Samples of Mathematics Short Tricks Book for JEE(Main) Volume - I

Magical Conceptual Trick - 1

Applications of The Concept of Identity

Q. The value of $(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$ is equals to :

(a) -3 (b) 0 (c) 1 *(d) 2

S. Here the value of the expression is independent of θ, therefore given expression is an identity in θ, so we can put any suitable value of θ to minimise the calculations.
here let $\theta=45^0$ then value of given expression is $(1+1-\sqrt{2})(1+1+\sqrt{2})=(2-\sqrt{2})(2+\sqrt{2})=2$.

Q. Let $k=4\cos x\cos 2x\cos 3x-\cos x-\cos 2x-\cos 3x$ then k is equals to :

(a) -1 (b) 0 *(c) 1 (d) 2

S. Again value of the expression is independent of x, so the expression is an identity in x,
so let $x=0^0$ then $k=4\cos 0^0\cos 0^0\cos 0^0-\cos 0^0-\cos 0^0-\cos 0^0=4-1-1-1=1$

Q. The value of the expression $\cos^2\alpha+\cos^2(\alpha+\beta)-2\cos\alpha\cos\beta\cos(\alpha+\beta)$ is ?

*(a) $\sin^2\beta$ (b) $\cos^2\beta$ (c) $\sin 2\beta$ (d) $\cos 2\beta$

S. Here options are in terms of β so the value of the expression is independent of α.
so we can put any suitable value of α to minimise the calculations.
put $\alpha=0^0$ then, $\cos^2\alpha+\cos^2(\alpha+\beta)-2\cos\alpha\cos\beta\cos(\alpha+\beta)$
$=\cos^2 0+\cos^2(0+\beta)-2\cos 0\cos\beta\cos(0+\beta)=1+\cos^2\beta-2\cos\beta\cos\beta=1-\cos^2\beta=\sin^2\beta$.

Q. If $\cos\alpha+\cos\beta+\cos\gamma=0$ and if $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=k\cos\alpha\cos\beta\cos\gamma$ then $k=$

(a) 1 (b) 8 *(c) 12 (d) 9

S. $\because\ \cos\alpha+\cos\beta+\cos\gamma=0$ so let $\alpha=0,\ \beta=120^0$ & $\gamma=120^0$ (satisfying given condition)

$\therefore \cos 3\alpha+\cos 3\beta+\cos 3\gamma=k\cos\alpha\cos\beta\cos\gamma$

$\Rightarrow \cos 0^0+\cos 360^0+\cos 360^0=k\cos 0^0\cos 120^0\cos 120^0$

$\Rightarrow 1+1+1=k.1.(-1/2).(-1/2)\Rightarrow k=12$

Q. If $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=k\sin^2\left(\frac{\alpha-\beta}{2}\right)$, then k is equals to:

(a) 0 (b) 1 (c) 3 *(d) 4

S. Let $\alpha=90^0,\beta=0^0$ then $(0-1)^2+(1-0)^2=k\times\frac{1}{2}\Rightarrow k=4$

Q. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x), x\in R$ and $k\geq 1$, then $f_4(x)-f_6(x)=$ **[JEE(Main) 2014]**

(a) $\frac{1}{4}$ *(b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

S. $\because f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)\Rightarrow f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$

As value of above expression is independent of x so put $x=0$ in above expression

$\therefore\ f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 0+\cos^4 0)-\frac{1}{6}(\sin^6 0+\cos^6 0)=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$.

The beauty of these short tricks is that many problems of this kind can be solved easily.

Q. If $\tan\alpha = (1+2^{-x})^{-1}$ and $\tan\beta = (1+2^{1+x})^{-1}$ then the value of $(\alpha+\beta)$ is equals to :

(a) 30^0 *(b) 45^0 (c) 60^0 (d) 90^0

S. from options it is clear that value of the expression is independent of x, so put $x = 0$

then $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$ and by using formula of $\tan(\alpha+\beta)$,

$$\tan(\alpha+\beta) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \frac{5/6}{5/6} = 1 \Rightarrow \alpha+\beta = 45^0.$$

Q. The value of $3\left\{\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^6(3\pi+\theta)\right\}-2\left\{\sin^6\left(\frac{\pi}{2}+\theta\right)+\sin^6(5\pi+\theta)\right\}$ is :

(a) 5 (b) 0 *(c) 1 (d) 3

S. Since the value of the expression is independent of θ, therefore put $\theta = 0^0$

in the expression. then, $3\left\{\sin^4\left(\frac{3\pi}{2}\right)+\sin^6(3\pi)\right\}-2\left\{\sin^6\left(\frac{\pi}{2}\right)+\sin^6(5\pi)\right\}+4$

$= 3\{1+0\}-2\{1+0\} = 3-2 = 1$, so option (c) is currect.

Q. The value of the expression $\cos\alpha\sin(\beta-\gamma)+\cos\beta\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)$ is ?

*(a) 0 (b) -1 (c) 1 (d) 2

S. $\because$ the value of the expression is independent of α, β and γ.

$\therefore$ put $\alpha = \beta = \gamma = 0$ in the expression, then, we will get $0+0+0 = 0$.

Q. $\frac{1}{\sin 3\alpha}\left(\sin^3\alpha+\sin^3\left(\frac{2\pi}{3}+\alpha\right)+\sin^3\left(\frac{4\pi}{3}+\alpha\right)\right)$ is equals to :

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ *(c) $-\frac{3}{4}$ (d) $-\frac{4}{3}$

S. $\because$ The value of above expressions is independent of α so put $\alpha = 30^0$.

then value of given expression $\frac{\left(\sin^3 30^0+\sin^3 150^0+\sin^3 270^0\right)}{\sin 90^0} = \frac{1}{8}+\frac{1}{8}-1 = -\frac{3}{4}$.

Q. Which of the followings is not equals to unity :

(a) $\cos^4\theta-\sin^4\theta+2\sin^2\theta$ (b) $\frac{\sin^2\theta}{2}(1+\cot^2\theta)+\frac{\cos\theta}{2}(1+\tan^2\theta)$

(c) $\sin^2\theta\cos^2\phi+\cos^2\theta\sin^2\phi+\sin^2\theta\sin^2\phi+\cos^2\theta\cos^2\phi$ *(d) $\sin^4\theta-\cos^4\theta-2\sin^2\theta$

S. $\because$ the value of the above expressions should be 1.

$\therefore$ put $\theta = \alpha = 45^0$ ($\because \sin 45^0 = \cos 45^0$ and also $\tan 45^0 = \cot 45^0$).

(a) $\cos^4\theta-\sin^4\theta+2\sin^2\theta = 0+2\times\frac{1}{2} = 1$

(b) $\frac{\sin^2\theta}{2}(1+\cot^2\theta)+\frac{\cos\theta}{2}(1+\tan^2\theta) = \frac{1}{4}.2+\frac{1}{4}.2 = \frac{1}{2}+\frac{1}{2} = 1$

(c) $\sin^2\theta\cos^2\phi+\cos^2\theta\sin^2\phi+\sin^2\theta\sin^2\phi+\cos^2\theta\cos^2\phi = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 1$

(d) $\sin^4\theta-\cos^4\theta-2\sin^2\theta = 0-2\times\frac{1}{2} = -1$ (not equals to unity).

The beauty of these short tricks is that many problems of this kind can be solved easily.

Magical Conceptual Trick - 2

Magical Method of Substitutions and Balancing

Q. If $\cos\alpha = 2\cos\beta$ then the value of $\tan\left(\frac{\alpha-\beta}{2}\right)\tan\left(\frac{\alpha+\beta}{2}\right)$ is equals to :

(a) $\frac{1}{3}$ *(b) $-\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$

S. **Magical Method of Substitution** : –

here by observation if we put $\alpha = 0^0$ and $\beta = 60^0$ in the given condition then

$\cos 0^0 = 2\cos 60^0 \Rightarrow 1 = 2.\frac{1}{2} \Rightarrow 1 = 1$ (satisfied)

then $\tan\left(\frac{\alpha-\beta}{2}\right)\tan\left(\frac{\alpha+\beta}{2}\right) = \tan\left(-30^0\right)\tan\left(30^0\right) = -\frac{1}{3}$.

Q. The value of the expression $\frac{\sin 5\theta + \sin 2\theta - \sin\theta}{\cos 5\theta + 2\cos 3\theta + 2\cos^2\theta + \cos\theta}$ is equals to :

*(a) $\tan\theta$ (b) $\cos\theta$ (c) $\cot\theta$ (d) $\sin\theta$

S. **Magical Method of Substitution and Balancing** : –

Let $\theta = 30^0$ then LHS = $\frac{\sin 150^0 + \sin 60^0 - \sin 30^0}{\cos 150^0 + 2\cos 90^0 + 2\cos^2 30^0 + \cos 30^0} = \frac{1}{\sqrt{3}}$

Now put $\theta = 30^0$ in options then (a) will match with L.H.S.

Q. The value of $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$ is :

(a) $\tan\alpha$ (b) $\tan 2\alpha$ *(c) $\cot\alpha$ (d) $\cot 2\alpha$

S. **Magical Method of Substitution and Balancing** : –

Let $\alpha = 15^\circ$ then LHS = $\tan 15^\circ + 2\tan 30^\circ + 4\tan 60^\circ + 8\cot 120^\circ$

$= 2-\sqrt{3} + 2\times\frac{1}{\sqrt{3}} + 4\sqrt{3} + 8\left(-\frac{1}{\sqrt{3}}\right) = 2+\sqrt{3} = \cot(15^\circ) = \cot\alpha$

Q. If $\tan A - \tan B = x$ and $\cot A - \cot B = y$ then $\cot(A-B)$ is equals to:

(a) $\frac{1}{x}+\frac{1}{y}$ *(b) $\frac{1}{x}-\frac{1}{y}$ (c) $x+y$ (d) $x-y$

S. **Magical Method of Substitution** : – Let $A = 60^0$ & $B = 30^0$ then

$x = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ and $y = \frac{1}{\sqrt{3}} - \sqrt{3} = -\frac{2}{\sqrt{3}}$, so L.H.S. $= \cot(60^0 - 30^0) = \cot 30^0 = \sqrt{3}$

now put the values of x and y in options then option (b) will gives $\sqrt{3}$ so it is R.H.S.

Q. If $\tan A = \frac{p}{q}$ and if $\alpha = 6\beta$ (α is acute angle) then $\frac{1}{2}(p\,\text{cosec}\,2\beta - q\sec 2\beta)$ is equals to:

(a) p^2+q^2 *(b) $2\sqrt{p^2+q^2}$ (c) $2\sqrt{p^2-q^2}$ (d) $\sqrt{p^2-q^2}$

S. **Magical Method of Substitution** : – Let $A = 45^0$ then $q = p$

and LHS $= p\,\text{cosec}15 - p\sec 15 = p\left(\frac{2\sqrt{2}}{\sqrt{3}-1} - \frac{2\sqrt{2}}{\sqrt{3}+1}\right) = 2\sqrt{2}\,p$

now put $q = p$ in options then (b) will give $2\sqrt{2}\,p$ so it is R.H.S.

☞ *The beauty of these short tricks is that many problems of this kind can be solved easily.*

Q. If $0 \le \left(\alpha_1, \alpha_2, \alpha_3, \ldots\ldots\ldots\ldots\ldots\alpha_n\right) \le \frac{\pi}{2}$ and if $\tan\alpha_1 \tan\alpha_2 \tan\alpha_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\tan\alpha_n = 1$ then $\cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\cos\alpha_n =$ **[IITJEE 2001]**

(a) $2^{n/2}$ *(b) $2^{-n/2}$ (c) $2^{n/4}$ (d) $2^{-n/4}$

S. **Magical Method of Substitution and Balancing** : –

Let $\alpha_1 = \alpha_2 = \alpha_3 = \ldots\ldots\ldots\ldots = \alpha_n = 45^0$ (Satisfying both given conditions)

Required value $= \cos 45^0 \times \cos 45^0 \times \cos 45^0 \ldots\ldots\ldots\ldots$ n terms

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{n tertms} = \frac{1}{(\sqrt{2})^n} = \frac{1}{2^{n/2}} = 2^{-n/2}$$

Q. If $X = \sin\left(\theta + \frac{7\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12}\right) + \sin\left(\theta + \frac{3\pi}{12}\right), Y = \cos\left(\theta + \frac{7\pi}{12}\right) + \cos\left(\theta - \frac{\pi}{12}\right) + \cos\left(\theta + \frac{3\pi}{12}\right)$

then $\frac{X}{Y} - \frac{Y}{X} =$

(a) $2\sin 2\theta$ (b) $2\cos 2\theta$ *(c) $2\tan 2\theta$ (d) $2\cot 2\theta$

S. **Magical Method of Substitution** : – Let $\theta = 15^0$.

then $X = \sin 120^0 + \sin 0^0 + \sin 60^0 = \frac{\sqrt{3}}{2} + 0 + \frac{\sqrt{3}}{2} = \sqrt{3}$

and $Y = \cos 120^0 + \cos 0^0 + \cos 60^0 = -\frac{1}{2} + 1 + \frac{1}{2} = 1$, then L.H.S. $= \frac{\sqrt{3}}{1} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

Now put $\theta = 15^0$ in the options then option (c) will gives $\frac{2}{\sqrt{3}}$ so it is R.H.S.

Q. If $a = \sin x + \sin y$, $b = \cos x + \cos y$, $c = \tan x + \tan y$ then $\frac{8ab}{(a^2 + b^2)^2 - 4a^2} =$

*(a) c (b) c^2 (c) 2c (d) $2c^2$

S. **Magical Method of Substitution** : – let $x = y = 45^0$, therefore $a = \sqrt{2}$, $b = \sqrt{2}$, $c = 2$

$$\text{then } \frac{8\sqrt{2}\sqrt{2}}{(\sqrt{2}^{\,2} + \sqrt{2}^{\,2})^2 - 4\sqrt{2}^{\,2}} = 2 = c$$

Q. If $\frac{(\cos 1^0 + \sin 1^0)(\cos 2^0 + \sin 2^0)(\cos 3^0 + \sin 3^0)\ldots\ldots\ldots\ldots(\cos 45^0 + \sin 45^0)}{\cos 1^0 \cos 2^0 \cos 3^0 \cos 4^0 \ldots\ldots\ldots\ldots \cos 44^0 \cos 45^0} = a^b$,

where a and b are prime numbers and $a < b$ then $a + b =$

(a) 22 (b) 23 (c) 24 *(d) 25

S. $\frac{(\cos 1^0 + \sin 1^0)}{\cos 1^0} \frac{(\cos 2^0 + \sin 2^0)}{\cos 2^0} \frac{(\cos 3^0 + \sin 3^0)}{\cos 3^0} \ldots\ldots\ldots \frac{(\cos 44^0 + \sin 44^0)}{\cos 44^0} \frac{(\cos 44^0 + \sin 44^0)}{\cos 45^0}$

$(1+\tan 1°)(1+\tan 2°)\ldots\ldots\ldots\ldots\ldots\ldots(1+\tan 43°)(1+\tan 44°)(1+\tan 45°)$

$\because\ 1° + 44° = 2° + 43° = 3° + 42° = \ldots\ldots\ldots = 45^0$

So first we find out if $\alpha + \beta = 45^0$ then $(1+\tan\alpha)(1+\tan\beta) = ?$

By method of substitution let $\alpha = 0$ and $\beta = 45°$ then $(1+\tan\alpha)(1+\tan\beta) = (1) \times 2 = 2$

$\underbrace{(1+\tan 1°)(1+\tan 44°)(1+\tan 2°)(1+\tan 43°)\ldots\ldots\ldots\ldots(1+\tan 22°)(1+\tan 23°).}_{\text{22 pairs}} \times (1+1) = a^b$

$\Rightarrow 2^{22} \times 2 = 2^k \Rightarrow 2^{23} = a^b \Rightarrow a = 2$ and $b = 23$ and hence $a + b = 25$

The beauty of these short tricks is that many problems of this kind can be solved easily.

Inverse Trigonometric Functions (I.T.F.)

Q. If $x^2+y^2+z^2=k^2$ then value of $\tan^{-1}\left(\frac{xy}{zk}\right)+\tan^{-1}\left(\frac{xz}{yk}\right)+\tan^{-1}\left(\frac{zy}{xk}\right)$ is equals to ?

*(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 0

S. **Method of Substitution** : – Let $x=y=z=1$, then $k^2=3\Rightarrow k=\sqrt{3}$

$$\therefore\ \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30^0+30^0+30^0=90^0=\frac{\pi}{2}$$

Q. If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$ then value of $1000(\alpha+\beta+\gamma)-\frac{300}{\alpha^2+\beta^2+\gamma^2}$ is :

(a) 0 (b) 2890 (c) 1900 *(d) 2900

S. **Method of Substitution** : –

$$\because \sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2},$$

$$\therefore\ \text{Let } \sin^{-1}\alpha=\sin^{-1}\beta=\sin^{-1}\gamma=\frac{\pi}{2}\Rightarrow\alpha=\beta=\gamma=\sin\left(\frac{\pi}{2}\right)=1$$

$$\text{L.H.S.}=1000(\alpha+\beta+\gamma)-\frac{300}{\alpha^2+\beta^2+\gamma^2}$$

$$=1000(1+1+1)-\frac{300}{1+1+1}=3000-100=2900$$

Q If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$ and if $9x^2-12xy\cos\theta+4y^2=k^2\sin^2\theta$ then $k=$

(a) ± 3 (b) ± 1 *(c) ± 6 (d) ± 2

S. **Method of Substitution** : – Let $x=1$ and $y=0$, $\theta=\cos^{-1}\left(\frac{1}{2}\right)+\cos^{-1}(0)=60°+90°=150°$

$$\therefore\ 9x^2-12xy\cos\theta+4y^2=k^2\sin^2\theta$$

$$\Rightarrow 9\times1-0+0=k^2\left(\frac{1}{2}\right)^2\Rightarrow 9=\frac{k^2}{4}\Rightarrow k^2=36\Rightarrow k=\pm 6.$$

Q. The value of $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{x}{y}\right)\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{x}{y}\right)\right\}$ is equals to :

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{2x}{y}$ *(d) $\frac{2y}{x}$

S. **Method of substitution :** - Let $x=1$ and $y=2$ $\left(\because \cos^{-1}\left(\frac{1}{2}\right)=30^0\right)$

$$\therefore\ \text{L.H.S.}=\tan\left\{\frac{\pi}{4}+\frac{1}{2}.30^0\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}.30^0\right\}$$

$$=\tan 75^0+\tan 15^0=\cot 15^0+\tan 15^0=2+\sqrt{3}+2-\sqrt{3}=4$$

Now put $x=1$ and $y=2$ in options then (d) will give 4 and (d) is right choice.

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Q Let $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$ then $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)} =$

*(a) 2abc (b) abc (c) $\frac{1}{2}abc$ (d) $\frac{abc}{3}$

S. **Method of Substitution** :–

$\because \sin^{-1}a + \sin^{-1}b + \sin^{-1}c = 180°$,

Let $\sin^{-1}a = \sin^{-1}b = \sin^{-1}c = 60° \Rightarrow a = b = c = \frac{\sqrt{3}}{2}$

L.H.S. $= a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$

$= \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} + \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} + \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} = \frac{3\sqrt{3}}{4}$

after putting the values of a, b, c in theoptions (a) will give the R.H.S.

Q If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$ then $4x^2 - 4xy\cos\alpha + y^2$ is equals to :

*(a) $4\sin^2\alpha$ (b) $-4\sin^2\alpha$ (c) $2\sin 2\alpha$ (d) 4

S. **Method of substitution :-** Let $x = 0$ and $y = 1, \alpha = \cos^{-1}0 - \cos^{-1}\frac{1}{2} = 90^0 - 60° = 30°$.

L.H.S. $= 4x^2 - 4xy\cos\alpha + y^2 = 0 + 0 + 1 = 1$

now put the value of α in options then only option (a) will match with L.H.S.

Q. If $0 < x < 1$, then $\sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1\right]^{1/2} =$

[IITJEE2008]

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) x *(c) $x\sqrt{1+x^2}$ (d) $\sqrt{1+x^2}$

S. **Method of Substitution** :–

$\because 0 < x < 1$ therefore let $x = \frac{1}{\sqrt{3}}$ $\left(\because \frac{1}{\sqrt{3}} = 0.57 \text{ approx.}\right)$

$\therefore \sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1\right]^{1/2}$

$= \sqrt{1+\frac{1}{3}}\sqrt{\left(\frac{1}{\sqrt{3}}\cos(60^0) + \sin(60^0)\right)^2 - 1}$

$= \frac{2}{\sqrt{3}}\sqrt{\left(\frac{1}{\sqrt{3}} \times \frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2 - 1} = \frac{2}{\sqrt{3}}\sqrt{\frac{1}{12} + \frac{3}{4} + 2\frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2} - 1}$

$= \frac{2}{\sqrt{3}}\sqrt{\frac{1}{12} + \frac{3}{4} - \frac{1}{2}} = \frac{2}{\sqrt{3}}\sqrt{\frac{1+9-6}{12}} = \frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{2}{3}$

now put $x = \frac{1}{\sqrt{3}}$ in options then (c) will give $\frac{2}{3}$, so it is right choice.

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Quadratic Equations

Q. If the roots of the equation $ax^2+bx+c=0$ are real and distinct then :

(a) both roots are greater than $-\frac{b}{2a}$. (b) both roots are less than $-\frac{b}{2a}$.

*(c) one of roots exceeds $-\frac{b}{2a}$. (d) both roots are exceeds $-\frac{b}{2a}$.

S. **Master quadratic equation** :– Let $\alpha=1,\ \beta=2$ then $x^2-3x+2=0 \Rightarrow a=1, b=-3, c=2$

$\therefore\ a=1, b=-3, c=2$ so $\frac{-b}{2a}=\frac{-(-3)}{2\times1}=\frac{3}{2}<2 \Rightarrow$ One of the roots exceeds $\left(-\frac{b}{2a}\right)$

Q. If α, β are the roots of $x^2-ax+b=0$ and if $\alpha^n+\beta^n=V_n$, then :

(a) $V_{n+1}=aV_n+bV_{n-1}$ *(b) $V_{n+1}=aV_n-bV_{n-1}$ (c) $V_{n+1}=aV_n+aV_{n-1}$ (d) $V_{n+1}=aV_{n-1}-bV_n$

S. **Method of substitution** :- Let $\alpha=1$ & $\beta=2$ then eq. is $x^2-3x+2=0 \Rightarrow a=3,\ b=2$

Let n=0 then $V_0=2$, n=1 then $V_1=\alpha+\beta=3$ n=2 then $V_2=\alpha^2+\beta^2=1+4=5$

Now check the options. Put n=1 in the options then

(a) $V_2=aV_1+bV_0 \Rightarrow 5=3\times3+2\times2 \Rightarrow 5=9+4=13$

(b) $V_2=aV_1-bV_0 \Rightarrow 5=3\times3-2\times2 \Rightarrow 9-4=5$ so (b) is the Right choice.

Q. If the roots of $x^2-bx+c=0$ are two consecutive integers, then $b^2-4c=$

*(a) 1 (b) 2 (c) 3 (d) 4

S. **Method of substitution** :-

$\because$ Roots are two consecutive integers so let $\alpha=1$ & $\beta=2$.

then equation is $x^2-3x+2=0 \Rightarrow b=3,\ c=2$ therefore $b^2-4c=(3)^2-4\times2=9-8=1$

Q. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of $ax^2+bx+c=0\ (a\neq0, a, b\in R)$ then the equation

$x(x+b^3)+(a^3-3abx)=0$ has roots :– **[JEE (Main) online 2014]**

(a) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$ *(b) $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$ (c) $\alpha\beta^{\frac{1}{2}}$ and $\alpha^{\frac{1}{2}}\beta$ (d) $\sqrt{\alpha\beta}$ and $\alpha\beta$

S. **Method of substitution** :- Let $\alpha=4$ and $\beta=9$ then the equation is $6x^2-5x+1=0$

and $a=6, b=-5, c=1$ so $x(x+b^3)+(a^3-3abx)=0$ becomes $x^2-35x+216=0$

and $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$ i.e. $4^{\frac{3}{2}}$ and $9^{\frac{3}{2}}$ or 8 & 27 satisfy this.

Q. If p and q are nonzero real numbers and $\alpha^3+\beta^3=-p, \alpha\beta=q$ then the quadratic equation whose roots are $\frac{\alpha^2}{\beta},\frac{\beta^2}{\alpha}$ is :– **[JEE (Main)online, 2014]**

*(a) $qx^2+px+q^2=0$ (b) $qx^2-px+q^2=0$ (c) $px^2-qx+p^2=0$ (d) $px^2+qx+p^2=0$

S. **Method of substitution** :- Let $\alpha=1$ and $\beta=2$ then the equation is $x^2-3x+2=0$

and $p=-9, q=2$ and roots of required equation are $\frac{\alpha^2}{\beta}=\frac{1}{2}$ and $\frac{\beta^2}{\alpha}=\frac{4}{1}$

and required equation is $2x^2-9x+4=0$ or $qx^2+px+q^2=0$

The beauty of these short tricks is that many problems of this kind can be solved easily.

Q. If α and β $(\alpha<\beta)$ are the roots of $x^2+bx+c=0$, where $c<0<b$, then **[IITJEE2000]**

(a) $0<\alpha<\beta$ *(b) $\alpha<0<\beta<|\alpha|$ (c) $\alpha<\beta<0$ (d) $\alpha<0<|\alpha|<\beta$

S. **Combination of methods of substitution and balancing : -**

let $c=-2, b=1$ ($\because c<0<b$) then given equation will be $x^2+x-2=0 \Rightarrow \beta=1, \alpha=-2$

After putting the values of α and β in options, option (b) will be satisfied.

Q. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has : **[IITJEE2000]**

(a) both roots in (a,b) (b) both roots in $(-\infty,a)$

(c) both roots in (b,∞) *(d) one root in $(-\infty,a)$ and the other in (b,∞)

S. **Methods of substitution : -** $\because$ $b>a$ so let $a=1, b=2$

$\therefore$ given equation will be $(x-1)(x-2)-1=0 \Rightarrow x^2-3x+1=0 \Rightarrow x=\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

$\because x=\frac{3+\sqrt{5}}{2}>2 \Rightarrow x>b \Rightarrow$ one root lies in (b,∞).

again $x=\frac{3-\sqrt{5}}{2}<1 \Rightarrow x<a \Rightarrow$ one root lies in $(-\infty,a)$ so option (d) is currect .

Q. If one root of the equation $x^2+px+q=0$ is square of the other then: **[IITJEE2004]**

*(a) $p^3=q(3p-1)+q^2$ (b) $p^3=q(3p+1)+q^2$ (c) $p^3+q(3p-1)+q^2=0$ (d) $p^3+q(3p+1)+q^2=0$

S. **Combination of methods of substitution and balancing : -**

$\because$ One roots is square of the other roots, therefore let $\alpha=1$ and $\beta=1$

then the equation is $x^2-2x+1=0$, $p=-2$ and $q=1$

Put these values of p & q in the options then (a) will satisfy so it is right option.

Q. If the equation $a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x=0$, $a_1\neq 0$, $n\geq 2$, has a positive root $x=\alpha$ then the equation $na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\ldots\ldots\ldots\ldots+a_1=0$ has a positive root, which is :

(a) Greater then or equal to α. (b) Equal to α. (c) Greater then α. *(d) Smaller then α.

S. **Method of substitution : -** Let $n=2$ then, $a_nx^n+a_{n-1}x^{n-1}+\ldots\ldots+a_1x=0 \Rightarrow a_2x^2+a_1x=0$

$a_2x^2+a_1x=0 \Rightarrow x=0$ and $x=-\frac{a_1}{a_2}=\alpha$ (let)

$na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\ldots+a_1=0$ will be $2a_2x+a_1=0 \Rightarrow x=-\frac{a_1}{2a_2}=\frac{\alpha}{2}$ (smaller than α).

Q. Let p and q be real numbers such that $p\neq 0, p^3\neq q$ and $p^3\neq q$ If α & β are non-zero complex numbers satisfying $\alpha+\beta=-p$ and $\alpha^3+\beta^3=q$ then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is :– **[IITJEE2010]**

(a) $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$. *(b) $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$.

(c) $(p^3-q)x^2-(5p^3-2q)x+(p^3-q)=0$. (d) $(p^3-q)x^2-(p^3+2q)x+(p^3-q)=0$.

S. **Combination of methods of substitution and balancing : -**

$\because$ α and β are nonzero complex numbers so let $\alpha=\omega$ and $\beta=\omega^2$

$\therefore$ $\alpha+\beta=\omega+\omega^2=-1=-p \Rightarrow p=1$ and $\alpha^3+\beta^3=\omega^3+(\omega^2)^3=2=q$

so required equation is $x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+1=0 \Rightarrow x^2+x+1=0$

put the value of $p\,\&\,q$ in options then (b) will give required result.

☞ *The beauty of these short tricks is that many problems of this kind can be solved easily.*

Samples of Mathematics Short Tricks Book for JEE(Main) Volume - I

Progressions

Arithmatic Progression

Master A.P.: – **1, 2, 3, 4, 5, 6, ...**

Q. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms of an A.P. then

$$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$$

*(a) 0 (b) 1 (c) $S_1S_2S_3$ (d) $n_1n_2n_3$

S. **Method of substitution : -** Let A.P. be :– **1, 2, 3, 4, 5, 6, ...(Master A.P.)**

Let $n_1 = 1$ then $S_1 = 1$, for $n_2 = 2$, $S_2 = 1 + 2 = 3$ and for $n_3 = 3$, $S_3 = 1+2+3=6$

$$\therefore \frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) = \frac{1}{1}(-1) + \frac{3}{2}(2) + \frac{6}{3}(-1) = -1 + 3 - 2 = 0.$$

Q. If S_n denotes the sum of first 'n' terms of a A.P., then $\dfrac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} =$

(a) $2n - 1$ *(b) $2n + 1$ (c) $2n$ (d) $\dfrac{n}{2}$

S. **Combination of methods of substitution and balancing : -**

Let the A.P. be **1, 2, 3, 4, 5, 6, ...(Master A.P.)**

Let n = 3, therefore LHS = $\dfrac{S_9 - S_2}{S_6 - S_5} = \dfrac{45 - 3}{21 - 15} = \dfrac{42}{6} = 7$

After putting the value of n = 3 in the option (b) will give 7, so it is R.H.S.

Q. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\, a_n$ are in A.P., then $\dfrac{1}{a_1a_2} + \dfrac{1}{a_2a_3} + \ldots\ldots\ldots\ldots + \dfrac{1}{a_{n-1}a_n} =$

(a) $\dfrac{1}{a_1a_n}$ (b) $\dfrac{n}{a_1a_n}$ *(c) $\dfrac{n-1}{a_1a_n}$ (d) $\dfrac{n+1}{a_1a_n}$.

S. **Combination of methods of substitution and balancing : -**

Let A.P. be **1, 2, 3, 4, 5, 6, ...(Master A.P.)**

and let n = 3 then $a_1 = 1, a_2 = 2, a_3 = 3$, $(a_n = 3)$ then

LHS = $\dfrac{1}{2} + \dfrac{1}{6} = \dfrac{2}{3}$.

Now put the values of $a_1=1$, $a_n=3$ in the option then (c) will give $\dfrac{2}{3}$, so it is R.H.S.

Q. The value of 0.573737373 ....or $0.5\dot{7}\dot{3}$. or $0.5\overline{73}$. is :

(a) $\dfrac{284}{497}$ (b) $\dfrac{284}{495}$ (c) $\dfrac{558}{990}$ (d) $\dfrac{567}{990}$

S. ***Short trick : - Remove all decimals i.e. 573 then subtract that number which has no recurring decimal, Then divide by as many 9's as many digits has recurring decimal and as many zero's as many digits has no recurring decimal.***

$\because$ 0.573737373...............∞ = $0.5\dot{7}\dot{3}$

$$= \frac{573-5}{990} = \frac{568}{990} = \frac{284}{495}.$$

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Harmonic Progression

Applications of master H.P. 3, 4, 6, 12.......

Q. If $H_1, H_2, H_3 \ldots\ldots\ldots\ldots H_{2n+1}$ are H.P., the $\sum_{i=1}^{2n}(-1)^i\left(\frac{H_i+H_{i+1}}{H_i-H_{i+1}}\right) =$

(a) $2n-1$ *(b) $2n+1$ (c) $2n$ (d) $2n+2$

S. **Combination of methods of substitution and balancing :-**

$\because$ $H_1, H_2, H_3, \ldots\ldots$ are in H.P. therefore **Let H.P. be master H.P. 3, 4, 6, 12........**

$\therefore$ $H_1 = 3,\ H_2 = 4,\ H_3 = 6,$ and Let $n = 1$

$$\text{LHS.} = \sum_{i=1}^{2n}(-1)^i\left[\frac{H_i+H_{i+1}}{H_i-H_{i+1}}\right]$$

$$= -\left[\frac{H_1+H_2}{H_1-H_2}\right] + \frac{H_2+H_3}{H_2-H_3} = -\left[\frac{3+4}{3-4}\right] + \left[\frac{4+6}{4-6}\right] = 2$$

Put $n = 1$ in the options then only (c) will match with L.H.S.

Q. If $a_1, a_2, a_3 \ldots\ldots\ldots\ldots, a_n$ are in H.P. then $a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots\ldots + a_{n-1}a_n$ is equals to:

(a) na_1a_n *(b) $(n-1)a_1a_n$ (c) $(n+1)a_1a_n$ (d) None of these

S. **Combination of methods of substitution and balancing :-**

$\because$ $a_1, a_2, a_3 \ldots\ldots, a_n$ are in H.P. therefore **Let H.P. be master H.P. 3, 4, 6, 12........**

also let $n=4$, then $a_1 = 3,\ a_2 = 4,\ a_3 = 6,\ a_4 = 12,$

$\text{LHS} = a_1a_2 + a_2a_3 + a_3a_4 = 3\times4 + 4\times6 + 6\times12 = 108$

After putting values of a_1 & a_n and n in the options (b) will match with LHS.

Q. If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, a_{100}$ are in H. P. then $\sum_{i=1}^{99}\frac{a_ia_{i+1}}{a_1a_{100}}$ is equals to :

(a) 100 *(b) 99 (c) 101 (d) 110

S. ***Master Result :- From above question If $a_1, a_2, a_3 \ldots\ldots\ldots\ldots, a_n$ are in H.P.***

then $a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{n-1}a_n = (n-1)a_1a_n$

$$\because \sum_{i=1}^{99}\frac{a_ia_{i+1}}{a_1a_{100}} = \frac{1}{a_1a_{100}}\sum_{i=1}^{99}a_ia_{i+1} = \frac{1}{a_1a_{100}}(a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots\ldots\ldots + a_{99}a_{100})$$

for $n = 100$, $(a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots + a_{99}a_{100}) = (100-1)a_{99}a_{100} = 99a_{99}a_{100}$

$$\therefore \sum_{i=1}^{99}\frac{a_ia_{i+1}}{a_1a_{100}} = \frac{1}{a_1a_{100}}.99\,a_{99}a_{100} = 99$$

Q. If $a_1, a_2, a_3, \ldots\ldots$ are in H.P. and if $f(k) = \sum_{r=1}^{n}a_r - a_k$, then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots\ldots\ldots\ldots \frac{a_n}{f(n)}$ are in:

(a) G.P. *(b) H.P. (c) A.P. (d) A.G.P.

S. **Method of substitution :- Let H.P. be master H.P. 3, 4, 6, 12........**

Let $\underset{\underset{3}{\downarrow}}{a_1}, \underset{\underset{4}{\downarrow}}{a_2}, \underset{\underset{6}{\downarrow}}{a_3}, \underset{\underset{12}{\downarrow}}{a_4}$ .... are in H.P.

$\because$ $f(k) = a_1 + a_2 + a_3 + \ldots\ldots + a_n - a_k$ let $n = 4$

$\therefore$ $f(k) + a_k = a_1 + a_2 + a_3 + a_4$

$\Rightarrow f(1) = 22,\ f(2) = 21,\ f(3) = 19$

$\therefore$ $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots\ldots\ldots\ldots \frac{a_n}{f(n)} \Rightarrow \frac{3}{22}, \frac{4}{21}, \frac{6}{19}, \ldots\ldots$ .are in H.P.

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Geometric Progression

Q. The sum of n terms of a G.P. is S, product is P and sum of their inverse is R, then the value of P^2 is equals to :

(a) $\frac{R}{S}$ (b) $\frac{S}{R}$ (c) $\left(\frac{R}{S}\right)^n$ *(d) $\left(\frac{S}{R}\right)^n$

S. **Combination of methods of substitution and balancing : -**

Let the master G.P. 1,2 ,4,8, 16...... Let n = 3 then S = 7, P = 8, $R = 1+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}$

L.H.S = $P^2 = 64$,Now put the values of n.S,R in options then (d) will match with L.H.S.

Q. If m is the A.M. of two distinct real numbers l and n ($l, n > 1$) and G_1, G_2, G_3 are three geometric means between l and n then $G_1^4 + 2G_2^4 + G_3^4 =$ **[JEE(Main) 2015]**

(a) $4l^2mn$ *(b) $4lm^2n$ (c) $4lmn^2$ (d) $4l^2m^2n^2$

S. **Combination of methods of substitution and balancing : -**

Let master G.P. $\underset{l}{2}, \underset{G_1}{4}, \underset{G_2}{8}, \underset{G_3}{16}, \underset{n}{32}$ ($\because l, n > 1$) and $m = \frac{l+n}{2} = \frac{2+32}{2} = 17$

$G_1^4 + 2G_2^4 + G_3^4 = 4^4 + 2.8^4 + 16^4 = 4^4 + 2^5.4^4 + 4^8 = 4^4(1+32+256) = 4^4.289 = 4lm^2n$

Q. If a, a_1, a_2..........a_{2n}, b are in arithmetic progression and a, g_1, g_2, ...g_{2n}, b are in gemettric progression and 'h' is the harmonic mean of a and b, then the value of the expression $\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n+1}} + \ldots\ldots + \frac{a_n + a_{n+1}}{g_n g_{n+1}}$ is :

(a) 2nh (b) $\frac{n}{h}$ (c) nh *(d) $\frac{2n}{h}$

S. **Combination of methods of substitution and balancing : -**

Let n = 1 then a, a_1, a_2, b are A.P. and a, g_1, g_2, b are in G.P. and $h = \frac{2ab}{a+b}$

$\because$ a, a_1, a_2, b are A.P. $\Rightarrow a + b = a_1 + a_2$ and a, g_1, g_2, b are in G.P. $\Rightarrow ab = g_1 g_2$.

L.H.S $= \frac{a_1 + a_2}{g_1 g_2} = \frac{a+b}{ab} = \frac{2}{h}$, now put n = 1 in option then (d) will give R.H.S.

Q. If the sum of first n terms of an A.P. is cn^2, then sum of squares of these n terms is : – **[IITJEE 2009]**

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$ *(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

S. **Method of substitution : -**

$\because T_n = S_n - S_{n-1} = cn^2 - c(n-1)^2 = c(2n-1) \Rightarrow T_n^2 = c^2(2n-1)^2$, $\because T_1^2 = S_1^2 = c^2$

If we put n = 1 in options the result should be c^2, so (c) is right option.

Determinants

Q. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

then k is equals to : – **[JEE(Main) 2014]**

(a) 1 (b) -1 (c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

S. **Method of substitution : -**

Let $\alpha = 2, \beta = -1$ then $f(n) = 2^n + (-1)^n$

$\therefore f(1) = 2^1 + (-1)^1 = 1,\ f(2) = 2^2 + (-1)^2 = 5,\ f(3) = 2^3 + (-1)^3 = 7,\ f(4) = 2^4 + (-1)^4 = 17$

$$\begin{vmatrix} 3 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{vmatrix} = K(1-2)^2(1+1)^2(2+1)^2 = 36K$$

$$= \begin{vmatrix} 3 & 2 & 1 \\ 2 & 6 & 0 \\ 6 & 8 & 4 \end{vmatrix} = 3(24-0) - 2(8-0) + 1(16-36) = 72 - 16 - 30 = 36 = 36K \Rightarrow K = 1$$

Q. For all real numbers x, y and z, the value of the determinant

$\begin{vmatrix} 2x & xy - xz & y \\ 2x+z+1 & xy - xz + yz - z^2 & 1+y \\ 3x+1 & 2xy - 2xz & 1+y \end{vmatrix}$ is equals: **(JEE(Main)P2, 2017)**

(a) $(y-xz)(z-x)$ (b) $(x-y)(y-z)(z-x)$ (c) 0 (d) $(x-yz)(y-z)$

S. **Combination of methods of substitution and balancing : -**

$\because x, y, z \in R$ so let $x = 1, y = 2, z = 3$

$$\therefore \begin{vmatrix} 2 & -1 & 2 \\ 6 & -4 & 3 \\ 4 & -2 & 3 \end{vmatrix} = 2(-12+6) + 1(18-12) + 2(-12+16) = -12 + 6 + 8 = 2$$

(a) $(y-xz)(z-x) = (2-3)(3-1) = -2$

(b) $(x-y)(y-z)(z-x) = (1-2)(2-3)(3-1) = 2.$

(d) $(1-6)(2-3) = 5$

so (b) is right choice.

Q. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} (\lambda \neq 0)$ then k =

(JEE (Main) online, 12April 2014)

(a) $4\lambda abc$ *(b) $4\lambda^2$ (c) $-4\lambda^2$ (d) $-4\lambda abc$

S. **Combination of methods of substitution and balancing : -**

Let $a = 1, b = 2$ and $c = 3$ and $\lambda = 1$

$$\therefore \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 0 & 1 & 4 \end{vmatrix} = k.1 \begin{vmatrix} 1 & 4 & 9 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} \Rightarrow k = 4$$

put the values $a = 1, b = 2$ and $c = 3$ and $\lambda = 1$ in options

then only (b) is right choice.

The beauty of these short tricks is that many problems of this kind can be solved easily.