JEE Class Companion Physics

for JEE Main and Advanced

Module-3

MOTION
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First Impression

Published by Pearson India Education Services Pvt. Ltd, CIN: U72200TN2005PTC057128, formerly known as TutorVista Global Pvt. Ltd, licensee of Pearson Education in South Asia.

Head Office: A-8(A), 7th Floor, Knowledge Boulevard, Sector 62, Noida 201 309, Uttar Pradesh, India.
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Compositor: Macrotex Solutions.
Printed in India by Rahul Print O Pack

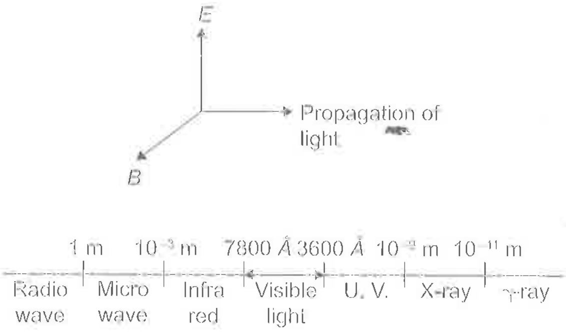
Contents

CHAPTER 1

Geometrical Optics

PROPERTIES OF LIGHT

1. Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.
2. Light is an electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.
3. Light carries energy and momentum.
4. The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

Figure 1.1

RAY OPTICS

Ray optics treats the propagation of light in terms of rays and is valid only if the size of the obstacle is much greater than the wavelength of light. It concerns with the image formation and deals with the study of the simple facts such as rectilinear propagation and laws of reflection and refraction by geometrical methods.

Ray

A ray can be defined as an imaginary line drawn in the direction in which light is travelling. Light behaves as a stream of energy propagated along the direction of the rays. The rays are directed outward from the source of light in straight lines.

Beam of Light

A beam of light is a collection of these rays. There are mainly three types of beams.

(i) Parallel Beam of Light

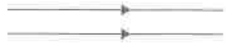


Figure 1.2

A search light and the headlight of a vehicle emit a parallel beam of light. The source of light at a very large distance like sun effectively gives a parallel beam.

(ii) Divergent Beam of Light

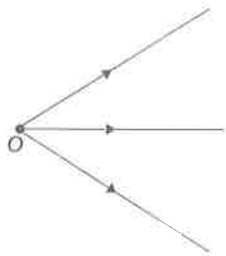


Figure 1.3

The rays going out from a point source generally form a divergent beam.

(iii) Convergent beam of light

A beam of light that is going to meet (or converge) at a point is known as a convergent beam. A parallel beam of light after passing through a convex lens becomes a convergent beam.

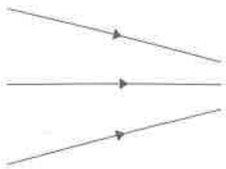


Figure 1.4

REFLECTION

When a ray of light is incident at a point on the surface, the surface throws partly or wholly the incident energy back into the medium of incidence. This phenomenon is called reflection.

Surfaces that cause reflection are known as mirrors or reflectors. Mirrors can be plane or curved.

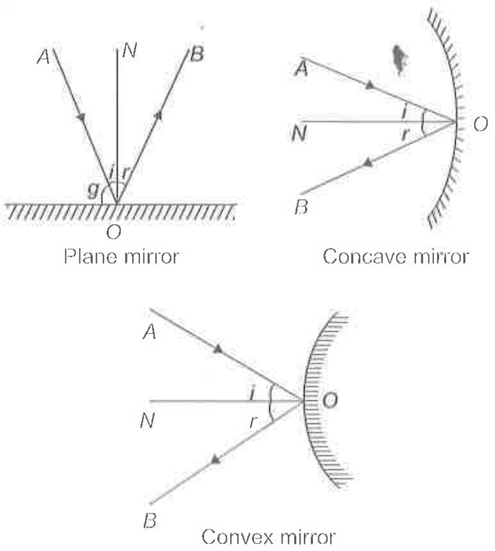


Figure 1.5

In Figure 1.5,

O is the point of incidence, and *AO* is the incident ray.

OB is the reflected ray, and *ON* is the normal at the incidence.

Angle of Incidence

The angle that the incident ray makes with the normal at the point of incidence is called the angle of incidence. It is generally denoted by i.

Angle of Reflection

The angle that the reflected ray makes with the normal at the point of incidence is called the angle of reflection. It is generally denoted by 'r'.

Glancing angle

The angle that the incident ray makes with the plane reflecting surface is called glancing angle. It is generally denoted by g.

$$g = 90^\circ - i \qquad (1)$$

Law of Reflection

1. The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
2. The angle of incidence is equal to the angle of reflection, i.e., $\angle i = \angle r$.

These laws hold good for all reflecting surfaces either plane or curved.

Some Important Points

1. If $\angle i = 0$, $\angle r = 0$, i.e., if a ray is incident normally on a boundary, after reflection it retraces its path.

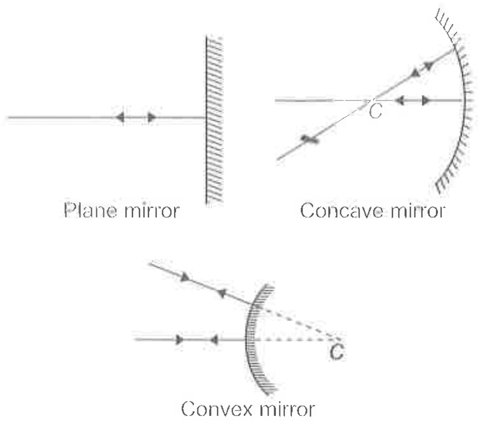


Figure 1.6

2. None of the frequency, wavelength and speed changes due to reflection. However, intensity and hence amplitude ($I \propto A^2$) usually decreases.

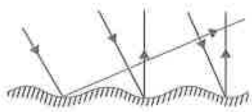


Figure 1.7

3. If the surface is irregular, the reflected rays on an incident beam of parallel light rays will be in random direction. Such an irregular reflection is called diffused reflection.

PLANE MIRROR

Plane mirror is formed by polishing one surface of a plane thin glass plate. It is also said to be silvered on one side.

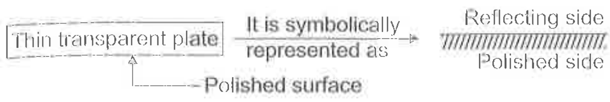


Figure 1.8

A beam of parallel rays of light incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Formation of Image by a Plane Mirror

From the argument of similar triangles,

$$OM = IM,$$

i.e., perpendicular distance of the object from the mirror = perpendicular distance of the image from the mirror.

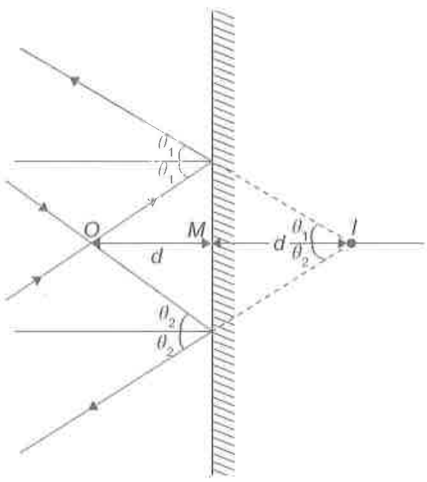


Figure 1.9

Steps to Draw the Image

1. Drop a perpendicular on the mirror and extend it on the back side of the mirror.
2. Image always lies on this extended line.
3. To exactly locate the image, use the following concept:

Perpendicular distance of the object from the mirror is equal to the perpendicular distance from the mirror of the image.

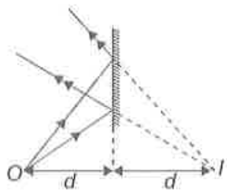


Figure 1.10

SOLVED EXAMPLE

EXAMPLE 1

A mirror is inclined at an angle of 45° with the horizontal and the mirror starts from the origin. An object is kept at $x = -2$ cm. Locate its image.

SOLUTION

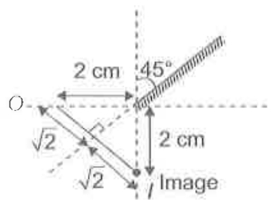


Image of an Extended Linear Object

Draw the images of the extreme points and join them with a straight line.

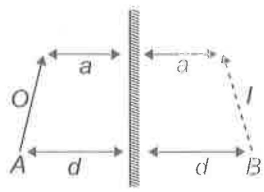


Figure 1.11

Properties of Image of an Extended Object, Formed by a Plane Mirror

1. Size of extended object = size of extended image.
2. The image is erect, if the extended object is placed parallel to the mirror.

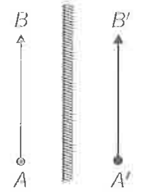


Figure 1.12

3. The image is inverted if the extended object lies perpendicular to the plane mirror.

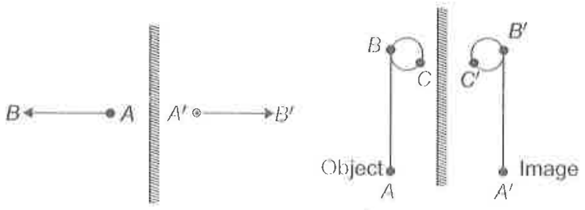


Figure 1.13

4. If an extended horizontal object is placed in front of a mirror inclined at 45° with the horizontal, the image formed will be vertical (Fig. 1.14).

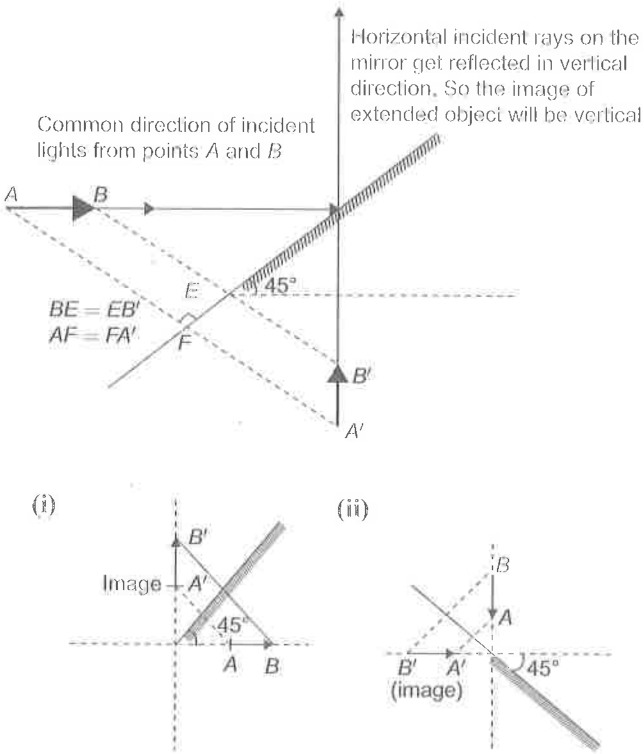


Figure 1.14

SOLVED EXAMPLES

EXAMPLE 2

An unnumbered wall clock shows time 8:12 where the first numeral represent hours and the second numerals represent minutes. What is the time that its image in a plane mirror show.

SOLUTION

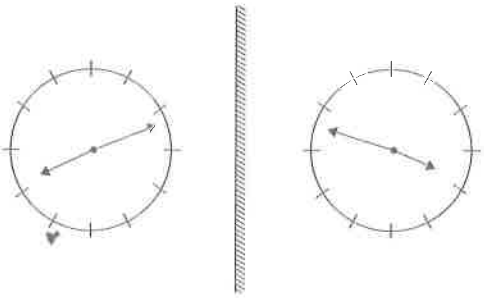


Image shows 3:48.

Short trick

Draw watch on a paper and then see it from the reverse side. ■

Field of View

The area in which reflected rays exist is called the field of view. It is the area from which an observer can see the image of an object. If the observer is outside this area, he will not be able to see the image, although the image will be there.

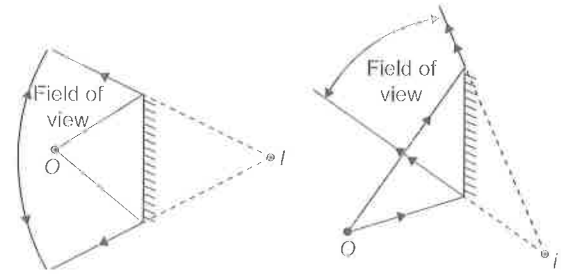
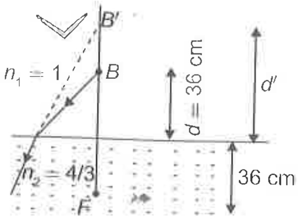


Figure 1.15

EXAMPLE 3

A man is travelling on the rod along AB. Find out the length of the road for which the image will be visible to him.

(b)

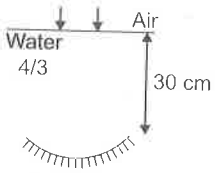


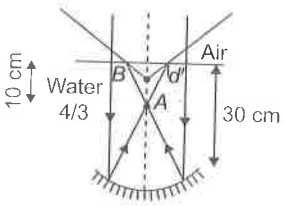
$$\frac{4/3}{1} = \frac{d'}{36}$$

$$\Rightarrow \quad d' = 48\,\text{cm}.$$

EXAMPLE 39

A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of the concave mirror. Find the position of the final image.





SOLUTION

The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, d = 10 cm.

$$\therefore \quad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)}$$

$$= \frac{10}{\left(\frac{4/3}{1}\right)}$$

$$= 7.5\,\text{cm}$$

Velocity of the image in case of plane refraction

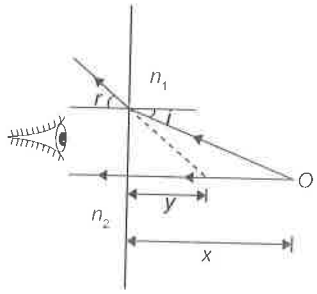


Figure 1.61

$$\frac{n_2}{n_1} = \frac{y}{x}$$

$$\Rightarrow \quad y = \frac{n_2}{n_1} \cdot x$$

$$\frac{dy}{dt} = \frac{n_2}{n_1}\frac{dx}{dt}$$

$$\Rightarrow \quad V_{IS} = \frac{n_2}{n_1} V_{OS}.$$

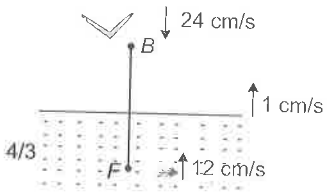
SOLVED EXAMPLE

EXAMPLE 40

Find out the following in the figure shown below.

(a) The apparent speed of the fish as observed by the bird.

(b) The apparent speed of the bird as observed by the fish.



SOLUTION

(a) $$V_{IS} = \frac{n_2}{n_1} V_{OS}$$

$$\Rightarrow \quad V_I - 1 = \frac{3}{4}(12 - 1)$$

$$V_I = \frac{3}{4} + 1$$

$$= \frac{37}{4}$$

$$V_{IB} = V_I - V_B$$

$$= \frac{37}{4} + 42$$

$$= \frac{133}{4} \text{ cm/s}$$

(b) $n_2 = 4/3$,

$$n_1 = 1$$

$$V_{IS} = \frac{n_2}{n_1} V_{OS}$$

$$v_0 = -24,$$

$$v_s = +1$$

$$\Rightarrow \quad v_I - 1 = \frac{4}{3}[-24 - 1]$$

$$v_I = -\frac{97}{3}$$

$$\Rightarrow \quad v_{If} = \frac{97}{3} + 12$$

$$= \frac{133}{3} \text{ cm/s.}$$

REFRACTION THROUGH A GLASS SLAB

When a light ray passes through a glass slab having parallel faces, it gets refracted twice before finally emerging out of it.

First refraction takes place from air to glass.

So, $$m = \frac{\sin i}{\sin r}, \quad (1)$$

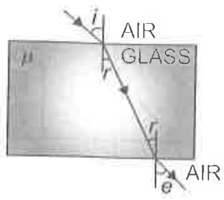


Figure 1.62

The second refraction takes place from glass to air.

So, $$\frac{1}{\mu} = \frac{\sin r}{\sin e}. \quad (2)$$

From Eqs. (1) and (2), we get

$$\frac{\sin i}{\sin r} = \frac{\sin e}{\sin r}$$

$$\Rightarrow \quad i = e.$$

Thus, the emergent ray is parallel to the incident ray.

Apparent shift due to slab when object is seen normally through the slab

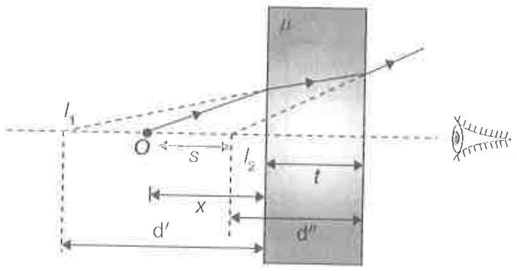


Figure 1.63

First refraction

Because of the refraction at the first surface, the image of O is formed at I_1. For this refraction, the real depth is x and the apparent depth is d'. Also, the first medium is air and the second is the slab. Thus,

$$\frac{m}{1} = \frac{d'}{x}$$

$\Rightarrow \quad \mu x$

Second refraction

The point I_1 acts as the object for the refraction at the second surface. Due to this refraction, the image of I_1 is formed at I_2. Thus,

$$\frac{1}{\mu} = \frac{d''}{\mu x + t}$$

$$\Rightarrow \qquad d' = x + \frac{t}{\mu}.$$

Shift $$S = x + t - x - \frac{t}{\mu}$$

$$S = t\left[1 - \frac{1}{\mu}\right].$$

If medium is not air outside the slab,

$$S = t\left[1 - \frac{\mu_{\text{surrounding}}}{\mu_{\text{slab}}}\right]$$

Important points

1. Rays should be paraxial.
2. Medium on both sides of the slab should be the same.
3. Shift comes out from the object.
4. Shift is independent of the distance of the object from the slab.
5. If shift comes out positive, then shift is towards the direction of incident rays and vice versa.

SOLVED EXAMPLES

EXAMPLE 41

Calculate the shift produced by the slab having thickness 15 cm and refractive index 1.5 which is kept in air.

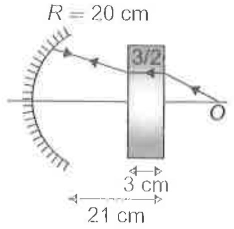
SOLUTION

Shift $$S = t\left[1 - \frac{1}{\mu}\right]$$

$$= 15\left[1 - \frac{2}{3}\right] = 5\,\text{cm}$$

EXAMPLE 42

See the figure. Find the distance of final image formed by the mirror.



SOLUTION

$$\text{Shift} = 3\left(1 - \frac{1}{3/2}\right)$$

$$= 3\left(1 - \frac{1}{3/2}\right)$$

For mirror, object is at a distance

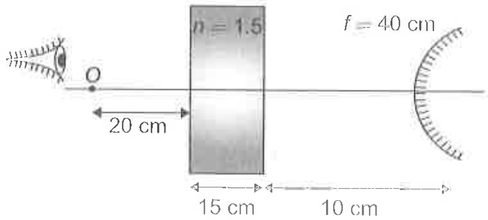
$$= 21 - 3\left(1 - \frac{1}{3/2}\right)$$

$$= 20\text{ cm}$$

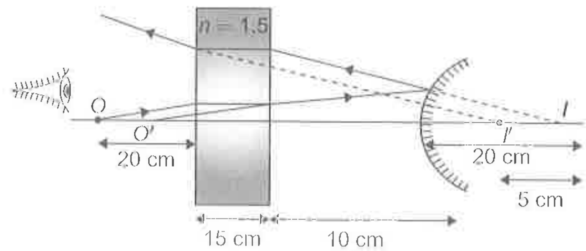
∴ Object is at the centre of curvature of the mirror. Hence, the light ray will retrace and the image will be formed on the object itself.

EXAMPLE 43

Find out the distance between image and the mirror as observed by the observer in the figure shown below



SOLUTION



$$\text{Shift} = t\left(1-\frac{1}{\mu}\right)$$

$$= 15\left(1-\frac{2}{3}\right)$$

$$= 5 \text{ cm}$$

$u = -40,$

$f = +40$

$v = +20$ cm

The distance between the mirror and the image as observed by the observer = 20 − shift = 15 cm. ■

Apparent distance between object and observer when both are in different medium

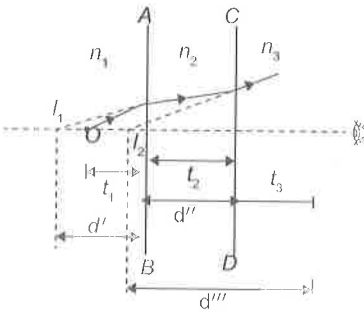


Figure 1.64

First refraction:

$$\frac{n_2}{n_1} = \frac{\text{Apparent distance of object from interface } AB}{\text{Real distance of object from interface } AB}$$

$$= \frac{d'}{t_1}$$

$$d' = \frac{n_2 t_1}{n_1}.$$

Second refraction:

$$\frac{n_3}{n_2} = \frac{\text{apparent distance of } I_1 \text{ from interface } CD}{\text{Real distance of } I_1 \text{ from interface } CD}$$

$$= \frac{d''}{\frac{n_2 t_1}{n_1} + t_2}$$

$$d'' = \frac{n_3}{n_2}\left[\frac{n_2}{n_1}t_1 + t_2\right]$$

$$= n_3\left[\frac{t_1}{n_1} + \frac{t_2}{n_2}\right].$$

Final distance of the image from the observer

$$= d'' + t_3$$

$$d''' = n_3\left[\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}\right].$$

Note

If the object and observer are in the same medium, then shift formula should be used and if both are in different medium, then the above formula of apparent distance should be used.

Lateral Shift

The perpendicular distance between the incident ray and the emergent ray, when the light is incident obliquely on a parallel sided refracting glass slab is called lateral shift.

In right-angled triangle *OBK*, we have

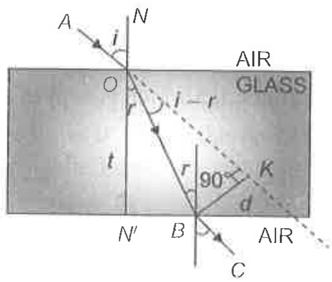


Figure 1.65

$$\angle BOK = i - r$$

$$\therefore \quad \sin(i - r) = \frac{d}{OB}$$

or, $$d = OB \sin(i - r). \quad (1)$$

In right-angled triangle $ON'B$, we have

$$\cos r = \frac{ON'}{OB}$$

or $$OB = \frac{t}{\cos r}.$$

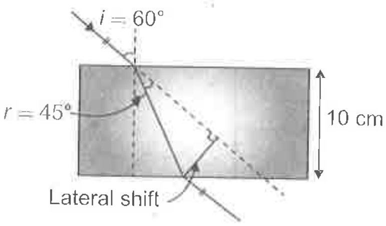
Substituting the above value of OB in Eq. (1), we get

$$d = \frac{t}{\cos r}\sin(i - r). \quad (13)$$

SOLVED EXAMPLE

EXAMPLE 44

Find the lateral shift of a light ray while is passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.



SOLUTION

$$d = \frac{t\sin(i - r)}{\cos r}$$

$$= \frac{10\sin(60° - 45°)}{\cos 45°}$$

$$= \frac{10\sin 15°}{\cos 45°}$$

$$= 10\sqrt{5}\sin 15°$$

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION (TIR)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90°. When angle in denser medium is more than the critical angle, the light ray reflects back in the denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror. In Fig. 1.66,

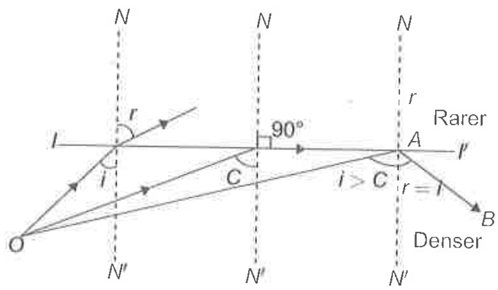


Figure 1.66

O = object

NN' = Normal to the interface

II' = Interface

C = Critical angle

AB = reflected ray due to TIR

When $i = C$,

then $r = 90°$

$$\therefore \quad C = \sin^{-1}\frac{n_r}{n_d}.$$

Conditions for TIR

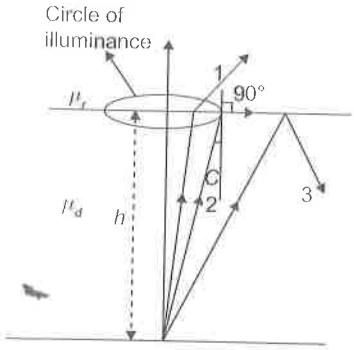


Figure 1.67

(a) Light is incident on the interface from a denser medium.

(b) Angle of incidence should be greater than the critical angle ($i > c$). Figure 1.67 shows a luminous object placed in a denser medium at a distance h from an interface separating the two media of refractive indices μ_r and μ_d. Subscripts r and d stand for rarer and denser medium, respectively.

In the figure, ray 1 strikes the surface at an angle less than the critical angle C and gets refracted in the rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than the critical angle and gets internally reflected. The locus of points where the ray strikes at critical angle is a circle, called the **circle of illuminance**. All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object cannot be seen from the rarer medium. Radius of COI can be easily found.

SOLVED EXAMPLES

EXAMPLE 45

Find the maximum angle that can be made in glass medium ($\mu = 1.5$) if a light ray is refracted from glass to vacuum.

SOLUTION

$$1.5 \sin C = 1 \sin 90°,$$

where C = critical angle.

$$\sin C = 2/3$$

$$\Rightarrow \quad C = \sin^{-1} 2/3$$

EXAMPLE 46

Find the angle of refraction in a medium ($n = 2$) if light is incident in vacuum, making angle equal to twice the critical angle.

SOLUTION

Since the incident light is in rarer medium, total internal reflection cannot take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30°$$

$$\Rightarrow \quad \therefore \quad i = 2C = 60°.$$

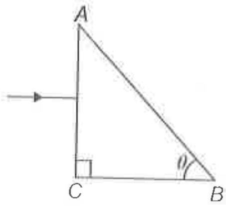
Applying Snell's law, $1 \sin 60° = 2 \sin r$

$$\sin r = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \quad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right).$$

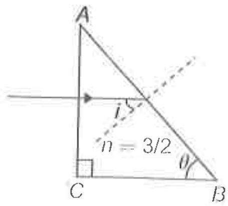
EXAMPLE 47

What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB.



SOLUTION

Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.



$$i = 90° - \theta$$

For the required condition,

$$90° - \theta > C$$

or $$\sin(90° - \theta) > \sin C$$

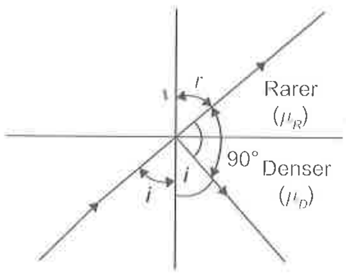
or $$\cos\theta > \sin C$$

$$= \frac{1}{3/2} = \frac{2}{3}$$

or $$\theta < \cos^{-1}\frac{2}{3}$$ ■

EXAMPLE 48

A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. If the reflected and the refracted rays are mutually perpendicular to each other, what is the value of the critical angle?



SOLUTION

From Snell's law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D}$$

or, $$\mu = \frac{\mu_D}{\mu_R}$$

$$= \frac{\sin r}{\sin i}. \quad (1)$$

According to the given problem,

$$i + r + 90° = 180°$$

or, $$r = 90° - i.$$

Substituting the above value of r in Eq. (1), we get

$$\mu = \frac{\sin(90° - i)}{\sin i}$$

or $$\mu = \cot i. \quad (2)$$

By definition,

$$C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

or $$C = \sin^{-1}\left(\frac{1}{\cot i}\right) \quad \text{(using Eq. (2)}$$

or $$C = \sin^{-1}(\tan i)).$$ ■

Optical Fibre Cable

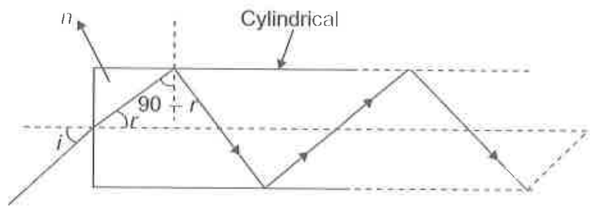


Figure 1.68

Find out the range of n for which ray will show T.I.R through curved surface.

SOLUTION

It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle.

Angle r' is minimum when r is maximum, i.e., C (Why?). Therefore, the minimum value of r' is $90° - C$. For TIR,

$$(90 - r)_{min} > C$$

$$90 - r_{max} > C$$

For r_{max}

$\Rightarrow$ $$i_{max} = 90°$$

when $$i = 90°, r = C,$$

$$90° - C > C$$

$\Rightarrow$ $$C < 45°$$

$\Rightarrow$ $$\sin C < \sin 45°$$

$$\Rightarrow \quad \frac{1}{n} < \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad n > \sqrt{2}.$$

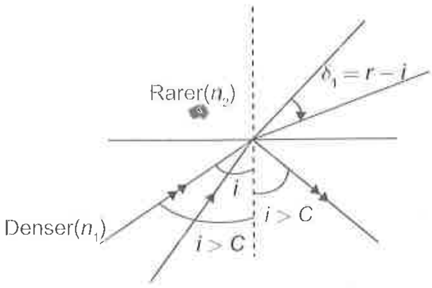


Figure 1.69

Graph Between δ and i

When $i \le C$

$$\delta = r - i$$

and $\quad n_1 \sin i = n_2 \sin r$

$$r = \sin^{-1}\left(\frac{n_1}{n_2}\sin i\right)$$

so $$\delta = \sin^{-1}\left(\frac{n_1}{n_2}\sin i\right) - i \qquad (1)$$

When $i > C$

Then, $$\delta = \pi - 2i \qquad (2)$$

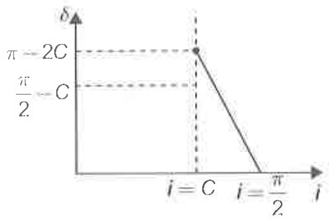


Figure 1.70

Note

If the angle δ is between $(0, \frac{\pi}{2} - C)$, then there are two possible values of i..

Variable Refractive Index

If RI is a function of y:

Take an small element of y of width dy.

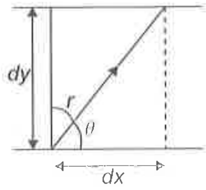
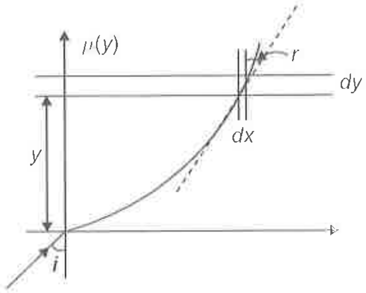


Figure 1.71

Here



Now $\quad 1 \sin i = \mu(y) \sin r$

$$\tan\theta = \frac{dy}{dx}$$

$$= \tan(90^\circ - r)$$

or $$\cot r = \frac{dy}{dx}$$

so $$\sin r = \frac{\sin i}{\mu(y)}$$

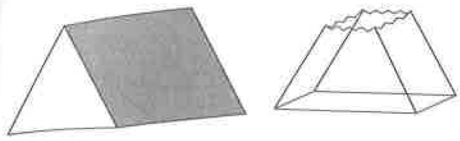
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{\mu^2(y) - \sin^2 i}}{\sin i}$$

$$\int_0^y \frac{dy}{\sqrt{\mu^2(y) - \sin^2 i}} = \int_0^x \frac{dx}{\sin i}.$$

PRISM

A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism.

3-D View



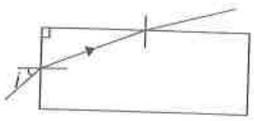


Figure 1.72

Refraction through a Prism

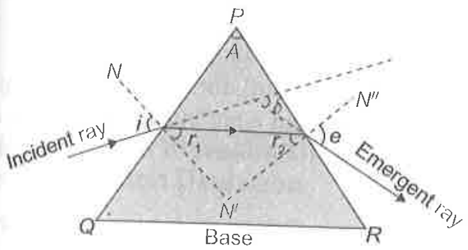


Figure 1.73

(a) PQ and PR are refracting surfaces.

(b) $\angle QPR = A$ is called the refracting angle or the angle of the prism (also called apex angle).

(c) δ = angle of deviation.

(d) For refraction of a monochromatic (single wave length) ray of light through a prism,

$$\delta = (i + e) - (r_1 + r_2)$$

and

$$r_1 + r_2 = A$$

$\therefore$

$$\delta = i + e - A.$$

Notes

I. If a ray crosses two surfaces which are inclined to each other, then we use the concept of prism

II. If a ray crosses two plain parallel surfaces, then we use the concept of slab.

SOLVED EXAMPLES

EXAMPLE 49

A ray of light is incident on one face of a prism ($\mu = 1.5$) at an angle of 60°. The refracting angle of the prism is also 60°. Find the angle of emergence and the angle of deviation. Is there any other angle of incidence, which will produce the same deviation?

SOLUTION

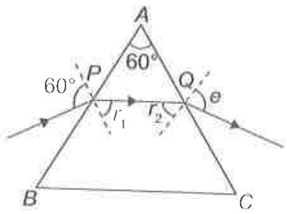
Angle of incidence = $i = 60°$.

At point P,

$$\frac{\sin 60°}{\sin r_1} = \frac{1.5}{1}$$

$\Rightarrow$ $$\sin r_1 = \frac{1}{\sqrt{3}}$$

or, $$r_1 \approx 35°6'.$$



Using $$r_1 + r_2 = A,$$

we get $$r_2 = A - r_1 = 60° - 35°6'$$

$$= 24°44'.$$

At point Q, $$\frac{\sin r_2}{\sin e} = \frac{1}{1.5}$$

$\Rightarrow$ $$\sin e = 1.5 \sin 24° 44'$$

$\Rightarrow$ $$\sin e = 0.63$$

$\Rightarrow$ $$e = 39°.$$

$\therefore$ Deviation = δ

$$= (i + e) - A$$

$= 60° + 39° - 60° = 39°.$

If i and e are interchanged, deviation remains the same. Hence, the same deviation is obtained for angles of incidences 60° and 39°. ■

EXAMPLE 50

A ray of light makes an angle of 60° on one of the faces of a prism and suffers a total deviation of 30° on emergence from the other face. If the angle of the prism is 30°, show that the emergent ray is perpendicular to the other face. Also calculate the refractive index of the material of the prism.

SOLUTION

The angle of deviation

$$\delta = (i_1 + i_2) - A$$

Here, $\delta = 30°$,

$i_1 = 60°$,

$A = 30°$.

Hence, $30° = 60° + i_2 - 30°$

$= 30° + i_2$

$\Rightarrow$ $i_2 = 0$.

The angle of emergence is zero. This means that the emergent ray is perpendicular to the second face.

Since $i_2 = 0$, the angle of incidence at the second face is zero.

$\therefore$ $r_2 = 0$.

Now, $r_1 + r_2 = A$

or, $r_1 = A = 30°$.

We know,

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$= \frac{\sin 60°}{\sin 30°}$$

$$= \frac{\sqrt{3}/2}{1/2}.$$

$$\sqrt{3} = 1.732.$$ ■

Graph Between ∠δ and ∠i

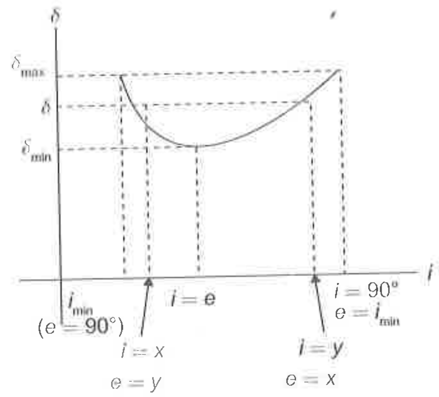


Figure 1.74

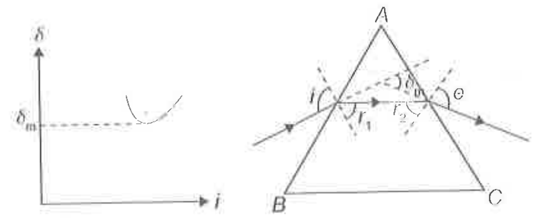
(1) Variation of δ versus i (shown in Fig. 1.74). For one δ (except δ min), there are two values of angle of incidence. If i and e are interchanged, then we get the same value of δ because of reversibility principle of light.

(2) There is one and only one angle of incidence for which the angle of deviation is minimum.

(3) Right-hand side part of the graph is more tilted than the left-hand side.

Minimum Deviation and Condition for Minimum Deviation

The angle of deviation depends on the angle of incidence in a particular way. When the angle of incidence is small, the deviation is large. As i increases, δ decreases rapidly and attains a minimum value and then increases slowly with increase of i. The minimum value of δ so attained is called the minimum deviation (δ_m).



Condition

Theory and experiment show that δ will be minimum when the path of the light ray through the prism is symmetrical.

■ Angle of incidence = angle of emergence or, $\angle i = \angle e$.

For the refraction at the face AB, we have

$$\frac{\sin i}{\sin r_1} = \mu \text{(Snell's law)}$$

or $\sin i = \mu \sin r_1$,

and $\frac{\sin e}{\sin r_2} = \mu$,

or $\sin e = \mu \sin r_2$

$\therefore$ $\mu \sin r_1 = \mu \sin r_2$

or, $r_1 = r_2$.

Hence, the condition for minimum deviation is

$$i = e \text{ and } r_1 = r_2$$

Relation Between Refractive Index and the Angle of Minimum Deviation

When $\delta = \delta_m$, we have

$$e = i$$

and $r_1 = r_2 = r$ (say).

We know,

$$A = r_1 + r_2$$

$$= r + r = 2r$$

or, $r = \frac{A}{2}$.

Also, $A + \delta = i + e$

or, $A + \delta_m = i + i$

or, $i = \frac{A + \delta_m}{2}$.

The refractive index of the material of the prism is given by

$$\mu = \frac{\sin i}{\sin r} \text{(Snell's law)},$$

or
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}.$$

If the surrounding medium has refractive index $= n_s$,

then
$$\frac{n_p}{n_s} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}.$$

SOLVED EXAMPLES

EXAMPLE 51

A ray of light incident at 49° on the face of an equilateral prism passes symmetrically. Calculate the refractive index of the material of the prism.

SOLUTION

As the prism is an equilateral one, $A = 60°$. As the ray of light passes symmetrically, the prism is in the position of minimum deviation.

So, $r = \frac{A}{2}$

$$= \frac{60°}{2} = 30°$$

also, $i = 49°$

$\therefore$ $\mu = \frac{\sin i}{\sin r}$

$$= \frac{\sin 49°}{\sin 30°}$$

$$= \frac{0.7547}{0.5} = 1.5$$

EXAMPLE 52

The refracting angle of the prism is 60° and the refractive index of the material of the prism is 1.632. Calculate the angle of minimum deviation.

SOLUTION

Here, $A = 60°$,

$$\mu = 1.632$$

Now, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

or, $$1.632 = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\frac{60°}{2}}$$

$$= \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin 30°}$$

or, $$= \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$= 1.632 \times \sin 30°$$

$$= 1.632 \times 0.5$$

or, $$\sin\left(\frac{60° + \delta_m}{2}\right) = 0.816,$$

or $$\frac{60° + \delta_m}{2} = 54°42'$$

$$\delta_m = 49°\ 27'.$$

Condition for Prism

(a) Relation between prism angle A and critical angle C is such that the ray will always show TIR at BC:

For this, $$(r_2)_{min} > C \quad (1)$$

For $(r_2)_{min}$, r_1 should be maximum and for $(r_1)_{max}$

$$\Rightarrow \quad i_{max} = 90°.$$

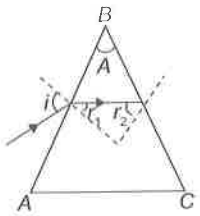


Figure 1.75

Now from Eq. (1),

$$A - C > C,$$

$$A > 2C,$$

i.e., $$A > 2C,$$

i.e., all rays are reflected back from the second surface.

(b) The relation between A and C is such that the ray will always cross the surface BC.

For this,

$$(r_2)_{max} < C$$

$$(A - r_1)_{max} < C$$

$$A - (r_1)_{min} < C \quad (2)$$

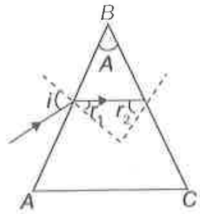


Figure 1.76

$$(r_1)_{min} = 0$$

when $$i_{min} = 0,$$

from Eq. (2),

$$A - 0 < C$$

$$A < C.$$

i.e., if $A \leq C$, no rays are reflected back from the second surface, i.e., all the rays are refracted from the second surface.

(c) If $2C \geq A > C$, some rays are reflected back from the second surface and some rays are refracted from the second surface, depending on the angle of incidence. δ is maximum for two values of i

$\Rightarrow$ i_{min} (corresponding to $e = 90°$) and $i = 90°$ (corresponding to e_{min}).

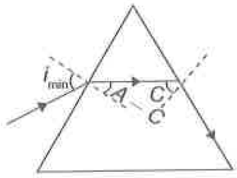


Figure 1.77

For $$i_{min},\ n_s \sin i_{min} = n_p \sin(A - C).$$

If $i < i_{min}$, then TIR takes place at the second refracting surface PR.

Condition for δ_{max}

$$i = 90^\circ$$

or

$$e = 90^\circ$$

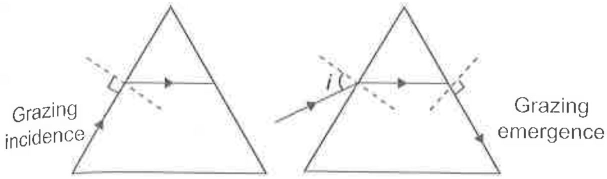


Figure 1.78

$$n\sin(A - C) = \sin e$$

$$e = \sin^{-1}[n\sin(A-C)]$$

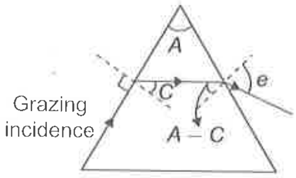


Figure 1.79

$$\delta_{max} = i + e - A$$

$$\delta_{max} = 90 + \sin^{-1}[n\sin(A-C)] - A,$$

SOLVED EXAMPLES

EXAMPLE 53

Find the minimum and maximum angles of deviation for a prism with angle $A = 60^\circ$ and $\mu = 1.5$.

SOLUTION

Minimum deviation

The angle of minimum deviation occurs when $i = e$ and $r_1 = r_2$ and is given by

$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \qquad \delta_m = 2\sin^{-1}\left(\mu\sin\frac{A}{2}\right) - A.$$

Substituting $\mu = 1.5$ and $A = 60^\circ$, we get

$$\delta_m = 2\sin^{-1}(0.75) - 60^\circ = 37^\circ.$$

Maximum deviation (Grazing incidence or Grazing emergence)

The deviation is maximum when $i = 90^\circ$ or $e = 90^\circ$, that is, at grazing incidence or grazing emergence.

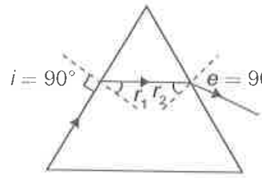
Let $\qquad i = 90^\circ$

$$\Rightarrow \qquad r_1 = C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \qquad r_1 = \sin^{-1}\left(\frac{2}{3}\right) = 42^\circ$$

$$\Rightarrow \qquad r_2 = A - r_1 = 60^\circ - 42^\circ = 18^\circ.$$

Using $\dfrac{\sin r_2}{\sin e} = \dfrac{1}{\mu}$, we have



$$\sin e = \mu\sin r_2 = 1.5 \times \sin 18^\circ$$

$$\Rightarrow \qquad \sin e = 0.463$$

$$\Rightarrow \qquad e = 28^\circ.$$

$$\therefore \qquad \text{Deviation} = \delta_{max} = (i + e) - A = 90^\circ + 28^\circ - 60^\circ = 58^\circ.$$

■

Deviation through a Prism of Small Angle

If the angle of the prism A is small, r_1 and r_2 (as $r_1 + r_2 = A$) and i and e will be small.

For the refraction at the face AB, we have

$$\mu = \frac{\sin i}{\sin r_1}$$

or, $\mu = \frac{i}{r_1}$ (since i and r_1 are small angles, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$).

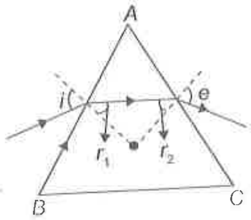


Figure 1.80

$\Rightarrow$ For refraction at the face AC, we have

$$\mu = \frac{\sin e}{\sin r_2}$$

or, $\mu = \frac{e}{r_2}$ ($\because$ e and r_2 are small angles, so $\sin e \simeq e$ and $\sin r_2 \simeq r_2$)

$$\Rightarrow \qquad e = \mu r_2.$$

Now, deviation produced by a prism

$$\delta = (i + e) - A$$

or, $$\delta = (\mu r_1 + \mu r_2) - A$$

or, $$\delta = \mu(r_1 + r_2) - A$$

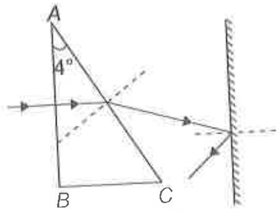
or, $$\delta = \mu A - A \; [\because r_1 + r_2 = A]$$

or, $$\delta = (\mu - 1)A$$

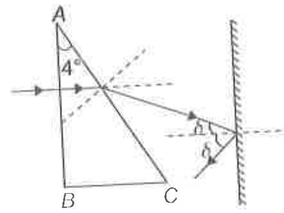
The above formula is valid for all positions of the prism provided the angle of the prism A is small (say $\leq 10°$).

EXAMPLE 54

A prism having a refracting angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown. A horizontal ray of light is incident on the prism. What is the angle of incidence at the mirror?



SOLUTION



The deviation suffered by refraction through the small-angled prism is given by

$$\delta = (\mu - 1)A$$

$$= (1.5 - 1) \times 4° = 2°.$$

This gives the angle of incidence 2° at the mirror.

EXAMPLE 55

Refracting angle of a prism $A = 60°$ and its refractive index is $n = 3/2$. What is the angle of incidence i to get minimum deviation. Also find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).

SOLUTION

For minimum deviation,

$$r_1 = r_2$$

$$= \frac{A}{2}$$

$$= 30°.$$

Applying Snell's law at I surface

$$1 \times \sin i = \frac{3}{2} \sin 30°$$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}.$$

EXAMPLE 56

For a prism, $A = 60°$, $n = \sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

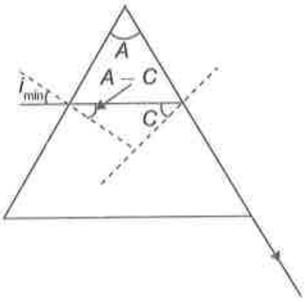
SOLUTION

In minimum incidence case, the angles will be as shown in the figure

Applying Snell's law,

$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin(A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$



$$= \sqrt{\frac{7}{3}}\left(\sin 60\sqrt{1 - \frac{3}{7}} - \cos 60\sqrt{\frac{3}{7}}\right)$$

$$= \frac{1}{2}$$

$$\therefore \quad i_{min} = 30°$$

$$\therefore \quad \delta_{max} = i_{min} + 90° - A$$

$$= 30° + 90° - 60° = 60°.$$

DISPERSION OF LIGHT

The angular splitting of a ray of white light into a number of components spreading in different directions is called Dispersion of Light. (It is for whole electromagnetic waves in totality.) This phenomenon takes place because waves of different wavelength move with the same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula

$$n(\lambda) = a + \frac{b}{\lambda^2},$$

where a and b are positive constants of a medium.

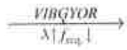


Figure 1.81

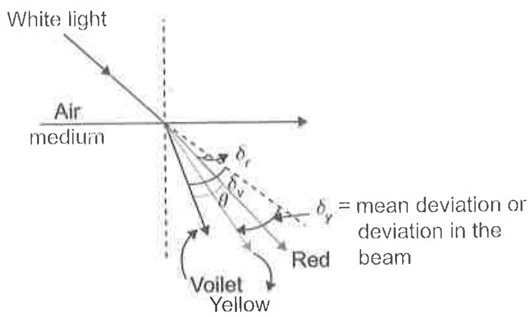
Notes

- Such phenomenon is not exhibited by sound waves.

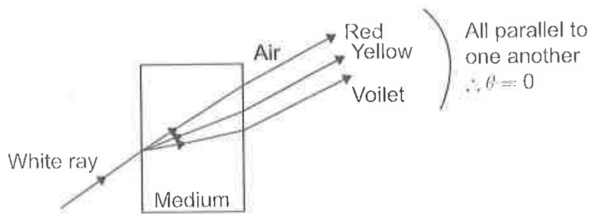
Angle between the rays of the extreme colour in the refracted (dispersed) light is called angle of dispersion.

$$\theta = \delta_v - \delta_r \qquad \text{(Fig. (a))}$$

Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



(a)



(b)

Notes (Cont'd)

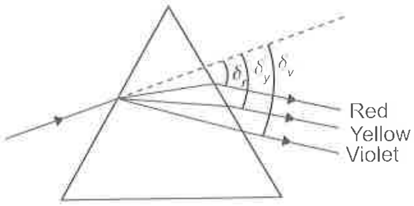


Figure 1.82

For prism of small A and with small i,

$\theta = \delta_v - \delta_r = (n_v - n_r)A.$

EXAMPLE 57

The refractive indices of flint glass for red and violet lights are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

SOLUTION

Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and the deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r$

$$= (\mu_v - \mu_r)A$$

$$= (1.632 - 1.613) \times 5°$$

$$= 0.095°.$$

Notes

Deviation of beam (also called mean deviation)

$$\delta = \delta_y = (n_y - 1)A,$$

n_v, n_r and n_y are R.I. of material for violet, red and yellow colours, respectively.

Numerical data reveal that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

Dispersive Power (ω)

Dispersive power of the medium of the material of prism is given by

$$\omega = \frac{n_v - n_r}{n_y - 1}.$$

- ω is the property of a medium.

For small-angled prism ($A \leq 10°$) with light incident at small angle i,

$$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y}$$

$$= \frac{\theta}{\delta_y}$$

$$= \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

[$n_y = \dfrac{n_v + n_r}{2}$, if n_y is not given in the problem].

- $N - 1$ = refractivity of the medium for the corresponding colour.

EXAMPLE 58

Refractive indexes of glass for red and violet colours are 1.50 and 1.60, respectively. Find

(a) the refractive index for yellow colour, approximately

(b) dispersive power of the medium.

SOLUTION

(a) $$\mu_Y \simeq \frac{\mu_V + \mu_R}{2} = \frac{1.50 + 1.60}{2} = 1.55$$

(b) $$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1} = \frac{1.60 - 1.50}{1.55 - 1} = 0.18.$$

EXAMPLE 59

Calculate the dispersive power of crown and flint glass prism from the following data. For crown glass,

$$\mu_V = 1.522$$

and $$\mu_R = 1.514.$$

For flint glass,

$$\mu_v = 1.662$$

and $$\mu_R = 1.644.$$

SOLUTION

For crown glass,

$$\mu_v = 1.522$$

and $$\mu_R = 1.514$$

$$\therefore \quad \mu_Y = \frac{\mu_V + \mu_R}{2}$$

$$= \frac{1.522 + 1.514}{2}$$

$$= 1.518.$$

Hence, the dispersive power of crown glass

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$$

$$= \frac{1.522 - 1.514}{(1.518 - 1)}$$

$$= 0.01544,$$

$$\therefore \quad \omega = 0.01544.$$

For flint glass:

$$\mu'_v = 1.662$$

and $$\mu'_R = 1.644$$

$$\therefore \quad \mu' = \frac{\mu'_v + \mu'_R}{2}$$

$$= \frac{1.662 + 1.644}{2}$$

$$= 1.653$$

$$\therefore \quad \omega' = \frac{\mu'_v - \mu'_R}{\mu' - 1}$$

$$= \frac{1.662 - 1.644}{(1.653 - 1)}$$

$$= 0.0276.$$ ■

Dispersion Without Average Deviation and Average Deviation Without Dispersion

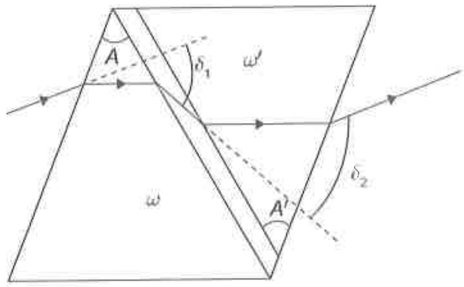


Figure 1.83

Figure 1.83 shows two thin prisms placed in contact in such a way that the two refracting angles are reversed with respect to each other. Suppose the refracting angles of the two prisms are A and A' and their dispersive power are ω and ω', respectively.

Consider a ray of light for which the refractive indices of the materials of the two prisms are μ and μ'. Assuming that the ray passes through the prisms in symmetrical situation, the deviations produced by the two prisms are

$$\delta_1 = (\mu - 1)A$$

and $$\delta_2 = (\mu' - 1)A'.$$

As the two deviations are opposite to each other, the net deviation is

$$\delta = \delta_1 - \delta_2$$

$$= (\mu - 1)A - (\mu' - 1)A'. \quad (1)$$

If white light passes through the combination, the net deviation of the violet ray is

$$\delta_v = (\mu_v - 1)A - (\mu'_v - 1)A'$$

and that of the red ray is

$$\delta_r = (\mu_r - 1)A - (\mu'_r - 1)A'.$$

The angular dispersion produced by the combination is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A' \quad (2)$$

The dispersive powers are given by

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

and $$\omega' = \frac{\mu'_v - \mu'_r}{\mu'_y - 1}.$$

Thus, by Eq. (2), the net angular dispersion is

$$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A'. \qquad (3)$$

The net deviation of the yellow ray, i.e., the average deviation, is, by Eq. (1),

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' \qquad (4)$$

Dispersion Without Average Deviation

If the combination is not to produce a net average deviation in the beam, δ_y should by 0. By Eq. (4), the required condition is

$$(\mu_y - 1)A = (\mu'_y - 1)A' \qquad (5)$$

Using this in Eq. (3), the net angular dispersion produced is

$$\delta_v - \delta_r = (\mu_y - 1)A(\omega - \omega') \qquad (6)$$

By choosing ω and ω' to be different and the refracting angles to satisfy Eq. (5), one can get dispersion without average deviation.

Average Deviation Without Dispersion

If the combination is not to produce a net dispersion, $\delta_v - \delta_r = 0$, By Eq. (3)

$$(\mu_y - 1)\omega A = (\mu'_y - 1)\omega' A' \qquad (7)$$

By Eq. (2), this condition may also be written as

$$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'. \qquad (8)$$

The net average deviation produced is, by Eq. (1),

$$\delta = (\mu_y - 1)A - (\mu'_y - 1)A'$$

$$= (\mu_y - 1)A\left[1 - \frac{\mu'_y - 1}{\mu_y - 1}\frac{A'}{A}\right].$$

By Eq. (7),

$$\frac{(\mu'_y - 1)}{(\mu_y - 1)}\frac{A'}{A} = \frac{\omega}{\omega'}$$

so that the net average deviation produced by the combination is

$$\delta = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right). \qquad (9)$$

SOLVED EXAMPLES

EXAMPLE 60

Find the angle of the flint glass prism which should be combined with a crown glass prism of 5° so as to give dispersion but no deviation.

For crown glass: $\mu_v = 1.523$; $\mu_R = 1.515$.

For flint glass: $\mu_{v'} = 1.688$; $\mu_{R'} = 1.650$.

SOLUTION

For no deviation

$$\frac{A'}{A} = \left(\frac{\mu - 1}{\mu' - 1}\right)$$

or, $$A' = \left(\frac{\mu - 1}{\mu' - 1}\right)A$$

Now,

$$\mu = \frac{\mu_v + \mu_R}{2}$$

$$= \frac{1.523 + 1.515}{2}$$

$$= 1.519$$

$$\mu' = \frac{\mu'_v + \mu'_R}{2}$$

$$= \frac{1.668 + 1.650}{2}$$

$$= 1.659$$

$$\therefore \quad A' = \left(\frac{1.519 - 1}{1.659 - 1}\right)5^\circ$$

$$= 3.94^\circ.$$

EXAMPLE 61

Find the angle of a prism of dispersive power 0.021 and refractive index 1.53 to form an achromatic combination with the prism of angle 4.2° and dispersive power 0.045 having refractive index 1.65. Also calculate the resultant deviation.

SOLUTION

$$\omega = 0.021;$$

$$\mu = 1.53\ ;$$

$$\omega' = 0.045;$$

$$\mu' = 1.65$$

$$A' = 4.2°.$$

For no dispersion,

$$\omega\delta + \omega'\delta' = 0$$

or,
$$\omega(\mu - 1)A + \omega'(\mu' - 1)A' = 0$$

or,
$$A = \frac{\omega' A'(\mu' - 1)}{\omega(\mu - 1)}$$

$$= \frac{0.045 \times 4.2° \times (1.65 - 1)}{0.021 \times (1.65 - 1)}.$$

Net deviation

$$\delta + \delta' = (\mu - 1)A + (\mu' - 1)A'$$

$$= -11.04°(1.53 - 1) + 4.2°(1.65 - 1)$$

$$= -3.12°.$$

REFRACTION FROM A SPHERICAL SURFACE

Consider two transparent media having indices of refraction μ_1 and μ_2, where the boundary between the two media is a spherical surface of radius R. We assume that $\mu_1 < \mu_2$. Let us consider a single ray leaving point O and focussing at point I. Snell's law applied to this refracted ray gives,

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

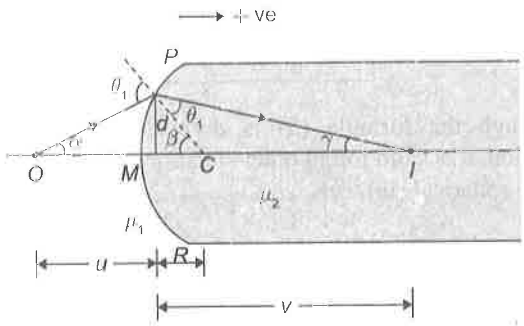


Figure 1.84

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation

$$\sin\theta \approx \theta$$

(angles in radians) and say that

$$\mu_1\theta_1 = \mu_2\theta_2. \quad (1)$$

From the geometry shown in the figure,

$$\theta_1 = \alpha + \beta \quad (2)$$

and
$$\beta = \theta_2 + \gamma. \quad (3)$$

Equations (1) and (3) can be combined to express θ_2 in terms of α and β. Substituting the resulting expression into Eq. (2) then yields,

$$\beta = \frac{\mu_1}{\mu_2}(\alpha + \beta) + \gamma\ .$$

So
$$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta\ . \quad (4)$$

Since, the arc PM (of length S) subtends an angle β at the centre of curvature

$$\beta = \frac{S}{R}.$$

Also in the paraxial approximation,

$$\alpha = \frac{S}{u}$$

and
$$\gamma = \frac{S}{v}.$$

Using these expressions in Eq. (4) with proper signs, we are left with

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or
$$\boxed{\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}}. \qquad (5)$$

Although the formula (5) is derived for a particular situation, it is valid for all other situations of refraction at a single spherical surface.

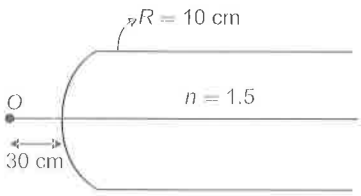
Important Point for Above Formula

- The above formula is valid only for paraxial ray.
- u, v and R should be put along with the sign.
- μ_2 is the refractive index of the medium through which the rays are going and μ_1 is the refractive index of the medium from which the rays are coming.

SOLVED EXAMPLES

EXAMPLE 62

Find out the position of the image formed and draw the appropriate ray diagram.



$$u = -30 \text{ cm}$$
$$R = +10 \text{ cm}$$

SOLUTION

$$\Rightarrow \quad \frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

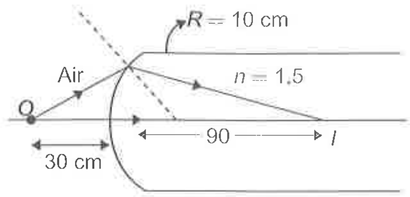
$$\Rightarrow \quad \frac{1.5}{v}+\frac{1}{30}=\frac{0.5}{10}$$

$$\Rightarrow \quad \frac{1.5}{v}=\frac{0.5}{10}-\frac{1}{30}$$

$$\frac{1.5}{v}=\frac{0.5}{30}$$

$$\Rightarrow \quad v=\frac{30\times 1.5}{0.5}$$

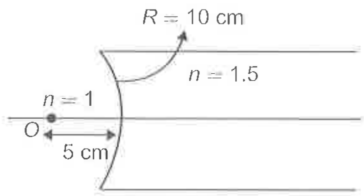
$$= 90 \text{ cm (Real)}$$



■

EXAMPLE 63

Find out the position of the image formed and draw the appropriate ray diagram.



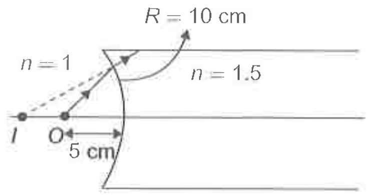
SOLUTION

$$n_2 = 1.5,$$
$$n_1 = 1$$
$$u = -5 \text{ cm}$$
$$R = -10 \text{ cm}$$

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{v}+\frac{1}{5}=-\frac{0.5}{10}$$

$$\Rightarrow \quad \frac{1.5}{v}=\frac{-0.5}{10}-\frac{1}{5}$$



$$\Rightarrow \quad \frac{1.5}{v}=\frac{-2.5}{10}$$

$$\Rightarrow \quad v=-6 \text{ cm}.$$

■

$$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{1.5}\frac{v^2}{u^2} \times 2$$

$$= +12 \text{ m/s.}$$

■

Transverse Magnification

If i and r are very small,

$$\tan i \approx \sin i \approx i$$

$$\tan r \approx \sin r \approx r$$

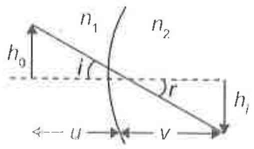


Figure 1.86

$$\Rightarrow \quad \tan r = \frac{h_i}{v}$$

$$\Rightarrow \quad r \approx \frac{h_i}{v} \qquad (1)$$

$$\tan i = \frac{h_0}{u}$$

$$\Rightarrow \quad i \approx \frac{h_0}{u}. \qquad (2)$$

Again, by applying Snell's law,

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow \quad n_1 i \approx n_2 r. \qquad (3)$$

$\Rightarrow$ From Eqs. (1), (2) and (3),

$$m = \frac{h_i}{h_0}$$

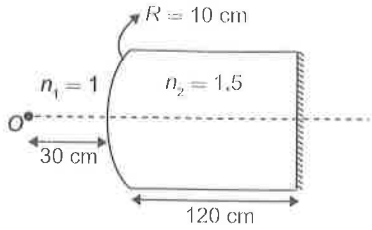
$$= \frac{n_1}{n_2}\left(\frac{v}{u}\right).$$

EXAMPLE 66

Find out the position of the image formed and draw the appropriate ray diagram.

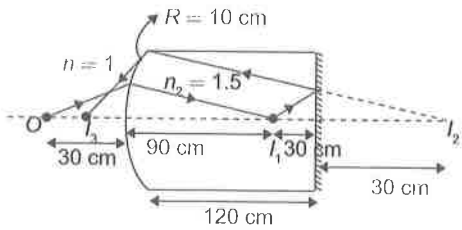
$$u = -30 \text{ cm}$$

$$R = +10 \text{ cm}$$



SOLUTION

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$



$$\Rightarrow \quad \frac{1.5}{v} + \frac{1}{30} = \frac{0.5}{10}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{10} - \frac{1}{30}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{30}$$

$$\Rightarrow \quad v = +90.$$

Mirror will form the image of I_1 30 cm behind it as shown in the figure.

For the second refraction:

$$u = -150 \text{ cm},$$

$$R = -10 \text{ cm},$$

$$n_1 = 1.5,$$

$$n_2 = 1$$

$$\frac{1}{v} + \frac{1.5}{150} = \frac{-0.5}{-10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{0.5}{10} - \frac{1}{100}$$

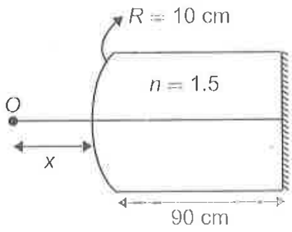
$$\Rightarrow \quad \frac{1}{v}=\frac{4}{100}$$

$$\Rightarrow \quad v=25 \text{ cm (Real).}$$

■

EXAMPLE 67

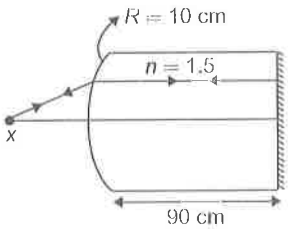
Find out the value of x for which the image is formed on the object itself.



SOLUTION

Case I: for first refraction

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$



$$\Rightarrow \quad \frac{1.5}{v}+\frac{1}{x}=\frac{0.5}{10}$$

$$\Rightarrow \quad \frac{1.5}{v}=\frac{0.5}{10}-\frac{1}{x}$$

$$\Rightarrow \quad \frac{1.5}{v}=\frac{0.5x-10}{10x}$$

$$\Rightarrow \quad v=\frac{15x}{0.5x-10}$$

$$v=\infty$$

$$\Rightarrow \quad x=20 \text{ cm}$$

Case II:

$$n_2=1.5,$$

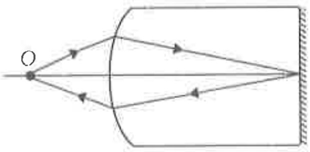
$$n_1=1$$

$$R=+10,$$

$$u=-x$$

$$v=90 \text{ cm},$$

$$x=30 \text{ cm}$$



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REFRACTION THROUGH THIN LENSES

Lens

A lens is a transparent medium bounded by two refracting surfaces such that at least one of the refracting surfaces is curved (or spherical).

Types of Lenses

Broadly, lenses are of the following types:

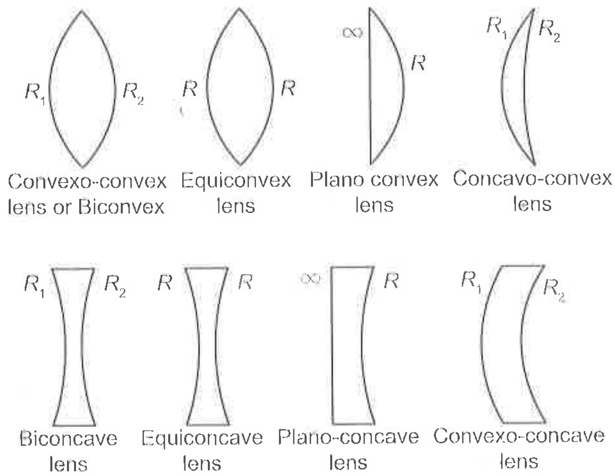


Figure 1.87

Principal Axis

The line joining the centres of curvature of the two bounding surfaces is called the principal axis.

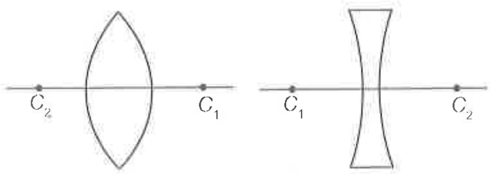


Figure 1.88

Lens Maker Formula

For first refraction:

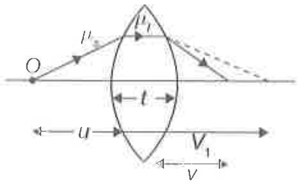


Figure 1.89

$$\frac{\mu_\ell}{v_1} - \frac{\mu_s}{u} = \frac{(\mu_\ell - \mu_s)}{R_1} \qquad (1)$$

thickness of the lens t is negligible.

For second refraction:

$$\frac{\mu_s}{v} - \frac{\mu_\ell}{v_1} = \frac{\mu_s - \mu_\ell}{R_2} \qquad (2)$$

Adding Eqs. (1) and (2), we get

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

Important Points for the Above Formula

1. Rays should be paraxial.
2. v, u, R_1 and R_2 should be put with the sign.
3. R_1 is the radius of curvature of that surface on which the ray strikes first.
4. Lens should be thin.
5. Medium on both sides of the lens should be the same.

Sign Convention (consider pole as origin)

(i) Whenever and wherever possible, rays of light are taken to travel from left to right.

(ii) Distances are measured along the principal axis from the optical centre of the lens.

(iii) Distances measured along the principal axis in the direction of the incident rays are taken as positive while those measured against the direction of the incident rays are taken as negative.

(iv) Distances measured above the principal axis are taken as positive and those measured below the principal axis are taken as negative.

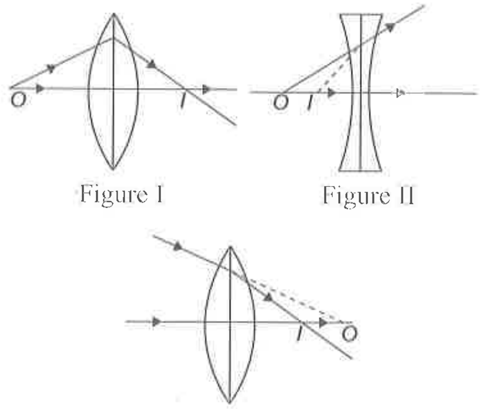


Figure 1.90

Figure	u	v	t	R_1	R_2
(i)	−ve	+ve	+ve	+ve	−ve
(ii)	−ve	−ve	−ve	−ve	+ve
(iii)	+ve	+ve	+ve	+ve	−ve

Focus

If the rays are parallel to optical axis and paraxial, then the point where they meet or appear to meet is known as focus of the system.

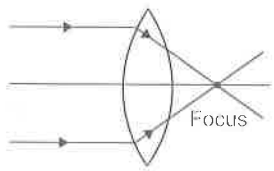


Figure 1.91

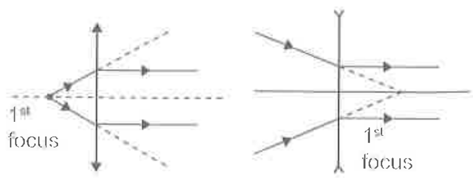
In the lens maker formula if $u \rightarrow \infty$, $v = f$,

$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Substituting in the lens maker formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ (lens formula).}$$

Lenses have two focii called first and second focus.



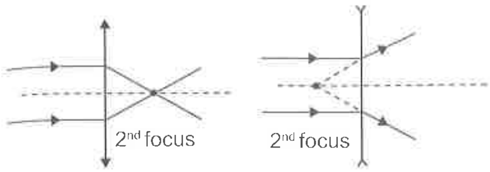


Figure 1.92

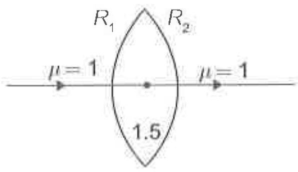
SOLVED EXAMPLES

EXAMPLE 68

Calculate the focal length of a biconvex lens in air if the radii of its surfaces are 60 cm and 15 cm. Refractive index of glass = 1.5.

SOLUTION

Consider a light ray going through the lens as shown. It strikes the convex side of 60 cm radius and concave side of 15 cm radius while coming out.



$$R_1 = +60 \text{ cm},$$

$$R_2 = -15 \text{ cm}$$

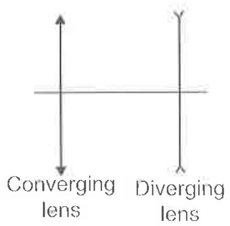
$$\therefore \quad \frac{1}{f} = \left[\frac{\mu_\ell}{\mu_s} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or,

$$\frac{1}{f} = \left[\frac{1.5}{1} - 1\right]\left[\frac{1}{60} + \frac{1}{15}\right]$$

$$\Rightarrow \quad f = +24 \text{ cm}$$

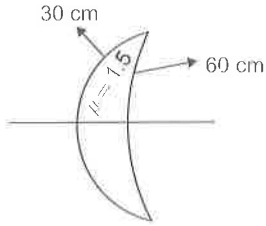
Notes



- For converging and diverging lenses, focal length of lens depends on surrounding medium.
- f+ve implies converging lens and f−ve implies diverging lens.

EXAMPLE 69

Calculate the focal length of the lens shown in the figure.

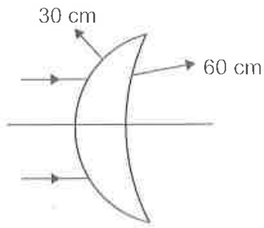


SOLUTION

$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5 - 1)\left(+\frac{1}{30} - \left(+\frac{1}{60}\right)\right)$$

$$\Rightarrow \quad f = 120 \text{ cm}$$



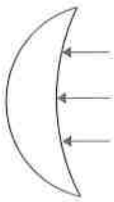
If in the above case, direction of rays is reversed,

$$\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5 - 1)\left(-\frac{1}{60} - \left(-\frac{1}{30}\right)\right)$$

$$f = +120 \text{ cm}.$$

This illustration shows that focal length does not depend on the incident ray direction.



Rules for image formation

(i) A ray passing through the optical centre of the lens proceeds undeviated through the lens (by definition of optical centre).

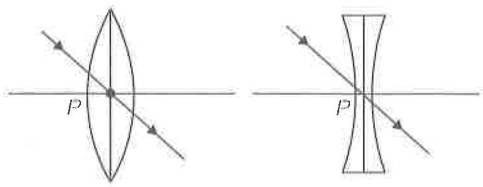


Figure 1.93

Pole is the intersection of the ray which goes undeviated through the lens and the optical axis.

(ii) A ray passing parallel to the principal axis after refraction through the lens passes or appears to pass through the focus (by the definition of the focus).

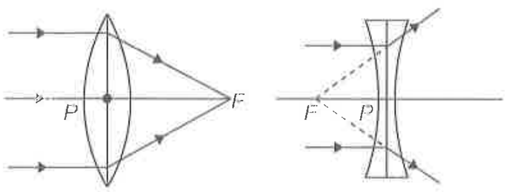


Figure 1.94

(iii) A ray through the focus or directed towards the focus, after refraction from the lens, becomes parallel to the principal axis (principle of reversibility of light).

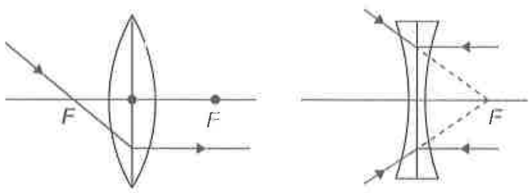


Figure 1.95

Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect is the image of the object. If they actually intersect each other, the image is real and if they appear to intersect, the image is said to be virtual.

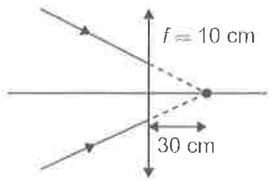
EXAMPLE 70

Find out the position of the image formed.

$$u = +30 \text{ cm}, f = +10 \text{ cm}$$

SOLUTION

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$



$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} - \frac{1}{30}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} + \frac{1}{30}$$

$$\Rightarrow \quad v = \frac{300}{40}$$

$$= 7.5 \text{ cm.}$$

Transverse Magnification Converging lens

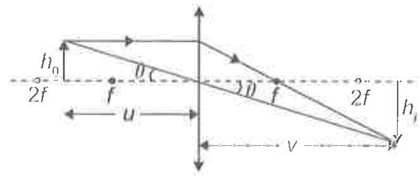


Figure 1.96

$$\tan\theta = \frac{h_0}{u} \quad (1)$$

$$\tan\theta = \frac{h_i}{v} \quad (2)$$

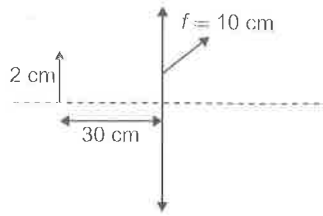
Dividing Eq. (2) by Eq. (1),

$$m = \frac{h_i}{h_0}$$

$$= \frac{v}{u}.$$

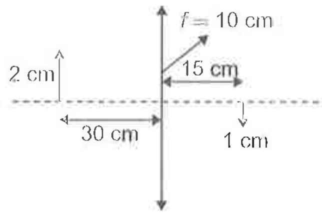
EXAMPLE 71

Find out the position, height and nature of the image formed.



SOLUTION

$$u = -30 \text{ cm},$$
$$f = +10 \text{ cm},$$
$$h_0 = +2 \text{ cm}$$



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{30}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{15}$$

$$\Rightarrow \quad v = +15$$

$$\frac{h_i}{h_0} = \frac{v}{u}$$

$$\Rightarrow \quad \frac{h_i}{2} = \frac{15}{-30}$$

$$\Rightarrow \quad h_i = -1 \text{ cm}.$$

$\therefore$ Real, inverted, diminished. ■

IMAGE FORMATION BY A CONVEX LENS OF THE LINEAR OBJECT

(i) *When the Object is at Infinity*

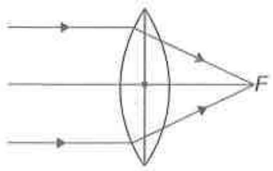


Figure 1.97

The image is formed at F. It is real, inverted and highly diminished.

(ii) *When the Object is Beyond 2F*

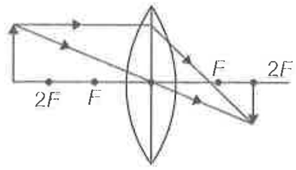


Figure 1.98

The image is formed between F and $2F$. It is real, inverted and diminished

(iii) *When the Object is at 2F*

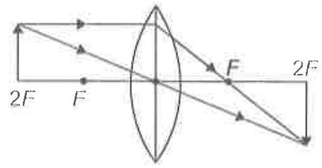


Figure 1.99

The image is formed at $2F$. It is real, inverted and the same size as the object.

(iv) *When the Object is Between F and 2F*

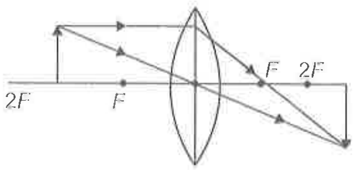


Figure 1.100

The image is formed beyond $2F$ (i.e., between $2F$ and ∞). It is real, inverted and enlarged.

(v) *When the Object is at F*

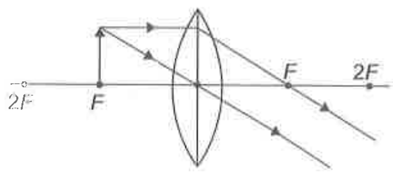


Figure 1.101

The image is formed at infinity. It is real, inverted and highly magnified.

(vi) When the Object is Between F and O

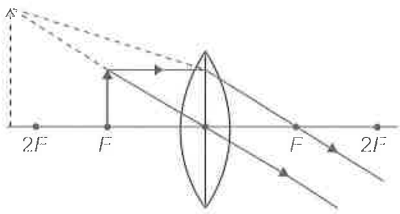


Figure 1.102

The image is on the same side as the object is. It is virtual, erect and magnified.

(vii) Virtual Object Case for Converging Lens

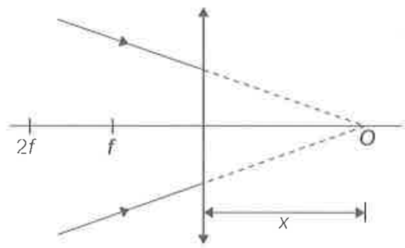


Figure 1.103

$$u = +x$$

$$f = +f$$

From lens formula,

$$\frac{1}{v} - \frac{1}{x} = \frac{1}{f}.$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{x}$$

If $x = 0,\ v \to \infty.$

If $x = \infty,\ v \to f.$

Graphs for Converging Lens

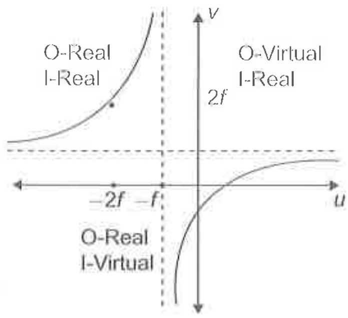


Figure 1.104

Image Formation by a Concave Lens of a Linear Object

(a) Real Object Case

$$u = -x, f = -f$$

From lens formula,

$$\frac{1}{v} + \frac{1}{x} = -\frac{1}{f}.$$

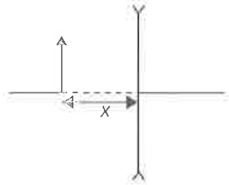


Figure 1.105

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{x}$$

If $x \to \infty$

$v \to -f.$

If $x \to 0$

$v \to 0'.$

(b) Virtual Object Case

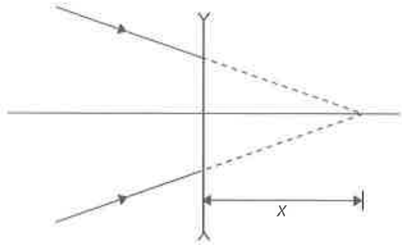


Figure 1.106

$$u = +x, f = -f$$

From lens formula,

$$\frac{1}{v} = \frac{1}{x} - \frac{1}{f}.$$

If $x = 0 \to v = 0$

If $x < f,\ v = +u.$

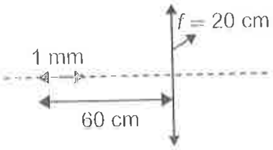
$$A'B' = (v_B - v_A)$$

$$= \frac{140}{3} - 40$$

$$= \frac{20}{3} \text{ cm.}$$

EXAMPLE 74

Find out the linear length of the image of the object as shown in figure.



SOLUTION

Here the length of the object is very small, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

By differentiating

$$\frac{dv}{du} = \frac{v^2}{u^2}$$

$$u = -60,$$

$$f = +20$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{60}$$

$$\Rightarrow \quad v = 30$$

$$\frac{dv}{du} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad \frac{dv}{1} = \frac{(30)^2}{(60)^2}$$

$$\Rightarrow \quad dv = \frac{1}{4} mm$$

= length of the image.

Note

Differentiating in solving examples 73 and 74 (Trick).

Velocity of the Image Formed by a Lens

From $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

differentiate the above equation

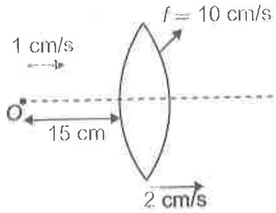
$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$$

$$v_{IL} = \frac{v^2}{u^2} V_{OL}.$$

EXAMPLE 75

Find out the velocity of the image of the object shown in the figure.



SOLUTION

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

$$\Rightarrow \quad v = \frac{150}{5} = 30$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

By differentiating :

$$0 = \frac{dv}{v^2 dt} - \frac{du}{dt\, u^2}$$

$$\Rightarrow \quad 0 = -\frac{dv}{v^2} + \frac{du}{u^2}$$

$$\Rightarrow \quad \frac{du}{u^2} = \frac{dv}{v^2}$$

$$\Rightarrow \quad \frac{dv}{du} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad dv = \frac{(30)^2}{(15)^2} \times (1-2)$$

$$= -4(1)$$

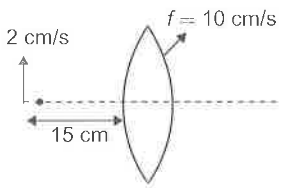
$$\Rightarrow \quad (v_I - v_L) = \frac{v^2}{u^2}(v_o - v_L)$$

$$\Rightarrow \quad (v_I - 2) = 4 \times -1$$

$$\Rightarrow \quad v_I = -2.$$

EXAMPLE 76

Find out the velocity of the image of the object shown in the figure.



SOLUTION

$$\frac{h_i}{h_0} = \frac{v}{u}$$

$$\Rightarrow \quad \frac{h_i}{h_0} = \frac{v}{u}$$

$$\Rightarrow \quad \frac{dh_i}{dt} = \frac{v}{u}\frac{dh_0}{dt}$$

$$\Rightarrow \quad \frac{30}{-15} \times 2 = -4 \text{ cm/sec(downwards)}$$

Combination of Lens

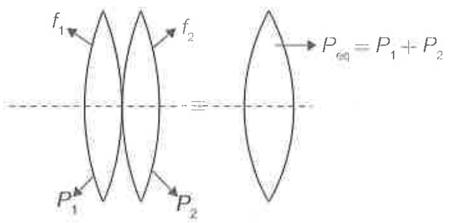


Figure 1.108

$$P_{eq} = P_1 + P_2$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

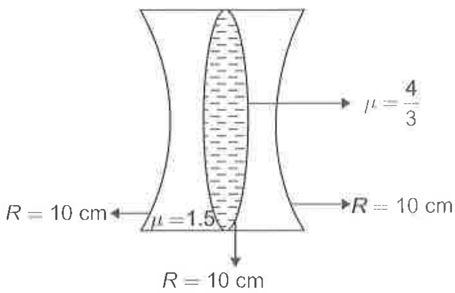
Important Points

1. Rays should be paraxial.
2. Lens should be thin.
3. Lenses should be kept in contact.
4. $f_1, f_2, f_3, \ldots$ should be put with sign.
5. $f_1, f_2, f_3, \ldots$ are the focal lengths of the lenses in the surrounding medium.
6. If f_{eq} is positive, then the system will behave as a converging system.

If f_{eq} is negative, then the system will behave as a diverging system.

EXAMPLE 77

Find out the equivalent focal length of the combination of lenses shown in the figure. Surrounding medium is air.



SOLUTION

$$\frac{1}{f_1} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(-\frac{1}{10} - \frac{1}{10}\right)$$

$$= 0.5 \times \frac{-2}{10}$$

$$\Rightarrow \quad f_1 = \frac{10}{0.5 \times -2}$$

$$= -10 \text{ cm}$$

$$\frac{1}{f_2} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{4}{3} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = \frac{1}{3}\left(\frac{2}{10}\right)$$

$$\Rightarrow \quad f_2 = \left(\frac{30}{2}\right)$$

$$= 15\,\text{cm}$$

$$f_3 = -10\text{ cm}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} - \frac{2}{10}$$

$$\Rightarrow \quad f_{eq} = \frac{150}{-20}$$

$$= -7.5\text{ cm}$$

CUTTING OF LENS

Parallel Cutting

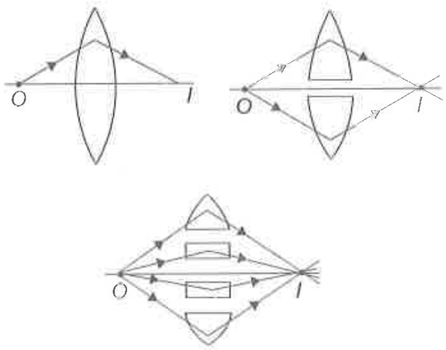


Figure 1.109

No. of images in all the cases = 1

∴ Principal axis does not shift.

(ii)

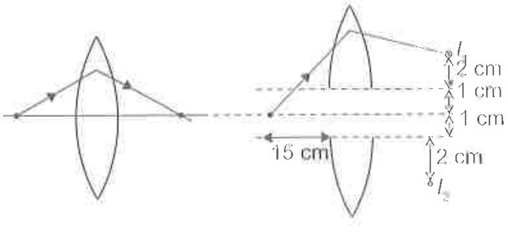


Figure 1.110

$$f = 10\text{ cm}$$

$$\frac{h_i}{h_0} = \frac{v}{u}$$

$$\Rightarrow \quad \frac{h_i}{-1} = \frac{30}{-15}$$

$$\Rightarrow \quad h = +2\text{ (upwards)}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

$$\Rightarrow \quad v = \frac{150}{5}$$

$$= 30\text{ cm.}$$

Total distance = 2 + 2 + 2 = 6 cm

(iii)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{5}$$

$$\Rightarrow \quad v = \frac{50}{5}$$

$$= -10\text{ cm}$$

$$\frac{h_i}{h_0} = \frac{v}{u}$$

$$\Rightarrow \quad \frac{h_i}{-1} = \frac{-10}{-5}$$

$$\Rightarrow \quad h_i = -2\text{ cm}$$

Distance = 1 + 1 = 2 cm

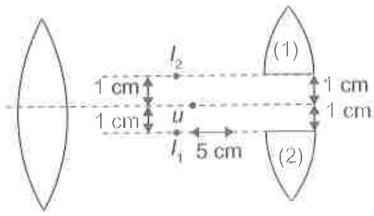


Figure 1.111

Power of a Lens

$$\text{Power} = \frac{1}{f} \text{ (dioptre)},$$

where $f \to$ meter

and f should be put with sign

power of converging lens = +ve

Power of diverging lens = −ve

Power of Mirror

$$\text{Power} = -\frac{1}{f} \text{ (dioptre)},$$

where $f \to$ meter

and f should be put with sign.

Power of converging mirror = +ve.

Power of diverging mirror = −ve.

Perpendicular Cutting

(1)

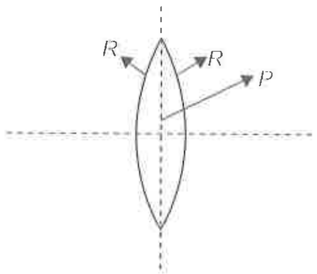


Figure 1.112

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \left(-\frac{1}{R}\right)\right)$$

$$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\frac{2}{R}$$

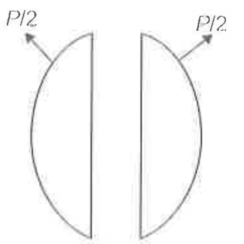


Figure 1.113

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - 0\right)$$

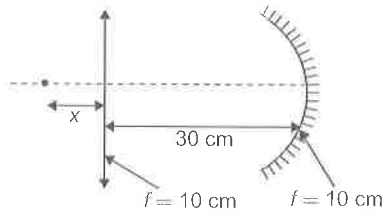
$$= \frac{(\mu - 1)}{R}$$

$$f' = 2f$$

SOLVED EXAMPLE

EXAMPLE 78

Find out the value of x so that the image is formed on the object itself.



SOLUTION

Case I: When the lens forms the image on the pole of the mirror,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{30} - \frac{1}{-x}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{10} - \frac{1}{30}$$

$$\Rightarrow \quad x = 15 \text{ cm}.$$

Case II: When the lens forms the image on the centre of curvature of the mirror,

$$\frac{1}{20} + \frac{1}{x} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{10} - \frac{1}{20}$$

$$\therefore \quad x = 20 \text{ cm}.$$

■

SILVERGING OF LENS

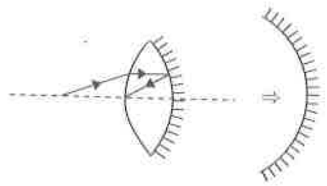


Figure 1.114

$$P_{eq} = 2P\ell + P_m$$

$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{2}{f_\ell} - \frac{1}{f_m}$$

$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_\ell}$$

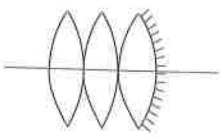


Figure 1.115

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - 2\left[\frac{1}{f_{\ell_1}} + \frac{1}{f_{\ell_2}} + \cdots\right]$$

Important Points

1. Rays should be paraxial.
2. Lenses should be thin.
3. All the lenses should be in contact.
4. $f\ell, f_m$ should be put along with the sign.
5. If $f_{eq} = -ve$

$\Rightarrow$ **concave,**

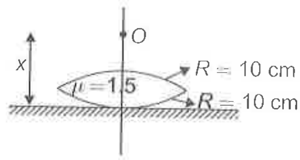
$f_{eq} = +ve \Rightarrow$ convex,

If $f_{eq} = \infty \Rightarrow$ plane mirror.

SOLVED EXAMPLE

EXAMPLE 79

Find out the value of x so that the image will form on object itself.



SOLUTION

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - 2\left\{(0.5)\left(\frac{1}{10} + \frac{1}{10}\right)\right\}$$

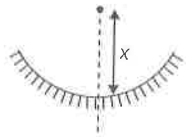
$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{\infty} - \frac{2}{10}$$

$$\Rightarrow \quad f_{eq} = -5\,\text{cm}.$$

The system is equivalent to a concave mirror of focal length 5 cm.

Object must be at the centre of curvature.

$$\therefore \quad x = 2(5) = 10\text{ cm}.$$



DISPLACEMENT METHOD

For the formation of the real image by convex lens, minimum distance between the object and the image is $4f$, f being the focal length of the lens.

If the distance between the object and screen (D) is greater than $4f$, then there are two positions of the lens for which the image of the object on the screen is distinct and clear. In these two positions, the distance of the object and image from the lens are interchange.

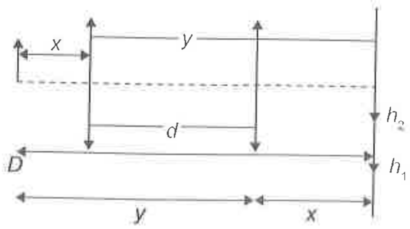


Figure 1.116

For First Refraction

$$u = -x$$

$$h_o = h_0$$
$$v = +y$$
$$h_i = -h_i$$
$$\frac{-h_1}{h_0} = \frac{-y}{x} \qquad (1)$$

For Second Refraction

$$u = -y,$$
$$v = +x$$
$$h_i = -h_2,$$
$$h_0 = h_0$$
$$\frac{-h_2}{h_0} = \frac{-x}{y} \qquad (2)$$
$$(1) \times (2)$$
$$h_0^2 = h_1 h_2$$
$$\Rightarrow \quad h_0 = \sqrt{h_1 h_2}.$$

Now $\qquad D = x + y, \qquad (3)$

$$d = y - x. \qquad (4)$$

After solving Eqs. (3) and (4),

$$y = \frac{D+d}{2}$$
$$x = \frac{D-d}{2}.$$

From $\qquad \frac{1}{y} + \frac{1}{x} = \frac{1}{f},$

$$\frac{2}{D+d} + \frac{2}{D-d} = \frac{1}{f}$$
$$\Rightarrow \quad f = \frac{D^2 - d^2}{4D}$$

CHROMATIC ABERRATION AND ACHROMATISM

The refractive index of the material of a lens varies slightly with the wavelength, and hence, the focal length is also different for different wavelengths. In the visible region, the focal length is maximum for red and minimum for violet. Thus, if white light is used, each colour forms a separate image of the object.

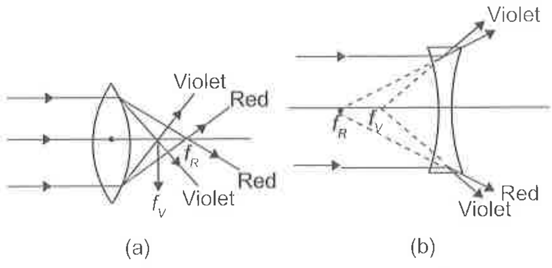


Figure 1.117

The violet rays are deviated more, and hence, they form an image closer to the lens as compared to the image formed by the red rays. If light is incident on the lens from left to right, the violet image is to the left of the red image for convex lens and it is to the right of the red image for the concave lens. In the first case, the chromatic aberration is called positive and in the second case, it is negative. Thus, a proper combination for a convex and a concave lens may result in no chromatic aberration. Such a combination is called an achromatic combination for the pair of wavelengths.

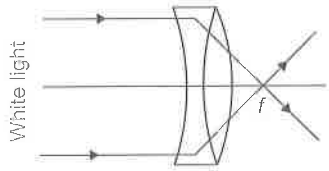


Figure 1.118

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (1)$$
$$\frac{1}{f_R} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (2)$$

Eq. (1) – Eq. (2) yields

$$\frac{1}{f_v} - \frac{1}{f_R} = (\mu_v - \mu_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{f_R - f_v}{f_R \cdot f_v} = \frac{(\mu_v - \mu_R)}{(\mu_v - 1)}(\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$f_R \cdot f_v = f_1^2$$
$$\Rightarrow \text{ and } \quad f_R - f_v = df$$

$$\frac{df}{f_y^2}=\frac{\omega}{f_y}$$

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \qquad -\frac{df_{eq}}{f^2}=-\frac{df_1}{f_1^2}-\frac{df_2}{f_2^2}.$$

For achromatism, $df_{eq}=0$,

$$\Rightarrow \qquad \frac{df_1}{f_1^2}+\frac{df_2}{f_2^2}=0$$

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0.$$

EXTRA PORTION OF JEE MAIN

OPTICAL INSTRUMENTS

Definition

Optical instruments are used primarily to assist the eye in viewing an object.

Types of Instruments

Depending upon the use, optical instruments can be categorized in the following way:

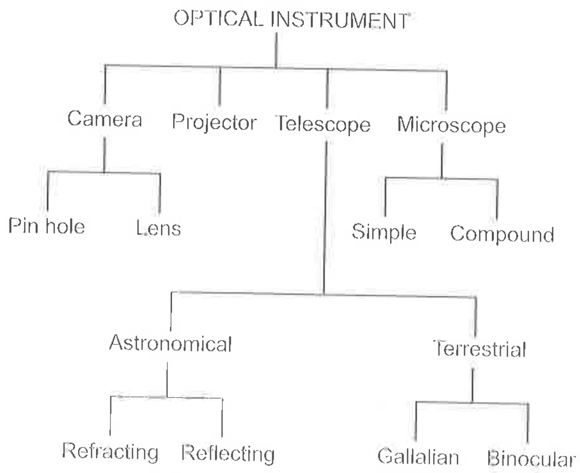


Figure 1.119

MICROSCOPE

It is an optical instrument used to increase the visual angle of near objects which are too small to be seen by the naked eye. Microscopes are of two types, viz., simple microscope and compound microscope.

Simple Microscope

It is also known as magnifying glass or **magnifier** and consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual, enlarged and on the same side of the lens between the object and infinity.

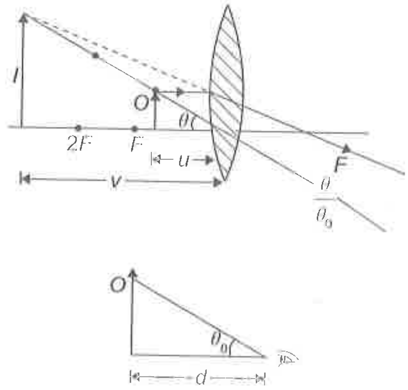


Figure 1.120

Here,

Magnifying power

$$=\frac{\text{visual angle with instrument}}{\text{Maximum visual angle for unaidedeye}}$$

$$=\frac{\theta}{\theta_0}.$$

Now, $$\theta=\frac{h_i}{v}=\frac{h_0}{u}$$

with, $$\theta_0=h_0/D$$

$$\text{M.P.}=\frac{\theta}{\theta_0}$$

$$=\frac{h_0}{u}\times\frac{D}{h_0}$$

$$= \frac{D}{u}.$$

Now, two possibilities are there.

(A) Image is at Infinity (Far Point)

If $v = \infty$, $u = f$ (from lens formula).

So, $$\text{M.P} = \frac{D}{u} = \frac{D}{f}$$

Note

Here, a parallel beam of light enters the eye, i.e., eye is least strained.

(B) Image is at D (Near Point)

In this situation, $v = -D$, so that,

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

or, $$\frac{D}{u} = 1 + \frac{D}{f}$$

So, $$\text{M.P.} = 1 + \frac{D}{f}.$$

Note

Here the final image is closest to the eye, i.e., eye is under maximum strain.

Compound Microscope

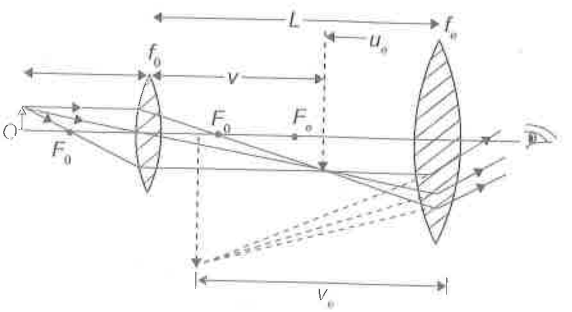


Figure 1.121

It consists of two convergent lenses of short focal lengths and apertures arranged co-axially.

Lens f_0 is the **objective** or **field lens** and f_e is the **eye-piece** or **ocular**. Objective has smaller aperture and focal length than the eye piece. The separation between the objective and the eye-piece can be varied.

$$\text{Magnifying Power} = \frac{\theta}{\theta_0}$$

$$= \frac{h_i}{u_e} \times \frac{D}{h_0}$$

$$= \frac{h_i}{h_0} \times \frac{D}{u_e}.$$

But for objective, $m = \frac{v}{u}$,

i.e., $$\frac{h_i}{h_0} = -\frac{v}{u},$$

So, $$\text{M.P.} = -\frac{v}{u}\left[\frac{D}{u_e}\right],$$

where, $u_e + u = L$.

Now two possibilities are there:

(A) Final Image is at Infinity (Far Point)

$$u_e = f_e$$

$$\Rightarrow \quad \text{M.P} = -\frac{v}{u}\left[\frac{D}{f_e}\right],$$

where $L = u + f_e$.

Note

A microscope is usually considered to operate in this mode unless stated otherwise.

(B) Final Image is at D (Near Point)

For eye-piece, $v_e = D$,

$$\Rightarrow \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{D}\left(1 + \frac{D}{f_e}\right)$$

$$\text{M.P.} = -\frac{v}{u}\left(1 + \frac{D}{f_e}\right)$$

with $\quad L = v + \frac{f_e D}{f_e + D}.$

Notes

In case, $u \cong f_0$

and $L = v + u_e \cong v$,

so that $|M.P| \cong \frac{L}{f_0} \times \frac{D}{f_e}$.

Important Points

1. As **magnifying power** is negative, the image seen in a microscope is always truly inverted, i.e., left is turned right while being upside down simultaneously.
2. **Resolving Power**: The minimum distance between two lines at which they are just distinct is called the limit of resolution and reciprocal of limit of resolution is called resolving power.

$$\text{RP} = \frac{1}{\Delta x} \propto \frac{1}{\lambda}$$

$$= 2\mu \sin/\theta/\lambda.$$

TELESCOPE

It is an optical instrument used to increase the visual angle of distant large objects. Telescopes mainly are of two types, viz., astronomical and terrestrial.

Astronomical Telescope

It consists of two converging lenses placed coaxially with the objective having a large aperture and a large focal length, while the eye piece has a smaller aperture and focal length. The separation between the eye piece and the objective can be varied.

Magnifying power

$$= \frac{\text{visual angle with instrument}}{\text{visual angle for unaided eye}}$$

$$= \frac{\theta}{\theta_0}$$

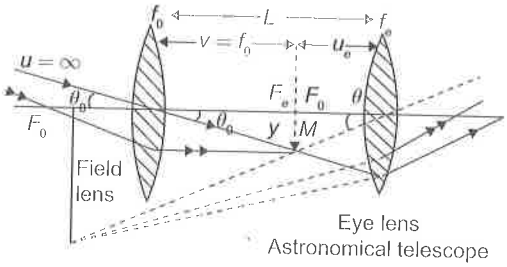


Figure 1.122

$$\theta_0 = \frac{h}{f_0}$$

and $\quad \theta = h/(-u_e)$

$$\Rightarrow \quad \text{MP} = -\left[\frac{f_0}{u_e}\right]$$

with $\quad L = f_0 + u_e.$

Now two possibilities are there.

(A) Final Image is at Infinity (Far Point)

Here, $\quad v = \infty$

$\Rightarrow \quad u_e = f_e.$

So, $\quad \text{M.P.} = -(f_0/f_e)$

with, $\quad L = f_0 + f_e$

Note

Usually, a telescope operates in this mode unless stated otherwise.

(B) *Final Image is at D (Near Point)*

Here, $v = D$

$$\Rightarrow \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$= \frac{1}{u_e}$$

$$= \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

So, $$\text{M.P.} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

with $$L = f_0 + \frac{f_e D}{f_e + D}$$

Notes

1. The above discussion is that of the **refracting telescope**.
2. **Reflecting telescope:** If the field lens of a refracting telescope is replaced by a converging mirror, then the telescope becomes a reflecting one, where MP $= f_0/f_e$

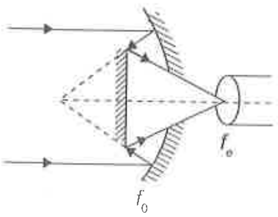


Figure 1.123

Terrestrial Telescope

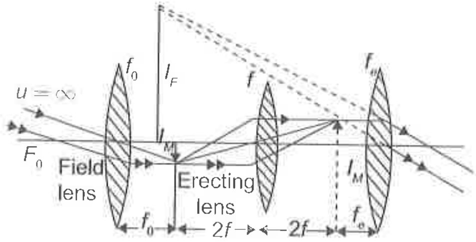


Figure 1.124

If a lens of short focal length f is placed at $2f$ from the intermediate image at a distance $2f$ on the other side of it, this image will act as an object for eye lens, which will produce an erect image with respect to the object. This lens is called an **erecting lens.** As $m = -1$ for an erecting lens, the MP and length of the telescope for a relaxed eye will be

$$\text{MP} = -\frac{f_0}{f_e}(-1)$$

$$= \frac{f_0}{f_e},$$

$$L = f_0 + f_e + 4f.$$

(A) *Galilean Telescope*

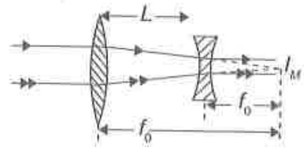


Figure 1.125

Here, the convergent eye piece of an astronomical telescope is replaced by a divergent lens.

Here MP $= f_0/f_e$ with $L = f_0 - f_e$

Notes

In this telescope as the intermediate image is outside the tube, the telescope cannot be used for measurements. This was not the case for all earlier telescopes.

Resolving power of a telescope $= \dfrac{D}{1.22\lambda}$.

EXERCISES

JEE Main

Section (A): Plane Mirror

1. A point source of light is placed in front of a plane mirror.
   (A) Only the reflected rays close to the normal meet at a point when produced backward.
   (B) All the reflected rays meet at a point when produced backward.
   (C) Only the reflected rays making a small angle with the mirror, meet at a point when produced backward.
   (D) Light of different colours make different images.

2. A point object is kept in front of a plane mirror. The plane mirror is doing SHM of amplitude 2 cm. The plane mirror moves along the x-axis and x-axis is normal to the mirror. The amplitude of the mirror is such that the object is always infront of the mirror. The amplitude of SHM of the image is
   (A) zero (B) 2 cm
   (C) 4 cm (D) 1 cm

3. A watch shows the time as 3:25. What will be the time that appears when seen through a plane mirror?
   (A) 8:35 (B) 9:35
   (C) 7:35 (D) 8:25

4. If a ray of light is incident on a plane mirror at an angle 60° from the mirror surface, then deviation produced by mirror is:
   (A) 30° (B) 60°
   (C) 90° (D) 120°

5. When light is reflected from a mirror a change occurs in its:
   (A) phase, (B) frequency,
   (C) wavelength, (D) speed

6. The images of clouds and trees in water always less bright than in reality
   (A) because water is forming the image dirty
   (B) because there is an optical illusion due to which the image appears to be less bright
   (C) because only a portion of the incident light is reflected and quite a large portion goes mid water
   (D) because air above the surface of water contains a lot of moisture

7. A rays is incident at an angle 38° on a mirror. The angle between normal and reflected ray is
   (A) 38° (B) 52°
   (C) 90° (D) 76°

8. Mark the correct options.
   (A) If the incident rays are converging, we have a real object.
   (B) If the final rays are converging, we have a real image.
   (C) The image of a virtual object is called a virtual image.
   (D) If the image is virtual, the corresponding object is called a virtual object.

9. A point source of light is placed in front of a plane mirror.
   (A) All the reflected rays meet at a point when produced backward.
   (B) Only the reflected rays close to the normal meet at a point when produced backward.
   (C) Only the reflected rays making a small angle with the mirror, meet at a point when produced backward.
   (D) Light of different colours make different images.

10. Which of the following letters do not surface lateral inversion:
    (A) HGA (B) HOX
    (C) VET (D) YUL

11. An object is initially at a distance of 100 cm from a plane mirror. If the mirror approaches the object at a speed of 5 cm/s. Then after 6 s the distance between the object and its image will be:
    (A) 60 cm (B) 140 cm
    (C) 170 cm (D) 150 cm

12. Two mirrors are placed perpendicular to each other. A ray strikes the first mirror and after reflection from the first mirror it falls on the second mirror. The ray after reflection from second mirror will emerge:
    (A) Perpendicular to the original ray
    (B) Parallel to the original ray
    (C) At 45° to the original ray
    (D) At 60° to the original ray

13. A person is in a room whose ceiling and two adjacent walls are mirrors. How many images are formed?
(A) 5 (B) 6
(C) 7 (D) 8

14. If an object is placed unsymmetrically between two plane mirrors, inclined at the angle of 60°, then the total number of images formed is
(A) 5 (B) 4
(C) 2 (D) infinite

Section (B): Spherical Mirror

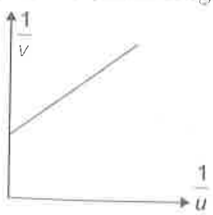
15. In image formation from spherical mirrors, only paraxial rays are considered because they:
(A) are easy to handle geometrically
(B) contain most of the intensity of the incident light
(C) form nearly a point image of a point source
(D) show minimum dispersion effect.

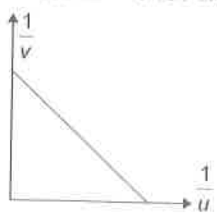
16. A concave mirror of radius of curvature 20 cm forms image of the sun. The diameter of the sun subtends an angle 1° on the earth. Then the diameter of the image is (in cm):
(A) $2\pi/9$ (B) $\pi/9$
(C) 20 (D) $\pi/18$

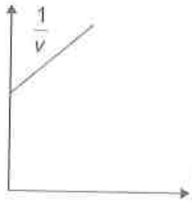
17. A convex mirror has a focal length f. A real object, placed at a distance f in front of it from the pole, produces an image at
(A) $2f$ (B) $f/2$
(C) f (D) ∞

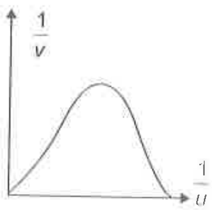
18. A convex mirror has a focal length = 20 cm. A convergent beam tending to converge to a point 20 cm behind convex mirror on principal axis falls on it. The image if formed at
(A) infinity (B) 40 cm
(C) 20 cm (D) 10 cm

19. An object is placed at a distance u from a concave mirror and its real image is received on a screen placed at a distance of v from the mirror. If f is the focal length of the mirror, then the graph between $1/v$ versus $1/u$ is

(A)


(B)


(C)


(D)


20. An infinitely long rod lies along the axis of a concave mirror of focal length f. The near end of the rod is at a distance $u > f$ from the mirror. Its image will have a length
(A) $f^2/(u-f)$ (B) $uf/(u-f)$
(C) $f^2/(u+f)$ (D) $uf/(u+f)$

21. A candle is kept at a distance equal to double the focal length from the pole of a convex mirror. its magnification will be:
(A) $-1/3$ (B) $1/3$
(C) $2/3$ (D) $-2/3$

22. A concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real, the focal length should be:
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 30 cm

23. If an object is 30 cm away from a concave mirror of focal length 15 cm, the image will be
(A) erect (B) virtual
(C) diminished (D) of same size

24. A concave mirror cannot form:
(A) virtual image of virtual object
(B) virtual image of a real object
(C) real image of a real object
(D) real image of a virtual object

25. The largest distance of the image of a real object from a convex mirror of focal length 20 cm can be:
(A) 20 cm
(B) infinite
(C) 10 cm
(D) depends on the position of the object.

26. A particle is moving towards a fixed spherical mirror. The image:
(A) must move away from the mirror
(B) must move towards the mirror.
(C) may move towards the mirror.
(D) will move towards the mirror, only if the mirror is convex.

27. A straight line joining the object point and image point is always perpendicular to the mirror
(A) if mirror is plane only
(B) if mirror is concave only
(C) if mirror is convex only
(D) none of these

28. The focal length of spherical mirror is
(A) Maximum for red light
(B) Maximum for blue light
(C) Maximum for white light
(D) Same for all lights

29. A virtual image, larger than the object can be produced by
(A) convex mirror
(B) concave mirror
(C) plane mirror
(D) concave lens

30. In case of concave mirror, the minimum distance between a real object and its real image is:
(A) f
(B) $2f$
(C) $4f$
(D) zero

31. The rear-view mirror of a car is:
(A) Plane
(B) Convex
(C) Concave
(D) None of the above

32. A candle flame of 3 cm is placed at 300 cm from a wall. A concave mirror is kept at distance x from the wall in such a way that image of the flame on the wall is 9 cm. Then x is
(A) 339 cm
(B) 900 cm
(C) 450 cm
(D) 423 cm

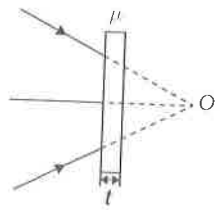
Section (C): Refraction in General, Refraction at Plane Surface and T.I.R.

33. When a wave is refracted:
(A) its path must change
(B) its amplitude must change
(C) its velocity must change
(D) its frequency must change

34. A ray incident at a point at an angle of incidence of 60° enters a glass sphere of $\mu = \sqrt{3}$ and it is reflected and refracted at the farther surface of the sphere. The angle between reflected and refracted rays at this surface is
(A) 50°
(B) 90°
(C) 60°
(D) 40°

35. A ray of light passes through a plane glass slab thickness t and refractive index $\mu = 1.5$. The an between incident ray and emergent ray will be
(A) 0°
(B) 30°
(C) 45°
(D) 60°

36. A beam of light is converging towards a point. A pla parallel plate of glass of thickness t refractive index is introduced in the path of the beam. The converge point is shifted by (assume near normal incidence):



(A) $t\left(1-\frac{1}{\mu}\right)$ away
(B) $t\left(1-\frac{1}{\mu}\right)$ away
(C) $t\left(1-\frac{1}{\mu}\right)$ nearer
(D) $t\left(1+\frac{1}{\mu}\right)$ nearer

37. A ray of light is incident on one face of a transparen slab of thickness 15 cm. The angle of incidence is 60°. If the lateral displacement of the ray on emerging from the parallel plane is $5\sqrt{3}$ cm, the refractive index of the material of the slab is
(A) 1.414
(B) 1.532
(C) 1.732
(D) none

38. The critical angle of light going from medium A to medium B is θ. The speed of light in medium A is v. The speed of light in medium B is:
(A) $\frac{v}{\sin\theta}$
(B) $v \sin\theta$
(C) $v \cot\theta$
(D) $v \tan\theta$

39. A ray of light from a denser medium strike a rarer medium. The angle of reflection is r and that of refraction is r'. The reflected and refracted rays make an angle of 90° with each other. The critical angle will be
(A) $\sin^{-1}(\tan r)$
(B) $\tan^{-1}(\sin r)$
(C) $\sin^{-1}(\tan r')$
(D) $\tan^{-1}(\sin r')$

40. Two transparent media A and B are separated by a plane boundary. The speed of light in medium A is 2.0×10^8 m s^{-1} and in medium B is 2.5×10^8 ms^{-1}. The critical angle for which a ray of light going from A to B is totally internally reflected is

(A) $\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\sin^{-1}\left(\frac{2}{5}\right)$

(C) $\sin^{-1}\left(\frac{4}{5}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

41. A small source of light is 4 m below the surface of a liquid of refractive index 5/3. In order to cut off all the light coming out of liquid surface, minimum diameter of the disc placed on the surface of liquid is

(A) 3 m (B) 4 m

(C) 6 m (D) ∞

42. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and fish is 12 cm below the surface, the radius of the circle in cm is

(A) $12 \times 3 \times \sqrt{5}$ (B) $4 \times \sqrt{5}$

(C) $12 \times 3 \times \sqrt{7}$ (D) $12 \times 3/\sqrt{7}$

Section (D): Refraction by Prism

43. A ray of light is incident at angle i on a surface of a prism of small angle A & emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equal to:

(A) $\frac{A}{\mu}$ (B) $\frac{A}{(2\mu)}$

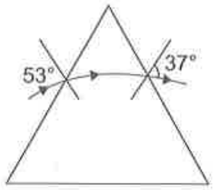
(C) μA (D) $\mu\frac{A}{2}$

44. A certain prism is found to produce a minimum deviation of 38°. It produces a deviation of 44° when the angle of incidence is either 42° or 62°. What is the angle of incidence when it is undergoing minimum deviation?

(A) 45° (B) 49°

(C) 40° (D) 55°

45. A ray incident at angle 53° on a prism emerges at an angle at 37° as shown. If the angle of incidence is made 50°, which of the following is a possible value of the angle of emergence.



(A) 35° (B) 42°

(C) 40° (D) 38°

46. A beam of monochromatic light is incident at i = 50° on one face of an equilateral prism, the angle of emergence is 40°, then the angle of minimum deviation is:

(A) 30° (B) $<$ 30°

(C) $\leq$ 30° (D) $\geq$ 30°

47. A prism has a refractive index $\sqrt{\frac{3}{2}}$ and refracting angle 90°. Find the minimum deviation produced by prism.

(A) 40° (B) 45°

(C) 30° (D) 49°

48. A prism is made up of material of refractive index $\sqrt{3}$. The angle of prism is A. If the angle of minimum deviation is equal to the angle of the prism, then the value of A is:

(A) 30° (B) 45°

(C) 60° (D) 75°

49. R.I. of a prism is $\sqrt{\frac{7}{3}}$ and the angle of prism is 60°. The limiting angle of incidence of a ray that will be transmitted through the prism is:

(A) 30° (B) 45°

(C) 15° (D) 50°

50. The angle of a prism is 60° and the index of refraction of glass with air is 1.5. If the angle of incidence on the first face is I_1 and the angle of emergence at the second face is I_2, then the prism produces minimum deviation when

(A) $I_1 = 0$ (B) $I_1 > I_2$

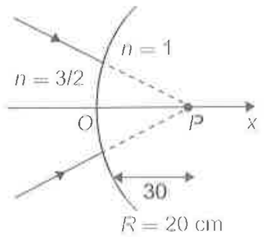
(C) $I_1 < I_2$ (D) $I_1 = I_2$

51. In a thin prism of glass (refractive index 1.5) which of the following relations between the angle of minimum deviation δ_m and angle of refraction r will be correct:

(A) $\delta_m = r$ (B) $\delta_m = 1.5\,r$
(C) $\delta_m = 2r$ (D) $\delta_m = r/2$

Section (E): Rafraction by Spherical Surface

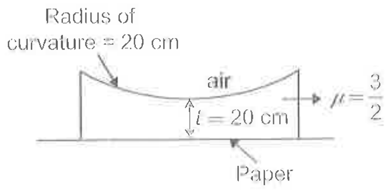
52. The image for the converging beam after refraction through the curved surface is formed at:



(A) $x = 40$ cm (B) $x = \frac{40}{3}$ cm

(C) $x = -\frac{40}{3}$ cm (D) $x = \frac{180}{7}$ cm

53. A planoconcave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be?



(A) 10 cm (B) 15 cm
(C) 50 cm (D) none

Section (F): Lens and Combination of Lenses/Lens and Mirrors

54. A thin lens of focal length f and its aperture diameter d, forms a real image of intensity I. Now the central part of the aperture upto diameter $(\frac{d}{2})$ is blocked by an opaque paper. The focal length and image intensity would change to

(A) $\frac{f}{2}, \frac{I}{2}$ (B) $f, \frac{I}{4}$

(C) $\frac{3f}{4}, \frac{I}{2}$ (D) $f, \frac{3I}{4}$

55. A thin symmetrical double convex lens of power P is cut into three part, as shown in the figure. Power of A is:

(A) $2P$ (B) $\frac{P}{2}$

(C) $\frac{P}{3}$ (D) P

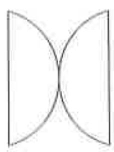
56. A plano convex lens has a curved surface of radius 100 cm. If $\mu = 1.5$, then the focal length of the lens is:

(A) 50 cm (B) 100 cm
(C) 200 cm (D) 500 cm

57. A lens of power $+2.0\ D$ is placed in contact with another lens of power $-1.0\ D$. The combination will behave like

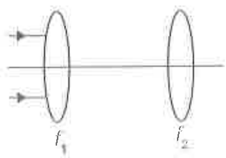
(A) a converging lens of focal length 100 cm
(B) a diverging lens of focal length 100 cm
(C) a converging lens of focal length 50 cm
(D) a diverging lens of focal length 50 cm.

58. A biconvex lens has a focal length of 10 cm. It is cut in half and two pieces are placed as shown. The focal length of the final combination is:



(A) 10 (B) 20
(C) 40 (D) Not a lens

59. Parallel beam of light is incident on a system of two convex lenses of focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated



(A) 60 cm (B) 30 cm
(C) 90 cm (D) 40 cm

60. A pin is placed 10 cm in front of convex lens of focal length 20 cm and refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature of 22 cm. How far from the lens is the final image formed?

(A) 11 cm (B) 12 cm
(C) 13 cm (D) 14 cm

61. When the object is at distances u_1 and u_2 the images formed by the same lens are real and virtual respectively and of the same size. Then focal length of the lens is:

(A) $\frac{1}{2}\sqrt{u_1u_2}$ (B) $\frac{1}{2}(u_1+u_2)$

(C) $\sqrt{u_1u_2}$ (D) $2(u_1+u_2)$

62. The height of the image formed by a converging lens on a screen is 8 cm. For the same position of the object and screen again an image of size 12.5 cm is formed on the screen by shifting the lens. The height of the object

(A) 625/32 cm (B) 64/12.5 cm

(C) 10 cm (D) none

Section (G): Dispersion of Light

63. The dispersion of light in a medium implies that:

(A) lights of different wavelengths travels with different speeds in the medium

(B) lights of different frequencies travel with different speeds in the medium

(C) the refractive index of medium is different for different wavelengths

(D) all of the above.

64. Critical angle of light passing from glass to air is minimum for

(A) red (B) green

(C) yellow (D) violet

65. A plane glass slab is placed over various coloured letters. The letter which appears to be raised the least is:

(A) violet (B) yellow

(C) red (D) green

66. A medium has $n_v = 1.56$, $n_r = 1.44$. Then its dispersive power is:

(A) $\frac{3}{50}$ (B) $\frac{6}{25}$

(C) 0.03 (D) none

67. The refractive index of flint glass for blue line is 1.6333 and red line is 1.6161, then dispersive power of the glass is:

(A) 0.0276 (B) 0.276

(C) 2.76 (D) 0.106

68. Indicate the correct statement in the following

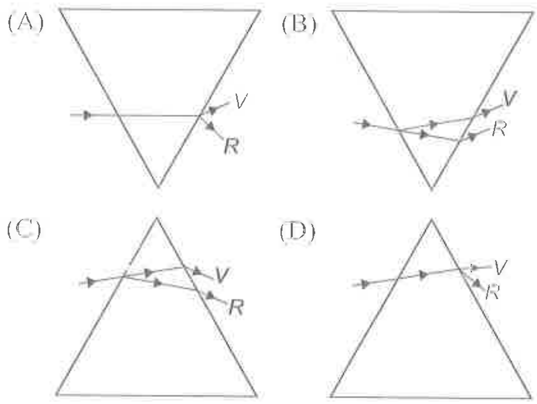
(A) The dispersive power depends upon the angle of prism

(B) The angular dispersion depends upon the angle of the prism

(C) The angular dispersion does not depend upon the dispersive power

(D) The dispersive power in vacuum is one

69. Which of the following diagrams shows correctly the dispersion of white light by a prism?



Reasoning Type

70. **Statement I:** If a source of light is placed in front of rough wall its image is not seen.
Statement II: The wall does not reflect light.

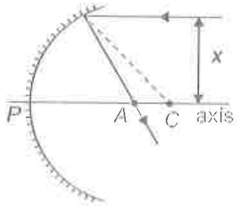
(A) Statement I is true, statement 2 is true; statement 2 is correct explanation for statement 1

(B) Statement 1 is true, statement 2 is true; statement 2 is NOT correct explanation for statement 1

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

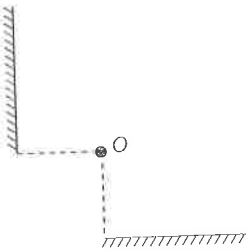
71. **Statement I:** As the distance x of a parallel ray from axis increases, focal length decreases
Statement II: As x increases, the distance from pole to the point of intersection of reflected ray with principal axis decreases



(A) Statement 1 is true, statement 2 is true; statement 2 is correct explanation for satement 1
(B) Statement 1 is true, statement 2 is true; statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

72. **Statement I:** When an object dipped in a liquid is viewed normally, the distance between the image and the object is independent of the height of the liquid above the object.
Statement II: The normal shift is independent of the location of the slab between the object and the observer.
(A) Statement 1 is true, statement 2 is true; statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true; statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

73. **Statement I:** When two plane mirrors are kept perpendicular to each other as shown (O si the point object), 3 image will be formed.
Statement II: In case of multiple reflection, image of one surface can act as an object for the next surface.



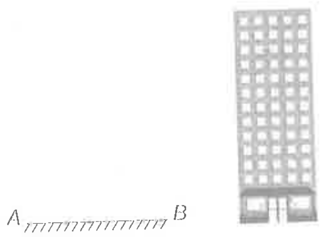
(A) Statement 1 is true, statement 2 is true; statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true; statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

74. **Statemen I:** A piece of paper placed at the position of a real image of a virtual object of intense light will burn after sufficient time.
Statement II: A virtual object is that point where the incident rays appear to converge and a real image is that point at which reflected/ refracted rays actually converge.
(A) Statement 1 is true, statement 2 is true; statement 2 is correct explanation for satement 1
(B) Statement 1 is true, statement 2 is true; statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

JEE Advanced

Section (A): Plane Mirror

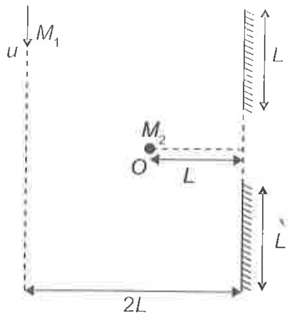
1. When a plane mirror AB is placed horizontally on level ground at a distance of 60 metres from the foot of a tower, the top of the tower and its image in the mirror subtends, an angle of 90° at B. The height of the tower is:



(A) 30 metre (B) 60 metre
(C) 90 metre (D) 120 metre.

2. A unnumbered wall clock shows time 04:25:37, where 1st term represents hours, 2nd represents minutes & the last term represents seconds, What time will its image in a plane mirror show.
(A) 08:35:23 (B) 07:35:23
(C) 07:34:23 (D) None of these

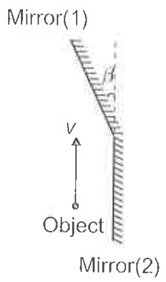
3. Two plane mirrors of length L are separated by distance L and a man M_2 is standing at distance L from the connecting line of mirrors as shown in figure. A man M_1 is walking in a straight line at distance $2L$ parallel to mirrors at speed u, then man M_2 at O will be able to see image of M_1 for total time:



(A) $\frac{4L}{u}$ (B) $\frac{3L}{u}$
(C) $\frac{6L}{u}$ (D) $\frac{9L}{u}$

4. A person is standing in a room of width 200 cm. A plane mirror of vertical length 10 cm is fixed on a wall in front of the person. The person looks into the mirror from distance 50 cm. How much width (height) of the wall behind him will he be able to see: (assume that he uses the full mirror)
(A) 30 cm (B) 40 cm
(C) 50 cm (D) None

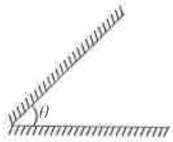
5. In the diagram shown, all the velocities are given with respect to earth. What is the relative velocity of the image in mirror (1) with respect to the image in the mirror (2)? The mirror (1) forms an angle β with the vertical.



(A) $2V \sin 2\beta$ (B) $2V \sin \beta$
(C) $2V/\sin 2\beta$ (D) none

6. Two plane mirrors are inclined to each other at an angle 60°. If a ray of light incident on the first mirror is parallel to the second mirror, it is reflected from the second mirror.
(A) Perpendicular to the first mirror
(B) Parallel to the first mirror
(C) Parallel to the second mirror
(D) Perpendicular to the second mirror.

7. Two mirrors are inclined at an angle θ as shown in the figure. Light ray is incident parallel to one of the mirrors. The ray will start retracing its path after third reflection if:



(A) $\theta = 45°$ (B) $\theta = 30°$
(C) $\theta = 60°$ (D) all three

8. There are two plane mirror with reflecting surface facing each other. Both the mirrors are moving with speed v away from each other. A point object is placed between the mirrors. The velocity of the image from due to n-th reflection will be
(A) nv (B) $2nv$
(C) $3nv$ (D) $4nv$

9. Two plane mirrors are placed parallel to each other at a distance L apart. A point object O is placed between them, at a distance $L/3$ from one mirror. Both mirrors form multiple images. The distance between any two images cannot be
(A) $3L/2$ (B) $2L/3$
(C) $2L$ (D) None

10. Images of an object placed between two plane mirrors whose reflecting surfaces make an angle of 90° with one another lie on a:
(A) straight line (B) zig-zag curve
(C) circle (D) ellipse

11. A person's eye is at a height of 1.5 m. He stands in front of a 0.3 m long plane mirror which is 0.8 m above the ground. The length of the image he sees of himself is:
(A) 1.5 m (B) 1.0 m
(C) 0.8 m (D) 0.6 m

12. A man of height 'h' is walking away from a street lamp with a constant speed 'v'. The height of the street lamp is $3h$. The rate at which of the length of the man's shadow is increasing when he is at a distance $10h$ from the base of the street lamp is:
(A) $v/2$ (B) $v/3$
(C) $2v$ (D) $v/6$

13. A boy of height 1.5 m with his eye level at 1.4 m stands before a plane mirror of length 0.75 m fixed on the wall. The height of the lower edge of the mirror above the floor is 0.8 m. Then:

(A) the boy will see his full image
(B) the boy cannot see his hair
(C) the boy cannot see his feet
(D) the boy cannot see neither his hair nor his feet.

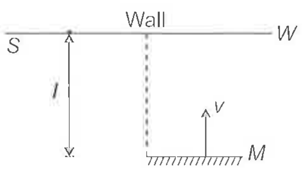
14. A plane mirror is moving with velocity $4\hat{i}+5\hat{j}+8\hat{k}$. A point object in front of the mirror moves with a velocity $3\hat{i}+4\hat{j}+5\hat{k}$. Here $\hat{k}$ is along the normal to the plane mirror and facing towards the object. The velocity of the image is

(A) $-3\hat{i}-4\hat{j}+5\hat{k}$ (B) $3\hat{i}+4\hat{j}+11\hat{k}$
(C) $-3\hat{i}-4\hat{j}+11\hat{k}$ (D) $7\hat{i}+9\hat{j}+11\hat{k}$

15. A man of height 170 cm wants to see his complete image in a plane mirror (while standing). His eyes are at a height of 160 cm from the ground.

(A) Minimum length of the mirror = 80 cm
(B) Minimum length of the mirror = 85 cm
(C) Bottom of the mirror should be at a height 80 cm or less
(D) Bottom of the mirror should be at a height 85 cm

16. A flat mirror M is arranged parallel to a wall W at a distance l from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light spot on the wall. The mirror moves with velocity v towards the wall.



(A) The spot of light will move with the speed v on the wall
(B) The spot of light will not move on the wall
(C) As the mirror comes closer the spot of light will become larger and shift away from the wall with speed larger then v
(D) The size of the light on the wall remains the same

Section (B): Spherical Mirror

17. The distance of an object from the focus of a convex mirror of radius of curvature 'a' is 'b'. Then the distance of the image from the focus is:

(A) $\frac{b^2}{4a}$ (B) $\frac{a}{b^2}$
(C) $\frac{a^2}{4b}$ (D) none

18. A boy 2 m tall stands 40 cm in front of a mirror. He sees an erect image, 1 m high. The mirror is:

(A) Concave, $f = 40$ cm
(B) Convex, $f = 40$ cm
(C) Plane
(D) Either convex or concave

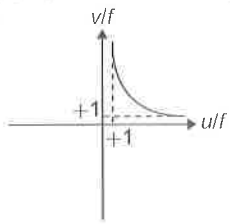
19. What is the distance of a needle from a concave mirror of focal length 10 cm for which a virtual image of twice its height is formed?

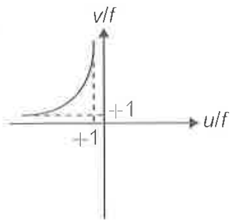
(A) 2.5 cm (B) 5 cm
(C) 8 cm (D) 9.1 cm

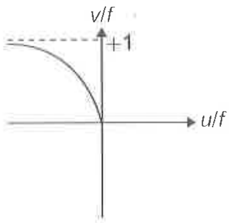
20. A convex mirror has a focal length f. An object of height h is placed in front of it. If an erect image of height h/n is formed. The distance of the object from the mirror is:

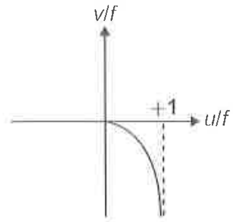
(A) nf (B) f/n
(C) $(n + 1) f$ (D) $(n - 1) f$

21. A real inverted image in a concave mirror is represented by (u, v, f are coordinates)

(A)


(B)


(C)


(D)


22. Which one of the following statements are **incorrect** for spherical mirrors.

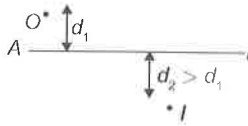
(A) a concave mirror forms only virtual images for any position of real object
(B) a convex mirror forms only virtual images for any position of a real object.

(C) a convex mirror forms only a virtual diminished image of an object placed between its pole and the focus

(D) a concave mirror forms a virtual magnified image of an object placed between its pole and the focus.

23. The distance of an object from a spherical mirror is equal to focal length of the mirror. Then the image:
(A) must be at infinity
(B) may be at infinity
(C) may be at the focus
(D) none

24. In the figure shown, the image of a real object is formed at point I. AB is the principal axis of the mirror. The mirror must be:



(A) concave & placed towards right I
(B) concave & placed towards left of I
(C) convex and placed towards right of I
(D) convex & placed towards left of I.

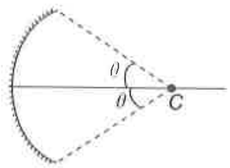
25. A point object at 15 cm from a concave mirror of radius of curvature 20 cm is made to oscillate along the principal axis with amplitude 2 mm. The amplitude of its image will be
(A) 2 mm (B) 4 mm
(C) 8 mm (D) none

26. A luminous point object is moving along the principal axis of a concave mirror of focal length 12 cm towards it. When its distance from the mirror is 20 cm its velocity is 4 cm/s. The velocity of the image in cm/s at that instant is
(A) 6, towards the mirror
(B) 6, away from the mirror
(C) 9, away from the mirror
(D) 9, towards the mirror.

27. A point object on the principal axis at a distance 15 cm in front of a concave mirror of radius of curvature 20 cm has velocity 2 mm/s perpendicular to the principal axis. The velocity of image at that instant will be:
(A) 2 mm/s (B) 4 mm/s
(C) 8 mm/s (D) none of these

28. The origin of x and y coordinates is the pole of a concave mirror of focal length 20 cm. The x-axis is the optical axis with $x > 0$ being the real side of mirror. A point object at the point (25 cm, 1 cm) is moving with a velocity 10 cm/s in positive x-direction. The velocity of the image in cm/s is approximately
(A) $-80\hat{i}+8\hat{j}$ (B) $160\hat{i}+8\hat{j}$
(C) $-160\hat{i}+8\hat{j}$ (D) $160\hat{i}-4\hat{j}$

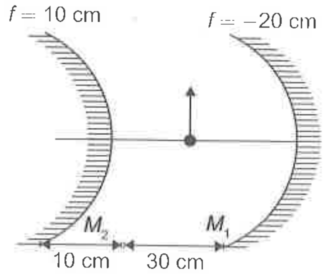
29. The circular boundary of the concave mirror subtends a cone of half angle θ at its centre of curvature. The minimum value of θ for which ray incident on this mirror parallel to the principle axis suffers reflection more than one is



(A) 30° (B) 45°
(C) 60° (D) 75°

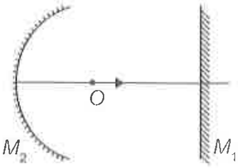
30. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no gap between the images formed by the two mirrors. The radius of the convex mirror is:
(A) 12.5 cm (B) 25 cm
(C) 50 cm (D) 100 cm

31. In the figure shown find the total magnification after two successive reflections first on M_1 & then on M_2



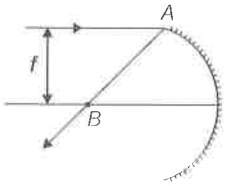
(A) +1 (B) −2
(C) +2 (D) −1

32. In the figure shown if the object 'O' moves towards the plane mirror, then the image I (which is formed after successive reflections from M_1 and M_2 respectively)



(A) towards right
(B) towards left
(C) with zero velocity
(D) cannot be determined

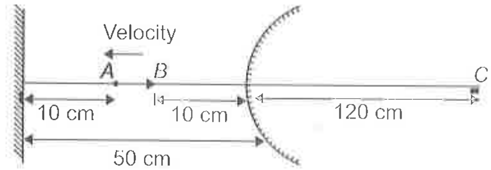
33. A ray of light is incident on a concave mirror. It is parallel to the principal axis and its height from principal axis is equal to the focal length of the mirror. The ratio of the distance of point B to the distance of the focus from the centre of curvature is (AB is the reflected by)



(A) $\frac{2}{\sqrt{3}}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{2}{3}$
(D) $\frac{1}{2}$

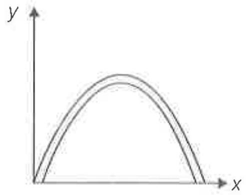
34. The image (of a real object) formed by a concave mirror is twice the size of the object. The focal length of the mirror is 20 cm. The distance of the object from the mirror is (are)
(A) 10 cm
(B) 30 cm
(C) 25 cm
(D) 15 cm

35. In the figure shown consider the first reflection at the plane mirror and second at the convex mirror. AB is object.



(A) the second image is real, inverted of 1/5th magnification
(B) the second image is virtual and erect with magnification 1/5
(C) the second image moves towards the convex mirror.
(D) the second image moves away from the convex mirror.

36. A reflecting surface is represented by the equation $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right)$, $0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point (s) where this ray is incident is



(A) $\left(\frac{L}{4}, \frac{\sqrt{2}L}{\pi}\right)$
(B) $\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right)$
(C) $\left(\frac{3L}{4}, \frac{\sqrt{2}L}{\pi}\right)$
(D) $\left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

37. A concave mirror cannot form
(A) virtual image of virtual object
(B) virtual image of a real object
(C) real image of a real object
(D) real image of a virtual object

Section (C): Refraction in General, Refraction at Plane Surface and T.I.R.

38. The x-z plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A and B. Its directions in the two media are given by the unit vectors, $\vec{r} = a\hat{i} + b\hat{j}$ and $\vec{r}_B = \alpha\hat{i} + \beta\hat{j}$ respectively where $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions. Then
(A) $\mu_1 a = \mu_2 \alpha$
(B) $\mu_1 \alpha = \mu_2 a$
(C) $\mu_1 b = \mu_2 \beta$
(D) $\mu_1 B = \mu_2 b$

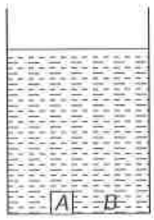
39. A ray of light moving along the unit vector $(-i - 2j)$ undergoes refraction at an interface of two media, which is the x-z plane. The refractive index for $y > 0$ is 2 while for $y < 0$, it is $\sqrt{5}/2$ The unit vector along which the refracted ray moves is:

(A) $\frac{(-3i - 5j)}{\sqrt{34}}$
(B) $\frac{(-4i - 3j)}{5}$
(C) $\frac{(-3i - 4j)}{5}$
(D) None of these

40. How much water should be filled in a container of 21 cm in height, so that it appears half filled (of total height of the container) when viewed from the top of the container?
(Assume near normal incidence and $\mu_w = 4/3$)
(A) 8.0 cm (B) 10.5 cm
(C) 12.0 cm (D) 14.0 cm

41. A mark at the bottom of a beaker containing liquid appears to rise by 0.1 m. The depth of the liquid is 1 m. the refractive index of liquid is:
(A) 1.33 (B) 9/10
(C) 10/9 (D) 1.5

42. A parallel sided block of glass of refractive index 1.5 which is 36 mm thick rests on the floor of a tank which is filled with water (refractive index = 4/3.) The difference between apparent depth of floor at A and B when seen from vertically above is equal to



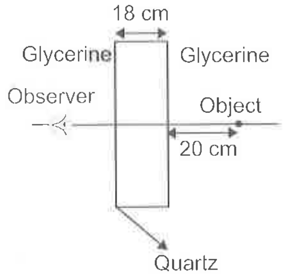
(A) 2 mm (B) 3 mm
(C) 4 mm (D) none

43. An underwater swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer, the bird appears to be at a distance X from the surface of water. (Refractive index of water is 4/3.) The value of X is
(A) 24 m (B) 12 m
(C) 18 m (D) 9 m

44. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. Its has a real image, also located at C (a condition called auto-collimation). If the mirror is now filled with water, the image will be:
(A) real, and will remain at C
(B) real, and located at a point between C and ∞
(C) virtual, and located at a point between C and O
(D) real, and located at a point between C and O

45. A bird is flying 3 m above the surface of water. If the bird is diving vertically down with speed = 6 m/s, his apparent velocity as seen by a stationary fish underwater is
(A) 8 m/s (B) 6 m/s
(C) 12 m/s (D) 4 m/s

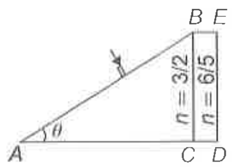
46. Given that, velocity of light in quartz = 1.5×10^8 m/s and velocity of light in glycerine = $\frac{9}{4} \times 10^8$ m/s. Now a slab made of quartz is placed in glycerine as shown. The shift of the object produced by slab is



(A) 6 cm (B) 3.55 cm
(C) 9 cm (D) 2 cm

47. A light ray is incident on a transparent sphere of index = $\sqrt{2}$, at an angle of incidence = 45°. What is the deviation of a tiny fraction of the ray, which enters the sphere, undergoes two internal reflections, and then refracts out into air?
(A) 270° (B) 240°
(C) 120° (D) 180°

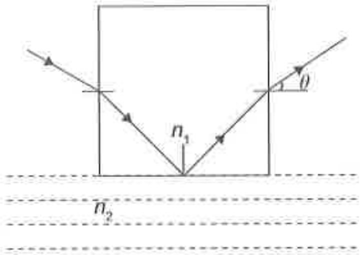
48. In the figure ABC is the cross section of a right angled prism and $BCDE$ is the cross section of a glass slab. The value of θ so that light incident normally on the face AB does not cross the face BC is (given $\sin^{-1}(3/5) = 37°$)



(A) $\theta \le 37°$ (B) $\theta > 37°$
(C) $\theta \le 53°$ (D) $\theta < 53°$

49. A cubical block of glass of refractive index n_1 is in contact with the surface of water of refractive index n_2.

A beam of light is incident on vertical face of the block (see figure). After refraction, a total internal reflection at the base and refraction at the opposite vertical face, the ray emerges out at an angle θ. The value of θ is given by

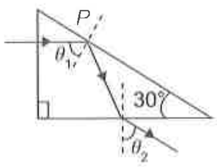


(A) $\sin\theta < \sqrt{n_1^2 - n_2^2}$ (B) $\tan\theta < \sqrt{n_1^2 - n_2^2}$

(C) $\sin\theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$ (D) $\tan\theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$

50. A vertical pencil of rays comes from bottom of a tank filled with a liquid, When it is accelerated with an acceleration of 7.5 m/s^2, the ray is seen to be totally reflected by liquid surface. What is minimum possible refractive index of liquid?
(A) slightly greater than 4/3
(B) slightly greater than 5/3
(C) slightly greater than 1.5
(D) slightly greater than 1.75

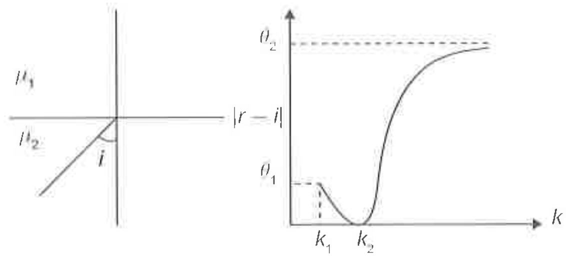
51. A ray of light is incident normally on one face of 30° – 60° – 90° prism of refractive index 5/3 immersed in water of refractive index 4/3 as shown in figure.



(A) The exit angle θ_2 of the ray is $\sin^{-1}(5/8)$
(B) The exit angle θ_2 of the ray is $\sin^{-1}(5/4\sqrt{3})$
(C) Total internal reflection at point P ceases if the refractive index of water is increased to $5/2\sqrt{3}$ by dissolving some substance.
(D) Total internal reflection at point P ceases if the refractive index of water is increased to 5/6 by dissolving some substance.

52. A ray of light in a liquid of refractive index 1.4, approaches the boundary surface between the liquid and air at an angle of incidence whose sine is 0.8. Which of the following statements is correct about the behaviour of the light
(A) It is impossible to predict the behaviour of the light ray on the basis of the information supplied
(B) The sine of the angle of refraction of the emergent ray will less than 0.8
(C) The ray will be internally reflected
(D) The sine of the angle of refraction of the emergent ray will be greater than 0.8.

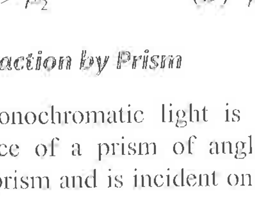
53. The figure shows ray incident at an angle $i = \pi/3$. If the plot drawn shown the variation of $|r - i|$ versus $\mu_1/\mu_2 = k$, (r = angle of refraction)



(A) the value of k_1 is $\frac{2}{\sqrt{3}}$ (B) the value of $\theta_1 = \pi/6$

(C) the value of $\theta_2 = \pi/3$ (D) the value of k_2 is 1

54. In the diagram shown, a ray of light is incident on the interface between 1 and 2 at angle slightly greater than critical angle. The light surfers total internal reflection at this interface. After that the light ray falls at the interface of 1 and 3, and again it suffers total internal reflection. Which of the following relations should hold true?



(A) $\mu_1 > \mu_2 < \mu_3$ (B) $\mu_1^2 - \mu_2^2 > \mu_3^2$

(C) $\mu_1^2 - \mu_3^2 > \mu_2^2$ (D) $\mu_1^2 + \mu_2^2 > \mu_3^2$

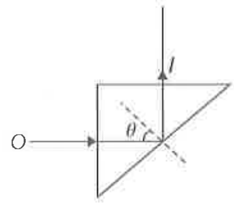
Section (D): Refraction by Prism

55. A ray of monochromatic light is incident on one refracting face of a prism of angle 75°. It passes through the prism and is incident on the other face a

the critical angle. If the refractive index of the material of the prism is $\sqrt{2}$, the angle of incidence on the first face of the prism is
(A) 30° (B) 45°
(C) 60° (D) 0°

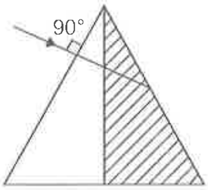
56. A prism having refractive index $\sqrt{2}$ and refracting angle 30°, has one of the refracting surface polished. A beam of light incident on the other refracting surface will retrace its path if the angle of incidence is:
(A) 0° (B) 30°
(C) 45° (D) 60°

57. A triangular prism of glass is shown in figure. A ray incident normally on one face is totally reflected. If θ is 45°, the index of refraction of glass is:



(A) Less than $\sqrt{2}$ (B) Equal to $\sqrt{2}$
(C) Greater than $\sqrt{2}$ (D) None of the above.

58. A ray of light is incident normally on the first refracting face of the prism of refracting angle A. The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will



(A) emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/2\right)$
(B) not emerge out of the prism
(C) emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/4\right)$
(D) None of these

59. A ray of light is incident at an angle 60° on the face of a prism having refractive angle 30°. The ray emerging out of the prism makes an angle 30° with the incident ray. θ through which it emerges from the surface.
(A) 0° (B) 30°
(C) 45° (D) 60°

60. A ray of light is incident normally on one face of an equilateral prism of refractive index 1.5. The angle of deviation is
(A) 30° (B) 45°
(C) 60° (D) 75°

61. Light ray is incident on a prism of angle $A = 60°$ and refractive index $\mu = \sqrt{2}$. The angle of incidence at which the emergent ray grazes the surface is given by
(A) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$
(C) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

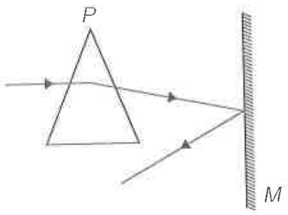
62. The angle of a prism is A and one of its refracting surfaces is silvered. Light rays falling at an angle of incidence $2A$ on the first surface return back through the same path after suffering reflection at the second (silvered) surface. The refractive index of the material of the prism is
(A) $2 \sin A$ (B) $2 \cos A$
(C) $1/2 \cos A$ (C) $\tan A$

63. A prism of refractive index $\sqrt{2}$ has a refracting angle of 30°. One of the refracting surfaces of the prism is polished. A beam of monochromatic light will retrace its path if its angle of incidence on the refracting surface is
(A) 0° (B) 30°
(C) 45° (D) 60°

64. The maximum refractive index of a material of a prism of apex angle 90° for which light will be transmitted is:
(A) $\sqrt{3}$ (B) 1.5
(C) $\sqrt{2}$ (D) None

65. A prism having an apex angle of 4° and refractive index of 1.50 is located in front of a vertical plane mirror as shown. A horizontal ray of light is incident

on the prism. The total angle through which the ray is deviated is



(A) 4° clockwise
(B) 178° clockwise
(C) 2° clockwise
(D) 8° clockwise

66. A thin prism of angle 5° is placed at a distance of 10 cm from object. What is the distance of the image from object? (Given μ of prism = 1.5)
(A) $\pi/8$cm
(B) $\pi/12$ cm
(C) $5\pi/36$ cm
(D) $\pi/7$ cm

67. An equilateral prism deviates a ray through 40° for two angles of incidence differing by 20°. The possible angle of incidences are:
(A) 40°
(B) 50°
(C) 20°
(D) 60°

68. A prism of refractive index $\sqrt{2}$ has refracting angle 60°. In order that a ray suffers minimum deviation it should be incident at an angle
(A) 45°
(B) 90
(C) 30°
(D) none

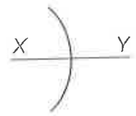
69. For refraction through a small angled prism, the angle of deviation:
(A) increases with the increase in R.I. of prism.
(B) will decrease with the increase in R.I. of prism.
(C) is directly proportional to the angle of prism.
(D) will be $2D$ for a ray of R.I. = 2.4 if it is D for a ray of R.I = 1.2

70. For the refraction of light through a prism
(A) For every angle of deviation there are two angles of incidence.
(B) The light travelling inside an equilateral prism is necessarily parallel to the base when prism is set for minimum deviation.
(C) There are two angles of incidence for maximum deviation.
(D) Angle of minimum deviation will increase if refractive index of prism is increased keeping the outside medium unchanged if $\mu_p > \mu_s$.

Section (E): Refraction by Spherical Surface

71. A fish is near the centre of a spherical water filled fish bowl. A child stands in air at a distance $2R$ (R is radius of curvature of the sphere) from the centre of the bowl. At what distance from the centre would the child's nose appear to the fish situated at the centre (R.I. of water = $\frac{4}{3}$)
(A) $4R$
(B) $2R$
(C) $3R$
(D) R

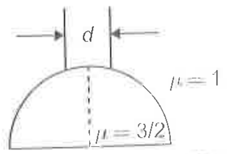
72. A concave spherical surface of radius of curvature 10 cm separates two medium x and y of refractive index 4/3 and 3/2 respectively. If the object is placed along principal axis in medium X then



(A) image is always real
(B) image is real if the object distance is greater than 90 cm
(C) image is always virtual
(D) image is virtual if the object distance is less than 90 cm

73. A spherical surface of radius of curvature 10 cm separates two media X and Y of refractive indices 3/2 and 4/3 respectively. Centre of the spherical surface lies in denser medium. An object is placed in medium X. For image to be real, the object distance must be
(A) greater than 90 cm
(B) less than 90 cm
(C) greater than 80 cm
(D) less than 80 cm

74. A beam of diameter 'd' is incident on a glass hemisphere as shown. If the radius of curvature of the hemisphere is very large in comparison to d, then the diameter of the beam at the base of the hemisphere will be



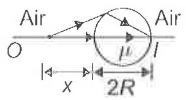
(A) 3/4 d
(B) d
(C) d/3
(D) 2/3 d

75 A concave spherical refracting surface separates two media glass and air ($\mu_{glass} = 1.5$). If the image is to be real at what minimum distance u should the object be placed in glass if R is the radius of curvature?

(A) $u > 3R$ (B) $u > 2R$
(C) $u < 2R$ (D) $u < R$

uestion No. 76 to 78

he figure, shows a transparent sphere of radius R and fractive index μ. An object O is placed at a distance x om the pole of the first surface so that a real image is rmed at the pole of the exactly opposite surface.

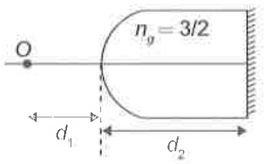


6. If $x = 2R$, then the value of μ is
(A) 1.5 (B) 2
(C) 3 (D) none

7. If $x = \infty$, then the value of μ is
(A) 1.5 (B) 2
(C) 3 (D) none

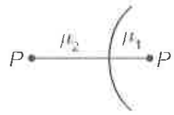
. If an object is placed at a distance R from the pole of first surface, then the real image is formed at a distance R from the pole of the second surface. The refractive index μ of the sphere is given by
(A) 1.5 (B) 2
(C) $\sqrt{2}$ (D) none

. In the figure shown a point object O is placed in air on the principal axis. The radius of curvature of the spherical is 60 cm. I_f is the final image formed after all the refractions and reflections.



(A) If $d_1 = 120$ cm, then the 'I_f' is formed on 'O' for any value of d_2.
(B) If $d_1 = 240$ cm, then the 'I_f' is formed on 'O' only if $d_2 = 360$ cm.
(C) If $d_1 = 240$ cm, then the 'I_f' is formed on 'O' for all value of d_2.
(D) If $d_1 = 240$ cm, then the 'I_f' cannot be formed on 'O'.

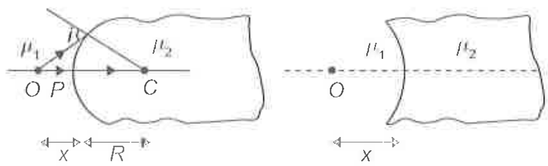
80. Two refracting media are separated by a spherical interface as shown in the figure. PP' is the principal axis, μ_1, and μ_2 are the refractive indices of medium of incidence and medium of refraction respectively. Then:



(A) if $\mu_2 > \mu_1$, then there cannot be a real image of real object.
(B) if $\mu_2 > \mu_1$, then there cannot be a real image of virtual object.
(C) if $\mu_1 > \mu_2$, then there cannot be a virtual image of virtual object.
(D) if $\mu_1 > \mu_2$, then there cannot be a real image of real object.

Question No. 81 to 83

A curved surface of radius R separates two medium of refractive indices μ_1 and μ_2 as shown in figures A and B



81. Choose the correct statement(s) related to the real image formed by the object O placed at a distance x, as shown in figure A
(A) Real image is always formed irrespective of the position of object if $\mu_2 > \mu_1$
(B) Real image is formed only when $x > R$
(C) Real image is formed due to the convex nature of the interface irrespective of μ_1 and μ_2
(D) None of these

82. Choose the correct statement(s) related to the virtual image formed by object O placed at a distance x, as shown in figure A
(A) Virtual image is formed for any position of O if $\mu_2 < \mu_1$
(B) Virtual image can be formed if $x > R$ and $\mu_2 < \mu_1$
(C) Virtual image is formed if $x < R$ and $\mu_2 > \mu_1$
(D) None of these

83. Identify the correct statement(s) related to the formation of images of a real object O placed at x from the pole of the concave surface, as shown in figure B

(A) If $\mu_2 > \mu_1$, then virtual image is formed for any value of x

(B) If $\mu_2 > \mu_1$, then virtual image is formed if $x < \frac{\mu_1 R}{\mu_1 - \mu_2}$

(C) If $\mu_2 < \mu_1$, then real image is formed for any value of x

(D) none of these

Section (F): Lens and Combination of Lenses/Lens and Mirrors.

84. wo symmetric double convex lenses A and B have same focal length, but the radii of curvature differ so that $R_A = 0.9\ R_B$. If $n_A = 1.63$, find n_B.

(A) 1.7 (B) 1.6
(C) 1.5 (D) 4/3

85. When a lens of power P (in air) made of material of refractive index μ is immersed in liquid of refractive index μ_0. Then the power of lens is:

(A) $\frac{\mu - 1}{\mu - \mu_0} P$ (B) $\frac{\mu - \mu_0}{\mu - 1} P$

(C) $\frac{\mu - \mu_0}{\mu - 1} \cdot \frac{P}{\mu_0}$ (D) none of these

86. An object is placed at 10 cm from a lens and real image is formed with magnification of 0.5. Then the lens is:

(A) concave with focal length of 10/3 cm
(B) convex with focal length of 10/3 cm
(C) concave with focal length of 10 cm
(D) convex with focal length of 10 cm

87. A thin linear object of size 1 mm is kept along the principal axis of a convex lens of focal length 10 cm. The object is at 15 cm from the lens. The length of the image is:

(A) 1 mm (B) 4 mm
(C) 2 mm (D) 8 mm

Question No. 88 to 90

A turnip sits before a thin converging lens, outside the focal point of the lens. The lens is filled with a transparent gel so that it is flexible; by squeezing its ends toward its center [as indicated in figure (a)], you can change the curvature of its front and rear sides.

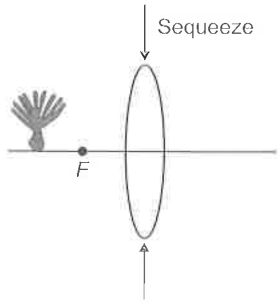


Figure (a)

88. When you squeeze the lens, the image

(A) moves towards the lens
(B) moves away from the lens
(C) shifts up
(D) remains as it is

89. The lateral height of image

(A) increases (B) decreases
(C) remains same (D) data insufficient

90. Suppose that the image must be formed on a card which is at a certain distance behind the lens [figure (b)] while you move the turnip away from the lens, then you should

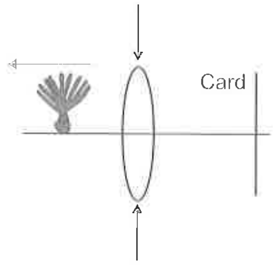
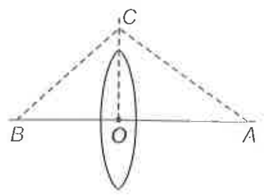


Figure (b)

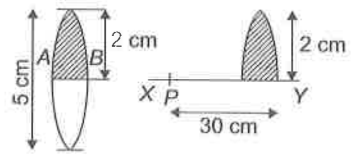
(A) decrease the squeeze of the lens
(B) increase the squeeze of the lens
(C) keep the card and lens as it is
(D) move the card away from the lens

91. If an object is placed at $A(OA > f)$; Where f is the focal length of the lens the image is found to be formed at B. A perpendicular is erected at O and C is chosen on it such that the angle $\angle BCA$ is a right angle. Then the value of f will be



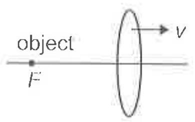
(A) AB/OC^2
(B) $(AC)(BC)/OC$
(C) $(OC)(AB)/AC + BC$
(D) OC^2/AB

92. A converging lens of focal length 20 cm and diameter 5 cm is cut along the line *AB*. The part of the lens shown shaded in the diagram is now used to form an image of a point *P* placed 30 cm away from it on the line *XY*. Which is perpendicular to the plane of the lens. The image of *P* will be formed.



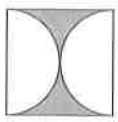
(A) 0.5 cm above *XY*
(B) 1 cm below *XY*
(C) on *XY*
(D) 1.5 cm below *XY*

93. A point object is kept at the first focus of a convex lens. If the lens starts moving towards right with a constant velocity, the image will



(A) always move towards right
(B) always move towards left
(C) first move towards right and then towards left.
(D) first move towards left and then towards right.

94. Two plano-convex lenses each of focal length 10 cm & refractive index $\frac{3}{2}$ are placed as shown. In the space left, water $\left(R.I = \frac{4}{3}\right)$ is filled. The whole arrangement is in air. The optical power of the system is (in dioptres):



(A) 6.67
(B) −6.67
(C) 33.3
(D) 20

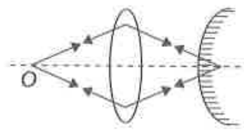
95. An object is placed at a distance of 10 cm from a co-axial combination of two lenses *A* and *B* in contact. The combination forms a real image three times the size of the object. If lens *B* is concave with a focal length of 30 cm, what is the nature and focal length of lens *A*?

(A) Convex, 12 cm
(B) Concave, 12 cm
(C) Convex, 6 cm
(D) Convex, 18 cm

96. The curvature radii of a concavo-convex glass lens are 20 cm and 60 cm. The convex surface of the lens is silvered. With the lens horizontal, the concave surface is filled with water. The focal length of the effective mirror is (μ of glass = 1.5, μ of water = 4/3)

(A) 90/13 cm
(B) 80/13 cm
(C) 20/3 cm
(D) 45/8 cm

97. An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is

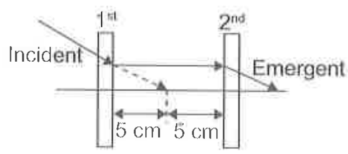


(A) 20 cm
(B) 10 cm
(C) 15 cm
(D) 30 cm

98. An object is placed in front of a thin convex lens of focal length 30 cm and a plane mirror is placed 15 cm behind the lens. If the final image of the object coincides with the object, the distance of the object from the lens is

(A) 60 cm
(B) 30 cm
(C) 15 cm
(D) 25 cm

99. Look at the ray diagram shown, what will be the focal length of the 1st and the 2nd lens, if the incident light ray passes without any deviation?



(A) −5 cm and −10 cm
(B) +5 cm and +10 cm
(C) −5 cm and +5 cm
(D) +5 cm and +5 cm

100. An object is placed in front of a symmetrical convex lens with refractive index 1.5 and radius of curvature 40 cm. The surface of the lens further away from the object is silvered. Under auto-collimation condition, the object distance is

(A) 20 cm (B) 10 cm
(C) 40 cm (D) 5 cm

101. A planoconvex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28 cm. When its curved surface is silvered and the plane surface not silverted, it is equivalent to a concave mirror of focal length 10 cm, then the refractive index of the material of the lens is:

(A) 9/14 (B) 14/9
(C) 17/9 (D) none

102. In the above question the radius of curvature of the curved surface of plano-convex lens is:

(A) $\frac{280}{9}$ cm (B) $\frac{80}{7}$ cm
(C) $\frac{39}{3}$ cm (D) $\frac{280}{11}$ cm

103. A screen is placed 90 cm from an object. The image of an object on the screen is formed by a convex lens at two different locations separated by 20 cm. The focal length of the lens is

(A) 18 cm (B) 21.4 cm
(C) 60 cm (D) 85.6 cm

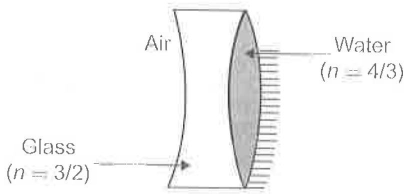
104. In the above problem, if the size of the image formed at the positions are 6 cm and 3 cm, then the highest of the object is

(A) 4.2 cm (B) 4.5 cm
(C) 5 cm (D) none of these

105. Which of the following cannot form real image of a real object?

(A) concave mirror (B) convex mirror
(C) plane mirror (D) diverging lens

106. The radius of curvature of the left & right surface of the concave lens are 10 cm & 15 cm respectively. The radius of curvature of the mirror is 15 cm.



(A) equivalent focal length of the combination is −18 cm
(B) equivalent focal length of the combination is +36 cm
(C) the system behaves like a concave mirror
(D) the system behaves like a convex mirror.

107. If a symmetrical biconcave thin lens is cut into two identical halves. They are placed in different ways as shown:

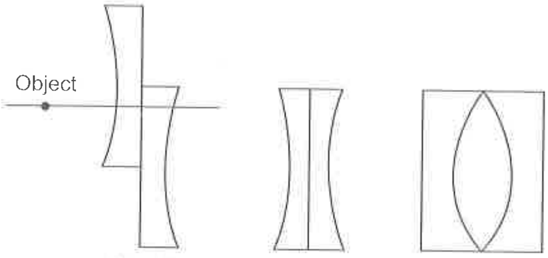


Fig (i) Fig (ii) Fig (iii)

(A) three images will be formed in case (i)
(B) two images will be formed in the case (i)
(C) the ratio of focal lengths in (ii) and (iii) is 1
(D) the ratio of focal lengths (in) (ii) and (iii) is 2

108. A convex lens forms an image of an object on screen. The height of the image is 9 cm. The lens is now displaced until an image is again obtained on the screen. The height of this image is 4 cm. The distance between the object and the screen is 90 cm.

(A) The distance between the two positions of the lens is 30 cm.
(B) The distance of the object from the lens is its first position is 36 cm.
(C) The height of the object is 6 cm.
(D) The focal length of the lens is 21.6 cm.

109. A thin lens with focal length f to be used as a magnifying glass. Which of the following statements regarding the situations is true?

(A) A converging lens may be used, and the object be placed at a distance greater than $2f$ from the lens.
(B) A diverging lens may be used, and the object be placed between f and $2f$ from the lens.
(C) A converging lens may be used, and the object be placed at a distance less than f from the lens.
(D) A diverging lens may be used, and the object be placed at any point other than the focal point.

110. Which of the following can form diminished, virtual and erect image of your face.
(A) Converging mirror (B) Diverging mirror
(C) Converging lens (D) Diverging lens

111. Which of the following quantities related to a lens depend on the wavelength of the incident light?
(A) Refractive index (B) Focal length
(C) Power (D) Radii of curvature

112. A man wishing to get a picture of a Zebra photographed a white donkey after fitting a glass with black streaks onto the objective of his camera.
(A) the image will look like a white donkey on the photograph.
(B) the image will look like a Zebra on the photograph.
(C) the image will be more intense compared to the case in which no such glass is used.
(D) the image will be less intense compared to the case in which no such glass is used.

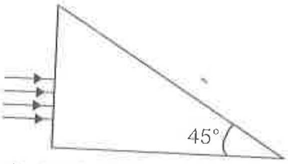
Section (G): Dispersion of Light

113. A thin prism P_1 with angle 4° made of glass of refractive index 1.54 is combined with another thin prism P_2 made of glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is:
(A) 3° (B) 2.6°
(C) 4° (D) 5.33°

14. Light of wavelength 4000 Å is incident at small angle on a prism of apex angle 4°. The prism has $n_v = 1.5$ and $n_r = 1.48$. The angle of dispersion produced by the prism in this light is:
(A) 0.2° (B) 0.08°
(C) 0.192° (D) none

5. Two lenses in contact made of materials with dispersive powers in the ratio 2:1, behaves as an achromatic lens of focal length 10 cm. The individual focal lengths of the lenses are:
(A) 5 cm, –10 cm (B) –5cm, 10 cm
(C) 10 cm, –20cm (D) –20 cm, 10 cm

A beam of light consisting of red, green and blue and is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will:



(A) separate part of the red colors from the green and blue colors.
(B) separate part of the blue colors from red and green colors.
(C) separate all the three colors from the other two colors.
(D) not separate even partially any colour from the other two colors.

117. It is desired to make an achromatic combination of two lenses (L_1 and L_2) made of materials having dispersive powers ω_1 and ω_2 ($<\omega_1$). If the combination of lenses is converging then
(A) L_1 is converging
(B) L_2 is converging
(C) Power of L_1 is greater than the power of L_2
(D) none of these

118. A narrow beam of white light goes through a slab having parallel faces
(A) The light never splits in different colour
(B) The emergent beam is white
(C) The light inside the slab is split into different colours
(D) The light inside the slab is white

119. By properly combining two prisms made of different materials, it is possible to
(A) have dispersion without average deviation
(B) have deviation without dispersion
(C) have both dispersion and average deviation
(D) have neither dispersion nor average deviation

120. Column -II shows the optical phenomenon that can be associated with optical components given in column-I. Note that column-I may have more than one matching options in column-II.

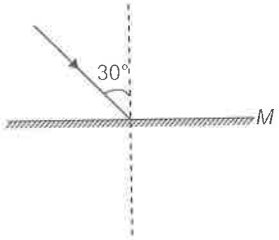
Column-I	Column-II
(i) Convex mirror	(A) Dispersion
(ii) Converging lens	(B) Deviation
(iii) Thin prism	(C) Real image of real object
(iv) Glass slab	(D) Virtual image of real object.

JEE Advanced

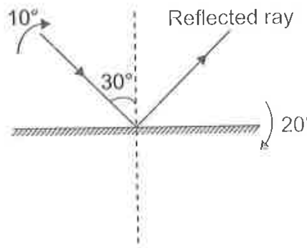
Level I

Section (A): Plane Mirror

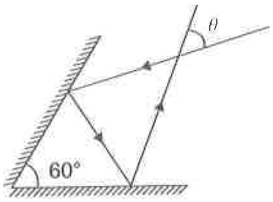
1. Find the angle of deviation (both clockwise and anticlockwise) suffered by a ray incident on a plane mirror, at an angle of incidence 30°.



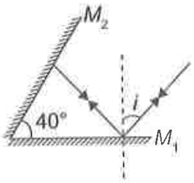
2. Figure shows a plane mirror onto which a light ray is incident. If the incident light ray is turned by 10° and the mirror by 20°, as shown, find the angle turned by the reflected ray.



3. A light ray is incident on a plane mirror, which after getting reflected strikes another plane mirror, as shown in figure. The angle between the two mirrors is 60°. Find the angle 'θ' shown in figure.

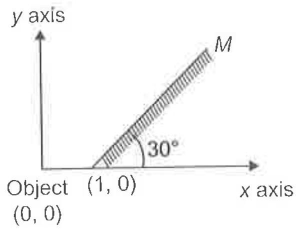


4. There are two plane mirror inclined at 40°, as shown. A ray of light is incident on mirror M_1. What should be the value of angle of incidence 'i' so that the light ray retraces its path after striking the mirror M_2.



5. Sun rays are incident at an angle of 24° to the horizon. How can they be directed parallel to the horizon using a plane mirror?

6. An object is placed at (0, 0) and a plane mirror is placed, inclined 30° with the x axis.



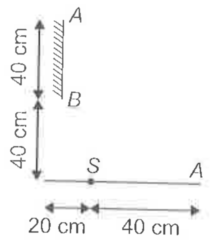
(a) Find the position of image.

(B) If the object starts moving with velocity $1\hat{i}$ m/ and the mirror is fixed find the velocity o image.

7. A point object is placed at (0, 0, 0) and a plane mirro is placed parallel to YZ plane at $x = 2$. Find th coordinate of image

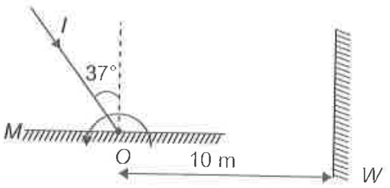
8. A plane mirror 50 cm long, is hung parallel to vertical wall of a room, with its lower edge 50 cn above the ground. A man stands infront of the mirro at a distance 2 m away from the mirror. If this eye are at a height 1.8 m above the ground, find the lengt of the floor between him & the mirror, visible to hir reflected from the mirror.

9. In figure shown AB is a plane mirror of length 40 c placed at a height 40 cm from ground. There is a ligl source S at a point on the ground. Find the minimu and maximum height of a man (eye height) required t see the image of the source if he is standing at a poi A on ground shown in figure.



10. A plane mirror of circular shape with radius $r = 20$ cm is fixed to the ceiling. A bulb is to be placed on the axis of the mirror. A circular area of radius $R = 1$ m on the floor is to be illuminated after reflection of light from the mirror. The height of the room is 3 m. What is maximum distance from the centre of the mirror and the bulb so that the required area is illuminated?

11. A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at frequency $9/\pi$ rev/sec. The light reflected the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37° find the speed of the spot (a point) on the wall?

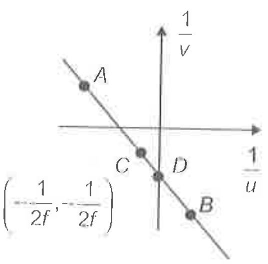


Section (B): Spherical Mirror

12. A rod of length 10 cm lies along the principal axis of a concave mirror of focal length 10 cm in such a way that the end farther from the pole is 20 cm away from it. Find the length of the image.

13. A concave spherical mirror forms a threefold magnified real image of a real object. The distance from the object of the image is 2.6 m. What is the radius of curvature of the mirror?

14. What does point A indicate? (f is magnitude of focal length, u and v are coordinates)



(i) Point A represents that the object is ________ (Real/Virtual) and the image is ________ (Real/Virtual)

(ii) Point A represents that $|u|$ is ________ (larger,/smaller) than $|v|$ and hence image size is ________ (larger/smaller) than the size of object.

15. Point B represents that the object is ________ (Real/Virtual) and the image is ________ (Real/Virtual)

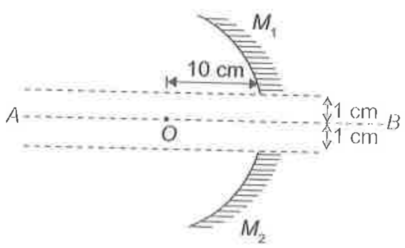
16. Point B represents that $|u|$ is ________ (larger,/smaller) than $|v|$ and hence image size is ________ (larger/smaller) than the size of object.

17. As we move from point C to D in the graph, the ________ (real/virtual) object moves from ________ to ________ and the ________ (real/virtual) image moves from ________ to ________ Show this movement in a diagram.

18. A point object is placed on the principal axis at 60 cm infront of a concave mirror of focal length 40 cm on the principal axis. If the object is moved with a velocity of 10 cm/s (a) along the principal axis, find the velocity of image (B) perpendicular to the principal axis, find the velocity of image at that moment.

19. A man uses a concave mirror for shaving. He keeps his face at a distance of 20 cm from the mirror and gets an image which is 1.5 times enlarged. Find the focal length of the mirror.

20. A concave mirror of focal length 20 cm is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1 cm from the previous principal axis AB. Find the distance between the images formed by the two parts?



21. A balloon is rising up along the axis of a concave mirror of radius of curvature 20 m. A ball is dropped

from the balloon at a height 15 m from the mirror when the balloon has velocity 20 m/s. Find the speed of the image of the ball formed by concave mirror after 4 seconds? [Take: $g = 10$ m/s^2]

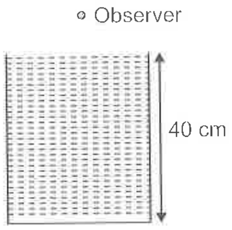
22. A thin rod of length $d/3$ is placed along the principal axis of a concave mirror of focal length $= d$ such that its image, which is real elongated, just touches the rod. Find the length of the image?

23. A point object is placed 33 cm from a convex mirror of curvature radius = 40 cm. A glass plate of thickness 6 cm and index 2.0 is placed between the object and mirror, close to the mirror. Find the distance of final image from the object?

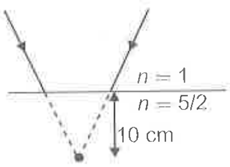
Section (C): Refraction in General, Refraction at Plane Surface and T.I.R.

24. A light ray falling at an angle of 60° with the surface of a clean slab of ice of thickness 1.00 m is refracted into it at an angle of 15°. Calculate the time taken by the light rays to cross the slab. Speed of light in vacuum = 3×10^8 m/s.

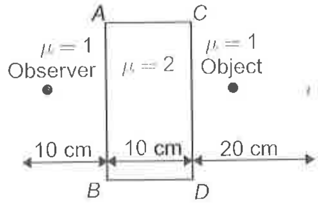
25. An observer in air ($n = 1$) sees the bottom of a beaker filled with water ($n = 4/3$) upto a height of 40 cm. What will be the depth felt by this observer.



26. Rays incident on an interface would converge 10 cm below the interface if they continued to move in straight lines without bending. But due to refraction, the rays will bend and meet somewhere else. Find the distance of meeting point of refracted rays below the interface, assuming the rays to be making small angles with the normal to the interface.

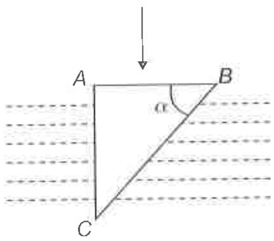


27. Find the apparent distance between the observer a the object shown in the figure and shift in the positi of object.

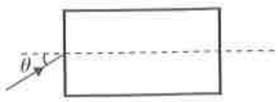


28. Light attempts to go from glass ($\mu = 3/2$) to air. Fi the angle of incidence for which the angle of deviati is 90°.

29. A rectangular glass wedge is lowered into water (μ 4/3). The refractive index of glass is $\mu_g = 1.5$. At wh angle α will the beam of light normally incident on reach AC entirely?

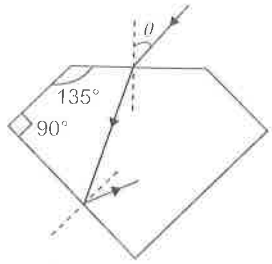


30. A long solid cylindrical glass rod of refractive in 3/2 is immersed in a liquid of refractive index $3\sqrt{3}$ The end of the rod are perpendicular to the central a of the rod. a light enters one end of the rod at central axis as shown in the figure. Find the maxim value of angle θ for which internal reflection occ inside the rod?

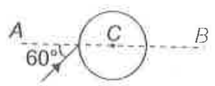


31. A slab of glass of thickness 6 cm and index 1. place somewhere in between a concave mirror an point object, perpendicular to the mirror's optical a The radius of curvature of the mirror is 40 cm. If reflected final image coincides with the object, t find the distance of the object from the mirror?

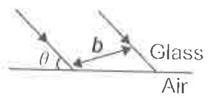
32. A ray of light enters a diamond ($n = 2$) from air and is being internally reflected near the bottom as shown in the figure. Find maximum value of angle θ possible?



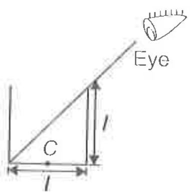
33. A ray of light on a transparent sphere with centre at C as shown in figure. The ray emerges from the sphere parallel to line AB. Find the refractive index of the sphere.



34. A beam of parallel rays of width b propagates in glass at an angle θ to its plane face. The beam width after it goes over to air through this face is ________ if the refractive index of glass is μ.



35. A cubical tank (of edge l) and position of an observer are shown in the figure. When the tank is empty, edge of the bottom surface of the tank is just visible. An insect is at the centre C of its bottom surface. To what height a transparent liquid of refractive index $\mu = \sqrt{5/2}$ must be poured in the tank so that the insect will become visible?



36. Light from a luminous point on the lower face of a 2 cm thick glass slab, strikes the upper face and the totally reflected rays outline a circle of radius 3.2 cm on the lower face. What is the refractive index of the glass.

Section (D): Refraction by Prism

37. A prism ($n = 2$) of apex angle 90° is placed in air ($n = 1$). What should be the angle of incidence so that light ray strikes the second surface at an angle of 60°

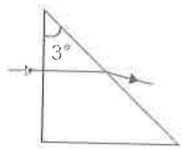
38. Ref. index of a prism ($A = 60°$) placed in air ($n = 1$) is $n = 1.5$. Light ray is incident on this prism at an angle of 60°. Find the angle of deviation. State whether this is a minimum deviation.

Given: $\sin^{-1}\frac{1}{\sqrt{3}} = 35°$, $\sin^{-1} 0.4 = 25°$, $\sin^{-1} 0.6 = 37°$

39. The cross section of a glass prism has the form of an equilateral triangle. A ray is incident onto one of the faces perpendicular to it. Find the angle θ between the incident ray and the ray that leaves the prism. The refractive index of glass is $\mu = 1.5$.

40. The angle of refraction of a prism is 60°. A light ray emerges from the prism at the same angle as it is incident on it. The refractive index of the prism is 1.5. Determine the angle by which the ray is deflected from its initial direction as a result of its passage through the prism.

41. Find the angle of deviation suffered by the light ray shown in figure for following two condition The refractive index for the prism material is $\mu = 3/2$.



(i) When the prism is placed in air ($\mu = 1$)
(ii) When the prism is placed in water ($\mu = 4/3$)

42. A prism of refractive index $\sqrt{2}$ has a refracting angle of 30°. One of the refracting surface of the prism is polished. For the beam of monochromatic light to retrace its path, find the angle of incidence on the refracting surface.

43. An equilateral prism deviates a ray through 23° for two angles of incidence differing by 23°. Find μ of the prism?

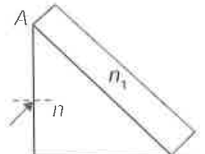
44. A equilateral prism provides the least deflection angle 46° in air. Find the refracting index of an unknown

liquid in which same prism gives least deflection angle of 30°.

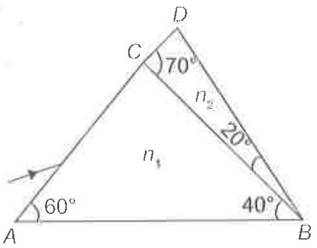
45. A right angle prism (45° – 90° – 45°) of refractive index n has a plate of refractive index $n_1 (n_1 < n)$ cemented to its diagonal face. The assembly is in air. a ray is incident on AB (see the figure).

(i) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(ii) Assuming $n = 1.352$. Calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.



46. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prisms are as shown n_1 and n_2 depend on λ, the wavelength of light according to $n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$ where λ is in nm.



(i) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

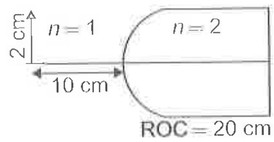
(ii) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms in minimum.

Section (E): Refraction by Spherical Surface

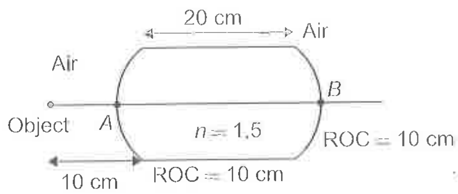
47. A spherical surface of radius 30 cm separates two transparent media A and B with refractive indices 4/3 and 3/2 respectively. The medium A is on the convex side of the surface. Where should a point object be placed in medium A so that the paraxial rays become parallel after refraction at the surface?

48. A narrow parallel beam of light is incident par axially on a solid transparent sphere of radius r. What should be the refractive index if the beam is to be focused (a) At the surface of the sphere, (B) at the centre of the sphere.

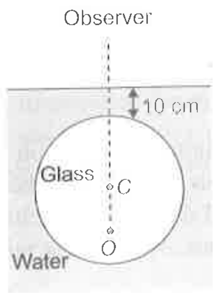
49. An extended object of size 2 cm is placed at a distance of 10 cm in air ($n = 1$) from pole, on the principal axis of a spherical curved surface. The medium on the other side of refracting surface has refractive index $n = 2$. Find the position, nature and size of image formed after single refraction through the curved surface.



50. An object is placed 10 cm away from a glass piece ($n = 1.5$) of length 20 cm bound by spherical surfaces of radii of curvature 10 cm. Find the position of final image formed after twice refractions.

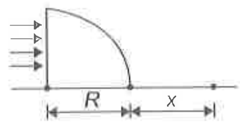


51. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 5 cm. The bubble is 7.5 cm below the surface of the glass. The sphere is placed inside water ($\mu = \frac{4}{3}$) such that the top surface of glass is 10 cm below the surface of water. The bubble is viewed normally from air. Find the apparent depth on the bubble.



52. A narrow parallel beam of light is incident on a transparent sphere of refractive index 'n'. If the beam finally gets focussed at a point situated at a distance = 2 × (radius of sphere) from the centre of the sphere, then find n?

53. A uniform, horizontal beam of light is incident upon a quarter cylinder of radius $R = 5$ cm, and has a refractive index $2/\sqrt{3}$. A patch on the table for a distance 'x' from the cylinder is unilluminated, find the value of 'x'?

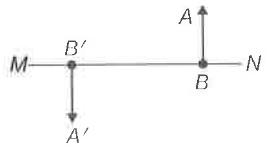


Section (F): Lens and Combination of Lenses/Lens and Mirrors.

54. A double convex lens has focal length 50 cm. The radius of curvature of one of the surfaces is double of the other. Find the radii, if the refractive index of the material of the lens is 2.

55. Lenses are constructed by a material of refractive index 1.50. The magnitude of the radii of curvature are 20 cm and 30 cm. Find the focal lengths of the possible lenses with the above specifications.

56. Given an optical axis MN & the positions of a real object $A B$ and its image $A' B'$, determine diagrammatically the position of the lens (its optical centre O) and its foci. Is it a converging or diverging lens? Is the image real or virtual?



57. A thin lens made of a material of refractive index μ_2 has a medium of refractive index μ_1 on one side and a medium of refractive index μ_3 on the other side. The lens is biconvex and the two radii of curvature has equal magnitude R. A beam of light travelling parallel to the principal axis is incident on the lens. Where will the image be formed if the beam is incident from (a) the medium μ_1 and (B) from the medium μ_3?

58. An object of height 6 cm is set at right angles to the optical axis of a double convex lens of optical power $5d$ & 25 cm away from the lens. Determine the focal length of the lens, the position of the image, the linear magnification of the lens, and the height of the image formed by it.

59. A 5.0 dioptre lens forms a virtual image which is 4 times the object placed perpendicularly on the principal axis of the lens. Find the distance of the object from the lens.

60. A converging lens and a diversing mirror are placed at a separation of 15 cm. The focal length of the lens is 25 cm and that of the mirror is 40 cm. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis?

61. A converging lens of focal length 15 cm and a converging mirror of length 10 cm are placed 50 cm apart. if a object of length 2.0 cm is placed 30 cm from the lens farther away from the mirror, where will the final image form and what will be the size of the final image?

62. 2 identical thin converging lenses brought in contact so that their axes coincide are placed 12.5 cm from an object. What is the optical power of the system & each lens, if the real image formed by the system of lenses is four times as large as the object?

63. A point object is placed at a distance of 15 cm from a convex lens. The image is formed on the other side at a distance of 30 cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal lengths of the two lenses.

64. A convex & a concave lens are brought it close contact along their optical axes. The focal length of the convex lens is 10 cm. When the system is placed at 40 cm from an object, a sharp image of the object is formed on a screen on the other side of the system. Determine the optical power of the concave lens if the distance between the object & the screen is 1.6 m.

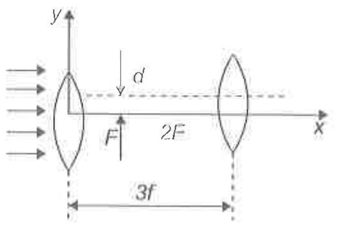
65. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of

thickness t and refractive index 1.5 is inserted between the lens and object. The image is formed at infinity. Find the thickness t?

66. An object is kept at a distance of 16 cm from the thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens the image formed is virtual. If the size of the image formed are equal, then find the focal length of the lens?

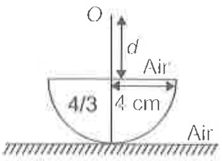
67. A thin convex lens forms a real image of a certain object 'p' times its size. The size of real image becomes 'q' times that of object when the lens is moved nearer to the object by a distance 'a' find focal length of the lens?

68. In the figure shown, the focal length of the two thin convex lenses is the same $= f$. They are separated by a horizontal distance $3f$ and their optical axes are displaced by a vertical separation 'd' ($d << f$), as shown. Taking the origin of coordinates O at the centre of the first lens, find the x and y coordinates of the point where a parallel beam of rays coming from the left finally get focussed?



69. A point source of light is kept at a distance of 15cm from a converging lens, on its optical axis. The focal length of the lens is 10 cm and its diameter is 3 cm. A screen is placed on the other side of the lens, perpendicular to the axis of lens, at a distance 20 cm from it. Then find the area of the illuminated part of the screen?

70. A glass hemisphere of refractive index 4/3 and of radius 4 cm is placed on a plane mirror. A point object is placed at distance 'd' on axis of this sphere as shown. If the final image be at infinity, find the value of 'd'.



71. A double convex lens has focal length 25.0 cm in air. The radius of one of the surfaces is double of the other. Find the radii of curvature it the refractive index of the material of the lens is 1.5.

72. A plano convex lens ($\mu = 1.5$) has a maximum thickness of 1 mm. If diameter of its aperture is 4 cm. Find

(i) Radius of curvature of curved surface

(ii) its focal length in air

73. A plano-convex lens, when silvered on the plane side, behaves like a concave mirror of focal length 30 cm. When it is silvered on the convex side, it behaves like a concave mirror of focal length 10 cm. Find the refractive index of the material of the lens.

Section (G): Dispersion of Light

74. A certain material has refractive indices 1.56, 1.60 and 1.68 for red, yellow and violet light respectively.

(a) Calculate the dispersive power.

(b) Find the angular dispersion produced by a thin prism of angle 6° made of this material.

75. A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1.620 and 1.518 respectively. If the refracting angle of the flint prism is 6.0°, what would be the refracting angle of crown prism?

Level II

Section (B): Spherical Mirror

1. An observer whose least distance of distinct vision is 'd', views his own face in a convex mirror of radius of curvature 'r'. Prove that magnification produced cannot exceed $\dfrac{r}{d+\sqrt{d^2+r^2}}$.

2. A thief is running away in a car with velocity of 20 m/s. A police jeep is following him, which is sighted by thief in his rear view mirror which is a convex mirror of focal length 10 m. He observes that the image of jeep is moving towards him with a velocity of 1 cm/s. If the magnification of the mirror for the jeep at that time is 1/10. Find

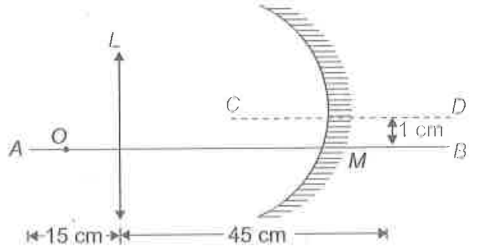
(a) actual speed of jeep

(b) rate at which magnification is changing
Assume that police jeep is on axis of the mirror.

3. A luminous point P is inside a circle. A ray enters from P and after two reflections by the circle, return to P. If θ be the angle of incidence, a the distance of P from the centre of the circle and b the distance of the centre from the point where the ray in its course crosses the diameter through P, prove that $\tan\theta = \sqrt{\dfrac{a-b}{a+b}}$.

4. An object is kept on the principal axis of a convex mirror of focal length 10 cm at a distance of 10 cm from the pole. The object starts moving at a velocity 20 mm/sec towards the mirror at angle 30° with the principal axis. What will be the speed of its image and direction with the principal axis at that instant?

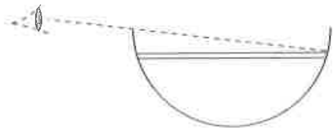
5. In the figure shown L is a converging lens of focal length 10 cm and M is a concave mirror of radius of curvature 20 cm. A point object O is placed in front of the lens at a distance 15 cm. AB and CD are optical axes of the lens and mirror respectively. Find the distance of the final image formed by this system from the optical centre of the lens. The distance between CD & AB is 1 cm.



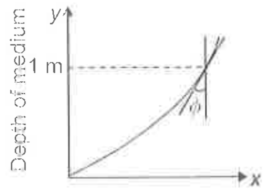
Section (C): Refraction in General, Refraction at Plane Surface and T.I.R.

6. A surveyor on one bank of canal observed the image of the 4 inch and 17 ft marks on a vertical staff, which is partially immersed in the water and held against the bank directly opposite to him, coincides. If the 17 ft mark and the surveyor's eye are both 6 ft above the water level, estimate the width of the canal, assuming that the refractive index of the water is 4/3.

7. A circular disc of diameter d lies horizontally inside a metallic hemispherical bowl radius a. The disc is just visible to an eye looking over the edge. The bowl is now filled with a liquid of refractive index μ. Now, the whole of the disc is just visible to the eye in the same position. Show that $d = 2a\dfrac{(\mu^2-1)}{(\mu^2+1)}$.



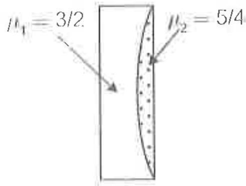
8. A ray of light travelling in air is incident at grazing angle (incidence angle = 90°) on a medium whose refractive index depends on the depth of the medium. The trajectory of the light in the medium is a parabola, $y = 2x^2$. Find, at a depth of 1 m in the medium.



(i) the refractive index of the medium and
(ii) angle of incidence ϕ.

9. Two thin similar watch glass pieces are joined together, front to front, with rear portion silvered and the combination of glass pieces is placed at a distance $a = 60$ cm from a screen. A small object is placed normal to the optical axis of the combination such that its two times magnified image is formed on the screen. If air between the glass pieces is replaced by water ($\mu = 4/3$), calculate the distance through which the object must be displaced so that a sharp image is again formed on the screen.

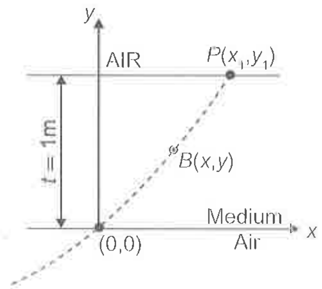
10. A thin plano-convex lens fits exactly into a plano concave lens with their plane surface parallel to each other as shown in the figure. The radius of curvature of the curved surface $R = 30$ cm. The lens are made of difference material having refractive index $\mu_1 = 3/2$ and $\mu_2 = 5/4$ as shown in figure.



(i) if plane surface of the plano-convex lens is silvered, then calculate the equivalent focal length of this system and also calculate the nature of this equivalent mirror.
(ii) An object having transverse length 5 cm in placed on the axis of equivalent mirror (in part 1), at a

distance 15 cm from the equivalent mirror along principal axis. Find the transverse magnification produced by equivalent mirror.

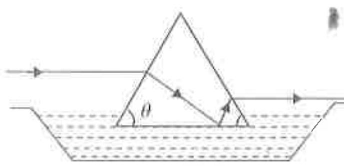
11. A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ (see figure). The point of incidence is the origin A (O, O). The medium has a variable index of refraction $n(y)$ given by: $n(y) = [ky^{3/2} + 1]^{1/2}$, where $k = 1.0\ m^{-3/2}$. The refractive index of air is 1.0. **[JEE 1995]**



(i) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incident angle at that point.
(ii) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.
(iii) Determine the coordinates (x_1, y_1) of the point P, where the ray the ray intersects the upper surface of the slab-air boundary.
(iv) Indicate the path of the ray subsequently.

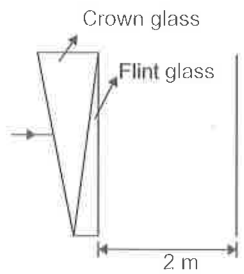
Section (D): Refraction by Prism

12. An isosceles triangular glass prism stands with its base in water as shown. The angles that its two equal sides make with the base are θ each. An incident ray of light parallel to the water surface internally reflects at the glass-water interface and subsequently re-emerges into the air. Taking the refractive indices of glass and water to be 3/2 and 4/3 respectively, show that θ must be at least $\tan^{-1}\frac{2}{\sqrt{17}}$ or 25.9°.



13. A parallel beam of light falls normally on the first face of a prism of small angle. At the second face it is partly transmitted and partly reflected, the reflected beam striking at the first face again, and emerging from it in a direction making an angle 6°30′ with the reversed direction of the incident beam. The refracted beam is found to have undergone a deviation of 1°15′ from the original direction. Find the refractive index of the glass and the angle of the prism.

14. The refractive indices of the crown glass for violet and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. A prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other thin flint glass prism is combined with a crown glass prism such that the net mean deviation is 1.5° anticlockwise.



(i) Determine the angle of the flint glass prism.
(ii) A screen is placed normal to the emerging beam at a distance of 2 m from the prism combination. Find the distance between red and violet spot on the screen. Which is the topmost colour on screen.

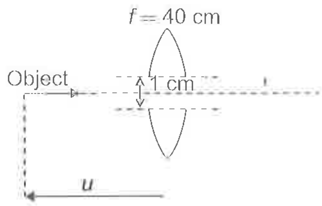
Section (E): Rafraction by Spherical Surface

15. A concave mirror has the form of a hemisphere with a radius of $R = 60$ cm. A thin layer of an unknown transparent liquid is poured into the mirror. The mirror-liquid system forms one real image and another real image is formed by mirror alone, with the source in a certain position. One of them coincides with the source and the other is at a distance of $l = 30$ cm from source. Find the possible value(s) refractive index μ of the liquid.

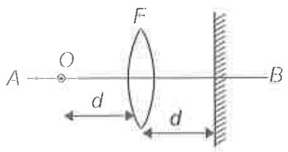
Section (F): Lens and Combination of Lenses/Lens and Mirrors

16. In the figure shown, find the relative speed of approach/separation of the two final images formed after the light rays pass through the lens, at the moment when $u = 30$ cm. The speed object = 4 cm/s. The two lens

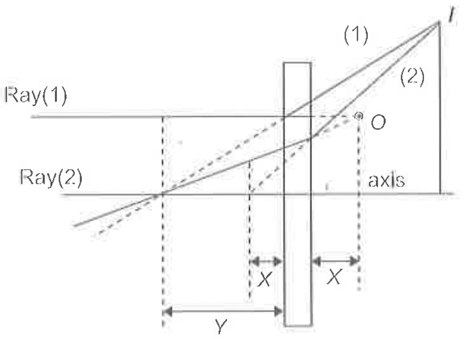
halves are placed symmetrically w.r.t. the moving object.



17. In the figure shown 'O' is point object. AB is principal axis of the converging lens of focal length F. Find the distance of the final image from the lens.

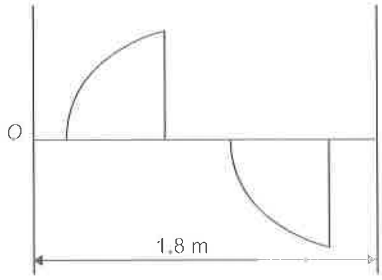


18. The rectangular box shown is the place of lens. By looking at the ray diagram, answer the following questions.
   (i) If X is 5 cm then what is the focal length of the lens?
   (ii) If the point O is 1 cm above the axis then what is the position of the image? Consider the optical center of the lens to be the origin.

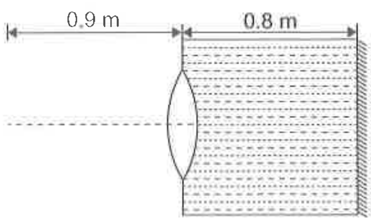


19. Two identical convex lenses L_1 and L_2 are placed at a distance of 20 cm from each other on the common principal axis. The focal length of each lens is 15 cm and the lens L_2 is to the right of lens L_1. A point object is placed at a distance of 20 cm on the left of lens L_1, on the common axis of two lenses. Find, where a convex mirror of radius of curvature 5 cm should be placed so that the final image coincides with the object?

20. A thin plano-convex. Lens of focal length F is split into two halves, one of the halves is shifted along the optical axis. The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the lens and separation between the two halves. Draw the ray diagram for image formation. **[JEE 1996]**



21. A thin equiconvex lens of glass of refractive index $\mu = 3/2$ & of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water ($\mu = 4/3$). On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system. **[JEE 1997]**



Previous Year Questions

JEE Main

1. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should to be in sharp focus of film? **[AIEEE 2012]**
(A) 7.2 m (B) 2.4 m
(C) 3.2 m (D) 5.6 m

2. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is **[AIEEE 2011]**

(A) $\left(1-\frac{1}{\mu_1}\right)h_2+\left(1-\frac{1}{\mu_2}\right)h_1$

(B) $\left(1+\frac{1}{\mu_1}\right)h_1+\left(1+\frac{1}{\mu_2}\right)h_2$

(C) $\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$

(D) $\left(1+\frac{1}{\mu_1}\right)h_2-\left(1+\frac{1}{\mu_2}\right)h_1$

3. When monochromatic red light is used instead of blue light in a convex lens, its focal length will **[AIEEE 2011]**
(A) does not depend on colour of light
(B) increase
(C) decrease
(D) remain same

4. **Statement I**: On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.
Statement II: The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. **[AIEEE 2011]**
(A) Statement I is false, Statement II is true
(B) Statement I is true, Statement II is true
(C) Statement I is true, Statement II is true, Statement II is the correct explanation of Statement I
(D) Statement I is true, Statement II is true; Statement II is not correct explanation of Statement I

5. Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is **[AIEEE 2011]**
(A) 45° (B) 60°
(C) 75° (D) 30°

6. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE 2011]**
(A) $\frac{1}{15}$ m/s (B) 10 m/s
(C) 15 m/s (D) $\frac{1}{10}$ m/s

7. As the beam enters the medium, it will **[AIEEE 2010]**
(A) diverge
(B) converge
(C) diverge near the axis and converge near the periphery
(D) travel as a cylindrical beam

8. The speed of light in the medium is **[AIEEE 2010]**
(A) minimum on the axis of the beam
(B) the same everywhere in the beam
(C) directly proportional to the intensity
(D) maximum on the axis of the beam

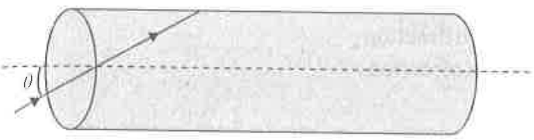
9. In an optics experiments, with the position of the object fixed, a student varies the positions of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between

the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be **[AIEEE 2009]**

(A) $(2f, 2f)$ (B) $\left(\frac{f}{2}, \frac{f}{2}\right)$

(C) (f, f) (D) $(4f, 4f)$

10. A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$ It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

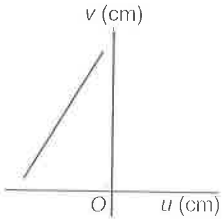


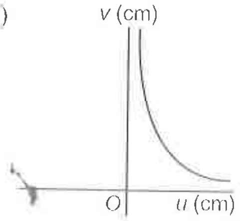
The incident angle θ for which the light ray grazes along the wall of the rod is **[AIEEE 2009]**

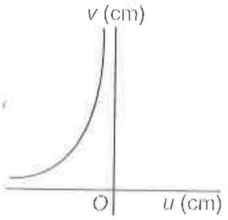
(A) $\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

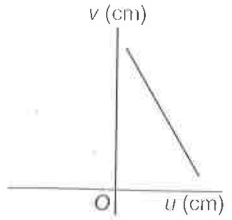
(C) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

11. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like **[AIEEE 2008]**

(A)


(B)


(C)


(D)


12. Two lenses of power $-15D$ and $+5D$ are in Cotact with each other. The focal length of the combination is **[AIEEE 2007]**

(A) -20 cm (B) -10 cm

(C) $+20$ cm (D) $+10$ cm

13. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then **[AIEEE 2006]**

(A) $D_1 < D_2$ (B) $D_1 = D_2$

(C) D_1 can be less than or greater than D_2 depending upon the angle of prism

(D) $D_1 > D_2$

14. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the water surface, the radius of this circle in cm is **[AIEEE 2005]**

(A) $36\sqrt{7}$ (B) $\frac{36}{\sqrt{7}}$

(C) $36\sqrt{5}$ (D) $4\sqrt{5}$

15. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm] **[AIEEE 2005]**

(A) 5 m (B) 1 m

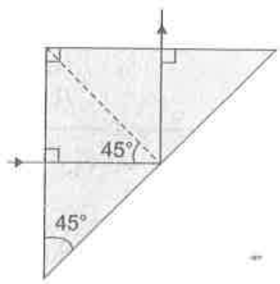
(C) 6 m (D) 3 m

16. A thin glass (refractive index 1.5) lens has optical power of $-5D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be **[AIEEE 2005]**

(A) $1D$ (B) $-1D$

(C) $25D$ (D) $-25D$

17. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that for the refractive index n as **[AIEEE 2005]**



(A) $n < \frac{1}{\sqrt{2}}$ (B) $n > \sqrt{2}$

(C) $n > \frac{1}{\sqrt{2}}$ (D) $n < \sqrt{2}$

18. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image the size of the object? **[AIEEE 2004]**
(A) 20 cm (B) 30 cm
(C) 60 cm (D) 80 cm

19. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is **[AIEEE 2004]**

(A) $\sin^{-1}(n)$ (B) $\sin^{-1}\left(\frac{1}{n}\right)$

(C) $\tan^{-1}\left(\frac{1}{n}\right)$ (D) $\tan^{-1}(n)$

20. The image formed by an objective of a compound microscope is **[AIEEE 2003]**
(A) virtual and diminished
(B) real and diminished
(C) real and enlarged
(D) virtual and enlarged

21. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by **[AIEEE 2003]**
(A) Rayleigh Jeans law
(B) Planck's law of radiation
(C) Stefan's law of radiation
(D) Wien's law

22. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**
(A) 60° (B) 90°
(C) 120° (D) 30°

23. If two mirrors are kept at 60° to each other, then the number of images formed by them is **[AIEEE 2002]**
(A) 5 (B) 6
(C) 7 (D) 8

24. Wavelength of light used in an optical instrument are $\lambda_1 = 4000\,\text{Å}$ and $\lambda_2 = 5000\,\text{Å}$, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is **[AIEEE 2002]**
(A) 16:25 (B) 9:1
(C) 4:5 (D) 5:4

25. An astronomical telescope has a large aperture to **[AIEEE 2002]**
(A) reduce spherical aberration
(B) have high resolution
(C) increase span of observation
(D) have low dispersion

26. Which of the following is used in optical fibres? **[AIEEE 2002]**
(A) Total internal reflection
(B) Scattering
(C) Diffraction
(D) Refraction

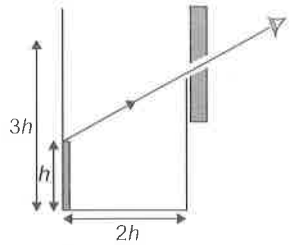
27. A thin convex lens made from crown grass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal length f_1 and f_2 respectively. The correct relation between the focal lengths is: **[JEE Main 2014]**
(A) $f_2 > f$ and f_1 becomes negative
(B) f_1 and f_2 both become negative
(C) $f_1 = f_2 < f$
(D) $f_1 > f$ and f_2 becomes negative

28. A green light is incident from the water to the air - water interface at the critical angle (θ). Select the correct statement. **[JEE Main 2014]**
(A) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(B) The entire spectrum of visible light will come out of the water at various angles to the normal.
(C) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(D) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

JEE Advanced

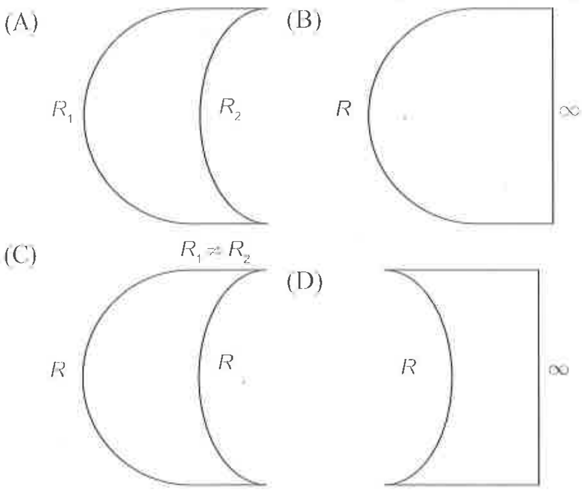
1. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the

lower end of the rod. Then the refractive index of the liquid is **[JEE (Scr) 2002]**

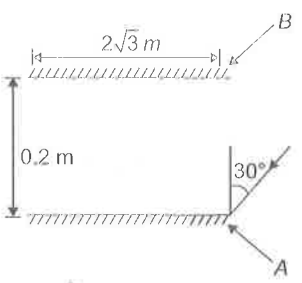


(A) 5/2 (B) $\sqrt{5/2}$
(C) $\sqrt{3/2}$ (D) 3/2

2. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surface of the lenses are as given in the diagrams. **[JEE (Scr)2002]**



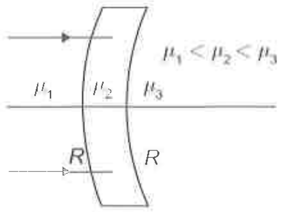
3. Two plane mirrors *A* and *B* are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of *A*. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **[JEE (Scr) 2002]**



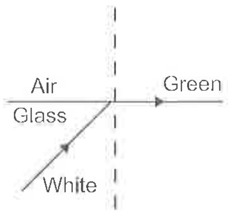
(A) 28 (B) 30
(C) 32 (D) 34

4. A convex lens of focal length 30 cm forms an image of height 2 cm for an object situated at infinity. If a concave lens of focal length 20 cm is placed coaxially at a distance of 26 cm is front of convex lens then size image would be **[JEE (Scr) 2003]**
(A) 2.5 cm (B) 5.0
(C) 1.25 (D) None

5. A meniscus lens is made of a material of refractive index μ_2. Both its surfaces have radii of curvature *R*. It has two different media of refractive indices μ_1 and μ_3 respectively, on its two sides (see figure). Calculate its focal length for $\mu_1 < \mu_2 < \mu_3$, when light is incident on it as shown. **[JEE 2003]**

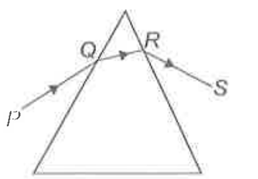


6. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains **[JEE (Scr) 2004]**



(A) yellow, orange, red
(B) violet, indigo, blue
(C) all colours
(D) all colour except green

7. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true? **[JEE (Scr) 2004]**

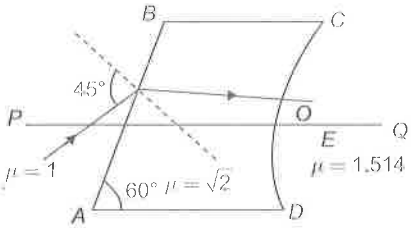


(A) PQ is horizontal
(B) QR is horizontal
(C) RS is horizontal
(D) Either PQ or RS is horizontal.

8. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is **[JEE (Scr) 2004]**

(A) 2 cm (B) 4 cm
(C) 6 cm (D) 12 cm

9. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal. **[JEE 2004]**

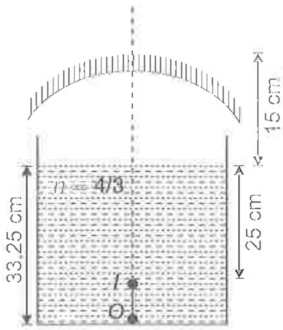


10. An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **[JEE 2004]**

11. The ratio of powers of a thin convex and thin concave lens is 3/2 and equivalent focal length of their combination is 30 cm. Then their focal lengths respectively are **[JEE (Scr) 2005]**

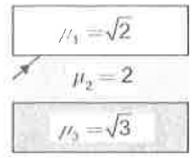
(A) 75, –50 (B) 75, 50
(C) 10, –15 (D) –75, 50

12. Figure shows object O. Final image I is formed after two refractions and one reflection is also shown in figure. Find the focal length of mirror. (in cm): **[JEE (Scr) 2005]**

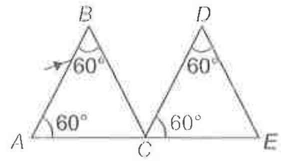


(A) 10 (B) 15
(C) 20 (D) 25

13. What will be the minimum angle of incidence su that the total internal reflection occurs on both t surfaces? **[JEE 200**

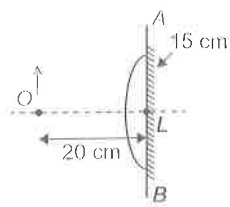


14. Two identical prisms of refractive index $\sqrt{3}$ are ke as shown in the figure. A light ray strikes the first pris at face AB. Find, **[JEE 200**



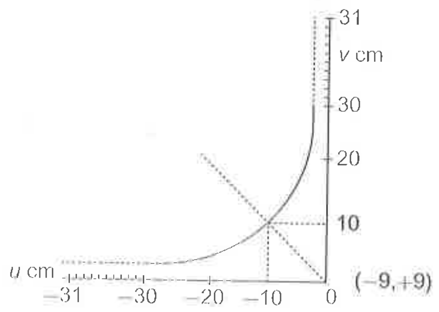
(a) the angle of incidence, so that the emergent r from the first prism has minimum deviation.
(b) through what angle the prism DCE should rotated about C so that the final emergent ray al has minimum deviation.

15. A point object is placed at a distance of 20 cm fro a thin plano-convex lens of focal length 15 cm, if t plane surface is silvered. The image will form at **[JEE 200**



(A) 60 cm left of AB (B) 30 cm left of AB
(C) 12 cm left of AB (D) 60 cm right of AB

16. Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is **[JEE 2006]**



(A) 0.50 ± 0.05 cm (B) 0.50 ± 0.10 cm
(C) 5.00 ± 0.05 cm (D) 5.00 ± 0.10 cm

17. Parallel rays of light from Sun falls on a biconvex lens of focal length f and the circular image of radius r is formed on the focal plane of the lens. Then which of the following statement is correct? **[JEE 2006]**
(A) Area of image πr^2 directly proportional to f
(B) Area of image πr^2 directly proportional to f^2
(C) Intensity of image increases if f is increased.
(D) If lower half of the lens is covered with black paper area of image will become half.

18. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_0, respectively. Then **[JEE 2006]**

Column 1	Column 2
(A) Intensity of light received by lens	(P) Radius of aperture R
(B) Angular magnification	(Q) Dispersion of lens
(C) Length of telescope	(R) focal length f_0, f_e
(D) Sharpness of image	(S) spherical aberration

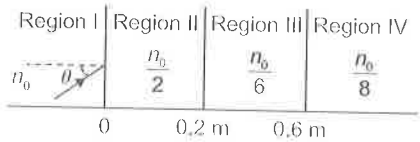
19. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be **[JEE 2007]**
(A) only a reflected ray and no refracted ray
(B) only a refracted ray and no reflected ray
(C) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(D) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

20. Statement I
The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose size are very small compared to their radii of curvature **[JEE 2007]**
because

Statement II
Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
(A) Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I
(B) Statement I is True, Statement II is True; Statement II is NOT correct explanation for Statement I
(C) Statement I is True, Statement II is False
(D) Statement I is False, Statement II is True

21. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **[JEE 2008]**
(A) 30° for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30° for both the colours

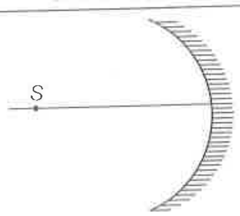
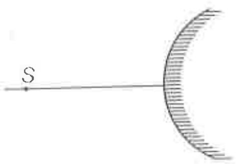
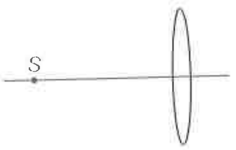
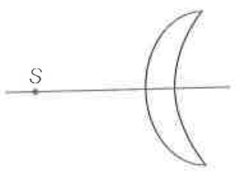
22. A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is Figure: **[JEE 2008]**



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$
(C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

23. An optical component and an object S placed along its optic axis are given in **Column-I**. The distance between the object and the component can be varied.

The properties of images are given in **Column -II.** Match all the properties of images from **Column-II** with the appropriate components given in **Column -I.** Indicate your answer by darkening the appropriate bubbles the 4 × 4 matrix given in the ORS. **[JEE 2008]**

	Column-I	Column-II
(A)		(p) Real image
(B)		(q) Virtual image
(C)		(r) Magnified image
(D)		(s) Image at infinity

24. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take g = 10 m/s^2] **[JEE 2009]**

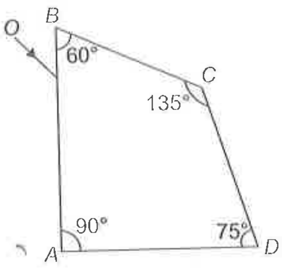
(A) 9 m/s (B) 12 m/s
(C) 16 m/s (D) 21.33 m/s

25. A student performed the experiment of determination of focal length of a concave mirror by $u-v$ method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are (42, 56) (48, 48), (60, 40), (66, 33) (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are) **[JEE 2009]**

(A) (42, 56) (B) (48, 48)
(C) (66, 38) (D) (78, 39)

26. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?

(A) The ray gets totally internally reflected at face CD
(B) The ray comes out through face AD
(C) The angle between the incident ray and the emergent ray is 90°
(D) The angle between the incident ray and the emergent ray is 120° **[JEE 2010]**



27. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm the magnification of its image changes from m_{25} to m_{50} The ratio $\frac{m_{25}}{m_{50}}$ is **[JEE 2010]**

28. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is: **[JEE 2010]**

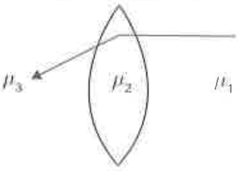
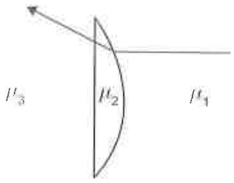
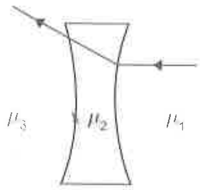
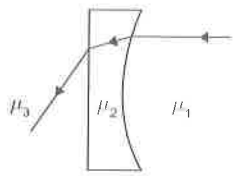
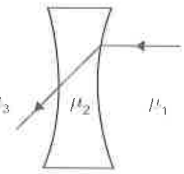
(A) virtual and at a distance of 16 cm from the mirror
(B) real and at a distance of 16 cm from the mirror
(C) virtual and at a distance of 20 cm from the mirror
(D) real and at a distance of 20 cm from the mirror

29. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **[JEE 2010]**

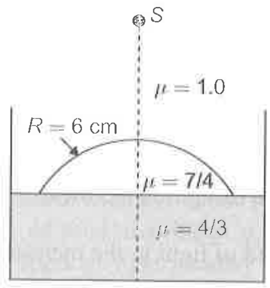
30. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds.

What is the speed of the object in km per hours?
[JEE 2010]

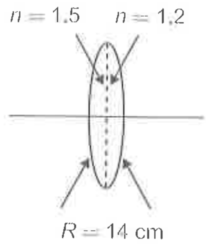
31. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figure in **Column - II.** A ray traversing these media is also shown in the figure. In **Column I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagrams shown in **Column II** **[JEE 2010]**

Column I	Column II
(A) $\mu_1 < \mu_2$	(P) 
(B) $\mu_1 > \mu_2$	(Q) 
(C) $\mu_2 = \mu_3$	(R) 
(D) $\mu_2 > \mu_3$	(S) 
	(T) 

32. Water (with refractive index $= \frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. then 'x' is **[JEE 2011]**



33. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be **[JEE 2012]**



(A) -280.0 cm (B) 40.0 cm
(C) 21.5 cm (D) 13.3 cm

Paragraph for Question Nos. 34 to 35

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occuring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively. In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[JEE 2012]**

34. Choose the correct statement.

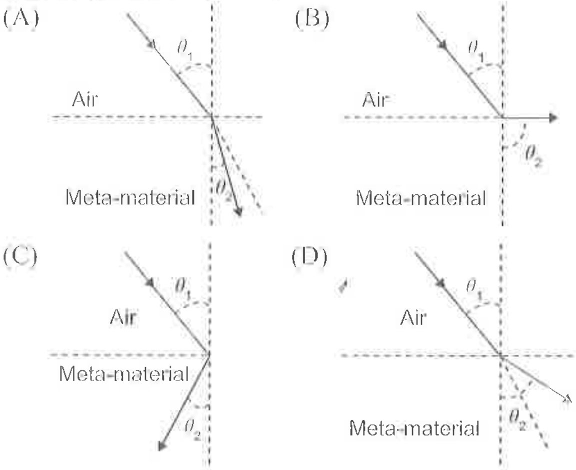
(A) the speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

35. For light incident from air on a meta-material, the appropriate ray diagram is



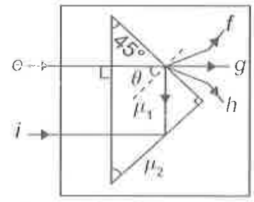
36. The image of an object, formed by a plano-convex lens at a distance of 8 m, behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is **[JEE 2013]**

(A) 1 m (B) 2 m
(C) 3 m (D) 6 m

37. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is: **[JEE 2013]**

(A) 30° (B) 45°
(C) 60° (D) 75°

38. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2, and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'.



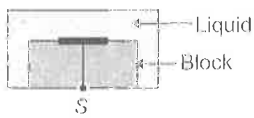
Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists: **[JEE 2013]**

List I	List II
(P) $e \to f$	1. $\mu_1 > \sqrt{2}\,\mu_2$
(Q) $e \to g$	2. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
(R) $e \to h$	3. $\mu_1 = \mu_2$
(S) $e \to i$	4. $\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

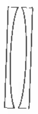
	P	Q	R	S
(A)	2	3	1	4
(B)	1	2	4	3
(C)	4	1	2	3
(D)	2	3	4	1

39. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **[JEE 2014]**



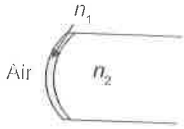
(A) 1.21 (B) 1.30
(C) 1.36 (D) 1.42

40. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surface is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists. **[JEE 2014]**

List I	**List II**
P. 	1. $2r$
Q. 	2. $r/2$
R. 	3. $-r$
S. 	4. r

Code:
(A) P-1, Q-2, R-3, S-4
(B) P-2, Q-4, R-3, S-1
(C) P-4, Q-1, R-2, S-3
(D) P-2, Q-1, R-3, S-4

41. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **[JEE 2014]**



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$
(C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

ANSWER KEYS

Exercises

JEE Main

1. B	2. C	3. A	4. D	5. A	6. C	7. A	8. B	9. A	10. B
11. B	12. B	13. C	14. A	15. C	16. D	17. B	18. A	19. B	20. A
21. B	22. B	23. D	24. A	25. A	26. C	27. D	28. D	29. B	30. D
31. B	32. C	33. C	34. B	35. A	36. A	37. C	38. A	39. A	40. C
41. C	42. D	43. C	44. B	45. D	46. B	47. C	48. C	49. A	50. D
51. A	52. A	53. A	54. D	55. D	56. C	57. A	58. A	59. B	60. B
61. B	62. C	63. D	64. D	65. C	66. B	67. A	68. B	69. B	70. C
71. D	72. D	73. D	74. A						

JEE Advanced

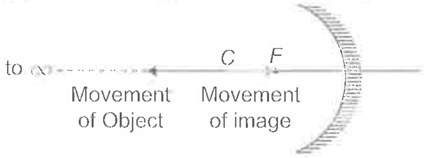
1. B	2. C	3. C	4. C	5. B	6. B	7. B	8. B	9. A	10. C
1. D	12. A	13. C	14. B	15. B, C	16. B, D	17. C	18. B	19. B	20. D
1. A	22. A	23. B	24. B	25. C	26. C	27. B	28. C	29. B	30. B
1. C	32. A	33. A	34. A, B	35. B, C	36. B, D	37. A	38. A	39. B	40. D
1. C	42. B	43. A	44. D	45. A	46. A	47. A	48. A	49. A	50. B
1. A, C	52. C	53. B, C, D	54. B, C, D	55. B	56. C	57. C	58. A	59. A	60. C

61. A **62.** B **63.** C **64.** C **65.** B **66.** C **67.** A, D **68.** A **69.** A, C
70. B, C ,D **71.** C **72.** C **73.** A **74.** D **75.** A **76.** C **77.** B **78.** B
79. A, B **80.** A, C **81.** D **82.** A, B **83.** A, B **84.** A **85.** C **86.** B **87.** B **88.** A
89. B **90.** A **91.** D **92.** D **93.** D **94.** A **95.** C **96.** A **97.** B **98.** B
99. C **100.** A **101.** B **102.** A **103.** B **104.** A **105.** B, D **106.** A, C **107.** A, C
108. B, C, D **109.** C **110.** B, D **111.** A, B, C **112.** A, D **113.** A **114.** D **115.** A
116. A **117.** B **118.** B, C **119.** A, B, C **120.** (i) B, D; (ii) A, B, C, D; (iii) A, B, D; (iv) D

JEE Advanced

Level I

1. 120° anticlockwise and 240° clockwise. **2.** 30° clockwise **3.** 60° **4.** 40°
5. Mirror should be placed on the path of the rays at an ∠ of 78° or 12° to the horizontal.
6. Position of image = (1 cos 60°, – 1 sin 60°), Velocity of image = 1 cos 60° $\hat{i}$, + 1 sin 60° $\hat{j}$ m/s
7. (4, 0, 0) **8.** 1.23 m **9.** 160 cm; 320 cm **10.** 75 cm **11.** 1000 m/s **12.** infinitely large. **13.** 1.95 m
14. (i) Real object, Virtual image, (ii) smaller, larger **15.** Virtual object, Real image **16.** larger, smaller

17. 

Real, $2f$, infinity: real $2f$, f

18. (a) 40 cm/s opposite to the velocity object., (b) 20 cm/s opposite along the velocity of object.
19. 60 cm **20.** 2cm **21.** 80 m/s **22.** $d/2$ **23.** 42 cm **24.** $\frac{2}{3}\times10^{-8}$ s **25.** 30 cm **26.** 25 cm
27. 35 cm, Shift() = 5 cm. **28.** 45° **29.** $\alpha > \sin\frac{-8}{9}$ **30.** $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **31.** 42 cm **32.** $\sin^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ **33.** $\sqrt{3}$
34. $\frac{b(1-m^2\cos^2\theta)^{1/2}}{\sin\theta}$ **35.** $h = l$ **36.** $\frac{\sqrt{41}}{4}$ **37.** 90° **38.** 37°, This deviation is not minimum.
39. $\theta = 60°$ **40.** $38° = \delta_m = 2\sin^{-1}\left(\frac{3}{4}\right) - 60°$ **41.** (i) 1.5°, (ii) $\frac{3°}{8}$ **42.** 45° **43.** $\frac{\sqrt{43}}{5}$ **44.** $\frac{8}{5\sqrt{2}}$
45. (i) $\sin^{-1}\left[\frac{1}{\sqrt{2}}\left(\sqrt{n^2-n_1^2}-n_1\right)\right]$ (ii) $r_1 = \sin^{-1}(n\sin45°) = 72.94°$
46. (i) $\lambda_0 = 600$ nm, $n = 1.5$, (ii) i = $\sin^{-1}(0.75)$ = 48.59°
47. 240 cm away from the separating surface
48. (a) 2, (b) not possible, it will focus close to the centre if the refractive index is large
49. 40 cm from pole in the medium of refractive index 1, virtual, erect and 4 cm in size.
50. 50 cm **51.** 27/2 = 13.5 cm below the surface of water
52. 4/3 **53.** 5 cm **54.** 75 cm, 150 cm **55.** ±24 cm, ±120 cm **56.** Converging
57. (a) $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$ (b) $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$ **58.** 20 cm, 1 m, –4, 24 cm **59.** 15 cm
60. 1.67 cm from the lens61. At the object itself, of the same size
62. $10D$, Optical power of each lens = $5D$ **63.** 10 cm for convex lens and 60 cm from concave lens.
64. $-\frac{20}{3}D = -6.7D$ **65.** 15 cm **66.** 11 cm **67.** $\frac{apq}{(q-p)}$ **68.** $(5f, 2d)$

69. $(\pi/4)$ cm^2 **70.** 3 cm **71.** 75/4 cm, 75/2 cm **72.** (i) 0.2 m, (ii) 0.4 m **73.** 1.5
74. (a) 1/5 = 0.2° (b) 0.72° **75.** 7.2°

Level II

2. (a) 21 m/s, (b) 1×10^{-3} /sec **4.** $\tan^{-1}\dfrac{2}{\sqrt{3}}$ with the principal axis, $\dfrac{\sqrt{7}}{4}$ cm/s **5.** $6\sqrt{26}$ cm **6.** 16 feet

8. $\mu = 3$, $\sin^{-1}(1/3)$ **9.** 15 cm towards the combination **10.** +60, +4/5

11. (a) $\tan\theta = \dfrac{dy}{dx} = \cot I$ (b) $y = k^2\left(\dfrac{x}{4}\right)^4$ (c) 4.0, 1 (d) It will become parallel to x-axis

13. $\mu = \dfrac{13}{8}$, $A = 2°$ **14.** (i) 2°, (ii) $\dfrac{4\pi}{9}$ mm **15.** 1.5 or $(\sqrt{5}-1)$ **16.** 8/5 cm/s

17. $I = \left|\dfrac{(3f-2d)fd}{4fd-2d^2-f^2}\right|$ **18.** 10 cm, 10, 2 **19.** 5.9 cm, 10.9 cm **20.** $f = 0.4$ m, separation = 0.6 m

21. −90 cm **21.** 90 cm from the lens towards right

Previous Year Questions

JEE Main

1. D	**2.** C	**3.** B	**4.** C	**5.** A	**6.** A	**7.** B	**8.** A	**9.** A	**10.** D
11. C	**12.** B	**13.** A	**14.** B	**15.** A	**16.** A	**17.** B	**18.** A	**19.** D	**20.** C
21. A	**22.** B	**23.** A	**24.** D	**25.** B	**26.** A	**27.** D	**28.** D		

JEE Advanced

1. B **2.** C **3.** B **4.** A **5.** $f = v = \dfrac{\mu_3 R}{\mu_3 - \mu_1}$ **6.** A **7.** B **8.** C

9. $\dfrac{1.514 \times 0.4}{0.1} = 6.06$ m correct upto two places of decimal.

10. 0.09 m/s; Magnitude of the rate of change of lateral magnification is 0.3 s^{-1}.
11. C **12.** C **13.** 60° **14.** (a) $i = 60°$, (b) 60° (anticlockwise) **15.** C **16.** C **17.** B
18. (A) P; (B) R; (C) R; (D) P, Q, S **19.** C **20.** C **21.** A **22.** B
23. (A) → p, q, r and s, (B) → q, (C) → p, q, r and s, (D) → p, q, r and s **24.** C **25.** C, D
26. A, B, C **27.** 6 **28.** B 29.6 cm **30.** 3 **31.** (A) → P, R; (B) → Q, S, T; (C) → P, R, T; (D) → Q, S
32. 2 cm **33.** B **34.** B **35.** C **36.** C **37.** A **38.** D **39.** C **40.** D **41.** A, C

Wave Optics

ELECTROMAGNETIC SPECTRUM

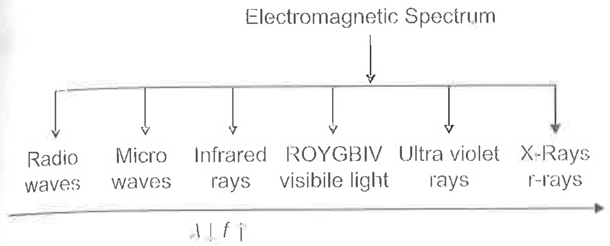


Figure 2.1

Visible light is the part of electromagnetic spectrum that is visible to us.

Light is studied under two sections.

1. Geometrical optics (If the dimensions of body are larger as compared to the wavelength of light)
2. Wave optics (If the dimensions of body are comparable to the wavelength of light)

WAVEFRONT

- Wavefront is a locus of particles having the same phase.
- Direction of propagation of wave is perpendicular to wavefront.
- Every particle of a wavefront acts as a new source and is known as secondary wavelet.

The shape of wavefronts varies from source to source.
Point source → Spherical wavefronts
Distant parallel rays → Planar wavefronts
Line source → Cylindrical wavefronts

Coherent Source

If the phase difference due to two sources at a particular point remains constant with time, then the two sources are considered as coherent source.

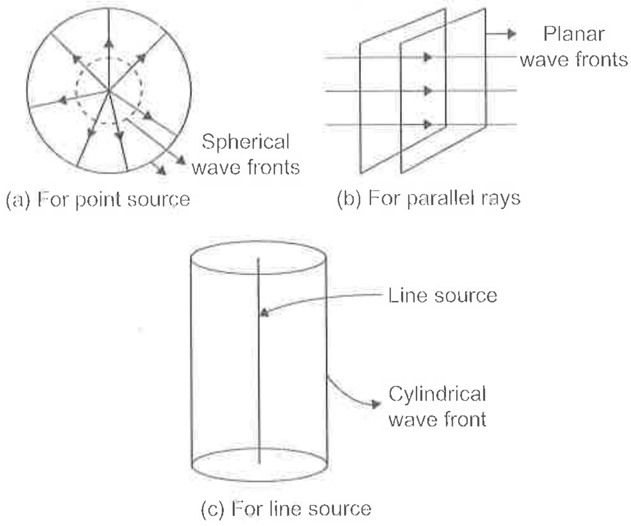


Figure 2.2

Note

Sources lying on the same wavefront are coherent in nature because their phase difference = 0.

$$\left.\begin{aligned} y_1 &= A_1 \sin(\omega t + kx) \\ y_2 &= A_2 \sin(\omega t + kx) \end{aligned}\right\} \text{(Coherent sources) Phase difference} = 0 \text{ (constant)}$$

$$\left.\begin{aligned} y_1 &= A_1 \sin(\omega t + kx + 30^\circ) \\ y_2 &= A_2 \sin(\omega t + kx + 60^\circ) \end{aligned}\right\} \text{(Coherent sources) Phase difference} = 30 \text{ (constant)}$$

PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance at the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement; in case of sound wave it means pressure change; in case of electromagnetic wave (EMW) it is electric field or magnetic field. Superposition of two light rays travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

Superposition of Two Sinusoidal Waves

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let $X_1(t) = a_1 \sin \omega t$

and, $X_2(t) = a_2 \sin(\omega t + \phi)$

$$X = A \sin(\omega t + \phi_0),$$

where $$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi \qquad (2.1)$$

(Refer topic: combination of SHM)

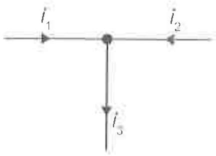
and $$\tan\phi_0 = \frac{a_2 \sin\phi}{a_1 + a_2\cos\phi} \qquad (2.2)$$

SOLVED EXAMPLES

EXAMPLE 1

If $i_1 = 3 \sin \omega t$ and $i_2 = 4 \cos \omega t$, then find i_3 which is given by $i_3 = i_1 + i_2$.

SOLUTION



$$i_3 = i_1 + i_2$$

$$= 3\sin\omega t + 4\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= 5\sin\left(\omega t + \tan^{-1}\frac{4}{3}\right)$$ ■

EXAMPLE 2

S_1 and S_2 are two sources of light which produce individually disturbance at a point P given by $E_1 = 3 \sin \omega t$ and $E_2 = 4 \cos \omega t$. Assuming $\vec{E_1}$ and $\vec{E_2}$ to be along the same line, find the result of their superposition.

SOLUTION

The two waves coming from two sources S_1 and S_2 are:

$$E_1 = 3\sin\omega t \quad [\text{Amplitude, } A_1 = 3]$$

$$E_2 = 4\sin\left(\omega t + \frac{\pi}{2}\right) \quad [\text{Amplitude, } A_2 = 4]$$

The phase difference 'ϕ' between the above two equations is $\frac{\pi}{2}$.

The resultant of super position of two sinusoidal waves:

$$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

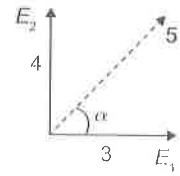
$$E_0 = \sqrt{3^2 + 4^2 + 2\cdot 3\cdot 4\cdot(0)}$$

$$E_0 = 5$$

We can also calculate the angle made by the resultant wave from one of the waves (Let us say E_1)

$$\tan\alpha = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$



Resultant equation $E_{res} = E_0 \sin(\omega t + \alpha)$

$$= 5\sin\left[\omega t + \tan^{-1}\left(\frac{4}{3}\right)\right]$$ ■

INTERFERENCE

Interference implies superposition of waves. Whenever two or more waves superimpose each other, they give sum of their individual displacement.

Let the two waves coming from sources S_1 and S_2 be

$$y_1 = A_1 \sin(\omega t + kx_1)$$

$$y_2 = A_2 \sin(\omega t + kx_2), \text{ respectively}$$

Due to superposition

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_1 \sin(\omega t + kx_1) + A_2 \sin(\omega t + kx_2)$$

Phase difference between y_1 and $y_2 = k(x_2 - x_1)$

i.e., $$\Delta\phi = k(x_2 - x_1)$$

As $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(where Δx = path difference and $\Delta\phi$ = phase difference)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\Rightarrow \quad A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\therefore \quad I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\theta \ (\text{as } I \propto A^2)$$

When the two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes and I_{net} will be maximum. This is known as constructive interference.

For I_{net} to be maximum,

$$\cos\phi = 1$$

$$\Rightarrow \quad \phi = 2n\pi,$$

where $n = \{0, 1, 2, 3, 4, 5, \ldots\}$

$$\frac{2\pi}{\lambda}\Delta x$$

$$\Rightarrow \quad \Delta x = n\lambda$$

For constructive interference,

$$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

When $\quad I_1 = I_2 = I$

$$I_{net} = 4I$$

$$A_{net} = A_1 + A_2$$

When superposing waves are in opposite phase, then the resultant amplitude is the difference of two amplitudes and I_{net} will be minimum. This is known as destructive interference.

For I_{net} to be minimum,

$$\cos\Delta\phi = -1$$

$$\Delta\phi = (2n+1)\pi,$$

where $n = \{0, 1, 2, 3, 4, 5, \ldots\}$

$$\frac{2\pi}{\lambda}\Delta x = (2n+1)\pi$$

$$\Rightarrow \quad \Delta x = (2n+1)\frac{\lambda}{2}$$

For destructive interference,

$$I_{net} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

If $\quad I_1 = I_2$

$$I_{net} = 0$$

$$A_{net} = A_1 - A_2$$

Generally,

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

If $\quad I_1 = I_2 = I$

$$I_{net} = 2I + 2I\cos\phi$$

$$I_{net} = 2I(1+\cos\phi) = 4I\cos^2\frac{\Delta\phi}{2}$$

$$\text{Ratio of } I_{max} \text{ and } I_{min} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

SOLVED EXAMPLES

EXAMPLE 3

If two light waves, having same frequency and travelling in the same direction, have the intensity ratio 4:1 and they interfere. Find the ratio of maximum to minimum intensity.

SOLUTION

$$\frac{I_{max}}{I_{max}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2$$

$$= 9:1$$

■

EXAMPLE 4

Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the interference is (a) coherent and (b) incoherent.

SOLUTION

The resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

(a) The sources are said to be coherent if they have constant phase difference between them. Then intensity will be maximum when $\phi = 2n\pi$; the sources are in same phase.

Thus,
$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Similarly, for n identical waves,
$$I_{max} = \left(\sqrt{I_0} + \sqrt{I_0} + \cdots\right)^2$$
$$= n^2 I_0$$

(b) The incoherent sources have phase difference that varies randomly with time.

Thus, $[\cos\phi]_{av} = 0$

Hence, $I = I_1 + I_2$

Hence for n identical waves,
$$I = I_0 + I_0 + \cdots = nI_0$$

YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)

In 1802, Thomas Young devised a method to produce a stationary interference pattern. This was based on division of a single wavefront into two; these two wavefronts acted as if they emitted from two sources having a fixed phase relationship. Hence, when they were allowed to interfere, stationary interference pattern was observed.

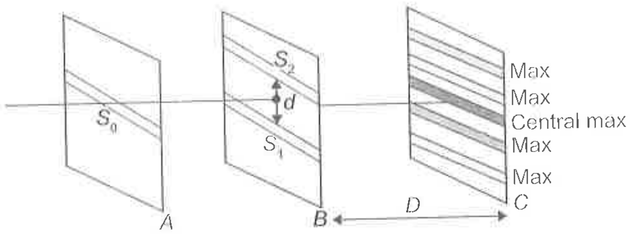


Figure 2.3

Young's arrangement to produce stationary interference pattern by division of wavefront S_0 into S_1 and S_2.

In Young's interference experiment, light diffracted from pinhole S_0 encounters pinholes S_1 and S_2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

The geometry of experiment is simple. Parallel wavefronts of a monochromatic wave are incident on two identical narrow slits, each of width is separated by a distance d. The slit widths and their separation are of the order of the wavelength of the incident monochromatic light. Monochromatic light after passing through two slits S_1 and S_2 acts as coherent sources of light waves that interfere constructively and destructively at different points on the screen to produce an interference pattern

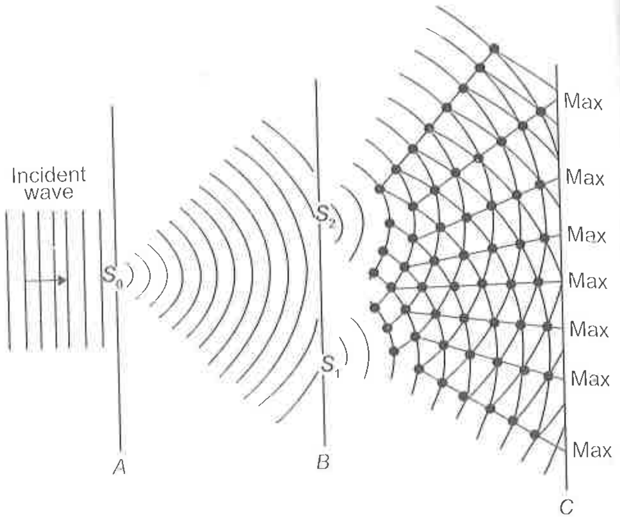


Figure 2.4

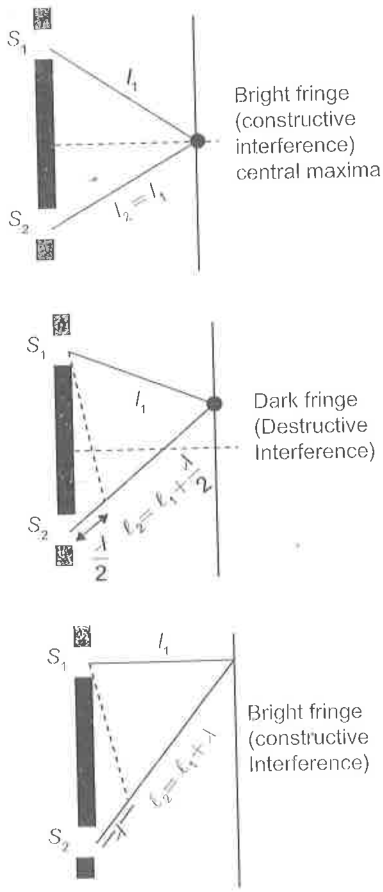


Figure 2.5

Analysis of Interference Pattern

We have ascertained from the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However, the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference. If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

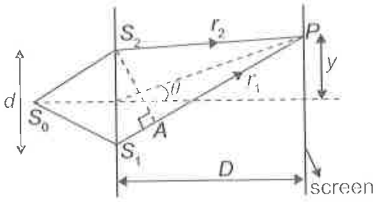


Figure 2.6

Path difference

$$Dp = S_1P - S_2P$$

$$= \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} \qquad (1)$$

Approximation I

For $D >> d$, we can approximate rays and, as being approximately parallel, at an angle θ to the principle axis.

Now, $\quad S_1P - S_2P = S_1A$

$$= S_1S_2 \sin\theta$$

$\Rightarrow \quad$ Path difference $= d \sin\theta \qquad (2)$

Approximation II

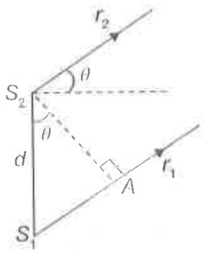


Figure 2.7

Further, if θ is small, i.e., $y << D$,

$$\sin\theta \simeq \tan\theta = \frac{y}{D}$$

and hence path difference $= \dfrac{dy}{D} \qquad (3)$

For maxima (constructive interference)

$$\Delta p = \frac{d \cdot y}{D} = n\lambda$$

$$\Rightarrow \quad y = \frac{n\lambda D}{d},$$

$$n = 0, \pm 1, \pm 2, \pm 3 \qquad (4)$$

Here, $n = 0$ corresponds to the central maxima,

$n = \pm 1$ corresponds to the first maxima,

$n = \pm 2$ corresponds to the second maxima and so on.

For minima (destructive interference)

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}$$

$$\Rightarrow \quad \Delta p = \begin{cases} (2n-1)\dfrac{\lambda}{2}, & n = 1, 2, 3, \ldots \\ (2n+1)\dfrac{\lambda}{2}, & n = -1, -2, -3, \ldots \end{cases}$$

Consequently,

$$y = \begin{cases} (2n-1)\dfrac{\lambda D}{2d}, & n = 1, 2, 3, \ldots \\ (2n+1)\dfrac{\lambda D}{2d}, & n = -1, -2, -3, \ldots \end{cases} \qquad (5)$$

Here, $n = \pm 1$ corresponds to the first minima,

$n = \pm 2$ corresponds to second minima and so on.

Fringe Width

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

$$\text{Fringe width } \beta = \frac{\lambda D}{d}$$

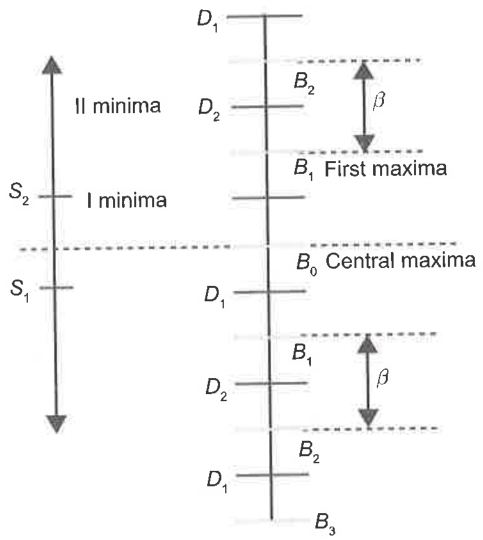


Figure 2.8 Fringe pattern in YDSE

Note that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

As vertical distance y is related to θ by

$$\theta = \frac{y}{D}$$

so $$\Delta\theta = \frac{\Delta y}{D},$$

which is referred as angular fringe width.

$$B_{0w} = \frac{\beta}{D} = \frac{\lambda}{d}$$

SOLVED EXAMPLES

EXAMPLE 5

In a YDSE performed with wavelength $\lambda = 5890$Å, the angular fringe width is 0.40°. What is the angular fringe width if the entire set-up is immersed in water?

SOLUTION

Angular fringe width is given by

$$\beta_\theta = \frac{\lambda}{d}$$

So, $$\beta_\theta^{\text{air}} = \frac{\lambda_{\text{air}}}{d},$$

$$\beta_\theta^{\text{water}} = \frac{\lambda_{\text{water}}}{d}$$

$$\frac{\beta_\theta^{\text{air}}}{\beta_\theta^{\text{water}}} = \frac{\lambda_{\text{air}}}{\lambda_{\text{water}}}$$

Thus, $$\beta_\theta^{\text{water}} = \frac{3}{4}\beta_\theta^{\text{air}}$$

$$= \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{3}{4}\beta_\theta^{\text{air}}$$

$$= 0.40° \times \frac{3}{4}$$

$$= 0.30°$$

EXAMPLE 6

A beam of light consisting of two wavelengths, 6500Å an 5200Å, is used to obtain interference fringes in a Young double slit experiment. What is the least distance from th central maximum where the bright fringes due to both th wavelengths coincide? The distance between the slits 2 mm and the distance between the plane of slits and th screen is 120 cm.

SOLUTION

The position of n^{th} bright fringe on the screen is

$$y_n = \frac{n\lambda D}{d}$$

Let the n^{th} bright fringe of 6500Å and the n^{th} brig fringe of 5200Å coincide; then

$$\frac{m \times 6500 \times D}{d} = \frac{n \times 5200 \times D}{d}$$

$$\frac{m}{n} = \frac{5200}{6500}$$

$$= \frac{4}{5}$$

Thus, the minimum values of m and n are 4 and respectively.

Hence, $$y = \frac{4 \times 6500 \times 120}{0.2}$$

$$= 0.156 \text{ cm}$$

$$= 1.56 \text{ mm}$$

EXAMPLE 7

In a YDSE, $D = 1$ m, $d = 1$ mm and $\lambda = 1/2$ mm.

(a) Find the distance between the first and the central maxima on the screen.

(b) Find the number of maxima and minima obtained on the screen.

SOLUTION

(a) $$D >> d$$

Hence, $$\Delta P = d \sin \theta$$

$$\frac{d}{\lambda} = 2,$$

Clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence, $\Delta p = \frac{dy}{D}$ cannot be used.

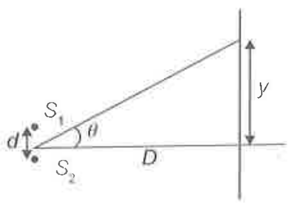
For the first maxima,

$$\Delta p = d \sin \theta = \lambda$$

$$\Rightarrow \sin \theta = \frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

Hence, $$y = D \tan \theta = \frac{1}{\sqrt{3}} \text{ m}$$



(b) Maximum path difference

$$\Delta P_{max} = d = 1 \text{ mm}$$

$\Rightarrow$ Highest order maxima,

$$n_{max} = \left[\frac{d}{\lambda}\right] = 2$$

and highest order minima,

$$n_{max} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$$

Total no. of maxima $= 2n_{max} + 1^* = 5$

*(central maxima)

Total no. of minima $= 2n_{min} = 4$ ■

EXAMPLE 8

Monochromatic light of wavelength 5000Å is used in YDSE, with the slit width $d = 1$ mm, distance between screen and slits $D = 1$ m. If intensities at the two slits are $I_1 = 4I_0$ and $I_2 = I_0$, find:

(a) Fringe width β,

(b) Distance of the 5th minima from the central maxima on the screen,

(c) Intensity at $y = \frac{1}{3}$ mm,

(d) Distance of the 1000th maxima,

(e) Distance of the 5000th maxima.

SOLUTION

(a) $$\beta = \frac{\lambda D}{d}$$

$$= \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}}$$

$$= 0.5 \text{ mm}$$

(b) $$y = (2n-1)\frac{\lambda D}{2d},$$

$$n = 5$$

$$\Rightarrow y = 2.25 \text{ mm}$$

(c) At $$y = \frac{1}{3} \text{ mm},$$

$$y << D$$

Hence, $$\Delta p = \frac{dy}{D}$$

$$\Delta \phi = \frac{2\pi}{\lambda}$$

$$\Delta p = 2\pi \frac{dy}{\lambda D}$$

$$= \frac{4\pi}{3}$$

Now resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$$

$$= 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi$$

$$= 5I_0 \cos\frac{4\pi}{3}$$

$$= 3I_0$$

(d) $$\frac{d}{\lambda} = \frac{10^{-3}}{0.5\times10^{-6}}$$

$$= 2000$$

$n = 1000$ is not $<<$ 2000
Hence, $\Delta p = d \sin\theta$ must be used.

Hence, $$d\sin\theta = n\lambda$$

$$= 1000\lambda.$$

$$\Rightarrow \quad \sin\theta = 1000\frac{\lambda}{d}$$

$$= \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

$$y = D\tan\theta = \frac{1}{\sqrt{3}}\text{ m}$$

(e) Highest order maxima

$$n_{max} = \left[\frac{d}{\lambda}\right]$$

$$= 2000$$

Hence, $n = 5000$ is not possible. ■

EXAMPLE 9

A beam of light consisting of wavelengths, 6000Å and 4500Å, is used in YDSE with $D = 1$m and $d = 1$ mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

SOLUTION

$$\beta_1 = \frac{\lambda_1 D}{d}$$

$$= \frac{6000\times10^{-10}\times1}{10^{-3}}$$

$$= 0.6 \text{ mm}$$

$$\beta_2 = \frac{\lambda_2 D}{d}$$

$$= 0.45 \text{ mm}$$

Let n_1^{th} maxima of λ_1 and n_2^{th} maxima of λ_2 coi at a position y.

Then, $y = n_1P_1 = n_2P_2 =$ LCM of β_1 and β_2

$\Rightarrow y =$ LCM of 0.6 cm and 0.45 mm

$$y = 1.8 \text{ mm}$$

At this point, the 3rd maxima for 6000Å and t maxima for 4500Å coincide.

GEOMETRICAL PATH AND OPTICAL PAT

Actual distance travelled by light in a medium is c geometrical path (Δx). Consider a light wave given b equation

$$E = E_0 \sin(\omega t - kx + \phi)$$

If the light travels by Δx, its phase change $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light doe depend on the medium, but v, the speed of light depen the medium as $v = \frac{c}{\mu}$

Consequently, change in phase,

$$\Delta\phi = k\Delta x$$

$$= \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx medium of refractive index μ suffers the same phase ch when it travels a distance $\mu\Delta x$ in vacuum, i.e., a path le of Δx in medium of refractive index μ is equivalent path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path le of light, Δx_{opt}. And in terms of optical path length, p difference would be given by

$$\Delta\phi = \frac{\omega}{c}\Delta x_{opt}$$

$$= \frac{2\pi}{\lambda_0}\Delta x_{opt}.$$

where λ_0 = wavelength of light in vacuum.

However, in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x)$$

$$= \frac{2\pi}{\lambda}\Delta x, \quad (2)$$

where λ = wavelength of light in the medium $\left(\lambda = \frac{\lambda_0}{\mu}\right)$.

Displacement of Fringe

On introduction of a glass slab in the path of the light coming out of the slits

On introduction of the thin glass slab of thickness t and refractive index m, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$$\Delta p = S_2P - (S_1P + t(\mu - 1))$$

$$= (S_2P - S_1P) - t(\mu - 1)$$

$$\Rightarrow \quad \Delta p = d\sin\theta - t(\mu - 1) \text{ if } d << D$$

and $$\Delta p = \frac{yd}{D} - t(\mu - 1) \text{ if } y << D \text{ as well}$$

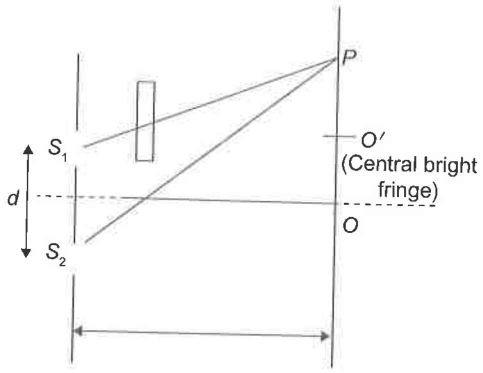


Figure 2.9

For central bright fringe,

$$\Delta p = 0$$

$$\frac{yd}{D} = t(\mu - 1)$$

$$y = OO'$$

$$= (\mu - 1)t\frac{D}{d}$$

$$= (\mu - 1)t.\frac{\beta}{\lambda}$$

The whole fringe pattern gets shifted by the same distance

$$\Delta = (\mu - 1)t\frac{D}{d}$$

$$= (\mu - 1)t\frac{\beta}{\lambda}$$

Note that this shift is in the direction of the slit before which the glass slab is placed. It happens so because S_2 compensates the path difference (arised due to optical path length covered by S_1) by covering more geometrical path length. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

SOLVED EXAMPLES

EXAMPLE 10

In a YDSE with $d = 1$ mm and $D = 1$ m, slabs of ($t = 1$ μm, $\mu = 3$) and ($t = 0.5$ μm, $\mu = 2$) are introduced in front of upper and lower slits, respectively. Find the shift in the fringe pattern.

SOLUTION

Optical path for light coming from upper slit S_1 is

$$S_1P + 1\ \mu\text{m}\ (2 - 1) = S_2P + 0.5\ \text{m}$$

Similarly, optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu\text{m}\ (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Path difference:

$$\Delta p = (S_2P + 0.5\ \mu\text{m}) - (S_1P + 2\mu\text{m})$$

$$= (S_2P - S_1P) - 1.5\ \mu\text{m}$$

$$= \frac{yd}{D} - 1.5\,\text{mm}$$

For central bright fringe $\Delta p = 0$

$$\Rightarrow \qquad y = \frac{1.5\ \mu m}{1\ mm} \times 1\ m = 1.5\ mm.$$

The whole pattern is shifted by 1.5 mm upwards. ■

EXAMPLE 11

Interference fringes were produced by the Young's double slit method; the wavelength of light used being 6000Å. The separation between the two slits is 2 mm. The distance between the slits and screen is 10 cm. When a transparent plate of thickness 0.5 mm is placed over one of the slits, the fringe pattern is displaced by 5 mm. Find the refractive index of the material of the plate.

SOLUTION

Here, $d = 2\ cm = 2 \times 10^{-3}\ m$,

$$D = 10\ cm = 0.10\ m,$$

$$t = 0.5\ mm = 0.5 \times 10^{-3}\ m,$$

$$\Delta x = 5\ mm = 5 \times 10^{-3}\ m,$$

$$\lambda = 6 \times 10^{-7}\ m$$

As $$x_0 = \frac{D}{d}(\mu - 1)t,$$

$$\mu - 1 = \frac{x_0 . d}{D \times t}$$

$$= \frac{5 \times 10^{-3} \times 2 \times 10^{-3}}{0.10 \times 0.5 \times 10^{-3}}$$

$$= 0.2$$

or $$\mu = 1 + 0.2$$

$$= 1.2$$ ■

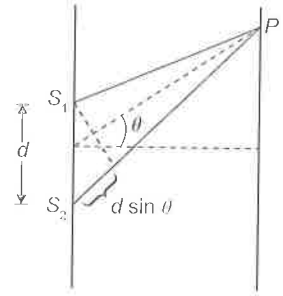
EXAMPLE 12

In a YDSE light of wavelength $\lambda = 5000$Å is used, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$ apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$ m, refractive index $n = 1.17$, is placed over one of the slits. Where do the central maxima of the interference now appear?

SOLUTION

The path difference introduced due to introduction of transparent sheet is given by

$$\Delta x = (\mu - 1)t.$$



If the central maxima occupy position of the n^{th} fringe,

then $$(\mu - 1)t = n\lambda$$

$$= d \sin \theta$$

$$\sin\theta = \frac{(\mu - 1)t}{d}$$

$$= \frac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}}$$

$$= 0.085$$

Hence, the angular position of the central maxima is

$$\theta = \sin^{-1}(0.085) = 4.88°$$

For small angles,

$$\sin\theta \simeq \theta \simeq \tan\theta$$

As $$\tan\theta = \frac{y}{D},$$

$$\frac{y}{D} = \frac{(\mu - 1)t}{d}$$

Shift of central maxima is

$$Y = \frac{D(\mu - 1)t}{d}.$$

This formula can be used if D is given. ■

YDSE WITH OBLIQUE INCIDENCE

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up for points above the central point on the screen (say for P_1).

$$\Delta p = d \sin \theta_0 + (S_2P_1 - S_1P_1)$$

$$\Rightarrow \quad \Delta p = d\sin\theta_0 + d\sin\theta_1 \ (\text{If } d << D)$$

For point O, $\Delta p = d\sin\theta_0$ (because $S_2O = S_1O$) and for points below O on the screen (say for P_2)

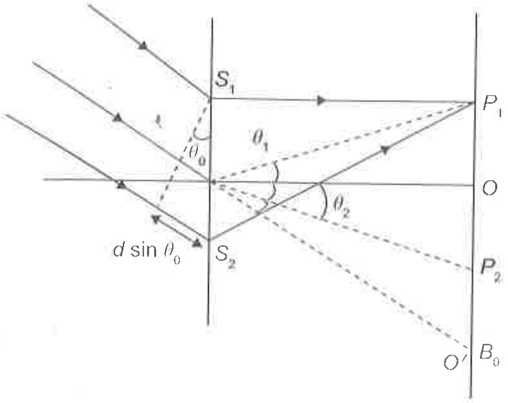


Figure 2.10

$$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2|$$

$$= |(d\sin\theta_0 - (S_1P_2 - S_2P_2)|$$

$$\Rightarrow \quad \Delta p = |d\sin\theta_0 - d\sin\theta_2| \ (\text{if } d << D)$$

We obtain central maxima at a point where $\Delta p = 0$

$$(d\sin\theta_0 - d\sin\theta_2) = 0$$

or $$\theta_2 = \theta_0$$

This corresponds to the point O' in Fig. 2.10.
Hence, we have finally for path difference,

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) \rightarrow \text{for points above } O \\ d(\sin\theta_0 - \sin\theta) \rightarrow \text{for points between } O \text{ and } O' \\ d(\sin\theta_0 - \sin\theta) \rightarrow \text{for points below } O' \end{cases}$$

SOLVED EXAMPLE

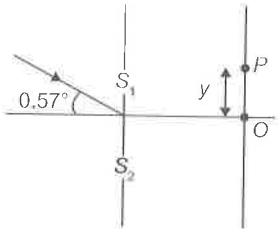
EXAMPLE 13

In YDSE with $D = 1$ m, $d = 1$ mm, light of wavelength 500 nm is incident at an angle of 0.57° w.r.t the axis of symmetry of the experimental set-up. If centre of symmetry of screen is O as shown, find:

(a) the position of central maxima,

(b) the intensity at point O in terms of intensity of central maxima I_0,

(c) the number of maxima lying between O and the central maxima.



SOLUTION

(a) $$\theta = \theta_0 = 0.57°$$

$$\Rightarrow \quad y = -D\tan\theta;$$

$$-D\theta = -1\text{ m} \times \left(\frac{0.57}{57}\text{ rad}\right)$$

$$\Rightarrow \quad y = -1\text{ cm}$$

(b) For point O, $\theta = 0$

Hence, $$\Delta p = d\sin\theta_0,$$

$$d\theta_0 = 1\text{ mm} \times (10^{-2}\text{ rad})$$

$$= 10{,}000\text{ nm}$$

$$= 20 \times (500\text{ nm})$$

$$\Rightarrow \quad \Delta p = 20\,\lambda$$

Hence, point O corresponds to the 20th maxima.

$$\Rightarrow \quad \text{Intensity at } O = I_0$$

(c) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima. ■

SHAPE OF INTERFERENCE PATTERN

1. Shape of the pattern when the interference takes place due to waves produced by two slits.

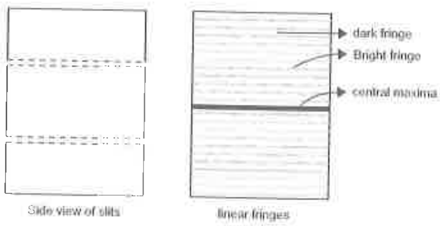


Figure 2.11

2. Shape of the pattern when the interference takes place due to waves produced by two point sources (where the line of sources is perpendicular to the screen).

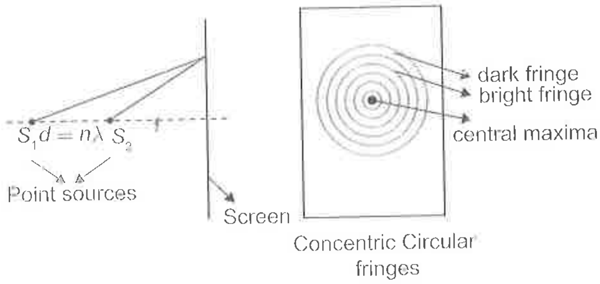


Figure 2.12

3. Shape of the pattern when the interference takes place due to waves produced by two point sources (where the line of sources is parallel to the screen).

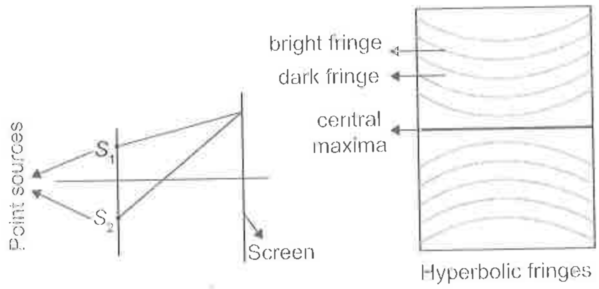


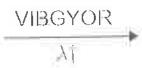
Figure 2.13

YDSE WITH WHITE LIGHT

Central maxima position (where phase difference = 0) is independent of the wavelength of light.

White light is used to find out the central maxima position of YDSE set-up because at this position only, all the wavelengths show constructive interference, i.e. why we get white spot at that position.

However, slightly below or above the position of central maxima fringes will be coloured.



And we know, $$y = \frac{\lambda D}{d}$$

As we move away from the central maxima, the first maxima and minima are of violet colour but in the nearby region of the central maxima reddish colour will dominate because in this region intensity of violet colour decreases at a faster rate as compared to red colour.

In usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such case.

SOLVED EXAMPLES

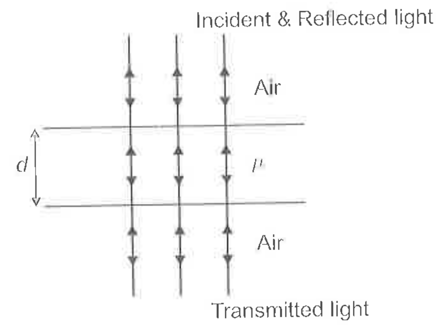
EXAMPLE 14

White light, with a uniform intensity across the visible wavelength ranged from 430 to 690 nm, is perpendicularly incident on a water film, of index of refraction $\mu = 1.33$ and thickness $d = 320$ nm, which is suspended in air. At what wavelength l is the light reflected by the film brightest to an observer?

SOLUTION

This situation is like that of the figure shown, for which equation written below gives the interference maxima.

$$2\mu d = (m + \frac{1}{2})\lambda \text{ for constructive interference.}$$



Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m + 1/2}$$

$$= \frac{(2)(1.33)(320\text{ nm})}{m + 1/2}$$

$$= \frac{851\text{ nm}}{m + 1/2}$$

For $m = 0$, this gives us $\lambda = 1700$ nm, which is in the infrared region. For $m = 1$, we find $l = 567$ nm, which is yellow–green light, near the middle of the visible spectrum.

For $m = 2$, $\lambda = 340$ nm, which is the ultraviolet region. So the wavelength, at which the light seen by the observer is the brightest, is

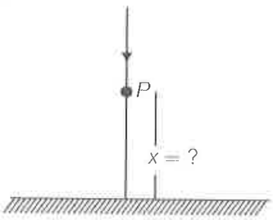
$$\lambda = 567 \text{ nm}.$$

Note

When a light gets reflected from a denser medium, there is an abrupt phase change of π and no phase change occurs when reflection takes place from a rarer medium.

EXAMPLE 15

ind the minimum value of x for which maxima is obtained t P.



OLUTION

or maxima, $\Delta x = \lambda$ (because x should be minimum)
ath difference between direct and reflected ray

$$= x + \frac{\lambda}{2} + x$$

ue to reflection, a phase change of π or path change of $\frac{\lambda}{2}$ kes place)

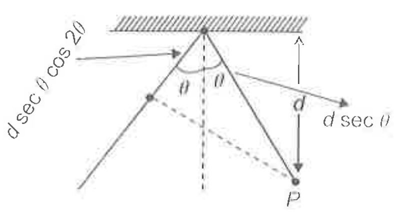
$$\lambda = x + \frac{\lambda}{2} + x$$

$$2x = \frac{\lambda}{2}$$

$$x = \frac{\lambda}{4}$$

AMPLE 16

d the value of θ for which maxima is obtained at P.



SOLUTION

For maxima at P, $\Delta x = \lambda$
Path difference between direct and reflected ray

$$= d \sec\theta \cos 2\theta + \frac{\lambda}{2} + d \sec\theta$$

$$= \lambda$$

$$d \sec\theta(1 + \cos^2\theta) = \frac{\lambda}{2}$$

$$d \sec\theta(2 \cos^2\theta) = \frac{\lambda}{2}$$

$$\cos\theta = \frac{\lambda}{4d}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\lambda}{4d}\right)$$

THIN FILM INTERFERENCE

When light passes the boundary between two transparent media, some light is reflected at the boundary.

*Some is refracted through the boundary. As shown in Fig. 2.14, some light is reflected from the first surface and some from the second surface. If we consider a monochromatic incident light, the two reflected waves are also monochromatic incident light wave via amplitude division. These waves interfere since they are superposed along the same normal line.

The phase difference between two waves arises due to

1. Optical path difference (due to distance travelled)
2. Reflection from a denser medium (the second factor is irrelevant for reflection at rarer medium)

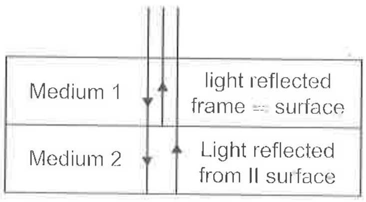


Figure 2.14

Three situations may arise:

1. Neither wave experience a phase change upon reflection.

$$(\mu_1 > \mu_2 > \mu_3)$$

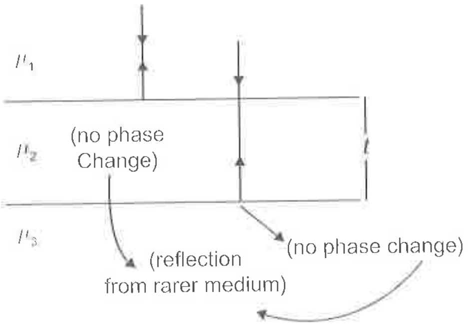


Figure 2.15

$$\Delta x = \frac{2\mu_2 t}{\mu_1}$$

2. Both the waves suffer a phase change upon reflection.

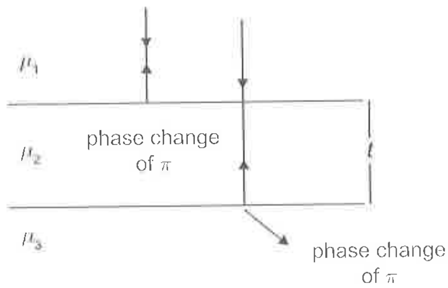


Figure 2.16

$$\mu_3 > \mu_2 > \mu_1$$

$$\Delta x = \frac{2\mu_2}{\mu_1} t$$

In either of the above cases, the phase change due to reflection is irrelevant, no difference in phase result due to reflection. In either of these cases, phase change is determined solely from path difference.

Condition for construction interference

$$\frac{2n_2 t}{n_1} = n\lambda$$

Condition for destructive interference

$$\frac{2n_2 t}{n_1} = \left(n + \frac{1}{2}\right)\lambda$$

3. One of the reflected waves experience a phase change of π radian upon reflection and the other wave does not.

$$\Delta x = 2\mu t - \frac{\lambda}{2}$$

Due to phase change of π (path change of $\frac{\lambda}{2}$), the conditions are reversed.

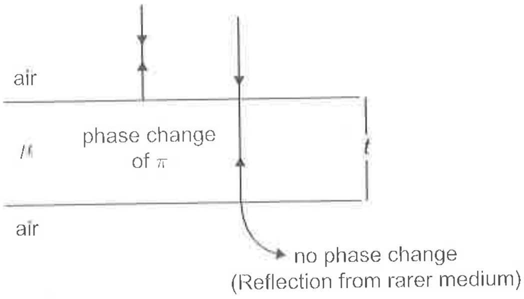


Figure 2.17

$$2\mu t = n\lambda$$

(for destructive interference)

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda$$

(for constructive interference)

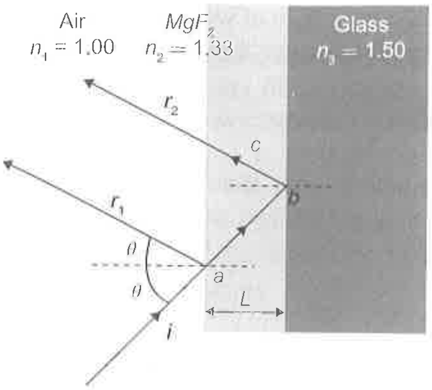
SOLVED EXAMPLES

EXAMPLE 17

A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38 and that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.

SOLUTION

The situation here differs from the figure; in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see the figure) Unwanted reflections from glass can be suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference $2L$ within the film must be equal to an odd number of half wavelengths:



$$2L = \left(m + \frac{1}{2}\right)\lambda_{n2'}$$

or, with

$$\lambda n_2 = \frac{\lambda}{n_{2'}}$$

$$2n_2 L = \left(m + \frac{1}{2}\right)\lambda$$

We want the least thickness for the coating, i.e., the smallest L. Thus, we choose $m = 0$, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2}$$

$$= \frac{550 \text{ nm}}{(4)(1.38)}$$

$$= 96.6 \text{ nm}$$

■

EXAMPLE 18

White light may be considered to have λ from 4000Å to 7500Å. If an oil film has thickness 10^{-6} m, deduce the wavelengths in the visible region for which the reflection along the normal direction will be (a) weak and (b) strong. Take m of the oil as 1.40.

SOLUTION

The condition for dark fringe or weak reflection when seen in reflected light is $2\mu t \cos r = n\lambda$, where n is an integer.

For normal incidence, $r = 0$ and $\cos r = 1$

so that $$2\mu t = n\lambda$$

or $$\lambda = \frac{2\mu t}{n}$$

Substituting the values of μ and t, we get

$$\lambda = \frac{2 \times 1.4 \times 10^{-6}}{n}$$

$$= \frac{28 \times 10^{-7}}{n} \text{ m}$$

For values of $n < 4$ or >7, the values of λ do not lie in the visible range 4000–7500Å. But for values of $n = 4$, 5, 6, 7, the following wavelengths lie in the visible region:

(a) $$\lambda = \frac{28 \times 10^{-7}}{4}$$

$$= 7.0 \times 10^{-7} \text{ m}$$

$$= 7000\text{Å}$$

(b) $$\lambda = \frac{28 \times 10^{-7}}{5}$$

$$= 5.6 \times 10^{-7} \text{ m}$$

$$= 5600\text{Å}$$

(c) $$\lambda = \frac{28 \times 10^{-7}}{6}$$

$$= 4.667 \times 10^{-7} \text{ m}$$

$$= 4667\text{Å}$$

(d) $$\lambda = \frac{28 \times 10^{-7}}{7}$$

$$= 4.0 \times 10^{-7} \text{ m}$$

$$= 4000\text{Å}$$

The condition for bright fringe or strong reflection is

$$2\mu t = \frac{(2n+1)\lambda}{2}$$

or $$\lambda = \frac{4\mu t}{(2n+1)}$$

Substituting the values of μ and t, we get

$$\lambda = \frac{4 \times 1.4 \times 10^{-6}}{2n+1}$$

$$= \frac{56+10^{-7}}{2n+1} \text{ m}$$

For values of $n < 4$ or >6, the values of λ do not lie in the visible range. But for $n = 4, 5, 6$ the following wavelengths lie in the visible range:

$$\lambda = \frac{56\times10^{-7}}{2\times4+7}$$

$$= 6.222 \times 10^{-7} \text{ m}$$

$$= 6222\text{Å}$$

LLOYD'S MIRROR EXPERIMENT

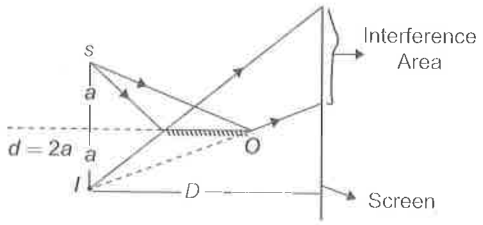


Figure 2.18

In this experiment, the light reflected from a long mirror and the light coming directly from the source without reflection produce interference on a screen, i.e. source and image behave as coherent sources.

An important feature of this experiment lies in the fact that when the screen is placed in contact with the end of the mirror, the edge of the reflecting surface comes at the centre of dark fringe instead of a bright fringe. The direct beam does not suffer any phase change; this means that the reflected beam undergoes a phase change of π radian.

Hence, at any point P on the screen the condition for minima and maxima is

$$S_2P - S_1P = n\lambda \text{ [For minima]}$$

$$S_2P - S_1P = \left(n + \frac{\lambda}{2}\right)\lambda \text{ [For maxima]}$$

SOLVED EXAMPLE

EXAMPLE 19

In Lloyd's interference experiment, 10 fringes occupy a space of 1.5 mm. The distance between the source and the screen is 1.25 m. If light of wavelength 6000Å is used, find the distance of the source from the plane minor.

SOLUTION

Here, $\beta = \frac{1.5}{10}$ mm

$= 0.15 \times 10^{-3}$ m

$D = 1.25$ m,

$\lambda = 6000$Å

$= 6 \times 10^{-7}$ m

As $\beta = \frac{D\lambda}{d}$

$\therefore$ $d = \frac{D\lambda}{\beta}$

$$= \frac{1.25\times6\times10^{-7}}{0.15\times10^{-3}} \text{ m}$$

$= 50 \times 10^{-4}$ m

$= 5.0$ mm

Hence, distance of source from the plane mirror

$$= \frac{d}{2} = 2.5 \text{ min.}$$

FRESENEL'S BIPRISM

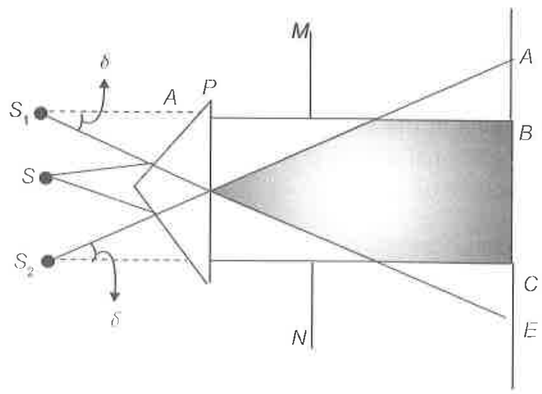


Figure 2.19

Figure 2.19 shows the Fresnel's biprism experiment schematically. The thin prism P refracts light from the slit source S into two beams AC and BE. When a screen MN

is placed as shown in the figure, the interference fringes are observed only in the region BC. If the screen MN is removed, the two beams will overlap over the whole region AE.

If A is the angle of refraction of thin prism and μ is the refractive index of its medium, then the angle of deviation produced by the prism is

$$\delta = A(\mu - 1)$$

If l_1 is the distance between the source and the prism, then the separation between virtual sources is

$$d = 2\delta$$

$$l_1 = 2A(\mu - 1)l_1$$

If l_2 is the distance between the prism and the screen, then the distance between virtual sources and the screen is given by

$$D = l_1 + l_2$$

Thus, by using the result of young's experiment, the fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \beta = \frac{\lambda(l_1 + l_2)}{2\delta l_1}$$

$$\beta = \frac{\lambda}{2\delta}\left[1 + \frac{l_2}{l_1}\right]$$

$$\Rightarrow \quad \beta = \frac{\lambda}{2A(\mu - 1)}\left[1 + \frac{l_2}{l_1}\right]$$

Fringes observed in the Fresnel's biprism experiment are vertical straight lines.

SOLVED EXAMPLES

EXAMPLE 20

In a biprism experiment, the slit is illuminated with light of wavelength 4800Å. The distance between the slit and biprism is 20 cm and that between biprism and eyepiece is 80 cm. If two virtual sources are 0.3 cm apart, determine the distance between the 5th bright band on one side of the central bright band and the 4th dark bank on the other side.

SOLUTION

Here,

$$\lambda = 4.8 \times 10^{-7}\text{ m},$$

$$d = 0.3 \times 10^{-2}\text{ m},$$

$$D = 20 + 80$$

$$= 100\text{ cm}$$

$$= 1\text{ m}$$

Distance of 5th bright from the central bright band is

$$x_5 = 5\frac{\Delta\lambda}{d}$$

■

EXAMPLE 21

In a biprism experiment, fringe width is measured as 0.4 mm. When the eyepiece is moved away from the biprism through 30 cm, the fringe width increases by 50%. If the two virtual sources are 0.6 mm apart, find the wavelength of light used.

SOLUTION

Here,

$$\beta_1 = 0.4\text{ mm}$$

$$= 0.4 \times 10^{-3}\text{ m},$$

$$\beta_2 = \frac{150}{100}$$

$$\beta_1 = 1.5\beta_1$$

$$D_2 = D_1 + 30\text{ cm}$$

$$= D_1 + 0.3\text{ m},$$

$$d = 0.6\text{ mm}$$

$$= 0.6 \times 10^{-3}\text{ m}$$

As

$$\beta = \frac{D_1\lambda}{d}$$

and

$$\beta = \frac{D_2\lambda}{d}$$

$$\frac{\beta_1}{\beta_2} = \frac{D_1}{D_2}$$

or $$\frac{\beta_1}{1.5\beta_1} = \frac{D_1}{D+0.3}$$

or $$D_1 = 0.6 \text{ m}$$

∴ Wavelength of light used,

$$\lambda = \frac{\beta_1 d}{D_1}$$

$$= \frac{0.4\times10^{-3}\times0.6\times10^{-3}}{0.6}$$

$$= 4 \times 10^{-7} \text{ m}$$

$$= 4000\text{Å}.$$

EXAMPLE 22

Interference fringes are produced by a Fresnel's biprism in the focal plane of reading microscope which is 100 cm from the slit. A lens interposed between the biprism and the microscope gives two images of the slit in two positions. If the images of the slits are 4.05 mm apart in one case, 2.90 mm in the other and the wavelength of light used is 5893Å, find the distance between two consecutive bands.

SOLUTION

Here, $$d_1 = 4.05 \text{ mm} = 0.405 \text{ cm},$$

$$d_2 = 2.09 \text{ mm} = 0.209 \text{ cm}$$

Distance between the two coherent sources will be

$$d = \sqrt{d_1 d_2} \text{ (Displacement method)}$$

$$= \sqrt{0.405\times0.209} \text{ cm}$$

$$= 0.2909 \text{ cm}$$

Also $$D = 100 \text{ cm},$$

$$\lambda = 5893 \times 10^{-8} \text{ cm}$$

∴ Fringe width,

$$\beta = \frac{D\lambda}{d}$$

$$= \frac{100\times5893\times10^{-8}}{0.2909} \text{ cm}$$

$$= 0.0203 \text{ cm}.$$

HUYGENS' PRINCIPLE

The following are the various postulates of Huyg principle:

1. Each source of light is a centre of disturbance fr which waves spread in all directions. All partic equidistant from the source and vibrating in sa phase lie on the surface known as wavefront.
2. Wave propagates perpendicular to wavefront.
3. Each ray takes same time to reach from one wavefr to another wavefront.
4. Every point on a wavefront is a source of n disturbance which produces secondary wavele These wavelets are spherical and travel with the spe of light in all directions in that medium.
5. Only forward envelope enclosing the tangents the secondary wavelets at any instant gives the n position of wavefront.

There is no backward flow of energy when a wa travels in the forward direction.

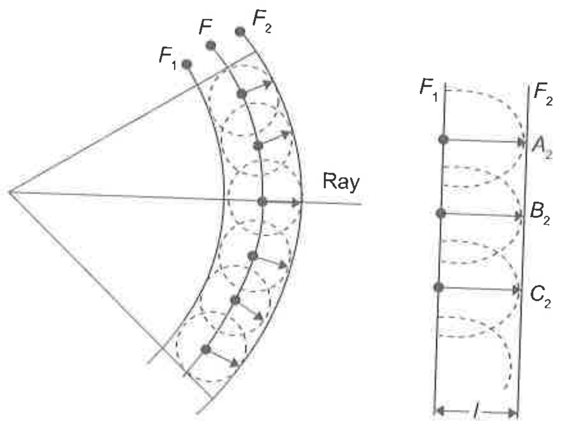
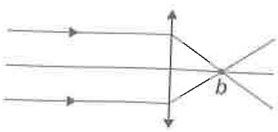


Figure 2.20

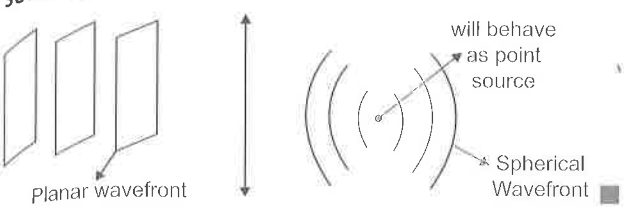
SOLVED EXAMPLE

EXAMPLE 23

For the given ray diagram, draw the wavefront.



SOLUTION



REFLECTION AND REFRACTION

We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure 2.21(a) shows an incident wavefront which makes an angle 'i' with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when point A on the incident wavefront strikes the surface, point B still has to travel a distance $BC = AC \sin i$, and this takes a time $t = BC/v_1 = AC(\sin i)/v_1$. After a time t, a secondary wavefront of radius $v_2 t$ with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e. would have zero radius. We also show a secondary wavelet originating from point D inbetween A and C. Its radius is less than $v_2 t$. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centred on A. We can see that the angle r' made by this refracted wavefront with the surface is given by $AE = v_2 t = AC \sin r'$. Hence, $t = AC(\sin r')/v_2$. Equating the two expressions for 't' gives us the law of refraction in the form $\sin i/\sin r' = v_1/v_2$. A similar picture is shown in Fig. 2.21(b) for the reflected wave, which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that $i = r$. Note that for both reflection and refraction, we see secondary wavelets starting at different times. Compare this with the earlier application (shown in the figure) where we start them at the same time.

The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. Figure 2.21 shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the reflected and incident wavefronts. The angle of incidence i and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in the figure, these are also the angles between the wavefront and the surface.

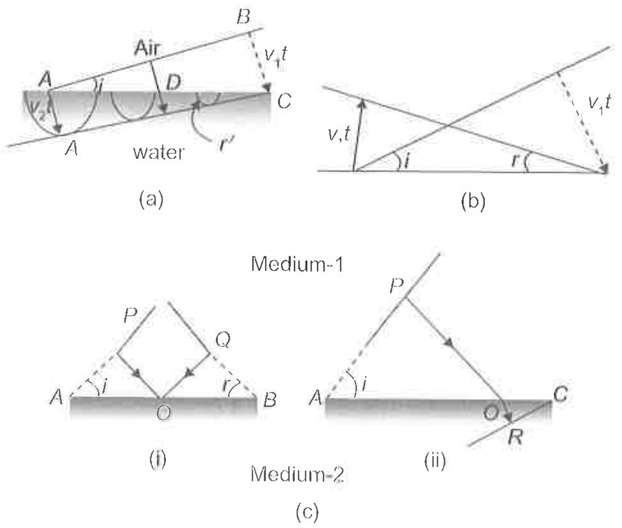


Figure 2.21 Huygens' construction for the (a) refracted wave, (b) reflected wave, and (c) calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.

We now calculate the total time to go from one wavefront to another along the rays. From Fig. 2.21(c), we have total time for light to reach from P to Q

$$= \frac{PO}{v_1} + \frac{OQ}{v_1}$$

$$= \frac{AO \sin i}{v_1} + \frac{OB \sin r}{v_1}$$

$$= \frac{OA \sin i + (AB - OA)\sin r}{v_1}$$

$$= \frac{AB \sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all the rays, the right side of equation must actually be independent of OA. The condition, for this to happen, is that the coefficient of OA in the equation should be zero, i.e., $\sin i = \sin r$. We, thus, have the law of reflection, $i = r$. Figure 2.21 also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles i and r' with the boundary. Angle r' is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the wavefronts along any ray.

$$\text{Time taken from } P \text{ to } R = \frac{PO}{v_1} + \frac{OR}{v_2}$$

$$= \frac{OA\sin i}{v_1} + \frac{(AC - OA)\sin r'}{v_2}$$

$$= \frac{AC\sin r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of which ray we consider. The coefficient of OA in the equation is, therefore, zero.

i.e.,
$$\frac{\sin i}{\sin r'} = \frac{v_1}{v_2} = n_{21}$$

where n_{21} is the refractive index of medium 2 with respect to medium 1. This is Snell's law of, refraction that we have already dealt with from the equation. n_{21} is the ratio of speed of light in the first medium (v_1) to that in the second medium (v_2). Equation is known as the Snell's law of refraction. If the first medium is vacuum, we have

$$\frac{\sin i}{\sin r'} = \frac{c}{v_2} = n_2,$$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From the equation, we then get

$$n_{21} = \frac{v_1}{v_2}$$

$$= \frac{(c/n_1)}{(c/n_2)}$$

$$= \frac{n_2}{n_1}$$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, $n_1 = 1.33$, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

Once we have the laws of reflection and refraction, the behaviour of prisms, lenses and mirrors can be understood. These topics are discussed in detail in the previous chapter. Here, the behaviour of the wavefronts in three cases (Fig. 2.21) is described.

1. Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.
2. A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.
3. Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig. 2.22). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because to the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.

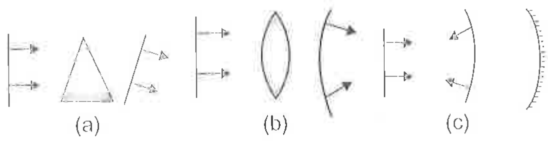


Figure 2.22

EXTRA PORTION FOR JEE MAIN

DIFFRACTION

1. **Meaning of Diffraction**
   It is the spreading of waves around the corners of an obstacle, of the order of wavelength.
2. **Definition of Diffraction**
   The phenomenon of bending of light waves around the sharp edges of opaque obstacles or aperture and their encroachment in the geometrical shadow of obstacle or aperture is defined as diffraction of light.
3. **Necessary Conditions of Diffraction of Waves**
   The size of the obstacle (a) must be of the order of the wavelength of the waves (λ).

$$\frac{a}{\lambda} \approx 1$$

Note

Greater the wavelength of the wave, the higher will be its degree of diffraction. This is the reason that diffraction of sound and radio waves is easily observed but for diffraction of light, additional arrangement have to be made.

$$\lambda_{sound} > \lambda_{light}$$

Wavelength of sound is nearly equal to size of obstacle. If the size of obstacle is a and the wavelength is λ, then

S.No.	a V/S λ	Diffraction
1	$a << \lambda$	Not possible
2	$a >> \lambda$	Not possible
3	$a \approx \lambda$	Possible

4. **Interpretation of Diffraction**
   As a result of diffraction, maxima and minima of light intensities are found, which have unequal intensities. Diffraction is the result of superposing of waves from infinite number of coherent sources on the same wavefront after the wavefront has been distorted by the obstacle.
5. **Example of Diffraction**
   (a) When an intense source of light is viewed with the partially opened eye, colours are observed in the light.
   (b) Sound produced in one room can be heard in the nearby room.
   (c) Appearance of a shining circle around the section of sun just before sun rise.
   (d) Coloured spectrum is observed if a light source at far distant is seen through a thin cloth.
6. **Fraunhofer Diffraction**
   Fraunhofer diffraction deals with wavefronts that are plane on arrival and an effective viewing distance of infinity. If follows that Fraunhofer diffraction is an important special case of Fresnel diffraction. In young's double slit experiment, we assume the screen to be relatively distant that we have Fraunhofer conditions.
7. **Difference Between Interference and Diffraction of Light**

Interference	Diffraction
1. Two coherent sources are necessary	One coherent source is necessary
2. All fringes has same width	Fringes have unequal width
3. Width of bright fringes is equal to other fringes	Width of bright fringes is just doubled to other fringes
4. Intensity of dark fringe is zero	Intensity of dark fringe is not zero
5. All bright fringes have equal intensity	As order of bright fringes increases, intensity goes down
6. For bright fringes:	For bright fringes:
(a) Path difference	$D = (2n-1)\lambda/2$
$\Delta = n\lambda$	$\delta = (2n-1)\pi$
(b) Phase difference $\delta = 2n\pi$	
7. For dark fringes:	For dark fringes:
(a) Path difference	$D = 2n\lambda/2$
$D = (2n-1)\lambda/2$	$d = 2n\pi$
(b) Phase difference	
$d = (2n-1)\pi$	

8. **Fraunhofer Diffraction for Single Slit**
   In this diffraction pattern, central maxima are bright on both sides of it; maxima and minima occur symmetrically

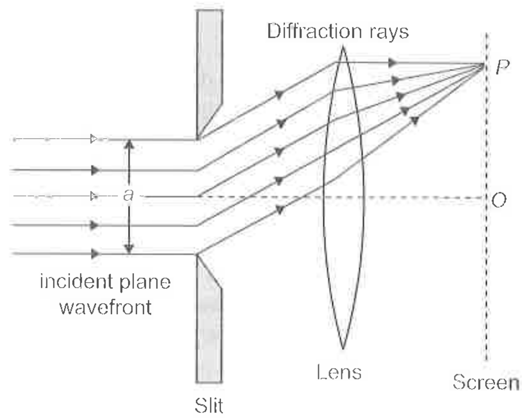


Figure 2.23

(A) For Diffraction Maxima:

$$a \sin \theta = (2n - 1)\ \lambda/2$$

(B) For Diffraction Minima:

$$a \sin \theta = n\lambda$$

(C) The maxima or minima are observed due to the superposition of waves emerging from infinite secondary sources between A and B points of slit.

(D) Fringe width:
The distance between two secondary minima formed on two sides of central maximum is known as the width of central maximum.

$$W = \frac{2f\lambda}{a}$$

f = focal distance of convex lens

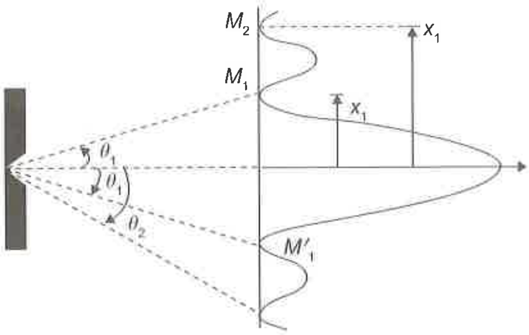


Figure 2.24

a = width of slit

$$\text{Angular width} = W_\theta = \frac{2\lambda}{a}$$

9. POINTS TO REMEMBER

1. Greymaldy discovered the diffraction.
2. Intensity of diffraction pattern decreases, if size of obstacle is increased.
3. Superposition of waves causes both diffraction and interference. Superposition of secondary waves originated from two coherent sources generates interference. Superposition of secondary wavelets generated from the same wavefront is called diffraction.
4. Interference fringes have equal width but diffraction fringes have unequal width.
5. In diffraction pattern, intensity of bright fringes is different but for interference it is equal.
6. Intensities of dark fringes of interference are zero but for diffraction it is not equal to zero.
7. Effect of diffraction can be observed in only geometrical shadow of end region.
8. In diffraction, bright central fringe has dou[ble] width, in comparison to others.

POLARIZATION

Unpolarized Light

1. An ordinary beam of light consists of a large num[ber] of waves emitted by the atoms or molecules of [the] light source. Each atom produces a wave with [its] own orientation of electric vector. However, beca[use] of all directions are equally probable the result[ant] electromagnetic wave is a superposition of wa[ves] produced by individual atomic sources. This wave [is] called unpolarized light.
2. All the vibrations of an unpolarized light at a gi[ven] instant can be resolved in two mutually perpendicu[lar] directions and hence an unpolarized light is equival[ent] to superposition of two mutually perpendicu[lar] identical plane polarized light.

Plane Polarized Light

1. If somehow we confine the vibrations of electric vec[tor] in one direction perpendicular to the direction of w[ave] motion, the light is said to be plane polarized and [the] plane containing the direction of vibration and w[ave] motion is called plane of polarization.
2. If an unpolarized light is converted into plane polari[zed] light, its intensity reduces to half.
3. Polarization is a convincing proof of wave nature [of] light.

Partially Polarized Light

If in case of unpolarized light, electric vector in some pl[ane] is either more or less; then in its perpendicular plane, [the] light is said to be partially polarized.

Methods of Polarization

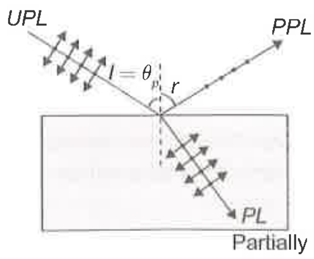


Figure 2.25

By Reflection: Brewster discovered that when light is incident at a particular angle on a transparent substance, the reflected light is completely plane polarized with vibrations in a plane perpendicular to the plane of incidence. This specific angle of incidence is called polarizing angle θ_p and is related to the refractive index μ of the material through the relation:

$$\tan \theta_p = \mu \qquad (1)$$

This is known as Brewster's law.

In case of polarization by reflection:

(a) For $I = \theta_p$, refracted light is plane polarized.

(b) For $I = \theta_p$, reflected and refracted rays are perpendicular to each other.

(c) For $I <$ or $> \theta_p$, both reflected and refracted light become partially polarized.

Intensity of Light Emerging from a Polaroid

If plane polarized light of intensity I_0 $(= KA^2)$ is incident on a polaroid and its vibrations of amplitude A make an angle θ with transmission axis, as polaroid will pass only those vibrations which are parallel to its transmission axis, i.e., $A \cos \theta$, so the intensity of emergent light will be

$$I = K(A \cos \theta)^2$$

$$= KA^2 \cos^2\theta$$

or

$$I = I_0 \cos^2\theta \quad [\text{as } I_0 = KA^2] \qquad (2)$$

This law is called **Malus law**. From this, it is clear that

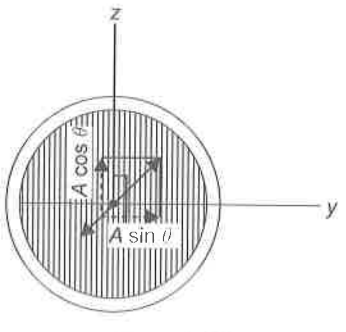


Figure 2.26

1. If the incident light is unpolarized, then as vibrations are equally probable in all directions (in a plane perpendicular to the direction of wave motion), θ can have any value from 0 to 2π and $(\cos^2 \theta)_{av} = \frac{1}{2}$

$$I = \frac{1}{2} I_0$$

i.e., If an unpolarized light is converted into plane polarized light, its intensity becomes half.

2. If light of intensity I_1 emerging from one polaroid called **polarizer** is incident on the second polaroid (usually called **analyzer**), the intensity of the light emerging from the second polaroid in accordance with Malus law will be given by

$$I_2 = I_1 \cos^2 \theta',$$

where θ' is the angle between the transmission axis of the two polaroids.

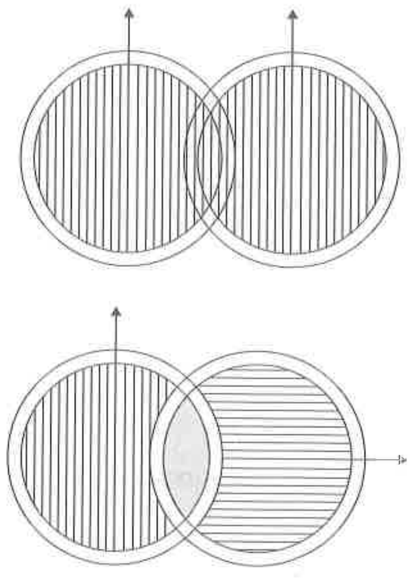


Figure 2.27

So if the two polaroids have their transmission axes parallel to each other, i.e., $\theta' = 0°$,

$$I_2 = I_1 \cos^2 0° = I_1$$

and if the two polaroids are crossed, i.e., have their transmission axes perpendicular to each other, $\theta' = 90°$

$$I_2 = I_1 \cos^2 90° = 0$$

So, if an analyzer is rotated from 0° to 90° with respect to polarizer, the intensity of emergent light changes from maximum value I_1 to minimum value zero.

EXERCISES

JEE Main

1. The path difference between two wave fronts emitted by coherent sources of wavelength 5460 Å is 2.1 micron. The phase difference between the wave fronts at that point is
   (A) 7.692 (B) 7.692π
   (C) $\frac{7.692}{\pi}$ (D) $\frac{7.692}{3\pi}$

2. Monochromatic light is that light in which
   (A) Single wavelength is present
   (B) Various wavelengths are present
   (C) Red and violet light is present
   (D) Yellow and red light is present

3. The path difference between two waves $y_1 = A_1 \sin wt$ and $y_2 = A_2 \cos(wt + \phi)$ will be
   (A) $(\lambda/2\pi)\,\phi$ (B) $(\lambda/2\pi)\,(\phi + \pi/2)$
   (C) $(2\pi/\lambda)\,(\phi - \pi/2)$ (D) $(2\pi/\lambda)\,\phi$

4. The resultant amplitude in interference with two coherent source depends upon -
   (A) Intensity
   (B) Only phase difference
   (C) On both the above
   (D) None of the above

5. The necessary condition for phenomenon of interference to occur is
   (A) There should be two coherent sources.
   (B) The frequency and amplitude of both the waves should be same
   (C) The propagation of waves should be simultaneously and in same direction
   (D) All of the above

6. Interference event is observed in
   (A) Only transverse waves
   (B) Only longitudinal waves
   (C) Both types of waves
   (D) Not observed in both type of waves

7. The nature of light which is verified by the interference event is
   (A) Particle nature (B) Wave nature
   (C) Dual nature (D) Quantum nature

8. The phenomenon of interference is based on the principle of
   (A) Diffraction (B) Superposition
   (C) Refraction (D) Polarisation

9. The equation for two waves obtained by two light sources are as given below: $y_1 = A_1 \sin 3wt$, $y_2 = A_2 \cos(3wt + \pi/6)$. What will be the value of phase difference at the time t -
   (A) $3\pi/2$ (B) $2\pi/3$
   (C) π (D) $\pi/2$

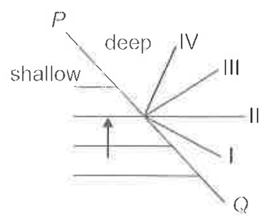
10. Two coherent sources have intensity ratio of 100:1, and are used for obtaining the phenomenon of interference. Then the ratio of maximum and minimum intensity will be
    (A) 100:1 (B) 121:81
    (C) 1:1 (D) 5:1

11. In coherent sources it is necessary that their
    (A) Amplitudes are same
    (B) Wavelengths are same
    (C) Frequencies are same
    (D) Initial phase remains constant

12. Two independent monochromatic sodium lamps cannot produce interference because
    (A) The frequencies of two sources are different
    (B) The phase difference between two sources changes with respect to time
    (C) The two sources become coherent
    (D) The amplitudes of two sources are different

A. Young's Double Slit Experiment

13. Figure, shows wave fronts in still water, moving in the direction of the arrow towards the interface PQ between a shallow region and a deep (denser) region. Which of the lines shown may represent one of the wave fronts in the deep region?



(A) I
(B) II
(C) III
(D) IV

14. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are:
(A) $5I$ and I
(B) $5I$ and $3I$
(C) $9I$ and I
(D) $9I$ and $3I$

15. When light is refracted into a denser medium,
(A) Its wavelength and frequency both increases
(B) Its wavelength increase but frequency remains unchanged
(C) Its wavelength decrease but frequency remains unchanged
(D) Its wavelength and frequency both decrease

16. In Young's double slit experiment 62 fringes are visible in the field of view with sodium light ($\lambda = 5893$ Å). If green light ($\lambda = 5461$ Å) is used then the number of visible fringes will be
(A) 62
(B) 67
(C) 85
(D) 58

17. In Young's double slit experiment, the distance of the n-th dark fringe from the centre is
(A) $n\left(\frac{\lambda D}{2d}\right)$
(B) $n\left(\frac{2d}{\lambda D}\right)$
(C) $(2n-1)\frac{\lambda D}{2d}$
(D) $(2n-1)\frac{4d}{\lambda D}$

18. If the yellow light is replaced by the violet light then the interference fringes-
(A) Will become fainter
(B) Will become brighter
(C) The fringe width will increase
(D) The fringe width will decrease

19. If the path difference between the interfering waves is $n\lambda$, then the fringes obtained on the screen will be
(A) Dark
(B) Bright
(C) coloured
(D) White

20. The fringe width in a Young's double slit experiment can be increased. If we decrease
(A) Width of the slits
(B) Separation of the slits
(C) Wavelength of the light used
(D) Distance between slits and screen

21. In young's double slit experiment, interference pattern is observed on the screen L distance apart from slits, average distance between adjacent fringes is x and slits separation is d, then the wavelength of light will be
(A) xd/L
(B) xL/d
(C) Ld/x
(D) Ldx

22. Plane microwaves from a transmitter are directed normally towards a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance 0.13 m. If the velocity of light is 3×10^8 m/s, find the frequency of the transmitter.
(A) 1.5×10^{10} Hz
(B) 10^{10} Hz
(C) 3×10^{10} Hz
(D) 6×10^{10} Hz

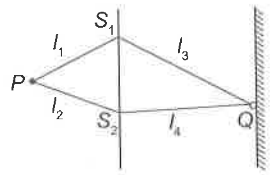
23. In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200 nm and $d =$ 700 nm.
(A) 12
(B) 7
(C) 18
(D) none of these

24. In a YDSE, the central bright fringe can be identified.
(A) as it has greater intensity than the other bright fringes
(B) as it is wider than the other bright fringes
(C) as it is narrower than the other bright fringes
(D) by using white light instead of single wavelength light.

25. In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is
(A) 1
(B) 2
(C) 3
(D) 4

26. If the Young's double slit experiment is performed with white light, then which of the following is not true
(A) the central maximum will be white
(B) there will not be a completely dark fringe
(C) the fringe next to the central will be red
(D) the fringe next to the central will be violet

27. Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P. If, as shown in the diagram above the light is then allowed to fall on a screen, and if n is a positive integer, the condition for destructive interference at Q is that



(A) $(l_1 - l_2) = (2n + 1)\ \lambda/2$
(B) $(l_3 - l_4) = (2n + 1)\ \lambda/2$
(C) $(l_1 + l_2) - (l_2 + l_4) = n\lambda$
(D) $(l_1 + l_3) - (l_2 + l_4) = (2n + 1)\ \lambda/2$

28. In a Young's double slit experiment, a small detector measures an intensity of illumination of I units at the centre of the fringe pattern. If one of the two (identical) slits is now covered, the measured intensity will be

(A) $2I$ (B) I
(C) $I/4$ (D) $I/2$

29. In a young double slit experiment D equals the distance of screen and d is the separation between the slit. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to

(A) $\frac{D\lambda}{d}$ (B) $\frac{D\lambda}{2d}$
(C) $\frac{D\lambda}{3d}$ (D) $\frac{2D\lambda}{d}$

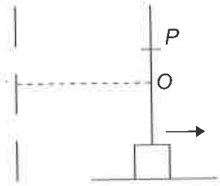
30. A beam of light consisting of two wavelength 6300 Å and λ Å is used to obtain interference fringes in a Young's double slit experiment. If 4th bright fringe of 6300 Å coincides with 5th dark fringe of λ Å, the value of λ (in Å) is

(A) 5200 (B) 4800
(C) 6200 (D) 5600

31. The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the fringe width from the centre is

(A) 2 (B) 1/2
(C) 4 (D) 16

32. In a Young's Double slit experiment, first maxima is observed at a fixed point P on the screen. Now the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (centre of the screen)



(A) remains constant
(B) keeps on decreasing
(C) first decreases and then increases
(D) First decreases and then becomes constant

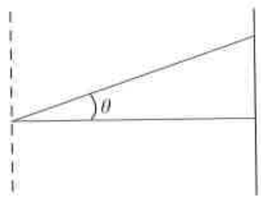
33. In a double slit experiment, the separation between th slits is $d = 0.25$ cm and the distance of the screen D = 100 cm from the slits. If the wavelength of light used i $\lambda = 6000$ Å and I_0 is the intensity of the central brigh fringe, the intensity at a distance $x = 4 \times 10^{-5}$ m fron the central maximum is

(A) I_0 (B) $I_0/2$
(C) $3I_0/4$ (D) $I_0/3$

34. In young's double slit experiment, the value of λ = 500 nm. The value of $d = 1$ mm, $D = 1$ m. Then th minimum distance from central maximum for whic the intensity is half the maximum intensity will be

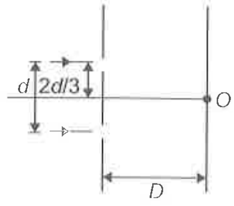
(A) 2.5×10^{-4} m (B) 2×10^{-4} m
(C) 1.25×10^{-4} (D) 10^{-4} m

35. Two slits are separated by 0.3 mm. A beam of 500 nı light strikes the slits producing an interference patter The number of maxima observed in the angular rang $-30° < \theta < 30°$.



(A) 300 (B) 150
(C) 599 (D) 149

36. In the figure shown if a parallel beam of white light incident on the plane of the slits then the distance the white spot on the screen from O is [Assume d < D, $\lambda << d$]



(A) 0 (B) $d/2$
(C) $d/3$ (D) $d/6$

37. In the above question if the light incident is monochromatic and point O is a maxima, then the wavelength of the light incident cannot be
(A) $d^2/3D$ (B) $d^2/6D$
(C) $d^2/12D$ (D) $d^2/18D$

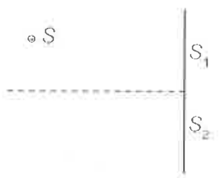
38. In Young's double slit arrangement, water is filled in the space between screen and slits. Then:
(A) fringe pattern shifts upwards but fringe width remains unchanged.
(B) fringe width decreases and central bright fringe shifts upwards.
(C) fringe width increases and central bright fringe does not shift.
(D) fringe width decreases and central bright fringe does not shift.

39. Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is:

(A) $\dfrac{2\pi\mu x}{\lambda}$ (B) $\dfrac{2\pi x}{\mu\lambda}$

(C) $\dfrac{2\pi(\mu-1)x}{\lambda}$ (D) $\dfrac{2\pi x}{(\mu-1)\lambda}$

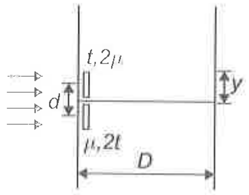
40. In YDSE, the source placed symmetrically with respect to the slit is now moved parallel to the plane of the slits so that it is closer to the upper slit, as shown. Then,



(A) the fringe width will increase and fringe pattern will shift down.
(B) the fringe width will remain same but fringe pattern will shift up.
(C) the fringe width will decrease and fringe pattern will shift down.
(D) the fringe width will remain same but fringe pattern will shift down.

41. Minimum thickness of a mica sheet having $\mu = 3/2$ which should be placed in front of one of the slits in YDSE is required to reduce the intensity at the centre of screen to half of maximum intensity is
(A) $\lambda/4$ (B) $\lambda/8$
(C) $\lambda/2$ (D) $\lambda/3$

42. In the YDSE shown the two slits are covered with thin sheets having thickness t and $2t$ and refractive index 2μ and μ. Find the position (y) of central maxima



(A) zero (B) tD/d
(C) $-tD/d$ (D) None

43. Two monochromatic and coherent point sources of light are placed at a certain distance from each other in the horizontal plane. The locus of all those points in the horizontal plane which have construct interference will be
(A) a hyperbola
(B) family of hyperbolas
(C) family of straight lines
(D) family of parabolas

B. Thin Film Interference

44. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate with the curved surface downwards. Monochromatic light is incident normally from the top. The observed interference fringes from this combination do not follow one of the following statements.

(A) the fringes are straight and parallel to the length of the piece.
(B) the line of contact of the cylindrical glass piece and the glass plate appears dark.
(C) the fringe spacing increases as we go outwards.
(D) the fringes are formed due to the interference of light rays reflected from the curved surface of the cylindrical piece and the top surface of the glass plate.

45. A circular planar wire loop is dipped in a soap solution and after taking it out, held with its plane vertical in air. Assuming thickness of film at the top very small, as sunlight falls on the soap film, and observer receive reflected light.
(A) the top portion appears dark while the first colour to be observed as one moves down is red.
(B) the top portion appears violet while the first colour to be observed as one moves down is indigo.
(C) the top portion appears dark while the first colour to be observed as one move down is violet.
(D) the top portion appears dark while the first colour to be observed as one move down depends on the refractive index of the soap solution.

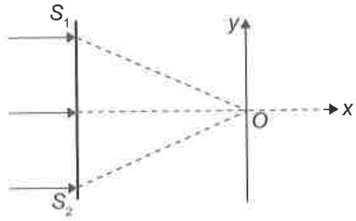
46. A thin film of thickness t and index of refraction 1.33 coats a glass with index of refraction 1.50. What is the least thickness t that will strongly reflect light with wavelength 600 nm incident normally?
(A) 225 nm (B) 300 nm
(C) 400 nm (D) 450 nm

47. In a biprism experiment the distance of source from biprism is 1 m and the distance of screen from biprism is 4 meters. The angle of refraction of biprism is 2×10^{-3} radians. μ of biprism is 1.5 and the wavelength of light used is 6000 Å. How many fringes will be seen on the screen?
(A) 4 (B) 5
(C) 3 (D) 6

Reasoning Type

48. Statement 1: In YDSE, as shown in figure, central bright fringe is formed at O. If a liquid is filled between plane of slits and screen, the central bright fringe is shifted in upward direction.

Statement 1: If path difference at O increases y-coordinate of central bright fringe will change



(A) Statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true and statement 2 is NOT the correct explanation for statement 1
(C) Statement 1 is true, statement -2 is false.
(D) Statement 1 is false, statement -2 is true.

49. Statement 1: In glass, red light travels faster than blue light.
Statement 2: Red light has a wavelength longer than blue
(A) Statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true and statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

50. Statement 1: In standard YDSE set up with visible light, the position on screen where phase difference is zero appears bright.
Statement 2: In YDSE set up magnitude of electromagnetic field at central bright fringe is not varying with time.
(A) Statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1
(B) Statement 1 is true, statement 2 is true and statement 2 is NOT correct explanation for statement 1
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

JEE Advanced

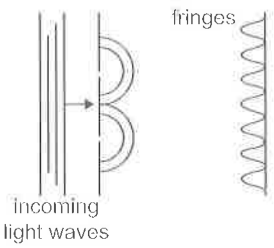
A. Young's Double Slit Experiment

1. To observe a stationary interference pattern formed by two light waves, it is not necessary that they must have:
   (A) the same frequency
   (B) same amplitude
   (C) a constant phase difference
   (D) the same intensity

2. In a YDSE apparatus, we use white light then:
   (A) the fringe next to the central will be red
   (B) the central fringe will be white.
   (C) the fringe next to the central will be violet
   (D) there will not be a completely dark fringe.

3. If the source of light used in a Young's Double Slit Experiment is changed from red to blue, then
   (A) the fringes will become brighter
   (B) consecutive fringes will come closer
   (C) the number of maxima formed on the screen increases
   (D) the central bright fringe will become a dark fringe.

4. In a Young's double slit experiment, green light is incident on the two slits. The interference pattern is observed on a screen. Which of the following changes would cause the observed fringes to be more closely spaced?



   (A) Reducing the separation between the slits
   (B) Using blue light instead of green light
   (C) Used red light instead of green light
   (D) Moving the light source further away from the slits.

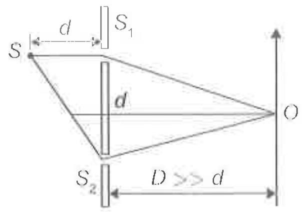
5. In a Young's double-slit experiment, let A and B be the two slits. A thin film of thickness t and refractive index μ is placed in front of A. Let β = fringe width. The central maximum will shift:
   (A) towards A (B) towards B
   (C) by $t(\mu-1)\beta/\lambda$ (D) by $\mu t\beta/\lambda$

6. In the previous question, films of thicknesses t_A and t_B and refractive indices μ_A and μ_B, are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will:
   (A) not shift
   (B) shift towards A
   (C) shift towards B
   (D) option (B), if $t_B > t_A$; option (C) if $t_B < t_A$

7. In a YDSE, if the slits are of unequal width.
   (A) fringes will not be formed
   (B) the positions of minimum intensity will not be completely dark
   (C) bright fringe will not be formed at the centre of the screen
   (D) distance between two consecutive bright fringes will not be equal to the distance between two consecutive dark fringes.

8. If one of the slits of a standard YDSE apparatus is covered by a thin parallel sided glass slab so that it transmit only one half of the light intensity of the other, then
   (A) the fringe pattern will get shifted towards the covered slit
   (B) the fringe pattern will get shifted away from the covered slit
   (C) the bright fringes will be less bright and the dark ones will be more bright.
   (D) the fringe width will remain unchanged

9. To make the central fringe at the centre O, a mica sheet of refractive index 1.5 is introduced. Choose the correct statements (s).



(A) the thickness of sheet is $2\left(\sqrt{2}-1\right)d$ infront of S_1

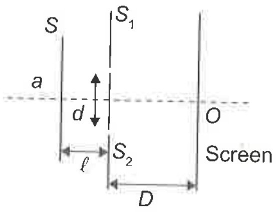
(B) the thickness of sheet is $\left(\sqrt{2}-1\right)d$ infront of S_2

(C) the thickness of sheet is $2\sqrt{2}d$ infront of S_1

(D) the thickness of sheet is $\left(2\sqrt{2}-1\right)d$ infront of S_1

Question No. 10 to 12

The figure shows a schematic diagram showing the arrangement of Young's Double Slit Experiment

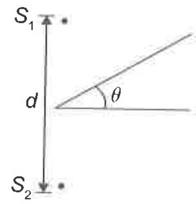


10. Choose the correct statement(s) related to the wavelength of light used
(A) Larger the wavelength of light larger the fringe width
(B) The position of central maxima depends on the wavelength of light used
(C) If white light is used in YDSE, then the violet colour forms its first maxima closest to the central maxima
(D) The central maxima of all the wavelengths coincide

11. If the distance D is varied, then choose the correct statement(s)
(A) the angular fringe width does not change
(B) the fringe width changes in direct proportion
(C) the change in fringe width is same for all wavelengths
(D) The position of central maxima remains unchanged

12. If the distance d is varied, then identify the correct statement
(A) the angular width does not change
(B) the fringe width changes in inverse proportion
(C) the positions of all maxima change
(D) the positions of all minima change

13. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity then $I(\theta)$ for $0 \le \theta \le 90°$ is given by



(A) $I(\theta) = \dfrac{I_0}{2}$ for $\theta = 30°$

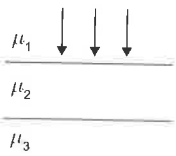
(B) $I(\theta) = \dfrac{I_0}{4}$ for $\theta = 90°$

(C) $I(\theta) = I_0$ for $\theta = 0°$

(D) $I(\theta)$ is constant for all values of θ

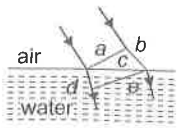
14. In a standard YDSE appratus a thin film ($\mu = 1.5$, $t = 2.1\ \mu$m) is placed in front of upper slit. How far above or below the centre point of the screen are two nearest maxima located? Take $D = 1$ m, $d = 1$ mm, $\lambda = 4500$ Å. (Symbols have usual meaning)
(A) 1.5 mm (B) 0.6 mm
(C) 0.15 mm (D) 0.3 mm

15. Consider a case of thin film interference as shown. Thickness of film is equal to wavelength of light is μ_2.



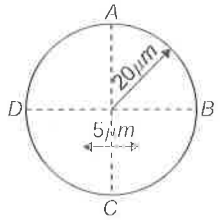
(A) Reflected light will be maxima if $\mu_1 < \mu_2 < \mu_3$
(B) Reflected light will be maxima if $\mu_1 < \mu_2 > \mu_3$
(C) Transmitted light will be maxima if $\mu_1 > \mu_2 > \mu_3$
(D) Transmitted light will be maxima if $\mu_1 > \mu_2 < \mu_3$

16. Figure shown plane waves refracted for air to water using Huygen's principle a, b, c, d, e are lengths on the diagram. The refractive index of water wrt air is the ratio.



(A) a/c (B) b/c
(C) b/d (D) d/b

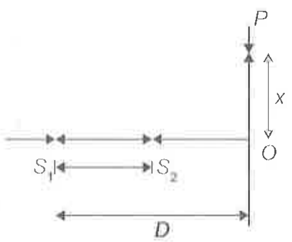
17. Two point source separated by $d = 5\mu m$ emit light of wavelength $\lambda = 2\mu m$ in phase. A circular wire of radius $20\mu m$ is placed around the source as shown in figure.



(A) Point A and B are dark and points C and D are bright
(B) Points A and B are bright and point C and D are dark
(C) Points A and C are dark and points B and D are bright
(D) Points A and C are bright and points B and D are dark

18. Two monochromatic (wavelength $= a/5$) and coherent sources of electromagnetic waves are placed on the x-axis at the points $(2a, 0)$ and $(-a, 0)$. A detector moves in a circle of radius $R(>>2a)$ whose centre is at the origin. The number of maximas detected during one circular revolution by the detector are
(A) 60 (B) 15
(C) 64 (D) None

19. Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 3λ. The light is collected on a screen S which is placed at a distance D ($>> \lambda$) from the slits. The smallest distance x such that the P is a maxima.

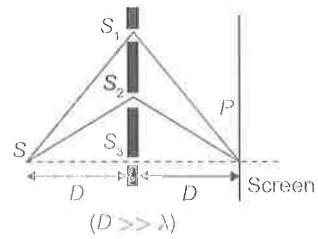


(A) $\sqrt{3}D$ (B) $\sqrt{8}D$
(C) $\sqrt{5}D$ (D) $\sqrt{5}\frac{D}{2}$

20. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same setup the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the average intensity of light at the midpoint of the screen in the first case to that in the second case is
(A) 1:1 (B) 2:1
(C) 4:1 (D) none of these

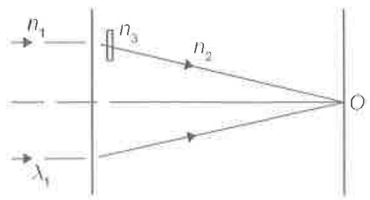
21. A beam of light consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in Young's double slit experiment. The distance between slits is 2 mm and the distance of screen from slits is 120 cm. What is the least distance from central maximum where the bright due to both wavelength coincide cm
(A) 0.156 cm (B) 0.312 cm
(C) 0.078 cm (D) 0.468 cm

22. A monochromatic light source of wavelength λ is placed at S. Three slits S_1, S_2 and S_3 are equidistant from the source S and the point P on the screen. $S_1P - S_2P = \lambda/6$ and $S_1P - S_3P = 2\lambda/3$. If I be the intensity at P when only one slit is open, the intensity at P when all the three slits are open is



(A) 3I (B) 5I
(C) 8I (D) zero

23. In the figure shown in YDSE, a parallel beam of light is incident on the slit from a medium of refractive index n_1. The wavelength of light in this medium is λ_1. A transparent slab to thickness 't' and refractive index n_3 is put infront of one slit. The medium between the screen and the plane of the slits is n_2. The phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits) is :

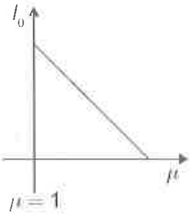


(A) $\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)t$ (B) $\frac{2\pi}{\lambda_1}(n_3 - n_2)t$

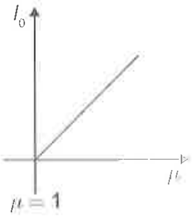
(C) $\frac{2\pi n_1}{n_2\lambda_1}\left(\frac{n_3}{n_2} - 1\right)t$ (D) $\frac{2\pi n_1}{\lambda_1}(n_3 - n_1)t$

24. In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with 'μ' will be best represented by ($\mu \geq 1$). [Assume slits of equal width and there is no absorption by slab]

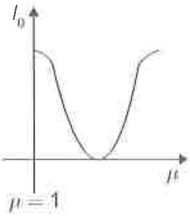
(A)



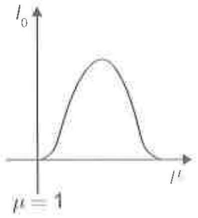
(B)



(C)



(D)



25. In a YDSE with two identical slits, when the upper slits is covered with a thin, perfectly transparent sheet of mica, the intensity at the centre of screen reduces to 75% of the initial value. Second minima is observed to be above this point and third maxima below it. Which of the following can not be a possible value of phase difference caused by the mica sheet.

(A) $\pi/3$ (B) $13\pi/3$

(C) $17\pi/3$ (D) $11\pi/3$

26. It is necessary to coat a glass with a non-reflecting layer. If the wave length of the light in the coating is λ, the best choice is a layer of material having an index of refraction between those of glass and air and a thickness of

(A) $\lambda/4$ (B) $\lambda/2$

(C) $3\lambda/8$ (D) λ

27. A parallel coherent beam of light falls on fresnel biprism of refractive index μ and angle α. The fringe width on a screen at a distance D from biprism will be (wavelength = λ)

(A) $\frac{\lambda}{2(\mu-1)\alpha}$ (B) $\frac{\lambda D}{2(\mu-1)\alpha}$

(C) $\frac{D}{2(\mu-1)\alpha}$ (D) none

JEE Advanced

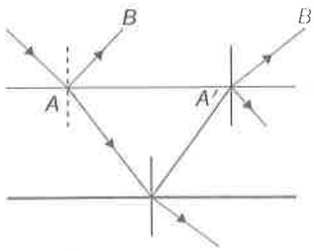
Level I

1. In a Young's double slit experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light (λ = 600 nm). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of index 4/3?

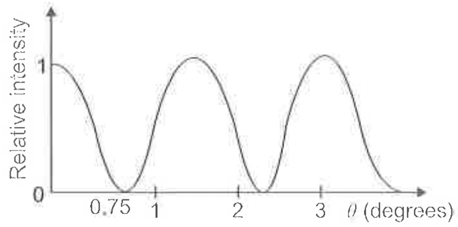
2. In Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of the light is changed to 400 nm, find the number of fringes observed in the same segment.

3. On slit of double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5^{th} bright fringe. Assuming that both the glass plates have same thickness and wavelength of light used is 4800 Å, find their thickness.

4. A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 20% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. Find the ratio I_{max}/I_{min}.



5. Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus deflection angle θ as shown in the figure. Find the separation d between the slits.



6. In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5 mm from the second dark fringe from the centre of the fringe pattern on same side. Find the wavelength of the light used.

7. In a YDSE apparatus, $d = 1$ mm, $\lambda = 600$ nm and $D = 1$ m. The slits produce same intensity on the screen Find the minimum distance between two points on the screen having 75% intensity of the maximum intensity.

8. The distance between two slits in a YDSE apparatus is 3 mm. The distance of the screen from the slits is 1 m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima.

9. A lens ($\mu = 1.5$) is coated with a thin film of refractive index 1.2 in order to reduce the reflection from its surface at $\lambda = 4800$ Å . Find the minimum thickness of the film which will minimize the intensity of the reflected light.

10. A long narrow horizontal slit lies 1 mm above a plane mirror. The interference pattern produced by the slit and its image is viewed on a screen distant 1 m from the slit. The wavelength of light is 600 nm. Find the distance of first maximum above the mirror.

11. A broad source of light of wavelength 680 nm illuminates normally two glass plates 120 mm long that meet at one end and are separated by a wire 0.048 mm in diameter at the other end. Find the number of bright fringes formed over the 120 mm distance.

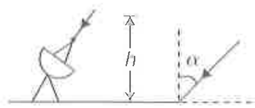
12. In a two-slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} . If the distance between the slits is 10^{-3}m, calculate the wavelength of the light used.

13. A monochromatic light of $\lambda = 5000$ Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern in seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness 1.5×10^{-6} m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate. Also find the lateral shift of the central maximum.

14. A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of the slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.
    (a) Calculate the fringe width.
    (b) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minima on the axis.

15. A young's double slit experiment is performed using light of wavelength $\lambda = 5000$ Å, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$ m apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$ m is placed over one of the slits. The refractive index of the material of this sheet is $\mu = 1.17$. Where does the central maximum of the interference pattern now appear ?

16. Radio waves coming at $<\alpha$ to vertical are recieved by a radar after reflection from a nearby water surface and directly. What should be height of antenna from water surface so that it records a maximum intensity. (wavelength = λ).



17. In a biprism experiment using sodium light $\lambda = 6000$ Å an interference pattern is obtained in which 20 fringes occupy 2 cm. On replacing sodium light by another source of wavelength λ_2 without making any other change 30 fringes occupy 2.7 cm on the screen. What is the value of λ_2?

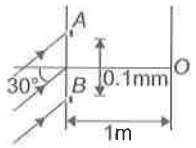
Level II

1. If the slits of the double slit were moved symmetrically apart with relative velocity v, calculate the rate at which fringes pass a point at a distance x from the centre of the fringe system formed on a screen y distance away from the double slits if wavelength of light is λ. Assume $y >> d$ and $d >> \lambda$.

2. **(a)** A thin glass plate of thickness t and refractive index μ is inserted between screen and one of the slits in a Young's experiment. If the intensity at the centre of the screen is I, what was the intensity at the same point prior to the introduction of the sheet.

   (b) One slit of a Young's experiment is covered by a glass plate ($\mu_1 = 1.4$) and the other by another glass plate ($\mu_2 = 1.7$) of the same thickness. The point of central maxima on the screen, before the plates were introduced is now occupied by the third bright fringe. Find the thickness of the plates, the wavelength of light used is 4000 Å.

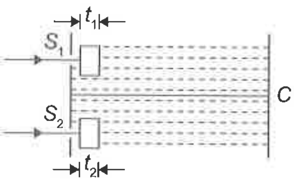
3. In a YDSE a parallel beam of light of wavelength 6000 Å is incident on slits at angle of incidence 30°. A and B are two thin transparent films each of refractive index 1.5. Thickness of A is 20.4 μm. Light coming through A and B have intensities I and $4I$ respectively on the screen. Intensity at point O which is symmetric relative to the slits is $3I$. The central maxima is above O.



   (a) What is the maximum thickness of B to do so. Assuming thickness of B to be that found in part (a) answer the following parts.
   (b) Find fringe width, maximum intensity and minimum intensity on screen.
   (c) Distance of nearest minima from O.
   (d) Intensity at 5 cm on either side of O.

4. In a YDSE experiment, the distance between the slits and the screen is 100 cm. For a certain distance between the slits, an interference pattern is observed on the screen with the fringe width 0.25 mm. When the distance between the slits is increased by $\Delta d = 1.2$ mm, the fringe width decreased to $n = 2/3$ of the original value. In the final position, a thin glass plate of refractive index 1.5 is kept in front of one of the slits and the shift of central maximum is observed to be 20 fringe width. Find the thickness of the plate and wavelength of the incident light.

5. A screen is at a distance $D = 80$ cm from a diaphragm having two narrow slits S_1 and S_2 which are $d = 2$ mm apart. Slit S_1 is covered by a transparent sheet of thickness $t_1 = 2.5$ μm and S_2 by another sheet of thickness $t_2 = 1.25$ μm as shown in figure. Both sheets are made of same material having refractive index $\mu = 1.40$. Water is filled in space between diaphragm and screen. A monochromatic light beam of wavelength $\lambda = 5000$ Å is incident normally on the diaphragm. Assuming intensity of beam to be uniform and slits of equal width, calculate ratio of intensity at C to maximum intensity of interference pattern obtained on the screen, where C is foot of perpendicular bisector of S_1S_2. (Refractive index of water, $\mu_w = 4/3$)

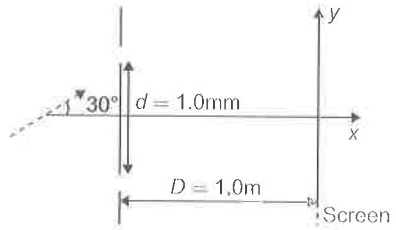


6. In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3}m to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7}m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength.

7. In a Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å . It is found that the point P on the screen where the central maximum ($n = 0$) fell before the glass plates were inserted now has 3/4 the original intensity. It is

further observed that what used to be the 5th maximum earlier, lies below the point P while the 6th minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected).

8. A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on screen placed parallel to the plane of the slits at a distance of 1.0 m from it, as shown in the figure.



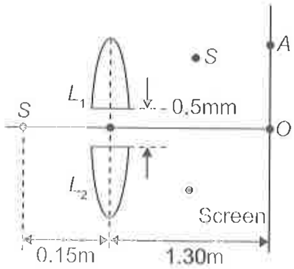
(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in the figure), find the y-coordinates of the first minima on either side of the central maximum.

9. In a Young's double slit arrangement, a source of wavelength 6000 Å is used. The screen is placed 1m from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes if they subtend an angle more than 1 minute of arc. Calculate the maximum distance between the slits so that the fringes are clearly visible. Using this information calculate the position of 3rd bright and 5th dark fringe from the centre of the screen.

10. A narrow monochromatic beam of light of intensity I is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25% of the light incident on it and transmits the remaining. Find the ratio of the minimum and the maximum intensities in the interference pattern formed by the two beams obtained after one reflection at each plate.

11. In the figure shown S is a monochromatic point source emitting light of wavelength = 500 nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m, while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO.

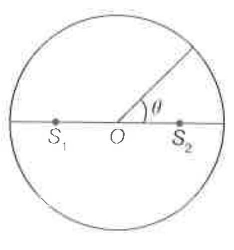


(i) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease or remain the same?

12. Two coherent sources S_1 and S_2 separated by distance 2λ emit light of wavelength λ in phase as shown in figure. A circular wire of radius 100λ is placed in such a way that S_1S_2 lies in its plane and the midpoint of S_1S_2 is at the centre of wire. Find the angular positions on the wire for which intensity reduces to half of its maximum value.

13. In a biprism experiment with sodium light, bands of width of 0.0195 cm are observed at 100 cm from slit. On introducing a convex lens 30 cm away from the slit between and screen, two images of the slit are seen 0.7 cm apart at 100 cm distance from the slit. Calculate the wavelength of sodium light.



evious Year Questions

: Main

1. To demonstrate the phenomenon of interference we require two sources which emit radiations of **(AIEEE 2003)**
   (a) nearly the same frequency
   (b) the same frequency
   (c) different wavelength
   (d) the same frequency and having a definite phase relationship

2. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is **(AIEEE 2004)**
   (a) infinite (b) five
   (c) three (d) zero

3. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is **(AIEEE 2005)**
   (a) $\frac{1}{2}I_0$ (b) $\frac{1}{4}i_0$
   (c) zero (d) I_0

4. If I_0 is the intersity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled ? **(AIEEE 2005)**
   (a) $2I_0$ (b) $4I_0$
   (c) I_0 (d) $\frac{I_0}{2}$

5. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **(AIEEE 2005)**
   (a) hyperbola (b) circle
   (c) straight line (d) parabola

6. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, is equal to **(AIEEE 2007)**
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

7. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is **(AIEEE 2009)**
   (a) 393.4 nm (b) 885.0 nm
   (c) 442.5 nm (d) 776.8 nm

8. The initial shape of the wave front of the beam is **(AIEEE 2010)**
   (a) convex
   (b) concave
   (c) convex near the axis and concave near the periphery
   (d) planer

9. This question has a paragraph followed by two statements, Statement I and Statement II. Of the given four alternatives after the statements, choose the one that describes the statements.A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film. **(AIEEE 2011)**
   Statement I : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change or π.
   Statement II : The centre of the inerference pattern is dark.
   (a) Statement I is true, Statement II is true Statement II is the correct explanationof Statement I
   (b) Statement I is true, Statement II is true, Statement II is not the correct explana tion of Statement I
   (c) Statement I is false, Statement II is true
   (d) Statement I is true, Statement II is false

10. In a Young's double slit experiment, the two slits act as coherent source of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in first

case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is

(AIEEE 2011)

(a) 4 (b) 2
(c) 1 (d) 0.5

11. At two points P and Q on screen in Young's double slit expereiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$ respectively. The ratio of intensities at P and Q will be **(AIEEE 2011)**

(a) 3:2 (b) 2:1
(c) $\sqrt{2}:1$ (d) 4:1

12. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when very interfere at phase difference ϕ, is given by **(AIEEE 2012)**

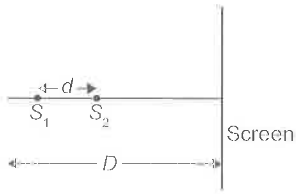
(A) $\frac{I_m}{9}(4+5\cos\varphi)$ (B) $\frac{I_m}{9}\left(1+2\cos^2\frac{\varphi}{2}\right)$

(C) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (D) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

13. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is : **(AIEEE 2013)**

(A) $I_0/4$ (B) $I_0/8$
(C) I_0 (D) $I_0/2$

14. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be : **(AIEEE 2013)**



(A) semi-circles (B) concentric circles
(C) points (D) straight lines

15. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then equals: **[JEE Main 2014]**

(A) 1 (B) $\frac{1}{3}$

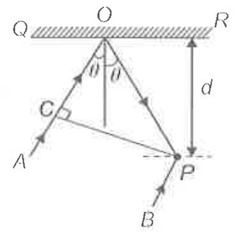
(C) 3 (D) $\frac{3}{2}$

JEE Advanced

1. In the ideal souble-slit experiment, when a glass - plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plote is: **[JEE 2002]**

(A) 2λ (B) $2\lambda/3$
(C) $\lambda/3$ (D) λ

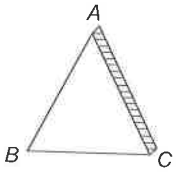
2. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP **[JEE (Scr) 2003]**



(A) $\cos\theta = \frac{3\lambda}{2d}$ (B) $\cos\theta = \frac{\lambda}{4d}$

(C) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (D) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

3. A prism ($\mu_p = \sqrt{3}$) has an angle of prism $A = 30°$. A thin film ($\mu_f = 2.2$) is coated on face AC as shown in the figure. Light of wavelength 550 nm is incident on the face AB at 60° angle of incidence. Find
   (i) the angle of its emergence from the face AC and
   (ii) the minimum thickness (in nm) of the film for which the emerging light is of maximum possible intensity.



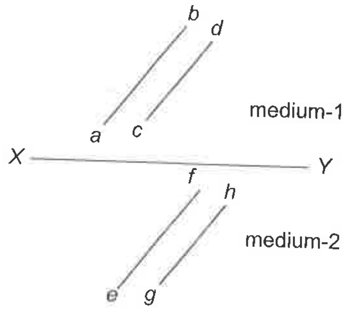
4. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 nm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is **[JEE (Scr) 2004]**
   (A) 4 mm (B) 5.6 mm
   (C) 14 mm (D) 28 mm

5. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maxima coincide again? Take $D/d = 10^3$. Symbols have their usual meanings. **[JEE 2004]**

6. In Young's double slit experiment maximum intensity is I than the angular position where the intensity becomes 1/4 is **[JEE (Scr) 2005]**
   (A) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (B) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$
   (C) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ (D) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

Question No. 7 to 9

The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave traveling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



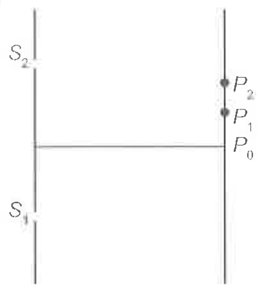
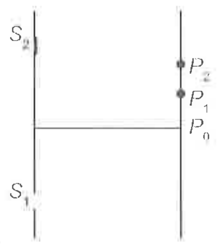
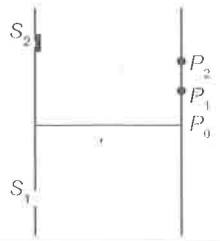
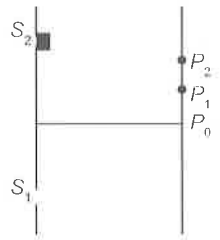
7. Light travels as a
   (A) parallel beam in each medium
   (B) convergent beam in each medium
   (C) divergent beam in each medium
   (D) divergent beam in one medium and convergent beam in the other medium **[JEE 2007]**

8. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.
   (A) ϕ_c cannot be equal to ϕ_d
   (B) ϕ_d can be equal to ϕ_e
   (C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
   (D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$ **[JEE 2007]**

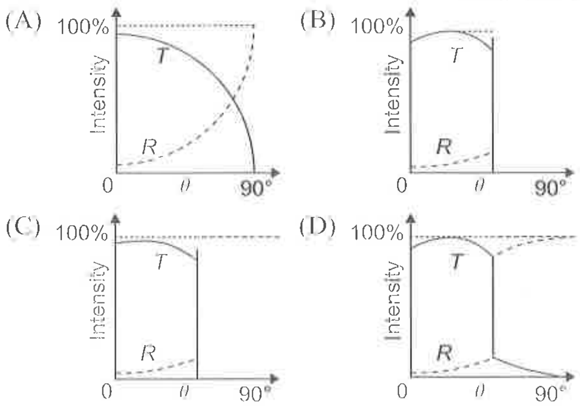
9. Speed of light is **[JEE 2007]**
   (A) the same in medium-1 and medium-2
   (B) larger in medium-1than in medium-2
   (C) larger in medium-2 than in medium-1
   (D) different at b and d

10. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).
    (A) If $d = \lambda$, the screen will contain only one maximum
    (B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
    (C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase.
    (D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark the bright fringes will increase. **[JEE 2008]**

11. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation. **[JEE 2009]**

Column I	Column II
(A) 	(P) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \dfrac{\lambda}{4}$ 	(Q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \dfrac{\lambda}{2}$ 	(R) $I(P_1) = 0$
(D) $(\mu - 1)t = \dfrac{3\lambda}{4}$ 	(S) $I(P_0) > I(P_1)$
	(T) $I(P_2) > I(P_1)$

12. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ are plotted. The correct sketch is **[JEE 2011]**



13. Young's double slit experiment is carried out by using green, red and bluelight, one color at a time. The fringe widths recorded are β_G, β_R and β_b, respectively. Then. **[JEE 20012]**

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$
(C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

14. In the young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is: **[JEE 2013]**

(A) $(2n+1)\dfrac{\lambda}{2}$ (B) $(2n+1)\dfrac{\lambda}{4}$

(C) $(2n+1)\dfrac{\lambda}{8}$ (D) $(2n+1)\dfrac{\lambda}{16}$

15. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes from them within a distance y on one side of the central maximum are m_1 and m_2 respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes from λ_1 is greater than λ_2 **[JEE 2014]**

ANSWER KEYS

Exercises

JEE Main

1. B	2. A	3. B	4. C	5. D	6. C	7. B	8. B	9. B	10. B
11. D	12. B	13. A	14. C	15. C	16. B	17. C	18. C	19. B	20. B
21. A	22. A	23. B	24. D	25. B	26. C	27. D	28. C	29. C	30. D
31. A	32. C	33. C	34. C	35. C	36. D	37. A	38. D	39. A	40. D
41. C	42. B	43. B	44. C	45. C	46. A	47. B	48. D	49. A	50. C

JEE Advanced

1. BD	2. BCD	3. BC	4. B	5. AC	6. D	7. B	8. ACD	9. A	10. ACD
11. ABD	12. BD	13. AC	14. CD	15. AD	16. C	17. D	18. A	19. D	20. B
21. A	22. A	23. A	24. C	25. A	26. A	27. A			

JEE Advanced

Level I

1. 0.225 mm 2. 18 3. 8 mm 4. 81:1 5. 1.98×10^{-2} mm 6. 5000 Å 7. 0.2 mm 8. 35.35 cm
9. 10^{-7} m 10. 0.15 mm 11. 141 12. 6000 Å 13. 0 1.5 mm 14. 0.63 mm, 1.575 mm
15. $y = 0.085\,D$; D = distance between screen and slits 16. $\dfrac{\lambda}{4\cos\alpha}$ 17. 5400 Å

Level II

1. $\dfrac{X}{\lambda Y}V$ 2. (a) $I_0 = I\sec^2\left[\dfrac{\pi(\mu-1)t}{\lambda}\right]$, (b) 4 mm 3. (a) t_B = 120 mm (b) b = 6 mm; $I_{max} = 9I$, $I_{min} = I$
(c) $b/6$ = 1 mm (d) I (at 5 cm above 0) = 9I, I (at 5 cm below 0) = 3I 4. l = 600 nm, t = 24 mm 5. 3/4
6. 7 mm, 1.6 mm (decrease) 7. 9.3 mm 8. (a) $\pm\dfrac{1}{\sqrt{15}}, \dfrac{3}{\sqrt{7}}$ (b) $+\dfrac{1}{\sqrt{15}}, \dfrac{3}{\sqrt{7}}$ 9. $\dfrac{6.48}{\pi}$; $\dfrac{\pi}{3.6}$ mm, $\dfrac{\pi}{2.4}$ mm
10. 1:49 11. (i) 1 mm (ii) increase 12. $\pm\cos^{-1}\left(\dfrac{2n+1}{8}\right)$, n = 0, 1, 2, 3 and $p \pm \left(\dfrac{2n+1}{8}\right)$ n = 0, 1, 2, 3 13. l = 5850 Å

Previous Year Questions

JEE Main

1. D	2. B	3. A	4. C	5. D	6. D	7. C	8. D	9. B	10. B
11. B	12. D	13. D	14. B	15. B					

JEE Advanced

1. A 2. B 3. 0.125 nm 4. D 5. 3.5 mm 6. B 7. A 8. C 9. B 10. A, B
11. (A) → (ps), (B) → (q), (C) → (t), (D) → (rst) 12. C 13. D 14. B
15. A, B, C

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CHAPTER 3

Electrostatics-1

INTRODUCTION

1. Introduction

Electromagnetism is a science of the combination of electrical and magnetic phenomenon. Electromagnetism can be divided into two parts:

Electrostatics: It deals with the study of charges at rest.

Electrodynamics: It deals with the study of charges in motion (discusses magnetic phenomenon).

In this chapter, we will be dealing with charges at rest, i.e. electrostatics.

2. Structure of Atom

An atom consists of two parts: (i) nucleus and (ii) extra nuclear part. Nucleus consists of neutrons and protons, and extra nuclear part has electrons revolving around nucleus.

In a neutral atom,

number of electrons = number of protons and

charge of electrons = charge of protons = 1.602×10^{-19} coulomb.

Normally, positive charges are positron, proton and positive ions. In nature, practically free-existing positive charges are positive ions and negative charges are electrons.

3. Electric Charge

Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of the naturally charged particles are electron, proton, α particle, etc.

4. Types of Charge

Positive charge: It is the deficiency of electrons compared to protons.

Negative charge: It is the excess of electrons compared to protons.

5. Units of Charge

Charge is a derived physical quantity. It is measured in coulomb in SI unit. In practice, we use mC (10^{-3} C), nC (10^{-6} C), nC (10^{-9}C), etc.

CGS unit of charge = electrostatic unit = *esu*.

1 coulomb = 3×10^9 *esu* of charge

Dimensional formula of charge = $[M^0L^0T^1I^1]$

6. Properties of Charge

(a) Charge is a scalar quantity: It adds algebraically and represents excess or deficiency of electrons.

(b) Charge is transferable: Charging a body implies transfer of charge (electrons) from one body to another positively charged body which means loss of electrons, i.e. deficiency of electrons. Negatively charged body means excess of electrons. This also shows that mass of a negatively charged body is greater than that of a positively charged identical body.

(c) Charge is conserved: In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.

(d) Charge is quantized: Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron ($1e = 1.6 \times 10^{-19}$ coulomb). So charge on any body $Q = \pm ne$, where n is an integer and e is the charge of the electron. Millikan's oil drop experiment proved the quantization of charge or atomicity of charge.

Note

Recently, the existence of particles of charge $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$ has been postulated. These particles are called quarks, but still this is not considered as the quantum of charge because these are unstable (They have very short span of life.)

(e) Like-point charges repel each other, while unlike point charges attract each other.

(f) Charge is always associated with mass, i.e. charge cannot exist without mass, though mass can exist without charge. The particle such as photon or neutrino, which has no (rest) mass, can never have a charge.

(g) **Charge is relativistically invariant:** This means that charge is independent of frame of reference, i.e. charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge of the mass of a body that depends on its speed and increases with increase in speed.

(h) A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself, while a charge having accelerated motion emits electromagnetic radiation.

7. Conductors and Insulators

Any object can be broadly classified in either of the following two categories:

(a) Conductors

(b) Insulators

Conductors: These are the materials that allow flow of charge through them. This category generally comprises metals but may sometimes contain non-metals too (e.g., carbon in the form of graphite).

Insulators: These are the materials which do not allow movement of charge through them.

8. Charging of Bodies

An object can be charged by addition or removal of electrons from it. In general, an object can either be a conductor or an insulator. Thus, we are going to discuss the charging of a conductor and charging of an insulator in brief.

(a) Charging of Conductors

Conductors can be charged by

(i) rubbing or frictional electricity,

(ii) conduction and induction (will be studied in later sections),

(iii) thermionic emission (will be studied under the topic "heat") and

(iv) photo electric emission (will be studied under the topic "modern physics").

(b) Charging of Insulators

Since charge cannot flow through insulators, neither conduction nor induction can be used to charge insulators, so in order to charge an insulator friction is used. Whenever an insulator is rubbed against a body, exchange of electrons takes place between the two. This results in the appearance of equal and opposite charges on the insulator and the other body. Thus, the insulator is charged. For example, rubbing of plastic with fur and silk with glass cause charging of these things.

To charge the bodies through friction, one of them has to be an insulator.

COLOUMB'S LAW

Coulomb, through his experiments, found out that the two charges 'q_1' and 'q_2' kept at distance 'r' in a medium as shown in Fig. 3.1 exert a force 'F' on each other. The value of force F is given by

$$F = \frac{Kq_1q_2}{r^2}$$

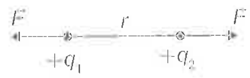


Fig. 3.1

This law gives the net force experienced by q_1 and q_2 taking in to account the medium surrounding them,
where

F gives the magnitude of electrostatic force;

q_1 and q_2 are the magnitudes of the two interacting charges;

K is the electrostatic constant that depends upon the medium surrounding the two charges.

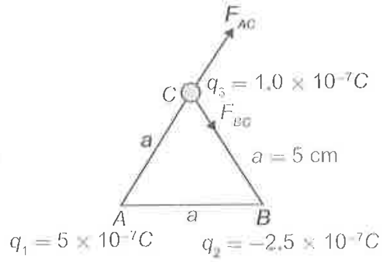
This force F acts along the line joining the two charges and is repulsive if q_1 and q_2 are of the same sign and is attractive if they are of opposite sign.

Let us consider some examples on application of Coulomb's law.

SOLVED EXAMPLES

EXAMPLE 1

Charges 5.0×10^{-7} C, -2.5×10^{-7} C and 1.0×10^{-7} C are fixed at the corners A, B and C of an equilateral triangle of side 5.0 cm. Find the electric force on the charge at C due to the rest two.



SOLUTION

$$F_{AC} = \frac{9\times10^9 \times 5\times10^{-7} \times 1\times10^{-7}}{(0.05)^2} = 0.18 \text{ Nt}$$

$$F_{BC} = \frac{9\times10^9 \times -2.5\times10^{-7} \times 1\times10^{-7}}{(0.05)^2} = -0.09 \text{ Nt}$$

Net force on C is

$$\vec{F}_{net} = \vec{F}_{AC} + \vec{F}_{BC}$$

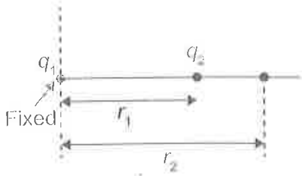
$$\left|\vec{F}_{net}\right| = \sqrt{(F_{AC})^2 + (F_{BC})^2 + 2(F_{AC})(F_{BC})\cos\theta}$$

$[\theta = 120°]$

$$= 0.15588 \text{ Nt}$$

EXAMPLE 2

If charge q_1 is fixed and q_2 is free to move, then find out the velocity of q_2 when it reaches distance r_2 after it is released from a distance of r_1 from q_1 as shown in the figure (assume friction is absent).



Find v of q_2 when it reaches distance r_2 after it is released from rest.

SOLUTION

$$a = \frac{Kq_1q_2}{mx^2}$$

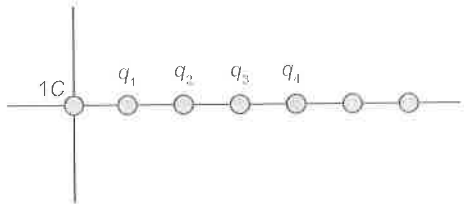
$$\int_0^v v\,dv = \frac{kq_1q_2}{m}\int_{r_1}^{r_2}\frac{dx}{x^2}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow \quad v = \sqrt{\frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

EXAMPLE 3

Ten charged particles are kept fixed on the x axis at point $x = 10$ mm, 20 mm, 30 mm, …, 100 mm. The first particle has a charge 10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8}C and so on. The tenth particle has a charge 1000×10^{-8}C. Find the magnitude of electric force acting on a 1 C charge placed at the origin.



SOLUTION

Force of 1 C charge

$$= \frac{Kq_1\times1}{(10\times10^{-3})^2} + \frac{Kq_2\times1}{(20\times10^{-3})^2} + \frac{Kq_3\times1}{(30\times10^{-3})^2} + \cdots$$

$$= \frac{K\times10^{-8}}{10^{-4}}\left[\frac{1^3}{1^2} + \frac{2^3}{2^2} + \frac{3^3}{3^2} + \cdots + \frac{10^3}{10^2}\right]$$

$$= 9\times10^9\times10^{-4}\times55 = 4.95\times10^7 \text{ Nt}$$

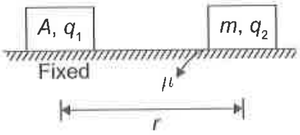
[This example explains that the concept of superposition holds in the case of electric forces. Net electric force at the origin is equal to the sum of the individual electric forces on the 1 C charge.]

EXAMPLE 4

A block 'A' of charge q_1 is fixed, and the second block of mass m and charge q_2 is allowed to be free on the floor. Find out the range of q_2 for which the particle is at rest.

SOLUTION

Maximum friction $= \mu mg$



$$\mu mg = \frac{kq_1q_2}{r^2}$$

$$\Rightarrow \qquad q_2 = \frac{\mu mgr^2}{kq_1}$$

$$-\frac{\mu mgr^2}{kq_1} < q < \frac{\mu mgr^2}{kq_1}$$

Vector Forms of Coloumb's Law

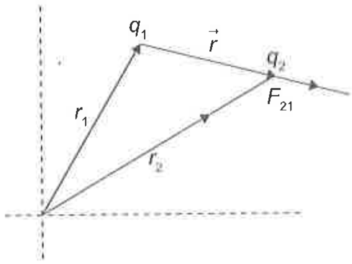


Fig. 3.2

(F_{21}: force on q_2 due to q_1)

$$\vec F_{21} = \frac{kq_1q_2}{r^2}\hat r = \frac{kq_1q_2}{r^3}\vec r$$

$$\vec F_{21} = \frac{kq_1q_2}{[\vec r_2 - \vec r_1]^3}(\vec r_2 - \vec r_1)$$

- Head of $\vec r$ points at that position where force has to be calculated.
- $\vec r_2$ and $\vec r_1$ depend on origin but does not.
- q_1 and q_2 should be put along with sign. ■

EXAMPLE 5

Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the seven charges.

SOLUTION

The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on the vertices of the cube.

For this, we use vector form of Coulomb's law

$$\vec F = \frac{Kq_1q_2}{|\vec r_1 - \vec r_2|^3}(\vec r_1 - \vec r_2)$$

From the figure, the different forces acting on A are given as

$$\vec F_{A1} = \frac{Kq^2(-a\hat k)}{a^3}$$

$$\vec F_{A2} = \frac{Kq^2(-a\hat j - a\hat k)}{(\sqrt2 a)^3}$$

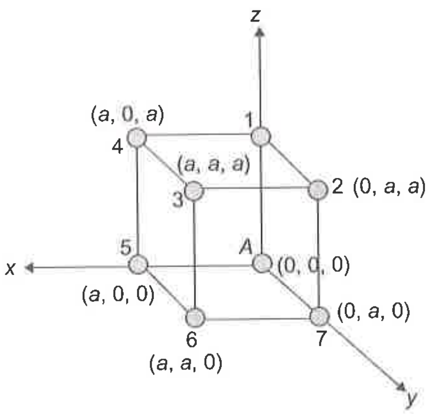
$$\vec F_{A3} = \frac{Kq^2(-a\hat i - a\hat j - a\hat k)}{(\sqrt3 a)^3}$$

$$\vec F_{A4} = \frac{Kq^2(-a\hat i - a\hat k)}{(\sqrt2 a)^3}$$

$$\vec F_{A5} = \frac{Kq^2(-a\hat i)}{a^3}$$

$$\vec F_{A5} = \frac{Kq^2(-a\hat i - a\hat j)}{(\sqrt2 a)^3}$$

$$\vec F_{A7} = \frac{Kq^2(-a\hat j)}{a^3}$$



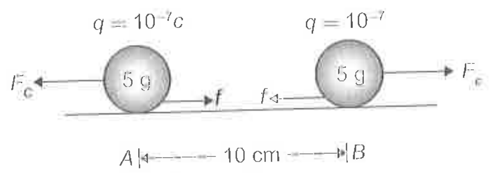
The net force experienced by A can be given as

$$\vec F_{net} = \vec F_{A1} + \vec F_{A2} + \vec F_{A3} + \vec F_{A4} + \vec F_{A5} + \vec F_{A6} + \vec F_{A7}$$

$$= \frac{-Kq^3}{a}\left[\left(\frac{1}{3\sqrt3} + \frac{1}{\sqrt2} + 1\right)(\hat i + \hat j + \hat k)\right]$$ ■

EXAMPLE 6

Two particles, each having a mass of 5 g and charge 1.0×10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.



SOLUTION

Consider particle A. **Forces** acting on A are coulombic force, and frictional force under limiting condition friction will be limiting and will be equal to coulombic force.

$$F_C = \frac{Kq^2}{r^2}$$

$$= \frac{9\times10^9 \times (10^{-7})^2}{(10\times10^{-2})^2} = 9\times10^{-3} \text{ N}$$

$$f = \mu N = \mu mg$$

$$= \mu(5\times10^{-3}\times10) = \mu(5\times10^{-2}\text{ N})$$

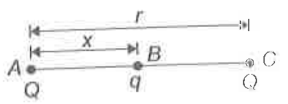
For equilibrium, we have $F_C = f$

$$9\times10^{-3} = \mu(5\times10^{-2})$$

$$\Rightarrow \qquad \mu = \frac{9\times10^{-3}}{5\times10^{-2}} = 0.18$$ ■

EXAMPLE 7

Two identical charges, Q each, are kept at a distance r from each other. A third charge q is placed on the line joining the above two charges such that all of these charges are in equilibrium. What is the magnitude, sign and position of the charge q?



SOLUTION

Suppose the three charges be placed in the manner, as shown in the figure.

The charge q will be in equilibrium if the forces exerted on it by the charges at A and C are equal and opposite.

$$k\cdot\frac{Qq}{x^2} = k\cdot\frac{Qq}{(r-x)^2}$$

or $\qquad x^2 = (r-x)^2$

or $\qquad x = r - x$

or $\qquad x = \frac{r}{2}$

Since the charge at A is repelled by the similar charge at C, it will be in equilibrium if it is attracted by the charge q at B, i.e. the sign of charge q should be opposite to that of charge Q.

$\therefore$ Force of repulsion between charges at A and C = Force of attraction between charges at A and B

$$k\frac{Q\cdot q}{(r/2)^2} = k\frac{Q\cdot Q}{r^2}$$

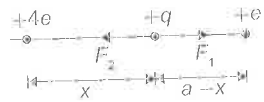
or $\qquad q = \frac{Q}{4}$ ■

EXAMPLE 8

Two point charges $+4e$ and $+e$ are fixed at a distance 'a' apart. Where should a third-point charge q be placed on the line joining the two charges so that it may be in equilibrium? In which case the equilibrium will be stable and in which it will be unstable?

SOLUTION

Suppose the three charges are placed as shown in the figure.



Let the charge q be positive.

For the equilibrium of charge $+q$, we must have Force of repulsion F_1 between $+4e$ and $+q$ = Force of repulsion F_2 between $+e$ and $+q$

or $\qquad \frac{1}{4\pi\varepsilon_0}\frac{4e\times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e\times q}{(a-x)^2}$

or $\qquad 4(a-x)^2 = x^2$

or $\qquad 2(a-x) = \pm x$

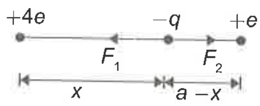
$$\therefore \qquad x = \frac{2a}{3} \text{ or } 2a.$$

As the charge q is placed between $+4e$ and $+e$, $x = 2a/3$ is possible. Hence for equilibrium, the charge q must be placed at a distance $2a/3$ from the charge $+4e$.

We have considered the charge q to be positive.

If we displace it slightly towards charge e, from the equilibrium position, then F_1 will decrease and F_2 will increase and a net force $(F_2 - F_1)$ will act on q towards left, i.e. towards the equilibrium position. Hence, the equilibrium of position q is stable.

Now if we take charge q to be negative, the forces F_1 and F_2 will be attractive, as shown in the figure.



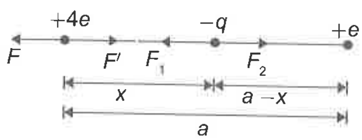
The charge $-q$ will still be in equilibrium at $x = 2a/3$. However, if we displace charge $-q$ slightly towards right, then F_1 will decrease and F_2 will increase. A net force $(F_2 - F_1)$ will act on $-q$ towards right, i.e. away from the equilibrium position. So the equilibrium of the negative q will be unstable. ■

EXAMPLE 9

Two 'free' point charges $+4e$ and $+e$ are placed a distance 'a' apart. Where should a third-point charge $-q$ be placed between them such that the entire system may be in equilibrium? What should be the magnitude and sign of q? What type of equilibrium will it be?

SOLUTION

Suppose the charges are placed as shown in the figure.



As the charge $+e$ exerts repulsive force F on charge $+4e$, so for the equilibrium of charge $+4e$, the charge $-q$ must exert attraction F' on $+4e$. This requires the charge q to be negative.

For equilibrium of charge $+4e$,

$$F = F'$$

$$\frac{1}{4\pi\varepsilon_0}\frac{4e \times e}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4e \times q}{x^2}$$

or

$$q = \frac{ex^2}{a^2}$$

For equilibrium of charge $-q$,

F_1 between $+4e$ and $-q$

F_2 between $+e$ and $-q$

$$\frac{1}{4\pi\varepsilon_0}\frac{4e \times e}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e \times q}{(a-x)^2}$$

or

$$x^2 = 4(a - x)^2$$

$$\therefore \qquad x = 2a/3$$

Hence,

$$q = \frac{ex^2}{a^2} = \frac{e}{a^2}\cdot\frac{4a^2}{9} = \frac{4e}{9}$$ ■

EXAMPLE 10

A charge Q is to be divided into two small objects. What should be the value of the charge on the objects so that the force between the objects will be maximum?

SOLUTION

Let one body have charge q and other $Q - q$

Here force between the charges

$$F = \frac{Kq(Q-q)}{r^2}$$

For F to be maximum

$$\frac{dF}{dq} = 0$$

$$\frac{d}{dq}\left(\frac{KqQ}{r^2} - \frac{Kq^2}{r^2}\right) = 0$$

$$\frac{KQ}{r^2} - \frac{2Kq}{r^2} = 0$$

$$q = \frac{Q}{2}$$

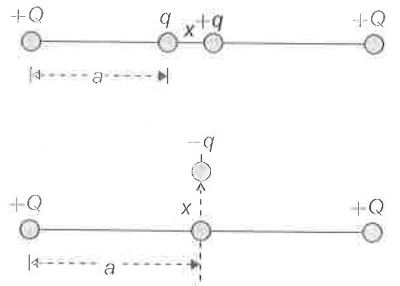
Thus, we have to divide charges equally on the objects. ■

EXAMPLE 11

Two identical positive point charges Q each are fixed apart at a distance $2a$. A point charge q lies midway between the fixed charges. Show that:

(a) For small displacement (relative to a) along line joining the fixed charges, the charge q executes SHM if it is positive and

(b) For small lateral displacement, it executes SHM if it is positive. Compare the frequencies of oscillation in the two cases.



SOLUTION

The two situations are shown in the figure.

(a) Let x be the displacement of the charge $+q$ from the mean position. Now net force acting on the charge q towards its equilibrium position is

$$F = \frac{KQq}{(a-x)^2} - \frac{KQq}{(a+x)^2}$$

$$= \frac{4KQqax}{(a^2-x^2)^2} \approx \frac{4KQqax}{a^4} \quad [\text{as } x << a]$$

Restoring acceleration $a = \frac{F}{m} = -\frac{4KQqx}{ma^3}$ [−ve sign shows restoring tendency]

$$a = -\omega^2 x$$

[where m is the mass of the charge]

As acceleration is directly proportional to displacement, the motion is SHM. Its time period T_1 is given by

$$T_1 = \frac{2\pi}{\omega}$$

$$T_1 = 2\pi\sqrt{\frac{ma^3}{4QqK}} = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{qQ}} \quad (1)$$

(b) Restoring force on $-q$ towards Q is given by

$$F = \frac{2KQq}{(a^2+x^2)} \cdot \frac{x}{\sqrt{(a^2+x^2)}}$$

$$= \frac{2KQq}{(a^2+x^2)^{3/2}} \approx \frac{2KQqx}{a^3} \quad [\text{as } x << a]$$

Restoring acceleration $a = \frac{F}{m} = -\frac{2KQq}{ma^3}x$

$$a = -\omega^2 x$$

Hence, the motion is SHM. Its time period T_2 is given by

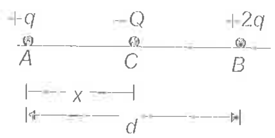
$$T_2 = \frac{2\pi}{\omega}$$

$$T_2 = 2\pi\sqrt{\frac{ma^3}{2QqK}} = 2\pi\sqrt{\frac{2\pi\varepsilon_0 ma^3}{qQ}} \quad (2)$$

Now, $\frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{2}$ ■

EXAMPLE 12

Two particles A and B having charges q and $2q$, respectively, are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?



SOLUTION

For the charges to be in equilibrium, forces should be balanced on A as well as on B.

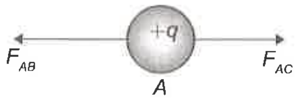
Balancing forces on A

$$F_{AB} = \frac{Kq(2q)}{d^2}$$

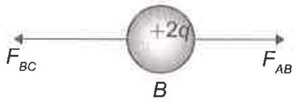
$$F_{AC} = \frac{KqQ}{x^2}$$

or $$\frac{2q}{d^2}=\frac{Q}{x^2}$$

or $$Q=\frac{2qx^2}{d^2} \quad (1)$$



Balancing force on B



$$F_{BC}=\frac{2Kq(Q)}{(d-x)^2}$$

or $$\frac{2Kq(Q)}{(d-x)^2}=\frac{Kq(2q)}{d^2}$$

or $$\frac{Q}{(d-x)^2}=\frac{q}{d^2} \quad (2)$$

Solving equation (1) and (2), we get

$$\frac{2qx^2}{d^2}=\frac{q}{d^2}(d-x)^2$$

or $$2x^2=(d-x)^2$$

or $$2x^2=d^2+x^2-2xd$$

or $$x^2+2xd-d^2=0$$

or $$x=\left(\sqrt{2}-1\right)d$$

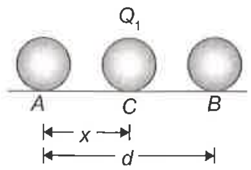
or $$-d\left(1+\sqrt{2}\right)$$

The negative value implies that the particle C will lie towards left of A at a distance $(\sqrt{2}-1)d$ from A (as x was measured from A).

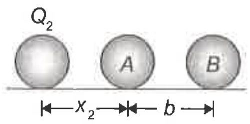
For the position $x=x_1=(\sqrt{2}-1)d$ $Q=Q_1=-q\left(6-\sqrt{2}\right)$

and for $x=x_2=-d\left(\sqrt{2}+1\right)$ $\quad Q=Q_2=-q(6+4\sqrt{2})$

Thus, the two possibilities are shown in the figure.



or

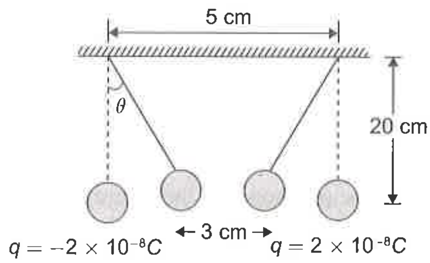


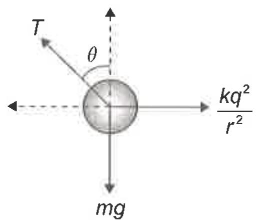
EXAMPLE 13

Two identical pitch balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each. The separation between the suspension points is being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the string. The charge on each ball has magnitude 2×10^{-8} C.

SOLUTION

As the balls are rubbed against each other, they will acquire equal and opposite charges. The FBD of left ball is shown in the figure which shows all the forces acting on ball in equilibrium position.





Here for equilibrium of each bob, we have

$$T\sin\theta=\frac{Kq^2}{r^2} \quad (1)$$

$$T\cos\theta=mg \quad (2)$$

or $$\tan\theta=\frac{kq^2}{r^2mg}$$

or $$\frac{1}{\sqrt{(20)^2-1^2}}=\frac{k\left(2\times10^{-8}\right)^2}{\left(3\times10^{-2}\right)^2 m\times10}$$

$$m = 7.99 \text{ gm}$$

or

From Eq. (2)

$$T = \frac{mg}{\cos\theta}$$

$$= \frac{m \times 10 \times 20}{\sqrt{(20)^2 - 1}}$$

$$= 8 \times 10^{-2} \text{ N}$$

EXAMPLE 14

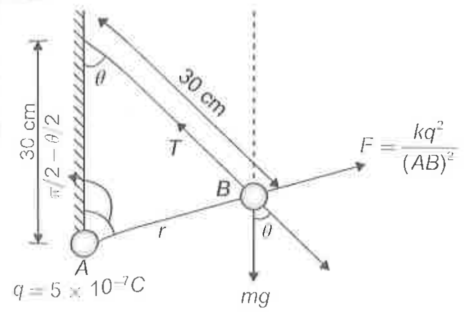
A particle A having a charge $q = 5 \times 10^{-7}$ C is fixed on a vertical wall. A second particle B of mass 100 g and having equal charge is suspended by a silk thread of length 30 cm from the wall. The point of suspension is 30 cm above the particle A. Find the angle of thread with vertical when it stays in equilibrium.

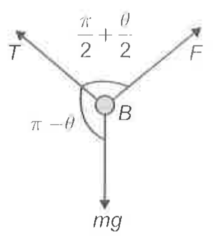
SOLUTION

The situation is shown in the figure.

Here, the forces acting on bob B can be shown as follows:

FBD of B is





Using Lami's theorem, we get

$$\frac{mg}{\sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right)} = \frac{F}{\sin(\pi - \theta)}$$

or

$$\frac{mg}{\cos\frac{\theta}{2}} = \frac{Kq^2}{2 \times 0.30 \times \sin\frac{\theta}{2} \times \sin\theta}$$

or

$$\frac{mg}{\cos\frac{\theta}{2}} = \frac{Kq^2}{2 \times 0.60 \sin\frac{\theta}{2} \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

or

$$\sin^2\frac{\theta}{2} = \frac{Kq^2}{2mg(0.60)}$$

On solving, we get $\theta = 17°$

Coloumb's Law in a MEDIUM

Relative Permittivity

When two charges are placed in vacuum or when the same set of charges is placed in a medium, the net force experienced by the charges will be different. The effect of the presence of medium is accounted in the proportionality constant K. This electrostatic constant K is defined as

$$K = \frac{1}{4\pi\varepsilon} \quad \text{where } \varepsilon = \varepsilon_0\varepsilon_r$$

where ε = absolute permittivity of medium

ε_0 = permittivity of free space, having a constant value = 8.85×10^{-12} C²/N-m²

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ = relative permittivity of medium with respect to free space, also termed as dielectric constant.

For free space $\varepsilon_r = 1$

and

$$K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2}$$

Force Dependency on Medium

We can say that when two charges are placed in vacuum (or air), the force experienced by the charges can be given as

$$F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

When these charges are submerged in a medium, having dielectric constant ε_r, the force becomes

$$F_{med} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

or

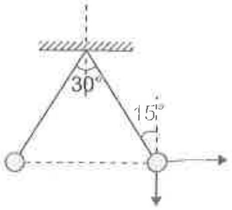
$$F_{med} = \frac{F_{air}}{\varepsilon_r} \text{ as } \varepsilon_r > 1$$

$$\Rightarrow \quad F_{med} < F_{air}$$

SOLVED EXAMPLES

EXAMPLE 15

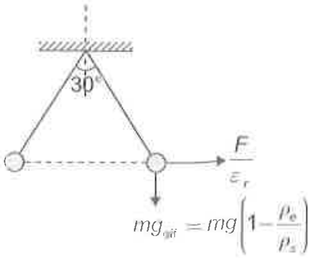
Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cc, the angle remains same. What is the dielectric constant of liquid? Density of sphere = 1.6 g/cc.



SOLUTION

When set-up shown in the figure is in air, we have

$$\tan 15^\circ = \frac{F}{mg}$$



When set-up is immersed in the medium as shown in the figure, the electric force experienced by the ball will reduce and will be equal to $\frac{F}{\varepsilon_r}$ and the effective gravitational force will become $mg\left(1-\frac{\rho_e}{\rho_s}\right)$

Thus, we have

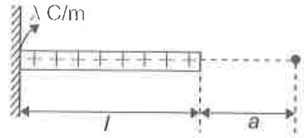
$$\tan 15^\circ = \frac{F}{mg\varepsilon_r\left(1-\frac{\rho_e}{\rho_s}\right)}$$

$$\varepsilon_r = \frac{1}{1-\frac{\rho_e}{\rho_s}} = 2$$

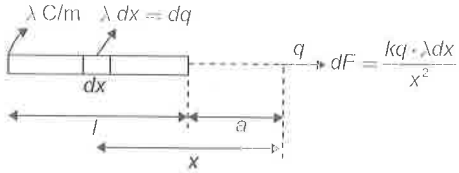
■

EXAMPLE 16

Find the total force on charge q due to a charge rod having linear charge density λ C/m placed as shown in the figure.



SOLUTION



$$\text{Force } F = \int dF = \int_a^{a+l} \frac{kq'\lambda \cdot dx}{x^2}$$

$$= kq\lambda\left[-\frac{1}{x}\right]_a^{a+l}$$

$$= kq\lambda\left[\frac{1}{a} - \frac{1}{a+l}\right]$$

■

ELECTRIC FIELD

Figure 3.3 shows a charge q that is lying in free space.

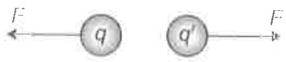


Figure 3.3

Now a charge q' is brought near it.

By Coulomb's law, we know that the charge q experiences a force and it exerts an equal force on q'.

How does q become aware of the presence of q'?

(We do not expect q to have sensory organs just as we have.)

The answer is electric field.

Electric field is the space surrounding an electric charge q in which another charge q' experiences a (electrostatic) force of attraction or repulsion.

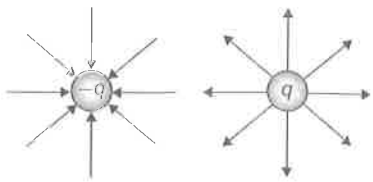


Figure 3.4

The direction of electric field is radially outwards for a positive charge and is radially inwards for a negative charge as shown in Fig. 3.4. There are some points always to be kept in mind. These are as follows:

1. Electric field can be defined as a space surrounding a charge in which another static charge experiences a force on it.
2. In a region, electric field is said to be exist if an electric force is exerted on a static charge placed at that point.
3. It is important to note that with every charge particle, there is an electric field associated which extends up to infinity.
4. No charged particle experiences force due to its own electric field.

$$E_p = \frac{F}{q_0}\,\frac{\text{N}}{\text{C}}$$

Note

A very small positive charge which does not produce its significant electric field is called a test charge.

Thus, electric field strength at a point can be defined generally as "Electric field strength at only point in space to be the electrostatic force per unit charge on a test charge".

If a charge q_0 placed at a point in electric field experiences a net force on it, then electric field strength at that point can be

or $$\vec{E} = \frac{\vec{F}}{q_0} \quad [q_0 \rightarrow \text{test charge}] \qquad (1)$$

Electric Field Strength due to Point Charge

As discussed earlier, if we find electric field due to a point charge at a distance x from it, its magnitude can be given as

$$E = \frac{Kq}{x^2} \qquad (2)$$

Vector Form of Electric field due to a Point Charge

As shown in Fig. 3.5, the direction of electric field strength at point P is along the direction of $\vec{x}$. Thus the value of can be written as

$$\vec{E} = \frac{Kq}{x^2}\cdot\hat{x}$$

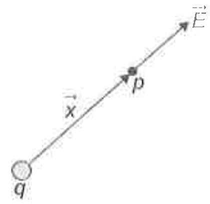


Figure 3.5

or $$\vec{E} = \frac{Kq}{x^3}\cdot\vec{x} \qquad (3)$$

Note

It should be noted that the expression in Eqs. (2) and (3) is only valid for point charges. We cannot find electric field strength due to charged extended bodies by concentrating their whole charge at geometric centre and using the result of a point charge.

SOLVED EXAMPLES

EXAMPLE 17

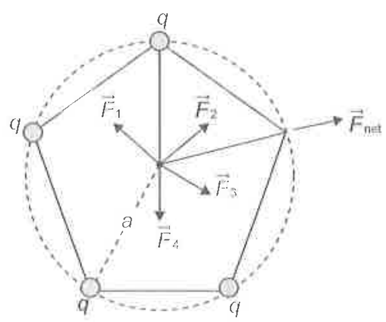
Four particles each having a charge q are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is 'a'. Find the electric field at the centre of pentagon.

SOLUTION

We can calculate the electric field at centre by the superposition method, i.e. by adding vectorially the electric field due to all these charges at centre which will come out to be:

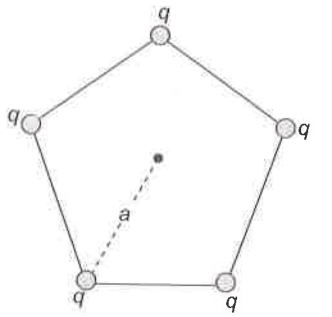
$$\vec{F}_{\text{centre}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \frac{Kq}{a^2}$$

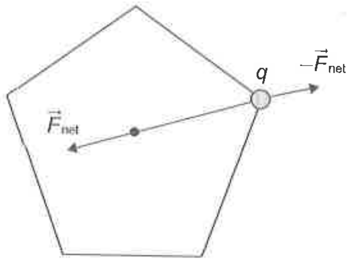
in the direction of the vector with no charge as shown in the figure.



Alternate:

Consider pentagon with charges on all vectors.
Now, electric field at centre must be zero due to symmetry.





Thus, electric field due to 4 charges + electric field due to 1 charge = 0

or electric field due to 4 charges = −electric field due to 1 charge

where − ve sign denotes that both the forces are in opposite direction.

Thus, electric field due to 4 charges = −electric field due to 1 charge = $\dfrac{Kq}{a^2}$

[Another good example of superposition theorem] ■

EXAMPLE 18

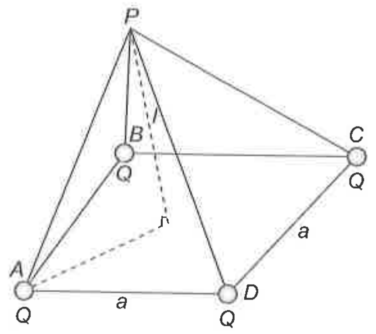
Four equal positive charges each of value Q are arranged at the four corners of a square of side a. A unit positive charge of mass m is placed at P, at a height h above the centre of the square. What should be the value of Q in order that this unit charge is in equilibrium?

SOLUTION

The situation is shown in Fig. (a).

Force experienced by unit positive charge placed at P due to a charge Q at A is given by

$$F = \frac{K(Q \times 1)}{\left(h^2 + \dfrac{a^2}{2}\right)}$$



(a)

Similarly, equal forces act on unit positive charge at P due to charge at B, C and D. When these forces are resolved in horizontal and vertical directions, the horizontal component ($F \sin\theta$) cancels each other and the net vertical force is $4F\cos\theta$.

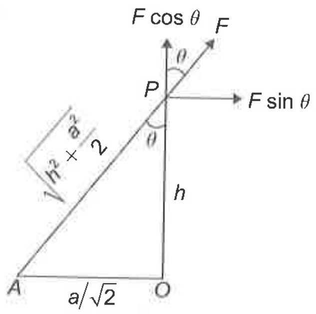
Thus, net upward force $= \dfrac{4KQ}{\left(h^2 + \dfrac{a^2}{2}\right)} \cdot \cos\theta$

For the equilibrium of unit positive charge at P,

upward force = weight of unit charge

$$\frac{4KQ}{\left(h^2 + \dfrac{a^2}{2}\right)} \cdot \cos\theta = mg$$

From Fig. (b),



(b)

$$\cos\theta = \left\{\frac{h}{\sqrt{(h^2 + a^2/2)}}\right\}$$

or

$$\frac{4KQh}{\left(h^2 + \dfrac{a^2}{2}\right)^{3/2}} = mg$$

or $$Q = \frac{mg}{4Kh}\left(h^2 + \frac{a^2}{2}\right)^{3/2}$$ ■

EXAMPLE 19

A particle of mass 9×10^{-31} kg and a negative charge of 1.6×10^{-19} C projected horizontally with a velocity of 10^5 m/s into a region between two infinite horizontal parallel plates of metal. The distance between the plates is 0.3 cm, and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected respectively to the positive and negative terminals of a 30 volt battery. Find the component of the velocity of the particle just before it hits one on the plates.

SOLUTION

We know that between two parallel plates electric field can be given as

$$E = \frac{V}{d}$$

Here, $V = 30$ V and $d = 0.3$ cm $= 3 \times 10^{-3}$ m

Thus, we have $E = 10^4$ N/C

Force on the particle of negative charge moving between the plates

$$F = e \times E$$
$$= 1.6 \times 10^{-19} \times 10^4$$
$$= 1.6 \times 10^{-15} \text{ N}$$

The direction of force will be towards the positive plate, i.e. upwards.

Now the acceleration of the particle is

$$a = \frac{eE}{m}$$

r $$a = (1.6 \times 10^{-15})/(9 \times 10^{-31})$$

r $$a = 1.78 \times 10^{15} \text{ m/s}^2$$

s the electric intensity E is acting in the vertical direction, e horizontal velocity v of the particle remains same. If y the displacement of the particle, in upward direction, we ve

$$y = \frac{1}{2}at^2$$

ere, $y = 0.1$ cm $= 10^{-3}$ m, $a = 1.78 \times 10^{15}$ m/s^2

us, $$10^{-3} = \frac{1}{2} \times (1.78 \times 10^{15})(t^2)$$

By solving, we get

$$t = 1.059 \times 10^{-9} \text{ s}$$

Component of velocity in the direction of field is given by

$$v_y = at$$
$$= (1.78 \times 10^{15})(1.059 \times 10^{-9})$$
$$= 1.885 \times 10^6 \text{ m/s}$$ ■

EXAMPLE 20

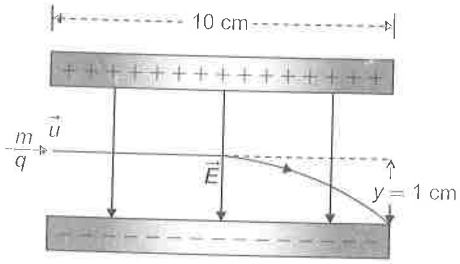
A particle having a charge of 1.6×10^{-19} C enters midway between the plates of a parallel plate condenser. The initial velocity of particle is parallel to the plates. A potential difference of 300 volts is applied to the capacitor plates. The length of the capacitor plates is 10 cm, and they are separated by 2 cm. Calculate the greatest initial velocity for which the particle will not be able to come out of the plates. The mass of the particle is 12×10^{-24} kg.

SOLUTION

The situation is shown in the figure.

Here, we know that the electric field can be given as

$$E = \frac{V}{d} = \frac{300}{2/100}$$
$$= 15{,}000 \text{ V/m}$$



As the particle does not come out, its maximum deflection in the vertical direction can be

$$y = 1 \text{ cm} = 10^{-2} \text{ m}.$$

We know that

$$y = \frac{1}{2}at^2$$
$$= \frac{1}{2}\cdot\frac{qE}{m}\left(\frac{l}{u}\right)^2$$

$$\left[\text{As } a = \frac{qE}{m} \text{ and } t = \frac{l}{u}\right]$$

or $$u^2 = \frac{1}{2}\cdot\frac{qE}{my}\cdot x^2$$

$$= \frac{1}{2}\frac{(1.6\times10^{-19})(15{,}000)}{(12\times10^{-24})(10^{-2})}\left(\frac{1}{10}\right)^2$$

$$= 10^8$$

$$u = 10^4 \text{ m/s}$$

EXAMPLE 21

A uniform electric field E is created between two parallel charged plates as shown in the figure. An electron enters the field symmetrically between the plates with a speed u. The length of each plate is l_1. Find the angle of deviation of the path of the electron as it comes out of the field.

SOLUTION

The situation is shown in the figure.

Here, we know that in the x-direction speed of electron remains uniform.

In the x-direction,

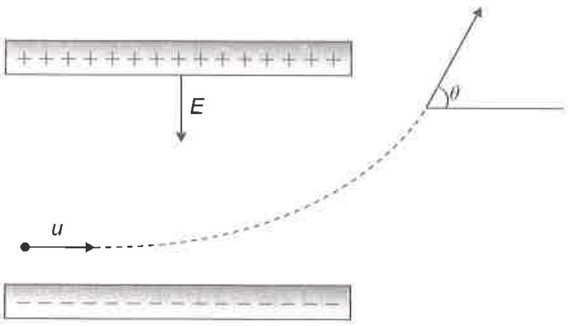
$$u_x = u$$

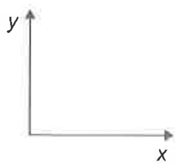
In the y-direction,

$$v_{y_{\text{initial}}} = 0$$

Acceleration in the y-direction of electron is

$$a = \frac{eE}{m}$$





$$v_{y_{\text{final}}} = u_{y_{\text{initial}}} + at$$

$$v_y = \left(\frac{eE}{m}\right)\left(\frac{l}{u}\right)$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mu}\Big/u$$

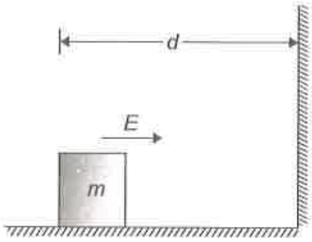
$$\theta = \tan^{-1}\left(\frac{eEl}{mu^2}\right)$$

EXAMPLE 22

A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in the figure. The distance of the block from the wall is d. A horizontal electric field 'E' towards right is switched on. Assuming elastic collision (if any) find the time period of resulting oscillatory motion. Is it a simple harmonic motion?

SOLUTION

Here, acceleration of block is $a = \dfrac{qE}{m}$



Time taken by the block to reach wall

$$d = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$t = \sqrt{\frac{2dm}{qE}}$$

Velocity at the time of impact is

$$v = \sqrt{2ad}$$

or $$v = \sqrt{\frac{2qEd}{m}}$$

When the block rebounds time taken by the block before it comes to rest

$$0 = \sqrt{\frac{2qEd}{m}} - \left(\frac{qE}{m}\right)t$$

$$t = \frac{\sqrt{\dfrac{2qEd}{m}}}{\dfrac{qE}{m}} = \sqrt{\frac{2md}{qE}}$$

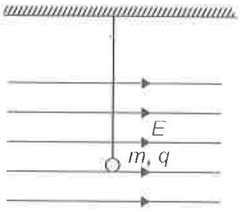
Thus, time period of oscillation of block is

$$T = 2t = 2\sqrt{\frac{2md}{qE}}$$

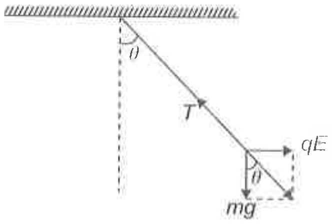
Since the restoring force is independent of x, the displacement from mean position, this is not a simple harmonic motion. ■

EXAMPLE 23

Find out the time period of oscillation when the bob is slightly shifted through an angle θ from its mean position.



SOLUTION



$$g_{eff} = \frac{T_{MD}}{m}$$

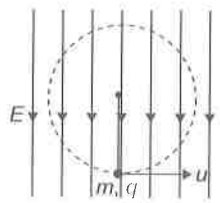
$$= \sqrt{(mg)^2 + (qE)^2}$$

$$= \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

$\Rightarrow$ $$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$ ■

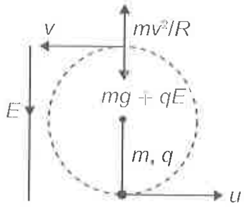
EXAMPLE 24

Find u_{min} so that particle will complete vertical circle.



SOLUTION

Applying force balance



$$T + mg + qE = \frac{mv^2}{R}$$

but for u_{min} $T = 0$

$\therefore$ $$mg + qE = \frac{mv^2}{R}$$

Using energy conservation

$$W_{mg} + W_T + W_{Ef} = \Delta k$$

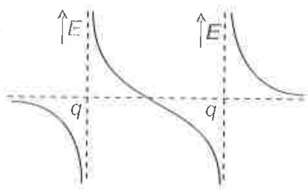
$$-mg \cdot 2R + 0 - qE \cdot 2R = mv^2 - mu^2$$

$$-4R(mg + qE) = R(mg + qE) - mu^2$$

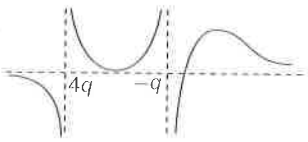
$$u = \sqrt{5\left(g + \frac{qE}{m}\right)R}$$ ■

Graph of Electric Field due to Binary Charge Configuration

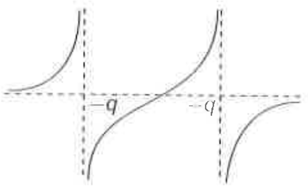
1.



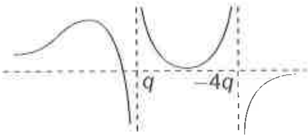
2.



3.



4.



Electric Field Strength at a General Point due to a Uniformly Charged Rod

As shown in Fig. 3.6, if P is any general point in the surrounding of rod, to find electric field strength at P, again we consider an element on rod of length dx at a distance x from point O as shown in Fig. 3.6.

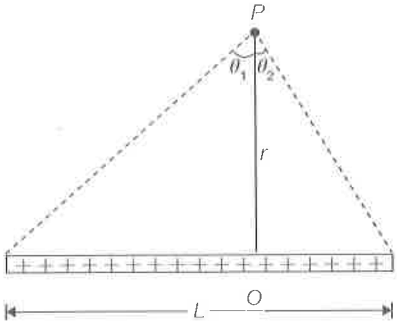


Figure 3.6

Now if dE be the electric field at P due to the element, then it can be given as

$$dE = \frac{Kdq}{(x^2 + r^2)}$$

Here $$dq = \frac{Q}{L}dx$$

Now we resolve electric field in components.

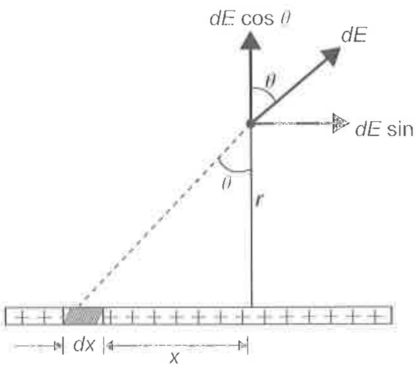


Figure 3.7

Electric field strength in the x-direction due to dq at P is

$$dE_x = dE \sin\theta$$

or $$dE_x = \frac{Kdq}{(x^2 + r^2)}\sin\theta$$

$$= \frac{KQ\sin\theta}{L(x^2 + r^2)}dx$$

Here we have

$$x = r\tan\theta$$

and $$dx = r\sec^2\theta\, d\theta$$

Thus, we have

$$dE_x = \frac{KQ}{L}\frac{r\sec^2\theta\, d\theta}{r^2\sec^2\theta}\sin\theta$$

$$\text{Strength} = \frac{KQ}{Lr}\sin\theta\, d\theta$$

Net electric field strength due to dq at point P in the x-direction is

$$E_x = \int dE_x = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\sin\theta\, d\theta$$

or $$E_x = \frac{KQ}{Lr}[-\cos\ \theta]_{-\theta_2}^{+\theta_1}$$

or $$E_x = \frac{KQ}{Lr}[\cos\theta_2 - \cos\theta_1]$$

Similarly, electric field strength at point P due to dq in the y-direction is

$$dE_y = dE\cos\theta$$

or $$dE_y = \frac{KQdx}{L(r^2 + x^2)} \times \cos\theta$$

Again we have $x = r\tan\theta$

and $$dx = r\sec^2\theta\, d\theta$$

Thus, we have

$$dE_y = \frac{KQ}{L}\cos\theta \times \frac{r\sec^2\theta}{r\sec^2\theta}d\theta = \frac{KQ}{Lr}\cos\theta\, d\theta$$

Net electric field strength at P due to dq in the y-direction is

$$E_y = \int dE_y = \frac{KQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta\, d\theta$$

or $$E_y = \frac{KQ}{Lr}[+\sin\theta]_{-\theta_2}^{+\theta_1}$$

$$E_y = \frac{KQ}{Lr}[\sin\theta_1 + \sin\theta_2]$$

ıus, electric field at a general point in the surrounding of uniformly charged rod which subtend angles θ_1 and θ_2 at e two corners of rod can be given as

||-direction

$$E_x = \frac{KQ}{Lr}(\cos\theta_2 - \cos\theta_1) = \frac{K\lambda}{r}(\cos\theta_2 - \cos\theta_1)$$

⊥-direction

$$E_y = \frac{KQ}{Lr}(\sin\theta_1 + \sin\theta_2) = \frac{K\lambda}{r}(\sin\theta_1 + \sin\theta_2)$$

Note

r is the perpendicular distance of the point from the wire.

θ_1 and θ_2 should be taken in opposite sense.

SOLVED EXAMPLES

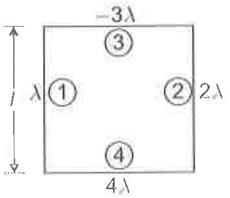
XAMPLE 25

the given arrangement of a charged square frame, find ld at centre. The linear charged density is as shown in e figure.

LUTION

$$\text{EF due to } = \frac{2K\lambda}{l}(\sin 45° + \sin 45°)i$$

$$= \frac{2\sqrt{2}K\lambda}{l}i$$



$$\text{EF due to 2} = -\frac{4\sqrt{4}K\lambda}{l}\hat{i}$$

$$\text{EF due to 3} = \frac{6\sqrt{2}K\lambda}{l}\hat{j}$$

$$\text{EF due to 4} = \frac{8\sqrt{2}K\lambda}{l}\hat{j}$$

$$\vec{E}_{net} = \vec{E}_{due\,to\,1} + \vec{E}_{due\,to\,2} + \vec{E}_{due\,to\,3} + \vec{E}_{due\,to\,4}$$

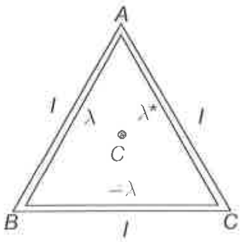
$$= \left(\frac{2\sqrt{2}K\lambda}{l} - \frac{4\sqrt{2}K\lambda}{l}\right)\hat{i} + \left(\frac{6\sqrt{2}K\lambda}{l} + \frac{8\sqrt{2}K\lambda}{l}\right)\hat{j}$$

$$= \frac{-2\sqrt{2}K\lambda}{l}\hat{i} + \frac{14\sqrt{2}K\lambda}{l}\hat{j}$$

■

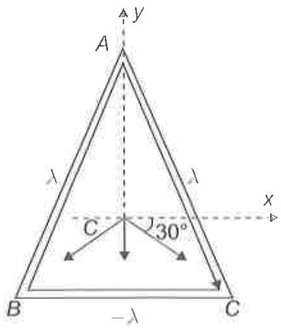
EXAMPLE 26

Given an equilateral triangle with side l. Find E at the centroid. The linear charge density is as shown in the figure.



SOLUTION

The electric field strength due to the three rods AB, BC and CA are as shown in the following figure.



$$\vec{E}_{AC} = \frac{-2K\lambda}{l/\sqrt{3}}(2\sin 30°)(\cos\theta\, i + \sin\theta\, j)$$

$$\vec{E}_{AB} = \frac{2K\lambda}{l/\sqrt{3}}(2\sin 30°)(\cos\theta\, i - \sin\theta\, j)$$

$$\vec{E}_{BC} = \frac{2K\lambda}{l/\sqrt{3}}(2\sin 30°)\, j$$

$$\vec{E}_{net} = \vec{E}_{AC} + \vec{E}_{AB} + \vec{E}_{BC}$$

$$\vec{E}_{net} = \frac{-\lambda}{2\pi\varepsilon_0 l}\hat{j}$$

Electric Field due to Infinite Wire ($l >> r$)

Here, we have to find the electric field at point p due to the given infinite wire. Using the formula learnt in the above section which are

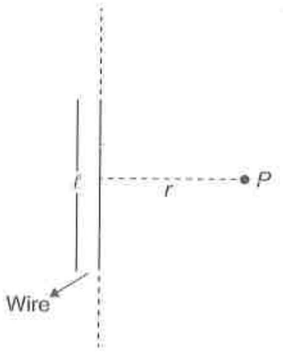


Figure 3.8

$$E_{\|} = \frac{K\lambda}{r}(\cos\theta_2 - \cos\theta_1)$$

$$E_{\perp} = \frac{K\lambda}{r}(\sin\theta_2 + \sin\theta_1)$$

For above case, $\theta_1 = \theta_2 = \frac{\pi}{2}$

$$\therefore \quad E_{net} \text{ at } P = \frac{K\lambda}{r}(1+1) = \frac{2K\lambda}{r}$$

Electric Field due to Semi-infinite Wire

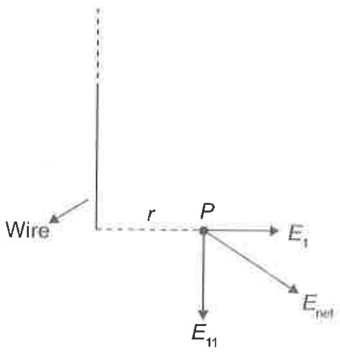


Figure 3.9

For this case,

$$\theta_1 = \frac{\pi}{2}, \quad \theta_2 = 0°$$

$$E_r = \frac{K\lambda}{r};$$

$$E_{\|} = \frac{K\lambda}{r}$$

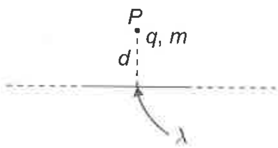
$$E_{net} \text{ at } P = \frac{\sqrt{2}K\lambda}{r}$$

SOLVED EXAMPLE

EXAMPLE 27

Consider the system shown below.

If the charge is slightly displaced perpendicular to the wire from its equilibrium position, then find out the time period of SHM.



SOLUTION

At equilibrium position, weight of the particle is balanced by the electric force

$$mg = q\frac{2k\lambda}{d} \quad (1)$$

Now if the particle is slightly displaced by a distance x, (where $x << d$) net force on the body,

$$F_{net} = \frac{2k\lambda q}{d+x} - mg$$

From (1),

$$F_{net} = \frac{2k\lambda q}{d+x} - \frac{2k\lambda q}{d} = \frac{-2k\lambda q}{d(d+x)}$$

As $x << d$,

$$F_{net} \approx \frac{-2k\lambda qx}{d^2}$$

$$\Rightarrow \quad a = \frac{-2k\lambda qx}{md^2}$$

For SHM,

$$a = -\omega^2 x$$

$$\omega^2 = \frac{2k\lambda q}{md^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2k\lambda q}{md^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md^2}{2k\lambda q}}$$

Electric Field due to Uniformly Charged Ring

Case I: At its Centre

Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. As shown in Fig. 3.10, the electric field strength at the centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is zero.

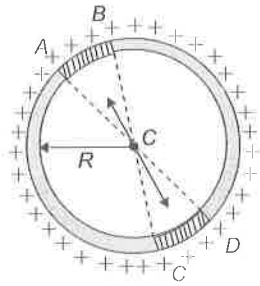


Figure 3.10

Case II: At a Point on the Axis of Ring

For this, see Fig. 3.11. There we will find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this, we consider a small section of length dl on a ring as shown. The charge on this elemental section is

$$dq = \frac{Q}{2\pi R}\, dl$$

[Q = total charge of ring]

Due to the element dq, electric field strength dE at point P can be given as

$$dE = \frac{Kdq}{(R^2 + x^2)}$$

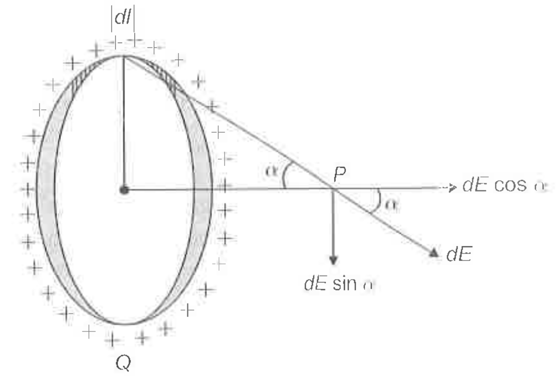


Figure 3.11

The component of this field strength $dE \sin\alpha$ which is normal to the axis of ring will be cancelled out due to the ring section opposite to dl. The component of electric field strength along the axis of ring $dE \cos\alpha$ due to all the sections will be added up. Hence, total electric field strength at point P due to the ring is

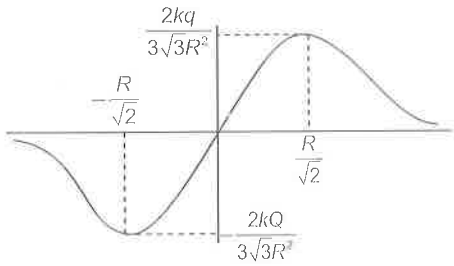


Figure 3.12

$$E_p = \int dE\cos\alpha$$

$$= \int_0^{2\pi R} \frac{Kdq}{(R^2+x^2)} \times \frac{x}{\sqrt{R^2+x^2}}$$

or

$$E_p = \int_0^{2\pi R} \frac{KQx}{2\pi R(R^2+x^2)^{3/2}}\, dl$$

$$= \frac{KQx}{2\pi R(R^2+x^2)^{3/2}} \int_0^{2\pi R} dl$$

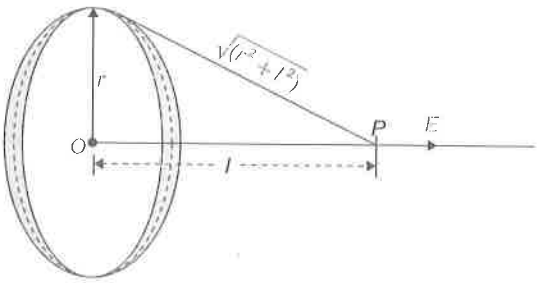
$$= \frac{KQx}{2\pi R(R^2+x^2)^{3/2}}(2\pi R)$$

$$E_P = \frac{KQx}{(R^2+x^2)^{3/2}}$$

SOLVED EXAMPLES

EXAMPLE 28

A thin wire ring of radius r carries a charge q. Find the magnitude of the electric field strength on the axis of the ring as a function of distance l from centre. Investigate the obtained function at $l >> r$. Find the maximum strength magnitude and the corresponding distance l.



SOLUTION

See the figure (Modify for maximum E)

We know due to ring electric field strength at a distance l from its centre on its axis can be given as

$$E = \frac{Kql}{(l^2 + r^2)3/2} \quad (1)$$

For $l >> r$, we have

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{l^2}$$

Thus, the ring behaves like a point charge.

For $\quad E_{max}, \frac{dE}{dl} = 0$

From the equation, we get

$$\frac{dE}{dl} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r^2+l^2)^{3/2}\cdot 1 - \frac{3l}{2}(r^2+l^2)^{1/2} \times 2l}{(r^2+l^2)^3}\right] = 0$$

or $\quad (r^2 + l^2)^{3/2} = \frac{3}{2}(r^2+l^2)^{1/2} \times 2l^2$

Solving we get,

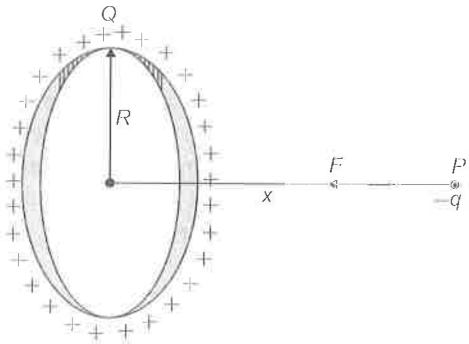
$$l = \frac{r}{\sqrt{2}} \quad (2)$$

Substituting the value of l in Eq. (1), we get

$$E = \frac{kq(r/\sqrt{2})}{(r^2 + r^2/2)^{3/2}} = \frac{2kq}{3\sqrt{3}r^2}$$

EXAMPLE 29

A thin fixed ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} C is placed on the axis at distance of 1 cm from the centre of the ring. Shown that the motion of the negatively charged particle as approximately simple harmonic. Calculate the time period of oscillations.



SOLUTION

Let us first find the force on $a - q$ charge placed at a distance x from the centre of ring along its axis.

Figure shows the respective situation.

In this case, force on particle P is

$$F_P = -qE = -q\cdot\frac{KQx}{(x^2+R^2)^{3/2}}$$

For small x, $x << R$, we can neglect x, compared to R, we have

$$F = -\frac{KqQx}{R^2}$$

Acceleration of particle is

$$a = -\frac{KqQ}{mR^3}x$$

[Here, we have $x = 1$ cm and $R = 1$ m; hence, $x << R$ can be used]

This shows that particle P executes SHM.

Now comparing this acceleration with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{KqQ}{mR^3}}$$

Thus, time period of SHM is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mR^3}{KqQ}}$$

$$= 2\pi\sqrt{\frac{0.9\times10^{-3}\times(1)^3}{9\times10^9\times10^{-5}\times10^{-6}}} = \frac{\pi}{5}\text{ s}$$

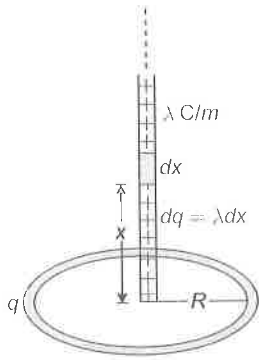
■

EXAMPLE 30

A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to λ. Find the interaction force between the ring and the thread.

SOLUTION

Force df on the wire $= dq\,\vec{E}$



$$= \frac{Kqx}{(x^2+R^2)^{3/2}}\cdot\lambda\,dx$$

$$F = Kq\lambda\int_0^{\infty}\frac{xdx}{(R^2+x^2)^{3/2}}$$

$$F = \frac{\lambda q}{4\pi\varepsilon_0 R}$$

Alternate:

Due to wire, electric field on the points of the ring in the y-direction is

$$E_y = \frac{K\lambda}{R}$$

Thus, force on the ring due to wire is

$$q\frac{K\lambda}{R} = \frac{Kq\lambda}{R} = \frac{\lambda q}{4\pi\varepsilon_0 R}$$

and $E_x = 0$ [As cancelled out]

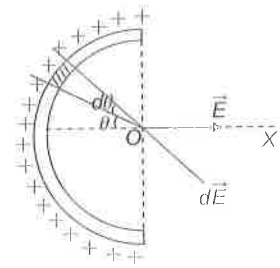
(Here x components of forces on small elements of rings are cancelled by the x component of diametrically opposite elements.) ■

EXAMPLE 31

A thin half-ring of radius $R = 20$ cm is uniformly charged with a total charge $q = 0.70$ nC. Find the magnitude of the electric field strength at the curvature centre of this half-ring.

SOLUTION

The situation is shown in the figure.



Here, the semi-circular wire subtends an angle π at the centre. We know that the electric field strength due to a circular arc subtending an angle ϕ at its centre can be given as

$$E = \frac{2Kq\sin\phi/2}{\phi R^2} = \frac{2Kq}{\pi R^2} \qquad [\text{Here } \phi = \pi]$$

$$= \frac{q}{2\pi^2\varepsilon_0 R^2}$$

Substituting the value, we get

$$= \frac{7\times10^{-10}}{2\times(3.14)^2\times(8.85\times10^{-12})\times(0.2)^2}$$

$$= 100\text{ V/m}$$

■

Electric Field Strength due to a Uniformly Surface Charged Disc

If there is a disc of radius R, charged on its surface with surface charge density σ C/m^2, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in Fig. 3.13.

To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in the figure. Now the charge on this elemental ring dq can be given as

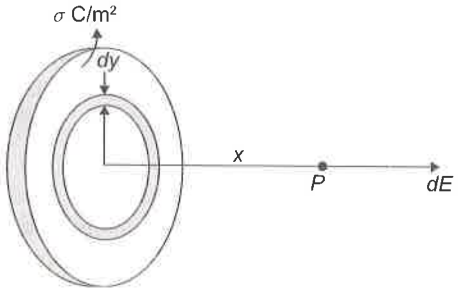


Figure 3.13

$$dq = \sigma 2\pi\, y\, dy$$

[Area of elemental ring $ds = 2\pi y\, dy$]

Now we know that electric field strength due to a ring of radius R. Charge Q at a distance x from its centre on its axis can be given as

$$E = \frac{KQx}{(x^2 + R^2)^{3/2}} \quad \text{[As done earlier]}$$

Here due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{Kdqx}{(x^2 + y^2)^{3/2}}$$

$$= \frac{K\sigma 2\pi y dqx}{(x^2 + y^2)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating the above expression from 0 to R as

$$E = \int dE = \int_0^R \frac{K\sigma 2\pi xy\, dy}{(x^2 + y^2)^{3/2}}$$

$$K\sigma\pi x \int_0^R \frac{2ydy}{(x^2 + y^2)^{3/2}} = 2K\sigma\pi x \left[-\frac{1}{\sqrt{x^2 + y^2}} \right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

Case (i): If $x >> R$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{\frac{R^2}{x^2} + 1}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 + \frac{R^2}{x^2}\right)^{-1/2}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} + \text{higher order terms}\right]$$

$$= \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

i.e., Behaviour of the disc is like a point charge.

Case (ii): If $x << R$

$$E = \frac{\sigma}{2\varepsilon_0}[1 - 0] = \frac{\sigma}{2\varepsilon_0}$$

i.e., Behaviour of the disc is like infinite sheet.

Electric Field Strength due to a Uniformly Charged Hollow Hemispherical Cup

Figure 3.14 shows a hollow hemisphere, uniformly charged with surface charge density σ C/m². To find electric field strength at its centre C, we consider an elemental ring on its surface of angular width $d\theta$ at an angle θ from its axis as shown. The surface area of this ring will be

$$ds = 2\pi R \sin\theta \times R d\theta$$

Charge on this elemental ring is

$$dq = \sigma ds = \sigma \bullet 2\pi R^2 \sin\theta\, d\theta$$

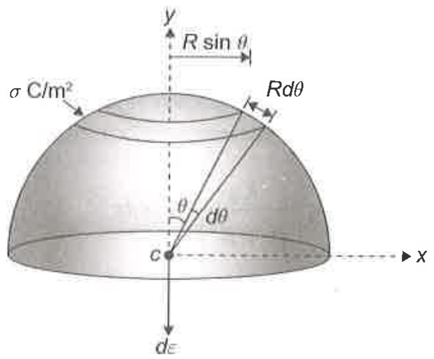


Figure 3.14

Now due to this ring electric field strength at centre C can be given as

$$dE = \frac{Kdq(R\cos\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$= \frac{K\sigma \cdot 2\pi R^2 \sin\theta\, d\theta \cdot R\cos\theta}{R^3}$$

$$= \pi K\sigma \sin 2\theta\, d\theta$$

Net electric field at the centre can be obtained by integrating this expression between limits 0 to $\frac{\pi}{2}$ as

$$E_0 = \int dE = \pi K\sigma \int_0^{\pi/2} \sin 2\theta \, d\theta$$

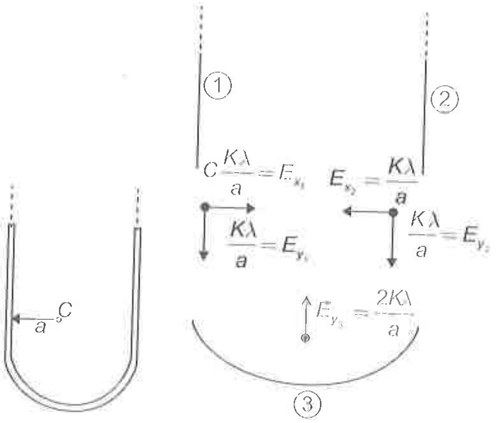
$$= \frac{\sigma}{4\varepsilon_0}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{\sigma}{4\varepsilon_0}\left[\frac{1}{2}+\frac{1}{2}\right] = \frac{\sigma}{4\varepsilon_0}$$

SOLVED EXAMPLE

EXAMPLE 32

In the given arrangement, find the electric field at C in Fig. (a). Here the U-shaped wire is uniformly charged with linear charge density λ.



(a) (b)

SOLUTION

The electric field due to the three parts of U-shaped wire are shown in Fig. (b). Thus we have

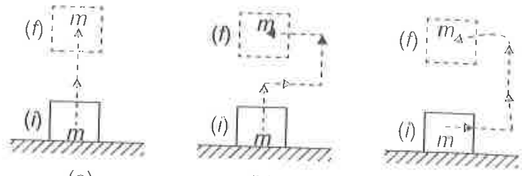
$$\vec{E}_{net} = \left(E_{x_1} + E_{x_2}\right)\hat{i} + \left(E_{y_1} + E_{y_2} + E_{y_3}\right)\hat{j}$$

$$\vec{E}_{net} = \left(\frac{K\lambda}{a} - \frac{K\lambda}{a}\right)\hat{i} + \left(\frac{2K\lambda}{a} - \frac{K\lambda}{a} - \frac{K\lambda}{a}\right)\hat{j} = 0$$

Thus, EF due to given arrangement at $C = 0$. ■

CONSERVATIVE FORCE

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



(a) (b) (c)

Figure 3.15

Consider a body of mass m being raised to a height h vertically upwards as shown in Fig. 3.15. The work done is mgh. Suppose we take the body along the path as in Fig. 3.15(b). The work done during horizontal motion is zero. Adding up the work done in the two vertical paths, we get the result mgh once again. Any arbitrary path like the one shown in Fig. 3.15(c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical paths add up to mgh. Thus, we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative Forces

1. Gravitational force, not only due to Earth in its general form as given by the universal law of gravitation, is a conservative force.
2. Elastic force in a stretched or compressed spring is a conservative force.
3. Electrostatic force between two electric charges is a conservative force.
4. Magnetic force between two magnetic poles is a conservative force.

   Forces acting along the line joining the centres of two bodies are called central forces. Gravitational and electrostatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative Forces

- Work done by or against a conservative force depends only on the initial and final positions of the body.
- Work done by or against a conservative force does not depend on the nature of the path between initial and final positions of the body.

  If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.
- Work done by or against a conservative force in a round trip is zero.

  If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

The concept of potential energy exists only in the case of conservative forces.

- The work done by a conservative force is completely recoverable.

  Complete recoverability is an important aspect of the work of a conservative force.

Work done by conservative forces

SOLVED EXAMPLES

I format: (When constant force is given)

EXAMPLE 33

Calculate the work done to displace the particle from (1, 2) to (4, 5), if $\vec{F} = 4\hat{i} + 3\hat{j}$.

SOLUTION

$$dw = \vec{F}\cdot d\vec{r} = (d\vec{r} = dxi + dyj + dzk)$$

$$dw = (4\hat{i} + 3\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow \quad dw = 4dx + 3dy$$

$$\int_0^w dw = \int_1^4 4dx + \int_2^5 3dy$$

$$\Rightarrow \quad w = [4x]_1^4 + [3y]_2^5$$

$$w = (16 - 4) + (15 - 6)$$

$$\Rightarrow \quad w = 12 + 9 = 21 \text{ Joule}$$

II format: (When F is given as a function of x, y, z)

If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

then

$$dw = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow \quad dw = F_x\,dx + F_y\,dy + F_z\,dz$$

EXAMPLE 34

An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})m$ to $\vec{r}_2 = (4\hat{i} + 6\hat{j})m$ under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$N. Find the work done by this force.

SOLUTION

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r}$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\,dx + 2y\,dy)$$

$$= [x^3 + y^2]_{(2,3)}^{(4,6)} = 83 \text{ J}$$

III format (Perfect differential format)

EXAMPLE 35

If $\vec{F} = y\hat{i} + x\hat{j}$, then find out the work done in moving the particle from position (2, 3) to (5, 6).

SOLUTION

$$dw = \vec{F}\cdot d\vec{s}$$

$$dw = (y\hat{i} + x\hat{j})\cdot(dx\hat{i} + dy\hat{j})$$

$$dw = ydx + xdy$$

Now $\quad ydx + xdy = d(xy)$

(perfect differential equation)

$$\Rightarrow \quad dw = d(xy)$$

For total work done we integrate both sides

$$\int dw = \int d(xy)$$

Put $\quad xy = k$

then at (2, 3)

$$k_i = 2 \times 3 = 6$$

at (5, 6)

$$k_f = 5 \times 6 = 30$$

Then $\quad w = \int_6^{30} dk = [k]_6^{30}$

$$\Rightarrow \quad w = (30 - 6) = 24 \text{ Joule}$$

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends on the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force, magnetic force etc., are non-conservative forces.

SOLVED EXAMPLE

EXAMPLE 36

Calculate the work done by the force $\vec{F} = y\hat{i}$ to move the particle from (0, 0) to (1, 1) in the following condition:
(a) $y = x$ and (b) $y = x^2$

SOLUTION

We know that

$$dw = \vec{F} \cdot d\vec{s}$$

$$\Rightarrow \quad dw = (y\hat{i}) \cdot (dx\hat{i})$$

$$dw = ydx$$

In Eq. (1), we can calculate work done only when we know the path taken by the particle.

either $y = x$ or $y = x^2$ so now

(a) when $y = x$

$$\int dw = \int_0^1 x\,dx$$

$$\Rightarrow \quad w = \frac{1}{2}\text{ Joule}$$

(b) when $y = x^2$

$$\int dw = \int_0^1 x^2 dx$$

$$\Rightarrow \quad w = \frac{1}{3}\text{ Joule}$$

Difference Between Conservative and Non-conservative Forces

S.No	Conservative forces	Non-Conservative forces
1	Work done does not depends on path	Work done depends on path
2	Work done in round trip is zero	Work done in a round trip is not zero
3	Central in nature	Forces are velocity dependent and retarding in nature
4	When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, does not change	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable	Work done is not completely recoverable

ELECTROSTATIC POTENTIAL ENERGY

Electrostatic Potential Energy

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Before proceeding further, we should keep in mind the following points, which are useful in understanding potential energy in electric fields.

1. Doing work implies supply of energy.
2. Energy can neither be transferred nor be transformed into any other form without doing work.
3. Kinetic energy implies utilization of energy whereas potential energy implies storage of energy
4. Whenever work is done on a system of bodies, the supplied energy to the system is either used in the form of KE of its particles or it will be stored in the system in some form, increases the potential energy of system.
5. When all particles of a system are separated far apart by infinite distance, there will be no interaction between them. This state we take as reference of zero potential energy.

Now potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles.

Electrostatic potential energy is defined in two ways,

(a) Interaction energy of charged particles of a system,
(b) Self-energy of a charged object (will be discussed later).

Electrostatic Interaction Energy

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to a given configuration.
When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system. Hence, final potential energy of system will be positive. If the force between the particles is attractive work will be done by the system and final potential energy of system will be negative.
Let us take some illustrations to understand this concept in detail.

Interaction Energy of a System of Two Charged Particles

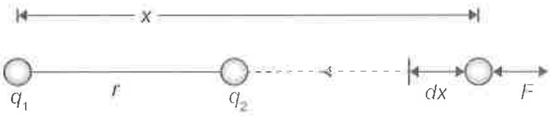


Figure 3.16

Figure 3.16 shows two +ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1. It can be calculated as

$$W = \int_{\infty}^{r} \vec{F} \cdot \overrightarrow{dx}$$

$$= -\int_{\infty}^{r} \frac{Kq_1q_2}{x^2} dx$$

[−ve sign shows that x is decreasing]

$$W = \frac{Kq_1q_2}{r} = U$$

[Interaction energy]

If the two charges here are of opposite sign, the potential energy will be negative as

$$U = -\frac{Kq_1q_2}{r}$$

SOLVED EXAMPLES

EXAMPLE 37

Find out speed of particles when separation between them is r.

SOLUTION

Energy conservation:

$$0 - \frac{Kq_1q_2}{2r} = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 - \frac{Kq_1q_2}{2r}$$

Momentum conservation (as EF is action–reaction pair)

$$mv_1 = 2mv_2$$

$$\Rightarrow \quad v_2 = \frac{v_1}{2}$$ ■

EXAMPLE 38

A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons in terms of mass of proton m and its charge e.

SOLUTION

As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with the same velocity. So if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two protons system.

$$mu = mv + mv, \text{ i.e. } v = \frac{1}{2}u$$

And by conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

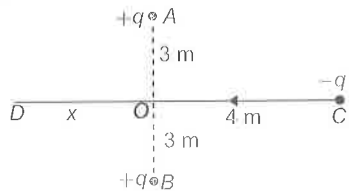
$$\Rightarrow \quad \frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad [\text{as } v = \frac{u}{2}]$$

$$\Rightarrow \quad \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$ ■

EXAMPLE 39

Two fixed equal positive charges, each of magnitude 5×10^{-5} C, are located at points A and B, separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge, when it reaches the point C at a distance of 4 m from O, has a kinetic energy of 4 J. Calculate the distance of the farthest point D which the negative charge will reach before returning towards C.



SOLUTION

The kinetic energy is lost and converted to electrostatic potential energy of the system as the negative charge goes from C to D and comes to rest at D instantaneously.

Loss of KE = Gain in potential energy

$4 = Uf - Ui$

$$\text{or} \quad 4 = \left[\frac{q\cdot q}{4\pi\varepsilon_0(6)^2} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+x^2}}\right] - \left[\frac{q\cdot q}{4\pi\varepsilon_0(6)^2} + \frac{2q(-q)}{4\pi\varepsilon_0\sqrt{9+16^2}}\right]$$

$$\text{or} \quad 4 = \frac{2q^2}{4\pi\varepsilon_0}\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

$$\text{or,} \quad 4 = 2 \times (5 \times 10^{-5})^2 \times (9 \times 10^9)\left[\frac{1}{5} - \frac{1}{\sqrt{9+x^2}}\right]$$

$$\text{or,} \quad 4 = 9 - \frac{45}{\sqrt{9+x^2}}$$

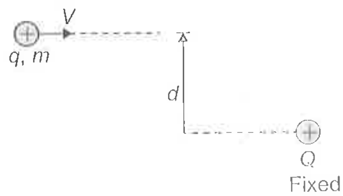
$$\Rightarrow \quad x = \sqrt{72} = 8.48 \text{ m}$$ ■

Motion of a Charge Particle and Angular Momentum Conservation

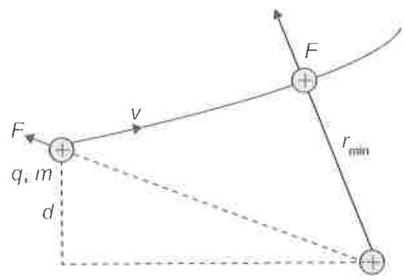
We know that a system of particles when no external torque acts, the total angular momentum of system remains conserved. Consider following examples which explains the concept for moving charged particles. ■

EXAMPLE 40

Figure shows a charge $+Q$ fixed at a position in space. From a large distance, another charge particle of charge $+q$ and mass m is thrown towards $+Q$ with an impact parameter d as shown with speed v. find the distance of closest approach of the two particles.



SOLUTION



Here, we can see that as $+q$ moves towards $+Q$, a repulsive force acts on $-q$ radially outwards $+Q$. As the line of action of force passes through the fix charge, no torque acts on $+q$ relative to the fix point charge $+Q$. Thus, we can say that with respect to $+Q$, the angular momentum of $+q$ must remain constant. Here, $+q$ will be closest to $+Q$ when it is moving perpendicularly to the line joining the two charges as shown.

If the closest separation in the two charges is r_{min}, from conservation of angular momentum, we can write as

$$mvd = mv_0 r_{min} \tag{1}$$

Now from energy conservation, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{KqQ}{r_{min}}$$

From Eq. (1),

$$v_0 = \frac{vd}{r_{min}}$$

$$\text{or} \quad \frac{1}{2}mu^2 = \frac{1}{2}mv^2\frac{d^2}{r_{min}^2} + \frac{KqQ}{r_{min}} \tag{2}$$

Solving Eq. (2), we will get the value of r_{min}. ■

Potential Energy for a System of Charged Particles

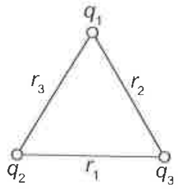


Figure 3.17

When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energy of all the pairs of particles. For example, if a system of three particles having charges q_1, q_2 and q_3 is given as shown in Fig. 3.17. The total interaction energy of this system can be given as

$$U = \frac{Kq_1q_2}{r_3} + \frac{Kq_1q_3}{r_2} + \frac{Kq_2q_3}{r_1}$$

Derivation for a System of Point Charges

1. Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. The PE of the system is algebraic sum of all the works.

   Let W_1 = work done in bringing first charge

   W_2 = work done in bringing second charge against force due to first charge

   W_3 = work done in bringing third charge against force due to first and second charges.

   $\text{PE} = W_1 + W_2 + W_3 + \cdots$

   (This will contain $\frac{n(n-1)}{2} = {}^nC_2$ terms)

2. Method of calculation (to be used in problems)

   U = sum of the interaction energies of the charges

   $= (U_{12} + U_{13} + \cdots + U_1n) + (U_{23} + U_{24} + \cdots + U_2n) + (U_{34} + U_{35} + \cdots U_3n)$

3. Method of calculation useful for symmetrical point charge systems.

   Find PE of each charge due to rest of the charges.

   If U_1 = PE of first charge due to all other charges

   $= (U_{12} + U_{13} + \cdots + U_1n)$

   U_2 = PE of second charges due to all other charges

   $= (U_{21} + U_{23} + \cdots + U_2n)$

   $$U = PE \text{ of the system} = \frac{U_1 + U_2 + \cdots}{2}$$

ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, electric potential is defined as the interaction energy of a unit positive charge.

If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

$$V = \frac{U}{q_0} \text{ J/C}$$

The potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly, we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces."

Properties

1. Potential is a scalar quantity, its value may be positive, negative or zero.
2. The SI unit of potential is volt $= \frac{\text{joule}}{\text{coulomb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.
3. Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e., infinity) to that point.
4. Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinity (taking $V_\infty = 0$).
5. Potential decreases in the direction of electric field.

Electric Potential due to a Point Charge in its Surrounding

We know that the region surrounding a charge is electric field. Thus, we can also define electric potential in the surrounding of a point charge.

The potential at a point P at a distance x from the charge q can be given as

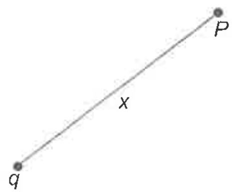


Figure 3.18

$$Vp = \frac{U}{q_0}$$

where U is the potential energy of charge q_0, if placed at point P, which can be given as

$$U = \frac{Kqq_0}{x}$$

Thus, potential at point P is

$$V_p = \frac{Kq}{x}$$

The above result is valid only for electric potential in the surrounding of a point charge. If we wish to find electric potential in the surrounding of a charged extended body, we first find the potential due to an elemental charge dq on a body by using the above result and then integrate the expression for the whole body.

Electric Potential due to a Charged Rod

Figure 3.18 shows a charged rod of length L, uniformly charged with a charge Q. Due to this, we will find electric potential at a point P at a distance r from one end of the rod shown in the figure.

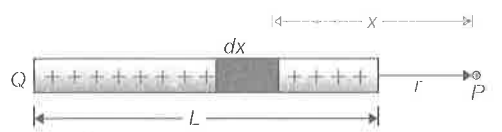


Figure 3.18

For this, we consider an element of width dx at a distance x from the point P. Charge on this element is

$$dq = \frac{Q}{L}dx$$

The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{Kdq}{x} = \frac{KQ}{Lx}dx$$

Net electric potential at point P can be given as

$$V = \int dV = \int_r^{r+L} \frac{KQ}{Lx}dx = \frac{KQ}{L}[\ln x]_r^{r+L} = \frac{KQ}{L}\ln\left(\frac{r+L}{r}\right)$$

Electric Potential due to a Charged Ring:

Case I: At its Centre

To find potential at the centre C of the ring, we first find potential dV at the centre due to an elemental charge dq on the ring which is given as

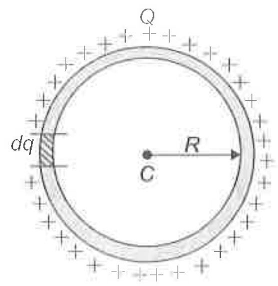


Figure 3.19

$$dV = \frac{Kdq}{R}$$

Total potential at C is

$$V = \int dV = \int \frac{Kdq}{R} = \frac{KQ}{R}$$

As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all is added as being a scalar quantity, we can directly say that the electric potential at ring centre is $\frac{KQ}{R}$. Here we can also state that even if charge Q is non-uniformly distributed on ring, the electric potential at C will remain same.

Case II: At a Point on Axis of Ring

If we wish to find the electric potential at a point P on the axis of ring as shown in Fig. 3.20, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2 + R^2}$.

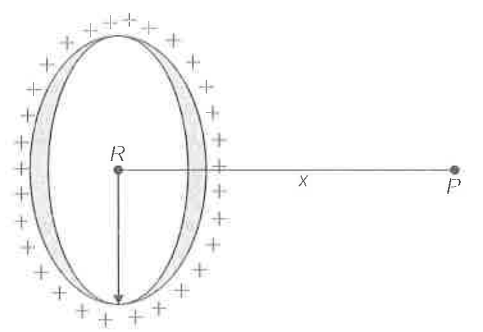
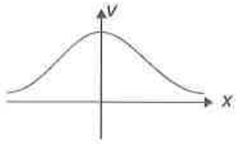


Figure 3.20

From the point P, the potential at P can be given as

$$V_P = \frac{KQ}{\sqrt{R^2 + x^2}}$$

Graph



Electric Potential due to a Uniformly Charged Disc

Figure 3.21 shows a uniformly charged disc of radius R with surface charge density σ C/m^2. To find electric potential at point P, we consider an elemental ring of radius y and width dy, charge on this elemental ring is

$$dq = \sigma \bullet 2\pi y\, dy$$

Due to this ring, the electric potential at point P can be given as

$$dV = \frac{Kdq}{\sqrt{x^2 + y^2}}$$

$$= \frac{K\sigma 2\pi y dy}{\sqrt{x^2 + y^2}}$$

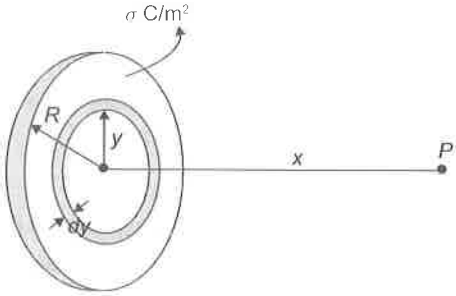


Figure 3.21

Net electric potential at point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\varepsilon_0} \cdot \frac{ydy}{\sqrt{x^2 + y^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + y^2}\right]_0^R$$

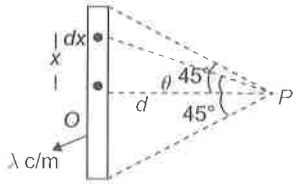
$$VP = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

SOLVED EXAMPLES

EXAMPLE 41

Consider the following rod and find the potential due to it at P.

SOLUTION



$OP = d$, $x = d \tan\theta$, $dx = d \sec^2\theta\, d\theta$

$$dV = \frac{k\lambda dx}{d \sec\theta}$$

$$\Rightarrow \quad \int dV = \int_{-\pi/4}^{+\pi/4} \frac{kd \sec^2 d\theta \lambda}{d \sec\theta}$$

$$V = k\lambda \int_{-\pi/4}^{\pi/4} \sec\theta\, d\theta$$

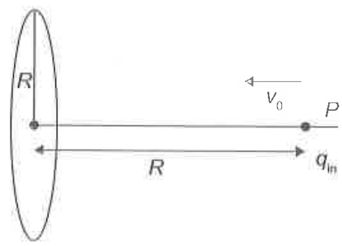
$$V = k\lambda\left[\ln(\sec\theta + \tan\theta)\right]_{-\pi/4}^{\pi/4}$$

$$V = k\lambda\left[\ln(\sqrt{2}+1)\right] - k\lambda\left[l\, n(\sqrt{2}-1)\right]$$

$$V = k\lambda \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = k\lambda \ln(\sqrt{2}+1)^2$$ ■

EXAMPLE 42

Find the minimum velocity v_0 such that particles cross the ring.



SOLUTION

Potential at $P = \dfrac{kQ}{\sqrt{2}R}$,

Applying energy conservation

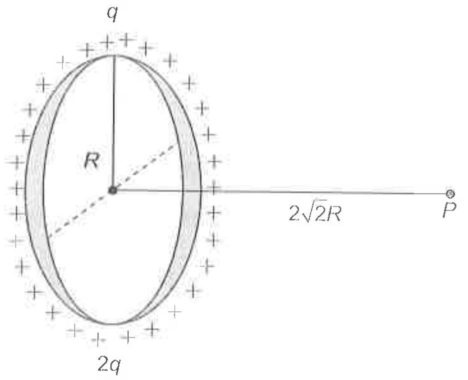
$$\frac{1}{2}mv_0^2 + \frac{kQq}{\sqrt{2}R} = 0 + \frac{kqQ}{R}$$

$$v_0 = \sqrt{\frac{2kQq}{mR}\left(1-\frac{1}{\sqrt{2}}\right)}$$

XAMPLE 43

ring of radius R is having two charges q and $2q$ distributed its two half parts. Find the electric potential at a point on axis at a distance from its centre.

LUTION



ance of P from periphery of ring is $\sqrt{R^2+(2\sqrt{2}R)^2} = 3R$

Electric potential = Potential due to upper half + Potential due to lower half

$$= \frac{Kq}{3R} + \frac{2Kq}{3R}$$

$$\frac{3Kq}{3R} = \frac{Kq}{R}$$

tric Potential Due to a Closed Disc at a Point he Edge

s calculate the potential at the edge of a thin disc of is 'R' carrying a uniformly distributed charge with ce density σ.

Let AB be a diameter and A be a point where the potential be calculated. From A as centre, we draw two arcs of radii $r + dr$ as shown in Fig. 3.22. The infinitesimal region een these two arcs is an element whose area is $dA = (2r\theta)$ here 2θ is the angle subtended by this element PQ at the A. Potential at A due to the element PQ is

$$dV = \frac{\sigma dA}{4\pi\varepsilon_0 r}$$

$$= \frac{2\sigma r\theta dr}{4\pi\varepsilon_0 r}$$

$$= \frac{2\sigma\theta dr}{4\pi\varepsilon_0}$$

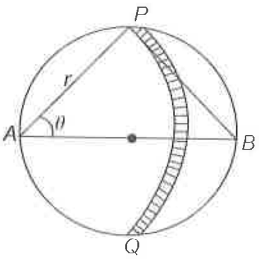


Figure 3.22

From $\Delta\, APB$, we have

$$r = 2R\cos\theta$$

or, $$dr = -2R\sin\theta\, d\theta$$

Hence,

$$dV = \frac{-4\sigma\theta R\sin\theta\, d\theta}{4\pi\varepsilon_0}$$

$$V = -\int_{\pi/2}^{0} \frac{\sigma R\theta\sin\theta}{\pi\varepsilon_0}\, d\theta$$

$$= -\frac{\sigma R}{\pi\varepsilon_0}\left|-\theta\cos\theta + \sin\theta\right|_{\pi/2}^{0}$$

$$= \frac{\sigma R}{\pi\varepsilon_0}$$

RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL

(a) For uniform electric field:

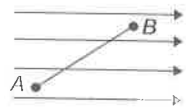


Figure 3.23

Potential difference between two points A and B

$$VB - VA = -\vec{E}\cdot\overrightarrow{AB}$$

(b) Non-uniform electric field:

$$E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right]$$

$$= -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V$$

$$= -\nabla V = -\text{grad } V$$

where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

(c) If electric potential and electric field depend only on one coordinate, say r:

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

where $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E}\cdot\vec{dr}$

$$\Rightarrow VB - VA = -\int_{r_A}^{r_B} \vec{E}\cdot\vec{dr}$$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point

$$V = -\int_{\infty}^{r} \vec{E}\cdot\vec{dr}$$

Note

Area under $E - x$ curve gives negative of change in potential.

Negative of slope of $V - x$ curve gives the electric field at that point.

EXAMPLE 44

$V = x^2 + y$, Find $\vec{E}$.

SOLUTION

$$\frac{\partial V}{\partial x} = 2x,\ \frac{\partial V}{\partial y} = 1 \text{ and } \frac{\partial V}{\partial z} = 0$$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$= -(2x\hat{i} + \hat{j})$$

Electric field is non-uniform.

EXAMPLE 45

For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$, find the potential at (x, y) if V at origin is 5 volts.

SOLUTION

$$\int_5^V dV = -\int \vec{E}\cdot\vec{dr}$$

$$= -\int_0^x E_x dx - \int_0^y E_y dy$$

$$\Rightarrow \quad V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2}$$

$$\Rightarrow \quad V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5$$

EXAMPLE 46

The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for the electric field strength.

SOLUTION

We know

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2x + 3y - z] = 2$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}[2x + 3y - z] = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}[2x + 3y - z] = -1$$

$$\therefore \quad \vec{E} = -(2\hat{i} + 3\hat{j} - \hat{k})$$

ELECTRIC LINES OF FORCE

The idea of electric lines of force or the electric field lines introduced by Michael Faraday is a way to visualize electrostatic field geometrically.

The properties of electric lines of force are the following:

1. The electric lines of force are continuous curves in an electric field starting from a positively charged body and ending on a negatively charged body.

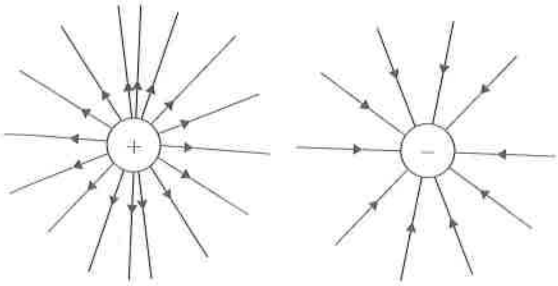
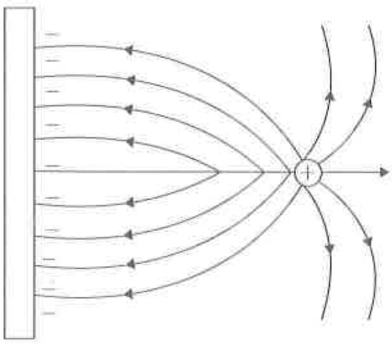


Figure 3.23

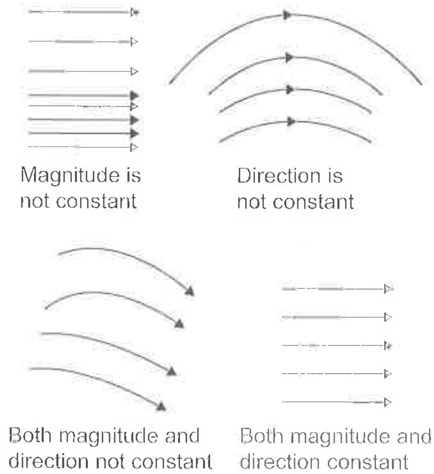
2. The tangent to the curve at any point gives the direction of the electric field intensity at that point.
3. Electric lines of force never intersect since if they cross at a point, electric field intensity at the point will have two directions, which is not possible.
4. Electric lines of force do not pass but leave or end on a charged conductor normally. Suppose the lines of force are not perpendicular to the conductor surface. In this situation, the component of electric field parallel to the surface would cause the electrons to move and hence conductor will not remain equipotential which is an absurd as in electrostatics conductor is an equipotential surface.



Fixed point charge near infinite metal plate

Figure 3.24

5. The number of electric lines of force that originate from or terminate on a charge is proportional to the magnitude of the charge.
6. As number of lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field. Further, if the lines of force are equidistant straight lines, the field is uniform.



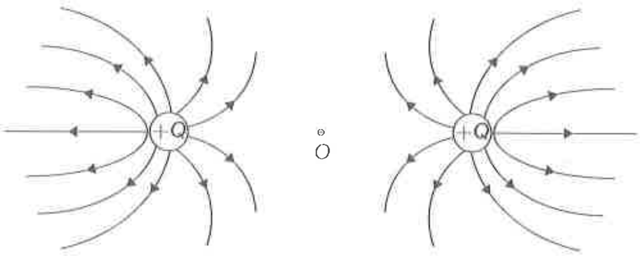


Figure 3.25

Electric lines of force due to two equal positive charges (field is zero at O). O is a null point.

Notes

A charged particle need not follow an ELOF.

Electric lines of force produced by static charges do not form closed loop.

SOLVED EXAMPLES

EXAMPLE 47

If the number of electric lines of force from charge q is 10, then find out number of electric lines of force from $2q$ charge.

SOLUTION

No. of ELOF μ charge

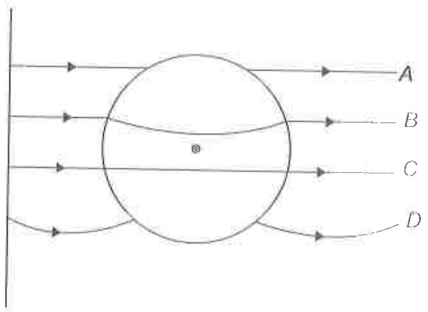
$$\frac{q'}{q} = \frac{N}{10}$$

$$\Rightarrow \qquad N' = \frac{2q}{q} \times 10 = 20$$

So the number of ELOF will be 20. ■

EXAMPLE 48

A solid metallic sphere is placed in a uniform electric field. Which of the lines A, B, C and D show the correct representation of lines of force and why?

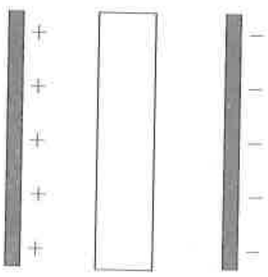


SOLUTION (D)

Line (A) is wrong as lines of force start or end normally on the surface of a conductor and here it is not so. Lines (B) and (C) are wrong as lines of force do not exist inside a conductor and here it is not so. Also lines of force are not normal to the surface of the conductor. Line (D) represents the correct situation, as here line of force does not exist inside the conductor and start and end normally on its surface. ■

EXAMPLE 49

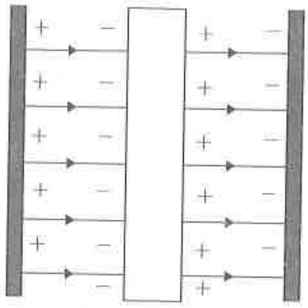
A metallic slab is introduced between the two charged parallel plates as shown in the figure. Sketch the electric lines of force between the plates.



SOLUTION

Keeping in mind that

(a) Electric lines of force start from positive charge and end on negative charge.



(b) Electric lines of force start and end normally on the surface of a conductor.

(c) Electric lines of force do not exist inside a conductor, the lines of force are shown in the adjacent figure. ■

EQUIPOTENTIAL SURFACES

As shown in Fig. 3.26, if a charge is shifted from a point A to B on a surface M which is perpendicular to the direction of electric field, the work done in shifting will obviously be zero as electric force is normal to the direction of displacement.

As no work is done in moving from A to B, we can say that A and B are at same potentials or we can say that all the points of surface M are at same potential or here we call surface M as equipotential surface.

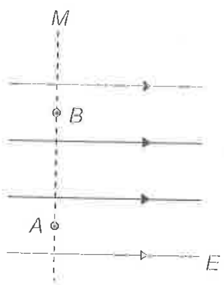
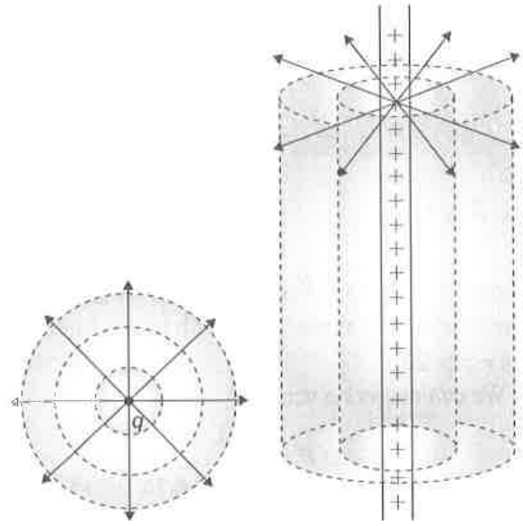


Figure 3.26

Figure 3.27 shows equipotential surfaces in the surrounding of point charge and a long charged wire.

Every surface in electric field in which at every point direction of electric field is normal to the surface can be regarded as equipotential surface.

Figure 3.28 shows two equipotential surfaces in a uniform electric field E. If we wish to find the potential difference between two points A and B shown in the



Point charge Spherical equipotential surfaces

Line charge Cylindrical equipotential surfaces

Figure 3.27

figure, we simply find the potential difference between the two equipotential surfaces on which the points lie, given as

$$VA - VB = Ed$$

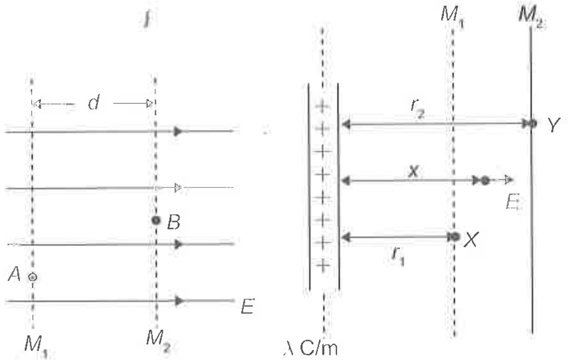


Figure 3.28

Figure 3.28 shows a line charge with linear charge density λ C/m. Here we wish to find potential difference between two points X and Y which lie on equipotential surfaces M_1 and M_2. To find the potential difference between these surfaces, we consider a point P at a distance x from wire as shown. The electric field at point P is

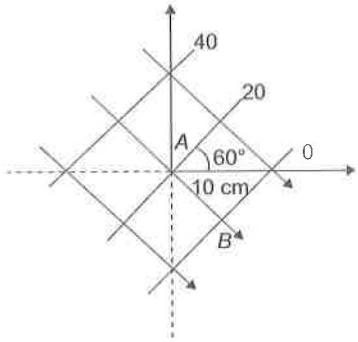
$$E = \frac{2K\lambda}{x}$$

Now the potential difference between surfaces M_1 and M_2 can be given as

$$VX - VY = 2K\lambda \ln\left(\frac{r_2}{r_1}\right)$$

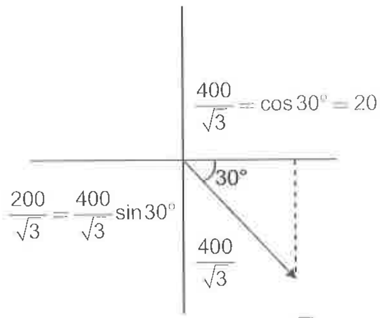
EXAMPLE 50

Write down the electric field in vector form.



SOLUTION

$$vA - vB = E \times 0.1 \times \cos 30^\circ$$



$$20 = E \times 0.1 \times \frac{\sqrt{3}}{2}$$

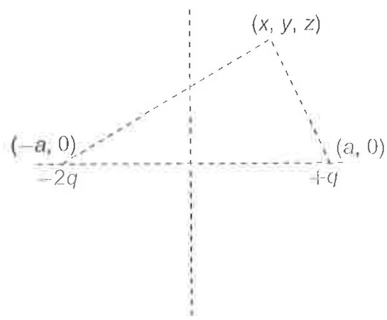
$$\Rightarrow \quad E = \frac{400}{\sqrt{3}}$$

$$EF = 200\hat{i} - \frac{200}{\sqrt{3}}\hat{j}$$

■

EXAMPLE 51

Find out equipotential surface where potential is zero.



SOLUTION

$$\frac{-2kq}{\sqrt{(x+a)^2+y^2+z^2}}+\frac{kq}{\sqrt{(x-a)^2+y^2+z^2}}=0$$

$$\Rightarrow \quad \frac{kq}{\sqrt{(x-a)^2+y^2+z^2}}=\frac{2kq}{\sqrt{(x+a)^2+y^2+z^2}}$$

Squaring both sides,

$$\frac{1}{(x-a)^2+y^2+z^2}=\frac{4}{(x+a)^2+y^2+z^2}$$ ■

ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in Fig. 3.29. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charges and the separation between the charges is given as $\vec{p}=q\vec{d}$

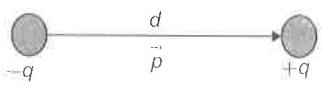


Figure 3.29

Dipole moment is a vector quantity and conventionally its direction is given from negative pole to positive pole.

Electric Field due to a Dipole

1. At an axial point
   Figure 3.30 shows an electric dipole placed on the x-axis at origin.

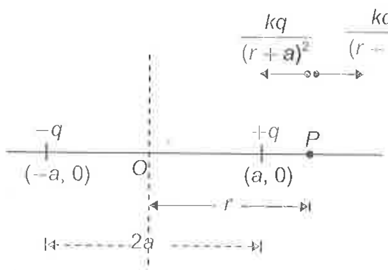


Figure 3.30

Here we wish to find the electric field at point P having coordinates $(r, 0)$ (where $r >> 2a$). Due to positive charge of dipole, electric field at P is in outward direction and due to negative charge it is in inward direction.

$$E_{\text{net}} \text{ at } P=\frac{kq}{(r-a)^2}-\frac{kq}{(r+a)^2}$$

$$=\frac{4kqar}{(r^2-a^2)^2}$$

As $\vec{P}=2aq$

$$\therefore \quad E_{\text{net}} \text{ at } P=\frac{2kpr}{(r^2-a^2)^2}$$

As $r >> 2a$

$\therefore$ We can neglect a w.r.t. r

$$E_{\text{net}} \text{ at } P=\frac{2kp}{r^3}$$

As we can observe that for axial point direction of field is in the direction of dipole moment.

$\therefore$ Vectorially,

$$\vec{E}=\frac{2k\vec{p}}{r^3}$$

2. At an equatorial point
   Again we consider the dipole placed along the x-axis and we wish to find electric field at point P which is situated equatorially at a distance r (where $r >> 2a$) from the origin.
   Vertical component of the electric field vectors cancel out each other.

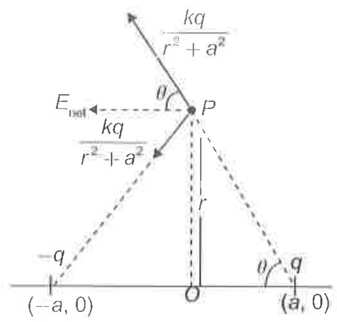


Figure 3.31

$$\therefore \quad E_{\text{net}} \text{ at } P=2E\cos\theta$$

$$\left[\text{where } E=\frac{kq}{r^2+a^2}\right]$$

$$E_{\text{net}} \text{ at } P=\frac{2kq}{r^2+a^2}\cdot\frac{a}{\sqrt{r^2+a^2}}$$

$$\left[\because \cos\theta=\frac{a}{\sqrt{r^2+a^2}}\right]$$

$$E_{net} = \frac{2kqa}{(r^2+a^2)^{3/2}} = \frac{kp}{(r^2+a^2)^{3/2}}$$

(As $p = 2aq$)

As we have already stated that $r >> 2a$

$$\therefore \quad E_{net} \text{ at } P = \frac{kp}{r^3}$$

We can observe that the direction of dipole moment and electric field due to dipole at P are in opposite direction.

$\therefore$ Vectorially,

$$\vec{E} = \frac{-k\vec{p}}{r^3}$$

Electric Field at a General Point due to a Dipole

Figure 3.31 shows an electric dipole placed on the x-axis at origin and we wish to find out the electric field at point P with coordinate (r, θ).

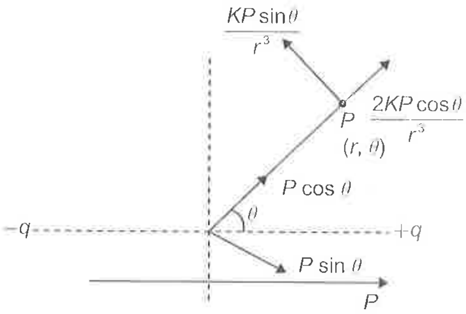


Figure 3.32

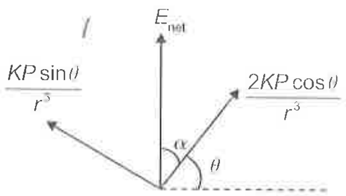


Figure 3.33

$$E_{net} = \sqrt{\left(\frac{2KP\cos\theta}{r^3}\right)^2 + \left(\frac{KP\sin\theta}{r^3}\right)^2}$$

$$= \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{\frac{Kp\sin\theta}{r^3}}{\frac{2Kp\cos\theta}{r^3}}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

Electric Potential due to a Dipole

1. At an axial point

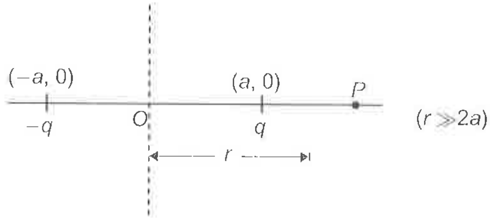


Figure 3.34

We wish to find out potential at P due to dipole (with $p = 2aq$)

$$V_{net} = \frac{kq}{(r-a)} - \frac{kq}{(r+a)}$$

$$V_{net} = \frac{2akq}{(r^2-a^2)}$$

$$V_{net} = \frac{kp}{r^2} \quad (\text{As } P = 2aq)$$

2. At a point on perpendicular bisector

At an equatorial point, electric potential due to dipole is always zero because potential due to +ve charge is cancelled by −ve charge.

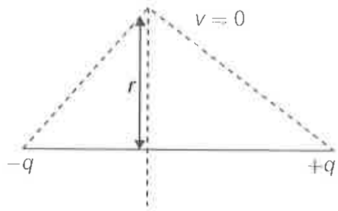


Figure 3.35

3. Potential due to dipole at a general point

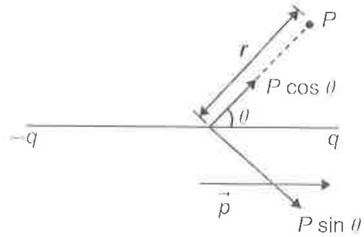


Figure 3.36

Potential at P due to dipole $= \dfrac{kp\cos\theta}{r^2}$

BASIC TORQUE CONCEPT

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- If the net translational force on the body is zero, then the torque of the forces may or may not be zero, but net torque of the forces about each point of universe is same.
- If we have to prove that a body is in equilibrium then first we will prove $\vec{F}_{\text{net}}$ is equal to zero and after that we will show τ_{net} about any point is equal to zero.
- If the body is free to rotate, then it will rotate about the axis passing through the centre of mass and parallel to torque vector direction and of the body is hinged; then it will rotate about hinged axis.

DIPOLE IN UNIFORM ELECTRIC FIELD

Figure 3.37 shows a dipole of dipole moment p placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces $q\varepsilon$ in opposite direction as shown in Fig. 3.37.

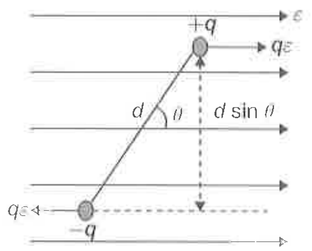


Figure 3.37

Thus, we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tends to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of action of forces

$= q\varepsilon \times d\sin\theta$

$= p\varepsilon\sin\theta$

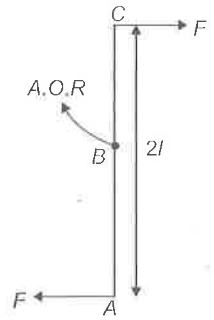
Or vectorially we can write the torque on the dipole is

$$\vec{\tau} = \vec{p} \times \vec{\varepsilon}$$

SOLVED EXAMPLE

EXAMPLE 52

Write down torque about A, B and C.



SOLUTION

$$\tau_A = 2Fl \otimes$$

$$\tau_C = 2Fl \otimes$$

$$\tau_B = Fl + Fl = 2Fl \otimes$$

Potential Energy of a Dipole in Uniform Electric Field

When a dipole is in an electric field at an angle θ, the torque on it due to electric field is

$$\tau = p\varepsilon\sin\theta$$

In Fig. 3.38, the torque is in the clockwise direction. If we rotate the dipole in anticlockwise direction from an angle θ_1 and θ_2 slowly, we have to apply an anticlockwise equal torque, then the work done in process will be given as

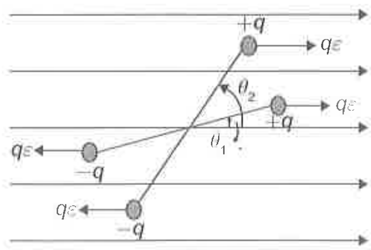


Figure 3.38

$$W = \int dW = \int \tau\, d\theta$$

$$= \int_{\theta_1}^{\theta_2} p\varepsilon \sin\theta\, d\theta$$

$$= p\varepsilon\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= p\varepsilon\left(\cos\theta_1 - \cos\theta_2\right)$$

$$W_{\text{external foce}} = -W_{\text{electric force}}$$

$$\therefore \quad W_{\text{electric field}} = -pE(\cos\theta_1 - \cos\theta_2)$$

As we know that for conservative forces

$$\text{WD} = -\Delta U$$

$$\Delta U = pE(\cos\theta_1 - \cos\theta_2)$$

$\therefore$ We can generalize that

$$U\theta = -pE\cos\theta$$

In vector notation, we can write potential energy of a dipole in an electric field is

$$U = -\vec{p}\cdot\vec{E}$$

[where potential energy at $\theta = 90° = 0$]

Stable and Unstable Equilibrium of a Dipole in an Electric Field

We have discussed that when a dipole in an electric field E, the potential energy of a dipole can be given as

$$U = -p\varepsilon\cos\theta$$

We also know that the net torque on a dipole in an electric field can be given as

$$\tau = p\varepsilon\sin\theta$$

It shows that the net torque on a dipole in an electric field is zero in two situations when $\theta = 0°$ and $\theta = 180°$ as shown in Fig. 3.39.

We can see that when $\theta = 0$ as shown in Fig. 3.39(a) when torque on a dipole is zero, the dipole is in equilibrium. We can verify that here equilibrium is stable. We slightly tilt the dipole from its equilibrium position in anticlockwise direction as shown by the dotted position. The dipole experiences a clockwise torque, which tends the dipole to rotate back to its equilibrium position. This shows that at $\theta = 0$, dipole is in stable equilibrium. We can also find the potential energy of dipole at $\theta = 0$, it can be given as

$$U = -p\varepsilon \text{ (minimum)}$$

Here at $\theta = 0$ potential energy of a dipole in an electric field is minimum, which favours the position of stable equilibrium.

Similarly when $\theta = 180°$, net torque on a dipole is zero and potential energy of a dipole in this state is given as

$$U = p\varepsilon \text{ (maximum)}$$

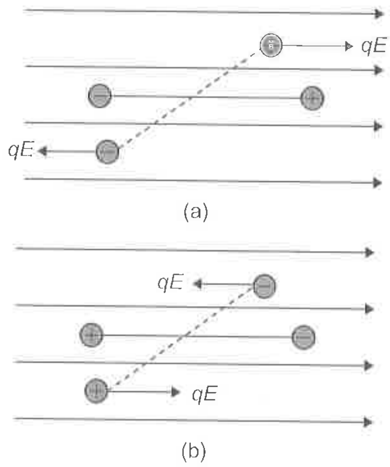


Figure 3.39

Thus at $\theta = 180°$, dipole is in unstable equilibrium. This can also be shown by Fig. 3.39(b). From equilibrium position if a dipole is slightly displaced in anti-clockwise direction, we can see that torque on the dipole also acts in anti-clockwise direction away from equilibrium position. Thus, dipole is in unstable equilibrium.

Angular SHM or Dipole

When a dipole is suspended in a uniform electric field, it will align itself parallel to the field.

Now if it is given a small angular displacement θ about its equilibrium, the (restoring) couple will be

$$C = -pE\sin\theta$$

or, $$C = -pE\theta$$

[as $\sin\theta \approx \theta$, for small θ]

or, $$I\frac{d^2\theta}{dt^2} = -pE\theta$$

or, $$\frac{d^2\theta}{dt^2} = -\frac{pE}{I}\theta$$

or, $$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

where $$\omega^2 = \frac{pE}{I}$$

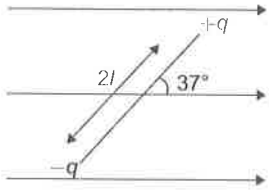
This is the standard equation of angular simple harmonic motion with time period $T\left(= \frac{2\pi}{\omega}\right)$. So the dipole will execute angular SHM with time period

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

SOLVED EXAMPLES

EXAMPLE 53

Find out the angular frequency of the dipole when it crosses the mean position.



SOLUTION

$$0 - PE\cos 37° = \frac{1}{2}I\omega^2$$

$$\frac{1}{2}I\omega^2 = \frac{PE}{5}$$

$$\frac{2ml^2}{2}\cdot\omega^2 = \frac{2ql\cdot E}{5}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{2qE}{5ml}}$$

Force on an Electric Dipole in a Non-uniform Electric Field

If in a non-uniform electric field, dipole is placed at a point where electric field is ε, the interaction energy of dipole at this point can be given as

$$U = -\vec{p}\cdot\vec{\varepsilon}$$

Now the force on the dipole due to electric field can be given as

$$F = -\Delta U$$

For unidirectional variation in electric field, we have

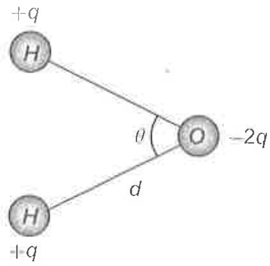
$$F = -\frac{d}{dx}(\vec{p}\cdot\vec{\varepsilon})$$

If dipole is placed in the direction of electric field, we have

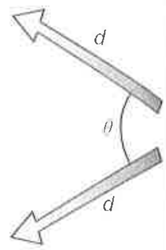
$$F = -p\frac{d\varepsilon}{dx}$$

EXAMPLE 54

A water molecule is placed at a distance λ from the line carrying linear charge density λ. Find the maximum force exerted on the water molecule. The shape of water molecule and the partial charges on H and O atoms are shown in the figure.



SOLUTION



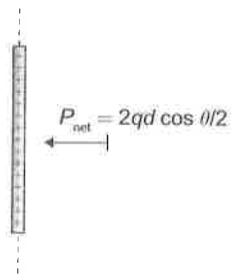
The figure can be resolved as the combination of two dipoles.

Dipole moments of each $p = qd$

Here that dipole moment of system is

$$P_{net} = 2\,qd\cos\theta/2$$

Now,
$$\vec{F} = \vec{P}_{net}\cdot\frac{d\vec{\varepsilon}}{dx}$$



For maximum force, the angle between $\vec{P}_{net}$ and $\dfrac{d\vec{\varepsilon}}{dx}$ is 0°

Or
$$F_{max} = 2qd\cos\frac{\theta}{2}\times\frac{d}{dx}\left(\frac{2k\lambda}{x}\right)$$

$$F_{max} = 2qd\cos\frac{\theta}{2}\times 2k\lambda\left(-\frac{1}{x}\right)$$

$$= \frac{-4kqd\lambda\cos\frac{\theta}{2}}{x^2}$$

$$\left|\vec{F}_{max}\right| = \frac{4kqd\lambda\cos\frac{\theta}{2}}{x^2}$$

$$\left|\vec{F}_{max}\right| = \frac{4kqd\lambda\cos\frac{\theta}{2}}{\ell^2}$$

EXERCISES

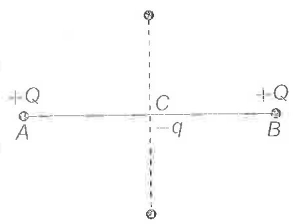
JEE Main

1. One quantum of charge should be at least be equal to the charge in coloumb:
   (A) 1.6×10^{-17} C (B) 1.6×10^{-19} C
   (C) 1.6×10^{-10} C (D) 4.8×10^{-10} C

2. An electron at rest has a charge of 1.6×10^{-19} C. It starts moving with a velocity $v = c/2$, where c is the speed of light, then the new charge on it is
   (A) 1.6×10^{-9} C
   (B) $1.6 \times 10^{-19}\sqrt{1-\left(\frac{1}{2}\right)^2}$ C
   (C) $1.6 \times 10^{-19}\sqrt{\left(\frac{2}{1}\right)^2 - 1}$ C
   (D) $\dfrac{1.6 \times 10^{-19}}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$ C

3. Which one of the following statement regarding electrostatics is wrong?
   (A) Charge is quantized
   (B) Charge is conserved
   (C) There is an electric field near an isolated charge at rest
   (D) A stationary charge produces both electric and magnetic fields

4. Two similar charge of $+Q$, as shown in figure are placed at A and B. $-q$ charge is placed at point C midway between A and B. $-q$ charge will oscillate if



   (A) It is moved towards A.
   (B) It is moved towards B.
   (C) It is moved upwards AB.
   (D) Distance between A and B is reduced.

5. When the distance between two charged particle is halved, the force between them becomes
   (A) One fourth (B) One half
   (C) Double (D) Four times

6. Two point charges in air at a distance of 20 cm. from each other interact with a certain force. At what distance from each other should these charges be placed in oil of relative permittivity 5 to obtain the same force of interaction
   (A) 8.94×10^{-2} m (B) 0.894×10^{-2} m
   (C) 89.4×10^{-2} m (D) 8.94×10^{2} m

7. A certain charge Q is divided at first into two parts, (q) and ($Q - q$). Later on the charges are placed at a certain distance. If the force of interaction between the two charges is maximum then
   (A) $(Q/q) = (4/1)$ (B) $(Q/q) = (2/1)$
   (C) $(Q/q) = (3/1)$ (D) $(Q/q) = (5/1)$

8. Two small balls having equal positive charge Q (Coulomb) on each are suspended by two insulating strings of equal length 'L' metre, from a hook fixed to a stand. The whole set up is taken in a satellite in to space where there is no gravity (state of weight lessness) Then the angle (θ) between the two strings is
   (A) $0°$ (B) $90°$
   (C) $180°$ (D) $0° < \theta < 180°$

9. Three equal charges (q) are placed at corners of a equilateral triangle. The force on any charge is
   (A) Zero (B) $\sqrt{3}\dfrac{Kq^2}{a^2}$
   (C) $\dfrac{Kq^2}{\sqrt{3}a^2}$ (D) $3\sqrt{3}\dfrac{Kq^2}{a^2}$

10. Two charges $4q$ and q are placed 30 cm apart. At what point the value of electric field will be zero
    (A) 10 cm away from q and between the charge.
    (B) 20 cm away from q and between the charge.
    (C) 10 cm away from q and outside the line joining the charge.
    (D) 10 cm away from $4q$ and outside the line joining them.

11. If $Q = 2$ coloumb and force on it is $F = 100$ newtons, Then the value of field intensity will be

(A) 100 N/C (B) 50 N/C
(C) 200 N/C (D) 10 N/C

12. Four equal but like charge are placed at four corners of a square. The electric field intensity at the center of the square due to any one charge is E, then the resultant electric field intensity at centre of square will be:
(A) Zero (B) $4E$
(C) E (D) $1/2\ E$

13. If mass of the electron $= 9.1 \times 10^{-31}$ Kg. Charge on the electron $= 1.6 \times 10^{-19}$ coulomb and $g = 9.8\ m/s^2$. Then the intensity of the electric field required to balance the weight of an electron is
(A) 5.6×10^{-9} N/C (B) 5.6×10^{-11} N/C
(C) 5.6×10^{-8} N/C (D) 5.6×10^{-7} N/C

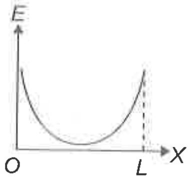
14. A point charge $50\mu C$ is located in the XY plane at the point of position vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. What is the electric field at the point of position vector $\vec{r} = 8\hat{i} - 5\hat{j}$
(A) 1200 V/m (B) 0.04 V/m
(C) 900 V/m (D) 4500 V/m

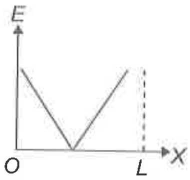
15. A point charge q is placed at origin. Let $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ be the electric field at three points $A(1, 2, 3)$, $B(1, 1, -1)$ and $C(2, 2, 2)$ due to charge q. Then

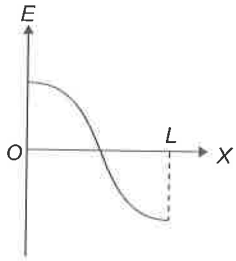
[i] $\vec{E}_A \perp \vec{E}_B$ [ii] $|\vec{E}_B| = 4|\vec{E}_C|$

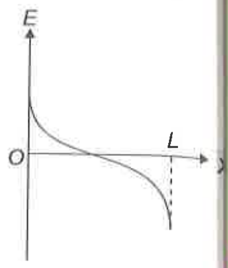
select the correct alternative
(A) only [i] is correct
(B) only [ii] is correct
(C) both [i] and [ii] are correct
(D) both [i] and [ii] are wrong

16. Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than l. Which of the following best represents the resulting curve ?

(A)


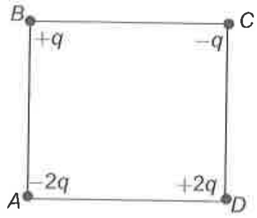
(B)


(C)


(D)


17. A particle of mass m and charge Q is placed in a electric field E which varies with time t ass $E = E$ sin ωt. It will undergo simple harmonic motion of amplitude
(A) $\frac{QE_0^2}{m\omega^2}$ (B) $\frac{QE_0}{m\omega^2}$
(C) $\sqrt{\frac{QE_0}{m\omega^2}}$ (D) $\frac{QE_0}{m\omega}$

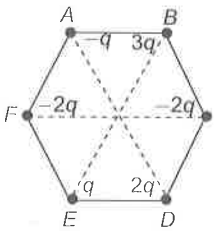
18. Four charges are arranged at the corners of a square $ABCD$, as shown. The force on +ve charge kept at the centre of the square is



(A) zero
(B) along diagonal AC
(C) along diagonal BD
(D) perpendicular to the side AB

19. Two free positive charges $4q$ and q are a distance l apart. What charge Q is needed to achieve equilibrium for the entire system and where should it be placed form charge q?
(A) $Q = \frac{4}{9}q$(negative) at $\frac{l}{3}$
(B) $Q = \frac{4}{9}q$(positive) at $\frac{l}{3}$
(C) $Q = q$ (positive) at $\frac{l}{3}$
(D) $Q = q$ (negative) at $\frac{l}{3}$

20. Six charges are placed at the corner of a regular hexagon as shown. If an electron is placed at its centre O, force on it will be



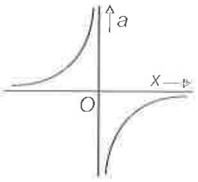
(A) Zero (B) Along OF
(C) Along OC (D) None of these

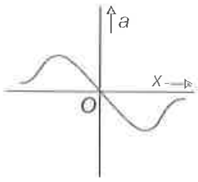
21. A charged particle of charge q and mass m is released from rest in an uniform electric field E. Neglecting the effect of gravity, the kinetic energy of the charged particle after time 't' seconds is

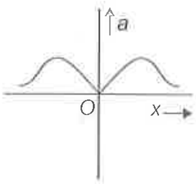
(A) $\frac{Eqm}{t}$ (B) $\frac{E^2q^2t^2}{2m}$

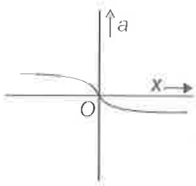
(C) $\frac{2E^2t^2}{mq}$ (D) $\frac{Eq^2m}{2t^2}$

22. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the $+x$-axis, passes through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-coordinate. Which of the following best represents the plot?

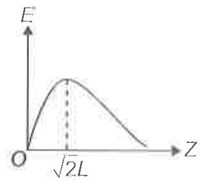
(A)


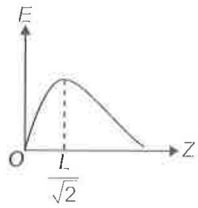
B)


(C)


(D)


23. Four equal positive charges are fixed at the vertices of a square of side L. Z-axis is perpendicular to the plane of the square. The point $z = 0$ is the point where the diagonals of the square intersect each other. The plot of electric field due to the four charges, as one moves on the z-axis.

(A)


(B)


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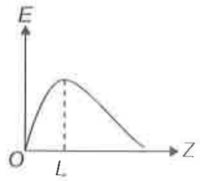

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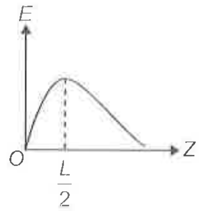


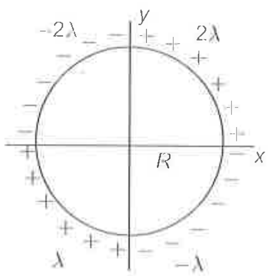
24. A small circular ring has a uniform charge distribution. On a far-off axial point distance x from the centre of the ring, the electric field is proportional to

(A) x^{-1} (B) $x^{-3/2}$
(C) x^{-2} (D) $x^{5/4}$

25. A nonconducting ring of radius R has uniformly distributed positive charge Q. A small part of the ring, of length d, is removed ($d << R$). The electric field at the centre of the ring will now be

(A) directed towards the gap, inversely proportional to R^3.
(B) directed towards the gap, inversely proportional to R^2.
(C) directed away from the gap, inversely proportional to R^3.
(D) directed away from the gap, inversely proportional to R^2.

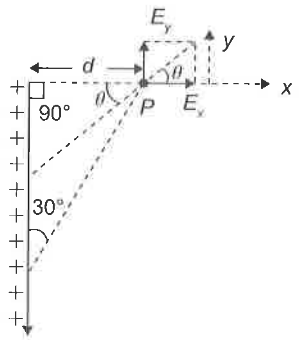
26. The charge per unit length of the four quadrant of the ring is 2λ, -2λ, λ and $-\lambda$ respectively. The electric field at the centre is



(A) $-\frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$ (B) $\frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$

(C) $\frac{\sqrt{2}\lambda}{2\pi\varepsilon_0 R}\hat{i}$ (D) None

27. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be

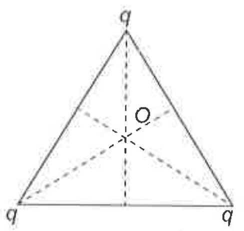


(A) at angle 30° from x-axis
(B) 45° from x-axis
(C) 60° from x-axis
(D) none of these

28. When charge of 3 coulomb is placed in a Uniform electric field, it experiences a force of 3000 newton, within this field, potential difference between two points separated by a distance of 1 cm is

(A) 10 V (B) 90 V
(C) 1000 V (D) 3000 V

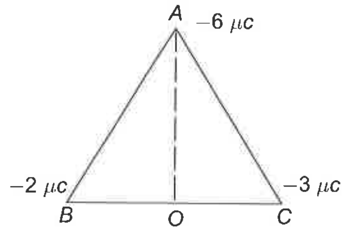
29. Three equal charges are placed at the three corners of an isosceles triangle as shown in the figure. The statement which is true for electric potential V and the field intensity E at the centre of the triangle is



(A) $V = 0, E = 0$ (B) $V = 0, E \neq 0$
(C) $V \neq 0, E = 0$ (D) $V \neq 0, E \neq 0$

30. Electric potential is a
(A) Vector quantity
(B) Scalar quantity
(C) Neither vector Nor scalar
(D) Fictious quantity

31. ABC is equilateral triangle of side 1 m. Charges are placed at its corners as shown in fig. O is the mid-point of side BC the potential at point (O) is

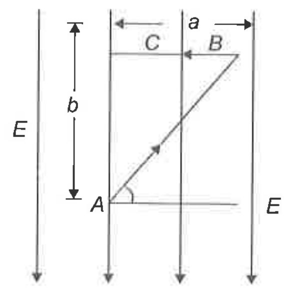


(A) 2.7×10^3 V (B) 1.52×10^5 V
(C) 1.3×10^3 V (D) -1.52×10^5 V

32. In a region where $E = 0$, the potential (V) varies with distance r as

(A) $V \alpha \frac{1}{r}$ (B) $V \alpha r$
(C) $V \alpha \frac{1}{r^2}$ (D) $V =$ const. independent of (r)

33. The potential difference between points A and B in the given uniform electric field is:



(A) Ea (B) $E\sqrt{(a^2 + b^2)}$
(C) Eb (D) $(Eb / \sqrt{2})$

34. An equipotential surface and a line of force:
(A) never intersect each other (B) intersect at 45°
(C) intersect at 60° (D) intersect at 90°

35. Which of the following is a volt
(A) Erg per cm
(B) Joule per coulomb
(C) Erg per ampere
(D) Newton/(coulomb × m²)

36. An infinite nonconducting sheet of charge has a surface charge density of 10^{-7} C/m². The separation between two equipotential surfaces near the sheet whose potential differ by 5 V is

(A) 0.88 cm (B) 0.88 mm
(C) 0.88 m (D) 5×10^{-7} m

37. In a uniform electric field, the potential is 10 V at the origin of coordinates, and 8 V at each of the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). The potential at the point (1, 1, 1) will be

(A) 0 (B) 4 V
(C) 8 V (D) 10 V

38. In a regular polygon of n sides, each corner is at a distance r from the centre. Identical charges are placed at $(n-1)$ corners. At the centre, the intensity is E and the potential is V. The ratio V/E has magnitude.

(A) rn (B) $r(n-1)$
(C) $(n-1)/r$ (D) $r(n-1)/n$

39. In a certain region of space, the potential is given by: $V = k[2x^2 - y^2 + z^2]$. The electric field at the point (1, 1, 1) has magnitude =

(A) $k\sqrt{6}$ (B) $2k\sqrt{6}$
(C) $2k\sqrt{3}$ (D) $4k\sqrt{3}$

40. When the seperation between two charges is increased, the electric potential energy of the charges

(A) increases
(B) decreases
(C) remains the same
(D) may increase or decrease

41. A point positive charge of Q' units is moved round another point positive charge of Q units in circular path. If the radius of the circle r is the work done on the charge Q' in making one complete revolution i

(A) $\frac{Q}{4\pi \in_0 r}$ (B) $\frac{QQ'}{4\pi \in_0 r}$
(C) $\frac{Q'}{4\pi \in_0 r}$ (D) 0

42. When a negative charge is released and moves in electric field, it moves toward a position of

(A) lower electric potential and lower potential energy
(B) lower electric potential and higher potential energy
(C) higher electric potential and lower potential energy
(D) higher electric potential and higher potential energy

43. Two identical thin rings, each of radius R meter are coaxially placed at distance R meter apart. If Q_1 and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

(A) zero
(B) $q(Q_1 - Q_2)(\sqrt{2}-1)/(\sqrt{2}.4\pi\varepsilon_0 R)$
(C) $q\sqrt{2}(Q_1 + Q_2)/4\pi\varepsilon_0 R$
(D) $q(Q_1 - Q_2)(\sqrt{2}+1)/(\sqrt{2}.4\pi\varepsilon_0 R)$

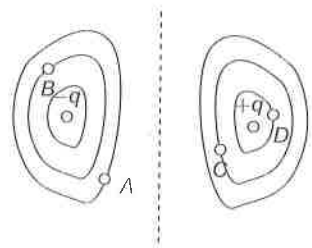
44. Two identical particles of mass m carry a charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards first particle from a large distance with speed v. The closed distance of approach be

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{mv}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{4Q^2}{mv^2}$
(C) $\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{mv^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{mv^2}$

45. An equipotential surface is that surface

(A) On which each and every point has the same potential
(B) Which has negative potential
(C) Which has positive potential
(D) Which has zero potential

46. Figure shows equi-potential surfaces for a two charges system. At which of the labeled points point will an electron have the highest potential energy?



(A) Point A (B) Point B
(C) Point C (D) Point D

47. The equation of an equipotential line in an electric field is $y = 2x$, then the electric field strength vector at (1, 2) may be

(A) $4\hat{i} + 3\hat{j}$ (B) $4\hat{i} + 8\hat{j}$
(C) $8\hat{i} + 4\hat{j}$ (D) $-8\hat{i} + 4\hat{j}$

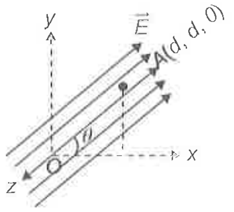
48. The electric field in region is given by: $E = (4axy\sqrt{z})\hat{i} + (2ax^2\sqrt{z})\hat{j} + (ax^2y/\sqrt{z})\hat{k}$, where a is a positive constant. The equation of an equipotential surface will be of the form

(A) $z = \text{constant}/[x^3y^2]$
(B) $z = \text{constant}/[xy^2]$
(C) $z = \text{constant}/[x^4y^2]$
(D) None

49. A charge 3 C experiences a force 3000 N when placed in a uniform electric field. The potential difference between two points separated by a distance of 1 cm along the field lines is

(A) 10 V (B) 90 V
(C) 1000 V (D) 9000 V

50. A uniform electric field having strength $\vec{E}$ is existing in x-y plane as shown in figure. Find the p.d. between origin O and A $(d, d, 0)$

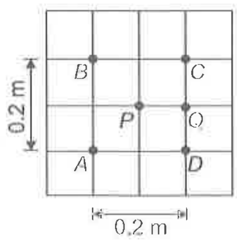


(A) $Ed(\cos\theta + \sin\theta)$
(B) $-Ed(\sin\theta - \cos\theta)$
(C) $\sqrt{2}Ed$
(D) None of these

51. Uniform electric field of magnitude 100 V/m in space is directed along the line $y = 3 + x$. Find the potential difference between point $A(3, 1)$ and $B(1, 3)$

(A) 100 V (B) $200\sqrt{2}$ V
(C) 200 V (D) 0

52. A, B, C, D, P and Q are points in a uniform electric field. The potentials at these points are $V(A) = 2$ V. $V(P) = V(B) = V(D) = 5$ V. $V(C) = 8$ V. The electric field at P is

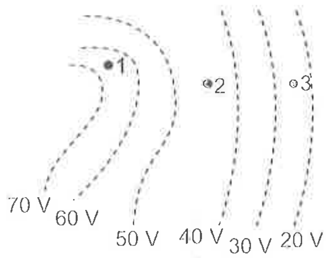


(A) 10 Vm^{-1} along PQ
(B) $15\sqrt{2}$ Vm^{-1} along PA
(C) 5 Vm^{-1} along PC
(D) 5 Vm^{-1} along PA

53. A and B are two points on the axis and the perpendicular bisector respectively of an electric dipole. A and B are far away from the dipole and at equal distance from it. The field at A and B are $\vec{E}_A$ and $\vec{E}_B$.

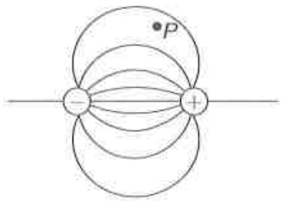
(A) $\vec{E}_A = \vec{E}_B$
(B) $\vec{E}_A = 2\vec{E}_B$
(C) $\vec{E}_A = -2\vec{E}_B$
(D) $|E_B| = \frac{1}{2}|E_A|$, and $\vec{E}_B$ is perpendicular to $\vec{E}_A$

54. Some equipotential lines are as shown is fig. E_1, E_2 and E_3 are the electric fields at points 1, 2 and 3 then



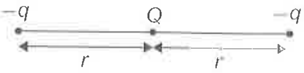
(A) $E_1 = E_2 = E_3$ (B) $E_1 > E_2 > E_3$
(C) $E_1 > E_2$, $E_2 < E_3$ (D) $E_1 < E_2 < E_3$

55. Figure shows the electric field lines around an electric dipole. Which of the arrows best represents the electric field at point P?



(A) ↑ (B) ↖
(C) ↗ (D) ↙

56. Three charges are placed as shown in figure if the electric potential energy of system is zero, then Q: q



(A) $\frac{Q}{q} = \frac{-2}{1}$ (B) $\frac{Q}{q} = \frac{2}{1}$

(C) $\frac{Q}{q} = \frac{-1}{2}$ (D) $\frac{Q}{q} = \frac{1}{4}$

57. In an electric field the work done in moving a unit positive charge between two points is the measures of
(A) Resistance
(B) Potential difference
(C) Intensity of electric field
(D) Capacitance

58. The K.E. in electron Volt gained by an α-particle when it moves from rest at point where its potential is 70 to a point where potential is 50 V, is
(A) 20 eV (B) 20 MeV
(C) 40 eV (D) 40 MeV

59. If an electric dipole is kept in a unifrom electric field, Then it will experience
(A) a force
(B) a couple and mover
(C) a couple and rotates
(D) a force and moves

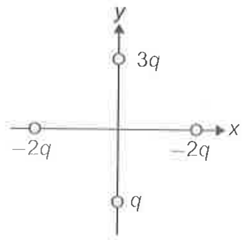
60. An electric dipole consists of two opposite charges each of magnitude 1×10^{-6} C separated by a distance 2 cm. The dipole is placed in an external field of 10×10^5 N/C. The maximum torque on the dipole is
(A) 0.2×10^{-3} N-m
(B) 1.0×10^{-3} N-m
(C) 2×10^{-2} N-m
(D) 4×10^{-3} N-m

61. The ratio of the electric field due to an electric dipole on its axis and on the perpendicular bisector of the dipole is
(A) 1:2 (B) 2:1
(C) 1:4 (D) 4:1

62. The region surrounding a stationary electric dipole has
(A) electric field only
(B) magnetic field only
(C) both electric and magnetic fields
(D) neither electric nor magnetic field

63. The electric potential at a point due to an electric dipole will be.
(A) $k\frac{\vec{p}.\vec{r}}{r^3}$ (B) $k\frac{\vec{p}.\vec{r}}{r^2}$
(C) $\frac{k(\vec{p}\times\vec{r})}{r}$ (D) $\frac{k(\vec{p}\times\vec{r})}{r^2}$

64. 4 charges are placed each at a distance 'a' from origin. The dipole moment of configuration is



(A) $2qa\hat{j}$ (B) $3qa\hat{j}$
(C) $2aq[\hat{i} + \hat{j}]$ (D) none

Reasoning Type

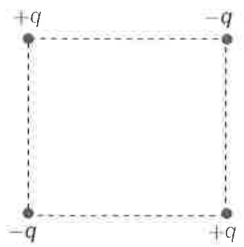
65. Statement 1: A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)
Statement 2: Electric lines of force represents path of charged particle which is released from rest in it.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

66. Statement 1 If electric potential while moving in a certain path is constant, then the electric field must be zero.

Statement 2 Component of electric field $E_r = -\frac{\partial V}{\partial r}$
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

67. **Statement 1:** For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.
Statement 2: For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on axis of the ring is zero.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

68. **Statement 1:** The electric potential and the electric field intensity at the centre of a square having four fixed point charges at their vertices as shown in figure are zero.



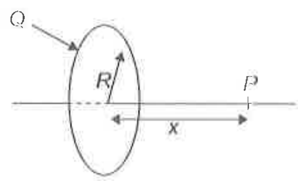
Statement 2: If electric potential at a point is zero then the magnitude of electric field at that point must be zero.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

JEE Advanced

Single Correct

1. Two equal negative charges are fixed at the points $[0, a]$ and $[0, -a]$ on the y-axis. A positive charge Q is released from rest at the points $[2a, 0]$ on the x-axis. The charge Q will
(A) execute simple harmonic motion about the origin
(B) move to the origin and remain at rest
(C) move to infinity
(D) execute oscillatory but not simple harmonic motion.

2. A small particle of mass m and charge $-q$ is placed at point P on the axis of uniformly charged ring and released. If $R >> x$, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to

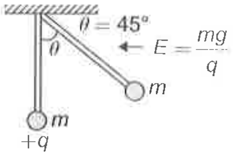


(A) $\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$ (B) $\sqrt{\dfrac{qQX}{4\pi\varepsilon_0 mR^4}}$

(C) $\dfrac{qQ}{4\pi\varepsilon_0 mR^3}$ (D) $\dfrac{qQX}{4\pi\varepsilon_0 mR^4}$

3. A charged particle having some mass is resting in equilibrium at a height H above the centre of a uniformly charged non-conducting horizontal ring of radius R. The force of gravity acts downwards. The equilibrium of the particle will be stable
(A) for all values of H (B) only if $H > R/\sqrt{2}$
(C) only if $H < R/\sqrt{2}$ (D) only if $H = R/\sqrt{2}$

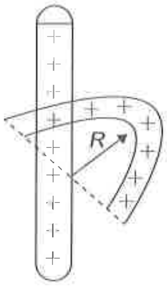
4. In space of horizontal $EF(E = (mg)/q)$ exist as shown in figure and a mass m attached at the end of a light rod. If mass m is released from the position shown in figure find the angular velocity of the rod when it passes through the bottom most position.



(A) $\sqrt{\frac{g}{l}}$ (B) $\sqrt{\frac{2g}{l}}$

(C) $\sqrt{\frac{3g}{l}}$ (D) $\sqrt{\frac{5g}{l}}$

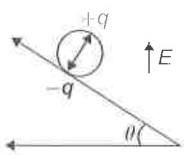
5. Find the force experienced by the semicircular rod charged with a charge q, placed as shown in figure. Radius of the wire is R and the infinitely long line of charge with linear density λ is passing through its centre and perpendicular to the plane of wire.



(A) $\frac{\lambda q}{2\pi^2\varepsilon_0 R}$ (B) $\frac{\lambda q}{\pi^2\varepsilon_0 R}$

(C) $\frac{\lambda q}{4\pi^2\varepsilon_0 R}$ (D) $\frac{\lambda q}{4\pi\varepsilon_0 R}$

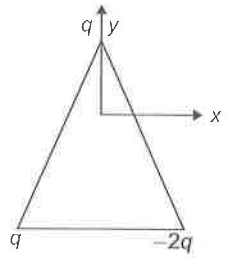
6. A wheel having mass m has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field $E =$



(A) $\frac{mg}{q}$ (B) $\frac{mg}{2q}$

(C) $\frac{mg\tan\theta}{2q}$ (D) none

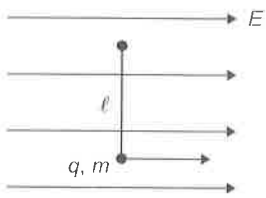
7. An equilateral triangle wire frame of side L having 3 point charges at its vertices is kept in x-y plane as shown. Component of electric field due to the configuration in z direction at $(0, 0, L)$ is [origin is centroid of triangle]



(A) $\frac{9\sqrt{3}kq}{8L^2}$ (B) zero

(C) $\frac{9\,kq}{8L^2}$ (D) None

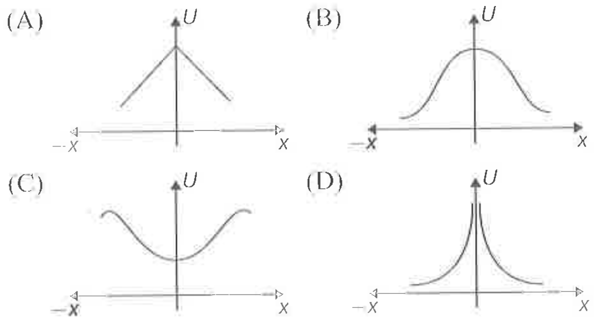
8. A simple pendulum has a length l, mass of bob m. The bob is given a charge q coulomb. The pendulum is suspended in a uniform horizontal electric field of strength E as shown in figure, then calculate the time period of oscillation when the bob is slightly displace from its mean position is



(A) $2\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\left\{\frac{\ell}{g+\frac{qE}{m}}\right\}}$

(C) $2\pi\sqrt{\left\{\frac{\ell}{g-\frac{qE}{m}}\right\}}$ (D) $2\pi\sqrt{\frac{\ell}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

9. Four equal charges $+q$ are placed at four corners of a square with its centre of origin and lying in yz plane. The electrostatic potential energy of a fifth charge $+q$ varies or x-axis as

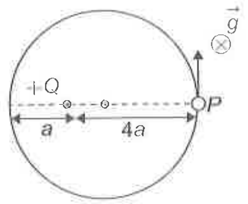


10. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.
(A) If Y is fixed, both p and E are conserved
(B) If Y is fixed, E is conserved, but not p
(C) If both are free to move, p is conserved but not E
(D) If both are free, E is conserved, but not p

11. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time, finally.
(A) X will stop, Y will move with velocity u
(B) X and Y will both move with velocities $u/2$ each
(C) X will stop, Y will move with velocity $< u$
(D) both will move with velocities $< u/2$

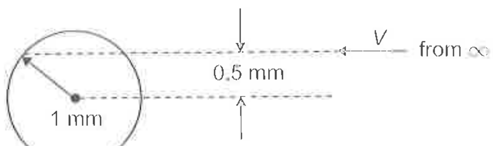
12. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y-z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(R\sqrt{3}, O, O)$ on the positive x-axis directly towards O, with an initial kinetic energy $\frac{\lambda q}{4\varepsilon_0}$.
(A) The particle crosses O and goes to infinity
(B) The particle returns to P
(C) The particle will just reach O
(D) The particle crosses O and goes to $-R\sqrt{3}$.

13. The diagram shows a small bead of mass m carrying charge q. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge $+Q$ has also been fixed as shown. The potential at the point P due to $+Q$ is V. The velocity with which the bead should projected from the point P so that it can complete a circle should be greater than



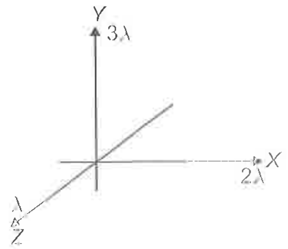
(A) $\sqrt{\frac{6qV}{m}}$ (B) $\sqrt{\frac{qV}{m}}$
(C) $\sqrt{\frac{3qV}{m}}$ (D) none

14. A particle of mass 1 kg and charge 1/3 μC is projected towards a non conducting fixed spherical shell having the same charge uniformly distributed on its surface. Find the minimum initial velocity of projection required if the particle just grazes the shell.



(A) $\sqrt{\frac{2}{3}}$ m/s (B) $2\sqrt{\frac{2}{3}}$ m/s
(C) $\frac{2}{3}$ m/s (D) none

15. The diagram shows three infinitely long uniform line charges placed on the X, Y and Z axis. The work done in moving a unit positive charge from (1, 1, 1) to (0, 1, 1) is equal to



(A) $(\lambda\, ln2)/2\pi\varepsilon_0$ (B) $(\lambda ln2)/\pi\varepsilon_0$
(C) $(3\lambda ln2)/2\pi\varepsilon_0$ (D) None

16. A charged particle of charge Q is held fixed and another charged particle of mass m and charge q (of the same sign) is released from a distance r. The impulse of the force exerted by the external agent on the fixed charge by the time distance between Q and q becomes $2r$ is

(A) $\sqrt{\frac{Qq}{4\pi \epsilon_0 mr}}$ (B) $\sqrt{\frac{Qqm}{4\pi \epsilon_0 r}}$

(C) $\sqrt{\frac{Qqm}{\pi \epsilon_0 r}}$ (D) $\sqrt{\frac{Qqm}{2\pi \epsilon_0 r}}$

17. Two point charges of $+Q$ each have been placed at the positions $(-a/2, 0, 0)$ and $(a/2, 0, 0)$. The locus of the points in YZ plane where $-Q$ charge can be placed such the that total electrostatic potential energy of the system can become equal to zero, is represented by which of the following equations?
(A) $Z^2 + (Y-a)^2 = 2a$
(B) $Z^2 + (Y-a)^2 = 27a^2/4$
(C) $Z^2 + Y^2 = 15a^2/4$
(D) None

18. A dipole consists of two particles one with charge $+1\ \mu C$ and mass 1 kg and the other with charge $-1\ \mu C$ and mass 2 kg separated by a distance of 3 m. For small oscillations about its equilibrium position, the angular frequency, when placed in a uniform electric field of 20 kV/m is
(A) 0.1 rad/s (B) 1.1 rad/s
(C) 10 rad/s (D) 2.5 rad/s

19. The dipole moment of a system of charge $+q$ distributed uniformly on an arc of radius R subtending an angle $\pi/2$ at its centre where another charge $-q$ is placed is.

(A) $\frac{2\sqrt{2}qR}{\pi}$ (B) $\frac{\sqrt{2}qR}{\pi}$

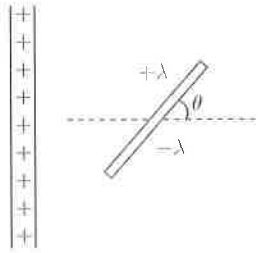
(C) $\frac{qR}{\pi}$ (D) $\frac{2qR}{\pi}$

20. An electric dipole is kept on the axis of a uniformly charged ring at distance $R/\sqrt{2}$ from the centre of the ring. The direction of the dipole moment is along the axis. The dipole moment is P, charge of the ring is Q and radius of the ring is R. The force on the dipole is nearly

(A) $\frac{4kPQ}{3\sqrt{3}R^2}$ (B) $\frac{4kPQ}{3\sqrt{3}R^3}$

(C) $\frac{2kPQ}{3\sqrt{3}P^3}$ (D) zero

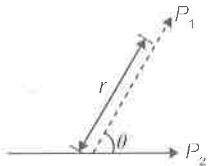
21. A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is



(A) 0 (B) $\frac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$

(C) $\frac{\sigma\lambda l^2}{\varepsilon_0}\sin\theta$ (D) $\frac{\sigma\lambda l}{2\varepsilon_0}$

22. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be



(A) $\frac{2kP_1P_2\cos\theta}{r^3}$ (B) $\frac{-2kP_1P_2\cos\theta}{r^3}$

(C) $\frac{-2kP_1P_2\sin\theta}{r^3}$ (D) $\frac{-4kP_1P_2\cos\theta}{r^3}$

23. Point P lies on the axis of a dipole. If the dipole is rotated by 90° anticlock wise, the electric field vector $\vec{E}$ at P will rotate by
(A) 90° clock wise
(B) 180° clock wise
(C) 90° anti clock wise
(D) none

Multiple Correct

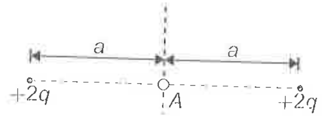
24. Select the correct alternative:
(A) The charge gained by the uncharged body from a charged body due to conduction is equal to half of the total charge initially present.

(B) The magnitude of charge increases with the increase in velocity of charge
(C) Charge can not exist without matter although matter can exist without charge
(D) Between two non-magnetic substances repulsion is the true test of electrification (electrification means body has net charge)

25. Two equal negative charges $-q$ are fixed at the point $(0, a)$ and $(0, -a)$ on the y-axis. A charge $+Q$ is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will:
(A) Execute simple harmonic motion about the origin.
(B) At origin velocity of particle is maximum
(C) Move to infinity
(D) Execute oscillatory but not simple harmonic motion.

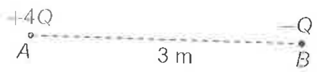
26. Mid way between the two equal and similar charges, we placed the third equal and similar charge. Which of the following statements is correct, concerned to the equilibrium along the line joining the charges?
(A) The third charge experienced a net force inclined to the line joining the charges
(B) The third charge is in stable equilibrium
(C) The third charge is in unstable equilibrium
(D) The third charge experiences a net force perpendicular to the line joining the charges

27. A negative point charge placed at the point A is



(A) in stable equilibrium along x-axis
(B) in unstable equilibrium along y-axis
(C) in stable equilibrium along y-axis
(D) in unstable equilibrium along x-axis

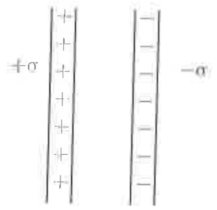
28. Two fixed charges $4Q$ (positive) and Q (negative) are located at A and B, the distance AB being 3 m.



(A) The point P where the resultant field due to both is zero is on AB outside AB.
(B) The point P where the resultant field due to both is zero is on AB inside AB.
(C) If a positive charge is placed at P and displaced slightly along AB it will execute oscillations.
(D) If a negative charge is placed at P and displaced slightly along AB it will execute oscillation.

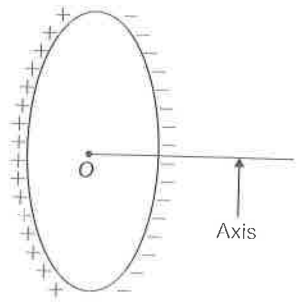
29. Select the correct statement: (Only force on a particle is due to electric field)
(A) A charged particle always moves along the electric line of force.
(B) A charged particle may move along the line of force
(C) A charge particle never moves along the line of force
(D) A charged particle moves along the line of force only if released from rest.

30. Two infinite sheets of uniform charge density $+\sigma$ and $-\sigma$ are parallel to each other as shown in the figure.



Electric field at the
(A) points to the left or to the right of the sheets is zero
(B) midpoint between the sheets is zero
(C) midpoint of the sheets is σ/ε_0 and is directed towards right
(D) midpoint of the sheets is $2\sigma/\varepsilon_0$ and is directed towards right

31. The figure shows a nonconducting ring which has positive and negative charge non uniformly distributed on it such that the total charge is zero. Which of the following statements is true?



(A) The potential at all the points on the axis will be zero.
(B) The electric field at all the points on the axis will be zero.
(C) The direction of electric field at all points on the axis will be along the axis
(D) If the ring is placed inside a uniform external electric field then net torque and force acting on the ring would be zero.

32. If we use permittivity ε, resistance R, gravitational constant G and voltage V as fundamental physical quantities, then
(A) [angular displacement] $= \varepsilon^0 R^0 G^0 V^0$
(B) [Velocity] $= \varepsilon^{-1} R^{-1} G^0 V^0$
(C) [dipole moment] $= \varepsilon^1 R^0 G^0 V^1$
(D) [force] $= \varepsilon^1 R^0 G^0 V^2$

33. A particle of mass m and charge q is thrown in a region where uniform gravitational field and electric field are present. The path of particle
(A) may be a straight line
(B) may be a circle
(C) may be a parabola
(D) may be a hyperbola

34. Two point charges Q and $-Q/4$ are separated by a distance x. Then

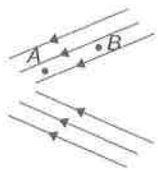
(A) potential is zero at a point on the axis which is $x/3$ on the right side of the charge $-Q/4$
(B) potential is zero at a point on the axis which is $x/5$ on the left side of the charge $-Q/4$
(C) electric field is zero at a point on the axis which is at a distance x on the right side of the charge $-Q/4$
(D) there exist two points on the axis where electric field is zero.

35. An electric charge 10^{-8} C is placed at the point (4 m, 7 m, 2 m). At the point (1 m, 3 m, 2 m), the electric
(A) potential will be 18 V
(B) field has no Y-component
(C) field will be along z-axis
(D) potential will be 1.8 V

36. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (–1, 0, 0) and (0, –1, 0).
(A) The potential at the origin is zero.
(B) The field at the origin is zero.
(C) The potential at all points on the z-axis, other than the origin, is zero.
(D) The field at all points on the z-axis, other than the origin acts along the z-axis.

37. A proton and a deuteron are initially at rest and are accelerated through the same potential difference. Which of the following is false concerning the final properties of the two particles?
(A) They have different speeds
(B) They have same momentum
(C) They have same kinetic energy
(D) None of these

38. Which of the following is true for the figure showing electric lines of force? (E is electrical field, V is potential)



(A) $E_A > E_B$ (B) $E_B > E_A$
(C) $V_A > V_B$ (D) $V_B > V_A$

39. Three point charges Q, $4Q$ and $16Q$ are placed on a straight line 9cm long. Charges are placed in such a way that the system has minimum potential energy. Then
(A) $4Q$ and $16Q$ must be at the ends and Q at a distance of 3 cm from the $16Q$
(B) $4Q$ and $16Q$ must be at the ends and Q at a distance of 6 cm from the $16Q$
(C) Electric field at the position of Q is zero
(D) Electric field at the position of Q is $\frac{Q}{4\pi\varepsilon_0}$

40. Potential at a point A is 3 V and at a point B is 7 V, an electron is moving towards A from B.
(A) It must have some K.E. at B to reach A
(B) It need not have any K.E. at B to reach A
(C) to reach A it must have more than or equal to 4eV K.E. at B.
(D) when it will reach A, it will have K.E. more then or at least equal to 4eV if it was released from rest at B.

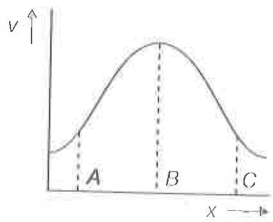
41. A particle of charge 1 μC and mass 1 gm moving with a velocity of 4 m/s is subjected to a uniform electric field of magnitude 300 V/m for 10 s. Then it's final speed cannot be:
(A) 0.5 m/s (B) 4 m/s
(C) 3 m/s (D) 6 m/s

42. Two particles of same mass and charge are thrown in the same direction along the horizontal with same velocity v from two different heights h_1 and h_2 ($h_1 < h_2$). Initially they were located on the same vertical line. Choose the correct alternative.
(A) Both the particles will lie on a vertical line until either of the ball hits the ground
(B) Acceleration of the centre of mass of two particles will be g downwards
(C) Horizontal displacement of the particle lying at h_1 is less and the particle lying at h_2 is more than the value, which would had been in the absence of charges on them.
(D) All of these

43. Let V be electric potential and E the magnitude of the electric field. At a given position, which of the statement is true?
(A) E is always zero where V is zero
(B) V is always zero where E is zero
(C) E can be zero where V is non zero
(D) E is always nonzero where V is nonzero

44. The electric potential decreases uniformly from V to $-V$ along x-axis in a coordinate system as we moves from a point $(-x_0, 0)$ to $(x_0, 0)$, then the electric field at the origin.
(A) must be equal to V/x_0
(B) may be equal to V/x_0
(C) must be greater than V/x_0
(D) may be less than V/x_0

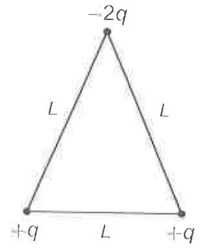
45. Variation of electrostatic potential along x-direction is shown in the graph. The correct statement about electric field is



(A) x component at point B is maximum
(B) x component at point A is towards positive x-axis
(C) x component at point C is along negative x-axis
(D) x component at point C is along positive x-axis

46. An electric dipole moment $\vec{p} = (2.0\hat{i} + 3.0\hat{j})$mC. m is placed in a uniform electric field
$\vec{E} = (3.0\hat{i} + 2.0\hat{k})\ 10^5\,\text{NC}^{-1}$.
(A) The torque that $\vec{E}$ exerts on $\vec{p}$ is $(0.6\hat{i} - 0.4\hat{j} - 0.9\hat{k})$ Nm
(B) The potential energy of the dipole is -0.6 J.
(C) The potential energy of the dipole is 0.6 J
(D) If the dipole is rotated in the electric field, the maximum potential energy of the dipole is 1.3 J.

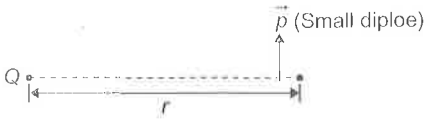
47. Three points charges are placed at the corners of an equilateral triangle of side L as shown in the figure.



(A) The potential at the centroid of the triangle is zero
(B) The electric field at the centroid of the triangle is zero.
(C) The dipole moment of the system is $\sqrt{2}\,qL$
(D) The dipole moment of the system is $\sqrt{3}\,qL$

48. Particle A having positive charge is moving directly head-on towards initially stationary positively charged particle B. At the instant when A and B are closest together.
(A) the momenta of A and B must be equal
(B) the velocities of A and B must be equal
(C) B would have gained less kinetic energy than A would have lost.
(D) B would have gained the same momentum as A would have lost.

49. For the situation shown in the figure below (assume $r >>$ lenth of dipole) mark out the correct statement(s).



(A) Force acting on the dipole is zero

(B) Force acting on the dipole is approximately $\frac{pQ}{4\pi\varepsilon_0 r^3}$ and is acting upward.

(C) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in clockwise direction

(D) Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in anti-clockwise direction

JEE Advanced

Level I

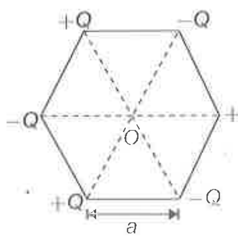
1. The distance between two fixed positive charges $4e$ and e is ℓ. How should a third charge 'q' be arranged for it to be in equilibrium? Under what condition will equilibrium of the charge 'q' be stable (for displacement on the line joining $4e$ and e) or will it be unstable?

2. Two particles A and B, each having a charge Q are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of the maximum force?

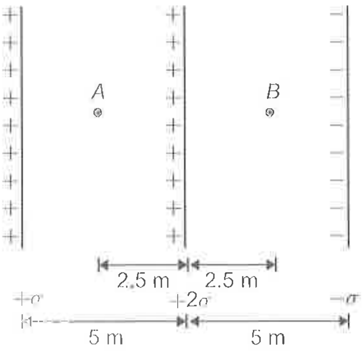
3. A negative point charge $2q$ and a positive charge q are fixed at a distance l apart. Where should a positive test charge Q be placed on the line connecting the charge for it to be in equilibrium? What is the nature of the equilibrium with respect to longitudinal motions?

4. A charge $+10^{-9}$ C is located at the origin in free space and another charge Q at (2, 0, 0). If the X-component of the electric field at (3, 1, 1) is zero, calculate the value of Q. Is the Y-component zero at (3, 1, 1)?

5. Six charges are placed at the vertices of a regular hexagon as shown in the figure. Find the electric field on the line passing through O and perpendicular to plane of the figure as a function of distance x from point O.

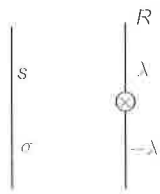


6. The figure shows three infinite non-conducting plates of charge perpendicular to the plane of the paper with charge per unit area $+\sigma$, $+2\sigma$ and $-\sigma$. Find the ratio of the net electric field at that point A to that at point B.



7. A thin circular wire of radius r has a charge Q. If a point charge q is placed at the centre of the ring, then find the increase in tension in the wire.

8. In the figure shown S is a large nonconducting sheet of uniform charge density σ. A rod R of length l and mass 'm' is parallel to the sheet and hinged at its mid point. The linear charge densities on the upper and lower half of the rod are shown in the figure. Find the angular acceleration of the rod just after it is released.

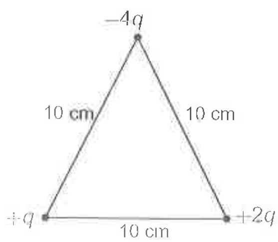


9. A charge of 8 mC is located at the origin. Calculate the work done by external agent in taking a small charge

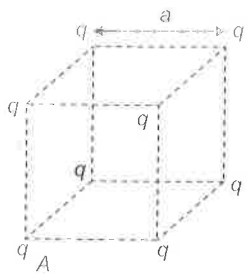
of -2×10^{-9} C from a point A (0, 0, 0.03 m) to a point B(0, 0.04 m, 0) via a point C(0, 0.06 m, 0.09 m).

10. A particle of mass m, charge $q > 0$ and initial kinetic energy K is projected from infinity toward a heavy nucleus of charge Q assumed to have a fixed position.
(a) If the aim is perfect, how close to the centre of the nucleus is the particle when it comes instantaneously to rest ?
(b) With a particular imperfect aim the particle's closest approach to nucleus is twice the distance determined in (a) Determine speed of particle at the closest distance of approach.

11. Three point charges are arranged at the three vertices of a triangle as shown in figure. Given: $q = 10^{-7}$C. Calculate the electrostatic potential energy of the system.

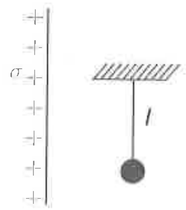


12. Eight equal point charges each of charge 'q' and mass 'm' are placed at eight corners of a cube of side 'a'.



(i) Find out potential energy of charge system
(ii) Find out work done by external agent against electrostatic forces and by electrostatic forces to increase all sides of cube from a to $2a$
(iii) If all the charges are released at rest then find out their speed when they are at the corners of cube of side $2a$.
(iv) If keeping all other charges fix, charge of corner 'A' is released then find out its speed when it is at infinite distance ?
(v) If all charges are released at rest then find out their speed when they are at a very large distance from each other.

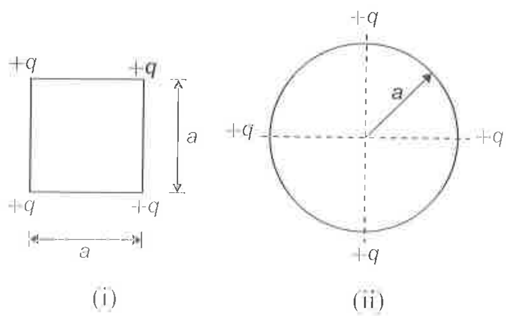
13. A simple pendulum of length l and bob mass m is hanging in front of a large nonconducting sheet having surface charge density σ. If suddenly a charge $+q$ is given to the bob and it is released from the position shown in figure. Find the maximum angle through which the string is deflected from vertical.



14. A charge $+Q$ is uniformly distributed over a thin ring with radius R. A negative point charge $-Q$ and mass m starts from rest at a point far away from the centre of the ring and moves towards the centre. Find the velocity of this particle at the moment it passes through the centre of the ring.

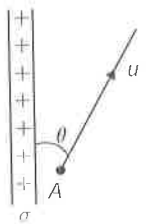
15. A point charge $+q$ and mass 100 gm experiences a force of 100 N at a point a distance 20 cm from a long infinite uniformly charged wire. If it is released find its speed when it is at a distance 40 cm from wire

16. Consider the configuration of a system of four charges each of value $+q$. Find the work done by external agent in changing the configuration of the system from figure (i) to figure (ii).



17. Two identical particles of mass m carry charge Q each. Initially one is at rest on a smooth horizontal plane and the other is projected along the plane directly towards the first from a large distance with an initial speed V. Find the closest distance of approach.

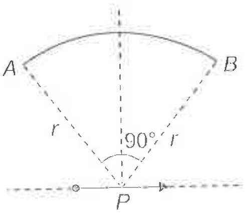
18. A particle of mass m and negative charge q is thrown in a gravity free space with speed u from the point A on the large non conducting charged sheet with surface charge density σ, as shown in figure. Find the maximum distance from A on sheet where the particle can strike.



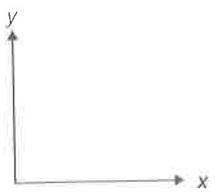
19. Three charges 0.1 coulomb each are placed on the corners of an equilateral triangle of side 1 m. If the energy is supplied to this system at the rate of 1 kW, how much time would be required to move one of the charges onto the midpoint of the line joining the other two?

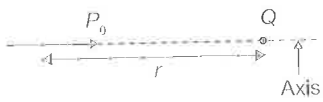
20. Two identical nonconducting spherical shells having equal charge Q are placed at a distance d apart. When they are released find out kinetic energy of each sphere when they are at a large distance.

21. A charge 'q' is carried from a point A (r, 135°) to point B(r, 45°) following a path which is a quadrant of circle of radius 'r'. If the dipole moment is $\vec{P}$, then find out the work done by external agent ?



22. Find out force experienced by short dipole $\vec{P}_0$ is following different arrangements as shown in figures. [Assume point charge is Q, $\vec{P}_0 = q_0(2a)$ and $\vec{P} = q(2a)$]



(i)


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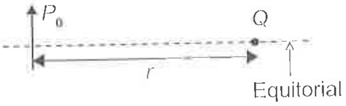

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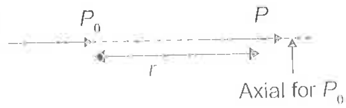

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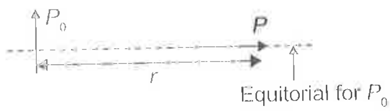
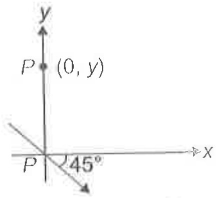
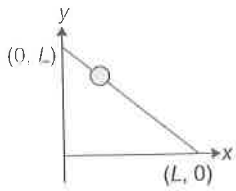


23. Find out the magnitude of electric field intensity at point (2, 0, 0) due to a dipole of dipole moment, $\vec{P} = \hat{i} + \sqrt{3}\hat{j}$ kept at origin? Also find out the potential at that point.

24. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point $P(0, y)$.

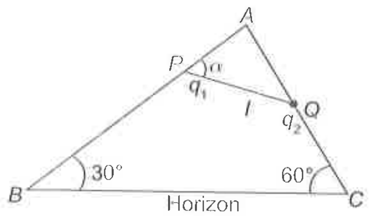


25. Electric field given by the vector $\vec{E} = x\hat{i} + y\hat{j}$ is present in the XY plane. A small ring carrying charge $+Q$, which can freely slide on a smooth non conducting rod, is projected along the rod from the point $(0, L)$ such that it can reach the other end of the rod. What minimum velocity should be given to the ring? (Assume zero gravity)



Level II

1. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane as shown. Two bead of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and slide without friction on the wires. Considering the case when the beads are stationary, determine.
   (a) The angle α.
   (b) The tension in the cord and
   (c) The normal reaction on the beads. If the cord is now cut, what are the values of the charges for which the beads continue to remain stationary.

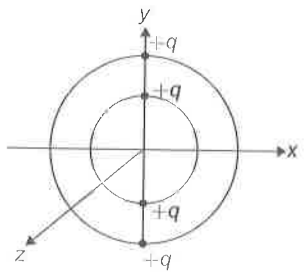


2. A clock face has negative charges $-q$, $-2q$, $-3q$, ..., $-12q$ fixed at the position of the corresponding numerals on the dial. The clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

3. A circular ring of radius R with uniform positive charge density λ per unit length is fixed in the Y-Z plane with its centre at the origin O. A particle of mass m and positive charge q is projected from the point $P(\sqrt{3}\,R, 0, 0)$ on the positive X-axis directly towards O, with initial velocity v. Find the smallest value of the speed v such that the particle does not return of P.

4. 2 small balls having the same mass and charge and located on the same vertical at heights h_1 and h_2 are thrown in the same direction along the horizontal at the same velocity v. The 1st ball touches the ground at a distance l from the initial vertical. At what height will the 2nd ball be at this instant? The air drag and the charges induced should be neglected.

5. Two concentric rings of radii r and $2r$ are placed with centre at origin. Two charges $+q$ each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle 90° about Z-axis then it is again rotated by 90° about Y-axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by 90° about X-axis, find the total work required to perform all three steps.



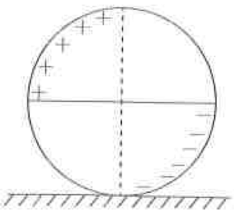
6. Two identical balls of charges q_1 and q_2 initially have equal velocity of the same magnitude and direction. After a uniform electric field is applied for some time, the direction of the velocity of the first ball changes by 60° and the magnitude is reduced by half. The direction of the velocity of the second ball changes there by 90°. In what proportion will the velocity of the second ball changes?

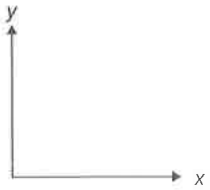
7. Small identical balls with equal charges are fixed at vertices of regular 2004 - gon with side a. At a certain instant, one of the balls is released and a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by K at a sufficiently long distance from the polygon. Determine the charge q of each part.

8. The electric field in a region is given by $\vec{E} = \dfrac{E_0 x}{l}\vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$. Take $E_0 = 5\times10^3$ N/C, $l = 2$ cm and $a = 1$ cm.

9. 2 small metallic balls of radii R_1 and R_2 are kept in vacuum at a large distance compared to the radii. Find the ratio between the charges on the 2 balls at which electrostatic energy of the system is minimum. What is the potential difference between the 2 balls? Total charge of balls is constant.

10. A nonconducting ring of mass m and radius R is charged as shown. The charged density i.e. charge per unit length is λ. It is then placed on a rough nonconducting horizontal surface plane.

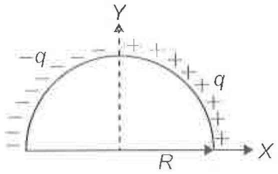




At time $t = 0$, a uniform electric field $\vec{E} = \vec{E}_0$ is switched on and the ring start rolling without sliding. Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.

11. Two spherical bobs of same mass and radius having equal charges are suspended from the same point by strings of same length. The bobs are immersed in a liquid of relative permittivity ε_r and density ρ_0. Find the density σ of the bob for which the angle of divergence of the strings to be the same in the air and in the liquid?

12. Find the electric field at centre of semicircular ring shown in figure.



Previous Year Questions

JEE Main

1. If a charge q is placed at the centre of the line joining two equal charge Q such that the system is in equilibrium then the value of q is: **(AIEEE 2002)**

(A) $\frac{Q}{2}$ (B) $-\frac{Q}{2}$

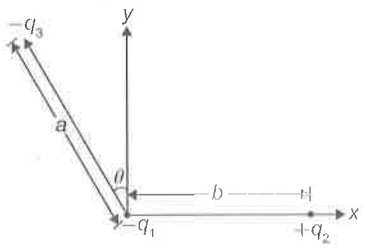
(C) $\frac{Q}{4}$ (D) $-\frac{Q}{4}$

2. On moving a charge of 20 C by 2 cm, 2 J of work is done, then the potential differance between the points is: **(AIEEE 2002)**

(A) 0.1 V (B) 8 V

(C) 2 V (D) 0.5 V

3. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to: **(AIEEE 2003)**



(A) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (B) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(C) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (D) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

4. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be: **(AIEEE 2004)**

(A) r (B) $2r$

(C) $\frac{r}{2}$ (D) $\frac{r}{4}$

5. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is: **(AIEEE 2004)**

(A) $-\frac{Q}{4}(1+2\sqrt{2})$ (B) $\frac{Q}{4}(1+2\sqrt{2})$

(C) $-\frac{Q}{2}(1+2\sqrt{2})$ (D) $\frac{Q}{2}(1+2\sqrt{2})$

6. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero, is: **(AIEEE 2005)**

(A) $2L$ (B) $\frac{L}{4}$

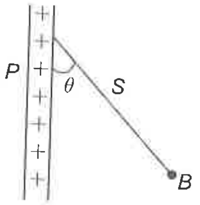
(C) $8L$ (D) $4L$

7. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is: **(AIEEE 2005)**

(A) $\frac{qR}{4\pi\varepsilon_0 d^2}$ (B) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

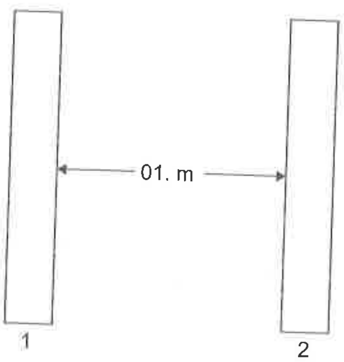
(C) zero (D) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

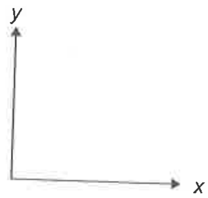
8. A charged ball B hangs from a silk thread S. which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to: **(AIEEE 2005)**



(A) $\cos\theta$ (B) $\cot\theta$
(C) $\sin\theta$ (D) $\tan\theta$

9. Two insulating plates are both uniformly charged in such a way that the potential difference between them $V_2 - V_1 = 20V$ (ie, plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? **(AIEEE 2006)**
($e = 1.6 \times 10^{-19}$ C, $m_0 = 9.11 \times 10^{-31}$ kg)





(A) 2.65×10^6 ms^{-1} (B) 7.02×10^{12} ms^{-1}
(C) 1.87×10^6 ms^{-1} (D) 32×10^{-19} ms^{-1}

10. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **(AIEEE 2006)**

(A) a translational force only in the direction of the field.
(B) a translational force only in a direction normal to the direction of the field.
(C) a torque as well as a translational force
(D) a torque only

11. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by volt. $V(x) = 20/(x^2 - 4)$V. **(AIEEE 2007)**
The electric field E at $x = 4$ μm is given by

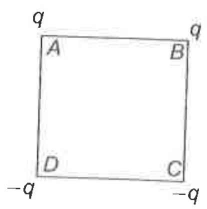
(A) $\frac{5}{3}$ V/μm and in the $-$ve x direction

(B) $\frac{5}{3}$ V/μm and in the +ve x direction

(C) $\frac{10}{9}$ V/μm and in the $-$ve x direction

(D) $\frac{10}{9}$ V/μm and in the +ve x direction

12. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **(AIEEE 2007)**



(A) $\vec{E}$ remains unchanged, V changes
(B) both $\vec{E}$ and V change

(C) $\vec{E}$ and V remain unchanged

(D) $\vec{E}$ change, V remains unchanged

13. An electric charge 10^{-3} is placed at the origin (0, 0) of X-Y coordinate system. Two points A and B are situated at ($\sqrt{2}$, $\sqrt{2}$) and (2, 0) the respectively. The potential difference between the points A and B will be **(AIEEE 2007)**

(A) 9 V (B) zero
(C) 2 V (D) 4.5 V

14. The questions contains Statements I and Statement II. of the four choice given after the statements, choose the one that best describes the two statements.

Statements I For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the paticle is independent of the path connecting point P to point Q.

Statements II The net work done by a conservative force on an object moving along a closed loop is zero **(AIEEE 2009)**

(A) Statement I is true, Statement II is true.
(B) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I.
(C) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I.
(D) Statement I is false, Statement II is true.

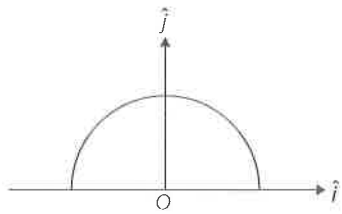
15. Two points P and Q are maintained at the potentials of 10 V and –4V respectively. The work done in moving 100 electrons from P to Q is **(AIEEE 2009)**

(A) -19×10^{-17} J (B) 9.60×10^{-17}J
(C) -2.24×10^{-16} J (D) 2.24×10^{-16}J

16. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the $\frac{Q}{q}$ equals **(AIEEE 2009)**

(A) $-2\sqrt{2}$ (B) -1
(C) 1 (D) $-\frac{1}{\sqrt{2}}$

17. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is **(AIEEE 2010)**



(A) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (B) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(C) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (D) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

18. Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **(AIEEE 2011)**

(A) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}(1-\frac{1}{\sqrt{5}})$

(B) zero

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}(1+\frac{1}{\sqrt{5}})$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}(1-\frac{2}{\sqrt{5}})$

19. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as fuctions of distance x between them, **(AIEEE 2011)**

(A) $v \propto x^{-1}$ (B) $v \propto x^{1/2}$
(C) $v \propto x$ (D) $v \propto x^{-1/2}$

20. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is: **[JEE Main 2014]**

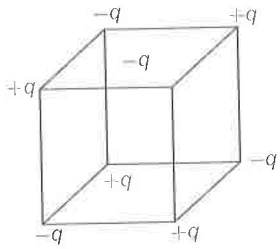
(A) –80 J/m (B) 80 J/m
(C) 120 J/m (D) – 120 J/m

JEE Advanced

1. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to **[JEE (Scr) 2002]**

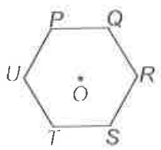
   (A) x (B) x^2
   (C) x^3 (D) $1/x$

2. Charges $+q$ and $-q$ are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance. **[JEE 2003]**



3. A charge $+Q$ is fixed at the origin of the co-ordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.
   (a) calculate the K.E. of the dipole when it reaches to a point $(d, 0)$ **[JEE 2003]**
   (b) calculate the force on the charge $+Q$ at this moment.

4. Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R. Which of the following arrangements of charges is possible for P, Q, R, S, T and U respectively ? **[JEE (Scr) 2004]**



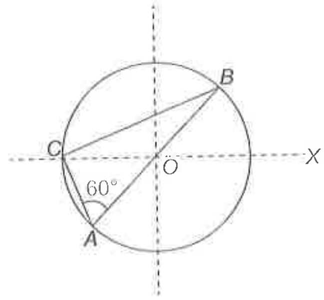
   (A) +, −, +, −, −, +
   (B) +, −, +, −, +, −
   (C) +, +, −, +, −, −
   (D) −, +, +, −, +, −

5. Two uniformly charged infinitely large planar sheet S_1 and S_2 are held in air parallel to each other with separation d between them. The sheets have charge distribution per unit area σ_1 and σ_2 (Cm^{-2}), respectively, with $\sigma_1 > \sigma_2$. Find the work done by the electric field on a point charge Q that moves from S_1 towards S_2 along a line of length a ($a < d$) making an angle $\pi/4$ with the normal to the sheets. Assume that the charge Q does not affect the charge distributions of the sheets. **[JEE 2004]**

6. Which of the following groups do not have same dimensions
   (A) Young's modulus, pressure stress
   (B) work, heat, energy
   (C) electromotive force, potential difference, voltage
   (D) electric dipole, electric flux, electric field

7. Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, -\frac{a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is
   (A) positive
   (B) negative
   (C) zero
   (D) depends on the path connecting the initial and final positions **[JEE 2007]**

8. Consider a system of three charges and placed at points $\frac{q}{3}, \frac{q}{3}$ and $-\frac{2q}{3}$ A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$ **[JEE 2008]**

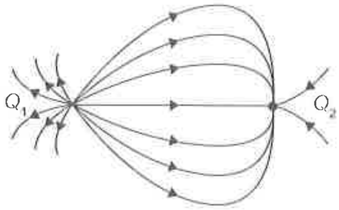


   (A) The electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis
   (B) The potential energy of the system is zero

(C) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$

(D) the potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$

9. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **[JEE 2010]**



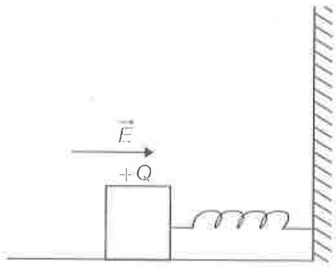
(A) $|Q_1| > |Q_2|$
(B) $|Q_1| < |Q_2|$
(C) at a finite distance to the left of Q_1 the electric field is zero.
(D) at a finite distance to the right of Q_2 the electric field is zero.

10. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s) **[JEE 2010]**
(A) The angular momentum of the charge $-q$ is constant
(B) The linear momentum of the charge $-q$ is constant
(C) The angular velocity of the charge $-q$ is constant
(D) The linear speed of the charge $-q$ is constant

11. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5\,\text{Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1} Given $g = 9.8$ ms^{-2}, viscosity of the air $= 1.8 \times 10^{-5}$ Ns m^{-2} and the density of oil $=$ 900 kg m^{-3}, the magnitude of q is: **[JEE 2010]**
(A) 1.6×10^{-19} C (B) 3.2×10^{-19} C
(C) 4.8×10^{-19} C (D) 8.0×10^{-19} C

12. Four point charges, each of $+q$ are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is 'γ'. the system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$ 'k' is a constant. Then N is **[JEE 2011]**

13. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be **[JEE 2011]**

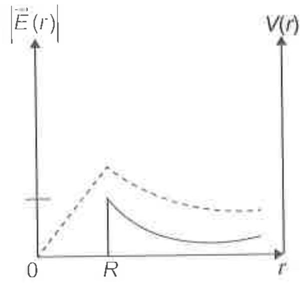


(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of changed frequency and with the same mean position

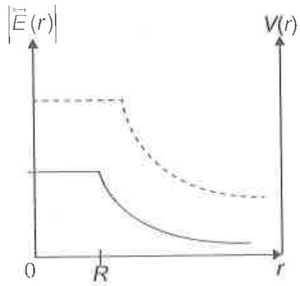
14. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical $JUST$ after release. Then X is nearly **[JEE 2012]**
(A) 1×10^{-5} V (B) 1×10^{-7} V
(C) 1×10^{-9} V (D) 1×10^{-10} V

15. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph? **[JEE 2012]**

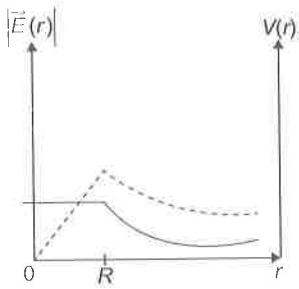
(A)



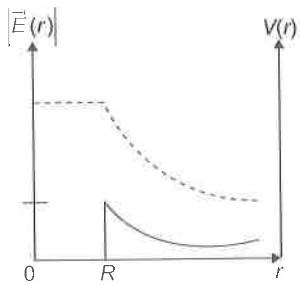
(B)



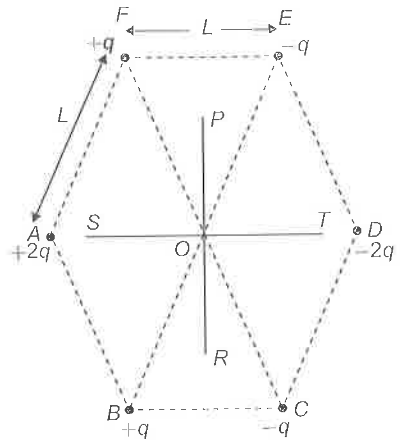
(C)



(D)

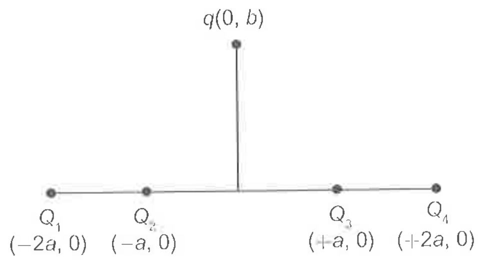


16. Six point charges are kept at the vertices of a regular hexagon of side L, and centtre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is(are) correct? **[JEE 2012]**



(A) The electric field at O is $6K$ along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

17. For charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists **[JEE 2014]**



List I	**List II**
P. Q_1, Q_2, Q_3, Q_4 all positive	1. $+x$
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. $-x$
R. Q_1, Q_4 positive; Q_2, Q_3 negative	3. $+y$
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. $-y$

Code:
(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

ANSWER KEYS

Exercises

JEE Main

1. B	2. A	3. D	4. C	5. D	6. A	7. B	8. C	9. B	10. A
11. B	12. A	13. B	14. D	15. C	16. D	17. B	18. D	19. A	20. D
21. B	22. B	23. D	24. C	25. A	26. A	27. A	28. A	29. C	30. B
31. D	32. D	33. C	34. D	35. B	36. B	37. B	38. B	39. B	40. D
41. D	42. C	43. B	44. B	45. A	46. B	47. D	48. C	49. A	50. A
51. D	52. B	53. C	54. C	55. B	56. D	57. B	58. C	59. C	60. C
61. B	62. A	63. A	64. A	65. C	66. D	67. C	68. C		

JEE Advanced

1. D	2. A	3. B	4. B	5. B	6. B	7. B	8. D	9. B	10. B
11. A	12. C	13. A	14. B	15. B	16. B	17. C	18. A	19. A	20. D
21. B	22. B	23. A	24. C, D	25. B, D	26. B	27. C, D	28. A, D	29. B	30. A, C
31. A	32. A, B, D	33. A, C	34. A, B, C	35. A	36. B, D	37. B	38. A, D	39. B, C	40. A, C
41. A	42. D	43. C	44. B	45. D	46. A, B, D	47. A, D	48. B, C, D	49. B, C	

JEE Advanced

Level I

1. $\frac{2\ell}{3}$ from charge $4e$ (If q is positive stable, If q is negative unstable) 2. $\frac{d}{\sqrt{2}}, \frac{4}{3\sqrt{3}}, \frac{Qq}{n\varepsilon_0 d^2}$

3. $a = l(1 + \sqrt{2})$, the equilibrium will be stable 4. $-\left[\frac{3}{11}\right]^{3/2} 3 \times 10^{-9}$C 5. 0 6. 0 7. $\frac{qQ}{8\pi^2\varepsilon_0 r^2}$ 8. $\frac{3\sigma\lambda}{2m\in_0}$

9. $W = Kqq_0\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = 1.2$ J 10. (a) $\frac{Qq}{4\pi\varepsilon_0 K}$ (b) $\sqrt{\frac{K}{m}}$ 11. -9.0×10^{-3} J

12. (i) $\frac{4Kq^2}{a}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$ (ii) $W_{ext} = -\frac{2Kq^2}{a}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]; W_{el} = \frac{2Kq^2}{a}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$

(iii) $\sqrt{\frac{Kq^2}{2ma}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]}$ (iv) $\sqrt{\frac{2Kq^2}{ma}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]}$ (v) $\sqrt{\frac{Kq^2}{ma}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]}$

13. $2\tan^{-1}\left(\frac{\sigma q_0}{2\varepsilon_0 mg}\right)$ 14. $\sqrt{\frac{2kQ^2}{mR}}$ 15. $20\sqrt{l\,n\,2}$ 16. $-\frac{kq^2}{a}(3 - \sqrt{2})$ 17. $\frac{Q^2}{m\pi\varepsilon_0 V^2}$

18. $\frac{2\varepsilon_0 u^2 m}{q\sigma}$ 19. 1.8×10^5 s 20. K.E. $= \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 d}$ 21. $\frac{\sqrt{2}qp}{4\pi\varepsilon_0 r^2}$

22. (i) $\frac{2KP_0Q}{r^3}(-\hat{i})$ (ii) $\frac{KP_0Q}{r^3}\hat{j}$ (iii) $\frac{6KP_0P}{r^4}\hat{i}$ (iv) $\frac{3KP_0P}{r^4}(+\hat{j})$

23. $|E| = \frac{\sqrt{7}K}{8}$, $V = \frac{K}{4}$ [where $K = 1/4\pi\varepsilon_0$] **24.** $\frac{kP}{\sqrt{2}y^3}(-\hat{i}-2\hat{j})$ **25.** $(QL^2/2\text{ m})^{1/2}$

Level II

1. (a) 60° (b) $mg + \frac{kq_1q_2}{\ell^2}$ (c) $\sqrt{3}$ mg, mg. q_1 and q_2 should have unlike charges for the beads to remain stationaly and $q_2q_2 = -mgl^2/\text{k}$

2. 9.30 **3.** $\sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$ **4.** $H_2 = h_1 + h_2 - g\left(\frac{\ell}{V}\right)^2$ **5.** $W_{\text{first step}} = \left(\frac{8}{3} - \frac{4}{\sqrt{5}}\right)\frac{Kq^2}{r}$, $W_{\text{second step}} = 0$, $W_{\text{total}} = 0$ **6.** $\frac{V}{\sqrt{3}}$

7. $\sqrt{4\pi\varepsilon_0 Ka}$ **8.** 2.2×10^{-12}C **9.** $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$ **10.** $\lambda RE_0\hat{i}$ **11.** $\sigma = \frac{\varepsilon_r\rho_0}{\varepsilon_r - 1}$ **12.** $-\frac{4kq}{\pi R^2}\hat{i}$

Previous Year Questions

JEE Main

1. D	**2.** A	**3.** B	**4.** D	**5.** B	**6.** A	**7.** B	**8.** D	**9.** A	**10.** C
11. D	**12.** D	**13.** B	**14.** B	**15.** D	**16.** A	**17.** C	**18.** A	**19.** D	**20.** A

JEE Advanced

1. B **2.** $-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}[3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$ **3.** (a) K.E. $= \frac{P}{4\pi\varepsilon_0}\frac{Q}{d^2}$, (b) $\frac{QP}{2\pi\varepsilon_0 d^3}$ along positive x-axis

4. D **5.** $\frac{(\sigma_1 - \sigma_2)Qa}{2\sqrt{2}\,\varepsilon_0}$ **6.** D **7.** C **8.** C **9.** A, D **10.** A **11.** D **12.** 0003 **13.** A **14.** C **15.** D

16. A, B, C **17.** A

CHAPTER 4

Electrostatics-2

ELECTRIC FLUX

Any group of electric lines of forces passing through a given surface is called electric flux and it is denoted by ϕ.

Area as a Vector

Till now, we have considered area of a surface as a scalar quantity but for further analysis we treat area of a surface as a vector quantity whose direction is along the normal to the surface. The area vector $\vec{S}$ of a surface which has surface area S can be written as

$$\vec{S} = S\hat{n},$$

where $\hat{n}$ is the unit vector in the direction along the normal to the surface.

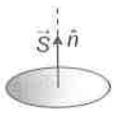


Figure 4.1

If a surface is three dimensional, we consider a small elemental area dS on this surface and direction of this elemental area vector is along the local normal of the surface at the point where elemental area is chosen as shown. Thus,

$$d\vec{S} = dS\,\hat{a}$$

Here, $\hat{a}$ is the unit vector in the direction along the normal at elemental area dS.

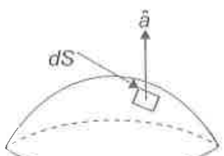


Figure 4.2

Electric Field Strength in Terms of Electric Flux

Earlier we have defined that the density of electric lines gives the magnitude of electric field strength. Mathematically, the numerical value of electric field strength at a point in the region of electric field can be given as the electric flux passing through a unit normal area at that point.

$$\text{Flux} = \phi = \int \vec{E} \cdot \vec{dA}$$

If $\vec{E}$ is constant, $\phi = \vec{E} \cdot \vec{A}$

In a uniform electric field shown in Fig. 4.3, if ϕ be the flux passing through an area S which is normal to the electric field lines, the value of electric field strength at this surface can be given as

$$E = \frac{\phi}{S}$$

or flux through the surface can be given as

$$\phi = ES$$

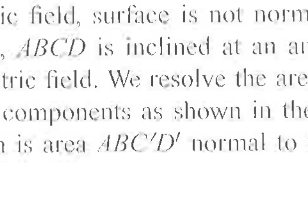


Figure 4.3

If in an electric field, surface is not normal as shown in Fig. 4.4. Here, $ABCD$ is inclined at an angle θ from the normal to electric field. We resolve the area $ABCD$ in two perpendicular components as shown in the figure. One is $S\cos\theta$, which is area $ABC'D'$ normal to the direction of electric field and the other is $S \sin\theta$, which is area $CDC'D'$ along the direction of electric field.

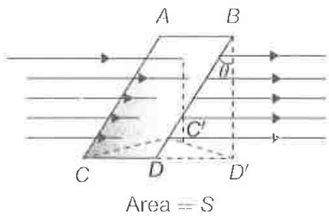


Figure 4.4

Here, the total flux passing through the given area $ABCD$ is same which is passing through its normal component $S\cos\theta$; thus, the flux ϕ through the area can be given as

$$\phi = ES\cos\theta$$

$$[S\cos\theta = \text{area of } ABC'D']$$

If we consider the direction of area vector normal to the area surface, as shown in Fig. 4.5, θ would be the angle between $\vec{S}$ and $\vec{E}$. Thus, flux through the surface $ABCD$ can be given as

$$\phi = \vec{E}\cdot\vec{S}$$

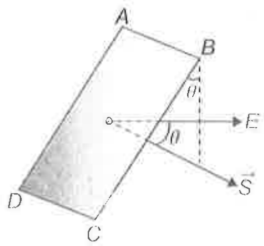


Figure 4.5

(a) Electric Flux in Non-uniform Electric Field

In non-uniform electric field, we can calculate electric flux through a given surface by integrating the above expression for elemental surface area of the given surface.

For this, consider the situation shown in Fig. 4.6, if we wish to calculate electric flux through the surface M.

Consider an elemental area dS on the surface M as shown in the figure. At this position, if the electric field is E, then the electric flux through this elemental area dS can be given as

$$d\phi = EdS\cos\theta$$

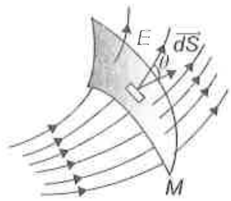


Figure 4.6

Total flux through the surface M can be given as

$$\phi = \int d\phi = \int_M EdS\cos\theta$$

(b) Electric Flux Through a Circular Disc

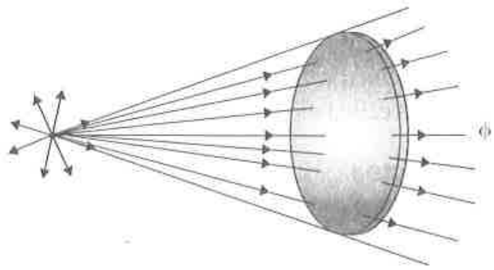


Figure 4.7

Figure 4.8 shows a point charge q placed at a distance ℓ from a disc of radius R. Here, we wish to find the electric flux through the disc surface due to the point charge q. We know that a point charge q originates electric flux in radially outward direction. The flux of q, which is originated in the cone shown in the figure, passes through the disc surface.

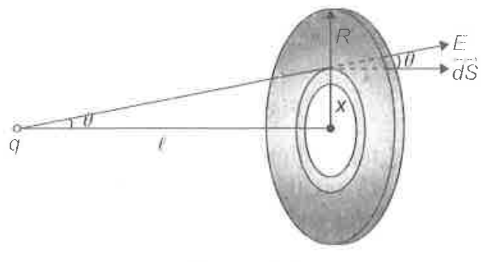


Figure 4.8

To calculate this flux, we consider an elemental ring on the disc surface of radius x and width dx as shown. The area of this ring (strip) is given as

$$dS = 2\pi x\, dx$$

The electric field due to q at this elemental ring is given as

$$E = \frac{Kq}{(x^2 + \ell^2)}$$

If $d\phi$ is the flux passing through this elemental ring, we have

$$d\phi = EdS\cos\theta$$

$$= \frac{Kq}{(x^2+\ell^2)} \times 2\pi x\,dx \times \frac{\ell}{\sqrt{x^2+\ell^2}}$$

$$= \frac{2\pi\,Kq\ell x dx}{(\ell^2+x^2)^{3/2}}$$

Total flux through the disc surface can be given by integrating this expression over the whole area of disc; thus, total flux can be given as

$$\phi = \int d\phi = \int_0^R \frac{q\ell}{2\varepsilon_0} \frac{x\,dx}{(\ell)}$$

$$= \frac{q\ell}{2\varepsilon_0} \int_0^R \frac{x\,dx}{(\ell^2+x^2)^{3/2}}$$

$$= \frac{q\ell}{2\varepsilon_0}\left[-\frac{1}{\sqrt{\ell^2+x^2}}\right]_0^R$$

$$= \frac{q\ell}{2\varepsilon_0}\left[\frac{1}{\ell} - \frac{1}{\sqrt{\ell^2+R^2}}\right]$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law, which will be discussed later.

(c) Electric Flux Through the Lateral Surface of a Cylinder due to a Point Charge

Figure 4.9 shows a cylindrical surface of length L and radius R. On its axis at its centre, a point charge q is placed. Here, we wish to find the flux coming out from the lateral surface of this cylinder due to the point charge q.

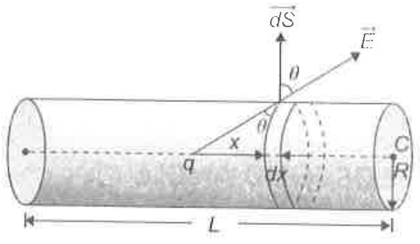


Figure 4.9

For this, we consider an elemental strip of width dx on the surface of cylinder as shown. The area of this strip is given as

$$dS = 2\pi R \cdot dx$$

The electric field due to the point charge on the strip can be given as

$$E = \frac{Kq}{(x^2+R^2)}$$

If $d\phi$ is the electric flux through the strip, we can write as

$$d\phi = E\,dS\cos\theta$$

$$= \frac{Kq}{(x^2+R^2)} \times 2\pi\,Rdx \times \frac{R}{\sqrt{x^2+R^2}}$$

$$= 2\pi\,KqR^2 \times \frac{dx}{(x^2+R^2)^{3/2}}$$

Total flux through the lateral surface of the cylinder can be given by integrating the above result for the complete lateral surface, which can be given as

$$\phi = \int d\phi = \frac{qR^2}{2\varepsilon_0}\int_{-L/2}^{+L/2} \frac{dx}{(x^2+R^2)^{3/2}}$$

$$\phi = \frac{q}{\varepsilon_0} \cdot \frac{\ell}{\sqrt{\ell^2+4R^2}}$$

The solution of the above integration is left for students as exercise. This situation can also be easily handled by using the concepts of Gauss's law, which will be discussed in the next section.

(d) Electric Flux Produced by a Point Charge

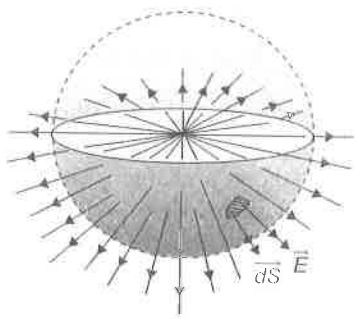


Figure 4.10

Figure 4.10 shows a point charge placed at the centre of a spherical surface of radius R from which electric lines are originated and coming out of the surface of the sphere. For clarity and convenience, only lower half of sphere is drawn. As the charge q is inside the sphere, whatever flux it originates will come out from the spherical surface. To find the total flux, we consider an elemental area dS on the surface. The electric field on the points on surface of the sphere can be given as

$$E = \frac{Kq}{R^2}$$

The electric flux coming out from the surface dS is

$$d\phi = \vec{E} \cdot \vec{dS} = EdS$$

[As $\theta = 0$ shown in the figure]

Thus, $$d\phi = \frac{Kq}{R^2} dS$$

Total flux coming out from the spherical surface is

$$\phi = \int d\phi = \int \frac{Kq}{R^2} dS$$

At every point of the spherical surface, the magnitude of electric field remains same. Hence, we have

$$\phi = \frac{Kq}{R^2} \int dS$$

or $$\phi = \frac{Kq}{R^2} \times 4\pi R^2 \quad [\text{as} \int dS = 4\pi R^2]$$

$$\phi = \frac{q}{\varepsilon_0}$$

Thus, total flux, the charge q originates is $\frac{q}{\varepsilon_0}$. Similarly, a charge $-q$ absorbs $\frac{q}{\varepsilon_0}$ electric lines (flux) into it.

Figure 4.11 shows a charge q enclosed in a closed surface S of random shape. Here, we can say that the total electric flux emerging out from the surface S is the complete flux which charge q is originating. Hence, flux emerging from the surface is

$$\phi_S = \frac{q}{\varepsilon_0}$$

The above result is independent of the shape of surface and it only depends on the amount of charge enclosed by the surface.

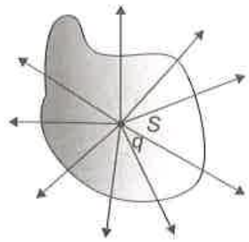


Figure 4.11

(e) Flux Calculation in the Region of Varying Electric Field

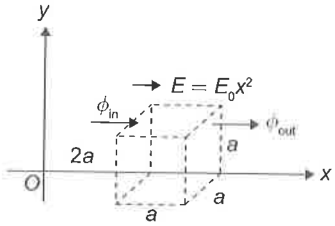


Figure 4.12

In a region, electric field depends on the x direction as

$$E = E_0 x^2$$

In the cube of edge as shown in Fig. 4.12 from the front face, electric flux goes in, which can be given as

$$\phi_{in} = E_0 (2a)^2 \cdot a^2 = 4E_0 a^4$$

From the other surface, flux coming out can be given as

$$\phi_{out} = E_0 (3a)^2 \cdot a^2 = 9E_0 a^4$$

Here, $\phi_{out} > \phi_{in}$ for the cubical surface.

Hence, net flux $= \phi_{out} - \phi_{in}$

$= 5E_0 a^4$.

CONCEPT OF SOLID ANGLE

Solid angle is the three-dimensional angle enclosed by the lateral surface of a cone at its vertex as shown in Fig. 4.13. It can also be defined as the three-dimensional angle subtended by a spherical section at its centre of curvature. As shown in the figure, point A is the centre of curvature of a spherical section S of radius R, which subtends a solid angle ω (omega) at point A.

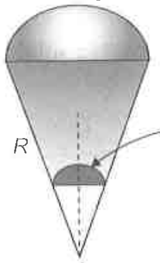


Figure 4.13

(a) Relation in Half Angle of Cone and Solid Angle at Vertex

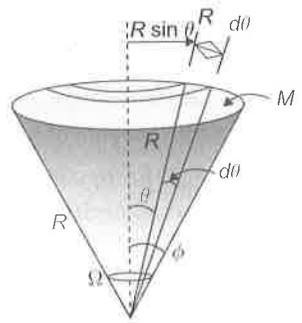


Figure 4.14

Consider a spherical section M of radius R, which subtends a half angle ϕ (radian) at the centre of curvature. To find the area of this section, we consider an elemental strip on this section of radius $R \sin\theta$ and angular width $d\theta$ as shown in Fig. 4.14. The surface area of this strip can be given as

$$dS = 2\pi R \sin\theta \times R d\theta$$

The total area of the spherical section can be given by integrating the area of this elemental strip within limits from O to ϕ.

Total area of the spherical section is given as

$$S = \int dS = \int_0^{\phi} 2\pi R^2 \sin\theta \, d\theta$$

$$= 2\pi R^2 \left[-\cos\theta\right]_0^{\phi}$$

$$= 2\pi R^2 (1 - \cos\phi) \quad (1)$$

If solid angle subtended by this section at its centre O is Ω, then its area can be given as

$$S = \Omega R^2$$

From Eq. (1), we have

$$\Omega R^2 = 2\pi R^2 (1 - \cos\phi)$$

$$\Omega = 2\pi (1 - \cos\phi) \quad (2)$$

Equation (2) gives the relation in half angle of a cone ϕ and the solid angle enclosed by the lateral surface of the cone at its vertex.

(b) Electric Flux Calculation due to a Point Charge Using Solid Angle

Figure 4.15 shows a point charge q placed at a distance λ from the centre of a circular disc of radius R. Now we wish to find the electric flux passing through the disc surface due to the charge q.

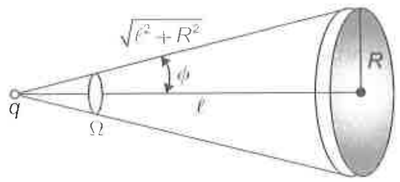


Figure 4.15

We know from a point charge q, total flux originated is $\frac{q}{\varepsilon_0}$ in all directions or we can say that from a point charge q, flux originated is 4π solid angle.

Here, the solid angle enclosed by the cone subtended by the disc at the point charge can be given as

$$\Omega = 2\pi(1 - \cos\phi)$$

$$= 2\pi\left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right)$$

Now we can easily calculate the flux of q which is passing through the disc surface as

$$\phi_{\text{disc}} = \frac{q/\varepsilon_0}{4\pi} \times \Omega$$

$$= \frac{q/\varepsilon_0}{4\pi} \times 2\pi\left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right)$$

or

$$\phi_{\text{disc}} = \frac{q}{2\varepsilon_0}\left(1 - \frac{\ell}{\sqrt{\ell^2 + R^2}}\right)$$

SOLVED EXAMPLES

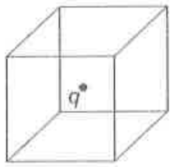
EXAMPLE 1

Find the electric flux coming out from one face of a cube of edge a, centre of which a point charge q is placed.

SOLUTION

Here, the total solid angle subtended by the cube surface at the point charge q is 4π. As q is at the centre of cube, we can say that each face of cube subtends equal solid angle at the centre. Thus, solid angle subtended by each force at point charge is

$$\Omega_{\text{face}} = \frac{4\pi}{6} \text{ steradian}$$



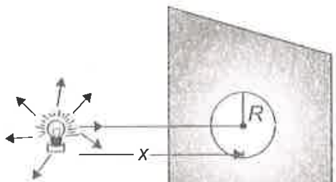
Thus, electric flux through each face is

$$\phi_{\text{face}} = \frac{q/\varepsilon_0}{4\pi} \times \Omega_{\text{face}}$$

$$= \frac{q/\varepsilon_0}{4\pi} \times \frac{4\pi}{6} = \frac{q}{6\varepsilon_0}$$

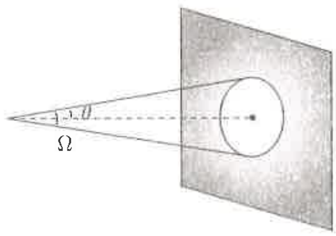
EXAMPLE 2

A point light source of 100 W is placed at a distance x from the centre of a hole of radius R in a sheet as shown in the figure. Find the power passing through the hole in the sheet.



SOLUTION

From the figure, the solid angle of the cone can be given as



$$\Omega = 2\pi(1 - \cos\theta)$$

$$= 2\pi\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

Power in hole = power given in solid angle Ω

$$P = \frac{100}{4\pi} \times \Omega$$

$$= \frac{100}{4\pi} \times 2\pi\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$= 50\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) \text{watt}$$

GAUSS'S LAW

This law is the mathematical analysis of the relation between the electric flux from a closed surface and its enclosed charge.

It states that the total flux emerging out from a closed surface is equal to the product of sum of enclosed charge by the surface and the constant $\frac{1}{\varepsilon_0}$.

Mathematically, Gauss's law is written as

$$\oint_M \vec{E} \cdot \overrightarrow{dS} = \frac{\sum q_{\text{encl}}}{\varepsilon_0}$$

Here, the sign $\oint$ represents the integration over a closed surface M which encloses a total charge $\sum q_{\text{encl}}$.

Let us consider a surface M shown in Fig. 4.16 which encloses three charges $q_1 - q_2$ and q_5. For the surface M if we find surface integral of electric field $\oint_M \vec{E} \cdot \overrightarrow{dS}$, it gives the total electric flux coming out from the surface, which can be given as

$$\oint_M \vec{E} \cdot \overrightarrow{dS} = \frac{q_1 + q_5 - q_3}{\varepsilon_0} \quad \text{[Gauss's law]}$$

Here, electric field $\vec{E}$ is the net electric field at the points on the surface of M. Remember that the electric field we use to find the flux must be the net electric field of the system due to all the charges but the total flux coming out from the surface is the flux originated by the charges enclosed in the closed surface.

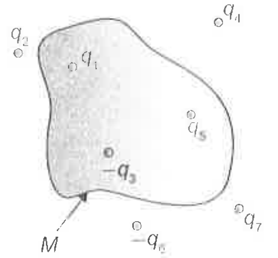


Figure 4.16

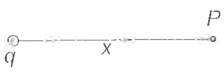
Using Gauss law, we can find electric field strength due to some symmetrical distribution of charges. For application of Gauss's law, we choose a closed surface over which we apply Gauss law, called Gaussian surface.

Gauss law can be used to calculate electric field strength, for this we first choose a proper Gaussian surface on which the electric field strength is to be calculated. Sometimes a random Gaussion surface is chosen, then the integral $\oint \vec{E} \cdot \overrightarrow{dS}$ involves complex calculations. To make these calculations easier, we choose a Gaussian surface keeping following points in mind.

1. The Gaussion surface should be chosen in such a way that at every point of surface the magnitude of electric field is either uniform or zero.
2. The surface should be chosen in such a way that at every point of surface electric field strength is either parallel or perpendicular to the surface.

The following example will illustrate the applications of Gauss's law in calculation of electric field in the surrounding of some charge configurations.

Gauss law is a very helpful tool in finding the electric field strength due to various distributions of charges. We start with a very simple example. Now we try to find the electric field strength due to a point charge q at a distance x, using Gauss's law.



To find electric field strength at P, we first consider a Gaussian surface so that point P will be on its surface. But the question is what should be the shape of Gaussion surface. Look at the following figure (Fig. 4.17).

If we apply Gauss's law to the above two cases, it will require laborious calculations to find $\oint \vec{E} \cdot \overrightarrow{dS}$. The Gaussion surface should be chosen in such a way to minimize the calculations. Now consider a spherical surface shown in Fig. 4.17. At every point of this surface, electric field due to the charge q is

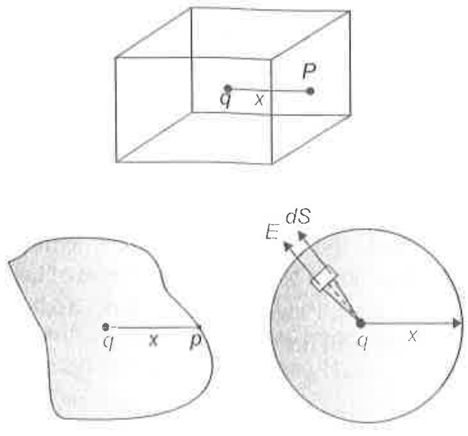


Figure 4.17

$$E = \frac{Kq}{x^2}$$

Here, if we use Gauss's law for the spherical surface, we have

$$\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E\int dS = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi x^2 = \frac{q}{\varepsilon_0}$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{Kq}{x^2}$$

Here, we can see that at every point of sphere electric field vector is parallel to $\overrightarrow{dS}$ and also the magnitude of $\vec{E}$ is uniform at every point. Thus, the integral $\oint \vec{E} \cdot \overrightarrow{dS}$ can be easily evaluated.

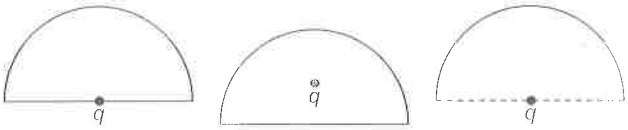
Notes

1. Basically flux is the count of number of lines of electric field crossing an area.
2. For open surface, we choose one direction as an area vector and stick to it for the whole problem.

SOLVED EXAMPLES

EXAMPLE 3

In figure (a), a charge q is placed just outside the centre of a closed hemisphere. In figure (b), the same charge q is placed just inside the centre of the closed hemisphere and in figure (c) the charge is placed at the centre of hemisphere open from the base. Find the electric flux passing through the hemisphere in all the three cases.



(a) (b) (c)

SOLUTION

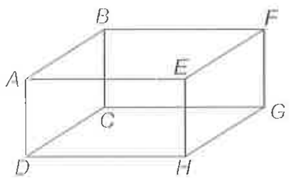
In figure (a), $\phi = 0$

In figure (b), $\phi = q/\varepsilon_0$

In figure (c) $\phi = q/2\varepsilon_0$ ■

EXAMPLE 4

A charge q is placed at point D of the cube. Find the electric flux passing through the faces $EFGH$ and $AEHD$.



SOLUTION

$$\phi = q/6\varepsilon_0$$ ■

(a) Electric Field Due to a Point Charge

The electric field due to a point charge is everywhere radial. We wish to find the electric field at a distance r from the charge q. We select Gaussain surface, a sphere at distance r from the charge. At every point of this sphere, the electric field has the same magnitude E and it is perpendicular to the surface itself. Hence, we can apply the simplified form of Gauss law,

$$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, S = area of sphere = $4\pi r^2$ and q_{in} = net charge enclosing the Gaussian surface = q

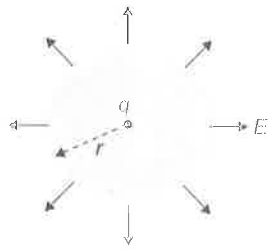


Figure 4.18

$$\therefore \quad E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

(b) Electric Field Strength Due to a Long Charged Wire

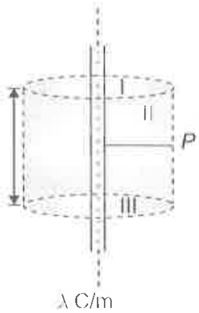


Figure 4.19

If we wish to find electric field strength due to a long charged wire having a linear charge density λ C/m at a point P situated at a distance x from the wire.

For this application of Gauss's law, we consider a cylindrical Gaussian surface of length λ and radius x as shown Fig. 4.19. If we apply Gauss's law on this surface, we have

$$\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q_{encl}}{\varepsilon_0} \quad (1)$$

The closed Gaussian surface is made of three parts, I, II and III, two flat circular faces and one cylindrical lateral surface. We split the closed surface integration in three parts as

$$\oint \vec{E} \cdot \overrightarrow{dS} = \int_{I} \vec{E} \cdot \overrightarrow{dS} + \int_{II} \vec{E} \cdot \overrightarrow{dS} + \int_{III} \vec{E} \cdot \overrightarrow{dS}$$

Here we know for parts I and III, electric field strength vector is perpendicular to the area vector as shown in

Fig. 4.19. Hence, no flux will come out of these parts. Thus, we have

$$\int_{I} \vec{E} \cdot \overrightarrow{dS} = \int_{III} \vec{E} \cdot \overrightarrow{dS} = 0$$

Now from Eq. (1), we have

$$E \int_{II} dS = \frac{\lambda \ell}{\varepsilon_0}$$

[As enclosed charged is $q_{encl} = \lambda\lambda$]

For lateral surface as $\vec{E}$ is parallel to $\vec{dS}$ is parallel

$$E \int_{II} dS = \frac{\lambda \ell}{\varepsilon_0}$$

or $$E \cdot 2\pi \times \ell = \frac{\lambda \ell}{\varepsilon_0}$$

or $$E = \frac{\lambda}{2\pi\varepsilon_0 x} = \frac{2K\lambda}{x}$$

(c) Electric Field Strength Due to a Long Uniformly Charged Cylinder

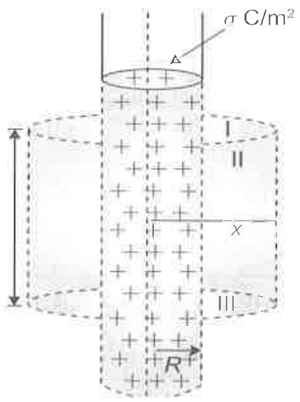


Figure 4.20

Case I: Conducting Cylinder

Figure 4.20 shows a long cylinder of radius R which is uniformly charged on its surface with surface charge density σ C/m^2.

We know that at interior points of a metal body electric field strength is zero. For finding electric field strength at outer points at a distance x from the axis of the cylinder, we consider a cylindrical Gaussian surface of radius x and length λ as shown in the figure. Now we apply Gauss's law on this surface. We have

$$\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q_{encl}}{\varepsilon_0}$$

Here enclosed charge in the cylindrical Gaussian surface can be given as

$$q_{encl} = \sigma \cdot 2\pi R\lambda$$

Here also similar to the previous case, the electric flux through the circular faces is zero. Hence, according to Gauss's law, we have

$$\int_{II} \vec{E} \cdot \overrightarrow{dS} = \frac{\sigma \cdot 2\pi R\ell}{\varepsilon_0}$$

or $$E \int_{II} dS = \frac{\sigma \cdot 2\pi R\ell}{\varepsilon_0}$$

or $$E = \frac{\sigma R}{\varepsilon_0 x}$$

Case II: Uniformly Charged Non-conducting Cylinder

Figure 4.21 shows a long cylinder of radius R, charged uniformly with volume charge density ρ C/m^3. To find electric field strength at a distance x from the cylinder axis, we again consider a cylindrical Gaussian surface shown in the figure.

If we apply Gauss law on this surface, we have

$$\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q_{encl}}{\varepsilon_0}$$

or $$\int_{II} \vec{E} \cdot \overrightarrow{dS} = \frac{\rho \cdot \pi R^2 \ell}{\varepsilon_0}$$

[As $q_{encl} = \rho\pi R^2\lambda$]

or $$E \int dS = \frac{\rho\pi R^2 \ell}{\varepsilon_0}$$

or $$E \cdot 2\pi x\ell = \frac{\rho\pi R^2 \ell}{\varepsilon_0}$$

To find electric field inside the cylinder at a distance x from the axis, we consider a small cylindrical Gaussian surface of radius x and length λ. If we apply Gauss law for this surface, we have

$$\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q_{encl}}{\varepsilon_0}$$

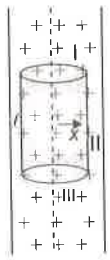


Figure 4.21

or $$E\int_{II} dS = \frac{\rho\pi x^2 \ell}{\varepsilon_0}$$

or $$E \cdot 2\pi x\ell = \frac{\rho\pi x^2 \ell}{\varepsilon_0}$$

or $$E = \frac{\rho R^2}{2\varepsilon_0 x}$$

or $$E = \frac{\rho x}{2\varepsilon_0}$$

(d) Electric Field Strength due to a Non-conducting Uniformly Charged Sheet

To find the electric field strength at a point P in front of the charged sheet, we consider a cylindrical Gaussian surface as shown in Fig. 4.22 of face area S. If we apply Gauss law for this surface, we have

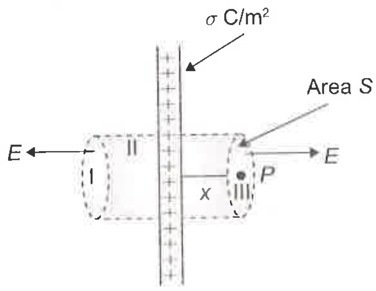


Figure 4.22

$$\oint \vec{E}\cdot\overrightarrow{dS} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

or $$\int_{I}\vec{E}\cdot\overrightarrow{dS} + \int_{II}\vec{E}\cdot\overrightarrow{dS} + \int_{III}\vec{E}\cdot\overrightarrow{dS} = \frac{\sigma S}{\varepsilon_0}$$

[As $q_{\text{encl}} = \sigma S$]

In this case, $\int_{II}\vec{E}\cdot\overrightarrow{dS} = 0$ as the lateral surface of cylinder is parallel to the direction of electric field strength; no flux is coming out from the lateral surface. Hence, we have

$$\int_{I} EdS + \int_{II} EdS = \frac{\sigma S}{2\varepsilon_0}$$

or $$2ES = \frac{\sigma S}{\varepsilon_0}$$

[As electric field is uniform on both sides]

or $$E = \frac{\sigma}{2\varepsilon_0}$$

(e) Electric Field Stength Due to a Charged Conducting Sheet

Figure 4.23 shows a large charged conducting sheet, charged on both the surfaces with surface charge density σ C/m^2. As we know that in the metal sheet there is no charge within the volume of the sheet and also the electric field inside the metal sheet is zero. To find electric field strength at a point P in front of the sheet, we consider a cylindrical Gaussian surface having one face at point P where electric field is required and the other face is within the volume of sheet. If we apply Gauss's law on this surface, we have

$$\oint \vec{E}\cdot\overrightarrow{dS} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

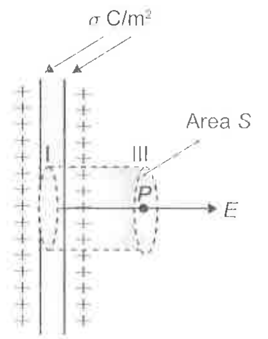
or $$\int_{I}\vec{E}\cdot\overrightarrow{dS} + \int_{II}\vec{E}\cdot\overrightarrow{dS} + \int_{III}\vec{E}\cdot\overrightarrow{dS} = \frac{\sigma S}{\varepsilon_0}$$

[As $q_{\text{encl}} = \sigma S$]

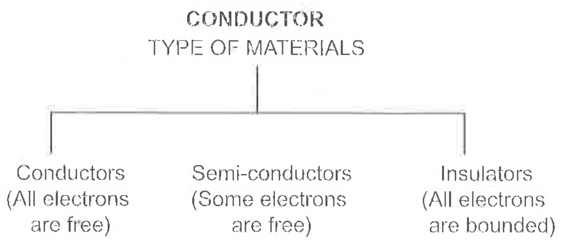
Here on surface I of the Gaussian surface $E = 0$. Hence, $\int_{I}\vec{E}\cdot\overrightarrow{dS} = 0$ and $\int_{II}\vec{E}\cdot\overrightarrow{dS} = 0$ as no electric flux is coming out from the lateral surface of cylinder ($\vec{E}$ is perpendicular to area vector of curved surface). Hence, we have total flux coming out is

$$\int_{II}\vec{E}\cdot\overrightarrow{dS} = \frac{\sigma S}{\varepsilon_0}$$

or $$ES = \frac{\sigma S}{\varepsilon_0}$$



or $E = \dfrac{\sigma}{\varepsilon_0}$



CONDUCTORS

A conductor contains free electrons, which can move freely in the material, but cannot leave it.

By applying an external electric field on a conductor, charges of a conductor adjust themselves in such a fashion that the net electric field inside the conductor is zero under electrostatics conditions.

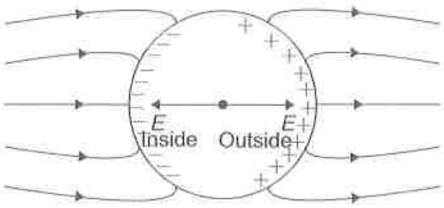


Figure 4.24

Net $\vec{E} = 0 \Rightarrow$ Potential is constant

- **Conductor behaves as an equipotential surface**

Being an equipotential surface, electric field lines will terminate or originate perpendicularly.

Let us now consider the interior of a charged conducting object. Since it is a conductor, the electric field in the interior is everywhere zero. Let us analyze a Gaussian surface inside the conductor as close as possible to the surface of the conductor. Since the electric intensity is zero everywhere inside the conductor, it must be zero for every point of the Gaussian surface. Hence, the flux through the surface $\oint \vec{E} \cdot \overrightarrow{dS}$ will be zero. Therefore, according to Gauss's law $\left(\oint \vec{E} \cdot \overrightarrow{dS} = \dfrac{q}{\varepsilon_0}\right)$, the net charge inside the Gaussian surface and hence inside the conductor must be zero. Since there can be no charge in the interior of the conductor charge given to the conductor will reside on the surface of the conductor.

- **All the charges given to the conductor reside on the surface of the conductor**

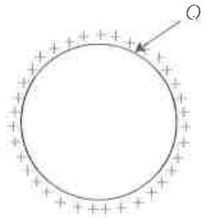


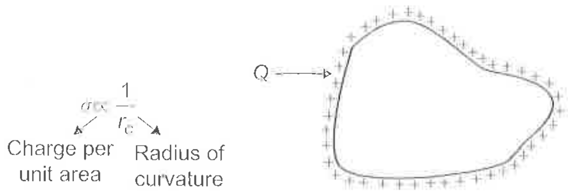
Figure 4.25

Till now we have only discussed the case of uniform-shaped bodies on which the charge distributes itself uniformly.

But what about the charge distribution on irregular-shaped bodies?

Does in this case also uniform charge distribution take place? **NO**

In this case



- Let us consider a random shaped body and find Electric field due to a small portion of this body. However the σ is not uniform everywhere but for a small area dA, we can assume that σ is constant. Considering a cylindrical gaussian surface, we will calculate flux passing through the cross-section dA.

$$\phi_{net} = \oint \vec{E} \cdot \overrightarrow{dS} = \frac{q_{in}}{\varepsilon_0}$$

$$\phi_{net} = \phi_{curved\ surface} + \phi_{outer\ flat\ surface} + \phi_{inner\ flat\ surface}$$

$$\phi_{curved\ surface} = 0$$

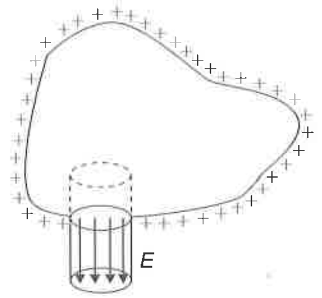


Figure 4.26

Because no flux is passing through lateral surface (electric field lines are perpendicular to area vector).

$$\vec{E} \cdot \overrightarrow{dS} = 0$$

$$\phi_{\text{inner flat surface}} = 0$$

because $\vec{E}$ inside conductor $= 0$

$$\therefore \quad \frac{q_{\text{in}}}{\varepsilon_0} = \phi_{\text{outer flat surface}}$$

$$\frac{\sigma dA}{\varepsilon_0} = \vec{E} \cdot d\vec{A}$$

$$\Rightarrow \quad \vec{E} = \frac{\sigma}{\varepsilon_0}$$

ELECTRIC PRESSURE

Electric Pressure on a Charged Metal Surface

We know when some charge is given to a metal body it will spread on the outer surface of the body due to mutual repulsion in the charge. When on surface every charge experiences an outward repulsive force due to remaining charges, every part of body experiences an outward pressure. This pressure which acts on every part of charged metal body surface due to remaining charges on the body is called electric pressure.

To calculate this, we consider a small segment AB on body surface of area dS as shown in Fig. 4.27. If σ be the surface charge density on AB, charge on it is

$$dq = \sigma dS$$

Now we consider two points M and N just outside and inside of section AB as shown in Fig. 4.27. At the two points if E_1 be the electric field due to section AB then the direction of the electric fields at M and N can be given as

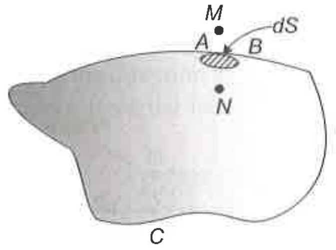


Figure 4.27

shown in the figure. If we remove section AB from the body, then due to removing body ACB, if E_2 be the electric field strength at points M and N, the direction of E_2 can be given as shown in Fig. 4.28.

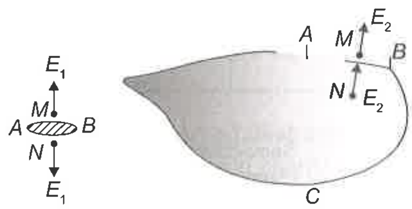


Figure 4.28

Due to complete body we know that net electric fields at just outside and inside points can be given as

$$E_M = E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \qquad (1)$$

and $$E_N = E_1 - E_2 = 0 \qquad (2)$$

Solving Eqs. (1) and (2), we get

$$E_1 = E_2$$

and $$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

Thus, electric field at the location of the section AB due to the remaining body ACB is $\frac{\sigma}{2\varepsilon_0}$, using which we can find the outward force on the section AB, due to the rest of the body ACB as

Force on AB is $dF = dq\, E_2$

$$= \sigma\, dS \times \frac{\sigma}{2\varepsilon_0}$$

Thus, pressure experienced by the section AB can be given as

$$P_e = \frac{df}{ds} = \frac{\sigma^2}{2\varepsilon_0}$$

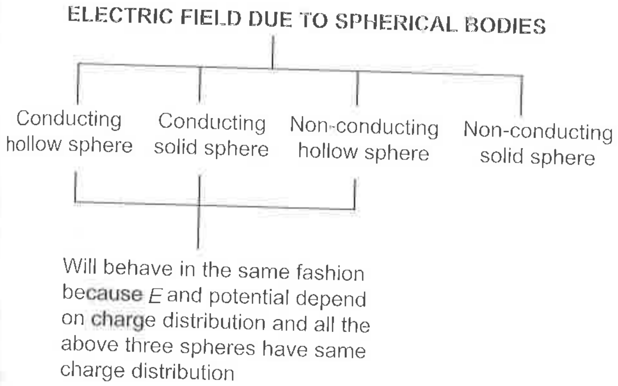
As net electric field outside the surface is

$$E_{net} = \frac{\sigma}{\varepsilon_0}$$

Thus, we have

$$P_e = \frac{(E_{net})^2}{2\varepsilon_0}$$

$$P_e = \frac{1}{2}\varepsilon_0 E_{net}^2$$



$\vec{E}$ Due to Conducting Hollow Sphere, Conducting Solid Sphere and Non-conducting Hollow Sphere

For the above-mentioned bodies, any excess charge given to the body gets distributed uniformly over its outer surface. Since the charge lines must point radially outwards and also the field strength will have the same value at all points on any imaginary spherical surface concentric with the charged conducting sphere or the shell this is the symmetry which leads us to choose the Gaussian surface to be a sphere. Any arbitrary element of area $d\vec{S}$ is parallel to the local $\vec{E}$ so $\vec{E} \cdot d\vec{S} = EdS$ at all points on the surface.

lectric Field Strength Due to a Conducting olid and Hollow) Sphere, Non-conducting ollow Sphere

ase I: $x > R$

find electric field at an outer point at a distance x from centre of sphere, we cosider a spherical Gaussian surface of radius x. If electric field strength at every point of this surface is E, using Gauss's law we have

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{encl}}{\varepsilon_0}$$

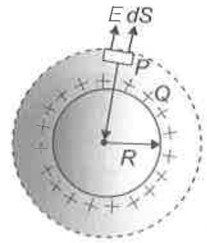


Figure 4.29

Here, we have

$$E\oint dS = \frac{Q}{\varepsilon_0}$$

or

$$E \cdot 4\pi x^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2}$$

Simlarly for surface points we can consider a spherical Gaussian surface of radius R which gives electric field strength on the sphere surface as

$$E_s = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

To find electric field strength at an interior point of the sphere, we consider an inner spherical Gaussian surface of radius x $(x < R)$.

Here if we apply Gauss law for this surface, we have

$$\oint \vec{E}.\overrightarrow{dS} = \frac{q_{encl}}{\epsilon_0} = 0$$

[As all charge is on surface]

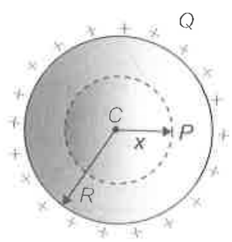


Figure 4.30

Thus, $E = 0 \quad [\text{As } dS \neq 0]$

Note

For points outside the sphere, the field is same as that of a point charge at the centre of sphere.

Case II: $x < R$

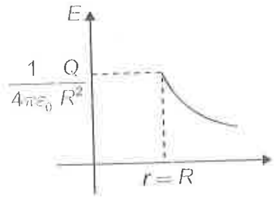


Figure 4.31

Non-conducting Uniformly Charged Sphere

For outer and surface points, the electric field strength can be calculated using Gauss's law similar to the previous case of conducting sphere.

For interior points of sphere, we consider a spherical Gaussian surface of radius x as shown in Fig. 4.32. If we apply Gauss's law for this surface, we have

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{encl}}{\varepsilon_0}$$

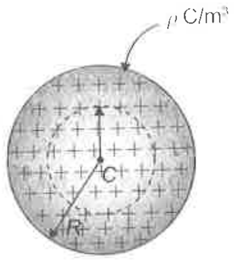


Figure 4.32

Here, enclosed charge can be given as

$$q_{encl} = \rho \times \frac{4}{3}\pi x^3$$

Thus,
$$E \cdot 4\pi x^2 = \frac{\rho \times \frac{4}{3}\pi x^3}{\varepsilon_0}$$

or
$$E = \frac{\rho x}{3\varepsilon_0}$$

At an external point ($r > R$)

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius r ($r > R$). This surface encloses the entire charged sphere. So, from Gauss's law, we have

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

or
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

The field at points outside the sphere is the same as that of a point charge at the centre.

Variation of E with the distance from the centre (r)

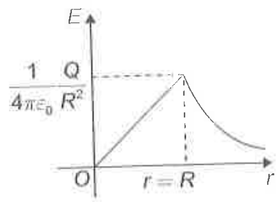


Figure 4.33

Electric Potential Inside a Metal Body

As we have already discussed whenever charge is given to a metal body, it is distributed on its outer surface in such a way that net electric field at every interior point of body is zero. Thus if inside a metal body, a charge is displaced, no work is done in the process as electric field at every point is zero. Hence, we can say that the whole metal body is equipotential.

Note

On the basis of the above explanation, we can state that a region, in which at every point electric field is zero, can be regarded as equipotential region.

Electric Potential Due to a Charged Sphere

Case I: Conducting Sphere

As we know for outer points of a charged sphere we can assume that whole charge is concentrated at its centre. Thus, electric potential at a distance x from the centre of sphere outside can be given as

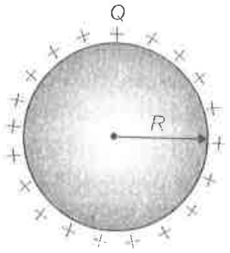


Figure 4.34

$$V = \frac{KQ}{x}$$

At the points on surface of the sphere, the potential can be given as

$$V_S = \frac{KQ}{R}$$

At the interior points of sphere as at every point electric field is zero, we can state that this is an equipotential region; thus at every interior point potential is same as that of its surface. Hence, we have

$$V_{in} = \frac{KQ}{R}$$

Variation of Potential with Distance from Centre of Sphere

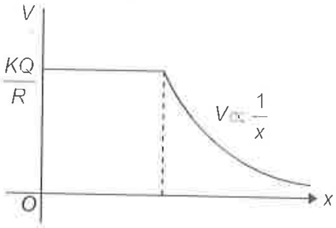


Figure 4.35

Note: Above results are also valid for a uniformly charged hollow sphere.

Case II: Non-conducting Uniformly Charged Sphere

For outer and surface points here also we can say that the potnetial remains same as that of a conducting sphere as

$$V_{out} = \frac{KQ}{x} \text{ (for } x > R)$$

$$V_S = \frac{KQ}{R} \text{ (for } x = R)$$

For an interior point unlike to a conducting sphere, potential will not remain uniform as electric field exists inside the region. We know that inside a uniformly charged sphere electric field is in radially outward direction; thus as we move away from the centre, in the direction of electric field potential decreases.

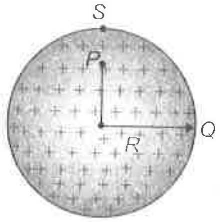


Figure 4.36

As shown in Fig. 4.36 if there is a point P at a distance x from the centre of sphere, the potential difference between points P and S can be given as

$$V_P - V_S = \int_x^r \frac{KQx}{R^3}dx$$

or
$$V_P - \frac{KQ}{R} = \frac{KQ}{2R^3}\left(R^2 - x^2\right)$$

or
$$V_P = \frac{KQ}{2R^3}\left(R^2 - x^2\right) + \frac{KQ}{R}$$

$$V_P = \frac{KQ}{2R^3}\left(3R^2 - x^2\right)$$

Here, at $x = 0$, we have potential at centre of sphere,

$$V_c = \frac{3KQ}{2R} = \frac{3}{2}V_S$$

Thus at centre, potential is maximum and is equal to 3/2 times that on the surface.

Variation of Potential in a Uniformly Charged Sphere with Distance

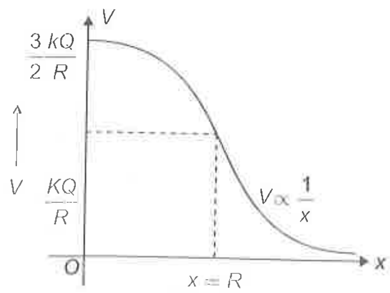


Figure 4.37

FIELD ENERGY OF ELECTROSTATIC FIELD

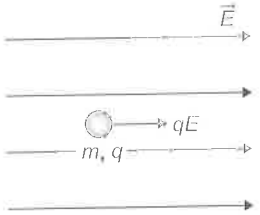


Figure 4.38

Consider a situation shown in Fig. 4.38. A small body of mass m and charge q placed in an electric field E. When the body is released, it starts moving in the direction of electric field due to the electric field qE acting on it. The body will gain some kinetic energy due to its motion. Which is giving energy to this particle? The answer is simple-electric field. This shows that electric field must posses some energy in the region where field exists due to which it can do work on any charged body placed in it. This energy is called field energy of electric field. Wherever electric field exists, field energy also exsits in space. Let us calculate the amount of energy stored in the space where electric field exists.

(a) Field Energy Density of Electric Field

As discussed in the previous section in every region where electric field is present, energy must exist. We can calculate this field energy by the following example.

Consider a charged conducting body shown in Fig. 4.39. Its surface M is having a charge distributed on it. We know that the electric field just outside the surface M at a point can be given as

$$E = \frac{\sigma}{\varepsilon_0}$$

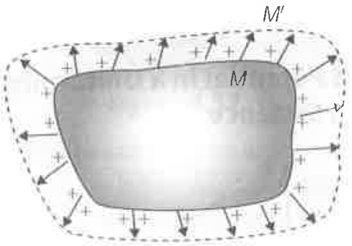


Figure 4.39

We also know that on the surface of metal body experience an outward electric pressure which is given as

$$P_e = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2$$

Now if we consider that the metal surface M is flexible and allowed to expand due to electric pressure upto a small limit to M'. Here if we check electric field associated with the body, we know that inside the body there is no electric field. Initially, electric field only exists from surface M to infinity. Hence, the field energy also exists from the surface M to infinity. When the surface expand to M' then in the final stage the electric field as well as field energy exist from surfaec M' to infinity. This implies that during expansion of surface field energy in the shaded volume (say dV) vanishes as before expansion there was electric field in this region and after expansion electric field becomes zero in the region as there is no electric field inside the body.

We also know that the expansion is done by electric force in the body (electric pressure). Hence, the work done by electric field during expansion is equal to the loss in field energy in the shaded volume dV.

If P_e is the electric pressure on the body surface, then in the small expansion in body volume dV, work done can be given as

$$dW = P_e dV$$

And if dU is the field energy stored in this volume dV then we can use

$$dU = dW = P_e dV$$

or
$$\frac{dU}{dV} = P_e$$

$$u = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \text{ J/m}^3$$

Here, $u = \dfrac{dU}{dV}$ is the field energy stored per unit volume in the space where electric field E exists and is called field energy density of electric field.

If in a region electric field is uniform, the total field energy stored in a given volume V of space can be given as

$$U = \frac{1}{2}\varepsilon_0 E^2 \times V$$

If electric field in a region is non-uniform, the total field energy stored in a given volume of space can be calculated by integrating the field energy in an elemental volume dV of space as

$$dU = \frac{1}{2}\varepsilon_0 E^2 \times dV$$

And the total field energy in a given volume can be given as

$$U = \int dU = \int \frac{1}{2}\varepsilon_0 E^2\, dV$$

(b) Self Energy of a Hollow, Conducting, Solid Conducting and Hollow Non-conducting Sphere

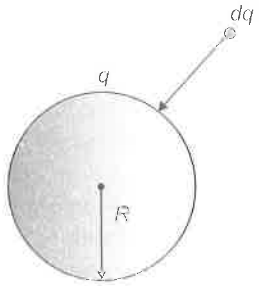


Figure 4.40

We have discussed whenever a system of charges is assembled, some work is done and this work is stored in the form of electrical potential energy of the system. Now we consider an example of charging a conducting sphere of radius R.

In the process of charging we bring charge to the sphere from infinity in steps of elemental charges dq. The charge on sphere opposes the elemental charge being brough to it. Let us assume that at an instant sphere has charge q, due to which it has a potential given as

$$V = \frac{Kq}{R}$$

If now a charge dq is brought to its surface from infinity work done in this process can be given as

$$dW = dqV = \frac{Kq}{R}dq$$

Total work done in charging the sphere can be given as

$$W = \int dW = \int_0^Q \frac{Kq}{R}dq$$

$$W = \frac{KQ^2}{2R} \qquad (1)$$

Equation (1) gives the total work done in charging the sphere of radius R.

We have discussed that in space wherever electric field exists, there must be some field energy stored which has energy density, given as

$$u = \frac{1}{2}\varepsilon_0 E^2 \text{ J/m}^3$$

Here, we can see that when the sphere was uncharged, there was no electric field in its surroundings. But when the sphere is fully charged, electric field exists in its surrounding from its surface to infinity. Let us calculate the field energy associated with this charged conducting sphere.

We know that electric field due to a sphere at outer points varies with distance from centre as

$$E = \frac{KQ}{x^2}$$

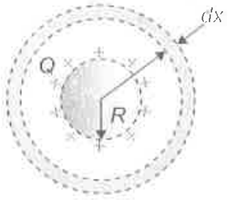


Figure 4.41

To find the total field energy due to this sphere, we consider an elemental spherical shell of radius x and width dx as shown in Fig. 4.41. The volume enclosed in this shell is

$$dV = 4\pi x^2 dx$$

Thus, the field energy stored in the volume of this elemental shell is

$$dU = \frac{1}{2}\varepsilon_0 E^2 \cdot dV$$

$$= \frac{1}{2}\varepsilon_0\left[\frac{KQ}{x^2}\right]^2 \times 4\pi x^2 dx$$

$$= \frac{KQ^2}{2x^2}dx$$

Thus, the total field energy associated with the sphere can be calculated by integrating this expression from surface of sphere to infinity as electric field inside the sphere is zero.

Total field energy in the surrounding of sphere is

$$U = \int dU = \int_R^\infty \frac{KQ^2}{2x^2}\, dx$$

$$= \frac{KQ^2}{2}\left[-\frac{1}{x}\right]_R^\infty$$

$$= \frac{KQ^2}{2R} \qquad (2)$$

Here, we can see that this result is same as Eq. (1). We can conclude that by this total whatever work is done in charging a body is stored in its surrounding in the form of its field energy and can be regarded as self-energy of that body. Once a body is charged in a given configuration, its self-energy is fixed; if the body is now displaced or moved in any manner keeping its shape and charge distribution constant, its self-energy does not charge. As discussed above, we can say that

"Self-energy of a charged body is the total field energy, associated with the electric field due to this body in its surrounding.

(c) Self-energy of a Uniformly Charged Non-conducting Sphere

We know that in outside region of a non-conducting uniformly charged sphere, every point is same as that of a conducting sphere of same radius. Thus, field energy in the surrounding of this sphere from surface to infinity can be given as

$$U_{R\to\infty} = \frac{KQ^2}{2R}$$

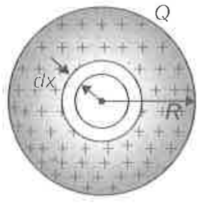


Figure 4.42

Unlike to the case of conducting sphere, in non-conducting sphere at interior point $E \neq 0$. Thus, field energy also exists in the interior region. This can be calculated by considering an elemental shell inside the sphere as shown below.

Here, the field energy in the volume of this elemental shell can be given as

$$dU = \frac{1}{2}\epsilon_0 \left[\frac{KQx}{R^3}\right]^2 \times 4\pi x^2 dx$$

$$\left[\text{As } E_{in} = \frac{KQx}{R^3}\right]$$

$$= \frac{KQ^2}{2R^6} x^4 dx$$

Total field energy inside the sphere can be given as

$$U = \int dU = \frac{KQ^2}{2R^6}\int_0^R x^4\, dx$$

$$\Rightarrow \quad U = \frac{KQ^2}{2R^6}\left[\frac{x^5}{5}\right]_0^R$$

$$U_{0\to R} = \frac{KQ^2}{10R}$$

Thus, the total self-energy of this sphere can be given as

$$U_{self} = U_{0\to R} + U_{R\to\infty}$$

$$U_{self} = \frac{KQ^2}{10R} + \frac{KQ^2}{2R}$$

$$\Rightarrow \quad U_{self} = \frac{3}{5}\frac{KQ^2}{R}$$

CHARGE INDUCTION IN METAL CAVITIES

We have discussed that there can never be any electric field inside a conductor due to static charges. Hence, no electric line of force can enter into a conducting body. Consider a point charge $+q$ inside a spherical cavity at centre within a metal body shown in Fig. 4.43.

The total electric flux originated by $+q$ is $\frac{q}{\varepsilon_0}$. Due to this charge at the inner surface of cavity, a charge $-q$ is induced on which this complete flux will terminate and no electric line of force exists into the metal body. A point, A inside the metal volume we know that net electric field is zero. Thus, the electric field at A due to the point charge $+q$ is nullified by the electric field due to the negative induced charges on the inner surface of cavity and the positive charge induced on the outer surface is automatically distributed on the surface in such a way that it does not produce any electric field within the metal body.

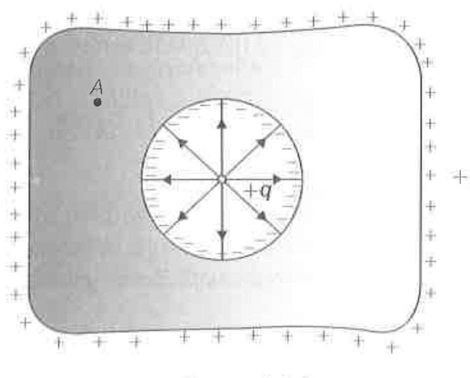


Figure 4.43

From the above analysis, we can conclude some points about the charge induction when a charge is placed inside the cavity of a metal body. These are as follows.

1. Whenever a charge is placed inside a metal cavity, an equal and opposite charge is induced on the inner surface of cavity.
2. A similar charge is induced on the outer surface of the body with surface charge density inversely proportional to the radius of curvature of body.
3. When the charge inside is displaced, the induced charge distribution on the inner surface of body changes in such a way that its centre of charge can be assumed to be at the point charge so as to nullify the electric field in the outer region.
4. Due to the movement in the point charge inside the body, the charge distribution on the outer surface of body does not change as shown in Fig. 4.44.

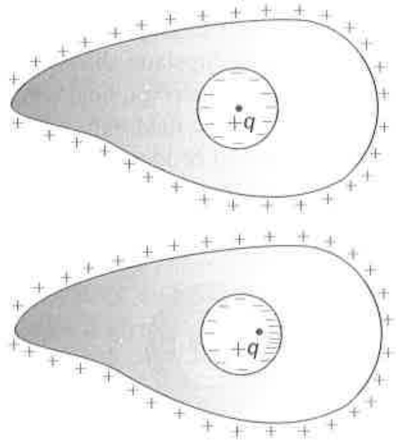


Figure 4.44

5. If another charge is brought to the body from outside, it will only affect the outer distribution of charges not on the charge distribution inside the cavity as shown in Fig. 4.45.

Now consider the situation shown in Fig. 4.46. Inside a conducting spherical shell of inner radius R_1 and outer radius R_2, a point charge q is placed at a distance x from the centre as shown. The electric potential at the centre due to this system can be given as

$$V_C = \frac{kq}{r} - \frac{Kq}{R_1} + \frac{Kq}{R_2}$$

If we find electric field and potential at a distance r from the centre outside the shell, it will be only due to the charge on the outer surface as induced charge on the inner surface of cavity always nullifies the effect of point charge inside it. Thus, it can be given as

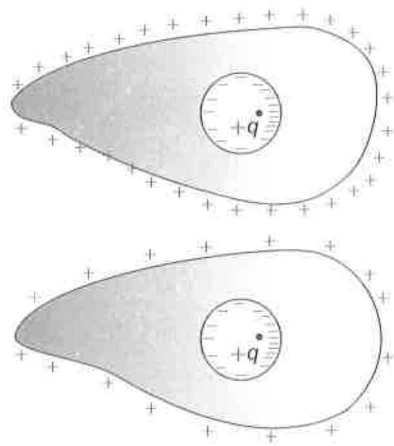


Figure 4.45

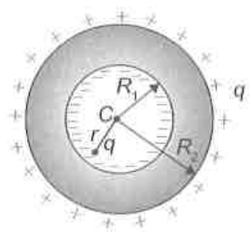


Figure 4.46

$$E_{out} = \frac{kq}{r^2}$$

and $$V_P = \frac{Kq}{r}$$

Cavity in a Conducting Material

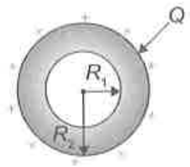


Figure 4.47

Consider the system shown in Fig. 4.47. As we know that when a charge is given to a conductor it resides on its outer surface.

Let us find $\vec{E}$ at a point distanced at r.

Case I

When $r < R_1$

$$\vec{E} = 0$$

[because net charge within this region = 0]

Case II

When $R_1 < r < R_2$

$$\vec{E} = 0$$

$$[q_{net} = 0, \phi = \frac{q_{in}}{\varepsilon_0} = 0 \Rightarrow E = 0]$$

Case III

When $r > R_2$

$$\vec{E} = \frac{kQ}{r^2}$$

[it is similar to case of hollow charged sphere]

Now we consider a case when charge is placed inside a conductor. For such case, charge distribution will be as follows.

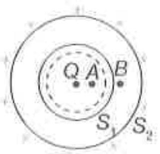


Figure 4.48

For simplicity in the calculation, we could bifurgate the above system as

- A point charge
- A hollow sphere S_1 with charge $-Q$
- A hollow sphere S_2 with charge $+Q$

Electric field at A

$$\text{Due to } Q = \frac{KQ}{r^2}$$

$$\text{Due to } S_1 = 0$$

[$\because$ Point lies inside the hollow sphere]

$$\text{Due to } S_2 = 0$$

[$\because$ Point lies inside the hollow sphere]

Electric field at B

$$\text{Due to } Q = \frac{KQ}{r^2} \rightarrow$$

$$\text{Due to } S_1 = \frac{KQ}{r^2} \leftarrow$$

$$\text{Due to } S_2 = 0$$

$$\vec{E}_{net} \text{ at } B = 0$$

Electric field at C

$$\text{Due to } Q = \frac{KQ}{r^2} \rightarrow$$

$$\text{Due to } S_1 = \frac{KQ}{r^2} \leftarrow$$

$$\text{Due to } S_2 = \frac{KQ}{r^2} \rightarrow$$

$$\vec{E}_{net} \text{ at } C = \frac{KQ}{r^2} \rightarrow$$

At point C, net due to S_1 and Q is zero. $\vec{E}$ at C is only due to outside charge (S_2). If we place an external charge at point C, then effect of S_1 and Q on external charge is zero or we can say that effect of external charge on S_1 and Q is zero.

Or we can say that charge placed inside the conductor and the charge induced on the inner surface of the conductor does not get affected by any external electric field. This is known as electrostatic shielding; that is why, equipment sensitive towards electric field are placed inside a conductor. External electric field only affects the charge distributed on the surface of conductor.

We again go back to the case when a charge was placed in the conductor.

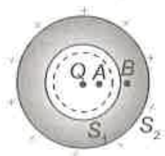


Figure 4.49

Potential at A

$$\text{Due to } Q = \frac{KQ}{r}$$

$$\text{Due to } S_1 = \frac{-KQ}{R_1}$$

$$\text{Due to } S_2 = \frac{KQ}{R_2}$$

$$\Rightarrow \quad V_{net} = KQ\left[\frac{1}{r} - \frac{1}{R_1} + \frac{1}{R_2}\right]$$

Potential at B

$$\text{Due to } Q = \frac{KQ}{r}$$

$$\text{Due to } S_1 = \frac{-KQ}{r}$$

$$\text{Due to } S_2 = \frac{KQ}{R_2}$$

$$\Rightarrow \quad V_{net} = \frac{KQ}{R_2}$$

Potential at C

$$\text{Due to } Q = \frac{KQ}{r}$$

$$\text{Due to } S_1 = \frac{-KQ}{r}$$

$$\text{Due to } S_2 = \frac{KQ}{r}$$

$$V_{net} = \frac{KQ}{r}$$

Note

While writing potential at various points is case of cavity in a conducting material first distribute charge on various surfaces and then the potential due to induced charges is also considered.

CAVITY IN A NON-CONDUCTING SPHERE

(a) Electric Field due to a Non-uniformly Radially Charged Solid Non-conducting Sphere

If a sphere of radius R is charged with a non-uniform charge density which varies with the distance x from centre as

$$\rho = \frac{\rho_0}{x}\ \mathrm{C/m^3}$$

Here if we wish to find electric field strength at a point situated at a distance r from centre of sphere outside it, at point P shown in Fig. 4.50. This can be given as

$$E_P = \frac{KQ}{r^2} \quad \text{(where } Q \text{ is the total charge of the sphere)}$$

For outer points, we can assume that whole charge of sphere to be at its centre. Now Q can be calculated by integrating the charge of an elemental shell of radius x and width dx

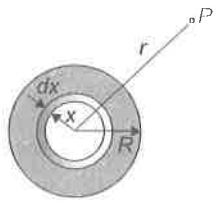


Figure 4.50

as shown in Fig. 4.50. The charge dq in this shell can be given as

$$dq = \rho 4\pi x^2 dx$$

$$= \frac{\rho_0}{x} \times 4\pi x^2 dx$$

$$= 4\pi\rho_0 x dx$$

Total charge of sphere can be given as

$$Q = \int dq$$

$$= \int_0^R 4\pi\rho_0 x dx$$

$$= 4\pi\rho_0 \left[\frac{x^2}{2}\right]_0^R$$

$$= 2\pi\rho_0 R^2$$

Thus, electric field strength at outer points can be given as

$$E_P = \frac{K(2\pi\rho_0 R^2)}{r^2}$$

$$= \frac{\rho_0 R^2}{2\varepsilon_0 r^2}$$

To find electric field strength at an interior point at a distance r from the centre of sphere, we first find the charge enclosed within the inner sphere of radius r of which point P is on the surface. Thus enclosed charge can be given as

$$q_{encl} = \int_0^r \frac{\rho_0}{R} 4\pi x^2 dx$$

$$= 2\pi\rho_0 r^2$$

Here, electric field strength at point P can be given as

$$E_P = \frac{Kq_{encl}}{r^2}$$

$$= \frac{K(2\pi\rho_0 r^2)}{r^2}$$

$$E_P = \frac{\rho_0}{2\varepsilon_0}$$

Note

Here we can see that the above expression is independent of distance from centre.

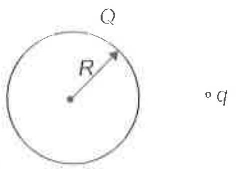
SOLVED EXAMPLES

EXAMPLE 5

Figure shows a uniformly charged sphere of radius R and total charge Q. A point charge q is situated outside the sphere at a distance r from the centre of sphere. Find out the following:

(a) Force acting on the point charge q due to the sphere.

(b) Force acting on the sphere due to the point charge.



SOLUTION

(a) Electric field at the position of point charge

$$\vec{E} = \frac{KQ}{r^2}\hat{r}$$

so,

$$\vec{F} = \frac{KqQ}{r^2}\hat{r}$$

$$|\vec{F}| = \frac{KqQ}{r^2}$$

(b) Since we know that every action has equal and opposite reaction so

$$\vec{F}_{\text{sphere}} = -\frac{KqQ}{r^2}\hat{r}$$

$$\left|\vec{F}_{\text{sphere}}\right| = \frac{KqQ}{r^2}$$ ■

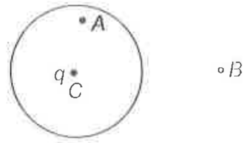
EXAMPLE 6

Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere. Find out the following:

(a) Force on charge q.

(b) Electric field intensity at A.

(c) Electric field intensity at B.



SOLUTION

(a) Electric field at the centre of the uniformly charged hollow sphere = 0

So force on charge $q = 0$

(b) Electric field at A

$$\vec{E}_A = \vec{E}_{\text{Sphere}} + \vec{E}_q$$

$$= 0 + \frac{Kq}{r^2};\ r = CA$$

E due to sphere = 0, because point lies inside the charged hollow sphere.

(c) Electric field $\vec{E}_B$ at point $B = \vec{E}_{\text{Sphere}} + \vec{E}_q$

$$= \frac{KQ}{r^2}\hat{r} + \frac{Kq}{r^2}\hat{r}$$

$$= \frac{K(Q+q)}{r^2}\cdot r\ ;\ r = CB$$ ■

Note

Here we can also assume that the total charge of sphere is concentrated at the centre, for the calculation of electric field at B.

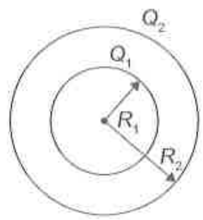
EXAMPLE 7

Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2, respectively. Derive an expression of electric field as a function of r for the following positions.

(a) $r < R_1$

(b) $R_1 \leq r < R_2$

(c) $r \geq R_2$



SOLUTION

(a) For $r < R_1$

Therefore, point lies inside both the spheres

$$E_{net} = E_{inner} + E_{outer} = 0$$

(b) For $R_1 \le r < R_2$

Therefore, point lies outside inner sphere but inside outer sphere:

$$E_{net} = E_{inner} + E_{outer}$$
$$= \frac{KQ_1}{r^2}\hat{r} + 0$$
$$= \frac{KQ_1}{r^2}\hat{r}$$

(c) For $r \ge R_2$

Therefore, point lies outside inner as well as outer sphere.

$$E_{net} = E_{inner} + E_{outer}$$
$$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r}$$
$$= \frac{K(Q_1 + Q_2)}{r^2}\hat{r}$$

EXAMPLE 8

A solid non-conducting sphere of radius R and uniform volume charge density ρ has its centre at origin. Find out electric field intensity in vector form at the following positions:

(a) $(\frac{R}{2}, 0, 0)$, (b) $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$, (c) $(R, R, 0)$

SOLUTION

(a) At $(\frac{R}{2}, 0, 0)$

Distance of point from the centre $= \sqrt{\left(\frac{R}{2}\right)^2 + 0^2 + 0^2} = \frac{R}{2}$

$< R$, so the point lies inside the sphere

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left[\frac{R}{2}\hat{i}\right]$$

(b) At $\left(\frac{R}{\sqrt{2}}, \frac{R}{\sqrt{2}}, 0\right)$

Distance of point from the centre $= \sqrt{\left(\frac{R}{\sqrt{2}}\right)^2 + \left(\frac{R}{\sqrt{2}}\right)^2 + 0^2}$

$= R = R$, so the point lies at the surface of sphere.

$$\therefore \quad \vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{R^3}\left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right]$$
$$= \frac{\rho}{3\varepsilon_0}\left[\frac{R}{\sqrt{2}}\hat{i} + \frac{R}{\sqrt{2}}\hat{j}\right]$$

(c) The point is outside the sphere

$$\therefore \quad \vec{E} = \frac{KQ}{r^3}\vec{r} = \frac{K\frac{4}{3}\pi R^3\rho}{\left(\sqrt{2}R\right)^3}\left[R\hat{i} + R\hat{j}\right]$$
$$= \frac{\rho}{6\sqrt{2}\varepsilon_0}\left[R\hat{i} + R\hat{j}\right]$$

(b) Electric Field Inside a Cavity of Non-conducting Charged Body

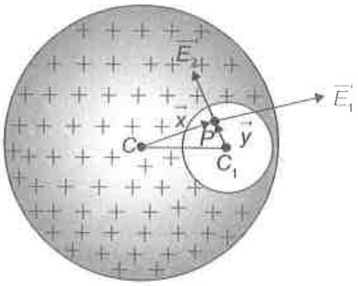


Figure 4.51

Consider the sphere shown in Fig. 4.51 charged uniformly with charge density ρ C/m³. Inside the sphere a spherical cavity is created with centre at C.

Now we find electric field strength inside the cavity. For this, we consider a point P in the cavity at a position vector $\vec{x}$ from the centre of sphere and at a position vector $\vec{y}$ from the centre of cavity as shown.

If $\vec{E}_1$ be the electric field strength at P due to the complete charge of the sphere (inside cavity also), then we know that electric field strength inside a uniformly charged sphere is given as

$$\vec{E} = \frac{\rho\vec{X}}{3\varepsilon_0}$$

Similarly if we assume that charge is only there in the region of cavity, this will also be a uniformly charged small sphere. If $\vec{E}_2$ be the electric field only due to the cavity charge, it can be given as

$$E_2 = \frac{\rho \vec{Y}}{3\varepsilon_0}$$

Now the electric field due to the charged sphere in the cavity at point P can be given as

$$\vec{E}_{net} = \vec{E}_1 - \vec{E}_2$$

[as charge of cavity is removed]

$$= \frac{\rho \vec{a}}{3\varepsilon_0} \quad [\text{as } \vec{x} - \vec{y} = \vec{a}]$$

This shows that the net electric field inside the cavity is uniform and in the direction of $\vec{a}$, i.e., along the line joining the centre of spheres and cavity.

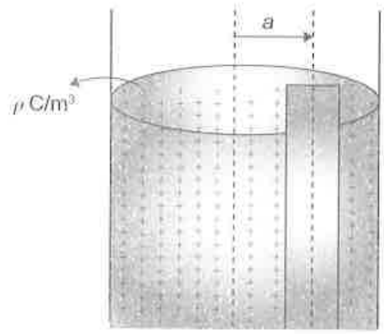


Figure 4.52

Similarly we can find the electric field strength inside a cyclindrical cavity of a long uniformly charged cylinder. If cavity axis is displaced from the axis of cylinder by a displacement vector, by the analysis we have done for a sphere, we can say that the electric field strength inside the cavity is also uniform and can be given as

$$\vec{E} = \frac{\rho \vec{a}}{2\varepsilon_0}$$

(c) Some Other Important Results for a Closed Conductor

1. If a charge q is kept in the cavity, then $-q$ will be induced on the inner surface and $+q$ will be induced on the outer surface of the conductor (it can be proved using Gauss theorem)

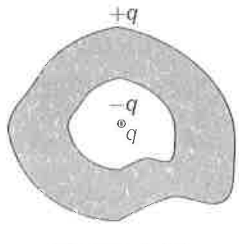


Figure 4.53

2. If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q, then $-q$ charge will be induced on the inner surface and the total charge on the outer surface will be $q + Q$. (It can be proved using Gauss theorem)

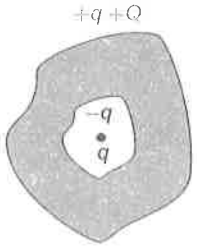


Figure 4.54

3. Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B, C) is zero. Resultant field due to $q + Q$ on S_2 and any other charge outside S_2, at any point inside of surface S_2 (like A, B) is zero.

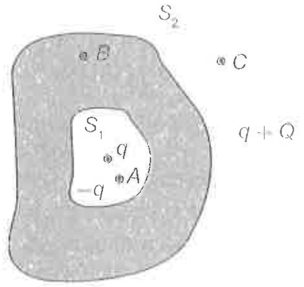
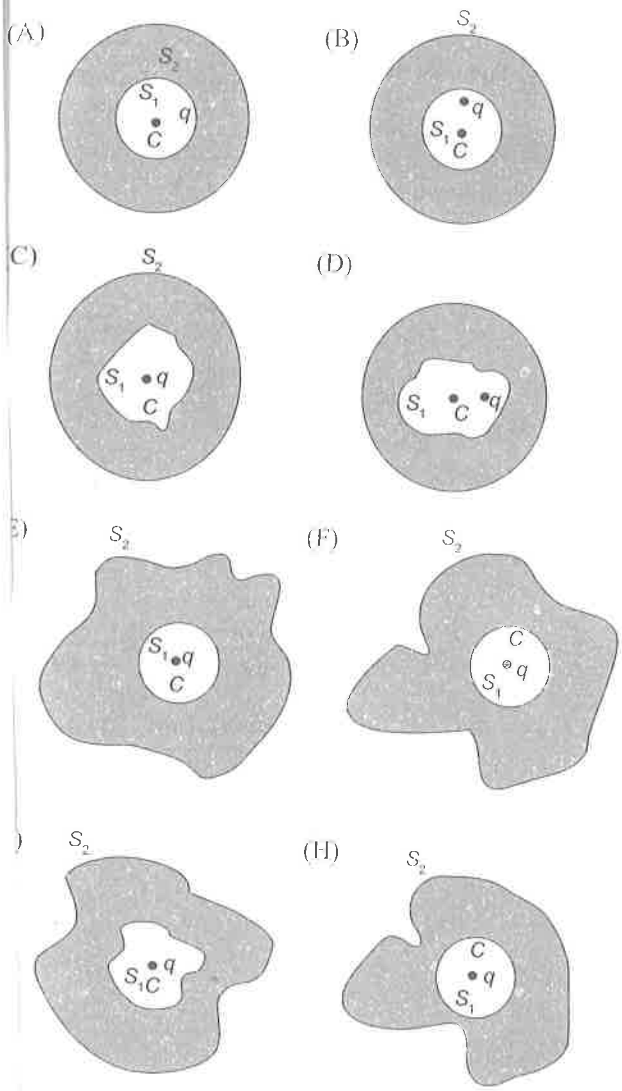


Figure 4.55

4. Resultant field in a charge-free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the innermost surface of the conductor.

Figure 4.56

5. Charge distribution for different types of cavities in conductors 0



Notes

In all cases, charge on inner surface $S_1 = -q$ and on outer surface $S_2 = i$. The distribution of charge on 'S_1' will not change even if some charges are kept outside the conductor (i.e., outside the surface S_2). But the charge distribution on 'S_2' may change if some charges(s) is/are kept outside the conductor.

Electric field at 'A' due to $-q$ of S_1 and $+q$ of S_2 is zero individually because they are uniformly distributed.

At point B:

$$V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2},\ E_B = 0$$

At point C:

$$V_C = \frac{Kq}{OC},\ \vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$$

Force on point charge Q:

Here force on 'Q' will be only due to 'q' of S_2; see result (3).

$$\vec{F}_Q = \frac{KqQ}{r^2}\hat{r}$$

(r = distance of 'Q' from centre 'O')

Force on point charge q:

$$\vec{F}_q = 0$$

(using result (3) and charge on S_1 uniform)

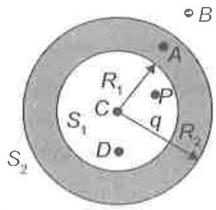
Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (3) we can show that

SOLVED EXAMPLES

EXAMPLE 9

An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in the figure.

...e	A	B	C	D	E	F	G	H
	Uniform	Non-uniform	Non-uniform	Non-uniform	Uniform	Non-uniform	Non-uniform	Non-uniform
	Uniform	Uniform	Uniform	Uniform	Non-uniform	Non-uniform	Non-uniform	Non-uniform



Find out the following:
(a) V_C, (b) V_A, (c) V_B, (d) E_A, (e) E_B, (f) force on charge Q if it is placed at B.

SOLUTION

(a) $V_C = \dfrac{Kq}{CP} + \dfrac{K(-q)}{R_1} + \dfrac{Kq}{R_2}$

Note

$-q$ on S_1 is non-uniformly distributed still it produces potential $\dfrac{K(-q)}{R_1}$ at 'C' because 'C' is at distance 'R_1' from each point of 'S_1'.

(b) $V_A = \dfrac{Kq}{R_2}$

(c) $V_B = \dfrac{Kq}{CB}$

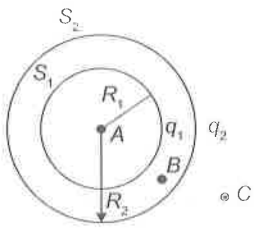
(d) $E_A = 0$ (point is inside metallic conductor)

(e) $E_B = \dfrac{Kq}{CB^2}\hat{CB}$

(f) $F_Q = \dfrac{KQq}{CB^2}\hat{CB}$ ■

Combination of Conducting Spherical Shells

Let us consider a system of concentric conducting shell with charge q_1 on inner shell and q_2 on outer shell.



Potential at $A = \dfrac{Kq_1}{R_1} + \dfrac{Kq_2}{R_2}$

Potential at $B = \dfrac{Kq_1}{r} + \dfrac{Kq_2}{R_2}$

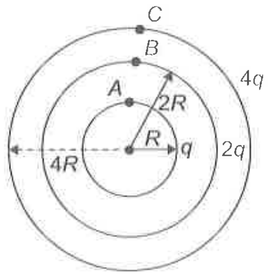
Potential at $C = \dfrac{Kq_1}{r} + \dfrac{Kq_2}{r}$

Potential of $S_1 = \dfrac{Kq_1}{R_1} + \dfrac{Kq_2}{R_2}$

Potential of $S_2 = \dfrac{Kq_1}{R_1} + \dfrac{Kq_2}{R_2}$

EXAMPLE 10

Consider the following system and find $V_C - V_A$.



SOLUTION

$$V_C = \frac{R(4q)}{4R} + \frac{K(2q)}{4R} + \frac{K(q)}{4R} = \frac{7Kq}{4R}$$

$$V_A = \frac{Kq}{R} + \frac{K(2q)}{2R} + \frac{K(4q)}{4R} = \frac{3Kq}{R}$$

$$V_C - V_A = \frac{7Kq}{4r} - \frac{3Kq}{R}$$

$$= \frac{7Kq - 12Kq}{4R}$$

$$= -\frac{5Kq}{4R}$$ ■

CONNECTION OF TWO CONDUCTING MATERIALS

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2, respectively, and placed separately by large distance, are joined by a conducting wire.

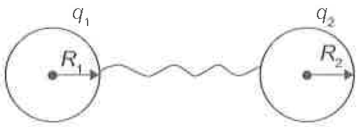


Figure 4.57

Let final charges on spheres are q_1 and q_2, respectively.

Potential on both spherical shells become equal after joining, and therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2} \quad (1)$$

and by charge conservation,

$$q_1 + q_2 = Q_1 + Q_2 \quad (2)$$

From (1) and (2)

$$q_1 = \frac{(Q_1 + Q_2)R_1}{R_1 + R_2},$$

$$q_2 = \frac{(Q_1 + Q_2)R_2}{R_1 + R_2}$$

Ratio of charges

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\Rightarrow \quad \frac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \frac{R_1}{R_2}$$

Ratio of surface charge densities

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

Ratio of final charges

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

Ratio of final surface charge densities

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

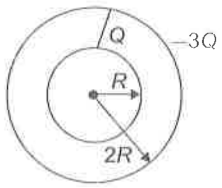
Note

If two concentric hollow spheres are connected by a wire, then all the charges from the inner sphere will reside to the outer sphere.

SOLVED EXAMPLES

EXAMPLE 11

The two conducting spherical shells are joined by a conducting wire and cut after sometime when charge stops flowing. Find out the charge on each sphere after that.



SOLUTION

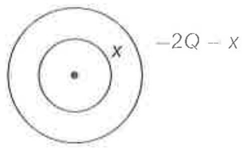
After cutting the wire, the potential of both the shells is equal.

Thus, potential of inner shell

$$V_{in} = \frac{Kx}{R} + \frac{K(-2Q - x)}{2R} = \frac{K(x - 2Q)}{2R}$$

and potential of outer shell

$$V_{out} = \frac{Kx}{2R} + \frac{K(-2Q - x)}{2R} = \frac{-KQ}{R}$$



As $V_{out} = V_{in}$

$$\Rightarrow \quad \frac{-KQ}{R} = \frac{K(x - 2Q)}{2R}$$

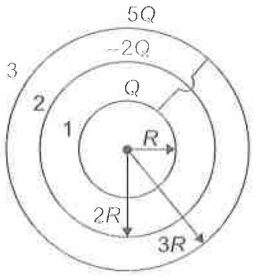
$$\Rightarrow \quad -2Q = x - 2Q$$

$$\Rightarrow \quad x = 0$$

So charge on inner spherical shell = 0 and outer spherical shell = $-2Q$. ■

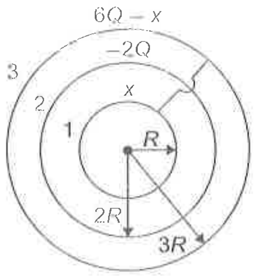
EXAMPLE 12

Find charge on each spherical shell after joining the innermost and outermost shell by a conducting wire. Also find charges on each surface.



SOLUTION

Let the charge on the innermost sphere be x.
Finally, potential of shell 1 = Potential of shell 3



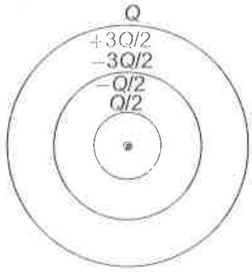
$$\frac{Kx}{R}+\frac{K(-2Q)}{2R}+\frac{K(6Q-x)}{3R}=\frac{Kx}{3R}+\frac{K(-2Q)}{3R}+\frac{K(6Q-x)}{3R}$$

$$3x-3Q+6Q-x=4Q;\ 2x=Q;\ x=\frac{Q}{2}$$

Charge on innermost shell $=\frac{Q}{2}$

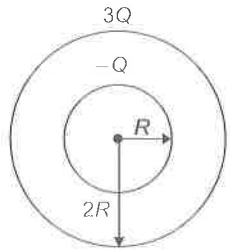
Charge on outermost shell $=\frac{5Q}{2}$

Middle shell $=-2Q$
Final charge distribution is as shown in the following figure.



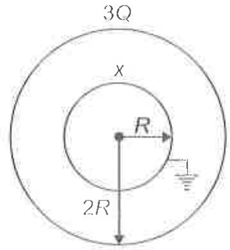
EXAMPLE 13

Two conducting hollow spherical shells of radii R and $2R$ carry charges $-Q$ and $3Q$, respectively. How much charge will flow into the earth if inner shell is grounded?



SOLUTION

When inner shell is grounded to the earth, then the potential of inner shell will become zero because potential of the earth is taken to be zero.



$$\frac{Kx}{R}+\frac{K3Q}{2R}=0$$

$$x=\frac{-3Q}{2},$$

The charge that has increased

$$=\frac{-3Q}{2}-(-Q)$$
$$=\frac{-Q}{2}$$

Hence, charge flows into the earth $=\frac{Q}{2}$

EXAMPLE 14

An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss?

SOLUTION

When two conducting spheres of equal radius are connected charge is equally distributed on them (Result 6). So we can say that heat loss of system

$$\Delta H = U_i - U_f$$

$$\left(\frac{Q^2}{8\pi\varepsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\varepsilon_0 R} + \frac{Q^2/4}{8\pi\varepsilon_0 R}\right) = \frac{Q^2}{16\pi\varepsilon_0 R}$$ ■

EARTHING OF CHARGED OR UNCHARGED METAL BODIES

In electrical analysis, earth is assumed to be a very large conducting sphere of radius 6400 km. If some charge Q is given to the earth, its potential becomes

$$V_i = \frac{KQ}{R_e}$$

As R_e is very large, V_e comes out to be a negligible value. Thus for very small bodies whose dimensions are negligible compared to earth we can assume that earth is always at zero potential.

Keeping the above fact in mind if we connect a small body to earth, charge flow takes place between the earth and the body till both will be at same potential, zero potential as potential of earth will always remain zero, no matter if charge flows into earth or from earth. This implies that if a body at some positive potential is connected to earth, earth will supply some negative charge to this body so that the final potential of body will become zero.

Consider a solid uncharged conducting sphere shown in Fig. 4.58. A point charge q is placed in front of the sphere centre at a distance x as shown. Here due to q, the potential at sphere is

$$V = \frac{Kq}{x}$$

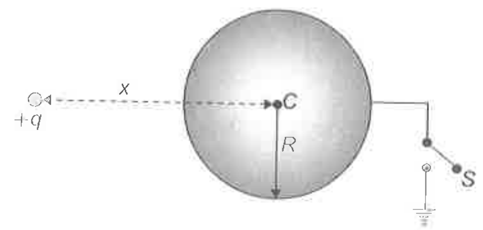


Figure 4.58

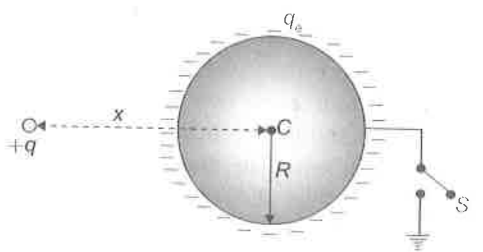


Figure 4.59

Here we ignore induced charges due to q because potential due to induced charges on sphere is zero. It we close the switch S, earth supplies a charge q_e on the the sphere to make its final potential zero. Thus the final potential on sphere can be taken as

$$V = \frac{Kq}{x} + \frac{Kq_e}{R}$$

or

$$q_e = \frac{-qR}{x}$$

Now it is obvious that earth has supplied a negative charge to develop a negative potential on sphere to nullify the initial positive potential on it due to q.

It is noted that whenever a metal body is connected to earth, we consider that earth supplies a charge to it (say q_e) to make its final potential zero due to all the charges including the charge on the body and the charges in its surrounding.

CONDUCTOR AND ITS PROPERTIES [FOR ELECTROSTATIC CONDITION]

1. Conductors are materials which contain a large number of free electrons that can move freely inside the conductor.
2. In electrostatics, conductors are always equipotential surfaces.
3. Charge always resides on outer surface of conductor.
4. If there is a cavity inside the conductor having no charge, then charge will always reside only on outer surface of conductor.
5. Electric field is always perpendicular to conducting surface.
6. Electric lines of force never enter into conductors.
7. Electric field intensity near the conducting surface is given by the formula

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

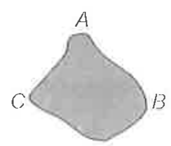


Figure 4.60

$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n};\ \vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n}$$

and $$\vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

8. When a conductor is grounded, its potential becomes zero.

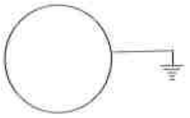


Figure 4.61

9. When an isolated conductor is grounded, then its charge becomes zero.
10. When two conductors are connected, there will be charge flow till their potential becomes equal.
11. Electric pressure: Electric pressure at the surface of a conductor is given by the formula $P = \frac{\sigma^2}{2\varepsilon_0}$, where σ is the local surface charge density.

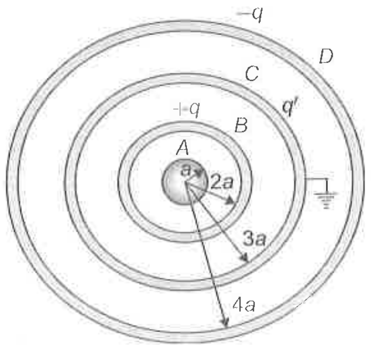
SOLVED EXAMPLE

EXAMPLE 14

There are four concentric shells A, B, C and D of radii of a, $2a$, $3a$ and $4a$, respectively. Shells B and D are given charges $+q$ and $-q$, respectively. Shell C is now earthed. Find the potential difference $V_A - V_C$.

SOLUTION

Let shell C acquires charge 'q' which will be such that final potential of C is zero.



$$V_C = \frac{Kq}{3a} + \frac{Kq'}{3a} + \left(\frac{-Kq}{4a}\right) = 0$$

$$\frac{Kq}{3a} + \frac{Kq'}{3a} = \frac{Kq}{4a}$$

$$\Rightarrow \quad q' = 3q\left(\frac{1}{4} - \frac{1}{3}\right)$$

$$q' = -\frac{q}{4}$$

As $V_C = 0$

$$V_A - V_C = V_A$$

Now calculating V_A, we get

$$V_A = \frac{Kq}{2a} - \frac{K(q/4)}{3a} - \frac{Kq}{4a}$$

$$\Rightarrow \quad V_A = \frac{Kq}{6a}$$

or $$V_A - V_C = \frac{Kq}{6a}$$

■

COMBINATION OF CONDUCTING PLATES

Let us consider two conducting plates placed parallel to each other.

I plate is given a charge Q_1 and II plate is given a charge Q_2, which distributes itself as shown in Fig. 4.62.

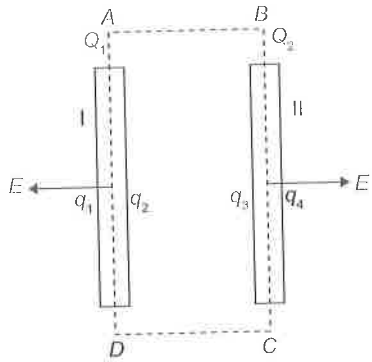


Figure 4.62

where $$q_1 + q_2 = Q_1$$
$$q_3 + q_4 = Q_2$$

Now we take a rectangular Gaussian surface $ABCD$.

Among the four faces, two faces AD and BC of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface AB and CD which are outside the conductor are parallel to the electric field, i.e. their area vector is perpendicular to $\vec{E}$

and hence the flux through these parts is also zero. The total flux of the electric field through the closed surface is therefore zero. From Gauss's law, the total charge inside the closed surface should be zero. The charge on the inner surface of I should be equal and opposite to that on the inner surface of II.

So $$q_2 = -q_3$$

Now to find further relations between the charges distributed we find electric field at point P

Electric field at point P

$$\text{due to } q_1 \text{ charge layer} = \frac{q_1}{2A\varepsilon_0} \text{(towards right)}$$

$$\left[\text{As } \vec{E} \text{ due to a single layer of charge is } \frac{\sigma}{2e_0} = \frac{q_1}{2A\varepsilon_0}\right]$$

$$\text{due to } q_2 \text{ charge layer} = \frac{q_1}{2A\varepsilon_0} \text{ [towards left]}$$

$$\text{due to } q_3 \text{ charge layer} = \frac{q_3}{2A\varepsilon_0} \text{ [towards left]}$$

$$\text{due to } q_4 \text{ charge layer} = \frac{q_4}{2A\varepsilon_0} \text{ [towards left]}$$

$$\vec{E}_{\text{net}} \text{ at } P = \frac{q_1}{2A\varepsilon_0} - \frac{q_2}{2A\varepsilon_0} - \frac{q_3}{2A\varepsilon_0} - \frac{q_4}{2A\varepsilon_0} \text{ [towards right]}$$

As the point P lies inside the conductor, the field should be zero.

Hence,

$$\frac{q_1}{2A\varepsilon_0} = \frac{q_2 + q_3 + q_4}{2A\varepsilon_0}$$

$$q_1 = q_2 + q_3 + q_4 (\text{but } q_2 = -q_3)$$

$$q_1 = q_4$$

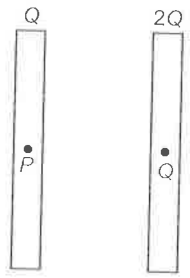
Substituting in the above equation, we get

$$q_1 = q_4 = \frac{Q_1 + Q_2}{2}$$

SOLVED EXAMPLES

XAMPLE 15

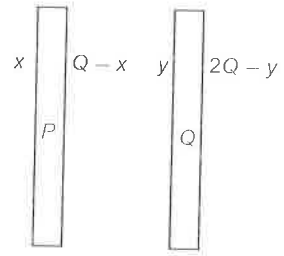
o large parallel conducting sheets (placed at finite tance) are given charges Q and $2Q$, respectively. Find charges appearing on all the surfaces.



SOLUTION

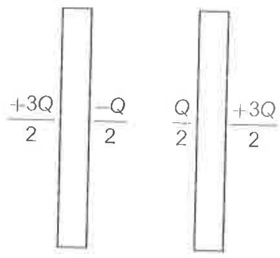
Let there be x amount of charge on the left side of the first plate, so on its right side charge will be $Q - x$. Similarly for the second plate there is y charge on the left side and $2Q - y$ charge is on the right side of the second plate

$E_p = 0$ (by property of conductor)



$$\Rightarrow \quad \frac{x}{2A\varepsilon_0} - \left\{\frac{Q-x}{2A\varepsilon_0} + \frac{y}{2A\varepsilon_0} + \frac{2Q-y}{2A\varepsilon_0}\right\} = 0$$

We can also say that charge on left side of P = charge on right side of P



$$x = Q - x + y + 2Q - y$$

$$\Rightarrow \quad x = \frac{3Q}{2},$$

$$Q - x = \frac{-Q}{2}$$

Similarly for point Q:

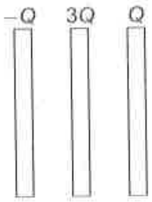
$$x + Q - x + y = 2Q - y$$

$\Rightarrow \qquad y = Q/2, 2Q - y = 3Q/2$

So final charge distribution of plates is: ■

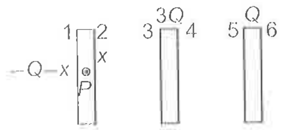
EXAMPLE 16

Figure shows three large metallic plates with charges $-Q$, $3Q$ and Q, respectively. Determine the final charges on all the surfaces.



SOLUTION

We assume that the charge on surface 2 is x. Following conservation of charge, we see that surface 1 has charge $(-Q - x)$. The electric field inside the metal plate is zero and so field at P is zero.



Resultant field at P

$$E_P = 0$$

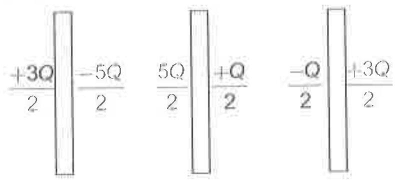
$$\Rightarrow \quad \frac{-Q - x}{2A\varepsilon_0} = \frac{x + 3Q + Q}{2A\varepsilon_0}$$

$$\Rightarrow \quad -Q - x = x + 4Q$$

$$\Rightarrow \quad x = \frac{-5Q}{2}$$ ■

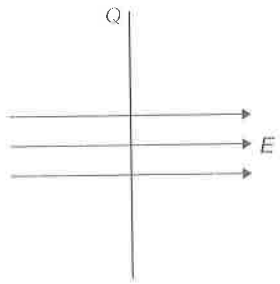
Note

We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by Gauss theorem also. Remember that this it is an important result. Thus, the final charge distribution on all the surfaces is as shown in the following figure:



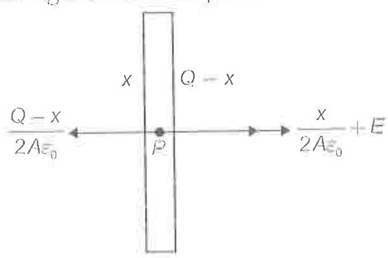
EXAMPLE 17

An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.



SOLUTION

Let there be x charge on the left side of the plate and $Q - x$ charge on the right side of the plate



$$E_P = 0$$

$$\frac{x}{2A\varepsilon_0} + E = \frac{Q - x}{2A\varepsilon_0}$$

$$\frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

$$x = \frac{Q}{2} - EA\varepsilon_0$$

and $$Q - x = \frac{Q}{2} + EA\varepsilon_0$$

So charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and the other side is $\frac{Q}{2} + EA\varepsilon_0$ ■

Note

Solve this question for $Q = 0$ without using the above answer and match that answers with the answers that you will get by putting $Q = 0$ in the above answer.

Earthing of a System of Parallel Plates

Consider a large plate shown in Fig. 4.63 charged with a charge Q. This is connected to earth with a switch S as shown. If switch S is closed, whole charge will flow to earth and the plate will become neutral as in the surrounding of a single earthed body no electric field exists.

Now consider the system of two plates A and B shown here. Plate A is given a charge Q and plate B is neutral. The charge distribution on plates is as shown in the figure. If the switch S is now closed, the total charge on the outer surface of the system of plates after earthing should become zero. Hence whole charge on plate A will transfer to its inner surface, and therefore on the inner surface of plate B an equal and opposite charge $-Q$ is developed which is given by earth as shown in the figure.

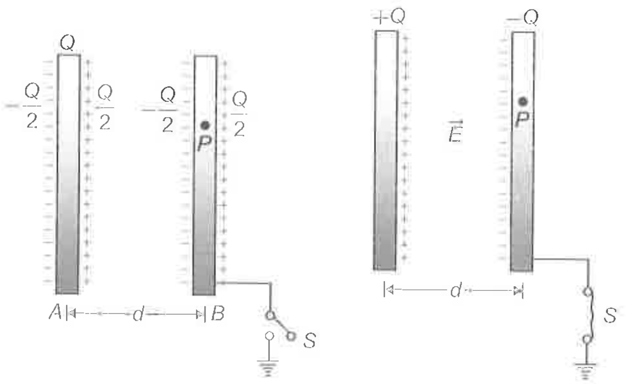


Figure 4.63

If the area of plates is A, the electric field between the system of plates can be given as

$$E_f = \frac{Q}{A\varepsilon_0}$$

Before earthing this electric field was

$$E_i = \frac{Q}{2A\varepsilon_0} = \frac{E_f}{2}$$

Thus just after earthing the electric field between the plates is doubled and the potential difference between the two plates will also be doubled. As plate B is earthed, its potential is zero. The potential of plate A can be given as

$$V_A = \frac{Q}{A\varepsilon_0}d$$

Now consider another example shown in Fig. 4.64. In a system of three parallel plates A, B and C, the middle plate B is given a charge Q due to which charges are induced on plates A and C as shown. On the basis of discussion done in the previous section, we can say that if switch S_1 is closed whole charge of plate B will shift on its left surface and a charge $-Q$ is flown through S_1 towards plate A and final situation will be as shwon in Fig. 4.64(b).

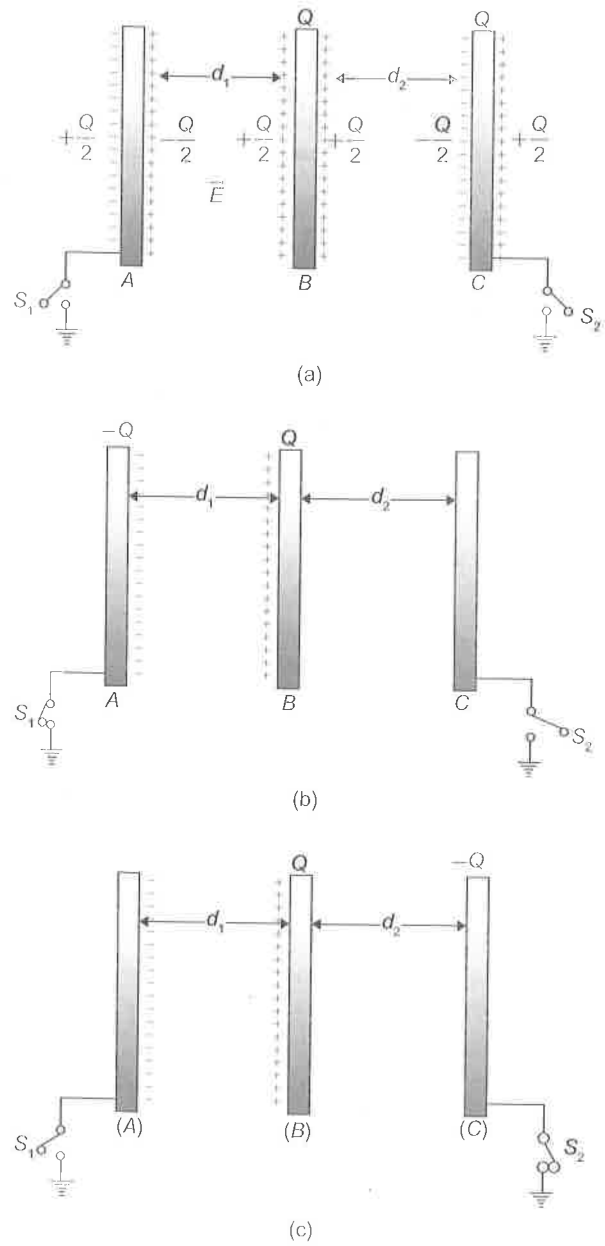


Figure 4.64

If instead of switch S_1, S_2 is closed in the beginning the distribution of charges on the system will be obviously as shown according to Fig. 4.64(c) and a charge $-Q$ now flows through switch S_2 from earth to plate C.

If we close both the switches simultaneously, the situation will be according to the figure shown. Now the charge on plate B is distributed on the two surfaces as shown and equal and oppsoite charges $-q_1$ and $-q_2$ are developed on the inner surfaces of plates A and C.

Here charges q_1 and q_2 can be calculated by equating the potential difference of plates A and B and C and B as

$$V_B - V_A = V_B - V_C$$

Here the electric field between plates can be given as

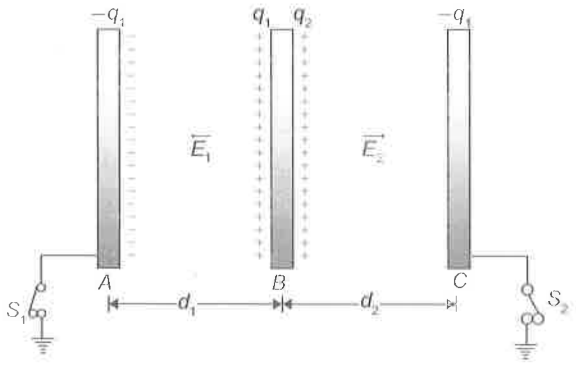


Figure 4.65

Between plates A and B

$$E_1 = \frac{q_1}{A\varepsilon_0}$$

Between plates B and C

$$E_2 = \frac{q_2}{A\varepsilon_0}$$

Now we have

$$V_{BA} = V_{BC}$$

$$\frac{q_1}{A\varepsilon_0}d_1 = \frac{q_2}{A\varepsilon_0}d_2$$

or $$q_1 d_1 = q_2 d_2$$

And we have

$$q_1 + q_2 = Q$$

Thus on solving we get

$$q_1 = \frac{Qd_2}{d_1 + d_2}$$

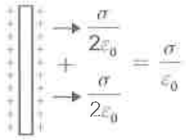
and $$q_2 = \frac{Qd_1}{d_1 + d_2}$$

Thus if both the switches are closed simultaneously, charges $-q_1$ and $-q_2$ will flow through the switches S_1 and S_2 from each of plates A and C.

SOLVED EXAMPLES

EXAMPLE 18

When a charge is given to a conducting plate, the charge distributes itself on two surfaces.



SOLUTION

$\frac{\sigma}{2\varepsilon_0}$ is the $\vec{E}$ due to a single layer of charge but as in the case of conducting sheet there is generation of two surfaces or two layers of charges.

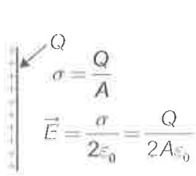
$\therefore$ Electric field outside the conducting plate is $\frac{\sigma}{\varepsilon_0}$. ■

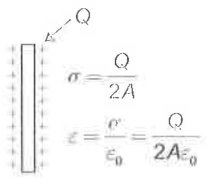
EXAMPLE 19

When a charge Q is given to non-conducting and conducting plates, find the ratio of electric field produced by them?

SOLUTION

For non-conducting plate For conducting plate





$$\text{Ratio} = \frac{\frac{Q}{2A\varepsilon_0}}{\frac{Q}{2A\varepsilon_0}} = 1:1$$ ■

TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGES

Total electrostatic potential energy of system of charges can be given as

$U = \Sigma$ self-energy of all charged bodies $+ \ \Sigma$ interaction energy of all pairs of charged bodies

Let us consider some cases to understand this concept. Figure 4.66 shows two uniformly charged non-conducting spheres of radii R_1 and R_2 and charged with charges Q_1 and Q_2, respectively, separated by a distance r. If we find the total electrostatic energy of this system, we can write as

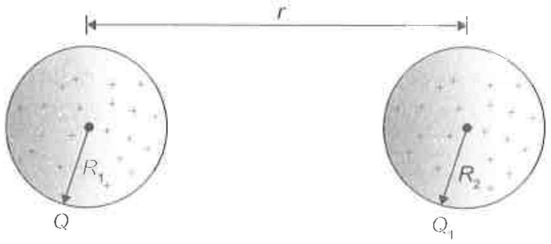


Figure 4.66

$$U = U_{\text{self}} + U_{\text{interaction}}$$

$$U = \frac{3KQ_1^2}{5R_1} + \frac{3KQ_2^2}{5R_2} + \frac{KQ_1Q_2}{r}$$

ELECTROSTATIC ENERGY OF A SYSTEM OF CONCENTRIC SHELLS

Figure 4.67 shows two concentric shells of radii a and b charged uniformly with charges q_1 and q_2. Here the total energy of this system can be given as

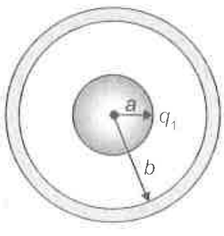


Figure 4.67

U_{total} = self-energy of inner shell + self-energy of outer shell + interaction energy of the two shells

$$= \frac{Kq_1^2}{2a} + \frac{Kq_2^2}{2b} + \frac{Kq_1q_2}{b}$$

Alternative Method:

We know that the total electrostatic energy of a system is stored in the form of field energy of the system. Hence we can calculate the total electrostatic energy of the system by integrating the field energy density in the space surrounding the shells where electric field exists.

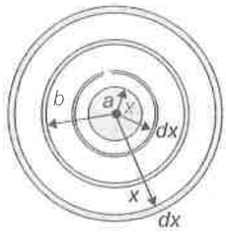


Figure 4.68

Total field energy in the electric field associated with the system shown in Fig. 4.68 can be given as

$$U = \int_0^b \frac{1}{2}\varepsilon_0 \left(\frac{Kq_1}{x^2}\right)^2 4\pi x^2 dx$$

$$+ \int_0^\infty \frac{1}{2}\varepsilon_0 \left(\frac{K(q_1+q_2)}{r^2}\right)^2 4\pi x^2 dx$$

$$= \frac{1}{2}Kq_1^2\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{1}{2}K(q_1+q_2)^2\left[\frac{1}{b}\right]$$

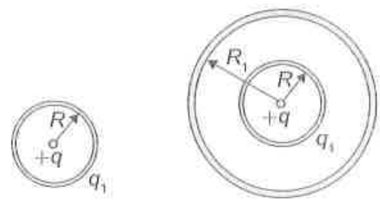
$$= \frac{1}{2}\frac{Kq_1^2}{a} - \frac{Kq_1^2}{2b} + \frac{Kq_1^2}{b} + \frac{Kq_2^2}{2b} + \frac{Kq_2^2}{b}$$

$$= \frac{Kq_1^2}{2b} + \frac{Kq_2^2}{2b} + \frac{Kq_1q_2}{b}$$

SOLVED EXAMPLES

EXAMPLE 20

Figure shows a shell of radius R having charge q_1 uniformly distributed over it. A point charge q is placed at the centre of the shell. Find work required to increase radius of shell from R to R_1 as shown in Fig. (b).



(a) (b)

SOLUTION

$$\text{Work} = U_i - U_f$$

$$U_i = SE_q + SE_{q_1} + IE$$

$$= SE_q + \frac{Kq_1^2}{2R} + \frac{Kq_1q}{R}$$

$$U_f = SE_q + \frac{Kq_1^2}{2R_1} + \frac{Kq_1q}{R_1}$$

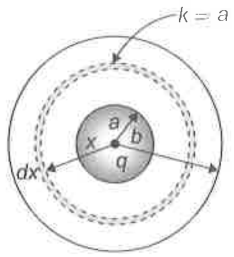
$$\text{Work done} = U_i - U_f$$

$$= \frac{Kq_1^2}{2R} + \frac{Kq_1q}{R} - \frac{Kq_1^2}{2R_1} - \frac{Kq_1q}{R_1}$$

(Try this problem by yourself using the energy density formula)

EXAMPLE 21

A point charge $q = 3\mu C$ is located at the centre of the spherical layer of uniform isotropic dielectric with relative permittivity $k = 3$. The inside radius of the layer is equal to $a = 250$ mm and the outside radius is $b = 500$ mm. Find the electrostatic energy inside the dielectric layer.



SOLUTION

Consider a small elemental shell of thickness dx.

$$\text{Volume} = dV = 4\pi x^2 dx$$

$$\text{Electric field at } x = \frac{Kq}{x^2 K}$$

$$\text{Electric energy density} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{K^2q^2}{x^4K^2}$$

Thus, energy content in the element shell is

$$= dE = \frac{1}{2}\varepsilon_0 \frac{K^2q^2}{x^4K^2} 4\pi x^2 dx$$

$$E = \int_a^b \frac{4}{2}\varepsilon_0 \frac{K^2\pi q^2}{K^2x^4} x^2\, dx$$

$$= \frac{q^2}{2}\frac{K}{R^2}\int_a^b \frac{1}{x^2} dx$$

$$= \frac{Kq^2}{2R^2}\left[\frac{1}{b} - \frac{1}{a}\right]$$

EXAMPLE 22

Find the electrostatic energy stored in a cylindrical shell of length λ, inner radius a and outer radius b, coaxial with a uniformly charged wire with linear charge density λ C/m.

SOLUTION

For this, we consider an elemental shell of radius x and width dx. The volume of this shell dV can be given as

$$dV = 2\pi x\lambda \cdot dx$$

The electric field due to the wire at the shell is

$$E = \frac{2K\lambda}{x}$$

The electrostatic field energy stored in the volume of this shell is

$$dU = \frac{1}{2}\varepsilon_0 E^2 \cdot dv$$

or $$dU = \frac{1}{2}\varepsilon_0 \left(\frac{2K\lambda}{x}\right)^2 2\pi x\lambda\, dx$$

The total electrostatic energy stored in the above mentioned volume can be obtained by integrating the above expression within limits from a to b as

$$U = \int dU$$

$$= \int_b^a \frac{1}{2}\varepsilon_0 \left(\frac{2K\lambda}{x}\right) 2\pi x\ell\, dx$$

or $$U = \frac{\lambda^2\ell}{4\pi\varepsilon_0}\int_a^b \frac{1}{x} dx$$

or $$U = \frac{\lambda^2\ell}{4\pi\varepsilon_0}\int_a^b \frac{1}{x} dx$$

or $$U = \frac{\lambda^2\ell}{4\pi\varepsilon_0}\ell n\left(\frac{b}{a}\right)$$

EXERCISES

JEE Main

1. In a region of space, the electric field is in the x direction and is given as. $\vec{E} = E_0 x\hat{i}$ Consider an imaginary cubical volume of edge a, with its edges parallel to the axes of coordinates. The charge inside this volume is

(A) zero (B) $\varepsilon_0 E_0 a^3$

(C) $\frac{1}{\varepsilon_0} E_0 a^3$ (D) $\frac{1}{6}\varepsilon_0 E_0 a^2$

2. Electric flux through a surface of area 100 m^2 lying in the xy plane is (in $V\alpha\ m$) if $\vec{E} = \hat{i} + \sqrt{2}\hat{j} + \sqrt{3}\hat{k}$

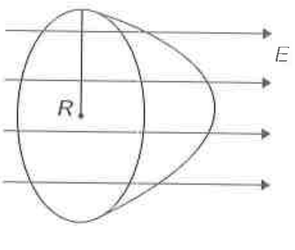
(A) 100 (B) 141.4
(C) 173.2 (D) 200

3. A cylinder of radius (R) and length (L) is placed in a uniform electrical field (E) parallel to the axis of the cyclinder. the total flux for the surface of the cylinder is given by

(A) $2\pi R^2 E$ (B) $\pi R^2 E$

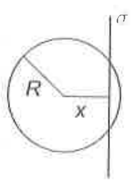
(C) $\frac{\pi R^2 + \pi R^2}{E}$ (D) zero

4. A hemisphere (radius R) is placed in electric field as shown in figure. Total outgoing flux is



(A) $\pi R^2 E$ (B) $2\pi R^2 E$
(C) $4\pi R^2 E$ (D) $(\pi R^2 E)/2$

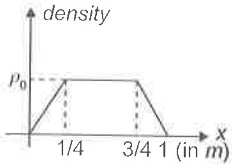
5. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux Φ through the Gaussian surface is



(A) $\frac{\pi R^2 \sigma}{\varepsilon_0}$ (B) $\frac{2\pi(R^2 - x^2)}{\varepsilon_0}$

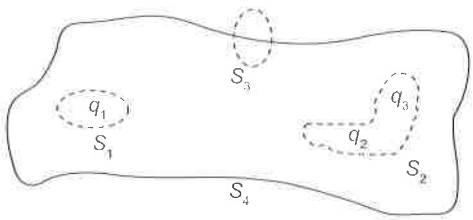
(C) $\frac{\pi(R-x)^2 \sigma}{\varepsilon_0}$ (D) $\frac{\pi(R^2 - x^2)\sigma}{\varepsilon_0}$

6. The volume charge density as a function of distance X from one face inside a unit cube is varying as shown in the figure. Then the total flux (in S.I. units) through the cube if ($r_0 = 8.85 \times 10^{-12}$ C/m^3) is



(A) 1/4 (B) ½
(C) 3/4 (D) 1

7. Three charges $q_1 = 1$, $q_2 = 2$ and $q_3 = -3$ and four surfaces S_1, S_2, S_3 and S_4 are shown. The flux emerging through surface S_2 in N-m^2/ C is



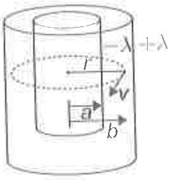
(A) $36\pi \times 10^3$ (B) $-36\pi \times 10^3$
(C) $36\pi \times 10^9$ (D) $-36\pi \times 10^9$

8. A surface enclosed an electric dipole, the flux through the surface is

(A) Infinite (B) Positive
(C) Negative (D) Zero

9. Figure shows two large cylindrical shells having uniform linear charge densities $+\lambda$ and $-\lambda$. Radius

of inner cylinder is 'a' and that of outer cylinder is 'b'. A charged particle of mass m, charge q revolves in a circle of radius r, Then its speed 'v' is: (Neglect gravity and assume the radii of both the cylinders to be very small in comparison to their length.)

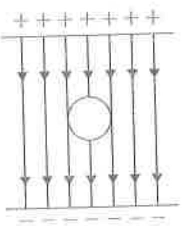


(A) $\sqrt{\dfrac{\lambda q}{2\pi \in_0 m}}$ (B) $\sqrt{\dfrac{2\lambda q}{\pi \in_0 m}}$

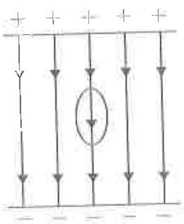
(C) $\sqrt{\dfrac{\lambda q}{\pi \in_0 m}}$ (D) $\sqrt{\dfrac{\lambda q}{4\pi \in_0 m}}$

10. An uncharged sphere of metal is placed in a uniform electric field produced by two large conducting parallel plates having equal and opposite charges, then lines of force look like:

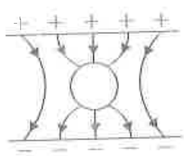
(A)



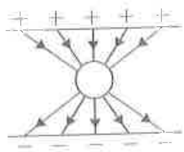
(B)



(C)



(D)



11. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is

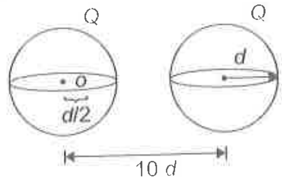
(A) 0 V

(B) 10 V

(C) same as at point 5 cm away from the surface out side sphere

(D) same as a point 25 cm away from the surface

12. Two spherical, nonconducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance $10d$ from each other. A positive point charge q is placed inside one of the shells at a distance $d/2$ from the center, on the line connecting the centers of the two shells, as shown in the figure. What is the net force on the charge q?



(A) $\dfrac{qQ}{361\pi\varepsilon_0 d^2}$ to the left

(B) $\dfrac{qQ}{361\pi\varepsilon_0 d^2}$ to the right

(C) $\dfrac{362qQ}{361\pi\varepsilon_0 d^2}$ to the left

(D) $\dfrac{360qQ}{361\pi\varepsilon_0 d^2}$ to the right

13. Potential difference between centre and the surface of sphere of radius R and uniform volume charge density ρ within it will be

(A) $\dfrac{\rho R^2}{6 \in_0}$ (B) $\dfrac{\rho R^2}{4 \in_0}$

(C) 0 (D) $\dfrac{\rho R^2}{2 \in_0}$

14. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre ?

(A) R (B) $R/2$

(C) $R/3$ (D) $2R$

15. Two similar conducting spherical shells having charges $40\ \mu C$ and $-20\mu C$ are some distance apart. Now they are touched and kept at same distance. The ratio of the initial to the final force between them is:

(A) 8:1 (B) 4:1

(C) 1:8 (D) 1:1

16. n small drops of same size are charged to V volts each. If they coalesce to form a signal large drop, then its potential will be -

(A) V/n (B) Vn

(C) $Vn^{1/3}$ (D) $Vn^{2/3}$

17. 1000 identical drops of mercury are charged to a potential of 1 V each. They join to form a single drop. The potential of this drop will be

(A) 0.01 V (B) 0.1 V

(C) 10 V (D) 100 V

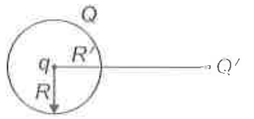
18. A positively charged body 'A' has been brought near a neutral brass sphere B mounted on a glass stand as shown in the figure. The potetial of B will be:

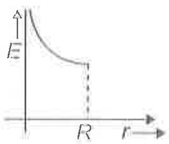
(A) Zero (B) Negative

(C) Positive (D) Infinite

19. A charge 'q' is placed at the centre of a conducting spherical shell of radius R, which is given a charge Q. An external charge Q' is also present at distance R' ($R' > R$) from 'q'. Then the resultant field will be best represented for region $r > R$ by:

[where r is the distance of the point from q]



(A)


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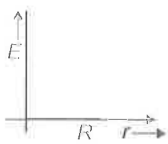
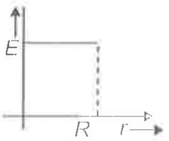


(C)


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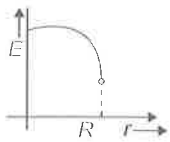
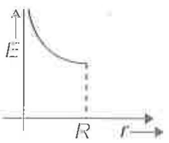
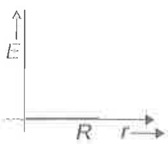
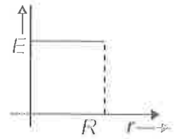
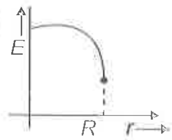


20. In the above questins, if Q' is removed then which option is correct:

(A)


(B)


(C)


(D)


21. The net charge given to an isolated conducting solid sphere:

(A) must be distributed uniformly on the surface
(B) may be distributed uniformly on the surface
(C) must be distributed uniformly in the volume
(D) may be distributed uniformly in the volume.

22. The net charge given to a solid insulating sphere:

(A) must be distributed uniformly in its volume
(B) may by distributed uniformly in its volume.
(C) must be distributed uniformly on its surface.
(D) the distribution will depend upon whether other charges are present or not.

23. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $3Q$ the new potential difference between the same two surfaces is

(A) V (B) 2V

(C) 4V (D) −2V

24. Three concentric conducting spherical shells carry charges as follows $+4Q$ on the inner shell, $-2Q$ on the middle shell and $-5Q$ on the outer shell. The charge on the inner surface of the outer shell is:

(A) 0 (B) $4Q$

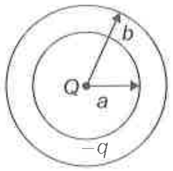
(C) $-Q$ (D) $-2Q$

25. Three concentric metallic spherical shell A, B and C or radii a, b and c ($a < b < c$) have surface charge densities $-\sigma$, $+\sigma$, and $-\sigma$ respectively. The potential of shell A is

(A) $(\sigma/e_0)[a + b - c]$ (B) $(\sigma/e_0)[a - b + c]$

(C) $(\sigma/e_0)[b - a - c]$ (D) none

26. Both question (a) and (b) refer to the system of charges as shown in the figure. A spherical shell with an inner radius 'a' and an outer radius 'b' is made of conducting material. A point charge $+Q$ is placed at the centre of the spherical shell and a total charge $-q$ is placed on the shell.

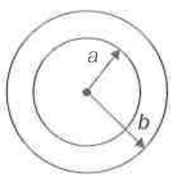


(i) charge $-q$ is distributed on the surfaces as
(A) $-Q$ on the inner surface, $-q$ on outer surface
(B) $-Q$ on the inner surface, $-q+Q$ on the outer surface
(C) $+Q$ on the inner surface, $-q-Q$ on the outer surface
(D) The charge $-q$ is spread uniformly between the inner and outer surface

(ii) Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance $R(a < R < b)$ from the centre of the shell is
(A) 0
(B) $\frac{KQ}{a}$
(C) $K\frac{Q-q}{R}$
(D) $K\frac{Q-q}{b}$ (where $K = \frac{1}{4\pi \in_0}$)

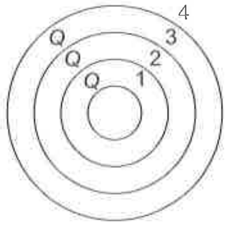
27. A positive charge q is placed in a spherical cavity made in a positively charged sphere. The centres of sphere and cavity are displaced by a small distance $\vec{l}$. Force on charge q is:
(A) in the direction parallel to vector $\vec{l}$
(B) in radial direction
(C) in a direction which depends on the magnitude of charge density in sphere
(D) direction cannot be determined

28. If the electric potential of the inner metal sphere is 10 V and that of the outer shell is 5 V, then the potential at the centre will be



(A) 10 V
(B) 5 V
(C) 15 V
(D) 0

29. An infinite number of concentric rings carry a charge Q each alternately positive and negative. Their radii are 1, 2, 4, 8... meters in geometric progression as shown in the figure. The potential at the centre of the rings will be



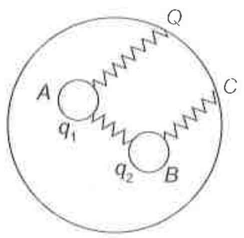
(A) zero
(B) $\frac{Q}{12\pi\varepsilon_0}$
(C) $\frac{Q}{8\pi\varepsilon_0}$
(D) $\frac{Q}{6\pi\varepsilon_0}$

30. A charge Q is kept at the centre of a conducting sphere of inner radius R_1 and outer radius R_2. A point charge q is kept at a distance $r(>R_2)$ from the centre. If q experiences an electrostatic force 10 N then assuming that no other charges are present, electrostatic force experienced by Q will be:
(A) -10 N
(B) 0
(C) 20 N
(D) none of these

31. A solid metallic sphere has a charge $+3Q$. Concentric with this sphere is a conducting spherical shell having charge $-Q$. The radius of the sphere is a and that of the spherical shell is b $(>a)$. What is the electric field at a distance r $(a < r < b)$ from the centre?
(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
(B) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{r}$
(C) $\frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$
(D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

32. Two identical conducting spheres, having charges of opposite sign, attract each other with a force of 0.108 N when separated by 0.5 m. The spheres are connected by a conducting wire, which is then removed, and thereafter, they repel each other with a force of 0.036 N. The initial charges on the spheres are
(A) $\pm 5 \times 10^{-6}$ C and $\mp 15 \times 10^{-6}$ C
(B) $\pm 1.0 \times 10^{-6}$ C and $\mp 3.0 \times 10^{-6}$ C
(C) $\pm 2.0 \times 10^{-6}$ C and $\mp 6.0 \times 10^{-6}$ C
(D) $\pm 0.5 \times 10^{-6}$ C and $\mp 1.5 \times 10^{-6}$ C

33. Two small conductors A and B are given charges q_1 and q_2 respectively. Now they are placed inside a hollow metallic conductor (C) carrying a charge Q. If all the three conductors A, B and C are connected by a conducting wire as shown, the charges on A, B and C will be respectively.



(A) $\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2}, Q$

(B) $\frac{Q+q_1+q_3}{3}, \frac{Q+q_1+q_2}{3}, \frac{Q+q_1+q_2}{3}$

(C) $\frac{q_1+q_2+Q}{2}, \frac{q_1+q_2+Q}{3}, 0$

(D) $0, 0, Q+q_1+q_2$

34. There are four concentric shells A, B, C and D of radii a, $2a$, $3a$ and $4a$ respectively. Shells B and D are given charges $+q$ and $-q$ respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is:

(A) $\frac{Kq}{2a}$ (B) $\frac{Kq}{3a}$

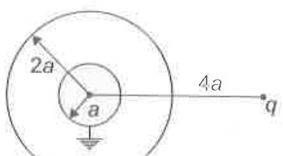
(C) $\frac{Kq}{4a}$ (D) $\frac{Kq}{6a}$

35. You are travelling in a car during a thunder storm. In order to protect yoursefl from lightening, would you prefer to:

(A) Remain in the car
(B) Take shelter under a tree
(C) Get out and be flat on the ground
(D) Touch the nearest electrical pole

Comprehension

A solid conducting sphere of radius 'a' is surronded by a thin uncharged concentric conducting shell of radius $2a$. A point charge q is placed at a distance $4a$ from common centre of conducting sphere and shell. The inner sphere is then grounded.



36. The charge on solid sphere is:

(A) $-\frac{q}{2}$ (B) $-\frac{q}{4}$

(C) $-\frac{q}{8}$ (D) $-\frac{q}{16}$

37. Pick up the correct statement:

(A) Charge on surface on inner sphere is non-uniformly distributed
(B) Charge on inner surface of outer shell in non-uniformly distributed.
(C) Charge on outer surface of outer shell is non-uniformly distributed.
(D) All the above statement are false.

38. The potential of outer shell is:

(A) $\frac{q}{32\pi\varepsilon_0 a}$ (B) $\frac{q}{16\pi\varepsilon_0 a}$

(C) $\frac{q}{8\pi\varepsilon_0 a}$ (D) $\frac{q}{4\pi\varepsilon_0 a}$

Reasoning Type Question

39. **Statement 1:** If a concentric spherical Gaussian surface is drawn inside thin spheical shell of charge, electric field (E) at each point of surface must be zero.

Statement 2: In accordance with Gauss's law

$$\phi_E = \oint \vec{E}.d\vec{A} = \frac{Q_{\text{net enclosed}}}{\varepsilon_0}$$

$$Q_{\text{net enclosed}} = 0 \text{ implies } \phi_E = 0$$

(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

40. **Statement 1:** Electric field of a dipole can't be found using only Gauss law. (i.e. without using superposition principle)

Statement 2: Gauss law is valid only for symmetrical charge distribution

(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.

(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

41. **Statement 1:** In a given situation of arrangement of charges, an extra charge is placed outside the Gaussian surface. In the Gauss Theorem

$$\oint \vec{E}.d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Q_{in} remains unchanged whereas electric field $\vec{E}$ at the site of the element is changed.

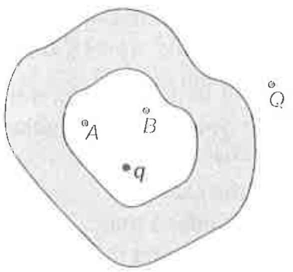
Statement 2: Electric field $\vec{E}$ at any point on the Gaussian surface is due to inside charge only.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

42. **Statement 1:** The flux crossing through a closed surface is independent of the location of encloses charge.
Statement 2: Upon the displacement of charges within a closed surface, the $\vec{E}$ at any point on surface does not change.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

43. The electrostatic potential on the surface of a charged solid conducting sphere is 100 V. Two statements are made in this regard
Statement 1: At any point inside the sphere, electro-static potetial is 100 V.
Statement 2: At any point inside the sphere, electric field is zero.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

44. When two charged concentric spherical conductors have electric potential V_1 and V_2 respectively
Statement 1: The potential at centre is $V_1 + V_2$
Statement 2: Potential is scalar quantity.
(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

45. **Statement 1:** A point charge q is placed inside a cavity of conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q pushed away from conductor, the potential difference $(V_A - V_B)$ between two point A and B within the cavity of sphere remains constant.
Statement 2: The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor

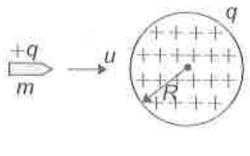


(A) Statement 1 is true, Statement 2 is true and statement 2 is correct explanation for statement 1.
(B) Statement 1 is true, Statement 2 is true and statement 2 is NOT correct explanation for statement 1.
(C) Statement 1 is true, statement 2 is false.
(D) Statement 1 is false, statement 2 is true.

JEE Advanced

Single Correct

1. A bullet of mass m and charge q is fired towards a solid uniformly charged sphere of radius R and total charge $+q$. If it strikes the surface of sphere with speed u, find the minimum speed u so that it can penetrate through the sphere. (Neglect all resistance forces or friction acting on bullet except electrostatic forces)



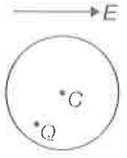
(A) $\frac{q}{\sqrt{2\pi\varepsilon_0 mR}}$ (B) $\frac{q}{\sqrt{4\pi\varepsilon_0 mR}}$

(C) $\frac{q}{\sqrt{8\pi\varepsilon_0 mR}}$ (D) $\frac{\sqrt{3}q}{\sqrt{2\pi\varepsilon_0 mR}}$

2. A unit positive point charge of mass m is projected with a velocity V inside the tunnel as shown. The tunnel has been made inside a uniformly charged nonconducting sphere. The minimum velocity with which the point charge should be projected such it can it reach the opposite end of the tunnel, is equal to

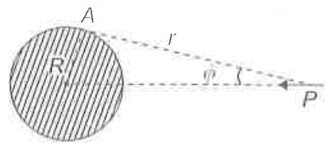
(A) $[rR^2/4me_0]^{1/2}$
(B) $[rR^2/24me_0]^{1/2}$
(C) $[rR^2/6me_0]^{1/2}$
(D) zero because the initial and the final points are at same potential.

3. A positive point charge Q is kept (as shown in the figure) inside a neutral conducting shell whose centre is at C. An external uniform electric field E is applied. Then:



(A) Force on Q due to E is zero
(B) Net force on Q is zero
(C) Net force acting on Q and conducting shell considered as a system is zero
(D) Net force acting on the shell due to E is zero.

4. A dipole having dipole moment p is placed in front of a solid uncharged conducting sphere as shown in the diagram. The net potential at point. A lying on the surface of the sphere is;



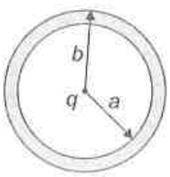
(A) $\frac{kp\cos\phi}{r^2}$ (B) $\frac{kp\cos^2\phi}{r^2}$

(C) zero (D) $\frac{2kp\cos^2\phi}{r^2}$

5. Two uniformly charged non-conducting hemispherical sheels each having uniform charge density σ and radius R form a complete sphere (not stuck together) and surround a concentric spherical conducting shell of radius $R/2$. If hemispherical parts are in equilibrium then minimum surface charge density of inner conducting shell is:

(A) -2σ (B) $-\sigma/2$
(C) $-\sigma$ (D) 2σ

6. A point charge q is borught from infinity (slowly so that heat developed in the shell is negligible) and is placed at the centre of a conducting neutral spherical shell of inner radius a and outer radius b, then work done by external agent is:



(A) 0 (B) $\frac{kq^2}{2b}$

(C) $\frac{kq^2}{2b}-\frac{kq^2}{2a}$ (D) $\frac{kq^2}{2a}-\frac{kq^2}{2b}$

Multiple Correct

7. Units of electric flux are

(A) $\frac{N - m^2}{Coul^2}$ (B) $\frac{N}{Coul^2 - m^2}$

(C) volt–m (D) Volt–m^3

8. An electric dipole is placed at the centre of a sphere. Mark the correct answer
   (A) the flux of the electric field through the sphere is zero
   (B) the electric field is zero at every point of the sphere.Ex
   (C) the electric potential is zero everywhere on the sphere.
   (D) the electric potential is zero on a circle on the surface.

9. Which of the following statements are correct?
   (A) Electric field calculated by Gauss law is the field due to only those charges which are enclosed inside the Gaussian surface.
   (B) Gauss law is applicable only when there is a symmetrical distribution of charge.
   (C) Electric flux through a closed surface will depends only on charges enclosed within that surface only.
   (D) None of these

10. Mark the correct options
    (A) Gauss's law is valid only for uniform charge distributions.
    (B) Gauss's law is valid only for charges placed in vacuum.
    (C) The electric field calculated by Gauss's law is the field due to all the charges.
    (D) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

11. Charges Q_1 and Q_2 lies inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S.
    (A) If Q_1 changes, both E and ϕ will change.
    (B) If Q_2 changes, E will change but ϕ will not change.
    (C) If $Q_1 = 0$ and $q_2 \neq 0$ then $E \neq 0$ but $\phi = 0$.
    (D) If $Q_1 \neq 0$ and $Q_2 = 0$ then $E = 0$ but $\phi \neq 0$.

12. An electric field converges at the origin whose magnitude is given by the expression $E = 100r$ Nt/Coul, where r is the distance measured from the origin.
    (A) total charge contained in any spherical volume with its centre at origin in negative.
    (B) total charge contained at any spherical volume, irrespective of the location of its centre, is negative.
    (C) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-13}C.
    (D) total charge contained in a spherical volume of radius 3 cm with its centre at origin has magnitude 3×10^{-9} Coul.

13. A conducting sphere of radius r has a charge. Then
    (A) The charge is uniformly distributed over its surface, if there is an external electric field.
    (B) Distribution of charge over its surface will be non unifrom if no external electric field exist in space.
    (C) Electric field strength inside the sphere will be equal to zero only when no external electric field exists.
    (D) Potential at every point of the sphere must be same

14. For a spherical shell
    (A) If potential inside it is zero then it necessarily electrically neutral
    (B) electric field in a charged conducting spherical shell can be zero only when the charge is uniformly distributed
    (C) electric potential due to induced charges at a point inside it will always be zero
    (D) none of these

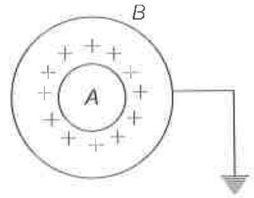
15. At distance of 5 cm and 10 cm outwards from the surface of a uniformly charged solid sphere, the potentials are 100 V and 75 V respectively. Then
    (A) potential at its surface is 150 V
    (B) the charge on the sphere is $(5/3) \times 10^{-10}$ C
    (C) the electric field on the surface is 1500 V/m
    (D) the electric potential at its centre is 225 V

16. A thin-walled, spherical conducting shell S of radius R is given charge Q. The same amount of charge is also placed at its centre C. Which of the following statements are correct ?
    (A) On the outer surface of S, the charge density is $\frac{Q}{2\pi R^2}$

(B) The electric field is zero at all points inside S.
(C) At a point just outside S, the electric field is double the field at a point just inside S.
(D) At any point inside S, the electric field is inversely proportional to the square of its distance from C.

17. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct ?
(A) The entire charge will appear on its outer surface.
(B) All points on the conductor will have the same potential
(C) All points on its surface will have the same charge density.
(D) All points near its surface and outside it will have the same electric intensity.

18. A and B are two conducting concentric spherical shells. A is given a charge Q while B is uncharged. If now B is earthed as shown in figure. Then:



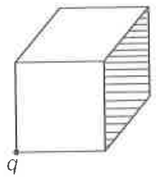
(A) The charge appearing on inner surface of B is $-Q$
(B) The field inside the outside A is zero.
(C) The field between A and B is not zero.
(D) The charge appearing on outer surface of B is zero.

JEE Advanced

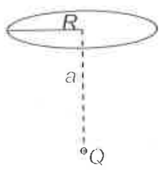
Level I

1. What do you predict by the given statement about the nature of charge (positive or negative) enclosed by the close surface. "In a close surface lines which are leaving the surface are double then the lines which are entering in it."

2. The length of each side of a cubical closed surface is l. If charge q is situated on one of the vertices of the cube, then find the flux passing through shaded face of the cube.



3. A point charge Q is located on the axis of a disc of radius R at a distance a from the plane of the disc. If one fourth ($1/4^{th}$) of the flux from the charge passes through the disc, then find the relation between a and R.



4. A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

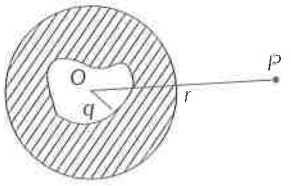
5. A very long uniformly charged thread oriented along the axis of a circle of radius R rests on its centre with one of the ends. The charge on the thread per unit length is equal to λ. Find the flux of the vector E through the circle area.

6. A particle of mass m and charge $-q$ moves along a diameter of a uniformly charged sphere of radius R and carrying a total charge $+Q$. Find the frequency of S.H.M. of the particle if the amplitude does not exceed R.

7. There are 27 drops of a conducting fluid. Each has radius r and they are charged to a potential V_0. They are then combined to form a bigger drop. Find its potential.

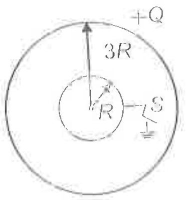
8. There are two concentric metal shells of radii r_1 and r_2 ($> r_1$). If initially the outer shell has a charge q and the inner shell is having zero charge. Now inner shell is grounded. Find:
(i) Charge on the inner surface of outer shell.
(ii) Final charges on each sphere.
(iii) Charge flown through wire in the ground.

9. A point charge 'q' is within an electrically neutral conducting shell whose other surface has spherical shape. Find potential V at point P lying outiside shell at a distance 'r' from centre O of outer sphere.



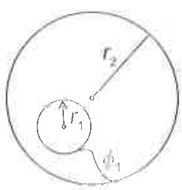
10. Consider two concentric conducting spheres of radii a and b ($b > a$). Inside sphere has a positive charge q_1. What charge should be given to the outer sphere so that potential of the inner sphere becomes zero? How does the potential varies between the two spheres and outside ?

11. Two thin conducting shells of radii R and $3R$ are shown in figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of switch S. Find the charge attained by the inner shell.

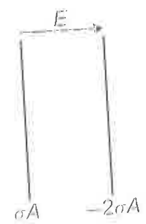


12. Consider three identical metal spheres A, B and C. Spheres A carries charge $+6q$ and sphere B carries charge $-3q$. Sphere C carries no charge. Spheres A and B are touched together and then separated. Sphere C is then touched to sphere A and separated from it. Finally the sphere C is touched to sphere B and separated from it. Find the final charge on the sphere C.

13. A metal sphere of radius r_1 charged to a potential V_1 is than placed in a thin-walled uncharged conducting spherical shell of radius r_2. Determine the potential acquired by the spherical shell after it has been connected for a short time to the sphere by a conductor.



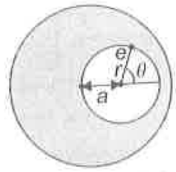
14. Two thin conducting plates (very large) parallel to each other carrying total charges σA and $-2\sigma A$ respectively (where A is the area of each plate), are placed in a uniform external electric field E as shown. Find the surface charge on each surface.



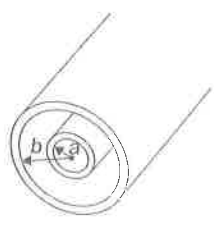
Level II

1. A positive charge Q is uniformly distributed throughout the volume of a dielectric sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity v from a point at distance r ($r > R$) from the centre of the sphere. Find the minimum velocity v so that it can penetrate $R/2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

2. A cavity of radius r is present inside a solid dielectric sphere of radius R, having a volume charge density of ρ. The distance between the centres of the sphere and the cavity is a. An electron e is kept inside the cavity at an angle $\theta = 45°$ as shown. How long will it take to touch the sphere again ?



3. Figure shows a section through two long thin concentric cylinders of radii a and b with $a < b$. The cylinders have equal and opposite charges per unit length λ. Find the electric field at a distance r from the axis for -



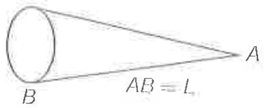
(A) $r < a$ (B) $a < r < b$

(C) $r > b$

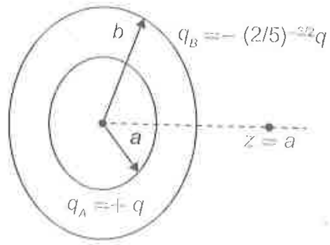
4. A solid non conducting sphere of radius R has a non-uniform charge distribution of volume charge density, $\rho = \rho_0 \frac{r}{R}$, where ρ_0 is a constant and r is the distance from the centre of the sphere. Show that
   (a) the total charge on the sphere is $Q = \pi\rho_0 R^3$ and
   (b) the electric field inside the sphere has a magnitude given by, $E = \frac{KQr^2}{R^4}$

5. An electron beam after being accelerated from rest through a potential difference of 500 V in vacuum is allowed to impinge normally on a fixed surface. If the incident current is 100 μA, determine the force exerted on the surface assuming that it brings the electrons to rest. ($e = 1.6 \times 10^{-19}$ C; $m = 9.0 \times 10^{-31}$ kg)

6. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to take a test charge q from infinity to apex A of cone. The slant length is L.



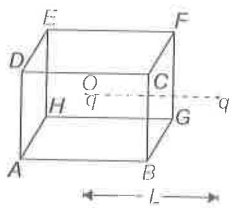
7. Two concentric rings, one of radius 'a' and the other of radius 'b' have the charges $+q$ and $-(2/5)^{-3/2}\,q$ respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at $z = a$ is in equilibrium.



Previous Year Questions

JEE Main

1. A charged particle q is placed at the centre O of cube of length $L(ABCDEFGH)$. Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is **(AIEEE 2002)**



(A) $\frac{q}{4\pi\varepsilon_0 L}$ (B) zero

(C) $\frac{q}{2\pi\varepsilon_0 L}$ (D) None

2. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **(AIEEE 2003)**

(A) $(\phi_2 - \phi_2)\varepsilon_0$ (B) $\frac{(\phi_1 + \phi_2)}{\varepsilon_0}$

(C) $\frac{(\phi_2 - \phi_1)}{\varepsilon_0}$ (D) $(\phi_1 + \phi_2)\varepsilon_0$

3. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre for the shell is **(AIEEE 2003)**

(A) $\frac{2Q}{4\pi\varepsilon_0 R}$ (B) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$

(C) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (D) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$

4. Two sperical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **(AIEEE 2004)**

(A) $\frac{F}{4}$ (B) $\frac{3F}{4}$

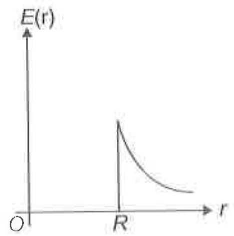
(C) $\frac{F}{8}$ (D) $\frac{3F}{8}$

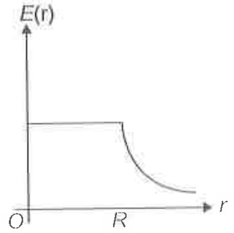
5. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **(AIEEE 2006)**

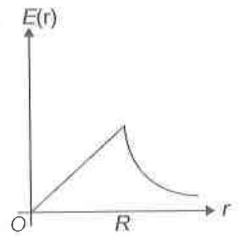
(A) 4:1 (B) 1:2

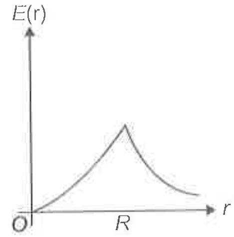
(C) 2:1 (D) 1:4

6. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \leq r < x$, where r is the distance from the centre of the shell ? **(AIEEE 2008)**

(A)


(B)


(C)


(D)


7. Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is **(AIEEE 2009)**

(A) zero (B) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$

(C) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (D) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

8. Let three be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by **(AIEEE 2010)**

(A) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (B) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(C) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (D) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

9. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre a, b are constants. Then the charge density inside the ball is **(AIEEE 2011)**

(A) $-6a\varepsilon_0 r$ (B) $-24\pi a\varepsilon_0$

(C) $-6a\varepsilon_0$ (D) $-24\pi a\varepsilon_0 r$

10. This question has statement 1 and statement 2 of the four choices given after the statements, choose the one that best describes the two statements

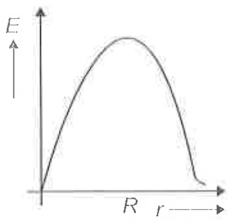
An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero **(AIEEE 2012)**

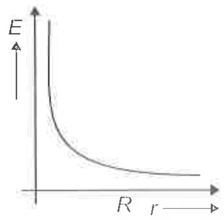
Statement 1 When a charge q is taken from the centre of the surface of the sphere its potential energy changes by. $\frac{q\rho}{3\varepsilon_0}$

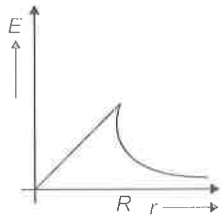
Statement 2 The electric field at a distance r ($r < R$) from the centre of the sphere is. $\frac{\rho r}{3\varepsilon_0}$

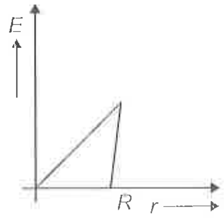
(A) Statement 1 is false, Statement 2 is true.
(B) Statement 1 is true, Statement 2 is false.
(C) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(D) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

11. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be **(AIEEE 2012)**

(A) 

(B) 

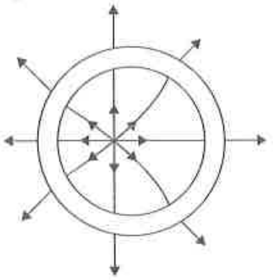
(C) 

(D) 

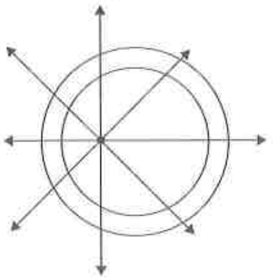
JEE Advanced

1. A point charge 'q' is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct ?

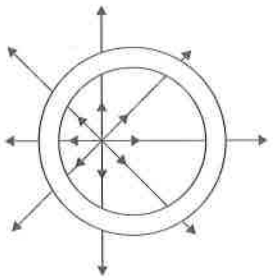
(A)



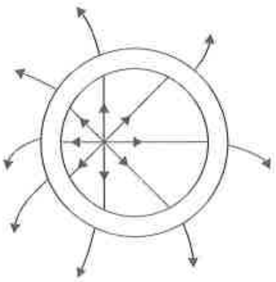
(B)



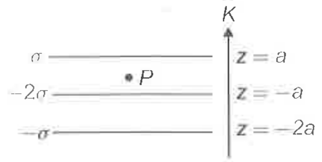
(C)



(D)



2. Three large parallel plates have uniform surface charge densities as shown in the figure. What is the electric field at P. **[JEE (Scr) 2005]**

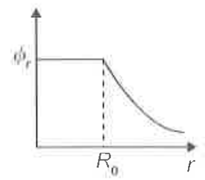


(A) $-\dfrac{4\sigma}{\epsilon_0}\hat{K}$ (B) $\dfrac{4\sigma}{\epsilon_0}\hat{K}$

(C) $-\dfrac{2\sigma}{\epsilon_0}\hat{K}$ (D) $\dfrac{2\sigma}{\epsilon_0}\hat{K}$

3. A conducting liquid bubble of radius a and thickness t ($t << a$) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **[JEE 2005]**

4. The electrostatic potential (f_r) of a spherical symmetric system, kept at origin, is shown in the adjacent figure, and given as **[JEE 2006]**



$$\phi = \frac{q}{4\pi \epsilon_0 r}(r \geq R_0) \quad \phi_r = \frac{q}{4\pi \epsilon_0 R_0}(r \leq R_0)$$

Which of the following option(s) is / are correct ?

(A) For spherical region $r \leq R_0$, total electrostatic energy stored is zero.

(B) Within $r = 2R_0$, total charge is q.

(C) There will be no charge anywhere except at $r = R_0$.

(D) Electric field is discontinuous at $r = R_0$

5. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. **[JEE 2007]**
   (A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder
   (B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder
   (C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinder
   (D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

6. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then, **[JEE 2007]**
   (A) negative and distributed uniformly over the surface of the sphere
   (B) negative and appears only at the point on the sphere closest to the point charge
   (C) negative and distributed non-uniformly over the entire surface of the sphere
   (D) Zero

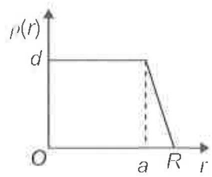
7. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is **[JEE 2007]**

   (A) zero everywhere
   (B) non-zero and uniform
   (C) non-uniform
   (D) zero only at its center

8. **Statement 1**
   For practical purposes, the earth is used as a refrence at zero potential in electrical circuits. **[JEE 2008]**
   and Statement 2
   The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by. $\frac{Q}{4\pi\varepsilon_0 R}$
   (A) Statement 1 is True, Statement 2 is True; Statement 2 **is** a correct explanation for Statement 1
   (B) Statement 1 is True, Statement 2 is True' Statement 2 **is NOT** a correct explanation for Statement 1
   (C) Statement 1 is True, Statement 2 is False
   (D) Statement 1 is False, Statement 2 is True

Question No. 9 to 11

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure The electric field is only along the radial direction. **[JEE 2008]**



Figure

9. The electric field at $r = R$ is
   (A) independent of a
   (B) directly proportional to a
   (C) directly proportional to a^2
   (D) inversely proportional to a

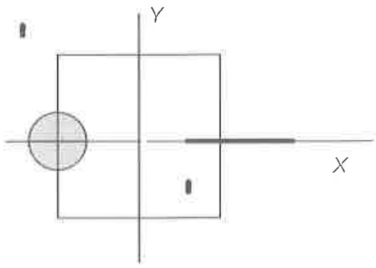
10. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is
   (A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$
   (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

11. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies.
   (A) $a = 0$ (B) $a = \frac{R}{2}$
   (C) $a = R$ (D) $a = \frac{2R}{3}$

12. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charge Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells $Q_1:Q_2:Q_3$, is **[JEE 2009]**
   (A) 1:2:3 (B) 1:3:5
   (C) 1:4:9 (D) 1:8:18

13. A solid sphere of radius R has a charge Q distributed in its volume with a charge dinsity $\rho = Kr^a$, where K and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a. **[JEE 2009]**

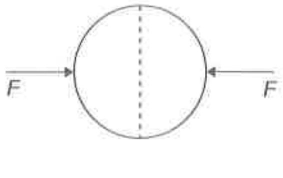
14. A disk of radius $a/4$ having a uniformly distributed charge $6\ C$ is placed in the $x-y$ plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge $8\ C$ is place on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7\ C$ and $3\ C$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is **[JEE 2009]**



(A) $\frac{-2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$

(C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

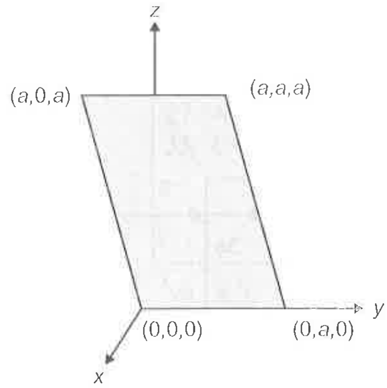
15. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to **[JEE 2010]**



(A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (B) $\frac{1}{\varepsilon_0}\sigma^2 R$

(C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

16. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **[JEE 2011]**



(A) $2E_0a^2$ (B) $\sqrt{2}\,E_0a^2$

(C) E_0a^2 (D) $\frac{E_0a^2}{\sqrt{2}}$

17. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($<R_A$) are kept far apart and each is given charge '$+Q$'. Now they are connected by a thin metal wire. Then **[JEE 2011]**

(A) $E_A^{\text{inside}} = 0$

(B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(D) $E_A^{\text{onsurface}} < E_B^{\text{onsurface}}$

18. Which of the following statement(s) is/are correct? **[JEE 2012]**

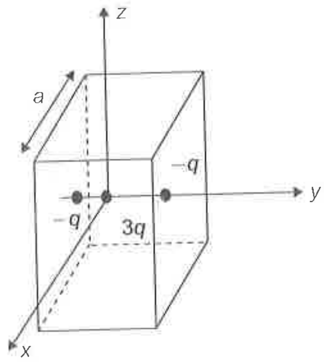
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid

(B) The Gauss law can be used to calculate the field distribution around an electric dipole

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same

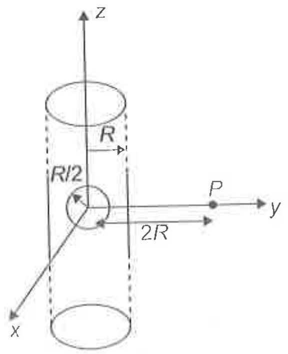
(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

19. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0,-a/4,0)$, $+3q$ at $(0,0,0)$ and $-q$ at $(0,+a/4,0)$. Choose the correct option(s). **[JEE 2012]**

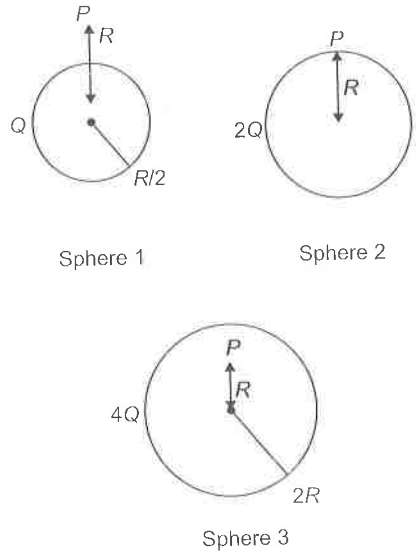


(A) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$.
(B) The net electric flux crossing the plane $y = +a/2$ is more then to the net electric flux crossing the plane $y = -a/2$.
(C) The net electric flux crossing the entire region is q/ε_0
(D) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x=+a/2$.

20. An infinitely long solid cylinder of radius R has a uniform volume charge density. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16\kappa\varepsilon_0}$. The value of is **[JEE 2012]**



21. Charges Q, $2Q$ and $4Q$ are uniformly distributed in three dielectric solid spheres 1,2 and 3 of radii $R/2$, R and $2R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1,2 and 3 are E_1, E_2 and E_3 respectively, the **[JEE 2014]**



(A) $E_1>E_2>E_3$
(B) $E_3>E_1>E_2$
(C) $E_2>E_1>E_3$
(D) $E_3>E_2>E_1$

22. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, Then **[JEE 2014]**

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

ANSWER KEYS

Exercises

JEE Main

1. B	2. C	3. D	4. A	5. D	6. C	7. B	8. D	9. A	10. C
11. B	12. A	13. A	14. C	15. A	16. D	17. D	18. C	19. A	20. A
21. A	22. B	23. A	24. D	25. C	26. (i) B, (ii) D		27. A	28. A	29. D
30. B	31. C	32. B	33. D	34. D	35. A	36. B	37. C	38. A	39. D
40. C	41. C	42. C	43. A	44. D	45. A				

JEE Advanced

1. B	2. A	3. D	4. B	5. A	6. C	7. C	8. A, C	9. C	10. C, D
11. A, B, C	12. A, B, C		13. D	14. D	15. A, C, D	16. A, C, D	17. A,B	18. A, C, D	

JEE Advanced

Level I

1. There is a positive charge in the close surface.

2. $\dfrac{q}{24\epsilon_0}$ 3. $a = \dfrac{R}{\sqrt{3}}$ 4. $\dfrac{Q}{2\varepsilon_0}$ 5. $\dfrac{R\lambda}{2\varepsilon_0}$ 6. $\dfrac{1}{2\pi}\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$ 7. $9V_0$

8. (i) $\left(\dfrac{r_1}{r_2}\right)q$ (ii) Charge on inner shell $= -\left(\dfrac{r_1}{r_2}\right)q$ and charge on the outer shell $= q$

(iii) Charge flown in to the earth $= \left(\dfrac{r_1}{r_2}\right)q$

9. $v = \dfrac{kq}{r}$ 10. (i) $q_2 = -\dfrac{b}{a}q_1$; (ii)

$$\begin{cases} V_r = \dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{a}\right) & ; a \le r \le b \\ V_b = \dfrac{q_1}{4\pi\varepsilon_0}\left(\dfrac{1}{b} - \dfrac{1}{a}\right) & ; r = b \\ V_r = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{r} + \dfrac{q_2}{a}\right) & ; r \ge b \end{cases}$$

11. $-Q/3$ 12. $1.125\,q$ 13. $v_2 = \dfrac{v_1 r_1}{r_2}$

14. $(\sigma - x)A$, xA, $-xA$, $(x - 2\sigma)A$, where $x = (2\epsilon_0 E + 3\sigma)/2$

Level II

1. $\left[\dfrac{2KQq}{mR}\left(\dfrac{r-R}{r} + \dfrac{3}{8}\right)\right]^{1/2}$ 2. $\sqrt{\dfrac{6\sqrt{2}mr\epsilon_0}{epa}}$ 3. $0, \dfrac{2K\lambda}{r}, 0$ 4. Proof 5. 7.5×10^{-9} N 6. $\dfrac{Qq}{2\pi\epsilon_0 L}$ 7. 2

Previous Year Questions

JEE Main

1. D 2. A 3. C 4. D 5. C 6. A 7. C 8. B 9. C
10. A 11. C

JEE Advanced

1. A 2. C 3. $V' = \left(\frac{a}{3t}\right)^{1/3} V$ 4. A, B, C, D 5. A 6. D 7. B
8. B 9. A 10. B 11. C 12. B 13. 2 14. A 15. A 16. C
17. A, B, C, D 18. C, D 19. A, C, D 20. 6 21. C 22. C

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