JEE Class Companion Physics

for JEE Main and Advanced

Module-2

MOTION
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CHAPTER 1

Simple Harmonic Motion

PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular intervals of time, its motion is said to be periodic motion and the interval of time is called time period (T). The path of periodic motion may be linear, circular, elliptical or any other curve; for example, rotation of the earth around the sun.

Oscillatory Motion

To and fro type of motion is called oscillatory motion. A particle has oscillatory motion when it moves about stable equilibrium position. It need not be periodic and need not have fixed extreme positions.

The oscillatory motions in which energy is conserved are also periodic, for example, motion of pendulum of a wall clock.

The force/torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force/torque. **Damped oscillations** are those in which energy consumed due to some resistive forces and hence total mechanical energy decreases and after some time oscillation will stop.

Oscillatory Equation

Consider a particle free to move on the x-axis that is being acted upon by a force given by

$$F = -kx^n$$

The above equation is called oscillatory equation. Here, k is a positive constant and x is the displacement from the mean position.

Now, the following cases are possible depending on the value of n.

1. If n is an even integer (0, 2, 4, ...), force is always along the negative x-axis whether x is positive or negative. Hence, the motion of the particle is not oscillatory. If the particle is released from any position on the x-axis (except $x = 0$), a force in the negative direction of x-axis acts on it and it moves rectilinearly along the negative x-axis.
2. If n is an odd integer (1, 3, 5, ...), force is along the negative x-axis for $x > 0$ and along the positive x-axis for $x < 0$ and zero for $x = 0$. Thus, the particle will oscillate about the stable equilibrium position $x = 0$. The force in this case is called the restoring force.

If $n = 1$, i.e., $F = -kx$, the motion is said to be SHM (simple harmonic motion).

If the restoring force/torque acting on the body in oscillatory motion is directly proportional to the displacement of the body/particle with reference to the mean position and is always directed towards equilibrium position, then the motion is called simple harmonic motion (SHM). It is the simplest form of oscillatory motion.

TYPES OF SHM

Linear SHM

A particle is said to be in linear simple harmonic motion when it moves to and fro about an equilibrium point, along a straight line. In Fig. 1.1, A and B are extreme positions and M is the mean position, so $AM = MB$ = amplitude.

Figure 1.1

Angular SHM

A body/particle is said to be in angular simple harmonic motion when it is free to rotate about a given axis and executes angular oscillations.

ANALYSIS OF MOTION IN LINEAR SHM

When the particle is moved away from the mean position or equilibrium position and released, a force ($-kx$) comes into play to pull it back towards the mean position. By the time it attains the mean position, it has picked up some kinetic energy and so it overshoots, stopping somewhere on the other side and is again pulled back towards the mean position.

It is necessary to study the change in speed and acceleration of a particle during SHM. Let us consider a particle whose position is $x = 0$ at $t = 0$ and $v = v_0$ (Fig. 1.2). Then we divide the motion of the particle in one time period into four parts:

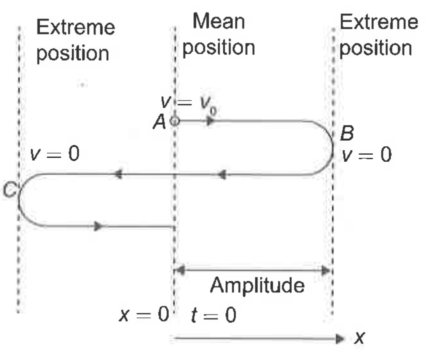


Figure 1.2

(A) from A to B **(B)** from B to A
(C) from A to C **(D)** from C to A

Note

As shown in Fig. 1.2, the path of the particle is a straight line.

(1) *Motion of a Particle from A to B*

Initially, the particle is at A (mean position) and is moving towards the positive x-direction with speed v_0. As the particle is moving towards B, force acting on it towards A is increasing. Consequently, its acceleration towards A is increasing in magnitude, while its speed decreases and finally it comes to rest momentarily at B.

(2) *Motion of a Particle from B to A*

Now the particle starts moving towards A with initial speed $v = 0$. As the particle is moving towards A, force is acting on it towards A and decreasing as it approaches A. Consequently, its acceleration towards A is decreasing in magnitude while its speed increases and finally it comes to A with the same speed $v = v_0$.

(3) *Motion of a Particle from A to C*

The motion of a particle from A to C is qualitatively the same as the motion of a particle from A to B.

(4) *Motion of a Particle from C to A*

It is qualitatively the same as the motion of a particle from B to A.

Summary

Motion from	Velocity (direction/ magnitude)	Acceleration (direction/magnitude)
A→B	V→↓	a←↑
B→A	V←↑	a←↓
A→C	V←↓	a→↑
C→A	V→↑	a→↓

CHARACTERISTICS OF SHM

Mean Position

It is the position where the net force on the particle is zero.

Extreme Point

It is the point where the speed of the particle is zero.

Displacement

It is defined as the distance of the particle from the mean position at that instant.

Amplitude

It is the maximum value of displacement of the particle from its mean position.

Extreme position − Mean position = Amplitude

It depends on the energy of the system.

Frequency

The frequency of SHM is equal to the number of complete oscillations per unit time.

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \text{s}^{-1} \text{ or Hz}$$

Time Period

Smallest time interval after which the oscillatory motion gets repeated is called the time period.

$$T = \frac{2\pi}{\omega}$$

SOLVED EXAMPLE

EXAMPLE 1

Describe the motion of a particle acted upon by a force.

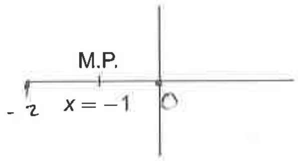
(a) $F = 3x + 3$ **(b)** $F = -3x - 3$
(c) $F = -3x + 3$ **(d)** $F = 3x - 3$

SOLUTION

(a) Given

$$F = 3x + 3 \quad (1)$$

We find the mean position at which the net force on the particle is zero.



$$\Rightarrow \quad 3x + 3 = 0$$

$$\Rightarrow \quad x = -1.$$

If we substitute $x = 0$ in Eq. (1), then

$$F = 3\text{ N} \quad (2)$$

(away from M.P.)

Now substituting $x = -2$ in Eq. (1)

$$F = -3\text{ N} \quad (3)$$

(away from M.P.)

From Eqs. (1) and (2), we conclude that the particle does not perform SHM.

(b) Given

$$F = -3x - 3 \quad (4)$$

at M.P. $F = 0$

$$\Rightarrow \quad x = -1.$$

Now put $x = 0$ in Eq. (4)

$$\Rightarrow \quad F = -3\text{ N (towards M.P.)}$$

If $x = -2,$

$$F = 3\text{ N (towards M.P.)}$$

We conclude from the above calculation that in every case (whether the particle is left from M.P. or right from M.P.), force acts towards M.P. so the particle performs S.H.M.

(c) Given

$$F = -3x + 3$$

when $F = 0$

$$x = 1\text{ (M.P.)}.$$

Now put $x = 0$.

Then $F = 3$ N (towards M.P.).

If $x = 2$

$$F = -3\text{ (towards M.P.)}.$$

i.e., Particle performs SHM.

(d) Given

$$F = 3x - 3$$

Mean position at $x = 1$.
When $x = 0$,

$$F = -3\text{ N (away from M.P.)}.$$

If $x = 2$,

$$F = 3\text{ N (away from M.P.)}.$$

Particle does not perform SHM. ■

EQUATION OF SIMPLE HARMONIC MOTION

The necessary and sufficient condition for SHM is given as

$$F = -kx.$$

We can write the above equation in the following way:

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (1)$$

Equation (1) is double differential equation of SHM.

Now $$\frac{d^2x}{dt^2} + \omega^2 x = 0.$$

The solution for the above equation is

$$x = A\sin(\omega t + \phi),$$

where

$$\omega = \text{angular frequency} = \sqrt{\frac{k}{m}}$$

x = displacement from mean position

k = SHM constant.

The equality $(\omega t + \phi)$ is called the phase angle or simply the phase of the SHM and ϕ is the initial phase, i.e., the phase at $t = 0$ and depends on the initial position and direction of velocity at $t = 0$.

To understand the role of ϕ in SHM, we take two particles performing SHM in the following condition:

Suppose we choose $t = 0$ at an instant when the particle is passing through its mean position towards right (i.e., positive direction) as shown in Fig. 1.3, then

In Fig. 1.3, at $t = 0$ $x = 0$

i.e., $$x = A \sin \omega t.$$

$\therefore$ The particle is at its mean position.

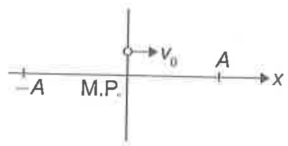


Figure 1.3

In Fig. 1.4 at $t = 0$, $x = A$, and the particle is moving towards the mean position,

i.e., $$x = A \sin(\omega t + \pi/2).$$

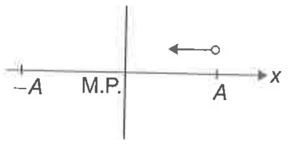


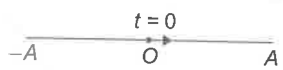
Figure 1.4

Here, $\pi/2$ is the only phase possible.

SOLVED EXAMPLES

EXAMPLE 2

A particle starts from the mean position and moves towards the positive extreme as shown below. Find the equation of the SHM. Amplitude of SHM is A.



SOLUTION

General equation of SHM can be written as

$$x = A \sin(\omega t + \phi).$$

At $t = 0$, $x = 0$,

$$\therefore \quad 0 = A \sin \phi$$

$$\therefore \quad \phi = 0, \pi$$

$$\phi \in [0, 2\pi).$$

Also, at $t = 0$, $v = +\text{ve}$.

$$\therefore \quad A\omega \cos \phi = +\text{ve}$$

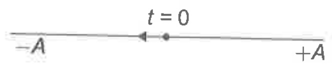
or $$\phi = 0.$$

Hence, if the particle is at mean position at $t = 0$ and is moving towards the positive extreme, then the equation of the SHM is given by

$$x = A \sin \omega t.$$

Similarly, for a particle moving towards the negative extreme,

$$\phi = \pi.$$



$\therefore$ Equation of SHM is

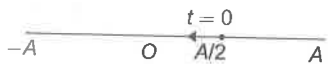
$$x = A \sin(\omega t + \pi)$$

or $$x = -A \sin \omega t.$$

■

EXAMPLE 3

Write the equation of SHM for the situation shown below:



SOLUTION

General equation of SHM can be written as

$$x = A \sin(\omega t + \phi).$$

At $t = 0$, $x = A/2$,

$$\Rightarrow \quad \frac{A}{2} = A \sin \phi$$

$$\Rightarrow \quad \phi = 30^\circ, 150^\circ.$$

Also at $t = 0$, $v = -\text{ve}$,

$$A\omega \cos \phi = -\text{ve}$$

$$\Rightarrow \quad \phi = 150^\circ.$$

■

VELOCITY

It is the rate of change of particle displacement with respect to time at that instant.

Let the displacement from the mean position is given by

$$x = A\sin(\omega t + \phi)$$

Velocity $$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$v = A\omega\cos(\omega t + \phi)$$

$$v = \omega\sqrt{A^2 - x^2}.$$

At the mean position ($x = 0$), velocity is maximum,

$$v_{max} = \omega A.$$

At the extreme position ($x = A$), velocity is minimum,

$$v_{min} = \text{zero}.$$

Graph of Velocity (*v*) versus Displacement (*x*)

$$v = \omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

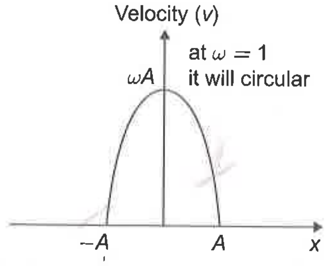


Figure 1.5

The graph would be a half ellipse (Fig. 1.5).

ACCELERATION

It is the rate of change of a particle's velocity with respect to time at that instant.

Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[A\omega\cos(\omega t + \phi)\right]$$

$$a = -\omega^2 A\sin(\omega t + \phi)$$

$$a = -\omega^2 x.$$

Notes

Negative sign shows that acceleration is always directed towards the mean position. At the mean position ($x = 0$), acceleration is minimum,

$$a_{min} = \text{zero}.$$

At the extreme position ($x = A$), acceleration is maximum,

$$|a_{max}| = \omega^2 A.$$

Graph of Acceleration (*A*) versus Displacement (*x*)

$$a = -\omega^2 x$$

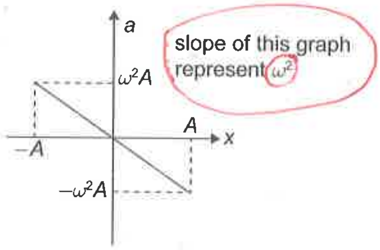


Figure 1.6

GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY AND ACCELERATION IN SHM

Displacement

$$x = A\sin\omega t$$

Velocity,

$$v = A\omega\cos\omega t$$

$$= A\omega\sin\left(\omega t + \frac{\pi}{2}\right)$$

or $$v = \omega\sqrt{A^2 - x^2}$$

Acceleration,

$$a = -\omega^2 A\sin\omega t$$

$$= \omega^2 A\sin(\omega t + \pi)$$

or $$a = -\omega^2 x.$$

Note

$v = \omega\sqrt{A^2 - x^2}$

$a = -\omega^2 x.$

These relations are true for any equation of x.

Time, t	0	$T/4$	$T/2$	$3T/4$	T
Displacement, x	0	A	0	$-A$	0
Velocity, v	$A\omega$	0	$-A\omega$	0	$A\omega$
Acceleration, a	0	$-\omega^2 A$	0	$\omega^2 A$	0

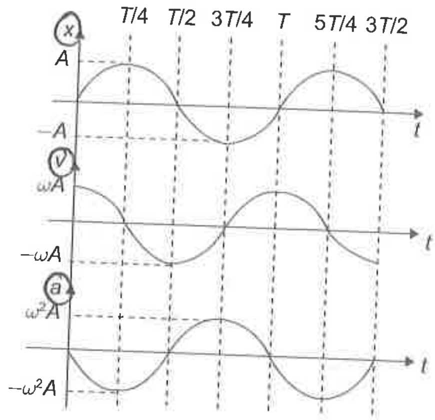


Figure 1.7

1. All the three quantities displacement, velocity and acceleration vary harmonically with time, having the same period.
2. The maximum velocity is ω times the amplitude ($V_{max} = \omega A$).
3. The acceleration is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
4. In SHM, the velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$.
5. In SHM, the acceleration is ahead of velocity by a phase angle of $\frac{\pi}{2}$.

SOLVED EXAMPLES

EXAMPLE 4

The equation of a particle executing simple harmonic motion is $x = (5\,\text{m})\sin\left[(\pi s^{-1})t + \frac{\pi}{3}\right]$. Write down the amplitude, time period and maximum speed. Also find the velocity at $t = 1$ s.

SOLUTION

Comparing with equation $x = A\sin(\omega t + \phi)$, we see that the amplitude = 5 m

and time period $= \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\pi s^{-1}} = 2\text{ s}$$

The maximum speed $= A\omega$

$$= 5\text{ m} \times \pi\text{ s}^{-1}$$

$$= 5\pi\text{ m/s.}$$

The velocity at time t

$$\Rightarrow \quad \frac{dx}{dt} = A\omega\cos(\omega t + \delta).$$

At $t = 1$ s,

$$v = (5\text{ m})(\pi\text{ s}^{-1})\cos\left(\pi + \frac{\pi}{3}\right)$$

$$= -\frac{5\pi}{2}\text{ m/s.}$$

EXAMPLE 5

A particle executing simple harmonic motion has angular frequency 6.28 s^{-1} and amplitude 10 cm. Find (a) the time period, (b) the maximum speed, (c) the maximum acceleration, (d) the speed when the displacement is 6 cm from the mean position, (e) the speed at $t = 1/6$ s assuming that the motion starts from rest at $t = 0$.

SOLUTION

(a) Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{6.28}\text{s}$

$$= 1\text{ s.}$$

(b) Maximum speed $= A\omega$

$$= (0.1\text{ m})(6.28\text{ s}^{-1}).$$

(c) Maximum acceleration $= A\omega^2$

$$= (0.1\text{ m})(6.28\text{ s}^{-1})^2$$

$$= 4\text{ m/s}^2.$$

(d)
$$v = \omega\sqrt{A^2 - x^2}$$
$$= (6.28\ \text{s}^{-1})\sqrt{(10\ \text{cm})^2 - (6\ \text{cm})^2}$$
$$= 50.2\ \text{cm/s}$$

(e) At $t = 0$, the velocity is zero, i.e., the particle is at an extreme. The equation for displacement may be written as

$$x = A\cos\omega t.$$

The velocity is

$$v = -A\omega\sin\omega t.$$

At $t = \frac{1}{6}\text{s}$,

$$v = -(0.1\ \text{m})\ (6.28\ \text{s}^{-1})\sin\left(\frac{6.28}{6}\right)$$
$$= (-0.628\ \text{m/s})\ \sin\frac{\pi}{3}$$
$$= -54.4\ \text{cm/s}$$

(towards mean position). ■

Note

If the mean position is not at the origin, then we can replace x by $x - x_0$ and the equation becomes $x - x_0 = -A\sin\omega t$, where x_0 is the position co-ordinate of the mean position.

EXAMPLE 6

A particle of mass 2 kg is moving on a straight line under the action force $F = (8 - 2x)$ N. It is released at rest from $x = 6$ m.

(a) Is the particle moving simple harmonically?
(b) Find the equilibrium position of the particle.
(c) Write the equation of motion of the particle.
(d) Find the time period of SHM.

SOLUTION

$$F = 8 - 2x$$

or
$$F = -2(x - 4).$$

For equilibrium position, $F = 0$,

$\Rightarrow$ $x = 4$ m is in equilibrium position.

Hence, the motion of the particle is SHM with force constant 2 and equilibrium position $x = 4$.

(a) Yes, motion is SHM.
(b) Equilibrium position is $x = 4$ m.
(c) At $x = 6$ m, particle at rest, i.e., it is one of the extreme position. Hence, amplitude is $A = 2$ m and initially the particle at the extreme position.

$\therefore$ Equation of SHM can be written as

$$x - 4 = 2\cos\omega t,$$

where
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1(\text{s})^{-1},$$

i.e.,
$$x = 4 + 2\cos t.$$

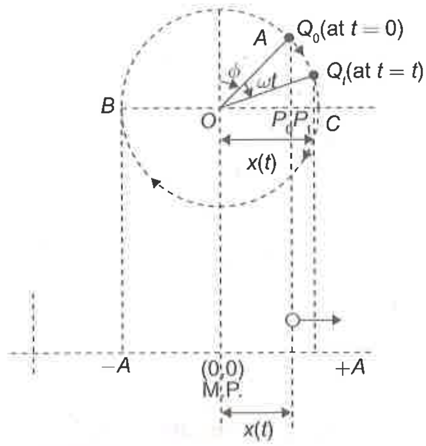
(d) Time period,
$$T = \frac{2\pi}{\omega} = 2\pi\ \text{s}.$$ ■

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle Q, moving on a circle of radius A with a constant angular velocity ω. The projection of Q on a diameter BC is P. It is clear from the figure that as Q moves around the circle, the projection P executes a simple harmonic motion on the x-axis between B and C. The angle that the radius OQ makes with the positive vertical in the clockwise direction in at $t = 0$ is equal to phase constant (ϕ).

Let the radius OQ_0 makes an angle ωt with the OQ_t at time t. Then,

$$x(t) = A\sin(\omega t + \phi)$$



In the above discussion, the foot of projection is x-axis. So it is called a horizontal phasor. Similarly, the foot of perpendicular on y-axis will also execute SHM of amplitude A and angular frequency $\omega[y(t) = A\cos\omega t]$. This is called vertical phasor. The phases of the two SHM differ by $\pi/2$.

Problem Solving Strategy in Horizontal Phasor

1. First assume a circle of radius equal to amplitude of SHM.
2. Assume a particle is rotating in a circular path moving with a constant ω same as that of SHM in clockwise direction.
3. Angle made by the particle at $t = 0$ with the upper vertical is equal to phase constant.
4. Horizontal component of velocity of particle gives you the velocity of particle performing SHM, for example.

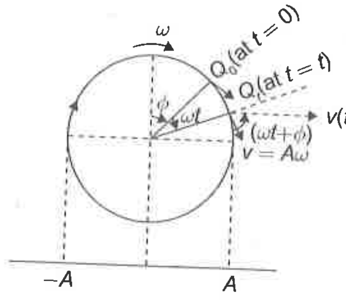


Figure 1.8

From Fig. 1.8,

$$v(t) = A\omega \cos(\omega t + \phi).$$

5. Component of acceleration of a particle in the horizontal direction is equal to the acceleration of particle performing SHM. The acceleration of a particle in uniform circular motion is only centripetal and has a magnitude $a = \omega^2 A$.

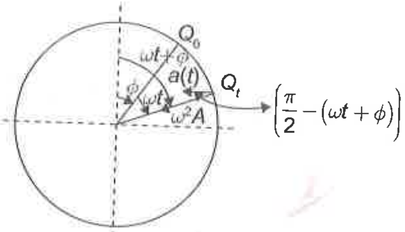


Figure 1.9

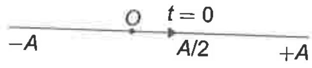
From Fig. 1.9,

$$a(t) = -\omega^2 A \sin(\omega t + \phi).$$

SOLVED EXAMPLES

EXAMPLE 7

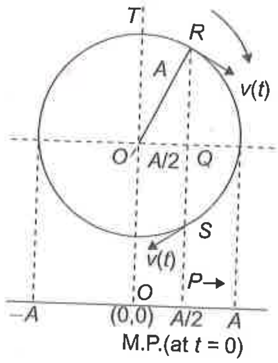
A particle starts from $A/2$ and moves towards the positive extreme as shown below. Find the equation of the SHM. Given amplitude of SHM is A.



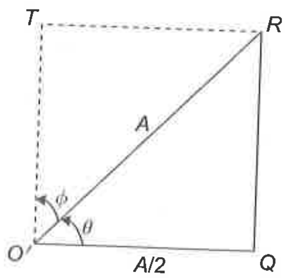
SOLUTION

We will solve the above problem with the help of horizontal phasor.

Step 1: Draw a perpendicular line in the upward direction from point P on the circle, which cuts it at point R and S.



Step 2: Horizontal component of $v(t)$ at R gives the direction P to A while at S gives P to O. So at $t = 0$ particle is at R.



Step 3: In ΔORQ,

$$\cos\theta = \frac{A/2}{A} = 60°$$

$\Rightarrow$

$$\phi = 30°.$$

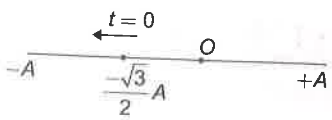
So, equation of the SHM is

$$x = A\sin(\omega t + 30°).$$

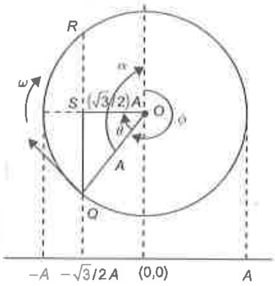
■

EXAMPLE 8

A particle starts from point $x = \frac{-\sqrt{3}}{2}A$ and move towards negative extreme as shown.



(a) Find the equation of the SHM.
(b) Find the time taken by the particle to go directly from its initial position to negative extreme.
(c) Find the time taken by the particle to reach at mean position.



SOLUTION

The figure shows the solution of the problem with the help of phase. Horizontal component of velocity at Q gives the required direction of velocity at $t = 0$.

In ΔOSQ,

$$\cos\theta = \frac{\sqrt{3}/2A}{A} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \qquad \theta = \frac{\pi}{6}.$$

Now

$$\phi = \frac{3\pi}{2} - \frac{\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

So, equation of SHM is

$$x = A\sin\left(\omega t + \frac{4\pi}{3}\right).$$

(b) Now to reach the particle at left extreme point, it will travel angle θ along the circle. So time taken,

$$t = \frac{\theta}{\omega} = \frac{\pi}{6\omega}$$

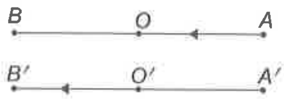
$$\Rightarrow \qquad t = \frac{T}{12}\text{ s}.$$

(c) To reach the particle at mean position, it will travel an angle $\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$.

So, time taken $= \frac{\alpha}{\omega} = \frac{T}{3}$ s.

EXAMPLE 9

Two particles undergo SHM along the parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time.

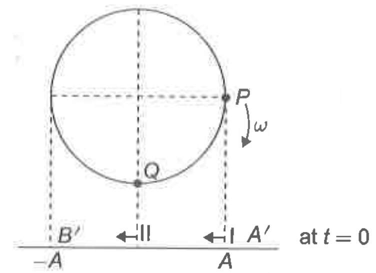


(a) $T/8$ (b) $3T/8$
(c) $T/6$ (d) $4T/3$

SOLUTION

This problem is easy to solve with the help of a phasor diagram.

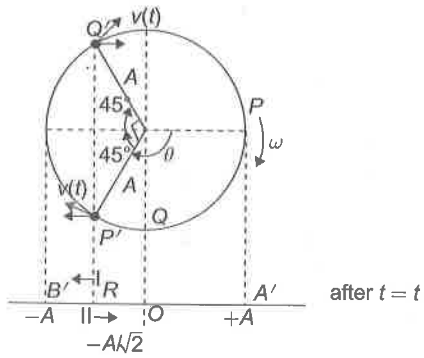
First, we draw the initial position of both the particles on the phasor as shown in the figure.



From the above figure, phase difference between both the particles is $\pi/2$.

They will cross each other when their projection from the circle on the horizontal diameter meets at one point.

Let after time t both will reach at $P'Q'$ point having phase difference $\pi/2$ as shown in the figure.



Both will meet at $-A/\sqrt{2}$.

When they meet, angular displacement of P is

$$\theta = \pi/2 + \pi/4 = 3\pi/4.$$

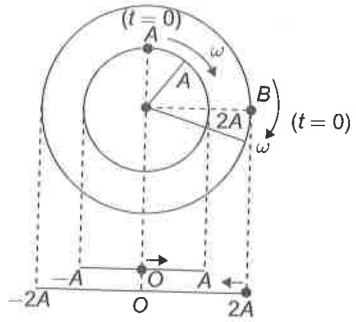
So they will meet after time

$$t = \frac{3\pi}{4\times\omega},$$

$$t = \frac{3\pi}{4\times 2\pi}\times T$$

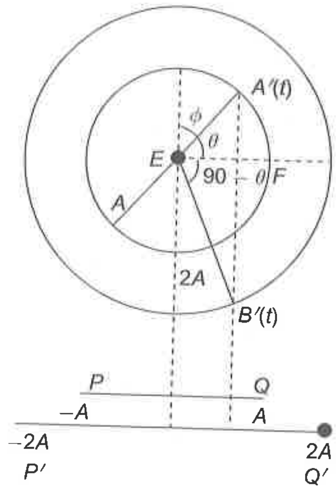
$$= \frac{3T}{8}\text{ s}$$

■



From the figure, phase difference between both the particles is $\pi/2$.

They will meet each other when their projection from the circle on the horizontal diameter meets at one point.



Now from the figure,

$$EF = A\cos\theta = 2A\sin\theta$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right).$$

So time taken by the particle to cross each other

$$t = \frac{\text{angle travelled by } A}{\omega}$$

$$\Rightarrow \quad t = \frac{\pi/2 - \theta}{\omega}.$$

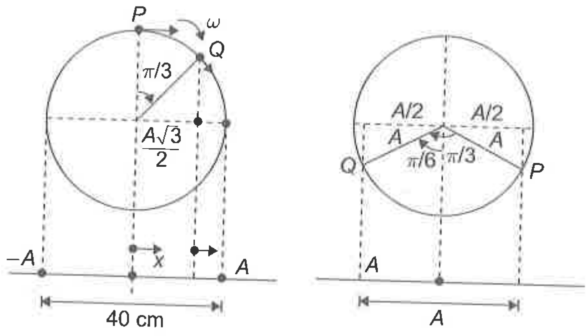
■

EXAMPLE 10

Two particles execute SHM of the same amplitude of 20 cm with the same period along the same line about the same equilibrium position. If the phase difference is $\pi/3$, then find out the maximum distance between these two.

SOLUTION

Let us assume that one particle starts from the mean position and another starts at a distance x having $\phi = \pi/3$. This condition is shown in the figure.



(i) (ii)

The above figures show the situation of maximum distance between them.

So maximum distance $= A = 10$ cm (as $2A = 20$ cm).

■

EXAMPLE 11

Two particles execute SHM of the same time period but different amplitudes along the same line. One starts from the mean position having amplitude A and other starts from the extreme position having amplitude $2A$. Find out the time when they both will meet.

SOLUTION

We solve the above problem with the help of a phasor diagram.

First, we draw the initial position of both the particles on the phasor.

EXAMPLE 12

Two particles have the time periods T and $5T/4$. They start SHM at the same time from the mean position. After how many oscillations of the particle having smaller time period, will they be again in the same phase?

SOLUTION

They will be again at m.p. and moving in same direction when the particle having smaller time period makes n_1 oscillations and the other one makes n_2 oscillations.

$$\Rightarrow \quad n_1 T = \frac{5T}{4} \times n_2$$

$$\frac{n_1}{n_2} = \frac{5}{4}$$

$$\Rightarrow \quad n_1 = 5, n_2 = 4.$$ ■

ENERGY OF SHM

Kinetic Energy

$$\text{Kinetic energy (KE)} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} mA^2\omega^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m\omega^2(A^2 - x^2)$$

$$\therefore \quad \omega^2 = \frac{k}{m}$$

$$\Rightarrow \quad \text{KE} = \frac{1}{2} K\left(A^2 - x^2\right)$$

$$\text{KE}_{max} = \frac{1}{2} KA^2 \text{ (at } x = 0)$$

$$\text{KE}_{min} = 0 \quad (\text{at } x = A);$$

$$\langle KE \rangle_{0-T} = \frac{1}{4} kA^2;$$

$$\langle KE \rangle_{0-A} = \frac{1}{3} kA^2.$$

Frequency of KE = 2 × (frequency of SHM).

Potential Energy

Simple harmonic motion is defined by the equation

$$F = -kx.$$

The work done by the force F during a displacement from x to $x + dx$ is

$$dW = Fdx = -kxdx.$$

The work done in a displacement from $x = 0$ to x is

$$W = \int_0^x (-kx)dx$$

$$= -\frac{1}{2} kx^2$$

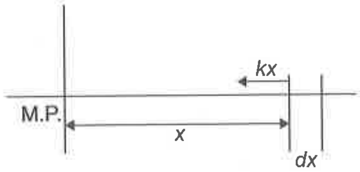


Figure 1.10

Let $U(x)$ be the potential energy of the system when the displacement is x. As the change in potential energy corresponding to a conservative force is the negative of the work done by that force,

$$U(x) - U_{M.P.} = -W = \frac{1}{2} kx^2.$$

Let us choose the potential energy to be zero when the particle is at the mean oscillation position, $x = 0$.

$$\text{Then} \quad U_{M.P.} = 0$$

$$\text{and} \quad U(x) = \frac{1}{2} kx^2 \qquad < PE >_{0-1} = \frac{1}{4} ka^2$$

$$\because \quad k = m\omega^2$$

$$\therefore \quad U(x) = \frac{1}{2} m\omega^2 x^2 \qquad < PE >_{0-x} = \frac{1}{6} ka^2$$

$$U = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi).$$

$$\text{But} \quad x = A \sin(\omega t + \phi).$$

Kinetic energy of the particle at any instant is

$$K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} mA^2\omega^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m\omega^2 (A^2 - x^2).$$

So the total mechanical energy at time t is

$$E = U + K$$

$$\Rightarrow \quad E = \frac{1}{2} m\omega^2 A^2.$$

Note

$U_{min} = U_{M.P.}$ (which is not always = 0).

(i)

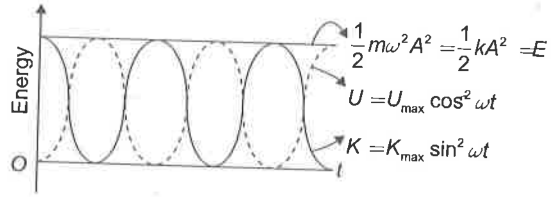


Figure 1.11

Potential, kinetic and total energy plotted as a function of time.

(ii)

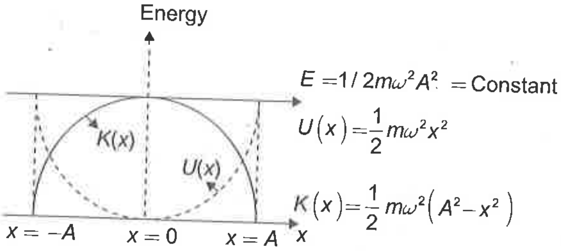


Figure 1.12

Potential, kinetic and total energy are plotted as a function of displacement from the mean position.

SOLVED EXAMPLES

EXAMPLE 13

A particle of mass 0.50 kg executes a simple harmonic motion under a force $F = -(50\ \text{N/m})x$. If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of the motion.

SOLUTION

The kinetic energy of the particle when it is at the centre of oscillation is

$$E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(0.60\ \text{kg})(10\ \text{m/s})^2$$

$$= 2.5\ \text{J}$$

The potential energy is zero here. At the maximum displacement $x = A$, the speed is zero and hence the kinetic energy is zero. The potential energy here is $\frac{1}{2}kA^2$. As there is no loss of energy,

$$\frac{1}{2}kA^2 = 2.5\ \text{J}.$$

The force on the particle is given by

$$F = -(50\ \text{N/m})\,x.$$

Thus, the spring constant is $k = 50$ N/m.

Equation (1) gives

$$\frac{1}{2}(60\ \text{N/m})\,A^2 = 2.5\ \text{J}$$

or,
$$A = \frac{1}{\sqrt{10}}\ \text{m}.$$

■

METHOD TO DETERMINE TIME PERIOD AND ANGULAR FREQUENCY IN SIMPLE HARMONIC MOTION

To understand the steps which are usually followed to find out the time period, we will take one example.

EXAMPLE 14

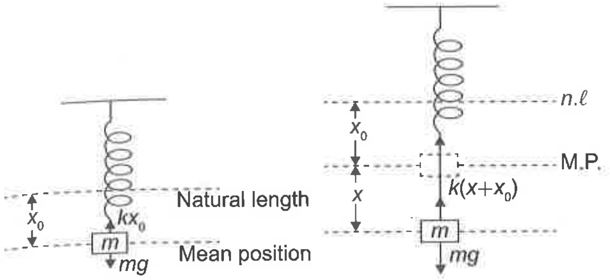
A mass m is attached to the free end of a massless spring of spring constant k with its other end fixed to a rigid support as shown in the figure. Find out the time period of the mass, if it is displaced slightly by an amount x downward.

SOLUTION

The following steps are usually followed in this method:

Step 1: Find the stable equilibrium position which is usually known as the mean position. Net force or torque on the particle at this position is zero. Potential energy is minimum.

In our example, initial position is the mean position.



Step 2: Write down the mean position–force relation. In the above figure, at the mean position,

$$kx_0 = mg \tag{1}$$

Step 3: Now displace the particle from its mean position by a small displacement x (in linear SHM) or angle θ (in case of an angular SHM) as shown in the figure.

Step 4: Write down the net force on the particle in the displaced position.

From the above figure,

$$F_{\text{net}} = mg - k(x + x_0) \tag{2}$$

Step 5: Now try to reduce this net force equation in the form of $F = -kx$ (in linear SHM) or $\tau = -k\theta$ (in angular SHM) using mean position–force relation in step 2 or binomial theorem.

From Eq. (2),

$$F_{\text{net}} = mg - kx - kx_0$$

Using Eq (1) in the above equation,

$$F_{\text{net}} = -kx \tag{3}$$

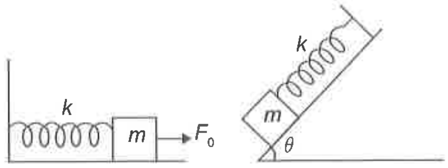
Equation (3) shows that the net force acts towards the mean position and is proportional to x, but in this SHM, constant K_{SHM} is replaced by the spring constant k. So,

$$T = 2\pi\sqrt{\frac{m}{K_{\text{SHM}}}} = 2\pi\sqrt{\frac{m}{k}}$$ ■

Note

If we apply constant force on the string, then time period T is always the same $T = 2\pi\sqrt{\dfrac{m}{K_{\text{SHM}}}}$.

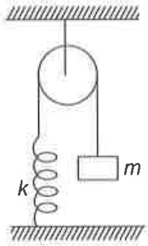
Note (contd)



In the above, both cases $T = \left(2\pi\sqrt{\dfrac{m}{k}}\right)$

EXAMPLE 15

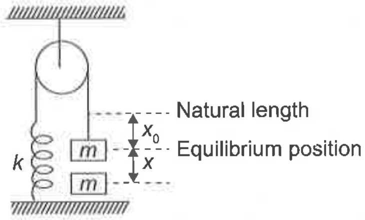
The string, the spring and the pulley shown in the figure are light. Find the time period of the mass m.



SOLUTION

Let in equilibrium position of the block, extension in spring is x_0.

$$\therefore \quad kx_0 = mg \tag{1}$$



Now if we displace the block by x in the downward direction, net force on the block towards mean position is

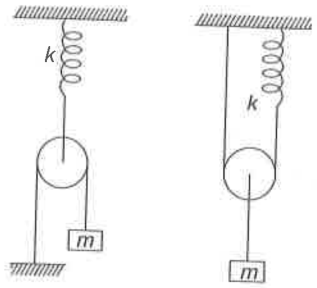
$$F = k(x + x_0) - mg = kx \quad \text{using Eq. (1).}$$

Hence, the net force is acting towards the mean position and is also proportional to x. So, the particle will perform SHM and its time period would be

$$T = 2\pi\sqrt{\frac{m}{k}}.$$ ■

EXAMPLE 16

The figure shows a system consisting of a massless pulley, a spring of force constant k and a block of mass m. If the block is slightly displaced vertically down from its equilibrium position and then released, find the period of its vertical oscillation in cases (a) and (b).

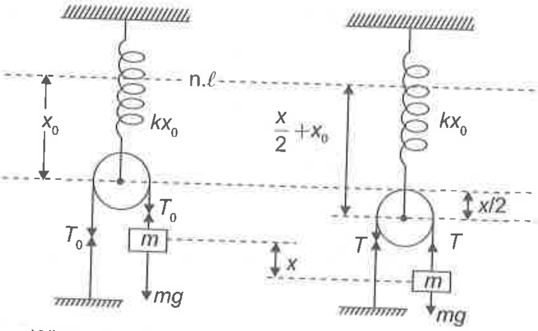


(a) (b)

SOLUTION

Let us assume that in equilibrium condition, spring is x_0 elongated from its natural length

Case (a)



When equillibrium When displaced block by '*x*'

In equilibrium $T_0 = mg$

and $kx_0 = 2T_0$

$\Rightarrow$ $kx_0 = 2mg$ (1)

If the mass m moves down a distance x from its equilibrium position, then the pulley will move down by $\frac{x}{2}$. So the extra force in the spring will be $\frac{kx}{2}$. From the figure,

$$F_{net} = mg - T$$

$$= mg - \frac{k}{2}\left(x_0 + \frac{x}{2}\right)$$

$$F_{net} = mg - \frac{kx_0}{2} - \frac{kx}{4}.$$

From Eq. (1),

$$F_{net} = \frac{-kx}{4} \quad (3)$$

Now compare Eq. (3) with $F = -K_{SHM}x$,

then $K_{SHM} = \frac{K}{4}$.

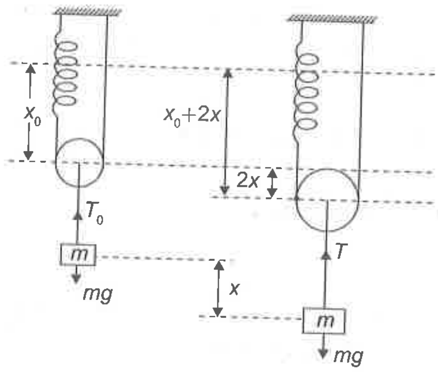
$\Rightarrow$ $T = 2\pi\sqrt{\frac{m}{K_{SHM}}} = 2\pi\sqrt{\frac{4\,m}{K}}$.

Case (b)

In this situation, if the mass m moves down distance x from its equilibrium position, then the pulley will also move by x and so the spring will stretch by $2x$.

At equilibrium,

$$kx_0 = \frac{T_0}{2} = \frac{mg}{2}$$



When the block is displaced,

$$F_{net} = mg - T$$

$$= mg - 2k(x_0 + 2x)$$

$$= -4kx$$

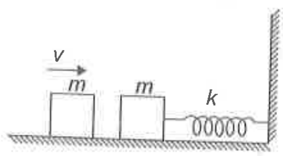
Now $F = -K_{SHM}x$,

then $K_{SHM} = 4K$.

So the time period $T = 2\pi\sqrt{\frac{m}{4k}}$. ■

EXAMPLE 17

The left block in figure collides inelastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.



SOLUTION

The collision is for a small interval only, we can apply the principal of conservation of momentum. The common velocity after the collision is $\frac{v}{2}$. The kinetic energy $= \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$. This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is A, the total energy can also be written as $\frac{1}{2}kA^2$

Thus, $$\frac{1}{2}kA^2 = \frac{1}{4}mv^2,$$

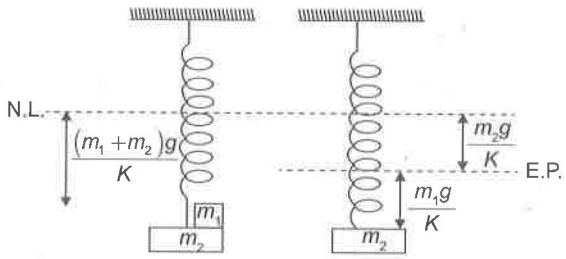
giving $$A = \sqrt{\frac{m}{2k}}v.$$ ■

EXAMPLE 18

The system is in equilibrium and at rest. Now mass m_1 is removed from m_2. Find the time period and amplitude of the resultant motion. (Given spring constant is K.)

SOLUTION

Initial extension in the spring



$$x = \frac{(m_1+m_2)g}{k}$$

Now, if we remove m_1, equilibrium position (EP)of m_2 will be $\frac{m_2g}{K}$ below natural length of the spring.

At the initial position, since velocity is zero, i.e., it is the extreme position.

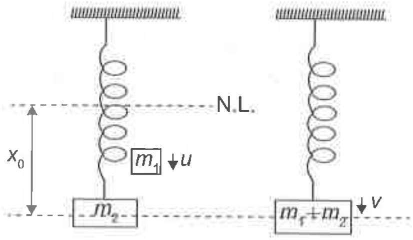
Hence, amplitude $= \frac{m_1g}{K}$

$$\text{Time period} = 2\pi\sqrt{\frac{m_2}{K}}.$$ ■

EXAMPLE 19

A block of mass m_2 is in equilibrium and at rest. The mass m_1 moving with velocity u vertically downwards collides with m_2 and sticks to it. Find the energy of oscillation.

SOLUTION



At equilibrium position,

$$m_2g = kx_0$$

$$\Rightarrow \quad x_0 = \frac{m_2g}{K}.$$

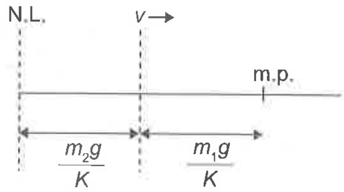
After collision, m_2 sticks to m_1.

∴ By momentum conservation,

$$m_1u = (m_1 + m_2)v$$

$$v = \frac{m_1u}{m_1+m_2}$$

Now both the blocks are executing SHM, which can be interpreted as follows:



Now, we know that

$$v^2 = \omega^2(A^2 - x^2) \tag{1}$$

$$\omega^2 = \frac{k}{m_1+m_2}$$

$$\Rightarrow \quad x = \frac{m_1g}{k}.$$

Put the values of v, ω^2 and x in Eq. (1),

$$\left(\frac{m_1u}{m_1+m_2}\right)^2 = \left(\frac{k}{m_1+m_2}\right)\left[A^2 - \left(\frac{m_1g}{k}\right)^2\right]$$

$$\Rightarrow \quad kA^2 = \left[\left(\frac{m_1^2u^2}{m_1+m_2}\right) + \left(\frac{m_1g}{k}\right)^2\right]$$

$$\Rightarrow \text{Energy of oscillation} = \frac{1}{2}kA^2$$

$$= \frac{1}{2}\left[\left(\frac{m_1^2u^2}{m_1+m_2}\right) + \left(\frac{m_1^2g^2}{k}\right)\right].$$ ■

EXAMPLE 20

A body of mass m falls from a height h on to the pan of a spring balance. The masses of the pan and spring are negligible. The spring constant of the spring is k. Having stuck to the pan, the body starts performing harmonic oscillations in the vertical direction. Find the amplitude and energy of oscillation.

SOLUTION

Suppose by falling down through a height h, the mass m compresses the spring balance by a length x.

$$x = \frac{mg}{k},$$

$$\omega = \sqrt{\frac{k}{m}}$$

Velocity at $\qquad Qv = \sqrt{2gh}$

$\because \qquad v = \omega\sqrt{A^2 - x^2}$

$$\sqrt{2gh} = \sqrt{\frac{k}{m}}\sqrt{A^2 - \left(\frac{mg}{k}\right)^2}$$

$$\Rightarrow \qquad A = \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

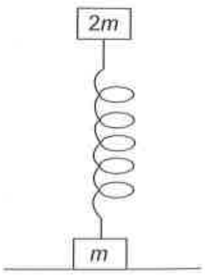
$$\text{Energy of oscillation } = \frac{1}{2}kA^2$$

$$= \frac{1}{2}k\left(\frac{mg}{k}\right)^2\left(1 + \frac{2kh}{mg}\right)$$

$$= mgh + \frac{(mg)^2}{2k}.$$ ■

EXAMPLE 21

A body of mass $2m$ is connected to another body of mass m as shown in the figure. The mass $2m$ performs vertical SHM. Then find out the maximum amplitude of $2m$ such that the mass m does not lift up from the ground.



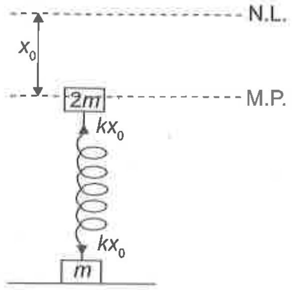
SOLUTION

In the given situation, $2m$ mass is in equilibrium condition.

Let us assume that the spring is compressed x_0 distance from its natural length.

$$\Rightarrow \qquad kx_0 = 2mg$$

$$\Rightarrow \qquad x_0 = \frac{2mg}{k}$$

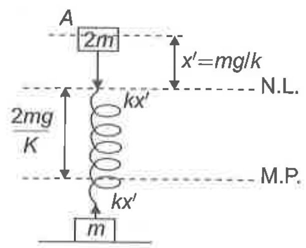


The lower block will be lift up, only in the case when the spring force on it will be greater than equal to mg and in the upward direction.

$$\Rightarrow \qquad kx' = mg$$

$$\Rightarrow \qquad x' = \frac{mg}{k}$$

The above situation arises when the $2m$ block moves upward mg/k from the natural length as shown in the figure.



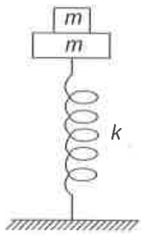
Block m does not lift up if the maximum amplitude of the $2m$ block is

$$= \frac{2mg}{k} + \frac{mg}{k}$$

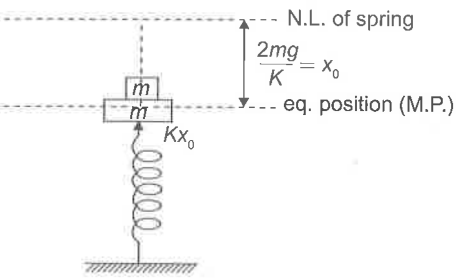
$$= \frac{3mg}{k}$$ ■

EXAMPLE 22

A block of mass m is at rest on the another block of the same mass as shown in the figure. Lower block is attached to the spring, then determine the maximum amplitude of motion so that both the block will remain in contact.



SOLUTION



The blocks will remain in contact till the blocks do not go above the natural length of the spring, because after this condition the deceleration of the lower block becomes more than the upper block due to spring force. So they will get separated.

So maximum possible amplitude $= x_0$

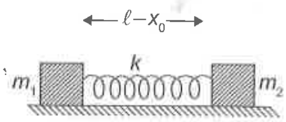
$$= \frac{2mg}{k}.$$

Two Block Systems

EXAMPLE 23

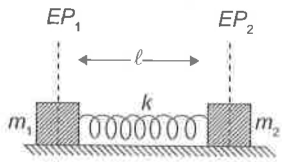
Two blocks of mass m_1 and m_2 are connected with a spring of natural length ℓ and spring constant k. The system is lying on a smooth horizontal surface. Initially, spring is compressed by x_0 as shown in the figure.

Show that the two blocks will perform SHM about their equilibrium position. Also (a) find the time period, (b) find the amplitude of each block and (c) the length of the spring as a function of time.

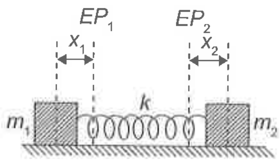


SOLUTION

(a) Here both the blocks will be in equilibrium at the same time when the spring is in its natural length. Let EP_1 and EP_2 be the equilibrium positions of blocks A and B as shown in the figure.



Let at any time during oscillation, blocks are at a distance of x_1 and x_2 from their equilibrium positions.



As no external force is acting on the spring block system,

$$\therefore \quad (m_1 + m_2)\Delta x_{cm} = m_1 x_1 - m_2 x_2 = 0$$

or $$m_1 x_1 = m_2 x_2.$$

For the first particle, force equation can be written as

$$k(x_1 + x_2) = -m_1 \frac{d^2 x_1}{dt^2}$$

or $$k\left(x_1 + \frac{m_1}{m_2} x_1\right) = -m_1 a_1$$

or $$a_1 = -\frac{k(m_1 + m_2)}{m_1 m_2} x_1$$

$$\therefore \quad \omega^2 = \frac{k(m_1 + m_2)}{m_1 m_2}$$

Hence, $$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$

$$= 2\pi\sqrt{\frac{\mu}{K}}$$

where $$\mu = \frac{m_1 m_2}{(m_1 + m_2)},$$

which is known as reduced mass.

(b) Let the amplitude of blocks be A_1 and A_2.

$$m_1 A_1 = m_2 A_2$$

By energy conservation,

$$\frac{1}{2}k(A_1 + A_2)^2 = \frac{1}{2}kx_0^2$$

or $$A_1 + A_2 = x_0$$

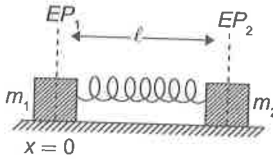
or $$A_1 + A_2 = x_0$$

or $$A_1 + \frac{m_1}{m_2} A_1 = x_0$$

or $$A_1 = \frac{m_2 x_0}{m_1 + m_2}$$

Similarly, $$A_2 = \frac{m_1 x_0}{m_1 + m_2}.$$

(c) Let equilibrium position of the first particle be the origin, i.e., $x = 0$.



The x co-ordinate of particles can be written as

$$x_1 = A_1 \cos \omega t$$

and $$x_2 = \ell - A_2 \cos \omega t$$

Hence, length of spring can be written as

$$\text{length} = x_2 - x_1$$

$$= \ell - (A_1 + A_2) \cos \omega t.$$ ■

COMBINATION OF SPRINGS

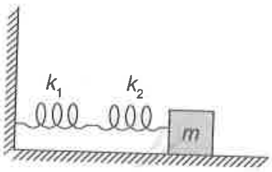


Figure 1.13

Series Combination

Total displacement,

$$x = x_1 + x_2$$

Tension in both the springs $= k_1 x_1 = k_2 x_2$.

$\therefore$ Equivalent constant in series combination k_{eq} is given by

$$1/k_{eq} = 1/k_1 + 1/k_2$$

$$\Rightarrow \quad T = 2\pi \sqrt{\frac{m}{k_{eq}}}$$

In series combination, tension is the same in all the springs and extension will be different. (If k is same, then deformation is also the same.)

Parallel Combination

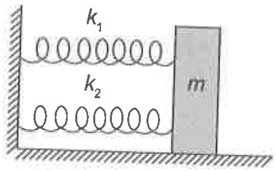


Figure 1.14

Extension is the same for both springs, but the force acting will be different.

Force acting on the system $= F$

$$\therefore \quad F = -(k_1 x + k_2 x)$$

$$\Rightarrow \quad F = -(k_1 + k_2)x$$

$$\Rightarrow \quad F = -k_{eq} x$$

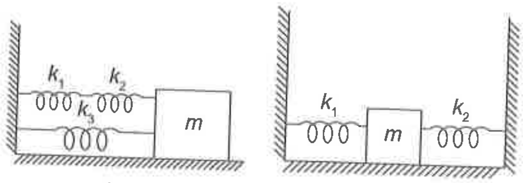
$$\therefore \quad k_{eq} = k_1 + k_2$$

$$\Rightarrow \quad T = 2\pi \sqrt{\frac{m}{k_{eq}}}$$

SOLVED EXAMPLES

EXAMPLE 24

Find the time period of the oscillation of mass m in figure a and$b$. What is the equivalent spring constant of the spring in each case?

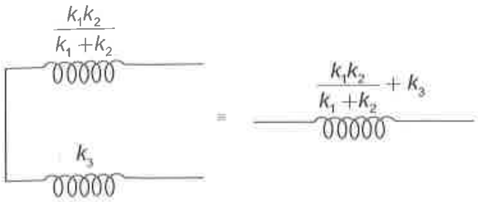


(a) (b)

SOLUTION

In Figure (a),

which gives



$$k_{eq} = \frac{k_1k_2}{k_1+k_2} + k_3$$

$$= \frac{k_1k_2 + k_2k_3 + k_1k_3}{k_1+k_2}$$

Now
$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$= 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2 + k_2k_3 + k_1k_3}}$$

In Figure (b), if the block is displaced slightly by an amount x, then both the springs are displaced by x from their natural length so it is a parallel combination of springs, which gives

$$k_{eq} = k_1 + k_2$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$= 2\pi\sqrt{\frac{m}{k_1+k_2}}$$ ■

Notes

1. In series combination, extension of springs will be reciprocal of its spring constant.

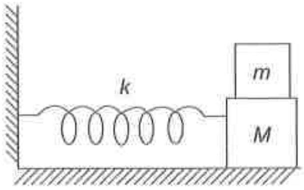
   $\therefore \quad k \propto 1/\ell$

   $\therefore \quad k_1\ell_1 = k_2\ell_2 = k_3\ell_3$

2. If a spring is cut in n equal pieces, then the spring constant of one piece will be nk.

EXAMPLE 25

The friction coefficient between the two blocks shown in the figure is μ and the horizontal plane is smooth. (a) If the system is slightly displaced and released, find the time period. (b) Find the magnitude of the frictional force between the blocks when the displacement from the mean position is x. (c) What can be the maximum amplitude if the upper block does not slip relative to the lower block?



SOLUTION

(a) For small amplitude, the two blocks oscillate together.

The angular frequency is $\omega = \sqrt{\frac{k}{M+m}}$, and so the time period

$$T = 2\pi\sqrt{\frac{M+m}{k}}.$$

(b) The acceleration of the blocks at displacement x from the mean position is

$$a = -\omega^2 x = \left(\frac{-kx}{M+m}\right).$$

The resultant force on the upper block is, therefore,

$$ma = \left(\frac{-mkx}{M+m}\right).$$

This force is provided by the friction of the lower block. Hence, the magnitude of the frictional force is $\left(\frac{mk|x|}{M+m}\right)$.

(c) Maximum force of friction required for simple harmonic motion of the upper block is $\frac{mkA}{M+m}$ at the extreme positions. But the maximum frictional force can only be μmg. Hence,

$$\frac{mkA}{M+m} = \mu mg$$

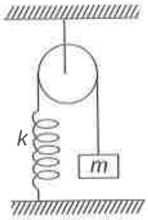
or
$$A = \frac{\mu(M+m)g}{k}$$

ENERGY METHOD

Another method of finding time period of SHM is energy method. To understand this method, we will consider the following example.

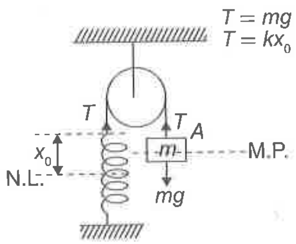
EXAMPLE 26

Figure shows a system consisting of pulley having radius R, a spring of force constant k and a block of mass m. Find the period of its vertical oscillation.



SOLUTION

The following steps are usually followed in this method:



Step 1: Find the mean position. In following figure, point A shows the mean position.

Step 2: Write down the mean position force relation from the figure.

$$mg = kx_0$$

Step 3: Assume that the particle is performing SHM with amplitude A. Then displace the particle from its mean position.

Step 4: Find the total mechanical energy (E) in the displaced position since mechanical energy in SHM remains constant $\frac{dE}{dt} = 0$.

$$* \quad E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x+x_0)^2 - mgx$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} + \frac{1}{2}k(x+x_0)^2 - mgx$$

$$\frac{dE}{dt} = \frac{2mv}{2}\frac{dv}{dt} + \frac{2Iv}{2R^2}\frac{dv}{dt} + \frac{2k(x+x_0)}{2}\frac{dx}{dt} - mg\frac{dx}{dt} \quad (1)$$

Put $\frac{dx}{dt} = v$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$ in Eq. (1),

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \quad mv\frac{d^2x}{dt^2} + \frac{Iv}{R^2}\frac{d^2x}{dt^2} + kxv + kx_0v - mgv = 0,$$

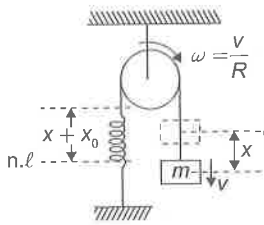
which gives $\left(m + \frac{I}{R^2}\right)\frac{d^2x}{dt^2} + kx = 0$

$$\frac{d^2x}{dt^2} + \frac{k}{\left(m + \frac{I}{R^2}\right)}x = 0 \quad (2)$$

Compare Eq. (2) with SHM equation, then

$$\omega^2 = \frac{k}{\left(m + \frac{I}{R^2}\right)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(m + I/R^2)}{k}}$$



ANGULAR SHM

If the restoring torque acting on the body in oscillatory motion is directly proportional to the angular displacement of the body from its equilibrium position, i.e.,

$$\tau = -k\theta$$

k = SHM constant

θ = angular displacement from M.P.

SHM equation is given by

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

Here $\omega = \sqrt{\frac{k}{I}}$

where I is the moment of inertia of the body/particle about a given axis.

SIMPLE PENDULUM

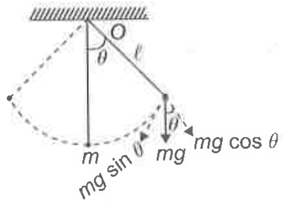


Figure 1.15

If a heavy point-mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum.

Time period of a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$.

(Sometimes we can take $g = \pi^2$ for making the calculation simple.)

Proof

Now taking moment of forces acting on the bob about point O.

$$\tau = \tau_T + \tau_{mg}$$

$$\tau_T = 0$$

$$\Rightarrow \quad \tau = -(mg\sin\theta)\ell.$$

If θ is very small, then $\sin\theta \simeq \theta$

$$\Rightarrow \quad \tau = -mg\theta\ell \qquad (1)$$

Now compare Eq. (1) with

$$\tau_{net} = -K_{SHM\theta},$$

which gives $K_{SHM} = mg\ell$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{K_{SHM}}}$$

$$= 2\pi\sqrt{\frac{m\ell^2}{mg\ell}} = 2\pi\sqrt{\frac{\ell}{g}}$$

Notes

1. Time period of second pendulum is 2 seconds.
2. Simple pendulum performs angular SHM but due to small angular displacement, it is considered as linear SHM
3. If time period of a clock based upon simple pendulum increases, then the clock will become slow, but if time period decreases, then the clock will become fast.

Time period of pendulam of very large length is

$$2\pi\sqrt{\frac{\ell}{g\left(\frac{1}{\ell}+\frac{1}{R}\right)}}$$

TIME PERIOD OF A SIMPLE PENDULUM IN ACCELERATING REFERENCE FRAME

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}},$$

where g_{eff} = effective acceleration due to gravity in the reference system = $|\vec{g}-\vec{a}|$. $\vec{a}$ = acceleration of the point of suspension with respect to the ground.

Condition for applying this formula,

$$|\vec{g}-\vec{a}| = \text{constant}.$$

If the acceleration $\vec{a}$ is upwards,

then $$|\vec{g}_{eff}| = g + a$$

and $$T = 2\pi\sqrt{\frac{\ell}{g+a}}.$$

Time lost or gained in time t is given by

$$\Delta T' = \frac{\Delta T}{T}t.$$

SOLVED EXAMPLES

EXAMPLE 27

If $T = 2$ s, $T_{new} = 3$ s, then

$$\Delta T = 1 \text{ s}.$$

Since time lost by the clock in 3 s is = 1 s,

then time lost by clock in 1 s = $\frac{1}{3}$ s.

$\therefore$ Time lost by the clock in an hour

$$= \frac{1}{3}\times 3600 = 1200 \text{ s}.$$

■

EXAMPLE 28

A simple pendulum is suspended from the ceiling of a car which is accelerating uniformly on a horizontal road. The acceleration of the car is a_0 and the length of the pendulum is ℓ_1. Then find the time period of small oscillations of the pendulum about the mean position.

SOLUTION

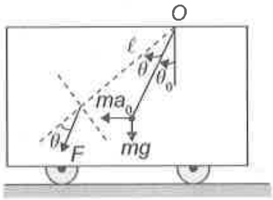
We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force ma_0 on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ_0 be the angle made by the string with the vertical, the tension, weight and the pseudo force will add to zero in this position.

Hence, resultant of mg and ma_0 (say $F = m\sqrt{g^2 + a_0^2}$) has to be along the string.

$$\therefore \qquad \tan\theta_0 = \frac{ma_0}{mg} = \frac{a_0}{g}.$$

Now, suppose the string is further deflected by an angle θ as shown in the figure.



Now, restoring torque about point O can be given by

$$\tau = I\alpha.$$

$$(F \sin\theta)\ell = -m\ell^2\alpha$$

Substituting F and using $\sin\theta = \theta$, for small θ,

$$(m\sqrt{g^2 + a_0^2})\ell\theta = -m\ell^2\alpha$$

or $$\alpha = -\frac{\sqrt{g^2 + a_0^2}}{\ell}\theta$$

so $$\omega^2 = \frac{\sqrt{g^2 + a_0^2}}{\ell}$$

This is an equation of simple harmonic motion with time period

$$T = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{\ell}}{(g^2 + a_0^2)^{1/4}}.$$ ■

COMPOUND PENDULUM/PHYSICAL PENDULUM

When a rigid body is suspended from an axis and made to oscillate about that, then it is called a compound pendulum.

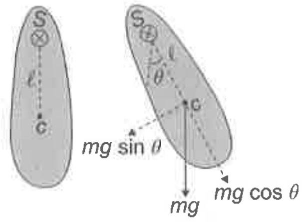


Figure 1.16

C = position of centre of mass

S = point of suspension

ℓ = distance between the point of suspension and centre of mass.

(It remains constant during motion for a small angular displacement θ from mean position.)

The restoring torque is given by

$$\tau = -mg\ell \sin\theta$$

$$\tau = -mg\ell\theta$$

$\therefore$ For small θ,

$$\sin\theta = \theta$$

or, $$I\alpha = -mg\ell\theta,$$

where,

I = moment of inertia about the point of suspension

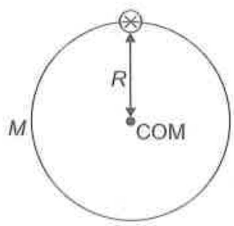
or $$a = -\frac{mg\ell}{I}\theta$$

or $$\omega^2 = \frac{mg\ell}{I}$$

Time period, $$T = 2\pi\sqrt{\frac{I}{mg\ell}}.$$

EXAMPLE 29

A ring is suspended at a point on its rim and it behaves as a second's pendulum when it oscillates such that its centre moves in its own plane. The radius of the ring would be ($g = \pi^2$).



SOLUTION

Time period of second pendulum $T = 2$ cm.

$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

Moment of inertia with respect to axis O

$$I = MR^2 + MR^2 = 2MR^2.$$

The distance between the centre of mass and the axis O

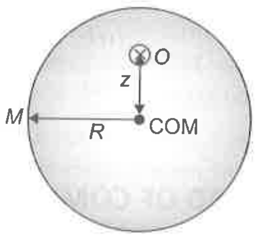
$$d = R$$

$$2 = 2\pi\sqrt{\frac{2MR^2}{MgR}}$$

$\Rightarrow$ $$R = 0.5 \text{ m}.$$

EXAMPLE 30

A circular disc has a tiny hole in it, at a distance z from its centre. Its mass is M and radius R ($R > 2$). Horizontal shaft is passed through the hole and held fixed so that the disc can freely swing in the vertical plane. For small disturbance, the disc performs SHM whose time period is minimum for z. Find the value of z.



SOLUTION

The time period with respect to the axis $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where

I = moment of inertia with respect to the axis O

d = distance between COM and O

$\Rightarrow$ $$I = \frac{MR^2}{2} + Mz^2$$

$$d = z$$

$\Rightarrow$ $$T = 2\pi\sqrt{\frac{\frac{MR^2}{2} + Mz^2}{Mgz}}$$

$$= 2\pi\sqrt{\frac{R^2}{2gz} + \frac{z}{g}}$$

The time period will be minimum when

$$\frac{R^2}{2z} + z = \text{minimum}.$$

Let us say

$$f = \frac{R^2}{2z} + z$$

f will be minimum when

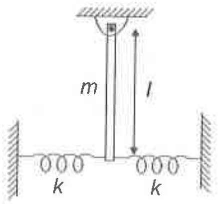
$$\frac{df}{dz} = 0$$

$\Rightarrow$ $$-\frac{R^2}{2z^2} + 1 = 0$$

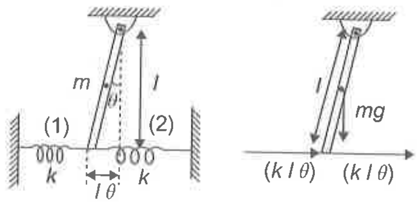
$\Rightarrow$ $$z = \frac{R}{\sqrt{2}}$$

EXAMPLE 31

Find out the angular frequency of the small oscillation about the axis O.



SOLUTION



The compression in spring (1) $= l\theta$

and the extension in spring (2) $= l\theta$.

Net torque opposite to the mean position

$$= -(2kl\theta)l - mg\frac{l}{2}\sin\theta = \tau_{net}.$$

θ is small

$\Rightarrow$ $$\sin\theta \simeq \theta.$$

$$\tau_{net} = -I\omega^2\theta$$

$$= -(2kl\theta)l - mg\frac{l}{2}\sin\theta = \tau_{net}$$

$$I = \frac{ml^2}{3}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{3}{2}\frac{(4kl + mg)}{ml}}$$ ■

TORSIONAL PENDULUM

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount, the wire applies a restoring torque causing the body to oscillate rotationally when released.

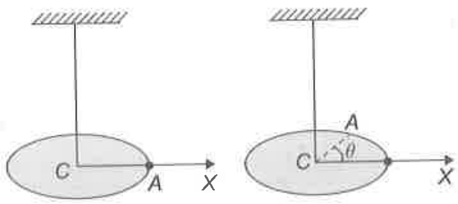


Figure 1.17

The restoring torque produced is given by

$$\tau = -C\theta,$$

where, C = torsional constant

or $$I\alpha = -C\theta,$$

where, I = moment of inertia about the vertical axis.

or $$\alpha = -\frac{C}{I}\theta$$

∴ Time period, $$T = 2\pi\sqrt{\frac{I}{C}}.$$

Note

The above concept of torsional pendulum is used in inertia table to calculate the moment of inertia of an unknown body.

SOLVED EXAMPLE

EXAMPLE 32

A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

SOLUTION

The situation is shown in the figure. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\,\text{kg})(5.0\times10^{-2}\,\text{m})^2}{2}$$

$$= 2.5 \times 10^{-4}\ \text{kgm}^2.$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

or $$C = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4\pi^2(2.5\times10^{-4}\ \text{kgm}^2)}{(0.02\ \text{s})^2}$$

$$= 0.25\ \frac{\text{kgm}^2}{\text{s}^2}$$ ■

VECTOR METHOD OF COMBINING TWO OR MORE SIMPLE HARMONIC MOTIONS

A simple harmonic motion is produced when a force (called restoring force) proportional to the displacement acts on a particle. If a particle is acted upon by two such forces, the resultant motion of the particle is a combination of two simple harmonic motions.

In the Same Direction

(a) Having the Same Frequencies

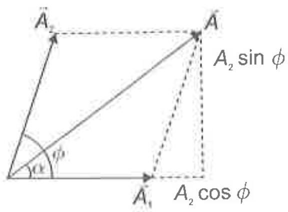


Figure 1.18

Suppose the two individual motions are represented by,

$$x_1 = A_1 \sin \omega t$$

and $$x_2 = A_2 \sin(\omega t + \phi).$$

Both the simple harmonic motions have the same angular frequency ω.

$$x = x_1 + x_2$$

$$= A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$

$$= A \sin(\omega t + \alpha)$$

Here, $$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

and $$\tan\alpha = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Thus, we can see that this is similar to the vector addition. The same method of vector addition can be applied to the combination of more than two simple harmonic motions.

Important Points to Remember Before Solving the Questions:

1. Convert all the trigonometric ratios into **sine** form and ensure that ωt term is with positive sign.
2. Make the sign between two terms positive.
3. A_1 is the amplitude of that SHM whose phase is small.
4. Then, resultant $x = A_{net}\sin(\text{phase of } A_1 + \alpha)$,

where A_{net} is the vector sum of A_1 and A_2 with angle between them is the phase difference between two SHMs.

SOLVED EXAMPLES

EXAMPLE 33

$x_1 = 3\sin\omega t$; $x_2 = 4\cos\omega t$. Find (a) the amplitude of the resultant SHM and (b) equation of the resultant SHM.

SOLUTION

First write all SHMs in terms of sine functions with positive amplitude. Keep ωt with positive sign.

$\therefore$ $$x_1 = 3\sin\omega t$$

$$x_2 = 4\sin(\omega t + \pi/2)$$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}}$$

$$= \sqrt{9+16} = \sqrt{25}$$

$$= 5$$

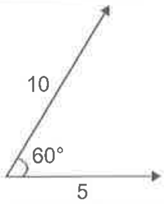
$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3}\phi$$

$$= 53°$$

Equation $$x = 5\sin(\omega t + 53°).$$ ■

EXAMPLE 34

$x_1 = 5\sin(\omega t + 30°)$; $x_2 = 10\cos(\omega t)$. Find the amplitude of the resultant SHM.



Phasor diagram

SOLUTION

$$x_1 = 5\sin(\omega t + 30°)$$

$$x_2 = 10\sin(\omega t + \frac{\pi}{2})$$

$$A = \sqrt{5^2 + 10^2 + 2\times5\times10\cos 60°}$$

$$= \sqrt{25+100+50}$$

$$= \sqrt{175} = 5\sqrt{7}$$ ■

EXAMPLE 35

A particle is subjected to two simple harmonic motions

$$x_1 = A_1\sin\omega t$$

and $$x_2 = A_2\sin(\omega t + \pi/3).$$

Find (a) the displacement at $t = 0$, (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.

SOLUTION

(a) At $t = 0$, $x_1 = A_1\sin\omega t = 0$

and $$x_2 = A_2\sin(\omega t + \pi/3)$$

$$= A_2\sin(\pi/3) = \frac{A_2\sqrt{3}}{2}$$

Thus, the resultant displacement at $t = 0$ is

$$x = x_1 + x_2$$

$$= A_2\frac{\sqrt{3}}{2}$$

(b) The resultant of the two motion is a simple harmonic motion of the same angular frequency ω. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/3)}$$

$$= \sqrt{A_1^2 + A_2^2 + A_1 A_2}$$

The maximum speed is

$$u_{max} = A\omega$$

$$= \omega\sqrt{A_1^2 + A_2^2 + A_1 A_2}$$

(c) The maximum acceleration is

$$a_{max} = A\omega^2$$

$$= \omega^2 \sqrt{A_1^2 + A_2^2 + A_1 A_2}$$

(b) Having different frequencies

$$x_1 = A_1 \sin \omega_1 t$$

$$x_2 = A_2 \sin \omega_2 t$$

Then the resultant displacement, $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$. This resultant motion is not SHM. ■

In Two Perpendicular Directions

$$x = A_1 \sin \omega t \quad (1)$$

$$y = A_2 \sin(\omega t + \phi) \quad (2)$$

The amplitudes A_1 and A_2 may be different and phase difference ϕ and ω is the same.

So equation of the path may be obtained by eliminating t from Eqs. (1) and (2)

$$\sin \omega t = \frac{x}{A_1} \quad (3).$$

$$\cos \omega t = \sqrt{1 - \frac{x^2}{A_1^2}} \quad (4)$$

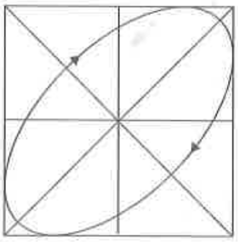


Figure 1.19

On rearranging we get

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\phi}{A_1 A_2} = \sin^2 \phi \quad (5)$$

(general equation of ellipse)

Special case

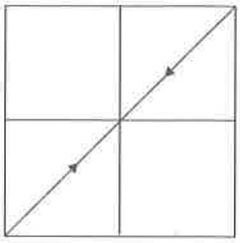


Figure 1.20

(1) If $\phi = 0$

$$\therefore \quad \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2} = 0$$

$$\therefore \quad y = \frac{A_2}{A_1} x \text{ (equation of a straight line).}$$

(2) If $\phi = 90°$

$$\Rightarrow \quad \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1 \text{ (equation of ellipse).}$$

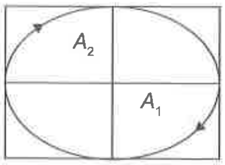


Figure 1.21

(3) If $\phi = 90°$

and $\quad A_1 = A_2 = A,$

then $\quad x^2 + y^2 = A^2$ (equation of a circle).

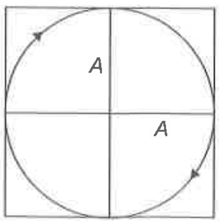


Figure 1.22

The above figures are called Lissajous figures.

MIND MAP

SHM

1. Equation of S.H.M

(a) **Linear**: $a = -\omega^2 x$

(b) **Angular**: $\alpha = -\omega^2\theta$

2. Linear SHM

(a) **Displacement of particle**

$$x = A\sin(\omega t + \phi)$$

(b) **Velocity**

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$$

(c) **Acceleration**

$$\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 x$$

(d) **Phase**: $\omega t + \phi$

(e) **Phase constant**: ϕ

3. Angular S.H.M

(a) **Displacement**:

$$\theta = \theta_0\sin(\omega t + \phi)$$

(b) **Angular velocity**

$$\frac{d\theta}{dt} = \theta_0\omega\cos(\omega t + \phi)$$

(c) **Acceleration**

$$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t + \phi) = -\omega^2\theta$$

(d) **Phase**: $\omega t + \phi$

(e) **Phase constant**: ϕ

4. Energy in SHM

(a) $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

(b) $U = \frac{1}{2}m\omega^2 x^2$

(c) $E = K + U = \frac{1}{2}m\omega^2 A^2$ = constant

5. Time period:

Pendulums:

(a) **Simple pendulum:**

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(b) **Physical pendulum:**

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

(c) **Torsional pendulum:**

$$T = 2\pi\sqrt{\frac{I}{C}}$$

6. Mass-spring system

(a) $T = 2\pi\sqrt{\frac{m}{k}}$

(b) **Two bodies system**:

$$T = 2\pi\sqrt{\frac{\mu}{k}};$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Combination of springs:

(a) **Series**: $\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2}$

(b) **Parallel**: $K_{eff} = K_1 + K_2$

(c) **Spring cut into two parts *m*:*n***

$$K_1 = \frac{(m+n)K}{m},\ K_2 = \frac{(m+n)K}{n}$$

7. Composition of 2 SHMs:

$$x_1 = A_1\sin\omega t$$

$$x_2 = A_2\sin(\omega t + \phi)$$

$$x = x_1 + x_2$$

$$x = A\sin(\omega t + \delta)$$

where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\tan\delta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

EXERCISES

JEE Main

Linear SHM

1. For a particle executing simple harmonic motion, the acceleration is proportional to
   (A) displacement from the mean position
   (B) distance from the mean position
   (C) distance travelled since $t = 0$
   (D) speed

2. The distance moved by a particle in simple harmonic motion in one time period is
   (A) A (B) $2A$
   (C) $4A$ (D) zero

3. The time period of a particle in simple harmonic motion is equal to the time between consecutive appearance of the particle at a particular point in its motion. This point is
   (A) the mean position
   (B) an extreme position
   (C) between the mean position and the positive extreme
   (D) between the mean position and the negative extreme

4. A particle is executing SHM of amplitude 'a' and time period = 4 seconds. Then the time taken by it to move from the extreme position to half the amplitude is
   (A) 1 s (B) $\frac{1}{3}$ s
   (C) $\frac{2}{3}$ s (D) $\frac{4}{3}$ s

5. Equations $y = 2A\cos^2\omega t$ and $y = A(\sin\omega t + \sqrt{3}\cos\omega t)$ represent the motion of two particles.
   (A) Only one of these is S.H.M
   (B) Ratio of maximum speeds is 2:1
   (C) Ratio of maximum speeds is 1:1
   (D) Ratio of maximum accelerations is 1:4

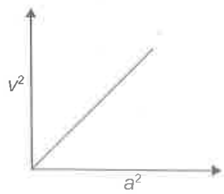
6. The displacement of a body executing SHM is given by $x = A\sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is
   (A) 0.33 s (B) 0.16 s
   (C) 0.25 s (D) 0.5 s

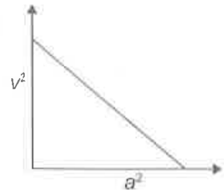
7. A simple harmonic motion having an amplitude A and time period T is represented by the equation $y = 5\sin\pi(t + 4)$ m. Then the values of A (in metre) and T (in seconds) are
   (A) $A = 5; T = 2$ (B) $A = 10; T = 1$
   (C) $A = 5; T = 1$ (D) $A = 10; T = 2$

8. Two particles are in SHM on same straight line with amplitudes A and $2A$ and with the same angular frequency ω. It is observed that when the first particle is at a distance $A/\sqrt{2}$ from the origin and going towards mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles.
   (A) 45° (B) 90°
   (C) 135° (D) 180°

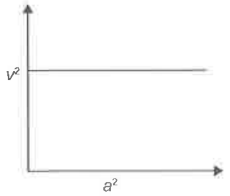
9. The time period of a particle in simple harmonic motion is equal to the smallest time between the particle acquiring a particular velocity $\vec{v}$. The value of v is
   (A) v_{max}
   (B) 0
   (C) between 0 and v_{max}
   (D) between 0 and $-v_{max}$

10. The average acceleration in one time period in a simple harmonic motion is
    (A) $A\omega^2$ (B) $A\omega^2/2$
    (C) $A\omega^2/\sqrt{2}$ (D) zero

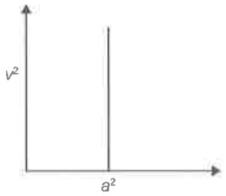
11. A mass m is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is
    (A) 

    (B) 

(C)



(D)



12. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8

(C) 6 (D) 4

13. A simple pendulum performs SHM about $x = 0$ with an amplitude a and time period T. The speed of the pendulum at $x = a/2$ will be

(A) $\frac{\pi a\sqrt{3}}{T}$ (B) $\frac{\pi a\sqrt{3}}{2T}$

(C) $\frac{\pi a}{T}$ (D) $\frac{3\pi^2 a}{T}$

14. A particle executes SHM given by the equation $y = 0.45 \sin 2t$ where y is in metre and t is in seconds. What is the speed of the particle when its displacement is 7.5 cm?

(A) $0.075\sqrt{3}$ ms^{-1} (B) $7.5\sqrt{3}$ ms^{-1}

(C) $0.15\sqrt{3}$ ms^{-1} (D) $15\sqrt{3}$ ms^{-1}

15. The maximum displacement of a particle executing SHM is 1 cm and the maximum acceleration is $(1.57)^2$ cm/s^2. Then the time period is

(A) 0.25 s (B) 4.00 s

(C) 1.57 s (D) $(1.57)^2$ s

16. A particle performing SHM is found at its equilibrium at $t = 1$ s and it is found to have a speed of 0.25 m/s at $t = 2$ s. If the period of oscillation is 6 s. Calculate the amplitude of oscillation.

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m

(C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$ m

17. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$

(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

18. Two particles are in SHM in a straight line about same equilibrium position. Amplitude A and time period T of both the particles are equal. At time $t = 0$, one particle is at displacement $y_1 = +A$ and the other at $y_2 = -A/2$, and they are approaching towards each other. After what time they cross each other?

(A) $T/3$ (B) $T/4$

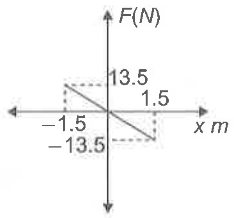
(C) $5T/6$ (D) $T/6$

19. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

20. A particle of mass 1 kg is undergoing SHM, for which graph between force and displacement (from mean position) as shown. Its time period, in seconds, is



(A) $\pi/3$ (B) $2\pi/3$

(C) $\pi/6$ (D) $3/\pi$

21. A point particle of mass 0.1 kg is executing SHM of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. The equation of motion of this particle when the initial phase of oscillation is 45° can be given by

(A) $0.1\cos\left(4t + \frac{\pi}{4}\right)$ (B) $0.1\sin\left(4t + \frac{\pi}{4}\right)$

(C) $0.4\sin\left(t + \frac{\pi}{4}\right)$ (D) $0.2\sin\left(\frac{\pi}{2} + 2t\right)$

22. A particle executes SHM of period 1.2 s and amplitude 8 cm. Find the time it takes to travel 3 cm from the positive extremely of its oscillation.

(A) 0.28 s (B) 0.32 s
(C) 0.17 s (D) 0.42 s

23. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance $a/2$ from the extreme position is

(A) a/T (B) $2a/T$
(C) $3a/T$ (D) $a/2T$

24. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position, the ribbons are underformed. Vibration period is

(A) $2\pi\sqrt{\frac{m(2k)}{k^2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$

(C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{k+k}}$

25. A spring mass system oscillates with a frequency v. If it is taken in an elevator slowly accelerating upwards, the frequency will

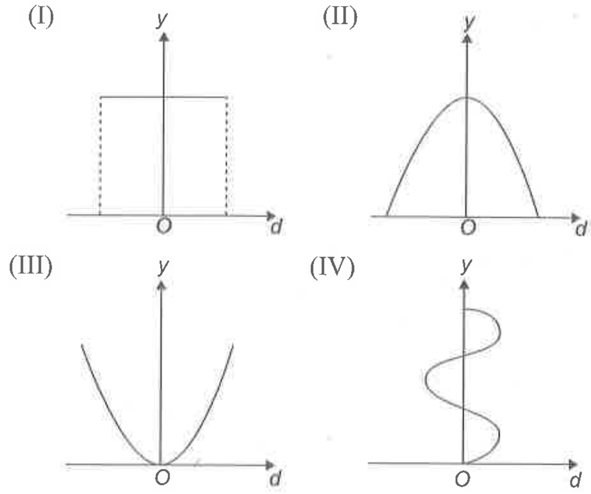
(A) increase (B) decrease
(C) remain same (D) become zero

26. A hollow metal sphere is filled with water and hung by a long thread. A small hole is drilled at the bottom through which water slowly flows out. Now the sphere is made to oscillate, the period of oscillation of the pendulum

(A) remains constant
(B) continuously decreases
(C) continuously increases
(D) first increases and then decreases

Question No. 27 to 29

The graph in the figure shows that a quantity y varies with displacement d in a system undergoing simple harmonic motion.



Which graphs best represents the relationship obtained when y is

27. The total energy of the system

(A) I (B) II
(C) III (D) IV

28. The time

(A) I (B) II
(C) III (D) IV

29. The unbalanced force acting on the system

(A) I (B) II
(C) III (D) None of these

30. The potential energy of a simple harmonic oscillator of mass 2 kg in its mean position is 5 J. If its total energy is 9 J and its amplitude is 0.01 m, its time period would be

(A) $\frac{\pi}{10}$ s (B) $\frac{\pi}{20}$ s

(C) $\frac{\pi}{50}$ s (D) $\frac{\pi}{100}$ s

31. Find the ratio of time periods of two identical springs if they are first joined in series and then in parallel and a mass m is suspended from them.

(A) 4 (B) 2
(C) 1 (D) 3

32. Two bodies P and Q of equal masses are suspended from two separate massless springs of force constants k_1 and k_2, respectively. If the maximum velocity of them are equal during their motion, the ratio of amplitude of P to Q is

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$

(C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

33. A spring mass system performs SHM. If the mass is doubled keeping amplitude same, then the total energy of SHM will become

(A) double (B) half
(C) unchanged (D) 4 times

34. A mass at the end of a spring executes harmonic motion about an equilibrium position with an amplitude A. Its speed as it passes through the equilibrium position is V. If extended $2A$ and released, the speed of the mass passing through the equilibrium position will be

(A) $2V$ (B) $4V$

(C) $\frac{V}{2}$ (D) $\frac{V}{4}$

35. A particle is subjected to two mutually perpendicular simple harmonic motions such that its x and y coordinates are given by $x = 2\sin \omega t$; $y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$. The path of the particle will be

(A) an ellipse (B) a straight line
(C) a parabola (D) a circle

36. The amplitude of the vibrating particle due to superposition of two SHMs, $y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin \omega t$ is

(A) 1 (B) $\sqrt{2}$

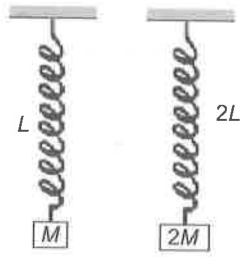
(C) $\sqrt{3}$ (D) 2

37. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is

(A) $\frac{1}{2} m\omega^2 A^2$ (B) $m\omega^2 A^2$

(C) $\frac{1}{4} m\omega^2 A^2$ (D) zero

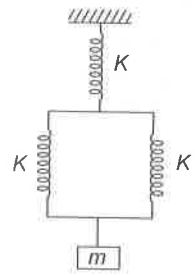
38. Two springs of the same material but of lengths L and $2L$ are suspended with masses M and $2M$ attached at their lower ends. Their time periods when they are allowed to oscillate will be in the ratio



(a) (b)

(A) 1:2 (B) 2:1
(C) 1:4 (D) 4:1

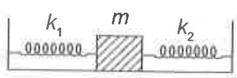
39. A body of mass 'm' hangs from three springs, each of spring constant 'k' as shown in the figure. If the mass is slightly displaced and let go, the system will oscillate with the time period



(A) $2\pi\sqrt{\frac{m}{3k}}$ (B) $2\pi\sqrt{\frac{3m}{2k}}$

(C) $2\pi\sqrt{\frac{2m}{3k}}$ (D) $2\pi\sqrt{\frac{3k}{m}}$

40. A block of mass m is connected between two springs (constants K_1 and K_2) as shown in the figure and is made to oscillate, the frequency of oscillation of the system will be



(A) $\frac{1}{2\pi}\left(\frac{m}{K_1 + K_2}\right)^{1/2}$ (B) $\frac{1}{2\pi}\left(\frac{K_1 K_2}{(K_1 + K_2)m}\right)^{1/2}$

(C) $\frac{1}{2\pi}\left(\frac{K_1 + K_2}{m}\right)^{1/2}$ (D) $\frac{1}{2\pi}\left(\frac{(K_1 + K_2)m}{K_1 K_2}\right)^{1/2}$

41. A particle of mass 4 kg moves between two points A and B on a smooth horizontal surface under the action of two forces such that when it is at a point P, the forces are $2\vec{PA}$ N and $2\vec{PB}$ N. If the particle is released from rest at A, find the time it takes to travel a quarter of the way from A to B.

(A) $\frac{\pi}{2}$ s (B) $\frac{\pi}{3}$ s

(C) πs (D) $\frac{\pi}{4}$ s

42. In an elevator, a spring clock of time period TS (mass attached to a spring) and a pendulum clock of time period TP are kept. If the elevator accelerates upwards

(A) TS as well as TP increases

(B) TS remains same, TP increases

(C) TS remains same, TP decreases

(D) TS as well as TP decreases

43. Two pendulums have time periods T and $5T/4$. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase?

(A) 5 (B) 4

(C) 11 (D) 9

44. A simple pendulum is oscillating in a lift. If the lift is going down with constant velocity, the time period of the simple pendulum is T_1. If the lift is going down with some retardation its time period is T_2, then

(A) $T_1 > T_2$

(B) $T_1 < T_2$

(C) $T_1 = T_2$

(D) depends upon the mass of the pendulum bob

45. A simple pendulum with length ℓ and bob of mass m executes SHM of small amplitude A. The maximum tension in the string will be

(A) $mg\,(1 + A/\ell)$ (B) $mg\,(1 + A/\ell)^2$

(C) $mg\,[1 + (A/\ell)^2]$ (D) $2mg$

46. A ring is suspended at a point on its rim and it behaves as a second's pendulum when it oscillates such that its centre moves in its own plane. The radius of the ring would be ($g = \pi^2$)

(A) 0.5 m (B) 1.0 m

(C) 0.67 m (D) 1.5 m

JEE Advanced

Single Correct

1. The maximum acceleration of a particle in SHM is made two times keeping the maximum speed to be constant. It is possible when

(A) amplitude of oscillation is doubled while frequency remains constant

(B) amplitude is doubled while frequency is halved

(C) frequency is doubled while amplitude is halved

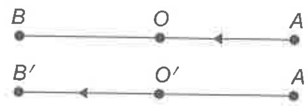
(D) frequency is doubled while amplitude remains constant

2. A small mass executes linear SHM about O with amplitude a and period T. Its displacement from O at time $T/8$ after passing through O is

(A) $a/8$ (B) $a/2\sqrt{2}$

(C) $a/2$ (D) $a/\sqrt{2}$

3. Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time



(A) $T/8$ (B) $3T/8$

(C) $T/6$ (D) $4T/3$

4. A particle executes SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$

(C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

5. Time period of a particle executing SHM is 8 s. At $t = 0$, it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$

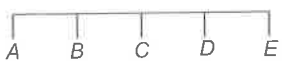
(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

6. Two particles P and Q describe simple harmonic motions of the same period, same amplitude, along the same line about the same equilibrium position O. When P and Q are on opposite sides of O at the same distance from O, they have the same speed of 1.2 m/s in the same direction, when their displacements are the same. They have the same speed of 1.6 m/s in opposite directions. The maximum velocity in m/s of either particle is

(A) 2.8 (B) 2.5

(C) 2.4 (D) 2

7. A body performs simple harmonic oscillations along the straight line $ABCDE$ with C as the mid-point of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If $AE = 2R$, the distance between B and D is



(A) $\frac{\sqrt{3}R}{2}$ (B) $\frac{R}{\sqrt{2}}$

(C) $\sqrt{3}\,R$ (D) $\sqrt{2}\,R$

8. A body at the end of a spring executes SHM with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(A) $T = t_1 + t_2$ (B) $T^2 = t_1^2 + t_2^2$

(C) $\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$ (D) $\frac{1}{T^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

9. A particle moves along the x-axis according to $x = A[1 + \sin \omega t]$. What distance does it travel between $t = 0$ and $t = 2.5\pi/\omega$?

(A) $4A$ (B) $6A$

(C) $5A$ (D) None of these

10. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in second^{-1}) is

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

11. Vertical displacement of a plank with a body of mass 'm' on it is varying according to the law $y = \sin \omega t + \sqrt{3}\cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after $t = 0$ are given by (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$

(C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

12. Two particles A and B perform SHM along the same straight line with the same amplitude 'a', same frequency 'f' and same equilibrium position 'O'. The greatest distance between them is found to be $3a/2$. At some instant of time, they have the same displacement from mean position. What is the displacement?

(A) $a/2$ (B) $a\sqrt{7}/4$

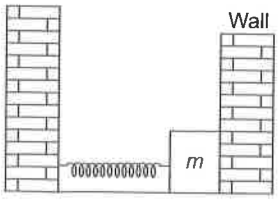
(C) $\sqrt{3}\,a/2$ (D) $3\,a/4$

13. A particle starts oscillating simple harmonically from its equilibrium position. Then the ratio of kinetic energy and potential energy of the particle at the time $T/12$ is (T = time period)

(A) 2:1 (B) 3:1

(C) 4:1 (D) 1:4

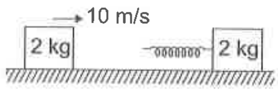
14. In the figure, the block of mass m, attached to the spring of stiffness k is in contact with the completely elastic wall, and the compression in the spring is 'e'. The spring is compressed further by 'e' by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic, then the time period of oscillations of the block will be



(A) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{k}}$

(C) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$

15. A 2 kg block moving with 10 m/s strikes a spring of constant π^2 N/m attached to 2 kg block at rest kept on a smooth floor. The time for which rear moving block remain in contact with spring will be



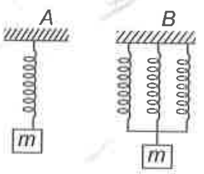
(A) $\sqrt{2}$ s (B) $\frac{1}{\sqrt{2}}$ s

(C) 1 s (D) $\frac{1}{2}$ s

16. In the above question, the velocity of the rear 2 kg block after it separates from the spring will be

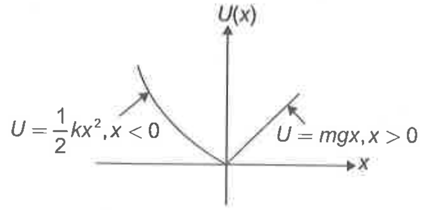
(A) 0 m/s (B) 5 m/s
(C) 10 m/s (D) 7.5 m/s

17. The springs in Figures A and B are identical but length in A is three times each of that in B. The ratio of period TA/TB is



(A) $\sqrt{3}$ (B) 1/3
(C) 3 (D) $1/\sqrt{3}$

18. A particle of mass m moves in the potential energy U as shown in the figure. The period of the motion when the particle has total energy E is



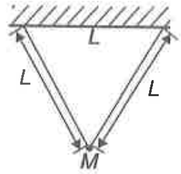
(A) $2\pi\sqrt{m/k} + 4\sqrt{2E/mg^2}$

(B) $2\pi\sqrt{m/k}$

(C) $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

(D) $2\sqrt{2E/mg^2}$

19. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is



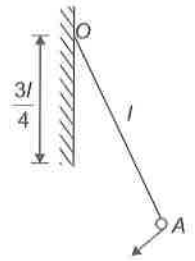
(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$

(C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

20. A ring of diameter 2 m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre moves in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is

(A) 2 m (B) 4 m
(C) 1.5 m (D) 3 m

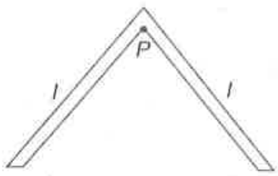
21. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $\frac{3\ell}{4}$ (as in the figure). If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be



(A) $\frac{3T}{4}$ (B) $\frac{T}{2}$

(C) T (D) $2T$

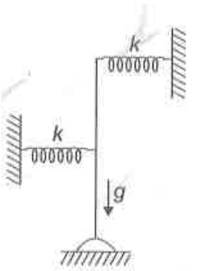
22. A system of two identical rods (L-shaped) of mass m and length l is resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations.



(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$

(C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

23. In the figure, the springs are connected to the rod at one end and at the mid-point. The rod is hinged at its lower end. Rotational SHM of the rod (mass m, length L) will occur only if



(A) $k > mg/3L$ (B) $k > 2mg/3L$
(C) $k > 2mg/5L$ (D) $k > 0$

24. What is the angular frequency of oscillations of the rod in the above problem if $k = mg/L$?
(A) $(3/2)\,[k/m]^{1/2}$ (B) $(3/4)\,[k/m]^{1/2}$
(C) $[2\,k/5\,m]^{1/2}$ (D) None of these

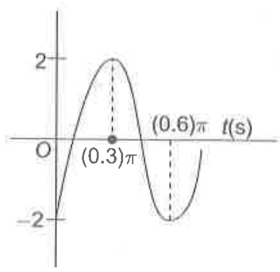
25. A rod whose ends are A and B of length 25 cm is hanged in vertical plane. When hanged from points A and B, the time periods calculated are 3 s and 4 s, respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in the ratio 9:4 at points A and B. Find the distance of the centre of mass from point A.
(A) 9 cm (B) 5 cm
(C) 25 cm (D) 20 cm

Multiple Correct

26. A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45 cm.
(A) The block will perform SHM of amplitude 5 cm.
(B) The block will have maximum velocity $30\sqrt{5}$ cm/s.
(C) The block will have maximum acceleration 15 m/s^2.
(D) The minimum potential energy of the spring will be zero.

27. A particle executing a simple harmonic motion of period 2 s. When it is at its extreme displacement from its mean position, it receives an additional energy equal to what it had in its mean position. Due to this, in its subsequent motion,
(A) its amplitude will change and become equal to $\sqrt{2}$ times its previous amplitude
(B) its periodic time will become doubled, i.e., 4 s
(C) its potential energy will be decreased
(D) it will continue to execute simple harmonic motion of the same amplitude and period as before receiving the additional energy

28. A part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this SHM is



(A) $y = 4\cos(0.6t)$

(B) $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

(C) $y = 4\sin\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

(D) $y = 2\cos\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

29. Two particles execute SHM with amplitudes A and $2A$ and angular frequencies ω and 2ω, respectively. At $t = 0$, they start with some initial phase difference. At $t = \frac{2\pi}{3\omega}$, they are in same phase. Their initial phase difference is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$ (D) π

30. Two particles are in SHM with the same angular frequency and amplitudes A and $2A$, respectively, along the same straight line with the same mean position. They cross each other at position $A/2$ distance from mean position in opposite direction. The phase between them is

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$

(C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

31. The equation of motion for an oscillating particle is given by $x = 3\sin(4\pi t) + 4\cos(4\pi t)$, where x is in mm and t is in seconds.
(A) The motion is simple harmonic.
(B) The period of oscillation is 0.5 s.
(C) The amplitude of oscillation is 5 mm.
(D) The particle starts its motion from the equilibrium.

32. A particle is executing SHM of amplitude A, about the mean position $X = 0$. Which of the following cannot be a possible phase difference between the positions of the particle at $x = +A/2$ and $x = -A/\sqrt{2}$?
(A) 75° (B) 165°
(C) 135° (D) 195°

33. Speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 108 - 9x^2$ (all quantities in SI unit). Then
(A) the motion is uniformly accelerated along the straight line
(B) the magnitude of the acceleration at a distance 3 cm from the fixed point is 0.27 m/s²
(C) the motion is simple harmonic about $x = \sqrt{12}$ m
(D) the maximum displacement from the fixed point is 4 cm

34. A block is placed on a horizontal plank. The plank is performing SHM along a vertical line with amplitude of 40 cm. The block just loses contact with the plank when the plank is momentarily at rest. Then
(A) the period of its oscillations is $2\pi/5$ s
(B) the block weights on the plank double its weight, when the plank is at one of the positions of momentary rest
(C) the block weights 1.5 times its weight on the plank halfway down from the mean position
(D) the block weights its true weight on the plank, when velocity of the plank is maximum

35. The potential energy of a particle of mass 0.1 kg, moving along the x-axis, is given by $U = 5x(x - 4)$ J where x is in metres. It can be concluded that
(A) the particle is acted upon by a constant force
(B) the speed of the particle is maximum at $x = 2$ m
(C) the particle executes simple harmonic motion
(D) the period of oscillation of the particle is $\pi/5$ s

36. A particle is executing SHM with amplitude A, time period T, maximum acceleration a_0 and maximum velocity v_0. It starts from mean position at $t = 0$ and at time t, it has the displacement $A/2$, acceleration a and velocity v then
(A) $t = T/12$ (B) $a = a_0/2$
(C) $v = v_0/2$ (D) $t = T/8$

37. The amplitude of a particle executing SHM about O is 10 cm. Then
(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6 cm from O.
(B) When the displacement is 5 cm from O its K.E. is 0.75 of its max. P.E.
(C) Its total energy at any point is equal to its maximum K.E.
(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

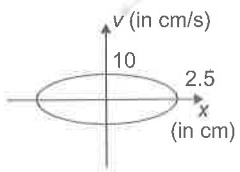
38. The displacement of a particle varies according to the relation $x = 3\sin 100t + 8\cos^2 50t$. Which of the following is/are correct about this motion?
(A) The motion of the particle is not SHM.
(B) The amplitude of the SHM of the particle is 5 units.
(C) The amplitude of the resultant SHM is $\sqrt{73}$ units.
(D) The maximum displacement of the particle from the origin is 9 units.

39. In SHM, acceleration versus displacement (from mean position) graph
(A) is always a straight line passing through origin and slope $-\omega^2$
(B) is always a straight line passing through origin and slope $+\omega^2$
(C) is a straight line not necessarily passing through origin
(D) none of these

40. A particle moves in the xy plane according to the law $x = a \sin \omega t$ and $y = a(1 - \cos \omega t)$, where a and ω are constants. The particle traces
(A) a parabola
(B) a straight line equally inclined to x and y axes
(C) a circle
(D) a distance proportional to time

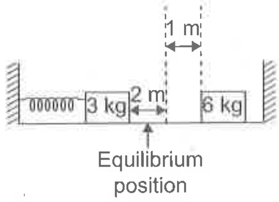
41. For a particle executing SHM, x = displacement from equilibrium position, v = velocity at any instant and a = acceleration at any instant, then
(A) v–x graph is a circle
(B) v–x graph is an ellipse
(C) a–x graph is a straight line
(D) a–v graph is an ellipse

42. The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM.



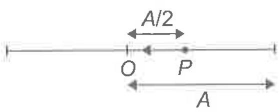
(A) The time period of the particle is 1.57 s.
(B) The maximum acceleration will be 40 cm/s^2.
(C) The velocity of particle is $2\sqrt{21}$ cm/s when it is at a distance 1 cm from the mean position.
(D) None of these

43. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to a spring with a force constant $k = 900$ Nm^{-1} which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1 m from mean position, 3 kg mass strikes the 6 kg mass and the two stick together.



(A) Velocity of the combined masses immediately after the collision is 10 ms^{-1}.
(B) Velocity of the combined masses immediately after the collision is 5 ms^{-1}.
(C) Amplitude of the resulting oscillation is $\sqrt{2}$ m.
(D) Amplitude of the resulting oscillation is $\sqrt{5}/2$ m.

44. A particle starts from a point P at a distance of $A/2$ from the mean position O and travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A, then the equation of motion of particle is



(A) $x = A \sin\left(\dfrac{2\pi}{T}t + \dfrac{\pi}{6}\right)$

(B) $x = A \sin\left(\dfrac{2\pi}{T}t + \dfrac{5\pi}{6}\right)$

(C) $x = A \cos\left(\dfrac{2\pi}{T}t + \dfrac{\pi}{6}\right)$

(D) $x = A \cos\left(\dfrac{2\pi}{T}t + \dfrac{\pi}{3}\right)$

45. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement.

(A) The amplitude of the block is $\frac{v_0}{\omega_0}$.
(B) The initial phase of the block is π.
(C) The equation of motion for the block is $\frac{v_0}{\omega_0}$ $\sin \omega_0 t$.
(D) The maximum speed of the block is v_0.

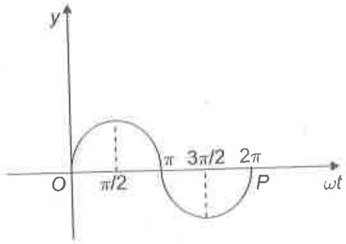
46. A disc of mass $3m$ and a disc of mass m are connected by a massless spring of stiffness k. The heavier disc is placed on the ground with the spring vertical and lighter disc on top. From its equilibrium position, the upper disc is pushed down by a distance δ and released. Then
(A) if $\delta > 3mg/k$, the lower disc will bounce up
(B) if $\delta = 2mg/k$, maximum normal reaction from ground on lower disc $= 6mg$
(C) if $\delta = 2mg/k$, maximum normal reaction from ground on lower disc $= 4\ mg$
(D) if $\delta > 4mg/k$, the lower disc will bounce up

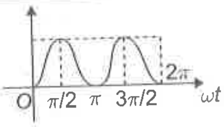
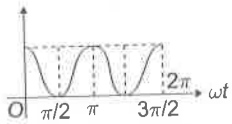
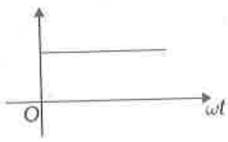
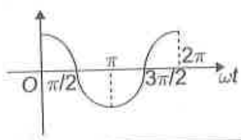
47. A system is oscillating with undamped simple harmonic motion. Then the
(A) average total energy per cycle of the motion is its maximum kinetic energy
(B) average total energy per cycle of the motion is $\frac{1}{\sqrt{2}}$ times its maximum kinetic energy
(C) root mean square velocity is $\frac{1}{\sqrt{2}}$ times its maximum velocity
(D) mean velocity is 1/2 of maximum velocity

48. A particle of mass m performs SHM along a straight line with frequency f and amplitude A.
(A) The average kinetic energy of the particle is zero.
(B) The average potential energy is $m\pi^2 f^2 A^2$.
(C) The frequency of oscillation of kinetic energy is $2f$.
(D) Velocity function leads acceleration by $\pi/2$.

49. A linear harmonic oscillator of force constant $2 \times 10^6\ \text{Nm}^{-1}$ and amplitude 0.01 m has a total mechanical energy of 160 J. Its
(A) maximum potential energy is 100 J
(B) maximum kinetic energy is 100 J
(C) maximum potential energy is 160 J
(D) minimum potential energy is zero

50. The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown in the figure. Then the best matching is



Column A	Column B
(A) K.E. versus phase angle curve	(i) 
(B) P.E. versus phase angle curve	(ii) 
(C) T.E. versus phase angle curve	(iii) 
(D) Velocity versus phase angle curve	(iv) 

(A) (A) - (i), (B) - (ii), (C) - (iii) and (D) - (iv)
(B) (A) - (ii), (B) - (i), (C) - (iii) and (D) - (iv)
(C) (A) - (ii), (B) - (i), (C) - (iv) and (D) - (iii)
(D) (A) - (ii), (B) - (iii), (C) - (iv) and (D) - (i)

JEE Advanced

Level I

Linear SHM

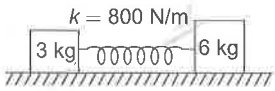
1. The equation of a particle executing SHM is $x = (5\,\text{m})\sin\left[(\pi s^{-1})t + \dfrac{\pi}{6}\right]$. Write down the amplitude, phase constant, time period and maximum speed.

2. A particle having mass 10 g oscillates according to the equation x = (2.0 cm) sin[100 s^{-1}] t + $\dfrac{\pi}{6}$]. Find (A) the amplitude, the time period and the force constant; (B) the position, the velocity and the acceleration at $t = 0$.

3. The equation of motion of a particle started at $t = 0$ is given by $x = 5\sin(20\,t + \pi/3)$ where x is in centimetre and t in seconds. When does the particle
   (A) first come to rest?
   (B) first have zero acceleration?
   (C) first have maximum speed?

4. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1:3 and again oscillated from the two parts separately, then the periods are T_1 and T_2. Find T_1/T_2.

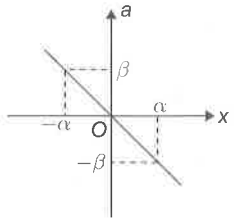
5. The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6 cm and then released. Find:



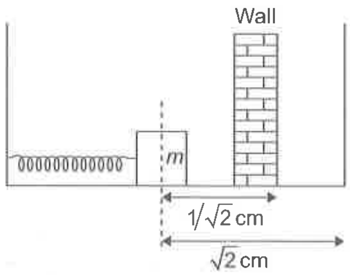
   (A) Time period
   (B) Amplitude of 3 kg block
   (C) Maximum momentum of 6 kg block

6. A body undergoing SHM about the origin has its equation is given by $x = 0.2\cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ s.

7. The acceleration–displacement ($a - x$) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



8. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it make elastic collision with the wall and its period of motion is 0.2 s. Find the approximate value of k.



9. A force $f = -10x + 2$ acts on a particle of mass 0.1 kg, where 'k' is in m and F in Newton. If it is released from rest at $x = -2$ m, find:
   (A) Amplitude
   (B) Time period
   (C) Equation of motion

10. Potential energy (U) of a body of unit mass moving in a one-dimensional conservative force field is given by, $U = (x^2 - 4x + 3)$. All units are in SI.
    (A) Find the equilibrium position of the body.
    (B) Show that oscillations of the body about this equilibrium position is simple harmonic motion and find its time period.
    (C) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

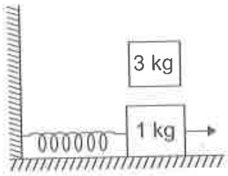
11. The resulting amplitude A' and the phase of the vibrations δ

$$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{4}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta)$$

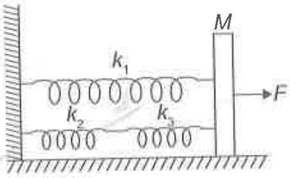
are ________ and ________ respectively.

12. A body is executing SHM under the action of force whose maximum magnitude is 50 N. Find the magnitude of force acting on the particle at the time when its energy is half kinetic and half potential.

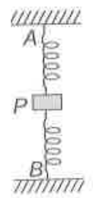
13. A 1 kg block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it at the instant it passes through the mean position. Assuming that the two blocks move together, find the frequency and the amplitude of the motion.



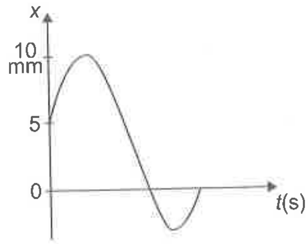
14. The springs shown in the figure are all unstretched in the beginning when a man starts pulling the block. The man exerts a constant force F on the block. Find the amplitude and the frequency of the motion of the block.



15. Two identical springs are attached to a small block P. The other ends of the springs are fixed at A and B. When P is in equilibrium, the extension of top spring is 20 cm and extension of bottom spring is 10 cm. Find the period of small vertical oscillations of P about its equilibrium position. (Take $g = 9.8$ m/s^2)

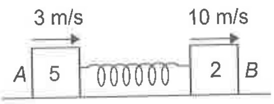


16. The figure shows the displacement–time graph of a particle executing SHM. If the time period of oscillation is 2 s, then the equation of motion is given by $x =$ ________.



17. Two particles A and B execute SHM along the same line with the same amplitude a, same frequency and same equilibrium position O. If the phase difference between them is $\phi = 2\sin^{-1}(0.9)$, then find the maximum distance between the two.

18. Two blocks A (5 kg) and B (2 kg) attached to the ends of a spring constant 1120 N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3 m/s and 10 m/s along the line of the spring in the same direction are imparted to A and B then



(A) Find the maximum extension of the spring.

(B) When does the first maximum compression occurs after start.

19. The motion of a particle is described by $x = 30\sin(\pi t + \pi/6)$, where x is in cm and t in seconds. Potential energy of the particle is twice of kinetic energy for the first time after $t = 0$ when the particle is at position ________ after ________ time.

20. A particle is performing SHM with acceleration $a = 8\pi^2 - 4\pi^2 x$, where x is the coordinate of the particle w.r.t. the origin. The parameters are in SI units. The particle is at rest at $x = -2$ at $t = 0$. Find coordinate of the particle w.r.t. origin at any time.

21. (A) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$ J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.

(B) A simple pendulum of length $l = 0.5$ m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

22. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point $x = 0.04$ m, the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ________.

23. A body of mass 1 kg is suspended from a weightless spring having force constant 600 N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0 m/s and gets embedded in it. Find the frequency of oscillations and amplitude of motion.

24. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.

25. A particle of mass m moves in a one-dimensional potential energy $U(x) = -ax^2 + bx^4$, where 'a' and 'b' are positive constants. Then what is the angular frequency of small oscillations about the minima of the potential energy.

26. A pendulum having time period equal to 2 seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a seconds pendulum at a place where $g = \pi^2$ m/s^2.

27. The angle made by the string of a simple pendulum with the vertical depends on time as $\theta = \frac{\pi}{90}\sin[(\pi s^{-1})\,t]$. Find the length of the pendulum if $g = \pi^2$ m/s^2.

28. A pendulum is suspended in a lift and its period of oscillation is T_0 when the lift is stationary.

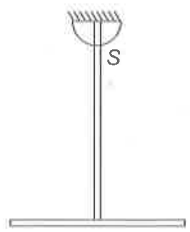
(A) What will the period T of oscillation of pendulum be, if the lift begins to accelerate downwards with an acceleration equal to $\frac{3g}{4}$?

(B) What must be the acceleration of the lift for the period of oscillation of the pendulum to be $\frac{T_0}{2}$?

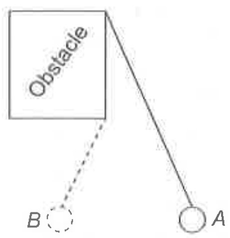
29. A simple pendulum of length ℓ is suspended through the ceiling of an elevator. Find the time period of small oscillations if the elevator (A) is going up with an acceleration a_0 (B) is going down with an acceleration a_0 and (C) is moving with a uniform velocity.

30. A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3.99 seconds. Making an approximate analysis, find the acceleration of the car.

31. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



32. A simple pendulum has a time period $T = 2$ s when it swings freely. The pendulum is hung as shown in the figure, so that only one-fourth of its total length is free to swing to the left of obstacle. It is displaced to position A and released. How long does it take to swing to extreme displacement B and return to A? Assume that displacement angle is always small.

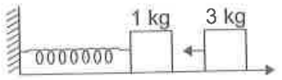


Level II

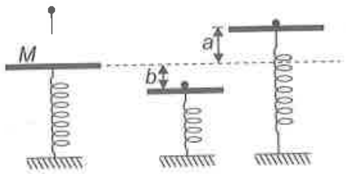
1. A point particle of mass 0.1 kg is executing SHM with amplitude of 0.1 m. When the particle passes through the mean position, its K.E. is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

2. The particle executing SHM in a straight line has velocities 8 m/s, 7 m/s, 4 m/s at three points distant 1 m from each other. What will be the maximum velocity of the particle?

3. One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to the x-axis. A block of mass $m = 1$ kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, t is in seconds and x in cm. Another block of mass $M = 3$ kg, moving towards the origin with velocity 30 cm/s collides with the block performing SHM at $t = 0$ and gets stuck to it, calculate:
   (A) new amplitude of oscillations,
   (B) new equation for position of the combined body,
   (C) loss of energy during collision. Neglect friction.

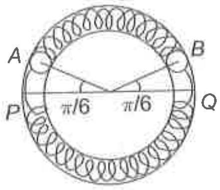


4. A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales and depth 'b' below it.



   (A) Find the force constant of the spring.
   (B) Find the oscillation frequency.
   (C) What is the height above the initial level from which the mass m was dropped?

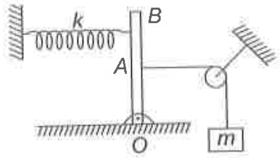
5. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in the figure. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06 m. Each spring has a natural length 0.06π m and force constant 0.1 N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest.



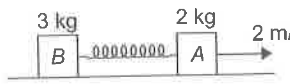
   (A) Calculate the frequency of oscillation of ball B.
   (B) What is the total energy of the system?
   (C) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

6. An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass m. The piston and the cylinder have equal cross-sectional area A, atmospheric pressure is P_0 and when the piston is in equilibrium position. Show that the piston executes SHM and find the frequency of oscillation (system is completely isolated from the surrounding). $\gamma = Cp/Cv$. Height of the gas in equilibrium position is h.

7. A massless rod is hinged at O. A string carrying a mass m at one end is attached to point A on the rod so that $OA = a$. At another point B ($OB = b$) of the rod, a horizontal spring of force constant k is attached as shown in the figure. Find the period of small vertical oscillations of mass m around its equilibrium position.



8. Two blocks A (2 kg) and B (3 kg) rest up on a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially, the spring is underformed. A is imparted a velocity of 2 m/s along the line of the spring away from B. Find the displacement of A t second later.



9. Consider a fixed ring shaped uniform body of linear mass density ρ and radius R. A particle at the centre of ring is displaced along the axis by a small distance, show that the particle will execute SHM under gravitation of ring and find its time period neglecting other forces.

Previous Year Questions

JEE Main

1. If a spring has time period T and is cut into n equal parts, then the time period of each part will be **[AIEEE 2002]**

(A) $T\sqrt{n}$ (B) $\frac{T}{\sqrt{n}}$

(C) nT (D) T

2. In a simple harmonic oscillator, at the mean position **[AIEEE 2002]**

(A) kinetic energy is minimum, potential energy is maximum

(B) both kinetic and potential energies are maximum

(C) kinetic energy is maximum, potential energy is minimum

(D) both kinetic and potential energies are minimum

3. A child swinging on a swing in sitting position, stands up, then the time period of the swing will **[AIEEE 2003]**

(A) increase

(B) decrease

(C) remain same

(D) increase if the child is long and decrease if the child is short

4. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as a function of displacement x. Which of the following statement is true? **[AIEEE 2003]**

(A) KE is maximum when $x = 0$

(B) TE is zero when $x = 0$

(C) KE is maximum when x is maximum

(D) PE is maximum when $x = 0$

5. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[AIEEE 2003]**

(A) 11% (B) 21%

(C) 42% (D) 10.5%

6. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is **[AIEEE 2003]**

(A) $\sqrt{\frac{k_1}{k_2}}$ (B) $\frac{k_1}{k_2}$

(C) $\sqrt{\frac{k_2}{k_1}}$ (D) $\frac{k_2}{k_1}$

7. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$, then the ratio of $\frac{m}{M}$ is **[AIEEE 2003]**

(A) $\frac{3}{5}$ (B) $\frac{25}{9}$

(C) $\frac{16}{9}$ (D) $\frac{5}{3}$

8. The total energy of a particle, executing simple harmonic motion is **[AIEEE 2004]**

(A) μx (B) μx^2

(C) independent of x (D) $\mu x^{1/2}$

where x is displacement from the mean position

9. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then **[AIEEE 2004]**

(A) $T = t_1 + t_2$ (B) $T^2 = t_1^2 + t_2^2$

(C) $T^{-1} = t_1^{-1} + t_2^{-1}$ (D) $T^{-2} = t_1^{-2} + t_2^{-2}$

10. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kgm^{-3}. What relationship between t and t_0 is true? **[AIEEE 2004]**

(A) $t = t_0$ (B) $t = \frac{t_0}{2}$

(C) $t = 2t_0$ (D) $t = 4t_0$

11. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x$, its time period is **[AIEEE 2005]**

(A) $\frac{2\pi}{\alpha}$ (B) $\frac{2\pi}{\sqrt{\alpha}}$

(C) $2\pi\alpha$ (D) $2\pi\sqrt{\alpha}$

12. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos \pi t$.

The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is **[AIEEE 2005]**

(A) $\frac{-\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{-\pi}{3}$ (D) $\frac{\pi}{3}$

13. The function $\sin^2(\omega t)$ represents **[AIEEE 2005]**

(A) a periodic, but not simple harmonic, motion with a period $2\pi/\omega$

(B) a periodic, but not simple harmonic with a period π/ω

(C) a simple harmonic motion with a period $2\pi/\omega$

(D) a simple harmonic motion with a period π/ω

14. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[AIEEE 2006]**

(A) $\frac{1}{6}$ s (B) $\frac{1}{4}$ s

(C) $\frac{1}{3}$ s (D) $\frac{1}{12}$ s

15. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1}. The period of oscillation is **[AIEEE 2006]**

(A) 0.01 s (B) 10 s

(C) 0.1 s (D) 100 s

16. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2\times10^{-2}\cos\pi t$ metre. The time at which the maximum speed first occurs is **[AIEEE 2007]**

(A) 0.5 s (B) 0.75 s

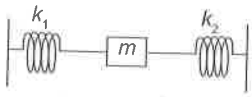
(C) 0.125 s (D) 0.25 s

17. A particle of mass m executes simple harmonic motion with amplitude a and frequency v. The average kinetic energy during its motion from the position of equilibrium to the end is **[AIEEE 2007]**

(A) $\pi^2 ma^2 v^2$ (B) $\frac{1}{4} ma^2 v^2$

(C) $4\pi^2 ma^2 v^2$ (D) $2\pi^2 ma^2 v^2$

18. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown in the figure. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[AIEEE 2007]**



(A) $\frac{f}{2}$ (B) $\frac{f}{4}$

(C) $4f$ (D) $2f$

19. A point mass oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then **[AIEEE 2007]**

(A) $A = x_0, \delta = -\frac{\pi}{4}$ (B) $A = x_0\omega^2, \delta = \frac{\pi}{4}$

(C) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$ (D) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$

20. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing, simple harmonic motion of time period of T, then which of the following does not change with time? **[AIEEE 2009]**

(A) $a^2T^2 + 4\pi^2v^2$ (B) $\frac{aT}{x}$

(C) $aT + 2\pi v$ (D) $\frac{aT}{v}$

21. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is **[AIEEE 2011]**

(A) $\frac{M+m}{M}$ (B) $\left(\frac{M}{M+m}\right)^{1/2}$

(C) $\left(\frac{M+m}{M}\right)^{1/2}$ (D) $\frac{M}{M+m}$

22. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 $(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is **[AIEEE 2011]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

23. If a spring of stiffness k is cut into two parts A and B of length $lA{:}lB = 2{:}3$, then the stiffness of spring A is given by **[AIEEE 2011]**

(A) $\frac{5}{2}k$ (B) $\frac{3k}{5}$

(C) $\frac{2k}{5}$ (D) k

24. This question has statement 1 and statement 2. Of the four choices given after the statement, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants K_1 and K_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2. **[AIEEE 2012]**

Statement 1: If stretched by the same amount, work done on S_1 will be more than that on S_2.

Statement 2: $k_1 < k_2$

(A) Statement 1 is false, Statement 2 is true.
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

25. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$, in the same direction, then **[JEE Main 2014]**

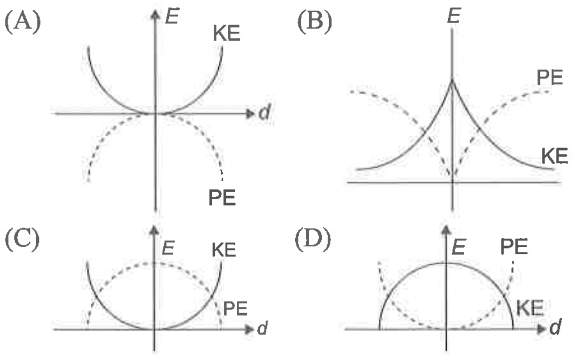
(A) amplitude of motion is $4a$
(B) time period of oscillations is 6τ
(C) amplitude of motion is $3a$
(D) time period of oscillations is 8τ

26. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to TM. If the Young's modulus of the material of the wire is Y, then $1/Y$ is equal to (g = gravitational acceleration) **[JEE Main 2015]**

(A) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (B) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(C) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (D) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

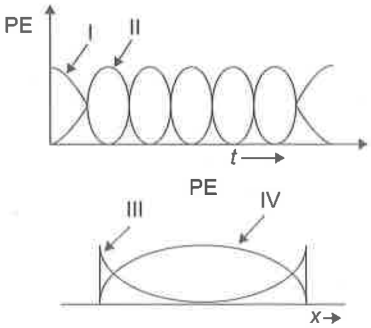
27. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **[JEE Main 2015]**



JEE Advanced

1. A particle is executing SHM according to $y = a\cos\omega t$. Then which of the graphs represent variations of potential energy? **[JEE (Scr)' 2003]**

(A) (I) and (III)
(B) (II) and (IV)
(C) (I) and (IV)
(D) (II) and (III)



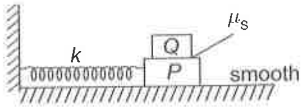
2. Two masses m_1 and m_2 connected by a light spring of natural length l_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along the +ve x-axis pass through the origin at $t = 0$. At this position, the string snaps. Position of mass m_1 at time is given by the equation $x_1(t) = v_0t - A(1 - \cos \omega t)$.

Calculate:

(A) Position of the particle m_2 as a function of time.

(B) l_0 in terms of A. **[JEE 2003]**

3. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of the same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is **[JEE 2004]**

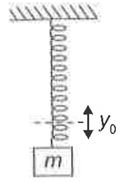


(A) kA
(B) $\frac{kA}{2}$
(C) zero
(D) $\mu_s mg$

4. A simple pendulum has time period T_1. When the point of suspension moves vertically up according to the equation $y = kt^2$ where $k = 1$ m/s² and 't' is time then the time period of the pendulum is T_2. Then $\left(\frac{T_1}{T_2}\right)^2$ is **[JEE 2005(Scr)]**

(A) $\frac{5}{6}$
(B) $\frac{11}{10}$
(C) $\frac{6}{5}$
(D) $\frac{5}{4}$

5. A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency ω and amplitude a. If at a height y^* from the mean position, the body gets detached from the spring, calculate the value of y^* so that the height H attained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment $(aw^2 > g)$. **[JEE 2005]**



6. Function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM **[JEE 2006]**

(A) for any value of A, B and C (except $C = 0$)

(B) if $A = -B$, $C = 2B$, amplitude $= |B\sqrt{2}|$

(C) if $A = B$, $C = 0$

(D) if $A = B$, $C = 2B$, amplitude $= |B|$

7. A student performs an experiment for determination of $g\left(= \frac{4\pi^2 l}{T^2}\right)$ $l \approx 1$ m and he commits an error of Δl.

For T, he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 s. For which of the following data, the measurement of g will be most accurate? **[JEE 2006]**

	Δl	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 s	10	5 mm
(B)	5 mm	0.2 s	20	5 mm
(C)	5 mm	0.1 s	20	1 mm
(D)	1 mm	0.1 s	50	1 mm

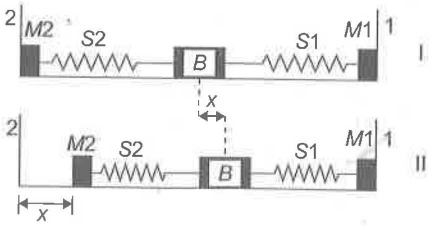
8. **Column I** describes some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situations in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[JEE 2007]**

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(P) The object executes a SHM.

Column I	Column II
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(Q) The object does not change its direction.
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(R) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where M_e is the mass of the earth and R^e is the radius of the earth. Neglect forces from objects other than the earth.	(S) The object can change its direction only once.

9. A block (B) is attached to two unstretched springs $S1$ and $S2$ with spring constants k and $4k$, respectively (see Figure I). The other ends are attached to identical supports $M1$ and $M2$ not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. Block B is displaced towards wall 1 by a small distance x (Figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of block B.

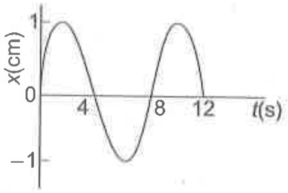
The ratio $\frac{y}{x}$ in the figure is **[JEE 2008]**



(A) 4 (B) 2

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

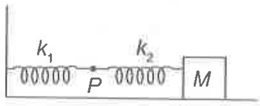
10. The x–t graph of the particle undergoing simple harmonic motion is shown in the figure. The acceleration of the particle at $t = 4/3$ s is **[JEE 2009]**



(A) $\frac{\sqrt{3}}{32}\pi^2 \text{ cm/s}^2$ (B) $\frac{-\pi^2}{32} \text{ cm/s}^2$

(C) $\frac{\pi^2}{32} \text{ cm/s}^2$ (D) $-\frac{\sqrt{3}}{32}\pi^2 \text{ cm/s}^2$

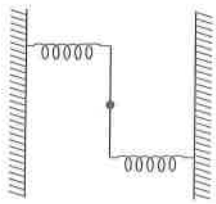
11. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[JEE 2009]**



(A) $\frac{k_2 A}{k_2}$ (B) $\frac{k_2 A}{k_2}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

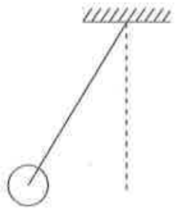
12. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is **[JEE 2009]**



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

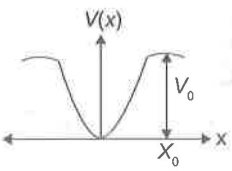
13. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disc of mass 'M' and radius 'R' ($< L$) is attached at its centre to the free end of the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position which of the following statement(s) is/are true? **[JEE 2011]**



(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A < Restoring torque in case B
(C) Angular frequency for case A > Angular frequency for case B
(D) Angular frequency for case A < Angular frequency for case B

Question No. 14 to 16

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see the figure). **[JEE 2010]**



14. If the total energy of the particle is E, it will perform periodic motion only if
(A) $E < 0$ (B) $E > 0$
(C) $V_0 > E > 0$ (D) $E > V_0$

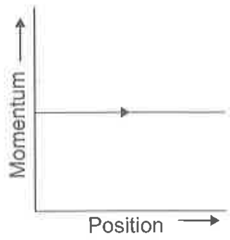
15. For periodic motion of small amplitude A, the time period T of this particle is proportional to
(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
(C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

16. The acceleration of this particle for $|x| > X_0$ is
(A) proportional to V_0
(B) proportional to $\frac{V_0}{mX_0}$
(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$
(D) zero

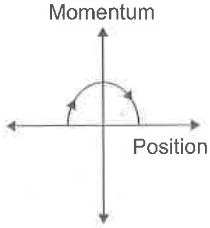
17. A point mass is subjected to two simultaneous sinusoidal displacements in the x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **[JEE 2011]**
(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$
(C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

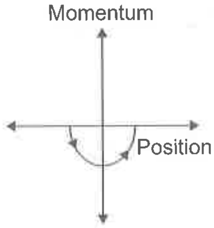
Question No. 18 to 20

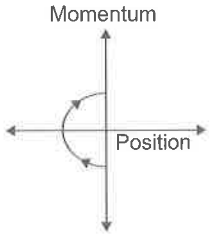
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here, we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along the horizontal axis and momentum is plotted along the vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position of momentum upwards (or to right) is positive and downwards (or to left) is negative. **[JEE 2012]**

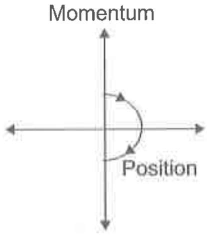


18. The phase space diagram for a ball thrown vertically up from the ground is

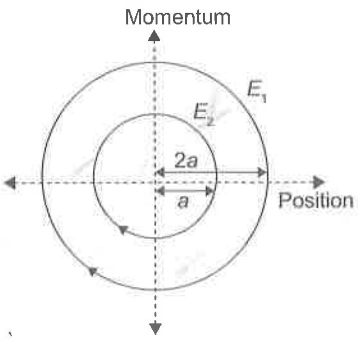
(A)


(B)


(C)


(D)


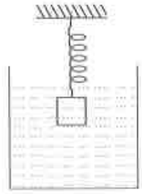
19. The phase space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

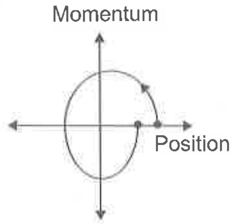


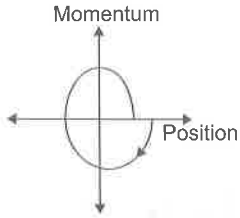
(A) $E_1 = \sqrt{2}\, E_2$ (B) $E_1 = 2E_2$

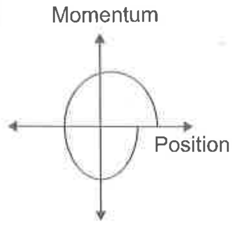
(C) $E_1 = 4E_2$ (D) $E_1 = 15E_2$

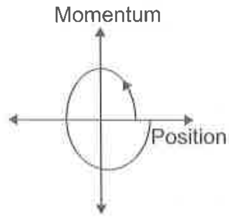
20. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


(B)


(C)


(D)


21. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\ u_0$, it collides elastically with a rigid wall. After this collision, **[JEE Advanced 2013]**

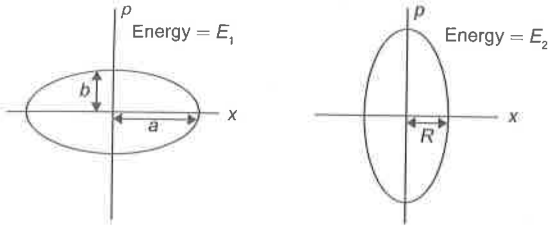
(A) the speed of the particle when it returns to its equilibrium position is u_0

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\dfrac{m}{k}}$

(C) the time at which the maximum compression of the spring occurs is $t = \dfrac{4\pi}{3}\sqrt{\dfrac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is $t = \dfrac{5\pi}{3}\sqrt{\dfrac{m}{k}}$

22. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their moments p with positions x are shown in the figures. If $a/b = n^2$ and $a/R = n$, then the correct equation(s) is (are) **[JEE Advanced 2015]**



(A) $E_1\omega_2 = E_2\omega_2$

(B) $\dfrac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$

(D) $\dfrac{E_1}{\omega_1} = \dfrac{E_2}{\omega_2}$

ANSWER KEYS

Exercises

JEE Main

1. A	**2.** C	**3.** B	**4.** C	**5.** C	**6.** A	**7.** A	**8.** C	**9.** A	**10.** D
11. B	**12.** B	**13.** A	**14.** C	**15.** B	**16.** A	**17.** C	**18.** D	**19.** C	**20.** B
21. B	**22.** C	**23.** C	**24.** C	**25.** C	**26.** D	**27.** A	**28.** D	**29.** D	**30.** D
31. B	**32.** B	**33.** C	**34.** A	**35.** A	**36.** C	**37.** B	**38.** A	**39.** B	**40.** C
41. B	**42.** C	**43.** A	**44.** A	**45.** C	**46.** A				

JEE Advanced

1. C	**2.** D	**3.** B	**4.** D	**5.** D	**6.** D	**7.** C	**8.** B	**9.** C	**10.** C
11. A	**12.** B	**13.** B	**14.** A	**15.** C	**16.** A	**17.** C	**18.** C	**19.** B	**20.** C
21. A	**22.** B	**23.** C	**24.** A	**25.** D	**26.** B, C, D	**27.** A	**28.** B	**29.** B, C	**30.** A
31. A, B, C	**32.** C	**33.** B	**34.** A, B, C, D		**35.** B, C, D	**36.** A, B	**37.** A, B, C	**38.** B, D	
39. A	**40.** C, D	**41.** B, C, D	**42.** A, B, C	**43.** A, C	**44.** B, D	**45.** B	**46.** B, D	**47.** A, C	**48.** B, C
49. B, C	**50.** B								

JEE Advanced

Level I

Linear SHM

1. Amplitude = 5 m, Initial phase = $\pi/6$, Maximum speed = 5π m/s

2. (A) 2.0 cm, $\pi/50$ s, 100 N/m, (B) 1 cm, $\sqrt{3}$ m/s, 100 m/s^{-1}

3. (A) $\dfrac{\pi}{120}$ s, (B) $\dfrac{\pi}{30}$ s, (C) $\dfrac{\pi}{30}$ s **4.** $\dfrac{1}{\sqrt{3}}$ **5.** (A) $\dfrac{\pi}{10}$ s, (B) 4 cm, (C) 2.40 kg m/s

6. 2 m/s **7.** $\dfrac{1}{2\pi}\sqrt{\dfrac{\beta}{\alpha}}$ **8.** 100 Nm^{-2} **9.** (A) $\dfrac{11}{5}$ m, (B) $\dfrac{\pi}{5}$ s, (C) $x = 0.2 - \dfrac{11}{5}\cos\omega t$

10. (A) $x_0 = 2$ m, (B) $T = \sqrt{2\pi}$ s, (C) $2\sqrt{3}$ **11.** $\dfrac{3\sqrt{5}}{8}A$, $\tan^{-1}\left(\dfrac{1}{2}\right)$ **12.** $25\sqrt{2}$ N **13.** $\dfrac{5}{2\pi}$ Hz, 5 cm

14. $\dfrac{F(K_2+K_3)}{K_1K_2+K_2K_3+K_3K_1}$, $\dfrac{1}{2\pi}\sqrt{\dfrac{K_1K_2+K_2K_3+K_3K_1}{M(K_2+K_3)}}$ **15.** $\dfrac{\pi}{7}$ **16.** $X = 10\sin(\pi t + \pi/6)$ **17.** $1.8a$

18. (A) 25 cm, (B) $\frac{3\pi}{56}$ s **19.** $10\sqrt{6}$ cm, $\frac{1}{\pi}\sin^{-1}\sqrt{\frac{2}{3}}-\frac{1}{6}$ s **20.** $2 - 4\cos 2\pi t$

21. (A) 2 s, (B) $T=\frac{2}{5^{1/4}}$ s **22.** 0.06 m **23.** $\frac{10}{\pi}$ Hz, $\frac{5\sqrt{37}}{6}$ cm **24.** 2 cm **25.** $2\sqrt{\frac{a}{m}}$

Angular SHM

26. 1 m **27.** 1 m **28.** (A) $2T_0$, (B) 3 g upwards **29.** (A) $2\pi\sqrt{\frac{\ell}{a_0+g}}$, (B) $2\pi\sqrt{\frac{\ell}{g-a_0}}$, (C) $2\pi\sqrt{\frac{\ell}{g}}$

30. $\frac{g}{10}$ **31.** $2\pi\sqrt{\frac{17L}{18g}}$ **32.** $\frac{3}{2}$ s

Level II

1. $y = 0.1\sin(4t + \pi/4)$ **2.** $\sqrt{65}$ m / s **3.** 3 cm, $x = 10 - 3\sin 5t$; $\Delta E = 0.135$ J

4. (A) $K=\frac{2mg}{b-a}$; (C) $\left(\frac{M+m}{m}\right)\frac{ab}{b-a}$, $\frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$ **5.** $f=\frac{1}{\pi}$; $E = 4\pi^2 \times 10^{-5}$ J; $v = 2\pi \times 10^{-2}$ m/s

6. $f=\frac{1}{2\pi}\sqrt{\frac{\gamma(P_0+mg/A)A}{mh}}$ **7.** $(2\pi a/b)(m/k)^{1/2}$ **8.** $0.8t + 0.12\sin 10t$ **9.** $\sqrt{\frac{2\pi R^2}{Gp}}$

Previous Year Questions

JEE Main

1. B	**2.** C	**3.** B	**4.** A	**5.** D	**6.** C	**7.** C	**8.** C	**9.** B	**10.** C
11. B	**12.** A	**13.** B	**14.** A	**15.** A	**16.** A	**17.** A	**18.** D	**19.** D	**20.** B
21. C	**22.** A	**23.** A	**24.** A	**25.** B	**26.** C	**27.** D			

JEE Advanced

1. A **2.** (A) $x_2 = v_0 t + \frac{m_1}{m_2}A(1-\cos\omega t)$, (B) $l_0 = \left(\frac{m_1}{m_2}+1\right)A$ **3.** B **4.** C **5.** $y^* = \frac{mg}{k} = \frac{g}{\omega^2} < a$

6. A, B, D **7.** D **8.** (A) P; (B) Q, R; (C) P; (D) Q, R or (A) P; (B) Q, R; (C) P; (D) R

9. C	**10.** D	**11.** D	**12.** C	**13.** A,D	**14.** C	**15.** B
16. D	**17.** B	**18.** D	**19.** C	**20.** B	**21.** A, D	**22.** B, D

CHAPTER 2

Waves

WAVES

Waves are distributed energy or distributed 'disturbance (force)'.

Important Points Regarding Waves

1. The disturbance (force) is transmitted from one point to another.
2. The energy is transmitted from one point to another.
3. The energy or disturbance passes in the form of waves without any net displacement of the medium.
4. The oscillatory motion of the preceding particle is imparted to the adjacent particle following it.
5. We need to keep creating disturbances in order to propagate waves (energy or disturbance) continuously.

WAVES CLASSIFICATION

Waves are classified under two high-level headings.

Mechanical Waves

The motion of the particle constituting the medium follows mechanical laws, i.e., Newton's laws of motion. Mechanical waves originate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium. The forces between the atoms in the medium are responsible for the propagation of mechanical waves. Each atom exerts a force on the atoms near it, and through this force, the motion of the atom is transmitted to the others. The atoms in the medium do not experience any net displacement.

Mechanical waves are further classified in two categories:

1. Transverse waves (waves on a string)
2. Longitudinal waves (sound waves)

Non-mechanical Waves

These are electromagnetic waves. Electromagnetic waves do not require a medium for propagation. Speed of electromagnetic waves in vacuum is a universal constant. The motion of the electromagnetic waves in a medium depends on the electromagnetic properties of the medium.

Transverse Waves

If the disturbance travels in the x-direction, but the particles move in a direction perpendicular to the x-axis as the wave passes, then those waves are called as transverse waves.

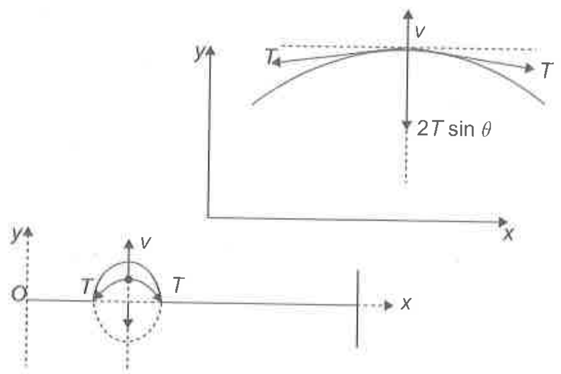


Figure 2.1

Consider a sinusoidal harmonic wave travelling through a string and the motion of a particle as shown in Fig. 2.1 (only one unit of wave is shown for illustration purpose). Since the particle is displaced from its natural (mean) position, the tension in the string arising from the deformation tends to restore the position of the particle. On the other hand, velocity of the particle (kinetic energy) which is moving it farther is zero. Therefore, the particle is pulled down due to tension towards mean position. In the process, it acquires kinetic energy (greater speed) and overshoots the mean position in the downward direction. The cycle of restoration of position continues as vibration (oscillation) of the particle takes place.

Longitudinal Waves

Longitudinal waves are characterized by the direction of vibration (disturbance) and wave motion. They are along the same direction. It is clear that vibration in the same direction needs to be associated with a 'restoring' mechanism in the longitudinal direction.

MATHEMATICAL DESCRIPTION OF WAVES

We shall attempt here to evolve a mathematical model of a travelling transverse wave. For this, we choose a specific set up of a string and associated transverse waves travelling through it. The string is tied to a fixed end, while disturbance is imparted at the free end by up and down motion. For our purpose, we consider that pulse is small in dimension and the string is light, elastic and homogeneous. The assumptions are required as we visualize a small travelling pulse, which remains undiminished when it moves through the strings. We also assume that the string is long enough so that our observation is not subjected to pulse reflected at the fixed end.

For understanding purpose, we first consider a single pulse as shown in Fig. 2.2 (irrespective of whether we can realize such pulse in practice or not). Our objective here is to determine the nature of a mathematical description which will enable us to determine displacement (disturbance) of the string as the pulse passes through it. We visualize two snapshots of the travelling pulse at two close time instants t and '$t + \Delta t$'. The single pulse is moving towards right in the positive x-direction.

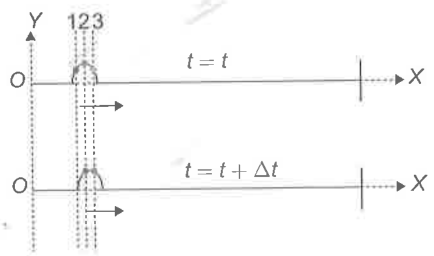


Figure 2.2

The vibration and wave motion are at right angle to each other.

Three positions along the x-axis indicated as 1, 2 and 3 are marked with three vertical dotted lines. At either of the two instants as shown, the positions of string particles have different displacements from the undisturbed position on the horizontal x-axis. We can conclude from this observation that displacement in the y-direction is a function of the positions of the particle in the x-direction. As such, the displacement of a particle constituting the string is a function of x.

Let us now observe the positions of a given particle, say 1. It has certain positive displacement at time $t = t$. At the next snapshot at $t = t + \Delta t$, the displacement has reduced to zero. The particle at 2 has maximum displacement at $t = t$, but the same has reduced at $t = t + \Delta t$. The third particle at 3 has certain positive displacement at $t = t$. At $t = t + \Delta t$, it acquires additional positive displacement and reaches the position of maximum displacement. From these observations, we conclude that displacement of a particle at any position along the string is a function of t.

Combining the two observations, we conclude that displacement of a particle is a function of both position of the particle along the string and time,

$$y = f(x, t).$$

We can further specify the nature of the mathematical function by association the speed of the wave in our consideration. Let v be the constant speed with which the wave travels from the left end to the right end. We notice that wave function at a given position of the string is a function of time only as we are considering displacement at a particular value of x. Let us consider the left-hand end of the string as the origin of reference ($x = 0$ and $t = 0$). The displacement in the y-direction (disturbance) at $x = 0$ is a function of time, t only:

$$y = f(t) = A \sin \omega t.$$

The disturbance travels to the right at a constant speed v. Let it reach a point specified as $x = x$ after time t. If we visualize to describe the origin of this disturbance at $x = 0$, then the time elapsed for the disturbance to move from the origin ($x = 0$) to the point ($x = x$) is x/v. Therefore, if we want to use the function of displacement at $x = 0$ as given above, then we need to subtract the time elapsed and set the equation as

$$y = f\left(t - \frac{x}{v}\right)$$

$$= A \sin \omega\left(t - \frac{x}{v}\right)$$

This can also be expressed as

$$\Rightarrow \qquad f\left(\frac{vt - x}{v}\right)$$

$$-f\left(\frac{x-vt}{v}\right)$$

$$\Rightarrow \quad y(x, t) = g(x - vt),$$

using any fixed value of t (i.e., at any instant), this shows the shape of the string.

If the wave is travelling in the $-x$ direction, the wave equation is written as

$$y(x, t) = f\left(t + \frac{x}{v}\right).$$

The quantity $x - vt$ is called the phase of the wave function. As phase of the pulse has a fixed value,

$$x - vt = \text{constant}.$$

Taking the derivative w.r.t. time,

$$\frac{dx}{dt} = v,$$

where v is the phase velocity, often called the wave velocity. It is the velocity at which a particular phase of the disturbance travels through space.

In order for the function to represent a wave travelling at speed v, the quantities x, v and t must appear in the combination $(x + vt)$ or $(x - vt)$. Thus, $(x - vt)^2$ is acceptable, but $x^2 - v^2t^2$ is not.

DESCRIBING WAVES

Two kinds of graphs may be drawn: displacement–distance and displacement–time.

A displacement–distance graph for a transverse mechanical waves shows the displacement y of the vibrating particles of the transmitting medium at different distances x from the source at a certain instant, i.e., it is like a photograph showing the shape of the wave at that particular instant.

The maximum displacement of each particle from its undisturbed position is the amplitude of the wave.

In Fig. 2.3, it is OA or OB.

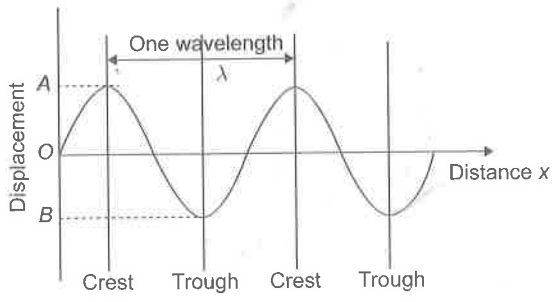


Figure 2.3

The wavelength λ of a wave is generally taken as the distance between two successive crests or two successive troughs. To be more specific, it is the distance between two consecutive points on the wave which have the same phase.

A displacement–time graph may also be drawn for a wave motion, showing how the displacement of one particle at a particular distance from the source varies with time. If this is simple harmonic variation, then the graph is a sine curve.

Wave Length, Frequency and Speed

If the source of a wave makes f vibrations per second, they will the particles of the transmitting medium. That is, the frequency of the waves equals frequency of the source.

When the source makes one complete vibration, one wave is generated and the disturbance spreads out a distance λ from the source. If the source continues to vibrate with constant frequency f, then f waves will be produced per second and the wave advances a distance $f\lambda$ in 1 s. If v is the wave speed, then

$$v = f\lambda.$$

This relationship holds for all wave motions.

Note

Frequency depends on the source (not on the medium), v depends on the medium (not on the source frequency), but wavelength depends on both the medium and the source.

Initial Phase

At $x = 0$ and $t = 0$, the sine function evaluates to zero and as such y-displacement is zero. However, a wave form can be such that y-displacement is not zero at $x = 0$ and $t = 0$. In such a case, we need to account for the displacement by introducing an angle like

$$y(x, t) = A \sin(kx - \omega t + \phi),$$

where ϕ is the initial phase.

At $x = 0$ and $t = 0$,

$$y(0, 0) = A \sin(\phi).$$

The measurement of an angle is determined by the following two aspects of wave form at $x = 0$, $t = 0$: (i) whether the displacement is positive or negative and (ii) whether the wave form has a positive or negative slope.

For a harmonic wave represented by a sine function, there are two values of initial phase angle for which displacement at reference origin ($x = 0$, $t = 0$) is positive and has equal magnitude. We know that the sine values of angles in the first and second quadrants are positive. A pair of initial phase angles, say $\phi = \pi/3$ and $2\pi/3$ correspond to equal positive sine values:

$$\sin\theta = \sin(\pi - \theta)$$

$$\sin\frac{\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{2\pi}{3}\right) = \frac{1}{2}.$$

To choose the initial phase between the two values $\pi/3$ and $2\pi/3$, we can look at a wave motion in yet another way. A wave form at an instant is displaced by a distance Δx in a very small time interval Δt then the speed of the particle at $t = 0$ and $x = 0$ is in the upward positive direction in further time Δt.

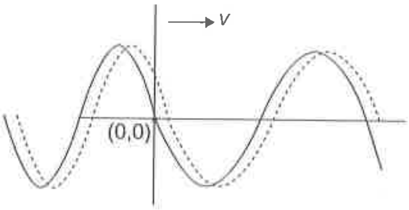


Figure 2.4

SOLVED EXAMPLES

EXAMPLE 1

Find out the expression of the wave equation which is moving in the positive x-direction and at $x = 0$, $t = 0$, $y = \frac{A}{\sqrt{2}}$.

SOLUTION

Let $$y = A\sin(\omega t - kx + \phi)$$

at $t = 0$ and $x = 0$.

$$\frac{A}{\sqrt{2}} = A\sin\phi$$

$$\Rightarrow \quad \sin\phi = \frac{1}{\sqrt{2}}$$

$$\phi = \frac{\pi}{4}, \frac{3\pi}{4}.$$

To choose the correct phase angle ϕ we displaced to wave. Slightly in the positive x-direction such that,

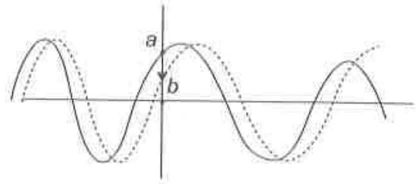


Figure 2.5

In Figure 2.5, particle at a is moving downward towards point b, i.e., particle at $x = 0$ and $y = \frac{A}{\sqrt{2}}$ have negative velocity, which gives

$$\frac{\partial y}{\partial t} = \omega A\cos(\omega - kx + \phi),$$

At $t = 0$, $x = 0$ is

$$\cos\phi = \text{negative (from Fig. 2.5)} \qquad (2)$$

From the above discussion, $3\pi/4$ gives $\sin\phi$ is positive and $\cos\phi$ is negative, i.e.,

$$\phi = \frac{3\pi}{4}.$$

■

Note

Equation of the wave which is moving in the negative x-direction is shown in Fig. 2.6.

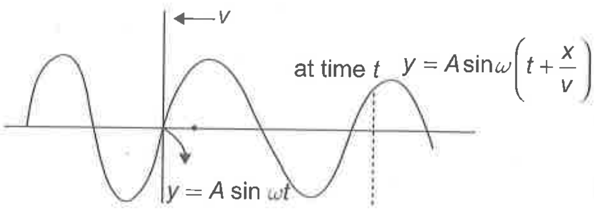


Figure 2.6

$$y = A\sin(\omega t + kx + \phi).$$

EXAMPLE 2

If (ωt) and (kx) terms have the same sign, then the wave moves towards the negative x-direction and vice versa and with a different initial phase.

$$\left.\begin{aligned} y &= A\sin(\omega t - kx) \\ y &= A\sin(-kx + \omega t) \end{aligned}\right] \text{Wave moves towards the + ve } x\text{-direction}$$

$$\left.\begin{aligned} y &= A\sin(-kx - \omega t) \\ &= A\sin(kx + \omega t + \pi) \\ y &= A\sin(kx + \omega t) \end{aligned}\right] \text{Wave moves towards the – ve } x\text{-direction.}$$

PARTICLE VELOCITY AND ACCELERATION

Particle velocity at a given position $x = x$ is obtained by differentiating the wave function with respect to time t. We need to differentiate the equation by treating x as a constant. The partial differentiation yields particle velocity as

$$v_p = \frac{\partial}{\partial t} y(x,t)$$

$$= \frac{\partial}{\partial t} A \sin(kx - \omega t)$$

$$= -\omega A \cos(kx - \omega t).$$

We can use the property of cosine function to find the maximum velocity. We obtain the maximum speed when the cosine function evaluates to -1:

$$\Rightarrow \qquad v_{p_{max}} = \omega A.$$

The acceleration of the particle is obtained by differentiating the expression of velocity partially with respect to time:

$$\Rightarrow \qquad a_p = \frac{\partial}{\partial t} v_p$$

$$= \frac{\partial}{\partial t}\{-\omega A \cos(kx - \omega t)\}$$

$$= -\omega^2 A \sin(kx - \omega t)$$

$$= -\omega^2 y.$$

Again the maximum value of the acceleration can be obtained using the property of sine function:

$$\Rightarrow \qquad a_{p_{max}} = \omega^2 A.$$

DIFFERENT FORMS OF WAVE FUNCTIONS

The different forms of wave functions give rise to a bit of confusion about the form of wave function. The forms used for describing waves are

$$y(x, t) = A \sin(kx - \omega t)$$

$$y(x, t) = A \sin(\omega t - kx + \pi)$$

Which of the two forms is correct? In fact, both are correct so long as we are in a position to accurately interpret the equation. Starting with the first equation and using trigonometric identity, we have,

$$\Rightarrow \qquad A \sin(kx - \omega t) = A \sin(\pi - kx + \omega t)$$

$$= A \sin(\omega t - kx + \pi).$$

Thus, we see that the two forms represent waves having the same speed $\left(v = \frac{\omega}{k}\right)$. They differ, however, in phase. There is a phase difference of π. This has implications on the waveform and in the manner particles oscillate at any given time instant and position. Let us consider two waveforms at $x = 0$, $t = 0$. The slopes of the waveforms are given as

$$\frac{\partial}{\partial x} y(x,t) = kA \cos(kx - \omega t)$$

$$= kA$$

$$= \text{a positive number}$$

and

$$\frac{\partial}{\partial x} y(x,t) = -kA \cos(\omega t - kx)$$

$$= -kA$$

$$= \text{a negative number.}$$

Forms of Wave Functions

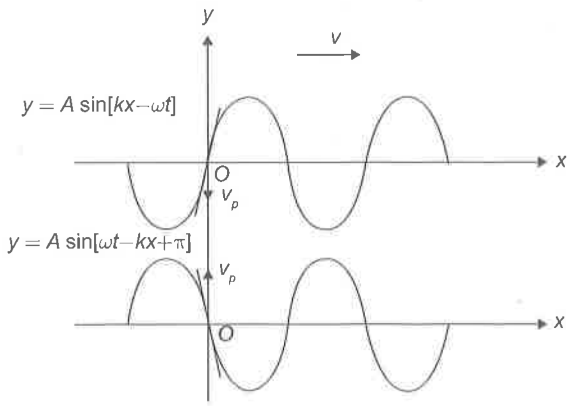


Figure 2.7

Exchange of terms in the argument of a sine function results in a phase difference of π.

In the first case, the slope is positive, and hence, particle velocity is negative. It means the particle is moving from the reference origin or mean position to negative extreme position. In the second case, the slope is negative, and hence, the particle velocity is positive. It means the particle is moving from the positive extreme position to the reference origin or mean position. Thus, the two forms represent waves that differ in the direction in which the particle is moving at a given position.

Once we select the appropriate wave form, we can write the wave equation in other forms as given here:

$$y(x,t) = A\sin(kx - \omega t)$$
$$= A\sin k\left(x - \frac{\omega t}{k}\right)$$
$$= A\sin\frac{2\pi}{\lambda}(x - vt).$$

Further, substituting for k and ω in the wave equation, we have,

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$
$$= A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right).$$

If we want to represent the waveform moving in the negative x-direction, then we need to replace t by $-t$.

LINEAR WAVE EQUATION

By using the wave function $y = A\sin(\omega t - kx + \phi)$, we can describe the motion of any point on the string. Any point on the string moves only vertically, and so its x-coordinate remains constant. The transverse velocity v_y of the point and its transverse acceleration a_y are therefore:

$$v_y = \left[\frac{dy}{dt}\right]_{x\,=\,\text{constant}}$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = \omega A\cos(\omega t - kx + \phi) \qquad (1)$$

$$a_y = \left[\frac{dv_y}{dt}\right]_{x\,=\,\text{constant}}$$

$$\Rightarrow \quad \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2}$$
$$= -\omega^2 A\sin(\omega t - kx + \phi) \qquad (2)$$

and hence,

$$v_{y\text{-max}} = \omega A$$
$$a_{y\text{-max}} = \omega^2 A$$

The transverse velocity and transverse acceleration of any point on the string do not reach their maximum value simultaneously. In fact, the transverse velocity reaches its maximum value (ωA) when the displacement $y = 0$, whereas the transverse acceleration reaches its maximum magnitudes ($\omega^2 A$) when $y = \pm A$.

Further,

$$\left[\frac{dy}{dx}\right]_{t\,=\,\text{constant}}$$

$$\Rightarrow \quad \frac{\partial y}{\partial x} = -kA\cos(\omega t - kx + \phi) \qquad (3)$$

$$= \frac{\partial^2 y}{\partial x^2}$$
$$= -k^2 A\sin(\omega t - kx + \phi) \qquad (4)$$

From Eqs. (1) and (3),

$$\frac{\partial y}{\partial t} = -\frac{\omega}{k}\frac{\partial y}{\partial x}$$

$$\Rightarrow \quad v_p = -v_w \times \text{slope},$$

i.e., if the slope at any point is negative, particle velocity and vice versa, for a wave moving along the positive x-axis, i.e., v_w is positive.

For example, consider two points A and B on the y-curve for a wave, as shown. The wave is moving along the positive x-axis.

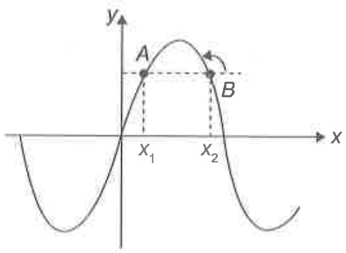


Figure 2.8

The slope at A is positive; therefore, at the given moment, its velocity is negative, which indicates that the wave is moving in the downward direction. For the particle at point B, the situation is the opposite.

Now using Eqs. (2) and (4),

$$\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}.$$

This is known as the linear wave equation or differential equation representation of the travelling wave model. We have developed the linear wave equation from a sinusoidal mechanical wave travelling through a medium. But it is much more general. The linear wave equation successfully describes waves on strings, sound waves, and also electromagnetic waves.

Thus, the above equation can be written as,

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \qquad (1)$$

The general solution of this equation is of the form

$$y(x, t) = f(ax \pm bt) \qquad (2)$$

Thus, any function of x and t which satisfies Eq. (1) or which can be written as Eq. (2) represents a wave. The only condition is that it should be finite everywhere and at all times. Further, if these conditions are satisfied, then speed of wave (v) is given by,

$$v = \frac{\text{coefficient of } t}{\text{coefficient of } x} = \frac{b}{a}.$$

Thus, plus (+) sign between ax and bt implies that the wave is travelling along the negative x-direction and minus (−) sign shows that it is travelling along the positive x-direction.

SOLVED EXAMPLES

EXAMPLE 3

Verify that the wave function

$$y = \frac{2}{(x-3t)^2 + 1}$$

is a solution to the linear wave equation x and y are in cm.

SOLUTION

By taking partial derivatives of this function w.r.t x and t,

$$\frac{\partial^2 y}{\partial x^2} = \frac{12(x-3t)^2 - 4}{[(x-3t)^2+1]^3},$$

and

$$\frac{\partial^2 y}{\partial t^2} = \frac{108(x-3t)^2 - 36}{[(x-3t)^2+1]^3}$$

or

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{9}\frac{\partial^2 x}{\partial t^2}.$$

Comparing with the linear wave equation, we see that the wave function is a solution to the linear wave equation if the speed at which the pulse moves is 3 cm/s. It is apparent from the wave function; therefore, it is a solution to the linear wave equation. ■

EXAMPLE 4

A wave pulse is travelling on a string at 2 m/s. Displacement y of the particle at $x = 0$ at any time t is given by

$$y = \frac{2}{t^2 + 1}.$$

Find:

(a) the expression of the function $y = (x, t)$, i.e., displacement of a particle position x and time t.

(b) the shape of the pulse at $t = 0$ and $t = 1$ s.

SOLUTION

(a) By replacing t by $\left(t - \frac{x}{v}\right)$, we can get the desired wave function, i.e.,

$$y = \frac{2}{\left(t - \frac{x}{2}\right)^2 + 1}$$

(b) We can use wave function at a particular instant, say $t = 0$, to find the shape of the wave pulse using different values of x.

at $\quad t = 0 \quad y = \dfrac{2}{\dfrac{x^2}{4} + 1}$

at $\quad x = 0 \quad y = 2$

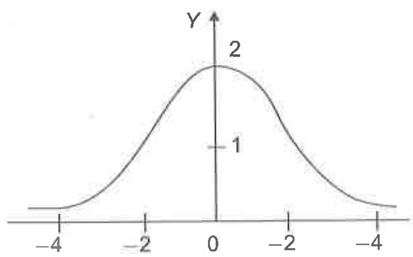
$x = 2 \quad y = 1$

$x = -2 \quad y = 1$

$x = 4 \quad y = 0.4$

$x = -4 \quad y = 0.4$

Using these values, the shape is drawn as shown below.



Similarly, for $t = 1$ s, the shape can drawn. What do you conclude about the direction of motion of the wave from the graphs? Also check how much the pulse has move in

1-s time interval. This is equal to the wave speed. Here is the procedure.

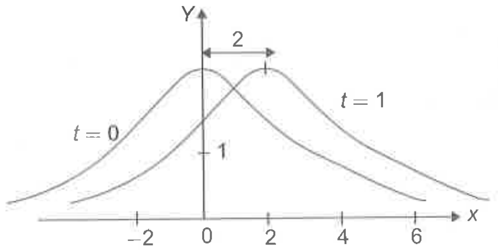
$$y = \frac{2}{\left(1-\frac{x}{2}\right)^2 + 1}$$

at $t = 1$ s

at $x = 2 \quad y = 2$ (maximum value)

at $x = 0 \quad y = 1$

at $x = 4 \quad y = 1$



The pulse has moved to the right by 2 units in 1 s interval.

Also as $t - \frac{x}{2}$ = constant,

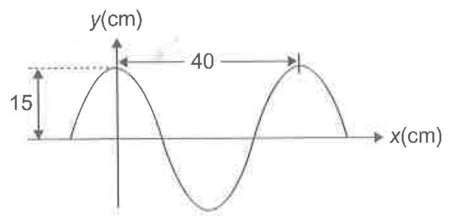
Differentiating w.r.t time

$$1 - \frac{1}{2}\cdot\frac{dx}{dt} = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = 2.$$

EXAMPLE 5

A sinusoidal wave travelling in the positive x-direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15 cm, as shown.



(a) Find the angular wave number, period angular frequency and speed of the wave.

(b) Determine the phase constant ϕ and write a general expression for the wave function.

SOLUTION

(a)
$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40 \text{ cm}} = \frac{\pi}{20} \text{ rad / cm}$$

$$T = \frac{1}{f} = \frac{1}{8} \text{ s}$$

$$\omega = 2\pi f = 16 \text{ s}^{-1}$$

$$v = f\lambda = 320 \text{ cm/s}.$$

(b) It is given that $A = 15$ cm and also $y = 15$ cm at $x = 0$ and $t = 0$.

Then using

$$y = A \sin(\omega t - kx + \phi)$$

$$15 = 15 \sin \phi$$

$$\Rightarrow \quad \sin \phi = 1.$$

Therefore, the wave function is

$$y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

$$= (15\,\text{cm})\sin\left[(16\pi\ \text{s}^{-1})t - \left(\frac{\pi \text{ rad}}{20 \text{ cm}}\right)x + \frac{\pi}{2}\right].$$

SPEED OF A TRANSVERSE WAVE ON A STRING

Consider a pulse travelling along a string with a speed v to the right. If the amplitude of the pulse is small compared to the length of the string, the tension T will be approximately constant along the string. In the reference frame moving with speed v to the right, the pulse is stationary and the string moves with a speed v to the left. Figure 2.9 shows a small segment of the string of length Δl. This segment forms part of a circular arc of radius R. Instantaneously, the segment is moving with speed v in a circular path, so it has centripetal acceleration v^2/R. The forces acting on the segment are the tension T at each end. The horizontal components of these forces are equal and opposite and thus cancel. The vertical component of these forces point radially inward towards the centre of the circular arc. These radial forces provide centripetal acceleration. Let the angle subtended by the segment at the centre be 2θ. The net radial force acting on the segment is

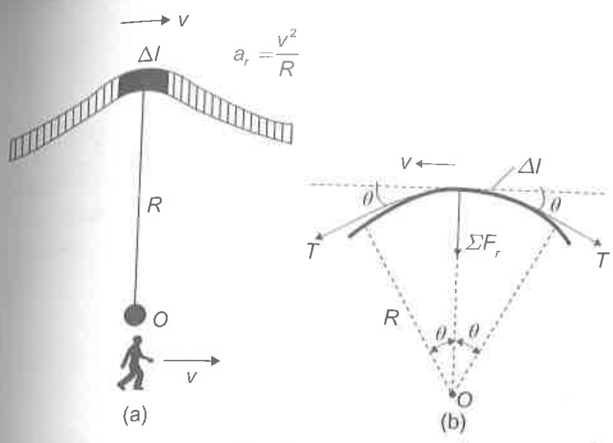


Figure 2.9

Figure 2.9(a). To obtain the speed v of a wave on a stretched string. It is convenient to describe the motion of a small segment of the string in a moving frame of reference.

Figure 2.9(b). In the moving frame of reference, the small segment of length Δl moves to the left with speed v. The net force on the segment is in the radial direction because the horizontal components of the tension force cancel.

$$\sum F_r = 2T\sin\theta = 2T\theta,$$

where we have used the approximation $\sin\theta \approx \theta$ for small θ.

If μ is the mass per unit length of the string, the mass of the segment of length Δl is

$$m = \mu\Delta l$$

$$= 2\mu R\theta \quad (\text{as } \Delta l = 2R\theta).$$

From Newton's second law,

$$\Sigma F_r = ma = \frac{mv^2}{R}$$

or

$$2T\theta = (2\mu R\theta)\frac{v^2}{R}$$

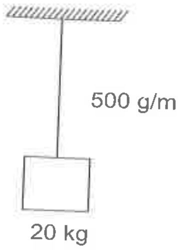
$\therefore$

$$v = \sqrt{\frac{T}{\mu}}.$$

SOLVED EXAMPLES

EXAMPLE 6

Find the speed of the wave generated in the string as in the situation shown. Assume that the tension is not affected by the mass of the cord.



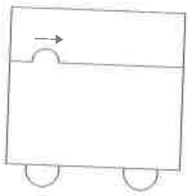
SOLUTION

$$T = 20 \times 10 = 200 \text{ N}$$

$$v = \sqrt{\frac{200}{0.5}} = 20 \text{ m/s}.$$

EXAMPLE 7

A taut string having tension 100 N and linear mass density 0.25 kg/m is used inside a cart to generate a wave pulse starting at the left end as shown. What should be the velocity of the cart so that pulse remains stationary w.r.t the ground.



SOLUTION

$$\text{Velocity of pulse} = \sqrt{\frac{T}{\mu}} = 20 \text{ m/s}.$$

Now

$$\vec{v}_{PG} = \vec{v}_{PC} + \vec{v}_{CG}$$

$$0 = 20\hat{i} + \vec{v}_{CG}$$

$$\vec{v}_{CG} = -20\hat{i} \text{ m/s}.$$

EXAMPLE 8

One end of a 12.0-m-long rubber tube with a total mass of 0.9 kg is fastened to a fixed support. A cord attached to the other end passes over a pulley and supports an object with a mass of 5.0 kg. The tube is struck a transverse blow at one end. Find the time required for the pulse to reach the other end ($g = 9.8$ m/s^2).

SOLUTION

Tension in the rubber tube AB,

$$T = mg$$

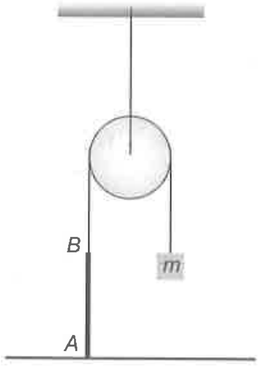
$$T = (5.0)(9.8) = 49 \text{ N}$$

or

Mass per unit length of the rubber tube,

$$\mu = \frac{0.9}{12}$$

$$= 0.075 \text{ kg/m}$$



∴ Speed of the wave on the tube,

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{49}{0.075}} = 25.56 \text{ m/s}$$

∴ The required time is

$$t = \frac{AB}{v}$$

$$= \frac{12}{25.56} = 0.47 \text{ s.}$$ ■

EXAMPLE 9

A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling.

(a) Find the speed of transverse wave in the rope at a point at 0.5 m distance from the lower end.

(b) Calculate the time taken by a transverse wave to travel the full length of the rope.

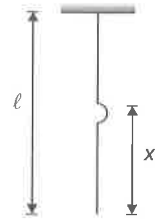
SOLUTION

(a) As the string has mass and it is suspended vertically, tension in it will be different at different points. For a point at a distance x from the free end, tension will be due to the weight of the string below it. So, if m is the mass of string of length l, the mass of length x of the string will be $\left(\frac{m}{l}\right)x$

∴ $$T = \left(\frac{m}{l}\right)xg = \mu xg$$

$$\left(\frac{m}{l} = \mu\right)$$

∴ $$\frac{T}{\mu} = xg$$



or $$v = \sqrt{\frac{T}{\mu}} = \sqrt{xg} \qquad (1)$$

At $x = 0.5$ m,

$$v = \sqrt{0.5 \times 9.8}$$

$$= 2.21 \text{ m/s.}$$

(b) From Eq. (1), we see that velocity of the wave is different at different points. So, if at point x the wave travels a distance dx in time dt, then

$$dt = \frac{dx}{v} = \frac{dx}{\sqrt{gx}}$$

∴ $$\int_0^t dt = \int_0^l \frac{dx}{\sqrt{gx}}$$

or $$t = 2\sqrt{\frac{l}{g}}$$

$$= 2\sqrt{\frac{2.45}{9.8}} = 1.0 \text{ s.}$$ ■

ENERGY CALCULATION IN WAVES

Kinetic Energy per Unit Length

The velocity of a string element in transverse direction is greatest at mean position and zero at the extreme positions of waveform. We can find an expression of the transverse velocity by differentiating displacement with respect to time. Now, the y-displacement is given by

$$y = A \sin(kx - \omega t).$$

Differentiating partially with respect to time, the expression of particle velocity is

$$v_p = \frac{\partial y}{\partial t}$$

$$= -\omega A \cos(kx - \omega t).$$

In order to calculate the kinetic energy, we consider a small string element of length dx having mass per unit length μ. The kinetic energy of the element is given by

$$dK = \frac{1}{2} dm v_p^2$$

$$= \frac{1}{2} \mu dx \omega^2 A^2 \cos^2(kx - \omega t).$$

This is the kinetic energy associated with the element in motion. Since it involves squared of cosine function, its value is greatest for a phase of zero (mean position) and zero for a phase of $\frac{\pi}{2}$ (maximum displacement).

Now, we get kinetic energy per unit length, K_L, by dividing this expression with the length of a small string considered

$$K_L = \frac{dK}{dx}$$

$$= \frac{1}{2} \mu \omega^2 A^2 \cos^2(kx - \omega t).$$

Rate of Transmission of Kinetic Energy

The rate, at which kinetic energy is transmitted is obtained by dividing the expression of kinetic energy by a small time element, dt:

$$\frac{dK}{dt} = \frac{1}{2} \mu \frac{dx}{dt} \omega^2 A^2 \cos^2(kx - \omega t).$$

But, wave or phase speed, v, is the time rate of position, i.e., $\frac{dx}{dt}$. Hence,

$$\frac{dK}{dt} = \frac{1}{2} \mu v \omega^2 A^2 \cos^2(kx - \omega t).$$

Here, kinetic energy is a periodic function. We can obtain the average rate of transmission of kinetic energy by integrating the expression for integral wavelengths. Since only $\cos^2(kx - wt)$ is the varying entity, we need to find average of this quantity only. Its integration over integral wavelengths gives a value of $\frac{1}{2}$. Hence, the average rate of transmission of kinetic energy is

$$\frac{dK}{dt} I_{\text{avg}} = \frac{1}{2} \times \frac{1}{2} \mu v \omega^2 A^2$$

$$= \frac{1}{4} \mu v \omega^2 A^2.$$

Elastic Potential Energy

The elastic potential energy of the string element results as a string element is stretched during its oscillation. The extension or stretching is maximum at the mean position. We can see in Fig. 2.10 that the length of the string element of equal x-length dx is greater at the mean position than at the extreme. As a matter of fact, the elongation depends on the slope of the curve. Greater the slope, greater is the elongation. The string has the least length when the slope is zero. For illustration purpose, the curve is purposely drawn in such a manner that the elongation of the string element at mean position is highlighted.

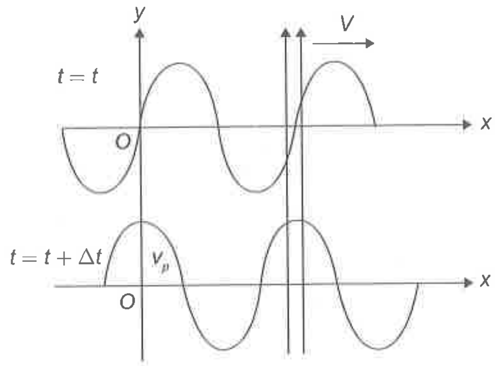


Figure 2.10 The string element stretched most at equilibrium position.

Greater extension of the string element corresponds to a greater elastic energy. As such, it is greatest at the mean position and zero at the extreme position. This deduction is contrary to the case of SHM in which potential energy is greatest at the extreme position and zero at the mean position.

Potential Energy per Unit Length

When the string segment is stretched from the length dx to the length ds, an amount of work $= T(ds - dx)$ is done. This is equal to the potential energy stored in the stretched string segment. So the potential energy in this case is

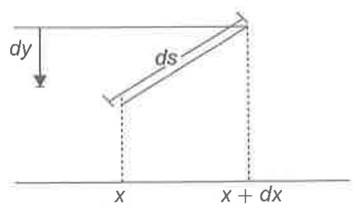


Figure 2.11

$$U = T(ds - dx).$$

Now,
$$ds = \sqrt{(dx^2 + dy^2)}$$
$$= dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

from the binomial expansion.

So,
$$ds \approx dx + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx$$
$$U = T(ds - dx)$$
$$\approx \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 dx$$

or the potential energy density

$$\frac{dU}{dx} = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 \quad (1)$$

$$\frac{dy}{dx} = kA\cos(kx - \omega t)$$

and
$$T = v^2\mu$$

Substituting the above value in Eq. (1), we get

$$\frac{dU}{dx} = \frac{1}{2}\mu\omega^2 A^2\cos^2(kx - \omega t)$$

Rate of Transmission of Elastic Potential Energy

The rate, at which elastic potential energy is transmitted is obtained by dividing expression of kinetic energy by a small time element, dt. This expression is the same as that for kinetic energy.

$$\frac{dU}{dt} = \frac{1}{2}\mu v\omega^2 A^2\cos^2(kx - \omega t)$$

and average rate of transmission of elastic potential energy is

$$\frac{dU}{dt}I_{\text{avg}} = \frac{1}{2}\times\frac{1}{2}\mu v\omega^2 A^2$$
$$= \frac{1}{4}\mu v\omega^2 A^2$$

Mechanical Energy per Unit Length

Since the expression of elastic potential energy is the same as that of kinetic energy, we obtain the mechanical energy expression by multiplying the expression of kinetic energy by 2. The mechanical energy associated with a small string element dx is

$$dE = 2xdK$$
$$= 2\times\frac{1}{2}dmv_p^2$$
$$= \mu dx\omega^2 A^2\cos^2(kx - \omega t).$$

Similarly, the mechanical energy per unit length is

$$E_L = \frac{dE}{dx}$$
$$= 2\times\frac{1}{2}\omega^2 A^2\cos^2(kx - \omega t)$$
$$= \mu\omega^2 A^2\cos^2(kx - \omega t).$$

Average Power Transmitted

The average power transmitted by a wave is equal to the time rate of transmission of mechanical energy over integral wavelengths. It is equal to

$$P_{\text{avg}} = \frac{dE}{dt}I_{\text{avg}}$$
$$= 2\times\frac{1}{4}\mu v\omega^2 A^2$$
$$= \frac{1}{2}\mu v\omega^2 A^2.$$

If the mass of the string is given in terms of mass per unit volume ρ, then we make appropriate changes in the derivation. We exchange μ by rs, where s is the cross-section of the string

$$P_{\text{avg}} = \frac{1}{2}\rho s v\omega^2 A^2.$$

Energy Density

Since there is no loss of energy involved, it is expected that energy per unit length is uniform throughout the string. As much energy enters that much energy goes out for a given length of string. This average value along unit length of the string length is equal to the average rate at which energy is being transferred.

The average mechanical energy per unit length is equal to integration of expression over integral wavelength

$$E_L I_{\text{avg}} = 2\times\frac{1}{4}\mu v w^2 A^2$$
$$= \frac{1}{2}\mu v w^2 A^2.$$

We have derived this expression for harmonic wave along a string. The concept, however, can be extended to two-or three-dimensional transverse waves. In the case of three-dimensional transverse waves, we consider a small volumetric element. We, then, use density ρ in place of mass per unit length, μ. The corresponding average energy per unit volume is referred to as energy density u:

$$u = \frac{1}{2}\rho v\omega^2 A^2.$$

Intensity

Intensity of wave (I) is defined as power transmitted per unit cross-section area of the medium:

$$I = \rho s v\omega^2 \frac{A^2}{2s}$$

$$= \frac{1}{2}\rho v\omega^2 A^2.$$

Intensity of wave (I) is a very useful concept for three-dimensional waves radiating in all directions from the source. This quantity is usually referred in the context of light waves, which is transverse harmonic waves in three dimensions. Intensity is defined as the power transmitted per unit cross-sectional area. Since light spreads uniformly all around, intensity is equal to power transmitted, divided by spherical surface drawn at that point with source at its centre.

Phase Difference Between Two Particles in the Same Wave

The general expression for a sinusoidal wave travelling in the positive x-direction is

$$y(x, t) = A\sin(\omega t - kx).$$

Equation of a particle at x_1 is given by

$$y_1 = A\sin(\omega t - kx_1).$$

Equation of a particle which is at x_2 from the origin

$$y_2 = A\sin(\omega t - kx_2).$$

Phase difference between particles is

$$k(x_2 - x_1) = \Delta\phi$$

$$K\Delta x = \Delta\phi$$

$$\Rightarrow \quad \Delta x \Rightarrow \frac{\Delta\phi}{k}.$$

PRINCIPLE OF SUPERPOSITION

This principle defines the displacement of a medium particle when it is oscillating under the influence of two or more than two waves. The principle of superposition is stated as:

When two or more waves superpose on a medium particle, then the resultant displacement of that medium particle is given by the vector sum of the individual displacements produced by the component waves at that medium particle independently.

Let $\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N$ be the displacements produced by N independent waves at a medium particle in the absence of others, then the displacement of that medium, when all the waves are superposed at that point, is given as

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \cdots + \vec{y}_N.$$

If all the waves producing oscillations at that point are collinear, then the displacement of the medium particle where superposition is taking place can be simply given by the algebraic sum of the individual displacement. Thus, we have

$$y = y_1 + y_2 + \cdots + y_N.$$

The above equation is valid only if all the individual displacements $y_1, y_2, \ldots, y_N$ are along the same straight line.

A simple example of superposition can be understood from Fig. 2.12. Suppose two wave pulses are travelling simultaneously in opposite directions as shown. When they overlap each other, the displacement of the particle on the string is the algebraic sum of the two displacements as the displacements of the two pulses are in the same direction. Fig. 2.12 (b) also shows the similar situation when the wave pulses are in the opposite side.

Applications of Principle of Superposition of Waves

There are several different phenomena that take place during the superposition of two or more waves depending on the wave characteristics, which are being superposed. We will discuss some standard phenomena:

1. Interference of waves
2. Stationary waves
3. Beats
4. Lissajou's figures (not discussed here in detail)

Lets discuss these in detail.

Interference of Waves

Suppose two sinusoidal waves of the same wavelength and amplitude travel in the same direction along the same straight line (may be on a stretched string), then superposition principle can be used to define the resultant displacement of every medium particle. The resultant wave in the medium depends on the extent to which the waves are in phase with respect to

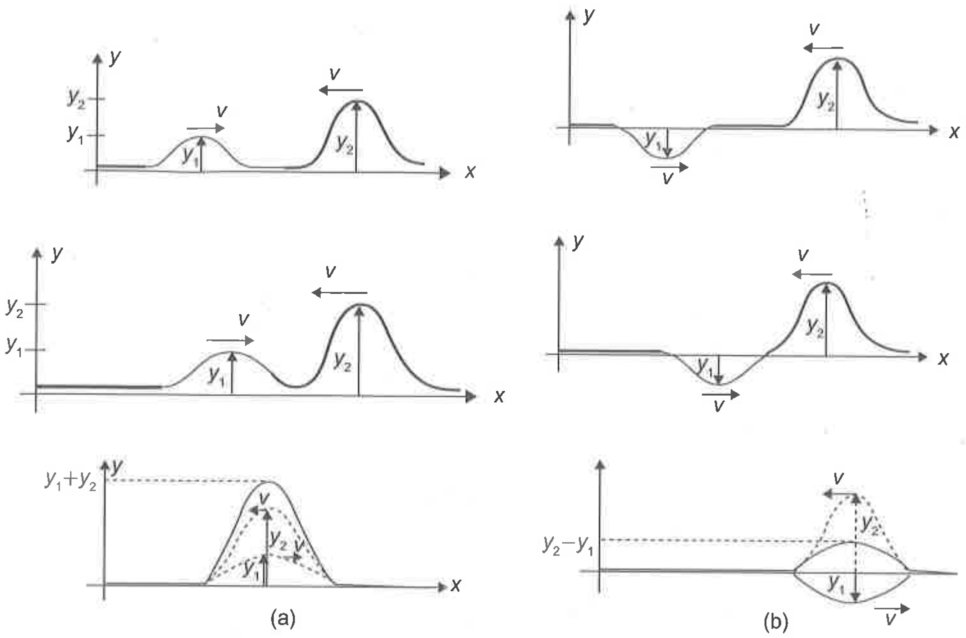


Figure 2.12

each other, that is, how much one wave form is shifted from the other waveform. If the two waves are exactly in the same phase, that is, the shape of one wave exactly fits on to the other wave, then they combine to double the displacement of every medium particle as shown in Fig. 2.13(a). We call this phenomenon as constructive interference. If the superposing waves are exactly out of phase or in opposite phase, then they combine to cancel all the displacements at every medium particle and medium remains in the form of a straight line as shown in Fig. 3.13(b).

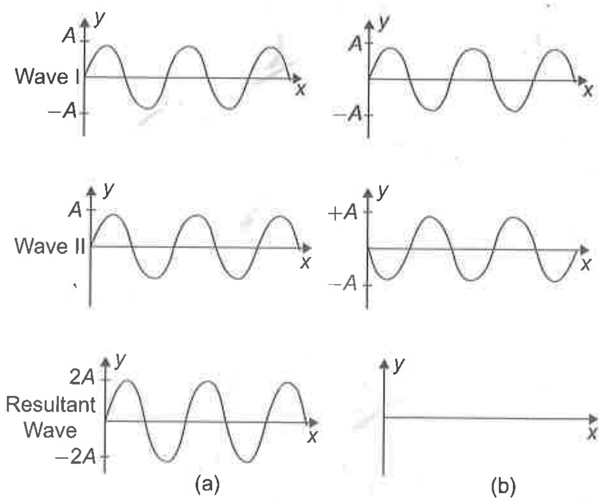


Figure 2.13

We call this phenomenon as destructive interference. Thus, we can state that when waves meet, they interfere constructively if they meet in the same phase and destructively if they meet in the opposite phases. In either case, the wave patterns do not shift relative to each other as they propagate. Such superposing waves which have the same form and wavelength and have a fixed phase relation to each other are called coherent waves. Sources of coherent waves are called coherent sources. Two independent sources can never be coherent in nature due to practical limitations of manufacturing process. Generally, all coherent sources are made either by splitting of the wave forms of a single source or the different sources are fed by a single main energy source.

In simple words, interference is the phenomenon of superposition of two coherent waves travelling in the same direction.

We have discussed that the resultant displacement of a medium particle when two coherent waves interfere at that point is the sum or difference of the individual displacements by the two waves if they are in the same phase (phase difference $= 0, 2\pi,\ldots$) or opposite phase (phase difference $= \pi, 3\pi,\ldots$), respectively. But the two waves can also meet at a medium particle with phase difference other than 0 or 2π. If the phase difference ϕ is such that $0 < \phi < 2\pi$, then how is the displacement of the point of superposition given? Now we discuss the interference of waves in detail analytically.

Analytical Treatment of Interference of Waves

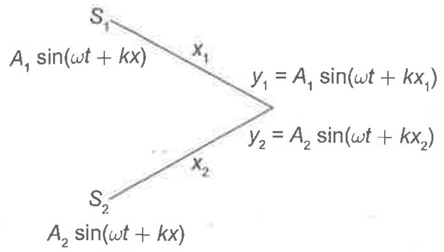


Figure 2.14

Interference implies super position of waves. Whenever two or more than two waves superimpose each other, they give sum of their individual displacements.

Let the two waves coming from sources S_1 and S_2 be

$$y_1 = A_1 \sin(\omega t + kx_1)$$

$$y_2 = A_2 \sin(\omega t + kx_2), \text{ respectively.}$$

Due to superposition,

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2).$$

Phase difference between y_1 and $y_2 = k(x_2 - x_1)$

i.e., $\Delta\phi = k(x_2 - x_1)$.

As $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

(where Δx = path difference and $\Delta\phi$ = phase difference)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\Rightarrow \quad A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\therefore \quad I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

(as $I \propto A^2$).

When the two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes and I_{net} will be maximum. This is known as constructive interference.

For I_{net} to be maximum,

$$\cos\phi = 1$$

$$\Rightarrow \quad \phi = 2n\pi,$$

where $n = \{0, 1, 2, 3, 4, 5, \ldots\}$

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi$$

$$\Rightarrow \quad \Delta x = n\lambda.$$

For constructive interference,

$$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2.$$

When $I_1 = I_2 = I$,

$$I_{net} = 4I$$

$$A_{net} = A_1 + A_2.$$

When superposing waves are in opposite phase, the resultant amplitude is the difference of two amplitudes and I_{net} is minimum; this is known as destructive interference.

For I_{net} to be minimum,

$$\cos\Delta\phi = -1$$

$$\Delta\phi = (2n + 1)\pi,$$

where $n = \{0, 1, 2, 3, 4, 5, \ldots\}$

$$\frac{2\pi}{\lambda}\Delta x = (2n + 1)\pi$$

$$\Rightarrow \quad \Delta x = (2n+1)\frac{\lambda}{2}.$$

For destructive interference,

$$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

If $I_1 = I_2$

$$I_{net} = 0$$

$$A_{net} = A_1 - A_2.$$

Ratio of I_{max} and $I_{min} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

Generally,

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

If $I_1 = I_2 = I$

$$I_{net} = 2I + 2I\cos\phi$$

$$I_{net} = 2I(1 + \cos\phi)$$

$$= 4I\cos^2\frac{\Delta\phi}{2}$$

SOLVED EXAMPLES

EXAMPLE 10

Wave from two sources, each of the same frequency and travelling in the same direction, but with intensity in the ratio 4:1 interfere. Find the ratio of maximum to minimum intensity.

SOLUTION

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

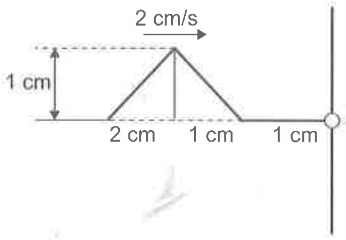
$$= \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2$$

$$= \left(\frac{2+1}{2-1}\right)^2$$

$$= 9:1$$

EXAMPLE 11

A triangular pulse moving at 2 cm/s on a rope approaches an end at which it is free to slide on a vertical pole.

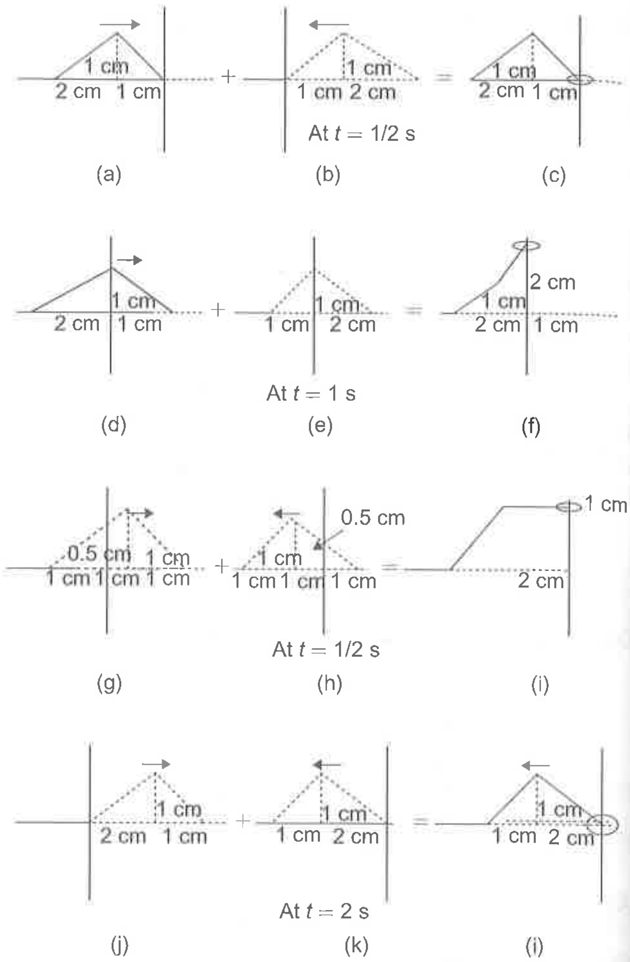


(a) Draw the pulse at $\frac{1}{2}$ s interval until it is completely reflected.

(b) What is the particle speed on the trailing edge at the instant depicted?

SOLUTION

(a) Reflection of a pulse from a free boundary is really the superposition of two identical waves travelling in the opposite directions. This can be shown as below.



In every $\frac{1}{2}$ s, each pulse (one real moving towards right and one imaginary moving towards left) travels at a distance of 1 cm, as the wave speed is 2 cm/s.

(b) Particle speed,

$$v_p = |-v\,(\text{slope})|.$$

Here, v = wave speed

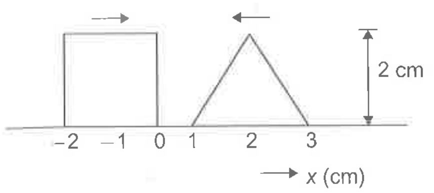
$= 2$ cm/s

and slope $= \frac{1}{2}$.

$\therefore$ Particle speed = 1 cm/s

EXAMPLE 12

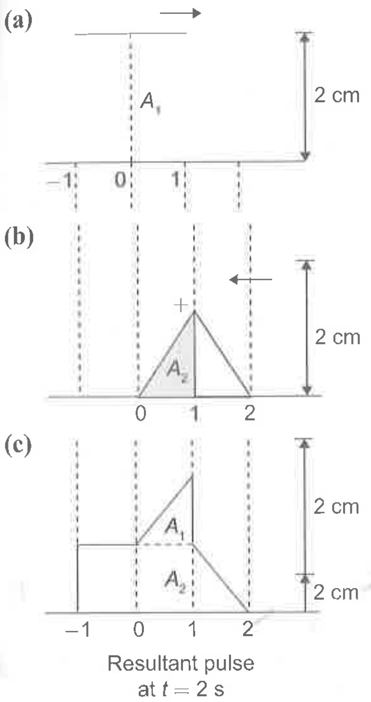
The figure shows a rectangular pulse and triangular pulse approaching each other. The pulse speed is 0.5 cm/s. Sketch the resultant pulse at $t = 2$ s.



SOLUTION

In 2 s, each pulse will travel a distance of 1 cm.

The two pulses overlap between 0 and 1 cm as shown in the figure. So, A_1 and A_2 can be added as shown in Fig. (c).



REFLECTION AND TRANSMISSION IN WAVES

1. When a pulse travelling along a string reaches the end, it is reflected. If the end is fixed as shown in Fig. 2.15 (a), the pulse returns inverted. This is because as the leading edge reaches the wall, the string pulls up the wall. According to Newton's third law, the wall will exert an equal and opposite force on the string at all instants. This force is, therefore, directed first down and then up. It produces a pulse that is inverted but otherwise identical to the original.

The motion of the free end can be studied by letting a ring at the end of the string slide smoothly on the rod. The ring and rod maintain the tension but exert no transverse force.

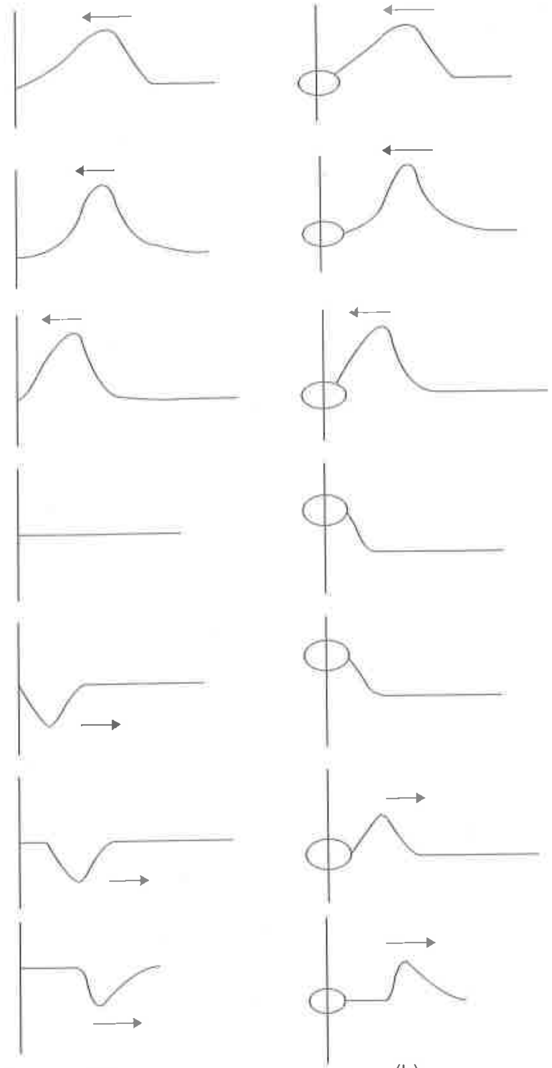
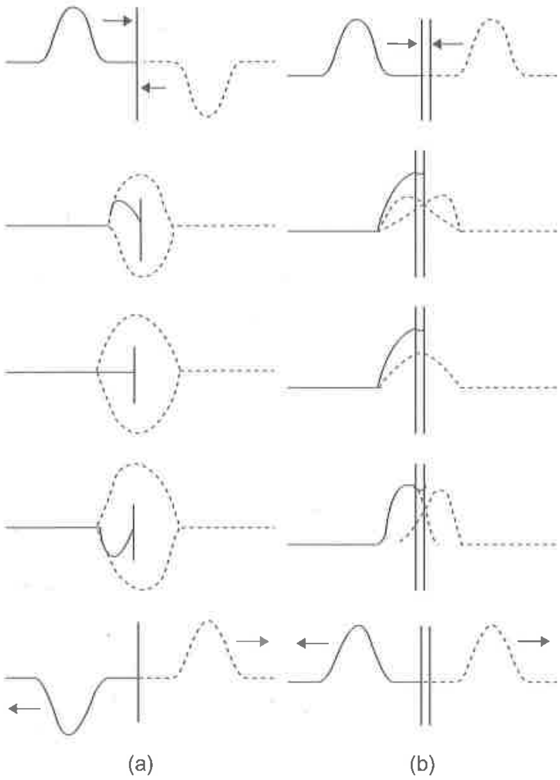


Figure 2.15 Reflection of wave pulse (a) at a fixed end of a string and (b) at a free end. Time increases from top to bottom in each figure

When a wave arrives at this free end, the ring slides the rod. The ring reaches a maximum displacement. At this position, the ring and string come momentarily to rest as in the fourth drawing from the top in Fig. 3.15(b). But the string is stretched in this position, giving increased tension, so the free end of the string is pulled back down, and again a reflected pulse is produced, but now the direction of the displacement is the same as for the initial pulse.

2. The formation of the reflected pulse is similar to the overlap of two pulses travelling in the opposite directions. The net displacement at any point is given by the principle of superposition.



Figurer 3.16

Figure 2.16(a) shows two pulses with the same shape, one inverted with respect to the other, travelling in the opposite directions. Because these two pulses have the same shape, the net displacement of the point where the string is attached to the wall is zero at all times.

Figure 2.16(b) shows two pulses with the same shape, travelling in the opposite directions but not inverted relative to each other. Note that at one instant, the displacement of the free end is double the pulse height.

REFLECTION AND TRANSMISSION BETWEEN TWO STRINGS

Here, we are dealing with the case where the end point is neither completely fixed nor completely free to move. As we consider an example where a light string is attached to a heavy string as shown in Fig. 2.17 (a).

If a wave pulse is produced on a light string moving towards the friction, a part of the wave is reflected and a part is transmitted on the heavier string. The reflected wave is inverted with respect to the original one.

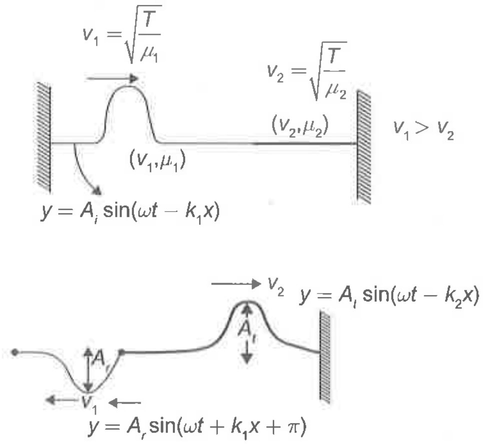


Figure 2.17 (a)

On the other hand if the wave is produced on the heavier string which moves toward the junction, a part will be reflected and a part will be transmitted. No inversion in wave shape will take place.

The wave velocity is smaller for the heavier string than for lighter string.

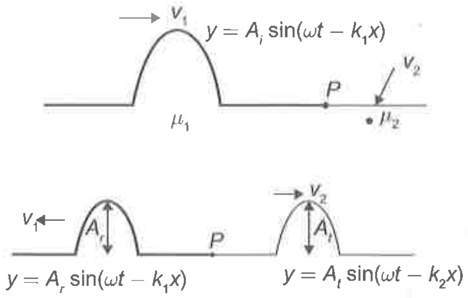


Figure 2.17 (b)

Now to find the relation between A_i, A_r and A_t, we consider Fig. 2.17 (b).

Incident power = Reflected power + Transmitted power

$$P_i = P_r + P_t$$

$$2\pi^2 f^2 A_i^2 \mu_1 v_1 = 2\pi^2 f^2 A_r^2 \mu_1 v_1 + 2\pi^2 f^2 A_t^2 \mu_1 v_1 \quad (1)$$

Put $$\mu_1 = \frac{T}{v_1^2}$$

and $$\mu_2 = \frac{T}{v_2^2}$$

in Eq. (1), then

$$\frac{A_i^2}{v_1} = \frac{A_r^2}{v_1} + \frac{A_t^2}{v_2}$$

$$A_i^2 - A_r^2 = \frac{v_1}{v_2} A_t^2 \qquad (2)$$

Maximum displacement of joint particle P (as shown in the figure) is due to left string

$$= A_i + A_r.$$

Maximum displacement of joint particle due to right string

$$= A_t.$$

At the boundary (at point P), the wave must be continuous, that is, there are no kinks in it. Then we must have

$$A_i + A_r = A_t \qquad (3)$$

From Eqs. (2) and (3),

$$A_i - A_r = \frac{v_1}{v_2} A_t \qquad (4)$$

From Eqs. (3) and (4),

$$A_t = \left[\frac{2v_2}{v_1 + v_2}\right] A_i$$

$$A_r = \left[\frac{v_2 - v_1}{v_1 + v_2}\right] A_i.$$

STANDING WAVES

In the previous section, we have discussed that when two coherent waves superpose on a medium particle, interference takes place. Similarly, when two coherent waves travelling in the opposite directions superpose, then simultaneous interference of all the particles in the medium takes place. These waves interfere to produce a pattern of all the particles in the medium, which is what we call a stationary wave. If the two interfering waves which travel in the opposite directions carry equal energies, then no net flow of energy takes place in the region of superposition. Within this region, redistribution of energy takes place between the particles in the medium. There are some medium particles where constructive interference takes place and hence energy increases and on the other hand there are some medium particles where destructive interference takes place and energy decreases. Now we will discuss the stationary waves analytically.

Let two waves of equal amplitude travel in opposite directions along the x-axis. The wave equation of the two waves can be given as

$$y_1 = A \sin(\omega t - kx) \qquad (1)$$

[wave travelling in $+x$ direction]

and $$y_2 = A \sin(\omega t + kx) \qquad (2)$$

[wave travelling in $-x$ direction].

When the two waves superpose on medium particles, the resultant displacement of the medium particles can be given as

$$y = y_1 + y_2$$

or $$y = A \sin(\omega t - kx) + A \sin(\omega t + kx)$$

or $$y = A\,[\sin \omega t \cos kx - \cos \omega t \sin kx + \sin \omega t \cos kx + \cos \omega t \sin kx]$$

or $$y = 2A \cos kx \sin \omega t \qquad (3)$$

Equation (3) can be rewritten as

$$y = R \sin \omega t \qquad (4)$$

where $$R = 2A \cos kx \qquad (5)$$

Here, Eq. (4) is an equation of SHM. It implies that after superposition of the two waves, the medium particles execute SHM with the same frequency ω and amplitude R, which is given by Eq. (5). Here, we can see that the oscillation amplitude of the medium particles depends on x, i.e., the position of medium particles. Thus, on superposition of two coherent waves travelling in the opposite directions, the resulting interference pattern, we call stationary waves, the oscillation amplitude of the medium particle at different positions is different.

At some point on the medium, the resultant amplitude is maximum, which is given as

R is maximum when $\cos kx = \pm 1$

or $$\frac{2\pi}{\lambda} x = N\pi \quad [N \in I]$$

or $$x = \frac{N\lambda}{2}$$

or $$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$$

and the maximum value of R is given as

$$R_{max} = \pm 2A \tag{6}$$

Thus, in the medium at position $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots,$ the waves interfere constructively and the amplitude of oscillations becomes $2A$. Similarly, at some points on the medium, the waves interfere destructively, the oscillation amplitude become minimum i.e. zero in this case. These are the points where R is minimum, when

$$\cos kx = 0$$

or $$\frac{2\pi x}{\lambda} = (2N+1)\frac{\pi}{2}$$

or $$x = (2N+1)\frac{\lambda}{4} \quad [N \in I]$$

or $$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

and the minimum value of R is given as

$$R_{min} = 0 \tag{7}$$

Thus, in the medium at position $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ the waves interfere destructively and the amplitude of oscillation becomes zero. These points always remain at rest. Figure 2.18(a) shows the oscillation amplitude of different medium particles in a stationary wave.

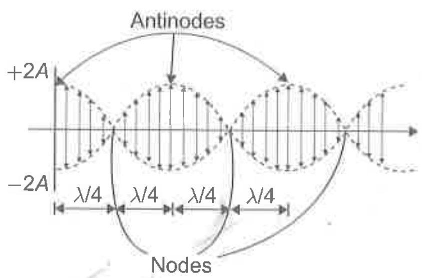


Figure 2.18 (a)

In Fig. 2.18(a) we can see that the medium particles at which constructive interference takes place are called antinodes of the stationary wave and the points of destructive interference are called nodes of stationary waves, which always remain at rest.

Figure 2.18(b) explains the movement of the medium particles with time in the region where stationary waves are formed. Let us assume that at an instant $t = 0$, all the medium particles are at their extreme positions as shown in Fig. 2.18(b)-(1). Here, points $ABCD$ are the nodes of stationary waves where medium particles remains at rest. All other starts moving towards their mean positions and at $t = T/4$, all the particles cross their mean position as shown in Fig. 2.18(b)-(3). It can be seen from the figure that the particles at nodes are not moving. Now, the medium crosses its mean position and starts moving to the other side of the mean position towards the other extreme position. At time $t = T/2$, all the particles reach the other extreme position as shown in Fig. 2.18(b)-(5) and at time $t = 3T/4$ all these particles again cross their mean position in the opposite direction as shown in Fig. 2.18(b)-(7).

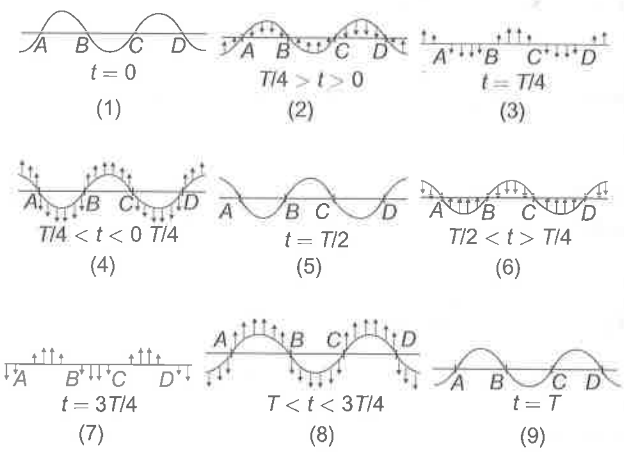


Figure 2.18 (b)

Based on the above analysis of one complete oscillation of the medium particles, we can make some interference for stationary waves. These are

1. In oscillations of stationary wave in a region, some points are always at rest (nodes) and some oscillate with maximum amplitudes (antinodes). All other medium particles oscillate with amplitudes less than those of the antinodes.
2. All medium particles between two successive nodes oscillate in the same phase and all medium particles on one side of a node oscillate in opposite phase with those on the other side of the same node.
3. In the region of a stationary wave during one complete oscillation all the medium particles come in the form of a straight line twice.
4. If the component wave amplitudes are equal, then in the region where stationary wave is formed, no net flow of energy takes place. Only redistribution of energy takes place in the medium.

Different Equations for a Stationary Wave

Consider two equal-amplitude waves travelling in the opposite directions as

$$y_1 = A\sin(\omega t - kx) \qquad (11)$$

and

$$y_2 = A\sin(\omega t + kx) \qquad (12)$$

The result of superposition of these two waves is

$$y = 2A\cos kx \sin \omega t, \qquad (13)$$

which is the equation of a stationary wave where $2A\cos kx$ represents the amplitude of the medium particle situated at position x and $\sin wt$ is the time sinusoidal factor. Equation (13) can be written in several ways depending on the initial phase differences in the component waves given by Eqs. (11) and (12). If the superposing waves are having an initial phase difference π, then the component waves can be expressed as

$$y_1 = A\sin(\omega t - kx) \qquad (14)$$

$$y_2 = -A\sin(\omega t - kx) \qquad (15)$$

Superposition of the above two waves will result in

$$y = 2A\sin kx \cos \omega t \qquad (16)$$

Equation (16) is also an equation of stationary wave but here amplitude of different medium particles in the region of interference is given by

$$R = 2A\sin kx \qquad (17)$$

Similarly, the possible equations of a stationary wave can be written as

$$y = A_0 \sin kx \cos(\omega t + \phi) \qquad (18)$$

$$y = A_0 \cos kx \sin(\omega t + \phi) \qquad (19)$$

$$y = A_0 \sin kx \sin(\omega t + \phi) \qquad (20)$$

$$y = A_0 \cos kx \cos(\omega t + \phi) \qquad (21)$$

Here, A_0 is the amplitude of antinodes. In a pure stationary wave, it is given as

$$A_0 = 2A,$$

where A is the amplitude of the component waves. If we carefully look at Eqs. (18)–(21), we can see that in Eqs. (18) and (20), the particle amplitude is given by

$$R = A_0 \sin kx \qquad (22)$$

Here, at $x = 0$, the nodes are $R = 0$ and in Eqs. (19) and (21), the particle amplitude is given as

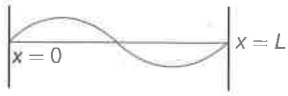
$$R = A_0 \cos kx \qquad (23)$$

Here, at $x = 0$, there is an antinode at $R = A_0$. Thus, we can state that in a given system of co-ordinates when the origin of the system is at a node, we use either Eq. (18) or Eq. (20) for analytical representation of a stationary wave and we use Eq. (19) or Eq. (21) for the same when an antinode is located at the origin of the system.

SOLVED EXAMPLES

EXAMPLE 13

Find out the equation of the standing waves for the following standing wave pattern.



(a) $A\sin\dfrac{2\pi}{L}x\cos\omega t$ **(b)** $A\sin\dfrac{\pi x}{L}\cos\omega t$

(c) $A\cos\dfrac{\pi x}{2L}\cos\omega t$ **(d)** $A\cos\dfrac{\pi x}{L}\cos\omega t$

SOLUTION

General equation of a standing wave

$$y = A'\cos\omega t,$$

where

$$A' = A\sin(kx + \theta).$$

Here, $\lambda = L$

$\Rightarrow$ $k = \dfrac{2\pi}{L}$

$$A' = A\sin(kx + \theta)$$

$$= A\sin\left(\frac{2\pi}{L}x + \theta\right).$$

At $x = 0$ node,

$\Rightarrow$ $A' = 0$ at $x = 0$

$\Rightarrow$ $\theta = 0$

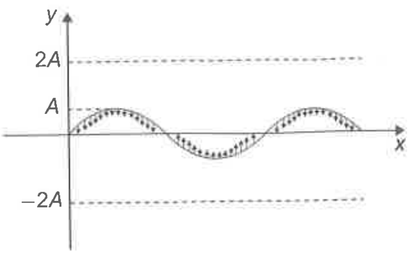
Equation of a standing wave

$$= A\sin\frac{2\pi}{L} \times \cos\omega t.$$

■

EXAMPLE 14

The figure shows the standing waves pattern in a string at $t = 0$. Find out the equation of the standing wave, where the amplitude of the antinode is $2A$.



SOLUTION

Let we assume the equation of standing waves is $A' \sin(\omega t + \phi)$,

where $$A' = 2A \sin(kx + \theta)$$

$\therefore$ $x = 0$ is node

$\Rightarrow$ $$A' = 0, \text{ at } x = 0$$

$$2A \sin\theta = 0$$

$\Rightarrow$ $$\theta = 0$$

at $t = 0$ particle at is at $y = A$ and going towards the mean position,

$\Rightarrow$ $$\phi = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}.$$

So equation of the standing waves is

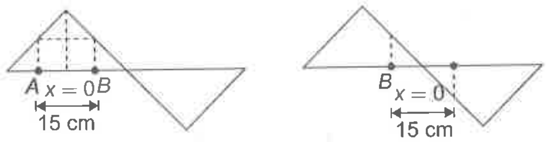
$$y = 2A \sin kx \sin\left(\omega t + \frac{5\pi}{6}\right).$$ ■

EXAMPLE 15

A string 120 cm in length sustains standing wave with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0 cm. The maximum displacement amplitude is $x = 95$ mm, then find out the value of x.

SOLUTION

In this problem, two cases are possible:



Case 1 is that A and B have the same displacement amplitude and case 2 is that C and D have the same amplitude, viz., 3.5 mm. In case 1, if $x = 0$ is taken at the antinode, then

$$A = a \cos kx.$$

In case 2, if $x = 0$ is taken at the node, then

$$A = a \sin kx.$$

But since nothing is given in the question. Hence from both the cases, the result should be the same. This is possible only when

$$a \cos kx = a \sin kx$$

or $$kx = \frac{\pi}{4}$$

or $$a = \frac{A}{\cos kx}$$

$$= \frac{3.5}{\cos \pi/4} = 4.95 \text{ mm.}$$ ■

Energy of a Standing Wave in One Loop

When all the particles of one loop are at the extreme position, then the total energy in the loop is in the form of potential energy only. When the particles reach their mean position, the total potential energy converts into kinetic energy of the particles so we can say total energy of the loop remains constant.

Total kinetic energy at the mean position is equal to the total energy of the loop because potential energy at the mean position is zero.

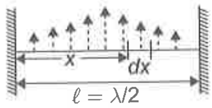


Figure 2.19

Small kinetic energy of the particle which is in element dx is

$$d(KE) = \frac{1}{2} dmv^2$$

$$dm = \mu dx.$$

Velocity of the particle at the mean position

$$= 2A \sin kx\omega.$$

Then, $$d(KE) = \frac{1}{2}\mu dx \cdot 4A^2\omega^2 \sin^2 kx$$

$$\Rightarrow \quad d(KE) = 2A^2\omega^2\mu \cdot \sin^2 kx dx$$

$$\int d(\text{KE}) = 2A^2\omega^2\mu \int_0^{\lambda/2} \sin^2 kx\, dx$$

$$\text{Total} \quad \text{KE} = A^2\omega^2\mu \int_0^{\lambda/2} (1-\cos 2kx)dx$$

$$= A^2\omega^2\mu \left[x - \frac{\sin 2kx}{2k}\right]_0^{\lambda/2}$$

$$= \frac{1}{2}\lambda A^2\omega^2\mu.$$

STATIONARY WAVES IN STRINGS

When Both Ends of a String Are Fixed

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the ends are always nodes, if both ends of a string are fixed.

Fundamental Mode

In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and the frequency of vibration is known as the fundamental frequency or first harmonic.

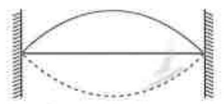


Figure 2.20

Since the distance between the consecutive nodes is $\frac{\lambda}{2}$,

$$L = \frac{\lambda_1}{2}$$

$$\therefore \quad l_1 = 2L.$$

If f_1 is the fundamental frequency of vibration, then the velocity of the transverse waves is given as,

$$v = \lambda_1 f_1$$

or

$$f_1 = \frac{v}{2L} \tag{1}$$

First Overtone

The same string under the same conditions may also vibrate in two loops, such that the centre is also the node

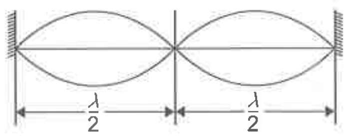


Figure 2.21

$$\therefore \quad L = \frac{2\lambda_2}{2}$$

$$\therefore \quad l_2 = L.$$

If f_2 is the frequency of vibrations,

$$\therefore \quad f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$\therefore \quad f_2 = \frac{v}{L} \tag{2}$$

The frequency f_2 is known as the second harmonic or first overtone.

Second overtone

The same string under the same conditions may also vibrate in three segments.

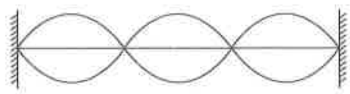


Figure 2.22

$$\therefore \quad L = \frac{3\lambda_3}{2}$$

$$\therefore \quad \lambda_3 = \frac{2}{3}L.$$

If f_3 is the frequency in this mode of vibration, then

$$f_3 = \frac{3v}{2L} \tag{3}$$

The frequency f_3 is known as the third harmonic or second overtone.

Thus, a stretched string vibrates with frequencies that are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

The velocity of a transverse wave in stretched string is given as

$$v = \sqrt{\frac{T}{\mu}},$$

where

T = tension in the string.

μ = linear density or mass per unit length of the string.

If the string fixed at two ends vibrates in its fundamental mode, then

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad (4)$$

In general, $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ n^{th} harmonic, $(n-1)^{th}$ overtone

Note

In general, any integral multiple of the fundamental frequency is an allowed frequency. These higher frequencies are called overtones. Thus, $v_1 = 2v_0$ is the first overtone, $v_2 = 3v_0$ is the second overtone, etc. An integral multiple of a frequency is called its harmonic. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and all the harmonics of the fundamental frequency are overtones.

When One End of the String is Fixed and Other is Free: Free End Acts as Antinode

1.

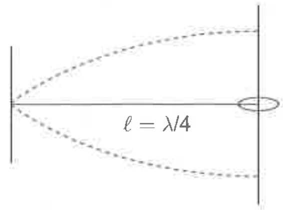


Figure 2.23

$$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$$ fundamental or I harmonic.

2.

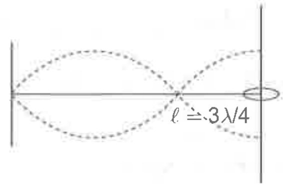


Figure 2.24

$$f = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$$ III harmonic or I overtone.

In general,

$$f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$$ (($(2n+1)^{th}$ harmonic, n^{th} overtone).

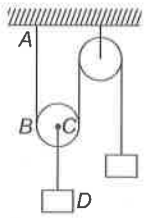
S.No.	Travelling waves	Stationary waves
1	These waves advance in a medium with a definite velocity.	These waves remain stationary between two boundaries in the medium.
2	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3	At any instant phase of vibration varies continuously from one particle to the other, i.e., phase difference between two particles can have any value between 0 and 2π.	At any instant the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e, phase difference between any two particles can be either 0 or π.
4	In these wave, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves all particles of the medium pass through their mean position simultaneously twice in each time period.
5	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

EXERCISES

JEE Main

1. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi(ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if
   (A) $\lambda = \pi Y_0/4$
   (B) $\lambda = \pi Y_0/2$
   (C) $\lambda = \pi Y_0$
   (D) $\lambda = 2\pi Y_0$

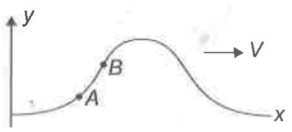
2. Both the strings, show in figure, are made of the same material and have same cross section. The pulleys are light. The wave speed of a transverse wave in the string AB is v_1 and in CD it is v_2. The v_1/v_2 is



   (A) 1
   (B) 2
   (C) $\sqrt{2}$
   (D) $1/\sqrt{2}$

3. A transverse wave of amplitude 0.50 m, wavelength 1 m and frequency 2 Hz is propagating in a string in the negative x-direction. The expression form of the wave is
   (A) $y(x, t) = 0.5 \sin(2px - 4pt)$
   (B) $y(x, t) = 0.5 \cos(2px + 4pt)$
   (C) $y(x, t) = 0.5 \sin(px - 2pt)$
   (D) $y(x, t) = 0.5 \cos(2px - 2pt)$

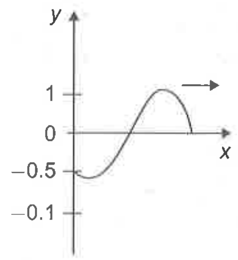
4. A wave pulse is generated in a string that lies along the x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratio of wave speed to the particle speed, respectively, then



   (A) $R_A > R_B$
   (B) $R_B > R_A$
   (C) $R_A = R_B$
   (D) Information is not sufficient to decide

5. A wave is propagating along x-axis. The displacement of particles of the medium in the Z-direction at $t = 0$ is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At $t = 1$ s, the same wave disturbance is given by: $z = \exp[-(2-x)^2]$, Then the wave propagation velocity is
   (A) 4 m/s in $+x$ direction
   (B) 4 m/s in $-x$ direction
   (C) 2 m/s in $+x$ direction
   (D) 2 m/s in $-x$ direction

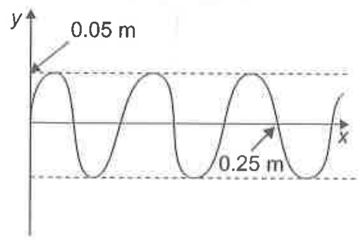
6. The equation of a wave travelling along the positive x-axis, as shown in figure at $t = 0$ is given by



   (A) $\sin\left(kx - \omega t + \frac{\pi}{6}\right)$
   (B) $\sin\left(kx - \omega t - \frac{\pi}{6}\right)$
   (C) $\sin\left(\omega t - kx + \frac{\pi}{6}\right)$
   (D) $\sin\left(\omega t - kx - \frac{\pi}{6}\right)$

7. The velocity of a wave propagating along a stretched string is 10 m/s and its frequency is 100 Hz. The phase difference between the particles situated at a distance of 2.5 cm on the string will be
   (A) $\pi/8$
   (B) $\pi/4$
   (C) $3\pi/8$
   (D) $\pi/2$

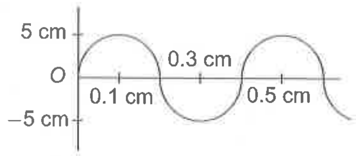
8. If the speed of the wave shown in the figure is 330 m/s in the given medium, then the equation of the wave propagating in the positive x-direction will be (all quantities are in MKS units)



(A) $y = 0.05 \sin 2\pi(4000t - 12.5x)$
(B) $y = 0.05 \sin 2\pi(4000t - 122.5x)$
(C) $y = 0.05 \sin 2\pi(3300t - 10x)$
(D) $y = 0.05 \sin 2\pi(3300x - 10t)$

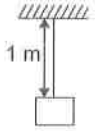
9. The displacement y (in cm) produced by a simple harmonic wave is : $y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right)$ cm. The time period and maximum velocity of the particle will be, respectively, are
(A) 10^{-3} second and 200 m/s
(B) 10^{-2} second and 2000 m/s
(C) 10^{-3} second and 330 m/s
(D) 10^{-4} second and 20 m/s

10. The figure shows the shape of part of a long string in which transverse waves are produced by attaching one end of the string to tuning for k of frequency 250 Hz. What is the velocity of the waves?



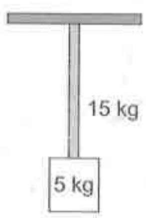
(A) $1.0\ ms^{-1}$
(B) $1.5\ ms^{-1}$
(C) $2.0\ ms^{-1}$
(D) $2.5\ ms^{-1}$

11. A block of mass 1 kg is hanging vertically from a string of length 1 m and Mass/length = 0.001 kg/m. A small pulse is generated at its lower end. The pulse reaches the top end in approximately.



(A) 0.2 s
(B) 0.1 s
(C) 0.02 s
(D) 0.01 s

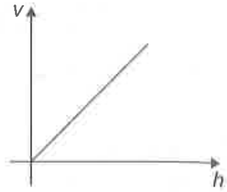
12. A uniform rope of length 10 m and mass 15 kg hangs vertically from a rigid support. A block of mass 5 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.08 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope will be-

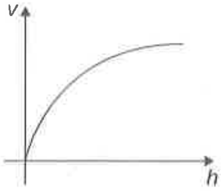


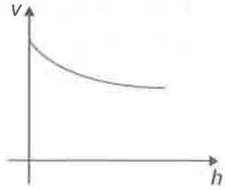
(A) 0.08 m
(B) 0.04 m
(C) 0.16 m
(D) 0 m

13. A uniform rope having some mass hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A)


(B)


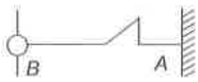
(C)


(D)


14. A wire of 10^{-2} kg m^{-1} passes over a frictionless light pulley fixed on the top of a frictionless inclined plane, which makes an angle of 30° with the horizontal. Masses m and M are tied at two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 ms^{-1}.

(A) $M = 5$ kg
(B) $\frac{m}{M} = \frac{1}{4}$
(C) $m = 20$ kg
(D) $\frac{m}{M} = 4$

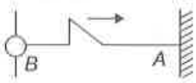
15. A pulse shown here is reflected from the rigid wall A and then from free end B. The shape of the string after these two reflection will be



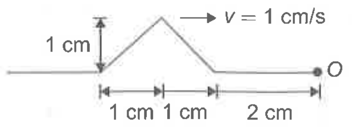
(A)


(B)


(C)

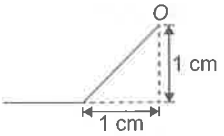

(D)


16. A wave pulse on a string has the dimension shown in the figure. The waves speed is $v = 1$ cm/s. If point O is a free end. The shape of wave at time $t = 3$ s is

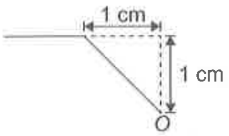


(A)

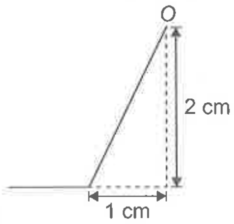
(B)



(C)



(D)



17. A string 1 m long is drawn by a 300 Hz vibrator attached to its end. The string vibrates in three segments. The speed of transverse waves in the string is equal to

(A) 100 m/s (B) 200 m/s
(C) 300 m/s (D) 400 m/s

18. For a wave displacement amplitude is 10^{-8} m, density of air is 1.3 kg m^{-3}, velocity in air is 340 ms^{-1} and frequency is 2000 Hz. The intensity of wave is

(A) 5.3×10^{-4} Wm^{-2} (B) 5.3×10^{-6} Wm^{-2}
(C) 3.5×10^{-8} Wm^{-2} (D) 3.5×10^{-6} Wm^{-2}

19. Two waves of equal amplitude A, and equal frequency travel in the same direction in a medium. The amplitude of the resultant wave is

(A) 0 (B) A
(C) $2A$ (D) between 0 and $2A$

20. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), interfere, then

(A) their resultant amplitude will be twice that of a single wave but the frequency will be same
(B) their resultant amplitude and frequency will both be twice that of a single wave
(C) their resultant amplitude will depend on the phase angle while the frequency will be the same
(D) the frequency and amplitude of the resultant wave will depend upon the phase angle.

21. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has wavelength λ and the transmitted wave λ', then

(A) $\lambda' > \lambda$
(B) $\lambda' = \lambda$
(C) $\lambda' < \lambda$
(D) nothing can be said about the relation of λ and λ'

22. The rate of transfer of energy in a wave depends

(A) directly on the square of the wave amplitude and square of the wave frequency
(B) directly on the square of the wave amplitude and square root of the wave frequency
(C) directly on the wave frequency and square of the wave amplitude
(D) directly on the wave amplitude and square of the wave frequency

23. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse is inverted with respect to the other.

(A) The pulses will collide with each other and vanish after collision.
(B) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(C) The pulses will pass through each other but their shapes will be modified.
(D) The pulses will pass through each other without any change in their shape.

24. A wave is represented by the equation

$$y = 10\sin 2\pi(100t - 0.02X) + 10\sin 2\pi(100t + 0.02X).$$

The maximum amplitude and loop length are respectively

(A) 20 units and 30 units
(B) 20 units and 25 units
(C) 30 units and 20 units
(D) 25 units and 20 units

25. A wire of linear mass density 9×10^{-3} kg/m is stretched between two rigid supports under a tension of 360 N. The wire resonates at frequency 210 Hz. The next higher frequency at which the same wire resonates is 280 Hz. The number of loops produced in first case will be

(A) 1 (B) 2
(C) 3 (D) 4

26. The resultant amplitude due to superposition of two waves

$$Y_1 = 5\sin(wt - kx)$$

and $\quad y_2 = -5\cos(wt - kx - 150°)$ is

(A) 5 (B) $5\sqrt{3}$
(C) $5\sqrt{2-\sqrt{3}}$ (D) $5\sqrt{2+\sqrt{3}}$

27. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is :

(A) 350 Hz (B) 5 Hz
(C) 70 Hz (D) 170 Hz

28. In a stationary wave represented by $y = a \sin wt \cos kx$ the amplitude of the component progressive wave is

(A) $\frac{a}{2}$ (B) a
(C) $2a$ (D) none of these

JEE Advanced

Single Correct

1. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the fundamental frequency of vibration of A relative to that of B is
(A) 1:1 (B) 1:2
(C) 1:4 (D) 1:8

2. A copper wire is held at the two ends by rigid supports. At 30°C the wire is just taut, with negligible tension. The speed of transverse waves in this wire at 10°C is ($\alpha = 1.7 \times 10^{-5}$/°C, $Y = 1.3 \times 10^{11}$ N/m², $d = 9 \times 10^3$ kg/m³).
(A) 80 m/s (B) 90 m/s
(C) 100 m/s (D) 70 m/s

3. A composition string is made up by joining two strings of different masses per unit length $\rightarrow \mu$ and 4μ the composite string is under the same tension. A transverse wave pulse: $Y = (6 \text{ mm}) \sin(5t + 40x)$, where '$t$' is in seconds and '$x$' in metres, is sent along the lighter string towards the joint. The joint is at $x = 0$. The equation of the wave pulse reflected from the joint is
(A) (2 mm) $\sin(5t - 40x)$
(B) (4 mm) $\sin(40x - 5t)$
(C) $-$(2 mm) $\sin(5t - 40x)$
(D) (2 mm) $\sin(5t - 10x)$

4. In the previous question, the percentage of power transmitted to the heavier string through the joint is approximately
(A) 33% (B) 89%
(C) 67% (D) 75%

5. The frequency of a sonometer wire is f, but when the weights producing the tensions are completely immersed in water the frequency becomes $f/2$ and on immersing the weights in a certain liquid the frequency becomes $f/3$. The specific gravity of the liquid is:

(A) $\frac{4}{3}$ (B) $\frac{16}{9}$
(C) $\frac{15}{12}$ (D) $\frac{32}{27}$

6. A wave moving with constant speed on a uniform string passes the point $x = 0$ with amplitude A_0, angular frequency w_0 and average rate of energy transfer P_0. As the wave travels down the string it gradually loses energy and at the point $x = \ell$, the average rate of energy transfer becomes $\frac{P_0}{2}$. At the point $x = \ell$, angular frequency and amplitude are respectively.
(A) w_0 and $A_0/\sqrt{2}$
(B) $w_0/\sqrt{2}$ and A_0
(C) less than w_0 and A_0
(D) $w_0/\sqrt{2}$ and $A_0/\sqrt{2}$

7. A harmonic wave is travelling on string 1. At a junction with string 2, it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at $x = 0$. If the expression for the incident wave is, $y_i = A_i \cos(k_1 x - w_1 t)$, then find out the expression for the transmitted wave.

(A) $\frac{1}{3} A_i \cos(2k_1 x - \omega_1 t)$

(B) $\frac{3}{2} A_i \cos(2k_1 x - \omega_1 t)$

(C) $\frac{2}{3} A_i \cos(2k_1 x - \omega_1 t)$

(D) None of these

8. A taut string at both ends vibrates in its n^{th} overtone. The distance between adjacent node and antinode is found to be 'd'. If the length of the string is L, then
(A) $L = 2d(n + 1)$
(B) $L = d(n + 1)$
(C) $L = 2dn$
(D) $L = 2d(n - 1)$

9. A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference ΔT (Y = young's modulus, ρ = density, α = coefficient of linear expansion), then the frequency of transverse vibration is proportional to

(A) $\frac{\alpha}{\sqrt{\rho Y}}$
(B) $\sqrt{\frac{Y\alpha}{\rho}}$

(C) $\frac{\rho}{\sqrt{Y\alpha}}$
(D) $\sqrt{\frac{\rho\alpha}{Y}}$

10. A standing wave $y = A\sin(\frac{20}{3}\pi x)\cos(1000\pi t)$ is maintained in a taut string where y and x are expressed in metres. The distance between the successive points oscillating with the amplitude $A/2$ across a node is equal to
(A) 2.5 cm
(B) 25 cm
(C) 5 cm
(D) 10 cm

11. A string of length 1 m and linear mass density $0.01\,\text{kgm}^{-1}$ is stretched to a tension of 100 N. When both ends of the string are fixed, the three lowest frequencies for standing wave are f_1, f_2 and f_3. When only one end of the string is fixed, the three lowest frequencies for standing wave are n_1, n_2 and n_3. Then
(A) $n_3 = 5n_1 = f_3 = 125$ Hz
(B) $f_3 = 5f_1 = n_2 = 125$ Hz
(C) $f_3 = n_2 = 3f_1 = 150$ Hz
(D) $n_2 = \frac{f_1 + f_2}{2} = 75$ Hz

12. A wave represented by the equation $y = a\cos(kx - wt)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for other wave is
(A) $a\sin(kx + wt)$
(B) $-a\cos(kx + wt)$
(C) $-a\cos(kx - wt)$
(D) $-a\sin(kx - wt)$

Multiple Correct

13. A wave equation which gives the displacement along the y direction is given by $Y = 10^{-4}\sin(60t + 2x)$, where x and y are in metres and t is time in seconds. This represents a wave
(A) travelling with a velocity of 30 m/s in the negative x direction
(B) of wavelength π metre
(C) of frequency $30/\pi$ hertz
(D) of amplitude 10^{-4} metre travelling along the negative x direction

14. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A\sin(at - bx)$, where t = time, and α and β are constants.
(A) The frequency of the wave is α.
(B) The frequency of the wave is $\alpha/2\pi$.
(C) The wavelength is $2\pi/\beta$.
(D) The velocity of the wave is α/β.

15. A sinusoidal progressive wave is generated in a string. It's equation is given by $y = (2\text{ mm})\sin(2px - 100pt + \pi/3)$. The time when particle at $x = 4$ m first passes through mean position, will be

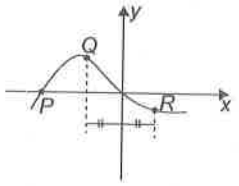
(A) $\frac{1}{150}$ s
(B) $\frac{1}{12}$ s

(C) $\frac{1}{300}$ s
(D) $\frac{1}{100}$ s

16. A transverse wave is described by the equation $y = A\sin[2\pi(ft - x/\lambda)]$. The maximum particle velocity is equal to four times the wave velocity if
(A) $\lambda = \pi A/4$
(B) $\lambda = \pi A/2$
(C) $\lambda = \pi A$
(D) $\lambda = 2\pi A$

17. A wave equation is given as $y = \cos(500t - 70x)$, where y is in mm, x in m and t is in seconds.
(A) The wave must be a transverse propagating wave.
(B) The speed of the wave is 50/7 m/s
(C) The frequency of oscillations 1000π Hz
(D) Two closest points which are in same phase have separation $20\pi/7$ cm.

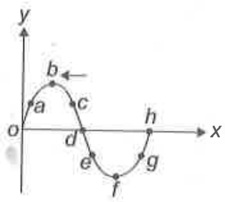
18. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string?



(A) v_P is upwards
(B) $v_Q = -v_R$
(C) $|v_P| > |v_Q| = |v_R|$
(D) $v_Q = v_R$

Question No. 19 to 22

The figure represents the instantaneous picture of a transverse harmonic wave traveling along the negative x-axis. Choose the correct alternative(s) related to the movement of the nine points shown in the figure.



19. The points moving upward is/are
(A) a
(B) c
(C) f
(D) g

20. The points moving downwards is/are
(A) o
(B) b
(C) d
(D) h

21. The stationary points is/are
(A) o
(B) b
(C) f
(D) h

22. The points moving with maximum speed is/are
(A) b
(B) c
(C) d
(D) h

23. A perfectly elastic uniform string is suspended vertically with its upper end fixed to the ceiling and the lower end loaded with the weight. If a transverse wave is imparted to the lower end of the string, the pulse will
(A) not travel along the length of the string
(B) travel upwards with increasing speed
(C) travel upwards with decreasing speed
(D) travelled upwards with constant acceleration

24. One end of a string of length L is tied to the ceiling of a lift accelerating upwards with an acceleration $2g$. The other end of the string is free. The linear mass density of the string varies linearly from 0 to λ from bottom to top.
(A) The velocity of the wave in the string will be 0.
(B) The acceleration of the wave on the string will be $3g/4$ everywhere.
(C) The time taken by a pulse to reach from bottom to top will be $\sqrt{8L/3g}$.
(D) The time taken by a pulse to reach from bottom to top will be $\sqrt{4L/3g}$.

25. A plane wave $y = A \sin \omega\left(t - \frac{x}{v}\right)$ undergo a normal incidence on a plane boundary separating medium M_1 and M_2 and splits into a reflected and transmitted wave having speeds v_1 and v_2 then
(A) for all values of v_1 and v_2 the phase of transmitted wave is same as that of incident wave
(B) for all values of v_1 and v_2 the phase of reflected wave is same as that of incident wave
(C) the phase of transmitted wave depends upon v_1 and v_2
(D) the phase of reflected wave depends upon v_1 and v_2

26. Two waves of equal frequency f and velocity v travel in opposite directions along the same path. The waves have amplitudes A and $3A$. Then
(A) the amplitude of the resulting wave varies with position between maxima of amplitude $4A$ and minima of zero amplitude
(B) the distance between a maxima and adjacent minima of amplitude is $V/2f$
(C) at point on the path the average displacement is zero
(D) the position of a maxima or minima of amplitude does not change with time

27. The vibration of a string fixed at both ends are described by $Y = 2\sin(px)\sin(100pt)$ where Y is in mm, x is in cm, t in seconds, then

(A) Maximum displacement of the particle at $x = 1/6$ cm would be 1 mm.

(B) velocity of the particle at $x = 1/6$ cm at time $t = 1/600$ sec will be $157\sqrt{3}$ mm/s

(C) If the length of the string is 10 cm, number of loop in it would be 5

(D) None of these

28. In a standing wave on a string,

(A) in one time period all the particles are simultaneously at rest twice

(B) all the particles must be at their positive extremes simultaneously once in one time period

(C) all the particles may be at their positive extremes simultaneously once in a time period

(D) all the particles are never at rest simultaneously

29. A standing wave pattern of amplitude A in a string of length L shows 2 nodes (plus those at two ends). If one end of the string corresponds to the origin and v is the speed of progressive wave, the disturbance in the string, could be represented (with appropriate phase) as

(A) $y(x, t) = A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi vt}{L}\right)$

(B) $y(x, t) = A\cos\left(\frac{3\pi x}{L}\right)\sin\left(\frac{2\pi vt}{L}\right)$

(C) $y(x, t) = A\cos\left(\frac{4\pi x}{L}\right)\cos\left(\frac{4\pi vt}{L}\right)$

(D) $y(x, t) = A\sin\left(\frac{3\pi x}{L}\right)\cos\left(\frac{3\pi vt}{L}\right)$

30. The length, tension, diameter and density of a wire B are double than the corresponding quantities for another stretched wire A. Then.

(A) the fundamental frequency of B is $\frac{1}{2\sqrt{2}}$ times that of A.

(B) the velocity of wave in B is $\frac{1}{\sqrt{2}}$ times that of velocity in A.

(C) the fundamental frequency of A is equal to the third overtone of B.

(D) The velocity of wave in B is half that of velocity in A.

31. A string is fixed at both ends vibrates in a resonant mode with a separation 2.0 cm between the consecutive nodes. For the next higher resonant frequency, this separation is reduced to 1.6 cm. The length of the string is

(A) 4.0 cm (B) 8.0 cm
(C) 12.0 cm (D) 16.0 cm

32. A clamped string is oscillating in nth harmonic, then

(A) total energy of oscillations will be n^2 times that of fundamental frequency

(B) total energy of oscillations will be $(n - 1)^2$ times that of fundamental frequency

(C) average kinetic energy of the string over a complete oscillations is half of that of the total energy of the string

(D) none of these

33. The figure, shows a stationary wave between two fixed points P and Q. Which point(s) of 1, 2 and 3 are in phase with the point x?

(A) 1, 2 and 3 (B) 1 and 2 only
(C) 2 and 3 only (D) 3 only

34. The equation of a wave disturbance is given as $y = 0.02\cos\left(\frac{\pi}{2} + 50\pi t\right)\cos(10\pi x)$, where x and y are in meters and t in seconds. Choose the wrong statement.

(A) Antinode occurs at $x = 0.3$ m.

(B) The wavelength is 0.2 m.

(C) The speed of the constituent waves is 4 m/s.

(D) Node occurs at $x = 0.15$ m.

35. In a stationary wave,

(A) all the particles of the medium vibrate in phase

(B) all the antinodes vibrate in phase

(C) the alternate antinodes vibrate in phase

(D) all the particles between consecutive nodes vibrate in phase

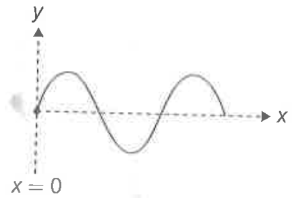
JEE Advanced

Level I

1. Consider the wave $y = (5\text{ mm})\sin[(1\text{ cm}^{-1})x - (60\text{ s}^{-1})t]$ Find (A) the amplitude (B) the wave number, (C) the wavelength, (D) the frequency, (e) the time period and (f) the wave velocity.

2. The wave function for a traveling wave on a taut string is (in SI unit)
$$y(x, t) = (0.350\text{ m})\sin(10pt - 3px + \pi/4).$$
(A) What are the speed and direction of travel of the wave?
(B) What is the vertical displacement of the string at $t = 0$, $x = 0.100$ m?
(C) What are wavelength and frequency of the wave?
(D) What is the maximum magnitude of the transverse speed of a particle of the string?

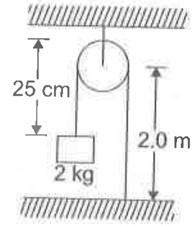
3. The string shown in figure is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine (A) the angular frequency and (B) wave number for this wave. (C) Write an expression for the wave function. Calculate (D) the maximum transverse speed and (e) the maximum transverse acceleration of a point on the string.



4. Two strings A and B with $\mu = 2$ kg/m and $\mu = 8$ kg/m respectively are joined in series and kept on a horizontal table with both the ends fixed. The tension in the string is 200 N. If a pulse of amplitude 1 cm travels in A towards the junction, then find the amplitude of reflected and transmitted pulse.

5. A parabolic pulse given by equation $y(\text{in cm}) = 0.3 - 0.1(x - 5t)^2$ $(y \geq 0)x$ in metre and t in seconds travelling in a uniform string. The pulse passes through a boundary beyond which its velocity becomes 2.5 m/s. What will be the amplitude of pulse in this medium after transmission?

6. In the arrangement shown in figure, the string has mass of 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley? Take $g = 10\text{ m/s}^2$



7. A uniform rope of length 12 m and mass 6 kg hanged vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?

8. A particle on stretched string supporting a travelling wave, takes 5.0 ms to move from its mean position to the extreme position. The distance between two consecutive particles, which are at their mean position, is 2.0 cm. Find the frequency, the wavelength and the wave speed.

9. A 6.00 m segment of a long string has a mass of 180 g. A high-speed photograph shows that the segment contains four complete cycles of wave. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" displacement is the vertical distance from the farthest positive displacement to the farthest negative displacement.) (A) Write the function that describes this wave traveling in the positive x direction. (B) Determine the power being supplied to the string.

10. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 g/m kept under a tension of 60 N. (A) Find the average power transmitted across a given point on the string. (B) Find the total energy associated with the wave in a 2.0 m long portion of the string.

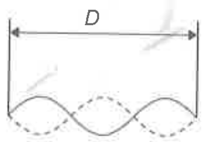
11. The equation of a plane wave travelling along the positive direction of x-axis is $y = a \sin \frac{2\pi}{\lambda}(vt - x)$. When this wave is reflected at a rigid surface and its amplitude becomes 80%, then find the equation of the reflected wave.

12. A travelling wave of amplitude $5A$ is partially reflected from a boundary with the amplitude $3A$. Due to superposition of two waves with different amplitudes in opposite directions a standing wave pattern is formed. Determine the amplitude at node and antinodes.

13. Two waves are described by $y_1 = 0.30 \sin[\pi(5x - 200)t]$ and $y_2 = 0.30 \sin[\pi(5x - 200t) + \pi/3]$, where y_1, y_2 and x are in metres and t is in seconds. When these two waves are combined, a traveling wave is produced. What are the (A) amplitude, (B) wave speed, and (C) wave length of that traveling wave?

14. What are (A) the lowest frequency, (B) the second lowest frequency, and (C) the third lowest frequency for standing waves on a wire that is 10.0 m long has a mass of 100 g. and is stretched under a tension of 250 N which is fixed at both ends?

15. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance $D = 90.0$ cm apart. The string is oscillating in the standing wave pattern shown in the figure. Calculate the (A) speed wavelength, and (C) frequency of the traveling waves whose superposition gives this standing wave.



16. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz with no intermediate resonant frequencies. What are
(A) the lowest resonant frequencies and
(B) the wave speed ?

17. A string oscillates according to the equation

$$y' = (0.50 \text{ cm}) \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)x\right] \cos[(40\pi \text{ s}^{-1})t]$$

What are the (A) amplitude and (B) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation ? (C) What is the distance between nodes ? (D) What is the transverse speed of a particle of the string at the position $x =$ 1.5 cm when $t = 9/8$ s?

18. In an experiment of standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. What tension must the string be under (weights are attached to the other end) if it is to oscillate in four loops?

19. A string vibrates in 4 loops with a frequency of 400 Hz.
(A) What is its fundamental frequency?
(B) What is frequency will cause it to vibrate into 7 loops?

20. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its lengths from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement.

21. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz?

22. A 2.00 m long rope, having a mass of 80 g, is fixed at one end and is tied to a light string at the other end. The tension in the string is 256 N. (A) Find the frequencies of the fundamental and the first two overtones. (B) Find the wavelength in the fundamental and the first two overtones.

23. A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning for k.

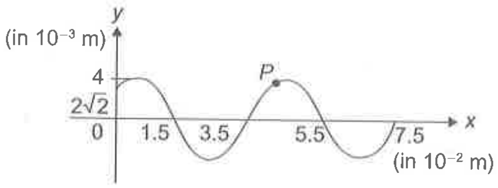
24. A sonometer wires resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by M, the wire resonates with the same tuning fork forming three antinodes for the same position of bridges. Find the value of M.

25. A 40 cm long wire having a mass 3.2 gm and area of c.s. 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode. It vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire.

26. A steel rod having a length of 1m is fastened at its middle. Assuming young's modulus to be 2×10^{11} Pa, and density to be 8 g/cm^3, find the fundamental frequency of the longitudinal vibration and frequency of first overtone.

Level II

1. The figure shows a snap photograph of a vibrating string at $t = 0$. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with the x-axis at P is 6°.



(A) Find the direction in which the wave is moving.
(B) The equation of the wave.
(C) The total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 g/m.

2. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

3. A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At $t = 0$ the pulse is entirely located between $x = 0$ and $x = 1$ m. Draw a graph of the transverse velocity of particle of string versus time at $x = +1$ m.

4. In a stationary wave pattern that forms as a result of reflection of waves from an obstacle the ratio of the amplitude at an antinode and a node is $\beta = 1.5$. What percentage of the energy passes across the obstacle?

5. A string, 25 cm long, having a mass of 0.25 gm/cm, is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone, and the air in the pipe in its fundamental frequency, 8 beats/s are heard. It is observed that decreasing the tension in the string, decreases the beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string.

6. A metal rod of length $l = 100$ cm is clamped at two points. Distance of each clamp from nearer end is $a = 30$ cm. If density and Young's modulus of elasticity of rod material are $\rho = 9000$ kgm^{-3} and $Y = 144$ GPa, respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

Previous Year Questions

JEE Main

1. A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is **[AIEEE 2002]**

(A) $y = a \sin(\omega t + kx)$
(B) $y = -a \sin(\omega t + kx)$
(C) $y = a \sin(\omega t - kx)$
(D) $y = -a \sin(\omega t - kx)$

2. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is **[AIEEE 2002]**

(A) 20
(B) 80
(C) 40
(D) 120

3. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[AIEEE 2003]**

(A) −4
(B) 4
(C) $4\sqrt{2}$
(D) 8

4. A metal wire of linear mass density of 9.8 gm^{-1} is stretched with a tension of 10 kg-wt between two rigid supports 1 m apart. The wire passes at its middle

point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[AIEEE 2003]**

(A) 50 Hz (B) 100 Hz
(C) 200 Hz (D) 25 Hz

5. The displacement y of a wave travelling in the x-direction is given by **[AIEEE 2003]**

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{m},$$

where, x is expressed in metres and t in seconds. The speeds of the wave-motion, in ms^{-1} is

(A) 300 (B) 600
(C) 1200 (D) 200

6. The displacement y of a particle in a medium can be expressed as

$$y = 10^{-6}\sin\left(100t - 20x + \frac{\pi}{4}\right)\text{m},$$

where t is in second and x in metre. The speed of the wave is **[AIEEE 2004]**

(A) 2000 ms^{-1} (B) 5 ms^{-1}
(C) 20 ms^{-1} (D) 5π ms^{-1}

7. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[AIEEE 2006]**

(A) 105 Hz (B) 1.05 Hz
(C) 1050 Hz (D) 10.5 Hz

8. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively. then α and β in appropriate units are

[AIEEE 2008]

(A) $\alpha = 25.00\pi, \beta = \pi$

(B) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(C) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

(D) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

9. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02\ (\text{m}) \sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right]$ The tension in the string is **[AIEEE 2009]**

(A) 4.0 N (B) 12.5 N
(C) 0.5 N (D) 6.25 N

10. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$. This represents a **[AIEEE 2011]**

(A) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(B) standing wave of frequency $\sqrt{b}$

(C) standing wave of frequency $\frac{1}{\sqrt{b}}$

(D) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

11. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$ The resultant is **[AIEEE 2011]**

(A) a standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$

(B) a wave travelling along $+x$ direction

(C) a wave travelling along $-x$ direction

(D) a standing wave having nodes at $x = \frac{n\lambda}{2}$; $n = 0, 1, 2$

12. **Statement I:** Two longitudinal waves given by equations $y_1(x, t) = 2a\sin(\omega t - kx)$ and $y_2(x, t) = a\sin(2\omega t - 2kx)$ will have equal intensity. **[AIEEE 2011]**

Statements II: Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

(A) Statement I is true, Statement II is true.
(B) Statement I is true, Statement II is false.
(C) Statement I is true, Statement II true; Statement II is the correct explanation of Statement I.
(D) Statement I is true, Statement II is true; Statement II is not correct explanation of Statement I.

JEE Advanced

1. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **[JEE-2002(Scr), 3]**

   (A) 25 kg (B) 5 kg
   (C) 12.5 kg (D) 1/25 kg

2. A stringer between $x = 0$ and $x = l$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[JEE-2003]**

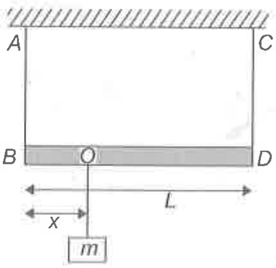
3. A string fixed at both ends is in resonance in its 2nd harmonic with a tuning fork of frequency f_1. Now it's one end becomes free. If the frequency of the tuning fork is increased slowly from f_1 then again a resonance is obtained when the frequency is f_2. If in this case the string vibrates in nth harmonic then **[JEE 2005(Scr)]**

   (A) $n = 3,\ f_2 = \frac{3}{4} f_1$ (B) $n = 3,\ f_2 = \frac{5}{4} f_1$

   (C) $n = 5,\ f_2 = \frac{5}{4} f_1$ (D) $n = 5,\ f_2 = \frac{3}{4} f_1$

4. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[JEE 2005]**

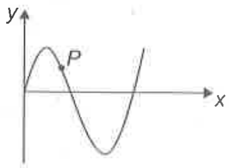
5. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of the first harmonic (fundamental frequency) in AB is equal to the second harmonic frequency in CD. Then, length of BO is **[JEE 2006]**



   (A) $\frac{L}{5}$ (B) $\frac{L}{4}$

   (C) $\frac{4L}{5}$ (D) $\frac{3L}{4}$

6. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is Figure: **[JEE-2008]**



   (A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

   (C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

7. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. **[JEE 2009]**

8. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, then find the amplitude of the resultant wave. **[JEE 2010]**

9. One end of a taut string of length 3 m along the x axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the direction so that stationary waves are set up in the string. The possible waveform of these stationary waves is (are) **[JEE Advanced 2014]**

   (A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

   (B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

   (C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

   (D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250pt$

ANSWER KEYS

Exercises

JEE Main

1. B 2. D 3. B 4. A 5. A 6. D 7. D 8. C 9. A 10. A
11. D 12. C 13. C 14. C 15. A 16. D 17. B 18. D 19. D 20. C
21. C 22. A 23. D 24. B 25. C 26. A 27. C 28. A

JEE Advanced

1. B 2. D 3. C 4. B 5. D 6. A 7. C 8. A 9. B 10. C
11. D 12. B 13. A, B, C, D 14. B, C, D 15. C 16. B 17. A, B, D 18. C, D
19. A, D 20. C 21. B, C 22. C, D 23. B, D 24. B, C 25. A, D 26. C, D 27. A, B 28. A, C
29. D 30. C, D 31. B 32. A, C 33. C 34. C 35. C, D

JEE Advanced

Level I

1. (A) Amplitude $A = 5$ mm (B) Wave number $k = 1\ \text{cm}^{-1}$ (C) Wavelength $\lambda = \dfrac{2\pi}{k} = 2\pi$ cm
(D) Frequency $v = \dfrac{\omega}{2\pi} = \dfrac{60}{2\pi}$ Hz (e) Time period $T = \dfrac{1}{v} = \dfrac{\pi}{30}$ s (f) Wave velocity $u = n\lambda = 60$ cm/s

2. (A) $\dfrac{10}{3}\hat{i}$ m / s (B) -5.48 cm (C) 0.667 m, 5.00 Hz (D) 11.0 m/s

3. (A) 10π rad/s (B) $\pi/2$ rad/m (C) $y = (0.120\ \text{m}) \sin(1.57x - 31.4t)$ (D) 1.2π m/s (e) 118 m/s^2

4. $A_r = -\dfrac{1}{3}$cm $A_t = \dfrac{2}{3}$cm **5.** 0.2 cm **6.** 0.02 s **7.** 0.12 m 8. 50 Hz, 4.0 cm, 2.0 m/s

9. (A) $y = (7.50\ \text{cm}) \sin(4.19x - 314t)$ (B) 625 W **10.** (A) 0.47 W, (B) 9.4 mJ

11. $y' = 0.8\, a \sin\dfrac{2\pi}{\lambda}(vt + x + \dfrac{\lambda}{2})$ **12.** $2A, 8A$ **13.** (A) 0.52 m; (B) 40 m/s; (C) 0.40 m

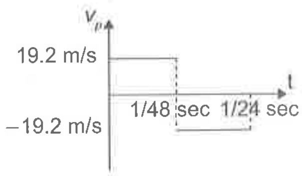
14. (A) $\dfrac{5\sqrt{10}}{2}$ Hz; (B) $5\sqrt{10}$ Hz; (C) $\dfrac{5\sqrt{10}}{2}$ Hz **15.** (A) 144 m/s; (B) 60.0 cm; (C) 241 Hz

16. (A) 105 Hz; (B) 158 m/s **17.** (A) 0.25 cm (B) 1.2×10^2 cm/s; (C) 3.0 cm; (D) 0
18. 36 N **19.** (A) 100 Hz (B) 700 Hz **20.** 300 Hz **21.** 60 cm from an end
22. (A) 10 Hz, 30 Hz, 50 Hz (B) 8.00 m, 1.60 m **23.** 1% **24.** 25 kg **25.** 1×10^9 Nm^2 **26.** 2.5 kHz, 7.5 kHz

Level II

1. (A) Negative x; (B) $y = 4 \times 10^{-3} \sin 100\pi\left(3t + 0.5x + \dfrac{1}{400}\right)$ (x, y in metre); (C) $144p^2 \times 10^{-5}$ J **2.** $\dfrac{\pi}{\sqrt{2}\omega}$

3.



4. 96% **5.** 67.6 N **6.** 10 kHz, 30 kHz

Previous Year Questions

JEE Main

1. B **2.** B **3.** C **4.** A **5.** A **6.** B **7.** A **8.** A **9.** D **10.** A
11. A **12.** B

JEE Advanced

1. A **2.** $E = \frac{A^2\pi^2T}{4l}$ **3.** C **4.** $y = (10 \text{ cm}) \sin(30t \pm \frac{3}{2}x + f)$ **5.** A **6.** A **7.** 5
8. 5 **9.** A, C, D

CHAPTER 3

Sound Waves

SOUND WAVES

Sound is a type of longitudinal wave. In general, majority of the longitudinal waves are termed as sound waves. Sound is produced by a vibrating source, like when a gong of a bell is struck with a hammer, sound is produced. The vibrations produced by a gong are propagated through air. Through air, these vibrations reach the ear and the ear drum is set into vibrations. These vibrations are communicated to the human brain. By touching the gong of the bell by hand, we can feel the vibrations.

PROPAGATION OF SOUND WAVES

Sound is a mechanical three-dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.

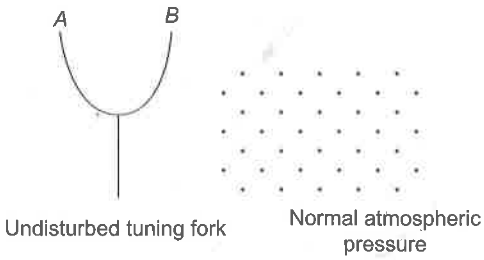


Figure 3.1

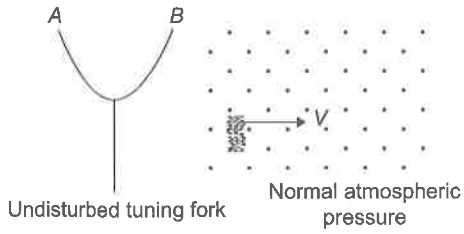


Figure 3.2

When prong *B* moves outwards towards right, it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound.

After producing the compression pulse, the prong *B* reverses its motion and moves inwards. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.

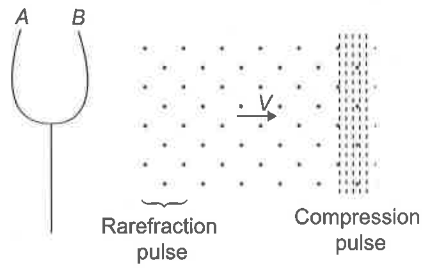


Figure 3.3

A longitudinal wave in a fluid is described in terms of the longitudinal displacements suffered by the particles of the medium.

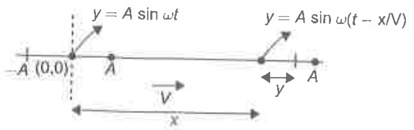


Figure 3.4

Consider a wave going in the x-direction in a fluid. Suppose that at a time t, the particle at the undisturbed position x suffers a displacement y in the x-direction.

$$y = A \sin \omega \left(t - \frac{x}{V}\right) \qquad (1)$$

Position of any particle from the origin at any time $= x + y$, where x = distance of the mean position of the particle from the origin, y = displacement of the particle from its mean position.

General equation is given as

$(0, 0) \Rightarrow \quad y = A \sin(\omega t + \phi)$

$(0, x) \Rightarrow \quad y = A \sin[(t - x/V) + \phi]$

Displacement wave, $y = A \sin(\omega t - kx + \phi)$

If we fix $x = x_0$, then we are dealing with the particle whose mean position is at a distance x_0 from the origin and this particle is performing SHM of amplitude A with time period T and phase difference $= -kx + \phi$.

COMPRESSION WAVES

When a longitudinal wave is propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically. In the region where compression occurs, the pressure is more than the normal pressure of the medium. Thus, we can also describe longitudinal waves in a gaseous medium as pressure waves and these are also termed as compression waves in which the pressure at different points of the medium also varies periodically with their displacements. Let us discuss the propagation of excess pressure in a medium in longitudinal wave analytically.

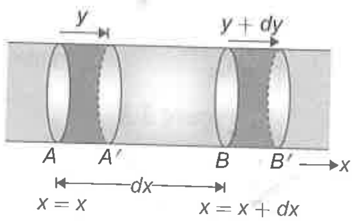


Figure 3.5

Consider a longitudinal wave propagating in the positive x-direction as shown in Fig. 3.5. The figure shows a segment AB of the medium of width dx. In this medium, let a longitudinal wave is propagating whose equation is given as

$$y = A \sin(\omega t - kx), \qquad (1)$$

where y is the displacement of the medium particle situated at a distance x from the origin, along the direction of propagation of the wave. In Fig. 3.5, AB is the medium segment whose medium particle is at position $x = x$ and B is at $x = x + dx$ at an instant. If after some time t, medium particle at A reaches a point A', which is displaced by y, and the medium particle at b reaches point B', which is at a displacement $y + dy$ from B. Here, dy is given by Eq. (3.1) as

$$dy = -Ak \cos(\omega t - kx)dx$$

Here, due to displacement of section AB to $A'B'$, the change in volume of its section is given as

$$dV = -Sdy \qquad [S \rightarrow \text{area of cross-section}]$$

$$= SAk \cos(\omega t - kx)dx$$

The volume of section AB is $V = Sdx$.

Thus, the volume strain in section AB is

$$\frac{dV}{V} = \frac{-SAk \cos(\omega t - kx)dx}{S\,dx}$$

or
$$\frac{dV}{V} = -Ax \cos(\omega t - kx)$$

If B is the bulk modulus of the medium, then the excess pressure in the section AB can be given as

$$\Delta P = -\ B\left(\frac{dV}{V}\right) \qquad (2)$$

$$\Delta P = BAk \cos(\omega t - kx)$$

or
$$\Delta P = \Delta P_0 \cos(\omega t - kx) \qquad (3)$$

Here, ΔP_0 is the pressure amplitude at a medium particle at position x from the origin and ΔP is the excess pressure at that point. Equation (3) shows that excess pressure varies periodically at every point of the medium with pressure amplitude ΔP_0, which is given as

$$\Delta P_0 = BAk = \frac{2\pi}{\lambda} AB \qquad (4)$$

Equation (4) is also termed as the equation of pressure wave in gaseous medium. We can also see that the pressure wave differs in phase by $\pi/2$ from the displacement wave and pressure maxima occurs where the displacement is zero and displacement maxima occurs when the pressure is at its normal level. Remember that pressure maxima implies that the pressure at a point is pressure amplitude times more or less than the normal pressure level of the medium.

SOLVED EXAMPLE

EXAMPLE 1

A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 2.0×10^{-3} N/m², find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m².

SOLUTION

The pressure amplitude is

$$P_0 = \frac{2.0\times10^{-3}\,\text{N/m}^2}{2} = 10^{-3}\,\text{N/m}^2$$

The displacement amplitudes S_0 is given by

$$P_0 = Bks_0$$

or,

$$S_0 = \frac{P_0}{Bk} = \frac{P_0\lambda}{2\pi B}$$

$$= \frac{10^{-3}\,\text{N/m}^2\times(40\times10^{-2}\,\text{m})}{2\times\pi\times14\times10^4\,\text{N/m}^2}$$

$$= \frac{100}{7\pi}\,\text{Å} = 6.6\,\text{Å}$$

Density Wave

In this section, we will find the relation between pressure wave and density wave.

According to the definition of bulk modulus (B),

$$B = \left(-\frac{d\rho}{dV/V}\right) \qquad (1)$$

Further, volume $= \dfrac{\text{mass}}{\text{density}}$

or $$V = \frac{m}{\rho}$$

or $$dV = -\frac{m}{\rho^2}d\rho = -\frac{V}{\rho}d\rho$$

or $$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

Substituting in Eq. (1), we get

$$d\rho = \frac{\rho(dP)}{B} = \frac{d\rho}{V^2}$$

$$\left(\frac{\rho}{B} = \frac{1}{V^2}\right)$$

Or this can be written as,

$$\Delta\rho = \frac{\rho}{B}\Delta\rho = \frac{1}{V^2}\Delta\rho$$

So, this relation relates the pressure equation with the density equation. For example, if

$$\Delta P = (\Delta P)_m \sin(kx - \omega t),$$

then $$\Delta\rho = (\Delta\rho)_m \sin(kx - \omega t)$$

where, $$(\Delta\rho)_m = \frac{\rho}{B}(\Delta P)_m = \frac{(\Delta P)_m}{V^2}$$

Thus, density equation is in phase with the pressure equation and this is 90° out of phase with the displacement equation.

Velocity and Acceleration of a Particle

General equation of a wave is given by

$$y = A\sin(\omega t - kx)$$

$$V_p = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx) \qquad (1)$$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx) \qquad (2)$$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) \qquad (3)$$

Here, $\dfrac{\partial y}{\partial x}$ = slope of (y, x) curve.

Now again differentiating Eq. (3),

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx) \qquad (4)$$

From Eqs. (2) and (4),

$$\frac{\partial^2 y}{\partial t^2} = V^2\frac{\partial^2 y}{\partial x^2}$$

VELOCITY OF SOUND/LONGITUDINAL WAVES IN SOLIDS

Consider a section AB of medium as shown in Fig. 3.6(a) of cross-sectional area S. Let A and B be two cross-sections as shown. Let in this medium sound propagation is from left to right. If wave source is at origin O and when it oscillates, the oscillations at that point propagate along the rod.

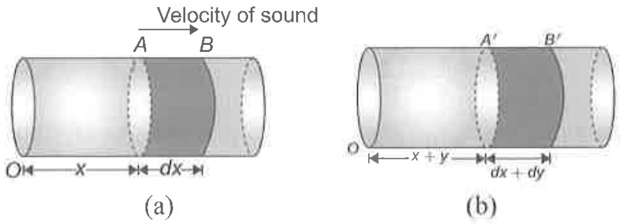


Figure 3.6

Here, we say an elastic wave has propagated along the rod with a velocity determined by the physical properties of the medium. Due to oscillations, say a force F is developed at every point of the medium which produces a stress in the rod and is the cause of strain or propagation of disturbance along the rod. This stress at any cross-sectional area can be given as

stress, $$S_1 = \frac{F}{S} \quad (1)$$

Let us consider the section AB of the medium at a general instant of time t. The end A is at a distance x from O and B is at a distance $x + dx$ from O. Let in time dt due to oscillations, medium particles at a are displaced along the length of the medium by y and those at B by $y + dy$. The resulting position of the sections and A' and B' are shown in Fig. 3.6(b). Here, we can say that the section AB is deformed (elongated) by a length dy. Thus, strain produced in it is

Strain in section AB, $$E = \frac{dy}{dx} \quad (2)$$

If Young's modulus of the material of medium is Y, we have

Young's modulus, $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{S_1}{E}$$

From Eqs. (1) and (2), we have $Y = \dfrac{F/S}{dy/dx}$

or $$F = YS\frac{dy}{dx} \quad (3)$$

If net force acting on section AB is dF, then it is given as

$$dF = dma, \quad (4)$$

where dm is the mass of the section AB and a is its acceleration, which can be given as for a medium of density ρ.

$$dm = \rho S dx$$

and $$a = \frac{d^2y}{dt^2}$$

From Eq. (4), we have $dF = (\rho S dx)\dfrac{d^2y}{dx^2}$

or $$\frac{dF}{dx} = \rho S\frac{d^2y}{dt^2} \quad (5)$$

From Eq. (3) on differentiating with respect to x, we can write

$$\frac{dF}{dx} = YS\frac{d^2y}{dt^2} \quad (6)$$

From Eqs. (5) and (6), we get

$$\frac{d^2y}{dx^2} = \left(\frac{Y}{\rho}\right)\frac{d^2y}{dx^2} \quad (7)$$

Equation (7) is the differential form of the wave equation, comparing it with previous equation, we get the wave velocity in the medium as

$$V = \sqrt{\frac{Y}{\rho}}$$

Similar to the case of a solid in fluid, instead of Young's modulus we use bulk modulus of the medium. Hence, the velocity of longitudinal waves in a fluid medium is given as

$$V = \sqrt{\frac{B}{\rho}},$$

where B is the bulk modulus of the medium.

For a gaseous medium, bulk modulus is defined as

$$B = \frac{dp}{(-dV/V)}$$

or $$B = -V\frac{dP}{dV}$$

Newton's Formula for Velocity of Sound in Gases

Newton assumed that during sound propagation, temperature of a medium remains constant. Hence, he stated that propagation of sound in a gaseous medium is an isothermal phenomenon. Boyle's law can be applied in the process. So for a section of medium, we use

$$PV = \text{constant}.$$

Differentiating, we get

$$PdV + VdP = 0$$

or $$-V\frac{dP}{dV} = P$$

or bulk modulus of the medium can be given as $B = P$ (pressure of the medium)

Newton found that during isothermal propagation of sound in a gaseous medium, bulk modulus of the medium is equal to the pressure of the medium. Hence, sound velocity in a gaseous medium can be given as

$$V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{P}{\rho}} \qquad (1)$$

From gas law, we have

$$\frac{P}{\rho} = \frac{RT}{M} \qquad (2)$$

From Eqs. (1) and (2), we have

$$V = \sqrt{\frac{RT}{M}} \qquad (3)$$

From the expression in Eq. (1), if we find the sound velocity in air at normal temperature and atmospheric pressure, we have

Normal atmospheric pressure,

$$P = 1.01 \times 10^5 \text{ Pa}$$

Density of air at NTP,

$$\rho = 1.293 \text{ kg/m}^3$$

Now from Eq. (1),

$$V = \sqrt{\frac{P}{\rho}}$$

$$\Rightarrow \quad V = \sqrt{\frac{10.1 \times 10^5}{1.293}}$$

$$= 279.45 \text{ m/s.}$$

But the experimental value of velocity of sound determined from various experiments gives the velocity of sound at NTP as 332 m/s. Therefore, there is a difference of about 52 m/s between the theoretical and experimental values. This large difference cannot be attributed to the experimental errors. Newton was unable to explain the error in his formula. This correction was explained by the French scientist Laplace.

Laplace Correction

Laplace explained that when sound waves propagated in a gaseous medium, there is compression and rarefaction in the particles of the medium. Where there is compression, particles come near each other and are heated up, where there is rarefaction, the medium expands and there is a fall of temperature. Therefore, the temperature of the medium at every point does not remain constant so the process of sound propagation is not isothermal. The total quantity of heat of the system as a whole remains constant. The medium does not gain or lose any heat to the surrounding. Thus, in a gaseous medium, sound propagation is an adiabatic process. For adiabatic process, the relation in pressure and volume of a section of medium can be given as

$$PV^{\gamma} = \text{constant} \qquad (1)$$

Here, $\gamma = \frac{C_P}{C_V}$ is the ratio of specific heats of the medium.

Differentiating Eq. (1) we get,

$$dPV^{\gamma} + \gamma V^{\gamma-1} dVP = 0$$

or

$$dP + \gamma \frac{PdV}{dV} = 0$$

or

$$-V\frac{dP}{dV} = \gamma P$$

Bulk modulus of medium, $B = \gamma P$.

Thus, Laplace found that during adiabatic propagation of sound, the bulk modulus of gaseous medium is equal to the product of ratio of specific heats and the pressure of the medium. Thus, the velocity of sound propagation can be given as

$$V = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

From gas law,

$$V = \sqrt{\frac{\gamma RT}{M}}$$

From the above equation, we find the sound velocity in air at NTP. We have,

Normal atmospheric pressure,

$$P = 1.01 \times 10^5 \text{ Pa}$$

Density of air at NTP,

$$P = 1.293 \text{ kg/m}^3$$

Ratio of specific heat of air,

$$\gamma = \frac{C_P}{C_V} = 1.42$$

$$\Rightarrow \quad V = \sqrt{\frac{\gamma p}{\rho}}$$

$$= \sqrt{\frac{1.42 \times 1.01 \times 10^5}{1.293}}$$

$$= 333.04 \text{ m/s.}$$

This value is in agreement with the experimental value. Now at any temperature t°C, the velocity of sound,

$$V_t = \sqrt{\frac{\gamma R(273+t^\circ)}{M}}$$

$$= \sqrt{\frac{\gamma R273}{M}}\left(1+\frac{t}{273}\right)^{1/2}$$

$$V_t = V_0\left(1+\frac{t}{546}\right) = V_0 + 0.61\,\mathrm{T}$$

Velocity of sound increases by 0.61 m/s. When temperature change by 1°C

Effect of Temperature on Velocity of Sound

We have velocity of sound propagation in a gaseous medium as

$$V = \sqrt{\frac{\gamma RT}{M}}$$

For a given gaseous medium γ, R and M remain constant. Thus, the velocity of sound is directly proportional to the square root of absolute temperature of the medium. Thus,

$$V \propto \sqrt{T}$$

If at two different temperatures T_1 and T_2, sound velocities in the medium are V_1 and V_2. Then, from the above equation, we have

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

Effect of Pressure on Velocity of Sound

We know from the gas law,

$$\frac{P}{\rho} = \frac{RT}{M}$$

If the temperature of a medium remains constant, then on changing pressure, density of the medium proportionally changes so that the ratio $\frac{P}{\rho}$ remains constant.

Hence, if in a medium,

$$T = \text{constant},$$

then, $$\frac{P}{\rho} = \text{constant}.$$

Thus, velocity of sound, $V = \sqrt{\frac{\gamma P}{\rho}} = \text{constant}.$

Therefore, the velocity of sound in air or in a gas is independent of change in pressure.

Effect of Humidity on the Velocity of Sound

The density of water vapour at NTP is 0.8 kg/m^3, whereas the density of dry air at NTP is 1.293 kg/m^3. Therefore, water vapour has a density less than the density of dry air. As atmospheric pressure remains approximately the same, the velocity of sound is more in moist air than the velocity of sound in dry air.

$$V_{\text{moist air}} > V_{\text{dry air}} \quad \text{(from the previous equation).}$$

Effect of Wind on Velocity of Sound

If wind is blowing in the direction of propagation of sound, it will increase the velocity of sound. On the other hand, if wave propagation is opposite to the direction of propagation of wind, wave velocity is decreased. If wind blows at speed V_ω, then sound velocity in the medium can be given as

$$\vec{V} = \vec{V}_S + \vec{V}_w,$$

where $\vec{V}_S$ is the velocity of sound in still air.

APPEARANCE OF SOUND TO HUMAN EAR

The appearance of sound to a human ear is characterized by three parameters: (a) pitch, (b) loudness and (c) quality.

Pitch and Frequency

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice and a female voice. We say that a buffalo voice is of low pitch, a male voice has higher pitch and a female voice has still higher pitch. This sensation primarily depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa.

Loudness and Intensity

The loudness that we sense is related to the intensity of sound, though it is not directly proportional to it. Our perception of loudness is better correlated with the sound level measured in decibels (abbreviated as dB) and defined as follows.

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right),$$

where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2.The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level $\beta = 0$. Maximum tolerable I is 1 W/m^2 and maximum level is 120 dB

$$0 < \beta < 120$$

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure 3.7 shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.

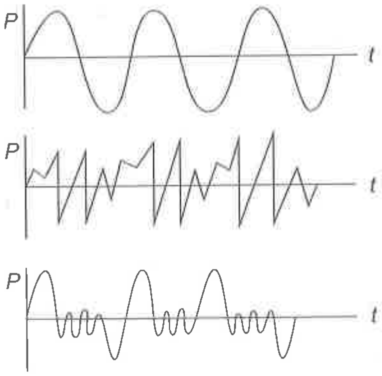


Figure 3.7

We differentiate between the sound from a table and that from a midrange by saying that they have different quality.

Energy in Sound Waves

$$P_{avg} = 2\pi^2 f^2 A^2 \mu V$$

$$\text{Intensity} = 2\pi^2 A^2 f^2 \rho V$$

$$\Delta P = \Delta P_0 \cos(\omega t - kx)$$

$$\Delta P_0 = \beta^3 kA$$

$$\Rightarrow \quad A = \frac{\Delta P_0}{\beta K}$$

$$\text{Intensity} = \frac{\omega^2 A^2 \rho V}{2}$$

$$= \frac{\omega^2 \Delta P_0^2 \rho V}{2\beta^2 k^2} = \frac{\omega^2 \Delta P_0^2 \rho V^3}{2\beta^2 \omega^2}$$

$$= \frac{\Delta P_0^2 \rho V^2 \cdot V}{2\beta^2} = \frac{\Delta P_0^2 \rho \beta \cdot V}{2\beta^2 \rho}$$

$$\left(V = \sqrt{\frac{\beta}{\rho}} \right)$$

$$I = \frac{\Delta P_0^2 V}{2\beta}$$

ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES

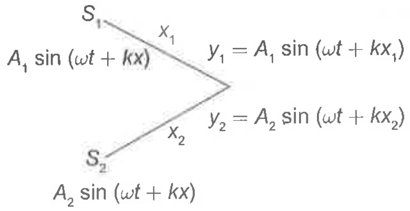


Figure 3.8

Interference implies super position of waves. Whenever two or more than two waves superimpose each other at some position, then the resultant displacement of the particle is given by the vector sum of the individual displacements.

Let the two waves coming from sources S_1 and S_2 be

$$y_1 = A_1 \sin(\omega t + kx_1)$$

$$y_2 = A_2 \sin(\omega t + kx_2), \text{ respectively.}$$

Due to superposition,

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$$

Phase difference between y_1 and $y_2 = k(x_2 - x_1)$,

i.e., $\Delta\phi = k(x_2 - x_1)$.

As $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

(where Δx = path difference and $\Delta\phi$ = phase difference)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\Rightarrow \quad A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\therefore \quad I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad (\text{as } I \propto A^2)$$

When the two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes and I_{net} will be maximum. This is known as constructive interference.

For I_{net} to be maximum,

$$\cos\phi = 1$$

$$\Rightarrow \quad \phi = 2n\pi, \text{ where } n = \{0, 1, 2, 3, 4, 5, \ldots\}$$

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi$$

$$\Rightarrow \quad \Delta x = n\lambda.$$

For constructive interference

$$I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $I_1 = I_2 = I$,

$$I_{net} = 4I$$
$$A_{net} = A_1 + A_2$$

When superposing waves are in opposite phase, the resultant amplitude is the difference of two amplitudes and I_{net} is minimum. This is known as destructive interference.

For I_{net} to be minimum,

$$\cos \Delta\phi = -1$$
$$\Delta\phi = (2n + 1)\pi$$

where $n = \{0, 1, 2, 3, 4, 5, \ldots\}$

$$\frac{2\pi}{\lambda}\Delta x = (2n+1)\pi$$

$\Rightarrow$ $$\Delta x = (2n+1)\frac{\lambda}{2}$$

For destructive interference,

$$I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If $$I_1 = I_2,$$
$$I_{net} = 0$$
$$A_{net} = A_1 - A_2$$

Generally,

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

If $$I_1 = I_2 = I$$
$$I_{net} = 2I + 2I\cos\phi$$
$$I_{net} = 2I(1 + \cos\phi)$$
$$= 4I\cos^2\frac{\Delta\phi}{2}$$

Ratio of I_{max} and $I_{min} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

LONGITUDINAL STANDING WAVES

Two longitudinal waves of the same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering waves are given by

$$P_1 = P_0 \sin(\omega t - kx)$$

and $$P_2 = P_0 \sin(\omega t + kx + \phi),$$

then the equation of the resultant standing wave would be given by

$$P = P_1 + P_2 = 2P_0 \cos(kx + \frac{\phi}{2}) \sin(\omega t + \frac{\phi}{2})$$

$\Rightarrow$ $$P = P_0' \sin(\omega t + \frac{\phi}{2}) \quad (1)$$

This is equation of SHM* in which the amplitude p_0' depends on position as

$$P_0' = 2P_0 \cos(kx + \frac{\phi}{2}) \quad (2)$$

Points where pressure remains permanently at its average value, i.e., pressure amplitude is zero is called a pressure node, and the condition for a pressure node would be given by

$$P_0' = 0,$$

i.e., $$\cos(kx + \frac{\phi}{2}) = 0$$

i.e., $$kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2},$$
$$n = 0, 1, 2, \ldots$$

Similarly, points where pressure amplitude is maximum is called a pressure anti-node and condition for a pressure anti-node would be given by

$$P_0' = \pm 2p_0,$$

i.e., $$\cos(kx + \frac{\phi}{2}) = \pm 1$$

or $$(kx + \frac{\phi}{2}) = n\pi,$$
$$n = 0, 1, 2, \ldots$$

Notes

1. Note that a pressure node in a standing wave would correspond to a displacement anti-node; and a pressure anti-node would correspond to a displacement node.
2. When we label Eq. (1) as SHM, what we mean is that excess pressure at any point varies simple harmonically. If the sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to Eq. (1) would be

$$S = S_0' \cos(\omega t + \frac{\phi}{2}),$$

where $$S_0' = 2S_0 \sin(kx + \frac{\phi}{2}).$$

Notes (Cont'd)

This can be easily observed to be an equation of SHM. It represents the medium particles moving simple harmonically about their mean position at x.

REFLECTION OF SOUND WAVES

Reflection of sound waves from a rigid boundary (e.g., closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion, i.e., an abrupt phase change of P. This is consistent with the requirement of displacement if amplitude remains zero at the rigid end, since a medium particle at the rigid end cannot vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in terms of phase, a displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure waves from the rigid boundary will have the same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse

On the other hand, reflection of sound wave from a low-pressure region (like the open end of an organ pipe) is analogous to reflection of string wave from a free end. This point corresponds to a displacement maxima, so that the incident and reflected displacement waves at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e., pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave, i.e., a compression pulse is reflected as a rarefaction pulse and vice versa.

WAVES IN A VIBRATING AIR COLUMN

Hollow pipes have long been used for making musical sounds. A hollow pipe is called as an organ pipe. To understand how these work, first we examine the behaviour of air in a hollow pipe that is open at both ends. If we blow air across one end, the disturbance due to the moving air at that end propagates along the pipe to the far end. When it reaches the far end, a part of the wave is reflected, similar to the case when a wave is reflected along a string whose end point is free to move. Since the air particles are free to move at the open end, the end point is an anti-node. If one end of the pipe is closed off, the air is not free to move any further in that direction and the closed end becomes a node. Now, the resonant behaviour of the pipe is completely changed. Similar to the case of a string, here also all harmonic frequencies are possible and resonance may take place if the frequency of external source matches with any of the harmonic frequency of the pipe. Let us discuss this in detail.

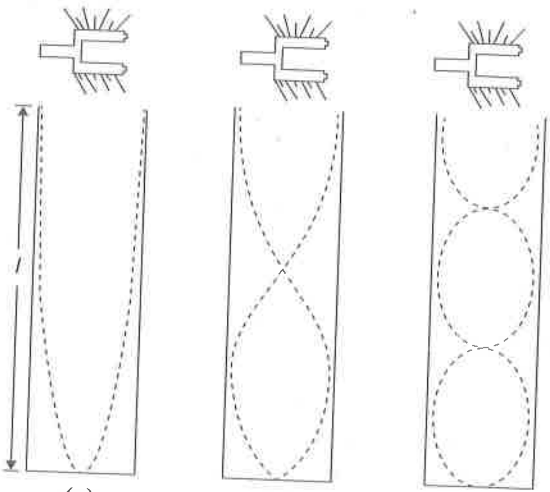
VIBRATION OF AIR IN A CLOSED-ORGAN PIPE

When a tuning fork is placed near the open end of a pipe, the air in the pipe oscillates with the same frequency as that of a tuning fork. Here, the open end should be an anti-node and closed and should be a node for perfect reflection of waves from either end or for formation of stationary waves. Since one end is a node and other is an anti-node, the lowest frequency (largest wavelength) vibration has no other nodes or anti-nodes between the ends as shown in Fig. 3.9(a). This is the fundamental (minimum) frequency at which stationary waves can be formed in a closed-organ pipe. Thus, if the wavelength is λ, then we can see from Fig. 3.9(a), which shows the displacement wave of the longitudinal waves in the closed-organ pipe.

$$l = \frac{\lambda}{4} \qquad (1)$$

or

$$\lambda = 4l$$



(a) (b) (c)

Figure 3.9

Thus, fundamental frequency of oscillations of a closed-organ pipe of length l can be given as

$$n_1 = \frac{V}{\lambda} = \frac{V}{4l} \qquad (2)$$

Similarly, first overtone of a closed-pipe vibration is shown in Fig. 3.9(b). Here, wavelength λ' and pipe length l are related as

$$l = \frac{3\lambda'}{4} \qquad (3)$$

or $$\lambda' = \frac{4l}{3}$$

Thus, frequency of the first overtone oscillations of a closed-organ pipe of length l can be given as

$$n_2 = \frac{V}{\lambda'} = \frac{3V}{4l} \qquad (4)$$
$$= 3n_1.$$

This is three times the fundamental frequency. Thus, after the fundamental frequency only third harmonic frequency exists for a closed-organ pipe at which resonance can take place or stationary waves can be formed in it.

Similarly, the next overtone, second overtone is shown in Fig. 3.9(c). Here, the wavelength λ'' and pipe length l are related as

$$l = \frac{5\lambda''}{4}$$

or $$\lambda'' = \frac{4l}{5}$$

Thus, the frequency of second overtone oscillation of a closed-organ pipe of length l can be given as

$$n_3 = \frac{V}{\lambda''}$$
$$= \frac{5V}{4l} = 5n_1 .$$

This is the fifth harmonic frequency of fundamental oscillations.

In general, $$f = \frac{(2n-1)V}{4\ell}$$

Here, frequency of oscillation is called $(2n - 1)^{th}$ harmonic and $(n - 1)^{th}$ overtone.

From the above analysis, it is clear that the resonant frequencies of the closed-organ pipe are only odd harmonics of the fundamental frequency.

Vibration of Air in an Open-Organ Pipe

Figure 3.10 shows the resonant oscillations of an open-organ pipe. The least frequency at which an open-organ pipe resonates is the one with the longest wavelength when at both the open ends of a pipe anti-nodes are formed and there is one node in between as shown in Fig. 3.10(a). In this situation, the wavelengths of sound in air λ is related to length of organ pipe as

$$l = \frac{\lambda}{2}$$

or $$\lambda = 2l. \qquad (1)$$

Thus, the fundamental frequency of organ pipe can be given as

$$n_1 = \frac{V}{\lambda} = \frac{V}{2l}$$

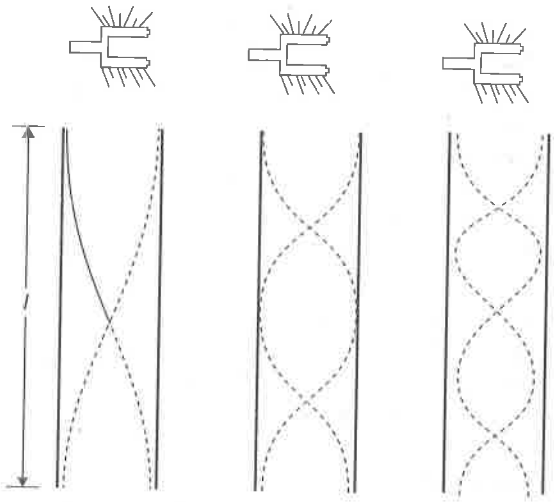


Figure 3.10

Similarly, the next higher frequency at which the open-organ pipe resonates is shown in Fig. 3.10(b), which we call the first overtone. Here, the wavelength λ' is related to the length of the pipe as

$$l = \lambda' \qquad (2)$$

Thus, here the resonant frequency for the first overtone is given as

$$n_2 = \frac{V}{\lambda'} = \frac{V}{l}, \qquad (3)$$

which is the second harmonic of fundamental frequency. Similarly, as shown in Fig. 3.10(c), in second overtone oscillations, the wavelength λ'' of sound is related to the length of pipe as

$$l = \frac{3\lambda''}{2} \qquad (4)$$

or $$\lambda'' = \frac{2l}{3} \qquad (5)$$

Thus, the frequency of second overtone oscillations of an open-organ pipe can be given as

$$n_3 = \frac{V}{\lambda''} = \frac{3V}{2l} \qquad (6)$$

$$= 3n_1, \qquad (7)$$

which is the third harmonic of fundamental frequency.

In general, $f = \dfrac{nV}{2\ell}$.

We can say the frequency of oscillation is called n-th harmonic and $(n-1)^{\text{th}}$ overtone.

The above analysis shows that resonant frequencies for the formation of stationary waves include all the possible harmonic frequencies for an open-organ pipe.

End Correction

As mentioned earlier, the displacement anti-node at an open end of an organ pipe lies slightly outside the open lend. The distance of the anti-node from the open end is called end correction and its value is given by

$$e = 0.6r;$$

where r = radius of the organ pipe.

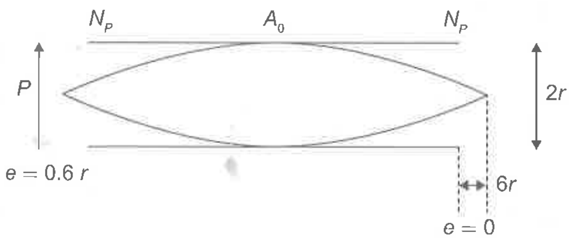


Figure 3.11

With end correction, the fundamental frequency of a closed pipe (f_e) and an open organ pipe (f_0) will be given by

$$f_e = \frac{V}{4(\ell + 0.6r)}$$

and

$$f_0 = \frac{V}{2(\ell + 1.2r)}$$

Resonance Tube

This an apparatus used to determine velocity of sound in air experimentally and also to compare frequencies of two tuning forks.

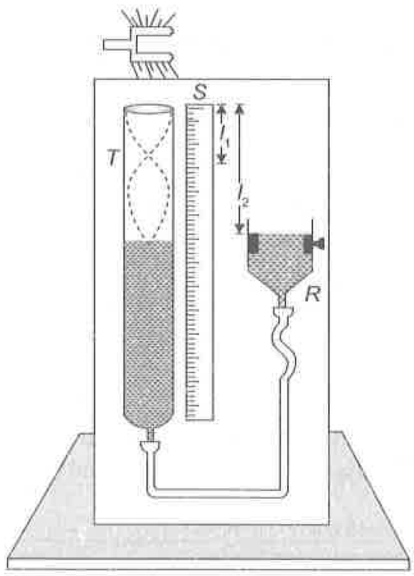


Figure 3.12

Figure 3.12 shows the setup of a resonance experiment. There is a long tube T in which initially water is filled up to the top and the water level can be changed by moving a reservoir R up and down.

A tuning fork of known frequency n_0 is struck gently on a rubber pad and brought near the open tube T due to which oscillations are transferred to the air column in the tube above water level. Now, we gradually decrease the water level in the tube. This air column behaves like a closed-organ pipe and the water level as the closed end of pipe. As soon as water level reaches a position where there is a node of corresponding stationary wave, in air column, resonance takes place and maximum sound intensity is detected. Let at this position length of air column be l_1. If water level is further decreased, again maximum sound intensity is observed when water level is at another node, i.e., at a length l_2 as shown in Fig. 3.12. Here, if we find two successive resonance lengths l_1 and l_2, we can get the wavelength of the wave as

$$l_2 - l_1 = \frac{\lambda}{2}$$

or

$$\lambda = 2(l_2 - l_1).$$

Thus, sound velocity in air can be given as

$$V = n_0\lambda$$

$$= 2n_0(l_2 - l_1)$$

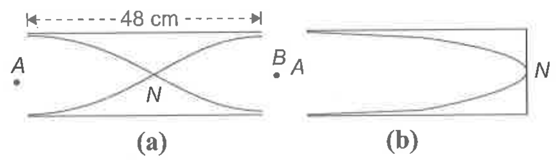
SOLVED EXAMPLE

EXAMPLE 2

A tube of certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.

SOLUTION

The displacement curves of longitudinal waves in a tube open at both ends is shown in Figs. (a) and (b).



Let r be the radius of the tube. We know the anti-nodes occur slightly outside the tube at a distance $0.6r$ from the tube end.

The distance between two anti-nodes is given by

$$\frac{\lambda}{2} = 48 + 2 \times 0.6r$$

We have, $$\lambda = \frac{V}{n}$$

$$= \frac{32000}{320} = 100 \text{ cm}$$

or $$50 = 48 + 1.2r$$

or $$r = \frac{2}{1.2} = 1.67 \text{ cm.}$$

Thus, diameter of the tube is $D = 2r = 3.33$.

When one end is closed, then

$$\frac{\lambda}{4} = 48 + 0.6r$$

$$= 48 + 0.6 \times 1.67 = 49$$

or $$\lambda = 4 \times 49 = 196 \text{ cm.}$$

Now, $$n = \frac{V}{\lambda}$$

$$= \frac{32000}{196} = 163.3 \text{ Hz.}$$ ■

Quinck's Tube

This is an apparatus used to demonstrate the phenomenon of interference and also used to measure velocity of sound in air. This is made up of two *U*-tubes *A* and *B* as shown in Fig. 3.13. Here, the tube *B* can slide in and out from the tube *A*. There are two openings *P* and *Q* in the tube *A*. At opening *P*, a tuning fork or a sound source of known frequency n_0 is placed, and at the other opening, a detector is placed to detect the resultant sound of interference occurring due to the superposition of two sound waves coming from the tubes *A* and *B*.

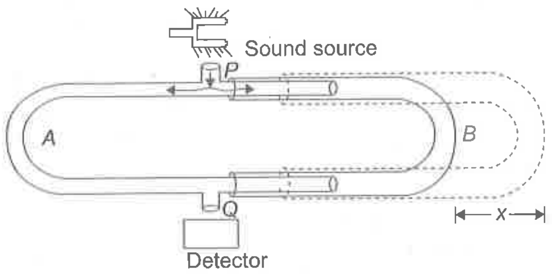


Figure 3.13

Initially, tube *B* is adjusted so that the detector detects a maximum. At this instant, if the length of the paths covered by the two waves from *P* to *Q* from the side *A* and side *B* are l_1 are l_2, respectively, then for constructive interference, we must have

$$l_2 - l_2 = N\lambda \quad (1)$$

If now tube B is further pulled out by a distance x so that the next maximum is obtained and the length of the path from the side of *B* is l_2', then we have

$$l_2' = l_2 + 2x, \quad (2)$$

where x is the displacement of the tube. For the next constructive interference of sound at point Q, we have

$$l_2' - l_1 = (N+1)\lambda \quad (3)$$

From Eqs. (1), (2) and (3), we get

$$x = \frac{\lambda}{2} \quad (4)$$

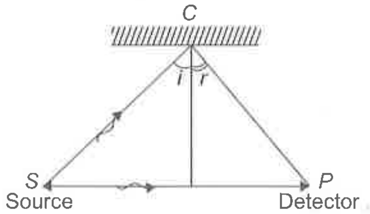
Thus, by experiment we get the wavelength of sound as for two successive points of constructive interference, the path difference must be λ. As the tube *B* is pulled out by x, this introduces a path difference $2x$ in the path of sound wave through tube *B*. If the frequency of the source is known, n_0, the velocity of sound in the air filled in tube can be given as

$$V = n_0\lambda = 2n_0x \quad (5)$$

SOLVED EXAMPLES

EXAMPLE 3

In a large room, a person receives direct sound waves from a source 120 m away from him. He also receives waves from the same source which reaches him, being reflected from the 25-m-high ceiling at a point halfway between them. For which wavelength will these two sound waves interference be constructive?



SOLUTION

As shown in the figure for reflection from the ceiling,

$$\text{Path } SCP = SC + CP = 2SC$$

(as $\angle i = \angle r$, $SC = CP$)

or $$\text{path } SCP = 2\sqrt{60^2 + 25^2} = 130 \text{ cm.}$$

So the path difference between the interfering waves along path SCP and SP,

$$\Delta x = 130 - 120 = 10 \text{ m.}$$

Now for constructive interference at P,

$$\Delta x = n\lambda, \text{ i.e., } 10 = n\lambda$$

or $$\lambda = \frac{10}{n},$$

with $$n = 1, 2, 3, \ldots,$$

i.e., $$\lambda = 10 \text{ m}, 5 \text{ m}, (10/3) \text{ m and so on.}$$ ■

EXAMPLE 4

Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semi-circular part has a radius of 20.0 cm. The frequency of the sound source can be varied electronically between 1000 and 4000 Hz. Find the frequencies at which the maxima of intensity are detected. The speed of sound in air = 340 m/s.

SOLUTION

The sound wave reaches the detector by two paths simultaneously in a straight as well as a semi-circular track. The wave through the straight path travels a distance $l_1 = 2 \times 20$ cm and the wave through the curved part travels a distance $l_2 = \pi$ (20 cm) = 62.8 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore,

$$\Delta l = l_2 - l_1 = 62.8 \text{ cm} - 40 \text{ cm}$$
$$= 22.8 \text{ cm} = 0.228 \text{ m.}$$

The wavelength of either wave is $\frac{V}{n} = \frac{340}{n}$. For constructive interference, $\Delta l = N\lambda$, where N is an integer

or $$0.228 = N\left(\frac{340}{n}\right)$$

or, $$n = N\left(\frac{340}{0.228}\right)$$
$$= N(1491.2) \text{ Hz}$$
$$= N(1490) \text{ Hz.}$$

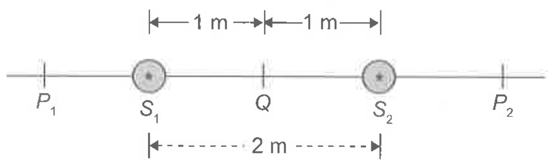
Thus, the frequencies within the specific range which cause maxima of intensity are 1490 Hz and 2980 Hz. ■

EXAMPLE 5

Two sources S_1 and S_2, separated by 2.0 m, vibrate according to equation $y_1 = 0.03 \sin \pi t$ and $y_2 = 0.02 \sin \pi t$, where y_1, y_2 and t are in MKS unit. They send out waves of velocity 1.5 m/s. Calculate the amplitude of the resultant motion of the particle co-linear with S_1 and S_2 and located at a point (a) to the right of S_2, (b) to the left of S_2 and (C) in the middle of S_1 and S_2.

SOLUTION

The situation is shown in the figure.



The oscillations y_1 and y_2 have amplitudes $A_1 = 0.03$ m and $A_2 = 0.02$, respectively.

The frequency of both sources in

$$n = \frac{\omega}{2\pi} \text{ s}$$

$$= \frac{1}{2} = 0.5 \text{ Hz.}$$

Now wavelength of each wave

$$\lambda = \frac{V}{n}$$

$$= \frac{1.5}{0.5} = 3.0 \text{ m.}$$

(a) The path difference for all points P_2 to the right of S_2 is

$$\Delta = (S_1P_2 - S_2P_2)$$

$$= S_1S_2 = 2 \text{ m.}$$

Phase difference $\phi = \frac{2\pi}{\lambda} \times$ Path difference

$$= \frac{2\pi}{3} \times 2.0 = \frac{4\pi}{3}$$

The resultant amplitude for this point is given by

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

$$= \sqrt{(0.03)^2 + (0.02)^2 + 2 \times 0.03 \times 0.02 \times \cos(4\pi/3)}$$

Solving, we get $R = 0.0265$ m.

(b) The path difference for all points P, to the left of S_1

$$\Delta = (S_2P - S_1P)$$

$$= S_1S_2 = 2.0 \text{ m.}$$

Hence, the resultant amplitude for all points to the left of S_1 is also 0.0265 m.

(c) For a point Q, between S_1 and S_2, the path difference is zero, i.e., $\phi = 0$. Hence, constructive interference take place at Q. Thus, amplitude at this point is maximum and given as

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$$

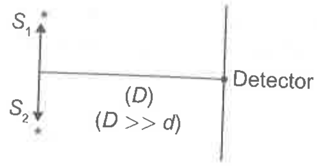
$$= A_1 + A_2$$

$$= 0.03 + 0.02$$

$$= 0.05 \text{ m.}$$ ■

EXAMPLE 6

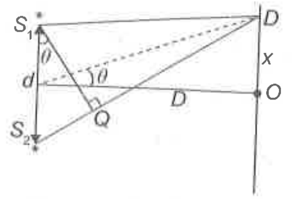
Two point sources of sound are placed at a distance d and a detector moves on a straight line parallel to the line joining the sources as shown in the figure. At a dista D, away from sources. Initially, detector is situated on line so that it is equidistant from both the sources. F the displacement of the detector when it detects the maximum sound and also find its displacement whe detects n-th minimum sound.



SOLUTION

The situation is shown in the figure.

Let us consider the situation when the detector mov by a distance x as shown. Let at this position the pa difference between the waves from S_1 and S_2 to detector Δ, then we have



$$\Delta = S_2D - S_1D \simeq S_2Q$$

(where S_1Q is perpendicular on line S_2D).

Here, if θ is small angle as $D \gg d$, we have

$$S_2Q = d \sin\theta \simeq$$

$$d \tan\theta = d\frac{x}{D}$$

Thus, at the position of detector, path deference is

$$\Delta = \frac{dx}{D} \quad (1)$$

The expression for path difference in Eq. (1) is an important formula for such problems. Students are advised to keep this formula in mind for future use.

When the detector was at point O, path difference was zero and it detects a maxima. Now, if the detector detects the n-th maximum, then its path difference at a distance x from O can be given as

$$\Delta = n\lambda$$

or $$\frac{dx}{D} = n\lambda$$

or $$x = \frac{n\lambda D}{d}$$

Similarly, if the detector detects the n-th minima, then the path difference between the two waves at the detector can be given as

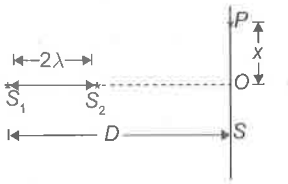
$$\Delta = (2n + 1)\frac{\lambda}{2}$$

or $$\frac{dx}{D} = (2n+1)\frac{\lambda}{2}$$

or $$x = \frac{(2n+1)\lambda D}{2d}$$

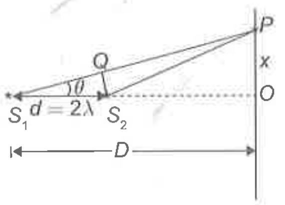
EXAMPLE 7

Two coherent narrow slits emitting wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The sound is detected by moving a detector on the screen S at a distance D ($\gg\lambda$) from the slit S_1 as shown in the figure. Find the distance x such that the intensity at P is equal to the intensity at O.



SOLUTION

When the detector is at O, we can see that the path difference in the two waves reaching O is $d = 2\lambda$. Thus at O, the detector receives a maximum sound. When it reaches P and again there is a maximum sound detected at P, the path difference between the two waves must be $\Delta = \lambda$. Thus, from the figure, the path difference at P can be given as



$$\Delta = S_1P - S_2P \simeq S_1Q$$
$$= d\cos\theta = 2\lambda\cos\theta.$$

And we have at point P, path difference $\Delta = \lambda$. Thus,

$$\Delta = 2\lambda\cos\theta = \lambda$$

or, $$\cos\theta = \frac{1}{2}$$

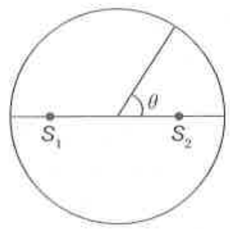
or, $$\theta = \frac{\pi}{3}$$

Thus, the value of x can be written as

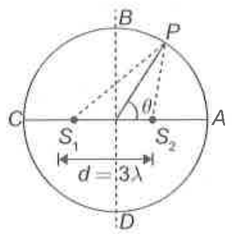
$$x = D\tan\theta = D\tan\theta$$
$$= D\tan\left(\frac{\pi}{3}\right)$$
$$= \sqrt{3}D$$

EXAMPLE 8

Figure shows two coherent sources S_1 and S_2 which emit sound waves of wavelength λ in phase. The separation between the sources is 3λ. A circular wire of large radius is placed in such a way that S_1S_2 lies in its plane and the middle point of S_1S_2 is at the centre of the wire. Find the angular position θ on the wire for which constructive interference takes place.



SOLUTION



From the previous question, we can say that for a point P on the circle shown in figure, the path difference in the two waves at P is

$$\Delta = S_1P - S_2P = d\cos\theta$$
$$= 3\lambda\cos\theta$$

We know for constructive interference at P, the path difference must be an integral multiple of wavelength λ. Thus, for a maxima at P, we have,

$$3\lambda\cos\theta = 0;$$

$$3\lambda \cos\theta = \lambda;$$

$$3\lambda \cos\theta = 2\lambda;$$

$$3\lambda \cos\theta = 3\lambda;$$

or, $$\theta = \frac{\pi}{2}$$

or $$\theta = \cos^{-1}\frac{1}{3}$$

or $$\theta = \cos^{-1}\frac{2}{3}$$

or $$\theta = 0$$

There are four points A, B, C and D on the circle at which $\theta = 0$ or $\frac{\pi}{2}$ and there are two points in each quadrant at $\theta = \cos^{-1}\frac{1}{3}$ and $\theta = \cos^{-1}\frac{2}{3}$ at which constructive interference takes place. Thus, there are a total of 12 points on circle at which maxima occurs. ■

Vibrations of Clamped Rod

We have discussed the resonant vibrations of a string clamped at two ends. Now, we discuss the oscillations of a rod clamped at a point on its length. Figure 3.14 shows a rod AB clamped at its middle point. If we gently hit the rod at its one end, it begins to oscillate and in the natural oscillations the rod vibrates at its lowest frequency and maximum wavelength, which we call fundamental mode of oscillations. With maximum wavelength, when transverse stationary waves set up in the rod, the free ends vibrate as anti-nodes and the clamped end is a node as shown in Fig. 3.14. Here, if λ be the wavelength of the wave, we have

$$l = \frac{\lambda}{2}$$

or $$\lambda = 2l$$

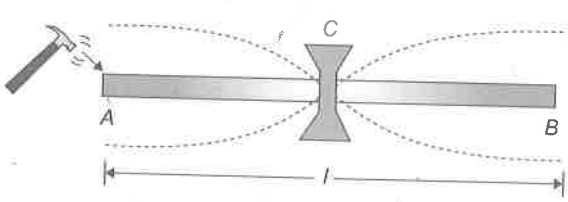


Figure 3.14

Thus, the frequency of fundamental oscillations of a rod damped at mid-point can be given as

$$n_0 = \frac{V}{\lambda}$$

$$= \frac{1}{2l}\sqrt{\frac{Y}{\rho}}, \quad (1)$$

where Y is the Young's modulus of the material of the rod and ρ is the density of the material of rod.

Next, the higher frequency at which the rod vibrates will be the one when wave length is decreased to a value so that one node is inserted between the mid-point and an end of rod as shown in Fig. 3.15.

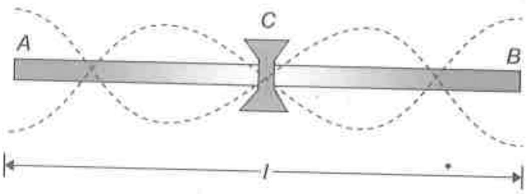


Figure 3.15

In this case, if λ is the wavelength of the waves in rod, we have

$$l = \frac{3\lambda}{2}$$

or $$\lambda = \frac{2l}{3} \quad (2)$$

Thus, in this case the oscillation frequency of the rod can be given as

$$n_1 = \frac{V}{\lambda}$$

$$= \frac{3}{2l}\sqrt{\frac{Y}{\rho}} = 3n_0 \quad (3)$$

This is called the first overtone frequency of the damped rod or third harmonic frequency. Similarly, the next higher frequency of oscillation, i.e., the second overtone of the oscillating rod can be shown in Fig. 3.16. Here, if λ is the wavelength of the wave, then it can be given as

$$l = \frac{5\lambda}{2}$$

or $$\lambda = \frac{2l}{5} \quad (4)$$

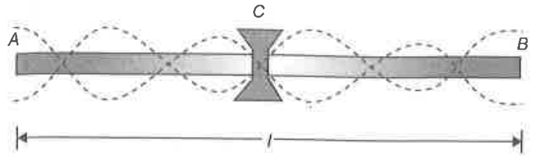


Figure 3.16

Thus, the frequency of oscillation of the rod can be given as

$$n_2 = \frac{V}{\lambda}$$

$$= \frac{5}{2l}\sqrt{\frac{Y}{\rho}} = 5n_0 \quad (5)$$

Thus, the second overtone frequency is the fifth harmonic of the fundamental oscillation frequency of the rod. We can also see from the above analysis that the resonant frequencies at which stationary waves are set up in a damped rod are only odd harmonics of fundamental frequency.

Thus, when an external source of frequency matches with any of the harmonic of the damped rod, then stationary waves are set up in the rod.

Natural Oscillation of Organ Pipes

When we initiate some oscillations in an organ pipe, harmonics excited in the pipe depends on how the initial disturbance is produced in it. For example, if you gently blow across the top of an organ pipe, it resonates softly at its fundamental frequency. But if you blow must harder you hear the higher pitch of an overtone because the faster airstream higher frequencies in the exciting disturbance. This sound effect can also be achieved by increasing the air pressure in an organ pipe.

Kundt's Tube

This is an apparatus used to find the velocity of sound in a gaseous medium or in different materials. It consists of a glass tube as shown in Fig. 3.17. One end of the piston B is fitted, which is attached to a wooden handle H and can be moved inside and outside the tube and fixed. The rod M of the required material is fixed at clamp C in which the velocity of sound is required, at one end of the rod, a disc A is fixed as shown.

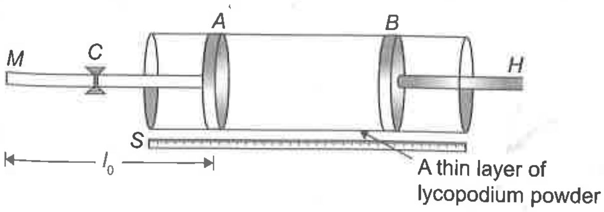


Figure 3.17

In the tube, air is filled at room temperature and a thin layer of lycopodium powder is put along the length of the tube. It is a very fine powder, particles of which can be displaced by the air particles also.

When rod M is gently rubbed with a resin cloth or hit gently, it starts oscillating in the fundamental mode as shown in Fig. 3.18, frequency of which can be given as

$$n_{rod} = \frac{V}{\lambda}$$

$$= \frac{1}{2l_0}\sqrt{\frac{Y}{\rho}}$$

(as $\lambda_0 = \frac{\lambda}{2}$)

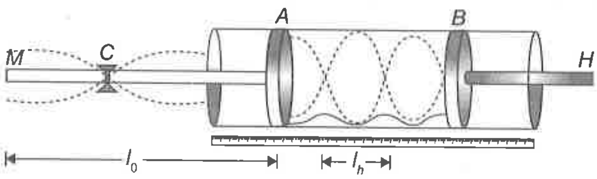


Figure 3.18

BEATS

When two sources of sound that have almost the same frequency are sounded together, an interesting phenomenon occurs. A sound with a frequency average of the two is heard and the loudness of sound repeatedly grows and then decays, rather than being constant. Such a repeated variation in amplitude of sound is called 'beats'.

If the frequency of one of the source is changed, there is a corresponding change in the rate at which the amplitude varies. This rate is called beat frequency. As the frequencies come close together, the beat frequency becomes slower. A musician can tune a guitar to another source by listening for the beats while increasing or decreasing the tension in each string. Eventually, the beat frequency becomes very low so that effectively no beats are heard, and the two sources are then in tune.

We can also explain the phenomenon of beat mathematically. Let us consider the two superposing waves have frequencies n_1 and n_2, then their respective equations of oscillation are

$$y_1 = A \sin 2\pi n_1 t \quad (1)$$

and

$$y_2 = A \sin 2\pi n_2 t \quad (2)$$

On superposition at a point, the displacement of the medium particle is given as

$$y = y_1 + y_2$$

$$y = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$$

$$y = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right) t \sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \quad (3)$$

$$y = R\sin 2\pi\left(\frac{n_1 + n_2}{2}\right)t \quad (4)$$

Then Eq. (4) gives the displacement of the medium particle where superposition takes place. It shows that the particle executes SHM with frequency $\frac{n_1 + n_2}{2}$, average of the two ssuperposing frequencies, and with amplitude R which varies with time, given as

$$R = 2A\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t \quad (5)$$

Here, R becomes maximum when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = \pm 1$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = N\pi \quad (N \in I)$$

or $$t = \frac{N}{n_1 - n_2}$$

or at time $t = 0, \frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2}, \ldots$

At all the above time instants, the sound of maximum loudness is heard. Similarly, we can find the time instants when the loudness of sound is minimum, it occurs when

$$\cos 2\pi\left(\frac{n_1 - n_2}{2}\right)t = 0$$

or $$2\pi\left(\frac{n_1 - n_2}{2}\right)t = (2N + 1)\frac{\pi}{2} \quad (N \in I)$$

or $$t = \frac{2N + 1}{2(n_1 + n_2)}$$

or at time instants

$$t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \ldots$$

Here, we can see that these time instants are exactly lying in the middle of the instants, when loudest sound is heard. Thus, on superposition of the above two frequencies at a medium particle, the sound will be increasing, decreasing, again increasing and decreasing and so on. This effect is called beats. Here, the time between two successive maximum or minimum sounds is called beat period, which is given as beat period, T_B = time between two successive maxima = time between two successive minima $= \frac{1}{n_1 - n_2}$.

Thus, beat frequency or number of beats heard per second can be given as

$$f_B = \frac{1}{T_B} = n_1 - n_2$$

The superposition of two waves of slightly different frequencies is graphically shown in Fig. 3.19(a). The resulting envelope of the wave formed after superposition is also shown in Fig. 3.19(b). Such a wave when propagates produces 'beat' effect at the medium particles.

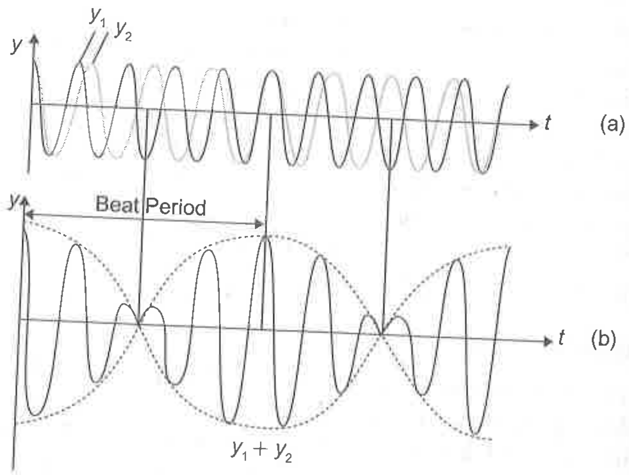


Figure 3.19

ECHO

The repetition of sound produced due to reflection by a distant extended surface like a different, hill well, building, etc., is called an echo. The effect of sound on human ear remains for approximately one-tenth of a second. If the sound is reflected back in a time less than 1/10 of a second, no echo is heard. Hence, the human ears are not able to distinguish a beat frequency of 10 Hz or more than 10 Hz.

DOPPLER'S EFFECT

When a car at rest on a road sounds its high-frequency horn and you are also standing on the road nearby, you will hear the sound of the same frequency it is sounding but when the car approaches you with its horn sounding, the pitch (frequency) of its sound seems to drop as the car passes. This phenomenon was first described by an Austrian Scientist Christine Doppler and is called the Doppler effect. He explained that when a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. Let us discuss the Doppler effect in detail for different cases.

Stationary Source and Stationary Observer

Figure 3.20 shows a stationary source of frequency n_0 which produces sound waves in air of wavelength λ_0 given as

$$\lambda_0 = \frac{V}{n_0} \qquad (V = \text{speed of sound in air})$$

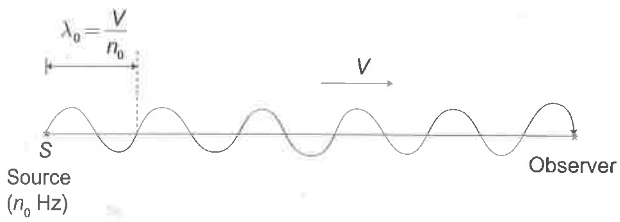


Figure 3.20

Although sound waves are longitudinal, here we represent sound waves by the transverse displacement curve as shown in Fig. 3.20 to understand the concept in a better way. As the source produces waves, these waves travel towards the stationary observer O in the medium (air) with speed V and wavelength λ_0. As observer is at rest here, he/she will observe the same wavelength λ_0 approaching him/her with a speed v so that he/she will hear the frequency n given as

$$n = \frac{V}{\lambda_0} = n_0 \quad \text{(same as that of the source)} \qquad (1)$$

This is why when a stationary observer hears the sound from a stationary source of sound, he/she detects the same frequency sound which the source is producing. Thus, no Doppler effect takes place if there is no relative motion between the source and observer.

Stationary Source and Moving Observer

Figure 3.21 shows the case when a stationary sources of frequency n_0 produces sound waves which have wavelength in air given as

$$\lambda_0 = \frac{V}{n_0}$$

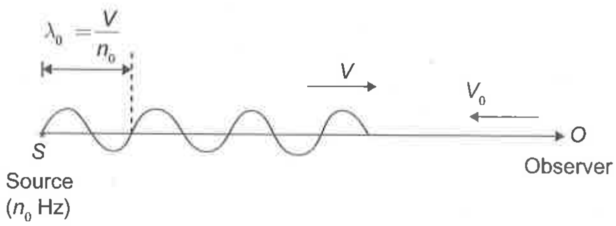


Figure 3.21

These waves travel towards the moving observer with velocity V_0 towards the source. When the sound waves approach the observer, it will receive the waves of wavelength λ_0 with speed $V + V_0$ (relative speed). Thus, the frequency of sound heard by the observer can be given as

$$\text{Apparent frequency,} \quad n_{ap} = \frac{V + V_0}{\lambda_0} = \frac{V + V_0}{\left(\frac{V}{n_0}\right)} = n_0\left(\frac{V + V_0}{V}\right) \qquad (2)$$

Similarly, we can say that if the observer is receding away from the source, the apparent frequency heard by the observer will be given as

$$n_{ap} = n_0\left(\frac{V - V_0}{V}\right) \qquad (3)$$

Moving Source and Stationary Observer

Figure 3.22 shows the situation when a moving source S of frequency n_0 produces sound waves in medium (air) and the waves travel towards the observer with velocity V.

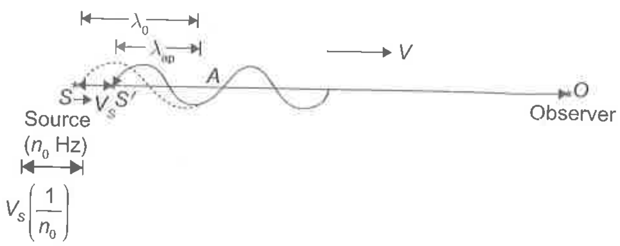


Figure 3.22

Here, if we carefully look at the initial situation, the source starts moving with velocity V_S and also starts producing waves. The period of one oscillation is $\left(\frac{1}{n_0}\right)$ s, and in this duration, the source emits one wavelength λ_0 in the direction of propagation of waves with speed V, but in this duration, the source will also move forward by a distance $V_S\left(\frac{1}{n_0}\right)$. Thus, the effective wavelength of emitted sound in air is slightly compressed by this distance as shown in Fig. 3.22. This is termed as apparent wavelength of sound in medium (air) by the moving source. This is given as

apparent wavelength, $$\lambda_{ap} = \lambda_0 - V_S\left(\frac{1}{n_0}\right) \quad (1)$$

$$= \frac{\lambda_0 n_0 - V_S}{n_0}$$

$$= \frac{V - V_S}{n_0}$$

Now this wavelength will approach the observer with speed V (O is at rest). Thus, the frequency of sound heard by the observer can be given as

apparent frequency, $$n_{ap} = \frac{V}{\lambda_{ap}}$$

$$= \frac{V}{(V - V_S)/n_0}$$

$$= n_0\left(\frac{V}{V - V_S}\right) \quad (2)$$

Similarly, if the source is receding away from the observer, the apparent wavelength emitted by the source in air towards the observer will be slightly expanded and the apparent frequency heard by the stationary observer can be given as

$$n_{ap} = n_0\left(\frac{V}{V + V_S}\right) \quad (3)$$

Moving Source and Moving Observer

Let us consider the situation when both the source and the observer are moving in the same direction as shown in Fig. 3.23 at speeds V_S and V_0, respectively.

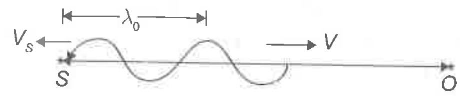


Figure 3.23

In this case, the apparent wavelength emitted by the source behind it is given as

$$\lambda_{ap} = \frac{V + V_S}{n_0}$$

Now this wavelength will approach the observer at relative speed $V + V_0$. Thus, the apparent frequency of sound heard by the observer is given as

$$n_{ap} = \frac{V + V_0}{\lambda_{ap}} = n_0\left(\frac{V + V_0}{V + V_S}\right) \quad (1)$$

By looking at the expression of apparent frequency given by the above equation, we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to a moving source as

$$n_{ap} = n_0\left[\frac{V \pm V_0}{V \mp V_S}\right] \quad (2)$$

Here, + and – signs are chosen according to the direction of motion of source and observer. The sign convention related to the motion direction can be stated as

1. For both source and observer V_0 and V_S are taken in equation with –ve sign if they are moving in the direction of $\vec{V}$, i.e., the direction of propagation of sound from source to observer.
2. For both source and observer V_0 and V_S are taken in Eq. (2) with +ve sign if they are moving in the direction opposite to $\vec{V}$, i.e., opposite to the direction of propagation of sound from source to observer.

Doppler Effect in Reflected Sound

When a car is moving towards a stationary wall as shown in Fig. 3.24, if the car sounds a horn, wave travels towards the wall and is reflected from the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch. If we wish to measure the frequency of reflected sound, then the problem must be handled in two steps.

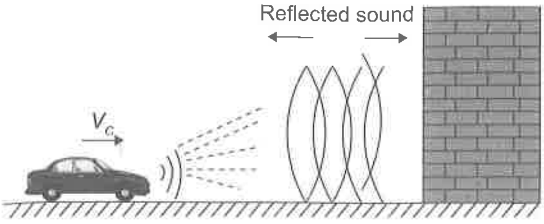


Figure 3.24

First, we treat the stationary wall as stationary observer and car as a moving source of sound of frequency n_0. In this case, the frequency received by the wall is given as

$$n_1 = n_0\left(\frac{V}{V - V_c}\right) \qquad (1)$$

Now the wall reflects this frequency and behaves like a stationary source of sound of frequency n_1 and the car (driver) behaves like a moving observer with velocity V_c. Here, the apparent frequency heard by the car driver can be given as

$$n_{ap} = n_1\left(\frac{V + V_c}{V}\right)$$

$$= n_0\left(\frac{V}{V - V_c}\right) \times \left(\frac{V + V_c}{V}\right)$$

$$= n_0\left(\frac{V + V_c}{V - V_c}\right) \qquad (2)$$

The same problem can also be solved in a different manner by using method of soundimages. In this procedure, we assume the image of the sound source behind the reflector. In the previous example, we can explain this situation as shown in Fig. 3.25.

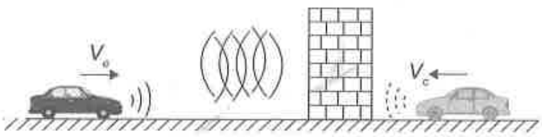


Figure 3.25

Here, we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming towards it with velocity V_c. Now the frequency of sound heard by the car driver can directly be given as

$$n_{ap} = n_0\left[\frac{V + V_c}{V - V_c}\right] \qquad (3)$$

This method of images for solving problems of Doppler effect is very convenient but is used only for velocities of source and observer, which are very small compared to the speed of sound and it should not be used frequently when the reflector of sound is moving.

Doppler's Effect for Accelerated Motion

For the case of a moving source and a moving observer, we known the apparent frequency the observer can be given as

$$n_{ap} = n_0\left[\frac{V \pm V_0}{V \mp V_S}\right] \qquad (4)$$

Here, V is the velocity of sound and V_0 and V_S are the velocities of the observer and source, respectively.

When a source of observer has accelerated or retarded motion, then in Eq. (4), we use that value of V_0 at which the observer receives the sound and for source, we use that value of V_S at which it has emitted the wave.

The alternative method of solving this case is by the traditional method of compressing or expanding the wavelength of sound by motion of source and using relative velocity of sound with respect to the observer

Doppler's Effect When the Source and Observer Are Not in the Same Line of Motion

Consider the situation shown in Fig. 3.26. Two cars 1 and 2 are moving along perpendicular roads at speeds V_1 and V_2. When car 1 sounds a horn of frequency n_0, it emits sound in all directions and say car 2 is at the position as shown in Fig. 3.26, when it receives the sound. In such cases, we use velocity components of the cars along the line joining the source and the observer. Thus, the apparent frequency of sound heard by car 2 can be given as

$$n_{ap} = n_0\left[\frac{V + V_2\cos\theta_2}{V - V_1\cos\theta_1}\right]$$

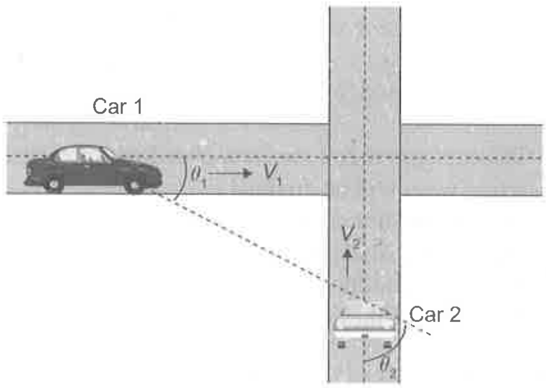


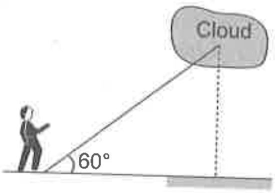
Figure 3.26

1. Doppler effect for circular motion.
2. Doppler effect of light $\Delta\lambda = \pm\lambda V$ (Read and Blue shift).

EXERCISES

JEE Main

1. The elevation of a cloud is 60° above the horizon. A thunder is heard 8 s after the observation of lighting. The speed of sound is 330 ms^{-1}. The vertical height of cloud from ground is



(A) 2826 m (B) 2682 m
(C) 2286 m (D) 2068 m

2. The ratio of speed of sound in neon to that in water vapours at any temperature (when molecular weight of neon is 2.02×10^{-2} kg mol^{-1} and for water vapours is 1.8×10^{-2} kg mol^{-1})
(A) 1.06 (B) 1.60
(C) 6.10 (D) 15.2

3. A Firecracker exploding on the surface of a lake is heard as two sounds a time interval t apart by a man on a boat close to water surface. Sound travels with a speed u in water and a sped v in air. The distance from the exploding firecracker to the boat is
(A) $\frac{uvt}{u+v}$ (B) $\frac{t(u+v)}{uv}$
(C) $\frac{t(u-v)}{uv}$ (D) $\frac{uvt}{u-v}$

4. The energy per unit area associated with a progressive sound wave will be doubled if
(A) the amplitude of the wave is doubled
(B) the amplitude of the wave is increased by 50%
(C) the amplitude of the wave is increased by 41%
(D) None of these

5. A sound level I is greater by 3.0103 dB from another sound of intensity $10n$ Wcm^{-2}. The absolute value of intensity of sound level I in Wm^{-2} is
(A) 2.5×10^{-4} (B) 2×10^{-4}
(C) 2.0×10^{-2} (D) 2.5×10^{-2}

6. A wave travels uniformly in all directions from a point source in an isotropic medium. The displacement of the medium at any point at a distance r from the source may be represented by (A is a constant representing strength of source)
(A) $[A/\sqrt{r}]\sin(kr - \omega t)$ (B) $[A/r]\sin(kr - \omega t)$
(C) $[Ar]\sin(kr - \omega t)$ (D) $[A/r^2]\sin(kr - \omega t)$

7. How many times more intense is 90 dB sound than 40 dB sound?
(A) 5 (B) 50
(C) 500 (D) 10^5

8. When two waves with same frequency and constant phase difference interfere,
(A) there is a gain of energy
(B) there is a loss of energy
(C) the energy is redistributed and the distribution changes with time
(D) the energy is redistributed and the distribution remains constant in time

9. Sound waves from a tuning fork F reach a point P by two separate routes FAP and FBP (when FBP is greater than FAP by 12 cm there is silence at P). If the difference is 24 cm the sound becomes maximum at P but at 36 cm there is silence again and so on. If the velocity of sound in air is 330 ms^{-1}, the least frequency of tuning fork is
(A) 1537 Hz (B) 1735 Hz
(C) 1400 Hz (D) 1375 Hz

10. S_1 and S_2 are two sources of sound emitting sine waves. The two sources are in phase. The sound emitted by the two sources interfere at point F. The waves of wavelength

(A) 1 m will result in constructive interference
(B) $\frac{2}{3}$ m will result in constructive interference
(C) 4 m will result in destructive interference
(D) All the above

11. The ratio of intensities between two coherent sound sources is 4:1 The difference of loudness in dB

between maximum and minimum intensities when they interfere in space is
(A) 10 log 2 (B) 20 log 3
(C) 10 log 3 (D) 20 log 2

12. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:
(A) 10/9 cm (B) 1 cm
(C) 1/2 cm (D) 5/9 cm

13. Two waves of sound having intensities I and $4I$ interfere to produce interference pattern. The phase difference between the waves $\frac{\pi}{2}$ is at point A and π at point B. Then the difference between the resultant intensities at A and B is
(A) $2I$ (B) $4I$
(C) $5I$ (D) $7I$

14. A cylindrical tube, open at one end and closed at the other, is in acoustic unison with an external source of frequency held at the open end of the tube, in its fundamental note. Then
(A) the displacement wave from the source gets reflected with a phase change of π at the closed end
(B) the pressure wave from the source get reflected without a phase change at the closed end
(C) the wave reflected from the closed end again gets reflected at the open end
(D) All of these

15. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particle has maximum amplitude of vibration is (velocity of sound in air is 330 m/s)
(A) 0.125 m (B) 0.5 m
(C) 0.25 m (D) 2 m

16. At the closed end of an organ pipe,
(A) the displacement is zero
(B) the displacement is maximum
(C) the wave pressure is zero
(D) None of these

17. An open organ pipe of length L vibrates in its fundamental mode. The pressure variation is maximum
(A) at the two ends
(B) at the middle of the pipe
(C) at distance $L/4$ inside the ends
(D) at distance $L/8$ inside the ends

18. The effect of making a hole exactly at ($1/3^{rd}$) of the length of the pipe from its closed end is such that:
(A) its fundamental frequency is an octave higher than the open pipe of same length
(B) its fundamental frequency is thrice that before making a hole
(C) the fundamental alone is changed while the harmonics expressed as ratio of fundamentals remain the same
(D) All the above

19. An open organ pipe of length L vibrates in second harmonic mode. The pressure vibration is maximum
(A) at the two ends
(B) at a distance $L/4$ from either end inside the tube
(C) at the mid-point of the tube
(D) none of these

20. An open organ pipe of length l is sounded together with another organ pipe of length $l + x$ in their fundamental tones ($x << l$). The beat frequency heard will be (speed of sound is v)
(A) $\frac{vx}{4l^2}$ (B) $\frac{vl^2}{2x}$
(C) $\frac{vx}{2l^2}$ (D) $\frac{vx^2}{2l}$

21. A sufficiently long close organ pipe has a small hole at its bottom. Initially the pipe is empty. Water is poured into the pipe at a constant rate. The fundamental frequency of the air column in the pipe
(A) continuously increasing
(B) first increases and them becomes constant
(C) continuously decreases
(D) first decreases and them become constant

22. A tuning fork of frequency 340 Hz is vibrated just above a cylindrical tube of length 120 cm. Water is slowly poured in the tube. If the speed of sound is 340 ms^{-1}, then the minimum height of water required for resonance is
(A) 95 cm (B) 75 cm
(C) 45 cm (D) 25 cm

23. An organ pipe P_1 closed at one end vibrating in its first overtone. Another pipe P_2 open at both ends is

vibrating in its third overtone. They are in a resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is:

(A) 8/3 (B) 3/8
(C) 1/2 (D) 1/3

24. A pipe's lower end is immersed in water such that the length of air column from the top open end has a certain length 25 cm. The speed of sound in air is 350 m/s. The air column is found to resonate with a tuning fork of frequency 1750 Hz. By what minimum distance should the pipe be raised in order to make the air column resonate again with the same tuning fork

(A) 7 cm (B) 5 cm
(C) 35 cm (D) 10 cm

25. In case of closed organ pipe which harmonic the p^{th} overtone will be

(A) $2p + 1$ (B) $2p - 1$
(C) $p + 1$ (D) $p - 1$

26. A closed organ pipe has length 'l'. The air in it is vibrating in the third overtone with maximum displacement amplitude 'a'. The displacement amplitude at distance $l/7$ from closed end of the pipe is

(A) 0 (B) a
(C) $a/2$ (D) none of these

27. The number of beats heard per second if there are three sources of frequencies $(n - 1)$, n and $(n + 1)$ of equal intensities sounded together is

(A) 2 (B) 1
(C) 4 (D) 3

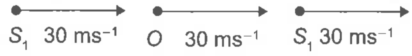
28. A tuning fork of frequency 280 Hz produces 10 beats/s when sounded with a vibrating sonometer string. When the tension in the string increases slightly, it produces 11 beats/s. The original frequency of the vibrating sonometer string is

(A) 269 Hz (B) 291 Hz
(C) 270 Hz (D) 290 Hz

29. The speed of sound in a gas, in which two waves of wavelength 1.0 m and 1.02 m produce 6 beats/s, is approximately

(A) 350 m/s (B) 300 m/s
(C) 380 m/s (D) 410 m/s

30. Consider two sound sources S_1 and S_2 having same frequency 100 Hz and the observer O located between them as shown in the figure. All the three are moving with same velocity in same direction. The beat frequency of the observer is



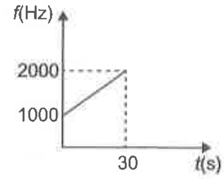
(A) 50 Hz (B) 5 Hz
(C) zero (D) 2.5 Hz

31. A source S of frequency f_0 and an observer O, moving with speeds v_1 and v_2 respectively, are moving away from each other. When they are separated by distance a ($t = 0$), a pulse is emitted by the source. This pulse is received by O at time t_1, then t_1 is equal to

(A) $\dfrac{a}{v_s + v_2}$ (B) $\dfrac{a}{v_1 + v_s}$

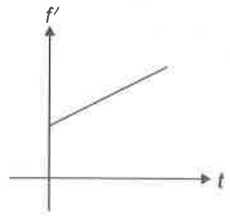
(C) $\dfrac{a}{v_s - v_2}$ (D) $\dfrac{a}{v_1 + v_2 + v_s}$

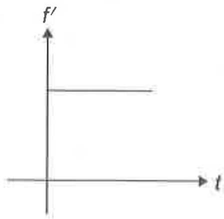
32. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is ($g = 10$ m/s^2)

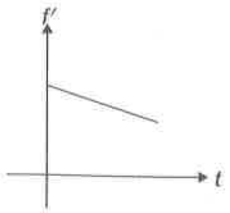


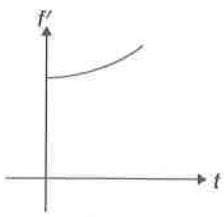
(A) 330 m/s (B) 350 m/s
(C) 300 m/s (D) 310 m/s

33. An observer starts moving with uniform acceleration 'a' towards a stationary sound source of frequency f. As the observer approaches the source, the apparent frequency f' heard by the observer varies with time t as:

(A)


(B)


(C)


(D)


34. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If the speed of sound with respect to air is C, the wavelength of sound detected by O is

(A) $\frac{C+u}{f}$ (B) $\frac{C-u}{f}$

(C) $\frac{C(C+u)}{(C-u)f}$ (D) $\frac{C}{f}$

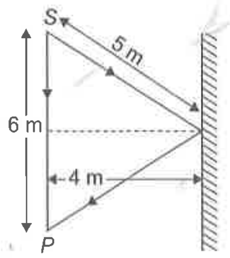
JEE Advanced

Single Correct

1. In a test of subsonic Jet flies over head at an altitude of 100 m. The sound intensity on the ground as the jet passes overhead is 160 dB. At what altitude should the plane fly so that the ground noise is not greater than 120 dB.
   (A) Above 10 km from ground
   (B) Above 1 km from ground
   (C) Above 5 km from ground
   (D) Above 8 km from ground

2. Three coherent waves of equal frequencies having amplitude 10 μm, 4 μm, and 7 μm respectively, arrive at a given point with successive phase difference of the amplitude of the resulting wave in μm is given by
   (A) 5 (B) 6
   (C) 3 (D) 4

3. A person standing at a distance of 6 m from a source of sound receives sound wave In two ways, one directly from the source and other after reflection from a rigid boundary as shown in figure. The maximum wavelength for which, the person will receive maximum sound intensity, is

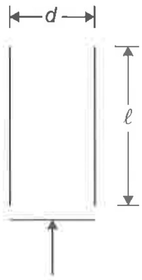


   (A) 4 m (B) $\frac{16}{3}$ m
   (C) 2 m (D) $\frac{8}{3}$ m

4. The ratio of maximum to minimum intensity due to superposition of two waves is $\frac{49}{9}$. Then the ratio of the intensity of component waves is
   (A) $\frac{25}{4}$ (B) $\frac{16}{25}$
   (C) $\frac{4}{49}$ (D) $\frac{9}{49}$

5. The displacement sound wave in a medium is given by the equation $Y = A\cos(ax + bt)$, where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. Tick the statement among the following that is incorrect.
   (A) The wavelength and frequency of the wave are $2\pi/a$ and $b/2\pi$ respectively.
   (B) The amplitude of the reflected wave is 0.8 A.
   (C) The resultant wave formed after reflection is $y = A\cos(ax + bt) + [-0.8A\cos(ax - bt)]$ and V_{max} (maximum particle speed) is 1.8 bA.
   (D) The equation of the standing wave so formed is $y = 1.6A\sin ax \cos bt$.

6. A tube of diameter d and of length ℓ unit is open at both ends. Its fundamental frequency of resonance is found to be ν_1. The velocity of sound in air is 330 m/s. One end of tube is now closed. The lowest frequency of resonance of tube is ν_2. Taking into consideration the end correction, $\frac{\nu_2}{\nu_1}$ is



   (A) $\frac{(\ell+0.6d)}{(\ell+0.3d)}$ (B) $\frac{1}{2}\frac{(\ell+0.3d)}{(\ell+0.6d)}$

(C) $\frac{1}{2}\frac{(\ell+0.6d)}{(\ell+0.3d)}$ (D) $\frac{1}{2}\frac{(d+0.3\ell)}{(d+0.6\ell)}$

7. In a closed end pipe of length 105 cm, standing waves are set up corresponding to the third overtone. What distance from the closed end, amongst the following, is a pressure Node?
(A) 20 cm (B) 60 cm
(C) 85 cm (D) 45 cm

8. A closed organ pipe of radius r_1 and an open organ pipe of radius r_2 and having same length 'L' resonate when excited with a given tuning fork. Closed organ pipe resonates in its fundamental mode whereas open organ pipe resonates in its first overtone, then
(A) $r_2 - r_1 = L$ (B) $r_2 - r_1 = L/2$
(C) $r_2 - 2r_1 = 2.5L$ (D) $2r_2 - r_1 = 2.5L$

9. First overtone frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. Further nth harmonic of closed organ pipe is also equal to the mth harmonic of open pipe, where n and m are
(A) 5, 4 (B) 7, 5
(C) 9, 6 (D) 7, 3.

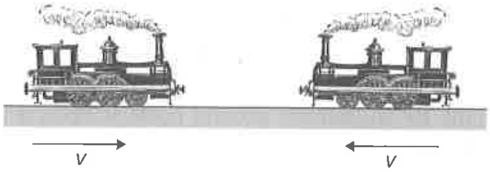
10. If I_1 and I_2 are the lengths of air column for the first and second resonance when a tuning fork of frequency n is sounded on a resonance tube, then the distance of the displacement antinode from the top end of the resonance tube is
(A) $2(l_2 - l_1)$ (B) $\frac{1}{2}(2I_1 - I_2)$
(C) $\frac{I_2 - 3I_1}{2}$ (D) $\frac{I_2 - I_1}{2}$

11. The first resonance length of a resonance tube is 40 cm and the second resonance length is 122 cm. The third resonance length of the tube will be
(A) 200 cm (B) 202 cm
(C) 203 cm (D) 204 cm

12. The tuning forks A & B produce notes of frequencies 256 Hz & 262 Hz respectively. An unknown note sounded at the same time as A produces beats. When the same note is sounded with B, beat frequency is twice as large. The unknown frequency could be
(A) 268 Hz (B) 250 Hz
(C) 260 Hz (D) none of these

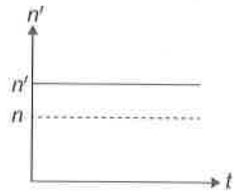
13. A closed organ pipe and an open pipe of same length produce 4 beats when they are set into vibrations simultaneously. If the length of each of them were twice their initial lengths, the number of beats produced will be
(A) 2 (B) 4
(C) 1 (D) 8

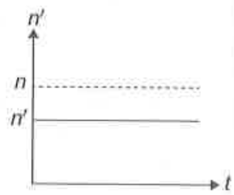
14. Two trains move towards each other with the same speed. The speed of sound is 340 ms^{-1}. If the pitch of the tone of the whistle of one when heard on the other changes by 9/8 times, then the speed of each train is

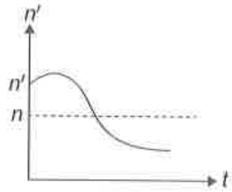


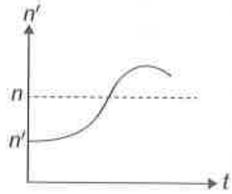
(A) 2 ms^{-1} (B) 40 ms^{-1}
(C) 20 ms^{-1} (D) 100 ms^{-1}

15. Source and observer both start moving simultaneously from origin, one along X-axis and the other along the Y-axis with speed of source equal to twice the speed of observer. The graph between the apparent frequency (n') observed by observer and time t would be (n is the frequency of the source)

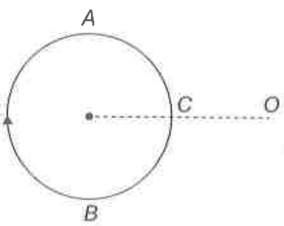
(A)


(B)


(C)


(D)


16. A small source of sound moves on a circle as shown in the figure and an observer is sitting at O. Let at v_1, v_2 and v_3 be the frequencies heard when the source is at A, B, and C respectively.



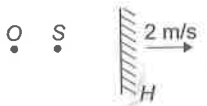
(A) $v_1 > v_2 > v_3$
(B) $v_1 = v_2 > v_3$
(C) $v_2 > v_3 > v_1$
(D) $v_1 > v_3 > v_2$

17. The frequency changes by 10% as a sound source approaches a stationary observer with constant speed v_s. What would be the percentage change in frequency as the source recedes the observer with the same speed. Given that $v_s < v$. (v = speed of sound in air)
(A) 14.3%
(B) 20%
(C) 10.0%
(D) 8.5%

18. An engine whistling at a constant frequency n_0 and moving with a constant velocity goes past a stationary observer. As the engine crosses him, the frequency of the sound heard by him changes by a factor f. The actual difference in the frequencies of the sound heard by him before and after the engine crosses him is

(A) $\frac{1}{2}n_0(1-f^2)$
(B) $\frac{1}{2}n_0\left(\frac{1-f^2}{f}\right)$
(C) $n_0\left(\frac{1-f}{1+f}\right)$
(D) $\frac{1}{2}n_0\left(\frac{1-f}{1+f}\right)$

19. A stationary sound source 's' of frequency 334 Hz and a stationary observer 'O' are placed near a reflecting surface moving away from the source with velocity 2 m/sec as shown in the figure. If the velocity of the sound waves in air is V = 330 m/sec, the apparent frequency of the echo is



(A) 332 Hz
(B) 326 Hz
(C) 334 Hz
(D) 330 Hz

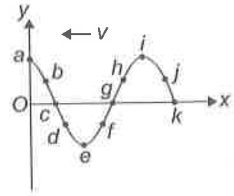
20. A sounding body of negligible dimension emitting a frequency of 150 Hz is dropped from a height. During its fall under gravity it passes near a balloon moving up with a constant velocity of 2 m/s one second after it started to fall. The difference in the frequency observed by the man in balloon just before and just after crossing the body will be (Given that, velocity of sound = 300 m/s ; g = 10 m/s^2)
(A) 12
(B) 6
(C) 8
(D) 4

21. Two sound sources each emitting sound of wavelength λ are fixed some distance apart. A listener moves with a velocity u along the line joining the two sources. The number of beats heard by him per second is

(A) $\frac{2u}{\lambda}$
(B) $\frac{u}{\lambda}$
(C) $\frac{u}{3\lambda}$
(D) $\frac{2\lambda}{u}$

Multiple Correct

The figure represents the instantaneous picture of a longitudinal harmonic wave travelling along the negative x-axis. Identify the correct statement(s) related to the movement of the points shown in the figure.



22. The points moving in the direction of wave are
(A) b
(B) c
(C) f
(D) i

23. The points moving opposite to the direction of propagation are
(A) a
(B) d
(C) f
(D) j

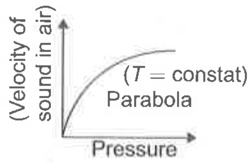
24. The stationary points are
(A) a
(B) c
(C) g
(D) k

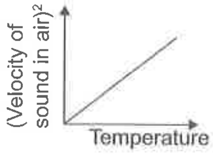
25. The maximum displaced points are
(A) a
(B) e
(C) g
(D) i

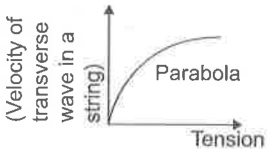
26. The points of maximum compression are
(A) c
(B) g
(C) e
(D) k

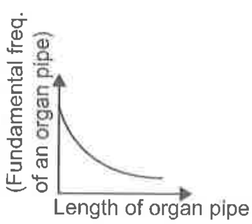
27. The points of maximum rarefaction are
(A) a
(B) e
(C) g
(D) i

28. Which of the following graphs is/are correct.

(A)


(B)


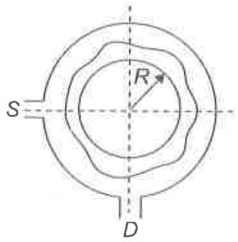
(C)


(D)


29. Which of the following statements are wrong about the velocity of sound in air :
(A) Decreases with increases in temperature
(B) Increases with decrease in temperature
(C) Decreases as humidity increases
(D) Independent of density of air

30. Two interfering waves have the same wavelength, frequency, and amplitude, They are traveling in the same direction but are 90° out of phase. Compared to the individual waves, the resultant wave will have the. same.
(A) Amplitude and velocity but different wavelength
(B) Amplitude and wavelength but different velocity
(C) Wavelength and velocity but different amplitude
(D) Amplitude and frequency but different velocity

Question No. 31 to 35



A narrow tube is bent in the form of a circle of radius R, as shown in the figure. Two small holes S and D are made in the tube at the positions right angle to each other. A source placed at S generated a wave of intensity I_0 which is equally divided into two parts: One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at the point D where a detector is placed

31. If a maxima is formed at the detector, then the magnitude of wavelength λ of the wave produced is given by
(A) πR (B) $\frac{\pi R}{2}$
(C) $\frac{\pi R}{4}$ (D) $\frac{2\pi R}{3}$

32. If the minima is formed at the detector, then the magnitude of wavelength λ of the wave produced is given by
(A) $2\pi R$ (B) $\frac{3\pi R}{2}$
(C) $\frac{2\pi R}{3}$ (D) $\frac{2\pi R}{5}$

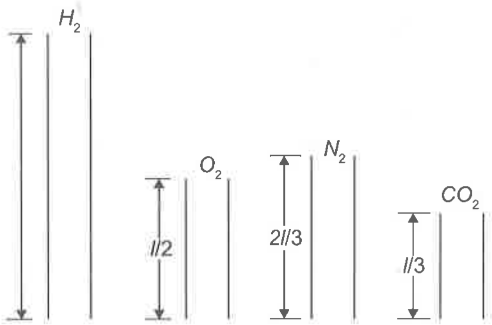
33. The maximum intensity produced at D is given by
(A) $4I_0$ (B) $2I_0$
(C) I_0 (D) $3I_0$

34. The maximum value of λ to produce a maxima at D is given by
(A) πR (B) $2\pi R$
(C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

35. The maximum value of λ to produce a minima at D is given by
(A) πR (B) $2\pi R$
(C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

36. The second overtone of an open organ pipe A and a closed pipe B have the same frequency at a given temperature. If follows that the ratio of the
(A) length of A and B is 4:3
(B) fundamental frequencies of A & B is 5:6
(C) lengths of B to that of A is 5:6
(D) frequencies of first overtone of A & B is 10:9

37. Four open organ pipes of different lengths and different gases at same temperature as shown in figure. Let f_A, f_B, f_C and f_D be their fundamental frequencies, then [Take $\gamma_{co_2} = 7/5$]



(A) $f_A / f_B = \sqrt{2}$ (B) $f_B / f_C = \sqrt{72/28}$
(C) $f_C / f_D = \sqrt{11/28}$ (D) $f_D / f_A = \sqrt{76/11}$

38. A gas is filled in an organ pipe and it is sounded with an organ pipe in fundamental mode. Choose the correct statement(s). (T = constant)
(A) If gas is changed from H_2 to O_2, the resonant frequency will increase.
(B) If gas is changed from O_2 to N_2, the resonant frequency will increase.
(C) If gas is changed from N_2 to He, the resonant frequency will decrease.
(D) If gas is changed from He to CH_4, the resonant frequency will decrease.

39. A closed organ pipe of length 1.2 m vibrates in its first overtone mode. The pressure variation is maximum at
(A) 0.8 m from the open end
(B) 0.4 m from the open end
(C) at the open end
(D) 1.0 m from the open end

40. For a certain organ pipe three successive resonance frequencies are observed at 425 Hz, 595 Hz and 765 Hz, respectively. If the speed of sound in air is 340 m/s, then the length of the pipe is
(A) 2.0 m (B) 0.4 m
(C) 1.0 m (D) 0.2 m

41. In an organ pipe whose one end is at $x = 0$, the pressure is expressed by $p = p_0 \cos\frac{3\pi x}{2}\sin 300\pi t$, where x is in metre and t in second. The organ pipe can be
(A) closed at one end, open at another with length = 0.5 m
(B) open at both ends, length = 1 m
(C) closed at both ends, length = 2 m
(D) closed at one end, open at another with length = $\frac{2}{3}$ m

42. Two whistles A and B each have a frequency of 500 Hz. A is stationary and B is moving towards the right (away from A) at a speed of 50 m/s. An observer is between the two whistles moving towards the right with a speed of 25 m/s. The velocity of sound in air is 350 m/s. Assume there is no wind. Then which of the following statements are true?
(A) The apparent frequency of whistle B as heard by A is 444 Hz approximately.
(B) The apparent frequency of whistle B as heard by the observer is 469 Hz approximately.
(C) The difference in the apparent frequencies of A and B as heard by the observer is 4.5 Hz.
(D) The apparent frequencies of the whistles of each other as heard by A and B are the same.

43. A source of sound moves towards an observer
(A) the frequency of the source is increased
(B) the velocity of sound in the medium is increased
(C) the wavelength of sound in the medium towards the observer is decreased
(D) the amplitude of vibration of the particles is increased

44. A car moves towards a hill with speed v_c. It blows a horn of frequency f which is heard by an observer following the car with speed v_0. The speed of sound in air is v.
(A) The wavelength of sound reaching the hill is $\frac{v}{f}$.
(B) The wavelength of sound reaching the hill is $\frac{v - v_c}{f}$.
(C) The beat frequency observed by the observer is $\left(\frac{v + v_0}{v - v_c}\right) f$.
(D) The beat frequency observed by the observer is $\frac{2v_c(v + v_0)f}{v^2 - v_c^2}$.

JEE Advanced

Level I

1. A sound wave of frequency 100 Hz is travelling in air. The speed of sound in air is 350 m/s. (A) By how much is the phase changed at a given point in 2.5 ms? (B) What is the phase difference at a given instant between two points separated by a distance of 10.0 cm along the direction of propagation?

2. The equation of a travelling sound wave is $y = 6.0 \sin(600t - 1.8x)$, where y is measured in 10^{-5} m, t in seconds and x in metres. (A) Find the ratio of the displacement amplitude of the particles to the wavelength of the wave. (B) Find the ratio of the velocity amplitude of the particles to the wave speed.

3. A man stands before a large wall at a distance of 100.0 m and claps his hands at regular intervals, in such a way that echo of a clap merges with the next clap. If he has to clap 5 times during every 3 s, find the velocity of sound in air.

4. Calculate the speed of sound in oxygen from the following data. The mass of 22.4 litre of oxygen at STP ($T = 273K$ and $p = 1.0 \times 10^5$ N/m^2) is 32 g, the molar heat capacity of oxygen at constant volume is $C_v = 2.5R$ and that at constant pressure is $C_p = 3.5R$.

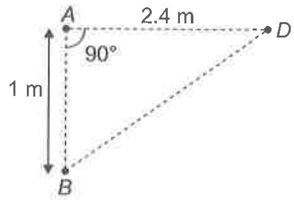
5. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is c. Find the velocity of sound in the gas.

6. Find the intensity of sound wave whose frequency is 250 Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is 1 kg/m^3, and the bulk modulus of elasticity of the medium is 400 N/m^2.

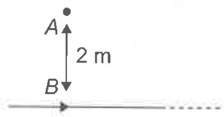
7. Two identical sounds A and B reach a point in the same phase. The resultant sound is C. The loudness of C is n dB higher than the loudness of A. Find the value of n.

8. The loudness level at a distance R from a long linear source of sound is found to be 40 dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level and amplitude at a point located at a distance '$10R$' from the source.

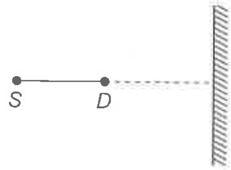
9. Two point sound sources A and B each of power 25π W and frequency 850 Hz are 1 m apart.
   (A) Determine the phase difference between the waves emitting from A and B received by detector D as in figure. B
   (B) Also determine the intensity of the resultant sound wave as recorded by detector D. Velocity of sound = 340 m/s.



10. Two identical loudspeakers are located at points A and B, 2 m apart. The loudspeakers are driven by the same amplifier. A small detector is moved out from point B along a line perpendicular to the line connecting A and B. Taking speed of sound in air as 332 m/s. Find the frequency below which there will be no position along the line BC at which destructive interference occurs.



11. A source of sound S and a detector D are placed at some distance from one another. A big cardboard is placed near the detector and perpendicular to the line SD as shown in figure. It is gradually moved away and it is found that the intensity changes from a maximum to a minimum as the board is moved through a distance of 20 cm. Find the frequency of the sound emitted. Velocity of sound in air is 336 m/s.



12. Sound of wavelength λ passes through a Quincke's tube, which is adjusted to give a maximum intensity I_0. Find the distance through the sliding tube should be moved to give an intensity $I_0/2$.

13. The stationary wave $y = 2a \sin kx \cos wt$ in a closed organ pipe is the result of the superposition of $y = a \sin(wt - kx)$.

14. The equation of a longitudinal standing wave due to superposition of the progressive waves produced by Two sources of sound is $s = -20\sin 10\pi x \sin 100\pi t$ where s is the displacement from mean position measured in mm, x is in metres and t in seconds. The specific gravity of the medium is 10^{-3}. Density of water $= 10^3$ kg/m^3. Find:
(A) Wavelength, frequency and velocity of the progressive waves.
(B) Bulk modulus of the medium and the pressure amplitude .
(C) Minimum distance between pressure antinode and a displacement antinode.
(D) Intensity at the displacement nodes.

15. A tube 1.0 m long is closed at one end. A wire of length 0.3 m and mass 1×10^{-2} kg is stretched between two fixed ends and is placed near the open end. When the wire is plucked at its mid-point the air column resonates in its first overtone. Find the tension in the wire if it vibrates in its fundamental mode.[V_{sound} = 330 m/s]

16. A closed organ pipe of length $\ell = 100$ cm is cut into two unequal pieces. The fundamental frequency of the new closed organ pipe piece is found to be same as the frequency of first overtone of the open organ pipe piece. Determine the length of the two pieces and the fundamental tone of the open pipe piece. Take velocity of sound = 320 m/s.

17. Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below v_0 = 1250 Hz. The length of the pipe is $\ell = 85$ cm. The velocity of sound is $v = 340$ m/s.
Consider the two cases:
(A) The pipe is closed from one end
(B) The pipe is opened from both ends

The open ends of the pipe are assumed to be the antinodes of displacement.

18. The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2:1. Find the ratio of the length of pipe to the length of string.

19. In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level act as a closed organ pipe. A vibrating tuning fork is held above the open end, first and the second resonances occur when the water level is 24.1 cm and 74.1 cm respectively below the open end. Find the diameter of the tube. [Hint : end correction is $0.3d$]

20. An open organ pipe filled with air has a fundamental frequency 500 Hz. The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbon dioxide to be 330 and 264 m/s respectively.

21. Two identical piano wires have a fundamental frequency of 600 vib/s, when kept under the same tension. What fractional increase in the tension of one wire will lead to the occurrence of 6 beats/s when both wires vibrate simultaneously?

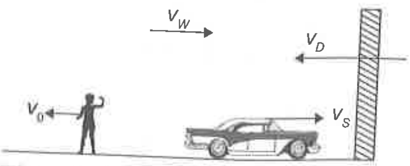
22. A metal wire of diameter 1 mm, is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vibrating in its fundamental frequency and a Vibrating tuning for k together produces 5 beats/s. The tension in the wire is then reduced to 81 N. When the two are excited, beats are again at the same rate. Calculate:
(A) the frequency of the fork,
(B) the density of the material of the wire.

23. Two stationary sources A and B are sounding notes of frequency 680 Hz. An observer moves from A to B with a constant velocity u. If the speed of sound is 340 ms^{-1}, what must be the value of u so that he hears 10 beats/s?

24. Tuning fork A when sounded with a tuning fork B of frequency 480 Hz gives 5 beats/s. When the prongs of A are loaded with wax, it gives 3 beats/s. Find the original frequency of A.

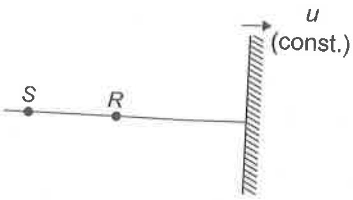
25. A, B and C are three tuning forks. Frequency of A is 350 Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax in put on A beat frequency between A and B is 2 Hz and between A and C is 6 Hz. Then, find the frequency of B and C respectively.

26. S, O & W represent source of sound (of frequency f), observer & wall respectively. V_0, V_s, V_D, V are velocity of observer, source, wall and sound (in still air) respectively. V_W is the velocity of wind. They are moving as shown in the figure. Find:



(i) The wavelength of the waves coming towards the observer from source.
(ii) The wavelength of the waves incident on the wall.
(iii) The wavelength of the waves coming towards observer from the wall.
(iv) Frequency of the waves (as detected by O) coming from the wall after reflection.

27. S is source R is receiver. R and S are at rest. Frequency of sound from S is f. Find the beat frequency registered by R. Velocity of sound is u.



28. A car moving towards a vertical wall sounds a horn. The driver hears that the sound of the horn reflected from the cliff has a pitch half-octave higher than the actual sound. Find the ratio of the velocity of the car and the velocity of sound.

29. The loudness level at a distance R from a long linear source of sound is found to be 40 dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level and the amplitude at a point located at a distance '$10R$' from the source.

30. A fixed source of sound emitting a certain frequency appears as f_a when the observer is approaching the source with speed v and frequency f_r when the observer recedes from the source with the same speed. Find the frequency of the source.

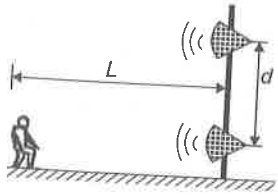
31. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Velocity of sound = 330 m/s.

Level II

1. The displacement of the medium in a sound wave is given by the equation, $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave.
   (A) What are the wavelength and frequency of the incident wave?
   (B) Write the equation for the reflected wave.
   (C) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

2. (A) A standing wave in second overtone is maintained in a open organ pipe of length l. The distance between consecutive displacement node and pressure node is ______.
   (B) Two consecutive overtones produced by a narrow air column closed at one end and open at the other are 750 Hz and 1050 Hz. Then the fundamental frequency from the column is ______.
   (C) A standing wave of frequency 1100 Hz in a column of methane at 20°C produces nodes that are 20 cm apart. What is the ratio of the heat capacity at constant pressure to that at constant volume.

3. Two speakers are driven by the same oscillator with frequency of 200 Hz. They are located 4 m apart on a vertical pole. A man walks straight towards the lower speaker in a direction perpendicular to the pole, as shown in figure.



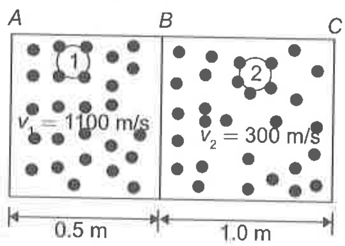
(A) How many times will he hear a minimum in sound intensity?
(B) How far is he from the pole at these moments?

Take the speed of sound to be 330 m/s, and ignore any sound reflections coming off the ground.

4. A cylinder *ABC* consists of two chambers 1 and 2 which contains two different gases. The wall *C* is rigid but the walls *A* and *B* are thin diaphragms. A vibrating tuning fork approaches the wall *A* with velocity u = 30 m/s and air columns in chamber 1 and 2 vibrates with minimum frequency such that there is node (displacement) at *B* and antinode (displacement) at *A*. Find

(i) the fundamental frequency of air column,
(ii) the frequency of tuning fork.

Assume thet the velocity of sound in the first and second chamber be 1100 m/s and 300 m/s, respectively. Velocity of sound in air is 330 m/s.



5. A source emits sound waves of frequency 1000 Hz. The source moves to the right with a speed of 32 m/s relative to ground. On the right a reflecting surface moves towards left with a speed of 64 m/s relative to the ground. The speed of sound in air is 332 m/s. Find

(A) the wavelength of sound in air incident on reflecting surface,
(B) the number of waves arriving per second which meet the reflecting surface,
(C) the speed of reflected waves,
(D) the wavelength of reflected waves.

6. A supersonic jet plane moves parallel to the ground at speed v = 0.75 mach (1 mach = speed of sound). The frequency of its engine sound is ν_0 = 2kHz and the height of the jat plane is h = 1.5 km. At some instant, an observer on the ground hears a sound of frequency $\nu = 2\nu_0$. Find the instant prior to the instant of hearing when the sound wave received by the observer was emitted by the jet plane. Velocity of sound wave in the condition of observer = 340 m/s.

7. A train of length l is moving with a constant speed v along a circular track of radius R, The engine of the train emits a whistle of frequency f. Find the frequency heard by a guard at the rear end of the train. Make suitable assumption.

8. A bullet travels horizontally at 660 m/s at a height of 5 m from a man. How far is the bullet from the man when he hears its whistle? Velocity of sound in air = 340 m/s.

Previous Year Questions

JEE Main

1. A tuning fork arrangement (pair) produces 4 beats/s with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/s. The frequency of the unknown fork is **[AIEEE 2002]**

(A) 286 cps (B) 292 cps
(C) 294 cps (D) 288 cps

2. Tube *A* has both ends open while tube *B* has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes *A* and *B* is **[AIEEE 2002]**

(A) 1:2 (B) 1:4
(C) 2:1 (D) 4:1

3. When temperature increases, the frequency of a tuning fork **[AIEEE 2002]**

(A) increases
(B) decreases
(C) remains same
(D) increases or decreases depending on the material

4. A tuning fork of known frequency 256 Hz makes 5 beats/s with the vibrating string of a piano. The beat frequency decreases to 2 beats/s when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[AIEEE 2003]**
(A) (256 + 2)Hz (B) (256 – 2)Hz
(C) (256 – 5)Hz (D) (256 + 5)Hz

5. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency? **[AIEEE 2005]**
(A) zero (B) 0.5%
(C) 5% (D) 20%

6. When two tuning forks (forks 1 and 2) are sounded simultaneously, 4 beats/s are heard. Now, some tape is attached on the prong of fork 2. When the tuning forks are sounded again, 6 beats/s are heard. If the frequency of fork is 200 Hz, then what was the original frequency of fork 2? **[AIEEE 2005]**
(A) 200 Hz (B) 202 Hz
(C) 196 Hz (D) 204 Hz

7. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v upto which he can hear the whistle is **[AIEEE 2006]**
(A) $15\sqrt{2}$ms^{-1} (B) $15/\sqrt{2}$ms^{-1}
(C) 15 ms^{-1} (D) 30 ms^{-1}

8. A sound absorber attenuates the sound level by 20 db. The intensity decreases by a factor of **[AIEEE 2007]**
(A) 1000 (B) 10000
(C) 10 (D) 100

9. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer she measures the column length to be x cm for the second resonance. Then **[AIEEE 2008]**
(A) $18 > x$ (B) $x > 54$
(C) $54 > x > 36$ (D) $36 > x > 18$

10. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal) **[AIEEE 2008]**
(A) 1420 ms^{-1} (B) 500 ms^{-1}
(C) 650 ms^{-1} (D) 330 ms^{-1}

11. Three sound waves of equal amplitudes have frequencies (v – 1), v and (v + 1). They superpose to give beat. The number of beats produced per second will be **[AIEEE 2009]**
(A) 4 (B) 3
(C) 2 (D) 1

12. A motor cycle starts from rest & accelerates along a straight path at 2 ms^{-2}. At the starting point of the motorcycle, there is a stationary electric siren. How far has the motorcycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (speed of sound = 330 ms^{-1}) **[AIEEE 2009]**
(A) 49 m (B) 98 m
(C) 147 m (D) 196 m

13. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[AIEEE 2012]**
(A) f (B) $\frac{f}{2}$
(C) $\frac{3f}{4}$ (D) $2f$

14. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? **[JEE Main 2013]**
(A) 200.5 Hz (B) 770 Hz
(C) 188.5 Hz (D) 178.2 Hz

15. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE Main 2014]**
(A) 6 (B) 4
(C) 12 (D) 8

16. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency

heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to **[JEE Main 2015]**

(A) 18% (B) 24%
(C) 6% (D) 12%

JEE Advanced

1. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz, while approaching the same siren. The ratio of the velocity of train B to that of train A is **[JEE 2002(Scr), 3]**

(A) 242/252 (B) 2
(C) 5/6 (D) 11/6

2. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of malar mass M_A. Pipe B is open at one end and closed at the other end and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[JEE 2002, 3 + 2]**

(A) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
(B) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

3. A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats, then his velocity is **[JEE 2003 (Scr)]**

(A) 22 m/s (B) 24 m/s
(C) 20 m/s (D) 18 m/s

4. A cylindrical tube when sounded with a tuning for k gives, first resonance when length of air column is 0.1 and gives second resonance when the length of air column is 0.35 m. Then end correction is **[JEE 2003 (Scr)]**

(A) 0.025 m (B) 0.020 m
(C) 0.018 m (D) 0.012 m

5. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length, 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. **[JEE 2003]**

6. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is **[JEE 2004 (Scr)]**

(A) $\frac{L}{3}$ (B) $\frac{4L}{3}$

(C) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (D) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

7. A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 m/s and in air is 300 m/s. The frequency of sound recorded by an observer who is standing in air is **[JEE 2004 (Scr)]**

(A) 200 Hz (B) 3000 HZ
(C) 120 Hz (D) 600 Hz

8. In a resonance column method, resonance occurs at two successive level of l_1 = 30.7 cm and l_2 = 63.2 cm using a tuning fork of f = 512 Hz. What is the maximum error in measuring speed of sound using relations $v = f\lambda$ and $\lambda = 2(l_2 - l_1)$ **[JEE 2005 (Sc)]**

(A) 256 cm/sec (B) 92 cm/sec
(C) 128 cm/sec (D) 102.4 cm/sec

9. A whistling train approaches a junction. An observer standing at junction observers the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed sound = 300 m/s). **[JEE 2005]**

Question No. 10 to 12

Two plane harmonic sound waves are expressed by the equations

$$y_1(x, t) = A\cos(\pi x - 100\pi t)$$
$$y_2(x, t) = A\cos(0.46\pi x - 92\pi t)$$

(All parameters are in MKS.) **[JEE 2006]**

10. How many times does an observer hear maximum intensity in one second?

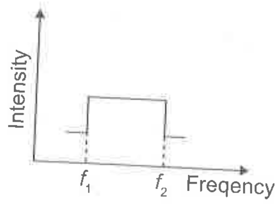
(A) 4
(B) 10
(C) 6
(D) 8

11. What is the speed of the second?
(A) 200 m/s
(B) 180 m/s
(C) 192 m/s
(D) 96 m/s

12. At $x = 0$ how many times the amplitude of $y_1 + y_2$ is zero in one second?
(A) 192
(B) 48
(C) 100
(D) 96

Question No. 13 to 15

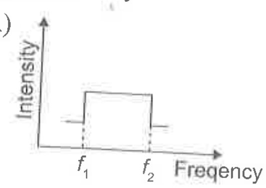
Two trains A and B are moving with speeds 20 m/s and 30 m/s, respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle. **[JEE 2007]**

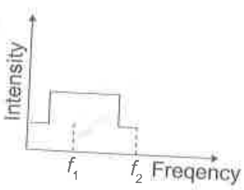


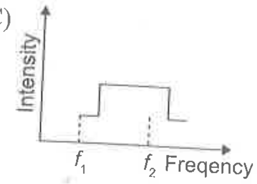
Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency−lowest frequency) is thus 320 Hz. The speed of sound in air is 340 m/s.

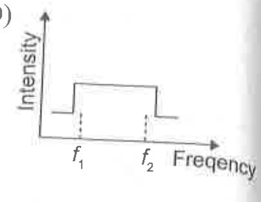
13. The speed of sound of the whistle is
(A) 340 m/s for passengers in A and 310 m/s for passengers in B.
(B) 360 m/s for passengers in A and 310 m/s for passengers in B.
(C) 310 m/s for passengers in A and 360 m/s for passengers in B.
(D) 340 m/s for passengers in both the trains.

14. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(A)


(B)


(C)


(D)


15. The spread of frequency as observed by the passengers in train B is
(A) 310 Hz
(B) 330 Hz
(C) 350 Hz
(D) 290 Hz

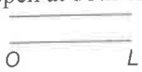
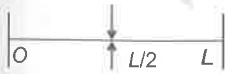
16. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **[JEE 2008]**
(A) 344
(B) 336
(C) 117.3
(D) 109.3

17. A student performed the experiment to measure the speed of sound in air using resonance air column method. Two resonances in the air column ware obtained by lowering the water level. The resonance with the shorter air column is the first resonance and that with the longer air column is the second resonance. Then, **[JEE 2009]**
(A) the intensity of the sound heard at the first resonance was more than that at the second resonance
(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(D) the length of the air column at the first resonance was somewhat shorter than $\frac{1}{4}$ th of the wavelength of the sound in air

18. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2 % of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. **[JEE 2010]**

19. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is: **[JEE 2010]**
(A) 5 g (B) 10 g
(C) 20 g (D) 40 g

20. Column *I* shows four systems, each of the same length *L*, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statement given in column II describing the nature and wavelength of the standing waves. **[JEE 2011]**

Column I	Column II
(A) Pipe closed at one end 	(P) Longitudinal waves
(B) Pipe open at both ends 	(Q) Transverse waves
(C) Stretched wire clamped at both ends 	(R) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(S) $\lambda_f = 2L$
	(T) $\lambda_f = 4L$

21. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **[JEE 2011]**
(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHz

22. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, **[JEE 2012]**
(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(B) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(C) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

23. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are) **[JEE 2013]**
(A) If the wind blows from the observer to the source, $f_2 > f_1$.
(B) If the wind blows from the source to the observer, $f_2 > f_1$.
(C) If the wind blows from observer to the source, $f_2 < f_1$.
(D) If the wind blows from the source to the observer, $f_2 < f_1$.

24. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is
(Useful information: $\sqrt{167RT} = 640\,J^{1/2}$ $mole^{-1/2}$, $\sqrt{140RT} = 590\,J^{1/2}$ $mole^{-1/2}$. The molar masses M is grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

[JEE Advanced 2014]

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

ANSWER KEYS

Exercises

JEE Main

1. C	**2.** A	**3.** D	**4.** D	**5.** B	**6.** B	**7.** D	**8.** D	**9.** D	**10.** D
11. B	**12.** B	**13.** B	**14.** D	**15.** A	**16.** A	**17.** B	**18.** D	**19.** B	**20.** C
21. B	**22.** C	**23.** B	**24.** D	**25.** A	**26.** B	**27.** B	**28.** D	**29.** B	**30.** C
31. C	**32.** C	**33.** A	**34.** A						

JEE Advanced

1. A	**2.** A	**3.** A	**4.** A	**5.** B	**6.** C	**7.** D	**8.** C	**9.** C	**10.** D
11. D	**12.** B	**13.** A	**14.** C	**15.** B	**16.** D	**17.** D	**18.** B	**19.** D	**20.** A
21. A	**22.** B	**23.** C	**24.** A	**25.** A, B, C	**26.** A, D	**27.** C	**28.** B, C	**29.** A, B, C, D	
30. C	**31.** A, B, D	**32.** A, B, D	**33.** B	**34.** A	**35.** B	**36.** C, D	**37.** C	**38.** B, D	**39.** B
40. C	**41.** C	**42.** C	**43.** C	**44.** B, D					

JEE Advanced

Level I

1. (A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{35}$ **2.** (A) 1.7×10^{-5} (B) 1.08×10^{-4} **3.** 333 m/s **4.** 310 m/s **5.** $2c/3$

6. $\frac{\pi^2 \times 10^{-9}}{4}$ W/m^2 **7.** 6 **8.** 30 dB, $10\sqrt{10}$ mm **9.** (A) p (B) $I = \left(\sqrt{I_A} - \sqrt{I_B}\right)^2 = (25/312)^2$ **10.** 83 Hz

11. 420 Hz **12.** $\lambda/8$ **13.** $a[\sin(kx + \omega t) + 2\sin(kx - \omega t)]$

14. (A) $f = 50$ Hz, $\lambda = 0.2$ m, $v = 10$ ms^{-1} (B) $P_m = 62.8$ Nm^{-2} $= 20\pi$ Nm^{-2} $B = 100$ Nm^{-2}
(C) $\lambda/4 = 0.05$ m (D) $I = 20\pi^2 \cong 200$ wm^{-2} **15.** 735 N **16.** 20, 80 cm, 200 Hz

17. (A) $v_n = \frac{v}{4\ell}(2n+1)$; six oscillations (B) $v_n = \frac{v}{2\ell}(n+1)$, also six oscillations; Here $n = 0, 1, 2, \ldots$

18. 1:1 **19.** 3 cm **20.** 33 cm and 13.2 cm **21.** 2% **22.** (A) 95% (B) $\frac{40}{\pi} \times 10^3$ kg/m^3 **23.** 2.5 ms^{-1}

24. 485 Hz **25.** 345, 341 or 349 Hz

26. (A) $(V - V_w + V_s)/f$ (B) $(V + V_w - V_s)/f$ (C) $(V - V_w - V_D)/f_r$;
where $f_r = (V + V_w + V_0/v + V_w - V_s)f$ (iv) $(V - V_w - V_0/V - V_w - V_D)f_r$

27. $f_b = \frac{2fu}{v+u}$ **28.** 1:5 **29.** 30 dB, $10\sqrt{10}$ mm **30.** $\frac{f_r + f_a}{2}$ **31.** $L_c = 0.75$ m, $L_o = \frac{150}{151}$ m, 1.006 m

Level II

1. (A) $2\pi/a, b/2\pi$, (B) $y_2 = \pm 0.8A\cos(ax - bt)$, (C) $\max = 1.8bA, \min = 0$, **2.** (A) $l/6$; (B) 150 Hz; (C) 1.28
3. (A) 2; (B) 9.28 m and 1.99 m 4. 1650 Hz, 1500 Hz **5.** (A) 0.3 m, (B) 1320, (C) 332 m/s, (D) 0.2 m
6. 5.9 s **7.** f **8.** 9.7 m

Previous Year Questions

JEE Main

1. B 2. C 3. B 4. C 5. D 6. C 7. C 8. D 9. B 10. A
11. C 12. B 13. A 14. D 15. A 16. D

JEE Advanced

1. B 2. (A) 2.116, (B) ¾ 3. A 4. A 5. 336 m/s 6. C 7. D 8. D
9. $V_s = 30$ m/s 10. A 11. A 12. C 13. B 14. A 15. A 16. A
17. A, C, D 18. 7 19. B 20. A → PT; B → PS; C → QS; D → QR 21. A
22. B, D 23. A, B 24. D

CHAPTER 4

Heat-1

CALORIMETRY

HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of receiving body. It should be clearly understood that the word 'heat' is meaningful only as long as the energy is being transferred. Thus, expressions like 'heat in a body' or 'heat of body' are meaningless.

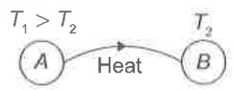


Figure 4.1

When we say that a body is heated it means that its molecules begin to move with greater kinetic energy.

The SI unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

Mechanical Equivalent of Heat

In early days, heat was not recognized as a form of energy. The heat was supposed to be something needed to raise the temperature of a body or to change its phase. The calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$W = JH$$

where J is called the mechanical equivalent of heat. J is expressed in joule/calories. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

1 calorie: The amount of heat needed to increase the temperature of 1 g of water from 14.5°C to 15.5°C at one atmospheric pressure is 1 calorie.

$$1 \text{ cal} = 4.186 \text{ J}$$

Specific Heat

Specific heat of a substance is equal to heat gained or released by that substance to raise or fall its temperature by 1°C for a unit mass of the substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

1. the mass of the body $\Delta Q \propto m$.
2. rise or fall of temperature of the body $\Delta Q \propto \Delta T$:

$$\Delta Q \propto m\Delta T$$

or $$\Delta Q \propto ms\Delta T$$

or $$dQ \propto msdT$$

or $$Q = m\int s\,dT$$

where s is a constant and is known as the specific heat of the body $s = \dfrac{Q}{m\Delta T}$. The SI unit of s is J/kg-K and the CGS unit is cal/g°C.

Specific heat of water:

$$s = 4200 \text{ J/kg°C}$$
$$= 1000 \text{ cal/kg°C}$$
$$= 1 \text{ Kcal/kg°C}$$
$$= 1 \text{ cal/g°C}$$

Specific heat of steam = half of specific heat of water

= specific heat of ice

SOLVED EXAMPLE

EXAMPLE 1

Heat required to increase the temperature of 1 kg water by 20°C.

SOLUTION

$$\text{Heat required} = \Delta Q$$
$$= ms\Delta\theta$$
$$= 1 \times 20$$
$$= 20 \text{ Kcal}$$
$$\therefore \quad S = 1 \text{ cal/g°C}$$
$$= 1 \text{ Kcal/kg°C}$$

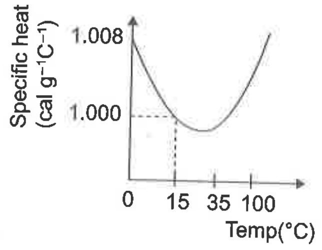
Important Points:

(a) We know, $s = \dfrac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), $s = Q/0 = \infty$. Thus, the specific heat of a substance when it melts or boils at constant temperature is infinite.

(b) If the temperature of the substance changes without the transfer of heat ($Q = 0$), then $s = \dfrac{Q}{m\Delta T} = 0$. Thus, when a liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.

(c) To raise the temperature of saturated water vapour, heat (Q) is withdrawn. Hence, the specific heat of saturated water vapour is negative. (This is for your information only and not in the course.)

(d) The slight variation of specific heat of water with temperature is shown in the following graph at 1 atmosphere pressure. Its variation is less than 1% over the interval from 0°C to 100°C.



Heat Capacity or Thermal Capacity

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°C. If m is the mass and s the specific heat of the body, then heat capacity = ms.

Units of heat capacity in the CGS system is cal°C^{-1}. The SI unit is JK^{-1}.

Relation Between Specific Heat and Water Equivalent

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object:

$$ms\Delta T = m_w S_w \Delta T$$

$$\Rightarrow \quad m_w = \frac{ms}{s_w}$$

In calorie, $\quad s_w = 1$

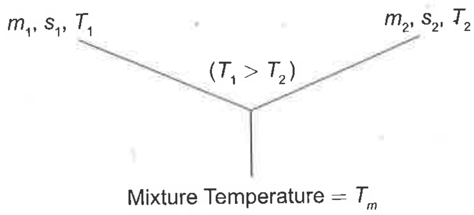
$$\therefore \quad m_w = ms,$$

m_w is also represented by W, so $W = ms$.

LAW OF MIXTURE

When two substances at different temperatures are mixed together, the exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of the mixture. Here, heat taken by one substance = heat given by another substance

$$\Rightarrow \quad m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$



SOLVED EXAMPLES

EXAMPLE 2 Similar NEET I (2016)

An iron block of mass 2 kg, fall from a height of 10 m. After colliding with the ground, it loses 25% energy to surroundings. Then find the temperature rise of the block (take specific heat of iron = 470 J/kg°C).

SOLUTION

$$mS\Delta\theta = \frac{1}{4}mgh$$

$$\Rightarrow \quad \Delta\theta = \frac{10\times10}{4\times470}.$$ ■

EXAMPLE 3

The temperatures of equal masses of three different liquids A, B and C are 10°C, 15°C and 20°C, respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

SOLUTION

When A and B are mixed

$$mS_1 \times (13-10) = m \times S_2 \times (15-13)$$

$$3S_1 = 2S_2 \qquad (1)$$

When B and C are mixed

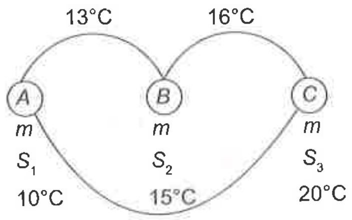
$$S_2 \times 1 = S_3 \times 4 \qquad (2)$$

When C and A are mixed

$$S_1(\theta - 10) = S_3 \times (20-\theta) \; (3)$$

By using Eqs. (1)–(3), we get

$$\theta = \frac{140}{11}\,°\text{C}$$



■

EXAMPLE 4

If three different liquids of different masses, specific heats and temperatures are mixed with each other and then what is the temperature mixture at thermal equilibrium?

$m_1, s_1, T_1 \rightarrow$ specification for liquid,

$m_2, s_2, T_2 \rightarrow$ specification for liquid,

$m_3, s_3, T_3 \rightarrow$ specification for liquid.

SOLUTION

Total heat loss or gain by all substances is equal to zero:

$$\Delta Q = 0.$$

$$m_1s_1(T-T_1) + m_2s_2(T-T_2) + m_3s_3(T-T_3) = 0$$

so $$T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}.$$ ■

PHASE CHANGE

Heat required for the change of phase or state,

$$Q = mL,$$

$$L = \text{latent heat}.$$

1. *Latent heat* (L): The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.
2. *Latent heat of fusion* (L_f): The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion.
3. *Latent heat of vaporization* (L_v): The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization.

If in question latent heat of water is not mentioned and to solve the problem it requires assuming that we should consider the following values.

Latent heat of ice:

$$L = 80 \text{ cal/g}$$

$$= 80 \text{ Kcal/kg}$$

$$= 4200 \times 80 \text{ J/kg}$$

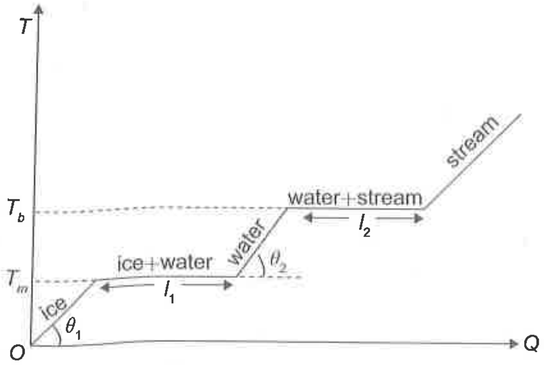
Latent heat of steam:

$$L = 540 \text{ cal/g}$$

$$= 540 \text{ Kcal/kg}$$

$$= 4200 \times 540 \text{ J/kg}$$

The given figure represents the change of state by different lines:



T_m = Melting temperature

T_b = Boiling temperature

$\theta_2 > \theta_1$

$l_2 > l_1$

Figure 4.2

Note

If we increase the temperature of liquid (phase) KE ↑ and temperature ↑ but at a later time KE stop increasing and the phase of the liquid starts changing.

SOLVED EXAMPLES

EXAMPLE 5

Find the amount of heat released if 100 g ice at −10°C is converted into 120°C, 100 g steam

−10°C, 100 gm ice ⟶ 120° 100 gm steam

$Q = ms\Delta T$

$= \frac{1}{2} \times 100 \times 20$

$= 500$ cal.

0°, 100 gm ice

$Q = mL_f$

$= 100 \times 80$

$= 8000$ cal.

0°, 100 gm water

$Q = ms\Delta T$

$= 1 \times 100 \times 100$

$= 10000$ cal.

100°, 100 gm water

$Q = mL_v$

$= 100 \times 540$

$= 54000$ cal

100°, 100 gm Steam

$Q = ms\Delta T$

$= \frac{1}{2} \times 100 \times 20$

$= 1000$ cal.

$Q_{net} = 73.5$ Kcal.

EXAMPLE 6

Five hundred grams of water at 80°C is mixed with 100 g of steam at 120°C. Find out the final mixture.

SOLUTION

120°C steam ⟶ 100°C steam

$$\text{Req. heat} = 100 \times \frac{1}{2} \times 20$$

$$= 1 \text{ Kcal}$$

80°C water ⟶ 100°C water

$$\text{Req. heat} = 500 \times 1 \times 20$$

$$= 10 \text{ Kcal}$$

100 g steam ⟶ 100 g water at 100°C

$$\text{Req. heat} = 100 \times 540$$

$$= 54 \text{ Kcal}$$

$$\text{Total heat} = 55 \text{ Kcal}$$

$$\text{Remaining heat} = 55 - 10$$

$$= 45 \text{ Kcal}$$

Now, we have 600 g water at 100°C

$$\Rightarrow \quad 4500 = m \times 540$$

$$\Rightarrow \quad m = \frac{250}{3} \text{ g}$$

Finally, we have $\frac{250}{3}$ g steam and $\left(600 - \frac{250}{3}\right)$g of water ■

HEAT TRANSFER

INTRODUCTION

Heat is energy in transit which flows due to temperature difference, from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes: (i) Conduction, (ii) Convection and (iii) Radiation

Conduction

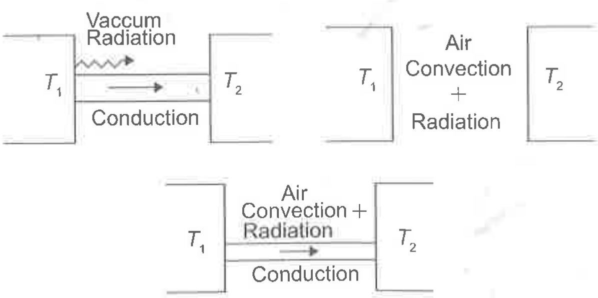
1. Requires medium
2. Energy is transmitted from one particle to another particle without displaced particle.
3. No transfer of particle

Convection

1. Requires medium
2. Energy is transferred through movement of the particle of a medium

Radiation

1. Does not require any medium
2. Energy is transferred through electromagnetic waves



CONDUCTION

Figure 4.3 shows a rod whose ends are in thermal contact with a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2. The sides of the rod are covered with insulating medium, and so the transport of heat is along the rod, not through the sides. The molecules at the hot reservoir have greater vibrational energy. This energy is transferred by collisions to the atoms at the end face of the rod. These atoms in turn transfer energy to their neighbours further along the rod. Such transfer of heat through a substance in which heat is transported without direct mass transport is called conduction.

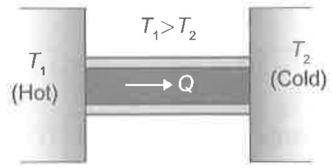


Figure 4.3

Most metals use another, more effective mechanism to conduct heat. The free electrons, which move throughout the metal, can rapidly carry energy from the hotter to cooler regions; so metals are generally good conductors of heat. The presence of 'free' electrons also causes most metals to be good electrical conductors. A metal rod at 5°C feels colder than a piece of wood at 5°C because heat can flow more easily from your hand into the metal.

Heat transfer occurs only between regions that are at different temperatures, and the rate of heat flow is $\frac{dQ}{dt}$. This rate is also called the heat current, denoted by H. Experiments show that the heat current is proportional to the cross-sectional area A of the rod and to the temperature gradient $\frac{dT}{dx}$, which is the rate of change of temperature with distance along the bar. In general,

$$H = \frac{dQ}{dt}$$

$$= -kA\frac{dT}{dx} \quad (1)$$

The negative sign is used to make $\frac{dQ}{dt}$, a positive quantity since $\frac{dT}{dt}$ is negative. The constant k, called the thermal conductivity, is a measure of the ability of a material to conduct heat.

A substance with a large thermal conductivity k is a good conductor of heat. The value of k depends on the temperature, increasing slightly with increasing temperature, but k can be taken to be practically constant throughout the substance if the temperature difference between its ends is not too high.

Let us apply Eq. (1) to a rod of length L and constant cross-sectional area A in which a steady state has been reached. In a steady state, the temperature at each point is constant in time. Hence,

$$-\frac{dT}{dt} = T_1 - T_2 \tag{2}$$

Therefore, the heat ΔQ transferred in time Δt is

$$\Delta Q = kA\left(\frac{T_1 - T_2}{L}\right)\Delta t \tag{3}$$

Here, ΔT = temperature difference (TD)

and $R = \dfrac{l}{kA}$ = thermal resistance of the rod.

Important Points in Conduction

1. Consider a section ab of a rod as shown in Fig. 4.4. Suppose Q_1 heat enters into the section at a and Q_2 leaves at b, then $Q_2 < Q_1$. Part of the energy $Q_1 - Q_2$ is utilized in raising the temperature of section ab and the remaining is lost to the atmosphere through ab. If heat is continuously supplied from the left end of the rod, a stage comes when temperature of the section becomes constant. In that case, $Q_1 = Q_2$ if rod is insulated from the surroundings (or loss through ab is zero). This is called the *steady-state* condition. Thus, in steady-state temperature of different sections of the rod becomes constant (but not same).

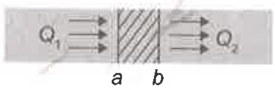


Figure 4.4

Hence, in Fig. 4.5,

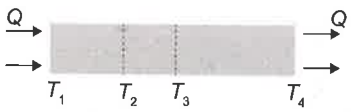


Figure 4.5

T_1 = constant, T_2 = constant, etc.

and $T_1 > T_2 > T_3 > T_4$

Now, a natural question arises, why the temperature of the whole rod does not become equal when heat is being continuously supplied? The answer is: there must be a temperature difference in the rod for the heat flow, same as we require a potential difference across a resistance for the current flow through it.

In steady state, the temperature varies linearly with distance along the rod if it is insulated.

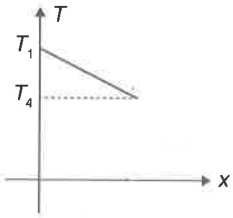


Figure 4.6

2. Comparing Eq. (3), i.e., heat current

$$H = \frac{dQ}{dt} = \frac{\Delta T}{R},$$

where $R = \dfrac{l}{kA}$

with the equation of current flows through a resistance,

$$i = \frac{dq}{dt} = \frac{\Delta V}{R},$$

where $R = \dfrac{l}{\sigma A}$.

We find the following similarities in heat flow through a rod and current flow through a resistance.

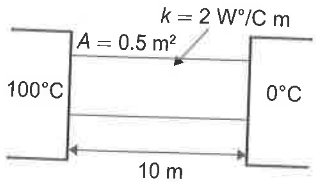
Heat flow through a conducting rod	Current flow through a resistance
Heat current, $H = \dfrac{dQ}{dt}$ = rate of heat flow	Electric current $i = \dfrac{dq}{dt}$ = rate of heat flow
$H = \dfrac{\Delta T}{R} = \dfrac{TD}{R}$	$i = \dfrac{\Delta V}{R} = \dfrac{PD}{R}$
$R = \dfrac{l}{kA}$	$R = \dfrac{l}{\sigma A}$
k = thermal conductivity	σ = electrical conductivity

From the above table, it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

SOLVED EXAMPLES

EXAMPLE 7

Find out the heat current and temperature at any distance x.



SOLUTION

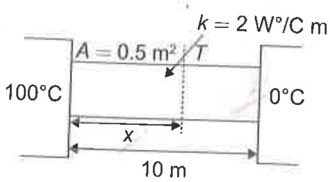
$$R=\frac{10}{2\times 0.5}$$

$$=10=\frac{l}{kA}$$

$$i=\frac{100}{10}$$

$$=10=\frac{kA\Delta T}{l}$$

and temperature at any distance x



$$q=\frac{kA(100-T)}{x}$$

$$=\frac{kA\Delta T}{l}$$

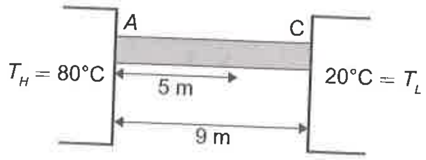
$$\Rightarrow \quad \frac{(100-T)}{x}=\frac{(100-0)}{l}$$

$$100l - Tl = 100x$$

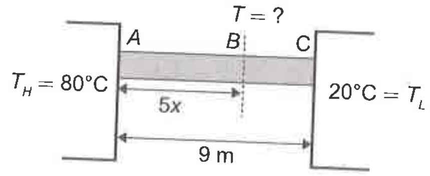
$$T=\frac{100(l-x)}{l}$$ ■

EXAMPLE 8

Find out the temperature at distance 5 m.



SOLUTION



Heat current is same. so,

$$\frac{T_H-T_L}{\ell}=\frac{T_H-T}{x}$$

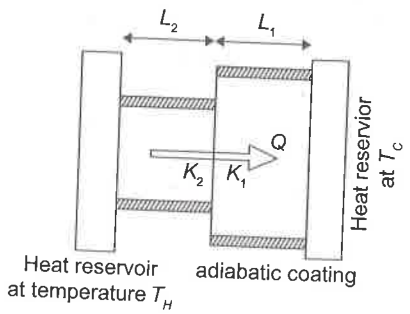
$$\Rightarrow \quad \frac{80-20}{9}=\frac{80-T}{5}$$

$$T=\frac{140}{3}\,°\mathrm{C}$$ ■

Slabs in Parallel and Series

(a) Slabs in Series (in Steady State)

Consider a composite slab consisting of two materials having different thicknesses L_1 and L_2, different cross-sectional areas A_1 and A_2, and different thermal conductivities K_1 and K_2. The temperature at the outer surface of the states is maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.



Let the temperature at the junction be T; since the steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab

$$i = \frac{Q}{t}$$

$$= \frac{T_H - T}{R_1}$$

or $$T_H - T = iR_1 \quad (1)$$

and that through the second slab

$$i = \frac{Q}{t}$$

$$= \frac{T - T_C}{R_2}$$

or $$T - T_C = iR_2 \quad (2)$$

Adding Eqs. (1) and (2), we get

$$T_H - T_L = (R_1 + R_2)\, i$$

or $$i = \frac{T_H - T_L}{(R_1 + R_2)}$$

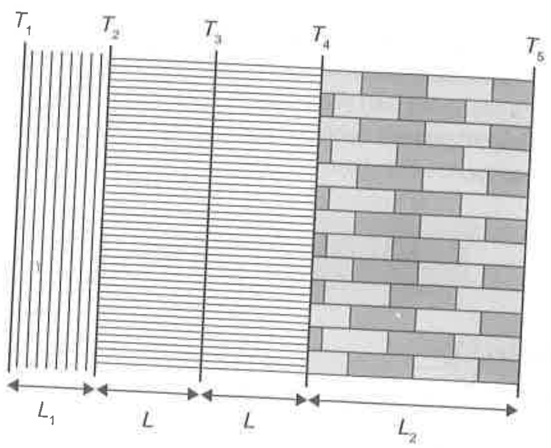
Thus, these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$.

If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + \cdots \quad (3)$$

EXAMPLE 9

The figure shows the cross-section of the outer wall of a house built in a hill-resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwiching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat conduction through the wall has reached a steady state with the temperature of three surfaces being known. ($T_1 = 25°C$, $T_2 = 20°C$ and $T_5 = -20°C$.) Find the interface temperature T_4 and T_3.



SOLUTION

Let the interface area be A. The thermal resistance of wood

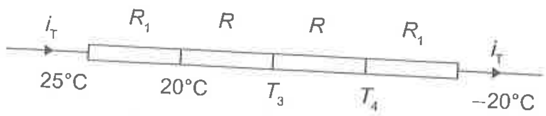
$$R_1 = \frac{L_1}{K_1 A}$$

and that of brick wall

$$R_2 = \frac{L_2}{K_2 A}$$

$$= \frac{5L_1}{5K_1 A} = R_1$$

Let the thermal resistance of each sandwich layer $= R$. Then the above wall can be visualized as a circuit.



Thermal current through each wall is same.

Hence,

$$\frac{25 - 20}{R_1} = \frac{20 - T_3}{R}$$

$$= \frac{T_3 - T_4}{R}$$

$$= \frac{T_4 + 20}{R_1}$$

$$\Rightarrow \quad 25 - 20 = T_4 + 20$$

$$\Rightarrow \quad T_4 = -15°C$$

also, $$20 - T_3 = T_3 - T_4$$

$$\Rightarrow \qquad T_3 = \frac{20 + T_4}{2} = 2.5°C$$

EXAMPLE 10

In Example 3, $K_1 = 0.125$ W/m°C, $K_2 = 5K_1 = 0.625$ W/m°C and thermal conductivity of the unknown material is $k = 0.25$ W/m°C. $L_1 = 4$ cm, $L_2 = 5$ cm, $L_1 = 20$ cm and $L = 10$ cm. If the house consists of a single room of total wall area of 100 m², then find the power of the electric heater being used in the room.

SOLUTION

$$R_1 = R_2$$

$$= \frac{(4\times10^{-2}\,\text{m})}{(0.125\,\text{W/m C})(100\,\text{m}^2)}$$

$$= 32 \times 10^{-4}\ °\text{C/W}$$

$$R = \frac{(10\times10^{-2}\,\text{m})}{(0.25\,\text{W/m C})(100\,\text{m}^2)}$$

$$= 40 \times 10^{-4}\ °\text{C/W}$$

Equivalent thermal resistance of the entire wall $= R_1 + R_2 + 2R = 144 \times 10^{-4}$ °C/W

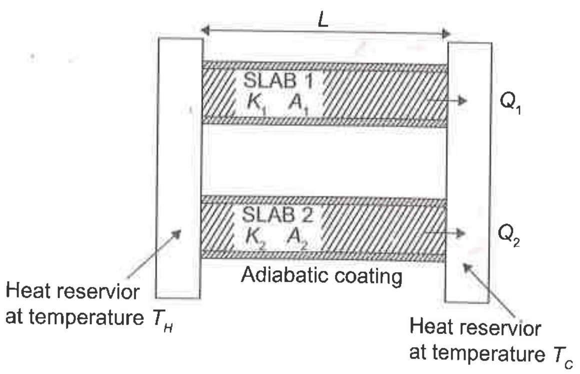
∴ Net heat current, i.e., amount of heat flowing out of the house per second $= \dfrac{T_H - T_C}{R}$

$$= \frac{25°\text{C} - (-20°\text{C})}{144\times10^{-4}°\text{C/W}}$$

$$= \frac{45\times10^4}{144}\,\text{W}$$

Hence, the heater must supply 3.12 kW to compensate for the outflow of heat.

(b) Slabs in Parallel



Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional areas A_1 and A_2.

Then
$$R_1 = \frac{L}{K_1 A_1},$$

$$R_2 = \frac{L}{K_2 A_2}$$

Thermal current through slab 1

$$i_1 = \frac{T_H - T_C}{R_1}$$

and that through slab 2

$$i_2 = \frac{T_H - T_C}{R_2}$$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2$$

$$= (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with $i = \dfrac{T_H - T_C}{R_{eq}}$, we get

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (5.4)$$

EXAMPLE 11

Two thin concentric shells made from copper with radii r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C, respectively, by keeping a heater of power P at the centre of the two spheres. Find the value of P.

SOLUTION

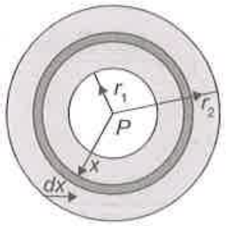
Heat flowing per second through each cross-section of the sphere $= P = i$

Thermal resistance of the spherical shell of radius x and thickness dx,

$$dR = \frac{dx}{K \cdot 4\pi x^2}$$

$$\Rightarrow \quad R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2 \cdot K}$$

$$= \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



Thermal current

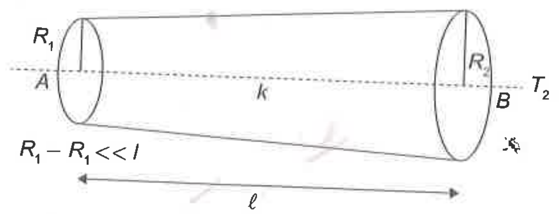
$$i = P$$

$$= \frac{T_H - T_C}{R}$$

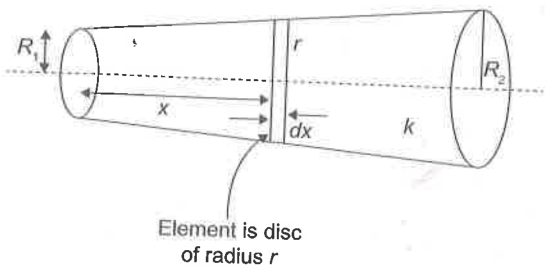
$$= \frac{4\pi K(T_H - T_C) r_1 r_2}{(r_2 - r_1)}$$ ■

EXAMPLE 12

Find out the equivalent thermal resistance between points *A* and *B*.



SOLUTION



$$dR = \frac{dx}{k\pi r^2}$$

$$\frac{r_1}{y} = \frac{r_2}{y+\ell}$$

$$= \frac{r}{y+x}$$

$$\Rightarrow \quad r_1 y + r_1 \ell = r_2 y$$

$$y = \frac{-r_1 \ell}{(r_1 - r_2)}$$

$$= \frac{r_1 \ell}{(r_2 - r_1)}$$

$$\Rightarrow \quad r = \frac{r_2(y+x)}{y+\ell}$$

$$= \frac{r_2\left[(r_1\ell/(r_2 - r_1)) + x\right]}{\left[(r_1\ell/(r_2 - r_1)) + \ell\right]}$$

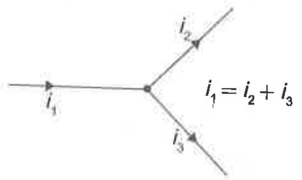
$$dR = \frac{dx}{k\pi r^2}$$

$$\Rightarrow \quad \int_0^{\ell} \frac{dx}{k \cdot \pi r^2} = \int_0^{\ell} \frac{dx}{k \cdot \pi\left(r_1 + \frac{(r_2 - r_1)x}{\ell}\right)^2}$$ ■

Junction Law

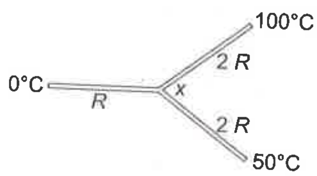
Heat current is a Tensor quantity, because it does not follow the vector laws but the direction of heat current matter.

According to the Junction law, the sum of all the heat current directed towards a point is equal to the sum of all the heat currents directed away from the points.

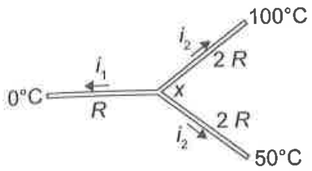


EXAMPLE 13

Find out the temperature at point *x*.



SOLUTION



$$i_1 + i_2 + i_3 = 0$$

$$i_1 = \frac{(x-0)}{R},$$

$$i_2 = \frac{(x-100)}{2R},$$

$$i_3 = \frac{(x-50)}{2R}$$

$$4x = 150$$

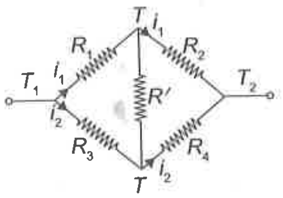
$$\Rightarrow \quad x = 37.5°\text{C}$$

■

WHEAT STONE BRIDGE

EXAMPLE 14

Find out the relation between R_1, R_2, R_3 and R_4, so that there is no current in R'.



SOLUTION

$$\Delta T = T_1 - T_2$$

$$T_1 - T = i_1 R_1 \qquad (1)$$

$$T_1 - T = i_2 R_3 \qquad (2)$$

Eq. (1)/Eq. (2)

$$\Rightarrow \quad \frac{i_1 R_1}{i_2 R_3} = 1 \qquad (3)$$

$$T - T_2 = i_1 R_2 \qquad (4)$$

$$T - T_2 = i_2 R_4 \qquad (5)$$

Eq. (4)/Eq. (5)

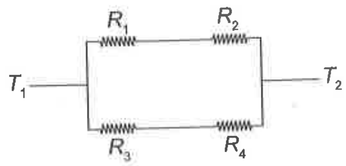
$$i_1 R_2 = i_2 R_4 \qquad (6)$$

Eq. (3)/Eq. (6)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

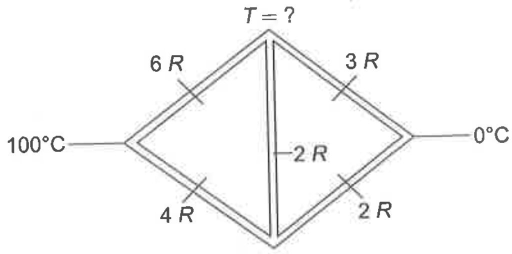
$$\Rightarrow \quad R_1 R_4 = R_2 R_3$$

Now,

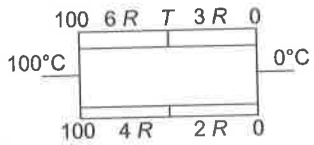


■

EXAMPLE 15



SOLUTION

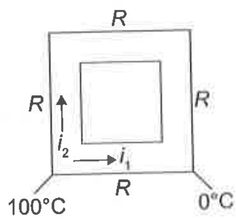


$$\frac{100 - T}{6R} = \frac{T - 0}{3R}$$

$$\Rightarrow \quad T = \frac{100}{3°\text{C}}$$

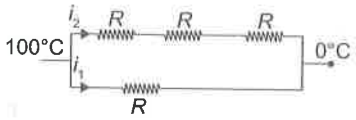
■

EXAMPLE 16



Find $i_2 / i_1 = ?$

SOLUTION



$$i_1 : i_2 = \frac{1}{R} : \frac{1}{3R} = 3:1$$

$$i_1 = \frac{3}{4} \times 100 = 75$$

$$\Rightarrow \quad i_2 = \frac{1}{4} \times 100 = 25$$

EXAMPLE 17

A container of negligible heat capacity contains 1 kg of water. It is connected by a steel rod of length 10 m and area of cross-section 10 cm² to a large steam chamber which is maintained at 100°C. If initial temperature of water is 0°C, find the time after which it becomes 50°C. (Neglect heat capacity of steel rod and assume no loss of heat to surroundings.) (take specific heat of water = 4180 J/kg°C)

SOLUTION

Let the temperature of water at time t be T, then thermal current at time t,

$$i = \left(\frac{100 - T}{R}\right)$$

This increases the temperature of water from T to $T + dT$

$$\Rightarrow \quad i = \frac{dH}{dt}$$

$$= ms\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{100 - T}{R} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad \int_0^{50} \frac{dT}{100 - T} = \int_0^t \frac{dT}{Rms}$$

$$\Rightarrow \quad -\ell n\left(\frac{1}{2}\right) = \frac{t}{Rms}$$

or $$t = Rms\ell n\, 2 \text{ sec}$$

$$= \frac{L}{KA} ms\ell n\, 2 \text{ sec}$$

$$= \frac{(10\,\text{m})(1\,\text{kg})(4180\,\text{J/kg}\,°\text{C})}{46(\text{W/m}\,°\text{C}) \times (10 \times 10^{-4}\,\text{m}^2)}$$

$$= \frac{418}{46}(0.69) \times 10^5$$

$$= 6.27 \times 10^5 \text{ s}$$

$$= 174.16 \text{ hours}$$

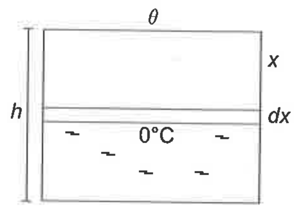
EXAMPLE 18

On a cold winter day, the atmospheric temperature is $-\theta$ (on Celsius scale) which is below 0°C. A cylindrical drum of height h made of a bad conductor is completely filled with water at 0°C and is kept outside without any lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is k and its latent heat of fusion is L. Neglect expansion of water on freezing.

SOLUTION

Suppose, the ice starts forming at time $t = 0$ and a thickness x is formed at time t. The amount of heat flown from the water to the surrounding in the time interval t to $t + dt$ is

$$\Delta Q = \frac{KA\theta}{x} dt$$



The mass of the ice formed due to the loss of this amount of heat is

$$dm = \frac{\Delta Q}{L}$$

$$= \frac{KA\theta}{xL} dt$$

The thickness dx of ice formed in time dt is

$$dx = \frac{dm}{A\rho}$$

$$= \frac{K\theta}{\rho x L} dt$$

or $$dt = \frac{\rho L}{K\theta} x\, dx.$$

Thus, the time T taken for the whole mass of water to freeze is given by

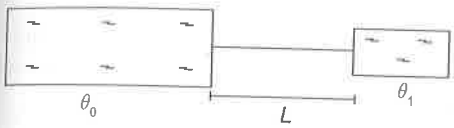
$$\int_0^T dt = \frac{\rho L}{K\theta}\int_0^h x\,dx$$

or

$$T = \frac{\rho L h^2}{2K\theta}.$$

EXAMPLE 19

The figure shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass m of water at an initial temperature θ_1 ($<\theta_0$). A metal rod of length L, an area of cross-section A and thermal conductivity k, connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel to become θ_2 ($\theta_1 < \theta_2 < \theta_0$). Specific heat capacity of water is s and all other heat capacities are negligible.



SOLUTION

Suppose, the temperature of the water in the smaller vessel is θ at time t. In the next time interval dt, a heat ΔQ is transferred to it where

$$\Delta Q = \frac{KA}{L}(\theta_0 - \theta)\,dt \qquad (1)$$

This heat increases the temperature of the water of mass m to $\theta + d\theta$ where

$$\Delta Q = ms\,d\theta \qquad (2)$$

From (1) and (2),

$$\frac{KA}{L}(\theta_0 - \theta)dt = ms\,d\theta$$

or

$$dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta}$$

or

$$\int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta},$$

where T is the time required for the temperature of the water to become θ_2.

Thus,

$$T = \frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}$$

RADIATION

The process of transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation

1. All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.
2. Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.
3. More radiations are emitted at the higher temperature of a body and lesser at the lower temperature.
4. The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this, the colour of a body appears to be changing. Radiations from a body at NTP have predominantly infrared waves.
5. Thermal radiations travel with the speed of light and move in a straight line.
6. Radiations are electromagnetic waves and can also travel through the vacuum.
7. Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.
8. Radiation from a point source obeys inverse square law (intensity $\propto \frac{1}{r^2}$).

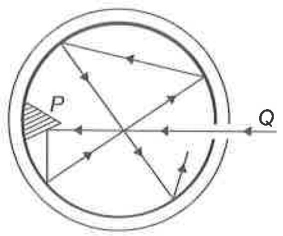
Prevost Theory of Exchange

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings, it radiates at the same rate as it absorbs.

Perfectly Black Body and Black Body Radiation (Fery's Black Body)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and, therefore, appears black whatever be the colour of the incident radiation.

In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp black and platinum black are a good approximation of black body. They absorb about 99% of the incident radiation. The most simple and commonly used black body was designed by Ferry. It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.



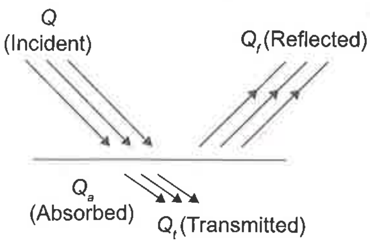
Absorption, Reflection and Emission of Radiations

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

where r = reflecting power, a = absorptive power and t = transmission power.

1. $r = 0, t = 0, a = 1$, perfect black body
2. $r = 1, t = 0, a = 0$, perfect reflector
3. $r = 0, t = 1, a = 0$, perfect transmitter



(a) Absorptive Power

In particular, absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body:

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

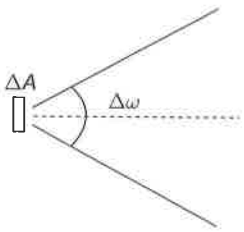
As all the radiations incident on a black body are absorbed, $a = 1$ for a black body.

(b) Emissive Power

Consider a small area ΔA of a body emitting thermal radiation, and a small solid angle $\Delta\omega$ about the normal to the radiating surface. Let the energy radiated by the area ΔA of the surface in the solid angle $\Delta\omega$ in time Δt be ΔU. We define emissive power of the body as

$$E = \frac{\Delta U}{(\Delta A)(\Delta\omega)(\Delta t)}$$

Thus, emissive power denotes the energy radiated per unit area per unit time per unit solid angle along the normal to the area.



(c) Spectral Emissive Power (Eλ)

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, $E\lambda$. If E is the total emissive power and $E\lambda$ is the spectral emissive power, they are related as follows:

$$E = \int_0^\infty E_\lambda d\lambda$$

and

$$\frac{dE}{d\lambda} = E\lambda$$

(d) Emissivity

$$e = \frac{\text{Emissive power of a body to a temperature } T}{\text{Emissive power of a black body to a same temperature } T}$$

$$= \frac{E}{E_0}$$

STEFAN-BOLITZMANN'S LAW

Consider a hot body at temperature T placed in an environment at a lower temperature T_0. The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations and this rate

$$\frac{d\theta}{dt} \propto T^4,$$

$$\frac{d\theta}{dt} \propto A,$$

$$\frac{d\theta}{dt} \propto e$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \sigma e A T^4$$

$$P_1 = eA\sigma T^4$$

and is receiving energy by absorbing radiations and this absorption rate

$$\frac{d\theta}{dt} = P_2$$

$$= aA\sigma T_0^4$$

Here, a is a pure number between 0 and 1 indicating the relative ability of the surface to absorb radiation from its surroundings. Note that this a is different from the absorptive power a. In thermal equilibrium, both the body and the surrounding have the same temperature (say T_c) and $P_1 = P_2$

or $$eA\sigma T_c^4 = aA\sigma T_c^4$$

or $$e = a$$

Thus, when $T > T_0$, the net rate of heat transfer from the body to the surroundings is

$$\text{Net heat loss} = \frac{dQ}{dt}$$

$$= eA\sigma(T^4 - T_0^4)$$

or $$ms\left(\frac{dT}{dt}\right) = eA\sigma(T^4 - T_0^4)$$

$\Rightarrow$ Rate of cooling

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4)$$

or $$\frac{dT}{dt} \propto (T^4 - T_0^4)$$

NEWTON'S LAW OF COOLING

According to this law, if the temperature T of the body is not very different from that of the surroundings T_0, then the rate of cooling $-\frac{dT}{dt}$ is proportional to the temperature difference between them. To prove it, let us assume that

$$T = T_0 + \Delta t$$

$$\frac{d\theta}{dt} = \sigma Ae[(T + \Delta T)^4 - T_0^4]$$

$$\frac{d\theta}{dt} = \sigma AeT_0^4\left[1 + \frac{4\Delta T}{T_0} - 1\right]$$

$$= 4\sigma AT_0^3\Delta T$$

if the temperature difference is small.

Thus, the rate of cooling

$$-\frac{dT}{dt} \propto \Delta T$$

or $$-\frac{d\theta}{dt} \propto \Delta\theta$$

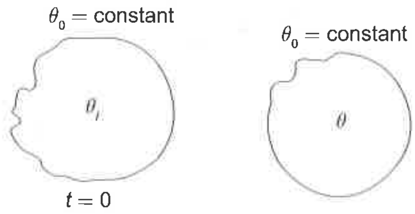
as $$dT = d\theta$$

or $$\Delta T = D\theta$$

Variation of Temperature of a Body According to Newton's Law

Suppose a body has a temperature θ_i at time $t = 0$. It is placed in an atmosphere whose temperature is θ_0. We are interested in finding the temperature of the body at time t, assuming Newton's law of cooling to hold good or by assuming that the temperature difference is small. As per this law,

rate of cooling $\propto$ temperature difference



or $$\left(-\frac{d\theta}{dt}\right) = \left(\frac{eA\sigma}{mc}\right)(4\theta_0^3)(\theta - \theta_0)$$

or $$-\frac{d\theta}{dt} = \alpha(\theta - \theta_0)$$

Here, $\alpha = \left(\frac{4eA\sigma\theta_0^3}{mc}\right)$ is a constant

$$\therefore \quad \int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -\alpha \int_0^t dt$$

$$\therefore \quad \theta = \theta_0 + (\theta_i - \theta_0)e^{-\alpha t}$$

From this expression, we see that $\theta = \theta_i$ at $t = 0$ and $\theta = \theta_0$ at $t = \infty$, i.e., temperature of the body varies exponentially with time from θ_i to θ_0 ($< \theta_i$). The temperature versus time graph is as shown in Fig. 4.6.

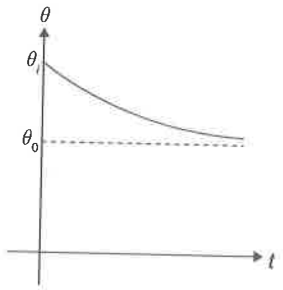


Figure 4.6

Notes

If the body cools by radiation from θ_1 to θ_2 in time t, then taking the approximation

$$\left(-\frac{d\theta}{dt}\right) = \frac{\theta_i - \theta_2}{t}$$

and $$\theta = \theta_{av} = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$ becomes

$$\frac{\theta_i - \theta_2}{t} = \alpha\left(\frac{\theta_i - \theta_2}{2} - \theta_0\right)$$

This form of the law helps in solving numerical problems related to Newton's law of cooling.

If the body cools by radiation from q_1 to q_2 in time t, then taking the approximation

$$\left(-\frac{d\theta}{dt}\right) = \frac{\theta_i - \theta_2}{t}$$

and $$\theta = \theta_{av} = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$ becomes

$$\frac{\theta_i - \theta_2}{t} = \alpha\left(\frac{\theta_i - \theta_2}{2} - \theta_0\right)$$

This form of the law helps in solving numerical problems related to Newton's law of cooling.

Limitations of Newton's Law of Cooling

1. The difference in temperature between the body and surroundings must be small.
2. The loss of heat from the body should be radiation only.
3. The temperature of surroundings must remain constant during the cooling of the body.

SOLVED EXAMPLE

EXAMPLE 20

A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 min. Find how much more time will it take for the body to attain a temperature of 30°C.

SOLUTION

$$\Delta\theta_f = \Delta\theta_i e^{-kt}$$

for the interval in which temperature falls from 40°C to 35°C

$$(35 - 20) = (40 - 20)\, e^{-k10}$$

$$\Rightarrow \quad e^{-10k} = \frac{3}{4}$$

$$\Rightarrow \quad k = \frac{\ell n(4/3)}{10}$$

for the next interval $(30 - 20) = (35 - 20)\, e^{-kt}$

$$\Rightarrow \quad e^{-10k} = \frac{2}{3}$$

$$\Rightarrow \quad kt = \ell n\frac{3}{2}$$

$$\Rightarrow \quad \frac{(\ell n(4/3))t}{10} = \ell n\frac{3}{2}$$

$$\Rightarrow \quad t = 10\frac{(\ell n(3/2))}{(\ell n(4/3))} \text{ min}$$

$$= 14.096 \text{ min}$$

Alter: (by approximate method)
For the interval in which temperature falls from 40°C to 35°C

$$\langle\theta\rangle = \frac{40+35}{2} = 37.5°C$$

From Eq. (14.4),

$$\left\langle\frac{d\theta}{dt}\right\rangle = -k(\langle\theta\rangle - \theta_0)$$

$$\Rightarrow \quad \frac{(35°C-40°C)}{10\,\text{min}} = -K(37.5°C-20°C)$$

$$\Rightarrow \quad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35°C to 30°C

$$\langle\theta\rangle = \frac{35+30}{2} = 32.5°C$$

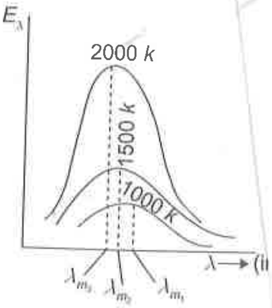
$$\Rightarrow \quad \frac{(30°C-35°C)}{t} = -(32.5°C-20°C)$$

$$\Rightarrow \quad \text{Required time,} \quad t = \frac{5}{12.5}\times 35\,\text{min} = 14\,\text{min}$$

NATURE OF THERMAL RADI... (WIEN'S DISPLACEMEN...

From the energy distribution cu... the following conclusions can b...

1. The higher the temperature ... area under the curve, i.e., ... emitted by the body at a hig...
2. The energy emitted by ... temperatures is not uniform. ... wavelengths, the energy emitt...



3. For a given temperature, there is a pa... (l_m) for which the energy emitted (...
4. With an increase in the tempera... body, the maxima of the curves shi... wavelengths.

From the study of the energy distribution of black body radiation as discussed above, it was established experimentally that the wavelength (l_m) corresponding to a maximum intensity of emission decreases inversely with the increase in the temperature of the black body. i.e.,

$$l_m \propto \frac{1}{T} \text{ or } l_m T = b$$

This is called Wien's displacement law.
Here, $b = 0.282$ cmK is the Wien's constant.

SOLVED EXAMPLE

EXAMPLE 21

The earth receives solar radiation at a rate of 8.2 J/cm² min. Assuming that the sun radiates like a black body, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is 0.53° and the Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/m²-K⁴.

SOLUTION

(B) $\frac{(K_1+K_2)}{2}$

(D) $\frac{2K_1K_2}{K_1+K_2}$

...a steel bottom 1.2 cm thick rests on a hot ... area of the bottom of the pot is 0.150 m². ...ater inside the pot is at 100°C and 0.440 kg ...ourise in every 5 minutes. The temperature of the ...wer surface of the pot, which is in contact with the stove is (Given: $L_v = 2.256 \times 10^6$ J/kg and $K_{steel} =$ 50.2 W/m-K)

(A) 105.3°C
(B) 205.3°C
(C) 185.3°C
(D) 115.3°C

18. A lake surface is exposed to an atmosphere where the temperature is < 0°C. If the thickness of the ice layer

or

$$-\frac{d\theta}{dt} = \alpha(\theta - \theta_0)$$

Here, $\alpha = \left(\frac{4eA\sigma\theta_0^3}{mc}\right)$ is a constant

$$\therefore \quad \int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -\alpha \int_0^t dt$$

$$\therefore \quad \theta = \theta_0 + (\theta_i - \theta_0)e^{-\alpha t}$$

From this expression, we see that $\theta = \theta_i$ at $t = 0$ and $\theta = \theta_0$ at $t = \infty$, i.e., temperature of the body varies exponentially with time from θ_i to θ_0 ($< \theta_i$). The temperature versus time graph is as shown in Fig. 4.6.

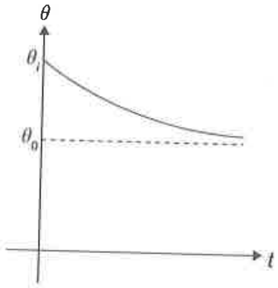


Figure 4.6

Notes

If the body cools by radiation from θ_1 to θ_2 in time t, then taking the approximation

$$\left(-\frac{d\theta}{dt}\right) = \frac{\theta_i - \theta_2}{t}$$

and

$$\theta = \theta_{av} = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$ becomes

$$\frac{\theta_i - \theta_2}{t} = \alpha\left(\frac{\theta_i - \theta_2}{2} - \theta_0\right)$$

This form of the law helps in solving numerical problems related to Newton's law of cooling.

If the body cools by radiation from q_1 to q_2 in time t, then taking the approximation

$$\left(-\frac{d\theta}{dt}\right) = \frac{\theta_i - \theta_2}{t}$$

and

$$\theta = \theta_{av} = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right) = \alpha(\theta - \theta_0)$ becomes

$$\frac{\theta_i - \theta_2}{t} = \alpha\left(\frac{\theta_i - \theta_2}{2} - \theta_0\right)$$

This form of the law helps in solving numerical problems related to Newton's law of cooling.

Limitations of Newton's Law of Cooling

1. The difference in temperature between the body and surroundings must be small.
2. The loss of heat from the body should be radiation only.
3. The temperature of surroundings must remain constant during the cooling of the body.

SOLVED EXAMPLE

EXAMPLE 20

A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 min. Find how much more time will it take for the body to attain a temperature of 30°C.

SOLUTION

$$\Delta\theta_f = \Delta\theta_i e^{-kt}$$

for the interval in which temperature falls from 40°C to 35°C

$$(35 - 20) = (40 - 20)\, e^{-k10}$$

$$\Rightarrow \quad e^{-10k} = \frac{3}{4}$$

$$\Rightarrow \quad k = \frac{\ell n(4/3)}{10}$$

for the next interval $(30 - 20) = (35 - 20)\, e^{-kt}$

$$\Rightarrow \quad e^{-10k} = \frac{2}{3}$$

$$\Rightarrow \quad kt = \ell n\frac{3}{2}$$

$$\Rightarrow \quad \frac{(\ell n(4/3))t}{10} = \ell n\frac{3}{2}$$

$$\Rightarrow \quad t = 10\frac{(\ell n(3/2))}{(\ell n(4/3))}\text{ min}$$

$$= 14.096 \text{ min}$$

Alter: (by approximate method)
For the interval in which temperature falls from 40°C to 35°C

$$\langle\theta\rangle = \frac{40+35}{2} = 37.5°C$$

From Eq. (14.4),

$$\left\langle\frac{d\theta}{dt}\right\rangle = -k(\langle\theta\rangle - \theta_0)$$

$$\Rightarrow \quad \frac{(35°C - 40°C)}{10\,\text{min}} = -K(37.5°C - 20°C)$$

$$\Rightarrow \quad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35°C to 30°C

$$\langle\theta\rangle = \frac{35+30}{2} = 32.5°C$$

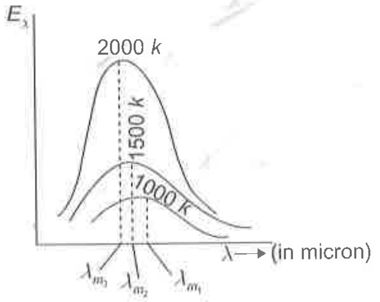
$$\Rightarrow \quad \frac{(30°C - 35°C)}{t} = -(32.5°C - 20°C)$$

$\Rightarrow$ Required time, $t = \frac{5}{12.5} \times 35\,\text{min} = 14\,\text{min}$ ■

NATURE OF THERMAL RADIATIONS: (WIEN'S DISPLACEMENT LAW)

From the energy distribution curve of black body radiation, the following conclusions can be drawn:

1. The higher the temperature of a body, the higher is the area under the curve, i.e., more amount of energy is emitted by the body at a higher temperature.
2. The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.



3. For a given temperature, there is a particular wavelength (l_m) for which the energy emitted ($E\lambda$) is maximum.
4. With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

From the study of the energy distribution of black body radiation as discussed above, it was established experimentally that the wavelength (l_m) corresponding to a maximum intensity of emission decreases inversely with the increase in the temperature of the black body. i.e.,

$l_m \propto \frac{1}{T}$ or $l_m T = b$

This is called Wien's displacement law.

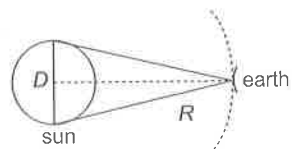
Here, $b = 0.282$ cmK is the Wien's constant.

SOLVED EXAMPLE

EXAMPLE 21

The earth receives solar radiation at a rate of 8.2 J/cm² min. Assuming that the sun radiates like a black body, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is 0.53° and the Stefan constant $\sigma = 5.67 \times 10^{-8}$ W/m²-K⁴.

SOLUTION



Let the diameter of the sun be D and its distance from the earth be R. From the questions,

$$\frac{D}{R} \approx 0.53 \times \frac{\pi}{180} = 9.25 \times 10^{-3} \qquad (1)$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^3 \sigma T^4 = \pi D^2 \sigma T^4$$

At distance R, this radiation falls on an area $4pR^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2$$

Thus, $$\frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2 = 8.2\ \text{J/cm}^2\ \text{min}$$

or $$\frac{1}{4} \times \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 - \text{K}^4}\right) T^4 \times (9.25 \times 10^{-3})^2 = \frac{8.2}{10^{-4} \times 60} \frac{\text{W}}{\text{m}^2}$$

or $$T = 5794\ K \approx 5800\ K.$$ ■

EXERCISES

JEE Main

1. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)
(A) 6200 cal
(B) 7200 cal
(C) 13600 cal
(D) 8200 cal

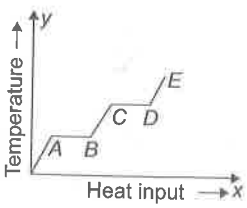
2. Heat is being supplied at a constant rate to a sphere of ice which is melting at the rate of 0.1 gm/sec. It melts completely in 100 sec. The rate of rise of temperature thereafter will be (Assume no loss of heat)
(A) 0.8°C/sec
(B) 5.4°C/sec
(C) 3.6°C/sec
(D) will change with time

3. A 2100 W continuous flow geyser (instant geyser) has water inlet temperature = 10°C while the water flows out at the rate of 20 g/sec. The outlet temperature of water must be about
(A) 20°C
(B) 30°C
(C) 35°C
(D) 40°C

4. A continuous flow water heater (geyser) has an electrical power rating = 2 kW and efficiency of conversion of electrical power into heat = 80%. If water is flowing through the device at the rate of 100 cc/sec, and the inlet temperature is 10°C, the outlet temperature will be
(A) 12.2°C
(B) 13.8°C
(C) 20°C
(D) 16.5°C

5. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted the temperature of the flask and contents is 40°C. When a further 80 g of ice has been added and has all metled, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice. [Take $S_w = 1$ cal/gm°C]
(A) 3.8×10^5 J/kg
(B) 1.2×10^5 J/kg
(C) 2.4×10^5 J/kg
(D) 3.0×10^5 J/kg

6. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent.

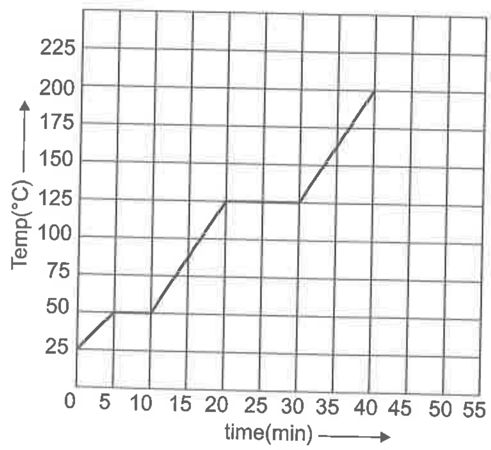


(A) latent heat of liquid
(B) latent heat of vapour
(C) heat capacity of vapour
(D) inverse of heat capacity of vapour

7. A block of ice with mass m falls into a lake. After impact, a mass of ice $m/5$ melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is
(A) $\frac{L}{5g}$
(B) $\frac{5L}{g}$
(C) $\frac{gL}{5m}$
(D) $\frac{mL}{5g}$

8. The specific heat of a metal at low temperatures varies according to $S = aT^3$ where a is a constant and T is abute temperature. The heat energy needed to raise unit mass of the metal from $T = 1$ K to $T = 2$ K is
(A) $3a$
(B) $\frac{15a}{4}$
(C) $\frac{2a}{3}$
(D) $\frac{12a}{5}$

9. The graph shown in the figure represent change in the temperature of 5 kg of a substance as it abosrbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vapourization of the substance is:



(A) 630 kJ kg^{-1} (B) 126 kJ kg^{-1}
(C) 84 kJ kg^{-1} (D) 12.6 kJ kg^{-1}

10. The density of a material A is 1500 kg/m^3 and that of another material B is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volumes of B. The ratio of specific heats of A and B will be

(A) 1:2 (B) 3:1
(C) 3:2 (D) 2:1

11. Find the amount of heat supplied to decrease the volume of an ice water mixture by 1 cm^3 without any change in temperature. ($\rho_{ice=}$ 0.9 $_{\rho water}$, L_{ice} = 80 cal/gm)

(A) 360 cal (B) 500 cal
(C) 720 cal (D) none of these

12. Some steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C so that the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensing. (in kg)

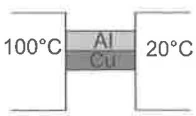
(A) 0.130 (B) 0.065
(C) 0.260 (D) 0.135

13. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B. Under steady state, the temperature difference across the whole wall is 36°C. Then the temperature difference across the layer A is

(A) 6°C (B) 12°C
(C) 18°C (D) 24°C

14. Two metal Cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure. (K_{CU} = 385 W/m-K, K_{AL} = 209 W/m-K) (K_{CU} = 385 W/m-K, K_{AL} = 209 W/m-K)

(A) The total thermal current from one reservoir to the other is:

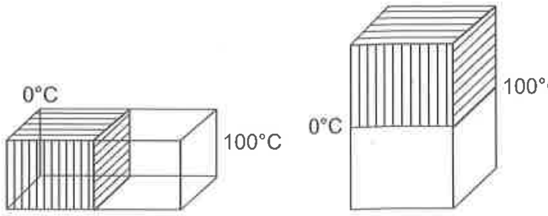


(A) 1.43 × 10^3 W (B) 2.53 × 10^3 W
(C) 1.53 × 10^4 W (D) 2.53 × 10^4 W

(B) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube is

(A) 1.79 (B) 1.69
(C) 1.54 (D) 1.84

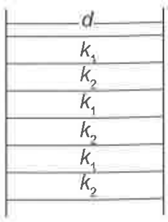
15. Two identical square rods of metal are welded end to end as shown in figure (A). Assume that 10 cal of heat flows through the rods in 2 min. Now the rods are welded as shown in figure. (B) The time it would take for 10 cal to flow through the rods now, is:



(A) (B)

(A) 0.75 min (B) 0.5 min
(C) 1.5 min (D) 1 min

16. A wall consists of alternating blocks with length 'd' and coefficient of thermal conductivity k_1 and k_2. The cross sectional area of the blocks are the same. The equivalent coefficient of thermal conductivity of the wall between left and right is



(A) $K_1 + K_2$ (B) $\frac{(K_1 + K_2)}{2}$

(C) $\frac{K_1 K_2}{K_1 + K_2}$ (D) $\frac{2K_1 K_2}{K_1 + K_2}$

17. A pot with a steel bottom 1.2 cm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m^2. The water inside the pot is at 100°C and 0.440 kg vapourise in every 5 minutes. The temperature of the lower surface of the pot, which is in contact with the stove is (Given: L_v = 2.256 × 10^6 J/kg and K_{steel} = 50.2 W/m-K)

(A) 105.3°C (B) 205.3°C
(C) 185.3°C (D) 115.3°C

18. A lake surface is exposed to an atmosphere where the temperature is < 0°C. If the thickness of the ice layer formed on the surface grows from 2 cm to 4 cm in 1 hour, The atmospheric temperature, T_a will be (Thermal conductivity of ice $K = 4 \times 10^{-3}$ cal/cm/s/°C; density of ice = 0.9 gm/cc. Latent heat of fusion of ice = 80 cal/gm. Neglect the change of density during the state change. Assume that the water below the ice has 0°c temperature every where)

(A) −20°C
(B) 0°C
(C) −30°C
(D) −15°C

19. One end of a 2.35 m long and 2.0 cm radius aluminium rod ($K = 235$ W.m^{-1}K^{-1}) is held at 20°C. The other end of the rod is in contact with a block of ice at its melting point. The rate in kg.s^{-1} at which ice melts is

(A) $48\pi \times 10^{-6}$
(B) $24\pi \times 10^{-6}$
(C) $2.4\pi \times 10^{-6}$
(D) $4.8\pi \times 10^{-6}$

[Take latent heat of fusion for ice as $\frac{10}{3} \times 10^5$ J.kg^{-1}]

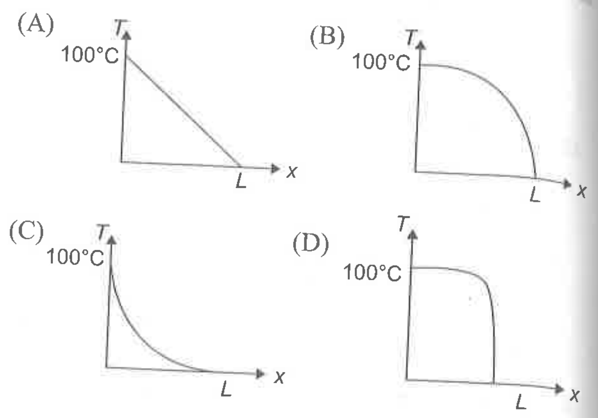
20. Four rods of same material with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat?

(A) $r = 2$ cm, $l = 0.5$ m
(B) r = 2 cm, $l = 2$ m
(C) $r = 0.5$ cm, $l = 0.5$ m
(D) r = 1 cm, $l = 1$ m

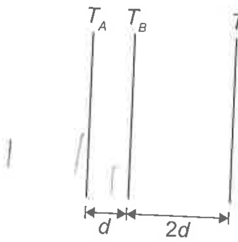
21. A cylinder of radius R made of a material of thermal conductivity k_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity k_2. The two ends of the combined system are maintained at different temperatures. There is no loss of heat from the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is

(A) $k_1 + k_2$
(B) $\frac{k_1 k_2}{k_1 + k_2}$
(C) $\frac{1}{4}(k_1 + 3k_2)$
(D) $\frac{1}{4}(3k + k_2)$

22. A rod of length L and uniform cross-sectional area has varying thermal conductivity which changes linearly from 2 K at end A to K at the other end B. The ends A and B of the rod are maintained at constant temperature 100°C and 0°C, respectively. At steady state, the graph of temperature: $T = T(x)$ where x = distance from end A will be

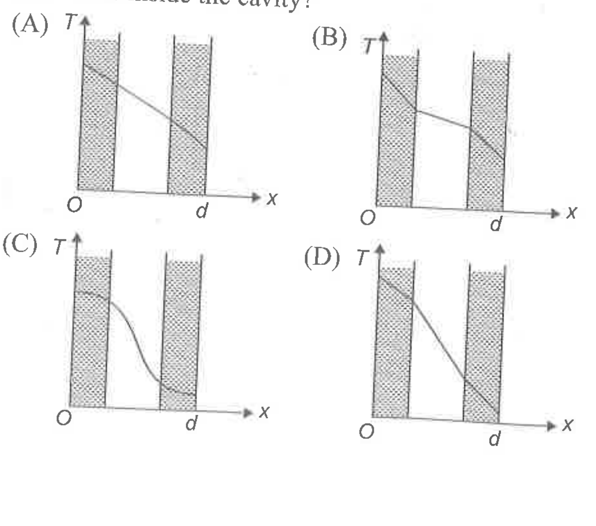


23. Two sheets of thickness d and $2d$ and same area are touching each other on their face. Temperature T_A, T_B, T_C shown are in geometric progression with common ratio $r = 2$. Then ratio of thermal conductivity of thinner and thicker sheet are

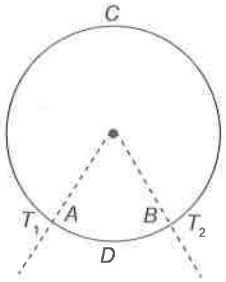


(A) 1
(B) 2
(C) 3
(D) 4

24. The wall with a cavity consists of two layers of brick separated by a layer of air. All three layers have the same thickness and the thermal conductivity of the brick is much greater than that of air. The left layer is at a higher temperature than the right layer and steady state condition exists. Which of the following graphs predicts correctly the variation of temperature T with distance d inside the cavity?



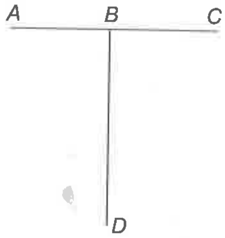
25. A ring consisting of two parts ADB and ACB of same conductivity k carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to $2H$. What should be the conductivity of the new ADB part? Given $\frac{ACB}{ADB} = 3$:



(A) $\frac{7}{3}k$ (B) $2k$

(C) $\frac{5}{2}k$ (D) $3k$

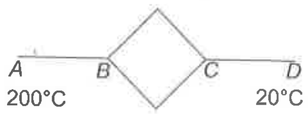
26. Three conducting rods of same material and cross-section are shown in figure. Temperatures of A, D and C are maintained at 20°C, 90°C and 0°C. The ratio of lengths of BD and BC if there is no heat flow in AB is:



(A) 2/7 (B) 7/2

(C) 9/2 (D) 2/9

27. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperature of 200°C and 20°C respectively. The temperature of junction B will be:



(A) 120°C (B) 100°C

(C) 140°C (D) 80°C

28. A metallic rod of cross-sectional area 9.0 cm² and length 0.54 m, with the surface insulated to prevent heat loss, has one end immersed in boiling water and the other in ice-water mixture. The heat conducted through the rod melts the ice at the rate of 1 gm for every 33 sec. The thermal conductivity of the rod is

(A) 330 $Wm^{-1}K^{-1}$ (B) 60 $Wm^{-1}K^{-1}$

(C) 600 $Wm^{-1}K^{-1}$ (D) 33 $Wm^{-1}K^{-1}$

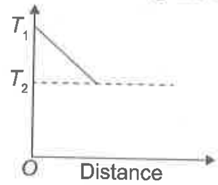
29. A hollow sphere of inner radius R and outer radius $2R$ is made of a material of thermal conductivity K. It is surrounded by another hollow sphere of inner radius $2R$ and outer radius $3R$ made of same material of thermal conductivity K. The inside of smaller sphere is maintained at 0°C and the outside of bigger sphere at 100°C. The system is in steady state. The temperature of the interface will be:

(A) 50°C (B) 70°C

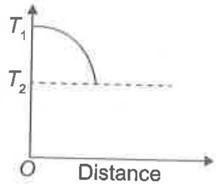
(C) 75°C (D) 45°C

30. The ends of a metal bar of constant cross-sectional area are maintained at temperatures T_1 and T_2 which are both higher than the temperature of the surroundings. If the bar is unlagged, which one of the following sketches best represents the variation of temperature with distance along the bar?

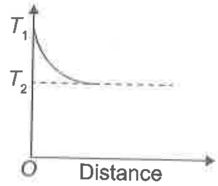
(A)



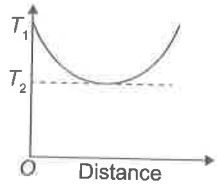
(B)



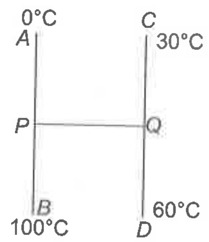
(C)



(D)

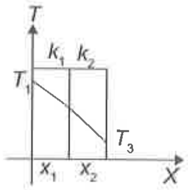


31. Three identical rods AB, CD and PQ are joined as shown. P an Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is



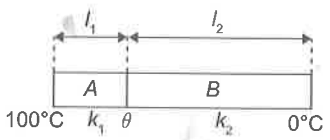
(A) from P to Q
(B) from Q to P
(C) heat does not flow in PQ
(D) data not sufficient

32. The temperature drop through each layer of two layer furnace wall is shown in figure. Assume that the external temperature T_1 and T_3 are maintained constant and $T_1 > T_3$. If the thickness of the layers x_1 and x_2 are the same, which of the following statements are correct.



(A) $k_1 > k_2$
(B) $k_1 < k_2$
(C) $k_1 = k_2$ but heat flow through material (1) is larger then through (2)
(D) $k_1 = k_2$ but heat flow through material (1) is less than that through (2)

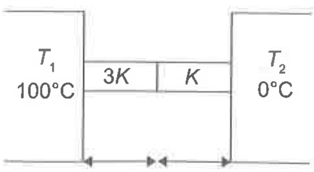
33. Two rods A and B of different materials but same cross section are joined as in figure. The free end of A is maintained at 100°C and the free end of B is maintained at 0°C. If $l_2 = 2l_1$, $K_1 = 2K_2$ and rods are thermally insulated from sides to prevent heat losses then the temperature θ of the junction of the two rods is



(A) 80°C
(B) 60°C
(C) 40°C
(D) 20°C

Question No. 34. to 36

Two rods A and B of same cross-sectional are A and length l connected in series between a source (T_1 = 100°C) and a sink (T_2 = 0°C) as shown in figure. The rod is laterally insulated



34. The ratio of the thermal resistance of the rod is

(A) $\frac{R_A}{R_B} = \frac{1}{3}$
(B) $\frac{R_A}{R_B} = 3$
(C) $\frac{R_A}{R_B} = \frac{3}{4}$
(D) $\frac{4}{3}$

35. If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B} = \frac{3}{1}$
(B) $\frac{T_A}{T_B} = \frac{1}{3}$
(C) $\frac{T_A}{T_B} = \frac{3}{4}$
(D) $\frac{T_A}{T_B} = \frac{4}{3}$

36. If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B} = \frac{3}{1}$
(B) $\frac{G_A}{G_B} = \frac{1}{3}$
(C) $\frac{G_A}{G_B} = \frac{3}{4}$
(D) $\frac{G_A}{G_B} = \frac{4}{3}$

37. Two sheets of thickness d and $3d$, are touching each other. The temperature just outside the thinner sheet side is A, and on the side of the thicker sheet is C. The interface temperature is B. A, B and C are in arithmetic progressing, the ratio of thermal conductivity of thinner sheet and thicker sheet is

(A) 1:3
(B) 3:1
(C) 2:3
(D) 1:9

38. A cylindrical rod with one end in a steam chamber and the outer end in ice results in melting of 0.1 gm of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is 1/4 that of first, the rate at which ice melts is gm/sec will be

(A) 3.2
(B) 1.6
(C) 0.2
(D) 0.1

39. A composite rod made of three rods of equal length and cross-section as shown in the fig. The thermal conductivities of the materials of the rods are $K/2$, $5K$ and K respectively. The end A and end B are at constant temperatures. All heat entering the face A goes out of the end B there being no loss of heat from the sides of the bar. The effective thermal conductivity of the bar is

(A) 15 K/16 (B) 6 K/13
(C) 5 K/16 (D) 2 K/13.

40. A rod of length L with sides fully insulated is of a material whose thermal conductivity varies with temperature as $K = \frac{\alpha}{T}$, where α is a constant. The ends of the rod are kept at temperature T_1 and T_2. The temperature T at x, where x is the distance from the end whose temperature is T_1 is

(A) $T_1\left(\frac{T_2}{T_1}\right)^{\frac{x}{L}}$ (B) $\frac{x}{L} ln \frac{T_2}{T_1}$

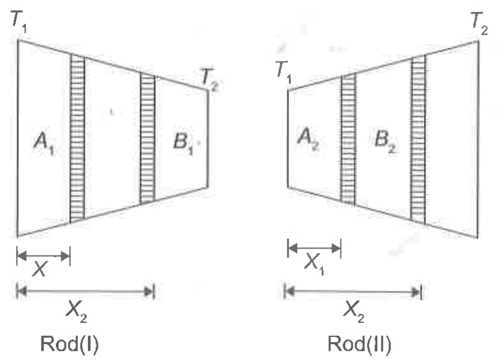
(C) $T_1 e^{\frac{T_2 x}{T_1 L}}$ (D) $T_1 + \frac{T_2 - T_1}{L}x$

41. Heat flows radially outward through a spherical shell of outside radius R_2 and inner radius R_1. The temperature of inner surface of shell is θ_1 and that of outer is θ_2. The radial distance from centre of shell where the temperature is just half way between θ_1 and θ_2 is:

(A) $\frac{R_1 + R_2}{2}$ (B) $\frac{R_1 R_2}{R_1 + R_2}$

(C) $\frac{2R_1 R_2}{R_1 + R_2}$ (D) $R_1 + \frac{R_2}{2}$

42. The two ends of two similar non-uniform rods of length λ each and thermal conductivity 'K' are maintained at different but constant temperature. The temperature gradient at any point on the rod is $\frac{\Delta T}{\Delta \ell}$. The heat flow per unit time through the rod is I: Given $T_1 > T_2$. Then which of the following is true:

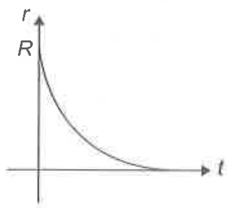


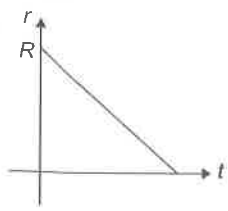
(A) I of Rod (I) = I of Rod (II)
(B) I of Rod (I) > I of Rod (II)
(C) I of Rod (I) < I of Rod (II)
(D) data is insufficient

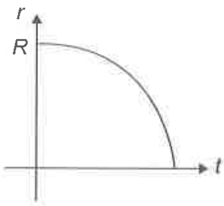
43. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

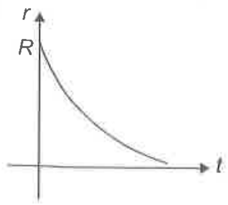
(A) 100 J/°C (B) 300 J/°C
(C) 750 J/°C (D) 1500 J/°C

44. A sphere of ice at 0°C having initial radius R is placed in an environment having ambient temperature > 0°C. The ice melts uniformly, such that shape remains spherical. After a time 't' the radius of the sphere has reduced to r. Assuming the rate of heat absorption is proportional to the surface area of the sphere at any moment, which graph best depicts $r(t)$.

(A)


(B)


(C)


(D)


45. The power radiated by a black body is P and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around wavelength $3/4\lambda_0$, the power radiated by it will increase by a factor of

(A) 4/3 (B) 16/9
(C) 64/27 (D) 256/81

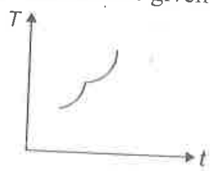
46. A black metal foil is warmed by radiation from a small sphere at temperature 'T' and at a distance 'd'. It is found that the power received by the foil is P. If both the temperature and distance are doubled, the power received by the foil will be:

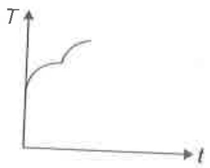
(A) $16P$ (B) $4P$
(C) $2P$ (D) P

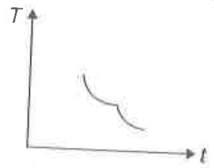
47. Star S_1 emits maximum radiation of wavelength 420 nm and the star S_2 emits maximum radiation of wavelength 560 nm, what is the ratio of the temperature of S_1 and S_2:
(A) 4/3
(B) $(4/3)^{1/4}$
(C) 3/4
(D) $(3/4)^{1/2}$

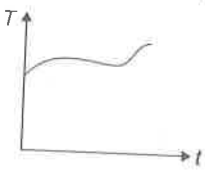
48. Spheres P and Q are uniformly constructed from the same material which is a good conductor of heat and the radius of Q is thrice the radius of P. The rate of fall of temperature of P is x times that of Q when both are at the same surface temperature. The value of x is:
(A) 1/4
(B) 1/3
(C) 3
(D) 4

49. An ice cube at temperature −20°C is kept in a room at temperature 20°C. The variation of temperature of the body with time is given by

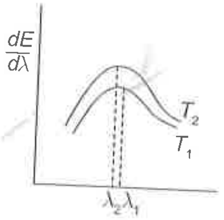
(A) 

(B) 

(C) 

(D) 

50. The spectral emissive power $E\lambda$ for a body at temperature T_1 is plotted against the wavelength and area under the curve is found to be A. At a different temperature T_2 the area is found to be $9A$ Then $\lambda_1/\lambda_2 =$



(A) 3
(B) 1/3
(C) $1/\sqrt{3}$
(D) $\sqrt{3}$

51. The intensity of radiation emitted by the Sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies then the ratio of the surface temperature of the Sun and the North Star is
(A) 1.46
(B) 0.69
(C) 1.21
(D) 0.83

52. Two bodies P and Q have thermal emissivities of ε_P and ε_Q respectively. Surface areas of these bodies are same and the total radiant power is also emitted at the same rate. If temperature of P is θ_P kelvin then temperature of Q i.e. θ_Q is

(A) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\theta_P$
(B) $\left(\frac{\varepsilon_P}{\varepsilon_Q}\right)^{1/4}\theta_P$
(C) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{1/4}\times\frac{1}{\theta_P}$
(D) $\left(\frac{\varepsilon_Q}{\varepsilon_P}\right)^{4}\theta_P$

53. A black body calorimeter filled with hot water cools from 60°C to 50°C in 4 min and 40°C to 30°C in 8 min. The approximate temperature of surrounding is
(A) 10°C
(B) 15°C
(C) 20°C
(D) 25°C

54. The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be
(A) $\frac{E}{16}$
(B) $\frac{E}{4}$
(C) $\frac{E}{8}$
(D) 0

55. A body cools from 75°C to 65°C in 5 minutes. If the room temperature is 25°, then the temperature of the body at the end of next 5 minutes is:
(A) 57°C
(B) 55°C
(C) 54°C
(D) 53°

56. The temperature of a body falls from 40°C to 36°C in 5 minutes. when placed in a surrounding of constant temperature 16°C. Then the time taken for the temperature of the body to become 32°C is
(A) 5 min
(B) 4.3 min
(C) 6.1 min
(D) 10.2 min.

JEE Advanced

1. From a black body, radiation is not:
   (A) emitted (B) absorbed
   (C) reflected (D) refracted

2. In accordance with Kirchhoff's law:
   (A) bad absorber is bad emitter
   (B) bad absorber is good reflector
   (C) bad reflector is good emitter
   (D) bad emitter is good absorber

3. The energy radiated by a body depends on:
   (A) area of body
   (B) nature of surface
   (C) mass of body
   (D) temperature of body

4. A hollow and a solid sphere of same material and identical outer surface are heated to the same temperature:
   (A) in the beginning both will emit equal amount of radiation per unit time.
   (B) in the beginning both will absorb equal amount of radiation per unit time
   (C) both spheres will have same rate of fall of temperature (dT/dt)
   (D) both spheres will have equal temperatures at any moment.

5. The rate of cooling of a body by radiation depends on:
   (A) area of body
   (B) mass of body
   (C) specific heat of body
   (D) temperature of body and surrounding.

6. A polished metallic piece and a black painted wooden piece are kept in open in bright sun for a long time:
   (A) the wooden piece will absorbs less heat than the metallic piece
   (B) the wooden piece will have a lower temperature than the metallic piece
   (C) if touched, the metallic piece will feel hotter than the wooden piece
   (D) when the two pieces are removed from the open to a cold room, the wooden piece will lose heat at a faster rate than the metallic piece

7. An experiment is perfomed to measure the specific heat of copper. A lump of copper is heated in an oven, then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all of the quantities below EXCEPT the
   (A) heat capacity of water and beaker
   (B) original temperature of the copper and the water
   (C) final (equilibrium) temperature of the copper and the water
   (D) time taken to achieve equilibrium after the copper is dropped into the water

8. One end of a conducting rod is maintained at temperature 50°C and at the other end, ice is melting at 0°C. The rate of melting of ice is doubled if:
   (A) the temperature is made 200°C and the area of cross-section of the rod is doubled
   (B) the temperature is made 100°C and length of rod is made four times
   (C) area of cross-section of rod is halved and length is doubled
   (D) the temperature is made 100°C and the area of cross-section of rod and length both are doubled.

9. Two metallic sphere A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.
   (A) The ratio of heat loss of A to that of B is $2^{4/3}$
   (B) The ratio of heat loss of A to that of B is $2^{2/3}$
   (C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$
   (D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$

10. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

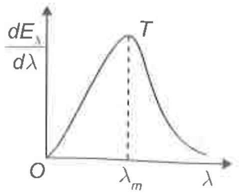
(A) the temperature of B is 1934 K
(B) $\lambda_B = 1.5\ \mu m$
(C) the temperature of B is 11604 K
(D) the temperature of B is 2901 K

11. Three bodies A, B and C have equal surface area and thermal emissivities in the ratio $e_A : e_B : e_C = 1 : \frac{1}{2} : \frac{1}{4}$. All the three bodies are radiating at same rate. Their wavelengths corresponding to maximum intensity are λ_A, λ_B and λ_C respectively and their temperature are T_A, T_B and T_C on kelvin scale, then select the incorrect statement.

(A) $\sqrt{T_A T_C} = T_B$
(B) $\sqrt{\lambda_A \lambda_C} = \lambda_B$
(C) $\sqrt{e_A T_A}\sqrt{e_C T_C} = e_B T_B$
(D) $\sqrt{e_A \lambda_A T_A \cdot e_B \lambda_B T_B} = e_C \lambda_C T_C$

Question No. 12 to 14

The figure shows a radiant energy spectrum graph for a black body at a temperature T.



12. Choose the correct statement(s)

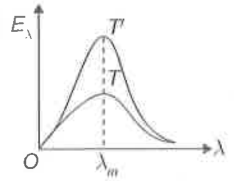
(A) The radiant energy is not equally distributed among all the possible wavelengths
(B) For a particular wavelength the spectral intensity is maximum
(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature
(D) None of these

13. If the temperature of the body is raised to a higher temperature T', then choose the correct statement(s)

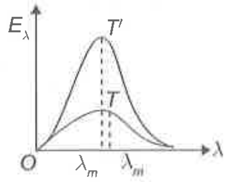
(A) The intensity of radiation for every wavelength increases
(B) The maximum intensity occurs at a shorter wavelength
(C) The area under the graph increases
(D) The area under the graph is proportional to the fourth power of temperature

14. Identify the graph which correctly represents the spectral intensity versus wavelength graph at two temperatures T' and $T(T < T')$

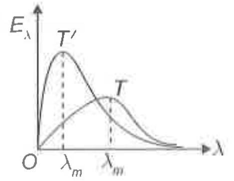
(A)



(B)



(C)



(D) none of these

JEE Advanced

Level I

1. In following equation calculate the value of H. 1 kg steam at 200°C = H + 1 Kg water at 100°C (S_{steam} = Constant = .5 cal/gm°C)

2. From what height should a piece of ice (0°C) fall so that it melts completely? Only one quarter of the heat produced is absorbed by the ice. The latent heat of ice is 3.4×10^5 J kg^{-1} and g is 10 N kg^{-1}.

3. A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down through 60 cm. Specific heat capacity of copper = 420 J/kg-K.

4. 10 gm ice at −10°C, 10 gm water at 20°C and 2 g steam at 100°C are mixed with each other then final equilibrium temperature.

5. Materials A, B and C are solids that are at their melting temperatures. Material A requires 200 J to melt 4 kg, material B requires 300 J to melt 5 kg, and material C requires 300 J to melt 6 kg. Rank the materials according to their heats of fusion, greatest first.

6. In a thermally iated container, material A of mass m is placed against material B, also of mass m but at higher temperature. When thermal equilibrium is reached, the temperature changes ΔT_A and ΔT_B of A and B are recorded. Then the experiment is repeated, using A with other materials. All of the same mass m. The results are given in the table. Rank the four materials according to their specific heats, greatest first.

Experiment	**Temperature**	**Changes**
1.	$\Delta T_A = +50C°$	$\Delta T_B = -50C°$
2.	$\Delta T_A = +10C°$	$\Delta T_C = -20C°$
3.	$\Delta T_A = +2C°$	$\Delta T_D = -40C°$

7. Indian style of cooling drinking water is to keep it in a pitcher having porous walls. Water comes to the outer surface very slowly and avaporates. Most of the energy needed for evaporation is taken from the water itself and the water is cooled down. Assume that a pitcher contains 10 kg of water and 0.2 g of water comes out per second. Assuming no backward heat transfer from the atmosphere to the water, calculate the time in which the temperature decreases by 5°C. Specific heat capacity of water = 4200 J/kg-°C and latent heat of vaporization of water = 2.27×10^6 J/kg.

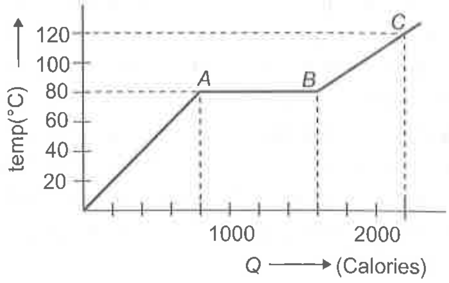
8. An aluminium container of mass 100 gm contains 200 gm of ice at −20°C. Heat is added to the system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice = 0.5 and $L = 80$ cal/gm, specific heat of $Al = 0.2$ cal/gm/°C)

9. A volume of 120 ml of drink (half alcohol + half water by mass) originally at a temperature of 25°C is cooled by adding 20 gm ice at 0°C. If all the ice melts, find the final temperature of the drink.(density of drink = 0.833 gm/cc, specific heat of alcohol = 0.6 cal/gm/°C)

10. Two identical calorimeter A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal $g^{-1}(C°)^{-1}$ is dropped into A and a 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metals is 40°C. Find the specific heat of metal Y in cal $g^{-1}(C°)^{-1}$.

11. Two 50 gm ice cubes are dropped into 250 gm of water into a glass. If the water was initially at a temperature of 25°C and the temperature of ice −15°C. Find the final temperature of water. (specific heat of ice = 0.5 cal/gm/°C and $L = 80$ cal/gm). Find final amount of water and ice.

12. A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph

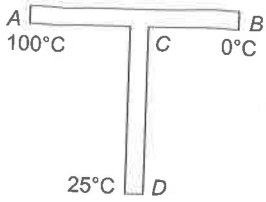


(i) the mass of the substance;
(ii) the specific latent heat of the melting process, and
(iii) the specific heat of the substance in the liquid state.

13. A uniform slab of dimension 10 cm × 10 cm × 1 cm is kept between two heat reservoirs at temperatures 10°C and 90°C. The larger surface areas touch the reservoirs. The thermal conductivity of the material is 0.80 W/m-°C. Find the amount of heat flowing through the slab per second.

14. One end of a steel rod ($K = 42$ J/m-s-°C) of length 1.0 m is kept in ice at 0°C and the other end is kept in boiling water at 100°C. The area of cross-section of the rod is 0.04 cm². Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice = 3.36×10^5 J/kg.

15. A rod CD of thermal resistance 5.0 K/W is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100°C, 0°C and 25°C respectively. Find the heat current in CD.



(A) 5°C
(B) 6°C
(C) 4°C
(D) 7°C

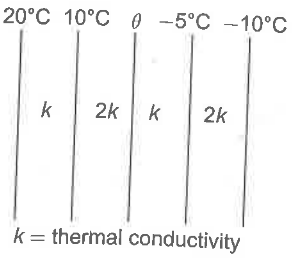
16. A semicircular rod is joined at its end to a straight rod of the same material and same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time.

17. One end of copper rod of uniform cross-section and of length 1.45 m is in contact with ice at 0°C and the other end with water at 100°C. Find the position of point along its length where a temperature of 200°C should be maintained so that in steady state the mass of ice melting is equal to that of steam produced in the same interval of time [Assume that the whole system is insulated from surroudings]. [take $L_v = 540$ cal/g $L_f = 80$ cal/g]

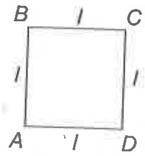
18. Three slabs of same surface area but different conductivities k_1, k_2, k_3 and different thickness t_1, t_2, t_3 are placed in close contact. After steady state his combination behaves as a single slab. Find is effective thermal conductivity.

19. A thin walled metal tank of surface area 5 m² is filled with water tank and contains an immersion heater dissipating 1 kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25°C. Find the temperature of the tank in the steady state.

20. The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.

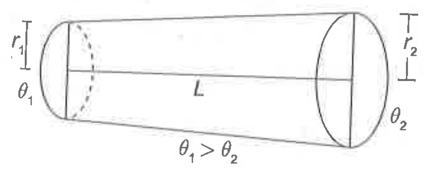


21. In the square frame of side l of metallic rods, the corners A and C are maintained at T_1 and T_2 respectively. The rate of heat flow from A to C is ω. If A and D are instead maintained T_1 and T_2 respectively find, find the total rate of heat flow.



22. A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 160π Joule of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

23. Find the rate of heat flow through a cross-section of the rod shown in figure ($\theta_2 > \theta_1$). Thermal conductivity of the material of the rod is K.



24. A metal rod of cross-sectional area 1.0 cm² is being heated at one end. At one time, the temperature gradient is 5.0°C/cm at cross-section A and is 2.6°C/cm at cross-section B. Calculate the rate at which the temperature is increasing in the part AB of the rod. The heat capacity of the part AB = 0.40 J/°C, thermal conductivity of the material of the rod = 200 W/m-°C. Neglect any loss of heat to the atmosphere.

25. A rod of negligible heat capacity has length 20 cm, area of cross-section 1.0 cm² and thermal conductivity 200 W/m-°C. The tempeature of one end is maintained at 0°C and that of the other end is slowly and linearly varied from 0°C to 60°C in 10 minutes. Assuming no loss of heat through the sides, find the total heat transmitted through the rod in these 10 minutes.

26. A pan filled with hot food cools from 50.1°C to 49.9°C in 5 sec. How long will it take to cool from 40.1°C to 39.9°C if room temperature is 30°C ?

27. A solid copper cube and sphere, both of same mass and emissivity are heated to same initial temperature and kept under identical conditions. What is the ratio of their initial rate of fall of temperature?

28. Two spheres of same radius R have their densities in the ratio 8:1 and the ratio of their specific heats are 1:4. If by radiation their rates of fall of temperature are same, then find the ratio of their rates of losing heat.

29. The maximum wavelength in the energy distribution spectrum of the sun is at 4753Å and its temperature is 6050 K. What will be the temperature of the star whose energy distribution shows a maximum at 9506Å.

30. A black body radiates 5 watts per square cm of its surface area at 27°C. How much will it radiate per square cm at 327°C.

31. A 100 W bulb has tungsten filament of total length 1.- m and radius 4×10^{-5} m. The emissivity of the filament is 0.8 and $\sigma = 6.0 \times 10^{-8}$ W/m^2-K^4. Calculate the temperature of the filament when the bulb is operating at correct wattage.

32. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

33. During a certain duration in the day, the earth is in radiative equilibrium with the sun. Find the surface temperature of the earth during that duratian.

[**Given,** radius of sun = 6.9×10^8 m surface temperature of sun = 6000 K and the distance of earth from the sun = 1.49×10^{11} m. Assume that the sun and earth behave as black bodies.]

34. Estimate the temperature at which a body may appear blue or red. The values of λ_{mean} for these are 5000 and 7500Å respectively. [Given Wein's constant b = 0.3 cm K]

35. Find the quantity of energy radiated from 1 cm^2 of a surface in one second by a black body if the maximum energy density corresponds to a wavelength of 5000Å (b = 0.3 cm K and $\sigma = 5.6 \times 10^{-8}$ w/m^{2}k^4)

36. The following observations have been noted for a black body spectrum, taken for T = 500 K. Calculate the value of λ_m at T = 1000 K.

λ (in μm)	10	8	6	4
E_λ (in SI units)	10	14	16	12

37. A liquid cools from 70°C to 60°C is 5 minutes. Find the time` in which it will further cool down to 50°C, if its surrounding is held at a constant temperature of 30°C

38. A body cools down from 50°C to 45°C in 5 minutes and to 40°C in another 8 minutes. Find the temperature of the surrounding.

Level II

1. A copper calorimeter of mass 100 gm contains 200 gm of a mixture of ice and water. Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50°C. If the mass of the calorimeter and its contents is now 330 gm, what was the ratio of ice and water in beginning? Neglect heat losses.

Given:

Specific heat capacity of copper

$= 0.42 \times 10^3$ J kg^{-1} K^{-1},

Specific heat capacity of water

$= 4.2 \times 10^3$ J kg^{-1} K^{-1},

Specific heat of fusion of ice

$= 3.36 \times 10^5$ J kg^{-1}

Latent heat of condensation of steam

$= 22.5 \times 10^5$ J kg^{-1}

2. A solid substance of mass 10 gm at −10°C was heated to −2°C (still in the solid state). The heat required was 64 calories. Another 880 calories was required to raise the temperature of the substance (now in the liquid state) to 1°C, while 900 calories was required to raise the temperature from $-2°C$ to $3°C$. Calculate the specific heat capacities of the substance in the solid and liquid state in calories per kilogram per kelvin. Show that the latent heat of fusion L is related to the melting point temperature t_m by $L = 85400 + 200t_m$.

3. A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If the entire mechanical work is used up in producing heat and the rate of raise in temperature of the block and the drill is 0.5°C/s. Find
(A) the rate of working of the drill in watts, and
(B) the torque required to drive the drill.

Specific heat of steel = 0.1 and $J = 4.2$ J/cal. Use: $P = \tau\omega$

4. A flow calorimeter is used to measure the specific heat of a liquid. Heat is added at a known rate to a stream of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow points of the liquid stream enables us to compute the specific heat of the liquid. A liquid of density 0.2 g/cm³ flows through a calorimeter at the rate of 10 cm³/s. Heat is added by means of a 250-W electric heating coil, and a temperature difference of 25°C is established in steady-state conditions between the inflow and the outflow points. Find the specific heat of the liquid.

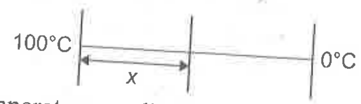
5. Ice at $-20°C$ is filled upto height $h = 10$ cm in a uniform cylindrical vessel. Water at temperature θ°C is filled in another identical vessel upto the same height $h = 10$ cm. Now, water from second vessel is poured into first vessel and it is found that level of upper surface falls through $\Delta h = 0.5$ cm when thermal equilibrium is reached. Neglecting thermal capacity of vessels, change in density of water due to change in temperature and loss of heat due to rediation, calculate initial temperature θ of water.
Given, Density of water, $\rho_w = 1$ gm cm^{-3}
Density of ice, $\rho_i = 0.9$ gm/cm³
Specific heat of water, $s_w = 1$ cal/gm°C
Specific heat of ice $s_i = 0.5$ cal/gm°C
Specific latent heat of ice, $L = 80$ cal/gm

6. A composite body consists of two rectangular plates of the same dimensions but different thermal conductivities K_A and K_B. This body is used to transfer heat between two objects maintained at different temperatures. The composite body can be placed such that flow of heat takes place either parallel to the interface or perpendicular to it. Calculate the effective thermal conductivities $K_{||}$ and $K_{\perp}$ of the composite body for the parallel and perpendicular orientations. Which orientation will have more thermal conductivity ?

7. A highly conducting solid cylinder of radius a and length l is surrounded by a co-axial layer of a material having thermal conductivity K and negligible heat capacity. Temperature of surrounding space (out side the layer) is T_0, which is higher than temperature of the cylinder. If heat capacity per unit volume of cylinder material is s and outer radius of the layer is b, calculate time required to increase temperature of the cylinder from T_1 to T_2. Assume end faces to be thermally insulated.

8. A vertical brick duct (tube) is filled with cast iron. The lower end of the duct is maintained at a temperature T_1 which is greater than the melting point T_m of cast iron and the upper end at a temperature T_2 which is less than the temperature of the melting point of cast iron. It is given that the conductivity of liquid cast iron is equal to k times the conductivity of solid cast iron. Determine the fraction of the duct filled with molten metal.

9. A lagged stick of cross section area 1 cm² and length 1 m is initially at a temperature of 0°C. It is then kept between 2 reservoirs of temperature 100°C and 0°C. Specific heat capacity is 10 J/kg°C and linear mass density is 2 kg/m. Find



(A) temperature gradient along the rod in steady state.
(B) total heat absorbed by the rod to reach steady state.

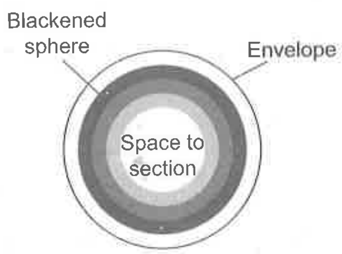
10. A cylindrical block of length 0.4 m an area of cross-section 0.04 m² is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purposes of calculation, the

thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

11. A solid copper sphere cools at the rate of 2.8°C per minute, when its temperature is 127°C. Find the rate at which another solid copper sphere of twice the radius lose its temperature at 327°C, if in both the cases, the room temperature is maintained at 27°C.

12. End A of a rod AB of length $L = 0.5$ m and of uniform cross-sectional area is maintained at some constant temperature. The heat conductivity of the rod is $k =$ 17 J/s-m°K. The other end B of this rod is radiating energy into vaccum and the wavelength with maximum energy density emitted from this end is $\lambda_0 = 75000$Å. If the emissivity of the end B is $e = 1$, determine the temperature of the end A. Assuming that except the ends, the rod is thermally insulated.

13. The shell of a space station is a blackened sphere in which a temperature $T = 500$ K is maintained due to operation of appliances of the station. Find the temperature of the shell if the station is enveloped by a thin spherical black screen of nearly the same radius as the radius of the shell.

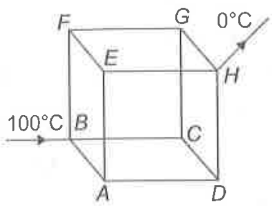


14. A liquid takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of surrounding is 20°C. Use exact method.

15. A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the faulty barometer reads 69 cm and 70 cm respectively.
    (i) What is the total length of the barometer tube ?
    (ii) What is the true reading when the faulty barometer reads 69.5 cm?
    (iii) What is the faulty barometer reading when the true barmeter reads 74 cm?

16. A vessel of volume $V = 30l$ is separated into three equal parts by stationary semipermeable thin membrances as shown in the Figure. The left, middle and right parts are filled with $m_{H_2} = 30\,g$ of hydrogen, $m_{O_2} = 160\,g$ of oxygen, and $m_{N_2} = 70\,g$ of nitrogen respectively. The left partition lets through only hydrogen, while the right partition lets through hydrogen and nitrogen. What will be the pressure in each part of the vessel after the equilibrium has been set in if the vessel is kept at a constant temperature $T =$ 300 K?

17. Twelve conducting rods form the riders of a uniform cube of side 'l'. If in steady state, B and H ends of the rod are at 100°C and 0°C. Find the temperature of the junction A.



Previous Year Questions

JEE Main

1. Which of the following is more close to a black body ?
   (A) Black board paint
   (B) Green leaves
   (C) Black holes
   (D) Red roses

2. Infrared radiations are detected by **[AIEEE 2002]**
   (A) spectrometer
   (B) pyrometer
   (C) nanometer
   (D) photometer

3. Heat given to a body which raises its temperature by 1°C is **[AIEEE 2002]**
(A) water equivalent
(B) thermal capacity
(C) specific heat
(D) temperature gradient

4. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will **[AIEEE 2002]**
(A) Increase
(B) Decrease
(C) Remain same
(D) Decrease for some, while increase for others

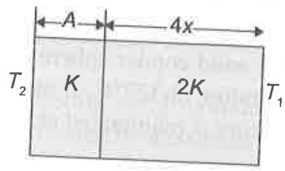
5. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is: **[AIEEE 2002]**
(A) 1:1
(B) 16:1
(C) 4:1
(D) 1:9

6. If mass-energy equivalence is taken ito account, when water is cooled to form ice, the mass of water should **[AIEEE 2002]**
(A) increase
(B) remain unchanged
(C) decrease
(D) first increase then decrease

7. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference of the temperature of the body and the surrounding, the n is equal to **[AIEEE 2003]**
(A) 2
(B) 3
(C) 4
(D) 1

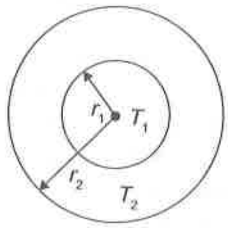
8. If the termperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be **[AIEEE 2004]**
(A) 4
(B) 16
(C) 32
(D) 64

9. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 $(T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equal to **[AIEEE 2004]**



(A) 1
(B) 1/2
(C) 2/3
(D) 1/3

10. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperature T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres, is proportional to **[AIEEE 2005]**

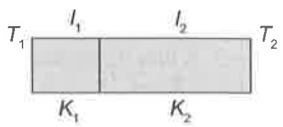


(A) $\frac{(r_2 - r_1)}{(r_1 r_2)}$
(B) $In\left(\frac{r_2}{r_1}\right)$
(C) $\frac{r_1 r_2}{(r_2 - r_1)}$
(D) $(r_2 - r_1)$

11. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun **[AIEEE 2006]**
(A) $\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$
(B) $\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$
(C) $\frac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$
(D) $\frac{R^2 \sigma T^4}{r^2}$

where r_0 is the radius of the earth and σ is Stefan's constant.

12. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of length l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is **[AIEEE 2007]**



(A) $(K_2l_2T_1 + K_1l_1T_2)/(K_1l_1 + K_2l_2)$

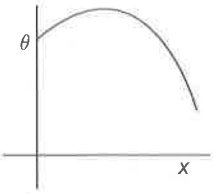
(B) $(K_2l_1T_1 + K_1l_2T_2)/(K_2l_1 + K_1l_2)$

(C) $(K_1l_2T_1 + K_2l_1T_2)/(K_1l_2 + K_2l_1)$

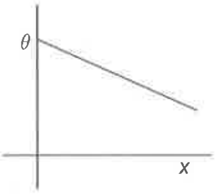
(D) $(K_1l_1T_1 + K_2l_2T_2)/(K_1l_1 + K_2l_2)$

13. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figure. **[AIEEE 2009]**

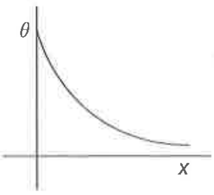
(A)



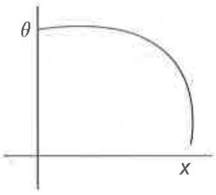
(B)



(C)

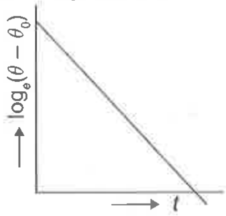


(D)

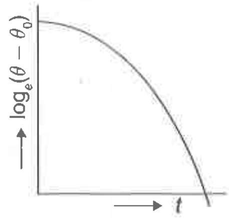


14. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surrounding, then according to Newton's law of cooling, the correct graph between $\log_e(\theta - \theta_0)$ and t is **[AIEEE 2012]**

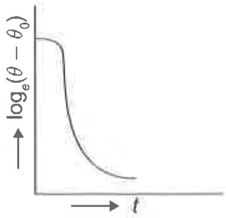
(A)



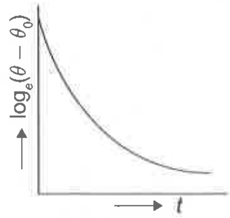
(B)



(C)



(D)



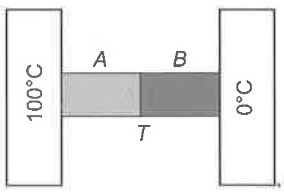
15. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is: **[JEE Main 2014]**

(A) 4.8 cal/s (B) 6.0 cal/s
(C) 1.2 cal/s (D) 2.4 cal/s

JEE Advanced

1. The temperature of 100 gm of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose. **[JEE 1996]**

2. Two metal cubes A and B of same size are arranged as shown in figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C respectively. After steady state is reached the temperature T of the interface will be ________



[JEE 1996]

3. A double pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass interface and the glass outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat flow through the window pane. Also find the temperatures of other interfaces. Given thermal conductivities of glass and air as 0.8 and 0.08 Wm^{-1}K^{-1} respectively. **[JEE 1997]**

4. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(A) 225 (B) 450
(C) 900 (D) 1800

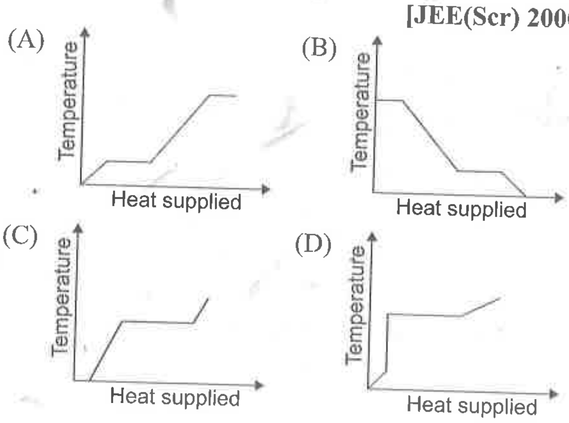
5. Earth receives 1400 W/m^2 of ar power. If all the ar energy falling on a lens of area 0.2 m^2 is focussed on to a block of ice of mass 280 grams, the time taken to melt the ice will be ________ minutes. (Latent heat of fusion of ice = 3.3×10^5 J/kg) **[JEE 1997]**

6. A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time $t = 0$, the temperature of X is $T_0 = 400$ K. It cools according to Newton's law of cooling. At time t_1 its temperature is found to be 350 K. At this time t_1, the body X is connected to a larger body Y at atmospheric temperature T_A, through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$ **[JEE 1998]**

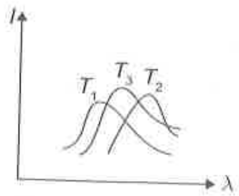
7. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant $b = 2.88 \times 10^6$ nm K. Then **[JEE 1998]**

(A) $U_1 = 0$ (B) $U_3 = 0$
(C) $U_1 > U_2$ (D) $U_2 > U_1$

8. A block of ice at -10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively? **[JEE(Scr) 2000]**

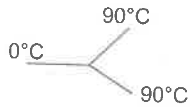


9. The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that **[JEE(Scr) 2000]**



(A) $T_1 > T_2 > T_3$ (B) $T_1 > T_3 > T_2$
(C) $T_2 > T_3 > T_1$ (D) $T_3 > T_2 > T_1$

10. Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be **[JEE(Scr) 2001]**



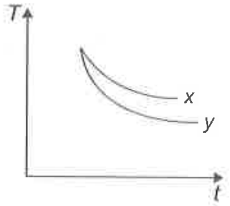
(A) 45°C (B) 60°C
(C) 30°C (D) 20°C

11. An ideal black body at room temperature is thrown into a furnace. It is observed that **[JEE(Scr) 2002]**
(A) initially it is the darkest body and at later times the brightest.
(B) it the darkest body at all times
(C) it cannot be distinguished at all times.
(D) initially it is the darkest body and at later times it cannot be distinguished.

12. An ice cube of mass 0.1 kg at 0°C is placed in an iated container which is at 227°C. The specific heat S of the container varies with temperature T according the empirical relation$s = A + BT$, where $A = 100$ cal/kg-K and $B = 2 \times 10^{-2}$ cal/kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion for water = 8×10^4 cal/kg. Specific heat of water = 10^3 cal/kg-K) **[JEE 2001]**

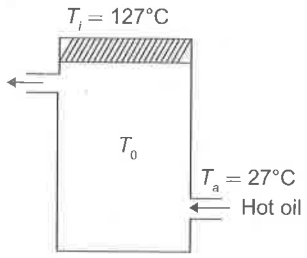
13. 2 kg ice at -20°C is mixed with 5 kg water at 20°C. Then final amount of water in the mixture would be; Given specific heat of ice = 0.5 cal/g°C, specific heat of water = 1 cal/g°C, **[JEE(Scr) 2003]**
Latent heat of fusion of ice = 80 cal/g.
(A) 6 kg (B) 5 kg
(C) 4 kg (D) 2 kg

14. If emissivity of bodies X and Y are e_x and e_y and absorptive power are A_x and A_y then



(A) $e_y > e_x ; A_y > A_x$
(B) $e_y < e_x ; A_y < A_x$
(C) $e_y > e_x ; A_y < A_x$
(D) $e_y = e_x ; A_y = A_x$

15. Hot oil is circulated through an insulated container with a wooden lid at the top whose $t = 5$ mm, emissivity = 0.6. Temperature of the top of the lid in steady state is at $T_l = 127°$. If the ambient temperature $T_a = 27°C$. Calculate **[JEE 2003]**

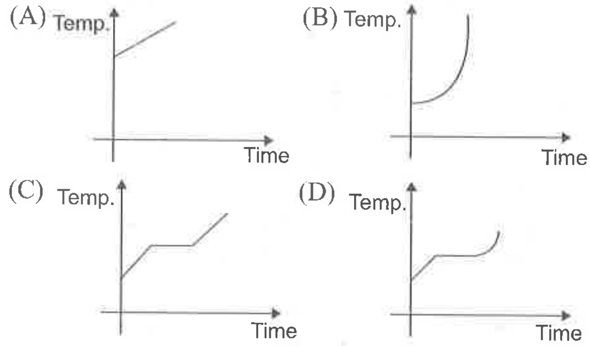


(A) rate of heat loss per unit area due to rediation from the lid.
(B) temperature of the oil. (Given $\sigma = \frac{17}{3} \times 10^{-8}$)

16. Three discs A, B, and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively. **[JEE(Scr) 2004]**
(A) Q_A is maximum
(B) Q_B is maximum
(C) Q_C is maximum
(D) $Q_A = Q_B = Q_C$

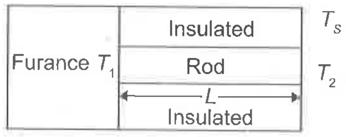
17. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of of ice in the two cases respectively. The ratio q_2/q_1 is **[JEE(Scr) 2004]**
(A) 1/2
(B) 2/1
(C) 4/1
(D) 1/4

18. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the variation of temperature with time **[JEE(Scr) 2004]**

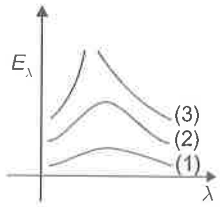


19. A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l, showing all the steps. **[JEE 2004]**

20. One end of a Rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$ where $\Delta T << T_S$, T_S being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_S)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2. **[JEE 2004]**



21. Three graphs marked as 1, 2, 3 representing the variation of maximum emissive power and wavelength of radiation of the sun, a welding arc and a tungsten filament. Which of the following combination is correct **[JEE(Scr) 2005]**



(A) 1-bulb, 2 → welding arc, 3 → sun
(B) 2-bulb, 3 → welding arc, 1 → sun
(C) 3-bulb, 1 → welding arc, 2 → sun
(D) 2-bulb, 1 → welding arc, 3 → sun

22. In which of the following phenomenon heat convection does not take place **[JEE(Scr) 2005]**
(A) land and sea breeze
(B) boiling of water
(C) heating of glass surface due to filament of the bulb
(D) air around the furance

23. 2 litre water at 27°C is heated by a 1 kW heater in an open container. On an average heat is lost to surroundings at the rate 160 J/s. The time required for the temperature to reach 77°C is **[JEE(Scr) 2005]**
(A) 8 min 20 sec (B) 10 min
(C) 7 min (D) 14 min

24. A spherical body of area A, and emissivity $e = 0.6$ is kept inside a black body. What is the rate at which energy is radiated per second at temperature T **[JEE(Scr) 2005]**
(A) $0.6\,\sigma AT^4$ (B) $0.4\,\sigma AT^4$
(C) $0.8\,\sigma AT^4$ (D) $1.0\,\sigma AT^4$

25. 1 calorie is the heat required to increased the temperature of 1 gm of water by 1°C from **[JEE(Scr) 2005]**
(A) 13.5° C to 14.5°C at 76 mm of Hg
(B) 14.5°C to 15.5°C at 760 mm of Hg
(C) 0°C to 1°C at 760 mm of Hg
(D) 3°C to 4°C to 760 mm of Hg

26. In a dark room with ambient temperature T_0, a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room Assuming that there is no change in the ambient temperature of the room, which of the following statement(s) is/are correct? **[JEE 2006]**
(A) The quantity of radiation absorbed by the black body in unit time will increase.
(B) Since emissivity = absorptivity, hence the quantity of radiation emitted by black body in unit time will increase.
(C) Black body radiates more energy in unit time in the visible spectrum.
(D) The reflected energy in unit time by the black body remains same.

27. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Then, find the final temperature of the mixture.
Given, $L_{fusion} = 80$ cal/g $= 336$ J/g, $L_{vaporization} =$ 540 cal/g $= 2268$ J/g,
$S_{ice} = 2100$ J/kg K $= 0.5$ cal/gK and $S_{water} = 4200$ J/kg K $= 1$ cal/gK **[JEE 2006]**

28. Column I gives some devices and Column II gives some processes on which the functioning of these devices depend. Match the devices in Column I with the processes in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the *ORS*. **[JEE 2007]**

Column I	Column II
(A) Bimetallic strip	(P) Radiation from a hot body
(B) Steam engine	(Q) Energy conversion
(C) Incandescent lamp	(R) Melting
(D) Electric fuse	(S) Thermal exapansion of solids

29. A metal rod AB of length $10x$ has its one end A in ice at 0°C, and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal g^{-1} and latent heat of melting of ice is 80 cal g^{-1} . If the point P is at a distance of lx from the ice end A, find the value of l. [Neglect any heat loss to the surrounding. **[JEE 2009]**

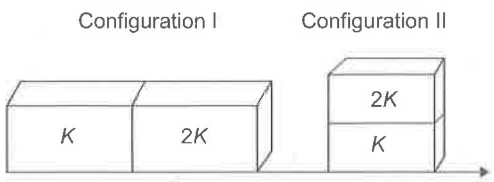
30. A piece of ice (heat capacity $= 2100$ J kg^{-1} °C^{-1} and latent heat $= 3.36 \times 10^5$ J kg^{-1}) of mass m grams is at -5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is: **[JEE 2010]**

31. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[JEE 2012]**

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}} T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}} T$

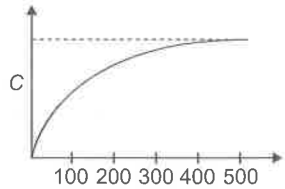
(C) $\left(\frac{97}{2}\right)^{\frac{1}{4}} T$ (D) $(97)^{\frac{1}{4}} T$

32. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity K and the other 2 K. The temperature difference between the ends along the x-axis is the same in both the configurations. it takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amout of heat in the configuration II is **[JEE 2013]**



(A) 2.0 s (B) 3.0 s
(C) 4.5 s (D) 6.0 s

33. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **[JEE 2013]**



(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.
(B) heat absorbed in increasing the temperature from 0 to 100 K is less than the heat required for increasing the temperature from 400 to 500 K.
(C) there is no change in the rate of heat absorption in the range 400–500 K.
(D) the rate of heat absorption increases in the range 200–300 K.

34. Parallel rays of light of intensity $I = 912\ \text{Wm}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temeprature of the black body is close to
(A) 330 K (B) 660 K
(C) 990 K (D) 1550 K

ANSWER KEYS

Exercises

JEE Main

1. D	2. A	3. C	4. B	5. A	6. D	7. A	8. B	9. C	10. D
11. C	12. A	13. B	14. (A) A	(B) D	15. B	16. B	17. A	18. C	19. C
20. A	21. C	22. B	23. A	24. D	25. A	26. B	27. C	28. B	29. C
30. C	31. A	32. A	33. A	34. A	35. B	36. B	37. A	38. C	39. A
40. A	41. B	42. A	43. D	44. B	45. D	46. B	47. A	48. C	49. B
50. D	51. B	52. B	53. B	54. A	55. A	56. C			

JEE Advanced

1. C, D	2. A, B, C	3. A, B, D	4. A, B	5. A, B, C, D	6. C, D	7. D	8. D
9. A, C	10. A, B	11. D	12. A, B	13. A, B, C, D	14. B		

JEE Advanced

Level I

1. $H = 590$ Kcal. **2.** 136 km **3.** 8.6×10^{-3} °C **4.** $\Delta\theta = \frac{315}{11}$ °C = 28.66°C **5.** $L_B > L_{A} = L_C$
6. $S_A = S_B > S_C > S_D$ **7.** $\frac{1050}{2.27}$s = 7.7 min **8.** 25.5°C **9.** 4°C **10.** 27/85
11. 0°C, 125/4 g ice, 1275/4 g water **12.** (i) 0.02 kg, (ii) 40,000 cal kg^{-1}, (iii) 750 cal kg^{-1} K^{-1} **13.** 64 J
14. 5×10^{-5} g/s **15.** 4.0 W **16.** 2:π **17.** 10 cm from end in contact with water at
18. $\dfrac{t_1 + t_2 + t_3}{\dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \dfrac{t_3}{k_3}}$ **19.** 65°C **20.** 5°C **21.** 4/3 ω **22.** 15 W/m-°C **23.** $\dfrac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$ **24.** 12 °C/s
25. 1800 J **26.** 10 s **27.** $(6/\pi)^{\frac{1}{3}}$ **28.** 2:1 **29.** 3025 K **30.** 80 Watt. **31.** 1700 K **32.** 0.3 **33.** 15°C
34. 6×10^3 K; 4×10^3 K **35.** 7.31×10^{10} erg/cm^2 s **36.** $\lambda_m = 3\ \mu$m **37.** 7 minutes. **38.** 34°C

Level II

1. 1:1.26 **2.** 800 cal kg^{-1} K^{-1}, 1000 cal kg^{-1} K^{-1} **3.** (A) 37.8 J/s (Watts), (B) 2.005 N-m **4.** 5000 J/°C kg
5. 45°C **6.** $K_{11} > K\perp$, $K_{\parallel} = \dfrac{K_A + K_B}{2}$, $K_{\perp} = \dfrac{2K_A K_B}{K_A + K_B}$ **7.** $\dfrac{a^2 s}{2K}\log_e\left(\dfrac{b}{a}\right)\log_e\left(\dfrac{T_0 - T_1}{T_0 - T_2}\right)$
8. $\dfrac{l_1}{l} = \dfrac{k(T_1 - T_m)}{k(T_1 - T_m) + (T_m - T_2)}$ **9.** (A) -100°C/m, (B) 1000 J **10.** 166.3 s **11.** 9.72°C/min **12.** $T_A = 423$K
13. $T'' = 4\sqrt{2} \times 500 = 600$ K **14.** 10 minutes **15.** (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm
16. (i) $p_1 = pH_2 \simeq 1.25 \times 10^6$ Pa; $p_2 = pH_2 + pO_2 + pN_2 \simeq 2.8125 \times 10^6$ Pa; $p_3 = pH_2 + pN_2 \simeq 1.5625 \times 10^6$ Pa
17. 60°C

Previous Year Questions

JEE Main

1. A **2.** B **3.** B **4.** C **5.** A **6.** A **7.** D **8.** D **9.** D **10.** C
11. B **12.** C **13.** B **14.** D **15.** A

JEE Advanced

1. 12 gm **2.** 60°C **3.** 41.53 Watt; 26.48 °C; 0.55 °C **4.** D **5.** 5.5 min
6. $k = \dfrac{\log_e 2}{t_1}$; $T = 300 + 50\exp\left[-\left\{\dfrac{KA}{LC} + \dfrac{\log_e 2}{L}\right\}2t_1\right]$ **7.** D **8.** A **9.** B **10.** B **11.** D
12. 0.5 kg **13.** A **14.** A **15.** (A) 595 watt/m^2, (B) $T_0 \approx 420$ K **16.** B **17.** D **18.** C
19. $\gamma_l = 2\alpha$ **20.** $\dfrac{K}{4e\sigma LT_S^3 + K}$ **21.** A **22.** C **23.** A **24.** A **25.** B **26.** A,D
27. 273K **28.** (A) S, Q; (B) Q; (C) P, Q; (D) Q, R or (A) S, (B) Q, (C) P, (D) R **29.** 9 **30.** 8 g
31. C **32.** A **33.** A, B, C, D **34.** A

CHAPTER 5

Heat-2

CONCEPT OF AN IDEAL GAS

A gas has no shape and size and can be contained in a vessel of any size or shape. It expands indefinitely and uniformly to fill the available space. It exerts pressure on its surroundings.

The gases whose molecules are point masses (mass without volume) and do not attract each other are called **ideal** or **perfect** gases. It is a hypothetical concept which cannot exist in reality. The gases such as hydrogen, oxygen and helium which cannot be liquefied easily are called **permanent gases**. An actual gas behaves as ideal gas most closely at low pressure and high temperature.

Ideal Gas Equation

According to this equation,

$$PV = nRT = \frac{m}{M}RT\ .$$

In this equation,

n = number of moles of the gas $= \dfrac{m}{M}$

m = total mass of the gas

M = molecular mass of the gas

R = Universal gas constant

= 8.31 J/mol-K

= 2.0 cal/mol-K

KINETIC THEORY OF GASES

Kinetic theory of gases is based on the following basic assumptions:

1. A gas consists of a very large number of molecules. These molecules are identical, perfectly elastic and hard spheres. They are so small that the volume of molecules is negligible as compared with the volume of the gas.
2. Molecules do not have any preferred direction of motion. Motion is completely random.
3. These molecules travel in straight lines and are in free motion most of the time. The time of the collision between any two molecules is very small.
4. The collision between molecules and the wall of the container is perfectly elastic. It means kinetic energy is conserved in each collision.
5. The path travelled by a molecule between two collisions is called free path and the mean of this distance travelled by a molecule is called mean free path.
6. The motion of molecules is governed by Newton's law of motion.
7. The effect of gravity on the motion of molecules is negligible.

Note

At higher temperature and low pressure or at higher temperature and low density, a real gas behaves as an ideal gas.

EXPRESSION FOR THE PRESSURE OF A GAS

Let us suppose that a gas is enclosed in a cubical box having length ℓ. Let there are N identical molecules, each having mass m. Since the molecules are of the same mass and are perfectly elastic, their mutual collisions result in the interchange of velocities only. Only collisions with the walls of the container contribute to the pressure by the gas molecules. Let us focus on a molecule having velocity v_1

and components of velocity v_{x_1}, v_{y_1}, v_{z_1} along the x-, y- and z-axis as shown in Fig. 5.1.

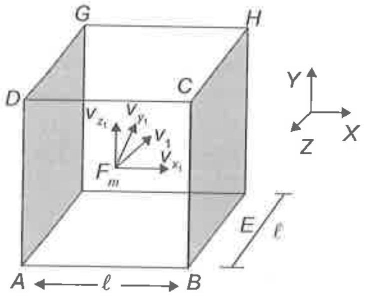


Figure 5.1

Here, $$v_1^2 = v_{x_1}^2 + v_{y_1}^2 + v_{z_1}^2.$$

The change in momentum of the molecule after one collision with wall *BCHE*

$$= mv_{x_1} - (-v_{x_2})$$

$$= 2mv_{x_1}.$$

The time taken between the successive impacts on the face *BCHE*

$$= \frac{\text{distance}}{\text{velocity}}$$

$$= \frac{2\ell}{v_{x_1}}.$$

Time rate of change of momentum due to collision

$$= \frac{\text{change in momentum}}{\text{time taken}}$$

$$= \frac{2mv_{x_1}}{2\ell / v_{x_1}}$$

$$= \frac{mv_{x_1}^2}{\ell}.$$

Hence, the net force on the wall *BCHE* due to the impact of nN molecules of the gas is

$$F_x = \frac{mv_{x_1}^2}{\ell} + \frac{mv_{x_2}^2}{\ell} + \frac{mv_{x_3}^2}{\ell} + \cdots + \frac{mv_{x_n}^2}{\ell}$$

$$= \frac{m}{\ell}\left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + \cdots + v_{x_n}^2\right)$$

$$= \frac{mN}{\ell} <v_x^2>,$$

where $<v_x^2>$ = mean square velocity in the x-direction. Since the molecules do not favour any particular direction, therefore $<v_x^2> = <v_y^2> = <v_z^2>$.

But $$<v^2> = <v_x^2> + <v_y^2> + <v_z^2>$$

$$\Rightarrow \quad <v_x^2> = \frac{<v^2>}{3}.$$

Pressure is equal to force divided by area. Hence,

$$P = \frac{F_x}{\ell^2}$$

$$= \frac{M}{3\ell^3} <v^2>$$

$$= \frac{M}{3V} <v^2>.$$

Pressure is independent of x-, y- and z-directions,

Here,

ℓ^3 = volume of the container = V

M = total mass of the gas

$<v^2>$ = mean square velocity of molecules

$$\Rightarrow \quad P = \frac{1}{3}\rho <v^2>$$

From $$PV = nRT,$$

$$\because \quad n = \frac{\text{Mass}}{\text{Molecular Weight}}$$

$$= \frac{M}{M_0} \text{(in kg/mole)}$$

$$P = \frac{M}{M_0 V} RT$$

$$= \frac{\rho RT}{M_0}$$

$$\Rightarrow \quad \frac{\rho RT}{M_0} = \frac{1}{3}\rho V_{rms}^2$$

$$\Rightarrow \quad V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$= \sqrt{\frac{3RT}{mN_A}}$$

$$= \sqrt{\frac{3Kt}{m}}$$

K = Boltzman's constant

$$= \frac{R}{N_A}.$$

Co-ordinate of the Gases

(P, V, T) is the coordinate of the gas.

If the initial condition of gas is given by $(P_1V_1T_1)$ and final condition of the gas is given by (P_2, V_2, T_2), then

$$(P_1V_1T_1) \Rightarrow (P_2V_2T_2).$$

Then (P, V, T) defines the situation of gas. When a gas changes from one coordinate system to another co-ordinate system, then we have to follow a process.

GAS LAWS

Assuming permanent gases to be ideal, through experiments, it was established that gases irrespective of their nature obey the following laws:

Boyle's Law

According to this law, for a given mass of a gas, the volume of a gas at constant temperature (called **isothermal** process) is inversely proportional to its pressure, i.e.,

$$V \propto \frac{1}{P} \quad (T = \text{constant})$$

or $$PV = \text{constant}$$

or $$P_iV_i = P_fV_f$$

Thus, P–V graph in an isothermal process is a rectangular hyperbola. Or PV versus P or V graph is a straight line parallel to P or V axis.

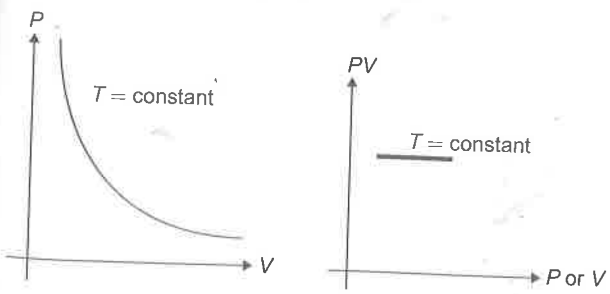


Figure 5.2

Charle's Law

According to this law, for a given mass of a gas the volume of a gas at constant pressure (called **isobaric** process) is directly proportional to its absolute temperature, i.e.,

$$V \propto T$$

or $$\frac{V}{T} = \text{constant}$$

or $$\frac{V_i}{T_i} = \frac{V_f}{T_f}.$$

Thus, V-T graph in an isobaric process is a straight line passing through the origin. Or V/T versus V or T graph is a straight line parallel to V or T axis.

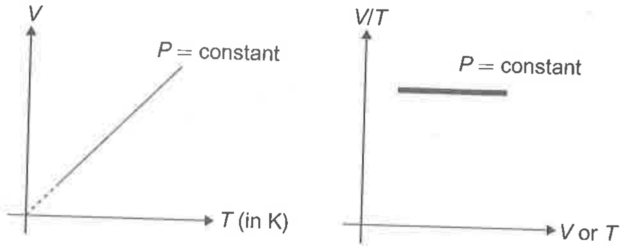


Figure 5.3

Gay Lussac's Law or Pressure Law

According to this law, for a given mass of a gas the pressure of a gas at constant volume (called **isochoric** process) is directly proportional to its absolute temperature, i.e.,

$$P \propto T$$

or $$\frac{P}{T} = \text{constant}$$

or $$\frac{P_i}{T_i} = \frac{P_f}{T_f}.$$

Thus, P-T graph in an isochoric process is a straight line passing through the origin or P/T versus P or T graph is a straight line parallel to P or T axis.

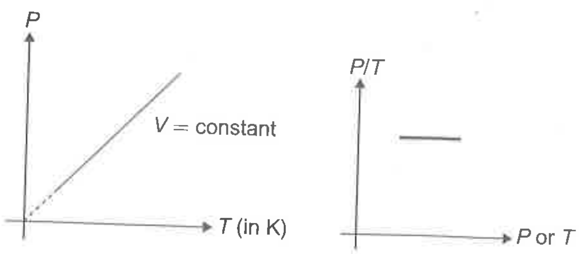


Figure 5.4

Avogadro's Law

Two gases at same volume, pressure and temperature contain equal amount of moles (mass of gas may be different) or we can say contain equal number of particles:

$$1 \text{ mole} = 6.023 \times 10^{-23} \text{ particles.}$$

Reading of *P-V* Diagram

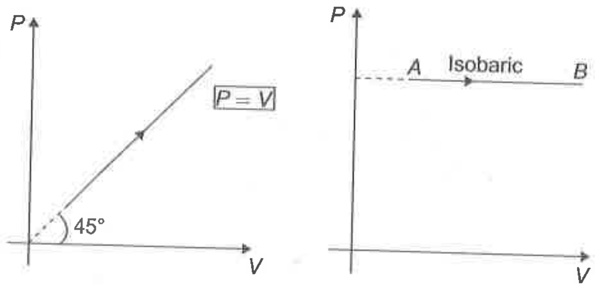


Figure 5.5

From $PV = nRT$

$P = \text{constant}$

$V = \uparrow$

$T = \uparrow$

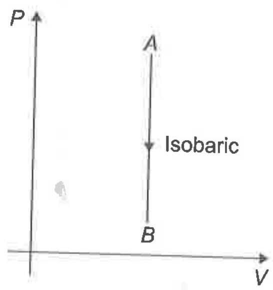


Figure 5.6

$V = \text{constant}$

$P = \downarrow$

$T = \downarrow$

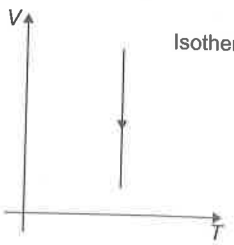


Figure 5.7

$T = \text{constant}$

$V = \downarrow$

$P = \uparrow$

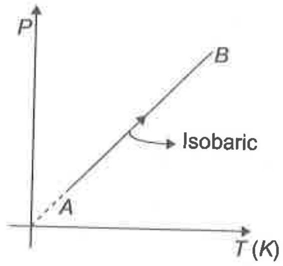


Figure 5.8

$PV = nRT$

$$A \rightarrow B \begin{cases} P = \text{const} \\ V \uparrow \\ T \uparrow \end{cases}$$

When T in $°C$

$\Rightarrow$ $PV = nR(T + 273)$

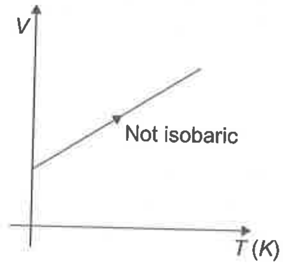


Figure 5.9

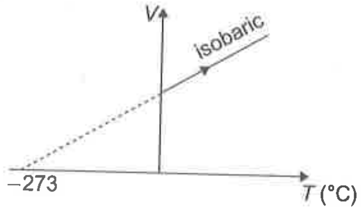


Figure 5.10

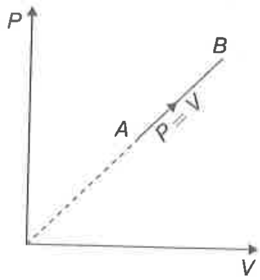


Figure 5.11

$A \rightarrow B$

$P = \uparrow$

$V = \uparrow$

$T = \uparrow$

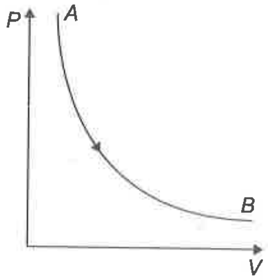


Figure 5.12

$PV = \text{constant}$

$T = C$

$P = \downarrow$

$V = \uparrow$

- ***ISOTHERMAL***

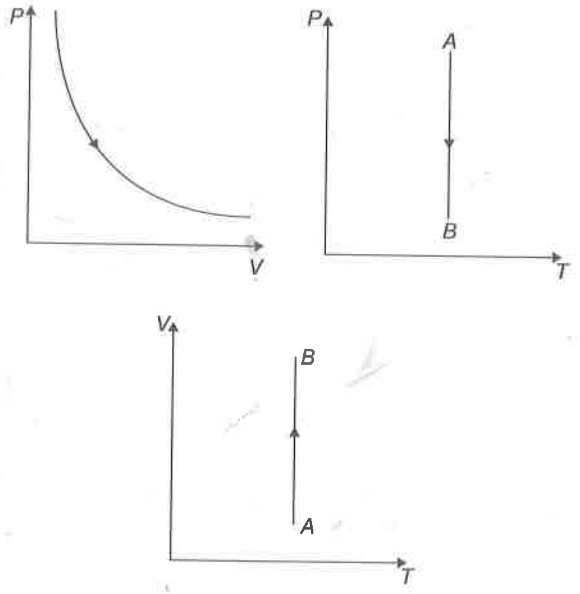


Figure 5.13

- ***ISOBARIC***

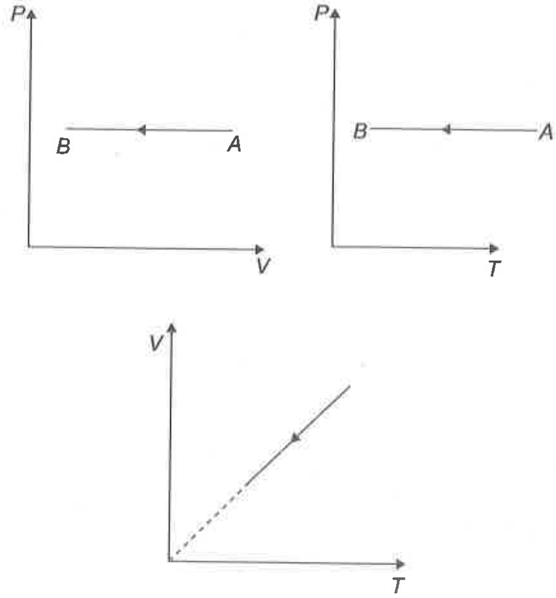


Figure 5.14

- ***ISOCHORIC***

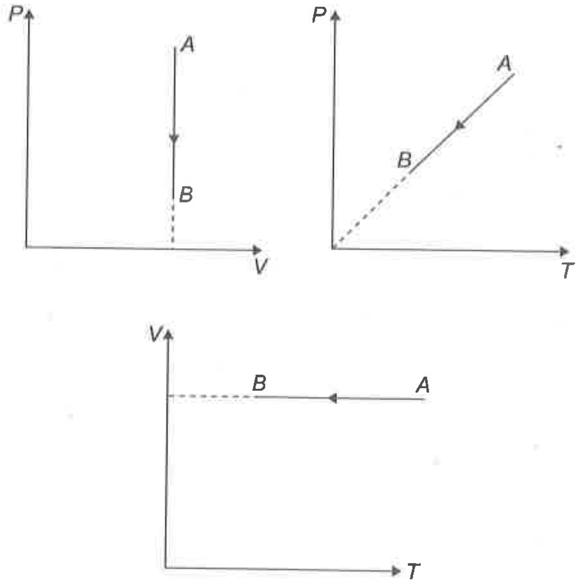
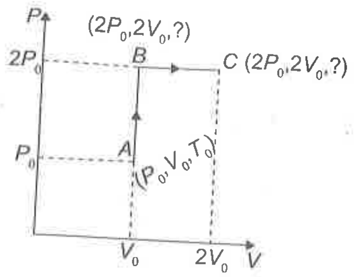


Figure 5.15

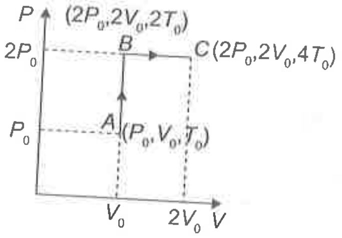
SOLVED EXAMPLES

EXAMPLE 1

Find out the values of co-ordinates at point A, B and C in terms of pressure, volume and temperature and draw the curve.



SOLUTION



$A \rightarrow B \quad V = \text{constant}$

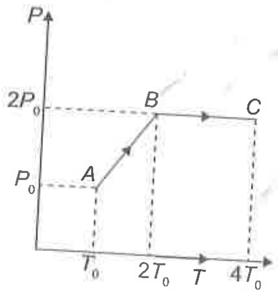
$$\frac{P_0}{T_0} = \frac{2P_0}{T_B}$$

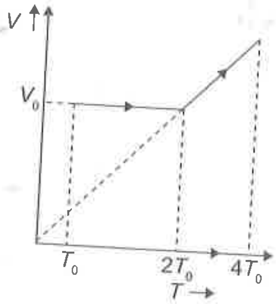
$$T_B = 2T_0$$

$B - C \quad P = \text{constant}$

$$\frac{V_0}{2T_0} = \frac{2V_0}{T_C}$$

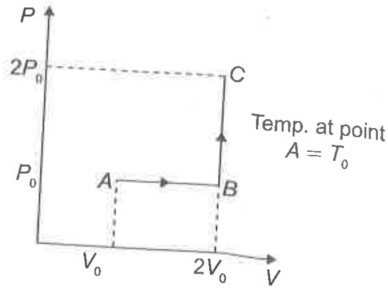
$$T_C = 4T_0$$



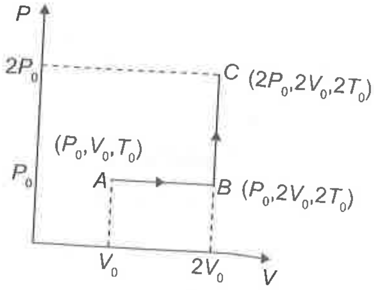


EXAMPLE 2

Find out the values of co-ordinates at points A, B and C in terms of pressure, volume and temperature and draw the curve.



SOLUTION



$A \rightarrow B$ (Isobaric)

$$\frac{V_0}{T_0} = \frac{2V_0}{T_B}$$

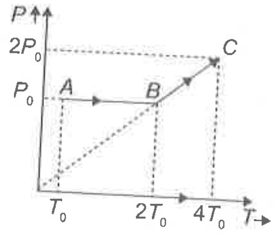
$$T_B = 2T_0$$

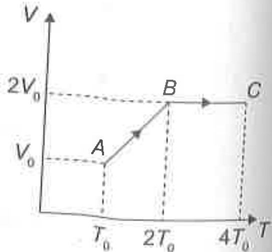
$B \rightarrow C$ (Isochoric)

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad \frac{P_0}{2T_0} = \frac{2P_0}{T_2}$$

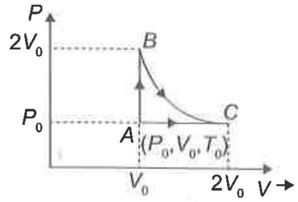
$$\Rightarrow \quad T_2 = 4T_0$$



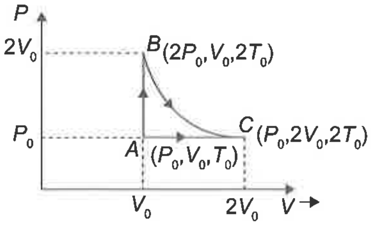


EXAMPLE 3

Find out the values of co-ordinates at points A, B and C in terms of pressure, volume and temperature and draw the curve.



SOLUTION



$A \rightarrow B$: Volume is constant (Isochoric),

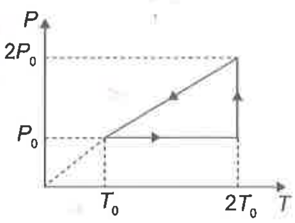
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}.$$

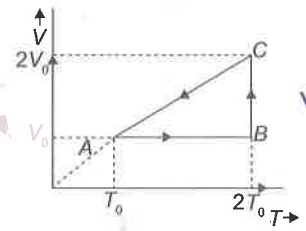
$B \rightarrow C$: Temperature is constant (Isothermal),

$$P_1V_1 = P_2V_2.$$

$C \rightarrow A$: Pressure is constant (Isobaric),

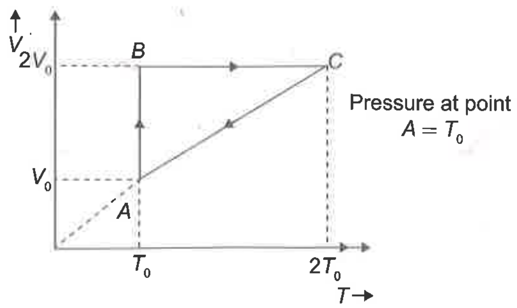
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$



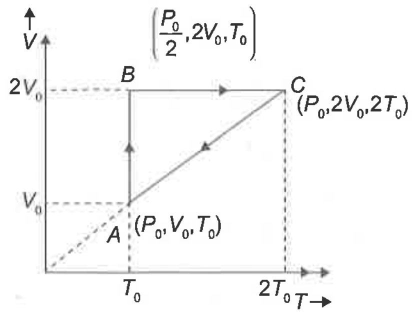


EXAMPLE 4

Find out the values of co-ordinates at points A, B and C in terms of pressure, volume and temperature and draw the curve.



SOLUTION



$A \rightarrow B$: Temperature is constant (isothermal).

$$P_1V_1 = P_2V_2$$

$$2P_0V_0 = 2V_0P_2$$

$$\Rightarrow \qquad P_2 = \frac{P_0}{2}$$

$B \rightarrow C$: Volume is constant (Isochoric)

$$\frac{P_B}{T_B} = \frac{P_C}{T_C}$$

$$\Rightarrow \qquad \frac{P_0}{2T_0} = \frac{P_C}{2T_0}$$

$$\Rightarrow \qquad P_C = P_0$$

Notes

Some commonly used terms

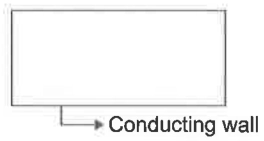


Figure 5.16

There is heat transfer from gas to surrounding and final temperature is the same.

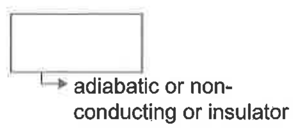


Figure 5.17

There is no heat transfer.

Notes (contd)

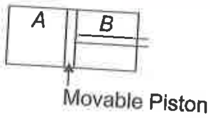


Figure 5.18

If it is a diathermic separator, then the final temperature is also the same on both sides (finally, pressure is the same).

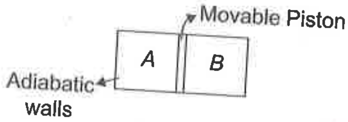
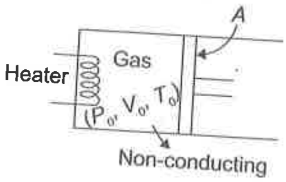


Figure 5.19

Finally, if the pressure on both sides is the same, then it does not move (massless).

EXAMPLE 5

If the temperature increases slowly from T_0 to $2T_0$, then what distance will the piston move?



SOLUTION

$$\text{Pressure is the same} = \frac{V_0}{T_0} = \frac{Vt}{2T_0}$$

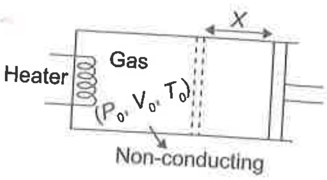
$$V_f = 2V_0$$

The distance moved is estimated as shown below:

$$\text{Change in volume} = Ax$$

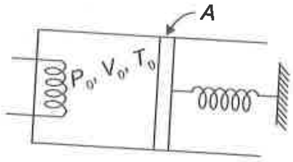
$$2V_0 - V_0 = Ax$$

$$x = \frac{V_0}{A}.$$



EXAMPLE 6

If the temperature of the gas changes slowly from T_0 to $2T_0$, then find out the displacement of the piston.



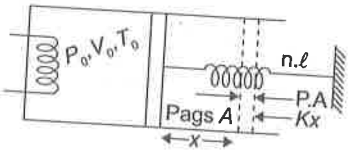
SOLUTION

$$P_{gas}A = Kx + P_0A$$

$$P_{gas} = P_f$$

$$= \left(\frac{Kx}{A} + P_0\right)P$$

$$\frac{P_0V_0}{T_0} = \frac{P_f(V_0 + A_x)}{T_f}.$$



Pressure Variation

Notes

Pressure of the liquid $\frac{N}{A} = \rho_\ell hg$

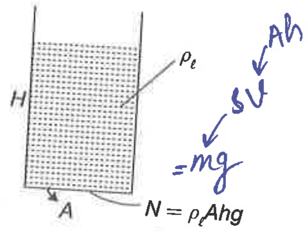


Figure 5.20

Notes (contd)

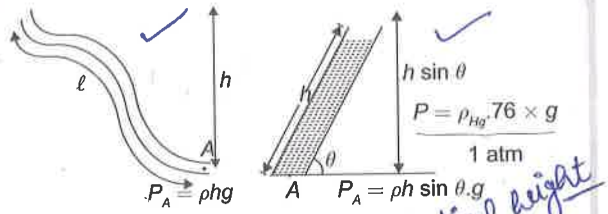


Figure 5.21

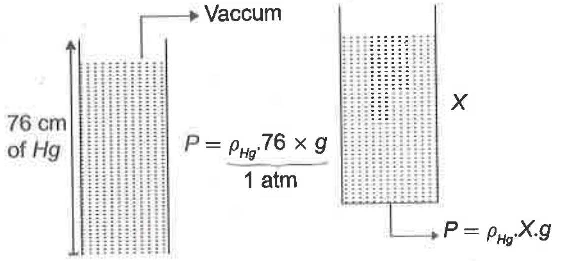


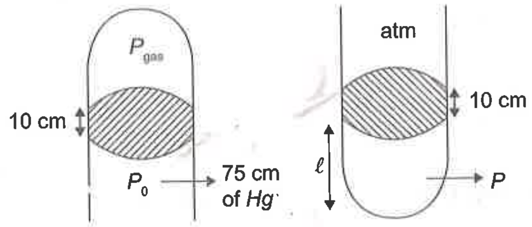
Figure 5.22

x cm of Hg pressure means if we placed a straight tube vertically in vacuum, fill the tube with Hg up to height x.

Then the pressure exerted by Hg at the bottom of the tube equals the pressure of the gas.

EXAMPLE 7

Find the new length of gas column in the tube if the tube is inverted (assume temperature is constant).



SOLUTION

Initially,

$$P_{gas} + 10 = 75$$

$$\Rightarrow \quad P_{gas} = 65.$$

Finally,

$$75 + 10 = P_{gas}$$

$$\Rightarrow \quad P_{gas} = 85 \text{ cm}$$

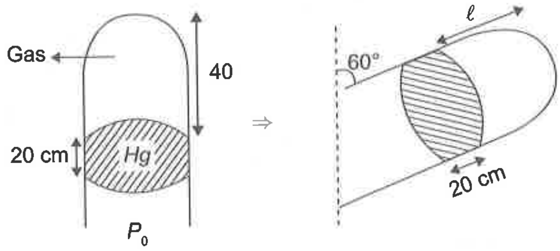
$$P_1V_1 = P_2V_2$$

$$85 \times A \times \ell = 65 \times 30 \times A$$

$$\Rightarrow \quad \ell = \frac{1950}{85}.$$

EXAMPLE 8

Find the new length of the gas column in the tube if the tube is rotated at an angle 60° as shown. (Assume constant temperature.)



$$P_{gas} = 75 - 20 = 55$$

$$P_1V_1 = P_2V_2$$

$$55 \times 40 \times A = 65 \times \ell \times A$$

$$\ell = \frac{55 \times 40}{65}$$

$[P + 20 \cos 60 = 75$

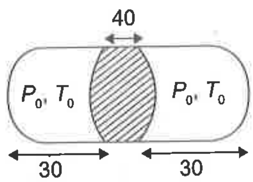
$2P + 20 = 150$

$2P = 130$

$P = 65]$

EXAMPLE 9

Assume constant temperature if the tube is changed to vertical position and the pallet comes down by 5 cm, then find out P_0.



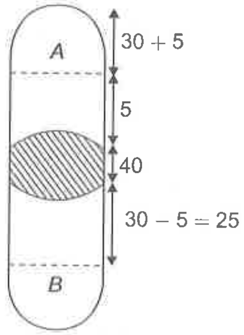
SOLUTION

For the upper part,

$$P_1V_1 = P_0V_0$$

$$P_1 35A = P_0 30A$$

$$P_1 = \frac{30}{35} P_0 \quad (1)$$



For the lower part,

$$P_2V_2 = P_0V_0$$

$$P_0\,30A = P_2\,25A$$

$$P_2 = \frac{30}{25}P_0 \qquad (2)$$

Again,

$$P_1 + 40 = P_2 \qquad (3)$$

From (1) and (2)

$$\frac{30}{35}P_0 + 40 = \frac{30}{25}P_0.$$

Pressure Variation in Atmosphere

Assuming temperature to be constant,

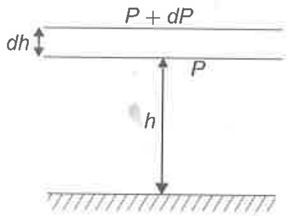


Figure 5.23

$$-dP = dH\rho g$$

$$\Rightarrow \quad -dP = dh\left(\frac{PM}{RT}\right)g$$

$$PV = nRT$$

$$\Rightarrow \quad P = \frac{\rho RT}{M}$$

$$\rho = \frac{PM}{RT}$$

$$-\int_{-P_0}^{P}\frac{dP}{P} = \int_0^h \frac{Mg}{RT}.dh$$

$$P_n\frac{P}{P_0} = -\frac{Mg}{RT}h$$

$$\Rightarrow \quad P = P_0 e^{\frac{Mg}{RT}} h.$$

Pressure Variation in Rotating Rod

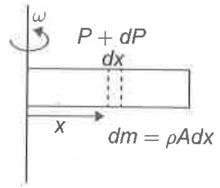


Figure 5.24

$$(P + dP)A - PA = dmw^2x$$

$$AdP = \rho Aw^2ddx$$

$$dP = \rho w^2xdx$$

$$\int_{P_0}^{P}\frac{dP}{P} = \frac{W^2M}{RT}\int_0^x x\,dx$$

$$[\ln P]_{P_0}^{P} = \frac{W^2M}{RT}\left[\frac{x^2}{2}\right]_0^x$$

$$ln\frac{P}{P_0} = \frac{W^2M}{RT}\frac{x^2}{2}$$

$$P = P_0\varepsilon\frac{W^2MX^2}{2RT}.$$

MAXWELL'S DISTRIBUTION LAW

Distribution Curve

A plot of $\frac{dN(v)}{dv}$ (number of molecules per unit speed interval) against c is known as Maxwell's distribution curve. The total area under the curve is given by the integral $\int_0^\infty \frac{dN(v)}{dv}dv = \int_0^\infty dN(v) = N$.

Note

The actual formula of $\frac{dN(v)}{dv}$ is not in JEE syllabus.

Figure 5.25 shows the distribution curves for two different temperatures. At any temperature, the number of molecules in a given speed interval dv is given by the area under the curve in that interval (shown shaded). This number increases, as the speed increases, up to a maximum and then decreases asymptotically towards zero. Thus, the maximum number of the molecules have speed lying within a small range centred about the speed corresponding to the peak (A) of the curve. This speed is called the 'most probable speed' v_p or v_{mp}.

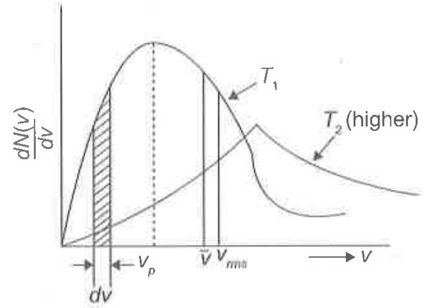


Figure 5.25

The distribution curve is asymmetrical about its peak (the most probable speed v_p) because the lowest possible speed is zero, whereas there is no limit to the upper speed a molecule can attain. Therefore, the average speed $\bar{v}$ is slightly larger than the most probable speed v_p. The root-mean-square speed, v_{rms}, is still larger ($v_{rms} > \bar{v} > v_p$).

(a) Average (or Mean) Speed

$$\bar{v} = \sqrt{\frac{8}{\pi}\frac{RT}{M_0}}$$

$$= \sqrt{\frac{8RT}{\pi M_0}}$$

$$= 1.59\sqrt{kT/m}$$

(derivation is not in the course)

(b) RMS Speed

$$v_{rms} = \sqrt{\langle v^2 \rangle}$$

$$= \sqrt{\frac{3RT}{M_0}}$$

$$= \sqrt{\frac{3kT}{m}}$$

$$= 1.73\sqrt{\frac{kT}{m}}$$

(c) Most Probable Speed

The most probable speed v_p or v_{mp} is the speed possessed by the maximum number of molecules and corresponds to the maximum (peak) of the distribution curve. Mathematically, it is obtained by the condition.

$\frac{dN(v)}{dv} = 0$ [by substitution of formula of $dN(v)$ (which is not in the course)].

Hence, the most probable speed is

$$v_p = \sqrt{\frac{2kT}{m}}$$

$$= \sqrt{\frac{2RT}{M_0}}.$$

From the above expression, we can see that

$$v_{rms} > \bar{v} < v_p.$$

$R = 8.314$ J/mole

k = Boltzmann constant ($k = 1.38 \times 10^{-23}$JK^{-1}).

DEGREE OF FREEDOM

Total number of independent co-ordinates which must be known to completely specify the position and configuration of dynamical system is known as 'degree of freedom f'. Maximum possible translational degrees of freedom are three, i.e.,

$$\left(\frac{1}{2}mV_x^2 + \frac{1}{2}mV_y^2 + \frac{1}{2}mV_z^2\right).$$

Maximum possible rotational degrees of freedom are three, i.e.,

$$\left(\frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2\right).$$

Vibrational degrees of freedom are two, i.e. (kinetic energy of vibration and potential energy of vibration).

Monoatomic

For example (all intent gases, He, Ar, etc.)

$f = 3$ (translational)

(V_x, V_y, V_z)

Diatomic

For example (gases like H_2, N_2, O_2, etc.)

$f = 5$ (3 translational + 2 rotational)

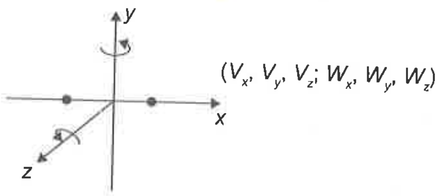


Figure 5.26

If temperature < 70 K for diatomic molecules, then

$f = 3.$

If the temperature is in between 250 K and 5000 K, then

$f = 5.$

If the temperature is very high (> 5000 K)

$f = 7$

[3 translational + 2 rotational + 2 vibrational].

Triatomic

(Non-linear)

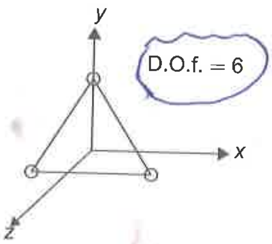


Figure 5.27

$\underbrace{V_x, V_y, V_z}_{\text{3 Trans.}}, \underbrace{W_x, W_y, W_z}_{\text{3 Rotational}}$

If linear (CO_2)

same as Diatomic

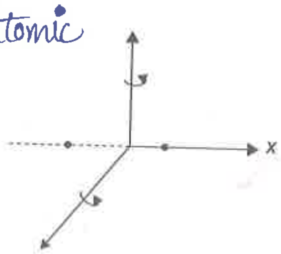


Figure 5.28

Total D.O.F. = 5

$\underbrace{V_x V_y V_z}_{\text{3 Trans}}, \underbrace{W_x, W_z}_{\text{2 Rotational}}$

Maxwell's law of equipartition of energy

Energy associated with each degree of freedom $= \frac{1}{2}KT.$

of one particle is same and $= \frac{1}{2}KT.$

If the degree of freedom of a molecule is f, then total kinetic energy of that molecule $= \frac{f}{2}KT.$

Monoatomic

Energy of one particle $= \frac{3}{2}KT,$

one mole $= \frac{3}{2}RT,$

n mole $= \frac{3}{2}nRT.$

Diatomic

Energy of one particle $= \frac{5}{2}KT,$

one mole $= \frac{5}{2}RT,$

n mole $= \frac{5}{2}nRT.$

General degree of freedom

Energy of one particle $= \frac{f}{2}KT,$

one mole $= \frac{f}{2}RT,$

n mole $= \frac{f}{2}nRT.$

Internal energy of a gas only depends on the temperature of the gas and does not depend on the process taken by the gas to reach the temperature.

INTERNAL ENERGY

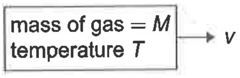
The internal energy of a system is the sum of kinetic and potential energies of the molecules of the system. It is denoted by U. Internal energy (U) of the system is the function of its absolute temperature (T) and its volume (V), i.e., $U = f(T, V)$.

In case of an ideal gas, intermolecular force is zero. Hence, its potential energy is also zero. In this case, the internal energy is only due to kinetic energy, which depends on the absolute temperature of the gas, i.e., $U = f(T)$. For an ideal gas, internal energy $U = \frac{f}{2}nRT$.

SOLVED EXAMPLES

EXAMPLE 10

A light container having a diatomic gas enclosed within is moving with velocity v. Mass of the gas is M and the number of moles is n.



(i) What is the kinetic energy of the gas with respect to the centre of mass of the system?

(ii) What is K.E. of the gas with respect to the ground?

SOLUTION

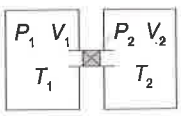
(i) $$\text{K.E.} = \frac{5}{2}nRT$$

(ii) Kinetic energy of gas with respect to the ground = Kinetic energy of the centre of mass with respect to the ground + Kinetic energy of the gas with respect to the centre of mass.

$$\text{K.E.} = \frac{1}{2}Mv^2 + \frac{5}{2}nRT$$ ■

EXAMPLE 11

Two non-conducting containers having volume V_1 and V_2 contain monoatomic and diatomic gases, respectively. The pressure and temperature of the containers are P_1, T_1 and P_2, T_2, respectively. Initially, the stop cock is closed. If the stop cock is opened, find the final pressure and temperature.



SOLUTION

$$n_1 = \frac{P_1V_1}{RT_1}$$

$$n_2 = \frac{P_2V_2}{RT_2}$$

$$n = n_1 + n_2$$

(number of moles are conserved).

Finally, pressure in both parts and temperature of both the gases will be equal.

$$\frac{P(V_1+V_2)}{RT} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}.$$

From energy conservation,

$$\frac{3}{2}n_1RT_1 + \frac{5}{2}n_2RT_2 = \frac{3}{2}n_1RT + \frac{5}{2}n_2RT$$

$$\Rightarrow \quad T = \frac{(3P_1V_1 + 5P_2V_2)T_1T_2}{3P_1V_1T_2 + 5P_2V_2T_1}$$

$$P = \left(\frac{3P_1V_1 + 5P_2V_2}{3P_1V_1T_2 + 5P_2V_2T_1}\right)\left(\frac{P_1V_1T_2 + P_2V_2T_2}{V_1 + V_2}\right)$$ ■

THERMODYNAMICS

Thermodynamics is mainly the study of exchange of heat energy between bodies and conversion of the same into mechanical energy and vice versa.

Thermodynamic System

Collection of an extremely large number of atoms or molecules confined within certain boundaries such that it has a certain value of pressure (P), volume (V) and temperature (T) is called a thermodynamic system. Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings. Taking into consideration the interaction between a system and its surroundings, a thermodynamic system is divided into three classes.

(a) **Open system:** A system is said to be an open system if it can exchange both energy and matter with its surroundings.

(b) **Closed system:** A system is said to be a closed system if it can exchange only energy (not matter) with its surroundings.

(c) **Isolated system:** A system is said to be isolated if it can exchange neither energy nor matter with its surroundings.

Zeroth Law of Thermodynamics

If two systems (B and C) are separately in thermal equilibrium with a third one (A), then they themselves are in thermal equilibrium with each other.

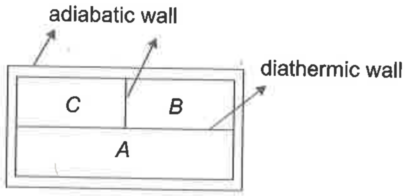


Figure 5.29

Equation of State (for Ideal Gases)

The relation between the thermodynamic variables (P, V, T) of a system is called equation of state. The equation of state for an ideal gas of n moles is given by

$$PV = nRT$$

Work Done by a Gas

Let P and V be the pressure and volume of the gas. If A be the area of the piston, then force exerted by the gas on the piston is

$$F = P \times A.$$

Let the piston move through a small distance dx during the expansion of the gas. Work done for a small displacement dx is

$$dW = Fdx = PAdx.$$

Since $Adx = dV$, increase in volume of the gas is dV

$$\Rightarrow \quad dW = PdV$$

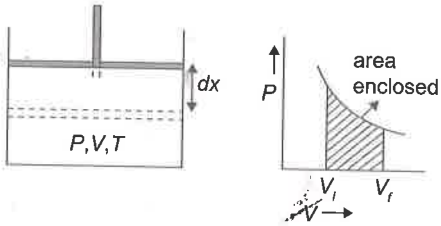


Figure 5.30

or
$$W = \int dw = \int Pdv.$$

Area enclosed under the P–V curve gives work done during the process.

Different Types of Processes

(a) Isothermal Process

T = constant [Boyle's law applicable],

PV = constant.

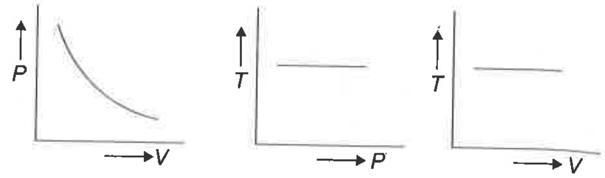


Figure 5.31

There is exchange of heat between the system and the surroundings. System should be compressed or expanded very slowly so that there is sufficient time for exchange of heat to keep the temperature constant.

Slope of *P-V* Curve in Isothermal Process

$$PV = \text{constant} = C$$

$$\Rightarrow \quad \frac{dp}{dv} = -\frac{P}{V}.$$

Work Done in Isothermal Process

$$W = nRT\ln\frac{V_f}{V_i} \quad \left[\begin{array}{l}\text{If } V_f > V_i \text{ then } W \text{ is positive} \\ \text{If } V_f > V_i \text{ then } W \text{ is negative}\end{array}\right]$$

$$W = \left[2.303\, nRT \log_{10}\frac{V_f}{V_i}\right]$$

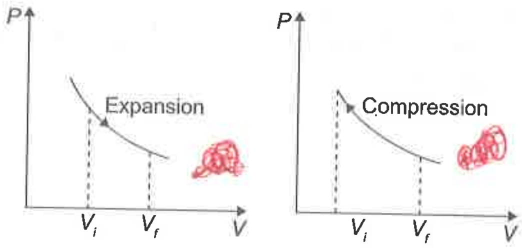


Figure 5.32

Internal Energy in Isothermal Process

$$U = f(T)$$

$$\Rightarrow \quad \Delta U = 0$$

(b) Isochoric Process (Isometric Process)

$$V = \text{constant}$$

$\Rightarrow$ Change in volume is zero

$\Rightarrow$ $\frac{P}{T}$ is constant

$$\frac{P}{T} = \text{constant (Gay–Lussac law).}$$

Work Done in Isochoric Process

Since change in volume is zero,

$$dW = pdV = 0.$$

Indicator Diagram of Isochoric Process

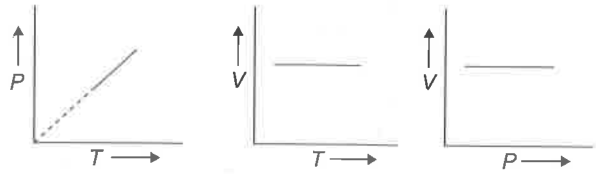


Figure 5.33

Change in Internal Energy in Isochoric Process

$$\Delta U = n\frac{f}{2}R\,\Delta T$$

Heat Given in an Isochoric Process

$$\Delta Q = \Delta U = n\frac{f}{2}R\Delta T$$

(c) *Isobaric Process*

Pressure remains constant in isobaric process

$\therefore$ $P = \text{constant}$

$\Rightarrow$ $\frac{V}{T} = \text{constant}.$

Indicator Diagram of Isobaric Process

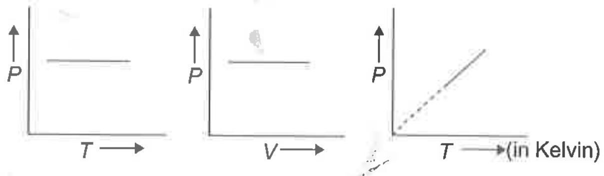


Figure 5.34

Work Done in Isobaric Process

$$\Delta W = P\Delta V$$

$$= P(V_{\text{final}} - V_{\text{initial}})$$

$$= nR(T_{\text{final}} - T_{\text{initial}}).$$

Change in Internal Energy in Isobaric Process

$$\Delta U = n\frac{f}{2}R\Delta T$$

Heat Given in Isobaric Process

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = n\frac{f}{2}R\,\Delta T + P[V_f - V_i]$$

$$= n\frac{f}{2}R\,\Delta T + nR\Delta T.$$

The above expression gives an idea that to increase temperature by ΔT in isobaric process, heat required is more than that in the isochoric process.

(d) *Cyclic Process*

In the cyclic process, initial and final states are the same; therefore, initial state = final state

Work done = Area enclosed under P-V diagram.

Change in internal energy

$$\Delta U = 0$$

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \Delta Q = \Delta W.$$

If the process on the P-V curve is clockwise, then net work done is positive and vice versa.

The graphs shown below explain when work is positive and when it is negative

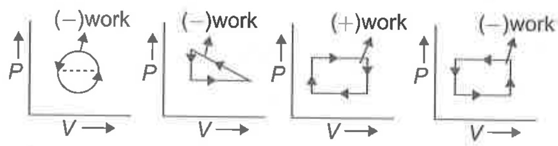


Figure 5.35

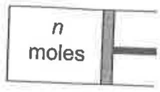
SOLVED EXAMPLES

EXAMPLE 12

The cylinder shown in the figure has conducting walls and temperature of the surrounding is T. The piston is initially in equilibrium, the cylinder contains n moles of a gas. Now the piston is displaced slowly by an external agent to make the volume double of the initial. Find the work done by an external agent in terms of n, R and T.

SOLUTION

First Method



Work done by an external agent is positive, because F_{ext} and displacement are in the same direction. Since the walls are conducting, the temperature remains constant. Applying equilibrium condition when pressure of the gas is P,

$$PA + F_{ext} = P_{atm}A$$

$$F_{ext} = P_{atm}A - PA$$

$$W_{ext} = \int_0^d F_{ext}\,dx$$

$$= \int_0^d P_{atm}A\,dx - \int_0^d PA\,dx$$

$$= P_{atm}A\int_0^d dx - \int_v^{2v} \frac{nRT}{V}dv$$

$$= P_{atm}Ad - nRT\ln 2$$

Second Method

Applying work–energy theorem, on the piston

$$\Delta k = 0$$

$$W_{all} = \Delta k$$

$$W_{gas} + W_{atm} + W_{ext} = 0$$

$$nRT\ln\frac{V_f}{V_i} - nRT + W_{ext} = 0$$

$$W_{ext} = nRT(1 - \ln 2).$$ ■

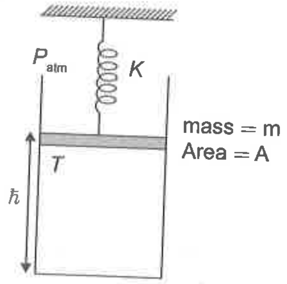
EXAMPLE 13

A non-conducting piston of mass m and area of cross-section A is placed on a non-conducting cylinder as shown in the figure. Temperature, spring constant, height of the piston are given by T, K and h, respectively. Initially, spring is relaxed and piston is at rest. Find

(i) Number of moles

(ii) Work done by gas to displace the piston by distance d when the gas is heated slowly.

(iii) Find the final temperature.



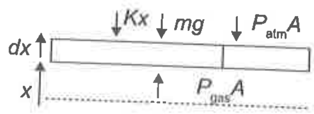
SOLUTION

(i)
$$PV = nRT$$

$$\Rightarrow \quad \left(P_{atm} + \frac{mg}{A}\right)Ah = nRT$$

$$\Rightarrow \quad n = \frac{\left(P_{atm} + \frac{mg}{A}\right)Ah}{RT}.$$

(2) **First method**



Applying Newton's law on the piston,

$$mg + P_{atm}A + Kx = P_{gas}A$$

$$W_{gas} = \int_0^d P_g A\,dx$$

$$= \int_0^d (mg + P_{atm}A + Kx)dx$$

$$\Rightarrow \quad W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$

Second method

Applying work–energy theorem on the piston

$$W_{all} = \Delta KE$$

Since the piston moves slowly, therefore

$$\Delta KE = 0,$$

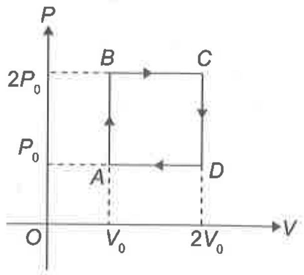
$$W_{gravity} + W_{gas} + W_{atm} + W_{spring} = 0$$

$$-mgd + W_{gas} + (-P_{atm}Ad) + [-(\frac{1}{2}Kd^2 - 0)] = 0$$

$$\Rightarrow \quad W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$ ■

EXAMPLE 14

Find out the work done in the given graph. Also draw the corresponding T–V curve and P–T curve.

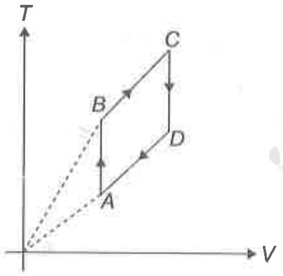


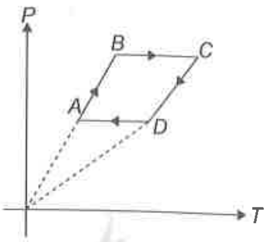
SOLUTION

Since in P–V curves, area under the cycle is equal to work done; therefore, work done by the gas is equal to P_0V_0.

Line AB and CD are isochoric line, line BC and DA are isobaric line.

∴ The T–V curve and P–T curve are drawn as shown.

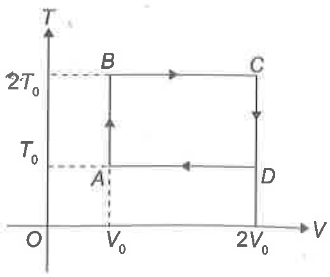




■

EXAMPLE 15

T–V curve of the cyclic process is as shown below, number of moles of the gas n. Find the total work done during the cycle.



SOLUTION

Since path AB and CD are isochoric, work done is zero during path AB and CD. Process BC and DA are isothermal; therefore,

$$W_{BC} = nR2T_0 \ln \frac{V_C}{V_B}$$

$$= 2nRT_0 \ln 2$$

$$W_{DA} = nRT_0 \ln \frac{V_A}{V_D}$$

$$= -nRT_0 \ln 2$$

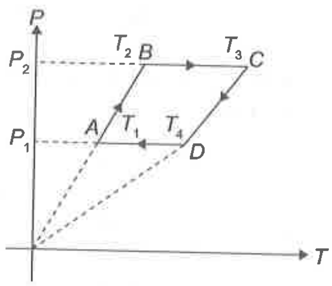
$$\text{Total work done} = W_{BC} + W_{DA}$$

$$= 2nRT_0 \ln 2 - nRT_0 \ln 2$$

$$= nRT_0 \ln 2.$$ ■

EXAMPLE 16

P-T curve of a cyclic process is shown. Find out the work done by the gas in the given process if number of moles of the gas are n.



SOLUTION

Since path AB and CD are isochoric; therefore, work done during AB and CD is zero. Path BC and DA are isobaric.

Hence, $$W_{BC} = nR\Delta T = nR(T_3 - T_2)$$

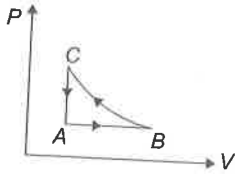
$$W_{DA} = nR(T_1 - T_4).$$

$$\text{Total work done} = W_{BC} + W_{DA}$$

$$= nR(T_1 + T_3 - T_4 - T_2).$$ ■

EXAMPLE 17

Consider the cyclic process $ABCA$ on a sample of 2.0 mol of an ideal gas as shown in the figure. The temperatures of the gas at A and B are 300 K and 500 K, respectively. A total of 1200 J heat is withdrawn from the sample in the process. Find the work done by the gas in part BC. Take $R = 8.3$ J/mol-K.



SOLUTION

The change in internal energy during the cyclic process is zero. Hence, the heat supplied to the gas is equal to the work done by it. Hence,

$$W_{AB} + W_{BC} + W_{CA} = -1200 \text{ J} \quad (1)$$

The work done during the process AB is

$$W_{AB} = P_A(V_B - V_A)$$

$$= nR(T_B - T_A)$$

$$= (2.0 \text{ mol})(8.3 \text{ J/mol-K})(200 \text{ K})$$

$$= 3320 \text{ J.}$$

The work done by the gas during the process CA is zero as the volume remains constant. From (1)

$$3320 \text{ J} + W_{BC} = -1200 \text{ J}$$

or

$$W_{BC} = -4520 \text{ J}$$

$$= -4520 \text{ J.}$$

FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is the law of conservation of energy. It states that if a system absorbs heat dQ and as a result the internal energy of the system changes by dU and the system does a work dW, then

$$dQ = dU + dW.$$

But,

$$dW = PdV$$

$$dQ = dU + PdV,$$

which is the mathematical statement of the first law of thermodynamics.

Heat gained by a system, work done by a system and increase in internal energy are taken as positive.

Heat lost by a system, work done on a system and decrease in internal energy are taken as negative.

SOLVED EXAMPLES

EXAMPLE 18

1 gm of water at 100°C is heated to convert into steam at 100°C at 1 atm. Find out the change in internal energy of water. It is given that volume of 1 gm water at 100°C = 1 cc. volume of 1 gm steam at 100°C = 1671 cc. Latent heat of vaporization = 540 cal/g. (Mechanical equivalent of heat J = 4.2 J/cal.)

SOLUTION

From the first law of thermodynamics

$$\Delta Q = \Delta u + \Delta w$$

$$\Delta Q = mL$$

$$= 1 \times 540 \text{ cal.}$$

$$= 540 \text{ cal.}$$

$$\Delta W = P\Delta V$$

$$= \frac{10^5(1671-1)\times 10^{-6}}{4.2}$$

$$= \frac{10^5(1670-1)\times 10^{-6}}{4.2}$$

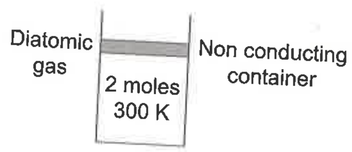
$$= 40 \text{ cal.}$$

$$\Delta U = 540 - 40$$

$$= 500 \text{ cal.}$$

EXAMPLE 19

Two moles of a diatomic gas at 300 K are kept in a non-conducting container enclosed by a piston. Gas is now compressed to increase the temperature from 300 K to 400 K. Find the work done by the gas.



SOLUTION

$$\Delta Q = \Delta u + \Delta W$$

Since the container is non-conducting,

$$\Delta Q = 0$$

$$= \Delta u + \Delta w$$

$$\Rightarrow \quad \Delta W = -\Delta u$$

$$= -n\frac{f}{2}R\Delta T$$

$$= -2 \times \frac{5}{2}R(400 - 300)$$

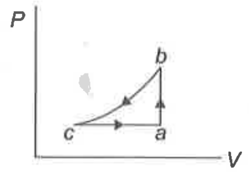
$$= -5 \times 8.314 \times 100 \text{ J}$$

$$= -5 \times 831.4 \text{ J}$$

$$= -4157 \text{ J}.$$

EXAMPLE 20

A sample of an ideal gas is taken through the cyclic process *abca*. It absorbs 50 J of heat during the part *ab*, no heat during *bc* and reflects 70 J of heat during *ca*. 40 J of work is done on the gas during the part *bc*. (a) Find the internal energy of the gas at *b* and *c* if it is 1500 J at *a*. (b) Calculate the work done by the gas during the part *ca*.



SOLUTION

(a) In the part *ab*, the volume remains constant. Thus, the work done by the gas is zero. The heat absorbed by the gas is 50 J. The increase in internal energy from *a* to *b* is

$$\Delta U = \Delta Q = 50 \text{ J}.$$

As the internal energy is 1550 J at *a*, it will be 1550 J at *b*. In the part *bc*, the work done by the gas is $\Delta W = -40$J and no heat is given to the system. The increase in internal energy from *b* to *c* is

$$\Delta U = -\Delta W = 40 \text{ J}.$$

As the internal energy is 1550 J at *b*, it will be 1590 J at *C*.

(b) The change in internal energy, from *c* to *a* is

$$\Delta U = 1500 \text{ J} - 1590 \text{ J}$$

$$= -90 \text{ J}.$$

The heat given to the system is

$$\Delta Q = -70 \text{ J}.$$

Using $\quad \Delta Q = \Delta U + \Delta W,$

$$\Delta W_{ca} = \Delta Q - \Delta U$$

$$= -70 \text{ J} + 90 \text{ J} = 20 \text{ J}.$$

EXAMPLE 21

The internal energy of a monatomic ideal gas is 1.5 nRT. One mole of helium is kept in a cylinder of cross-section 8.5 cm^2. The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2°C, find the distance moved by the piston. Atmospheric pressure = 100 kPa.

SOLUTION

The change in internal energy of the gas is

$$\Delta U = 1.5\, nR\, (\Delta T)$$

$$= 1.5 \text{ (1 mol) (8.3 J/mol-K) (2K)}$$

$$= 24.9 \text{ J}.$$

The heat given to the gas = 42 J.

The work done by the gas is

$$\Delta W = \Delta Q - \Delta U$$

$$= 42 \text{ J} - 24.9 \text{ J}$$

$$= 17.1 \text{ J}.$$

If the distance moved by the piston is *x*, the work done is

$$\Delta W = (100 \text{ kPa})(8.5 \text{ cm}^2)x.$$

Thus, $\quad (10^5 \text{ N/m}^2)(8.5 \times 10^{-4} \text{ m}^2)x = 17.1 \text{ J}$

or, $\quad x = 0.2 \text{ m} = 20 \text{ cm}.$

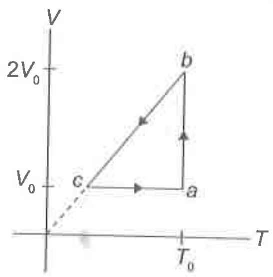
EXAMPLE 22

A sample of an ideal gas has pressure p_0, volume v_0 and temperature T_0. It is isothermally expanded to twice its original volume. It is then compressed at constant pressure to have the original volume V_0. Finally, the gas is heated at constant volume to get the original temperature. (a) Show the process in a V–T diagram. (b) Calculate the heat absorbed in the process.

SOLUTION

(a) The V–T diagram for the process is shown in the figure. The initial state is represented by the point a. In the first step, it is isothermally expanded to a volume $2V_0$. This shown by ab. Then the pressure is kept constant and the gas is compressed to the volume V_0. From the ideal gas equation, V/T is constant at constant pressure. Hence, the process is shown by a line bc which passes through the origin. At point c, the volume is V_0. In the final step, the gas is heated at constant volume to a temperature T_0. This is shown by ca. The final state is the same as the initial state.

(b) The process is cyclic so that the change in internal energy is zero. The heat supplied is, therefore, equal to the work done by the gas. The work done during ab is



$$W_1 = nRT_0 \ln \frac{2v_0}{v_0}$$

$$= nRT_0 \ln 2$$

$$= p_0V_0 \ln 2.$$

Also, from the ideal gas equation,

$$p_aV_a = p_bV_b$$

or,

$$p_b = \frac{p_aV_a}{V_b}$$

$$= \frac{p_0V_0}{2V_0}$$

$$= \frac{p_0}{2}.$$

In the step bc, the pressure remains constant. Hence, the work done is

$$W_2 = \frac{p_0}{2}(V_0 - 2V_0)$$

$$= -\frac{p_0V_0}{2}.$$

In the step ca, the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$$W = W_1 + W_2$$

$$= p_0V_0[\ln 2 - 0.5]$$

$$= 0.193p_0V_0.$$

Hence, the heat supplied to the gas is $0.193p_0V_0$. ■

EXAMPLE 23

A sample of ideal gas ($f = 5$) is heated at constant pressure. If an amount 140 J of heat is supplied to the gas, find (a) the change in internal energy of the gas and (b) the work done by the gas.

SOLUTION

Suppose the sample contains n moles. Also suppose the volume changes from V_1 to V_2 and the temperature changes from T_1 to T_2.

The heat supplied is

$$\Delta Q = \Delta U + P\Delta V$$

$$= \Delta U + nR\Delta T$$

$$= \Delta U + \frac{2\Delta U}{f}.$$

(a) The change is internal energy is

$$\Delta U = n\frac{f}{2}R(T_2 - T_1)$$

$$= \frac{f}{2}Rn(T_2 - T_1)$$

$$= \frac{f}{2+f}\Delta Q$$

$$= \frac{140J}{1.4} = 100\text{ J}.$$

(b) The work done by the gas is

$$\Delta W = \Delta Q - \Delta U$$

$$= 140\ \text{J} - 100\ \text{J} = 40\ \text{J}.$$

EXAMPLE 24

There are two vessels. Each of them contains one mole of a monoatomic ideal gas. Initial volume of the gas in each vessel is 8.3×10^{-3} m^3 at 27°C. Equal amount of heat is supplied to each vessel. In one of the vessels, the volume of the gas is doubled without change in its internal energy, whereas the volume of the gas is held constant in the second vessel. The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system.

SOLUTION

369.3 K, 2.462×10^5 N/m^2

Efficiency of cycle (η):

$$\eta = \frac{\text{Total mechanical work done by the gas in the whole process}}{\text{Heat absorbed by the gas (only +ve)}}$$

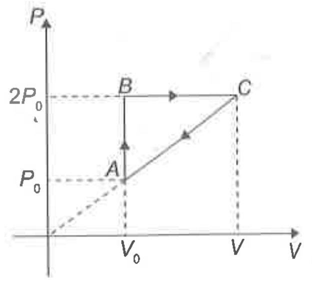
$$= \frac{\text{area under the cycle in } P\text{-}V \text{ curve}}{\text{Heat injected into the system}}$$

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \text{ for heat engine,}$$

$$\Rightarrow \quad \eta = \left(1 - \frac{T_2}{T_1}\right) \text{ for Carnot cycle.}$$

EXAMPLE 25

n moles of a diatomic gas has undergone a cyclic process ABC as shown in the figure. Temperature at A is T_0. Find



(i) Volume at C
(ii) Maximum temperature
(iii) Total heat given to gas
(iv) is heat rejected by the gas, if yes how much heat is rejected?
(v) Find out the efficiency.

SOLUTION

(i) Since the triangles OAV_0 and OCV are similar,

$$\frac{2P_0}{V} = \frac{P_0}{V_0}$$

$$\Rightarrow \quad V = 2V_0.$$

(ii) Since the process AB is isochoric,

$$\frac{P_A}{T_A} = \frac{P_B}{T_B}$$

$$\Rightarrow \quad T_B = 2T.$$

Since the process BC is isobaric,

$$\frac{T_B}{V_B} = \frac{T_C}{V_C}$$

$$\Rightarrow \quad T_C = 2T_B = 4T_0.$$

(iii) Since the process is cyclic,

$$\Delta Q = \Delta W$$

$$= \text{area under the cycle}$$

$$= \frac{1}{2} p_0 V_0.$$

(iv) Since Δu and ΔW both are negative in process CA, ΔQ is negative in the process CA and heat is rejected in the process CA

$$\Delta Q_{CA} = \Delta w_{CA} + \Delta u_{CA}$$

$$= -\frac{1}{2}[P_0 + 2P_0]V_0 - \frac{5}{2} nR(T_c - T_a)$$

$$= -\frac{1}{2}[P_0 + 2P_0]V_0 - \frac{5}{2} nR\left(\frac{4p_0V_0}{nR} - \frac{p_0V_0}{nR}\right)$$

$$= -9P_0V_0 = \text{Heat injected.}$$

(v) η = efficiency of the cycle

$$= \frac{\text{Work done by the gas}}{\text{heat injected}}$$

$$= \eta = \frac{P_0V_0/2}{Q_{\text{injected}}} \times 100$$

$$\Delta Q_{\text{inj}} = \Delta Q_{AB} + \Delta Q_{BC}$$

$$= \left[\frac{5}{2}nR(2T_0 - T_0)\right] + \left[\frac{5}{2}nR(2T_0) + 2P_0(2V_0 - V_0)\right]$$

$$= \frac{19}{2}P_0V_0\,\eta$$

$$= \frac{100}{19}\%.$$

SPECIFIC HEAT

The specific heat capacity of a substance is defined as the heat supplied per unit mass of the substance per unit rise in the temperature. If an amount ΔQ of heat is given to a mass m of the substance and its temperature rises by ΔT, the specific heat capacity s is given by the equation

$$s = \frac{\Delta Q}{m\Delta T}$$

The molar heat capacities of a gas are defined as the heat given per mole of the gas per unit rise in the temperature. The molar heat capacity at constant volume, denoted by C_v, is

$$C_v = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant volume}}$$

$$= \frac{f}{2}R$$

and the molar heat capacity at constant pressure, denoted by C_p, is

$$C_p = \left[\frac{\Delta Q}{n\Delta T}\right]_{\text{constant pressure}} = \left[\frac{f}{2}+1\right]R,$$

where n is the amount of the gas in number of moles and f is the degree of freedom. Quite often, the term specific heat capacity or specific heat is used for molar heat capacity. It is advised that the unit be carefully noted to determine the actual meaning. The unit of specific heat capacity is J/kg-K, whereas that of molar heat capacity is J/mol-K.

Molar Heat Capacity of Ideal Gas in Terms of R

(i) For a monoatomic gas,

$$f = 3$$

$$C_V = \frac{3}{2}R,$$

$$C_P = \frac{5}{2}R$$

$$\Rightarrow \quad \frac{C_p}{C_v} = \gamma$$

$$= \frac{5}{3} = 1.67$$

(ii) For a diatomic gas,

$$f = 5$$

$$C_V = \frac{5}{2}R,$$

$$C_P = \frac{7}{2}R\,\gamma$$

$$= \frac{C_p}{C_v} = 1.4$$

(iii) For a triatomic gas,

$$f = 6$$

$$C_V = 3R,\ C_P = 4R$$

$$\gamma = \frac{C_p}{C_v}$$

$$= \frac{4}{3} = 1.33.$$

[Note for CO_2, $f = 5$, it is linear.]

In general, if f is the degree of freedom of a molecule, then

$$C_V = \frac{f}{2}R,$$

$$C_p = \left(\frac{f}{2}+1\right)R,$$

$$\gamma = \frac{C_p}{C_v} = \left[1+\frac{2}{f}\right].$$

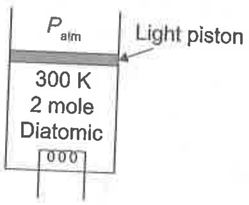
For any general process,

$$C = \frac{fR}{2} + \frac{\text{Work done by gas}}{n\Delta T}.$$

SOLVED EXAMPLES

EXAMPLE 26

Two moles of a diatomic gas at 300 K are enclosed in a cylinder as shown in the figure. The piston is light. Find out the heat given if the gas is slowly heated to 400 K in the following three cases:



(i) Piston is free to move
(ii) If piston does not move
(iii) If piston is heavy and movable.

SOLUTION

(i) Since the pressure is constant,

$$\Delta Q = nC_p\Delta T$$

$$= 2 \times \frac{7}{2} \times R \times (400 - 300)$$

$$= 700R.$$

(ii) Since volume is constant,

$$\Delta W = 0$$

and $\Delta Q = \Delta u$ (from the first law)

$$\Delta Q = \Delta u$$

$$= nC_v\Delta T$$

$$= 2 \times \frac{5}{2} \times R \times (400 - 300)$$

$$= 500R.$$

(iii) Since pressure is constant,

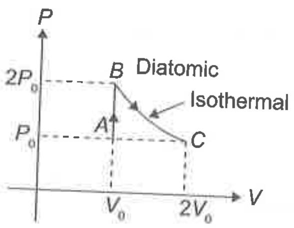
$$\Delta Q = nC_p\Delta T$$

$$= 2 \times \frac{7}{2} \times R \times (400 - 300)$$

$$= 700R.$$

EXAMPLE 27

P–V curve of a diatomic gas is shown in the figure. Find the total heat given to the gas in the process AB and BC.



SOLUTION

From the first law of thermodynamics,

$$\Delta Q_{ABC} = \Delta u_{ABC} + \Delta W_{ABC}$$

$$\Delta W_{ABC} = \Delta W_{AB} + \Delta W_{BC}$$

$$= 0 + nRT_B \ln\frac{V_C}{V_B}$$

$$= nRT_B \ln\frac{2V_0}{V_0}$$

$$= nRT_B \ln 2$$

$$= 2P_0V_0 \ln 2$$

$$\Delta u = nC_v\Delta T$$

$$= \frac{5}{2}(2P_0V_0 - P_0V_0)$$

$$\Rightarrow \Delta Q_{ABC} = \frac{5}{2}P_0V_0 + 2P_0V_0 \ln 2.$$

EXAMPLE 28

Calculate the value of mechanical equivalent of heat from the following data. Specific heat capacity of air at constant volume = 170 cal/kg-K, g = C_p/C_v = 1.4 and the density of air at STP is 1.29 kg/m³. Gas constant R = 8.3 J/mol-K.

SOLUTION

Using $pV = nRT$, the volume of 1 mole of air at STP is

$$V = \frac{nRT}{p}$$

$$= \frac{(1\text{mol})\times(8.3J/\text{mol-k})\times(273\text{K})}{1.0\times10^5 N/m^2}$$

$$= 0.0224 \text{ m}^3.$$

The mass of 1 mole is, therefore,

$$(1.29 \text{ kg/m}^3) \times (0.0224 \text{ m}^3) = 0.029 \text{ kg}.$$

The number of moles in 1 kg is $\frac{1}{0.029}$. The molar heat capacity at constant volume is

$$C_V = \frac{170\,\text{cal}}{(1/0.029)\,\text{mol-k}}$$

$$= 4.93 \text{ cal/mol-K}.$$

Hence, $Cp = \gamma C_v$

$$= 1.4 \times 4.93 \text{ cal/mol-K}$$

or, $C_p - C_v = 0.4 \times 4.93$ cal/mol-K

$$= 1.97 \text{ cal/mol-K}.$$

Thus, 8.3 J = 1.97 cal.

The mechanical equivalent of heat is

$$\frac{8.3\,\text{J}}{1.97\,\text{cal}} = 4.2 \text{ J/cal}.$$

Average Molar Specific Heat of Metals [Dulong and Petit Law]

At room temperature, average molar specific heat of all metals is the same and is nearly equal to 3R. (6 cal. mol⁻¹K⁻¹)

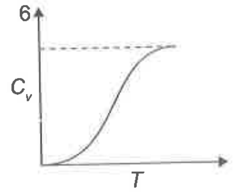


Figure 5.36

Notes

Temperature above which the metals have constant C_v is called Debye temperature.
Mayer's equation is given as $C_P - C_V = R$ (for ideal gases only).

Adiabatic Process

When no heat is supplied or extracted from the system, the process is called adiabatic. Process is sudden so that there is no time for exchange of heat. If walls of a container are thermally insulated, no heat can cross the boundary of the system and the process is adiabatic.

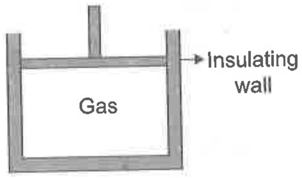


Figure 5.37

Equation of adiabatic process is given by

PV^γ = constant [Poisson law]

$T^\gamma P^{1-\gamma}$ = constant

$TV^{\gamma-1}$ = constant

Slope of P-V curve in adiabatic process

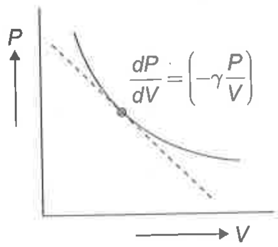


Figure 5.38

Since PV^{γ} is constant,

$$\frac{dp}{dv} = -\gamma\left(\frac{p}{V}\right)$$

Slope of P-T curve in adiabatic process

Since $T^{\gamma}P^{1-\gamma}$ is a constant,

$$\frac{dV}{dT} = -\frac{\gamma}{(1-\gamma)}\frac{p}{T}$$

$$= \frac{(\gamma)}{(\gamma-1)}\frac{p}{T}$$

$$\frac{dP}{dT} = \frac{\gamma}{(\gamma-1)}\frac{P}{T}$$

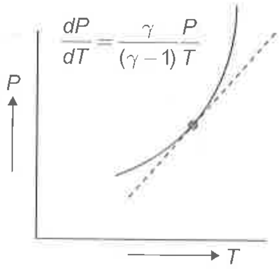


Figure 5.39

Slope of T-V curve

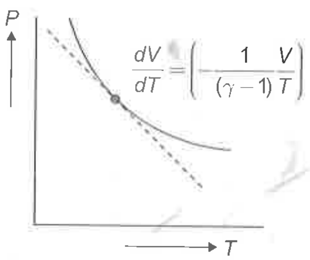


Figure 5.40

$$\frac{dV}{dT} = -\frac{1}{(\gamma-1)}\frac{V}{T}$$

Work Done in Adiabatic Process

$$\Delta W = -\Delta U$$

$$= nC_v(T_i - T_f)$$

$$= \frac{p_iV_i - p_fV_f}{(\gamma-1)}$$

$$= \frac{nR(T_i - T_f)}{\gamma - 1}$$

Work done by the system is positive, if $T_i > T_f$ (hence expansion).

Work done on the system is negative if $T_i < T_f$ (hence compression).

SOLVED EXAMPLES

EXAMPLE 29

A quantity of air is kept in a container having walls which are slightly conducting. The initial temperature and volume are 27°C (equal to the temperature of the surroundings) and 800 cm^3, respectively. Find the rise in the temperature if the gas is compressed to 200 cm^3 (a) in a short time (b) in a long time. Take $\gamma = 1.4$.

SOLUTION

(i) When the gas is compressed in a short time, the process is adiabatic. Thus,

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

or
$$T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

$$= (300\text{ K}) \times \left[\frac{800}{200}\right]^{0.4}$$

$$= 522\text{ K}.$$

Rise in temperature $= T_2 - T_1 = 222$ K.

(ii) When the gas is compressed for a long time, the process is isothermal. Thus, the temperature remains equal to the temperature of the surroundings, that is, 27°C. The rise in temperature = 0. ■

EXAMPLE 30

A monoatomic gas is enclosed in a non-conducting cylinder having a piston which can move freely. Suddenly, gas is compressed to 1/8 of its initial volume. Find the final pressure and temperature if the initial pressure and temperature are P_0 and T_0, respectively.

SOLUTION

Since the process is adiabatic,

$$P_0V^{\frac{5}{3}} = P_{\text{final}}\left(\frac{V}{8}\right)^{5/3}$$

$$\gamma = \frac{C_P}{C_V}$$

$$= \frac{5R}{2} / \frac{3R}{2}$$

$$= \frac{5}{3}.$$

Since the process is adiabatic,

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

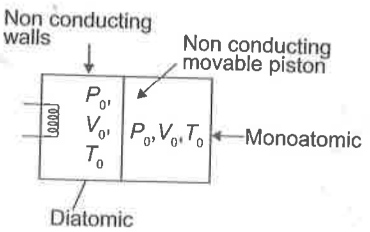
$$\Rightarrow \quad T_0V_0^{2/3} = T_{\text{final}}\left(\frac{V_0}{8}\right)^{2/3}$$

$$\Rightarrow \quad T = 4T_0.$$

■

EXAMPLE 31

A cylindrical container having non-conducting walls is partitioned in two equal parts such that the volume of the each part is equal to V_0. A movable non-conducting piston is kept between the two parts. Gas on the left is slowly heated so that the gas on the right is compressed up to volume $\frac{V_0}{8}$. Find the pressure and temperature on both sides if the initial pressure and temperature were P_0 and T_0, respectively. Also, find the heat given by the heater to the gas. (Number of moles in each part is n.)



SOLUTION

Since the process on the right is adiabatic,

$$PV^{\gamma} = \text{constant}.$$

$$\Rightarrow \quad P_0V_0^{\gamma} = P_{\text{final}}(V_0/8)^{\gamma}$$

$$\Rightarrow \quad P_{\text{final}} = 32\,P_0$$

$$T_0V_0^{\gamma-1} = T_{\text{final}}(V_0/8)^{\gamma-1}.$$

Let volume of the left part is V_1

$$\Rightarrow \quad 2V_0 = V_1 + \frac{V_0}{8}$$

$$\Rightarrow \quad V_1 = \frac{15V_0}{8}.$$

Since the number of moles on the left part remains constant, for the left part PV/T = constant.

Final pressure on both sides will be the same

$$\Rightarrow \quad \frac{P_0V_0}{T_0} = \frac{P_{\text{final}}V_1}{T_{\text{final}}}$$

$$\Rightarrow \quad T_{\text{final}} = 60T_0.$$

$$\Delta Q = \Delta u + \Delta w$$

$$\Delta Q = n\frac{5R}{2}(60T_0 - T_0) + n\frac{3R}{2}(4T_0 - T_0)$$

$$\Delta Q = \frac{5nR}{2} \times 59T_0 + \frac{3nR}{2} \times 3T_0.$$

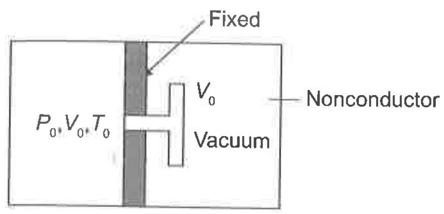
■

Free Expansion

If a system, say a gas, expands in such a way that no heat enters or leaves the system and also no work is done by or on the system, then the expansion is called 'free expansion'. $\Delta Q = 0$, $\Delta U = 0$ and $\Delta W = 0$. Temperature in the free expansion remains constant.

EXAMPLE 32

A non-conducting cylinder having volume $2V_0$ is partitioned by a fixed non-conducting wall in two equal parts. Partition is attached with a valve. Right side of the partition is a vacuum and left part is filled with a gas having pressure and temperature P_0 and T_0, respectively. If the valve is opened, find the final pressure and temperature of the two parts.



SOLUTION

From the first law of thermodynamics,

$$\Delta Q = \Delta u + \Delta W.$$

Since gas expands freely, $\Delta W = 0$. Since no heat is given to the gas,

$$\Delta Q = 0.$$

$\Rightarrow \quad \Delta u = 0$ and temperature remains constant.

$$T_{\text{final}} = T_0.$$

Since the process is isothermal,

$$P_0 \times V_0 = P_{\text{final}} \times 2V_0$$

$$\Rightarrow \quad P_{\text{final}} = P_0/2$$

Reversible and Irreversible Processes

A process is said to be reversible when the various stages of an operation in which it is subjected can be traversed back in the opposite direction in such a way that the substance passes through exactly the same conditions at every step in the reverse process as in the direct process.

Comparison of Slopes of Isothermal and Adiabatic Curves

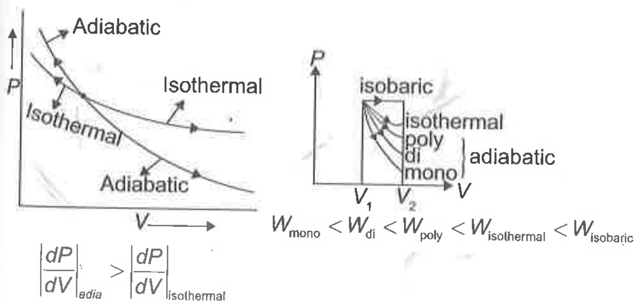


Figure 5.41

In compression up to the same final volume,

$$|W_{\text{adia}}| > |W_{\text{isothermal}}|.$$

In expansion up to the same final volume,

$$W_{\text{isothermal}} > W_{\text{adia}}.$$

Limitations of the First Law of Thermodynamics

The first law of thermodynamics tells us that heat and mechanical work are interconvertible. However, this law fails to explain the following points:

1. It does not tell us about the direction of transfer of heat.
2. It does not tell us about the conditions under which heat energy is converted into work.
3. It does not tell us whether some process is possible or not.

Mixture of non-reacting gases:

1. Molecular weight $= \dfrac{n_1M_1 + n_2M_2}{n_1 + n_2}$

   where M_1 and M_2 are molar masses.

2. Specific heat,

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2},$$

$$C_p = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1 + n_2}$$

3. For mixture,

$$\gamma = \frac{C_{p_{mix}}}{C_{v_{mix}}} = \frac{n_1C_{p_1} + n_2C_{p_2} + \cdots}{n_1C_{v_1} + n_2C_{v_2} + \cdots}$$

4. Degree of freedom for mixture,

$$f = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$$

EXERCISES

JEE Main

Kinetic Theory of Gases

1. The temperature at which the r.m.s velocity of oxygen molecules equal that of nitrogen molecules at 100°C is nearly:
   (A) 426.3 K (B) 456.3 K
   (C) 436.3 K (D) 446.3 K

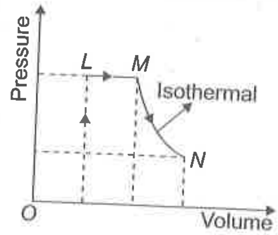
2. The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the O_2 molecule dissociated into oxygen atoms, the rms speed will become
   (A) v (B) $v\sqrt{2}$
   (C) $2v$ (D) $4v$

3. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2, the average speed of the O_2 molecules in vessel C will be:
   (A) $(V_1 + V_2)/2$ (B) V_1
   (C) $(V_1V_2)^{1/2}$ (D) $\sqrt{3kT/M}$

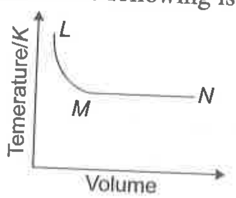
4. $N(< 100)$ molecules of a gas have velocities 1, 2, 3… N/km/s respectively. Then
   (A) rms speed and average speed of molecules is same.
   (B) ratio of rms speed to average speed is $\sqrt{(2N+1)(N+1)}/6\text{ N}$
   (C) ratio of rms speed to average speed is $\sqrt{(2N+1)(N-1)}/6\text{ N}$
   (D) ratio of rms speed to average speed of a molecules is $2/\sqrt{6} \times \sqrt{(2N+1)/(N+1)}$

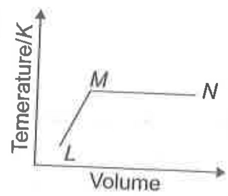
5. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is:
   (A) 1:1
   (B) 1:2
   (C) 2:1
   (D) depends on the moments of inertia of the two molecules

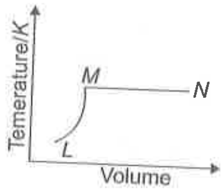
6. A fixed mass of ideal gas undergoes changes of pressure and volume starting at L, as shown in figure.

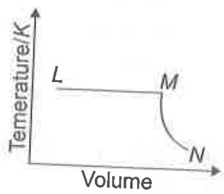


Which of the following is correct:

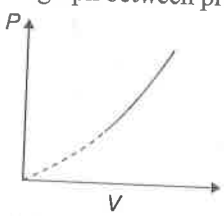
(A)


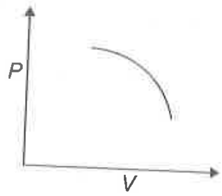
(B)


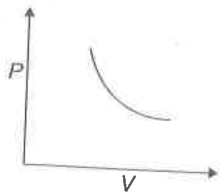
(C)


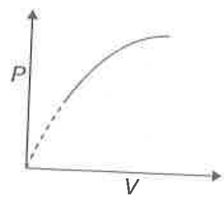
(D)


7. An ideal gas follows a process PT = constant. The correct graph between pressure and volume is

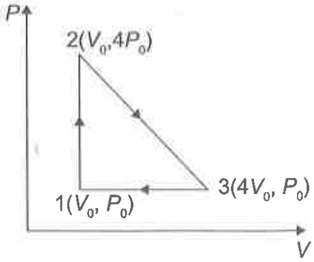
(A)


(B)


(C)


(D)


8. One mole of an ideal diatomic gas is taken through the cycle as shown in the figure.



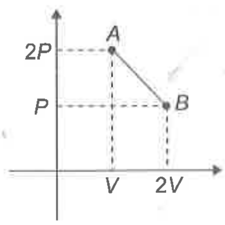
1 → 2: isochoric process
2 → 3: straight line on P-V diagram
3 → 1: isobaric process
The average of molecular speed of the gas in the states 1, 2 and 3 are in the ratio
(A) 1:2:2 (B) $1:\sqrt{2}:\sqrt{2}$
(C) 1:1:1 (D) 1:2:4

9. The pressure of an ideal gas is written as $P = \dfrac{2E}{3V}$. Here E refers to
(A) translational kinetic energy
(B) rotational kinetic energy
(C) vibrational kinetic energy
(D) total kinetic energy.

10. Which of the following quantities is the same for all ideal gases at the same temperature?
(A) the kinetic energy of 1 mole
(B) the kinetic energy of 1 g
(C) the number of molecules in 1 mole
(D) the number of molecules in 1 g

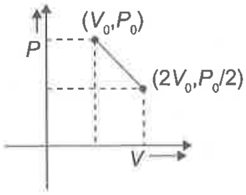
11. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal
(A) 2.4×10^{23} (B) 3×10^{23}
(C) 6×10^{23} (D) 4.8×10^{23}

12. The process AB is shown in the diagram. As the gas is taken from A to B, its temperature



(A) initially increases then decreases
(B) initially decreases then increases
(C) remains constant
(D) variation depends on type of gas

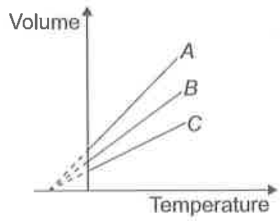
13. One mole of a gas expands obeying the relation as shown in the P/V diagram. The maximum temperature in this process is equal to



(A) $\dfrac{P_0V_0}{R}$ (B) $\dfrac{3P_0V_0}{R}$
(C) $\dfrac{9P_0V_0}{8R}$ (D) *None*

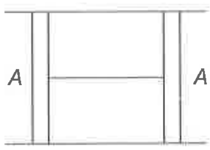
14. 12 gms of gas occupy a volume of 4×10^{-3} m³ at a temperature of 7°C. After the gas is heated at constant pressure its density becomes 6×10^{-4} gm/cc. What is the temperature to which the gas was heated.
(A) 1000 K (B) 1400 K
(C) 1200 K (D) 800 K

15. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then the expansion of the same ideal gas of mass $2m$ at a pressure $2P$ is given by the straight line



(A) C (B) A
(C) B (D) none

16. A cylindrical tube of cross-sectional area A has two air tight frictionless pistons at its two ends. The pistons are tied with a straight two ends. The pistons are tied with a straight piece of metallic wire. The tube contains a gas at atmospheric pressure P_0 and temperature T_0. If temperature of the gas is doubled then the tension in the wire is



(A) $4P_0A$
(B) $P_0A/2$
(C) P_0A
(D) $2P_0A$

17. An ideal gas of Molar mass M is contained in a vertical tube of height H, closed at both ends. The tube is accelerating vertically upwards with acceleration g. Then, the ratio of pressure at the bottom and the mid point of the tube will be
(A) $\exp[2MgH/RT]$
(B) $\exp[-2MgH/RT]$
(C) $\exp[MgH/RT]$
(D) MgH/RT

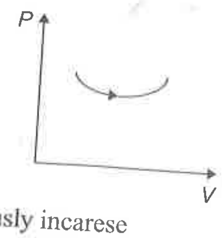
18. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43 m. The length of air column above mercury level in the tube will be: Take P_{atm} = 76 cm of mercury
(A) 0.215 m
(B) 0.2 m
(C) 0.1 m
(D) 0.4 m

19. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube ?
(A) 33.3 kPa
(B) 66.7 kPa
(C) 3.33 MPa
(D) 6.67 MPa

Thermodynamics

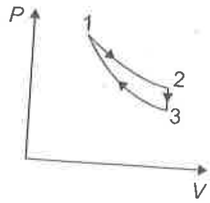
20. One mole of an ideal gas at temperature T_1 expends according to the law $\frac{P}{V^2} = a$ (constant). The work done by the gas till temperature of gas becomes T_2 is:
(A) $\frac{1}{2}R(T_2 - T_1)$
(B) $\frac{1}{3}R(T_2 - T_1)$
(C) $\frac{1}{4}R(T_2 - T_1)$
(D) $\frac{1}{5}R(T_2 - T_1)$

21. Consider the process on a system shown in fig. During the process, the cumulative work done by the system



(A) continuously incarese
(B) continuously decreases
(C) first increases then decreases
(D) first decreases then increases

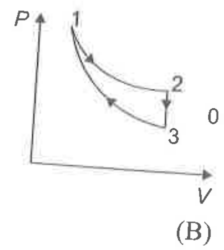
22. Three processes from a thermodynamic cycle as shown on P-V diagram for an ideal gas. Process 1 → 2 takes place at constant temperature (300 K). Process 2 → 3 takes place at constant volume. During this process 40 J of heat leaves the system. Process 3 → 1 is adiabatic and temperature T_3 is 275 K. Work done by the gas during the process 3 → 1 is



(A) −40 J
(B) −20 J
(C) +40 J
(D) +20 J

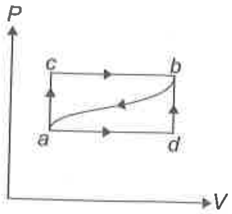
23. When unit mass of water boils to become steam at 100°C, it absorbs Q amount of heat. The densities of water and steam at 100°C are ρ_1 and ρ_2 respectively and the atmospheric pressure is p_0. The increase in internal energy of the water is
(A) Q
(B) $Q + p_0\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$
(C) $Q + p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$
(D) $Q - p_0\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

24. Three processes compose a thermodynamics cycle shown in the PV diagram. Process 1 → 2 takes place at constant temperature. Process 2 → 3 takes place at constant volume, and process 3 → 1 is adiabatic. During the complete cycle, the total amount of work done is 10 J. During process 2 → 3, the internal energy decrease by 20 J and during process 3 → 1, 20 J of work is done on the system. How much heat is added to the system during process 1 → 2 ?



(A) 0
(B) 10 J
(C) 20 J
(D) 30 J

25. When a system is taken from state '*a*' to state '*b*' along the path '*acb*', it is found that a quantity of heat $Q = 200$ J is absorbed by the system and a work $W = 80$ J is done by it. Along the path '*adb*', $Q = 144$ J. The work done along the path '*adb*' is



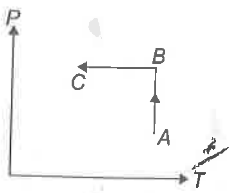
(A) 6 J (B) 12 J
(C) 18 J (D) 24 J

26. In the above question, if the work done on the system along the curved path '*ba*' is 52 J, heat absorbed is
(A) −140 J (B) −172 J
(C) 140 J (D) 172 J

27. In above question, if $U_a = 40$ J, value of U_b will be
(A) −50 J (B) 100 J
(C) −120 J (D) 160 J

28. In above question, if $U_d = 88$ J, heat absorbed for the path '*db*' is
(A) −72 J (B) 72 J
(C) 144 J (D) −144 J

29. Ideal gas is taken through process shown in figure:



(A) In process *AB*, work done by system is positive.
(B) In process *AB*, heat is rejected out of the system.
(C) In process *AB*, internal energy increases
(D) In process *AB* internal energy decreases and in process *BC* internal energy increases.

30. A thermodynamic cycle takes in heat energy at a high temperature and rejects energy at a lower temperature. If the amount of energy rejected at the low temperature is 3 times the amount of work done by the cycle, the efficiency of the cycle is
(A) 0.25 (B) 0.33
(C) 0.67 (D) 0.9

31. A diatomic gas of molecular weight 30 gm/mole is filled in a container at 27°C. It is moving at a velocity 100 m/s. If it is suddenly stopped, the rise in temperature of gas is:
(A) $60/R$ (B) $\frac{600}{R}$
(C) $\frac{6\times10^4}{R}$ (D) $\frac{6\times10^5}{R}$

Question No. 32 to 35

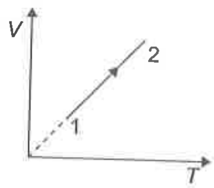
Five moles of helium are mixed with two moles of hydrogen to form a mixture. Take molar mass of helium $M_1 = 4$ g and that of hydrogen $M_2 = 2$ g

32. The equivalent molar mass of the mixture is
(A) 6 g (B) 13 g/7
(C) 18 g/7 (D) none

33. The equivalent degree of freedom *f* of the mixture is
(A) 3.57 (B) 1.14
(C) 4.4 (D) none

34. The equivalent value of γ is
(A) 1.59 (B) 1.53
(C) 1.56 (D) none

35. An ideal gas undergoes the process $1 \rightarrow 2$ as shown in the figure, the heat supplied and work done in the process is ΔQ and ΔW respectively. The ratio $\Delta Q{:}\Delta W$ is



(A) $\gamma:\gamma - 1$ (B) γ
(C) $\gamma - 1$ (D) $\gamma - 1/\gamma$

36. A reversible adiabatic path on a *P-V* diagram for an ideal gas passes through state *A* where $P = 0.7 \times 10^5$ N/m^{-2} and $v = 0.0049$ m^3. The ratio of specific heat of the gas is 1.4. The slope of path at *A* is:
(A) 2.0×10^7 Nm^{-5} (B) 1.0×10^7 Nm^{-5}
(C) -2.0×10^7 Nm^{-5} (D) -1.0×10^7 Nm^{-5}

37. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature

change. When the temperature of the gas changes by 300 K, the internal energy of the gas changes by

(A) 5000 J (B) 12600 J
(C) 17600 J (D) 22600 J

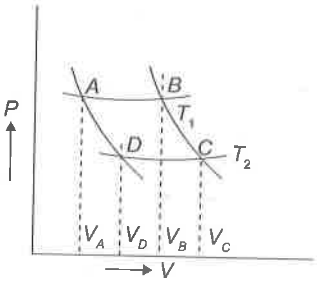
38. One mole of an ideal monoatomic gas at temperature T_0 expands slowly according to the law P/V = constant. If the final temperature is $2T_0$, heat supplied to the gas is:

(A) $2RT_0$ (B) $\frac{3}{2}RT_0$
(C) RT_0 (D) $\frac{1}{2}RT_0$

39. 2 moles of a diatomic gas undergoes the process: PT^2/V = constant. Then, the molar heat capacity of the gas during the process will be equal to

(A) $5R/2$ (B) $9R/2$
(C) $3R$ (D) $4R$

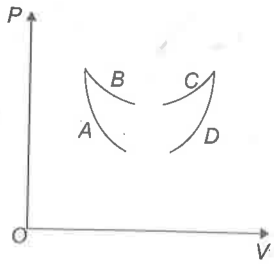
40. In the following P-V diagram of an ideal gas, two adiabates cut two isotherms at T_1 and T_2. The value of V_B/V_C is



$AB \rightarrow T_1$, $DC \rightarrow T_2$

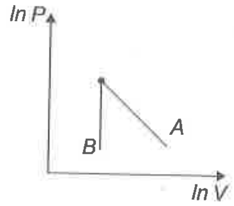
(A) $= V_A/V_D$ (B) $< V_A/V_D$
(C) $> V_A/V_D$ (D) cannot say

41. Four curves A, B, C and D are drawn in the fig. for a given amount of gas. The curves which represent adiabatic and isothermal changes are



(A) C and D respectively
(B) D and C respectively
(C) A and B respectively
(D) B and A respectively

42. The figure, shows the graph of logarithmic reading of pressure and volume for two ideal gases A and B undergoing adiabatic process. From figure it can be concluded that

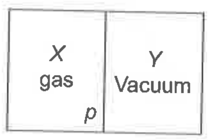


(A) gas B is diatomic
(B) gas A and B both are diatomic
(C) gas A is monoatomic
(D) gas B is monoatomic & gas A is diatomic

43. An ideal gas expands from volume V_1 to V_2. This may be achieved by either of the three processes : isobaric, isothermal and adiabatic, Let ΔU be the change in internal energy of the gas, Q be the quantity of heat added to the system and W be the work done by the system on the gas. Identify which of the following statements is false for ΔU?

(A) ΔU is least under adiabatic process
(B) ΔU is greatest under adiabatic process.
(C) ΔU is greatest under the isobaric process
(D) ΔU in isothermal process lies in-between the values obtained under isobaric and adiabatic processes.

44. A closed container is fully insulated from outside. One half of it is filled with an ideal gas X separated by a plate P from the other half Y which contains a vacuum as shown in figure. When P is removed, X moves into Y. Which of the following statements is correct ?



(A) No work is done by X
(B) X decreases in temperature
(C) X increases in internal energy
(D) X doubles in pressure.

45. 1 kg of a gas does 20 kJ of work and receives 16 kJ of heat when it is expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kJ. The work done by the gas in the second expansion is:

(A) 32 kJ (B) 5 kJ
(C) −4 kJ (D) 13 kJ

46. An ideal gas undergoes an adiabatic process obeying the relation $PV^{4/3}$ = constant. If its initial temperature is 300 K and then its pressure is increased upto four times its initial value, then the final temperature (in Kelvin):

(A) $300\sqrt{2}$ (B) $300\sqrt[3]{2}$
(C) 600 (D) 1200

JEE Advanced

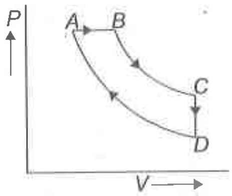
Kinetic Theory of Gases

1. According to kinetic theory of gases,
(A) The velocity of molecules decreases for each collision
(B) The pressure exerted by a diatomic gas is proportional to the mean velocity of the molecule.
(C) The K.E. of the gas decreases on expansion at constant temperature.
(D) The mean translational K.E. of a diatomic gas increases with increase in absolute temperature.

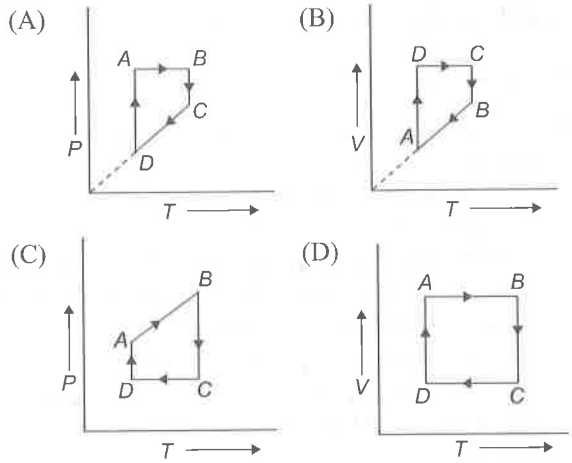
2. Consider a collision between an oxygen molecule and a hydrogen molecules in a mixture of oxygen and hydrogen kept at room temperature. Which of the following are possible ?
(A) The kinetic energies of both the molecules increase.
(B) The gas is not isotropic and the constant (1/3) in equation $P = (1/3)\rho v^2_{rms}$ is result of this property
(C) The time during which a collision lasts is negligible compared to the time of free path between collisions.
(D) There is no force of interaction between molecules among themselves or between molecules and the wall except during collision.

3. In case of hydrogen and oxygen at N.T.P., which of he following quantities is/are the same ?
(A) average momentum per molecule
(B) average kinetic energy per molecule
(C) kinetic energy per unit volume
(D) kinetic energy per unit mass

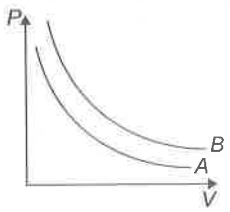
4. A cylic process $ABCD$ is shown in the P-V diagram. (BC and DA are isothermal)



Which of the following curves represents the same process ?



5. Figure shows the pressure P versus volume V graphs for two different gas sample at a given temperature. M_A and M_B are masses of two samples, n_A and n_B are numbers of moles. Which of the following **must be incorrect.**



(A) $M_A > M_B$
(B) $M_A < M_B$
(C) $n_A > n_B$
(D) $n_A < n_B$

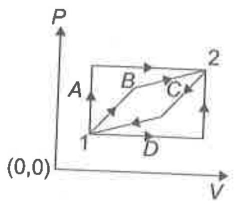
6. During an experiment, an ideal gas is found to obey a condition P^2/ρ = constant [ρ = density of the gas]. The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\rho/2$.
(A) The pressure of the gas changes to $\sqrt{2}P$
(B) The temperature of the gas changes to $\sqrt{2}T$
(C) The graph of above process on the P-T diagram is parabola
(D) The graph of the above process on the P-T diagram is hyperbola.

7. Two vessels of the same volume contain the same gas at same temperature. If the pressure in the vessel be in the ratio of 1:2, then
(A) the ratio of the average kinetic energy is 1:2
(B) the ratio of the root mean square velocity is 1:1
(C) the ratio of the average velocity is 1:2
(D) the ratio of number of molecules is 1:2

Thermodynamics

8. A vertical cylinder with heat-conducting walls is closed at the bottom and is fitted with a smooth light piston. It contains one mole of an ideal gas. The temperature of the gas is always equal to the surrounding\s temperature, T_0. The piston is moved up slowly to increase the volume of the gas to η times. Which of the following is incorrect ?
(A) Work done by the gas is $RT_0 \ln \eta$.
(B) Work done against the atmosphere is $RT_0(\eta - 1)$
(C) There is no change in the internal energy of the gas.
(D) The final pressure of the gas is $\frac{1}{(\eta - 1)}$ times it's initial pressure.

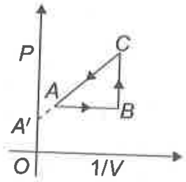
9. An ideal gas is taken from state 1 to state 2 through optional path A, B, C & D as shown in P-V diagram. Let Q, W and U represent the heat supplied, work done & internal energy of the gas respectively. Then



(A) $Q_B - W_B > Q_C - W_C$
(B) $Q_A - Q_D = W_A - W_D$
(C) $W_A < W_B < W_C < W_D$
(D) $Q_A > Q_B > Q_C > Q_D$

10. Two moles of monoatomic gas is expanded from (P_0, V_0) to $(P_0, 2V_0)$ under isobaric condition. Let ΔQ_1, be the heat given to the gas, ΔW_1 the work done by the gas and ΔU_1 the change in internal energy. Now the monoatomic gas is replaced by a diatomic gas. Other conditions remaining the same. The corresponding values in this case are ΔQ_2, ΔW_2, ΔU_2 respectively, then
(A) $\Delta Q_1 - \Delta Q_2 = \Delta U_1 - \Delta U_2$
(B) $\Delta U_2 + \Delta W_2 > \Delta U_1 + \Delta W_1$
(C) $\Delta U_2 > \Delta U_1$
(D) All of these

11. An enclosed ideal gas is taken through a cycle as shown in the figure. Then



(A) Along AB, temperature decrease while along BC temperature increases
(B) Along AB, temperature increases while along BC the temperature decreases
(C) Along CA work is done by the gas and the internal energy remains constant
(D) Along CA work is done on the gas and internal energy of the gas increases

12. A rigid container of neligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C if 3.0 cal of heat is added to it. The gas may be
(A) helium
(B) argon
(C) oxygen
(D) carbon dioxide

13. A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2. Then
(A) equivalent molecular weight of the mixture is 36.
(B) equivalent molecular weight of the mixture is 18.
(C) γ for the mixture is 5/2
(D) γ for the mixture is 47/35
(Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

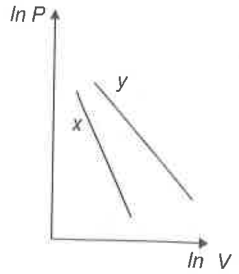
14. What is/are the same for O_2 and NH_3 in gaseous state
(A) ratio of specific heats
(B) average velocity
(C) maximum no. of vibrational degree of freedom
(D) None of these

15. moles of a monoatomic gas are expanded to double its initial volume, through a process P/V = constant. If its initial temperature is 300 K, then which of the following is not true.
(A) $\Delta T = 900$ K
(B) $\Delta Q = 3200\ R$
(C) $\Delta Q = 3600\ R$
(D) $W = 900\ R$

16. A gas kept in a container of finite conductivity is suddenly compressed. The process
(A) must be very nearly adiabatic
(B) must be very nearly isothermal
(C) may be very nearly adiabatic
(D) may be very nearly isothermal

17. When an enclosed perfect gas is subjected to an adiabatic process:
(A) Its total internal energy does not change
(B) Its temperature does not change
(C) Its pressure varies inversely as a certain power of its volume
(D) The product of its pressure and volume is directly proportional to its absolute temperature.

18. For two different gases X and Y, having degrees of freedom f_1 and f_2 and molar heat capacities at constant volume C_{v_1} and C_{v_2} respectively, the ln P versus ln V graph is plotted of adiabatic process, as shown



(A) $f_1 > f_2$
(B) $f_2 > f_1$
(C) $C_{v_2} > C_{v_1}$
(D) $C_{v_1} > C_{v_2}$

JEE Advanced

Level I

Kinetic Theory of Gases

1. Consider a sample of oxygen at 300 K. Find the average time taken by a molecule to travel a distance equal to the diameter of the earth. (Diameter of earth = 12800 km)

2. Find the average magnitude of linear momentum of a helium molecule in a sample of helium gas at 0°C Mass of helium molecule = 6.64×10^{-27} kg and Boltazmann constant = 1.38×10^{-23} J/K.

3. The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

4. Find the ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.

5. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, find the total internal energy of the system ?

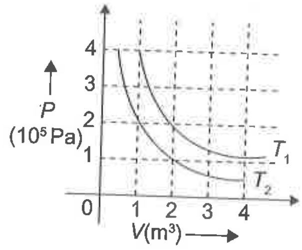
6. 0.040 g of He is kept in a closed container initially at 100.0°C. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12 J.

7. Show that the internal energy of the air (treated as an ideal gas) contained in a room remains constant as the temperature changes between day and night. Assume that the atmospheric pressure around remains constant and the air in the room maintains this pressure by communicating with the surrounding through the windows etc.

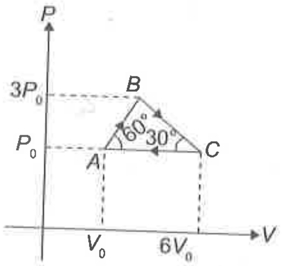
8. Total translational kinetic energy per mole of an ideal gas at 0° C is nearly ________ joules.

9. During an experiment, an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at a temperature T, and volume V. When it expands to a volume $2V$, the temperature becomes ________.

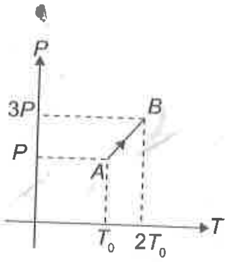
10. The following graph shows two isotherms for a fixed mass of an ideal gas. Find the ratio of r.m.s. speed of the moelcules at temperatures T_1 and T_2?



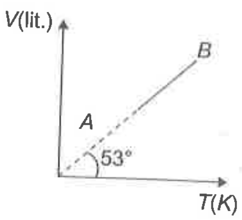
11. Two moles of an ideal monoatomic gas undergone a cyclic process $ABCA$ as shown in figure. Find the ratio of temperature at B and A.



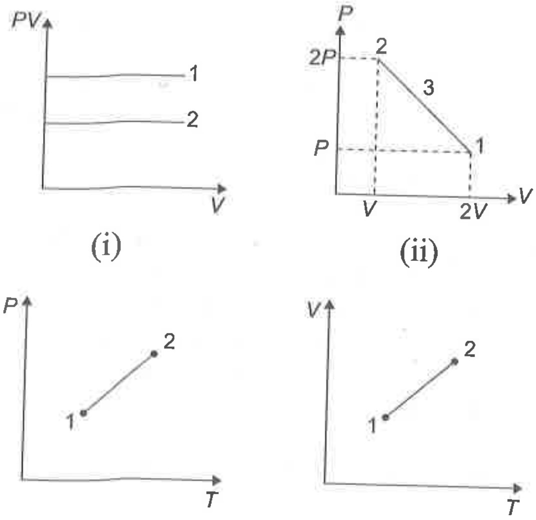
12. Pressure versus temperature graph of an ideal gas is shown. Density of gas at point A is r_0. Find the density of gas at B.



13. V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



14. Examine the following plots and predict whether in
 (i) $P_1 < P_2$ and $T_1 > T_2$ in
 (ii) $T_1 = T_2 < T_3$, in
 (iii) $V_1 > V_2$, in
 (iv) $P_1 > P_2$ or otherwise.



(i) (ii) (iii) (iv)

15. The height of mercury in a faulty barometer is 75 cm and the tube above mercury having air is 10 cm long. The correct barometer reading is 76 cm. If the faulty barometer reads 74 cm, find the true barometer reading.

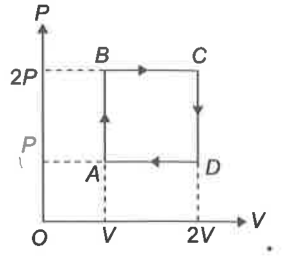
Thermodynamics

16. One mole of gas expands with temperature T such that its volume, $V = kT^2$, where k is a constant. If the temperature of the gas changes by 60°C then find the work done by the gas?

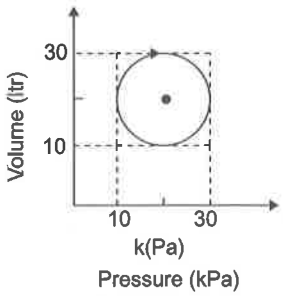
17. One mole of an idel monoatomic gas is at 360 K and a pressure of 10^5 pa. It is compressed at constant pressure until its volume is halved. Taking R as 8.3 J mol^{-1}K^{-1} and the initial volume of the gas as 3.0×10^{-2} m^3, find the work done on the gas ?

18. When 1 g of water at 0°C and 1×10^5 Nm^{-2} pressure is converted into ice of volume 1.091 cm^3, find the work done by water ?

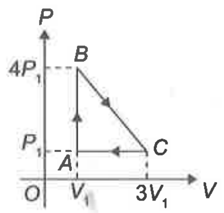
19. An ideal monoatomic gas is taken round the cycle $ABCDA$ as shown in the P-V diagram. Find the work done by the gas during the cycle ?



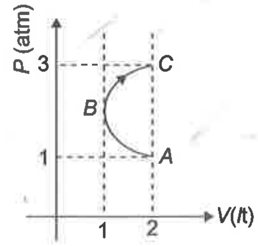
20. Find the work done by gas going through a cyclic process shown in figure ?



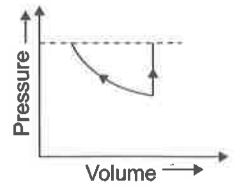
21. An ideal gas taken around the cycle $ABCA$ shown in P-V diagram. Find the net work done by the gas during the cycle ?



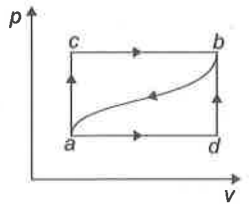
22. In the P-V diagram shown in figure, ABC is a semicircle. Find the workdone in the process ABC.



23. A sample of an ideal gas initially having internal energy U_1 is allowed to expand adiabatically performing work W. Heat Q is then supplied to it, keeping the volume constant at its new value, until the pressure rised to its original value. The internal energy is then U_2. (See Fig.) Find the increase in internal enery $(U_2 - U_1)$?

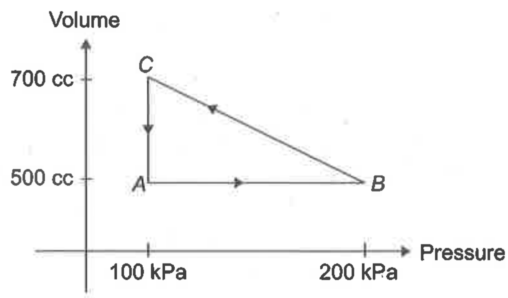


24. When a system is taken from state 'a' to state 'b' along the path 'acb', it is found that a quantity of heat $Q = 200$ J is absorbed by the system and work $W = 80$ J is done by it. Along the pathy 'adb', $Q = 144$ J. Find work done along the path 'adb'?

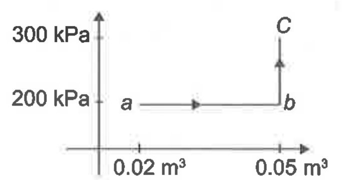


25. Find the change in the internal energy of 2 kg of water as it is heated from 0°C to 4°C. The specific heat capacity of water is 4200 J/kg-K and its densities at 0°C and 4°C 999.9 kg/m³ and 1000 kg/m³ respectively. Atmospheric pressure = 10^5 Pa.

26. A gas is taken through a cyclic process $ABCA$ as shown in figure. If 2.4 cal of heat is given in the process, what is the value of J.



27. A substance is taken through the process abc as shown in figure. If the internal energy of the substance increases by 5000 J and a heat of 2625 cal is given to the system, calculate the value of J.



28. Calculate the change in internal energy of a gas kept in a rigid container when 100 J of heat is supplied to it.

29. A mixture of 4 gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume 300°K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules.
(R = Universal gas constant)

30. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding works involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively.
(i) Find the value of W_4
(ii) What is the efficiency of the cycle ?

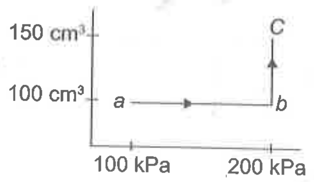
31. An amount Q of heat is added to a monoatomic ideal gas in a process in which the gas perform a work $Q/2$ on its surrounding. Find the molar heat capacity for the process.

32. An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation $p = kV$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + R/2$.

33. An ideal gas ($C_p/C_v = \gamma$) is taken through a process in which the pressure and the volume vary as $p = aV^b$. Find the value of b for which the molar specific heat capacity in the process is zero.

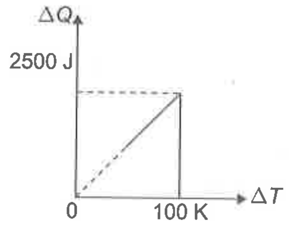
34. Two ideal gases have the same value of $C_p/C_v = \gamma$. What will be the value of this ratio for a mixture of the two gases in the ratio 1:2 ?

35. A mixture contains 1 mole of helium ($C_p = 2.5R$, $C_v = 1.5R$) and 1 mole of hydrogen ($C_p = 3.5R$, $C_v = 2.5R$). Calculate the value of C_p, C_v and γ for mixture.



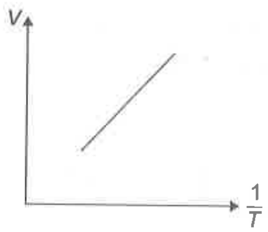
36. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. Find the ratio $\frac{C_p}{C_v}$ of the mixture?

37. One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.



38. An ideal gas has a molar heat capacity C_v at constant volume. Find the molar heat capacity of this gas as a function of volume, if the gas undergoes the process : $T = T_0^{\alpha V}$

39. One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.



40. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K. Determine the change is the internal energy of the gas.

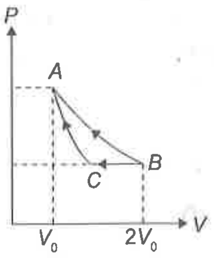
41. 70 calorie of heat is required to raise the temperautre of 2 mole of an ideal gas at constant pressure from 40°C to 45°C. Find the amount of heat required to raise the temperature of the same through the same range at constant volume ($R = 2$ cal/mol-K)

42. The volume of one mole of an ideal gas with specific heat ratio γ is varied according to the law $V = \frac{a}{T^2}$, where a is a constant. Find the amount of heat obtained by the gas in this process if the gas temperature is increased by ΔT.

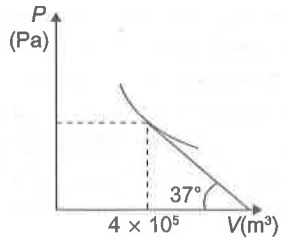
43. Find the molecular mass of a gas if the specific heats of the gas are $C_p = 0.2$ cal/gm°C and $C_v = 0.15$ cal/gm°C. [Take $R = 2$ cal/mol°C]

44. A gas at NTP is suddenly compressed to one-fourth of its original volume. If γ is supposed to be 3/2, then find final pressure ?

45. In a cycle $ABCA$ consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle
(Given : $T_A = T_B = 400$ K, $\gamma = 1.5$)



46. P-V graph for an ideal gas undergoing polytropic process PV^m = constant is shown here.
Find the value of m.



47. Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

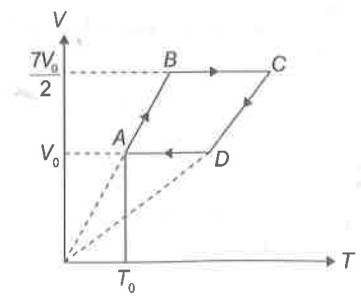
48. A gas undergoes a process in which the pressure and volume are related by VP^n = constant. Find the buk modulus of the gas.

49. A container of volume 1 m³ is divided into two equal compartment by a partition. One of these compartments contains an ideal gas at 300 K. The other compartment is vaccum. The whole system is thermally isolated from its surroundings. The partition is removed and the gas expands to occupy the whole volume of the container. Find the new temperature ?

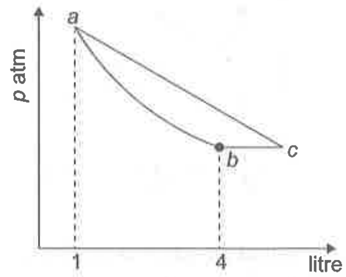
Level II

1. A freely moving piston divides a vertical cylinder, closed at both ends, into two parts each containing 1 mole of air. In equilibrium, at $T = 300$ K, volume of the upper part in $\eta = 4$ times greater than the lower part. At what temperature will the ratio of these volumes be equal to $\eta' = 2$?

2. A sample of an ideal non linear tri-atomic gas has a pressure P_0 and temperature T_0 taken through the cycle as shown starting from A. Pressure for process $C \to D$ is 3 times P_0. Calculate the heat absorbed in the cycle and work done.

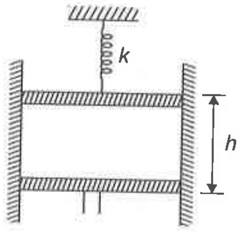


3. Figure shown three processes for an ideal gas. The tempeature at 'a' is 600 K, pressure 16 atm and volume 1 litre. The volume at 'b' is 4 litre. Out of the two process ab and ac, one is adiabatic and he other is isothermal. The ratio of specific heats of the gas is 1.5. Answer the following :

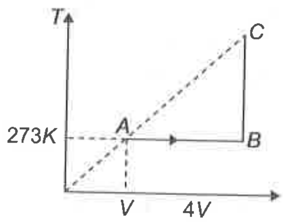


(i) Which of ab and ac processes is adiabatic. Why ?
(ii) Compute the pressure of th gas at b and c.
(iii) Compute the temperature and b and c.
(iv) Compute the volume at c.

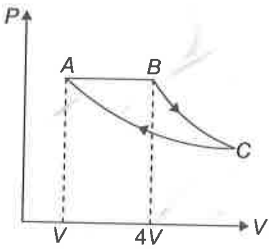
4. An ideal gas NTP is enclosed in a adiabatic vertical cylinder having area of cross section $A = 27$ cm², between two light movable pistons as shown in th figure. Spring with force constant $k = 3700$ N/m is in a relaxed state initially. Now the lower piston is moved upwards a height $h/2$, h being the initial length of gas column. It is observed that the upper piston moves up by a distance $h/16$. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.



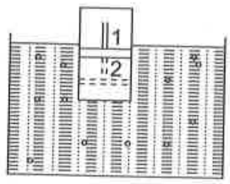
5. At a temperature of $T_0 = 273°$K, two mols of an ideal gas undergoes a process as shown. The total amount of heat imparted to the gas equals $Q = 27.7$ kJ. Determine the ratio of molar specific heat capacities.



6. A fixed mass of a gas is taken through a process $A \to B \to C \to A$. Here $A \to B$ is isobaric. $B \to C$ is adiabatic and $C \to A$ is isothemal. Find efficiency of the process (take $\gamma = 1.5$)

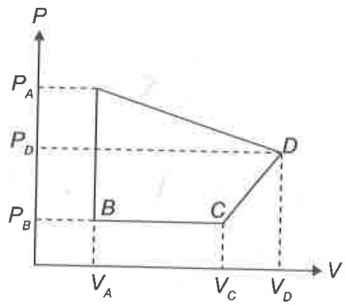


7. A cylinder containing a gas is closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position 1 to position 2. The piston is held at position 2 until the gas is again at 0°C and then slowly raised back to position 1. Represent the whole process on P-V diagram. If $m = 100$ gm of ice are melted during the cycle, how much work is done on the gas. Latent heat of ice = 80 cal/gm.



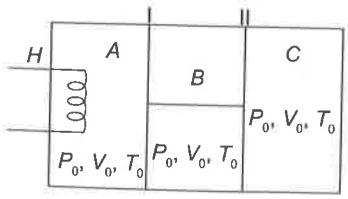
8. A parallel beam of particles of mass m moving with velocities v impings on a wall at an angle θ to its normal. The number of particles per unit volume in the beam is n. If the collision of particles with the wall is elastic, then find the pressure exerted by this beam on the wall.

9. For the thermodynamic process shown in the figure.
$P_A = 1 \times 10^5$ Pa; $P_B = 0.3 \times 10^5$ Pa
$P_D = 0.6 \times 10^5$ Pa; $V_A = 0.20$ litre
$V_D = 1.30$ litre.



(A) Find the work performed by the system along path AD.
(B) In the total work done by the system along the path ADC is 85 J find the volume a point C.
(C) How much work is perfomed by the system along the path CDA ?

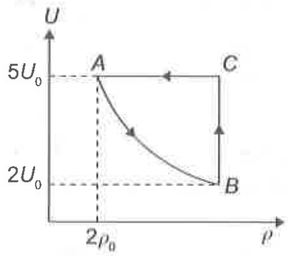
10. The figure shows an insulated cylinder divided into three parts A, B and C. Pistons I and II are connected by a rigid rod and can move without friction inside the cylinder. Piston I is perfectly conducting while piston II si pefectly insulating. The initial state of the gas ($\gamma = 1.5$) present in each compartment A, B and C is as shown. Now, compartment A is slowly given heat through a heater H such that the final volume of C becomes $\frac{4V_0}{9}$. Assume the gas to be ideal and find.



(A) Final pressurs in each compartment A, B and C
(B) Final temperatures in each compartment A, B and C
(C) Heat supplied by the heater
(D) Work done by gas in A and B.
(E) Heat flowing across piston I.

11. How many atoms do the molecules of gas consist of if γ increases 1.20 times when the vibrational degrees of freedom are "frozen" ? Assume that molecules are non linear.

12. Figure shows the variation of the internal energy U with the density ρ of one mole of idel monoatomic gas for a thermodynamic cycle $ABCA$.
Here process AB is a part of rectangular hyperbola.

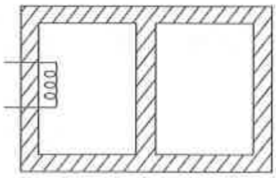


(A) Draw the P-V diagram for the above process.
(B) Find the net amount of heat absorbed by the system for the cyclic process.
(C) Find the work done in the process AB.

13. An ideal monoatomic gas undergoes a process where its pressur is inversely proportional to its tempeature.
(i) Calculate the specific heat for process.
(ii) Find the work done by two moles of gas if the temperature changes from T_1 to T_2.

14. An ideal diatomic gas undergoes a process in which its internal energy ralates to the volume as $U = a\sqrt{V}$ where α is a constant.
(A) Find the work performed by the gas and the amount of heat to be transferred to this gas to increase its internal energy by 100 J.
(B) Find the molar specific heat of the gas for this process.

15. Two rectangular boxes shown in figures has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a monoatomic ideal gas (γ = 5/3) at a pressure p_0 volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of tho box and the partitions are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands, pushing the partition until the final pressure in both chambers becomes 243 P_0/32 . Determine



(i) the final temperature of the gas in each chamber and
(ii) the work-done by the gas in the right chamber.

Previous Year Questions

JEE Main

1. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C? **[AIEEE 2002]**
(A) 80 K (B) −73 K
(C) 3 K (D) 20 K

2. 1 mole of a gas with $\gamma = \frac{7}{5}$ is mixed with 1 mole of a gas with $\gamma = \frac{5}{3}$, then the value of γ for the resulting mixture is **[AIEEE 2002]**
(A) $\frac{7}{5}$ (B) $\frac{2}{5}$
(C) $\frac{24}{16}$ (D) $\frac{12}{7}$

3. Which statement is incorrect ? **[AIEEE 2002]**
(A) All reversible cycles have same efficiency.
(B) Reversible cycle has more efficiency than an irreversible one.

(C) Carnot cycle is a reversible one.
(D) Carnot cycle has the maximum efficiency in all cycles.

4. Even Carnot engine cannot give 100% efficiency because we cannot **[AIEEE 2002]**
(A) prevent radiation
(B) find ideal sources
(C) reach absolute zero temperature
(D) eliminate friction

5. A carnot engine taken 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is **[AIEEE 2002]**
(A) 4.2×10^6 J
(B) 8.4×10^6 J
(C) 16.8×10^6 J
(D) zero

6. Which of the following parameters does not characterise the thermodynamics state of matter? **[AIEEE 2003]**
(A) Temperature
(B) Pressure
(C) Work
(D) Volume

7. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is **[AIEEE 2003]**
(A) 4/3
(B) 2
(C) 5/3
(D) 3/2

8. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of **[AIEEE 2003]**
(A) second law of thermodynamics
(B) conservation of momentum
(C) conservation of mass
(D) first law of thermodynamics

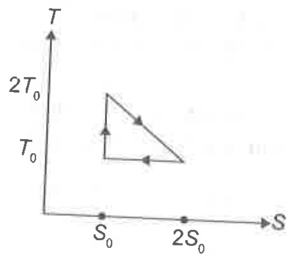
9. One mole of ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$. What is γ for the mixture ? γ denotes the ratio of specific heat at constant pressure, to that at constant volume **[AIEEE 2004]**
(A) $\frac{3}{2}$
(B) $\frac{23}{15}$
(C) $\frac{35}{23}$
(D) $\frac{4}{3}$

10. Which of the following statement is correct for any thermodynamic system ? **[AIEEE 2004]**
(A) The internal energy changes in all processes.
(B) Internal energy and entropy are state functions.
(C) The change in entropy can never be zero.
(D) The work done in an adiabatic process is always zero.

11. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is **[AIEEE 2005]**
(A) 1.59
(B) 1.62
(C) 1.4
(D) 1.54

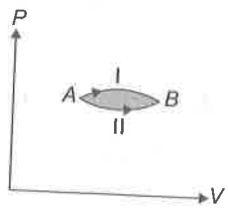
12. Which of the following is incorrect regarding the first law of thermodynamics? **[AIEEE 2005]**
(A) It is not applicable to any cyclic process
(B) It is a restatement of the principle of conservation of energy
(C) It introduces the concept of the internal energy
(D) It introduces the concept of the entropy

13. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is **[AIEEE 2005]**



(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{2}{3}$

14. A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then **[AIEEE 2005]**



(A) $\Delta U_1 = \Delta U_2$

(B) relation between ΔU_1 and ΔU_2 can not be determined

(C) $\Delta U_2 > \Delta U_1$

(D) $\Delta U_2 < \Delta U_1$

15. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature (Ignore the heat reach a capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is **[AIEEE 2006]**

(A) $T_f = \frac{3}{7}T_0$ (B) $T_f = \frac{7}{3}T_0$

(C) $T_f = \frac{3}{2}T_0$ (D) $T_f = \frac{5}{2}T_0$

16. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is ($R = 8.3$ J mol^{-1}K^{-1}) **[AIEEE 2006]**

(A) diatomic

(B) triatomic

(C) a mixture of monoatomic and diatomic

(D) monoatomic

17. If C_p and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[AIEEE 2007]**

(A) $C_p - C_v = \frac{R}{28}$ (B) $C_p - C_v = \frac{R}{14}$

(C) $C_p - C_v = R$ (D) $C_p - C_v = 28\,R$

18. A carnot engine, having an efficiency of $\eta = \frac{1}{10}$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is **[AIEEE 2007]**

(A) 99 J (B) 90 J

(C) 1 J (D) 100 J

19. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilbrium temperature of the gas in the container will be **[AIEEE 2008]**

(A) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_2 + p_2V_2T_2}$

(B) $\frac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

(C) $\frac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$

(D) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$

20. When a system is taken from state i to state f along the path iaf, it it found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf, $Q = 36$ cal. W along the path ibf is **[AIEEE 2008]**

(A) 6 cal (B) 16 cal

(C) 66 cal (D) 14 cal

21. One kg of a diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kgm^{-3}. What is the energy of the gas due to its thermal motion? **[AIEEE 2009]**

(A) 3×10^4 J (B) 5×10^4 J

(C) 6×10^4 J (D) 7×10^4 J

22. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is **[AIEEE 2011]**

(A) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(B) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(C) $\dfrac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

(D) $\dfrac{(T_1+T_2+T_3)}{3}$

23. A carnot engine operating between temperatures T_1 and T_2 has efficiency 1/6. When T_2 is lowered by 62 K its efficiency increases to 1/3. Then T_1 and T_2 are, respectively. **[AIEEE 2011]**

(A) 372 K and 330 K (B) 330 K and 268 K
(C) 310 K and 248 K (D) 372 K and 310 K

24. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increase by **[AIEEE 2011]**

(A) $\dfrac{(\gamma-1)}{2\gamma R}Mv^2\text{K}$ (B) $\dfrac{\gamma Mv^2}{2R}\text{K}$

(C) $\dfrac{(\gamma-1)}{2R}Mv^2\text{K}$ (D) $\dfrac{(\gamma-1)}{2(\gamma+1)R}Mv^2\text{K}$

25. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure p and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be- **[AIEEE 2011]**

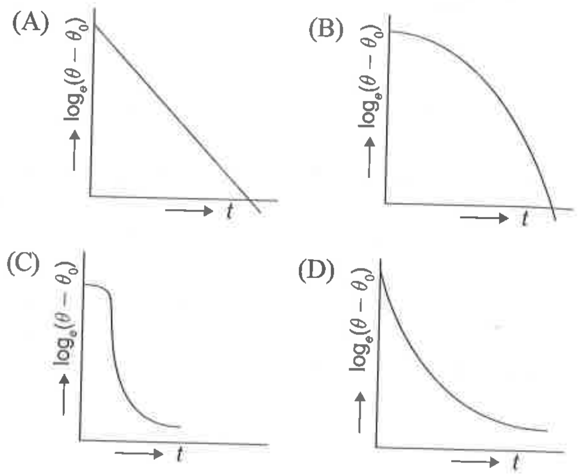
(A) $\dfrac{p}{2}, T$ (B) $\dfrac{p}{2}, \dfrac{T}{2}$

(C) P, T (D) $p, \dfrac{T}{2}$

26. The specific heat capacity of a metal at low temperature (T) is given as $C_p\left(\text{kJK}^{-1}\text{Kg}^{-1}\right)=32\left(\dfrac{T}{400}\right)^3$. A 100 g vessel of this metal is to be cooled from 20 K to 4 K by a special refrigerator operating at room temperture (27°C). The amount of work required to cool the vessel is **[AIEEE 2011]**

(A) equal to 0.002 kJ
(B) greater than 0.148 kJ
(C) between 0.148 kJ and 0.028 kJ
(D) less than 0.028 kJ

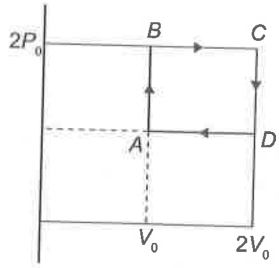
27. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surrounding, then according to Newton's law of cooling, the correct graph between $\log_e(\theta-\theta_0)$ and t is **[AIEEE 2012]**



28. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K, It is desired to have an engine of efficiency 60% Then, the intake temperature for the same exhaust (sink) temperature must be **[AIEEE 2012]**

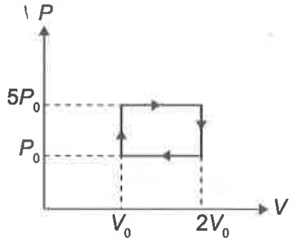
(A) Efficiency of Carnot engine cannot be made larger than 50%
(B) 1200 K
(C) 750 K
(D) 600 K

29. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) **[AIEEE 2012]**



(A) 15.4% (B) 9.1%
(C) 10.5% (D) 12.5%

30. The above P-V diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is: **[JEE Main 2013]**



(A) $\left(\frac{11}{2}\right)P_0V_0$ (B) $4P_0V_0$

(C) P_0V_0 (D) $\left(\frac{13}{2}\right)P_0V_0$

31. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency : **[JEE Main 2013]**

(A) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

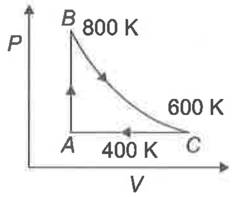
(C) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0M}$ (D) $\frac{1}{2\pi}\frac{V_0MP_0}{A^2\gamma}$

32. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? **[JEE Main 2014]**

(Atmospheric pressure = 76 cm of Hg)

(A) 38 cm (B) 6 cm
(C) 16 cm (D) 22 cm

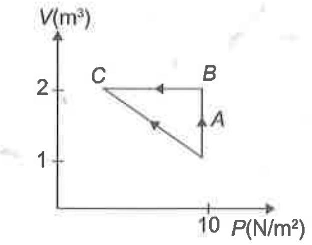
33. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperature at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement:



(A) The change in internal energy in the process AB is $-350\,R$.
(B) The change in internal energy in the process BC is $-500\,R$.
(C) The change in internal energy in whole cyclic process is $250\,R$.
(D) The change in internal energy in the process CA is $700\,R$. **[JEE Main 2014]**

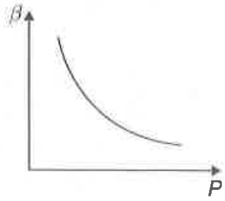
JEE Advanced

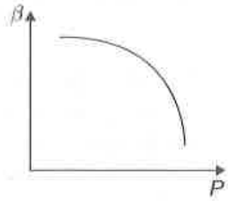
1. An ideal gas is taken through the cycle $A \rightarrow B \rightarrow C \rightarrow A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \rightarrow A$ is **[JEE(Scr) 2002]**

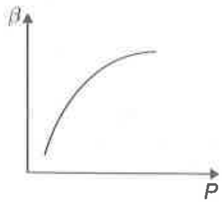


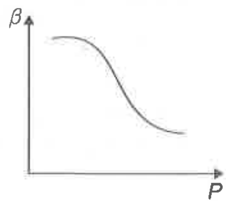
(A) -5 J (B) -10 J
(C) -15 J (D) -20 J

2. Which of the following graphs correctly represents the variation of $\beta = -(dV/dP)/V$ with P for an ideal gas at constant temperature?

(A)


(B)


(C)


(D)


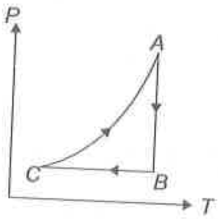
3. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m². During an observation time of 1 second, an atom travelling with the root mean square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = 25/3$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K. **[JEE 2002]**

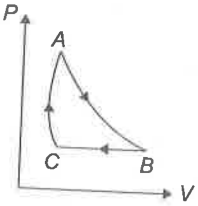
   (A) Evaluate the temperature of the gas

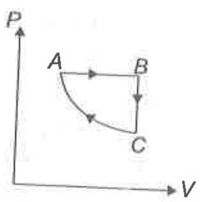
   (B) Evaluate the average kinetic energy per atom

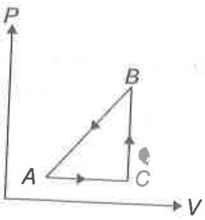
   (C) Evaluate the total mass of helium gas in the box.

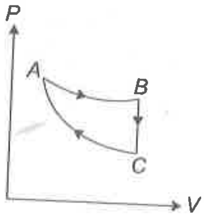
4. In the figure *AC* represent Adiabatic process. The corresponding *PV* graph is **[JEE(Scr) 2003]**



(A) 

(B) 

(C) 

(D) 

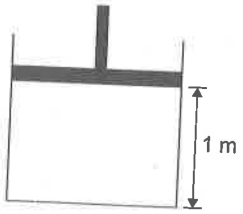
5. An insulated container containing monoatomic gas of molar *m* is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[JEE 2003]**

6. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then **[JEE(Scr) 2004]**

   (A) $P_3 > P_1, W > 0$ (B) $P_3 < P_1, W < 0$

   (C) $P_3 > P_1, W < 0$ (D) $P_3 = P_1, W = 0$

7. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m². Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer is fraction. **[JEE 2004]**



8. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 *A* for 5 minutes supplies heat to the gas. The change in internal energy of the gas is **[JEE(Scr) 2004]**

   (A) 10 KJ (B) 20 KJ

   (C) 30 KJ (D) 0 KJ

9. When the pressure is changed from $p_1 = 1.01 \times 10^5$ Pa to $P_2 = 1.165 \times 10^5$ Pa then the volume changes by 10% the bulk modulus is **[JEE(Scr) 2004]**

   (A) 1.55×10^5 Pa (B) 0.0015×10^5 Pa

   (C) 0.015×10^5 Pa (D) none of these

10. A cylinder of mass 1 kg is given heat of 20000 J at atmospheric pressure. If initially temperature of cylinder is 20°C, find **[JEE 2005]**

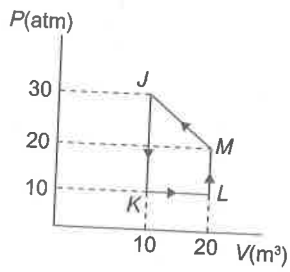
    (A) final temperature of the cylinder

    (B) work done by the cylinder

    (C) change in internal energy of the cylinder.

    (Given that specific heat of cylinder = 400 J kg^{-1}°C^{-1}, Coefficient of volume expansion = 9×10^{-5}°C^{-1}, Atmospheric pressure = 10^5 N/m² and density of cylinder = 9000 kg/m³)

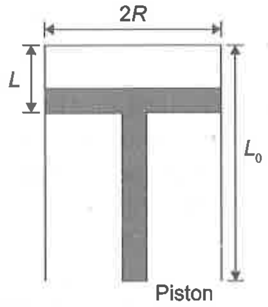
11. Match the following for the given process: **[JEE 2006]**



Column I	Column II
(A) Process $J \rightarrow K$	(P) $w > 0$
(B) Process $K \rightarrow L$	(Q) $w < 0$
(C) Process $L \rightarrow M$	(R) $Q > 0$
(D) Process $M \rightarrow J$	(S) $Q < 0$

Question No. 12 to 14

A fixed thermally conducting cylinder has radius R and length L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



12. The piston is now pulled out slowly and held at a distnace $2L$ from the top. The pressure in the cylinder between its top and the piston will then be **[JEE 2007]**

(A) P_0 (B) $P_0/2$

(C) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

13. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is **[JEE 2007]**

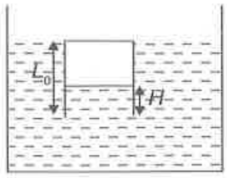
(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$

(B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$

(D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

14. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies **[JEE 2007]**



(A) $\rho g(L_0 - H)^2 + P_0(L_0 - H) + L_0P_0 = 0$
(B) $\rho g(L_0 - H)^2 - P_0(L_0 - H) - L_0P_0 = 0$
(C) $\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$
(D) $\rho g(L_0 - H)^2 - P_0(L_0 - H) + L_0P_0 = 0$

15. **Statement 1**

The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume **[JEE 2007]**
because

Statement 2

The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.

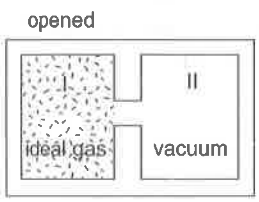
(A) Statement 1 is True, Statement 2 is True, Statement 2 is a correct explanation for Statement 1
(B) Statement 1 is True, Statement 2 is True, Statement 2 **is NOT** a correct explanation for Statement 1
(C) Statement 1 is True, Statement 2 is False
(D) Statement 1 is False, Statement 2 is True

16. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is **[JEE 2008]**

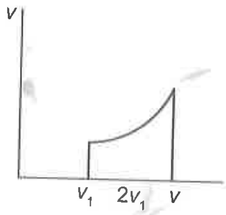
(A) $\frac{1}{T}$ (B) $\frac{2}{T}$

(C) $\frac{3}{T}$ (D) $\frac{4}{T}$

17. **Column I** Contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4×4 matrix given in the ORS. **[JEE 2008]**



Column I	Column II
(A) An insulated container has two chambers separated a valve. Chamber I contains an ideal gas the Chamber II has vacaum. The valve is opened.	(p) The temperature of the by gas decreases
(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas	(q) The temperature of the gas increase or remains constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume	(r) The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph	(s) The gas gains heat



18. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[JEE 2009]**

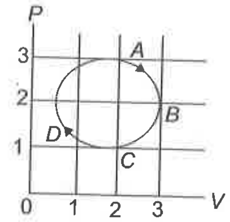
(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p/C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p.C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

19. The figure shows the *P*-*V* plot of an ideal gas taken through a cycle *ABCDA*. The part *ABC* is a semicircle and *CDA* is half of an ellipse. Then, **[JEE 2009]**



(A) the process during the path $A \to B$ is isothermal

(B) heat flows out of the gas during the path $B \to C \to D$

(C) work done during the path $A \to B \to C$ is zero

(D) positive work is done by the gas in the cycle *ABCDA*

20. This section contains 2 questions. Each questions contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled *A*, *B*, *C* and *D*, while the statements in **Column II** are labelled *p*, *q*, *r*, *s* and *t*. Any given statement in **Column I** can have correct matching with one or more statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example: If the correct matches are *A* - *p*, *s* and *t*; *B* - *q* and *r*; *C* - *p* and *q*; and *D* - *s* and *t*; then the correct darkening of bubbles will look like the following.
Column II gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II**. **[JEE 2009]**

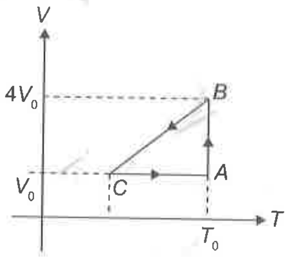
Column I	Column II
(A) The energy of the system is increased	(P) System: A capacitor Initially uncharged increased Process: It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts compressed by pushing the piston	(Q) System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is piston

Column I	Column II
(C) Internal energy of system is converted into its mechanical energy.	(R) System: a gas in a the rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(S) System: A heavy is decreased nucleus initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(T) System: A resistive wire loop Process: The loop is placed in a time varying magnetic field perpendicular to its plane.

21. A real gas behaves like an ideal gas if its **[JEE 2010]**

(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high

22. One mole of an ideal gas in initial sate A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option (s) from the following. **[JEE 2010]**



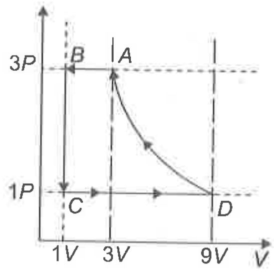
(A) Internal energies at A and B are the same.
(B) Work done by the gas in process AB is $P_0V_0 \ell n\, 4$
(C) Pressure at C is $P_0/4$
(D) Temperature at C is $T_0/4$

23. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. In the initial temperature of the gas is T_1 (in Kelvin) and the final temperature is aT_1. the value of a is **[JEE 2010]**

24. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is **[JEE 2011]**

(A) $9/8RT_1$ (B) $3/2RT_1$
(C) $15/8RT_1$ (D) $9/2RT_1$

25. One mole of a monatomic ideal gas is taken through a cycle $ABCDA$ as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I:



Column I	Column II
(A) Process $A \to B$	(P) Internal energy decreases
(B) Process $B \to C$	(Q) Internal energy increases
(C) Process $C \to D$	(R) Heat is lost
(D) Process $D \to$ A	(S) Heat is gained
	(T) Work is done on the gas

[JEE 2011]

26. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is **[JEE 2012]**

(A) 0.32 (B) 0.45
(C) 2.24 (D) 3.16

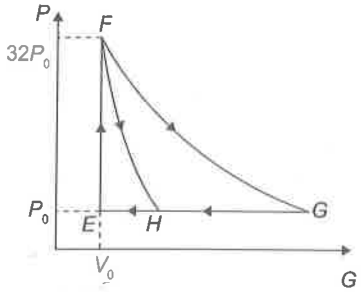
27. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. the amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol. K) **[JEE 2012]**

(A) 62 J (B) 104 J
(C) 124 J (D) 208 J

28. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is **[JEE 2013]**

(A) 1:4 (B) 1:2
(C) 6:9 (D) 8:9

29. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the *PV* digram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists. **[JEE 2013]**

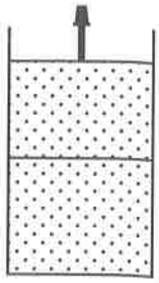
List I	List II
(P) $G \to E$	1. $160\,P_0V_0 \ln 2$
(Q) $G \to H$	2. $36\,P_0V_0$
(R) $F \to H$	3. $24\,P_0V_0$
(S) $F \to G$	4. $31\,P_0V_0$

Codes :

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

Question No. 30 and 31

In the figure a containner is shown to have a movalble (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_v = \frac{3}{2}R, C_p = \frac{5}{2}R$, and those for an ideal diatomis gas are $C_v = \frac{5}{2}R, C_p = \frac{7}{2}R$.



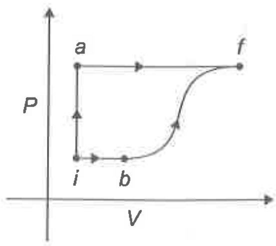
30. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature to the gases will be

(A) 550 K (B) 525 K
(C) 513 K (D) 490 K

31. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 *R* (B) 200 *R*
(C) 100 *R* (D) −100 *R*

32. A thermodynamic system is taken from an initial state *i* with internal energy $U_i = 100$ J to the final state *f* slong two different paths *iaf* and *ibf*, as schematically shown in the figure. The work done by the system along the paths *af*, *ib* and *bf* are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path *iaf*, *ib* and *bf* are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state *b* is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{id} is



ANSWER KEYS

Exercises

JEE Main

1. A	**2.** C	**3.** B	**4.** D	**5.** A	**6.** B	**7.** C	**8.** A	**9.** A	**10.** C
11. A	**12.** A	**13.** C	**14.** B	**15.** B	**16.** C	**17.** C	**18.** C	**19.** A	**20.** B
21. A	**22.** A	**23.** B	**24.** D	**25.** D	**26.** B	**27.** D	**28.** B	**29.** B	**30.** A
31. A	**32.** D	**33.** A	**34.** C	**35.** A	**36.** C	**37.** B	**38.** A	**39.** D	**40.** C
41. C	**42.** D	**43.** B	**44.** A	**45.** D	**46.** A				

JEE Advanced

1. D	**2.** C, D	**3.** A, B, C	**4.** A, B	**5.** C	**6.** A, D	**7.** B, D	**8.** D	**9.** B, D
10. D	**11.** A	**12.** A, B	**13.** A, D	**14.** B	**15.** B	**16.** C, D	**17.** C, D	**18.** B, C

JEE Advanced

Level I

1. 28.7236×10^3 s **2.** $\sqrt{\dfrac{20012.428}{\pi}} \times 10^{-25}$ kg-m/s **3.** 1:2 **4.** $\sqrt{14}$ **5.** $11\,RT$ **6.** 196°C

7. PROOF **8.** 3.3×10^3 **9.** $\sqrt{2}T$ **10.** $1:\sqrt{2}$ **11.** 27:4 **12.** $\dfrac{3}{2}\rho_0$ **13.** 1.25×10^4 N/m^2

14. (i) $P_1 < P_2, T_1 < T_2$; (ii) $T_1 = T_2 < T_3$; (iii) $V_2 > V_1$; (iv) $P_2 > P_1$ **15.** 74.9 cm **16.** 120 R

17. 1500 J **18.** 0.0091 J **19.** PV **20.** $-100\,\pi$ J **21.** $3P_1V_1$ **22.** $\pi/2$ atm-lt **23.** $Q - W$ **24.** 24 J

25. $(33600 + 0.02)$ J **26.** $\dfrac{25}{6}$ J/cal **27.** $\dfrac{88}{21}$ J/cal **28.** 100 J **29.** 3600 R **30.** (i) 765 J (ii) $\dfrac{208}{1921}$

31. $3R$ **32.** PROOF **33.** $-\gamma$ **34.** γ **35.** $3R, 2R, 1.5$ **36.** 47/29 **37.** 1.5

38. $C_V + \dfrac{R}{\alpha V}$ **39.** $\dfrac{R}{2}$ **40.** 12600 J **41.** 50 calorie **42.** $R\Delta T\left(\dfrac{3-2\gamma}{\gamma-1}\right)$

43. the molar mass of the gas is 40 gm, the numbe of degrees of freedom of the gas molecules is 6

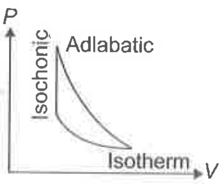
44. 8 atmosphere **45.** $1 - \dfrac{3\left(1-\dfrac{1}{2^{1/3}}\right)}{ln2}$ **46.** 1.5 **47.** $5R$ **48.** P/n **49.** 300 K

Level II

1. 750 K **2.** $31P_0V_0$; $-5P_0V_0$ **3.** (ii) $P_b = P_c = 2$ atm, (iii) $T_b = 300$ K, $T_C = 600$ K, (iv) $V_C = 8$ litre

4. 1.6 m, 364 K **5.** 1.63 **6.** $\dfrac{3-2ln2}{3}$

7. 8000 cal.

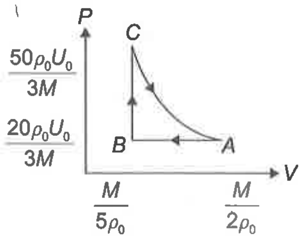


8. $2\,mnv^2\cos^{20}$ **9.** (A) $W_{AD} = 88$ J, (B) $V_C = 1.223$ litre, (C) $W_{CDA} = -85$ J

10. (A) Final pressure in $A = \dfrac{27}{8}P_0$ = Final pressure in C, Final pressure in $B = \dfrac{21}{4}P_0$

(B) Final temperature in A (and B) $= \frac{21}{4}T_0$, Final temperature in $C = \frac{3}{2}T_0$
(C) $18P_0V_0$, (D) work done by gas in $A = +P_0V_0$, work done by gas in $B = 0$ (e) $\frac{17}{2}P_0V_0$ **11.** four

12. (A),



(B) $Q = \left(\frac{10}{3}ln2.5 - 2\right)U_0$, (C) $-2U_0$

13. $\frac{7R}{2M}$, $4R(T_2 - T_1)$ **14.** (A) 80 J, 180 J, (B) $4.5R$ **15.** $T_1 = (207/16)T_0$; $T_2 = \frac{9}{4}T_0$, $-\frac{15}{8}P_0V_0$

Previous Year Questions

JEE Main

1. D	**2.** C	**3.** A	**4.** C	**5.** B	**6.** C	**7.** D	**8.** A	**9.** A	**10.** B
11. B	**12.** A, C	**13.** C	**14.** A	**15.** C	**16.** A	**17.** A	**18.** B	**19.** A	**20.** A
21. B	**22.** A	**23.** D	**24.** C	**25.** A	**26.** C	**27.** D	**28.** C	**29.** A	**30.** A
31. A	**32.** C	**33.** B							

JEE Advanced

1. A **2.** A **3.** 160 K, **3.** 3×10^{-21} J, 0.3 gm **4.** A **5.** $\Delta T = \frac{mv_0^2}{3R}$ **6.** C

7. $T_3 = 400\left(\frac{4}{3}\right)^{0.4}$ K **8.** C **9.** A **10.** **(A)** $T_{real} = 70°C$, **(B)** 0.05 J, **(C)** 19999.95

11. (A) → S; (B) → P and R; (C) → R; (D) → Q and S **12.** A **13.** D **14.** C **15.** B
16. C **17.** (A) → q, (B) → p and r, (C) → p and s, (D) → q and s **18.** B, D **19.** B, D
20. (A) → (PQST), (B) → (Q), (C) → (S), (D) → (S) **21.** D **22.** A, B **23.** 4 **24.** A
25. (A) → p, t, r; (B) → p, r; (C) → q, s; (D) → r, t **26.** D **27.** D **28.** D **29.** A
30. D **31.** D **32.** 2

CHAPTER 6

Elasticity and Thermal Expansion

DEFINITION

Elasticity is that property of the material of a body by virtue of which the body opposes any change in its shape or size when deforming forces are applied to it and recovers its original state as soon as the deforming forces are removed.

On the basis of definition, bodies may be classified in two types.

Perfectly Elastic (PE)

If a body regains its original shape and size completely after the removal of force, it is said to be a perfectly elastic body.
Nearest approach PE: quartz-fibre.

Perfectly Plastic (PP)

If a body does not have the tendency to recover its original shape and size, it is said to be a perfectly plastic body.
Nearest approach PP: Putty.

Limit of Elasticity

The maximum deforming force up to which a body retains its property of elasticity is called the limit of elasticity of the material of the body.

STRESS

When a deforming force is applied to a body, it reacts to the applied force by developing a reaction (or restoring force which, from Newton's third law, is equal in magnitude and opposite in direction to the applied force). *The reaction force per unit area of the body which is called into play due to the action of the applied force is called stress.* Stress is measured in units of force per unit area, i.e., Nm^{-2}. Thus,

$$\text{stress} = \frac{F}{A},$$

where F is the applied force and A is the area over which it acts.

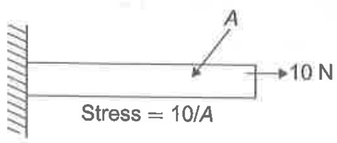


Figure 6.1

Unit of stress: N/m^2
Dimension of stress: $M^1L^{-1}T^{-2}$

Types of Stress

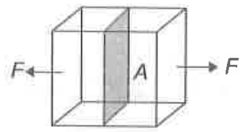


Figure 6.2

There are three types of stress.

(a) Tensile Stress

It is the pulling force per unit area.

It is applied parallel to the length.

It causes increase in length or volume.

(b) Compressive Stress

It is the pushing force per unit area.

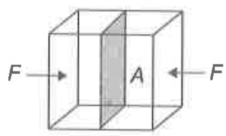


Figure 6.3

It is applied parallel to the length.

It causes decrease in length or volume.

(c) Tangential Stress

It is the tangential force per unit area.

It causes shearing of bodies.

Notes

1. If the stress is normal to the surface, it is called normal stress.
2. Stress is always normal to the surface in case of change in length of a wire or volume of a body.
3. When an external force compresses the body, the nature of the atomic force will be repulsive.
4. When external forces causes the body to expand, the nature of the atomic force will be attractive.

Difference between Pressure Versus Stress

S.No.	Pressure	Stress
1	Pressure is always normal to the area.	Stress can be normal or tangential.
2	It is always compressive in nature.	It may be compressive or tensile in nature.
3	It is a scalar quantity	It is a tensor quantity

SOLVED EXAMPLE

EXAMPLE 1

A 4.0-m-long copper wire of cross-sectional area 1.2 cm^2 is stretched by a force of 4.8×10^3 N. The stress will be

(A) 4.0×10^7 N/mm^2 **(B)** 4.0×10^7 kN/m^2
(C) 4.0×10^7 N/m^2 **(D)** none of these

SOLUTION [C]

$$\text{Stress} = \frac{F}{A} = \frac{4.8\times10^3\,\text{N}}{1.2\times10^{-4}\,\text{m}^2}$$
$$= 4.0 \times 10^7\ \text{N/m}^2$$

STRAIN

When a deforming force is applied to a body, it may suffer a change in size or shape. Strain is defined as the ratio of the change in size or shape to the original size or shape of the body. Strain is a number; it has no units or dimensions.

The ratio of the change in length to the original length is called *longitudinal strain*. The ratio of the change in volume to the original volume is called *volume strain*. The strain resulting from a change in shape is called a shearing strain.

$$\text{Strain} = \frac{\Delta L}{L_0} = \frac{\text{final length} - \text{original length}}{\text{orginal length}}$$
$$= \alpha \Delta T.$$

Note

Original and final length should be at the same temperature.

Types of Strain

There are three types of strain.

(a) Linear Strain

The change in length per unit length is called linear strain.

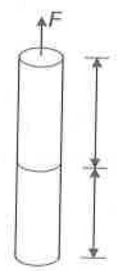


Figure 6.4

$$\text{Linear strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}.$$

(b) Volume Strain

Change in volume per unit volume is called volume strain.

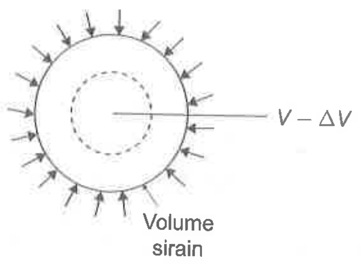


Figure 6.5

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}.$$

(c) Shear Strain

The angle through which a line originally normal to the fixed surface is turned is known as shear strain.

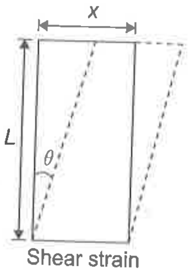


Figure 6.6

$$\phi = \frac{x}{L}.$$

Note

Strain is unitless.

SOLVED EXAMPLE

EXAMPLE 2

A copper rod 2 m long is stretched by 1 mm. Strain will be
(A) 10^{-4}, volumetric **(B)** 5×10^{-4}, volumetric
(C) 5×10^{-4}, longitudinal **(D)** 5×10^{-3}, volumetric

SOLUTION [C]

$$\text{Strain} = \frac{\Delta \ell}{\ell} = \frac{1\times 10^{-3}}{2} = 5 \times 10^{-4}, \text{ longitudinal.}$$ ■

THERMAL STRESS

If the ends of a rod are rigidly fixed and its temperature is changed, then compressive stresses are set up in the rod. These developed stress are called thermal stress.

$$\text{Thermal stress} = Y\alpha\Delta t,$$

where Y is the modulus of elasticity, α is the coefficient of linear expansion and Δt is the change in temperature.

WORK DONE IN STRETCHING A WIRE

In stretching a wire, work is done against internal restoring forces. This work is stored in the body as elastic potential energy or strain energy.

If L = length of wire and
A = cross-sectional area,

$$Y = \frac{F/A}{x/L}$$

$$\Rightarrow \quad F = \frac{YA}{L}x.$$

Work done to increase dx length,

$$dW = Fdx = \frac{YA}{L}xdx.$$

$$\text{Total work done} = W = \int_0^{\Delta L} \frac{YA}{L}x\,dx$$

$$= \frac{1}{2}\frac{YA}{L}(\Delta L)^2.$$

$$\text{Work done per unit volume} = \frac{W}{V} = \frac{1}{2}Y\left(\frac{\Delta L}{L}\right)^2$$

$$[\because \; V = AL]$$

$$\frac{W}{V} = \frac{1}{2}Y(\text{strain})^2$$

$$\frac{W}{V} = \frac{1}{2}\times \text{stress} \times \text{strain}$$

$$\left[\because \; Y = \frac{\text{Stress}}{\text{Strain}}\right]$$

$$\frac{W}{V} = \frac{1}{2}\frac{(\text{stress})^2}{Y}$$

$$\Rightarrow \quad \frac{W}{AL} = \frac{1}{2}\frac{F}{A}\times\frac{\Delta L}{L}$$

$$W = \frac{1}{2}F \times \Delta L$$

$$= \frac{1}{2}\text{ load} \times \text{elongation.}$$

STRESS-STRAIN CURVE

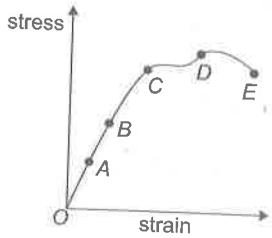


Figure 6.7

If we increase the load gradually on a vertically suspended metal wire, the following can be observed.

Region OA

Strain is small (<2%).

Stress $\propto$ strain $\Rightarrow$ Hook's law is valid.

Slope of line *OA* gives Young's modulus *Y* of the material.

Region AB

Stress is not proportional to the strain, but the wire will still regain its original length after removing the stretching force.

Region BC

If the wire yields, strain increases rapidly with a small change in stress. This behaviour is shown up to point *C*, known as the yield point.

Region CD

Point *D* corresponds to maximum stress, which is called the point of breaking or tensile strength.

Region DE

The wire literally flows. In this region, there is maximum stress corresponding to *D* after which the wire begins to flow.

In this region, strain increases even if the wire is unloaded and ruptures at *E*.

HOOKE'S LAW

Hooke's law states that within the elastic limit, the stress developed in a body is proportional to the strain produced in it. Thus, the ratio of stress to strain is a constant. This constant is called the modulus of elasticity. Thus,

$$\text{modulus of elasticity} = \frac{\text{stress}}{\text{strain}}.$$

Since strain has no unit, the unit of the modulus of elasticity is the same as that of stress, namely, Nm^{-2}.

YOUNG'S MODULUS

Suppose that a rod of length *l* and a uniform cross-sectional area *a* is subjected to a longitudinal pull. In other words, two equal and opposite forces are applied at its ends.

$$\text{Stress} = \frac{F}{A}.$$

The stress in the present case is called linear stress, tensile stress or extensional stress. If the direction of the force is reversed so that ΔL is negative, we speak of compressional strain and compressional stress. If the elastic limit is not exceeded, then from Hooke's law,

$$\text{stress} \propto \text{strain}$$

or

$$\text{stress} = Y \times \text{strain}$$

or

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{A} \cdot \frac{L}{\Delta L}, \quad (1)$$

where *Y*, the constant of proportionality, is called the Young's modulus of the material of the rod and may be defined as the ratio of the linear stress to linear strain, provided the elastic limit is not exceeded. Since strain has no units, the unit of *Y* is Nm^{-2}.

Consider a rod of length ℓ_0 which is fixed between two rigid ends separated by a distance ℓ_0. Now, if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be:

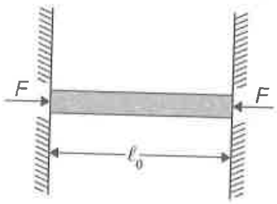


Figure 6.8

$$\text{Strain} = \frac{\text{length of the rod at new temperature} - \text{natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}.$$

$\because$ α is very small, strain $= -\alpha\Delta\theta$ (negative sign in the answer represents that the length of the rod is less than the natural length, which means it is compressed by the ends).

We know that $\gamma = \frac{\text{stress}}{\text{strain}}$, then $F = \gamma\alpha\Delta TA$.

Notes

1. For loaded wire

$$\Delta L = \frac{FL}{\pi r^2 Y}$$

$$\left[\because Y = \frac{FL}{A\Delta L} \text{ and } A = \pi r^2\right].$$

Notes (Cont'd)

For a rigid body, $\Delta L = 0$, so $Y = \infty$, i.e., elasticity of the rigid body is infinite.

2. **If the same stretching force is applied to different wires of the same material,**

$$\Delta L \propto \frac{L}{r^2} \quad \text{[as } F \text{ and } Y \text{ are constants].}$$

Greater the value of ΔL, greater will be the elongation.

3. **Elongation of a wire by its own weight**
In this case, $F = Mg$ acts at CG of the wire. So length of wire which is stretched will be $L/2$

$$\Delta L = \frac{FL}{AY} = \frac{(Mg)\times L/2}{\pi r^2 Y}$$

$$= \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

$$[\because \; M = \rho AL]$$

$$\Delta L = \frac{\rho g L^2}{2Y}$$

SOLVED EXAMPLE

EXAMPLE 3

A wire of length 1 m and area of cross-section 4×10^{-8} m² increases in length by 0.2 cm when a force of 16 N is applied. Value of Y for the material of the wire will be

(A) 2×10^{6} N/m² (B) 2×10^{11} kg/m²
(C) 2×10^{11} N/mm² (D) 2×10^{11} N/m²

SOLUTION [D]

By Hook's law,

$$Y = \frac{F/A}{\ell/L} = \frac{FL}{A\ell}$$

$$Y = \frac{16\times 1}{(4\times 10^{-8})(0.2\times 10^{-2})}$$

$$= 2 \times 10^{11} \text{ N/m}^2$$

Bulk Modulus

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

$$\Rightarrow \quad B = -\frac{V\Delta P}{\Delta V}.$$

Compressibility

$$k = \frac{1}{B}$$

$$= -\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right)$$

Modulus of Rigidity

$$\eta = \frac{\text{tangential stress}}{\text{tangential strain}}$$

$$\Rightarrow \quad \eta = \frac{F/A}{\phi}.$$

Only a solid can have shearing as these have a definite shape.

POISSION'S RATIO

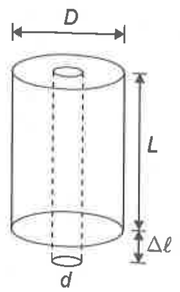


Figure 6.9

$$\sigma = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

$$= \frac{d/D}{\Delta L/L}$$

$$\Rightarrow \quad \sigma = \frac{dL}{\Delta LD}$$

Interatomic force constant = Young modulus × interatomic distance.

THERMAL EXPANSION

Most substances expand when they are heated. Thermal expansion is a consequence of the change in average separation between the constituent atoms of an object. Atoms of an object can be imagined to be connected to one another by stiff springs as shown in Figure 6.10. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately 10^{-11} m. The average spacing between the atom is about 10^{-10} m. As the temperature of solid increases, the atoms oscillate with greater amplitudes, as a result the average separation between them increases, and consequently, the object expands.

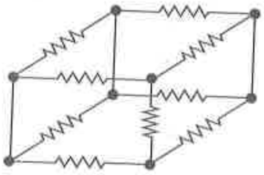


Figure 6.10

Linear Expansion

When the rod is heated, the increase in length ΔL is proportional to its original length L_0 and change in temperature is indicated as ΔT, where ΔT is in °C or K.

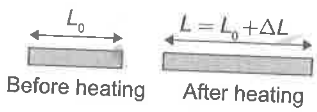


Figure 6.11

$$dL = \alpha L_0 dT$$

$\Rightarrow$ $$\Delta L = \alpha L_0 \Delta T.$$

If $$a\Delta T = 1,$$

$$\alpha = \frac{\Delta L}{L_0 \Delta T},$$

where α is called the coefficient of linear expansion, whose unit is °C^{-1} or K^{-1}.

$L = L_0(1 + \alpha\Delta T)$, where L is the length after heating the rod.

Variation of α with Temperature and Distance

1. If α varies with distance, $\alpha = ax + b$.

   Then, total expansion $= \int (ax + b)\Delta T\, dx$.

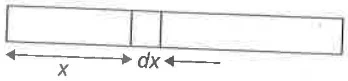


Figure 6.12

2. If α varies with temperature, $\alpha = f(T)$.

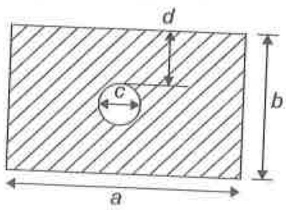
   Then $$\Delta L = \int \alpha L_0\, dT.$$

> **Note**
>
> Actually, thermal expansion is always three-dimensional expansion. When the other two dimensions of an object are negligible with respect to one, then the observations are significant only in one dimension. This is known as linear expansion.
>
> Avery linear dimensions of the object changes in the same fashion.

SOLVED EXAMPLES

EXAMPLE 4

A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature, then which dimension will increase in the figure.

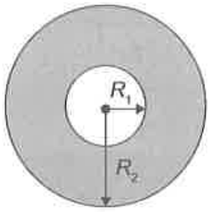


SOLUTION

Distance between any two points on an object increases with an increase in temperature. So, all dimensions a, b, c and d will increase. ■

EXAMPLE 5

In the given figure, when temperature is increased, then which of the following increases:



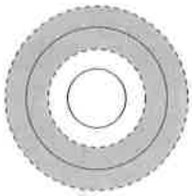
(A) R_1 (B) R_2
(C) $R_2 - R_1$

SOLUTION

All of the above

__________ represents expanded boundary

__________ represents original boundary



As the intermolecular distance between the atoms increases on heating, the inner and outer perimeters increase. Also if the atomic arrangement in the radial direction is observed, then we can say that it also increases. Hence, all A, B and C are true. ■

EXAMPLE 6

A small ring having a small gap is shown in the figure. On heating, what will happen to the size of the gap.

SOLUTION

The gap size will also increase. The reason is the same as in the above example.

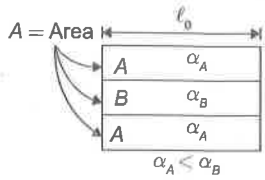
■

Note

Original and final lengths should be at the same temperature.

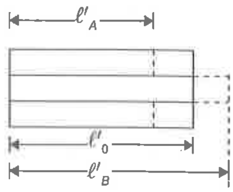
EXAMPLE 7

Find the equilibrium length for the system after increasing temperature by ΔT.



SOLUTION

Here, ℓ'_A and ℓ'_B are the natural lengths of the rods A and B after an increase in temperature by ΔT. ℓ'_0 is the actual length after the temperature increases by ΔT.



$$\text{So strain in } A = \frac{\ell'_0 - \ell'_A}{\ell'_A}$$

$$\text{and that in } B = \frac{\ell'_B - \ell'_0}{\ell'_B}.$$

Now force balance,

$$\frac{F}{A} = \gamma_A \frac{\ell'_0 - \ell'_A}{\ell' A} \quad (1)$$

and

$$\frac{2F}{A} = \gamma_B \frac{\ell'_B - \ell'_0}{\ell'_B}. \quad (2)$$

Dividing (1) ÷ (2),

$$\frac{1}{2} = \frac{\gamma_A\left[\ell'_0 - \ell_0(1+\alpha_A\Delta T)\right]\ell_0(1+\alpha_B\Delta T)}{\gamma_B\,\ell_0(1+\alpha_A\Delta T)\left[\ell_0(1+\alpha_B\Delta T) - \ell'_0\right]}$$

$$\ell'_0 = \frac{\ell_0(\gamma_B + 2\gamma_A)\left[1+(\alpha_B+\alpha_A)\Delta T\right)}{2\ell_A(1+\alpha_B\Delta T) + \gamma_B(1+\alpha_A\Delta T)}.$$

■

Measurement of Length by Metallic Scale

Case (i)

When the object is expanded only,

$$\ell_2 = \ell_1\{1+\alpha_0(\theta_2-\theta_1)\},$$

ℓ_1 = actual length of the object at θ_1°C = measure length of object at θ_1°C

ℓ_2 = actual length of the object at θ_2°C = measured length of object at θ_2°C

α_0 = linear expansion coefficient of the object

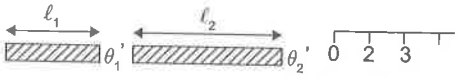


Figure 6.13

Case (ii)

When only measure instrument is expanded, actual length of the object will not change, but the measured value (MV) decreases.

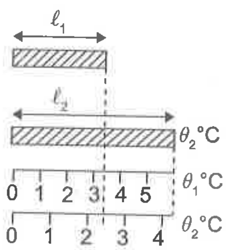


Figure 6.14

$$MV = \ell_1\{1-\alpha_s(\theta_2-\theta_1)\},$$

where α_s = linear expansion coefficient of the measuring instrument.

Case (iii)

If both are expanded simultaneously,

$$MV = l_1\{1+(\alpha_0-\alpha_s)(\theta_2-\theta_1)\},$$

1. if $\alpha_0 > \alpha_s$, then the measured value is more than actual value at θ_1°C
2. If $\alpha_0 < \alpha_s$, then the measured value is less than actual value at θ_1°C

At θ_1°C, $MV = 3.4$, θ_2°C, $MV = 4.1$.

Effect of Temperature on the Time Period of a Pendulum

The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or $$T \propto \sqrt{l}.$$

As the temperature is increased, length of the pendulum, and hence, time period gets increased or a pendulum clock becomes slow and it loses the time,

$$\frac{T'}{T} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l+\Delta l}{l}}.$$

Here, we put $\Delta l = l\alpha\Delta\theta$ in place of $l\alpha\Delta T$ so as to avoid the confusion with change in time period. Thus,

$$\frac{T'}{T} = \sqrt{\frac{l+l\alpha\Delta\theta}{l}} = (1+\alpha\Delta\theta)^{1/2}$$

or $$T' \approx T\left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

or $$\Delta T = T' - T$$

$$= \frac{1}{2}T\alpha\Delta\theta.$$

Time lost in time t (by a pendulum clock whose actual time period is T and the changed time period at some higher temperature T') is

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t.$$

Similarly, if the temperature is decreased the length, and hence, the time period gets decreased. A pendulum clock in this case runs fast and it gains the time.

$$\frac{T'}{T} = \sqrt{\frac{l'}{l}}$$

$$= \sqrt{\frac{l-l\alpha\Delta\theta}{l}}$$

$$\approx 1-\frac{1}{2}\alpha\Delta\theta$$

or $$T' = T\left(1-\frac{1}{2}\alpha\Delta\theta\right)$$

$$\Delta T = T - T' = \frac{1}{2} T\alpha\Delta\theta$$

and time gained in time t is the same, i.e.,

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t.$$

SOLVED EXAMPLE

EXAMPLE 8

A second's pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock lose or gain in one week when the temperature is increased to 30°C? $\alpha_{steel} = 1.2 \times 10^{-5}$ (°C)$^{-1}$.

SOLUTION

The time period of second's pendulum is 2 s. As the temperature increases, length and, hence, time period increases. Clock becomes slow and it loss the time. The change in time period is

$$\Delta T = \frac{1}{2} T\alpha\Delta\theta$$
$$= \left(\frac{1}{2}\right)(2)(1.2\times10^{-5})(30° - 20°)$$
$$= 1.2 \times 10^{-4} \text{ s}$$

∴ New time period is

$$T' = T + \Delta T$$
$$= (2 + 1.2 \times 10^{-4})$$
$$= 2.0012 \text{ s.}$$

∴ Time lost in one week,

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t$$
$$= \frac{(1.2\times10^{-4})}{(2.00012)}(7\times24\times3600)$$
$$= 36.28 \text{ s.}$$

Superficial or Aerial Expansion

When a solid is heated and its area increases, then the thermal expansion is called superficial or aerial expansion. Consider a solid plate of side l_0 and linear expansion coefficient α_s. Then, $A_i = ab$.

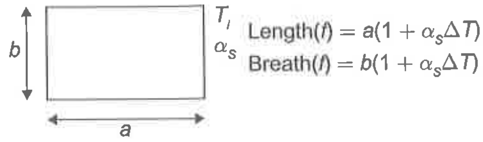


Figure 6.15

$$\text{Final area} = l \times b$$
$$= ab(1 + \alpha_s\Delta T)^2$$
$$= ab(1 + 2\,\alpha_s\Delta T)$$
$$= ab(1 + \beta\Delta T)$$
$$A_f = Ai(1 + \beta\Delta T)$$
$$\beta = 2\alpha$$
$$\beta = \text{coefficient of area of expansion.}$$

(a) Isotropic Material

An isotropic material is a material that has the same coefficient of linear expansion in all the directions.

(b) Anisotropic Material

Anisotropic material has different coefficients of linear expansion in different directions.

Notes

Most of the time, we consider a material as an isotropic material.
For an isotropic material,

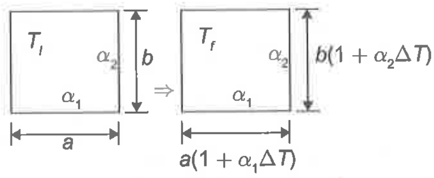


Figure 6.16

$$A_i = ab$$
$$A_f = ab(1 + \alpha_1\Delta T)(1 + \alpha_2\Delta T)$$
$$= ab(1 + (\alpha_1 + \alpha_2)\Delta T + \alpha_1\alpha_2\Delta T^2)$$
$$= ab(1 + (\alpha_1 + \alpha_2)\Delta T)$$
$$= A_i(1 + (\alpha_1 + \alpha_2)\Delta T).$$

Volume or Cubical Expansion

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion.

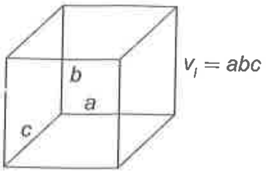


Figure 6.17

Notes

Now, after an increase in temperature by ΔT,

$$v_f = a'b'c'$$
$$= a[1+\alpha\Delta T]^3 bc$$
$$= abc[1 + 3\alpha\Delta T] \quad \therefore \ \alpha\Delta T = 1$$
$$v_f = v_i[1 + 3\alpha\Delta T].$$

So, $3\alpha = \gamma =$ coefficient of volume expansion.

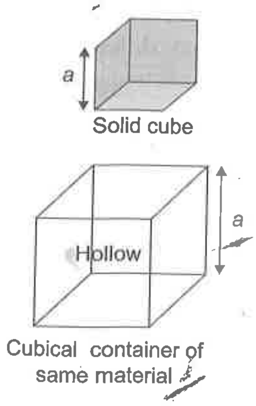


Figure 6.18

1. When the temperature changes, the volume of the container and the volume of the cube changes in the same manner because a changes in the same manner.
2. In volume expansion of container, we use γ of the container material.

   For an isotropic material,

   $$v_f = v_i(1 + 3\alpha\Delta T)$$

   For anisotropic material

   $$v_f = v_i\left[1+(\alpha_1+\alpha_2+\alpha_3)\Delta T\right]$$

Notes

(i) $\alpha:\beta:\gamma = 1:2:3$
(ii) They are dependent on the temperature.

Effect of Temperature on Density

If the initial density of the body is ρ_i, mass is m and volume is v, then

$$\rho_i = \frac{m}{v}.$$

If the temperature increases, then volume should change and the final volume is given by

$$v_f = v(1 + \gamma\Delta T).$$

So the final density

$$\rho_f = \frac{m}{v_f}$$

$$\Rightarrow \quad \rho_f = \frac{m}{v(1+\gamma\Delta T)}$$

$$\rho_f = \rho_i(1 + \gamma\Delta T)^{-1}.$$

From the binomial theorem,

$$\rho_f = \rho_i(1 - \gamma\Delta T).$$

Temperature Scale

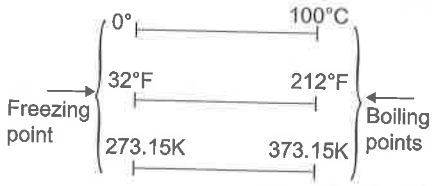


Figure 6.19

Relation between different scales is given as follows:
K = Kelvin
C = Centigrade
F = Fahrenheit
100°C difference = 180°F difference

$$1°\text{C difference} = \frac{9°}{5}\text{F difference}$$

$$\frac{9\text{C}}{5} = \text{F} - 32$$

$$\text{K} = \text{C} + 273.15.$$

Relation between temperature on two difference scales is given as follows

L.F. value = Lower fixed value

U.F. value = Upper fixed value

$$\frac{\text{Temperature on } S_1 \text{ scale} - \text{L.F. value of } S_1}{\text{U.F. value of } S_1 - \text{L.F. value of } S_1}$$

$$= \frac{\text{Temp. on } S_2 \text{ scale} - \text{L.F. value of } S_2}{\text{U.F. value of } S_2 - \text{L.F. value of } S_2}$$

SOLVED EXAMPLE

EXAMPLE 9

A faulty thermometer reads 5° at freezing point and 95° at boiling point. Then find out the original reading in °C when it reads 50°.

SOLUTION

$$\frac{50-5}{95-5} = \frac{x-0}{100}$$

$$\Rightarrow \quad \frac{45}{90} = \frac{x}{100}$$

$$\Rightarrow \quad x = 50.$$

(a) *Effect of Temperature on Buoyancy Force*

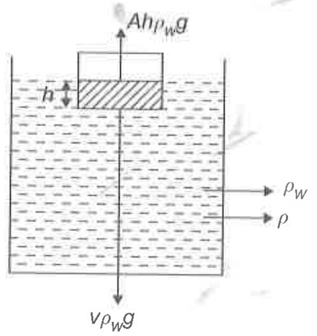


Figure 6.20

Initially, at temperature T,

$$F_B = v\rho_\ell g$$

If the temperature increases to ΔT, then

$$F_B = \frac{V(1+\gamma_B \Delta T)\rho_\ell g}{(1+\gamma_\ell \Delta T)}$$

$$= v\rho_\ell g \frac{(1+\gamma_B \Delta T)}{(1+\gamma_\ell \Delta T)}.$$

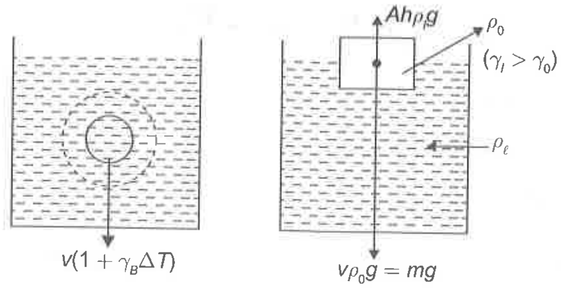


Figure 6.21

(a) If $\gamma_B > \gamma_L$ and $T\uparrow$, then $F_B\downarrow$.

(b) If $\gamma_L > \gamma_B$ and $T\uparrow$, then $F_B\downarrow$ and $T\downarrow$ then $F_B\uparrow$. ■

Barometer

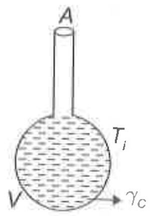


Figure 6.22

There is a capillary tube which have coefficient of linear expansion α_c and a liquid of volume v of volume expansion coefficient v of volume expansion coefficient of γ_l at temperature T_i and given $3\alpha_c < \gamma_\ell$. The area of cross-section of the capillary tube is A.

Now the temperature increases to T_f, so the volume of the liquid rises in the capillary. Let it increase to height H'. So the volume increase in the tube = ΔV.

$$\Delta V = V[1 + \gamma_\ell \Delta T] - V[1 + 3\alpha_c \Delta T]$$

$$= V(\gamma_\ell - 3\alpha_c)\Delta T$$

Area of cross-section of capillary = A'

$$= A[1 + 2\alpha_c \Delta T].$$

So, height in the capillary tube

$$H' = \frac{\Delta V}{A'}$$

$$= \frac{V\Delta T(\gamma_\ell - 3\alpha_c)}{A(1+2\alpha_c\Delta T)}.$$

SOLVED EXAMPLE

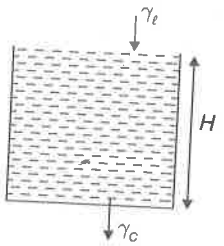
EXAMPLE 10

What will happen to the water level if the vessel is heated?

SOLUTION

(i) If $\gamma_\ell > \gamma_c$, then overflow occurs and is given as

$$= AH = (1+\gamma_\ell\Delta t) - AH(1+\gamma_c\Delta T)$$



(ii) If $\gamma_\ell < \gamma_c$,

final volume $V_{fc} = AH(1+\gamma_c\Delta T)$

final volume $V_{fl} = AH(1+\gamma_\ell\Delta T)$.

Now, $A_F = A[1+2\alpha_c\Delta T]$.

So, H' = final height

$$= \frac{H[1+Y_\ell\Delta T]}{[1+2\alpha_c\Delta T]}.$$ ■

Note

If two strips of equal length but of different metals are placed on each other and riveted, the single strip so formed is called a bimetallic strip (Fig. 5.23). This strip

Note (Cont'd)

has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side, i.e., convex side. This strip finds its application in auto-cut or thermostat in electric heating circuits. It has also been used as a *thermometer* by calibrating its bending.

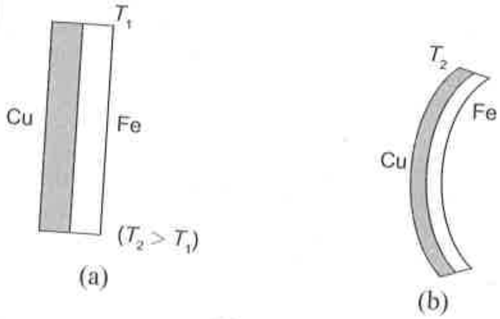
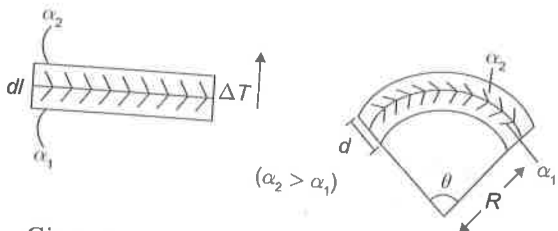


Figure 6.23

EXAMPLE 11

When the two rods having expansion coefficients α_1, α_2 $(\alpha_2 > \alpha_1)$ and width d are heated, find the radius of the rod after expansion.



Given $\alpha_2 > \alpha_1$,

$$R = \frac{d}{(\alpha_2 - \alpha_1)\Delta T}$$

SOLUTION

$$\ell_2 = \ell(1+\alpha_2\Delta T) = (R+d)\theta$$

$$\Rightarrow \quad \ell_1 = \ell(1+\alpha_1\Delta t) = R\theta$$

$$\frac{R+d}{R} = \frac{(1+\alpha_2\Delta T)}{(1+\alpha_1\Delta T)}$$ from binomial theorem

$$R = \frac{d}{(\alpha_2 - \alpha_1)\Delta T}.$$ ■

EXERCISES

JEE Main

1. A steel scale is to be prepared such that the millimetre intervals are to be accurate within 6×10^{-5} mm. The maximum temperature variation from the temperature of calibration during the reading of the millimetre marks is ($\alpha = 12 \times 10^{-6}\, k^{-1}$)
(A) 4.0°C (B) 4.5°C
(C) 5.0°C (D) 5.5°C

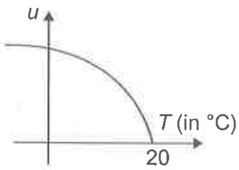
2. A steel rod 25 cm long has a cross-sectional area of 0.8 cm². Force that would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C is:
(Coefficient of linear expansion of steel is 10^{-5}/°C and Young's modulus of steel is 2×10^{10} N/m².)
(A) 160 N (B) 360 N
(C) 106 N (D) 260 N

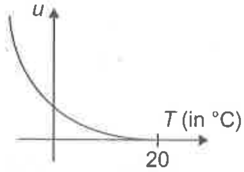
3. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2:3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to
(A) 2:3 (B) 1:1
(C) 3:2 (D) 4:9

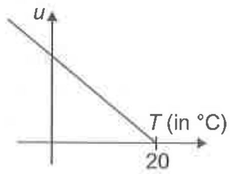
4. If I is the moment of inertia of a solid body having α-coefficient of linear expansion then the change in I corresponding to a small change in temperature ΔT is
(A) $\alpha I \Delta T$ (B) $\frac{1}{2}\alpha I \Delta T$
(C) $2\alpha I \Delta T$ (D) $3\alpha I \Delta T$

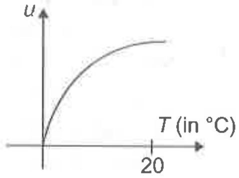
5. A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference ΔT(Y = young's modulus, ρ = density, α = coefficient of linear expansion) then the frequency of transverse vibration is proportional to :
(A) $\frac{\alpha}{\sqrt{\rho Y}}$ (B) $\sqrt{\frac{Y\alpha}{\rho}}$
(C) $\frac{\sigma}{\sqrt{Y\alpha}}$ (C) $\sqrt{\frac{\sigma\alpha}{Y}}$

6. A metal wire is clamped between two vertical walls. At 20°C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between elastic energy density (u) and temperature (T) of the wire is

(A)


(B)


(C)


(D)


7. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:
(A) 25 cm (B) < 25 cm
(C) >25 cm (D) cannot say

8. A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature, a third rod of the same length is composed of two parts one of metal A and the other of metal B. Thus rod expand by 0.06 cm for the same change in temperature. The portion made of metal A has the length.
(A) 20 cm (B) 10 cm
(C) 15 cm (D) 18 cm

9. A sphere of diameter 7 cm and mass 266.5 gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the coefficient of cubical expansion of the liquid is f:
(A) 8.486×10^{-4} per °C
(B) 8.486×10^{-5} per °C

(C) 8.486×10^{-6} per °C
(D) 8.486×10^{-3} per °C

10. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. The coefficient of linear expansion of glass is a_g per °C and the cubical expansion of mercury γ_m per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C.

(A) $\frac{V_0T(\gamma_m+3a_g)}{A_0(1+2a_gT)}$ (B) $\frac{V_0T(\gamma_m-3a_g)}{A_0(1+2a_gT)}$

(C) $\frac{V_0T(\gamma_m+2a_g)}{A_0(1+3a_gT)}$ (D) $\frac{V_0T(\gamma_m-2a_g)}{A_0(1+3a_gT)}$

11. A metallic rod 1 cm long with a square cross-section is heated through 1°C. If Young's modulus of elasticity of the metal is E and the mean coefficient of linear expansion is α per degree Celsius, then the compressional force required to prevent the rod from expanding along its length is:(Neglect the change of cross-sectional area)
(A) $EA\alpha t$ (B) $EA\alpha t/(1 + \alpha t)$
(C) $EA\ \alpha t/(1 - \alpha t)$ (D) $E/\alpha t$

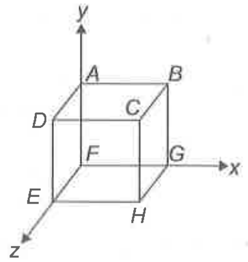
12. The loss in weight of a solid when immersed in a liquid at 0°C is W_0 and at t°C is W. If cubical coefficient of expansion of the solid and the liquid by γ_s and γ_1 respectively, then W is equal to:
(A) $W_0[1 + (\gamma_s - \gamma_1)t]$ (B) $W_0[1 - (\gamma_s - \gamma_1)t]$
(C) $W_0[(\gamma_s - \gamma_1)t]$ (D) $W_0t/(\gamma_s - \gamma_1)$

13. A thin walled cylindrical metal vessel of linear coefficient of expansion 10^{-3}°C^{-1} contains benzenr of volume expansion coefficient 10^{-3}°C^{-1}. If the vessel and its contents are now heated by 10°C, the pressure due to the liquid at the bottom.
(A) increases by 2%
(B) decreases by 1%
(C) decreases by 2%
(D) remains unchanged

14. A rod of length 2 m at 0°C and having expansion coefficient $\alpha = (3x + 2) \times 10^{-6}$°C^{-1} where x is the distance (in cm) from one end of rod. The length of rod at 20°C is:
(A) 2.124 m (B) 3.24 m
(C) 2.0120 m (D) 3.124 m

15. A copper ring has a diameter of exactly 25 mm at its temperature of 0°C. An aluminium sphere has a diameter of exactly 25.05 mm at its temperature of 100°C. The sphere is placed on top of the ring and two are allowed to come to thermal equilibrium, no heat being lost to the surrounding. The sphere just passes through the ring at the equilibrium temperature. The ratio of the mass of the sphere & ring is :
(given: $\alpha_{Cu} = 17 \times 10^{-6}$/°C, $\alpha_{Al} = 2.3 \times 10^{-5}$/°C, specific heat of $Cu = 0.0923$ Cal/g°C and specific heat of $Al = 0.215$ cal/g°C)
(A) 1/5 (B) 23/108
(C) 23/54 (D) 216/23

16. A cuboid $ABCDEFGH$ is anisotropic with $\alpha_x = 1 \times 10^{-5}$/°C, $\alpha_y = 2 \times 10^{-5}$/°C, $\alpha_z = 3 \times 10^{-5}$/°C. Coefficient of superficial expansion of faces can be



(A) $\beta_{ABCD} = 5 \times 10^{-5}$/°C
(B) $\beta_{BCGH} = 4 \times 10^{-5}$/°C
(C) $\beta_{CDEH} = 3 \times 10^{-5}$/°C
(D) $\beta_{EFGH} = 2 \times 10^{-5}$/°C

17. An open vessel is filled completely with oil which has same coefficient of volume expansion as that of the vessel. On heating both oil and vessel,
(A) the vessel can contain more volume and more mass of oil
(B) the vessel can contain same volume and same mass of oil
(C) the vessel can contain same volume but more mass of oil
(D) the vessel can contain more volume but same mass of oil

18. A metal ball immersed in Alcohol weights W_1 at 0°C and W_2 at 50°C. The coefficient of cubical expansion of the metal $(\gamma)_m$ is less than that of alcohol $(\gamma)_{Al}$. Assuming that density of metal is large compared to that of alcohol, it can be shown that
(A) $W_1 > W_2$
(B) $W_1 = W_2$
(C) $W_1 < W_2$
(D) any of (A), (B) or (C)

19. A solid ball is completely immersed in a liquid. The coefficients of volume expansion of the ball and liquid are 3×10^{-6} and 8×10^{-6} per °C respectively. The percentage change in upthrust when the temperature is increased by 100°C is

(A) 0.5 % (B) 0.11 %
(C) 1.1% (D) 0.05 %

20. A thin copper wire of length L increase in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions $2L \times L$ is heated from T_1 to T_2?

(A) 1% (B) 2%
(C) 3% (D) 4%

21. If two rods of length L and $2L$ having coefficients of linear expansion α and 2α respectively are connected so that total length becomes $3L$, the average coefficient of linear expansion of the composition rod equals:

(A) $\frac{3}{2}\alpha$ (B) $\frac{5}{2}\alpha$
(C) $\frac{5}{3}\alpha$ (D) none of these

22. The bulk modulus of copper is 1.4×10^{11} Pa and the coefficient of linear expansion is $1.7 \times 10^{-5}(C°)^{-1}$. What hydrostatic pressure is necessary to prevent a copper block from expanding when its temperature is increased from 20°C to 30°C ?

(A) 6.0×10^5 Pa (B) 7.1×10^7 Pa
(C) 5.2×10^6 Pa (D) 40 atm

23. The coefficients of thermal expansion of steel and a metal X are respectively 12×10^{-6} and 2×10^{-6} per °C, At 40°C, the side of a cube of metal X was measured using a steel vernier callipers. The reading was 100 mm. Assuming that the calibration of the vernier was done at 0°C, then the actual length of the side of the cube at 0°C will be

(A) > 100 mm
(B) < 100 mm
(C) = 100 mm
(D) data insufficient to conclude

24. A glass flask contains some mercury at room temperature. It is found that at different temperature the volume of air inside the flask remains the same. If the volume of mercury in the flask is 300 cm³, then volume of the flask is (given that coefficient of volume expansion of mercury and coefficient of linear expansion of glass are $1.8 \times 10^{-4}(°C)^{-1}$ and $9 \times 10^{-6}(°C)^{-1}$ respectively)

(A) 4500 cm³ (B) 450 cm³
(C) 2000 cm³ (D) 6000 cm³

Question No. 25 to 29

Solids and liquids both expand on heating. The density of substance decreases on expanding according to the relation

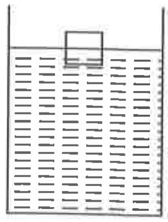
$$\rho_2 = \frac{\rho_1}{1+\gamma(T_2 - T_1)}$$

where, $\rho_1 \rightarrow$ density at T_1
$\rho_2 \rightarrow$ density at T_2
$\gamma \rightarrow$ coeff. of volume expansion of substances

when a solid is submerged in a liquid, liquid exerts an upward force on solid which is equal to the weight of liquid displaced by submerged part of solid.

Solid will float or sink depends on relative densities of solid and liquid.

A cubical block of solid floats in a liquid with half of its volume submerged in liquid as shown in figure (at temperature T)



$\alpha_s \rightarrow$ coeff. of linear expansion of solid
$\gamma_L \rightarrow$ coeff. of volume expansion of liquid
$\rho_s \rightarrow$ density of solid at temp. T
$\rho_L \rightarrow$ density of liquid at temp. T

25. The relation between densities of solid and liquid at temperature T is

(A) $\rho_S = 2\rho_L$ (B) $\rho_S = (1/2)\rho_L$
(C) $\rho_S = \rho_L$ (D) $\rho_S = (1/4)\rho_L$

26. If temperature of system increases, then fraction of solid submerged in liquid

(A) increases
(B) decreases
(C) remains the same
(D) inadequate information

27. Imagine fraction submerged does not change on increasing temperature the relation between γ_L and α_S is

(A) $\gamma_L = 3\alpha_S$ (B) $\gamma_L = 2\alpha_S$
(C) $\gamma_L = 4\alpha_S$ (D) $\gamma_L = (3/2)\alpha_S$

28. Imagine the depth of the block submerged in the liquid does not change on increasing temperature then

(A) $\gamma_L = 2\alpha$ (B) $\gamma_L = 3\alpha$
(C) $\gamma_L = (3/2)\alpha$ (D) $\gamma_L = (4/3)\alpha$

29. Assume block does not expand on heating. The temperature at which the block just begins to sink in liquid is

(A) $T + 1/\gamma_L$ (B) $T + 1/(2\gamma_L)$
(C) $T + 2/\gamma_L$ (D) $T + \gamma_L/2$

30. The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel is S. The coefficient of volume expansion of copper is γ_C. What is the coefficient of linear expansion of silver ?

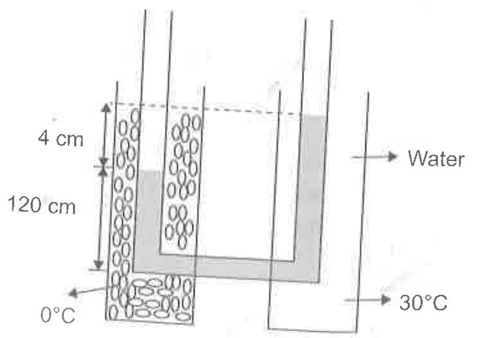
(A) $\frac{(C+\gamma_c+S)}{3}$ (B) $\frac{(C-\gamma_c+S)}{3}$

(C) $\frac{(C+\gamma_c-S)}{3}$ (D) $\frac{(C-\gamma_c-S)}{3}$

31. An aluminium container of mass 100 gm contains 200 gm of ice at −20°C. Heat is added to the system at the rate of 100 cal/s. The temperature of the system after 4 minutes will be (specific heat of ice = 0.5 and $L = 80$ cal/gm, specific heat of $Al = 0.2$ cal/gm/°C)

(A) 40.5°C (B) 25.5°C
(C) 30.3°C (D) 35.0°C

32. Two vertical glass tubes filled with a liquid are connected by a capillary tube as shown in the figure. The tube on the left is put in an ice bath at 0°C while the tube on the right is kept at 30°C in a water bath. The difference in the levels of the liquid in the two tubes is 4 cm while the height of the liquid column at 0°C is 120 cm. The coefficient of volume expansion of liquid is (Ignore expansion of glass tube)



(A) 22×10^{-4}/°C (B) 1.1×10^{-4}/°C
(C) 11×10^{-4}/°C (D) 2.2×10^{-4}/°C

33. A difference of temperature of 25°C is equivalent to a difference of:

(A) 45°F (B) 72°F
(C) 32°F (D) 25°F

34. Two thermometers x and y have fundamental intervals of 80° and 120°. When immersed in ice, they show the reading of 20° and 30°. If y measures the temperature of a body as 120°, the reading of x is:

(A) 59° (B) 65°
(C) 75° (D) 80°

Multiple Choice Questions

35. When an enclosed perfect gas is subjected to an adiabatic process:

(A) Its total internal energy does not change
(B) Its temperature does not change
(C) Its pressure varies inversely as a certain power of its volume
(D) The product of its pressure and volume is directly proportional to its absolute temperature.

36. Four rods A, B, C, D of same length and material but of different radii r, $r\sqrt{2}$, $r\sqrt{3}$ and $2r$ respectively are held between two rigid walls. The temperature of all rods is increased by same amount. If the rods donot bend, then

(A) the stress in the rods are in the ratio 1:2:3:4
(B) the force on the rod exerted by the wall are in the ratio 1:2:3:4
(C) the energy stored in the rods due to elasticity are in the ratio 1:2:3:4
(D) the strains produced in the rods are in the ratio 1:2:3:4

37. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is l.

(A) Loss in gravitational potential energy of M is Mgl
(B) The elastic potential energy stored in the wire is Mgl
(C) The elastic potential energy stored in the wire is $1/2\ Mgl$
(D) Heat produced is $1/2\ Mgl$

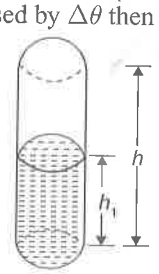
38. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.
(A) Percentage rise in the area of a face is 0.4%
(B) Percentage rise in the thickness is 0.4%
(C) Percentage rise in the volume is 0.6%
(D) Coefficient of linear expansion of copper is $0.25 \times 10^{-4} C^{\circ -1}$.

JEE Advanced

1. We have a hollow sphere and a solid sphere of equal radii and of the same material. They are heated to raise their temperature by equal amounts. How will the change in their volumes, due to volume expansions, be related? Consider two cases (i) hollow sphere is filled with air, (ii) there is vaccum inside the hollow sphere.

2. The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reach to its extreme position the second hand of the clock advances by one second that means second hand move by two second when one oscillation in complete
(A) How many number of oscillations completed by pendulum of clock in 15 minutes at calibrated temperature 20°C
(B) How many number of oscillations are completed by a pendulum of clock in 15 minute at temperature of 40°C if $\alpha = 2 \times 10^{-5} c$
(C) What time is represented by the pendulum clock at 40°C after 15 minutes if the initial time shown by the clock is 12:00 pm ?
(D) If the clock gains two second in 15 minutes then find (i) Number of extra oscillation (ii) New time period (iii) change in temperature.

3. Consider a cylindrical container of cross section area 'A', length 'h' having coefficient of linear expansion α_c. The container is filled by liquid of real expansion coefficient γ_L up to height h_1. When temperature of the system is increased by $\Delta\theta$ then



(A) Find out new height, area and volume of cylindrical container and new volume of liquid.
(B) Find the height of liquid level when expansion of container is neglected.
(C) Find the relation between γ_L and α_c for which volume of container above the liquid level.
(i) increases (ii) decreases
(iii) remains constant.
(D) If $Y_\ell > 3\alpha_C$ and $h = h_1$ then calculate, the volume of liquid overflow
(E) What is the relation between γ_ℓ and α_c for which volume of empty space becomes independent of change of temp.
(F) If the surface of a cylindrical container is marked with numbers for the measurement of liquid level of liquid filled inside it. If we increase the temperature of the system be $\Delta\theta$ then
(i) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of liquid
(ii) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of container
(iii) Find relation between γ_L and α_c so that height of liquid level with respect to ground
(1) increases (2) decreases
(3) remains constant.

4. A loaded glass bulb weighs 156.25 g in air. When the bulb is immersed in a liquid at temperature 15°C, it weighs 56.25 g. On heating the liquid, for a temperature upto 52°C the apparent weight of the bulb becomes 66.25 g. Find the coefficient of real expansion of the liquid. (Given coefficient of linear expansion of glass = 9×10^{-6}/°C).

5. A body is completely submerged inside the liquid. It is in equilibrium and in rest condition at certain temperature. It γ_L volumetric expansion coefficient of liquid

α_s = linear expansion coefficient by of body. It we increases temperature by $\Delta\theta$ amount than find

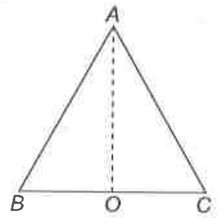
(A) New thrust force if initial volume of body is V_0 and density of liquid is d_0.

(B) Relation between α_s and γ_L so body will (i) move upward (ii) down ward (iii) remains are rest

6. A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, aporoximately. How much fast or slow will the clock run in 10^6 sec.
($\alpha_{invar} = 1 \times 10^{-6}/°C$)

7. An iron bar (Young's modulus = 10^{11} N/m², $\alpha = 10^{-6}/°C$) 1 m long and 10^{-3} m² in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

8. Three aluminium rods of equal length form an equilateral triangle ABC. Taking O (mid point of rod BC) as the origin.Find the increase in Y-coordinate of center of mass per unit change in temperature of the system. Assume the length of the each rod is 2 m, and $\alpha_{al} = 4\sqrt{3} \times 10^{-6}/°C$



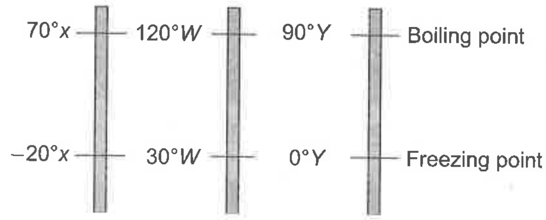
9. If two rods of length L and $2L$ having coefficients of linear expansion α and 2α respectively are connected so that total length becomes $3L$, determine the average coefficient of linear expansion of the composite rod.

10. A thermostatted chamber at small height h above earth's surface maintained at 30°C has a clock fitted in it with an uncompensated pendulum. The clock designer correctly designs it for height h, but for temperature of 20°C. If this chamber is taken to earth's surface, the clock in it would click correct time. Find the coefficient of linear expansion of material of pendulum. (earth's radius is R)

11. The coefficient of volume expansion of mercury is 20 times the coefficient of linear expansion of glass Find the volume of mercury that must be poured into a glass vessel of volume V so that the volume above mercury may remain constant at all temperature.

12. A metal rod A of 25 cm lengths expands by 0.050 cm. When its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40 cm expands by 0.040 cm for the same rise in temperature. A third rod C of 50 cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0° C to 50°C. Find the lengths of each portion of the composite rod.

13. The figure shows three temperature scales with the freezing and boiling points of water indicated.



(A) Rank the size of a degree on these scales, greatest first.

(B) Rank the following temperatures, highest first 50°X, 50°W and 50°Y.

14. What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scales ?

Previous Year Questions

JEE Main

1. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. The the elastic energy stored in the wire is **[AIEEE 2003]**

(A) 0.2 J
(B) 10 J
(C) 20 J
(D) 0.1 J

2. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is **[AIEEE 2004]**

(A) $\frac{F}{2l}$ (B) Fl

(C) $2Fl$ (D) $\frac{Fl}{2}$

3. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is **[AIEEE 2005]**

(A) $2S^2Y$ (B) $\frac{S^2}{2Y}$

(C) $\frac{2Y}{S^2}$ (D) $\frac{S}{2Y}$

4. A wire elongates by l mm when a load w is hanged from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) **[AIEEE 2006]**

(A) l (B) $2l$

(C) zero (D) $\frac{l}{2}$

5. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire-2 has cross-sectinal area $3A$. If the length of wire 1 increases by ΔX on applying force F, how much force is needed to stretch wire 2 by the same amount ? **[AIEEE 2009]**

(A) F (B) $4F$

(C) $6F$ (D) $9F$

6. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear strees developed in it is **[AIEEE 2011]**

(A) $\frac{\alpha t}{Y}$ (B) $Y\alpha t$

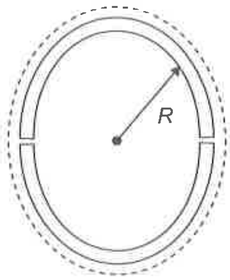
(C) $\frac{Y}{\alpha t}$ (D) $\frac{1}{Y\alpha t}$

7. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23\times10^{-6}$ /°C **[AIEEE 2011]**

(A) 28.9 cc (B) 2.89 cc

(C) 9.28 cc (D) 49.8 cc

8. A wooden wheel of radius R is made of two semi-circular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y, the force that one part of the wheel applies on the other part is **[AIEEE 2012]**

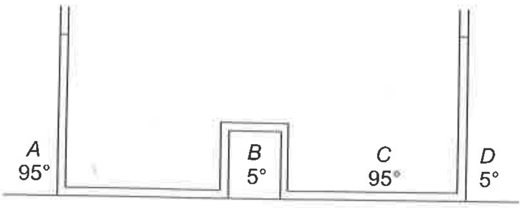


(A) $2\pi sy\alpha\Delta T$ (B) $sy\alpha\Delta T$

(C) $\pi sy\alpha\Delta T$ (D) $2sy\alpha\Delta T$

JEE Problems

1. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A & D are open to the atmosphere. A and C are maintained at a temperature of 95°C while the columns B and d are maintained at 5°C. The height of the liquid in A and D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. **[JEE 1997]**



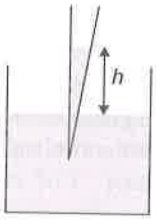
2. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is : **[JEE 1999]**
(A) proportional at ΔT
(B) inversely proportional to ΔT
(C) proportional to $|\alpha_B - \alpha_C|$
(D) inversely proportional to $|\alpha_B - \alpha_C|$

3. Two rods one of aluminium of length l_1 having coefficient of linear expansion α_a, and other steel of length l_2 having coefficient of linear expansion α_s are joined end to end. The expansion in both the rods is same on variation of temperature. Then the value of is $\frac{l_1}{l_1 + l_2}$ **[JEE(Scr) 2003]**
(A) $\frac{\alpha_s}{\alpha_a + \alpha_s}$
(B) $\frac{\alpha_s}{\alpha_a - \alpha_s}$
(C) $\frac{\alpha_a + \alpha_s}{\alpha_s}$
(D) None of these

4. A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l, showing all the steps. **[JEE 2004]**

5. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 red s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is **[JEE 2010]**

6. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. when dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)



(A) $\frac{2s}{b\rho g}\cos(\theta - \alpha)$
(B) $\frac{2s}{b\rho g}\cos(\theta + \alpha)$
(C) $\frac{2s}{b\rho g}\cos(\theta + \alpha/2)$
(D) $\frac{2s}{b\rho g}\cos(\theta + \alpha/2)$

7. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is **[JEE Main 2014]**
(For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})
(A) 2.2×10^7 Pa
(B) 2.2×10^6 Pa
(C) 2.2×10^8 Pa
(D) 2.2×10^9 Pa

ANSWER KEYS

Exercises

JEE Main

1. C	2. A	3. C	4. C	5. B	6. B	7. B	8. B	9. A	10. B
11. B	12. A	13. C	14. C	15. C	16. C	17. D	18. C	19. D	20. B

21. C **22.** B **23.** A **24.** C **25.** B **26.** A **27.** A **28.** A **29.** A **30.** C
31. B **32.** C **33.** A **34.** D **35.** C, D **36.** B, C **37.** A, C, D **38.** A, C, D

JEE Advanced

1. (i) hollow sphere > solid sphere, (ii) hollow sphere = solid sphere
2. (A) 450 (B) 449 (C) 12:14:59 (D) (i) 1 (ii) $\frac{900}{451}$ s (iii) $\frac{1}{450\times10^{-5}}$ s
3. (A) $h^1 = h\{1 + \alpha_c \Delta\theta\}, A^1 = A\{1 + 2\alpha_s \Delta\theta\}, v^1 = Ah\{1 + 3\alpha_s \Delta\theta\}$
   volume of liquid $V_w = Ah_1(1 + \gamma_L \Delta\theta)$
   (B) $h^1 = h\{1 + \gamma_L \Delta\theta\}$ (C) (i) $\gamma_L < 3\alpha_c$ (ii) $\gamma_L > 3\alpha_c$ (iii) $\gamma_L = 3\alpha_c$.
   (D) $\Delta V = Ah(\gamma_L - 3\alpha_c)\Delta\theta$ (E) $3h\alpha_c = h_1\gamma_L$
   (F) (i) $h_1(1 - 3\alpha_c\Delta\theta)$, (ii) $h_1(1 + \gamma_L \Delta\theta)$, (iii) (1) $\gamma_L > 2\alpha_c$ (2) $\gamma_L < 2\alpha_c$ (3) $\gamma_L = 2\alpha_c$
4. $Y_R = \left(\frac{1}{9} + 27\times37\times10^{-6}\right)/°C$ **5.** (A) $V_0 d_0 g\left(\frac{1+3\alpha_s\Delta\theta}{1+\gamma_L\Delta\theta}\right)$ (B) (i) $\gamma_L < 3\alpha_s$ (ii) $\gamma_L > 3\alpha_s$ (iii) $\gamma_L = 3\alpha_s$.
6. 5 sec slow **7.** 10000 N **8.** 4×10^{-6} m/°C **9.** $5\alpha/3$ **10.** $h/5R$ **11.** $3V/20$ **12.** 10 cm, 40 cm
13. (A) All tie (B) 50°X, 50°Y, 50°W. **14.** −40°C or −40°F

Previous Year Questions

JEE Main

1. D **2.** D **3.** B **4.** A **5.** D **6.** C **7.** A **8.** D

JEE Problems

1. $2 \times 10^{-4} C$ **2.** B, D **3.** A **4.** $\gamma_l = 2\alpha_s$ **5.** 4 **6.** D **7.** C

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