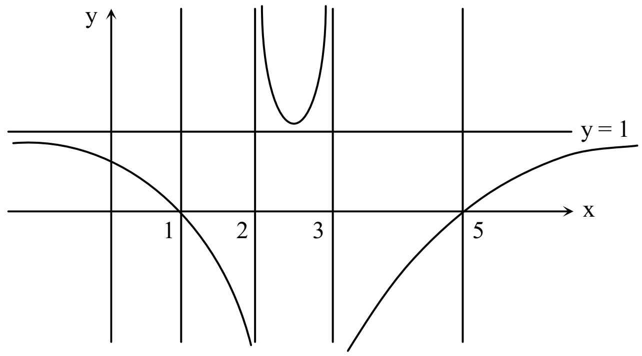
The graph of f(x) is shown

(A) If $-1 < x < 1$
$\Rightarrow 0 < f(x) < 1$

(B) If $1 < x < 2 \Rightarrow f(x) < 0$

(C) If $3 < x < 5 \Rightarrow f(x) < 0$

(D) If $x > 5 \Rightarrow 0 < f(x) < 1$



*65. Let (x, y) be such that

$$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}.$$

Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) If a = 1 and b = 0, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If a = 1 and b = 1, then (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If a = 1 and b = 2, then (x, y)	(r) lies on $y = x$
(D) If a = 2 and b = 2, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

Sol. **A → p ; B → q ; C → p ; D → s**

(A) If a = 1, b = 0
then $\sin^{-1}x + \cos^{-1}y = 0$
$\Rightarrow \sin^{-1}x = -\cos^{-1}y$
$\Rightarrow x^2 + y^2 = 1.$

(B) If a = 1 and b = 1, then

$\sin^{-1}x + \cos^{-1}y + \cos^{-1}xy = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x - \cos^{-1}y = \cos^{-1}xy$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{1-y^2} = xy$ (taking sine on both the sides)

(C) If a = 1, b = 2

$\Rightarrow \sin^{-1}x + \cos^{-1}y + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x + \cos^{-1}y = \sin^{-1}(2xy)$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{1-y^2} = 2xy$

$\Rightarrow x^2 + y^2 = 1$ (on squaring).

(D) If a = 2 and b = 2 then

$\sin^{-1}(2x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$

$\Rightarrow 2xy + \sqrt{1-4x^2}\sqrt{1-y^2} = 2xy$

$\Rightarrow (4x^2 - 1)(y^2 - 1) = 0.$

*66. Match the statements in **Column I** with the properties **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the **ORS**.

Column I		Column II	
(A)	Two intersecting circles	(p)	have a common tangent
(B)	Two mutually external circles	(q)	have a common normal
(C)	two circles, one strictly inside the other	(r)	do not have a common tangent
(D)	two branches of a hyperbola	(s)	do not have a common normal

Sol. **A → p, q ; B → p, q ; C → q, r ; D → q, r**

(A) When two circles are intersecting they have a common normal and common tangent.
(B) Two mutually external circles have a common normal and common tangent.
(C) When one circle lies inside of other then, they have a common normal but no common tangent.
(D) Two branches of a hyperbola have a common normal but no common tangent.

Nitin M Sir (physics-iitjee.blogspot.com)

FIITJEE Solutions to

IIT - JEE – 2008

(Paper – 1, Code–7)

Time: 3 hours ***M. Marks: 246***

Note: ***(i)*** The question paper consists of 3 parts (Part I : Mathematics, Part II : Physics, Part III : Chemistry). Each part has 4 sections.

(ii) **Section I** contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

(iii) **Section II** contains 4 multiple correct answer type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **one or more answers** are correct.

(vi) **Section III** contains 4 Reasoning type questions. Each question contains STATEMENT–1 and STATEMENT–2.
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

(iv) **Section IV** contains 3 sets of Linked Comprehension type questions. Each set consists of a paragraph followed by 3 questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

Marking Scheme:

(i) For each question in **Section I**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(ii) For each question in **Section II**, you will be awarded **4 Marks** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** for all other cases. It may be noted that there is **no negative marking** for **wrong answer**.

(iii) For each question in **Section III**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(iv) For each question in **Section IV**, you will be awarded **4 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

Mathematics

PART – I

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents
(A) four straight lines, when c = 0 and a, b are of the same sign
(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a
(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Sol. **(B)**

$(ax^2 + by^2 + c)\ (x^2 - 5xy + 6y^2) = 0$

$\Rightarrow ax^2 + by^2 + c = 0$ or $x^2 - 5xy + 6y^2 = 0$

$\Rightarrow x^2 + y^2 = \left(-\dfrac{c}{a}\right)$ iff $a = b$, $x - 2y = 0$ and $x - 3y = 0$

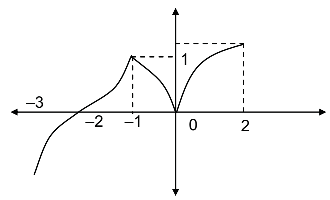
Hence the given equation represents two straight lines and a circle, when a = b and c is of sign opposite to that of a.

2. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

(A) 0
(B) 1
(C) 2
(D) 3

Sol. **(C)**

Local maximum at x = – 1
and local minimum at x = 0
Hence total number of local maxima and local minima is 2.



3. Let $g(x) = \dfrac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2$, m and n are integers, $m \ne 0$, $n > 0$, and let p be the left hand derivative of $|x - 1|$ at x = 1. If $\lim\limits_{x\to 1^+} g(x) = p$, then

(A) n = 1, m = 1
(B) n = 1, m = – 1
(C) n = 2, m = 2
(D) n > 2, m = n

Sol. **(C)**

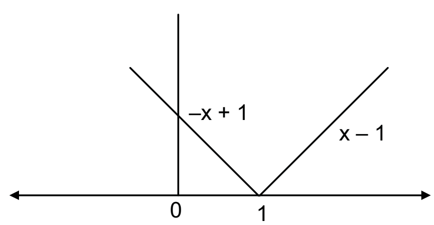
From graph, p = – 1

$\Rightarrow \lim\limits_{x\to 1^+} g(x) = -1$

$\Rightarrow \lim\limits_{h\to 0} g(1+h) = -1$

$\Rightarrow \lim\limits_{h\to 0}\left(\dfrac{h^n}{\log\cos^m h}\right) = -1$

$\Rightarrow \lim\limits_{h\to 0}\dfrac{n\cdot h^{n-1}}{m\cdot(-\tan h)} = -\left(\dfrac{n}{m}\right)\lim\limits_{h\to 0}\left(\dfrac{h^{n-1}}{\tan h}\right) = -1$, which holds if n = m = 2.



4. If $0 < x < 1$, then $\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{1/2}$ is equal to

(A) $\dfrac{x}{\sqrt{1+x^2}}$
(B) x
(C) $x\sqrt{1+x^2}$
(D) $\sqrt{1+x^2}$

Sol. **(C)**

$$\sqrt{1+x^2}\left[\left(x\cos\cot^{-1}x + \sin\cot^{-1}x\right)^2 - 1\right]^{1/2}$$

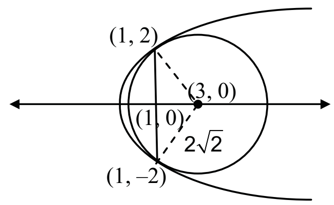
$$= \sqrt{1+x^2}\left[\left(x\cos\cos^{-1}\frac{x}{\sqrt{1+x^2}} + \sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\,(x^2+1-1)^{1/2} = x\sqrt{1+x^2}.$$

5. Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

Sol. **(B)**
The circle and the parabola touch each other at x = 1 i.e. at the points (1, 2) and (1, – 2) as shown in the figure.



6. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = 1/2$. Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol. **(A)**

$$\text{Volume} = \left|\hat{a}\cdot\left(\hat{b}\times\hat{c}\right)\right| = \sqrt{\begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}.$$

SECTION – II

Multiple Correct Answers Type

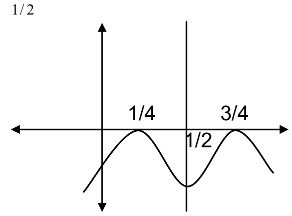
This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

7. Let f (x) be a non-constant twice differentiable function defined on (– ∞, ∞) such that f (x) = f (1 – x) and $f'\left(\frac{1}{4}\right) = 0$.
Then

(A) $f''(x)$ vanishes at least twice on [0, 1] (B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\, dx = 0$ (D) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t}\, dt$

Sol. **(A, B, C, D)**
f (x) = f (1 – x)
Put x = 1/2 + x



$$f\left(\frac{1}{2} + x\right) = f\left(\frac{1}{2} - x\right)$$

Hence f (x + 1/2) is an even function or f (x + 1/2) sin x is an odd function.
Also, $f'(x) = -f'(1 - x)$ and for x = 1/2, we have $f'(1/2) = 0$.

Also, $\int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt = -\int_{1/2}^{0} f(y)e^{\sin \pi y}dy$ (obtained by putting, 1 – t = y).

Since f′(1/4) = 0, f′ (3/4) = 0. Also f′ (1/2) = 0
⇒ f″ (x) = 0 atleast twice in [0, 1] (Rolle's Theorem)

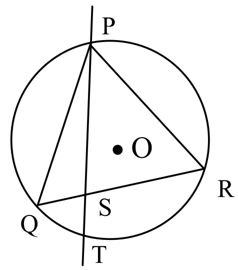
Nitin M Sir (physics-iitjee.blogspot.com)

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Sol. **(B, D)**



$PS \times ST = QS \times SR$

$$\frac{\frac{1}{PS}+\frac{1}{ST}}{2}>\sqrt{\frac{1}{PS}\times\frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$$

$$\frac{QS+SR}{2}>\sqrt{QS\times SR}$$

$$\frac{QR}{2}>\sqrt{QS\times SR} \quad \Rightarrow \quad \frac{1}{\sqrt{QS\times SR}}>\frac{2}{QR}$$

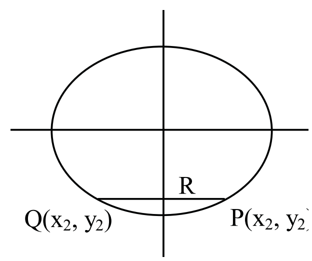
$$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}.$$

9. Let P (x_1, y_1) and Q (x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2+2\sqrt{3}\,y=3+\sqrt{3}$ (B) $x^2-2\sqrt{3}\,y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}\,y=3-\sqrt{3}$ (D) $x^2-2\sqrt{3}\,y=3-\sqrt{3}$

Sol. **(B, C)**



$$\frac{x^2}{4}+\frac{y^2}{1}=1$$

$b^2 = a^2(1 - e^2)$

$$\Rightarrow e=\frac{\sqrt{3}}{2}$$

$\Rightarrow P\left(\sqrt{3}, -\frac{1}{2}\right)$ and $Q\left(-\sqrt{3}, -\frac{1}{2}\right)$ (given y_1 and y_2 less than 0).

Co-ordinates of mid-point of PQ are

$R \equiv \left(0, -\frac{1}{2}\right)$.

$PQ = 2\sqrt{3}$ = length of latus rectum.

$\Rightarrow$ two parabola are possible whose vertices are $\left(0, -\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$ and $\left(0, \frac{\sqrt{3}}{2}-\frac{1}{2}\right)$.

Hence the equations of the parabolas are $x^2 - 2\sqrt{3}y = 3+\sqrt{3}$

and $x^2 + 2\sqrt{3}y = 3-\sqrt{3}$.

10. Let $S_n=\sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n=\sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n = 1, 2, 3, …. Then,

(A) $S_n<\frac{\pi}{3\sqrt{3}}$ (B) $S_n>\frac{\pi}{3\sqrt{3}}$

(C) $T_n<\frac{\pi}{3\sqrt{3}}$ (D) $T_n>\frac{\pi}{3\sqrt{3}}$

Sol. **(A, D)**

$$S_n < \lim_{n\to\infty} S_n = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\frac{1}{1+k/n+(k/n)^2}$$

$$= \int_0^1 \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$$

Now, $T_n > \frac{\pi}{3\sqrt{3}}$ as $h\sum_{k=0}^{n-1} f(kh) > \int_0^1 f(x)dx > h\sum_{k=1}^{n} f(kh)$

SECTION – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

---

11. Consider the system of equations $ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

STATEMENT – 1: The probability that the system of equations has a unique solution is 3/8.

and

STATEMENT – 2: The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

For unique solution $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$ where $a, b, c, d \in \{0, 1\}$

Total cases = 16.

Favorable cases = 6 (Either ad = 1, bc = 0 or ad = 0, bc = 1).

Probability that system of equations has unique solution is $\frac{6}{16} = \frac{3}{8}$ and system of equations has either unique solution or infinite solutions so that probability for system to have a solution is 1.

12. Consider the system of equations

$$x - 2y + 3z = -1$$
$$-x + y - 2z = k$$
$$x - 3y + 4z = 1$$

STATEMENT -1 : The system of equations has no solution for $k \neq 3$.

and

STATEMENT -2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. **(A)**

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

and $D_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = (3-k) = 0$ if $k = 3$

$$D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = (k-3) = 0 \text{ , if } k = 3$$

$$D_3 = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = (k-3) = 0 \text{ , if } k = 3$$

$\Rightarrow$ system of equations has no solution for $k \neq 3$.

13. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

STATEMENT -1 : $\lim\limits_{x\to 0}[g(x)\cot x - g(0)\text{cosec}x] = f''(0)$.

and

STATEMENT -2 : $f'(0) = g(0)$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

$f'(x) = g(x)\cos x + \sin x.g'(x)$
$\Rightarrow f'(0) = g(0)$
$f''(x) = 2g'(x)\cos x - g(x)\sin x + \sin x\, g''(x)$
$\Rightarrow f''(0) = 2g'(0) = 0$

But $\lim\limits_{x\to 0}\left[g(x)\cot x - g(0)\text{cosec } x\right] = \lim\limits_{x\to 0}\dfrac{g(x)\cos x - g(0)}{\sin x} = \lim\limits_{x\to 0}\dfrac{g'(x)\cos x - g(x)\sin x}{\cos x} = g'(0) = 0 = f''(0)$.

14. Consider three planes

$P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2$.

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT -1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT -2 : The three planes do not have a common point.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

The direction cosines of each of the lines L_1, L_2, L_3 are proportional to (0, 1, 1).

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 17

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.
If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.
If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

15. If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ (B) $-\frac{4\sqrt{2}}{7^3 3^2}$
(C) $\frac{4\sqrt{2}}{7^3 3}$ (D) $-\frac{4\sqrt{2}}{7^3 3}$

Sol. **(B)**
Differentiating the given equation, we get
$3y^2y' - 3y' + 1 = 0$
$\Rightarrow y'\left(-10\sqrt{2}\right) = -\frac{1}{21}$
Differentiation again we get $6yy'^2 + 3y^2y'' - 3y'' = 0$
$\Rightarrow f''\left(-10\sqrt{2}\right) = -\frac{6.2\sqrt{2}}{(21)^4} = -\frac{4\sqrt{2}}{7^3 3^2}$.

16. The area of the region bounded by the curves $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx + bf(b) - af(a)$ (B) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$ (D) $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$

Sol. **(A)**

The required area $= \int_a^b f(x)dx = xf(x)\Big|_a^b - \int_a^b xf'(x)dx$

$$= bf(b) - af(a) + \int_a^b \frac{x}{3\left[(f(x)^2 - 1)\right]}dx.$$

17. $\int_{-1}^{1} g'(x)dx =$

(A) $2g(-1)$ (B) 0
(C) $-2g(1)$ (D) $2g(1)$

Sol. **(D)**
We have $y' = \frac{1}{3\left(1 - \left(f(x)^2\right)\right)}$ which is even

Hence $\int_{-1}^{1} g'(x) = g(1) - g(-1) = 2g(1)$.

Paragraph for Question Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

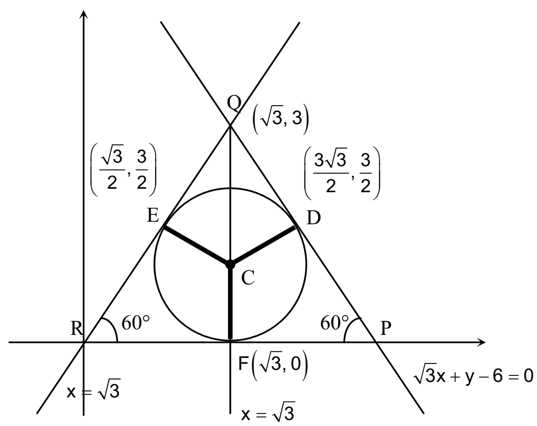
(A) $\left(x-2\sqrt{3}\right)^2 + \left(y-1\right)^2 = 1$

(B) $\left(x-2\sqrt{3}\right)^2 + \left(y+\frac{1}{2}\right)^2 = 1$

(C) $\left(x-\sqrt{3}\right)^2 + \left(y+1\right)^2 = 1$

(D) $\left(x-\sqrt{3}\right)^2 + \left(y-1\right)^2 = 1$

Sol. **(D)**



Equation of CD is $\dfrac{x-\frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{y-\frac{3}{2}}{\frac{1}{2}} = -1$

$\Rightarrow C \equiv \left(\sqrt{3}, 1\right)$

Equation of the circle is $\left(x-\sqrt{3}\right)^2 + \left(y-1\right)^2 = 1$.

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$

(B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Sol. **(A)**

Since the radius of the circle is 1 and C $\left(\sqrt{3}, 1\right)$, coordinates of $F \equiv \left(\sqrt{3}, 0\right)$

Equation of CE is $\dfrac{x-\sqrt{3}}{-\frac{\sqrt{3}}{2}} = \dfrac{y-1}{\frac{1}{2}} = 1$

$\Rightarrow E \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$.

20. Equation of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$
(B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$
(C) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$
(D) $y = \sqrt{3}x,\ y = 0$

Sol. **(D)**
Equation of QR is $y - 3 = \sqrt{3}\left(x - \sqrt{3}\right)$
$\Rightarrow\ y = \sqrt{3x}$
Equation of RP is y = 0.

Paragraph for Question Nos. 21 to 23

Let A, B, C be three sets of complex numbers as defined below
$A = \{z : \text{Im}z \ge 1\}$
$B = \{z : |z - 2 - i| = 3\}$
$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$.

21. The number of elements in the set $A \cap B \cap C$ is
(A) 0
(B) 1
(C) 2
(D) ∞

Sol. **(B)**
A = Set of points on and above the line y = 1 in the Argand plane.
B = Set of points on the circle $(x - 2)^2 + (y - 1)^2 = 3^2$
$C = \text{Re}(1 - i)\,z = \text{Re}((1 - i)(x + iy))$
$\Rightarrow x + y = \sqrt{2}$
Hence $(A \cap B \cap C)$ = has only one point of intersection.

22. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between
(A) 25 and 29
(B) 30 and 34
(C) 35 and 39
(D) 40 and 44

Sol. **(C)**
The points (– 1, 1) and (5, 1) are the extremities of a diameter of the given circle .
Hence $|z + 1 - i|^2 + |z - 5 - i|^2 = 36$.

23. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between
(A) –6 and 3
(B) –3 and 6
(C) –6 and 6
(D) –3 and 9

Sol. **(D)**
$\left||z| - |w|\right| < |z - w|$
and $|z - w|$ = Distance between z and w
z is fixed. Hence distance between z and w would be maximum for diametrically opposite points.
$\Rightarrow |z - w| < 6$
$\Rightarrow -6 < |z| - |w| < 6$
$\Rightarrow -3 < |z| - |w| + 3 < 9$.

Physics

PART - II

Useful Data:

Plank's constant $h = 4.1 \times 10^{-15}$ eV.s

Velocity of light $c = 3 \times 10^{8}$ m/s

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

24. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(A) 30° for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30° for both the colours

Sol. **(A)**

At minimum deviation for any wavelength

$r_1 = r_2 = A/2$, Because $r_1 + r_2 = A$

25. Which one of the following statements is **WRONG** in the context of X-rays generated from a X-ray tube?

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.
(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

Sol. **(B)**

$\lambda_{cutoff} = \dfrac{hc}{eV}$ (independent of atomic number)

26. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\dfrac{1}{T}$
(B) $\dfrac{2}{T}$
(C) $\dfrac{3}{T}$
(D) $\dfrac{4}{T}$

Sol. **(C)**

$$\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right)$$

PT^2 = constant

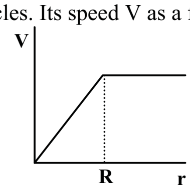
$$\frac{nRT}{V}T^2 = \text{constant}$$

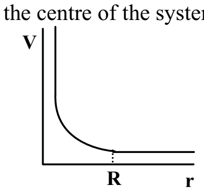
$$\therefore \quad \gamma = \frac{3}{T}$$

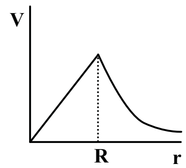
27. A spherically symmetric gravitational system of particles has a mass density

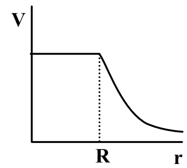
$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A)


(B)


(C)


(D)


Sol. **(C)**

$G\frac{4}{3}\pi\rho_0 r = \frac{v^2}{r}$, $\quad r \le R$

Hence, $v \propto r$

$\left(\frac{G\frac{4}{3}\pi\rho_0 R^3}{r^2}\right) = \left(\frac{v^2}{r}\right)$, $\quad r \ge R$

Hence $\quad v \propto \frac{1}{\sqrt{r}}$

28. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and /or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II and III, respectively,

(A) $E_I = 0$

(B) E_I is minimum

(C) $E_I = E_{II}$

(D E_{II} is maximum

Sol. **(B)**

$g = 4\pi^2\left(\frac{\ell}{T^2}\right)$

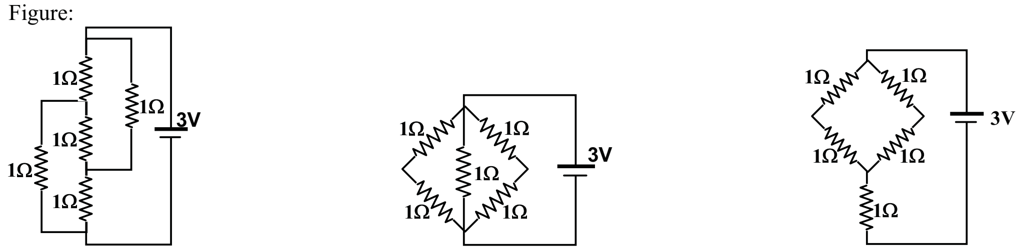
$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

$\Rightarrow E = \frac{\Delta \ell}{\ell} + 2\frac{\Delta t}{t}$, greater the value of t, lesser the error

Hence, fractional error in the Ist observation is minimum

29. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power dissipated by the configuration R1, R2 and R3 is P1, P2 and P3, respectively, then

Figure:



R1　　R2　　R3

(A) P1 > P2 >P3　　(B) P1 > P3 >P2

(C) P2 > P1 >P3　　(D) P3 >P2 > P1

Sol. **(C)**

$P = \frac{V^2}{R}$

$R_1 = 1\ \Omega$, $R_2 = 1/2\ \Omega$, $R_3 = 2\ \Omega$

$\therefore \quad P_2 > P_1 > P_3$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

30. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

(A) If $d = \lambda$, the screen will contain only one maximum

(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Sol. **(A) & (B)**

For at least one maxima, $\sin\theta = \lambda/d$

If $\lambda = d$, $\sin\theta = 1$ and $y \to \infty$

If $\lambda < d < 2d$, $\sin\theta$ exists and y is finite

31. The balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option(s) is (are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{p}_1' = c_1\hat{k}$
$\vec{p}_2' = c_2\hat{k}$

(C) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$
$\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

Sol. **(A) & (D)**

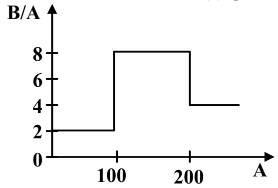
$\vec{P} = \vec{P}_1 + \vec{P}_2 = \vec{P}_1' + \vec{P}_2'$

$F_{ext} = 0$

$|\vec{P}| = 0$

32. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.

Figure:

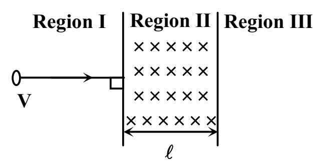


(A) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy
(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments
(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

Sol. **(B) & (D)**
If $(BE)_{final} - (BE)_{initial} > 0$
Energy will be released.

33. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s).

Figure:



(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) Time spent in Region II is same for any velocity V as long as the particle returns to Region I

Sol. **(A), (C) & (D)**

SECTION – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

34. STATEMENT-1
The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT-2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(A)**

35. STATEMENT-1
Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2
By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(D)**

$$a = \frac{mgR^2 \sin\theta}{I_{cm} + mR^2}$$

36. STATEMENT-1
In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT-2
Resistance of a metal increases with increase in temperature.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(D)**

$$R_{unknown} = \frac{R(100-\ell)}{\ell}$$

37. STATEMENT-1
An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT-2
An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(A)**

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 38 to 40

In a mixture of H–He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

38. The quantum number n of the state finally populated in He^+ ions is
(A) 2 (B) 3 (C) 4 (D) 5

Sol. **(C)**

39. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m
(C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

Sol. **(C)**

$E_4 - E_3 = \frac{hc}{\lambda}$ [λ: visible region]

40. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol. **(A)**

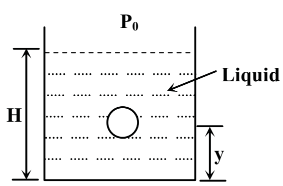
$KE \propto Z^2/n^2$

$$\frac{KE_H}{KE_{He}} = \left(\frac{Z_H}{2}\Big/\frac{Z_{He}}{2}\right)^2 = \frac{1}{4}$$

Paragraph for Question Nos. 41 to 43

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure:



41. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it
(A) Only the force of gravity
(B) The force due to gravity and the force due to the pressure of the liquid
(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
(D) The force due to gravity and the force due to viscosity of the liquid

Sol. **(D)**

Buoyant force is resultant of pressure-force of liquid.

42. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{2/5}$ (B) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\frac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{3/5}$ (D) $T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{3/5}$

Sol. **(B)**

$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$

$P_1 = P_0 + \rho_\ell gH, \quad T_1 = T_0$

$P_2 = P_0 + \rho_\ell g(H - y)$

43. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \dfrac{(P_0 + \rho_\ell gH)^{2/5}}{(P_0 + \rho_\ell gy)^{7/5}}$

(B) $\dfrac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{2/5}[P_0 + \rho_\ell g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0 \dfrac{(P_0 + \rho_\ell gH)^{3/5}}{(P_0 + \rho_\ell gy)^{8/5}}$

(D) $\dfrac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell gH)^{3/5}[P_0 + \rho_\ell g(H-y)]^{2/5}}$

Sol. **(B)**

$\rho_\ell Vg = \text{Buoyancy force} = \rho_\ell g \dfrac{nRT_2}{P_2}$

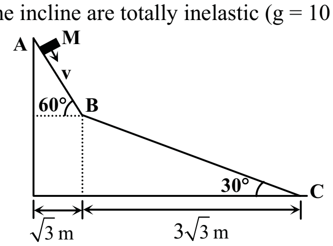
$T_2 = T_0\left[\dfrac{P_0 + \rho_\ell g(H-y)}{P + \rho_\ell gH}\right]^{2/5}$

$P_2 = P_0 + \rho_\ell g(H - y)$

Paragraph for Question Nos. 44 to 46

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$).

Figure:



44. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

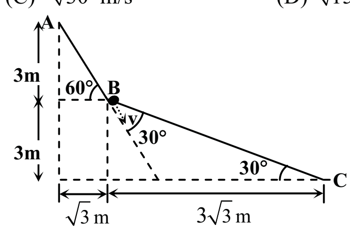
Sol. **(B)**

Along the plane velocity just before collision

$v = \sqrt{2g(3)} = \sqrt{60}$ m/s

Along the plane velocity just after collision

$v_B = v \cos 30° = \sqrt{45}$ m/s



45. The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

Sol. **(B)**

$mg(3) = \dfrac{1}{2}m\left(v_C^2 - v_B^2\right) \Rightarrow \quad v_C = \sqrt{105}$ m/s

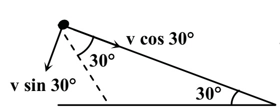
46. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

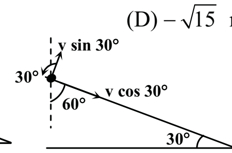
Sol. **(C)**

$v_y = v \sin 30° \cos 30° - v \cos 30° \cos 60°$

$v_y = 0$



Before collision



After collision

Chemistry

PART – III
SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of
(A) nitrogen (B) oxygen
(C) carbon dioxide (D) argon

Sol. **(B)**
Ag dissociates in a solution of NaCN in the presence of air, and forms sodium argentocyanide.

$$4Ag + 8NaCN + 2H_2O + O_2 \longrightarrow 4Na\left[Ag(CN)_2\right] + 4NaOH$$

48. 2.5 mL of $\frac{2}{5}$M weak monoacidic base $\left(K_b = 1\times10^{-12} \text{ at } 25^0C\right)$ is titrated with $\frac{2}{15}$M HCl in water at 25°C. The concentration of H^+ at equivalence point is $\left(K_w = 1\times10^{-14} \text{at } 25^0C\right)$
(A) 3.7×10^{-13}M (B) 3.2×10^{-7}M
(C) 3.2×10^{-2}M (D) 2.7×10^{-2}M

Sol. **(D)**

$$BOH + HCl \longrightarrow \underset{C}{BCl} + H_2O$$

$$\underset{C(1-h)}{B^+} + H_2O \rightleftharpoons \underset{Ch}{BOH} + \underset{Ch}{H^+}$$

Volume of HCl used $= \dfrac{2.5\times\frac{2}{5}}{2/15} = 7.5\,\text{ml}$

Concentration of Salt, C $= \dfrac{2.5\times\frac{2}{5}}{10} = 0.1\,M$

$$\therefore \frac{Ch^2}{1-h} = \frac{K_w}{K_b}$$

Solving h = 0.27

$\left[H^+\right] = Ch = 0.1\times0.27 = 2.7\times10^{-2}$ M

49. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reactions is
(A) 0.5 mol^{-1} dm^3 (B) 1.0 mol dm^{-3}
(C) 1.5 mol dm^{-3} (D) 2.0 mol^{-1} dm^3

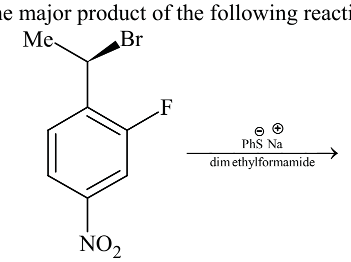
Sol. **(A)**

$$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40}$$

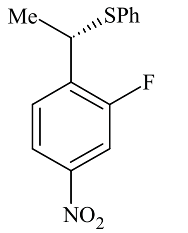
$$k_0 = \frac{A_0}{2t_{1/2}} = \frac{1.386}{2\times20}$$

$$\frac{k_1}{k_0} = \frac{0.693}{40}\times\frac{40}{1.386} = \frac{0.693}{1.386} = 0.5\ \text{mol}^{-1}\text{litre}$$

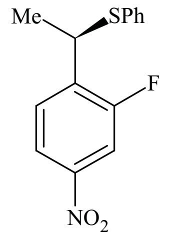
50. The major product of the following reaction is



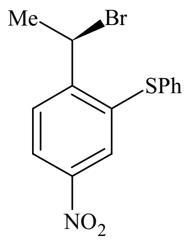
(A)



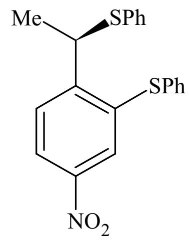
(B)



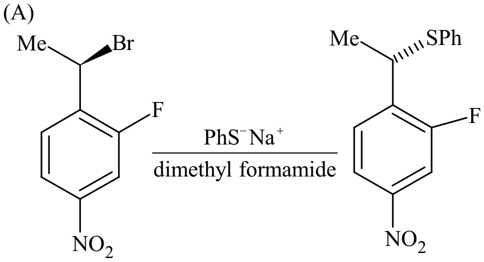
(C)



(D)



Sol. (A)



It is easier to do nucleophilic substitution on alkyl halides than on aryl halides.

51. Hyperconjugation involves overlap of the following orbitals

(A) σ - σ (B) σ - p

(C) p – p (D) π - π

Sol. **(B)**

Hyperconjugation involves overlap of $\sigma - p$ orbitals.

52. Aqueous solutions of $Na_2S_2O_3$ on reaction with Cl_2 gives

(A) $Na_2S_4O_6$ (B) $NaHSO_4$

(C) NaCl (D) NaOH

Sol. **(B)**

$$Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY OR MORE** is/are correct.

53. A gas described by van der Waal's equation
(A) behaves similar to an ideal gas in the limit of large molar volumes
(B) behaves similar to an ideal gas in the limit of large pressures
(C) is characterized by van der Waal's coefficients that are dependent on the identify of the gas but are independent of the temperature
(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally

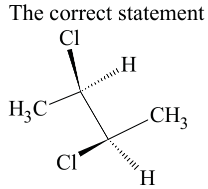
Sol. **(A, C, D)**

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

At low pressure, when the sample occupies a large volume, the molecules are so far apart for most of the time that the intermolecular forces play no significant role, and the gas behaves virtually perfectly.

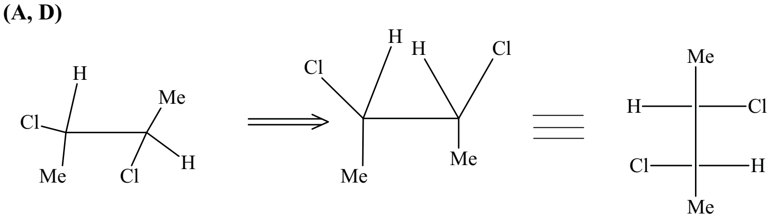
a and b are characteristic of a gas and are independent of temperature. The term $\left(P+\frac{n^2a}{V^2}\right)$ represents the pressure exerted by an ideal gas while P represents the pressure exerted by a real gas.

54. The correct statement (s) about the compound given below is (are)

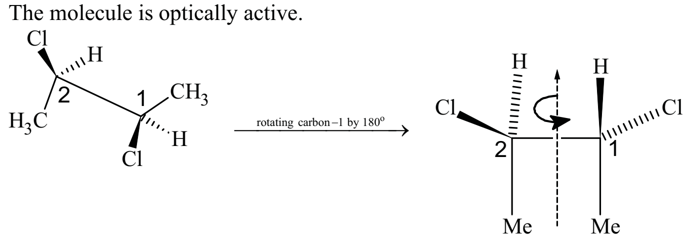


(A) The compound is optically active
(B) The compound possesses centre of symmetry
(C) The compound possesses plane of symmetry
(D) The compound possesses axis of symmetry

Sol. **(A, D)**

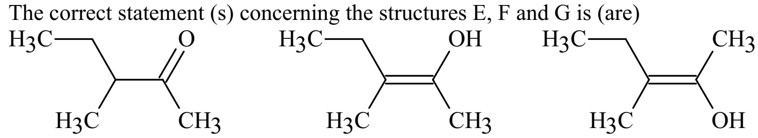


The molecule is optically active.



The molecule possesses an axis of symmetry (C_2) perpendicular to the C – C bond.

55. The correct statement (s) concerning the structures E, F and G is (are)



(E) (F) (G)

(A) E, F and G are resonance structures
(B) E, F and E, G are tautomers
(C) F and G are geometrical isomers
(D) F and G are diasteromers

Sol. **(B), (C), (D)**

56. A solution of colourless salt **H** on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt (s) **H** is (are)

(A) NH_4NO_3
(B) NH_4NO_2
(C) NH_4Cl
(D) $(NH_4)_2SO_4$

Sol. **(A, B)**

$$NH_4NO_3 + NaOH \longrightarrow NH_3 + NaNO_3 + H_2O$$
$$7NaOH + NaNO_3 + 4Zn \rightarrow 4Na_2ZnO_2 + NH_3 + 2H_2O$$
$$NH_4NO_2 + NaOH \rightarrow NaNO_2 + NH_3 + H_2O$$
$$3Zn + 5NaOH + NaNO_2 \rightarrow 3Na_2ZnO_2 + NH_3 + H_2O$$

SECTION – III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

57. **STATEMENT-1:** For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

and

STATEMENT-2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A) STATEMENT – 1 is True, STATEMENT-2 is True; STATEMENT -2 is correct explanation for STATEMENT-1
(B) STATEMENT – 1 is True, STATEMENT-2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT – 1 is True, STATEMENT-2 is False
(D) STATEMENT – 1 is False, STATEMENT-2 is True

Sol. **(D)**

At equilibrium $\Delta G = 0$, ΔG^{o} of a reaction may or may not be zero.

For a spontaneous process $\Delta G < 0$

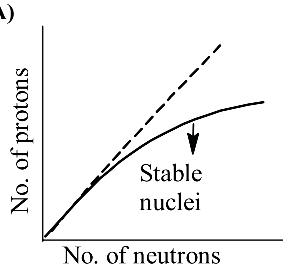
58. **STATEMENT-1:** The plot of atomic number (y-axis) versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from the line of 45° slope as the atomic number is increased.

and

STATEMENT-2: Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons and neutrons in heavier nuclides.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(A)**



If the curve does not bend down towards the x axis then the proton-proton repulsion would overcome the attractive force of proton and neutron. Therefore, the curve bends down.

Nitin M Sir (physics-iitjee.blogspot.com)

59. **STATEMENT-1:** Bromobenzene upon reaction with Br_2/Fe gives 1, 4 – dibromobenzene as the major product.

and

STATEMENT-2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(C)**
In bromobenzene, it is the mesomeric effect which directs the incoming electrophile.

60. **STATEMENT-1:** Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.
and
STATEMENT-2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(C)**
The lower oxidation states for the group 14 elements are more stable for the heavier member of the group due to inert pair effect.

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 61 to 63

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water. Nitrates are difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorus.

61. Among the following, the correct statement is
(A) Phosphates have no biological significance in humans
(B) Between nitrates and phosphates, phosphates are less abundant in earth's crust
(C) Between nitrates and phosphates, nitrates are less abundant in earth's crust
(D) Oxidation of nitrates is possible in soil

Sol. **(C)**

62. Among the following, the correct statement is
(A) Between NH_3 and PH_3, NH_3 is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional
(B) Between NH_3 and PH_3, PH_3 is better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(D) Between NH_3 and PH_3, PH_3 is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional

Sol. **(C)**
On going from top to bottom in nitrogen group the bond angle decreases due to more p-character in the bond pair and subsequently more s-character in lone pair orbital.

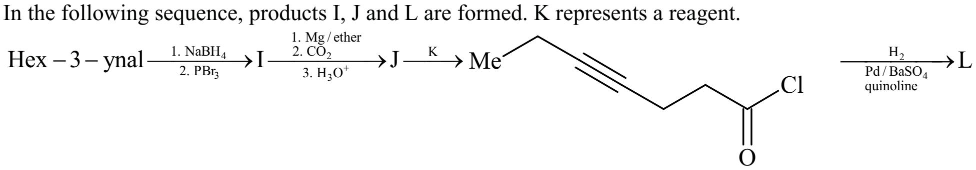
Nitin M Sir (physics-iitjee.blogspot.com)

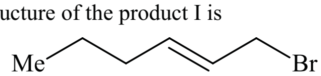
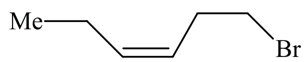
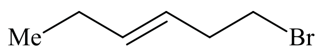
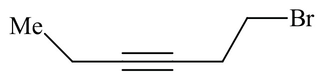
63. White phosphorus on reaction with NaOH gives PH_3 as one of the products. This is a
(A) dimerization reaction
(B) disproportionation reaction
(C) condensation reaction
(D) precipitation reaction

Sol. **(B)**
$$P_4 + 3NaOH + 3H_2O \longrightarrow 3NaH_2PO_2 + PH_3$$

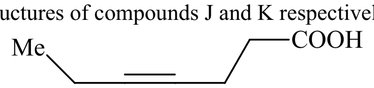
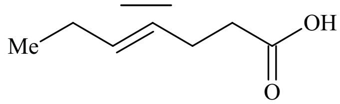
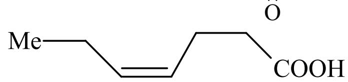
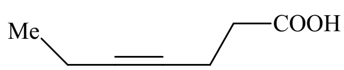
Paragraph for Question Nos. 64 to 66

In the following sequence, products I, J and L are formed. K represents a reagent.

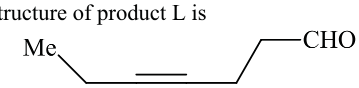
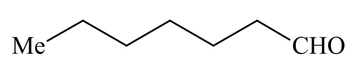
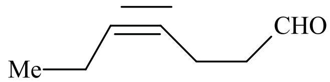
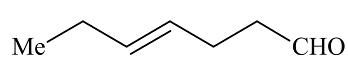


64. The structure of the product I is
(A) 
(B) 
(C) 
(D) 

Sol. **(D)**
$$CH_3-CH_2-C\equiv C-CH_2-CHO \xrightarrow[\text{2. PBr}_3]{\text{1. NaBH}_4} CH_3-CH_2-C\equiv C-CH_2-CH_2Br$$

65. The structures of compounds J and K respectively are
(A)  and $SOCl_2$
(B)  and SO_2Cl_2
(C)  and $SOCl_2$
(D)  and CH_3SO_2Cl

Sol. **(A)**
$$CH_3-CH_2-C\equiv C-CH_2-CH_2Br \xrightarrow[\text{3. H}_3\text{O}^+]{\text{1. Mg / Ether, 2. CO}_2} J \xrightarrow{K} CH_3-CH_2-C\equiv C-CH_2-COCl$$
$J = CH_3-CH_2-C\equiv C-CH_2-COOH$
$K = SOCl_2$
Hence, (A) is the correct answer.

66. The structure of product L is
(A) 
(B) 
(C) 
(D) 

Sol. **(C)**

Paragraph for Question Nos. 67 to 69

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute molecules are added to get homogeneous solution. These are called colligative properties. Applications of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol and water mixture as anti-freezing liquid in the radiator of automobiles.

A solution M is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9.

Given: Freezing point depression constant of water $\left(K_f^{water}\right) = 1.86\ K\ kg\ mol^{-1}$

Freezing point depression constant of ethanol $\left(K_f^{ethanol}\right) = 2.0\,K\ kg\ mol^{-1}$

Boiling point elevation constant of water $\left(K_b^{water}\right) = 0.52\,K\ kg\ mol^{-1}$

Boiling point elevation constant of ethanol $\left(K_b^{ethanol}\right) = 1.2\,K\ kg\ mol^{-1}$

Standard freezing point of water = 273 K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be non-volatile and non-dissociative.

67. The freezing point of the solution **M** is

(A) 268.7 K
(B) 268.5 K
(C) 234.2 K
(D) 150.9 K

Sol. **(D)**

$$\Delta T_f = K_f \times m$$

$$2 \times \frac{0.1}{0.9 \times 46} \times 1000 = 4.83\ K$$

Freezing point of solution M = 155.7 – 4.83 = 150.87 K $\approx$ 150.9 K

68. The vapour pressure of the solution **M** is

(A) 39.3 mm Hg
(B) 36.0 mm Hg
(C) 29.5 mm Hg
(D) 28.8 mm Hg

Sol. (B)

P = 0.9 × 40 = 36 mm Hg

69. Water is added to the solution **M** such that the fraction of water in the solution becomes 0.9. The boiling point of this solution is

(A) 380.4 K
(B) 376.2 K
(C) 375.5 K
(D) 354.7 K

Sol. **(B)**

$$\Delta T_b = K_b \times m$$

$$= 0.52 \times \frac{0.1}{0.9 \times 18} \times 1000 = 3.2\ K$$

T_b = 373 + 3.2 = 376.2 K

FIITJEE Solutions to

IIT - JEE – 2008

(Paper – 2, Code–4)

Time: 3 hours | ***M. Marks: 243***

Note: (i) The question paper consists of 3 parts (Part I : Mathematics, Part II : Physics, Part III : Chemistry). Each part has 4 sections.

(ii) **Section I** contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

(iii) **Section II** contains 4 questions. Each question contains STATEMENT–1 and STATEMENT–2.
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

(iv) **Section III** contains 3 sets of Linked Comprehension type questions. Each set consists of a paragraph followed by 3 questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

(v) **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column. The answers to these questions have to be appropriately bubbled in the ORS as per the instructions given at the beginning of the section.

Marking Scheme:

(i) For each question in **Section I**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(ii) For each question in **Section II**, you will be awarded **3 Marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.

(iii) For each question in **Section III**, you will be awarded **4 Marks** if you darken only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(iv) For each question in **Section IV**, you will be awarded **6 Marks** if you have darken ALL the bubble corresponding ONLY to the correct answer or **awarded 1 mark each** for correct bubbling of answer in any row. **No negative mark will be awarded for an incorrectly bubbled answer**.

Mathematics

PART – I

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is
(A) 2, 4 or 8 (B) 3, 6 or 9

(C) 4 or 8 (D) 5 or 10

Sol. **(D)**

$P(A \cap B) = \frac{4}{10} \times \frac{p}{10} = \frac{2p/5}{10}$

$\Rightarrow \frac{2p}{5}$ is an integer

$\Rightarrow p = 5$ or 10.

2. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ (B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

Sol. **(B)**

$$\int_0^{\pi/4}\left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$$

$$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} - \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}}\right)dx = \int \frac{\left(1+\tan\frac{x}{2}\right)-\left(1-\tan\frac{x}{2}\right)}{\sqrt{1-\tan^2\frac{x}{2}}}dx$$

$$= \int_0^{\pi/4} \frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx = \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt \text{ as } \tan\frac{x}{2} = t.$$

3. Consider three points $P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$ and $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment QP (D) P, Q, R are non-collinear

Sol. **(D)**

$P \equiv (-\sin(\beta-\alpha), -\cos\beta) \equiv (x_1, y_1)$

$Q \equiv (\cos(\beta-\alpha), \sin\beta) \equiv (x_2, y_2)$

and $R \equiv (x_2\cos\theta + x_1\sin\theta, y_2\cos\theta + y_1\sin\theta)$

We see that $T \equiv \left(\frac{x_2\cos\theta + x_1\sin\theta}{\cos\theta+\sin\theta}, \frac{y_2\cos\theta + y_1\sin\theta}{\cos\theta+\sin\theta}\right)$

and P, Q, T are collinear

$\Rightarrow$ P, Q, R are non-collinear.

4. Let $I = \int\frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J = \int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. Then, for an arbitrary constant C, the value of J – I equals

(A) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$ (B) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$ (D) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

Sol. **(C)**

$$J - I = \int\frac{e^x(e^{2x}-1)}{e^{4x}+e^{2x}+1}dx = \int\frac{(z^2-1)}{z^4+z^2+1}dz \quad \text{where } z = e^x$$

$$= \int \frac{\left(1-\frac{1}{z^2}\right)dz}{\left(z+\frac{1}{z}\right)^2 - 1} = \frac{1}{2}\ln\left(\frac{e^x + e^{-x} - 1}{e^x + e^{-x} + 1}\right) + c$$

$$\therefore\ J - I = \frac{1}{2}\ln\left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right) + c\ .$$

5. Let $g(x) = \log(f(x))$ where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = x\, f(x)$. Then, for $N = 1, 2, 3, \ldots$,

$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$

(A) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$ (B) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$ (D) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

Sol. **(A)**

$g(x + 1) = \log(f(x + 1)) = \log x + \log(f(x))$

$\qquad = \log x + g(x)$

$\Rightarrow g(x + 1) - g(x) = \log x$

$\Rightarrow g''(x + 1) - g''(x) = -\frac{1}{x^2}$

$g''\left(1+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4$

$g''\left(2+\frac{1}{2}\right) - g''\left(1+\frac{1}{2}\right) = -\frac{4}{9}$

….

….

$g''\left(N+\frac{1}{2}\right) - g''\left(N-\frac{1}{2}\right) = -\frac{4}{(2N-1)^2}$

Summing up all terms

Hence, $g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left(1+\frac{1}{9}+\cdots+\frac{1}{(2N-1)^2}\right)$.

6. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$ (B) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+\hat{a}\cdot\hat{b})^{1/2}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$ (D) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1+2\hat{a}\cdot\hat{b})^{1/2}$

Sol. **(A)**

$\left|\overrightarrow{OP}\right| = \left|\hat{a}\cos t + \hat{b}\sin t\right|$

$= \left(\cos^2 t + \sin^2 t + 2\cos t\sin t\ \hat{a}\cdot\hat{b}\right)^{1/2}$

$= \left(1 + 2\cos t\ \sin t\,\hat{a}\cdot\hat{b}\right)^{1/2}$

$= \left(1 + \sin 2t\,\hat{a}\cdot\hat{b}\right)^{1/2}$

Nitin M Sir (physics-iitjee.blogspot.com)

$\therefore \left|\overline{OP}\right|_{max} = \left(1 + \hat{a}\cdot\hat{b}\right)^{1/2}$ when, $t = \frac{\pi}{4}$

$$\hat{u} = \frac{\hat{a}+\hat{b}}{\sqrt{2}\frac{\left|\hat{a}+\hat{b}\right|}{\sqrt{2}}}$$

$$\Rightarrow \hat{u} = \frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|}.$$

7. Let the function $g: (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is

(A) even and is strictly increasing in $(0, \infty)$
(B) odd and is strictly decreasing in $(-\infty, \infty)$
(C) odd and is strictly increasing in $(-\infty, \infty)$
(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

Sol. **(C)**

$g(u) = 2\tan^{-1}\left(e^u\right) - \frac{\pi}{2}$

$= 2\tan^{-1}e^u - \tan^{-1}e^u - \cot^{-1}e^u = \tan^{-1}e^u - \cot^{-1}e^u$

$g(-x) = -g(x)$

$\Rightarrow g(x)$ is odd

and $g'(x) > 0 \Rightarrow$ increasing.

8. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}\,x - 4\sqrt{2}\,y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}} - 1$

(C) $1 + \sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}} + 1$

Sol. **(B)**

Hyperbola is $\frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$

$a = 2,\ b = \sqrt{2}$

$e = \sqrt{\frac{3}{2}}$

Area $= \frac{1}{2}a(e-1)\times\frac{b^2}{a} = \frac{1}{2}\frac{(\sqrt{3}-\sqrt{2})\times 2}{\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})}{\sqrt{2}}$

$\Rightarrow$ Area $= \left(\sqrt{\frac{3}{2}} - 1\right)$.

9. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) $6 + 7i$ (B) $-7 + 6i$
(C) $7 + 6i$ (D) $-6 + 7i$

Sol. **(D)**

$z_0 \equiv (1 + 2i)$
$z_1 \equiv (6 + 5i)$
$z_2 \equiv (-6 + 7i)$.

SECTION – II

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

10. Consider
$L_1 : 2x + 3y + p - 3 = 0$
$L_2 : 2x + 3y + p + 3 = 0$,
where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.
STATEMENT–1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.
and
STATEMENT–2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1.
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol. **(C)**
Circle $\equiv (x + 3)^2 + ((y - 5)^2 = 4$
Distance between L_1 and L_2
$\Rightarrow \dfrac{6}{\sqrt{13}} <$ radius
$\Rightarrow$ statement (2) is false
But statement (1) is correct.

11. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \dfrac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

STATEMENT–1 : $(p^2 - q)(b^2 - ac) \geq 0$
and
STATEMENT–2 : $b \neq pa$ or $c \neq qa$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1.
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol. **(B)**
Suppose roots are imaginary then $\beta = \bar{\alpha}$
and $\dfrac{1}{\beta} = \bar{\alpha} \Rightarrow \beta = \dfrac{1}{\beta}$ not possible
$\Rightarrow$ roots are real $\Rightarrow (p^2 - q)(b^2 - ac) \geq 0$
$\Rightarrow$ statement (1) is correct.
$\dfrac{-2b}{a} = \alpha + \dfrac{1}{\beta}$ and $\dfrac{\alpha}{\beta} = \dfrac{c}{a}$, $\alpha + \beta = -2p$, $\alpha\beta = q$
If $\beta = 1$, then $\alpha = q \Rightarrow c = qa$(not possible)
also $\alpha + 1 = \dfrac{-2b}{a} \Rightarrow -2p = \dfrac{-2b}{a} \Rightarrow b = ap$(not possible)
$\Rightarrow$ statement (2) is correct but it is not the correct explanation.

Nitin M Sir (physics-iitjee.blogspot.com)

12. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT–1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.
and
STATEMENT–2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1.
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol. **(C)**
$b_1 = a_1$, $b_2 = a_1 + a_2$, $b_3 = a_1 + a_2 + a_3$, $b_4 = a_1 + a_2 + a_3 + a_4$
Hence b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. nor in H.P.

13. Let a solution $y = y(x)$ of the differential equation
$x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

STATEMENT–1 : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$

and

STATEMENT–2 : $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

(A) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is** a correct explanation for STATEMENT–1
(B) STATEMENT–1 is True, STATEMENT–2 is True; STATEMENT–2 **is NOT** a correct explanation for STATEMENT–1.
(C) STATEMENT–1 is True, STATEMENT–2 is False
(D) STATEMENT–1 is False, STATEMENT–2 is True

Sol. **(C)**

$$\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{dy}{y\sqrt{y^2-1}}$$

$$\sec^{-1}x = \sec^{-1}y + c$$

$$\sec^{-1}2 = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) + c$$

$$c = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\sec^{-1}x = \sec^{-1}y + \frac{\pi}{6}$$

$$y = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$$

$$\cos^{-1}\frac{1}{x} = \cos^{-1}\frac{1}{y} + \frac{\pi}{6}$$

$$\cos^{-1}\frac{1}{y} = \cos^{-1}\frac{1}{x} - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{y} = \frac{\sqrt{3}}{2x} - \sqrt{1-\frac{1}{x^2}}\left(\frac{1}{2}\right)$$

$$\frac{2}{y} = \frac{\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}\ .$$

SECTION – III

Linked Comprehension Type

This section contains 2paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 14 to 16

Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by $f(x) = \dfrac{x^2 - ax + 1}{x^2 + ax + 1}$, $0 < a < 2$.

14. Which of the following is true?

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$
(B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$
(C) $f'(1) f'(-1) = (2 - a)^2$
(D) $f'(1) f'(-1) = -(2 + a)^2$

Sol. **(A)**

$$f''(x) = \frac{4ax(x^2 + ax + 1)^2 - 4ax(x^2 - 1)(2x + a)(x^2 + ax + 1)}{(x^2 + ax + 1)^4}$$

$$f''(1) = \frac{4a}{(2 + a)^2} \qquad f''(-1) = \frac{-4a}{(2 - a)^2}$$

$(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0.$

15. Which of the following is true?

(A) f(x) is decreasing on (–1, 1) and has a local minimum at x = 1
(B) f(x) is increasing on (–1, 1) and has a local maximum at x = 1
(C) f(x) is increasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1
(D) f(x) is decreasing on (–1, 1) but has neither a local maximum nor a local minimum at x = 1

Sol. **(A)**

$$f'(x) = \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}$$

Decreasing (–1, 1) and minima at x = 1

16. Let $g(x) = \displaystyle\int_0^{e^x} \frac{f'(t)}{1 + t^2} dt$

which of the following is true?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
(B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
(D) $g'(x)$ does not change sign on $(-\infty, \infty)$

Sol. **(B)**

$$g'(x) = \frac{f'(e^x)e^x}{1 + e^{2x}}$$

Hence positive for $(0, \infty)$ and negative for $(-\infty, 0)$.

Paragraph for Question Nos. 17 to 19

Consider the line

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}, \; L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

17. The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

Sol. **(B)**

$$\begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -i - 7j + 5k$$

Hence unit vector will be $\frac{-i-7j+5k}{5\sqrt{3}}$.

18. The shortest distance between L_1 and L_2 is

(A) 0 (B) $\frac{17}{\sqrt{3}}$

(C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

Sol. **(D)**

$$\text{S. D} = \frac{(1+2)(-1)+(2-2)(-7)+(1+3)(5)}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}.$$

19. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$

(C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

Sol. **(C)**

Plane is given by $-(x+1) - 7(y+2) + 5(z+1) = 0$

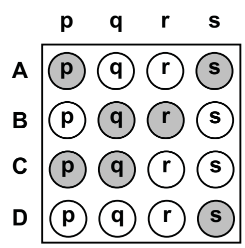
$\Rightarrow x + 7y - 5z + 10 = 0$

$\Rightarrow$ distance $= \frac{1+7-5+10}{\sqrt{75}} = \frac{13}{\sqrt{75}}$.

SECTION – IV

Matrix-Match Type

This contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.
If the correct match are A-p, A-s, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



20. Consider the lines given by

$$L_1 : x + 3y - 5 = 0$$
$$L_2 : 3x - ky - 1 = 0$$
$$L_3 : 5x + 2y - 12 = 0$$

Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I	Column II
(A)	L_1, L_2, L_3 are concurrent, if	(p) $k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(r) $k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

Sol. **(A) → (s); (B) → (p, q); (C) → (r); (D) → (p, q, s)**

$x + 3y - 5 = 0$ and $5x + 2y - 12 = 0$ intersect at (2, 1)

Hence $6 - k - 1 = 0 \quad k = 5$

for L_1, L_2 to be parallel

$$\frac{1}{3} = \frac{3}{-k} \Rightarrow k = -9$$

for L_2, L_3 to be parallel

$$\frac{3}{5} = \frac{-k}{2} \Rightarrow k = \frac{-6}{5}.$$

for $k \neq 5, -9, \frac{-6}{5}$ they will form triangle

for $k = 5$ $k = -9, \frac{-6}{5}$ they will not form triangle

21. Consider all possible permutations of the letters of the word ENDEANOEL.
Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I	Column II
(A)	The number of permutations containing the word ENDEA is	(p) 5!
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	(q) 2 × 5!
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r) 7 × 5!
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s) 21 × 5!

Sol. **(A) → (p); (B) → (s); (C) → (q); (D) → (q)**

(A) ENDEA, N, O, E, L are five different letter, then permutation = 5!

(B) If E is in the first and last position then $\frac{(9-2)!}{2!} = 7 \times 3 \times 5! = 21 \times 5!$

(C) for first four letters = $\frac{4!}{2!}$

for last five letters = 5!/3!

Hence $\frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

(D) For A, E and O 5!/3! and for others 4!/2!

hence $\frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$.

22. Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I	Column II
(A)	The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(B)	Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and (A + B) (A – B) = (A – B) (A + B). If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C)	Let a = $\log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(r) 2
(D)	If sinθ = cosϕ, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are	(s) 3

Sol. **(A) → (r); (B) → (q, s); (C) → (r, s); (D) → (p, r)**

(A) $y = \frac{x^2+2x+4}{x+2}$

$\Rightarrow x^2 + (2-y)x + 4 - 2y = 0$

$\Rightarrow y^2 + 4y - 12 \geq 0$

$y \leq -6$ or $y \geq 2$

minimum value is 2.

(B) (A + B) (A – B) = (A – B) (A + B)

⇒ AB = BA

as A is symmetric and B is skew symmetric

$\Rightarrow (AB)^t = -AB$

⇒ k = 1 and k = 3

(C) $a = \log_3 \log_3 2 \;\Rightarrow\; 3^{-a} = \log_2 3$

Now $1 < 2^{-k+\log_2^3} < 2$

$\Rightarrow 1 < 3.2^{-k} < 2$

$\Rightarrow \log_2\left(\frac{3}{2}\right) < k < \log_2(3)$

⇒ k = 1 or k < 2 and k < 3.

(D) $\sin\theta = \cos\phi \Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi$

$\frac{\pi}{2} - \theta = 2n\pi \pm \phi$

$\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n$

$\Rightarrow$ 0 and 2 are possible.

Physics

PART – II

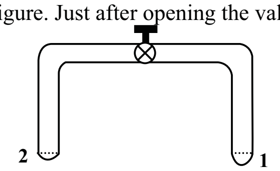
SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

23. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve,

Figure:



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(C) no changes occurs
(D) air from end 2 flows towards end 1. volume of the soap bubble at end 1 increases

Sol. **(B)**

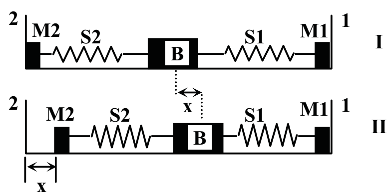
P_1 = pressure just inside the bubble at the end 2 = $P_0 + \frac{4T}{R}$

P_2 = pressure just inside the bubble at the end 1 = $P_0 + \frac{4T}{r}$

$R > r \Rightarrow P_2 < P_1 \Rightarrow$ Air will flow from end 1 to end 2

24. A block (B) is attached to two unstretched springs S1 and S2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is

Figure:



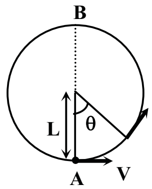
(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Sol. **(C)**

$\frac{1}{2}kx^2 = \frac{1}{2}4ky^2 \Rightarrow y = x/2$

25. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

Figure:



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

Sol. **(D)**

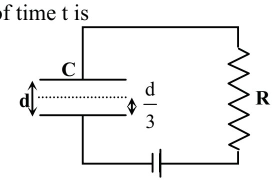
$$\frac{1}{2}5mg\ell = \frac{1}{2}m\frac{5g\ell}{4} + mg\ell(1 - \cos\theta)$$

$$\cos\theta = -\frac{7}{8}$$

Hence, $3\pi/4 < \theta < \pi$

26. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

Figure:



(A) $\frac{6\varepsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(C) $\frac{6\varepsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

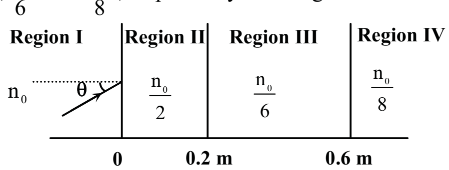
Sol. **(A)**

$$C_{\text{equivalent}} = \frac{\dfrac{2\varepsilon_0}{\frac{d}{3} - vt} \cdot \dfrac{\varepsilon_0}{\frac{2d}{3} + vt}}{\dfrac{2\varepsilon_0}{\frac{d}{3} - vt} + \dfrac{\varepsilon_0}{\frac{2d}{3} + vt}}$$

$\therefore\ \tau = C_{\text{equivalent}} R$

27. A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are n_0, $\frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is

Figure:



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$ (C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

Sol. **(B)**

Total internal reflection occurs at the interface of region III and IV.

Because mediums are parallel

$n_0 \sin\theta = \frac{n_0}{8}\sin\left(\frac{\pi}{2}\right)$

$\sin\theta = 1/8$

28. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is

(A) 344 (B) 336 (C) 117.3 (D) 109.3

Sol. **(A)**

$n_s = \frac{3}{4}\left(\frac{340}{0.75}\right) = n - 4$

$\therefore$ n = 344 Hz

29. A radioactive sample S1 having an activity 5μCi has twice the number of nuclei as another sample S2 which has an activity of 10 μCi. The half lives of S1 and S2 can be

(A) 20 years and 5 years, respectively

(B) 20 years and 10 years, respectively

(C) 10 years each

(D) 5 years each

Sol. **(A)**

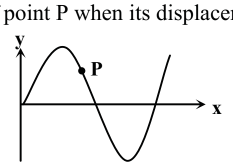
$5\mu Ci = \frac{\ln 2}{T_1}(2N_0)$

$10\ \mu Ci = \frac{\ln 2}{T_2}(N_0)$

Dividing we get $T_1 = 4T_2$

30. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap –shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is

Figure:



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

(C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

Sol. **(A)**

y = 5 cm and V = +ve

$y = A\sin(\omega t \pm \phi)$ $V = A\omega\cos(\omega t \pm \phi)$

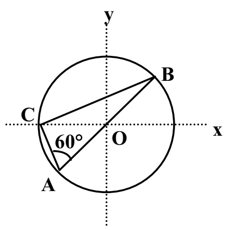
We get $\omega t \pm \phi = 30°$

$\omega = 2\pi\frac{v}{\lambda} = \frac{2\pi}{5}$

$v = A\omega\cos(\omega t + \phi) = \left(\frac{10}{100}\right)\times\left(\frac{2\pi}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi\sqrt{3}}{50}$ m/s

31. Consider a system of three charges $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60°

Figure:



(A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis

(B) The potential energy of the system is zero

(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$

(D The potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

Sol. **(C)**

$$F_{BC} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{\left(R\sqrt{3}\right)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

SECTION – II

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

32 STATEMENT-1

For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

and

STATEMENT-2

The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(B)**

33 STATEMENT-1

It is easier to pull a heavy object than to push it on a level ground.

and

STATEMENT-2

The magnitude of frictional force depends on the nature of the two surfaces in contact.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(B)**
In pushing Normal contact force is greater than in pulling.

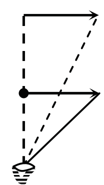
34 STATEMENT-1
For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

and

STATEMENT-2
If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(B)**
Distance appeared to move depends upon angle subtended on eye.



35 STATEMENT-1
The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

STATEMENT-2
Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(C)**

SECTION – III

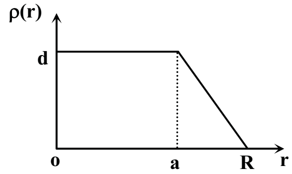
Linked Comprehension Type

This section contains 2paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 36 to 38

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ (r) [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure The electric field is only along rhe radial direction.

Figure:



Nitin M Sir (physics-iitjee.blogspot.com)

36. The electric field at r = R is
(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a

Sol. **(A)**

37. For a = 0, the value of d (maximum value of ρ as shown in the figure) is
(A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

Sol. **(B)**

$q = \int_0^R \frac{d}{R}(R-x)4\pi x^2 dx = Ze$

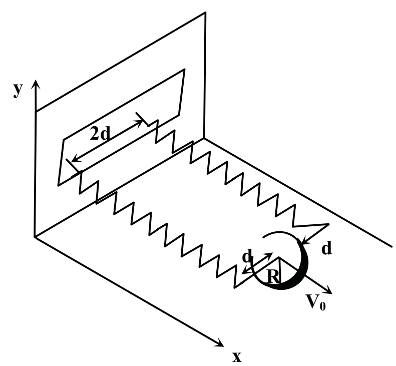
$d = \frac{3Ze}{\pi R^3}$

38. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies.
(A) a = 0 (B) $a = \frac{R}{2}$ (C) a = R (D) $a = \frac{2R}{3}$

Sol. **(C)**
If within a sphere ρ is constant $E \propto r$

Paragraph for Question Nos. 39 to 41

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ.
Figure:



39. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is
(A) –kx (B) –2kx (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

Sol. **(D)**
2kx – f = ma
⇒ f.R = Iα
a = Rα
⇒ $ma = \frac{4kx}{3}$

40. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to
(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

Sol. **(D)**

$-(2kx)R = I_P\alpha$

$$\alpha = -\frac{4kR}{3mR^2}(R\theta) = -\frac{4k}{3m}\theta$$

41. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

Sol. **(C)**

$2kx - f_{max} = ma$

$2kx.r = I_P\alpha$

$f_{max} = \mu mg$

$$\Rightarrow \quad x = \frac{3}{2}\frac{\mu mg}{k}$$

$$\Rightarrow \quad \frac{1}{2}(2k)x^2 = \frac{1}{2}I_P\omega^2$$

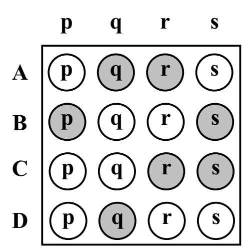
$$\Rightarrow \quad \gamma = \mu g\sqrt{\frac{3m}{k}}$$

SECTION – IV

Matrix-Match Type

*This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements in **Column I** are labelled as A, B, C and D whereas statements in **Column II** are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

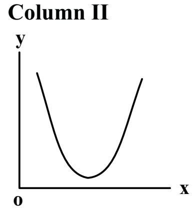
If the correct matches are A-p, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will be look like the following:



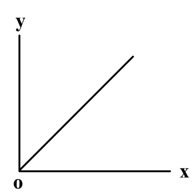
42. **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graph given in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I / **Column II**

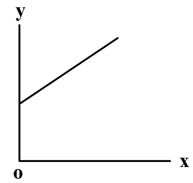
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

(p)


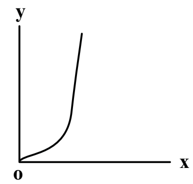
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

(q)


(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle

(r)


(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

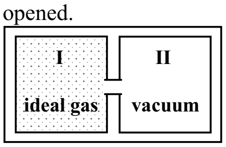
(s)


Sol. **(A) → s, (B) → q & s, (C) → s, (D) → q**

43. **Column I** Contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I

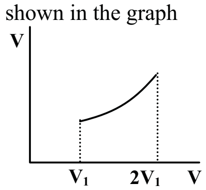
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.



(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas

(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume

(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph



Column II

(p) The temperature of the gas decreases

(q) The temperature of the gas increases or remains constant

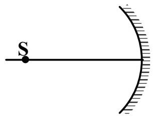
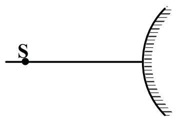
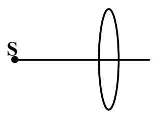
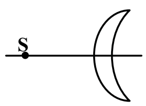
(r) The gas loses heat

(s) The gas gains heat

Sol. **(A) → q, (B) → p & r, (C) → p & s, (D) → q & s**

Nitin M Sir (physics-iitjee.blogspot.com)

44. An optical component and an object S placed along its optic axis are given in **Column I**. The distance between the object and the component can be varied. The properties of images are given in **Column II**. Match all the properties of images from **Column II** with the appropriate components given in **Column I**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) 	(p) Real image
(B) 	(q) Virtual image
(C) 	(r) Magnified image
(D) 	(s) Image at infinity

Sol. **(A) → p, q, r & s, (B) → q, (C) → p, q, r & s, (D) → p, q, r & s**

Chemistry

PART – III
SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

45. The IUPAC name of $[Ni(NH_3)_4][NiCl_4]$ is
(A) Tetrachloronickel (II) – tetraamminenickel (II)
(B) Tetraamminenickel (II) – tetrachloronickel (II)
(C) Tetraamminenickel (II) – tetrachloronickelate (II)
(D) Tetrachloronickel (II) – tetraamminenickelate (0)

Sol. **(C)**
IUPAC name is tetraamminenickel (II) – tetrachloronickelate (II)

46. Among the following the coloured compound is
(A) $CuCl$
(B) $K_3[Cu(CN)_4]$
(C) CuF_2
(D) $[Cu(CH_3CN)_4]BF_4$

Sol. **(C)**
In the crystalline form CuF_2 is blue coloured.

47. Both $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ are diamagnetic. The hybridization of nickel in these complexes, respectively, are
(A) sp^3, sp^3
(B) sp^3, dsp^2
(C) dsp^2, sp^3
(D) dsp^2, dsp^2

Sol. **(B)**
$Ni(CO)_4 = sp^3$
$[Ni(CN)_4]^{2-} = dsp^2$

48. Among the following, the surfactant that will form micelles in aqueous solution at the lowest molar concentration at ambient conditions is

(A) $CH_3(CH_2)_{15}N^+(CH_3)_3Br^-$
(B) $CH_3(CH_2)_{11}OSO_3^-Na^+$
(C) $CH_3(CH_2)_6COO^-Na^+$
(D) $CH_3(CH_2)_{11}N^+(CH_3)_3Br^-$

Sol. **(A)**

Critical concentration for micelle formation decreases as the molecular weight of hydrocarbon chain of surfactant grows because in this case true solubility diminishes and the tendency of surfactant molecule to associate increases.

49. Solubility product constant (K_{sp}) of salts of types MX, MX_2 and M_3X at temperature 'T' are 4.0×10^{-8}, 3.2×10^{-14} and 2.7×10^{-15}, respectively. Solubilities (mole dm^{-3}) of the salts at temperature 'T' are in the order

(A) $MX > MX_2 > M_3X$
(B) $M_3X > MX_2 > MX$
(C) $MX_2 > M_3X > MX$
(D) $MX > M_3X > MX_2$

Sol. **(D)**

Solubility of (MX) = $\sqrt{4\times10^{-8}} = 2\times10^{-4}$

Solubility of (MX_2) = 8×10^{-5}

Solubility of (M_3X) = 1×10^{-4}

$\therefore MX > M_3X > MX_2$

50. Electrolysis of dilute aqueous NaCl solution was carried out by passing 10 milli ampere current. The time required to liberate 0.01 mol of H_2 gas at the cathode is (1 Faraday = 96500 C mol^{-1})

(A) 9.65×10^4 sec
(B) 19.3×10^4 sec
(C) 28.95×10^4 sec
(D) 38.6×10^4 sec

Sol. **(B)**

$Q = i \times t$

$Q = 10 \times 10^{-3} \times t$

$$2H_2O + 2e^- \longrightarrow H_2 + 2OH^-$$

To liberate 0.01 mole of H_2, 0.02 Faraday charge is required

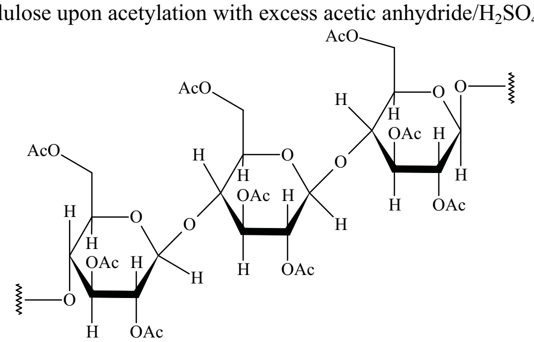
$Q = 0.02 \times 96500$ C

$\therefore 0.02\times96500 = 10^{-2}\times t$

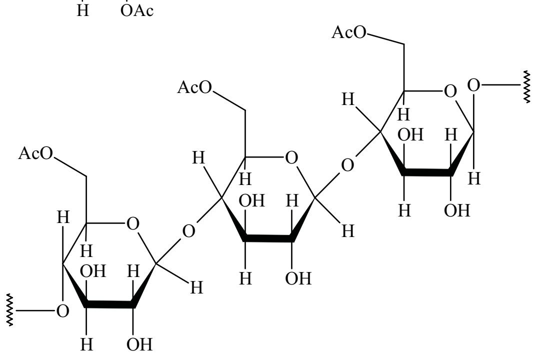
$t = 19.30 \times 10^4$ sec

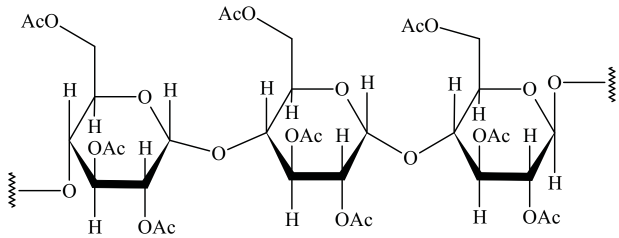
51. Cellulose upon acetylation with excess acetic anhydride/H_2SO_4 (catalytic) gives cellulose triacetate whose structure is

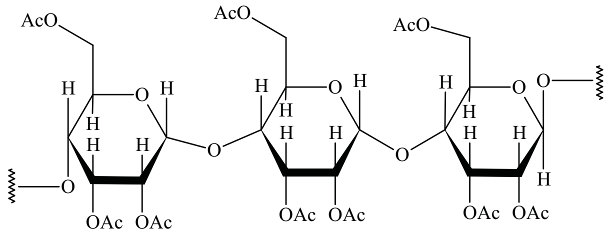
(A)



(B)

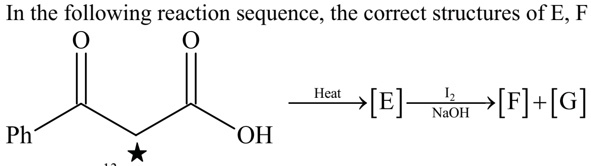


(C) 

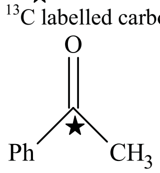
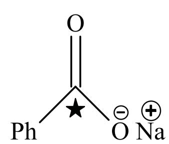
(D) 

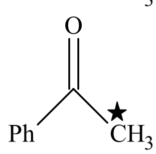
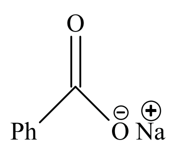
Sol. **(A)**
As in cellulose β 1-4 glycosidic linkage is present.

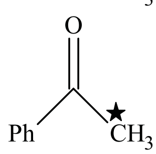
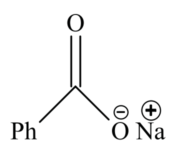
52. In the following reaction sequence, the correct structures of E, F and G are

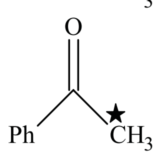
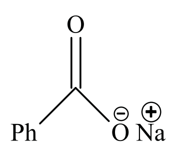


(* implies ^{13}C labelled carbon)

(A) E =  F =  G = CHI_3

(B) E =  F =  G = CHI_3

(C) E =  F =  G = $\overset{\star}{C}HI_3$

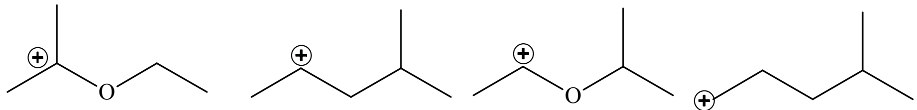
(D) E =  F =  G = $\overset{\star}{C}H_3I$

Sol. **(C)**

$$Ph-\overset{O}{\overset{||}{C}}-\overset{*}{C}H_2-\overset{O}{\overset{||}{C}}-OH \xrightarrow{\Delta} \underset{(E)}{Ph-\overset{O}{\overset{||}{C}}-\overset{*}{C}H_3} + CO_2$$

$$Ph-\overset{O}{\overset{||}{C}}-\overset{*}{C}H_3 \xrightarrow[NaOH]{I_2} \underset{(F)}{Ph-\overset{O}{\overset{||}{C}}-COONa} + \underset{(G)}{\overset{*}{C}HI_3}$$

53. The correct stability order for the following species is

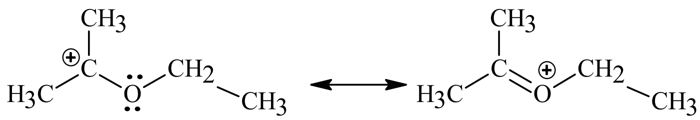


(I) (II) (III) (IV)

(A) (II) > (IV) > (I) > (III)
(B) (I) > (II) > (III) > (IV)
(C) (II) > (I) > (IV) > (III)
(D) (I) > (III) > (II) > (IV)

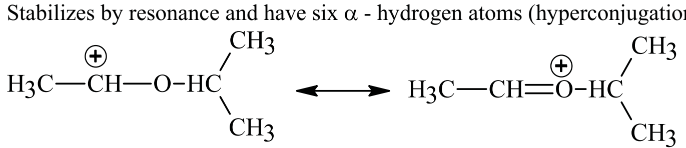
Sol. **(D)**

(I)



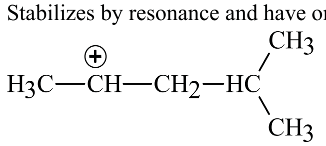
Stabilizes by resonance and have six α - hydrogen atoms (hyperconjugation)

(III)



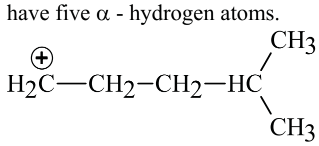
Stabilizes by resonance and have only three α - hydrogen atoms.

(II)



have five α - hydrogen atoms.

(IV)



have only two α - hydrogen atoms.

∴ I > III > II > IV

SECTION – II

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

54. STATEMENT-1: The geometrical isomers of the complex $[M(NH_3)_4Cl_2]$ are optically inactive.
and
STATEMENT-2: Both geometrical isomers of the complex $[M(NH_3)_4Cl_2]$ possess axis of symmetry.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(B)**
The molecule should not posses alternate axis of symmetry to be optically active.

55. STATEMENT-1: $[Fe(H_2O)_5NO]SO_4$ is paramagnetic.
and
STATEMENT-2: The Fe in $[Fe(H_2O)_5NO]SO_4$ has three unpaired electrons.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

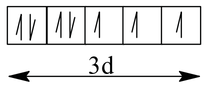
Sol. **(A)**

$\left[Fe(H_2O)_5\ NO\right]SO_4$

Here Fe has +1 oxidation state.

$Fe^+ = 3d^6 4s^1$ in presence of NO^+ $4s^1$ electron are paired in 3d sub shell.

So electronic configuration of Fe^+ is



56. STATEMENT-1: Aniline on reaction with $NaNO_2/HCl$ at 0^oC followed by coupling with β-naphthol gives a dark blue coloured precipitate.

and

STATEMENT-2: The colour of the compound formed in the reaction of aniline with $NaNO_2/HCl$ at 0^oC followed by coupling with β-naphthol is due to the extended conjugation.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(D)**

$C_6H_5\overset{\oplus}{N_2}\overset{\ominus}{Cl}$ gives scarlet red coloured dye with β - naphthol.

57. STATEMENT-1: There is a natural asymmetry between converting work to heat and converting heat to work.

and

STATEMENT-2: No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(B)**

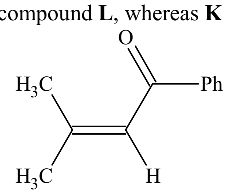
SECTION – III

Linked Comprehension Type

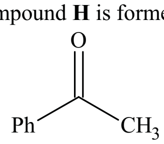
This section contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 60

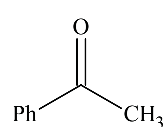
A tertiary alcohol **H** upon acid catalysed dehydration gives a product **I**. Ozonolysis of **I** leads to compounds **J** and **K**. Compound **J** upon reaction with KOH gives benzyl alcohol and a compound **L**, whereas **K** on reaction with KOH gives only **M**,

M = 

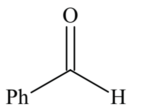
58. Compound **H** is formed by the reaction of

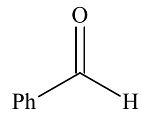
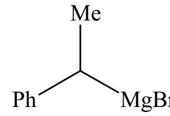
(A) 

+ PhMgBr

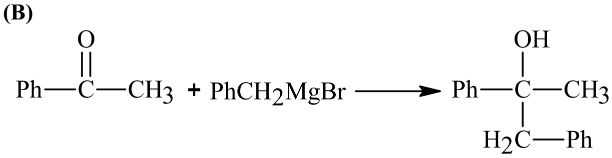
(B) 

+ $PhCH_2MgBr$

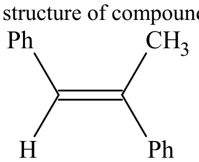
(C)  + $PhCH_2MgBr$

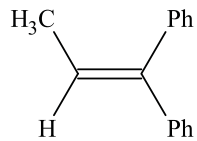
(D)  + 

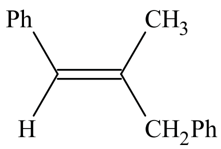
Sol. **(B)**

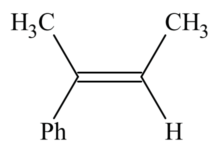


59. The structure of compound **I** is

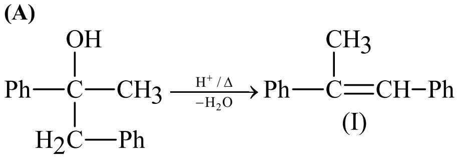
(A) 

(B) 

(C) 

(D) 

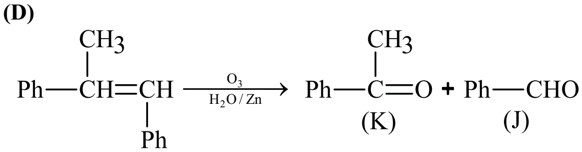
Sol. **(A)**



60. The structures of compounds **J**, **K** and **L**, respectively, are

(A) $PhCOCH_3$, $PhCH_2COCH_3$ and $PhCH_2COO^-K^+$
(B) $PhCHO$, $PhCH_2CHO$ and $PhCOO^-K^+$
(C) $PhCOCH_3$, $PhCH_2CHO$ and $CH_3COO^-K^+$
(D) $PhCHO$, $PhCOCH_3$ and $PhCOO^-K^+$

Sol. **(D)**



Hence, (D) is the correct answer.

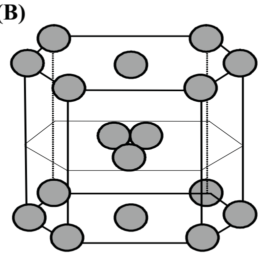
Paragraph for Question Nos. 61 to 63

In hexagonal systems of crystals, a frequently encountered arrangement of atoms is described as a hexagonal prism. Here, the top and bottom of the cell are regular hexagons and three atoms are sandwiched in between them. A space-filling model of this structure, called hexagonal close-packed (HCP), is constituted of a sphere on a flat surface surrounded in the same plane by six identical spheres as closely as possible. Three spheres are then placed over the first layer so that they touch each other and represent the second layer. Each one of these three spheres touches three spheres of the bottom layer. Finally, the second layer is covered with a third layer that is identical to the bottom layer in relative position. Assumer radius of every sphere to be 'r'.

61. The number of atoms on this HCP unit cell is

(A) 4
(B) 6
(C) 12
(D) 17

Sol. **(B)**



Total effective number of atoms = $12\times\frac{1}{6}+2\times\frac{1}{2}+3=6$

62. The volume of this HCP unit cell is

(A) $24\sqrt{2}r^3$ (B) $16\sqrt{2}r^3$

(C) $12\sqrt{2}r^3$ (D) $\frac{64r^3}{3\sqrt{3}}$

Sol. **(A)**

Height of unit cell = $4r\sqrt{\frac{2}{3}}$

Base area = $6\times\frac{\sqrt{3}}{4}(2r)^2$

Volume = height ×base area

$= 24\sqrt{2}r^3$

63. The empty space in this HCP unit cell is

(A) 74% (B) 47.6%

(C) 32% (D) 26%

Sol. **(D)**

Packing fraction = 74%

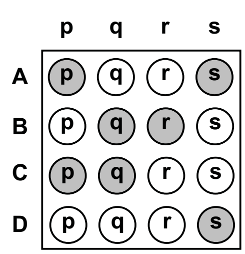
Empty space = 26%

SECTION – IV

Matrix-Match Type

This contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

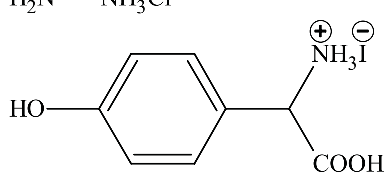
If the correct match are A-p, A-s, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:

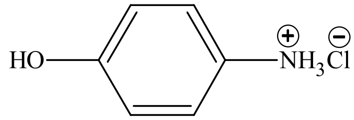


64. Match the compounds in **Column I** with their characteristic test(s)/ reaction(s) given in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix gives in the ORS.

Column I

(A) $H_2N-\overset{\oplus}{N}H_3\overset{\ominus}{Cl}$

(B) 

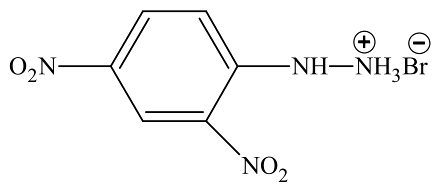
(C) 

Column II

(p) sodium fusion extract of the compound gives Prussian blue colour with $FeSO_4$

(q) gives positive $FeCl_3$ test

(r) gives white precipitate with $AgNO_3$

(D) 

(s) reacts with aldehydes to form the corresponding hydrazone derivative

Sol. **A – r, s**
B – p, q
C – p, q, r
D – p, s

65. Match the entries in **Column I** with the correctly related quantum number(s) in **Column II.** Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Orbital angular momentum of the electron in a hydrogen-like atomic orbital	(p) Principal quantum number
(B) A hydrogen-like one-electron wave function obeying Pauli principle	(q) Azimuthal quantum number
(C) Shape, size and orientation of hydrogen-like atomic orbitals	(r) Magnetic quantum number
(D) Probability density of electron at the nucleus in hydrogen-like atom	(s) Electron spin quantum number

Sol. **A – q**
B – s
C – p, q, r
D – p, q, r

66. Match the conversions in **Column I** with the type(s) of reaction(s) given in **Column II**. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) $PbS \rightarrow PbO$	(p) roasting
(B) $CaCO_3 \rightarrow CaO$	(q) calcination
(C) $ZnS \rightarrow Zn$	(r) Carbon reduction
(D) $Cu_2S \rightarrow Cu$	(s) self reduction

Sol. **A – p**
B – q
C – p, r
D – p, s

FIITJEE Solutions to

IIT – JEE - 2009

(PAPER – 1, CODE – 1)

Time: 3 Hours **Maximum Marks: 240**

A. Question paper format:

1. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part has 4 sections.
2. **Section I** contains **8** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**
3. **Section II** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct.**
4. **Section III** contains **2** groups of questions. Each group has 3 questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**
5. **Section IV** contains **2** questions. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in column I can have correct matching with **one or more** statements(s) given in column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

B. Marking scheme:

6. For each question in **Section I** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (-1) mark** will be awarded.
7. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **Minus one (-1) mark** will be awarded.
8. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded.
9. For each question in **Section IV**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8** marks. There is **no negative marking** for incorrect answer(s) for this section.

IITJEE 2009 (PAPER-1, CODE-1)

PART I: CHEMISTRY

SECTION-I
Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

1. The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is

(A) 4.0×10^{-4} (B) 4.0×10^{-5}
(C) 5.0×10^{-4} (D) 4.0×10^{-6}

Sol. **(A)**

$P = K_H\ \chi_{N_2}$

$0.8 \times 5 = 1 \times 10^5 \times \chi_{N_2}$

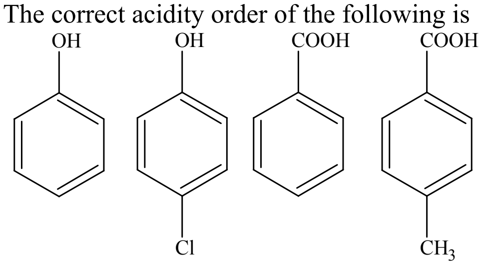
$\chi_{N_2} = 4 \times 10^{-5}$ (in 10 moles of water)

$$\Rightarrow 4\times10^{-5} = \frac{n_{N_2}}{n_{N_2}+10}$$

$$n_{N_2} \times 5\times10^{-5} + 4\times10^{-4} = n_{N_2}$$

$$\Rightarrow n_{N_2} = 4\times10^{-4}$$

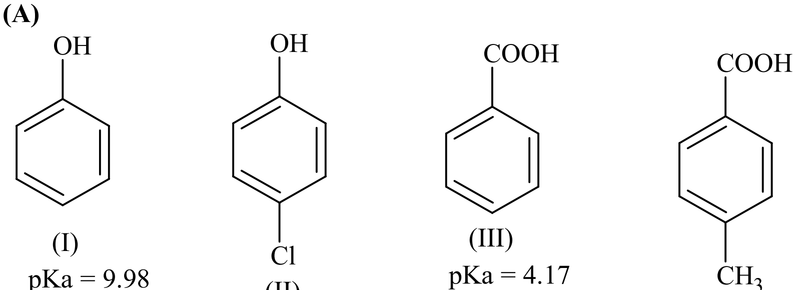
2. The correct acidity order of the following is



(I) (II) (III) (IV)

(A) (III) > (IV) > (II) > (I) (B) (IV) > (III) > (I) > (II)
(C) (III) > (II) > (I) > (IV) (D) (II) > (III) > (IV) > (I)

Sol. **(A)**



(I) pKa = 9.98

(II) pKa = 9.38

(III) pKa = 4.17

(IV) pKa = 4.37

Decreasing order of acidic strength: III > IV > II > I

Nitin M Sir (physics-iitjee.blogspot.com)

3. The reaction of P_4 with **X** leads selectively to P_4O_6. The **X** is
(A) Dry O_2
(B) A mixture of O_2 and N_2
(C) Moist O_2
(D) O_2 in the presence of aqueous NaOH

Sol. **(B)**
$P_4 + 3O_2 \xrightarrow{N_2} P_4O_6$ (exclusively)
(N_2 is used to retard the further oxidation.)

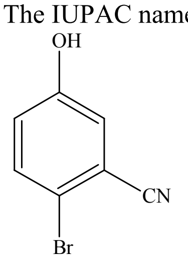
4. Among cellulose, poly(vinyl chloride), nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is
(A) Nylon
(B) Poly(vinyl chloride)
(C) Cellulose
(D) Natural Rubber

Sol. **(D)**
As chain of natural rubber involves weak van der Waal force of interaction.

5. Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5% respectively, the atomic mass of Fe is
(A) 55.85
(B) 55.95
(C) 55.75
(D) 56.05

Sol. **(B)**
$$\bar{A} = \frac{\sum A_i x_i}{\sum x_i}$$
$\bar{A} = 54 \times 0.05 + 56 \times 0.90 + 57 \times 0.05$ (where $\bar{A}$ is atomic mass of Fe)
$\bar{A} = 55.95$

6. The IUPAC name of the following compound is



(A) 4-Bromo-3-cyanophenol
(B) 2-Bromo-5-hydroxybenzonitrile
(C) 2-Cyano-4-hydroxybromobenzene
(D) 6-Bromo-3-hdyroxybenzonitrile

Sol. **(B)**
Priority of CN is highest.

7. Among the electrolytes Na_2SO_4, $CaCl_2$, $Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_3 sol is
(A) Na_2SO_4
(B) $CaCl_2$
(C) $Al_2(SO_4)_3$
(D) NH_4Cl

Sol. **(C)**
As Sb_2S_3 is a negative sol, so, $Al_2(SO_4)_3$ will be the most effective coagulant due to higher charge density on Al^{3+} in accordance with Hardy-Schulze rule.
Order of effectiveness of cations: $Al^{3+} > Ca^{++} > Na^{+} > NH_4^{+}$

8. The term that corrects for the attractive forces present in a real gas in the van der Waals equation is

(A) nb (B) $\frac{an^2}{V^2}$

(C) $-\frac{an^2}{V^2}$ (D) $-nb$

Sol. **(B)**

The measure of force of attraction for 'n' moles of real gas $\left(\frac{n^2a}{V^2}\right)$

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

SECTION-II
Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONE OR MORE** is/are correct.

9. The compound(s) formed upon combustion of sodium metal in excess air is(are)

(A) Na_2O_2 (B) Na_2O

(C) NaO_2 (D) NaOH

Sol. **(A, B) in dry air**

10. The correct statement(s) about the compound $H_3C(HO)HC-CH=CH-CH(OH)CH_3$ (**X**) is(are)

(A) The total number of stereoisomers possible for **X** is 6

(B) The total number of diastereomers possible for **X** is 3

(C) If the stereochemistry about the double bond in **X** is *trans*, the number of enantiomers possible for **X** is 4

(D) If the stereochemistry about the double bond in **X** is *cis*, the number of enantiomers possible for **X** is 2

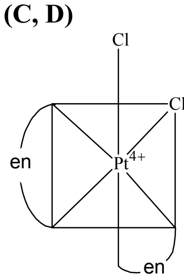
Sol. **(A, D)**

11. The compound(s) that exhibit(s) geometrical isomerism is(are)

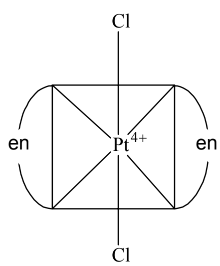
(A) $[Pt(en)Cl_2]$ (B) $[Pt(en)_2]Cl_2$

(C) $[Pt(en)_2Cl_2]Cl_2$ (D) $[Pt(NH_3)_2Cl_2]$

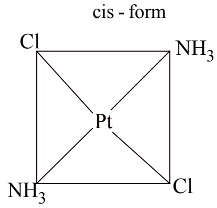
Sol. **(C, D)**



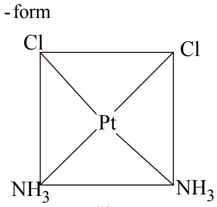
cis - form



trans - form



Trans



Cis

12. The correct statement(s) regarding defects in solids is(are)
(A) Frenkel defect is usually favoured by a very small difference in the sizes of cation and anion
(B) Frenkel defect is a dislocation defect
(C) Trapping of an electron in the lattice leads to the formation of F-center
(D) Schottky defects have no effect on the physical properties of solids

Sol. **(B, C)**

SECTION-III
Comprehension Type

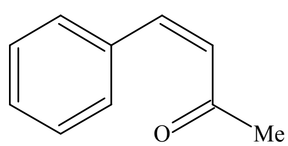
This section contains 2 groups of questions. Each group has 3 multiple choice question based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

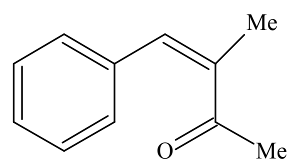
Paragraph for Question Nos. 13 to 15

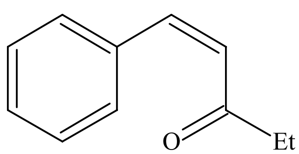
A carbonyl compound **P**, which gives positive iodoform test, undergoes reaction with MeMgBr followed by dehydration to give an olefin **Q**. Ozonolysis of **Q** leads to a dicarbonyl compound **R**, which undergoes intramolecular aldol reaction to give predominantly **S**.

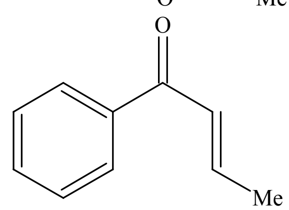
$$\mathbf{P} \xrightarrow[\text{2. } H^+,\, H_2O \quad \text{3. } H_2SO_4,\, \Delta]{\text{1. MeMgBr}} \mathbf{Q} \xrightarrow[\text{2. Zn, } H_2O]{\text{1. } O_3} \mathbf{R} \xrightarrow[\text{2. } \Delta]{\text{1. } OH^-} \mathbf{S}$$

13. The structure of the carbonyl compound **P** is

(A) 

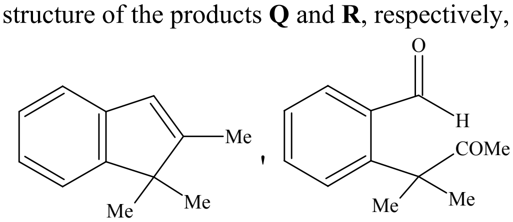
(B) 

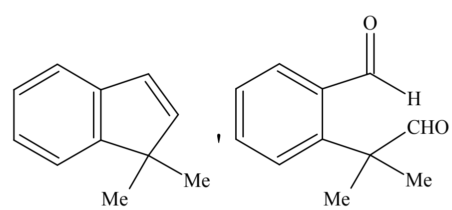
(C) 

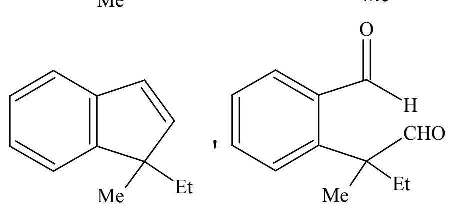
(D) 

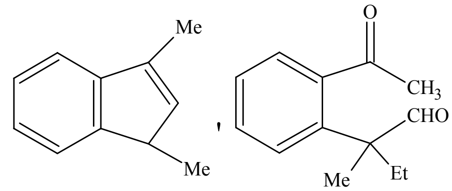
Sol. **(B)**

14. The structure of the products **Q** and **R**, respectively, are

(A) 

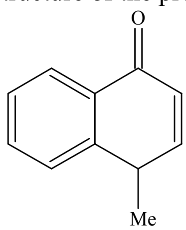
(B) 

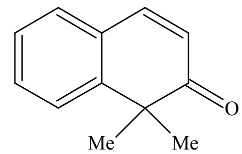
(C) 

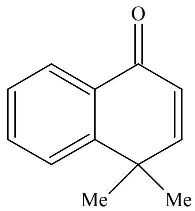
(D) 

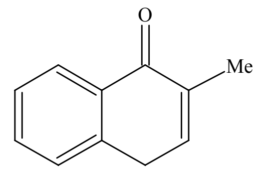
Sol. **(A)**

15. The structure of the product **S** is

(A) 

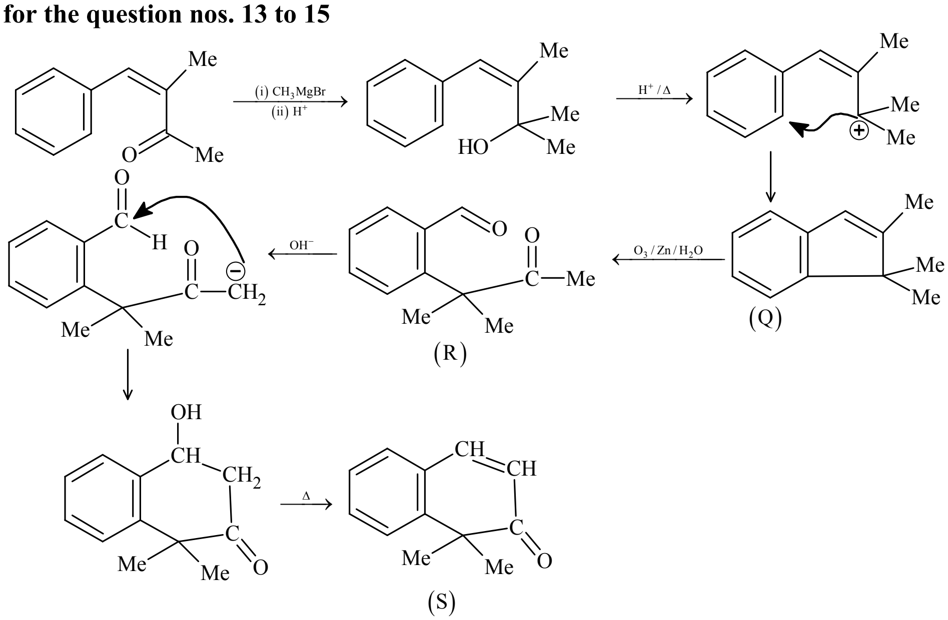
(B) 

(C) 

(D) 

Sol. **(B)**

Solution for the question nos. 13 to 15



Paragraph for Question Nos. 16 to 18

p-Amino-*N*, *N*-dimethylaniline is added to a strongly acidic solution of **X**. The resulting solution is treated with a few drops of aqueous solution of **Y** to yield blue coloration due to the formation of methylene blue. Treatment of the aqueous solution of **Y** with the reagent potassium hexacyanoferrate (II) leads to the formation of an intense blue precipitate. The precipitate dissolves on excess addition of the reagent. Similarly, treatment of the solution of **Y** with the solution of potassium hexacyanoferrate (III) leads to a brown coloration due to the formation of **Z**.

16. The compound **X** is

(A) $NaNO_3$ (B) NaCl

(C) Na_2SO_4 (D) Na_2S

Sol. **(D)**

17. The compound Y is

(A) $MgCl_2$ (B) $FeCl_2$

(C) $FeCl_3$ (D) $ZnCl_2$

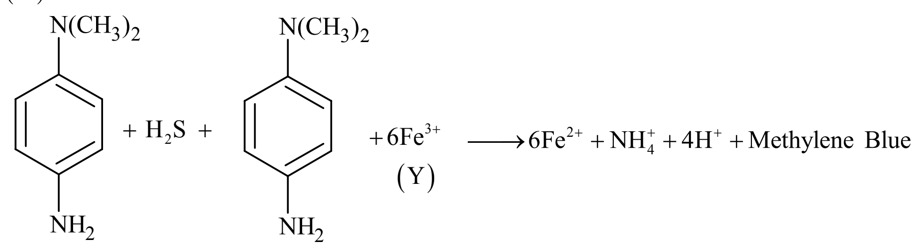
Sol. **(C)**

18. The compound Z is
(A) $Mg_2[Fe(CN)_6]$ (B) $Fe[Fe(CN)_6]$
(C) $Fe_4[Fe(CN)_6]_3$ (D) $K_2Zn_3[Fe(CN)_6]_2$

Sol. **(B)**

Solution for the question nos. 16 to 18

$$\underset{(X)}{Na_2S} + 2H^+ \longrightarrow H_2S + 2Na^+$$



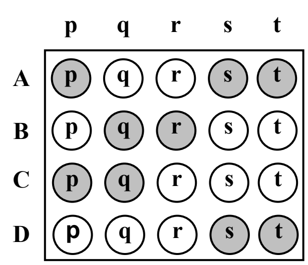
$$4FeCl_3 + 3K_4\left[\overset{II}{Fe}(CN)_6\right] \longrightarrow \underset{\text{Intense blue}}{Fe_4\left[Fe(CN)_6\right]_3} + 2KCl$$

$$FeCl_3 + K_3\left[Fe(CN)_6\right] \longrightarrow \underset{\text{Brown coloration}}{Fe\left[Fe(CN)_6\right]} + 3KCl$$

(X) – Na_2S
(Y) – $FeCl_3$
(Z) – $Fe[Fe(CN)_6]$

SECTION – IV
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



19. Match each of the compounds in **Column I** with its characteristic reaction(s) in **Column II**.

Column – I	**Column – II**
(A) $CH_3CH_2CH_2CN$	(p) Reduction with Pd–C/H_2
(B) $CH_3CH_2OCOCH_3$	(q) Reduction with $SnCl_2$/HCl
(C) $CH_3–CH=CH–CH_2OH$	(r) Development of foul smell on treatment with chloroform and alcoholic KOH
(D) $CH_3CH_2CH_2CH_2NH_2$	(s) Reduction with diisobutylaluminium hydride (DIBAL-H)
	(t) Alkaline hydrolysis

Sol. **((A – p, q, s, t) (B – s, t) (C – p) (D – r))**

20. Match each of the diatomic molecules in **Column I** with its property/properties in **Column II**.

Column – I		**Column – II**	
(A)	B_2	(p)	Paramagentic
(B)	N_2	(q)	Undergoes oxidation
(C)	O_2^-	(r)	Undergoes reduction
(D)	O_2	(s)	Bond order ≥ 2
		(t)	Mixing of 's' and 'p' orbitals

Sol. **((A – p, r, t) (B – s, t) (C – p, q) (D – p, q, s))** [According to MOT]

PART II: MATHEMATICS

SECTION–I
Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

21. Let z = x + iy be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$ is

(A) 48 (B) 32
(C) 40 (D) 80

Sol. **(A)**
$z\bar{z}\left(\bar{z}^2 + z^2\right) = 350$
Put z = x + iy
$(x^2 + y^2)(x^2 - y^2) = 175$
$(x^2 + y^2)(x^2 - y^2) = 5 \cdot 5 \cdot 7$
$x^2 + y^2 = 25$
$x^2 - y^2 = 7$
$x = \pm 4, y = \pm 3$
$x, y \in I$
Area = 8 × 6 = 48 sq. unit.

22. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right) = 1$ and $\vec{a}\cdot\vec{c} = \frac{1}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
(C) $\vec{b}, \vec{d}$ are non-parallel (D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

Sol. **(C)**
$\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right) = 1$ possible only when $\left|\vec{a}\times\vec{b}\right| = \left|\vec{c}\times\vec{d}\right| = 1$
and $\left(\vec{a}\times\vec{b}\right) \parallel \left(\vec{c}\times\vec{d}\right)$
Since $\vec{a}\cdot\vec{c} = 1/2$ and $\vec{b}\parallel\vec{d}$, then $\left|\vec{c}\times\vec{d}\right| \neq 1$.

23. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$
(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Sol. **(D)**
Equation of line AM is x + 3y – 3 = 0
Perpendicular distance of line from origin = $\frac{3}{\sqrt{10}}$

Length of AM = $2\sqrt{9-\frac{9}{10}} = 2\times\frac{9}{\sqrt{10}}$

$\Rightarrow$ Area = $\frac{1}{2}\times 2\times\frac{9}{\sqrt{10}}\times\frac{3}{\sqrt{10}} = \frac{27}{10}$ sq. units.

24. Let z = cosθ + i sinθ. Then the value of $\sum_{m=1}^{15}\text{Im}\left(z^{2m-1}\right)$ at θ = 2° is

(A) $\frac{1}{\sin 2°}$ (B) $\frac{1}{3\sin 2°}$
(C) $\frac{1}{2\sin 2°}$ (D) $\frac{1}{4\sin 2°}$

Sol. **(D)**
X = sinθ + sin3θ + … + sin29θ
2(sinθ)X = 1 – cos2θ + cos2θ – cos4θ + … + cos28θ – cos30θ
$X = \frac{1-\cos 30\theta}{2\sin\theta} = \frac{1}{4\sin 2°}$

25. Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r} = \left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1 is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$
(C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Sol. **(A)**
Any point on the line can be taken as
$Q \equiv \{(1-3\mu),\ (\mu-1),\ (5\mu+2)\}$
$\overrightarrow{PQ}$ = {– 3μ – 2, μ – 3, 5μ – 4}
Now, 1(– 3μ – 2) – 4(μ – 3) + 3 (5μ – 4) = 0
⇒ – 3μ – 2 – 4μ + 12 + 15μ – 12 = 0
8μ = 2 ⇒ μ = 1/4.

26. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is
(A) 55 (B) 66
(C) 77 (D) 88

Sol. **(C)**
Coefficient of x^{10} in $(x + x^2 + x^3)^7$

coefficient of x^3 in $(1 + x + x^2)^7$
coefficient of x^3 in $(1 - x^3)^7 (1 - x)^{-7}$
$= {}^{7+3-1}C_3 - 7$
$= {}^{9}C_3 - 7$
$= \dfrac{9\times8\times7}{6} - 7 = 77.$

Alternate:
The digits are 1, 1, 1, 1, 1, 2, 3
or 1, 1, 1, 1, 2, 2, 2
Hence number of seven digit numbers formed
$= \dfrac{7!}{5!} + \dfrac{7!}{4!\,3!} = 77.$

27. Let f be a non-negative function defined on the interval [0, 1]. If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$, and $f(0) = 0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Sol. **(C)**
$f' = \pm\sqrt{1-f^2}$
$\Rightarrow f(x) = \sin x$ or $f(x) = -\sin x$ (not possible)
$\Rightarrow f(x) = \sin x$
Also, $x > \sin x \ \forall\ x > 0$.

28. Tangents drawn from the point P (1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is
(A) $x^2 + y^2 + 4x - 6y + 19 = 0$
(B) $x^2 + y^2 - 4x - 10y + 19 = 0$
(C) $x^2 + y^2 - 2x + 6y - 29 = 0$
(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Sol. **(B)**
The centre of the circle is C (3, 2).
Since CA and CB are perpendicular to PA and PB, CP is the diameter of the circumcircle of triangle PAB.
Its equation is
$(x - 3)(x - 1) + (y - 2)(y - 8) = 0$
or $x^2 + y^2 - 4x - 10y + 19 = 0$.

SECTION–II
Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

29. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then
(A) $b + c = 4a$
(B) $b + c = 2a$
(C) locus of point A is an ellipse
(D) locus of point A is a pair of straight lines

Sol. **(B, C)**

$$2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 4\sin^2\frac{A}{2}$$

$$\cos\left(\frac{B-C}{2}\right) = 2\sin(A/2)$$

$$\Rightarrow \frac{\cos\left(\frac{B-C}{2}\right)}{\sin A/2} = 2$$

$$\Rightarrow \frac{\sin B + \sin C}{\sin A} = 2$$

$\Rightarrow b + c = 2a$ (constant).

30. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

Sol. **(A, B)**

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$

$$3\sin^4 x + 2\left(1-\sin^2 x\right)^2 = \frac{6}{5}$$

$\Rightarrow 25\sin^4 x - 20\sin^2 x + 4 = 0$

$\Rightarrow \sin^2 x = \frac{2}{5}$ and $\cos^2 x = \frac{3}{5}$

$\therefore \tan^2 x = \frac{2}{3}$ and $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

31. Let $L = \lim\limits_{x\to 0}\frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

(A) $a = 2$

(B) $a = 1$

(C) $L = \frac{1}{64}$

(D) $L = \frac{1}{32}$

Sol. **(A, C)**

$$L = \lim_{x\to 0}\frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4} = \lim_{x\to 0}\frac{1}{x^2\left(a + \sqrt{a^2 - x^2}\right)} - \frac{1}{4x^2}$$

$$= \lim_{x\to 0}\frac{(4-a) - \sqrt{a^2 - x^2}}{4x^2\left(a + \sqrt{a^2 - x^2}\right)}$$

numerator $\to 0$ if $a = 2$ and then $L = \frac{1}{64}$.

32. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

Nitin M Sir (physics-iitjee.blogspot.com)

(A) e – 1

(B) $\int_1^e \ln(e+1-y)\,dy$

(C) $e - \int_0^1 e^x dx$

(D) $\int_1^e \ln y\,dy$

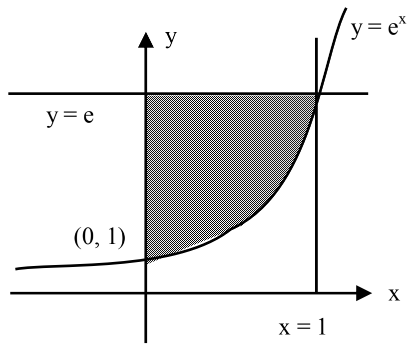
Sol. **(B, C, D)**

Required Area = $\int_1^e \ln y\,dy$

$= (y\ln y - y)_1^e = (e - e) - \{-1\} = 1.$

Also, $\int_1^e \ln y\,dy = \int_1^e \ln(e+1-y)\,dy$

Further the required area $= e \times 1 - \int_0^1 e^x dx$.



SECTION–III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for question Nos. 33 to 35

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

33. The probability that X = 3 equals

(A) $\frac{25}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{125}{216}$

Sol. **(A)**

$P(X = 3) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6} = \frac{25}{216}$.

34. The probability that X ≥ 3 equals

(A) $\frac{125}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{25}{216}$

Sol. **(B)**

$P(X \le 2) = \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{11}{36}$

Required probability $= 1 - \frac{11}{36} = \frac{25}{36}$.

35. The conditional probability that X ≥ 6 given X > 3 equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$

(C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Sol. **(D)**

For X ≥ 6, the probability is

$$\frac{5^5}{6^6}+\frac{5^6}{6^7}+...\infty=\frac{5^5}{6^6}\left(\frac{1}{1-5/6}\right)=\left(\frac{5}{6}\right)^5$$

For X > 3

$$\frac{5^3}{6^4}+\frac{5^4}{6^5}+\frac{5^5}{6^6}+...\infty=\left(\frac{5}{6}\right)^3$$

Hence the conditional probability $\frac{(5/6)^6}{(5/6)^3}=\frac{25}{36}$.

Paragraph for question Nos. 36 to 38

Let $\mathscr{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

36. The number of matrices in $\mathscr{A}$ is

(A) 12 (B) 6

(C) 9 (D) 3

Sol. **(A)**

If two zero's are the entries in the diagonal, then

${}^3C_2 \times {}^3C_1$

If all the entries in the principle diagonal is 1, then

${}^3C_1$

⇒ Total matrix = 12.

37. The number of matrices A in $\mathscr{A}$ for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is

(A) less than 4 (B) at least 4 but less than 7

(A) atleast 7 but less than 10 (D) at least 10

Sol. **(B)**

$$\begin{bmatrix}0 & a & b\\ a & 0 & c\\ b & c & 1\end{bmatrix}$$

either b = 0 or c = 0 ⇒ |A| ≠ 0

⇒ 2 matrices

$$\begin{bmatrix}0 & a & b\\ a & 1 & c\\ b & c & 0\end{bmatrix}$$

either a = 0 or c = 0 ⇒ |A| ≠ 0

⇒ 2 matrices

Nitin M Sir (physics-iitjee.blogspot.com)

$$\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$$

either a= 0 or b = 0 $\Rightarrow$ |A| $\neq$ 0

$\Rightarrow$ 2 matrices.

$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

If a = b = 0 $\Rightarrow$ |A| = 0

If a = c = 0 $\Rightarrow$ |A| = 0

If b = c = 0 $\Rightarrow$ |A| = 0

$\Rightarrow$ there will be only 6 matrices.

38. The number of matrices A in $\mathscr{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2

(C) 2 (D) 1

Sol. **(B)**

The six matrix A for which |A| = 0 are

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$ inconsistent.

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow$ inconsistent.

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow$ infinite solutions.

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ inconsistent.

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow$ inconsistent.

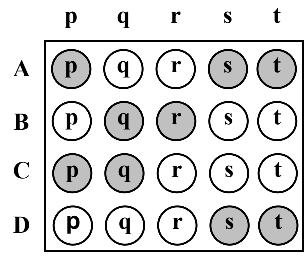
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow$ infinite solutions.

SECTION – IV
Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statement in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement (s) in

Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D –s and t; then the correct darkening of bubbles will look like the following.



39. Match the conics in **Column I** with the statements/expressions in **Column II.**

Column I		**Column II**	
(A)	Circle	(p)	The locus of the point (h, k) for which the line hx + ky = 1 touches the circle $x^2 + y^2 = 4$
(B)	parabola	(q)	Points z in the complex plane satisfying $\|z + 2\| - \|z - 2\| = \pm 3$
(C)	Ellipse	(r)	Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right)$, $y = \frac{2t}{1+t^2}$
(D)	Hyperbola	(s)	The eccentricity of the conic lies in the interval $1 \le x < \infty$
		(t)	Points z in the complex plane satisfying $\text{Re}(z + 1)^2 = \|z\|^2 + 1$

Sol. **(A) → (p)** **(B) → (s, t)** **(C) → (r)** **(D) → (q, s)**

(p). $\frac{1}{k^2} = 4\left(1 + \frac{h^2}{k^2}\right)$

$\Rightarrow 1 = 4(k^2 + h^2)$

$\therefore h^2 + k^2 = \left(\frac{1}{2}\right)^2$ which is a circle.

(q). If $|z - z_1| - |z - z_2| = k$ where $k < |z_1 - z_2|$

the locus is a hyperbola.

(r). Let $t = \tan\alpha$

$\Rightarrow x = \sqrt{3}\cos 2\alpha$ and $y = \sin 2\alpha$

or $\cos 2\alpha = \frac{x}{\sqrt{3}}$ and $\sin 2\alpha = y$

$\therefore \frac{x^2}{3} + y^2 = \sin^2 2\alpha + \cos^2 2\alpha = 1$ which is an ellipse.

(s). If eccentricity is $[1, \infty)$, then the conic can be a parabola (if e = 1) and a hyperbola if $e \in (1, \infty)$.

(t). Let $z = x + iy$; $x, y \in R$

$\Rightarrow (x + 1)^2 - y^2 = x^2 + y^2 + 1$

$\Rightarrow y^2 = x$; which is a parabola.

40. Match the statements/expressions in **Column I** with the open intervals in **Column II.**

Column I		**Column II**	
(A)	Interval contained in the domain of definition of non-zero solutions of the differential equation $(x - 3)^2 y' + y = 0$	(p)	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B)	Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q)	$\left(0, \frac{\pi}{2}\right)$
(C)	Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r)	$\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D)	Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s)	$\left(0, \frac{\pi}{8}\right)$
		(t)	$(-\pi, \pi)$

Sol. **(A) → (p, q, s)** **(B) → (p, t)** **(C) → (p, q, r, t)** **(D) → (s)**

(A). $(x-3)^2 \frac{dy}{dx} + y = 0$

$$\int \frac{dx}{(x-3)^2} = -\int \frac{dy}{y}$$

$$\Rightarrow \frac{1}{x-3} = \ln|y| + c$$

so domain is $R - \{3\}$.

(B). Put $x = t + 3$

$$\int_{-2}^{2} (t+2)(t+1)t(t-1)(t-2)\,dt = \int_{-2}^{2} t(t^2-1)(t^2-4)\,dt = 0 \text{ (being odd function)}$$

(C). $f(x) = \frac{5}{4} - \left(\sin x - \frac{1}{2}\right)^2$

Maximum value occurs when $\sin x = \frac{1}{2}$

(D). $f'(x) > 0$ if $\cos x > \sin x$.

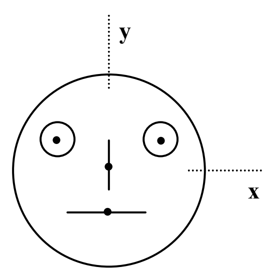
PART III: PHYSICS

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

41. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0) left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$

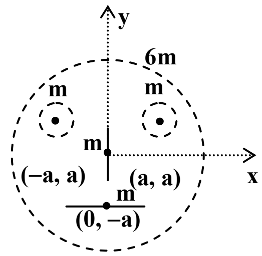
(C) $\frac{a}{12}$ (D) $\frac{a}{3}$

Nitin M Sir (physics-iitjee.blogspot.com)

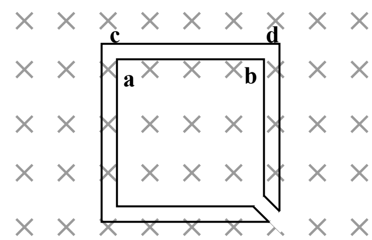
Sol. **(A)**

$$Y_{cm} = \frac{6m\times 0 + m\times a + m\times a + m\times 0 + m\times(-a)}{10\times m}$$

$$Y_{cm} = \frac{a}{10}$$



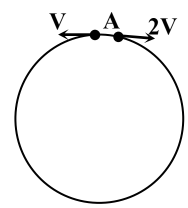
42. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then,



(A) $I_1 > I_2$
(B) $I_1 < I_2$
(C) I_1 is in the direction **ba** and I_2 is in the direction **cd**
(D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

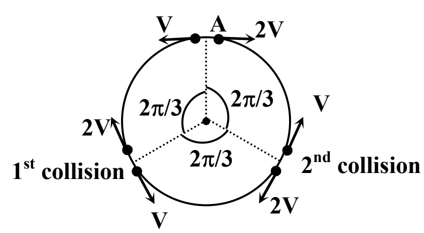
Sol. **(D)**
According to Lenz's law, current will be in anticlockwise sense as magnetic field is increasing into the plane of paper.

43. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

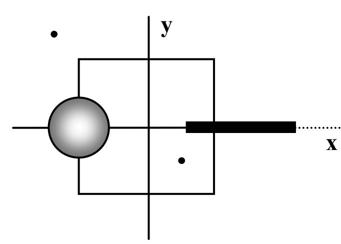


(A) 4 (B) 3
(C) 2 (D) 1

Sol. **(C)**
Velocity will exchange after each collision



44. A disk of radius a/4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at (–a/2, 0, 0). A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from x = a/4 to x = 5a/4. Two point charges –7C and 3C are placed at (a/4, –a/4, 0) and (–3a/4, 3a/4, 0), respectively. Consider a cubical surface formed by six surfaces x = ± a/2, y = ± a/2, z = ± a/2. The electric flux through this cubical surface is



(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$
(C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

Sol. **(A)**

Total charge enclosed by cube is –2C. Hence electric flux through the cube is $\frac{-2C}{\varepsilon_0}$.

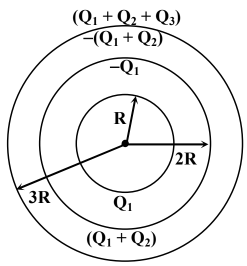
45. Three concentric metallic spherical shells of radii R, 2R, 3R, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) 1 : 2 : 3
(B) 1 : 3 : 5
(C) 1 : 4 : 9
(D) 1 : 8 : 18

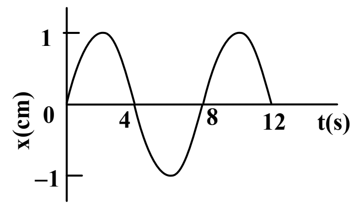
Sol. **(B)**

$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$\Rightarrow$ $Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$



46. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2
(B) $\frac{-\pi^2}{32}$ cm/s^2
(C) $\frac{\pi^2}{32}$ cm/s^2
(D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

Sol. **(D)**

The given motion is represented by

x = 1 sin (π/4) t

$$\frac{d^2x}{dt^2} = \frac{-\pi^2}{16}\ \sin(\pi/4)\ t$$

At t = 4/3 sec,

$$\frac{d^2x}{dt^2} = -\frac{\sqrt{3}}{32}\pi^2\ \text{cm/s}^2.$$

47. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(A) 9 m/s
(B) 12 m/s
(C) 16 m/s
(D) 21.33 m/s

Sol. **(C)**

$v_{ball}^2 = 2 \times 10 \times 7.2$

$\Rightarrow$ v = 12 m/s

$$x_{\text{image of ball}} = \frac{4}{3} x_{ball}$$

$$v_{\text{image of ball}} = \frac{4}{3} v_{ball} = \frac{4}{3} \times 12 = 16\ \text{m/s}$$

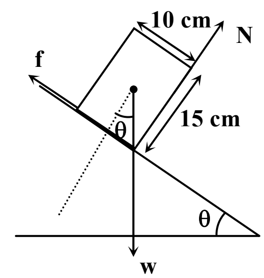
48. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(A) at θ = 30°, the block will start sliding down the plane
(B) the block will remain at rest on the plane up to certain θ and then it will topple
(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles
(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

Sol. **(B)**

For sliding, $\tan\theta \geq \sqrt{3}$ (=1.732)

For toppling, $\tan\theta \geq \frac{2}{3}$ (=0.67)



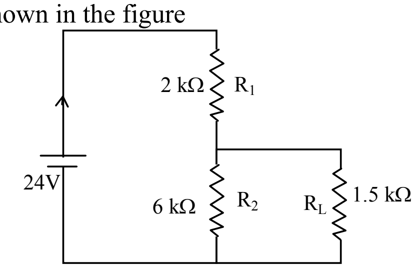
F.B.D. at just toppling condition.

SECTION – II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is /are correct.

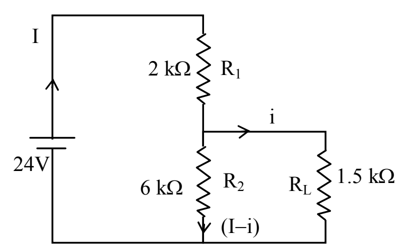
49. For the circuit shown in the figure



(A) the current I through the battery is 7.5 mA
(B) the potential difference across R_L is 18 V
(C) ratio of powers dissipated in R_1 and R_2 is 3
(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9.

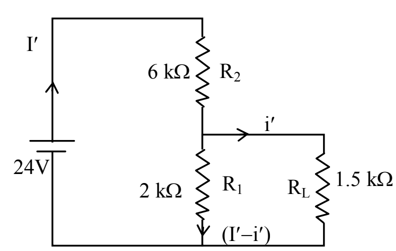
Sol. **(A & D)**

$24 - 2 \times 10^3\, I - 6 \times 10^3\,(I - i) = 0$
$24 - 2 \times 10^3\, I - 1.5 \times 10^3\, i = 0$
Hence I = 7.5 mA
i = 6mA



$24 - 6 \times 10^3 I' - 2 \times 10^3\,(I' - i') = 0$
$24 - 6 \times 10^3\, I' - 1.5 \times 10^3\, i' = 0$
I′ = 3.5 mA
i′ = 2mA

$$\frac{P_1}{P_2} = \frac{6^2}{2^2} = 9$$



50. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then
(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.
(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.
(C) C_p/C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas.
(D) $C_p . C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas.

Sol. **(B & D)**

For Monoatomic gas

$$C_v = \frac{3}{2}R\ ,\ C_p = \frac{5}{2}R$$

For diatomic gas

$$C_v = \frac{5}{2}R \quad C_p = \frac{7}{5}R$$

51. A student performed the experiment of determination of focal length of a concave mirror by u – v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that **cannot** come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56)
(B) (48, 48)
(C) (66, 33)
(D) (78, 39)

Sol. **(C & D)**

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad \text{(mirror formula)}$$

$f = -24$ cm

52. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

Sol. **(A)**

Linear momentum remains constant if net external force on the system of particles is zero.

SECTION –III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 53 to 55

When a particle is restricted to move along x-axis between x = 0 and x = a, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends x=0 and x=a. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = p^2/2m$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, …. (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line x = 0 to x = a. Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C.

53. The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2}
(B) $a^{-3/2}$
(C) a^{-1}
(D) a^2

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. **(A)**

$$a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$$

$$\lambda_{deBroglie} = \frac{h}{p}$$

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8a^2m}$$

$$\Rightarrow E \propto 1/a^2$$

54. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to
(A) 0.8 meV (B) 8 meV
(C) 80 meV (D) 800 meV

Sol. **(B)**

$$E = \frac{h^2}{8a^2m} = \frac{(6.6\times10^{-34})^2}{8\times(6.6\times10^{-9})^2\times10^{-30}\times1.6\times10^{-19}} = 8 \text{ meV.}$$

55. The speed of the particle that can take discrete values is proportional to
(A) $n^{-3/2}$ (B) n^{-1}
(C) $n^{1/2}$ (D) n

Sol. **(D)**

$$mv = \frac{nh}{2a}$$

$$v = \frac{nh}{2am}$$

$$v \propto n$$

Paragraph for Question Nos. 56 to 58

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1H$ known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D-D reaction is ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n$ + energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\frac{e^2}{4\pi\varepsilon_0} = 1.44\times10^{-9}$ eVm.

56. In the core of nuclear fusion reactor, the gas becomes plasma because of
(A) strong nuclear force acting between the deuterons
(B) Coulomb force acting between the deuterons
(C) Coulomb force acting between deuteron-electron pairs
(D) the high temperature maintained inside the reactor core

Sol. **(D)**

57. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) 1.0×10^9 K < T < 2.0×10^9 K
(B) 2.0×10^9 K < T < 3.0×10^9 K
(C) 3.0×10^9 K < T < 4.0×10^9 K
(D) 4.0×10^9 K < T < 5.0×10^9 K

Sol. **(A)**

$2 \times 1.5\ kT = \frac{e^2}{4\pi\varepsilon_0 d}$ (conservation of energy)

$T = 1.4 \times 10^9$ K

58. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3} s
(B) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1} s
(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11} s
(D) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12} s

Sol. **(B)**

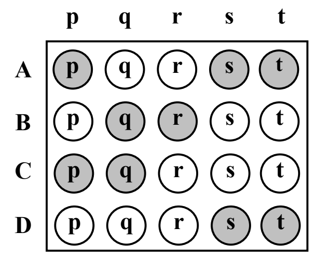
$nt_0 > 5 \times 10^{14}$ (as given)

Nitin M Sir (physics-iitjee.blogspot.com)

SECTION – IV
Matrix-Match Type

*This section contains 2 questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t; then the correct darkening of bubbles will look like the following.



59. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II**.

Column – I

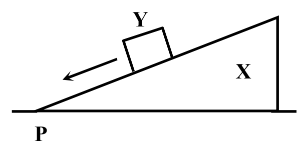
(A) The force exerted by X on Y has a magnitude Mg.

(B) The gravitational potential energy of X is continuously increasing.

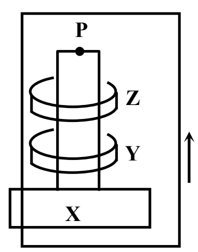
(C) Mechanical energy of the system X + Y is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

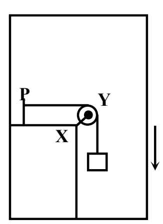
Column II

(p) 

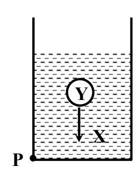
Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q) 

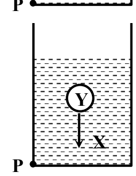
Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(r) 

A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity

(s) 

A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

(t) 

A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Sol. **(A) → (p, t), (B) → (q, s, t), (C) → (p, r, t), (D) → (q)**

60. Six point charges, each of the same magnitude q, are arranged in different manners as shown in **Column II**. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column – I

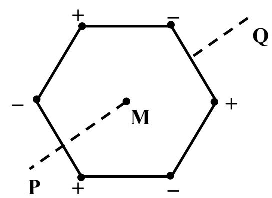
(A) $E = 0$

(B) $V \neq 0$

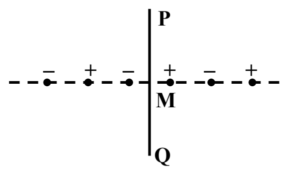
(C) $B = 0$

(D) $\mu \neq 0$

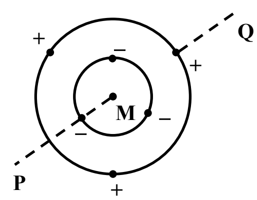
Column II

(p) 

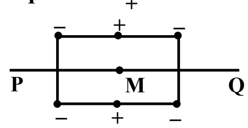
Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon.

(q) 

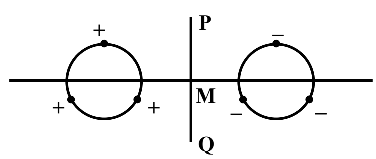
Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.

(r) 

Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.

(s) 

Charges are placed at the corners of a rectangle of sides a and 2a and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides.

(t) 

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid points between the centres of the rings. PQ is perpendicular to the line joining the centers and coplanar to the rings.

Sol. **(A) → (p, r, s) , (B) → (r, s), (C) → (p, q, t), (D) → (r, s)**

* * * * *

FIITJEE Solutions to

IIT – JEE - 2009

(PAPER – 2, CODE – 1)

Time: 3 Hours **Maximum Marks: 240**

A. Question paper format:

1. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part has 4 sections.
2. **Section I** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**
3. **Section II** contains **5** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct.**
4. **Section III** contains **2** questions. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with **one or more** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.
5. **Section IV** contains **8** questions. The answer to each of the questions is a **single – digit integer,** ranging from 0 to 9. The answer will have to be appropriately bubbled in the ORS as per the instructions given at the beginning of the section.

B. Marking scheme:

6. For each question in **Section I** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles is darkened. In case of bubbling of incorrect answer, **minus one (-1) mark** will be awarded.
7. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **Minus one (-1) mark** will be awarded.
8. For each question in **Section III**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8** marks. There is **no negative marking** for incorrect answer(s) for this section.
9. For each question in **Section IV**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded.

IITJEE 2009 (PAPER-2, CODE-1)

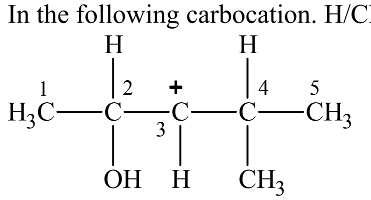
PART I: CHEMISTRY

SECTION-I

Single Correct Choice Type

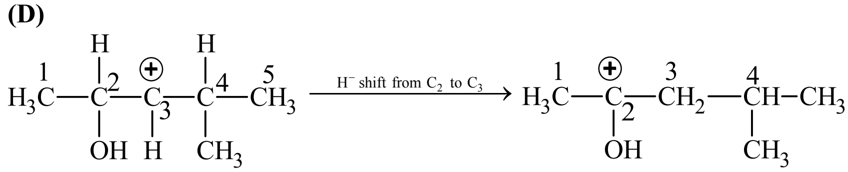
This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

1. In the following carbocation. H/CH_3 that is most likely to migrate to the positively charged carbon is



(A) CH_3 at C-4 (B) H at C-4
(C) CH_3 at C-2 (D) H at C-2

Sol. **(D)**



(driving force is conjugation from oxygen)

2. The correct stability order of the following resonance structures is

$\underset{(I)}{H_2C=\overset{+}{N}=\overset{-}{N}}$ $\underset{(II)}{H_2\overset{+}{C}-N=\overset{-}{N}}$ $\underset{(III)}{H_2\overset{-}{C}-\overset{+}{N}\equiv N}$ $\underset{(IV)}{H_2\overset{-}{C}-N=\overset{+}{N}}$

(A) (I) > (II) > (IV) > (III) (B) (I) > (III) > (II) > (IV)
(C) (II) > (I) > (III) > (IV) (D) (III) > (I) > (IV) > (II)

Sol. **(B)**
On the basis of stability of resonating structures.

3. The spin only magnetic moment value (in Bohr magneton units) of $Cr(CO)_6$ is
(A) 0 (B) 2.84
(C) 4.90 (D) 5.92

Sol. **(A)**
$[Cr(CO)_6]$
$Cr(24) = [Ar]3d^5\, 4s^1$
Since (CO) is strong ligand, in $Cr(CO)_6$ no unpaired electron is present. So 'spin only' magnetic moment is zero.

4. For a first order reaction $A \rightarrow P$, the temperature (T) dependent rate constant (k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are
(A) $1.0 \times 10^6\ s^{-1}$ and 9.2 $kJmol^{-1}$ (B) 6.0 s^{-1} and 16.6 $kJmol^{-1}$
(C) $1.0 \times 10^6\ s^{-1}$ and 16.6 $kJmol^{-1}$ (D) $1.0 \times 10^6\ s^{-1}$ and 38.3 $kJmol^{-1}$

Sol. **(D)**

Given, $\log K = 6 - \dfrac{2000}{T}$

Since, $\log K = \log A - \dfrac{Ea}{2.303RT}$ So, $A = 10^6\ \text{sec}^{-1}$ and Ea = 38.3 kJ/mole

SECTION-II

Multiple Correct Choice Type

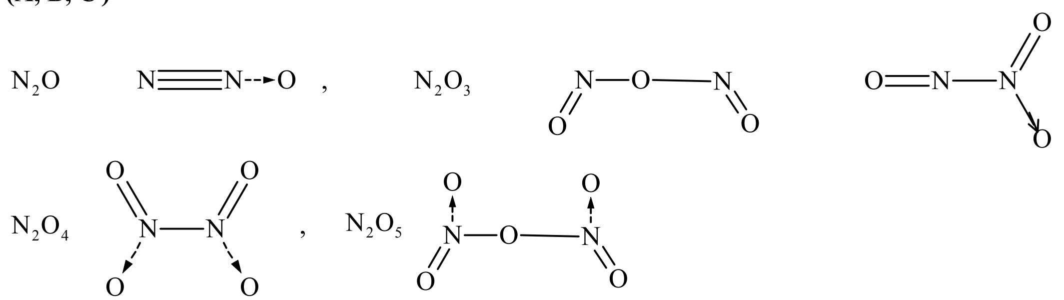
This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONE OR MORE** is/are correct.

5. The nitrogen oxide(s) that contain(s) N–N bond(s) is(are)

(A) N_2O (B) N_2O_3

(C) N_2O_4 (D) N_2O_5

Sol. **(A, B, C)**



6. In the reaction

$2\mathbf{X} + B_2H_6 \rightarrow \left[BH_2(\mathbf{X})_2\right]^+ \left[BH_4\right]^-$

the amine(s) **X** is(are)

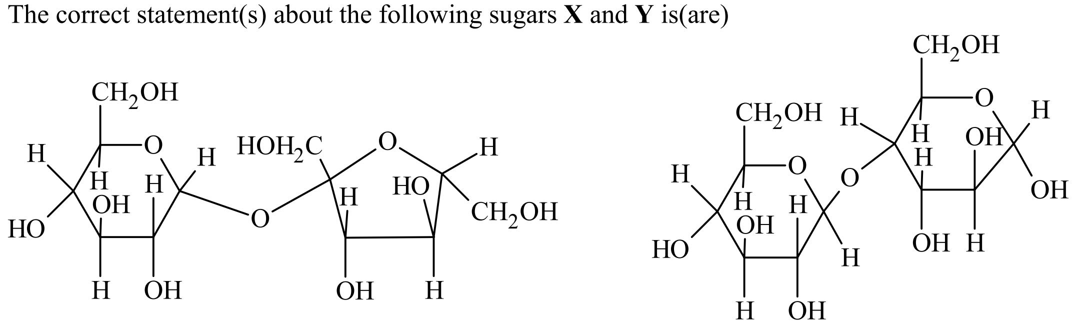
(A) NH_3 (B) CH_3NH_2

(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

Sol. **(A, B, C)**

Due to bulkiness of trimethylamine, it does not react.

7. The correct statement(s) about the following sugars **X** and **Y** is(are)



X **Y**

(A) **X** is a reducing sugar and **Y** is a non-reducing sugar

(B) **X** is a non-reducing sugar and **Y** is a reducing sugar

(C) The glucosidic linkages in **X** and **Y** are α and β, respectively

(D) The glucosidic linkages in **X** and **Y** are β and α, respectively

Sol. **(B, C)**

8. Among the following, the state function(s) is(are)

(A) Internal energy
(B) Irreversible expansion work
(C) Reversible expansion work
(D) Molar enthalpy

Sol. **(A, D)**

9. For the reduction of NO_3^- ion in an aqueous solution, E^0 is +0.96 V. Values of E^0 for some metal ions are given below

$V^{2+}(aq) + 2e^- \rightarrow V \qquad E^0 = -1.19\ V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe \qquad E^0 = -0.04\ V$

$Au^{3+}(aq) + 3e^- \rightarrow Au \qquad E^0 = +1.40\ V$

$Hg^{2+}(aq) + 2e^- \rightarrow Hg \qquad E^0 = +0.86\ V$

The pair(s) of metals that is(are) oxidized by NO_3^- in aqueous solution is(are)

(A) V and Hg
(B) Hg and Fe
(C) Fe and Au
(D) Fe and V

Sol. **(A, B, D)**

$E^o_{\substack{NO_3^-(aq)\\(SRP)}} = 0.96\ V$ All V, Fe, Hg have less SRP w.r.t. NO_3^-.

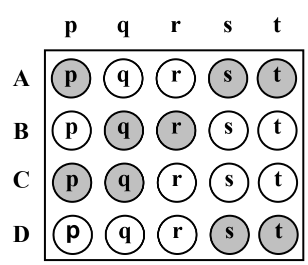
So, V, Fe, Hg can be oxidized by NO_3^- in aqueous solution.

SECTION – III

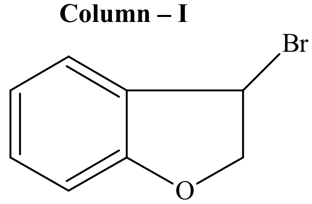
Matrix – Match Type

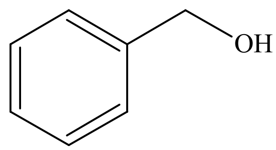
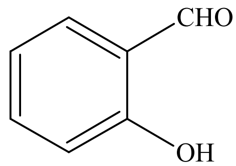
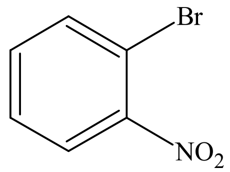
This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



10. Match each of the compounds given in **Column I** with the reaction(s), that they can undergo given in **Column II**.

Column – I	Column – II
(A) 	(p) Nucleophilic substitution

(B) 	(q) Elimination
(C) 	(r) Nucleophilic addition
(D) 	(s) Esterification with acetic anhydride
	(t) Dehydrogenation

Sol. **((A – p, q, t) (B – p, s, t) (C – r, s) (D – p))**

11. Match each of the reactions given in **Column I** with the corresponding product(s) given in **Column II**.

Column – I	**Column – II**
(A) Cu + dil HNO_3	(p) NO
(B) Cu + conc HNO_3	(q) NO_2
(C) Zn + dil HNO_3	(r) N_2O
(D) Zn + conc HNO_3	(s) $Cu(NO_3)_2$
	(t) $Zn(NO_3)_2$

Sol. **((A – p, s) (B – q, s) (C – r, t) (D – q, t))**

$$3Cu + dil.8HNO_3 \rightarrow 3Cu(NO_3)_2 + 4H_2O + 2NO$$

$$Cu + conc.\ 4HNO_3 \rightarrow Cu(NO_3)_2 + 2H_2O + 2NO_2$$

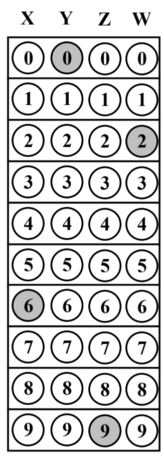
$$4Zn + dil.\ 10HNO_3 \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O$$

$$Zn + conc.\ 4HNO_3 \rightarrow Zn(NO_3)_2 + 2H_2O + 2NO_2$$

SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



12. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is

Sol. **6**

$2MnO_2 + 4KOH + O_2 \rightarrow \underset{\text{(Potassium manganate)}}{2K_2MnO_4} + 2H_2O$ O. S. of Mn = +6 in K_2MnO_4

13. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 \cdot 5H_2O$ is

Sol. **4**

$CuSO_4 \cdot 5H_2O \rightarrow \left[Cu\left(H_2O\right)_4\right]SO_4 \cdot H_2O$

So, water molecules directly attached to Cu are 4.

14. The coordination number of Al in the crystalline state of $AlCl_3$ is

Sol. **6**

Coordination number of Al is 6. It exists in ccp lattice with 6 coordinate layer structure.

15. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in $kJmol^{-1}$ is

Sol. **9**

Energy release at constant volume due to combustion of 3.5 gm of a gas = 2.5×0.45

Hence energy released due to the combustion of 28 gm (i.e., 1 mole) of a gas $= 2.5 \times 0.45 \times \frac{28}{3.5} = 9\ kJmol^{-1}$

16. The dissociation constant of a substituted benzoic acid at 25°C is 1.0×10^{-4}. The pH of a 0.01 M solution of its sodium salt is

Sol. **8**

K_a (C_6H_5COOH) = 1 × 10^{-4}

pH of 0.01 M C_6H_5COONa

$$\underset{0.01(1-h)}{C_6H_5COO^-} + H_2O \rightleftharpoons \underset{0.01\text{ h}}{C_6H_5COOH} + \underset{0.01\text{ h}}{OH^{-1}}$$

$$K_h = \frac{K_w}{K_a} = \frac{0.01\ h^2}{1-h}$$

$$\frac{10^{-14}}{10^{-4}} = \frac{10^{-2}h^2}{1-h} \quad (1-h \simeq 1)$$

$[OH^-]$ = 0.01 h = 0.01 × 10^{-4} = 10^{-6}

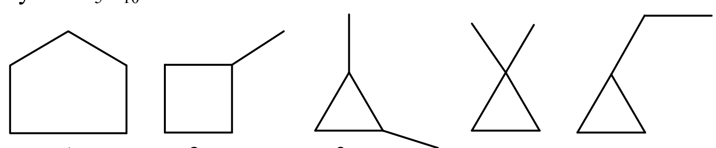
$[H^+]$ = 10^{-8}

pH = 8

17. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

Sol. **7**

Cyclic C_5H_{10}



1 2 3 4 5

For 3rd structure 2 cis-trans and 1 optical isomer are possible.

Total 7 isomers.

18. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb$ is

Sol. **8**

$${}_{92}U^{238} \xrightarrow{-6\alpha} {}_{80}X^{214} \xrightarrow{-2\beta} {}_{82}Pb^{214}$$ (6α, 2β), total 8 paritcles.

19. At 400 K, the root mean square (rms) speed of a gas **X** (molecular weight = 40) is equal to the most probable speed of gas **Y** at 60 K. The molecular weight of the gas **Y** is

Sol. **4**

$$V_{rms_{(X\ gas)(400\ K)}} = V_{mp_{(Y\ gas)(60\ K)}}$$

M.W. (X gas) = 40; M.W. (Y gas) = x

$$\sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{2RT_2}{M_2}}$$

$$\sqrt{\frac{400\times 3}{40}} = \sqrt{\frac{2\times 60}{x}}$$

$$30 = \frac{120}{x}$$

x = 4

Nitin M Sir (physics-iitjee.blogspot.com)

PART II: MATHEMATICS

SECTION–I

Single Correct Choice Type

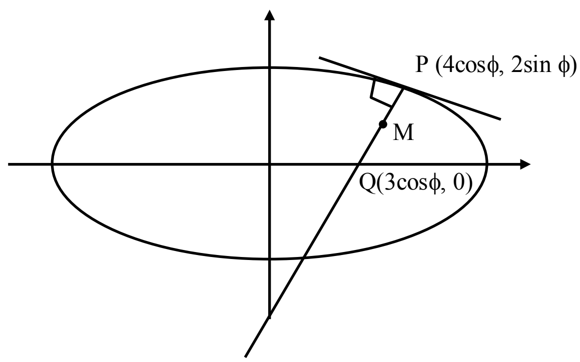
This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

20. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Sol. **(C)**



Normal is $4x \sec\phi - 2y \operatorname{cosec}\phi = 12$

$Q \equiv (3\cos\phi, 0)$

$M \equiv (\alpha, \beta)$

$\alpha = \frac{3\cos\phi + 4\cos\phi}{2} = \frac{7}{2}\cos\phi$

$\Rightarrow \cos\phi = \frac{2}{7}\alpha$

$\beta = \sin\phi$

$\cos^2\phi + \sin^2\phi = 1$

$\Rightarrow \frac{4}{49}\alpha^2 + \beta^2 = 1 \Rightarrow \frac{4}{49}x^2 + y^2 = 1$

$\Rightarrow$ latus rectum $x = \pm\, 2\sqrt{3}$

$\frac{48}{49} + y^2 = 1 \Rightarrow y = \pm\frac{1}{7}$

$(\pm 2\sqrt{3}, \pm 1/7)$.

21. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

Sol. **(D)**

Intersection point of $y = 0$ with first line is $B(-p, 0)$

Intersection point of $y = 0$ with second line is $A(-q, 0)$

Intersection point of the two lines is $C(pq, (p + 1)(q + 1))$

Altitude from C to AB is $x = pq$

Altitude from B to AC is $y = -\frac{q}{1+q}(x+p)$

Solving these two we get $x = pq$ and $y = -pq$

$\therefore$ locus of orthocentre is $x + y = 0$.

Nitin M Sir (physics-iitjee.blogspot.com)

22. A line with positive direction cosines passes through the point P(2, – 1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals

(A) 1
(B) $\sqrt{2}$
(C) $\sqrt{3}$
(D) 2

Sol. **(C)**

D.C of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Any point on the line at a distance t from P(2, – 1, 2) is $\left(2+\frac{t}{\sqrt{3}},\ -1+\frac{t}{\sqrt{3}},\ 2+\frac{t}{\sqrt{3}}\right)$

which lies on $2x + y + z = 9 \Rightarrow t = \sqrt{3}$.

23. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$
(B) $\frac{n(4n^2+1)c^2}{3}$
(C) $\frac{n(4n^2-1)c^2}{3}$
(D) $\frac{n(4n^2+1)c^2}{6}$

Sol. **(C)**

$t_n = c\ \{n^2 - (n-1)^2\}$
$= c\ (2n - 1)$
$\Rightarrow t_n^2 = c^2(4n^2 - 4n + 1)$
$\Rightarrow \sum_{n=1}^{n} t_n^2 = c^2\left\{\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right\}$
$= \frac{c^2 n}{6}\{4(n+1)(2n+1) - 12(n+1) + 6\}$
$= \frac{c^2 n}{3}\{4n^2 + 6n + 2 - 6n - 6 + 3\} = \frac{c^2}{3}n(4n^2 - 1)$.

SECTION–II

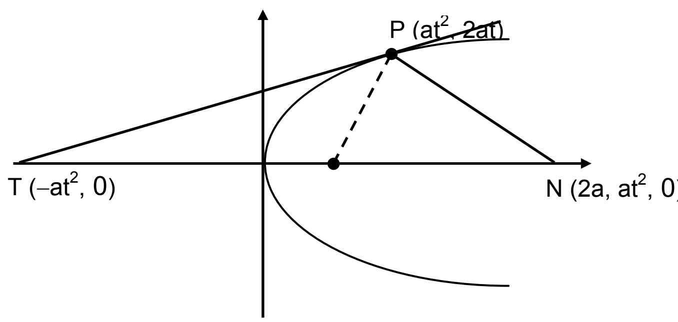
Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

24. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3},\ 0\right)$
(B) directrix is x = 0
(C) latus rectum is $\frac{2a}{3}$
(D) focus is (a, 0)

Sol. **(A, D)**



$G \equiv (h, k)$

$\Rightarrow h = \dfrac{2a + at^2}{3}$, $k = \dfrac{2at}{3}$

$\Rightarrow \left(\dfrac{3h - 2a}{a}\right) = \dfrac{9k^2}{4a^2}$

$\Rightarrow$ required parabola is

$\dfrac{9y^2}{4a^2} = \dfrac{(3x - 2a)}{a} = \dfrac{3}{a}\left(x - \dfrac{2a}{3}\right)$

$\Rightarrow y^2 = \dfrac{4a}{3}\left(x - \dfrac{2a}{3}\right)$

Vertex $\equiv \left(\dfrac{2a}{3}, 0\right)$; Focus $\equiv (a, 0)$

25. For function $f(x) = x \cos\dfrac{1}{x}$, $x \geq 1$,

(A) for atleast one x in interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim\limits_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Sol. **(B, C, D)**

For $f(x) = x\cos\left(\dfrac{1}{x}\right)$, $x \geq 1$

$f'(x) = \cos\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\sin\left(\dfrac{1}{x}\right) \to 1$ for $x \to \infty$

also $f''(x) = \dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) - \dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) - \dfrac{1}{x^3}\cos\left(\dfrac{1}{x}\right)$

$= -\dfrac{1}{x^3}\cos\left(\dfrac{1}{x}\right) < 0 \quad$ for $x \geq 1$

$\Rightarrow f'(x)$ is decreasing for $[1, \infty)$

$\Rightarrow f'(x + 2) < f'(x)$. Also, $\lim\limits_{x\to\infty} f(x+2) - f(x) = \lim\limits_{x\to\infty}\left[(x+2)\cos\dfrac{1}{x+2} - x\cos\dfrac{1}{x}\right] = 2$

$\therefore f(x+2) - f(x) > 2 \;\forall\, x \geq 1$

26. For $0 < \theta < \dfrac{\pi}{2}$, the solution(s) of $\sum\limits_{m=1}^{6} \operatorname{cosec}\left(\theta + \dfrac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \dfrac{m\pi}{4}\right) = 4\sqrt{2}$ is(are)

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{12}$ (D) $\dfrac{5\pi}{12}$

Sol. **(C, D)**

Given solutions

$$\frac{1}{\sin(\pi/4)}\left[\frac{\sin(\theta + \pi/4 - \theta)}{\sin\theta \cdot \sin(\theta + \pi/4)} + \frac{\sin(\theta + \pi/2 - (\theta + \pi/4))}{\sin(\theta + \pi/4)\cdot\sin(\theta + \pi/2)} + \ldots + \frac{\sin((\theta + 3\pi/2) - (\theta + 5\pi/4))}{\sin(\theta + 3\pi/2)\cdot\sin(\theta + 5\pi/4)}\right] = 4\sqrt{2}$$

$$\Rightarrow \sqrt{2}\left[\cot\theta - \cot(\theta + \pi/4) + \cot(\theta + \pi/4) - \cot(\theta + \pi/2) + \ldots + \cot(\theta + 5\pi/4) - \cot(\theta + 3\pi/2)\right] = 4\sqrt{2}$$

$\Rightarrow \tan\theta + \cot\theta = 4 \Rightarrow \tan\theta = 2 \pm \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$.

27. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$
(B) the foci of ellipse are $(\pm 1, 0)$
(C) equation of ellipse is $x^2 + 2y^2 = 4$
(D) the foci of ellipse are $(\pm \sqrt{2}, 0)$

Sol. **(A, B)**

Ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

$\Rightarrow \left(\pm a \times \frac{1}{\sqrt{2}},\ 0\right) \equiv (\pm 1, 0)$

$\Rightarrow a = \sqrt{2}$ and $e = \frac{1}{\sqrt{2}}$

$\Rightarrow b^2 = a^2(1 - e^2) \Rightarrow b^2 = 1$

$\therefore$ Equation of ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$.

28. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, n = 0, 1, 2, …, then

(A) $I_n = I_{n+2}$
(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
(C) $\sum_{m=1}^{10} I_{2m} = 0$
(D) $I_n = I_{n+1}$

Sol. **(A, B, C)**

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$$

$$= \int_0^{\pi}\left(\frac{\sin nx}{(1+\pi^x)\sin x} + \frac{\pi^x \sin nx}{(1+\pi^x)\sin x}\right)dx = \int_0^{\pi}\frac{\sin nx}{\sin x}$$

$$\text{Now, } I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$$

$$= \int_0^{\pi} \frac{2\cos(n+1)x \cdot \sin x}{\sin x} dx = 0$$

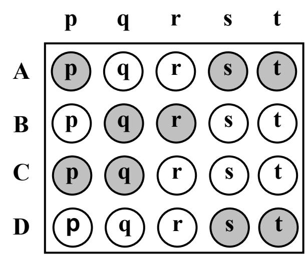
$$\Rightarrow I_1 = \pi,\ I_2 = \int_0^{\pi} 2\cos x\, dx = 0$$

SECTION – III

Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statement in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement (s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D –s and t; then the correct darkening of bubbles will look like the following.



29. Match the statements/expressions in **Column I** with the values given in **Column II.**

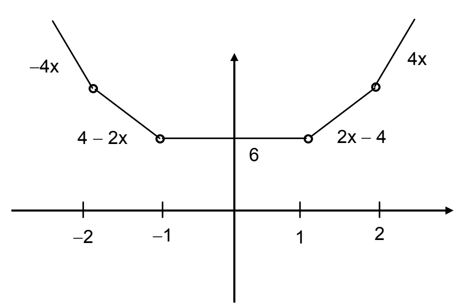
Column I		**Column II**	
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
(B)	Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q)	2
(C)	Value(s) of k for which $\|x - 1\| + \|x - 2\| + \|x + 1\| + \|x + 2\| = 4k$ has integer solution(s)	(r)	3
(D)	If $y' = y + 1$ and $y(0) = 1$ then value(s) of $y(\ln 2)$	(s)	4
		(t)	5

Sol. **(A) → (p)** **(B) → (q, s)** **(C) → (q, r, s, t)** **(D) → (r)**

(A). $f'(x) > 0, \forall x \in (0, \pi/2)$
$f(0) < 0$ and $f(\pi/2) > 0$
so one solution.

(B). Let (a, b, c) is direction ratio of the intersected line, then
$ak + 4b + c = 0$
$4a + kb + 2c = 0$
$$\frac{a}{8-k} = \frac{b}{4-2k} = \frac{c}{k^2-16}$$
We must have $2(8 - k) + 2(4 - 2k) + (k^2 - 16) = 0$
$\Rightarrow k = 2, 4.$

(C). Let $f(x) = |x + 2| + |x + 1| + |x - 1| + |x - 2|$
$\Rightarrow$ k can take value 2, 3, 4, 5.



Nitin M Sir (physics-iitjee.blogspot.com)

(D). $\int \frac{dy}{y+1} = \int dx$

$\Rightarrow f(x) = 2e^x - 1$

$\Rightarrow f(\ln 2) = 3$

30. Match the statements/expressions in **Column I** with the values given in **Column II.**

Column I		Column II	
(A)	Root(s) of the expression $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q)	$\frac{\pi}{4}$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j},\ \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r)	$\frac{\pi}{3}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c} = \vec{0}$	(s)	$\frac{\pi}{2}$
		(t)	π

Sol. **(A) → (q, s) (B) → (p, r, s, t) (C) → (t) (D) → (r)**

(A). $2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$

$\sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$

$3\sin^2\theta - 2\sin^4\theta - 1 = 0 \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}, \pm 1$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}$.

(B). Let $y = \frac{3x}{\pi}$

$\Rightarrow \frac{1}{2} \le y \le 3\ \forall\ x \in \left[\frac{\pi}{6}, \pi\right]$

Now f(y) = [2y] cos[y]

Critical points are $y = \frac{1}{2}, y = 1, y = \frac{3}{2}, y = 3$

$\Rightarrow$ points of discontinuity $\left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$.

(C). $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi \Rightarrow$ volume of parallelopiped $= \pi$

(D). $|\vec{a}+\vec{b}| = \sqrt{3}$

$\Rightarrow \sqrt{2+2\cos\alpha} = \sqrt{3}$

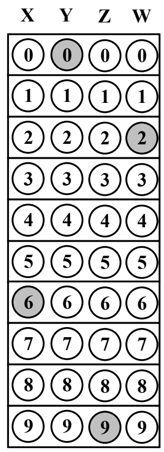
$\Rightarrow 2 + 2\cos\alpha = 3$

$\Rightarrow \alpha = \frac{\pi}{3}$.

SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



31. Let f: R → R be a continuous function which satisfies $f(x) = \int_0^x f(t)\,dt$. Then the value of f(ln5) is

Sol. **0**

$$f(x) = \int_0^x f(t)\,dt \Rightarrow f(0) = 0$$

also, $f'(x) = f(x),\ x > 0$
$\Rightarrow f(x) = ke^x,\ x > 0$
$\because$ f(0) = 0 and f(x) is continuous $\Rightarrow f(x) = 0\ \forall\ x > 0$
$\therefore$ f(ln5) = 0.

32. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

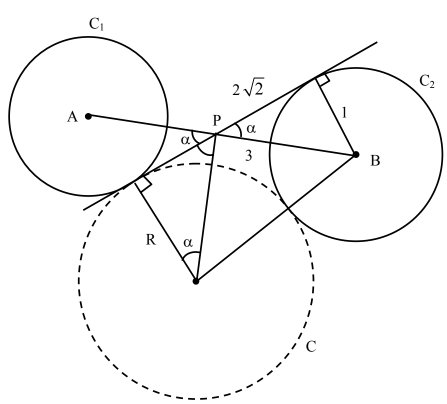
Sol. **8**

$$\cos\alpha = \frac{2\sqrt{2}}{3}$$

$$\sin\alpha = \frac{1}{3}$$

$$\tan\alpha = \frac{2\sqrt{2}}{R}$$

$$\Rightarrow R = \frac{2\sqrt{2}}{\tan\alpha} = 8 \text{ units.}$$



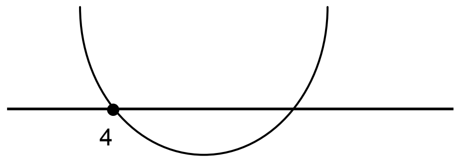
Alternate:

$(R + 1)^2 = (R - 1)^2 + \left(4\sqrt{2}\right)^2$

$\Rightarrow R = 8.$

33. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Sol. **2**



$x^2 - 8kx + 16(k^2 - k + 1) = 0$

$D > 0 \Rightarrow k > 1$ … (1)

$\frac{-b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4$

$\Rightarrow k > 1$ … (2)

$f(4) \ge 0 \Rightarrow 16 - 32k + 16(k^2 - k + 1) \ge 0$

$k^2 - 3k + 2 \ge 0$

$k \le 1 \cup k \ge 2$ … (3)

Using (1), (2) and (3)

$k_{min} = 2.$

34. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x | x^2 + 20 \le 9x|\}$ is

Sol. **7**

$f'(x) = 6(x - 2)(x - 3)$

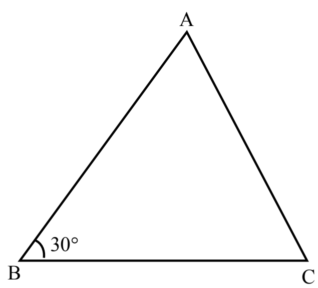
so $f(x)$ is increasing in $(3, \infty)$

Also, $A = \{4 \le x \le 5\}$

$\therefore f_{max} = f(5) = 7.$

35. Let ABC and ABC′ be two non–congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

Sol. **4**



$\cos\beta = \frac{a^2 + 16 - 8}{2 \times a \times 4}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2 + 8}{8a}$

$\Rightarrow a^2 - 4\sqrt{3}\,a + 8 = 0$

$\Rightarrow a_1 + a_2 = 4\sqrt{3}, \; a_1a_2 = 8$

$\Rightarrow |a_1 - a_2| = 4$

$\Rightarrow |\Delta_1 - \Delta_2| = \frac{1}{2} \times 4 \sin 30° \times 4 = 4.$

36. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Sol. **2**

$f(0) = 1,\; f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$

$\Rightarrow f'(g(x))\, g'(x) = 1$

Put $x = 0 \Rightarrow g'(1) = \frac{1}{f'(0)} = 2.$

Nitin M Sir (physics-iitjee.blogspot.com)

37. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is

Sol. **0**

Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$P'(1) = P'(2) = 0$

$\lim_{x\to 0}\left(\frac{x^2 + P(x)}{x^2}\right) = 2$

$\Rightarrow P(0) = 0 \Rightarrow e = 0$

$\lim_{x\to 0}\left(\frac{2x + P'(x)}{2x}\right) = 2$

$\Rightarrow P'(0) = 0 \Rightarrow d = 0$

$\lim_{x\to 0}\left(\frac{2 + P''(x)}{2}\right) = 2$

$\Rightarrow c = 1$

On solving, a = 1/4, b = – 1

So $P(x) = \frac{x^4}{4} - x^3 + x^2$

$\Rightarrow P(2) = 0.$

38. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

$3x - y - z = 0$

$-3x + z = 0$

$-3x + 2y + z = 0.$

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is

Sol. **7**

$3x - y - z = 0$

$-3x + z = 0$

$\Rightarrow y = 0$

and $z = 3x$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + z^2 = x^2 + 9x^2 = 10x^2 \le 100$

$\Rightarrow x^2 \le 10$

$\Rightarrow x = 0, \pm 1, \pm 2, \pm 3$

There are such seven points.

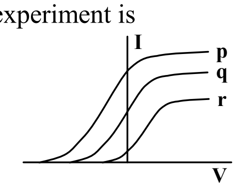
PART III: PHYSICS

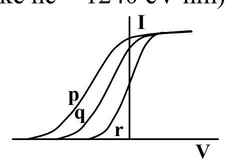
SECTION-I

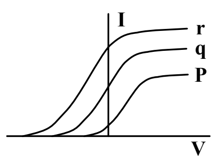
Single Correct Choice Type

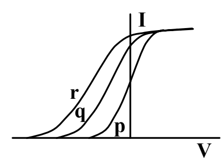
This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

39. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is (Take hc = 1240 eV nm)

(A) 

(B) 

(C) 

(D) 

Sol. **(A)**

$V_B = (1/e)[(hc/\lambda) - \phi]$

$V_P = (1/e)[(1240/550) - 2]eV = 0.2545$ V

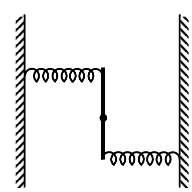
$V_q = (1/e)[(1240/450) - 2.5]eV = 0.255$ V

$V_r = (1/e)[(1240/350) - 3]eV = 0.5428$ V

If n is the number of photons in unit time then $nhc/\lambda = I$

$\Rightarrow\ i_P : i_q : i_r = n_P : n_q : n_r = \lambda_P : \lambda_q : \lambda_r$

40. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Sol. **(C)**

Restoring torque $= -2 \times k\left(\frac{\ell}{2}\theta\right)\frac{\ell}{2} = \frac{Id^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\frac{k\ell^2}{2}(-\theta)}{\frac{M\ell^2}{12}}$$

$$\Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

Nitin M Sir (physics-iitjee.blogspot.com)

41. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stays at rest with respect to the wire, from the y-axis is

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$

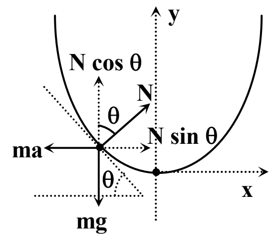
(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

Sol. (B)

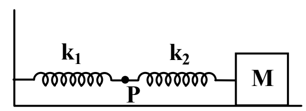
$\tan\theta = \frac{a}{g}$

$\tan\theta = \frac{dy}{dx} = 2kx$

$\Rightarrow \quad x = \frac{a}{2gk}$



42. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Sol. **(D)**

$x_1 + x_2 = A$

$k_1 x_1 = k_2 x_2$

Hence $x_1 = \frac{k_2 A}{k_1 + k_2}$

SECTION -II

Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

43. Under the influence of the coulomb field of charge +Q, a charge −q is moving around it in an elliptical orbit. Find out the correct statement(s).

(A) The angular momentum of the charge −q is constant

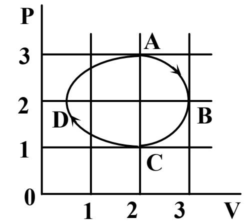
(B) The linear momentum of the charge −q is constant

(C) The angular velocity of the charge −q is constant

(D) The linear speed of the charge −q is constant

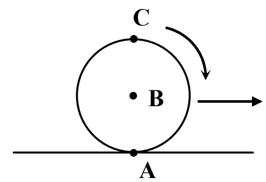
Sol. **(A)**

44. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,
(A) the process during the path $A \to B$ is isothermal
(B) heat flows out of the gas during the path $B \to C \to D$
(C) work done during the path $A \to B \to C$ is zero
(D) positive work is done by the gas in the cycle ABCDA



Sol. **(B) & (D)**
$\Delta Q = \Delta U + W$
For process $B \to C \to D$
ΔU is negative as well as W is also negative

45. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



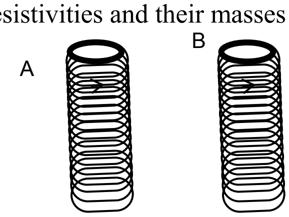
(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

Sol. **(B) & (C)**

46. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then,
(A) the intensity of the sound heard at the first resonance was more than that at the second resonance
(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(D) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

Sol. **(A) & (D)**
Larger the length of air column, feebler is the intensity

47. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



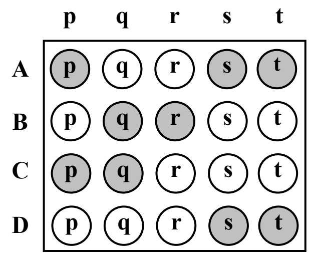
(A) $\rho_A > \rho_B$ and $m_A = m_B$ (B) $\rho_A < \rho_B$ and $m_A = m_B$
(C) $\rho_A > \rho_B$ and $m_A > m_B$ (D) $\rho_A < \rho_B$ and $m_A < m_B$

Sol. **(B) & (D)**
As $-\frac{d\phi}{dt}$ = emf is same, the current induced in the ring will depend upon resistance of the ring. Larger the resistivity smaller the current.

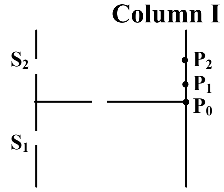
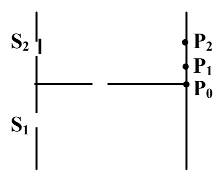
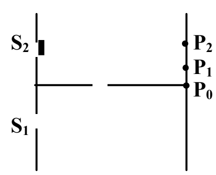
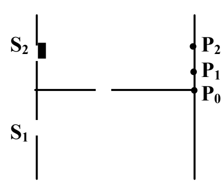
SECTION – III

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t; then the correct darkening of bubbles will look like the following.



48. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by I(P). Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column I		**Column II**
(A)		(p) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \lambda/4$		(q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \lambda/2$		(r) $I(P_1) = 0$
(D) $(\mu - 1)t = 3\lambda/4$		(s) $I(P_0) > I(P_1)$
		(t) $I(P_2) > I(P_1)$

Sol. **A → (p, s); B → (q); C → (t); D → (r, s, t)**

49. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II.**

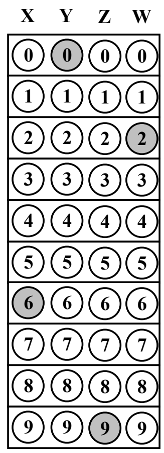
Column I		**Column II**	
(A) The energy of the system is increased	(p)	System:	A capacitor, initially uncharged
		Process :	It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q)	System:	A gas in an adiabatic container fitted with an adiabatic piston
(C) Internal energy of the system is converted in to its mechanical energy		Process:	The gas is compressed by pushing the piston
(D) Mass of the system is decreased	(r)	System:	A gas in a rigid container
		Process:	The gas gets cooled due to colder atmosphere surrounding it
	(s)	System :	A heavy nucleus, initially at rest
		Process:	The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t)	System:	A resistive wire loop
		Process:	The loop is placed in a time varying magnetic field perpendicular to its plane

Sol. **A → (p, q, t); B → (q); C → (s); D → (s)**

SECTION –IV

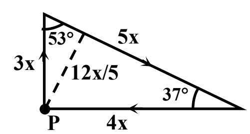
Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following



50. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

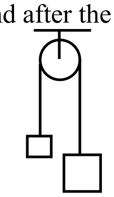
Sol. **7**



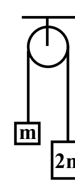
$$B = \frac{\mu_0 I}{4\pi \frac{12x}{5}}[\cos 53° + \cos 37°] = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

k = 7

51. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (**in joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Sol. **8**



2mg – T = 2ma

T – mg = ma

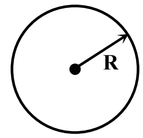
$\Rightarrow$ a = g/3

T = 4mg/3

$W = T.s = T.\frac{1}{2}at^2 = 8$ Joules

52. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at r = R/2 is 1/8 times that at r = R, find the value of a.

Sol. **2**



$\rho = kr^a$

$E\left(r = \frac{R}{2}\right) = \frac{1}{8}E(r = R)$

$\frac{q_{enclosed}}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$

$32q_{enclosed} = Q$

$q_{enclosed} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)}$

$Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$

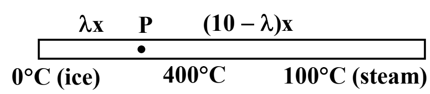
$\frac{Q}{q_{enclosed}} = 2^{a+3}$

$2^{a+3} = 32$

$a = 2$

53. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding.]

Sol. **9**



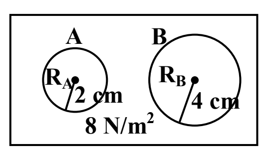
$\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$

$\frac{400kS}{\lambda x L_{ice}} = \frac{300kS}{(100-\lambda)xL_{vapour}}$

$\lambda = 9$

54. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Sol. **6**



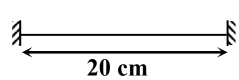
$P_A = P_0 + \frac{4T}{R_A} = 16 \text{ N/m}^2$

$P_B = P_0 + \frac{4T}{R_B} = 12 \text{ N/m}^2$

$\frac{n_B}{n_A} = \frac{P_B}{P_A}\left(\frac{R_B}{R_A}\right)^3 = 6$

55. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string.

Sol. **5**

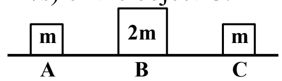


$v = \sqrt{\frac{T}{\mu}} = 10 \text{ m/s}$

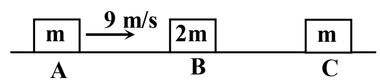
$$\lambda = \frac{v}{f} = \frac{10}{100} = 10 \text{ cm}$$

Distance between the successive nodes = $\lambda/2$ = 5 cm

56. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C.



Sol. 4



After 1st collision

$mv_A = mv'_A + 2mv'_B$

$$-1 = \frac{v'_B - v'_A}{0 - v_A} \Rightarrow v'_B = 6 \text{ m/s}$$

After 2nd collision

$$2mv'_B = (2m + m)v_C \Rightarrow \quad v_C = \frac{2}{3}v'_B \Rightarrow v_C = 4 \text{ m/s}$$

57. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (**in mm**) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Sol. **6**

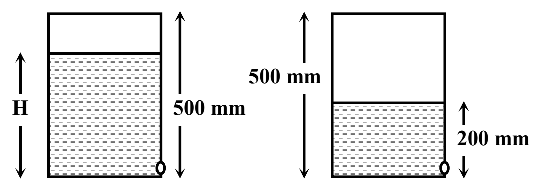
$P = P_0 - \rho gh = 98 \times 10^3$ N/m^2

$P_0V_0 = PV$

$10^5[A(500 - H)] = 98 \times 10^3[A(500 - 200)]$

H = 206 mm

Level fall = 206 – 200 = 6 mm



FIITJEE Solutions to IIT-JEE-2010

CODE 3

PAPER 1

Time: 3 Hours

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This Question Paper contains 32 pages having 84 questions.
2. The **question paper** CODE is printed on the right hand top corner of this sheet and also on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
5. The answer sheet, a machine-gradable Objective Response Sheet **(ORS)**, is provided separately.
6. Do not Tamper / mutilate the ORS or this booklet.
7. Do not break the seals of the question – paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS:

8. The ORS has CODE printed on its lower and upper Parts.
9. Make sure the CODE on the **ORS** is the same as that on this booklet. **If the Codes do not match, ask for a change of the Booklet.**
10. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. **Do not write these any where else.**
11. Darken the appropriate bubbles below your registration number with **HB Pencil.**

C. Question paper format and Marking scheme:

12. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists of four Sections.
13. For each question in **Section I,** you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus **one (–1) mark** will be awarded.
14. For each question in **Section II,** you will be awarded **3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. **Partial marks** will be awarded for partially correct answers. No negative marks will be awarded in this Section.
15. For each question in **Section III,** you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus one (–1)** mark will be awarded.
16. For each question in **Section IV,** you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. No negative marks will be awarded for in this Section

Write your name, registration number and sign in the space provided on the back page of this booklet.

Useful Data

Atomic Numbers: Be 4; C 6; N 7; O 8; Al 13; Si 14; Cr24; Fe 26; Zn 30; Br 35.

1 amu	=	1.66×10^{-27} kg	R	=	0.082 L-atm K^{-1} mol^{-1}
h	=	6.626×10^{-34} J s	N_A	=	6.022×10^{23}
m_e	=	9.1×10^{-31} kg	e	=	1.6×10^{-19} C
c	=	3.0×10^{8} m s^{-1}	F	=	96500 C mol^{-1}
R_H	=	2.18×10^{-18} J	$4\pi\epsilon_0$	=	1.11×10^{-10} J^{-1} C^2 m^{-1}

IITJEE 2010 PAPER-1 [Code – 3]

PART - I: CHEMISTRY

SECTION – I (Single Correct Choice Type)

This Section contains **8 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

1. The synthesis of 3 – octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

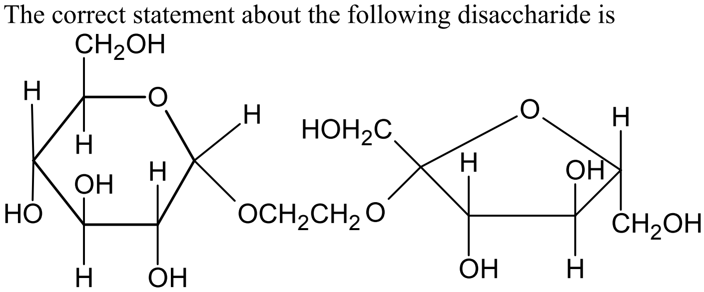
A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C \equiv CH$
B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C \equiv CH$
C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C \equiv CH$
D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C \equiv CH$

Sol. **(D)**

$$CH_3-CH_2-C\equiv CH \xrightarrow{NaNH_2} CH_3-CH_2-C\equiv C^-Na^+$$

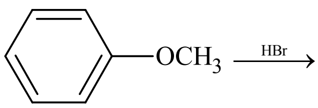
$$CH_3-CH_2-CH_2-CH_2\boxed{Br+Na^+}\ {}^-C\equiv C-CH_2-CH_3 \longrightarrow \underset{\text{3-octyne}}{CH_3-CH_2-CH_2-CH_2-C\equiv C-CH_2-CH_3}$$

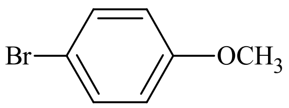
2. The correct statement about the following disaccharide is

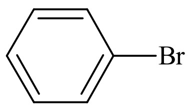


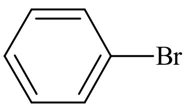
A) Ring (a) is pyranonse with α - glycosidic link
B) Ring (a) is furanonse with α - glycosidic link
C) Ring (b) is furanonse with α - glycosidic link
D) Ring (b) is pyranonse with β - glycosidic link

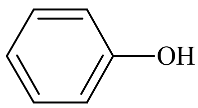
Ans. **(A)**

3. In the reaction  the products are

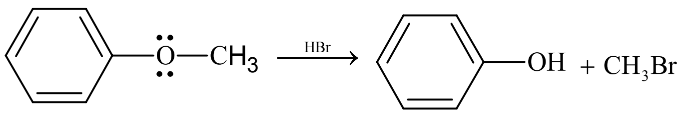
A)  and H_2

B)  and CH_3Br

C)  and CH_3OH

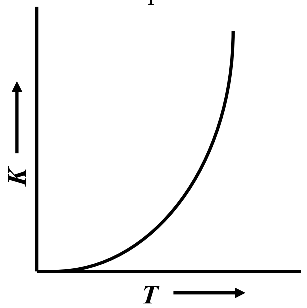
D)  and CH_3Br

Sol. **(D)**

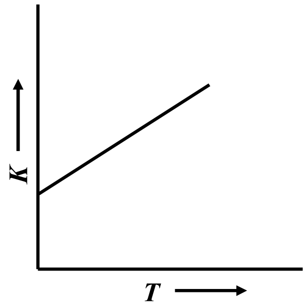


4. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is

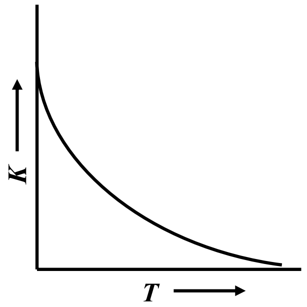
A)



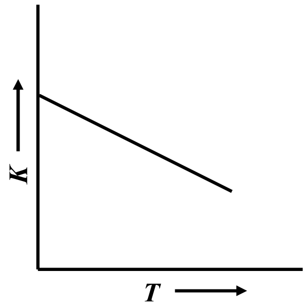
B)



C)



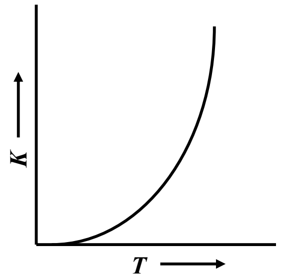
D)



Sol. **(A)**

$k = Ae^{-Ea/RT}$

As T ↑; k ↑ exponentially.



5. The species which by definition has ZERO standard molar enthalpy of formation at 298 K is

A) Br_2 (g) B) Cl_2 (g)
C) H_2O (g) D) CH_4 (g)

Sol. **(B)**

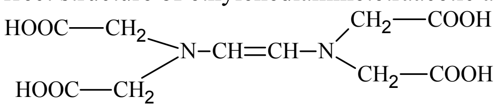
Cl_2 is gas at 298 K while Br_2 is a liquid.

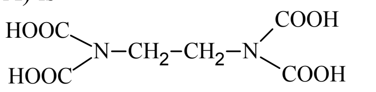
6. The bond energy (in **kcal mol^{-1}**) of a C – C single bond is approximately

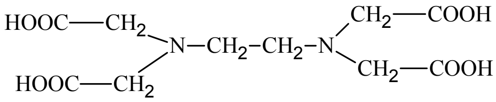
A) 1 B) 10
C) 100 D) 1000

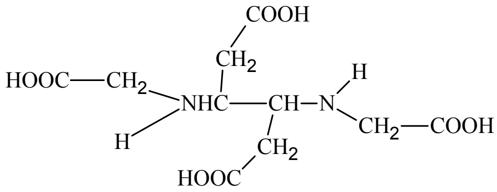
Ans. **(C)**

7. The correct structure of ethylenediaminetetraacetic acid (EDTA) is

A) 

B) 

C) 

D) 

Ans. **(C)**

8. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is

A) $[Cr(H_2O)_4(O_2N)]Cl_2$
B) $[Cr(H_2O)_4Cl_2](NO_2)$
C) $[Cr(H_2O)_4Cl(ONO)]Cl$
D) $[Cr(H_2O)_4Cl_2(NO_2)]H_2O$

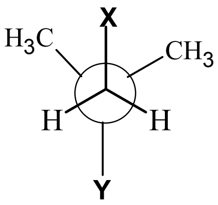
Sol. **(B)**

Cl^- is replaced by NO_2^- in ionization sphere.

SECTION – II (Multiple Correct Choice Type)

This section contains **5 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONE OR MORE** may be correct.

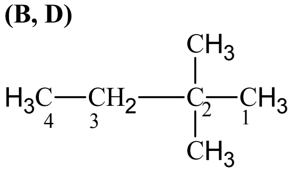
9. In the Newman projection for 2,2-dimethylbutane



X and **Y** can respectively be

A) H and H
B) H and C_2H_5
C) C_2H_5 and H
D) CH_3 and CH_3

Sol. **(B, D)**



On $C_2 – C_3$ bond axis

$X = CH_3$

$Y = CH_3$

On $C_1 – C_2$ bond axis

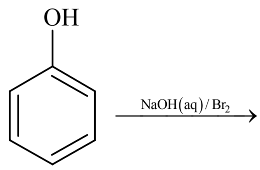
$X = H$

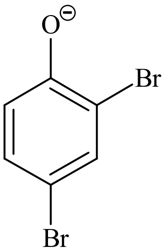
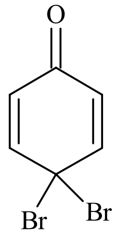
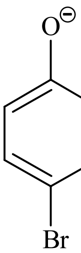
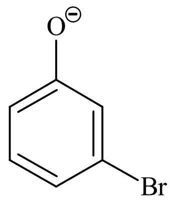
$Y = C_2H_5$

10. Aqueous solutions of HNO_3, KOH, CH_3COOH, and CH_3COONa of identical concentrations are provided. The pair (s) of solutions which form a buffer upon mixing is(are)

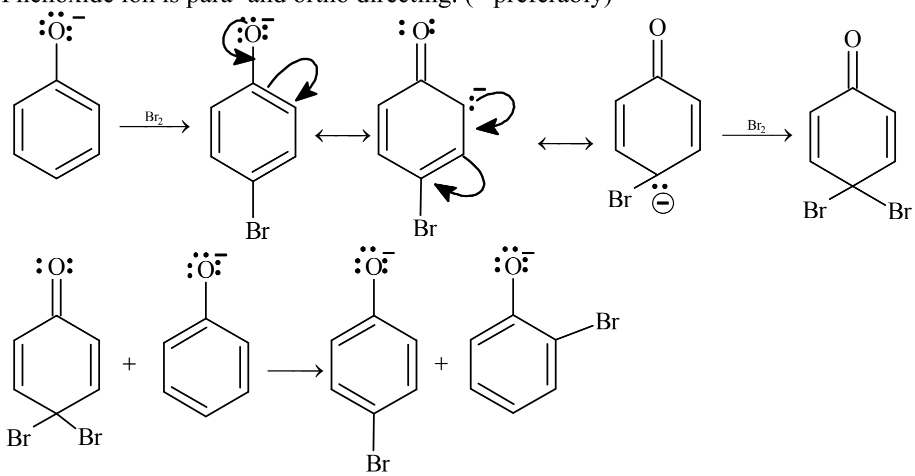
A) HNO_3 and CH_3COOH B) KOH and CH_3COONa
C) HNO_3 and CH_3COONa D) CH_3COOH and CH_3COONa

Sol. **(C, D)**
In option (C), if HNO_3 is present in limiting amount then this mixture will be a buffer. And the mixture given in option (D), contains a weak acid (CH_3COOH) and its salt with strong base NaOH, i.e. CH_3COONa.

11. In the reaction  the intermediate(s) is(are)

A) 
B) 
C) 
D) 

Sol. **(A, B, C)**
Phenoxide ion is para* and ortho directing. (* preferably)



12. The reagent(s) used for softening the temporary hardness of water is(are)

A) $Ca_3(PO_4)_2$ B) $Ca(OH)_2$
C) Na_2CO_3 D) NaOCl

Sol. **(B, C, D)**

$$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 \downarrow + 2H_2O$$

[Clarke's method]

$$NaOCl + H_2O \rightleftharpoons HOCl + NaOH$$

$$OH^- + HCO_3^- \longrightarrow CO_3^{2-} + H_2O$$

$$Ca(HCO_3)_2 + Na_2CO_3 \longrightarrow CaCO_3 \downarrow + 2NaHCO_3$$

13. Among the following, the intensive property is (properties are)
A) molar conductivity
B) electromotive force
C) resistance
D) heat capacity

Sol. **(A, B)**
Resistance and heat capacity are mass dependent properties, hence extensive.

SECTION-III (Paragraph Type)

This section contains **2 paragraphs.** Based upon the first paragraph 2 multiple choice questions and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices A), B), C) and D) out of **WHICH ONLY ONE CORRECT.**

Paragraph for Question Nos. 14 to 15

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is:
$M(s) \mid M^+(aq;\ 0.05\ \text{molar}) \mid\mid M^+(aq),\ 1\ \text{molar}) \mid M(s)$
For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

14. For the above cell
A) $E_{cell} < 0;\ \Delta G > 0$
B) $E_{cell} > 0;\ \Delta G < 0$
C) $E_{cell} < 0;\ \Delta G^o > 0$
D) $E_{cell} > 0;\ \Delta G^o > 0$

Sol. **(B)**

$$M(s) + M^+_{(aq)1M} \longrightarrow M^+_{(aq).05M} + M(s)$$

According to Nernst equation ,

$$E_{cell} = 0 - \frac{2.303RT}{F}\log\frac{M^+_{.05M}}{M^+_{1M}}$$

$$= 0 - \frac{2.303RT}{F}\log\left(5\times10^{-2}\right)$$

$= +ve$

Hence, $|E_{cell}| = E_{cell} = 0.70$ V and $\Delta G < 0$ for the feasibility of the reaction.

15. If the 0.05 molar solution of M^+ is replaced by 0.0025 molar M^+ solution, then the magnitude of the cell potential would be
A) 35 mV
B) 70 mV
C) 140 mV
D) 700 mV

Sol. **(C)**

From above equation $\frac{2.303RT}{F} = 0.0538$

So, $E_{cell} = E^o_{cell} - \frac{0.0538}{1}\log 0.0025$

$= 0 - \frac{0.0538}{1}\log 0.0025$

≈ 0.13988 V

≈ 140 mV

Paragraph for Question Nos. 16 to 18

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite $(CuSO_4 \cdot 5H_2O)$, atacamite $(Cu_2Cl(OH)_3)$, cuprite (Cu_2O), copper glance (Cu_2S) and malachite $(Cu_2(OH)_2CO_3)$. However, 80% of the world copper production comes from the ore of chalcopyrite $(CuFeS_2)$. The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

16. Partial roasting of chalcopyrite produces

A) Cu_2S and FeO
B) Cu_2O and FeO
C) CuS and Fe_2O_3
D) Cu_2O and Fe_2O_3

Sol. **(B)**

$2CuFeS_2 + O_2 \rightarrow Cu_2S + 2FeS + SO_2 \uparrow$

$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2 \uparrow$

$2FeS + 3O_2 \rightarrow 2FeO + 2SO_2 \uparrow$

17. Iron is removed from chalcopyrite as

A) FeO
B) FeS
C) Fe_2O_3
D) $FeSiO_3$

Sol. **(D)**

$FeO + SiO_2 \rightarrow \underset{(slag)}{FeSiO_3}$

18. In self-reduction, the reducing species is

A) S
B) O^{2-}
C) S^{2-}
D) SO_2

Sol. **(C)**

$Cu_2S + 2Cu_2O \rightarrow \underset{\text{(Blister copper)}}{6Cu} + SO_2 \uparrow$

$S^{2-} \rightarrow S^{4+}$ is oxidation, i.e., S^{2-} is reducing agent.

SECTION-IV (Integer Type)

This section contains **TEN** questions. The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the **ORS** is to be bubbled.

19. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL and 25.0 mL. The number of significant figures in the average titre value is

Ans. **3**

20. The concentration of R in the reaction R $\rightarrow$ P was measured as a function of time and the following data is obtained:

[R] (molar)	1.0	0.75	0.40	0.10
t (min.)	0.0	0.05	0.12	0.18

The order of the reaction is

Sol. **0**

From two data, (for zero order kinetics)

$$K_I = \frac{x}{t} = \frac{0.25}{0.05} = 5$$

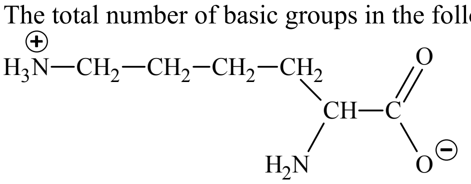
$$K_{II} = \frac{x}{t} = \frac{0.60}{0.12} = 5$$

21. The number of neutrons emitted when ${}^{235}_{92}U$ undergoes controlled nuclear fission to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ is

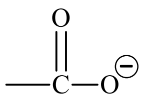
Sol. **3**

$${}_{92}U^{235} \rightarrow {}_{54}Xe^{142} + {}_{38}Sr^{90} + 3\,{}_0n^1$$

22. The total number of basic groups in the following form of lysine is



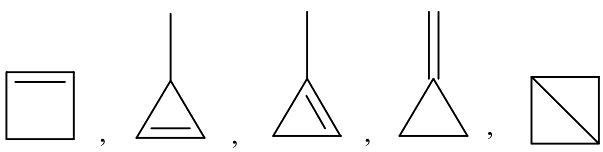
Sol. **2**

 and —NH_2 are basic groups in lysine.

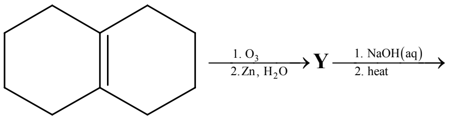
23. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is

Sol. **5**

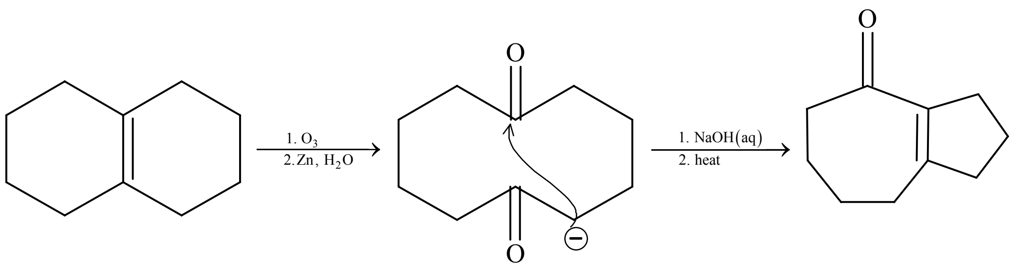
In C_4H_6, possible cyclic isomers are



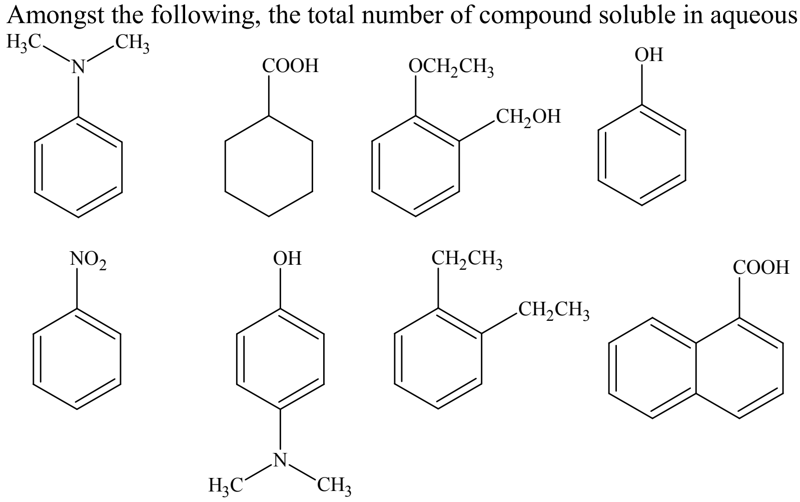
24. In the scheme given below, the total number of intra molecular aldol condensation products formed from 'Y' is



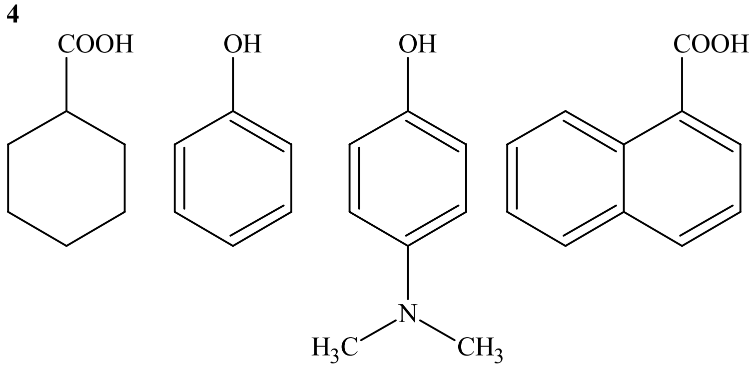
Sol. **1**



25. Amongst the following, the total number of compound soluble in aqueous NaOH is



Sol. **4**



These four are soluble in aqueous NaOH.

26. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

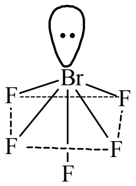
KCN K_2SO_4 $(NH_4)_2C_2O_4$ NaCl $Zn(NO_3)_2$ $FeCl_3$ K_2CO_3 NH_4NO_3 LiCN

Sol. **3**

KCN, K_2CO_3, LiCN are basic in nature and their aqueous solution turns red litmus paper blue.

27. Based on VSEPR theory, the number of 90 degree F–Br–F angles in BrF_5 is

Sol. **0**



All four planar bonds (F–Br–F) will reduce from 90^o to 84.8^o after $\ell p - bp$ repulsion.

28. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

Sol. **3**

$Be_3Al_2Si_6O_{18}$ (Beryl)
(according to charge balance in a molecule)
$2n + 6 + 24 - 36 = 0$
$n = 3$

PART - II: MATHEMATICS

SECTION – I (Single Correct Choice Type)

This Section contains **8 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

29. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol. **(C)**

$r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$

r_1, r_2, r_3 are of the form 3k, 3k + 1, 3k + 2

Required probability $= \frac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \frac{6 \times 8}{216} = \frac{2}{9}$.

30. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

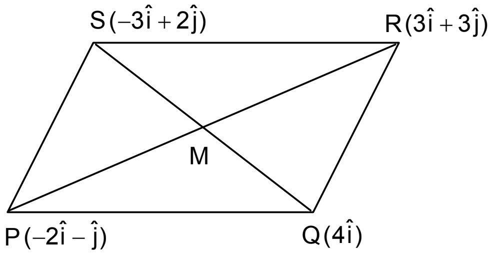
(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

Sol. **(A)**

Evaluating midpoint of PR and QS which gives $M \equiv \left[\frac{\hat{i}}{2} + \hat{j}\right]$, same for both.



$\overrightarrow{PQ} = \overrightarrow{SR} = 6\hat{i} + \hat{j}$

$\overrightarrow{PS} = \overrightarrow{QR} = -\hat{i} + 3\hat{j}$

$\Rightarrow \overrightarrow{PQ} \cdot \overrightarrow{PS} \neq 0$

$\overrightarrow{PQ} \parallel \overrightarrow{SR}$, $\overrightarrow{PS} \parallel \overrightarrow{QR}$ and $|\overrightarrow{PQ}| = |\overrightarrow{SR}|$, $|\overrightarrow{PS}| = |\overrightarrow{QR}|$

Hence, PQRS is a parallelogram but not rhombus or rectangle.

31. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$

(C) 168 (D) 2

Sol. **(A)**

Three planes cannot intersect at two distinct points.

32. The value of $\lim\limits_{x\to 0}\frac{1}{x^3}\int\limits_0^x \frac{t\ln(1+t)}{t^4+4}dt$ is

(A) 0
(B) $\frac{1}{12}$
(C) $\frac{1}{24}$
(D) $\frac{1}{64}$

Sol. **(B)**

$$\lim_{x\to 0}\frac{1}{x^3}\int\limits_0^x \frac{t\ln(1+t)dt}{t^4+4} = \lim_{x\to 0}\frac{x\ln(1+x)}{(x^4+4)3x^2}$$

$$= \lim_{x\to 0}\frac{1}{3}\frac{\ln(1+x)}{x(x^4+4)} = \frac{1}{12}.$$

33. Let p and q be real numbers such that p ≠ 0, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$
(B) $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$
(C) $(p^3-q)x^2-(5p^3-2q)x+(p^3-q)=0$
(D) $(p^3-q)x^2-(5p^3+2q)x+(p^3-q)=0$

Sol. **(B)**

$\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$\Rightarrow -p^3 + 3p\alpha\beta = q \Rightarrow \alpha\beta = \frac{q+p^3}{3p}$

$$x^2 - \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 0$$

$$x^2 - \frac{(\alpha^2+\beta^2)}{\alpha\beta}x + 1 = 0$$

$$\Rightarrow x^2 - \left(\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$$

$$\Rightarrow x^2 - \frac{p^2 - 2\left(\frac{p^3+q}{3p}\right)}{\frac{p^3+q}{3p}}x + 1 = 0$$

$\Rightarrow (p^3 + q)x^2 - (3p^3 - 2p^3 - 2q)x + (p^3 + q) = 0$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0.$

34. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) a = b and c ≠ b
(B) a = c and a ≠ b
(C) a ≠ b and c ≠ b
(D) a = b = c

Sol. **(D)**

$f(x) = e^{x^2} + e^{-x^2} \Rightarrow f'(x) = 2x\left(e^{x^2} - e^{-x^2}\right) \geq 0 \ \forall\ x \in [0, 1]$

Clearly for $0 \leq x \leq 1$ $\quad f(x) \geq g(x) \geq h(x)$

$\because f(1) = g(1) = h(1) = e + \frac{1}{e}$ and f(1) is the greatest

$\therefore a = b = c = e + \frac{1}{e} \Rightarrow a = b = c.$

35. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) 1 (D) $\sqrt{3}$

Sol. **(D)**

$B = 60°$

$\therefore \frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A = 2\sin A\cos C + 2\sin C\cos A$

$= 2\sin(A + C) = 2\sin B = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$.

36. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Sol. **(C)**

Plane 1: $ax + by + cz = 0$ contains line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$

$\therefore 2a + 3b + 4c = 0$ …(i)

Plane 2: $a'x + b'y + c'z = 0$ is perpendicular to plane containing lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

$\therefore 3a' + 4b' + 2c' = 0$ and $4a' + 2b' + 3c' = 0$

$\Rightarrow \quad \frac{a'}{12-4} = \frac{b'}{8-9} = \frac{c'}{6-16}$

$\Rightarrow \quad 8a - b - 10c = 0$ …(ii)

From (i) and (ii)

$$\frac{a}{-30+4} = \frac{b}{32+20} = \frac{c}{-2-24}$$

$\Rightarrow$ Equation of plane $x - 2y + z = 0$.

SECTION – II (Multiple Correct Choice Type)

This section contains **5 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONE OR MORE** may be correct.

37. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)\, z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg (w) denotes the principal argument of a non-zero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(B) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(D) $\text{Arg}\,(z - z_1) = \text{Arg}\,(z_2 - z_1)$

Sol. **(A), (C), (D)**

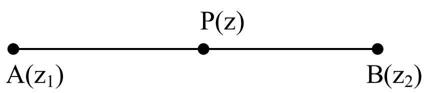
Given $z = (1 - t)\, z_1 + tz_2$

$$\Rightarrow \frac{z - z_1}{z_2 - z_1} = t \;\Rightarrow \arg\left(\frac{z - z_1}{z_2 - z_1}\right) = 0 \qquad \ldots (1)$$

$\Rightarrow \arg\,(z - z_1) = \arg\,(z_2 - z_1)$

$$\frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1}$$

$$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$$



AP + PB = AB

$\Rightarrow |z - z_1| + |z - z_2| = |z_1 - z_2|$.

38. The value(s) of $\displaystyle\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$ is (are)

(A) $\dfrac{22}{7} - \pi$

(B) $\dfrac{2}{105}$

(C) 0

(D) $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Sol. **(A)**

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

$$= \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}\right) dx$$

$$= \left[\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x\right]_0^1 - \pi$$

$$= \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - \pi = \frac{22}{7} - \pi$$

39. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-(2+\sqrt{3})$ (B) $1 + \sqrt{3}$
(C) $2 + \sqrt{3}$ (D) $4\sqrt{3}$

Sol. **(B)**

Using cosine rule for $\angle C$

$$\frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2 + (x^2-1)^2 - (2x+1)^2}{2(x^2+x+1)(x^2-1)}$$

$$\Rightarrow \sqrt{3} = \frac{2x^2+2x-1}{x^2+x+1}$$

$$\Rightarrow (\sqrt{3}-2)x^2 + (\sqrt{3}-2)x + (\sqrt{3}+1) = 0$$

$$\Rightarrow x = \frac{(2-\sqrt{3}) \pm \sqrt{3}}{2(\sqrt{3}-2)}$$

$$\Rightarrow x = -(2+\sqrt{3}),\ 1+\sqrt{3} \Rightarrow x = 1+\sqrt{3} \text{ as } (x > 0).$$

40. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$
(C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Sol. **(C), (D)**

$A = (t_1^2,\ 2t_1)$, $B = (t_2^2,\ 2t_2)$

$$\text{Centre} = \left[\frac{t_1^2+t_2^2}{2},\ (t_1+t_2)\right]$$

$t_1 + t_2 = \pm r$

$$m = \frac{2(t_1-t_2)}{t_1^2-t_2^2} = \frac{2}{t_1+t_2} = \pm\frac{2}{r}.$$

41. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$
(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$
(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$
(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

Sol. **(B), (C)**

$$f'(x) = \frac{1}{x} + \sqrt{1+\sin x}$$

$f'(x)$ is not differentiable at $\sin x = -1$ or $x = 2n\pi - \frac{\pi}{2}, n \in N$

In $x \in (1, \infty)$ $f(x) > 0$, $f'(x) > 0$

Consider f(x) – f′(x)

$= \ln x + \int_0^x \sqrt{1+\sin t}\,dt - \frac{1}{x} - \sqrt{1+\sin x}$

$= \left(\int_0^x \sqrt{1+\sin t}\,dt - \sqrt{1+\sin x}\right) + \ln x - \frac{1}{x}$

Consider $g(x) = \int_0^x \sqrt{1+\sin t}\,dt - \sqrt{1+\sin x}$

It can be proved that $g(x) \ge 2\sqrt{2} - \sqrt{10}\quad \forall\, x \in (0, \infty)$

Now there exists some $\alpha > 1$ such that $\frac{1}{x} - \ln x \le 2\sqrt{2} - \sqrt{10}$ for all $x \in (\alpha, \infty)$ as $\frac{1}{x} - \ln x$ is strictly decreasing function.

$\Rightarrow g(x) \ge \frac{1}{x} - \ln x$.

SECTION – III (Paragraph Type)

This section contains **2 paragraphs.** Based upon the first paragraph 2 multiple choice questions and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices A), B), C) and D) out of **WHICH ONLY ONE CORRECT.**

Paragraph for Questions 42 to 43

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

42. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$
(B) $2x - \sqrt{5}y + 4 = 0$
(C) $3x - 4y + 8 = 0$
(D) $4x - 3y + 4 = 0$

Sol. **(B)**

A tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is $y = mx + \sqrt{9m^2 - 4},\ m > 0$

It is tangent to $x^2 + y^2 - 8x = 0$

$\therefore\ \frac{4m + \sqrt{9m^2 - 4}}{\sqrt{1+m^2}} = 4$

$\Rightarrow 495m^4 + 104m^2 - 400 = 0$

$\Rightarrow m^2 = \frac{4}{5}$ or $m = \frac{2}{\sqrt{5}}$

$\therefore$ the tangent is $y = \frac{2}{\sqrt{5}}m + \frac{4}{\sqrt{5}}$

$\Rightarrow 2x - \sqrt{5}y + 4 = 0$.

43. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$
(B) $x^2 + y^2 + 12x + 24 = 0$
(C) $x^2 + y^2 + 24x - 12 = 0$
(D) $x^2 + y^2 - 24x - 12 = 0$

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. **(A)**

A point on hyperbola is $(3\sec\theta, 2\tan\theta)$

It lies on the circle, so $9\sec^2\theta + 4\tan^2\theta - 24\sec\theta = 0$

$\Rightarrow 13\sec^2\theta - 24\sec\theta - 4 = 0 \Rightarrow \sec\theta = 2, -\frac{2}{13}$

$\therefore \sec\theta = 2 \Rightarrow \tan\theta = \sqrt{3}$.

The point of intersection are $A\left(6, 2\sqrt{3}\right)$ and $B\left(6, -2\sqrt{3}\right)$

$\therefore$ The circle with AB as diameter is

$(x-6)^2 + y^2 = \left(2\sqrt{3}\right)^2 \Rightarrow x^2 + y^2 - 12x + 24 = 0.$

Paragraph for Questions 44 to 46

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, \dots, p-1\}\right\}$$

44. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$

(C) $(p-1)^2 + 1$ (D) $2p - 1$

Sol. **(D)**

We must have $a^2 - b^2 = kp$

$\Rightarrow (a + b)(a - b) = kp$

$\Rightarrow$ either $a - b = 0$ or $a + b$ is a multiple of p

when $a = b$; number of matrices is p

and when $a + b$ = multiple of $p \Rightarrow a, b$ has $p - 1$

$\therefore$ Total number of matrices $= p + p - 1$

$= 2p - 1$.

45. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[Note: The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2 - p + 1)$ (B) $p^3 - (p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2 - 2)$

Ans. **(C)**

46. The number of A in T_p such that det (A) is not divisible by p is

(A) $2p^2$ (B) $p^3 - 5p$

(C) $p^3 - 3p$ (D) $p^3 - p^2$

Ans. **(D)**

SECTION – IV (Integer Type)

This section contains **TEN** questions. The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the **ORS** is to be bubbled.

47. Let S_k, k = 1, 2, ….. , 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is

Sol. **(3)**

$$S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!}$$

$$\sum_{k=2}^{100}\left|(k^2-3k+1)\frac{1}{(k-1)!}\right|$$

$$= \sum_{k=2}^{100}\left|\frac{(k-1)^2-k}{(k-1)!}\right|$$

$$= \sum\left|\frac{k-1}{(k-2)!}-\frac{k}{(k-1)!}\right|$$

$$= \left|\frac{2}{1!}-\frac{3}{2!}\right|+\left|\frac{3}{2!}-\frac{4}{3!}\right|+\cdots$$

$$= \frac{2}{1!}-\frac{1}{0!}+\frac{2}{1!}-\frac{3}{2!}+\frac{3}{2!}-\frac{4}{3!}+\cdots+\frac{99}{98!}-\frac{100}{99!}$$

$$= 3-\frac{100}{99!}.$$

48. The number of all possible values of θ, where 0 < θ < π, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$
$$x\sin 3\theta = \frac{2\cos 3\theta}{y}+\frac{2\sin 3\theta}{z}$$
$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0z_0 \neq 0$, is

Sol. **(3)**

$(y + z)\cos 3\theta - (xyz)\sin 3\theta = 0$ … (1)

$xyz\sin 3\theta = (2\cos 3\theta)z + (2\sin 3\theta)y$ … (2)

$\therefore (y + z)\cos 3\theta = (2\cos 3\theta)z + (2\sin 3\theta)y = (y + 2z)\cos 3\theta + y\sin 3\theta$

$y(\cos 3\theta - 2\sin 3\theta) = z\cos 3\theta$ and

$y(\sin 3\theta - \cos 3\theta) = 0 \Rightarrow \sin 3\theta - \cos 3\theta = 0 \Rightarrow \sin 3\theta = \cos 3\theta$

$\therefore 3\theta = n\pi + \pi/4$

49. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to

Sol. **(9)**

$y - y_1 = m(x - x_1)$

Put x = 0, to get y intercept

$y_1 - mx_1 = x_1^3$

$y_1 - x_1\dfrac{dy}{dx} = x_1^3$

$x\dfrac{dy}{dx} - y = -x^3$

$\dfrac{dy}{dx} - \dfrac{y}{x} = -x^2$

$e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \dfrac{1}{x}$

$y \times \dfrac{1}{x} = \int -x^2 \times \dfrac{1}{x}dx$

$\dfrac{y}{x} = -\int xdx \Rightarrow \dfrac{y}{x} = -\dfrac{x^2}{2} + c$

$\Rightarrow f(x) = -\dfrac{x^3}{2} + \dfrac{3}{2}x \;\; \therefore \; f(-3) = 9.$

50. The number of values of θ in the interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ such that $\theta \neq \dfrac{n\pi}{5}$ for n = 0, ±1, ±2 and tanθ = cot 5θ as well as sin 2θ = cos 4θ is

Sol. **(3)**

$\tan\theta = \cot 5\theta$

$\Rightarrow \cos 6\theta = 0$

$4\cos^3 2\theta - 3\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = 0 \text{ or } \pm\dfrac{\sqrt{3}}{2}$

$\sin 2\theta = \cos 4\theta$

$\Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$2\sin^2 2\theta + 2\sin 2\theta - \sin 2\theta - 1 = 0$

$\sin 2\theta = -1 \text{ or } \sin 2\theta = \dfrac{1}{2}$

$\cos 2\theta = 0 \text{ and } \sin 2\theta = -1$

$\Rightarrow 2\theta = -\dfrac{\pi}{2} \;\; \Rightarrow \theta = -\dfrac{\pi}{4}$

$\cos 2\theta = \pm\dfrac{\sqrt{3}}{2}, \sin 2\theta = \dfrac{1}{2}$

$\Rightarrow 2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6} \;\; \Rightarrow \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

$\therefore \; \theta = -\dfrac{\pi}{4}, \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

51. The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol. **(2)**

$$\frac{1}{4\cos^2\theta+1+\frac{3}{2}\sin 2\theta}$$

$$\Rightarrow \frac{1}{2[1+\cos 2\theta]+1+\frac{3}{2}\sin 2\theta}$$

lies between $\frac{1}{2}$ to $\frac{11}{2}$

$\therefore$ maximum value is 2.

Minimum value of $1 + 4\cos^2\theta + 3\sin\theta\cos\theta$

$1 + \frac{4(1+\cos 2\theta)}{2}+\frac{3}{2}\sin 2\theta$

$= 1+ 2 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta$

$= 3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta$

$\therefore = 3 - \sqrt{4+\frac{9}{4}} = 3-\frac{5}{2}=\frac{1}{2}$.

So maximum value of $\frac{1}{4\cos^2\theta+1+\frac{3}{2}\sin 2\theta}$ is 2.

52. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b})\cdot[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$ is

Sol. **(5)**

E = $(2\vec{a}+\vec{b})\cdot\left[2|\vec{b}|^2\,\vec{a}-2(\vec{a}\cdot\vec{b})\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}+|\vec{a}|^2\,\vec{b}\right]$

$\vec{a}\cdot\vec{b}=\frac{2-2}{\sqrt{70}}=0$

$|\vec{a}|=1$

$|b| = 1$

$\vec{a}\cdot\vec{b}=0$

E = $(2\vec{a}+\vec{b})\cdot\left[2|\vec{b}|^2\,\vec{a}+|\vec{a}|^2\,\vec{b}\right]$

$= 4|\vec{a}|^2|\vec{b}|^2+|\vec{a}|^2(\vec{a}\cdot\vec{b})+2|\vec{b}|^2(\vec{b}\cdot\vec{a})+|\vec{a}|^2|\vec{b}|^2$

$= 5|\vec{a}|^2|\vec{b}|^2=5$

53. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Sol. **(2)**

Substituting $\left(\frac{a}{e},\ 0\right)$ in y = – 2x + 1

$0 = -\frac{2a}{e}+1$

Nitin M Sir (physics-iitjee.blogspot.com)

$\frac{2a}{e} = 1$

$a = \frac{e}{2}$

Also, $1 = \sqrt{a^2m^2 - b^2}$

$1 = a^2m^2 - b^2$

$1 = 4a^2 - b^2$

$1 = \frac{4e^2}{4} - b^2$

$b^2 = e^2 - 1.$

Also, $b^2 = a^2 (e^2 - 1)$

$\therefore$ a = 1, e = 2

54. If the distance between the plane Ax − 2y + z = d and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

Sol. **(6)**

$2l + 3m + 4n = 0$

$3l + 4m + 5n = 0$

$\frac{l}{-1} = \frac{m}{2} = \frac{n}{-1}$

Equation of plane will be

$a (x - 1) + b (y - 2) + c (z - 3) = 0$

$-1 (x - 1) + 2 (y - 2) - 1 (z - 3) = 0$

$-x + 1 + 2y - 4 - z + 3 = 0$

$-x + 2y - z = 0$

$x - 2y + z = 0$

$\frac{|d|}{\sqrt{6}} = \sqrt{6}$

d = 6.

55. For any real number x, let |x| denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [−10, 10] by

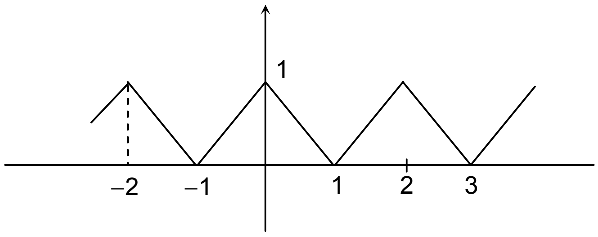
$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x)\ \cos \pi x\ dx$ is

Sol. **(4)**

$$f(x) = \begin{cases} x - 1, & 1 \le x < 2 \\ 1 - x, & 0 \le x < 1 \end{cases}$$

Nitin M Sir (physics-iitjee.blogspot.com)



f(x) is periodic with period 2

$\therefore\ I = \int_{-10}^{10} f(x)\cos\pi x\, dx$

$= 2\int_0^{10} f(x)\cos\pi x\, dx = 2\times 5\int_0^2 f(x)\cos\pi x\, dx$

$= 10\left[\int_0^1 (1-x)\cos\pi x\, dx + \int_1^2 (x-1)\cos\pi x\, dx\right] = 10(I_1 + I_2)$

$I_2 = \int_1^2 (x-1)\cos\pi x\, dx$ put x − 1 = t

$I_2 = -\int_0^1 t\cos\pi t\, dt$

$I_1 = \int_0^1 (1-x)\cos\pi x\, dx = -\int_0^1 x\cos(\pi x)dx$

$\therefore\ I = 10\left[-2\int_0^1 x\cos\pi x\, dx\right]$

$= -20\left[x\frac{\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^1$

$= -20\left[-\frac{1}{\pi^2} - \frac{1}{\pi^2}\right] = \frac{40}{\pi^2} \quad \therefore\ \frac{\pi^2}{10} I = 4$

56. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to

Sol. **(1)**

$\omega = e^{i2\pi/3}$

$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$\Rightarrow z\left[(z+\omega^2)(z+\omega)-1-\omega(z+\omega-1)+\omega^2(1-z-\omega^2)\right]=0$

$\Rightarrow z^3 = 0$

$\Rightarrow z = 0$ is only solution.

Nitin M Sir (physics-iitjee.blogspot.com)

PART - III: PHYSICS

SECTION – I (Single Correct Choice Type)

This Section contains **8 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

57. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$

B) $R_{100} = R_{40} + R_{60}$

C) $R_{100} > R_{60} > R_{40}$

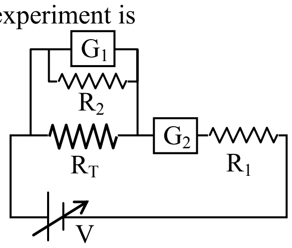
D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

Sol. ***(D)***

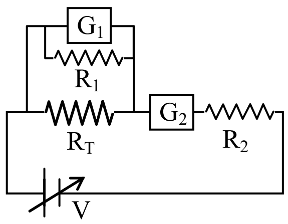
Power $\propto$ 1/R

58. To Verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

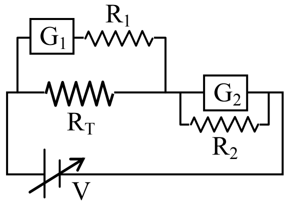
A)



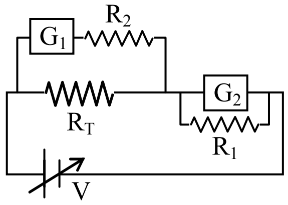
B)



C)



D)



Sol. ***(C)***

G_1 is acting as voltmeter and G_2 is acting as ammeter.

59. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

A) the bulb glows dimmer

B) the bulb glows brighter

C) total impedance of the circuit is unchanged

D) total impedance of the circuit increases

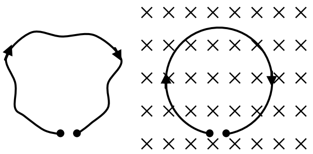
Sol. ***(B)***

Impedance $Z = \sqrt{\frac{1}{(\omega C)^2} + R^2}$

as ω increases, Z decreases.

Hence bulb will glow brighter

60. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



A) IBL

B) $\frac{IBL}{\pi}$

C) $\frac{IBL}{2\pi}$

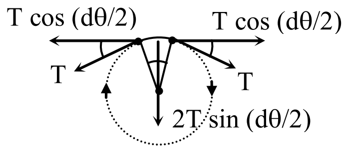
D) $\frac{IBL}{4\pi}$

Sol. ***(C)***

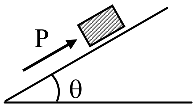
$2T\sin\frac{d\theta}{2} = BiRd\theta$

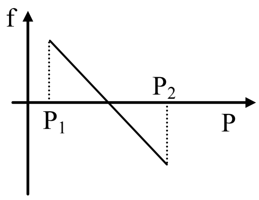
$Td\theta = BiRd\theta$ (for θ small)

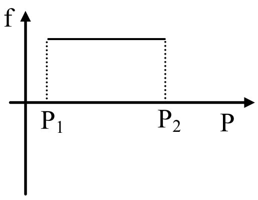
$T = BiR = \frac{BiL}{2\pi}$

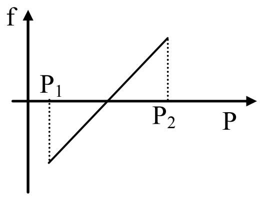


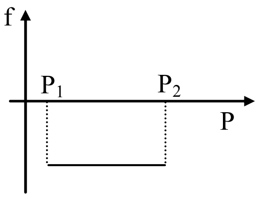
61. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tan θ > μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



A)


B)


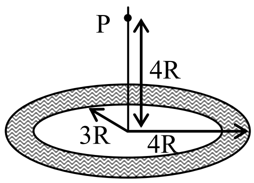
C)


D)


Sol. ***(A)***

Initially the frictional force is upwards as P increases frictional force decreases.

62. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



A) $\frac{2GM}{7R}(4\sqrt{2}-5)$

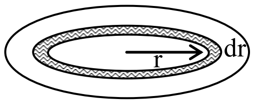
B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

C) $\frac{GM}{4R}$

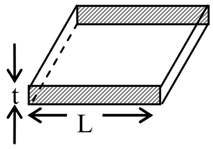
B) $\frac{2GM}{5R}(\sqrt{2}-1)$

Sol. ***(A)***

$$V = -\int_{3R}^{4R} \frac{\sigma 2\pi r dr G}{\sqrt{r^2 + 16R^2}}$$



63. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



A) directly proportional to L
B) directly proportional to t
C) independent of L
D) independent of t

Sol. ***(C)***

$$R = \frac{\rho L}{Lt}$$

64. A real gas behaves like an ideal gas if its
A) pressure and temperature are both high
B) pressure and temperature are both low
C) pressure is high and temperature is low
D) pressure is low and temperature is high

Sol. ***(D)***

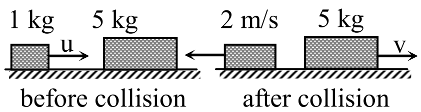
SECTION – II (Multiple Correct Choice Type)

This section contains **5 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONE OR MORE** may be correct.

65. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?
A) Total momentum of the system is 3 kg ms^{-1}
B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}
C) Kinetic energy of the centre of mass is 0.75 J
D) Total kinetic energy of the system is 4J

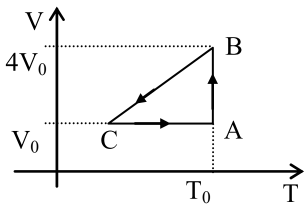
Sol. ***(A, C)***

By conservation of linear momentum
$u = 5v - 2$ …(i)
By Newton's experimental law of collision
$u = v + 2$ …(ii)
using (i) and (ii) we have
$v = 1$ m/s and $u = 3$ m/s



Kinetic energy of the centre of mass $= \frac{1}{2} m_{system} v_{cm}^2 = 0.75$ J

66. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following



A) Internal energies at A and B are the same
B) Work done by the gas in process AB is $P_0V_0 \ell n\, 4$
C) Pressure at C is $\frac{P_0}{4}$
D) Temperature at C is $\frac{T_0}{4}$

Sol. *(A, B)*
Process AB is isothermal process

67. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true?
A) Error ΔT in measuring T, the time period, is 0.05 seconds
B) Error ΔT in measuring T, the time period, is 1 second
C) Percentage error in the determination of g is 5%
D) Percentage error in the determination of g is 2.5%

Sol. *(A, C)*

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{40}$$

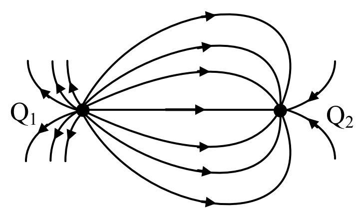
$\Delta T = 0.05$ sec

$$g = \frac{4\pi^2 L n^2}{t^2}$$

$$\frac{\Delta g}{g} = \frac{2\Delta t}{t}$$

$$\Rightarrow \quad \%\ \text{error} = \frac{2\Delta t}{t} \times 100 = 5\%$$

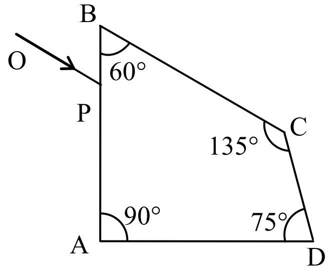
68. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



A) $|Q_1| > |Q_2|$
B) $|Q_1| < |Q_2|$
C) at a finite distance to the left of Q_1 the electric field is zero
D) at a finite distance to the right of Q_2 the electric field is zero

Sol. *(A, D)*
No. of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2

69. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?

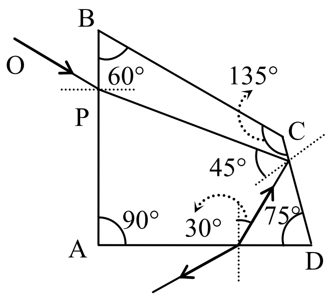


A) The ray gets totally internally reflected at face CD
B) The ray comes out through face AD
C) The angle between the incident ray and the emergent ray is 90°
D) The angle between the incident ray and the emergent ray is 120°

Sol. ***(A, B, C)***
Using snell's law

$$\sin^{-1}\frac{1}{\sqrt{3}} < \sin^{-1}\frac{1}{\sqrt{2}}$$

Net deviation is 90°

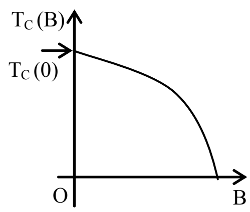


SECTION –III (Paragraph Type)

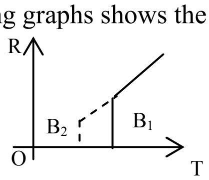
This section contains **2 paragraphs.** Based upon the first paragraph 2 multiple choice questions and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices A), B), C) and D) out of **WHICH ONLY ONE CORRECT.**

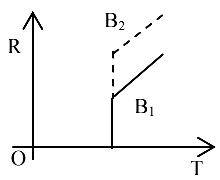
Paragraph for Questions 70 to 71

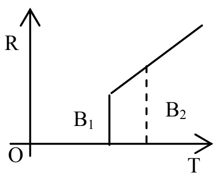
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature T_c (B) is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in the figure.

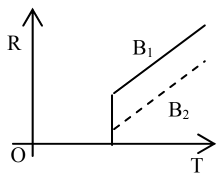


70. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1 which of the following graphs shows the correct variation of R with T in these fields?

A) 

B) 

C) 

D) 

Sol. ***(A)***
Larger the magnetic field smaller the critical temperature.

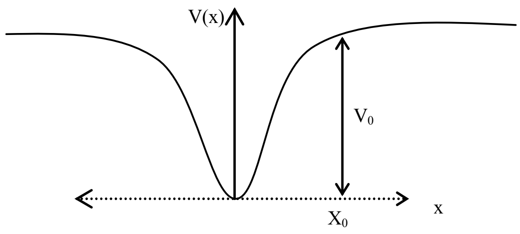
71. A superconductor has T_C (0) = 100 K. When a magnetic field of 7.5 Tesla is applied, its T_c decreases to 75 K. For this material one can definitely say that when

A) B = 5 Tesla, T_c (B) = 80 K
B) B = 5 Tesla, 75 K < T_c (B) < 100 K
C) B = 10 Tesla, 75K < T_c < 100 K
D) B = 10 Tesla, T_c = 70K

Sol. ***(B)***

Paragraph for Questions 72 to 74

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



72. If the total energy of the particle is E, it will perform periodic motion only if

A) E < 0
B) E > 0
C) $V_0 > E > 0$
D) $E > V_0$

Sol. ***(C)***
Energy must be less than V_0

73. For periodic motion of small amplitude A, the time period T of this particle is proportional to

A) $A\sqrt{\frac{m}{\alpha}}$
B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
C) $A\sqrt{\frac{\alpha}{m}}$
D) $A\sqrt{\frac{\alpha}{m}}$

Sol. ***(B)***
$[\alpha] = ML^{-2}T^{-2}$
Only (B) option has dimension of time
Alternatively

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + kx^4 = kA^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2k}{m}\left(A^4 - x^4\right)$$

$$4\sqrt{\frac{m}{2k}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \int dt = T$$

$$4\sqrt{\frac{m}{2k}}\frac{1}{A}\int_0^1 \frac{du}{\sqrt{1-u^4}} = T$$ Substitute x = Au

74. The acceleration of this particle for $|x| > X_0$ is

A) proportional to V_0

B) proportional to $\frac{V_0}{mX_0}$

C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

D) zero

Sol. ***(D)***

As potential energy is constant for $|x| > X_0$, the force on the particle is zero hence acceleration is zero.

SECTION –IV (Integer Type)

This section contains **TEN** questions. The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the **ORS** is to be bubbled.

75. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$ g, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

Sol. ***3***

$\frac{g'}{g} = \frac{\sqrt{6}}{11}$; $\frac{\rho'}{\rho} = \frac{2}{3}$

Hence, $\frac{R'}{R} = \frac{3\sqrt{6}}{22}$

$\frac{v'_{esc}}{v_{esc}} \propto \sqrt{\frac{R'^2\rho'}{R^2\rho}} = \frac{3}{11}$

$v'_{esc} = 3$ km/s.

76. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5J kg^{-1}) of mass m grams is at -5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Sol. ***8***

$420 = (m \times 2100 \times 5 + 1 \times 3.36 \times 10^5) \times 10^{-3}$

where m is in gm.

77. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Sol. ***7***

$f_{app} = f_0 \frac{c+v}{c-v}$

$$df = \frac{2f_0 c}{(c-v)^2} dv$$

where c is speed of sound

$$df = \frac{1.2}{100} f_0$$

hence dv ≈ 7 km/hr.

78. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Sol. ***6***

$$m = \frac{f}{f+u}$$

79. An α- particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

Sol. ***3***

$$\frac{1}{2}mv^2 = qV$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \sqrt{8} \simeq 3 .$$

80. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is

Sol. ***4***

$$J_1 = \left(\frac{2E}{R+2}\right)^2 R$$

$$J_2 = \left(\frac{E}{R+1/2}\right)^2 R \quad \text{since } J_1/J_2 = 2.25$$

R = 4 Ω.

81. Two spherical bodies A (radius 6 cm) and B(radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Sol. ***9***

$\lambda_m T$ = constant

$\lambda_A T_A = \lambda_B T_B$

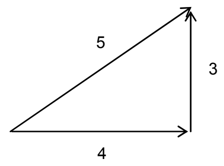
Rate of total energy radiated $\propto AT^4$

Nitin M Sir (physics-iitjee.blogspot.com)

82. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Sol. ***5***

Two waves have phase difference $\pi/2$.



83. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

Sol. ***4***

$$\omega = \sqrt{\frac{YA}{mL}}$$

84. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

Sol. ***6***

$$\frac{L_{total}}{L_B} = \frac{m_1 r_1^2}{m_2 r_2^2} + 1$$

****

Nitin M Sir (physics-iitjee.blogspot.com)

FIITJEE Solutions to IIT-JEE-2010

CODE 0

PAPER 2

Time: 3 Hours

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This Question Paper contains 32 pages having 57 questions.
2. The **question paper CODE** is printed on the right hand top corner of this sheet and also on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
5. Log and Antilog tables are given in page numbers 30 and 31 respectively.
6. The answer sheet, a machine-gradable Objective Response Sheet **(ORS)**, is provided separately.
7. Do not Tamper / Mutilate the ORS or this booklet.
8. Do not break the seals of the question – paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS:

9. The ORS has CODE printed on its lower and upper Parts.
10. Make sure the CODE on the **ORS** is the same as that on this booklet. If the Codes do not match, ask **for a change of the Booklet.**
11. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes**.** Do not write these anywhere else.
12. Darken the appropriate bubbles under each digit of your Registration No. with HB Pencil.

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists **of four** Sections.
14. For each question in **Section I:** you will be awarded **5 marks if you darken only the bubble** corresponding to the correct answer and **zero** mark if no bubbles are darkened. In all other cases, **minus two (–2) mark** will be awarded.
15. For each question in **Section II:** you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this Section.
16. For each question in **Section III:** you will be **awarded 3 marks** if you darken **only** the bubble corresponding to the correct answer and **zero** mark if no bubbles are darkened. In all other cases, **minus one (–1)** mark will be awarded.
17. For each question in Section IV: you will be awarded 2 marks for each row in which your darkened the bubbles(s) corresponding to the correct answer. Thus each question in this section carries a maximum of 8 marks. There is no negative marks awarded for incorrect answer(s) in this Section.

Write your name, registration number and sign in the space provided on the back page of this booklet.

Useful Data

Atomic Numbers: B 5; C 6; N 7; O 8; F 9; Na 11; Si14; P 15; S 16; Cl 17; Ti 22; V 23; Cr 24; Ni 28; Cu 29; Br 35; Rh 45; Sn 50; Xe 54; Tl 81.

1 amu	=	1.66×10^{-27} kg	e	=	1.6×10^{-19} C
R	=	0.082 L-atm K^{-1} mol^{-1}	c	=	3.0×10^{8} m s^{-1}
h	=	6.626×10^{-34} J s	F	=	96500 C mol^{-1}
N_A	=	6.022×10^{23}	R_H	=	2.18×10^{-18} J
m_e	=	9.1×10^{-31} kg	$4\pi\epsilon_0$	=	1.11×10^{-10} J^{-1} C^2 m^{-1}

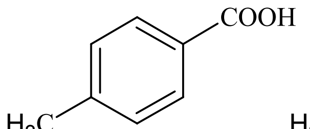
IITJEE 2010 PAPER-2 [Code – 0]

PART - I: CHEMISTRY

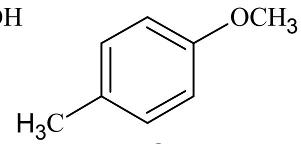
SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

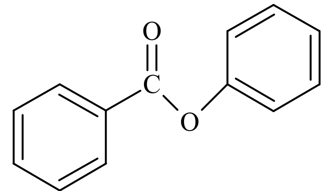
*1. The compounds P, Q and S



P



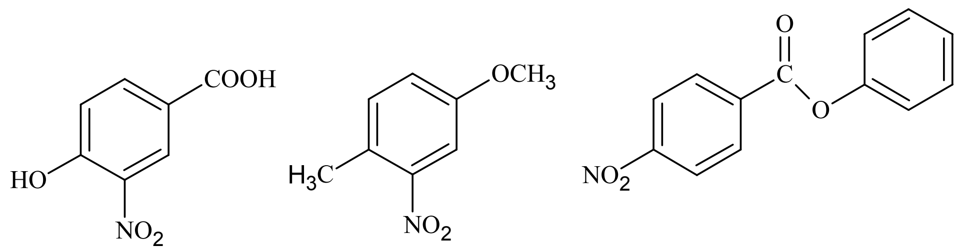
Q



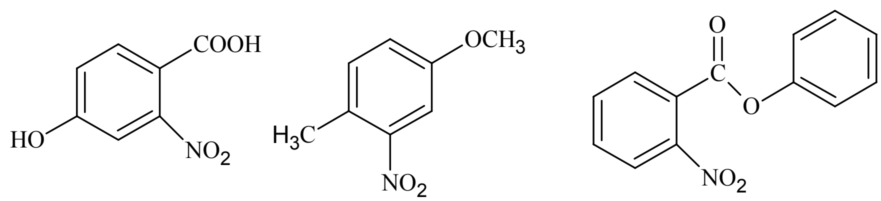
S

were separately subjected to nitration using HNO_3/H_2SO_4 mixture. The major product formed in each case respectively, is

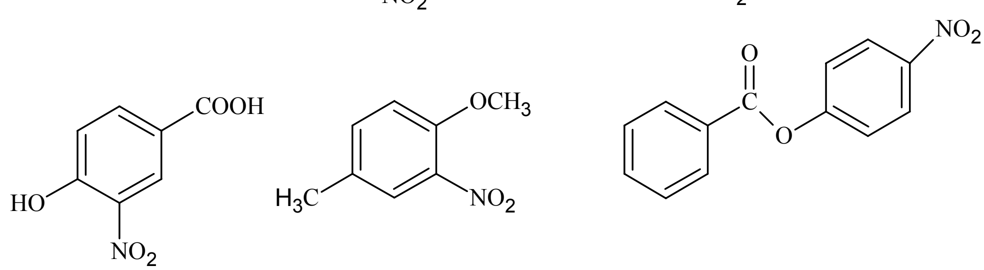
A)



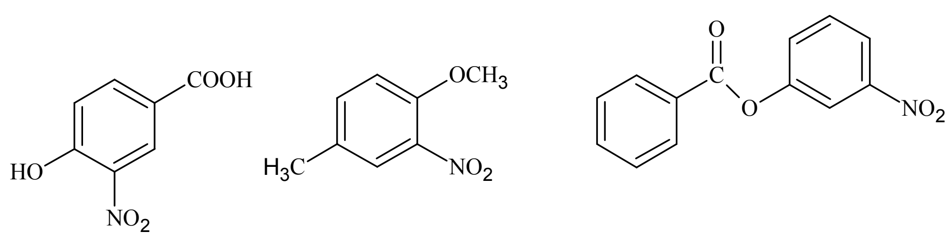
B)



C)

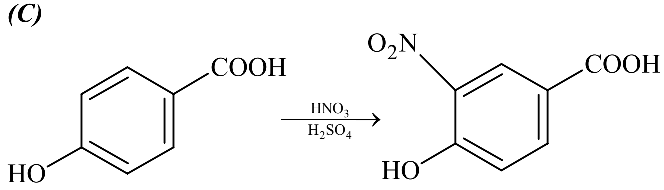


D)



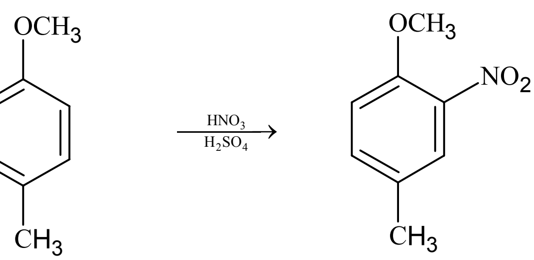
Nitin M Sir (physics-iitjee.blogspot.com)

Sol. ***(C)***



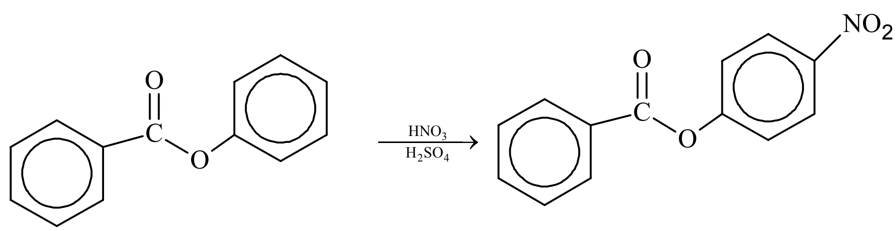
(- OH gp is more activating)

(P)



(- OCH_3 gp is more activating)

(Q)



*2. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is

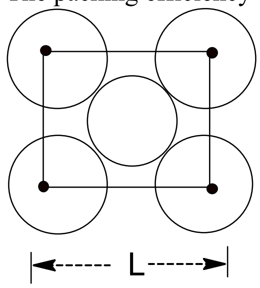
A) 1 and diamagnetic
B) 0 and diamagnetic
C) 1 and paramagnetic
D) 0 and paramagnetic

Sol. ***(A)***

B_2 (10) = $\sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\pi_{2p_x^2}$

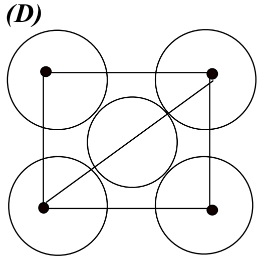
Bond order = $\dfrac{6-4}{2} = 1$ (nature diamagnetic as no unpaired electron)

3. The packing efficiency of the two-dimensional square unit cell shown below is



A) 39.27%
B) 68.02%
C) 74.05%
D) 78.54%

Sol. ***(D)***



$$a = \left(2\sqrt{2}r\right)$$

$$\text{P.F.} = \frac{2 \times \pi r^2}{\left(2\sqrt{2}r\right)^2} = \frac{2\pi r^2}{8r^2} = \frac{\pi}{4}$$

*4. The complex showing a spin-only magnetic moment of 2.82 B.M. is

A) $Ni(CO)_4$
B) $[NiCl_4]^{2-}$
C) $Ni(PPh_3)_4$
D) $[Ni(CN)_4]^{2-}$

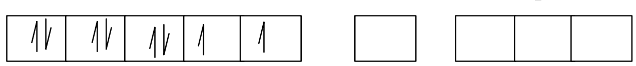
Sol. ***(B)***

$[NiCl_4]^{-2}$, O.S. of Ni = +2

Ni(28) = $3d^8\ 4s^2$

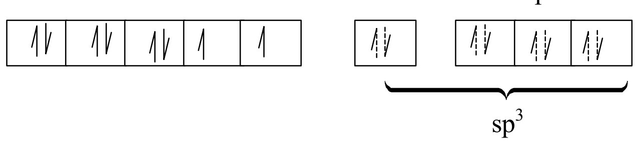
$Ni^{+2} = 3d^8$

3d 4s 4p



Cl^- being weak ligand

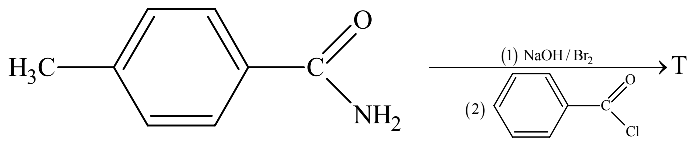
3d 4s 4p

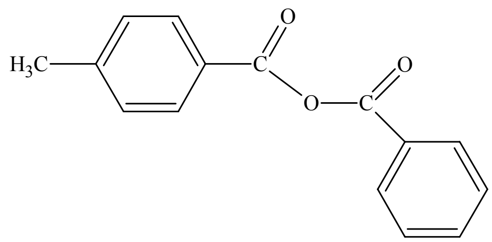


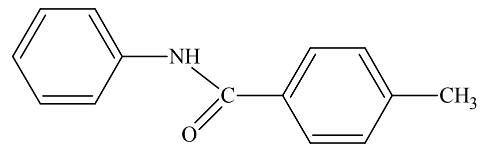
No. of unpaired electrons = 2

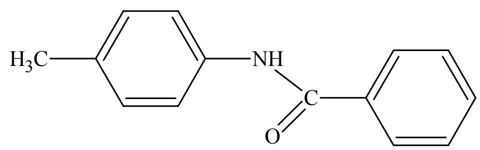
Magnetic moment $\mu = 2.82$ BM.

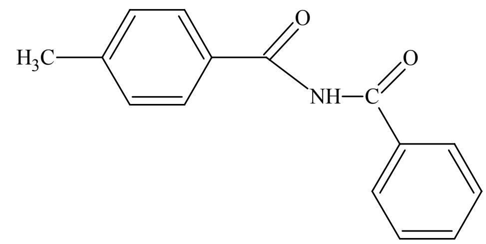
Nitin M Sir (physics-iitjee.blogspot.com)

5. In the reaction  the structure of the Product T is

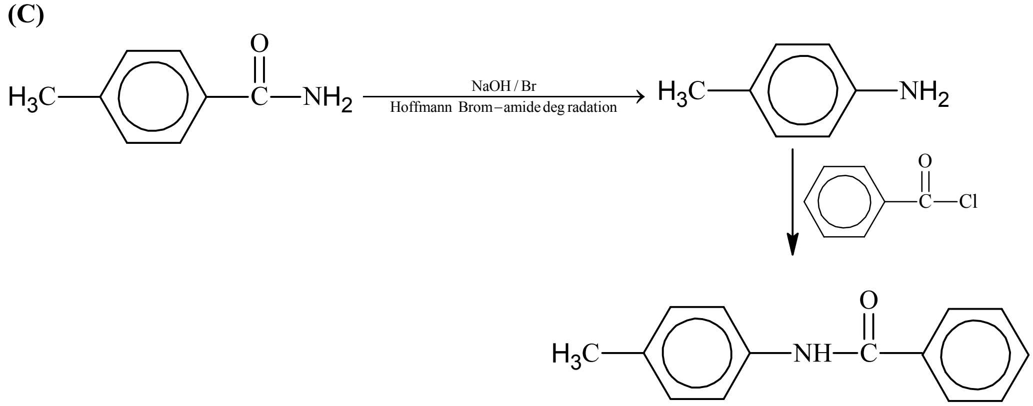
A) 

B) 

C) 

D) 

Sol. **(C)**



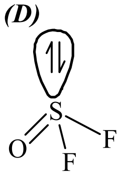
(T)

*6. The species having pyramidal shape is

A) SO_3 B) BrF_3

C) SiO_3^{2-} D) OSF_2

Sol. ***(D)***



SECTION-II (Integer Type)

This Section contains **5 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

*7. Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 g cm^{-3}. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as $y \times 10^x$. The value of x is

Sol. *7*

$d = \frac{\text{mass}}{V} \Rightarrow$ 10.5 g/cc means in 1 cc $\Rightarrow$ 10.5 g of Ag is present.

Number of atoms of Ag in 1 cc $\Rightarrow \frac{10.5}{108} \times N_A$

In 1 cm , number of atoms of Ag $= \sqrt[3]{\frac{10.5}{108} N_A}$

In 1 cm^2, number of atoms of Ag $= \left(\frac{10.5}{108} N_A\right)^{2/3}$

In $10^{-12} m^2$ or 10^{-8} cm^2, number of atoms of Ag $= \left(\frac{10.5}{108} N_A\right)^{2/3} \times 10^{-8} = \left(\frac{1.05 \times 6.022 \times 10^{24}}{108}\right)^{2/3} \times 10^{-8}$

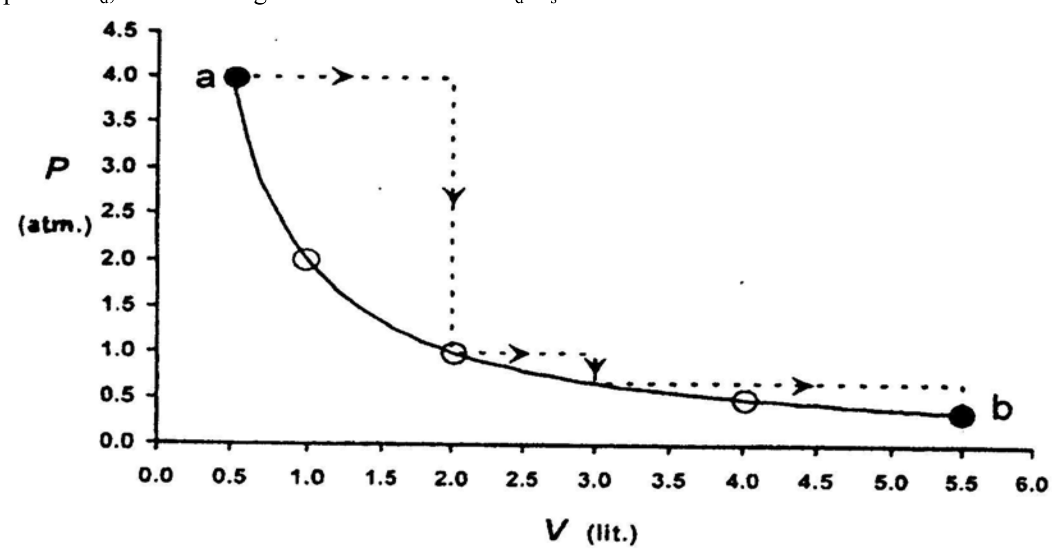
$= 1.5 \times 10^7$

Hence x = 7

*8. Among the following , the number of elements showing only one non-zero oxidation state is
O, Cl, F, N, P, Sn, Tl, Na, Ti

Sol. *2*
Na, F show only one non-zero oxidation state.

*9. One mole of an ideal gas is taken from a to b along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is w_s and that along the dotted line path is w_d, then the integer closest to the ratio w_d/w_s is



Sol. ***2***

$w_d = 4 \times 1.5 + 1 \times 1 + 2.5 \times 2/3 = 8.65$

Process is isothermal

$$w_s = 2 \times 2.303 \log \frac{5.5}{0.5} = 2 \times 2.303 \times \log 11 = 2 \times 2.303 \times 1.0414 = 4.79$$

$$\frac{w_d}{w_s} = \frac{8.65}{4.79} = 1.80 \simeq 2$$

*10. The total number of dirpotic acids among the following is

H_3PO_4 H_2SO_4 H_3PO_3 H_2CO_3 $H_2S_2O_7$

H_3BO_3 H_3PO_2 H_2CrO_4 H_2SO_3

Sol. ***6***

H_2SO_4, H_2CO_3, $H_2S_2O_7$, H_2CrO_4, H_3PO_3, H_2SO_3

11. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is

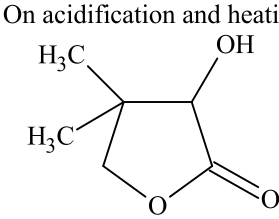
Sol. ***3***

SECTION-III (Paragraph Type)

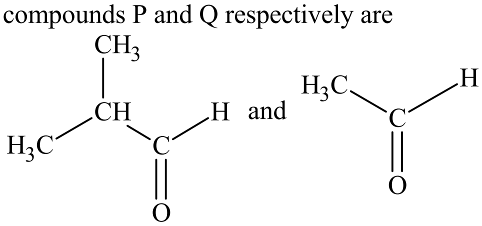
This Section contains **2 paragraphs.** Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

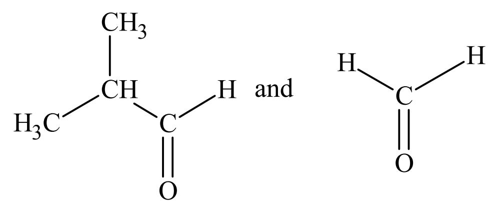
Paragraph for questions 12 to 14

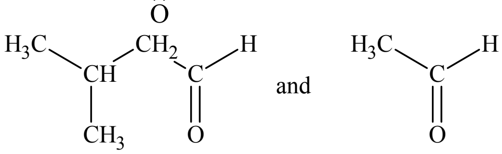
Two aliphatic aldehydes P and Q react in the presence of aqueous K_2CO_3 to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below:

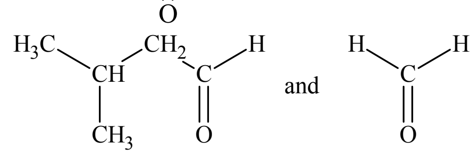


12. The compounds P and Q respectively are

A) 

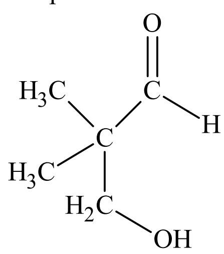
B) 

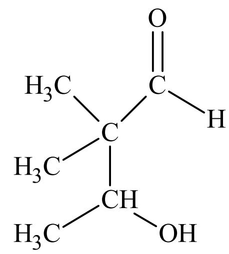
C) 

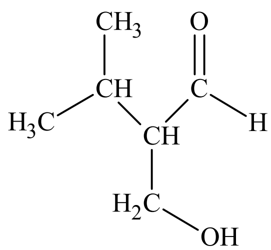
D) 

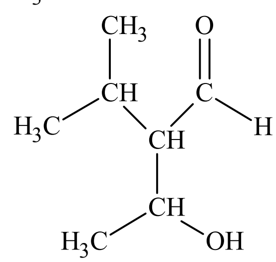
Sol. ***(B)***

13. The compound R is

A) 

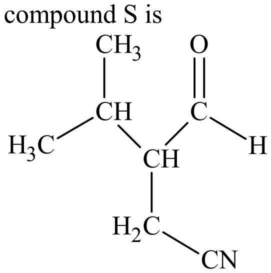
B) 

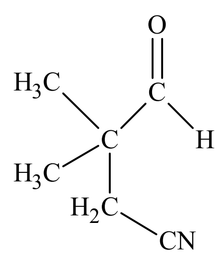
C) 

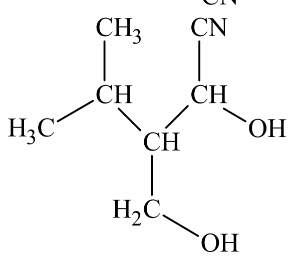
D) 

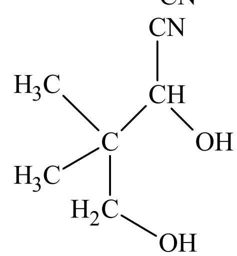
Sol. ***(A)***

14. The compound S is

A) 

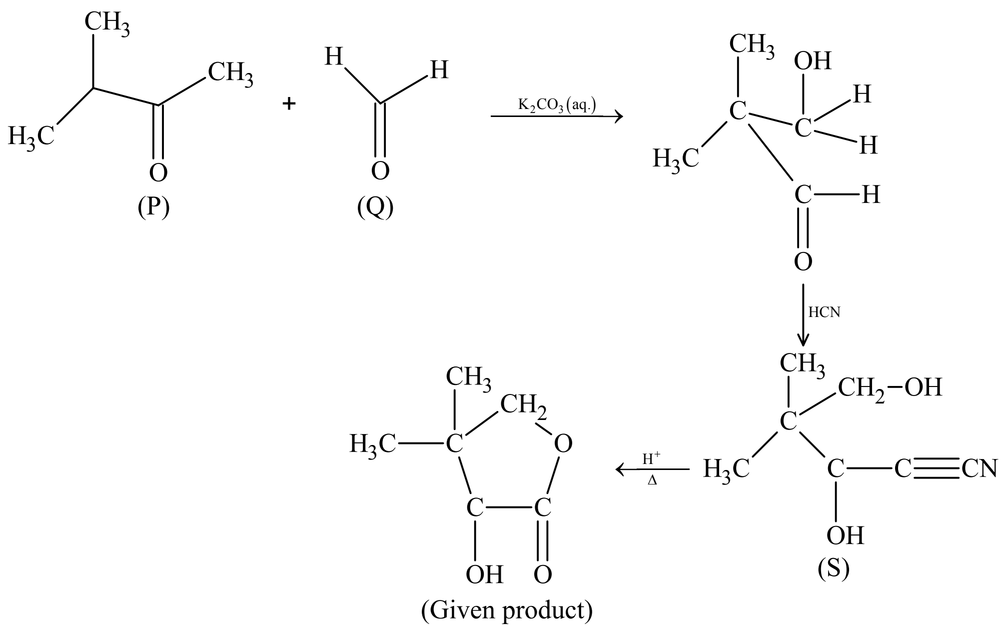
B) 

C) 

D) 

Sol. ***(D)***

Sol: ***(12 to 14)***



Paragraph for questions 15 to 17

The hydrogen-like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

*15. The state S_1 is

A) 1s B) 2s
C) 2p D) 3s

Sol. ***(B)***

For, S_1 (spherically symmetrical)

node = 1

$\Rightarrow$ $n - 1 = 1$

$n = 2$

For S_2, radial node = 1

$$E_{S_2} = \frac{-13.6 \times z^2}{n^2} = E_H \text{ in ground state} = -13.6$$

$$E = \frac{-13.6 \times 9}{n^2} \Rightarrow n = 3$$

So, state S_1 is 2s and S_2 is 3p.

*16. Energy of the state S_1 in units of the hydrogen atom ground state energy is

A) 0.75 B) 1.50
C) 2.25 D) 4.50

Sol. ***(C)***

$$\frac{E_{S_1}}{E_{H(\text{ground})}} = \frac{-13.6 \times 9}{4 \times (-13.6)} = 2.25$$

*17. The orbital angular momentum quantum number of the state S_2 is

A) 0 B) 1
C) 2 D) 3

Sol. ***(B)***

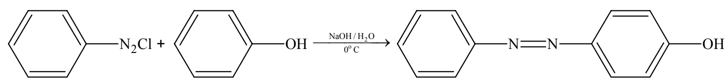
Azimuthal quantum number for $S_2 = \ell = 1$

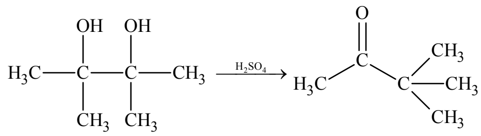
SECTION-IV (Matrix Type)

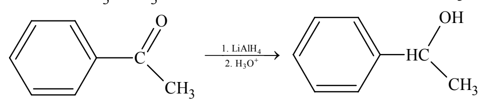
This Section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

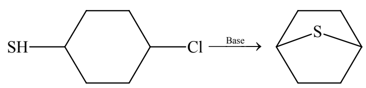
18. Match the reactions in Column I with appropriate options in Column II.

Column I

A) 

B) 

C) 

D) 

Column II

p) Racemic mixture

q) Addition reaction

r) Substitution reaction

s) Coupling reaction

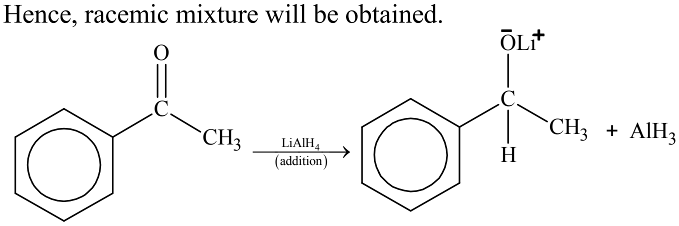
t) Carbocation intermediate

Sol. ***(A – r, s, t); (B – t); (C – p, q); (D – r)***

(A) It is an example of electrophilic substitution reaction which results in coupled product hence it is coupling reaction also.

(B) Pinacole-pinacolone rearrangement. In this reaction intermediate is carbocation.

(C) It is an example of addition reaction by carbonyl compounds and both enantiomers will be formed. Hence, racemic mixture will be obtained.



(D) It is an example of nucleophilic substitution.

19. All the compounds listed in Column I react with water. Match the result of the respective reactions with the appropriate options listed in Column II.

Column I	Column II
A) $(CH_3)_2SiCl_2$	p) Hydrogen halide formation
B) XeF_4	q) Redox reaction
C) Cl_2	r) Reacts with glass
D) VCl_5	s) Polymerization
	t) O_2 formation

Sol. ***(A – p, s); (B – p, q, r, t); (C – p, q, t); (D – p)***

(A) $(CH_3)_2SiCl_2 + 2H_2O \rightarrow (CH_3)_2Si(OH)_2 + 2HCl$

$(CH_3)_2Si(OH)_2$ can undergo polymerization to form silicones.

(B) $3XeF_4 + 6H_2O \rightarrow XeO_3 + 2Xe + 12HF + 1\frac{1}{2}O_2$

(C) $Cl_2 + H_2O \rightarrow 2HCl + \frac{1}{2}O_2$

(D) $VCl_5 + H_2O \rightarrow VOCl_3 + 2HCl$

Nitin M Sir (physics-iitjee.blogspot.com)

PART - II: MATHEMATICS

SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

20. If the distance of the point P (1, – 2, 1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is

(A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Sol. ***(A)***

Distance of point (1, – 2, 1) from plane x + 2y – 2z = α is 5 ⇒ α = 10.

Equation of PQ $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} = t$

$Q \equiv (t + 1, 2t - 2, -2t + 1)$ and PQ = 5 ⇒ $t = \frac{5+\alpha}{9} = \frac{5}{3} \Rightarrow Q \equiv \left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$.

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$

(C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Sol. ***(C)***

Event G = original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

P(signal received by B is green) = $P(GE_1E_2) + P(G\bar{E}_1\bar{E}_2) + P(\bar{G}E_1\bar{E}_2) + P(\bar{G}\bar{E}_1E_2)$

$P(E) = \frac{46}{5\times16}$

$P(G/E) = \frac{40/5\times16}{46/5\times16} = \frac{20}{23}$.

22. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′ makes a right angle with the side AB, then the cosine of the angle α is given by

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$

(C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. ***(B)***

$\overrightarrow{AD} = \overrightarrow{AB}\times\left(\overrightarrow{AB}\times\overrightarrow{AD}\right) = 5\left(61\hat{i}-10\hat{j}-21\hat{k}\right) \Rightarrow \cos\alpha = \dfrac{\left|\overrightarrow{AD'}\cdot\overrightarrow{AD}\right|}{\left|\overrightarrow{AD'}\right|\left|\overrightarrow{AD}\right|} = \dfrac{\sqrt{17}}{9}$.

*23. For r = 0, 1, …, 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$ is equal to

(A) $B_{10} - C_{10}$
(B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$
(C) 0
(D) $C_{10} - B_{10}$

Sol. ***(D)***

Let $y = \sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$

$\sum_{r=1}^{10} A_rB_r$ = coefficient of x^{20} in $((1 + x)^{10}(x + 1)^{20}) - 1$

$= C_{20} - 1 = C_{10} - 1$ and $\sum_{r=1}^{10}(A_r)^2$ = coefficient of x^{10} in $((1 + x)^{10}(x + 1)^{10}) - 1 = B_{10} - 1$

$\Rightarrow y = B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10}$.

24. Let f be a real-valued function defined on the interval (– 1, 1) such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

(A) 1
(B) 1/3
(C) 1/2
(D) 1/e

Sol. ***(B)***

$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt \quad \ldots(i)$

$f(f^{-1}(x)) = x$

$\Rightarrow f'(f^{-1}(x))\,(f^{-1}(x))' = 1 \Rightarrow (f^{-1}(2))' = \dfrac{1}{f'(f^{-1}(2))} \Rightarrow f(0) = 2 \Rightarrow f^{-1}(2) = 0$

$(f^{-1}(2))' = \dfrac{1}{f'(0)}$

$e^{-x}(f'(x) - f(x)) = \sqrt{x^4+1}$

Put $x = 0 \Rightarrow f'(0) - 2 = 1 \Rightarrow f'(0) = 3$

$(f^{-1}(2))' = 1/3$.

*25. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 25
(B) 34
(C) 42
(D) 41

Sol. ***(D)***

Total number of unordered pairs of disjoint subsets

$= \dfrac{3^4+1}{2} = 41$.

SECTION – II (Integer Type)

This Section contains **5 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

*26. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, …, 11. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Sol. ***(0)***

$a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \ldots, a_{11}$ are in A.P.

$\therefore \frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$

$\Rightarrow 225 + 35d^2 + 150d = 90$

$35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$

Given $a_2 < \frac{27}{2}$ $\therefore$ d = – 3 and d ≠ – 9/7 $\Rightarrow \frac{a_1 + a_2 + \ldots + a_{11}}{11} = \frac{11}{2}[30 - 10 \times 3] = 0$.

27. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010\,(x - 2009)\,(x - 2010)^2\,(x - 2011)^3\,(x - 2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that f (x) = ln (g (x)), for all $x \in R$, then the number of points in R at which g has a local maximum is

Sol. ***(1)***

$f(x) = \ln\{g(x)\}$

$g(x) = e^{f(x)}$

$g'(x) = e^{f(x)} . f'(x)$

$g'(x) = 0 \Rightarrow f'(x) = 0$ as $e^{f(x)} \neq 0$

$\Rightarrow 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4 = 0$

so there is only one point of local maxima.

28. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$. If det (adj A) + det (adj B) = 10^6, then [k] is equal to

[Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

Sol. ***(5)***

$|A| = (2k + 1)^3$, $|B| = 0$ (Since B is a skew–symmetric matrix of order 3)

$\Rightarrow$ det (adj A) = $|A|^{n-1} = ((2k + 1)^3)^2 = 106 \Rightarrow 2k + 1 = 10 \Rightarrow 2k = 9$

[k] = 4.

*29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[Note : [k] denotes the largest integer less than or equal to k].

Sol. ***(2)***

$$2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1$$

Nitin M Sir (physics-iitjee.blogspot.com)

$$\cos\frac{\pi}{2k} + \cos\frac{\pi}{k} = \frac{\sqrt{3}+1}{2}$$

Let $\frac{\pi}{k} = 0$, $\cos\theta + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$

$\cos\frac{\theta}{2} = t \qquad 2t^2 + t - \frac{\sqrt{3}+3}{2} = 0$

$t = \frac{-1 \pm \sqrt{1+4(3+\sqrt{3})}}{4} = \frac{-1 \pm \left(2\sqrt{3}+1\right)}{4} = \frac{-2-2\sqrt{3}}{4}, \frac{\sqrt{3}}{2} \qquad \because t \in [-1, 1],\ \cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$

$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3.$

*30. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Sol. ***(3)***

$\Delta = \frac{1}{2}\,ab\sin C \Rightarrow \sin C = \frac{2\Delta}{ab} = \frac{2\times 15\sqrt{3}}{6\times 10} = \frac{\sqrt{3}}{2} \Rightarrow C = 120°$

$\Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos C}$

$= \sqrt{6^2 + 10^2 - 2\times 6\times 10\times \cos 120°} = 14$

$\therefore r = \frac{\Delta}{s} \Rightarrow r^2 = \frac{225\times 3}{\left(\frac{6+10+14}{2}\right)^2} = 3.$

SECTION – III (Paragraph Type)

This Section contains **2 paragraphs.** Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for questions 31 to 33.

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of f(x) and let t = |s|.

31. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$

(C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

Sol. ***(C)***

Since, $f\left(-\frac{1}{2}\right)\cdot f\left(-\frac{3}{4}\right) < 0 \Rightarrow$ S lie in $\left(-\frac{3}{4}, -\frac{1}{2}\right)$.

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$

(C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

Sol. ***(A)***

$$-\frac{3}{4} < s < -\frac{1}{2}$$

$$\frac{1}{2} < t < \frac{3}{4}$$

$$\int_0^{1/2} (4x^3 + 3x^2 + 2x + 1)dx < \text{area} < \int_0^{3/4} (4x^3 + 3x^2 + 2x + 1)\,dx$$

$$[x^4 + x^3 + x^2 + x]_0^{1/2} < area < [x^4 + x^3 + x^2 + x]_0^{3/4}$$

$$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} < area < \frac{81}{256} + \frac{27}{64} + \frac{9}{16} + \frac{3}{4}$$

$$\frac{15}{16} < area < \frac{525}{256}.$$

33. The function f′(x) is

(A) increasing in $\left(-t,\ -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},\ t\right)$

(B) decreasing in $\left(-t,\ -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},\ t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

Sol. ***(B)***

$f''(x) = 2[12x + 3] = 0 \Rightarrow x = -1/4$.

Paragraph for questions 34 to 36.

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

*34. The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Sol. ***(D)***

$$y = mx + \sqrt{9m^2 + 4}$$

$$4 - 3m = \sqrt{9m^2 + 4}$$

$$16 + 9m^2 - 24m = 9m^2 + 4 \Rightarrow m = \frac{12}{24} = \frac{1}{2}$$

Equation is $y - 4 = \frac{1}{2}(x - 3)$

$2y - 8 = x - 3 \Rightarrow x - 2y + 5 = 0$

Let B = (α, β) $\Rightarrow \frac{x\alpha}{9} + \frac{y\beta}{4} - 1 = 0 \Rightarrow \frac{\alpha/9}{1} = \frac{\beta/4}{-2} = \frac{-1}{5} \Rightarrow \alpha = -\frac{9}{5}, \beta = \frac{8}{5}$

$$B \equiv \left(-\frac{9}{5}, \frac{8}{5}\right).$$

*35. The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$

(C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Sol. ***(C)***

Slope of BD must be 0

$\Rightarrow y - \frac{8}{5} = 0 \left(x + \frac{9}{5}\right) \Rightarrow y = \frac{8}{5}$

Hence y coordinate of D is 8/5.

*36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Sol. ***(A)***

Locus is parabola

Equation of AB Is $\frac{3x}{9} + \frac{4y}{4} = 1 \Rightarrow \frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$

$(x - 3)^2 + (y - 4)^2 = \frac{(x + 3y - 3)^2}{10}$

$10x^2 + 90 - 60x + 10y^2 + 160 - 80y = x^2 + 9y^2 + 9 + 6xy - 6x - 18y$

$\Rightarrow 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0.$

SECTION – IV (Matrix Type)

This Section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

*37. Match the statements in column-I with those in column-II.

[Note: Here z takes the values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z]

	Column – I		**Column – II**
(A)	The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to	(p)	an ellipse with eccentricity $\frac{4}{5}$
(B)	The set of points z satisfying $\lvert z + 4\rvert + \lvert z - 4\rvert = 10$ is contained in or equal to	(q)	the set of points z satisfying Im z = 0
(C)	If $\lvert\omega\rvert = 2$, then the set of points $z = \omega - 1/\omega$ is contained in or equal to	(r)	the set of points z satisfying $\lvert \text{Im } z\rvert \le 1$
(D)	If $\lvert\omega\rvert = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to	(s)	the set of points z satisfying $\lvert \text{Re } z\rvert \le 1$
		(t)	the set of points z satisfying $\lvert z\rvert \le 3$

Sol. (A) (q)

$\left|\frac{z}{|z|} - i\right| = \left|\frac{z}{|z|} + i\right|, z \neq 0$

$\frac{z}{|z|}$ is unimodular complex number

and lies on perpendicular bisector of i and –i

$\Rightarrow \frac{z}{|z|} = \pm 1 \Rightarrow z = \pm 1|z| \Rightarrow$ a is real number $\Rightarrow$ Im(z) = 0.

(B) ***(p)***

$|z + 4| + |z - 4| = 10$

z lies on an ellipse whose focus are (4, 0) and (–4, 0) and length of major axis is 10

$\Rightarrow 2ae = 8$ and $2a = 10 \Rightarrow e = 4/5$

$|\text{Re}(z)| \le 5$.

(C) ***(p), (t)***

$|w| = 2 \Rightarrow w = 2(\cos\theta + i\sin\theta)$

$x + iy = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos\theta - i\sin\theta)$

$= \frac{3}{2}\cos\theta + i\frac{5}{2}\sin\theta \Rightarrow \frac{x^2}{(3/2)^2} + \frac{y^2}{(5/2)^2} = 1$

$e^2 = 1 - \frac{9/4}{25/4} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$

(D) ***(q), (t)***

$|w| = 1 \Rightarrow x + iy = \cos + i\sin\theta + \cos\theta - i\sin\theta$

$x + iy = 2\cos\theta$

$|\text{Re}(z)| \le 1$, $|\text{Im}(z) = 0$.

38. Match the statements in column-I with those in column-II.

	Column – I		**Column – II**
(A)	A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x - \frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p)	– 4
*(B)	The values of x satisfying $\tan^{-1}(x + 3) - \tan^{-1}(x - 3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q)	0
(C)	Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}\cdot\vec{b} = 0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c}) = 0$ and $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(r)	4
(D)	Let f be the function on [–π, π] given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for x ≠ 0. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	(s)	5
		(t)	6

Sol. (A). (t)

Let the line be $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ intersects the lines

$\Rightarrow$ S. D = 0 $\Rightarrow a + 3b + 5c = 0$ and $3a + b - 5c = 0 \Rightarrow a : b : c :: 5r : -5r : 2r$

on solving with given lines we get points of intersection P ≡ (5, – 5, 2) and Q ≡ $\left(\frac{10}{3}, -\frac{10}{3}, \frac{8}{3}\right)$

⇒ $PQ^2 = d^2 = 6$.

(B) *(p), (r)*

$\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}(3/5)$

$\Rightarrow \tan^{-1}\frac{(x+3)-(x-3)}{1+(x^2-9)} = \tan^{-1}\frac{3}{4} \Rightarrow \frac{6}{x^2-8} = \frac{3}{4}$

$\therefore x^2 - 8 = 8$

or $x = \pm 4$.

(C) *(q), (s)*

As $\vec{a} = \mu\vec{b} + 4\vec{c} \Rightarrow \mu(|\vec{b}|) = -4\vec{b}\cdot\vec{c}$ and $|\vec{b}|^2 = 4\vec{a}\cdot\vec{c}$ and $|\vec{b}|^2 + \vec{b}\cdot\vec{c} - \vec{d}\cdot\vec{c} = 0$

Again, as $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$

Solving and eliminating $\vec{b}\cdot\vec{c}$ and eliminating $|\vec{a}|^2$

we get $(2\mu^2 - 10\mu)|\vec{b}|^2 = 0 \Rightarrow \mu = 0$ and 5.

(D) *(r)*

$$I = \frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx = \frac{2}{\pi}\times 2\int_{0}^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx$$

$x/2 = \theta \Rightarrow dx = 2d\theta$

$x = 0, \theta = 0$

$x = \pi\ \theta = \pi/2$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 9\theta}{\sin\theta}d\theta$$

$$= \frac{8}{\pi}\int_0^{\pi/2}\frac{(\sin 9\theta - \sin 7\theta)}{\sin\theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin\theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin\theta} + \frac{(\sin 3\theta - \sin\theta)}{\sin\theta} + \frac{\sin\theta}{\sin\theta}d\theta$$

$$= \frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1)d\theta + \frac{8}{\pi}\int_0^{\pi/2}d\theta$$

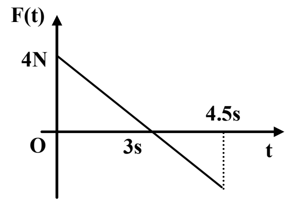
$$= \frac{16}{\pi}\left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2}\right] + \frac{8}{\pi}[\theta]_0^{\pi/2} = 0 + \frac{8}{\pi}\times\left[\frac{\pi}{2} - 0\right] = 4$$

PART - III: PHYSICS

SECTION – I (Single Correct Choice Type)

This Section contains **6 multiple choice questions.** Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

39. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force F(t) in the x direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



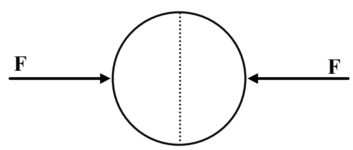
A) 4.50 J B) 7.50 J C) 5.06 J D) 14.06 J

Sol. ***(C)***

Area under F-t curve = 4.5 kg-m/sec

$$\text{K.E.} = \frac{1}{2}(2)\left(\frac{4.5}{2}\right)^2 = 5.06\text{ J}$$

40. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ B) $\frac{1}{\varepsilon_0}\sigma^2 R$ C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

Sol. ***(A)***

Pressure = $\frac{\sigma^2}{2\varepsilon_0}$ and force = $\frac{\sigma^2}{2\varepsilon_0}\times\pi R^2$

41. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times10^5\ \text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2\times10^{-3}\ \text{ms}^{-1}$. Given g = 9.8 ms^{-2}, viscosity of the air = $1.8\times10^{-5}\ \text{Ns m}^{-2}$ and the density of oil = 900 kg m^{-3}, the magnitude of q is

A) 1.6×10^{-19} C B) 3.2×10^{-19} C C) 4.8×10^{-19} C D) 8.0×10^{-19} C

Sol. ***(D)***

$$\frac{4}{3}\pi R^3\rho g = qE = 6\pi\eta R v_T$$

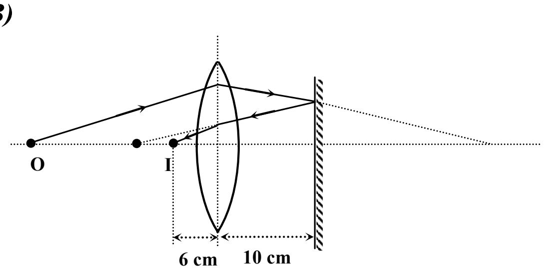
$\therefore$ q = 8.0×10^{-19} C

42. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
A) 0.02 mm B) 0.05 mm C) 0.1 mm D) 0.2 mm

Sol. ***(D)***
L.C. = 1 M.S.D – 1 V.S.D
$$= \left(1 - \frac{16}{20}\right) \text{M.S.D}$$
$$= \left(1 - \frac{4}{5}\right)(1\,\text{mm}) = 0.2\ \text{mm}$$

43. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
A) virtual and at a distance of 16 cm from the mirror
B) real and at a distance of 16 cm from the mirror
C) virtual and at a distance of 20 cm from the mirror
D) real and at a distance of 20 cm from the mirror

Sol. ***(B)***



44. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is
A) 5 grams B) 10 grams C) 20 grams D) 40 grams

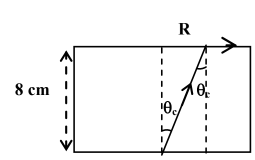
Sol. ***(B)***
$$\frac{v_S}{4L_P} = \frac{2\sqrt{\frac{T}{\mu}}}{2\ell_S}$$
$$\mu\ell_s = 10\ \text{gm}$$

SECTION –II (Integer Type)

This Section contains **5 questions.** The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question no. in the **ORS** is to be bubbled.

45. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

Sol. ***6***
sin θ_c = 3/5
$\therefore$R = 6 cm.



Nitin M Sir (physics-iitjee.blogspot.com)

46. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour ?

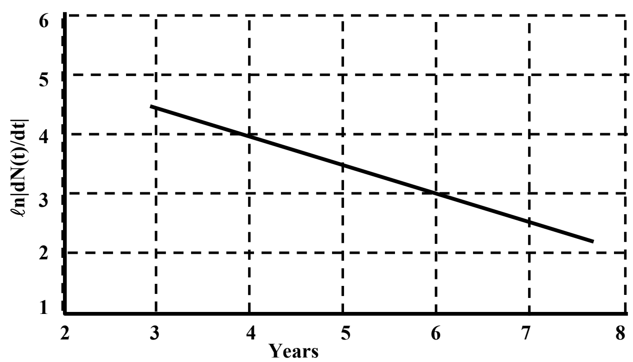
Sol. *3*

For $v_1 = \frac{50}{7}$ m, $u_1 = -25$ m

$v_2 = \frac{25}{3}$ m, $u_2 = -50$ m

Speed of object $= \frac{25}{30} \times \frac{18}{5} = 3$ kmph.

47. To determine the half life of a radioactive element, a student plots a graph of $\ell n\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is



Sol. *8*

$N = N_0 e^{-\lambda t}$

$\ell n|dN/dt| = \ell n(N_0\lambda) - \lambda t$

From graph, $\lambda = \frac{1}{2}$ per year

$t_{1/2} = \frac{0.693}{1/2} = 1.386$ year

4.16 yrs = $3t_{1/2}$

$\therefore$ p = 8

48. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

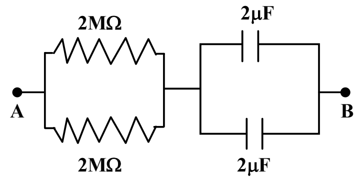
Sol. *4*

$TV^{\gamma-1}$ = constant

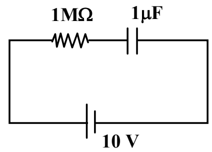
$TV^{7/5-1} = aT\left(\frac{v}{32}\right)^{7/5-1}$

$\therefore$ a = 4.

49. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt?
[take $\ell n\ 5 = 1.6$, $\ell n\ 3 = 1.1$]



Sol. ***2***
$4 = 10\,(1 - e^{-t/4})$
$\therefore\ t = 2$ sec.



SECTION –III (Paragraph Type)

This Section contains **2 paragraphs.** Based upon each of the paragraphs **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

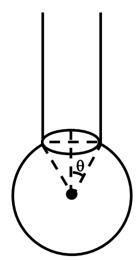
Paragraphs for Question 50 To 52

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is

A) $2\pi rT$ B) $2\pi RT$ C) $\frac{2\pi r^2 T}{R}$ D) $\frac{2\pi R^2 T}{r}$

Sol. ***(C)***
Surface Tension force $= 2\pi r\,T\,\frac{r}{R} = \frac{2\pi r^2 T}{R}$



51. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, g = 10 m/s^2, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

A) 1.4×10^{-3} m B) 3.3×10^{-3} m C) 2.0×10^{-3} m D) 4.1×10^{-3} m

Sol. ***(A)***
$$\frac{2\pi r^2 T}{R} = mg = \frac{4}{3}\pi R^3 \rho g$$

52. After the drop detaches, its surface energy is

A) 1.4×10^{-6} J B) 2.7×10^{-6} J C) 5.4×10^{-6} J D) 8.1×10^{-6} J

Sol. ***(B)***

Surface energy = $T(4\pi R^2) = 2.7 \times 10^{-6}$ J

Paragraph for questions 53 to 55.

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Sol. ***(D)***

$$L = \frac{nh}{2\pi}$$

$$K.E. = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I}$$

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi \times 10^{-34}$ Js)

A) 2.76×10^{-46} kg m^2 B) 1.87×10^{-46} kg m^2 C) 4.67×10^{-47} kg m^2 D) 1.17×10^{-47} kg m^2

Sol. ***(B)***

$h\nu = k.E_{n=2} - kE_{n=1}$

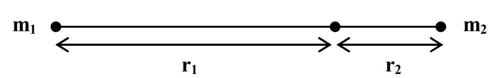
$I = 1.87 \times 10^{-46}$ kg m^2

55. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. $= \frac{5}{3} \times 10^{-27}$ kg, is close to

A) 2.4×10^{-10} m B) 1.9×10^{-10} m C) 1.3×10^{-10} m D) 4.4×10^{-11} m

Sol. ***(C)***

$$r_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 d}{m_1 + m_2}$$



$I = m_1 r_1^2 + m_2 r_2^2$

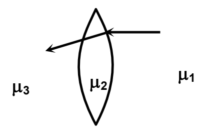
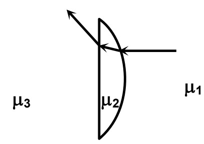
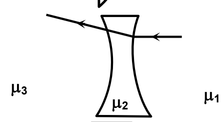
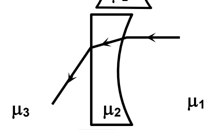
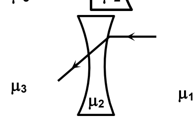
$\therefore$ $d = 1.3 \times 10^{-10}$ m.

Nitin M Sir (physics-iitjee.blogspot.com)

SECTION – IV (Matrix Type)

This Section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the **ORS**.

56. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I		Column II	
A)	$\mu_1 < \mu_2$	p)	
B)	$\mu_1 > \mu_2$	q)	
C)	$\mu_2 = \mu_3$	r)	
D)	$\mu_2 > \mu_3$	s)	
		t)	

Sol. *(A) $\to$ (p, r), (B) $\to$ (q, s, t), (C) $\to$ (p, r, t), (D) $\to$ (q, s)*

57. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in Column I. Match the two

Column I

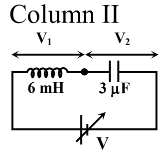
A) $I \neq 0$, V_1 is proportional to I

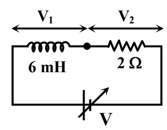
B) $I \neq 0$, $V_2 > V_1$

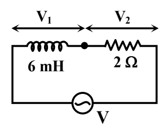
C) $V_1 = 0$, $V_2 = V$

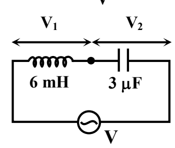
D) $I \neq 0$, V_2 is proportional to I

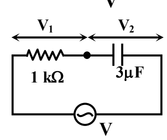
Column II

p) 

q) 

r) 

s) 

t) 

Sol. *(A) → (r, s, t), (B) → (q, r, s, t), (C) → (p, q), (D) → (q, r, s, t)*

****

FIITJEE Solutions to IIT-JEE-2011

CODE **8**

PAPER 1

Time: 3 Hours **Maximum Marks: 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 36) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This question Paper contains 36 pages having 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign with pen in the boxes provided**. Do not write them anywhere else. Darken the appropriate bubble UNDER** each digit of your Registration No. with a **good quality HB pencil.**

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 21), for each question you will be awarded 3 marks if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero** marks other wise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 15), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
17. In **Section IV** (Total Marks: 28), for each question you will be awarded 4 marks if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.

Write your name, registration number and sign in the space provided on the back of this booklet.

PAPER-1 [Code – 8]
IITJEE 2011
PART - I: CHEMISTRY

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Extra pure N_2 can be obtained by heating
(A) NH_3 with CuO
(B) NH_4NO_3
(C) $(NH_4)_2Cr_2O_7$
(D) $Ba(N_3)_2$

Sol. **(D)**

$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$$

2. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
(A) 1.78 M
(B) 2.00 M
(C) 2.05 M
(D) 2.22 M

Sol. **(C)**

Total mass of solution = 1000 + 120 = 1120 g

Total volume of solution in (L) $= \frac{1120}{1.15} \times 10^3$

$$M = \frac{W}{M} \times \frac{1}{V(\text{in L})} = \frac{120}{60} \times \frac{1.15 \times 10^3}{1120} = 2.05 \text{ M}$$

3. Bombardment of aluminium by α-particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are,

$$\begin{array}{ccc} {}^{27}_{13}Al & \xrightarrow{(ii)} & {}^{30}_{15}P + Y \\ (i)\downarrow & & \downarrow \\ {}^{30}_{14}Si + X & & {}^{30}_{14}Si + Z \end{array}$$

(A) proton, neutron, positron
(B) neutron, positron, proton
(C) proton, positron, neutron
(D) positron, proton, neutron

Sol. **(A)**

$${}^{27}_{13}Al + {}_2\alpha^4 \rightarrow {}^{30}_{14}Si + {}_1p^1 (X)$$
$${}^{27}_{13}Al + {}_2\alpha^4 \rightarrow {}^{30}_{15}P + {}_0n^1 (Y)$$
$${}^{30}_{15}P \rightarrow {}^{30}_{14}Si + {}_{+1}\beta^0 (Z)$$

4. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively, are
(A) octahedral, tetrahedral and square planar
(B) tetrahedral, square planar and octahedral
(C) square planar, tetrahedral and octahedral
(D) octahedral, square planar and octahedral

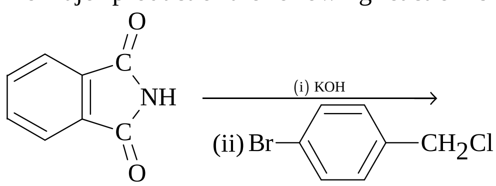
Sol. **(B)**

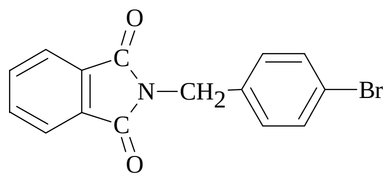
$\left[NiCl_4\right]^{2-} \rightarrow$ Tetrahedral

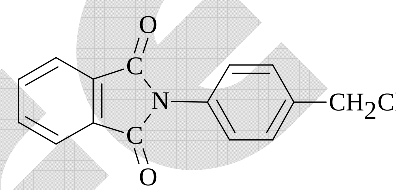
$\left[Ni(CN)_4\right]^{2-} \rightarrow$ Square Planar

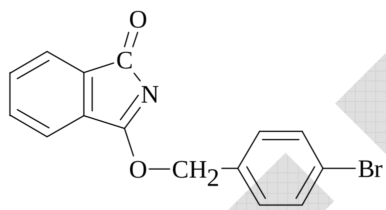
$\left[Ni(H_2O)_6\right]^{2+} \rightarrow$ Octahedral

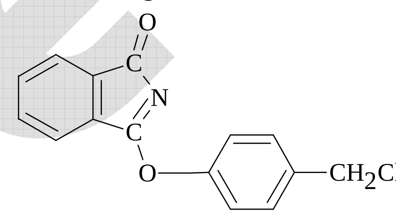
5. The major product of the following reaction is



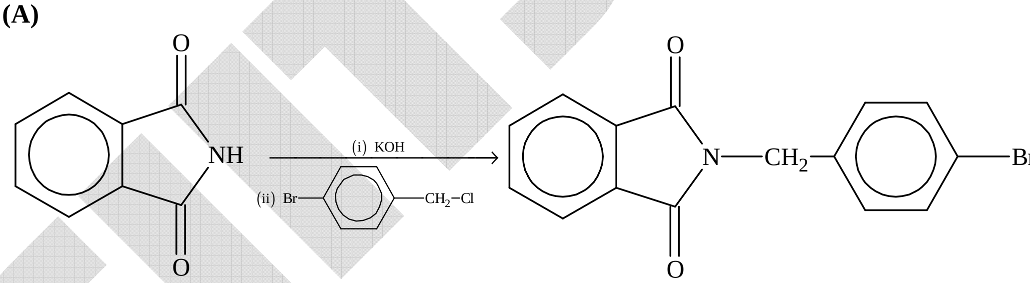
(A)


(B)


(C)


(D)


Sol. **(A)**



[**Reason:** Due to partial double bond character along C–Br bond prevents the attack of nucleophile at phenylic position]

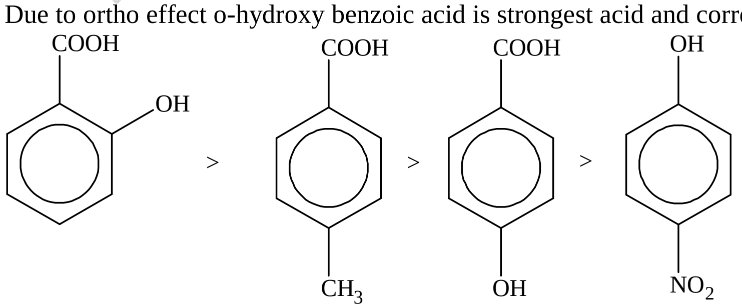
6. Among the following compounds, the most acidic is

(A) p-nitrophenol (B) p-hydroxybenzoic acid

(C) o-hydroxybenzoic acid (D) p-toluic acid

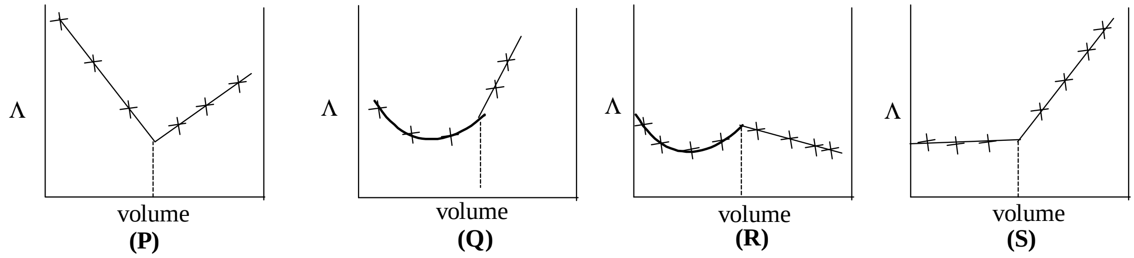
Sol. **(C)**

Due to ortho effect o-hydroxy benzoic acid is strongest acid and correct order of decreasing K_a is



Nitin M Sir (physics-iitjee.blogspot.com)

7. $AgNO_3$(aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(A) (P) (B) (Q)
(C) (R) (D) (S)

Sol. **(D)**

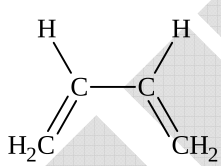
$AgNO_3 + KCl(aq) \rightarrow AgCl(s) + KNO_3(aq)$

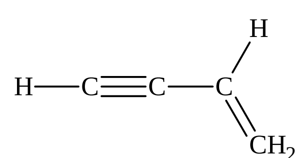
Initially there is aq. KCl solution now as solution of $AgNO_3$ is added, AgCl(s) is formed. Hence conductivity of solution is almost compensated (or slightly increase) by the formation of KNO_3. After end point conductivity increases more rapidly because addition of excess $AgNO_3$ solution.

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

8. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

(A)


(B)


(C) $H_2C=C=O$

(D) $H_2C=C=CH_2$

Sol. **(B, C)**

Along C–C single bond conformations are possible in butadiene in which all the atoms may not lie in the same plane.

9. Extraction of metal from the ore **cassiterite** involves
(A) carbon reduction of an oxide ore
(B) self-reduction of a sulphide ore
(C) removal of copper impurity
(D) removal of iron impurity

Sol. **(A, C, D)**

$SnO_2 + 2C \rightarrow 2CO + Sn$

The ore cassiterite contains the impurity of Fe, Mn, W and traces of Cu.

10. According to kinetic theory of gases
(A) collisions are always elastic
(B) heavier molecules transfer more momentum to the wall of the container
(C) only a small number of molecules have very high velocity
(D) between collisions, the molecules move in straight lines with constant velocities

Sol. **(A, B, C, D)**

11. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)
(A) Adsorption is always exothermic
(B) Physisorption may transform into chemisorption at high temperature
(C) Physiosorption increases with increasing temperature but chemisorption decreases with increasing temperature
(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation

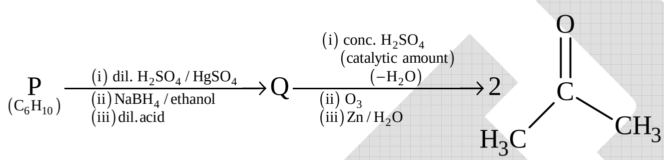
Sol. **(A, B, D)**

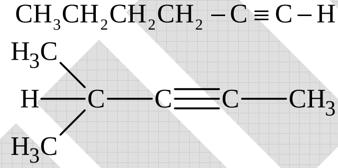
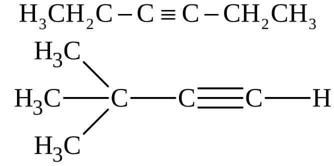
SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

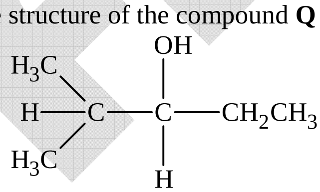
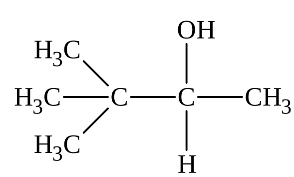
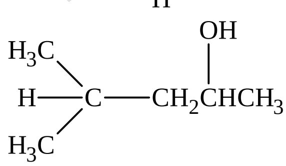
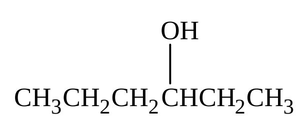
Paragraph for Question Nos. 12 and 13

An acyclic hydrocarbon **P**, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of reaction, in which **Q** is an intermediate organic compound.



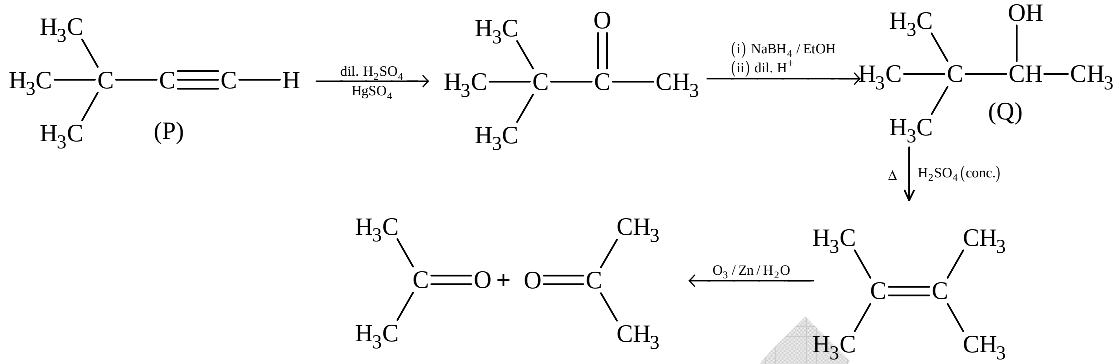
12. The structure of compound **P** is
(A) $CH_3CH_2CH_2CH_2-C\equiv C-H$
(B) $H_3CH_2C-C\equiv C-CH_2CH_3$
(C) 
(D) 

Sol. **(D)**

13. The structure of the compound **Q** is
(A) 
(B) 
(C) 
(D) 

Sol. **(B)**

Solution for the Q. No. 12 to 13



Paragraph for Question Nos. 14 to 16

When a metal rod **M** is dipped into an aqueous colourless concentrated solution of compound **N**, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate **O**. Addition of aqueous NH_3 dissolves **O** and gives an intense blue solution.

14. The metal rod **M** is
(A) Fe
(B) Cu
(C) Ni
(D) Co

Sol. **(B)**

$$\underset{M}{Cu} + 2\underset{N}{AgNO_3} \rightarrow \underset{Blue}{Cu(NO_3)_2} + 2Ag$$

While Cu partially oxidizes to $Cu(NO_3)_2$ and remaining $AgNO_3$ reacts with NaCl.

15. The compound **N** is
(A) $AgNO_3$
(B) $Zn(NO_3)_2$
(C) $Al(NO_3)_3$
(D) $Pb(NO_3)_2$

Sol. **(A)**

16. The final solution contains
(A) $\left[Pb(NH_3)_4\right]^{2+}$ and $\left[CoCl_4\right]^{2-}$
(B) $\left[Al(NH_3)_4\right]^{3+}$ and $\left[Cu(NH_3)_4\right]^{2+}$
(C) $\left[Ag(NH_3)_2\right]^{+}$ and $\left[Cu(NH_3)_4\right]^{2+}$
(D) $\left[Ag(NH_3)_2\right]^{+}$ and $\left[Ni(NH_3)_6\right]^{2+}$

Sol. **(C)**

$$\underset{(N)}{AgNO_3} + NaCl \rightarrow \underset{(O)}{AgCl}\downarrow + NaNO_3$$

$$AgCl + 2NH_3 \rightarrow \left[Ag(NH_3)_2\right]^{+} Cl^{-}$$

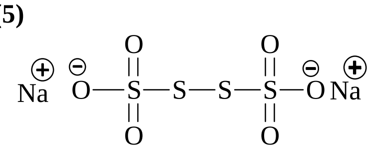
$$Cu(NO_3)_2 + 4NH_4OH \rightarrow \left[Cu(NH_3)_4\right]^{2+}$$

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

17. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

Sol. **(5)**



S will have oxidation number = +5, 0
Difference in oxidation number = 5

18. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

Sol. **(6)**
For n-units of glycine,

$$\frac{n\times 75}{(796+9\times 18)}\times 100 = 47$$

$\Rightarrow$ n = 6

19. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Sol. **(4)**

The energy associated with incident photon = $\frac{hc}{\lambda}$

$$\Rightarrow E = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{300\times 10^{-9}}\ J$$

$$E \text{ in eV} = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{300\times 10^{-9}\times 1.6\times 10^{-19}} = 4.16\ eV$$

So, number of metals showing photo-electric effects will be (4), i.e., Li, Na, K, Mg

20. The maximum number of electrons that can have principal quantum number, n = 3, and spin quantum number, $m_s = -\frac{1}{2}$, is

Sol. **(9)**
For principal quantum number (n = 3)
Number of orbitals = n^2 = 9

So, number of electrons with $m_s = -\frac{1}{2}$ will be 9.

21. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

Sol. **(5)**

$$3Br_2 + 3Na_2CO_3 \longrightarrow 5NaBr + NaBrO_3 + 3CO_2$$

So, number of NaBr molecules = 5

22. To an evacuated vessel with movable piston under external pressure of 1 atm, 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0^oC) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0^oC is close to

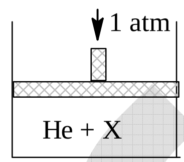
Sol. **(7)**

For any ideal gas, PV = nRT

$0.32 \times V = 0.1 \times 0.0821 \times 273$

V = 7 litre

(unknown compound X will not follow ideal gas equation)

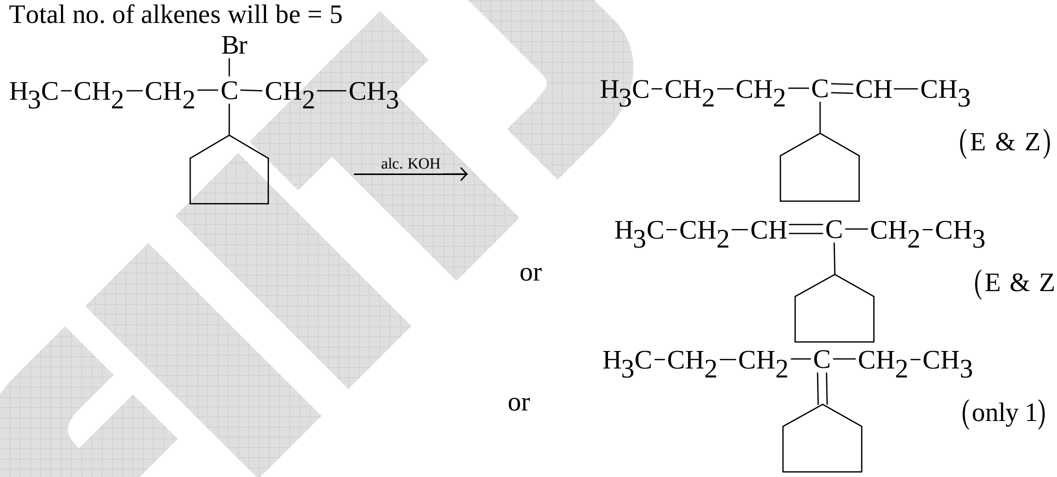


For He, n = 0.1, P = 0.32 atm., V = ?, T = 273

23. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

Sol. **(5)**

Total no. of alkenes will be = 5



PART - II: PHYSICS

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

24. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is
(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHZ

Sol. **(A)**

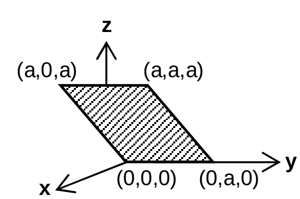
$$f = \frac{320}{320-10}\times 8\times 10^3 \times \frac{320+10}{320}$$
$= 8.5$ kHz

25. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 A^0. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is
(A) 1215 A^0 (B) 1640 A^0
(C) 2430 A^0 (D) 4687 A^0

Sol. **(A)**

$$\frac{1}{6561} = R\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5R}{36}$$
$$\frac{1}{\lambda} = 4R\left(\frac{1}{4}-\frac{1}{16}\right) = \frac{3R\times 4}{16}$$
$$\lambda = 1215A^0$$

26. Consider an electric field $\vec{E} = E_0\hat{x}$ where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$
(C) $E_0\,a^2$ (D) $\frac{E_0a^2}{\sqrt{2}}$

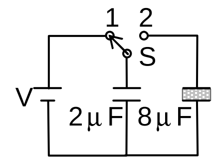
Sol. **(C)**
(E_0) (Projected area) $= E_0a^2$

27. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is
(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$
(C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

Sol. **(A)**
$TV^{\gamma-1} = C$
$T_1(5.6)^{2/3} = T_2\,(0.7)^{2/3} \quad \Rightarrow \quad T_2 = T_1(8)^{2/3} = 4T_1$

$$\therefore \quad \Delta w(\text{work done on the system}) = \frac{nR\Delta T}{\gamma - 1} = \frac{9}{8}RT_1$$

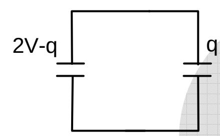
28. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(A) 0 % (B) 20 %
(C) 75 % (D) 80 %

Sol. **(D)**

$U_i = \frac{1}{2} \times 2 \times V^2 = V^2$



$q_i = 2V$

Now, switch S is turned to position 2

$$\frac{2V - q}{2} = \frac{q}{8}$$

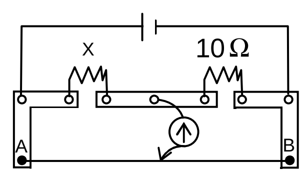
$8V - 4q = q$

$$\Rightarrow q = \frac{8V}{5}$$

$$\Delta H = V^2 - \left(\frac{64V^2}{2\times25\times8} + \frac{4V^2}{2\times25\times2}\right)$$

$$= \frac{4V^2}{5}$$

29. A meter bridge is set up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



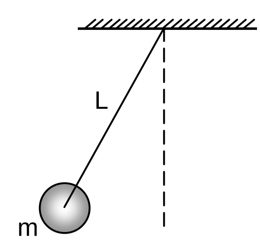
(A) 10.2 ohm (B) 10.6 ohm
(C) 10.8 ohm (D) 11.1 ohm

Sol. **(B)**

$X(48 + 2) = (10)(52 + 1)$

$$X = \frac{530}{50} = 10.6\Omega$$

30. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9 (B) 18
(C) 27 (D) 36

Sol. **(D)**

$324 = 0.5\,\omega^2\,(0.5)$

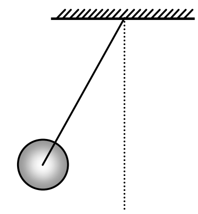
$\omega^2 = 324 \times 4$

$\omega = \sqrt{1296} = 36 \text{ rad/s}$

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

31. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B
(B) restoring torque in case A < Restoring torque in case B
(C) Angular frequency for case A > angular frequency for case B.
(D) Angular frequency for case A < Angular frequency for case B.

Sol. **(A, D)**

In case A

$$mg(\ell/2)\sin\theta + Mg\,\ell\sin\theta = \left(\frac{m\ell^2}{3}+\frac{MR^2}{2}+M\ell^2\right)\alpha_A$$

In case B

$$mg\,(\ell/2)\sin\theta + Mg\,\ell\sin\theta = \left(\frac{m\ell^2}{3}+M\ell^2\right)\alpha_B$$

$\tau_A = \tau_B$, $\omega_A < \omega_B$

32. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($<R_A$) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

(A) $E_A^{inside} = 0$
(B) $Q_A > Q_B$
(C) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$
(D) $E_A^{on\ surface} < E_B^{on\ surface}$

Sol. **(A, B, C, D)**

$R_B < R_A$

$Q_A + Q_B = 2Q$

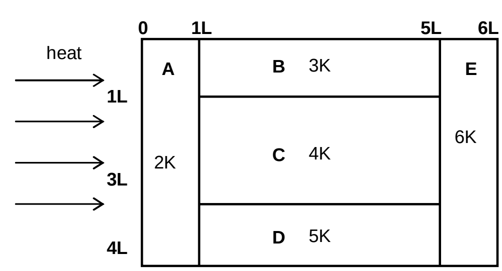
$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$\sigma_A R_A = \sigma_B R_B$

$$Q_A = \frac{2QR_A}{R_A + R_B}$$

$$Q_B = \frac{2QR_B}{R_A + R_B}$$

33. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same.
(B) heat flow through slab E is maximum.
(C) temperature difference across slab E is smallest.
(D) heat flow through C = heat flow through B + heat flow through D.

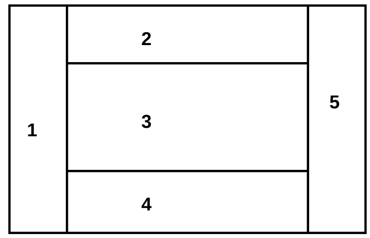
Sol. **(A, C, D) or (A, B, C, D)**

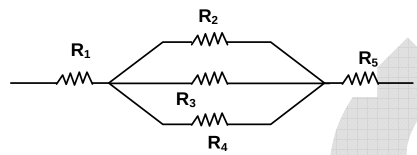
Let width of each rod is d

$R_1 = \dfrac{1}{8kd}$, $R_2 = \dfrac{4}{3kd}$

$R_3 = \dfrac{1}{2kd}$, $R_4 = \dfrac{4}{5kd}$

$R_5 = \dfrac{1}{24kd}$,





34. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is / are true?

(A) They will never come out of the magnetic field region.
(B) They will come out travelling along parallel paths.
(C) They will come out at the same time.
(D) They will come out at different times.

Sol. **(B, D)**

Both will travel in semicircular path

t = $E_{ind} = -\dfrac{d\phi}{dt} = -\dfrac{\mu_0(\pi r^2)}{L}\dfrac{dI}{dt}$ time period of semi circular path

m is different, hence time will be different

SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

Paragraph for Question Nos. 35 and 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0'. Use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

Sol. **(C)**

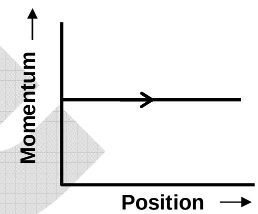
36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons N $\approx 4 \times 10^{27}$ m^{-3}. Taking $\varepsilon_0 = 10^{-11}$ and mass m $\approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm
(C) 300 nm (D) 200 nm

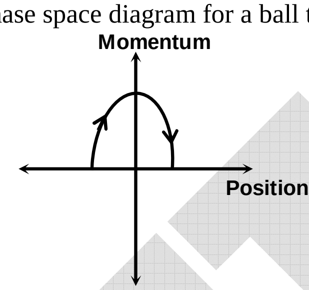
Sol. **(B)**
$\omega = 2\pi c / \lambda$

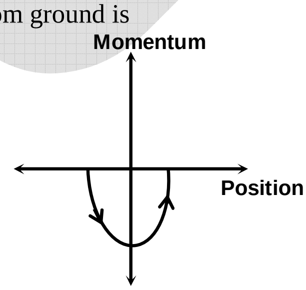
Paragraph for Question Nos. 37 to 39

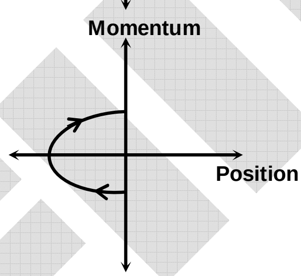
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momenum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which positon or momentum upwards (or to right) is positive and downwards (or to left) is negative.

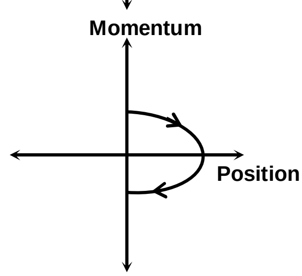


37. The phase space diagram for a ball thrown vertically up from ground is

(A)


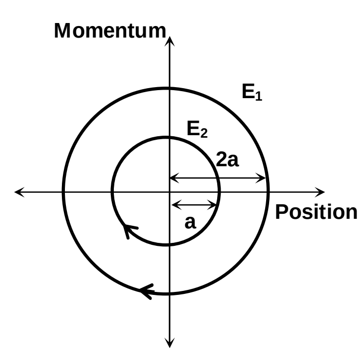
(B)


(C)


(D)


Sol. **(D)**

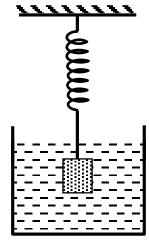
38. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



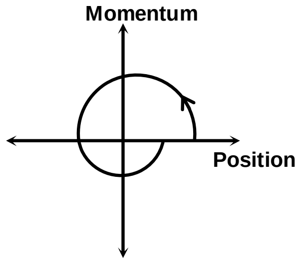
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$
(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

Sol. **(C)**
$E \propto A^2$

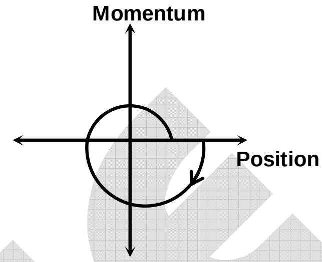
39. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



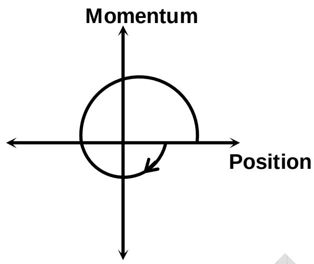
(A)



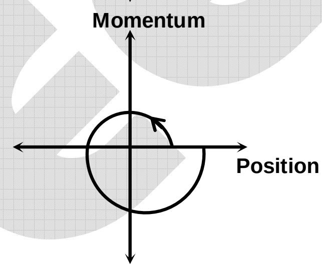
(B)



(C)



(D)

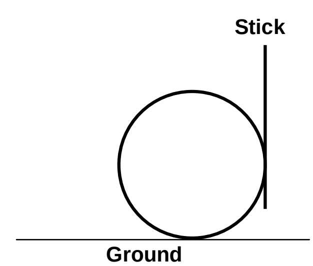


Sol. **(B)**

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

40. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficeint of friction between the stick and the ring is (P/10). The value of P is



Sol. **(4)**

Now

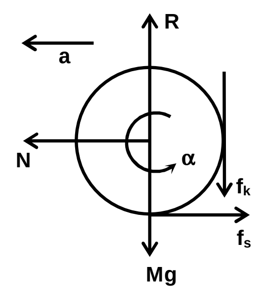
$N - f_s = ma$ $\therefore f_s = 1.4$ N

and $(f_s - f_K)R = I\alpha$, $a = R\alpha$

$\therefore f_K = 0.8$ N

So, $\mu = \dfrac{P}{10} = 0.4$

P = 4



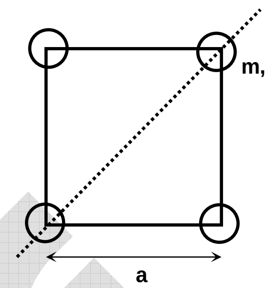
41. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is

Sol. **(9)**

$$I = 2\left(\frac{2}{5}\times mr^2\right) + 2\left[\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right]$$

$I = 9 \times 10^{-4}$ kg-m^2

$\therefore$ N = 9



42. Steel wire of lenght 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L $\gg$ diameter of the wire. Then the value of 'm' in kg is nearly

Sol. **(3)**

Change in length $\Delta L = \dfrac{MgL}{YA} = L\alpha\Delta T$

$\therefore$ m $\approx$ 3 kg

43. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\dfrac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is

Sol. **(3)**

Since $F_{electric} \propto \dfrac{q^2}{a^2} \propto \gamma a$

$\therefore a = k\left(\dfrac{q^2}{\gamma}\right)^{1/3}$

$\therefore$ N = 3

44. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is

Sol. **(5)**

mg(sin θ + μcosθ) = 3mg(sinθ – μcosθ)

$\therefore \mu = 0.5$

$\therefore$ N = 5

45. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Sol. **(1)**

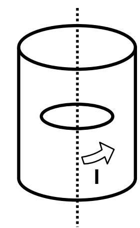
Nitin M Sir (physics-iitjee.blogspot.com)

$$\frac{dN}{dt} = \lambda N$$

$$\therefore\ N = \frac{\left(\frac{dN}{dt}\right)}{\lambda} = 10^{10} \times 10^{9} = 10^{19} \text{ atoms}$$

$m_{sample} = 10^{-25} \times 10^{19}$ kg = 1 mg

46. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300\ t)$, then 'N' is



Sol. **(6)**

$$B_{inside} = \mu_0 ni = \frac{\mu_o I}{L}$$

$$\therefore\ E_{ind} = -\frac{d\phi}{dt} = -\frac{\mu_0(\pi r^2)}{L}\frac{dI}{dt}$$

So magnetic moment = $\left(\frac{E_{ind}}{R}\right)\pi r^2$

$= 6\mu_0 I_0 \sin(300t)$

Therefore, n = 6

PART - III:

SECTION – I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

47. Let (x_0, y_0) be solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) 6

Sol. **(C)**

$(2x)^{\ln 2} = (3y)^{\ln 3}$ …(i)

$3^{\ln x} = 2^{\ln y}$ …(ii)

$\Rightarrow$ (logx) (log3) = (logy)log2

$\Rightarrow \log y = \frac{(\log x)(\log 3)}{\log 2}$ …(iii)

In (i) taking log both sides

(log2) {log2 + logx} = log3{log3 + logy}

$(\log 2)^2 + (\log 2)(\log x) = (\log 3)^2 + \frac{(\log 3)^2(\log x)}{\log 2}$ from (iii)

$\Rightarrow (\log 2)^2 - (\log 3)^2 = \frac{(\log 3)^2 - (\log 2)^2}{\log 2}(\log x) \Rightarrow -\log 2 = \log x$

$\Rightarrow x = \frac{1}{2} \Rightarrow x_0 = \frac{1}{2}$.

48. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) P = Q

Sol. **(D)**

In set P, $\sin\theta = \left(\sqrt{2}+1\right)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1$

In set Q, $\left(\sqrt{2}-1\right)\sin\theta = \cos\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1 \Rightarrow P = Q$.

49. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}+\hat{k}$

(C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

Sol. **(C)**

$\vec{v} = \lambda\bar{a} + \mu\bar{b}$

$= \lambda\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

Projection of $\vec{v}$ on $\bar{c}$

$\dfrac{\bar{v}\cdot\bar{c}}{|\bar{c}|} = \dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{\left[(\lambda+\mu)\hat{i}+(\lambda-\mu)\hat{j}+(\lambda+\mu)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \lambda + \mu - \lambda + \mu - \lambda - \mu = 1 \Rightarrow \mu - \lambda = 1 \Rightarrow \lambda = \mu - 1$

$\bar{v} = (\mu-1)\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right) \;= \mu\left(2\hat{i}+2\hat{k}\right)-\hat{i}-\hat{j}-\hat{k}$

$\bar{v} = (2\mu-1)\hat{i}-\hat{j}+(2\mu-1)\hat{k}$

At $\mu = 2$, $\bar{v} = 3\hat{i}-\hat{j}+3\hat{k}$.

50. The value of $\displaystyle\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2+\sin\left(\ln 6-x^2\right)}dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$
(B) $\frac{1}{2}\ln\frac{3}{2}$
(C) $\ln\frac{3}{2}$
(D) $\frac{1}{6}\ln\frac{3}{2}$

Sol. **(A)**

$x^2 = t \Rightarrow 2x\,dx = dt$

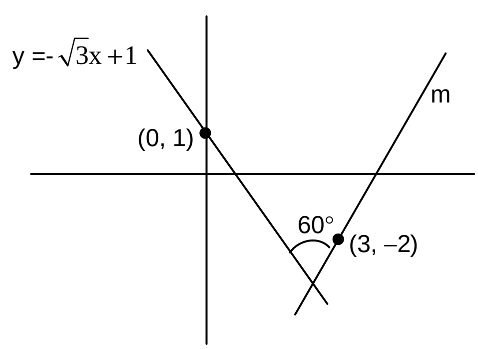
$I = \dfrac{1}{2}\displaystyle\int_{\ln 2}^{\ln 3}\frac{\sin t}{\sin t+\sin(\ln 6-t)}dt$ and $I = \dfrac{1}{2}\displaystyle\int_{\ln 2}^{\ln 3}\frac{\sin(\ln 6-t)}{\sin(\ln 6-t)+\sin t}dt$

$2I = \dfrac{1}{2}\displaystyle\int_{\ln 2}^{\ln 3}1dt \Rightarrow I = \dfrac{1}{4}\ln\dfrac{3}{2}$.

51. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(A) $y+\sqrt{3}x+2-3\sqrt{3}=0$
(B) $y-\sqrt{3}x+2+3\sqrt{3}=0$
(C) $\sqrt{3}y-x+3+2\sqrt{3}=0$
(D) $\sqrt{3}y+x-3+2\sqrt{3}=0$

Sol. **(B)**



$\left|\dfrac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

$\Rightarrow m+\sqrt{3} = \pm\left(\sqrt{3}-3m\right)$

$\Rightarrow 4m = 0 \Rightarrow m = 0$

or $2m = 2\sqrt{3} \Rightarrow m = \sqrt{3}$

$\therefore$ Equation is $y + 2 = \sqrt{3}(x-3)$

$\Rightarrow \sqrt{3}x - y - \left(2+3\sqrt{3}\right) = 0$

52. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \ge 1$, then the value of $\dfrac{a_{10}-2a_8}{2a_9}$ is

(A) 1
(B) 2
(C) 3
(D) 4

Sol. **(C)**

$a_n = \alpha^n - \beta^n$

$\alpha^2 - 6\alpha - 2 = 0$

Multiply with α^8 on both sides

$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ …(i)

similarly $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$ …(ii)

(i) and (ii)

$\Rightarrow \alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) = 2(\alpha^8 - \beta^8)$

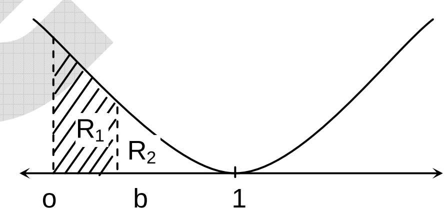
$\Rightarrow a_{10} - 6a_9 = 2a_8 \Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3$.

53. Let the straight line x = b divides the area enclosed by $y = (1 - x)^2$, y = 0 and x = 0 into two parts $R_1(0 \le x \le b)$ and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \dfrac{1}{4}$. Then b equals

(A) $\dfrac{3}{4}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{1}{3}$ (D) $\dfrac{1}{4}$

Sol. **(B)**



$\because \int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \dfrac{1}{4}$

$\Rightarrow \left.\dfrac{(x-1)^3}{3}\right|_0^b - \left.\dfrac{(x-1)^3}{3}\right|_b^1 = \dfrac{1}{4}$

$\Rightarrow \dfrac{(b-1)^3}{3} + \dfrac{1}{3} - \left(0 - \dfrac{(b-1)^3}{3}\right) = \dfrac{1}{4}$

$\Rightarrow \dfrac{2(b-1)^3}{3} = -\dfrac{1}{12} \Rightarrow (b-1)^3 = -\dfrac{1}{8} \Rightarrow b = \dfrac{1}{2}$.

SECTION – II (Total Marks : 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

54. Let f: R → R be a function such that f(x + y) = f(x) + f(y), ∀ x, y ∈ R. If f(x) is differentiable at x = 0, then

(A) f(x) is differentiable only in a finite interval containing zero

(B) f(x) is continuous ∀ x ∈ R

(C) f′(x) is constant ∀ x ∈ R

(D) f(x) is differentiable except at finitely many points

Sol. **(B, C)**

$\because f(0) = 0$

and $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= = \lim\limits_{h\to 0} \dfrac{f(h)}{h} = f'(0) = k(\text{say})$

$\Rightarrow f(x) = kx + c \Rightarrow f(x) = kx \ (\because f(0) = 0)$.

55. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$
(B) $-\hat{i}+\hat{j}$
(C) $\hat{i}-\hat{j}$
(D) $-\hat{j}+\hat{k}$

Sol. **(A, D)**

Any vector in the plane of $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ is $\vec{r}=(\lambda+\mu)\hat{i}+(\lambda+2\mu)\hat{j}+(2\lambda+\mu)\hat{k}$

Also, $\vec{r}\cdot\vec{c}=0 \Rightarrow \lambda+\mu=0$

$\Rightarrow \left[\vec{r}\ \vec{a}\ \vec{b}\right]=0$.

56. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$
(B) a focus of the hyperbola is (2, 0)
(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$
(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Sol. **(B, D)**

Ellipse is $\frac{x^2}{2^2}+\frac{y^2}{1^2}=1$

$1^2 = 2^2(1 - e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$

$\therefore$ eccentricity of the hyperbola is $\frac{2}{\sqrt{3}} \Rightarrow b^2 = a^2\left(\frac{4}{3}-1\right) \Rightarrow 3b^2 = a^2$

Foci of the ellipse are $\left(\sqrt{3},\ 0\right)$ and $\left(-\sqrt{3},\ 0\right)$.

Hyperbola passes through $\left(\sqrt{3},\ 0\right)$

$\frac{3}{a^2}=1 \Rightarrow a^2 = 3$ and $b^2 = 1$

$\therefore$ Equation of hyperbola is $x^2 - 3y^2 = 3$

Focus of hyperbola is $(ae, 0) \equiv \left(\sqrt{3}\times\frac{2}{\sqrt{3}},\ 0\right) \equiv (2, 0)$.

57. Let M and N be two 3 × 3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2
(B) $-N^2$
(C) $-M^2$
(D) MN

Sol. **(C)**

MN = NM

$M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^T.M^T$

$= M^2N.(M^T)^{-1}(N^{-1})^TM^T = -M^2.N(M)^{-1}(N^T)^{-1}M^T$

$= +M^2NM^{-1}N^{-1}M^T = -M.NMM^{-1}N^{-1}M = -MNN^{-1}M = -M^2$.

Nitin M Sir (physics-iitjee.blogspot.com)

SECTION-III (Total Marls : 15)
(Paragraph Type)

This section contains **2 paragraphs.** Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$

(C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Sol. **(B)**

$H \to 1$ ball from U_1 to U_2

$T \to 2$ ball from U_1 to U_2

E : 1 ball drawn from U_2

$$\text{P/W from } U_2 = \frac{1}{2}\times\left(\frac{3}{5}\times 1\right)+\frac{1}{2}\times\left(\frac{2}{5}\times\frac{1}{2}\right)+\frac{1}{2}\times\left(\frac{{}^3C_2}{{}^5C_2}\times 1\right)+\frac{1}{2}\times\left(\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}\right)+\frac{1}{2}\times\left(\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right)=\frac{23}{30}.$$

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$

(C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Sol. **(D)**

$$P\left(\frac{H}{W}\right)=\frac{P(W/H)\times P(H)}{P(W/T)\cdot P(T)+(W/H)\cdot P(H)}=\frac{\frac{1}{2}\left(\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right)}{23/30}=\frac{12}{23}.$$

Paragraph for Question Nos. 60 to 62

Let a, b and c be three real numbers satisfying $\begin{bmatrix} a & b & c\end{bmatrix}\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\end{bmatrix}$ ……(E)

60. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is

(A) 0 (B) 12

(C) 7 (D) 6

Sol. **(D)**

a + 8b + 7c = 0

9a + 2b + 3c = 0

a + b + c = 0
Solving these we get
b = 6a ⇒ c = – 7a
now 2x + y + z = 0
⇒ 2a + 6a + (–7a) = 1 ⇒ a = 1, b = 6, c = – 7.

61. Let ω be a solution of $x^3 - 1 = 0$ with Im(ω) > 0. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to

(A) –2
(B) 2
(C) 3
(D) – 3

Sol. **(A)**
a = 2, b and c satisfies (E)
b = 12, c = – 14

$$\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}} = -2.$$

62. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is

(A) 6
(B) 7
(C) $\frac{6}{7}$
(D) ∞

Sol. **(B)**
$ax^2 + bx + c = 0 \Rightarrow x^2 + 6x - 7 = 0$
⇒ α = 1, β = – 7

$$\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{1} - \frac{1}{7}\right)^n = 7.$$

SECTION-IV (Total Marks : 28)
(Integer Answer Type)

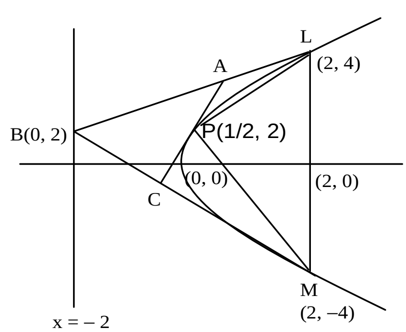
This section contains **7** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. The bubble corresponding to the correct is to be darkened in the ORS.

63. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Sol. **(2)**
$y^2 = 8x = 4.2.x$

$$\frac{\Delta LPM}{\Delta ABC} = 2$$

$$\frac{\Delta_1}{\Delta_2} = 2$$



Nitin M Sir (physics-iitjee.blogspot.com)

64. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Sol. **(1)**

$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{2\cos^2\theta - 1}}\right)\right)$

$= \sin\left(\sin^{-1}(\tan\theta)\right) = \tan\theta.$

$\frac{d(\tan\theta)}{d(\tan\theta)} = 1.$

65. Let f: $[1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

Sol. **(6)**

$6\int_1^x f(t)\,dt = 3x f(x) - x^3 \Rightarrow 6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow 3f(x) = 3xf'(x) - 3x^2 \Rightarrow xf'(x) - f(x) = x^2$

$\Rightarrow x\frac{dy}{dx} - y = x^2 \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x \quad \ldots(i)$

I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log_e x}$

Multiplying (i) both sides by $\frac{1}{x}$

$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 1 \Rightarrow \frac{d}{dx}\left(y.\frac{1}{x}\right) = 1$

integrating

$\frac{y}{x} = x + c$

Put $x = 1$, $y = 2$

$\Rightarrow 2 = 1 + c \Rightarrow c = 1 \Rightarrow y = x^2 + x$

$\Rightarrow f(x) = x^2 + x \Rightarrow f(2) = 6.$

66. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Sol. **(7)**

$$\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} \Rightarrow \frac{\sin\frac{3\pi}{n} - \sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} \quad \frac{\left(2\sin\frac{\pi}{n}\cos\frac{2\pi}{n}\right)\sin\frac{2\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = 1$$

$\Rightarrow \sin\frac{4\pi}{n} = \sin\frac{3\pi}{n} \Rightarrow \frac{4\pi}{n} + \frac{3\pi}{n} = \pi \Rightarrow n = 7.$

67. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

Sol. **(9)**

$a_1, a_2, a_3, \ldots a_{100}$ is an A.P.

$a_1 = 3$, $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 1\,00$

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}\left(6+(5n-1)d\right)}{\frac{n}{2}\left(6-d+nd\right)}$$

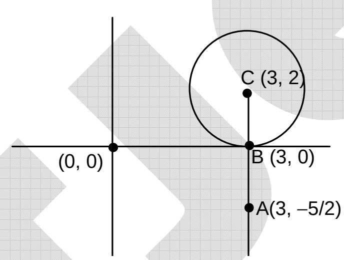
$\frac{S_m}{S_n}$ is independent of n of $6 - d = 0 \Rightarrow d = 6$.

68. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is

Sol. **(5)**

Length AB = $\frac{5}{2}$

$\Rightarrow$ Minimum value = 5.



69. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

Sol. **(8)**

$$\frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+a^{8}+a^{10}+1}{8} \ge 1$$

minimum value = 8.

FIITJEE Solutions to IIT-JEE-2011

CODE 5

PAPER 2

Time: 3 Hours

Time: 3 Hours

Maximum Marks: 240

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page No. 36) of this booklet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This question Paper contains 36 pages having 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign with pen in the boxes provided**. Do not write them anywhere else. Darken the appropriate bubble UNDER** each digit of your Registration No. with a **good quality HB pencil.**

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**
14. In **Section I** (Total Marks: 24), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1)** mark will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero** marks other wise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 24), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise There are **no negative marks** in this section.
17. In **Section IV** (Total Marks: 16), for each question you will be awarded **2 marks** for each row in which you have darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. Thus each question in this section carries **a maximum of 8** Marks. There are **no negative marks** in this section.

PAPER-2 [Code – 5]
IITJEE 2011
PART - I: CHEMISTRY

SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. The freezing point (in ^oC) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g of water (K_f = 1.86 K kg mol^{-1}) is

(A) -2.3×10^{-2}
(B) -5.7×10^{-2}
(C) -5.7×10^{-3}
(D) -1.2×10^{-2}

Sol. **(A)**

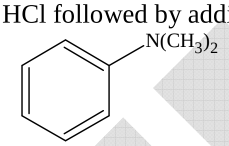
$$K_3\left[Fe(CN)_6\right] \rightarrow 3K^+ + \left[Fe(CN)_6\right]^{3-}$$

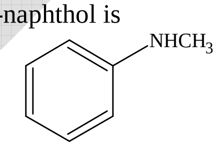
i = 4

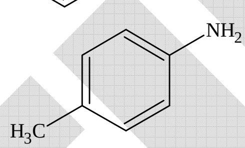
$$\Delta T_f = K_f \times i \times \frac{m}{M} \times \frac{1000}{W} = 1.86 \times 4 \times \frac{0.1}{329} \times \frac{1000}{100} = 2.3 \times 10^{-2}$$

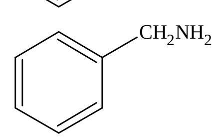
$$T_f^{'} = -2.3 \times 10^{-2}$$

2. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

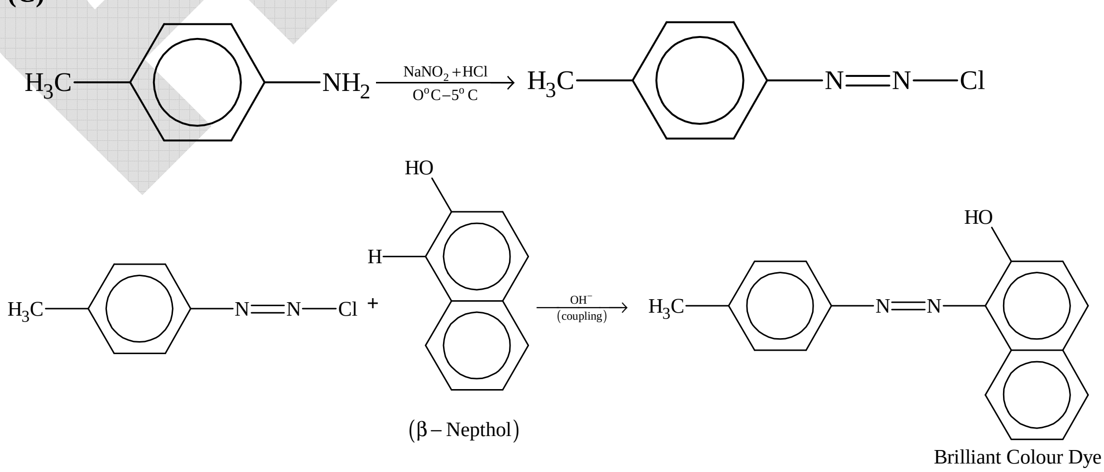
(A) 

(B) 

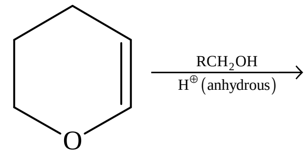
(C) 

(D) 

Sol. **(C)**

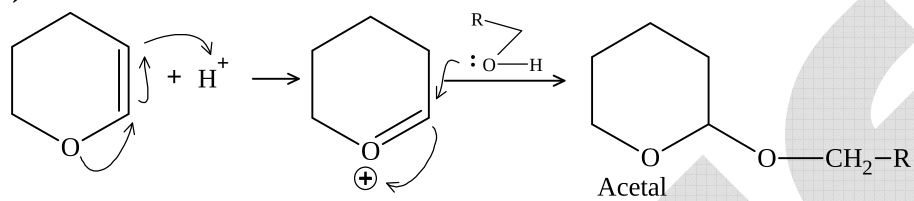


3. The major product of the following reaction is

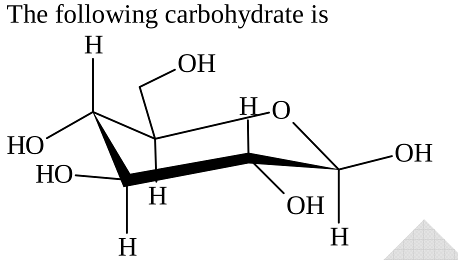


(A) a hemiacetal (B) an acetal
(C) an ether (D) an ester

Sol. **(B)**



4. The following carbohydrate is



(A) a ketohexose (B) an aldohexose
(C) an α-furanose (D) an α-pyranose

Sol. **(B)**

5. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are
(A) II, III in haematite and III in magnetite (B) II, III in haematite and II in magnetite
(C) II in haematite and II, III in magnetite (D) III in haematite and II, III in magnetite

Sol. **(D)**
Haematite : Fe_2O_3 : $2x + 3 \times (-2) = 0$
$x = 3$
Magnetite : Fe_3O_4 [an equimolar mixture of FeO and Fe_2O_3]
FeO : $x - 2 = 0 \Rightarrow x = 2$
Fe_2O_3 : $x = 3$

6. Among the following complexes (**K–P**)
$K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_6]Cl_2$ (**N**), $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**)
(A) **K, L, M, N** (B) **K, M, O, P**
(C) **L, M, O, P** (D) **L, M, N, O**

6. **(C)**
Following compounds are diamagnetic.
L : $[Co(NH_3)_6]Cl_3$
M : $Na_3[Co(Ox)_3]$
O : $K_2[Pt(CN)_4]$
P : $[Zn(H_2O)_6](NO_3)_2$

7. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates
(A) CuS and HgS
(B) MnS and CuS
(C) MnS and NiS
(D) NiS and HgS

Sol. **(A)**
H_2S in presence of aqueous acidified solution precipitates as sulphide of Cu and Hg apart from Pb^{+2}, Bi^{+3}, Cd^{+2}, As^{+3}, Sb^{+3} and Sn^{+2}.

8. Consider the following cell reaction:
$$2Fe_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + 2H_2O_{(\ell)} \quad E^\circ = 1.67\ V$$
At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is
(A) 1.47 V
(B) 1.77 V
(C) 1.87 V
(D) 1.57 V

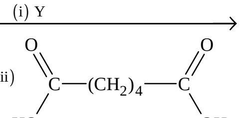
Sol. **(D)**
$$2Fe(s) + O_2(g) + 4H^+(aq) \longrightarrow 2Fe^{+2}(aq) + 2H_2O(\ell)$$
N = 4 (no. of moles of electron involved)
From Nernst's equation,
$$E_{cell} = E^o_{cell} - \frac{0.0591}{n}\log Q$$
$$= 1.67 - \frac{0.0591}{4}\log\frac{(10^{-3})^2}{0.1\times(10^{-3})^4} \qquad \{\because [H^+] = 10^{-pH}\}$$
$= 1.67 - 0.106$
$= 1.57$ V

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

9. Reduction of the metal centre in aqueous permanganate ion involves
(A) 3 electrons in neutral medium
(B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium
(D) 5 electrons in acidic medium

Sol. **(A, D)**
In acidic medium
$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$
In neutral medium
$$MnO_4^- + 2H_2O + 3e^- \longrightarrow MnO_2 + 4OH^-$$
Hence, number of electron loose in acidic and neutral medium 5 and 3 electrons respectively.

10. The correct functional group X and the reagent/reaction conditions Y in the following scheme are
$$X-(CH_2)_4-X \xrightarrow{\text{(i) Y}} \text{condensation polymer}$$
(ii)

heat

(A) X = $COOCH_3$, Y = H_2/Ni/heat
(B) X = $CONH_2$, Y = H_2/Ni/heat
(C) X = $CONH_2$, Y = Br_2/NaOH
(D) X = CN, Y = H_2/Ni/heat

Sol. **(A, B, C, D)**

Condensation polymers are formed by condensation of a diols or diamine with dicarboxylic acids.

Hence, X may be $-\overset{\overset{\displaystyle O}{\|}}{C}-OR$ or $-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2$ or $-C\equiv N$

$$H_3CO-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_4-\overset{\overset{\displaystyle O}{\|}}{C}-OCH_3 \xrightarrow{H_2/Ni} OH-H_2C(CH_2)_4-CH_2-OH \xrightarrow{HO-\overset{\overset{O}{\|}}{C}-(CH_2)_4-\overset{\overset{O}{\|}}{C}-OH} \text{Poly}-\text{ester}$$

$$H_2N-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_4-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \xrightarrow{H_2/Ni} H_2N-CH_2(CH_2)_4-CH_2-NH_2 \xrightarrow{HO-\overset{\overset{O}{\|}}{C}-(CH_2)_4-\overset{\overset{O}{\|}}{C}-OH} \text{Polyamide}$$

$\downarrow Br_2/OH^-, \Delta$ Hoffmann Bromamide Reaction

$$H_2N-(CH_2)_4-NH_2 \xrightarrow{HO-\overset{\overset{O}{\|}}{C}-(CH_2)_4-\overset{\overset{O}{\|}}{C}-OH} \text{Polyamide}$$

$$N\equiv C-(CH_2)_4-C\equiv N \xrightarrow{H_2/Ni} H_2N-CH_2-(CH_2)-CH_2-NH_2 \xrightarrow{HO-\overset{\overset{O}{\|}}{C}-(CH_2)_4-\overset{\overset{O}{\|}}{C}-OH} \text{Polyamides}$$

11. For the first order reaction

$$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$$

(A) the concentration of the reactant decreases exponentially with time
(B) the half-life of the reaction decreases with increasing temperature
(C) the half-life of the reaction depends on the initial concentration of the reactant
(D) the reaction proceeds to 99.6% completion in eight half-life duration

Sol. **(A, B, D)**

For first order reaction

$[A] = [A]_0 e^{-kt}$

Hence concentration of $[NO_2]$ decreases exponentially.

Also, $t_{1/2} = \frac{0.693}{K}$. Which is independent of concentration and $t_{1/2}$ decreases with the increase of temperature.

$$t_{99.6} = \frac{2.303}{K}\log\left(\frac{100}{0.4}\right)$$

$$t_{99.6} = \frac{2.303}{K}(2.4) = 8\times\frac{0.693}{K} = 8\ t_{1/2}$$

12. The equilibrium

$$2Cu^{I} \rightleftharpoons Cu^{o} + Cu^{II}$$

in aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^-
(C) SCN^- (D) CN^-

Sol. **(B, C, D)**

Cu^{2+} ions will react with CN^- and SCN^- forming $[Cu(CN)_4]^{3-}$ and $[Cu(SCN)_4]^{3-}$ leading the reaction in the backward direction.

$$Cu^{2+} + 2CN^- \rightarrow Cu(CN)_2$$

$$2Cu(CN)_2 \rightarrow 2CuCN + (CN)_2$$

$$CuCN + 3CN^- \rightarrow [Cu(CN)_4]^{3-}$$

$$Cu^{2+} + 4SCN^- \rightarrow [Cu(SCN)_4]^{3-}$$

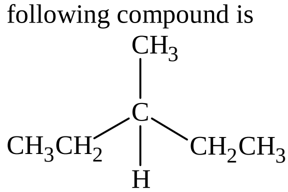
Cu^{2+} also combines with $CuCl_2$ which reacts with Cu to produce CuCl pushing the reaction in the backward direction.

$$CuCl_2 + Cu \rightarrow 2CuCl\downarrow$$

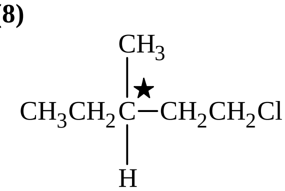
SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

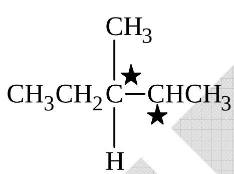
13. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound is



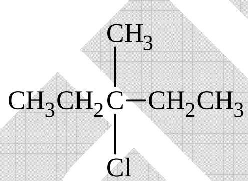
Sol. **(8)**



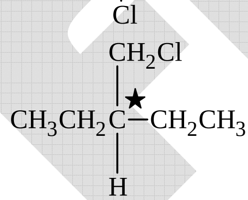
Enantiomeric pair = 2



Two Enantiomeric pairs = 4



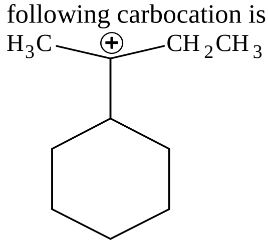
1



1

Total = 2 + 4 + 1 + 1 = 8

14. The total number of contributing structure showing hyperconjugation (involving C−H bonds) for the following carbocation is



Sol. **(6)**
$6 \times$ H–atoms are there

15. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is
O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

Sol. **(5)**

16. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Sol. **(6)**
Number of ionisable Cl^- in $[Cr(H_2O)_5Cl]Cl_2$ is 2
$\therefore$ Millimoles of Cl^- = $30 \times 0.01 \times 2 = 0.6$
$\therefore$ Millimoles of Ag^+ required = 0.6
$\therefore$ 0.6 = 0.1 V
V = 6 ml

17. In 1 L saturated solution of AgCl [$K_{sp}(AgCl) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{sp}(CuCl) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

Sol. **(7)**
Let the solubility of AgCl is x mollitre^{-1}
and that of CuCl is y mollitre^{-1}

$$AgCl \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{Cl^-}$$

$$CuCl \rightleftharpoons \underset{y}{Cu^+} + \underset{y}{Cl^-}$$

$\therefore K_{sp}$ of AgCl = $[Ag^+][Cl^{-1}]$
$1.6 \times 10^{-10} = x(x + y)$ … (i)
Similarly K_{sp} of CuCl = $[Cu^+][Cl^-]$
$1.6 \times 10^{-6} = y(x + y)$ … (ii)
On solving (i) and (ii)
$[Ag^+] = 1.6 \times 10^{-7}$
$\therefore$ x = 7

18. The number of hexagonal faces that are present in a truncated octahedron is

Sol. **(8)**

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

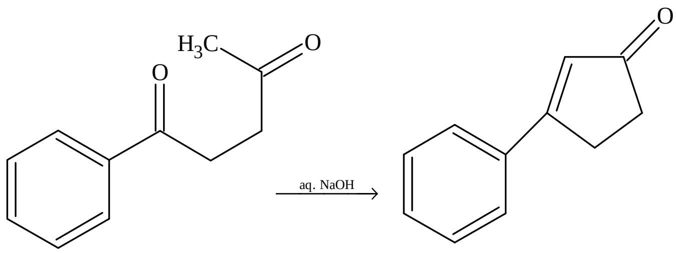
19. Match the transformations in **column I** with appropriate options in **column II**

	Column I		**Column II**
(A)	$CO_2(s) \rightarrow CO_2(g)$	(p)	phase transition
(B)	$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$	(q)	allotropic change
(C)	$2H\bullet \rightarrow H_2(g)$	(r)	ΔH is positive
(D)	$P_{(white,\ solid)} \rightarrow P_{(red,\ solid)}$	(s)	ΔS is positive
		(t)	ΔS is negative

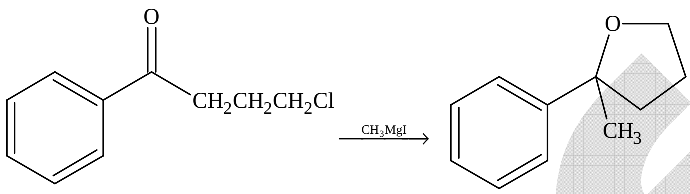
Sol. **(A) → (p, r, s)** **(B) → (r, s)** **(C) → (t)** **(D) → (p, q, t)**

20. Match the reactions in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**

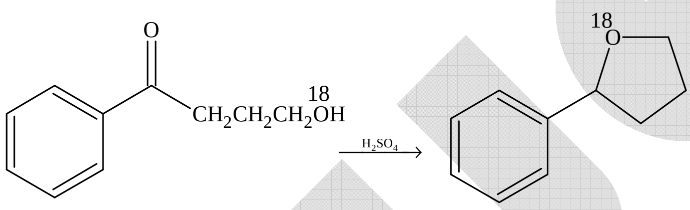
Column I **Column II**

(A) 

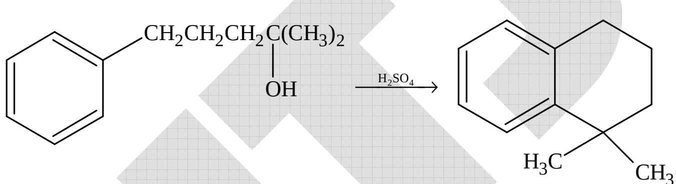
(p) Nucleophilic substitution

(B) 

(q) Electrophilic substitution

(C) 

(r) Dehydration

(D) 

(s) Nucleophilic addition

(t) Carbanion

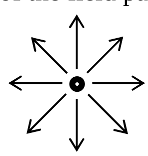
Sol. **(A) → (r, s, t)** **(B) → (p, s)** **(C) → (r, s)** **(D) → (q, r)**

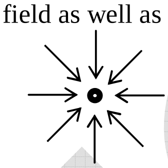
PART - II:

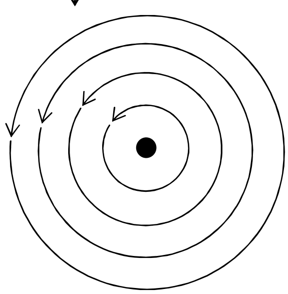
SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

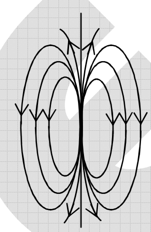
This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A)


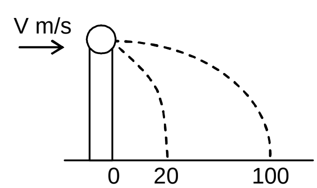
(B)


(C)


(D)


Sol. **(C)**

22. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is



(A) 250 m/s
(B) $250\sqrt{2}$ m/s
(C) 400 m/s
(D) 500 m/s

Sol. **(D)**

$5 = \frac{1}{2}(10)t^2 \Rightarrow t = 1$ sec

$v_{ball} = 20$ m/s

$v_{bullet} = 100$ m/s

$0.01\ V = 0.01 \times 100 + 0.2 \times 20$

$v = 100 + 400 = 500$ m/s

23. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2 %, the relative percentage error in the density is

(A) 0.9 %
(B) 2.4 %
(C) 3.1 %
(D) 4.2 %

Sol. **(C)**

diameter $= 2.5 + \frac{0.5}{50} \times 20$

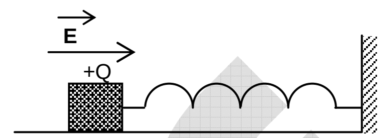
$= 2.70$ mm

$\%\ \text{error} = \left(\frac{dm}{m} + 3\frac{dr}{r}\right) \times 100$

$= 2 + 3 \times \frac{0.01}{2.70} \times 100$

$= 3.1\ \%$

24. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be

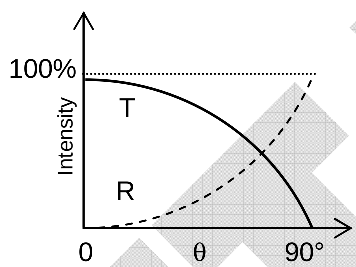


(A) of the same frequency and with shifted mean position.
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position.
(D) of changed frequency and with the same mean position.

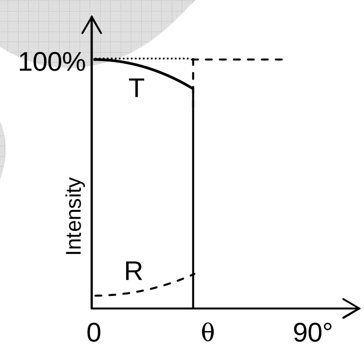
Sol. **(A)**

25. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

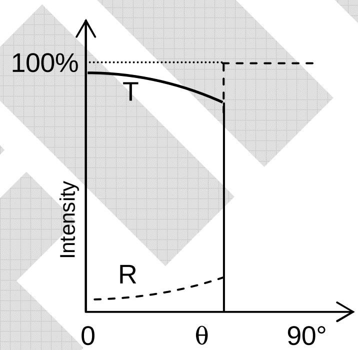
(A)



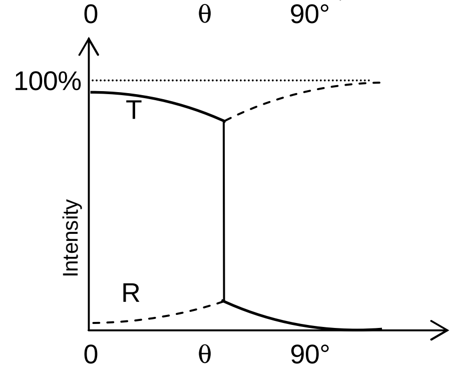
(B)



(C)



(D)



Sol. **(C)**
After total internal reflection, there is no refracted ray.

26. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

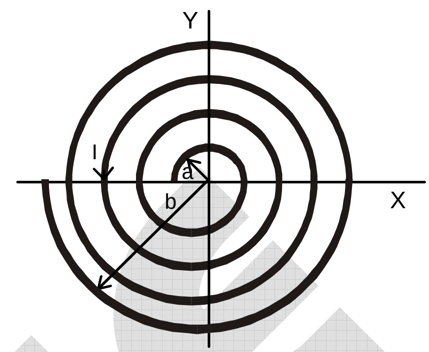
(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Sol. **(B)**

$$\frac{mV^2}{r} = \frac{GMm}{r^2} \quad \therefore mV^2 = \frac{GMm}{r}$$

27. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the XY plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the centre of the spiral is



(A) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b}{a}\right)$

(D) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b+a}{b-a}\right)$

Sol. **(A)**

$$\int_a^b \frac{\mu_0 IN}{2r(b-a)} dr$$

$$= \frac{\mu_0 IN}{2(b-a)} \int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

28. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

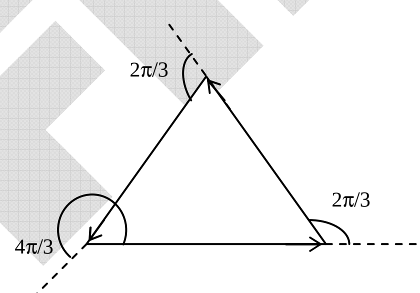
(A) $\sqrt{2}$ A, $\frac{3\pi}{4}$

(B) A, $\frac{4\pi}{3}$

(C) $\sqrt{3}$ A, $\frac{5\pi}{6}$

(D) A, $\frac{\pi}{3}$

Sol. **(B)**

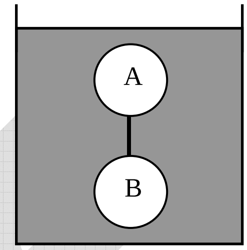


Nitin M Sir (physics-iitjee.blogspot.com)

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

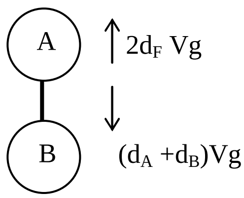
This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

29. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

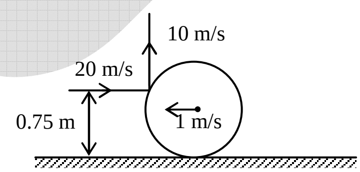


(A) $d_A < d_F$
(B) $d_B > d_F$
(C) $d_A > d_F$
(D) $d_A + d_B = 2d_F$

Sol. **(A, B, D)**



30. A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM.
(B) the ring comes to a complete stop.
(C) friction between the ring and the ground is to the left.
(D) there is no friction between the ring and the ground.

Sol. **(C)**
During collision friction is impulsive and immediately after collision the ring will have a clockwise angular velocity hence friction will be towards left.

31. Which of the following statement(s) is/are correct?
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
(B) The Gauss law can be used to calculate the field distribution around an electric dipole
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

Sol. **(C) or (C, D*)**
(D) is correct if we assume it is work done against electrostatic force

32. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?
(A) $I_R^A > I_R^B$
(B) $I_R^A < I_R^B$
(B) $V_C^A > V_C^B$
(D) $V_C^A < V_C^B$

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. **(B, C)**

$$I = \frac{V}{Z}$$

$$V^2 = V_R^2 + V_C^2 = (IR)^2 + \left(\frac{I}{\omega C}\right)^2$$

$$z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$$

SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

33. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Sol. **(4)**

$$R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

RC = 4 ms

34. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of 'Z' is

Sol. **(7)**

Stopping potential = $\frac{hc}{\lambda} - W$

= 6.2 eV – 4.7 eV
= 1.5 eV

$$V = \frac{Kq}{r} = 1.5$$

$$n = \frac{1.5\times10^{-2}}{9\times10^{9}\times1.6\times10^{-19}} = 1.05\times10^{7}$$

Z = 7

35. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Sol. **(5)**

$$0 = 10\frac{\sqrt{3}}{2}t - \frac{1}{2}10t^2 \qquad \ldots\text{(i)}$$

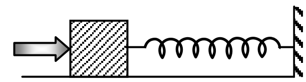
$t = \sqrt{3}$ sec

$$\Rightarrow 1.15 = 5\sqrt{3} - \frac{3}{2}a \qquad \ldots\text{(ii)}$$

$\Rightarrow a \approx 5$ m/s^2

Nitin M Sir (physics-iitjee.blogspot.com)

36. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



Sol. **(4)**

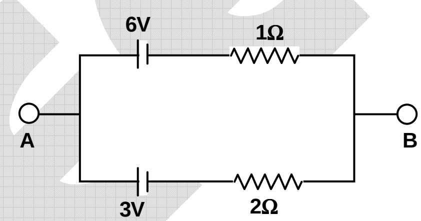
Applying work energy theorem

$$-\frac{1}{2}kx^2 - \mu mgx = -\frac{1}{2}mV^2$$

$$\Rightarrow V = \frac{4}{10}$$

N = 4

37. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



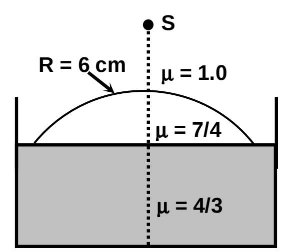
Sol. **(5)**

$$i = \frac{3}{3} = 1 \text{ ampere}$$

$$V_A - 6 + 1 - V_B = 0$$

$$V_A - V_B = 5$$

38. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Sol. **(2)**

$$\frac{7}{4V_1} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6} \Rightarrow V_1 = 21 \text{ cm}$$

$$\frac{4/3}{V_2} - \frac{7/4}{21} = 0$$

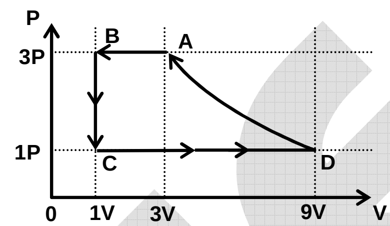
$V_2 = 16$ cm

x = 18 – 16 = 2 cm

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I		Column II	
(A)	Process A $\rightarrow$ B	(p)	Internal energy decreases
(B)	Process B $\rightarrow$ C	(q)	Internal energy increases.
(C)	Process C $\rightarrow$ D	(r)	Heat is lost
(D)	Process D $\rightarrow$ A	(s)	Heat is gained
		(t)	Work is done on the gas

Sol. **(A) $\rightarrow$ (p, r, t) (B) $\rightarrow$ (p, r) (C) $\rightarrow$ (q, s) (D) (r, t)**

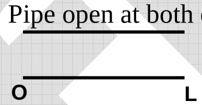
Process A $\rightarrow$ B $\rightarrow$ Isobaric compression
Process B $\rightarrow$ C $\rightarrow$ Isochoric process
Process C $\rightarrow$ D $\rightarrow$ Isobaric expansion
Process D $\rightarrow$ A $\rightarrow$ Polytropic with $T_A = T_D$

40. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

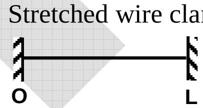
Column I

(A) Pipe closed at one end

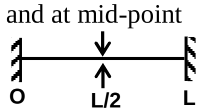
(B) Pipe open at both ends



(C) Stretched wire clamped at both ends



(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves
(q) Transverse waves
(r) $\lambda_f = L$
(s) $\lambda_f = 2L$
(t) $\lambda_f = 4L$

Sol. **(A) $\rightarrow$ (p, t) (B) $\rightarrow$ (p, s) (C) $\rightarrow$ (q, s) (D) (q, r)**

Nitin M Sir (physics-iitjee.blogspot.com)

PART - III:

SECTION – I (Total Marks : 24)
(Single Correct Answer Type)

This Section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. If $\lim_{x\to 0}\left[1+x\ln\left(1+b^2\right)\right]^{1/x} = 2b\sin^2\theta$, b > 0 and $\theta \in (-\pi, \pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Sol. **(D)**

$e^{\ln\left(1+b^2\right)} = 2b\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1+b^2}{2b}$

$\Rightarrow \sin^2\theta = 1$ as $\frac{1+b^2}{2b} \ge 1$

$\theta = \pm\pi/2$.

42. Let f : [–1, 2] → [0, ∞) be a continuous function such that f(x) = f(1 – x) for all x ∈[–1, 2]. Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by y = f(x), x = –1, x = 2, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$
(C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Sol. **(C)**

$R_1 = \int_{-1}^{2} xf(x)dx = \int_{-1}^{2}(2-1-x)f(2-1-x)dx$

$= \int_{-1}^{2}(1-x)f(1-x)dx = \int_{-1}^{2}(1-x)f(x)dx$

Hence $2R_1 = \int_{-1}^{2} f(x)dx = R_2$.

43. Let $f(x) = x^2$ and g(x) = sinx for all $x \in \mathbb{R}$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = f(g(x)), is

(A) $\pm\sqrt{n\pi},\ n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi},\ n \in \{1, 2, ...\}$

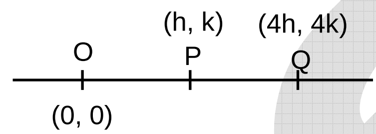
(C) $\frac{\pi}{2}+2n\pi,\ n \in \{...,-2,-1,0,1,2,....\}$ (D) $2n\pi,\ n \in \{...,-2,-1,0,1,2,....\}$

Sol. **(A)**
(fogogof) (x) = $\sin^2(\sin x^2)$
(gogof) (x) = $\sin(\sin x^2)$
$\therefore \sin^2(\sin x^2) = \sin(\sin x^2)$
$\Rightarrow \sin(\sin x^2)\,[\sin(\sin x^2) - 1] = 0$

$\Rightarrow \sin(\sin x^2) = 0$ or 1
$\Rightarrow \sin x^2 = n\pi$ or $2m\pi + \pi/2$, where m, $n \in I$
$\Rightarrow \sin x^2 = 0$
$\Rightarrow x^2 = n\pi \Rightarrow x = \pm\sqrt{n\pi}$, $n \in \{0, 1, 2, \ldots\}$.

44. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is
(A) $x^2 = y$ (B) $y^2 = 2x$
(C) $y^2 = x$ (D) $x^2 = 2y$

Sol. **(C)**



$y^2 = 4x$
and Q will lie on it
$\Rightarrow (4k)^2 = 4 \times 4h$
$\Rightarrow k^2 = h$
$\Rightarrow y^2 = x$ (replacing h by x and k by y)

45. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is
(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$
(C) $\sqrt{2}$ (D) $\sqrt{3}$

Sol. **(B)**
Equation of normal is $(y - 3) = \frac{-a^2}{2b^2}(x - 6) \Rightarrow \frac{a^2}{2b^2} = 1 \Rightarrow e = \sqrt{\frac{3}{2}}$.

46. A value of b for which the equations
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$,
have one root in common is
(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$
(C) $i\sqrt{5}$ (D) $\sqrt{2}$

Sol. **(B)**
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$ … (1)
Common root is
$(b - 1)x - 1 - b = 0$
$\Rightarrow x = \frac{b+1}{b-1}$
This value of x satisfies equation (1)
$\Rightarrow \frac{(b+1)^2}{(b-1)^2} + \frac{b+1}{b-1} + b = 0 \Rightarrow b = \sqrt{3}i,\ -\sqrt{3}i,\ 0$.

47. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6
(C) 4 (D) 8

Sol. **(A)**
For being non-singular

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$\Rightarrow ac\omega^2 - (a + c)\omega + 1 \neq 0$
Hence number of possible triplets of (a, b, c) is 2.
i.e. $(\omega, \omega^2, \omega)$ and (ω, ω, ω).

48. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$
(C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

Sol. **(D)**
Circle touching y-axis at (0, 2) is $(x - 0)^2 + (y - 2)^2 + \lambda x = 0$
passes through (– 1, 0)
$\therefore 1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$
$\therefore x^2 + y^2 + 5x - 4y + 4 = 0$
Put $y = 0 \Rightarrow x = -1, -4$
$\therefore$ Circle passes through (– 4, 0)

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

49. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$, then

(A) f(x) is continuous at x = – π/2 (B) f(x) is not differentiable at x = 0
(C) f(x) is differentiable at x = 1 (D) f(x) is differentiable at x = –3/2

Sol. **(A, B, C, D)**

$$\lim_{x \to -\frac{\pi}{2}^-} f(x) = 0 = f(-\pi/2)$$

$$\lim_{x \to -\frac{\pi}{2}^+} f(x) = \cos\left(-\frac{\pi}{2}\right) = 0$$

Nitin M Sir (physics-iitjee.blogspot.com)

$$f'(x) = \begin{cases} -1, & x \le -\pi/2 \\ \sin x, & -\pi/2 < x \le 0 \\ 1, & 0 < x \le 1 \\ 1/x, & x > 1 \end{cases}$$

Clearly, f (x) is not differentiable at x = 0 as $f'(0^-) = 0$ and $f'(0^+) = 1$.
f (x) is differentiable at x = 1 as $f'(1^-) = f'(1^+) = 1$.

50. Let $f : (0, 1) \to \mathbb{R}$ be defined by $f(x) = \frac{b-x}{1-bx}$, where be is a constant such that 0 < b < 1. Then

(A) f is not invertible on (0, 1)
(B) $f \ne f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(C) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(D) f^{-1} is differentiable on (0, 1)

Sol. **(A)**

$f(x) = \frac{b-x}{1-bx}$

Let $y = \frac{b-x}{1-bx} \Rightarrow x = \frac{b-y}{1-by}$

$0 < x < 1 \Rightarrow 0 < \frac{b-y}{1-by} < 1$

$\frac{b-y}{1-by} > 0 \Rightarrow y < b \text{ or } y > \frac{1}{b}$

$\frac{b-y}{1-by} - 1 < 0 \Rightarrow -1 < y < \frac{1}{b}$

$\therefore -1 < y < b$.

51. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$
(B) $y + 3x - 33 = 0$
(C) $y + x - 15 = 0$
(D) $y - 2x + 12 = 0$

Sol. **(A, B, D)**

$y^2 = 4x$
Equation of normal is $y = mx - 2m - m^3$.
It passes through (9, 6)
$\Rightarrow m^3 - 7m + 6 = 0$
$\Rightarrow m = 1, 2, -3$
$\Rightarrow y - x + 3 = 0,\ y + 3x - 33 = 0,\ y - 2x + 12 = 0$.

52. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$
(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$
(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$
(D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Sol. **(A, D)**

Let P (E) = e and P (F) = f

$P(E \cup F) - P(E \cap F) = \frac{11}{25}$

$\Rightarrow e + f - 2ef = \frac{11}{25}$ … (1)

$P(\bar{E} \cap \bar{F}) = \frac{2}{25}$

$\Rightarrow (1 - e)(1 - f) = \frac{2}{25}$

$\Rightarrow 1 - e - f + ef = \frac{2}{25}$ … (2)

From (1) and (2)

$ef = \frac{12}{25}$ and $e + f = \frac{7}{5}$

Solving, we get

$e = \frac{4}{5},\ f = \frac{3}{5}$ or $e = \frac{3}{5},\ f = \frac{4}{5}$.

SECTION-III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the **ORS**.

53. Let $\vec{a} = -\hat{i} - \hat{k},\ \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is

Sol. **(9)**

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

taking cross with a

$\vec{a} \times (\vec{r} \times \vec{b}) = \vec{a} \times (\vec{c} \times \vec{b})$

$(\vec{a} \cdot \vec{b})\vec{r} - (\vec{a} \cdot \vec{r})\vec{b} = \vec{a} \times (\vec{c} \times \vec{b})$

$\Rightarrow \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{r} \cdot \vec{b} = 3 + 6 = 9$.

54. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

Sol. **(2)**

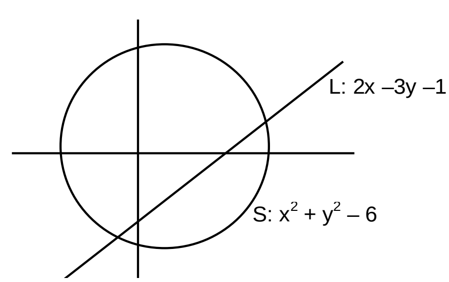
L: 2x – 3y – 1

S: $x^2 + y^2 - 6$

If $L_1 > 0$ and $S_1 < 0$

Then point lies in the smaller part.

$\therefore \left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ lie inside.



55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Sol. **(3)**

The expression may not attain integral value for all a, b, c

If we consider a = b = c, then

$x = 3a$

$y = a\,(1 + \omega + \omega^2) = a\left(1 + i\sqrt{3}\right)$

$z = a\,(1 + \omega^2 + \omega) = a\left(1 + i\sqrt{3}\right)$

$\therefore |x|^2 + |y|^2 + |z|^2 = 9\,|a|^2 + 4\,|a|^2 + 4\,|a|^2 = 17\,|a|^2$

$\therefore \frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{17}{3}$

Note: However if $\omega = e^{i(2\pi/3)}$, then the value of the expression = 3.

56. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Sol. **(2)**

Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1 = 0$

$f'(x) = 4x^3 - 12x^2 + 24x + 1 = 4\,(x^3 - 3x^2 + 6x) + 1$

$f''(x) = 12x^2 - 24x + 24 = 12\,(x^2 - 2x + 2)$

$f''(x)$ has 0 real roots

f (x) has maximum 2 distinct real roots as f (0) = –1.

57. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on $\mathbb{R}$ with $g(0) = g(2) = 0$. Then the value of y(2) is

Sol. **(0)**

$y'(x) + y(x)\,g'(x) = g(x)\,g'(x)$

$\Rightarrow e^{g(x)}\,y'(x) + e^{g(x)}\,g'(x)\,y(x) = e^{g(x)}\,g(x)\,g'(x)$

$\Rightarrow \frac{d}{dx}\left(y(x)e^{g(x)}\right) = e^{g(x)}\,g(x)\,g'(x)$

$\therefore y(x) = e^{g(x)} = \int e^{g(x)} g(x) g'(x)\,dx$

$= \int e^t\,dt$, where $g(x) = t$

$= (t - 1)\,e^t + c$

$\therefore y(x)\,e^{g(x)} = (g(x) - 1)\,e^{g(x)} + c$

Put $x = 0 \Rightarrow 0 = (0 - 1)\,.\,1 + c \Rightarrow c = 1$

Put $x = 2 \Rightarrow y(2)\,.\,1 = (0 - 1)\,.\,(1) + 1$

$y(2) = 0.$

58. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix},\ \text{and } M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is

Sol. **(9)**

Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow b = -1,\ e = 2, \quad h = 3$

$M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a = 0, \quad d = 3, \quad g = 2$

$M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow g + h + i = 12 \Rightarrow i = 7$

$\therefore$ Sum of diagonal elements = 9.

SECTION-IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in Column I with the values given in Column II

	Column – I		Column – II
(A)	If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)\,dx$ is	(r)	$\frac{\pi}{3}$
(D)	The maximum value of $\left\lvert \operatorname{Arg}\left(\frac{1}{1-z}\right) \right\rvert$ for $|z| = 1, z \neq 1$ is given by	(s)	π
		(t)	$\frac{\pi}{2}$

Sol. **(A) → (q)** **(B) → (p)** **(C) → (s)** **(D) → (t)**

(A). $\vec{a} \cdot \vec{b} = -1 + 3 = 2$

$|\vec{a}| = 2,\ |\vec{b}| = 2$

$\cos\theta = \frac{2}{2 \times 2} = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ but its $\frac{2\pi}{3}$ as its opposite to side of maximum length.

(B). $\int_a^b (f(x) - 3x)dx = a^2 - b^2$

$\int_a^b f(x)dx = \frac{3}{2}(b^2 - a^2) + a^2 - b^2 = \frac{-a^2 + b^2}{2}$

$\Rightarrow f(x) = x.$

(C). $\frac{\pi^2}{\ln 3}\left(\frac{\ln|(\sec \pi x + \tan \pi x)|_{7/6}^{5/6}}{\pi}\right) = \frac{\pi}{\ln 3}\left(\ln\left|\sec\frac{5\pi}{6} + \tan\frac{5\pi}{6}\right| - \ln\left|\sec\frac{7\pi}{6} + \tan\frac{7\pi}{6}\right|\right) = \pi.$

(D). Let $u = \frac{1}{1-z} \Rightarrow z = 1 - \frac{1}{u}$

$|z| = 1 \Rightarrow \left|1 - \frac{1}{u}\right| = 1$

$\Rightarrow |u - 1| = |u|$

$\therefore$ locus of u is perpendicular bisector of line segment joining 0 and 1

$\Rightarrow$ maximum arg u approaches $\frac{\pi}{2}$ but will not attain.

60. Match the statements given in Column I with the intervals/union of intervals given in Column II

	Column – I		Column – II
(A)	The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, |z| = 1, z \neq \pm 1\right\}$ is	(p)	$(-\infty, -1) \cup (1, \infty)$
(B)	The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty, 0) \cup (0, \infty)$
(C)	If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	(r)	$[2, \infty)$
(D)	If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then f(x) is increasing in	(s)	$(-\infty, -1] \cup [1, \infty)$
		(t)	$(-\infty, 0] \cup [2, \infty)$

Sol. **(A) → (s)** **(B) → (t)** **(C) → (r)** **(D) → (r)**

(A). $z = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}$

Using $1 - x^2 = y^2$

$Z = \frac{2ix - 2y}{2y^2 - 2ixy} = -\frac{1}{y}.$

$\because\ -1 \le y \le 1 \Rightarrow -\frac{1}{y} \le -1$ or $-\frac{1}{y} \ge 1$.

(B). For domain

$-1 \le \frac{8\cdot 3^{x-2}}{1-3^{2(x-1)}} \le 1$

$\Rightarrow -1 \le \frac{3^x - 3^{x-2}}{1-3^{2x-2}} \le 1$.

Case –I: $\frac{3^x - 3^{x-2}}{1-3^{2x-2}} - 1 \le 0$

$\Rightarrow \frac{(3^x-1)(3^{x-2}-1)}{(3^{2x-2}-1)} \ge 0$

$\Rightarrow x \in (-\infty, 0] \cup (1, \infty)$.

Case –II: $\frac{3^x - 3^{x-2}}{1-3^{2x}-2} + 1 \ge 0$

$\Rightarrow \frac{(3^{x-2}-1)(3^x+1)}{(3^x \cdot 3^{x-2}-1)} \ge 0$

$\Rightarrow x \in (-\infty, 1) \cup [2, \infty)$

So, $x \in (-\infty, 0] \cup [2, \infty)$.

(C). $R_1 \to R_1 + R_3$

$f(\theta) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$= 2(\tan^2\theta + 1) = 2\sec^2\theta$.

(D). $f'(x) = \frac{3}{2}(x)^{1/2}(3x-10) + (x)^{3/2} \times 3 = \frac{15}{2}(x)^{1/2}(x-2)$

Increasing, when $x \ge 2$.

FIITJEE Solutions to IIT-JEE-2012

CODE 8

PAPER 1

Time: 3 Hours **Maximum Marks: 210**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This booklet is your Question paper. Do not break the seats of his booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4 Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator.
6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains 28 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS:

9. The ORS also has a **CODES** printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET.** Sign at the place provided on the ORS affirming that you have verified that all the code are same.
11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble **UNDER** each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS(R_4).

C. Question paper format and Marking scheme:

The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. **Section I** contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
13. **Section II** contains **5 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE are correct**.
14. **Section III** contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

D. Marking Scheme

15. For each question in **Section I**, you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero marks** if no bubbles are darkened. In all other cases, **minus one (–1)** mark will be awarded in this section.
16. For each question in **Section II**, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.
17. For each question in **Section III**, you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

Write your Name, Registration Number and sign in the space provided on the back page of this booklet.

PAPER-1 [Code – 8]
IITJEE 2012
PART - I: PHYSICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement
(A) due to the errors in the measurements of d and ℓ are the same.
(B) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.
(C) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.
(D) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

Sol. **(A)**

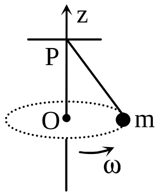
$$\text{L.C.} = \frac{0.5}{100} = 0.005\,\text{mm}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta(d)}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\times10^{-3}}{0.25\times10^{-3}} = \frac{1}{50}$$

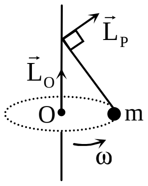
$$2\frac{\Delta(d)}{d} = \frac{2\times0.005\times10^{-3}}{0.5\times10^{-3}} = \frac{1}{50}$$

2. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then

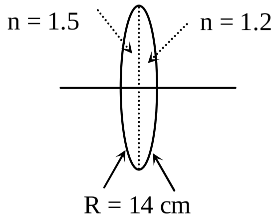


(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.
(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.
(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

Sol. **(C)**



3. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Sol. **(B)**

$$P_T = (1.5-1)\left(\frac{1}{14}-0\right)+(1.2-1)\left(0-\frac{1}{-14}\right) = \frac{0.5}{14}+\frac{0.2}{14}=\frac{1}{20}$$

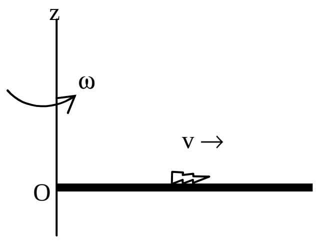
f = +20 cm

$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

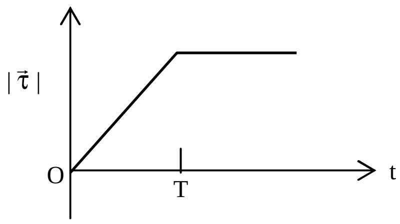
$$\frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}$$

$\therefore$ v = 40 cm

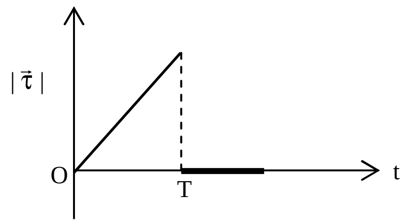
4. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) about O, as a function of time is best represented by which plot?



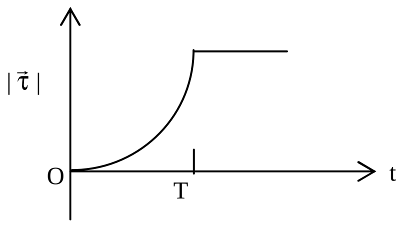
(A)



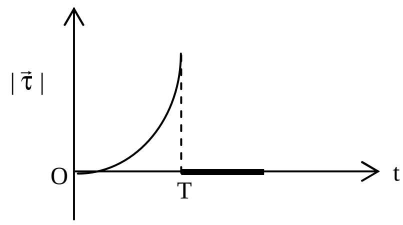
(B)



(C)



(D)



Sol. **(B)**

$$\tau = \omega\frac{dI}{dt}=\omega\frac{d}{dt}(C+mv^2t^2)$$

$= m\omega v^2\, 2t.$

5. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Sol. **(D)**

$v_{rms} = \sqrt{\frac{3RT}{M}}$

Required ratio = $\sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10}$

= 3.16.

6. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45^0 to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Sol. **(C)**

qE = mg

q(V/d) = mg

$V = \frac{mgd}{q}$

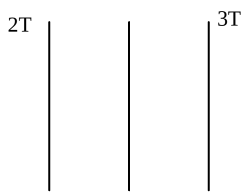
$= \frac{1.67\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}}$

$= \frac{10^{-28}}{10^{-19}} = 10^{-9}\,V$

7. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{1/4}T$ (B) $\left(\frac{97}{4}\right)^{1/4}T$ (C) $\left(\frac{97}{2}\right)^{1/4}T$ (D) $(97)^{1/4}T$

Sol. **(C)**



$\sigma A\,(2T)^4 + \sigma A\,(3T)^4 = \sigma\,2A(T')^4$

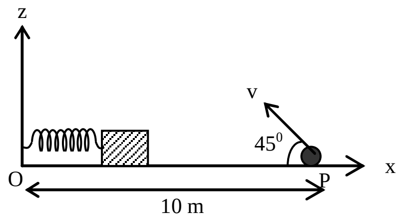
$16T^4 + 81T^4 = 2\,(T')^4$

$97\,T^4 = 2\,(T')^4$

$(T')^4 = \frac{97}{2}\,T^4$

$\therefore\ T' = \left(\frac{97}{2}\right)^{1/4}T$

8. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v form point P at an angle of 45^0 as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take $g = 10\ m/s^2$)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Sol. **(A)**

$$\frac{2v\sin 45^0}{g} = 1$$

$\therefore v = \sqrt{50}$ m/s.

9. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

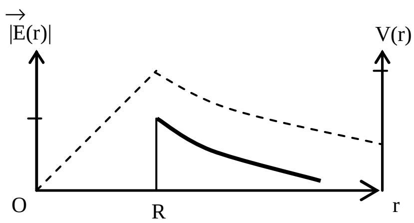
Sol. **(D)**

$\lambda_R > \lambda_G > \lambda_B$

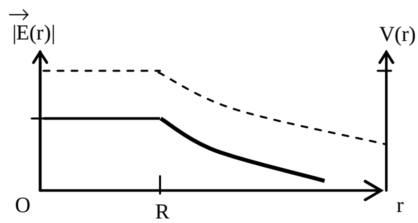
$\therefore \beta_R > \beta_G > \beta_B$

10. Consider a thin spherical shell of radius R with centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

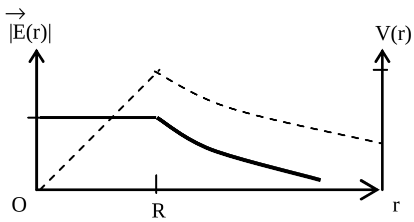
(A)



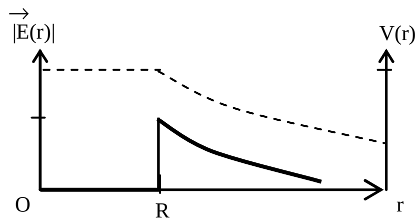
(B)



(C)



(D)



Sol. **(D)**

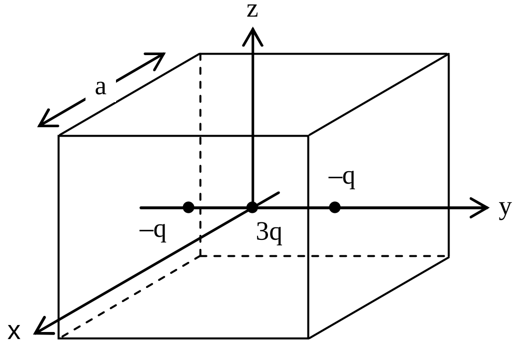
SECTION II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

11. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time t > 0?
(A) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(B) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.
(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

Sol. **(C, D)**
If $\theta = 90°, \vec{B}$ exerts no force on q.
If $\theta = 0°, 10°$; the charge particle moves in helix with increasing pitch due to $\vec{E}$ along y-axis.

12. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), +3q at (0, 0, 0) and –q at (0, +a/4, 0). Choose the correct options(s)



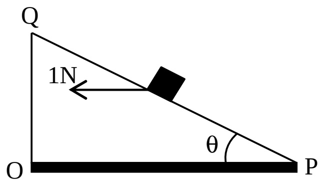
(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = –a/2
(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2.
(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.
(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

Sol. **(A, C, D)**
Net flux through the cubical region $= \frac{-q+3q-q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$
The flux passing through the faces $x = \frac{-a}{2}$, $x = +\frac{a}{2}$ and $z = +\frac{a}{2}$ are same due to symmetry.

13. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,
(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(B) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(C) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

Sol. **(B, D)**
At the open end, the phase of a pressure wave changes by π radian due to reflection. At the closed end, there is no change in the phase of a pressure wave due to reflection.

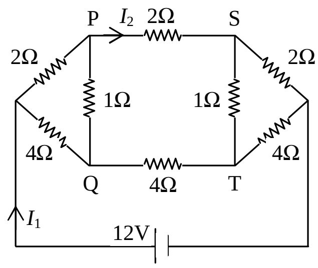
14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2)



(A) θ = 45°
(B) θ > 45° and a frictional force acts on the block towards P.
(C) θ > 45° and a frictional force acts on the block towards Q.
(D) θ < 45° and a frictional force acts on the block towards Q.

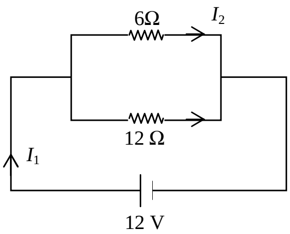
Sol. **(A, C)**
At $\theta = 45°$, mg sin $\theta = 1\times\cos\theta$
At $\theta > 45°$, mg sin $\theta > 1\times\cos\theta$ (friction acts upward)
At $\theta < 45°$, mg sin $\theta < 1\times\cos\theta$ (friction acts downward)

15. For the resistance network shown in the figure, choose the correct option(s)



(A) The current through PQ is zero.
(B) I_1 = 3A
(C) The potential at S is less than that at Q.
(D) I_2 = 2A

Sol. **(A, B, C, D)**
Nodes P and Q are equipotential and nodes S and T are equipotential from wheatstone bridge, no current passes through PQ and ST.



$$I_1 = \frac{12}{4} = 3A$$

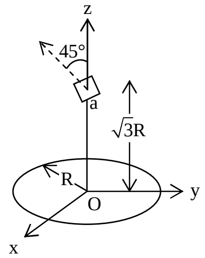
$$I_2 = I_1\left(\frac{12}{6+12}\right) = 2A$$

Nitin M Sir (physics-iitjee.blogspot.com)

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

16. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a(a<<R) having two turns is placed with its centre at z = $\sqrt{3}$R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is

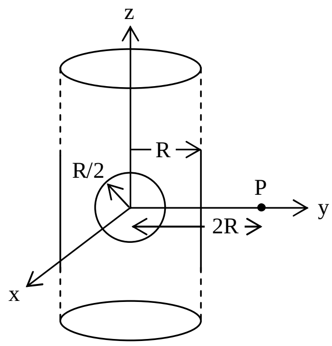


Sol. **(7)**

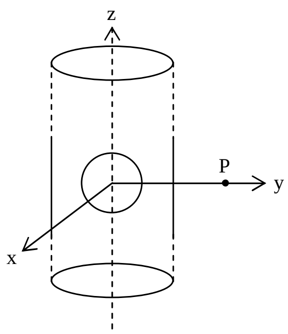
$$M = \frac{N\phi}{I} = \frac{2\left[\frac{\mu_0 IR^2}{2(8R^3)}\right]a^2 \cos 45°}{I} \frac{\mu_0 a^2}{8R2^{1/2}} = \frac{\mu_0 a^2}{R2^{7/2}}$$

So P = 7

17. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Sol. **(6)**



$$\vec{E} = \frac{\lambda(\hat{j})}{2\pi\varepsilon_0(2R)} + \frac{K\left(\rho\frac{4}{3}\pi\frac{R^3}{8}\right)(-\hat{j})}{4R^2}$$

$$\vec{E} = \frac{\rho\pi R^2(\hat{j})}{4\pi\varepsilon_0 R} + \frac{K\pi\rho R(-\hat{j})}{24}$$

$$\vec{E} = K\rho\pi R(\hat{j}) + \frac{K}{24}\rho\pi R(-\hat{j})$$

$$\vec{E} = K\rho\pi R\frac{23}{24}(\hat{j}) = \frac{23}{96\varepsilon_0}\rho R(\hat{j})$$

$\Rightarrow$ k = 6

Nitin M Sir (physics-iitjee.blogspot.com)

18. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, m_p = (5/3) × 10^{-27} kg; h/e = 4.2 × 10^{-15} J.s / C; $\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$ m/F; 1 fm = 10^{-15})

Sol. **(7)**

$$0+\frac{1}{2}mv^2 = \frac{K(Q)e}{10\times10^{-15}} = \frac{K(120e)e}{10\times10^{-15}}$$

$$\frac{1}{2}\times\frac{5}{3}\times10^{-27}v^2 = \frac{9\times10^{9}\times120\times(1.6\times10^{-19})^2}{10\times10^{-15}}$$

$$v = \frac{9\times6\times10^{9}\times120\times2.56\times10^{-38}}{50\times10^{-42}}$$

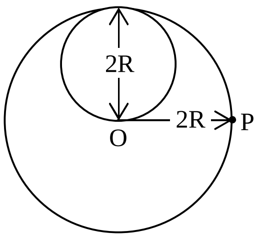
$$v = \sqrt{331.776\times10^{13}}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{4.2\times10^{-15}\times1.6\times10^{-19}}{\frac{5}{3}\times10^{-27}\times\sqrt{331.776\times10^{13}}} = \frac{4.2\times4.8\times10^{-34}}{57.6\times5\times10^{-21}} = 0.07\times10^{-13}$$

$$\lambda = 7\times10^{-15} = 7\text{fm}$$

19. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P / I_O to the nearest integer is



Sol. **(3)**

$$I_p = \left[\frac{4mR^2}{2}+m\left(4R^2\right)\right]-\left[\frac{m}{4}\frac{R^2}{2}+\frac{m}{4}5R^2\right]$$

$$I_p = mR^2\left[(2+4)-\left(\frac{1}{8}+\frac{5}{4}\right)\right]$$

$$I_p = mR^2\left(6-\frac{11}{8}\right) = \frac{37}{8}mR^2 \qquad ...(1)$$

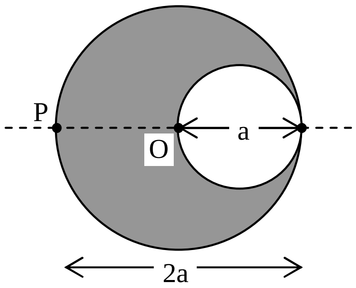
$$I_0 = \left(\frac{4mR^2}{2}\right)-\left(\frac{m}{4}\frac{R^2}{2}+\frac{m}{4}R^2\right)$$

$$I_0 = mR^2\left[2-\left(\frac{1}{8}+\frac{1}{4}\right)\right] = mR^2\left[2-\frac{3}{8}\right] = mR^2\left(\frac{13}{8}\right) \qquad ...(2)$$

So $\frac{I_p}{I_0} = \frac{37/8}{13/8} \approx 3$ (Nearest Integer)

Nitin M Sir (physics-iitjee.blogspot.com)

20. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinity long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Sol. **(5)**

$$B = \frac{\mu_0(J\pi a^2)}{2\pi a} - \frac{\mu_0(J\pi a^2/4)}{2\pi\left(\frac{3a}{2}\right)}$$

$$B = \frac{5\mu_0 Ja}{12} = \frac{\mu_0 NJa}{12}$$

So N = 5

PAPER-1 [Code – 8]

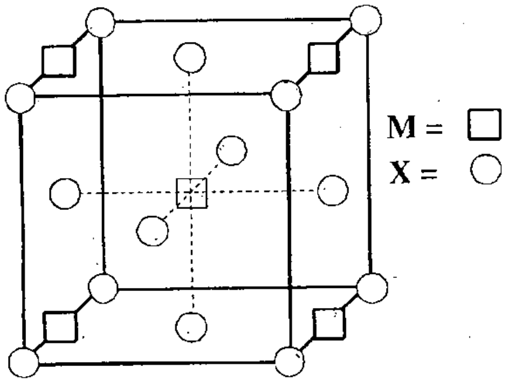
IITJEE 2012

PART - II: CHEMISTRY

SECTION – I: Single Correct Answer Type

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

21. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



(A) MX
(B) MX_2
(C) M_2X
(D) M_5X_{14}

Sol. **(B)**

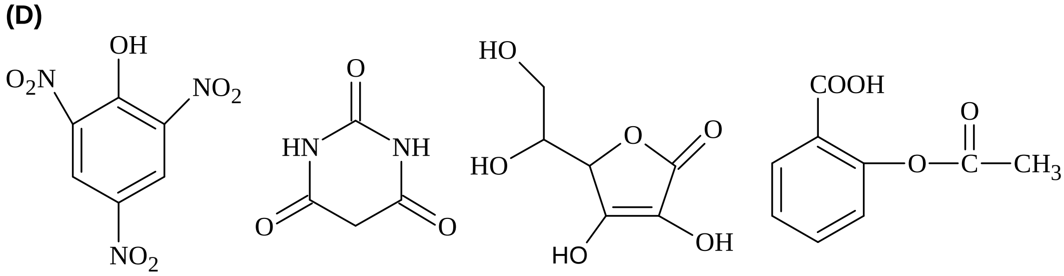
$$X = 8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

$$M = 4\times\frac{1}{4}+1=2$$

So, unit cell formula of the compound is M_2X_4 and the empirical formula of the compound is MX_2.

22. The carboxyl functional group (–COOH) is present in
(A) picric acid
(B) barbituric acid
(C) ascorbic acid
(D) aspirin

Sol. **(D)**



(Picric Acid) (Barbituric Acid) (Ascorbic Acid) (Asprin)

23. As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is
(A) Tetraaquadiaminecobalt (III) chloride
(B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacobalt (III) chloride
(D) Diamminetetraaquacobalt (III) chloride

Sol. **(D)**

$[Co(H_2O)_4(NH_3)_2]Cl_3$
Dia**mm**inetetraaquacobalt (III) chloride

24. In allene (C_3H_4), the type(s) of hybridization of the carbon atoms is (are)
(A) *sp* and sp^3
(B) *sp* and sp^2
(C) only sp^2
(D) sp^2 and sp^3

Sol. **(B)**

$$\overset{sp^2}{H_2C} = \overset{sp}{C} = \overset{sp^2}{CH_2}$$

25. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]
(A) $\frac{h^2}{4\pi^2 m a_0^2}$
(B) $\frac{h^2}{16\pi^2 m a_0^2}$
(C) $\frac{h^2}{32\pi^2 m a_0^2}$
(D) $\frac{h^2}{64\pi^2 m a_0^2}$

Sol. **(C)**

As per Bohr's postulate,

$$mvr = \frac{nh}{2\pi}$$

So, $v = \frac{nh}{2\pi mr}$

$$KE = \frac{1}{2}mv^2$$

So, $KE = \frac{1}{2}m\left(\frac{nh}{2\pi mr}\right)^2$

Since, $r = \frac{a_o \times n^2}{z}$

So, for 2nd Bohr orbit

$$r = \frac{a_o \times 2^2}{1} = 4a_o$$

$$KE = \frac{1}{2}m\left(\frac{2^2 h^2}{4\pi^2 m^2 \times (4a_0)^2}\right)$$

$$KE = \frac{h^2}{32\pi^2 m a_0^2}$$

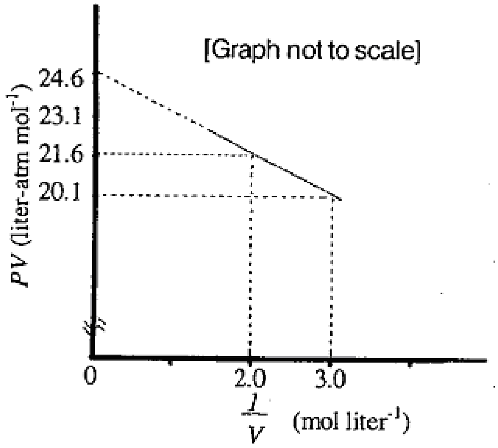
Nitin M Sir (physics-iitjee.blogspot.com)

26. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen?
(A) HNO_3, NO, NH_4Cl, N_2 (B) HNO_3, NO, N_2, NH_4Cl
(C) HNO_3, NH_4Cl, NO, N_2 (D) NO, HNO_3, NH_4Cl, N_2

Sol. **(B)**

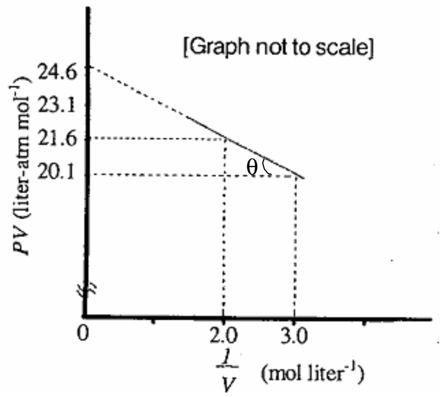
$H\overset{+5}{N}O_3,\ \overset{+2}{N}O,\ \overset{0}{N}_2,\ \overset{-3}{N}H_4Cl$

27. For one mole of a van der Waals gas when $b = 0$ and $T = 300$ K, the PV vs. $1/V$ plot is shown below. The value of the van der Waals constant a (atm.litre^{2}mol^{-2}) is



(A) 1.0 (B) 4.5
(C) 1.5 (D) 3.0

Sol. **(C)**



van der Waals equation for 1 mole of real gas is,

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

but, b = 0 (given)

$$\Rightarrow\left(P+\frac{a}{V^2}\right)(V)=RT$$

$$\therefore\quad PV=-a\times\frac{1}{V}+RT \qquad \text{.....(i)}$$

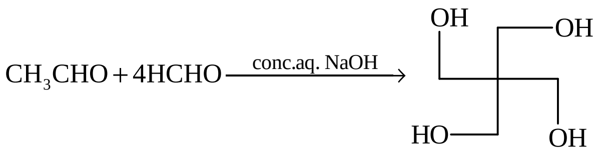
y = mx + c

Slope = tan (π – θ) = –a

So, $\tan\theta = a = \dfrac{21.6-20.1}{3-2} = 1.5$

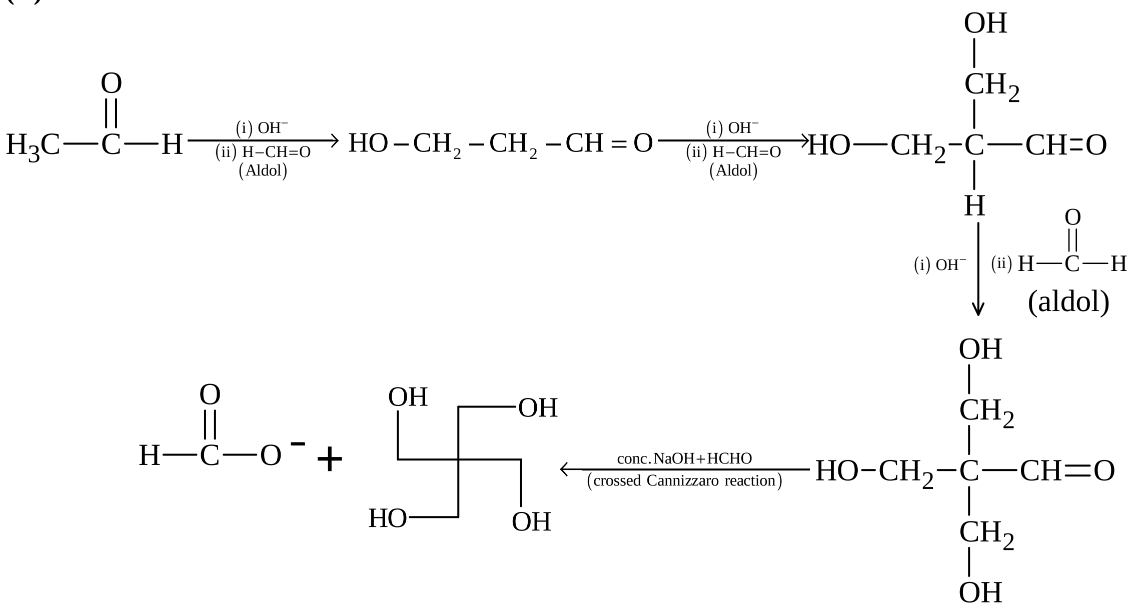
or, $\tan\theta=\dfrac{24.6-20.1}{3-0}=1.5$

28. The number of aldol reaction(s) that occurs in the given transformation is



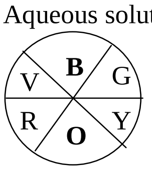
(A) 1
(B) 2
(C) 3
(D) 4

Sol. **(C)**

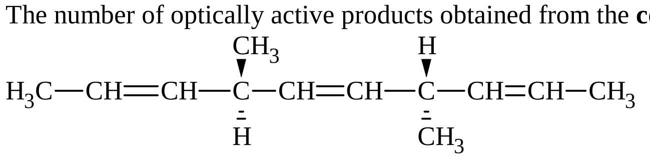


29. The colour of light absorbed by an aqueous solution of $CuSO_4$ is
(A) orange- red
(B) blue-green
(C) yellow
(D) violet

Sol. **(A)**
Aqueous solution of copper sulphate absorbs orange red light and appears blue (complementary colour).



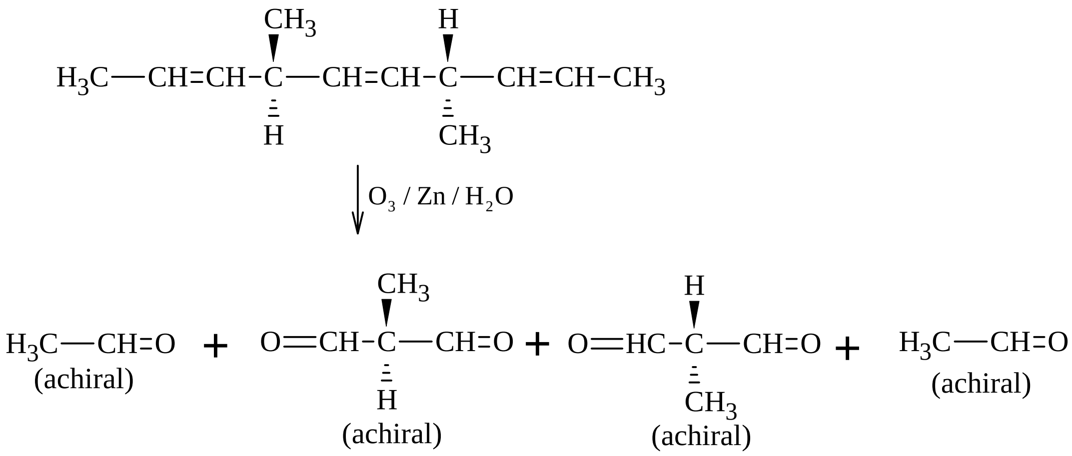
30. The number of optically active products obtained from the **complete** ozonolysis of the given compound is



(A) 0
(B) 1
(C) 2
(D) 4

Sol. **(A)**

Nitin M Sir (physics-iitjee.blogspot.com)



SECTION II : Multiple Correct Answer (s) Type

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

31. Which of the following hydrogen halides react(s) with $AgNO_3$(aq) to give a precipitate that dissolves in $Na_2S_2O_3$(aq)?

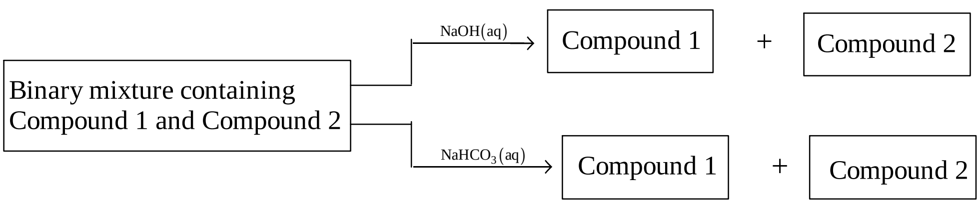
(A) HCl (B) HF
(C) HBr (D) HI

Sol. **(A, C, D)**

$$HX + AgNO_3 \rightarrow AgX \downarrow + HNO_3 \left(X = Cl, Br, I\right)$$

$$AgX + 2Na_2S_2O_3 \rightarrow Na_3\left[Ag\left(S_2O_3\right)_2\right] + NaX$$

32. Identify the binary mixture(s) that can be separated into individual compounds, by differential extraction, as shown in the given scheme.

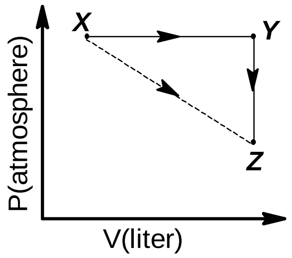


(A) C_6H_5OH and C_6H_5COOH (B) C_6H_5COOH and $C_6H_5CH_2OH$
(C) $C_6H_5CH_2OH$ and C_6H_5OH (D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Sol. **(B, D)**

(A) Both are soluble in NaOH, hence **inseparable**.
(B) Only benzoic acid (C_6H_5COOH) is soluble in NaOH and $NaHCO_3$, while benzyl alcohol ($C_6H_5CH_2OH$) is not. Hence, **separable.**
(C) Although NaOH can enable separation between benzyl alcohol ($C_6H_5CH_2OH$) and phenol (C_6H_5OH) as only the later is soluble in NaOH. However, in $NaHCO_3$, both are insoluble. Hence, **inseparable.**
(D) α-phenyl acetic acid ($C_6H_5CH_2COOH$) is soluble in NaOH and $NaHCO_3$. While benzyl alcohol ($C_6H_5CH_2OH$) is not. Hence, **separable.**

33. For an ideal gas, consider only *P-V* work in going from an initial state ***X*** to the final state ***Z.*** The final state ***Z*** can be reached by either of the two paths shown in the figure. Which of the following choice(s) is(are) correct? [Take ΔS as change in entropy and *w* as work done]



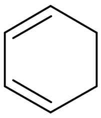
(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$
(B) $w_{x\to z} = w_{x\to y} + w_{y\to z}$
(C) $w_{x\to y\to z} = w_{x\to y}$
(D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

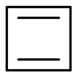
Sol. **(A, C)**

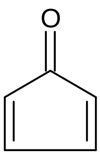
$\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$ [entropy (S) is a state function, hence additive]

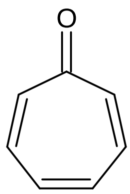
$w_{X\to Y\to Z} = w_{X\to Y}$ (work done in $Y \to Z$ is zero as it is an isochoric process)

34. Which of the following molecules, in pure form, is (are) **unstable** at room temperature?

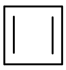
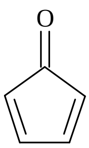
(A)


(B)


(C)


(D)


Sol. **(B, C)**

Compound  and  being antiaromatic are unstable at room temperature.

35. Choose the correct reason(s) for the stability of the **lyophobic** colloidal particles.
(A) Preferential adsorption of ions on their surface from the solution
(B) Preferential adsorption of solvent on their surface from the solution
(C) Attraction between different particles having opposite charges on their surface
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

Sol. **(A, D)**
Lyophobic colloids are stable due to preferential adsorption of ions on their surface from solution and potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles that makes lyophobic sol stable.

SECTION III: Integer Answer Type

This section contains **5 questions.** The answer to each question is a **single digit integer,** ranging from 0 to 9 (both inclusive).

36. 29.2 % (w/w) HCl stock solution has density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is

Sol. **(8)**

Stock solution of HCl = 29.2% (w/w)

Molarity of stock solution of HCl $= \dfrac{29.2\times1000\times1.25}{36.5\times100}$

If volume of stock solution required = V ml

$$V\times\frac{29.2}{36.5}\times\frac{1000}{80} = 200\times0.4$$

$\Rightarrow$ V = 8 ml

37. The substituents $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0?

$$\overset{\oplus}{H_3N}—\underset{H}{\underset{|}{CH}}—CO—NH—\underset{\mathbf{R_1}}{\underset{|}{CH}}—CO—NH—\underset{\mathbf{R_2}}{\underset{|}{CH}}—CO—NH—\underset{H}{\underset{|}{CH}}—\overset{\ominus}{COO}$$

Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2`	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$`	CH_3

Sol. **(4)**

Peptides with isoelectric point (pI) > 7, would exist as cation in neutral solution (pH = 7).

IV, VI, VIII and IX

38. An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\dfrac{[t_{1/8}]}{[t_{1/10}]}\times10$? (take $\log_{10}2$ = 0.3)

Sol. **(9)**

$$t_{1/8} = \frac{2.303\log8}{k} = \frac{2.303\times3\log2}{k}$$

$$t_{1/10} = \frac{2.303}{k}\log10 = \frac{2.303}{k}$$

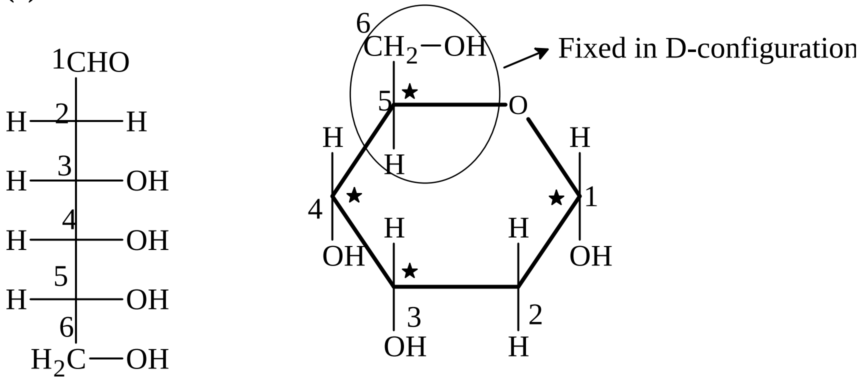
$$\left[\frac{t_{1/8}}{t_{1/10}}\right]\times10 = \frac{\left(\dfrac{2.303\times3\log2}{k}\right)}{\left(\dfrac{2.303}{k}\right)}\times10 = 9$$

39. When the following aldohexose exists in its **D**-configuration, the total number of stereoisomers in its pyranose form is

$$\begin{array}{l} CHO \\ | \\ CH_2 \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CHOH \\ | \\ CH_2OH \end{array}$$

Nitin M Sir (physics-iitjee.blogspot.com)

Sol. **(8)**



Hence total number of stereoisomers in pyranose form of D-configuration = $2^3 = 8$

40. The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element **X** as shown below. To which group, element **X** belongs in the periodic table?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + X$$

Sol. **(8)**

$${}^{63}_{29}Cu + {}_{1}H^{1} \rightarrow 6\,{}_{0}n^{1} + {}^{4}_{2}He + 2\,{}_{1}H^{1} + {}^{A}_{Z}X$$

Mass number: $63 + 1 = 1 \times 6 + 4 + 1 \times 2 + A$

$A = 64 – 12 = 52$

Atomic number: $29 + 1 = 6 \times 0 + 2 + 2 \times 1 + Z$

$Z = 30–4 = 26$

${}^{A}_{Z}X = {}^{52}_{26}Fe$

Hence X is in group ‘8’ in the periodic table.

Nitin M Sir (physics-iitjee.blogspot.com)

PAPER-1 [Code – 8]

IITJEE 2012

PART - III: MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. If $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) a = 1, b = 4
(B) a = 1, b = –4
(C) a = 2, b = – 3
(D) a = 2, b = 3

Sol. **(B)**

Given $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$

$\Rightarrow \lim\limits_{x\to\infty}\frac{x^2+x+1-ax^2-ax-bx-b}{(x+1)}=4 \Rightarrow \lim\limits_{x\to\infty}\frac{(1-a)x^2+(1-a-b)x+(1-b)}{(x+1)}=4$

$\Rightarrow$ 1 – a = 0 and 1 – a – b = 4 $\Rightarrow$ b = –4, a = 1.

42. Let P = $[a_{ij}]$ be a 3 × 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10}
(B) 2^{11}
(C) 2^{12}
(D) 2^{13}

Sol. **(D)**

$$|Q| = \begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix} \Rightarrow |Q| = 2^2\cdot2^3\cdot2^4\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33} \end{vmatrix}$$

$$|Q| = 2^9\cdot2\cdot2^2\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\Rightarrow |Q| = 2^{12}|P|$
$|Q| = 2^{13}$.

43. The locus of the mid–point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2+y^2)-36x+45y=0$
(B) $20(x^2+y^2)+36x-45y=0$
(C) $36(x^2+y^2)-20x+45y=0$
(D) $36(x^2+y^2)+20x-45y=0$

Sol. **(A)**

Equation of the chord bisected at P (h, k)
$hx + ky = h^2 + k^2$ …(i)

Let any point on line be $\left(\alpha,\ \frac{4}{5}\alpha-4\right)$

Equation of the chord of contact is

$\Rightarrow \alpha x + \left(\frac{4}{5}\alpha - 4\right)y = 9$ …(ii)

Comparing (i) and (ii)

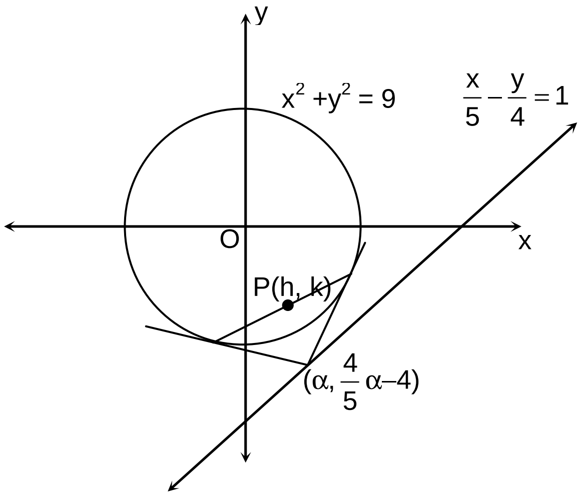
$\frac{h}{\alpha} = \frac{k}{\frac{4}{5}\alpha - 4} = \frac{h^2 + k^2}{9}$

$\alpha = \frac{20h}{4h - 5k}$

Now, $\frac{h(4h-5k)}{20h} = \frac{h^2 + k^2}{9}$

$20(h^2 + k^2) = 9(4h - 5k)$

$20(x^2 + y^2) - 36x + 45y = 0.$



44. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150
(C) 210 (D) 243

Sol. **(B)**

Number of ways

$= 3^5 - {}^3C_1.2^5 + {}^3C_2 1^5$

$= 243 - 96 + 3 = 150.$

45. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$ (B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$ (D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Sol. **(C)**

$I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$

Let sec x + tan x = t

$\Rightarrow$ sec x – tan x = 1/t

Now (sec x tan x + $\sec^2$ x) dx = dt

sec x (sec x + tan x) dx = dt

sec x dx = $\frac{dt}{t}$, $\frac{1}{2}\left(t + \frac{1}{t}\right)$ = sec x

$I = \frac{1}{2}\int \frac{\left(t + \frac{1}{t}\right)}{t^{9/2}} \frac{dt}{t}$

$= \frac{1}{2}\int (t^{-9/2} + t^{-13/2}) dt$

$= \frac{1}{2}\left[\frac{t^{-9/2+1}}{-\frac{9}{2}+1} + \frac{t^{-13/2+1}}{-\frac{13}{2}+1}\right]$

$$= \frac{1}{2}\left[\frac{t^{-7/2}}{-\frac{7}{2}} + \frac{t^{-11/2}}{-\frac{11}{2}}\right]$$

$$= -\frac{1}{7}t^{-7/2} - \frac{1}{11}t^{-11/2}$$

$$= -\frac{1}{7}\frac{1}{t^{7/2}} - \frac{1}{11}\frac{1}{t^{11/2}}$$

$$= -\frac{1}{t^{11/2}}\left(\frac{1}{11} + \frac{t^2}{7}\right) = -\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + k$$

46. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

(C) 2 (D) $2\sqrt{2}$

Sol. **(A)**

D. R. of QR is 1, 4, 1

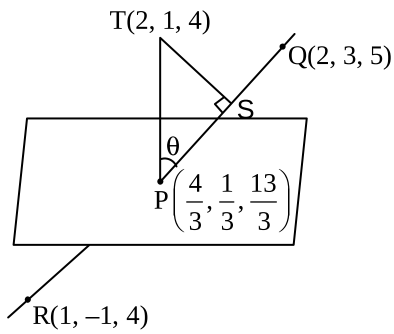
Coordinate of P $\equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

D. R. of PT is 2, 2, –1

Angle between QR and PT is 45º

And PT = 1

$\Rightarrow$ PS = TS $= \frac{1}{\sqrt{2}}$



47. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}, \ x \in \mathbb{R}$, then f is

(A) differentiable both at x = 0 and at x = 2

(B) differentiable at x = 0 but not differentiable at x = 2

(C) not differentiable at x = 0 but differentiable at x = 2

(D) differentiable neither at x = 0 nor at x = 2

Sol. **(B)**

$$f'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h^2\left|\cos\frac{\pi}{h}\right| - 0}{h}$$

$$= \lim_{h\to 0} h\cos\left(\frac{\pi}{h}\right) = 0$$

so, f(x) is differentiable at x = 0

$$f'(2^+) = \lim_{h\to 0}\frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0}\frac{(2+h)^2\left|\cos\frac{\pi}{2+h}\right| - 0}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 \cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$f'\left(2^+\right) = \lim_{h\to 0} \frac{(2+h)^2}{h} \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)$$

$$= \lim_{h\to 0} \frac{(2+h)^2}{h} \sin\left[\frac{\pi \cdot h}{2(2+h)}\right]$$

$$= \lim_{h\to 0} \frac{(2+h)^2}{\frac{\pi h}{2(2+h)}} \sin\frac{\pi h}{2(2+h)} \times \frac{\pi}{2(2+h)} = \pi$$

Again, $f'\left(2^-\right) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$

$$= \lim_{h\to 0} \frac{(2-h)^2 \left|\cos\left(\frac{\pi}{2-h}\right)\right|}{-h}$$

$$= \lim_{h\to 0} \frac{-(2-h)^2 \cos\left(\frac{\pi}{2-h}\right)}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2 \sin\left[\frac{\pi}{2} - \frac{\pi}{2-h}\right]}{h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2}{h} \cdot \sin\left[\frac{-\pi h}{2(2-h)}\right]$$

$$= -\lim_{h\to 0} \frac{(2-h)^2}{\frac{\pi h}{2(2-h)}} \cdot \sin\frac{\pi h}{2(2-h)} \times \frac{\pi}{2(2-h)} = -\pi$$

48. Let z be a complex number such that the imaginary part of z is nonzero and a = z^2 + z + 1 is real. Then a cannot take the value

(A) –1
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$

Sol. **(D)**
Given equation is $z^2 + z + 1 - a = 0$
Clearly this equation do not have real roots if
$D < 0$
$\Rightarrow 1 - 4(1 - a) < 0$
$\Rightarrow 4a < 3$
$a < \frac{3}{4}$.

49. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$
(B) $\frac{\sqrt{3}}{2}$

Nitin M Sir (physics-iitjee.blogspot.com)

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

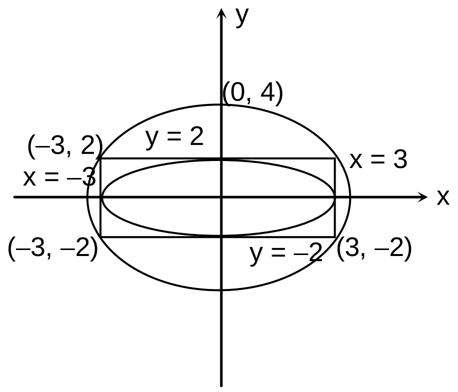
Sol. **(C)**

Equation of ellipse is (y + 2) (y – 2) + λ(x + 3) (x – 3) = 0

It passes through (0, 4) ⇒ $\lambda = \frac{4}{3}$

Equation of ellipse is $\frac{x^2}{12}+\frac{y^2}{16}=1$

$e = \frac{1}{2}$.



Alternate

Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as it is passing through (0, 4) and (3, 2).

So, $b^2 = 16$ and $\frac{9}{a^2}+\frac{4}{16}=1$

⇒ $a^2 = 12$

So, $12 = 16(1 - e^2)$

⇒ e = 1/2.

50. The function f : [0, 3] → [1, 29], defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one–one and onto (B) onto but not one–one
(C) one–one but not onto (D) neither one–one nor onto

Sol. **(B)**

$f(x) = 2x^3 - 15x^2 + 36x + 1$
$f'(x) = 6x^2 - 30x + 36$
$= 6(x^2 - 5x + 6)$
$= 6(x - 2)(x - 3)$
f (x) is increasing in [0, 2] and decreasing in [2, 3]
f (x) is many one
f (0) = 1
f (2) = 29
f (3) = 28
Range is [1, 29]
Hence, f (x) is many-one-onto

SECTION II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

51. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line 2x – y = 1. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
(C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

Sol. **(A, B)**

Slope of tangent = 2

The tangents are y = 2x ± $\sqrt{9\times 4-4}$
i.e., 2x – y = ± $4\sqrt{2}$

$\Rightarrow \frac{x}{2\sqrt{2}}-\frac{y}{4\sqrt{2}}=1$ and $-\frac{x}{2\sqrt{2}}+\frac{y}{4\sqrt{2}}=1$

Comparing it with $\frac{xx_1}{9}-\frac{yy_1}{4}=1$

We get point of contact as $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Alternate:

Equation of tangent at P (θ) is $\left(\frac{\sec\theta}{3}\right)x-\left(\frac{\tan\theta}{2}\right)y=1$

$\Rightarrow$ Slope = $\frac{2\sec\theta}{3\tan\theta}=2$

$\Rightarrow \sin\theta=\frac{1}{3}$

$\Rightarrow$ points are $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

52. Let θ, φ ∈ [0, 2π] be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1$, $\tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then φ **cannot** satisfy

(A) $0<\varphi<\frac{\pi}{2}$
(B) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$
(C) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$
(D) $\frac{3\pi}{2}<\varphi<2\pi$

Sol. **(A, C, D)**

2 cos θ (1 – sin φ) = $\frac{2\sin^2\theta}{\sin\theta}\cos\varphi-1=2\sin\theta\cos\varphi-1$

2 cos θ – 2 cos θ sin φ = 2 sin θ cos φ – 1
2 cos θ + 1 = 2 sin (θ + φ)

$\tan(2\pi-\theta)>0 \Rightarrow \tan\theta<0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$

$\Rightarrow \theta\in\left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

$\frac{1}{2}<\sin(\theta+\varphi)<1$

$\Rightarrow 2\pi+\frac{\pi}{6}<\theta+\varphi<\frac{5\pi}{6}+2\pi$

$2\pi+\frac{\pi}{6}-\theta_{max}<\varphi<2\pi+\frac{5\pi}{6}-\theta_{min}$

$\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$.

53. If y(x) satisfies the differential equation y′ – ytanx = 2x secx and y(0) = 0, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$
(B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Sol. **(A, D)**

$\frac{dy}{dx}-y\tan x=2x\sec x$

$\cos x\frac{dy}{dx}+(-\sin x)y=2x$

$\frac{d}{dx}(y\cos x)=2x$

y (x) cos x = x^2 + c, where c = 0 since y (0) = 0

when x = $\frac{\pi}{4}$, $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$, when x = $\frac{\pi}{3}$, $y\left(\frac{\pi}{3}\right)=\frac{2\pi^2}{9}$

when x = $\frac{\pi}{4}$, $y'\left(\frac{\pi}{4}\right)$ = $\frac{\pi^2}{8\sqrt{2}}+\frac{\pi}{\sqrt{2}}$

when x = $\frac{\pi}{3}$, $y'\left(\frac{\pi}{3}\right)=\frac{2\pi^2}{3\sqrt{3}}+\frac{4\pi}{3}$

54. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2},\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true ?

(A) $P\left[X_1^c\middle|X\right]=\frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(C) $P\left[X\middle|X_2\right]=\frac{5}{16}$ (D) $P\left[X\middle|X_1\right]=\frac{7}{16}$

Sol. **(B, D)**

$P(X_1)=\frac{1}{2},\ P(X_2)=\frac{1}{4},\ P(X_3)=\frac{1}{4}$

P(X) = $P(X_1\cap X_2\cap X_3^C)+P(X_1\cap X_2^C\cap X_3)+P(X_1^C\cap X_2\cap X_3)+P(X_1\cap X_2\cap X_3)$ = $\frac{1}{4}$

(A) $P(X_1^C/X)$ = $\frac{P(X\cap X_1^C)}{P(X)}$ = $\frac{\frac{1}{32}}{\frac{1}{4}}=\frac{1}{8}$

(B) P [exactly two engines of the ship are functioning | X] = $\frac{\frac{7}{32}}{\frac{1}{4}}=\frac{7}{8}$

(C) $P\left(\frac{X}{X_2}\right)$ = $\frac{\frac{5}{32}}{\frac{1}{4}}=\frac{5}{8}$

(D) $P\left(\frac{X}{X_1}\right)$ = $\frac{\frac{7}{32}}{\frac{1}{2}}=\frac{7}{16}$

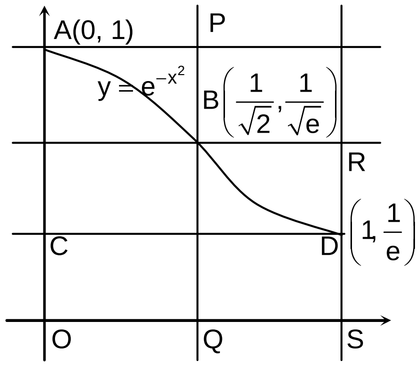
Nitin M Sir (physics-iitjee.blogspot.com)

55. Let S be the area of the region enclosed by $y = e^{-x^2}$, y = 0, x = 0 and x = 1. Then

(A) $S \ge \frac{1}{e}$ (B) $S \ge 1 - \frac{1}{e}$

(C) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (D) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Sol. **(A, B, D)**



$S > \frac{1}{e}$ (As area of rectangle OCDS = 1/e)

Since $e^{-x^2} \ge e^{-x}$ $\forall$ x $\in$ [0, 1]

$\Rightarrow S > \int_0^1 e^{-x} dx = \left(1-\frac{1}{e}\right)$

Area of rectangle OAPQ + Area of rectangle QBRS > S

$S < \frac{1}{\sqrt{2}}(1) + \left(1-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{e}}\right)$.

Since $\frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right) < 1 - \frac{1}{e}$

Hence, (C) is incorrect.

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

56. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Sol. **(3)**

As, $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 3\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right) - |\vec{a}+\vec{b}+\vec{c}|^2$

$\Rightarrow 3 \times 3 - |\vec{a}+\vec{b}+\vec{c}|^2 = 9$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 0 \Rightarrow \vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \vec{b}+\vec{c} = -\vec{a}$

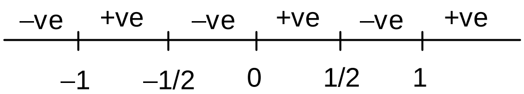
$\Rightarrow |2\vec{a}+5(\vec{b}+\vec{c})| = |-3\vec{a}| = 3|\vec{a}| = 3$.

57. Let f : IR $\to$ IR be defined as f(x) = $|x| + |x^2-1|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. **(5)**

$$f'(x) = \frac{|x|}{x} + \frac{|x^2-1|}{x^2-1}\cdot(2x)$$

$$= \begin{cases} 2x-1 & , \quad x<-1 \\ -(2x+1) & , \quad -1<x<0 \\ 1-2x & , \quad 0<x<1 \\ 2x+1 & , \quad x>1 \end{cases}$$



So, $f'(x)$ changes sign at points

$x = -1,\ -\frac{1}{2}, 0, \frac{1}{2}, 1$

so, total number of points of local maximum or minimum is 5.

58. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Sol. **(4)**

The parabola is $x = 2t^2$, $y = 4t$

Solving it with the circle we get :

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$\Rightarrow t^4 + 3t^2 - 4t = 0 \ \Rightarrow t = 0, 1$

so, the points P and Q are (0, 0) and (2, 4) which are also diametrically opposite points on the circle. The focus is $S \equiv (2, 0)$.

The area of $\Delta PQS = \frac{1}{2}\times 2\times 4 = 4$.

59. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then $p'(0)$ is

Sol. **(9)**

Let $p'(x) = k(x-1)(x-3)$

$\Rightarrow p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

Now, $p(1) = 6 \Rightarrow \frac{4}{3}k + c = 6$

also, $p(3) = 2 \Rightarrow c = 2$

so, $k = 3$, so, $p'(0) = 3k = 9$.

60. The value of $6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}}\right)$ is

Sol. **(4)**

Let $\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}} = y$

So, $4-\frac{1}{3\sqrt{2}}y = y^2$ $\qquad (y > 0)$

$\Rightarrow y^2 + \frac{1}{3\sqrt{2}}y - 4 = 0 \Rightarrow y = \frac{8}{3\sqrt{2}}$

so, the required value is $6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right)$

$= 6+\log_{3/2}\frac{4}{9} = 6 - 2 = 4$.

Nitin M Sir (physics-iitjee.blogspot.com)

FIITJEE Solutions to IIT-JEE-2012

CODE 8

PAPER 2

Time: 3 Hours Maximum Marks: 198

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This booklet is your Question paper. Do not break the seats of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.
3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.
4 Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.
5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
6. **Using a black ball point pen, darken the bubbles on the upper original sheet**. Apply sufficient pressure so that the impression is created on the bottom sheet.
7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.
8. On breaking the seals of the booklet check that it contains 36 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS:

9. The ORS has CODES printed on its left and right parts.
10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET**. Sign at the place provided on the ORS affirming that you have verified that all the codes are same.
11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else**. Darken the appropriate bubble **UNDER** each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS(R_4).

C. Question Paper Format:

The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. **Section I** contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.
13. **Section II** contains **3 paragraphs** each describing theory, experiment, data etc. There are **6 multiple choice questions** relating to three paragraphs with **2 questions on each paragraph**. Each question of a particular paragraph has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.
14. **Section III** contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

D. Marking Scheme

15. For each question in **Section I and Section II**, you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero (0) marks** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded in these sections.
16. For each question in **Section III**, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY**. In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer(s) in this section.

Write your Name, Registration Number and sign in the space provided on the back page of this booklet.

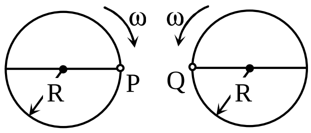
PAPER-2 [Code – 8]
IITJEE 2012

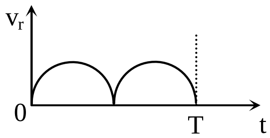
PART - I: PHYSICS

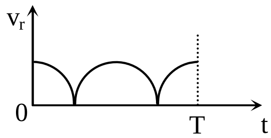
SECTION I : Single Correct Answer Type

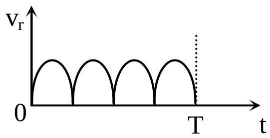
This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

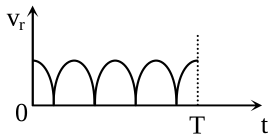
1. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



(A)


(B)


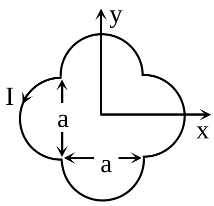
(C)


(D)


Sol. **(A)**

In each rotation relative speed becomes zero twice and becomes maximum twice.

2. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

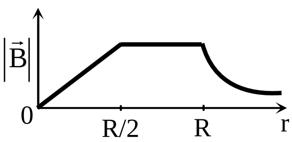


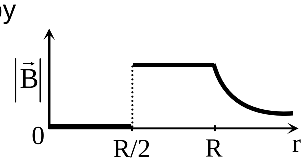
(A) $a^2 I\,\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$ (D) $(2\pi+1)a^2 I\,\hat{k}$

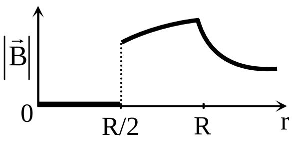
Sol. **(B)**

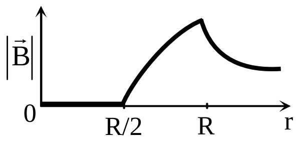
Magnetic moment, $\vec{M} = I\vec{A} = I\left(\frac{\pi}{2}+1\right)a^2\hat{K}$

3. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(A)


(B)


(C)


(D)


Sol. **(D)**

Inside the cavity, B = 0

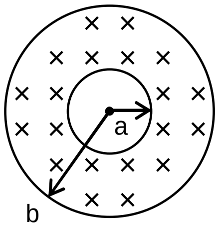
Outside the cylinder, $B = \frac{\mu_0 I}{2\pi r}$

In the shaded region

$$B = \frac{\mu_0 I}{2\pi r(b^2 - a^2)}\left(r - \frac{a^2}{r}\right)$$

at r = a, B = 0

at r = b, $B = \frac{\mu_0 I}{2\pi b}$



4. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is
(A) more than half-filled if ρ_C is less than 0.5. (B) more than half-filled if ρ_C is more than 1.0.
(C) half-filled if ρ_C is more than 0.5. (D) less than half-filled if ρ_C is less than 0.5.

Sol. **(A)**

$$\frac{V_m + V_a + V_w}{2}\rho_w g = V_m \rho_c \rho_w g + V_w \rho_w g$$

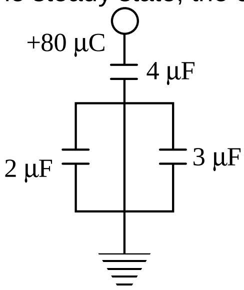
$$V_w = V_m(1 - 2\rho_C) + V_a$$

if $\rho_C > \frac{1}{2} \Rightarrow V_w < V_a$

if $\rho_C < \frac{1}{2} \Rightarrow V_w > V_a$,

where, V_w = volume occupied by water in the shell
V_a = volume occupied by air in the shell
V_m = volume of the material in the shell

5. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(A) +32 μC (B) +40 μC (C) +48 μC (D) +80 μC

Sol. **(C)**

Let 'q' be the final charge on 3μ F capacitor then

$$\frac{80 - q}{2} = \frac{q}{3} \Rightarrow q = 48\mu C$$

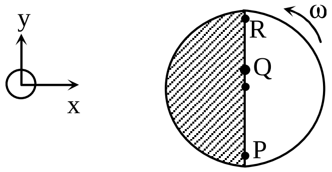
6. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the

Nitin M Sir (physics-iitjee.blogspot.com)

balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)
(A) 62 J (B) 104 J (C) 124 J (D) 208 J

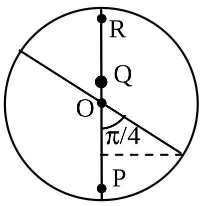
Sol. **(D)**
$\Delta Q = nC_p\Delta T$ (Isobaric process)
$= 2\times\frac{5}{2}R\times(35-30)$
$= 208\,J$

7. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

Sol. **(C)**
At t = $\frac{1}{8}\times\frac{2\pi}{\omega} = \frac{\pi}{4\omega}$



x –coordinate of P = $\omega R\left(\frac{\pi}{4\omega}\right)$
$= \frac{\pi R}{4} > R\cos 45°$
$\therefore$ Both particles P and Q land in unshaded region.

8. A student is performing the experiment of resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is
(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

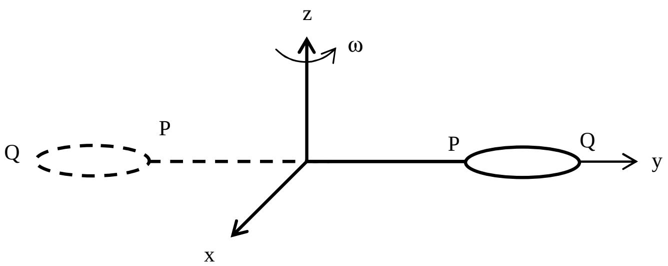
Sol. **(B)**
L + e = $\frac{\lambda}{4}$
$\Rightarrow$ L = $\frac{\lambda}{4} - e$
= 16.4 – 1.2 = 15.2 cm

SECTION II : Paragraph Type

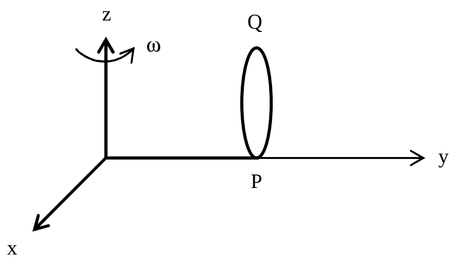
This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Paragraph for Questions 9 and 10

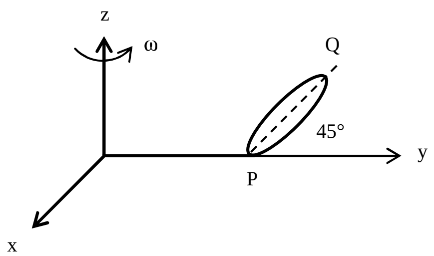
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case



Now consider two similar systems as shown in the figure: Case(a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

9. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
   (A) It is vertical for both the cases (a) and (b)
   (B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).
   (C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).
   (D) It is vertical for case (a); and is 45° to the x-z plane and is normal to the plane of the disc for case (b).

Sol. **(A)**

10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?
   (A) It is $\sqrt{2}\omega$ for both the cases.
   (B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).
   (C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).
   (D) It is ω for both the cases.

Sol. **(D)**

Paragraph for Questions 11 and 12

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

11. If the anti-neutrino had a mass of $3\text{eV}/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
(A) $0 \le K \le 0.8 \times 10^6\,\text{eV}$
(B) $3.0\,\text{eV} \le K \le 0.8 \times 10^6\,\text{eV}$
(C) $3.0\,\text{eV} \le K < 0.8 \times 10^6\,\text{eV}$
(D) $0 \le K < 0.8 \times 10^6\,\text{eV}$

Sol. **(D)**

12. What is the maximum energy of the anti-neutrino?
(A) Zero
(B) Much less than $0.8 \times 10^6\,\text{eV}$.
(C) Nearly $0.8 \times 10^6\,\text{eV}$
(D) Much larger than $0.8 \times 10^6\,\text{eV}$

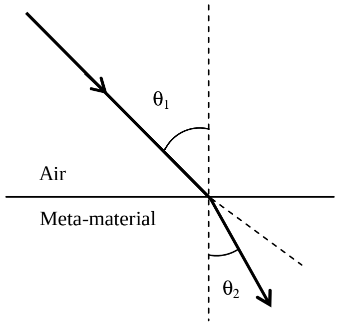
Sol. (C)

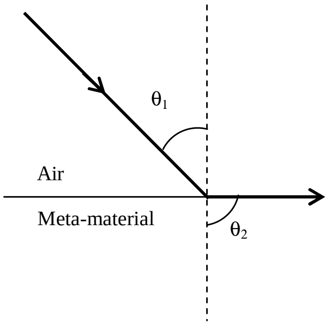
Paragraph for Questions 13 and 14

Most materials have the refractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

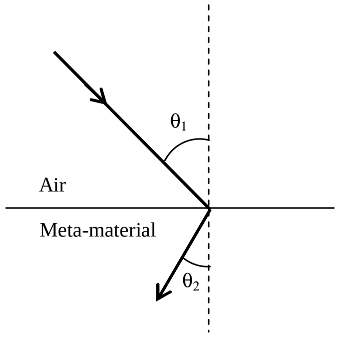
In normal materials, both ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

13. For light incident from air on a meta-material, the appropriate ray diagram is

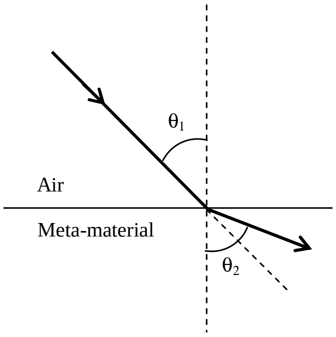
(A)


(B)


(C)



(D)



Sol. **(C)**

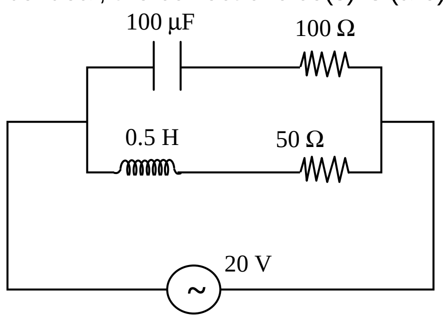
14. Choose the correct statement.
(A) The speed of light in the meta-material is v = c|n|
(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$
(C) The speed of light in the meta-material is v = c.
(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is wavelength of the light in air.

Sol. **(B)**

SECTION III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

15. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)



(A) The current through the circuit, I is 0.3 A. (B) The current through the circuit, I is $0.3\sqrt{2}$ A.
(C) The voltage across 100 Ω resistor = $10\sqrt{2}$ V. (D) The voltage across 50 Ω resistor = 10 V.

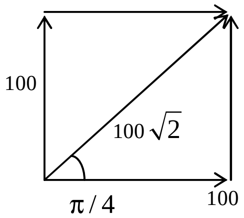
Sol. **(A, C)**

$I_{upper} = \frac{20}{100\sqrt{2}}$; $+\frac{\pi}{4}$ ahead of voltage

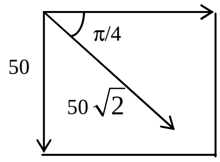
$I_{lower} = \frac{20}{50\sqrt{2}}$; $-\frac{\pi}{4}$ behind voltage

$I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{10}} \approx 0.3A$

$V_{100\,\Omega} = \frac{20}{100\sqrt{2}} \times 100 = 10\sqrt{2}$.



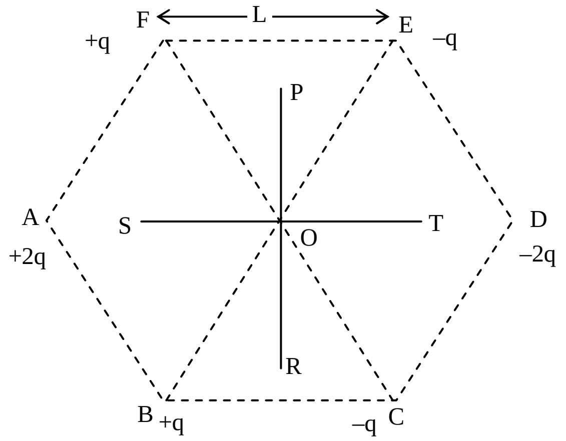
Capacitive circuit



Inductive circuit

Nitin M Sir (physics-iitjee.blogspot.com)

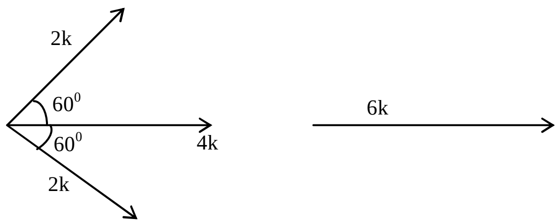
16. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K=\frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct?



(A) The electric field at O is 6K along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

Sol. **(A, B, C)**
Line PR is perpendicular bisector of all the dipoles.

At point O: $\frac{1}{4\pi\varepsilon_0}\Sigma\frac{Q_i}{r_i}=0$



17. Two spherical planets P and Q have the same uniform density ρ, masses M_p and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is (M_P + M_Q). The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then

(A) $V_Q > V_R > V_P$
(B) $V_R > V_Q > V_P$
(C) $V_R / V_P = 3$
(D) $V_P / V_Q = \frac{1}{2}$

Sol. **(B, D)**
By calculation, if Mass of P = m
and Radius of P = R
Then Mass of Q = 8M
and radius of Q = 2R
and Mass of R = 9M
and radius of R = $9^{1/3}$R

$$V_P\sqrt{\frac{2GM}{R}}$$

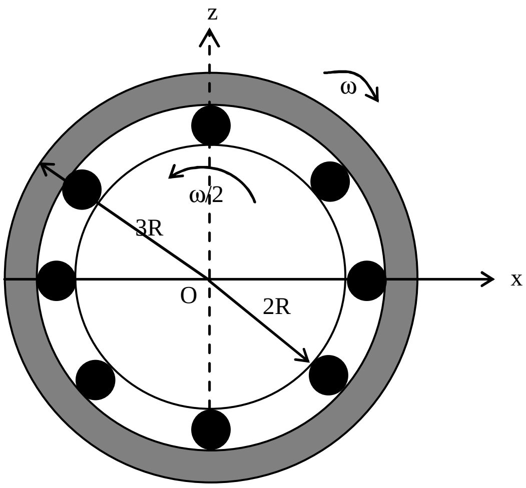
$$V_Q=\sqrt{\frac{2G8M}{2R}}=2V_p$$

$$V_R=\sqrt{\frac{2G8M}{9^{1/3}R}}=9^{1/3}V_p$$

$$\therefore \quad V_R > V_Q = V_p$$

$$\frac{V_Q}{V_P}=2$$

18. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30^0 with the horizontal. Then with respect to the horizontal surface,



(A) the point O has linear velocity $3R\omega\hat{i}$

(B) the point P has linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$.

(C) the point P has linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

Sol. **(A, B)**

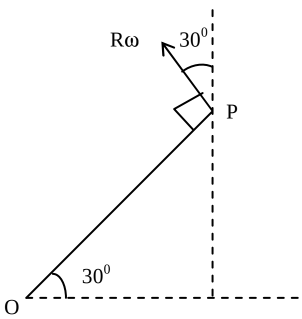
$\vec{V}_0(3R)\omega\hat{i}=0$

$\therefore\ \vec{v}_0=3R\omega\hat{i}$

$\vec{v}_{P,O}=\frac{-R\omega}{4}\hat{i}+\frac{R\omega\sqrt{3}}{4}\hat{j}$

$\therefore\ \vec{v}_P=\vec{v}_{P,O}+\vec{v}_O$

$=\frac{11}{4}R\omega\hat{i}+R\omega\frac{\sqrt{3}}{4}\hat{j}$



19. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?
(A) Both cylinders P and Q reach the ground at the same time.
(B) Cylinders P has larger linear acceleration than cylinder Q.
(C) Both cylinders reach the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

Sol. **(D)**

a = $\frac{Mg\sin\theta}{M+\frac{I}{R^2}}$

$$a_P = \frac{Mg\sin\theta}{M + \frac{MR^2}{R^2}} \approx \frac{g}{2}$$

$a_Q = g \sin\theta$ as $I_Q \sim 0$

$$\therefore \quad \omega_P = \frac{\sqrt{2.\frac{g}{2}.\ell}}{R}$$

$$\omega_Q = \frac{\sqrt{2.g.\ell}}{R}$$

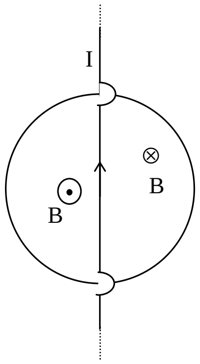
$\therefore \quad \omega_Q > \omega_P$

20. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is(are)
(A) The emf induced in the loop is zero if the current is constant.
(B) The emf induced in the loop is finite if the current is constant.
(C) The emf induced in the loop is zero if the current decreases at a steady rate.
(D) The emf induced in the loop is infinite if the current decreases at a steady rate.

Sol. **(A, C)**
ϕ = zero

$\therefore \frac{d\phi}{dt} = \text{zero}$

$\therefore$ A, C are correct.



Nitin M Sir (physics-iitjee.blogspot.com)

PAPER-2 [Code – 8]
IITJEE 2012

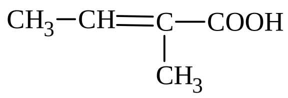
PART - II: CHEMISTRY

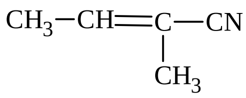
SECTION I : Single Correct Answer Type

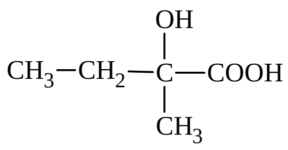
This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

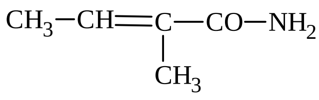
21. The major product **H** of the given reaction sequence is

$CH_3-CH_2-CO-CH_3 \xrightarrow{\overset{\ominus}{C}N} G \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} H$

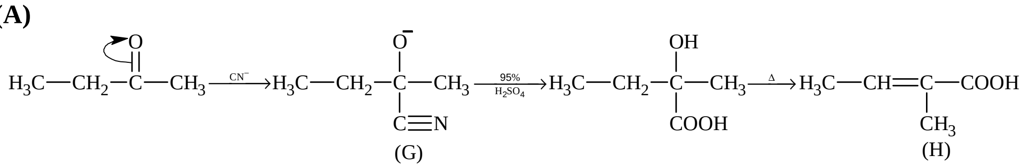
(A) 

(B) 

(C) 

(D) 

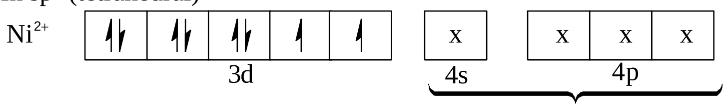
Sol. **(A)**



22. $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behaviour (paramagnetic/diamagnetic). The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

(A) tetrahedral and tetrahedral
(B) square planar and square planar
(C) tetrahedral and square planar
(D) square planar and tetrahedral

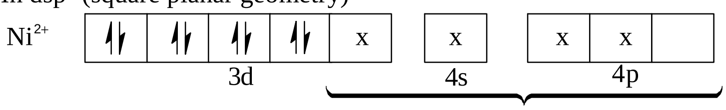
Sol. **(C)**
In both complexes Ni exists as Ni^{2+}.
In sp^3 (tetrahedral)



x represents electron pair donated by the ligands

sp^3 & paramagnetic

In dsp^2 (square planar geometry)



dsp^2 & hence diamagnetic

23. In the cyanide extraction process of silver from argentite ore, the oxidising and reducing agents used are

(A) O_2 and CO respectively
(B) O_2 and Zn dust respectively
(C) HNO_3 and Zn dust respectively
(D) HNO_3 and CO respectively

Sol. **(B)**
The reactions involved in cyanide extraction process are:

$$\underset{(\text{argentite ore})}{Ag_2S} + 4NaCN \rightleftharpoons 2Na\left[Ag(CN)_2\right] + Na_2S$$

Nitin M Sir (physics-iitjee.blogspot.com)

$$4Na_2S + \underset{(\text{oxidising agent})}{5[O_2]} + 2H_2O \longrightarrow 2Na_2SO_4 + 4NaOH + 2S$$

$$2Na\left[Ag(CN)_2\right] + \underset{(\text{reducing agent})}{Zn} \longrightarrow Na_2\left[Zn(CN)_4\right] + 2Ag\downarrow$$

24. The reaction of white phosphorous with aqueous NaOH gives phosphine along with another phosphorous containing compound. The reaction type; the oxidation states of phosphorus in phosphine and the other product are respectively

(A) redox reaction; –3 and –5
(B) redox reaction; +3 and +5
(C) disproportionation reaction; –3 and +5
(D) disproportionation reaction; –3 and +3

Sol. **(C)**

The balanced disproportionation reaction involving white phosphorus with aq. NaOH is

Oxidation

$$\overset{0}{P_4} + 3NaOH + 3H_2O \longrightarrow \overset{-3}{P}H_3 + 3NaH_2\overset{+1}{P}O_2$$

Reduction

* However, as the option involving +1 oxidation state is completely missing, one might consider that NaH_2PO_2 formed has undergone thermal decomposition as shown below:

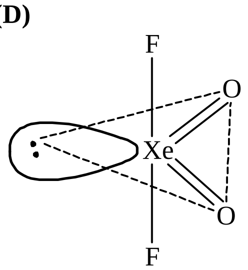
$$2NaH_2PO_2 \xrightarrow{\Delta} Na_2H\overset{+5}{P}O_4 + PH_3$$

Although heating is nowhere mentioned in the question, the "other product" as per available options seems to be Na_2HPO_4 (oxidation state = +5).

*25. The shape of XeO_2F_2 molecule is

(A) trigonal bipyramidal
(B) square planar
(C) tetrahedral
(D) see–saw

Sol. **(D)**



Hybridization = sp^3d
Shape = see – saw

26. For a dilute solution containing 2.5 g of a non–volatile non–electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2°C. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take K_b = 0.76 K kg mol^{-1})

(A) 724
(B) 740
(C) 736
(D) 718

Sol. **(A)**

B → Solute; A → Solvent
W_B = 2.5 g, W_A = 100 g

$$\Delta T_b = 2^o$$

$$\frac{p^o - p_s}{p^o} = X_B = \frac{n_B}{n_B + n_A}$$

$$\frac{p^o - p_s}{p^o} = \frac{n_B}{n_A} \quad \because n_B << n_A$$

$$\frac{p^o - p_s}{p^o} = \frac{n_B}{n_A}$$

$$\frac{760 - P_{soln}}{760} = \frac{2.5/M}{\frac{100}{18} \times \frac{1000}{1000}} = \frac{m \times 18}{1000} \qquad \ldots(i)$$

and from boiling point elevation,

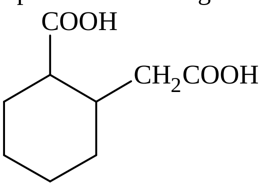
$2 = 0.76 \times m$

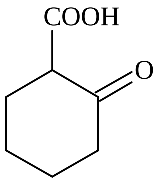
$$m = \frac{2}{0.76} \qquad \ldots(ii)$$

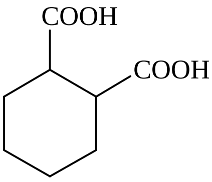
on equating (i) and (ii)

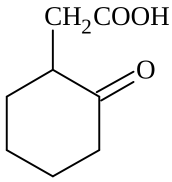
P_{soln} = 724 mm

27. The compound that undergoes decarboxylation most readily under mild condition is

(A) 

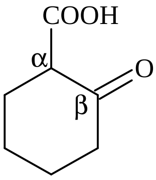
(B) 

(C) 

(D) 

Sol. **(B)**

β - keto acids undergoes decarboxylation easily.



$(\beta - \text{keto acid})$

*28. Using the data provided, calculate the multiple bond energy (kJ mol^{-1}) of a C≡C bond in C_2H_2. That energy is (take the bond energy of a C–H bond as 350 kJ mol^{-1})

$2C(s) \longrightarrow 2C(g) \qquad \Delta H = 1410 \text{ kJmol}^{-1}$

$2C(s) \longrightarrow 2C(g) \qquad \Delta H = 1410 \text{ kJmol}^{-1}$

$H_2(g) \longrightarrow 2H(g) \qquad \Delta H = 330 \text{ kJmol}^{-1}$

(A) 1165 (B) 837

(C) 865 (D) 815

Sol. **(D)**

(i) $2C(s) + H_2(g) \longrightarrow H-C\equiv C-H(g) \qquad \Delta H = 225 \text{ kJmol}^{-1}$

(ii) $2C(s) \longrightarrow 2C(g) \qquad \Delta H = 1410 \text{ kJmol}^{-1}$

(iii) $H_2(g) \longrightarrow 2H(g) \qquad \Delta H = 330 \text{ kJmol}^{-1}$

From equation (i):

$$225 = \left[2\times\Delta H_{C(s)\longrightarrow C(g)} + 1\times BE_{H-H}\right] - \left[2\times BE_{C-H} + 1\times BE_{C\equiv C}\right]$$

$$225 = [1410 + 1\times 330] - [2\times 350 + 1\times BE_{C\equiv C}]$$

$$225 = [1410 + 330] - [700 + BE_{C\equiv C}]$$

$$225 = 1740 - 700 - BE_{C\equiv C}$$

Nitin M Sir (physics-iitjee.blogspot.com)

$225 = 1040 - BE_{C\equiv C}$
$BE_{C\equiv C} = 1040 - 225 = 815 \text{ kJ mol}^{-1}$

SECTION II : Paragraph Type

This section contains 6 **multiple choice questions** relating to three paragraphs with **two questions on each paragraph**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

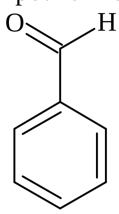
Paragraph for Questions 29 and 30

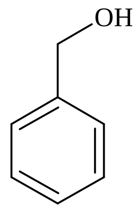
In the following reaction sequence, the compound **J** is an intermediate.

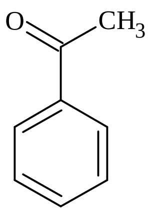
$$\mathbf{I} \xrightarrow[CH_3COONa]{(CH_3CO)_2O} \mathbf{J} \xrightarrow[\substack{\text{(ii) } SOCl_2 \\ \text{(iii) anhyd. } AlCl_3}]{\text{(i) } H_2,\ Pd/C} \mathbf{K}$$

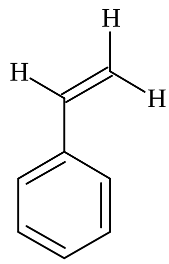
J($C_9H_8O_2$) gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test.

29. The compound **I** is

(A) 

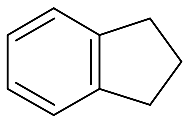
(B) 

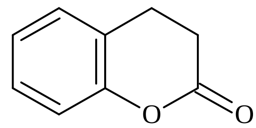
(C) 

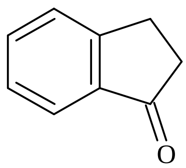
(D) 

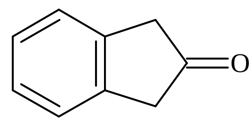
Ans. **(A)**

30. The compound **K** is

(A) 

(B) 

(C) 

(D) 

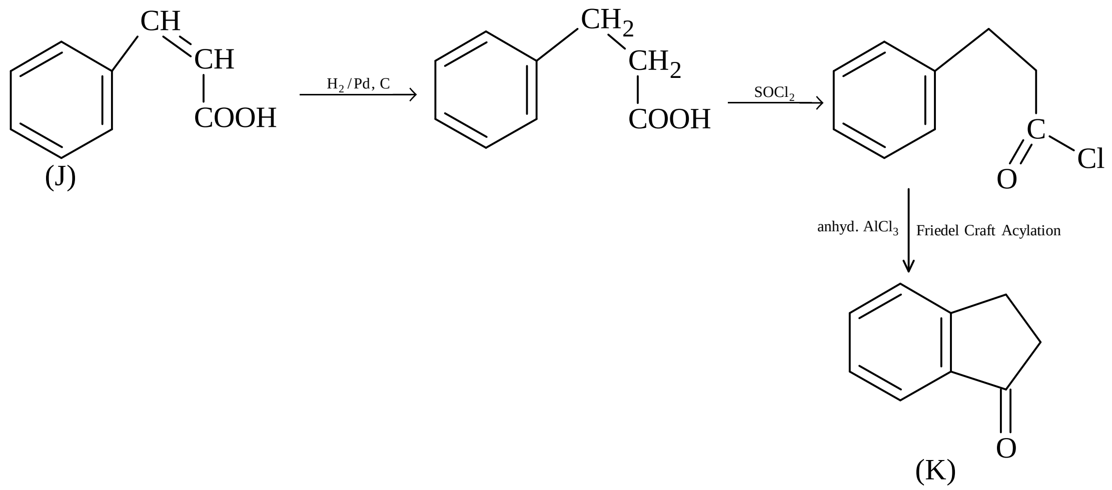
Ans. **(C)**

Sol. **29–30**

$$\underset{(I)}{Ph-CHO} \xrightarrow[\substack{CH_3COONa \\ \text{(Perkin condensation)}}]{(CH_3CO)_2O} \underset{\substack{(J) \\ \text{(Cinnamic Acid)}}}{Ph-CH=CH-COOH} \xrightarrow{NaHCO_3} (\text{Effervescence})$$

$$\xrightarrow[\text{(Bayer's reagent)}]{\text{1\% cold, alk; } KMnO_4} (\text{Positive Test})$$

Nitin M Sir (physics-iitjee.blogspot.com)



(J) (K)

Paragraph for Questions 31 and 32

The electrochemical cell shown below is a concentration cell.
$M \mid M^{2+}$ (saturated solution of a sparingly soluble salt, MX_2) $\mid\mid M^{2+}$ (0.001 mol dm^{-3}) $\mid M$
The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

31. The value of ΔG (kJ mol^{-1}) for the given cell is (take 1F = 96500 C mol^{-1})
(A) –5.7
(B) 5.7
(C) 11.4
(D) –11.4

Sol. **(D)**
At anode: $M(s) + 2X^{-}(aq) \rightarrow MX_2(aq) + 2e^{-}$
At cathode: $M^{+2}(aq) + 2e^{-} \rightarrow M(s)$
n–factor of the cell reaction is 2.
$\Delta G = -nFE_{cell} = -2 \times 96500 \times 0.059 = -113873 / \text{mole} = -11.387\ \text{KJ / mole} \approx -11.4\ \text{KJ / mole}$

32. The solubility product (K_{sp}; $mol^3\ dm^{-9}$) of MX_2 at 298 K based on the information available for the given concentration cell is (take $2.303 \times R \times 298/F = 0.059$ V)
(A) 1×10^{-15}
(B) 4×10^{-15}
(C) 1×10^{-12}
(D) 4×10^{-12}

Sol. **(B)**
$M \mid \underset{(K_{sp}=?)}{M^{+}(sat.)} \mid\mid M^{2+}(0.001\ M)$
emf of concentration cell,
$$E_{cell} = \frac{-0.059}{n} \log \frac{[M^{+2}]_a}{[M^{+2}]_c}$$
$$0.059 = \frac{0.059}{2} \log \frac{[0.001]}{[M^{+2}]_a}$$
$[M^{+2}]_a = 10^{-5}$ = S (solubility of salt in saturated solution)
$$\underset{(S)}{MX_2} \rightleftharpoons \underset{(S)}{M^{+2}} + \underset{(2S)}{2x^{-}(aq)}$$
$K_{sp} = 4S^3 = 4 \times (10^{-5})^3 = 4 \times 10^{-15}$

Paragraph for Questions 33 and 34

Bleaching powder and bleach solution are produced on a large scale and used in several household products. The effectiveness of bleach solution is often measured by iodometry.

*33. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is
(A) Cl_2O (B) Cl_2O_7
(C) ClO_2 (D) Cl_2O_6

Sol. **(A)**

$$\underset{\text{(Bleaching Powder)}}{Ca(OCl)Cl} \rightarrow Ca^{+2} + \overset{-}{O}Cl + Cl^-$$

$$\underset{\text{(oxo acid)}}{HOCl} \rightarrow H^+ + OCl^-$$

$$2HOCl \xrightarrow{\Delta} H_2O + Cl_2O$$

Anhydride of oxoacid (HOCl) is Cl_2O.

*34. 25 mL of household solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is
(A) 0.48 M (B) 0.96 M
(C) 0.24 M (D) 0.024 M

Sol. **(C)**

$$\underset{\substack{25\text{ mL}\\(M)\text{molar}}}{CaOCl_2(aq)} + \underset{\substack{30\text{ mL}\\0.5(M)}}{2KI} \rightarrow I_2 + Ca(OH)_2 + KCl$$

$$I_2 + \underset{\substack{48\text{ mL}\\0.25(N)=0.25\text{ M}}}{2Na_2S_2O_3} \rightarrow Na_2S_4O_6 + 2NaI$$

So, number of millimoles of I_2 produced $= 48 \times \frac{0.25}{2} = 24 \times 0.25 = 6$

In reaction;

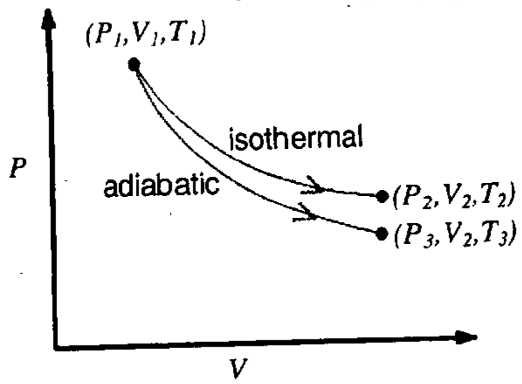
Number of millimoles of bleaching powder $\left(n_{CaOCl_2}\right) = n_{I_2-\text{produced}} = \frac{1}{2} \times n_{Na_2S_2O_3}$ used = 6

So, (M) = $\frac{n_{CaOCl_2}(\text{millimoles})}{V(\text{in mL})} = \frac{6\text{ millimoles}}{25\text{ mL}} = 0.24$

SECTION III : Multiple Correct Answer(s) Type

The section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

*35. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct?



(A) $T_1 = T_2$ (B) $T_3 > T_1$
(C) $w_{\text{isothermal}} > w_{\text{adiabatic}}$ (D) $\Delta U_{\text{isothermal}} > \Delta U_{\text{adiabatic}}$

Sol. **(A, C, D)**

$T_1 = T_2$ because process is isothermal.

Work done in adiabatic process is less than in isothermal process because area covered by isothermal curve is more than the area covered by the adiabatic curve.

In adiabatic process expansion occurs by using internal energy hence it decreases while in isothermal process temperature remains constant that's why no change in internal energy.

36. For the given aqueous reactions, which of the statement(s) is (are) true?

$$\text{excess KI} + K_3\left[Fe(CN)_6\right] \xrightarrow{\text{dilute } H_2SO_4} \text{brownish-yellow solution}$$

$$\downarrow ZnSO_4$$

$$\text{white precipitate} + \underbrace{\text{brwonish-yellow filtrate}}$$

$$\downarrow Na_2S_2O_3$$

$$\text{colourless solution}$$

(A) The first reaction is a redox reaction.
(B) White precipitate is $Zn_3[Fe(CN)_6]_2$.
(C) Addition of filtrate to starch solution gives blue colour.
(D) White precipitate is soluble in NaOH solution.

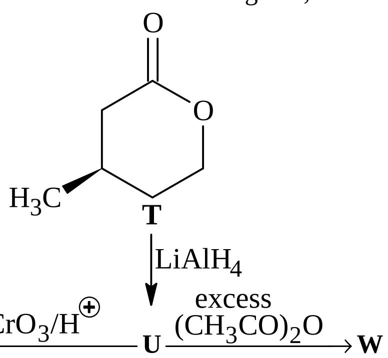
Sol. **(A, C, D)**

$$K_3\left[\overset{+3}{Fe}(CN)_6\right] + KI(\text{excess}) \rightarrow K_4\left[\overset{+2}{Fe}(CN)_6\right] + \underset{\text{Brownish yellow solution}}{KI_3} \quad (\text{redox reaction})$$

$$\underset{(\text{Brownish yellow filterate})}{I_3^-} + 2Na_2S_2O_3 \rightarrow \underset{\text{Clear solution}}{Na_2S_4O_6} + +2NaI + I^-$$

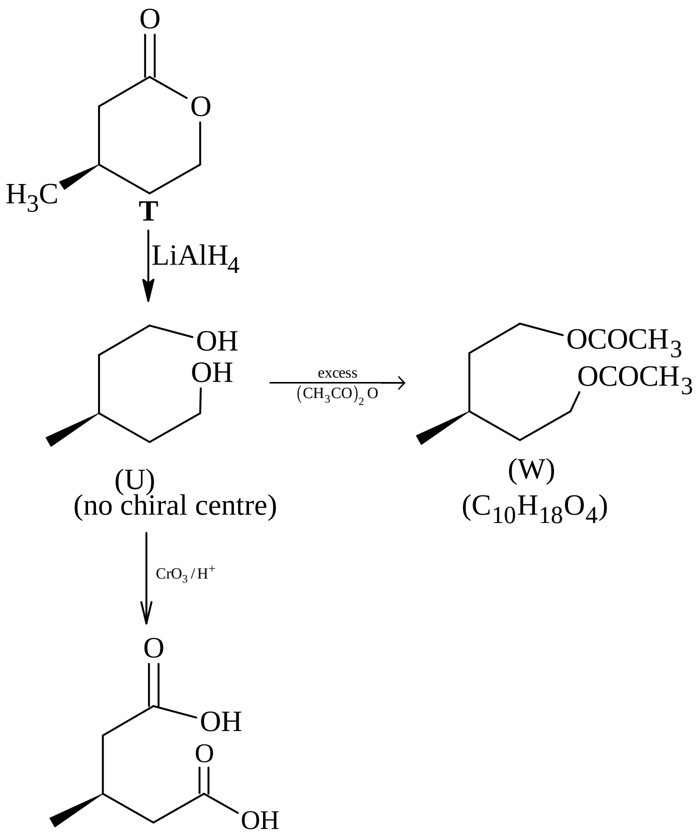
$$K_4\left[Fe(CN)_6\right] + ZnSO_4 \rightarrow \underset{(\text{White ppt.})}{K_2Zn_3\left[Fe(CN)_6\right]_3} \xrightarrow{NaOH} \underset{\text{Soluble}}{Na_2\left[Zn(OH)_4\right]}$$

37. With reference to the scheme given, which of the given statement(s) about **T, U, V** and **W** is (are) correct?



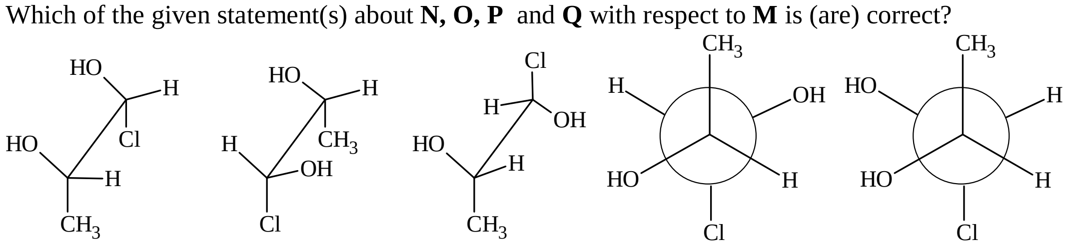
(A) **T** is soluble in hot aqueous NaOH
(B) **U** is optically active
(C) Molecular formula of **W** is $C_{10}H_{18}O_4$
(D) **V** gives effervescence on treatment with aqueous $NaHCO_3$

Sol. **(A, C, D)**



(V)
(Effervescence with $NaHCO_3$)

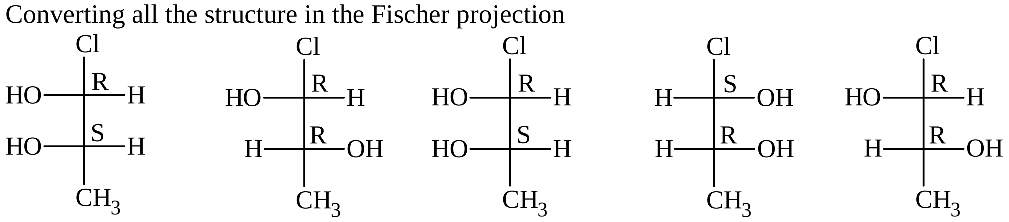
38. Which of the given statement(s) about **N, O, P** and **Q** with respect to **M** is (are) correct?



M **N** **O** **P** **Q**

(A) **M** and **N** are non–mirror image stereoisomers
(B) **M** and **O** are identical
(C) **M** and **P** are enantiomers
(D) **M** and **Q** are identical

Sol. **(A, B, C)**
Converting all the structure in the Fischer projection



M N O P Q

M and N are diastereoisomers
M and O are identical
M and P are enantiomers
M and Q are diastereoisomers
Hence, the correct options are A, B, C.

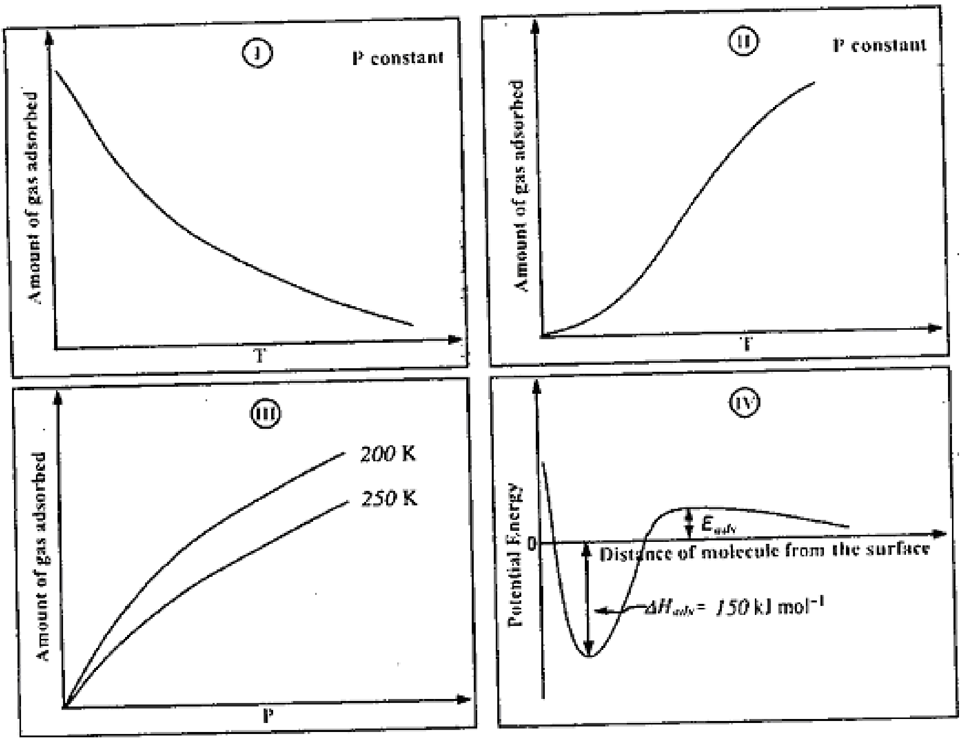
39. With respect to graphite and diamond, which of the statement(s) given below is (are) correct?

Nitin M Sir (physics-iitjee.blogspot.com)

(A) Graphite is harder than diamond.
(B) Graphite has higher electrical conductivity than diamond.
(C) Graphite has higher thermal conductivity than diamond.
(D) Graphite has higher C–C bond order than diamond.

Sol. **(B, D)**
⇒ Diamond is harder than graphite.
⇒ Graphite is good conductor of electricity as each carbon is attached to three C-atoms leaving one valency free, which is responsible for electrical conduction, while in diamond, all the four valencies of carbon are satisfied, hence insulator.
⇒ Diamond is better thermal conductor than graphite. Whereas electrical conduction is due to availability of free electrons; thermal conduction is due to transfer of thermal vibrations from atom to atom. A compact and precisely aligned crystal like diamond thus facilitates fast movement of heat.
⇒ In graphite, C – C bond acquires double bond character, hence higher bond order than in diamond.

40. The given graphs / data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about I, II, III and IV is (are) correct?



(A) **I** is physisorption and **II** is chemisorption
(B) **I** is physisorption and **III** is chemisorption
(C) **IV** is chemisorption and **II** is chemisorption
(D) **IV** is chemisorption and **III** is chemisorption

Sol. **(A, C)**
Graph (I) and (III) represent physiosorption because, in physiosorption, the amount of adsorption decreases with the increase of temperature and increases with the increase of pressure.
Graph (II) represent chemisorption, because in chemisorption amount of adsorption increase with the increase of temperature. Graph (IV) is showing the formation of a chemical bond, hence chemisorption.

PAPER-2 [Code – 8]
IITJEE 2012
PART - III: MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. Let a_1, a_2, a_3, ….. be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$

(A) 22
(B) 23
(C) 24
(D) 25

Sol. **(D)**

a_1, a_2, a_3, are in H.P.

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ are in A.P.

$\Rightarrow \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d < 0$, where $\frac{\frac{1}{25} - \frac{5}{25}}{19} = d = \left(\frac{-4}{9 \times 25}\right)$

$\Rightarrow \frac{1}{5} + (n-1)\left(\frac{-4}{19 \times 25}\right) < 0$

$\frac{4(n-1)}{19 \times 5} > 1$

$n - 1 > \frac{19 \times 5}{4}$

$n > \frac{19 \times 5}{4} + 1 \Rightarrow n \ge 25.$

42. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) $5x - 11y + z = 17$
(B) $\sqrt{2}x + y = 3\sqrt{2} - 1$
(C) $x + y + z = \sqrt{3}$
(D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Sol. **(A)**

Equation of required plane is

$P \equiv (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$

Its distance from (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$\Rightarrow \frac{2}{\sqrt{3}} = \frac{|3(1+\lambda) + (2-\lambda) - (3+\lambda) - (2+3\lambda)|}{\sqrt{(\lambda+1)^2 + (2-\lambda)^2 + (3+\lambda)^2}}$

$= \frac{4}{3} = \frac{(-2\lambda)^2}{3\lambda^2 + 4\lambda + 14} \Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2$

$\Rightarrow \lambda = -\frac{7}{2} \Rightarrow -\frac{5}{2}x + \frac{11}{2}y - \frac{z}{2} + \frac{17}{2} = 0$

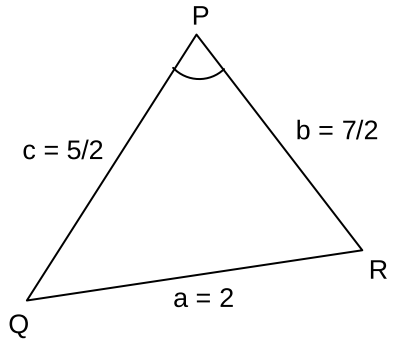
$-5x + 11y - z + 17 = 0$.

43. Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b, and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$

(C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Sol. **(C)**



$$\frac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \frac{1-\cos P}{1+\cos P} = \frac{2\sin^2\frac{P}{2}}{2\cos^2\frac{P}{2}} = \tan^2\frac{P}{2}$$

$$= \frac{(s-b)(s-c)}{s(s-a)}$$

$$= \frac{((s-b)(s-c))^2}{\Delta^2} = \frac{\left(\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\right)^2}{\Delta^2} = \left(\frac{3}{4\Delta}\right)^2$$

44. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}| = \sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0 (B) 3

(C) 4 (D) 8

Sol. **(C)**

$\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right) = \left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$

$\left(\vec{a}+\vec{b}\right)\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right) = \vec{0}$

$\Rightarrow \vec{a}+\vec{b} = \pm\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ (as $\left|\vec{a}+\vec{b}\right| = \sqrt{29}$)

$\Rightarrow \left(\vec{a}+\vec{b}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$

$= \pm(-14 + 6 + 12) = \pm 4$.

45. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$

(C) $PX = 2X$ (D) $PX = -X$

Sol. **(D)**

Give $P^T = 2P + I$

$\Rightarrow P = 2P^T + I = 2(2P + I) + I$

$\Rightarrow P + I = 0$

$\Rightarrow PX + X = 0$

$PX = -X$.

Nitin M Sir (physics-iitjee.blogspot.com)

46. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a > -1$. Then $\lim\limits_{a\to o^+}\alpha(a)$ and $\lim\limits_{a\to o^+}\beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1

(C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

Sol. **(B)**

Let $1 + a = y$

$\Rightarrow (y^{1/3} - 1)\, x^2 + (y^{1/2} - 1)\, x + y^{1/6} - 1 = 0$

$$\Rightarrow \left(\frac{y^{1/3}-1}{y-1}\right)x^2+\left(\frac{y^{1/2}-1}{y-1}\right)x+\frac{y^{1/6}-1}{y-1}=0$$

Now taking $\lim\limits_{y\to 1}$ on both the sides

$$\Rightarrow \frac{1}{3}x^2+\frac{1}{2}x+\frac{1}{6}=0$$

$\Rightarrow 2x^2 + 3x + 1 = 0$

$x = -1, -\frac{1}{2}$.

47. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5, and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$

(C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Sol. **(A)**

Required probability $= 1 - \frac{6\cdot 5^3}{6^4} = 1-\frac{125}{216}=\frac{91}{216}$.

48. The value of the integral $\int\limits_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\, dx$ is

(A) 0 (B) $\frac{\pi^2}{2}-4$

(C) $\frac{\pi^2}{2}+4$ (D) $\frac{\pi^2}{2}$

Sol. **(B)**

$$\int\limits_{-\pi/2}^{\pi/2}\left\{x^2+\ln\left(\frac{\pi+x}{\pi-x}\right)\right\}\cos xdx$$

$$= \int\limits_{-\pi/2}^{\pi/2} x^2\cos xdx+\int\limits_{-\pi/2}^{\pi/2}\ln\left(\frac{\pi+x}{\pi-x}\right)\cos xdx$$

$$= 2\int\limits_{0}^{\pi/2} x^2\cos xdx$$

$$= 2\left[x^2\sin x+2x\cos x-2\sin x\right]_0^{\pi/2}$$

Nitin M Sir (physics-iitjee.blogspot.com)

$$= 2\left[\frac{\pi^2}{4} - 2\right] = \frac{\pi^2}{2} - 4.$$

SECTION II : Paragraph Type

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Paragraph for Questions 49 and 50

A tangent *PT* is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line *L*, perpendicular to *PT* is a tangent to the circle $(x-3)^2 + y^2 = 1$.

49. A possible equation of *L* is

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$

(C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

Sol. **(A)**

Equation of tangent at $P\left(\sqrt{3}, 1\right)$

$\sqrt{3}x + y = 4$

Slope of line perpendicular to above tangent is $\frac{1}{\sqrt{3}}$

So equation of tangents with slope $\frac{1}{\sqrt{3}}$ to $(x - 3)^2 + y^2 = 1$ will be

$$y = \frac{1}{\sqrt{3}}(x-3) \pm 1\sqrt{1+\frac{1}{3}}$$

$\sqrt{3}\,y = x - 3 \pm (2)$

$\sqrt{3}\,y = x - 1$ or $\sqrt{3}\,y = x - 5$.

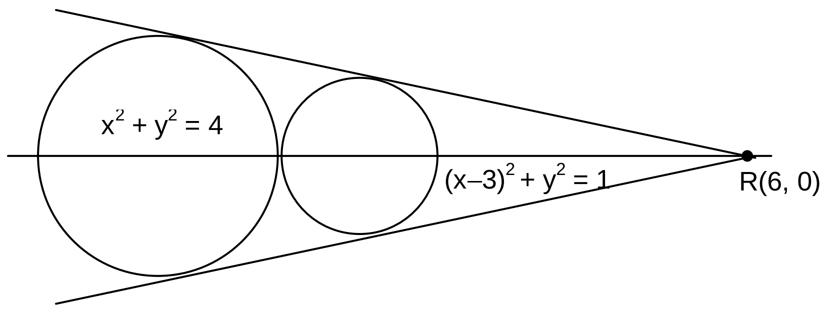
50. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$

(C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{2}y = 6$

Sol. **(D)**

Point of intersection of direct common tangents is (6, 0)



so let the equation of common tangent be

$y - 0 = m(x - 6)$

as it touches $x^2 + y^2 = 4$

$$\Rightarrow \left|\frac{0 - 0 + 6m}{\sqrt{1+m^2}}\right| = 2$$

$9m^2 = 1 + m^2$

$m = \pm\dfrac{1}{2\sqrt{2}}$

So equation of common tangent

$y = \dfrac{1}{2\sqrt{2}}(x-6)$, $y = -\dfrac{1}{2\sqrt{2}}(x-6)$ and also $x = 2$

Paragraph for Questions 51 and 52

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in IR$, and let $g(x) = \displaystyle\int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt$ for all $x \in (1, \infty)$.

51. Consider the statements:

P : There exists some $x \in$ □ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in$ □ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) both **P** and **Q** are true
(B) **P** is true and **Q** is false
(C) **P** is false and **Q** is true
(D) both **P** and **Q** are false

Sol. **(C)**

$f(x) = (1-x)^2\sin^2 x + x^2 \quad \forall\, x \in R$

$g(x) = \displaystyle\int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt \quad \forall\, x \in (1, \infty)$

For statement P :

$f(x) + 2x = 2(1 + x^2) \quad \ldots(i)$

$(1-x)^2\sin^2 x + x^2 + 2x = 2 + 2x^2$

$(1-x)^2\sin^2 x = x^2 - 2x + 2 = (x-1)^2 + 1$

$(1-x)^2 (\sin^2 x - 1) = 1$

$-(1-x)^2\cos^2 x = 1$

$(1-x)^2.\cos^2 x = -1$

So equation (i) will not have real solution

So, P is wrong.

For statement Q :

$2(1-x)^2\sin^2 x + 2x^2 + 1 = 2x + 2x^2 \quad \ldots(ii)$

$2(1-x)^2\sin^2 x = 2x - 1$

$2\sin^2 x = \dfrac{2x-1}{(1-x)^2}$ Let $h(x) = \dfrac{2x-1}{(1-x)^2} - 2\sin^2 x$

Clearly $h(0) = -$ve, $\displaystyle\lim_{x\to 1^-} h(x) = +\infty$

So by IVT, equation (ii) will have solution.

So, Q is correct.

52. Which of the following is true?

(A) g is increasing on $(1, \infty)$
(B) g is decreasing on $(1, \infty)$
(C) g is increasing on (1, 2) and decreasing on $(2, \infty)$
(D) g is decreasing on (1, 2) and increasing on $(2, \infty)$

Sol. **(B)**

$g'(x) = \left(\dfrac{2(x-1)}{x+1} - \ln x\right) f(x)$. For $x \in (1, \infty)$, $f(x) > 0$

Let $h(x) = \left(\dfrac{2(x-1)}{x+1} - \ln x\right) \Rightarrow h'(x) = \left(\dfrac{4}{(x+1)^2} - \dfrac{1}{x}\right) = \dfrac{-(x-1)^2}{(x+1)^2 x} < 0$

Also $h(1) = 0$ so, $h(x) < 0 \quad \forall\, x > 1$

$\Rightarrow g(x)$ is decreasing on $(1, \infty)$.

Nitin M Sir (physics-iitjee.blogspot.com)

Paragraph for Questions 53 and 54

Let a_n denote the number of all n–digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n–digit integers ending with digit 1 and c_n = the number of such n–digit integers ending with digit 0.

53. The value of b_6 is
(A) 7
(B) 8
(C) 9
(D) 11

Sol. **(B)**
$a_n = b_n + c_n$
$b_n = a_{n-1}$
$c_n = a_{n-2} \Rightarrow a_n = a_{n-1} + a_{n-2}$
As $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 5$, $a_5 = 8 \Rightarrow b_6 = 8$.

54. Which of the following is correct?
(A) $a_{17} = a_{16} + a_{15}$
(B) $c_{17} \neq c_{16} + c_{15}$
(C) $b_{17} \neq b_{16} + c_{16}$
(D) $a_{17} = c_{17} + b_{16}$

Sol. **(A)**
As $a_n = a_{n-1} + a_{n-2}$
for $n = 17$
$\Rightarrow a_{17} = a_{16} + a_{15}$.

SECTION III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

55. For every integer n, let a_n and b_n be real numbers. Let function f : IR → IR be given by
$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}$, for all integers n. If f is continuous, then which of the following hold(s) for all n?
(A) $a_{n-1} - b_{n-1} = 0$
(B) $a_n - b_n = 1$
(C) $a_n - b_{n+1} = 1$
(D) $a_{n-1} - b_n = -1$

Sol. **(B, D)**
At x = 2n
L.H.L. = $\lim_{h\to 0}\left(b_n + \cos \pi(2n-h)\right) = b_n + 1$
R.H.L. = $\lim_{h\to 0}\left(a_n + \sin \pi(2n+h)\right) = a_n$
$f(2n) = a_n$
For continuity $b_n + 1 = a_n$
At $x = 2n + 1$
L.H.L = $\lim_{h\to 0}\left(a_n + \sin \pi(2n+1-h)\right) = a_n$
R.H.L = $\lim_{h\to 0}\left(b_{n+1} + \cos\left(\pi(2n+1-h)\right)\right) = b_{n+1} - 1$
$f(2n + 1) = a_n$
For continuity
$a_n = b_{n+1} - 1$
$a_{n-1} - b_n = -1$.

Nitin M Sir (physics-iitjee.blogspot.com)

56. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$ (B) $y + z = -1$
(C) $y - z = -1$ (D) $y - 2z = -1$

Sol. **(B, C)**
For given lines to be coplanar, we get
$$\begin{vmatrix} 2 & k & 2 \\ 5 & 2 & k \\ 2 & 0 & 0 \end{vmatrix} = 0 \Rightarrow k^2 = 4,\ k = \pm 2$$
For $k = 2$, obviously the plane $y + 1 = z$ is common in both lines
For $k = -2$, family of plane containing first line is $x + y + \lambda (x - z - 1) = 0$.
Point $(-1, -1, 0)$ must satisfy it
$-2 + \lambda(-2) = 0 \Rightarrow \lambda = -1$
$\Rightarrow y + z + 1 = 0$.

57. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

(A) -2 (B) -1
(C) 1 (D) 2

Sol. **(A, D)**
$|\text{Adj } P| = |P|^2$ as $(|\text{Adj }(P)| = |P|^{n-1})$
Since $|\text{Adj } P| = 1(3 - 7) - 4(6 - 7) + 4(2 - 1)$
$= 4$
$|P| = 2$ or -2.

58. Let $f : (-1, 1) \to \mathbb{R}$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\frac{3}{2}}$ (B) $1+\sqrt{\frac{3}{2}}$
(C) $1-\sqrt{\frac{2}{3}}$ (D) $1+\sqrt{\frac{2}{3}}$

Sol. **(A, B)**
For $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.
Let $\cos 4\theta = 1/3$
$\Rightarrow \cos 2\theta = \pm\sqrt{\frac{1+\cos 4\theta}{2}} = \pm\sqrt{\frac{2}{3}}$
$$f\left(\frac{1}{3}\right) = \frac{2}{2-\sec^2\theta} = \frac{2\cos^2\theta}{2\cos^2\theta - 1} = 1 + \frac{1}{\cos 2\theta}$$
$$f\left(\frac{1}{3}\right) = 1 - \sqrt{\frac{3}{2}} \text{ or } 1 + \sqrt{\frac{3}{2}}.$$

59. Let X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y|X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct?

Nitin M Sir (physics-iitjee.blogspot.com)

(A) $P(X \cup Y) = \frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^C \cap Y) = \frac{1}{3}$

Sol. **(A, B)**

$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$ and $\frac{P(X \cap Y)}{P(X)} = \frac{1}{3}$

$P(X \cap Y) = \frac{1}{6} \Rightarrow$ P (Y) = $\frac{1}{3}$ and P (X) = $\frac{1}{2}$

Clearly, X and Y are independent

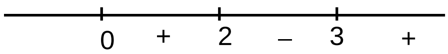
Also, $P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$.

60. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x \in (0, \infty)$, then

(A) f has a local maximum at $x = 2$ (B) f is decreasing on (2, 3)

(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$ (D) f has a local minimum at $x = 3$

Sol. **(A, B, C, D)**



f′ (x) = $e^{x^2}(x-2)(x-3)$

Clearly, maxima at $x = 2$, minima at $x = 3$ and decreasing in $x \in (2, 3)$.

$f'(x) = 0$ for $x = 2$ and $x = 3$ (Rolle's theorem)

so there exist c $\in$ (2, 3) for which

$f''(c) = 0$.